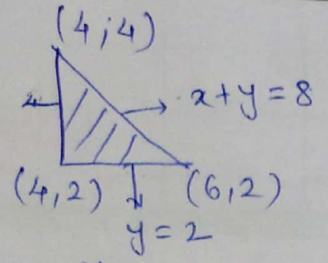
Q.1 (a).



$$\text{Area} = \iint 1 \cdot dA$$

$$= \int_{4}^{6} \int_{y=2}^{-x+8} dy\, dx \qquad \text{1- mks}$$

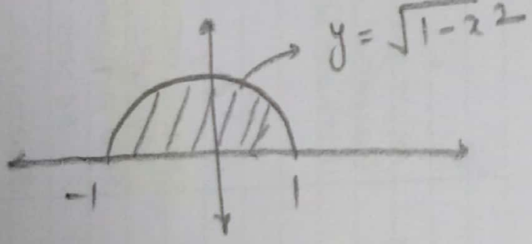
$$= \int_{4}^{6} (-x+8-2)\, dx$$

$$= \int_{4}^{6} (-x+6)\, dx$$

$$= \left(-\frac{x^2}{2} + 6x\right)_4^6$$

$= 2$ Sq. units — 1 mks.

(b) (I) →



– ½ mks

(II) → Given, $\rho(x,y) = 1$,

$$\bar{x} = \frac{\iint x\, dA}{\iint dA}, \quad \bar{Y} = \frac{\iint y\, dA}{\iint dA}.$$

From sketch, $\bar{x} = 0$, since sym. y-axis.

Now, $\iint dA = \int_0^1 \int_0^{\pi} r\, dr\, d\theta = \pi/2$ = Area of semi circle.

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and. $\iint y \, dA$

$$= \int_{-1}^{1} \int_{0}^{y=\sqrt{1-x^2}} y \, dy \, dx.$$

$$= \int_{-1}^{1} \left(\frac{y^2}{2} \right)_0^{\sqrt{1-x^2}} dx$$

$$= \frac{1}{2} \int_{-1}^{1} (1-x^2) \, dx$$

$$= \int_{0}^{1} (1-x^2) \, dx$$

$$= \left(x - \frac{x^3}{3} \right)_0^1$$

$$= 1 - \frac{1}{3}$$

$$= \frac{2}{3}$$

$$\therefore \bar{Y} = \frac{2}{3} \times \frac{2}{\pi} = \frac{4}{3\pi}$$

$$\therefore (\bar{X}, \bar{Y}) = \left(0, \frac{4}{3\pi}\right).$$

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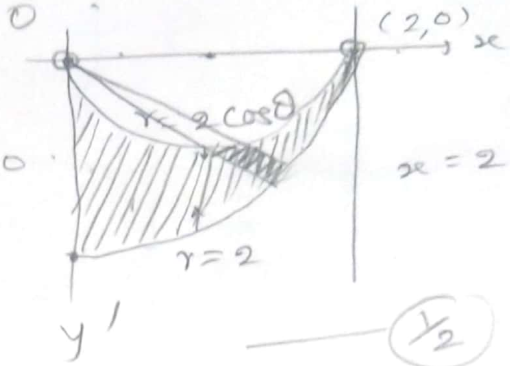
Test 1 . ABD .

a) $I = \int_0^2 \int_{-\sqrt{4-x^2}}^{-\sqrt{2x-x^2}} \frac{dy\,dx}{\sqrt{4-x^2-y^2}}$ ——— (CO2) 2 mrks.



——— 1/2

$x = r\cos\theta$, $y = r\sin\theta$

$x^2+y^2 = r^2$ $dA = r\,dr\,d\theta$

$x^2+y^2 = 4 \implies r = 2$

$x^2+y^2 = 2x \implies r = 2\cos\theta$

$\therefore I = \int_{3\pi/2}^{2\pi} \int_{2\cos\theta}^{2} \frac{r\,dr\,d\theta}{\sqrt{4-r^2}}$ ——— 1/2

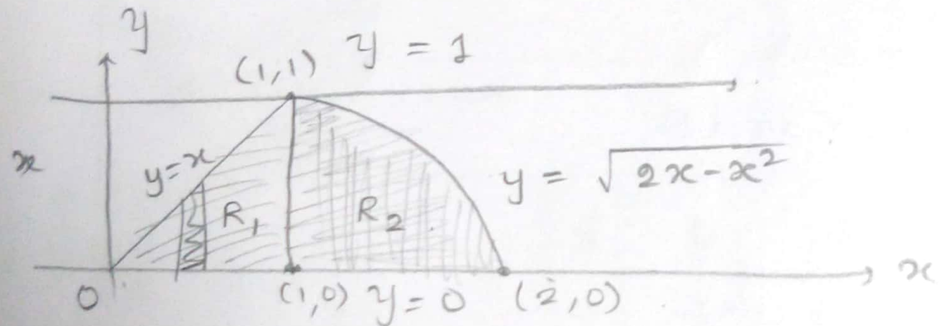
$= \int_{3\pi/2}^{2\pi} \left(-\frac{1}{2}\right) \sqrt{4-r^2}\,\Big|_{r=2\cos\theta}^{2} d\theta$ ——— 1/2

$= +\frac{1}{2}\int_{3\pi/2}^{2\pi} \cancel{2}\sin\theta\,d\theta$ ——— 1/2

$= -\cos\theta\,\Big|_{3\pi/2}^{2\pi} = -(1-0) = -1$ Ans.

b). change the order of Integration. (CO 2mr)

$$\int_{0}^{1} \int_{y}^{1+\sqrt{1-y^2}} f(x,y)\, \cancel{dy\,dx}\; dx\,dy$$

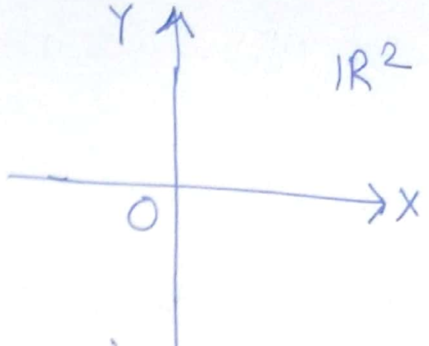


①

$$\therefore\ I = \int_{x=0}^{1} \int_{y=0}^{x} f(x,y)\, dy\, dx + \int_{x=1}^{2} \int_{y=0}^{\sqrt{2x-x^2}} f(x,y)\, dy\, dx$$

(1/2) (1/2)

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)}\,dx\,dy$$



Using polar coordinates $x = r\cos\theta,\ y = r\sin\theta,\ |J| = r$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)}\,dx\,dy = \int_{r=0}^{\infty}\int_{\theta=0}^{2\pi} e^{-r^2}\, r\, d\theta\, dr$$

$$—||— = 2\pi \int_{r=0}^{\infty} e^{-r^2} r\, dr$$

$$—||— = -\pi \int_{r=0}^{\infty} e^{-r^2}(-2r)\, dr$$

$$—||— = -\pi \lim_{R\to\infty} \int_{0}^{R} e^{-r^2}(-2r)\, dr$$

$$—||— = -\pi \left[\lim_{R\to\infty}\left(e^{-r^2}\right)_{r=R} - \left(e^{-r^2}\right)_{r=0}\right]$$

$$—||— = -\pi\,(0-1) = \pi$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)}\,dx\,dy = \pi$$

$\square$

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Q3

(b) step (iii) gives us $I^2 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)} dx dy = \pi$

$$I^2 = \pi \Rightarrow I = \sqrt{\pi}$$

Hence $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$ $\square$

(c) $$\int_0^1\int_0^2\int_0^3 (x^2+y^2+z^2)\, dz\, dy\, dx = \int_0^1\int_0^2 (3x^2+3y^2+9)\, dy\, dx$$

$$—''— = \int_0^1 \left(3x^2y + y^3 + 9y\right)_0^2 dx$$

$$—''— = \int_0^1 (6x^2+26)\, dx$$

$$—''— = \left(\frac{6x^3}{3} + 26x\right)_0^1$$

$$—''— = \left(2x^3 + 26x\right)_0^1 = 28$$

$$\int_0^1\int_0^2\int_0^3 (x^2+y^2+z^2)\, dz\, dy\, dx = 28$$ $\square$

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4)

$$I = \iiint_E y \, dV$$

E - region - below by plane $z = x+2$

above by xy-plane -

betn cylinders, $x^2+y^2=1$

$x^2+y^2=4$

Consider, cylindrical co-ordinates

$x = r\cos\theta$, $y = r\sin\theta$, $z = z$

$|J| = r$

as we are above the xy plane, so we are above the plane $z=0$ $\Rightarrow$ $0 \le z \le x+2$

$0 \le z \le r\cos\theta + 2$

Now, projection in xy-plane betn two circles

$x^2+y^2=1$, & $x^2+y^2=4$

so, ranges $0 \le \theta \le 2\pi$, $1 \le r \le 2$

$$V = \iiint_E y \, dV = \int_0^{2\pi}\int_1^2\int_0^{r\cos\theta+2} (r\sin\theta)\, r \, dz\, dr\, d\theta$$

$$= \int_0^{2\pi}\int_1^2 r^2\sin\theta\,(r\cos\theta+2)\, dr\, d\theta$$

$$= \int_0^{2\pi}\int_1^2 [\tfrac{1}{2}r^3\sin(2\theta) + 2r^2\sin\theta)\, dr\, d\theta$$

$$= \int_0^{2\pi}\left(\frac{1}{8}r^4\sin(2\theta) + \frac{2}{3}r^3\sin\theta\right)_1^2 d\theta$$

$$= \int_0^{2\pi} \frac{15}{8}\sin(2\theta) + \frac{14}{3}\sin\theta \, d\theta$$

$$= \left(-\frac{15}{6}\cos 2\theta - \frac{14}{3}\cos\theta\right)_0^{2\pi} = 0.$$

Q.5 Let ϕ represents a vertical angle measured from the +ve z-axis, θ represents a horizontal angle measured from the +ve x-axis.

(a) In spherical coordinate system, what does the eq^n $\phi = \pi/4$ represent

→ Note that ϕ is measured from +ve z-axis, thus $\phi = \frac{\pi}{4}$ represents all the points on the cone whose surface makes $\frac{\pi}{4}$ angle with +ve z-axis

$$z = \sqrt{x^2+y^2}$$

(b) In spherical coordinate system, what solid does the eq^n $\rho = 4\sin\phi\sin\theta$ represent? What is the center of that solid.

→ $\rho = 4\cos\phi\sin\theta \Rightarrow \rho^2 = 4\rho\sin\phi\sin\theta$

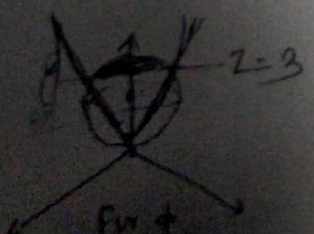
$$x^2+y^2+z^2 = 4y$$

$$\Rightarrow x^2+(y-2)^2+z^2 = 4$$

Sphere of radius 2
Center at $(0,2,0)$

(c) D is the part of sphere $x^2+y^2+(z-2)^2=4$ inside the paraboloid $z = x^2+y^2$ bounded below by the plane $z=3$.

→ Limits of D in spherical coordinates



Intersection of cone & sphere

$z+(z-2)^2 = 4 \Rightarrow z+z^2-4z+4=0$
$\Rightarrow z(z-3)=0 \Rightarrow z=0$ or $z=3$

$0 \le \theta \le 2\pi$

$\tan(\phi) = \frac{\sqrt{3}}{3}$
$\Rightarrow \phi = \frac{\pi}{6}$
$\Rightarrow 0 \le \phi \le \frac{\pi}{6}$

For ρ: $z=3 \Rightarrow \rho\cos\phi = 3$

$$\frac{3}{\cos\phi} \le \rho \le 4\cos\phi$$