

# DC\_Notes\_Ip\_Academy

## Line Coding

05:14

**LINE CODING** (Digital Data  $\xleftrightarrow{LC}$  Digital Signals)  
1001101

**I Unipolar Scheme:** (Only one voltage level)

Amplitude  $\uparrow$  1 0 1 0 0 1 1  
Time

\* Only NRZ is there in unipolar generally  
\* It is also called Unipolar NRZ.  
Q 1010011 UNRZ

**II Polar Schemes:** (two voltage levels)

(a) NRZ-L (Non Return to zero - Level)

A  $\uparrow$  1 1 0 0 1 1 0 1 0 1  
-A  
Time

\* 0 is drawn above and 1 is drawn below

DD  $\rightarrow$  DS  
 $\downarrow$   
DD

NRZ L

09:42

(b) NRZ-I (Non-Return to zero - Invert)

0 0 0 1 1 1 0 0  
1 0 0 1 1 1 0 0

Next bit 0  $\Rightarrow$  No transition  
Next bit 1  $\Rightarrow$  Transition

\* It has an advantage over NRZ-L.

NRZ  
P NRZ-L  
P NRZ-I

Why to learn more line coding Schemes?  
Receiver don't know when one bit ended and next bit started. Although, it only arises when sender and receiver are not synchronized.

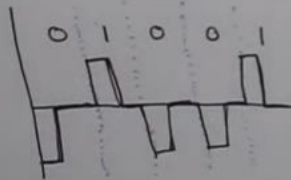
1 1 1 1 1 1 1 1  
1 1 1 1 1 1 1 1  
1 1 1 1 1 1 1 1  
1 1 1 1 1 1 1 1

## Polar (RZ & Manchester) and Bipolar

01:23

**II Polar Schemes:**

(c) RZ



Legend:

$$1 \equiv \text{high}$$

$$0 \equiv \text{low}$$

Disadvantages:

- (i) Requires greater bandwidth.
- (ii) Increased complexity.

(d) Biphasic.

- (i) Manchester (RZ + NRZ-L)
- (ii) Differential Manchester (RZ + NRZ-I)

Legend:

$$0 = \text{No}$$

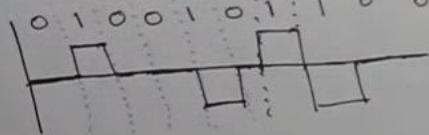
$$1 = \text{Yes}$$

03:19

08:13

**III Bipolar Schemes:**

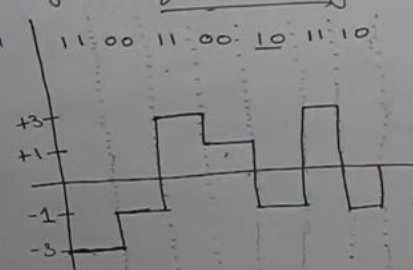
(a) Alternate Mark Inversion (AMI):



(b) 2B1Q (Two binary one quaternary)

	+	-
00	+1	-1
01	+3	-3
10	-1	+1
11	-3	+3

Legend:

$$A \mid 11 \ 00 \ 11 \ 00 \ 10 \ 11 \ 10$$


Legend:

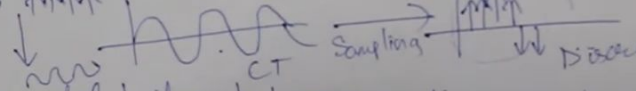
$$110010010110$$

$$10101010$$

# Sampling Theorem with proof

02:32

**SAMPLING THEOREM**



It is used as a fundamental bridge between continuous-time signals and discrete time signals.

$f_s \geq 2 f_m$

- \* Sampling is a process of converting a signal (continuous time) into a sequence of values (discrete time).
- \* Sampling theorem states if a continuous time signals contains no frequency component higher than  $f_m$  Hz, then it can be completely determined by uniform samples taken at a rate  $f_s$  samples per second where

$f_s \geq \frac{2W}{\text{atm}}$

or

$T \leq \frac{1}{2W}$

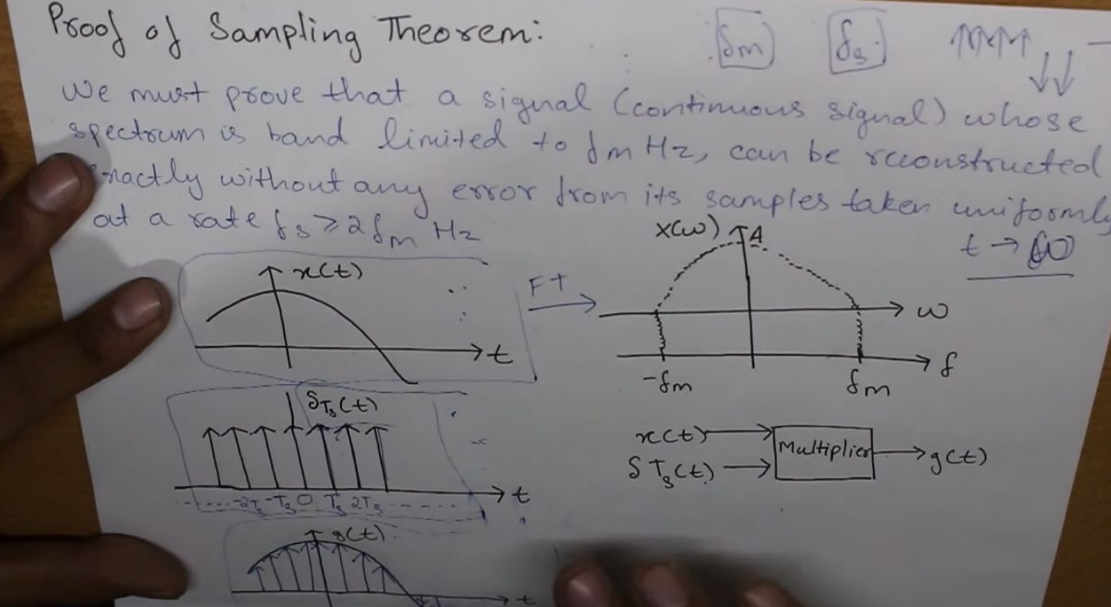
$f_s \equiv \text{Sampling Frequency}$   
 $W \equiv f_m \equiv \text{Maximum Frequency}$

Minimum Sampling rate  $\equiv$  Nyquist Rate  $\equiv 2W$   
 and Nyquist Interval  $\equiv \frac{1}{f_s}$

06:53

**Proof of Sampling Theorem:**

We must prove that a signal (continuous signal) whose spectrum is band limited to  $f_m$  Hz, can be reconstructed exactly without any error from its samples taken uniformly at a rate  $f_s \geq 2 f_m$  Hz



The diagram illustrates the proof of the sampling theorem. It shows a continuous signal  $x(t)$  being transformed into the frequency domain  $X(\omega)$  via the Fourier Transform (FT). The spectrum  $X(\omega)$  is shown as a band-limited signal from  $-f_m$  to  $f_m$ . The signal is then sampled to produce a discrete signal  $S T_s(t)$ . This sampled signal is multiplied by the original signal  $x(t)$  to produce the reconstructed signal  $g(t)$ . The diagram also shows the sampling process in the time domain with a sampling clock  $T_s(t)$  and the resulting sampled signal  $S T_s(t)$ .

08:38

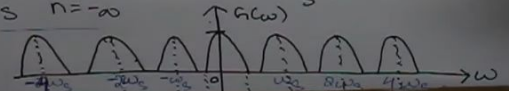
$$g(t) = x(t) \delta_{T_s}(t)$$

$$\delta_{T_s}(t) = \frac{1}{T_s} [1 + 2\cos\omega_s t + 2\cos 2\omega_s t + 2\cos 3\omega_s t + \dots]$$

$$\omega_s = \frac{2\pi}{T_s} = 2\pi f_s$$

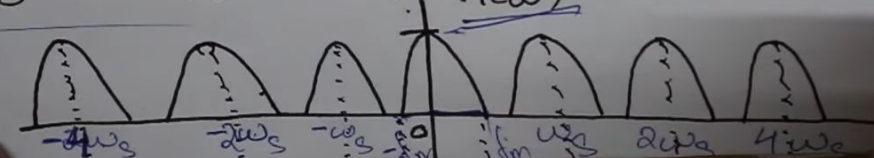
$$g(t) = \frac{1}{T_s} [x(t) + 2x(t)\cos\omega_s t + 2x(t)\cos 2\omega_s t + 2x(t)\cos 3\omega_s t + \dots]$$

$$G(\omega) = \frac{1}{T_s} [X(\omega) + X(\omega - \omega_s) + X(\omega + \omega_s) + X(\omega - 2\omega_s) + X(\omega + 2\omega_s) + \dots]$$

$$G(\omega) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} X(\omega - n\omega_s)$$


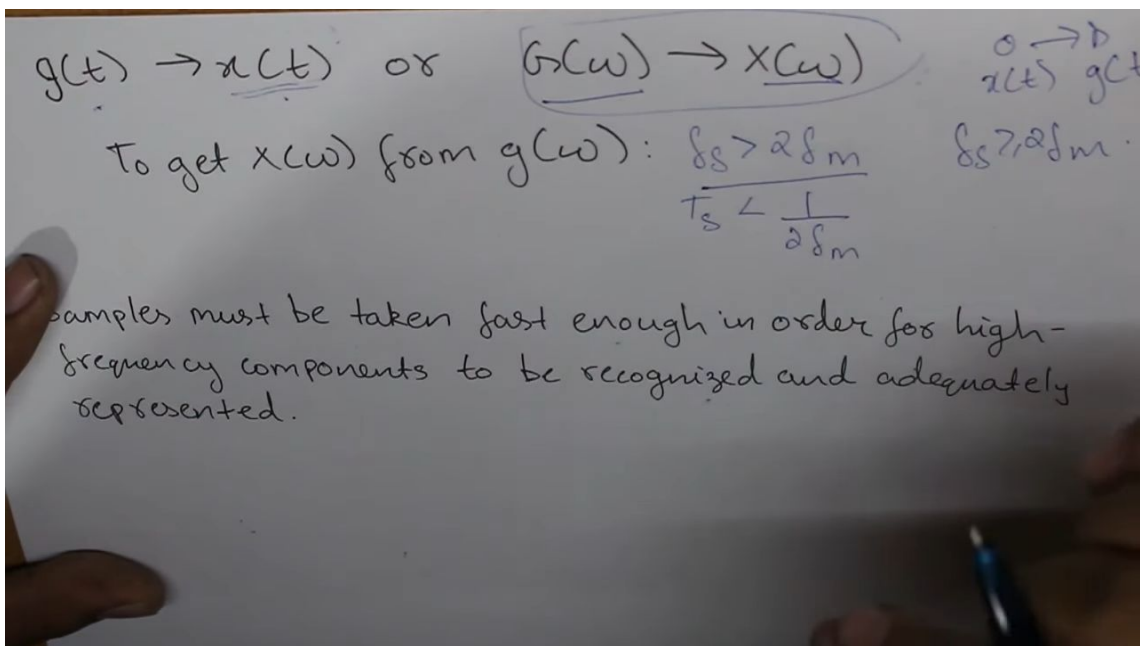
09:53

$$\frac{1}{T_s} [X(\omega) + X(\omega - \omega_s) + X(\omega + \omega_s) + X(\omega - 2\omega_s) + X(\omega + 2\omega_s) + \dots]$$

$$\frac{1}{T_s} \sum_{n=-\infty}^{\infty} X(\omega - n\omega_s)$$


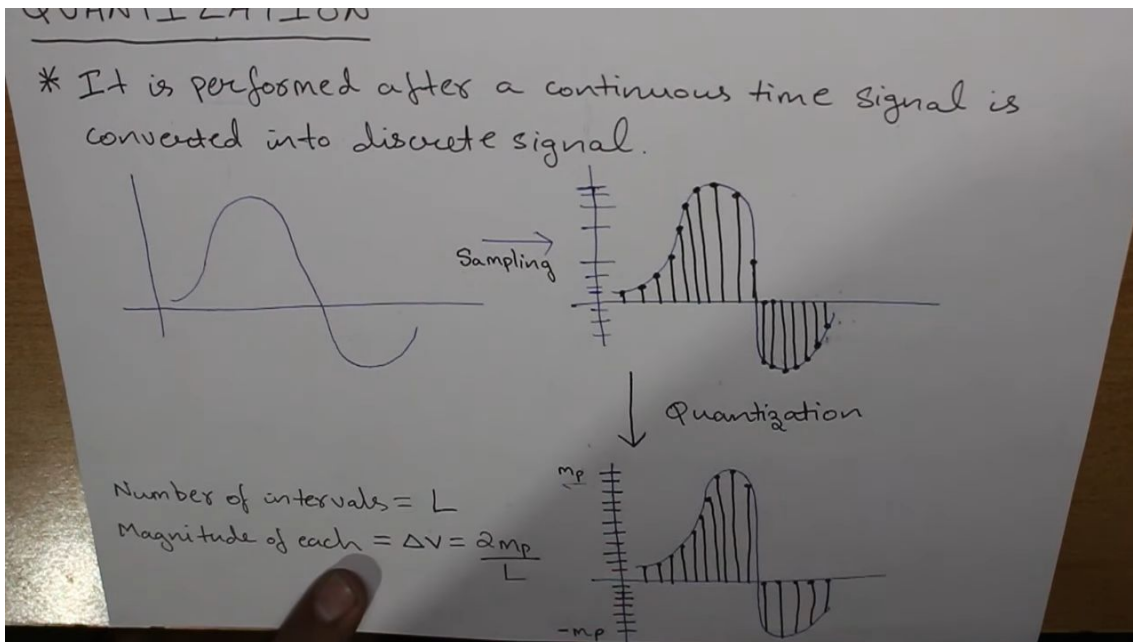
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## What is Quantization?

01:49

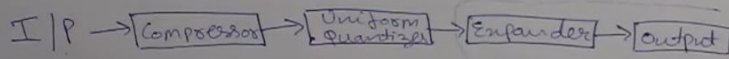


## Companding

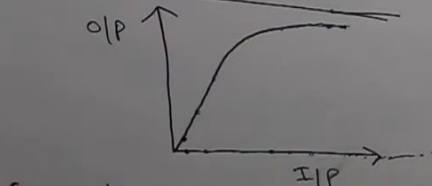
01:57

## COMPANDING (Compressing + Expanding)

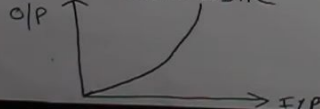
\* Signal is compressed at transmitter end and expanded at receiver end.



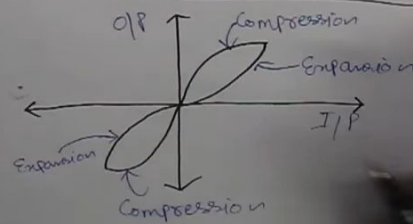
### Compressor Characteristic



### Expander characteristic



### Compander Characteristic



03:42

Types:

- (i)  $\mu$ -law companding
- (ii) A-law companding

I  $\mu$ -law companding.

$$Z(x) = (\text{sgn } x) \ln \left( 1 + \frac{\mu |x|}{x_{\max}} \right)$$

$$\ln(1 + \mu)$$

$\pm$

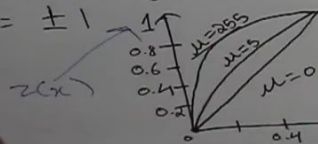
$\mu$  = extent of compression.

$Z(x)$  = Output or Normalized output

$x$  = Input

$|x|/x_{\max}$  = Normalized value of input w.r.t  $x_{\max}$ .

$(\text{sgn } x) = \pm 1$

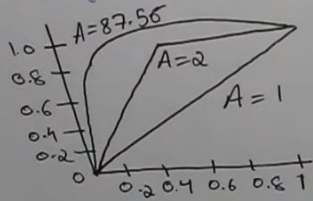


\* Practically used value of  $\mu$  is 255  
 \* Used in pcr telephone in Canada & Japan  
 \* Used for speech & music signals

05:45

## A-Law Companding

\* It is used for PCM telephone systems in Europe.



\* Practically used value for 87.56.

$$\frac{z(x)}{x_{\max}} = \begin{cases} \frac{A|x|/x_{\max}}{1 + \log_e A} & ; 0 \leq \frac{|x|}{x_{\max}} \leq \frac{1}{A} \\ \frac{1 + \log_e [A|x|/x_{\max}]}{1 + \log_e A} & ; \frac{1}{A} \leq \frac{|x|}{x_{\max}} \leq 1 \end{cases}$$

The most important difference between the two types is that A-law compressor has mid-riser at origin and  $\mu$ -law has mid-tread at origin. Thus, A-law has no zero value.

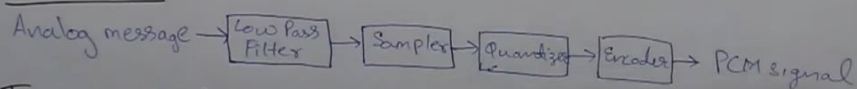
## **Pulse Code Modulation (PCM)**

01:21

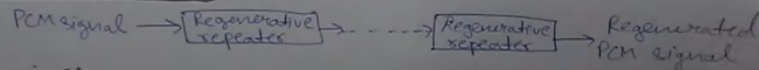
### PULSE CODE MODULATION (PCM)

- \* It is a digital pulse modulation technique.
- \* It is more complex than analog pulse modulation technique in the sense that the message signal is subjected to a greater number of operations.
- \* It has three parts: Transmitter, Path and Receiver.

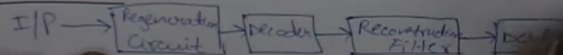
#### Transmitter



#### Transmission Path

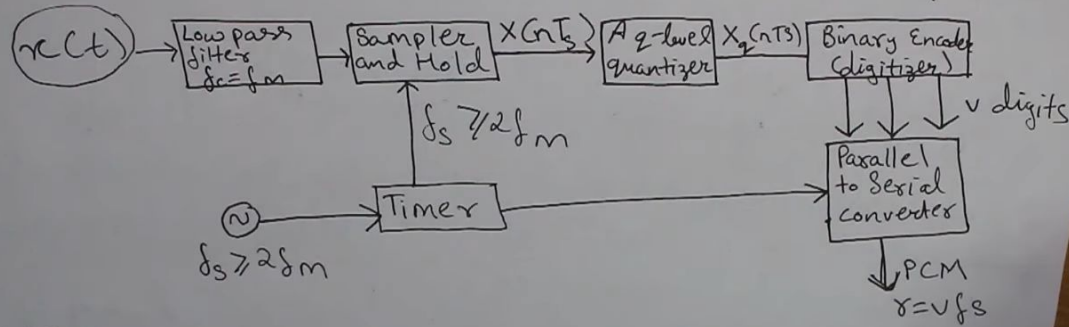


#### Receiver



04:40

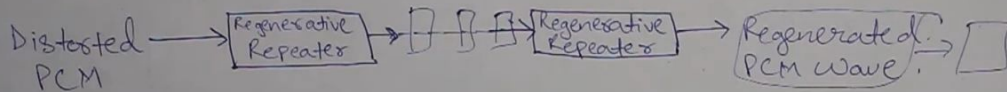
## # PCM Generator or Transmitter.



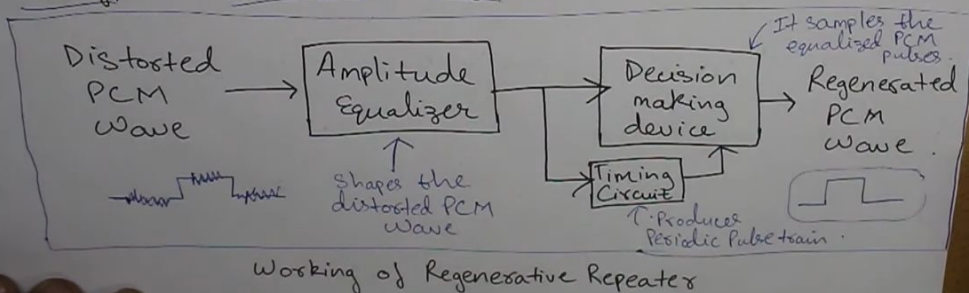
LPF: Blocks all the frequency components lying above  $f_m$  Hz.  
 Sampler: Samples the signal  $f_s \geq 2f_m$   
 $x(nT_s)$ : Discrete time signal.

07:49

## PCM Transmission Path



\* Regenerative repeater is primarily used for reduction of distortion and noise. It does so by performing equalization, timing, and decision-making.

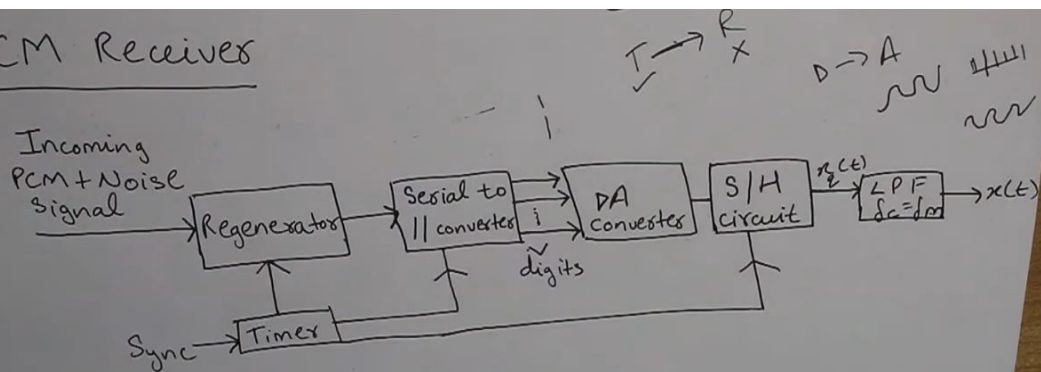


Working of Regenerative Repeater

09:51



## PCM Receiver



Regenerator  $\equiv$  Reshapes the pulse and removes the noise

$x_2(t) \equiv$  Final analog signal after conversion.

$x(t) \equiv$  original message signal

===

00:57

00:57

## Applications of PCM:

1. PCM can be used in telephony using fibre optic cables.
2. In space communication (Transmission power is very small and distance is very large. Due to high noise immunity only PCM can be used here)

## Advantages of PCM:

1. High noise immunity.
2. Signal can be stored due to its digital nature.
3. Line coding can be used.
4. Repeaters can be used.

## Drawbacks of PCM:

1. Requires large bandwidth.
2. Complex circuit.

05:45

## # QUANTIZATION NOISE / ERROR IN PCM

1.  $\epsilon = x_q(nT_s) - x(nT_s)$  ← quantization error.
2.  $\Delta = \frac{2x_{\max}}{q}$  where  $\Delta$  = step size  
 $q$  = levels.
3.  $BW \geq v f_m$  where  $v$  = no. of binary digits to represent each level ( $q = 2^v$ )
4.  $\Delta = \frac{2}{2^v}$  (for normalized signal)  $x_{\max} = 1$  Max QE =  $\left|\frac{\Delta}{2}\right|$

$\text{Quantization Error} = \frac{\Delta^2}{12} = \frac{\text{Normalized noise power}}{\text{Quantization noise power}}$

06:53

## # SIGNAL TO QUANTIZATION NOISE RATIO

(linear)

$$\frac{S}{N} = \frac{\text{Normalized signal power}}{\text{Normalized noise power}}$$

$q = 2^v$  where  $v$  = number of bits or code word length.

$\frac{S}{N} = \frac{NSP}{\frac{\Delta^2}{12}} = \frac{NSP}{\left(\frac{2x_{\max}^2}{2^v}\right) \frac{1}{12}} = \frac{3P}{x_{\max}^2} 2^v$

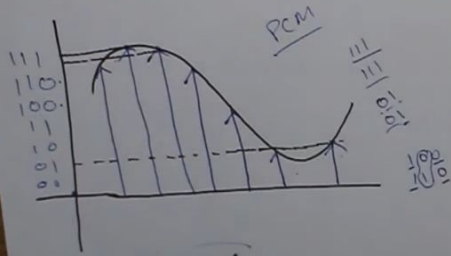
If  $x_{\max} = 1$  and  $P \leq 1$

$\frac{S}{N} \leq (4.8 + 6v)$

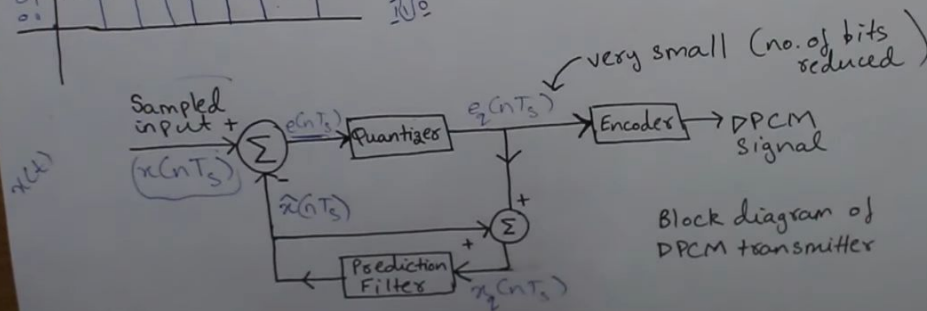
## Differential Pulse Code Modulation (DPCM)

03:28

## Differential Pulse Code Modulation (DPCM)



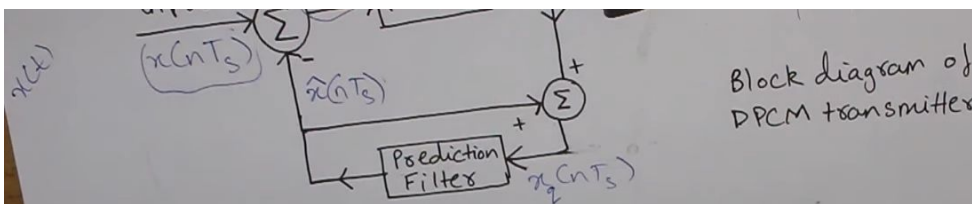
\* It is completely based on the principle of prediction.



Block diagram of DPCM transmitter

$$\text{Prediction error} = e(nT_s) = x(nT_s) - \hat{x}(nT_s)$$

07:16



Block diagram of DPCM transmitter

$$\text{Prediction error} = e(nT_s) = x(nT_s) - \hat{x}(nT_s)$$

$$e_q(nT_s) = e(nT_s) + q(nT_s) \leftarrow \text{quantization error}$$

$$\hat{x}_q(nT_s) = \hat{x}(nT_s) + e_q(nT_s)$$

$$x_q(nT_s) = \hat{x}(nT_s) + e(nT_s) + q(nT_s)$$

$$x_q(nT_s) = x(nT_s) + q(nT_s)$$

\*\* Quantized version of the signal  $x_q(nT_s)$  is the sum of original sample value and quantization error  $q(nT_s)$ .

08:45

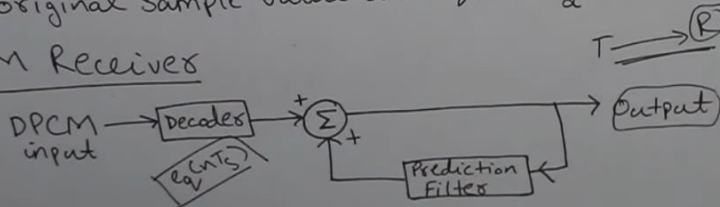
$$x_q(nT_s) = \hat{x}(nT_s) + e_q(nT_s)$$

$$x_q(nT_s) = \hat{x}(nT_s) + e(nT_s) + q(nT_s)$$

$$\boxed{x_q(nT_s) = \hat{x}(nT_s) + q(nT_s)}$$

\*\* Quantized version of the signal  $x_q(nT_s)$  is the sum of original sample value and quantization error  $q(nT_s)$ .

### DPCM Receiver



Decoder: reconstructs the quantized error signal from incoming binary signal.

\* Signal at the receiver differs from actual signal by quantization error  $q(nT_s)$ .

10:25

### Advantages of DPCM:

1. Bandwidth is lesser than PCM ✓
2. Less no. of quantization levels. ✓
3. Less no. of bits required to represent quantization levels ✓

### # Evaluation of Output Signal to Noise (S/N) ratio

$$\text{SNR} = \frac{\text{Mean square value of a signal}}{\text{Mean square value of quantization noise}}$$

$$\boxed{\text{SNR} = \frac{\sigma_x^2}{\sigma_q^2}}$$

$$\text{SNR} = \left( \frac{\sigma_x^2}{\sigma_E^2} \right) \times \left( \frac{\sigma_E^2}{\sigma_q^2} \right) \quad \text{where } \sigma_E^2 = \text{Variance of prediction error } e(nT_s).$$

↓  
Prediction gain
↓  
Prediction error to quantization noise

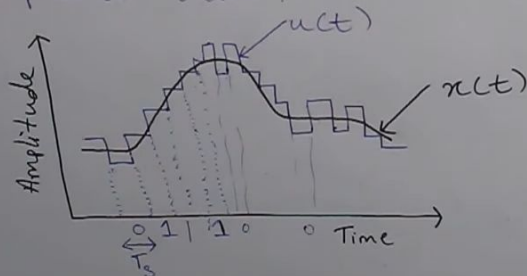
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03:37



## Delta Modulation (PCM $\rightarrow$ DPCM $\rightarrow$ DM $\rightarrow$ ADM CT $\rightarrow$ DT)

It transmits only one bit per sample, depending upon whether the present value has increased or decreased as compared to the previous value.



+	1
-	0

$T_s \equiv$  sampling interval

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s)$$

$e(nT_s) \equiv$  error in present sample

$x(nT_s) \equiv$  sampled signal of  $x(t)$

$u[(n-1)T_s] = \hat{x}(nT_s) \equiv$  last sample approximation of the staircase

05:16

$u(nT_s) =$  present sample approximation of staircase output

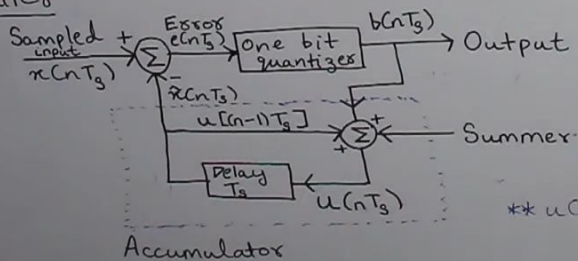
$u[(n-1)T_s] = \hat{x}(nT_s) =$  last sample approximation of staircase waveform.

$$b(nT_s) = \Delta \operatorname{sgn}[e(nT_s)]$$

$$= \begin{cases} +\Delta & \text{if } x(nT_s) \geq \hat{x}(nT_s) \\ -\Delta & \text{if } x(nT_s) < \hat{x}(nT_s) \end{cases}$$

1 is transmitted  
0 is transmitted

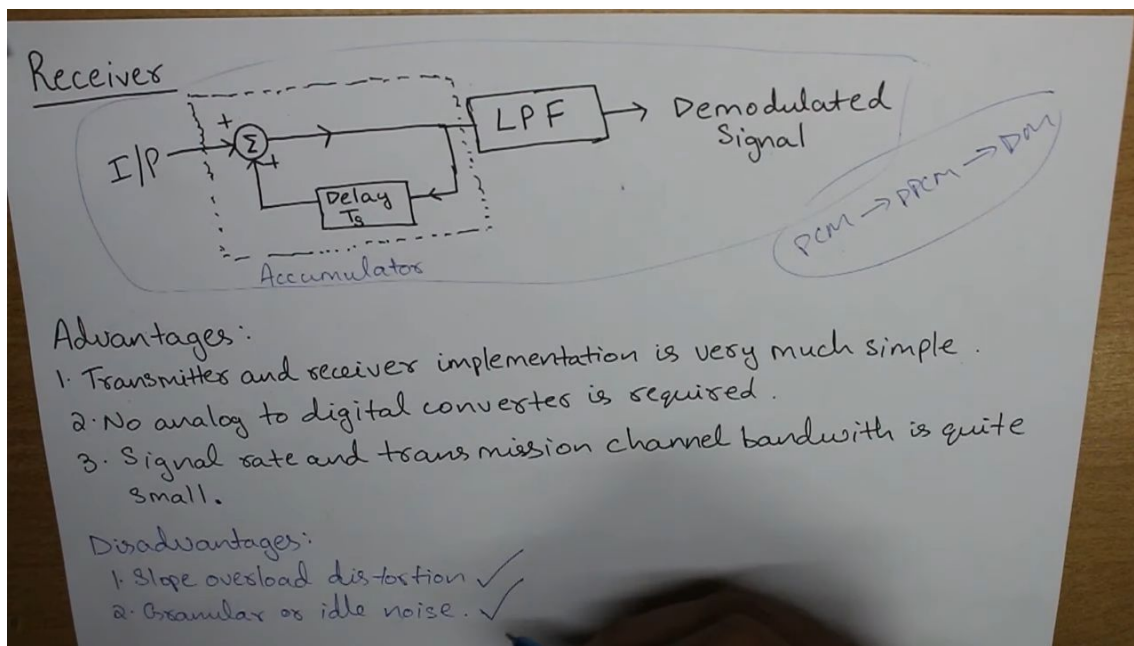
### Transmitter



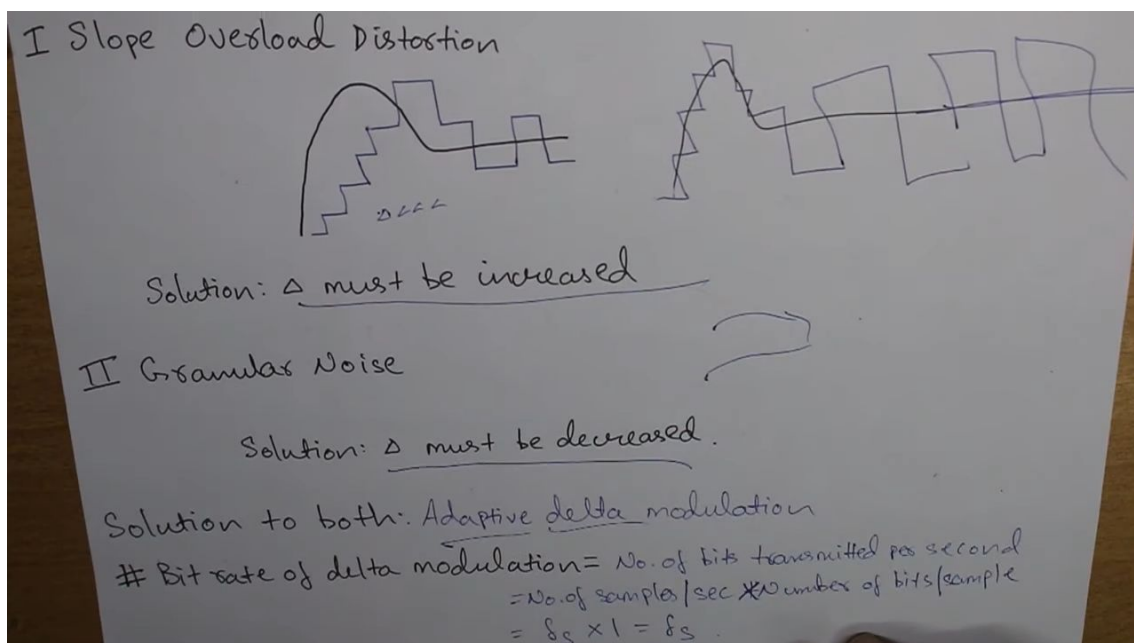
$$** u(nT_s) = u[(n-1)T_s] + [\pm \Delta]$$

$$= u[(n-1)T_s] + b(nT_s)$$

09:56

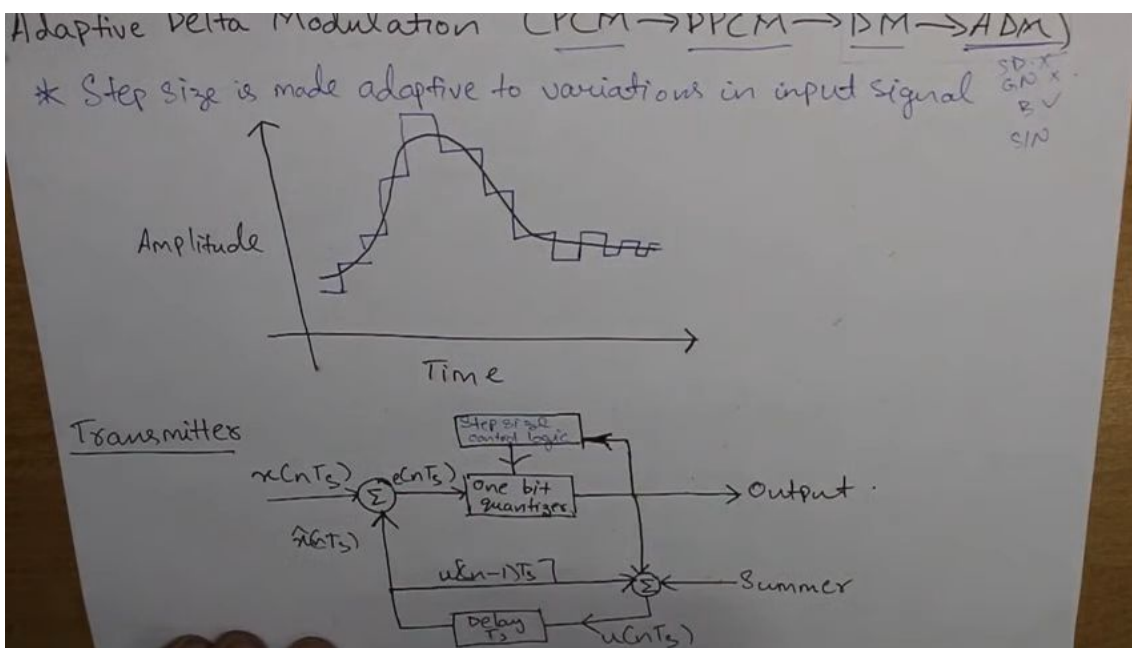


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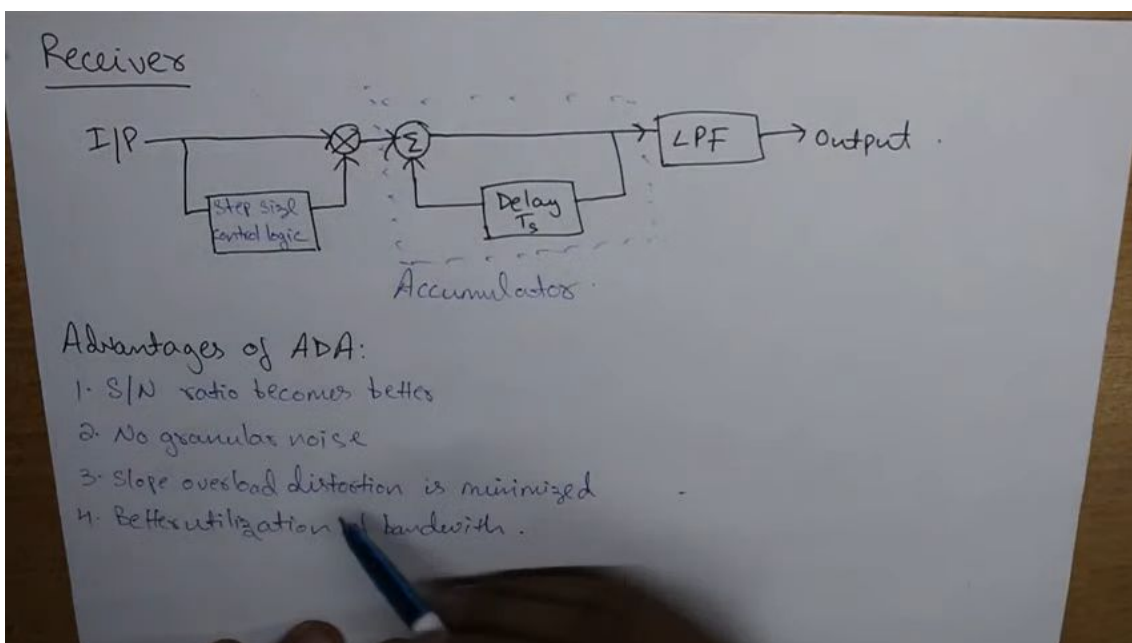


## Adaptive Delta Modulation (ADM): Analysis

02:52



03:08



## Random Variable | Random Signal Theory

01:53

## RANDOM SIGNAL THEORY

Probability:  $\frac{\text{Favourable Outcomes}}{\text{Total Outcomes}}$

Deterministic Signals ✓  
Non-Deterministic P · X

Deterministic Signals: They are described by fixed mathematical equations.

Random Signals: Their behaviour cannot be predicted.  
(Noise Interferences)

\*\* Questions of Basic Probability are already covered in other playlists.

05:37

## RANDOM VARIABLES

Types: 1. Discrete Random Variable .. 5 Billion 6 Billions  
2. Continuous Random Variable .. (50, 90)  
(200, 400)  
500 trillion 200.01  
905

(50, 90)  
60.102  
60.000001 kg

## WSS & SSS Random Process | Random Signal Theory

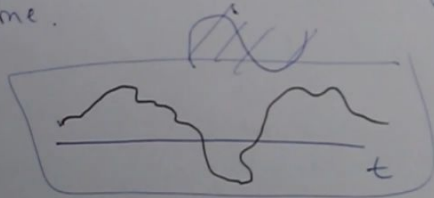
02:09



Random Process:  $X(t)$ .

$X = 1, 2, 3, \dots$

A random variable which is a function of time or indexed by time.



random signal or random noise at the receiver.

$$X: \delta_X(x), F_X(x)$$

PDF of  $X(t)$ :  $\delta_X(x, t) \leftarrow$  PDF at every time instant  $t$ .

Mean of  $X(t)$ :  $E\{X(t)\} = \int_{-\infty}^{\infty} x \underbrace{\delta_X(x, t)}_{\text{PDF at time } t} dx = \underbrace{\mu_X(t)}_{\text{Function of time}}$

06:20

Special Random Process:  $(S, NS, WSS, SSS)$

I Wide Sense Stationary (WSS) (I moment + II moment)

$$E\{X(t)\} = \mu_X = \text{constant}$$

Does not depend on  $t$ .  
 $\Rightarrow$  It is irrespective of  $t$ .

1. I, II, III, ... WSS  
 Mean  
 2.  $(t_0 - t_1)$

# Stationary Vs Non-Stationary Process

Stationary: A random process is stationary if its statistics (mean, variance, covariance) are not affected by any shift in time origin.

II Strict Sense Stationary

Non-Stationary: opposite of stationary.

## Cumulative Distribution Function (CDF)

03:16

## CUMULATIVE DISTRIBUTION FUNCTION (CDF)

CDF of a random variable  $X$  is the probability that a random variable takes value less than or equal to  $x$ .

$$\text{CDF: } F_X(x) = P(X \leq x)$$

$$P(X \leq x) \\ f_X(x)$$

Properties:

1. It can be defined for discrete as well as continuous random variables.
2. It is also called probability distribution function or simply, Cumulative probability distribution function.

$$3. 0 \leq F_X(x) \leq 1.$$

$$4. F_X(\infty) = 1 \text{ and } F_X(-\infty) = 0.$$

$$5. F_X(x_1) \leq F_X(x_2) \text{ if } x_1 \leq x_2.$$

$\Rightarrow$  CDF is a monotone non-decreasing function.

$$\begin{array}{l} \text{PDF} \\ \text{PDF: } P_{\infty} \text{ Density Function} \\ \frac{P(X \leq \infty)}{1} \quad 1, 2, 3, 4 \\ F_X(-\infty) = P(X \leq -\infty) = 0 \end{array}$$

04:12

$$\begin{array}{l} \text{If } X \in [x_1, x_n] \\ \text{then } F_X(x) = 0; -\infty \leq x \leq x_1 \\ = \sum_{j=1}^n P(x = x_j); x_1 \leq x \leq x_n \\ = 1; x_n < x < \infty. \end{array}$$

$$\begin{array}{l} X \in [x_1, x_n] \\ [1, 5] \\ 1, 1 \end{array}$$

## Probability Density Function (PDF)

01:19

## Probability Density Function (PDF)

CDF

$$\text{PDF: } f_x(x) = \frac{d}{dx} \underbrace{F_x(x)}_{\text{CDF}}$$

$$f_x(x) =$$

CDF  
PDF

\* It is more convenient for continuous random variable.

Properties:

1.  $f_x(x) \geq 0$  for every  $x$  [because CDF is non-decreasing]

2.  $\int_{-\infty}^{\infty} f_x(x) dx = 1$ .

3.  $F_x(x) = \int_{-\infty}^x f_x(x) dx$ .

4.  $P(x_1 < x \leq x_2) = \int_{x_1}^{x_2} f_x(x) dx$ .

04:53

PDF is given by  $f_x(x) = \frac{a}{b} e^{-b|x|}$  where  $X$  is a random variable whose value lie in the range  $x = -\infty$  to  $x = +\infty$ .

Determine:

(i) The relationship b/w  $a$  &  $b$ .

(ii) CDF.

(iii) The probability that outcome lies b/w 1 and 2.

$$\rightarrow \int_{-\infty}^{\infty} f_x(x) dx = 1$$

$$f_x(x) = \frac{a}{b} e^{-b|x|}$$

$$\Rightarrow \int_{-\infty}^0 a e^{-b(-x)} dx + \int_0^{\infty} a e^{-bx} dx = 1 \quad \checkmark$$

$$\frac{a}{b} [e^x]_{-\infty}^0 + -\frac{a}{b} [e^{-bx}]_0^{\infty} = 1$$

$$\frac{a}{b} [1] - \frac{a}{b} [-1] = 1$$

$$\Rightarrow \boxed{b = 2a}$$

$$X \in (-\infty, \infty)$$

$$P(1 \leq x \leq 2) = \int_1^2 f_x(x) dx$$

06:13

$$\begin{aligned}
 F_X(x) &= \int_{-\infty}^x f_X(x) dx \\
 &= \int_{-\infty}^x a e^{-b|x|} dx \\
 &= \int_{-\infty}^x a e^{-b(-x)} dx + \int_{0^+}^x a e^{-bx} dx + \int_{x=0} a e^{-bx} dx \\
 &= \frac{a}{b} [e^{bx}]_{-\infty}^x + \left(-\frac{a}{b}\right) [e^{-bx}]_{0^+}^x + \frac{a}{b} [e^0] \\
 &= \frac{a}{b} e^{bx} + \left(-\frac{a}{b}\right) (e^{-bx} - 1) + \frac{1}{2} \\
 &\quad \text{Putting } b = 2a \\
 &= \frac{1}{2} e^{bx} + \frac{1}{2} + \frac{1}{2} - \frac{1}{2} e^{-bx} \\
 &= \left( \frac{1}{2} e^{bx} + 1 - \frac{1}{2} e^{-bx} \right) \\
 &\quad \quad \quad x < 0 \quad \quad \quad x > 0
 \end{aligned}$$

$\delta_X(x) = \frac{d}{dx} F_X(x)$

06:35

$$\begin{aligned}
 P(x_1 < X \leq x_2) &= \int_{x_1}^{x_2} f_X(x) dx \\
 \text{Put } x_1 &= 1 \text{ and } x_2 = 2 \\
 P(1 \leq X \leq 2) &= \int_1^2 a e^{-bx} dx \\
 &= \frac{a}{b} (e^{-bx})_1^2 \\
 &= -\frac{a}{b} [e^{-2b} - e^{-b}] \\
 &= \boxed{\frac{1}{2} [e^b - e^{-2b}]}
 \end{aligned}$$

$P(1 \leq X \leq 2)$

## Joint PDF & Joint CDF

00:59



## JOINT CDF

It is defined for two random variables  $X$  and  $Y$  and gives the probability that a random variable  $X$  is less than or equal to a specified value  $x$  and that the random variable  $Y$  is less than or equal to specified value of  $y$ .

$$F_{X,Y}(x,y) = P(X \leq x, Y \leq y) \quad \frac{F_{X,Y}(x,y)}{F_{X,Y}(x,y)} = \frac{P(X \leq x)}{F_{X,Y}(x,y)}$$

Properties:

1.  $F_{X,Y}(x,y) \geq 0$  ✓
2. Non-decreasing ✓
3. Always continuous in the  $xy$  plane ✓

01:56

## JOINT PDF

Joint PDF of two random variables  $X$  and  $Y$  may be defined as the partial derivative of the joint CDF with respect to the variables  $x$  and  $y$ .

$$f_{X,Y}(x,y) = \frac{\partial^2 F_{X,Y}(x,y)}{\partial x \partial y}$$

Properties:

1.  $f_{X,Y}(x,y) \geq 0$ .
2.  $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{X,Y}(x,y) dx dy = 1$ .
3. Everywhere continuous...

$$f_X(x) = \frac{\partial}{\partial x} F_X(x)$$
$$\frac{\partial^2 F_{X,Y}(x,y)}{\partial x \partial y} = \frac{\partial}{\partial x} \left( \frac{\partial F_{X,Y}(x,y)}{\partial y} \right)$$

03:35

Relationship between Probability and Joint PDF.

$$P(x_1 < X \leq x_2) = \int_{x_1}^{x_2} f_X(x) dx.$$

$$P(x_1 < X \leq x_2, y_1 < Y \leq y_2) = \int_{y_1}^{y_2} \int_{x_1}^{x_2} f_{XY}(x, y) dx dy.$$

If  $X$  &  $Y$  are statistically independent.

$$f_{XY}(x, y) = f_X(x) f_Y(y).$$

$$\begin{aligned} \Rightarrow P(x_1 < X \leq x_2, y_1 < Y \leq y_2) &= \int_{y_1}^{y_2} \int_{x_1}^{x_2} f_X(x) f_Y(y) dx dy. \\ &= \int_{x_1}^{x_2} f_X(x) dx \int_{y_1}^{y_2} f_Y(y) dy. \end{aligned}$$

05:18

Q  $f_{XY}(x, y) = e^{-(x+y)}$  for  $x \geq 0, y \geq 0$   
 $= 0$  otherwise.

Determine:

(i)  $P(X < 1)$  (ii)  $P(X > Y)$  (iii)  $P(X + Y < 1)$ .

$$\rightarrow f_{XY}(x, y) = e^{-x} e^{-y} = f_X(x) f_Y(y)$$

$$(i) \underline{P(X < 1)} = \int_0^1 e^{-x} dx = \left[ \frac{e^{-x}}{-1} \right]_0^1 = -[e^{-x}]_0^1 \checkmark$$

$$(ii) \underline{P(X > Y)} \checkmark$$

## Marginal PDF or Marginal Densities

03:43

## Marginal PDF or Marginal Densities

PDF CDF  
JPDF JCDF

When  $f_x(x)$  and  $f_y(y)$  for any random variable are obtained from  $f_{xy}(x,y)$  then  $f_x(x)$  and  $f_y(y)$  are called marginal PDF.

$$F_{xy}(x,y) = \int_{-\infty}^y \int_{-\infty}^x f_{xy}(x,y) dx dy$$

$$PDF = \frac{d}{dx} CDF$$

$$F_x(x) = P(X \leq x) = P(X \leq x, -\infty \leq y \leq \infty) = \int_{-\infty}^{\infty} \int_{-\infty}^x f_{xy}(x,y) dx dy$$

$$f_x(x) = \frac{d}{dx} F_x(x) = \frac{d}{dx} \left[ \int_{-\infty}^{\infty} \int_{-\infty}^x f_{xy}(x,y) dx dy \right]$$

$$f_x(x) = \int_{-\infty}^{\infty} f_{xy}(x,y) dy$$

$$\text{Similarly } f_y(y) = \int_{-\infty}^{\infty} f_{xy}(x,y) dx$$

## Mean, Moments and Central Moment

00:47

### STATISTICAL AVERAGES OF RANDOM VARIABLES <

#### Mean (Average)

Mean =  $\frac{\text{Sum of all values}}{\text{Total no. of values.}}$

$$E(p_i x_i)$$

$$= \sum_{i=1}^n p_i x_i \quad (\text{Discrete})$$

$$p_i x_i$$

$$= \int_{-\infty}^{\infty} x f_x(x) dx \quad (\text{Continuous}).$$

If  $g(x)$  transforms  $x \rightarrow x'$  then mean of  $g(x)$

$$\text{Mean} = \int_{-\infty}^{\infty} g(x) f_x(x) dx$$

03:40

## Moments

$N^{\text{th}}$  moment of any random variable  $X = \text{mean}(X^n)$

$$g(n) = x^n \Rightarrow \text{mean} = \overline{g(x)}$$

$$\overline{x^n} = \overline{g(x)} = \int_{-\infty}^{\infty} x^n f_x(x) dx = \overline{x^n} = E(X^n)$$

$$n=1: \overline{x} = E(X) = \int_{-\infty}^{\infty} x f_x(x) dx = \text{mean}$$

$$n=2: \overline{x^2} = E(X^2) = \int_{-\infty}^{\infty} x^2 f_x(x) dx \leftarrow$$

$\vdots$   
Central moment: Moments of diff. b/w random variable and its mean  $m_x$ .

$$E[(X - m_x)^n] = \int_{-\infty}^{\infty} (x - m_x)^n f_x(x) dx$$

$$\text{Variance} = \sigma_x^2 = \text{Var}(X) = E[(X - m_x)^2] = \int_{-\infty}^{\infty} (x - m_x)^2 f_x(x) dx.$$

05:13

$$\sigma_x^2 = E[X^2 - 2m_x X + m_x^2]$$

$$\sigma_x^2 = E(X^2) - m_x^2 \checkmark$$

$$\sigma_x^2 = \overline{x^2} - (\overline{m_x})^2 \checkmark$$

$$\text{Standard deviation} = \sqrt{\text{variance}} = \sqrt{\sigma_x^2} = \sigma_x$$

## Correlation Function and its types

01:43



## CORRELATION FUNCTION

X Y

### I Autocorrelation Function

It is defined as a measure of similarity between a signal and its replica by a variable amount.

$X \leftrightarrow X'$

$$R_x(t_j - t_i) = E[X(t_j) X(t_i)]$$

autocorrelation  
function of stationary  
process  $X(t)$ .

correlation b/w two times.

$X(t_j)$  &  $X(t_i)$  = Random variable obtained by observing the process  $X(t)$  at times  $t_j$  and  $t_i$  respectively.

\* It only depends on time difference ( $t_j - t_i$ ) for stationary process

$$R_x(\tau) = E[X(t) X(t-\tau)]$$

$\tau$  = Time lag or time delay parameter.

03:07

### II Cross-Correlation Functions

X Y

It is defined for two random processes.

Let us consider two random processes  $X(t)$  and  $Y(t)$  with autocorrelation functions  $R_x(t, u)$  and  $R_y(t, u)$  respectively.

$$R_{xy}(t, u) = E[X(t) Y(u)]$$

$$R_{yx}(t, u) = E[Y(t) X(u)]$$

\*  $t$  and  $u$  are two values of time at which processes are observed.

### # Spectral Densities

They are used to represent random process in frequency domain.

## **Power Spectral Density (PSD)**

01:07

## I Power Spectral Density (PSD)

$$S_X(\omega) = \text{Fourier transform of } R_X(\tau) \quad \checkmark$$

$$= \int_{-\infty}^{\infty} R_X(\tau) e^{-j\omega\tau} d\tau$$

PSD of WSS random process  $X(t)$ .

$R_X(\tau)$  = Autocorrelation function of random process  $X(t)$ .

Einstein-Weiner Khinchine eq<sup>n</sup> (EWK)

\* PSD is used to measure energy of the signal in frequency domain.

# Cross Power Spectral Density (CPSD)

$$S_{XY}(\omega) = \int_{-\infty}^{\infty} R_{XY}(\tau) e^{-j\omega\tau} d\tau$$

#  $S_X(\omega) = \int_{-\infty}^{\infty} R_X(\tau) \cos \omega\tau d\tau$  ← using Euler's formula

03:31

Characteristics of  $S_X(\omega)$

- (i)  $S_X(\omega) = S_X(-\omega)$  ✓
- (ii)  $S_X(\omega) \geq 0$  for all  $\omega$  ✓
- (iii) It is a real function of  $\omega$  ✓

# Energy Spectral Density

\* Not in Course

$$E = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Psi_X(\omega) d\omega$$

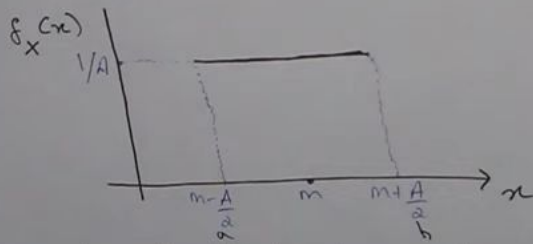
$\Psi_X(\omega)$  = ESD = Density of energy contained in random process  $X(t)$  in Joules per Hz.

$E$  = Total Energy.

## All Distributions in Brief

01:01

## Uniform Distribution



$x$  vs  $f_x(x)$

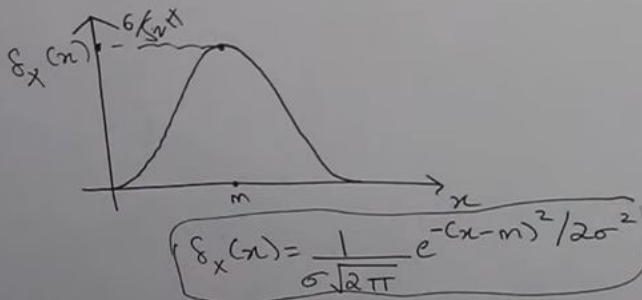
$\frac{1}{A}$  = Amplitude of all values.

$$f_x(x) = \frac{1}{b-a} = \frac{1}{m+\frac{A}{2} - m + \frac{A}{2}} = \frac{1}{A}$$

$$\text{Area} = 1$$

03:22

## Gaussian Distribution / Normal Distribution.



$$f_x(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-m)^2/2\sigma^2}$$

$m$  = mean value of random variable.

$\sigma^2$  = variance of random variable.

Properties:

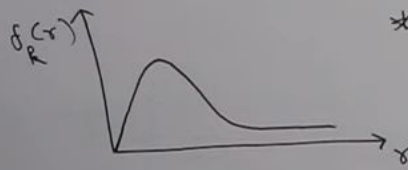
(i) Peak at  $x=m$  and  $f_x(m) = \frac{1}{\sigma\sqrt{2\pi}}$

(ii) Symmetrical around mean.

(iii)  $P(x \leq m) = P(x > m) = 1/2$ .

04:57

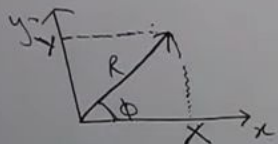
## Rayleigh Distribution



\* It is defined for two random variables  $X$  and  $Y$ .

\*  $m_x = m_y = 0$

\*  $\sigma_x^2 = \sigma_y^2 = \sigma^2$

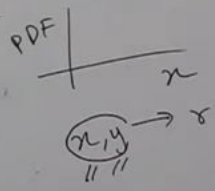


$\phi = \tan^{-1}\left(\frac{Y}{X}\right)$

$R = \sqrt{X^2 + Y^2}$

$f_R(r) = \frac{r}{\sigma^2} e^{-r^2/2\sigma^2}$  for  $r \geq 0$

$f_R(r) = 0$  for  $r < 0$



06:50

## Binomial Distribution

$P(X) = {}^nC_x p^x (1-p)^{n-x}$

Q A die is tossed 5 times. What is the probability of getting exactly two fours.

$\rightarrow p = \frac{1}{6}$

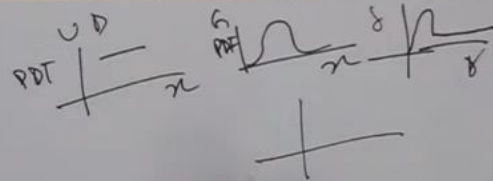
$x = 2$

$P(2) = {}^5C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^3$

Mean =  $np$

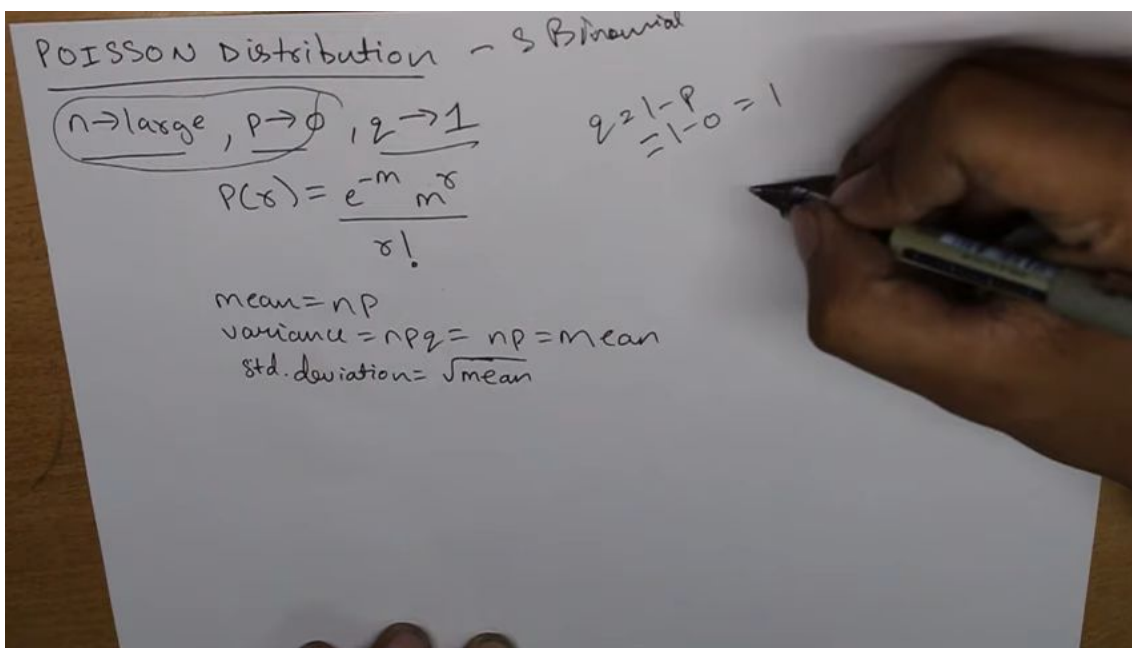
Variance =  $npq$

Std. Deviation =  $\sqrt{npq}$



08:47





## Central Limit Theorem

03:35

