EOE038

Roll No:

B. Tech.
(SEM III) ODD SEMESTER
THEORY EXAMINATION 2013 - 2014

DISCRETE MATHEMATICS

Time: 3 Hrs. **Max. Marks100**

Note: Attempt all questions.

Q.1 Attempt any four parts of the following: **4 x 5 = 20**

(a) If $f(x) = \sqrt{81 - x^2}$, then find range and domain of $f(x)$. If $g(x) = x^2 - 2$, then find $g^{-1}(14)$.

(b) If A, Band C are sets, then prove that $A - (B \cap C) = (A - B) \cup (A - C)$.

(c) If $f(x) = x^2 + 3$ and $g(x) = 4x - 7$, then find fog, gof and $g^{-1}(4)$.

(d) Prove that $R = \{(x, y): x \text{ divides } y, x \in Z, y \in Z\}$ is transitive but not an equivalence relation.

(e) Show that the mapping $f: R \to R, f(x) = \frac{1}{x}, x \neq 0$ and $x \in R$ is one-one and onto, where R is set of non-zero real numbers.

(f) If $f: X \to Y$ and $g: Y \to Z$ are one-one and onto, then prove that $gof: X \to Z$ is one-one onto and $(gof)^{-1} = f^{-1}og^{-1}$.

Q.2 Attempt any two parts of the following: **2 x 10 = 20**

(a) By finding truth table verify whether $(p \vee q) \wedge (\sim p) \wedge (\sim q)$ is a tautology or not.

(b) By Mathematical induction, show that $n(n^2 - 1)$ is divisible by 24, where n is any odd positive integer.

(c) Find truth table of $(p \Leftrightarrow q) \wedge (r \vee q)$ and $(p \Leftrightarrow q \wedge r) \Rightarrow (\sim r \Rightarrow \sim p)$.

Q.3 Attempt any two parts of the following: **2 x 10 = 20**

(a) Using Generating function method, solve the recurrence relation:
$a_n - 9a_{n-1} + 20a_{n-2} = 0;\ a_0 = 5, a_1 = 22$. Also, find the sequence $\{a_n\}$ whose generating function is $\frac{1}{5-6z+z^2}$.

(b) Solve the recurrence relations:

i. $a_n + 2a_{n-1} - 15a_{n-2} = 0;\ a_0 = 0, a_1 = 1$.

ii. $a_r - 5a_{r-1} + 6a_{r-2} = 7^r$.

(c) Prove that ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$. State and prove Pigeon hole principle. If 7 colours are used to paint 50 cars, find at least how many cars will have the same colour.

Q.4 Attempt any two parts of the following: **2 x 10 = 20**

(a) Define a commutative ring with unit element. Show that every field is an integral domain.

(b) Prove the following:

i. A subgroup H of a group G is normal iff $xH^{-1}x = H, \forall x \in G$.

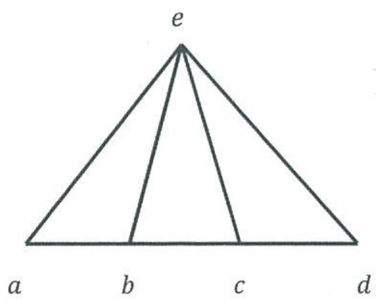
ii. The intersection of any two normal subgroup of a group is a normal subgroup.

(c) Prove that the order of each subgroup of a finite group is a divisor of the order of group.

Q.5 **Attempt any two parts of the following:** **2 x 10 = 20**

(a) Define Hamiltonian circuit. Prove that a planar graph with e edges and v vertices will have Hamiltonian circuit if $2(e - 3) \geq v(v - 3)$.

(b) Define proper coloring and chromatic polynomial of a graph. Find chromatic polynomial of graph given below:



(c) Write short notes on:

i. Enumeration of graphs;

ii. Posets and Lattices;

iii. Kuratowski's graphs.