Unit 6: Network Analysis

Network Analysis: It is a technique used for planning and scheduling large projects, in the field of constructed, maintenance, fabrication and purchasing of computer systems etc.

Projects: A project is defined as defined as combination of interrelated activities, all of which must be executed in a certain order for its completion.

Phase of Project Management:

(a) Planning:

(i) Divide the project into distinct activities.

(ii) Estimate time and cost requirement for various activities of project completion.

(iii) Establish the precedence relationship.

(iv) Construct arrow diagram.

(v) Establish control channels for project personnel.

(b) Scheduling:

(i) Determine the start and end time for activity.

(ii) Determine the critical path on which the activity requires attention.

(iii) Determine the slack and float for non-critical path.

(iv) Develop a network diagram, showing the sequential interrelationship between various activities.

(c) Controlling:

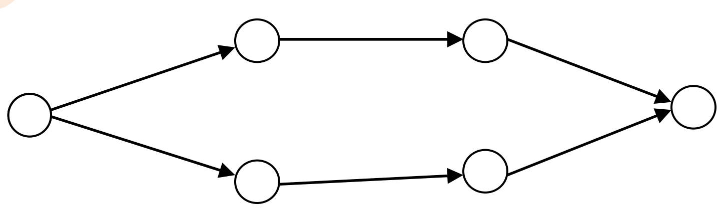
(i) Making the periodical progress report.

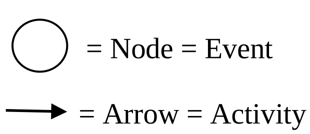
(ii) Reviewing the progress.

(iii) Analyzing the status of project.

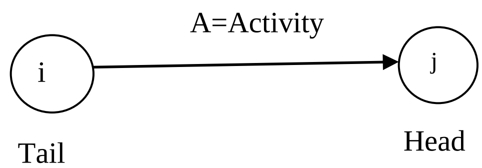
Basic Terms:

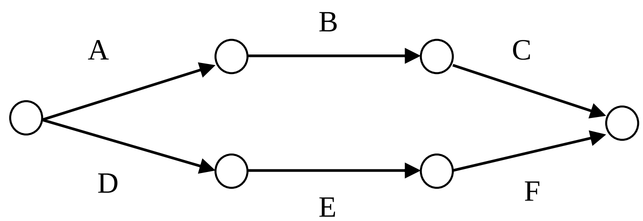
(a) Network diagram: It is the graphical representation of logically and sequentially connected **arrow** and **node**, representing activities and events in a project.





(b) Activity: It represents some action and is a time consuming effort necessary to complete a particular part of overall project.



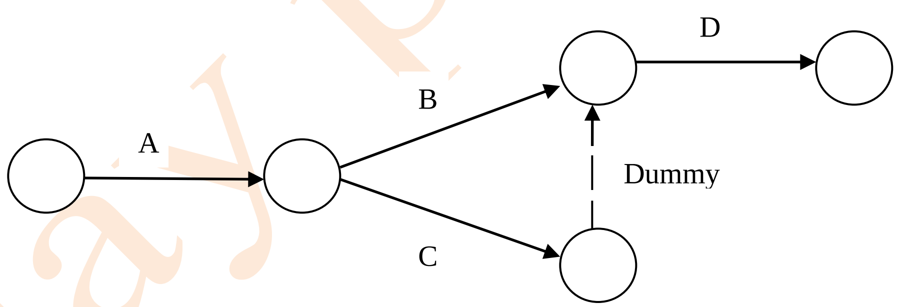


(i) Predecessor Activity: An activity which must be completed before one or more other activities start is known as predecessor activity.

(ii) Successor Activity: An activity which starts immediately after one or more of other activities are completed is known as successor activity.

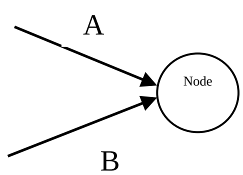
(iii) Concurrent Activity: An activity taking place at the same time or in the same location.

(iv) Dummy Activity: An activity which neither consume time nor resources but are used simply to represent a connection or a link between the events are called dummy activity. It is shown in network by a dotted lines.

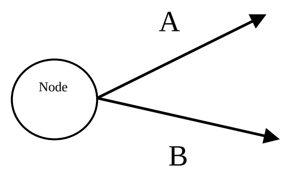


(c) Event: Beginning and end points of an activity are called events or nodes.

(i) Merge event: An event which represents the joint completion of more than one activity is known as a merge event.

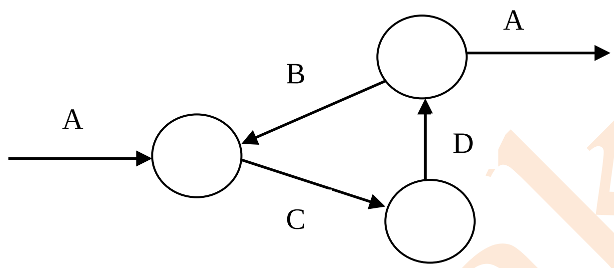


(ii) Burst event: An event that represents the initiation (beginning) of more than one activity is known as burst event.

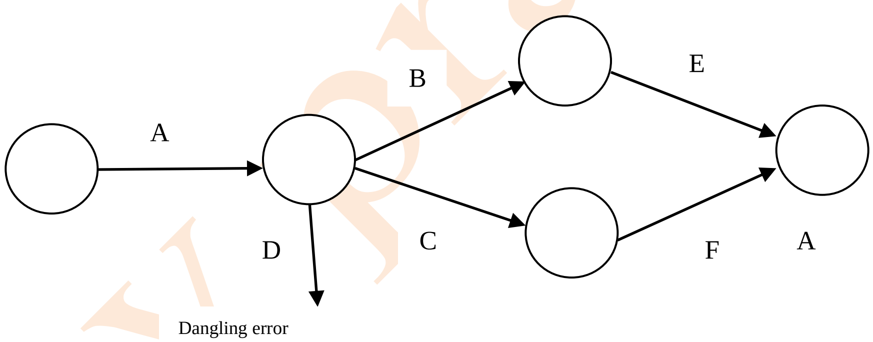


Common Errors:

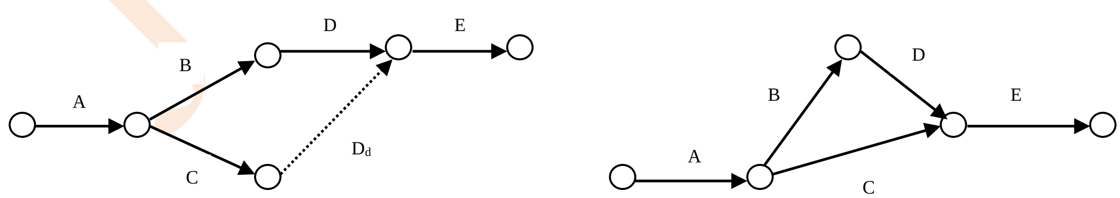
(a) Looping (Cycling): Drawing an endless loop in a network is known as error of looping.



(b) Dangling Error: To disconnect an activity before the completion of all activities called error of dangling.



(c) Redundancy: If a dummy activity is the only activity emanating from event and can be eliminated called redundancy error.



Rules of Network Construction:

(i) In network diagram, arrows represent activities and circles the events.

(ii) Each activity should be represented only by one arrow and must start and end in a circle called event.

(iii) Network has only one entry point called start event and one point of emergence called end point.

(iv) Try to avoid the arrow that cross each other.

(v) Use straight arrow.

(vi) A network should have only one initial event and only one terminal event.

(vii) The general rule for numbering the event is that the head event should always be numbered larger than the number at its tail.

(viii) An event number should not get repeated or duplicated.

(ix) Two activities should not be identified by the same completion event.

(x) An event cannot occur twice.

(xi) Use arrow left to right. Avoiding mixing two directions.

(xii) No event can occur until every activity preceding it has been completed.

(xiii) Dummies should be introduced only, if it is extremely necessary.

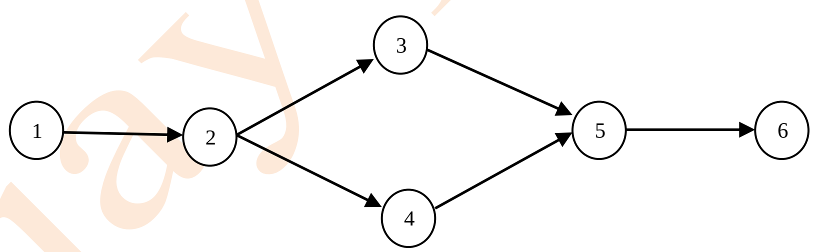
Numbering the events:

(i) Number must be unique.

(ii) Number should be carried out on a sequential basis, from left to right.

(iii) Initial event must be numbered as 1

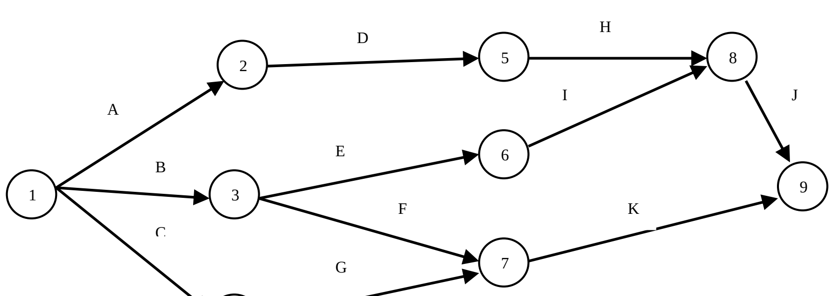
(iv) Number all new start event 2, 3 and so on.



Construction of Network:

Construct a network for the project whose activities and precedence relationships are as given below.

Activity	A	B	C	D	E	F	G	H	I	J	K
Predecessor	-	-	-	A	B	B	C	D	E	H, I	F, G



Time Analysis:

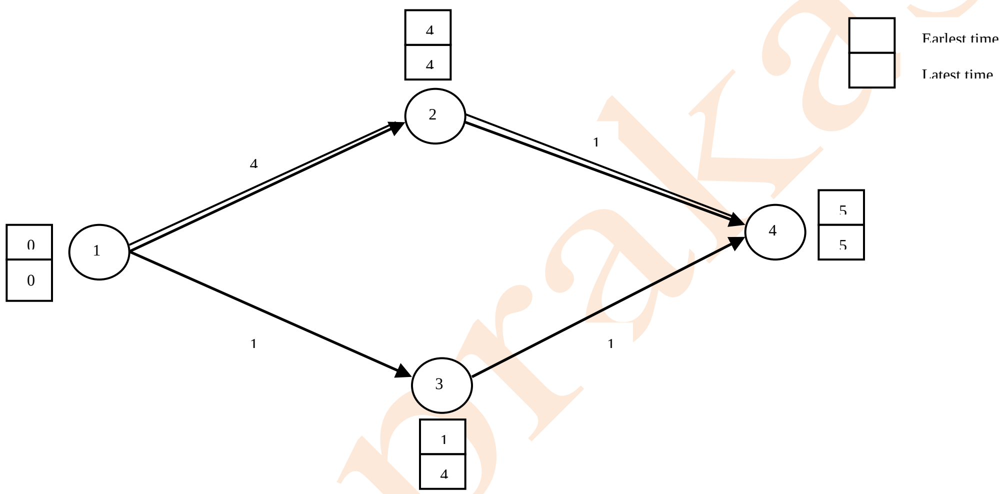
T_{ij} = Estimate completion time of activity (i, j)

ES_{ij} = Earliest starting time of activity (i, j)

EF_{ij} = Earliest finishing time of activity (i, j)

LS_{ij} = Latest starting time of activity (i, j)

LF_{ij} = Latest finishing time of activity (i, j)



1 —— 2 —— 4 Critical path

4 + 1 = 5 days

(a) Forward pass computation:

(i) Zero is the starting time for the project.

(ii) $(EF)_{i\,j} = (ES)_{i\,j} + t_{i\,j}$

(iii) E_j = Max $\left(E_i + t_{ij}\right)$

(b) Backward pass computation:

(i) For ending event assume, E = L

(ii) $(LS)_{i\,j} = (LF)_{i\,j} - t_{i\,j} = L_j - t_{ij}$

(iii) L_j = Min $\left(L_j - t_{ij}\right)$

(c) Determination of floats and slack time:

Float: It is defined as the difference between the latest and earliest activity time.

Slack: Slack is defined as difference between the latest and the earliest event time.

Total Float:

$$(TF)_{i\,j} = (LS)_{i\,j} - (ES)_{i\,j}$$

$$or,\ (TF)_{i\,j} = \left(L_j - E_i\right) - t_{i\,j}$$

Where,

E_i = Earliest time for tail event.

L_j = Latest time for head event.

t_{ij} = Normal time for activity (i, j).

Free Float:

$$FF_{i\,j} = \left(E_j - E_i\right) - t_{i\,j}$$

$FF_{i\,j}$ = Total float – Head event slack

Head event slack = $L_j - E_j$

Independent Float:

$$IF_{i\,j} = \left(E_j - L_i\right) - t_{i\,j}$$

$IF_{i\,j}$ = Free float – Tail event slack

Tail event slack = $L_i - E_i$

Activity	Normal time	Earliest		Latest	
		Start (ES)	Finish (EF)	Start (LS)	Finish (LF)
	t_{ij}	ES (E_i)	EF = $E_i + t_{ij}$	LS = $L_j - t_{ij}$	LF (L_j)
1 – 2	4	0	4	0	4
1 – 3	1	0	1	3	4
2 – 4	1	4	5	4	5
3 – 4	1	1	2	4	5

Total Float	Free Float	Independent Float	E_j	L_i
TF = LS – ES	FF = $E_j - E_i - t_{i\,j}$	IF = $E_j - L_i - t_{i\,j}$		
0	0	0	4	0
3	0	0	1	0
0	0	0	5	4
3	3	0	5	4

Critical Activity: An activity is said to be critical, if the total float TF_{ij} for any activity (i, j) is zero.

Critical Path Conditions:

(i) $\mathrm{ES_i} = \mathrm{LF_i}$

(ii) $\mathrm{ES_j} = \mathrm{LF_j}$

(iii) $\mathrm{ES_j} - ES_i = \mathrm{LF_j} - \mathrm{LF_i} = t_{ij}$

Critical Path Method (CPM):

CPM is a technique of network analysis which is effectively used for the planning, scheduling and controlling commercial project. It is generally used for repetitive type of projects like building, bridge, road etc. It includes the both cost and completion time for each activity of the planning, scheduling and controlling of the project. The longest path of the network is called critical path. The critical path of network gives the shortest time in which the whole project can be completed.

Some of steps of CPM:

(i) The completion time of each activity is known with certainty that too unique.

(ii) CPM analysis explicitly estimates the cost of project in addition to the completion time.

(iii) It is suitable for establishing a trade – off for optimum balancing between schedule time and cost of the project.

(iv) It is used for completing of projects that involve activities of repetitive nature.

Question: A project schedule has the following characteristics.

Activity	1 – 2	1 – 3	2 – 4	3 – 4	3 – 5	4 – 9	5 – 6	5 – 7	6 – 8	7 – 8	8 – 10	9 – 10
Time (day)	4	1	1	1	6	5	4	8	1	2	5	7

(a) Construct network diagram.

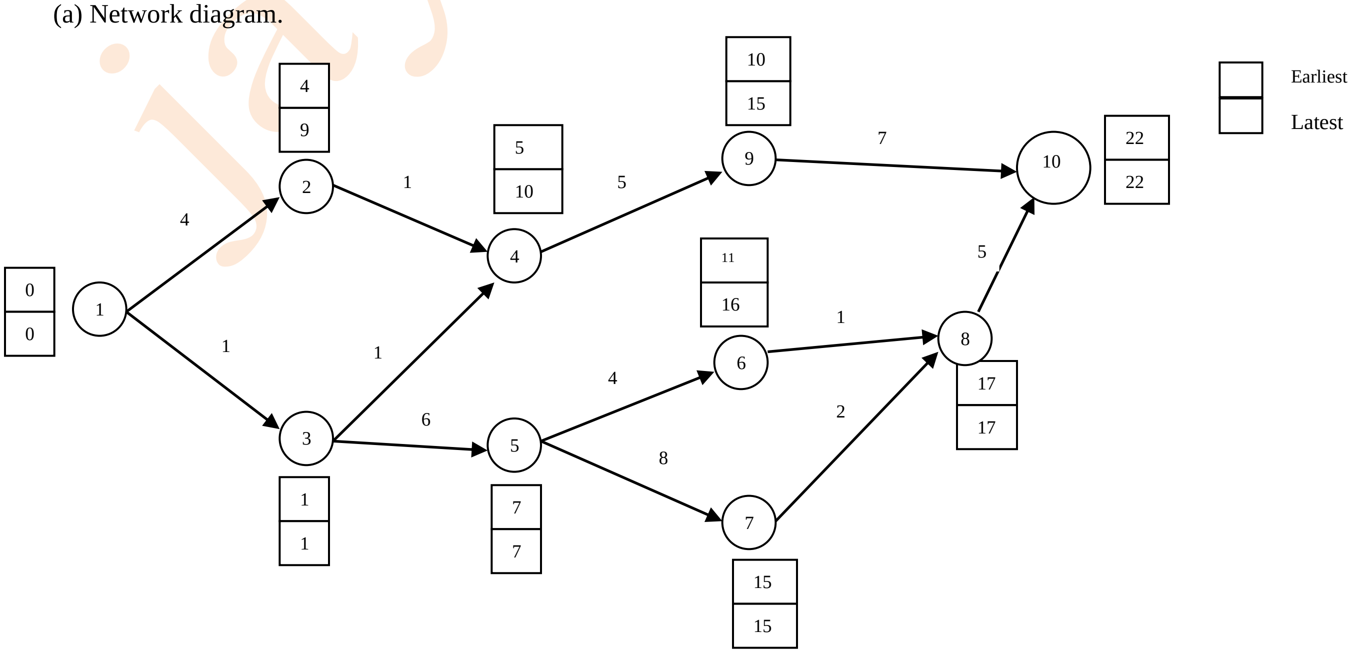
(b) Complete the earliest event time and latest event time.

(c) Determine the critical path and total project duration.

(d) Complete total and free float for each activity.

Solution: We have,

(a) Network diagram.



(b) Following table gives the information:

Activity	Time day	Earliest time		Latest time		Total	Free	
	t_{ij}	Start (ES) E_i	Finish(EF) $E_i + t_{ij}$	Start (LS) $L_j - t_{ij}$	Finish(LF) L_j	Float (TF) LS - ES	Float (FF) = $E_j - E_i - t_{i\,j}$	E_j
1 – 2	4	0	4	5	9	5	0	4
1 – 3	1	0	1	0	1	0	0	1
2 – 4	1	4	5	9	10	5	0	5
3 – 4	1	1	2	9	10	8	3	5
3 – 5	6	1	7	1	7	0	0	7
4 – 9	5	5	10	10	15	5	0	10
5 – 6	4	7	11	12	16	5	0	11
5 – 7	8	7	15	7	15	0	0	15
6 – 8	1	11	12	16	17	5	5	17
7 – 8	2	15	17	15	17	0	0	17
8 – 10	5	17	22	17	22	0	0	22
9 – 10	7	10	17	15	22	5	5	22

Critical Path:

$$1 \rightarrow 3 \rightarrow 5 \rightarrow 7 \rightarrow 8 \rightarrow 10$$

Total project duration = 1 + 6 + 8 + 2 + 5 = 22

Program Evaluation and Review Technique (PERT):

PERT is a probabilistic method, where the activity times are represented by a probability distribution. This distribution of activity of times is based on three different time estimates made for each activity, which are follows.

(i) Optimistic time estimate (t_0)

(ii) Most likely time estimate (t_m)

(iii) Pessimistic time estimate (t_p)

For these three estimates, we have to calculate the expected time of an activity.

$$t_e = \frac{t_0 + 4t_m + t_p}{6}$$

Variance of an activity is given by

$$\sigma^2 = \left(\frac{t_p - t_0}{6}\right)^2$$

Probability that the project will be completed = $p(Z \leq Z_{cal})$

Where,

$$Z = Z_{cal} = \frac{T_s - T_e}{\sigma} = \text{Standard normal variable}$$

T_s = Schedule time to complete the project

T_e = Normal expected project length

σ = Expected standard deviation of the project length

Question: The following table shows the jobs of a network along with their time estimates.

Activity	Estimated duration		
	Optimistic (t_0)	Most likely (t_m)	Pessimistic (t_p)
1 – 2	1	7	13
1 – 6	2	5	14
2 – 3	2	14	26
2 – 4	2	5	8
3 – 5	7	10	19
4 – 5	5	5	17
6 – 7	5	8	29
5 – 8	3	3	9
7 – 8	8	17	32

(i) Draw the project network.

(ii) Find the expected duration and variance of each activity.

(iii) Calculate the earliest and latest occurrences for each event.

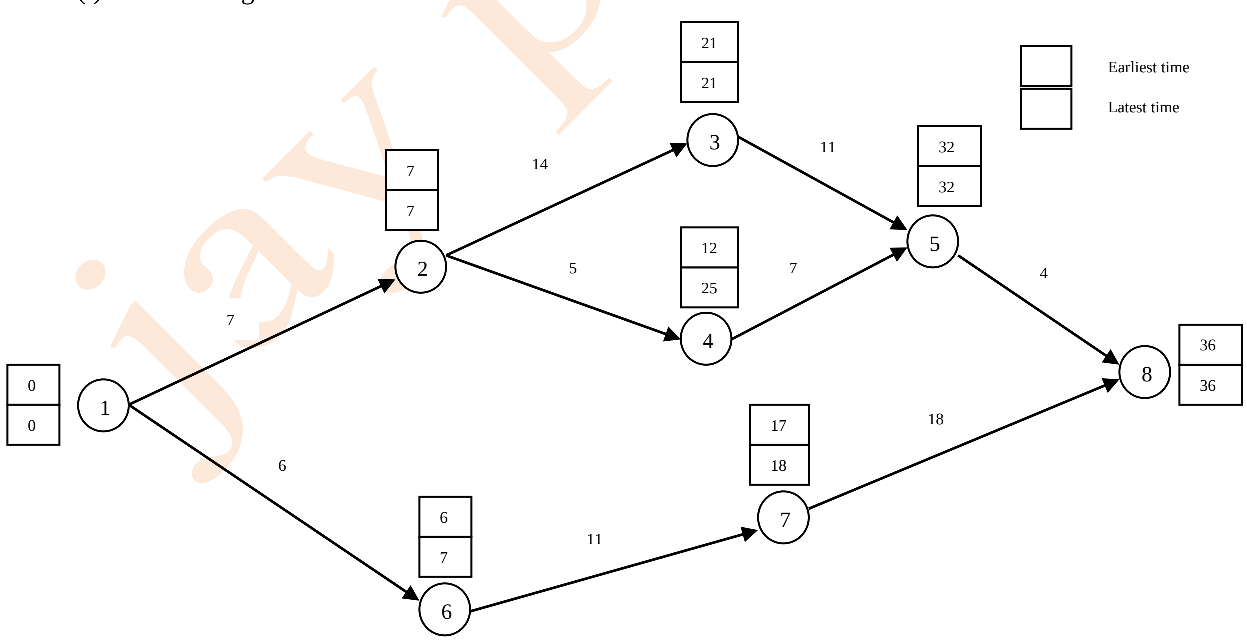
(iv) Calculate expected project length.

(v) Calculate the variance and standard deviations of project length.

(vi) Find the probability of the project completing in 40 days.

Solution:

(i) Network diagram:



(ii) Calculation of expected time and variance.

	Estimated duration	$t_e = \frac{t_0+4t_m+t_p}{6}$	$\sigma^2 = \left(\frac{t_p-t_0}{6}\right)^2$

Activity	t_0	t_m	t_p		
1 – 2	1	7	13	7	4
1 – 6	2	5	14	6	4
2 – 3	2	14	26	14	16
2 – 4	2	5	8	5	1
3 – 5	7	10	19	11	4
4 – 5	5	5	17	7	4
6 – 7	5	8	29	11	16
5 – 8	3	3	9	4	1
7 – 8	8	17	32	18	16

Critical Path = 1 → 2 → 3 → 5 → 8

Expected project duration = 7 + 14 + 11 + 4 = 36

Project length variance = 4 + 16 + 4 + 1 = 25

Project length (σ) = $\sqrt{25}$ = 5

(iv) The probability that the project will be completed in 40 days is given by

$$p(Z \leq Z_{cal}) = p\left(Z \leq \frac{T_s - T_e}{\sigma}\right)$$

$$= p\left(Z \leq \frac{40-36}{5}\right)$$

$$= p(Z \leq 0.8)$$

= 0.7881

= 78.81%

Difference between PERT and CPM:

PERT	CPM
(i) Stands for program evaluation and review technique.	(i) Stands for critical path method.
(ii) Event oriented.	(ii) Activity oriented.
(iii) Associated with probabilistic activity.	(iii) Associated with deterministic activity,
(iv) Based on three time estimate namely; (a) Optimistic (b) Most likely (c) Pessimistic	(iv) Based on single time to complete.
(v) Resources such as labor, equipments, material etc are limited.	(v) No limitation of resources.
(vi) Mainly used for research and development project.	(vi) Mainly used for construction project.
(vii) It is a technique of planning and control of time.	(vii) It is a method to control costs and time.
(viii) PERT technique is suited for a high precision time estimate.	(viii) CPM is appropriate for a reasonable time estimate.
(ix) PERT is used where the nature of the job is non – repetitive.	(ix) CPM involves the job of repetitive nature.
(x) Crashing concept is not applicable to PERT.	(x) Crashing is a compression technique applied to CPM, to shorten the project duration, along with least additional cost.

Program Evaluation and Review Technique (PERT):

In the most of projects the activity times are not known with certainty and they may be assumed as random variables. In such cases where activity times are not known, PERT can be used for planning, scheduling and controlling

In PERT for each activity, three times activity are obtained:

(i) Optimistic time estimate (t_0)

(ii) Most likely time estimate (t_m)

(iii) Pessimistic time estimate (t_p)

Critical Path Method (CPM):

CPM is a network diagramming technique used to predict total project duration. A critical path for a project is series of activities that determines the earliest time by which the project can be completed.

1. An assembly is to be made from two parts X and Y. Both parts must be turned on a lathe. Y must be polished whereas X need not be polished. The sequence of activities, together with their predecessors, is given below.

Activity	Description	Predecessor Activity
A	Open work order	–
B	Get material for X	A
C	Get material for Y	A
D	Turn X on lathe	B
E	Turn Y on lathe	B, C
F	Polish Y	E
G	Assemble X and Y	D, F
H	Pack	G

Draw a network diagram of activities for the project.

2. Listed in the table are the activities and sequencing necessary for a maintenance job on the heat exchangers in a refinery.

Activity	Description	Predecessor Activity
A	Dismantle pipe connections	—
B	Dismantle heater, closure, and floating front	A
C	Remove tube bundle	B
D	Clean bolts	B

E	Clean heater and floating head front	B
F	Clean tube bundle	C
G	Clean shell	C
H	Replace tube bundle	F, G
I	Prepare shell pressure test	D, E, H
J	Prepare tube pressure test and reassemble	I

Draw a network diagram of activities for the project.

3. Listed in the table are the activities and sequencing necessary for the completion of a recruitment procedure for management trainees (MT) in an organization.

Activity	Description	Predecessor Activity
A	Asking units for requirements	–
B	Ascertaining management trainees (MTs) requirements for commercial functions	A
C	Ascertaining MTs requirement for Accounts/Finance functions	A
D	Formulating advertisement for MT(A/C)	C
E	Formulating advertisement for MT (Commercial)	B
F	Calling applications from the successful candidates passing through the Institute of Chartered Accountants (ICA)	C
G	Releasing the advertisement	D, E
H	Completing applications received	G
I	Screening of applications against advertisement	H
J	Screening of applications received from ICA	F
K	Sending of personal forms	I, J
L	Issuing interview/regret letters	K
M	Preliminary interviews	L
N	Preliminary interviews of outstanding candidates from ICA	J
O	Final interview	M, N

Draw a network diagram of activities for the project.

Critical Path Analysis:

4. An established company has decided to add a new product to its line. It will buy the product from a manufacturing concern, package it, and sell it to a number of distributors that have been selected on a geographical basis. Market research has already indicated the volume expected and the size of sales force required. The steps shown in the following table are to be planned.

Activity	Description	Predecessors	Duration (days)
A	Organize sales office	–	6
B	Hire salesmen	A	4
C	Train salesmen	B	7
D	Select advertising agency	A	2
E	Plan advertising campaign	D	4
F	Conduct advertising campaign	E	10
G	Design package	–	2
H	Setup packaging facilities	G	10
I	Package initial stocks	J, H	6
J	Order stock from manufacturer	–	13
K	Select distributors	A	9
L	Sell to distributors	C, K	3
M	Ship stocks to distributors	I, L	5

5. An insurance company has decided to modernize and refit one of its branch offices. Some of the existing office equipments will be disposed of but the remaining will be returned to the branch after the completion of the renovation work. Tenders are invited from a number of selected contractors. The contractors would be responsible

for all the activities in connection with the renovation work excepting the prior removal of the old equipment and its subsequent replacement. The major elements of the project have been identified, as follows, along with their durations and immediately preceding elements.

Activity	Description	Duration(weeks)	Immediate Predecessors
A	Design new premises	141	–
B	Obtain tenders from the contractors	4	A
C	Select the contractor	2	B
D	Arrange details with selected contractor	1	C
E	Decide which equipment is to be used	2	A
F	Arrange storage of equipment	3	E
G	Arrange disposal of other equipment	2	E
H	Order new equipment	4	E
I	Take delivery of new equipment	3	H, L
J	Renovations take place	12	K
K	Remove old equipment for storage or disposal	4	D, F, G
L	Cleaning after the contractor has finished	2	J
M	Return old equipment for storage	2	H, L

(a) Draw the network diagram showing the interrelations between the various activities of the project. (b) Calculate the minimum time that the renovation can take from the design stage. (c) Find the effect on the overall duration of the project if the estimates or tenders can be obtained in two weeks from the contractors by reducing their numbers. (d) Calculate the 'independent float' that is associated with the non-critical activities in the network diagram.

PERT:

6. The following network diagram represents activities associated with a project:

Activities	A	B	C	D	E	F	G	H	I
Optimistic time, (t_o)	5	18	26	16	15	6	7	7	3
Pessimistic time, (t_p)	10	22	40	20	25	12	12	9	5
Most likely time, (t_m)	8	20	33	18	20	9	10	8	4

Determine the following:

(a) Network diagram

(b) Expected completion time and variance of each activity

(c) The earliest and latest expected completion times of each event.

(d) The critical path.

(e) The probability of expected completion time of the project if the original scheduled time of completing the project is 41.5 weeks.

(f) The duration of the project that will have 95 per cent chance of being completed.

7. A small project involves 7 activities, and their time estimates are listed in the following table. Activities are identified by their beginning (i) and ending (j) node numbers.

Activity (i – j)	Estimated Duration (weeks)		
	Optimistic	Most likely	Pessimistic
1 – 2	1	1	7
1 – 3	1	4	7
1 – 4	2	2	8

2 – 5	1	1	1
3 – 5	2	5	14
4 – 6	2	5	8
5 – 6	3	6	15

(a) Draw the network diagram of the activities in the project. (b) Find the expected duration and variance for each activity. What is the expected project length? (c) Calculate the variance and standard deviation of the project length. What is probability that the project will be completed: (i) at least 4 weeks earlier than expected times. (ii) no more than 4 weeks later than expected time. (d) If the project due date is 19 weeks, what is the probability of not meeting the due date.

Time cost trade-off analysis:

The starting point for crashing is when all critical activity are completed with their normal time, and crashing is stopped when all critical activities are crashed.

The crashing of normal completion time of critical activities will increase total time cost of the project. But, the decision maker will always look trade-off between the total cost of project and the total time required to complete it.

The method of establishing time cost trade-off for the completion of a project can be summarized as follows:

(i) Determine the normal project completion time and associated critical path.

(ii) Identify critical activities and complete the cost slope for each of these by using the relationship

Cost slope = $\frac{Crash\ cost - Normal\ cost}{Normal\ time - Crash\ time}$

The values of cost slope for critical activities indicate the direct extra cost required to execute an activity per unit of time.

(iii) For reducing the total project completion time, identify and crash an activity time on the critical path with lowest cost slope value to the point. Where,

(a) Another path in the network becomes critical.

(b) The activity has been crash to its lowest possible time.

(iv) If the critical path under crashing is still critical return to step (iii). However, if due to crashing of an activity time in step (iii), other paths in the network also become critical, then indentify and crash the activities on the critical paths with the minimum joint cost slope.

(v) Terminate the procedure when each critical activity has been crashed to its lowest possible time. Determine total project cost (indirect cost plus direct cost) corresponding to different project duration.

Question: Following table shows, activity, their normal time and cost, and crash time and cost for a small construction project.

Activity	Normal time (day)	Normal cost (Rs)	Crash time (day)	Crash cost (Rs)
1 – 2	9	640	6	700
1 – 3	8	500	5	575
1 – 4	15	400	10	550
2 – 4	5	100	3	120
3 – 4	10	200	6	260
4 – 5	2	100	1	140

Indirect cost Rs 60 per day.

(i) Draw the Network diagram.

(ii) Determine minimum total time and corresponding cost.

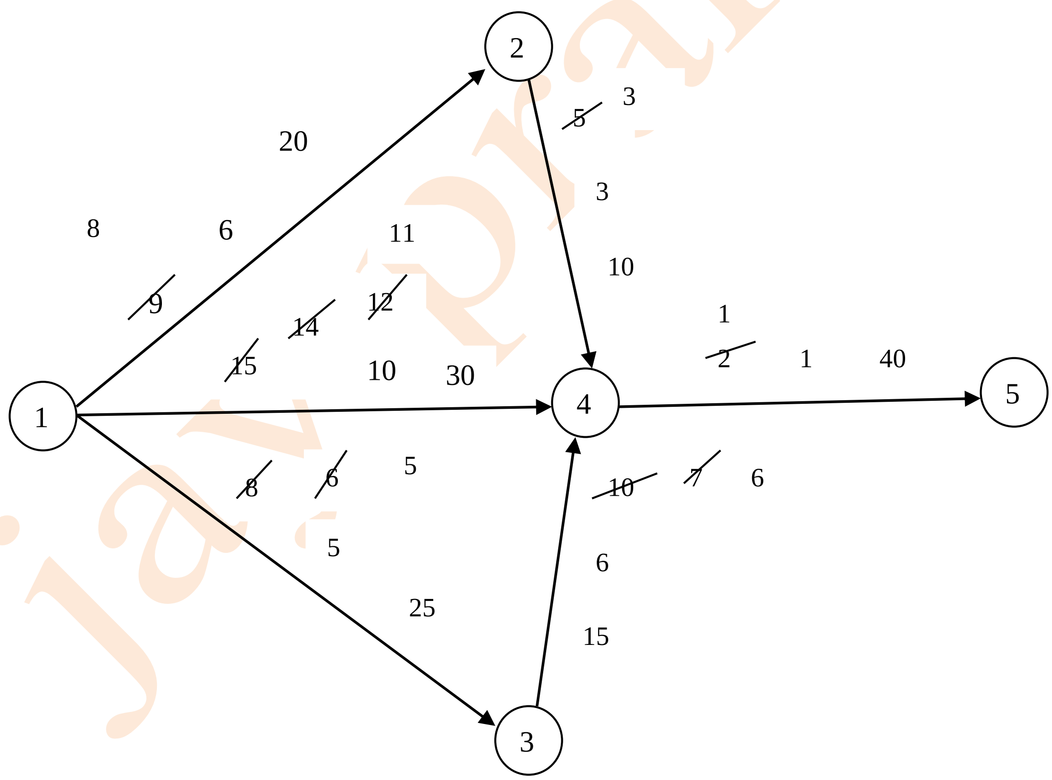
Solution: Here,

$$\text{Cost slope (crash per day)} = \frac{Crash\ cost - Normal\ cost}{Normal\ time - Crash\ time}$$

Activity	Normal		Crash		Shorten time	Increment cost	Cost slope	Rank
	Time	Cost	Time	Cost				
1 – 2	9	640	6	700	3	60	20	3
1 – 3	8	500	5	575	3	75	25	4
1 – 4	15	400	10	550	5	150	30	5
2 – 4	5	100	3	120	2	20	10	1
3 – 4	10	200	6	260	4	60	15	2
4 – 5	2	100	1	140	1	40	40	6
		1940						

Now,

Network diagram



<u>Various paths</u> <u>Project duration</u> <u>Crash time</u>

(i) 1 – 2 – 4 – 5 = 9 + 5 + 2 = 16 $\rightarrow$ 16 $\rightarrow$ 15 $\rightarrow$ 15 $\rightarrow$ 13 $\rightarrow$ 12   6 + 3 + 1 = 10

(ii) 1 – 4 – 5 = 15 + 2 = 17 $\rightarrow$ 17 $\rightarrow$ 16 $\rightarrow$ 15 $\rightarrow$ 13 $\rightarrow$ 12   10 + 1 = 11

(iii) 1 – 3 – 4 – 5 = 8 + 10 + 2 = 20 $\rightarrow$ 17 $\rightarrow$ 16 $\rightarrow$ 15 $\rightarrow$ 13 $\rightarrow$ 12   5 + 6 + 1 = 12

Therefore,

Project duration = 20 days

Minimum project duration = 12 days

∴ Critical path = 1 – 3 – 4 – 5

Project duration = 20 days

Total direct cost of project under normal condition = Rs 1940

Indirect cost = 20×60 = Rs 1200

Total normal cost = Direct cost + Crash cost + overhead cost

$= 1940 + 0 + 20 \times 60 =$ Rs 3140

Table value

Z	0.50	0.67	1.00	1.33	2.00
Probability	0.3085	0.2514	0.1587	0.0918	0.0228

44444444 444444