Turing Machines and Decidability

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I want to Begin With... J K Rowling's Quote

It is impossible to live without failing at something, unless you live so cautiously that you might has well not have lived at all, in which case you have failed by default.

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- We skip over the extensive theory of Turing machines.
- We do not spend much time on the low-level programming of Turing machines.
- Believe that Turing machines capture all algorithms.

Standardizing the way to Describe Turing machine Algorithms

Initial Question

What is the right level of detail to give when describing such algorithms?

Three possibilities

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- Once you feel confident, high-level descriptions are sufficient.

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- A Turing machine may be programmed to decode the representation so that it can be interpreted in the way we intend.

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- The encoding itself can be done in many reasonable.
- It doesnt matter which one we pick because a Turing machine can always translate one such encoding into another.

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  - We write

$$A = \{< G > | G \text{ is a connected undirected graph}\}.$$

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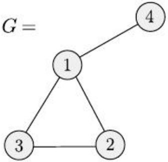
- Consider an encoding that is a list of the nodes of G followed by a list of the edges of G.
- Each node is a decimal number, and each edge is the pair of decimal numbers that represent the nodes at the two endpoints of the edge.

Focus

$G =$



$\langle G \rangle =$

(1,2,3,4)((1,2),(2,3),(3,1),(1,4))

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- To do so, M scans the tape to be sure that there are two lists and that they are in the proper form.
- The first list should be a list of distinct decimal numbers.
- And the second should be a list of pairs of decimal numbers.
- Then M checks several things.

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- This verification completes the input check, and M goes on to stage 1.

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- If they arent, M checks the next edge on the list.

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- For stage 4, M scans the list of nodes to determine whether all are dotted.
- If they are, it enters the accept state; otherwise, it enters the reject state.

THANK YOU