12-1. A bicyclist starts from rest and after traveling along a straight path a distance of 20 m reaches a speed of 30 km/h. Determine his acceleration if it is *constant*. Also, how long does it take to reach the speed of 30 km/h?

$v_2 = 30 \text{ km/h} = 8.33 \text{ m/s}$

$v_2^2 = v_1^2 + 2a_c(s_2 - s_1)$

$(8.33)^2 = 0 + 2a_c(20-0)$

$a_c = 1.74 \text{ m/s}^2$ **Ans**

$v_2 = v_1 + a_c t$

$8.33 = 0 + 1.74(t)$

$t = 4.80 \text{ s}$ **Ans**

12-2. A car starts from rest and reaches a speed of 80 ft/s after traveling 500 ft along a straight road. Determine its constant acceleration and the time of travel.

$v_2^2 = v_1^2 + 2a_c(s_2 - s_1)$

$(80)^2 = 0 + 2a_c(500-0)$

$a_c = 6.40 \text{ ft/s}^2$ **Ans**

$v_2 = v_1 + a_c t$

$80 = 0 + 6.4(t)$

$t = 12.5 \text{ s}$ **Ans**

12-3. A baseball is thrown downward from a 50-ft tower with an initial speed of 18 ft/s. Determine the speed at which it hits the ground and the time of travel.

$v_2^2 = v_1^2 + 2a_c(s_2 - s_1)$

$v_2^2 = (18)^2 + 2(32.2)(50-0)$

$v_2 = 59.532 = 59.5 \text{ ft/s}$ **Ans**

$v_2 = v_1 + a_c t$

$59.532 = 18 + 32.2(t)$

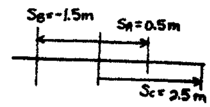
$t = 1.29 \text{ s}$ **Ans**

*12-4. A particle travels along a straight line such that in 2 s it moves from an initial position $s_A = +0.5$ m to a position $s_B = -1.5$ m. Then in another 4 s it moves from s_B to $s_C = +2.5$ m. Determine the particle's average velocity and average speed during the 6-s time interval.

Total displacement $(s_C - s_A) = 2$ m

Total distance traveled $(0.5 + 1.5 + 1.5 + 2.5) = 6$ m

Total time traveled $(2 + 4) = 6$ s



$v_{avg} = \frac{2}{6} = 0.333$ m/s **Ans**

$v_{sp} = \frac{6}{6} = 1$ m/s **Ans**

12-5. Traveling with an initial speed of 70 km/h, a car accelerates at 6000 km/h² along a straight road. How long will it take to reach a speed of 120 km/h? Also, through what distance does the car travel during this time?

$v = v_1 + a_c t$

$120 = 70 + 6000(t)$

$t = 8.33(10^{-3})$ hr $= 30$ s **Ans**

$v^2 = v_1^2 + 2a_c(s - s_1)$

$(120)^2 = 70^2 + 2(6000)(s - 0)$

$s = 0.792$ km $= 792$ m **Ans**

12-6. A freight train travels at $v = 60(1 - e^{-t})$ ft/s, where t is the elapsed time in seconds. Determine the distance traveled in three seconds, and the acceleration at this time.

Prob. 12–6

$v = 60(1 - e^{-t})$

$\int_0^s ds = \int v\,dt = \int_0^3 60(1 - e^{-t})dt$

$s = 60(t + e^{-t})|_0^3$

$s = 123$ ft **Ans**

$a = \frac{dv}{dt} = 60(e^{-t})$

At $t = 3$ s

$a = 60e^{-3} = 2.99$ ft/s² **Ans**

12-7. The position of a particle along a straight line is given by $s = (t^3 - 9t^2 + 15t)$ ft, where t is in seconds. Determine its maximum acceleration and maximum velocity during the time interval $0 \le t \le 10$ s.

$s = t^3 - 9t^2 + 15t$

$v = \dfrac{ds}{dt} = 3t^2 - 18t + 15$

$a = \dfrac{dv}{dt} = 6t - 18$

a_{max} occurs at $t = 10$ s,

$a_{max} = 6(10) - 18 = 42 \text{ ft/s}^2$ **Ans**

v_{max} occurs when $t = 10$ s

$v_{max} = 3(10)^2 - 18(10) + 15 = 135$ ft/s **Ans**

***12-8.** From approximately what floor of a building must a car be dropped from an at-rest position so that it reaches a speed of 80.7 ft/s (55 mi/h) when it hits the ground? Each floor is 12 ft higher than the one below it. (*Note:* You may want to remember this when traveling 55 mi/h.)

$(+\downarrow) \quad v^2 = v_0^2 + 2a_c(s - s_0)$

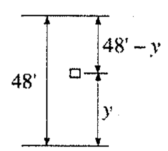
$80.7^2 = 0 + 2(32.2)(s - 0)$

$s = 101.13$ ft

$\# \text{ of floors} = \dfrac{101.13}{12} = 8.43$

The car must be dropped from the 9th floor. **Ans**

12-9. A car is to be hoisted by elevator to the fourth floor of a parking garage, which is 48 ft above the ground. If the elevator can accelerate at 0.6 ft/s, decelerate at 0.3 ft/s, and reach a maximum speed of 8 ft/s, determine the shortest time to make the lift, starting from rest and ending at rest.



$+\uparrow \; v^2 = v_1^2 + 2a_c(s - s_1)$

$v_{max}^2 = 0 + 2(0.6)(y - 0)$

$0 = v_{max}^2 + 2(-0.3)((48 - y) - 0)$

$0 = 1.2y - 0.6(48 - y)$

$y = 16.0$ ft, $\quad v_{max} = 4.382 \text{ ft/s} < 8$ ft/s

$+\uparrow \; v = v_1 + a_c t$

$4.382 = 0 + 0.6t_1$

$t_1 = 7.303$ s

$0 = 4.382 - 0.3t_2$

$t_2 = 14.61$ s

$t = t_1 + t_2 = 21.9$ s **Ans**

12-10. A particle travels in a straight line such that for a short time $2\text{ s} \le t \le 6\text{ s}$ its motion is described by $v = (4/a)$ ft/s, where a is in ft/s^2. If $v = 6$ ft/s when $t = 2$ s, determine the particle's acceleration when $t = 3$ s.

$$a = \frac{dv}{dt} = \frac{4}{v}$$

$$\int_6^v v\,dv = \int_2^t 4\,dt$$

$$\frac{1}{2}v^2 - 18 = 4t - 8$$

$$v^2 = 8t + 20$$

At $t = 3$ s, choosing the positive root

$v = 6.63$ ft/s

$$a = \frac{4}{6.63} = 0.603 \text{ ft/s}^2 \qquad \textbf{Ans}$$

12-11. The acceleration of a particle as it moves along a straight line is given by $a = (2t - 1)$ m/s^2, where t is in seconds. If $s = 1$ m and $v = 2$ m/s when $t = 0$, determine the particle's velocity and position when $t = 6$ s. Also, determine the total distance the particle travels during this time period.

$$\int_2^v dv = \int_0^t (2t - 1)\,dt$$

$$v = t^2 - t + 2$$

$$\int_1^s ds = \int_0^t (t^2 - t + 2)\,dt$$

$$s = \frac{1}{3}t^3 - \frac{1}{2}t^2 + 2t + 1$$

When $t = 6$ s,

$v = 32$ m/s **Ans**

$s = 67$ m **Ans**

Since $v \neq 0$ then

$d = 67 - 1 = 66$ m **Ans**

*12-12. When a train is traveling along a straight track at 2 m/s, it begins to accelerate at $a = (60\,v^{-4})$ m/s^2, where v is in m/s. Determine its velocity v and the position 3 s after the acceleration.

Prob. 12–12

$$a = \frac{dv}{dt}$$

$$dt = \frac{dv}{a}$$

$$\int_0^3 dt = \int_2^v \frac{dv}{60v^{-4}}$$

$$3 = \frac{1}{300}\left(v^5 - 32\right)$$

$$v = 3.925 \text{ m/s} = 3.93 \text{ m/s} \qquad \text{Ans}$$

$$a\,ds = v\,dv$$

$$ds = \frac{v\,dv}{a} = \frac{1}{60}v^5\,dv$$

$$\int_0^s ds = \frac{1}{60}\int_2^{3.925} v^5\,dv$$

$$s = \frac{1}{60}\left(\frac{v^6}{6}\right)\Bigg|_2^{3.925}$$

$$= 9.98 \text{ m} \qquad \text{Ans}$$

12-13. The position of a particle along a straight line is given by $s = (1.5t^3 - 13.5t^2 + 22.5t)$ ft, where t is in seconds. Determine the position of the particle when $t = 6$ s and the total distance it travels during the 6-s time interval. *Hint:* Plot the path to determine the total distance traveled.

Position : The position of the particle when $t = 6$ s is

$$s|_{t=6\,s} = 1.5(6^3) - 13.5(6^2) + 22.5(6) = -27.0 \text{ ft} \qquad \text{Ans}$$

Total Distance Traveled : The velocity of the particle can be determined by applying Eq. 12–1.

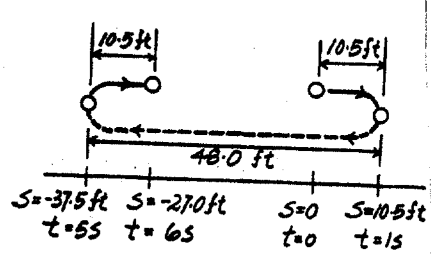
$$v = \frac{ds}{dt} = 4.50t^2 - 27.0t + 22.5$$

The times when the particle stops are

$$4.50t^2 - 27.0t + 22.5 = 0$$
$$t = 1 \text{ s} \quad \text{and} \quad t = 5 \text{ s}$$

The position of the particle at $t = 0$ s, 1 s and 5 s are

$$s|_{t=0\,s} = 1.5(0^3) - 13.5(0^2) + 22.5(0) = 0$$
$$s|_{t=1\,s} = 1.5(1^3) - 13.5(1^2) + 22.5(1) = 10.5 \text{ ft}$$
$$s|_{t=5\,s} = 1.5(5^3) - 13.5(5^2) + 22.5(5) = -37.5 \text{ ft}$$



From the particle's path, the total distance is

$$s_{tot} = 10.5 + 48.0 + 10.5 = 69.0 \text{ ft} \qquad \text{Ans}$$

12-14. The position of a particle on a straight line is given by $s = (t^3 - 9t^2 + 15t)$ ft, where t is in seconds. Determine the position of the particle when $t = 6$ s and the total distance it travels during the 6-s time interval. *Hint:* Plot the path to determine the total distance traveled.

$s = t^3 - 9t^2 + 15t$

$v = \frac{ds}{dt} = 3t^2 - 18t + 15$

$v = 0$ when $t = 1$ s and $t = 5$ s

$t = 0, \quad s = 0$

$t = 1$ s, $\quad s = 7$ ft

$t = 5$ s, $\quad s = -25$ ft

$t = 6$ s, $\quad s = -18$ ft **Ans**

$s_T = 7 + 7 + 25 + (25 - 18) = 46$ ft **Ans**

12-15. A particle travels to the right along a straight line with a velocity $v = [5/(4 + s)]$ m/s, where s is in meters. Determine its position when $t = 6$ s if $s = 5$ m when $t = 0$.

$\frac{ds}{dt} = \frac{5}{4+s}$

$\int_5^s (4 + s)\, ds = \int_0^t 5\, dt$

$4s + 0.5 s^2 - 32.5 = 5t$

When $t = 6$ s,

$s^2 + 8s - 125 = 0$

Solving for the positive root

$s = 7.87$ m **Ans**

***12-16.** A particle travels to the right along a straight line with a velocity $v = [5/(4 + s)]$ m/s, where s is in meters. Determine its deceleration when $s = 2$ m.

$v = \frac{5}{4+s}$

$v\, dv = a\, ds$

$dv = \frac{-5\, ds}{(4+s)^2}$

$\frac{5}{(4+s)}\left(\frac{-5\, ds}{(4+s)^2}\right) = a\, ds$

$a = \frac{-25}{(4+s)^3}$

When $s = 2$ m,

$a = -0.116 \text{ m/s}^2$ **Ans**

12-17. Two particles A and B start from rest at the origin $s = 0$ and move along a straight line such that $a_A = (6t - 3)$ ft/s^2 and $a_B = (12t^2 - 8)$ ft/s^2, where t is in seconds. Determine the distance between them when $t = 4$ s and the total distance each has traveled in $t = 4$ s.

Velocity: The velocity of particles A and B can be determine using Eq. 12-2.

$$dv_A = a_A dt$$

$$\int_0^{v_A} dv_A = \int_0^t (6t - 3)dt$$

$$v_A = 3t^2 - 3t$$

$$dv_B = a_B dt$$

$$\int_0^{v_B} dv_B = \int_0^t (12t^2 - 8)dt$$

$$v_B = 4t^3 - 8t$$

The times when particle A stops are

$3t^2 - 3t = 0 \qquad t = 0$ s and $= 1$ s

The times when particle B stops are

$4t^3 - 8t = 0 \quad t = 0$ s and $t = \sqrt{2}$ s

Position: The position of particles A and B can be determine using Eq. 12-1.

$$ds_A = v_A dt$$

$$\int_0^{s_A} ds_A = \int_0^t (3t^2 - 3t)dt$$

$$s_A = t^3 - \frac{3}{2}t^2$$

$$ds_B = v_B dt$$

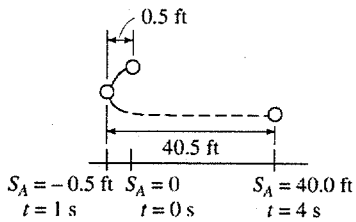
$$\int_0^{s_B} ds_B = \int_0^1 (4t^3 - 8t)dt$$

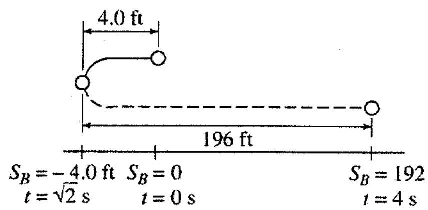
$$s_B = t^4 - 4t^2$$

The positions of particle A at $t = 1$ s and 4 s are

$$s_A|_{t=1s} = 1^3 - \frac{3}{2}(1^2) = -0.500 \text{ ft}$$

$$s_A|_{t=4s} = 4^3 - \frac{3}{2}(4^2) = 40.0 \text{ ft}$$





Particle A has traveled

$d_A = 2(0.5) + 40.0 = 41.0$ ft **Ans**

The positions of particle B at $t = \sqrt{2}$ s and 4 s are

$$s_B|_{t=\sqrt{2}} = (\sqrt{2})^4 - 4(\sqrt{2})^2 = -4 \text{ ft}$$

$$s_B|_{t=4} = (4)^4 - 4(4)^2 = 192 \text{ ft}$$

Particle B has traveled

$d_B = 2(4) + 192 = 200$ ft **Ans**

At $t = 4$ s, the distance between A and B is

$\Delta s_{AB} = 192 - 40 = 152$ ft **Ans**

12-18. A car starts from rest and moves along a straight line with an acceleration of $a = (3s^{-1/3})\ \text{m/s}^2$, where s is in meters. Determine the car's acceleration when $t = 4$ s.

$$a = 3s^{-\frac{1}{3}}$$

$$a\,ds = v\,dv$$

$$\int_0^s 3s^{-\frac{1}{3}}\,ds = \int_0^v v\,dv$$

$$\frac{3}{2}(3)s^{\frac{2}{3}} = \frac{1}{2}v^2$$

$$v = 3s^{\frac{1}{3}}$$

$$\frac{ds}{dt} = 3s^{\frac{1}{3}}$$

$$\int_0^s s^{-\frac{1}{3}}\,ds = \int_0^t 3\,dt$$

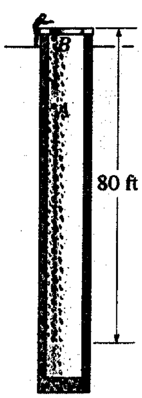
$$\frac{3}{2}s^{\frac{2}{3}} = 3t$$

$$s = (2t)^{\frac{3}{2}}$$

$$s|_{t=4} = (2(4))^{\frac{3}{2}} = 22.62 = 22.6\ \text{m}$$

$$a|_{t=4} = 3(22.62)^{-\frac{1}{3}} = 1.06\ \text{m/s}^2 \qquad \textbf{Ans}$$

12-19. A stone A is dropped from rest down a well, and in 1 s another stone B is dropped from rest. Determine the distance between the stones another second later.



$$+\downarrow\ s = s_1 + v_1 t + \frac{1}{2}a_c t^2$$

$$s_A = 0 + 0 + \frac{1}{2}(32.2)(2)^2$$

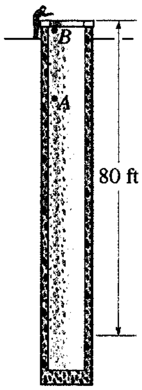
$$s_A = 64.4\ \text{ft}$$

$$s_B = 0 + 0 + \frac{1}{2}(32.2)(1)^2$$

$$s_B = 16.1\ \text{ft}$$

$$\Delta s = 64.4 - 16.1 = 48.3\ \text{ft}$$

*12-20. A stone A is dropped from rest down a well, and in 1 s another stone B is dropped from rest. Determine the time interval between the instant A strikes the water and the instant B strikes the water. Also, at what speed do they strike the water?



B is dropped one second after A, so that

$\Delta t = 1$ s **Ans**

$+\downarrow s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$80 = 0 + 0 + \frac{1}{2}(32.2)(t^2)$

$t = 2.2291$ s

$+\downarrow v = v_0 + a_c t$

$v = 0 + 32.2(2.2291)$

$v = 71.8$ ft/s **Ans**

Also,

$v^2 = v_0^2 + 2a_c s$

$v^2 = 0^2 + 2(32.2)(80)$

$v = 71.8$ ft/s **Ans**

12-21. A particle travels in a straight line with accelerated motion such that $a = -ks$, where s is the distance from the starting point and k is a proportionality constant which is to be determined. For $s = 2$ ft the velocity is 4 ft/s, and for $s = 3.5$ ft the velocity is 10 ft/s. What is s when $v = 0$?

$a = -ks$

$a\,ds = v\,dv$

$-k\int_2^s s\,ds = \int_4^v v\,dv$

$-k(\frac{s^2}{2} - \frac{(2)^2}{2}) = (\frac{v^2}{2} - \frac{(4)^2}{2})$

$-k(\frac{s^2}{2} - 2) = \frac{v^2}{2} - 8$

Set $s = 3.5$ ft, $v = 10$ ft/s,

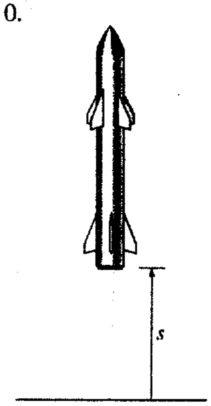
Then $k = -10.18 = -10.2\ s^{-2}$ **Ans**

When $v = 0$

$10.18(\frac{s^2}{2} - 2) = -8$

$s = 1.56$ ft **Ans**

12-22. The acceleration of a rocket traveling upward is given by $a = (6 + 0.02s)$ m/s^2, where s is in meters. Determine the rocket's velocity when $s = 2$ km and the time needed to reach this altitude. Initially, $v = 0$ and $s = 0$ when $t = 0$.



$$a\,ds = v\,dv$$

$$\int_0^s (6+0.02\,s)\,ds = \int_0^v v\,dv$$

$$6\,s + 0.01\,s^2 = \frac{1}{2}v^2$$

$$v = \sqrt{12\,s + 0.02\,s^2}$$

When $s = 2000$ m,

$v = 322$ m/s **Ans**

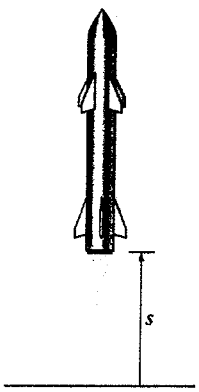
$$\int_0^s \frac{ds}{\sqrt{12\,s + 0.02\,s^2}} = \int_0^t dt$$

$$\frac{1}{\sqrt{0.02}}\ln\left(\sqrt{12\,s + 0.02\,s^2} + s\sqrt{0.02} + \frac{12}{2\sqrt{0.02}}\right|_0^s = t$$

Set $s = 2000$ m

$t = 19.3$ s **Ans**

12-23. The acceleration of a rocket traveling upward is given by $a = (6 + 0.02s)$ m/s^2, where s is in meters. Determine the time needed for the rocket to reach an altitude of $s = 100$ m. Initially, $v = 0$ and $s = 0$ when $t = 0$.



$$a\,ds = v\,dv$$

$$\int_0^s (6+0.02\,s)\,ds = \int_0^v v\,dv$$

$$6\,s + 0.01\,s^2 = \frac{1}{2}v^2$$

$$v = \sqrt{12\,s + 0.02\,s^2}$$

$$ds = v\,dt$$

$$\int_0^{100} \frac{ds}{\sqrt{12\,s + 0.02\,s^2}} = \int_0^t dt$$

$$\frac{1}{\sqrt{0.02}}\ln\left[\sqrt{12\,s + 0.02\,s^2} + s\sqrt{0.02} + \frac{12}{2\sqrt{0.02}}\right]_0^{100} = t$$

$t = 5.62$ s **Ans**

*12-24. At $t = 0$ bullet A is fired vertically with an initial (muzzle) velocity of 450 m/s. When $t = 3$ s, bullet B is fired upward with a muzzle velocity of 600 m/s. Determine the time t, after A is fired, as to when bullet B passes bullet A. At what altitude does this occur?

$$+\uparrow s_A = (s_A)_0 + (v_A)_0\, t + \frac{1}{2}a_c\, t^2$$

$$s_A = 0 + 450\,t + \frac{1}{2}(-9.81)\, t^2$$

$$+\uparrow s_B = (s_B)_0 + (v_B)_0 t + \frac{1}{2}\, a_c\, t^2$$

$$s_B = 0 + 600(t-3) + \frac{1}{2}(-9.81)(t-3)^2$$

Require $s_A = s_B$

$$450\,t - 4.905\,t^2 = 600\,t - 1800 - 4.905\,t^2 + 29.43\,t - 44.145$$

$t = 10.3$ s **Ans**

$s_A = s_B = 4.11$ km **Ans**

■12-25. A particle moves along a straight line with an acceleration of $a = 5/(3s^{1/3} + s^{5/2})$ m/s^2, where s is in meters. Determine the particle's velocity when $s = 2$ m, if it starts from rest when $s = 1$ m. Use Simpson's rule to evaluate the integral.

$$a = \frac{5}{\left(3s^{\frac{1}{3}} + s^{\frac{5}{2}}\right)}$$

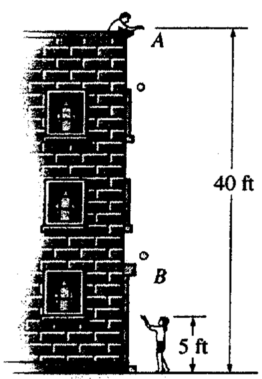
$$a\, ds = v\, dv$$

$$\int_1^2 \frac{5\, ds}{\left(3s^{\frac{1}{3}} + s^{\frac{5}{2}}\right)} = \int_0^v v\, dv$$

$$0.8351 = \frac{1}{2}v^2$$

$v = 1.29$ m/s **Ans**

12-26. Ball A is released from rest at a height of 40 ft at the same time that a second ball B is thrown upward 5 ft from the ground. If the balls pass one another at a height of 20 ft, determine the speed at which ball B was thrown upward.



For ball # 1:

$$(+\downarrow)\, s = s_0 + v_0 t + \tfrac{1}{2}a_c t^2$$

$$20 = 0 + 0 + \tfrac{1}{2}(32.2)t^2$$

$$t = 1.1146 \text{ s}$$

For ball # 2:

$$(+\uparrow)\, s = s_0 + v_0 t + \tfrac{1}{2}a_c t^2$$

$$15 = 0 + v_B(1.1146) + \tfrac{1}{2}(-32.2)(1.1146)^2$$

$v_B = 31.4$ ft/s **Ans**

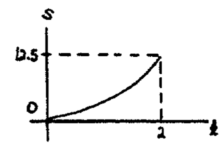
■12-27. A projectile, initially at the origin, moves vertically downward along a straight-line path through a fluid medium such that its velocity is defined as $v = 3(8e^{-t} + t)^{1/2}$ m/s, where t is in seconds. Plot the position s of the projectile during the first 2 s. Use the Runge-Kutta method to evaluate s with incremental values of $h = 0.25$ s.

$v = 3\,(8\,e^{-t} + t)^{1/2}$

$s_0 = 0$ at $t = 0$

Using the Runge–Kutta method:

s	t
0	0
2.01	0.25
3.83	0.50
5.49	0.75
7.03	1.00
8.48	1.25
9.87	1.50
11.2	1.75
12.5	2.00



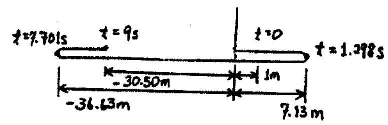
*12-28. The acceleration of a particle along a straight line is defined by $a = (2t - 9)$ m/s^2, where t is in seconds. At $t = 0$, $s = 1$ m and $v = 10$ m/s. When $t = 9$ s, determine (a) the particle's position, (b) the total distance traveled, and (c) the velocity.

$a = 2t - 9$

$\int_{10}^{v} dv = \int_0^t (2\,t - 9)\,dt$

$v - 10 = t^2 - 9\,t$

$v = t^2 - 9\,t + 10$



$\int_1^s ds = \int_0^t (t^2 - 9\,t + 10)\,dt$

$s - 1 = \frac{1}{3}t^3 - 4.5\,t^2 + 10\,t$

$s = \frac{1}{3}t^3 - 4.5\,t^2 + 10\,t + 1$

Note when $v = 0$ at $t^2 - 9\,t + 10 = 0$

$t = 1.298$ s and $t = 7.701$ s

When $t = 1.298$ s, $\quad s = 7.13$ m

When $t = 7.701$ s, $\quad s = -36.63$ m

When $t = 9$ s, $\quad s = -30.50$ m

(a) $\quad s = -30.5$ m $\qquad$ **Ans**

(b) $\quad s_{Tot} = (7.13 - 1) + 7.13 + 36.63 + (36.63 - 30.50)$

$\qquad s_{Tot} = 56.0$ m $\qquad$ **Ans**

(c) $\quad v = 10$ m/s $\qquad$ **Ans**

12-29. A particle is moving along a straight line such that when it is at the origin it has a velocity of 4 m/s. If it begins to decelerate at the rate of $a = (-1.5v^{1/2})$ m/s^2, where v is in m/s, determine the distance it travels before it stops.

$$a = \frac{dv}{dt} = -1.5v^{\frac{1}{2}}$$

$$\int_4^v v^{-\frac{1}{2}}\,dv = \int_0^t -1.5\,dt$$

$$2v^{\frac{1}{2}}\Big|_4^v = -1.5t\Big|_0^t$$

$$2\left(v^{\frac{1}{2}} - 2\right) = -1.5t$$

$$v = (2 - 0.75t)^2 \text{ m/s} \quad (1)$$

$$\int_0^s ds = \int_0^t (2-0.75t)^2\,dt = \int_0^t \left(4 - 3t + 0.5625t^2\right) dt$$

$$s = 4t - 1.5t^2 + 0.1875t^3 \quad (2)$$

From Eq. (1), the particle will stop when

$$0 = (2 - 0.75t)^2$$

$$t = 2.667 \text{ s}$$

$$s\big|_{t=2.667} = 4(2.667) - 1.5(2.667)^2 + 0.1875(2.667)^3 = 3.56 \text{ m} \quad \textbf{Ans}$$

12-30. A particle moves along a straight line with an acceleration of $a = 5/(3s^{1/3} + s^{5/2})$ m/s^2, where s is in meters. Determine the particle's velocity when $s = 2$ m, if it starts from rest when $s = 1$ m. Use Simpson's rule to evaluate the integral.

$$a = \frac{5}{\left(3s^{\frac{1}{3}} + s^{\frac{5}{2}}\right)}$$

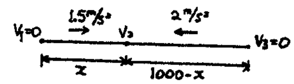
$$a\,ds = v\,dv$$

$$\int_1^2 \frac{5\,ds}{\left(3s^{\frac{1}{3}} + s^{\frac{5}{2}}\right)} = \int_0^v v\,dv$$

$$0.8351 = \frac{1}{2}v^2$$

$$v = 1.29 \text{ m/s} \quad \textbf{Ans}$$

12-31. Determine the time required for a car to travel 1 km along a road if the car starts from rest, reaches a maximum speed at some intermediate point, and then stops at the end of the road. The car can accelerate at 1.5 m/s^2 and decelerate at 2 m/s^2.



Using formulas of constant acceleration:

$$v_2 = 1.5\,t_1$$

$$x = \frac{1}{2}(1.5)(t_1^2)$$

$$0 = v_2 - 2\,t_2$$

$$1000 - x = v_2 t_2 - \frac{1}{2}(2)(t_2^2)$$

Combining equations:

$$t_1 = 1.33\,t_2; \quad v_2 = 2\,t_2$$

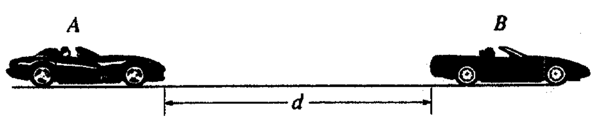
$$x = 1.33\,t_2^2$$

$$1000 - 1.33\,t_2^2 = 2\,t_2^2 - t_2^2$$

$$t_2 = 20.702 \text{ s} \quad ; t_1 = 27.603 \text{ s}$$

$$t = t_1 + t_2 = 48.3 \text{ s}$$ **Ans**

***12-32.** When two cars A and B are next to one another, they are traveling in the same direction with speeds v_A and v_B, respectively. If B maintains its constant speed, while A begins to decelerate at a_A, determine the distance d between the cars at the instant A stops.



Motion of car A :

$$v = v_0 + a_c t$$

$$0 = vv_A - a_A t \qquad t = \frac{v_A}{a_A}$$

$$v^2 = v_0^2 + 2a_c(s - s_0)$$

$$0 = v_A^2 + 2(-a_A)(s_A - 0)$$

$$s_A = \frac{v_A^2}{2a_A}$$

Motion of car B :

$$s_B = v_B t = v_B\left(\frac{v_A}{a_A}\right) = \frac{v_A v_B}{a_A}$$

The distance between cars A and B is

$$s_{BA} = |s_B - s_A| = \left|\frac{v_A v_B}{a_A} - \frac{v_A^2}{2a_A}\right| = \left|\frac{2v_A v_B - v_A^2}{2a_A}\right|$$ **Ans**

12-33. If the effects of atmospheric resistance are accounted for, a freely falling body has an acceleration defined by the equation $a = 9.81[1 - v^2(10^{-4})]$ m/s^2, where v is in m/s and the positive direction is downward. If the body is released from rest at a *very high altitude*, determine (a) the velocity when $t = 5$ s, and (b) the body's terminal or maximum attainable velocity (as $t \rightarrow \infty$).

(a) $a = \frac{dv}{dt} = 9.81\left[1 - v^2\left(10^{-4}\right)\right]$

$$\int_0^v \frac{dv}{\left[10^4 - v^2\right]} = \int_0^t 9.81\left(10^{-4}\right)dt \qquad (1)$$

$$\frac{1}{100}\tanh^{-1}\left(\frac{v}{100}\right)\Big|_0^v = 9.81\left(10^{-4}\right)t$$

$$\tanh^{-1}\left(\frac{v}{100}\right) = \left(9.81\left(10^{-2}\right)t\right) \qquad (2)$$

$$v = 100\tanh\left(9.81\left(10^{-2}\right)(5)\right)$$

$$= 100\tanh(0.4905) = 45.5 \text{ m/s} \qquad \textbf{Ans}$$

(b) From Eq. (2), with $t \rightarrow \infty$,

$$v = 100\tanh\infty = 100 \text{ m/s} \qquad \textbf{Ans}$$

Also note that Eq. (1) can be written as

$$10^4\int_0^v \frac{dv}{\left[10^4 - v^2\right]} = 9.81t$$

$$10^4\left[\left(\frac{1}{2(10^2)}\right)\ln\left(\frac{10^2 + v}{10^2 - v}\right)\right]_0^v = 9.81t$$

$$50\left[\ln\left(\frac{100+v}{100-v}\right) - \ln 1\right] = 9.81t$$

When $t = 5$ s,

$$\frac{100+v}{100-v} = e^{0.981} = 2.667$$

$$100 + v = 266.7 - 2.667v$$

$$v = \frac{166.7}{3.667} = 45.5 \text{ m/s} \qquad \textbf{Ans}$$

12-34. As a body is projected to a high altitude above the earth's *surface*, the variation of the acceleration of gravity with respect to altitude y must be taken into account. Neglecting air resistance, this acceleration is determined from the formula $a = -g_0[R^2/(R + y)^2]$, where g_0 is the constant gravitational acceleration at sea level, R is the radius of the earth, and the positive direction is measured upward. If $g_0 = 9.81$ m/s^2 and $R = 6356$ km, determine the minimum initial velocity (escape velocity) at which a projectile should be shot vertically from the earth's surface so that it does not fall back to the earth. *Hint:* This requires that $v = 0$ as $y \rightarrow \infty$.

$$v\,dv = a\,dy$$

$$\int_v^0 v\,dv = -g_0R^2\int_0^\infty \frac{dy}{(R+y)^2}$$

$$\frac{v^2}{2}\Big|_v^0 = \frac{g_0R^2}{R+y}\Big|_0^\infty$$

$$v = \sqrt{2g_0R}$$

$$= \sqrt{2(9.81)(6356)(10)^3}$$

$$= 11167 \text{ m/s} = 11.2 \text{ km/s} \qquad \textbf{Ans}$$

12-35. Accounting for the variation of gravitational acceleration a with respect to altitude y (see Prob. 12-34), derive an equation that relates the velocity of a freely falling particle to its altitude. Assume that the particle is released from rest at an altitude y_0 from the earth's surface. With what velocity does the particle strike the earth if it is released from rest at an altitude $y_0 = 500$ km? Use the numerical data in Prob. 12-34.

From Prob. 12-34,

$$(+\uparrow) \quad g = -g_0 \frac{R^2}{(R+y)^2}$$

Since $g\,dy = v\,dv$

then

$$-g_0 R^2 \int_{y_0}^{y} \frac{dy}{(R+y)^2} = \int_0^v v\,dv$$

$$g_0 R^2 \left[\frac{1}{R+y}\right]_{y_0}^{y} = \frac{v^2}{2}$$

$$g_0 R^2 \left[\frac{1}{R+y} - \frac{1}{R+y_0}\right] = \frac{v^2}{2}$$

Thus

$$v = -R\sqrt{\frac{2g_0(y_0-y)}{(R+y)(R+y_0)}}$$

When $y_0 = 500$ km, $y = 0$,

$$v = -6356(10^3)\sqrt{\frac{2(9.81)(500)(10^3)}{6356(6356+500)(10^6)}}$$

$v = -3016$ m/s $= 3.02$ km/s $\downarrow$ **Ans**

***12-36.** When a particle falls through the air, its initial acceleration $a = g$ diminishes until it is zero, and thereafter it falls at a constant or terminal velocity v_f. If this variation of the acceleration can be expressed as $a = (g/v_f^2)(v_f^2 - v^2)$, determine the time needed for the velocity to become $v < v_f$. Initially the particle falls from rest.

$$\frac{dv}{dt} = a = \left(\frac{g}{v_f^2}\right)(v_f^2 - v^2)$$

$$\int_0^v \frac{dv}{v_f^2 - v^2} = \frac{g}{v_f^2}\int_0^t dt$$

$$\frac{1}{2v_f}\ln\left(\frac{v_f+v}{v_f-v}\right)\Bigg|_0^v = \frac{g}{v_f^2}t$$

$$t = \frac{v_f}{2g}\ln\left(\frac{v_f+v}{v_f-v}\right)$$ **Ans**

12-37. An airplane starts from rest, travels 5000 ft down a runway, and after uniform acceleration, takes off with a speed of 162 mi/h. It then climbs in a straight line with a uniform acceleration of 3 ft/s^2 until it reaches a constant speed of 220 mi/h. Draw the *s–t*, *v–t*, and *a–t* graphs that describe the motion.

$$v_1 = 0$$

$$v_2 = 162\,\frac{\text{mi}}{\text{h}}\,\frac{(1\text{ h})\,5280\text{ ft}}{(3600\text{ s})(1\text{ mi})} = 237.6\text{ ft/s}$$

$$v_2^2 = v_1^2 + 2a_c(s_2 - s_1)$$

$$(237.6)^2 = 0^2 + 2(a_c)(5000-0)$$

$$a_c = 5.64538\text{ ft/s}^2$$

$$v_2 = v_1 + a_c t$$

$$237.6 = 0 + 5.64538\,t$$

$$t = 42.09 = 42.1\text{ s}$$

$$v_3 = 220\,\frac{\text{mi}}{\text{h}}\,\frac{(1\text{ h})\,5280\text{ ft}}{(3600\text{ s})(1\text{ mi})} = 322.67\text{ ft/s}$$

$$v_3^2 = v_2^2 + 2a_c(s_3 - s_2)$$

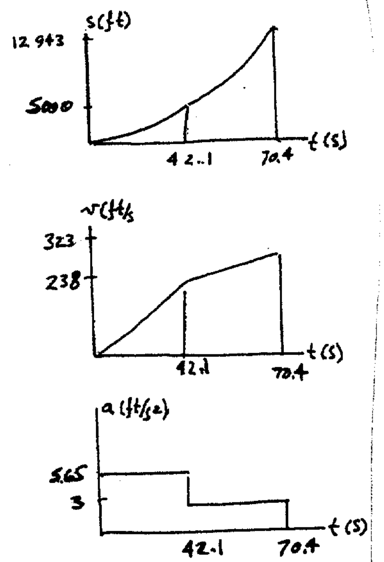
$$(322.67)^2 = (237.6)^2 + 2(3)(s - 5000)$$

$$s = 12\,943.34\text{ ft}$$

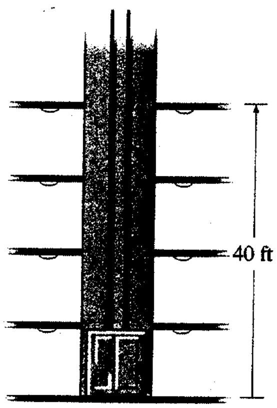
$$v_3 = v_2 + a_c t$$

$$322.67 = 237.6 + 3t$$

$$t = 28.4\text{ s}$$



12-38. The elevator starts from rest at the first floor of the building. It can accelerate at 5 ft/s^2 and then decelerate at 2 ft/s^2. Determine the shortest time it takes to reach a floor 40 ft above the ground. The elevator starts from rest and then stops. Draw the *a–t*, *v–t*, and *s–t* graphs for the motion.



$+\uparrow\ v_2 = v_1 + a_c t_1$

$v_{max} = 0 + 5 t_1$

$+\uparrow\ v_3 = v_1 + a_c t$

$0 = v_{max} - 2 t_2$

Thus

$t_1 = 0.4 t_2$

$+\uparrow\ s_2 = s_1 + v_1 t + \frac{1}{2} a_c t_1^2$

$h = 0 + 0 + \frac{1}{2}(5)(t_1^2) = 2.5 t_1^2$

$+\uparrow\ 40 - h = 0 + v_{max} t_2 - \frac{1}{2}(2)\, t_2^2$

$+\uparrow\ v^2 = v_1^2 + 2 a_c (s - s_1)$

$v_{max}^2 = 0 + 2(5)(h - 0)$

$v_{max}^2 = 10 h$

$0 = v_{max}^2 + 2(-2)(40 - h)$

$v_{max}^2 = 160 - 4h$

Thus,

$10h = 160 - 4h$

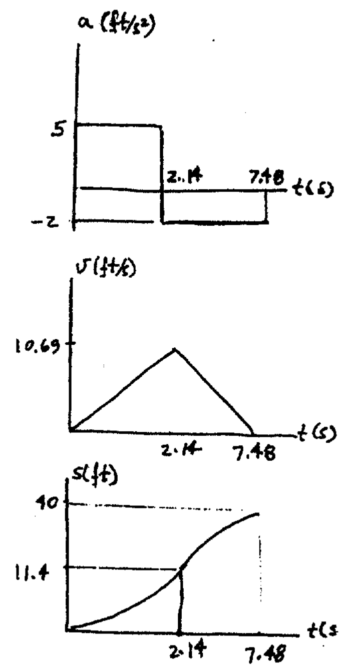
$h = 11.429$ ft

$v_{max} = 10.69$ ft/s

$t_1 = 2.138$ s

$t_2 = 5.345$ s

$t = t_1 + t_2 = 7.48$ s **Ans**



12-39. A freight train starts from rest and travels with a constant acceleration of 0.5 ft/s^2. After a time t' it maintains a constant speed so that when t = 160 s it has traveled 2000 ft. Determine the time t' and draw the v-t graph for the motion.

Total Distance Traveled : The distance for part one of the motion can be related to time $t = t'$ by applying Eq. 12−5 with $s_0 = 0$ and $v_0 = 0$.

$$(\xrightarrow{+}) \qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$s_1 = 0 + 0 + \frac{1}{2}(0.5)(t')^2 = 0.25(t')^2$$

The velocity at time t can be obtained by applying Eq. 12−4 with $v_0 = 0$.

$$(\xrightarrow{+}) \qquad v = v_0 + a_c t = 0 + 0.5t = 0.5t \qquad [1]$$

The time for the second stage of motion is $t_2 = 160 - t'$ and the train is travelling at a constant velocity of $v = 0.5t'$ (Eq. [1]). Thus, the distance for this part of motion is

$$(\xrightarrow{+}) \qquad s_2 = vt_2 = 0.5t'(160 - t') = 80t' - 0.5(t')^2$$

If the total distance traveled is $s_{Tot} = 2000$, then

$$s_{Tot} = s_1 + s_2$$

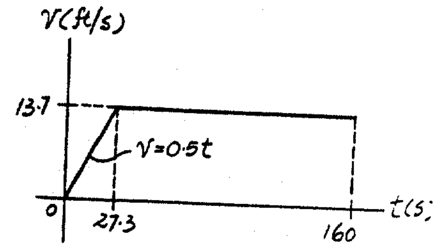
$$2000 = 0.25(t')^2 + 80t' - 0.5(t')^2$$

$$0.25(t')^2 - 80t' + 2000 = 0$$

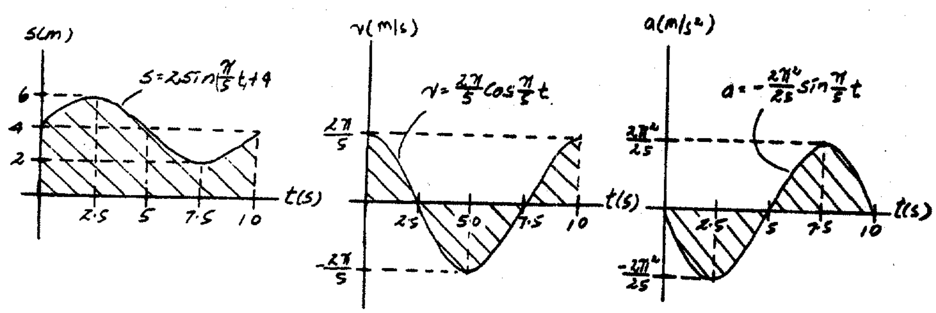
Chose a root that is less than 160 s, then

$$t' = 27.34 \text{ s} = 27.3 \text{ s} \qquad \textbf{Ans}$$

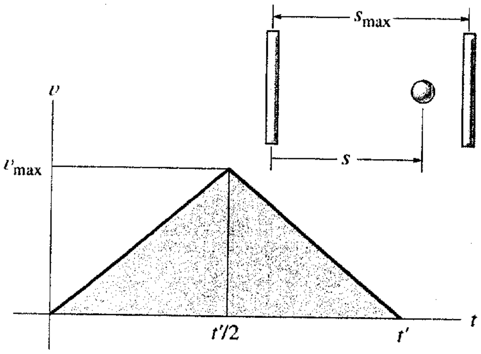
v − t Graph : The equation for the velocity is given by Eq. [1]. When $t = t'$ = 27.34 s, $v = 0.5(27.34) = 13.7$ ft/s.



***12-40.** If the position of a particle is defined by $s = [2 \sin (\pi/5)t + 4]$ m, where t is in seconds, construct the s–t, v–t, and a–t graphs for $0 \le t \le 10$ s.



12-41. The v–t graph for a particle moving through an electric field from one plate to another has the shape shown in the figure. The acceleration and deceleration that occur are constant and both have a magnitude of 4 m/s². If the plates are spaced 200 mm apart, determine the maximum velocity v_{max} and the time t' for the particle to travel from one plate to the other. Also draw the s–t graph. When $t = t'/2$ the particle is at $s = 100$ mm.



$a_c = 4 \text{ m/s}^2$

$\frac{s}{2} = 100 \text{ mm} = 0.1 \text{ m}$

$v^2 = v_0^2 + 2\,a_c(s - s_0)$

$v_{max}^2 = 0 + 2(4)(0.1 - 0)$

$v_{max} = 0.89442 \text{ m/s}$ **Ans**

$v = v_0 + a_c\,t'$

$0.89442 = 0 + 4(\frac{t'}{2})$

$t' = 0.44721 \text{ s}$ **Ans**

$s = s_0 + v_0\,t + \frac{1}{2}\,a_c\,t^2$

$s = 0 + 0 + \frac{1}{2}(4)(t)^2$

$s = 2\,t^2$

When $t = \frac{0.44721}{2} = 0.2236 = 0.224$ s,

$s = 0.1$ m

$\int_{0.894}^{v} dv = -\int_{0.2235}^{t} 4\,dt$

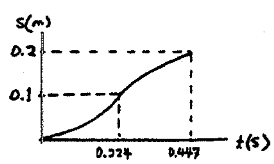
$v = -4\,t + 1.788$

$\int_{0.1}^{s} ds = \int_{0.2235}^{t} (-4t + 1.788)\,dt$

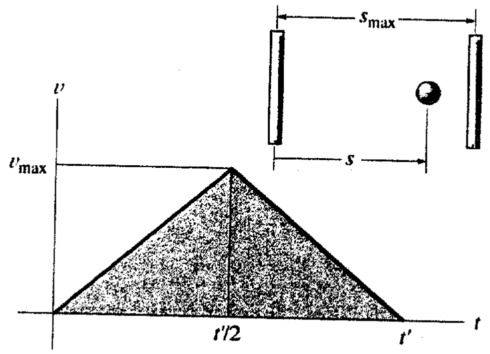
$s = -2\,t^2 + 1.788\,t - 0.2$

When $t = 0.447$ s,

$s = 0.2$ m



12-42. The v–t graph for a particle moving through an electric field from one plate to another has the shape shown in the figure, where $t' = 0.2$ s and $v_{max} = 10$ m/s. Draw the s–t and a–t graphs for the particle. When $t = t'/2$ the particle is at $s = 0.5$ m.



For $0 < t < 0.1$ s

$v = 100\,t$

$a = \dfrac{dv}{dt} = 100$

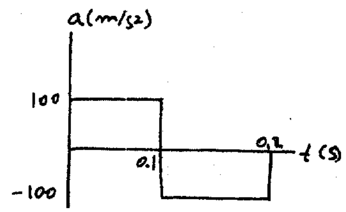
$ds = v\,dt$

$\int_0^s ds = \int_0^t 100\,t\,dt$

$s = 50\,t^2$

When $t = 0.1$ s,

$s = 0.5$ m



For 0.1 s $< t < 0.2$ s

$v = -100\,t + 20$

$a = \dfrac{dv}{dt} = -100$

$ds = v\,dt$

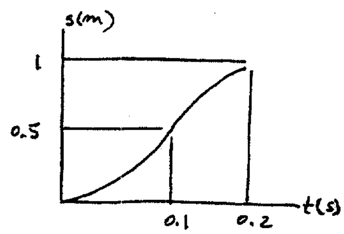
$\int_{0.5}^s ds = \int_{0.1}^t (-100\,t + 20)dt$

$s - 0.5 = (-50\,t^2 + 20\,t - 1.5)$

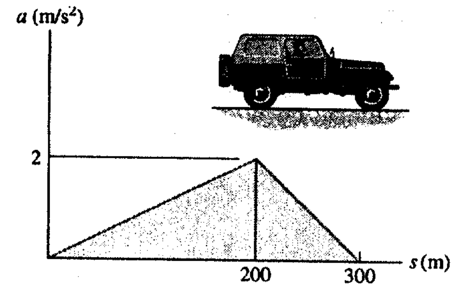
$s = -50\,t^2 + 20\,t - 1$

When $t = 0.2$ s,

$s = 1$ m



12-43. The a-s graph for a jeep traveling along a straight road is given for the first 300 m of its motion. Construct the v-s graph. At $s = 0$, $v = 0$.



a − s Graph : The function of acceleration a in terms of s for the interval $0\ \text{m} \le s < 200\ \text{m}$ is

$$\frac{a-0}{s-0} = \frac{2-0}{200-0} \qquad a = (0.01s)\ \text{m/s}^2$$

For the interval $200\ \text{m} < s \le 300\ \text{m}$,

$$\frac{a-2}{s-200} = \frac{0-2}{300-200} \qquad a = (-0.02s + 6)\ \text{m/s}^2$$

v − s Graph : The function of velocity v in terms of s can be obtained by applying $v\,dv = a\,ds$. For the interval $0\ \text{m} \le s < 200\ \text{m}$,

$$v\,dv = a\,ds$$

$$\int_0^v v\,dv = \int_0^s 0.01s\,ds$$

$$v = (0.1s)\ \text{m/s}$$

At $s = 200$ m, $\qquad v = 0.100(200) = 20.0\ \text{m/s}$

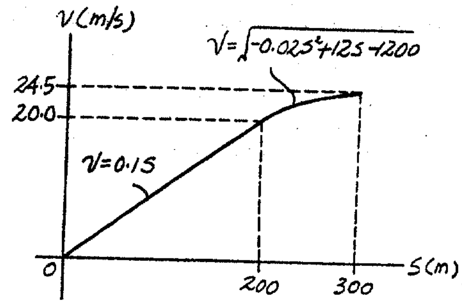
For the interval $200\ \text{m} < s \le 300\ \text{m}$,

$$v\,dv = a\,ds$$

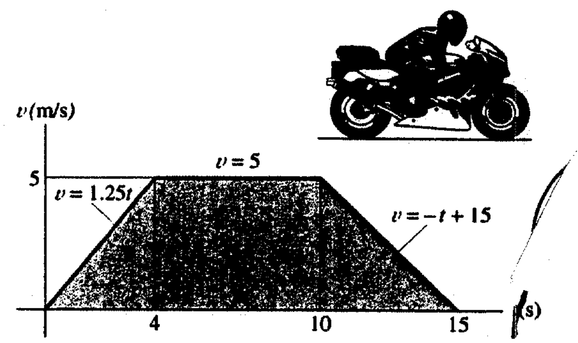
$$\int_{20.0\,\text{m/s}}^{v} v\,dv = \int_{200\,\text{m}}^{s} (-0.02s + 6)\,ds$$

$$v = \left(\sqrt{-0.02s^2 + 12s - 1200}\right)\ \text{m/s}$$

At $s = 300$ m, $\qquad v = \sqrt{-0.02(300^2) + 12(300) - 1200} = 24.5\ \text{m/s}$



***12-44.** A motorcycle starts from rest at $s = 0$ and travels along a straight road with the speed shown by the v–t graph. Determine the motorcycle's acceleration and position when $t = 8$ s and $t = 12$ s.



At $t = 8$ s

$$a = \frac{dv}{dt} = 0 \qquad \textbf{Ans}$$

$$\Delta s = \int v\,dt$$

$$s - 0 = \frac{1}{2}(4)(5) + (8-4)(5) = 30$$

$$s = 30\text{ m} \qquad \textbf{Ans}$$

At $t = 12$ s

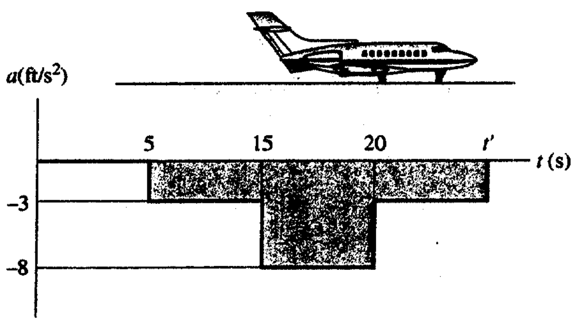
$$a = \frac{dv}{dt} = \frac{-5}{5} = -1\text{ m/s}^2 \qquad \textbf{Ans}$$

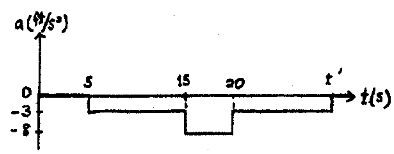
$$\Delta s = \int v\,dt$$

$$s - 0 = \frac{1}{2}(4)(5) + (10-4)(5) + \frac{1}{2}(15-10)(5) - \frac{1}{2}(\frac{3}{5})(5)(\frac{3}{5})(5)$$

$$s = 48\text{ m} \qquad \textbf{Ans}$$

12-45. An airplane lands on the straight runway, originally traveling at 110 ft/s when $s = 0$. If it is subjected to the decelerations shown, determine the time t' needed to stop the plane and construct the s–t graph for the motion.



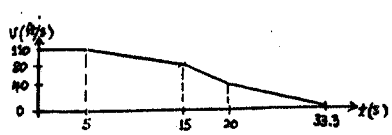


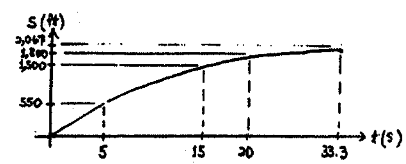
$v_0 = 110$ ft/s

$$\Delta v = \int a\,dt$$

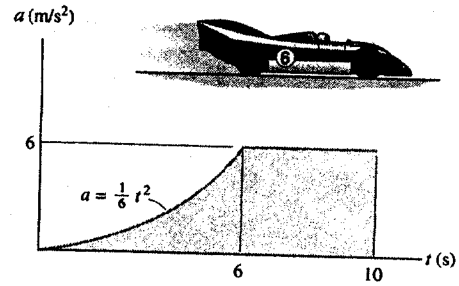
$$0 - 110 = -3(15-5) - 8(20-15) - 3(t' - 20)$$

$$t' = 33.3\text{ s} \qquad \textbf{Ans}$$





12-46. A race cars starting from rest travels along a straight road and for 10 s has the acceleration shown. Construct the v-t graph that describes the motion and find the distance traveled in 10 s.



$v-t$ Graph : The velocity function in terms of time t can be obtained by applying formula $a = \frac{dv}{dt}$. For time interval $0\ \text{s} \le t < 6\ \text{s}$,

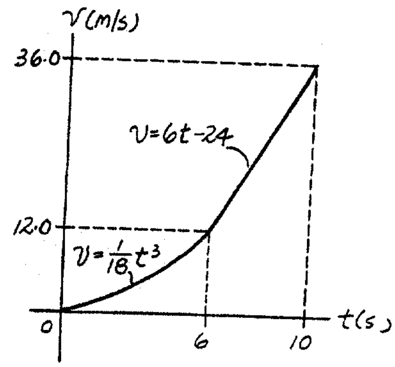
$$dv = adt$$
$$\int_0^v dv = \int_0^t \frac{1}{6}t^2 dt$$
$$v = \left(\frac{1}{18}t^3\right)\ \text{m/s}$$

At $t = 6$ s, $\quad v = \frac{1}{18}(6^3) = 12.0$ m/s

For time interval $6\ \text{s} < t \le 10\ \text{s}$,

$$dv = adt$$
$$\int_{12.0\text{m/s}}^{v} dv = \int_{6\text{s}}^{t} 6dt$$
$$v = (6t - 24)\ \text{m/s}$$

At $t = 10$ s, $\quad v = 6(10) - 24 = 36.0$ m/s



Position : The position in terms of time t can be obtained by applying $v = \frac{ds}{dt}$. For time interval $0\ \text{s} \le t < 6\ \text{s}$,

$$ds = vdt$$
$$\int_0^s ds = \int_0^t \frac{1}{18}t^3 dt$$
$$s = \left(\frac{1}{72}t^4\right)\ \text{m}$$

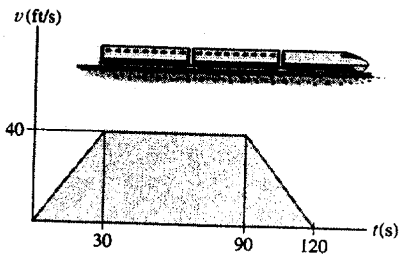
When $t = 6$ s, $\quad s = \frac{1}{72}(6^4) = 18.0$ m

For time interval $6\ \text{s} < t \le 10\ \text{s}$,

$$ds = vdt$$
$$\int_{18.0\text{m}}^{s} dv = \int_{6\text{s}}^{t} (6t - 24)\,dt$$
$$s = (3t^2 - 24t + 54)\ \text{m}$$

When $t = 10$ s, $\quad s = 3(10^2) - 24(10) + 54 = 114$ m $\quad$ **Ans**

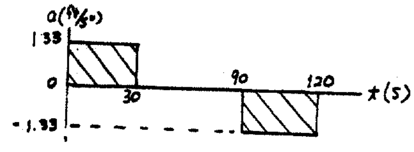
12-47. The v–t graph for the motion of a train as it moves from station A to station B is shown. Draw the a–t graph and determine the average speed and the distance between the stations.



For $0 \le t < 30$ s $\qquad a = \dfrac{\Delta v}{\Delta t} = \dfrac{40}{30} = 1.33 \text{ ft/s}^2$

For $30 < t < 90$ s $\qquad a = \dfrac{\Delta v}{\Delta t} = 0$

For $90 < t < 120$ s $\qquad a = \dfrac{\Delta v}{\Delta t} = \dfrac{0-40}{120-90} = -1.33 \text{ ft/s}^2$



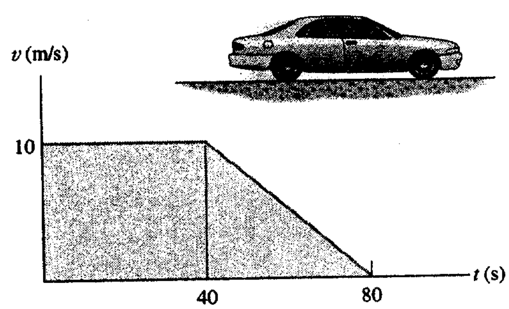
$$\Delta s = \int v\,dt$$

$$s - 0 = \frac{1}{2}(40)(30) + 40(90-30) + \frac{1}{2}(40)(120-90)$$

$$s = 3600 \text{ ft} \qquad \textbf{Ans}$$

$$(v_{sp})_{Avg} = \frac{\Delta s}{\Delta t} = \frac{3600}{120} = 30 \text{ ft/s} \qquad \textbf{Ans}$$

***12-48.** The velocity of a car is plotted as shown. Determine the total distance the car moves until it stops (t = 80 s). Construct the a-t graph.



Distance Traveled : The total distance traveled can be obtained by computing the area under the $v-t$ graph.

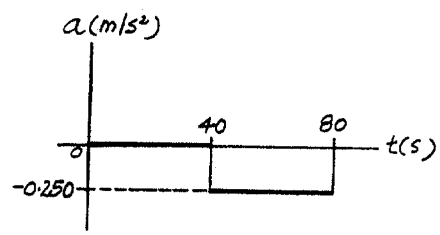
$$s = 10(40) + \frac{1}{2}(10)(80-40) = 600 \text{ m} \qquad \textbf{Ans}$$

a – t Graph : The acceleration in terms of time t can be obtained by applying $a = \dfrac{dv}{dt}$. For time interval $0 \text{ s} \le t < 40$ s,

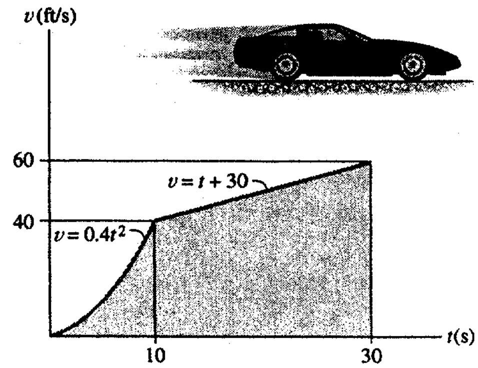
$$a = \frac{dv}{dt} = 0$$

For time interval $40 \text{ s} < t \le 80$ s, $\dfrac{v-10}{t-40} = \dfrac{0-10}{80-40}$, $v = \left(-\dfrac{1}{4}t + 20\right)$ m/s.

$$a = \frac{dv}{dt} = -\frac{1}{4} = -0.250 \text{ m/s}^2$$



12-49. The v–t graph for the motion of a car as it moves along a straight road is shown. Draw the a–t graph and determine the maximum acceleration during the 30-s time interval. The car starts from rest at $s = 0$.



For $t < 10$ s :

$$v = 0.4t^2$$

$$a = \frac{dv}{dt} = 0.8t$$

At $t = 10$ s :

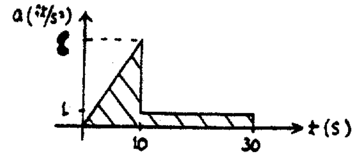
$$a = 8 \text{ ft/s}^2$$

For $10 < t \leq 30$ s :

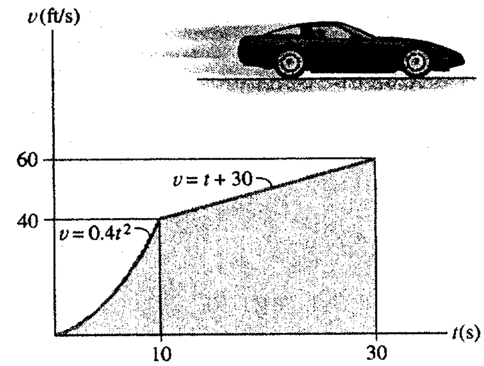
$$v = t + 30$$

$$a = \frac{dv}{dt} = 1$$

$$a_{max} = 8 \text{ ft/s}^2 \quad \textbf{Ans}$$



12-50. The v–t graph for the motion of a car as it moves along a straight road is shown. Draw the s–t graph and determine the average speed and the distance traveled for the 30-s time interval. The car starts from rest at $s = 0$.



For $t < 10$ s,

$$v = 0.4t^2$$

$$ds = v\,dt$$

$$\int_0^s ds = \int_0^t 0.4t^2\,dt$$

$$s = 0.1333t^3$$

At $t = 10$ s,

$$s = 133.3 \text{ ft}$$

For $10 < t < 30$ s,

$$v = t + 30$$

$$ds = v\,dt$$

$$\int_{133.3}^s ds = \int_{10}^t (t+30)\,dt$$

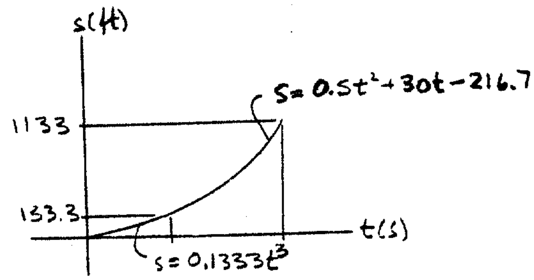
$$s = 0.5t^2 + 30t - 216.7$$

At $t = 30$ s,

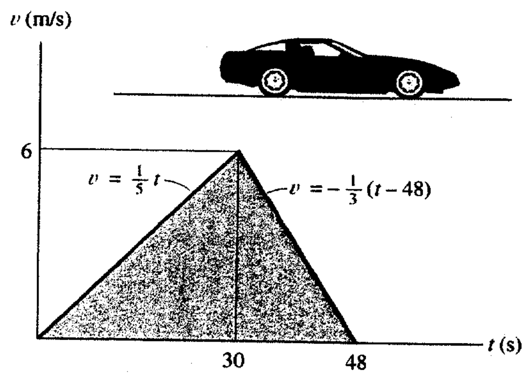
$$s = 1133 \text{ ft}$$

$$(v_{sp})_{Avg} = \frac{\Delta s}{\Delta t} = \frac{1133}{30} = 37.8 \text{ ft/s} \qquad \textbf{Ans}$$

$$s_T = 1133 \text{ ft} = 1.13(10^3) \text{ ft} \qquad \textbf{Ans}$$



12-51. A car travels along a straight road with the speed shown by the v–t graph. Determine the total distance the car travels until it stops when $t = 48$ s. Also plot the s–t and a–t graphs.



$0 \le t < 30$

$$v = \frac{1}{5}t$$

$$a = \frac{dv}{dt} = \frac{1}{5}$$

$$\int_0^s ds = \int_0^t \frac{1}{5}t\,dt$$

$$s = \frac{1}{10}t^2$$

When $t = 30$ s, $\quad s = 90$ m

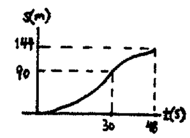
$$v = -\frac{1}{3}(t-48)$$

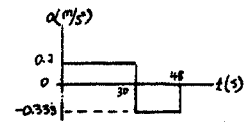
$$a = \frac{dv}{dt} = -\frac{1}{3}$$

$$\int_{90}^s ds = \int_{30}^t -\frac{1}{3}(t-48)\,dt$$

$$s = -\frac{1}{6}t^2 + 16\,t - 240$$

When $t = 48$ s, $\quad s = 144$ m $\qquad$ **Ans**





Also, from $v-t$ graph

$$\Delta s = \int v\,dt$$

$$s - 0 = \frac{1}{2}(6)(48) = 144 \text{ m} \qquad \textbf{Ans}$$

***12-52.** A man riding upward in a freight elevator accidentally drops a package off the elevator when it is 100 ft from the ground. If the elevator maintains a constant upward speed of 4 ft/s, determine how high the elevator is from the ground the instant the package hits the ground. Draw the v–t curve for the package during the time it is in motion. Assume that the package was released with the same upward speed as the elevator.

For package :

$(+\uparrow) \quad v^2 = v_0^2 + 2a_c(s_2 - s_0)$

$$v^2 = (4)^2 + 2(-32.2)(-100-0)$$

$$v = 80.35 \text{ ft/s} \downarrow$$

$(+\uparrow) \quad v = v_0 + a_c t$

$$-80.35 = 4 + (-32.2)t$$

$$t = 2.620 \text{ s}$$

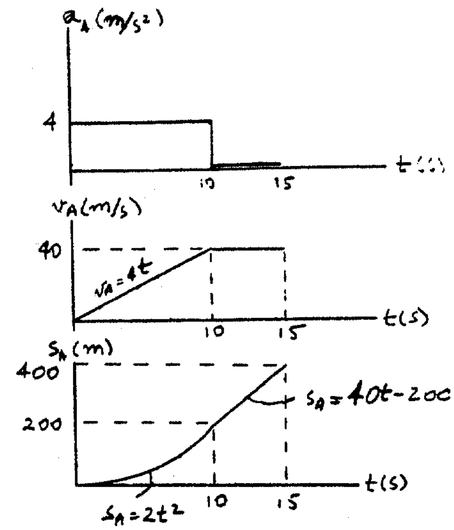
For elevator :

$(+\uparrow) \quad s_2 = s_0 + vt$

$$s = 100 + 4(2.620)$$

$$s = 110 \text{ ft} \qquad \textbf{Ans}$$

12-53. Two cars start from rest side by side and travel along a straight road. Car A accelerates at 4 m/s^2 for 10 s and then maintains a constant speed. Car B accelerates at 5 m/s^2 until reaching a constant speed of 25 m/s and then maintains this speed. Construct the a–t, v–t, and s–t graphs for each car until $t = 15$ s. What is the distance between the two cars when $t = 15$ s?



Car A :

$v = v_0 + a_c t$

$v_A = 0 + 4t$

At $t = 10$ s, $v_A = 40$ m/s

$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$s_A = 0 + 0 + \frac{1}{2}(4)t^2 = 2t^2$

At $t = 10$ s, $s_A = 200$ m

$t > 10$ s, $ds = v\,dt$

$$\int_{200}^{s_A} ds = \int_{10}^{t} 40\,dt$$

$s_A = 40t - 200$

At $t = 15$ s, $s_A = 400$ m

Car B :

$v = v_0 + a_c t$

$v_B = 0 + 5t$

When $v_B = 25$ m/s, $t = \frac{25}{5} = 5$ s

$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$s_B = 0 + 0 + \frac{1}{2}(5)t^2 = 2.5t^2$

At $t = 5$ s, $s_B = 62.5$ m

$t > 5$ s, $ds = v\,dt$

$$\int_{62.5}^{s_B} ds = \int_{5}^{t} 25\,dt$$

$s_B - 62.5 = 25t - 125$

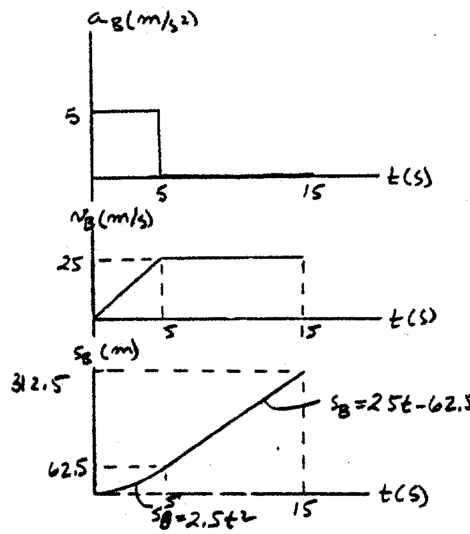
$s_B = 25t - 62.5$

When $t = 15$ s, $s_B = 312.5$

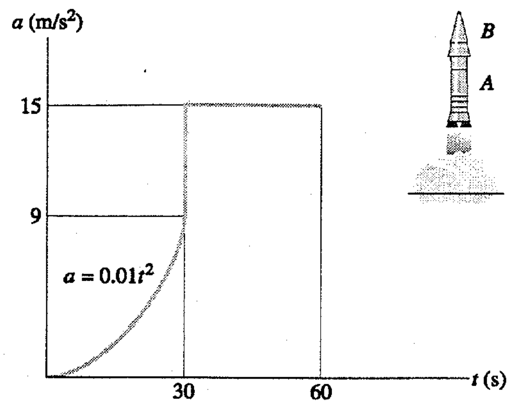
Distance between the cars is

$\Delta s = s_A - s_B = 400 - 312.5 = 87.5$ m **Ans**

Car A is ahead of car B.



12-54. A two-stage rocket is fired vertically from rest at $s = 0$ with an acceleration as shown. After 30 s the first stage A burns out and the second stage B ignites. Plot the $v-t$ and $s-t$ graphs which describe the motion of the second stage for $0 \leq t \leq 60$ s.



For $0 \leq t < 30$ s

$$\int_0^v dv = \int_0^t 0.01t^2 dt$$

$$v = 0.00333t^3$$

When $t = 30$ s, $v = 90$ m/s

$$\int_0^s ds = \int_0^t 0.00333t^3 dt$$

$$s = 0.000833t^4$$

When $t = 30$ s, $s = 675$ m

For $30 \text{ s} \leq t < 60$ s

$$\int_{90}^v dv = \int_{30}^t 15dt$$

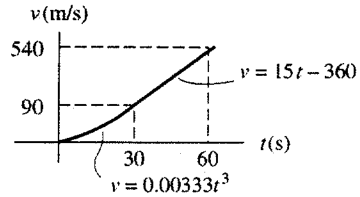
$$v = 15t - 360$$

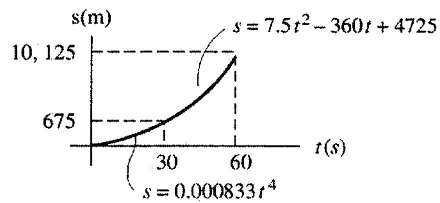
When $t = 60$ s, $v = 540$ m/s

$$\int_{675}^t ds = \int_{30}^t (15t - 360)dt$$

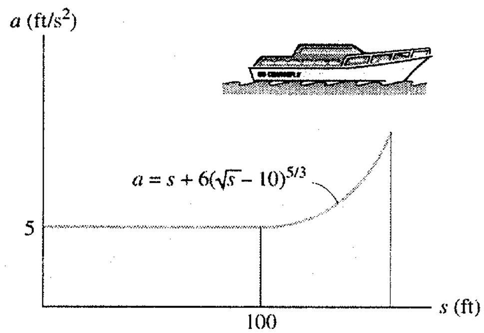
$$s = 7.5t^2 - 360t + 4725$$

When $t = 60$ s, $s = 10\,125$ m





■12-55. The $a-s$ graph for a boat moving along a straight path is given. If the boat starts at $s = 0$ when $v = 0$, determine its speed when it is at $s = 75$ ft, and 125 ft, respectively. Use Simpson's rule with $n = 100$ to evaluate v at $s = 125$ ft.



Velocity: The velocity v in terms of s can be obtained by applying $vdv = ads$. For the interval **0 ft ≤ s < 100 ft,**

$$vdv = ads$$

$$\int_0^v vdv = \int_0^s 5ds$$

$$v = (\sqrt{10s}) \text{ ft/s}$$

At $s = 75$ ft, $v = \sqrt{10(75)} = 27.4$ ft/s **Ans**

At $s = 100$ ft, $v = \sqrt{10(100)} = 31.62$ ft/s **Ans**

For the interval **100 ft < s ≤ 125 ft,**

$$vdv = ads$$

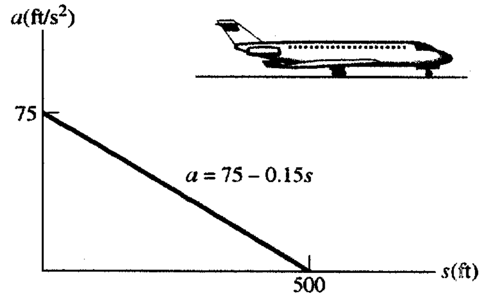
$$\int_{31.62 \text{ ft/s}}^v vdv = \int_{100 \text{ ft}}^{125 \text{ ft}} \left[s + 6(\sqrt{s} - 10)^{5/3}\right] ds$$

Evaluating the integral on the right using Simpson's rule, we have

$$\left.\frac{v^2}{2}\right|_{31.62 \text{ ft/s}}^{v} = 2888.53$$

$$v = 82.3 \text{ ft/s} \quad \textbf{Ans}$$

*12-56. The jet plane starts from rest at $s = 0$ and is subjected to the acceleration shown. Determine the speed of the plane when it has traveled 200 ft. Also, how much time is required for it to travel 200 ft?



$a = 75 - 0.15s$

$\int_0^v v\,dv = \int_0^s (75 - 0.15s)\,ds$

$v = \sqrt{150\,s - 0.15\,s^2}$

At $s = 200$ ft

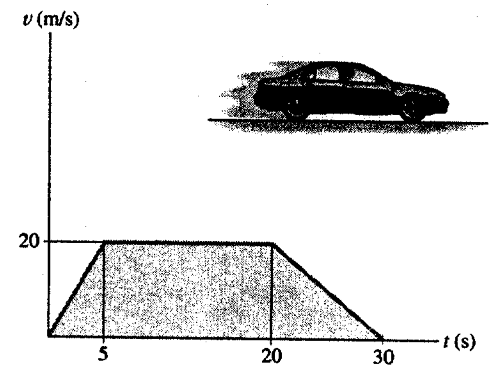
$v = \sqrt{150(200) - 0.15(200)^2} = 155$ ft/s **Ans**

$v = \dfrac{ds}{dt}$

$\int_0^t dt = \int_0^{200} \dfrac{ds}{\sqrt{150\,s - 0.15\,s^2}}$

$t = 2.582 \sin^{-1}\left(\dfrac{0.3\,s - 150}{150}\right)\Big|_0^{200} = 2.39$ s **Ans**

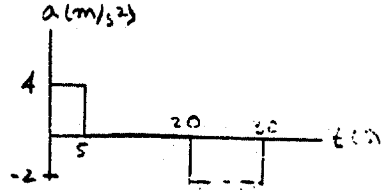
12-57. The v–t graph of a car while traveling along a road is shown. Draw the s–t and a–t graphs for the motion.



$0 \le t \le 5$ $\qquad a = \dfrac{\Delta v}{\Delta t} = \dfrac{20}{5} = 4\ \text{m/s}^2$

$5 \le t \le 20$ $\qquad a = \dfrac{\Delta v}{\Delta t} = \dfrac{20-20}{20-5} = 0\ \text{m/s}^2$

$20 \le t \le 30$ $\qquad a = \dfrac{\Delta v}{\Delta t} = \dfrac{0-20}{30-20} = -2\ \text{m/s}^2$

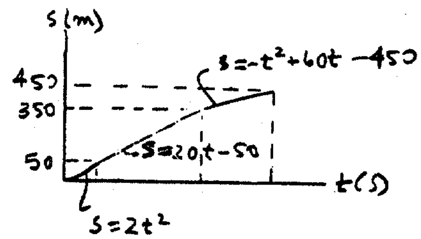


From the $v-t$ graph at $t_1 = 5\ s$, $t_2 = 20\ s$, and $t_3 = 30\ s$,

$s_1 = A_1 = \frac{1}{2}(5)(20) = 50\ \text{m}$

$s_2 = A_1 + A_2 = 50 + 20(20-5) = 350\ \text{m}$

$s_3 = A_1 + A_2 + A_3 = 350 + \frac{1}{2}(30-20)(20) = 450\ \text{m}$



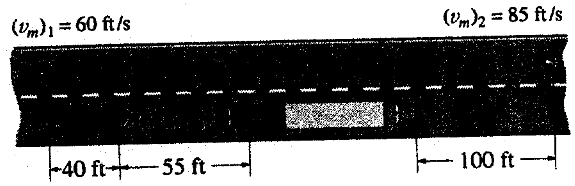
The equations defining the portions of the $s-t$ graph are

$0 \le t \le 5\ \text{s}$ $\qquad v = 4t; \quad ds = v\,dt; \quad \int_0^s ds = \int_0^t 4t\,dt; \quad s = 2t^2$

$5 \le t \le 20\ \text{s}$ $\qquad v = 20; \quad ds = v\,dt; \quad \int_{50}^s ds = \int_5^t 20\,dt; \quad s = 20t - 50$

$20 \le t \le 30\ \text{s}$ $\qquad v = 2(30-t); \quad ds = v\,dt; \quad \int_{350}^s ds = \int_{20}^t 2(30-t)\,dt; \quad s = -t^2 + 60t - 450$

12-58. A motorcyclist at A is traveling at 60 ft/s when he wishes to pass the truck T which is traveling at a constant speed of 60 ft/s. To do so the motorcyclist accelerates at 6 ft/s^2 until reaching a maximum speed of 85 ft/s. If he then maintains this speed, determine the time needed for him to reach a point located 100 ft in front of the truck. Draw the v–t and s–t graphs for the motorcycle during this time.



Motorcycle:

Time to reach 85 ft/s,

$$v = v_0 + a_c t$$

$$85 = 60 + 6t$$

$$t = 4.167 \text{ s}$$

$$v^2 = v_0^2 + 2a_c(s - s_0)$$

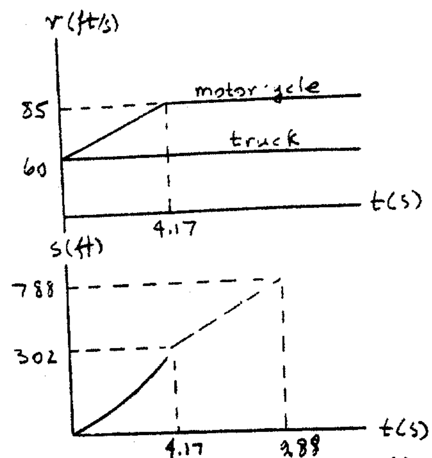
Distance traveled,

$$(85)^2 = (60)^2 + 2(6)(s_m - 0)$$

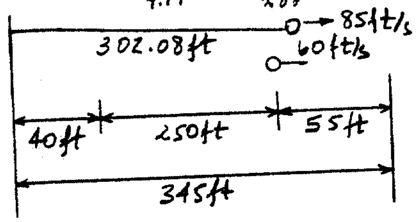
$$s_m = 302.08 \text{ ft}$$

In $t = 4.167$ s, truck travels

$$s_t = 60(4.167) = 250 \text{ ft}$$



Further distance for motorcycle to travel: $40 + 55 + 250 + 100 - 302.08 = 142.92$ ft



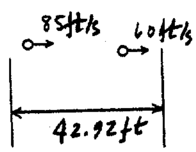
Motorcycle:

$$s = s_0 + v_0 t$$

$$(s + 142.92) = 0 + 85t'$$

Truck:

$$s = 0 + 60t'$$



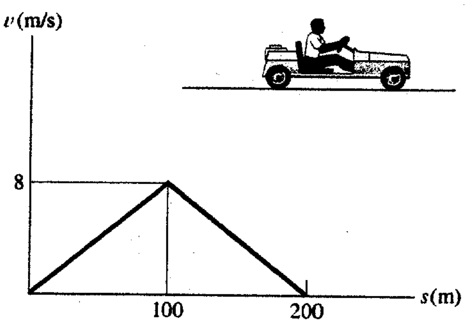
Thus $t' = 5.717$ s

$t = 4.167 + 5.717 = 9.88$ s **Ans**

Total distance motorcycle travels

$$s_T = 302.08 + 85(5.717) = 788 \text{ ft}$$

12-59. The v–s graph for a go-cart traveling on a straight road is shown. Determine the acceleration of the go-cart at $s = 50$ m and $s = 150$ m. Draw the a–s graph.



For $0 \leq s < 100$

$v = 0.08\,s, \quad dv = 0.08\,ds$

$a\,ds = (0.08\,s)(0.08\,ds)$

$a = 6.4(10^{-3})\,s$

At $s = 50$ m, $\quad a = 0.32$ m/s^2 **Ans**

For $100 < s < 200$

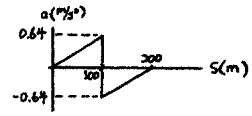
$v = -0.08\,s + 16,$

$dv = -0.08\,ds$

$a\,ds = (-0.08\,s + 16)(-0.08\,ds)$

$a = 0.08(0.08\,s - 16)$

At $s = 150$ m, $\quad a = -0.32$ m/s^2 **Ans**



Also,

$v\,dv = a\,ds$

$a = v(\frac{dv}{ds})$

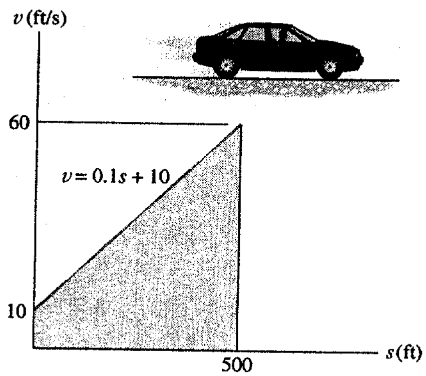
At $s = 50$ m,

$a = 4(\frac{8}{100}) = 0.32$ m/s^2 **Ans**

At $s = 150$ m,

$a = 4(\frac{-8}{100}) = -0.32$ m/s^2 **Ans**

***12-60.** The v-s graph for the car is given for the first 500 ft of its motion. Construct the a-s graph for $0 \leqslant s \leqslant 500$ ft. How long does it take to travel the 500-ft distance? The car starts at $s = 0$ when $t = 0$.



***a*−*s* Graph :** The acceleration a in terms of s can be obtained by applying $v\,dv = a\,ds$.

$$a = v\frac{dv}{ds} = (0.1s + 10)(0.1) = (0.01s + 1)\ \text{ft/s}^2$$

At $s = 0$ and $s = 500$ ft, $a = 0.01(0) + 1 = 1.00$ ft/s^2 and $a = 0.01(500) + 1 = 6.00$ ft/s^2, respectively.

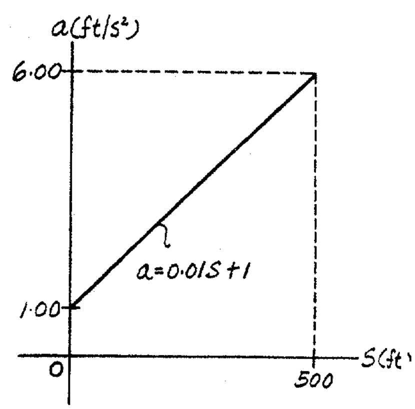
***Position* :** The position s in terms of time t can be obtained by applying $v = \frac{ds}{dt}$.

$$dt = \frac{ds}{v}$$

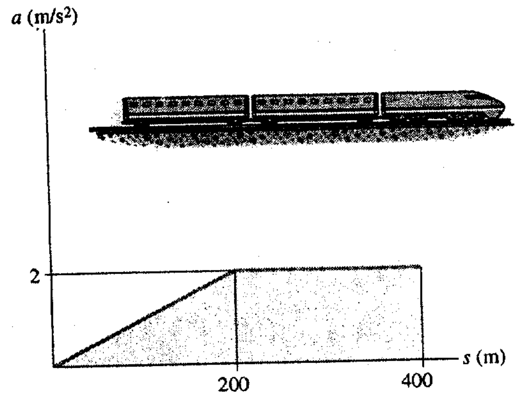
$$\int_0^t dt = \int_0^s \frac{ds}{0.1s + 10}$$

$$t = 10\ln(0.01s + 1)$$

When $s = 500$ ft, $\quad t = 10\ln[0.01(500) + 1] = 17.9$ s **Ans**



12-61. The a–s graph for a train traveling along a straight track is given for the first 400 m of its motion. Plot the v–s graph. $v = 0$ at $s = 0$.



$0 \le s \le 200$: $\quad a = \frac{1}{100}s$

$$a\,ds = v\,dv$$

$$\int_0^s \frac{1}{100}s\,ds = \int_0^v v\,dv$$

$$\frac{1}{200}s^2 = \frac{1}{2}v^2$$

$$v = 0.1s$$

At $s = 200$, $\quad v = 20$ m/s

$200 \le s \le 400$: $\quad a = 2$

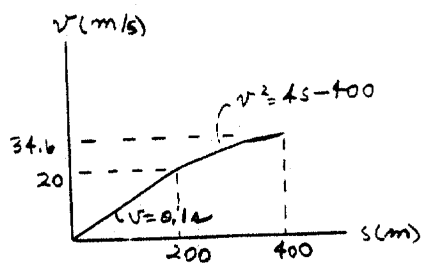
$$a\,ds = v\,dv$$

$$\int_{200}^s 2\,ds = \int_{20}^v v\,dv$$

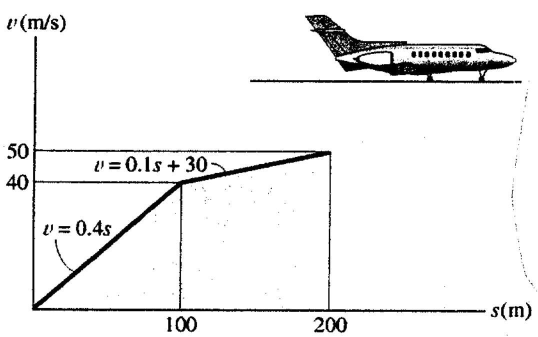
$$2(s - 200) = \frac{1}{2}(v^2 - 400)$$

$$v^2 = 4s - 400$$

At $s = 400$ m, $\quad v = \sqrt{4(400) - 400} = 34.6$ m/s



12-62. The v–s graph for an airplane traveling on a straight runway is shown. Determine the acceleration of the plane at $s = 100$ m and $s = 150$ m. Draw the a–s graph.



For $0 \le s < 100$ m

$a\,ds = v\,dv$

$a\,ds = 0.4\,s\,(0.4\,ds)$

$a = 0.16\,s$

At $s = 100$ m, $\quad a = 16$ m/s² $\quad$ **Ans**

For 100 m $< s \le 200$ m

$a\,ds = v\,dv$

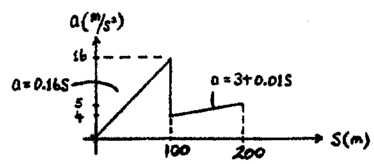
$a\,ds = (30 + 0.1\,s)(0.1\,ds)$

$a = 3 + 0.01\,s$

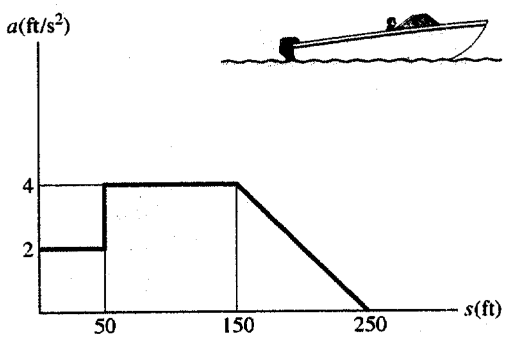
At $s = 150$ m, $\quad a = 4.5$ m/s² $\quad$ **Ans**

At $s = 100$ m, $\quad a = 4$ m/s²

At $s = 200$ m, $\quad a = 5$ m/s²



12-63. Starting from rest at $s = 0$, a boat travels in a straight line with an acceleration as shown by the a–s graph. Determine the boat's speed when $s = 40$, 90, and 200 ft.



Since $\int a\,ds = \int_0^v v\,dv$

$$\int a\,ds = \frac{1}{2}v^2$$

$$v = \sqrt{2\int a\,ds}$$

$\int a\,ds$ = area under $a-s$ graph.

For $s = 40$ ft

$$v = \sqrt{2(2)(40)} = 12.7 \text{ ft/s} \qquad \textbf{Ans}$$

For $s = 90$ ft

$$v = \sqrt{2[2(50) + 4(40)]} = 22.8 \text{ ft/s} \qquad \textbf{Ans}$$

For $s = 200$ ft

$$v = \sqrt{2[2(50) + 4(100) + \frac{1}{2}(50)(4+2)]}$$

$$v = 36.1 \text{ ft/s} \qquad \textbf{Ans}$$

Also,

For $0 \le s < 50$ ft

$a = 2, \quad v\,dv = a\,ds$

$$\int_0^s 2\,ds = \int_0^v v\,dv$$

$$v = \sqrt{4s}$$

When $s = 40$ ft, $v = \sqrt{4(40)} = 12.7$ ft/s **Ans**

When $s = 50$ ft, $v = \sqrt{4(50)} = 14.14$ ft/s

For 50 ft $< s <$ 150 ft

$a = 4,$

$$\int_{50}^s 4\,ds = \int_{14.14}^v v\,dv$$

$$v = \sqrt{8s - 200}$$

When $s = 90$ ft, $v = \sqrt{8(90) - 200} = 22.8$ ft/s **Ans**

When $s = 150$ ft, $v = \sqrt{8(90) - 200} = 31.62$ ft/s

For 150 ft $< s <$ 250 ft

$$a = -\frac{4}{100}s + 10$$

$$\int_{150}^s \left(-\frac{1}{25}s + 10\right) ds = \int_{31.62}^v v\,dv$$

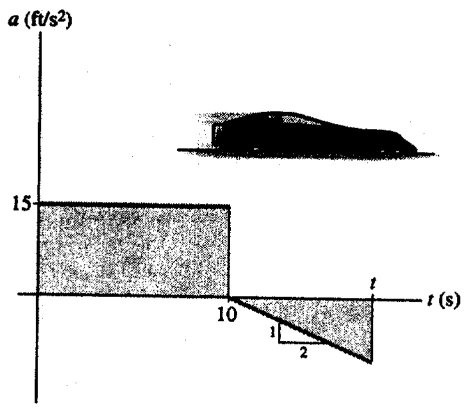
$$v = \left(-\frac{1}{25}s^2 + 20s - 1100\right)^{1/2}$$

When $s = 200$ ft

$$v = \left[-\frac{1}{25}(200)^2 + 20(200) - 1100\right]^{1/2}$$

$$v = 36.1 \text{ ft/s} \qquad \textbf{Ans}$$

*12-64. The test car starts from rest and is subjected to a constant acceleration of $a_c = 15\ \text{ft/s}^2$ for $0 \le t < 10$ s. The brakes are then applied, which causes a deceleration at the rate shown until the car stops. Determine the car's maximum speed and the time t when it stops.



$v_{max} = A_1 = (15)(10) = 150$ ft/s **Ans**

From the graph, for $t > 10$ s, $a = -\frac{1}{2}(t-10)$

$$dv = a\,dt$$

$$\int_{150}^{v} dv = \int_{10}^{t} -\frac{1}{2}(t-10)\,dt$$

$$v - 150 = -\frac{1}{2}\left[\frac{1}{2}t^2 - 10t\right]_{10}^{t}$$

$$v = 150 - \frac{1}{4}t^2 + 5t + \frac{1}{4}(10)^2 - \frac{1}{2}(10)^2$$

$$= -\frac{1}{4}t^2 + 5t + 125$$

When the car stops, $v = 0 = -\frac{1}{4}t^2 + 5t + 125$ (1)

Solving for the positive root,

$t = 34.5$ s **Ans**

Using the $a-t$ graph, we can obtain the same result by requiring

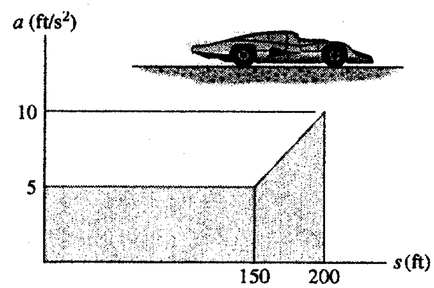
$$A_1 + A_2 = (15)(10) + \frac{1}{2}(a)(t-10) = 0$$

$$150 + \frac{1}{2}\left[-\frac{1}{2}(t-10)\right](t-10) = 0$$

$$-\frac{1}{4}t^2 + 5t + 125 = 0$$

Which is the same as Eq. (1).

12-65. The a-s graph for a race car moving along a straight track has been experimentally determined. If the car starts from rest at $s = 0$, determine its speed when $s = 50$ ft. 150 ft, and 200 ft, respectively.



Velocity **:** The velocity v in terms of s can be obtained by applying $vdv = ads$. For the interval $0\ \text{ft} \le s < 150\ \text{ft}$,

$$v\,dv = a\,ds$$

$$\int_0^v v\,dv = \int_0^s 5\,ds$$

$$v = \left(\sqrt{10s}\right)\ \text{m/s}$$

At $s = 50$ ft, $v = \sqrt{10(50)} = 22.4$ ft/s **Ans**

At $s = 150$ ft, $v = \sqrt{10(150)} = 38.7$ ft/s **Ans**

For the interval $150\ \text{ft} < s \le 200\ \text{ft}$, $\frac{a-5}{s-150} = \frac{10-5}{200-150}$, $a = \left(\frac{1}{10}s - 10\right)\ \text{ft/s}^2$.

$$v\,dv = a\,ds$$

$$\int_{38.7\text{ft/s}}^{v} v\,dv = \int_{150\text{ft}}^{s} \left(\frac{1}{10}s - 10\right)ds$$

$$v = \left(\sqrt{\frac{1}{10}s^2 - 20s + 2250}\right)\ \text{ft/s}$$

At $s = 200$ ft, $v = \sqrt{\frac{1}{10}(200^2) - 20(200) + 2250} = 47.4$ ft/s **Ans**

12-66. A particle, originally at rest and located at point (3 ft, 2 ft, 5 ft), is subjected to an acceleration of $\mathbf{a} = \{6t\mathbf{i} + 12t^2\mathbf{k}\}$ ft/s^2. Determine the particle's position (x, y, z) at $t = 1$ s.

Velocity : The velocity express in Cartesian vector form can be obtained by applying Eq. 12–9.

$$d\mathbf{v} = \mathbf{a}dt$$

$$\int_0^{\mathbf{v}} d\mathbf{v} = \int_0^t \left(6t\mathbf{i} + 12t^2\mathbf{k}\right) dt$$

$$\mathbf{v} = \{3t^2\mathbf{i} + 4t^3\mathbf{k}\}\ \text{ft/s}$$

Position : The position express in Cartesian vector form can be obtained by applying Eq. 12–7.

$$d\mathbf{r} = \mathbf{v}dt$$

$$\int_{\mathbf{r}_1}^{\mathbf{r}} d\mathbf{r} = \int_0^t \left(3t^2\mathbf{i} + 4t^3\mathbf{k}\right) dt$$

$$\mathbf{r} - (3\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}) = t^3\mathbf{i} + t^4\mathbf{k}$$

$$\mathbf{r} = \left\{\left(t^3 + 3\right)\mathbf{i} + 2\mathbf{j} + \left(t^4 + 5\right)\mathbf{k}\right\}\ \text{ft}$$

When $t = 1$ s, $\mathbf{r} = \left(1^3 + 3\right)\mathbf{i} + 2\mathbf{j} + \left(1^4 + 5\right)\mathbf{k} = \{4\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}\}$ ft.

The coordinates of the particle are

(4, 2, 6) ft **Ans**

12-67. The velocity of a particle is given by $v = \{16t^2\mathbf{i} + 4t^3\mathbf{j} + (5t + 2)\mathbf{k}\}$ m/s, where t is in seconds. If the particle is at the origin when $t = 0$, determine the magnitude of the particle's acceleration when $t = 2$ s. Also, what is the x, y, z coordinate position of the particle at this instant?

Acceleration : The acceleration express in Cartesian vector form can be obtained by applying Eq. 12–9.

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \{32t\mathbf{i} + 12t^2\mathbf{j} + 5\mathbf{k}\}\ \text{m/s}^2$$

When $t = 2$ s, $\mathbf{a} = 32(2)\mathbf{i} + 12\left(2^2\right)\mathbf{j} + 5\mathbf{k} = \{64\mathbf{i} + 48\mathbf{j} + 5\mathbf{k}\}$ m/s^2. The magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2} = \sqrt{64^2 + 48^2 + 5^2} = 80.2\ \text{m/s}^2 \qquad \textbf{Ans}$$

Position : The position express in Cartesian vector form can be obtained by applying Eq. 12–7.

$$d\mathbf{r} = \mathbf{v}dt$$

$$\int_0^{\mathbf{r}} d\mathbf{r} = \int_0^t \left(16t^2\mathbf{i} + 4t^3\mathbf{j} + (5t + 2)\mathbf{k}\right) dt$$

$$\mathbf{r} = \left[\frac{16}{3}t^3\mathbf{i} + t^4\mathbf{j} + \left(\frac{5}{2}t^2 + 2t\right)\mathbf{k}\right]\ \text{m}$$

When $t = 2$ s,

$$\mathbf{r} = \frac{16}{3}\left(2^3\right)\mathbf{i} + \left(2^4\right)\mathbf{j} + \left[\frac{5}{2}\left(2^2\right) + 2(2)\right]\mathbf{k} = \{42.7\mathbf{i} + 16.0\mathbf{j} + 14.0\mathbf{k}\}\ \text{m}.$$

Thus, the coordinate of the particle is

(42.7, 16.0, 14.0) ft **Ans**

■*12-68. A particle is traveling with a velocity of $\mathbf{v} = \{3\sqrt{t}e^{-0.2t}\mathbf{i} + 4e^{-0.8t^2}\mathbf{j}\}$ m/s, where t is in seconds. Determine the magnitude of the particle's displacement from $t = 0$ to $t = 3$ s. Use Simpson's rule with $n = 100$ to evaluate the integrals. What is the magnitude of the particle's acceleration when $t = 2$ s?

$$ds = v\,dt$$

$$\Delta s_x = \int_0^3 3\sqrt{t}\,e^{-0.2t}\,dt = 7.341$$

$$\Delta s_y = \int_0^3 4\,e^{-0.8t^2}\,dt = 3.963$$

Thus,

$$\Delta s = \sqrt{(7.341)^2 + (3.963)^2} = 8.34 \text{ m} \qquad \textbf{Ans}$$

$$a_x = \dot{v}_x = 3\left(\frac{1}{2}\right)t^{-\frac{1}{2}}e^{-0.2t} + 3\sqrt{t}\,e^{-0.2t}(-0.2)\Big|_{t=2} = 0.1422$$

$$a_y = \dot{v}_y = 4\,e^{-0.8t^2}(-0.8)(2t)\Big|_{t=2} = -0.5218$$

$$a = \sqrt{(0.1422)^2 + (-0.5218)^2} = 0.541 \text{ m/s}^2 \qquad \textbf{Ans}$$

12-69. The position of a particle is defined by $r = \{5(\cos 2t)\,\mathbf{i} + 4(\sin 2t)\mathbf{j}\}$ m, where t is in seconds and the arguments for the sine and cosine are given in radians. Determine the magnitudes of the velocity and acceleration of the particle when $t = 1$ s. Also, prove that the path of the particle is elliptical.

Velocity : The velocity expressed in Cartesian vector form can be obtained by applying Eq. 12–7.

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \{-10\sin 2t\mathbf{i} + 8\cos 2t\mathbf{j}\} \text{ m/s}$$

When $t = 1$ s, $\mathbf{v} = -10\sin 2(1)\mathbf{i} + 8\cos 2(1)\mathbf{j} = \{-9.093\mathbf{i} - 3.329\mathbf{j}\}$ m/s. Thus, the magnitude of the velocity is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(-9.093)^2 + (-3.329)^2} = 9.68 \text{ m/s} \qquad \textbf{Ans}$$

Acceleration : The acceleration express in Cartesian vector form can be obtained by applying Eq. 12–9.

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \{-20\cos 2t\mathbf{i} - 16\sin 2t\mathbf{j}\} \text{ m/s}^2$$

When $t = 1$ s, $\mathbf{a} = -20\cos 2(1)\mathbf{i} - 16\sin 2(1)\mathbf{j} = \{8.323\mathbf{i} - 14.549\mathbf{j}\}$ m/s^2. Thus, the magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{8.323^2 + (-14.549)^2} = 16.8 \text{ m/s}^2 \qquad \textbf{Ans}$$

Travelling Path : Here, $x = 5\cos 2t$ and $y = 4\sin 2t$. Then,

$$\frac{x^2}{25} = \cos^2 2t \qquad [1]$$

$$\frac{y^2}{16} = \sin^2 2t \qquad [2]$$

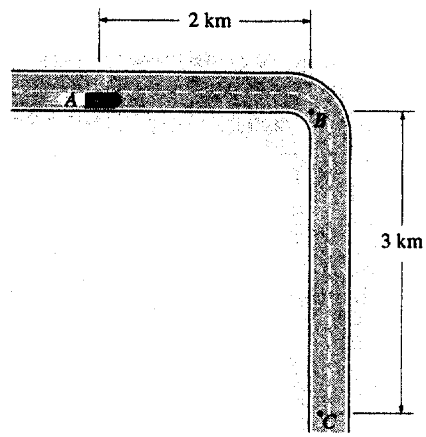
Adding Eqs[1] and [2] yields

$$\frac{x^2}{25} + \frac{y^2}{16} = \cos^2 2t + \sin^2 2t$$

However, $\cos^2 2t + \sin^2 2t = 1$. Thus,

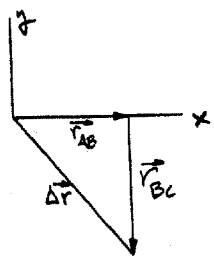
$$\frac{x^2}{25} + \frac{y^2}{16} = 1 \text{ (Equation of an Ellipse)} \quad (Q.E.D.)$$

12-70. The car travels from A to B, and then from B to C, as shown in the figure. Determine the magnitude of the displacement of the car and the distance traveled.



Displacement: $\Delta \mathbf{r} = \{2\mathbf{i} - 3\mathbf{j}\}$ km

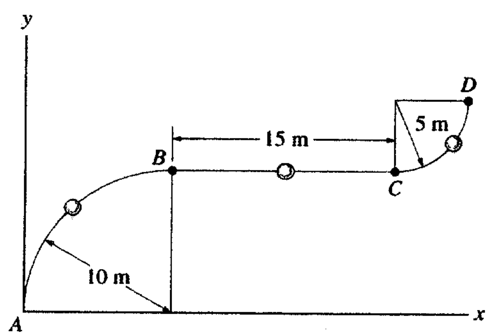
$$\Delta r = \sqrt{2^2 + 3^2} = 3.61 \text{ km} \qquad \textbf{Ans}$$



Distance traveled:

$d = 2 + 3 = 5$ km **Ans**

12-71. A particle travels along the curve from A to B in 2 s. It takes 4 s for it to go from B to C and then 3 s to go from C to D. Determine its average speed when it goes from A to D.



$$s_T = \frac{1}{4}(2\pi(10)) + 15 + \frac{1}{4}(2\pi(5)) = 38.56$$

$$v_{sp} = \frac{s_T}{t_t} = \frac{38.56}{2+4+3} = 4.28 \text{ m/s} \qquad \textbf{Ans}$$

***12-72.** A car travels east 2 km for 5 minutes, then north 3 km for 8 minutes. and then west 4 km for 10 minutes. Determine the total distance traveled and the magnitude of displacement of the car. Also, what is the magnitude of the average velocity and the average speed?

Total Distance Traveled and Displacement: The total distance traveled is

$$s = 2 + 3 + 4 = 9 \text{ km} \qquad \textbf{Ans}$$

and the magnitude of the displacement is

$$\Delta r = \sqrt{2^2 + 3^2} = 3.606 \text{ km} = 3.61 \text{ km} \qquad \textbf{Ans}$$

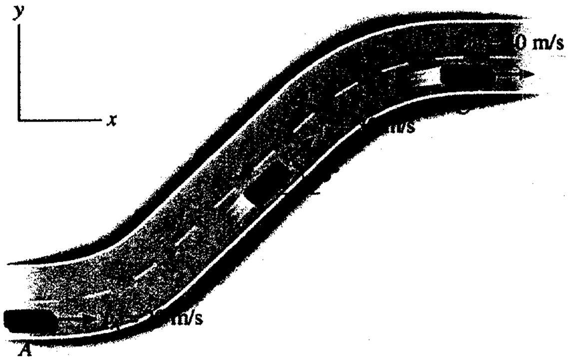
Average Velocity and Speed: The total time is $\Delta t = 5 + 8 + 10 = 23$ min $= 1380$ s. The magnitude of average velocity is

$$v_{avg} = \frac{\Delta r}{\Delta t} = \frac{3.606(10^3)}{1380} = 2.61 \text{ m/s} \qquad \textbf{Ans}$$

and the average speed is

$$(v_{sp})_{avg} = \frac{s}{\Delta t} = \frac{9(10^3)}{1380} = 6.52 \text{ m/s} \qquad \textbf{Ans}$$

12-73. A car traveling along the straight portions of the road has the velocities indicated in the figure when it arrives at points A, B, and C. If it takes 3 s to go from A to B, and then 5 s to go from B to C, determine the average acceleration between points A and B and between points A and C.



$$\mathbf{v}_A = 20\,\mathbf{i}$$

$$\mathbf{v}_B = 21.21\,\mathbf{i} + 21.21\,\mathbf{j}$$

$$\mathbf{v}_C = 40\,\mathbf{i}$$

$$\mathbf{a}_{AB} = \frac{\Delta \mathbf{v}}{\Delta t} = \frac{21.21\,\mathbf{i} + 21.21\mathbf{j} - 20\,\mathbf{i}}{3}$$

$$\mathbf{a}_{AB} = \{0.404\,\mathbf{i} + 7.07\,\mathbf{j}\}\ \mathrm{m/s^2} \qquad \textbf{Ans}$$

$$\mathbf{a}_{AC} = \frac{\Delta \mathbf{v}}{\Delta t} = \frac{40\,\mathbf{i} - 20\,\mathbf{i}}{8}$$

$$\mathbf{a}_{AC} = \{2.50\,\mathbf{i}\ \}\ \mathrm{m/s^2} \qquad \textbf{Ans}$$

12-74. A particle moves along the curve $y = e^{2x}$ such that its velocity has a constant magnitude of $v = 4$ ft/s. Determine the x and y components of velocity when the particle is at $y = 5$ ft.

Velocity **:** Taking the first derivative of the path $y = e^{2x}$, we have

$$\dot{y} = 2e^{2x}\dot{x} \qquad [1]$$

However, $\dot{x} = v_x$ and $\dot{y} = v_y$. Thus, Eq. [1] becomes

$$v_y = 2e^{2x}v_x \qquad [2]$$

Here, $v = 4$ ft/s. Then

$$v^2 = v_x^2 + v_y^2$$
$$v_x^2 + v_y^2 = 16 \qquad [3]$$

Solving Eqs. [2] and [3] yields

$$v_x = 4\sqrt{\frac{1}{1+4e^{4x}}} \quad \text{and} \quad v_y = 8\sqrt{\frac{e^{4x}}{1+4e^{4x}}}$$

At $y = 5$ ft, $5 = e^{2x}$, $x = 0.8047$ ft. Thus,

$$v_x = 4\sqrt{\frac{1}{1+4e^{4(0.8047)}}} = 0.398 \text{ ft/s} \qquad \textbf{Ans}$$

$$v_y = 8\sqrt{\frac{e^{4(0.8047)}}{1+4e^{4(0.8047)}}} = 3.98 \text{ ft/s} \qquad \textbf{Ans}$$

12-75. The path of a particle is defined by $y^2 = 4kx$, and the component of velocity along the y axis is $v_y = ct$, where both k and c are constants. Determine the x and y components of acceleration.

$$y^2 = 4kx$$

$$2yv_y = 4kv_x$$

$$2v_y^2 + 2ya_y = 4ka_x$$

$$v_y = ct$$

$$a_y = c \qquad \textbf{Ans}$$

$$2(ct)^2 + 2yc = 4ka_x$$

$$a_x = \frac{c}{2k}\left(y + ct^2\right) \qquad \textbf{Ans}$$

*12-76. A particle is moving along the curve $y = x - (x^2/400)$, where x and y are in ft. If the velocity component in the x direction is $v_x = 2$ ft/s and remains *constant*, determine the magnitudes of the velocity and acceleration when $x = 20$ ft.

Velocity **:** Taking the first derivative of the path $y = x - \dfrac{x^2}{400}$, we have

$$\dot{y} = \dot{x} - \frac{1}{400}(2x\dot{x})$$

$$\dot{y} = \dot{x} - \frac{x}{200}\dot{x} \qquad [1]$$

However, $\dot{x} = v_x$ and $\dot{y} = v_y$. Thus, Eq.[1] becomes

$$v_y = v_x - \frac{x}{200}v_x \qquad [2]$$

Here, $v_x = 2$ ft/s at $x = 20$ ft. Then, From Eq.[2]

$$v_y = 2 - \frac{20}{200}(2) = 1.80 \text{ ft/s}$$

Also,

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{2^2 + 1.80^2} = 2.69 \text{ ft/s} \qquad \textbf{Ans}$$

Acceleration **:** Taking the second derivative of the path $y = x - \dfrac{x^2}{400}$, we have

$$\ddot{y} = \ddot{x} - \frac{1}{200}\left(\dot{x}^2 + x\ddot{x}\right) \qquad [3]$$

However, $\ddot{x} = a_x$ and $\ddot{x} = a_y$. Thus, Eq.[3] becomes

$$a_y = a_x - \frac{1}{200}\left(v_x^2 + xa_x\right) \qquad [4]$$

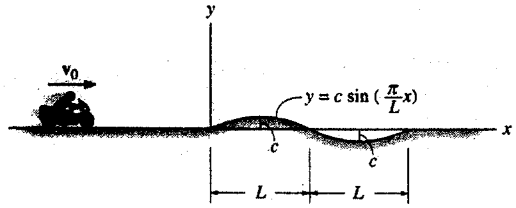
Since $v_x = 2$ ft/s is constant, hence $a_x = 0$ at $x = 20$ ft. Then, From Eq.[4]

$$a_y = 0 - \frac{1}{200}\left[2^2 + 20(0)\right] = -0.020 \text{ ft/s}$$

Also,

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{0^2 + (-0.020)^2} = 0.0200 \text{ ft/s}^2 \qquad \textbf{Ans}$$

12-77. The motorcycle travels with constant speed v_0 along the path that, for a short distance, takes the form of a sine curve. Determine the x and y components of its velocity at any instant on the curve.



$$y = c\sin\left(\frac{\pi}{L}x\right)$$

$$\dot{y} = \frac{\pi}{L}c\left(\cos\frac{\pi}{L}x\right)\dot{x}$$

$$v_y = \frac{\pi}{L}c\, v_x\left(\cos\frac{\pi}{L}x\right)$$

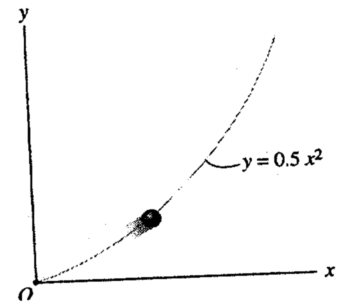
$$v_0^2 = v_y^2 + v_x^2$$

$$v_0^2 = v_x^2\left[1+\left(\frac{\pi}{L}c\right)^2\cos^2\left(\frac{\pi}{L}x\right)\right]$$

$$v_x = v_0\left[1+\left(\frac{\pi}{L}c\right)^2\cos^2\left(\frac{\pi}{L}x\right)\right]^{-\frac{1}{2}}$$ **Ans**

$$v_y = \frac{v_0\pi c}{L}\left(\cos\frac{\pi}{L}x\right)\left[1+\left(\frac{\pi}{L}c\right)^2\cos^2\left(\frac{\pi}{L}x\right)\right]^{-\frac{1}{2}}$$ **Ans**

12-78. The particle travels along the path defined by the parabola $y = 0.5x^2$. If the component of velocity along the x axis is $v_x = (5t)$ ft/s, where t is in seconds, determine the particle's distance from the origin O and the magnitude of its acceleration when $t = 1$ s. When $t = 0$, $x = 0$, $y = 0$.



Position : The x position of the particle can be obtained by applying the $v_x = \dfrac{dx}{dt}$.

$$dx = v_x\,dt$$
$$\int_0^x dx = \int_0^t 5t\,dt$$
$$x = \left(2.50t^2\right)\ \text{ft}$$

Thus, $y = 0.5\left(2.50t^2\right)^2 = \left(3.125t^4\right)$ ft. At $t = 1$ s, $x = 2.5\left(1^2\right) = 2.50$ ft and $y = 3.125\left(1^4\right) = 3.125$ ft. The particle's distance from the origin at this moment is

$$d = \sqrt{(2.50-0)^2+(3.125-0)^2} = 4.00\ \text{ft} \qquad \textbf{Ans}$$

Acceleration : Taking the first derivative of the path $y = 0.5x^2$, we have $\dot{y} = x\dot{x}$. The second derivative of the path gives

$$\ddot{y} = \dot{x}^2 + x\ddot{x} \qquad [1]$$

However, $\dot{x} = v_x$, $\ddot{x} = a_x$ and $\ddot{y} = a_y$. Thus, Eq. [1] becomes

$$a_y = v_x^2 + xa_x \qquad [2]$$

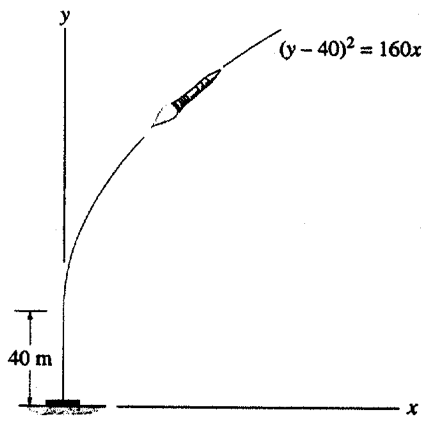
When $t = 1$ s, $v_x = 5(1) = 5$ ft/s, $a_x = \dfrac{dv_x}{dt} = 5$ ft/s and $x = 2.50$ ft. Then, from Eq. [2]

$$a_y = 5^2 + 2.50(5) = 37.5\ \text{ft/s}^2$$

Also,

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{5^2 + 37.5^2} = 37.8\ \text{ft/s}^2 \qquad \textbf{Ans}$$

12-79. When a rocket reaches an altitude of 40 m it begins to travel along the parabolic path $(y - 40)^2 = 160x$, where the coordinates are measured in meters. If the component of velocity in the vertical direction is constant at $v_y = 180$ m/s, determine the magnitudes of the rocket's velocity and acceleration when it reaches an altitude of 80 m.



$$v_y = 180 \text{ m/s}$$

$$(y-40)^2 = 160x$$

$$2(y-40)v_y = 160 v_x \qquad (1)$$

$$2(80-40)(180) = 160 v_x$$

$$v_x = 90 \text{ m/s}$$

$$v = \sqrt{90^2 + 180^2} = 201 \text{ m/s} \qquad \textbf{Ans}$$

$$a_y = \frac{d v_y}{dt} = 0$$

From Eq. 1,

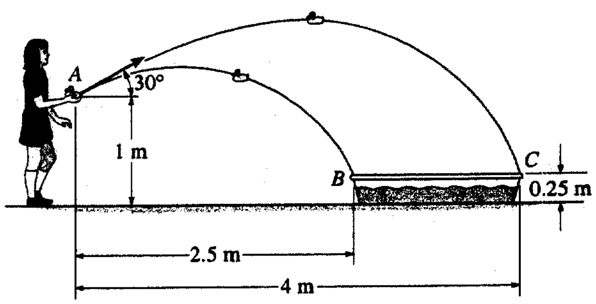
$$2 v_y^2 + 2(y-40)a_y = 160 a_x$$

$$2(180)^2 + 0 = 160 a_x$$

$$a_x = 405 \text{ m/s}^2$$

$$a = 405 \text{ m/s}^2 \qquad \textbf{Ans}$$

***12-80.** The girl always throws the toys at an angle of 30° from point A as shown. Determine the time between throws so that both toys strike the edges of the pool B and C at the same instant. With what speed must she throw each toy?



To strike B:

$$(\xrightarrow{+})\ s = s_0 + v_0 t$$

$$2.5 = 0 + v_A \cos 30^\circ\, t$$

$$(+\uparrow)\ s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$0.25 = 1 + v_A \sin 30^\circ\, t - \frac{1}{2}(9.81)t^2$$

Solving

$$t = 0.6687 \text{ s}$$

$$(v_A)_B = 4.32 \text{ m/s} \qquad \textbf{Ans}$$

To strike C:

$$(\xrightarrow{+})\ s = s_0 + v_0 t$$

$$4 = 0 + v_A \cos 30^\circ\, t$$

$$(+\uparrow)\ s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$0.25 = 1 + v_A \sin 30^\circ\, t - \frac{1}{2}(9.81)t^2$$

Solving

$$t = 0.790 \text{ s}$$

$$(v_A)_C = 5.85 \text{ m/s} \qquad \textbf{Ans}$$

Time between throws:

$$\Delta t = 0.790 \text{ s} - 0.6687 \text{ s} = 0.121 \text{ s} \qquad \textbf{Ans}$$

12-81. The nozzle of a garden hose discharges water at the rate of 15 m/s. If the nozzle is held at ground level and directed $\theta = 30°$ from the ground, determine the maximum height reached by the water and the horizontal distance from the nozzle to where the water strikes the ground.

$(v_0)_x = 15\cos 30° = 12.99$ m/s

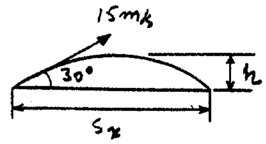
$(v_0)_y = 15\sin 30° = 7.5$ m/s

Maximum height :

$(+\uparrow) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$

$0 = (7.5)^2 + 2(-9.81)(h - 0)$

$h = 2.87$ m **Ans**



Time of travel to top of path :

$(+\uparrow) \qquad v = v_0 + a_c t$

$0 = 7.5 + (-9.81)t$

$t = 0.7645$ s

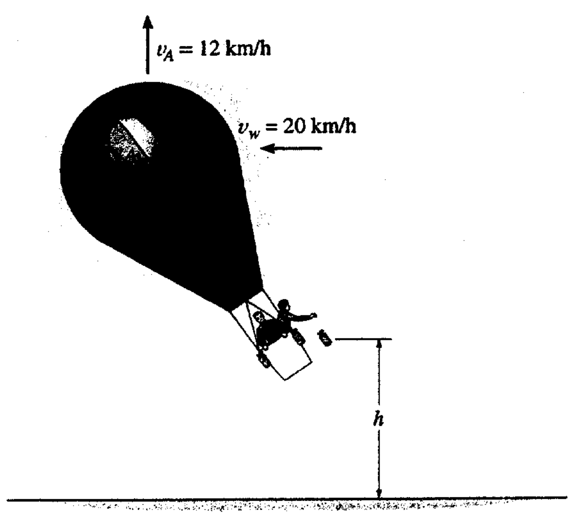
Total time along path

$t = 2(0.7645) = 1.529$ s

Range

$s_x = v_x t = (12.99)(1.529) = 19.9$ m **Ans**

12-82. The balloon A is ascending at the rate $v_A = 12$ km/h and is being carried horizontally by the wind at $v_w = 20$ km/h. If a ballast bag is dropped from the balloon at the instant $h = 50$ m, determine the time needed for it to strike the ground. Assume that the bag was released from the balloon with the same velocity as the balloon. Also, with what speed does the bag strike the ground?



$(+\uparrow) \quad v^2 = v_0^2 + 2a_c(s - s_0)$

$v_y^2 = (3.33)^2 + 2(-9.81)(-50 - 0)$

$v_y = 31.50$ m/s

$(+\uparrow) \quad v = v_0 + a_c t$

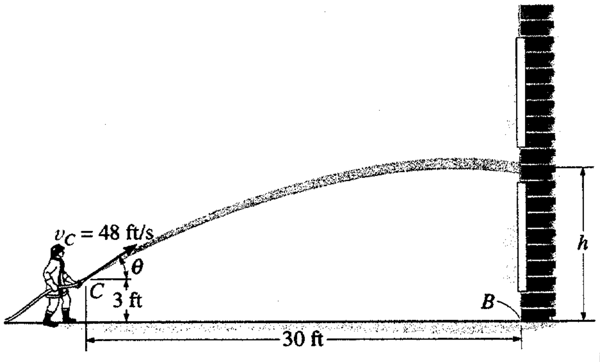
$-31.50 = 3.33 - 9.81t$

$t = 3.55$ s **Ans**

$v_x = 20$ km/h $= 5.556$ m/s

$v = \sqrt{(31.50)^2 + (5.556)^2} = 32.0$ m/s **Ans**

12-83. Determine the maximum height on the wall to which the firefighter can project water from the hose, if the speed of the water at the nozzle is $v_C = 48$ ft/s.



$$(+\uparrow)\ v = v_0 + a_c t$$

$$0 = 48 \sin\theta - 32.2t$$

$$(\overset{+}{\rightarrow})\ s = s_0 + v_0 t$$

$$30 = 0 + 48(\cos\theta)(t)$$

$$48 \sin\theta = 32.2 \frac{30}{48 \cos\theta}$$

$$\sin\theta\cos\theta = 0.41927$$

$$\sin 2\theta = 0.83854$$

$$\theta = 28.5^\circ$$

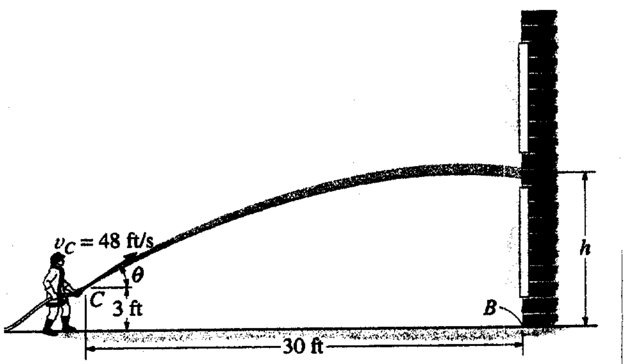
$$t = 0.7111 \text{ s}$$

$$(+\uparrow)\ s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$h - 3 = 0 + 48 \sin 28.5^\circ (0.7111) + \frac{1}{2}(-32.2)(0.7111)^2$$

$$h = 11.1 \text{ ft} \qquad \textbf{Ans}$$

■*12-84. Determine the smallest angle θ, measured above the horizontal, that the hose should be directed so that the water stream strikes the bottom of the wall at B. The speed of the water at the nozzle is $v_C = 48$ ft/s.



$$(\overset{+}{\rightarrow})\ s = s_0 + v_0 t$$

$$30 = 0 + 48 \cos\theta\ t$$

$$t = \frac{30}{48\cos\theta}$$

$$(+\uparrow)\ s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$0 = 3 + 48 \sin\theta\ t + \frac{1}{2}(-32.2)t^2$$

$$0 = 3 + \frac{48 \sin\theta\ (30)}{48 \cos\theta} - 16.1\left(\frac{30}{48\cos\theta}\right)^2$$

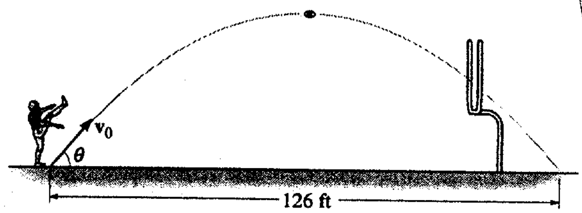
$$0 = 3\cos^2\theta + 30\sin\theta\cos\theta - 6.2891$$

$$3\cos^2\theta + 15\sin 2\theta = 6.2891$$

Solving

$$\theta = 6.41^\circ \qquad \textbf{Ans}$$

12-85. From a videotape, it was observed that a pro football player kicked a football 126 ft during a measured time of 3.6 seconds. Determine the initial speed of the ball and the angle θ at which it was kicked.



$(\overset{+}{\rightarrow})\quad s = s_0 + v_0 t$

$$126 = 0 + (v_0)_x(3.6)$$

$$(v_0)_x = 35 \text{ ft/s}$$

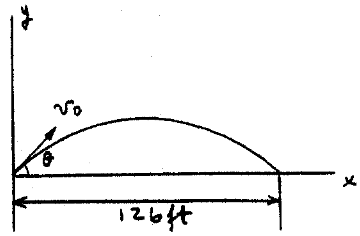
$(+\uparrow)\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

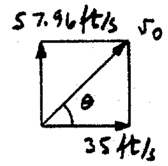
$$0 = 0 + (v_0)_y(3.6) + \frac{1}{2}(-32.2)(3.6)^2$$

$$(v_0)_y = 57.96 \text{ ft/s}$$

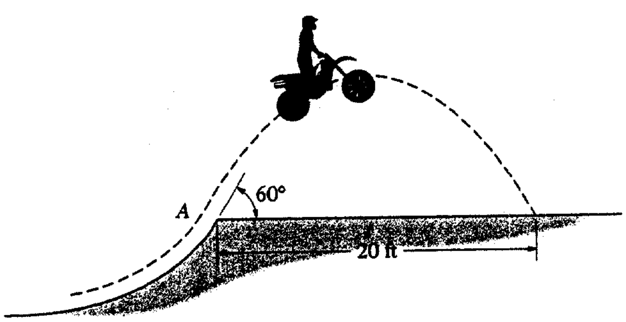
$$v_0 = \sqrt{(35)^2 + (57.96)^2} = 67.7 \text{ ft/s} \qquad \textbf{Ans}$$

$$\theta = \tan^{-1}\left(\frac{57.96}{35}\right) = 58.9° \qquad \textbf{Ans}$$





12-86. During a race the dirt bike was observed to leap up off the small hill at A at an angle of 60° with the horizontal. If the point of landing is 20 ft away, determine the approximate speed at which the bike was traveling just before it left the ground. Neglect the size of the bike for the calculation.



$(\overset{+}{\rightarrow})\; s = s_0 + v_0 t$

$$20 = 0 + v_A \cos 60° t$$

$(+\uparrow)\; s = s_0 + v_0 + \frac{1}{2}a_c t^2$

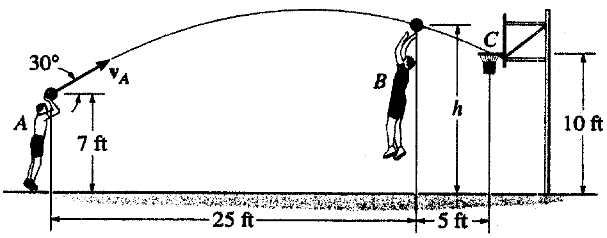
$$0 = 0 + v_A \sin 60° t + \frac{1}{2}(-32.2)t^2$$

Solving

$$t = 1.4668 \text{ s}$$

$$v_A = 27.3 \text{ ft/s} \qquad \textbf{Ans}$$

12-87. Measurements of a shot recorded on a videotape during a basketball game are shown. The ball passed through the hoop even though it barely cleared the hands of the player B who attempted to block it. Neglecting the size of the ball, determine the magnitude v_A of its initial velocity and the height h of the ball when it passes over player B.



$(\overset{+}{\rightarrow})$ $\quad s = s_0 + v_0 t$

$$30 = 0 + v_A \cos 30°\, t_{AC}$$

$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$10 = 7 + v_A \sin 30°\, t_{AC} - \frac{1}{2}(32.2)(t_{AC}^2)$$

Solving

$v_A = 36.73 = 36.7$ ft/s **Ans**

$t_{AC} = 0.943$ s

$(\overset{+}{\rightarrow})$ $\quad s = s_0 + v_0 t$

$$25 = 0 + 36.73 \cos 30°\, t_{AB}$$

$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

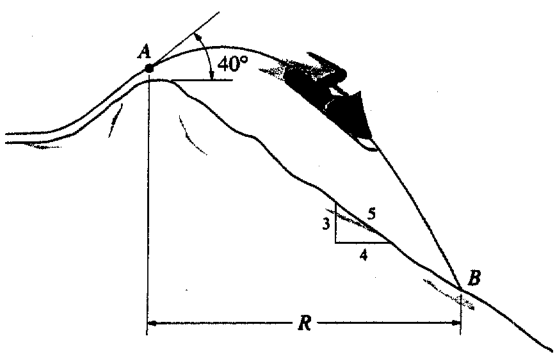
$$h = 7 + 36.73 \sin 30°\, t_{AB} - \frac{1}{2}(32.2)(t_{AB}^2)$$

Solving

$t_{AB} = 0.786$ s

$h = 11.5$ ft **Ans**

***12-88.** The snowmobile is traveling at 10 m/s when it leaves the embankment at A. Determine the time of flight from A to B and the range R of the trajectory.



$(\overset{+}{\rightarrow})$ $\quad s_B = s_A + v_A t$

$$R = 0 + 10 \cos 40°\, t$$

$(+\uparrow)$ $\quad s_B = s_A + v_A t + \frac{1}{2} a_c t^2$

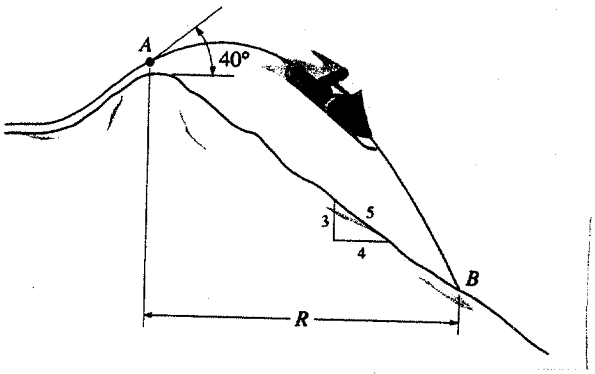
$$-R\left(\frac{3}{4}\right) = 0 + 10 \sin 40°\, t - \frac{1}{2}(9.81) t^2$$

Solving :

$R = 19.0$ m **Ans**

$t = 2.48$ s **Ans**

12-89. The snowmobile is traveling at 10 m/s when it leaves the embankment at A. Determine the speed at which it strikes the ground at B and its maximum acceleration along the trajectory AB.



At all times $a = g = -9.81 \text{ m/s}^2$ **Ans**

From Prob. 12–98 :

$R = 19.0 \text{ m}$

$t = 2.48 \text{ s}$

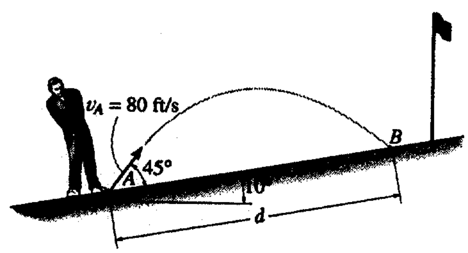
$(v_B)_x = (v_A)_x = 10 \cos 40° = 7.6604 \text{ m/s}$

$(+\uparrow)$ $(v_B)_y = (v_A)_y + a_c t$

$(v_B)_y = 10 \sin 40° - 9.81\,(2.48) = -17.901 \text{ m/s}$

$v_B = \sqrt{(7.6604)^2 + (-17.901)^2} = 19.5 \text{ m/s}$ **Ans**

12-90. A golf ball is struck with a velocity of 80 ft/s as shown. Determine the distance d to where it will land.



Horizontal Motion : The horizontal component of velocity is $(v_0)_x = 80\cos 55° = 45.89$ ft/s. The initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = d\cos 10°$, respectively.

$$(\overset{+}{\rightarrow}) \qquad s_x = (s_0)_x + (v_0)_x t$$

$$d\cos 10° = 0 + 45.89t \qquad [1]$$

Vertical Motion : The vertical component of initial velocity is $(v_0)_y = 80\sin 55° = 65.53$ ft/s. The initial and final vertical positions are $(s_0)_y = 0$ and $s_y = d\sin 10°$, respectively.

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

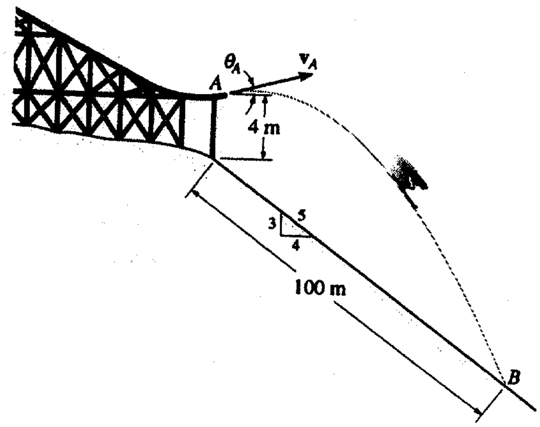
$$d\sin 10° = 0 + 65.53t + \frac{1}{2}(-32.2)t^2 \qquad [2]$$

Solving Eqs. [1] and [2] yields

$d = 166$ ft **Ans**

$t = 3.568$ s

12-91. It is observed that the skier leaves the ramp A at an angle $\theta_A = 25°$ with the horizontal. If he strikes the ground at B, determine his initial speed v_A and the time of flight t_{AB}.

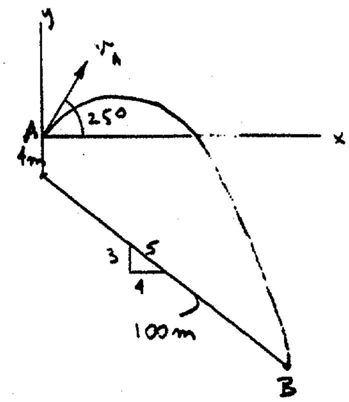


$(\xrightarrow{+})$ $\quad s = v_0 t$

$$100\left(\frac{4}{5}\right) = v_A \cos 25° t_{AB}$$

$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$-4 - 100\left(\frac{3}{5}\right) = 0 + v_A \sin 25° t_{AB} + \frac{1}{2}(-9.81) t_{AB}^2$$

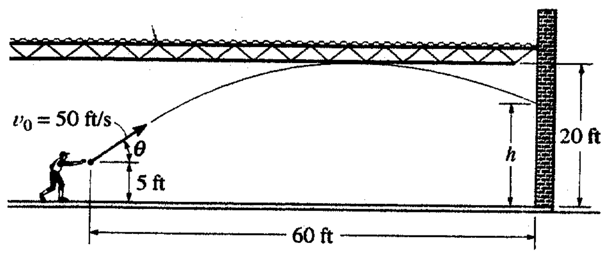


Solving,

$v_A = 19.4$ m/s **Ans**

$t_{AB} = 4.54$ s **Ans**

*12-92. The man stands 60 ft from the wall and throws a ball at it with a speed $v_0 = 50$ ft/s. Determine the angle θ at which he should release the ball so that it strikes the wall at the highest point possible. What is this height? The room has a ceiling height of 20 ft.



$v_x = 50\cos\theta$

$(\xrightarrow{+})\ s = s_0 + v_0 t$

$x = 0 + 50\cos\theta\ t \qquad (1)$

$(+\uparrow)\ v = v_0 + a_c t$

$v_y = 50\sin\theta - 32.2t \qquad (2)$

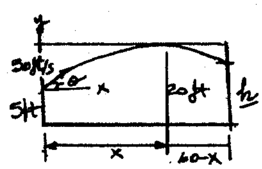
$(+\uparrow)\ s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$y = 0 + 50\sin\theta\ t - 16.1t^2 \qquad (3)$

$(+\uparrow)\ v^2 = v_0^2 + 2a_c(s - s_0)$

$v_y^2 = (50\sin\theta)^2 + 2(-32.2)(s - 0)$

$v_y^2 = 2500\sin^2\theta - 64.4s \qquad (4)$



Require $v_y = 0$ at $s = 20 - 5 = 15$ ft

$0 = 2500\sin^2\theta - 64.4(15)$

$\theta = 38.433° = 38.4°$ **Ans**

From Eq. (2)

$0 = 50\sin 38.433° - 32.2t$

$t = 0.9652$ s

From Eq.(1)

$x = 50\cos 38.433°(0.9652) = 37.8$ ft

Time for ball to hit wall

From Eq. (1),

$60 = 50(\cos 38.433°)t$

$t = 1.53193$ s

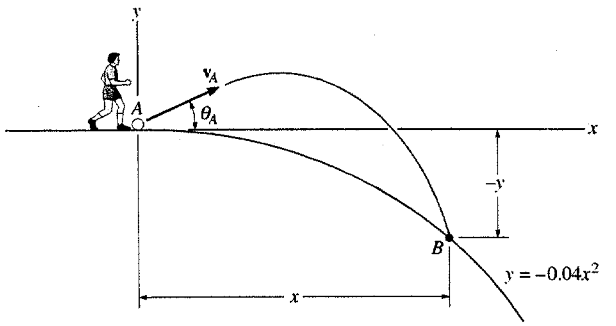
From Eq. (3)

$y = 50\sin 38.433°(1.53193) - 16.1(1.53193)^2$

$y = 9.830$ ft

$h = 9.830 + 5 = 14.8$ ft **Ans**

12-93. The ball at A is kicked with a speed $v_A = 80$ ft/s and at an angle $\theta_A = 30°$. Determine the point $(x, -y)$ where it strikes the ground. Assume the ground has the shape of a parabola as shown.



$(v_A)_x = 80 \cos 30° = 69.28$ ft/s

$(v_A)_y = 80 \sin 30° = 40$ ft/s

$(\xrightarrow{+})\ s = s_0 + v_0 t$

$x = 0 + 69.28\,t \qquad (1)$

$(+\uparrow)\ s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$-y = 0 + 40\,t + \frac{1}{2}(-32.2)t^2 \qquad (2)$

$y = -0.04\,x^2$

From Eqs. (1) and (2) :

$-y = 0.5774\,x - 0.003354\,x^2$

$0.04\,x^2 = 0.5774x - 0.003354\,x^2$

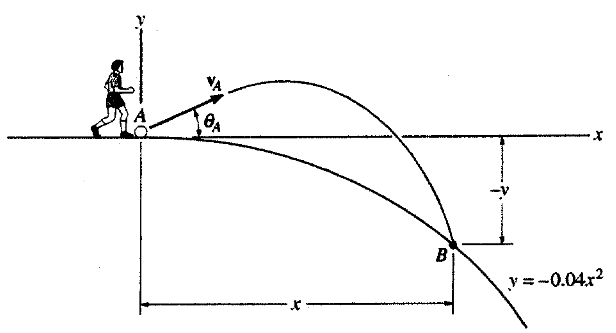
$0.04335x^2 = 0.5774x$

$x = 13.3$ ft **Ans**

Thus

$y = -0.04(13.3)^2 = -7.09$ ft **Ans**

12-94. The ball at A is kicked such that $\theta_A = 30°$. If it strikes the ground at B having coordinates $x = 15$ ft, $y = -9$ ft, determine the speed at which it is kicked and the speed at which it strikes the ground.



$(\xrightarrow{+})\ s = s_0 + v_0 t$

$15 = 0 + v_A \cos 30°\, t$

$(+\uparrow)\ s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$-9 = 0 + v_A \sin 30°\, t + \frac{1}{2}(-32.2)t^2$

$v_A = 16.5$ ft/s **Ans**

$t = 1.047$ s

$(\xrightarrow{+})\ (v_B)_x = 16.54 \cos 30° = 14.32$ ft/s

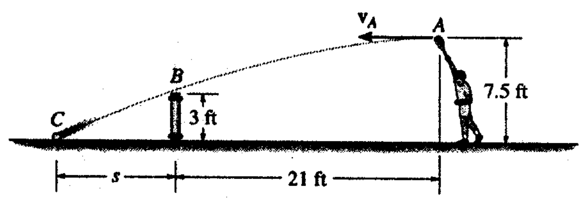
$(+\uparrow)\ v = v_0 + a_c t$

$(v_B)_y = 16.54 \sin 30° + (-32.2)(1.047)$

$= -25.45$ ft/s

$v_B = \sqrt{(14.32)^2 + (-25.45)^2} = 29.2$ ft/s **Ans**

12-95. Determine the horizontal velocity v_A of a tennis ball at A so that it just clears the net at B. Also, find the distance s where the ball strikes the ground.



Vertical Motion : The vertical component of initial velocity is $(v_0)_y = 0$. For the ball to travel from A to B, the initial and final vertical positions are $(s_0)_y = 7.5$ ft and $s_y = 3$ ft, respectively.

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$
$$3 = 7.5 + 0 + \frac{1}{2}(-32.2)t_1^2$$
$$t_1 = 0.5287 \text{ s}$$

For the ball to travel from A to C, the initial and final vertical positions are $(s_0)_y = 7.5$ ft and $s_y = 0$, respectively.

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$
$$0 = 7.5 + 0 + \frac{1}{2}(-32.2)t_2^2$$
$$t_2 = 0.6825 \text{ s}$$

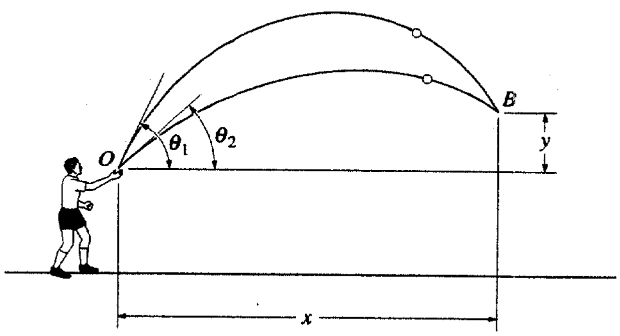
Horizontal Motion : The horizontal component of velocity is $(v_0)_x = v_A$. For the ball travel from A to B, the initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = 21$ ft, respectively. The time is $t = t_1 = 0.5287$ s.

$$(\overset{+}{\leftarrow}) \qquad s_x = (s_0)_x + (v_0)_x t$$
$$21 = 0 + v_A(0.5287)$$
$$v_A = 39.72 \text{ ft/s} = 39.7 \text{ ft/s} \qquad \textbf{Ans}$$

For the ball travel from A to C, the initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = (21 + s)$ ft, respectively. The time is $t = t_2 = 0.6825$ s.

$$(\overset{+}{\leftarrow}) \qquad s_x = (s_0)_x + (v_0)_x t$$
$$21 + s = 0 + 39.72(0.6825)$$
$$s = 6.11 \text{ ft} \qquad \textbf{Ans}$$

***12-96.** A boy at O throws a ball in the air with a speed v_O at an angle θ_1. If he then throws another ball at the same speed v_O at an angle $\theta_2 < \theta_1$, determine the time between the throws so the balls collide in mid air at B.



Time of flight

$$(\xrightarrow{+})\ s = s_0 + v_0 t$$

$$x_1 = 0 + v_0 \cos\theta_1\, t_1 \qquad (1)$$

$$x_2 = 0 + v_0 \cos\theta_2\, t_2 \qquad (2)$$

Thus,

$$x_1 = x_2$$

$$\Delta t = t_1 - t_2 = \frac{x_1}{v_0}\left(\frac{\cos\theta_2 - \cos\theta_1}{\cos\theta_1 \cos\theta_2}\right) \qquad (3)$$

$$(+\uparrow)\ s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$y = 0 + (v_0 \sin\theta_1)(t_1) - \frac{1}{2} g t_1^2 \qquad (4)$$

Use Eq. (1)

$$y = x_1 \tan\theta_1 - \frac{1}{2} g \frac{x_1^2}{v_0^2 \cos^2\theta_1}$$

In the same way :

$$y = x_2 \tan\theta_2 - \frac{1}{2} g \frac{x_2^2}{v_0^2 \cos^2\theta_2}$$

Equating and solving for $x_1 = x_2 = x$

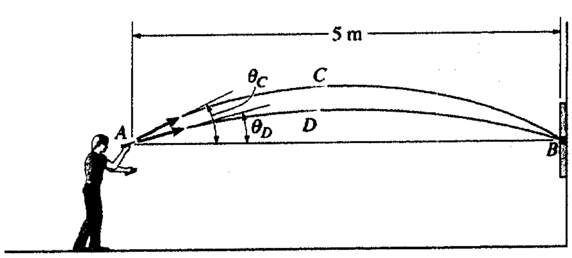
$$x = \frac{2v_0^2}{g}\left[\frac{(\cos^2\theta_1 \cos^2\theta_2)(\tan\theta_1 - \tan\theta_2)}{(\cos^2\theta_2 - \cos^2\theta_1)}\right]$$

Substituting into Eq. (3) yields

$$\Delta t = \frac{2v_0}{g}\left[\frac{(\cos\theta_1 \cos\theta_2)(\tan\theta_1 - \tan\theta_2)}{(\cos\theta_2 + \cos\theta_1)}\right]$$

$$\Delta t = \frac{2v_0}{g}\left[\frac{\sin(\theta_1 - \theta_2)}{\cos\theta_2 + \cos\theta_1}\right] \qquad \textbf{Ans}$$

12-97. The man at A wishes to throw two darts at the target at B so that they arrive at the *same time*. If each dart is thrown with a speed of 10 m/s, determine the angles θ_C and θ_D at which they should be thrown and the time between each throw. Note that the first dart must be thrown at $\theta_C (> \theta_D)$, then the second dart is thrown at θ_D.



$$(\xrightarrow{+})\ s = s_0 + v_0 t$$

$$5 = 0 + (10 \cos\theta)\, t \qquad (1)$$

$$(+\uparrow)\ v = v_0 + a_c t$$

$$-10 \sin\theta = 10 \sin\theta - 9.81\, t$$

$$t = \frac{2(10 \sin\theta)}{9.81} = 2.039 \sin\theta$$

From Eq. (1),

$$5 = 20.39 \sin\theta \cos\theta$$

Since $\quad \sin 2\theta = 2 \sin\theta \cos\theta$

$$\sin 2\theta = 0.4905$$

The two roots are $\theta_D = 14.7°$ **Ans**

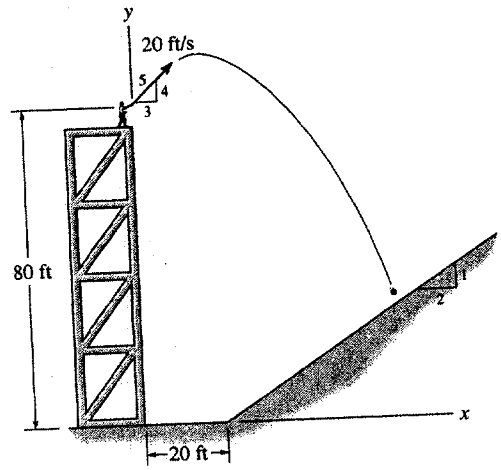
$\theta_C = 75.3°$ **Ans**

From Eq.(1) : $\quad t_D = 0.517$ s

$t_C = 1.97$ s

So that $\quad \Delta t = t_C - t_D = 1.45$ s **Ans**

12-98. The ball is thrown from the tower with a velocity of 20 ft/s as shown. Determine the x and y coordinates to where the ball strikes the slope. Also, determine the speed at which the ball hits the ground.



Assume ball hits slope.

$(\xrightarrow{+})$ $\quad s = s_0 + v_0 t$

$$x = 0 + \frac{3}{5}(20)t = 12t$$

$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$y = 80 + \frac{4}{5}(20)t + \frac{1}{2}(-32.2)t^2 = 80 + 16t - 16.1t^2$$

Equation of slope : $\quad y - y_1 = m(x - x_1)$

$$y - 0 = \frac{1}{2}(x - 20)$$

$$y = 0.5x - 10$$

Thus,

$$80 + 16t - 16.1t^2 = 0.5(12t) - 10$$

$$16.1t^2 - 10t - 90 = 0$$

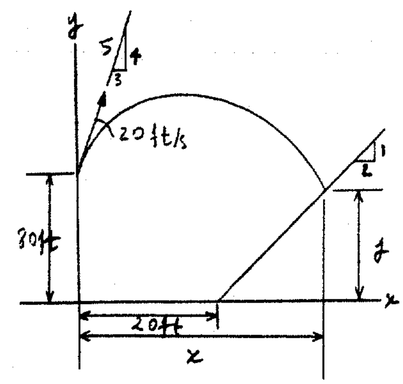
Choosing the positive root :

$t = 2.6952$ s

$x = 12(2.6952) = 32.3$ ft **Ans**

Since 32.3 ft > 20 ft, assumption is valid.

$y = 80 + 16(2.6952) - 16.1(2.6952)^2 = 6.17$ ft **Ans**

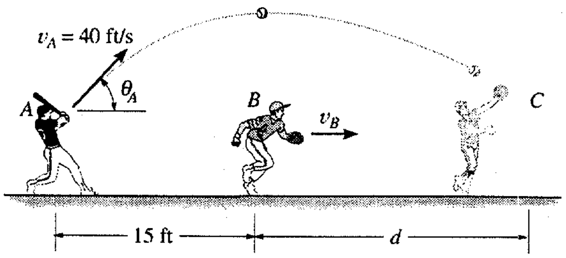


$(\xrightarrow{+})$ $\quad v_x = (v_0)_x = \frac{3}{5}(20) = 12$ ft/s

$(+\uparrow)$ $\quad v_y = (v_0)_y + a_c t = \frac{4}{5}(20) + (-32.2)(2.6952) = -70.785$ ft/s

$v = \sqrt{(12)^2 + (-70.785)^2} = 71.8$ ft/s **Ans**

12-99. The baseball player A hits the baseball at $v_A = 40$ ft/s and $\theta_A = 60°$ from the horizontal. When the ball is directly overhead of player B he begins to run under it. Determine the constant speed at which B must run and the distance d in order to make the catch at the same elevation at which the ball was hit.



Vertical Motion: The vertical component of initial velocity for the football is $(v_0)_y = 40 \sin 60° = 34.64$ ft/s. The initial and final vertical positions are $(s_0)_y = 0$ and $s_y = 0$, respectively.

$$(+\uparrow) \quad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$0 = 0 + 34.64t + \frac{1}{2}(-32.2)t^2$$

$$t = 2.152 \text{ s}$$

Horizontal Motion: The horizontal component of velocity for the baseball is $(v_0)_x = 40 \cos 60° = 20.0$ ft/s. The initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = R$, respectively.

$$(\overset{+}{\rightarrow}) \quad s_x = (s_0)_x + (v_0)_x t$$

$$R = 0 + 20.0(2.152) = 43.03 \text{ ft}$$

The distance for which player B must travel in order to catch the baseball is

$d = R - 15 = 43.03 - 15 = 28.0$ ft **Ans**

Player B is required to run at a same speed as the horizontal component of velocity of the baseball in order to catch it.

$v_B = 40 \cos 60° = 20.0$ ft/s **Ans**

***12-100.** A car is traveling along a circular curve that has a radius of 50 m. If its speed is 16 m/s and is increasing uniformly at 8 m/s^2, determine the magnitude of its acceleration at this instant.

$v = 16$ m/s

$a_t = 8$ m/s^2

$r = 50$ m

$$a_n = \frac{v^2}{\rho} = \frac{(16)^2}{50} = 5.12 \text{ m/s}^2$$

$$a = \sqrt{(8)^2 + (5.12)^2} = 9.50 \text{ m/s}^2 \qquad \textbf{Ans}$$

12-101. A car moves along a circular track of radius 250 ft such that its speed for a short period of time $0 \le t \le 4$ s, is $v = 3(t + t^2)$ ft/s, where t is in seconds. Determine the magnitude of its acceleration when $t = 3$ s. How far has it traveled in $t = 3$ s?

$v = 3(t + t^2)$

$$a_t = \frac{dv}{dt} = 3 + 6t$$

When $t = 3$ s, $\quad a_t = 3 + 6(3) = 21$ ft/s^2

$$a_n = \frac{[3(3+3^2)]^2}{250} = 5.18 \text{ ft/s}^2$$

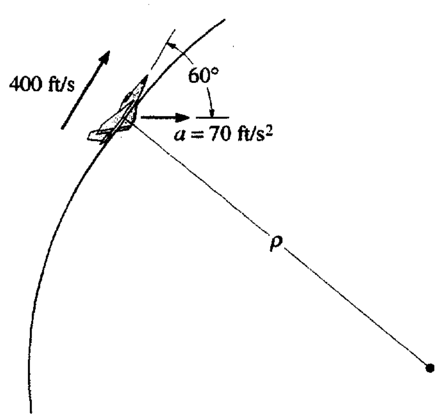
$$a = \sqrt{(21)^2 + (5.18)^2} = 21.6 \text{ ft/s}^2 \qquad \textbf{Ans}$$

$$\int ds = \int_0^3 3(t + t^2)\,dt$$

$$\Delta s = \frac{3}{2}t^2 + t^3\Big|_0^3$$

$$\Delta s = 40.5 \text{ ft} \qquad \textbf{Ans}$$

12-102. At a given instant the jet plane has a speed of 400 ft/s and an acceleration of 70 ft/s^2 acting in the direction shown. Determine the rate of increase in the plane's speed and the radius of curvature ρ of the path.



$a_t = 70\cos 60° = 35.0$ ft/s^2 $\qquad$ **Ans**

$$a_n = \frac{(400)^2}{\rho} = 70\sin 60°$$

$\rho = 2.64(10^3)$ ft $\qquad$ **Ans**

12-103. A boat is traveling along a circular curve having a radius of 100 ft. If its speed at $t = 0$ is 15 ft/s and is increasing at $\dot{v} = (0.8t)$ ft/s^2, determine the magnitude of its acceleration at the instant $t = 5$ s.

$$\int_{15}^{v} dv = \int_0^5 0.8t\,dt$$

$$v = 25 \text{ ft/s}$$

$$a_n = \frac{v^2}{\rho} = \frac{25^2}{100} = 6.25 \text{ ft/s}^2$$

At $t = 5$ s, $\quad a_t = \dot{v} = 0.8(5) = 4 \text{ ft/s}^2$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{4^2 + 6.25^2} = 7.42 \text{ ft/s}^2$$ **Ans**

***12-104.** A boat is traveling along a circular path having a radius of 20 m. Determine the magnitude of the boat's acceleration when the speed is $v = 5$ m/s and the rate of increase in the speed is $\dot{v} = 2$ m/s^2.

$$a_t = 2 \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{5^2}{20} = 1.25 \text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{2^2 + 1.25^2} = 2.36 \text{ m/s}^2$$ **Ans**

■**12-105.** Starting from rest, a bicyclist travels around a horizontal circular path, $\rho = 10$ m, at a speed of $v = (0.09t^2 + 0.1t)$ m/s, where t is in seconds. Determine the magnitudes of his velocity and acceleration when he has traveled $s = 3$ m.

$$\int_0^s ds = \int_0^t (0.09t^2 + 0.1t)\,dt$$

$$s = 0.03t^3 + 0.05t^2$$

When $s = 3$ m, $\quad 3 = 0.03t^3 + 0.05t^2$

Solving,

$t = 4.147$ s

$$v = \frac{ds}{dt} = 0.09t^2 + 0.1t$$

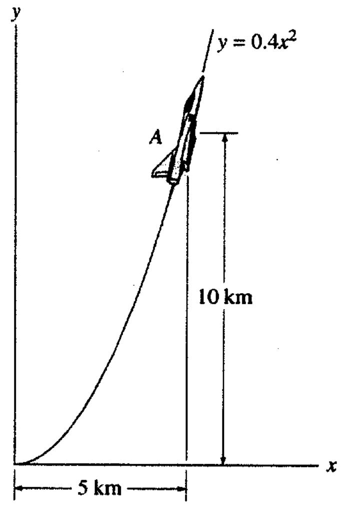
$$v = 0.09(4.147)^2 + 0.1(4.147) = 1.96 \text{ m/s}$$ **Ans**

$$a_t = \frac{dv}{dt} = 0.18t + 0.1\Big|_{t=4.147\text{ s}} = 0.8465 \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{1.96^2}{10} = 0.3852 \text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(0.8465)^2 + (0.3852)^2} = 0.930 \text{ m/s}^2$$ **Ans**

12-106. The jet plane travels along the vertical parabolic path. When it is at point A it has a speed of 200 m/s, which is increasing at the rate of $0.8\ \text{m/s}^2$. Determine the magnitude of acceleration of the plane when it is at point A.



$$y = 0.4x^2$$

$$\frac{dy}{dx} = 0.8x\Big|_{x=5\text{ km}} = 4$$

$$\frac{d^2y}{dx^2} = 0.8$$

$$\rho = \frac{[1+(4)^2]^{3/2}}{0.8} = 87.62\text{ km}$$

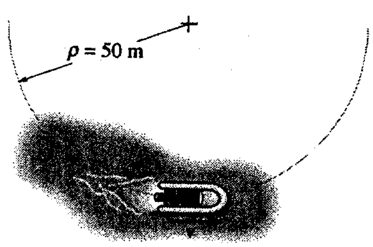
$$a_t = 0.8\text{ m/s}^2$$

$$a_n = \frac{(0.200)^2}{87.62} = 0.457(10^{-3})\text{ km/s}^2$$

$$a_n = 0.457\text{ m/s}^2$$

$$a = \sqrt{(0.8)^2 + (0.457)^2} = 0.921\text{ m/s}^2 \qquad \textbf{Ans}$$

12-107. Starting from rest, motorboat travels around the circular path, $\rho = 50$ m, at a speed $v = (0.8t)$ m/s, where t is in seconds. Determine the magnitudes of the boats velocity and acceleration when it has traveled 20 m.



Velocity : The time for which the boat to travel 20 m must be determined first.

$$ds = v\,dt$$

$$\int_0^{20\text{m}} ds = \int_0^t 0.8t\,dt$$

$$t = 7.071\text{ s}$$

The magnitude of the boat's velocity is

$$v = 0.8(7.071) = 5.657\text{ m/s} = 5.66\text{ m/s} \qquad \textbf{Ans}$$

Acceleration : The tangential acceleration is

$$a_t = \dot{v} = 0.8\text{ m/s}^2$$

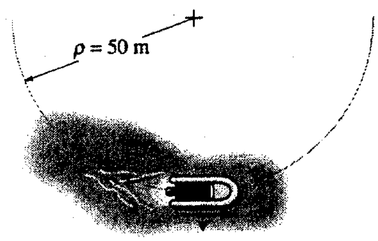
To determine the normal acceleration, apply Eq. 12 – 20.

$$a_n = \frac{v^2}{\rho} = \frac{5.657^2}{50} = 0.640\text{ m/s}^2$$

Thus, the magnitude of acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0.8^2 + 0.640^2} = 1.02\text{ m/s}^2 \qquad \textbf{Ans}$$

***12-108.** Starting from rest, the motorboat travels around the circular path, $\rho = 50$ m, at a speed $v = (0.2t^2)$ m/s, where t is in seconds. Determine the magnitudes of the boats velocity and acceleration at the instant $t = 3$ s.



Velocity : When $t = 3$ s, the boat travels at a speed of

$$v = 0.2(3^2) = 1.80 \text{ m/s} \qquad \textbf{Ans}$$

Acceleration : The tangential acceleration is $a_t = \dot{v} = (0.4t)$ m/s^2. When $t = 3$ s,

$$a_t = 0.4(3) = 1.20 \text{ m/s}^2$$

To determine the normal acceleration, apply Eq. 12 – 20.

$$a_n = \frac{v^2}{\rho} = \frac{1.80^2}{50} = 0.0648 \text{ m/s}^2$$

Thus, the magnitude of acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{1.20^2 + 0.06048^2} = 1.20 \text{ m/s}^2 \qquad \textbf{Ans}$$

12-109. A car moves along a circular track of radius 250 ft, and its speed for a short period of time $0 \le t \le 2$ s is $v = 3(t + t^2)$ ft/s, where t is in seconds. Determine the magnitude of its acceleration when $t = 2$ s. How far has it traveled in $t = 2$ s?

$$v = 3(t + t^2)$$

$$a_t = \frac{dv}{dt} = 3 + 6t$$

When $t = 2$ s,

$$a_t = 3 + 6(2) = 15 \text{ ft/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{[3(2 + 2^2)]^2}{250} = 1.296 \text{ ft/s}^2$$

$$a = \sqrt{(15)^2 + (1.296)^2} = 15.1 \text{ ft/s}^2 \qquad \textbf{Ans}$$

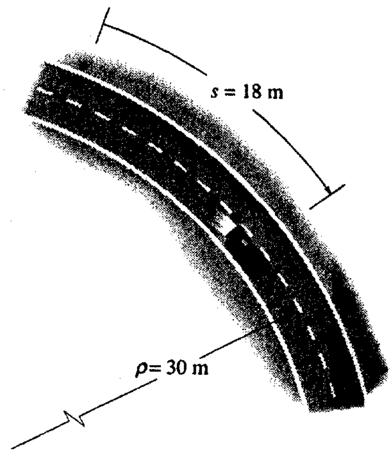
$$ds = v\,dt$$

$$\int ds = \int_0^2 3(t + t^2)\,dt$$

$$\Delta s = \frac{3}{2}t^2 + t^3 \Big]_0^2$$

$$\Delta s = 14 \text{ ft} \qquad \textbf{Ans}$$

•12-110. The car travels along the curved path such that its speed is increased by $\dot{v} = (0.5e^t)$ m/s^2, where t is in seconds. Determine the magnitudes of its velocity and acceleration after the car has traveled $s = 18$ m starting from rest. Neglect the size of the car.



$$\int_0^v dv = \int_0^t 0.5e^t dt$$

$$v = 0.5(e^t - 1)$$

$$\int_0^{18} ds = 0.5\int_0^t (e^t - 1)dt$$

$$18 = 0.5(e^t - t - 1)$$

Solving, $t = 3.7064$ s

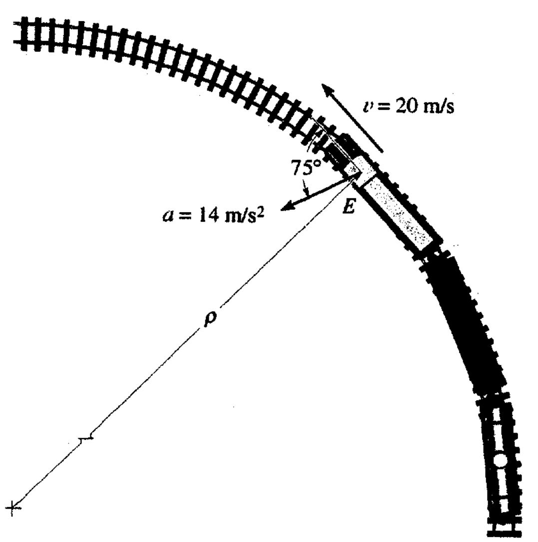
$$v = 0.5(e^{3.7064} - 1) = 19.85 \text{ m/s} = 19.9 \text{ m/s}$$ **Ans**

$$a_t = \dot{v} = 0.5e^t\big|_{t=3.7064\text{ s}} = 20.35 \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{19.85^2}{30} = 13.14 \text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{20.35^2 + 13.14^2} = 24.2 \text{ m/s}^2$$ **Ans**

12-111. At a given instant the train engine at E has a speed of 20 m/s and an acceleration of 14 m/s^2 acting in the direction shown. Determine the rate of increase in the train's speed and the radius of curvature ρ of the path.



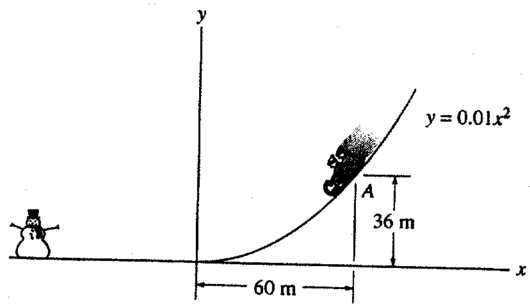
$$a_t = 14\cos 75° = 3.62 \text{ m/s}^2$$ **Ans**

$$a_n = 14\sin 75°$$

$$a_n = \frac{(20)^2}{\rho}$$

$$\rho = 29.6 \text{ m}$$ **Ans**

***12-112.** A toboggan is traveling down along a curve which can be approximated by the parabola $y = 0.01x^2$. Determine the magnitude of its acceleration when it reaches point A, where its speed is $v_A = 10$ m/s, and it is increasing at the rate of $\dot{v}_A = 3$ m/s^2.



Acceleration : The radius of curvature of the path at point A must be determined first. Here, $\frac{dy}{dx} = 0.02x$ and $\frac{d^2y}{dx^2} = 0.02$, then

$$\rho = \frac{\left[1+(dy/dx)^2\right]^{3/2}}{|d^2y/dx^2|} = \left.\frac{\left[1+(0.02x)^2\right]^{3/2}}{|0.02|}\right|_{x=60\text{m}} = 190.57 \text{ m}$$

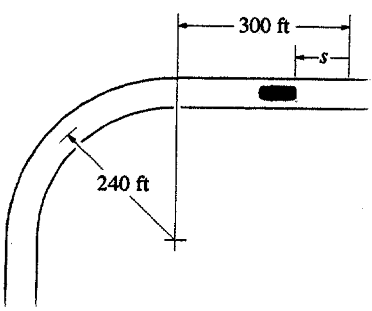
To determine the normal acceleration, apply Eq. 12 – 20.

$$a_n = \frac{v^2}{\rho} = \frac{10^2}{190.57} = 0.5247 \text{ m/s}^2$$

Here, $a_t = \dot{v}_A = 3$ m/s. Thus, the magnitude of acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{3^2 + 0.5247^2} = 3.05 \text{ m/s}^2 \qquad \text{Ans}$$

12-113. The automobile is originally at rest at $s = 0$. If its speed is increased by $\dot{v} = (0.05t^2)$ ft/s^2, where t is in seconds, determine the magnitudes of its velocity and acceleration when $t = 18$ s.



$a_t = 0.05\,t^2$

$\int_0^v dv = \int_0^t 0.05\,t^2\,dt$

$v = 0.0167\,t^3$

$\int_0^s ds = \int_0^t 0.0167\,t^3\,dt$

$s = 4.167(10^{-3})\,t^4$

When $t = 18$ s, $\quad s = 437.4$ ft

Therefore the car is on a curved path.

$v = 0.0167(18)^3 = 97.2$ ft/s $\qquad$ **Ans**

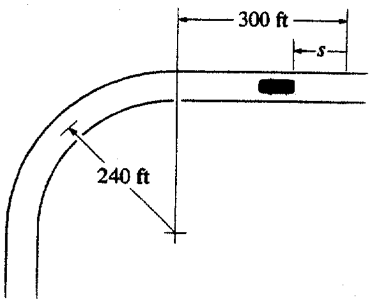
$a_n = \frac{(97.2)^2}{240} = 39.37$ ft/s^2

$a_t = 0.05(18)^2 = 16.2$ ft/s^2

$a = \sqrt{(39.37)^2 + (16.2)^2}$

$a = 42.6$ ft/s^2 $\qquad$ **Ans**

12-114. The automobile is originally at rest $s = 0$. If it then starts to increase its speed at $\dot{v} = (0.05t^2)$ ft/s^2, where t is in seconds, determine the magnitudes of its velocity and acceleration at $s = 550$ ft.



The car is on the curved path.

$$a_t = 0.05\,t^2$$

$$\int_0^v dv = \int_0^t 0.05\,t^2\,dt$$

$$v = 0.0167\,t^3$$

$$\int_0^s ds = \int_0^t 0.0167\,t^3\,dt$$

$$s = 4.167(10^{-3})\,t^4$$

$$550 = 4.167(10^{-3})\,t^4$$

$$t = 19.06\text{ s}$$

So that

$$v = 0.0167\,(19.06)^3 = 115.4$$

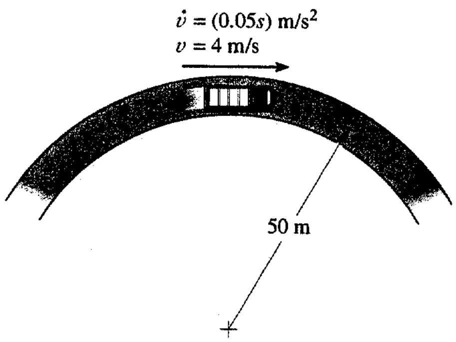
$v = 115$ ft/s **Ans**

$$a_n = \frac{(115.4)^2}{240} = 55.48\text{ ft/s}^2$$

$$a_t = 0.05(19.06)^2 = 18.16\text{ ft/s}^2$$

$a = \sqrt{(55.48)^2 + (18.16)^2} = 58.4$ ft/s^2 **Ans**

12-115. The truck travels in a circular path havng a radius of 50 m at a speed of 4 = m/s. For a short distance from $s = 0$, its speed is increased by $\dot{v} = (0.05s)$ m/s^2, where s is in meters. Determine its speed and the magnitude of its acceleration when it has moved $s = 10$ m.



$$v\,dv = a_t\,ds$$

$$\int_4^v v\,dv = \int_0^{10} 0.05s\,ds$$

$$0.5v^2 - 8 = \frac{0.05}{2}(10)^2$$

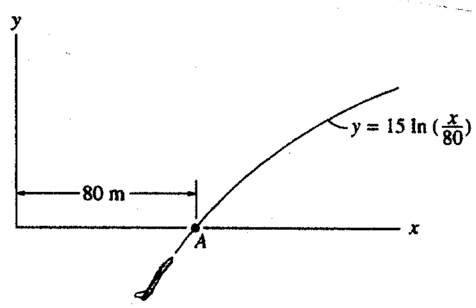
$v = 4.583 = 4.58$ m/s **Ans**

$$a_n = \frac{v^2}{\rho} = \frac{(4.583)^2}{50} = 0.420\text{ m/s}^2$$

$$a_t = 0.05(10) = 0.5\text{ m/s}^2$$

$a = \sqrt{(0.420)^2 + (0.5)^2} = 0.653$ m/s^2 **Ans**

*12-116. The jet plane is traveling with a constant speed of 110 m/s along the curved path. Determine the magnitude of the acceleration of the plane at the instant it reaches point A ($y = 0$).



$$y = 15\ln\left(\frac{x}{80}\right)$$

$$\frac{dy}{dx} = \frac{15}{x}\bigg|_{x=80\text{ m}} = 0.1875$$

$$\frac{d^2y}{dx^2} = -\frac{15}{x^2}\bigg|_{x=80\text{ m}} = -0.002344$$

$$\rho\bigg|_{x=80\text{ m}} = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}\Bigg|_{x=80\text{ m}}$$

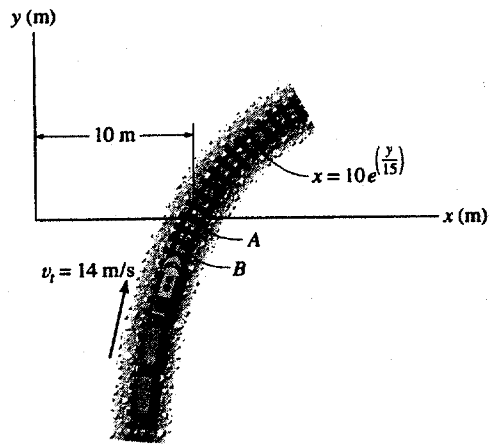
$$= \frac{\left[1+(0.1875)^2\right]^{3/2}}{|-0.002344|} = 449.4\text{ m}$$

$$a_n = \frac{v^2}{\rho} = \frac{(110)^2}{449.4} = 26.9\text{ m/s}^2$$

Since the plane travels with a constant speed, $a_t = 0$. Hence

$a = a_n = 26.9\text{ m/s}^2$ **Ans**

12-117. A train is traveling with a constant speed of 14 m/s along the curved path. Determine the magnitude of the acceleration of the front of the train, B, at the instant it reaches point A ($y = 0$).



$$x = 10e^{\left(\frac{y}{15}\right)}$$

$$y = 15\ln\left(\frac{x}{10}\right)$$

$$\frac{dy}{dx} = 15\left(\frac{10}{x}\right)\left(\frac{1}{10}\right) = \frac{15}{x}$$

$$\frac{d^2y}{dx^2} = -\frac{15}{x^2}$$

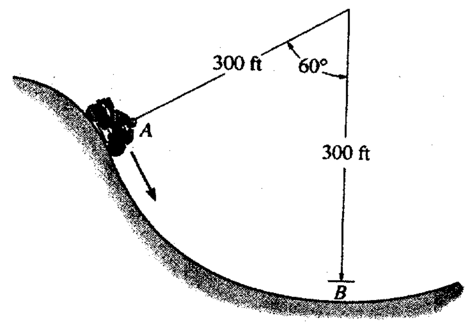
At $x = 10$,

$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1+(1.5)^2\right]^{\frac{3}{2}}}{|-0.15|} = 39.06\text{ m}$$

$$a_t = \frac{dv}{dt} = 0$$

$a_n = a = \frac{v^2}{\rho} = \frac{(14)^2}{39.06} = 5.02\text{ m/s}^2$ **Ans**

12-118. When the motorcyclist is at A, he increases his speed along the vertical circular path at the rate of $\dot{v} = (0.3t)$ ft/s^2, where t is in seconds. If he starts from rest at A, determine the magnitudes of his velocity and acceleration when he reaches B.



$\int_0^v dv = \int_0^t 0.3t\,dt$

$v = 0.15t^2$

$\int_0^s ds = \int_0^t 0.15t^2\,dt$

$s = 0.05t^3$

When $s = \frac{\pi}{3}(300)$ ft, $\quad \frac{\pi}{3}(300) = 0.05t^3 \quad t = 18.453$ s

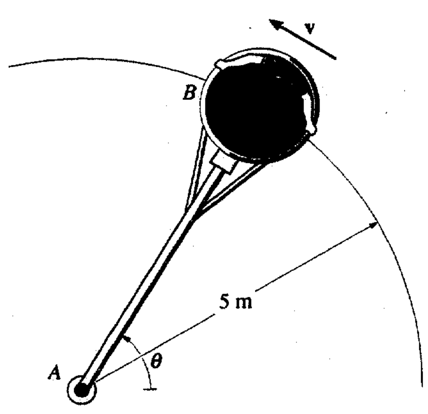
$v = 0.15(18.453)^2 = 51.08$ ft/s $= 51.1$ ft/s **Ans**

$a_t = \dot{v} = 0.3t|_{t=18.453\,s} = 5.536$ ft/s^2

$$a_n = \frac{v^2}{\rho} = \frac{51.08^2}{300} = 8.696 \text{ ft/s}^2$$

$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(5.536)^2 + (8.696)^2} = 10.3$ ft/s^2 **Ans**

■12-119. The car B turns such that its speed is increased by $\dot{v}_B = (0.5e^t)$ m/s^2, where t is in seconds. If the car starts from rest when $\theta = 0°$, determine the magnitudes of its velocity and acceleration when the arm AB rotates $\theta = 30°$. Neglect the size of the car.



$$\frac{dv_B}{dt} = 0.5e^t$$

$$\int_0^{v_B} dv_B = \int_0^t 0.5e^t\,dt$$

$v_B = 0.5(e^t - 1)$

$$\int_0^{s_B} ds_B = \int_0^t 0.5(e^t - 1)\,dt$$

$s_B = 0.5(e^t - t)|_0^t = 0.5(e^t - t - 1)$

At $\theta = 30°$,

$$s_B = \left(\frac{30°}{180°}\pi\right)(5) = 2.618 \text{ m}$$

Thus,

$6.236 = (e^t - t)$

Solving by trial and error,

$t = 2.123$ s

Thus,

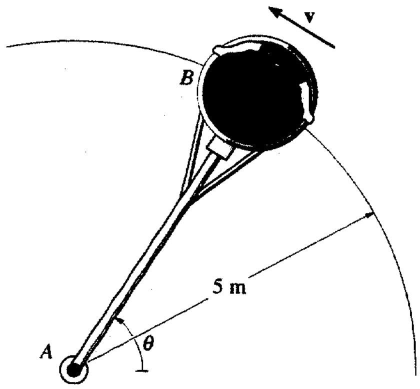
$v_B = 0.5(e^{2.123} - 1) = 3.678 = 3.68$ m/s **Ans**

$(a_B)_t = \dot{v}_B = 0.5(e^{2.123}) = 4.178$ m/s^2

$$(a_B)_n = \frac{v_B^2}{5} = \frac{(3.678)^2}{5} = 2.706 \text{ m/s}^2$$

$a_B = \sqrt{(4.178)^2 + (2.706)^2} = 4.98$ m/s^2 **Ans**

*12-120. The car B turns such that its speed is increased by $\dot{v}_B = (0.5e^t)\ \text{m/s}^2$, where t is in seconds. If the car starts from rest when $\theta = 0°$, determine the magnitudes of its velocity and acceleration when $t = 2$ s. Neglect the size of the car. Also, through what angle θ has it traveled?



$$\int_0^v dv = \int_0^t 0.5\,e^t\,dt$$

$$v = 0.5\,e^t\big|_0^t = 0.5\,(e^t - 1)$$

$t = 2$ s,

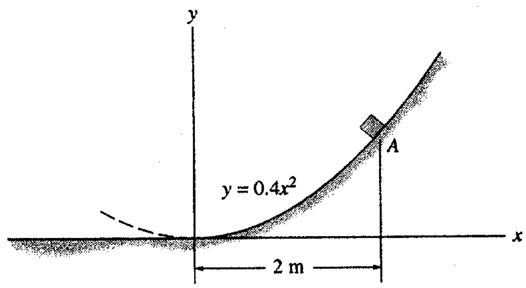
$$v = 0.5(e^2 - 1) = 3.1945 = 3.19\ \text{m/s} \qquad \textbf{Ans}$$

$$a_t = 0.5\,e^2 = 3.6945\ \text{m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{(3.1945)^2}{5} = 2.041\ \text{m/s}^2$$

$$a = \sqrt{(3.6945)^2 + (2.041)^2} = 4.22\ \text{m/s}^2 \qquad \textbf{Ans}$$

12-121. The box of negligible size is sliding down along a curved path defined by the parabola $y = 0.4x^2$. When it is at A ($x_A = 2$ m, $y_A = 1.6$ m), the speed is $v_B = 8$ m/s and the increase in speed is $dv_B/dt = 4\ \text{m/s}^2$. Determine the magnitude of the acceleration of the box at this instant.



$$y = 0.4\,x^2$$

$$\left.\frac{dy}{dx}\right|_{x=2\text{ m}} = 0.8x\Big|_{x=2\text{ m}} = 1.6$$

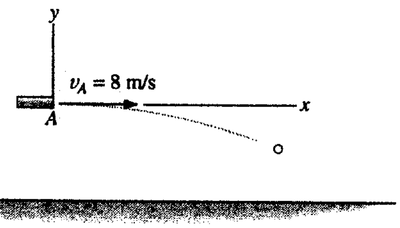
$$\left.\frac{d^2y}{dx^2}\right|_{x=2\text{ m}} = 0.8$$

$$\rho = \left.\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}\right|_{x=2\text{ m}} = \frac{\left[1+(1.6)^2\right]^{3/2}}{|0.8|} = 8.396\ \text{m}$$

$$a_n = \frac{v_B^{\,2}}{\rho} = \frac{8^2}{8.396} = 7.622\ \text{m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(4)^2 + (7.622)^2} = 8.61\ \text{m/s}^2 \qquad \textbf{Ans}$$

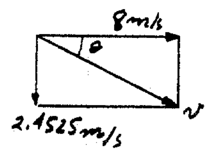
12-122. The ball is ejected horizontally from the tube with a speed of 8 m/s. Find the equation of the path, $y = f(x)$, and then find the ball's velocity and the normal and tangential components of acceleration when $t = 0.25$ s.



$v_x = 8$ m/s

$(\overset{+}{\rightarrow})$ $\quad s = v_0 t$

$x = 8t$

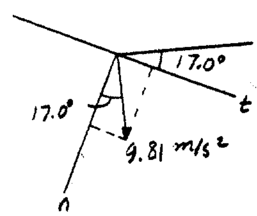


$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$y = 0 + 0 + \frac{1}{2}(-9.81)t^2$

$y = -4.905t^2$

$y = -4.905\left(\frac{x}{8}\right)^2$



$y = -0.0766x^2$ (Parabola) **Ans**

$v = v_0 + a_c t$

$v_y = 0 - 9.81t$

When $t = 0.25$ s,

$v_y = -2.4525$ m/s

$v = \sqrt{(8)^2 + (2.4525)^2} = 8.37$ m/s **Ans**

$\theta = \tan^{-1}\left(\frac{2.4525}{8}\right) = 17.04°$

$a_x = 0 \quad a_y = 9.81$ m/s^2

$a_n = 9.81\cos 17.04° = 9.38$ m/s^2 **Ans**

$a_t = 9.81\sin 17.04° = 2.88$ m/s^2 **Ans**

12-123. The motion of a particle is defined by the equations $x = (2t + t^2)$ m and $y = (t^2)$ m, where t is in seconds. Determine the normal and tangential components of the particle's velocity and acceleration when $t = 2$ s.

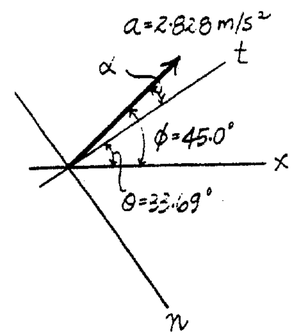
Velocity : Here, $\mathbf{r} = \{(2t+t^2)\mathbf{i} + t^2\mathbf{j}\}$ m. To determine the velocity **v**, apply Eq. 12-7.

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \{(2+2t)\mathbf{i} + 2t\mathbf{j}\} \text{ m/s}$$

When $t = 2$ s, $\mathbf{v} = [2+2(2)]\mathbf{i} + 2(2)\mathbf{j} = \{6\mathbf{i} + 4\mathbf{j}\}$ m/s. Then $v = \sqrt{6^2+4^2}$ $= 7.21$ m/s. Since the velocity is always directed tangent to the path,

$$v_n = 0 \quad \text{and} \quad v_t = 7.21 \text{ m/s} \qquad \textbf{Ans}$$

The velocity **v** makes an angle $\theta = \tan^{-1}\frac{4}{6} = 33.69°$ with the x axis.



Acceleration : To determine the acceleration **a**, apply Eq. 12-9.

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \{2\mathbf{i} + 2\mathbf{j}\} \text{ m/s}^2$$

Then

$$a = \sqrt{2^2+2^2} = 2.828 \text{ m/s}^2$$

The acceleration **a** makes an angle $\phi = \tan^{-1}\frac{2}{2} = 45.0°$ with the x axis. From the figure, $\alpha = 45° - 33.69 = 11.31°$. Therefore,

$$a_n = a\sin\alpha = 2.828\sin 11.31° = 0.555 \text{ m/s}^2 \qquad \textbf{Ans}$$
$$a_t = a\cos\alpha = 2.828\cos 11.31° = 2.77 \text{ m/s}^2 \qquad \textbf{Ans}$$

***12-124.** The motorcycle travels along the elliptical track at a constant speed v. Determine the greatest magnitude of the acceleration if $a > b$.

Acceleration : Defferentiating twice the expression $y = \frac{b}{a}\sqrt{a^2 - x^2}$, we have

$$\frac{dy}{dx} = -\frac{bx}{a\sqrt{a^2 - x^2}}$$

$$\frac{d^2y}{dx^2} = -\frac{ab}{(a^2 - x^2)^{3/2}}$$

The radius of curvature of the path is

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + \left(-\frac{bx}{a\sqrt{a^2 - x^2}}\right)^2\right]^{3/2}}{\left|-\frac{ab}{(a^2 - x^2)^{3/2}}\right|} = \frac{\left[1 + \frac{b^2x^2}{a^2(a^2 - x^2)}\right]^{3/2}}{\frac{ab}{(a^2 - x^2)^{3/2}}} \qquad [1]$$

To have the maximum normal acceleration, the radius of curvature of the path must be a minimum. By observation, this happens when $y = 0$ and $x = a$. When $x \to a$, $\frac{b^2x^2}{a^2(a^2 - x^2)} >>> 1$. Then, $\left[1 + \frac{b^2x^2}{a^2(a^2 - x^2)}\right]^{3/2} \to \left[\frac{b^2x^2}{a^2(a^2 - x^2)}\right]^{3/2}$ $= \frac{b^3x^3}{a^3(a^2 - x^2)^{3/2}}$. Substituting this value into Eq. [1] yields $\rho = \frac{b^2}{a^4}x^3$. At $x = a$,

$$\rho = \frac{b^2}{a^4}(a^3) = \frac{b^2}{a}$$

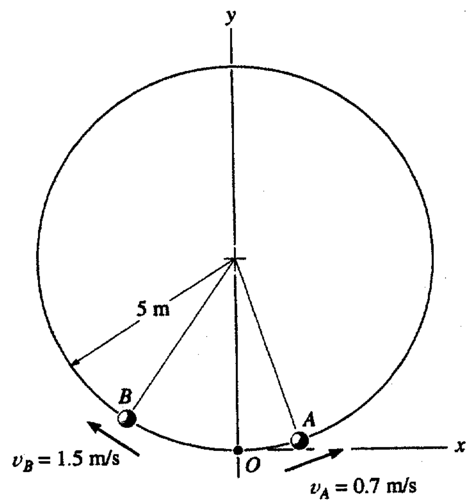
To determine the normal acceleration, apply Eq. 12 – 20.

$$(a_n)_{\max} = \frac{v^2}{\rho} = \frac{v^2}{b^2/a} = \frac{a}{b^2}v^2$$

Since the motorcycle is traveling at a constant speed, $a_t = 0$. Thus,

$$a_{\max} = (a_n)_{\max} = \frac{a}{b^2}v^2 \qquad \textbf{Ans}$$

12-125. The two particles A and B start at the origin O and travel in opposite directions along the circular path at constant speeds $v_A = 0.7$ m/s and $v_B = 1.5$ m/s, respectively. Determine in $t = 2$ s, (a) the displacement along the path of each particle, (b) the position vector to each particle, and (c) the shortest distance between the particles.

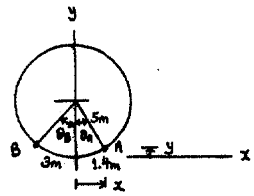


(a) $s_A = 0.7(2) = 1.40$ m **Ans**

$s_B = 1.5(2) = 3$ m **Ans**

(b) $\theta_A = \dfrac{1.40}{5} = 0.280$ rad. $= 16.04°$

$\theta_B = \dfrac{3}{5} = 0.600$ rad. $= 34.38°$



For A

$x = 5\sin 16.04° = 1.382 = 1.38$ m

$y = 5(1 - \cos 16.04°) = 0.1947 = 0.195$ m

$\mathbf{r}_A = \{1.38\,\mathbf{i} + 0.195\,\mathbf{j}\}$ m **Ans**

For B

$x = -5\sin 34.38° = -2.823 = -2.82$ m

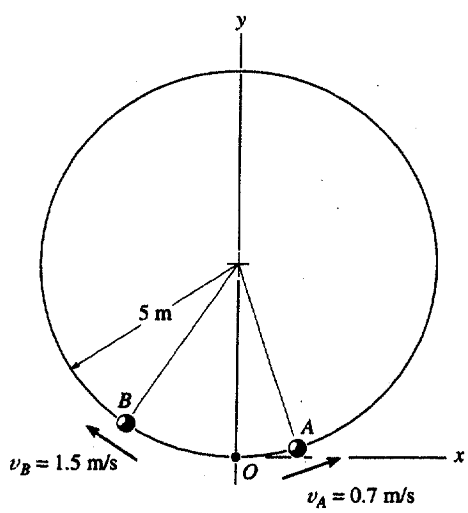
$y = 5(1 - \cos 34.38°) = 0.8734 = 0.873$ m

$\mathbf{r}_B = \{-2.82\,\mathbf{i} + 0.873\,\mathbf{j}\}$ m **Ans**

(c) $\Delta\mathbf{r} = \mathbf{r}_B - \mathbf{r}_A = \{-4.20\,\mathbf{i} + 0.678\,\mathbf{j}\}$ m

$\Delta r = \sqrt{(-4.20)^2 + (0.678)^2} = 4.26$ m **Ans**

12-126. The two particles A and B start at the origin O and travel in opposite directions along the circular path at constant speeds $v_A = 0.7$ m/s and $v_B = 1.5$ m/s, respectively. Determine the time when they collide and the magnitude of the acceleration of B just before this happens.



$s_t = 2\pi(5) = 31.4159$ m

$s_A = 0.7\,t$

$s_B = 1.5\,t$

Require

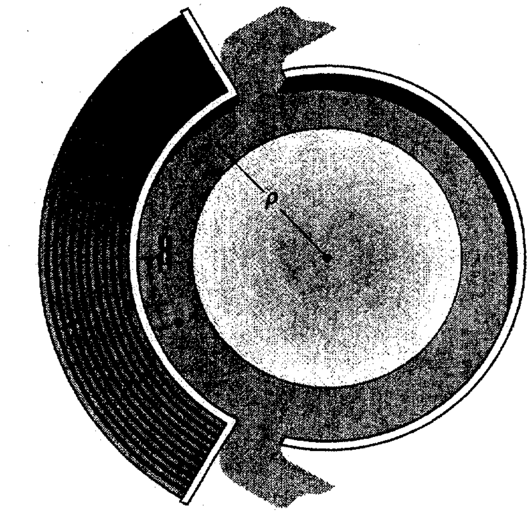
$s_A + s_B = 31.4159$

$0.7\,t + 1.5\,t = 31.4159$

$t = 14.28$ s $= 14.3$ s **Ans**

$a_n = \dfrac{v_B^2}{\rho} = \dfrac{(1.5)^2}{5} = 0.45$ m/s^2 **Ans**

12-127. The race car has an initial speed $v_A = 15$ m/s at A. If it increases its speed along the circular track at the rate $a_t = (0.4s)$ m/s^2, where s is in meters, determine the time needed for the car to travel 20 m. Take $\rho = 150$ m.



$$a_t = 0.4s = \frac{v\,dv}{ds}$$

$$a\,ds = v\,dv$$

$$\int_0^s 0.4s\,ds = \int_{15}^v v\,dv$$

$$\frac{0.4s^2}{2}\Big|_0^s = \frac{v^2}{2}\Big|_{15}^v$$

$$\frac{0.4s^2}{2} = \frac{v^2}{2} - \frac{225}{2}$$

$$v^2 = 0.4s^2 + 225$$

$$v = \frac{ds}{dt} = \sqrt{0.4s^2 + 225}$$

$$\int_0^s \frac{ds}{\sqrt{0.4s^2 + 225}} = \int_0^t dt$$

$$\int_0^s \frac{ds}{\sqrt{s^2 + 562.5}} = 0.632\,456t$$

$$\ln\left(s + \sqrt{s^2 + 562.5}\right)\Big|_0^s = 0.632\,456t$$

$$\ln\left(s + \sqrt{s^2 + 562.5}\right) - 3.166\,196 = 0.632\,456t$$

At $s = 20$ m,

$t = 1.21$ s **Ans**

***12-128.** A boy sits on a merry-go-round so that he is always located at $r = 8$ ft from the center of rotation. The merry-go-round is originally at rest, and then due to rotation the boy's speed is increased at 2 ft/s^2. Determine the time needed for his acceleration to become 4 ft/s^2.

$$a = \sqrt{a_n^2 + a_t^2}$$

$$a_t = 2$$

$$v = v_0 + a_c t$$

$$v = 0 + 2t$$

$$a_n = \frac{v^2}{\rho} = \frac{(2t)^2}{8}$$

$$4 = \sqrt{(2)^2 + \left(\frac{(2t)^2}{8}\right)^2}$$

$$16 = 4 + \frac{16t^4}{64}$$

$t = 2.63$ s **Ans**

12-129. A particle travels along the path $y = a + bx + cx^2$, where a, b, c are constants. If the speed of the particle is constant, $v = v_0$, determine the x and y components of velocity and the normal component of acceleration when $x = 0$.

$$y = a + bx + cx^2$$

$$\dot{y} = b\dot{x} + 2cx\dot{x}$$

$$\ddot{y} = b\ddot{x} + 2c(\dot{x})^2 + 2cx\ddot{x}$$

When $x = 0$, $\dot{y} = b\dot{x}$

$$v_0^2 = \dot{x}^2 + b^2\dot{x}^2$$

$$v_x = \dot{x} = \frac{v_0}{\sqrt{1 + b^2}} \quad \textbf{Ans}$$

$$v_y = \frac{v_0 b}{\sqrt{1 + b^2}} \quad \textbf{Ans}$$

$$a_n = \frac{v_0^2}{\rho}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|}$$

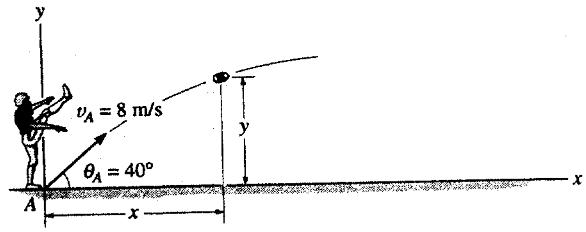
$$\frac{dy}{dx} = b + 2cx$$

$$\frac{d^2y}{dx^2} = 2c$$

At $x = 0$, $\rho = \frac{(1 + b^2)^{3/2}}{2c}$

$$a_n = \frac{2cv_0^2}{(1 + b^2)^{3/2}} \quad \textbf{Ans}$$

■12-130. The ball is kicked with an initial speed $v_A = 8$ m/s at an angle $\theta_A = 40°$ with the horizontal. Find the equation of the path, $y = f(x)$, and then determine the ball's velocity and the normal and tangential components of its acceleration when $t = 0.25$ s.



Horizontal Motion : The horizontal component of velocity is $(v_0)_x = 8\cos 40° = 6.128$ m/s and the initial horizontal and final positions are $(s_0)_x = 0$ and $s_x = x$, respectively.

$$(\overset{+}{\rightarrow}) \qquad s_x = (s_0)_x + (v_0)_x t$$

$$x = 0 + 6.128t \qquad [1]$$

Vertical Motion : The vertical component of initial velocity is $(v_0)_y = 8\sin 40° = 5.143$ m/s. The initial and final vertical positions are $(s_0)_y = 0$ and $s_y = y$, respectively.

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$y = 0 + 5.143t + \frac{1}{2}(-9.81)(t^2) \qquad [2]$$

Eliminate t from Eqs[1] and [2], we have

$$y = \{0.8391x - 0.1306x^2\}\text{ m} = \{0.839x - 0.131x^2\}\text{ m} \qquad \textbf{Ans}$$

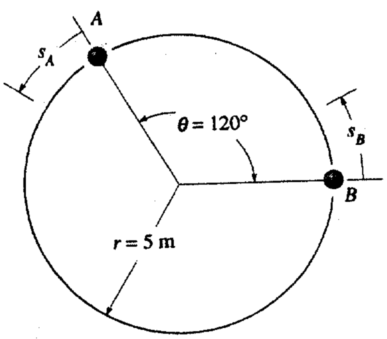
The tangent of the path makes an angle $\theta = \tan^{-1}\frac{3.644}{4} = 42.33°$ with the x axis.

Acceleration : When $t = 0.25$ s, from Eq.[1], $x = 0 + 6.128(0.25) = 1.532$ m. Here, $\frac{dy}{dx} = 0.8391 - 0.2612x$. At $x = 1.532$ m, $\frac{dy}{dx} = 0.8391 - 0.2612(1.532) = 0.4389$ and the tangent of the path makes an angle $\theta = \tan^{-1}0.4389 = 23.70°$ with the x axis. The magnitude of the acceleration is $a = 9.81$ m/s² and is directed downward. From the figure, $\alpha = 23.70°$. Therefore,

$$a_t = a\sin\alpha = 9.81\sin 23.70° = 3.94\text{ m/s}^2 \qquad \textbf{Ans}$$

$$a_n = a\cos\alpha = 9.81\cos 23.70° = 8.98\text{ m/s}^2 \qquad \textbf{Ans}$$

***12-131.** Particles A and B are traveling counterclockwise around a circular track at a constant speed of 8 m/s. If at the instant shown the speed of A is increased by $\dot{v}_A = (4s_A)$ m/s^2, where s_A is in meters, determine the distance measured counterclockwise along the track from B to A when $t = 1$ s. What is the magnitude of the acceleration of each particle at this instant?



Distance Traveled : Initially the distance between two particles is $d_0 = \rho\theta = 5\left(\frac{120°}{180°}\pi\right) = 10.47$ m. When $t = 1$ s, particle B travels a distance of $s_B = 8(1) = 8$ m. The distance traveled by particle A can be obtained as follows

$$v_A\,dv_A = a_A\,ds_A$$
$$\int_{8\,\text{m/s}}^{v_A} v_A\,dv_A = \int_0^{s_A} 4s_A\,ds_A$$
$$v_A = 2\sqrt{s_A^2 + 16} \qquad [1]$$

$$dt = \frac{ds_A}{v_A}$$
$$\int_0^{1\,\text{s}} dt = \int_0^{s_A} \frac{ds_A}{2\sqrt{s_A^2 + 16}}$$
$$1 = \frac{1}{2}\sinh^{-1}\left(\frac{s_A}{4}\right)$$
$$s_A = 14.51 \text{ m}$$

Thus, the distance between two cyclists after $t = 1$ s is

$$d = d_0 + s_A - s_B = 10.47 + 14.51 - 8 = 17.0 \text{ m} \qquad \textbf{Ans}$$

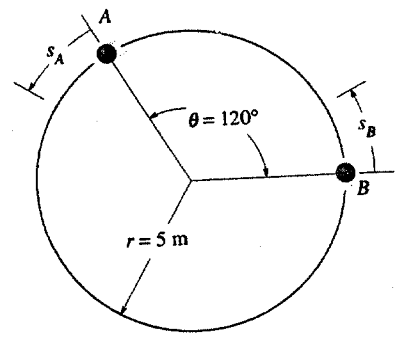
Acceleration : The tangential acceleration for cyclist A and B when $t = 1$ s is $(a_t)_A = 4s_A = 4(14.51) = 58.03$ m/s^2 and $(a_t)_B = 0$ (particle B travels at constant speed), respectively. When $t = 1$ s, from Eq. [1], $v_A = 2\sqrt{14.51^2 + 16} = 30.10$ m/s. To determine the normal acceleration, apply Eq. 12 – 20.

$$(a_n)_A = \frac{v_A^2}{\rho} = \frac{30.10^2}{5} = 181.17 \text{ m/s}^2$$
$$(a_n)_B = \frac{v_B^2}{\rho} = \frac{8^2}{5} = 12.80 \text{ m/s}^2$$

The magnitudes of the acceleration for particle A and B are

$$a_A = \sqrt{(a_t)_A^2 + (a_n)_A^2} = \sqrt{58.03^2 + 181.17^2} = 190 \text{ m/s}^2 \qquad \textbf{Ans}$$
$$a_B = \sqrt{(a_t)_B^2 + (a_n)_B^2} = \sqrt{0^2 + 12.80^2} = 12.8 \text{ m/s}^2 \qquad \textbf{Ans}$$

***12-132.** Particles A and B are traveling around a circular track at a speed of 8 m/s at the instant shown. If the speed of B is increased by $\dot{v}_B = 4\ \text{m/s}^2$, and at the same instant A has an increase in speed $\dot{v}_A = 0.8t\ \text{m/s}^2$, determine how long it takes for a collision to occur. What is the magnitude of the acceleration of each particle just before the collision occurs?



Distance Traveled : Initially the distance between the two particles is $d_0 = \rho\theta$ $= 5\left(\frac{120°}{180°}\pi\right) = 10.47$ m. Since particle B travels with a constant acceleration, distance can be obtained by applying equation

$$s_B = (s_0)_B + (v_0)_B\, t + \frac{1}{2}a_c t^2$$

$$s_B = 0 + 8t + \frac{1}{2}(4)\,t^2 = \left(8t + 2t^2\right)\ \text{m} \qquad [1]$$

The distance traveled by particle A can be obtained as follows.

$$dv_A = a_A\, dt$$

$$\int_{8\ \text{m/s}}^{v_A} dv_A = \int_0^t 0.8t\,dt$$

$$v_A = \left(0.4t^2 + 8\right)\ \text{m/s} \qquad [2]$$

$$ds_A = v_A\, dt$$

$$\int_0^{s_A} ds_A = \int_0^t \left(0.4t^2 + 8\right) dt$$

$$s_A = 0.1333t^3 + 8t$$

In order for the collision to occur

$$s_A + d_0 = s_B$$

$$0.1333t^3 + 8t + 10.47 = 8t + 2t^2$$

Solving by trial and error $\quad t = 2.5074\ \text{s} = 2.51\ \text{s}$ **Ans**

Note : If particle A strikes B then, $s_A = 5\left(\frac{240°}{180°}\pi\right) + s_B$. This equation will result in $t = 14.6\ \text{s} > 2.51\ \text{s}$.

Acceleration : The tangential acceleration for particle A and B When $t = 2.5074$ are $(a_t)_A = 0.8t = 0.8(2.5074) = 2.006\ \text{m/s}^2$ and $(a_t)_B = 4\ \text{m/s}^2$, respectively. When $t = 2.5074$ s, from Eq. [1], $v_A = 0.4\left(2.5074^2\right) + 8 = 10.51$ m/s and $v_B = (v_0)_B + a_c t = 8 + 4(2.5074) = 18.03$ m/s. To determine the normal acceleration, apply Eq. 12 – 20.

$$(a_n)_A = \frac{v_A^2}{\rho} = \frac{10.51^2}{5} = 22.11\ \text{m/s}^2$$

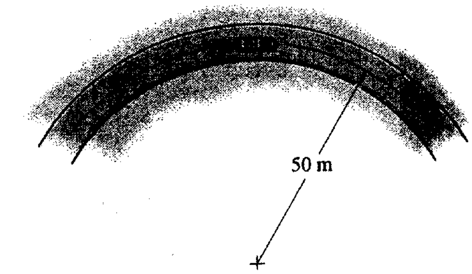
$$(a_n)_B = \frac{v_B^2}{\rho} = \frac{18.03^2}{5} = 65.01\ \text{m/s}^2$$

The magnitude of the acceleration for particles A and B just before collision are

$$a_A = \sqrt{(a_t)_A^2 + (a_n)_A^2} = \sqrt{2.006^2 + 22.11^2} = 22.2\ \text{m/s}^2 \qquad \textbf{Ans}$$

$$a_B = \sqrt{(a_t)_B^2 + (a_n)_B^2} = \sqrt{4^2 + 65.01^2} = 65.1\ \text{m/s}^2 \qquad \textbf{Ans}$$

12-133. The truck travels at a speed of 4 m/s along a circular road that has a radius of 50 m. For a short distance from $s = 0$, its speed is then increased by $\dot{v} = (0.05s)$ m/s², where s is in meters. Determine its speed and the magnitude of its acceleration when it has moved $s = 10$ m.



Velocity **:** The speed v in terms of position s can be obtained by applying $vdv = ads$.

$$vdv = ads$$

$$\int_{4\,\mathrm{m/s}}^{v} vdv = \int_0^s 0.05s\,ds$$

$$v = \left(\sqrt{0.05s^2 + 16}\right)\ \mathrm{m/s}$$

At $s = 10$ m, $\quad v = \sqrt{0.05(10^2) + 16} = 4.583\ \mathrm{m/s} = 4.58\ \mathrm{m/s}$ **Ans**

Acceleration **:** The tangential acceleration of the truck at $s = 10$ m is $a_t = 0.05(10) = 0.500\ \mathrm{m/s^2}$. To determine the normal acceleration, apply Eq. 12 – 20.

$$a_n = \frac{v^2}{\rho} = \frac{4.583^2}{50} = 0.420\ \mathrm{m/s^2}$$

The magnitude of the acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0.500^2 + 0.420^2} = 0.653\ \mathrm{m/s^2} \qquad \textbf{Ans}$$

■12-134. A go-cart moves along a circular track of radius 100 ft such that its speed for a short period of time, $0 \le t \le 4$ s, is $v = 60(1 - e^{-t^2})$ ft/s. Determine the magnitude of its acceleration when $t = 2$ s. How far has it traveled in $t = 2$ s? Use Simpson's rule with $n = 50$ to evaluate the integral.

$$v = 60(1 - e^{-t^2})$$

$$a_t = \frac{dv}{dt} = 60(-e^{-t^2})(-2t) = 120\,t\,e^{-t^2}$$

$$a_t\big|_{t=2} = 120(2)e^{-4} = 4.3958$$

$$v\big|_{t=2} = 60(1 - e^{-4}) = 58.9011$$

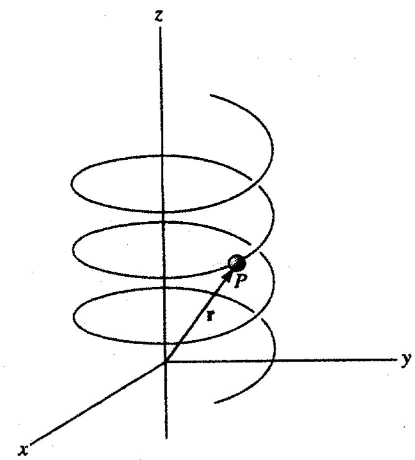
$$a_n = \frac{(58.9011)^2}{100} = 34.693$$

$$a = \sqrt{(4.3958)^2 + (34.693)^2} = 35.0\ \mathrm{m/s^2} \qquad \textbf{Ans}$$

$$\int_0^s ds = \int_0^2 60(1 - e^{-t^2})\,dt$$

$$s = 67.1\ \mathrm{ft} \qquad \textbf{Ans}$$

12-135. A particle P travels along an elliptical spiral path such that its position vector $\mathbf{r}$ is defined by $\mathbf{r} = \{2\cos(0.1t)\mathbf{i} + 1.5\sin(0.1t)\mathbf{j} + (2t)\mathbf{k}\}$ m, where t is in seconds and the arguments for the sine and cosine are given in radians. When $t = 8$ s, determine the coordinate direction angles α, β, and γ, which the binormal axis to the osculating plane makes with the x, y, and z axes. *Hint:* Solve for the velocity $\mathbf{v}_P$ and acceleration $\mathbf{a}_P$ of the particle in terms of their $\mathbf{i}, \mathbf{j}, \mathbf{k}$ components. The binormal is parallel to $\mathbf{v}_P \times \mathbf{a}_P$. Why?



$\mathbf{r}_P = 2\cos(0.1t)\mathbf{i} + 1.5\sin(0.1t)\mathbf{j} + 2t\mathbf{k}$

$\mathbf{v}_P = \dot{\mathbf{r}} = -0.2\sin(0.1t)\mathbf{i} + 0.15\cos(0.1t)\mathbf{j} + 2\mathbf{k}$

$\mathbf{a}_P = \ddot{\mathbf{r}} = -0.02\cos(0.1t)\mathbf{i} - 0.015\sin(0.1t)\mathbf{j}$

When $t = 8$ s,

$\mathbf{v}_P = -0.2\sin(0.8\text{rad})\mathbf{i} + 0.15\cos(0.8\text{rad})\mathbf{j} + 2\mathbf{k} = -0.143\,47\mathbf{i} + 0.104\,51\mathbf{j} + 2\mathbf{k}$

$\mathbf{a}_P = -0.02\cos(0.8\text{rad})\mathbf{i} - 0.015\sin(0.8\text{rad})\mathbf{j} = -0.013\,934\mathbf{i} - 0.010\,76\mathbf{j}$

Since the binormal vector is perpendicular to the plane containing the $n-t$ axis, and $\mathbf{a}_P$ and $\mathbf{v}_P$ are in this plane, then by the definition of the cross product,

$$\mathbf{b} = \mathbf{v}_P \times \mathbf{a}_P = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.143\,47 & 0.104\,51 & 2 \\ -0.013\,934 & -0.010\,76 & 0 \end{vmatrix} = 0.021\,52\mathbf{i} - 0.027\,868\mathbf{j} + 0.003\mathbf{k}$$

$b = \sqrt{(0.02152)^2 + (-0.027868)^2 + (0.003)^2} = 0.035\,338$

$\mathbf{u}_b = 0.608\,99\mathbf{i} - 0.788\,62\mathbf{j} + 0.085\mathbf{k}$

$\alpha = \cos^{-1}(0.608\,99) = 52.5°$ **Ans**

$\beta = \cos^{-1}(-0.788\,62) = 142°$ **Ans**

$\gamma = \cos^{-1}(0.085) = 85.1°$ **Ans**

Note: The direction of the binormal axis may also be specified by the unit vector $\mathbf{u}_{b'} = -\mathbf{u}_b$, which is obtained from $\mathbf{b}' = \mathbf{a}_P \times \mathbf{v}_P$.
For this case, $\alpha = 128°$, $\beta = 37.9°$, $\gamma = 94.9°$ **Ans**

***12-136.** The time rate of change of acceleration is referred to as the *jerk*, which is often used as a means of measuring passenger discomfort. Calculate this vector, $\dot{\mathbf{a}}$, in terms of its cylindrical components, using Eq. 12–32.

$$\mathbf{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\mathbf{u}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\mathbf{u}_\theta + \ddot{z}\mathbf{u}_z$$

$$\dot{\mathbf{a}} = \left(\dddot{r} - \dot{r}\dot{\theta}^2 - 2r\dot{\theta}\ddot{\theta}\right)\mathbf{u}_r + \left(\ddot{r} - r\dot{\theta}^2\right)\dot{\mathbf{u}}_r + \left(\dot{r}\ddot{\theta} + r\dddot{\theta} + 2\ddot{r}\dot{\theta} + 2\dot{r}\ddot{\theta}\right)\mathbf{u}_\theta + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\dot{\mathbf{u}}_\theta + \dddot{z}\mathbf{u}_z + \ddot{z}\dot{\mathbf{u}}_z$$

But, $\dot{\mathbf{u}}_r = \dot{\theta}\mathbf{u}_\theta \quad \dot{\mathbf{u}}_\theta = -\dot{\theta}\mathbf{u}_r \quad \dot{\mathbf{u}}_z = 0$

Substituting and combining terms yields

$$\dot{\mathbf{a}} = \left(\dddot{r} - 3\dot{r}\dot{\theta}^2 - 3r\dot{\theta}\ddot{\theta}\right)\mathbf{u}_r + \left(3\dot{r}\ddot{\theta} + r\dddot{\theta} + 3\ddot{r}\dot{\theta} - r\dot{\theta}^3\right)\mathbf{u}_\theta + \left(\dddot{z}\right)\mathbf{u}_z \qquad \textbf{Ans}$$

12-137. If a particle's position is described by the polar coordinates $r = 4(1 + \sin t)$ m and $\theta = (2e^{-t})$ rad, where t is in seconds and the argument for the sine is in radians, determine the radial and tangential components of the particle's velocity and acceleration when $t = 2$ s.

When $t = 2$ s,

$r = 4(1 + \sin t) = 7.637$

$\dot{r} = 4\cos t = -1.66459$

$\ddot{r} = -4\sin t = -3.6372$

$\theta = 2e^{-t}$

$\dot{\theta} = -2e^{-t} = -0.27067$

$\ddot{\theta} = 2e^{-t} = 0.270665$

$v_r = \dot{r} = -1.66$ m/s **Ans**

$v_\theta = r\dot{\theta} = 7.637(-0.27067) = -2.07$ m/s **Ans**

$a_r = \ddot{r} - r(\dot{\theta})^2 = -3.6372 - 7.637(-0.27067)^2 = -4.20$ m/s^2 **Ans**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 7.637(0.270665) + 2(-1.66459)(-0.27067) = 2.97$ m/s^2 **Ans**

12-138. A particle is moving along a circular path having a radius of 4 in. such that its position as a function of time is given by $\theta = \cos 2t$, where θ is in radians and t is in seconds. Determine the magnitude of the acceleration of the particle when $\theta = 30°$.

When $\theta = \frac{\pi}{6}$ rad, $\quad \frac{\pi}{6} = \cos 2t \quad t = 0.5099$ s

$$\dot{\theta} = \frac{d\theta}{dt} = -2\sin 2t\Big|_{t=0.5099\text{ s}} = -1.7039 \text{ rad/s}$$

$$\ddot{\theta} = \frac{d^2\theta}{dt^2} = -4\cos 2t\Big|_{t=0.5099\text{ s}} = -2.0944 \text{ rad/s}^2$$

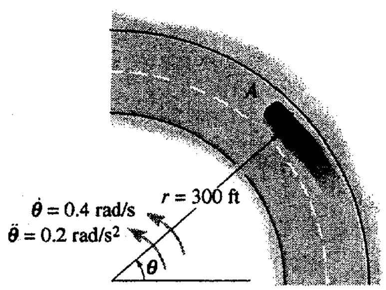
$r = 4 \qquad \dot{r} = 0 \qquad \ddot{r} = 0$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 4(-1.7039)^2 = -11.6135 \text{ in./s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 4(-2.0944) + 0 = -8.3776 \text{ in./s}^2$$

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-11.6135)^2 + (-8.3776)^2} = 14.3 \text{ in./s}^2 \qquad \textbf{Ans}$$

12-139. A car is traveling along the circular curve of radius $r = 300$ ft. At the instant shown, its angular rate of rotation is $\dot{\theta} = 0.4$ rad/s, which is increasing at the rate of $\ddot{\theta} = 0.2$ rad/s^2. Determine the magnitudes of the car's velocity and acceleration at this instant.



Velocity : Applying Eq. 12–25, we have

$$v_r = \dot{r} = 0 \qquad v_\theta = r\dot{\theta} = 300(0.4) = 120 \text{ ft/s}$$

Thus, the magnitude of the velocity of the car is

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{0^2 + 120^2} = 120 \text{ ft/s} \qquad \textbf{Ans}$$

Acceleration : Applying Eq. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 300(0.4^2) = -48.0 \text{ ft/s}^2$$
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 300(0.2) + 2(0)(0.4) = 60.0 \text{ ft/s}^2$$

Thus, the magnitude of the acceleration of the car is

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-48.0)^2 + 60.0^2} = 76.8 \text{ ft/s}^2 \qquad \textbf{Ans}$$

***12-140.** If a particle moves along a path such that $r = (2\cos t)$ ft and $\theta = (t/2)$ rad, where t is in seconds, plot the path $r = f(\theta)$ and determine the particle's radial and transverse components of velocity and acceleration.

$$r = 2\cos t \qquad \dot{r} = -2\sin t \qquad \ddot{r} = -2\cos t$$

$$\theta = \frac{t}{2} \qquad \dot{\theta} = \frac{1}{2} \qquad \ddot{\theta} = 0$$

$$v_r = \dot{r} = -2\sin t \qquad \textbf{Ans}$$

$$v_\theta = r\dot{\theta} = 2\cos t\left(\frac{1}{2}\right) = \cos t \qquad \textbf{Ans}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -2\cos t - 2\cos t\left(\frac{1}{2}\right)^2 = -\frac{5}{2}\cos t \qquad \textbf{Ans}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2\cos t(0) + 2(-2\sin t)\left(\frac{1}{2}\right) = -2\sin t \qquad \textbf{Ans}$$

12-141. If a particle's position is described by the polar coordinates $r = (2\sin 2\theta)$ m and $\theta = (4t)$ rad, where t is in seconds, determine the radial and tangential components of its velocity and acceleration when $t = 1$ s.

When $t = 1$ s,

$$\theta = 4t = 4$$

$$\dot{\theta} = 4$$

$$\ddot{\theta} = 0$$

$$r = 2\sin 2\theta = 1.9787$$

$$\dot{r} = 4\cos 2\theta\,\dot{\theta} = -2.3280$$

$$\ddot{r} = -8\sin 2\theta(\dot{\theta})^2 + 8\cos 2\theta\,\ddot{\theta} = -126.638$$

$$v_r = \dot{r} = -2.33 \text{ m/s} \qquad \textbf{Ans}$$

$$v_\theta = r\dot{\theta} = 1.9787(4) = 7.91 \text{ m/s} \qquad \textbf{Ans}$$

$$a_r = \ddot{r} - r(\dot{\theta})^2 = -126.638 - (1.9787)(4)^2 = -158 \text{ m/s}^2 \qquad \textbf{Ans}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 1.9787(0) + 2(-2.3280)(4) = -18.6 \text{ m/s}^2 \qquad \textbf{Ans}$$

12-142. A particle is moving along a circular path having a 400-mm radius. Its position as a function of time is given by $\theta = (2t^2)$ rad, where t is in seconds. Determine the magnitude of the particle's acceleration when $\theta = 30°$. The particle starts from rest when $\theta = 0°$.

$r = 400$ mm $\quad \dot{r} = 0 \quad \ddot{r} = 0$

$\theta = 2t^2 \quad \dot{\theta} = 4t \quad \ddot{\theta} = 4$

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 400(4t)^2 = -6400t^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 400(4) + 0 = 1600$

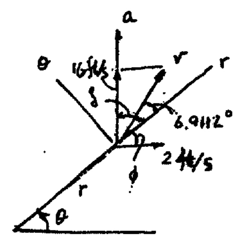
When $\theta = 30° = 30°\left(\frac{\pi}{180°}\right) = 0.5236$ rad, then,

$0.5236 = 2t^2, \quad t = 0.5117$ s

Hence,

$a = \sqrt{(-6400(0.5117)^2)^2 + (1600)^2} = 2316.76 \text{ mm/s}^2 = 2.32 \text{ m/s}^2$ **Ans**

12-143. A particle moves in the x–y plane such that its position is defined by $\mathbf{r} = \{2t\mathbf{i} + 4t^2\mathbf{j}\}$ ft, where t is in seconds. Determine the radial and tangential components of the particle's velocity and acceleration when $t = 2$ s.



$\mathbf{r} = 2t\mathbf{i} + 4t^2\mathbf{j}|_{t=2} = 4\mathbf{i} + 16\mathbf{j}$

$\mathbf{v} = 2\mathbf{i} + 8t\mathbf{j}|_{t=2} = 2\mathbf{i} + 16\mathbf{j}$

$\mathbf{a} = 8\mathbf{j}$

$\theta = \tan^{-1}(\frac{16}{4}) = 75.964°$

$v = \sqrt{(2)^2 + (16)^2} = 16.1245$ ft/s

$\phi = \tan^{-1}(\frac{16}{2}) = 82.875°$

$a = 8$ ft/s^2

$\phi - \theta = 6.9112°$

$v_r = 16.1245\cos 6.9112° = 16.0$ ft/s **Ans**

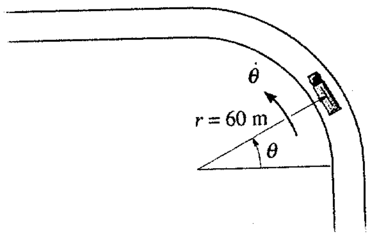
$v_\theta = 16.1245\sin 6.9112° = 1.94$ ft/s **Ans**

$\delta = 90° - \theta = 14.036°$

$a_r = 8\cos 14.036° = 7.76$ ft/s^2 **Ans**

$a_\theta = 8\sin 14.036° = 1.94$ ft/s^2 **Ans**

*12-144.** A truck is traveling along the horizontal circular curve of radius $r = 60$ m with a constant speed $v = 20$ m/s. Determine the angular rate of rotation $\dot{\theta}$ of the radial line r and the magnitude of the truck's acceleration.



$r = 60$

$\dot{r} = 0$

$\ddot{r} = 0$

$v = 20$

$v_r = \dot{r} = 0$

$v_\theta = r\dot{\theta} = 60\,\dot{\theta}$

$v = \sqrt{(v_r)^2 + (v_\theta)^2}$

$20 = 60\,\dot{\theta}$

$\dot{\theta} = 0.333$ rad/s **Ans**

$a_r = \ddot{r} - r(\dot{\theta})^2$

$= 0 - 60(0.333)^2$

$= -6.67$ m/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

$= 60\,\ddot{\theta}$

Since

$v = r\dot{\theta}$

$\dot{v} = \dot{r}\dot{\theta} + r\ddot{\theta}$

$0 = 0 + 60\,\ddot{\theta}$

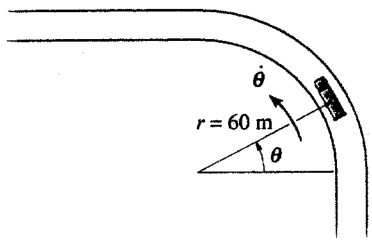
$\ddot{\theta} = 0$

Thus,

$a_\theta = 0$

$a = a_r = 6.67$ m/s^2 **Ans**

12-145. A truck is traveling along the horizontal circular curve of radius $r = 60$ m with a speed of 20 m/s which is increasing at 3 m/s^2. Determine the truck's radial and transverse components of acceleration.



$r = 60$

$a_t = 3\ \text{m/s}^2$

$a_n = \dfrac{v^2}{r} = \dfrac{(20)^2}{60} = 6.67\ \text{m/s}^2$

$a_r = -a_n = -6.67\ \text{m/s}^2$ **Ans**

$a_\theta = a_t = 3\ \text{m/s}^2$ **Ans**

12-146. A particle is moving along a circular path having a radius of 6 in. such that its position as a function of time is given by $\theta = \sin 3t$, where θ is in radians, the argument for the sine is in degrees, and t is in seconds. Determine the acceleration of the particle at $\theta = 30°$. The particle starts from rest at $\theta = 0°$.

$r = 6$ in., $\quad \dot{r} = 0, \quad \ddot{r} = 0$

$\theta = \sin 3t$

$\dot{\theta} = 3\cos 3t$

$\ddot{\theta} = -9\sin 3t$

At $\theta = 30°$,

$\dfrac{30°}{180°}\pi = \sin 3t$

$t = 10.525$ s

Thus,

$\dot{\theta} = 2.5559$ rad/s

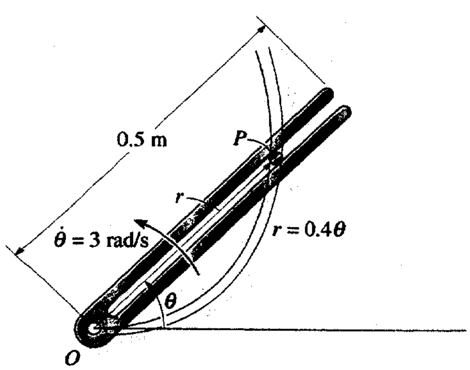
$\ddot{\theta} = -4.7124$ rad/s^2

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 6(2.5559)^2 = -39.196$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 6(-4.7124) + 0 = -28.274$

$a = \sqrt{(-39.196)^2 + (-28.274)^2} = 48.3\ \text{in./s}^2$ **Ans**

12-147. The slotted link is pinned at O, and as a result of the constant angular velocity $\dot{\theta} = 3$ rad/s it drives the peg P for a short distance along the spiral guide $r = (0.4\,\theta)$ m, where θ is in radians. Determine the radial and transverse components of the velocity and acceleration of P at the instant $\theta = \pi/3$ rad.



$\dot{\theta} = 3$ rad/s $\qquad r = 0.4\,\theta$

$\dot{r} = 0.4\,\dot{\theta}$

$\ddot{r} = 0.4\,\ddot{\theta}$

At $\theta = \dfrac{\pi}{3}$, $\qquad r = 0.4189$

$\dot{r} = 0.4(3) = 1.20$

$\ddot{r} = 0.4(0) = 0$

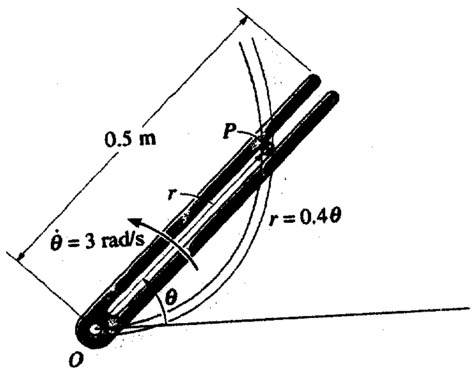
$v_r = \dot{r} = 1.20$ m/s **Ans**

$v_\theta = r\dot{\theta} = 0.4189(3) = 1.26$ m/s **Ans**

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 0.4189(3)^2 = -3.77\ \text{m/s}^2$ **Ans**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(1.20)(3) = 7.20\ \text{m/s}^2$ **Ans**

***12-148.** Solve Prob. 12-147 if the slotted link has an angular acceleration $\ddot{\theta} = 8\ \text{rad/s}^2$ when $\dot{\theta} = 3\ \text{rad/s}$ at $\theta = \pi/3\ \text{rad}$.



$\dot{\theta} = 3\ \text{rad/s} \qquad r = 0.4\,\theta$

$\dot{r} = 0.4\,\dot{\theta}$

$\ddot{r} = 0.4\,\ddot{\theta}$

$\theta = \dfrac{\pi}{3}$

$\dot{\theta} = 3$

$\ddot{\theta} = 8$

$r = 0.4189$

$\dot{r} = 1.20$

$\ddot{r} = 0.4(8) = 3.20$

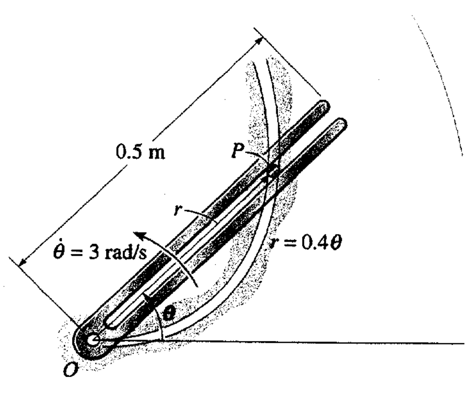
$v_r = \dot{r} = 1.20\ \text{m/s}$ **Ans**

$v_\theta = r\dot{\theta} = 0.4189(3) = 1.26\ \text{m/s}$ **Ans**

$a_r = \ddot{r} - r\dot{\theta}^2 = 3.20 - 0.4189(3)^2 = -0.570\ \text{m/s}^2$ **Ans**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.4189(8) + 2(1.20)(3) = 10.6\ \text{m/s}^2$ **Ans**

12-149. The slotted link is pinned at O, and as a result of the constant angular velocity $\dot{\theta} = 3\ \text{rad/s}$ it drives the peg P for a short distance along the spiral guide $r = (0.4\,\theta)$ m, where θ is in radians. Determine the velocity and acceleration of the particle at the instant it leaves the slot in the link, i.e., when $r = 0.5$ m.



$r = 0.4\,\theta$

$\dot{r} = 0.4\,\dot{\theta}$

$\ddot{r} = 0.4\,\ddot{\theta}$

$\dot{\theta} = 3$

$\ddot{\theta} = 0$

At $r = 0.5$ m,

$\theta = \dfrac{0.5}{0.4} = 1.25\ \text{rad}$

$\dot{r} = 1.20$

$\ddot{r} = 0$

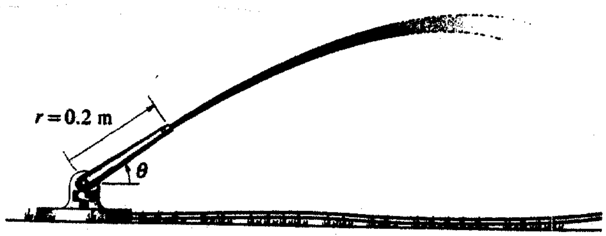
$v_r = \dot{r} = 1.20\ \text{m/s}$ **Ans**

$v_\theta = r\dot{\theta} = 0.5(3) = 1.50\ \text{m/s}$ **Ans**

$a_r = \ddot{r} - r(\dot{\theta})^2 = 0 - 0.5(3)^2 = -4.50\ \text{m/s}^2$ **Ans**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(1.20)(3) = 7.20\ \text{m/s}^2$ **Ans**

***12-152.** At the instant shown, the watersprinkler is rotating with an angular speed $\dot{\theta} = 2$ rad/s and an angular acceleration $\ddot{\theta} = 3$ rad/s^2. If the nozzle lies in the vertical plane and water is flowing through it at a constant rate of 3 m/s, determine the magnitudes of the velocity and acceleration of a water particle as it exits the open end, $r = 0.2$ m.



$r = 0.2$

$\dot{r} = 3 \qquad \dot{\theta} = 2$

$\ddot{r} = 0 \qquad \ddot{\theta} = 3$

$v_r = 3$

$v_\theta = 0.2(2) = 0.4$

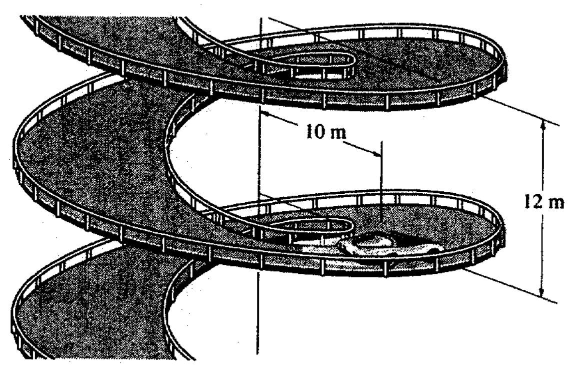
$v = \sqrt{(3)^2 + (0.4)^2} = 3.03$ m/s **Ans**

$a_r = 0 - (0.2)(2)^2 = -0.80$ m/s^2

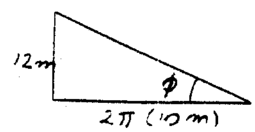
$a_\theta = 0.2(3) + 2(3)(2) = 12.6$ m/s^2

$a = \sqrt{(-0.80)^2 + (12.6)^2} = 12.6$ m/s^2 **Ans**

12-153. The automobile is traveling from a parking deck down along a cylindrical spiral ramp at a constant speed of $v = 1.5$ m/s. If the ramp descends a distance of 12 m for every full revolution, $\theta = 2\pi$ rad, determine the magnitude of the car's acceleration as it moves along the ramp, $r = 10$ m. *Hint:* For part of the solution, note that the tangent to the ramp at any point is at an angle of $\phi = \tan^{-1}(12/[2\pi(10)]) = 10.81°$ from the horizontal. Use this to determine the velocity components v_θ and v_z, which in turn are used to determine $\dot{\theta}$ and $\dot{z}$.



$\phi = \tan^{-1}\left(\frac{12}{2\pi(10)}\right) = 10.81°$



$v = 1.5$ m/s

$v_r = 0$

$v_\theta = 1.5\cos 10.81° = 1.473$ m/s

$v_z = -1.5\sin 10.81° = -0.2814$ m/s

Since

$r = 10 \qquad \dot{r} = 0 \qquad \ddot{r} = 0$

$v_\theta = r\dot{\theta} = 1.473 \qquad \dot{\theta} = \frac{1.473}{10} = 0.1473$

Since $\ddot{\theta} = 0$

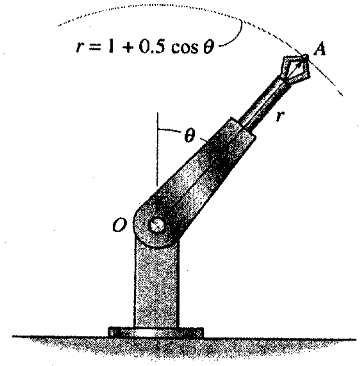
$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 10(0.1473)^2 = -0.217$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 10(0) + 2(0)(0.1473) = 0$

$a_z = \ddot{z} = 0$

$a = \sqrt{(-0.217)^2 + (0)^2 + (0)^2} = 0.217$ m/s^2 **Ans**

12-154. Because of telescopic action, the end of the industrial robotic arm extends along the path of the limaçon $r = (1 + 0.5\cos\theta)$ m. At the instant $\theta = \pi/4$, the arm has an angular rotation $\dot{\theta} = 0.6$ rad/s, which is increasing at $\ddot{\theta} = 0.25$ rad/s^2. Determine the radial and transverse components of the velocity and acceleration of the object A held in its grip at this instant.



$\theta = \frac{\pi}{4} = 45°$

$\dot{\theta} = 0.6$

$\ddot{\theta} = 0.25$

$r = 1 + 0.5\cos\theta$

$\dot{r} = -0.5\sin\theta\dot{\theta}$

$\ddot{r} = -0.5\left(\cos\theta\dot{\theta}^2 + \sin\theta\ddot{\theta}\right)$

At $\theta = 45°$

$r = 1.354$ $\quad \dot{r} = -0.2121$ $\quad \ddot{r} = -0.2157$

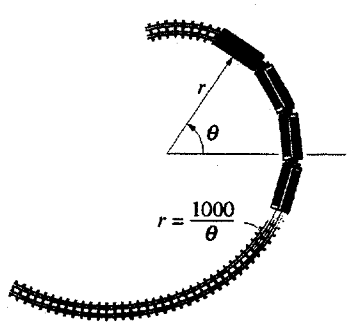
$v_r = \dot{r} = -0.212$ m/s **Ans**

$v_\theta = r\dot{\theta} = 1.354(0.6) = 0.812$ m/s **Ans**

$a_r = \ddot{r} - r\dot{\theta}^2 = -0.2157 - (1.354)(0.6)^2 = -0.703$ m/s^2 **Ans**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 1.354(0.25) + 2(-0.2121)(0.6) = 0.0838$ m/s^2 **Ans**

12-155. For a short distance the train travels along a track having the shape of a spiral, $r = (1000/\theta)$ m, where θ is in radians. If it maintains a constant speed $v = 20$ m/s, determine the radial and transverse components of its velocity when $\theta = (9\pi/4)$ rad.



$r = \frac{1000}{\theta}$

$\dot{r} = -\frac{1000}{\theta^2}\dot{\theta}$

Since

$v^2 = (\dot{r})^2 + (r\dot{\theta})^2$

$(20)^2 = \frac{(1000)^2}{\theta^4}(\dot{\theta})^2 + \frac{(1000)^2}{\theta^2}(\dot{\theta})^2$

$(20)^2 = \frac{(1000)^2}{\theta^4}(1 + \theta^2)(\dot{\theta})^2$

Thus,

$\dot{\theta} = \frac{0.02\theta^2}{\sqrt{1 + \theta^2}}$

At $\theta = \frac{9\pi}{4}$

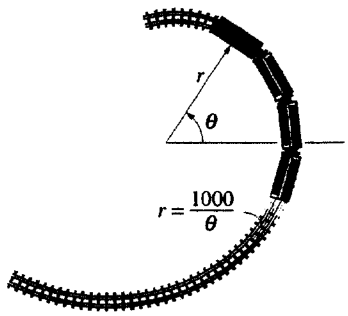
$\dot{\theta} = 0.140$

$\dot{r} = \frac{-1000}{(9\pi/4)^2}(0.140) = -2.80$

$v_r = \dot{r} = -2.80$ m/s **Ans**

$v_\theta = r\dot{\theta} = \frac{1000}{(9\pi/4)}(0.140) = 19.8$ m/s **Ans**

*12-156. For a short distance the train travels along a track having the shape of a spiral, $r = (1000/\theta)$ m, where θ is in radians. If the angular rate is constant, $\dot{\theta} = 0.2$ rad/s, determine the radial and transverse components of its velocity and acceleration when $\theta = (9\pi/4)$ rad.



$\dot{\theta} = 0.2$

$\ddot{\theta} = 0$

$r = \dfrac{1000}{\theta}$

$\dot{r} = -1000(\theta^{-2})\dot{\theta}$

$\ddot{r} = 2000(\theta^{-3})(\dot{\theta})^2 - 1000(\theta^{-2})\ddot{\theta}$

When $\theta = \dfrac{9\pi}{4}$

$r = 141.477$

$\dot{r} = -4.002812$

$\ddot{r} = 0.226513$

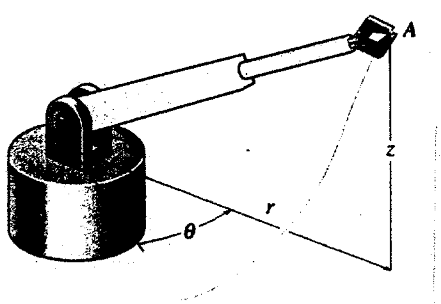
$v_r = \dot{r} = -4.00$ m/s **Ans**

$v_\theta = r\dot{\theta} = 141.477(0.2) = 28.3$ m/s **Ans**

$a_r = \ddot{r} - r(\dot{\theta})^2 = 0.226513 - 141.477(0.2)^2 = -5.43$ m/s^2 **Ans**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(-4.002812)(0.2) = -1.60$ m/s^2 **Ans**

12-157. The arm of the robot has a fixed length so that $r = 3$ ft and its grip A moves along the path $z = (3\sin 4\theta)$ ft, where θ is in radians. If $\theta = (0.5t)$ rad, where t is in seconds, determine the magnitudes of the grip's velocity and acceleration when $t = 3$ s.



$\theta = 0.5t$ $\quad r = 3$ $\quad z = 3\sin 2t$

$\dot{\theta} = 0.5$ $\quad \dot{r} = 0$ $\quad \dot{z} = 6\cos 2t$

$\ddot{\theta} = 0$ $\quad \ddot{r} = 0$ $\quad \ddot{z} = -12\sin 2t$

At $t = 3$ s,

$z = -0.8382$

$\dot{z} = 5.761$

$\ddot{z} = 3.353$

$v_r = 0$

$v_\theta = 3(0.5) = 1.5$

$v_z = 5.761$

$v = \sqrt{(0)^2 + (1.5)^2 + (5.761)^2} = 5.95$ ft/s **Ans**

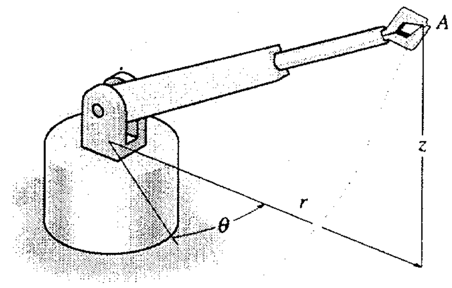
$a_r = 0 - 3(0.5)^2 = -0.75$

$a_\theta = 0 + 0 = 0$

$a_z = 3.353$

$a = \sqrt{(-0.75)^2 + (0)^2 + (3.353)^2} = 3.44$ ft/s^2 **Ans**

12-158. For a short time the arm of the robot is extending at a constant rate such that $\dot{r} = 1.5$ ft/s when $r = 3$ ft, $z = (4t^2)$ ft, and $\theta = 0.5t$ rad, where t is in seconds. Determine the magnitudes of the velocity and acceleration of the grip A when $t = 3$ s.



$\theta = 0.5t$ rad $\quad r = 3$ ft $\quad z = 4t^2$ ft

$\dot{\theta} = 0.5$ rad/s $\quad \dot{r} = 1.5$ ft/s $\quad \dot{z} = 8t$ ft/s

$\ddot{\theta} = 0 \quad \ddot{r} = 0 \quad \ddot{z} = 8$ ft/s^2

At $t = 3$ s,

$\theta = 1.5 \quad r = 3 \quad z = 36$

$\dot{\theta} = 0.5 \quad \dot{r} = 1.5 \quad \dot{z} = 24$

$\ddot{\theta} = 0 \quad \ddot{r} = 0 \quad \ddot{z} = 8$

$v_r = 1.5$

$v_\theta = 3(0.5) = 1.5$

$v_z = 24$

$v = \sqrt{(1.5)^2 + (1.5)^2 + (24)^2} = 24.1$ ft/s **Ans**

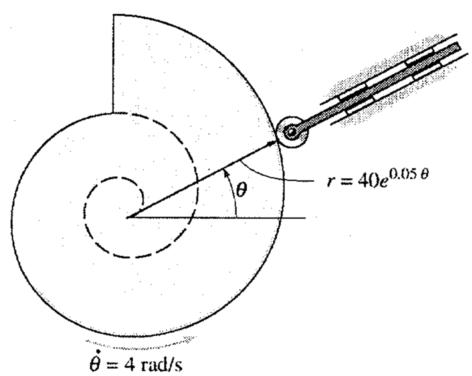
$a_r = 0 - 3(0.5)^2 = -0.75$

$a_\theta = 0 + 2(1.5)(0.5) = 1.5$

$a_z = 8$

$a = \sqrt{(-0.75)^2 + (1.5)^2 + (8)^2} = 8.17$ ft/s^2 **Ans**

12-159. The partial surface of the cam is that of a logarithmic spiral $r = (40e^{0.05\theta})$ mm, where θ is in radians. If the cam is rotating at a constant angular rate of $\dot{\theta} = 4$ rad/s, determine the magnitudes of the velocity and acceleration of the follower rod at the instant $\theta = 30°$.



$r = 40e^{0.05\theta} \quad r = 40e^{0.05\left(\frac{\pi}{6}\right)} = 41.0610$

$\dot{r} = 2e^{0.05\theta}\dot{\theta} \quad \dot{r} = 2e^{0.05\left(\frac{\pi}{6}\right)}(4) = 8.21$ mm/s **Ans**

$\ddot{r} = 0.1e^{0.05\theta}(\dot{\theta})^2 + 2e^{0.05\theta}\ddot{\theta}$

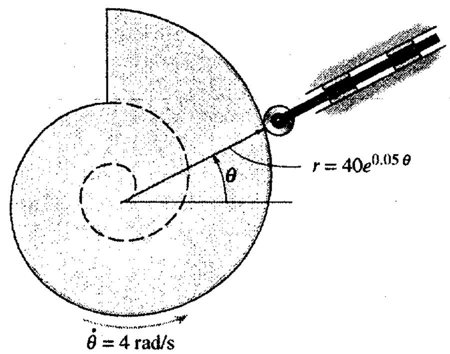
$\ddot{r} = 0.1e^{0.05\left(\frac{\pi}{6}\right)}(4)^2 + 0 = 1.64$ mm/s^2 **Ans**

$\theta = \dfrac{\pi}{6}$

$\dot{\theta} = 4$

$\ddot{\theta} = 0$

***12-160.** Solve Prob. 12-159, if the cam has an angular acceleration of $\ddot{\theta} = 2\ \text{rad/s}^2$ when its angular velocity is $\dot{\theta} = 4$ rad/s at $\theta = 30°$.



$r = 40e^{0.05\theta}$

$\dot{r} = 2e^{0.05\theta}\dot{\theta}$

$\ddot{r} = 0.1e^{0.05\theta}\left(\dot{\theta}\right)^2 + 2e^{0.05\theta}\ddot{\theta}$

$\theta = \dfrac{\pi}{6}$

$\dot{\theta} = 4$

$\ddot{\theta} = 2$

$r = 40e^{0.05\left(\frac{\pi}{6}\right)} = 41.0610$

$\dot{r} = 2e^{0.05\left(\frac{\pi}{6}\right)}(4) = 8.2122$

$\ddot{r} = 0.1e^{0.05\left(\frac{\pi}{6}\right)}(4)^2 + 2e^{0.05\left(\frac{\pi}{6}\right)}(2) = 5.749$

$v_r = \dot{r} = 8.2122$

$v_\theta = r\dot{\theta} = 41.0610(4) = 164.24$

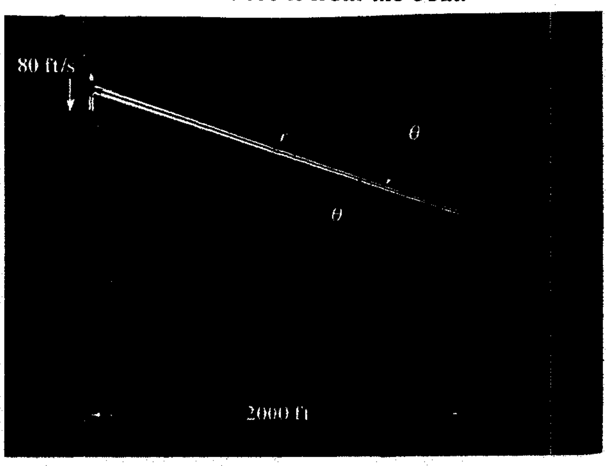
$v = \sqrt{(8.2122)^2 + (164.24)^2} = 164$ mm/s **Ans**

$a_r = \ddot{r} - r\dot{\theta}^2 = 5.749 - 41.0610(4)^2 = -651.2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 41.0610(2) + 2(8.2122)(4) = 147.8197$

$a = \sqrt{(-651.2)^2 + (147.8197)^2} = 668\ \text{mm/s}^2$ **Ans**

12-161. The searchlight on the boat anchored 2000 ft from shore is turned on the automobile, which is traveling along the straight road at a constant speed of 80 ft/s. Determine the angular rate of rotation of the light when the automobile is r = 3000 ft from the boat.



$r = 2000 \csc\theta$

$\dot{r} = -2000 \csc\theta \operatorname{ctn}\theta\ \dot{\theta}$

At r = 3000 ft, $\theta = 41.8103°$

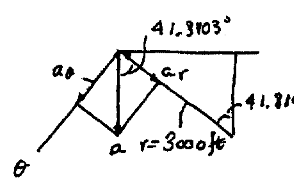
$\dot{r} = -3354.102\ \dot{\theta}$

$v = \sqrt{(\dot{r})^2 + (r\dot{\theta})^2}$

$(80)^2 = [(-3354.102)^2 + (3000)^2](\dot{\theta})^2$

$\dot{\theta} = 0.0177778 = 0.0178$ rad/s **Ans**

12-162. If the car in Prob. 12–161 is accelerating at $15\ \text{ft/s}^2$ at the instant r = 3000 ft, determine the required angular acceleration $\ddot{\theta}$ of the light at this instant.



$r = 2000 \csc\theta$

$\dot{r} = -2000 \csc\theta \operatorname{ctn}\theta\ \dot{\theta}$

At r = 3000 ft, $\theta = 41.8103°$

$\dot{r} = -3354.102\ \dot{\theta}$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

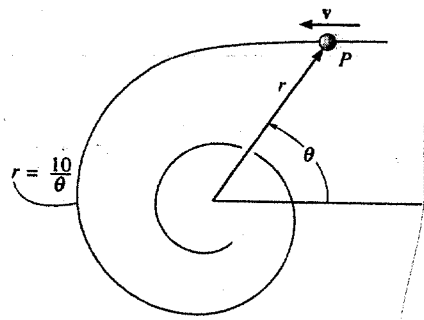
$a_\theta = 3000\ddot{\theta} + 2(-3354.102)(0.0177778)^2$

Since $a_\theta = 15 \sin 41.8103° = 10$ m/s

Then,

$\ddot{\theta} = 0.00404\ \text{rad/s}^2$ **Ans**

12-163. A particle P moves along the spiral path $r = (10/\theta)$ ft, where θ is in radians. If it maintains a constant speed of $v = 20$ ft/s, determine the magnitudes v_r and v_θ as functions of θ and evaluate each at $\theta = 1$ rad.



$$r = \frac{10}{\theta}$$

$$\dot{r} = -\left(\frac{10}{\theta^2}\right)\dot{\theta}$$

Since $v^2 = \dot{r}^2 + \left(r\dot{\theta}\right)^2$

$$(20)^2 = \left(\frac{10^2}{\theta^4}\right)\dot{\theta}^2 + \left(\frac{10^2}{\theta^2}\right)\dot{\theta}^2$$

$$(20)^2 = \left(\frac{10^2}{\theta^4}\right)\left(1+\theta^2\right)\dot{\theta}^2$$

Thus, $\dot{\theta} = \dfrac{2\theta^2}{\sqrt{1+\theta^2}}$

$$v_r = \dot{r} = -\left(\frac{10}{\theta^2}\right)\left(\frac{2\theta^2}{\sqrt{1+\theta^2}}\right) = -\frac{20}{\sqrt{1+\theta^2}} \qquad \textbf{Ans}$$

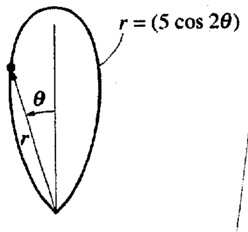
$$v_\theta = r\dot{\theta} = \left(\frac{10}{\theta}\right)\left(\frac{2\theta^2}{\sqrt{1+\theta^2}}\right) = \frac{20\theta}{\sqrt{1+\theta^2}} \qquad \textbf{Ans}$$

When $\theta = 1$ rad,

$$v_r = \left(-\frac{20}{\sqrt{2}}\right) = -14.1 \text{ ft/s} \qquad \textbf{Ans}$$

$$v_\theta = \left(\frac{20}{\sqrt{2}}\right) = 14.1 \text{ ft/s} \qquad \textbf{Ans}$$

***12-164.** A particle travels along the portion of the "four-leaf rose" defined by the equation $r = (5 \cos 2\theta)$ m. If the angular velocity of the radial coordinate line is $\dot{\theta} = (3t^2)$ rad/s, where t is in seconds, determine the radial and transverse components of the particle's velocity and acceleration at the instant $\theta = 30°$. When $t = 0$, $\theta = 0$.



$\dot{\theta} = 3t^2$

$\ddot{\theta} = 6t$

$\int_0^\theta d\theta = \int_0^t 3t^2\, dt$

$\theta = t^3$

At $\theta = 30° = \frac{\pi}{6}$

$t = (\pi/6)^{1/3} = 0.806$

$\dot{\theta} = 1.95$

$\ddot{\theta} = 4.84$

$r = 5 \cos 2\theta$

$\dot{r} = -10\sin 2\theta\, \dot{\theta}$

$\ddot{r} = -10(2\cos 2\theta(\dot{\theta})^2 + \sin 2\theta\, \ddot{\theta})$

At $\theta = 30°$

$r = 2.5$

$\dot{r} = -16.88$

$\ddot{r} = -79.87$

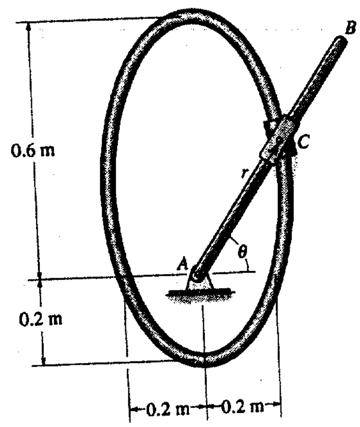
$v_r = \dot{r} = -16.9$ m/s **Ans**

$v_\theta = r\dot{\theta} = 2.5(1.95) = 4.87$ m/s **Ans**

$a_r = \ddot{r} - r(\dot{\theta})^2 = -79.86 - 2.5(1.95)^2 = -89.4$ m/s^2 **Ans**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2.5(4.84) + 2(-16.88)(1.95) = -53.7$ m/s^2 **Ans**

■12-165. The double collar C is pin connected such that one collar slides over a fixed rod and the other slides over a rotating rod AB. If the angular velocity of AB is given as $\dot{\theta} = (e^{0.5t^2})$ rad/s, where t is in seconds, and the path defined by the fixed rod is $r = |0.4 \sin \theta + 0.2|$ m, determine the radial and transverse components of the collar's velocity and acceleration when $t = 1$ s. When $t = 0$, $\theta = 0°$. Use Simpson's rule to determine θ at $t = 1$ s.



$\dot{\theta} = e^{0.5t^2}\big|_{t=1\,s} = 1.649$ rad/s

$\ddot{\theta} = e^{0.5t^2}(t)\big|_{t=1\,s} = 1.649$ rad/s^2

$\theta = \int_0^1 e^{0.5t^2}\,dt = 1.195$ rad $= 68.47°$

$r = 0.4\sin\theta + 0.2$

$\dot{r} = 0.4\cos\theta\dot{\theta}$

$\ddot{r} = -0.4\sin\theta\dot{\theta}^2 + 0.4\cos\theta\ddot{\theta}$

At $t = 1$ s,

$r = 0.5721$

$\dot{r} = 0.2421$

$\ddot{r} = -0.7694$

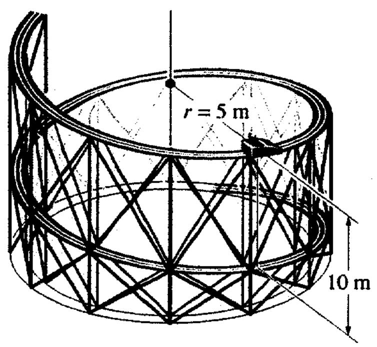
$v_r = \dot{r} = 0.242$ m/s **Ans**

$v_\theta = r\dot{\theta} = 0.5721(1.649) = 0.943$ m/s **Ans**

$a_r = \ddot{r} - r\dot{\theta}^2 = -0.7694 - 0.5721(1.649)^2 = -2.32$ m/s^2 **Ans**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5721(1.649) + 2(0.2421)(1.649) = 1.74$ m/s^2 **Ans**

12-166. The roller coaster is traveling down along the spiral ramp with a constant speed $v = 6$ m/s. If the track descends a distance of 10 m for every full revolution, $\theta = 2\pi$ rad, determine the magnitude of the roller coaster's acceleration as it moves along the track, $r = 5$ m. *Hint:* For part of the solution, note that the tangent to the ramp at any point is at an angle $\phi = \tan^{-1}[10/2\pi(5)] = 17.66°$ from the horizontal. Use this to determine the velocity components v_θ and v_z, which in turn are used to determine $\dot{\theta}$ and $\dot{z}$.



$\phi = 17.66°$

$v = 6$ m/s

$v_z = -6 \sin 17.66° = -1.820$ m/s

$v_\theta = 6 \cos 17.66° = 5.717$ m/s

Since $r = 5$

$\dot{r} = 0$

$\ddot{r} = 0$

$r\dot{\theta} = v_\theta = 5.717$

$\dot{\theta} = \dfrac{5.717}{5} = 1.143$

$v^2 = (\dot{r})^2 + (r\dot{\theta})^2$

$0 = 2\dot{r}\ddot{r} + 2(r\dot{\theta})(\dot{r}\dot{\theta} + r\ddot{\theta})$

$\ddot{\theta} = 0$

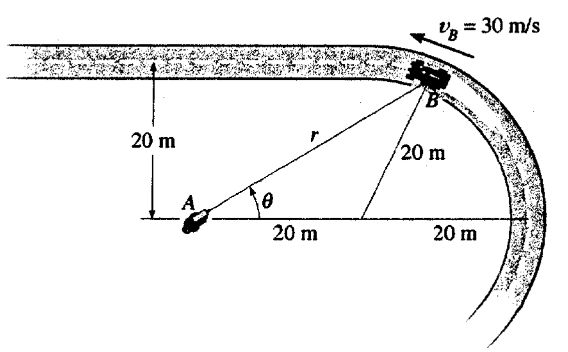
$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 5(1.143)^2 = -6.537$ m/s^2

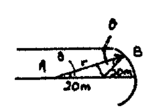
$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0$

$a_z = \ddot{z} = \dot{v}_z = 0$

$a = 6.54$ m/s^2 **Ans**

12-167. A cameraman standing at A is following the movement of a race car, B, which is traveling around a curved track at a constant speed of 30 m/s. Determine the angular rate $\dot{\theta}$ at which the man must turn in order to keep the camera directed on the car at the instant $\theta = 30°$.





$r = 2(20\cos\theta) = 40\cos\theta$

$\dot{r} = -40\sin\theta\,\dot{\theta}$

$\mathbf{v} = \dot{r}\mathbf{u}_r + r\dot{\theta}\mathbf{u}_\theta$

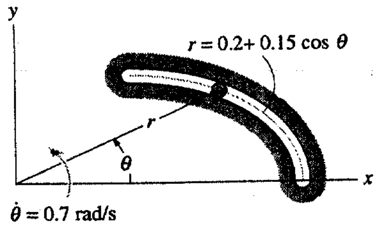
$v = \sqrt{(\dot{r})^2 + (r\dot{\theta})^2}$

$(30)^2 = (-40\sin\theta)^2(\dot{\theta})^2 + (40\cos\theta)^2(\dot{\theta})^2$

$(30)^2 = (40)^2[\sin^2\theta + \cos^2\theta](\dot{\theta})^2$

$\dot{\theta} = \dfrac{30}{40} = 0.75$ rad/s **Ans**

***12-168.** The pin follows the path described by the equation $r = (0.2 + 0.15\cos\theta)$ m. At the instant $\theta = 30°$, $\dot{\theta} = 0.7$ rad/s and $\ddot{\theta} = 0.5$ rad/s^2. Determine the magnitudes of the pin's velocity and acceleration at this instant. Neglect the size of the pin.



$$r = 0.2 + 0.15\cos\theta = 0.2 + 0.15\cos 30° = 0.3299 \text{ m}$$

$$\dot{r} = -0.15\sin\theta\dot{\theta} = -0.15\sin 30°(0.7) = -0.0525 \text{ m/s}$$

$$\ddot{r} = -0.15\left[\cos\theta\dot{\theta}^2 + \sin\theta\ddot{\theta}\right] = -0.15\left[\cos 30°(0.7)^2 + \sin 30°(0.5)\right] = -0.10115 \text{ m/s}^2$$

$$v_r = \dot{r} = -0.0525 \text{ m/s}$$

$$v_\theta = r\dot{\theta} = 0.3299(0.7) = 0.2309 \text{ m/s}$$

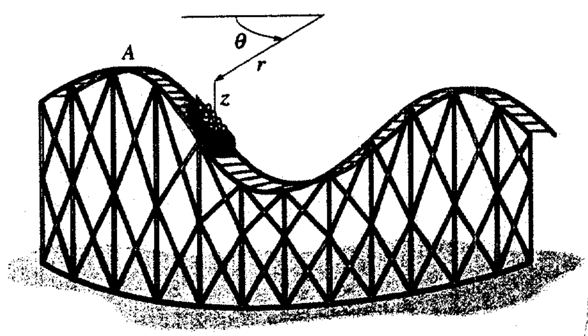
$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{(-0.0525)^2 + (0.2309)^2} = 0.237 \text{ m/s} \qquad \textbf{Ans}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -0.10115 - 0.3299(0.7)^2 = -0.2628 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.3299(0.5) + 2(-0.0525)(0.7) = 0.09145 \text{ m/s}^2$$

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-0.2628)^2 + (0.09145)^2} = 0.278 \text{ m/s}^2 \qquad \textbf{Ans}$$

12-169. For a short time the position of the roller-coaster car along its path is defined by the equations $r = 25$ m, $\theta = (0.3t)$ rad, and $z = (-8\cos\theta)$ m, where t is in seconds. Determine the magnitude of the car's velocity and acceleration when $t = 4$ s.



$r = 25$ m $\qquad \theta = 0.3t|_{t=4\text{ s}} = 1.2$ rad

$\dot{r} = 0 \qquad \dot{\theta} = 0.3$ rad/s

$\ddot{r} = 0 \qquad \ddot{\theta} = 0$

$z = -8\cos\theta \qquad \dot{z} = 8\sin\theta\dot{\theta}\big|_{\theta=1.2\text{ rad}} = 2.2369$ m/s

$\ddot{z} = 8\left[\cos\theta\dot{\theta}^2 + \sin\theta\ddot{\theta}\right]\big|_{\theta=1.2\text{ rad}} = 0.2609$ m/s^2

$v_r = \dot{r} = 0 \qquad v_\theta = r\dot{\theta} = 25(0.3) = 7.5$ m/s $\qquad v_z = \dot{z} = 2.2369$ m/s

$$v = \sqrt{v_r^2 + v_\theta^2 + v_z^2} = \sqrt{0^2 + 7.5^2 + 2.2369^2} = 7.83 \text{ m/s} \qquad \textbf{Ans}$$

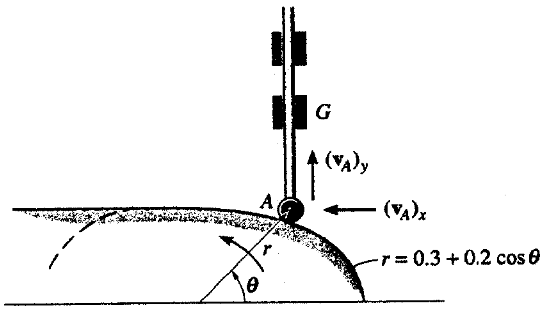
$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 25(0.3)^2 = -2.25$ m/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 25(0) + 2(0)(0.3) = 0$

$a_z = \ddot{z} = 0.2609$ m/s^2

$$a = \sqrt{a_r^2 + a_\theta^2 + a_z^2} = \sqrt{(-2.25)^2 + 0^2 + 0.2609^2} = 2.27 \text{ m/s}^2 \qquad \textbf{Ans}$$

12-170. The mechanism of a machine is constructed so that the roller at A follows the surface of the cam described by the equation $r = (0.3 + 0.2\cos\theta)$ m. If $\dot{\theta} = 0.5$ rad/s and $\ddot{\theta} = 0$, determine the magnitudes of the roller's velocity and acceleration when $\theta = 30°$. Neglect the size of the roller. Also compute the velocity components $(\mathbf{v}_A)_x$ and $(\mathbf{v}_A)_y$ of the roller at this instant. The rod to which the roller is attached remains vertical and can slide up or down along the guides while the guides translate horizontally to the left.



$\dot{\theta} = 0.5$

$\ddot{\theta} = 0$

$r = (0.3 + 0.2\cos\theta)$

$\dot{r} = -0.2\sin\theta\,\dot{\theta}$

$\ddot{r} = -0.2(\cos\theta\,\dot{\theta}^2 + \sin\theta\,\ddot{\theta})$

At $\theta = 30°$

$r = 0.473$

$\dot{r} = -0.05$

$\ddot{r} = -0.0433$

$v_r = \dot{r} = -0.05$

$v_\theta = r\dot{\theta} = 0.473(0.5) = 0.237$

$v = \sqrt{(-0.05)^2 + (0.237)^2} = 0.242$ m/s **Ans**

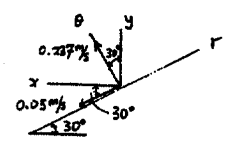
$a_r = \ddot{r} - r\dot{\theta}^2 = -0.0433 - 0.473(0.5)^2$

$a_r = -0.162$ m/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(-0.05)(0.5)$

$a_\theta = -0.05$ m/s^2

$a = \sqrt{(-0.162)^2 + (-0.5)^2} = 0.169$ m/s^2 **Ans**



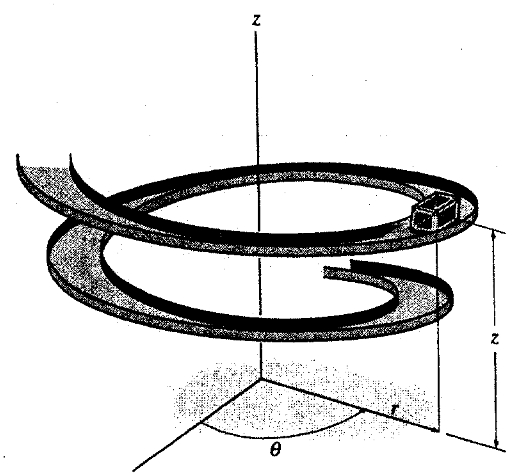
$(\overset{+}{\leftarrow})$ $v_x = 0.05\cos 30° + 0.237\sin 30°$

$v_x = 0.162$ m/s **Ans**

$(+\uparrow)$ $v_y = -0.05\sin 30° + 0.237\cos 30°$

$v_y = 0.180$ m/s **Ans**

12-171. The crate slides down the section of the spiral ramp such that $r = (0.5z)$ ft and $z = (100 - 0.1t^2)$ ft, where t is in seconds. If the rate of rotation about the z axis is $\dot{\theta} = 0.04\pi t$ rad/s, determine the magnitudes of the velocity and acceleration of the crate at the instant $z = 10$ ft.



$r = 0.5z$

$z = 100 - 0.1t^2$

Thus,

$r = 50 - 0.05t^2$

$\dot{r} = -0.1t$

$\ddot{r} = -0.1$

$\dot{\theta} = 0.04\pi t$ rad/s $= 0.12566t$ rad/s

$\ddot{\theta} = 0.12566$

$\dot{z} = -0.2t$

$\ddot{z} = -0.2$

At $z = 10$ ft,

$10 = 100 - 0.1t^2$

$t = 30$ s

$r = 50 - 0.05(30)^2 = 5$

$\dot{r} = -0.1(30) = -3$

$\ddot{r} = -0.1$

$\dot{\theta} = 0.12566(30) = 3.76991$

$\ddot{\theta} = 0.12566$

$\dot{z} = -0.2(30) = -6$

$\ddot{z} = -0.2$

$v_r = \dot{r} = -3$

$v_\theta = r\dot{\theta} = 5(3.76991) = 18.850$

$v_z = \dot{z} = -6$

$v = \sqrt{(-3)^2 + (18.850)^2 + (-6)^2} = 20.0$ ft/s **Ans**

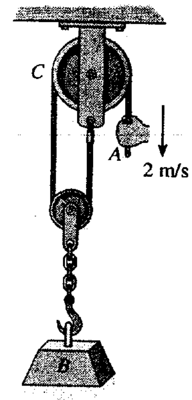
$a_r = \ddot{r} - r\dot{\theta}^2 = -0.1 - 5(3.76991)^2 = -71.16$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 5(0.12566) + 2(-3)(3.76991) = -21.99$

$a_z = \ddot{z} = -0.2$

$a = \sqrt{(-71.16)^2 + (-21.99)^2 + (-0.2)^2} = 74.5$ ft/s^2 **Ans**

*12-172. If the end of the cable at A is pulled down with a speed of 2 m/s, determine the speed at which block B rises.

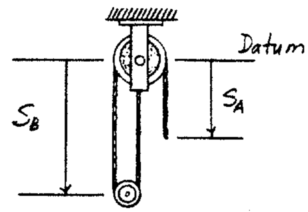


Position - Coordinate Equation : Datum is established at fixed pulley C. The position of point A and block B with respect to datum are s_A and s_B, respectively.

$$2s_B + s_A = l$$

Time Derivative : Taking the time derivative of the above equation yields

$$2v_B + v_A = 0 \qquad [1]$$

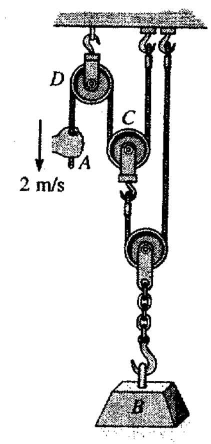


Since $v_A = 5$ m/s, from Eq.[1]

$$(+\downarrow) \qquad 2v_B + 2 = 0$$

$$v_B = -1 \text{ m/s} = 1 \text{ m/s} \uparrow \qquad \textbf{Ans}$$

12-173. If the end of the cable at A is pulled down with a speed of 2 m/s, determine the speed at which block B rises.



Position - Coordinate Equation : Datum is established at fixed pulley D. The position of point A, block B and pulley C with respect to datum are s_A, s_B and s_C respectively. Since the system consists of two cords, two position - coordinate equations can be derived.

$$(s_A - s_C) + (s_B - s_C) + s_B = l_1 \qquad [1]$$

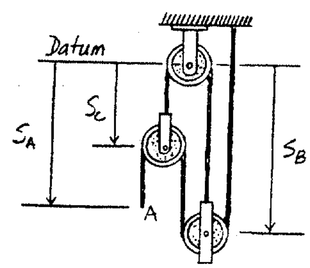
$$s_B + s_C = l_2 \qquad [2]$$

Eliminating s_C from Eqs.[1] and [2] yields

$$s_A + 4s_B = l_1 + 2l_2$$

Time Derivative : Taking the time derivative of the above equation yields

$$v_A + 4v_B = 0 \qquad [3]$$

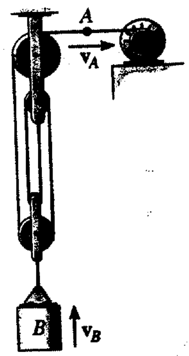


Since $v_A = 2$ m/s, from Eq.[3]

$$(+\downarrow) \qquad 2 + 4v_B = 0$$

$$v_B = -0.5 \text{ m/s} = 0.5 \text{ m/s} \uparrow \qquad \textbf{Ans}$$

12-174. Determine the constant speed at which the cable at A must be drawn in by the motor in order to hoist the load at B 15 ft in 5 s.



$$v_B = \frac{-15}{5} = -3 \text{ ft/s} = 3 \text{ ft/s} \uparrow$$

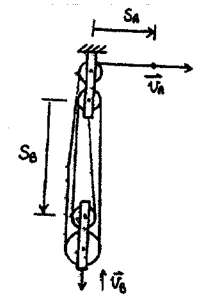
$$4s_B + s_A = l$$

$$4\Delta s_B = -\Delta s_A$$

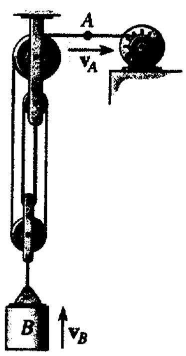
$$4v_B = -v_A$$

$$4(-3) = -v_A$$

$$v_A = 12 \text{ ft/s} \rightarrow \qquad \textbf{Ans}$$



12-175. Determine the time needed for the load at B to attain a speed of 8 m/s, starting from rest, if the cable is drawn into the motor with an acceleration of 0.2 m/s^2.



$$4s_B + s_A = l$$

$$4v_B = -v_A$$

$$4a_B = -a_A$$

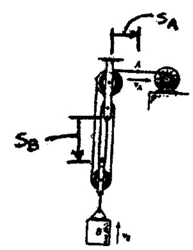
$$4a_B = -0.2$$

$$a_B = -0.05 \text{ m/s}^2$$

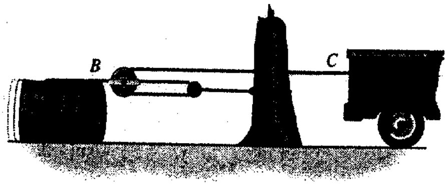
$$(+\downarrow) \quad v_B = (v_B)_0 + a_B t$$

$$-8 = 0 - (0.05)(t)$$

$$t = 160 \text{ s} \qquad \textbf{Ans}$$



*12-176. Determine the displacement of the log if the truck at C pulls the cable 4 ft to the right.



$$2s_B + (s_B - s_C) = l$$

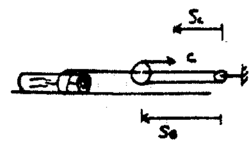
$$3s_B - s_C = l$$

$$3\Delta s_B - \Delta s_C = 0$$

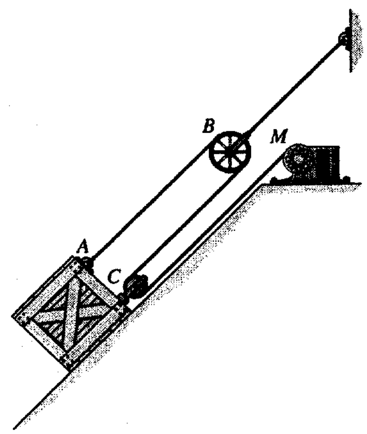
Since $\Delta s_C = -4$, then

$$3\Delta s_B = -4$$

$\Delta s_B = -1.33 \text{ ft} = 1.33 \text{ ft} \rightarrow$ **Ans**

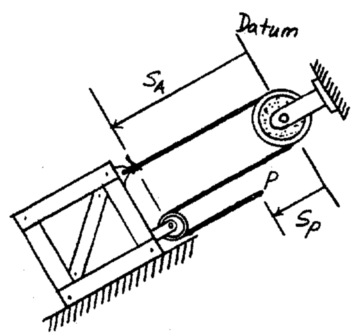


12-177. The crate is being lifted up the inclined plane using the motor M and the rope and pulley arrangement shown. Determine the speed at which the cable must be taken up by the motor in order to move the crate up the plane with a constant speed of 4 ft/s.



Position - Coordinate Equation : Datum is established at fixed pulley B. The position of point P and crate A with respect to datum are s_P and s_A, respectively.

$$2s_A + (s_A - s_P) = l$$
$$3s_A - s_P = 0$$



Time Derivative : Taking the time derivative of the above equation yields

$$3v_A - v_P = 0 \qquad [1]$$

Since $v_A = 4$ ft/s, from Eq.[1]

(+) $\quad 3(4) - v_P = 0$

$v_P = 12$ ft/s **Ans**

12-178. Determine the displacement of the block at B if A is pulled down 4 ft.

$$2s_A + 2s_C = l_1$$

$$\Delta s_A = -\Delta s_C$$

$$s_B - s_C + s_B = l_2$$

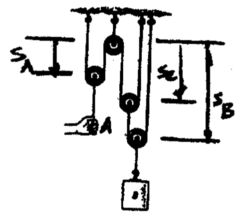
$$2\Delta s_B = \Delta s_C$$

Thus,

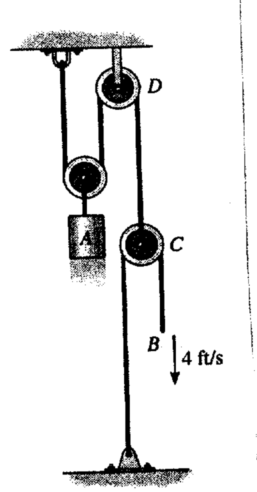
$$2\Delta s_B = -\Delta s_A$$

$$2\Delta s_B = -4$$

$$\Delta s_B = -2\text{ ft} = 2\text{ ft}\uparrow \qquad \textbf{Ans}$$



12-179. The cable at B is pulled downwards at 4 ft/s, and is slowing at 2 ft/s^2. Determine the velocity and acceleration of block A at this instant.



$$2s_A + (h - s_C) = l$$

$$2v_A = v_C$$

$$s_C + (s_C - s_B) = l$$

$$2v_C = v_B$$

$$v_B = 4v_A$$

$$a_B = 4a_A$$

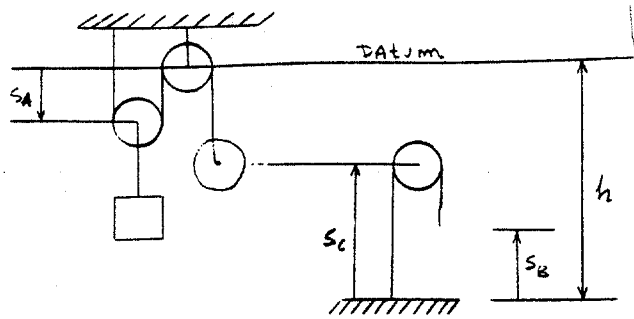
Thus,

$$-4 = 4v_A$$

$$v_A = -1\text{ ft/s} = 1\text{ ft/s}\uparrow \qquad \textbf{Ans}$$

$$2 = 4a_A$$

$$a_A = 0.5\text{ ft/s} = 0.5\text{ ft/s}^2\downarrow \qquad \textbf{Ans}$$



***12-180.** The pulley arrangement shown is designed for hoisting materials. If *BC remains fixed* while the plunger *P* is pushed downward with a speed of 4 ft/s, determine the speed of the load at *A*.

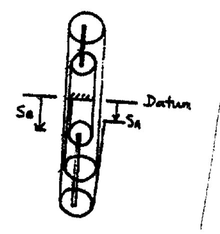
$$5s_B + (s_B - s_A) = l$$

$$6s_B - s_A = l$$

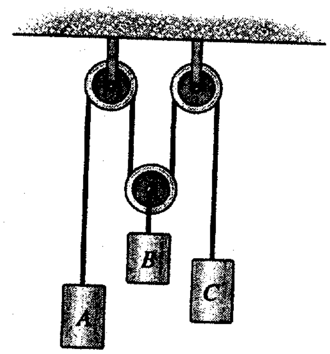
$$6v_B - v_A = 0$$

$$6(4) = v_A$$

$$v_A = 24 \text{ ft/s} \qquad \textbf{Ans}$$



12-181. If block *A* is moving downward with a speed of 4 ft/s while *C* is moving up at 2 ft/s, determine the speed of block *B*.

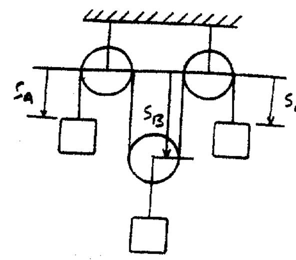


$$s_A + 2s_B + s_C = l$$

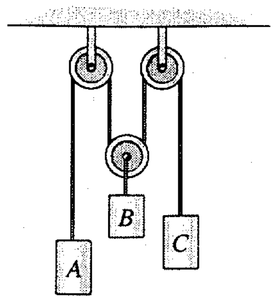
$$v_A + 2v_B + v_C = 0$$

$$4 + 2v_B - 2 = 0$$

$$v_B = -1 \text{ ft/s} = 1 \text{ ft/s} \uparrow \qquad \textbf{Ans}$$



12-182. If block A is moving downward at 6 ft/s while block C is moving down at 18 ft/s, determine the relative velocity of block B with respect to C.



$s_A + 2s_B + s_C = l$

$v_A + 2v_B + v_C = 0$

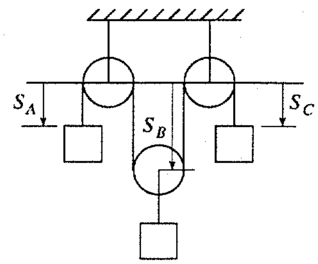
$6 + 2v_B + 18 = 0$

$v_B = -12 \text{ ft/s} = 12 \text{ ft/s} \uparrow$

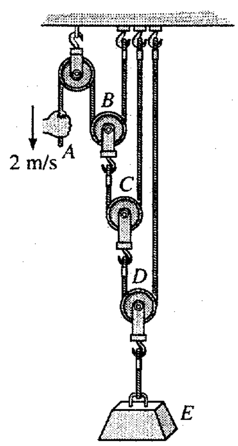
$v_B = v_C + v_{B/C}$

$[12 \uparrow] = [18 \downarrow] + [v_{B/C} \uparrow]$

$v_{B/C} = 30 \text{ ft/s} \uparrow$ **Ans**



12-183. If the end of the cable at A is pulled down with a speed of 2 m/s, determine the speed at which block E rises.



Position-Coordinate Equation: Datum is established at fixed pulley. The position of point A, pulley B and C and block E with respect to datum are s_A, s_B, s_C and s_E, respectively. Since the system consists of three cords, three position–coordinate equations can be developed.

$2s_B + s_A = l_1$ [1]

$s_C + (s_C - s_B) = l_2$ [2]

$s_E + (s_E - s_C) = l_3$ [3]

Eliminating s_C and s_B from Eqs. [1], [2] and [3], we have

$s_A + 8s_E = l_1 + 2l_2 + 4l_3$

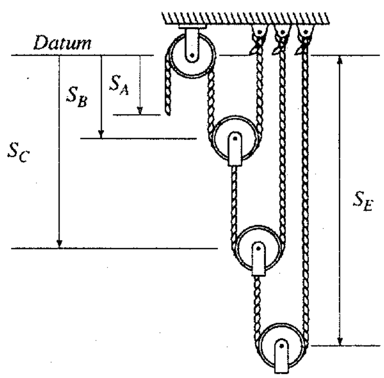
Time Derivative: Taking the time derivative of the above equation yields

$v_A + 8v_E = 0$ [4]

Since $v_A = 2$ m/s, from Eq. [3]

$(+\downarrow)$ $2 + 8v_E = 0$

$v_E = -0.250 \text{ m/s} = 0.250 \text{ m/s} \uparrow$ **Ans**



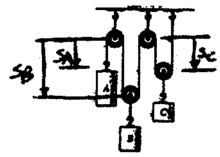
*12-184. If block A of the pulley system is moving downward with a speed of 4 ft/s while block C is moving up at 2 ft/s, determine the speed of block B.

$$s_A + 2s_B + 2s_C = l$$

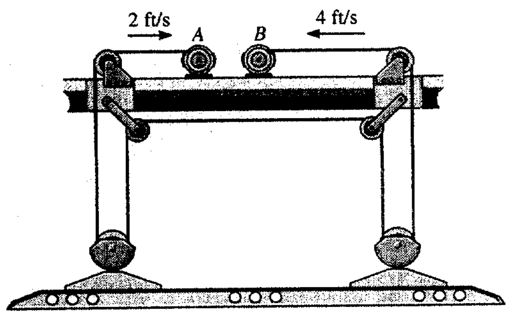
$$v_A + 2v_B + 2v_C = 0$$

$$4 + 2v_B + 2(-2) = 0$$

$$v_B = 0 \qquad \textbf{Ans}$$



12-185. The crane is used to hoist the load. If the motors at A and B are drawing in the cable at a speed of 2 ft/s and 4 ft/s, respectively, determine the speed of the load.



Position - Coordinate Equation : Datum is established as shown. The position of point A and B and load C with respect to datum are s_A, s_B and s_C. respectively.

$$4s_C + s_A + s_B + 2h = l$$

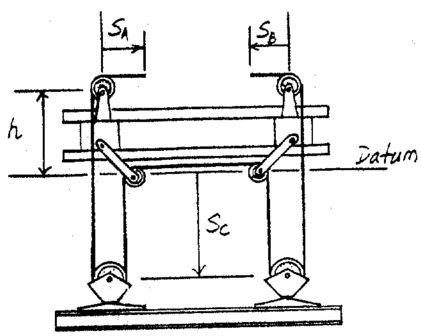
Time Derivative : Since h is a constant. Taking the time derivative of the above equation yields

$$4v_C + v_A + v_B = 0 \qquad [1]$$

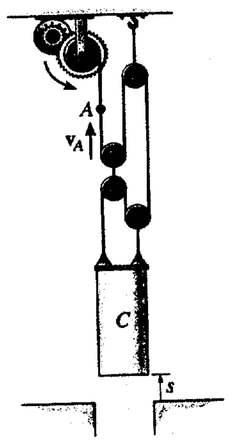
Since $v_A = 2$ ft/s and $v_B = 4$ ft/s, from Eq.[1]

$$4v_C + 2 + 4 = 0$$

$$v_C = -1.50 \text{ ft/s} = 1.50 \text{ ft/s} \uparrow \qquad \textbf{Ans}$$



12-186. The cylinder C is being lifted using the cable and pulley system shown. If point A on the cable is being drawn toward the drum with a speed of 2 m/s, determine the speed of the cylinder.

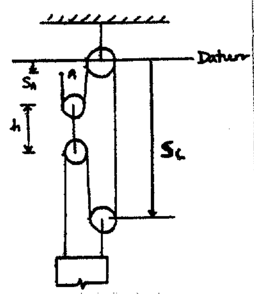


$$l = s_C + (s_C - h) + (s_C - h - s_A)$$

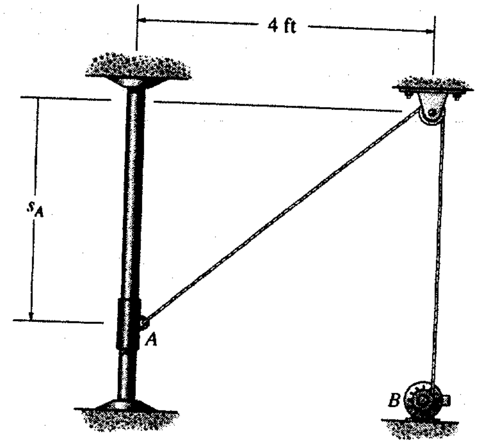
$$l = 3s_C - 2h - s_A$$

$$0 = 3v_C - v_A$$

$$v_C = \frac{v_A}{3} = \frac{-2}{3} = -0.667 \text{ m/s} = 0.667 \text{ m/s} \uparrow \qquad \textbf{Ans}$$



12-187. The motion of the collar at A is controlled by a motor at B such that when the collar is at $s_A = 3$ ft it is moving upwards at 2 ft/s and slowing down at 1 ft/s^2. Determine the velocity and acceleration of the cable as it is drawn into the motor B at this instant.



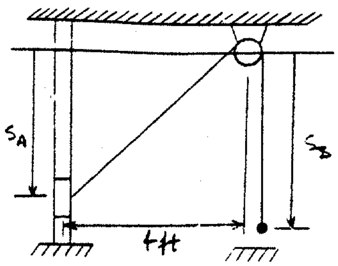
$$\sqrt{s_A^2+4^2}+s_B=l$$

$$\frac{1}{2}(s_A^2+16)^{-\frac{1}{2}}(2s_A)\dot{s}_A+\dot{s}_B=0$$

$$\dot{s}_B=-s_A\dot{s}_A(s_A^2+16)^{-\frac{1}{2}}$$

$$\ddot{s}_B=-\left[(\dot{s}_A)^2(s_A^2+16)^{-\frac{1}{2}}+s_A\ddot{s}_A(s_A^2+16)^{-\frac{1}{2}}+s_A\dot{s}_A\left(-\frac{1}{2}\right)(s_A^2+16)^{-\frac{3}{2}}(2s_A\dot{s}_A)\right]$$

$$\ddot{s}_B=\frac{(s_A\dot{s}_A)^2}{(s_A^2+16)^{\frac{3}{2}}}-\frac{(\dot{s}_A)^2+s_A\ddot{s}_A}{(s_A^2+16)^{\frac{1}{2}}}$$

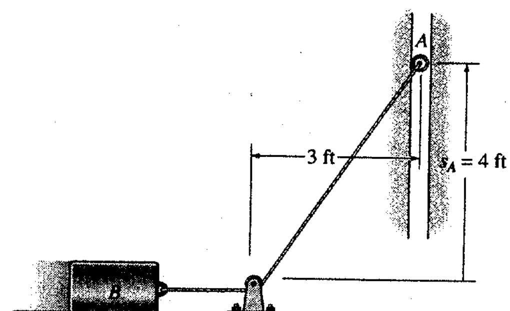


Evaluating these equations :

$\dot{s}_B=-3(-2)\left((3)^2+16\right)^{-\frac{1}{2}}=1.20$ ft/s $\downarrow$ **Ans**

$\ddot{s}_B=\dfrac{((3)(-2))^2}{((3)^2+16)^{\frac{3}{2}}}-\dfrac{(-2)^2+3(1)}{((3)^2+16)^{\frac{1}{2}}}=-1.11\text{ ft/s}^2=1.11\text{ ft/s}^2\uparrow$ **Ans**

***12-188.** The roller at A is moving upward with a velocity of $v_A = 3$ ft/s and has an acceleration of $a_A = 4$ ft/s^2 when $s_A = 4$ ft. Determine the velocity and acceleration of block B at this instant.



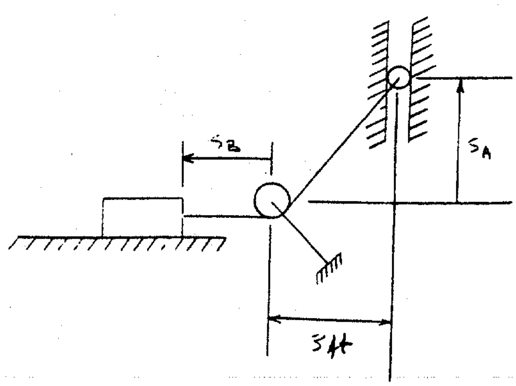
$$s_B+\sqrt{(s_A)^2+3^2}=l$$

$$\dot{s}_B+\frac{1}{2}\left[(s_A)^2+3^2\right]^{-\frac{1}{2}}(2s_A)\dot{s}_A=0$$

$$\dot{s}_B+\left[s_A^2+9\right]^{-\frac{1}{2}}\left(s_A\dot{s}_A\right)=0$$

$$\ddot{s}_B-\left[(s_A)^2+9\right]^{-\frac{3}{2}}\left(s_A^2\dot{s}_A^2\right)+\left[s_A^2+9\right]^{-\frac{1}{2}}\left(\dot{s}_A^2\right)+\left[s_A^2+9\right]^{-\frac{1}{2}}\left(s_A\ddot{s}_A\right)=0$$

At $s_A=4$ ft, $\dot{s}_A=3$ ft/s, $\ddot{s}_A=4$ ft/s^2



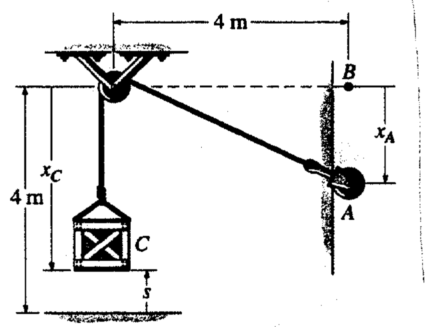
$$\dot{s}_B+\left(\frac{1}{5}\right)(4)(3)=0$$

$v_B=-2.4$ ft/s $=2.40$ ft/s $\rightarrow$ **Ans**

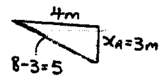
$$\ddot{s}_B-\left(\frac{1}{5}\right)^3(4)^2(3)^2+\left(\frac{1}{5}\right)(3)^2+\left(\frac{1}{5}\right)(4)(4)=0$$

$a_B=-3.85$ ft/s$^2=3.85$ ft/s^2 $\rightarrow$ **Ans**

12-189. The crate C is being lifted by moving the roller at A downward with a constant speed of $v_A = 2$ m/s along the guide. Determine the velocity and acceleration of the crate at the instant $s = 1$ m. When the roller is at B, the crate rests on the ground. Neglect the size of the pulley in the calculation. *Hint:* Relate the coordinates x_C and x_A using the problem geometry, then take the first and second time derivatives.



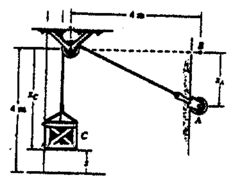
$$x_C + \sqrt{x_A^2 + (4)^2} = l$$



$$\dot{x}_C + \frac{1}{2}(x_A^2 + 16)^{-1/2}(2x_A)(\dot{x}_A) = 0$$

$$\ddot{x}_C - \frac{1}{2}(x_A^2 + 16)^{-3/2}(2x_A^2)(\dot{x}_A^2) + (x_A^2 + 16)^{-1/2}(\dot{x}_A)^2 + (x_A^2 + 16)^{-1/2}(x_A)(\ddot{x}_A) = 0$$

When $s = 1$ m, $l = 8$ m,



$x_C = 3$ m

$x_A = 3$ m

$v_A = \dot{x}_A = 2$ m/s

$a_A = \ddot{x}_A = 0$

Thus,

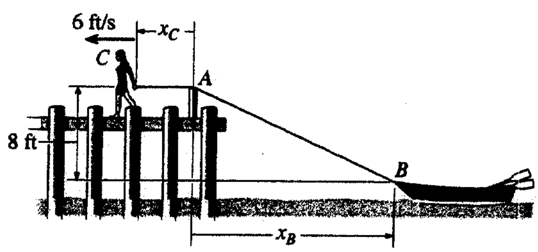
$v_C + [(3)^2 + 16]^{-1/2}(3)(2) = 0$

$v_C = -1.2$ m/s $= 1.2$ m/s $\uparrow$ **Ans**

$a_C - [(3)^2 + 16]^{-3/2}(3)^2(2)^2 + [(3)^2 + 16]^{-1/2}(2)^2 + 0 = 0$

$a_C = -0.512$ m/s^2 $= \underline{0.512 \text{ m/s}^2} \uparrow$ **Ans**

12-190. The girl at C stands near the edge of the pier and pulls in the rope *horizontally* at a constant speed of 6 ft/s. Determine how fast the boat approaches the pier at the instant the rope length AB is 50 ft.



The length l of cord is

$\sqrt{(8)^2 + x_B^2} + x_C = l$

Taking the time derivative:

$\frac{1}{2}[(8)^2 + x_B^2]^{-1/2}\, 2x_B\dot{x}_B + \dot{x}_C = 0$ (1)

$\dot{x}_C = 6$ ft/s

When $AB = 50$ ft,

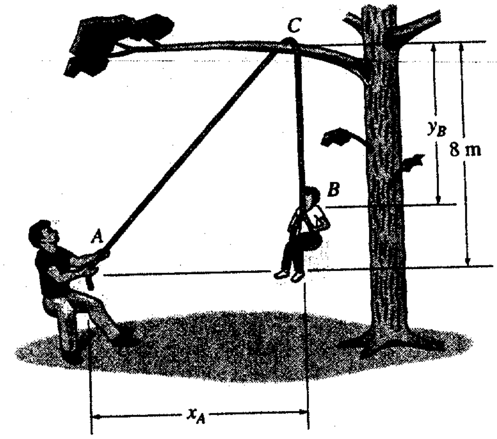
$x_B = \sqrt{(50)^2 - (8)^2} = 49.356$ ft

From Eq. (1)

$\frac{1}{2}[(8)^2 + (49.356)^2]^{-1/2}\, 2(49.356)(\dot{x}_B) + 6 = 0$

$\dot{x}_B = -6.0783 = 6.08$ ft/s $\leftarrow$ **Ans**

12-191. The man pulls the boy up to the tree limb C by walking backward. If he starts from rest when $x_A = 0$ and moves backward with a constant acceleration $a_A = 0.2\ \text{m/s}^2$, determine the speed of the boy at the instant $y_B = 4$ m. Neglect the size of the limb. When $x_A = 0$, $y_B = 8$ m, so that A and B are coincident, i.e., the rope is 16 m long.



Position - Coordinate Equation : Using the Pythagorean theorem to determine l_{AC}, we have $l_{AC} = \sqrt{x_A^2 + 8^2}$. Thus,

$$l = l_{AC} + y_B$$
$$16 = \sqrt{x_A^2 + 8^2} + y_B$$
$$y_B = 16 - \sqrt{x_A^2 + 64} \qquad [1]$$

Time Derivative : Taking the time derivative of Eq. [1] where $v_A = \frac{dx_A}{dt}$ and $v_B = \frac{dy_B}{dt}$, we have

$$v_B = \frac{dy_B}{dt} = -\frac{x_A}{\sqrt{x_A^2 + 64}} \frac{dx_A}{dt}$$
$$v_B = -\frac{x_A}{\sqrt{x_A^2 + 64}} v_A \qquad [2]$$

At the instant $y_B = 4$ m, from Eq. [1], $4 = 16 - \sqrt{x_A^2 + 64}$, $x_A = 8.944$ m. The velocity of the man at that instant can be obtained.

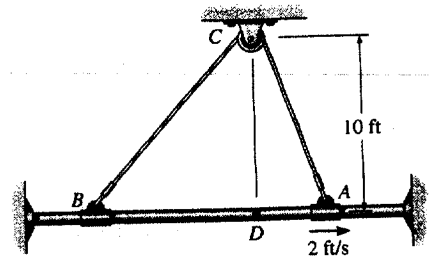
$$v_A^2 = (v_0)_A^2 + 2(a_c)_A[s_A - (s_0)_A]$$
$$v_A^2 = 0 + 2(0.2)(8.944 - 0)$$
$$v_A = 1.891 \text{ m/s}$$

Substitute the above results into Eq. [2] yields

$$v_B = -\frac{8.944}{\sqrt{8.944^2 + 64}}(1.891) = -1.41 \text{ m/s} = 1.41 \text{ m/s} \uparrow \qquad \textbf{Ans}$$

Note : The negative sign indicates that velocity v_B is in the opposite direction to that of positive y_B.

***12-192.** Collars A and B are connected to the cord that passes over the small pulley at C. When A is located at D, B is 24 ft to the left of D. If A moves at a constant speed of 2 ft/s to the right, determine the speed of B when A is 4 ft to the right of D.

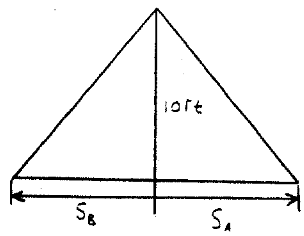


$$l = \sqrt{(24)^2 + (10)^2} + 10 = 36 \text{ ft}$$

$$\sqrt{(10)^2 + s_B^2} + \sqrt{(10)^2 + s_A^2} = 36$$

$$\frac{1}{2}\left(100 + s_B^2\right)^{-\frac{1}{2}}\left(2 s_B \dot{s}_B\right) + \frac{1}{2}\left(100 + s_A^2\right)^{-\frac{1}{2}}\left(2 s_A \dot{s}_A\right) = 0$$

$$\dot{s}_B = -\left(\frac{s_A \dot{s}_A}{s_B}\right)\left(\frac{100 + s_B^2}{100 + s_A^2}\right)^{\frac{1}{2}}$$



At $s_A = 4$,

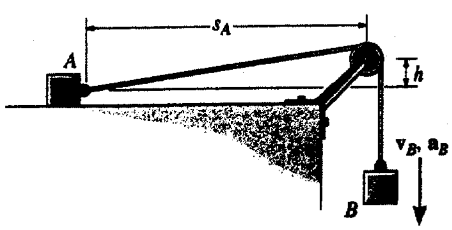
$$\sqrt{(10)^2 + s_B^2} + \sqrt{(10)^2 + (4)^2} = 36$$

$$s_B = 23.163 \text{ ft}$$

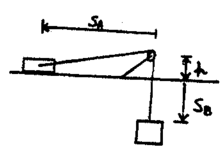
Thus,

$$\dot{s}_B = -\left(\frac{4(2)}{23.163}\right)\left(\frac{100 + (23.163)^2}{100 + 4^2}\right)^{\frac{1}{2}} = -0.809 \text{ ft/s} = 0.809 \text{ ft/s} \rightarrow \quad \textbf{Ans}$$

12-193. If block B is moving down with a velocity v_B and has an acceleration a_B, determine the velocity and acceleration of block A in terms of the parameters shown.



$$l = s_B + \sqrt{s_A^2 + h^2}$$



$$0 = \dot{s}_B + \frac{1}{2}(s_A^2 + h^2)^{-1/2}\, 2 s_A \dot{s}_A$$

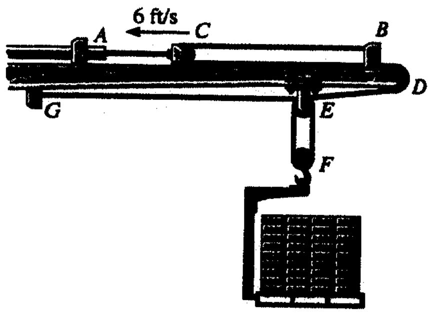
$$v_A = \dot{s}_A = \frac{-\dot{s}_B (s_A^2 + h^2)^{1/2}}{s_A}$$

$$v_A = -v_B\left(1 + \left(\frac{h}{s_A}\right)^2\right)^{1/2} \quad \textbf{Ans}$$

$$a_A = \dot{v}_A = -\dot{v}_B\left(1 + \left(\frac{h}{s_A}\right)^2\right)^{1/2} - v_B\left(\frac{1}{2}\right)\left(1 + \left(\frac{h}{s_A}\right)^2\right)^{-1/2}(h^2)(-2)(s_A)^{-3}\dot{s}_A$$

$$a_A = -a_B\left(1 + \left(\frac{h}{s_B}\right)^2\right)^{1/2} + \frac{v_A v_B h^2}{s_A^3}\left(1 + \left(\frac{h}{s_A}\right)^2\right)^{-1/2} \quad \textbf{Ans}$$

12-194. Vertical motion of the load is produced by movement of the piston at A on the boom. Determine the distance the piston or pulley at C must move to the left in order to lift the load 2 ft. The cable is attached at B, passes over the pulley at C, then D, E, F, and again around E, and is attached at G.

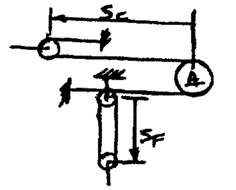


$$2s_C + 2s_F = l$$

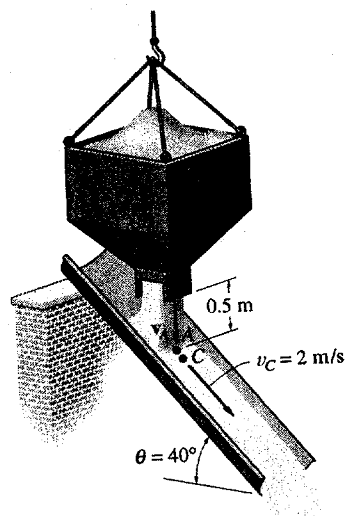
$$2\Delta s_C = -2\Delta s_F$$

$$\Delta s_C = -\Delta s_F$$

$$\Delta s_C = -(-2\text{ ft}) = 2\text{ ft} \qquad \textbf{Ans}$$



12-195. Sand falls from rest 0.5 m vertically onto a chute. If the sand then slides with a velocity of $v_C = 2$ m/s down the chute, determine the relative velocity of the sand just falling on the chute at A with respect to the sand sliding down the chute. The chute is inclined at an angle of 40° with the horizontal.



$(+\uparrow)\qquad v_A^2 = (v_A)_1^2 + 2a_c(s_A - s_{A1})$

$$v_A^2 = 0 + 2(-9.81)(0.5 - 0)$$

$$v_A = -3.1321 \text{ m/s}$$

$$\mathbf{v}_A = \mathbf{v}_C + \mathbf{v}_{A/C}$$

$$-3.1321\mathbf{j} = 2\cos 40°\mathbf{i} - 2\sin 40°\mathbf{j} + (v_{A/C})_x\mathbf{i} + (v_{A/C})_y\mathbf{j}$$

$$0 = 2\cos 40° + (v_{A/C})_x$$

$$-3.1321 = -2\sin 40° + (v_{A/C})_y$$

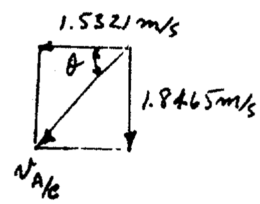
Solving,

$$(v_{A/C})_x = -1.5321$$

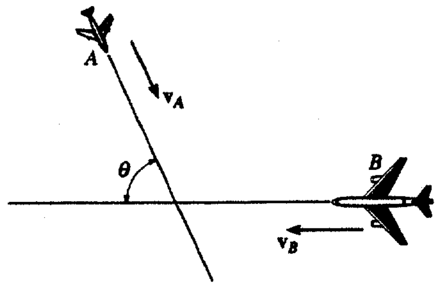
$$(v_{A/C})_y = -1.8465$$

$$v_{A/C} = \sqrt{(-1.5321)^2 + (-1.8465)^2} = 2.40 \text{ m/s} \qquad \textbf{Ans}$$

$$\theta = \tan^{-1}\left(\frac{1.8465}{1.5321}\right) = 50.3° \qquad \textbf{Ans}$$



*12-196. Two planes, A and B, are flying at the same altitude. If their velocities are $v_A = 600$ km/h and $v_B = 500$ km/h such that the angle between their straight-line courses is $\theta = 75°$, determine the velocity of plane B with respect to plane A.



$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$[500 \leftarrow] = [600 \overset{75°}{\searrow}] + \mathbf{v}_{B/A}$$

$(\overset{+}{\leftarrow})$ $\quad 500 = -600\cos 75° + (v_{B/A})_x$

$$(v_{B/A})_x = 655.29 \leftarrow$$

$(+\uparrow)$ $\quad 0 = -600\sin 75° + (v_{B/A})_y$

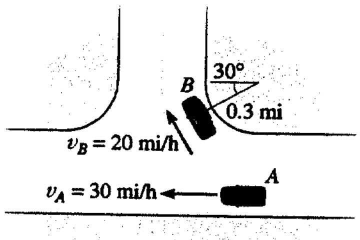
$$(v_{B/A})_y = 579.56 \uparrow$$

$$(v_{B/A}) = \sqrt{(655.29)^2 + (579.56)^2}$$

$$v_{A/B} = 875 \text{ km/h} \qquad \text{Ans}$$

$$\theta = \tan^{-1}\left(\frac{579.56}{655.29}\right) = 41.5° \qquad \text{Ans}$$

12-197. At the instant shown, cars A and B are traveling at speeds of 30 mi/h and 20 mi/h, respectively. If B is increasing its speed by 1200 mi/h^2, while A maintains a constant speed, determine the velocity and acceleration of B with respect to A.



$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$20 \overset{30°}{\nwarrow} = \underset{\leftarrow}{30} + \underset{\rightarrow}{(v_{B/A})_x} + \underset{\uparrow}{(v_{B/A})_y}$$

$(\overset{+}{\rightarrow})$ $\quad -20\sin 30° = -30 + (v_{B/A})_x$

$(+\uparrow)$ $\quad 20\cos 30° = (v_{B/A})_y$

Solving

$(v_{B/A})_x = 20 \rightarrow$

$(v_{B/A})_y = 17.32 \uparrow$

$$v_{B/A} = \sqrt{(20)^2 + (17.32)^2} = 26.5 \text{ mi/h} \qquad \text{Ans}$$

$$\theta = \tan^{-1}\left(\frac{17.32}{20}\right) = 40.9° \qquad \text{Ans}$$

$$(a_B)_n = \frac{(20)^2}{0.3} = 1333.3$$

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

$$1200 \overset{30°}{\nwarrow} + \underset{30°}{\overset{1333.3}{\swarrow}} = 0 + \underset{\rightarrow}{(a_{B/A})_x} + \underset{\uparrow}{(a_{B/A})_y}$$

$(\overset{+}{\rightarrow})$ $\quad -1200\sin 30° + 1333.3\cos 30° = (a_{B/A})_x$

$(+\uparrow)$ $\quad 1200\cos 30° + 1333.3\sin 30° = (a_{B/A})_y$

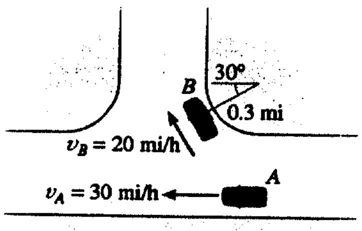
Solving

$(a_{B/A})_x = 554.7 \rightarrow \quad ; \quad (a_{B/A})_y = 1705.9 \uparrow$

$$a_{B/A} = \sqrt{(554.7)^2 + (1705.9)^2} = 1.79(10^3) \text{ mi/h}^2 \qquad \text{Ans}$$

$$\theta = \tan^{-1}\left(\frac{1705.9}{554.7}\right) = 72.0° \qquad \text{Ans}$$

12-198. At the instant shown, cars A and B are traveling at speeds of 30 mi/h and 20 mi/h, respectively. If A is increasing its speed at 400 mi/h^2 whereas the speed of B is decreasing at 800 mi/h^2, determine the velocity and acceleration of B with respect to A.



$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

$$\left[\frac{20^2}{0.3} = 1333.3\right]_{30^\circ} + [800]_{30^\circ} = [400]_{\leftarrow} + [(a_{B/A})_x]_{\rightarrow} + [(a_{B/A})_y]_{\uparrow}$$

$(\overset{+}{\rightarrow})$ $\quad 1333.3\cos 30^\circ + 800\sin 30^\circ = -400 + (a_{B/A})_x$

$$(a_{B/A})_x = 1954.7 \rightarrow$$

$(+\uparrow)$ $\quad 1333.3\sin 30^\circ - 800\cos 30^\circ = (a_{B/A})_y$

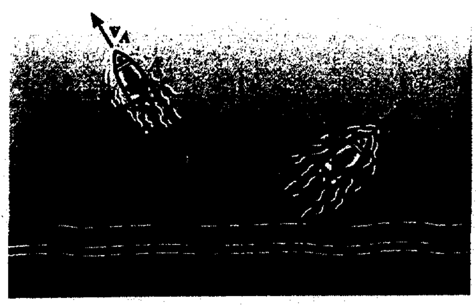
$$(a_{B/A})_y = -26.154 = 26.154 \downarrow$$

$$(a_{B/A}) = \sqrt{(1954.7)^2 + (26.154)^2}$$

$a_{B/A} = 1955$ mi/h^2 **Ans**

$\theta = \tan^{-1}\left(\frac{26.154}{1954.7}\right) = 0.767^\circ$ **Ans**

12-199. Two boats leave the shore at the same time and travel in the directions shown. If $v_A = 20$ ft/s and $v_B = 15$ ft/s, determine the speed of boat A with respect to boat B. How long after leaving the shore will the boats be 800 ft apart?



$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$-20\sin 30^\circ\mathbf{i} + 20\cos 30^\circ\mathbf{j} = 15\cos 45^\circ\mathbf{i} + 15\sin 45^\circ\mathbf{j} + \mathbf{v}_{A/B}$$

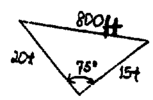
$$\mathbf{v}_{A/B} = \{-20.61\mathbf{i} - 6.714\mathbf{j}\}\ \text{ft/s}$$

$v_{A/B} = \sqrt{(-20.61)^2 + (-6.714)^2} = 21.7$ ft/s **Ans**

$\theta = \tan^{-1}\left(\frac{6.714}{20.61}\right) = 18.0^\circ$ **Ans**

$$(800)^2 = (20\,t)^2 + (15\,t)^2 - 2(20\,t)(15\,t)\cos 75^\circ$$

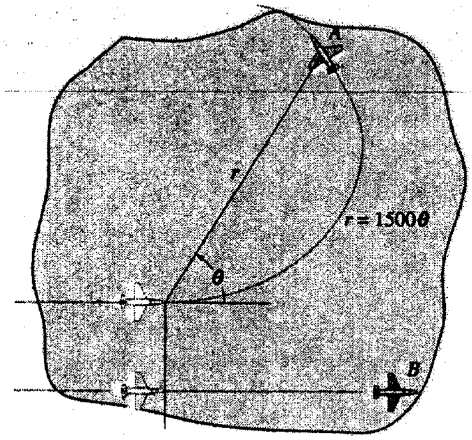
$t = 36.9$ s **Ans**



Also

$t = \frac{800}{v_{A/B}} = \frac{800}{21.68} = 36.9$ s **Ans**

***12-200.** Two planes A and B are flying side by side at a constant speed of 900 km/h. Maintaining this speed, plane A begins to travel along the spiral path $r = (1500\theta)$ km, where θ is in radians, whereas plane B continues to fly in a straight line. Determine the speed of plane A with respect to plane B when $r = 750$ km.



Relative Velocity : At $r = 750$ km, $750 = 1500\theta$, $\theta = 0.5$ rad $= 28.64°$. Here, $\tan\psi = \dfrac{r}{dr/d\theta} = \dfrac{1500\theta}{1500} = \theta = 0.5$, $\psi = 26.57°$ and $\theta + \psi = 55.21°$. Applying Eq. 12 − 34 gives

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$900\cos 55.21°\mathbf{i} + 900\sin 55.21°\mathbf{j} = 900\mathbf{i} + \mathbf{v}_{A/B}$$

$$\mathbf{v}_{A/B} = \{-386.52\mathbf{i} + 739.15\mathbf{j}\}\ \text{km/h}$$

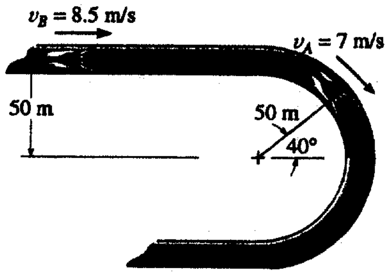
Thus, the magnitude of the relative velocity $\mathbf{v}_{A/B}$ is

$$v_{A/B} = \sqrt{(-386.52)^2 + 739.15^2} = 834\ \text{km/h} \qquad \textbf{Ans}$$

The direction of the relative velocity is the same as the drection of that for relative acceleration. Thus

$$\theta = \tan^{-1}\frac{739.15}{386.52} = 62.4° \qquad \textbf{Ans}$$

12-201. At the instant shown, the bicyclist at A is traveling at 7 m/s around the curve on the race track while increasing his speed at 0.5 m/s^2. The bicyclist at B is traveling at 8.5 m/s along the straight-a-way and increasing his speed at 0.7 m/s^2. Determine the relative velocity and relative acceleration of A with respect to B at this instant.



$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$

$[7 \underset{40°}{\searrow}] = [8.5 \rightarrow] + [(v_{A/B})_x \rightarrow] + [(v_{A/B})_y \downarrow]$

$(\overset{+}{\rightarrow})\quad 7\sin 40° = 8.5 + (v_{A/B})_x$

$(+\downarrow)\quad 7\cos 40° = (v_{A/B})_y$

Thus,

$(v_{A/B})_x = 4.00 \text{ m/s} \leftarrow$

$(v_{A/B})_y = 5.36 \text{ m/s} \downarrow$

$(v_{A/B}) = \sqrt{(4.00)^2 + (5.36)^2}$

$v_{A/B} = 6.69$ m/s **Ans**

$\theta = \tan^{-1}\left(\frac{5.36}{4.00}\right) = 53.3°$ **Ans**

$(a_A)_n = \frac{7^2}{50} = 0.980 \text{ m/s}^2$

$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$

$[0.980 \underset{40°}{\swarrow}] + [0.5 \underset{40°}{\searrow}] = [0.7 \rightarrow] + [(a_{A/B})_x \rightarrow] + [(a_{A/B})_y \downarrow]$

$(\overset{+}{\rightarrow})\quad -0.980\cos 40° + 0.5\sin 40° = 0.7 + (a_{A/B})_x$

$(a_{A/B})_x = 1.129 \text{ m/s}^2 \leftarrow$

$(+\downarrow)\quad 0.980\sin 40° + 0.5\cos 40° = (a_{A/B})_y$

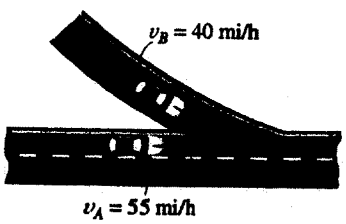
$(a_{A/B})_y = 1.013 \text{ m/s}^2 \downarrow$

$(a_{A/B}) = \sqrt{(1.129)^2 + (1.013)^2}$

$a_{A/B} = 1.52 \text{ m/s}^2$ **Ans**

$\theta = \tan^{-1}\left(\frac{1.013}{1.129}\right) = 41.9°$ **Ans**

12-202. At the instant shown, cars A and B are traveling at speeds of 55 mi/h and 40 mi/h, respectively. If B is increasing its speed by 1200 mi/h^2, while A maintains a constant speed, determine the velocity and acceleration of B with respect to A. Car B moves along a curve having a radius of curvature of 0.5 mi.



$\mathbf{v}_B = -40\cos 30°\mathbf{i} + 40\sin 30°\mathbf{j} = \{-34.64\mathbf{i} + 20\mathbf{j}\}$ mi/h

$\mathbf{v}_A = \{-55\mathbf{i}\}$ mi/h

$\mathbf{v}_{B/A} = \mathbf{v}_B - \mathbf{v}_A$

$= (-34.64\mathbf{i} + 20\mathbf{j}) - (-55\mathbf{i}) = \{20.36\mathbf{i} + 20\mathbf{j}\}$ mi/h

$v_{B/A} = \sqrt{20.36^2 + 20^2} = 28.5$ mi/h **Ans**

$\theta = \tan^{-1}\dfrac{20}{20.36} = 44.5°$ **Ans**

$(a_B)_n = \dfrac{v_A^2}{\rho} = \dfrac{40^2}{0.5} = 3200$ mi/h^2 $\quad (a_B)_t = 1200$ mi/h^2

$\mathbf{a}_B = (3200\cos 60° - 1200\cos 30°)\mathbf{i} + (3200\sin 60° + 1200\sin 30°)\mathbf{j}$
$= \{560.77\mathbf{i} + 3371.28\mathbf{j}\}$ mi/h^2
$\mathbf{a}_A = 0$

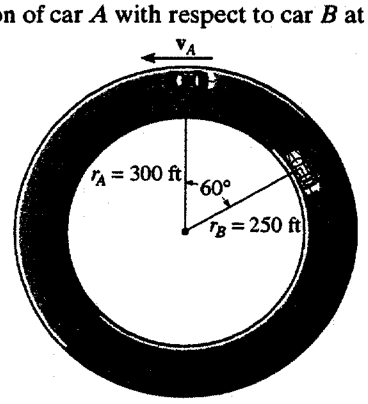
$\mathbf{a}_{B/A} = \mathbf{a}_B - \mathbf{a}_A$

$= \{560.77\mathbf{i} + 3371.28\mathbf{j}\} - 0 = \{560.77\mathbf{i} + 3371.28\mathbf{j}\}$ mi/h^2

$a_{B/A} = \sqrt{(560.77)^2 + (3371.28)^2} = 3418$ mi/h^2 **Ans**

$\theta = \tan^{-1}\dfrac{3371.28}{560.77} = 80.6°$ **Ans**

12-203. Cars A and B are traveling around the circular race track. At the instant shown, A has a speed of 90 ft/s and is increasing its speed at the rate of 15 ft/s^2, whereas B has a speed of 105 ft/s and is decreasing its speed at 25 ft/s^2. Determine the relative velocity and relative acceleration of car A with respect to car B at this instant.



$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$

$-90\mathbf{i} = -105\sin 30°\mathbf{i} + 105\cos 30°\mathbf{j} + \mathbf{v}_{A/B}$

$\mathbf{v}_{A/B} = \{-37.5\mathbf{i} - 90.93\mathbf{j}\}$ ft/s

$v_{A/B} = \sqrt{(-37.5)^2 + (-90.93)^2} = 98.4$ ft/s **Ans**

$\theta = \tan^{-1}(\dfrac{90.93}{37.5}) = 67.6°$ **Ans**

$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$

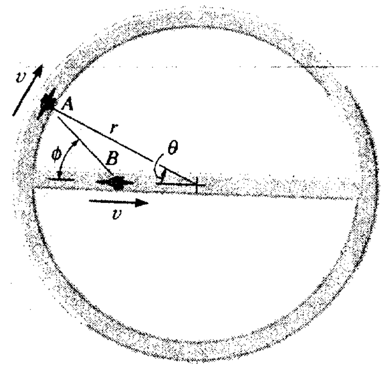
$-15\mathbf{i} - \dfrac{(90)^2}{300}\mathbf{j} = 25\cos 60°\mathbf{i} - 25\sin 60°\mathbf{j} - 44.1\sin 60°\mathbf{i} - 44.1\cos 60°\mathbf{j} + \mathbf{a}_{A/B}$

$\mathbf{a}_{A/B} = \{10.69\mathbf{i} + 16.70\mathbf{j}\}$ ft/s^2

$a_{A/B} = \sqrt{(10.69)^2 + (16.70)^2} = 19.8$ ft/s^2 **Ans**

$\theta = \tan^{-1}(\dfrac{16.70}{10.69}) = 57.4°$ **Ans**

***12-204.** The two cyclists A and B travel at the same constant speed v. Determine the speed of A with respect to B if A travels along the circular track, while B travels along the diameter of the circle.



$$\mathbf{v}_A = v\sin\theta\mathbf{i} + v\cos\theta\mathbf{j} \qquad \mathbf{v}_B = v\mathbf{i}$$

$$\mathbf{v}_{A/B} = \mathbf{v}_A - \mathbf{v}_B$$

$$= (v\sin\theta\mathbf{i} + v\cos\theta\mathbf{j}) - v\mathbf{i}$$

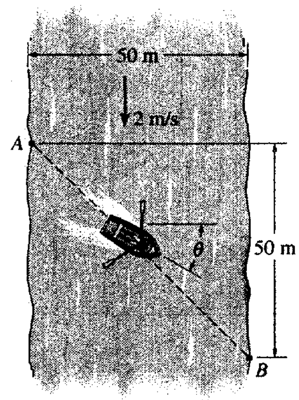
$$= (v\sin\theta - v)\mathbf{i} + v\cos\theta\mathbf{j}$$

$$v_{A/B} = \sqrt{(v\sin\theta - v)^2 + (v\cos\theta)^2}$$

$$= \sqrt{2v^2 - 2v^2\sin\theta}$$

$$= v\sqrt{2(1-\sin\theta)} \qquad \textbf{Ans}$$

12-205. A man can row a boat at 5 m/s in still water. He wishes to cross a 50-m-wide river to point B, 50 m downstream. If the river flows with a velocity of 2 m/s, determine the speed of the boat and the time needed to make the crossing.



Relative Velocity :

$$\mathbf{v}_b = \mathbf{v}_r + \mathbf{v}_{b/r}$$
$$v_b \sin 45°\mathbf{i} - v_b\cos 45°\mathbf{j} = -2\mathbf{j} + 5\cos\theta\mathbf{i} - 5\sin\theta\mathbf{j}$$

Equating **i** and **j** component, we have

$$v_b \sin 45° = 5\cos\theta \qquad [1]$$
$$-v_b\cos 45° = -2 - 5\sin\theta \qquad [2]$$

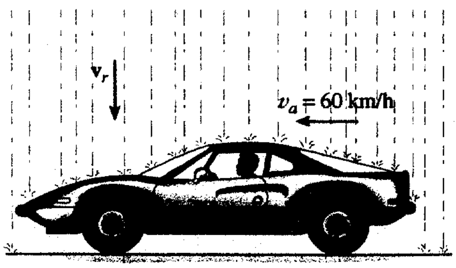
Solving Eqs.[1] and [2] yields

$$\theta = 28.57°$$
$$v_b = 6.210 \text{ m/s} = 6.21 \text{ m/s} \qquad \textbf{Ans}$$

Thus, the time t required by the boat to travel from point A to B is

$$t = \frac{s_{AB}}{v_b} = \frac{\sqrt{50^2 + 50^2}}{6.210} = 11.4 \text{ s} \qquad \textbf{Ans}$$

12-206. A passenger in an automobile observes that raindrops make an angle of 30° with the horizontal as the auto travels forward with a speed of 60 km/h. Compute the terminal (constant) velocity $\mathbf{v}_r$ of the rain if it is assumed to fall vertically.



$\mathbf{v}_r = \mathbf{v}_a + \mathbf{v}_{r/a}$

$-v_r\mathbf{j} = -60\mathbf{i} + v_{r/a}\cos 30°\mathbf{i} - v_{r/a}\sin 30°\mathbf{j}$

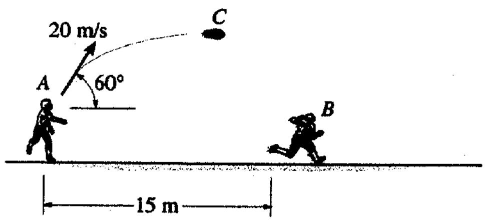
$(\overset{+}{\rightarrow})\quad 0 = -60 + v_{r/a}\cos 30°$

$(+\uparrow)\quad -v_r = 0 - v_{r/a}\sin 30°$

$v_{r/a} = 69.3$ km/h

$v_r = 34.6$ km/h **Ans**

12-207. At a given instant the football player at A throws a football C with a velocity of 20 m/s in the direction shown. Determine the constant speed at which the player at B must run so that he can catch the football at the same elevation at which it was thrown. Also calculate the relative velocity and relative acceleration of the football with respect to B at the instant the catch is made. Player B is 15 m away from A when A starts to throw the football.



Ball:

$(\overset{+}{\rightarrow})\ s = s_0 + v_0 t$

$s_C = 0 + 20\cos 60°\, t$

$(+\uparrow)\quad v = v_0 + a_c t$

$-20\sin 60° = 20\sin 60° - 9.81\, t$

$t = 3.53$ s

$s_C = 35.31$ m

Player B:

$(\overset{+}{\rightarrow})\quad s = s_0 + v_0 t$

$s_B = 0 + v_B\, t$

Require,

$35.31 = 15 + v_B(3.53)$

$v_B = 5.75$ m/s **Ans**

At the time of the catch

$(v_C)_x = 20\cos 60° = 10$ m/s $\rightarrow$

$(v_C)_y = 20\sin 60° = 17.32$ m/s $\downarrow$

$\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C/B}$

$10\mathbf{i} - 17.32\mathbf{j} = 5.75\mathbf{i} + (v_{C/B})_x\mathbf{i} + (v_{C/B})_y\mathbf{j}$

$(\overset{+}{\rightarrow})\quad 10 = 5.75 + (v_{C/B})_x$

$(+\uparrow)\quad -17.32 = (v_{C/B})_y$

$(v_{C/B})_x = 4.25$ m/s $\rightarrow$

$(v_{C/B})_y = 17.32$ m/s $\downarrow$

$v_{C/B} = \sqrt{(4.25)^2 + (17.32)^2} = 17.8$ m/s **Ans**

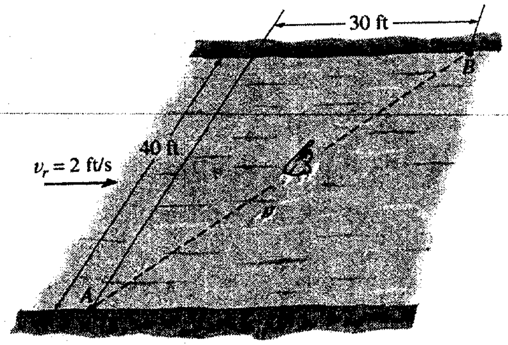
$\theta = \tan^{-1}\left(\frac{17.32}{4.25}\right) = 76.2°$ **Ans**

$\mathbf{a}_C = \mathbf{a}_B + \mathbf{a}_{C/B}$

$-9.81\mathbf{j} = 0 + \mathbf{a}_{C/B}$

$a_{C/B} = 9.81$ m/s^2 $\downarrow$ **Ans**

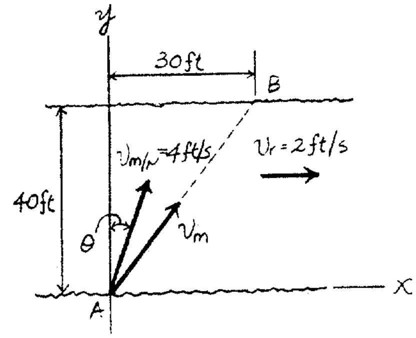
*12-208. A man can swim at 4 ft/s in still water. He wishes to cross the 40-ft-wide river to point B, 30 ft downstream. If the river flows with a velocity of 2 ft/s, determine the speed of the man and the time needed to make the crossing. *Note*: While in the water he must not direct himself towards point B to reach this point. Why?



Relative Velocity :

$$\mathbf{v}_m = \mathbf{v}_r + \mathbf{v}_{m/r}$$

$$\frac{3}{5}v_m\mathbf{i} + \frac{4}{5}v_m\mathbf{j} = 2\mathbf{i} + 4\sin\theta\mathbf{i} + 4\cos\theta\mathbf{j}$$



Equating the **i** and **j** components, we have

$$\frac{3}{5}v_m = 2 + 4\sin\theta \qquad [1]$$

$$\frac{4}{5}v_m = 4\cos\theta \qquad [2]$$

Solving Eqs. [1] and [2] yields

$$\theta = 13.29°$$

$$v_b = 4.866 \text{ ft/s} = 4.87 \text{ ft/s} \qquad \textbf{Ans}$$

Thus, the time t required by the boat to travel from point A to B is

$$t = \frac{s_{AB}}{v_b} = \frac{\sqrt{40^2 + 30^2}}{4.866} = 10.3 \text{ s} \qquad \textbf{Ans}$$

In oder for the man to reached point B, the man has to direct himself at an angle $\theta = 13.3°$ with y axis.