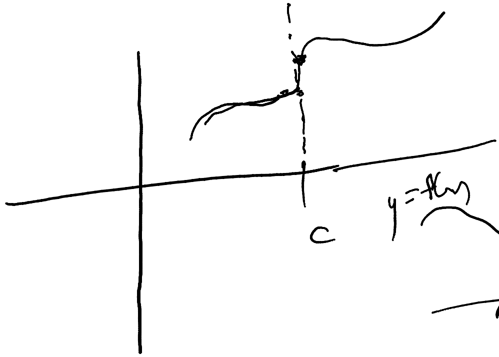
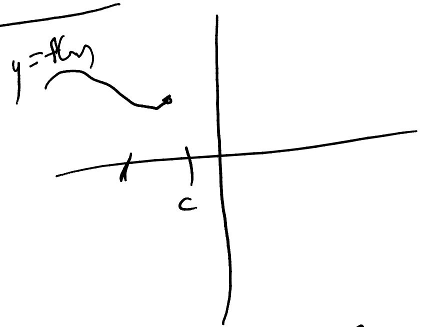
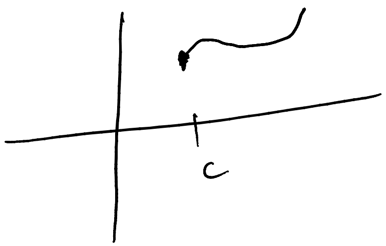
Precise definition of left limit





$$\lim_{x \to c^-} f(x) = L$$



$$\lim_{x \to c^+} f(x) = L$$

<u>left limit</u>

$$\lim_{x \to c^-} f(x) = L$$

For given $\epsilon > 0$ then there exist a corresponding number $\delta > 0$
Such that $|f(x) - L| < \epsilon$ whenever $c - \delta < x < c$.

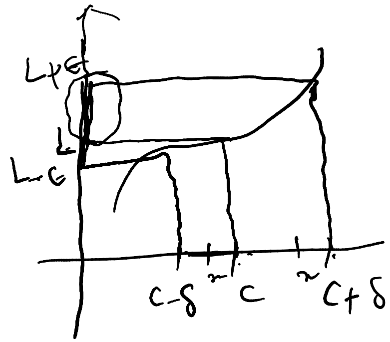
Right limit

$$\lim_{x \to c^+} f(x) = L$$

Def

If for given $\epsilon > 0$ then their exist a number $\delta > 0$ such that

$|f(x) - L| < \epsilon$ whenever $c < x < c + \delta$.



$f(x) - L$

Theorem : limit laws

If L, M, c and K are real numbers and

$$\lim_{x \to c} f(x) = L \quad \text{and} \quad \lim_{x \to c} g(x) = M.$$

(i) $$\lim_{x \to c} [f(x) \pm g(x)] = L \pm M$$

(ii) $$\lim_{x \to c} [K \cdot f(x)] = K \cdot L$$

(iii) $$\lim_{x \to c} [f(x) \cdot g(x)] = L \cdot M$$

(iv) $$\lim_{x \to c} \left[\frac{f(x)}{g(x)}\right] = \frac{L}{M} \quad \text{if } M \neq 0$$

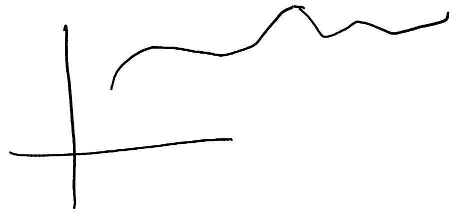
v) $$\lim_{x \to c} [f(x)]^n = L^n \quad n \text{ +ve integer}$$

(vi) $$\lim_{x \to c} \left[\sqrt[n]{f(x)}\right] = \sqrt[n]{L} \quad n \text{ is +ve integer}$$

$c \in (a, b)$

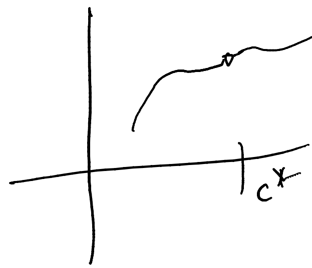
Continuous function

$y = f(x)$



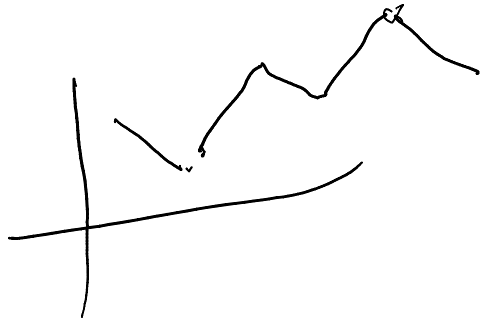
The function $f(x)$ is continuous at $x = c$ if the following conditions must satisfy

(i) $f(c)$ defined.



(ii) $\lim\limits_{x \to c} f(x)$ exist

(iii) $\lim\limits_{x \to c} f(x) = f(c)$

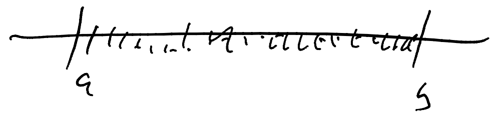


$f(x) \longrightarrow (a\ b)$

(i) $f(a) = f(b)$ — ?

$c \in (a\ b)$ — (ii)

(iii) $\lim\limits_{x \to a^+} f(x) = \lim\limits_{x \to b^-} f(x)$ — ?



(iv) $\lim\limits_{x \to a} f(x) = \lim\limits_{x \to b} f(x)$

Ex

$$f(x) = \frac{x^2-1}{x-1}$$

Is it Continuous at $x = 1$

No

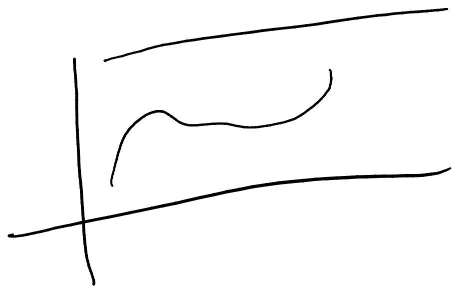
$f(1) \Rightarrow$ Not defined

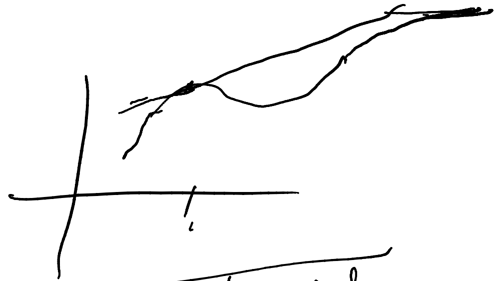
(ii) $f(x) = \begin{cases} \dfrac{x^2-1}{x-1} & \text{if } x \neq 1 \\ 3 & \text{if } x = 1 \end{cases}$ No

(iii) $f(x) = \begin{cases} \dfrac{x^2-1}{x-1} & \text{if } x \neq 1 \\ 2 & \text{if } x = 1 \end{cases}$ Yes

Asymptotes

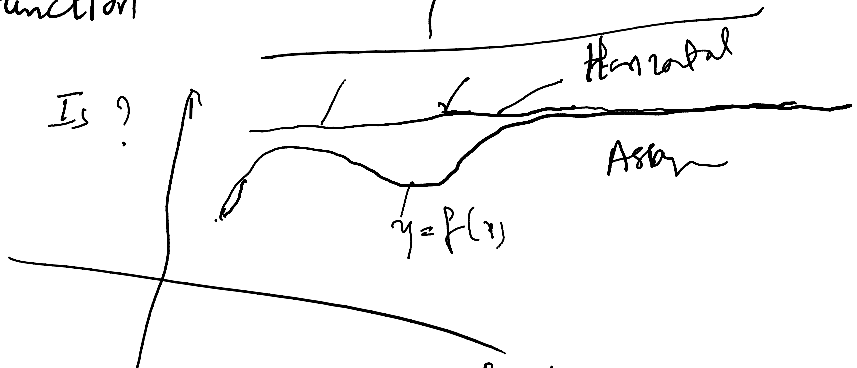
- Horizontal Asymptotes
- Vertical Asymptotes
- Oblique Asymptotes.





Asymptotes

If the distance between the graph of the function and line approaches to zero as a functions tendings to infinity.



Horizontal Asymptotes

A line $y=b$ is a horizontal asymptote of the graph of the function $y=f(x)$ if either

$$\lim_{x \to \infty} f(x) = b \quad \text{or} \quad \lim_{x \to -\infty} f(x) = b$$

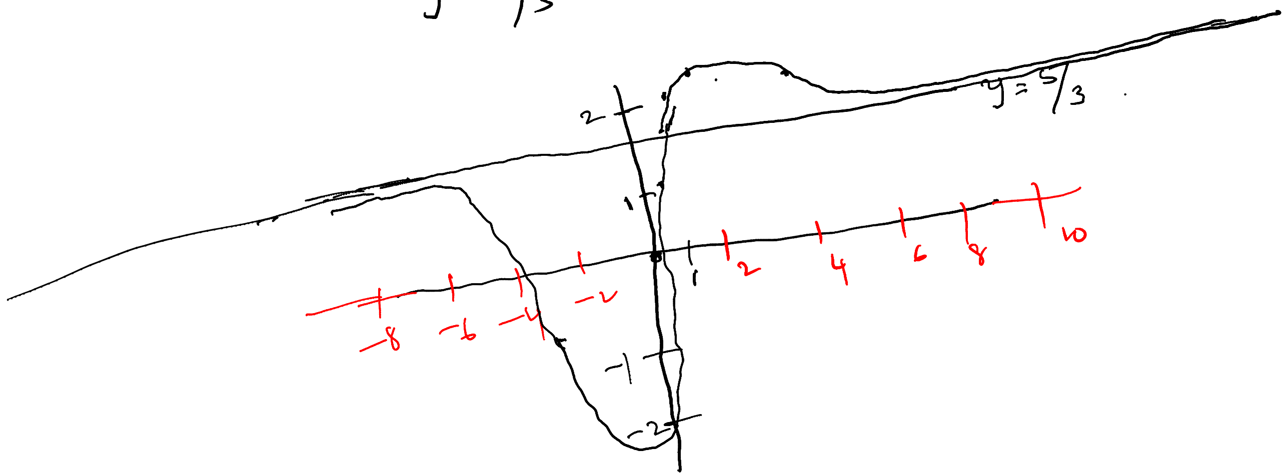
Ex :-

$$f(x) = \frac{5x^2+8x-3}{3x^2+2}$$

$y = 5/3$??.

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{5x^2+8x-3}{3x^2+2} = \frac{5}{3} \checkmark$$

$y = 5/3$ is a Horizontal Asymptote.



Vertical Asymptotes:-

A line $x = a$ is a vertical asymptote of the graph of a function $y = f(x)$ if either

$$\lim_{x \to c^-} f(x) = \pm\infty \quad \text{(or)} \quad \lim_{x \to c^+} f(x) = \pm\infty$$

Ex $y = f(x) = \dfrac{x+3}{x+2}$

$y = 1$ Horizontal Asymptote & ??

$x = -2$ Vertical Asymptote.

(ii) $f(x) = \dfrac{x}{(x-1)(x-2)}$ $y = 1$

? — $y = 0$ Horizontal Asym & ?

$x = 1, 2$ Vertical Asym

$$f(x) = \frac{x}{(x-1)(x-2)}$$

$x = 1, 2$

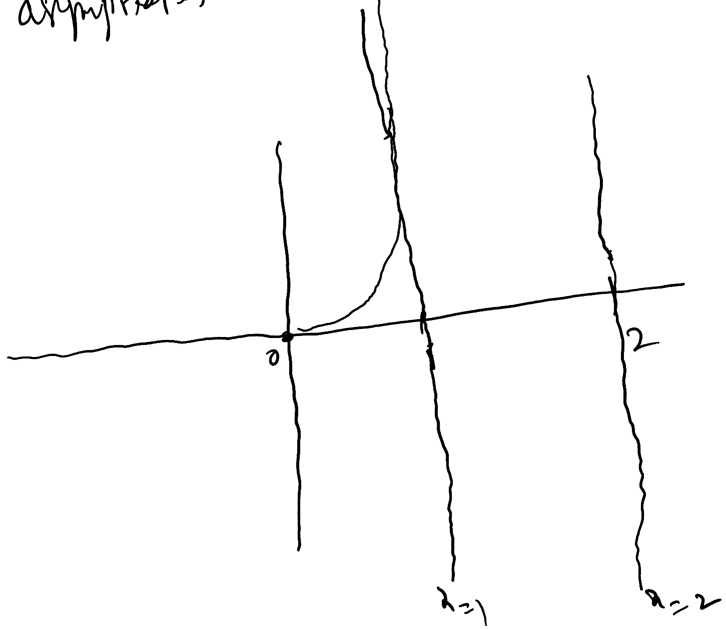
V

Vertical asymptotes $x = 1$ and 2

$f''(x)$

$+$ concave up.

$-$ c down



$$f(x) = \frac{-8}{x^2-4}$$

$x = \pm 2$ Vertical asymptotes

$y = 0$

$$\downarrow$$

$$y = mx + c$$

Oblique Asymptotes:-

$$y = f(x)$$

Degree of Numerator $\overset{+1}{>}$ Degree of Denominator. ??

If the degree of the numerator of a rational function is 1 greater than the degree of the denominator, the graph has an oblique

$$y = f(x) = \frac{\phi(x)}{\psi(x)}$$

express ~~it~~ a combination of linear function

+ Remainder that goes to zero as $x \to \pm\infty$

(i) $f(x) = \dfrac{x^2 - 3}{2x - 4}$ ✓

Home work

Try yourself.