Turing Machines and Decidability

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I want to Begin With... J K Rowling's Quote

It is impossible to live without failing at something, unless you live so cautiously that you might has well not have lived at all, in which case you have failed by default.

Turing Recognizable

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A language is Turing-recognizable if some Turing machine recognizes it.

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- Looping may entail any simple or complex behavior that never leads to a halting state.

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- A **decider** that recognizes some language also is said to **decide** that language.

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- Every decidable language is Turing-recognizable.

Multi-tape Turing Machine

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Every multitape Turing machine has an equivalent single-tape Turing machine.

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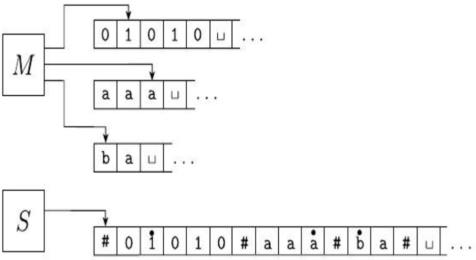
Representing three tapes with Single tape



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  2. The formatted tape contains

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    - To update the tapes according to the way that M's transition function dictates.

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- Then it continues the simulation as before.

Complexity

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The time taken by the one-tape TM N to simulate n moves of the $k - tape$ TM M is $O(n^2)$.

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Exercise

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- Ancient mathematical literature contains descriptions of algorithms for a variety of tasks,
  - Finding prime numbers and greatest common divisors.

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- The notion of algorithm itself was not defined precisely until the twentieth century.
- Before that, mathematicians had an intuitive notion of what algorithms were.
- And relied upon that notion when using and describing them.
- But that intuitive notion was insufficient for gaining a deeper understanding of algorithms.

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- **The tenth problem on his list concerned algorithms.**

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  - Some polynomials have an integral root and some do not.

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- Proving that an algorithm does not exist requires having a clear definition of algorithm.
- Progress on the tenth problem had to wait for that definition.

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- These two definitions were shown to be equivalent.
- This connection between the informal notion of algorithm and the precise definition has come to be called the **ChurchTuring thesis.**

Church-Turing Thesis

Intuitive notion of algorithms	equals	Turing machine algorithms

THANK YOU