Angle modulation

- unlike AM, here we change/modify the instantaneous phase or frequency of ~~modul~~ carrier as per $m(t)$

general form:

$$s(t) = A_c \cos[\overbrace{\omega_c t + \phi(t)}^{\theta_i(t)}]$$

$$= \mathrm{Re}\{A \cdot e^{\wedge}(j(\omega_c t + \phi(t)))\}$$

↳ exponential modulation

$\theta_i t$: Instantaneous phase of carrier

thus, $\omega_i(t) = \dfrac{d\,\theta_i(t)}{dt} \triangleq$ instantaneous freq.

now, $\theta_i(t) = \omega_c t + \phi(t)$ $\qquad \phi \Rightarrow$ phase deviation (instantaneous)

$\therefore \quad \omega_i(t) = \omega_c + \dfrac{d\phi}{dt}$ ⟶ instantaneous freq. deviation

Phase modulation

- We say carrier is phase modulated, when

$$\underline{\phi(t) = K_p\, m(t)} \qquad K_p : \text{rad}/V$$

Frequency modulation

- Freq. modulated when,

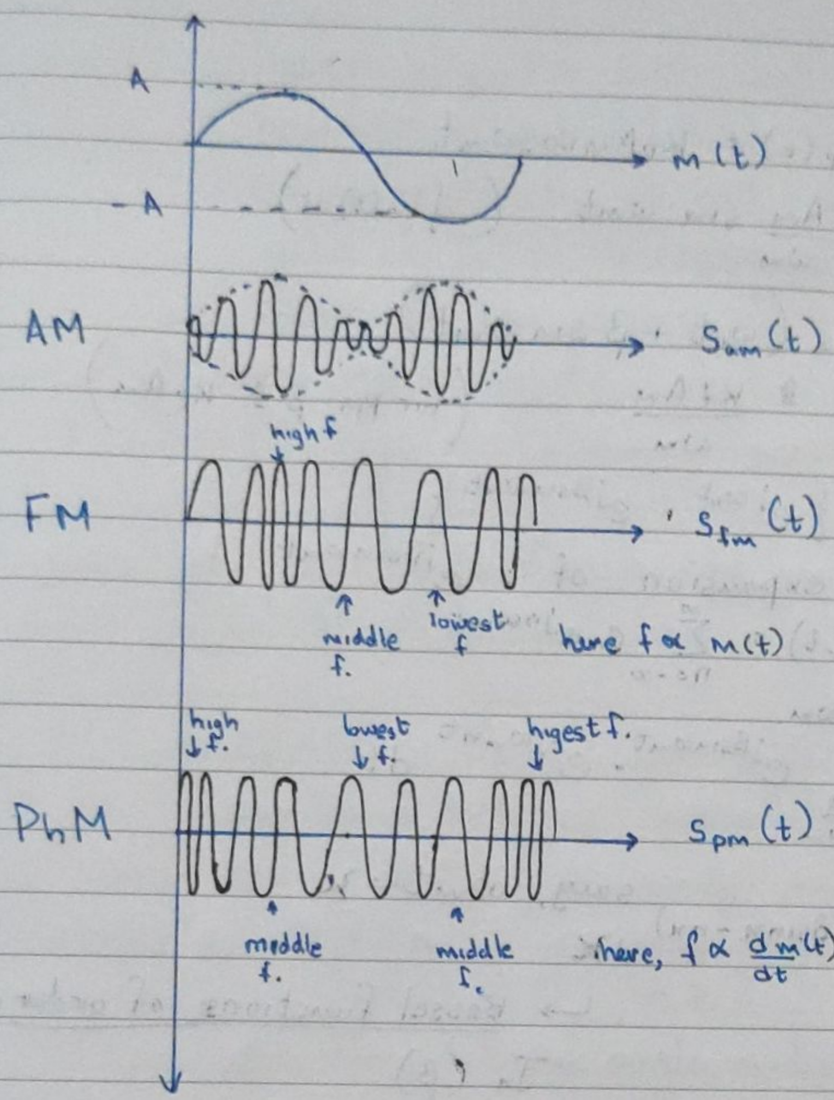
$$\frac{d\phi(t)}{dt} = K_f\, m(t) \qquad K_f : \text{rad}/sV$$

$$\Rightarrow \phi(t) = K_f \int_{-\infty}^{t} m(t)\,dt$$

$\therefore$ PM $\Rightarrow$ $s(t) = A\cos[\omega_c t + K_p\, m(t)]$

FM $\rightarrow$ $s(t) = A\cos[\omega_c t + K_f \int_{-\infty}^{t} m(t)\,dt]$

Waveforms



here, $f \propto \frac{dm(t)}{dt}$, $\therefore m(t) = \sin \Rightarrow m'(t) = \cos$

hence, f varies as per cos signal

we do not have over/under modulation like AM, but some other implications are involved.

Spectrum of $\angle$ mod. (+ B.W. & Power req.)

- FM & PM are non-linear modulations (not following superposiⁿ)
- Exact spectrum calculation is difficult for general message signals
- Possible to study spectrum when

$$m(t) = A_m \cdot \cos \omega_m t$$

thus,

PM: $\cancel{s_{pm}(t)} = \phi_p(t) = K_p A_m \cos\omega_m t$

FM: $\phi_f(t) = K_f \frac{A_m}{\omega_m} \sin\omega_m t \quad \left(\int m(t)\,dt\right)$

$$\therefore \quad s_{fm}(t) = A\cos(\omega_c t + \beta\sin\omega_m t)$$

$$\beta \triangleq \frac{K_f A_m}{\omega_m} \qquad (\text{for PM, } \beta \triangleq K_p A_m)$$

$$= A\,Re\{e^{j\omega_c t} \cdot e^{j\beta\sin\omega_m t}\}$$

Now, Fourier series expansion of : $e^{j\beta\sin\omega_m t}$

$e^{j\beta\sin\omega_m t}$: $f(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_m t}$

$$C_n = \frac{\omega_m}{2\pi}\int_{-\frac{\pi}{\omega_m}}^{\pi/\omega_m} e^{j\beta\sin\omega_m t} \cdot e^{-j\omega_m n t}\,dt$$

say, $\omega_m t = x$

$$C_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(\beta\sin x - nx)}\,dx$$

↳ Bessel functions of order n

$J_n(\beta)$

$$\therefore C_n \triangleq J_n(\beta)$$

This cannot be quantified & is available in tables.

Thus

$$s_{fm}(t) = A\,Re\left\{e^{j\omega_c t} \cdot \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_m t}\right\}$$

$$= A\,Re\left\{\sum_{n=-\infty}^{\infty} J_n(\beta)\,e^{j(\omega_c t + n\omega_m)}\right\}$$

$$= A \cdot \sum_{n=-\infty}^{\infty} J_n(\beta)\cos(\omega_c + n\omega_m)t$$

: A series expansion of FM
we get harmonics of $\omega_c \pm n\omega_m$

Theoretically, the B.W. will be ∞.
however, practical B.W. is reasonable & depends on $\beta \rightarrow$ which we call modulation index.
value of $J_n(\beta)$ decreases as n increases, hence we can ignore after certain n.

useful facts on Bessel f_n

1. $J_n(\beta) = (-1)^n J_{-n}(\beta)$

$$= \begin{cases} J_n(\beta) & n\text{ even} \\ -J_{-n}(\beta) & n\text{ - odd} \end{cases}$$

2. $\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$

3. For $\beta << 1$, $J_0(\beta)$ dominates.

ie.: $J_0(\beta) \approx 1$
$J_1(\beta) \approx \frac{\beta}{2}$
these will be the only ones mattering to spectrum

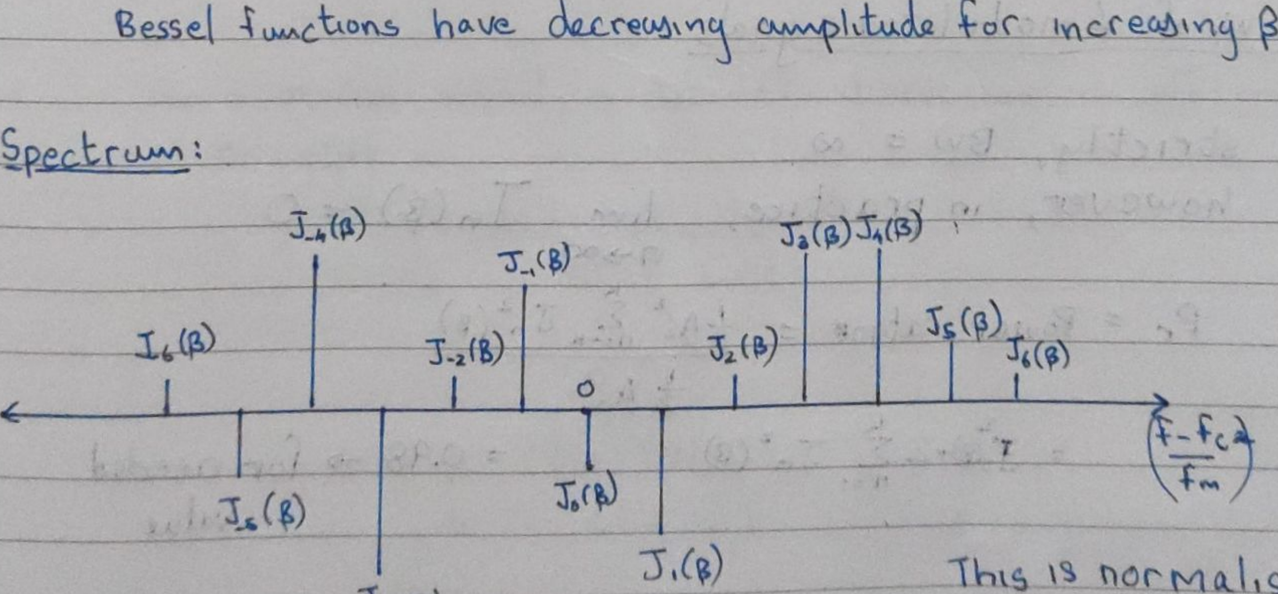
$J_n(\beta) \approx 0$, $n \geq 2$

↳ Narrow band angle modulation

4. $J_{n+1}(\beta) + J_{n-1}(\beta) = \frac{2n}{\beta} J_n(\beta)$

Bessel functions have decreasing amplitude for increasing β

Spectrum:



This is normalised!
$\therefore 0 \Rightarrow \frac{f-f_c}{f_m} = 0 \Rightarrow f = f_c$

By spectrum, we get a great amount of BW for large value of β.

Thus, spectral properties:

1. Spectrum contains carrier + ∞ Sidebands.
   ie $f_c \pm nf_m$ where $n \to 0 - \infty$

2. Relative amp. & no. of significant spectral components depends on β, the mod. index.

3. for $\beta << 1$, only J_0, $J_{\pm 1}$ are significant
   ∴ spectrum similar to AM but has $180°$ phase reversal of LSB component
   this is called Narrow Band FM (NBFM)

4. For $\beta >> 1$, no. of significant components are very large. ∴ Wideband FM & has large BW

B.W. of ∠modulated signals

strictly, BW = ∞
however, in practice, $\lim_{n \to \infty} J_n(\beta) = 0$

$$P_r = \text{Power ratio} = \frac{\frac{1}{2} A_c^2 \sum_{n=-K}^{K} J_n^2(\beta)}{\frac{1}{2} A_c^2}$$

$$= J_0^2(\beta) + 2\sum_{n=1}^{K} J_n^2(\beta) \quad = 0.98 \to \text{for needed value}$$

We consider 98% BW (ie, $P_r = 0.98$) for appropriate consideration & find K accordingly.

thus,

$$B.W. = 2Kf_m$$

For $P_r \geq 0.98$,

K will depend on β.

however, emperically, (ie, not proven, but usable)

$$K = \text{int}(\beta) + 1$$

ie if $\beta = 1.3$, $\Rightarrow K = 1 + 1 = 2$

$$\therefore BW = 2(\text{int}(\beta)+1) f_m \approx 2(\beta+1) f_m \rightarrow \text{a good approximation.}$$

now, by original formula, max freq. deviation $= \beta\omega_m$

$$\therefore BW = 2(\Delta f + f_m)$$

$$\Delta f \triangleq \beta f_m$$

↳ max freq. deviation

The above is for a single tone sinusoidal signal. For a general signal, a detailed discussion is off limits.

However,

$$D = \frac{\text{peak freq. devia}^n}{\text{BW of } m(t)} = \frac{\Delta f}{W}$$

$$\therefore BW \approx 2(D+1)W$$

$$\approx 2(\Delta f + W)$$

This is similar to earlier formula

CARSON'S FORMULAE.