

11/11/18

Queue:

It is a linear data structure. Follows FIFO. Insertion and deletion from opposite ends.

Deletion:

- ① Both F and R are -1

Insertion:

- ① If $F = R = -1$
- ② If $R = \text{max} - 1 \Rightarrow \text{overflow}$
- ③ $R++ \Rightarrow \text{insert value}$

* Insertion and deletion: Array based:

```
#define max 30
```

```
int F = R = -1;
```

```
int arr[max];
```

```
void insertQ(int arr[], int num)
```

```
{
```

```
    if (R == max - 1)
```

```
    { printf("Overflow");
```

```
      return();
```

```
    }
```

```
    else if (F == -1)
```

```
    {
```

```
        F = R = 0;
```

```
        arr[R] = num;
```

```
    }
```

```
    else
```

```
    {
```

```
        R = R + 1;
```

```
        arr[R] = num;
```

```
    }
```

```
}
```

```
int delQ(int arr[])
```

```
{
```

```
    if (F == -1)
```

```
    {
```

```
        printf("Empty queue.");
```

```
        return();
```

```
    }
```

```
    else if (L == R)
```

```

    if (item == arr[F]);
    F = -1;
    R = -1;
}
else
{
    item = arr[F];
    return (item);
}
return item;
}

```

Linked List based:

```

typedef struct node
{
    int data;
    struct node *next;
} Queue;
Queue * Front = NULL;

```

Circular Queue :

```

void insertQueue(int value)
{
    if (F == R || R == max-1 || (R == F-1))
    {
        printf("Queue is full.");
        return;
    }
    else if (F == -1)
    {
        F = R = 0;
        arr[R] = value;
    }
    else if (R == max-1 || F == 0)
    {
        F = 0;
        arr[F] = value;
    }
    else
    {
        R = R+1;
        arr[R] = value;
    }
}

```

```

int *q;
if (q == NULL)

```

```

int *q;
if (q == NULL)

```


Insertion from both ends, deletion from

1. Output restricted (deletion from one end) if
2. Input restricted (insertion from one end) if

```

if (q == NULL)

```



```

int *q;

```

```

int *q;

```

if (q == NULL) return -1; else return *q;

```

void enqueue(int x)

```

```

void enqueue(int x)

```

```

void enqueue(int x)

```

```

void enqueue(int x)

```

```

int main()

```

```

{

```

```

    enqueue(x);

```

```

    case 1: enqueue(x);

```

```

}

```

```

void enqueue(int x, int front)

```

```

{

```

```

    if (front == 0)

```

```

    if (front == 0)

```

```

{

```

```

    printf("Queue is full");

```

```

    return;

```

```

}

```

```

else if (front == 0)

```

```

{

```

```

    front = 0;

```

```

    front = 0;

```

```

    front = 0;

```

```

}

```

```

else
{
    p->next = p->next + 1;
    p->arr[p->next] = item;
}
}

```

```

void InsDeqF ( deque *p, int item)
{

```

```

    if (p->front == 0)
    {
        printf("Queue is full");
        return;
    }
    else if (p->front == -1)
    {
        p->front = p->next = 0;
        p->arr[p->front] = item;
    }
    else
    {
        p->front = p->front - 1;
        p->arr[p->front] = item;
    }
}

```

```

void DelDeqR ( deque *p)
{

```

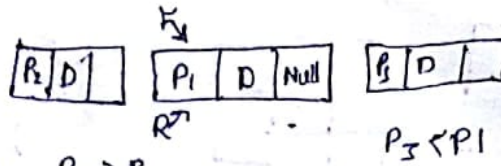
```

    int item;
    if (p->front == -1)
    {
        printf("Underflow");
        return;
    }
    else if (p->front == p->next)
    {
        item = p->arr[p->front];
        p->front = p->next = -1;
    }
    else
    {
        item = p->arr[p->next];
        p->next = p->next - 1;
    }
    return item;
}

```


Priority Queue:

If the new node has priority higher than current, it is inserted to the left of the node, otherwise to the right, $P_2 > P_1$.



Insertion can be done sorted or unsorted.

For deletion, we need to find out the highest priority node each time. That element is deleted first.

13/9/18

More Examples on Recursion:

* Draw n concentric circles.

DrawCircle(int x , int y , int m , int n)

{

if ($n == 0$)

return;

else

{

DrawCircle(x , y , $m-10$, $n-1$);

Circle(x , y , m);

}

}

To draw the bigger circle first, call the in-built function, circle before recursion of DrawCircle().

* Without using in-built function:

main()

{

int gd = DETECT, gm;

initgraph(&gd, &gm, "c:\\");

* Draw n rectangles/squares:

DrawRectangle(int x , int y , int l , int n)

{

if ($n == 0$)

return;

else

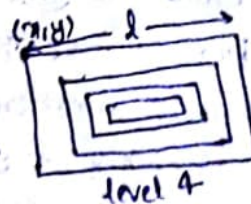
{

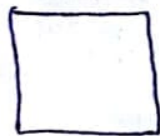
square
DrawRectangle($x+l/4$, $y+l/4$, $l/2$, $n-1$);

square
rectangle(x , y , l);

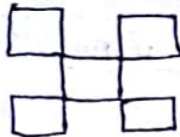
}

}

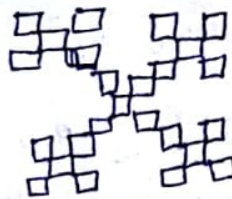




L=0



L=1



L=2

* Complex Geometrical Shapes:

• DrawFractal(int x, int y, int l, int n)

{

if (n == 0)

Square(x, y, l); Delay(100);

else

{

DrawFractal(x, y, l/3, n-1); // 1

DrawFractal(x + 2 * l/3, y, l/3, n-1); // 2

DrawFractal(x + l/3, y + l/3, l/3, n-1); // 3

DrawFractal(x, y + 2 * l/3, l/3, n-1); // 4

DrawFractal(x + 2 * l/3, y + 2 * l/3, l/3, n-1); // 5

}

}



• Sierpinsky Triangle:

DrawTriangle(int x1, int y1, int x2, int y2,
int x3, int y3, int n)

{

if (n == 0)

return;

else

{

DrawTriangle(x1, y1, (x1 + x2)/2, (y1 + y2)/2, (x1 + x3)/2, (y1 + y3)/2, n-1);

DrawTriangle((x1 + x2)/2, (y1 + y2)/2, x2, y2, (x2 + x3)/2, (y2 + y3)/2, n-1);

DrawTriangle((x1 + x2)/2, (y1 + y2)/2, (x2 + x3)/2, (y2 + y3)/2, x3, y3, n-1);

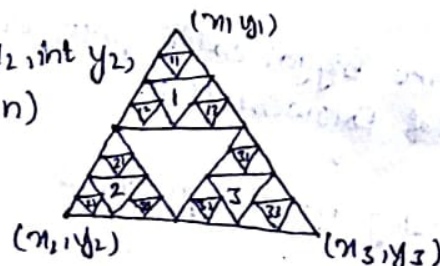
line(x1, y1, x2, y2);

line(x2, y2, x3, y3);

line(x1, y1, x3, y3);

}

}



Koch Curve:



18/9/2018

KoshCurve(int x_1 , int y_1 , int x_2 , int y_2 , int n)

{

int $x_3, y_3, x_4, y_4, n_1, y_1$;

if ($n == 1$)

line(x_1, y_1, x_2, y_2);

else

{

$x_3 = (2x_1 + x_2) / 3$;

$y_3 = (2y_1 + y_2) / 3$;

$x_4 = (x_1 + 2x_2) / 3$;

$y_4 = (y_1 + 2y_2) / 3$;

$n =$

$y =$

KoshCurve($x_1, y_1, x_3, y_3, n-1$);

KoshCurve($x_3, y_3, x_1, y_1, n-1$);

KoshCurve($x_1, y_1, x_4, y_4, n-1$);

KoshCurve($x_4, y_4, x_2, y_2, n-1$);

* Backtracking:

n Queen Problem:

n queens are to be placed on a $n \times n$ board such that no ² queens are in a vertically, horizontally or diagonally in the same line.

No place is left for Q_6 , so now start backtracking. Move Q_5 to new position, still no place found, bt again and place Q_4 to new position. Move Q_4 to position (ii)
Now place Q_5

Move Q_4 to position (iii). Now Q_5 has two possible positions. Place it on (i). Now Q_6 has one possible position and so does Q_7 . But now Q_8 can not be placed. Back-track again

8x8

Q_1	x		x				
	x	x	Q_4	Q_5			
	Q_2						
				Q_5	Q_6		
		Q_3					

4x4

Q_1		Q_3	
Q_1			
	Q_2		Q_4
	Q_2		

* Rat in a maze Problem:

R								
	X		X		X			X
X		X		X				
				X		X	X	
		X	X		X	X	X	
					X	X	X	
X					X	X	X	
X								F

The rat has to reach its destination, ie, the food using four movements only- up, down, right and left. There are certain blockages in the path.

19/8/18

Insertion Sort:

If $A[] = \{a_1, a_2, a_3, \dots, a_n\}$

1. Sort the first element.
2. Compare a_2 and a_1 , swap if $a_2 < a_1$. {for ascending order}
3. Compare a_3 and a_2 . If $a_3 < a_2$ compare a_3 with a_1 . If $a_3 < a_1$
4. Store a_3 in a variable and shift both to right
5. $a_4 < a_3 \Rightarrow$ compare with a_2

eg. 5, 10, 2, 1, 15, 20, 4

```

5 10 2 1 15 20 4
2 5 10 1 15 20 4
1 2 5 10 15 20 4
1 2 5 10 15 20 4
1 2 4 5 10 15 20
  
```

No. of comparison = 13

- * Best case: already sorted, No. of comparisons = no. of elements = n
- * Worst case: sorted in reverse order, No. of comparisons = n^2

Quick Sort:

Divide and conquer algorithm

$A[] = \{a_1, a_2, a_3, a_4, \dots, a_n\}$

Consider an element as pivot and find its right place. The elements on the right are bigger and on the left are smaller but are not sorted among themselves

a_1 is pivot. If $a_2 < a_1$, swap start from right. Compare a_n with a_1 ,
 a_{n-2}, \dots, a_1 Compare a_n and a_1 . If suppose we find $a_{n-1} < a_1 \Rightarrow$ swap. Else
 continue comparison of a_n with
 $a_{n-2}, a_{n-3}, \dots, a_1$.

* Worst case: already sorted array

No. of comparisons = n^2

* All elements have to be compared.

* Best case: If pivot divides the list in two equal halves.

Time complexity = $n \log n$

eg. 2 10 5 15 1 6 4 20 12

```

2 10 5 15 1 6 4 20 12
1 2 10 5 15 1 6 4 20 12
1 2 5 15 10 6 4 20 12
1 2 4 15 10 6 5 20 12
1 2 4 5 10 6 15 20 12
  
```

$a_1, a_2, a_3, \dots, a_n$

$n/2 \quad a_1 \quad n/2$

TC = $n + \frac{n}{2} + \frac{n}{4} + \frac{n}{8} + \dots$ $(1 + \frac{1}{2} + \frac{1}{4} + \dots)$

$2n \log n$

3/10/2018

Tree:

It is a non-linear data structure used to represent data in hierarchical order.

Root: Every tree has a unique root.

Note: The data elements in a tree are represented by nodes.

Edges: Lines which connect two nodes.

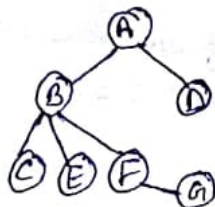
Parent: If there are two nodes connected with an edge, the ^{predecessor node} (source) of connection is called parent.

* Except root, all nodes must have a parent.

Child: The immediate successor node of any node is called child.

Degree: Degree of a node is the maximum no. of children.

Degree of tree is the ^{degree of node having} max no. of children of any node.



Level: Level of root is 0. Its child has level 1 and so on.

Siblings: All children of a node are siblings. eg B, D, C, E, F.

Height of a tree: Max no. of edges in the longest path starting from the root.

Degree of tree = 3 Height of given tree = 3
Height of D = 1

Path: Combination of edges from root to a particular node.

Depth of a tree: ~~max~~ No. of edges from the leaf. eg Depth of this tree = 3
Depth of B = 2, Depth of D = 0.

Leaf: The nodes which do not have any child. eg D, E, G, C.

or
External nodes or Terminal nodes

Internal nodes: Nodes which have at least one child. eg A, B, F.

Subtrees: Nodes attached to left/right child constitute left/right subtree.

Binary Tree:

A tree having max two children at any node.

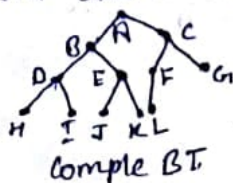
Full Binary Tree: A BT in which all levels have max possible no. of nodes.

* No. of nodes possible at each level = 2^l

~~Complete Binary Tree:~~ No. of internal nodes = $n-1$ $\nexists n$ = no. of leaves
No. of external nodes = $e+1$ $\nexists e$ = no. of internal nodes

Complete Binary Tree: It is a subset of full binary tree.

All levels have max possible no. of nodes except the last level and in the last level, nodes are aligned as much left aligned as possible.



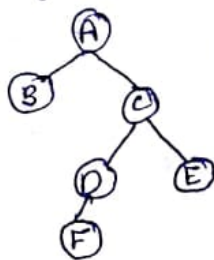
2 Tree: A BT in which each node has either two or zero child.

Representation of Binary Tree;

1. Sequential (array based) representation
2. Linked list representation

Sequential Representation:

Declare a 1D array. Assign first place to root. If there is a node stored at i th location, then its left child is stored at $(2*i)$ th location and right child at $(2*i+1)$ th location.



1	2	3	4	5	6	7	8	9	10	11	12
A	B	C			D	E					F

If there is a node at i th location, its parent should be at $\lceil i/2 \rceil$ th location.

Greatest Integer

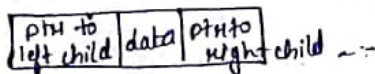
If $i=6$, parent is at $\lceil 6/2 \rceil = 3$

If $i=7$, parent is at $\lceil 7/2 \rceil = 3$

Drawbacks:

- i) Wastage of memory
- ii) Insertion, deletion is difficult.

Linked List Representation:



Tree Traversal:

There are three methods of traversing a tree:

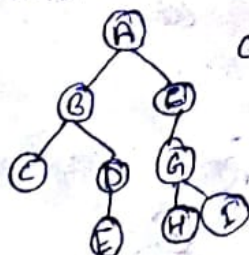
- i) In-order
- ii) Pre-order
- iii) Post-order

i) In order Traversal:

Steps to traverse a tree ~~by any method~~:

1. Traverse the left subtree.
2. Visit root.
3. Traverse right subtree.

Start traversing from left subtree. Print the node encountered second time.

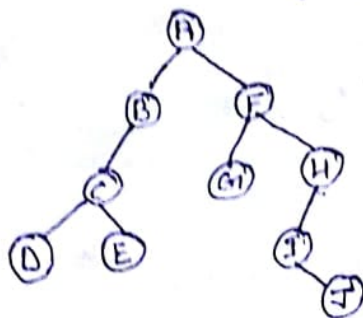


C B E D A H G I F

4-10-2020

(ii) Pre-order Traversal:

1. Process the root.
2. Traverse left subtree in pre-order.
3. Traverse right subtree in pre-order.



A B C D E F G H I J

* Follow the path and print the node first encountered.

(iii) Post-order Traversal:

1. Traverse left subtree in post-order.
2. Traverse right subtree in post-order.
3. Process the root.

In the above binary tree, post-order traversal gives following results:

DECBG I J H F A

* The node is printed when you do not get a chance to revisit that node ~~again~~. (in the last visit of the node)

In-order: D C E B A G I F J H

Construction of a Tree using two given traversal orders:

* A unique tree can be drawn provided either of following is known:

- i) In-order and pre-order
- ii) In-order and post-order

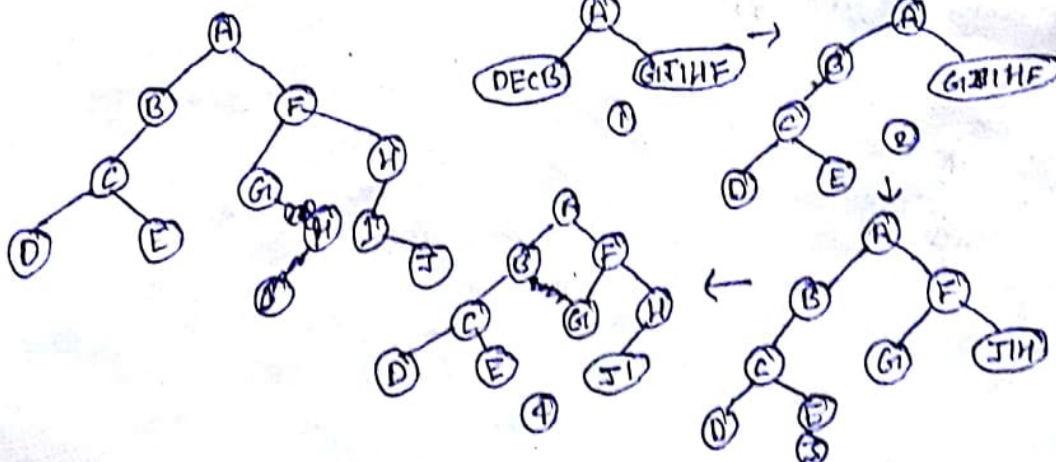
* The first node of pre-order and last node of post-order gives root.

* In-order separates the left subtree from the right one.

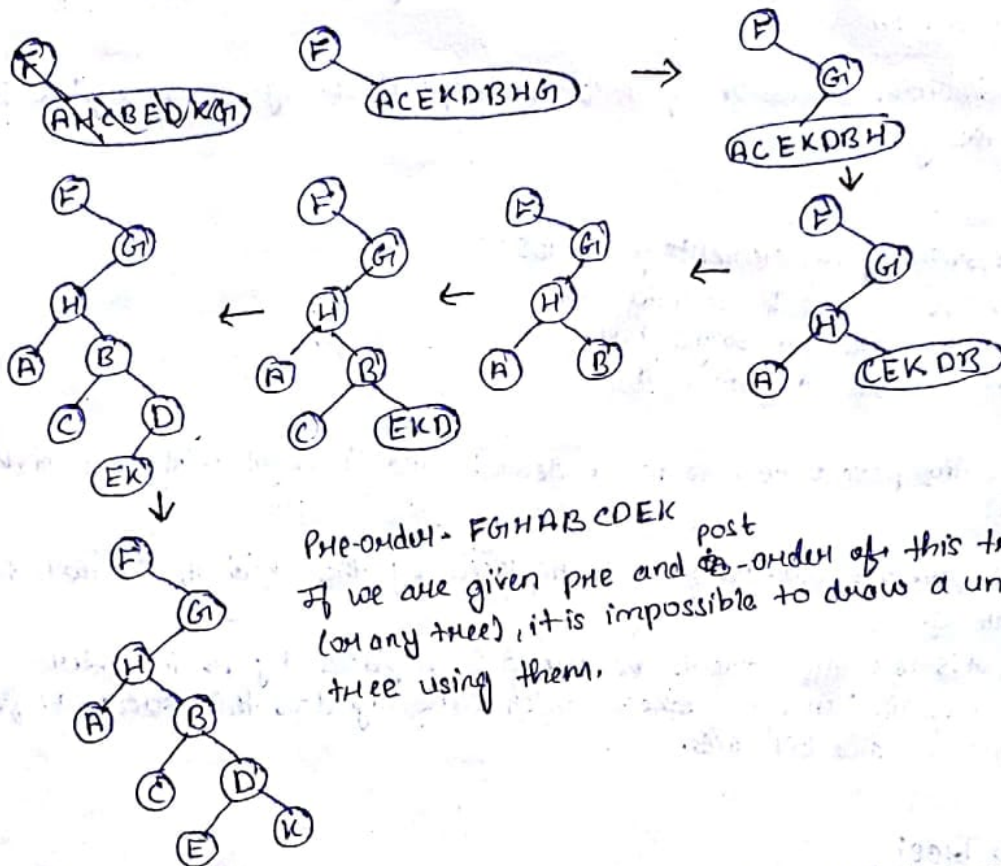
* Pre-order of left subtree separates the right and left children.

eg. Pre - A B C D E F G H I J
In - D C E B A G I F J H

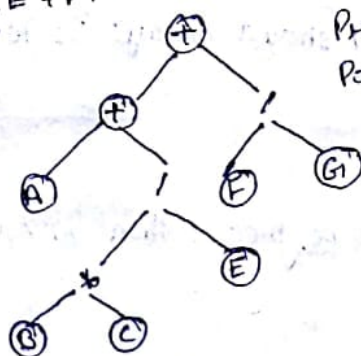
Post - D E C B G I J H F A
In - D C E B A G I F J H



eg. In- F A H C B E D K G I , Post- A C E K D B H G I F



* We can draw a binary tree for a given expression (which does not include unary operators).
Operators - ~~leaf~~ internal nodes, operands - ~~internal~~ leaf nodes.
eg. $A + B * C / E + F / G$



Prefix: $++A/*BCE/FG$
Postfix: $ABC*E/+FG/+$

Binary Search Tree (BST):

For any node, left subtree always contains smaller nodes and the right subtree contains ~~smaller~~ equal or greater nodes.

eg. 15 12 5 27 20 30 18

Make first element root. 12 is < 15 , so

left. $5 < 15, 12 \Rightarrow$ left

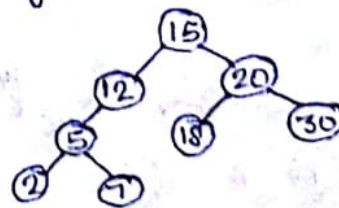
$2 < 15, 12, 5 \Rightarrow$ left

$7 < 15, 12$ but $> 5 \Rightarrow$ right to 5

$20 > 15 \Rightarrow$ right

$30 > 15, 20 \Rightarrow$ right

$18 < 30, 20 \Rightarrow$ left to 20



* If BST is balanced, we can perform binary search in time complexity of order n .

* The in-order successor of node can replace it. eg. 18 can replace 15 in the given tree.

Deletion

Deletion of an element from BST:

- i) The node without child
- ii) The node has single child
- iii) The node has two child

i) Find the parent of node to be deleted. Find its right child and make it null.

ii) The parent's address field is replaced by the child of the node to be deleted.

iii) In this case the node to be deleted is replaced by its in-order successor, then check wherein which category does this successor fall b/w i) and ii) cases.

Heap Tree:

It is of two types:

- i) Min heap
- ii) Max heap

Min Heap Tree:

i) It is a complete binary tree.

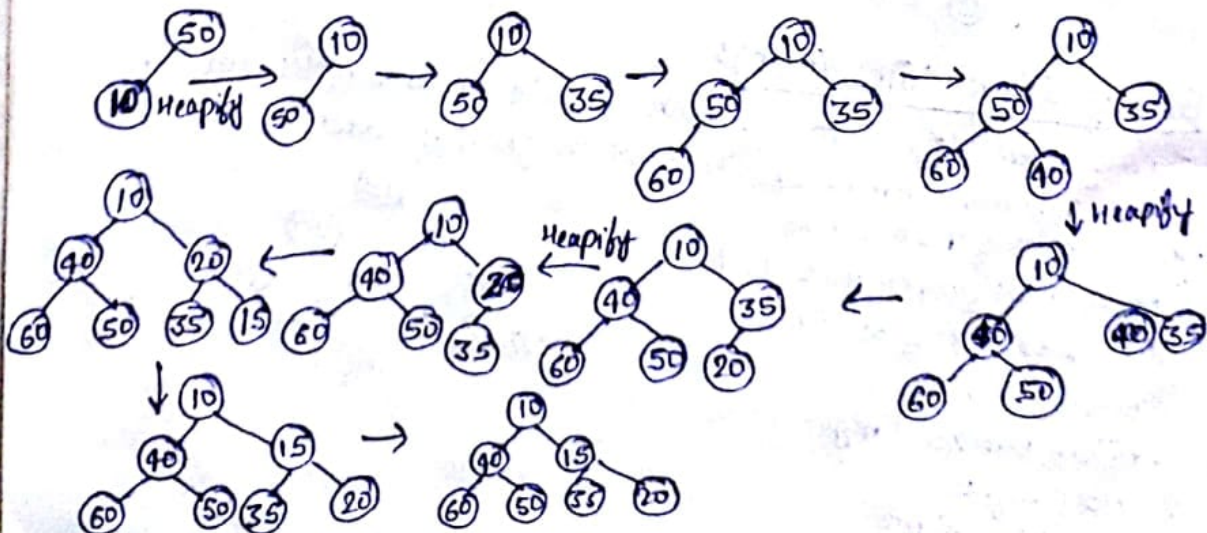
ii) The parent node (including root) should always be less than or equal to its children.

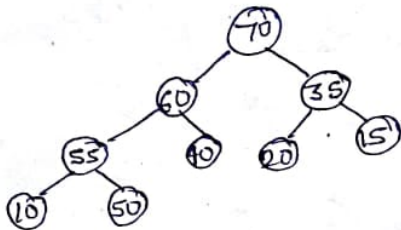
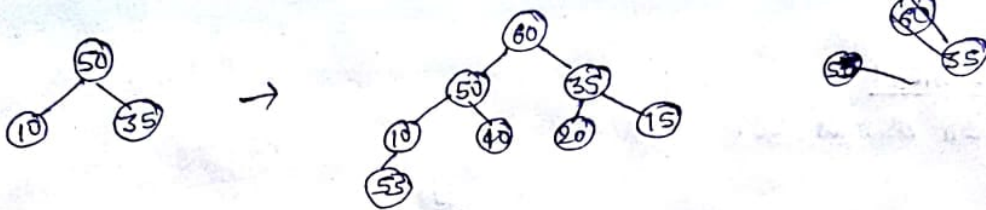
Max Heap Tree:

* It is a complete binary tree.

* The parent node should always be greater than or equal to its children.

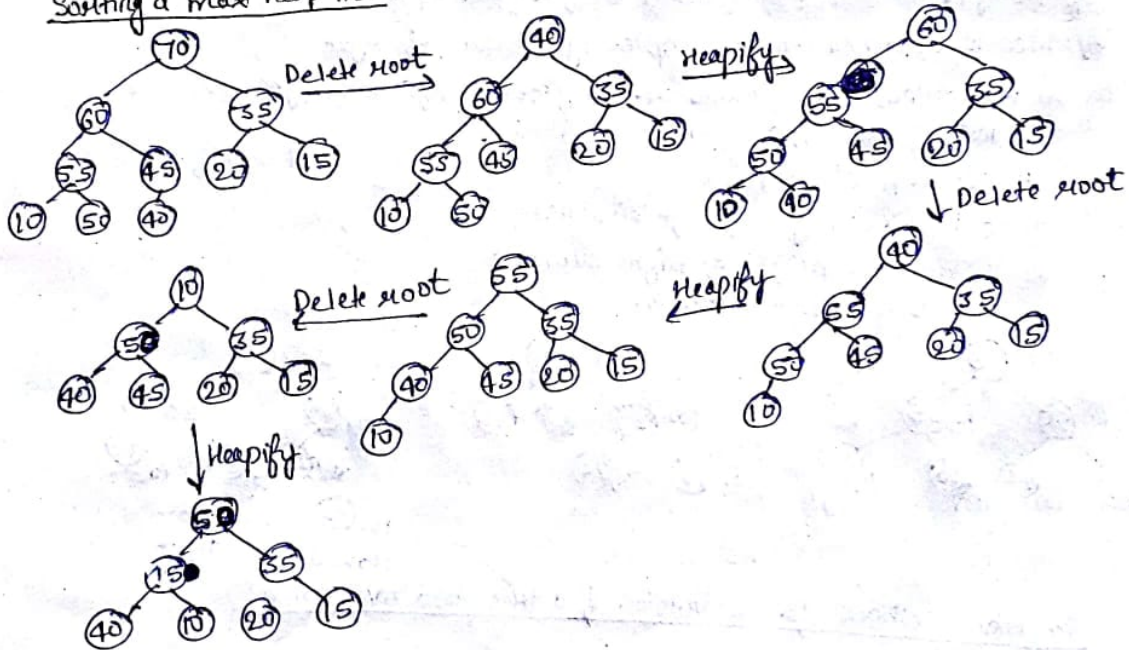
50, 10, 35, 60, 40, 20, 15, 55, 70





Max heap

Sorting a max heap tree:

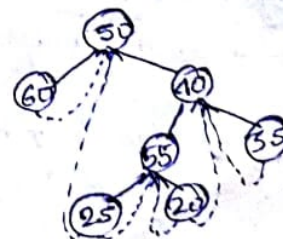
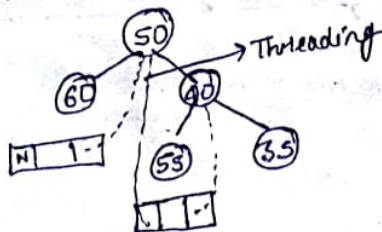


Threaded Binary Tree:

It is a 2 tree an extended tree

Left pointer \Rightarrow In order predecessor

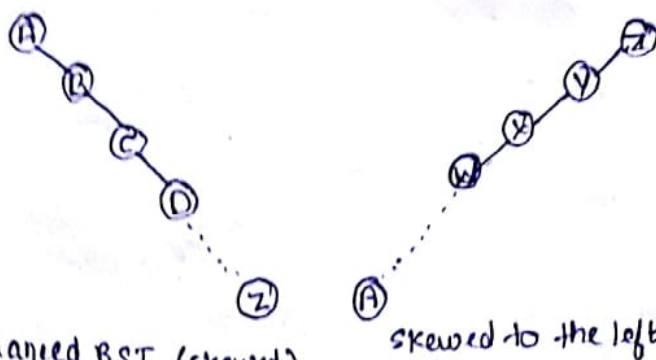
Right pointer \Rightarrow In order successor



Module 2

AVL Tree:

It is a balanced BST



Balanced BST (skewed)

If no. of elements = n , Time taken is of order of n .

If tree is balanced, time complexity = order of $\log n$.

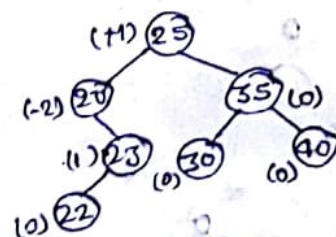
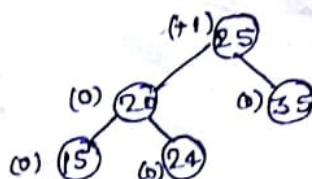
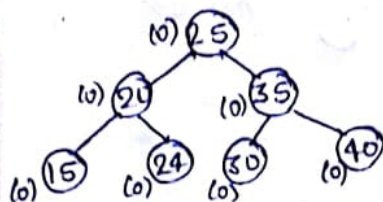
* In an AVL tree, the difference of heights of left and right subtree should always be +1, 0 or -1.

$$BF = h(LT) - h(RT) \leq 1$$

where $h(LT)$ = height of left subtree

$h(RT)$ = height of right subtree

BF = Balance Factor



Not an AVL tree

Rotation methods for conversion of a tree into AVL tree:

(a) LL

Find a node A such that it is the first ancestor of the node that we insert to make the tree non-AVL.

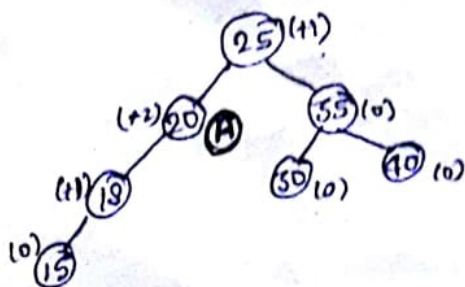
Start from inserted node towards root, find node that does not have its BF as 0, or +1, or -1.

Rotate the non-AVL tree around the ancestor.

Based on the ancestor, there are four kinds of rotations.

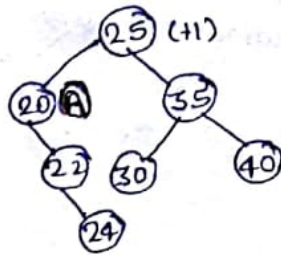
i) LL

LL = Left subtree of left subtree to A.



ii) RR

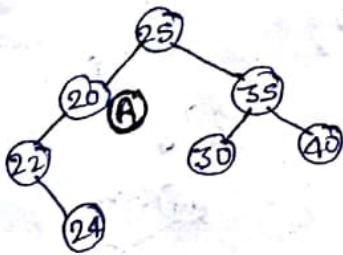
RR = Rotate along right subtree of right subtree from A.



iii) LR

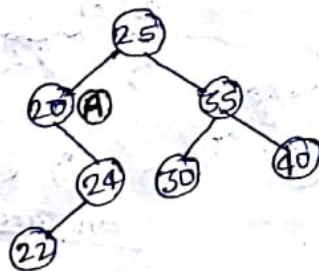
LR = Rotate along right subtree of left subtree from A.

Parent of inserted node becomes root, and the inserted node replaces its parent.

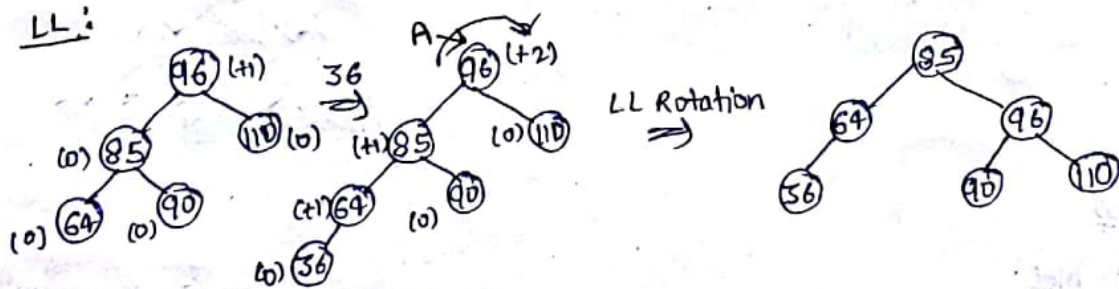


iv) RL

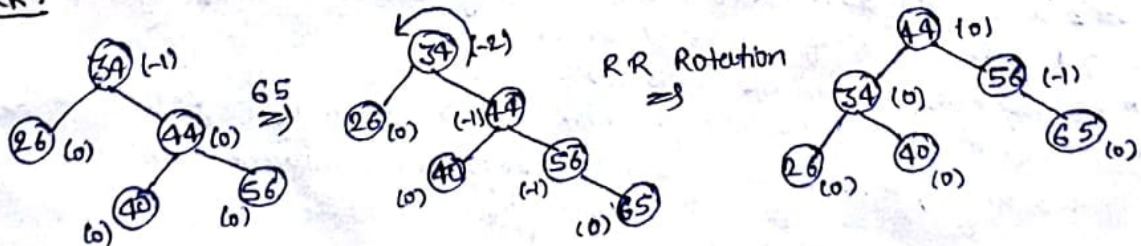
RL = Rotate along left subtree of right subtree from A.



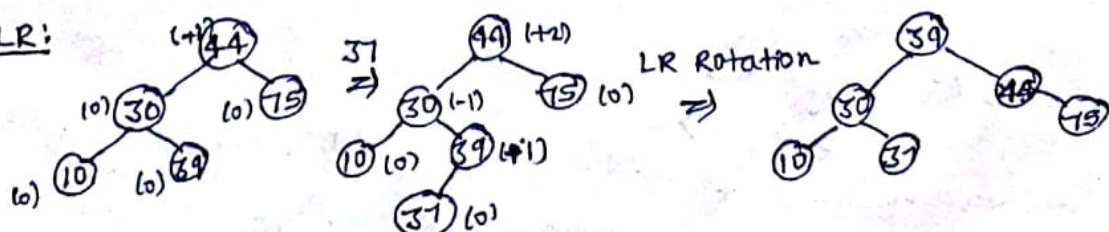
LL:



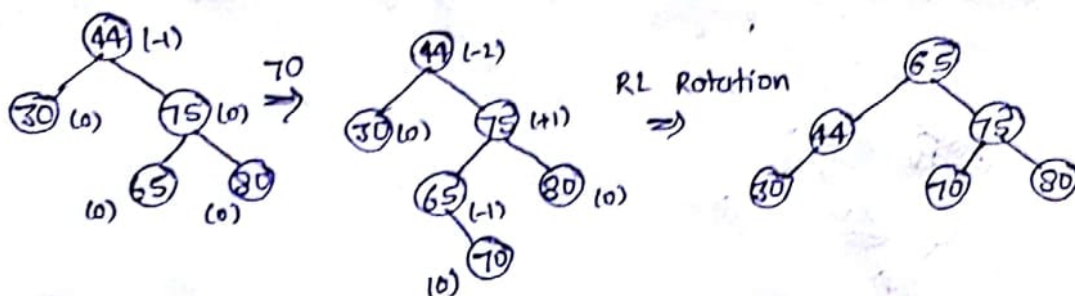
RR:



LR:

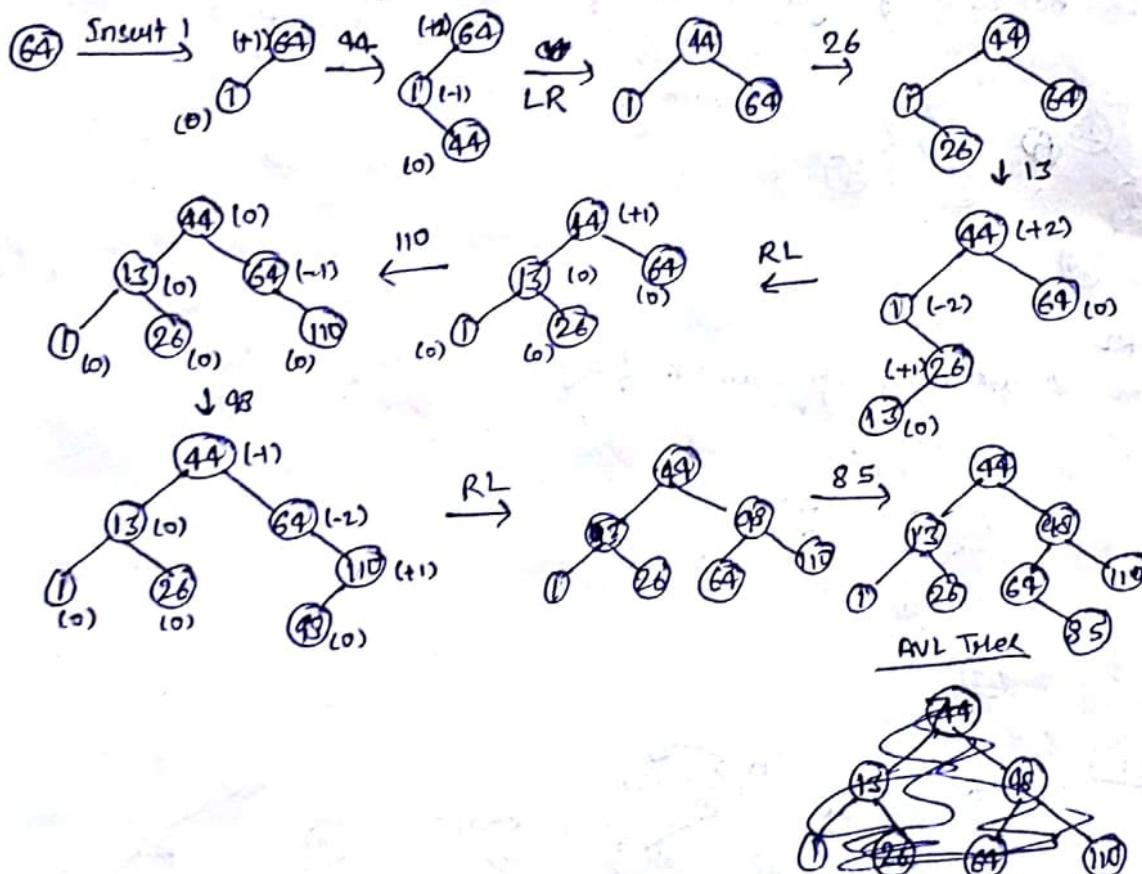


RL:



Construction of an AVL Tree:

Construct an AVL tree from 64, 1, 44, 26, 13, 110, 98, 85



22/10/2018

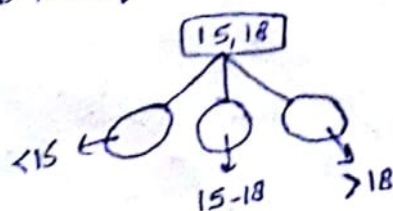
B-Tree:

It is a balanced BST. A b-tree is an m-way tree that satisfies the properties of a BST. (m = no. of possible children)

* All leaf nodes are on the same level.

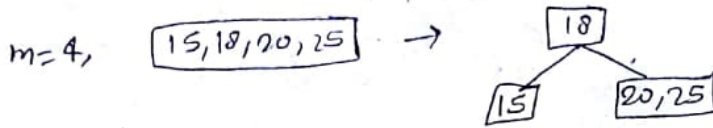
* Root can have minimum two and maximum m children (m-1 keys/elements)

eg. A B-tree of order 3 can have max 2 keys and 3 children. For root node (child)

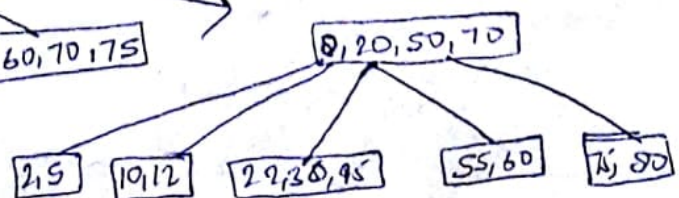
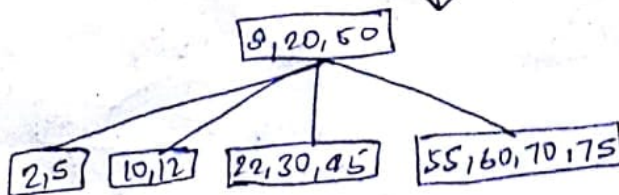
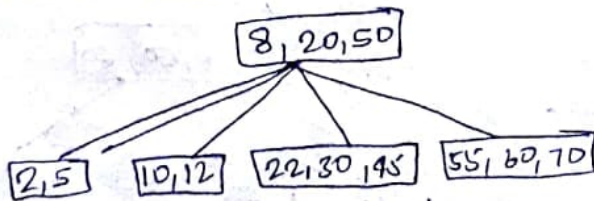
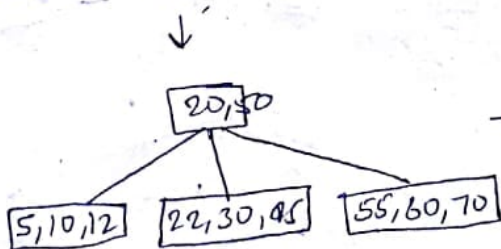
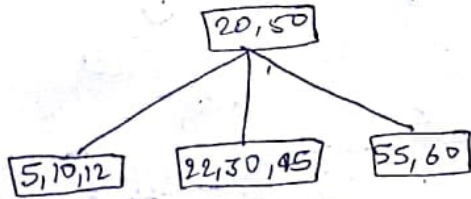
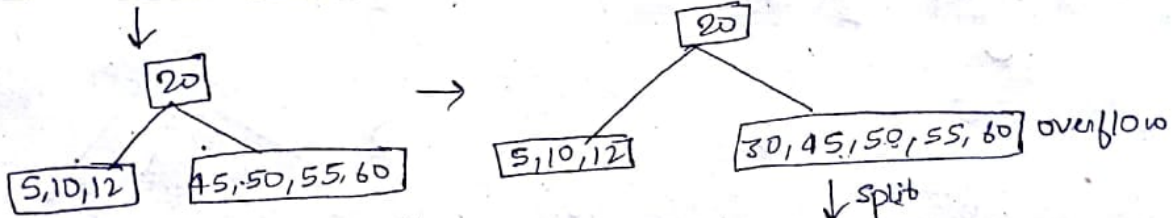
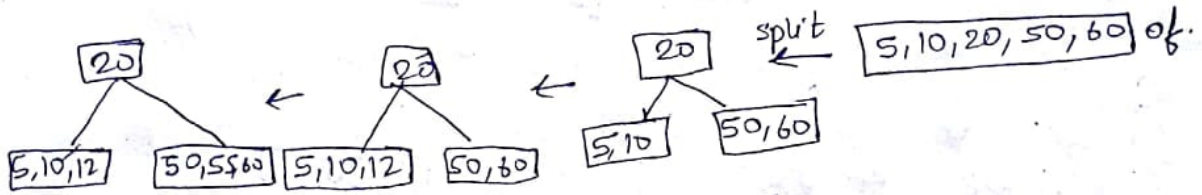
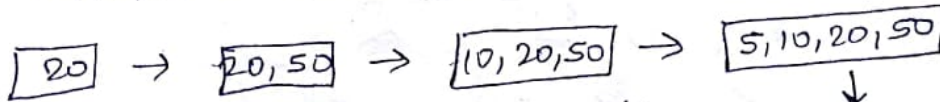


Root can have minimum 2 Non-Root node $\lceil \frac{m}{2} \rceil$ children $(\lceil \frac{m}{2} \rceil - 1)$ ← at least
at most m child, (m-1) keys

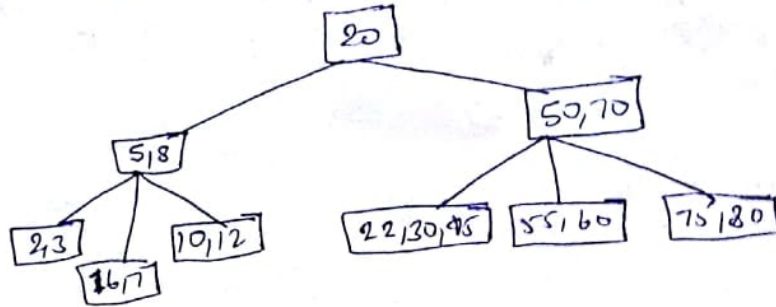
In case of overflow, the node splits. eg if $m=2$,



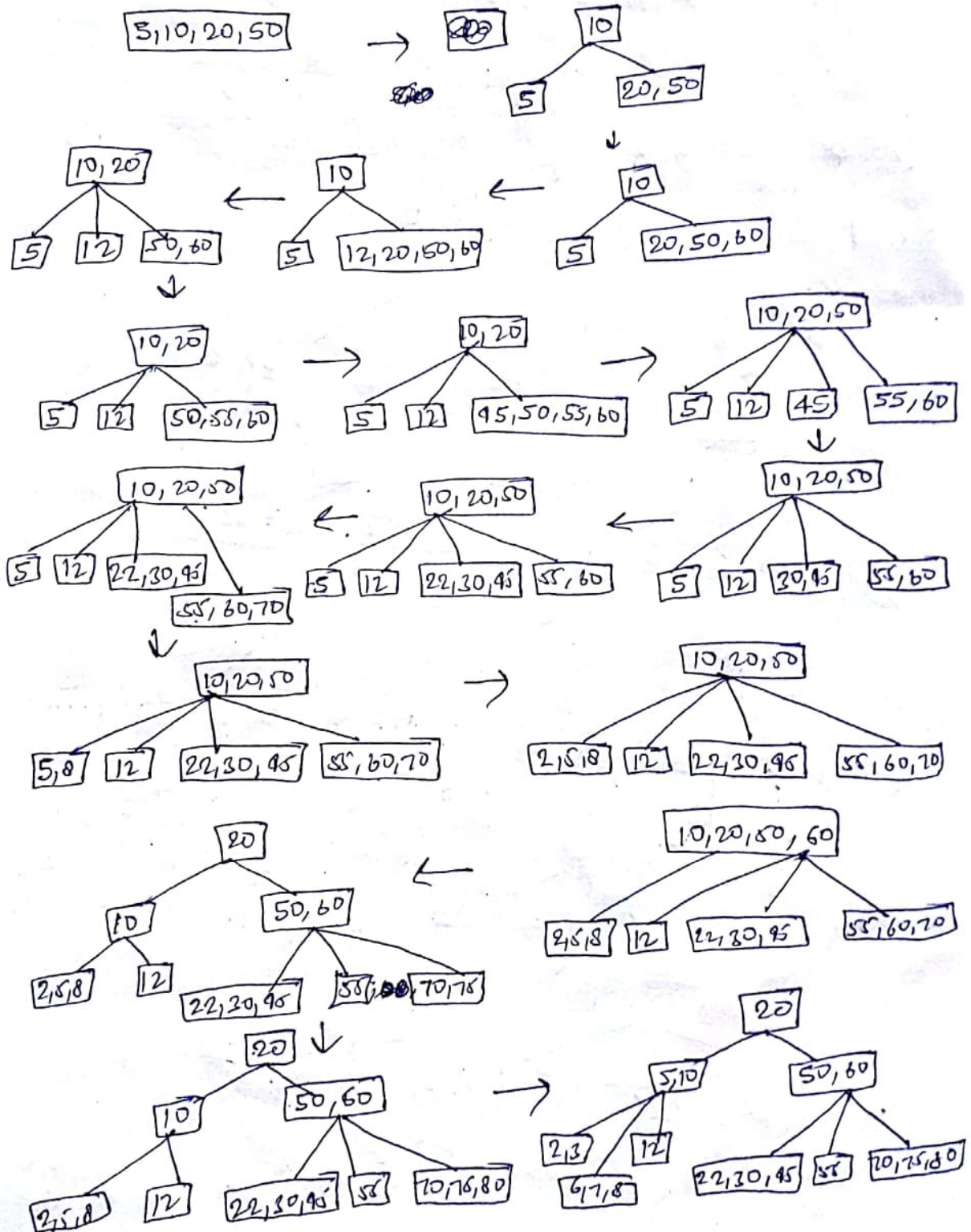
eg Create a B-tree of order 5 with elements
20, 50, 10, 5, 60, 12, 55, 45, 30, 22, 70, 8, 2, 75, 80



A Now add 6, 3, 7



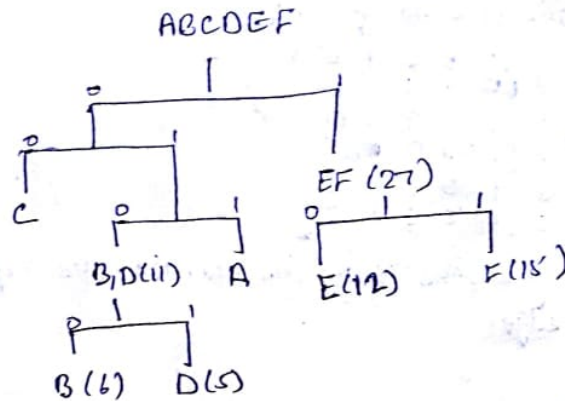
B-tree of order 4:-



Huffman Tree:

Arrange in decreasing order of frequency.
Form the least frequency in a B-tree.
Delete them from list and combine them.
Again, form pair of least frequencies,
combine and place in correct order.

Symbol	Frequency
A	10
B	5
C	20
D	6
E	12
F	15

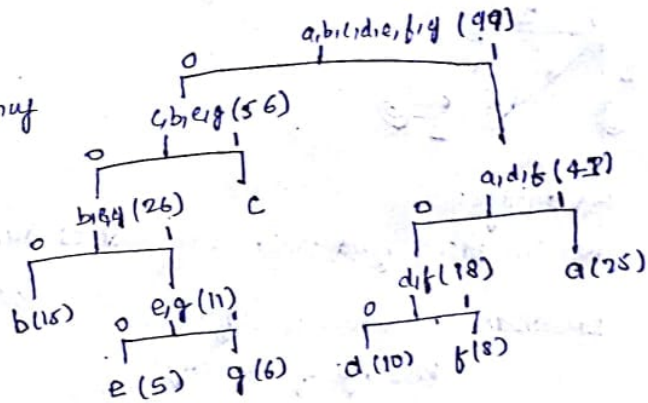


A = 011
B = 0100
C = 00
D = 0101
E = 10
F = 11

25/10/2018

5,60

Symbol/Data	Frequency
a	25
b	15
c	30
d	10
e	5
f	8
g	6
h	5



160

0,70

0,70

0,75,80

Draw
Step by Step

c	30
a	25
b	15
d	10
f	8
g	6
e	5

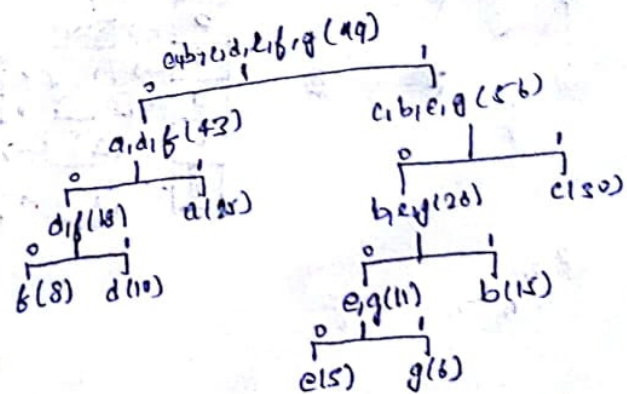
c	30
a	25
d,f	18
b	15
e,g	11

c	30
a	25
d,f	18
b	15
e,g	11

a,d,f	43
c	30
b,e,g	26

a,b,c,d,e,f,g,h	56
a,d,f	43

a - 01
b - 010
c - 011
d - 001
e - 001000
f - 000
g - 001001



30/10/2018

Graph:

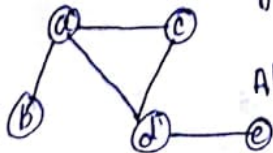
Non-linear data structure, can be directed or undirected

Representation of Graph:

- i.) Adjacency matrix
- ii.) Adjacency list

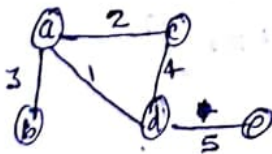
Adjacency Matrix: (Undirected graph)

Consider a $n \times n$ matrix, $n = \text{no. of vertices}$
if $n = 5$



$$A[S][S] = \begin{matrix} & \begin{matrix} a & b & c & d & e \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{matrix}$$

If the graph is weighted, the 1s in the matrix are replaced by the weight of the edge.



Advantage:

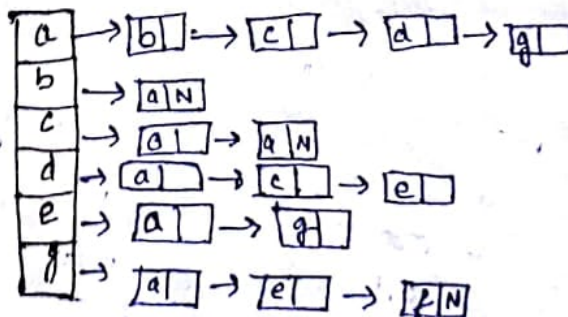
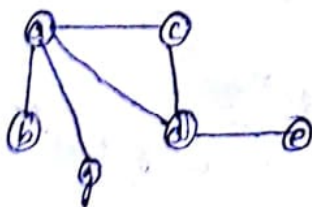
We can easily find which two vertices are connected.

Drawback: Insertion is difficult.

Wastage of memory.

Adjacency List:

Make an array of structures. Form singly linked list to show connections.



Pros:

Insertion is easy.

Cons:

Not easy to find the existence of an edge b/w two vertices.

Applications of graphs:

- To find the optimum length of wire required to connect computers.
- Find route b/w two places.

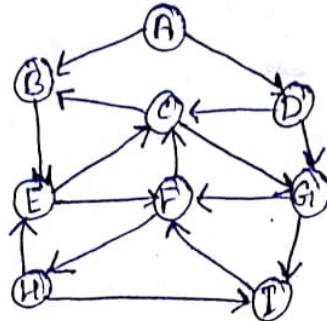
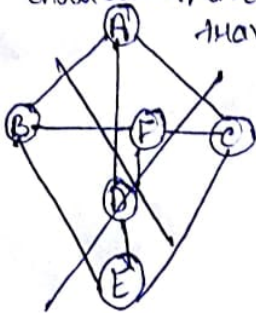
Graph Traversal:

Methods of graph traversal:

- Breadth First Search (BFS)
- Depth First Search (DFS)

BFS:

Choose source, traverse the neighbours first, then their neighbours. One path is traversed only once.



Q. empty initially 0e
visited[] = 0 0 0 0 0 0 0 0

A visited[]

ABCD 1111

ABCDE 11111

ABCDEF 1111101

ABCDEF G 1111111

ABCDEF G F 111111101

ABCDEF G F H 111111111

Print: ABCDEFGFH

DFS:

Stack

E
H
I
F
G
D
C
B
A

visited[]

↓
1111
1111001
111101101
111101111
111111111

Pop Print

A ↓ A

D ↓ AD

G ADG

E ADGE

F ADGEF

H ADGEFH

E ADGEFHE

C ADGEFHEC

B ADGEFHECB