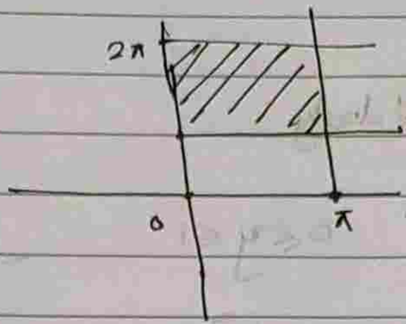
Tutorial :-

1] (a) $I = \int_{\pi}^{2\pi}\int_{0}^{\pi} (\sin x + \cos y)\,dx\,dy$



$0 \le x \le \pi \quad \pi \le y \le 2\pi$

$I = \int_{\pi}^{2\pi} \left[-\cos x + x\cos y\right]_0^{\pi} dy$

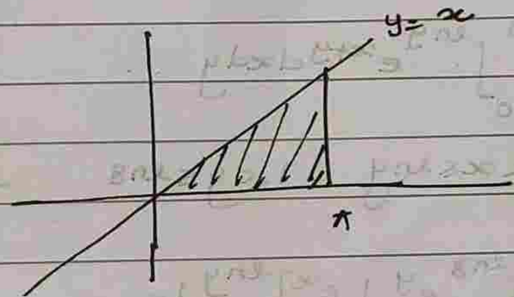
$= \int_{\pi}^{2\pi} \left[1 + \pi\cos y + 1\right] dy$

$= \left[2y + \pi\sin y\right]_{\pi}^{2\pi}$

$= 4\pi - 2\pi$

$I = 2\pi$

(b) $I = \int_0^{\pi}\int_0^{x} x\sin y\,dy\,dx.$



$0 \le y \le x \quad 0 \le x \le \pi$

$I = \int_0^{\pi} \left[-x\cos y\right]_0^{x} dx$

$= \int_0^{\pi} (-x\cos x + x)\, d\cancel{g}x$

$= \int_0^{\pi} x(1 - \cos x)\,dx$

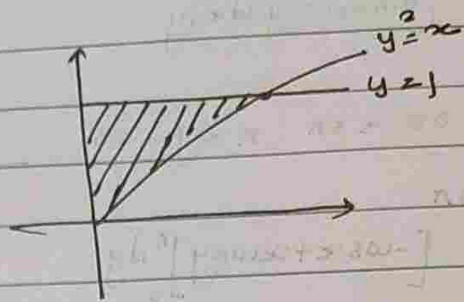
$= \left[x\left[x - \sin x\right]\right]_0^{\pi} - \int_0^{\pi} \cancel{(1-\cos x)}\,(x - \sin x)\,dx$

$= \pi\left[\pi\right] - \left[\frac{x^2}{2} + \cos x\right]_0^{\pi}$

$= \pi^2 - \left[\frac{\pi^2}{2} - 1 - 1\right]$

$I = 2 + \frac{\pi^2}{2}$

c] $I = \int_0^1 \int_0^{y^2} 3y^3 e^{xy}\, dx\, dy$



$0 \le x \le y^2 \quad 0 \le y \le 1$

$I = \int_0^1 3y^2 \left[e^{xy} \right]_0^{y^2} dy$

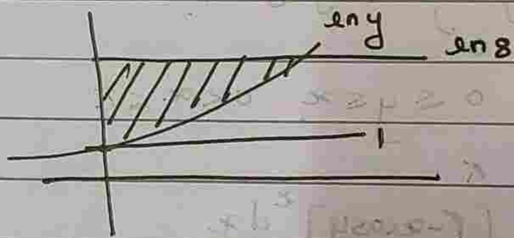
$= \int_0^1 3y^2 \left[e^{y^3} - 1 \right] dy$

$= \int_0^1 3y^2 e^{y^3} dy - \int_0^1 3y^2 dy$

$= e - 1 - 1$

$I = e - 2$

d] $I = \int_1^{\ln 8} \int_0^{\ln y} e^{x+y}\, dx\, dy$



$0 \le x \le \ln y \quad 1 \le y \le \ln 8$

$I = \int_1^{\ln 8} e^y \left[e^x \right]_0^{\ln y} dy$

$= \int_1^{\ln 8} e^y \left[y - 1 \right] dy$

$= \int_1^{\ln 8} e^y \left[(y-1) e^y \right]_1^{\ln 8} \cancel{+} - \int_1^{\ln 8} e^y dy$

$= (\cancel{\ln} 8 - \cancel{1})\, 8 \cancel{+ e}$

$I = (\ln 8 - 1)\, 8 - (8 - e)$

$= 8 \ln 8 - 16 + e$

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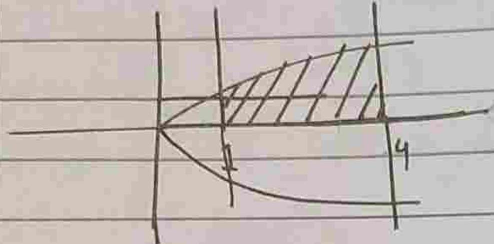
e] $I = \int_1^4 \int_0^{\sqrt{x}} \frac{3}{2} e^{y/\sqrt{x}} dy\, dx$

$0 \leq y \leq \sqrt{x} \qquad 1 \leq x \leq 4$



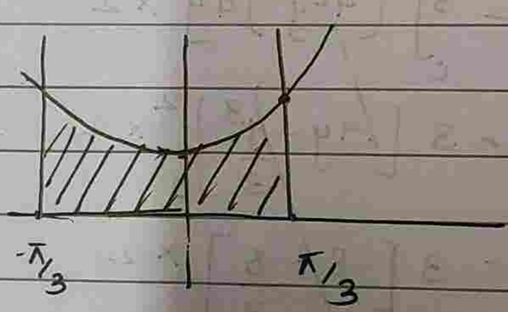
$I = \int_1^4 \frac{3}{2} \times \sqrt{x} \left[e^{y/\sqrt{x}} \right]_0^{\sqrt{x}} dx = \int_1^4 \frac{3}{2} \sqrt{x}(e-1) dx$

$= (e-1) \times \frac{3}{2} \times \frac{\left[x^{3/2} \right]_1^4}{3/2}$

$I = (e-1)\left[8-1\right] = 7(e-1)$

f] $\int_{-\pi/3}^{\pi/3} \int_0^{\sec t} 3\cos t\, du\, dt.$

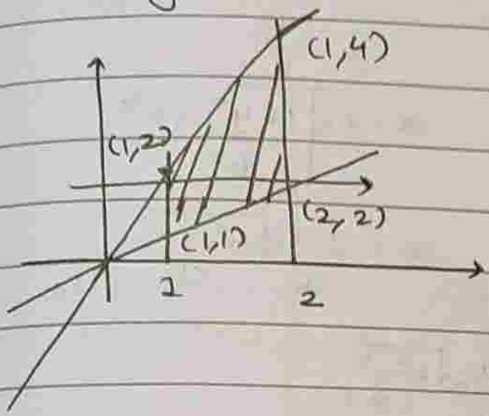
$0 \leq u \leq \sec t \qquad \frac{-\pi}{3} \leq t \leq \frac{\pi}{3}$



$I = \int_{-\pi/3}^{\pi/3} \left[3u\cos t \right]_0^{\sec t} dt$

$= \int_{-\pi/3}^{\pi/3} 3\cos t \left[\sec t \right] dt = 3 \times \frac{2\pi}{3} = 2\pi.$

2] (a) $f(x,y) = \frac{x}{y}$ $\quad y=x \quad y=2x, \; x=1, \; x=2$



$$\cancel{I = \int_1^4 \int_y^{y/2} \frac{x}{y}\, dx\, dy}$$

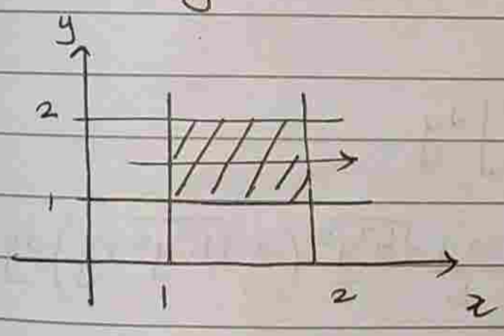
$$\cancel{= \int_1^4 \frac{1}{2y} \left[x^2\right]_y^{y/2} dy}$$

$$\cancel{= \int_1^4 \frac{1}{2y} \left[\right.}$$

$$I = \int_1^2 \int_x^{2x} \frac{x}{y}\, dy\, dx = \int_1^2 x \left(\ln y\right)_x^{2x} dx = \int_1^2 x \ln 2\, dx$$

$$= \frac{\ln 2}{2}\left[4-1\right] = \frac{3}{2}\ln 2$$

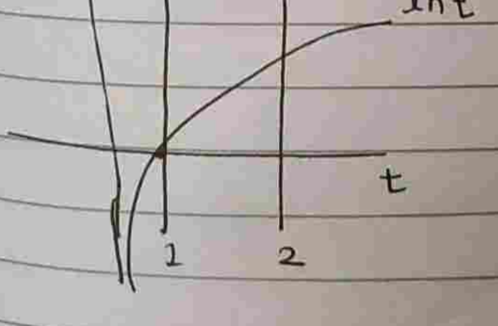
(b) $f(x,y) = \frac{1}{xy}$ $\quad 1 \le x \le 2 \quad 1 \le y \le 2.$



$$I = \int_1^2 \int_1^2 \frac{1}{xy}\, dx\, dy$$

$$I = \int_1^2 \frac{1}{y}\left[\ln x\right]_1^2 dy = \int_1^2 \frac{1}{y}\left[\ln 2\right] dy = (\ln 2)^2$$

(c) $f(s,t) = e^s \ln t$ $\quad s = \ln t \quad t=1$ to $t=2.$



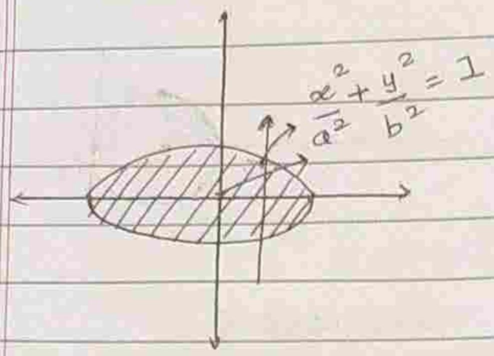
$$I = \int_1^2 \int_0^{\ln t} e^s \ln t\, ds\, dt$$

$$= \int_1^2 \ln t\left[e^s\right]_0^{\ln t} dt = \int_1^2 \ln t\left[t-1\right] dt$$

$$= \left[\ln t\left[\frac{t^2}{2} - t\right]\right]_1^2 - \int_1^2 \left(\frac{t}{2} - 1\right) dt$$

$$= \left[\ln t\left(\frac{t^2}{2} - t\right)\right]_1^2 - \left(\frac{t^2}{4} - t\right)_1^2 = -\frac{3}{4} + 1 = \boxed{\frac{1}{4} = I}$$

Q.2 d]



$$= \iint_R (x+y)^2 \, dx\, dy$$

$$= \int_{x=-a}^{a} \int_{y=-b\sqrt{1-\frac{x^2}{a^2}}}^{y=b\sqrt{1-x^2/a^2}} (x+y)^2 \, dy\, dx$$

OR

$x = ar\cos\theta \qquad y = br\sin\theta$

$r = \sqrt{\frac{x^2}{a^2} + \frac{y^2}{b^2}} \qquad \theta =$

$$J = \frac{\partial(x,y)}{\partial(r,\theta)} = abr.$$

$$dx\,dy = abr\,dr\,d\theta.$$

$$I = \int_{\theta=0}^{2\pi} \int_{0=r}^{1} (ar\cos\theta + br\sin\theta)^2 \, abr\,dr\,d\theta.$$

$$= \int_0^{2\pi} \int_0^1 r^3 (a^2\cos^2\theta + b\sin\theta)^2 ab \, dr\, d\theta$$

$$= \int_0^{2\pi} \frac{ab(a\cos\theta + b\sin\theta)^2}{4} \left[r^4 \right]_0^1 d\theta$$

$$= \int_0^{2\pi} \frac{ab}{4} (a\cos\theta + b\sin\theta)^2 d\theta$$

$s^2\theta - 1 = \cos 2\theta$

$$= \frac{ab}{4} \int_0^{2\pi} (a^2\cos^2\theta + b^2\sin^2\theta + ab\sin 2\theta)\, d\theta$$

$$= \frac{ab}{4} \int_0^{2\pi} \left(a^2\left(\frac{1+\cos 2\theta}{2}\right) + b^2\left(\frac{1-\cos 2\theta}{2}\right) + ab\sin 2\theta \right) d\theta$$

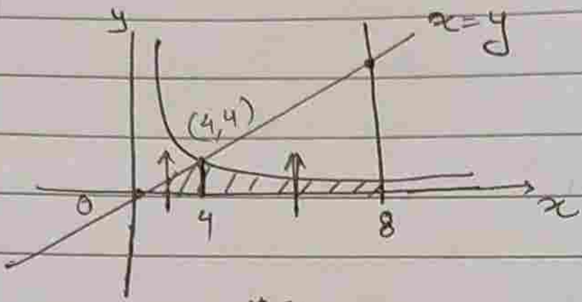
$$= \frac{ab}{4} \left[\frac{a^2}{2}\left(\theta + \frac{\sin 2\theta}{2}\right) + \frac{b^2}{2}\left(\theta - \frac{\sin 2\theta}{2}\right) \mp \frac{ab\cos 2\theta}{2} \right]_0^{2\pi}$$

$$= \frac{ab}{4} \left[\frac{a^2}{2}(2\pi) + \frac{b^2}{2}(2\pi) - \frac{ab}{2} + \frac{ab}{2} \right] = \frac{\pi ab}{4}(a^2+b^2)$$

e]

$\Rightarrow$ $f(x,y) = x^2$ $\quad xy = 16$

$x = y \quad y = 0 \quad x = 8.$



$$\cancel{I = \int \int_{0}^{x} x^2 \, dy \, dx}$$

$$I = \int_0^4 \int_0^x x^2 \, dy \, dx + \int_4^8 \int_0^{16/x} x^2 \, dy \, dx$$

$$= \int_0^4 x^3 \, dx + \int_4^8 16x \, dx$$

$$= \left[\frac{x^4}{4}\right]_0^4 + 8\left[x^2\right]_4^8$$

$$= (4)^3 + 8\left[64 - 16\right]$$

$$= 64 + 8\left[64 - 16\right]$$

$$I = 448$$

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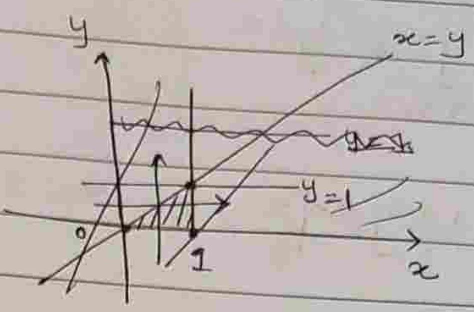
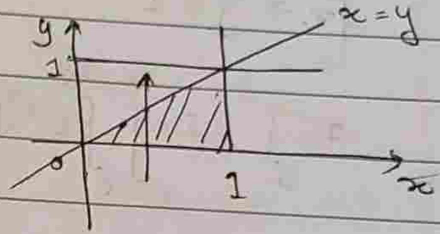
3.

(a) $\int_0^1 \int_y^1 x^2 e^{xy}\, dx\, dy$

$= y \le x \le 1 \quad 0 \le y \le 1$



$I = \int_0^1 \int_y^1 \quad I = \int_0^1 \int_0^x x^2 e^{xy}\, dy\, dx$



$I = \int_0^1 \cancel{x^2}\, x \left[e^{xy} \right]_0^x dx$

$I = \int_0^1 x \left[e^{x^2} - 1 \right] dx$

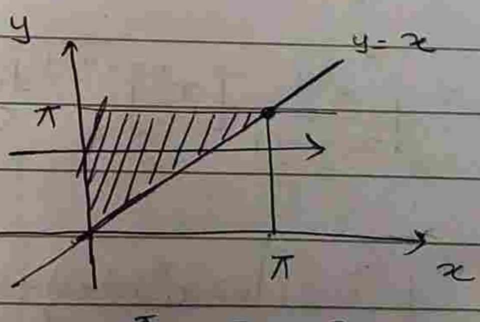
$= \frac{1}{2} \left[e^{x^2} \right]_0^1 - \left[\frac{x^2}{2} \right]_0^1$

$= \frac{e}{2} - \frac{1}{2} - \frac{1}{2}$

$\boxed{I = \frac{e}{2} - 1}$

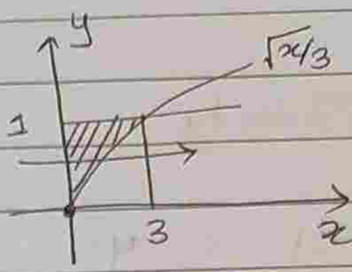
(b) $\int_0^\pi \int_x^\pi \frac{\sin y}{y}\, dy\, dx$

$x \le y \le \pi \quad 0 \le x \le \pi$



$I = \int_0^\pi \int_0^y \frac{\sin y}{y}\, dx\, dy = \int_0^\pi \sin y\, dy = -[\cos y]_0^\pi = -[-1-1] = 2$

(c) $\int_0^3 \int_{\sqrt{x/3}}^{1} e^{y^3}\, dy\, dx$



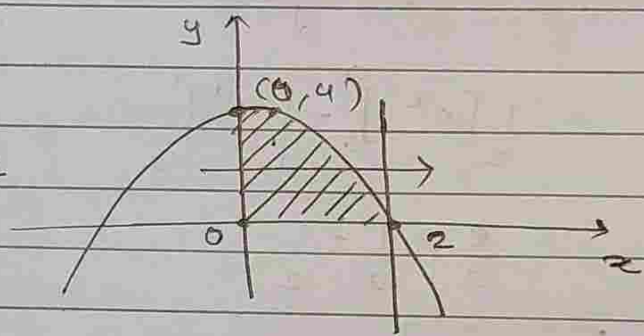
$\cancel{1 \le y \le \sqrt{\tfrac{x}{3}}}$ $\quad 0 \le x \le 3$

$\sqrt{\tfrac{x}{3}} \le y \le 1$

$\cancel{I = \int_0^3}$ $\quad I = \int_0^1 \int_0^{3y^2} e^{y^3}\, dx\, dy = \int_0^1 e^{y^3}(3y^2)\, dy = \left[e^{y^3}\right]_0^1 = e - 1$

$\cancel{= \int_0}$

(d) $\int_0^2 \int_0^{4-x^2} \frac{x e^{2y}}{4-y}\, dy\, dx$

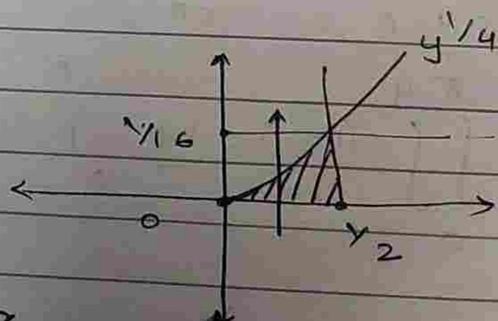


$0 \le y \le 4 - x^2 \quad 0 \le x \le 2$

$I = \int_0^4 \int_0^{\sqrt{4-y}} \frac{x e^{2y}}{4-y}\, dx\, dy$

$= \int_0^4 \frac{e^{2y}}{2(4-y)} \left[x^2\right]_0^{\sqrt{4-y}} dy = \int_0^4 \frac{e^{2y}}{2}\, dy = \frac{1}{4}\left[e^8 - 1\right]$

(e) $\int_0^{1/16} \int_{y^{1/4}}^{1/2} \cos(16\pi x^5)\, dx\, dy$



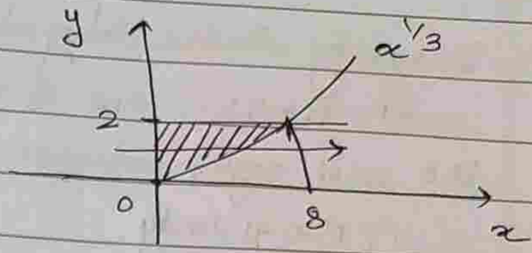
$y^{1/4} \le x \le 1/2 \quad 0 \le y \le 1/16$

$\cancel{I = \int_0^{16}}$ $\quad I = \int_0^{1/2} \int_0^{x^4} \cos(16\pi x^5)\, dy\, dx$

$= \int_0^{1/2} \cos(16\pi x^5)\, x^4\, dx = \frac{1}{16\pi \times 5}\left[\sin(16\pi x^5)\right]_0^{1/2}$

$I = \frac{1}{16\pi \times 5}[1] = \frac{1}{80\pi}$

(f) $\int_0^8 \int_{x^{1/3}}^{2} \frac{1}{y^4+1}\,dy\,dx = I$



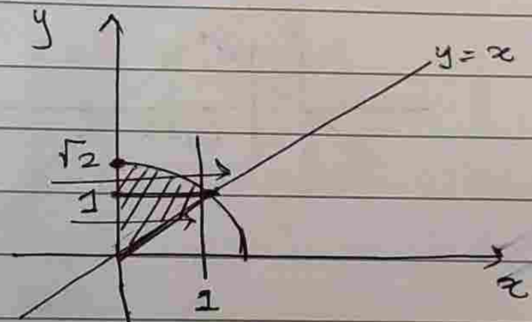
$x^{1/3} \le y \le 2 \quad 0 \le x \le 8$

$$I = \int_0^2 \int_0^{y^3} \frac{1}{y^4+1}\,dx\,dy$$

$$= \int_0^2 \frac{y^3}{y^4+1}\,dy = \frac{1}{4}\Big[\ln(y^4+1)\Big]_0^2$$

$$\boxed{I = \frac{\ln 17}{4}}$$

(g) $\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2+y^2}}\,dy\,dx$



$x \le y \le \sqrt{2-x^2} \quad 0 \le x \le 1$

$$I = \int_1^{\sqrt2} \int_0^{\sqrt{2-y^2}} \frac{x}{\sqrt{x^2+y^2}}\,dx\,dy + \int_0^1 \int_0^y \frac{x}{\sqrt{x^2+y^2}}\,dx\,dy$$

$$= \int_1^{\sqrt2} \frac{2}{2}\Big[\sqrt{x^2+y^2}\Big]_0^{\sqrt{2-y^2}} dy + \int_0^1 \frac{2}{2}\Big[\sqrt{x^2+y^2}\Big]_0^y dy$$

$$= \int_1^{\sqrt2} (\sqrt2 - y)\,dy + \int_0^1 \cancel{\sqrt2 y}(\sqrt2 - 1)y\,dy$$

$$= \Big[\sqrt2 y - \frac{y^2}{2}\Big]_1^{\sqrt2} + \frac{(\sqrt2-1)}{2}\Big[y^2\Big]_0^1$$

$$I = \Big[2 - 1 - \sqrt2 + \frac{1}{2}\Big] + \frac{\sqrt2-1}{2}[1] = 1 - \sqrt2 + \frac{1}{\sqrt2} = \frac{\sqrt2 - 1}{\sqrt2} = 1 - \frac{1}{\sqrt2}$$

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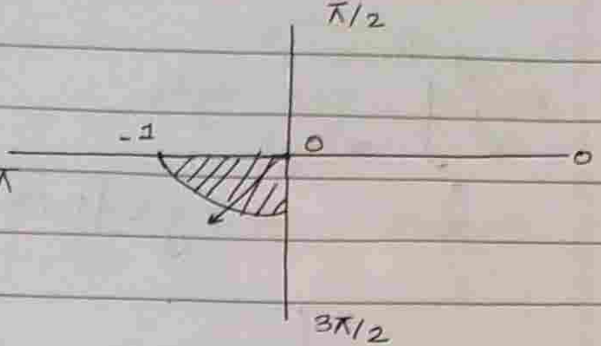
(17)

(a) $\displaystyle\int_{-1}^{0}\int_{-\sqrt{1-x^2}}^{0} \frac{2}{1+\sqrt{x^2+y^2}}\,dy\,dx$



$\cancel{0 \le y \le}\ -1 \le x \le 0 \quad -\sqrt{1-x^2} \le y \le 0$

$x = r\cos\theta \quad y = r\sin\theta$

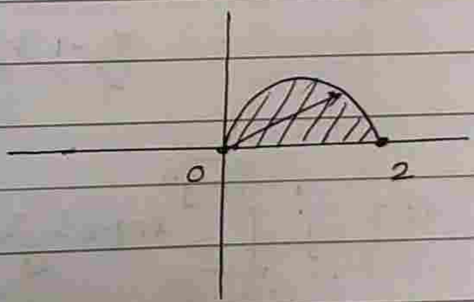
$J = dx\,dy = \begin{vmatrix} x_r & x_\theta \\ y_r & y_\theta \end{vmatrix} = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix} = r\,dr\,d\theta = \frac{\partial(x,y)}{\partial(r,\theta)}$

$I = \displaystyle\int_{\pi}^{3\pi/2}\int_{0}^{1} \frac{2r\,dr\,d\theta}{1+r} = 2\int_{\pi}^{3\pi/2}\int_{0}^{1}\left(1 - \frac{1}{1+r}\right)dr\,d\theta$ $\qquad r = u-1$ $\quad \frac{2(u-1)}{u}$

$= 2\displaystyle\int_{\pi}^{3\pi/2}\Big[r - \ln(1+r)\Big]_0^1 d\theta$ $\qquad 2\left(1-\frac{1}{u}\right)$

$= 2\displaystyle\int_{\pi}^{3\pi/2}(1-\ln 2)\,d\theta = (1-\ln 2)\left[\pi - \frac{3\pi}{2}\right]\times 2 = (1-\ln 2)\pi$

(b) $\displaystyle\int_{0}^{2}\int_{0}^{\sqrt{1-(x-1)^2}} \frac{x+y}{x^2+y^2}\,dy\,dx$



$0 \le x \le 2 \qquad 0 \le y \le \sqrt{1-(x-1)^2}$

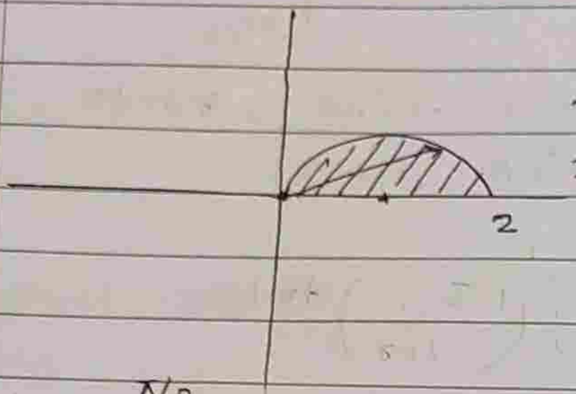
$(x-1)^2 + y^2 = 1$

$I = \displaystyle\int_{0}^{\pi/2}\int_{0}^{2} \frac{r(\cos\theta+\sin\theta)\times r\,dr\,d\theta}{r^2} = \int_{0}^{\pi/2} 2\left[\cos\theta + \sin\theta\right]d\theta$

$= 2\Big[-\sin\theta + \cos\theta\Big]_0^{\pi/2} = 2\Big[-1-1\Big] = -4$

(b) $I=\int_0^2\int_0^{\sqrt{1-(x-1)^2}} \frac{x+y}{x^2+y^2}\,dy\,dx$

$0\le x\le 2 \qquad 0\le y\le\sqrt{1-(x-1)^2} \qquad (x-1)^2+y^2=1$



$I=\int_0^{\pi/2}\int_0^{2\cos\theta}(\cos\theta+\sin\theta)\,dr\,d\theta$

$I=2\int_0^{\pi/2}(\cos\theta+\sin\theta)\,\cancel{d\theta}\cos\theta\,d\theta$

$=\cancel{2\left[\sin\theta - \cos\theta\right]_0^{\pi/2}}$

$=\cancel{2[1+1]=4}.$

$I=2\int_0^{\pi/2}\cos^2\theta+\sin\theta\cos\theta\,d\theta$

$=\int_0^{\pi/2}\left(\frac{\cos2\theta+1}{\cancel{2}}+\sin2\theta\right)d\theta$

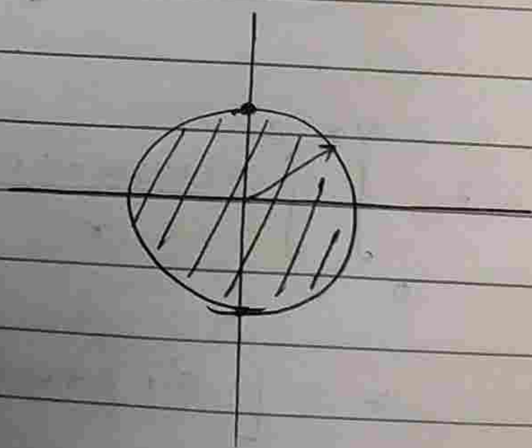
$=\left[\frac{\sin2\theta}{\cancel{2}}+\frac{\theta}{\cancel{2}}-\frac{\cos2\theta}{2}\right]_0^{\pi/2}$

$=\cancel{\frac{\pi}{4}-1+\frac{1}{2}}\quad \cancel{\frac{1}{3}}\quad \frac{\pi}{2}+\frac{1}{2}+\frac{1}{2}=\frac{\pi}{2}+1$

(c) $\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}}\ln(x^2+y^2+1)\,dx\,dy$

$-\sqrt{1-y^2}\le x\le\sqrt{1-y^2} \qquad -1\le y\le 1$

$x^2+y^2=1$



$I=\int_0^{2\pi}\int_0^1 \ln(r^2+1)(r\,dr\,d\theta)$

$=\frac{1}{2}\int_0^{2\pi}2\left[\left[\ln(r^2+1)\cdot r\right]_0^1-\int_0^1\frac{2r^2}{r^2+1}\,dr\right]$

$2-\frac{2}{r^2+1}$

$=\frac{1}{2}\int_0^{2\pi}2\left[\ln2-\left(2r-2\tan^{-1}r\right)_0^1\right]d\theta$

$=\frac{1}{2}\pi\left[\ln2-\left(2-\frac{\pi}{2}\right)\right]-\pi/[$

$I = \int_0^{2\pi}\int_0^1 \ln(r^2+1)(r\,dr\,d\theta)$ $\qquad r - \frac{r}{r^2+1}$

$= \int_0^{2\pi}\left[\ln(r^2+1)\cdot\frac{r^2}{2}\Big]_0^1 - \int_0^1 \frac{2r}{r^2+1}\times\frac{r^2}{2}\,dr\right]d\theta$

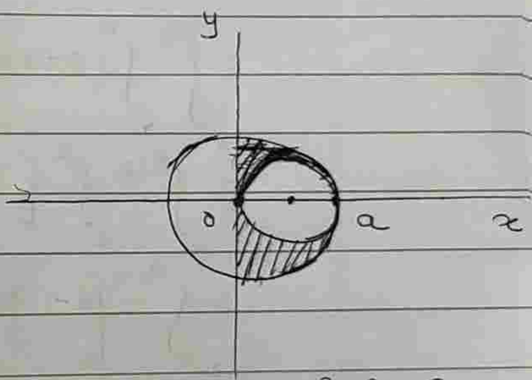
$= 2\pi \times \left[\frac{\ln 2}{2} - \int_0^1 \frac{r^3}{r^2+1}\,dr\right]$

$= 2\pi\left[\frac{\ln 2}{2} - \left[\frac{r^2}{2} - \frac{1}{2}\ln(r^2+1)\right]_0^1\right]$

$= 2\pi\left[\frac{\ln 2}{2} - \left[\frac{1}{2} - \frac{\ln 2}{2}\right]\right]$

$I = 2\pi\left[\ln 2 - \frac{1}{2}\right] = \pi(\ln 4 - 1)$

(d) $\int_0^a \int_{\sqrt{ax-x^2}}^{\sqrt{a^2-x^2}} \frac{dx\,dy}{\sqrt{a^2-x^2-y^2}}$



$0 \le x \le a \qquad \sqrt{ax-x^2} \le y \le \sqrt{a^2-x^2}$

$x^2+y^2=r^2$

$ax-x^2=y^2$

$x^2+y^2=ax \qquad x^2+y^2=a^2$

$a^2-r^2=u^2$

$-2r\,dr=2u\,du$

$\left(x-\frac{a}{2}\right)^2+y^2=\frac{a^2}{4} \qquad r^2=ar\cos\theta$

$r=a\cos\theta$

$I = \int_{3\pi/2}^{2\pi}\int_0^{a\cos\theta} \frac{r\,dr\,d\theta}{\sqrt{a^2-r^2}} = -\int_{3\pi/2}^{2\pi}\left[a\cos\theta\right]d\theta = +a\left[\sin\theta\right]_{3\pi/2}^{2\pi} = +a$

$$I = \int_0^{2\pi}\int_0^1 \ln(r^2+1)\,(r\,dr\,d\theta) \qquad r - \frac{r}{r^2+1}$$

$$= \int_0^{2\pi} \left[\ln(r^2+1)\cdot\frac{r^2}{2}\Big]_0^1 - \int_0^1 \frac{2r}{r^2+1} \times \frac{r^2}{2}\,dr \right] d\theta$$

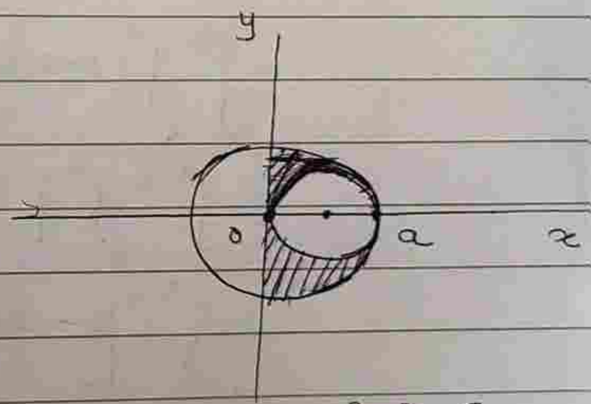
$$= 2\pi \times \left[\frac{\ln 2}{2} - \int_0^1 \frac{r^3}{r^2+1}\,dr \right]$$

$$= 2\pi \left[\frac{\ln 2}{2} - \left[\frac{r^2}{2} - \frac{1}{2}\ln(r^2+1) \right]_0^1 \right]$$

$$= 2\pi \left[\frac{\ln 2}{2} - \left[\frac{1}{2} - \frac{\ln 2}{2} \right] \right]$$

$$I = 2\pi \left[\ln 2 - \frac{1}{2} \right] = \pi(\ln 4 - 1)$$

(d) $$\int_0^a \int_{\sqrt{ax-x^2}}^{\sqrt{a^2-x^2}} \frac{dx\,dy}{\sqrt{a^2-x^2-y^2}}$$



$0 \le x \le a \qquad \sqrt{ax-x^2} \le y \le \sqrt{a^2-x^2}$

$x^2+y^2 = r^2$

$ax - x^2 = y^2$

$x^2+y^2 = ax \qquad x^2+y^2 = a^2$

$a^2 - r^2 = u^2$

$-2r\,dr = 2u\,du$

$\left(x-\frac{a}{2}\right)^2 + y^2 = \frac{a^2}{4} \qquad r^2 = ar\cos\theta$

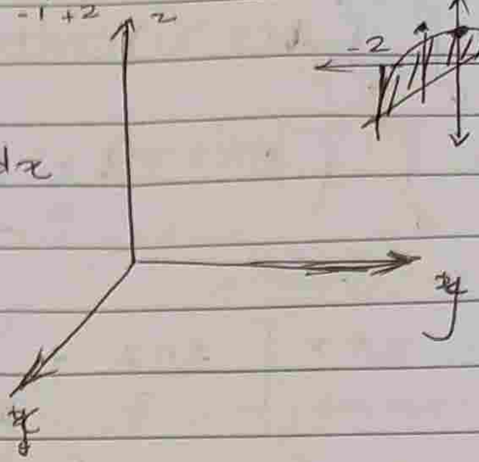
$r = a\cos\theta$

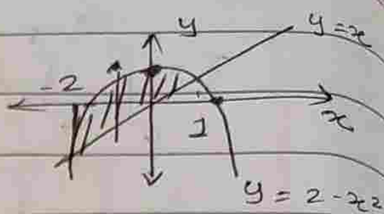
$$I = \int_{3\pi/2}^{2\pi}\int_0^{a\cos\theta} \frac{r\,dr\,d\theta}{\sqrt{a^2-r^2}} = -\frac{1}{2}\int_{3\pi/2}^{2\pi}\int_0^{r} \left[a\cos\theta\right] d\theta = \mp a\left[\sin\theta\right]_{3\pi/2}^{2\pi} = \mp a$$

$x^2+x-2=0$

$x=1 \quad x=-2$

(4) $z=x^2 \quad y=2-x^2 \quad y=x$

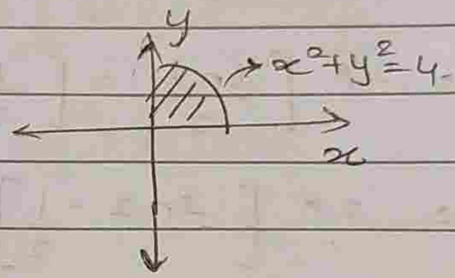




$$V=\int_{-2}^{1}\int_{y=x}^{2-x^2} x^2\,dy\,dx=\int_{-2}^{1} x^2\left[2-x^2-x\right]dx$$

$$=\left[\frac{2x^3}{3}-\frac{x^5}{5}-\frac{x^4}{4}\right]_{-2}^{1}=\frac{63}{20}$$

(5) $x^2+y^2=4 \quad z+y=3$



$$V=\iint(3-y)\,dy\,dx$$

$$=\int_0^{\pi/2}\int_0^2(3-r\sin\theta)\,r\,dr\,d\theta$$

$$=\int_0^{\pi/2}\cancel{\int_0^2}\left[\frac{3r^2}{2}-\frac{\sin\theta\, r^3}{3}\right]_0^2 d\theta$$

$$=\int_0^{\pi/2}\left[6-\frac{8}{3}\sin\theta\right]d\theta$$

$$=\left[6\theta+\frac{8}{3}\cos\theta\right]_0^{\pi/2}$$

$$=\frac{6\pi}{2}-\frac{8}{3}$$

$$V=\frac{9\pi-8}{3}$$

$y_2 - y_1$

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x}$$

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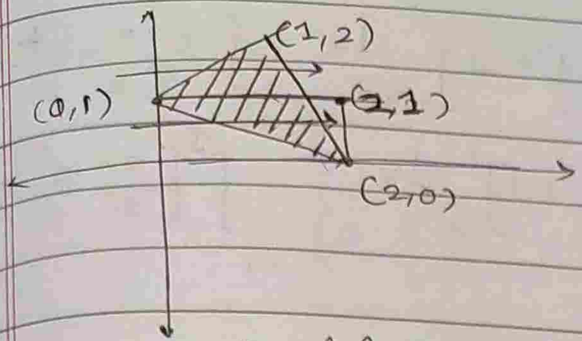
(6)



$$1 \qquad y-1 = (x-0)\times\left(\frac{1}{}\right)$$

$$y-1 = x$$

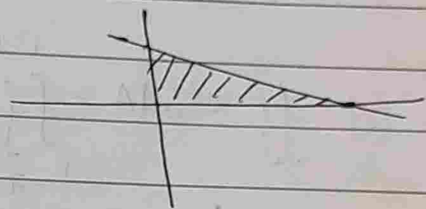
$$y = x-2 \times(-2)$$

$$y = (x-2)\times\left(\frac{-1}{2}\right)$$

$$I = \iint_R f(x,y)\,dx\,dy = \int_0^1 \int_{x=-2y+2}^{-y/2+2} f(x,y)\,dx\,dy + \int_1^2 \int_{x=y-1}^{-y/2+2} f(x,y)\,dx\,dy.$$

(7)

$z = 12 - 3y^2 \qquad x+y=2$



$$V = \int_0^2 \int_0^{2-y} (12-3y^2)\,dx\,dy$$

$$= \int_0^2 (12-3y^2)(2-y)\,dy = 3\int_0^2 (4-y^2)(2-y)\,dy = 3\int_0^2 \cancel{(2-y)(4-y^2)}$$

$$= 3\int_0^2 (8-2y^2-4y+y^3)\,dy$$

$$= 3\left[8y - \frac{2y^3}{3} - 2y^2 + \frac{y^4}{4}\right]_0^2 = 20$$

$$\bar{x}\bar{y} = \frac{\int_{-1}^{1}\int_{x^2}^{1} x(y+1)\,dy\,dx}{\int_{-1}^{1}\int_{x^2}^{1}(y+1)\,dy\,dx} = \frac{\int_{-1}^{1} x\left[\frac{y^2}{2}+y\right]_{+x^2}^{1}dx}{\int_{-1}^{1}\left[\frac{y^2}{2}+y\right]_{+x^2}^{1}dx} = \frac{\int_{-1}^{1} x\left[\frac{3}{2}-\frac{x^4}{2}-x^2\right]dx}{\int_{-1}^{1}\left[\frac{3}{2}-\frac{x^4}{2}+x^2\right]dx}$$

$$\bar{x} = \frac{\left[\frac{3x^2}{4}-\frac{x^6}{12}-\frac{x^4}{4}\right]_{-1}^{1}}{\left[\frac{3x}{2}-\frac{x^5}{10}-\frac{x^3}{3}\right]_{-1}^{1}} = 0$$

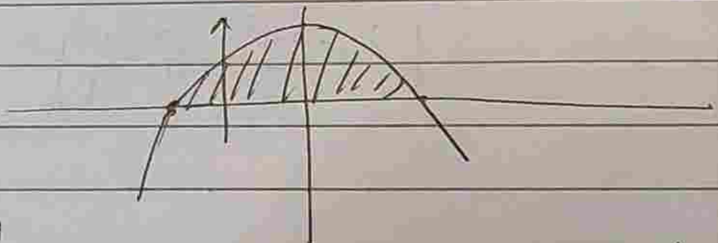
$$I_y = \iint x^2\delta\,dA = \int_{-1}^{1}\int_{x^2}^{1} x^2(y+1)\,dy\,dx = \int_{-1}^{1} x^2\left[\frac{y^2}{2}+y\right]_{x^2}^{1}dx$$

$$= \int_{-1}^{1} x^2\left[\frac{3}{2}-\frac{x^4}{2}-x^2\right]dx = \left[\frac{x^3}{2}-\frac{x^7}{14}-\frac{x^5}{5}\right]_{-1}^{1} = \frac{16}{35}$$

$$R_y = \sqrt{\frac{I_y}{M}} = \sqrt{\frac{\cancel{16}}{\cancel{35}_{7}} \times \frac{\cancel{15}^{3}}{\cancel{32}_{2}}} = \sqrt{\frac{3}{14}}$$

(g) $x^2+4y-16=0$

$$y = \frac{16-x^2}{4}$$



$$\bar{x} = \frac{\int_{-4}^{4}\int_{0}^{(16-x^2)/4} x\,dy\,dx}{\int_{-4}^{4}\int_{0}^{(16-x^2)/4} dy\,dx} = \frac{\int_{-4}^{4}\frac{x}{4}\times(16-x^2)\,dx}{\int_{-4}^{4}\frac{(16-x^2)}{4}dx} = \frac{\left[\frac{16x^2}{2}-\frac{x^4}{4}\right]_{-4}^{4}}{\left[16x-\frac{x^3}{3}\right]_{-4}^{4}} = 0$$

$$\bar{y} = \frac{\int_{-4}^{4}\int_{0}^{\frac{(16-x^2)}{4}} y\,dy\,dx}{\int_{-4}^{4}\int_{0}^{\frac{(16-x^2)}{4}} dy\,dx} = \frac{\frac{1}{2}\int_{-4}^{4}\left[\frac{(16-x^2)^2}{16}\right]dx}{\int_{-4}^{4}\left[\frac{16-x^2}{4}\right]dx} = \frac{\frac{1}{8}\left[256x+\frac{x^5}{5}-\frac{32x^3}{3}\right]_{-4}^{4}}{\left[16x-\frac{x^3}{3}\right]_{-4}^{4}}$$

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$\bar{y} = \cancel{\frac{1}{3}}\frac{8}{5}$

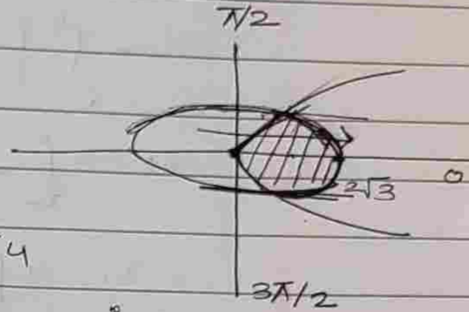
$$\boxed{(\bar{x}, \bar{y}) = \left(0, \frac{8}{5}\right)}$$

(10) $x^2+4y^2=12 \qquad x=4y^2 \qquad \delta(x,y)=5x$

$x^2+x=12$

$M = \iint (5x)\,dy\,dx \qquad x^2+x-12=0$

$x = 3, -4 \qquad -12 \to -3, +4$



$x = r\cos\theta \quad \cancel{y = r\sin\theta.} \qquad x = r\cos\theta \quad y = r\sin\theta$

$\cancel{r^2\cos^2\theta + 4r^2\sin^2\theta = 12} \qquad r^2\cos^2\theta + 4r^2\sin^2\theta = 12$

$\cancel{r^2 = \frac{12}{(}} \qquad r^2 = 12/$

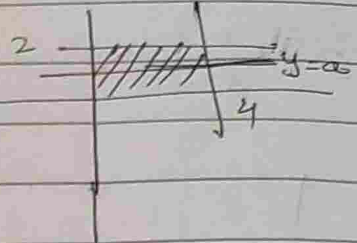
$\cancel{M = \int_{3\pi/2}^{2\pi}\int (5x)\,dy\,dx = \int_{3\pi/2}^{2\pi}\int_0^{2\sqrt{3}}}$

$$M = \int_{-\sqrt{3/4}}^{+\sqrt{3/4}}\int_{4y^2}^{\sqrt{12-4y^2}} (5x)\,dx\,dy = \int_{-\sqrt{3/4}}^{\sqrt{3/4}} \frac{5}{2}\left[12-4y^2-16y^4\right] dy$$

$$= \frac{5\times 4}{2}\left[3y - \frac{y^3}{3} - \frac{4\times y^5}{5}\right]_{\sqrt{3/4}}^{\sqrt{3/4}} = 10\times\left[3\times\frac{\sqrt{3}}{2} - \frac{1}{3}\times\left(\frac{\sqrt{3}}{2}\right)^3 - \frac{4}{5}\times\left(\frac{\sqrt{3}}{2}\right)^5\right]$$

$$\boxed{M = 23\sqrt{3}}$$

11] $\delta(x,y) = 1 \qquad x=4,\ y=2$



$$I_a = \int_0^4 \int_0^2 (y-a)^2\,dy\,dx$$

$$I_a = \int_0^4 \int_0^2 \left[y^2 + a^2 - 2ay\right] dy\,dx$$

$$= \int_0^4 \left[\frac{y^3}{3} + a^2 y - ay^2\right]_0^2 dx$$

$$I_a = 4\left[\frac{2^3}{3} + a^2 \times 2 - a(2)^2\right]$$

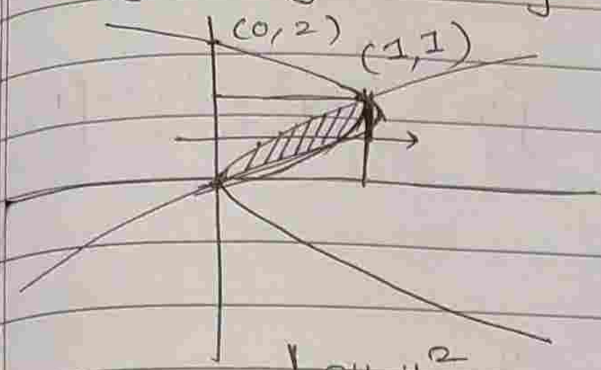
$$I_a = 4\left[\frac{8}{3} + 2a^2 - 4a\right] = 8\left[\frac{4}{3} + a^2 - 2a\right]$$

$$\frac{dI_a}{da} = 8\left[2a - 2\right] = 0$$

$$\boxed{a = 1}$$

12

$f(x,y) = 100(y+1)$ $\quad x = y^2 \quad x = 2y - y^2 = y^2 \quad 2y^2 - 2y = 0$



$y(y-1) = 0$

$y = 0,\ y = 1$

$2y - y^2 = 0$

$y(2-y) = 0$

$y = 0, 2$

$\cancel{x =}\ x = -(y^2 - 2y)$

$x = -(y-1)^2 + 1$

$x - 1 = -(y-1)^2$

$$I = \int_0^1 \int_{y^2}^{2y-y^2} 100(y+1)\, d\cancel{y}x\, dy$$

$$= 100\int_0^1 [\cancel{x^2}(y+1)[2y - y^2 - y^2]\,dy$$

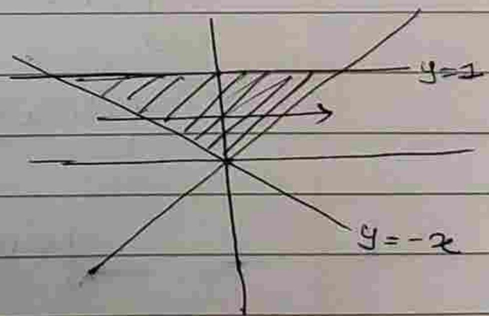
$$= 200\int_0^1 (y+1)(y - y^2)\,dy$$

$$= 200\int_0^1 \cancel{y^2} - y^3 + y - \cancel{y^2}\,dy = 200\int_0^1 y - y^3\,dy = 200\left[\frac{y^2}{2} - \frac{y^4}{4}\right]_0^1$$

$$I = 200\left[\frac{1}{4}\right] = 50 \qquad \boxed{I = 50}$$

13

$y = x \quad y = -x \quad y = 1 \qquad \delta(x,y) = 3x^2 + 1$



$$I_0 = \iint (x^2 + y^2)(3x^2 + 1)\,dx\,dy$$

$$= \int_0^1 \int_{x=-y}^{y} (x^2 + y^2)(3x^2 + 1)\,dx\,dy$$

$$= \int_0^1 \int_{-y}^{y} (3x^4 + x^2 + 3x^2y^2 + y^2)\,dx\,dy$$

$$= \int_0^1 \left[\frac{3}{5}x^5 + \frac{x^3}{3} + y^2x^3 + y^2x\right]_{-y}^{y} dy = 2\int_0^1 \left(\frac{3}{5}y^5 + \frac{y^3}{3} + y^5 + y^3\right)dy$$

$$= 2\times\left[\frac{3}{5}\times\frac{1}{6} + \frac{1}{3}\times\frac{1}{4} + \frac{1}{6} + \frac{1}{4}\right] = \frac{6}{5} \qquad \boxed{I_0 = \frac{6}{5}}$$

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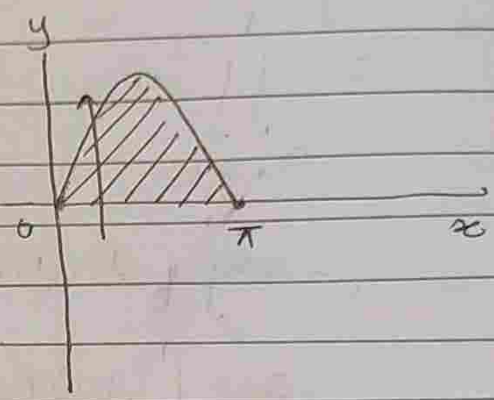
$$R_0 = \sqrt{\frac{I_0}{m}} = \sqrt{\frac{6}{5}\times\frac{1}{3}} = \sqrt{\frac{2}{5}}$$

$$m = \int_0^1\int_{-y}^{y} (3x^2+1)\,dx\,dy = \int_0^1 \left[x^3 + x\right]_{-y}^{y} dy = 2\int_0^1 \left[y^3+y\right] dy = 2\left[\frac{1}{4}+\frac{1}{2}\right]$$

$$m = 3$$

14] $y = \sin^2 x \quad y = 0 \quad 0 \le x \le \pi$



$$\bar{x} = \frac{\int_0^{\pi}\int_0^{\sin^2 x} x\,dy\,dx}{\int_0^{\pi}\int_0^{\sin^2 x} dy\,dx} = \frac{\int_0^{\pi} x\sin^2 x\,dx}{\int_0^{\pi}\sin^2 x\,dx}$$

$$= \frac{\left[x\int \sin^2 x\,dx\right]_0^{\pi} - \int_0^{\pi}\left(\int \sin^2 x\,dx\right)dx}{\left[\sin x(-\cos x)\right]_0^{\pi} + \int_0^{\pi}\cos^2 x\,dx}$$

$$\int \sin^2 x\,dx = \sin x(-\cos x) + \int \cos^2 x\,dx$$

$$\int \sin^2 x\,dx = -\sin x\cos x + x - \int \sin^2 x\,dx$$

$$\int \sin^2 x\,dx = \frac{-\sin x\cos x + x}{2}$$

$$\bar{x} = \frac{\left[x \times \frac{x - \sin x\cos x}{2}\right]_0^{\pi} - \int_0^{\pi}\frac{x - \sin x\cos x}{2}\,dx}{\left[\frac{-\sin x\cos x + x}{2}\right]_0^{\pi}}$$

$$= \frac{\frac{\pi^2}{2} - \frac{1}{2}\left[\frac{x^2}{2} - \frac{\sin^2 x}{2}\right]_0^{\pi}}{\frac{\pi}{2}} = \frac{\frac{\pi^2}{2} - \frac{1}{2}\left[\frac{\pi^2}{2}\right]}{\pi/2}$$

$$\bar{x} = \pi - \frac{\pi}{2} = \frac{\pi}{2}$$

$$\bar{x} = \frac{\pi}{2}$$

$$\bar{y} = \frac{\int_0^{\pi}\int_0^{\sin^2 x} y\,dy\,dx}{\int_0^{\pi}\int_0^{\sin^2 x} dy\,dx} = \int_0^{\pi} \sin^4 x\,dx \times \frac{2}{\pi} \times \frac{1}{2}$$

$$-1$$

$$= \frac{1}{\pi}\int_0^{\pi} \sin^4 x\,dx$$

$$\sin^4 x = \sin^2 x\,(1-\cos^2 x) = \sin^2 x - (\sin x \cos x)^2 = \sin^2 x - \frac{1}{4}\sin^2 2x$$

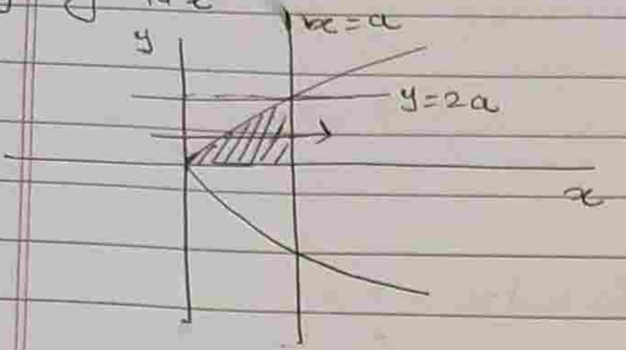
$$= \frac{1}{\pi}\left[\int_0^{\pi}\left(\sin^2 x - \frac{1}{4}\sin^2 2x\right)dx\right]$$

$$= \frac{1}{\pi}\left[\frac{\pi}{2} - \frac{1}{4} \times \frac{\pi}{2}\right] = \frac{1}{2} - \frac{1}{8} = \frac{3}{8}$$

$$\bar{y} = \frac{3}{8}$$

$$\boxed{(\bar{x}, \bar{y}) = \left(\frac{\pi}{2}, \frac{3}{8}\right)}$$

15] $y^2=4ax$



$\bar{x} = \iint x\,dy\,dx$

$$\bar{x} = \frac{\int_0^{2a}\int_{y^2/4a}^{a} x\,dx\,dy}{\int_0^{2a}\int_{y^2/4a}^{a} dx\,dy}$$

$$\bar{x} = \frac{\int_0^{2a}\left[a^2 - \frac{y^4}{16a^2}\right]dy \times \frac{1}{2}}{\int_0^{2a}\left[a-\frac{y^2}{4a}\right]dy} = \frac{\left[a^2y - \frac{y^5}{80a^2}\right]_0^{2a}\times\frac{1}{2}}{\left[ay-\frac{y^3}{12a}\right]_0^{2a}}$$

$$= \frac{\left[2a^3 - \frac{2^5a^3}{80}\right]\times\frac{1}{2}}{\left[2a^2-\frac{2^3a^2}{12}\right]} = \frac{3a}{5}$$

$$\bar{y} = \frac{\int_0^{2a}\int_{y^2/4a}^{a} y\,dx\,dy}{\int_0^{2a}\int_{y^2/4a}^{a} dx\,dy} = \frac{\int_0^{2a}\left[a-\frac{y^2}{4a}\right]y\,dy}{\left[2a^2-\frac{2^3a^2}{12}\right]}$$

$$= \frac{\left[\frac{ay^2}{2}-\frac{y^4}{16a}\right]_0^{2a}}{\left[2a^2-\frac{2^3a^2}{12}\right]} = \frac{\left[\frac{4a^3}{2}-\frac{8a^3}{\text{[illegible]}}\right]}{\left[2a^2-\frac{2a^2}{3}\right]} = \frac{3a}{4}$$

$$(\bar{x},\bar{y}) = \left(\frac{8a}{5},\frac{3a}{4}\right)$$

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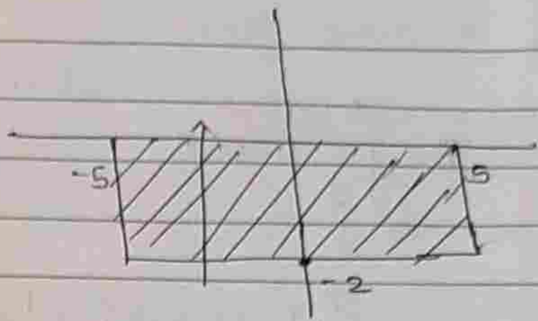
16] $f(x,y) = \dfrac{10000\, e^{y}}{1 + \frac{|x|}{2}}$



$$I = \int_{-5}^{5}\int_{-2}^{0} \frac{10000\, e^{y}}{1+\frac{|x|}{2}}\, dy\, dx$$

$$= \int_{-5}^{5} \frac{10000}{1+|x|/2}\left[1 - \frac{1}{e^{2}}\right] dx$$

$$= \left(1-\frac{1}{e^{2}}\right)\left[\int_{0}^{5} \frac{10000}{1+x/2}\, dx + \int_{-5}^{0} \frac{10000\, dx}{1-x/2}\right]$$

$$= 20000\left(1-\frac{1}{e^{2}}\right)\left[\left[\ln(2+x)\right]_{0}^{5} - \left[\ln(2-x)\right]_{-5}^{0}\right]$$

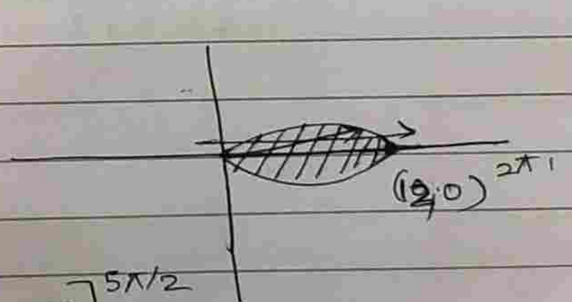
$$= 20000\left(1-\frac{1}{e^{2}}\right)\left[\ln 7/2 - \ln 2/7\right]$$

$$= 40000\left(1-\frac{1}{e^{2}}\right)\left[\ln 7/2\right]$$

$$\boxed{I = 43329}$$

18] $r = 12\cos 3\theta$

$$A = \int_{3\pi/2}^{5\pi/2}\int_{0}^{12\cos 3\theta} r\, dr\, d\theta = \int_{3\pi/2}^{5\pi/2} \frac{\cancel{144}\ 72 \times \cos^{2} 3\theta}{\cancel{2}}\, d\theta$$



$$= 72 \int_{3\pi/2}^{5\pi/2} \cos^{2} 3\theta\, d\theta = \frac{72}{3}\left[\frac{\theta + \sin\theta\cos\theta}{2}\right]_{3\pi/2}^{5\pi/2}$$

$2\pi + \pi/2 = 5\pi/2$

$$= 12\,[\pi]$$

$$\boxed{A = 12\pi}$$

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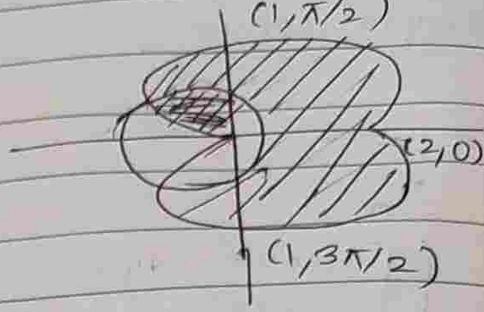
19) $r = 1+\cos\theta \quad r = 1$

$\cos\theta = 0$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$



$$A = \int_0^{2\pi}\int_1^{1+\cos\theta} r\,dr\,d\theta$$

$$= \int_0^{2\pi} \frac{1}{2}\left[(1+\cos\theta)^2 - 1\right] d\theta$$

$$= \frac{1}{2}\int_0^{2\pi}\left[1+\cos^2\theta + 2\cos\theta - 1\right] d\theta$$

$$= \frac{1}{2}\int_0^{2\pi}(\cos^2\theta + 2\cos\theta)\,d\theta$$

$$= \frac{1}{2}\left[\frac{\theta + \sin\theta\cos\theta}{2} + 2\sin\theta\right]_{\cancel{0}\,3\pi/2}^{\cancel{2\pi}\,5\pi/2}$$

$$= \cancel{\frac{1}{2}}\left[\cancel{\frac{2\pi}{}} - \cancel{\frac{\pi}{2}}\right] \frac{1}{2}\left[\frac{5\pi}{4} - \frac{3\pi}{4} + 2 + 2\right]$$

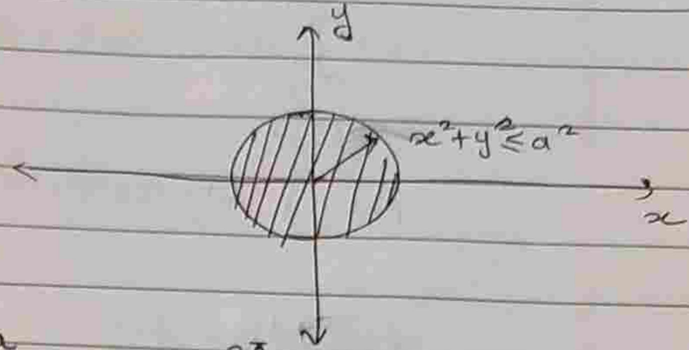
$$= \frac{1}{2}\left[\frac{\pi}{2} + 4\right]$$

$$= \frac{\pi}{4} + 2$$

$$A = \frac{\pi + 8}{4}$$

20] $z=\sqrt{x^2+y^2}$ $\quad x^2+y^2\le a^2$

$\quad\quad\quad\quad\quad\quad\quad r$



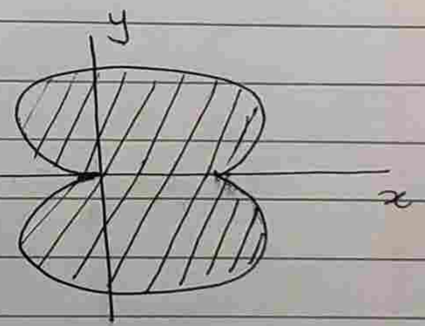
$V = \iint$

$$V=\iint \sqrt{x^2+y^2}\,dx\,dy=\int_{0}^{2\pi}\int_{0}^{a} r^2\,dr\,d\theta=\int_{0}^{2\pi}\frac{a^3}{3}\,d\theta=\frac{a^3}{3}\times 2\pi$$

$$A=\cancel{\int_0^{2\pi}\int_0^{a} r^2\times r\,dr\,d\theta}\int_0^{2\pi}\int_0^{a} r\,dr\,d\theta=\int_0^{2\pi}\frac{a^2}{2}\,d\theta=\frac{a^2}{2}\times 2\pi$$

$$\text{Average height}=\frac{V}{A}=\frac{\frac{a^3}{3}\times 2\pi}{\frac{a^2}{2}\times 2\pi}=\frac{2a}{3}$$

21] $r=1+\cos\theta$



$$\bar{x}=\frac{\iint x\,dx\,dy}{\iint dx\,dy}=\frac{\int_0^{2\pi}\int_0^{1+\cos\theta} r^2\cos\theta\,dr\,d\theta}{\int_0^{2\pi}\int_0^{1+\cos\theta} r\,dr\,d\theta}$$

$$=\frac{\int_0^{2\pi}\frac{\cos\theta}{3}\left[(1+\cos\theta)^3\right]d\theta}{\int_0^{2\pi}\frac{(1+\cos\theta)^2}{2}\,d\theta}=\frac{\int_0\frac{\cos\theta}{3}\left[1+\cos^3\theta+3\cos\theta(1+\cos\theta)\right]d\theta}{\int_0^{2\pi}\left(\frac{1+\cos^2\theta+2\cos\theta}{2}\right)d\theta}$$

$$=\frac{\int_0^{2\pi}\left[\frac{\cos\theta}{3}+\frac{\cos^4\theta}{3}+\cos^2\theta+\cos^3\theta\right]d\theta\times 2}{\int_0^{2\pi}\left(1+\cos^2\theta+2\cos\theta\right)d\theta}$$

$$= \frac{2 \times \int_0^{2\pi} \left[\frac{\cos\theta}{3} + \cos^2\theta + \cos^3\theta + \frac{\cos^4\theta}{3} \right] d\theta}{\int_0^{2\pi} (1 + \cos^2\theta + 2\cos\theta)\, d\theta}$$

$$\int \cos^2\theta\, d\theta = \frac{\theta + \sin\theta\cos\theta}{2} \qquad \int \sin^2\theta\, d\theta = \frac{\theta - \sin\theta\cos\theta}{2}$$

$$\int \cos^4\theta\, d\theta = \int \cos^2\theta(1-\sin^2\theta)\, d\theta = \int \frac{\cos^2\theta}{d\theta} - \int (\cos\theta\sin\theta)^2 d\theta$$

$$= \int \cos^2\theta\, d\theta - \frac{1}{4}\int \sin^2 2\theta\, d\theta$$

$$= \frac{\theta + \sin\theta\cos\theta}{2} - \frac{1}{4} \times \frac{1}{2} \times \frac{2\theta - \sin 2\theta \cos 2\theta}{2}$$

$$\int \cos^4\theta\, d\theta = \frac{\theta + \sin\theta\cos\theta}{2} - \frac{1}{16} \times (2\theta - \sin 2\theta\cos 2\theta)$$

$$\int \cos^3\theta\, d\theta = \int \cos\theta(1-\sin^2\theta)\, d\theta = \int \frac{\cos\theta}{d\theta} - \cancel{\int \sin^2\theta\, d\theta} \int \cos\theta\sin^2\theta\, d\theta$$

$$= \sin\theta - \frac{\sin^3\theta}{3}$$

$$\bar{x} = \frac{2 \times \left[\frac{\sin\theta}{3} + \frac{\theta + \sin\theta\cos\theta}{2} + \sin\theta - \frac{\sin^3\theta}{3} + \frac{1}{3}\left[\frac{\theta + \sin\theta\cos\theta}{2} - \frac{1}{16} \times (2\theta - \sin 2\theta\cos 2\theta) \right) \right]_0^{2\pi}}{\left[\theta + \frac{\theta + \sin\theta\cos\theta}{2} + 2\sin\theta \right]_0^{2\pi}}$$

$$= \frac{2 \cancel{\times} \left[\pi + \frac{\cancel{\pi}}{3} - \frac{1}{16} \times 4\pi \right]}{2\cancel{\pi} + \pi} = \frac{2}{3}\left[1 + \frac{1}{3} - \frac{1}{4} \right] =$$

$$= \frac{2 \times \left[\pi + \frac{1}{3}\left(\pi - \frac{1}{16} \times 4\pi \right) \right]}{2\pi + \pi} = \frac{2}{3} \times \left[1 + \frac{1}{3} \times \frac{3}{4} \right] = \frac{5}{6}$$

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$\bar{x} = \frac{5}{6}$

$$\bar{y} = \frac{\int_0^{2\pi}\int_0^{1+\cos\theta} r^2 \sin\theta \, dr \, d\theta}{3\pi} = \frac{\int_0^{2\pi} \frac{\sin\theta\,(1+\cos\theta)^3}{3} d\theta}{3\pi}$$

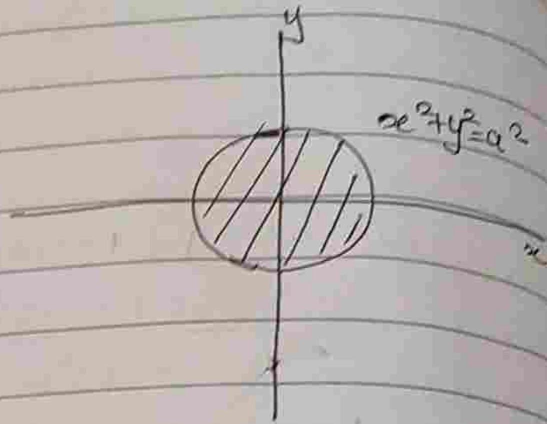
$$= \cancel{\int_0^{2\pi}} \cancel{\frac{1}{36\pi}}$$

$$= \frac{-1}{36\pi}\left[(1+\cos\theta)^4\right]_0^{2\pi}$$

$$= -\frac{1}{36\pi}\left[2^4 - 2^4\right] = 0$$

$\bar{y} = 0$

$$\boxed{(\bar{x}, \bar{y}) = \left(\frac{5}{6}, 0\right)}$$



23] $\delta(x,y) = k(x^2+y^2)$

$$I_x = \iint y^2 k(x^2+y^2)\,dx\,dy$$

$$= \int_0^{2\pi}\int_0^{a} r^2\sin^2\theta \times k r^2 \times r\,dr\,d\theta$$

$$= k\int_0^{2\pi}\sin^2\theta \int_0^{a} r^5\,dr\,d\theta$$

$$= k\int_0^{2\pi}\sin^2\theta \left[\frac{a^6}{6}\right] d\theta$$

$$= k\times\frac{a^6}{6}\times\left[\frac{\theta-\sin\theta\cos\theta}{2}\right]_0^{2\pi}$$

$$= k\times\frac{a^6}{6}\times\pi$$

$$I_0 = \iint (x^2+y^2)\,k\,(x^2+y^2)\,dx\,dy$$

$$= \int_0^{2\pi}\int_0^{a} k\times r^5\,dr\,d\theta = \int_0^{2\pi} k\times\frac{a^6}{6}\,d\theta = \frac{k\times a^6}{6}\times 2\pi = \frac{k a^6\pi}{3}$$