Property of Photon

PEE

Compton

1. $E = h\nu$

2. $m = \frac{h\nu}{c^2}$

3. $P = \frac{h\nu}{c}$

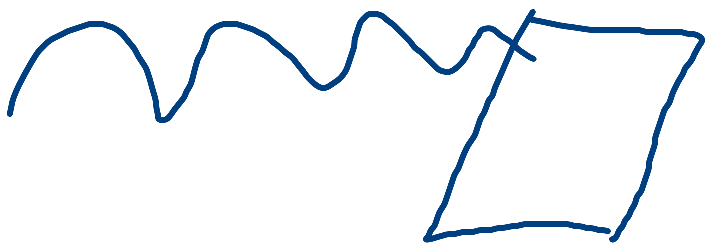
4. Charge — No charge

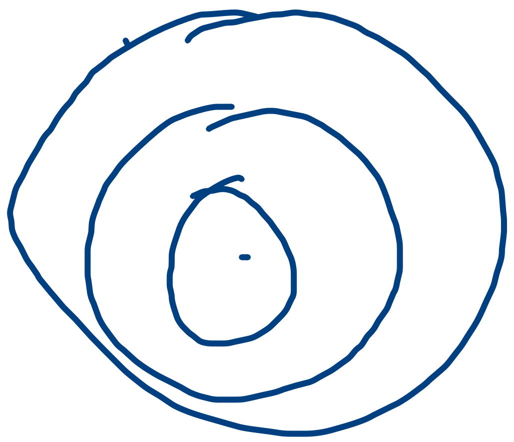
Compton Effect

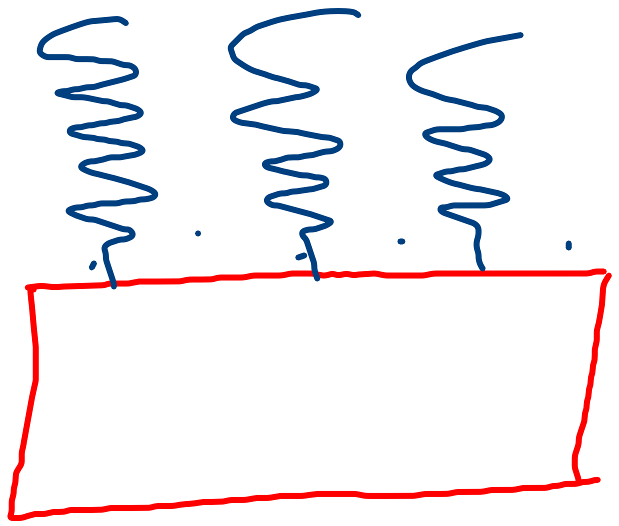
↳ 1922 A. H. Compton

$\nu' < \nu$

$\lambda' > \lambda$







$I \to I$

mc^2

$E = h\nu = \frac{hc}{\lambda}$

$E' = h\nu' = \frac{hc}{\lambda'}$

$p = \frac{h\nu}{c}$

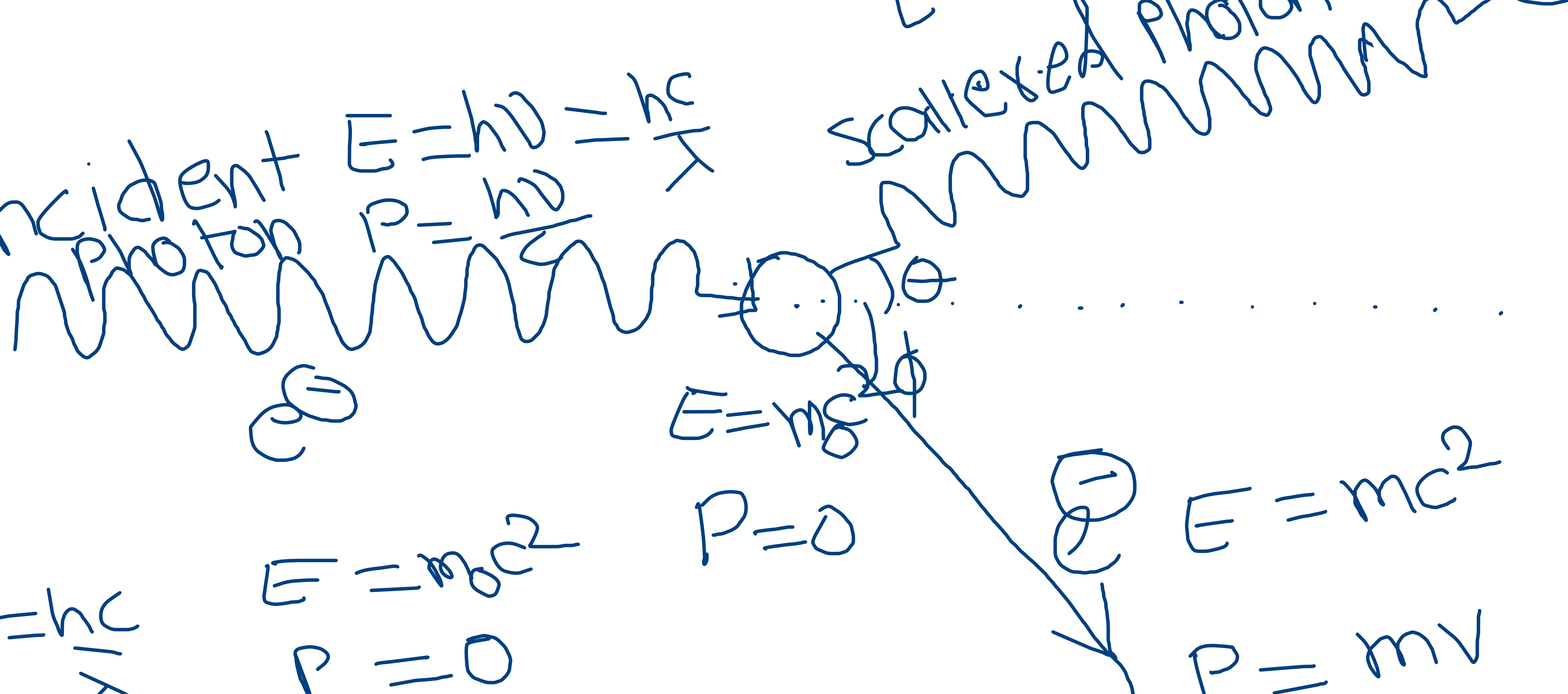
$h\nu = h\nu_0 + \frac{1}{2}mv^2$

$4h\nu = 2h\nu + 2h\nu$

$E' = h\nu' = \frac{hc}{\lambda'}$ $\quad p' = \frac{h\nu'}{c}$

Scattered photon

Incident photon $\quad E = h\nu = \frac{hc}{\lambda}$ $\quad p = \frac{h\nu}{c}$



e^{-} $\quad E = m_0c^2$ $\quad P = 0$

e^{-} $\quad E = mc^2$

$P = mv = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}$

Photon

B.C.

$E = h\nu = \frac{hc}{\lambda}$

$P = \frac{h\nu}{c}$

A.C.

$E = h\nu' = \frac{hc}{\lambda'}$

$P = \frac{h\nu'}{c}$

e^{-}

$E = m_0c^2$

$P = 0$

$E = mc^2$

$P = mv = \frac{m_0 v}{\sqrt{1 - v^2/c^2}}$

1. Conservation of Energy

B.C.E = A.C.E

$$h\nu + m_0c^2 = h\nu' + mc^2$$

$$h\nu - h\nu' = mc^2 - m_0c^2 \quad \text{--- (1)}$$

2. Conservation of momentum

$$\frac{h\nu}{c} + 0 = \frac{h\nu'}{c}\cos\theta + mv\cos\phi$$

$$\frac{h\nu}{c} - \frac{h\nu'}{c}\cos\theta = mv\cos\phi = \frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}} \quad \text{--- (2)}$$

$$0 + 0 = -\frac{h\nu'}{c}\sin\theta + mv\sin\phi \quad \perp^{r}$$

$$\frac{h\nu'}{c}\sin\theta = mv\sin\phi \quad -$$

$$\boxed{\frac{h\nu'}{c}\sin\theta = \frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}}\sin\phi} \quad - \quad 3$$

$$h\nu - h\nu' = mc^2 - m_0c^2 \quad \text{— } (1)$$

$$\frac{h\nu}{c} - \frac{h\nu'}{c}\cos\theta = mv\cos\phi \quad \text{— } (2)$$

$$\frac{h\nu'}{c}\sin\theta = mv\sin\phi \quad \text{— } (3)$$

Compton Shift

$$\lambda' > \lambda \Rightarrow \Delta\lambda = \lambda' - \lambda$$

$$h\nu - h\nu' = mc^2 - m_0c^2$$

$$\underbrace{(h\nu - h\nu')}_{a} + \underbrace{m_0c^2}_{b} = mc^2$$

Squiring on both side

$$(h\nu - h\nu')^2 + m_0^2c^4 + 2(h\nu - h\nu')m_0c^2 = m^2c^4$$

$$\frac{h^2(\nu^2+\nu'^2-2\nu\nu')}{c^2} + \frac{m_0^2c^4}{c^2} + \frac{2hm_0\cancel{c^2}(\nu-\nu')}{\cancel{c^2}} = \frac{m^2c^4}{c^2}$$

$$\frac{h^2}{c^2}(\nu^2+\nu'^2-2\nu\nu') + m_0^2c^2 + 2hm_0(\nu-\nu') = m^2c^2$$

$$\frac{h^2}{c^2}\left(\nu^2+\nu'^2-2\nu\nu'\right)+2m_0h(\nu-\nu')=\frac{m_0^2v^2}{1-\frac{v^2}{c^2}} \quad (4)$$

R.H.S

$$=\frac{m_0^2c^2}{1-\frac{v^2}{c^2}}-m_0^2c^2 \qquad m_0^2c^2\left(1-\frac{v^2}{c^2}\right)$$

$$=\frac{\cancel{m_0^2c^2}-\cancel{m_0^2c^2}+\frac{m_0^2\cancel{c^2}v^2}{\cancel{c^2}}}{1-\frac{v^2}{c^2}}=\frac{m_0^2v^2}{1-\frac{v^2}{c^2}}$$

$(2)^2 + (3)^2$

$$\frac{h\nu}{c} - \frac{h\nu'}{c}\cos\theta = mv\cos\phi \quad (2)$$

$$\frac{h\nu'}{c}\sin\theta = mv\sin\phi \quad (3)$$

$(2)^2 \Rightarrow$

$$\frac{h^2}{c^2}\left(\nu^2 + \nu'\cos^2\theta - 2\nu\nu'\cos\theta\right) = m^2v^2\cos^2\phi$$

$(3)^2 \Rightarrow$

$$\frac{h^2\nu'^2}{c^2}\sin^2\theta = m^2v^2\sin^2\phi$$

$(2)^2 + (3)^2 \Rightarrow$

$$\frac{h^2}{c^2}\left(\nu^2+\nu'^2\cos^2\theta-2\nu\nu'\cos\theta+\nu'^2\sin^2\theta\right)=m^2v^2\left(\sin^2\phi+\cos^2\phi\right)$$

$$\frac{h^2}{c^2}\left(\nu^2+\nu'^2-2\nu\nu'\cos\theta\right)=m^2v^2$$

$$\frac{h^2}{c^2}\left(\nu^2+\nu'^2-2\nu\nu'\cos\theta\right)=\frac{m_0^2v^2}{1-\frac{v^2}{c^2}} \quad \text{— } (5)$$

$$(5)-(4) \Rightarrow$$

$$\frac{h^2}{c^2}\left[\nu^2+\nu'^2-2\nu\nu'\cos\theta\right] = \cancel{\frac{m_0^2 v^2}{1-v^2/c^2}} \quad \text{— } (5)$$

$$\frac{h^2}{c^2}\left(\nu^2+\nu'^2-2\nu\nu'\right) + 2m_0h(\nu-\nu') = \cancel{\frac{m_0 v^2}{1-\frac{v^2}{c^2}}}$$

$(5) - (4)$

$$\frac{h^2}{c^2}\left[\cancel{\nu^2}+\cancel{\nu'^2}-2\nu\nu'\cos\theta-\cancel{\nu^2}-\cancel{\nu'^2}+2\nu\nu'\right] - 2m_0h(\nu-\nu') = 0$$

$$\frac{h^{\cancel{2}\,1}}{c^2}\left[\cancel{2}\nu\nu'(1-\cos\theta)\right] = \cancel{2}m_0\cancel{h}(\nu-\nu')$$

$$\nu-\nu' = \frac{h\nu\nu'}{m_0c^2}(1-\cos\theta) \quad \text{— } (6)$$

6 $\rightarrow$ $$\nu - \nu' = \frac{h\nu\nu'}{m_0c^2}(1-\cos\theta)$$

$$\nu = \frac{c}{\lambda} \quad \& \quad \nu' = \frac{c}{\lambda'}$$

$$\frac{c}{\lambda} - \frac{c}{\lambda'} = \frac{h\cancel{c^2}}{m_0\cancel{c^2}\lambda\lambda'}(1-\cos\theta)$$

$$c\left[\frac{\lambda'-\lambda}{\cancel{\lambda\lambda'}}\right] = \frac{h}{m_0\cancel{\lambda\lambda'}}(1-\cos\theta)$$

$$\boxed{\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0c}(1-\cos\theta)}$$ Compton Shift

He-Ne 6328 Å

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0 c}(1-\cos\theta)$$

$\lambda = 6000$ Å $\theta = 90^\circ$ $\lambda' = 6000.0242$ Å

$\Delta\lambda =$

θ	$\Delta\lambda$	λ'
$\theta = 0^\circ$	$\Delta\lambda = 0$	$\lambda' = 6000$ Å
$= 45^\circ$	$\Delta\lambda = 0.007$ Å	$\lambda' = 6000.007$ Å
$= 90$	$\Delta\lambda = 0.0242$ Å	$\lambda' = 6000.0242$ Å
$= 135$	$\Delta\lambda = 0.041$ Å	$\lambda' = 6000.041$ Å
$= 180^\circ$	$\Delta\lambda = 0.0484$ Å	$\lambda' = 6000.0484$ Å

$\lambda' > \lambda$

$\lambda = 6000$ Å $\lambda' = 6000.0241$ Å

$\theta = 0$ $\Delta\lambda = 0$

$\theta = 90$ $\Delta\lambda = 0.0242$ Å

$$\Delta\lambda = \lambda' - \lambda = \checkmark$$

$$\Delta\lambda = \frac{h}{m_0 c}(1-\cos\theta)$$

$$\Delta\lambda = \frac{h}{60 \times 15\,\text{km/hr}}$$

$\theta = 90^\circ$

e $\quad \Delta\lambda = 0.0242\,\text{Å}$

H $\quad \Delta\lambda = 0.0000013\,\text{Å}$

C $\quad \Delta\lambda = 0.12 \times 10^{-30}\,\text{Å}$

Y $\quad \Delta\lambda = 0.034 \times 10^{-30}\,\text{Å}$

B $\quad \Delta\lambda = 3.52 \times 10^{-32}\,\text{Å}$

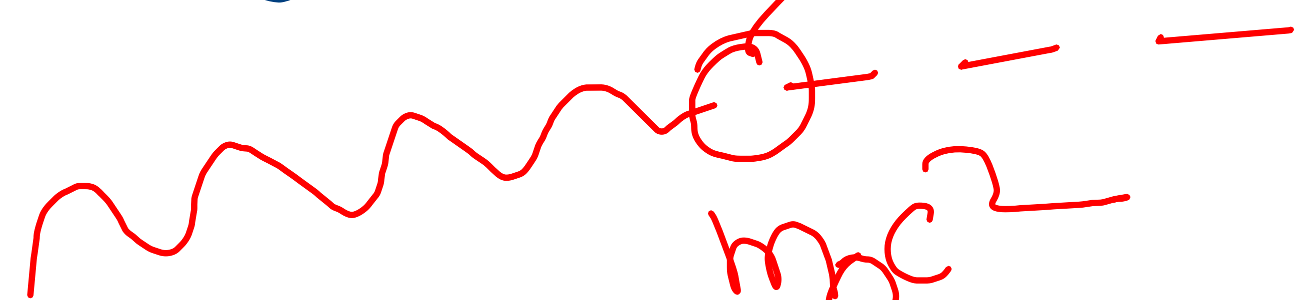
$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0 c}(1-\cos\theta)$$

$$\lambda_K = \frac{h}{m_0 c} \qquad \theta = 90^\circ$$

$$\lambda_K = \frac{h}{m_0 c} = \frac{6.625\times10^{-34}\,\mathrm{J\,s}}{9.1\times10^{-31}\,\mathrm{kg}\times3\times10^{8}\,\mathrm{m/s}} = 0.0242\,\text{Å}$$

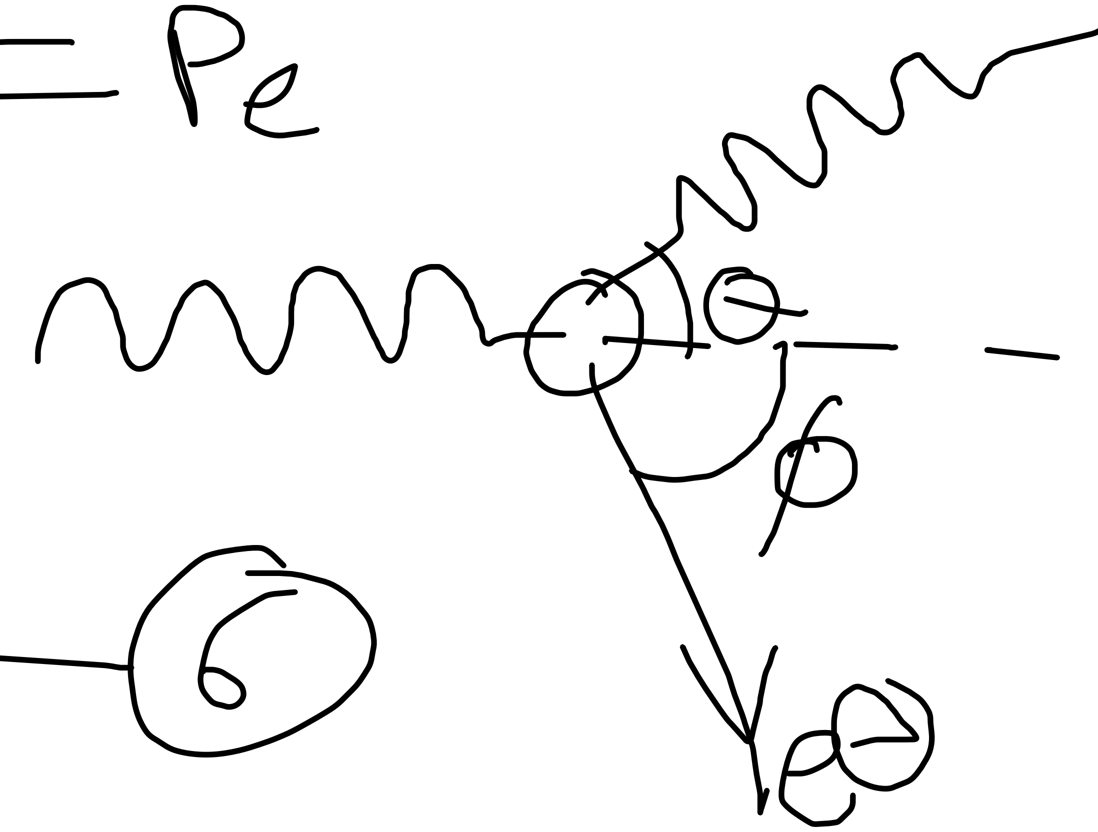
$$m_0 c = \frac{h}{\lambda_K} \quad \Big|\; c$$

$$m_0 c^2 = \frac{hc}{\lambda_K} = h\nu_K$$



m_0c^2

$\frac{h\nu}{c} = p$ & $\frac{h\nu'}{c} = p'$ $\qquad mv = P_e$



(2) $\Rightarrow$ $$\frac{h\nu}{c} - \frac{h\nu'}{c}\cos\theta = mv\cos\phi$$

$$p - p'\cos\theta = P_e\cos\phi \quad \text{— (6)}$$

(3) $\Rightarrow$ $$\frac{h\nu'}{c}\sin\theta = mv\sin\phi$$

$$p'\sin\theta = P_e\sin\phi \quad \text{— (7)}$$

$$\frac{(7)}{(6)} \Rightarrow \tan\phi = \frac{p'\sin\theta}{p - p'\cos\theta} \times \frac{c}{c}$$

$$\tan\phi = \frac{p'c\sin\theta}{pc - p'c\cos\theta}$$

$$\tan\phi = \frac{E'\sin\theta}{E - E'\cos\theta} \Rightarrow \phi = \tan^{-1}\left[\frac{E'\sin\theta}{E - E'\cos\theta}\right]$$

Angle of recoil e^{-}

Conclusions from P.E.E & C.E.

Matter wave

$e \rightarrow$ ~~~ $\hookrightarrow$ YDE

Photon $<$ Particle $<$ PEE, CE
Photon $<$ Wave — Y.D.E.

$e <$ Particle — $\frac{e}{m}$
$e <$ Wave — $q <$ D G, G.P.

de-Broglie → matter wavy

e/m → e^- particly └→ e^-. p, n

light showed wave & particle like

e^-. protons. ne - . ele

$\lambda = \frac{h}{p}$

$= \frac{h}{mv}$ $e^- \rightarrow$ ∿∿∿∿∿∿∿ → de-Broglie

(or)

matter wavy

$e^- \longrightarrow$ ∿∿∿∿ $\longrightarrow \boxed{\lambda = \dfrac{h}{p} = \dfrac{h}{mv}}$

$E = h\nu$

If the $\longrightarrow$ m $\longrightarrow$

As photon travels

$\longrightarrow p = mc$ — (2)

$E = mc^2$

$h\nu = mc^2$ — (1)

$\dfrac{(1)}{(2)} \longrightarrow \dfrac{h\nu}{p} = \dfrac{\cancel{m}c^{\cancel{2}}}{\cancel{m}\cancel{c}}$

$\dfrac{h\nu}{p} = c$

$p \Rightarrow \dfrac{h\nu}{c} = \dfrac{h}{\lambda}$

$p = \dfrac{h}{\lambda}$

$\boxed{\lambda = \dfrac{h}{p}}$

$P = mv$

$$\boxed{\lambda = \frac{h}{p} = \frac{h}{mv}}$$

① de-Broglie w.l in terms of K.E.

$$E = \frac{1}{2}mv^2 \times \frac{m}{m} = \frac{m^2v^2}{2m}$$

$$\lambda = \frac{h}{p}$$

$$E = \frac{p^2}{2m} \Rightarrow p = \sqrt{2mE}$$

$$\boxed{\lambda = \frac{h}{\sqrt{2mE}}}$$

② de-Broglie w.l. in terms of absolute Temp

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$$

$$E = \frac{3}{2}KT$$

$$\lambda = \frac{h}{\sqrt{2m \times \frac{3}{\cancel{2}}KT}} = \frac{h}{\sqrt{3mKT}}$$

$$\boxed{\lambda = \frac{h}{\sqrt{3mKT}}}$$

3. de Broglie wl. in terms of PD.

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

$\lambda = 3.8\times10^{-9}$ m

V = 10V
100V
1000V

V = 0V λ =

= 10V $\lambda = 3.8\times10^{-10}$ m

= 100V = 1.2×10^{-10} m

= 1000V = 3.8×10^{-9} m

= 10^4 V =

= 10^5 V

(1) e^{-} $\longrightarrow$ 10^7 m/s $\lambda = 0.728$ Å

(2) 1000kg rocket $v = 3000$ km/hr $\lambda =$

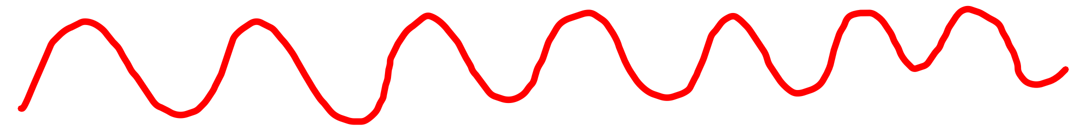
(3) what is the $\lambda = ?$ of thermal neutron at $T = 27°C$

(4) Cricket ball $m = 170$ g $\xrightarrow{\lambda = ?}$ 150 m/hr

(5) Bullet mass 1g $v = 10^3$ m/s $\lambda = ?$

Matter waves

↳ e^-. protons. n — .



$\lambda =$

de-Broglie eqn

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

— p

— w — E — m

PD

T

$$\lambda = \frac{h}{\sqrt{3mkT}}$$

KE

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$\lambda = \frac{h}{\sqrt{2meV}}$$

① $\lambda = ?$ $e^{\ominus}$ $\longrightarrow 10^{7}$ m/s

$\lambda = 7.27 \times 10^{-11}$ m

$\lambda =$

$\lambda = \frac{h}{mv}$

② $\lambda = ?$ 1000 kg rocket $\longrightarrow$ 3000 km/hr

$\lambda = 0.792 \times 10^{-40}$ m

e $\lambda : 10^{-11}$ m

③ $\lambda = ?$ thermal neutron at 27°C

you $\lambda = 10^{-36}$ m

$\lambda = 1.46 \times 10^{-10}$ m

④ $\lambda = ?$ C.B m = 170 gm $\longrightarrow$ 150 m/hr

$\lambda = 1.615 \times 10^{-35}$ m

⑤ $\lambda = ?$ bullet mass 1 gm $\longrightarrow 10^{3}$ m/s

$\lambda = 6.626 \times 10^{-34}$ m

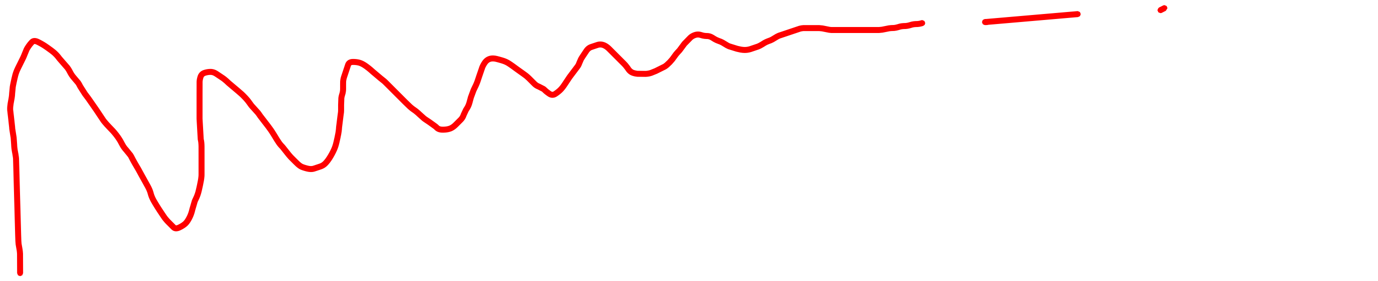
⑥ $\lambda = ?$ m = 75 kg, v = 20 km/hr

$\lambda = 1.5 \times 10^{-36}$ m

The Quantum Particle

Ideal Particle

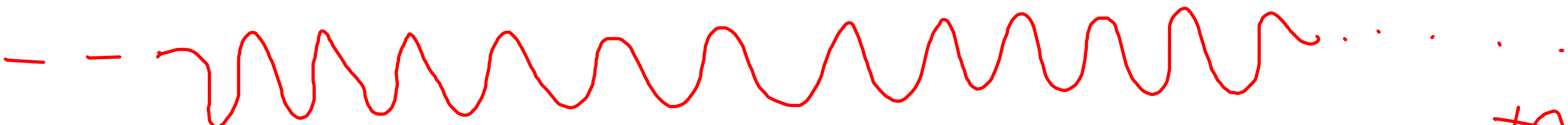
1. It has zero size
2. it is <u>localized</u> in space



Ideal wave

1. it has a single freq & infinitely long
2. <u>unlocalized</u> in space

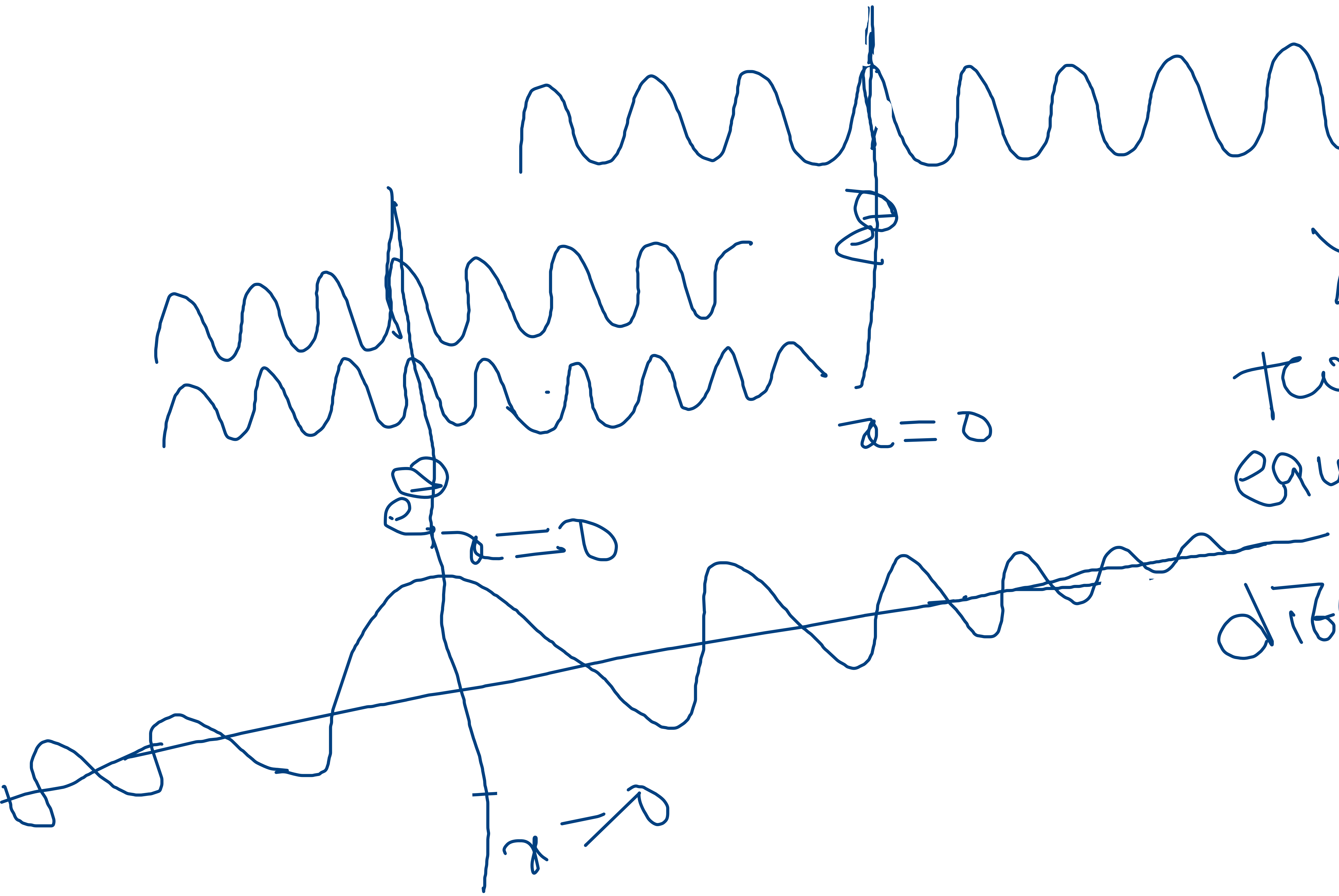
$-\infty$



$+\infty$

f = same at all the points

matter waves



$$Y = A\cos(kx - \omega t)$$

two waves with equal amplitude but different angular frequency

ω_1 ω_2

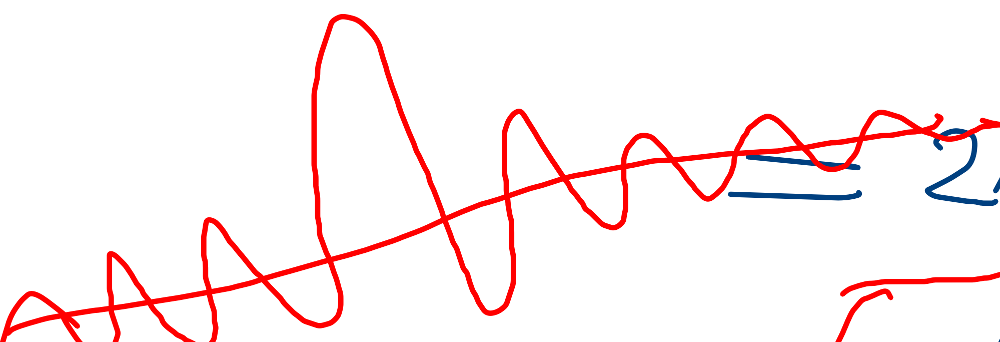
$$y_1 = A\cos(k_1 x - \omega_1 t) \qquad k = \frac{2\pi}{\lambda} \qquad \omega = 2\pi\nu$$

$$y_2 = A\cos(k_2 x - \omega_2 t)$$

$$y = y_1 + y_2 \Rightarrow A\cos(k_1 x - \omega_1 t) + A\cos(k_2 x - \omega_2 t)$$

$$\cos a + \cos b \Rightarrow a = k_1 x - \omega_1 t \ \ \& \ \ b = k_2 x - \omega_2 t$$

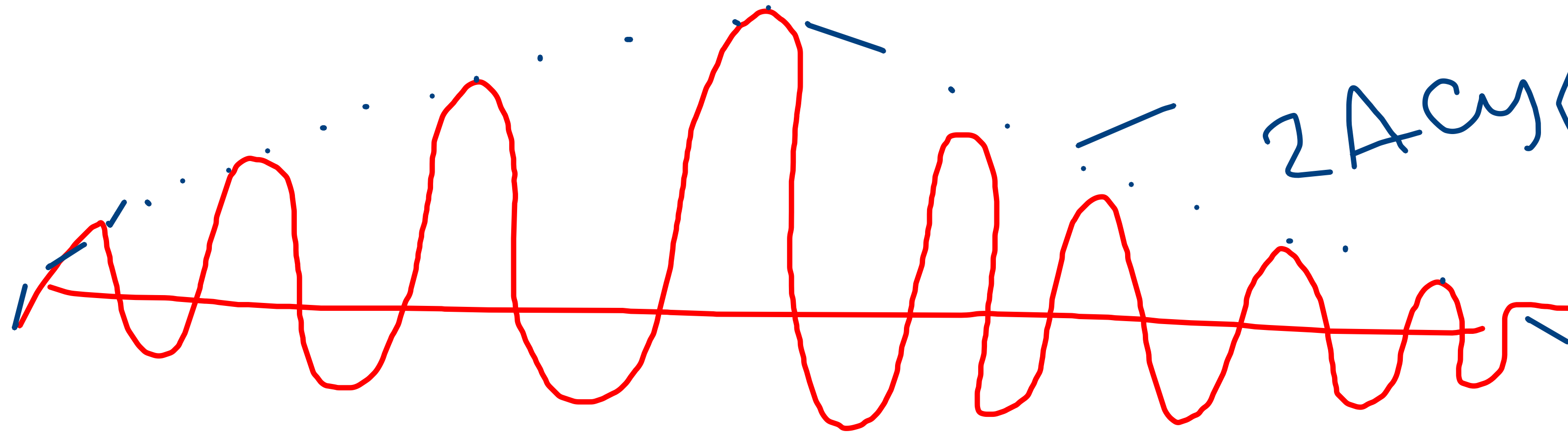
$$\cos a + \cos b = 2\cos\left(\frac{a-b}{2}\right)\cos\left(\frac{a+b}{2}\right)$$



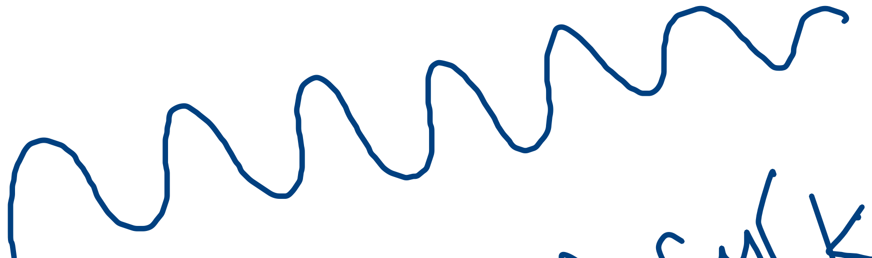
$$= 2A\cos\left(\frac{(k_1 x - \omega_1 t) - (k_2 x - \omega_2 t)}{2}\right)\cos\left(\frac{(k_1 x - \omega_1 t) + (k_2 x - \omega_2 t)}{2}\right)$$

$$= 2A\cos\left(\frac{\Delta k}{2}x - \frac{\Delta \omega}{2}t\right)\cos\left(\frac{k_1 + k_2}{2}x - \frac{\omega_1 + \omega_2}{2}t\right)$$

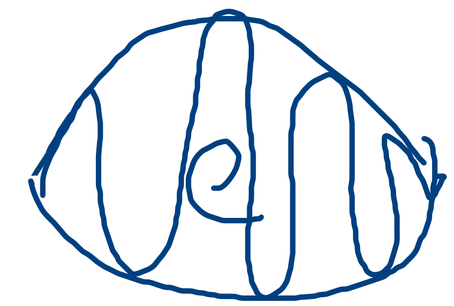
$$\Delta k = k_1 - k_2 \ \ \& \ \ \Delta\omega = \omega_1 - \omega_2$$



$$2A\cos\left(\frac{\Delta k}{2}x - \frac{\Delta\omega}{2}t\right)$$

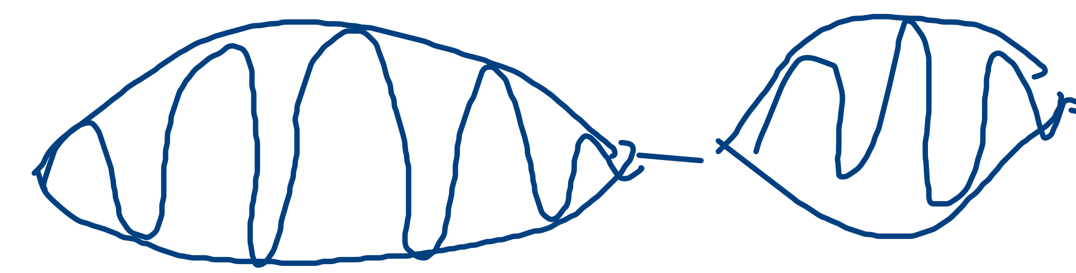


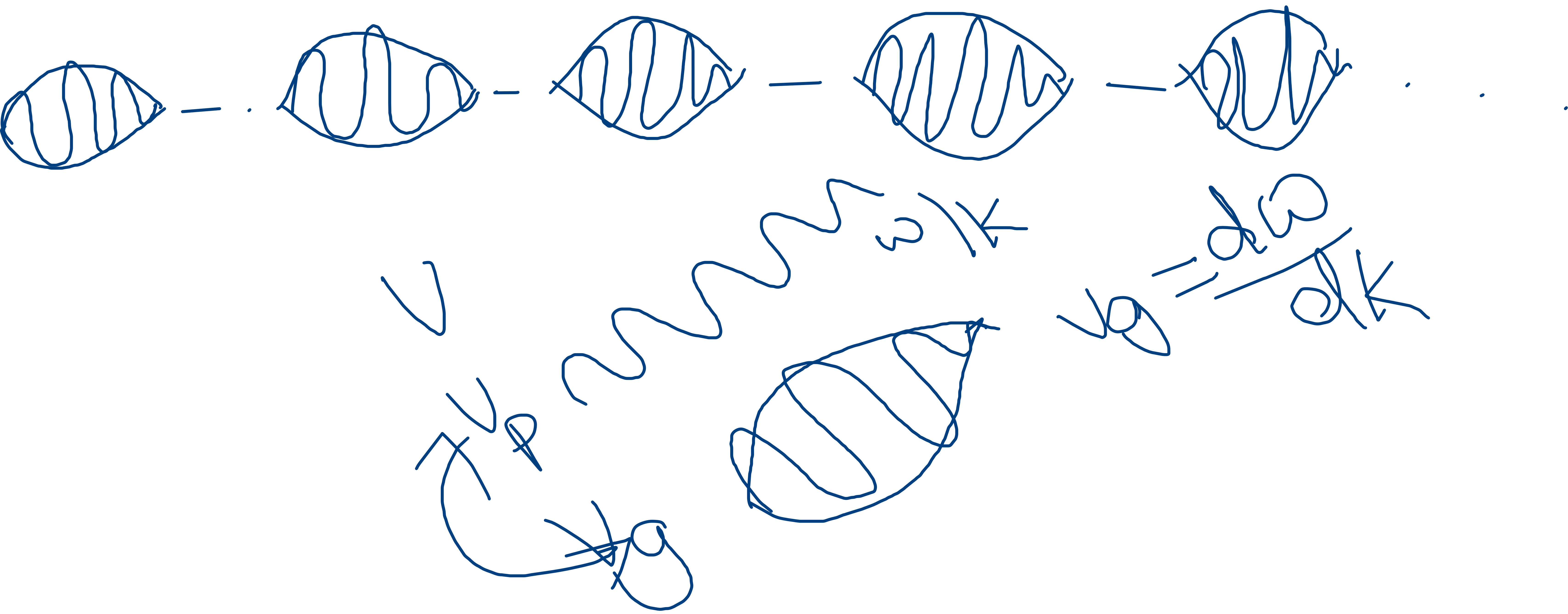
$$y = A\cos(kx - \omega t)$$



$$\boxed{v_{phase} = \frac{\omega}{k}}$$

$$\boxed{v_g = \frac{\frac{\Delta\omega}{2}}{\frac{\Delta k}{2}} = \frac{\Delta\omega}{\Delta k}}$$




v
v_p
ω/k
$v_g = \frac{d\omega}{dk}$

$$k = \frac{2\pi}{\lambda} \quad \& \quad \lambda = \frac{h}{p}$$

$$\lambda = \frac{2\pi}{k} \quad \& \quad \lambda = \frac{h}{p}$$

$$\frac{2\pi}{k} = \frac{h}{p}$$

$$\boxed{p = \frac{h}{2\pi} k = \hbar k}$$

momentum of
the particle

$$E = h\nu$$

$$\omega = 2\pi\nu$$

$$\nu = \frac{\omega}{2\pi}$$

$$E = \frac{h \cdot \omega}{2\pi}$$

$$\boxed{E = \hbar\omega}$$

Energy

$v = 3\times10^8$ m/s

Wave (Phase) & Group velocity (A wave packet)

$$E = h\nu \quad (\text{or}) \quad \nu = \frac{E}{h} \qquad \lambda = \frac{h}{mv}$$

$v_p > c$

$$E = mc^2$$

$$h\nu = mc^2 \Rightarrow \nu = \frac{mc^2}{h}$$

Velocity of the propagating wave $v_p = \nu \cdot \lambda$

$$v_p = \frac{\cancel{m}c^2}{\cancel{h}} \cdot \frac{\cancel{h}}{\cancel{m}v} \Rightarrow \boxed{v_p = \frac{c^2}{v}}$$

Phase velocity $v_p = \frac{\omega}{k}$ & G.V $\Rightarrow v_g = \frac{d\omega}{dk}$

$\omega = 2\pi\nu$ & $k = \frac{2\pi}{\lambda}$

$$v_g = \frac{d(2\pi\nu)}{d(2\pi/\lambda)} = \frac{2\pi\, d\nu}{-\frac{2\pi}{\lambda^2}\, d\lambda} = -\lambda^2 \frac{d\nu}{d\lambda}$$

$$\boxed{v_p = \frac{c^2}{v}}$$

$$\boxed{v_g = -\lambda^2 \frac{d\nu}{d\lambda}}$$

Relation b/w v_g and v_p

$$\frac{dE}{dp} = \frac{d\omega}{dk}$$

$$\boxed{\frac{dE}{dp} = v_g}$$

① According to Non-Relativistic

② 〃 Relativistic Pa:b

① $\Rightarrow$ $E = \hbar\omega$

$$E = \frac{p^2}{2m} \Rightarrow \boxed{\frac{dE}{dp} = \frac{2p}{2m} = \frac{p}{m} = v}$$

$$\boxed{v_g = v}$$

But $E = \hbar\omega$ & $p = \hbar k$

$dE = \hbar d\omega$ & $dp = \hbar dk \Rightarrow \frac{dE}{dp} = \frac{\hbar d\omega}{\hbar dk}$

$= \omega$

For a Relativistic Particle

$$E^2 = p^2c^2 + m_0^2c^4$$

$$\frac{dE}{dp} = \frac{pc^2}{E}$$

$$E = mc^2 = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$p = mv = \frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$v_g = \frac{dE}{dp} = \frac{pc^2}{E}$$

$$\Rightarrow \frac{\frac{\cancel{m_0} v}{\cancel{\sqrt{1-v^2/c^2}}} \cdot \cancel{c^2}}{\cancel{m_0} \cancel{c^2} / \cancel{\sqrt{1-v^2/c^2}}}$$

$$\boxed{v_g = v}$$

$$v_p = v\lambda$$

$$v_p = c^2/v$$

V_p is inde of k

$$\frac{d\omega}{dk} \longrightarrow k$$

R B/w V_p & V_g

$$V_p = \nu\lambda \Rightarrow \frac{\cancel{m}v_g^2}{2\cancel{h}} \cdot \frac{\cancel{h}}{\cancel{m v_g}}$$

$$\lambda = \frac{h}{mv_g}$$

$$E = \frac{1}{2}mv_g^2$$

$$\boxed{V_p = \frac{v_g}{2} = \frac{1}{2}v_g}$$

$$E = h\nu \Rightarrow \nu = \frac{E}{h} = \frac{mv_g^2}{2h}$$

$$v_p = \frac{\omega}{k} \quad (\text{or}) \quad \omega = k\, v_p$$

$$v_g = \frac{d\omega}{dk} = \frac{d}{dk}(k \cdot v_p)$$

$$v_g = v_p + k \cdot \frac{dv_p}{dk} \Longrightarrow \boxed{v_g = v_p - \lambda \frac{dv_p}{d\lambda}}$$

$$k = \frac{2\pi}{\lambda} \Rightarrow dk = -\frac{2\pi}{\lambda^2}\, d\lambda$$

$$\frac{k}{dk} = \frac{\frac{2\pi}{\lambda}}{-\frac{2\pi}{\lambda^2}\, d\lambda} = \frac{-\lambda}{d\lambda}$$