The introduction of Zernike subpixel edge detection algorithm

Ghosal and Mehrotal[1] first proposed to use Zernike orthogonal moments to detect subpixel edges. In their algorithm, an ideal edge step grayscale model was established like Fig. 1(a). Four parameters of the model were calculated by three Zernike moments of different orders, and the four parameters were used to determine the accurate sub-pixel coordinates of the edges.

As Fig. 1(b) shows, the shaded circle represents the 7×7 Zernike moment template and each grid represents one pixel.

L is the part of boundary that falls in the inner part of the circle. The center of the circle P_l is the integer pixel obtained by Canny operator. Obviously, there is a discrepancy between P_l with the integer pixel coordinate and the real edge L. So Zernike moment method is applied to calculate a more accurate sub-pixel edge point P_r based on the local grayscale information of the circular area.

Zernike moments induce a set of complex polynomials, which form a complete orthogonal set over the interior of circle $x^2 + y^2 = 1$. An ideal edge step can be defined by h, b, θ and d as Fig. 1(a) shows. h is the step height, b is the background grey value, θ is the angle between the normal of L and the positive x-axis, and d is the distance from the circle center P_l to L. Similar to Fig. 1(a), Fig. 1(c) is the two-dimensional diagram of step model and Fig. 1(d) is Fig. 1(c) after rotating θ clockwise around the circle center.

According to the orthogonality and rotation invariance of Zernike moments, Ghosal and Mehrotal proposed edge locating principles based on Zernike moments. The basic prindciple is to calculate three Zernike moments by convolution and determine the four parameters h, b, θ and d by solving a equation set (Equation (1.6), (1.8), (1.9) and (1.10)). Once all parameters of the edge step model are determined, the sub-pixel edge point P_r can be located accurately by Equation. (1.11).

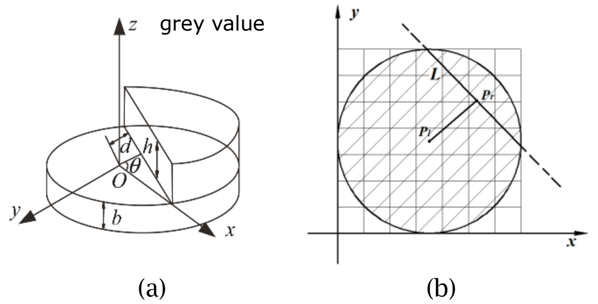

Fig. 1: Step model for sub-pixel edge, (a) Three-dimensional step model diagram, (b) Circular kernel defined for a 7×7 pixels area, (c) Two-dimensional step model diagram, (d) Two-dimensional rotation step model diagram

The Zernike moments Z_{nm} for a continuous image function $f(x,y)$ can be defined in polar coordinates

$$Z_{nm} = \frac{n+1}{\pi} \iint_{x^2+y^2 \le 1} f(x,y) V_{nm}^{*}(\rho,\theta) \rho d\rho d\theta \tag{1.1}$$

Where n and m are the order and repetition of the moment respectively. $(n+1)/\pi$ is a normalization factor and will be ignored later. For a discrete digital image, the Zernike moments can be defined as

$$Z_{nm} = \frac{n+1}{\pi}\sum_{x}\sum_{y} f(x,y)V_{nm}^{*}(\rho,\theta) \tag{1.2}$$

Where ρ is the length of vector, $*$ is the complex conjugate and $V_{nm}(\rho,\theta) = R_{nm}(\rho)e^{im\theta}$ is the orthogonal integral kernel function. Where $R_{nm}(\rho)$ is a complex polynomial and it can be expressed as Equation (1.3).

$$R_{nm}(\rho)=\sum_{i=0}^{(n-|m|)/2}[(-1)^{i}(n-i)^{n-2i}][i!(\frac{n+|m|}{2}-i)!(\frac{n-|m|}{2}-i)!]^{-1} \tag{1.3}$$

Where n is a non-negative integer and m is an integer that satisfies $|m| = (even)$ and $|m| \leq n$.

As shown in Fig. 1(d), if image is rotated clockwise by θ to the edge L parallel to the y-axis. The following formula can be obtained

$$\iint_{x^2+y^2\leq 1} f^{'}(x,y)ydxdy = 0 \tag{1.4}$$

Where $f^{'}(x,y)$ is the rotated image. The relationship between Zernike moments of the original image and the rotated image can be written as Equation (1.5).

$$Z_{00}^{'} = Z_{00},\ Z_{11}^{'} = Z_{11}e^{j\theta},\ Z_{20}^{'} = Z_{20} \tag{1.5}$$

So we can get $\mathrm{Im}[Z_{11}^{'}] = \mathrm{Re}[Z_{11}^{'}]\sin\theta - \mathrm{Im}[Z_{11}]\cos\theta = 0$. The $\mathrm{Im}[Z_{11}]$ and $\mathrm{Re}[Z_{11}]$ are the imaginary part and the real part of Z_{11} respectively. Therefore,

$$\theta=\arctan(\mathrm{Im}[Z_{11}]/\mathrm{Re}[Z_{11}]) \tag{1.6}$$

Refer to Fig. 1(d), the Zernike moments of the rotated image can be expressed as follows:

$$Z_{00}^{'} = b\pi + \frac{h\pi}{2} - k\arcsin(d) - kd(1-d^2)^{1/2},\ Z_{11}^{'} = 2h(1-d^2)^{3/2}/3,\ Z_{20}^{'} = 2hd(1-d^2)^{3/2}/3 \tag{1.7}$$

Using Equation (1.6) and solving above Equation (1.7), the other three edge parameters can be determined.

$$h = 3Z_{11}^{'}\left[2(1-d^2)^{3/2}\right]^{-1} = 3Z_{11}\left[2(1-d^2)^{3/2}\right]^{-1}e^{j\theta} \tag{1.8}$$

$$b = \left[Z_{00} - \frac{h\pi}{2} + h\sin^{-1}(d) + hd(1-d^2)^{\frac{1}{2}}\right]/\pi \tag{1.9}$$

$$d = Z_{20}/Z_{11}^{'} = Z_{20}e^{-j\theta}/Z_{11} \tag{1.10}$$

When a unit circle is used in the region of $N\times N$ pixels and the window is moving on the image for convolution calculation, the window is the template covering N^2 pixels surrounding the center and the radius unit becomes $N/2$. So the vertical distance l should be amplified $N/2$ times. So the sub-pixel coordinates of pixel calculation formula can be defined as follows:

$$\begin{bmatrix} x_s \\ y_s \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \frac{N}{2}d\begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} \tag{1.11}$$

References:

[1] Ghosal S; Mehrotra R.: Orthogonal moment operators for subpixel edge detection, Pattern Recognition, 26(2), 1993, 295-306. https://doi.org/10.1016/0031-3203(93)90038-x