Solve $u_{xx} - u = 0$ $\longrightarrow$ $u'' - u = 0$? $\frac{d^2u}{dx^2} - u = 0$

$\downarrow$

$u = \underline{A}\, e^{-x} + \underline{B}\, e^{x}$

$u(x,y) = A(\underline{y})\, e^{-x} + B(y)\, e^{x}$

A, B = Arbitrary Const

$u = X(x)\, Y(\underline{y})$.

(2) solve $u_{xy} = -u_x$

Set $u_x = p$

$u_{xy} = p_y$

$\Rightarrow$ $p_y = -u_x = -p$

$p_y = -p$ $\Rightarrow$ $\int \frac{p_y}{p} = \int -1\, dy$

$\ln p = -y + c_1(x)$

$p = e^{-y + c_1(x)}$ $\Rightarrow p = c(x)\, e^{-y}$ ✓

$u_x = p$

$u_x = c(x)\, e^{-y}$

$$\boxed{u(x,y) = e^{-y} \int c(x)\, dx + c_2(y)}$$

$u(x,y) = X(x)\, Y(y)$ — Method of separating variables

Solve $$\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$$

$$z(x,y) = X(x)\,Y(y).$$

$$\frac{\partial z}{\partial x} = X'Y \qquad \frac{\partial z}{\partial y} = XY'$$

$$\frac{\partial^2 z}{\partial x^2} = X''Y$$

Y'

Y''

$\frac{d^2Y}{dy^2}$

$$X''Y - 2X'Y + XY' = 0$$

$$(X'' - 2X')\,Y = -XY'$$

$$\frac{X'' - 2X'}{X} = -\frac{Y'}{Y} = K$$

$$\frac{X'' - 2X'}{X} = K \quad \text{and} \quad -\frac{Y'}{Y} = K$$

$$X'' - 2X' - KX = 0 \quad \text{and} \quad Y' + KY = 0$$

$$X'' - 2X' - K X = 0 \quad \text{and} \quad Y' + KY = 0$$

$$m^2 - 2m - K = 0 \qquad m + K = 0$$

$$m = 1 \pm \sqrt{K+1} \qquad m = -K$$

$$X(x) = c_1 e^{(1+\sqrt{K+1})x} + c_2 e^{(1-\sqrt{K+1})x} \qquad Y(y) = c_3 e^{-Ky}$$

$$z(x,y) = X(x)\,Y(y)$$

$$= \left[c_1 e^{(1+\sqrt{K+1})x} + c_2 e^{(1-\sqrt{K+1})x}\right] c_3 e^{-Ky}$$

$$= \left[A e^{(1+\sqrt{K+1})x} + B e^{(1-\sqrt{K+1})x}\right] e^{-Ky}$$

(4) Using the method of separation of variables, solve

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u \quad \text{where } u(x,0) = 6e^{-3x}$$

$$u(x,t) = X(x)\,T(t)$$

$$\frac{\partial u}{\partial x} = X'T \qquad \frac{\partial u}{\partial t} = XT'$$

$$X'T = 2XT' + XT$$

$$\frac{X' - X}{2X} = \frac{T'}{T} = K$$

$$\frac{X' - X}{2X} = K \quad \text{and} \quad \frac{T'}{T} = K$$

$$X' - X = 2KX \quad \& \quad T' = KT$$

$$X' - (1+2K)X = 0 \quad \& \quad T' = KT$$

$$X' - (1+2K)X \quad \text{and} \quad T' - KT = 0$$

$$X = c_1 e^{(1+2K)x} \quad \text{and} \quad T = c_2 e^{Kt}$$

$$u(x,t) = X(x)\,T(t)$$

$$u(x,t) = c_1 c_2 \, e^{(1+2K)x} e^{Kt} \quad \text{—— } (*)$$

$$u(x,0) = 6e^{-3x}$$

$$u(x,0) = 6e^{-3x} = c_1 c_2 \, e^{(1+2K)x}$$

$$c_1 c_2 = 6$$

$$1+2K = -3 \Rightarrow K = -2$$

$$(*) \Rightarrow \boxed{u(x,t) = 6\, e^{-(3x+2t)}}$$

(1) $u_{yy} = 0$

(2) $u_{xx} + 16\pi^2 u = 0$

(3) $2u_{xx} + 9u_x + 4u = -3\cos x - 9\sin x$.