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DEVELOPMENT OF HEURISTIC PROCEDURES  
FOR FLIGHT RESCHEDULING IN THE  
AFTERMATH OF IRREGULAR AIRLINE  
OPERATIONS

BY: MICHAEL D. D. CLARKE

# Development of Heuristic Procedures for Flight Rescheduling in the Aftermath of Irregular Airline Operations

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**ABSTRACT**

Airlines are constantly faced with operational problems which develop from severe weather patterns and unexpected aircraft or personnel failures. However, very little research has been done on the problem of addressing the impact of irregular operations, and developing potential decision systems which could aid in aircraft re-scheduling. The primary goal of this research project has been to develop and validate algorithms, procedures and new methodologies to be used to reschedule planned activities (flights) in the event of irregular operations in large scale scheduled transportation systems, such as airline networks.

A mathematical formulation of the Airline Schedule Recovery Problem is given, along with a decision framework which is used to develop efficient solution methodologies. These heuristic procedures and algorithms have been developed for potential use in a comprehensive real-time decision support systems (DSS), incorporating several aspects of the tactical operations of the transport system. These include yield management, vehicle routing, maintenance scheduling, and crew scheduling. The heuristic procedures developed will enable the carrier to recover from an irregular operation and maintain an efficient schedule for the remainder of a given resolution horizon.

The algorithms are validated using real-world operational data from a major US domestic carrier, and data from an international carrier based in the Asia Pacific region. A comprehensive case study was conducted on historical operational data to compare the output of the algorithms to what actually occurred at the airline operation control center in the aftermath of an irregularity. Some of the issues considered include the percentage of flights delayed, percentage of flights cancelled, and the overall loss in operating revenue. From these analyses, it was possible to assess the potential benefits of such algorithms on the operations of an airline.

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# Glossary of Terms

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<i>algorithm run-time</i>	total solution time required for the execution of an algorithm
<i>delay arc</i>	additional flight arc that represents potential delay alternative for a given flight
<i>duration of delay</i>	amount of delay time assigned to a given flight, or event (arrival or departure)
<i>flight arc</i>	representation of a scheduled flight in the schedule map, connecting a departure node to an arrival node
<i>flight coverage</i>	number of scheduled flights that are cancelled or delayed in the network
<i>ground arc</i>	connects chronological events (arrival or departures) in a schedule map
<i>irregular operation</i>	aftermath of unexpected events which have a significant impact on an airline's schedule, and requires the rescheduling of resources
<i>maintenance arc</i>	represents the period in which an aircraft is under maintenance (scheduled, or unplanned)
<i>minimum aircraft turn time</i>	minimum time required to process an arriving aircraft into a departing aircraft
<i>operational aircraft</i>	an aircraft that is flight ready, and contains adequate flying time
<i>passenger recapture rate</i>	percentage of spilled passengers reaccommodated on other scheduled flights operated by the carrier

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## Glossary of Terms

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<i>passenger spill</i>	number of passengers that are not accommodated on a given flight
<i>resolution horizon</i>	prescribed time period required for the resolution of a given irregularity
<i>sequence of flights</i>	series of flights that originate at specific location and terminates at a maintenance station
<i>schedule map</i>	time-space representation of the daily flight schedule of an airline
<i>tail number</i>	specific identification for a given aircraft, usually related to the vehicle's registration number

“Emancipate yourself from  
mental slavery, none but  
ourselves can free our minds”

The Right Honourable Robert Nesta Marley



# **Chapter 1**

## **Introduction**

### **1.1 Overview**

Airlines are constantly faced with operational problems which develop from severe weather patterns and unexpected aircraft or airport failures. A significant amount of computational time and effort is invested in developing efficient operational schedules for airlines which are impacted by these irregular events. Over the last decade, airlines have become more concerned with developing an optimal flight schedule, with very little slack left in the system to accommodate for any form of variation from the optimal solution. However, very little research has been done on the problem of addressing the impact of irregular operations, and developing potential decision support systems which could aid in short term aircraft rescheduling.

The primary objective of this research was to develop algorithms, procedures and new solution methodologies to be used to reschedule planned activities (flights) in the event of irregular operations in large scale scheduled airline systems. These heuristic procedures and algorithms would be developed for use in a comprehensive real-time decision support systems (DSS), incorporating several aspects of the tactical operations of the transport system. These include yield management, vehicle routing, maintenance scheduling, and crew scheduling. The heuristic procedures will enable the carrier to recover from an irregular operation and maintain an efficient schedule for the remainder of a given rotation period.

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Having been exposed to issues relevant to the problem of irregular operations, the author is confident that these procedures when developed and implemented, will have a substantial impact on future airline system operations.

The development of an airline's published flight schedule is one of the most important aspects of its strategic planning. Significant efforts are made to ensure that the airline has plans which efficiently make use of its resources in order to maximize revenue or operating profits. The overall schedule planning process depends on an extensive array of information, and it starts several months ahead of the actual operation of a given flight. The process of deciding which aircraft type is assigned to a given flight is called the fleet assignment problem, and the process of assigning a specific aircraft or "tail number" to a given flight is known as the aircraft rotation/routing problem. This is necessary as aircraft must rotate through the planned maintenance services available at limited number of locations in the network.

Throughout the course of daily operations, the airline is often faced with situations that may result in substantial variations from its planned operations, and then is required to make real-time decisions that can have a significant impact on the overall operations of the airline over the rest of the day, or next few days. These irregular operations impact all aspects of the airline's operations, but are most detrimental to the schedules for basic resources such as aircraft and flight crews. The cause of the irregularity may range from severe weather to aircraft breakdowns, and it may result in the need to reschedule flight services, and reroute aircraft and crews. These actions cause flight delays and cancellations, which affect passenger services.

Irregular operations impact will also have an effect on the aircraft maintenance routing decision process, and the scheduling of maintenance resources. The ability of the airline to

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recover from such unexpected irregularities will depend on its ability to effectively make use of operational information that is readily available throughout the airline's computer databases. The decision maker will be trying to assign operational (available) aircraft to the most valuable flights, while meeting maintenance requirements of all operational aircraft.

## 1.2 Motivation

Currently, the resolution of flight irregularities is primarily a manually driven decision process, wherein the airline controller assesses all the available information, and makes an informed decision about the airline's operations. In general, this decision process is sufficient to solve the existing irregularity; however, it may have a significant impact on other future activities which were not considered by the controller. The ability of a computer based decision support system to consider all relevant activities should have great benefit to the overall resolution process. It is important to underscore the role of the airline controller in the decision making process, as it is only with extensive experience in the Airline Operations Control Center, that the controller can effectively deal with resolving irregularities.

For a typical airline, approximately ten percent (10%) of its scheduled revenue flights are affected by irregularities, with a large percentage being caused by severe weather conditions and the associated loss of airport capacity. In an article published in the New York Times [January 21, 1997], it was noted that the financial impact of irregularities on the daily operations on a single major US domestic carrier can exceed \$440 million per annum in lost revenue, crew overtime pay, and passenger hospitality costs. During the late spring of 1995, a severe hailstorm over Dallas-Fort Worth resulted in the damage of nearly one hundred aircraft parked at the airport terminals [Aviation Week; May 8, 1995]. In fact, eighty of these damaged aircraft belonged to American Airlines, accounting for nearly nine percent of

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its total fleet. In the immediate aftermath of this irregularity, American had to cancel almost ten percent of its scheduled flights, and needed almost an entire month to return to normal operations.

In January of 1996, it was estimated that a single snow storm "The Blizzard of '96" costs the US airline industry between \$50 - \$100 million [Aviation Week; January 15, 1996]. On a daily basis, airlines have to cope with reduced fleet size, as a result of aircraft breakdowns, as well as external factors such as ATC flow management restrictions, which affect the planned operations of the carrier. It is important to point out that the causes of airline irregularities are not limited to severe weather patterns during the winter season. Based on data obtained from the US Department of Transportation, it was established that poor weather conditions were cited as the largest causes of irregularities in the airline system over the course of the entire year, as reported by the airlines themselves.

In recent years, airlines have invested significantly in the development of their Operations Control Centers, with extensive infrastructure improvements in communications channels, and new computer architectures which promote the free flow of information throughout the entire airline company. The presence of these centralized decision centers have allowed airline controllers to make better decisions regarding the carrier's operations, based on up-to-date and accurate information from numerous divisions within the airline, available to them on state-of-the-art information systems. But the existence of robust and efficient decision support tools to help airline controllers in the decision process is not apparent. The development of such methodology is warranted, as airlines will gain financially from the availability of such decision tools.

### **1.3 The Airline Schedule Recovery Problem**

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### 1.3.1 Problem Statement

Throughout the course of daily operations, an airline is faced with the potential of deviations in the planned flight schedule as a result of various unexpected events. The impact of these deviations on the three primary airline operational schedules (Flight Services, Crew Rotations, and Aircraft Rotations) will vary, depending on the specific irregularity, and the flexibility and robustness of the original schedules. As discussed in Grandeau [33], any changes which may occur to the three airline system schedules are often defined as “operational deviations”. Deviations that do not cause significant rerouting problems are defined here as “time deviations”, and deviations that lead to rerouting of airline resources are referred to as “irregular operations”.

Time deviations are defined as any variation from the original scheduled times in any of the system schedules, and often result from minor delays in the air traffic control (ATC) system. One of the main causes of time deviations is the variation in wind patterns, which affect the overall airborne time of a given flight. They usually do not have a large negative impact on the airline’s flight operations, but simply reflect small changes in the arrival and departure times during normal daily operations. Time deviations are distinguished from irregular operations since they do not generally require any aircraft or crew reassignment decisions. However, there may be rescheduling of gates and other ground resources.

An “irregular operation” is defined as the aftermath of unexpected events which have a significant impact on the carrier’s schedule. This often results from poor weather patterns and the resulting severe delays in ATC operations, airport closures, aircraft breakdowns, lack of adequate flight personnel (cockpit and cabin crew), problems in ground handling and support services, and/or equipment failures. Irregular operations generally result in aircraft rescheduling and rerouting, with the added impact of flight delays and cancellations. In

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addition, aircraft rescheduling will have an impact on the scheduling of maintenance resources for the carrier.

On a daily basis, airlines operating hub and spoke operations suffer from irregularities, which can have a significant impact on their profitability and ability to compete effectively. In fact, many carriers now see the need to address the problem of irregular operations as one issue necessary to maximize operating profit, by reducing additional operating expenses and loss of revenues, which result from such irregularities. However, robust decision support systems for the purpose of rescheduling operational aircraft do not readily exist, and very little research has been done on the topic to date. At the majority of airline operation centers throughout the world, irregular operations are dealt with manually, with a heavier reliance on the human controller and his past experience, and his knowledge of available spare aircraft and other resources such as terminal gates, regulations, and maintenance schedules. Given the complexity of the Airline Schedule Recovery Problem, the need for real-time decision making tools to assist in the event of irregular airline operations is therefore apparent.

There are several questions that have to be considered when trying to solve the problem of irregular airline operations. These include:

- How should flight schedules and aircraft rotations be revised in the aftermath of irregular airline operations?
  - What flights should be cancelled to minimize the loss of profit, based on available resources and the actual number of passengers on-board a given flight?
  - Is it possible to carry out the revised flight schedule with the available number of flight crews?
  - How does one develop new crew rotations in the aftermath of irregular operations?
-

- How will the revised flight schedule and corresponding aircraft rotations affect the scheduled maintenance program of the airline?

The availability of high-performance workstations, which are already in use in the strategic stage of airline planning will play a significant role in tactical planning. The use of these computers would give the airline controller the ability to incorporate demand and revenue data from the airline's computer reservation and yield management systems, and to interact with maintenance scheduling, crew scheduling, and other elements of airline operations.

Historically, little interaction exists during the tactical phase of operations between the various operational divisions (maintenance, fleet assignment, yield management, etc.), and the presence of irregular operations only adds to the problem. This has changed somewhat with the advent of the development of the centralized Airline Operations Control Center (AOCC). It should be possible to develop a decision support system whose primary goal would be to regain the strategic schedule of the airline within a given time period, minimizing the overall impact of cancellations and delays on profitability, and on the operational schedules. This can be called the Airline Schedule Recovery Problem (ASRP) and is the focus of this research.

The most severely impacted aspects of the planning process are fleet assignment and subsequent aircraft routing. Although these problems are generally developed independently in the strategic planning stage, the need to reschedule aircraft operations in real-time after an irregularity, causes both fleet assignment and routing to be considered concurrently. The utilization of a decision support system to solve ASRP, should provide significant benefit to the airline, and potentially to the traveller (through significant reduced flight delays, and/or cancellations).

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### **1.3.2 Model Development and Solution Approach**

In order to develop effective decision support tools to assist airline controllers in the resolution of irregularities, it is imperative for the researcher to establish a thorough understanding of the daily operations of the Airline Operations Control Center, and the role it plays in the airline operational activities. In addition, it is necessary to identify the operational requirements of any tool which will be developed and deployed in the AOCC. It is essential to incorporate the experience of the airline controller in the decision process, thereby dictating an interactive tool. Trade-offs have to be made in this and future research initiatives between the level of automation in the decision process versus flexibility, and the ability of the controller to guide the decision process.

Although the overall goal of the decision process is to fully resolve any irregularities, the sheer size of the airline network often dictates that the underlying problem has to be decomposed and considered in different phases. Decisions about rerouting aircraft will be affected by the availability of eligible flight crews at each station, as well as adequate ground resources to process aircraft and passengers at a station. Conversely, the allocation of these support services will be driven by the revised aircraft schedule. It was established in the early phases of this research, that the problem of resolving irregular airline operations would have to be addressed through a phased or sequential approach.

The basic decision that has to be made is the reassignment of aircraft to flights, within the constraints of crew availability, the number of landing slots at a given station, and the level of station resources. Primarily, the aircraft have to be reassigned to flights based on revenue data, while meeting maintenance requirements. Secondarily, issues such as the availability of flight crews, landing slots, and in some cases, limited ground resources and passenger flow requirements are considered. The allocation of crews, landing slots and ground

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resources is done after the primary aircraft reassignment problem has been solved, and if necessary, there then would be an iterative process implemented to improve upon the primary aircraft routing decision.

Based on discussions with airline controllers at major US carriers, it was established that one of the most important operational requirements of any decision support tool is the ability to provide real-time decision making. Throughout the course of this research project, this requirement was thus placed at the forefront of the design process. However, several other requirements were incorporated into the development of the solution methodology. These include the ability to consider switching between different types of aircraft in the fleet, crew scheduling considerations, and to make trade-offs between delaying and cancelling a given flight using a single decision model.

#### **1.4 Overview of the Airline Operations Control Center (AOCC)**

Airline operational planning is generally handled in two phases, strategic and tactical. Strategic planning is concerned with creating a flight schedule of services to be offered to passengers (called the Schedule of Services), and is established by the Commercial / Marketing department. The Operations group then generates the Nominal Operational Schedule (NOS) for the airline's generic resources such aircraft rotations and crew rotations. It subsequently schedules specific airline resources by assigning tail numbers, and individual crew members to a given flight. This second step creates the Resource Operational Schedule (ROS), and constitutes the resource allocation phase of the total scheduling process. The resource allocation steps are carried out by various airline groups. The reader is referred to Grandeau [33, 34] for a more comprehensive discussion of the overall airline scheduling process.

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Given these resource schedules, the tactical side of the Operations group is responsible for the final stage of the scheduling process: Execution Scheduling. Execution scheduling is the process of executing the system resource schedules on a daily basis. This involves three main activities: executing the pre-planned schedules, updating the schedules for minor operational deviations, and rerouting for irregular operations. The tactical operations of a regular scheduled air carrier are usually under the 24 hour/day control of a central organization often referred to in generic terms as the Airline Operational Control Center (AOCC), although it may have a different name at each airline.

This section presents a brief summary of a typical AOCC, outlining its organization, primary activities within the airline, and operational facilities. The facilities and personnel of a particular AOCC will vary considerably depending on the type and size of the airline. AOCC centers can range from a single controller/dispatcher on duty to several dispatchers and hundreds of other personnel handling flights throughout the carrier's entire global network. During the process of operation control, the AOCC is supported by the Maintenance Operations Control Center (MOCC) which controls airline maintenance activities, and by various Station Operations Control Centers (SOCC) which control station resources (gates, refuelers, catering, ramp handling, and passenger handling facilities).

Operations Control Centers are usually linked to the Aeronautical Radio Inc. (ARINC) and the Societe International Telecommunications Aeronautiques (SITA) networks to send and receive teletype/telex messages. Communications with maintenance and engineering, customer service, and airport services are maintained to facilitate prompt contact with the appropriate personnel. Teletype, telex, facsimile, telephone, leased lines, and public data networks combine to provide an effective medium for collecting information and communicating revised operational plans developed by the AOCC center. In some cases, the AOCC has communications systems connected to VHF, HF and Satcom radio links, air

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traffic control centers, and other relevant locations, allowing them to effectively gather and disseminate information instantaneously.

#### **1.4.1 Functional Groups Within the AOCC**

The AOCC is organized into three functional groups, each with a distinct responsibility within the schedule execution process. These are: 1) the Airline Controllers, 2) On-line Support, and 3) Off-line Support; as shown in Figure 1-1. The airline Operation Controllers are responsible for maintaining the current operational version of all the system resource schedules (crew, aircraft and flight), and for the management of irregular operations. The final operational decisions are made by one (or more) Operation Controller(s). The operation controllers at larger US airlines may have a dedicated airline Air Traffic Control (ATC) coordinator, to deal with Air Traffic Flow management advisories from the ATC system.

They are assisted by four types of on-line support personnel: the flight dispatch group, the crew dispatch group, MOCC, and SOCC. The Flight Dispatch group is responsible for flight planning, flight dispatch and enroute flight following. The Crew Operations group is responsible for tracking individual crew members as they move through the airline's route network, for maintaining up to date status for all crew members, and for calling in reserve crews as required. The airline controllers, flight and crew dispatch groups are usually located together in the AOCC. The later two support groups, the MOCC and the several SOCC's are usually not physically located at the central AOCC.

Ancillary off-line services such as the maintenance of the navigation database, meterology, and operations engineering (or flight technical services) are usually located at the operations control center, and serve to provide supporting resources for all AOCC personnel. In

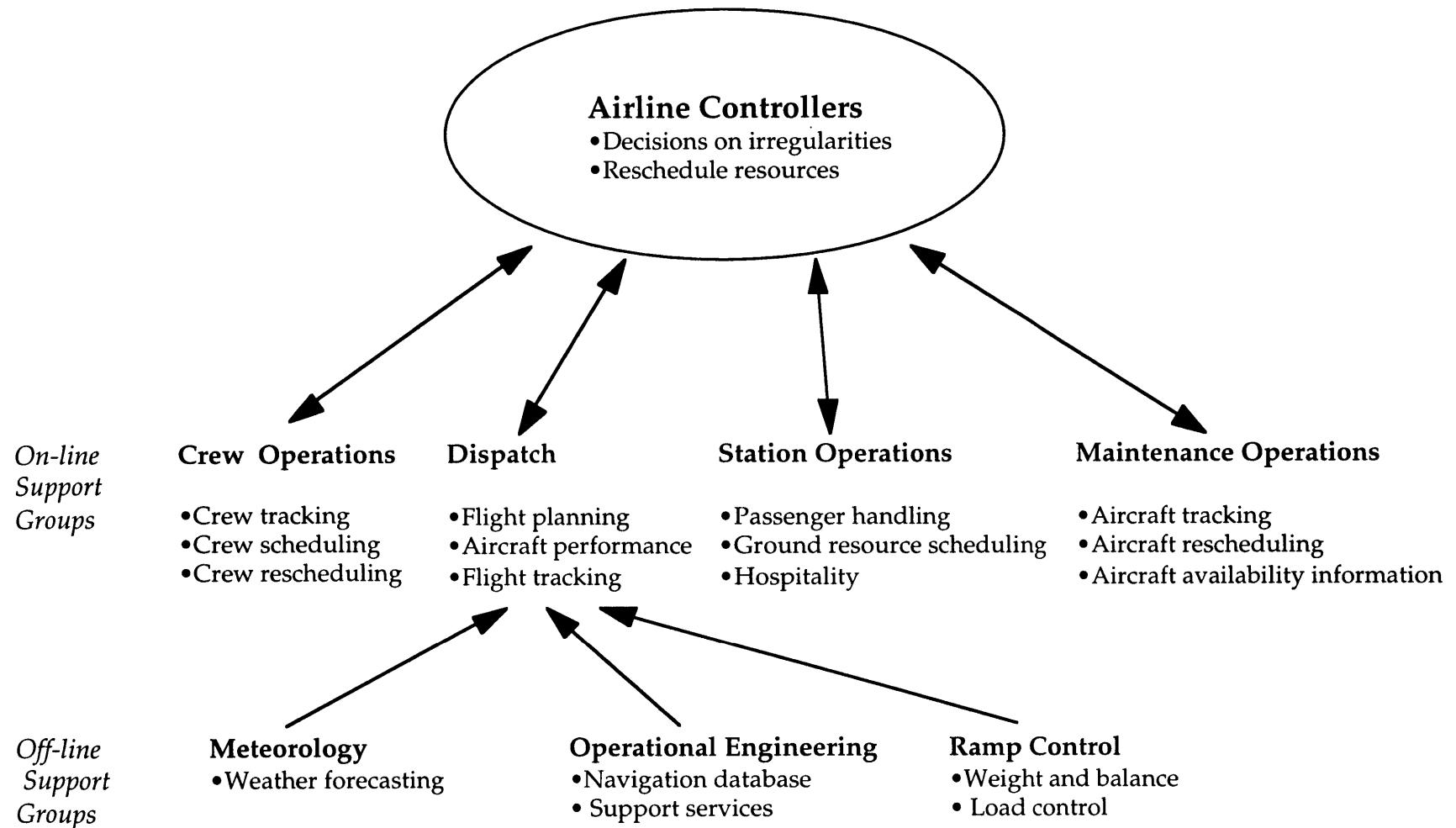
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addition, the crisis center which manages activities after an accident or incident is often an integrated part of the Airline's Operational Control Center.

#### **1.4.2 Information Flow within the AOCC**

The airline Operation Controllers are the center of the airline operation control process. They are the sole operational group within the AOCC with the authority and responsibility to resolve problems that develop during the course of both regular and irregular operations. Airline Operation Controllers receive information from every facet of the airline during operations, through established information channels as represented in Figure 1-1. From these inputs, the Controllers maintain an updated version of the airline system resource schedules which includes delays, irregular routings for aircraft and crews, and additional flights. These can be called the "Current Operational Schedules " (COS). As the focal point in the AOCC for flight and schedule management, controllers interact with key personnel and divisions.

During normal operations, Dispatchers are responsible for the successful release of a flight, depending on maintenance issues (deferred minimum equipment list [MEL] or configuration deviation list [CDL] items), aircraft restrictions (such as noise), the availability of required operational support (fuel, gates, ground power, airport facilities) at the departure, destination and alternate airports. During irregular operations and emergencies, the Dispatcher will inform the Operations Controller of the problem, and their role is to handle the additional coordination that such situations demand. If the airline is experiencing irregularities, the Operation Controllers have to devise modified



**Figure 1-1** Information Flow Diagram for the Airline Operations Control Center (AOCC)

operational schedules on a very short notice. The Current Operational Schedule is the plan that the airline will follow in order to return to the Nominal Schedule of Services. These modified schedules are disseminated to the relevant airline divisions, and stations of the system.

### **1.5 Thesis Outline**

In the next chapter, there is a discussion of the primary causes of irregularities and resulting flight delays and cancellations at major hub airports in the US domestic market, derived from information obtained from the US Department of Transportation. A review of existing decision support tools and solution methodologies currently in use at airline operations control centers of major US domestic carriers and an international carrier is presented, outlining the major characteristics of these systems. An extensive literature review of airline operations is given, summarizing research that has been done on the topic of irregular airline operations, as well as work on other closely related research topics.

In the first phase of the research program, the overall structure of the problem was defined, and a large-scale mathematical model was formulated to represent the decision process for aircraft rerouting. Based on discussions with airline controllers, potential solution methodologies were investigated, and the underlying operational requirements and capabilities of candidate decision procedures were established. In the second phase, a series of algorithms were developed to solve the established problem based on concepts of network flow theory and mathematical programming theory. These solution procedures have been developed and implemented in an UNIX operating system environment using the C++ programming language.

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In Chapter 3, the mathematical formulation of the airline schedule recovery problem is presented, outlining the decomposition of this highly complex problem. The primary problem considered is the reassignment of aircraft to scheduled flights in the aftermath of irregularities. Based on this output, the residual airline network and associated revised schedule map, are used as the basis to assign crews, terminal gates, ATC landing slots, and for solving the passenger reaccommodation problem. Each resulting sub-problem is outlined with a representative formulation of the problem.

Chapter 4 outlines the underlying mathematical programming theory and network flow theory which were used to develop the solution methodologies and procedures. This includes a brief overview of the implicit column generation procedure, and a review of a constrained shortest path algorithm, and a constrained minimum cost flow algorithm. In Chapter 5, the solution procedures developed are discussed, incorporating concepts presented in Chapter 4.

In the final phase of the research project, operational data from a US domestic carrier and an international carrier have been used to validate the algorithms, and establish the potential limitations of the solution methodology as a result of memory limitations and CPU processing capabilities. A comprehensive case study was conducted on historical operational data to compare the output of the algorithms to what actually occurred at the airline operation control center in the aftermath of an irregularity. From this analysis, it was possible to determine the potential benefits of such algorithms on the operations of an airline.

Chapter 6 presents the results of the case studies used to demonstrate the algorithms and solution procedures developed during the course of the research project. Several design parameters and implementation issues were considered including the effect of the size of the

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airline schedule map on the solution time of each algorithm. In particular, the case study considered the effects of several operational constraints, the number and positioning of delay arcs, passenger recapture rate, and minimum aircraft turn time. These affected the quality of the solution as measured by operating profit, flight coverage (percentage of flights delayed, and percentage of flights cancelled) and the overall solution time of each algorithm.

Chapter 7 summarizes the major contributions of this dissertation, and discusses the results of the case study and their implications to future research initiatives on the topic of irregular airline operations.

### **1.6 Contributions of the Thesis**

The Airline Schedule Recovery Problem (ASRP) developed in this dissertation provides a comprehensive framework that addresses how airlines can efficiently reassign operational aircraft to scheduled revenue flights in the aftermath of irregularities. The mathematical formulation of the problem enables flight delays and cancellations to be considered simultaneously, i.e., in the same decision model. The algorithms and solution methodologies developed in this dissertation have successfully demonstrated that it is possible to develop efficient procedures for flight rescheduling.

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“Its not an easy road, Many see the glamour and the  
glitter, and thinks it’s a bed of rose, Who feels it knows,  
Lord help us sustain these blows”

Mark Myrie, aka Buju Banton

# **Chapter 2**

## **Irregular Airline Operations**

### **2.1 Introduction**

In order to effectively model any physical system, it is imperative for the researcher to develop a thorough understanding of the underlying problem being considered, as well as all the major factors that may affect the system. In the initial stages of the research, a comprehensive review of flight delays in the US domestic airline system was conducted in an effort to accomplish this task. In addition, field trips were made to existing airline operations control centers to further help establish the state-of-the-practice procedures for dealing with irregularities. The reader is referred to the Appendices for a more detailed description of the survey questionnaire used on these field trips. In this chapter, a summary of the major findings of the delay study and a survey of current AOCC are given as a preamble to developing the decision model, and subsequent algorithms.

The daily operations of regularly scheduled airline carriers are prone to unexpected irregularities which develop from several factors ranging from severe weather conditions to the unavailability of eligible flight crew. In many cases, these factors can have a significant impact on an airline's operations, resulting in substantial deviation from the planned schedule of services. Since 1993, the US Department of Transportation has recorded information on flight delays throughout the domestic air travel market. The Air Traffic Operating Management System (ATOMS) database system contains the number of

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scheduled flights delayed more than fifteen minutes by cause of delay (e.g. weather, and air traffic control volume) and by airport. Flights which arrive within fifteen minutes of the scheduled arrival time are considered "on-time" by the DOT.

As part of the research effort, data from the ATOMS database has been used to assess the primary causes of flight delays at major hub airports in the US domestic system, as categorized by the DOT. The major findings of the analysis will be influenced by the way in which the data is collected, as it is the responsibility of the reporting airport to assign the delay cause to each scheduled flight when necessary. The following list summarizes the major categories of irregularities as established by the ATOMS program. They are:

- Weather - Wind, fog, thunderstorm, low cloud ceiling
- Equipment - Air traffic radar/computer outage
- Runway - Unavailable because of construction, surface repair, disabled aircraft
- Volume - Aircraft movement rate exceeds capacity of the airport at a given time
- Other - Anything excluding weather, volume, runway, and equipment

The airports considered in the study were hub complexes for the six largest US major passenger carriers (American AA, United UA, Delta DL, Continental CO, USAirways US, and Northwest NW).

Several important observations were made during the course of reviewing, and analysing the delay data obtained from the US Department of Transportation. The main points are listed below:

- Loss of capacity due to severe weather and traffic volume account for 93% of flight delays at hub airports.
  - There is a marginal correlation between the overall level of aircraft movement at an airport and the level of flight delay experienced.
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- The level of flight delay at an airport is affected by its geographical location, and the resulting meteorological conditions.
- The variation in the level of flight delay at a given station is closely related to the seasonal weather changes.
- The level of hub activity at an airport can have an impact on the level of flight delay.
- In the majority of the airports studied, the highest percentages of delays were experienced in January and July of a given year.

## **2.2 Implications for Algorithm Development**

It is evident from the empirical study that the majority of flight delays result from severe weather conditions. The ability of a given aircraft routing to absorb any delays is minimal, as most routings have been optimally determined, with very little slack time built into the flight sequence. Thus, a delay in flights early in the day may course continuing lateness unless the airline pro-actively rescheduled its resources. In order to effectively deal with irregularities, it is thus apparent that a system-wide approach should be applied to the problem, if one hopes to efficiently resolve airline irregularities. However, current practice generally takes a localized approach in dealing with irregularities. In the next section, a review of existing solution procedures and decision support tools used by the AOCC is given to highlight the need for more efficient methodologies to deal with abnormal operations.

## **2.3 Review of Existing Information Systems and Decision Support Tools**

The overall impact of irregularities on the daily operations of an airline will depend on the level of precautionary measures the carrier has built into its schedules to deal with typical irregularities. Many carriers have developed extensive resolution procedures which are generally implemented manually in the aftermath of irregularities, with little if any

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dependence on automated decision support systems. Decisions regarding future operational schedules and actual operations of the airline are made based on forecasted and often out-dated data and information, and this can have a significant effect on the value of the decision process. In some cases, the airline may decide to delay or even cancel flights, only to find out that these actions were unnecessary for the resolution of irregularities in the network.

Airlines have identified the need to improve the processes which assist airline controllers in the real-time operations of the carrier. They have invested heavily in state-of-the-art, Airline Operations Control Centers (AOCC), sometimes referred to as system operations control centers, which gather an extensive array of operational information and data. However, very little effort has been placed in developing solution procedures and methodologies which could complement the decision making capabilities of experienced airline controllers. In order to appreciate the need for such systems, the following is a summary of some of the resolution procedures and decision support systems, currently in use at Airline Operation Control Centers of major US domestic carriers, and an international carrier based in Asia.

United Airlines [10] has developed and deployed the "System Operations Advisor" (SOA), a real-time decision support system for use at its AOCC (which they refer to as the Operations Control Center [OCC]) to increase the effectiveness of its operational decisions. The SOA system consists of three primary components: the Status Monitor, the Delay and Swap Advisor, and the Delay or Cancellation Advisor. The purpose of the Status Monitor subsystem is to alert the airline controller of potential irregularities such as delays and cancellations through a graphical user interface. The interface provides mechanisms to launch tools such as the Delay and Swap Advisor for developing solutions to existing operational problems. The Delay or Cancellation Advisor can then be deployed in order to

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determine potential resolution procedures to problems which have developed from irregularities in the airline's network. It is important to note that decisions regarding delays and cancellations of scheduled flights are made independently of each other in this current system.

The AOCC at American Airlines is called the System Operations Control center (SOC), and relies on an array of decision support tools to make informed decisions about the operations of the carrier. The airline's primary goal in the aftermath of irregularities is to return to the operational schedule as soon as possible, regardless of its impact to potential revenues. The controllers consider the number of passengers booked on a given flight segment instead of the actual value of the flight. In resolving irregularities, the airline controllers subjectively incorporate passenger flow issues such as connectivity, goodwill, and volume of traffic, into the decision process.

The airline has identified crew scheduling as the important parameter in the resolution of irregularities in the network, and consequently, most aircraft substitutions are done within a given fleet. In the aftermath of an irregularity, the carrier first establishes a reduced flight schedule, and then figures out how to implement this schedule. It takes into consideration such issues as critical departure times, mission compatibility, and system balance in the daily flight cycle. American Airlines describes mission compatibility as any decision which minimizes downstream effects in schedule variation, and provides a feasible resolution in a timely fashion. Decisions are generally made to initially delay flights, and then if necessary determine flight cancellations.

Delta Air Lines recently opened its new operations control centre in Atlanta, responsible for monitoring weather, flight schedules and maintenance problems that may develop during the course of normal operations. The airline makes use of readily available operation data to

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fine tune its flight schedules to accommodate for prevailing weather conditions. It is apparent however, that most of the decision making regarding flight delays and cancellations at Delta is manually executed, with little if any reliance on automated decision support systems. The airline is currently in the process of developing such software, including a program named the Inconvenienced Passenger Rebooking System, which allows the airline to notify passengers of cancellations or delays and aid in passenger flow recommendations. In addition, they are reportedly in the middle of developing software to assist in the redeployment of flight crews in the aftermath of irregularities.

In recent years, many airlines have come to rely extensively on pre-emptive decision making, developing flight cancellation plans which are implemented long before an airport or region is actually impacted by severe weather conditions. At Continental Airlines, they have developed a resolution procedure referred to as the Severe Weather Action Plan, which is used to minimize the number of aircraft and crews remaining in a geographical region forecasted to have bad weather conditions. The airline controllers believe that such preemptive actions are beneficial to the carrier, as it makes schedule recovery easier, and greatly facilitates restarting normal operations. However, they may in fact compromise revenue operations, which could have occurred without the influence of the prevailing irregularities. Continental recently opened its new operations control centre, similar to those existing at American, United and Delta airlines.

Northwest Airlines is currently in the process of developing decision support systems for use in the carrier's operations control center. In the interim, the airline has developed and implemented several alternative aircraft "thinning" procedures that incorporate both operational and economic factors in the decision making process. "Thinning of flights" is defined as the response to irregular operations, based on forecasted adverse weather conditions that are expected to reduce the operational capacity of airports in the given

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region. The thinning process is designed to match operations with the level of reduced airport capacity, while ensuring that net revenue contributions are maximized, as well as minimizing customer inconvenience, and disruptions to crew and maintenance scheduling. The overall guidelines for thinning operations are to recover safely, and efficiently to normal operations as soon as physically possible, in the aftermath of the irregularity. Similar to Continental Airlines, it is Northwest's policy to pre-cancel flights in preparation for the reduced operational capacity.

At Garuda Indonesia, the AOCC is referred to as Operations Movement Control (EM), and it serves as the core of Garuda's operations. The primary information system is the Resource Management Operations Control (ROC) system , which is used for monitoring the actual operations of every Garuda flight. The airline's Nominal Operations Schedule which is generated by Operations Planning (EP) using the Airline Resource Planner (ARP) is electronically transferred (via floppy disk) to the ROC system. However, there is no direct line connection between the two computer systems.

Actual operational data in the form of a departure message from each airport station is transmitted via SITA telex, and automatically entered into the Resource Operations Control ROC database/graphical display system. The departure message includes information on actual arrival time at station, aircraft type, aircraft's next destination, departure time, estimated arrival time, delay status, passenger count, cargo, mail, captain in command, and fuel uplift data. The departure messages are stored for each flight leg in a centralized operations database in DBase 3 format. This data can be accessed and analyzed using the database management system Paradox. Any additional changes or modifications in flight schedules such as charter flights, special flights, etc. are manually entered into the ROC system via keyboard. A hard copy output of the flight schedules from the ARP program

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(prepared by EP) is used as a back-up to computer systems, as well as to manually record changes in the schedule in the event of an irregular operation.

At the Operations Control facility, four micro-computers serve as a platform for the ROC monitoring system. One computer acts as a dedicated server, with the remaining three units providing display capabilities and limited operational access to the stored data. The ARP/ROC systems have been in use at Garuda since 1990. Before that all operations were manual. In addition, Operations Control has access to the reservation system ARGA and the departure control system DCS database via a separate computer terminal. The information is used during irregular operations, to determine the impact of cancellations on revenue (manually).

## **2.4 Literature Review**

Mathaisel [8] reports on the development of a decision support system for AOCC which integrates computer science and operations research techniques. The application integrates real-time flight following, aircraft routing, maintenance, crew management, gate assignment and flight planning with dynamic aircraft rescheduling and fleet rerouting algorithms for irregular operations. As discussed by the author, the algorithms help airline controllers optimally reroute aircraft, crews and passengers when operational problems disrupt the execution of the schedule plan. The system includes a real-time, interactive, graphical aircraft routing displays; a rule system which provides warnings of constraint violations and usual conditions; and the ability to generate what-if solution scenarios. The integrated system is demonstrated by simulating a disruption to a planned schedule and by using one of the available tools, a network flow algorithm, to determine optimal rerouting alternatives.

The problem of irregular airline operations has only been recently considered in research projects conducted by Dusan Teodorovic, et al. and in work done by the Research and

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Development Department of United Airlines. Teodorovic and Gubernic [13] discuss the problem of minimizing overall passenger delays in the aftermath of a schedule perturbation. They attempt to determine the least expensive set of aircraft routings and schedule plan using a branch and bound procedure. Their methodology is based on the assumption that all the aircraft in the fleet have the same capacity, and they only considered a marginally sized fleet of three aircraft operating a total of eight scheduled flights. Teodorovic [14] presents research on the reliability of airline scheduling as it relates to meteorological conditions, the ability to identify an indicator for quantifying the adaptability of such airline schedules to weather conditions, and an overview of a potential solution procedure. The author outlines this heuristic algorithm for minimizing the number of aircraft required to accommodate a given traffic volume, while ensuring that aircraft are assigned to only one flight within a given time period.

Teodorovic and Stojkovic [11] discuss a greedy heuristic algorithm for solving a lexicographic optimization problem which considers aircraft scheduling and routing in a new daily schedule while minimizing the total number of cancelled flights in the network. The algorithm developed is based on dynamic programming, and is characterized by a sequential approach to solving the problem as flights are assigned to aircraft in sequences. The solutions obtained using this methodology are highly sensitive to the decision matrix, and the ranking of the various objective functions. The model does not consider the impact of crew scheduling in the aircraft scheduling process. Teodorovic and Stojkovic [12] outline a model for operational daily airline scheduling which incorporates all operational constraints, and is used to reduce airline schedule perturbations. Their heuristic model based on the FIFO principle and a sequential approach based on dynamic programming, is developed to facilitate and incorporate the work and experience of the dispatcher in the decision process regarding traffic management. The model developed is used to determine

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the aircraft rotations, as well as the crew rotations, while minimizing the number of cancelled flights.

The Research and Development Department at United Airlines has conducted several projects on the topic of irregular airline operations, and has presented material on its efforts at annual symposiums of AGIFORS (Airline Group of the International Federation of Operations Research Societies). The work at United is part of the development of a comprehensive decision support system for use in the carrier's operations control centre. Jarrah, et al. [4] present an overview of a decision support framework for airline flight cancellations and delays at United Airlines. Their underlying solution methodology is based on network flow theory, as the models cast some of the problems faced by flight controllers while dealing with irregularities into minimum-cost network flow problems.

Jarrah's paper outlines two separate network flow models which provide solutions in the form of a set of flight delays (the delay model) or a set of flight cancellations (the cancellation model), while allowing for aircraft swapping among flights and the utilization of spare aircraft. The models assume that a disutility can be assigned to each flight in order to reflect the lost revenue if the flight is cancelled, and that the disutility of delaying each flight is assessable. Both models are solved using Busacker-Gowen's dual algorithm for the minimum cost flow problem in which the shortest path is solved repeatedly to achieve the necessary flow in the network. The network models presented are solved independently of each other, and does not take into consideration crew and aircraft maintenance constraints. This solution framework is deficient in that it does not allow for a trade-off between cancelling and delaying a given flight in a single decision process. In addition, the solution methodology does not allow for potential substitution of aircraft with varying capacity, and operational capabilities.

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Yan and Yang [15] develop a decision support framework for handling schedule perturbations which incorporates concepts published by United Airlines. The framework is based on a basic schedule perturbation model constructed as a dynamic network (time-space network) from which several perturbed network models are established for scheduling following irregularities. The authors formulate both pure network flow problems which are solved using a network simplex algorithm, and network flow problem with side constraints, which are solved using Lagrangian relaxation with subgradient methods. They outline the basic schedule perturbation model which is designed to minimize the schedule-perturbed period after an incident, while maximizing profitability. In addition, they consider the effects of flight cancellations, flight delays and ferry flights as solution alternatives in the decision process. The framework is designed to aid airlines in handling schedule perturbations caused by aircraft breakdowns, and assumes scenarios with only one broken down aircraft and a single fleet type. In addition, the models do not incorporate aircraft maintenance and crew constraints in the formulation.

Cao and Kanafani [2] discuss a real-time decision support tool for the integration of airline flight cancellations and delays. This research is an extension of the work of Jarrah [4], using many of the concepts presented and discussed in Jarrah's paper. The authors present a quadratic 0-1 programming model for the integrated decision problem, which maximizes operating profit while taking into consideration both delay costs and penalties for flight cancellations. They discuss special properties of the Flight Operations Decision Problem (FODP) model which are exploited to develop a specialized algorithm to solve the problem in real-time. The model considers the airport network as a complete system, and traces the effect of delay and aircraft reassignment from one station to the next. The authors consider as an extension to their base model, issues of ferrying surplus aircraft and multiple aircraft type swapping capabilities. In a subsequent article, Cao and Kanafani [3] present an

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effective algorithm to solve the FODP model and discuss computational experiments with a continuous mathematical problem, derived from the 0-1 quadratic problem. In the case studies presented, aircraft ferrying, crew scheduling and airport capacity constraints are ignored in the solution procedure.

Arguello et. al [1] present a time-band optimization model for reconstructing aircraft routings in response to groundings and delays experienced in daily operations. This model is constructed by transforming the aircraft routing problem into a time-based network in which the time horizon is discretized, resulting in an integral minimum cost network flow problem with side constraints. The authors outline conditions in which exact solutions are attainable, and discuss the complexity of the problem relative to the size of the underlying airline network. In addition, they present computational results for a marginally sized case study of a single fleet of 27 similar aircraft, serving a network of 30 stations with 162 flights. The problem is initially solved as a relaxed linear programming problem, and if necessary a mixed integer problem, based on the underlying structure of the transformed network, is solved.

The ability of an airline to recover from severe weather conditions and resulting irregularities will depend on its interaction with the air traffic control (ATC) system. Under such conditions, ATC typically imposes restrictions on aircraft movements at affected airports and implements what is generally referred to as a slot allocation scheme, as well as ground-delay programs. The response of the airline to these imposed conditions will be based on available data in the system operations control center. The guidelines governing such slot substitutions have been recently changed to help accommodate the operating needs of carriers in the ATC system. Most of the published literature on the topic of slot allocation has been rendered obsolete, as changes to the substitution guidelines have now significantly altered recovery procedures in use at AOCC.

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The problem of crew reassignment (crew recovery) in the aftermath of irregular airline operations has been considered by researchers at the Logistics Institute of the Georgia Institute of Technology. Lettovsky et al. [5] have developed a mathematical programming based solution methodology which uses an integer programming model to optimally reassign crews to flight segments. In a presentation given at the INFORMS meeting in the fall of 1995, one of the researchers outlined a model which reassigns crews to flight legs, while minimizing the additional cost and operational difficulties to the airline. The solution strategy initially identifies a set of eligible crews, whose original assigned unflown flight segments are used to form new crew pairings which are then reassigned to individual crew members through a set covering problem.

During the normal operations of a carrier, situations often develop wherein modifications have to be made to the existing schedule plan. In addition, due to the inherent variation in passenger demand over the course of the week, airlines find it necessary to adjust their daily flight schedules to adequately meet demand. This will result in the need to make minor modifications to aircraft routings and possibly fleet assignments. Talluri [48] describes an algorithm for making aircraft swaps that will not affect the equipment type composition overnighting at various stations throughout the airline's network. The algorithm repeatedly calls a shortest-path algorithm, and the performance of the swapping algorithm is a reflection of the availability of very fast shortest path algorithms. He also outlines the application of the swapping procedure in the airline schedule development process.

Given a predetermined flight schedule, the fleet assignment problem is to determine which aircraft type is assigned to a given flight segment in the carrier's network. The aircraft routing problem is traditionally solved after the successful completion of the fleet assignment problem. It involves the allocation of candidate flight segments to a specific aircraft tail number within a given sub-fleet of the airline. The process of aircraft routing has

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traditionally been a manual activity at airlines, but in recent years, researchers have developed solution procedures that can be applied to the problem.

In all the published literature dealing with irregular airline operations, there is an underlying assumption that the fleet assignment problem is solved before considering the aircraft re-routing problem. There has been extensive work done on the topics of fleet assignment, aircraft routing and crew scheduling [16 - 53]. In recent years, there has been a trend towards addressing hybrid airline problems such as the combination of the aircraft assignment and routing problem, and the combined fleet assignment and crew scheduling problem. Researchers have started to explore these so-called hybrid strategic planning problems, combining different phases of the airline planning process, which have been traditionally considered in sequential order. However, these hybrid problems have been considered only for the strategic phase of the airline planning process.

One such problem is that of the combined aircraft fleeting and routing problem. Barnhart et. al [18] discuss a model and solution approach to solve simultaneously the fleet assignment and aircraft routing problems. The authors state that the methodology incorporates costs associated with aircraft connections, and complicating constraints (such as maintenance requirements, and aircraft utilization restrictions) which are usually ignored in traditional fleet assignment solution procedures. The model is string-based and a branch and price solution approach is used to solve the problem. This hybrid solution procedure combines the standard integer programming IP solution technique of branch and bound, and explicit column generation. As described by the authors, a string is a sequence of connected flights that begins and ends at a maintenance station, satisfies flow balance, and meets the required maintenance constraints. The methodology is validated using operational data from a long-haul carrier.

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Soumis et. al [44] present a model for large-scale aircraft routing and scheduling problems which incorporates passenger flow issues. The solution methodology proposed is a heuristic adaptation of the Frank-Wolfe algorithm for an integer problem with a special structure. The procedure involves solving alternatively the aircraft routing problem, and the passenger assignment problem until a prescribed criterion is satisfied. The authors discuss the technique used to transfer information from the passenger flow problem to the aircraft routing problem.

Throughout the course of daily operations, airlines face a major operational problem in assigning aircraft capacity to flight schedules to meet fluctuating market demands. Berge and Hopperstad [19] discuss the Demand Driven Dispatch (D3) operating concept that attempts to address this problem. Utilizing up-to-date and more accurate demand forecast for each scheduled departure, aircraft are dynamically assigned to flights in order to better meet anticipated passenger demand. The solution procedure requires the frequent solution of large aircraft assignment problems, which are formulated as multi-commodity network flow problems, and solved with heuristic algorithms. The authors outline case studies of actual airline systems in which increases in passenger loads are achieved, along with reductions in operating costs, resulting in a net improvement in operating profit. From a conceptual standpoint, the potential may exist to conduct aircraft swapping with multiple aircraft types (different crew rating). Some of the concepts used in Boeing's Demand Driven Dispatch methodology can be used as a foundation for incorporating the issue of dynamic aircraft assignment in the resolution of flight schedules in the aftermath of irregular operations.

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“A voice in my head . . . keep talking to me . . . It tells  
me the road is long, it tells me I must be strong, grow  
with the pain and strife, Today is the start of the rest of  
your life”

Edwin Yearwood

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# Chapter 3

## The Airline Schedule Recovery Problem

### 3.1 Discussion of the Airline Schedule Map

The overall framework of the mathematical model of the airline recovery problem is based on a time-space network called a “Schedule Map” which represents the published daily schedule of the airline’s network (Simpson [42]). The Schedule Map (SM) outlines the relationship between activities and events over space and time, and should be considered as a fundamental graphical representation of the airline’s operations. A representative diagram of such a Schedule Map is shown in Figure 3-1. The SM is drawn using vertical timelines, located over a horizontal space representing given stations. Each event (arrival or departure) at a given station is represented by a node for a specific time and location coordinate.

Each flight is represented by a “flight arc” which connects the corresponding nodes at the origin and destination of the scheduled flight. Additional flight arcs may exist in the network to represent potential delay alternatives for each flight during the resolution procedure. These arcs are referred to as “delay arcs” and are automatically generated based on parameter settings, prior to the implementation of the solution algorithms. “Ground arcs” in the network connect chronologically successive pairs of event nodes at a given station. These arcs are necessary in order to describe the flow of aircraft through the network and for the application of network flow algorithms. “Maintenance arcs”

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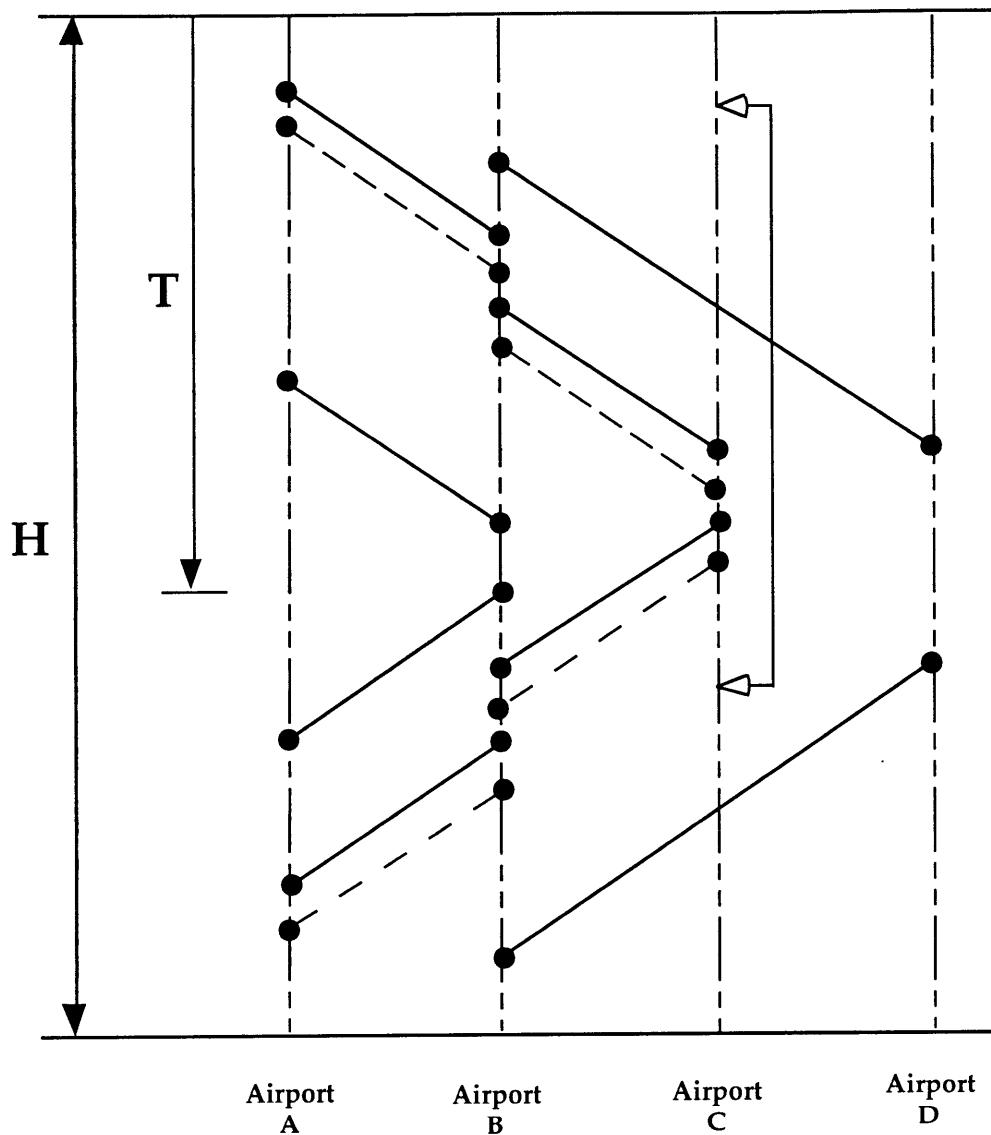


Figure 3-1 Schedule Map Representation

in the network represent the time period of a given aircraft undergoing a planned or unplanned maintenance check within the prescribed resolution horizon. The Resolution Horizon "H", is defined as the total time required to return the airline's operational schedule back to the originally planned schedule. The duration of H will depend on the overall dimensions of the recovery problem, incorporating issues such as the number of aircraft in the fleet, the average length of haul of each flight, and the number of scheduled flights being considered.

The development of the Airline Schedule Recovery Problem (ASRP) based on the schedule map allows the use of efficient tree-searching algorithms to quickly solve the underlying subproblem of finding the best possible aircraft routing, subject to one or more operating constraints. Based on concepts from network flow theory and linear programming theory, algorithms have been developed that can be used to solve the airline recovery problem in a real-time environment. In Chapter 4, a brief summary of these underlying theories will be discussed, since it relates to the development of the solution methodology. In addition, a more detailed description of the schedule map will be given in Chapter 5, incorporating certain aspects of the solution procedures.

### **3.2 Mathematical Formulation of ASRP**

#### **3.2.1 Sub-Problem: Rerouting Aircraft**

In the Airline Schedule Recovery Problem, a path-based formulation was developed in which the decision variable corresponds to the assignment of a specific aircraft tail number to a predetermined sequence of flights; i.e., a particular path in the Schedule Map. However, a specific aircraft would not be considered for a given sequence of flights unless it meets its maintenance requirements; that is, it must be delivered to a maintenance location within the remaining legal flying time. This forms the basic subproblem which must be

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solved quickly and easily. The approach to solving this subproblem relies on specialized tree-searching algorithms to generate the feasible sequence of flights. These include a modified version of the out-of-kilter algorithm for constrained minimum cost flow, and a constrained shortest path multi-labelling algorithm to solve the “constrained optimal path problem” which optimizes airline profitability.

In creating these optimal flight sequences, each tree-searching algorithm always incorporates maintenance constraints that limit the eligibility of a specific aircraft tail number and its ability to cover a given flight segment. In addition the maintenance constraint, several other operational constraints can be incorporated into the tree-searching algorithm such as restrictions on aircraft range, the ability to fly over water, and the level of anticipated passenger spill for assigning a given aircraft to a specific flight segment. In its current form, the sub-problem considered in this research does not explicitly incorporate these additional factors. However, the necessary mechanism for including such factors have already been designed into the solution procedure.

### 3.2.2 The Main Problem: ASRP

The complete model must solve the problem of aircraft reassignment for all operational aircraft in the fleet. It can be best described as a hybrid of the traditionally defined fleet assignment problem and the aircraft routing/rotation problem. The following terms are defined prior to the statement of the complete model:

#### *Indices*

F	set of all flights ij
$F(j,k)$	subset of flights that can be assigned to aircraft k at station j
$F(i,p)$	subset of flights departing from station i in time period p
$F(j,p)$	subset of flights arriving at station j in time period p

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$N$	set of all feasible flight sequences for all aircraft in the fleet
$N(k)$	subset of all feasible sequence of flights for aircraft $k$
$K$	set of all aircraft $k$ in the fleet
$K(t)$	subset of aircraft of type $t$ in the fleet
$K(i, p)$	subset of aircraft scheduled to arrive at station $i$ in time period $p$
$K(t, i, p)$	subset of aircraft of type $t$ , scheduled to arrive at station $i$ , in time period $p$

*Parameters*

$D_{ij}$	actual passenger demand for flight $(i,j)$
$f_{ij}$	average fare per passenger on flight $(i,j)$
$r_{ij}$	goodwill value per passenger on flight $(i,j)$
$t_{ij}$	flight time for flight segment $(i,j)$
$C_{ijk}$	operating cost of assigning aircraft $k$ to flight $(i,j)$
$C_{ijo}$	cost of cancelling flight $(i,j)$
$M_{jt}^T$	maintenance resource capacity for aircraft type $t$ at station $j$ at time $T$
$AC_{jt}^T$	number of aircraft type $t$ required at station $j$ at time $T$
$SLOTS_{jp}$	number of landing slots available at station $j$ during period $p$
$GATES_{jp}$	number of terminal gates available at station $j$ during period $p$
$CREWS_{tip}$	number of crews for aircraft type $t$ , available at station $i$ during period $p$
$CAP_k$	seating capacity of aircraft $k$
$TIME_k$	legal flight time remaining on aircraft $k$ before maintenance is required
$CYCLE_k$	maximum number of flight cycles permitted on aircraft $k$
$\alpha_{ijn}$	equals one if flight sequence $n$ contains flight segment $(i,j)$
$C_{nk}$	cost of assigning flight sequence $n$ to aircraft $k$
$S_{ij}$	amount of spilled passengers from flight $(i,j)$

The decision variables involved are:

$X_{nk} = 1$  if flight sequence  $n$  is assigned to aircraft  $k$ , 0 otherwise

$Y_{ij} = 1$  if flight  $(i,j)$  is cancelled, 0 otherwise

The model can be expressed as:

#### Objective Function

$$\min \sum_{n \in N} \sum_{k \in K} C_{nk} X_{nk} + \sum_{(i,j) \in F} C_{ij} Y_{ij}$$

where;

$$C_{nk} = \sum_{\eta \in n} \left\{ C_{\eta k} + r_{\eta} S_{\eta} - \min[D_{\eta}, CAP_k] \cdot f_{\eta} \right\} \forall k$$

subject to:

1) flight covering

$$\sum_{n \in N} \sum_{k \in K} \alpha_{\eta n} \cdot X_{nk} + Y_{ij} = 1 \quad \forall ij \in F$$

2) aircraft covering

$$\sum_{n \in N} X_{nk} \leq 1 \quad \forall k \in K$$

3) aircraft utilization

$$\sum_{n \in N} \sum_{(\iota, j)} t_\eta \alpha_{\eta n} X_{nk} \leq TIME_k \forall k$$

4) leg based demand covering

$$\sum_{n \in N} \sum_{k \in K} \alpha_{\eta n} \cdot CAP_k \cdot X_{nk} + S_\eta - D_\eta \geq 0 \forall ij, S_\eta \geq 0$$

and further, subject to additional “auxiliary” operational constraints:

A1) crew availability

$$\sum_{k \in K(t, \iota, p)} \sum_{n \in N} \sum_{\eta \in F(\iota, p)} \alpha_{\eta n} \cdot X_{nk} \leq CREWS_{tp} \forall t, i, p$$

A2) ATC slot allocation

$$\sum_{k \in K(j, p)} \sum_{n \in N} \sum_{\eta \in F(j, p)} \alpha_{\eta n} \cdot X_{nk} \leq SLOTS_{j, p} \forall j, p$$

A3) Gate allocation

$$\sum_{k \in K(j, p)} \sum_{n \in N} \sum_{\eta \in F(j, p)} \alpha_{\eta n} \cdot X_{nk} - \sum_{k \in K(i, p)} \sum_{n \in N} \sum_{\eta \in F(i, p)} \alpha_{\eta n} \cdot X_{nk} \leq GATES_{j, p} \forall j, p$$

A4) Aircraft Balance

$$\sum_{n \in N} \sum_{k \in K(t)} \sum_{\eta \in F(j, p)} \alpha_{\eta n} \cdot X_{nk} \geq AC_{jt}^T \forall j, \forall t$$

A5) Maintenance resource allocation

$$\sum_{n \in N} \sum_{k \in K(t)} \sum_{ij \in F(j,p)} \alpha_{ijn} \cdot X_{nk} \leq M_{jt}^T \forall j, \forall t$$

Over all the potential flight sequences (and scheduled flights implicitly), the objective function sums the costs associated with reassigning flights to operational aircraft within the confines of the available resources. These cost coefficients include aircraft direct operating costs, predetermined passenger revenue spill costs, and operating revenue. Operating revenue is determined based on the actual passenger loads for each scheduled flight, and incorporates the impact of schedule delays in terms of recapture, passenger retention, and lost passenger goodwill. Spill costs account for the impact of spilling passengers on a given flight. Direct operating costs include fuel, cockpit crew costs, direct maintenance and ownership costs, accounting for all costs that are generally allocated against the actual flying time of the aircraft.

The flight covering constraint sums over all candidate flight sequences and has a right hand side coefficient of one, to ensure that each flight is either covered (i.e. flown) by one aircraft at a given time, or is cancelled. The coefficients  $\alpha_{ijn}$  for each flight sequence are determined from the solution of the aircraft rerouting subproblem, and have value one if the given flight "ij" is part of the candidate sequence of flights denoted by "n".

The aircraft covering constraint sums over all flight sequences to ensure that each aircraft is assigned to no more than one sequence at a given time. The aircraft utilization constraint ensures that for each aircraft, the potential sequence of flights does not exceed the number of available flight time left on the aircraft before scheduled maintenance. The leg based demand constraint accounts for the accommodation of passengers on each flight segment. This constraint also serves as a definition of passenger spill in the model. These constraints on aircraft utilization and passenger demand covering are not considered in the solution of

the main ASRP problem, as they are implicitly considered in the solution of the underlying subproblem of aircraft rerouting.

In addition, there are five auxiliary operational constraints that have been considered for the complete ASRP. These include constraints on crew availability, ATC slot allocation, gate allocation, maintenance resource allocation, and aircraft balance at the end of the Resolution Horizon H. The crew availability constraint ensures that the number of outbound flights at a given station within a given time period does not exceed the number of crews available at the station. The ATC slot allocation constraint limits the number of arriving flights to an airport with a given period, based on restrictions provided by the ATC system. The gate allocation constraint limits the number of operational aircraft at the terminal based on the maximum number of gates available at the given airport. It is likely to be satisfied by the original Flight Service Schedule if all gates are available, but now arriving flights may be delayed.

Similarly, the maintenance resource allocation constraint ensures that the number of aircraft assigned to a given maintenance station (overnight) does not exceed the capacity of that station. The aircraft balance constraint ensures that the aircraft at each station at the end of the Resolution Horizon, corresponds to the number of aircraft “positioned” in the current maintenance routing plan.

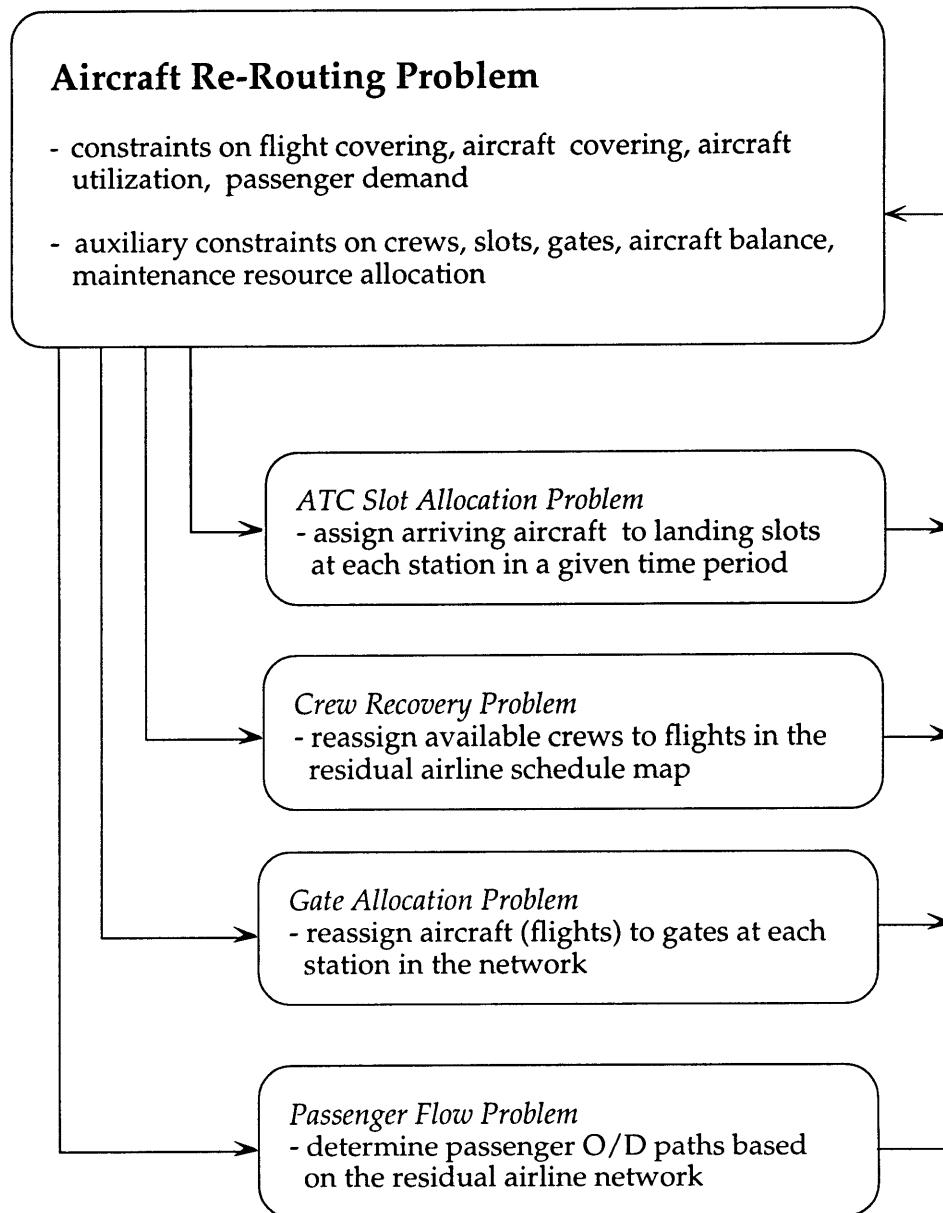
It is important to point out that these auxiliary constraints are best described as soft constraints, since ideally, the actual value of the right hand side coefficients should be ideally determined interactively during the solution process by the airline operation controllers.

### **3.3 Problem Decomposition and Auxiliary Problems**

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Each of these auxiliary constraints could lead to its own sub-problem for the reassignment of the given resource to each operational flight. The actual scheduled flights considered in each sub-problem would depend on the outcome of the primary Airline Schedule Recovery Problem. Significant research work has been done by other practitioners (see references [16] through [53]) on the topics of slot allocation, crew scheduling and recovery, and on the general topic of resource allocation.

The envisioned subproblems of this mathematical formulation would share many of the characteristics of decision models and corresponding solution methodologies developed in the various independent research initiatives. The overall framework of the decision model is outlined in Figure 3-2. The primary focus of this dissertation is to develop the formulation of the airline recovery problem with an emphasis on the aircraft rescheduling aspect of the problem.



**Figure 3-2** Decomposition of the Airline Schedule Recovery Problem

### 3.3.1 ATC Slot Allocation Problem

The ability of US domestic carriers to freely assign individual flights to prescribed landing slots under an ATC ground delay program is an underlying assumption in the overall airline recovery problem formulation. As such, each flight has a certain value associated with it, and the assignment of flights to slots can be modelled using the classical transportation assignment problem. The following model is a representative formulation of the slot allocation problem. Under a typical operating situation, several airport stations would be affected by ATC slot restrictions, and the assignment problem would incorporate each airport in the decision process. More elaborate decision models for this problem and an extensive overview on the slot allocation problem can be found in Carlson [22].

This model solves the problem of slot allocation for all operational flights in the airline's network. It can be expressed as:

$$\min \sum_{f \in F(j,t)} \sum_{t \in T} C_{fjt} X_{fjt}$$

subject to;

$$\sum_{t \in T(j)} X_{fjt} = 1 \forall j, t$$

$$\sum_{f \in F(j,t)} X_{fjt} \leq SLOTS_{jt} \forall j, t$$

where;

$X_{fjt}$  equal to one if flight f is assigned to slot t at station j, 0 otherwise

F set of all operational flights

T(j) set of all landing slots at station j

F(j,t) subset of flights arriving at station j that can be assigned to time slot t

$C_{ft}$	cost of assigning flight f to landing slot t at station j
SLOT <sub>j</sub>	number of arrivals possible at station j at time t

The cost parameter would reflect the value of a given flight to the airline based on issues such as the total passenger delay time, or the total operating costs. The actual form of this coefficient could be adjusted by the airline controller. The first constraint ensures that each flight is assigned to only one landing slot time, and the second places a limit on the number of flights assigned to slots at a given time t.

### 3.3.2 The Crew Recovery Problem

The rescheduling of flights in the airline network is affected by several operational constraints as outlined in the formulation, but it is important to point out the level of complexity which results from the crew constraints. Crew scheduling is by far the most complex aspect of the airline planning process, and the ability to reschedule crews will depend on the actual operational flights, which in turn, will depend on the availability of crews at each station. Unlike all other resources in this system, the movement of the crew members adds significant complexity in trying to solve the flight rescheduling problem. Again, this sub-problem would be solved iteratively, and the resulting number of legal flight crews at each station within a given time period would then be updated in the main problem after each iteration. The following formulation of the crew recovery problem is based on research of Lettovsky [5] on the topic.

This model solves the problem of crew rescheduling for all legal crew members “displaced” in the network. It is based on the assumption that the airline has the ability to reassign crew members to modified bidlines without the consent of each individual, provided the crew member is able to maintain legality throughout the network. The model can be expressed as:

---

$$\min \sum_{m \in M} \sum_{p \in P(m)} \sum_{f \in p} C_{fm} \delta_{fp} X_{pm}$$

subject to;

$$\sum_{p \in P(m)} X_{pm} \leq 1 \forall m \in M$$

$$\sum_{m \in M} \sum_{p \in P(m)} \delta_{fp} X_{pm} \geq CREW_f \forall f \in F$$

$$\sum_{p \in P(m)} \sum_{f \in p} \tau_f \delta_{fp} X_{pm} \leq \alpha_m \forall m \in M$$

$$\sum_{p \in P(m)} \sum_{f \in p} d_f \delta_{fp} X_{pm} \leq \beta_m \forall m \in M$$

where;

$X_{pm}$	equal to one if crew path p is assigned to crew m
$C_{fm}$	cost of assigning flight f to crew member m
$\delta_{fp}$	equal to one if crew path p contains flight f
F	set of all operational flights
M	set of all available crew members m
$P(m)$	set of all possible crew paths for crew member m
$\alpha_m$	amount of legal flying time remaining for crew member m
$\beta_m$	amount of legal duty time remaining for crew member m
$\tau_f$	total flying time for flight f
$d_f$	total duty time for flight f
$CREW_f$	number of crew members required for flight f

The primary objective of this subproblem is to minimize the cost of reassigning crews to operating flights in the residual airline network in the aftermath of the irregularity. The first constraint ensures that each crew member is assigned to only one crew path at a given time, and the second constraint ensures that all operating flights have the adequate number of crew members on-board the aircraft. Constraints three and four in this model ensure that each crew member does not violate established FAA operating safety requirements.

### 3.3.3 The Gate Allocation Problem

After the flight rescheduling problem has been completely solved, the reallocation of flights to terminal gates would then be addressed, as some flights have the potential of being delayed, thereby losing their originally scheduled time slot at a given gate. As the number of aircraft on the ground is restricted by the number of available gates at each station in the solution of the primary aircraft problem, all operational flights can be accommodated. The only required task would be to re-assign aircraft (flights) to gates, taking into consideration such issues as passenger connectivity, gates handling constraints, and the availability of ground support services. The following model of the gate allocation problem is solely for outlining the resulting subproblem. A more comprehensive discussion of this subproblem can be found in Svrcek [47]. It is based on the assumption that an airline has the ability to reassign aircraft to gates at will, provided the necessary airport operational regulations are satisfied. The model can be expressed as:

$$\min \sum_{f \in F} \sum_{g \in G(f)} C_{fg} X_{fg} \quad \forall j \in J, p \in P$$

subject to;

$$\sum_{f \in F(p,j)} X_{fg} \leq 1 \quad \forall g \in G(j), \forall p \in P$$

$$\sum_{g \in G(f,j,p)} X_{fg} = 1 \quad \forall f \in F$$

where;

$X_{fg}$	equal to one if flight f is assigned to gate g, zero otherwise
P	set of time periods p considered at a given station j
F	set of all operational flights f
$F(p,j)$	subset of flights on the ground at station j during time period p
$G(j)$	set of all gates at station j
$G(f,j,p)$	subset of gates eligible for flight f at station j during time period p
$C_{fg}$	"cost" index for assigning flight f to gate g

The objective of this model is to minimize the "cost" of the gate allocation decision. The actual content of such a cost function would depend on the operational philosophy of the airline, and would potentially take into consideration issues such as aircraft size, passenger walking distance, baggage transfer, and aircraft servicing requirements. The first constraint ensures that each gate is assigned to only one flight which is on the ground at a given station and time period. The second constraint ensures that each flight is assigned to only one gate at a time.

### 3.3.4 The Passenger Flow Problem

Although the actual passenger itinerary issues are not explicitly considered in this model formulation, the passenger flow problem has to be addressed in the aftermath of the flight rescheduling decision. Based on the residual Schedule Map, the airline has to reassign

passengers to flights in such a way that some prescribed criterion is minimized. The decision objective of the passenger flow model would depend on the operational philosophy of the carrier. Examples of such objectives range from minimizing overall passenger delay time, to maximizing the passenger revenue “recovered” in the modified flight schedule; since passengers could be potentially lost to competing carriers. The model is based on the assumption that all spilled passengers of a specific “high-valued” origin-destination itinerary are recaptured, provided there is adequate capacity to accommodate such passengers. In effect, priority is given in the model to accommodate as many valuable passengers as possible in the residual flight network. Again, the value of each passenger would depend on the operational directives of the carrier.

The following formulation of the passenger flow problem is based on research currently being done at MIT on the topic of an origin-destination based fleet assignment model by Barnhart and Kniker [36]. In this representative form, the primary objective of the model is to maximize the recovered passenger revenue in the residual flight network, through the optimal reassignment of seats to origin-destinations itineraries on each operational flight.

The model can be expressed as:

$$\max \sum_{i \in I} \sum_{p \in P(i)} f_i X_{ip}$$

subject to;

$$\sum_{i \in I} \sum_{p \in P(i)} \delta_f X_{ip} \leq CAP_f \quad \forall f \in F$$

$$\sum_{p \in P(i)} X_{ip} \leq D_i \quad \forall i \in I$$

where;

$X_{ip}$	number of passengers for itinerary i assigned to path p
F	set of all operational flights f in the residual network
I	set of all potential origin-destination itineraries i at a given time
P	set of all potential passenger travel paths p in the residual network
$P(i)$	subset of paths that can be considered for a passenger with itinerary i
$f_i$	average passenger revenue for itinerary i
$CAP_f$	capacity of the aircraft assigned to flight leg f
$D_i$	total number of passenger booked to travel on itinerary i
$\delta_{if}$	equal to one if itinerary i contains flight leg f, zero otherwise

The subset of passenger paths considered in the reallocation of passenger flows in the residual flight network would be generated depending on the operational constraints employed in the decision process (such as the maximum allowable delay for a given passenger). For each itinerary, it is assumed that one fare class exists; as in practice, ticketed passengers are not generally differentiated during this phase of the airline recovery process. The ability to accommodate as many revenue passengers as possible on the residual flight network could potentially influence flight reassignment decisions made in the main aircraft problem. For example, it may be possible to ensure that certain origin-destination markets are covered within a given time period, thereby guaranteeing that certain “valuable” passengers are taken to their destinations in a timely fashion.

“Some dreams live on in time forever, those dreams,  
you want with all your heart . . . If I could reach, higher,  
just for one moment touch the sky, from that one  
moment in my life, I’m gonna be stronger, know that  
I’ve tried my very best, I’d put my spirit to the test, If I  
could reach . . .”

Gloria Estefan

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# Chapter 4

## Review of Linear Programming and Network Flow Theory

### 4.1 Overview

The overall framework for the mathematical modelling and the corresponding solution methodologies for the airline schedule recovery problem are based on network flow theory. A comprehensive review of network theory can be found in *Network Flows : Theory, Algorithms and Applications* (Ahuja, Magnanti, Orlin: Prentice Hall). The following sections discuss several algorithms that have been adapted, and further enhanced by the author for solving the schedule recovery problem. These include a specialized multi-label shortest path algorithm, a multi-label out-of-kilter algorithm, and a column generation procedure which uses the revised simplex algorithm.

In Chapter 3, the underlying subproblem of aircraft rerouting was discussed, outlining the framework of the solution approach. The “constrained optimal path problem” can be modelled either as a “constrained minimum cost flow problem” or as a “constrained shortest path problem” and solved using specialized tree-searching algorithms. In this research project, a variation of the out-of-kilter algorithm is used to solve the constrained minimum cost flow problem, and the multi-label shortest path algorithm is used to solve the constrained shortest path problem. In the next chapter, there is an extensive discussion of the solution methodologies developed, but first it is necessary to give an introduction to the underlying theory used in creating such methodologies.

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#### 4.2 The Constrained Minimum Cost Flow Problem

The specialized algorithm developed to solve the constrained minimum cost flow problem is based on concepts of the out-of-kilter (OKF) algorithm, originally developed by Ford and Fulkerson [72] for circulation flows. The primary enhancement being a modified version of the tree-searching procedure within the OKF algorithm, in which multiple parameter labels are monitored during the execution process, and the resulting minimum cost flow satisfies additional constraints of the flow, such as time duration of the total flow in the network.

The name out-of-kilter reflects the fact that arcs in the network either satisfy the complementary slackness optimality conditions (in-kilter) or do not (out-of-kilter).

**Theorem** (Ahuja et. al, 1993) A feasible solution is an optimal solution of the minimum cost flow problem if and only if for some set of node potentials  $p$ , the reduced costs  $C_{ij}p$  and flow values  $X_{ij}$  satisfy the following complementary slackness optimality conditions for every arc  $(i,j)$  in the network:

- If  $C_{ij}p$  greater than zero, then  $X_{ij}$  equal zero
- If flow  $X_{ij}$  within arc limits, then  $C_{ij}p$  equal zero
- If  $C_{ij}p$  less than zero, then  $X_{ij}$  equal upper arc limit  $U_{ij}$

The out-of-kilter algorithm attempts to find the minimum cost cyclic flow in a network, within the prescribed constraints of the problem. The algorithm iteratively modifies arc flows and node potentials (later referred to as node prices) in a way that decreases the infeasibility of the solution and simultaneously moves the solution closer to optimality. The procedure concentrates on a particular out-of-kilter arc and attempts to put it in kilter. The algorithm does this in such a way that all in-kilter arcs stay in-kilter, whereas the state (kilter number) for any out-of-kilter arc either decreases or stays the same after each

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iteration. On each such iteration, the network is scanned, and the labelling process for increasing or decreasing a particular arc flow in the circulation is found.

**algorithm** Clarke-OKF

**begin**

    Out-of-Kilter scan

    scan all arcs in the network to determine if any out-of-kilter arc exists

    define the residual network  $G(x)$  and compute the kilter number of arcs;

**while** the network contains an out-of-kilter arc **do**

**begin**

            select an out-of-kilter arc  $(p, q)$  in  $G(x)$ ;

            identify target node for the labelling process;

**while** target node not labelled **do**

**begin**

                    constrained forward labelling from opened nodes in the network;

                    constrained reverse labelling from opened nodes in the network;

                    if target node labelled, break;

                    else if new labels, continue labelling;

                    else, update node prices;

                    if node price update not possible, STOP, infeasible flow;

**end;**

                augment flow cycle;

                update kilter number of arcs in the network;

**end;**

**end;**

#### Figure 4 - 1    Clarke-OKF Algorithm

It is possible to identify potential cost reduction arcs in the network, where a negative cost cycle could be found using a set of temporary node prices (potentials) and reduced arc costs ( $c_{\bar{b}}$ ) that can be determined using optimal tree construction techniques. If the flow in

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some arc is infeasible (i.e., exceeds upper/lower bounds), then the out-of-kilter arc can be scanned to bring it into feasibility. By scanning only the out-of-kilter arcs, and making the appropriate flow changes, it is possible to find a minimum cost, feasible circulation flow in the network for any values of the arc attributes. It is important to reiterate that the primary decision parameter in the minimum cost flow problem is cost, but the feasible flow has to also satisfy the time constraints of the problem, which is incorporated into the searching procedure of the algorithm.

In order to implement the modified OKF algorithm, it is necessary to define the various out-of-kilter states for arcs, based on the reduced arc cost, and the current arc flow relative to the flow constraints placed on the arc.

**Case 0    In-Kilter (no changes done to the network flow)**

- alpha       $c_{\bar{b}}$  greater than zero, and flow equal lower arc limit
- beta        $c_{\bar{b}}$  equal zero, and flow within arc flow range
- gamma       $c_{\bar{b}}$  less than zero, and flow equal upper arc limit

**Case 1    Out-of-Kilter (increase flow in arc if possible)**

- alpha 1      $c_{\bar{b}}$  greater than zero, and flow less than lower arc limit
- beta 1       $c_{\bar{b}}$  equal zero, and flow less than lower arc limit
- gamma 1      $c_{\bar{b}}$  less than zero, and flow less than upper arc limit

**Case 2    Out-of-Kilter (decrease flow in arc if possible)**

- alpha 2      $c_{\bar{b}}$  greater than zero, and flow greater than lower are limit
- beta 2       $c_{\bar{b}}$  equal zero, and flow greater than upper arc limit
- gamma 2      $c_{\bar{b}}$  less than zero, and flow greater than upper arc limit

If it is found that an arc is in states Case 1 or Case 2, it is required that the flow in the network be modified to bring the arc into kilter. For the states alpha one, and beta one, it is necessary to increase the arc flow to reach feasibility. In state gamma one, the negative value of the reduced cost indicates the potential for reducing the cost of the flow by increasing the

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arc flow. For these three states, it is necessary to determine the possibility of increasing the circulation flow in order to find a least cost feasible flow. If the arc is found to be in state alpha two, it has a positive cost, but the possibility of reducing its flow will allow a reduction of the network total flow cost. In states beta two, and gamma two, it is necessary to reduce the arc flow in order to bring it into feasibility. Figure 4-1 summarizes the modified Clarke-OKF algorithm, as it is used to solve the constrained minimum cost flow problem.

#### 4.3 The Constrained Shortest Path Problem

The shortest path problem is one of the fundamental problems studied in the operations research field. Extensive research has been done on the topic, and a comprehensive summary of such work can be found in an article by Deo and Pang [63]. In the case of the constrained shortest path problem, many researchers have attempted to solve this problem through the use of modified algorithms which were originally designed to solve the shortest path problem. These algorithms make use of linear programming concepts such as the relaxation of the additional and complicating constraints on the problem in order to achieve a solution to the problem. In reviewing existing solution methodology developed to solve complex problems such as the constrained shortest path problem, the generalized permanent labelling algorithm (Desrochers and Soumis, 1984) appeared to be the most efficient algorithm available to solve the problem.

The generalized permanent labelling (GPL) algorithm for the shortest path problem with time windows developed by Desrochers, et. al at the GERAD Institute, has been modified by the author to efficiently solve the shortest path problem with schedule time constraints. This algorithm is a variation of the Ford-Bellman algorithm for the shortest path problem, and assigns multiple labels to each node representing the cost and time constraint. During

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the solution procedure, the routes have to be compared based on the multiple criterion of the problem. Several labels have to be stored at each node in the network and they are used dynamically to calculate the labels of other nodes which satisfy all the side constraints on the problem, such as a maximum cumulative time on the routing.

The algorithm stores at each node multiple labels of time and cost, until a less costly and/or less travel time route arriving at the given node is found. At a given node, a new label is said to dominate an existing label if both its time and cost parameters are better than the “best” label to date. The set of labels stored at each node is dynamically managed in such a way that unnecessary or “dominated” labels are deleted from the linked list at each node in the network, and the label list is sorted in decreasing cost order. Each label corresponds to a different path through the network from the source to the given node, and is classified as being efficient (Desrochers and Soumis, 1988). An efficient path is defined as one such that all of its labels are efficient, and such paths are used to determine the constrained shortest path from source to sink in the network.

**algorithm**    Clarke-GPL

**begin**

    Initialize all label values at each node

    Set “dominance label” at each node to zero cost and zero time

    Open source node

**while** the network contains “opened” nodes **do**

**begin**

            Scan all arcs from all opened nodes in the network

            Establish candidate labels based on dominance test (cost and time parameters)

            If cost or time is less than dominant label, store label; else discard new label

            Open nodes whose labels satisfy dominance test

            Update multiple attribute label linked list at each open/unscanned node

---

```
        Close scanned nodes at end of iteration
end
Select shortest path from source to sink in the network that satisfies schedule
constraints
end
```

**Figure 4 - 2** Clarke - Generalized Permanent Labelling Algorithm

The underlying network used for the constrained shortest path problem is designed in such a way as to prevent any cycling in the solution procedure. It is important to point out that during the solution process, there is the possibility that all paths considered into a node result in efficient labels. Depending on the structure of the Schedule Map, there can be an exponential number of paths in the network, an exponential number of labels may exist, and as a result, the permanent labelling algorithm can take exponential time to solve. The exponential time issue has played a substantial role in the development and implementation of the modified algorithm, especially in the design of the data structures used in the labelling procedure. Figure 4-2 summarizes the modified version of the generalized permanent labelling algorithm based on this implementation.

#### 4.4 Algorithm Comparison

One of the driving design parameters in developing the solution procedures for solving the ASRP problem has been real-time solution capabilities. The ability to solve the subproblem of aircraft rerouting quickly is thus essential in achieving this goal. The modelling of the subproblem as a constrained minimum cost flow problem and as a constrained shortest path problem resulted in two separate solution algorithms for solving the subproblem.

**Table 4-1** Comparison of Solution Run-time in Seconds for the Clarke-OKF  
and Clarke-GPL Algorithms

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Problem	Clarke-OKF	Clarke-GPL
1	30.05	8.89
2	16.45	3.84
3	7.25	1.16
4	6.20	1.25

During the course of the research project, both algorithms were fully developed and tested to compare the performance of each algorithm. Table 4-1 summarizes the run-time in seconds for each algorithm using datasets derived from the case study analysis data. Based on these preliminary tests, it was established that the Clarke-GPL algorithm was the best choice for solving the aircraft rerouting subproblem.

#### 4.5 Column Generation Procedure

The column generation method is based on the decomposition principles of Dantzig-Wolfe, and it takes advantage of the premise that it is not necessary to store the complete constraint matrix during the solution process, and that columns can be generated only on a “as-needed” basis. The Dantzig-Wolfe decomposition technique was originally developed to solve large scale, structured linear programming problems. Based on the solution of the coordinating restricted master problem, the underlying subproblems are modified and iteratively solved until a prescribed criterion is satisfied in the problem.

The process of implicit column generation using the revised simplex method is based on the principle that the reduced cost of any feasible variable in the restricted master problem should be non-negative in any optimal solution to a minimization problem. The overall column generation procedure is more or less an extension of the simplex method, in which subproblems and the restricted master problem are iteratively solved until the optimal

solution is achieved. The form of the subproblem will depend on the underlying characteristics of the problem being considered, and it was established during the course of the research project that both the constrained minimum cost flow problem, and the constrained shortest path problem discussed above were applicable as subproblems to the flight rescheduling problem.

During the column generation procedure, the large scale linear programming problem is classified as the master problem MP and can be represented by the following mathematical formulation (Bradley, et. al) :

$$Z^*: \quad \text{Min } z = C_1X_1 + C_2X_2 + \dots + C_nX_n$$

subject to;

$$a_{i1}X_1 + a_{i2}X_2 + \dots + a_{in}X_n = b_i \quad (I = 1, 2, \dots, m)$$

$$X_j \geq 0 \quad (j = 1, 2, \dots, n).$$

As in decomposition, an assumption is made a priori that certain variables,  $X_{k+1}, X_{k+2}, \dots, X_n$  are non-basic variables with value zero. The resulting linear program is described as being a restricted problem, and is referred to as the restricted master problem RMP.

$$Z^K: \quad \text{Min } z = C_1X_1 + C_2X_2 + \dots + C_kX_k$$

subject to;

$$a_{i1}X_1 + a_{i2}X_2 + \dots + a_{ik}X_k = b_i \quad (I = 1, 2, \dots, m)$$

$$X_j \geq 0 \quad (j = 1, 2, \dots, K).$$

where;

$\Pi^k$  are the optimal shadow prices for each constraint equation

From linear programming theory, the solution to the restricted master problem if feasible, may be optimal to the master problem if and only if the simplex optimality conditions are satisfied. Let  $\Pi_1^k, \Pi_2^k, \dots, \Pi_m^k$  denote the optimal dual variables for the restricted master problem, and as such, the reduced cost  $C\text{-bar}$ , of variable  $j$  is defined by:

$$C\text{-bar}_j = C_j - \sum_m \left\{ \Pi_m^k a_{ij} \right\}$$

The simplex optimality conditions state that the solution is optimal if all reduced costs in the restricted master problem are non-negative, that is  $C\text{-bar}_j$  is greater than or equal to zero. If this condition is met, the original master problem has been solved without explicitly using all the constraint data or solving the full master problem. If any of the reduced costs are negative, the corresponding variable (column) would be introduced into the basis of the restricted master problem and re-optimized using the revised simplex method. The procedure used to determine the reduced cost of each variable is itself an optimization problem, and is generally referred to as the subproblem.

An overview of the complete column generation procedure for minimization problems is summarized in the Figure 4-3. The efficiency of the solution methodology is a result of its ability to take advantage of the underlying structure of the subproblems, and to obtain an optimal solution before numerous columns have been added to the restricted master problem. The application of the column generation procedure in solving the airline flight rescheduling problem is complicated by the fact that each aircraft in the fleet has to be represented as an individual commodity in the problem, and this has significant impact on the overall dimensions of the problem.

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The ability to solve such large-scale multi-commodity flow MCF problems calls for the reformulation of the generic assignment problem as a path based formulation instead of an arc based formulation, as was outlined in Chapter 3. Based on the flow decomposition theorem of network flows, it is possible to decompose optimal arc flows into path flows such that mass balance conditions are satisfied in the problem. A comprehensive discussion of the column generation procedure applied to multicommodity flow problems can be found in Network Flows: Theory, Algorithms and Applications [54].

```
algorithm    column generation using revised simplex method
begin
    establish a restricted master problem with a feasible subset R of columns;
    while simplex optimality conditions are not met do
        begin
            solve the RMP to optimality over the restricted subset;
            obtain dual variables from existing solution;
            using the dual variable, update subproblems and solve to determine new
            variable (columns) to be added to the restricted master problem;
            if minimum reduced cost column has a non-negative reduced cost,
            STOP, global optimality.
            otherwise, add minimum reduced cost column to the restricted subset R.
        end
    end
```

**Figure 4 -3** Column Generation Procedure

The underlying principles are the same for the path based formulation, but there are significant benefits through constraint size reduction, and the resulting solution time for the problem being shortened. For a network with  $n$  nodes,  $m$  arcs, and  $K$  commodities, the path formulation problem contains  $m + K$  constraints, in addition to any non-negativity

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restrictions imposed on the path flow variables. On the other hand, the arc based formulation will have  $m + nK$  constraints since it contains one mass balance constraint for every node and commodity combination. Based on the resulting structure of the constraint matrix, it is possible to apply a specialized version of the simplex method such as the generalized upper bounding (revised) simplex method to efficiently solve the path flow formulation of the problem.

It is important to point out that the immense number of potential path possibilities for each commodity in the problem may have a negative impact on the solution time, and overall algorithm efficiency. However, from linear programming theory, it is known that at most  $K + m$  paths carry positive flow in some optimal solution to the problem. The implementation of the generalized upper bounding linear programming procedure enables one to take advantage of this observation. At each step of the revised simplex method, a basis is maintained for the problem, which is used to determine the vector of simplex multipliers for each constraint.

In the path-based formulation, there will be a dual variable  $w_{ij}$  for each arc constraint in the matrix, as well as a dual variable  $\sigma_k$  for each commodity demand constraint in the problem. The resulting reduced cost expression for each path (P) flow variable will be given by;

$$C_{P_{\sigma,w}} = \sum_{(ij) \in P} \{ C_{ij}^k + w_{ij} \} - \sigma^k \quad \text{for each commodity } k$$

As in the arc based formulation case, it is required for all the reduced costs to be non-negative for optimality in any minimization problem. The complementary slackness conditions for optimality require that:

- 1) the dual variable  $w_{ij}$  of an arc  $(i,j)$  is zero if the optimal solution does not use all of the capacity of the arc.
- 2) the modified path cost " $\sum_{(i,j) \in P} (C_{ij}k + w_{ij})$ " for each path connecting the source node  $s_k$  and the sink node  $t_k$  of commodity  $k$  must be at least as large as the commodity cost  $\sigma^k$
- 3) the reduced cost must be zero for any path  $P$  that carries flow in the optimal solution.

Based on these optimality conditions, it can be stated (Ahuja et. al):

$\sigma^k$  is the shortest path distance from source  $s_k$  to node  $t_k$  with respect to the modified costs  $c_{ij}k + w_{ij}$ , and in the optimal solution every path from node  $s_k$  to node  $t_k$  that carries a positive flow must be a shortest path with respect to the modified costs.

This result shows that the arc price (dual variable)  $w_{ij}$  permits the decomposition of the multicommodity flow MCF problem into a set of independent "modified" cost shortest path problems.

# **Chapter 5**

## **Solution Methodology**

### **5.1 Overview**

In developing solution methodologies for the airline schedule recovery (ASRP) problem, the role of the airline operations controller was a constant factor in the design process. It was determined that any decision procedures and methodologies should have the ability to incorporate the high degree of uncertainty which exists in the daily operations of an airline, and that it must look at problems from a total system perspective, rather than on a localized decision level. During the development phases, several factors were considered including the ability to have switching of aircraft types, to combine the decision on flight delays and cancellations, to consider the effects of crew scheduling on the hybrid fleet assignment/aircraft routing problem, and be compatible with solution methodologies and resolution procedures currently in use at airline operation control centers.

The ability to solve the ASRP problem in real-time dictates very efficient solution procedures and methodologies which will provide the user with a number of good possible options. A trade-off has to be made between the optimality of the solution versus the solution time. Airline operation controllers will address several irregularities during a given shift period, so there is a sequence of decisions, and not just a single global decision. During the initial development phase, it was uncertain if the

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real-time decision requirements would demand heuristic procedures for the resolution process. The following section will present an overview of several solution methodologies that have been developed throughout the course of the research program, and that are validated and tested with real world case studies.

Each of the solution procedures, whether heuristic or optimization-based, was developed around the framework of a three-phase decision process. These are:

*Generate*

Potential flight sequences that meet all operational constraints, using modified tree search algorithms on a sub-graph of the overall network schedule map.

*Assign*

Sequence of flights to each operating aircraft while optimizing specified objective (e.g. maximize profit). If there are less aircraft than flight sequences, some flights are assigned to "cancellation" sequences.

*Revise*

Overall network structure, adjusting scheduled arrival and departure times of each flight, reflecting the output of the ASSIGN module.

The following solution procedures have been developed and implemented as computer algorithms using the C++ programming language. The optimization-based methodology was developed around the CPLEX callable programming library, which consists of a wide array of mathematical programming solution procedures such as the revised simplex method, and the branch and bound method. A comprehensive discussion of these solution procedures can be found in Applied Mathematical Programming (Bradley, Magnanti, Hax: Addison-Wesley 1983) and Network Flows: Theory, Algorithms and Applications (Ahuja, Magnanti, Orlin: Prentice Hall 1993). There are two options for the solution approach:

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*Option 1: Heuristic*

The flight rescheduling problem is solved using specialized tree-searching procedures, based on network flow theory. At each iteration, a possibly sub-optimal assignment of an aircraft to a generated sequence of flights is made using a prescribed decision matrix.

*Option 2: Optimization-Based*

The flight rescheduling problem is solved as a large scale set-packing problem, in which several feasible flight sequences are generated for each aircraft on an underlying structured sub-problem and optimally assigned to operational aircraft using the revised simplex method, and branch and bound method. This solution methodology is similar to state-of-the-art procedures used to solve the airline crew scheduling problem.

## 5.2 Schedule Map Generation

### 5.2.1 Pre-Processing Procedures

The implementation of the solution procedures includes the generation of flight delay arcs and ground arcs in the Schedule Map, based on information from the originally scheduled revenue flights in the airline network, and established operational philosophies and requirements of the carrier. These include, but would not be limited to operational limitations (such as the maximum allowable delay for flights at a given station and time period), passenger connectivity issues, arrival-departure bank integrity, the ability of a given aircraft to operate a specific flight based on range capability, over-water requirements, or type of aircraft originally assigned to the flight, and the ability to cancel a given flight in the resolution process. Information for all operational aircraft in the fleet and for scheduled revenue flights are input to the computer module, and the required arcs are automatically

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generated to create the Schedule Map consisting of flight, delay, ground, and maintenance arcs, which was described in Chapter 3.

The generation of the delay arcs in the Schedule Map enables the solution procedures to efficiently make trade-offs between cancelling and delaying each individual flight in a single decision process. The number of delay arcs for a given flight would be restricted such that cycling in the network would be prohibited, i.e., to prevent multiple covering of the same flight in a generated sequence of flights. This is accomplished by restricting the latest departure time of a given “delay arc” (delayed flight) to be within the total roundtrip timeframe of the originally scheduled flight segment. This approach to the flight delay issue was taken to allow the delay of individual flights, independent of upstream effects in the network, thereby minimizing delay propagation. In modelling flight delays in this manner, it is possible to absorb any delays in originally scheduled “slack” time in the Schedule Map. Concern was also given to the impact of the increase in the number of arcs in the network to the overall size of the problem, and the resulting solution time requirements.

Each delay arc would be coupled to the corresponding original flight arc such that any decisions about the flight would be reflected on all fleet duplicates of the network. The network generation procedure is summarized in Figure 5-1. It is important to re-iterate that one of the driving design parameters in developing these solution methodologies was the desire to provide “real-time” decision making capabilities to the airline controller.

**procedure**    delay arcs and ground arcs generation procedure

**begin**

    Read in flight information from data file, Edit if desired

    Generate delay arcs as desired, based on operational constraints

    Generate chronological event list of all potential aircraft movement activity at each station, including delay arcs

---

Generate ground arcs between consecutive “nodes” using sorted event lists

Build airline network of flight arcs, delay arcs, ground arcs and cycle arcs

Create specialized duplicate network for each aircraft in the fleet, based on that fleet’s operational capabilities and constraints.

**end**

**Figure 5-1** Network Generation Procedure

### 5.2.2 Maintenance Arcs

The presence of “maintenance arcs” in the Schedule Map provides the ability to model planned or unexpected maintenance checks within the resolution horizon, while determining feasible flight sequences to assign to a given operational aircraft. Each maintenance arc would be given an operating cost greater than zero, and a travel time of negative forty-hours (current industry average flying time between minimum planned maintenance “A” check). This represents the replenished flying time that would be available on the serviced aircraft until the next scheduled maintenance check. During the tree-searching procedure, a maintenance arc would only be considered if the aircraft required maintenance, as it would be more beneficial (profitable) for an aircraft to cover a “flight arc” or “delay arc” than to be assigned to the maintenance arc, provided it has the necessary flying time. As discussed in Chapter 4, the tree-searching algorithm is based on both time and operating profit (negative cost). The following paragraphs discuss each solution procedure developed, outlining the main phases of the solution process.

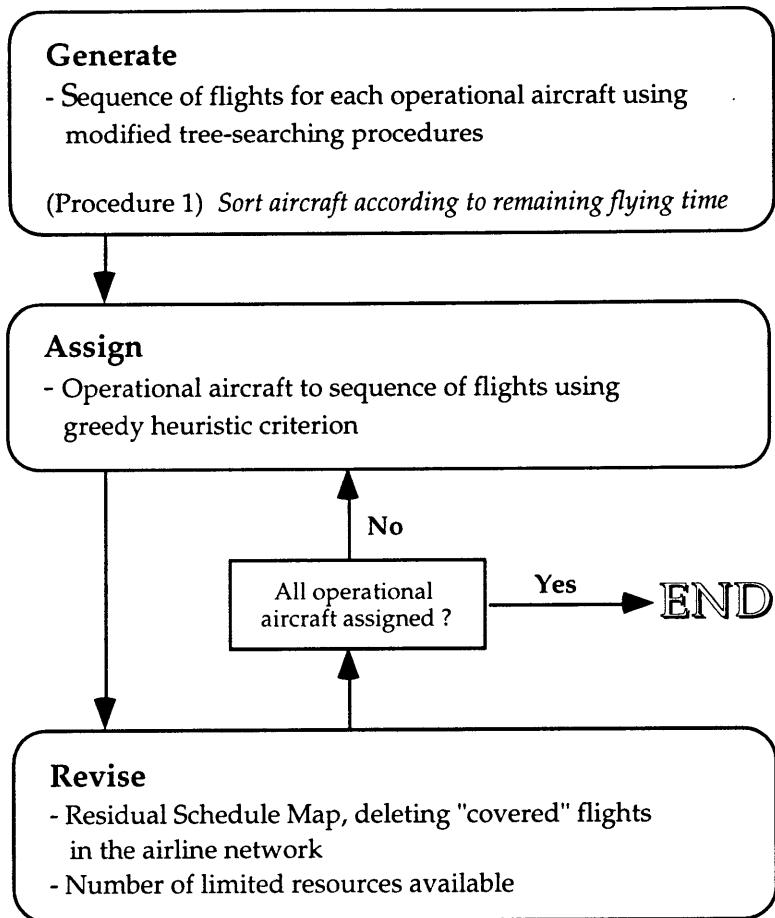
## 5.3 Greedy Heuristic Solution Procedures

The application of network based algorithms to solve the flight rescheduling problem is possible because of the underlying structure of the problem. As outlined in Chapter 3, the Schedule Map representing the airline’s flight network is acyclic and as such, the modified

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multiple criterion generalized permanent labelling algorithm for the constrained shortest path problem or the modified out-of-kilter minimum cost flow algorithm presented in Chapter 4, can be used effectively in the solution of the three dimensional assignment problem. In attempting to solve this complex problem in a real-time setting, a greedy heuristic methodology was initially developed. Subsequently, an alternative greedy heuristic procedure was developed from this initial method.

The overall functional flow diagram for each greedy heuristic procedure is shown in Figure 5-2. In the first case, the primary concern is to assign the most "maintenance critical" aircraft first, i.e., based on the amount of remaining flying time on the aircraft. In the second case, assign aircraft such as to maximize a prescribed decision criterion such as maximizing operating profits, including the costs of potential passenger spill. The decision criterion is defined as the primary operational objective that the airline controller will use in making any decisions regarding routing aircraft in the SM. The overall greedy heuristic methodologies are summarized in Figures 5-3 and 5-4.



**Figure 5-2** Functional Flow Diagram for Greedy Heuristic Solution Procedures

In the first phase of the procedure, the Schedule Map is developed based on a list of scheduled flights in the airline network. As discussed, the appropriate delay and ground arcs are automatically generated to complete the Schedule Map. Specialized Schedule Maps (SM) are then created for each operational aircraft, based on operational constraints such as range capabilities, over-water equipment requirements, and possibly noise restrictions. Flights that are not eligible for a given aircraft are “deleted” from the specialized SM, but may be covered by other aircraft in the fleet. The aircraft “structures” with specialized SM are then stored in a linked list, and if required are sorted based on a prescribed criterion such as remaining flight time.

In the second phase of the solution procedure, aircraft are systematically assigned to a sequence of flights, which has been determined using a modified tree-searching algorithm. A candidate sequence of flights is found that already satisfies the maintenance time restrictions, and is then assigned to a given aircraft so as to maximize operating profit. In the first greedy heuristic procedure, the most "maintenance critical" aircraft that has not been assigned to flights, is considered at each iteration of the solution procedure. During the execution of the second greedy heuristic procedure, all unassigned aircraft are considered at each iteration. The most profitable aircraft is then assigned to the sequence of flights.

In the final phase of the greedy heuristic procedure, the underlying Schedule Map is updated, removing all "covered" flights in the network, and adjusting the number of limited resources (such as crews, slots and gates) that have been used in the solution. The solution mechanism of the tree-searching algorithm is normalized and the procedure is repeated until all operational aircraft are assigned to a sequence of flights.

It is important to point out the role of the decision maker in implementing these solution methodologies as it is necessary for such a person to prescribe which objective is being used. As an example of a decision criterion, the primary objective of the problem could be to minimize the amount of wasted maintenance time left over on each aircraft at the end of the Resolution Horizon. In other cases, the airline controller who would serve as the decision maker might find it desirable to minimize the overall cost of resolving the flight irregularities over the prescribed time horizon.

**methodology      Greedy Heuristic Solution Procedure One**

**begin**

    Initialize parameters for tree-searching algorithms

    Input flight and aircraft data to the data structures

---

```
Create operational constraint decision criterion  
Create "specialized" Schedule Maps for each aircraft  
Sort aircraft based on remaining maintenance time available  
while any operational aircraft is not assigned to a flight sequence do  
begin  
    Determine candidate sequence of flights for most "critical" unassigned  
    aircraft which meets all operational constraints using modified tree-  
    searching algorithm.  
    Select aircraft assignment which maximizes the decision criterion  
    Delete "covered" flights from residual airline network  
    Update operational constraints information, e.g. gate utilization  
end  
end
```

**Figure 5-3** Greedy Heuristic Solution Procedure One

The assignment of operational aircraft to potential flights is restricted by several operational constraints as outlined in the mathematical formulation described in Chapter 3. These include conditions on the number of arriving flights at a given station within a given time period because of gate capacity, and landing slot availability. On the other hand, departing flights are constrained by availability of legal crew members to staff all operating flights. Once a decision has been made to assign an aircraft to a sequence of flights using the heuristic procedure, the number of resources available at each station has to be automatically updated. This is achieved by monitoring the flight assignment process, and keeping track of the resulting flight covering.

**methodology**     Greedy Heuristic Solution Procedure Two  
**begin**  
    Initialize parameters for tree-searching algorithms  
    Input flight and aircraft data to the data structures

---

```
Create operational constraint decision matrix  
Create "specialized" Schedule Maps for each aircraft  
while any operational aircraft is not assigned to a flight sequence do  
begin  
    Determine candidate sequence of flights which meet all operational constraints  
    for each unassigned operational aircraft in the fleet using modified tree-  
    searching algorithm  
    Select aircraft assignment which maximizes decision criterion  
    Delete "covered" flights from residual Schedule Map  
    Update operational constraints information, e.g. gate utilization  
end  
end
```

**Figure 5-4** Greedy Heuristic Solution Procedure Two

#### 5.4 Optimization-Based Solution Procedure

An alternative to the greedy heuristic procedure is a large-scale integer programming set-packing problem, which can be solved using the branch and bound procedure. Initially, a linear programming LP relaxation of the complex assignment problem is solved using the efficient implicit column generation solution methodology outlined in Chapter 4. The underlying structure of the problem allows the utilization of the constrained shortest path problem as the subproblem in the solution process, which is solved using the multi-labelling Clarke-GPL algorithm given in Chapter 4.

The output of each subproblem is a path (column) for addition to the Restricted Master Problem (RMP), provided it meets the necessary optimality conditions for inclusion. Each column contains information on the sequence of flights to be covered by an aircraft, and as well as information on the corresponding operational constraints within the problem, such

as landing slot utilization, gate utilization, and crew allocation. Figure 5-5 outlines the functional flow diagram for the optimization-based procedure.

The initial phase of this procedure is identical that of the greedy heuristic procedure. In the second phase of the optimization procedure, candidate flight sequences are generated for all operational aircraft in the fleet. These are transformed variables and are used in a large-scale set-packed problem. This problem is referred to as the "restricted master problem". Based on the solution of the initial RMP, dual variables (multipliers) are determined and used to update the structure of the underlying Schedule Map. An explicit column generation procedure then used to iteratively solve the restricted master problem, and the series of aircraft rerouting subproblems that are associated with the main problem. Each subproblem is solved using the specialized tree-searching algorithm. The column generation procedure is repeated until a pre-determined "sub-optimal" condition is satisfied. The final solution of this phase is then used as the root of a branch and bound method, to solve the airline schedule recovery ASRP problem. The overall solution procedure is summarized in Figure 5-6.

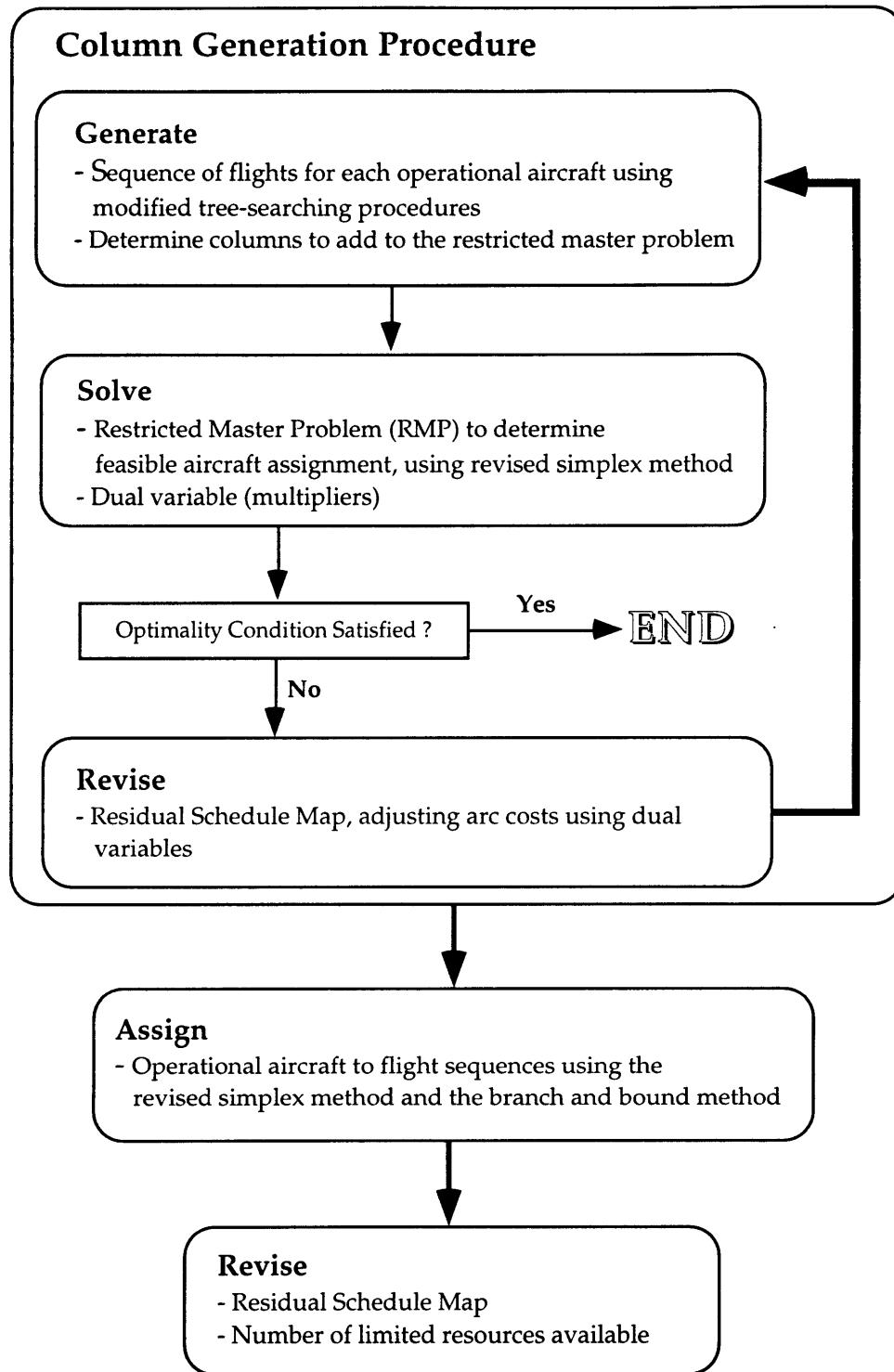


Figure 5-5 Functional Flow Diagram for the Optimization-Based Solution Procedure

**methodology** Integer Programming Optimization-Based Solution Procedure

**begin**

    Initialize parameters for tree-searching algorithms

    Input flight and aircraft data to the data structures

    Create “specialized” flight networks for each aircraft

    Solve initial restricted master problem to determine multipliers

**while** eligible columns exist for addition to the master problem **do**

**begin**

            Generate flight sequence for each aircraft fleet using modified tree-searching algorithm

            Determine “aircraft” column corresponding to each variable and add to the restricted master problem

            Using the revised simplex method, determine the aircraft-flight sequence assignments that will maximize decision criterion

            Using dual variables found in revised simplex procedure, adjust costs on each corresponding flight arcs in each specialized aircraft network

**end**

    Solve restricted master problem as an integer programming problem using the branch and bound solution procedure

    Determine final aircraft assignment based on output of the IP solution procedure

**end**

**Figure 5-6**     Optimization-Based Solution Procedure

#### 5.4.1 Column Generation Solution Procedure

During the column generation process, the dual variables (multipliers)  $w_{ij}$  are used to price out the non-basic variables (columns) by considering their reduced costs. The dual variables ensure that the reduced cost for every variable (path P) in the basis is zero. If any reduced cost is negative in a minimization problem, the method will introduce the corresponding non-basic variable into the basis in place of one of the current basic variables, and

---

recompute the simplex multipliers. In order to use column generation, the columns need to have structural characteristics which allows pricing out operations without explicitly considering every possible column in the problem.

The revised simplex procedure attempts to check if all reduced cost of variables are non-negative for optimality, such that:

$$\text{Min } \sum_{(ij) \in P} \{ C_{ij}^k + w_{ij} \} \geq \sigma^k$$

The left hand side of this expression is the length of the time constrained shortest path connecting the source and sink nodes of commodity k with respect to the modified costs  $c_{ij}^k + w_{ij}$ . If for all commodities k, the length of the constrained shortest path for that commodity is at least as large as its corresponding dual variable  $\sigma^k$ , the procedure will satisfy the complementary slackness conditions, and the solution will be optimal. Otherwise, based on the constrained shortest path on the modified network, the reduced cost of the column (path) is less than the length  $\sigma^k$  for a given commodity. By inserting this column into the basis, there will be an improvement to the objective function.

As a result, the changed basis will lead to new dual variables, and thus a modified shortest path distance  $\sigma^k$  between the source and sink nodes of the commodity k. At each iteration, the dual variables are found to ensure that the reduced cost of all basis columns is zero. Based on the new dual variables, the constrained shortest path problem would be resolved on the modified network, to determine whether any commodity path has a shorter length than its corresponding dual variable  $\sigma^k$ . If this occurs, the path is introduced into the problem basis, and the solution procedure will continue by alternatively finding new values for the dual variables for each arc constraint and for path length  $\sigma^k$ , and solving the

constrained shortest path problem for each commodity  $k$ . The process is thus repeated iteratively until the linear programming complementary conditions are satisfied.

#### 5.4.2 Column Generation Termination Mechanism

In order to effectively implement the column generation procedure in a real-time solution environment, the ability to prematurely stop the column generation phase can have a significant impact on the duration of the solution process. It is important for this mechanism to have a minimal effect on the quality of the LP relaxation solution of the problem, as this will be used as the lower bound for the integer programming branch and bound procedure. In reviewing the column generation procedure described in Chapter 4, one can identify several mechanisms which can be used to terminate the solution procedure, provided an *a priori* criterion is established within the solution module. For this research project, two such efficient stopping mechanisms were developed using concepts from linear programming theory; the first being the setting of a tolerance on the reduced cost optimality conditions (less than zero), and the second being a variation of the Lagrangian relaxation technique for the lower bound on the problem.

Based on Lagrangian relaxation theory, it is possible to establish both lower and upper bounds to the optimal solution of the resulting linear programming problem being solved by the column generation procedure, since this problem is equivalent to the LP problem that would exist during a Lagrangian relaxation solution procedure (Network Flows: Ahuja, 1993).  $Z^*$  is used to denote the optimal objective function value of the multi-commodity flow problem, and  $Z_{lp}$  to represent the optimal objective function value at any iteration in solving the path flow formulation of the problem by the revised simplex methodology. From linear programming theory,  $Z_{lp}$  corresponds to a feasible solution to the problem, such that  $Z^* \leq Z_{lp}$ . From Lagrangian relaxation theory, the optimal value  $L(w)$  of the Lagrangian

---

subproblem is a lower bound on  $Z^*$  for any value of the arc dual variables (prices)  $w$ . During the course of the column generation methodology developed to solve the ASRP problem, the solution of each modified constrained shortest path subproblem at each iteration corresponds to solving the Lagrangian subproblem with respect to the current arc prices  $w_{ij}$ .

The value of the Lagrangian subproblem can be expressed as:

$$L(w) = \sum_{k \in K} \{l^k(w)\} - \sum_{(ij) \in A} \{w_{ij} u_{ij}\}$$

where  $l^k(w)$  is the constrained shortest path length for all commodities  $k$  with respect to the modified costs  $c_{ij}k + w_{ij}$ , and  $u_{ij}$  is the upper bound on each arc. From the theory of Lagrangian relaxation;

$$L(w) \leq Z^* \leq Z_{lp}$$

For the purposes of this research project, the column generation stopping mechanism is derived from the static value of the lower and upper bound on  $Z^*$ . This stopping mechanism, later referred to as the "Lagrangian Gap", is defined as the percentage difference between the upper bound  $Z_{lp}$  and the lower bound value  $L(w)$ .

It is important to point out that this stopping mechanism is based on the lower bound of the objective function value which is determined as a by-product of finding the constrained shortest path distances  $l^k(w)$ , since the algorithm is pricing out columns during the course of the column generation procedure. Based on an apriori tolerance range, the solution procedure can be prematurely terminated to obtain a near optimal solution to the relaxed linear programming problem. The utilization of the revised simplex methodology guarantees that the objective value  $Z_{lp}$  of the LP problem (upper bound) is monotonically non-increasing

after each iteration of the algorithm. On the other hand, the value of the Lagrangian subproblem  $L(w)$  need not decrease at each iteration, and as such, the stopping mechanism would use the largest value of  $L(w)$  as the best lower bound.

#### 5.4.3 Branch and Bound Solution Procedure

After the successful completion of the column generation procedure, the resulting near optimal solution to the relaxed LP problem is then used as the root node to the branch and bound procedure for solving the original ASRP problem. As outlined in Chapter 3, this decision model has been formulated as an integer programming problem. The branch and bound solution procedure is based on the ability to use derived lower bounds to the optimal solution as an algorithmic tool in reducing the number of computations required to solve the problem to near optimality. This final phase of the solution methodology involves the solution of the integer programming problem which represents the combinatorial optimization nature of the complex reassignment problem.

During the branch and bound procedure, the feasible region  $F$  of the problem is systematically partitioned into subregions  $F_1, F_2, \dots, F_k$  (Network Flows: Ahuja, 1993). If  $\underline{X}$  denotes the best feasible objective function solution value after each iteration, either  $F_k$  is empty or  $X_k$  is a solution of a relaxation of the set  $F_k$  and  $C\underline{X} \leq C\underline{X}_k$  for each subregion  $k$ . If these conditions are satisfied, no point in any of the subregions can have a better objective function value than  $\underline{X}$ , and as such  $\underline{X}$  solves the original optimization problem. If  $C\underline{X} > C\underline{X}_k$  for any region  $F_k$ , it would be necessary to subdivide this region by “branching” on some of the variables (i.e. dividing a subregion into two by setting  $X_j = 0$  or  $X_j = 1$  for some variable  $j$  to define two new subregions in the original problem). The solution procedure would then continue until the necessary optimality conditions are met, and the optimal solution is determined.

The development and implementation of an efficient branch and bound procedure can be greatly influenced by many solution parameters including the branching strategy (order for choosing the subregions), the variable selection criterion for branching, the node selection in the branch and bound tree, an apriori objective solution optimality gap, the pricing algorithm, and the underlying solution algorithms. Each solution parameter listed above can have a significant impact on the quality of the final solution, as well as the solution time necessary for a particular problem. In the next Chapter, there is a discussion of a series of real-world case studies, using operational data from a major US domestic carrier and an international carrier to validate the solution procedures and algorithms developed. Trade-off comparisons are made for each solution parameter in order to establish the most efficient branch and bound solution procedure, based on the commercial optimization package CPLEX.

# **Chapter 6**

## **Case Study Analysis**

### **6.1 Introduction**

The ability to reassign operational aircraft to flights in the residual Schedule Map is influenced by many factors as outlined in the previous chapters. As part of the final phases of this research project, operational data from a major US domestic carrier, and data from an international carrier were used to validate and test the algorithms and solution procedures developed during the course of the research. Several parameters and important issues were considered including the effect of the size of the Schedule Map on the solution time of each algorithm. In particular, the case study considered the effects of number of operational constraints, the number and duration of delay arcs, and passenger recapture rate on the quality of the solution, flight coverage and the overall solution time of each algorithm. Flight coverage is defined as the number of scheduled flights which are delayed or cancelled in the final solution.

#### **6.1.1 Description of the Datasets**

The primary goal of this research project has been to develop solution procedures for flight rescheduling in a real-time environment. As such, operational data from two different carriers were studied in order to validate the algorithm, and attempt to establish a better understanding of this highly complex problem. The following paragraphs outline each airline's operations as used in the case studies.

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- Garuda Indonesia (GA)

Garuda Indonesia is the national carrier of the Republic of Indonesia, a country which consists of an archipelago of over 13,000 islands. It currently serves both an extensive domestic and international flight network, spanning four continents. In this study, only the domestic network is considered, consisting of fifteen airport stations, scattered across the country. Garuda's operations are centered around the country's capital city Jakarta, which is served by the international airport at Cengkareng (CGK). The airport in Denpasar, Bali (DPS) plays a major role as a second hub in the airline's operations. The carrier's domestic fleet is made up of four different types, totalling 35 aircraft. These include the 737-300, 737-400, A300-B4 and the A300-600R. Based on information from the carrier's published timetable, a Schedule Map of 180 flights is used in the study.

- Northwest Airlines (NW)

Northwest Airlines NW is the fifth largest major carrier in the US domestic network, with a fleet of over 475 aircraft. Its domestic network is based on the hub and spoke concept, with over 98% of scheduled flights either arriving or departing from a hub airport. The carrier operates three main hub airports at Detroit (DTW), Minneapolis (MSP), Memphis (MEM), with satellite hubs at Boston (BOS) and Tampa (TPA). The domestic network consists of 37 stations, served by 1591 scheduled flights per day. Northwest's domestic fleet consists of five aircraft types, namely the A320-200, 757-251, DC10-30, 727-200 and the DC9/M80 family. In this case study, a subset of the carrier's domestic network is considered due to memory limitations on available computer facilities at the time of the study. The final NW problem considered involved a network of 612 flights, and a fleet of four aircraft types (all except the DC9/M80 family).

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Table 6-1 summarizes each case study problem addressed, based on the operational data provided by the two carriers. Problem one corresponds to daily operations of Garuda's domestic network, and Problems two through five are derived from the US domestic operations of Northwest Airlines. Several important aspects are captured in these studies including the ability to consider multiple fleet type swapping in attempting to resolve irregularities.

Table 6-1      Summary of Operational Case Studies

Problem	Aircraft Types	Aircraft	Flights	Stations
1	4	35	180	15
2	1	49	201	37
3	1	50	192	37
4	2	99	393	37
5	4	177	612	37

### 6.1.2 Review of Actual Airline Operations

In order to compare actual operational data to results generated by each algorithm, data on aircraft operating costs and average passenger fares for each origin-destination pair were used to establish benchmarks for each study. Passenger fare data were determined using revenue data from the airlines, and on-board revenue data from the O/D Plus database. Operating costs were determined using published industry averages by aircraft type. From this data, representative operating profit values were determined for each flight segment, ignoring any network or connectivity effects on operating revenue. In addition, these figures assume 100% passenger recapture, that is, all passengers from cancelled or delayed flights are reaccommodated by flights flown by the carrier. In effect, this estimation ignores loss of

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passengers to other carriers, and any effects that actual flight delays may have on passenger levels for a given flight segment. In later paragraphs, distribution of actual delay times will be addressed, based on operational data collected by the carriers.

**Table 6-2** Summary of Estimated Operating Profit based on Actual Operating Data  
(Daily Normal Operations)

Problem	Flights Scheduled	Flights Flown	Percent Cancelled	Operating Profit (\$)
1	180	174	4.92	619,885
2	201	196	3.45	2,674,739
3	192	189	1.56	2,148,606
4	393	385	2.28	4,823,345
5	612	590	3.75	7,013,333

**Table 6-3** Summary of Estimated Operating Profit based on Actual Operating Data  
(Daily Irregular Operations)

Problem	Flights Scheduled	Flights Flown	Percent Cancelled	Operating Profit (\$)
1	180	n/a		
2	201	182	10.34	2,515,657
3	192	183	4.69	2,097,174
4	393	365	7.36	4,612,831
5	612	560	8.65	6,791,656

Table 6-2 and Table 6-3 summarizes operating profit for each case study, based on actual traffic levels as reported by the airlines in the study. In the first scenario, the airline's operations are not subject to any major disruptions and represents the "normal" operations of the carrier. In the second scenario, the carrier's operations are affected by various irregularities during the course of the day. It was not possible based on the format of the database to explicitly identify the nature of the irregularities in the study. As a result, the

impact of the irregularities on the operations of the carrier was modelled by restricting the number of the arrivals and departures within a given time period, (fifteen minute interval) based on the the actual levels of aircraft movement on the "irregular" day. The data presented for Problems 2 through 5 under "normal" conditions corresponds to the daily operations for NW on January 13, 1997. The "irregular" scenario corresponds to NW's operations on January 9, 1997. These two distinct days of operations were identified by the carrier for the purpose of the case study.

For each case, the estimated operating profit was calculated using Expression 6-1 outlined below. In forthcoming studies of the algorithms, this formula is used to determine the cumulative objective function value for each aircraft assignment, that is, the estimated value of assigning a given aircraft to a predetermined sequence of flights in the airline network.

#### Expression 6-1

$$\text{Operating Profit} = (\text{AHOC} * \text{BT}) - (\text{FARE} * \text{PAX})$$

where

AHOC	average hourly operating cost
BT	average flight block time
FARE	average passenger fare
PAX	actual number of passengers on leg

The ability to assess the impact of delay on passenger spill is a difficult task, and is a topic worth addressing in future research projects. Previous work on this topic has been reported by Mathaisel [8]. In this study, sensitivity analysis of both the delay duration and passenger recapture rate were done, in order to determine their importance in the mechanism of the solution procedures. As a preamble to these empirical studies, Table 6-4 summarizes the distribution of actual flight delays in each case study, based on the reported aircraft movement times. From these figures, candidate delay times were established for use in the case studies.

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**Table 6-4** Summary of Delay Time Distribution (Percentages)

Delay time (min)	None	0 - 14	15 - 29	30 - 44	45 - 59	60 - 119	120 - 179	> 180	Cancel
<b>Problem 2</b>									
Normal	52.00	35.00	6.31	2.91	0.00	2.91	0.49	0.00	0.97
Irregular	21.49	26.86	14.88	5.79	4.55	12.81	5.37	1.65	6.20
<b>Problem 3</b>									
Normal	55.28	29.15	7.04	3.52	0.50	2.51	0.00	0.50	1.51
Irregular	25.52	24.69	12.97	11.30	3.77	13.39	3.35	1.26	3.77
<b>Problem 4</b>									
Normal	53.61	31.85	6.67	3.20	0.25	2.72	0.25	0.25	1.24
Irregular	23.49	25.99	13.93	8.52	4.15	13.10	4.37	1.46	4.99
<b>Problem 5</b>									
Normal	51.88	30.40	6.74	3.13	0.31	3.60	0.63	0.16	3.13
Irregular	21.48	25.82	13.57	9.09	5.01	12.25	4.74	1.19	6.85

## 6.2 Simulation of Irregular Airline Operations

It was not possible from the existing format of the database of actual operational data to explicitly identify discrete irregularities. As a result, it was not possible to recreate the exact impact of these irregularities on the planned schedule of the airline in the case study. In an effort to conduct a proof-of-concept of the developed solution methodologies and algorithms, an attempt was made to simulate a series of potential irregularities over the course of the resolution horizon. Based on actual aircraft movement data from the operational database, the reduced number of arrivals and departures at each hub airport

(subject to a series of irregularities) in the network were determined for prescribed time intervals of fifteen minutes over the course of the day.

This information was then used to restrict aircraft movement in the network, thereby simulating the “end-effect” of the multiple irregularities, ignoring the actual cause of each event. Several operational scenarios were considered in the study, based on the level of restrictions, or more appropriately, the number of auxiliary operational constraints incorporated in the decision model. These are summarized below:

1. No auxiliary constraints are considered in the solution methodology
2. Landing slots constraints are considered by restricting the number of arrivals at each affected station within a given time period.
3. Crew constraints are considered by restricting the number of departures at a given station, within a given time period.
4. Both landing slots and crew constraints are incorporated into the decision model, by restricting all aircraft movement at a given station within a time period.

### **6.3     Important Issues and Assumptions**

In order to compare the results of each algorithm to the existing operational data, several parameters were varied in order to access the quality of the solution relative to the actual data. These included:

- Number of aircraft and flights in the airline network
- Number of operational constraints incorporated into the decision model
- Number of delay arcs, and the duration of the delay per flight
- Passenger recapture rate
- Minimum aircraft turn time

The quality of each solution was measured by the resulting operating profit, percentage of flights delayed, and the percentage of flights cancelled for each scenario. In addition, the overall solution time for each algorithm was recorded, in order to establish the applicability of these solution procedures for real-time decision making.

As outlined above, the current solution methodologies ignore the effects of passenger flow and connectivity issues in determining passenger revenue, thereby taking a segment-based approach. In the passenger flow sub-model presented in Chapter 3, such issues would be considered, and any relevant information could be incorporated in the main model through additional constraints on the aircraft movement. For example, an additional constraint could be used to ensure that a particular origin-destination market is serviced by at least one flight within a given time period. In each algorithm, it is assumed that each flight can be flown by any aircraft in the fleet, provided it satisfies a prescribed criterion such as a passenger “no-spill” condition or operational range capability. A minimum turn time of 30 minutes is allocated to all scheduled flights.

For the purposes of the case studies, it was assumed that each operational aircraft had twenty-five “flying” hours remaining, since it was not possible to ascertain these actual values from the available historical data. In addition, a maximum daily aircraft utilization of twelve hours was preset for the implementation of the tree-searching algorithm. It is assumed that all hub airports in the airline network are capable of serving as maintenance bases for all aircraft types in the fleet. The resolution horizon  $H$  was set at twenty-four (24) hours. In current airline scheduling planning, a planned aircraft rotation (equivalent to a flight sequence beginning and terminating at a maintenance base) is typically 72 hours in duration.

#### **6.4 Practical Decision Model**

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In order to utilize the ASRP model presented in Chapter 3, it is necessary to adapt the mathematical model to real-world problems by relaxing some of the operational constraints in the formulation. The overall framework of the solution procedure incorporates several factors in the main problem of rescheduling flights in the aftermath of irregularities. The primary constraint satisfied is the aircraft maintenance routing constraint. In the simplified model used in the case study, all operational constraints are included in the model except for the overnight aircraft balance constraint. In addition, the crew balance constraint is relaxed to restrict the number of aircraft departures across the entire fleet, thereby assuming crew commonality within the fleet. It is important to point out however, that the aircraft type specific constraint could be easily incorporated into the model, but it would have a marginal impact on the size of the network studied, and potentially the resulting solution time. In addition, crew legality issues would highly complicate such constraints in the rerouting problem.

#### **6.4.1 LP Lagrangian Gap for Column Generation**

As outlined in the previous Chapter, an optimization based algorithm has been developed for solving the airline recovery problem which employs an implicit column generation procedure. The ability to use this algorithm to solve real-world problems made it necessary to determine an appropriate LP Lagrangian gap, in order to achieve a practical solution quality. Based on preliminary analysis, a Lagrangian gap of 0.005 was determined as the candidate value to satisfy this criterion, while maintaining a real-time solution capability. Table 6-5 summarizes the effects of varying the Lagrangian gap on the solution time of the column generation portion of the optimization algorithm. These figures are based on Problem 2, consisting of 49 aircraft, 201 scheduled flights, and no replica delay arcs in the network. The solution times are reported in seconds for each scenario (specific parameter settings, and/or number of operational constraints), with runs on a Sun Sparc20

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workstation using the CPLEX callable library. An important observation of the results of the column generation procedure was the high level of integrality which existed in the solution to the linear relaxed problem studied. This resulted in relatively short branch and bound solution times for each scenario.

**Table 6-5** Effects of Lagrangian Gap on Solution Time (Secs) of Column Generation

Scenario	Types of constraints	0.0500	0.0375	0.0250	0.0125	0.0050	0.0005
1	flight, aircraft	88.00	88.82	88.84	91.83	100.68	114.08
2	flight, aircraft, landing slots	272.25	287.02	302.42	301.75	302.36	302.36
3	flight, aircraft, crew	274.16	286.33	286.58	286.58	286.58	286.58
4	flight, aircraft, slots, crew	495.90	495.90	545.24	545.24	567.96	567.96

#### 6.4.2 Integer Programming Solution Procedure

Based on preliminary analysis of the Schedule Map and resulting integer programming problem input to CPLEX, a test matrix was established to determine the appropriate settings for the CPLEX optimization module. In effect, an empirical study was conducted to determine the best IP solution procedure for the airline recovery problem. Based on run times, the following parameter settings were used for the mixed integer programming module of the CPLEX callable optimization library.

**Table 6-6** CPLEX Settings for Optimization-Based Algorithm

CPLEX Parameter	Setting
Start Algorithm	Primal Simplex
Sub Algorithm	Dual Simplex
Start pricing algorithm	Devex pricing
Sub pricing algorithm	Steepest edge/Automatic
Integrality heuristic	YES

Node selection	Best bound search
Variable Selection	Branch automatically selected
Branch strategy	Algorithm decides

The reader is referred to the CPLEX manual for a more detailed discussion of these parameters. Using these parameter settings, an extensive sensitivity analysis was conducted using Problem 2's dataset to determine the most efficient IP optimality gap setting for implementing optimization-based algorithm in a real-time environment. Table 6-7 summarizes the variation in solution quality and run time relative to the optimality gap. From this empirical study, an optimality gap of 0.005 was set for terminating the optimization algorithm.

**Table 6-7** Effects of IP Optimality Gap on Solution Quality and Algorithm Run Time (secs)

Scenario	Types of constraints	Factor	0.0500	0.0375	0.0250	0.0125	0.0050	0.0005
1	flight, aircraft	obj	2733476	2733476	2733476	2733476	2733476	2733476
		time	108.00	105.87	107.38	106.28	106.54	295.29
2	flight, aircraft,	obj	1497023	1497023	1497023	1497023	1497023	1497023
		slots	293.54	293.54	293.54	293.54	293.54	309.75
3	flight, aircraft,	obj	1873689	1873689	1873689	1873689	1873689	1873689
		crew	287.42	287.42	287.42	287.42	287.47	303.35

4	flight, aircraft,	obj	1175347	1175347	1175347	1175347	1175347	1175347
	slots, crew	time	573.10	573.10	573.10	573.10	573.10	573.10

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## 6.5 Review of Primary Findings

The following experimental results were obtained using the algorithms developed during the course of the research project. All experimental results reported in this section are based on computational runs conducted on a SunSparc 20 workstation. The underlying concepts of each solution methodology were discussed in Chapter 5. A summary of this discussion is now given, prior to presentation of the empirical results.

*Algorithm 1* corresponds to greedy heuristic procedure one, in which each aircraft is considered individually based on the amount of remaining flight time before scheduled maintenance. Each aircraft is assigned to a sequence of flights based on operating profit.

*Algorithm 2* corresponds to greedy heuristic procedure two, and attempts to establish a local optima at each phase of the solution process. Each aircraft is assigned to a sequence of flights based on operating profit.

*Algorithm 3* corresponds to the optimization-based solution procedure, wherein column generation is used to generate candidate flight sequences that meet aircraft maintenance conditions, while attempting to maximize operating profit.

In order to implement these algorithms, several assumptions were made regarding the underlying airline network and corresponding Schedule Map discussed in Chapter 3. The resulting problem parameters are now listed, as a preamble to the solution results.

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Additionally, a series of sensitivity analyses were conducted to assess the impact of varying such parameters on the quality of the solution and the corresponding algorithm run time. These results are reported in the next chapter.

### *Assumptions*

- Minimum aircraft turn time      30 minutes
- Passenger recapture rate      0.750
- Number of delay arcs      1
- Duration of delay      30 minutes
- Lagrangian gap      0.005
- IP optimality gap      0.005
- Problem size      49 aircraft, 201 scheduled flights

#### **6.5.1 Actual Airline Operations**

The following tables summarize operating parameters (characteristics) of actual airline data, and an assessment of the “operational schedule” generated by each algorithm during the simulation phase of the case study. Table 6-8 shows the operating results based actual airline data, using the Problem 2 dataset (49 aircraft, 201 scheduled flights, one aircraft type). This dataset was also used for the irregularity simulation study.

**Table 6-8**    Actual Airline Operations (49 aircraft, 201 scheduled flights)

Parameter	Normal	Irregular
Operating Profit (\$)	2674739	2515657
ASM (seat-miles)	40384752	39119384
RPM (pax-miles)	23910632	23542646
ALF	0.59	0.60
Variable Unit Cost (\$/mile)	0.032	0.032
Yield (\$/mile)	0.165	0.159
Aircraft Utilization (hrs)	10.78	10.44
Avg. Block Time (hrs)	2.69	2.81

Flight Delay (%)	48.00	78.50
Flight Cancelled (%)	3.45	10.34

Each parameter was used in an effort to accurately quantify each algorithm based on the airline industry's standard measures. These are now defined as a precursor to reviewing the data. The available seat miles "ASM" represents the available capacity in the airline network, based on the residual schedule map composition. The revenue passenger miles "RPM" is a measure of the total operating revenue achieved in operating the scheduled flights. The average load factor "ALF" is a measure of the percentage of seats occupied on each flight segment. The average aircraft utilization measures the number of flight hours flown by a given aircraft over the course of a day. The average block time represents the mean duration of a flight in the airline network. The average yield is the amount of operating revenue generated by carrying one passenger, one mile in the airline network. The variable unit cost is a measure of the additional variable costs required to carry one seat, one mile. The flight coverage in the network is indicated by the percentage of flights delayed and cancelled.

### 6.5.2 Simulation of Irregular Airline Operations

Table 6-9 through Table 6-12 outline the resulting operating values for each scenario described for the "irregularity" simulation. It can be observed from these results that each algorithm is capable of generating a schedule of flights that are comparable to the actual operations.

**Table 6-9**      Scenario 1      *No auxiliary operational constraints*

Parameter	Algorithm 1	Algorithm 2	Algorithm 3
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Operating Profit (\$)	2647527	2590156	2752362
ASM (seat-miles)	31545264	31236910	35253572
RPM (pax-miles)	20590180	19753926	22660084
ALF	0.65	0.63	0.64
Variable Unit Cost (\$/mile)	0.032	0.031	0.032
Yield (\$/mile)	0.177	0.180	0.171
Aircraft Utilization (hrs)	8.43	8.22	9.42
Avg. Block Time (hrs)	2.52	2.55	2.61
Flight Delay (%)	0.49	0.00	0.00
Flight Cancelled (%)	19.21	22.00	12.81

In the case of Scenario 1 (equivalent to normal operations), each algorithm creates a schedule that is equivalent, if not better than the actual airline schedule. In considering each parameter, one can observe that each schedule of flights generated by an algorithm is operationally practical, and beneficial to the carrier.

**Table 6-10**   Scenario 2   *Constraints on aircraft arrivals*

Parameter	Algorithm 1	Algorithm 2	Algorithm 3
Operating Profit (\$)	2156899	2319814	2520176
ASM (seat-miles)	29636202	29921202	31277682
RPM (pax-miles)	17821380	17913154	20467372
ALF	0.60	0.60	0.65
Variable Unit Cost (\$/mile)	0.032	0.031	0.032
Yield (\$/mile)	0.174	0.182	0.171
Aircraft Utilization (hrs)	7.92	7.87	8.35
Avg. Block Time (hrs)	2.55	2.47	2.59
Flight Delay (%)	26.60	20.00	0.50
Flight Cancelled (%)	25.12	23.00	22.17

**Table 6-11**   Scenario 3   *Constraints on aircraft departures*

Parameter	Algorithm 1	Algorithm 2	Algorithm 3
Operating Profit (\$)	2334368	2310077	2628115
ASM (seat-miles)	29354720	30448688	32088630
RPM (pax-miles)	18566216	18715436	21072076
ALF	0.63	0.61	0.66
Variable Unit Cost (\$/mile)	0.032	0.031	0.032
Yield (\$/mile)	0.176	0.174	0.173
Aircraft Utilization (hrs)	7.85	8.01	8.57
Avg. Block Time (hrs)	2.55	2.55	2.56
Flight Delay (%)	17.24	21.00	0.50
Flight Cancelled (%)	25.62	24.00	19.21

**Table 6-12 Scenario 4** *Constraints on all aircraft movement*

Parameter	Algorithm 1	Algorithm 2	Algorithm 3
Operating Profit (\$)	1955652	2059552	2393016
ASM (seat-miles)	27896194	28203332	29441276
RPM (pax-miles)	16372050	16765522	19377704
ALF	0.59	0.59	0.66
Variable Unit Cost (\$/mile)	0.032	0.031	0.032
Yield (\$/mile)	0.173	0.175	0.172
Aircraft Utilization (hrs)	7.46	7.42	7.86
Avg. Block Time (hrs)	2.54	2.47	2.53
Flight Delay (%)	33.50	34.00	0.50
Flight Cancelled (%)	29.06	27.00	25.12

The reader is referred to the appendices for sample output data files of Scenario 1 using Algorithm 1, and the actual aircraft rotations for the normal day of operations. In addition, the data input files containing the scheduled flights and aircraft, can be found in the appendices.

## 6.6 Summary and Conclusions

The main objective function considered in this study is based on an operating profit expression which only accounts for the variable operating costs, and the average passenger fare when determining the “value” of a given flight segment. In addition, since cancellation costs are not explicitly accounted for the current model, the number of cancelled flights are artificially inflated. As a result, aircraft utilization in the schedules generated by each algorithm for normal operating conditions (no operational constraints on aircraft movement) is slightly less (within 85%) than that of the actual airline operations. These artificially reduced aircraft utilization figures result in lower available seat miles for each algorithm, and associated revenue passenger miles. The average passenger yield achieved by each algorithm is better than the actual operations, as “less-beneficial”(small profit/loss margin) flights would be not flown, as there are no penalties to cancel these flights in the current implementation.

The results of the simulation have shown that it is possible to efficiently reschedule flights in the aftermath of irregularities. In each scenario considered, the value of majority of the operating parameters monitored is within the same order of magnitude as the baseline case of normal operating conditions (actual operations). For example, the average aircraft utilization for each scenario under an irregular operating condition is within 95% of that of normal operating conditions. Similarly, the average flight block time achieved in each scenario under irregularities is within 99% of the norm. The simulation of the irregular operations has successfully demonstrated a proof-of-concept, since the applicability of these algorithms to reschedule flights has clearly been shown from the operational results of this study. In the next chapter, a comprehensive sensitivity analysis study is discussed, in which the major modelling parameters identified in this chapter are varied, and their impact of algorithm run-time, solution quality, and flight coverage assessed.

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# **Chapter 7**

## **Sensitivity Analysis**

### **7.1 Solution Time and Quality of the Solution**

As outlined in the introduction to the previous chapter, several issues were considered during the course of the case study analysis, with a primary emphasis on the quality of the solution (profitability) and corresponding algorithm run time. The following tables summarize the major findings of the sensitivity analysis, by considering each issue individually. In each scenario, “obj” corresponds to the value of the objective function as defined in Chapter 5, and “time” corresponds to the CPU run time in seconds on a SunSparc20 workstation for each algorithm.

#### **7.1.1 Number of aircraft flights**

From the onset of the research project, it was anticipated that one of the most important factors to establish during the course of the validation phase of the project, was the functional limitation of the algorithms developed. As such, the first issue to be addressed in the case study analysis was the impact of problem size on the overall solution time of each algorithm. In each case, all additional operational constraints were excluded from the study. Table 7-1 outlines the run times in seconds for each case study problem dataset, based on their descriptions in Chapter 6. By varying the dimension of the underlying airline network, it is possible to assess the impact of problem size on the algorithm run-time.

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**Table 7-1** Summary of Effects of Problem Size on Solution Time (secs)

Problem	Aircraft	Flights	Factor	Algorithm 1	Algorithm 2	Algorithm 3
1	35	180	obj	716941	755835	784199
			time	2.70	35.49	234.40
2	49	201	obj	2647527	2603870	2734698
			time	2.29	27.84	105.72
3	50	192	obj	2092465	2104083	2141805
			time	2.71	22.73	51.69
4	99	393	obj	4713562	4811564	4943535
			time	9.65	277.11	707.04
5	177	612	obj	n/a	n/a	n/a
			time			

One of the primary observations from this experiment was the strong correlation between the problem dimensions and the overall solution run time. It was also apparent from these results, that the performance of each algorithm is affected by the actual composition of the underlying airline network. In a later sensitivity study, the impact of the duration of the minimum aircraft turn time is considered, in terms of its effects on the solution quality, flight coverage and algorithm run time. During the course of these computer runs, the issue of CPU memory and processing speed surfaced as major factors which would limit the actual case study problem used for the remainder of the validation phase of the research project. As a result, in order to complete the planned comparison study of all three algorithms, the problem size was limited to satisfy the memory capacity of the workstation. In subsequent studies, the dataset for each scenario corresponds to Problem 2 (49 aircraft, 201 flights).

### 7.1.2 Number of Additional Constraints

In this study, it is assumed that there are no delay arcs in the network, and as such, the algorithm results would report which flights to cancel in the event that flight delays are not considered in the decision process. Table 7-2 summarizes the variation in solution run time and solution quality, based on varying the number of constraints considered. An important observation in this study was the impact of the integrality requirement in Algorithm 3 on the quality of the solution for problems subject to additional operational constraints. The solution procedure initially solves the ASRP problem as a relaxed linear programming problem, and then transforms the result to an IP solution. As a result, the final solution of the algorithm is highly impacted by the number of constraints, which may result in higher instances of fractionality in the initial LP solution. In determining the IP solution, the quality of the solution is thus sub-optimal. It is apparent from the case study, that this issue will depend on the underlying structure of the Schedule Map being considered.

**Table 7-2** Effects of Additional Constraints on Solution Quality / Run Time (secs)

Scenario	Types of constraints	Factor	Algorithm 1	Algorithm 2	Algorithm 3
1	flight, aircraft	obj	2647527	2603870	2734698
		time	2.29	27.84	122.80
2	flight, aircraft, landing slots	obj	1464448	1777935	1497023
		time	4.12	28.56	293.54
3	flight, aircraft, crew	obj	1948904	1942234	1873689
		time	3.72	22.40	288.44
4	flight, aircraft, slots, crew	obj	1295528	1457813	1175347
		time	5.40	24.73	559.92

### 7.1.3 Number of Delay Arcs and Delay Time

The ability to efficiently reschedule flights in the aftermath of irregularities can be greatly influenced by the capability to accurately make a trade off between cancelling and delaying

a given flight in the network. In this study, the issue of the impact of delay arcs is considered in two separate scenarios. In the first case, there are no additional operational constraints considered during the decision process. In the later case, constraints on aircraft movement are imposed based on actual operational data from a particular “irregular” day which affected the operations of the airline.

**Table 7-3** Effects of Delay Arcs on Solution Quality and Algorithm Run Time [secs]  
(no additional constraints)

Scenario	Number of Delay Arc	Delay (mins)	Factor	Algorithm 1	Algorithm 2	Algorithm 3
1	1	15	obj	2638203	2590156	2750408
			time	8.71	131.29	550.60 e
2	1	30	obj	2647527	2590156	2752362
			time	8.45	120.79	511.15 e
3	1	45	obj	2647853	2607270	2736194
			time	8.34	115.85	384.15 e
4	1	1/2 flt time	obj	2652539	2604561	2734616
			time	8.71	114.78	424.65 e
5	2	15	obj	2638203	2590156	n/a
			time	16.78	281.30	
6	2	30	obj	2657066	2600518	n/a
			time	16.10	259.01	
7	2	45	obj	2628835	2587663	n/a
			time	14.86	n/a	
8	2	1/2 flt time	obj	2634016	2587077	n/a
			time	14.97	n/a	

Table 7-3 reports the solution times and the quality of the solution for each algorithm, for the case where there are no additional operational constraints. In contrast, Table 7-4 contains the figures for the real-world case, with constraints on aircraft movement based as a result of reduced landing slots and available flight crews.

**Table 7-4** Effects of Delay Arcs on Solution Quality and Algorithm Run Time [secs]  
(additional constraints on crew and landing slots )

Scenario	Number of Delay Arc	Delay (mins)	Factor	Algorithm 1	Algorithm 2	Algorithm 3
1	1	15	obj	1851038	1915756	1801499
			time	15.47	82.08	1782.40 e
2	1	30	obj	1955652	2059552	1967399
			time	15.66	89.02	2113.55 e
3	1	45	obj	1974784	2117415	2023848
			time	16.12	92.43	2124.55 e
4	1	1/2 flt time	obj	1949323	1994718	1954047
			time	15.71	73.48	2078.21 e
5	2	15	obj	1984528	2016319	n/a
			time	27.85	n/a	
6	2	30	obj	2014968	2091298	n/a
			time	28.42	n/a	
7	2	45	obj	2013300	2108169	n/a
			time	28.65	n/a	
8	2	1/2 flt time	obj	1939778	2063627	n/a
			time	28.23	n/a	

It was observed from these experiments that the addition of delay arcs to the time-space network had a significant impact on the solution time. For example, the addition of one delay arc for each flight causes a 300% increase in run-time for Algorithm 1. In addition, as the size of the problem increased, corresponding to the number of delay arcs considered in the problem, it was not possible to solve the candidate case study problem on the computer platform due solely to CPU memory limitations. However, it was possible to determine the actual solution to these scenarios on a more powerful machine, with adequate memory capacity but a slower processor time. The run times corresponding to these instances are reported as estimated values, indicated by an “e”.

#### 7.1.4 Passenger Recapture Rate

In developing algorithms for use in any decision support systems, it is important to establish a thorough understanding of all the underlying factors which may affect the quality of the solution generated by the system. One of the fundamental issues that affects airline operations is that of passenger recapture, and how this is incorporated into any fleeting decisions.

**Table 7-5** Effects of Passenger Recapture Rate on Solution Quality  
and Algorithm Run Time [secs]  
(additional constraints on crew and landing slots )

Scenario	Passenger Recapture Rate	Factor	Algorithm 1	Algorithm 2	Algorithm 3
1	0.500	obj	1619350	1719544	1557409
		time	15.66	73.52	2093.41 e
2	0.625	obj	1778868	1888817	1763671
		time	15.67	79.84	2385.83 e
3	0.750	obj	1955652	2059552	1967399
		time	16.00	78.29	2113.55 e
4	0.875	obj	2283686	2243996	2187794
		time	15.75	83.17	2271.71 e
5	1.000	obj	2561004	2498131	2412189
		time	16.02	89.72	2145.40 e

In this study, a sensitivity analysis is conducted in which the passenger recapture rate is varied, and its effects on profitability and algorithm run times are observed. Table 7-5 summarizes the results of the sensitivity study, in which a delay time of 30 minutes is assumed. It is apparent from this experiment, that the actual value of the recapture rate does not affect the solution time of the algorithm for a given duration of delay. On the other hand, the profitability of the solution is significantly affected by this parameter in the decision process.

### 7.1.5 Minimum Aircraft Turn Time

Based on the observations made during the initial phases of the case study, a sensitivity analysis was conducted on the minimum aircraft turn time assumed for the study. In this study, it is assumed that there is one delay arc for each scheduled flight, with a

**Table 7-6** Effects of Minimum Aircraft Turn Time on Solution Quality and Algorithm Run Time [secs]  
(no additional operational constraints)

Scenario	Minimum Aircraft Turn Time (min)	Factor	Algorithm 1	Algorithm 2
1	30	obj	2647527	2590156
		time	9.11	125.35
2	45	obj	2617542	2566682
		time	8.87	91.73
3	60	obj	2382182	2387878
		time	8.06	105.75
4	$\max(30, 1/4 \text{ flt time})$	obj	2606652	2549006
		time	8.93	137.35
5	$\max(30, 1/2 \text{ flt time})$	obj	2362666	2351759
		time	8.39	88.52

corresponding delay time of 30 minutes. Table 7-6 outlines the effects of the prescribed minimum aircraft turn time on the solution quality for the case with no additional constraints. Table 7-7 presents the results for the case in which additional constraints are incorporated into the decision process. Due to the computer memory limitations, figures for algorithm 3 are not reported. From these empirical tests, it was apparent that the assumed minimum aircraft turn time can have a significant impact of the solution quality and the level of flight coverage in the underlying airline network.

**Table 7-7** Effects of Minimum Aircraft Turn Time on Solution Quality and Algorithm Run Time [secs]

(additional constraints on crew and landing slots )

Scenario	Minimum Aircraft Turn Time (min)	Factor	Algorithm 1	Algorithm 2
1	30	obj	1955652	2059552
		time	15.82	110.65
2	45	obj	2056484	2041396
		time	15.63	118.93
3	60	obj	1881298	1874251
		time	16.09	115.55
4	$\max(30, 1/4 \text{ flt time})$	obj	1923115	2020063
		time	16.68	93.08
5	$\max(30, 1/2 \text{ flt time})$	obj	1773190	1893163
		time	16.08	89.54

The preceding tables of results have summarized the effects of various factors on the solution run time of each algorithm, and its corresponding solution quality. From an operational perspective, it is also important to assess the impact of these algorithms on the actual airline's operation in the residual flight network. The following tables outline the impact of each factor on the flight coverage in the airline network.

## 7.2 Flight Coverage

The existence of additional operational constraints in the airline recovery problem are a required component to accurately model any real-world situation. A study of the impact of such constraints on flight coverage in the network is thus warranted as a base case for looking at the impact of delay arcs in the network. Table 7-8 shows the flight coverage results for the baseline case of Problem 2, a network of 201 scheduled flights and 49 aircraft. In this study, there are no delay arcs in the network, and the impact of the operational constraints can be observed from the experimental results. As the number of

operational constraints increases, there is a corresponding increase in the level of flight cancellations in the network (with no delay options).

### 7.2.1 Number of Additional Constraints

As anticipated, the number of operational constraints in the problem does have a significant impact on the level of flight coverage in the network. An interesting

**Table 7-8** Effects of Additional Constraints on Flight Coverage [%]

Scenario	Types of constraints	Flight	Algorithm 1	Algorithm 2	Algorithm 3
1	flight, aircraft	Delay	0.00	0.00	0.00
		Cancel	18.23	22.00	13.79
2	flight, aircraft, landing slots	Delay	0.00	0.00	0.00
		Cancel	55.67	51.00	57.64
3	flight, aircraft, crew	Delay	0.00	0.00	0.00
		Cancel	41.38	46.00	46.80
4	flight, aircraft, slots, crew	Delay	0.00	0.00	0.00
		Cancel	61.08	60.00	65.52

observation regarding the level of cancellation was made. In the current model formulation of the airline recovery problem, the “cost” of flight cancellations are implicitly incorporated into the decision process, and as such, the true penalty (cost) for cancelling a given flight is not made accountable. As a result, there may be an artificially higher level of flight cancellations in the solutions generated by an algorithm, even under normal conditions. It is important to point out however, that the solution quality (profitability) of each algorithm under these conditions is comparable to the actual levels of the real world operations.

**Table 7-9** Effects of Delay Arcs on the Flight Coverage [%]  
(no additional operational constraints )

Scenario	Number of Delay Arcs	Delay (mins)	Flight	Algorithm 1	Algorithm 2	Algorithm 3
1	1	15	Delay	0.99	0.00	0.00
			Cancel	19.21	22.00	13.30
2	1	30	Delay	0.49	0.00	0.00
			Cancel	19.21	22.00	12.81
3	1	45	Delay	0.99	1.00	0.00
			Cancel	18.72	21.00	14.29
4	1	1/2 flt time	Delay	2.96	3.00	1.00
			Cancel	17.73	19.00	13.79
5	2	15	Delay	0.99	0.00	n/a
			Cancel	19.21	22.00	
6	2	30	Delay	2.96	1.00	n/a
			Cancel	17.73	21.00	
7	2	45	Delay	4.43	4.00	n/a
			Cancel	18.23	19.00	
8	2	1/2 flt time	Delay	2.96	4.00	n/a
			Cancel	17.73	20.00	

### 7.2.2 Number of Delay Arcs and Delay Time

The introduction of delay arcs into the Schedule Map increases the length of the solution run time, but does provide the decision maker the ability to make an efficient trade-off between cancelling and delaying a given flight. Table 7-9 shows the level of flight coverage for the baseline problem with the additional delay arcs in the network. Since the primary decision matrix is one of operating profit maximization, flights are intentionally delayed to help improve profitability.

**Table 7-10 Effects of Delay Arcs on the Flight Coverage [%]  
(constraints on crew and landing slots )**

Scenario	Number of	Delay	Flight	Algorithm 1	Algorithm 2	Algorithm 3
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Delay Arcs		(mins)				
1	1	15	Delay	26.11	27.00	0.00
			Cancel	36.45	35.00	36.95
2	1	30	Delay	33.50	34.00	0.50
			Cancel	29.06	27.00	25.12
3	1	45	Delay	41.38	36.00	4.50
			Cancel	24.14	25.00	24.63
4	1	1/2 flt time	Delay	37.44	33.00	7.00
			Cancel	29.06	31.00	30.54
5	2	15	Delay	36.45	36.00	n/a
			Cancel	27.59	25.00	
6	2	30	Delay	40.89	39.00	n/a
			Cancel	24.63	23.00	
7	2	45	Delay	45.32	39.00	n/a
			Cancel	21.18	23.00	
8	2	1/2 flt time	Delay	43.84	37.00	n/a
			Cancel	24.14	24.00	

In cases where additional operating constraints are imposed, the level of flight cancellations are greatly reduced by the presence of delay arcs, which in turn lead to significant levels of flight delays in the solution. Table 7-10 summarizes the level of flight coverage in the airline network under operational constraints on aircraft movement. In this study, these operational constraints included limitations on aircraft arrivals due to landing slot allocation, and restrictions on departing flights based on the number of crew available at a given airport station.

### 7.2.3 Passenger Recapture Rate

The level of passenger recapture in the decision matrix has been shown to sufficiently influence the level of flight coverage in the network. Table 7-11 outlines the flight coverage in the network in light of variations in the recapture rate.

**Table 7-11** Effects of Passenger Recapture Rate on Flight Coverage [%]  
(constraints on crew and landing slots )

Scenario	Passenger Recapture Rate	Flight	Algorithm 1	Algorithm 2	Algorithm 3
1	0.500	Delay	27.59	25.00	0.50
		Cancel	33.99	33.00	35.47
2	0.625	Delay	30.54	33.00	0.50
		Cancel	31.03	28.00	28.57
3	0.750	Delay	33.50	34.00	0.50
		Cancel	29.06	27.00	25.12
4	0.875	Delay	43.84	40.00	0.50
		Cancel	23.15	23.00	23.15
5	1.000	Delay	60.10	60.00	0.50
		Cancel	20.20	21.00	21.18

In this study, one delay arc is generated for each scheduled flight in the original airline network. The ability to accurately account for this factor in the current study is limited by the omission of network effects, as it relates to passenger flow and connectivity in the main problem.

#### 7.2.4 Minimum Aircraft Turn Time

As outlined in earlier discussions, the assumed minimum aircraft turn time substantially influences the underlying time-space network, and the resulting outcome of each algorithm. The ability to cover scheduled flights in the airline network will be dictated by the amount of “available” flight time across the fleet. By varying the minimum aircraft turn time (adjusting block times, and/or shifting arrival/departure times), it is possible to determine more efficient flight sequences with higher levels of aircraft utilization.

**Table 7-12** Effects of Minimum Aircraft Turn Time on Flight Coverage [%]

(no additional operational constraints)

Scenario	Minimum Aircraft Turn Time (min)	Flight	Algorithm 1	Algorithm 2
1	30	delay	0.49	0.00
		cancel	19.21	22.00
2	45	delay	3.45	4.00
		cancel	19.70	23.00
3	60	delay	10.84	11.00
		cancel	25.62	25.00
4	$\max(30, 1/4 \text{ flt time})$	delay	3.94	3.00
		cancel	20.20	21.00
5	$\max(30, 1/2 \text{ flt time})$	delay	5.42	5.00
		cancel	27.09	29.00

**Table 7-13** Effects of Minimum Aircraft Turn Time on Flight Coverage [%]  
(constraints on crew and landing slots)

Scenario	Minimum Aircraft Turn Time (min)	Flight	Algorithm 1	Algorithm 2
1	30	delay	33.50	34.00
		cancel	29.06	27.00
2	45	delay	34.98	35.00
		cancel	27.09	28.00
3	60	delay	28.57	30.00
		cancel	35.47	34.00
4	$\max(30, 1/4 \text{ ft time})$	delay	36.95	36.00
		cancel	28.57	26.00
5	$\max(30, 1/2 \text{ ft time})$	delay	31.03	31.00
		cancel	35.96	34.00

Table 7-12 and Table 7-13 summarize the flight coverage observed for the case with no additional constraints, and the case with additional constraints respectively. From this study, it is apparent that the assumed minimum aircraft turn time will marginally affect the flight coverage achieved in the network.

### 7.3 Validation of the Algorithms

The results of the case studies in this chapter are based on several assumptions that have been explicitly discussed. In an effort to validate the algorithm, an “approximate” comparison is made between the solution quality of each algorithm and the actual operations of the airline under normal operating conditions. Table 7-14 shows the comparison of the output of each algorithm to actual operating results under normal conditions.

**Table 7-14 Comparison of Solution Quality to Estimated Operating Results  
(Normal Operations)**

Problem	Algorithm 1	Algorithm 2	Algorithm 3	Actual Operations
1	716941 (115%)	755835 (122%)	784199 (127%)	619,885
2	2647527 (99%)	2603870 (97%)	2734698 (102%)	2,674,739
3	2092465 (97%)	2104083 (98%)	2141805 (99.6%)	2,148,606
4	4713562 (97.7%)	4811564 (99.8%)	4943535 (102.5%)	4,823,345

As discussed in Chapter 6, it is almost impossible to “recreate” the series of irregularities over the course of a day using one decision process. The “irregular” operating conditions have been simulated for the purpose of this case study by restricting the number of aircraft movement within a given time interval, as it was impossible to identify and model each individual “irregularity” in the study. Consequently, the quality of the solution of each algorithm for the problem under irregular conditions is not presented, since it is impossible to make an accurate comparison to the actual airline operations.

However, the comparison of the algorithms under normal operating conditions does support the validity of the algorithms for solving the airline schedule recovery ASRP problem. Future research initiatives could explore the validation of these algorithms for case study problems under irregular airline operations through in-field case studies at an airline operation control center of a marginally sized carrier.

#### 7.4 Summary and Conclusions

#### 7.4.1 Analysis

The primary purpose of the sensitivity analyses have been to further validate, and “beta-test” the greedy heuristic and optimization-based algorithms developed in the project.

Several operational issues were considered in the study, through a series of sensitivity analyses that were conducted to establish the importance of each parameter in the future development and implementation of these algorithms in a real-world environment. The major findings and observations of the sensitivity studies are now summarized.

- There is a strong correlation between the dimensions of the problem (number of aircraft and scheduled flights) and the overall algorithm run time. The underlying tree-searching algorithm runs in  $O(m)$  time.
- The performance of each algorithm appears to be affected by the actual composition and structure of the underlying airline network. The ability to efficiently solve the subproblem of aircraft rerouting will be driven by the number of possible flight sequence combinations in the network. If the network is highly connected, the number of possible routings will increase exponentially.
- The solution time of Algorithm 1 and Algorithm 2 are not significantly affected by the addition of operational constraints to the problem. The inclusion of these auxiliary constraints results in a corresponding “network truncation” prior to the execution of the tree-searching algorithm. In effect, the addition of these constraints actually improve the performance of Algorithm 1 and Algorithm 2.
- The solution time of Algorithm 3 is substantially impacted by the presence of additional operational constraints in the decision model. This is a direct result of the fractionality which exists in the initial LP solution to the problem.
- The duration of flight delays in an airline network which is subject to additional operational constraints, does not affect the solution run-time of each algorithm.

However, the delay duration affects the profit and the flight coverage achieved in the network. By varying the average flight delay in the network from 15 minutes to 45 minutes, there is a 7.0% increase in the overall operating profit using Algorithm 1. At the same time, there is a 33% reduction in the number of cancelled flights and a 60% increase in the number of delayed flights in the network. Similar results can be observed for Algorithm 2 and Algorithm 3.

- The assumed passenger recapture rate used in each algorithm does not affect the solution run-time, but significantly impacts the profit and the associated flight coverage in the network. By varying the passenger recapture rate from 0.50 to 0.75, the overall operating profit (Algorithm 1) increases by 20%. This is associated with a 15% reduction in the number of cancelled flights, and a 22% increase in the number of delayed flights, as it becomes more beneficial to delay flights. Similar results can be obtained using Algorithm 2 and Algorithm 3.
- The profitability and corresponding flight coverage is influenced by the assumed length of the minimum aircraft turn time. The aircraft turn time does not affect the algorithm run-time. For a network subject to operational constraints, an increase in the minimum aircraft turn time from 45 minutes to 60 minutes, results in a 9% reduction in the operating profit. This is associated with a 18% reduction in the number of delayed flights, but a 30% increase in the number of cancelled flights.
- The ability to efficiently trade-off between cancelling and delaying a given flight in an airline network using a single decision model, is beneficial for the resolution process. The presence of delay arcs in a network subject to operational constraints on landing slots and crews, results in a 53% reduction in the percentage of flights cancelled, and an associated 33% increase in the percentage of delayed flights. This improved flight coverage results in a 50% in profitability using Algorithm 1.

- The flight coverage achieved in the solution generated by each algorithm is affected by the manner in which passenger spill and the corresponding “value” of a given flight is incorporated into the decision model. In particular, there is an artificially higher level of the flight cancellations, as the true “cost” of cancelling a given flight and other “network” effects are not explicitly modelled.
- Under normal conditions, the quality of the solution (profit) generated by each algorithm is comparable to the estimated profit values of the actual airline operations, (please refer to prior discussions on the accuracy of these estimates).
- Under irregular operating conditions, it is very hard to make a meaningful validation of the model, as it is almost impossible to simulate the series of decisions made by a controller over the course of a day, using a single decision process.

Over the course of the case study phase of the project, many of the experiments considered implicitly underscore the importance of the airline controller in dictating the outcome of any resolution methodology implemented in an operations control center. By adjusting the number of fleets included in the solution process, the controller has the ability to control the effects of problem size (number of aircraft and corresponding flights) on solution time and the quality of the solution. Similarly, by considering only the appropriate operational constraints for a given situation, the controller is capable of limiting the effects of additional constraints on the solution time, and overall quality of the final flight rescheduling solution.

As observed in the sensitivity analyses presented in this chapter, the various parameter settings can significantly impact the outcome of the algorithm. An experienced airline controller would be able to accurately control the execution of the solution procedures, through varying the minimum aircraft turn time, passenger recapture rate, number of delay arcs, and the duration of flight delays in the underlying airline network considered in the solution process.

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#### 7.4.2 Computational Experience

During the course of the case analysis, one of the major limitations faced was that of computer memory capacity on the test platform. As a precursor to future research on the airline recovery problem, the algorithms and solution procedures which had been developed for the SunSparc workstation were ported to the UNIX environment running on an INTEL Pentium-Pro equipped computer. As shown in Table 7-15 and Table 7-16, there are significant gains in solution times from changing platforms, and in some cases considered, as much as ten fold. This reinforces the premise that it is possible to develop efficient real-time procedures to assist airline controllers in flight rescheduling in the aftermath of irregularities. In analysing the computational times of the SunSparc workstation, it was observed that almost 50% of the reported run time could be attributed to internal computer memory management, due to the physical size of the machine's RAM space, and the resulting need to swap memory between the hard-drive (virtual memory) and the actual RAM. In addition, the processing speed of the Pentium-Pro processor (266 Mhz) significantly exceeds that of the SunSparc 20 (75 MHz).

**Table 7-15** Summary of Effects of Problem Size on Solution Time (secs)

Case	Aircraft	Flights	Factor	Algorithm 1	Algorithm 2
1	35	180	obj	716941	755835
			time	0.27	3.00
2	49	201	obj	2647527	2603870
			time	0.41	2.45
3	50	192	obj	2092465	2104083
			time	0.21	2.71
4	99	393	obj	4713562	4811564
			time	1.21	15.72
5	177	612	obj	6823536	n/a

	time	3.03
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**Table 7-16 Effects of Delay Arcs on the Solution Quality and Algorithm Run Time  
(additional constraints on crew and landing slots )**

Scenario	Number of Delay Arcs	Duration (mins)	Factor	Algorithm 1	Algorithm 2
1	1	15	obj	1851038	1915756
			time	2.19	11.31
2	1	30	obj	1955652	2059552
			time	1.71	10.56
3	1	45	obj	1974784	2117415
			time	1.46	11.92
4	1	1/2 flt time	obj	1949323	1994718
			time	2.26	10.38
5	2	15	obj	1984528	2016319
			time	2.86	25.38
6	2	30	obj	2014968	2091298
			time	2.88	28.08
7	2	45	obj	2013300	2108169
			time	3.09	16.03
8	2	1/2 flt time	obj	1939778	2063627
			time	3.21	25.15

# **Chapter 8**

## **Summary and Conclusions**

### **8.1 Review of the Airline Schedule Recovery Problem**

The primary motivation of this dissertation has been the need to address flight rescheduling in the aftermath of irregular airline operations. The ability of an airline to address flight rescheduling depends on the availability of up-to-date, and accurate operational information from all divisions of the carrier. The underlying assumption of this research project has been that an efficient information flow mechanism already exists in the airline's operation control center, and that airline controllers have full access to all relevant information and corresponding databases, in order to make informed decisions about the operations of the carrier.

The rescheduling of flights after irregularities is modelled as the Airline Schedule Recovery Problem, and this is used as a foundation to develop efficient, robust and "real-time" solution methodologies for reassigning operational aircraft to flights and concurrently construct the residual airline network, and new "current" schedules. The development of the airline schedule recovery problem has been greatly influenced by previous work on related airline scheduling topics, as well as communications with airline controllers, the potential end-users of the envisioned decision support tool.

### **8.2 Discussion of the Case Studies**

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The algorithms developed during the course of this research were validated and tested using historical operational data from a major US domestic carrier, and data from the domestic network of an international airline. Several parameters and implementation issues were considered during the case study analysis, including the effect of the size of the airline Schedule Map on the solution time of each algorithm. In particular, the case study considered the effects of the number of operational constraints incorporated into the decision model, the number and duration of delay arcs generated and considered, and the passenger recapture rate on the quality of the solution, flight coverage and the overall solution time of each algorithm.

Based on the extensive computational experiences of the case studies, it is important to highlight the high level of sensitivity of the aircraft assignment results of each algorithm to initial assumptions and prescribed parameters in the decision process. The ability to use such algorithms to generate practical aircraft-flight assignments and corresponding aircraft routings will depend on the experience of the airline controller. The analysis presented in the previous chapter has demonstrated the flexibility and robustness of the algorithms in dealing with variations in the level of irregularity experienced by the carrier. In addition, results of the Case Studies have reinforced the need for such solution procedures, when one considers the impact of irregularities on the airline's profitability. Finally, the Case Studies identified limitations to potential "real-world" applications of these algorithms, in terms of the virtual CPU memory requirements.

### **8.3 Contributions of the Research**

The Airline Schedule Recovery decision model developed in this dissertation provides a comprehensive framework which addresses how airlines can efficiently reassigned operational aircraft to scheduled revenue flights in the aftermath of irregularities. The design of the

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decision model and resulting solution methodologies have been driven by real-world experiences in airline operations, and emphasize the role of the airline controller in the decision process. The model integrates various aspects of the airline's tactical planning processes, which are traditionally considered separately.

The mathematical formulation of the problem enables flight delays and cancellations to be considered simultaneously, i.e., in the same decision model. In a real-world scenario, airline controllers generally make this trade-off implicitly, but this fundamental mechanism has not been modelled in previous work. The decision model allows for multiple fleet type aircraft swapping in flight rescheduling, provided the candidate aircraft is capable of flying a given flight segment. In addition, the impact of air traffic control (ATC) traffic flow management initiatives and crew availability are incorporated into the model through restrictions on aircraft movement at affected airports in the network system.

The Airline Schedule Recovery problem is best described as a hybrid three dimensional decision model as it simultaneously solves the fleet assignment problem and the aircraft routing problem which are normally solved sequentially. As a result, aircraft maintenance requirements are implicitly satisfied in the aircraft assignment output from the implemented algorithms. This unique solution approach to the aircraft routing aspect of the problem is different from traditional procedures currently employed in the strategic phase of the planning process, and in the aftermath of irregularities.

The algorithms and solution methodologies developed and validated in this dissertation have successfully demonstrated that it is possible to develop efficient decision support procedures for flight rescheduling. These algorithms, which are based on Network Flow Theory and Mathematical Programming Theory, produce "real-time" solutions to highly complex assignment problems. During the course of the implementation of the algorithms, it

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was established that it is possible to incorporate many aspects of the tactical planning process into the decision process, thereby producing a “robust” solution to the main problem of rescheduled flights, and rerouting operational aircraft. Based on experiences from the case study, future research initiatives should explore the implementation of these algorithms with even larger sized airline networks.

The design and implementation of the solution methodologies are based on an object-oriented framework, and as a result, the various functional modules are interchangeable, which provides flexibility in the solution process. The execution of each algorithm is highly interactive, and requires an array of user-defined conditions and parameters, thereby incorporating the airline controller in the decision process. These solution procedures can be further enhanced and developed as the foundation of an operations control decision support tool, to assist airline controllers in dealing with irregularities. The state-of-the practice in AOCC generally involves manual resolution of irregularities.

#### **8.4 Directions for Future Research Initiatives**

##### **8.4.1 Modelling Issues**

In the current formulation of the airline schedule recovery problem, network effects on revenue are not explicitly considered in the derivation of the cost coefficient. This simplified version of revenue accounting in effect ignores leg-dependence effects in demand and revenue estimations. However, the prevalence of hub and spoke airline network operations does warrant such considerations, as only then can passenger connectivity effects be truly incorporated into the decision process. The related issue of passenger flow considerations are necessary in order to accurately determine spill, and the corresponding spill costs associated with each flight segment in the network.

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The existing model does not explicitly account for all aspects of crew scheduling, and its impact on aircraft assignment. The ability to incorporate such issues is limited by the potential impact on the tractability of the model. There exists a strong interdependence between the aircraft reassignment problem, and the crew rescheduling problem. It is important however, that future researchers accurately model the rescheduling of crew members to flights in the residual airline network. This can be highly complicated by real-world issues such as labour union contracts, which can be hard to incorporate into any discrete decision model, and are particular to each airline.

The solution methodologies presented in the dissertation deal solely with the main problem of reassigning aircraft to flights in the aftermath of irregularities. Currently, several independent research projects are studying one of the auxiliary problems, but it is necessary for future researchers to consider the interaction between these sub-problems, as decisions made in one problem can significantly impact another problem. The ability to efficiently capture such interaction could substantially improve the robustness of any solution methodology developed for dealing with irregularities.

The overall framework of the ASRP model involves the iterative solution on the main aircraft assignment problem, and associated sub-problems of ATC slot allocation, crew rescheduling, gate allocation, and passenger origin-destination flow problems. In its present form, the main problem of the airline schedule recovery model incorporates aspects of these sub-problems, but future research initiatives should explore improvements in the modelling of these constraints. In particular, it is important to assess the required information flow mechanism necessary for the successful implementation of the overall solution methodology.

As previously discussed, there is a fundamental assumption in this dissertation that the required information flow mechanism already exists. As a result, the further development

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and implementation of the airline schedule recovery problem is closely coupled to information flow considerations. Future research initiatives should explore how the current problem formulation affects information flow, and what implications this may have on future work on the topic of irregularities. In the next section, implementation issues are addressed in light of the computational experiences of the empirical studies.

#### **8.4.2 Implementation Issues**

As demonstrated in the case studies, real-time solution capabilities are possible with the existing problem and corresponding algorithms. However, it is necessary to ascertain how the issue of solution time will affect the applicability of these algorithms to larger airline networks. In addition, researchers should consider what impact the need for "lead-time" will have on the solution methodology as it relates to uncertainty in the available data, and the ability to retrieve real-time up-to-date information from the corresponding databases in the airline system. From a practical standpoint, the full benefits of any implementation of the developed algorithms would depend significantly on efficient interfacing between the front-end decision support tool and the back-office database systems. Researchers should also explore alternative decision frameworks, such as considering sequential decision mechanisms, and the inclusion of the probability of future irregularities.

The solution methodologies and procedures for dealing with irregularities presented in this dissertation are a departure from current state-of-the-practice of Airline Operations Control Centers (AOCC). In recent years, airlines have come to understand the importance of collaborative decision making in its tactical operations. Many questions will arise from this research, such as who would be responsible for the implementation of these algorithms in the AOCC? In addition, the issue of information flow, and the dissemination of decisions to the various divisions within the airline does warrant some consideration. For example,

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how would the deployment of a decision support system based on these developed algorithms affect the daily operations of an AOCC, and how they deal with irregular airline operations? In answering these and other important questions, future research initiatives will further advance the development of efficient algorithms for flight rescheduling, and other aspects of tactical airline planning.

*So what exactly is Flight Transportation?*

**flight** (flait) n. 1. the act, skill, or manner of flying. 2. a soaring mental journey above or beyond the normal everyday world. 3. the act of fleeing or running away, as from danger.

**transportation** (traenspor' teifen) n. 1. a means or system to carry or cause to go from one place to another, especially over some distance. 2. a system that provides ecstasy, rapture, or any powerful emotion.

**flight transportation** (flait traenspor' teifen) n. 1. a program of study that incorporates a broader education in the disciplines of engineering, economics, management, law, and operations research. 2. the ultimate frequent flyer program.

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# Bibliography

## Irregular Airline Operations

- [1] Arguello, Michael et. al. An Optimization Model for Aircraft Routing in Response to Groundings and Delays. Working paper, University of Texas - Austin, March 1997.
- [2] Cao, Jia-Ming and Adib Kanafani. Real-time Decision Support for Integration of Airline Flight Cancellations and Delays, Part I: Mathematical Formulation. *Transportation Planning and Technology*, Vol. 20, pg. 183-199 (1997).
- [3] Cao, Jia-Ming and Adib Kanafani. Real-time Decision Support for Integration of Airline Flight Cancellations and Delays, Part II: Algorithm and Computational Experiments. *Transportation Planning and Technology*, Vol. 20, pg. 201-17 (1997).
- [4] Jarrah, Ahmad et. al. A Decision Support Framework for Airline Flight Cancellations and Delays. *Transportation Science*, Vol. 27, No. 3, August 1993.
- [5] Lettovsky, Ladislav et. al. Airline Crew Recovery. INFORMS Aviation Applications Section Presentation, New Orleans, October 1995.
- [6] Luo, Songjun et. al. Airline Schedule Perturbation Problem: Ground Delay Program with Splitable Resources. Univeristy of Texas-Austin, August 23, 1994.
- [7] Luo, Songjun et. al. Airline Schedule Perturbation Problem: Landing and Take-off with Non-splitable Resources for the Ground Delay Program. Univeristy of Texas-Austin, August 8, 1994.
- [8] Mathaisel, Dennis. Decision Support for Airline System Operations Control and Irregular Operations. *Computers and Operations Research*, Vol. 23, No. 11, pg. 1083-98 (1996).
- [9] Mette, Matthias. Sequential Heuristic Algorithm for Minimum-Cost Rescheduling of Connecting Complexes in Airline Hub Operations. *Flight Transportation Operations Analysis* term paper, May 1994.

- [10] Rakshit, Ananda et. al. Systems Operations Advisor: A Real-time Decision Support System for Managing Airline Operations at United Airlines. *Interfaces* 26: 2 March - April 1996 (pp. 50-58).
- [11] Teodorovic, Dusan et. al. Model to Reduce Airline Schedule Disturbances. *Journal of Transportation Engineering*, Volume 121, Number 4, July-August 1995.
- [12] Teodorovic, Dusan et. al. Model for Operational Daily Airline Scheduling. *Transportation Planning and Technology* 1990, Volume 14, pp. 273-285.
- [13] Teodorovic, Dusan et. al. Optimal dispatching strategy on an airline network after a schedule perturbation. *European Journal of Operational Research* 15 (1984).
- [14] Teodorovic, Dusan. A Model for Designing the Meteorological most reliable Airline Schedule. *European Journal of Operational Research* 21 (1985), pp. 156-164.
- [15] Yan, Shangy and Dah-Hwei Yang. A Decision Support Framework for Handling Schedule Perturbation. *Transportation Research B*, Vol. 30, No. 6, pp. 405 - 419.

### Airline Operations

- [16] Airline Operational Control Overview. Prepared by the Airline Dispatchers Federation and Seagull Technology Inc. May 1995.
- [17] Aykin, Turgut. Networking Policies for Hub-and-Spoke Systems with Application to the Air Transportation System. *Transportation Science* Vol. 29, No. 3, Aug 1995.
- [18] Barnhart, Cynthia et. al. Flight String Models for Aircraft Fleeting and Routing. Working paper, MIT Center for Transportation Studies, January 1997.
- [19] Berge, Matthew and Craig Hopperstand. Demand Driven Dispatch: A Method for Dynamic Aircraft Capacity Assignment, Models and Algorithms. *Operations Research*, Vol. 41, No. 1, January - February 1993.
- [20] Bihr, Richard. A Conceptual Solution to the Aircraft Gate Assignment Problem using 0,1 Linear Programming. *Computers and Industrial Engineering*, Vol. 19, No. 1-4, 1990, pg. 280 - 284.
- [21] Cao, Jia-Ming and Adib Kanafani. The Value of Runway Time Slots for Airlines. Working paper, University of California - Berkeley, May 1997.
- [22] Carlson, Paul. Allocating Banks of Flights to Arrival Slots in Reduced-Capacity Situations. MIT Center for Transportation Studies, May 1997.
- [23] Chang, Ching et. al. Flight Sequencing in Airport Hub Operations. Presented at the 1995 Annual Meeting of the Transportation Research Board.

- [24] Clarke, L.W. et. al. Maintenance and Crew Considerations in Fleet Assignment. *Transportation Science* Vol. 30, No. 3, August 1996.
- [25] Deckwitz, Thomas. Interactive Dynamic Aircraft Scheduling. MIT Thesis, May 1984.
- [26] Dennis, Nigel. Scheduling Strategies for Airline Hub Operations. *Journal of Air Transportation Management* 1994 1(3), pp. 131-144.
- [27] Dennis, Nigel. Airline Hub Operations in Europe. *Journal of Transport Geography* 1994 2(4), pp. 219-233.
- [28] Etschmaier, Maximiliam and Dennis Mathaisel. Airline Scheduling: An Overview. *Transportation Science*, Vol. 19, No. 2, May 1985.
- [29] Farkas, Andras. The Influence of Network Effects and Yield Management on Airlne Fleet Assignment Decisions. MIT Flight Transportation Report R96-1.
- [30] Ghobrial, A. et. al. Future of Airline Hubbed Networks: Some Policy Implications. *Journal of Transportation Engineering*, Vol. 121, No. 2, March/April 1995.
- [31] Gillingwater, D. et. al. Information System for Operations at Medium-Sized Airports. *Transportation Research Board* 1461.
- [32] Gosling, Geoffrey. Design of an Expert System for Aircraft Gate Assignment. *Transportation Research A*, Vol. 24A, No. 1 (1990), pg. 59 - 69.
- [33] Grandea, Seth. The Processes of Airline Operation Control. MIT Flight Transportation Report R95-2, February 1995.
- [34] Grandea, Seth et. al. The Processes of Airline System Operations Control. MIT Flight Transportation Laboratory. Submitted for publication, March 1996.
- [35] Kyle, Jon S. Airline and Airport Control Systems - The Next Generation. Presented at the AGIFORS 1989 Symposium in Bymose Hegn, Denmark.
- [36] Kniker, Timonthy et. al. Fleet Assignment and the Passenger Mix Problem Incorporating Spill and Recapture. Working paper, MIT Center for Transportation Studies, Spring 1997.
- [37] Murray, Geoffrey. Dynamic Aircraft Reassignment: A Simulation. INFORMS Airline Scheduling Session, New Orleans October 1995.
- [38] Nakazawa, Shizuya. Dynamic Scheduling in Operation Control System. Presented at AGIFORS 31st Symposium 1991, Minnesota.

- [39] Riccio, Lawrence and Nathan Ron. Computer-Generated System Aids Airline's Passenger Flow and Routing of Aircraft. *Industrial Engineering* Vol. 17, No. 9, September 1985, pg. 52 -56.
  - [40] Rosenthal, Richard et. al. Optimizing Flight Operations for an Aircraft Carrier in Transit. *Operations Research* Vol. 44, No. 2, March - April 1996.
  - [41] Shumsky, Robert. Dynamic Statistical Models for the Prediction of Aircraft Take-off Times. MIT Thesis June 1995.
  - [42] Simpson, Robert. Cancelling and Switching Flights within Clusters of Aircraft in a Schedule. FTL Memorandum M88-8, November 1988.
  - [43] Simpson. Robert. Counting the "Turns" within a Cluster: The Turn Tree. FTL Memorandum M88-10, December 1988.
  - [44] Soumis, Francois, et. al. A Model for Large-Scale Aircraft Routing and Scheduling Problems. *Transportation Research B*, Vol. 14B, pp. 191-201.
  - [45] Su, Y.Y. and K. Srihari. A Knowledge Based Aircraft-Gate Assignment Advisor. *Computers and Industrial Engineering* Vol. 25, Nos 1 - 4, pg. 123 - 126, 1993.
  - [46] Sullivan, James. The Effects of Inclement Weather on Airline Operations, AIAA-89-0797
  - [47] Svrcek, Tom. Planning Level Decision Support for the Selection of Robust Configurations of Airport Passenger Buildings. MIT Flight Transportation Laboratory Report R94-6, May 1994.
  - [48] Talluri, Kalyan. Swapping Applications in a Daily Airline Fleet Assignment. *Transportation Science* Vol. 30, No. 3, August 1996.
  - [49] Teodorovic, Dusan. Multi-Attribute Aircraft Choice For Airline Network. *Journal of Transportation Engineering* Vol. 112, No. 6, November 1986.
  - [50] Vanderstraeten, Godelieve and Michel Bergeron. Automatic Assignment of Aircraft to Gates at a Terminal. *Computers and Industrial Engineering* Vol. 14, No. 1, pg. 15-25.
  - [51] Waldman, Gary. A Study of the Practicality and Profit Enhancement Potential of Demand Driven Dispatch in Airline Hub Operations. MIT Thesis June 1993.
  - [52] Well, Alexander. *Principles of Airline Scheduling, Air Transportation: A Management Perspective*, Wadsworth Books, 1995.
  - [53] Yau, C. Dynamic Flight Scheduling. *OMEGA International Journal of Management Science*, Vol. 17 (6) 1989, pp. 533 - 542.
-

**Network Flow Theory**

- [54] Ahuja, Ravindra et. al. **Network Flows: Theory, Algorithms and Applications.** Prentice Hall 1993.
- [55] Ahuja, Ravindra and James Orlin. A Capacity Scaling Algorithm for the Constrained Maximum Flow Problem. *Networks*, Vol. 25 (1995), pg. 89-98.
- [56] Aneja, Y. P. et al. Shortest Chain Subject to Side Constraints. *Networks*, Vol. 13 (1983), pg. 295-302.
- [57] Aneja, Y. P. and KPK Nair. The Constrained Shortest Path Problem. *Naval Research Logistics Quarterly*. Vol 25 (1978)
- [58] Azevedo, Jose Augusto de, and Ernesto Martins. An Algorithm for the Multi-Objective Shortest Path Problem on Acyclic Networks. *Investigacao Operacional*, Vol. 11, No. 1 (1991), pp. 52 - 69.
- [59] Beasley, J.E. et. al. An Algorithm for the Resource Constrained Shortest Path Problem. *Networks*, Vol. 19 (1989), pg. 379-394.
- [60] Calvette, Herminia and Pedro Mateo. An Approach for the Network Flow Problem with Multiple Objectives. *Computers and Operations Research*, Vol. 22, No. 9, pp. 971 - 983, 1995.
- [61] Climaco, J. C. and Ernesto Martins. A Bicriterion Shortest Path Algorithm. *European Journal of Operational Research* 11 (1982), pp. 399 - 404.
- [62] Current, John et al. Efficient Algorithms for Solving the Shortest Covering Path Problem. *Transportation Science*, Vol. 28, No. 4, November 1994.
- [63] Deo, Narsingh and Chi-yin Pang. Shortest-Path Algorithms: Taxanomy and Annotation. *Networks*, Vol. 14 (1984), pg. 275-323.
- [64] Desrochers, Martin et. al. A Reoptimization Algorithm for the Shortest Path Problem with Time Windows. *European Journal of Operational Research* 35 (1988), pp. 242 - 254.
- [65] Desrochers, Martin et. al. A Generalized Permanent Labelling Algorithm for the Shortest Path Problem with Time Windows. *Informs* Vol. 26, No. 3 (1988)
- [66] Desrochers, Martin et. al. A New Optimization Algorithm for the Vehicle Routing Problem with Time Windows. *Operations Research*, Vol. 40, No. 2, March/April 1992.

- [67] Desrosiers, Jacques et al. Time Constrained Routing and Scheduling. Handbooks In Operations Research and Management Science (1995), Vol. 8, Chapter 2.
- [68] Desrosiers, Jacques et al. Routing with Time Windows by Column Generation. Networks, Vol. 14 (1984) 545-565.
- [69] Desrosiers, Jacques et al. Methods for Routing with Time Windows. European Journal of Operational Research 23 (1986), pp. 236 - 245.
- [70] Desrosiers, Jacques et al. The Pickup and Delivery Problem with Time Windows. European Journal of Operational Research, Vol. 54, No. 1, Sep. 5, 1991.
- [71] Divoky, James and Ming Hung. Performance of Shortest Path Algorithms in Network Flow Problems. Management Science, Vol. 36, No. 6, June 1990, pp. 661 - 673.
- [72] Ford, L. R. and D. R. Fulkerson. **Flows in Networks**. Princeton University Press, 1962.
- [73] Handler, Gabriel et al. A Dual Algorithm for the Constrained Shortest Path Problem. Networks, Vol. 10 (1990) 293-310.
- [74] Hassan, Mohsen. Network Reduction for the Acyclic Constrained Shortest Path Problem. European Journal of Operational Research 63 (1992)
- [75] Henig, Mordechai. The Shortest Path Problem with Two Objective Functions. European Journal of Operational Research 25 (1985), pp. 282 - 291.
- [76] Jaffe, Jeffrey. Algorithms for Finding Paths with Multiple Constraints. Networks, Vol. 14 (1984), pg. 95-116.
- [77] Martins, Ernesto and Jose Luis Santos. An Algorithm for the Quickest Path Problem. Published on the WWW site of the Department de Matematica, Universidade de Coimbra, Portugal. March 1996.
- [78] Pallottino, Stefano. Shortest Path Methods: Complexity, Interrelations and New Propositions. Networks, Vol. 14 (1984), pg. 257-267.
- [79] Sheir, D.R. Iterative Methods for Determining the k Shortest Paths in a Network. Networks, Vol. 6 (1976) pg. 205-229.
- [80] Sheir, D.R. On Algorithms for Finding the k Shortest Path in a Network. Networks, Vol. 9 (1979), pg. 195-214.
- [81] Sivakumar, Raj and Rajan Batta. The Variance-Constrained Shortest Path Problem. Transportation Science, Vol. 28, No. 4, November 1994.

**Mathematical Programming Theory**

- [82] Barnhart, Cynthia. Dual-Ascent Heuristics for Large-Scale Multi-commodity Flow Problems. *Naval Research Logistics* 40 (1993), pg. 305 - 423.
  - [83] Barnhart, Cynthia et. al. A Column Generation and Partitioning Approach for Multi-commodity Network Problems. Submitted for publication to *Telecommunication Systems*.
  - [84] Barnhart, Cynthia et. al. Branch and Price: Column Generation for Solving Huge Integer Programs, COC-94-03. Working paper from Industrial and Systems Engineering at Georgia Tech.
  - [85] Barnhart, Cynthia et. al. Formulating A Mixed Integer Programming Problem to Improve Solvability. *Operations Research* 41 (1993), No. 6, pg. 1013 - 1019.
  - [86] Bradley, Stephen et. al. **Applied Mathematical Programming**. Addison-Wesley 1983.
  - [87] Dantzig, G.B. and P. Wolfe. The Decomposition Algorithm for Integer Programs. *Econometrica* 29 (1961), No. 4, pg. 767 - 778.
  - [88] Everett, Hugh. Generalized Lagrange Multiplier Method for Solving Problems of Optimum Allocation of Resources. *Management Science* 11 (1963), pg. 399 - 417.
  - [89] Fisher, Marshall. The Lagrangian Relaxation Method for Solving Integer Programming Problems. *Management Science*, Vol. 27:1 (1981), pg. 1 - 18.
  - [90] Fisher, Marshall. An Application Oriented Guide to Lagrangian Relaxation. *Interfaces*, Vol. 15:2 (March-April 1985), pg. 10 - 21.
  - [91] Fisher, Marshall and P. Kedia. Optimal Solution of Set Covering/Partitioning Problems using Dual Heuristics. *Management Science*, Vol. 36, No. 6, June 1990, pp. 674 - 88.
  - [92] Forrest, J.J and D. Goldfarb. Steepest Edge Simplex Algorithms for Linear Programming. *Mathematical Programming* 57 (1992), No. 3, pg. 341 - 374.
  - [93] Gilmore, P.C. and R.E. Gomory. A Linear Programming Approach to the Cutting-Stock Problem. *Operations Research* 9 (1961), pg. 849 - 859.
  - [94] Hearn, D.W. and S. Lawphongpanich. Lagrangian Dual Ascent by Generalized Linear Programming. *Operations Research Letters* 8 (1989), pg. 189 - 196.
  - [95] Jones, K.L. et. al. Multi-commodity Network Flows: The Impact of Formulation on Decomposition. *Mathematical Programming* 62 (1993), No. 1, pg. 95 - 118.
-

- [96] Magnanti, Tom et. al. Generalized Linear Programming Solves the Dual. Management Science 22 (1976), No. 11, pg. 1195 - 1203.
- [97] Parker, M. and J. Ryan. A Column Generation Algorithm for Bandwidth Packing. Submitted for publication to Telecommunications Systems.
- [98] Sherali, H.D. and D.C. Myers. Dual Formulations and Subgradient Optimization Strategies for Linear Programming Relaxations of Mixed-Integer Programs. Discrete Applied Mathematics 20 (1988), pg. 51 - 68.
- [99] Williams, H.P. **Model Building in Mathematical Programming**. John Wiley, 1990.

# **Appendices**

## **A1 Survey Questionnaire for AOCC Visit**

## **A2 Summary of Data Requirements**

## **A3 Sample Data Files for Case Study Analysis**

### 1. Scheduled flights

(Origin, departure time, destination, arrival time, flight number, average fare, distance, block time, number of passengers, type of aircraft originally assigned to flight)

### 2. Operating aircraft

(Tail number, aircraft type, capacity, remaining flying time before maintenance, range, hourly operating cost, crew, cabin, noise restriction)

### 3. Actual airline schedule of flights, and corresponding aircraft rotations

### 4. Schedule and flight sequences generated using Algorithm 1

## A1 Survey Questionnaire for Visit to AOCC

The primary purpose of these trips were to develop a better understanding of how actual AOCC deal with irregular airline operations, as well as to get an insight into the daily operations of the center. Several issues have been identified as being essential to effective resolution of such irregularities, and it is the hope of the investigator to see the relevance of each issue.

### 1. Information Flow

- types of communication channels currently in use at the center
- what is the most effective one
- areas for improvement
- how are decisions distributed to all relevant parties?

### 2. Information systems and databases accessible by AOCC

- how much access does AOCC have to other division's computer systems
- how much information is actually used from each system, accuracy
- which system is most important in the decision process
- what other databases do controllers want access to, why?

### 3. Interaction with other "operations" divisions

- during normal daily operations and irregularities
- how does the relationship between divisions change with irregularities
- how much consideration is given to passenger flow issues
- how much consideration is given to crew legality issues, who handles it?
- maintenance routing issues, and how is it dealt with in AOCC

### 4. Impact of external factors in the decision process

- how does ATC flow control programs affect resolution
- what role if any, does competitive concerns play in the decision process
- how is meteorological issues, flight planning issues incorporated

### 5. What are some of the current "rule-of-thumbs" used by irregularities

- which flights are considered for cancellation first, for delay

## A2 Summary of Data Requirement

In order to assess the heuristic procedures developed in the research program, it is necessary to gather detailed operational data from an airline carrier with an extensive route network, which is often subjected to severe weather patterns, resulting in irregularities. The following is a preliminary listing of such operational data for each scheduled flight required for the analysis.

### *Operational Data*

- scheduled arrival time
- actual arrival time
- scheduled departure time
- actual departure time
- passenger load and fare mix (from CRS system)
- passenger itinerary mix (connectivity)
- aircraft type assignment
- delay status/recorded cause of delay
- planned aircraft rotations (sequence of flights) for a given period
- actual aircraft rotations
- planned crew rotations for a given period
- actual crew rotations

In addition, it would be necessary to ascertain if deemed important, airline specific operating data in order to better assess the impact of recommended decisions on the cost of operating an effective flight schedule (as an example, crew costs which are strongly affected by labour contracts particular to the carrier). Maintenance planning data would also be necessary to better understand the airline's maintenance planning process and how it currently affects aircraft routing during irregularities. Establish a dataset of specific "irregularities", and try to incorporate other factors such as slot allotment in a given time period and its effects on operations. It would be necessary to quantify the cost of an irregularity and the resolution, for comparison purposes.

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