$$\langle E\rangle = \left\langle \frac{p^2}{2m}\right\rangle + \langle V\rangle$$

$$\langle E\rangle = \int \psi^* i\hbar \frac{\partial \psi}{\partial t}\, d\tau$$

$$\langle p\rangle = \int \psi^* (-i\hbar \nabla)\psi\, d\tau.$$

*) The wavefunction for a particle in a one dimensional box is given by $\psi(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$ Evaluate the expectation value for K.E for a particle in the nth quantum state.

$$\langle E\rangle = \int_{-\infty}^{\infty} \psi^* E \psi\, dx = \int_{-\infty}^{\infty} \psi^* \frac{p^2}{2m} \psi\, dx$$

$$= \frac{1}{2m}\int_{-\infty}^{\infty} \psi^* (-i\hbar)^2 \frac{\partial^2}{\partial x^2} \psi\, dx$$

$$= \frac{\hbar}{2m}\int_{-\infty}^{\infty} \psi^* \frac{\partial^2 \psi}{\partial x^2}\, dx$$

$$= \frac{\hbar}{2m}\int_0^L \frac{2}{L} \sin\left(\frac{n\pi x}{L}\right)\left(\frac{-n^2\pi^2}{L^2}\right) \sin \frac{n\pi x}{L}\, dx$$

$$= \frac{n^2\pi^2\hbar^2}{mL^3}\int_0^L \sin^2 \frac{n\pi x}{L}\, dx = \frac{n^2\pi^2\hbar^2}{2mL^3}\int_0^L \left(1-\cos \frac{2n\pi x}{L}\right) dx$$

$$= \frac{n^2\pi^2\hbar^2}{2mL^3} \cdot L = \frac{n^2\pi^2\hbar}{2mL^2} = \frac{n^2 h^2}{8mL^2}$$

*) At certain time, the normalized wave function of a particle moving along x-axis has the form given by

$$\psi(x) = \begin{cases} x+\beta & \text{for } -\beta < x < 0 \\ -x+\beta & \text{for } 0 < x < \beta \end{cases}$$

and zero elsewhere. Find the value of β and probability that the particle positions between $x = \beta/2$ and $x = \beta$.

Sol:

As $\psi(x)$ is normalized, $\therefore$

$$\int_{-\beta}^{\beta} |\psi(x)|^2 dx = 1$$

$$\int_{-\beta}^{0} |\psi(x)|^2 dx + \int_{0}^{\beta} |\psi(x)|^2 dx = 1$$

$$\int_{-\beta}^{0} (x+\beta)^2 dx + \int_{0}^{\beta} (-x+\beta)^2 dx = 1$$

$$\int_{-\beta}^{0} (x^2+\beta^2+2\beta x)\,dx + \int_{0}^{\beta} (x^2+\beta^2-2\beta x)\,dx = 1$$

$$\left[\frac{x^3}{3}+\beta x\right]_{-\beta}^{0} + \left[\frac{x^3}{3}-\beta x\right]_{0}^{\beta} = 1$$

$$\frac{2\beta^3}{3} = 1$$

$$\beta = \left(\frac{3}{2}\right)^{1/3}$$

The probability that the particle positions b/w $x = \beta/2$ and $x = \beta$

$$P = \int_{\beta/2}^{\beta} |\psi(x)|^2 dx = \int_{\beta/2}^{\beta} |-x+\beta|^2 dx =$$

$$= \int_{\beta/2}^{\beta} (x^2+\beta^2-2x\beta)\,dx = \beta^3/24$$

$$= \left[\left(\frac{3}{2}\right)^{1/3}\right]^3 \cdot \frac{1}{24}$$

$$= \frac{3}{2} \times \frac{1}{24} = \frac{1}{16}$$

*) The normalized wave-function for certain particle is $\psi(x) = \sqrt{\frac{3}{\pi}} \cos x$ $-\frac{\pi}{2} < x < \frac{\pi}{2}$. Calculate the probability of finding the particle between $0 < x < \frac{\pi}{4}$.

Sol:

The Probability of finding the particle b/w 0 & $\pi/4$

$$P = \int_0^{\pi/4} \psi^* \psi\,dx = \int_0^{\pi/4} \left|\sqrt{\frac{3}{\pi}} \cos x\right|^2 dx$$

$$= \frac{3}{\pi} \int_0^{\pi/4} \cos^2 x\,dx$$

$$= \frac{3}{\pi} \left[\frac{1}{2} \cos x \sin x + \frac{1}{2} \int dx\right]_0^{\pi/4}$$

$$= \frac{3}{2\pi} \left[\cos x \sin x + x\right]_0^{\pi/4}$$

$$= \frac{3}{2\pi} \left[\cos\frac{\pi}{4} \sin\frac{\pi}{4} + \frac{\pi}{4} - \cos 0 \sin 0 - 0\right]$$

$$= \frac{3}{2\pi} \left[\frac{1}{2} + \frac{\pi}{4}\right] = \frac{3}{4\pi} + \frac{3}{8}$$

$$= 0.614 \text{ (or) } 61.4\%.$$