ARITHMETIC DESIGN

- Adders
  - Single bit adder
  - n- bit addition

→ RCA | Subtraction

→ n - FAs

→ 2n

→ 2n+2

- CLA

→ Gi, Pi }

- B-cell for CLA

- n-bit CLA

→ n B-cells

→ 4-bit CLA

C_1, C_2, C_3, C_4

→ 8-bit (n=8)

$C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8$

↓

Gate fan-in

n - bit : (n+1)

→ 2 alternative.

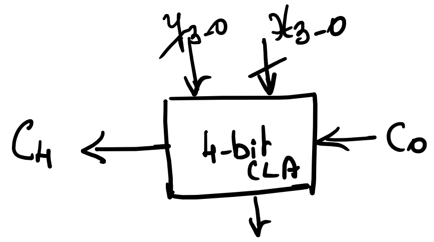
4-bit CLA

– 16 bit → 4 bit CLAs

→ Cascade style I (RCA)

→ Cascade " II

↳ Higher level Generator & Propa.

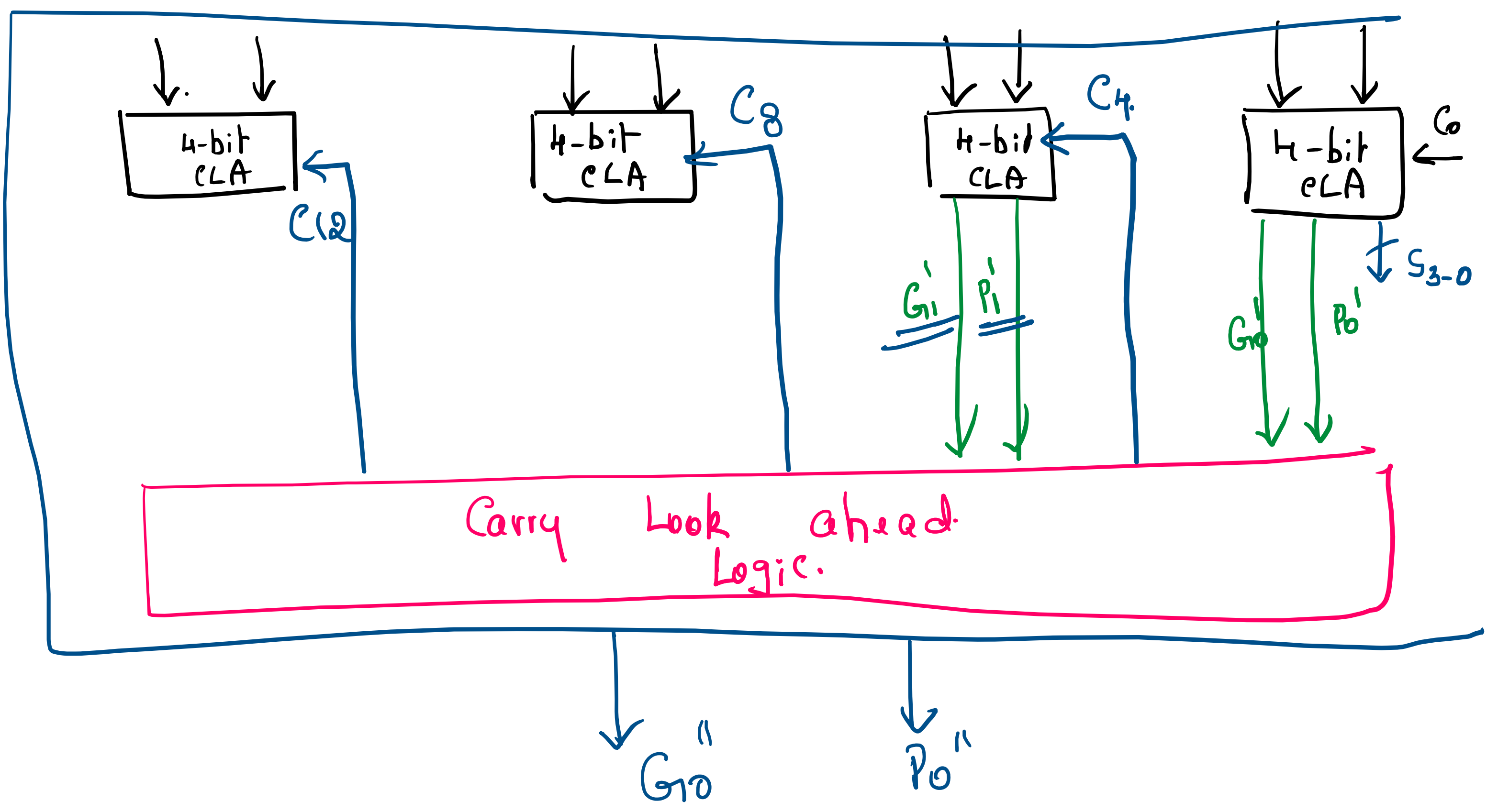


$$C_4 = \underbrace{G_3 + P_3 G_2 + P_3 P_2 G_1 + P_3 P_2 P_1 G_0}_{G_0'} + \underbrace{P_3 P_2 P_1 P_0}_{P_0'} C_0$$

$$C_4 = G_0' + P_0' C_0$$

P_i, G_i : First level

P_i', G_i' : Second level.



Multiplication

- Unsigned
  (Positive)

— Partial Product

4-bit, 8-bit

$P_0, P_1 \ldots P_7$

→ Combinational array multiplier.

Sequential Circuit Binary Multiplier

→ 10 × 3

→ Repeated addition : n-bit

→ Registers:

A : n-bit : Initially all 0s
Q : n-bit : To keep the multiplier Q
M : n-bit : Multiplicand M.

— Flipflop C : Carry out from the adder.

q_i : Controls whether the multiplicand M is added to the Partial Product or not

$= 1$ M is added to $PP_i \rightarrow PP_{i+1}$

$= 0$ M is not added to PP_i

M : 1101 Q : 1011

	C	A	Q	M
Initial Configuration. ($i=0$)	0	0000	1011	1101

C	A	Q		
0	1101	1011	Add.	First cycle.
0	0110	1101	Shift Right.	
1	0011	1101	Add.	Second cycle.
0	1001	1110	Shift Right.	
0	1001	1110	No Add.	Third cycle.
0	0100	1111	Shift Right.	
1	0001	1111	Add.	Fourth cycle.
0	1000	1111	Shift Right.	

Product: 1000 1111

n-bit, n-cycles

- q_0 / q_i
  - $= 1$ Add M
  - $= 0$ No add
- Shift Right

Example 2

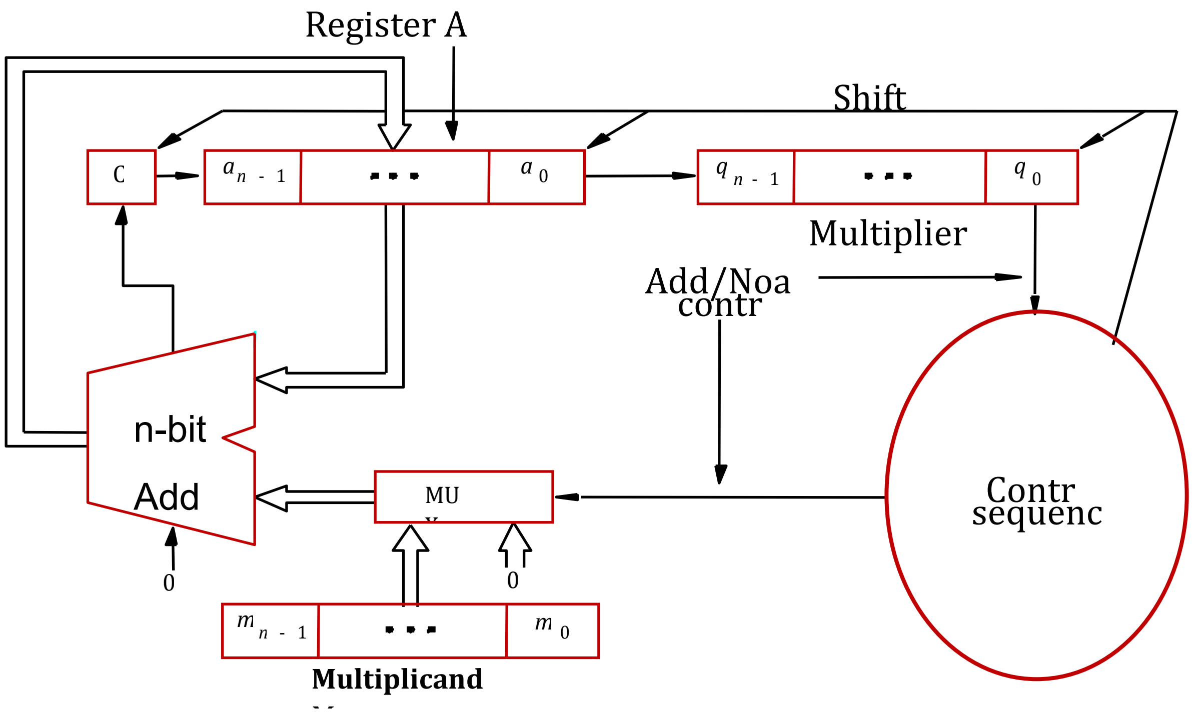
M : 11001 $\qquad$ n = 5 bits

Q : 11110

M: $\boxed{11001}$

C	A	Q		
$\boxed{0}$	$\boxed{00000}$	$\boxed{11110}$		
0	00000	11110	No Add.	I
0	00000	01111	Shift Right	
0	11001	01111	Add.	II
0	01100	10111	Shift Right	
1	00101	10111	Add.	III
0	10010	11011	Shift Right	
1	01011	11011	Add.	IV
0	10101	11101	Shift Right	
1	01110	11101	Add.	V
0	10111	01110	Shift Right	

PRODUCT: 1011101110



→ Faster Multiplication?

→ Unsigned/Positive numbers.

Signed Number Multiplication

→ 2's complement.

M : Multiplicand.
Q: Multiplier.

→ 4 Cases

1) +ve M, +ve Q
2) −ve M, +ve Q.
3) +ve M, −ve Q
4) −ve M, −ve Q.

Sign extension

- Representing a number in the 2's complement scheme by using a no of bits higher than required.

$+2 \longrightarrow$ 8 bit

0 0 0 0 0 0 1 0

-2 1 1 1 1 1 1 1 0

Case 1: +ve M, +ve Q

M = 13	0 1 1 0 1	5-bits
Q = 11.	0 1 0 1 1	

```
          0 1 1 0 1   (M)
          0 1 0 1 1   (Q)
------------------------
0 0 0 0 0 0 1 1 0 1
0 0 0 0 0 1 1 0 1
0 0 0 0 0 0 0 0
0 0 0 1 1 0 1
0 0 0 0 0 0
------------------------
0 0 1 0 0 0 1 1 1 1   (143)
```

Case 2 : −ve Multiplicand, +ve Q

M = −13   1 0 0 1 1
Q := 11   0 1 0 1 1

$\left.\begin{matrix} 10011 \\ 01011 \end{matrix}\right\}$ 5-bit.

```
                    1 0 0 1 1   (M)
                    0 1 0 1 1   (Q)
  ---------------------------
  1 1 1 1 1 1 0 0 1 1
  1 1 1 1 1 0 0 1 1
  0 0 0 0 0 0 0 0
  1 1 1 0 0 1 1
  0 0 0 0 0 0
  ---------------------------
    1 1 0 1 1 1 0 0 0 1
```

= −143 ✓

−13 × 11.

Case 3 : +ve M, −ve Q.

M = 13   0 1 1 0 1
Q = −11   1 0 1 0 1

$$
\begin{array}{rrrrrrrrrrl}
 & & & & & 0 & 1 & 1 & 0 & 1 & (M) \\
 & & & & & 1 & 0 & 1 & 0 & 1 & (Q) \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & & & & \\
0 & 0 & 1 & 1 & 0 & 1 & & & & & \\
\hline
\end{array}
$$

$\rightarrow$ 0 1 0 0 0 1 0 0 0 1

-143

~~151~~

273

1+16+

Case 4: -ve M , -ve Q

M = -13   1 0 0 1 1

Q = -11   1 0 1 0 1

$$
\begin{array}{rrrrr}
1 & 0 & 0 & 1 & 1 \\
1 & 0 & 1 & 0 & 1 \\
\hline
\end{array}
$$

⋮   ⋮

1 0 1 1 1 0 1 1 1 1.

-13×-11

143

Case 3: +ve M —ve Q.

$\downarrow$ 2's $\downarrow$ 2's

—ve M. +ve Q.

Case 3 $\longrightarrow$ Case 2.

Case 4: —ve M —ve Q.

$\downarrow$ 2's $\downarrow$ 2's

+ve M +ve Q $\rightarrow$ Case 1.

① +ve M, +ve Q $\rightarrow$ long hand multiplication with sign extension

② —ve M +ve Q $\rightarrow$ "

(3) +ve M -ve Q
(4) -ve M -ve Q

2's complement of M, Q.

Long hand multiplication approach - Sign extention.

Booth Encode Q.

1	1		
	0	1	1 0
	0	1	
	1	1	1 0
	0	1	
	1	0	