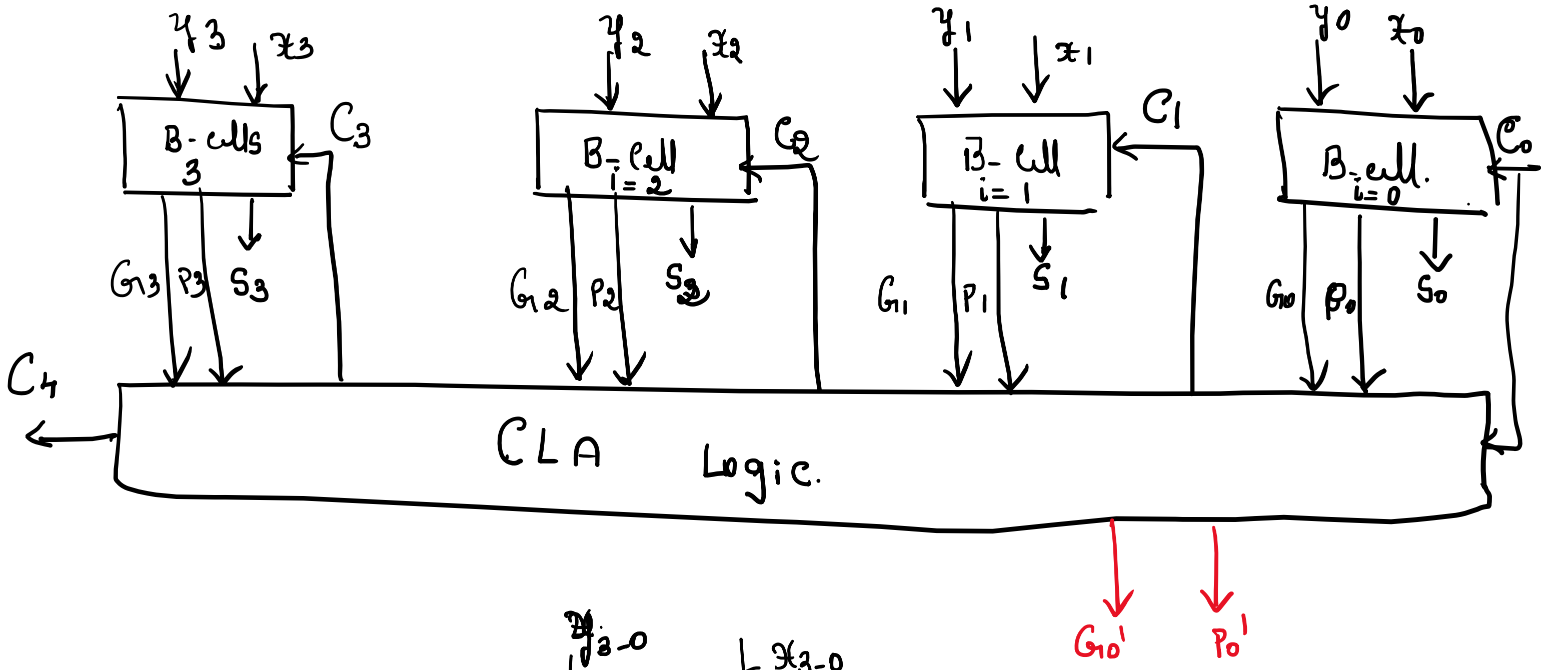
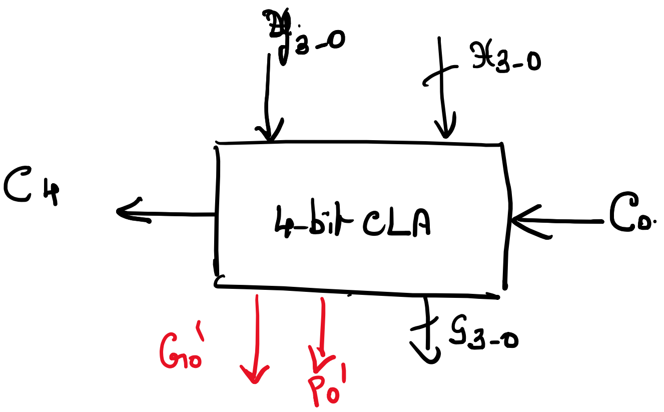
ARITHMETIC UNIT

→ Multiplier

→ CLA Cascode Style-II

G_i, P_i : Bit stage i generate or propagate a carry



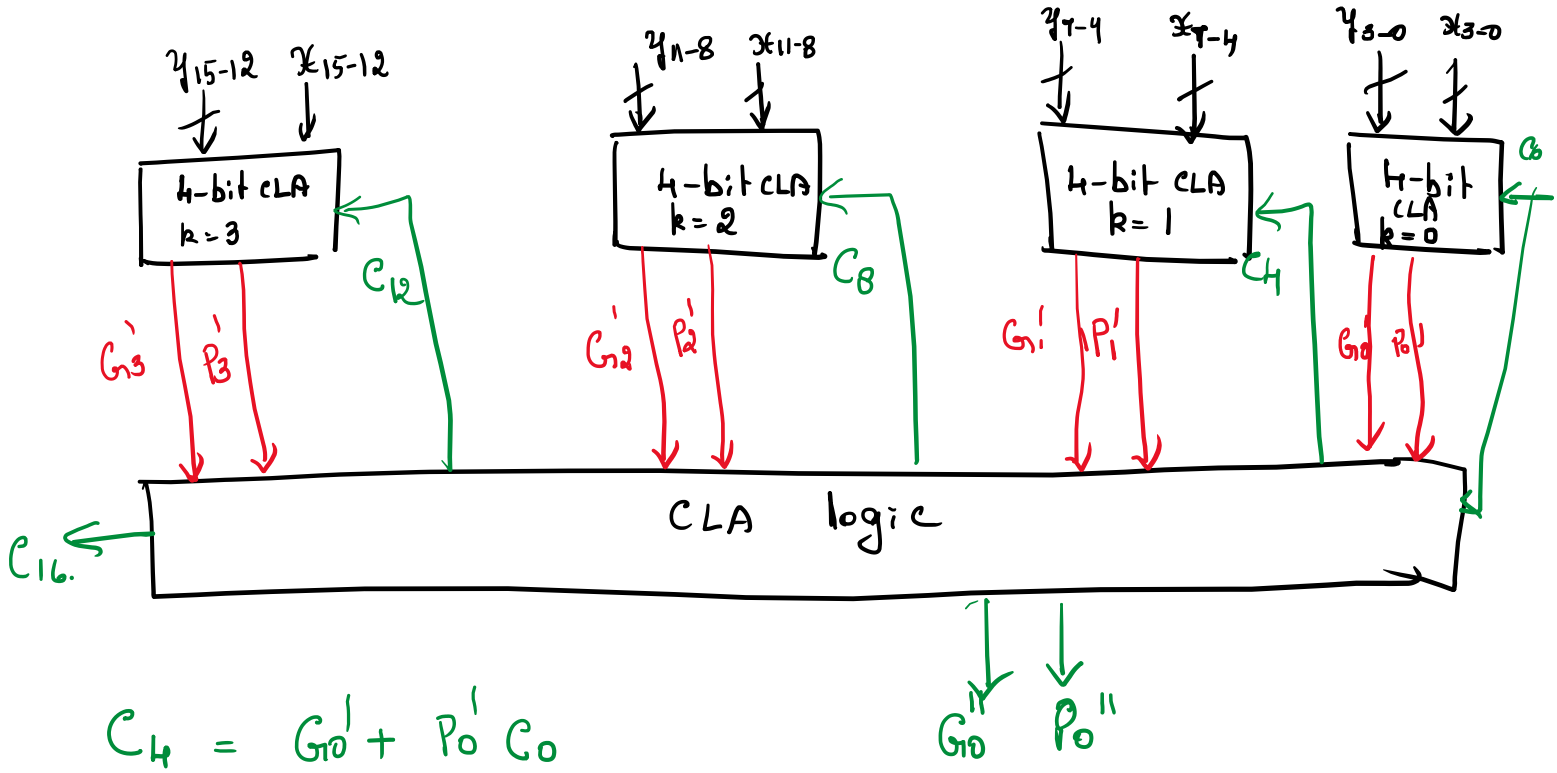


16-bit CLA : 4-bit CLA

$$C_4 = \underbrace{G_3 + P_3 G_2 + P_3 P_2 G_1 + P_3 P_2 P_1 G_0}_{G_0'} + \underbrace{P_3 P_2 P_1 P_0}_{P_0'} C_0.$$

$$C_4 = G_0' + P_0' C_0$$

G_i' P_i' : Second level Generator and Propagator



$C_4 = G_0' + P_0' C_0$

$C_8 = G_1' + P_1' C_4$

$C_8 = G_1' + P_1' G_0' + P_1' P_0' C_0$

$C_{12} = G_2' + P_2' C_8$

$C_{16} = G_3' + P_3' C_{12}$

Signed Number Multiplication

→ Long hand, with sign extension

1. +ve M, +ve Q ✓
2. −ve M, +ve Q ✓
3. +ve M, −ve Q
4. −ve M, −ve Q

(3), (4) → M, Q 2's complement ↓

−ve Multiplier

↓

Special treatment

↳ 2's complement

③ → −ve M" +ve Q" → ②

④ → +ve M" ~~+~~ −ve Q" → ①

⇓

Technique → treats positive and negative multiplier equally

⇓

BOOTH ALGORITH

→ Encodes the multiplier

Booth Encoding

BOOTH ALGORITHM

→ Recoding of multiplier.

→ Booth scheme

→ 3 kinds of bits in multiplier

→ 0 bit : Do nothing

→ +1 bit : Add shifted multiplicand.

→ −1 bit : (−1) times add shifted multiplicand

Add 2's complement of multiplicand.

Booth Scheme.

$$\begin{array}{ccccc} 3 & 2 & 1 & 0 & \\ 0 & 1 & 1 & 1 & \boxed{0} \end{array}$$

i   i-1

i   i-1

Multiplier		Version of Multiplicand selected by bit i	
Bit i	Bit i-1		
0	0	$0 \times M$	0
0	1	$+1 \times M$	+1
1	0	$-1 \times M$	-1
1	1	$0 \times M$	0

Ex :

1) M = 0 0 1 1 1 1 0 0

0 +1 0 0 0 -1 0

2) Q = 0 0 1 1 1 1 0 = 30

M = 0 1 0 1 1 0 1 = 45

0 1 0 1 1 0 1 (M)

0 0 +1 +1 +1 +1 0 (Q)

0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 1 0 1 1 0 1
0 0 0 0 0 0 1 0 1 1 0 1
0 0 0 0 0 1 0 1 1 0 1
0 0 0 0 1 0 1 1 0 1
0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0

0 0 0 1 0 0 0 1 0 0 0 1 1 0

Q = 0 0 1 1 1 1 0 0

$\bar{Q}$ = 0 +1 0 0 0 −1 0

M'' = 1010011

0 1 0 1 1 0 1 (M)

0 +1 0 0 0 −1 0 ($\bar{Q}$)

0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 1 1 1 1 1 0 1 0 0 1 1
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0
0 0 0 1 0 1 1 0 1
0 0 0 0 0 0 0 0

0 0 0 1 0 1 0 1 0 0 0 1 1 0

1 1 1 1
—
1

1 1 } S=0 C=1
1 1 } S=0 C=1
1
—
1

Case 1:

M = +13 0 1 1 0 1

Q = +11 0 1 0 1 1 0

$\bar{Q} \rightarrow$ +1 −1 +1 0 −1

$M'' =$ 1 0 0 1 0 + 1 = 1 0 0 1 1

```
                 0   1   1   0   1   (M)
                +1  −1  +1   0  −1   (Q̄)
--------------------------------------------
 1   1   1   1   1   1   0   0   1   1
 0   0   0   0   0   0   0   0   0
 0   0   0   0   1   1   0   1
 1   1   1   0   0   1   1
 0   0   1   1   0   1
--------------------------------------------
 0   0   1   0   0   0   1   1   1   1
```

Case 2:

M = −13 1 0 0 1 1

$M'' =$ 0 1 1 0 1

Q = 11 0 1 0 1 1

$\bar{Q} \rightarrow$ +1 −1 +1 0 −1

					1	0	0	1	1	(M)
					+1	−1	+1	0	−1	($\bar{Q}$)
0	0	0	0	0	0	1	1	0	1	
	0	0	0	0	0	0	0	0	0	
		1	1	1	1	0	0	1	1	
			0	0	0	1	1	0	1	
				1	1	0	0	1	1	
1	1	0	1	1	1	0	0	0	1	(−143)

Case 3:

M = 13 0 1 1 0 1 $M'' =$ 1 0 0 1 0

Q = −11 1 0 1 0 1

$\bar{Q}$ −1 +1 −1 +1 −1

					0	1	1	0	1	(M)
					−1	+1	−1	+1	−1	($\bar{Q}$)
1	1	1	1	1	1	0	0	1	1	
	0	0	0	0	0	1	1	0	1	
		1	1	1	1	0	0	1	1	
			0	0	0	1	1	0	1	
				1	1	0	0	1	1	
1	1	0	1	1	1	0	0	0	1	= −143

Case 4

$M = -13$ 1 0 0 1 1 $M'' = 0\,1\,1\,0\,1$

$Q = -11$ 1 0 1 0 1

$\bar{Q}$ $-1\ \ +1\ \ -1\ \ +1\ \ -1$

					1	0	0	1	1	(M)
					−1	+1	−1	+1	−1	($\bar{Q}$)
0	0	0	0	0	0	1	1	0	1	
1	1	1	1	1	0	0	1	1		
0	0	0	0	1	1	0	1			
1	1	1	0	0	1	1				
0	0	1	1	0	1					
0	0	1	0	0	0	1	1	1	1	= 143 ✓

Booth method : Positive and negative multipliers are treated alike.

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0 0 1 1 1 1 0

↓

0 +1 0 0 0 −1 0

1 1 0 1 0

0 −1 +1 −1 0

Worst Case.

0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 → 8

+1 −1 +1 −1 +1 −1 +1 −1 +1 −1 +1 −1 +1 −1 +1 −1 → 16

Ordinary

1 1 0 0 0 1 0 1 1 0 1 1 1 1 0 0

0 −1 0 0 +1 −1 +1 0 −1 +1 0 0 0 −1 0 0

(Good) Best Case.

0 0 0 0 1 1 1 1 1 0 0 0 0 1 1 1

0 0 0 +1 0 0 0 0 −1 0 0 0 +1 0 0 −1