# NATIONAL INSTITUTE OF TECHNOLOGY GOA

## Department of Computer Science and Engineering

## B.Tech V Semester Mid-Examination

Course Name: **Theory of Computation** Course Code: **CS303**
Date: **October 07, 2020** Duration: **90 Minutes**

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**Note:**

- Be legible. Strictly, keep the rough work separate from the space you write the answer.
- All the steps to arrive at the solution must be shown and should be part of the answer.
- Unnecessary details attracts penalty.
- The question paper is of *three* pages and is for **50 Marks**.

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1. If $M = (Q, \Sigma, \delta, q_0, F)$ is a DFA accepting $L$, then the DFA $M' = (Q, \Sigma, \delta, q_0, Q - F)$ accepts $\overline{L}$ (complement of $L$). Note that $M'$ is obtained by interchanging the final and non-final states of $M$. Does this still work if $M$ is an NFA? If so prove it. If not, find an counter example.**(3)**

2. The following is the "proof" for $2 = 1$. Find the error in the proof.**(2)**

$$-2 = -2$$
$$4 - 6 = 1 - 3$$
$$4 - 6 + 9/4 = 1 - 3 + 9/4$$
$$(2 - 3/2)^2 = (1 - 3/2)^2$$
$$2 - 3/2 = 1 - 3/2$$
$$2 = 1$$

3. Consider the statement: " There is no largest integer". Prove the statement by using proof by contradiction technique. **(3)**

4. Design a DFA to recognize the language of "all strings that begin or end with $aa$ or $bb$". You have to highlight the purpose of each state in the automata. Note: $\Sigma = \{a, b\}$. **(4)**

5. Design a DFA to recognize the language of "all strings containing both $bb$ and $aba$ as substrings". You have to highlight the purpose of each state in the automata. Note: $\Sigma = \{a, b\}$.**(4)**

6. Suppose $M = (Q, \Sigma, q_0, \delta, F)$ is an DFA, $q$ is a state of $Q$, and $x$ and $y$ are strings in $\Sigma^*$. Using structural induction on $y$, prove the formula

$$\delta^*(q, xy) = \delta^*(\delta^*(q, x), y) \qquad \textbf{(3)}$$

7. Consider the language consisting of the set of strings in $\{0, 1\}^*$ that are binary representations of integers divisible by 5. Design a DFA to recognize the language. Highlight the purpose of each of the states of your DFA.**(4)**

8. Let $M = (Q, \Sigma, q_0, \delta, F)$ be an NFA accepting a language $L$. Assume that there are no transitions to $q_0$, that $F$ has only one state, $q_f$, and that there are no transitions from $q_f$.

   **a)** Let $M_1$ be obtained from $M$ by adding $\epsilon$ -transitions from $q_0$ to every state that is reachable from $q_0$ in $M$. (If $p$ and $q$ are states, $q$ is reachable from $p$ if there is a string $x \in \Sigma^*$ such that $q \in \delta^*(p, x)$.) Describe (in terms of $L$ ) the language accepted by $M_1$.**(2)**

   **b)** Let $M_2$ be obtained from $M$ by adding $\epsilon$ -transitions to $q_f$ from every state from which $q_f$ is reachable in $M$. Describe in terms of $L$ the language accepted by $M_2$.**(2)**

9. Can every regular language not containing $\epsilon$ be accepted by an NFA having only one accepting state and no $\epsilon$-transitions? Prove your answer.**(3)**

10. Design a DFA for the language of strings of length at least two that begin and end with the same symbol. Assume $\Sigma = \{0, 1\}$. **(3)**

11. Give NFA's for the following languages. Each NFA should respect the specified limits on the number of states and transitions. (Transitions labeled with two symbols count as two transitions.) In all cases, the alphabet is $\{0, 1\}$.

    **a)** The language of strings of length at least two whose last two symbols are the same. No more than four states and six transitions.**(2)**

    **b)** The language of strings that contain exactly one 1. No more than two states and three transitions.**(2)**

    **c)** The language of strings of length at least two that begin with 0 and end in 1. No more than three states and four transitions.**(2)**

12. The number of substrings (of all lengths inclusive) that can be formed from a character string of length $n$ is ......... Justify. **(2)**

13. Consider a DFA over $\Sigma = \{a, b\}$ accepting all strings which have number of $a's$ divisible by 6 and number of $b's$ divisible by 8. What is the minimum number of states that the DFA will have? Justify. **(2)**

14. The smallest finite automaton which accepts the language $L = \{w|$ length of $w$ is divisible by $3\}$ has ......... states. Justify. **(2)**

15. What can be said about a regular language $L$ over $\{a\}$ whose minimal finite automaton has two states. **(1)**

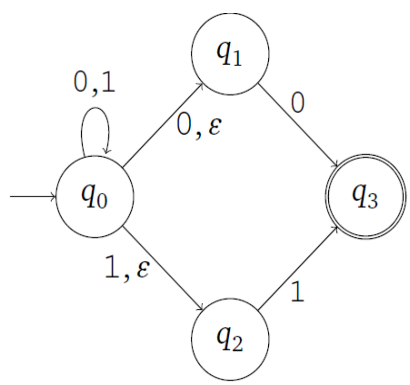
   **a)** Must be $\{a^n | n \text{ is odd }\}$.

   **b)** Must be $\{a^n | n \text{ is even }\}$.

   **c)** Must be $\{a^n | n \geq 0\}$.

   **d)** Either $L$ must be $\{a^n | n \text{ is odd }\}$ or $L$ must be $\{a^n | n \text{ is even }\}$.

16. Convert the following NFA to DFA. **(4)**




********************************All the Best********************************