Turing Machines and Decidability

Purushothama B R

Department of Computer Science and Engineering
National Institute of Technology Goa ,INDIA

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I want to Begin With... J K Rowling's Quote

It is impossible to live without failing at something, unless you live so cautiously that you might has well not have lived at all, in which case you have failed by default.

Investigate the Power of Algorithms

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- The unsolvability of certain problems may come as a surprise.

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    - Help you gain an important perspective on computation.

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  - Starting with examples where decidability is possible helps you to appreciate the undecidable examples.

DECIDABLE PROBLEMS CONCERNING REGULAR LANGUAGES

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  - Whether two finite automata are equivalent.

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- This language contains the encodings of all DFAs together with strings that the DFAs accept. Let

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Theorem

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Proof Idea

We simply need to present a TM M that decides A_{DFA}.

Algorithm

$M =$ "On input $\langle B, w\rangle$, where B is a DFA and w is a string:

1. Simulate B on input w.
2. If the simulation ends in an accept state, *accept*. If it ends in a nonaccepting state, *reject*."

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Theorem 4.2

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  - N first converts the NFA it receives as input to a DFA before passing it to M .

Algorithm

$N =$ "On input $\langle B, w\rangle$, where B is an NFA and w is a string:

1. Convert NFA B to an equivalent DFA C, using the procedure for this conversion given in Theorem 1.39.
2. Run TM M from Theorem 4.1 on input $\langle C, w\rangle$.
3. If M accepts, *accept*; otherwise, *reject*."

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$P =$ "On input $\langle R, w \rangle$, where R is a regular expression and w is a string:

1. Convert regular expression R to an equivalent NFA A by using the procedure for this conversion given in Theorem 1.54.
2. Run TM N on input $\langle A, w \rangle$.
3. If N accepts, *accept*; if N rejects, *reject*."

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- In the preceding three theorems we had to determine whether a finite automaton accepts a particular string.
- Now we must determine whether or not a finite automaton accepts any strings at all.

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$T =$ "On input $\langle A \rangle$, where A is a DFA:

1. Mark the start state of A.

2. Repeat until no new states get marked:

3. Mark any state that has a transition coming into it from any state that is already marked.

4. If no accept state is marked, *accept*; otherwise, *reject*."

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  - For proving the class of regular languages **closed under complementation, union, and intersection.**
- These constructions are algorithms that can be carried out by Turing machines.

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F = "On input $\langle A, B\rangle$, where A and B are DFAs:

1. Construct DFA C as described.
2. Run TM T from Theorem 4.4 on input $\langle C\rangle$.
3. If T accepts, *accept*. If T rejects, *reject*."

DECIDABLE PROBLEMS CONCERNING CONTEXT-FREE LANGUAGES

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Focus

- I describe algorithms
  - To determine whether a CFG generates a particular string w
  - To determine whether the language of a CFG is empty.
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- Only finitely many such derivations exist.
- We know how to can convert G to Chomsky normal form.

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S = "On input $\langle G, w\rangle$, where G is a CFG and w is a string:

1. Convert G to an equivalent grammar in Chomsky normal form.
2. List all derivations with $2n-1$ steps, where n is the length of w; except if $n=0$, then instead list all derivations with one step.
3. If any of these derivations generate w, *accept*; if not, *reject*."

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- More efficient algorithm for recognizing general context-free languages exist.
- Even greater efficiency is possible for recognizing deterministic context-free languages.

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  - We need to take a different approach

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- It determines for each variable,
  - whether that variable is capable of generating a string of terminals.
- When the algorithm has determined that a variable can generate some string of terminals,
  - The algorithm keeps track of this information by placing a mark on that variable.

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  - The algorithm knows that this variable can be marked, too.
- The algorithm continues in this way until it cannot mark any additional variables.

Proof

$R =$ "On input $\langle G \rangle$, where G is a CFG:

1. Mark all terminal symbols in G.
2. Repeat until no new variables get marked:
3. Mark any variable A where G has a rule $A \rightarrow U_1 U_2 \cdots U_k$ and each symbol $U_1, \ldots, U_k$ has already been marked.
4. If the start variable is not marked, *accept*; otherwise, *reject*."

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  - intersection.
- In fact, EQ_{CFG} **is not decidable.**
- I will prove it after going through the technique.

CFLs are Decidable

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Theorem 4.9

Every context-free language is decidable.

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  - And we know that **any nondeterministic TM can be converted into an equivalent deterministic TM.**
  - Yet, there is a difficulty!!!

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- A different Idea is necessary!!!

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- It works as follows.

M_G = "On input w:

1. Run TM S on input $\langle G, w \rangle$.
2. If this machine accepts, *accept*; if it rejects, *reject*."

Relationships

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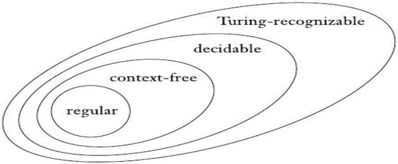

Undecidability

- I will prove one of the most philosophically important theorems of the theory of computation.
- There is a specific problem that is algorithmically unsolvable.
- Computers appear to be so powerful that you may believe that all problems will eventually yield to them.
- The theorem that I present strates that computers are limited in a fundamental way.

Unsolvable? What Sort of Problems?

- What sorts of problems are unsolvable by computer?
- Are they esoteric, dwelling only in the minds of theoreticians?
- **No!**
- Even some **ordinary problems turn out to be computationally unsolvable.**

Unsolvable Problem

- You are given a computer program and
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- For example, **sort a list of numbers**

Challenge

You need to verify that the program performs as specified (i.e., that it is correct).

What to Do?

- Both the program and the specification are mathematically precise objects.
- You hope to automate the process of verification by feeding these objects pause into a suitably programmed computer.
- However, you will be disappointed.

My Goal

- I will highlight several computationally unsolvable problems.
- We aim to help you develop a feeling for the types of problems that are unsolvable .
- To learn **techniques for proving unsolvability.**

Undecidability of a language

- Let us establishes the undecidability of a specific language.
- **The problem of determining whether a Turing machine accepts a given input string.**
- Let,

$$A_{TM} = \{< M, w > | M \text{ is a TM and } M \text{ accepts } w\}.$$

- Note: A_{DFA} and $A_C FG$ is decidable!!!.

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- Note: A_{DFA} and $A_C FG$ is decidable!!!. I have proved!!!

Theorem

Theorem 4.11

A_{TM} is undecidable.

Discussion

- Observe that A_{TM} is Turing-recognizable.
- It shows that recognizers are more powerful than deciders.
- Requiring a TM to halt on all inputs restricts

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- It shows that recognizers are more powerful than deciders.
- Requiring a TM to halt on all inputs restricts the kinds of languages that it can recognize.

Turing machine U to recognize A_{TM}

U = "On input $\langle M, w\rangle$, where M is a TM and w is a string:

1. Simulate M on input w.
2. If M ever enters its accept state, *accept*; if M ever enters its reject state, *reject*."

- If M loops on w, machine U loops on input $< M, w >$
- So machine does not decide A_{TM}.
- If the algorithm had some way to determine that M was not halting on w,
  - It could reject in this case.
- However, I will discuss that algorithm has no way to determine this.

- The Turing machine U is interesting.
- It is an example of the universal Turing machine first proposed by Alan Turing in 1936.
- This machine is called universal because
  - Tt is capable of simulating any other Turing machine from the description of that machine.
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Univeral Turing Machine

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- The UTM played an important early role in the development of **stored-program computers.**

THANK YOU