Unit 5: Decision Theory

Introduction:

Decision theory is primarily concerned with helping people and organizations in making decisions. It provides a meaningful conceptual frame work for important decision making. The decision making refers to the selection of an act from amongst various alternatives, the one which is judged to be the best under given circumstances.

The management has to consider phases like planning, organization, direction, command and control. While performing so many activities, the management has to face many situations from which the best choice is to be taken. This choice making is technically termed as "decision making" or decision taking. A decision is simply a selection from two or more courses of action. Decision making may be defined as - "a process of best selection from a set of alternative courses of action, that course of action which is supposed to meet objectives up to satisfaction of the decision maker."

The knowledge of statistical techniques helps to select the best action. The statistical decision theory refers to an optimal choice under condition of uncertainty. In this case probability theory has a vital role, as such, this probability theory will be used more frequently in the decision making theory under uncertainty and risk.

The statistical decision theory tries to reveal the logical structure of the problem into alternative action, states of nature, possible outcomes and likely pay-offs from each such outcome. Let us explain the concepts associated with the decision theory approach to problem solving.

Process of decision making:

Steps of the decision-making process

1. Identify the decision.
2. Gather relevant information.
3. Identify the alternatives.
4. Weigh the evidence.
5. Choose among the alternatives.
6. Take action.
7. Review your decision.

Terminology of decision making:

(i) Decision Alternatives or Alternative course of Actions:

Firstly, the decision maker has to select the various alternative courses of action from which the final decision is to be made to achieve his objectives. The decision maker has control over the choice of these alternatives. It is also called acts, actions or strategies.

(ii) States of Nature:

A future event which is not under control of the decision maker is called states of nature or outcomes. For example: If some manufacture takes decision to manufacture criteria product then the demand of that item in the market may be high, median, low or nil. These four outcomes are not in his control.

(iii) Certainty:

The decision environment in which only one state of nature exists is called certainty.

(iv) Uncertainty:

The decision environment in which more than one state of nature exists but in which the decision maker cannot assign probability to the various states is called uncertainty. It must be beyond the control the control of the decision maker who cannot be sure of the state of nature which will take place.

(v) Pay-off:

It is a numerical value (outcome) obtained due to the application of each possible combination of decision alternatives and states of nature.

The payoff values are always conditional values because of unknown states of nature. The payoffs in most decisions are monetary. Payoffs resulting from each possible combination of decision alternatives and states of natures are displayed in a pay-off matrix form as shown in figure below:

Pay-off table with no probability

Decision Alternatives (Course of action)	States of Nature (Events or outcome)			
	D_1	D_2	……………………	D_m
S_1	p_{11}	p_{12}	……………………	p_{1m}
S_2	p_{21}	p_{22}	……………………	p_{2m}
….	………………………………………………………………			
….	………………………………………………………………			
….	………………………………………………………………			
S_n	p_{n1}	p_{n2}	……………………	p_{nm}

Pay-off table with probability

Decision Alternatives (Course of action)	Probability	States of Nature (Events or outcome)			
		D_1	D_2	……………………	D_n
S_1	P_1	P_{11}	P_{12}	……………………	P_{1n}
S_2	P_2	P_{21}	P_{22}	……………………	P_{2n}
….	….	………………………………………………………………			
….	….	………………………………………………………………			
….	….	………………………………………………………………			
S_m	P_m	P_{m1}	P_{m2}	……………………	P_{mn}

Regret Value or Opportunity Loss:

The regret value is defined as the differences between the highest possible pay-off for an event (state of nature) and the actual pay-off made by a particular action with the same event. The regret value is calculated by each of the event-action cells by the following formula:

- Regret Value = Pay-off of the best act for an event – Pay-off of the particular act for the same event

Regret table

Decision Alternatives (Course of action)	States of Nature (Events or outcome)			
	D_1	D_2	……………………	D_m
S_1	L_{11}	L_{12}	……………………	L_{1m}
S_2	L_{21}	L_{22}	……………………	L_{2m}
….	………………………………………………………………			
….	………………………………………………………………			
….	………………………………………………………………			
S_n	L_{n1}	L_{n2}	……………………	L_{nm}

Where,

L_{ij} = Regret for the decision alternative S_i and state of nature D_j

Risk:

The decision environment in which the decision maker has information supporting the assignment of probability values to each of the possible states of nature is called the risk.

Computation of Pay-off Values:

If S and D denote the decision alternatives and state of nature respectively, then number of unit unsold will S – D and pay-off values obtained from the combination of alternatives and states of nature can be obtained by using following formula.

(a) When there is no salvage value:

The pay-off values of fixed variable from can be obtained by using following formula:

Pay-off = MP $\times$ no. of units supplied, if $S \leq D$

= MP $\times$ S

Pay-off = MP $\times$ no. of units sold – CP $\times$ no. of units unsold, if $S > D$

= MP $\times$ D – CP $\times$ (S – D)

Where,

MP = Marginal profit = SP – CP

S = Supply

D = Demand

ML = Marginal loss = CP

(b) When there is salvage value:

The pay-off values can be obtained by using following formula:

Pay-off = MP $\times$ units supplied, if $S \leq D$

= MP $\times$ S

Pay-off = SP $\times$ units sold – CP $\times$ units supplied + Salvage price $\times$ unit unsold, if $S > D$

= CP $\times$ D – CP $\times$ S + Salvage price $\times$ (S – D)

OR, Pay-off = MP $\times$ units sold – ML $\times$ units unsold, if S > D

= MP $\times$ D – ML $\times$ (S – D)

Where,

MP = Marginal profit = SP – CP

ML = Marginal loss = CP – Salvage price

(c) If per unit cost of holding items beyond the season and bargain price after the season are given, the marginal loss (ML) is given by

$$ML = \text{Marginal loss} = CP - \text{Salvage price}$$

Where,

$$\text{Salvage price} = \text{Bargain price after the season} - \text{Cost of holding beyond the season}$$

Then,

$$\text{Pay-off} = MP \times \text{units supplied, if } S \leq D$$

$$= MP \times S$$

$$\text{Pay-off} = MP \times \text{units sold} - ML \times \text{units unsold, if } S > D$$

$$= MP \times D - ML \times (S - D)$$

$$\text{OR, Pay-off} = SP \times \text{units sold} - CP \times \text{units supplied} + \text{Salvage price} \times \text{units unsold, if } S > D$$

$$= CP \times D - CP \times S + \text{Salvage price} \times (S - D)$$

(d) The expected pay-off values is computed by following formula:

$$\text{Expected pay-off} = \text{pay-off} \times \text{probability}$$

(e) The sum of expected pay-off values is called Expected Monetary value and computed by following formula:

$$\text{Expected Monetary Value (EMV)} = \text{Sum of (pay-off} \times \text{probability)}$$

$$= \sum \text{pay} - \text{off} \times \text{probability}$$

Decision making Environments (or situations):

Decision theory provides a rational approach to determine the best alternatives (or strategies) where a decision maker is faced with several decision alternatives. In this section three types of decision-making environments: certainty, uncertainty, and risk, have been discussed. The knowledge of these environments helps in choosing the quantitative approach for decision-making.

(i) Decision making under certainty:

In this decision-making environment, decision-maker has complete knowledge (perfect information) of outcome due to each decision-alternative (course of action). In such a case he would select a decision alternative that yields the maximum return (payoff) under known state of nature.

(ii) Decision making under uncertainty:

When probability of any outcome cannot be quantified, the decision-maker must arrive at a decision only on the actual conditional payoff values, keeping in view the criterion of effectiveness (policy). The following criteria of decision-making under uncertainty have been discussed in this section.

a. Optimism (Maximax or Minimin) criterion

b. Pessimism (Maximin or Minimax) criterion

(c) Equal probabilities (Laplace) criterion

(d) Coefficient of optimism (Hurwiez) criterion

(e) Regret (salvage) criterion

Numerical Problem: Decision Making Under Uncertainty

1. A food products' company is contemplating the introduction of a revolutionary new product with new packaging or replacing the existing product at much higher price (S1). It may even make a moderate change in the composition of the existing product, with a new packaging at a small increase in price (S2), or may mall a small change in the composition of the existing product, backing it with the word 'New' and a negligible increase in price (S3). The three possible states of nature or events are: (i) high increase in sales (N1), (ii) no change in sales (N2) and (iii) decrease in sales (N3). The marketing department of the company worked out the payoffs in terms of yearly net profits for each of the strategies of three events (expected sales). This is represented in the following table:

Alternative decision (Strategies)	State of Nature		
	N_1	N_2	N_3
S_1	7,00,000	3,00,000	1,50,000
S_2	5,00,000	4,50,000	0
S_3	3,00,000	3,00,000	3,00,000

Which strategy should the concerned executive choose on the basis of (a) Maximin criterion (b) Maximax criterion (c) Minimax regret criterion (d) Laplace criterion?

2. Consider the following pay-off (profit) matrix Action State.

Alternatives	State of Nature			
	N_1	N_2	N_3	N_4
A_1	5	10	18	25
A_2	8	7	8	23
A_3	21	18	12	21
A_4	30	22	19	15

Determine best action using maximin principle. [Ans: Alternative A_4 is best]

3. A business man has three alternatives open to him each of which can be followed by any of the four possible events. The conditional pay offs for each action - event combination are given below:

Action	Pay offs (Conditional events)			
	A	B	C	D
X	8	0	- 10	6
Y	- 4	12	18	- 2
Z	14	6	0	8

Max (–10,–4, 0) = 0. Since the maximum payoff is 0, the alternative Z is selected by the businessman.

4. Consider the following pay-off matrix

Alternatives	Pay offs (Conditional events)			
	A_1	A_2	A_3	A_4
E_1	7	12	20	27
E_2	10	9	10	25
E_3	23	20	14	23

E_4	32	24	21	17

Using minmax principle, determine the best alternative.

[Ans: Min(27, 25, 23, 32) = 23. Since the minimum cost is 23, the best alternative is E3 according to minimax principle.

5. Given the following pay-off matrix (in rupees) for three strategies and two states of nature.

Strategy	States of nature	
	E_1	E_2
S_1	40	60
S_2	10	- 20
S_3	- 40	150

Select a strategy using each of the following rule (i) Maximin (ii) Minimax

[Ans: (i) S1 (ii) S2]

6. A farmer wants to decide which of the three crops he should plant on his 100-acre farm. The profit from each is dependent on the rainfall during the growing season. The farmer has categorized the amount of rainfall as high medium and low. His estimated profit for each is shown in the table.

Railfall	Estimated Conditional Profit (Rs.)		
	Crop A	Crop B	Crop C
High	8000	3500	5000
Medium	4500	4500	5000
Low	2000	5000	4000

If the farmer wishes to plant only crop, decide which should be his best crop using (i) Maximin (ii) Minimax

[Ans: (a) Crop C (b) Crop B and Crop C]

7. The research department of Hindustan Ltd. has recommended to pay marketing department to launch a shampoo of three different types. The marketing types of shampoo to be launched under the following estimated pay-offs for various level of sales.

Types of Shampoo	Estimated Sales (In units)		
	15000	10000	5000
Egg Shampoo	30	10	10
Clinic Shampoo	40	15	5
Deluxe Shampoo	55	20	3

What will be the marketing manager's decision if (i) Maximin and (ii) Minimax principle applied?

[Ans: (i) Egg shampoo (ii) Egg Shampoo]

8. Following pay-off matrix, which is the optimal decision under each of the following rule (i) maxmin (ii) minimax

Action	States of Nature			
	S_1	S_2	S_3	S_4
A_1	14	9	10	5
A_2	11	10	8	7
A_3	9	10	10	11
A_4	8	10	11	13

[Ans: . (i) A1 and A3 (ii) A2 and A3

(iii) Decision making criteria under risk:

In this environment, more than one outcome (or states of nature) exist with each decision and the decision maker has sufficient information to assign probabilities to each of these states of nature (or outcome). For decision problems under risk, the most popular decision criterions for evaluating the alternative are:

(i) Expected Monetary Value (EMV) criterion or Expected Value criterions.

(ii) Expected Opportunity Loss (EOL) criterion or Expected Regret criterions.

Expected Monetary Value (EMV) criterion:

In this criterion, the Expected Monetary Value (EMV) for each alternative which is the sum of possible pay-off of the alternative, each weighted by the probability of that pay-off occurring, is calculated. Then the alternative corresponding to highest EMV (lowest cost table) is selected as a decision for the decision maker.

EMV (course of action S $_j$) = $\sum_{j=1}^{m} p_{ij} p_i$

Where,

m = number of possible states of nature

p_i = probability of occurrence of state of nature, N_i

p_{ij} = payoff associated with state of nature N_i and course of action, S_j

Procedure:

1. Construct a payoff matrix listing all possible courses of action and states of nature. Enter the conditional payoff values associated with each possible combination of course of action and state of nature along with the probabilities of the occurrence of each state of nature.

2. Calculate the EMV for each course of action by multiplying the conditional payoffs by the associated probabilities and adding these weighted values for each course of action.

3. Select the course of action that yields the optimal EMV.

Example: The Parker Flower Shop promises its customers delivery within four hours on all flowers orders. All flowers are purchased on the previous day and delivered to Parker by 8:00 AM. The next morning Parker's daily demand for roses is as follows:

Dozens of Roses	7	8	9	10
Probability	0.1	0.2	0.4	0.3

Parker purchases roses for Rs 10 per dozens and sells them for Rs 30. All unsold roses are donated to a local hospital. How many dozens of roses should parker order each evening to maximize its profits? What is optimal expected profit?

Solution: Given,

CP = Rs 10

SP = Rs 30

Salvage value = Rs 0

MP = SP – CP = 30 – 10 = Rs 20

ML = CP – Salvage value = 10 – 0 = Rs 10

$$\text{Pay off} = \begin{cases} \text{MP} \times \text{S} & ;\ \text{If S} \leq \text{D} \\ \text{MP} \times \text{D} - \text{CP}(\text{S} - \text{D}); & \text{If S} > D \end{cases}$$

Strategy	States of Nature				EMV
	D = 7	D = 8	D = 9	D = 10	
S = 7	140	140	140	140	140
S = 8	130	160	160	160	157
S = 9	120	150	180	180	168
S = 10	110	140	170	200	167
	0.1	0.2	0.4	0.3	

When S = 7 and D = 7, Pay off (P_{11}) = 20 × 7 = Rs 140; if S ≤ D

When S = 8 and D = 7, Pay off (P_{21}) = 20 × 7 – 10 × (8 – 7) = Rs 130; if S > D and so on.

Then,

EMV (S = 7) = 140×0.1 + 140×0.2 + 140×0.4 + 140×0.3 = 140

EMV (S = 8) = 130×0.1 + 160×0.2 + 160×0.4 + 160×0.3 = 157

EMV (S = 9) = 120×0.1 + 150×0.2 + 180×0.4 + 180×0.3 = 168

EMV (S = 10) = 110×0.1 + 140×0.2 + 170×0.4 + 200×0.3 = 167

Hence, maximum EMV = Rs 168, so, Parker's should ordered 9 dozens of roses and optimal expected profit is Rs 168.

Example: A retailer purchases cherries every morning at Rs 50 a case and sells them for Rs 80 a case. Any case remaining at the end of the day can be disposed of next day at a salvage value of Rs 20 per case (there after they have no value). Past sales have ranged from 15 to 18 cases per day. The following is the record of sales for the past 120 day.

Cases sold	15	16	17	18
Number of days	12	24	48	36

Find how many the retailer should purchases per day to maximize his profit.

Solution: Given,

CP = Rs 50

SP = Rs 80

Salvage value = Rs 20

MP = SP – CP = 80 – 50 = Rs 30

ML = CP – Salvage value = 50 – 20 = Rs 30

Conditional profit (Pay off) = MP × Unit sold = MP × S; if S ≤ D

= SP × Unit sold – CP × Unit supplied + Salvage value × Unit unsold; S > D

= SP × D– CP × S+ Salvage value × (S – D); if S > D

Or, Conditional profit = MP × Unit sold – ML × Unit unsold

Strategy	States of Nature				EMV
	D = 15	D = 16	D = 17	D = 18	
S = 15	450	450	450	450	450
S = 16	420	480	480	480	474
S = 17	390	450	510	510	486
S = 18	360	420	480	540	474
Probability	0.1	0.2	0.4	0.3	

When S = 15 and D = 15, Pay off (P_{11}) = 30 × 15 = Rs 450; if S ≤ D

When S = 16 and D = 15, Pay off (P_{21}) = 80 × 15 – 50 × 16 + 20 × (16 – 15) = Rs 420; if S > D and so on.

Then,

EMV (S = 7) = 450×0.1 + 450×0.2 + 450×0.4 + 450×0.3 = 450

EMV (S = 8) = 420×0.1 + 480×0.2 + 480×0.4 + 480×0.3 = 474

EMV (S = 9) = 390×0.1 + 450×0.2 + 510×0.4 + 510×0.3 = 486

EMV (S = 10) = 360×0.1 + 420×0.2 + 480×0.4 + 540×0.3 = 474

Hence, maximum EMV = Rs 486, so, retailer should purchases 17 cases and maximum expected profit is Rs 486.

Example: A grocer must decide how many crates of milk should be stocked each week to meet the demand. The probability distribution of demand is as under. Each crates costs the grocer Rs 100, it's selling price is Rs 120 and unsold crates are sold to local farmer for Rs 20 per crate. If shortage exists, a shortage cost of Rs 40 per crate incurred. Construct a matrix and calculate EMV.

Demand	15	16	17	18	19
Probability	0.15	0.25	0.4	0.15	0.05

Solution: Given,

CP = Rs 100

SP = Rs 120

Salvage value = Rs 20

Shortage cost = Rs 40

MP = SP – CP = 120 – 100 = Rs 20

ML = CP – Salvage value = 100 – 20 = Rs 80

Conditional profit (Pay off) = MP $\times$ Unit Supplied – Shortage cost $\times$ Unit shortage; if $S \leq D$

= MP $\times$ S – Shortage cost $\times$ (D – S); if $S \leq D$

= SP $\times$ Unit sold – CP $\times$ Unit supplied + Salvage value $\times$ Unit unsold; $S > D$

= SP $\times$ D – CP $\times$ S+ Salvage value $\times$ (S – D); if $S > D$

Or, Conditional profit = MP$\times$Unit sold–Shortage cost$\times$Unit shortage; if $S \leq D$

= MP$\times$S – Shortage Cost $\times$(D-S); if $S \leq D$

= MP $\times$ Unit sold – ML $\times$ Unit unsold = MP $\times$ D – ML (S – D); if $S > D$

Strategy	Demand					EMV
	15	16	17	18	19	
15	300	260	220	180	140	232
16	220	320	280	240	200	271
17	140	240	340	300	260	275
18	60	160	260	360	320	223
19	-20	80	180	280	380	150
	0.15	0.25	0.4	0.15	0.05	

Maximum EMV = Rs 275, so select alternative 17 and maximum expected profit is Rs 275.

Example: The probability of demand for hiring cars on any day in a given city is as follows:

No of cars demanded	0	1	2	3	4
Probability	0.1	0.2	0.3	0.2	0.2

Cars have a fixed cost of Rs 90 each day to keep the daily hire charges (variable costs of running) Rs 200. If the car-hire company owns 4 cars, what is its daily expectation? If the company is about to go into business and currently has no car, how many cars should it buy?

Solution: Given that Rs 90 is the fixed cost and Rs 200 is variable cost. The payoff values with 4 cars at the disposal of decision-maker are calculated as under.

Payoff = Variable cost $\times$ No. of cars; $S \leq D$

= $200 \times S$; $S \leq D$

= $200 \times S$ – Fixed cost $\times$ S; $S > D$

= $200 \times D - 90 \times S$; $S > D$

Payoff table

Strategy	Demand of cars (States of Nature)					EMV
	0	1	2	3	4	
0	0	0	0	0	0	0
1	-90	110	110	110	110	90
2	-180	20	220	220	220	140
3	-270	-70	130	330	330	130
4	-360	-160	40	240	440	80

	0.1	0.2	0.3	0.2	0.2	

Since, the maximum EMV is Rs 140 for the course of action 2, the company should buy 2 cars.

Assignment Problems:

1. The manager of a flower shop promises its customers delivery within four hours on all flower orders. All flowers are purchased on the previous day and delivered to Parker by 8.00 am the next morning. The daily demand for roses is as follows.

Dozens of roses	70	80	90	100
Probability	0.1	0.2	0.4	0.3

The manager purchases roses for Rs 10 per dozen and sells them for Rs 30. All unsold roses are donated to a local hospital. How many dozens of roses should Parker order each evening to maximize its profits? What is the optimum expected profit?

2. A farmer wants to decide which of the crops he should plant on his Farm. The profit from each crop is dependent on the rainfall during the season. He has categorized the rainfall as substantial, moderate or light he estimates his profit for each crop as shown in the table.

	Substantial	Moderate	Light
Crop A	7000	3500	1000
Crop B	2500	3500	4000
Crop C	4000	4000	3000

He estimates the probability of substantial rainfall as 0.2, of moderate rainfall as 0.3 and that of light rainfall as 0.5. Determine the optimal decision as to which crop to plant using EMV criterion.

Expected Opportunity Loss (EOL) Criterion:

Expected opportunity loss (EOL), also called expected value of regret, is an alternative decision criterion for decision making under risk. The EOL is defined as the difference between the highest profit (or payoff) and the actual profit due to choosing a particular course of action in a particular state of nature. Hence, EOL is the amount of payoff that is lost by not choosing a course of action resulting to the minimum payoff in a particular state of nature. A course of action resulting to the minimum EOL is preferred. Mathematically, EOL is stated as follows.

EOL (course of action S_j) = $\sum_{j=1}^{m} L_{ij} p_i$

Where,

m = number of possible states of nature

p_{ij} = probability of occurrence of state of nature N_i

L_{ij} = Conditional payoff (Opportunity Loss) associated with state of nature N_i and course of action S_j

= (Maximum pay-off – Actual Pay-off) of state of nature N_i; if payoff is profit.

= (Actual cost – Minimum cost) of state of nature N_i; if payoff is cost.

Procedure:

1. Prepare a conditional payoff values matrix for each combination of course of action and state of nature along with the associated probabilities.

2. For each state of nature calculate the conditional opportunity loss (COL) values by subtracting each payoff from the maximum payoff.

3. Calculate the EOL for each course of action by multiplying the probability of each state of nature with the COL value and then adding the values.

4. Select a course of action for which the minimum of EOL.

1: Under an employment promotion program it is proposed to allow sale of newspapers on buses during off-peak hour. The vender can purpose the papers at a special concessional rate of 25 paisa and sells it at 40 paisa. Any unsold copy is a dead loss. A vender has estimated the following probabilities for the number of copies demanded.

No. of copies	15	16	17	18	19	20
Probability	0.04	0.19	0.33	0.26	0.11	0.07

Prepare a pay of table and find out how many copies of newspapers should be ordered by using EOL criterion.

Solution: Given,

CP = 25 paisa

SP = 40 paisa

MP = SP – CP = 15 paisa

Payoff = MP × S; if S ≤ D

= MP × D – CP (S – D); if S > D

Then,

Strategy	States of Nature					
	15	16	17	18	19	20
15	225	225	225	225	225	225
16	200	240	240	240	240	240
17	175	215	255	255	255	255
18	150	190	230	270	270	270
19	125	165	205	245	285	285
20	100	140	180	220	260	300
Probability	0.04	0.19	0.33	0.26	0.11	0.07

L_{ij} = (Maximum pay-off – Actual Pay-off) of state of nature N_i

Regret table is

	States of Nature	

Strategy	15	16	17	18	19	20	EOL
15	0	15	30	45	60	75	36.30
16	25	0	15	30	45	60	22.90
17	50	25	0	15	30	45	17.10
18	75	50	25	0	15	30	24.50
19	100	75	50	25	0	15	42.30
20	125	100	75	50	25	0	64.50
Probability	0.04	0.19	0.33	0.26	0.11	0.07	

The minimum EOL is 17.10 paisa. Hence by EOL criterion 17 newspapers is the optimal stock level.

2. A company manufactures goods for a market in which the technology of the product is changing rapidly. The research and development department has produced a new product that appears to have potential for commercial exploitation. A further Rs 60,000 is required for development testing. The company has 100 customers and each customer might purchase, at the most, one unit of the product. Market research suggests that a selling price of Rs 6,000 for each unit, with total variable costs of manufacturing and selling estimate as Rs 2,000 for each unit. From previous experience, it has been possible to derive a probability distribution relating to the proportion of customers who will buy the product as follows:

Proportion of customers	0.04	0.08	0.12	0.16	0.20
Probability	0.10	0.10	0.20	0.40	0.20

Determine the expected opportunity losses, given no other information than that stated above, and state whether or not the company should develop the product.

Solution: If p is the proportion of customers who purchase the new product, the company's conditional profit is

Payoff = (6,000 – 2,000) × 100p – 60,000

= Rs (400000×p – 60,000)

Payoff table

Proportion of Customers (State of Nature	Probability	Strategy	
		Develop(S_1)	Do not develop(S_2)
0.04	0.1	-44000	0
0.08	0.1	-28000	0
0.12	0.2	-12000	0
0.16	0.4	4000	0
0.20	0.2	20000	0

L_{ij} = (Maximum pay-off – Actual Pay-off) of state of nature N_i

Regret table

Proportion of Customers (State of Nature	Probability	Strategy	
		S_1	S_2
0.04	0.1	44000	0
0.08	0.1	28000	0
0.12	0.2	12000	0
0.16	0.4	0	4000
0.20	0.2	0	20000
EOL		9600	5600

Since , the minimum expected opportunity loss = Rs 5600, the company should select course of action S_2 (do not develop the product) with minimum EOL.

3. Steve's Mountain Bicycle Shop is considered three options for its facility next year. Steve can expand his current shop, move to a larger facility, or make no change. With good market, the annual payoff would be \$76000 if he expands, \$ 90000 if he moves, and \$40000 if he does nothing. With an average market, his payoffs will be \$30000, \$41000, and \$15000, respectively. With a poor market, his payoff will be - \$17000, -\$28000, and \$4000, respectively. Steve has gathered some additional information. The probabilities of good, average and poor markets are 0.25, 0.45 and 0.3, respectively. (a) Using EMVs what option should Stave choose? What is the maximum EMV? (b) Using EOL, what option should Stave choose? What is the minimum EOL? (c) Compute the EVPI and show that it is the same as the minimum EOL.

Solution: (a) Given,

Calculation table for EMV

Alternatives	States of Nature			EMV = $\sum Probability \times payoff$
	Good	Average	Poor	
Expand	760000	30000	- 17000	27400
Move	90000	41000	- 28000	32550
No change	40000	15000	4000	17950
Probability	0.25	0.45	0.30	

Since, maximum EMV is \$32550, so select to move alternative option.

(b) We have,

Regret value (L_{ij}) = Maximum payoff – Actual payoff in each state of nature

Calculation table for EOL

Alternatives	States of Nature			EMV = $\sum Probability \times regret$
	Good	Average	Poor	
Expand	14000	11000	21000	14750
Move	0	0	32000	9600
No change	50000	26000	0	24200
Probability	0.25	0.45	0.30	

Since, minimum EOL is \$9600, so select to move alternative option.

(c) The expected profit with perfect information is

EPPI = $\sum$ Probability × maximum payoff of in each states of Nature

= 90000×0.25 + 41000×0.45 + 4000×0.30 = \$42150

Then, expected value of perfect information is

EVPI = EPPI – Maximum of EMV = 42150 – 32550 = \$ 9600

Hence, EVPI is equal to minimum EOL, proved.

Decision Tree Analysis Definition

Decision tree analysis is the process of drawing a decision tree, which is a graphic representation of various alternative solutions that are available to solve a given problem, in order to determine the most effective courses of action. Decision trees are comprised of nodes and branches - nodes represent a test on an attribute and branches represent potential alternative outcomes.

Decision tree analysis is helpful for solving problems, revealing potential opportunities, and, making complex decisions regarding cost management, operations management, organization strategies, project selection, and, production methods.

Decision-making problems discussed earlier were limited to arrive at a decision over a fixed period of time. That is, payoffs, states of nature, courses of action and probabilities associated with the occurrence of states of nature were not subject to change.

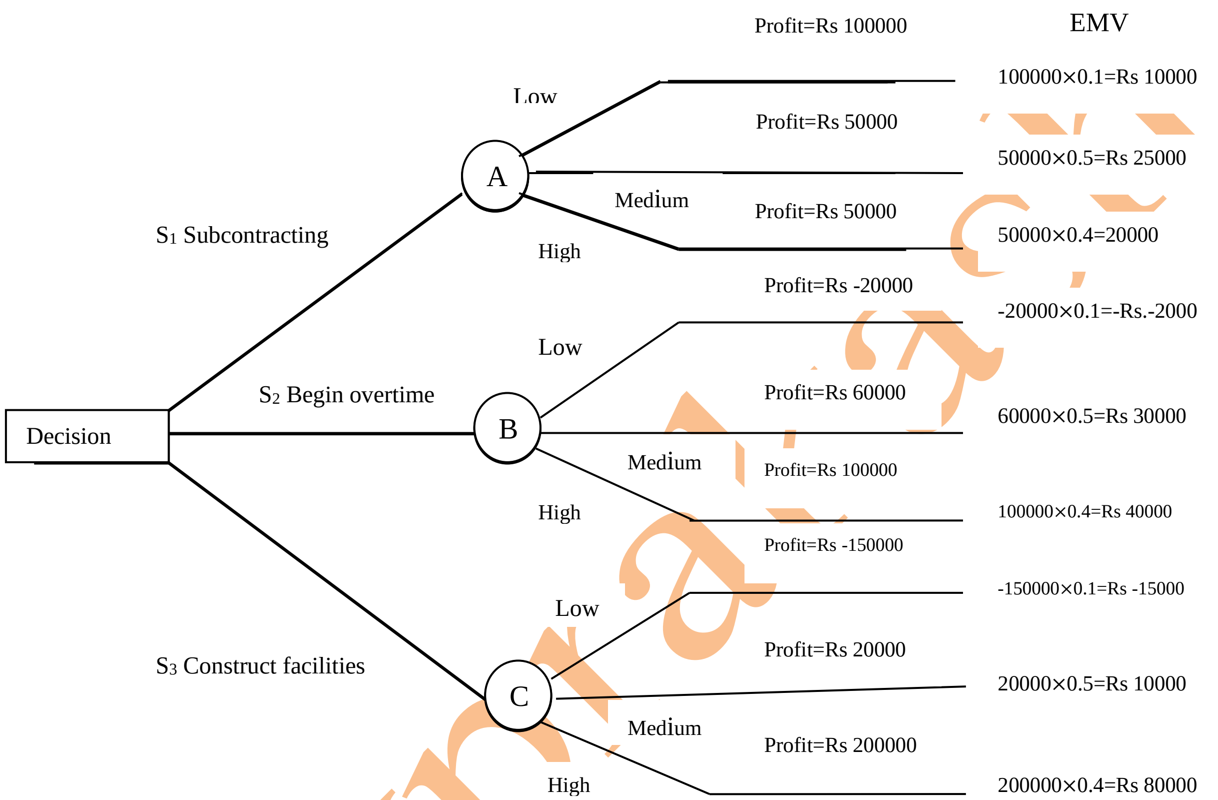
Decision tree approach

Example: A glass factory specializing in crystal is developing a substantial backlog and the firm's management is considering free courses of action. Average for sub-contracting (S_1), begin overtime productions (S_2) and construct new facilities (S_3). The correct choice depends largely upon future demand which may be low, medium or high. By consensus, management ranks the respective probabilities as 0.10, 0.50 and 0.40. A cost analysis reveals the effect upon the profits that is shown in the table below:

Demand	Probability	Course of action		
		S_1	S_2	S_2
Low	0.10	100000	- 20000	- 150000
Medium	0.50	50000	60000	20000
High	0.40	50000	100000	200000

Show this decision situation in the form of a decision tree and indicates the most preferred decision and corresponding expected value.

Solution: A decision tree which shows possible alternative and state of nature is shown in following figure.



The total expected monetary value (EMV) for mode A, B and C are calculated as follows.

$\sum$ EMV(A) = Rs (100000 × 0.1 + 50000 × 0.5 + 50000 × 0.4) = Rs 46000

$\sum$ EMV(B) = Rs (-20000 × 0.1 + 60000 × 0.5 + 100000 × 0.4) = Rs 68000

$\sum$ EMV (C) = Rs (-150000 × 0.1 + 20000 × 0.5 + 200000 × 0.4) = Rs 75000

Since node C has the highest EMV, the optimal decision will be to choose the course of action S_3.

Question: A business firm is considering opening its branch in either Bhaktapur or Lalitpur. From the preliminary study the following data has been obtained.

	Bhaktapur		Lalitpur	
	Probability	Profit (Rs.)	Probability	Profit (Rs.)
Low success	0.35	25000	0.28	18000
Medium success	0.40	36000	0.38	29000
High success	0.25	16000	0.34	23000

With the help of the decision the diagram which branch should be firm prefer.

Marginal Analysis:

Marginal analysis compares the additional benefits derived from an activity and extra cost incurred by the same activity. It serves as a decision-making tool in projecting the maximum potential profits for the company by comparing the costs and benefits of the activity.

(i) Marginal analysis a decision-making tool used to examine the additional benefit of an activity contrasted with the extra cost incurred by the same activity.

(ii) It is mostly used by companies to maximize efficiency and improve their decision-making processes.

(iii) The marginal analysis of costs and benefits is necessary, especially for a company planning to expand its business operations.

Marginal Analysis Approach

The marginal analysis is based on the fact that when an additional unit purchased will either be sold or remain unsold. If p represents the probability of selling one additional unit, then (1 – p) must be the probability of not selling it.

If the additional unit is sold, the conditional profit will increase as a result of the profit earned from the additional unit. Such an increase in conditional profit is known as incremental (marginal) profit. It is denoted by MP

MP = Selling price – cost price

If the additional unit is not sold, the conditional profit will reduces and the amount of reduction is called the incremental (marginal) loss. It is denoted by ML

ML = Cost price = CP; if no salvage value, and,

ML = Cost price – Salvage value; if salvage value

ML = CP – Salvage value + Cost of holding beyond season.

Expected marginal profit = p × MP

Expected marginal loss = (1 – p) × ML

Under the marginal analysis technique, the additional units should be stocked as long as the expected marginal profit from stocking each of them is greater than or equal to the expected marginal loss from stocking each. Thus, additional units should be stocked as long as:

$$p \times \text{MP} \geq (1 - \text{p}) \times \text{ML}$$

$$or, \quad p \times \text{MP} \geq \text{ML} - p \times \text{ML}$$

$$or, \quad p \times \text{MP} + p \times \text{ML} \geq \text{ML}$$

$$or, \quad p(\text{MP} + \text{ML}) \geq \text{ML}$$

$$\therefore \quad p \geq \frac{\text{ML}}{(\text{MP} + \text{ML})}$$

The symbol p represents the minimum required probability of selling at least one additional unit to justify the stocking of that additional unit. Additional units should be stocked so long as the probability of selling at least that additional unit is greater than p.

Steps of marginal analysis approach:

1. Compute value of p using formula,

$$p \geq \frac{\text{ML}}{(\text{MP} + \text{ML})}$$

2. Construct a cumulative probability table. The cumulative probability of greater than type is obtained by cumulating the given probability from bottom to top.

3. Locate the cumulative probability of stocking or buying greater than or equal to the calculated value of p. The corresponding stock is the optimum value.

Example: A veterinarian purchases rabies immunization vaccine on Monday of each week. Because of the characteristic of this vaccine, it must be used by Friday or disposed of. The vaccine cost Rs. 9 per dose and the veterinarian charges Rs. 16 per dose. In the past, the veterinarian has administered rabies vaccine in the following quantities.

Quantities used per week	No. of weeks this occurred	Probability of occurrence	Cumulative probability
2500	15	0.3	1.00
4000	20	0.4	0.70
5000	10	0.2	0.30
7500	5	0.1	0.10

Using marginal analysis determine how many does the veterinarian should order each week. If the veterinarian is offered a forecasting method costing Rs. 5000, should he purchase this model or not.

Solution: We have,

Cumulative probability table

Quantities used per week	Probability	Cumulative probability
2500	0.3	1.00
4000	0.4	0.70
5000	0.2	0.30
7500	0.1	0.10

Here,

SP per dose = Rs 16 and CP per dose = Rs 9

MP = SP –CP = 16 – 9 = Rs 7

ML = CP = Rs 9

Now,

$$p \geq \frac{\text{ML}}{(\text{MP} + \text{ML})} \geq \frac{9}{7+9} \; 0 \geq 0.56$$

Since the cumulative probability just greater than 0.56 is 0.70, the veterinarians should order 4000 doses per week

We know that an additional unit should be stocked as long as

$$p \times \text{MP} \geq (1 - \text{p}) \times \text{ML}$$

Now,

Expected marginal profit = $p \times$ MP = 0.7 × 7 = Rs 4.9

Expected marginal loss = (1 – p) × ML = (1 – 0.70) × 9 = Rs 2.7

Since $p \times$ MP ≥ (1 – p) × ML, the optimum stock is 4000 doses per week. Therefore, the conditional profit (pay-off) is calculated as follows:

Pay-off = MP × S; if S ≤ D

= Rs 7 × S

Pay-off = MP × D – CP (S – D); if S > D

= Rs 7 × D – 9 (S – D)

= Rs (16 D – 9 S)

If S denotes the optimum stock per week, then S = 4000 doses per week. The maximum EMV under uncertainty is calculated as follows:

Calculation of maximum EMV under uncertainty

Demand (D)	Conditional profit (Rs) (1)	Probability (2)	Expected profit (Rs) (1) × (2)
2500	16 × 2500 – 9 × 4000 = 4000	0.3	4000 × 0.3 = 1200
4000	7 × 4000 = 28000	0.4	28000 × 0.4 = 11200
5000	7 × 4000 = 28000	0.2	28000 × 0.2 = 5600
7500	7 × 4000 = 28000	0.1	28000 × 0.1 = 2800
			Rs = 20800

Hence, maximum EMV under uncertainty = Rs 20800

In order to find EVPI, we first need to calculate the expected profit with perfect information (EPPI).

Calculation table of EPPI

Demand (D)	Conditional profit (Rs) = MP×S (1)	Probability (2)	Expected profit (Rs) (1) × (2)
2500	7 × 2500 = 17500	0.3	17500 × 0.3 = 5250
4000	7 × 4000 = 28000	0.4	28000 × 0.4 = 11200
5000	7 × 5000 = 35000	0.2	35000 × 0.2 = 7000
7500	7 × 7500 = 52500	0.1	52500 × 0.1 = 5250
			$\sum EMV$ = Rs 28700

Hence, EPPI = $\sum EMV$ under certainty = Rs 28700

The expected value of perfect information (EVPI) is given by

EVPI = EPPI – Maximum EMV under uncertainty

= Rs (28700 – 20800)

= Rs 7900

Hence, the cost of uncertainty = Rs 7900

Since the cost of uncertainty is greater than the cost of forecasting model (i.e. Rs 5000), the veterinarian should purchase the model so that he will gain Rs (7900 – 5000 = 2900).

Question (1): A retailer purchases cherries every morning at Rs 50 a case and sells them for Rs 80 a case. Any case remaining unsold at the end of the day can be disposed of next day at a salvage value of Rs 20 per case there after they have no value. Past sales have ranged from 15 to 18 cases per day. The following is the record of sales for the past 120 days.

Cases sold	15	16	17	18
Number of days	12	24	48	36

Find how many cases the retailer should purchase per day to maximize the profit.

[Ans: 17 cases purchase to maximize his profit]

Question (2): A retailer has to decide as to the optimum number of units to be stocked of a certain item under following conditions. (a) Cost price in season is Rs 15, (b) Bargain price after season is Rs 9, (c) Cost of holding the item beyond the season is Rs 2, and, (d) Selling price in season is Rs 20.

Unit demand	12	13	14	15	16
Probability	0.20	0.20	0.25	0.20	0.15

[Ans: p = 0.615, optimum stock size = 13, maximum EMV = Rs 62.4, EPPI = Rs 69.5, and, EVPI = Rs 7.1]