Pg no 1	NAME: ADNAN SHAH MUZAVOR ROLL NO: 20C005 PR NO: 2020207863 SEAT NO: 2101305	SUBJECT: MATHS-1 REGULATION: RC 19-20 Total Pages: 21 Sign:	Date: 08/03/2021

Part A

A.1. a $\int_0^\infty \frac{x^4}{4^x}\,dx$

$\rightarrow \int_0^\infty x^4 4^{-x}\,dx$ — ①

$\rightarrow$ applying $e^{\log}$ to eq ①

$\rightarrow \int_0^\infty x^4 e^{\log 4^{-x}}\,dx$

$\rightarrow \int_0^\infty x^4 e^{-x\log 4}\,dx$ — ②

$\rightarrow$ we know $\Gamma n = \int_0^\infty e^{-x} x^{n-1}\,dx$ $\rightarrow$ gamma eq — ③

$\rightarrow$ Comparing ② with ③ we get substitution required

$\rightarrow \int_0^\infty x^4 e^{-x\log 4}\,dx$

$\rightarrow$ $t = x\log 4$

$x\rightarrow 0,\ t\rightarrow 0$

$x\rightarrow \infty,\ t\rightarrow \infty$

$x = \frac{t}{\log 4}$ — ④ , $dx = \frac{dt}{\log 4}$ — ⑤

substituting ④ and ⑤ in eq ② we get

$\rightarrow \int_0^\infty x^4 e^{-x\log 4}\,dx$

$\rightarrow \int_0^\infty \frac{(t)^4}{(\log 4)^4} e^{-t} \frac{dt}{\log 4}$

Continued →

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Part A

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A.1.a continued →

$$\int_0^\infty \frac{(t)^4}{(\log 4)^4} e^{-t} \frac{dt}{(\log 4)}$$

→ $\frac{1}{(\log 4)^5}\int_0^\infty (t)^4 e^{-t} dt$ — (6)

→ we know $\Gamma n = \int_0^\infty e^{-x} (x)^{n-1} dx$ → gamma function — (A)

→ Comparing (6) with (A)

→ we get $n-1 = 4$
$n = 5$

→ $\therefore \frac{1}{(\log 4)^5}\int_0^\infty (t)^4 e^{-t} dt$

→ $\frac{1}{(\log 4)^5} \times \Gamma n$

→ $\frac{1}{(\log 4)^5} \times \Gamma 5$ — (7)

→ we know $\Gamma n+1 = n!$
$\therefore \Gamma 5 = \Gamma 4+1 = 4!$ — (8)

→ sub (8) in (7)

→ $\frac{1}{(\log 4)^5} \times 4!$

→ $\frac{24}{(\log 4)^5}$

Asm.

08/03/2021

Part A

A.1. a) $\sqrt{9.12}$

we know $f(x+h) = \frac{f(x)h^0}{0!} + \frac{f'(x)(h)^1}{1!} + \frac{f^2(x)(h)^2}{2!} + \frac{f^3(x)h^3}{3!}, \ldots$

Comparing with $f(x+h)$, we get

$x = 9$, $h = 0.12$ and $f(x) = \sqrt{x}$

→ $f(x) = \sqrt{x}$

$f(9) = \sqrt{9} = 3$

→ $f'(x) = \frac{1}{2\sqrt{x}}$

$f'(9) = \frac{1}{2\sqrt{9}} = \frac{1}{6}$

→ $f'(x) = \frac{1}{2}(x)^{-1/2}$

$f^2(x) = \frac{-1}{4}(x)^{-3/2}$

$f^2(x) = \frac{-1}{4 \times \sqrt{x} \times x}$

$f^2(9) = \frac{-1}{4 \times \sqrt{9} \times 9}$

$f^2(9) = \frac{-1}{108}$

→ Continued

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Asm.

Part A

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A.1.a Contd

→ $f^2(x) = \frac{-1}{4}(x)^{-3/2}$

→ $f^3(x) = \frac{-1}{4} \times \frac{-3}{2} \times (x)^{-5/2}$

→ $f^3(x) = \frac{+3}{8 \times \sqrt{x} \times x \times x}$

→ $f^3(9) = \frac{\cancel{3} \times 1}{8 \times \sqrt{9} \times \cancel{3} \times 3 \times 9}$

$= \frac{1}{8 \times 3 \times 3 \times 9}$

$= \frac{1}{72} \quad \frac{1}{648}$

→ sub values in main eq

→ $f(x+h) = \frac{f(x)(h)^0}{0!} + \frac{f'(x)(h)^1}{1!} + \frac{f^2(x)(h)^2}{2!} + \frac{f^3(x)(h)^3}{3!} + \ldots$

→ $f(9+0.12) = f(9) + f'(9)(0.12) + \frac{f^2(9) \times (0.12)^2}{2} + \frac{f^3(9)(0.12)^3}{6} \ldots$

→ $= 3 + \frac{0.12}{6} + \frac{-1}{108} \times \frac{(0.12)^2}{2} + \frac{1}{648} \times \frac{(0.12)^3}{6} \ldots$

→ $= 3 + 0.02 + (-) 0.000066 + 0.00000044$

→ $= 3.0199$

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Asm.

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Part A

A.1.b.i) $\sum_{n=1}^{n=\infty} \sin(1/n)$

→ applying limit to given series

→ $\lim_{n\to\infty} \sin(1/n)$

→ Applying comparison theorem / test

→ $U_n = \sin(1/n)$

→ $V_n = \frac{1}{n}$

→ By theorem

→ $\lim_{n\to\infty} \frac{U_n}{V_n}$

→ $\lim_{n\to\infty} \sin(1/n) \times n$

→ $\lim_{n\to\infty} \frac{\sin(1/n)}{(1/n)}$

$\lim_{n\to\infty}$ hence $\frac{1}{\infty} \to 0$ we know $\frac{\sin(0)}{0} = 1$

~~we know 1/n when n→∞ is 0~~

→ $= 1$

→ 1 is non zero hence U_n and V_n behave alike

→ $\sum V_n = \frac{1}{n}$, n^1, power of $n = 1$, $1 \leq 1$

$\therefore$ By $\left[\frac{1}{n^p} \text{ test}\right]$ $\frac{1}{n}$ is divergent

→ Hence by comparison theorem

→ $\sum U_n = \sin(1/n)$ is also divergent

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Asm.

Part A

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A.1. b.(ii) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+\sqrt{n+1}}$

→ ~~By~~ Applying limit

→ $\lim_{n\to\infty} \frac{1}{\sqrt{n}+\sqrt{n+1}}$

→ Applying comparison test / theorem

→ $U_n = \frac{1}{\sqrt{n}+\sqrt{n+1}}$

→ $V_n = \frac{1}{n^{1/2-0}} = \frac{1}{\sqrt{n}}$

→ ∴ By comparison theorem

→ $\lim_{n\to\infty} \frac{U_n}{V_n} = \lim_{n\to\infty} \frac{1}{\sqrt{n}+\sqrt{n+1}} \times \frac{\sqrt{n}}{1}$

→ $= \lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n}+\sqrt{n+1}}$

→ $= \frac{\div}{\div}$ by $\sqrt{n}$

→ $= \lim_{n\to\infty} \frac{\sqrt{n/n}}{\sqrt{n/n}+\sqrt{1+1/n}}$

→ $= \lim_{n\to\infty} \frac{\sqrt{1}}{\sqrt{1}+\sqrt{1+1/n}}$

→ = apply limit $\boxed{1/n = 0}$

→ $= \frac{1}{1+1} = \frac{1}{2}$

→ $\frac{1}{2}$ is non zero hence U_n and V_n behave alike

→ $\sum V_n = \frac{1}{n^{1/2}}$, $n^{1/2}$, power of $n = 1/2$, $1/2 \leq 1$, ∴ By p-test $\left[\frac{1}{n^p}\right]$

→ $\sum V_n$ is divergent hence $\sum U_n = \frac{1}{\sqrt{n}+\sqrt{n+1}}$ is also divergent

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Part A

A.1.b.(iii) $\frac{3}{4} + \frac{3.4}{4.6} + \frac{3.4.5}{4.6.8} + \dots \infty$

$\frac{3}{4} + \frac{3.4}{4.6} + \frac{3.4.5}{4.6.8} + \dots +$

3.4.5.6...	4.6.8....
$d=1$	$d=2$
$a=3$	$a=4$
$a_n = a+(n-1)d$	$a_n = a+(n-1)d$
$= 3+(n-1)1$	$= 4+(n-1)2$
$= 3+n-1$	$= 4+2n-2$
$= 2+n$	$= 2+2n$

$\therefore \frac{3}{4} + \frac{3.4}{4.6} + \frac{3.4.5}{4.6.8} + \dots \frac{3.4.5\dots(2+n)}{4.6.8\dots(2+2n)}$

Applying DART (D'Alemberts Ratio Test)

$U_n = \frac{3.4.5 \dots (2+n)}{4.6.8 \dots (2+2n)}$

$U_{n+1} = \frac{3.4.5 \dots (2+n)(3+n)}{4.6.8 \dots (2+2n)(4+2n)}$

$2(2(n+1))$
$2+2n$
$2+2(n+1)$
$2+2n+2$
$4+2n$

By DART (D'Alemberts Ratio Test)

$\lim_{n\to\infty} \frac{U_{n+1}}{U_n}$

$\lim_{n\to\infty} \frac{\cancel{3.4.5\dots(2+n)}(3+n)}{\cancel{4.6.8\dots(2+2n)}(4+2n)} \times \frac{\cancel{4.6.8\dots(2+2n)}}{\cancel{3.4.5\dots}(2+n)}$

$\lim_{n\to\infty} \frac{3+n}{4+2n}$

continued

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Asm.

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A.1.b.(iii) Continued (Part n)

$\rightarrow \lim_{n\to\infty} \frac{3+n}{4+2n}$

$\rightarrow$ $\div$ by n

$\rightarrow \lim_{n\to\infty} \frac{3/n+n/n}{4/n+2n/n}$

$\rightarrow \lim_{n\to\infty} \frac{3/n+1}{4/n+2}$

applying limit

$\rightarrow \lim_{n\to\infty} \frac{0+1}{0+2}$

$\rightarrow \frac{1}{2}$

$\rightarrow = 0.5$

$0.5 < 1$ $\therefore$ by D'Alembert's ratio test given series is convergent

Part B

Q4. a.(i)

$\frac{dy}{dx} = \frac{xy^2}{\sqrt{1+x^2}}$

$\frac{dy}{y^2} = \frac{x\,dx}{\sqrt{1+x^2}}$

applying integral on both sides

$\int dy\, y^{-2} = \int \frac{x\,dx}{\sqrt{1+x^2}}$

Continued $\rightarrow$

Part B

08/03/2021

B.4.a-i Continued

$\int y^{-2} dy = \int \frac{x\,dx}{\sqrt{1+x^2}}$ — (A)

for RHS

$\int \frac{x\,dx}{\sqrt{1+x^2}}$ — (1)

Let $1+x^2 = t$ — (3)

$dt = 2x\,dx$ — (4)

$\frac{1}{2}\int \frac{2x\,dx}{\sqrt{1+x^2}}$ — (5)

sub (3) and (4) in (5)

$\frac{1}{2}\int \frac{2x\,dx}{\sqrt{1+x^2}}$

$\frac{1}{2}\int \frac{dt}{\sqrt{t}}$ — (6)

sub (6) again in eq (A)

$\int y^{-2} dy = \frac{1}{2}\int dt\,(t)^{-1/2}$

$\frac{y^{-2+1}}{-2+1} = \frac{1}{2} \times \frac{(t)^{-1/2+1}}{(-1/2+1)} + C$

$\frac{y^{-1}}{-1} = \frac{1}{\cancel{2}} \times \frac{\sqrt{t}}{1} \times \cancel{2} + C$

$-\frac{1}{y} = \sqrt{t} + C$

sub $t = 1+x^2$ back from eq (3)

$-\frac{1}{y} = \sqrt{1+x^2} + C$

$\sqrt{1+x^2} + \frac{1}{y} + C = 0 \rightarrow \boxed{y\sqrt{1+x^2} + 1 + yC = 0}$

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Part B

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B.4.a.ii

$$x\frac{dy}{dx} + y = y^2 \log_e x$$

$\div$ by x

$$\frac{dy}{dx} + \frac{y}{x} = y^2 \frac{\log_e x}{x} \quad — (A) \rightarrow \text{Bernoulli's equation}$$

$\div$ by y^2

$$\frac{1}{y^2}\frac{dy}{dx} + \frac{1}{x} \times \frac{y}{y^2} = \frac{\log_e x}{x}$$

$$\frac{1}{y^2}\frac{dy}{dx} + \frac{1}{x} \times \frac{1}{y^1} = \frac{\log_e x}{x} \quad — (1)$$

Let $z = y^{-1}$ — (2)

$$\frac{dz}{dx} = -1 y^{-2}\frac{dy}{dx}$$

$$-\frac{dz}{dx} = \frac{1}{y^2}\frac{dy}{dx} \quad — (3)$$

sub (2) and (3) in (1)

$$\frac{1}{y^2}\frac{dy}{dx} + \frac{1}{x} y^{-1} = \frac{\log_e x}{x}$$

$$-\frac{dz}{dx} + \frac{1}{x} \times z = \frac{\log_e x}{x} \quad \times(-1)$$

$$\frac{dz}{dx} + \left(\frac{-1}{x}\right) z = -\frac{\log_e x}{x}$$

$\hookrightarrow$ linear eq of type $\frac{dz}{dx} + P(x)z = Q(x)$

continued $\rightarrow$

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B.3.a.ii continued

Part B

Asm

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$$\frac{dz}{dx} + \left(\frac{-1}{x}\right)z = -\frac{\log_e x}{x} \rightarrow \text{linear equation}$$

~~Fin~~ Finding Integrating factor, $\boxed{P = -\frac{1}{x}}$

$$I.F = e^{\int P dx}$$

$$= e^{\int -\frac{dx}{x}}$$

$$= e^{-\int \frac{dx}{x}}$$

$$= e^{-\log x}$$

$$= e^{\log x^{-1}}$$

$$= x^{-1}$$

$$= 1/x$$

Hence solution is given by

$$(I.F)(z) = \int Q(x)(I.F)\,dx$$

$$Q(x) = \frac{-\log_e x}{x}, \quad I.F = 1/x$$

$$\frac{1}{x} \times z = -\int \frac{\log_e x}{x} \times \frac{1}{x}\,dx$$

$$\frac{z}{x} = -\int \log_e x \; x^{-2}\,dx \quad \text{—— (A)}$$

By Applying integration by parts

LIATE $\therefore \log_e x > x^{-2}$

Solving $\int x^{-2} \log x \, dx$ by parts

$\longrightarrow$

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Part B

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B.2.a.(ii) continued

$\rightarrow \int \log_e x \, x^{-2} dx$

$\rightarrow \log_e x \frac{x^{-1}}{-1} - \int \frac{x^{-1}}{(-1)} \times \frac{1}{x} dx$

$\rightarrow \log_e x \left(\frac{-1}{x}\right) + \int \frac{1}{x^2} dx$

$\rightarrow -\frac{\log_e x}{x} + \int x^{-2} dx$

$\rightarrow -\frac{\log_e x}{x} + \frac{x^{-1}}{-1} + C$

$\rightarrow -\frac{\log_e x}{x} - \frac{1}{x} + C$ — (1)

$\rightarrow$ sub (1) back in (A)

$\rightarrow \frac{z}{x} = -\int \frac{\log_e x}{x^2} dx$

$\rightarrow \frac{z}{x} = -\int \log_e x . x^{-2} dx$

$\rightarrow \frac{z}{x} = -\left[-\frac{\log_e x}{x} - \frac{1}{x} + C\right]$

$\rightarrow \frac{z}{x} = +\frac{\log_e x}{x} + \frac{1}{x} + C$

$\rightarrow \boxed{z = \log_e x + 1 + xC}$

we know $z = 1/y$

$\rightarrow \frac{1}{y} = \log_e x + 1 + xC$

$\rightarrow 1 = y \log_e x + y + xyC$

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Asm.

Part B

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3.4.a.(iii) $(x^2y^2+1)y\,dx + (3-2x^2y^2)dy\,x = 0$

$x^2y^3dx + y\,dx + 3x\,dy - 2x^3y^2\,dy = 0$

$x^2y^3dx - 2x^3y^2\,dy + y\,dx + 3x\,dy = 0$

$x^2y^2(y\,dx - 2x\,dy) + x^0y^0(y\,dx + 3x\,dy) = 0$

$m_1 = 2 \quad n_1 = 2 \quad a_1 = 1 \quad b_1 = -2$

$m_2 = 0 \quad n_2 = 0 \quad a_2 = 1 \quad b_2 = 3$

$\frac{2+h+1}{1} = \frac{2+k+1}{-2}$

$-4-2h-2 = 2+k+1$

$-2h-6 = 3+k$

$2h+k = -9 \quad - (1)$

$\frac{0+h+1}{1} = \frac{0+k+1}{3}$

$3h+3 = k+1$

$3h-k = -2 \quad - (2)$

solving (1) and (2)

$h = -\frac{11}{5}, \quad k = -\frac{23}{5}$

$\therefore IF = x^h y^k$

$= x^{-11/5}\, y^{-23/5}$

$\therefore$ solving eq $\rightarrow$

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Asm

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4. a.(iii) continued. →

$(x^2y^3 \cdot x^{-11/5} \cdot y^{-23/5} + y \cdot x^{-11/5} y^{-23/5})\,dx$
$+ (3x \cdot x^{-11/5} \cdot y^{-23/5} - 2x^3y^2 x^{-11/5} \cdot y^{-23/5})\,dy = 0$

$\rightarrow ((x)^{-1/5} y^{-8/5} + x^{-11/5} y^{-18/5})\,dx +$
$(3x^{-6/5} y^{-23/5} + -2x^{5/5} y^{-13/5})\,dy = 0$

$\rightarrow \int (y^{-8/5} x^{-1/5} + y^{-18/5} x^{-11/5})\,dx + \int 0\,dy = C$

$\rightarrow y^{-8/5} \times \frac{(x)^{4/5}}{4/5} + y^{-18/5} \frac{(x)^{-6/5}}{-6/5} + 0 = C$

$\rightarrow \boxed{\frac{5}{4} y^{-8/5} (x)^{4/5} + -\frac{5}{6} (x)^{-6/5} (y)^{-18/5} = C}$

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Part B

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B.4.6

$U = x - y$

$V = xy$

$\frac{dU}{dx} = +1$ | $\frac{dV}{dx} = y$

$\frac{du}{dy} = -1$ | $\frac{dV}{dy} = x$

$\frac{dz}{dx} = \frac{dz}{du} \cdot \frac{du}{dx} + \frac{dz}{dV} \cdot \frac{dV}{dx}$

$\frac{dz}{dx} = \frac{dz}{dU}(1) + \frac{dz}{dV} \cdot (y)$ —(1)

$\frac{d}{dx}\left(\frac{dz}{dx}\right) = \frac{d}{dx}\left(\frac{dz}{du}\right) + y\frac{d}{dx}\left(\frac{dz}{dV}\right)$

$= \frac{d}{du}\left(\frac{dz}{du}\right) \cdot \frac{du}{dx} + \frac{d}{dV}\left(\frac{dz}{du}\right) \cdot \frac{dV}{dx} +$

$y\left[\frac{d}{du}\left(\frac{dz}{dV}\right)\frac{du}{dx} + \frac{d}{dV}\left(\frac{dz}{dV}\right) \cdot \frac{dV}{dx}\right]$

$= \frac{d^2z}{du^2} + y\frac{d^2z}{du\,dV} + y\frac{d^2z}{du\,dV} + y^2\frac{d^2z}{dV^2}$

$\frac{d^2z}{dx^2} = \frac{d^2z}{du^2} + 2y\frac{d^2z}{du\,dV} + y^2\frac{d^2z}{dV^2}$ —(2)

$\frac{dz}{dy} = \frac{dz}{dU} \cdot \frac{du}{dy} + \frac{dz}{dV} \cdot \frac{dV}{dy}$

$= \frac{dz}{du}(-1) + \frac{dz}{dV}(x)$ —(3)

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Part B

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B.4.b continued

$$\frac{d}{dy}\left(\frac{dz}{dy}\right) = -\frac{d}{dy}\left(\frac{dz}{du}\right) + x\frac{d}{dy}\left(\frac{dz}{dv}\right)$$

$$= -\left[\frac{d}{du}\left(\frac{dz}{du}\right)\frac{du}{dy} + \frac{d}{dv}\left(\frac{dz}{du}\right)\frac{dv}{dy}\right]$$

$$+ x\left[\frac{d}{du}\left(\frac{dz}{dv}\right)\frac{du}{dy} + \frac{d}{dv}\left(\frac{dz}{dv}\right)\left(\frac{dv}{dy}\right)\right]$$

$$\rightarrow \frac{d^2z}{dy^2} = -\left[\frac{d^2z}{du^2}(-1) + \frac{d^2z}{dudv}(x)\right] + (-x)\frac{d^2z}{dudv} + x^2\frac{d^2z}{dv^2}$$

$$\rightarrow \frac{d^2z}{dy^2} = \frac{d^2z}{du^2} - 2x\frac{d^2z}{dudv} + x^2\frac{d^2z}{dv^2} \quad - (4)$$

$$\rightarrow x\frac{d^2z}{dx^2} = x\frac{d^2z}{du^2} + 2xy\frac{d^2z}{dudv} + xy^2\frac{d^2z}{dv^2} \quad - (5)$$

$$\rightarrow y\frac{d^2z}{dy^2} = y\frac{d^2z}{du^2} - 2xy\frac{d^2z}{dudv} + x^2y\frac{d^2z}{dv^2} \quad - (6)$$

Adding (5) and (6)

$$\rightarrow x\frac{d^2z}{dx^2} + y\frac{d^2z}{dy^2} = (x+y)\frac{d^2z}{du^2} + (xy^2 + x^2y)\frac{d^2z}{dv^2}$$

$$= (x+y)\frac{d^2z}{du^2} + xy(y+x)\frac{d^2z}{dv^2}$$

$$\rightarrow x\frac{d^2z}{dx^2} + y\frac{d^2z}{dy^2} = (x+y)\left[\frac{d^2z}{du^2} + xy\frac{d^2z}{dv^2}\right]$$

LHS = RHS Hence proved

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Part C

Q.7. a) $\beta(m,n) = \int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}}$

Part 1

→ We know $\beta(m,n) = \int_0^1 (x)^{m-1}(1-x)^{n-1}\,dx$

→ $\boxed{\text{sub} \rightarrow x = \frac{t}{1+t} \quad x \to 0,\ t \to 0 \quad x \to 1,\ t \to \infty \quad \Rightarrow dx = \frac{1}{(1+t)^2}dt}$

→ $\beta(m,n) = \int_0^\infty \frac{(t)^{m-1}}{(1+t)^{m-1}} \left(1 - \frac{t}{1+t}\right)^{n-1} \frac{(1)}{(1+t)^2} dt$

→ $= \int_0^\infty \frac{(t)^{m-1}}{(1+t)^{m-1+2}} \frac{(1)^{n-1}}{(1+t)^{n-1}} dt$

→ $= \int_0^\infty \frac{(t)^{m-1}}{(1+t)^{m-1+2+n-1}} dt$

→ $= \int_0^\infty \frac{(t)^{m-1}}{(1+t)^{m+n}} dt$

→ changing variable to $x \Rightarrow \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx$ — (A)

Part 2

Main proof

splitting (A) → $\int_0^1 \frac{x^{m-1}}{(1+x)^{m+n}} dx + \int_1^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx$ — (B)

→ Consider RHS → $\int_1^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx$

→ $\left.\begin{array}{l} \text{Let } x = \frac{1}{t},\ dx = \frac{-1}{(t)^2} dt \\ x \to 1,\ t \to 1 \\ x \to \infty \quad t \to 0 \end{array}\right\}$ — (1)

sub (1) in (A)

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7. a continued

$$RHS = \int_1^\infty \frac{x^{m-1}}{(1+x)^{m+n}}\,dx$$

→ after sub (1) in (A)

→ $$RHS = \int_1^0 \left(\frac{1}{t}\right)^{m-1} \frac{1}{\left(1+\frac{1}{t}\right)^{m+n}} \left(\frac{-1}{t^2}\right) dt$$

→ $$RHS = \cancel{-}\int_0^1 \frac{1}{t^{m-1}} \times \frac{(t)^{m+n}}{(1+t)^{m+n}} \cancel{-}\frac{1}{t^2}\, dt$$

→ $$= \int_0^1 \frac{(t)^{m+n-m+1}}{(1+t)^{m+n} \cdot (t)^2}\, dt$$

→ $$= \int_0^1 \frac{(t)^{n+1}}{t^2} \times \frac{1}{(1+t)^{m+n}}\, dt$$

→ $$= \int_0^1 \frac{(t)^{n+1-2}}{(1+t)^{m+n}}\, dt$$

→ $$= \int_0^1 \frac{(t)^{n-1}}{(1+t)^{m+n}}\, dt$$

→ changing variable t to x

→ $$= \int_0^1 \frac{(x)^{n-1}}{(1+x)^{m+n}}\, dx \quad — (2)$$

→ sub (2) in main eq (0)

→ $$\int_0^1 \frac{x^{m-1}}{(1+x)^{m+n}} + \int_1^\infty \frac{x^{m-1}}{(1+x)^{m+n}}\, dx$$

→ $$\int_0^1 \frac{x^{m-1}}{(1+x)^{m+n}} + \int_0^1 \frac{(x)^{n-1}}{(1+x)^{m+n}}\, dx$$

→ $$\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}\, dx$$ → hence proved

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C.7. b) $U = \dfrac{x^2+y^2}{\sqrt{x}+\sqrt{y}} + \dfrac{1}{5}\sin^{-1}\left(\dfrac{x^2+y^2}{x^2+2xy}\right)$

→ Let $U1 = \dfrac{x^2+y^2}{\sqrt{x}+\sqrt{y}}$ $U_2 = \dfrac{1}{5}\sin^{-1}\left(\dfrac{x^2+y^2}{2xy+x^2}\right)$

~~U1 is homogen~~ Solving for U1

$U1 = \dfrac{x^2[1+(y/x)^2]}{\sqrt{x}[1+\sqrt{y/x}]}$

→ $U1 = x^{3/2}\,\phi(y/x)$

→ ∴ U1 is homogeneous function of degree 3/2

→ ∴ By Euler's formula

→ $x\dfrac{dU_1}{dx} + y\dfrac{dU_1}{dy} = \dfrac{3}{2}U_1$

hence

→ $x^2\dfrac{d^2U_1}{dx^2} + 2xy\dfrac{d^2U_1}{dxdy} + y^2\dfrac{d^2U_1}{dy^2} = \left(\dfrac{3}{2}\right)\left(\dfrac{1}{2}\right)U_1$

Solving for U_2

→ $5U_2 = \sin^{-1}\left(\dfrac{x^2+y^2}{x^2+2xy}\right)$

→ $\sin(5U_2) = \dfrac{x^2+y^2}{x^2+2xy}$

→ $\sin(5U_2) = \dfrac{x^2[1+(y/x)^2]}{x^2[1+2(y/x)]}$

→ $\sin(5U) = x^0\,\phi(y/x) = 3$

Continued

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Aem.

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C.7.b continued

$\therefore U_2$ is homogeneous eq of degree '0'

$\therefore$ By Euler's eq

$$x\frac{dz}{dx} + y\frac{dz}{dy} = 0$$

$$x\left[\cos(5U_2)\times 5\times\frac{dU_2}{dx}\right] + y\left[\cos(5U_2)\cdot 5\frac{dU_2}{dy}\right] = 0$$

Dividing by $5\cos(2U)$

$$x\frac{dU_2}{dx} + y\frac{dU_2}{dy} = 0$$

here

$$x^2\frac{d^2U_2}{dx^2} + 2xy\frac{d^2U_2}{dx\,dy} + y^2\frac{d^2U_2}{dy^2} = 0$$

$$U = U_1 + U_2$$

$$\therefore U = x^2\frac{d^2U}{dx^2} + 2xy\frac{d^2U}{dx\,dy} + y^2\frac{d^2U}{dy^2} + x\frac{dU}{dx} + y\frac{dU}{dy}$$

$$U = \frac{3}{2}U_1 + \frac{3}{4}U_1$$

$$U = \frac{9}{4}U_1$$

$$x\frac{d^2U}{dx^2} + 2xy\frac{d^2U}{dx\,dy} + y^2\frac{d^2U}{dy^2} + x\frac{dU}{dx} + y\frac{dU}{dy}$$

$$= \frac{9}{4}\left(\frac{x^2+y^2}{\sqrt{x}+\sqrt{y}}\right)$$

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C.T.C. $\lim_{x \to 0} (\cos x)^{1/x^2}$

$\rightarrow (\cos(0))^{1/0}$

1^{∞} indefinite form

$\rightarrow$ Let $L = \lim_{x \to 0} (\cos x)^{1/x^2}$

$\rightarrow$ $\boxed{\text{Applying log}}$

$\rightarrow \log L = \lim_{x \to 0} \frac{1}{x^2} \log \cos x$

$\rightarrow \log L = \lim_{x \to 0} \frac{\log \cos x}{x^2}$

$\rightarrow$ $\boxed{\text{Applying L'opitals rule}}$

$\rightarrow \log L = \lim_{x \to 0} \frac{1 \, (-\sin x)}{\cos x \times 2x}$ $\rightarrow \frac{0}{0}$ indefinite form

$\rightarrow$ $\boxed{\text{Applying L'opitals rule}}$

$\rightarrow \log L = \lim_{x \to 0} \frac{-\cos x}{[2x(-\sin x) + 2(\cos x)]}$ $\rightarrow \frac{-1}{0+2}$ definite form

$\rightarrow \log L =$ $\boxed{\text{apply limits}}$

$\rightarrow \log L = \frac{-\cos(0)}{[2 \times 0 \times 0 + 2\cos(0)]}$

$\rightarrow \log L = -\frac{1}{2}$

$\rightarrow \therefore L = e^{-1/2}$

$\rightarrow L = 1/\sqrt{e}$