



Bitmask DP

Bharat Singla (singlabharat)

Expert on CodeForces

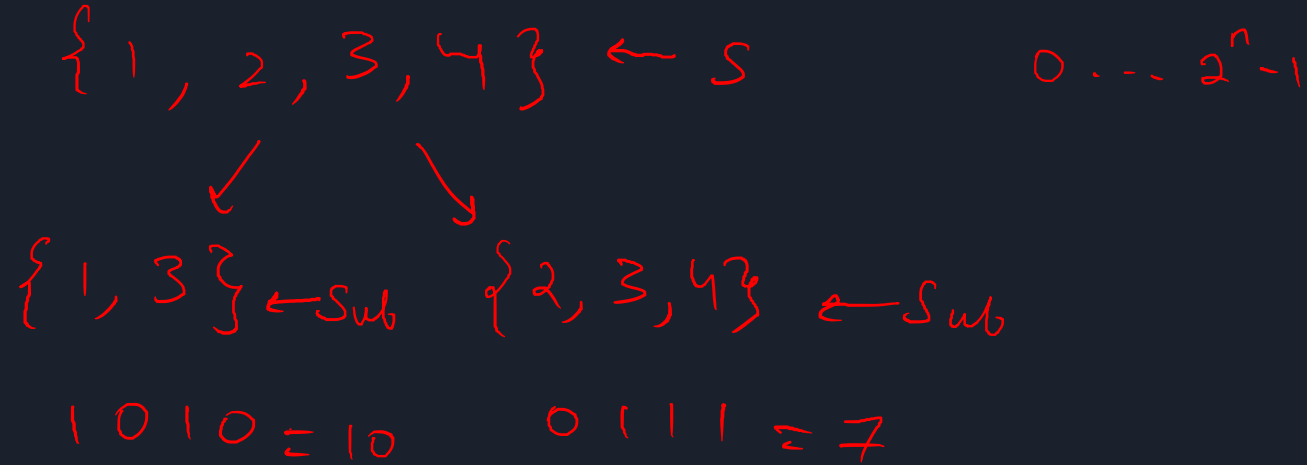
5★ on CodeChef



Today's plan :

- Bitmasking Revision
- Subset Sum Problem
- Task Assignment Problem
- Other Tips & Stuff

Revision Time



Subset Sum Problem

$\{ \overset{\downarrow}{\underset{=}{1}}, \overset{\downarrow}{\underset{=}{2}}, 3, 4 \}$

$S[\text{mask}] \rightarrow \text{Sum of subset}$

$T.C = O(\underset{=}{2^N} \cdot N)$

$S[3] \rightarrow 00\underline{11} \rightarrow \{1, 2\} \rightarrow 3$

$S[\overset{\uparrow}{1}\overset{\uparrow}{0}\overset{\uparrow}{1}\overset{\uparrow}{0}] = a[1] + S[1000]$

Min Cost Task Assignment Problem

There are N persons and N tasks, each task is to be allotted to a single person. We are also given a matrix *cost* of size $N \times N$, where *cost*[i][j] denotes, how much person i is going to charge for task j . Now we need to assign each task to a person in such a way that the total cost is minimum. Note that each task is to be allotted to a single person, and each person will be allotted only one task.

3	1	2	5
1	5	4	1
7	3	2	9
7	9	1	2

Min Cost Task Assignment Problem

$dp[\cancel{i}][mask]$

#set bits in mask = i

↓ ↓
first i tasks assigned
ppl

$dp[\cancel{i+1}][mask \text{ with } j^{\text{th}} \text{ bit set}]$

$= (dp[\cancel{i}][mask] + cost[i+1][j])_{\min}$

Min Cost Task Assignment Problem

```
assign(N, cost)
  for i = 0 to power(2,N)
    dp[i] = INFINITY
  dp[0] = 0
  for mask = 0 to power(2, N)
    x = count_set_bits(mask)
    for j = 0 to N
      if jth bit is not set in i
        dp[mask|(1<<j)] = min(dp[mask|(1<<j)],
dp[mask]+cost[x][j])
  return dp[power(2,N)-1]
```

-- builtin - popcount(mask)
task not assigned yet
all tasks assigned

Submasks of Masks

for mask : $[0, 2^n - 1]$

for sub \subseteq mask:

// code

$$\sum \binom{n}{k} \cdot 2^k = (1+2)^n$$

$$\begin{array}{r} 01010 \\ \hline 01110 \end{array} \} \text{OR}$$

$$\begin{aligned} \# \text{ masks : } k \text{ set bits} \\ = \binom{n}{k} \end{aligned}$$

$$\text{T.C. } O(3^n)$$