Computers, Complexity and Intractability

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I want to Begin With... Michael Jordan's Quote

I've always believed that if you put in the work, the results will come. I don't do things half-heartedly. Because I know if I do, then I can expect half-hearted results.

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  - If so, for constructing a design that meets them.
- Since you are the company's chief algorithm designer, your charge is to find an efficient algorithm for doing this.

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- So far you have not been able to come up with any algorithm substantially better than searching through all possible designs.
- This would involve years of computation time for just one set of specifications.

You certainly don't want to return to your boss's office and report

"I can't find an efficient algorithm, I guess I'm just too dumb."

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- It would be much better if you could prove that the bandersnatch problem is **inherently intractable**, that no algorithm could possibly solve it Quickly .

You stride Confidently into your boss office and proclaim:

"I can't find an efficient algorithm, because no such algorithm is possible!"

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- Unfortunately, proving **inherent intractability** can be just **as hard as finding efficient algorithms.**
- Even the best theoreticians have been stymied in their attempts to obtain such proofs for commonly encountered hard problems.

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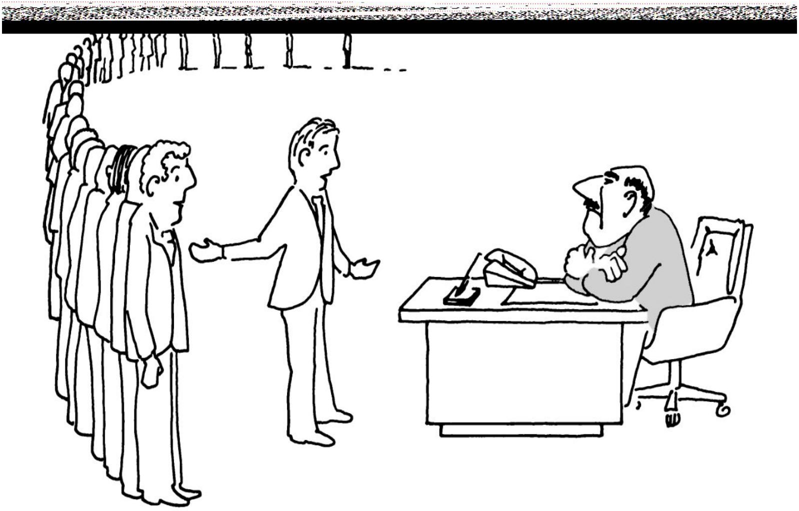
- Armed with these techniques,
  - You might be able to prove that the bandersnatch problem is **NP-complete**,and
  - It is equivalent to all these other hard problems.

You march into your boss's office and announce:



"I can't find an efficient algorithm, but neither can all these famous people."

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- At the very least, this would inform your boss that it would do no good to fire you and hire another expert on algorithms.

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- The needs of the bandersnatch department won't disappear overnight simply because their problem is known to be NP-complete.
- However, the knowledge that it is NP-complete does provide valuable information about what lines of approach have the potential of being most productive.

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- Look for efficient algorithms that solve various **special cases** of the general problem.
- You might even relax the problem somewhat, looking for a fast algorithm that merely finds designs that meet **most** of the component specifications.

Primary Application of theory of NP-Completeness

To assist the algorithm designers in directing their **problem solving** efforts toward those approaches that have the greatest likelihood of leading to **useful algorithms**.

Problems, Algorithms and Complexity

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  - A statement of what properties the answer, or solution, is required to satisfy.
- An **instance of a problem** is obtained by specifying particular values for all the problem parameters.

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- A solution is an ordering $< c_{\pi(1)}, c_{\pi(2)}, \ldots, c_{\pi(m)} >$ of the given cities that minimizes

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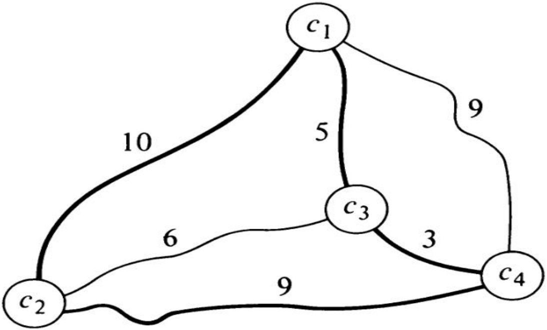
- This expression gives the length of the "tour" that starts at $c_{\pi(1)}$, visits each city in sequence, and then returns directly to $c_{\pi(1)}$ from the last city $c_{\pi(m)}$.

Instance of a Traveling Salesman Problem

$C = \{c_1, c_2, c_3, c_4\}$, $d(c_1, c_2) = 10$, $d(c_1, c_3) = 5$, $d(c_1, c_4) = 9$, $d(c_2, c_3) = 6$, $d(c_2, c_4) = 9$, $d(c_3, c_4) = 3$



The ordering $< c_1, c_2, c_4, c_3 >$ is a solution for this instance.

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- In its broadest sense, the notion of efficiency involves all the various computing resources needed for executing an algorithm.
- However, by the "most efficient" algorithm, normally means the fastest.
- Since time requirements are often a **dominant factor** determining whether or not a particular algorithm is efficient enough to be useful in practice, we concentrate primarily on this single resource.

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- This is convenient because we would expect the relative difficulty of problem instances to vary roughly with the size.

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- However, an $m - city$ problem instance includes, in addition to the labels of the m cities, a collection of $m(m-1)/2$ numbers defining the inter-city distances, and the sizes of these numbers also contribute to the amount of input data.
- To deal with time requirements in a precise, mathematical manner, we must take care to define instance size in such a way that all these factors are taken into account.

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  - that each problem has associated with it a fixed **encoding scheme.**

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- The **input length** for an instance I of a problem Π is defined to be the number of symbols in the description of I obtained from the encoding scheme for Π.
- It is this number, the input length, that is used as the **formal measure of instance size.**

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- If this were the encoding scheme associated with the traveling salesman problem, then the input length for our example would be 32.

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- However, the particular choices made for these will have little effect on the broad distinctions made in the theory of NP-completeness.
- Hence, fix in mind a particular encoding scheme for each problem and a particular computer or computer model, and to think in terms of time complexity as determined from the corresponding input lengths and execution times.

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- So, for this purpose a better model is the **Random Access Machine Model**.

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- One operation on a pointer machine corresponds to a constant number of operations on a random-access machine.

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Properties shared by all three models

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  - The future behavior of the machine is uniquely determined by its present configuration. Outside

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- We shall use running time as our complexity measure.

QUESTIONS?

THANK YOU