Modulation

Types of modulation

- Continuous wave modulation
  - amp modulation
  - angle mod^n
    - Freq mod
    - Phase mod
- Pulse modulation
  - Analog mod
    - Pulse Amp Mod
    - P width Mod
    - P Posi^n M
  - Digital mod
    - Delta Mod
    - Pulse Code Mod

Amp Modulation

Basic modula^n formula:

$$\boxed{s(t) = A_c\left[1 + \mu \cos \omega_m t\right] \cos \omega_c t}$$

μ = mod^n index. may/may not $= \frac{A_m}{A_c}$

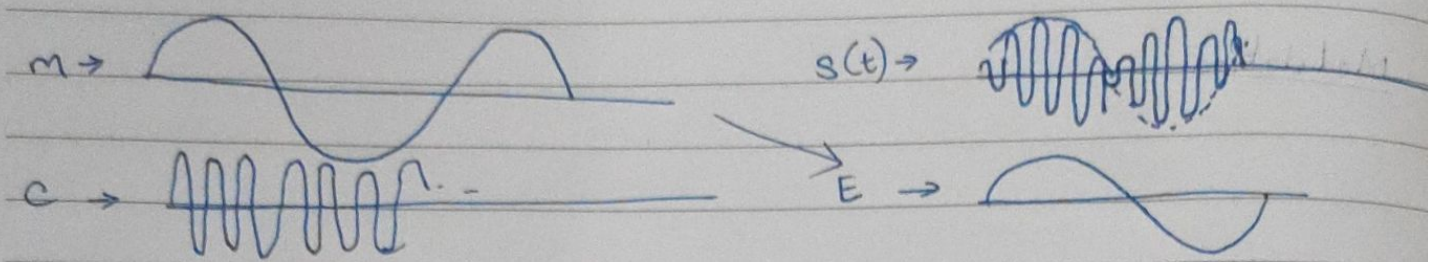
A_c = carrier amp

ω_c = " freq.

ω_m = msg "

$E = \cancel{A_c \cos}\ A_c\left[1 + \mu \cos \omega_m t\right]$

(Envelope)



$$E_{max} = A_c[1 + m_a]$$
$$+\quad E_{min} = A_c[1 - m_a]$$
$$\Rightarrow E_{mx} + E_{mN} = 2A_c$$

$$\boxed{A_c = \frac{E_{mx} + E_{mN}}{2}}$$

$$\div \Rightarrow \frac{E_{mx}}{E_{mn}} = \frac{1 + m_a}{1 - m_a}$$

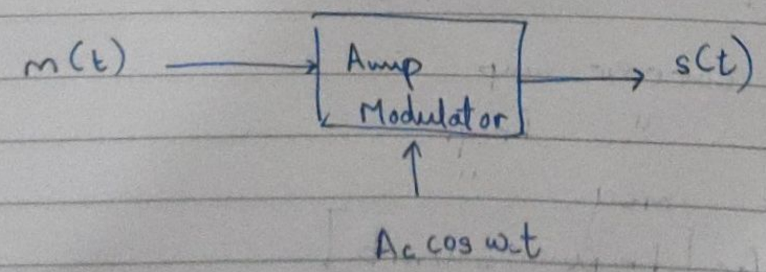
$$\Rightarrow \frac{E_{mx} - E_{mN}}{E_{mx} + E_n} = \frac{2m_a}{2} = m_a$$

$$\therefore m_a = \frac{E_{mx} - E_{mN}}{E_{mx} + E_{mN}} = \cancel{\frac{2A_c m_a}{2A_c}}$$

thus,

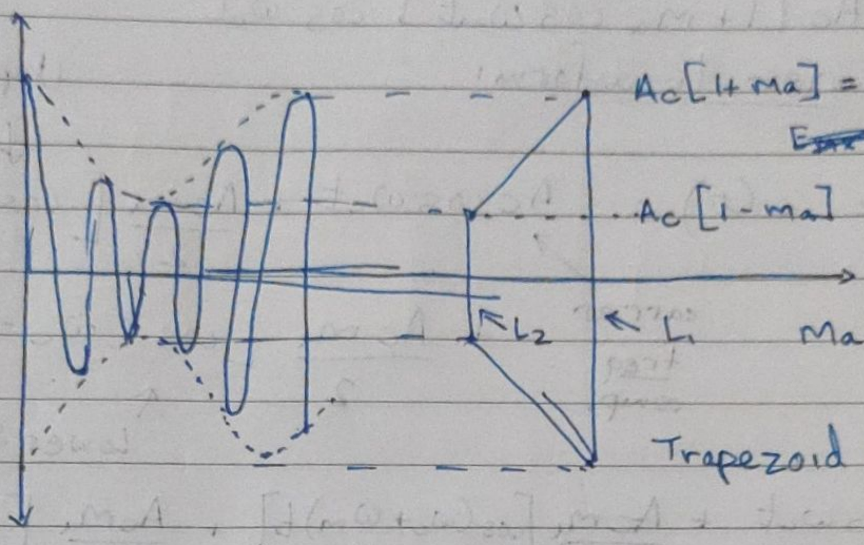
$$\mu = m_a = K_a A_m = \frac{A_m}{A_c} = \frac{E_{mx} - E_{mN}}{E_{mx} + E_{mn}}$$

(1) (2) (3)



Time domain, representa^n

Case ① $m_a < 1$: under modulation



$1 < K < 2$

$A_c[1+m_a] = A_c \cdot K$

$E_{\cancel{mx}}$

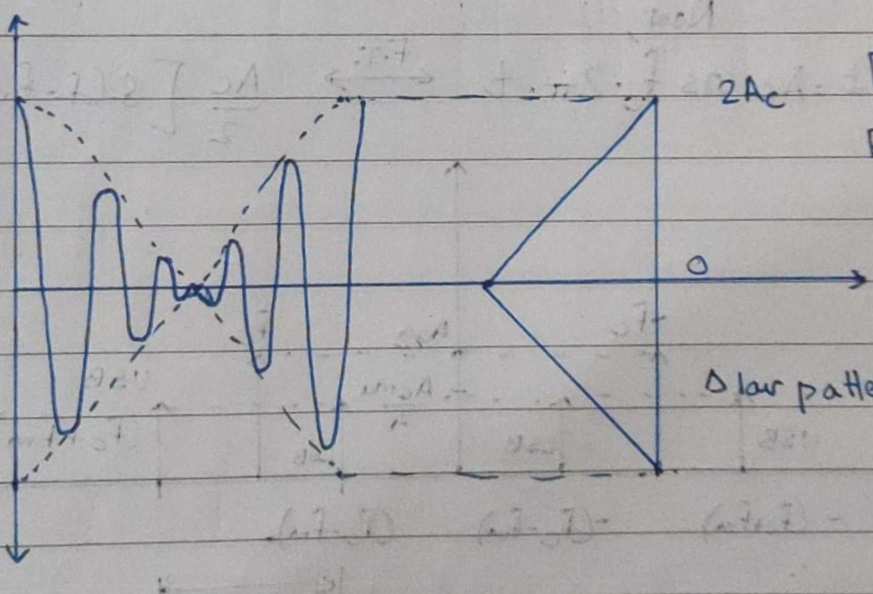
$A_c[1-m_a] = A_c \cdot M$

$0 < M < 1$

$m_a = \dfrac{L_1 - L_2}{L_1 + L_2}$

Trapezoid pattern

Case ② $m_a = 1$: ~~under~~ critical



$= 2A_c$

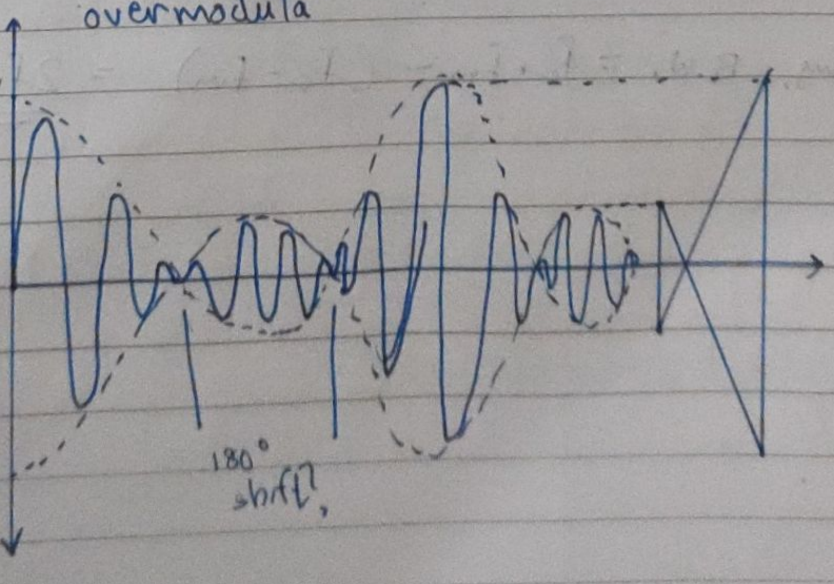
$E_{mx} = A_c[1+m_a]$

$E_{mn} = A\,C[1 - m_a]$

$= 0$

Δlar pattern

Case ③: $m_a > 1$ overmodula^n



$E_{mx} = A_c \cdot K$

$2 < K$

$E_{mN} = A_c \cdot M$

$M < 0$

-ve envelope

Freq domain representation

$s(t) = A_c [1 + m_a \cos \omega_m t] \cos \omega_c t$

Fourier transform:

Upper SideBand (USB) term

$~~S(s)~~ = s(t) = A_c \cos \omega_c t + \frac{A_c m_a}{2} [\cos(\omega_c + \omega_m) t]$

carrier freq comp.

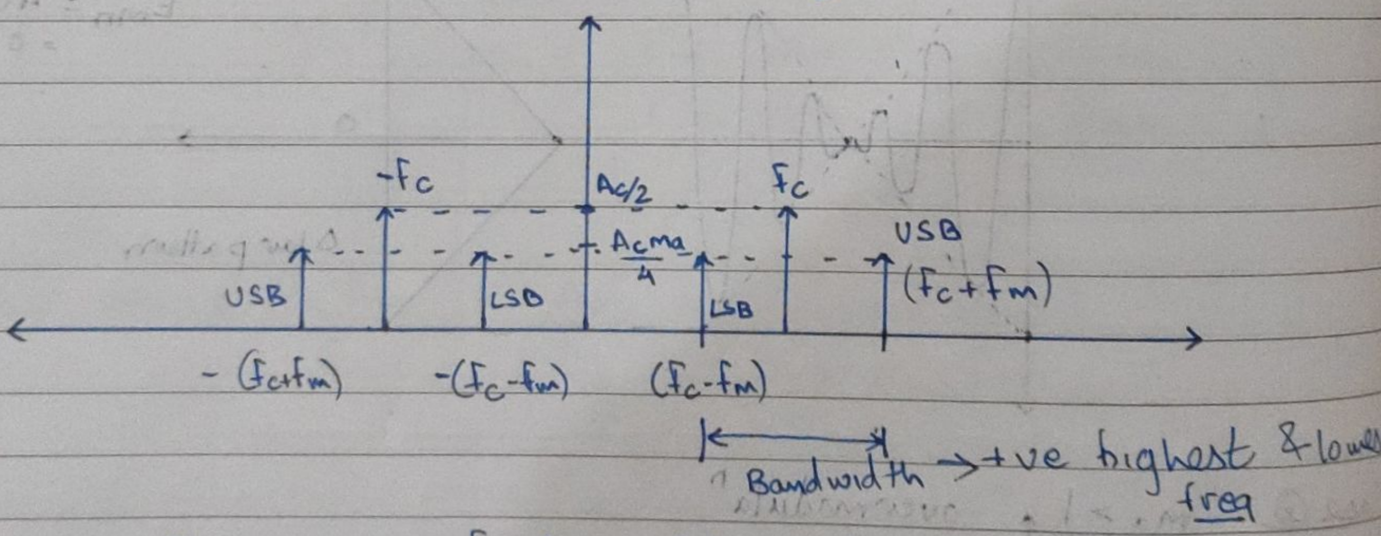
$+ \frac{A_c m_a}{2} [\cos(\omega_c - \omega_m) t]$

Lower SB (LSB) term

$$s(t) = A_c \cos \omega_c t + \frac{A_c m_a}{2} [\cos(\omega_c + \omega_m) t] + \frac{A_c m_a}{2} [\cos(\omega_c - \omega_m) t]$$

Now,

$$A_c \cos \omega_c t = A_c \cos f_c \cdot 2\pi \cdot t \xleftrightarrow{F.T.} \frac{A_c}{2} [\delta(f - f_c) + \delta(f + f_c)]$$



Thus, $B.W. = f_c + f_m - (f_c - f_m) = \underline{\underline{2 f_m}}$

Power calculation in AM

For single tone sinusoidal modulating signal

$$s(t) = \underset{①}{A_c \cos \omega_c t} + \underset{②}{\frac{A_c m_a}{2} \cos(\omega_c + \omega_m)t} + \underset{③}{\frac{A_c m_a}{2} \cos(\omega_c - \omega_m)t}$$

Carrier Power   USB power   LSB power

① C.P.

$$P_c = V_{rms} \cdot I_{rms}$$

$$= \frac{V_{rms}^2}{R} = \left(\frac{V_0}{\sqrt{2}}\right)^2 \cdot \frac{1}{R}$$

$$= \frac{V_0^2}{2R}$$

$$= \frac{A_c^2}{2R}$$

[If R not given, usually taken as 1Ω]

② USB P

$$P_{USB} = \frac{\left(A_c m_a/2\right)^2}{2R}$$

$$= \frac{A_c^2 M_a^2}{8R} = \frac{P_c M_a^2}{4}$$

③ LSB P

$$P_{LSB} = P_{USB} = \frac{A_c^2 m_a^2}{8R} = \frac{P_c M_a^2}{4}$$

Thus,

$$\text{total Power} = P_c + \underbrace{P_{USB} + P_{LSB}}_{\text{Total sideband Power}}$$

$$P_t = P_c + \frac{P_c m_a^2}{4} + \frac{P_c m_a^2}{4}$$

$$= P_c + \frac{P_c m_a^2}{2} \leftarrow \text{Sideband power}$$

$$P_t = P_c\left[1 + \frac{m_a^2}{2}\right]$$ for sinusoidal modulating signals

Transmission efficiency

$$\eta = \frac{P_{SB}}{P_t} \quad \left[\frac{\text{Sideband}}{\text{Total}}\right]$$

$$= \frac{P_c m_a^2/2}{P_c\left(1 + \frac{m_a^2}{2}\right)}$$

$$\eta = \frac{m_a^2}{2 + m_a^2} \quad [\text{for sinusoidal}]$$

For general,

$$s(t) = \underset{\text{carrier}}{A_c \cos \omega_c t} + \underset{\text{double sideband}}{A_c k_a m(t) \cos \omega_c t}$$

$$P_{SB} = \frac{1}{T}\int_T |x(t)|^2\, dt = \frac{1}{T}\int_{-T/2}^{T/2} A_c^2 k_a^2 m^2(t) \cos^2 \omega_c t\, dt$$

$$= \frac{1}{T}\int_{-T/2}^{T/2} \frac{A_c^2 k_a^2 m^2(t)}{2}(1 + \cos 2\omega_c t)\, dt$$

$$= \frac{1}{T}\int_{-T/2}^{T/2} \frac{A_c^2 k_a^2 m^2(t)}{2}\, dt + \frac{1}{T}\int_{-T/2}^{T/2} \frac{A_c^2 k_a^2 m^2(t)}{2} \cos 2\omega_c t$$

This is zero, due to cos

$$\therefore P_{SB} = \frac{1}{T}\int_{-T/2}^{T/2} \frac{A_c^2 K_a^2 m^2(t)}{2}\, dt$$

constants

$$= \frac{A_c^2 K_a^2}{2T}\int_{-T/2}^{T/2} m^2(t)\, dt$$

$$= \frac{A_c^2 K_a^2}{2} \cdot \underbrace{\frac{1}{T}\int_{-T/2}^{T/2} |m(t)|^2\, dt}_{}$$ ← Power of msg signal

For msg signal, say power is:

$$P_m = \frac{1}{T}\int_{-T/2}^{T/2} |m(t)|^2\, dt$$

$$\therefore P_{SB} = \frac{A_c^2 K_a^2}{2} \cdot P_m$$

Also, $P_c = \frac{A_c^2}{2R}$

$$\therefore P_t = P_c + P_{SB}$$

$$= \frac{A_c^2}{2R} + \frac{A_c^2 K_a^2 P_m}{2R}$$

earlier considered as 1, we now include R

$$\therefore P_t = P_c(1 + K_a^2 P_m)$$

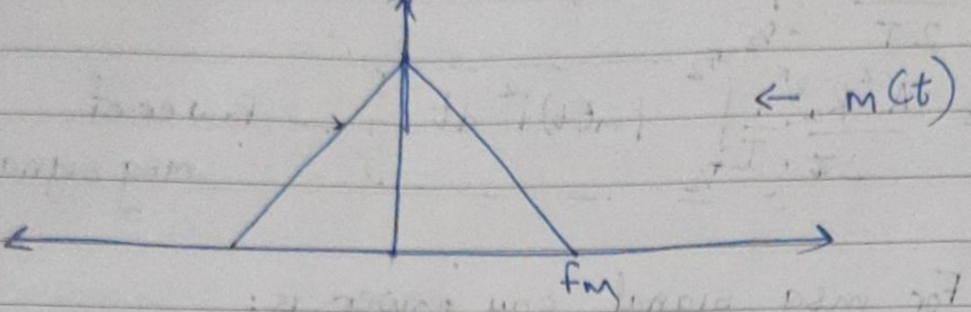
Transmission efficiency

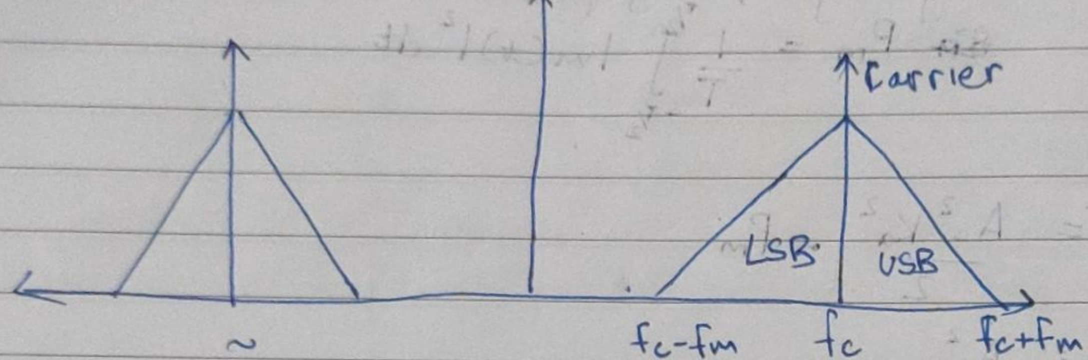
$$\eta = \frac{P_{SB}}{P_t} = \frac{\cancel{P_c} K_a^2 P_m}{\cancel{P_c}(1 + K_a^2 P_m)}$$

$$= \frac{K_a^2 P_m}{K_a^2 P_m + 1}$$

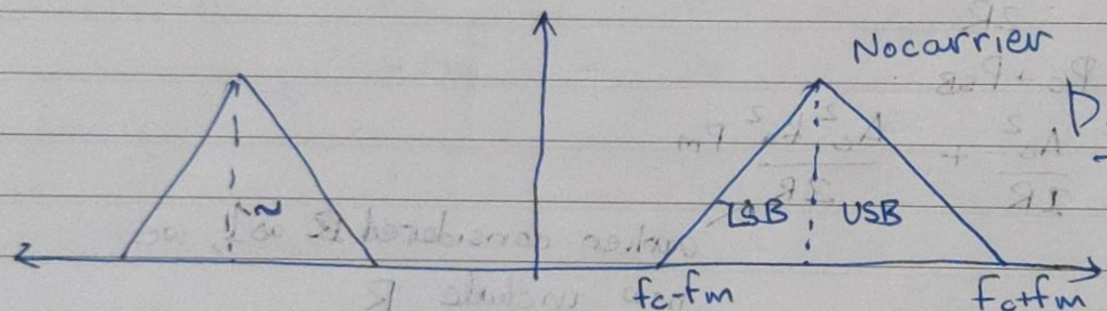
Types of AM

1) Double SideBand Suppressed Carrier → DSB-SC
2) Single " " " → SSB-SC
3) Vestigial " " " → VSB-SC



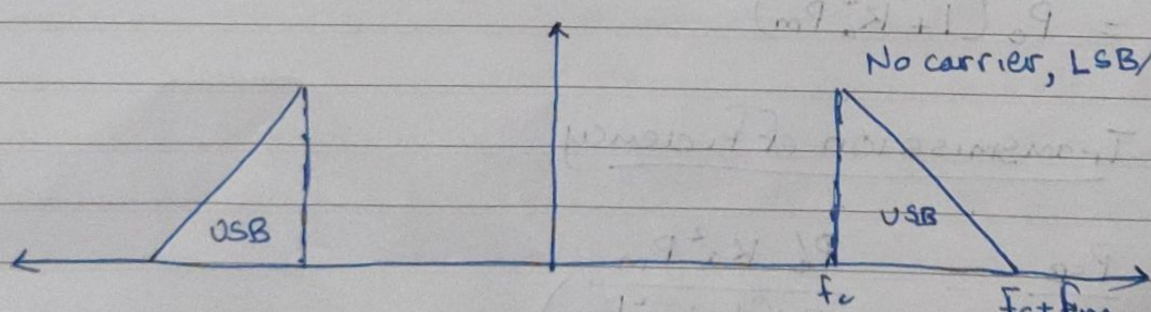


DSB-FC
∴ Full Carr
BW = 2fm



DSB-SC

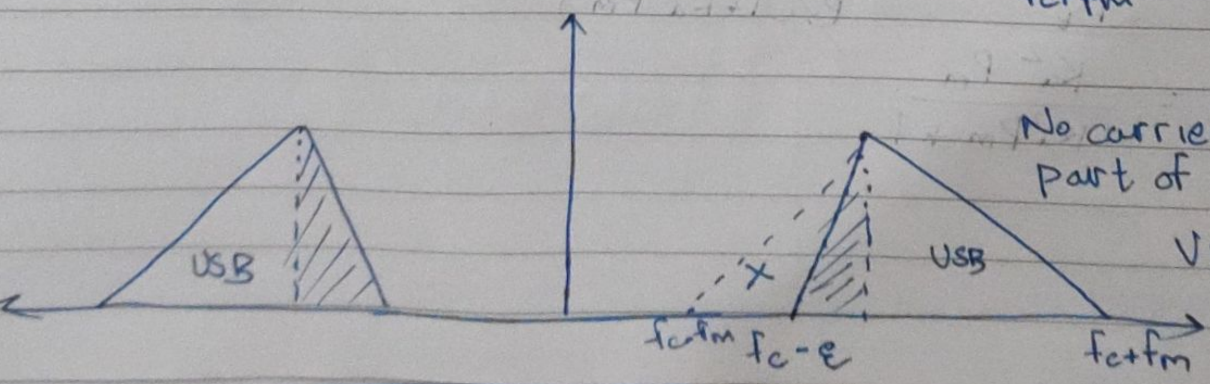
BW = 2fm



No carrier, LSB/USB (one at a time

SSB-SC

BW = fm



No carrier, only part of cutoff SB

VSB-SC

B = fm+ε

By B.W.,

DSB-SC = AM > VSB-SC > SSB-SC
(DSB-FC)

By Power req.,

AM > DSB-SC > VSB-SC > SSB-SC
(DSB-FC)

AM (DSB-FC)	DSB-SC	VSB-SC	SSB-SC
2SB + C	2SB	SB + ε part × SB	SB

Power saving

$$= \frac{\text{Power saved}}{\text{Total Power}}$$

1) for DSB-SC,

$$P.S. = \frac{P_c}{P_t} = \frac{P_c}{P_c(1+m_a^2/2)} = \frac{2}{2+m_a^2} \quad [\approx 70-80\%]$$

2) For SSB-SC

$$P.S. = \frac{P_c + P_{LSB}}{P_t} = \frac{P_c + P_c m_a^2/4}{P_c(1+m_a^2/2)} = \frac{1+m_a^2/4}{1+m_a^2/2}$$

In VSB, it is difficult to estimate. the saving is betn DSB & SSB

B.W. & P calcⁿ of AM

If,

$$m(t) = A_1 \cos \omega_1 t + A_2 \cos \omega_2 t$$

$$\therefore s(t) = A_c (1 + K_a A_1 \cos \omega_1 t + K_a A_2 \cos \omega_2 t) \cos \omega_c t$$

$$= A_c (1 + m_{a1} \cos \omega_1 t + m_{a2} \cos \omega_2 t) \cos \omega_c t$$

$$= A_c \cos \omega_c t + \left[\underbrace{\frac{A_c m_{a1}}{2} \cos(\omega_1 + \omega_c)t}_{USB1} + \underbrace{\frac{A_c m_{a1}}{2} \cos(\omega_c - \omega_1)t}_{LSB1} \right]$$

$$+ \left[\underbrace{\frac{A_c m_{a2}}{2} \cos(\omega_c + \omega_1)t}_{USB2} + \underbrace{\frac{A_c m_{a2}}{2} \cos(\omega_c - \omega_2)}_{LSB2} \right] \rightarrow (1)$$

$\Rightarrow$

$$P_t = \underbrace{\frac{A_c^2}{2R}}_{\text{Carrier Power}} + \underbrace{\frac{A_c^2 m_{a1}^2}{4R}}_{\text{B.1 Power SB}} + \underbrace{\frac{A_c^2 m_{a2}^2}{4R}}_{\text{B.2 Power SB}}$$

$$P_t = P_c \left[1 + \frac{m_{a1}^2}{2} + \frac{m_{a2}^2}{2} \right]$$

$$= P_c \left[1 + \frac{m_{a1}^2 + m_{a2}^2}{2} \right]$$

OR $P_T = P_c \left[1 + \frac{m_{aT}^2}{2} \right]$

where, $m_{aT}^2 = m_{a1}^2 + m_{a2}^2 \ldots$

$\therefore$ for $m(t) = A_1 \cos \omega_1 t + A_2 \cos \omega_2 t \ldots$

$$P_T = P_c \left[1 + \frac{m_{aT}^2}{2} \right]$$

where $m_{aT} = \sqrt{m_{a1}^2 + m_{a2}^2 \ldots}$

also,

$$\eta = \frac{P_{SB}}{P_t} = \frac{M_{aT}^2}{2+M_{aT}^2}$$

~~and~~, In general,

$$P_{SB} = \frac{A_c^2 K_a^2}{2} \overline{m^2 t} = \frac{A_c^2 K_a^2}{2} P_m \text{ (say)}$$

$$P_c = \frac{A_c^2}{2}$$

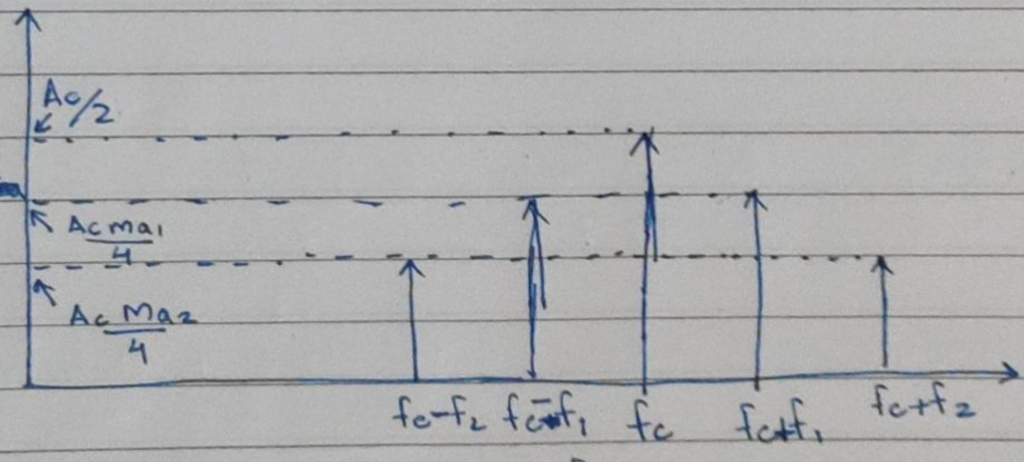
$$\Rightarrow \frac{P_{SB}}{P_c} = K_a^2 P_m$$ ← Power of msg signal

$\propto K_a^2 A_m^2$ ← general

~~$\Rightarrow P_T = P_c + P_{SB}$~~

$$\Rightarrow \frac{P_{SB}}{P_c} \propto M_a^2$$

Using F.T. of eq$_n$ ①,



← -ve part is mirrored from +ve

here we took $f_1 < f_2$ & $M_{a1} > M_{a2}$, however, that may change

Here, B.W. $= 2f_2$

However,

B.W. $= 2 \times$ [freq. of - Highest freq. component of $m(t)$]