Mathematics-I (MA100)

Differential Calculus

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Course Name : Mathematics - I

Course Code : MA100

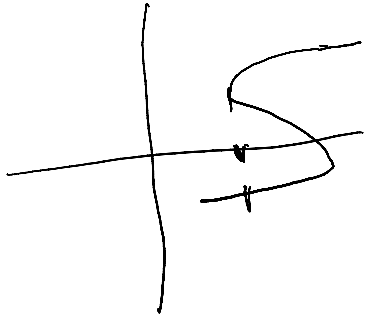
Credits : 4

Duration : 56 L

Module 1 :	Differential Calculus	12 H
Module 2 :	Integral Calculus	19 H
Module 3 :	Vector Calculus	14 H
Module 4 :	Sequences and Series	8 H
Module 5	Fourier Series and Fourier Transforms	10 H

Function:-

$f: X \to Y$



$f(x) \in Y$ to each $x \in X$

$y = f(x)$ — function

x ↳ Independent variable

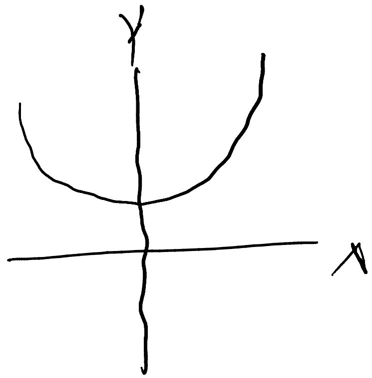
y ↓ Dependent variable

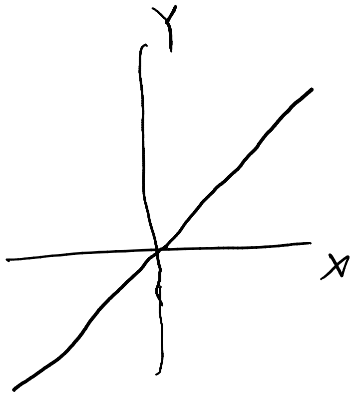
x	$f(x)$
1	3
1.2	3.2
1.6	4
1.8	2
2	3.1
2.1	6
2.2	3.5
2.5	4
2	5

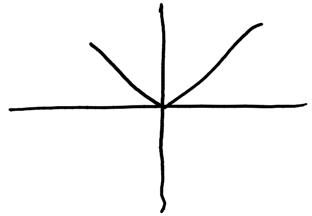
why?

No?

Not a function







Domain:- The set of all possible inputs

Range:- The set of all possible outputs.

Function	Domain	Range
$y = x^2$	$\mathbb{R}$	$\mathbb{R}^+$
$y = 1/x$	$R - \{0\}$	$R - \{0\}$
$y = \sqrt{x}$	$\mathbb{R}^+$	$\mathbb{R}^+$
$y = \sqrt{4-x}$	$(-\infty \ 4]$	$\mathbb{R}^+$
$y = \sqrt{1-x^2}$	$[-1 \ 1]$	$[0 \ 1]$?

$4 - x \geq 0$

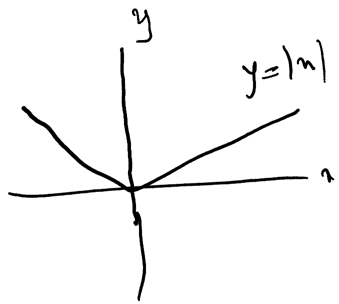
$4 \geq x$

$1 - x^2 \geq 0$

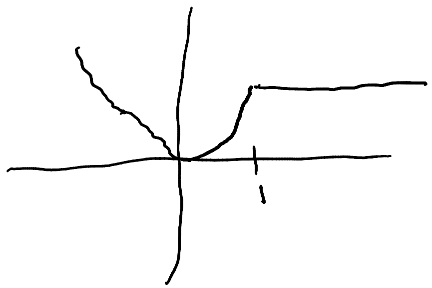
x^2

Piecewise - Functions

$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$



(i) $$f(x) = \begin{cases} -x & x < 0 \\ x^2 & 0 \leq x \leq 1 \\ 1 & x > 1 \end{cases}$$



Increasing functions

$f(x)$ I

$x_1, x_2 \in I$

$f(x_1) < f(x_2)$ wherever $x_2 > x_1$

f is an increasing on I

$f(x_1) > f(x_2)$ wherever $x_1 < x_2$

f is a decreasing fn on I

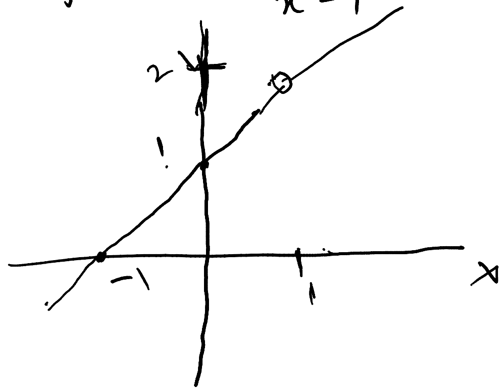
$\underline{\underline{y = f(x)}}$

Even $\quad f(-x) = f(x) \quad \forall x$

odd $\quad f(-x) = -f(x)$

Limit of a function:—

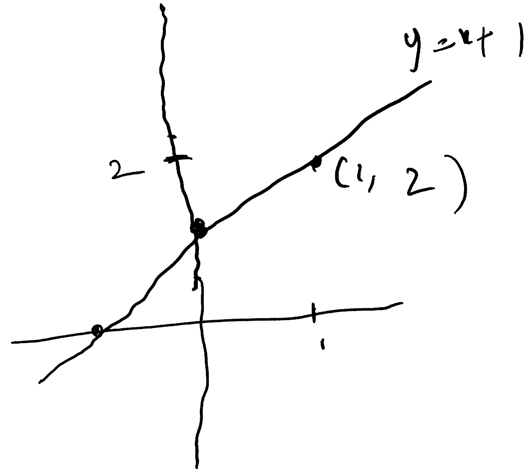
$$f(x) = \frac{x^2-1}{x-1} \quad \text{at } x=1$$



$$f(x) = \frac{x^2-1}{x-1} \quad x \neq 1$$

$$= \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}}$$

$$f(x) = x+1 \quad x \neq 1$$



x	$f(x) = \frac{x^2-1}{x-1}$
0.9	1.9
0.99	1.99
0.999	1.999
0.999999	1.999999
1.1	2.1
1.01	2.01
1.001	2.001
1.000001	2.000001

x	0.9	0.999	0.999999	1	1.000001	1.001	1.01
$f(x)$	1.9	1.999	1.999999		2.000001	2.001	2.01

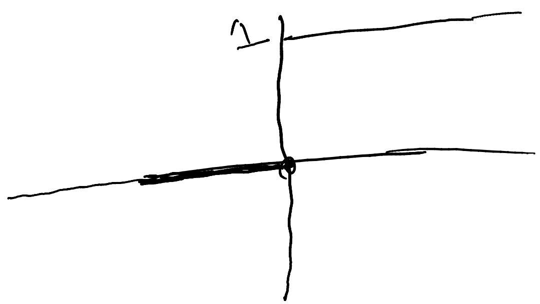
Def Suppose $f(x)$ defined on an open interval about c (except)

$$\lim_{x \to c} f(x) = L$$

$$\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) = L$$

then $\lim_{x \to c} f(x)$ is exist

$$\lim_{x \to c} f(x) \quad f(x) = L$$

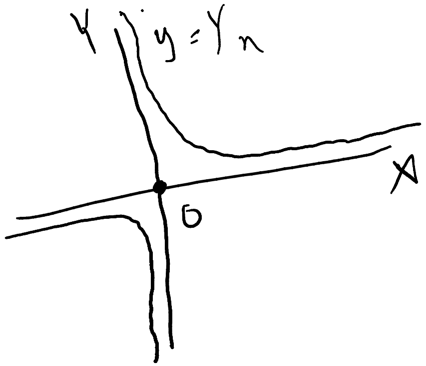


$$U(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

$$\lim_{x \to 0^-} U(x) \neq \lim_{x \to 0^+} U(x)$$

Does this limit of $U(x)$ exist?

Not exist



$$y = \begin{cases} \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

Theorem limit laws

If L, M, c and k are real numbers and

$\lim_{x \to c} f(x) = L$ and $\lim_{x \to c} g(x) = M$

(i) $\lim_{x \to c} \left[f(x) \pm g(x) \right] = L \pm M$

(ii) $\lim_{x \to c} \left[k\, f(x) \right] = k \left[\lim_{x \to c} f(x) \right] = kL$

(iii) $\lim_{x \to c} \left[f(x) \cdot g(x) \right] = L \cdot M$

(iv) $\lim_{x \to c} \frac{f(x)}{g(x)} = L/M$ if $M \neq 0$

v) $\lim_{x \to c} [f(x)]^n = L^n$ n is a positive integer

vi) $\lim_{x \to c} \sqrt[n]{f(x)} = \sqrt[n]{L}$ n is a positive integer

$\boxed{\text{P.T } \lim_{x \to 2} x^2 = 4}$ 2

3M

$\lim_{x \to 9} (x^3 + 4x^2 + 5)$

The precise definition of a limit :- ($\epsilon - \delta$ definition)

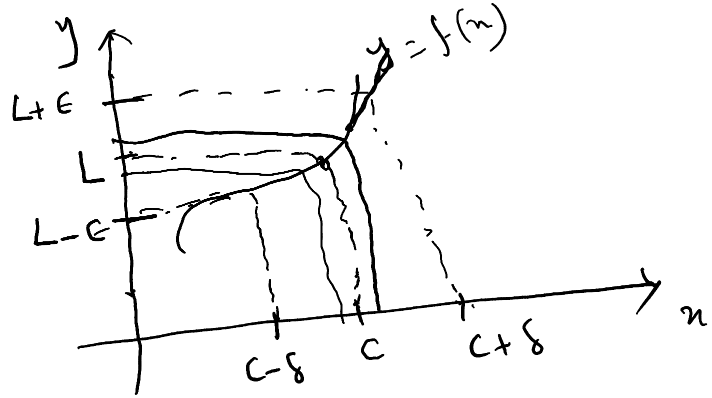
To show that $\lim_{x \to c} f(x) = L$

For given $\epsilon > 0$ then there exist a number $\delta > 0$ such that

$|f(x) - L| < \epsilon$ whenever $0 < |x - c| < \delta$

Relation b/w

ϵ and δ



$|f(x) - L| < \epsilon$ whenever $|x - c| < \delta$

$-\epsilon < f(x) - L < \epsilon$ when $-\delta < x - c < \delta$

$L - \epsilon < f(x) < L + \epsilon$ when $c - \delta < x < c + \delta$

Show that $\lim_{x \to 1} (5x - 3) = 2$

$$\lim_{x \to c} f(x) = L$$

$f(x) = 5x - 3$, $c = 1$ $L = 2$

if for given $\epsilon > 0$ $\exists$ $\delta > 0$ $\ni$

$|(5x-3) - 2| < \epsilon$ whenever $0 < |x-1| < \delta$

Consider, $|f(x) - L| = |(5x-3) - 2|$

$= |5(x-1)|$

$= 5|x-1|$

$\leq 5\delta$

$5\delta = \epsilon$

$$\boxed{\delta = \epsilon/5}$$

$|f(x) - L| = \epsilon$

$\delta = \epsilon/5$

$|f(x) - L| = |(5x-3) - 2|$

$= 5|x-1|$

$\leq \epsilon$

$5|x-1| \leq \epsilon$

$|x-1| \leq \epsilon/5$

$$\boxed{\delta \leq \epsilon/5}$$

$|5x - 3 - 2| = |5x - 5|$

$= 5|x-1|$

$= \cancel{5}\,\epsilon/\cancel{5}$

$= \epsilon$

True

$$\lim_{x \to 1} (5x-3) = 2$$

$f(x) = 5x - 3 \quad L = 2, \quad c = 1$

$|x-1| < \delta \implies |f(x) - L| < \epsilon$

given $\epsilon > 0 \; \exists \; \delta > 0 \ni$
$|f(x) - L| < \epsilon$ whenever $0 < |x - c| < \delta$

$\implies$

$0 < |x-c| < \delta \implies |f(x) - L| < \epsilon$

Consider

$$|f(x) - L| = |(5x-3) - 2|$$
$$= |5x - 5|$$
$$= 5|x-1|$$
$$< 5\delta = \epsilon$$

$$5\delta = \epsilon$$

with choice of $\boxed{\delta = \epsilon/5}$

Verifying

$$|f(x) - L| = |5x - 3 - 2|$$
$$= |5x - 5|$$
$$= 5|x-1|$$
$$< 5 \cdot \delta$$
$$< \cancel{5} \cdot \epsilon/\cancel{5}$$
$$< \epsilon$$

For given $\epsilon > 0 \; \exists \; \delta = \epsilon/5 > 0 \ni$

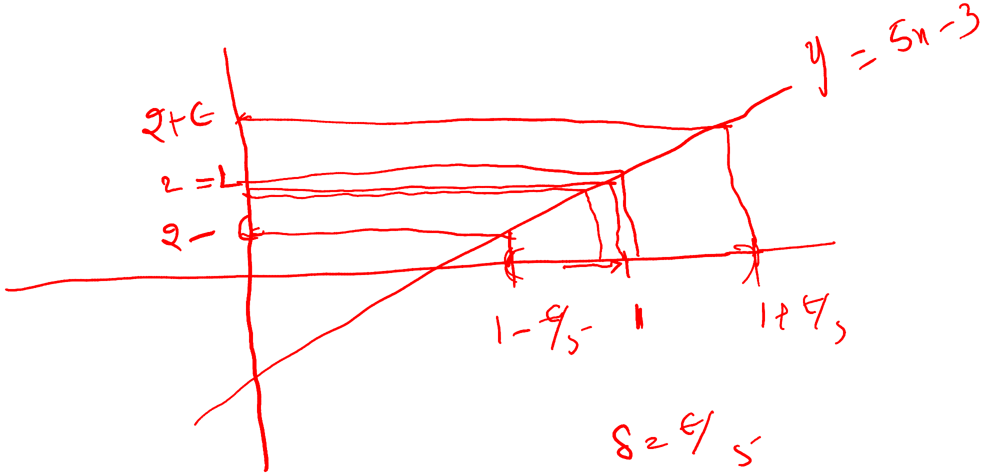
$|(5x-3) - 2| < \epsilon$ when $0 < |x-1| < \epsilon/5$

$$\lim_{x \to 1} (5x-1) = 2$$

$|x-1| < \epsilon/5$

$-\epsilon/5 < x-1 < \epsilon/5$

$1-\epsilon/5 < x < 1+\epsilon/5$



$\delta = \epsilon/5$