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Chapter

NON PARAMETRIC TEST

CHAPTER OUTLINE

After studying this chapter, students will be able to understand the:

- Parametric Vs. non-parametric test, Needs of applying non-parametric tests,
- One-sample test, Run test, Binomial test, Kolmogorov-Smirnov test
- Two independent sample test, Median test, Kolmogrov-Smirnov test, Wilconxon Mann Whitney test, Chi-square test
- Paired-sample test: Wilcoxon signed rank test, Cochran's Q test, Fredman two way analysof test variance test, Kruskal Wallis test
- Problems and illustrative examples related using software.

Most of the hypothesis testing procedures so far such as Z test, t test, F test are based upon the assumption that the random samples are selected from a normal population. If this is true, these methods can extract all the information that is available in sample and they usually give the best possible precision. Parametric tests depends on parameters, viz., mean or proportion or standard deviation of the population from which sample is taken.

In practice there are many circumstances in which sample are selected from non normal population. In such case, we can have no assumptions about parameters or normality about the population. In such special cases, Parametric are inevitable and are used for testing of hypothesis. Manynon parametric procedures are based upon ranked data or even categorical data. For such data no parametric tests are available.

Non parametric tests are often used in place of their parametric counterparts when certain assumptions about the underlying population are in weak state. However, if the sample size is large enough most non parametric tests can be viewed as (the usual normal theory based procedures applied to ranks- make this statement more clear).

The following table shows a comparative view of some normal distribution based tests (that is parametric tests) and its non parametric counterparts

Normal Distribution based Test	Corresponding non Parametric Test	Purpose of Test
t test for independent samples	Mann Whitney U test	Compare two independent samples
Paired t test	Wilcoxon matched pair signed rank test	Compare dependent samples
One way ANOVA	Kruskalwallis H test	Compare three or more groups
Two way ANOVA	Friedman test	Compare groups classified by two factors

Advantages of Non-parametric Test

- Can be applied to qualitative data (rank, ordinal, categorical data) as well as quantitative data.
- For relatively small sample it is the only possible test
- It is simple to understand, quicker an easier to apply.
- It is less time consuming.
- It needs no assumption about the population from which sample is selected.
- It has greater range of applicability because of milder assumption.
- It does not require complicated sampling theory.

Disadvantages of Non-parametric Test

- It can not be used to estimate the parameters of population.
- These tests are less reliable and less powerful than parametric tests.
- These tests are less efficient than parametric tests.
- In these test many times lot of information are discarded or unused.
- Lot of tables are needed for tests.

Uses of non parametric test

(i) When data size is small
(ii) When assumptions of parametric procedure are not satisfied.
(iii) When quick or preliminary data analysis is required.
(iv) When data are in weak scaled.
(v) When data show highly skewed nature (large positive/negative skewness)
(vi) When basic question of interest is distribution free in nature.

Measurement Scale

The measurement is the process of assigning number codes to observations with or without true numerical meaning. Measurement scales are used to categorize and/or quantify variables. Any statistical data must have measurement. There are four measurement scale in which any data is measured.

(i) Nominal scale
(ii) Ordinal scale
(iii) Interval scale
(iv) Ratio scale

Nominal Scale

It is a process of assigning numbers or symbols to events such that the numbers assigned have no numerical meaning. It is the weakest scale. It is numeric in name only because it does not satisfy ordinary arithmetic properties such as addition, subtraction, multiplication, division etc. eg. Numbers on football player's jerseys, numbers assigned to sex, numbers assigned to marital status.

Ordinal Scale

It is system of assigning numbers to events such that numbers assigned have grading to events. It is rank order of the events. In comparing one is higher or more than the other ordinal scale is used. e.g. grading system in government job , preference to different cold drinks such as Coca-Cola, Sprite, Fanta, Pepsi.

Interval Scale

It is extended form of ordinal scale in which distance (interval) between two objects are exactly known. In this scale the numerals with quantitative meaning are associated to the objects. In this scale ratio of any two interval is independent of unit of measurements and zero point(origin).eg. temperature recorded.

Ratio Scale

It is extended form of interval scale including a true zero point as it's origin. In this case ratio of any two points is independent of unit of measurement but not of zero point. The numbers associated to ratio scale are true numbers with true zero. eg. heights recorded, weights recorded.

Difference between Parametric and Non Parametric test

Parametric test	Non parametric test
1. It specifies certain condition about parameter of the population from which sample is selected.	1. It does not specify certain condition about parameter of the population from which sample is selected.
2. It is used in testing of hypothesis and estimation of parameters.	2. It is used in testing of hypothesis but not in estimation of parameters.
3. Mostly it is used in data measured in interval and ratio scale.	3. It is used in data measured in nominal and ordinal scale.
4. It is most powerful.	4. It is less powerful.
5. It requires complicated sampling technique.	5. It does not require complicated sampling technique.

Assumptions of Non-parametric Test

Non parametric tests do not depend upon assumption about parameter of the population from which sample is selected. However, following basic and weak assumptions are made in the non parametric tests.

(i) The sample observations are independent.

(ii) The variable under study is continuous.

(iii) Samplep.d.f. is continuous.

(iv) Lower order moments exists(mean and variance).

One Sample Test

Run Test

It is non parametric test used to determine the randomness of the selected samples.

Run is a set of identical or related symbols contained between two different symbols or none at all. In sequence HHHTTHHTHHHTTH HHHTT number of run is 8.

Let us consider a random sample of size n is selected from non normal population. Let x_1, x_2, x_3, x_4 ………… x_n be samples.

Different steps in the test are

Problem to test

H_0 : Sample observations are in random order

H_1: Sample observations are not in random order.

First find the median of sample and assign a symbol (say A) for x_i if $x_i > Md$, assign next symbol (say B) for x_i if $x_i < Md$ and omit if $x_i = Md$ (called tie)

Count the number of runs (r) of the symbols, number of observations of symbol A denoted by n_1 and number of observations of symbol B denoted by n_2.

For small sample size ($n_1, n_2 \leq 20$)

Test statistic

Number of runs (r)

Level of significance

Let α be the level of significance. Usually we take $\alpha = 0.05$ unless we are given.

Critical value

Critical or tabulated value $\underline{r}$ and $\bar{r}$ is obtained from table according to the level of significance α, degree of freedom n_1 and n_2 and alternative hypothesis.

Decision

Accept H_0 at α level of significance if $r \in (\underline{r}, \bar{r})$, reject otherwise.

For large sample size (n_1 or $n_2 > 20$)

In case of large sample size r is approximately normally distributed with mean $\mu_r = \frac{2n_1n_2}{n_1 + n_2} + 1$

And variance $\sigma_r^2 = \frac{2n_1n_2\,(2n_1n_2 - n_1 - n_2)}{(n_1 + n_2)^2\,(n_1 + n_2 - 1)}$

Test statistic

$$Z = \frac{r - \mu_r}{\sigma_r} \sim N(0, 1)$$

Level of significance

Let α be the level of significance. Usually we take $\alpha = .05$ unless we are given.

Critical value

Critical or tabulated value of Z is obtained from table according to the level of significance, and alternative hypothesis.

Decision

Reject H_0 at α level of significance if $|Z| > Z_{\text{tabulated}}$, accept otherwise.

Note: For α other than 5% even if sample size is small use Z test in which $Z = \frac{|r - \mu_r| - 0.5}{\sigma_r}$.

Example 1: In 15 toss of a coin the following sequence of heads (H) and tails (T) is obtained H H T TT H T H HH TT H H T .

Test at 0.05 level of significance whether the sequence is random.

Solution:

Here, Number of heads (n_1) = 8, Number of tails (n_2) = 7, Number of runs (r) = 8, Level of significance (α) = 0.05

Problem to test

H_0: The sequence is in random order

H_1: The sequence is not in random order

Test statistic

r = 8

Critical value

At $\alpha = 0.05$ level of significance, the critical value for n_1=8 and n_2 =7 degree of freedom are $\underline{r} = 4$ and $\bar{r} = 13$ for two tailed test.

Decision

$r = 8 \in (\underline{r} = 4, \bar{r} = 13)$, accept H_0 at 0.05 level of significance.

Conclusion

The sequence of H and T are in random order.

Example 2: The following is the arrangement of defective (d) and non defective (n) pieces of keyboard produced in the given order by a certain machine:

n n n n n d d d d n n n n n n n n n n d d n n d d d d n n n d d n n d n n n

Test the randomness at the 0.01 level of significance.

Solution:

Here, number of n (n_1) = 25

Number of d (n_2) = 13 number of runs (r) = 11

$$\mu_r = \frac{2n_1n_2}{n_1 + n_2} + 1 = \frac{2 \times 25 \times 13}{25 + 13} + 1 = 8.105$$

$$\sigma_r^2 = \frac{2n_1n_2\,(2n_1n_2 - n_1 - n_2)}{(n_1 + n_2)^2\,(n_1 + n_2 - 1)}$$

$$= \frac{2 \times 25 \times 13(2 \times 25 \times 13 - 25 - 13)}{(25 + 13)^2\,(25 + 13 - 1)}$$

$$= \frac{397800}{53428} = 7.44$$

$$\sigma_r = \sqrt{7.44} = 2.72.$$

Problem to test

H_0 : Arrangement is in random order

H_1 : Arrangement is not in random order.

Test statistic

$$Z = \frac{r - \mu_r}{\sigma_r} = \frac{11 - 8.105}{2.72} = 1.064$$

Critical value

At $\alpha = 0.01$ level of significance critical value is Ztabulated = $Z_{\alpha/2} = 2.576$.

Decision

$Z = 1.064 < Z_{tabulated} = 1.96$, accept H_0 at 0.01 level of significance

Conclusion

The defective and non defective pieces produced by machine are in random order.

Example 3: The height (in inches) of 16 BSC CSIT students are as follows:

68.2,71.6,69.3,71.6,70.4, 65.0, 63.6,64.7,65.3,64.2,67.6,68.6,66.8,68.9,66.8, 70.1

Test whether the order of the heights is random or not? Use 5% level of significance.

Solution:

Here, n = 16, to find run first of all find median. To find the median, arranging data in ascending order

63.6, 64.2, 64.7, 65.0, 65.3, 66.8, 66.8, 67.7, 68.2, 68.6, 68.9, 69.3, 70.1, 70.4, 71.6, 71.6

$$Md = \frac{(n+1)}{2}^{th}\text{ item} = \frac{16+1}{2} = 8.5^{th}\text{ item}$$

Hence, median = (8^{th} item + 9^{th} item)/2 = (67.7+68.2)/2 = 67.95

Let the number greater than 67.95 is denoted by A and number less than 67.95 is denoted by B, then the arrangement of given sample are

A AAAA B BBBBB A B A B A

Here, number of A (n_1) = 8, number of B (n_2) = 8, number of run (r) = 7

Problem to test

H_0: Order of heights are in random order

H_1: Order of heights are not in random order.

Test statistic

r = 7

Critical value

Let α = 0.05 be the level of significance, then critical value for n_1 = 8 and n_2 = 8 are $\underline{r}$ = 4 and $\bar{r}$ = 14

Decision

$r = 7 \in (\underline{r} = 4, \bar{r} = 14)$, accept H_0 at 0.05 level of significance.

Conclusion

The order of heights are in random order.

Example 4: The following are the no. of emails arrived within one hour in a communication room: 46, 58, 60, 56, 70, 66, 48, 54, 62, 41, 39, 52, 45, 62, 53, 69, 65, 65, 67, 76, 52, 52, 59, 59, 67, 51, 46, 61, 40, 43, 42, 77, 67, 63, 59, 63, 63, 72, 57, 59, 42, 56, 47, 62, 67, 70, 63, 66, 69 and 73. Given that median is 59.5

Test the randomness at the 0.05 level of significance.

Solution:

Here, Sample size (n) = 50 , Median (Md) = 59.5, Level of significance (α) = 0.05

Assign A for number greater than 59.5 and assign B for number less than 59.5 then arrangement of sample becomes

B B A B A A B B A B BBB A B AAAAA B BBB A B B A B BB A AA B A AA B BBBB A AAAAAA.

Number of B (n_1) = 25 , Number of A (n_2) = 25, Number of run (r) = 20

$$\mu_r = \frac{2n_1n_2}{n_1 + n_2} + 1 = \frac{2 \times 25 \times 25}{25 + 25} + 1 = 26$$

$$\sigma_r^2 = \frac{2n_1n_2(2n_1n_2 - n_1 - n_2)}{(n_1 + n_2)^2(n_1 + n_2 - 1)} = \frac{2 \times 25 \times 25(2 \times 25 \times 25 - 25 - 25)}{(25 + 25)^2(25 + 25 - 1)} = 12.2,$$

$$\sigma_r = \sqrt{12.2} = 3.5$$

Problem to test

H_0 : Emails are in random order

H_1: Emails are not in random order.

Test statistic

$$Z = \frac{r - \mu_r}{\sigma_r} = \frac{20 - 26}{3.5} = -1.71$$

Critical value

At $\alpha =$ 0.05 level of significance, critical value is $Z_{tabulated} = Z_{\alpha/2} = 1.96$.

Decision

$|Z| = 1.71 < Z_{tabulated} = 1.96$, accept H_0 at 0.05 level of significance.

Conclusion

The sample is in random order.

Binomial Test

It is non parametric test used for data present in either nominal or ordinal scale. It is used to test whether the binomial population has two distinct groups of two equal numbers of outcomes or not.

Let us consider a sample of size n is selected from binomial population (dichotomized population)

Let $x_1, x_2, x_3, \ldots\ldots x_n$ be independent sample of size n selected from binomial population having probability mass function $P(x) = c(n,x)\ p^x(1 - p)^{n-x}$ such that probability associated with each trial is equal. x is number of observations having certain characteristic and n - x is number of observations having next characteristic then $p = \frac{x}{n}$

Different steps in the test are

Problem to test

H_0: $P = P_0$

H_1: $P \neq P_0$ (two tailed) or H_1: $P > P_0$ one tail) or H_1: $P < P_0$ (one tail)

Let n_1 and n_2 be the number of observations belonging to two groups from n samples.

$X_0 = \min(n_1, n_2)$

Small sample size ($n \leq 25$)

Test statistic

$X_0 = \min\{n_1, n_2\}$

Level of significance

Let α be the level of significance. Generally fix α= 0.05 unless we are given.

Critical value

Using the binomial distribution $p = \text{Prob}(X \le x_0) = \sum_{x=0}^{x_0} C(n, x)\, p^x (1-p)^{n-x} = \sum_{x=0}^{x_0} C(n, x) \left(\frac{1}{2}\right)^n$

Decision

Accept H_0 at α level of significance if $p > \alpha$ for one tailed test and $2p > \alpha$ for two tailed test, reject otherwise

Large sample size (n > 25)

For large sample size, x_0 is normally distributed with mean np and variance npq

Test statistic

$$Z = \frac{X_0 - \mu}{\sigma} = \frac{X_0 - np}{\sqrt{npq}}$$

Since X_0 is discrete so that continuity correction is made as

$$Z = \frac{(x_0 \pm 0.5) - np}{\sqrt{npq}}$$, use + 0.5 if $x_0 < np$ and use – 0.5 if $x_0 > np$

Level of significance

Let α be the level of significance. Generally fix α = 0.05 unless we are given.

Critical value

Critical value $Z_{tabulated}$ is obtained from table according to level of significance and alternative hypothesis.

Decision

Reject H_0 at α level of significance if $Z > Z_{tabulated}$, Accept otherwise.

Example 5: The following are defective (D) and non defective (N) electronic items produced in the given order by a certain machine:

N N D D N D N N N D D N D D N N D N D N N N

Test whether defective and non defective are equally produced or not. Use Binomial test at the 0.01 level of significance.

Solution:

Here number of N (n_1) = 13

Number of D(n_2) = 9

Problem to test

$H_0 : P = \frac{1}{2}$ $\qquad$ $H_1 : P \neq \frac{1}{2}$

Test statistics

$X_0 = \min\{n_1, n_2\} = \min(13, 9) = 9$

Level of significance

$\alpha = 0.01$

Critical value

$$P = \sum_{x=0}^{x_0} C(n, x)\left(\frac{1}{2}\right)^n = \sum_{x=0}^{9} C(n, x)\left(\frac{1}{2}\right)^n = 0.262$$

$2p = 2 \times 0.262 = 0.524$

Decision

$2P = 0.524 > \alpha = 0.01$

Accept H_0 at 0.01 level of significance.

Conclusion

Defective and non defective are produced equally.

Example 6: Out of 50 students willing to express opinion of laptop 30 expressed preferences to brand Dell and 20 for brand Lenovo. Use binomial test to test hypothesis that both brand of laptop are equally popular

Solution:

Here number of female students prefer Dell laptop (n_1) = 30

Number of female students prefer Lenovo laptop (n_2) = 20

Problem to test

$H_0 : P = \frac{1}{2}$

$H_1 : P \neq \frac{1}{2}$

Test statistics

$X_0 = \min\{n_1, n_2\} = \min(20, 30) = 20$

$np = 50 \times \frac{1}{2} = 25$

$$Z = \frac{(x_0 \pm 0.5) - np}{\sqrt{npq}}$$

$$= \frac{(x_0 + 0.5) - np}{\sqrt{npq}}$$

$$= \frac{(20 + 0.5) - 25}{\sqrt{12.5}}$$

$= -1.27$

Level of significance

Let $\alpha = 0.05$

Critical value

$Z_{\alpha/2} = 1.96$

Decision

$|Z| = 1.27 < Z_{\alpha/2} = 1.96$

Accept H_0 at 0.05 level of significance.

Conclusion

Both brand of laptop are equally popular

Kolmogorov Smirnov Test

Kolmogorov Smirnov one sample test is a test of goodness of fit.It is alternate to chi square test for goodness of fit when sample size is small.

Let $x_1, x_2, x_3, \dots\dots\dots x_n$ be random sample of size n from population having distribution function F(x).

Let us consider n observations of random variable x is classified into k classes with their respective frequencies

Different steps in the test are;

Problem to test

H_0: Samples come from population with distribution $F_0(x)$

H_1: Samples do not come from population with distribution $F_0(x)$

Obtain the observed relative frequency F_o and expected relative frequency F_e respectively. Then find the absolute deviation of F_e and F_0 i.e., $|F_e - F_0|$

Test statistic

$D_0 = \text{Max } |F_e - F_0|$

Level of significance

Let α be the level of significance. Generally fix $\alpha = 0.05$ unless we are given.

Critical value

Critical value $D_{n,a}$ is obtained according to the level of significance α and sample size n

Decision

Reject H_0 at α level of significance if $D_0 \geq D_{n,a}$ accept otherwise.

Example 7: The number of laptop in 10 different department are given below. Test whether the laptops are uniformly distributed over the entire office use Kolmogorov smirnov test.

Department No.	1	2	3	4	5	6	7	8	9	10
No of laptop	8	10	9	12	15	7	5	12	13	9

Solution:

Her expected frequency for each class = Sum of frequencies/No of class

Department no.	No of laptop	Observed cf (cf_0)	Observed relative freq (F_0)	Expected freq (f_e)	Expected cf (cf_e)	Expected relative freq(F_e)	$\|F_e - F_0\|$
11	8	8	8/100	10	10	10/100	2/100
2	10	18	18/100	10	20	20/100	2/100
3	9	27	27/100	10	30	30/100	3/100
4	12	39	39/100	10	40	40/100	1/100
5	15	54	54/100	10	50	50/100	4/100
6	7	61	61/100	10	60	60/100	1/100
7	5	66	66/100	10	70	70/100	4/100
8	12	78	78/100	10	80	80/100	2/100
9	13	91	91/100	10	90	90/100	1/100
10	9	100	100/100	10	100	100/100	0/100

Problem to test

H_0: The disease infected plants are uniformly distributed over the entire area.

H_1: The disease infected plants are not uniformly distributed over the entire area.

Test statistic

$D_0 = \text{Max } |F_e - F_0| = 4/100 = 0.04$

Critical value

Let 5% be the level of significance then critical value is $D_{100,0.05} = \dfrac{1.36}{\sqrt{100}} = 0.136.$

Decision

$D_0 = 0.04 < D_{10,0.05} = 0.136$, accept H_0 at 5% level of significance.

Conclusion

The disease infected plants are uniformly distributed over the entire area.

Example 8: A random sample of 20 volume based internet connected have following speed of internet connection in mps:

2.7 , 2.9 , 3.0 , 3.1 , 2.8 , 3.0 , 2.9 , 3.0 , 2.6 , 3.1 , 3.2 , 3.1 , 3.0 , 2.9 , 3.3 , 3.0 , 2.8 , 2.9 , 3.0 , 2.9

Apply the Kolmogorov Smirnov test for testing that the internet speed are equally distributed.

Solution:

Expected frequency for each class = Σf_0/no of class = 20/8 = 2.5

Speed	Tally bar	No of internet connection	cf_0	F_0	Expected Freq (f_e)	cf_e	F_e	$\lvert F_e - F_0 \rvert$
2.6	I	1	1	1/20	2.5	2.5	2.5/20	1.5/20
2.7	I	1	2	2/20	2.5	5	5/20	3/20
2.8	II	2	4	4/20	2.5	7.5	7.5/20	3.5/20
2.9	卌	5	9	9/20	2.5	10	10/20	1/20
3.0	卌 I	6	15	15/20	2.5	12.5	12.5/20	2.5/20
3.1	III	3	18	18/20	2.5	15	15/20	3/20
3.2	I	1	19	19/20	2.5	17.5	17.5/20	1.5/20
3.3	I	1	20	20/20	2.5	20	20/20	0/20

Problem to test

H_0: Internet speeds are equally distributed.

H_1: Internet speed is not equally distributed.

Test statistic

$D_0 = \text{Max } \lvert F_e - F_0 \rvert = 3.5/20 = 0.175$

Critical value

Let 5% be the level of significance then critical value is $D_{20,0.05} = 0.294$

Decision

$D_0 = 0.175 < D_{20,\,0.05} = 0.294$, accept H_0 at 5% level of significance.

Conclusion

Children of different heights are equally enrolled.

EXERCISE

1. What do you mean by non parametric test? Write down advantages of non parametric test over parametric test.
2. Differentiate between parametric test and non parametric test.
3. Discuss assumptions of non parametric test.
4. Write down steps of non parametric test.
5. Describe the function and procedure of run test.
6. What is binomial text? Why is it used?
7. Describe the method of binomial test
8. Discuss process of kolmogorov Smirnov test.
9. In 31 toss of a coin the following sequence of heads (H) and tails (T) is obtained.

   H T T H T H HH T H H TT H T H T H H T H T T H T H H T H T H

   Test at 0.05 level of significance level whether the sequence is random. **Ans:** r = 23, sig.

10. A random samples of 15 adults living in a small town is selected to estimate the proportion of voting favoring a certain candidate for Mayer. Each individual was also asked if he or she was a college graduate. By letting Y and N designate the response of yes and no to the education question, the following sequence was obtained N NNN Y Y N Y Y N Y N NNN. Use the run test to determine if the sequence supports the condition that the sample was selected at random. **Ans:** r = 7, insig.
11. In production line touch screen are inspected periodically for defectives .The following is sequence of defective item D and non defective item N produced by production line.
D D N NN D N N D D N NNNN D DD N N D N NNN D N D.
Use run test with a significance level 0.05 to determine whether the defectives are occurring at random order. **Ans:** r = 13, insig.
12. The following are the number of IT students absent from a college on 24 consecutive college days: 29, 25, 31, 28, 30, 28, 33, 31, 35, 29, 31, 33, 35, 28, 36, 30, 33, 26, 30, 28, 32, 31, 38 and 27.Test for randomness at the 0.01 level of significance. **Ans:** z = 0, insig.
13. The height (in inches) of 15 football players are as follows:
65.3, 67.7, 68.4,71, 70.2, 67, 69.8, 64, 67.6, 71.5, 64.4, 63.6, 66.8, 70.4, 69.
Test whether the order of the heights is random or not .Use 1% level of significance.
Ans: z = 0, insig.
14. The following are defective (D) and non defective (N) electric cable produced in the given order by a certain machine in a manufacturing industry:
D N D D N D NDN N D N N D D N N D N D D N N N N
Test whether defective and non defective cables are equally produced. Use Binomial test at the 5% level of significance. **Ans:** p = 0.345, insig.
15. 100 workers of sales house willing to express opinion of cell phones, 40 expressed preferences to brand Nokia and 60 for brand Oppo. Are both brand of cell phone equally popular at 5% level of significance. Use Binomial test **Ans:** p = – 1.9, insig.
16. In a certain computer hardware manufacturing industry six different types of machines are working to cut pieces of wires. The number of wires of unequal length recorded in a day is as follows:

Machine	1	2	3	4	5	6
No of wire	2	0	4	8	5	11

Do these data provide sufficient evidence that the six machines equally cut the wires of unequal length? Apply Kolmogorov Smirnov test at 5% level of significance. **Ans:** D_0 = 0.3, sig.
17. The number of virus infected computers of five different capacity of hard disk is given below:

Capacity of hard disk (GB)	500	320	1000	2000	4000
No of virus infected	11	15	20	3	1

Test whether the computers of five hard disk are uniformly infected using Kolmogorov Smirnov test. **Ans:** D_0 =0.333, insig.
18. A game consists of four pairs of color cards. Twenty chimpanzees of same age were taught the matching game of color cards for a specified period of time. At the end of the training 4 pairs of color cards are given to each of the chimpanzee for matching. The result were as follows;

Matched set	0	1	2	3	4
Frequency	1	0	5	7	7

Does chimpanzees recognize colors? Use Kolmogorov Smirnov test at 5% level of significance. **Ans:** D = 0.35, insig.

Two Independent Sample Test

Median Test

It is non parametric test used to test the significance difference between two independent distribution.

Let $x_1, x_2, x_3 \ldots\ldots x_{n_1}$ and $y_1, y_2, y_3 \ldots\ldots y_{n_2}$ be two independent samples of sizes n_1 and n_2 respectively be selected from continuous populations with unknown medians Md_1 and Md_2 respectively.

Different steps in the test are

Problem to test

H_0: $Md_1 = Md_2$

H_1: $Md_1 \pm M_2$ (Two tailed test) or H_1: $Md_1 > Md_2$ (One tailed right) or H_1: $Md_1 < Md_2$ (one tailed left)

Small sample size ($n_1 \leq 10, n_2 \leq 10$)

Combine n_1 and n_2 such that $n = n_1 + n_2$ and obtain median of n observation Find number of observations in $x_i \leq Md$ and denote by a.

Test statistic

$$P(A=a) = \frac{c(n_1, a)\, c(n_2, k-a)}{c(n_1 + n_2, k)} : a = 0, 1, 2, \ldots\ldots \min(n_1, k), k = \frac{n_1 + n_2}{2} = \frac{n}{2}$$

Level of significance

Let α be the level of significance. Generally we fix $\alpha = 0.05$ unless we are given.

Critical value

Critical value is given by $p = P(A \geq a)$

Decision

Accept H_0 at α level of significance if $p > \alpha$ for one tailed and $2p > \alpha$ for two tailed test, reject otherwise.

Large sample size ($n_1 > 10, n_2 > 10$)

Combine n_1 and n_2 such that $n = n_1+n_2$ and obtain median of n observation. Find number of observations in $x_i \leq Md$ and denote by

a. Find number of observations in $y_i \leq Md$ and denote by b.

Also find the number of observations in x_i>Md an y_i>Md and denote by c and d respectively.

	No. of obs.≤ Md	No. of obs.>Md	Total
Sample x	a	c	a+c
Sample y	b	d	b+d
Total	a+b	c+d	N=a+b+c+d

Test statistic

$$\chi^2 = \frac{N(ad - bc)^2}{(a + c)\,(b + d)\,(a + b)\,(c + d)} \sim \chi^2_{(1)}$$

If any cell frequency is less than 5 then

$$\text{Corrected } \chi^2 = \frac{N(|ad - bc| - \frac{N}{2})^2}{(a + c)(b + d)(a + b)(c + d)} \sim \chi^2_{(1)}$$

Level of significance

Let α be the level of significance. Generally we fix $\alpha = 0.05$ unless we are given.

Critical value

At α level of significance for 1 degree of freedom critical value is $\chi^2_{\alpha(1)}$

Decision

Reject H_0 at α level of significance if $\chi^2 > \chi^2_{\alpha(1)}$, accept otherwise.

Example 1: Two independent samples are given below;

Sample I	10	11	8	8	14		
Sample II	9	12	13	9	15	9	17

Test whether the two samples have come from the same population with respect to their medians. Use median test at 0.05 level of significance.

Solution:

Here, to find the median of combined group, arranging data in ascending order,

8, 8, 9, 9, 9, 10, 11, 12, 13, 14, 15, 17.

$n = 12$, $Md = \frac{(n + 1)}{2}$ th item $= \frac{(12 + 1)}{2} = 6.5^{th}$ item.

Hence median $= \frac{10 + 11}{2} = 10.5$

Now no of observations in first sample less than or equal to median (a) = 3

First sample size $(n_1) = 5$

Second sample size $(n_2) = 7$, $k = \frac{n_1 + n_2}{2} = \frac{(5 + 7)}{2} = 6$

Let Let Md_1 and Md_2 be median of I population and II population respectively.

Problem to test

H0: There is no significant difference between median of I population and median of II population ($Md_1 = Md_2$)

H_1: There is significant difference between median of I population and II population ($Md_1 \neq Md_2$)

Test statistic

$$P(A=a) = \frac{c(n_1, a)\, c(n_2, k-a)}{c(n_1 + n_2, k)} = \frac{c(5, a)\, c(7, 6 - a)}{c(12, 6)}, \; a = 0, 1, 2, 3, 4, 5$$

Critical value

$$P = P(A \geq a) = P(A \geq 3) = \sum \frac{c(5, a)\, c(7, 6 - a)}{c(12, 6)}$$

$$= \frac{c(5, 3)\, c(7, 6 - 3)}{c(12, 6)} + \frac{c(5, 4)\, c(7, 6 - 4)}{c(12, 6)} + \frac{c(5, 5)\, c(7, 6 - 5)}{c(12, 6)}$$

$= 462/9 \quad 24 = 0.5$

Decision

$2P = 1 > \alpha = 0.05$, accept H_0 at 0.05 level of significance.

Conclusion

Two samples have come from same population with respect to median.

Example 2: An IQ test was given to a random sample of 15 male and 20 female students of a university. Their scores were recorded as follows;

Male: 56, 66, 62, 81, 75, 73, 83, 68, 48, 70, 60, 77, 86, 44, 72

Female: 63, 77, 65, 71, 74, 60, 76, 61, 67, 72, 64, 65, 55, 89, 45, 53, 68, 73, 50, 81

Use median test to determine whether IQ of male and female students is same in the university. (Given that the median of combined sample =68)

Solution:

Here Number os male (n_1) = 15, Number of female (n_2) = 20, $N = n_1 + n_2 = 15 + 20 = 35$,

a(no. of obs. of male ≤ Md) = 7 , b(no. of obs. of female ≤ Md) = 12

c(no. of obs. of male > Md) = 8, d(no. of obs. of female > Md) = 8

The 2 × 2 contingency table is

	Sample I	Sample II	Total
No. of obs. ≤ Md	7 (a)	12 (b)	19
No. of obs > Md	8 (c)	8 (d)	16
	15	20	35

Let Md_1 and Md_2 be median IQ of male and female respectively.

Problem to test

H_0: There is no significant difference between IQ of male and female ($Md_1 = Md_2$)

H_1: There is significant difference between IQ of male and female. ($Md_1 \neq Md_2$)

Test statistic

$$\chi^2 = \frac{N(ad - bc)^2}{(a + c)\ (b + d)\ (a + b)\ (c + d)} = \frac{35(7 \times 8 - 12 \times 8)^2}{15 \times 20 \times 19 \times 16} = 0.61$$

Critical value

Let $\alpha = 0.05$ be the level of significance , then critical value at 0.05 level of significance with 1 degree of freedom is $\chi^2_{(1)} = 3.84$.

Decision

$\chi^2 = 0.61 < \chi^2_{(1)} = 3.84$, accept H_0 at 0.05 level of significance.

Conclusion

IQ of male and female is same in the university.

Two Sample Kolmogorov Smirnov Test

It is non parametric test used to test whether two independent samples are from same population or not.

Let $x_1, x_2, x_3, \ldots\ldots x_{n_1}$ and $y_1, y_2, y_3, \ldots\ldots y_{n_2}$ be independent samples of size n_1 and n_2 drawn from continuous population having distribution function F(x) and F(y) respectively.

Different steps in the test are

Problem to test

H_0: F(x) = F(y)

H_1: F(x) ≠ F(y)(two tailed) or H_1: F(x) > F(y)one tail) or H_1: F(x) < F(y) (one tail)

Obtain cumulated distribution function of x and y separately after arranging in order.

$F(x) = \frac{k_1}{n_1}$, where k_1 is observed cumulative frequency of x

$F(y) = \frac{k_2}{n_2}$, where k_2 is observed cumulative frequency of y

Small sample test ($n_1 = n_2 < 40$, n_1 and $n_2 \le 20$ for $n_1 \neq n_2$)

Test statistic

D_0 = maximum {| F(x) – F(y) |

Level of significance

Let α be level of significance

Critical value

At α level of significance critical value for n_1 and n_2 is

$D n_1, n_2, \alpha$

Decision

Reject H_0 at α level of significance if $D_0 \ge D\, n_1, n_2, \alpha$

Accept otherwise

Large sample test ($n_1 = n_2 > 40$, n_1 and $n_2 > 20$ for $n_1 \neq n_2$)

Test statistic

D_0 = maximum {| F(x) – F(y) | for two tail test

$\chi^2 = 4D_0^2 \frac{n_1 n_2}{n_1 + n_2}$ follows chi square distribution with 2 degree of freedom. For one tail test

Level of significance

Let α be the level of significance

Critical value

At α level of significance critical value is

$D_\alpha = 1.36\sqrt{\frac{n_1 + n_2}{n_1 n_2}}$ for two tail with α = 5%

$\chi^2_{\alpha\, 2}$ for one tail.

Decision

Reject H_0 at α level of significance if $D_0 \ge D_\alpha$ for two tail

if $\chi^2 > \chi^2_{\alpha 2}$ for one tail

Accept otherwise.

Example 3: Life in years of two types of cells used in laptop are given below:

Cell X	4	6	5	6	3	4	5	3	5
Cell Y	2	5	4	3	4	2	4	3	5

Test whether life of two brands of cells are same or not? Use Kolmogorov Smirnov test at 0.05 level of significance.

Solution:

Problem to test

H_0: $F(x) = F(y)$

H_1: $F(x) \neq F(y)$

Combined ordered life	Frequency for x	Frequency for Y	F(x)	F(y)	|F(x) – F(y)|
2	0	2	0/9	2/9	2/9
3	2	2	2/9	4/9	2/9
4	2	3	4/9	7/9	3/9
5	3	2	7/9	1	2/9
6	2	0	1	1	0
Total	9	9			

Test statistics

D_0 = maximum $\{|F(x) - F(y)| = 3/9$

Critical value

At $\alpha = 0.05$ and $n_1 = 9$, $n_2 = 9$ from K – S table

$Dn_1, n_2, \alpha = 5/9$

Decision

$D = 3/9 < D\, n_1, n_2, \alpha = 5/9$

Accept H_0 at 0.05 level of significance.

Conclusion

Two brands of cell X and Y are same.

Example 4: Given below represents monthly income distribution of employees in a hardware company and software company

Income (000 rs)	Number of employees in hardware company	Number of employees in software company
20 – 30	6	12
30 – 40	10	18
40 – 50	11	16
50 – 60	13	12
60 – 70	25	10
70 – 80	15	12
80 – 90	10	10

Do the income distribution support that income of employees in Hardware Company is more than income of employees in Software Company? Use Kolmogorov Smirnov test.

Solution:

Let x be salary of employees in hardware company and y be salary of employees in software company

Problem to test:

H_0 : H_0: $F(x) = F(y)$

H_1 : $F(x) > F(y)$

Income (000 Rs)	Number of employees in hardware company (x)	Number of employees in software company (y)	F(x)	F(y)	$\lvert F(x) - F(y) \rvert$
20 – 30	6	12	6/100	12/90	66/900
30 – 40	10	18	16/100	30/90	156/900
40 – 50	11	16	27/100	46/90	57/900
50 – 60	13	12	40/100	58/90	220/900
60 – 70	25	10	65/100	68/90	95/900
70 – 80	20	12	85/100	80/90	35/900
80 – 90	15	10	1	1	0
Total	100	90			

Test statistic

$$D_0 = \text{maximum } \{ \lvert F(x) - F(y) \rvert = \frac{220}{900} = \frac{11}{45}$$

$$\chi^2 = 4D_0^2 \frac{n_1 n_2}{n_1 + n_2} = 4 \times \left(\frac{11}{45}\right)^2 \frac{100 \times 90}{100 + 90} = 11.32$$

Critical value

Let $\alpha = 0.05$ then critical value is $\chi^2_{\alpha,2} = 5.99$.

Decision

$\chi^2 = 11.32 > c^2_{a,\,2} = 5.99.$

Reject H_0 at $\alpha = 0.05$ level of significance.

Conclusion

Income of employees in Hardware Company is more than income of employees in Software Company.

Example 5: Life time of cell phones branded A and B recorded from a repair centre is given below, determine if there is any significant difference between two brand cell phones using Kolmogorov Smirnov test at 5% level of significance.

Life of cell phone in years	Cell phone A	Cell phone B
0 - 2	8	5
2 - 4	12	7
4 - 6	16	31
6 - 8	10	12
8 and above	4	5

Solution:

Let cell phone A = x and cell phone B = y

Problem to test

H_0: $F(x) = F(y)$

H_1: $F(x) \neq F(y)$

Life of cell phone in years	Cell phone A	Cell phone B	F(x)	F(y)	|F(x) – F(y)|
0 - 2	8	5	8/50	5/60	23/300
2 - 4	12	7	20/50	12/60	60/300
4 - 6	16	31	36/50	43/60	1/300
6 - 8	10	12	46/50	55/60	1/300
8 and above	4	5	1	1	0
Total	50	60			

Test statistic

$D_0 = \text{maximum} \{ |F(x) - F(y)| = \frac{60}{300} = 0.2$

Critical value

At $\alpha = 0.05$ critical value is $D_a = 1.36\sqrt{\frac{n_1 + n_2}{n_1 n_2}} = 1.36\sqrt{\frac{50 + 60}{50 \times 60}} = 0.26.$

Decision

$D_0 = 0.2 < D_a = 0.26$

Accept H_0 at 5% level of significance.

Conclusion

There is no significant difference between two brand cell phones.

Mann Whitney U Test

It is non parametric test used to determine whether two independent samples have been drawn from populations with same distribution.

Let us consider two independent samples of sizes n_1 and n_2 are drawn from continuous populations with unknown medians Md_1 and Md_2 respectively.

Let $x_1, x_2, x_3, \ldots\ldots, x_{n_1}$ and $y_1, y_2, y_3, \ldots\ldots, y_{n_2}$ be independent samples of size n_1 and n_2.

Different steps in the test are

Problem to test

H_0: $Md_1 = Md_2$

H_1: $Md_1 \neq Md_2$ (two tailed) or H_1: $Md_1 > Md_2$ (one tailed right)

or H_1: $Md_1 < Md_2$ (one tailed left)

Combine n_1 and n_2 such that $n_1+n_2 = n$ and rank these n observations in ascending order. If two or more observations are equal then assign average rank and is called tied. Sum the ranks of sample of sizes n_1 and n_2 separately to get R_1 and R_2. If two sample sizes are unequal then smaller one is n_1. Obtain U_1 and U_2 as $U_1 = n_1n_2 + \frac{n_1(n_1+1)}{2} - R_1$ and $U_2 = n_1n_2 + \frac{n_2(n_2+1)}{2} - R_2$.

such that $n_1n_2 = U_1 + U_2$. Finally get $U_0 = \min\{U_1, U_2\}$

Small sample size ($n_1 \leq 10$, $n_2 \leq 10$)

Test statistic

$U_0 = \min\{U_1, U_2\}$

Level of significance

Let α be the level of significance. Generally fix $\alpha = 0.05$ unless we are given.

Critical value

At α level of significance, we obtain critical value from Mann Whitney table as

$p = \text{Prob}(U \leq U_0)$

Alternately

At α level of significance, we obtain critical value from Mann Whitney table as

$U_{tabulated} = U_{\alpha(n1,n2)}$ for two tail and $U_{\alpha/2(n1,\,n2)}$ for one tail test.

Decision

Accept H_0 at α level of significance if $p > \alpha$ for one tailed test and $2p > \alpha$ for two tailed test, reject otherwise.

Alternately

Accept H_0 at α level of significance if $U_0 > U_{tabulated}$ reject otherwise.

Large sample size ($n_1 > 10$, $n_2 > 10$)

For large sample size U_0 is approximately normally distributed with mean $\mu_u = \frac{n_1n_2}{2}$ and variance $\sigma_u^2 = \frac{n_1n_2\,(n_1+n_2+1)}{12}$.

Test statistic

$$Z = \frac{U_0 - \mu_\alpha}{\sigma_n} = \frac{U_0 - \frac{n_1n_2}{2}}{\sqrt{\frac{n_1n_2\,(n_1+n_2+1)}{12}}}$$

it is used even if tied occurs within sample. If tied occurs between samples then standard deviation is corrected as $\sigma_u = \frac{n_1\,n_2}{n(n-1)}\left\{\frac{n^3-n}{12} - \frac{\Sigma t_i^3 - ti}{12}\right\}$

t_i = number of times i^{th} rank repeated between samples.

Level of significance

Let α be the level of significance. Generally fix $\alpha = 0.05$ unless we are given.

Critical value

Critical value $Z_{tabulated}$ is obtained from table according to level of significance and alternative hypothesis.

Decision

Reject H_0 at α level of significance if $Z > Z_{tabulated}$, Accept otherwise.

Example 6: The heart beating rate of 5 vegetarians and 5 non vegetarians are recorded below:

Vegetarians	56	67	82	60	75
Non vegetarians	53	42	75	58	65

Is the mean heart beating rate of non vegetarians significantly high. Use Mann Whitney U test.

Solution:

Vegetarians	Ranks	Non vegetarians	Ranks
56	3	53	2
67	7	42	1
82	10	75	8.5
60	5	58	4
75	8.5	65	6
	$R_1 = 33.5$		$R_2 = 21.5$

Here, Sample size of vegetarian $(n_1) = 5$

Sample size of Non vegetarian $(n_2) = 5$

Sum of ranks of vegetarian $(R_1) = 33.5$

Sum of ranks of non vegetarian $(R_2) = 21.5$

$$U_1 = n_1n_2 + \frac{n_1(n_1+1)}{2} - R_1 = 5 \times 5 + \frac{5 \times 6}{2} - 33.5 = 6.5$$

$$U_2 = n_1n_2 - U_1 = 5 \times 5 - 6.5 = 18.5$$

$$U_0 = \min\{U_1, U_2\} = 6.5$$

Let Md_1 and Md_2 be median heart beat rate of vegetarians and non vegetarians respectively.

Problem to test

H_0: There is no significant difference between heart beating rate of vegetarian and non vegetarian $(Md_1 = Md_2)$

H_1: Heart beating rate of non vegetarian is significantly high than vegetarian $(Md_1 < Md_2)$

Test statistic

$U_0 = 6.5$

Critical value

Let $\alpha = 0.05$ be the level of significance then critical value is $p = 0.111$

Decision

$p = 0.111 > \alpha = 0.05$, accept H_0 at 5% level of significance.

Conclusion

There is no significant difference between heart beating rate of vegetarians and non vegetarians.

Example 7: Test the hypothesis of no difference between the ages of male and female employees of a certain IT company, using the Mann- Whitney U test for the sample data below. Use $\alpha = 0.1$.

Male	35	43	26	44	40	42	33	38	25	26
Female	30	41	34	31	36	32	25	47	28	24

Solution:

Male	Ranks	Female	Ranks
35	12	30	7
43	18	41	16
26	4.5	34	11
44	19	31	8
40	15	36	13
42	17	32	9
33	10	25	2.5
38	14	47	20
25	2.5	28	6
26	4.5	24	1
	$R_1 = 116.5$		$R_2 = 93.5$

Here, Sample size of male $(n_1) = 10$

Sample size of female $(n_2) = 10$

Sum of ranks of male $(R_1) = 116.5$

Sum of ranks of female $(R_2) = 93.5$

$$U_1 = n_1n_2 + \frac{n_1(n_1+1)}{2} - R_1 = 10 \times 10 + \frac{10 \times 11}{2} - 116.5 = 38.5$$

$$U_2 = n_1n_2 - U_1 = 10 \times 10 - 38.5 = 61.5$$

$$U_0 = \min\{U_1, U_2\} = 38.5$$

Let Md_1 and Md_2 be median age of male and female employees respectively.

Problem to test

H_0: There is no significant difference between age of male and female employee $(Md_1 = Md_2)$

H_1: There is significant difference between age of male and female employee $(Md_1 \neq Md_2)$

Test statistic

$U_0 = 38.5$

Critical value

Let $\alpha = 0.10$ be the level of significance then critical value is $U_{tabulated} = U_{\alpha(n_1, n_2)} = 27$

Decision

$U_0 = 38.5 > U_{\alpha(n_1, n_2)} = 27$, accept H_0 at 0.10 level of significance.

Conclusion:

There is no significant difference between ages of males and female employees.

Example 8: Comparing 2 kinds of emergency flares, a consumer testing service obtained the following burning times (rounded to the nearest tenth of a minute):

Brand C	19.4	21.5	15.3	17.4	16.8	16.6	20.3	22.5	21.3	23.4	19.7	21.0
Brand D	16.5	15.8	24.7	10.2	13.5	15.9	15.7	14.0	12.1	17.4	15.6	15.8

Use Mann Whitney U test at the 0.01 level of significance to check whether it is reasonable to say that the population of burning times of the two kinds of flares are identical.

Solution:

Brand C	Rank	Brand D	Rank
19.4	16	16.5	11
21.5	21	15.8	8.5
15.3	5	24.7	24
17.4	14.5	10.2	1
16.8	13	13.5	3
16.6	12	15.9	10
20.3	18	15.7	7
22.5	22	14.0	4
21.3	20	12.1	2
23.4	23	17.4	14.5
19.7	17	15.6	6
21.0	19	15.8	8.5
	R_1=200.5		R_2= 99.5

Here,

Sample size of brand C (n_1) =12

Sample size of brand D (n_2) = 12

Sum of ranks of brand C (R_1) = 200.5

Sum of ranks of brand D (R_2) = 99.5

$$U_1 = n_1n_2 + \frac{n_1(n_1+1)}{2} - R_1 = 12 \times 12 + \frac{12 \times 13}{2} - 200.5 = 21.5$$

$$U_2 = n_1n_2 - U_1 = 12 \times 12 - 21.5 = 122.5$$

$$U_0 = \min\{U_1, U_2\} = \text{Min}\ \{21.5, 200.5\} = 21.5$$

$$\mu_u = \frac{n_1n_2}{2} = \frac{12 \times 12}{2} = 72$$

Here tied occurs between groups, hence

$$\sigma_u = \frac{n_1 n_2}{n(n-1)}\left\{\frac{n^3-n}{12} - \frac{\Sigma t_i^3 - t_i}{12}\right\} = \frac{12\times 12}{24(24-1)}\left\{\frac{24^3-24}{12} - \frac{2^3-2}{12}\right\} = 0.2608 \times 1149.5 = 299.789$$

$\therefore \quad \sigma_u = 17.3$

Problem to test

H_0: Population burning time of two kinds of flares are identical.

H_1: Population burning time of two kinds of flares are not identical.

Test statistic

$$Z = \frac{U_0 - \mu_v}{\sigma_u} = \frac{21.5 - 72}{17.3} = -2.91$$

Critical value

At $\alpha = 0.01$ level of significance critical value is $Z_{tabulated} = Z_{\alpha/2} = 2.58$

Decision

$|Z| = 2.91 > Z_{tabulated} = 2.58$, reject H_0 at 0.01 level of significance.

Conclusion

Burning time of two kind of flares are different.

Example 9: The following are the scores which random samples of students from 2 groups obtained on a random coding test.

Group I	73	82	39	68	91	75	89	67	50	86	57	65	70
Group II	51	42	36	53	88	59	49	66	25	64	18	76	74

Use Mann Whitney U test at the 0.05 level of significance to test whether or not students from the two groups can be expected to score equally well on the test

Solution:

Group I	Rank	Group II	Rank
73	18	51	8
82	22	42	5
39	4	36	3
68	16	53	9
91	26	88	24
75	20	59	11
89	25	49	6
67	15	66	14
50	7	25	2
86	23	64	12
57	10	18	1
65	13	76	21
70	17	74	19
	$R_1 = 216$		$R_2 = 135$

Here, sample size for group I (n_1) = 13

Sample size for group II (n_2) = 13

Sum of ranks of group I (R_1) = 216

Sum of ranks of group II (R_2) = 135

$$U_1 = n_1n_2 + \frac{n_1(n_1+1)}{2} - R_1 = 13 \times 13 + \frac{13 \times 14}{2} - 216 = 44$$

$$U_2 = n_1n_2 - U_1 = 13 \times 13 - 44 = 125$$

$$U_0 = \min\{U_1, U_2\} = \text{Min}\,\{44, 125\} = 44$$

$$\mu_u = \frac{n_1n_2}{2} = \frac{13 \times 13}{2} = 84.5$$

$$\sigma_u = \sqrt{\frac{n_1n_2(n_1+n_2+1)}{12}} = \sqrt{\frac{13 \times 13(13+13+1)}{12}} = \sqrt{380.25} = 19.5$$

Let Md_1 and Md_2 be median scores of group I and group II respectively.

Problem to test

H_0: Students from two groups score equally($Md_1 = Md_2$)

H_1: Students from two groups do not score equally ($Md_1 \neq Md_2$)

Test statistic

$$Z = \frac{U_0 - \mu_u}{\sigma_u} = \frac{44 - 84.5}{19.5} = -2.07$$

Critical value

At $\alpha = 0.05$ level of significance, critical value is $Z_{tabulated} = Z_{\alpha/2} = 1.96$.

Decision

$|Z| = 2.07 > Z_{0.05} = 1.96$, reject H_0 at 0.05 level of significance.

Conclusion

Students from two minority groups do not score equally well

Chi Square Test for Goodness of Fit

It is test used to test the significant difference between observed frequencies (O_i) and expected frequencies (E_i).

Let us consider n observations of random variable x is classified into k classes with their respective frequencies.

Different steps in the test are;

Problem to test

H_0: $O_i = E_i$

H_1 : $O_i \neq E_i$

Test statistic

$$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i} \sim \chi^2(k-1)$$

Level of significance

Let α be the level of significance. Generally fix $\alpha = 0.05$ unless we are given.

Critical value

Critical value $\chi^2_{\alpha\,(k-1)}$ is obtained according to the level of significance α and degree of freedom (k-1).

Decision

Reject H_0 at α level of significance if $\chi^2 > \chi^2_{\alpha(k-1)}$, accept otherwise.

Note

If any cell frequencies are less than 5 then combine the frequencies of adjoining cells till the resulting cell frequency is greater than or equal to 5.

Example 10: The following table gives the number of air crafts accidents that occurred during seven days of the week. Find whether the accidents are uniformly distributed over week.

Days	Sun	Mon	Tue	Wed	Thu	Fri	Sat
No. of accidents	14	16	8	12	11	9	14

Solution:

Here, $n = 14 + 16 + 8 + 12 + 11 + 9 + 14 = 84$, $k = 7$

Days	No. of accidents(O_i)	P_i	$E_i = NP_i$	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
Sun	14	1/7	12	4	0.333
Mon	16	1/7	12	16	1.333
Tue	8	1/7	12	16	1.333
Wed	12	1/7	12	0	0
Thu	11	1/7	12	1	0.0833
Fri	9	1/7	12	9	0.75
Sat	14	1/7	12	4	0.333
Total	$N = \Sigma O_i = 84$				$\sum \frac{(O_i - E_i)^2}{E_i} = 4.166$

Problem to test

H_0: Accidents are uniformly distributed over the week.

H_1: Accidents are not uniformly distributed over the week.

Test statistic

$$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i} = 4.166$$

Critical value

Let $\alpha = 0.05$ be the level of significance then critical value for k-1 = 6 degree of freedom at 0.05 level of significance is $\chi^2_{0.05(6)} = 12.59$

Decision:

$\chi^2 = 4.166 < \chi^2_{0.05(6)} = 12.59$, accept H_0 at 0.05 level of significance.

Conclusion

Accidents are uniformly distributed over the week.

Example 11: In an experiment on pea breeding, Mendal obtained the following frequencies of seeds: 315 round and yellow, 101 wrinkle and yellow, 108 round and green, 32 wrinkle and green in total of 556. Theory predicts that the frequencies should be in ratio 9:3:3:1 respectively. Set up the proper hypothesis and test it at 10% level of significance.

Solution:

Seeds	Frequencies (o_i)	P_i	$E_i = N P_i$	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
Round and yellow	315	9/16	312.75	5.0625	0.01618
Wrinkle and yellow	101	3/16	104.25	10.5625	0.10131
Round and green	108	3/16	104.25	14.0625	0.13489
Wrinkle and green	32	1/16	34.75	2.75	0.07913
Total	$N = \Sigma O_i = 556$				0.3315

Here k = 4

Problem to test

H_0: There is no significant difference between theory and experiment.

H_1: There is significant difference between theory and experiment.

Test statistic

$$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

$$= 0.3315$$

Critical value

At 0.10 level of significance and 3 degree of freedom, critical value is $\chi^2_{0.05(3)} = 4.166$

Decision: $\chi^2 = 0.3315 < \chi^2_{0.05(3)} = 4.166$, accept H_0 at 10% level of significance.

Conclusion: There is no significant difference between theory and experiment.

Example 12: A die is thrown 132 times with the following results:

Number turned up	1	2	3	4	5	6
Frequency	16	20	25	14	29	28

Is the die unbiased?

Solution:

No. turned up	Observed freq.(O_i)	P_i	Expected freq(E_i) $= NP_i$	$(O_i - E_i)^2$	$(O_i - E_i)^2/E_i$
1	16	1/6	22	36	36/22
2	20	1/6	22	4	4/22
3	25	1/6	22	9	9/22
4	14	1/6	22	64	64/22
5	29	1/6	22	49	49/22
6	28	1/6	22	36	36/22
Total	$N = \Sigma O_i = 132$				9

Here k = 6

Problem to test

H_0: Die is unbiased ($O_i = E_i$)

H_1: Die is biased ($O_i \neq E_i$)

Test statistic

$$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i} = 9$$

Critical value

Let $\alpha = 5\%$ be the level of significance, then critical value for 5 degree of freedom at 0.05 level of significance is $\chi^2_{0.05(5)} = 11.07$.

Decision

$\chi^2 = 9 < \chi^2_{0.05(5)} = 11.07$, accept H_0 at 0.05 level of significance.

Conclusion

The die is unbiased.

Example 13: Test whether binomial distribution fits the following data:

x	0	1	2	3	4
f	28	62	46	10	4

Solution:

Problem to test

H_0: Binomial distribution fits the data

H_1: Binomial distribution does not fit the data.

x	f	fx
0	28	0
1	62	62
2	46	92
3	10	30
4	4	16
	$\Sigma f = 150$	$\Sigma fx = 200$

$N = 150$, $n = 4$

$$\bar{x} = \frac{\Sigma fx}{N} = \frac{200}{150} = \frac{4}{3}$$

$$\bar{x} = np$$

or $$\frac{4}{3} = 4p$$

or $$p = \frac{1}{3} \text{ and } q = \frac{2}{3}$$

x	O_i		$P_i = c(n,x)p^x q^{n-x}$	$E_i = Np_i$		$(O_i - E_i)^2$	$(O_i - E_i)^2/E_i$
0	28		$C(4,0)(1/3)^0 (2/3)^4 = 0.1975$	$29.63 \approx 30$		4	0.13
1	62		$C(4,1)(1/3)^1 (2/3)^3 = 0.3951$	$59.26 \approx 59$		9	0.15
2	46		$C(4,2)(1/3)^2 (2/3)^2 = 0.2963$	$44.44 \approx 44$		4	0.09
3	10	14	$C(4,3)(1/3)^3 (2/3)^1 = 0.0988$	$14.81 \approx 15$	17	9	0.529
4	4		$C(4,4)(1/3)^4 (2/3)^0 = 0.0123$	$1.85 \approx 2$			
							$\Sigma(O_i - E_i)^2/E_i = 0.899$

Test statistic

$$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i} = 0.899$$

Critical value

Let 0.05 be the level of significance then critical value for 2 degree of freedom is $\chi^2_{0.05(2)} = 5.911$ (here 2 more df is reduced because of estimation of parameter p and combination of two classes)

Decision

$\chi^2 = 0.899 < \chi^2_{0.05(3)} = 5.911$, accept H_0 at 0.05 level of significance.

Conclusion

Binomial distribution fits the data.

Example 14: The number of cyber attack per month at a certain websites was checked for 50 months and the results shown are as follows:

X (no of cyber attack)	0	1	2
f	22	18	10

Assuming the observations are independent, test the hypothesis that the random variable has a poisson distribution.

Solution:

x	f	fx
0	22	0
1	18	18
2	10	20
	$\Sigma f = 50$	$\Sigma fx = 38$

N = 50

$\bar{X} = \frac{\Sigma fx}{N} = \frac{38}{50} = 0.76 \qquad \lambda = \bar{X} = 0.76$

Problem to test

H_0: The random variable has poisson distribution.

H_1: The random variable has not poisson distribution.

x	O_i	$P_i = e^{-\lambda}\lambda^x/x!$	$E_i = Np_i$	$(O_i-E_i)^2$	$(O_i-E_i)^2/E_i$
0	22	$e^{-0.76}(0.76)^0/0! = 0.4676$	$23.38 \approx 23$	1	0.0434
1	18	$e^{-0.76}(0.76)^1/1! = 0.3554$	$17.77 \approx 18$	0	0
2	10	$e^{-0.76}(0.76)^2/2! = 0.1350$	$6.75 \approx 7$	9	1.285
					$\Sigma(O_i-E_i)^2/E_i = 1.3284$

Test statistic

$$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i} = 1.3284$$

Critical value

Let 0.05 be the level of significance then critical value for 1 degree of freedom is $\chi^2_{0.05(1)} = 3.841$ (Here one more df is reduced due to estimation of parameter λ)

Decision

$\chi^2 = 1.3284 < \chi^2_{0.05(1)} = 3.841$, accept H_0 at 0.05 level of significance.

Conclusion

Random variable has poisson distribution.

Chi Square Test for Independence of Attributes

The characteristics which are capable of being measured qualitatively but not quantitatively are called attributes. This test is used to find any association or not between the attributes.

Let us consider a sample of size n is taken from population of unknown distribution. The observations are classified into attributes say A and B into $A_1, A_2, A_3 \ldots\ldots A_r$ and $B_1, B_2, B_3 \ldots\ldots B_c$ classes respectively. Let O_{ij} be the observed frequency of $(i, j)^{th}$ class.

The arrangement of observed frequencies O_{ij} in r x c contingency table is present as

A_i \ B_j	B_1	B_2	B_j	B_c	Total(O_i)
A_1	O_{11}	O_{12}	O_{1j}	O_{1c}	$O_{1.}$
A_2	O_{21}	O_{22}	O_{2j}	O_{2c}	$O_{2.}$
A_i	O_{i1}	O_{i2}	O_{ij}	O_{ic}	$O_{i.}$
A_r	O_{r1}	O_{r2}	O_{rj}	O_{rc}	$O_{r.}$
Total($O._j$)	$O._1$	$O._2$	$O._j$	$O._c$	N

Different steps in the test are

Problem to test

H_0: Attributes A and B are independent.

H_1: Attributes A and B are dependent.

Test statistic

$$\chi^2 = \sum_{i=1}^{r}\sum_{j=1}^{c} \frac{(O_{ij} - E_{ij})^2}{E_{ij}} \sim \chi^2_{(r-1)(c-1)}$$

$$E_{ij} = (O_{i.} \times O._j)/N$$

Level of significance

Let α be the level of significance. Generally fix $\alpha = 0.05$ unless we are given.

Critical value

At α level of significance and $(r - 1)(c - 1)$ degree of freedom the critical value is $\chi^2_{\alpha\,(r-1)(c-1)}$

Decision

Reject H_0 at α level of significance if $\chi^2 > \chi^2_{\alpha\,(r-1)(c-1)}$

When attributes A and B are arranged into two subgroups then the 2 x 2 contingency table will be

A_i \ B_j	B_1	B_2	Total
A_1	a	b	a+b
A_2	c	d	c+d
Total	a+c	b+d	N

Different steps in the test are

Problem to test

H_0: Attributes A and B are independent.

H_1: Attributes A and B are dependent.

Test statistic

$$\chi^2 = \frac{N(ad - bc)^2}{(a + c)\ (b + d)\ (a + b)\ (c + d)} \sim \chi^2_{(1)}$$

If any cell frequency is less than 5 then the test statistic is

$$\chi^2_{corrected} = \frac{N(|ad - bc| - \frac{N}{2})^2}{(a + c)\ (b + d)\ (a + b)\ (c + d)} \sim \chi^2_{(1)}$$

Level of significance

Let α be the level of significance. Generally fix $\alpha = 0.05$ unless we are given.

Critical value

At α level of significance and 1 degree of freedom the critical value is $\chi^2_{\alpha\,(1)}$

Decision

Reject H_0 at α level of significance if $\chi^2 > \chi^2_{\alpha\,(1)}$

Example 15: In an experiment to study the dependence of hypertension on smoking habits, the following data were taken on 180 individuals:

	No smokers	Moderate smokers	Heavy smokers
Hypertension	21	36	36
No hypertension	48	26	19

Test the hypothesis that presence or absence of hypertension is independent of smoking habit.

Solution:

	No smokers (B_1)	Moderate smokers (B_2)	Heavy smokers (B_3)	$O_{i.}$
Hypertension (A_1)	21	36	36	93
No hypertension(A_2)	48	26	19	93
$O_{.j}$	69	62	55	N=186

Now

Group	O_{ij}	$E_{ij} = (O_{i.} \times O_{.j})/N$	$(O_{ij} - E_{ij})$	$(O_{ij} - E_{ij})^2/E_{ij}$
A_1B_1	21	$(93 \times 69)/186 = 34.5$	-13.5	5.2826
A_1B_2	36	$(93 \times 62)/186 = 31$	5	0.8064
A_1B_3	36	$(93 \times 55)/186 = 27.5$	8.5	2.6272
A_2B_1	48	$(93 \times 69)/186 = 34.5$	13.5	5.2826
A_2B_2	26	$(93 \times 62)/186 = 31$	-5	0.8064
A_2B_3	19	$(93 \times 55)/186 = 27.5$	-8.5	2.6272
Total				17.4324

Problem to test

H_0: Hypertension is independent of smoking habit.

H_1: Hypertension is dependent of smoking habit.

Test statistic

$$\chi^2 = \Sigma\Sigma \frac{(O_{ij} - E_{ij})^2}{E_{ij}} = 17.4324$$

Critical value

Let $\alpha = 0.05$ be the level of significance, then critical value for (2-1)(3-1) =2 degree of freedom at 0.05 level of significance is $\chi^2_{\alpha\,(2)} = 5.99$

Decision

$\chi^2 = 17.4324 > \chi^2_{\alpha\,(2)} = 5.99$, reject H_0 at 5% level of significance.

Conclusion

Presence or absence of hypertension depends upon the smoking habit.

Example 16: A tobacco company claims that there is no relationship between smoking and lung ailments. To investigate the claims random sample of 300 males in age group of 40 to 50 is given medical test. The observed sample result are tabulated below:

	Lung ailment	No lung ailment	Total
Smokers	75	105	180
No smokers	25	95	120
Total	100	200	300

On the basis of this information, can it be concluded that smoking and lung ailment are independent?

Solution:

	Lung ailment(B)	No lung ailment(β)	Total
Smokers (A)	75 =a	105 =b	180=a+b
No smokers(α)	25 =c	95=d	120=c+d
Total	100 =a+c	200= b+d	300=N

Problem to test

H_0: Smoking and lung ailment are independent

H_1: Smoking and lung ailment are dependent

Test statistic

$$\chi^2 = \frac{N(ad - bc)^2}{(a + c)\,(b + d)\,(a + b)\,(c + d)}$$

$$= \frac{300(75 \times 95 - 105 \times 25)^2}{100 \times 200 \times 180 \times 120} = 14.063$$

Critical value

Let $\alpha = 0.05$ be the level of significance, then critical value for 1 degree of freedom at 0.05 level of significance is $\chi^2_{0.05(1)} = 3.84$.

Decision

$\chi^2 = 14.063 > \chi^2_{0.05(1)} = 3.84$, reject H_0 at 0.05 level of significance.

Conclusion

Smoking and lung ailment are dependent.

Example 17: In an experiment on immunization of cattle from tuberculosis, the following results were obtained.

	Affected	Not affected
Inoculated	2	10
Not inoculated	6	6

Examine whether the vaccine has an effect in controlling the disease.

Solution:

	Affected	Not affected	Total
Inoculated	2 =a	10 =b	12 =a+b
Not inoculated	6 =c	6 =d	12 =c+d
Total	8= a+c	16= b+d	N=24

Problem to test

H_0: The vaccine has no effect in controlling the disease.

H_1: The vaccine has effect on controlling the disease.

Test statistic

$$\chi^2_{\text{corrected}} = \frac{N(|ad - bc| - \frac{N}{2})^2}{(a + c)\,(b + d)\,(a + b)\,(c + d)}$$

$$= \frac{24(|12 - 60| - 12)^2}{8 \times 16 \times 12 \times 12} = 1.687$$

Critical value

Let $\alpha = 0.05$ be the level of significance, then for 1 degree of freedom at 0.05 level of significance critical value is $\chi^2_{0.05(1)} = 3.84$

Decision

$\chi^2 = 1.687 < \chi^2_{0.05(1)} = 3.84$, accept H_0 at 0.05 level of significance.

Conclusion

The vaccine has no effect in controlling the disease.

Example 18: The table given below shows the data obtained during outbreak of smallpox:

	Attacked	Not attacked
Vaccinated	31	469
Not vaccinated	185	1315

Test the effectiveness of vaccination in preventing the attack from smallpox.

Solution:

Here,

	Attacked(B)	Not attacked (β)	Total ($O_{i.}$)
Vaccinated (A)	31	469	500
Not vaccinated (α)	185	1315	1500
Total ($O_{.j}$)	216	1784	2000

Now,

Group	O_{ij}	$E_{ij} = (O_{i.} \times O_{.j})/N$	$(O_{ij} - E_{ij})$	$(O_{ij} - E_{ij})^2/E_{ij}$
AB	31	54	-23	9.796
Aβ	469	446	23	1.186
αB	185	162	23	3.265
αβ	1315	1338	-23	0.395
Total				14.642

Problem to test

H_0: Vaccination is not effective in preventing the attack from smallpox

H_1: Vaccination is effective in preventing the attack from smallpox.

Test statistic

$$\chi^2 = \Sigma\Sigma\frac{(O_{ij} - E_{ij})^2}{E_{ij}} = 14.642$$

Critical value

Let α = 0.05 be the level of significance, then for (r-1)(c-1) =1 degree of freedom at 0.05 level of significance, the critical value is $\chi^2_{0.05(1)} = 3.84$

Decision

$\chi^2 = 14.642 > \chi^2_{0.05(1)} = 3.84$, reject H_0 at 0.05 level of significance.

Conclusion

Vaccination is effective in preventing the attack from smallpox.

EXERCISE

1. What do you mean by median test? Describe the procedure of median test.
2. Describe Kolmogorov Smirnov two sample test.
3. Describe the function and procedure of Mann Whitney U test.
4. Describe function and procedure of chi square test for goodness of fit.
5. Discuss the rationale and method of chi square test for independence of attributes.
6. Differentiate between Chi square test for goodness of fit and Kolmogorov Smirnov one sample test.

7. The following are the performance score of 10 person under two software training X and Y.

X	46	45	32	42	39	48	49	30	51	34
Y	44	40	59	47	55	50	47	71	43	55

Use median test to test the effectiveness of two training. **Ans:** (0.328, insig.)

8. The length of life in kilowatt hours of some type of electronic Neon tube and Helium tube made by two manufacturers were as follows.

Tube	Length of life															
Neon	96	238	24	200	7	108	76	140	39	165	61	25	41	99		
Helium	11	125	47	20	34	101	25	68	17	59	178	30	83	75	45	28

Compare by using Median test at 5% level of significance, the median lives of the electronic tubes made by manufacturers Neon and Helium. **Ans:** (2.14, insig.)

9. The same C programming papers were marked by two teachers A and B. The final marks were recorded as follows:

Teacher A	73	89	82	43	80	73	66	45	93	36	77	60
Teacher B	88	78	91	48	85	74	77	31	78	62	76	77

Using median test at 5% level of significance to determine if the marks distributions of two teachers differ significantly. **Ans:** (0.66, insig.)

10. A quality controller wishes to determine whether there is a difference in outcome between two different tools of software I and II. The following data shows the outcome of two different tools. Can the controller conclude that a difference exists? Use median test at 5% level of significance.

Software I	24.0	16.7	22.8	19.8	18.9	
Software II	23.2	19.8	18.1	17.6	20.2	17.8

Ans: (p=0.6, insig.)

11. Life in years of two types of electric motor used in irrigation of farms are given below;

Motor M	3	6	7	4	3	5	5	3	4
Motor N	4	3	5	6	3	2	4	7	6

Is there any significant difference between two types of motor? Use Kolmogorov Smirnov test at 0.05 level of significance. **Ans:** $D_0 = 0.15$ insig.

12. Amount of time required to design website by software developers A and B are found as follows;

Time (hrs)	0 – 4	4 – 8	8 – 12	12 – 16	16- 20
Number of website designed by A	2	7	12	5	4
Number of web sires designed by B	6	9	8	4	3

Does A take more time than B to design website? Use Kolmogorov Smirnov test at 5% level of significance. **Ans:** $D_0 = 0.2$ insig.

13. Two independent samples of 26 junior programming and 25senior programming smokers selected from a software company used to smoke following number of cigarettes per day

Number of cigarettes	0 - 2	2 – 4	4 – 6	6 – 8	8 – 10	10 – 12	12 – 14
Number of junior programmer	7	6	4	2	2	3	2
Number of senior programmer	5	4	6	3	4	2	1

Using Kolmogorov Smirnov test identify if there is any significance between junior and senior programmer? Use 0.05 level of significance. **Ans:** $D_0 = 0.14$, insig.

14. Two groups of date managers, one group consisting of trained ones, another groups not trained have the following number of correction required.

Trained	78	64	75	45	82
Untrained	110	70	53	51	

Use Mann Whitney U test to test if there is a difference between the two average number of correction of trained and untrained data manger. **Ans:** Uo = 9, insig.

15. The nicotine contents of two brands of cigarettes, measured in milligrams was found to be as follows:

Brand A	2.1	4.0	6.3	5.4	4.8	3.7	6.1	3.3		
Brand B	4.1	0.6	3.1	2.5	4.0	6.2	1.6	2.2	1.9	5.4

If there any significance difference between two brands of cigarettes. Use Mann-whitney U-test. **Ans:** p=0.0729, insig.

16. A farmer wishes to determine whether there is a difference in yields between two different varieties of wheat I and II. The following data shows the production of wheat per unit area using the two varieties. Can the farmer conclude at significance level 0.01 that a difference exists?

Wheat I	15.9	15.3	16.4	14.9	15.3	16.0	14.6	15.3	14.5	16.6	16.0
Wheat II	16.4	16.8	17.1	16.9	18.0	15.6	18.1	17.2	15.4		

Use Mann Whitney U test. **Ans:** z=-2.89, sig.

17. Two independent random samples of unemployed men and women are drawn and the ages of 4 unemployed women and 5 unemployed men are recorded as follows:

Women	60	63	36	44	
Men	53	39	22	23	24

Do the data present sufficient evidence to conclude that there is a difference in the average age of unemployed men and women? Use Mann Whitney U test at $\alpha = 0.05$. (p = 0.055, insig.)

18. The following are the number of minutes it took a sample of 13 men and 12 women to complete the application form for a position:

Men	16.5	20.0	17.0	19.8	18.5	19.2	19.0	18.2	20.8	18.7	16.7	18.1	17.9
Women	18.6	17.8	18.3	16.6	20.5	16.3	19.3	18.4	19.7	18.8	19.9	17.6	

Use Mann Whitney U test at the 0.05 level of significance to test the hypothesis that the two samples come from identical population against the alternative that two populations are not identical. **Ans:** $z = -0.108$, insig.

19. A die was rolled for 60 times and observed the following outcomes:

Side	1	2	3	4	5	6	Total
Number of times observed	8	9	13	7	15	8	60

Is the die fair? Test the hypothesis at 5% level of significance. **Ans:** (5.2, insig.)

20. A certain chemical plant processes sea water to collect sodium chloride and magnesium. From scientific analysis, sea water is known to contain sodium chloride, magnesium and other elements in the ratio of 62:4:34. A sample of 200 tons of sea water resulted in 130 tons of sodium chloride and 6 tons of magnesium. Are these data consistent with the scientific model at 5% level of significance? **Ans:** (1.025, insig)

21. A group of 150 college students were asked to indicate their most liked mobile brand among six different well known film actors viz. , A, B, C, D, E and F in order to ascertain their relative popularity. The observed frequency data were as follows:

Brrand	A	B	C	D	E	F	Total
Frequencies	24	20	32	25	28	21	150

Test at 5% level of significance whether all brands are equally popular. **Ans:** (4, insig)

22. Genetic theory states that children having one parent of blood type A and other of blood type B will always be one of three types; A, AB, B and the proportion of three types will on an average be as 1:2:1. A report states that out of 300 children having one A parent and other B parent, 30% were found to be type A, 45% type AB and remainder type B. Test the hypothesis at 5% level of significance. **Ans:** (4.5, insig.)

23. A publishing house got a 500 page book composed for printing. Before final printing the first draft was sent for proof reading. The proof reader detected the number of misprints X on each pages tabulated as follows. Test whether poisson distribution fits the data.

No. of misprint page	0	1	2	3	4	5
No. of pages	221	167	70	30	7	5

Ans: (13.32, sig.)

24. In 50 random sample of a manufactured mice, the number of samples containing defective mice is noted below:

No. of defective mice	0	1	2	3	4	5
Frequencies	4	13	17	12	3	1

Can the binomial distribution be a good with $p = 0.30$? **Ans:** (11.11, sig.)

25. The number of telephone calls received per day over a period of 100 days is shown in the table below.

Number of calls	0	1	2	3
Number of days	45	25	20	10

Carry out chi square test to test the hypothesis that the number of calls received per day has a Poisson distribution at 5% level of significance. **Ans:** (8.01, sig)

26. The distribution of persons according to sex and blood groups are given below:

	Blood group			
Sex	O	A	B	AB
Male	100	40	45	10
Female	110	35	55	5

Is there any association between sex and blood group? **Ans:** (3.607, insig.)

27. A random sample of 200 married men, all retired was classified according to education and number of children.

Education	Number of children		
	0 – 1	2 - 3	Over 3
Elementary	14	37	32
Secondary	19	42	17
College	12	17	10

Test the hypothesis at the 0.05 level of significance, that the number of children is independent of the level of education attained by the father. **Ans:** (8.04, insig.)

28. Test whether the color of son's eyes is associated with that of the father's at 5% level of significance using the data available in the following table.

Father's eye color	Son's eye color	
	Not light	Light
Not light	230	148
Light	151	471

Ans: (133.32 sig.)

29. 88 workers of a IT company were interviewed during a sample survey for their smoking habit. Classification of respondents according to their gender and their smoking habit are found as

	Sex	
Habit	Male	Female
Smoker	40	33
Non-smoker	3	12

Do the smoking tea habit is associated with gender. **Ans:** 4.71, sig.

30. Out of a sample of 120 persons in a village, 76 were administered a new drug for preventing influenza and out of them 24 persons were attacked by influenza. Out of those who were not administered the new drug, 12 persons were not affected by influenza. Is the new drug effective in controlling influenza, test at 5% level of significance. **Ans:** 19.54, sig.

31. The following table gives the result of flower color and type of leaf.

Flower color	Type of leaf	
	Flat	Curled
Pink	3	22
Red	9	11

Test whether the flower color is independent of flatness of leaf or not.

Ans: 4.61, sig.

Paired Sample Test

Wilcoxon Matched Pair Signed Rank Test

It is non parametric test used to compare two populations for which observations are paired. Itis based upon magnitude and direction of difference between observations within each pair of related random samples.

Let us consider two related samples of size n drawn from continuous populations with unknown medians Md_1 and Md_2 respectively.

Let $x_1, x_2, x_3, \ldots\ldots x_n$ and $y_1, y_2, y_3, \ldots\ldots y_n$ be two related samples of size n

Different steps in the test are

Problem to test

H_0: $Md_1 = Md_2$

H1: $Md_1 \neq Md_2$ (two tailed) or H_1:$Md_1 > Md_2$ (one tailed right) or H_1: $Md_1 < Md_2$ (one tailed left)

Find $d_i = y_i - x_i$ or $x_i - y_i$ for each pair of observations(x_i, y_i) , $i = 1,2,3\ldots\ldots n$. Rank di irrespective of sign in ascending order but omit $d_i = 0$. If two or more d_i are equal then assign the average rank and is called tied. In such case corrected sample size $n_c = n - t$, t is number of tied occurred .Assign sign to the ranks with respect to the sign of d_i. Sum the ranks of + sign and - sign separately to get S(+) and S(-) respectively. Finally get T = min {S(+) , S(-)}.

Small sample size ($n \leq 25$)

Test statistic

T = min {S(+) , S(-)}.

Level of significance

Let α be the level of significance. Generally fix $\alpha = 0.05$ unless we are given.

Critical value

At α level of significance, we obtain critical value from Wilcoxon Matched pair signed rank test table

$T_{\alpha, n}$ where n is corrected sample size after omitting $d_i = 0$.

Decision

Reject H_0 at α level of significance if $T \leq T_{\alpha, n}$ accept otherwise.

Large sample size (n> 25)

For large sample size sampling distribution of T is approximately normally distributed with mean μ_T and variance σ_T^2.

$\mu_T = \dfrac{n(n+1)}{4}$ and $\sigma_T^2 = \dfrac{n(n+1)(2n+1)}{24}$ where n is corrected sample size if $d_i = 0$.

Test statistic

$$Z = \frac{T - \mu_T}{\sigma_T} = \frac{T - \frac{n(n+1)}{4}}{\sqrt{\frac{n(n+1)(2n+1)}{24}}} \sim N(0, 1) \qquad \text{(Here } n = n_c\text{)}$$

Level of Significance

Let α be the level of significance. Generally fix α= 0.05 unless we are given.

Critical Value

Critical value $Z_{tabulated}$ is obtained from table according to level of significance and alternative hypothesis.

Decision

Reject H_0 at α level of significance if $Z > Z_{tabulated}$, Accept otherwise.

Example 1: The number of server manufactured from each of two companies A and B was recorded daily for a period of 10 days with the following results;

Days	1	2	3	4	5	6	7	8	9	10
A	172	165	206	184	174	142	190	169	161	200
B	201	179	159	192	177	170	182	179	169	210

Assuming both companies produced same daily output, test the hypothesis that there is no difference between median defectives for two companies against the alternative hypothesis that company B produced more defective than company A using Wilkcoxon Matched pairs signed rank test.

Solution:

Days	Company A(x_i)	Company B(y_i)	$d_i = y_i - x_i$	R_i
1	172	201	29	9
2	165	179	14	7
3	206	159	-47	-10
4	184	192	8	3
5	174	177	3	1
6	142	170	28	8
7	190	182	-8	-3
8	169	179	10	5.5
9	161	169	8	3
10	200	210	10	5.5

Here, n = 10,Sum of ranks of + sign [S(+)] = 9+7+3+1+8+5.5+3+5.5 = 42 , Sum of ranks of – sign [S(-)]= 10 + 3 = 13

Let Md_A and Md_B be median defective of company A and B respectively.

Problem to test

H_0: There is no significant difference in median defect of company A and company B($Md_A = Md_B$)

H_1: There is significant increase in median defective in company B than company A ($Md_A < Md_B$)

Test statistic

$T = \min\{S(+), S(-)\}$

$= \min\{42, 13\} = 13$

Critical value

Let $\alpha = 5\%$ be the level of significance the critical value is $T_{0.05(10)} = 8$

Decision

$T = 13 > T_{0.05(10)} = 11$

Accept H_0 at 5% level of significance.

Conclusion

There is no difference between median defectives for two companies A and B.

Example 2: The scores under two conditions X and Y obtained by the respondents are given below:

X	12	16	8	6	4	8
Y	7	12	17	5	12	11

Use Wilcoxon Matched pair signed rank test to test the difference between scores under condition X and Y.

Solution:

X	Y	d = Y - X	R
12	7	-5	-4
16	12	-4	-3
8	17	9	6
6	5	-1	-1
4	12	8	5
8	11	3	2

Here, $n = 6$, Sum of ranks of + sign $[S(+)] = 6+5+2 = 13$, Sum of ranks of – sign $[S(-)] = 4+3+1 = 8$

Problem to test

H_0 : Distribution of X = Distribution of Y

H_1: Distribution of X $\neq$ Distribution of Y

Test statistic

$T = \min\{S(+), S(-)\} = \min\{13, 8\} = 8$

Critical value

Let $\alpha = 5\%$ be the level of significance then critical value is $T_{0.05\,(6)} = 0$

Decision

$T = 8 > T_{0.05\,(6)} = 1$

Accept H_0 at 5% level of significance.

Conclusion

There is no difference between scores under condition X and Y.

Example 3: The following data gives the additional hours of sleep gained by 31 patients in an experiment to test the effect of a drug. Do these data give evidence that the drug produces additional hours of sleep?

Sleep hours gained by patients are; 0.5 , 0.7 , 0.1 , -0.2 , 1.2 , 1.5 , -2 , 4 , 0.1 , 3.4 , 3.7, -1.1, 0.8 , - 0.8 , 1.3 , 2.7 , -3.4 , -1.9 , 3.4 ,0.1 , 0.6 , 2.3 , 0.1 , 2.7 , -0.9 , 3.1 , 2.0 , 1.2 , 1.2, -1.8 , 1.0

Solution:

Additional hours sleep (d_i)	Rank
0.5	6
0.7	8
0.1	2.5
-0.2	-5
1.2	15
1.5	18
-2	-21.5
4	31
0.1	2.5
3.4	28
3.7	30
-1.1	-13
0.8	9.5
-0.8	-9.5
1.3	17
2.7	24.5
-3.4	-28
-1.9	-20
3.4	28
0.1	2.5
0.6	7
2.3	23
0.1	2.5
2.7	24.5
-0.9	-11
3.1	26
2.0	21.5
1.2	15
1.2	15
-1.8	-19
1.0	12

Here, $n = 31$

Sum of - sign ranks $[S(-)] = 5+21.5+13+9.5+28+20+11+19 = 127$

Sum of + sign ranks $[S(+)] = S_n - S(-) = \frac{n(n+1)}{2} - S(-) = \frac{31(1+31)}{2} - 127 = 369$

$T = \min\{S(+), S(-)\} = \min\{369, 127\} = 127$

Problem to test

H_0 : drug does not produce additional hours of sleep

H_1: drug produces additional hours of sleep

Test statistic

$T = 127$

$$\mu_T = \frac{n(n+1)}{4} = \frac{31(31+1)}{4} = 248$$

$$\sigma_T^2 = \frac{n(n+1)(2n+1)}{24} = \frac{31(31+1)(62+1)}{24} = 2604$$

$$\Rightarrow \sigma_T = 51.029$$

Hence, $Z = \frac{T - \mu_T}{\sigma_T} = \frac{127 - 248}{51.029} = -2.37$

Critical value

Let $\alpha = 5\%$ be the level of significance then critical value is $Z_{0.05} = 1.96$

Decision

$|Z| = 2.37 > Z_{0.05} = 1.96$, reject H_0 at 5% level of significance.

Conclusion

Drug produces additional hours of sleep to the patients.

Cochran Q Test

It is non parametric test used for more than two related samples. It is used to test significant difference in frequencies or proportions of three or more related samples.

Let us consider k treatments (k>2) applied to the set of n objects dichotomized as Yes(Y) or No(N), Presence(P) or Absence(A), Pass(P) or Fail(F), Accept(A) or Reject (R), Increase (I) or Decrease (D) etc.

Treatments	Objects				
	1	2	3	J	N
T_1 T_2 T_3	Y N Y	N Y N	Y N Y	Y Y N	N Y N
. . .					
T_i	N	Y	N	Y	Y
T_k	Y	N	Y	N	Y

Different steps in the test are

Problem to test

H_0: All the treatments are equally effective

H_1: All the treatments are not equally effective.

Sum all the Y(Positive) according to treatment to get R_i (Row wise) and according to objects to get C_i (Column wise) , i = 1 , 2 , 3 , ... k and j = 1, 2, 3, ... n. Then get

$\sum_{i=1}^{k} R_i, \sum_{i=1}^{k} R_i^2, \sum_{j=1}^{n} C_j, \sum_{j=1}^{n} C_j^2$

Test statistic

$$Q = \frac{(k-1)\,[K\Sigma R_i^2 - (\Sigma Ri)^2]}{K\Sigma C_j - \Sigma C_j^2} \sim \chi^2$$

Level of significance

Let α be the level of significance .Generally fix $\alpha = 0.05$ unless we are given.

Critical value

$\chi^2_{\alpha\,(k-1)}$ is the critical value obtained according to level of significance and degree of freedom.

Decision

Reject H_0 at α% level of significance if $Q > \chi^2_{\alpha(k-1)}$, accept otherwise.

Example 4: Five laptop users were asked for the acceptability of four brands for daily use. The response of acceptability (A) and rejection (R) are given below;

Users	Brands			
	Alfa	Beta	Gamma	Delta
H_1	A	R	A	R
H_2	R	A	A	R
H_3	R	A	R	A
H_4	A	R	R	R
H_5	A	A	R	A

Test whether there is any significant difference between brands with respect to acceptability

Solution:

Lipstick Brands	Users					R_i	R_i^2
	H_1	H_2	H_3	H_4	H_5		
Alfa	A	R	R	A	A	3	9
Beta	R	A	A	R	A	3	9
Gamma	A	A	R	R	R	2	4
Delta	R	R	A	R	A	2	4
C_j	2	2	2	1	3	$\Sigma R_i = \Sigma C_j = 10$	$\Sigma R_i^2 = 26$
C_j^2	4	4	4	1	9	$\Sigma C_j^2 = 22$	

Here,

Number of brands (k) = 4 , Number of housewives (n) = 5

Problem to test

H_0 : There is no significant difference between brands

H_1: There is at least one significant difference between brands.

Test statistic

$$Q = \frac{(k-1)\left\{k\sum_{i=1}^{k} R_i^2 - \left(\sum_{i=1}^{k} R_i\right)^2\right\}}{k\sum_{j=1}^{n} C_j - \sum_{j=1}^{n} C_j^2} = \frac{(4-1)\{4\times 26-(10)^2\}}{4\times 10-22} = \frac{12}{18} = 0.666$$

Critical value

Let 5% be the level of significance then critical value is $\chi^2_{0.05\,(3)} = 7.81$.

Decision

$Q = 0.666 < \chi^2_{0.05\,(3)} = 7.81$, accept H_0 at 5% level of significance.

Conclusion:

There is no significant difference between brands according to acceptability.

Kruskal Wallis H Test

It is also called Kruskal Wallis one way ANOVA test

It is test used to test the significant difference between location of three or more independent populations.

Let us consider k independent samples of size n_i such that $\Sigma n_i = n$ drawn from continuous population with unknown medians $Md_1, Md_2 \ldots\ldots\ldots Md_k$ respectively.

Let $x_{11}, x_{12}, x_{13}, \ldots\ldots x_1n_1$ be samples of size n_1. $x_{21}, x_{22}, x_{23}, \ldots\ldots\ldots x_2n_2$ be sample of size n_2. $x_{31}, x_{32}, x_{33}, \ldots\ldots\ldots x_3n_3$ be sample of size n_3 ……… $xk_1, xk_2, xk_3, \ldots\ldots\ldots x_kn_k$ be sample of size n_k respectively.

Samples	Blocks				
	1	2	3	j	n
1	x_{11}	x_{12}	x_{13}	$x1j$	x_1n_1
2	x_{21}	x_{22}	x_{23}	x_{2j}	x_2n_2
3	x_{31}	x_{32}	x_{33}	x_{3j}	x_3n_3
.					
.					
i	x_{i1}	x_{i2}	x_{i3}	x_{ij} x_in_i	
.					
.					
k	x_{k1}	x_{k2}	$x_{k3}x_{kj}$	x_kn_k	

Different steps in the test are

Problem to test

H_0: $Md_1 = Md_2 = Md_3 = \ldots\ldots\ldots\ldots\ldots Md_k$

H_1: At least one Md_i is different $i = 1, 2, 3, \ldots\ldots\ldots k$.

Combine $n_1, n_2, n_3 \ldots\ldots\ldots$ and n_k such that $n_1 + n_2 + n_3 + \ldots\ldots\ldots + nk = n$ and rank these n observations in ascending order .If two or more observations are equal then assign average rank and is called tied. Sum the ranks of sample of sizes $n_1, n_2, n_3, \ldots\ldots\ldots$ and n_k separately to get R_1, $R_2, R_3, \ldots\ldots\ldots R_k$.

Test statistic

$$H = \frac{12}{n(n+1)} \sum \frac{R_i^2}{n_i} - 3(n+1) \sim \chi^2(k-1)$$

If tied occurs then corrected test statistic is

$$H = \frac{\frac{12}{n(n+1)} \sum \frac{R_i^2}{n_i} - 3(n+1)}{1 - \sum \frac{(t_i^3 - t_i)}{n^3 - n}}$$, t_i = number of times i^{th} rank is repeated.

Level of significance

Let α be the level of significance .Generally fix $\alpha = 0.05$ unless we are given.

Critical value

For $ni \leq 5$ and $k = 3$, critical value p is obtained from Kruskal Wallis table.

For $n_i > 5$ and $k > 3$, critical value is $\chi^2_{\alpha\,(k-1)}$.

Decision

Accept H_0 at α level of significance if $p > \alpha$, reject otherwise for $n_i \leq 5$ and $k=3$.

Reject H0 at α level of significance if $H > \chi^2\alpha\,(k-1)$, accept otherwise for $n_i > 5$ and $k > 3$.

Example 5: A bacteriologist was interested to study the number of plankton organism inhabiting the lake water. He made hauls of water from three lakes each and the following results were obtained.

Lake	Number of plankton organism				
Phewa	12	19	16		
Rara	4	8	3	2	3
Taudaha	14	12	20	12	

Do the data provide substantial evidence to conclude significant variation between lake water? Use Kruskal Wallis test at 0.05 level of significance.

Solution:

Lake	Number of plankton organism					R_i	R_i^2/n_i
Phewa	12	19	16				
Rank	7	11	10			28	261.33
Rara	4	8	3	2	3		
Rank	4	5	2.5	1	2.5	15	45
Taudaha	14	12	20	12			
Rank	9	7	12	7		35	306.25
							612.58

Sample size of Phewa (n_1) = 3

Sample size of Rara (n_2) = 5

Sample size of Taudha (n_3) = 4

Total sample size (n) = $n_1 + n_2 + n_3 = 3 + 5 + 4 = 12$

No of times rank 2.5 is repeated (t_1) = 2

No of times rank 7 is repeated (t_2) =3

$\Sigma (t_i^3 - t_i) = (2^3-2)+(3^3-3) = 6+24 = 30$

R_1, R_2, R_3 be sum of ranks for Phewa , Rara and Taudaha respectively.

Let Md_1, Md_2 and Md_3 be median of Phewa, Rara and Taudaha lake respectively.

Problem to test

H_0: There is no significant variation between lake water.($Md_1 = Md_2 = Md_3$)

H_1: There is at least one significant variation between lake water. (At least one Mdi is different, i = 1 , 2 , 3)

Test statistic

$$H = \frac{\frac{12}{n(n+1)} \Sigma \frac{R_i^2}{n_i} - 3(n+1)}{1 - \Sigma \frac{(t_i^3 - t_i)}{n^3 - n}} = \frac{\frac{12}{12\,(12+1)} \times 612.5 - 3(12+1)}{1 - \frac{30}{12^3 - 12}} = \frac{8.12}{0.98} = 8.285$$

Level of significance

$\alpha = 0.05$ is the level of significance.

Critical value

From Kruskal Wallis table critical value is p = 0.01

Decision

$P = 0.01 < \alpha = 0.05$, reject H_0 at 0.05 level of significance.

Conclusion

There is at least one significant variation between lake water.

Example 6: The following are the numbers of misprints counted on pages selected at random from three editions of a book

Edition I	4	10	2	6	4	12
Edition II	8	5	13	8	8	10
Edition III	7	9	11	2	14	7

Use Kruskal Wallis H test at the 0.05 level of significance to test the null hypothesis that the samples come from identical populations.

Solution:

Date	No. of misprints						R_i	R_i^2/n_i
April 11	4	10	2	6	4	12		
Rank	3.5	13.5	1.5	6	3.5	16	44	322.66
April 18	8	5	13	8	8	10		
Rank	10	5	17	10	10	13.5	65.5	715.041
April 25	7	9.	11	2	14	7		
Rank	7.5	12	15	1.5	18	7.5	61.5	630.375
Total								1668.076

Sample size of April 11 (n_1) = 6

Sample size of April 18 (n_2) = 6

Sample size of April 25 (n_3) = 6

Total sample size (n) $n = n_1 + n_2 + n_3 = 6 + 6 + 6 = 18$

No of times rank 1.5 is repeated (t_1) = 2 , No of times rank 3.5 is repeated (t_2) = 2

No of times rank 7.5 is repeated (t_3) = 2 , No of times rank 10 is repeated (t_4) = 3

No of times rank 13.5 is repeated (t_5) = 2

$\Sigma(t_i^3 - t_i) = (2^3 - 2) + (2^3 - 2) + (2^3 - 2) + (3^3 - 3) + (2^3 - 2) = 48$

R_1, R_2, R_3 be sum of ranks for April 11, April 18 and April 25 respectively.

Let Md_1, Md_2 and Md_3 be median misprint of April 11, April 18 and April 25 respectively.

Problem to test

H_0: Populations are identical. ($Md_1 = Md_2 = Md_3$)

H_1: Populations are not identical.(At least one Md_i is different, i = 1 , 2 , 3)

Test statistic

$$H = \frac{\frac{12}{n(n+1)}\Sigma\frac{R_i^2}{n_i} - 3(n+1)}{1 - \Sigma\frac{(t_i^3 - t_i)}{n^3 - n}}$$

$$= \frac{\frac{12}{18\,(18+1)} \times 1668.076 - 3(18+1)}{1 - \frac{48}{18^3 - 18}}$$

$$= \frac{58.528 - 57}{0.9917} = 1.5407$$

Critical value

Critical value at 0.05 level of significance for 2 degree of freedom is $\chi^2_{0.05\,(2)} = 5.99$.

Decision

$H = 1.54 < \chi^2_{0.05\,(2)} = 5.99$, accept H_0 at 0.05 level of significance.

Conclusion

The samples come from identical population.

Friedman F Test

It is also called Friedman two way ANOVA test.

It is test used to test the significant difference between location of three or more independent populations.

Let us consider k independent samples of size n each.

Let x_{ij} be the observations classified into k rows (treatments) and n columns(blocks) such that there are $N = n \times k$ observations.

Samples	Blocks				
	1	2	3	j	n
1	x_{11}	x_{12}	x_{13}	x_{1j}	x_{1n}
2	x_{21}	x_{22}	x_{23}	x_{2j}	x_{2n}
3	x_{31}	x_{32}	x_{33}	x_{3j}	x_{3n}
.					
.					
i	x_{i1}	x_{i2}	$x_{i3} x_{ij}\ x_{in}$		
.					
.					
k	x_{k1}	x_{k2}	$x_{k3} x_{kj}\ x_{kn}$		

Different steps in the test are

Problem to test

H_0: $Md_1 = Md_2 = Md_3 = \ldots\ldots\ldots\ldots\ Md_k$

H_1: At least one Md_i is different $i = 1, 2, 3, \ldots\ldots\ldots\ k$.

Rank k sample observations for each block separately from 1 to k. in ascending order. If two or more observations are same then assign average rank which is also called tied. Obtain sum of ranks for each sample to get R_i, $i = 1, 2, 3, \ldots\ldots\ldots k$

Test statistic

$$F_r = \frac{12}{nk(k+1)} \sum_{i=1}^{k} R_i^2 - 3n(k+1)$$

If tied occurs then corrected test statistic is

$$F_r = \frac{\frac{12}{nk(k+1)} \sum_{i=1}^{k} R_j^2 - 3n(k+1)}{1 - \Sigma \frac{(t_i^3 - t_i)}{n(k^3 - k)}}$$

t_i = number of times ith rank is repeated.

Level of significance

Let α be the level of significance .Generally fix $\alpha = 0.05$ unless we are given.

Critical value

For $2 \le n \le 9$ and $k = 3$, also $2 \le n \le 4$ and $k = 4$ critical value p is obtained from Friedman probability table.

For $n \ge 5$ and $k > 3$, critical value is $\chi^2_{\alpha\,(k-1)}$

Decision

Accept H_0 at α level of significance if $p > \alpha$, reject otherwise for $2 \le n \le 9$ and $k = 3$, also $2 \le n \le 4$ and $k = 4$.

Reject H_0 at α level of significance if $H > \chi^2\alpha\,(k-1)$, accept otherwise for $n \ge 5$ and $k > 3$.

Example 7: A survey was conducted in four hospitals in a particular city to obtain the number of babies born over a 12 months period. This time period was divided into four seasons to test the hypothesis that the birth rate is constant over all the four seasons. The results of the survey were as follows:

Hospital	No of births			
	Winter	Spring	Summer	Fall
A	92	72	94	77
B	15	16	10	17
C	58	71	51	62
D	19	26	20	18

Analyze the data using Friedman two way ANOVA test.

Solution:

Hospital	No of births in Hospital								R_i	R_i^2
	A	Rank	B	Rank	C	Rank	D	Rank		
Winter	92	3	15	2	58	2	19	2	9	81
Spring	72	1	16	3	71	4	26	4	12	144
Summer	94	4	10	1	51	1	20	3	9	81
Fall	77	2	17	4	62	3	18	1	10	100
$\Sigma R_i^2 = 406$										

Here,

Number of hospitals (n) = 4

Number of seasons (k) = 4

Let Md_1, Md_2, Md_3, Md_4 be median of winter, spring, summer and fall seasons respectively.

Problem to test

H_0: The birth rate is constant over all four seasons.($Md_1 = Md_2 = Md_3 = Md_4$)

H_1: The birth rate is not constant over all four seasons (At least one Md_i is different, i = 1, 2, 3, 4)

Test statistic

$$F_r = \frac{12}{nk(k+1)} \sum_{i=1}^{k} R_i^2 - 3n(k+1)$$

$$= \frac{12 \times 406}{4 \times 4 \times 5} - 3 \times 4 \times 5 = 0.9$$

Level of significance

Let $\alpha = 5\%$ be the level of significance

Critical value

The critical value is $p = P(F_r > 0.9) = 0.9$ for $n = 4$ and $k = 4$

Decision

$P = 0.9 > \alpha = 0.05$, accept H_0 at 5% level of significance.

Conclusion: Birth rate is constant over all four seasons.

Example 8: An investigator wants to study the scores of 3 matched groups under 5 conditions, each group contains five subjects, one being assigned to each of the five conditions. Let the score obtained are given in the following table.Is there any significant difference between three groups.

Group	Conditions				
	I	II	III	IV	V
A	9	8	5	1	7
B	6	7	5	2	8
C	9	7	5	2	6

Solution:

Ranking the different conditions separately

Group	Conditions					R_i	R_i^2
	I	II	III	IV	V		
A	2.5	3	2	1	2	10.5	110.25
B	1	1.5	2	2.5	3	10	100
C	2.5	1.5	2	2.5	1	9.5	90.25
							300.5

Here,

Number of conditions (n) = 5

Number of groups (k) = 3

Number of times rank 2.5 is repeated in I (t_1) = 2

Number of times rank 1.5 is repeated in II (t_2) = 2

Number of times rank 2 is repeated in III (t_3) = 3

Number of times rank 2.5 is repeated in IV (t_4) = 2

Let Md_A, Md_B, Md_c be median of group A, B and C respectively.

Problem to test

H_0: Scores of 3 matched groups is same ($Md_A = Md_B = Md_c$)

H_1: Scores of 3 matched groups is different (At least one Md_i is different, i= A, B, C)

Test statistic

$$F_r = \frac{\frac{12}{nk(k+1)}\sum_{i=1}^{k} R_j^2 - 3n(k+1)}{1 - \sum \frac{(t_i^3 - t_i)}{n(k^3-k)}} = \frac{\frac{12 \times 300.5}{5 \times 3 \times 4} - 3 \times 5 \times 4}{1 - \frac{\{(2^3-2)+(2^3-2)+(3^3-3)+(2^3-2)\}}{5(3^3-3)}} = \frac{60.1-60}{1-0.35} = 0.153$$

Level of significance

Let $\alpha = 5\%$ be the level of significance

Critical value

Critical value is $p = P(F_r > 0.153) = 0.4$ for n=5 and k=3

Decision

$p = 0.4 > \alpha = 0.05$, accept H_0 at 5% level of significance.

Conclusion

There is no significant difference between scores of three matched group.

Example 9: Three different advertising media T.V., Radio and News paper are being compared to study their effectiveness in promoting sales of WaiWai noodles. Each advertising media is exposed for specified period of time and sales (000 package) from 10 stores located at different areas are recorded.

Advertising Media	Stores									
	A	B	C	D	E	F	G	H	I	J
T.V.	20	21	15	12	14	17	21	16	20	18
Radio	7	9	11	12	10	10	14	12	8	7
News Paper	8	6	11	12	9	6	8	10	8	6

Are three advertising media equally effective, use Friedman two way ANOVA test.

Solution:

Ranking the sales from different stores separately

Advertising Media	Stores										R_i	R_i^2
	A	B	C	D	E	F	G	H	I	J		
T.V.	3	3	3	2	3	3	3	3	3	3	29	841
Radio	1	2	1.5	2	2	2	2	2	1.5	2	18	324
News Paper	2	1	1.5	2	1	1	1	1	1.5	1	13	169
$\Sigma R_i^2 = 1334$												

Here,

Number of stores (n) = 10

Number of advertising media (k) = 3

Number of times rank 1.5 is repeated in store C (t_1) = 2

Number of times rank 2 is repeated in store D (t_2) = 3

Number of times rank 1.5 is repeated in store I (t_3) = 2

Let Md_A, Md_B, Md_c be medians of advertising media TV, Radio and Newspaper respectively.

Problem to test

H_0: Three advertising media are equally effective ($Md_A = Md_B = Md_c$)

H_1: Three advertising media are not equally effective (At least one Md_i is different, i = A, B, C)

Test statistic

$$F_r = \frac{\frac{12}{nk(k+1)}\sum_{i=1}^{k} R_i^2 - 3n(k+1)}{1 - \sum \frac{(t_i^3 - t_i)}{n(k^3 - k)}}$$

$$= \frac{\frac{12 \times 1334}{10 \times 3 \times 4} - 3 \times 10 \times 4}{1 - \frac{\{(2^3 - 2) + (3^3 - 3) + (2^3 - 2)\}}{10(3^3 - 3)}} = \frac{133.4 - 120}{1 - 0.15} = 15.76$$

Critical value

Let $\alpha = 5\%$ be the level of significance then critical value is $\chi^2_{0.05\,(2)} = 5.99$

Decision

$F_r = 15.76 > \chi^2_{0.05\,(2)} = 5.99$, reject H_0 at 5% level of significance

Conclusion

Three advertising media T.V., Radio and News Paper are not equally effective in sales of WaiWai noodle.

EXERCISE

1. What is wilcoxon matched pair signed rank test? Why is it superior to sign test?
2. Discuss the rationale and method of Wilcoxon Matched pair signed rank test.
3. Describe rationale and procedure of Cochran Q test.
4. Discuss the function and procedure of Kruskal Wallis H test.
5. What do you mean by Friedman two way ANOVA test? Describe process of the test.
6. Differentiate between Kruskal Wallis One way ANOVA test and Friedman Two way ANOVA test.
7. The weight (kg) of 5 people before they stopped smoking are as follows:

Before	66	80	69	52	75
After	71	82	68	56	73

Use wilcoxon Matched pairs signed rank test for paired observations to test the hypothesis at 0.05 level of significance that giving up smoking has no effect on a person's weight against the alternative hypothesis that one's weight increases if he or she quits smoking.
Ans: T=3.5, insig.

8. To evaluate a speed of reading course, a group of 10 subjects were asked to read two comparable articles one before the course and one after the course. Their scores on reading test are as follows;

Before course(X)	57	80	64	70	90	59	76	98	70	83
After course(Y)	60	90	62	70	95	58	80	99	75	94

Test whether the course is beneficial using the Wilcoxon Matched pairs signed rank test at 5% level of significance.
Ans: T=4.5, sig.

9. Seven prospective graduate students took a test twice with the following scores.

First attempt	470	530	610	440	600	590	580
Second attempt	510	550	600	490	585	620	598

Test whether there is significant difference between scores in first attempt and second attempt using Wilcoxon Matched pair signed rank test. **Ans:** T=3, insig.

10. The following are the number of artifacts dug up by two archaeologists at an ancient cliff dwelling on 30 days.

Arch. X	1	0	2	3	1	0	2	2	3	0	1	1	4	1	2
Arch. Y	0	0	1	0	2	0	0	1	1	2	0	1	2	1	1
Arch . X	1	3	5	2	1	3	2	4	1	3	2	0	2	4	1
Arch. Y	0	2	2	6	0	2	3	0	2	1	0	1	0	1	5

Do these data present sufficient evidence to support the research hypothesis that archaeologist X is better than Y. Use Wilcoxon Matched pairs signed rank test. **Ans:** z = 1.86, sig.

11. Four objective questions are given to 5 students and the results of correct answer(1) and wrong answers(0) are arranged in the following table.

Objective Questions	Students				
	1	2	3	4	5
Q_1	1	0	0	1	1
Q_2	0	1	1	0	1
Q_3	1	1	1	0	0
Q_4	0	0	1	0	1

Apply Cochran q test for testing the hypothesis that there is no significant difference between four objective questions with respect to the correct answers. **Ans:** Q=0.529, insig.

12. Three diets P, Q and R are fed to 9 buffaloes, each diet for one month and the result of Increasing (I) and decreasing (D) of milk given by different buffaloes are given in the following table.

Diet	Buffaloes								
	1	2	3	4	5	6	7	8	9
P	I	D	I	D	I	I	D	D	I
Q	I	I	I	D	I	I	D	I	D
R	I	D	D	D	I	D	I	I	I

Test whether the three diets are equally effective or not using Cochran Q test at $\alpha = 5\%$. **Ans:** Q=0.33, insig.

13. Following are the final examination marks of three group of students who were taught computer by three different methods;

Method I	94	88	91	74	87	97	
Method II	85	82	79	84	61	72	80
Method III	89	67	72	76	69		

Are all three methods equally effective? Use H test at 0.05 level of significance. **Ans:** H=6.67, sig.

14. An agricultural experiment was conducted to compare the yield of wheat by using three types of chemical fertilizer nitrogen(N), phosphorous (P) and potash (K). Twelve plots of equal size were selected at random and divided into three groups of four each and planted wheat. Each group was randomly selected and the fertilizer was applied in the plots under the identical conditions. Then the yield of wheat recorded were given in the following table.

Chemical fertilizer		
N	P	K
122	81	80
80	80	82
138	79	65
121	65	58

Test whether the three types of fertilizers are equally effective or not. Use Kruskal Wallis test at 0.05 level of significance. **Ans:** P=0.008, insig.

15. For the following scores of 3 groups, apply Kruskal Wallis H test to test the hypothesis that the three groups are not significantly different:

Group	Scores				
A	96	128	83	61	101
B	82	124	132	135	109
C	115	149	166	147	

Ans: p=0.01, sig.

16. An experiment designed to compare three preventive methods against corrosion yielded the following maximum depths of pits (in thousandths of an inch)in pieces of wire subjected to the respective treatments:

Method I	77	54	67	74	71	66	
Method II	60	41	59	65	62	64	52
Method III	49	52	69	47	56		

Use the 0.05 level of significance to test the hypothesis that the three samples come from identical population. Use Kruskal Wallis H test.

Ans: H=6.66, sig.

17. The following data represent the operating times in hours for 3 types of scientific pocket calculators before a charge is required.

Calculator A	4.9	6.1	4.3	4.6	5.3		
Calculator B	5.5	5.4	6.2	5.8	5.5	5.2	4.8
Calculator C	6.4	6.8	5.6	6.5	6.3	6.6	

Use Kruskal Wallis test, at the 0.0 level of significance to test the hypothesis that the operating times for all three calculators are equal.

Ans: H=10.4, sig.

18. A researcher wants to compare the teaching standard of three English medium schools on the basis of performance of the student's final examination scores. The percentage of passers in I to IV grade in the schools are presented in the following table.

	Grade			
	I	II	III	IV
Alpha	89	98	70	80
Sigma	45	76	40	55
Gamma	20	58	35	67

Test the performances of the schools with respect to pass percentage using Friedman's test.

Ans: p=0.042, sig.

19. A survey was conducted in four Hospitals in a particular city to obtain the number of babies born over a 12 months period divided into four seasons to test the hypothesis that the birth rate is constant over all four seasons. The result of the survey are as follows:

Hospitals	Number of births			
	Winter	Spring	Summer	Fall
A	92	112	94	77
B	9	11	10	15
C	58	71	51	62
D	19	26	19	18

Analyze the data using Friedman's test.

Ans: p = 0.928, insig.

20. The scores of 3 matched groups under the six conditions are given below

Group	Condition					
	I	II	III	IV	V	VI
A	9	5	2	5	6	7
B	6	4	3	4	6	5
C	5	1	3	3	6	5

Apply the Friedman two way ANOVA test to identify if there is significantly difference in variation between matched groups. Use 5% level of significance.

Ans: 0.184, insig.

Using Software for Non parametric test:

Run test

In 30 toss of a coin the following sequence of heads (H) and tails (T) is obtained.

H T T H T H HH T H H TT H T H T H H T H T T H T H H T H T

Test at 0.05 level of significance level whether the sequence is random.

Using Excel

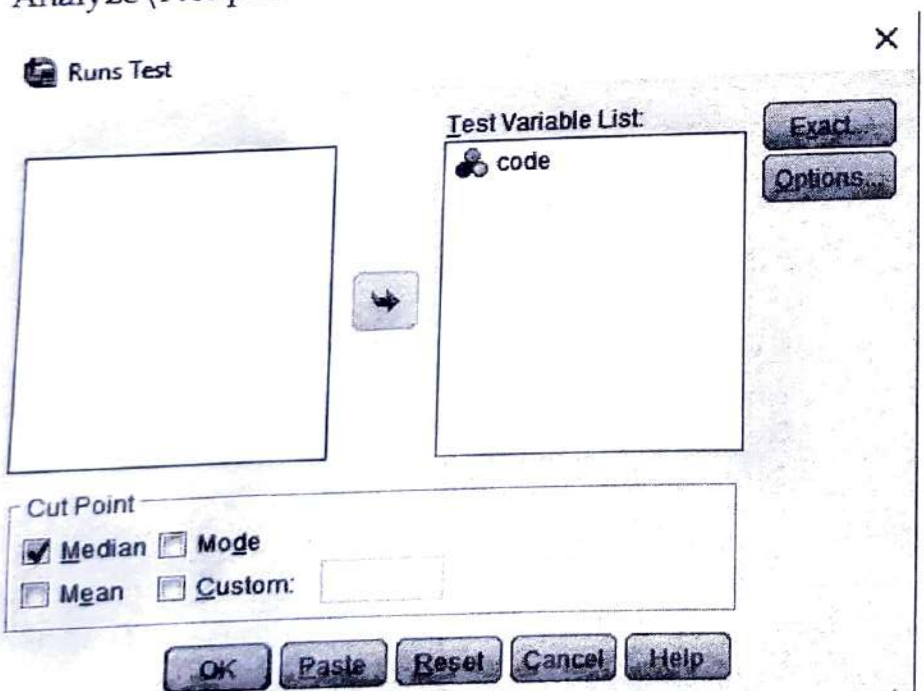
	A	B	C
1	outcome	code	runs
2	H	1	1
3	T	0	2
4	T	0	2
5	H	1	3
6	T	0	4
7	H	1	5
8	H	1	5
9	H	1	5
10	T	0	6
11	H	1	7
12	H	1	7
13	T	0	8
14	T	0	8
15	H	1	9
16	T	0	10
17	H	1	11
18	T	0	12
19	H	1	13
20	H	1	13
21	T	0	14
22	H	1	15
23	T	0	16
24	T	0	16
25	H	1	17
26	T	0	18
27	H	1	19
28	H	1	19
29	T	0	20
30	H	1	21
31	T	0	22
32			

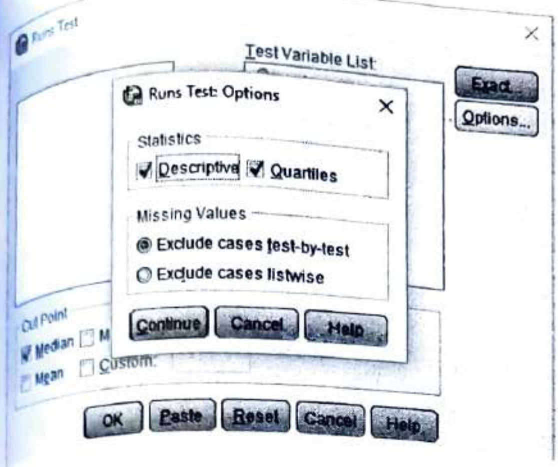
	A	B	C	D
		symbol	value	formula
34	cases	n	30	=COUNT(B2:B31)
35	no of observation	n1	16	=COUNTIF(B2:B31,1)
36	no of head	n2	14	=COUNTIF(B2:B31,0)
37	no of tail	R	22	=MAX(C2:C31)
38	no of runs	E(R)	15.93333	=((2*C36*C37)/(C36+C37))+1
39	mean			
40	$\mu_r = \frac{2n_1n_2}{n_1+n_2}+1$	$\sigma_r^2 = \frac{2n_1n_2(2n_1n_2-n_1-n_2)}{(n_1+n_2)^2(n_1+n_2-1)}$		
41				
42		V(R)	7.174866	=(2*C36*C37*(2*C36*C37-C36-C37))/(((C36+C37)^2)*(C36+C37-1))
43	variance			
44				
45	test hypothesis			
46	Null Hypothesis	H0: sample observation is random.		
47	Alternative hypothesis	H1: sample observation isnot random.		
48				
49	test statistic	z	2.26487	=(C38-C39)/SQRT(C43)
50	z statistic	α	0.05	
51	level of significance	$z_{\alpha/2}$	1.959964	=NORMSINV(1-C51/2)
52	tabulated z for two tailed test	p	0.01176	=1-NORMSDIST(C50)
53	p value			

	A	B	C	D
54				
55	Decision:			
56	*Significant approach*			=IF(ABS(C50)<C52,"There is no reason to reject Null Hypothesis H0","H0 is rejected")
57	H0 is rejected			
58	*p-value approach*			=IF(C51<C53,"It is insignificant","It is significant")
59	It is significant			

Using SPSS

Analyze\Nonparametric tests\Legacy Dialogs\Runs





Outputs

NPar Tests

activate

Descriptive Statistics

	N	Mean	Std. Deviation	Minimum	Maximum	Percentiles 25th	50th (Median)	75th
code	30	.53	.507	0	1	.00	1.00	1.00

Runs Test

	code
Test Value[a]	1
Cases < Test Value	14
Cases >= Test Value	16
Total Cases	30
Number of Runs	22
Z	2.078
Asymp. Sig. (2-tailed)	.038

a. Median

Using STATA

Run test using stata

Recode the data such that the head = 1 and tail = 2 in variable n1code.

Then use syntax

Runtest n1code

Output will be as follows;

```
. runtest n1code
N(n1code <= 1) = 16
N(n1code >  1) = 14
           obs = 30
       N(runs) = 22
             z = 2.26
      Prob>|z| = .02
```

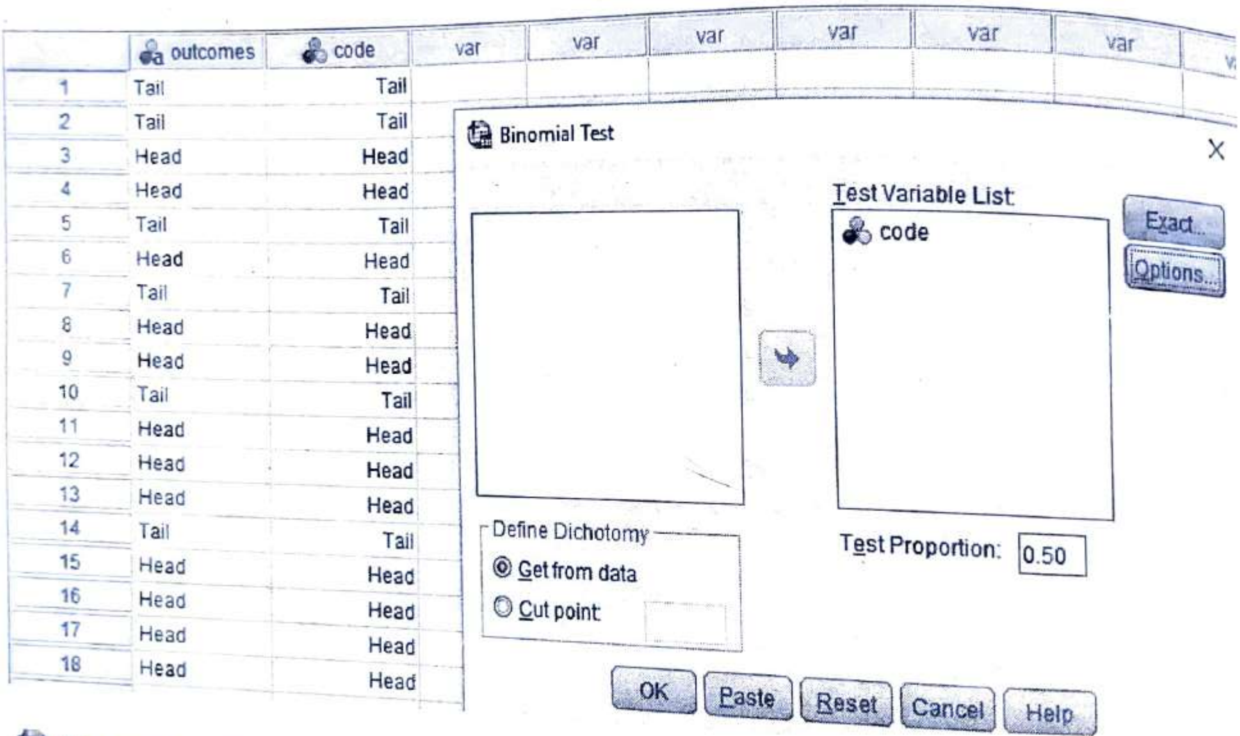
Binomial test

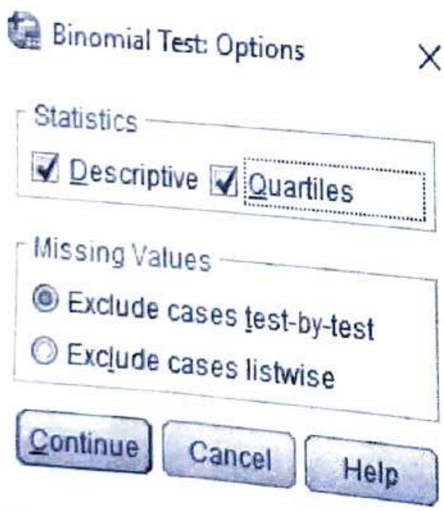
Test whether the coin is unbiased from following observations.

Tail	Tail	Head	Head	Tail	Head	Tail	Head	Head	Tail
Head	Head	Head	Tail	Head	Head	Head	Head	Head	Head
Tail	Tail	Tail	Tail	Head	Tail	Tail	Tail	Tail	Tail
Tail	Tail	Head	Tail	Tail	Tail	Head	Tail	Tail	Tail
Tail	Head	Tail	Tail	Head	Tail	Head	Tail	Tail	Tail

Using SPSS

Analyze\Nonparametric tests\Legacy Dialogs\Binomial





Outputs

NPar Tests

Descriptive Statistics

	N	Mean	Std. Deviation	Minimum	Maximum	Percentiles 25th	Percentiles 50th (Median)	Percentiles 75th
code	50	.40	.495	0	1	.00	.00	1.00

Binomial Test

		Category	N	Observed Prop.	Test Prop.	Exact Sig. (2-tailed)
code	Group 1	0 Tail	30	.60	.50	.203
	Group 2	1 Head	20	.40		
	Total		50	1.00		

Since p=0.203> 0.05, the null hypothesis of equality of proportion of head and tail can be accepted.

Using STATA

bitest code == 0.5

```
. bitest code == 0.5

    Variable |        N   Observed k   Expected k   Assumed p   Observed p
-------------+-------------------------------------------------------------
        code |       50        20           25       0.50000     0.40000

  Pr(k >= 20)            = 0.940540  (one-sided test)
  Pr(k <= 20)            = 0.101319  (one-sided test)
  Pr(k <= 20 or k >= 30) = 0.202639  (two-sided test)
```

Since p=0.203> 0.05, the null hypothesis of equality of proportion of head and tail can be accepted.

Kolmogorov Smirnov one sample

The number of disease infected tomato plants in 10 different plots of equal size are given below. Test whether the disease infected plants are uniformly distributed over the entire area use Kolmogorov Smirnov test.

Plot no.	1	2	3	4	5	6	7	8	9	10
No of infected plants	8	10	9	12	15	7	5	12	13	9

Using excel

	A	B	C	D	E	F	G	H
4	Plot no.	No of infected plants						
5	1	8						
6	2	10						
7	3	9						
8	4	12						
9	5	15						
10	6	7						
11	7	5						
12	8	12						
13	9	13						
14	10	9						
15								
16								
17	Problem to test							
18	H0: The disease infected plants are uniformly distributed over the entire area.							
19	H1: The disease infected plants are not uniformly distributed over the entire area.							

	A	B	C	D	E	F	G	H
21	Plot no.	No of infected plants	Observed cf (cf0)	Observed relative freq (F0)	Expected freq (fe)	Expected cf (cfe)	Expected relative freq(Fe)	$\lvert F_e - F_0 \rvert$ differenc e
22	1	8	8	0.08	10	10	0.1	0.02
23	2	10	18	0.18	10	20	0.2	0.02
24	3	9	27	0.27	10	30	0.3	0.03
25	4	12	39	0.39	10	40	0.4	0.01
26	5	15	54	0.54	10	50	0.5	0.04
27	6	7	61	0.61	10	60	0.6	0.01
28	7	5	66	0.66	10	70	0.7	0.04
29	8	12	78	0.78	10	80	0.8	0.02
30	9	13	91	0.91	10	90	0.9	0.01
31	10	9	100	1.00	10	100	1	0
32								
33	*Test statistic*		value	formula				
34		D_0 =	0.04	=MAX(H22:H31)				
35		n	=COUNT(B22:B31)					
36		α	0.05					
37	Critical value							
38	Let 5% be the level of significance then critical value is							
39		$D_{(n,\alpha)}$	0.40925	from Ks test D table				

	A	B	C	D	E
41	decision				
42					
43	*Significant approach*				
44	There is no reason to reject Null Hypothesis H0				

use Kolmogorov Smirnov test.

Test for normal distribution

weight									
19.5	20	26.9	27.1	28.1	30	31.6	32.7	34.4	37.2
37.5	37.9	38	38.4	38.6	38.8	38.9	40.1	41.6	42.6
42.9	45	45.2	45.5	46.5	46.8	47.3	48.1	48.3	48.4
48.8	49	49.1	49.3	49.4	49.5	49.9	50.4	51.8	54.4
54.9	55.3	55.6	57.3	57.4	57.5	58.7	58.8	59.7	59.9

Using excel

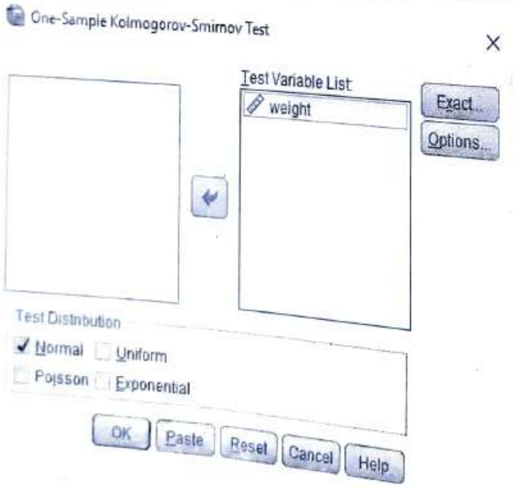
	A	B	C	D	E	F
1	weight	rank	expected	(rank-1)/n	actual	difference
2	19.5	1	0.02	0	0.007859	0.007859
3	20	2	0.04	0.02	0.008969	0.011031
4	26.9	3	0.06	0.04	0.044763	0.004763
5	27.1	4	0.08	0.06	0.046624	0.013376
6	28.1	5	0.1	0.08	0.056876	0.023124
7	30	6	0.12	0.1	0.081156	0.018844
8	31.6	7	0.14	0.12	0.10708	0.01292
9	32.7	8	0.16	0.14	0.128072	0.011928
10	34.4	9	0.18	0.16	0.165842	0.005842
11	37.2	10	0.2	0.18	0.242198	0.062198
12	37.5	11	0.22	0.2	0.251377	0.051377
13	37.9	12	0.24	0.22	0.263896	0.043896
14	38	13	0.26	0.24	0.267074	0.027074
15	38.4	14	0.28	0.26	0.279979	0.019979
16	38.6	15	0.3	0.28	0.286543	0.006543
17	38.8	16	0.32	0.3	0.293179	0.006821
18	38.9	17	0.34	0.32	0.296524	0.023476
19	40.1	18	0.36	0.34	0.337946	0.002054
20	41.6	19	0.38	0.36	0.392564	0.032564
21	42.6	20	0.4	0.38	0.430271	0.050271
22	42.9	21	0.42	0.4	0.441725	0.041725
23	45	22	0.44	0.42	0.522732	0.102732
24	45.2	23	0.46	0.44	0.53045	0.09045
25	45.5	24	0.48	0.46	0.542006	0.082006
26	46.5	25	0.5	0.48	0.580216	0.100216

[part of data is shown]

	A	B	C	D	E	F	G
55	Problem to test						
56	H0: weights are normally distributed over the entire area.						
57	H1: weights are not normally uniformly distributed over the entire area.						
58							
59		n	50				
60		mean	44.412				
61		st dev	10.3139				
62							
63	Test Statistics						
64			value	formula			
65		D	0.102732	=MAX(F2:F51)			
66		n	50	=COUNT(A2:A51)			
67		α	0.05				
68	Critical value						
69	Let 5% be the level of significance then critical value is						
70		$D_{(n,\alpha)}$	0.18845	From table			
71							
72	decision						
73							
74	*Significant approach*						
75	There is no reason to reject Null Hypothesis H0						

Using SPSS

Analyze\Nonparametric tests\Legacy Dialogs\1 sample K-S



NPar Tests

[DataSet1]

Descriptive Statistics

	N	Mean	Std. Deviation	Minimum	Maximum	Percentiles 25th	50th (Median)	75th
weight	50	44.4120	10.31390	19.50	59.90	37.9750	46.6500	50.7500

One-Sample Kolmogorov-Smirnov Test

		weight
N		50
Normal Parameters[a,b]	Mean	44.4120
	Std. Deviation	10.31390
Most Extreme Differences	Absolute	.103
	Positive	.067
	Negative	-.103
Test Statistic		.103
Asymp. Sig. (2-tailed)		.200[c,d]

a. Test distribution is Normal.

b. Calculated from data.

c. Lilliefors Significance Correction.

d. This is a lower bound of the true significance.

Mann- Whitney U test

Small sample

Test the hypothesis of no difference between the ages of male and female employees of a certain company, using the Mann- Whitney U test for the sample data below. Use $\alpha = 0.1$

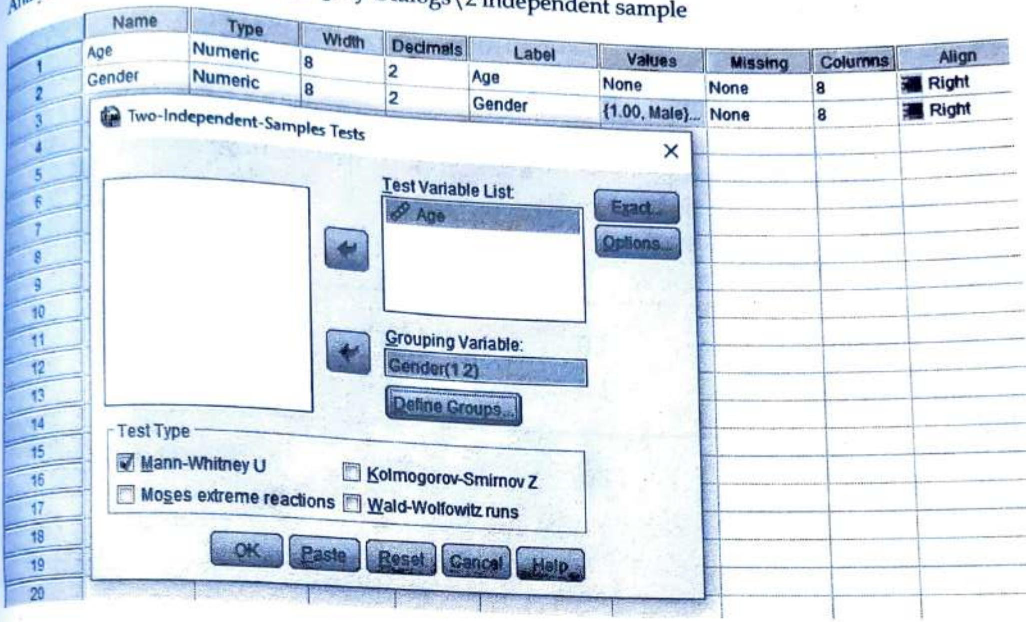
Male	35	43	26	44	40	42	33	38	25	26
Female	30	41	34	31	36	32	25	47	28	24

Using excel

	A	B	C
4			
5	Age	Gender	rank
6	24	2	1
7	25	1	2.5
8	25	2	2.5
9	26	1	4.5
10	26	1	4.5
11	28	2	6
12	30	2	7
13	31	2	8
14	32	2	9
15	33	1	10
16	34	2	11
17	35	1	12
18	36	2	13
19	38	1	14
20	40	1	15
21	41	2	16
22	42	1	17
23	43	1	18
24	44	1	19
25	47	2	20
26			

	A	B	C	D
27		symbol	value	formula
28	Sample size of male	n_1	10	=COUNTIF(B6:B25,1)
29	Sample size of female	n_2	10	=COUNTIF(B6:B25,2)
30	Sum of ranks of male	R_1	116.5	=SUMIF(B6:B25,1,C6:C25)
31	Sum of ranks of female	R_2	93.5	=SUMIF(B6:B25,2,C6:C25)
32	level of significance	α	0.01	
33–34	U_1	$n_1n_2 + \frac{n_1(n_1+1)}{2} - R_1$	38.5	=C28*C29+(C28*(C28+1)/2)-C30
35	U_2	$n_1n_2 - U_1$	61.5	=C28*C29-C33
36				
37	U0	$\min\{U_1,U_2\}$	38.5	=MIN(C33,C35)
38				
39	Let Md_1 and Md_2 be median age of male and female employees respectively.			
40	Problem to test			
41	H0: There is no significant difference between age of male and female employee ($Md_1 = Md_2$)			
42	H_1: There is significant difference between age of male and female employee ($Md_1 \neq Md_2$)			
43				
44	Test statistic			
45	U0	38.5		
46	Critical value			
47	Let $\alpha = 0.10$ be the level of significance then critical value is			
48	$U_{\alpha\,(n_1,n_2)}$	27	From table	
49				
50	Decision			
51	$U0 = 38.5 > U_{\alpha\,(n_1,n_2)} = 27,$		accept H_0 at 0.10 level of significance.	

Analyze\Nonparametric tests\Legacy Dialogs\2 independent sample



Outputs

Mann-Whitney Test

Ranks

	Gender Gender	N	Mean Rank	Sum of Ranks
Age Age	1.00 Male	10	11.65	116.50
	2.00 Female	10	9.35	93.50
	Total	20		

Test Statistics[a]

	Age Age
Mann-Whitney U	38.500
Wilcoxon W	93.500
Z	-.870
Asymp. Sig. (2-tailed)	.384
Exact Sig. [2*(1-tailed Sig.)]	.393[b]

a. Grouping Variable: Gender Gender

b. Not corrected for ties.

Using STATA

ranksum Age, by(Gender)

```
. ranksum Age, by(Gender)

Two-sample Wilcoxon rank-sum (Mann-Whitney) test

      Gender |      obs    rank sum    expected
-------------+---------------------------------
        Male |       10       116.5         105
      Female |       10        93.5         105
-------------+---------------------------------
    combined |       20         210         210

unadjusted variance      175.00
adjustment for ties       -0.26
                       ----------
adjusted variance        174.74

Ho: Age(Gender==Male) = Age(Gender==Female)
             z =   0.870
    Prob > |z| =   0.3843
```

Result: Analysis result shows the p-value for Z test is 0.3843. This indicates that our null hypothesis, of no difference in male female, is accepted.

Large sample

The following are the scores which random samples of students from 2 minority groups obtained on a current event test:

Minority Group I	73	82	39	68	91	75	89	67	50	86	57	65	70
Minority Group II	51	42	36	53	88	59	49	66	25	64	18	76	74

Use Mann Whitney U test at the 0.05 level of significance to test whether or not students from the two minority groups can be expected to score equally well on the test.

Using excel

	A	B	C	D	E
1					
2					
3					
4					
5	scores	Group	rank =RANK.AVG(A7:A32,A7:A32,1)		
6	73	1	18		
7	82	1	22		
8	39	1	4		
9	68	1	16		
10	91	1	26		
11	75	1	20		
12	89	1	25		
13	67	1	15		
14	50	1	7		
15	86	1	23		
16	57	1	10		
17	65	1	13		
18	70	1	17		
19	51	2	8		
20	42	2	5		
21	36	2	3		
22	53	2	9		
23	88	2	24		
24	59	2	11		
25	49	2	6		
26	66	2	14		
27	25	2	2		
28	64	2	12		
29	18	2	1		
30	76	2	21		
31	74	2	19		

	A	B	C	D
35		symbol	value	formula
36	Sample size of male	n_1	13	=COUNTIF(B7:B32,1)
37	Sample size of female	n_2	13	=COUNTIF(B7:B32,2)
38	Sum of ranks of male	R_1	216	=SUMIF(B7:B33,1,C7:C33)
39	Sum of ranks of female	R_2	135	=SUMIF(B7:B33,2,C7:C33)
40	level of significance	α	0.05	
41–42	U_1	$n_1n_2 + \frac{n_1(n_1+1)}{2} - R_1$	44	=C28*C29+(C28*(C28+1)/2)-C30
43	U_2	$n_1n_2 - U_1$	125	=C28*C29-C33
44				
45	U0	$\min\{U_1,U_2\}$	44	=MIN(C33,C35)
46				

47 Let Md_1 and Md_2 be median age of male and female employees respectively.

48 Problem to test

49 H0: There is no significant difference between age of male and female employee ($Md_1 = Md_2$)

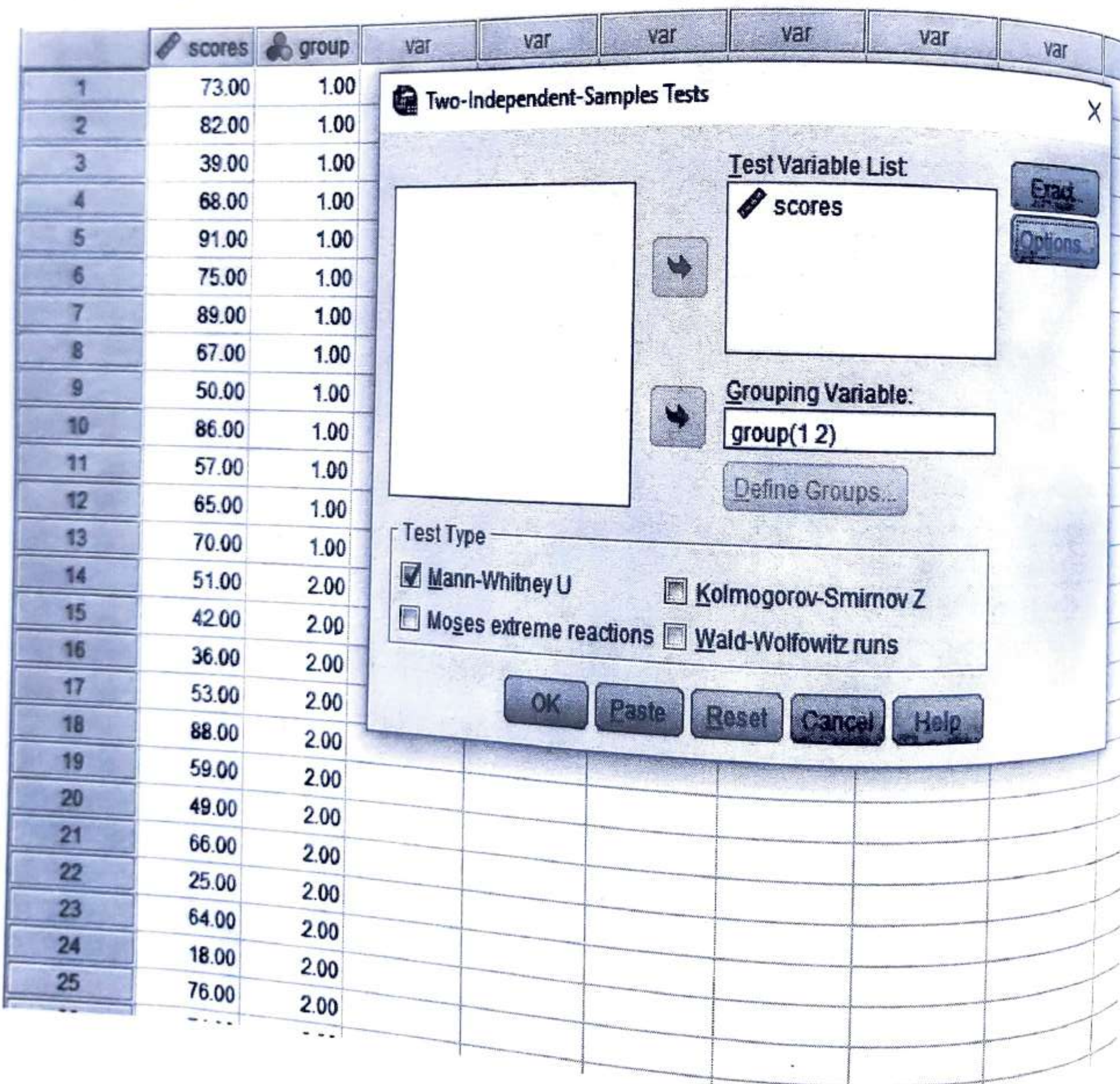
50 H_1: There is significant difference between age of male and female employee ($Md_1 \neq Md_2$)

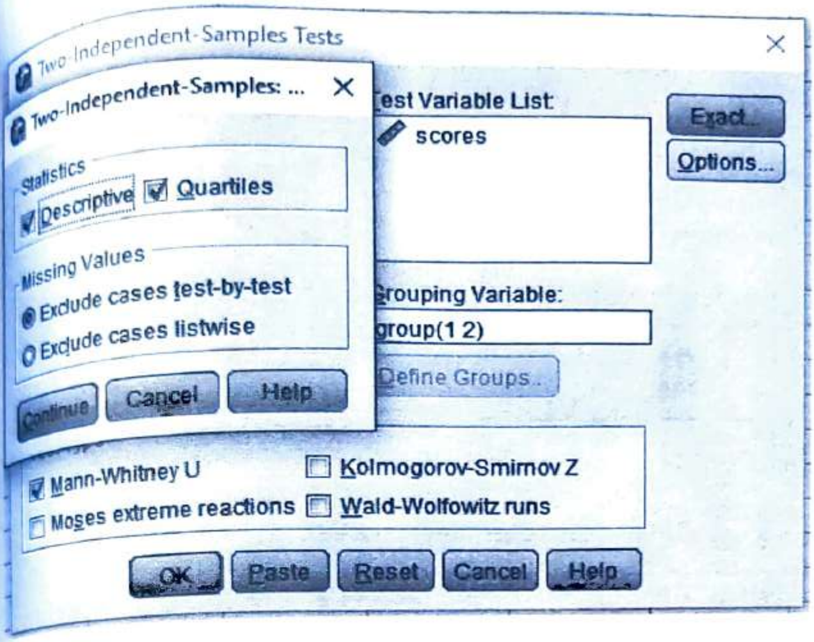
51

	A	B	C	D	E	F	G	H
51								$\mu u = \frac{n_1 n_2}{2}$
52			84.5	=C36*C37/2				
53	mean	μ_u						
54			19.5	=SQRT((C36*C37*(C36+C37+1)/12))				$\sigma_u = \sqrt{\frac{n_1 n_2(n_1+n_2+1)}{12}}$
55	st dev	σ_u						
56								
57	Test statistic							
58		Z =	-2.07692	=(C45-C53)/C55				
59								
60								
61	Critical value							
62		$Z_{\alpha/2}$	1.959964	=NORMSINV(1-C40/2)				
63								
64	Decision							
65	| Z | = 2.07 > Z0.05 = 1.96, reject H0 at 0.05 level of significance.							
66								

Using SPSS

Analyze\Nonparametric tests\Legacy Dialogs\





Outputs

Descriptive Statistics

	N	Mean	Std. Deviation	Minimum	Maximum	Percentiles 25th	50th (Median)	75th
scores	26	62.0385	19.45761	18.00	91.00	49.7500	65.5000	75.2500
group	26	1.5000	.50990	1.00	2.00	1.0000	1.5000	2.0000

Mann-Whitney Test

Ranks

	group	N	Mean Rank	Sum of Ranks
scores	1.00	13	16.62	216.00
	2.00	13	10.38	135.00
	Total	26		

Test Statistics[a]

	scores
Mann-Whitney U	44.000
Wilcoxon W	135.000
Z	-2.077
Asymp. Sig. (2-tailed)	.038
Exact Sig. [2*(1-tailed Sig.)]	.039[b]

a. Grouping Variable: group

b. Not corrected for ties.

Using STATA

ranksum scores, by(group)

```
. ranksum scores, by(group)

Two-sample Wilcoxon rank-sum (Mann-Whitney) test

       group |      obs    rank sum    expected
-------------+---------------------------------
           1 |       13         216       175.5
           2 |       13         135       175.5
-------------+---------------------------------
    combined |       26         351         351

unadjusted variance       380.25
adjustment for ties         0.00
                      ----------
adjusted variance         380.25

Ho: scores(group==1) = scores(group==2)
             z =   2.077
    Prob > |z| =   0.0378
```

Result: Analysis result shows the p-value for Z test is 0.0378. This indicates that our null hypothesis, of no difference in male female, is rejected.

Use chi-square test of independence to determine association between type of place of place of residence (V102) with source of drinking water(V113) and type of toilet facility(V116) using NPBR7HFL data.

Using Excel

	A	B	C	D	E
1		Observed Frequency			
2			V102 Type of place of residence		Total
3			1 Urban	2 Rural	
4	V113 Source of drinking water	11 Piped into dwelling	958	124	1082
5		12 Piped to yard/plot	3386	2523	5909
6		13 Piped to neighbor	283	173	456
7		14 Public tap/standpipe	2730	2818	5548
8		21 Tube well or borehole	6051	3937	9988
9		31 Protected well	148	38	186
10		32 Unprotected well	269	32	301
11		41 Protected spring	254	136	390
12		42 Unprotected spring	233	209	442
13		43 River/dam/lake/ponds/stream/canal/irrigation channel	458	283	741
14		51 Rainwater	9	0	9
15		61 Tanker truck	39	0	39
16		62 Cart with small tank	3	0	3
17		71 Bottled water	265	6	271
18		97 Not a dejure resident	423	240	663
19	Total		15509	10519	26028

Expected Frequency

=RT*CT/GT

V113 Source of drinking water	V102 Type of place of residence 1 Urban	2 Rural	Total	=((ABS(C4-C24)-0.5)^2)/C24		=((ABS(C4-C24))^2)/C24	
11 Piped into dwelling	645	437	1082	151.7439	223.7282	152.2295	224.444
12 Piped to yard/plot	3521	2388	5909	5.132327	7.566998	5.170577	7.623394
13 Piped to neighbor	272	184	456	0.428374	0.631586	0.469	0.691485
14 Public tap/standpipe	3306	2242	5548	100.125	147.6222	100.2991	147.8789
21 Tube well or borehole	5951	4037	9988	1.649068	2.431352	1.665756	2.455957
31 Protected well	111	75	186	12.13317	17.8889	12.4663	18.38006
32 Unprotected well	179	122	301	44.30988	65.32959	44.80832	66.06447
41 Protected spring	232	158	390	1.918604	2.82875	2.010543	2.964304
42 Unprotected spring	263	179	442	3.387559	4.994549	3.501921	5.163161
43 River/dam/lake/ponds/stream/canal/irrigation channel	442	299	741	0.577556	0.851537	0.614289	0.905696
51 Rainwater	5	4	9	1.835354	2.706008	2.466987	3.637275
61 Tanker truck	23	16	39	10.02278	14.77739	10.69028	15.76153
62 Cart with small tank	2	1	3	0.283932	0.418623	0.822329	1.212425
71 Bottled water	161	110	271	65.72809	96.90816	66.36764	97.8511
97 Not a dejure resident	395	268	663	1.906776	2.811312	1.976883	2.914676
Total	15509	10519	26028				

Here	Symbol	Value	Formula
No of rows	r	15	=COUNT(C24:C38)
No of columns	c	2	=COUNT(C24:D24)
No of observation	N=rc	30	=COUNT(C24:D38)
Level of significance	α	0.05	
Degrees of freedom	(r-1)*(c-1)	14	=(D42-1)*(D43-1)

Test Hypothesis:

H0: Type of residence and source of Drinking water are independent

H1: Type of residence and source of Drinking water are not independent

Test Statistics

$$\chi^2 = \frac{\left\{|O_i - E_i| - \frac{1}{2}\right\}^2}{E_i}$$

for Yates corrected cases

$$\chi^2 = \frac{\{|O_i - E_i|\}^2}{E_i}$$

for no Yates correction

	Yates corrected		No Yates correction	
calculated chi square	992.6774914	=SUM(H24:I38)	1003.508	=SUM(K24:L38)
			2.8E-205	=CHISQ.TEST(C4:D18,C24:D38)
p value	5.8268E-203	=CHISQ.DIST.RT(C56,D46)	2.8E-205	=CHISQ.DIST.RT(F56,D46)

Tabulated Chi square: 23.68479

decision:

significant approach

Reject H0 =IF(F56<F60,"there is no reason to reject null hypothesis H0","Reject H0")

p value approach

It is significant =IF(F59>D45,"it is insignificant","It is significant")

Using SPSS

Crosstab

Count

		V102 Type of place of residence		
		1 Urban	2 Rural	Total
V113 Source of drinking water	11 Piped into dwelling	958	124	1082
	12 Piped to yard/plot	3386	2523	5909
	13 Piped to neighbor	283	173	456
	14 Public tap/standpipe	2730	2818	5548
	21 Tube well or borehole	6051	3937	9988
	31 Protected well	148	38	186
	32 Unprotected well	269	32	301
	41 Protected spring	254	136	390
	42 Unprotected spring	233	209	442
	43 River/dam/lake/ponds/stream/canal/irrigation channel	458	283	741
	51 Rainwater	9	0	9
	61 Tanker truck	39	0	39
	62 Cart with small tank	3	0	3
	71 Bottled water	265	6	271
	97 Not a dejure resident	423	240	663
Total		15509	10519	26028

Chi Square test of type of residence vs source of drinking water

Chi-Square Tests

	Value	df	Asymptotic Significance (2-sided)
Pearson Chi-Square	1003.508[a]	14	.000
Likelihood Ratio	1174.554	14	.000
Linear-by-Linear Association	90.020	1	.000
N of Valid Cases	26028		

a. 3 cells (10.0%) have expected count le Double-click to activate minimum expected count is 1.21.

V102 Type of place of residence * V116 Type of toilet facility

Crosstab

Count

		V102 Type of place of residence		
		1 Urban	2 Rural	Total
V116 Type of toilet facility	11 Flush to piped sewer system	595	30	625
	12 Flush to septic tank	8919	4766	13685
	13 Flush to pit latrine	2226	2094	4320
	14 Flush to somewhere else	44	21	65
	15 Flush, don't know where	3	0	3
	21 Ventilated Improved Pit latrine (VIP)	569	663	1232
	22 Pit latrine with slab	479	533	1012
	23 Pit latrine without slab/open pit	168	138	306
	31 No facility/bush/field	1817	1870	3687
	41 Composting toilet	259	162	421
	96 Other	7	2	9
	97 Not a dejure resident	423	240	663
Total		15509	10519	26028

Chi-Square Tests

	Value	df	Asymptotic Significance (2-sided)
Pearson Chi-Square	954.042[a]	11	.000
Likelihood Ratio	1050.446	11	.000
Linear-by-Linear Association	45.574	1	.000
N of Valid Cases	26028		

a. 3 cells (12.5%) have expected count less than 5. The minimum expected count is 1.21.

Using STATA

Chi-square type of place of residence (V102) vs Source of drinking water (V113)

tabulate V113 V102, chi2

```
. tabulate V113 V102,chi2
```

Source of drinking water	Type of place of residence Urban	Rural	Total
Piped into dwelling	958	124	1,082
Piped to yard/plot	3,386	2,523	5,909
Piped to neighbor	283	173	456
Public tap/standpipe	2,730	2,818	5,548
Tube well or borehole	6,051	3,937	9,988
Protected well	148	38	186
Unprotected well	269	32	301
Protected spring	254	136	390
Unprotected spring	233	209	442
River/dam/lake/ponds/	458	283	741
Rainwater	9	0	9
Tanker truck	39	0	39
Cart with small tank	3	0	3
Bottled water	265	6	271
Not a dejure resident	423	240	663
Total	15,509	10,519	26,028

Pearson chi2(14) = 1.0e+03 Pr = 0.000

Chi-square type of place of residence (V102) vs Type of toilet facility (V116)

tab V116 V102, chi2

```
. tab V116 V102, chi2
```

Type of toilet facility	Type of place of residence Urban	Rural	Total
Flush to piped sewer	595	30	625
Flush to septic tank	8,919	4,766	13,685
Flush to pit latrine	2,226	2,094	4,320
Flush to somewhere el	44	21	65
Flush, don't know whe	3	0	3
Ventilated Improved P	569	663	1,232
Pit latrine with slab	479	533	1,012
Pit latrine without s	168	138	306
No facility/bush/fiel	1,817	1,870	3,687
Composting toilet	259	162	421
Other	7	2	9
Not a dejure resident	423	240	663
Total	15,509	10,519	26,028

Pearson chi2(11) = 954.0416 Pr = 0.000

Median Test

For Small sample size ($n_1 \leq 10, n_2 \leq 10$)

Data below shows one week growth(in cm) of maize plant from two different localities (Sample I &Sampel II);

Sample I	10	11	8	8	14		
Sample II	9	12	13	9	15	9	17

Test whether the two samples have come from the same population with respect to their medians. Use median test at 0.05 level of significance.

Using Excel

	A	B	C	D	E	F	G	H
1	Sample I	10	11	8	8	14		
2	Sample II	9	12	13	9	15	9	17
3								
4	sample	value						
5	1	10						
6	1	11						
7	1	8						
8	1	8						
9	1	14						
10	2	9						
11	2	12						
12	2	13						
13	2	9						
14	2	15						
15	2	9						
16	2	17						
17								
18		symbol	value	formula				
19	no of observation	n	12					
20	median	Md	10.5					
21	sample size 1	n1	5					
22	sample size 2	n2	13					
23		k	9					

	A	B	C	D	E	F	G	H	I	J
24	no of obs less than median									
25	in 1st sample	a	3							
26										

27 Let Let Md_1 and Md_2 be median of I population and II population respectively.

28

29 Problem to test

30 H0: There is no significant difference between median of I population and median of II population ($Md_1 = Md_2$)

31 H_1: There is significant difference between median of I population and II population ($Md_1 \neq Md_2$)

32 Test statistic

33 $P(A=a) = \dfrac{c(n_1,a)c(n_2,k-a)}{c(n_1+n_2,k)}$

34

35

36 level ov significance 0.05

37 Critical value

38 $P = P(A \geq a) = P(A \geq 3)$

39 0.5

40 decision

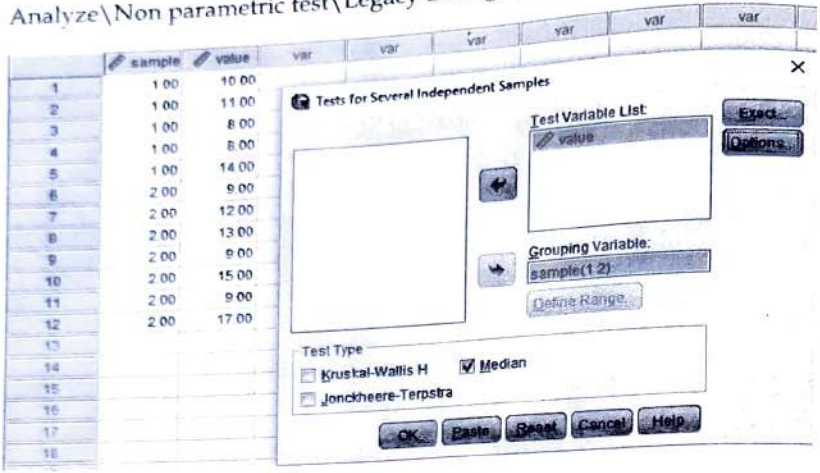
41 since 2*p=

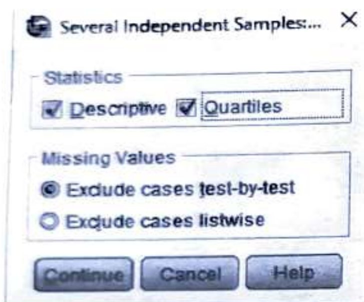
42 it is insignificant 1

43

Using SPSS

Analyze\Non parametric test\Legacy Dailogs\k-independent samples





Outputs

```
[DataSet1] D:\Desktop\1111111111111111\stat2_pra\md small.sav
```

Descriptive Statistics

	N	Mean	Std. Deviation	Minimum	Maximum	Percentiles 25th	50th (Median)	75th
value value	12	11.2500	2.95804	8.00	17.00	9.0000	10.5000	13.7500
sample sample	12	1.5833	.51493	1.00	2.00	1.0000	2.0000	2.0000

Median Test

Frequencies

		sample sample 1.00	2.00
value value	> Median	2	4
	<= Median	3	3

Test Statistics[a]

	value value
N	12
Median	10.5000
Exact Sig.	1.000

a. Grouping Variable: sample sample

Answer as the p value associated with the median test is about 1 it indicates that null hypothesis of no difference is accepted for the given data.

using STATA

median value, by(sample) medianties(below)

```
. median value, by(sample) medianties(below)

Median test
```

Greater than the median	sample 1	2	Total
no	3	3	6
yes	2	4	6
Total	5	7	12

```
          Pearson chi2(1) =   0.3429   Pr = 0.558

 Continuity corrected:
          Pearson chi2(1) =   0.0000   Pr = 1.000
```

Large sample size ($n_1 > 10$, $n_2 > 10$)

An IQ test was given to a randomly selected 15 male and 20 female students of a university. Their scores were recorded as follows;

Male: 56, 66, 62, 81, 75, 73, 83, 68, 48, 70, 60, 77, 86, 44, 72

Female: 63, 77, 65, 71, 74, 60, 76, 61, 67, 72, 64, 65, 55, 89, 45, 53, 68, 73, 50, 81

Use median test to determine whether IQ of male and female students is same in the university.
(Given that the median of combined sample = 68)

Using EXCEL

	A	B	C	D	E
5	scores	Gender			
6	56	Male			
7	66	Male			
8	62	Male			
9	81	Male			
10	75	Male			
11	73	Male			
12	83	Male			
13	68	Male			
14	48	Male			
15	70	Male			
16	60	Male			
17	77	Male			
18	86	Male			
19	44	Male			
20	72	Male			
21	63	Female			
22	77	Female			
23	65	Female			
24	71	Female			
25	74	Female			
26	60	Female			
27	76	Female			
28	61	Female			
29	67	Female			
30	72	Female			
31	64	Female			
32	65	Female			
33	55	Female			
34	89	Female			
35	45	Female			
36	53	Female			
37	68	Female			
38	73	Female			
39	50	Female			
40	81	Female			

Table using pivot table and grouping by 25 since minimum value is 44

Count of scores	Column Labels		
Row Labels	Male	Female	Grand Total
44-68	7	12	19
69-93	8	8	16
Grand Total	15	20	35

2X2 contingency table for below and above median values from pivot table

Row Labels	Male	Female	Grand Total
44-68	7	12	19
69-93	8	8	16
Grand Total	15	20	35

Here	Symbol	value	fomula
no of male	n1	15	=B52
no of female	n2	20	=C52
Total sample size	N=n1+n2	35	=D52
no. of obs. of male ≤ Md	a	7	=B50
no. of obs. of female ≤ Md	b	12	=C50
no. of obs. of male > Md	c	8	=B51
no. of obs. of female> Md	d	8	=C51

Let Md_1 and Md_2 be median IQ of male and female respectively.

Problem to test

H0: There is no significant difference between IQ of male and female ($Md_1 = Md_2$)

H_1: There is significant difference between IQ of male and female. ($Md_1 \neq Md_2$)

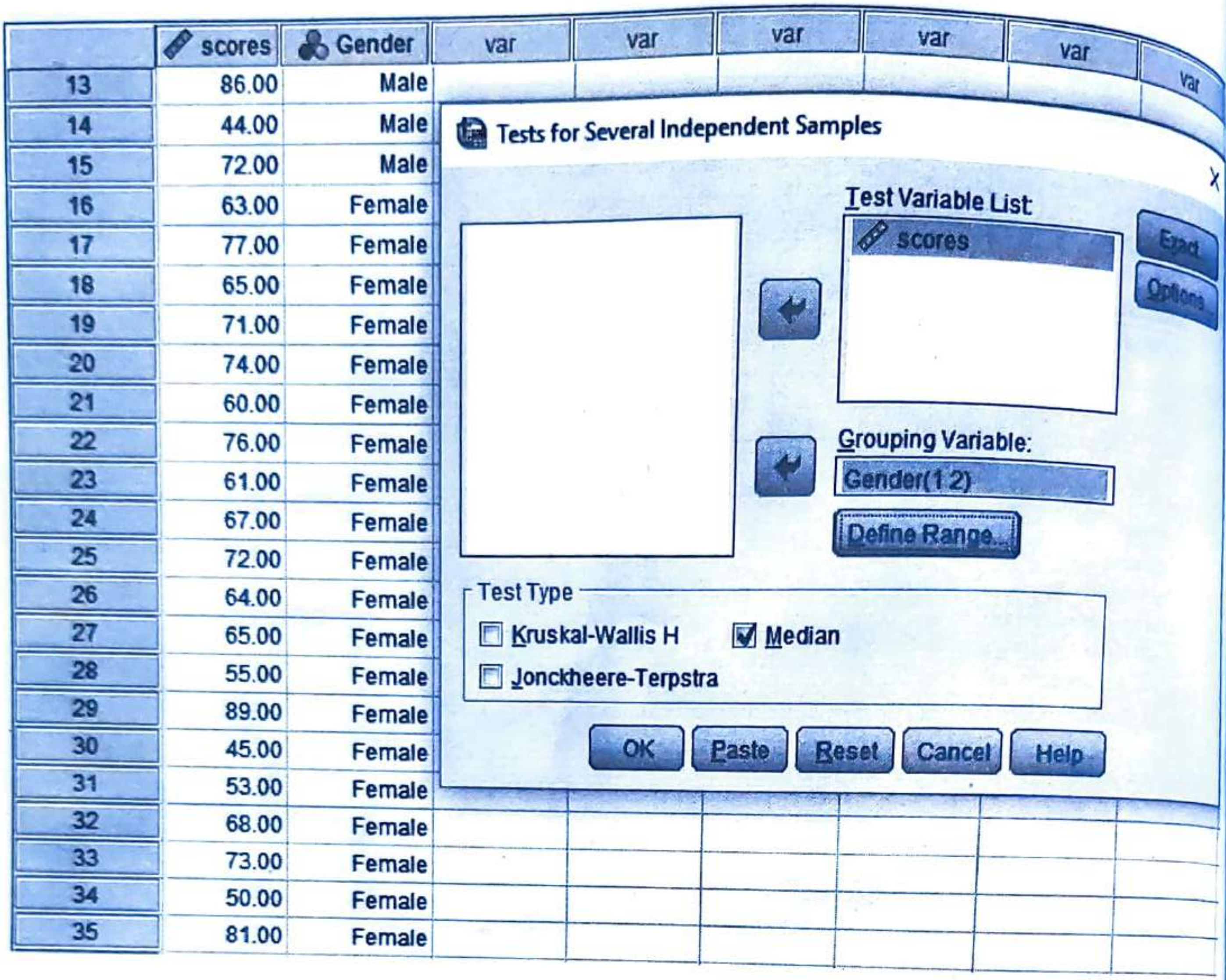
Test statistic

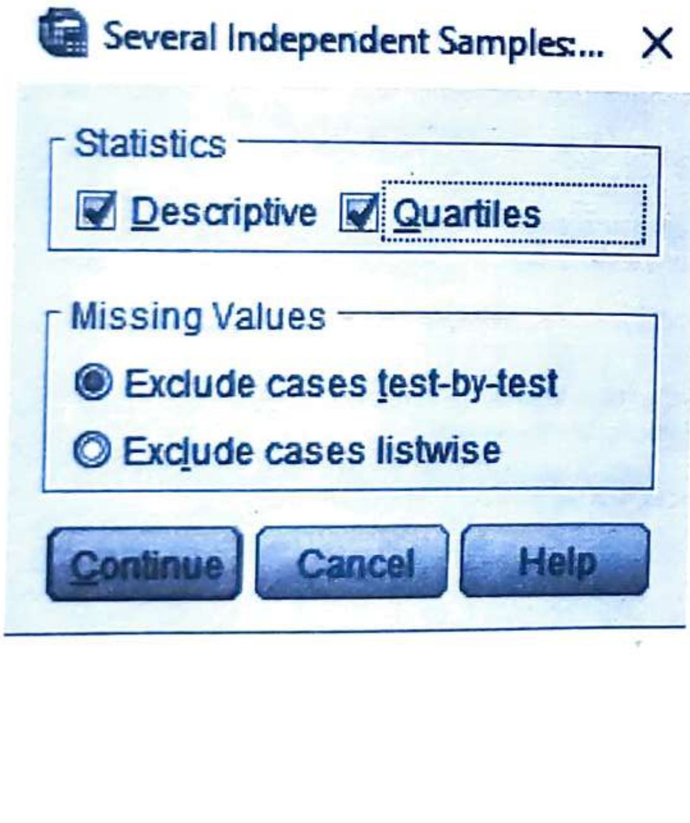
$$\chi^2 = \frac{N(ad-bc)^2}{(a+c)(b+d)(a+b)(c+d)}$$

χ^2	0.614035088		=(C57*(C58*C61-C59*C60)^2)/(D50*D51*B52*C52)
level of significance	α	0.05	
degrees of freedom	(r-1)(c-1)	1	=(2-1)*(2-1)
Critical value			
Tabulated chi-square	tab χ^2 0.05,(1)	3.8415	=CHISQ.INV.RT(C73,C74)
p value	p	0.4333	=CHISQ.DIST.RT(B71,C74)
Decision			
significant approach			
There is no reason to reject null hypothesis H0			=IF(B71<C76,"There is no reason to reject null hypothesis H0", "Reject H0.")
pvalue approach			
It is insignificant.			=IF(C77>C73,"It is insignificant.", "It is significant.")

Using SPSS

Analyze\Non parametric test\Legacy Dailogs\k-independent samples


scores
Gender
var
13 86.00 Male
14 44.00 Male
15 72.00 Male
16 63.00 Female
17 77.00 Female
18 65.00 Female
19 71.00 Female
20 74.00 Female
21 60.00 Female
22 76.00 Female
23 61.00 Female
24 67.00 Female
25 72.00 Female
26 64.00 Female
27 65.00 Female
28 55.00 Female
29 89.00 Female
30 45.00 Female
31 53.00 Female
32 68.00 Female
33 73.00 Female
34 50.00 Female
35 81.00 Female
Tests for Several Independent Samples
Test Variable List:
scores
Exact
Options
Grouping Variable:
Gender(1 2)
Define Range...
Test Type
Kruskal-Wallis H
Median
Jonckheere-Terpstra
OK
Paste
Reset
Cancel
Help


Several Independent Samples:...
Statistics
Descriptive
Quartiles
Missing Values
Exclude cases test-by-test
Exclude cases listwise
Continue
Cancel
Help

Descriptive Statistics

	N	Mean	Std. Deviation	Minimum	Maximum	Percentiles 25th	50th (Median)	75th
scores scores	35	67.1429	11.31408	44.00	89.00	60.0000	68.0000	75.0000
Gender Gender	35	1.57	.502	1	2	1.00	2.00	2.00

Median Test

Frequencies

		Gender Gender 1 Male	2 Female
scores scores	> Median	8	8
	<= Median	7	12

Test Statistics[a]

		scores scores
N		35
Median		68.0000
Chi-Square		.614
df		1
Asymp. Sig.		.433
Yates' Continuity Correction	Chi-Square	.194
	df	1
	Asymp. Sig.	.659

a. Grouping Variable: Gender Gender

Answer: As the asymptotic significance related with the Chi-square test is found to be 0.659 this indicates that the null hypothesis of no difference is accepted for the given data.

Using STATA

median scores, by(Gender) medianties(below)

```
. median scores, by(Gender) medianties(below)

Median test

 Greater |
 than the |       Gender
   median |    Male    Female |    Total
----------+-------------------+---------
       no |       7        12 |       19
      yes |       8         8 |       16
----------+-------------------+---------
    Total |      15        20 |       35

     Pearson chi2(1) =   0.6140   Pr = 0.433

 Continuity corrected:
     Pearson chi2(1) =   0.1943   Pr = 0.659
```

Wilcoxon Matched pair signed rank test

Small Sample (n<25)

Use Wilcoxon Matched pair signed rank test to determine the equality of effectiveness of two types of drugs in suppressing pain from following data.

Patient No.	Drug A	Drug B	Patient No.	Drug A	Drug B
1	6.5	3.5	11	5.4	5.5
2	3.7	3.7	12	4.0	4.1
3	3.9	4.7	13	5.7	4.1
4	6.7	5.0	14	3.9	4.2
5	6.2	5.6	15	3.6	3.7
6	6.7	4.3	16	4.9	4.1
7	6.1	5.4	17	3.9	5.4
8	4.3	5.8	18	5.8	3.7
9	5.5	4.3	19	4.9	4.1
10	6.8	4.3	20	4.9	4.1

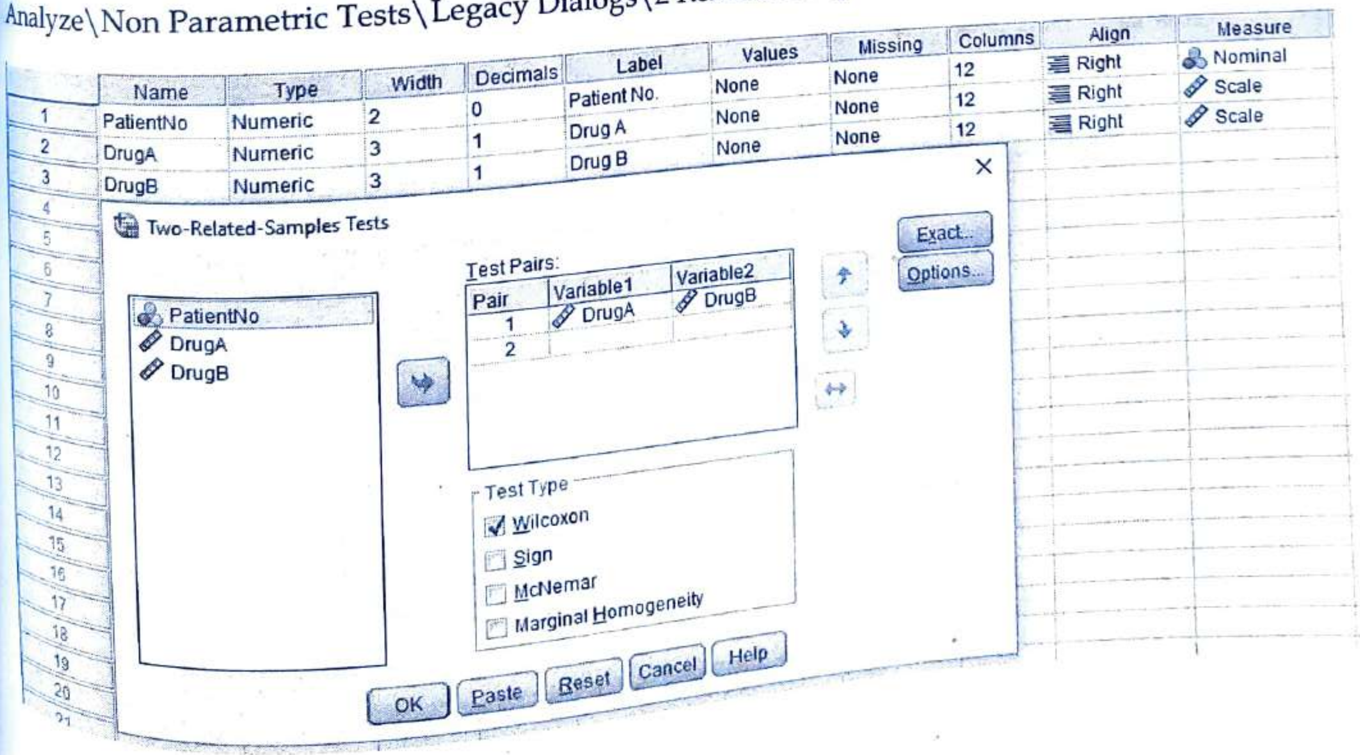
Using Excel

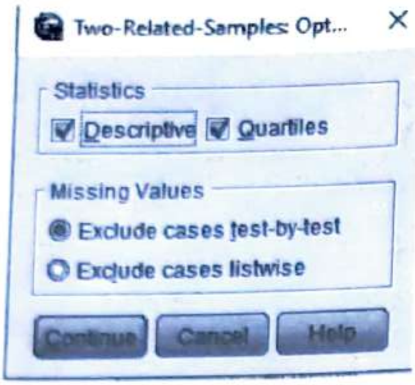
	A	B	C	D	E	F	G	H	I
1	Patient No.	Drug A	Drug B	difference	Sign	|difference|	Rank	Signed Rank	tied count list
2				=B3-C3	=IF(D3>0,1,IF(D3<0,-1,""))	=IF(OR(B3="",C3="",D3=0),"",ABS(D3))	=IF(F3="","",RANK.AVG(F3,$F$3:$F$22,1))	=IF(F3="","",E3*G3)	=IF(H3="","",IF(COUNTIF($G$2:G2,G3)>0,"",COUNTIF(G3:G22,G3)))
3	1	6.5	3.5	3.0	1	3	19	19	1
4	2	3.7	3.7	0.0					
5	3	3.9	4.7	-0.8	-1	0.8	7	-7	1
6	4	6.7	5.0	1.7	1	1.7	15	15	1
7	5	6.2	5.6	0.6	1	0.6	5	5	1
8	6	6.7	4.3	2.4	1	2.4	17	17	1
9	7	6.1	5.4	0.7	1	0.7	6	6	1
10	8	4.3	5.8	-1.5	-1	1.5	12	-12	1
11	9	5.5	4.3	1.2	1	1.2	11	11	1
12	10	6.8	4.3	2.5	1	2.5	18	18	1
13	11	5.4	5.5	-0.1	-1	0.1	1.5	-1.5	2
14	12	4.0	4.1	-0.1	-1	0.1	1.5	-1.5	
15	13	5.7	4.1	1.6	1	1.6	14	14	1
16	14	3.9	4.2	-0.3	-1	0.3	4	-4	1
17	15	3.6	3.7	-0.1	-1	0.1	3	-3	1
18	16	4.9	4.1	0.8	1	0.8	9	9	3
19	17	3.9	5.4	-1.5	-1	1.5	13	-13	1
20	18	5.8	3.7	2.1	1	2.1	16	16	1
21	19	4.9	4.1	0.8	1	0.8	9	9	
22	20	4.9	4.1	0.8	1	0.8	9	9	
23									

	A	B	C	D
		Symbol	Value	Formula
29	Sample Size of Drug A	n1	20	=COUNT(B3:B22)
30	Sample Size of Drug B	n2	20	=COUNT(C3:C22)
31	no of case with zero difference		1	=COUNTIF(D3:D22,0)
32	sum of positive ranks	S(+)	148	=SUMIF(E3:E22,">"&0,H3:H22)
33	sum of negative ranks	S(-)	42	=ABS(SUMIF(E3:E22,"<"&0,H3:H22))
34	count of postive ranks	n+	12	=COUNTIF(H3:H22,">"&0)
35	count of negative ranks	n-	7	=COUNTIF(H3:H22,"<"&0)
36	effective sample size	n'	19	=C31-C32
37	**Since sample size, n<25**			
38	**Test statistic**			
39	T = min {S(+) , S(-)}.	T	42	=MIN(C33:C34)
40	**Level of significance**	α	0.05	
41				
42	**Critical value**			
43	At α level of significance, we obtain critical value from Wilcoxon Matched pair signed rank test table	Tα , n = T0.05,19	46	From Wilcoxon Matched pair signed rank test table
44	Note:	Tα , n where n is corrected sample size after omitting di = 0.		
45	**Decision**			
46	Ho is rejected			
47	Note:			
48	Reject H0 at α level of significance if T < Tα , n, accept otherwise			

Using SPSS

Analyze\Non Parametric Tests\Legacy Dialogs\2 Related Samples





Outputs

NPar Tests

Descriptive Statistics

	N	Mean	Std. Deviation	Minimum	Maximum	Percentiles 25th	Percentiles 50th (Median)	Percentiles 75th
DrugA Drug A	20	5.170	1.1164	3.6	6.8	3.925	5.150	6.175
DrugB Drug B	20	4.480	.7157	3.5	5.8	4.100	4.250	5.300

Wilcoxon Signed Ranks Test

Ranks

		N	Mean Rank	Sum of Ranks
DrugB Drug B - DrugA Drug A	Negative Ranks	12[a]	12.21	146.50
	Positive Ranks	7[b]	6.21	43.50
	Ties	1[c]		
	Total	20		

a. DrugB Drug B < DrugA Drug A

b. DrugB Drug B > DrugA Drug A

c. DrugB Drug B = DrugA Drug A

Test Statistics[a]

	DrugB Drug B - DrugA Drug A
Z	-2.076[b]
Asymp. Sig. (2-tailed)	.038

a. Wilcoxon Signed Ranks Test

b. Based on positive ranks.

Since, $p<0.05$, it is significant.

Using STATA

signrankDrugA=DrugB

```
. signrank DrugA=DrugB

Wilcoxon signed-rank test

        sign |      obs   sum ranks    expected
-------------+---------------------------------
    positive |       12         160       104.5
    negative |        7          49       104.5
        zero |        1           1           1
-------------+---------------------------------
         all |       20         210         210

unadjusted variance        717.50
adjustment for ties         -0.63
adjustment for zeros        -0.25
                         ----------
adjusted variance          716.63

Ho: DrugA = DrugB
             z =   2.073
    Prob > |z| =   0.0382

.
```

Since, $p<0.05$, it is significant.

Large sample size (n> 25)

For large sample size sampling distribution of T is approximately normally distributed with mean μ_T and variance σ^2_T

$\therefore \mu T \frac{n(n+1)}{4}$

$\mu_T = \frac{n(n+1)}{4}$ and $\sigma_T^2 = \frac{n(n+1)(2n+1)}{24}$, where n is corrected sample size if $d_i = 0$.

Test statistic

$$Z = \frac{T - \mu_T}{\sigma_T} = \frac{T - \frac{n(n+1)}{4}}{\sqrt{\frac{n(n+1)(2n+1)}{24}}} \sim N(0, 1) \text{ (Here } n = n_c)$$

Level of significance

Let α be the level of significance. Generally fix $\alpha = 0.05$ unless we are given.

Critical value

Critical value $Z_{tabulated}$ is obtained from table according to level of significance and alternative hypothesis.

Decision

Reject H_0 at a level of significance if $Z > Z_{tabulated}$, Accept otherwise.

Use Wilcoxon Matched pair signed rank test to determine the equality of oxygen level of patients in ICU on the day of admission and 7 days after admission from following data.

Patient No	Day1	Day7	Patient No	Day1	Day7	Patient No	Day1	Day7
1	39.32	64.88	21	61.68	62.28	41	64.08	65.03
2	70.59	67.20	22	71.56	72.80	42	60.34	75.71
3	53.92	74.77	23	75.84	44.88	43	37.35	66.13
4	73.46	72.39	24	46.03	46.65	44	60.93	58.75
5	45.16	58.17	25	32.74	58.93	45	66.48	79.03
6	72.21	61.86	26	69.85	55.90	46	46.74	64.87
7	47.35	82.36	27	42.24	59.98	47	58.38	85.63
8	72.32	52.09	28	45.49	83.25	48	74.70	83.81
9	43.60	65.38	29	58.86	81.93	49	49.50	64.98
10	35.69	76.27	30	72.18	75.00	50	75.66	69.47
11	73.25	71.53	31	48.12	56.99	51	73.74	47.54
12	54.22	80.13	32	50.11	78.41	52	63.07	69.22
13	42.50	50.80	33	71.18	69.54	53	64.88	44.26
14	46.85	69.35	34	64.92	67.39	54	51.60	73.08
15	46.17	73.07	35	71.45	59.49	55	39.14	66.76
16	46.89	84.32	36	51.67	78.51	56	59.49	68.01
17	71.03	67.15	37	75.67	67.74	57	51.75	55.39
18	64.79	68.41	38	40.13	82.34	58	39.06	58.86
19	34.68	59.60	39	52.48	52.68	59	75.43	84.15
20	57.02	54.47	40	53.50	43.92			

Using EXCEL

	A	B	C	D	E	F	G	H	I
1	Patient No	Day1	Day7	difference =B3-C3	Sign =IF(D3>0, 1,IF(D3<0 ,-1,""))	|difference| =IF(OR(B3="" ,C3="",D3=0), "",ABS(D3))	Rank =IF(F3="" ,"",RANK. AVG(F3,$ F$3:$F$6 1,1))	Signed Rank =IF(F3="","", E3*G3)	tied count list =IF(H3="","",I F(COUNTIF($G $2:G2,G3)>0," ",COUNTIF(G3 :G22,G3)))
2	1	39.32	64.88	-25.56	-1	25.56	44	-44	1
3	2	70.59	67.20	3.39	1	3.39	13	13	1
4	3	53.92	74.77	-20.85	-1	20.85	38	-38	1
5	4	73.46	72.39	1.07	1	1.07	5	5	1
6	5	45.16	58.17	-13.01	-1	13.01	29	-29	1
7	6	72.21	61.86	10.35	1	10.35	26	26	1
8	7	47.35	82.36	-35.01	-1	35.01	55	-55	1
9	8	72.32	52.09	20.23	1	20.23	36	36	1
10	9	43.60	65.38	-21.78	-1	21.78	40	-40	1
11	10	35.69	76.27	-40.58	-1	40.58	58	-58	1
12	11	73.25	71.53	1.72	1	1.72	8	8	1
13	12	54.22	80.13	-25.91	-1	25.91	45	-45	1
14	13	42.50	50.80	-8.30	-1	8.3	20	-20	1
15	14	46.85	69.35	-22.50	-1	22.5	41	-41	1
16	15	46.17	73.07	-26.90	-1	26.9	49	-49	1
17	16	46.89	84.32	-37.43	-1	37.43	56	-56	1
18	17	71.03	67.15	3.88	1	3.88	16	16	1
19	18	64.79	68.41	-3.62	-1	3.62	14	-14	1
20	19	34.68	59.60	-24.92	-1	24.92	43	-43	1
21	20	57.02	54.47	2.55	1	2.55	11	11	1

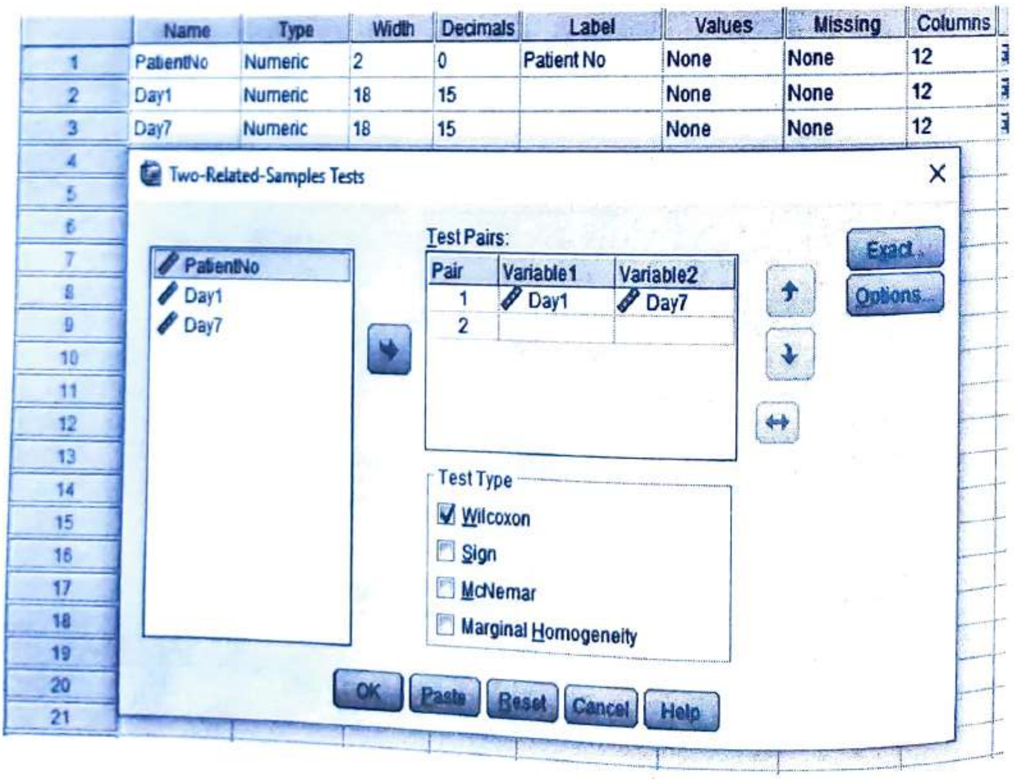
Only part of data is shown

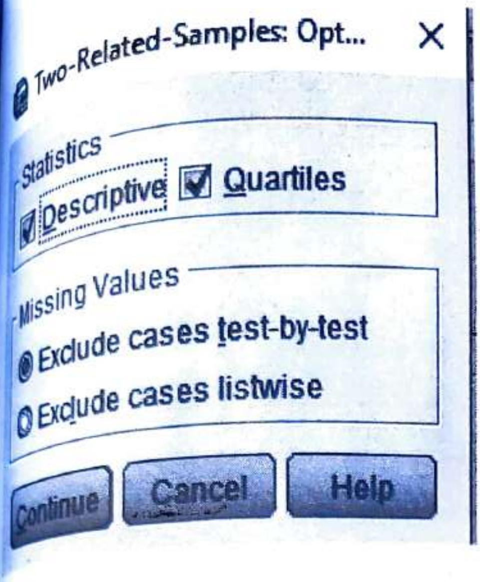
	A	B	C	D	E	F	G	H
64	H0: there is no significant difference between oxygen level							
65	H1: there is significant difference between osygen level							
66								
67		Symbol	Value	Formula				
68	Sample Size of Drug A	n1	59	=COUNT(B3:B22)				
69	Sample Size of Drug B	n2	59	=COUNT(C3:C22)				
70	no of case with zero difference		0	=COUNTIF(D3:D22,0)				
71	sum of positive ranks	S(+)	388	=SUMIF(E3:E22,">"&0,H3:H22)				
72	sum of negative ranks	S(-)	1382	=ABS(SUMIF(E3:E22,"<"&0,H3:H22))				
73	count of postive ranks	n+	17	=COUNTIF(H3:H22,">"&0)				
74	count of negative ranks	n-	42	=COUNTIF(H3:H22,"<"&0)				
75	effective sample size	n'	59	=C31-C32				
76		T	388	=MIN(C71:C72)				
77	Since Sample Size n>25							
78	For large sample size sampling distribution of T is approximately normally distributed with mean							
79								
80	μ_T *and variance* σ_T^2							
81								
82	mean	Symbol	Value	Formula				
83		μ_T	885	=C75*(C75+1)/4	$\frac{n(n+1)}{4}$			
84	Variance	σ_T^2	17552.5	=(C75*(C75+1)*(2*C75+1))/(24)		$\frac{n(n+1)(2n+1)}{24}$		

	A	B	C	D	E	F
85						
86	Test Statistics	$\frac{T-\mu_T}{\sigma_T}$		$\frac{T-\frac{n(n+1)}{4}}{\sqrt{\frac{n(n+1)(2n+1)}{24}}} \sim N(0,1)$		
87	Z =					
88						
89						
90	cal Z		-3.75134406	=(C76-C82)/SQRT(C84)		
91						
92	level of significance	α	0.05			
93	tabulated Z for Two tailed test	$z_{\alpha/2}$	1.959963985	=NORM.S.INV(1-C92/2)		
94						
95	tabulated Z for one tailed test	z_α	1.644853627	=NORM.S.INV(1-C92)		
96						
97	pvalue					
98	Two tailed		0.000175889	=2*NORMSDIST(C90)		
99	One tailed		8.79446E-05	=NORMSDIST(C90)		
100						
101	Decision:					
102	Significant approach for two taied test					
103	Null hypothesis H0 is rejected					
104						
105	p value approach					
106	It is significant					
107						

Using SPSS

Analyze\Non Parametric Tests\Legacy Dialogs\2 Related Samples





Outputs

NPar Tests

Descriptive Statistics

	N	Mean	Std. Deviation	Minimum	Maximum	Percentiles 25th	Percentiles 50th (Median)	Percentiles 75th
Day1	59	56.93288136	13.00609659	32.74000000	75.84000000	46.17000000	57.02000000	71.03000000
Day7	59	66.66932203	11.19511514	43.92000000	85.63000000	58.86000000	67.20000000	75.00000000

Wilcoxon Signed Ranks Test

Wilcoxon Signed Ranks Test

Ranks

		N	Mean Rank	Sum of Ranks
Day7 - Day1	Negative Ranks	17[a]	22.82	388.00
	Positive Ranks	42[b]	32.90	1382.00
	Ties	0[c]		
	Total	59		

a. Day7 < Day1

b. Day7 > Day1

c. Day7 = Day1

Test Statistics[a]

	Day7 - Day1
Z	-3.751[b]
Asymp. Sig. (2-tailed)	.000

a. Wilcoxon Signed Ranks Test

b. Based on negative ranks.

Since, $p<0.05$, it is significant.

Using STATA

signrank Day1=Day7

```
. signrank Day1=Day7

Wilcoxon signed-rank test

        sign |      obs   sum ranks    expected
-------------+---------------------------------
    positive |       17         388         885
    negative |       42        1382         885
        zero |        0           0           0
-------------+---------------------------------
         all |       59        1770        1770

unadjusted variance    17552.50
adjustment for ties        0.00
adjustment for zeros       0.00
                     ----------
adjusted variance      17552.50

Ho: Day1 = Day7
             z =  -3.751
    Prob > |z| =   0.0002
```

Since, $p<0.05$, it is significant.

Cochran Q test

Five housewives were asked for the acceptability of four brands of lipsticks for daily use. The response of acceptability (A) and rejection (R) are given below;

House wives	Lipstick Brands			
	Alfa	Beta	Gamma	Delta
H_1	A	R	A	R
H_2	R	A	A	R
H_3	R	A	R	A
H_4	A	R	R	R
H_5	A	A	R	A

Test whether there is any significant difference between brands with respect to acceptability

Solution: Using Excel

Cochran's Q test is a non-parametric statistical test to verify whether k treatments have identical effects. It is the nonparametric counterpart of the two-way ANOVA in RBD. The data for the test are binary that needed to be coded 0, 1 for analysis. In this test the null hypothesis is acceptability of four different lipstick brands are same among housewives.

A	B	C	D	E	F	G
	Lipstick Brands					
House wives	Alfa	Beta	Gamma	Delta		
H_1	A	R	A	R		
H_2	R	A	A	R		
H_3	R	A	R	A		
H_4	A	R	R	R		
H_5	A	A	R	A		
	Lipstick Brands					
House wives	Alfa	Beta	Gamma	Delta	C_j	C_j^2
H_1	1	0	1	0	2	4
H_2	0	1	1	0	2	4
H_3	0	1	0	1	2	4
H_4	1	0	0	0	1	1
H_5	1	1	0	1	3	9
Ri	3	3	2	2	10	22
Ri^2	9	9	4	4	26	

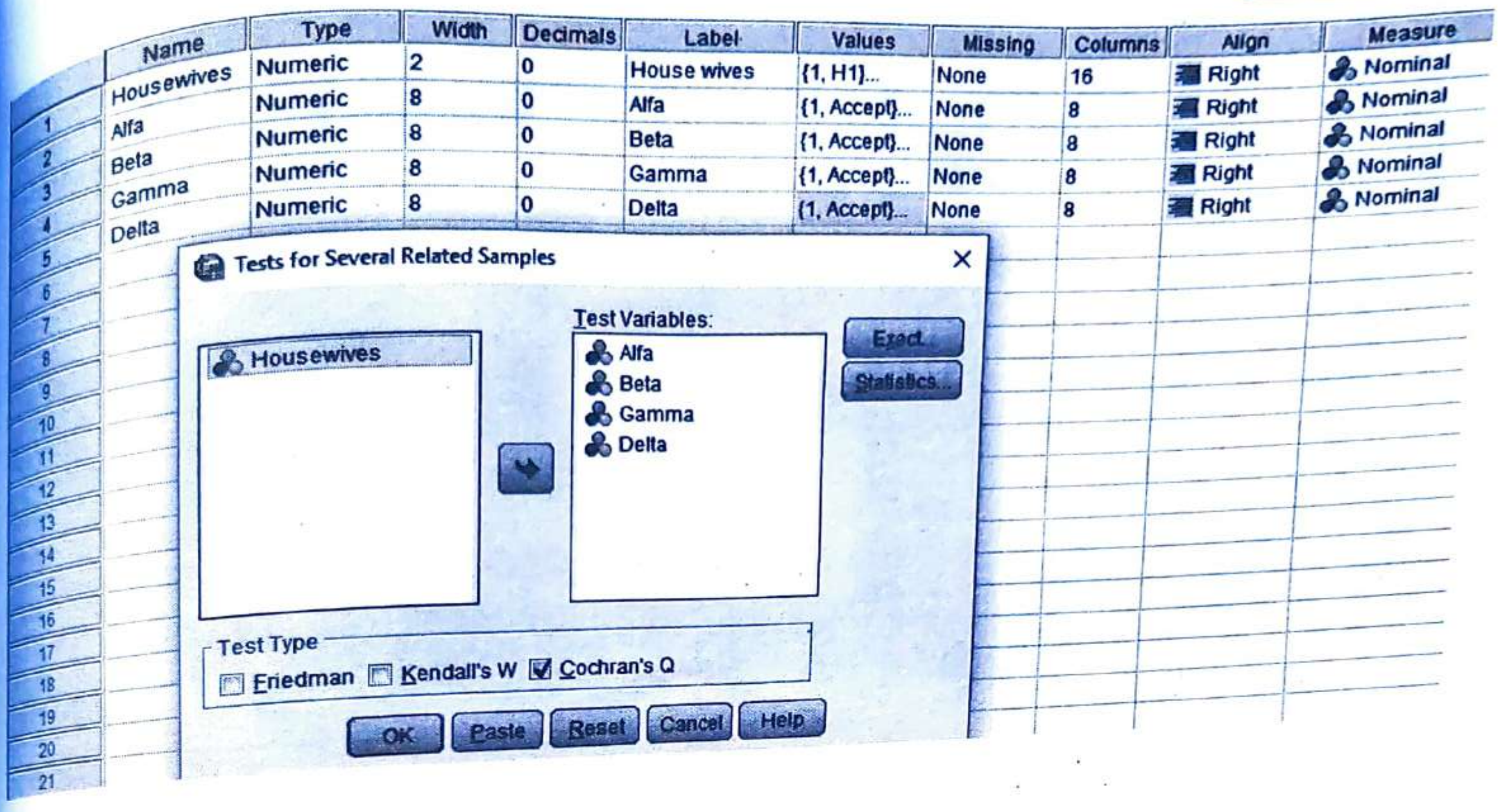
	A	B	C	D
19				
20		symbol	value	formula
21	Number of brands	k	4	=COUNT(B11:E11)
22	success	1		
23	Number of housewives	n	5	=COUNT(B11:B15)
24		ΣR_i	10	
25		ΣR_i^2	26	
26		ΣC_j	10	
27		ΣC_j^2	22	
28				
29	**Problem to test**			
30	H_0 : There is no significant difference between brands			
31	H_1: There is at least one significant difference between brands.			
32	**Test statistic**			
33–35	$Q = \frac{(k-1)\{k\sum_{i=1}^{k} R_i^2 - (\sum_{i=1}^{k} R_i)^2\}}{k\sum_{j=1}^{n} C_j - \sum_{j=1}^{n} C_j^2} \sim \chi^2_{k-1}$			
36				
37		Q	0.6666667	=((C21-1)*(C21*C25-(C24)^2))/(C21*C26-C27)
38	level of significance	α	0.05	
39	degrees of freedom	k-1	3	
40	Critical value	$\chi^2_{\alpha,(k-1)}$	7.8147279	
41	p value	p	0.8810148	
42				

	A	D
43	Decision	
44	Significant Approcah	
45	There if no reason to reject null hypothesis H0.	=IF(C37<C40,"There if no reason to reject null hypothesis H0.","Reject H0")
46		
47	p value Approach	
48	It is in significant.	
49	Conclusion:	=IF(C41>C38,"It is in significant."," It is significant.")
50	There is no significant difference between brands according to acceptability.	
51		

Answer: Here the analysis shows the p-value associated with the test statistics Q is 0.88 that indicates acceptance of the null hypothesis of no difference.

Solution: Using SPSS

Analyze\Nonparametric test\Legacy Dialogs\k Related Samples



Outputs

Cochran Test

Frequencies

	Value 0	1
Alfa Alfa	2	3
Beta Beta	2	3
Gamma Gamma	3	2
Delta Delta	3	2

Test Statistics

N	5
Cochran's Q	.667[a]
df	3
Asymp. Sig.	.881

a. 1 is treated as a success.

Kruskal Wallis H test

For non-repeated ranks

Following are the scores obtained by trainees in 3 different categories. Test whether 3 categories have performed equally.

categories	scores									
A	68	65	92	82	62	64	68	92	86	64
B	93	86	73	87	76	85	67	79	75	75
C	95	72	85	70	80	80	78	85	72	90

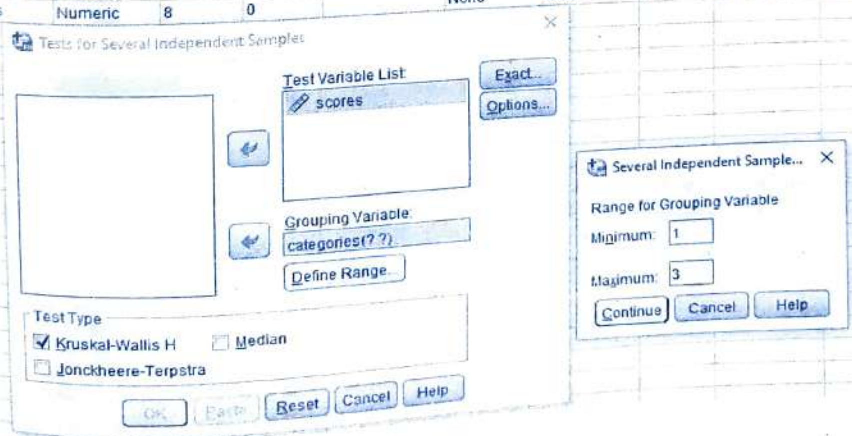
Using Excel

	A	B	C	D	E	F	G
6	categories	scores	rank =RANK.AVG(B8:B22,B8:B22,1)				
7							
8	1	67	9				
9	1	77	14				
10	1	79	15				
11	1	72	13				
12	1	64	7				
13	2	55	3				
14	2	50	1				
15	2	58	5				
16	2	57	4				
17	2	54	2				
18	3	66	8				
19	3	70	12				
20	3	69	11				
21	3	62	6				
22	3	68	10				
23			120				
24			=SUM(C8:C22)				
25							
26	test statistics						
27	H0: there is no significant difference betweenmedain score of three categories						
28	H1: there is significant difference betweenmedain score of three categories						

	A	B	C	D	E
32	cases	code	symbol	value	formula
33	no of categories		k	3	=MAX(A8:A22)
34	no of categories	1	n1	5	=COUNTIF($A$8:$A$22,B34)
35	no of categories	2	n2	5	=COUNTIF($A$8:$A$22,B35)
36	no of categories	3	n3	5	=COUNTIF($A$8:$A$22,B36)
37	no of observations		n	15	=COUNT(A8:A22)
38	sum of rank in	1	R1	58	=SUMIF($A$8:$A$22,B38,$C$8:$C$22)
39	sum of rank in	2	R2	15	=SUMIF($A$8:$A$22,B39,$C$8:$C$22)
40	sum of rank in	3	R3	47	=SUMIF($A$8:$A$22,B40,$C$8:$C$22)
41					
42					
43	For not tied observations				
44	H	$=\frac{12}{n(n+1)}\sum_{i=1}^{k}\frac{R_i^2}{n_i}-3(n+1)$			
45				9.98	=((12/(D37^2+D37))*((D38^2/D34)+(D39^2/D35)+(D40^2/D36))-3*(D37+1))
46					
47					
48					
49	level of significance		α	0.05	
50	degrees of freedom		k-1	2	=D33-1
51	critical value		$\chi^2_{\alpha,k-1}$	5.99146	=CHISQ.INV(1-D49,D50)
52	p value		p	0.99319	=CHISQ.DIST(D45,D50,TRUE)
53					
54	Decision				
55	Significant approach				
56	Reject Null Hypothesis H0				=IF(D45<D51,"There is no reason to reject null hypothesis H0","Reject Null Hypothesis H0")
57					
58	p value approach				
59	it is insignificant				=IF(D52>D49,"it is insignificant","it is significant")
60					

Using SPSS

	Name	Type	Width	Decimals	Label	Values	Missing	Columns	Align	Meas
1	categories	Numeric	8	0		{1, A}...	None	8	Right	Nomir
2	scores	Numeric	8	0		None	None	8	Right	Scale



Outputs

Descriptive Statistics

	N	Mean	Std. Deviation	Minimum	Maximum	Percentiles 25th	50th (Median)	75th
scores	15	64.53	8.493	50	79	57.00	66.00	70.00
categories	15	2.00	.845	1	3	1.00	2.00	3.00

Kruskal-Wallis Test

Ranks

	categories	N	Mean Rank
scores	1 A	5	11.60
	2 B	5	3.00
	3 C	5	9.40
	Total	15	

Test Statistics[a,b]

	scores
Kruskal-Wallis H	9.980
df	2
Asymp. Sig.	.007

a. Kruskal Wallis Test

Using STATA

kwallis scores, by(categories)

```
. kwallis scores, by(categories)

Kruskal-Wallis equality-of-populations rank test
```

catego~s	Obs	Rank Sum
A	5	58.00
B	5	15.00
C	5	47.00

```
chi-squared =     9.980 with 2 d.f.
probability =     0.0068

chi-squared with ties =     9.980 with 2 d.f.
probability =     0.0068
```

Answer: As we found that P value for Chi-square test with tie is 0.0068 which is smaller than 0.05 we can conclude that the sores among three categories is significantly different as null is rejected.

For repeated ranks

Following are the scores obtained by trainees in 3 different categories. Test whether 3 categories are equally performed.

categories	scores									
A	68	65	92	82	62	64	68	92	86	64
B	93	86	73	87	76	85	67	79	75	75
C	95	72	85	70	80	80	78	85	72	90

Using Excel

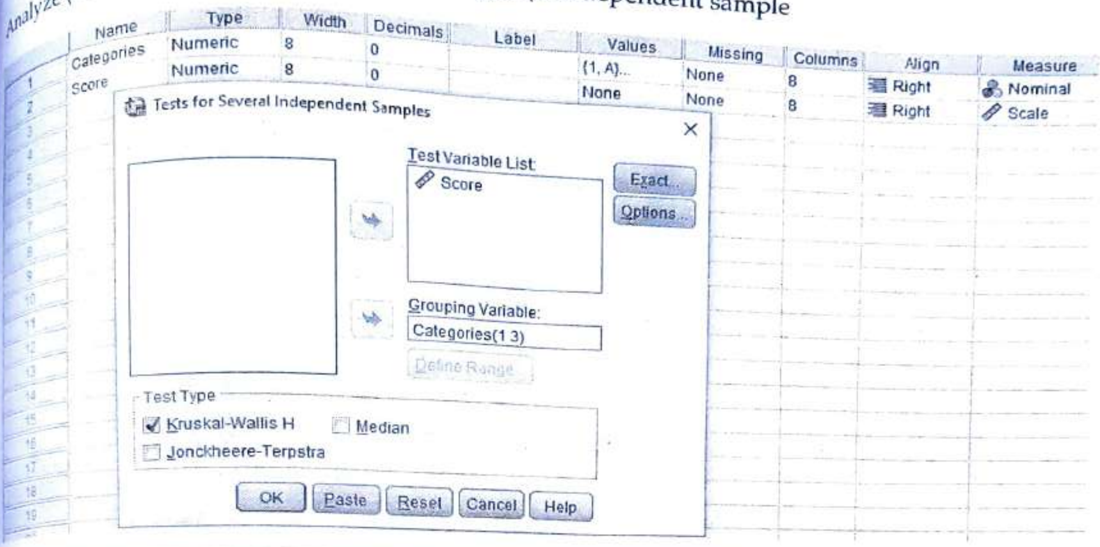
A	B	C	D	E
categories	scores	rank		
		=RANK.AVG(B8:B37,B8:B37,1)		
1	68	6.5		
1	65	4		
1	92	27.5		
1	82	19		
1	62	1		
1	64	2.5		
1	68	6.5		
1	92	27.5		
1	86	23.5		
1	64	2.5		
2	93	29		
2	86	23.5		
2	73	11		
2	87	25		
2	76	14		
2	85	21		
2	67	5		
2	79	16		
2	75	12.5		
2	75	12.5		
3	95	30		
3	72	9.5		
3	85	21		
3	70	8		
3	80	17.5		
3	80	17.5		
3	78	15		
3	85	21		
3	72	9.5		
3	90	26		
		465		
		=SUM(C8:C37)		

	A	B	C	D	E
41	test statistics				
42	H0: there is no significant difference betweenmedain score of three categories				
43	H1: there is significant difference betweenmedain score of three categories				
44					
45					
46					
47	cases	code	symbol	value	formula
48	no of categories		k	3	=MAX(A8:A37)
49	no of categories	1	n1	10	=COUNTIF($A$8:$A$37,B49)
50	no of categories	2	n2	10	=COUNTIF($A$8:$A$37,B50)
51	no of categories	3	n3	10	=COUNTIF($A$8:$A$37,B51)
52	no of observations		n	30	=COUNT(A8:A37)
53	sum of rank in	1	R1	120.5	=SUMIF($A$8:$A$37,B53,$C$8:$C$37)
54	sum of rank in	2	R2	169.5	=SUMIF($A$8:$A$37,B54,$C$8:$C$37)
55	sum of rank in	3	R3	175	=SUMIF($A$8:$A$37,B55,$C$8:$C$37)
56					
57	no of repeated ranks	2.5	t1	2	=COUNTIF($C$8:$C$37,B57)
58	no of repeated ranks	6.5	t2	2	=COUNTIF($C$8:$C$37,B58)
59	no of repeated ranks	9.5	t3	2	=COUNTIF($C$8:$C$37,B59)
60	no of repeated ranks	12.5	t4	2	=COUNTIF($C$8:$C$37,B60)
61	no of repeated ranks	17.5	t5	2	=COUNTIF($C$8:$C$37,B61)
62	no of repeated ranks	23.5	t6	2	=COUNTIF($C$8:$C$37,B62)
63	no of repeated ranks	27.5	t7	2	=COUNTIF($C$8:$C$37,B63)
64	no of repeated ranks	21	t8	3	=COUNTIF($C$8:$C$37,B64)

	A	B	C	D	E
65	$\sum T = \sum_{p=1}^{8} T_p = \sum_{i=1}^{8} (t_i^3 - t)$			66	=SUMPRODUCT(D57:D64^3)-SUM(D57:D64)
66					
67					
68					
69	For ties observations				
70	$H_{corrected}$	$= \dfrac{\frac{12}{n(n+1)} \sum_{i=1}^{k} \frac{R_i^2}{n_i} - 3(n+1)}{1 - \frac{\sum T}{n^3 - n}}$		2.328925	=((12/(D52^2+D52))*((D53^2/D49)+(D54^2/D50)+(D55^2/D51))-3*(D52+1))/(1-(D66/(D52^3-D52)))
71					
72					
73					
74					
75	level of significance		α	0.05	
76	degrees of freedom		k-1	2	=D48-1
77	critical value		$\chi^2_{\alpha,k-1}$	5.991465	=CHISQ.INV(1-D75,D76)
78	p value		p	0.68791	=CHISQ.DIST(D71,D76,TRUE)
79					
80	Decision				
81	Significant approach				
82	There is no reason to reject null hypothesis H0				=IF(D71<D77,"There is no reason to reject null hypothesis H0","Reject Null Hypothesis H0")
83					
84	p value approach				
85	it is insignificant				=IF(D78>D75,"it is insignificant","it is significant")

Using SPSS

Analyze\Non parametric test\Legacy dialogue\k independent sample



NPar Tests

Descriptive Statistics

	N	Mean	Std. Deviation	Minimum	Maximum	Percentiles 25th	50th (Median)	75th
Score	30	78.20	9.739	62	95	69.50	78.50	86.00
Categories	30	2.00	.830	1	3	1.00	2.00	3.00

Kruskal-Wallis Test

Ranks

	Categories	N	Mean Rank
Score	1 A	10	12.05
	2 B	10	16.95
	3 C	10	17.50
	Total	30	

Test Statistics[a,b]

	Score
Kruskal-Wallis H	2.329
df	2
Asymp. Sig.	.312

a. Kruskal Wallis Test

b. Grouping Variable: Categories

Using STATA

kwallis Score, by(Categories)

```
kwallis Score, by(Categories)

Kruskal-Wallis equality-of-populations rank test
```

Catego~s	Obs	Rank Sum
A	10	120.50
B	10	169.50
C	10	175.00

```
chi-squared =     2.323 with 2 d.f.
probability =     0.3130

chi-squared with ties =     2.329 with 2 d.f.
probability =     0.3121
```

Answer: As we found that P value with tie for the Chi-square test is 0.3121 which is larger than 0.05 we can conclude that the sores among three categories is not significantly different as null is accepted.

Friedman F test

A survey was conducted in four hospitals in a particular city to obtain the number of babies born over a 12 months' period. This time period was divided into four seasons to test the hypothesis that the birth rate is constant over all the four seasons. The results of the survey were as follows:

Hospital	No of births			
	Winter	Spring	Summer	Fall
A	92	72	94	77
B	15	16	10	17
C	58	71	51	62
D	19	26	20	18

Analyze the data using Friedman two way ANOVA test.

Using EXCEL

Here we assume that the depended variable (here birth) is normally distributed, which is the needed assumption for this test. Also, we perform Friedman test when we have one within-subjects independent variable(hospital) with two or more levels(seasons) and a dependent variable. We use this test to determine if there is a difference in the birth by season, so the null hypothesis tested will be H0: birth is distributed homogeneously in all seasons vs H1: at least in some season it is different from other.

Hospital	No of births			
	Winter	Spring	Summer	Fall
A	92	72	94	77
B	15	16	10	17
C	58	71	51	62
D	19	26	20	18

ranks						
Hospital	Winter	Spring	Summer	Fall		
A	3	1	4	2		formula
B	2	3	1	4		=RANK.AVG(B4:E4,B4:E4,1)
C	2	4	1	3		=RANK.AVG(B5:E5,B5:E5,1)
D	2	4	3	1		=RANK.AVG(B6:E6,B6:E6,1)
Ri	9	12	9	10		=RANK.AVG(B7:E7,B7:E7,1)
Ri^2	81	144	81	100		=SUM(E11:E14)
					406	=E15^2
						=SUM(B16:E16)

Test Hypothesis

H0: The birth rate is constant over all four seasons.

H1: The birth rate is not constant over all four seasons.

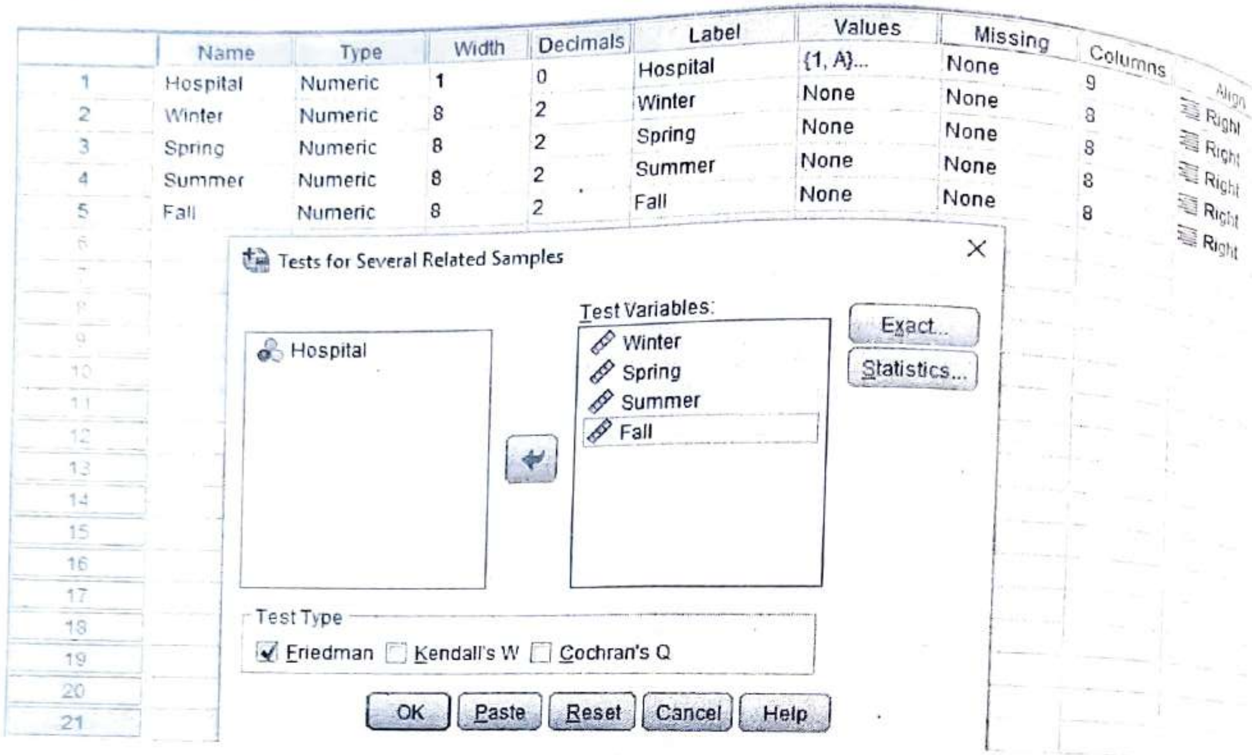
cases	symbols	value	formula
sample size	n	4	
no of samples	k	4	

cases	symbols	value	formula
sample size	n	4	=COUNT(B11:B14)
no of samples	k	4	=COUNT(B11:E11)
Test statistic	$F_r = \frac{12}{nk(k+1)}\sum_{i=1}^{k} R_i^2 - 3n(k+1)$		
	F_r	0.9	=(12/(C25*C26*(C26+1)))*F16-3*C25*(C26+1)
level of significance	α	0.05	
Critical value	p=P(Fr>0.9)	0.9	from table
decision			
There is no reason to reject Null hypothesis H0			

Answer: As from the Friedman test the p-value of the test is 0.9 we conclude that the null hypothesis is correct. This indicates there is no seasonal effect found in the birth rate in the data.

Using SPSS

Analyze\Nonparametric test\Legacy Dialogs\k Related Samples



Outputs

Friedman Test

Ranks

	Mean Rank
Winter Winter	2.25
Spring Spring	3.00
Summer Summer	2.25
Fall Fall	2.50

Test Statistics[a]

N	4
Chi-Square	.900
df	3
Asymp. Sig.	.825

a. Friedman Test

Friedman test for repeated ranks

Three different advertising media T.V., Radio and Newspaper are being compared to study their effectiveness in promoting sales of WaiWai noodles. Each advertising media is exposed for specified period of time and sales (000 package) from 10 stores located at different areas are recorded.

Advertising Media	Stores									
	A	B	C	D	E	F	G	H	I	J
T.V.	20	21	15	12	14	17	21	16	20	18
Radio	7	9	11	12	10	10	14	12	8	7
News Paper	8	6	11	12	9	6	8	10	8	6

Are three advertising media equally effective, use Friedman two-way ANOVA test.

Using EXCEL

Advertising Media	Stores									
	A	B	C	D	E	F	G	H	I	J
T.V.	20	21	15	12	14	17	21	16	20	18
Radio	7	9	11	12	10	10	14	12	8	7
News Paper	8	6	11	12	9	6	8	10	8	6

ranks												
Advertising Media	A	B	C	D	E	F	G	H	I	J	Ri	Ri^2
T.V.	3	3	3	2	3	3	3	3	3	3	29	841
Radio	1	2	1.5	2	2	2	2	2	1.5	2	18	324
News Paper	2	1	1.5	2	1	1	1	1	1.5	1	13	169
												1334

Test Hypothesis

H0: Three advertising media are equally effective.

H1: Three advertising media are not equally effective.

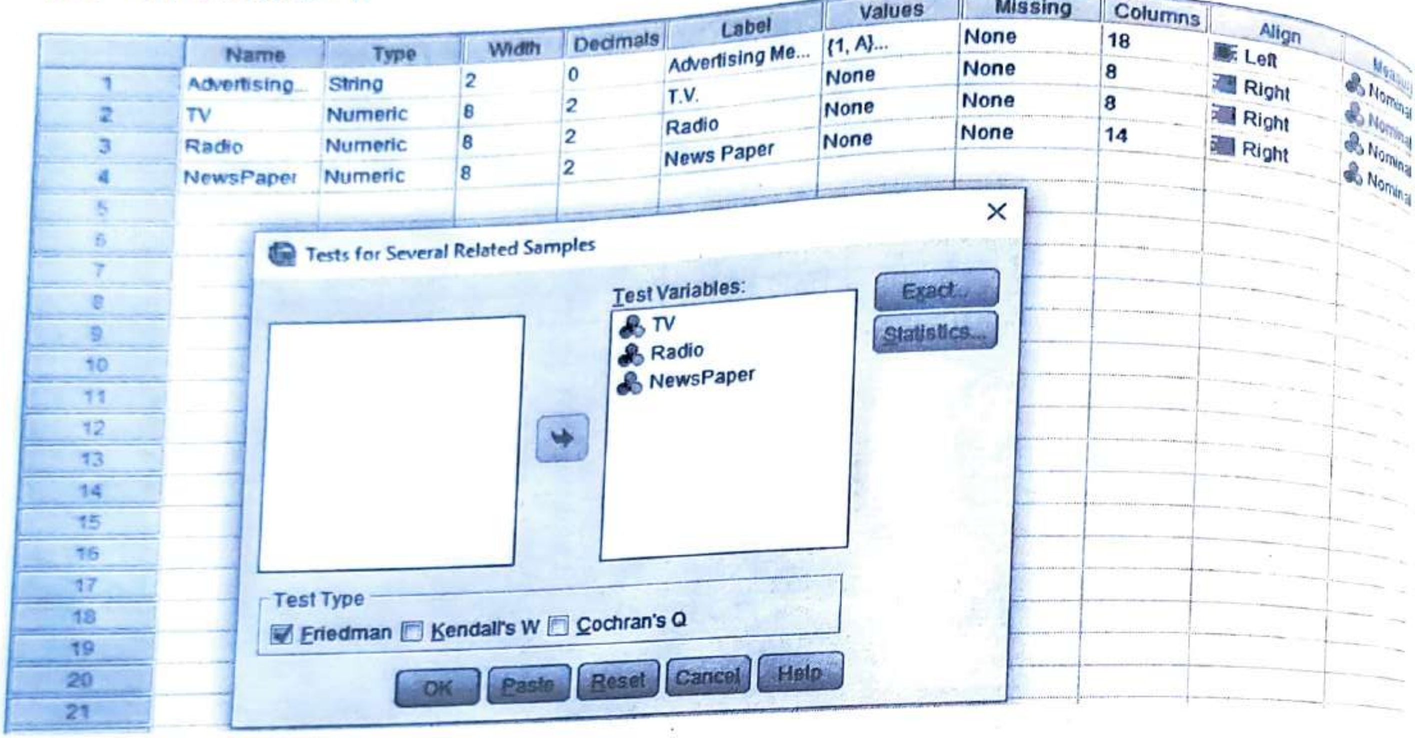
A		B	C	D	G
cases		symbols	value	formula	
sample size		n	10	=COUNT(B11:B14)	
no of samples		k	3	=COUNT(B11:E11)	
	1.5	t1	2		repeats of C
	2	t2	3		repeats of D
	1.5	t3	2		repeats of I

$$F_r = \frac{\frac{12}{nk(k+1)}\sum_{i=1}^{k} R_i^2 - 3n(k+1)}{1-\sum\frac{(t_i^3 - t_i)}{n(k^3-k)}}$$

A	B	C	D
Test statistic			
	F_r	15.76471	=((12/(C21*C22*(C22+1)))*M13-3*C21*(C22+1))/(1-(((C23^3-C23)+(C24^3-C24)+(C25^3-C25))/(C21*(C22^3-C22))))
level of significance	α	0.05	
degrees of freedom	df	2	=C22-1
Critical value	$\chi^2_{0.05(2)}$	5.991465	=CHISQ.INV.RT(C32,C33)
p value	p	0.000377	=CHISQ.DIST.RT(C30,C33)
decision			
significant approach			
Reject H0			=IF(C34>C30,"There is no reason to reject null hypothesis H0", "Reject H0")
p value approach			
It is significant			=IF(C35>C32,"It is insignificant","It is significant")

Using SPSS

Analyze\Nonparametric test\Legacy Dialogs\k Related Samples



OUTPUTS

Friedman Test

Ranks

	Mean Rank
TV T.V.	2.90
Radio Radio	1.80
NewsPaper News Paper	1.30

Test Statistics[a]

N	10
Chi-Square	15.765
df	2
Asymp. Sig.	.000

a. Friedman Test

□□□