FAST MULTIPLIERS

- Combinational array multiplier
- Sequential circuit binary multiplier
- Booth method
- Bit pair recoding

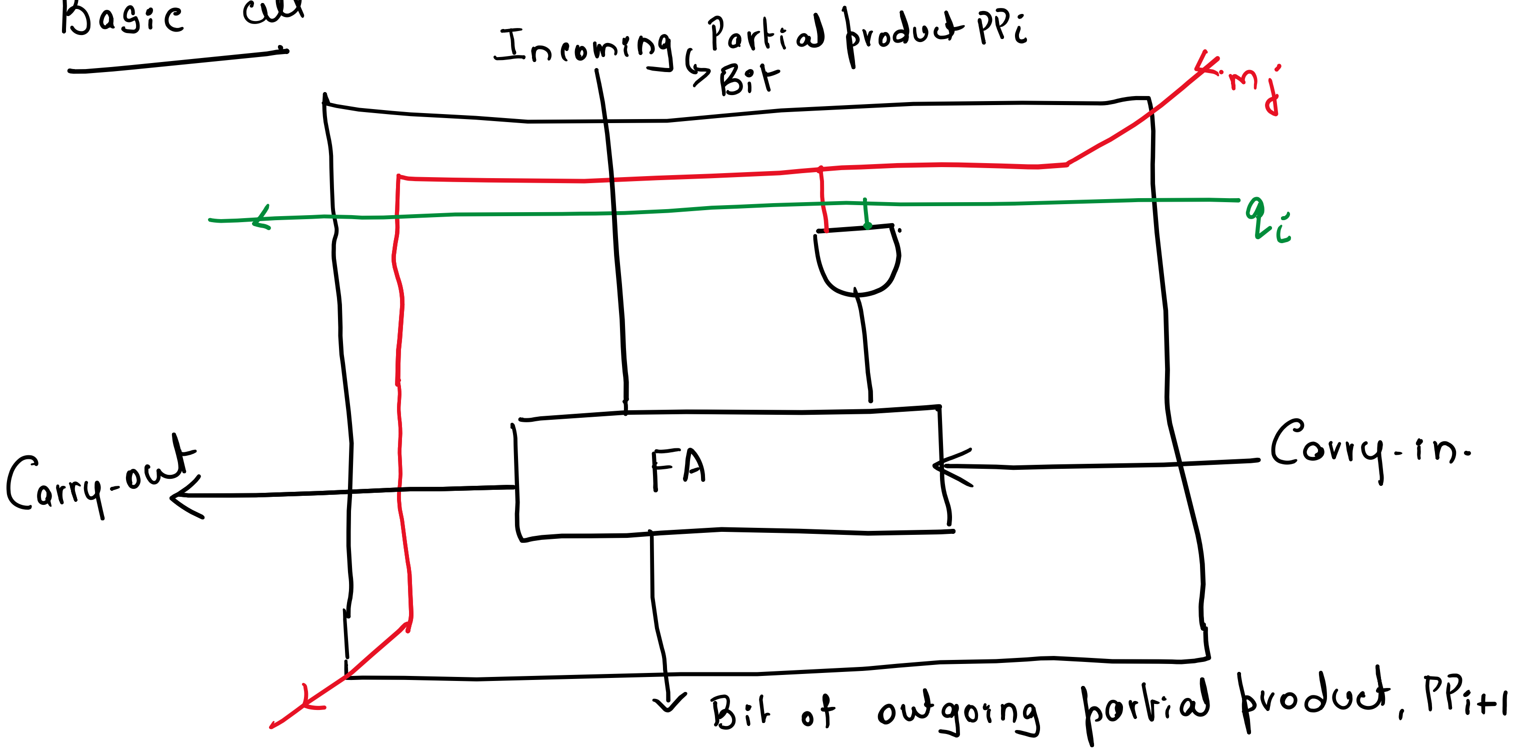
Carry save addition

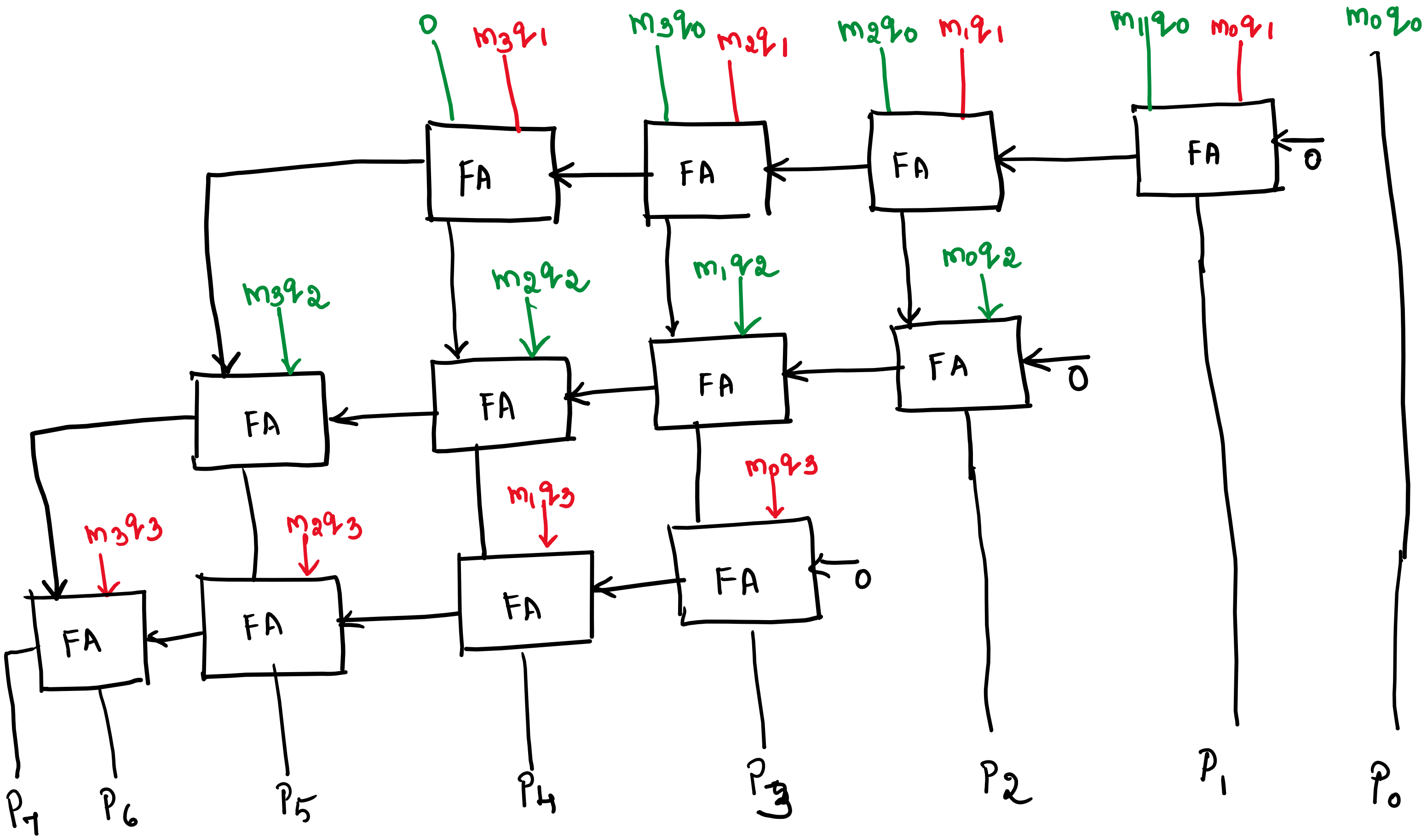
- Unsigned number multiplication
- $\underline{n=4}$ $\begin{cases} \text{Multiplicand - } M \\ \text{Multiplier - } Q \end{cases}$

$2n$ bits Product - $P = M \times Q$

				m_3	m_2	m_1	m_0	$\times\ q_3\ q_2\ q_1\ q_0$	
				m_3q_0	m_2q_0	m_1q_0	m_0q_0	$\longrightarrow PP_0 \rightarrow A$	
			m_3q_1	m_2q_1	m_1q_1	m_0q_1		$\longrightarrow PP_1 \rightarrow B$	
		m_3q_2	m_2q_2	m_1q_2	m_0q_2			$\longrightarrow PP_2 \rightarrow C$	
	m_3q_3	m_2q_3	m_1q_3	m_0q_3				$\longrightarrow PP_3 \rightarrow D$	
P_7	P_6	P_5	P_4	P_3	P_2	P_1	P_0		

Basic cell





Ripple-carry array

Save the carry for next level.

Carry Save Addition

X 0 0 1 1 0

Y 0 0 0 1 1

Normal binary addition

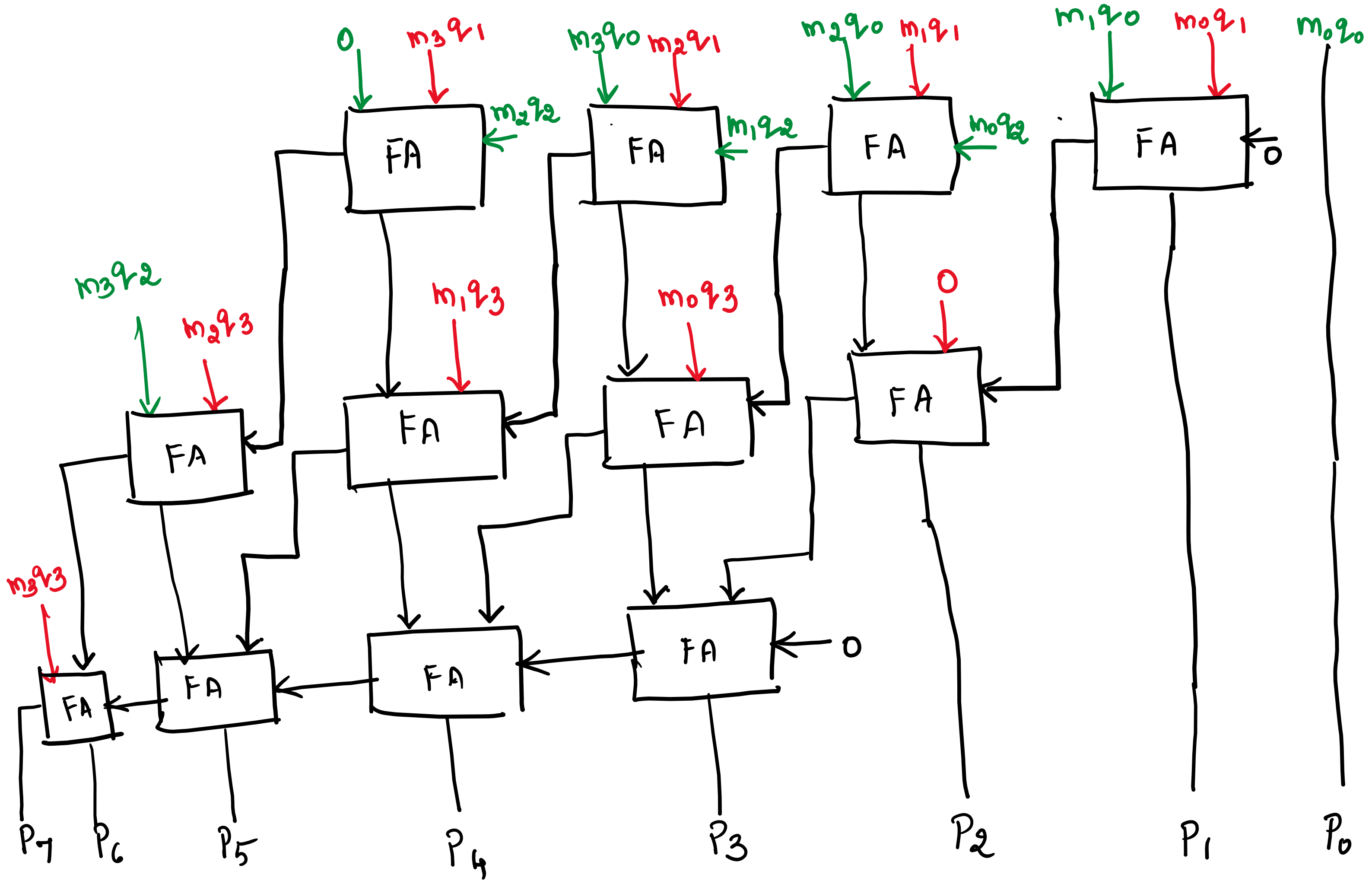
		1	1	0	
X	0	0	1	1	0
+ Y	0	0	0	1	1
Z	0	1	0	0	1

CSA

		0	0	1	1	0
		0	0	0	1	1
S →		0	0	1	0	1
C →	0	0	0	1	0	

S		0	0	1	0	1
	0	0	0	1	0	
		0	1	0	0	1

Array multiplier



I Row: PP_0, PP_1, PP_2

Sum Vector — S_1

Carry Vector — C_1

II Row: PP_3, S_1, C_1

Sum Vector and a Carry Vector

S_2 C_2

III Row

S_2

C_2 RCA

P

PP_0 PP_1 PP_2 PP_3 — 1 tpd.

A B C D

I level of CSA — 2tpd.

S_1 C_1 PP_3

II level of CSA — 2tpd.

S_2 C_2 — 2×8 tpd = 16

III Row RCA

P

21 tpd

Normal Addition
Not CSA
(2n-bit Addition)
8-bit

Operands n=4 bits : Unsigned numbers

① Combinational array multiplier: $6(n-1)-1$

$= 6(4-1)-1$

$= 18-1$

$= 17$ tpd

② CSA- array : RCA

Partial product bit generation = 1 tpd

I CSA = 2 tpd

II CSA = 2 tpd

Last level RCA - normal addition. (8-bit) = 2 × 8 = 16 tpd.

21 tpd

③ CSA array — CLA in last level.

Partial product bit generation = 1 tpd.

I CSA = 2 tpd.

II CSA = 2 tpd.

8-bit CLA } Built using Four bit CLA

K l

$= 2 \cdot \frac{K}{l} + 2$

$= 2 \times \frac{8}{4} + 2$

= 6 tpd.

Total = 11 tpd

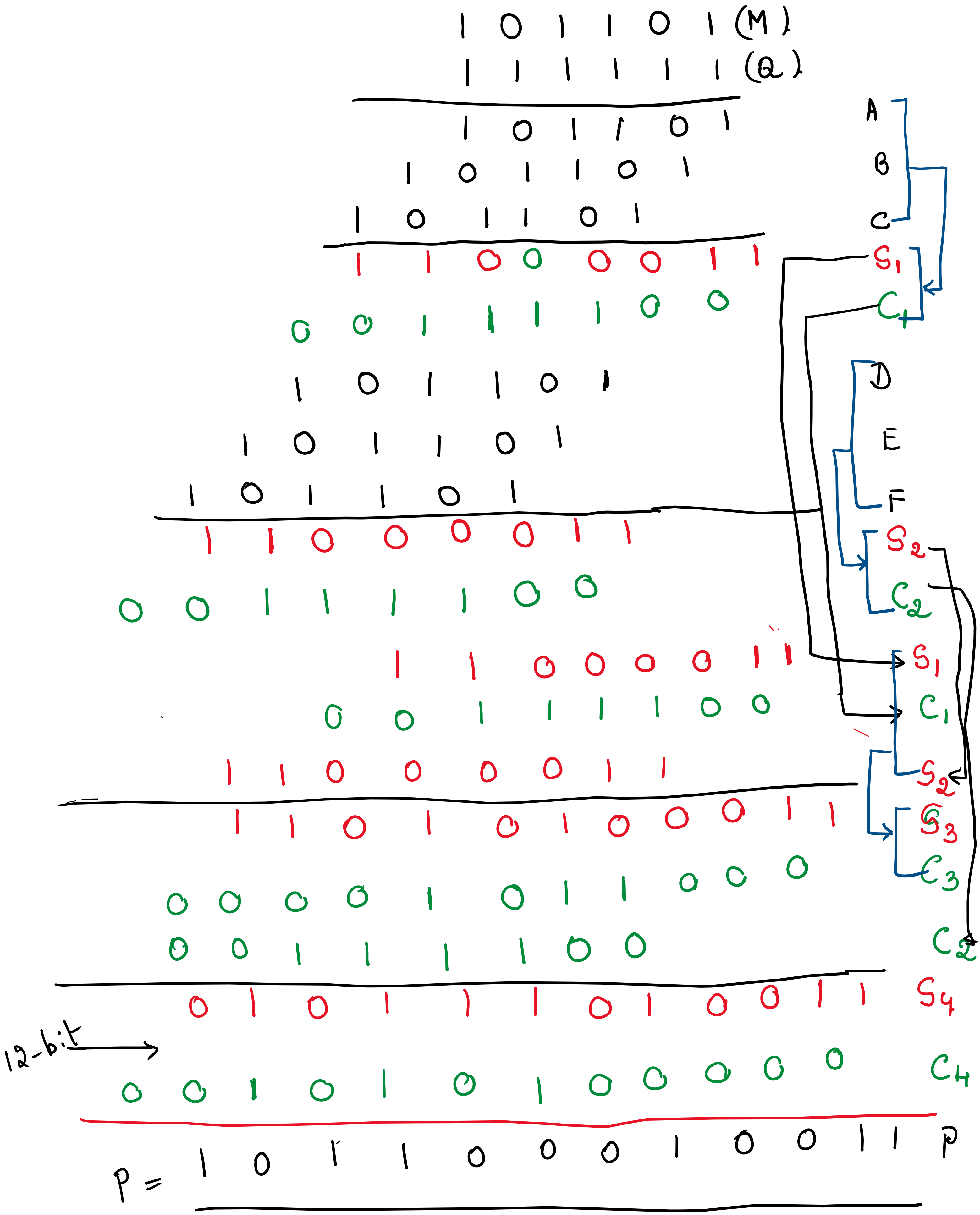
More significant reduction in delay can be achieved when having large number of summands.

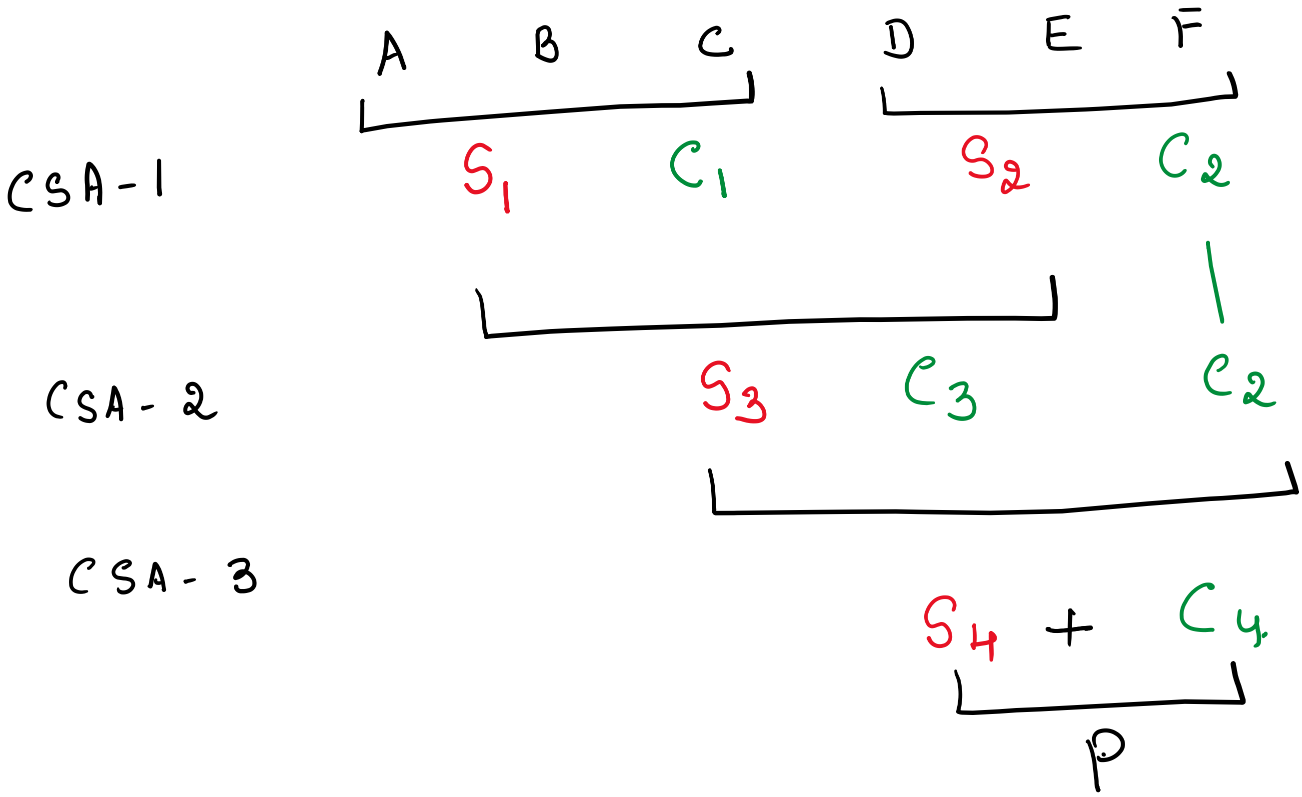
↳ Large n

M = 45
Q = 63 } 6-bit representation } 6 PPs, 6 Summands

P = 2835 ⟶ 12 bits

						1	0	1	1	0	1	M(45)
						1	1	1	1	1	1	Q(63)
						1	0	1	1	0	1	$\xrightarrow{PP_0}$ A
					1	0	1	1	0	1		$\xrightarrow{PP_1}$ B
				1	0	1	1	0	1			$\xrightarrow{PP_2}$ C
			1	0	1	1	0	1				$\xrightarrow{PP_3}$ D
		1	0	1	1	0	1					$\xrightarrow{PP_4}$ E
	1	0	1	1	0	1						$\xrightarrow{PP_5}$ F
1	0	1	1	0	0	0	1	0	0	1	1	


1 0 1 1 0 1 (M)
1 1 1 1 1 1 (Q)
1 0 1 1 0 1 A
1 0 1 1 0 1 B
1 0 1 1 0 1 C
1 1 0 0 0 0 1 1 S1
0 0 1 1 1 1 0 0 C1
1 0 1 1 0 1 D
1 0 1 1 0 1 E
1 0 1 1 0 1 F
1 1 0 0 0 0 1 1 S2
0 0 1 1 1 1 0 0 C2
1 1 0 0 0 0 1 1 S1
0 0 1 1 1 1 0 0 C1
1 1 0 0 0 0 1 1 S2
1 1 0 1 0 1 0 0 0 1 1 S3
0 0 0 0 1 0 1 1 0 0 0 C3
0 0 1 1 1 1 0 0 C2
0 1 0 1 1 1 0 1 0 0 1 1 S4
12-bit
0 0 1 0 1 0 1 0 0 0 0 0 C4
P = 1 0 1 1 0 0 0 1 0 0 1 1 P



<u>6-bit multiplication</u>

① Combinational array multiplier : $6(n-1)-1$

$= 6(6-1)-1$

$= \underline{29\ tpd}$

② CSA Using RCA (12-bit)

Partial product bit generation = 1 tpd.

I CSA = 2 tpd.

CSA - II = 2 tpd.

CSA - III = 2 tpd.

7 tpd

12-bit - RCA = 2 x 12 = 24 bit

Total = 31 tpd.

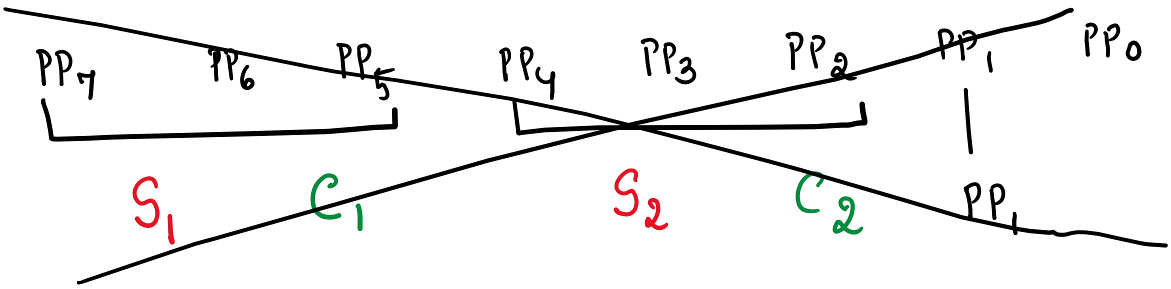
③. CSA – 12 bit addition CLA – 4 bit CLAs

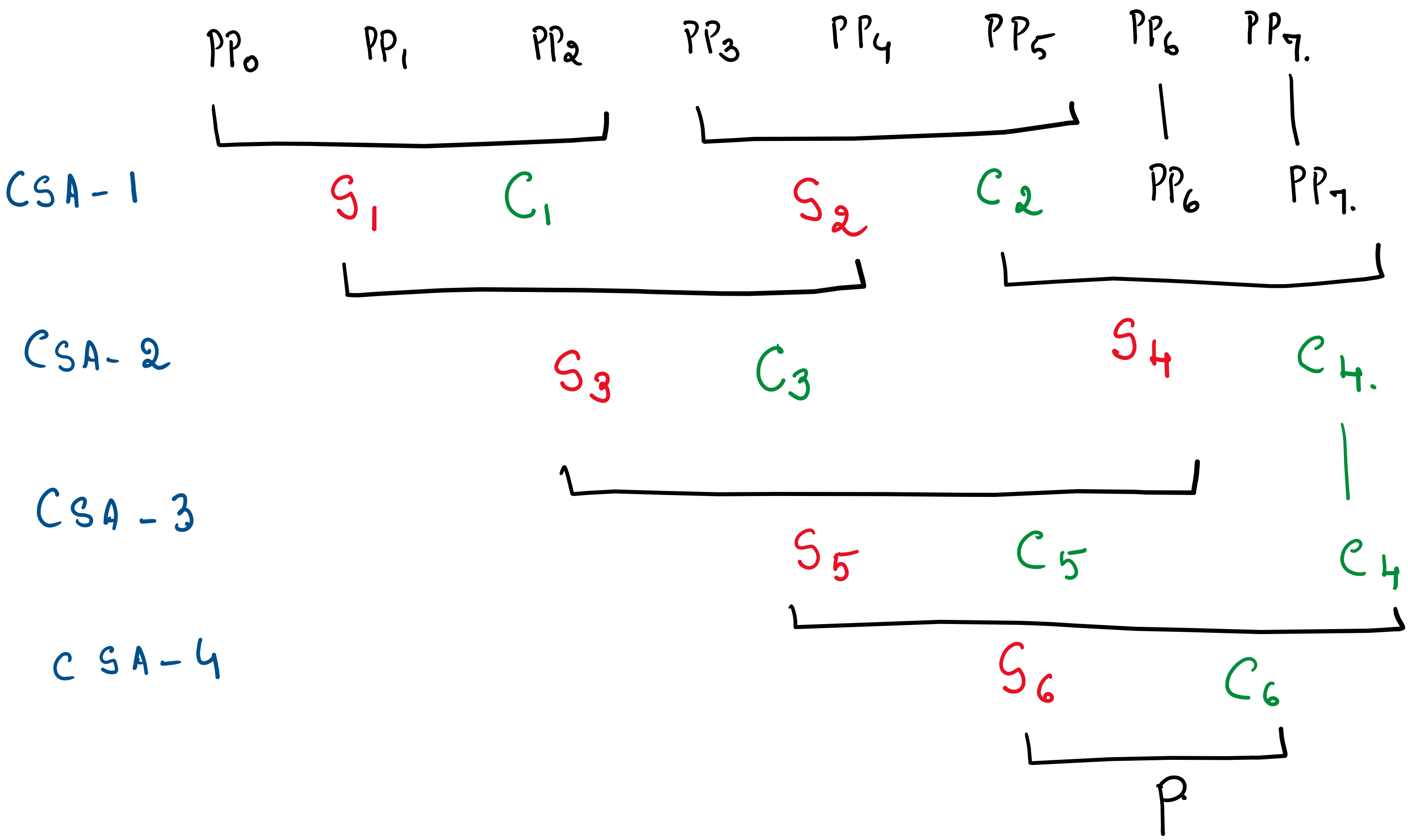
Delay in last layer $= 2 \cdot \frac{12}{4} + 2$

$= 8$ tpd.

Total $= 15$ tpd

8-bit Multiplication





Partial product bit generation : 1 tpd.

Number of CSA levels : 4.

Delay in 4 levels of CSA = 4x 2 = 8 tpd.

16-bit adder (CLA) - 4-bit CLAs :

$$2 \times \frac{\cancel{16}^{4}}{\cancel{4}_{1}} + 2 = 10 \text{ tpd}.$$

Total = 1+8+10

= $\underline{19 \text{ tpd}}$

Generalized delay computation

PP bit generation = 1 tpd.

Delay through 'm' CSA levels = 2x m tpd.

2n bit CLA built using l bit CLAs = $2 \cdot \frac{2n}{l} + 2$

$$\boxed{\text{Total} = 1 + 2m + 2 \cdot \frac{2n}{l} + 2}$$

How to decide m?

16-bit operands — 16 pps

Total.	Level	Grouped	Pending
16	I	15	1
11	II	9	2
8	III	6	2
6	IV	6	0
4	V	3	1
3	VI	1	
2			

$m = 6$

K sommands

$m = 1.7 \log_2 K - 1.7.$

$K = 16.$

$m = 1.7 \log_2 16 - 1.7$

$= 1.7 \log_2 2^4 - 1.7$

$= 1.7 \times 4 - 1.7$

$= 5.1 \approx 6$

$K = 8.$

$m = 1.7. \log_2 2^3 - 1.7$

$= 1.7 \times 3 - 1.7$

$= 3.4$

≈ 4