NATIONAL INSTITUTE OF TECHNOLOGY GOA

Department of Computer Science and Engineering

B.Tech V Semester End-Examination

Course Name: **Theory of Computation** Course Code: **CS303**

Date: **December 24, 2020** Duration: **3 Hours**

Note:

- Be legible. Strictly, keep the rough work separate from the space you write the answer.
- The steps to arrive at the solution should be shown and should be part of the answer.
- You can use the results discussed in the class.
- Unnecessary details attracts penalty.
- The question paper is of *five* pages and is for **100 Marks**.

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1. For all the following questions, for your answer justification should be provided. Just selecting the option will not fetch complete marks. When you choose an option in multiple choice questions you have to write the complete sentence of the selected option.

   a) Consider the following languages over the alphabet $\Sigma = \{0, 1, c\}$: $L_1 = \{0^n1^n | n \geq 0\}$, $L_2 = \{wcw^R | w \in \{0,1\}^*\}$, $L_3 = \{ww^R | w \in \{0,1\}^*\}$ Which of these languages are deterministic Context-free languages.**(2)**

   i) None of the languages
   ii) Only L_1
   iii) Only L_1 and L_2
   iv) All the three languages.

   b) Let $\Sigma = \{0, 1\}$. Write the DFA that recognizes the language $L = \emptyset$. **(2)**

   c) Let $\Sigma = \{0, 1\}$. Write a DFA that recognizes the language $L = \{w | w$ does not contain more than one $1\}$.**(2)**

   d) What is the minimum pumping length for the language $\{1010\}$? **(2)**

   e) I want to prove the relationship between problem A and problem B. If I prove that A is reducible to B and B is decidable, then which of the following is true?**(2)**

   i) A is undecidable
   ii) A need not be decidable
   iii) A is decidable
   iv) None of the above.

f) In the following identities, which is/are the wrong identities?**(2)**

i) $(a+b)^* = (a^*b^*)^*$

ii) $(a^* + b^*)^* = (a^* + b)^*$

iii) $(ab)^*a = a^*(ab^*)^*$

iv) $a(ba)^* = (ab)^*a$

g) Consider the following problems. Which of them are undecidable? **(1)**

i) Finiteness problem for Finite Automata

ii) Equivalence problem for Finite Automata

iii) Ambiguity problem for CFG

iv) Membership problem for CFG

h) In the regular expression $(c+d)^*d(a+b)^*$, what is the total number of strings generated by the language with length less than 3 and which are those strings?**(2)**

i) What is the minimum number of states in any DFA accepting the language $L = (111 + 11111)^*$?.**(2)**

j) Consider the statement, "Every subset of a regular language is regular". Prove or disprove.**(2)**

k) Give an example that proves the statement "It is possible that, if L_1 and L_2 are context free, $L_1 \cap L_2$ is not context free".**(2)**

l) Given an arbitrary non-deterministic finite automaton (NFA) with n states, the maximum number of states in an equivalent minimized DFA is at least **(1)**

i) n^2

ii) 2^n

iii) $2n$

iv) $n!$

m) What is the length of the shortest string not in the language over alphabet $\{0,1\}$ for regular expression $1^*(0+1)^*1^*$ and which is the string?**(1)**

n) Consider the following languages. Which languages are regular? **(2)**

i) $\{wxw^R |\ w, x \in (a+b)^+\}$

ii) $\{wxw^R |\ w \in (a+b)^*, x \in \{a,b\}\}$

iii) $\{ww^R x | w, x \in (a+b)^+\}$

iv) $\{ww^R | w \in (a+b)^*\}$

o) Let x be any string of length n in $\{0,1\}^*$. Let L be the set of all strings ending with at least n $0's$. What is the minimum number of states in non-deterministic finite automata (NFA) that accept L? **(2)**

p) What is the minimum number of states in deterministic finite automata (DFA) for string starting with 10^2 and ending with 0 over alphabet $\{a,b\}$? **(3)**

q) Consider the following CFG's over the alphabet $\{a, b\}$

$$G_1 : S \to aSa|aSb|\epsilon$$
$$G_2 : S \to aaS|bbS|\epsilon$$

What is the length of the shortest string which does not belong to $L(G_1)$ but belongs to $L(G_2)$? **(3)**

r) Consider the context-free grammar below

$$S \to \epsilon|aSb|bSa \mid SS$$

What language does it generate? **(3)**

(i) $(ab)^* + (ba)^*$

(ii) $(abba)^* + (baab)^*$

(iii) $(aabb)^* + (bbaa)^*$

(iv) Strings of the form $a^n b^n$ or $b^n a^n, n$ any positive integer

(v) Strings with equal numbers of a and b.

s) Let the language D be defined on the binary alphabet {0,1} as follows:
$D = \{w \in \{0, 1\}^* \mid$ substrings 01 and 10 occur an equal number of times in $w\}$
For example, $101 \in D$ while $1010 \notin D$. Which of the following must be TRUE of the language D? **(3)**

(i) D is regular

(ii) D is context-free but not regular

(iii)D is decidable but not context-free

(iv) D is undecidable

t) Consider the language $L \subseteq \{a, b, c\}^*$ defined as

$$L = \{a^p b^q c^r : p = q \text{ or } q = r \text{ or } r = p\}$$

Which of the following answers is TRUE this language? **(3)**

(i) L is regular but not context-free.

(ii) L is context-free but not regular.

(iii) L is decidable but not context-free.

(iv) L is regular, context-free and decidable.

u) Let a, b, c be regular expressions. Which of the following identities is correct? **(3)**

(i) $(a + b)^* = a^*b^*$

(ii) $a(b + c) = ab + c$

(iii) $(a + b)^* = a^* + b^*$

(iv) $(ab+a)^*a = a(ba+a)^*$

(v) None of the above

v) Let B consist of all binary strings beginning with a 1 whose value when converted to decimal is divisible by 7 **(2)**

(i) B can be recognised by a deterministic finite state automaton.

(ii) B can be recognised by a non-deterministic finite state automaton but not by a deterministic finite state automaton.

(iii) B can be recognised by a deterministic push-down automaton but not by a non-deterministic finite state automaton.

(iv) B can be recognised by a non-deterministic push-down automaton but not by a deterministic push-down automaton.

(v) B cannot be recognised by any push down automaton, deterministic or non-deterministic.

2. **a)** Show that the grammar $S \to a|abSb|aAb, A \to bS \mid aAAb$ is ambiguous. **(4)**

b) Consider the language $L = \{w \in \{a,b\}^* |$ the first, middle and last symbols of w are identical.$\}$

i) Design a CFG that generates L. **(3)**

ii) Design a PDA that accepts L either by final state or empty stack.**(4)**

c) Design PDA to accept the language $L = \{a^i b^j | i \leq j \leq 2i\}$.**(4)**

3. For all the designs of TM's (Turing Machines) below, you have to briefly write the algorithm in English to make its working easy to understand and then you have to give the TM (Transition diagram or transition table).

a) Let $\Sigma = \{0,1\}$. Construct a TM that accepts the language $01^* + 10^*$. **(4)**

b) Construct a TM that accepts $L = \left\{0^{2^n} \mid n \geq 0\right\}$. Show how the string 0000 is processed by the TM.**(4)**

c) Design a Turing machine that takes as input a number N and adds 1 to it in binary. To be precise, the tape initially contains a B followed by N in binary. The tape head is initially scanning the B in state q_0. Your TM should halt with $N+1$, in binary, on its tape, scanning the leftmost symbol of $N+1$, in state q_f. You may destroy the B in creating $N+1$, if necessary. For example $q_0B10011 \vdash^* Bq_f10100$, and $q_0B11111 \vdash^* Bq_f100000$.*Give the transitions of your Turing machine, and explain the purpose of each state. Show the sequence of ID's of your TM when given input* $B111$.**(6)**

4. **a)** Let $EQ_{\mathrm{DFA}} = \{\langle A, B\rangle \mid A$ and B are DFAs and $L(A) = L(B)\}$. Prove that EQ_{DFA} is a decidable language.**(4)**

   b) Let $A = \{\langle M\rangle \mid M$ is a DFA which doesn't accept any string containing an even number of $0's\}$. Show that A is decidable.**(4)**

   c) Prove that a language is decidable if and only if it is Turing-recognizable and co-Turing-recognizable. **(4)**

5. **a)** Let $HALT_{TM} = \{< M, w > |M$ is a TM and M halts on input $w\}$. $HALT_{TM}$ is the problem of determining whether a Turing machine halts (by accepting or rejecting) on a given input. Prove that $HALT_{TM}$ is undecidable. **(6)**

   b) Let $EQ_{\mathrm{TM}} = \{\langle M_1, M_2\rangle \mid M_1$ and M_2 are TM's and $L(M_1) = L(M_2)\}$. Prove that EQ_{TM} is undecidable. **(6)**

*********************************All the Best**********************************