Decision theory

Introduction:

Decision theory may be defined as "a process which results in the selection, from a set of alternatives courses of action, the courses (course) of ~~actitic~~ action which is considered to be more statisfactory than others or best to meet the objectives of the decision problem, as judged by the decision maker".

Decision making ~~because~~ becomes essential for the management of an organization or by an individual when he feels dis-satisfied with the existing decision or when alternative courses of action are available. Decision theory plays an important role in helping the decision maker to make better decision under uncertain future conditions. In the present advancing computer age, the decision makers of a business organization can not afford to make a decision by marely guessing as the wrong decision may lead to disastrous consequences.

The OR Technique that requires the basic knowledge of statistics, mathematics and computers help the decision maker to make effective decisions in the best interest of the organization.

Steps in decision theory approach

(1) To determine various alternative/strategies:

Firstly, the decision maker has to select the various alternative courses of action from which the final decision is to be made to achieve his objective. The decision maker has control over the choice of these.

(2) To know the effect of decision:

corresponding to each alternative there are various events (states) that can happen in future which should b identified by the decision maker beforehand. These event are often called " states of nature" or " outcomes and are not in control of the decision maker since they are affected by external reasons.

For example: If some manufacturer takes decision to manufacture criteria product, then the demand of that item in the market may be high, median, low or nil. These four outcomes are not in his control.

③ To determine pay-offs:

corresponding to each strategy and event, there is a consequence called the pay-off which measures the net benefit to the decision maker that occurs from a combination of decision alternative and event. These pay-off are known as conditional profit values or conditional costs.

④ Construction of pay-off table:

Let a problem under consideration has 'm' possible events denoted by $E_1, E_2, \ldots, E_m$ and n alternatives denoted by $A_1, A_2, \ldots, A_n$. Let P_{ij} be the pay-off to the decision maker corresponding to ith event E_i and jth alternative A_j. Thus we get $m \times n$ pay-offs which can be arranged in tabular form, which is known as pay-off table or conditional pay-off table

Sates of nature (events)	Alternatives / conditional pay-off						
	A_1	A_2	------	A_j	----	A_n
E_1	P_{11}	P_{12}	-----	P_{1j}	---	A_{1n}
E_2	P_{21}	P_{22}	-----	P_{2j}	---	P_{2n}
⋮	⋮	⋮		⋮		
E_i	P_{i1}	P_{i2}	-----	P_{ij}	---	P_{in}
⋮	⋮	⋮		⋮		⋮
E_m	P_{m1}	P_{m2}	-----	P_{mj}	---	P_{mn}

⑤ selection of optimum decision criterion:

Finally the decision maker will choose the alternative which results in largest pay-off or lowest cost.

Decision making environment (or situations):

Decision theory provides a rational approach to determine the best strategies where a decision maker is faced with several decision alternatives/strategies.

(i) Decision making under certainty:

Here, the decision maker knows with certainty the outcome resulting from the selection of a particular courcular course of action.

Examples of such environment are linear programming, transportation and assignment method, etc.

(ii) Decision making under uncertainity:

Here the decision maker lacks the sufficient knowledge to follow him to assess the probabilities to the various events associated with each decision.

(iii) Decision making under risk:

In this environment, more than one states of nature associated with each decision exist and the decision maker is supposed to have sufficient information, or exprerience to enable him to assign probabilities to the various events.

(iv) Decision making under conflict:

In this environment, the states of nature are neither known completely nor they are completely uncertain. In this situation more than one compelitors have conflicting interests on the samme issue and each of them try to take decisions in his interest.

Decision making under risk:

In this environment, more than one states of nature exist with each decision and the decision ma[ker] has sufficient information to assign probabilities to ea[ch] of those states

For decision problems under risk, the most pop[ular] decision criterions for evaluating the alternative a[re]:

(i) Expected Monetary value (EMV) criterion.

(ii) Expected oppotunity loss (EOL) criterion.

Expected Monetary Value (EMV) criterion (or expected value criterions):

In this criterion, the expected monetary valu[e] (EMV) for each alternative, which is the sum of possible pay-off of the alternative, each weighted by the probabi[lity] of that pay-off occuring, is calculated. Then the altern[ative] corresponding to highest EMV (lowest in cost table) is select[ed] as a decision for the decision maker.

$$EMV(S_i) = \sum_{j=1}^{n} P(O_j)\, a_{ij}$$

Where,

S_i = Course of action i

$P(O_j)$ = probability of occurence of O_j (event)

n = No. of possible states of nature (events)

a_{ij} = Pay-off associated with course of action i and state of nature O_j

In this EMV criterion, steps involved are:

Step 1: construct a pay-off table with all alternative decisions, events and the pay-off for each alternative and event combination. Also enter the probabilities associated to each events.

Step 2: Calculate EMV for each alternative, which is the sum of the product of pay-offs and the probabilities of events for that alternative.

Step 3: Select the alternative corresponding to the highest EMV in cost table.

Decision making under condition of Risk:

(1) Expected Monetary value (EMV) or expected value criterions.

(2) Expected opportunity loss (EOL) or expected regret criterions.

Expected Monetary criterion (EMV)

Expected value = pay-off × probability.

EMV for each decision alternative = Σ pay-off × probability.

EPPI = sum of (Maximum pay-off each demand × probability)
= Expected profit with perfect information.

EVPI = EPPI − Max of EMV.
= Expected value of perfect information.

Question: The parker flower shop promisses its customers delivery within four hours on all flowers orders. All flowers are purchased on the previous day and delivered to parker by 8:00 AM. The next morning parker's daily demand for roses is as follows:

Dozens of roses :	7	8	9	10
probability :	0.1	0.2	0.4	0.3

parker purchases roses for Rs 10 per dozen and sells them for Rs. 30. All unsold roses are donated to a local hospital. How many dozens of roses should parker order each eving to maximize its profits? What is optimum expected profit. Also find EPPI and EVPI.

Solution: cp = Rs. 10

sp = Rs. 30

mp = Rs. 30 − Rs 10 = Rs. 20

pay-off = MP × S ; if S≤D

= MP×D − CP(S−D) ; if S>D

EMV for Items that have a Salvage value:

If conditional profit = cp.

$\therefore$ CP = MP × unit supplied ; if $S \le D$

= MP × S

cp = sp × unit sold − cp × unit supplied + salvage price × unit unsold, if $S > D$.

= sp × D − cp × S + salvage price × (S−D) if $S > D$.

MP = sp − cp

EMV = Σ CP × probability.

Example: A retailer purchases cherries every morning at Rs. 50 a case and sells them for Rs. 80 a case. Any case remaining at the end of the day can be disposed of next day at a salvage value of Rs. 20 per case (thereafter they have no value). Past sales have ranged from 15 to 18 cases per day. The following is the record of sales for the past 120 day.

Cases sold :	15	16	17	18
Number of days :	12	24	48	36

Find how many the retailer should purchase per day to maximize his profit.

Solution: mp = sp − cp = 80 − 50 = Rs. 30

Salvage price = Rs. 20

Conditional profit (cp) = mp × unit supplied ; if $S \le D$

= Rs. 30 × S

$\therefore$ cp = sp × unit sold − cp × unit supplied + salvage price × unit unsold; if $S > D$.

= sp × D − cp × S + salvage price × (S−D).

Alternative	State of nature				EMV	
	15	16	17	18		
15 = S	450	420	390	360	450	
16 = S	450	480	450	420	474	
17 = S	450	480	510	480	$\boxed{486}$	
18 = S	450	480	510	540	474	
probability	0.1	0.2	0.4	0.3		

EMV(15) = 450×0.1 + 420×0.2 + 390×0.4 + 360×0.3 = 450

EPPI = 450×0.1 + 480×0.2 + 510×0.4 + 540×0.3 =

EVPI = EPPI − Max of EMV.

Highest EMV is 486 so decision maker select strategy 17 stocked.

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payoff (P_{11}) = S×mp = 15×30 = 450

Payoff (P_{21}) = D×mp − (S−D)ML = 15×30 − (16−15)×30 =

alternative method.

payoff = MP × unit sold − ML × unit unsold

MP = SP − CP

ML = CP − salvage value.

question: A grocer must decide how many crates of milk should be stocked each week to meet the demand. The probability distribution of demand is as under. (Each crates costs the grocer Rs. 100 & its selling price is Rs. 120) unsold crates are sold to a local farmer for Rs. 20 per crate. If ~~storage~~ shortage exist, a shortage cost of Rs 40 per crate incurred. Construct a matrix and calculate EMV, EPPI & EVPI.

Demand	15	16	17	18	19
probability	0.15	0.25	0.4	0.15	0.05

Solution: CP = 100

SP = 120

profit = 120 - 100 = 20

Loss = 100 - 20 = 80; when spply is more.

Shortage cost = 40 when demand is more

pay-off = MP × S if S ≤ D

pay-off = MP × S - 1×40 ; S ≤ D ; 1 unit shortage.

pay-off = MP × D - 1×80 ; S > D ; 1 unit unsold.

pay-off table

Action	Deman					EMV	
	15 = D	16 = D	17 = D	18 = D	19 = D		
15 = S	300	260	220	180	140	232	
16 = S	220	320	280	240	200	271	
17 = S	140	240	340	300	260	$\boxed{275}$	
18 = S	60	160	260	360	320	~~235~~ 223	
19 = S	-20	80	180	280	380	150	
P	0.15	0.25	0.4	0.15	0.05		

Higest EMV is 275 so ~~we~~ select 17.

~~EMV = 300×~~

EMV(15) = 300×0.15 + 260×0.25 + 220×0.4 + 180×0.15 + 140,
= 232

EMV(16) =

According to EMV Rule, higest EMV is 275, 17 carates should be stocked.

EPPI = Expected profit with perfect information
= Max × probability.
= 300×0.15 + 320×0.25 + 340×0.4 + 360×0.15 + 380×0.05
= 334

EVPI = Expected value of perfect information,
= EPPI − Max. EMV
= 334 − 275 = 59

Expected opportunity Loss (EOL) criterion:

An alternative useful ~~way~~ to maximize expected monetary value is to minimize the expected opportunity loss (EOL) or expected value of regret.

The EOL is the amount by which the maximum ~~Profit~~ possible profit will be reduced by taking other course of action (alternative). Thus the decision maker will choose the course of action that minimizes the expected opportunity loss.

$$EOL(\text{course of action } S_j) = \sum_{j=1}^{m} L_{ij} \times P_i$$

Where, m = No. of possible state of nature.

P_{ij} = probability of occurrance of state of nature N_i

L_{ij} = conditional pay-off (opportunity loss) associated with ~~the~~ state of nature N_i and course of action S_j.

= Maximum pay-off − Actual payoff; if payoff is profit.

= Actual cost − minimum cost ; if payoff is cost.

Question: under an employment promotion program it is proposed to allow sale of newspapers on buses during off-peak hour. The vendor can purchase the papers at a special concessional rate of 25 paise and sells it at 40 paise. Any unsold copy is a dead loss. A vendor has estimated the following probabilities for the number of copies demanded.

No. of copies :	15	16	17	18	19	20
Probability :	0.04	0.19	0.33	0.26	0.11	0.07

prepare a pay of table and find out how many copies of newspapers should be ordered by using EOL criterion.

Solution: Here,

$CP = 25$ paisa

$SP = 40$ paisa

$MP = SP - CP = 15$ paisa

Pay off $= MP \times S$ if $S \le D$

$= MP \times D - CP(S-D)$; if

Strategies	Probability	State of nature 15	16	17	18	19	20
15	0.04	225	225	225	225	225	225
16	0.19	200	240	240	240	240	240
17	0.33	175	215	255	255	255	255
18	0.26	150	190	230	270	270	270
19	0.11	125	165	205	245	285	285
20	0.07	100	140	180	220	260	300
Probability		0.04	0.19	0.33	0.26	0.11	0.07

Regret table

Strategy	P_i	State of nature 15	16	17	18	19	20	
15	0.04	0	15	30	45	60	75	EOL
16	0.19	25	0	15	30	45	60	36.30
17	0.33	50	25	0	15	30	45	22.90
18	0.26	75	50	25	0	15	30	17.10
19	0.11	100	75	50	25	0	15	24.50
20	0.07	125	100	75	50	25	0	42.30
P_i		0.04	0.19	0.33	0.26	0.11	0.07	64.50

The minimum EOL is 17.10 paisa. Hence by EOL criterion 17 Newspapers is the optimal stock level.

Multi-stage Decision problems:

So far we have discussed the problems which are single stage decision problems. In such problems the strategies, state of nature, probability distribution etc. all remain unchanged and the decision maker is not required to revlve his once taken decision. But there are problems which involve a long sequence of inter-related actions and outcomes. such problems are called multi-stage problems.

In these problems for a decision, there are a number of outcomes, which leads to another decision and so on.

Decision ~~tree~~ tree analysis:

A decision tree analysis is highly useful to a decision maker in multi-stage situations which involve a sequence of decisions each depending on the ~~producing one~~ preceding one.

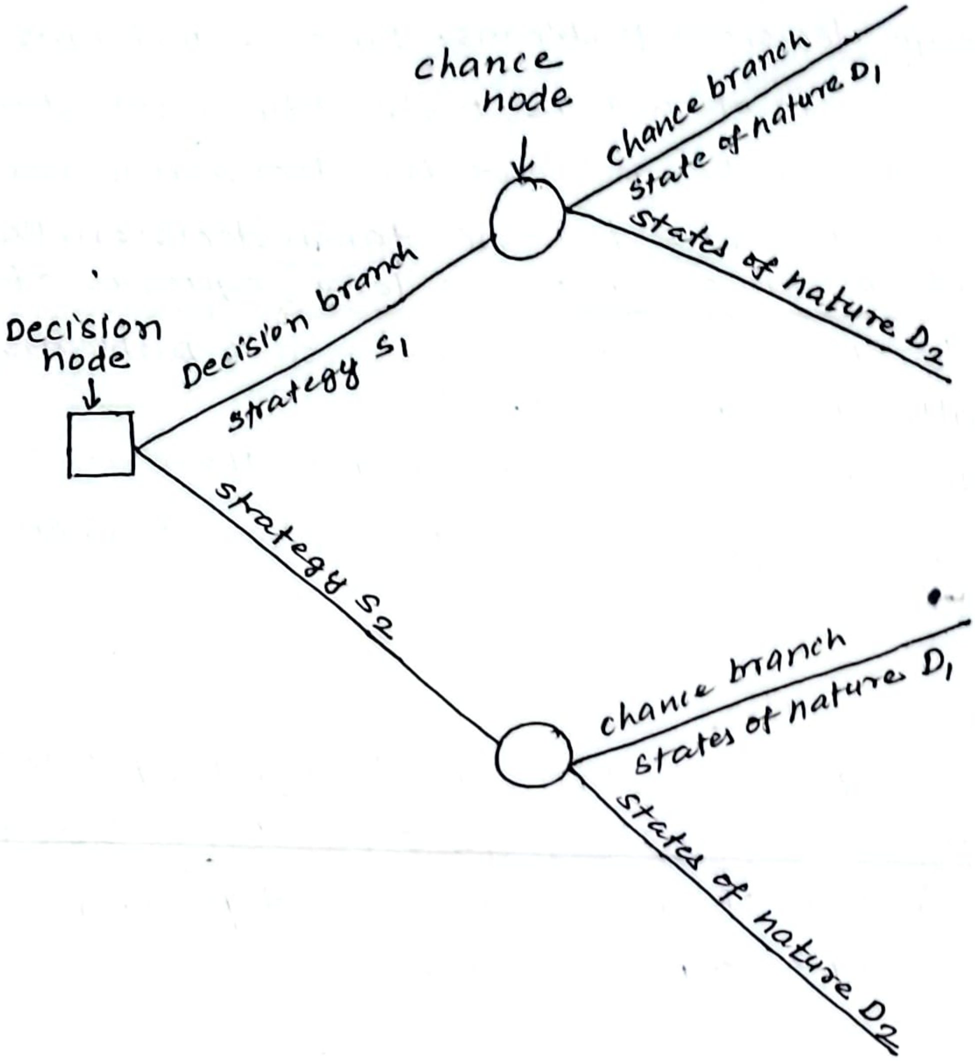
Decision Tree:

A decision tree is a graphical representation of the decision process involving various decisions alternatives and sequence of state of nature as if they are branches of a tree. It consists of a network of nodes and branches.

In the construction of a tree diagram, there are two types of nodes (i) the decision nodes; represented using □ and (ii) chance nodes; represented using O, are used.

The alternatives available at a decision node are represented using brances originating from that decision point (node) and are called decision branches. At the end of each branch there is an event (chance) node

O, represent all possible situations and are called branches.



Analysis of decision tree:

The general approach used here is the "Roll Back process" i.e. we move from right to left. The EMV is calculated at each node starting from the extreme right node. For a chance node the EMV is the sum of the products of the probabilities of chances branches emanating from this node and the respective payoffs.

The EMV of a decision node is the maximum of the EMV's along all decision branches emanating from this node. Thus starting from the right we move along the path that yields the maximum payoff (EMV) for each of the decisions.

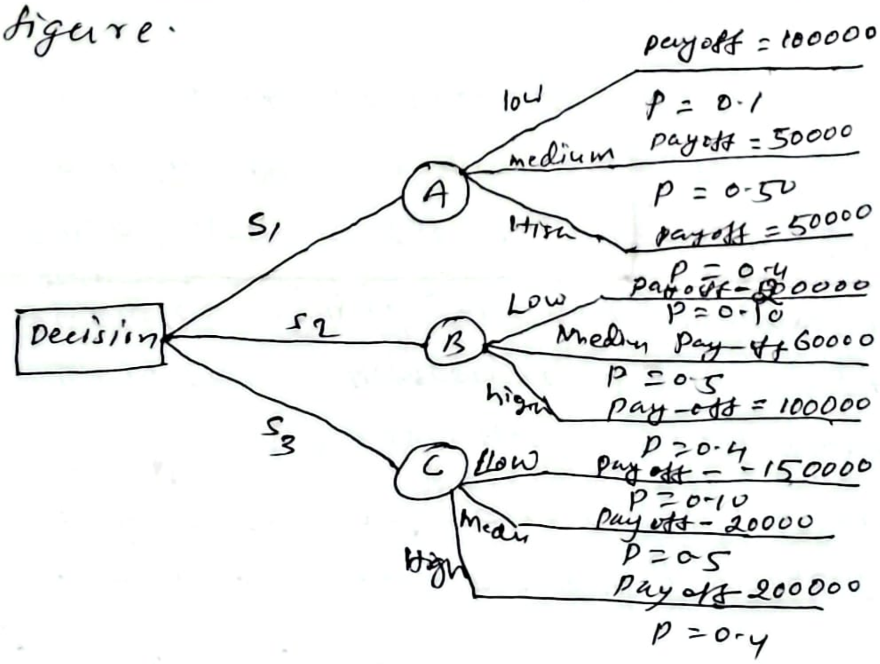
Decision tree approach:

Example: A glass factory specializing in crystal is developing a substantial backlog and the firm's management is considering free courses of action. Arrange for sub-contracting (S_1), Begin over time productions (S_2) and construct new facilities (S_3). The correct choice depends largely upon future demand which may be low, medium or high. By consensus, management ranks the respective probabilities as 0.10, 0.50 and 0.40. A cost analysis reveals the effect upon the profits that is shown in the table below:

Demand	probability	course of action		
		S_1	S_2	S_3
Low	0.10	100000	-20000	-150000
medium	0.50	50000	60000	20000
High	0.40	50000	100000	200000

Show this decision situation in the form of a decision tree and indicates the most preferred decision and corresponding expected value.

solution: A decision ~~three~~ tree which shows possible alternative and state of nature is shown in following figure.



EMV   ΣEMV

100000 × 0.1 = Rs. 10000
50000 × 0.5 = Rs. 2500   } 46000
50000 × 0.4 = Rs. 2000

−20000 × 0.1 = −Rs. 2000
60000 × 0.5 = Rs. 30000   } 68000
100000 × 0.4 = Rs. 40000

= −150000 × 0.1 = −Rs. 15000
20000 × 0.5 = Rs 10000   } Rs. 75000
20000 × 0.4 = Rs. 80000

Marginal Analysis:

In operation reseach, the term marginal is additional "thinking on the margin", or marginal analysis involves making decisions based on the additional benefit vs. the additional cost.

The marginal analysis is based on the fact that when an additional unit, purchased will either be sold or remain unsold. If P represents the probability of selling one additional unit, then (1-P) must be the probability of not selling it.

If the additional unit is sold, the conditional profit will increase as a result of the profit earned from the additional unit. such an increase profit is known as increament (marginal) profit. It is denoted by MP.

$$MP = \text{selling price} - \text{cost price}.$$

If the additional unit is not sold, the conditional profit will reduces and the amount of reduction is called the increamental (marginal) loss. It is denoted by ML.

$ML = \text{Cost price} = CP$; if no salvage value,

$ML =$ ~~cost price~~ $CP - \text{salvage value}$; if salvage value.

$ML = CP - \text{salvage value} + \text{cost of holding beyond season}$.

Expected marginal profit $= P \times MP$

Expected marginal loss $= (1-P) \times ML$.

Under the marginal analysis technique, the additional units should be stocked as long as the expected marginal profit from stocking each of them is greater than or equal to the expected marginal loss from stocking each. Thus additional units should be stocked as long as:

$$P \times MP \geq (1-P) \times ML$$

$$\text{or, } P \times MP \geq ML - P \times ML$$

or $p \times MP + p \times ML \geq ML$

or, $p(MP + ML) \geq ML$

$\therefore p \geq \frac{ML}{MP+ML}$

The symbol p represents the minimum required probability of selling at least one additional unit to justify the stocking of that additional unit. Additional units should be stocked so long as probability of selling at least that additional unit is greater than p.

Steps of marginal analysis approach:

(i) compute value of p using formula.

$$p \geq \frac{ML}{MP+ML}$$

(ii) construct a cumulative probability table. The cumulative of greater than type is obtained by cumulating the given probability from bottom to top

(iii) Locate the cumulative probability of stoking or buying greater than or equal to the calculated value of p. The corresponding stock is the optimum value.

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Question: A veterinarian purchases rabies immunization vaccine on Monday of each week. Because of the chara cteristic of this vaccine, it must be used by Friday or disposed of. The vaccine cost Rs. 9 per dose and the veterinarian charges Rs. 16 per dose. In the past, the veterinarian has administered rabies vaccine in the following quantities.

quantities used per week	No. of weeks this occured
2500	15
4000	20
5000	10
7500	5

using marginal analysis determine how many does the veterinarian should order each week. If the veterinarian is offered a forecasting method costing Rs. 5000, should be purchase this model or not.

Solution: We have,

Cumulative probability table

quantities used per week	Probability	Cumulative probability
2500	0.3	1.0
4000	0.4	0.7
5000	0.2	0.3
7500	0.1	0.1

Here,

$SP = Rs. 16$

$CP = Rs. 9$

$MP = SP - CP = 16 - 9 = Rs. 7$

$ML = CP = Rs. 9$

Now, $$P \geq \frac{ML}{MP + ML} \geq \frac{9}{7+9} \geq 0.56$$

since the cumulative probability just greater than 0.56 is 0.70, the veterinarian should order 4000 does per week.

We know that an additional unit should be stocked as long a

$p \times MP \geq (1-p) \times ML$

Now,

Expected marginal profit $= p \times MP = 0.7 \times 7 = Rs. 4.9$

Expected marginal loss $= (1-p) \times ML = (1-0.7) \times 9 = Rs. 2.7$

Since, $p \times MP \geq (1-p) \times ML$, the optimum stock is 4000 doses per week. Therefore, the conditional profit (payoff) is calculated as

$$\text{Payoff} = MP \times S \; ; \text{ if } S \leq D$$
$$= Rs. 7 \times S$$
$$\text{Payoff} = MP \times D - CP(S-D) ; \text{ if } S > D$$
$$= Rs. 7 \times D - 9(S-D)$$
$$= 7D - 9S + 9D$$
$$= 16D - 9S$$

If S denotes the optimum stock per week, then S = 4000 does per week. The maximum EMV under uncertainty is calculated as follows:

Calculation of maximum EMV under uncertainty

Demand	Conditional profit (Rs)	probability	Expected profit (Rs)
2500	16×2500 − 9×4000 = 4000	0.3	4000 × 0.3 = 1200
4000	7×4000 = 28000	0.4	28000 × 0.4 = 11200
5000	7×4000 = 28000	0.2	28000 × 0.2 = 5600
7500	7×4000 = 28000	0.1	28000 × 0.1 = 2800
			Rs. 20800

Hence, maximum EMV under uncertainty = Rs. 20800

In order to find EVPI, we first need to calculate the expected profit with perfect information (EPPI).

Calculation table of EPPI

Demand	Conditional profit (Rs) = MP×S	probability	Expected profit (Rs)
2500	7×2500 = 17500	0.3	17500×0.3 = 5250
4000	7×4000 = 28000	0.4	28000×0.4 = 11200
5000	7×5000 = 35000	0.2	35000×0.2 = 7000
7500	7×7500 = 52500	0.1	52500×0.1 = 5250
			ΣEMV = Rs. 28700

Hence, EPPI = ΣEMV under uncertainty = Rs. 28700

The expected value of perfect information (EVPI) is given by

EVPI = EPPI − Maximum EMV under uncertainty.

= 28700 − 20800 = Rs. 7900

Since the cost of uncertainty > cost of forecasting model 5000, the veterinarian should purchase the model, so that ~~we that~~ he will gain Rs. 7900 − 5000 = 29..

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