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An improved column generation algorithm for the disrupted flight recovery problem with discrete flight duration control and aircraft assignment constraints

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ABSTRACT

The disrupted flight recovery problem is well-studied in the literature owing to its significant impact on airlines and passengers. In this work, we consider the disrupted flight recovery problem with two new realistic aspects, i. e., a new implementation for changing flight duration as a recovery option and considering the aircraft assignment constraints. Firstly, we develop a new mixed-integer quadratic programming model encapsulating a functional relationship between the reduced flight duration and the changes in fuel consumption using a piecewise function. Secondly, we propose an improved column generation approach to solve some scenarios based on actual flight data obtained from an airline. Lastly, the experimental results show that the improved column generation algorithm can obtain the optimal solution for all scenarios. Compared with not considering changing the flight duration, considering changing the flight duration can save about 24% of recovery costs on average. In addition, we analyze the effect of enforcing aircraft assignment constraints and different fuel prices on recovery costs. Based on the experimental results, we discuss how to support practitioners in choosing the appropriate combination of recovery options.

1. Introduction

According to a McKinsey study, the year 2020 saw the revenues of the airline industry totalling $328 billion, which is just about 40 % of the previous year. In nominal terms, this value is the same as in 2000. The study expects the industry to be smaller in the coming years, with traffic volumes not expected to return to 2019 levels until 2024 (Jaap et al., 2021). Airlines worldwide had to tide off the COVID-19 crisis by borrowing vast sums of money to make ends meet and deal with daily cash burn. Most of them survived by taking advantage of state aid, lines of credit and bond issuance. The International Air Transport Association (IATA) estimates that the industry accumulated more than $180 billion in debt in 2020 (Alexandre, 2020). One of the crucial factors that drive the future of the aviation sector is customer demand. A Deloitte report forecasts that leisure trips primarily drive customer demand as fewer corporates are likely to make business trips because of remote work and other flexible working arrangements (Anisha, 2021). Given this scenario, airlines are under intense pressure to re-evaluate the economics of routes serviced, destinations, aircraft types, and fleet size to improve operational efficiencies and profitability.

One of the airline industry's most challenging problems is disruptions to airline schedules due to mechanical failures, bad weather, or crew absenteeism. Any of these events can have a devastating impact on the operations. Some direct and immediate consequences are capacity reduction, build-up of delays, cancellations, and changes in crew rostering decisions. These disruption costs are roughly 5 % of airline revenue or about $35 billion globally. And this figure is revised up to $60 billion when we factor in the cost of lost passenger productivity and lost business in support industries like hospitality, business services and tourism (Amadeus, 2020). When these unexpected events occur, airlines immediately initiate recovery operations. In practice, typically, recovery operations happen sequentially in stages. Airlines usually first focus on flight schedule recovery, followed by crew and passenger recovery.

This work is motivated by a real-life problem faced by an airline headquartered in China. When an unexpected event causes an interruption, the airline must recover the flight schedule within a stipulated

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time. To solve the flight recovery problem, we employ a variety of recovery operations such as flight delays, maintenance delays, flight swaps, and flight cancellations. Some recent studies, such as Arıkan et al., 2017, address the cruising speed's recovery option. Most early studies considering cruise speed recovery options were based on the EUROCONTROL aircraft performance model BADA (Base of Aircraft Data), a fuel estimation method based on physical model. Due to the unavailability of aircraft engine and model information, especially new engine and aircraft models (Yanto and Liem, 2018), and commercial sensitivity (Zhang and Mahadevan, 2020), this method may not be feasible in practice. Another fuel estimation method is based on data-driven model. In recent years, many studies have used machine learning methods to establish data-driven model for fuel estimation (Khan et al., 2019; Khan et al., 2021; etc.). Our approach can take advantage of the results based on data-driven models as an alternative when physical based models such as BADA are not available. For the sake of generality, we use a piecewise function to describe the discrete functional relationship between the reduction of flight duration and the changes of fuel consumption. This step obliviates the need for physical models.

Another crucial aspect that has received little attention in the literature is the importance of the aircraft assignment constraints. The aircraft assignment constraints means that the number of aircraft at each airport after the recovery period shall not be less than the required number. Enforcing the aircraft assignment constraints can ensure that there are enough aircraft to carry out the flights after the recovery period. The desired fleet size obviates the need to cancel or delay other flights after the recovery period. Fig. 1 illustrates an example of a recovery plan that does not consider aircraft assignment constraints. When the recovery plan ends, there are no aircraft at Airport 3, but there are flights departing from Airport 3 in the flight plan for the next day. Hence flight f_2 and f_3 may be cancelled due to lack of aircraft. In essence, the flight disruption is not solved by the recovery plan and spreads to the next day. On the other hand, if we consider the aircraft assignment constraints, flight f_1 should be canceled so that the flight plan for the next day can operate normally without cancelling flight f_2 andf_3. Note that aircraft assignment constraints differ from minimizing changes in flight plans because changes that can reduce costs are encouraged as long as aircraft assignment constraints are met. The aircraft cannot arrive/depart when the airport is closed due to inclement weather or other reasons. Therefore, a small number of severely closed airports chosen by the airline do not consider aircraft assignment constraints in this paper.

Given this background, we restrict the scope to the flight recovery problem in the following context: For a given flight schedule, a set of disruptions, a set of maintenances, and a discrete function between the reduced flight duration and the changes in fuel consumption, the problem is to address the following issues while minimizing the total costs, i.e., the sum of changed fuel consumption costs, carbon emission costs, delay costs, swap costs, cancellation costs and penalties for violation of aircraft assignment constraints:

- Extent of departure delays of different flights & maintenances
- Optimal duration of different flights
- Flights to be cancelled
- Reassignment of the available aircraft to the flights

Based on the above, the contributions of this paper are summarized as follows. First, we propose a new mathematical model for changing flight duration as a recovery option. By expressing the nonlinear functional relationship between flight duration reduction and fuel consumption change as a piecewise function, this model can be compatible with data-driven fuel estimation models in recent studies. Furthermore, this model can be used when methods by means of physics-based fuel estimation models such as BADA are not available in existing studies. We propose a mixed integer quadratic program that can be solved using commercial solvers. In order to solve larger-scale problems, an improved column generation algorithm including a master problem and multiple subproblems is proposed. This algorithm avoids the limitation of the column generation technique proposed by Liang et al. (2018) that only connections can be made when the departure time of the first flight is earlier than that of the second flight. In our algorithm, even if the departure time of the first flight is not earlier than the second flight, it is still possible to obtain a feasible connection by delaying the departure time of the second flight to avoid missing the optimal solution. A modified label-setting algorithm is used to solve the pricing subproblem, where the amount of flight duration change is determined during label expansion by solving a small-scale linear programming. The branch-and-bound framework is used after obtaining the optimal solution to the master problem to ensure that the optimal integer solution is obtained. In the experimental part, the performance of the algorithm is explored and analyzed, and the sensitivity analysis of some parameters involved in the fuel consumption function is carried out.

Second, in both mixed integer quadratic programming model and master problem model of column generation, aircraft assignment constraints are considered and added to the objective function in the form of penalty costs. When the modified label-setting algorithm obtains columns with negative reduced cost for each aircraft, the destination airport information of the corresponding aircraft after the end of the recovery plan is passed to the master problem for constraining in the master problem. In experiments, we report and analyze the effects of aircraft assignment constraints.

The remainder of the paper is as follows. Section 2 reviews the pertinent literature and highlight the research gap. In Section 3, we provide the problem description and mathematical formulation. Section 4 explains the column generation approach coupled with different acceleration strategies. We report the computational results in Section 5, followed by conclusions and ideas for future research in Section 6.

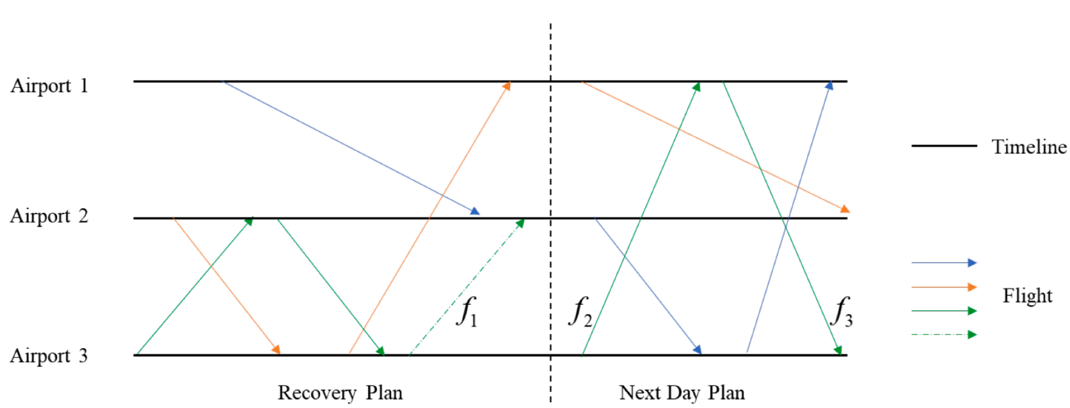

Fig. 1. Example for aircraft assignment constraints.

2. Literature review

We select a few representative articles published in this field in the last ten years as a point of reference. For a thorough review of the literature, interested readers can refer to the latest review work of Hassan et al. (2021) on airline disruption management for the years 2009 – 2020 and Clausen et al. (2010) for earlier years. We divide the literature review section into three sub-sections: First, we review the studies that do not consider cruise speed control as a recovery option. Second, we review based on the solution methodology and last, we analyze studies that accounted for the cruise speed as a recovery option.

When developing a recovery plan, the typical recovery options include flight delays (Brunner, 2014, etc.), flight cancellations (Hu et al., 2015, etc.), aircraft swaps (Hu et al., 2016, etc.), and flight creation (Jozefowiez et al., 2013; Sinclair et al., 2014; Zhang et al., 2016). In addition to this, some studies have considered airport closures (Eggenberg et al., 2010; Lee et al., 2020; Vink et al., 2020) or airport flow control (Zhang et al., 2016; Sinclair et al., 2016; Liang et al., 2018; Huang et al., 2022). This work considers flight delays, flight cancellations, aircraft swaps and maintenance delays as possible recovery options apart from cruise speed. We do not consider flight creation a recovery option due to the required complex approval process.

We can broadly divide the literature concerning the solution methodologies into heuristic and exact methods. The majority of the studies consider heuristic techniques. Eggenberg et al. (2010) proposed a heuristic-based column generation algorithm and discretized continuous timeline into time windows. Jozefowiez et al. (2013) introduced a heuristic method for aircraft recovery and passenger assignment. Similarly, Sinclair et al. (2014) studied integrated aircraft and passenger recovery problems using a large neighbourhood search (LNS) algorithm. Zhang et al. (2015) proposed a two-stage heuristic for the aircraft and crew recovery problem based on a multi-commodity network model. Hu et al. (2016) developed a heuristic methodology named “GRASP algorithm” for the aircraft and passenger recovery problem. Liang et al. (2018) proposed a column generation-based heuristic algorithm to solve the aircraft recovery problem. Vink et al. (2020) proposed a selection algorithm with MILP. Lee et al. (2020) developed an approach to optimize recovery decisions by exploiting partial and probabilistic predictions of future disruptions and proposed an approximation algorithm. Compared with heuristics, relatively, very few articles study the problem using exact methodologies. Brunner (2014) presented a linear integer programming to solve the inbound flights’ reassignment due to capacity shortage. Similarly, Hu et al. (2015) introduced an integrated integer programming model for aircraft and passenger recovery. In this work, we use exact methodologies by proposing a mixed-integer quadratic programming model and an improved column generation algorithm.

Lastly, a few studies consider cruise speed control as a recovery option. Aktürk et al. (2014) pioneered the usage of cruise speed as a decision variable while taking environmental constraints and cost into the route recovery optimization model. The model allows studying the trade-offs between flight delays and recovery costs. Based on conic mixed-integer programming, the authors formulate a nonlinear recovery optimization model that considers one swap per aircraft during the recovery period. Gürkan et al. (2016) incorporated cruise time control into a comprehensive model of scheduling, fleet assignment and aircraft routing problem within the scope of daily planning. Arıkan et al. (2016) developed a fuel cost function for flights based on Airbus’s technical report. Based on cruising speed, they comprehensively consider aircraft and passenger recovery options. The downside is that the proposed mathematical formulation is difficult to extend the model to other recovery actions flexibly. Arıkan et al. (2017) developed a flight network model based on aircraft, crew, and passenger flow for the integrated airline recovery problem. Besides, Marla et al. (2017) used a cost index-based flight planning mechanism to change the flight speed on long-haul flights. The authors define the cost index as the ratio of cost per unit time to fuel cost per unit mass. The fuel costs were based on Jeppesen’s flight planning engine. A significant drawback of this work is the consideration of fuel consumption costs for flights whose duration is only more than 3 h.

Most of the literature uses the fuel estimation model of physical model (Aktürk et al., 2014; Gürkan et al., 2016; Arıkan et al., 2016; Arıkan et al., 2017; etc.). These studies base their work on the EUROCONTROL aircraft performance model BADA (Base of Aircraft Data), which may have some limitations. The speed schedules that are part of the BADA model may differ from those used in reality (Turgut et al., 2014; Wasiuk et al., 2015; Wasiuk et al., 2016). In practice, parameters may be unavailable due to missing information (Trani and Wing-Ho, 1997). Information on some aircraft engines and types may not be available, especially new engines and types (Yanto and Liem, 2018). Unavailability of data may result in inaccurate default values for global parameters rather than proper local parameters, which may cause the results to be erroneous (Pagoni and Psaraki-Kalouptsidi, 2017; Khan et al., 2019). In the practice of an airline, the result of using physical model to estimate fuel consumption deviates from the actual situation (Khan et al., 2021). Due to commercial sensitivity, real-time aircraft status information cannot be obtained, which will make it difficult to implement the physical model (Zhang and Mahadevan, 2020). Our approach can use the fuel estimation results based on data-driven models (Yanto and Liem, 2018; Khan et al., 2019; Zhang and Mahadevan, 2020; Khan et al., 2021; etc.), which can be used as an alternative when physical models (such as BADA) are unavailable or have large deviations.

To summarize, this work fills a significant research gap in the literature by addressing two critical aspects. Firstly, without relying on the BADA model, we propose a new flexible approach for accounting for fuel consumption by discretizing the flight duration in units of one minute. This provision makes it convenient for the airlines, irrespective of the model, to obtain fuel consumption parameters based on historical flight operation data analysis, i.e., a data-driven model. This can also be used as a substitute when physical models for fuel estimation, such as BADA model, are unavailable on new aircraft or under other circumstances. Secondly, this work is the first to study the importance of aircraft assignment constraints. Quantifying the total recovery costs with and without the aircraft assignment constraints helps practitioners arrive at the most appropriate set of recovery options.

3. Problem description and formulation

Before describing the problem, we describe some of the elements involved.

Airports: The airports are the connecting nodes for the aircraft to carry out a set of consecutive flights. When an airport is unavailable, no aircraft can fly the original flight schedule from that airport. A closed airport means that the airport is closed or restricted for a period of time.

Aircraft: Each aircraft has a start and end time regarding its availability. After completing a flight, the plane will arrive at an airport and depart from the same airport for its next flight. We do not consider the option of ferrying the aircraft to other airports.

Flight: The aircraft travels from the departure airport to the arrival airport to complete a flight and has a scheduled departure and arrival time. Flights are of limited duration and delays are allowed.

Maintenance: This task requires a particular aircraft to stay at a specified airport with start and end times. The maintenance of an aircraft is identical to a specific type of flight whose origin and destination airports are the same with zero passengers. Some maintenance can be performed by replacing the aircraft, while others cannot (Vink et al., 2020).

The development of a flight recovery plan should broadly follow the flight operation mechanism. When two flights are flown sequentially by the same aircraft, the arrival airport of the previous one and the departure airport of the next one must be the same. In addition, after completing the last flight, the aircraft needs to remain on the ground for a minimum turnaround time to fly the next flight. A limited fleet of heterogeneous aircraft with different available time windows is located at different airports. Each aircraft has additional swap costs when performing flights other than the original plan. These costs may include the passenger cancellation costs because of the aircraft capacity restrictions. Different airlines may have different maintenance schedules. All the aircraft must comply with the regulations on regular maintenance and complete the maintenance within the specified time. Here, we assume that the aircraft's maintenance requirements are known a priori during the recovery period. Once a disruption occurs, one or a combination of the following five recovery options are activated to generate a recovery plan.

Changing flight duration: flight duration can be changed through aircraft speed control. Speeding up aircraft can reduce the actual duration of the flight, thereby reducing delays. As shown in Fig. 2, maintenance conflicts with the flight departure time. By delaying the flight departure time and reducing the flight duration, the actual arrival time of the flight will not be delayed. Flight duration varies with aircraft speed, along with changes in fuel consumption and carbon emissions.

Flight delays: The flight can be delayed appropriately within the maximum delay time constraint to avoid more significant costs caused by the immediate cancellation of the flight.

Maintenance delays: Maintenance can be delayed appropriately within the permissible time windows.

Flight swaps: As per the original flight plan, each flight is performed by specific aircraft. Other aircraft can execute any scheduled flight with additional swap costs in the recovery process. Since maintenance can be considered as a special flight at the same departure and arrival airports, the flight swaps include flexible maintenance swaps.

Flight cancellations: When the cost of cancelling a flight is lower than executing it with different recovery options, the airline can cancel the flight. However, flight passengers are entitled to adequate compensation in such cases.

Thus, using the five recovery options, the problem is to minimize the additional fuel consumption and carbon emission costs, costs associated with delays, costs of swapping the aircraft, flight cancellation costs and penalty costs. The next section proposes a mixed-integer quadratic programming (MIQP) model.

Mathematical model (MIQP).

Sets.

A Set of all airports;

A' Set of airports considering aircraft assignment constraints,$A' \subset A$;

P Set of all aircraft;

G Set of fleets;

F Set of all flights and maintenances;

M Set of all maintenances,$M \subset F$;

Index Set of the sequence of flights visited by the same aircraft,$Index = \{1, 2, \ldots, |F|\}$;

Tin_f Set of discrete values of allowable reduced flight duration of flight$f$, and $|Tin_f|$ is the number of discrete values of allowable reduced flight duration of flight$f$.

Indices.

a Airport index;

p Aircraft index;

g Fleet index;

f, f_1, f_2 Flight or maintenance index;

i,j Sequence of flights index;

t Discrete values index.

Parameters.

cl_f Cancellation cost of the flight f ,$\forall f \in F$;

h_{ag} Number of aircraft required of type g at the airport a at the end of the recovery period;

eh_{ag} Number of aircraft of type g at the airport a at the beginning of the original flight plan;

s_{pa} A boolean indicator equals 1 if airport a is the start airport of the aircraft p in the original plan and 0 otherwise,$\forall p \in P,\ a \in A$;

dt_f Departure time of the flight$f$, the meaning of dt_f is the start time of the maintenance if f indicates maintenance;

at_f Arrival time of the flight$f$, the meaning of at_f is the end time of the maintenance if f indicates maintenance;

gt_p Minimum turnaround time between two flights for aircraft$p$;

ft_p Type of aircraft$p$;

da_{fa} A boolean indicator equals 1 if airport a is the departure airport of the flight f and 0 otherwise,$\forall f \in F,\ a \in A$;

aa_{fa} A boolean indicator equals 1 if airport a is the arrival airport of the flight f and 0 otherwise,$\forall f \in F,\ a \in A$;

T_f^{up} Maximum flight arrival delay allowed;

$[st_p, et_p]$ Time window of the availability of the aircraft$p$;

Tcr_f Duration of the flight or maintenance f in the original plan;

$Tcrl_f$ Minimum allowed duration of the flight or maintenance f in the original plan, if f indicates maintenance, then$Tcrl_f = Tcr_f$;

ism_f A boolean indicator equals 1 if f represents maintenance and 0 otherwise;

fa_f Aircraft that is to perform flight f in the original plan, if f indicates maintenance, the meaning of fa_f is the aircraft due to for maintenance$f$;

SC_{pf} Swap cost of aircraft p to perform flight$f$, equals 0 if aircraft p is initially assigned flight$f$;

g_f Passenger delay cost of the flight f per unit time;

$cfuel$ Fuel price;

c_{co_2} Unit carbon emission cost;

rc Carbon dioxide emission constant;

fv_{pft} Changes in fuel consumption of the aircraft p performing flight f with reduced flight duration$t$;

$cruc_f$ Maximum flight duration allowed to be reduced for flight$f$;

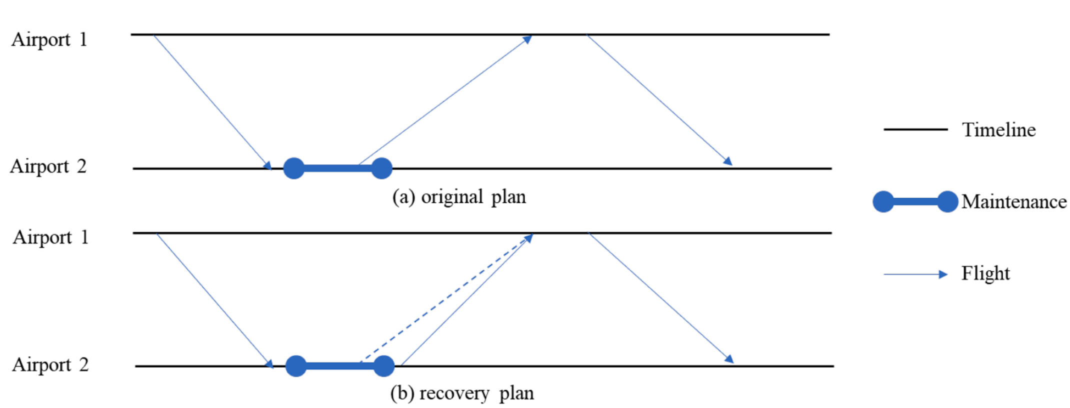

Fig. 2. An example of changing flight duration.

$bound_{pft}$ Dividing points of piecewise function of changes in fuel consumption for aircraft p performing flight$f$, and$bound_{pf0} = 0$;

$W1$ Penalties for violation of aircraft assignment constraints. When its value is large, it means that the constraints are enforced, and when it is0, it means that the constraints are ignored;

W A large number.

Decision variables.

z_{pfi} A boolean indicator equals 1 if f is the i th flight or maintenance executed by aircraft$p$, and 0 otherwise;

d_f Delay time of departure of flight$f$;

tcr_f Actual duration of flight$f$;

$wfuel_{pf}$ Actual changes in fuel consumption of aircraft p executing flight$f$, with the condition $\sum_{i\in Index} z_{pfi} = 1$ holding true;

QA_{ag} Shortage in the number of aircraft of type $g \in G$ at the airport a that does not meet the aircraft assignment constraints;

$fuel_{pf}$ Changes in fuel consumption of aircraft p executing flight$f$;

$fuely_{pft}$ A boolean indicator equals 1 if aircraft p performs flight f with reduced flight duration t and 0 otherwise.

Objective function and constraints.

$$Min \sum_{f\in F} cl_f \left(1 - \sum_{p\in P}\sum_{i\in Index} z_{pfi}\right) + \sum_{f\in F}\left(g_f\left(d_f - (Tcr_f - tcr_f)\right)\sum_{p\in P}\sum_{i\in Index} z_{pfi}\right) + \sum_{p\in P}\sum_{f\in F, ism_f\leqslant 0}(cfuel + c_{co_2} rc) wfuel_{pf} + \sum_{p\in P}\sum_{f\in F}\left(SC_{pf}\sum_{i\in Index} z_{pfi}\right) + W1\cdot\sum_{a\in A'}\sum_{g\in G} QA_{ag} \tag{1}$$

$$\sum_{f\in F} da_{fa} z_{pfi} = s_{pa}\sum_{f\in F} z_{pfi},\ \forall p\in P, a\in A, i = 1 \tag{2}$$

$$\left(at_{f_1} + d_{f_1} - (Tcr_{f_1} - tcr_{f_1}) + gt_p\right) - \left(dt_{f_2} + d_{f_2}\right) \leqslant W\left(2 - z_{pf_1 i} - z_{pf_2(i+1)}\right),\ \forall p\in P, f_1\in F, f_2\in F, f_1\neq f_2, i\in Index, i < |F| \tag{3}$$

$$eh_{ag} + \sum_{p\in P, ft_p = g}\sum_{f\in F}\left(aa_{fa}\sum_{i\in Index} z_{pfi}\right) - \sum_{p\in P, ft_p = g}\sum_{f\in F}\left(da_{fa}\sum_{i\in Index} z_{pfi}\right) + QA_{ag} \geqslant h_{ag}, \forall a\in A', g\in G \tag{4}$$

$$QA_{ag} \geqslant 0,\ \forall a\in A', g\in G \tag{5}$$

$$z_{pf_1 i} + z_{pf_2(i+1)} \leqslant 1,\ \forall p\in P, f_1\in F, f_2\in F, f_1\neq f_2, i\in Index, i<|F|, aa_{f_1 a} \neq da_{f_2 a} \tag{6}$$

$$d_f - (Tcr_f - tcr_f) \leqslant T_f^{up}, \forall f\in F \tag{7}$$

$$st_p\sum_{i\in Index} z_{pfi} \leqslant dt_f + d_f,\ \forall p\in P, f\in F \tag{8}$$

$$at_f + d_f - Tcr_f + tcr_f \leqslant et_p\sum_{i\in Index} z_{pfi} + W\left(1 - \sum_{i\in Index} z_{pfi}\right),\ \forall p\in P, f\in F \tag{9}$$

$$Tcrl_f\sum_{p\in P}\sum_{i\in Index} z_{pfi} \leqslant tcr_f \leqslant Tcr_f, \forall f\in F \tag{10}$$

$$d_f \geqslant 0, \forall f\in F \tag{11}$$

$$\sum_{i\in Index} z_{pfi} = 1, \forall p\in P, f\in F, ism_f = 1, p = fa_f \tag{12}$$

$$d_f - (Tcr_f - tcr_f) \geqslant W\cdot\left(\sum_{p\in P}\sum_{i\in Index} z_{pfi} - 1\right), \forall f\in F \tag{13}$$

$$\sum_{f\in F} z_{pfi} \leqslant 1,\ \forall p\in P, i\in Index \tag{14}$$

$$\sum_{f\in F} z_{pfj} \leqslant \sum_{f\in F} z_{pfi},\ \forall p\in P, i\in Index, j\in Index, i<j \tag{15}$$

$$tcr_f \leqslant W\cdot\sum_{p\in P}\sum_{i\in Index} z_{pfi},\ \forall f\in F, ism_f = 0 \tag{16}$$

$$d_f \leqslant W\cdot\sum_{p\in P}\sum_{i\in Index} z_{pfi},\ \forall f\in F, ism_f = 0 \tag{17}$$

$$wfuel_{pf} = \begin{cases} fv_{pf0}\sum_{i\in Index} z_{pfi} & Tcr_f\sum_{p\in P}\sum_{i\in Index} z_{pfi} - tcr_f = bound_{pf0} \\ fv_{pf1}\sum_{i\in Index} z_{pfi} & bound_{pf0} < Tcr_f\sum_{p\in P}\sum_{i\in Index} z_{pfi} - tcr_f \leqslant bound_{pf1} \\ \cdots & \cdots \\ fv_{pft}\sum_{i\in Index} z_{pfi} & bound_{pf(t-1)} < Tcr_f\sum_{p\in P}\sum_{i\in Index} z_{pfi} - tcr_f \leqslant bound_{pft} \\ \cdots & \cdots \\ fv_{pf|Tin_f|-1}\sum_{i\in Index} z_{pfi} & bound_{pf(|Tin_f|-2)} < Tcr_f\sum_{p\in P}\sum_{i\in Index} z_{pfi} - tcr_f \leqslant bound_{pf(|Tin_f|-1)} \end{cases}, \forall p\in P, f\in F, t\in Tin_f, t\leqslant cruc_f \tag{18}$$

$$z_{pfi} \in \{0,1\}, \forall p \in P, f \in F, i \in Index \tag{19}$$

The Objective function (1) minimizes the sum of the costs associated with flight cancellations, flight delays, changes in fuel consumption & carbon emission costs, the swap costs of executing unplanned flights, and the penalty costs for violation of aircraft assignment constraints. Constraint (2) stipulates that every aircraft departs for its first flight from its start airport. Constraint (3) forbids conflicts in flight schedules. When an aircraft executes a series of flights, these flights will not conflict with each other in time, which ensures the feasibility of the aircraft executing the flight plan. Equalities (4) and (5) report the shortage between the number of aircraft at each airport after the recovery plan ends and the actual number required. Constraint (6) ensures flow conservation. When the aircraft takes two flights in sequence, the arrival airport of the former flight should be the same as the departure airport of the latter flight. Constraint (7) implies that the delay associated with any flight is within its permissible limits. Constraints (8) and (9) account for the availability time windows of an aircraft. The upper and lower limits of any flight duration (actual) are accounted for by constraint (10). Constraint (11) ensures that no aircraft departs earlier than its stipulated time. Constraint (12) forbids maintenance tasks from cancellation. Constraint (13) prohibits the early arrival of any flight f. Constraints (14) and (15) enforce time–space constraints by assigning only one flight f to an aircraft p at any point in time. Constraints (16) and (17) imply that a cancelled flight's actual flight duration and departure delay are zero. Constraint (18) represents the functional relationship between the reduced flight duration and the changes in fuel consumption in terms of a piecewise function. The piecewise function is fitted according to the estimated fuel consumption data of the aircraft to determine the split point and function value. The reduced flight duration is the difference between the initially planned duration and the exact duration of the flight. Constraint (19) indicates the binary nature of the variables.

Since Constraint (18) is nonlinear, we linearize it to obtain the following constraints so that the model can be solved by commercial solver.

$$\sum_{t \in Tin_f, t \leqslant cruc_f} fuely_{pft} \leqslant 1, \ \forall p \in P, f \in F, ism_f \leqslant 0 \tag{20}$$

$$fuel_{pf} = \sum_{t \in Tin_f, t \leqslant cruc_f} fv_{pft} fuely_{pft}, \ \forall p \in P, f \in F, ism_f \leqslant 0 \tag{21}$$

$$Tcr_f \sum_{p \in P} \sum_{i \in Index} z_{pfi} - tcr_f \leqslant \sum_{t \in Tin_f, t \leqslant cruc_f} bound_{pft} fuely_{pft}, \ \forall p \in P, f \in F, ism_f \leqslant 0 \tag{22}$$

$$Tcr_f \sum_{p \in P} \sum_{i \in Index} z_{pfi} - tcr_f > \sum_{t \in Tin_f, 1 \leqslant t \leqslant cruc_f} bound_{pf(t-1)} fuely_{pft}, \ \forall p \in P, f \in F, ism_f \leqslant 0 \tag{23}$$

$$wfuel_{pf} \leqslant fuel_{pf}, \ \forall p \in P, f \in F, ism_f \leqslant 0 \tag{24}$$

$$wfuel_{pf} \geqslant fuel_{pf} - W \cdot \left(1 - \sum_{i \in Index} z_{pfi}\right), \ \forall p \in P, f \in F, ism_f \leqslant 0 \tag{25}$$

$$fuely_{pft} \in \{0,1\}, \ \forall p \in P, f \in F, t \in Tin_f, t \leqslant cruc_f, ism_f \leqslant 0 \tag{26}$$

$$wfuel_{pf} \geqslant 0, \ \forall p \in P, f \in F, ism_f \leqslant 0 \tag{27}$$

Constraint (20) means that the value of the change of flight duration can only be in one interval. Constraint (21) represents the changes in fuel consumption that will result when the aircraft p performs flight f and the change of flight duration is in the interval corresponding to $fuely_{pft} = 1$. Constraints (22) and (23) enforce the interval constraints of the change of flight duration. Constraint (24) and Constraint (25) represent the actual changes in fuel consumption for the flight f actually performed by the aircraft p. Constraint (26) indicates the binary nature of the variables. Constraint (27) states that the actual fuel consumption change is non-negative.

4. Improved column generation

After preliminary experiments, the proposed model could only solve small problem instances to optimality in real-time. We develop a column generation-based solution methodology to overcome this drawback for quickly solving even large problem instances. We model the master problem as a set-partitioning problem, with the subproblem being a variant of the shortest path problem. The detailed flow chart of the improved column generation algorithm is illustrated in Fig. 3.

4.1. Master problem formulation

Sets.

R Set of routes, where a route refers to a series of flights and maintenance executed by the same aircraft.

Indices.

r Route index.

Parameters.

α_{pfr} A boolean indicator equals 1 if flight f belongs to the route r of the aircraft p and 0 otherwise;

β_{par} A boolean indicator equals 1 if the end airport of the aircraft p along the route r is airport a and 0 otherwise;

c_{pr} Cost of the route r of the aircraft p.

Decision variables.

x_{pr} A boolean indicator equals 1 if the aircraft p performs route r and 0 otherwise;

y_f A boolean indicator equals 1 if the flight f is cancelled and 0 otherwise.

q_{ag} Shortage of aircraft that the recovery plan does not meet the aircraft assignment constraints.

Model.

$$Min \sum_{r \in R} c_{pr} x_{pr} + \sum_{f \in F} cl_f y_f + W1 \cdot \sum_{a \in A'} \sum_{g \in G} q_{ag} \tag{28}$$

$$\sum_{p \in P} \sum_{r \in R} \alpha_{pfr} x_{pr} + y_f = 1, \ \forall f \in F \tag{29}$$

$$\sum_{p \in P, ft_p = g} \sum_{r \in R} \beta_{par} x_{pr} + q_{ag} \geqslant h_{ag}, \ \forall a \in A', g \in G \tag{30}$$

$$\sum_{r \in R} x_{pr} \leqslant 1, \ \forall p \in P \tag{31}$$

$$x_{pr} \in \{0,1\}, \ \forall p \in P, \forall r \in R \tag{32}$$

$$y_f \in \{0,1\}, \ \forall f \in F \tag{33}$$

$$q_{ag} \geqslant 0, \forall a \in A', g \in G \tag{34}$$

The objective function (28) minimizes the route costs, flight cancellation costs and the penalty costs for violation of aircraft assignment constraints. The route costs account for flight delay costs, swap costs, changes in fuel consumption and carbon emissions costs. Equality (29) stipulates that each flight must be included in just one route when not cancelled. Constraint (30) enforces aircraft assignment constraints.

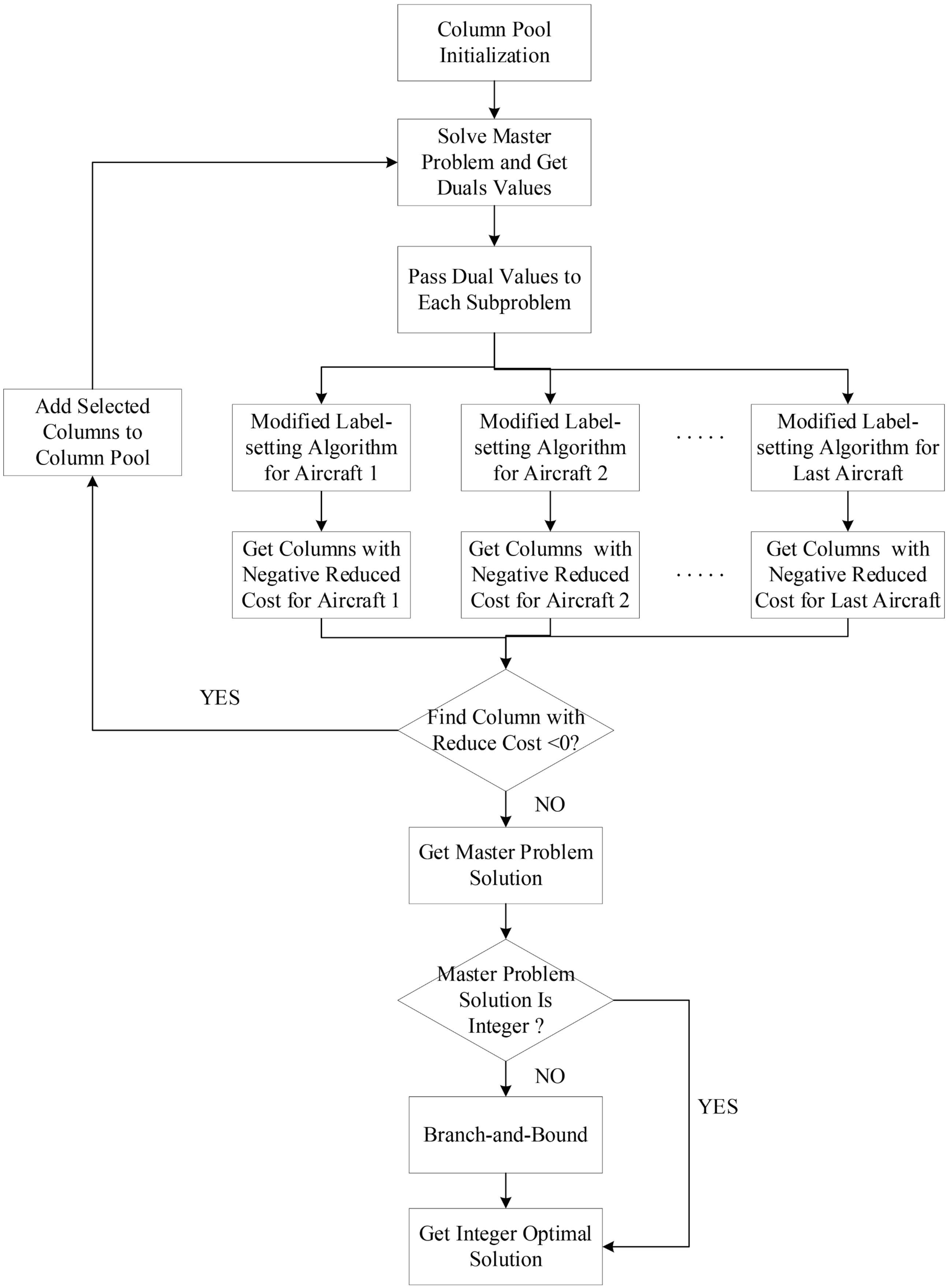

Fig. 3. The flow chart of the improved column generation algorithm.

Constraint (31) forbids any aircraft from flying more than one route. Constraints (32), (33) and (34) indicate the variables value constraints.

4.2. Pricing

Relaxing the integer constraints leads to a linear master problem (LMP). Let π_f^1 be the dual variable of the Constraints (29) for flight$f$, π_{ag}^2 be the dual variable of the Constraints (30) for the airport$a$, π_p^3 be the dual variable of the Constraints (31) for aircraft$p$. For the route r of aircraft p of type$g$, the reduced cost $\bar{c}_{pr}$ is.

$$\bar{c}_{pr} = c_{pr} - \sum_{f\in F}\alpha_{pfr}\pi_f^1 - \sum_{a\in A'}\beta_{par}\pi_{ag}^2 - \pi_p^3 \tag{35}$$

$$c_{pr} = \sum_{f\in F}\left(c_{pf}^{swap} + c_{pf}^{delay} + c_{pf}^{fuel}\right)\alpha_{pfr} \tag{36}$$

In Equation (36), the cost c_{pr} is the sum of the swap costs, delay costs, changes in fuel consumption and carbon emission costs. For a given aircraft$p$, c_{pf}^{swap} is the swap costs, c_{pf}^{delay} is the delay costs and c_{pf}^{fuel} is the additional fuel consumption and carbon emission costs of the flight$f$.

Inserting Equation (36) into Equation (35), we have.

$$\bar{c}_{pr} = \sum_{f\in F}\alpha_{pfr}\left(c_{pf}^{swap} + c_{pf}^{delay} + c_{pf}^{fuel} - \pi_f^1\right) - \sum_{a\in A'}\beta_{par}\pi_{ag}^2 - \pi_p^3 \tag{37}$$

Therefore, the objective function of the subproblem is.

$$\begin{aligned} Min\sum_{f\in F}\alpha_{pfr}SC_{pf} + \sum_{f\in F}\alpha_{pfr}g_f\left(d_f - \left(Tcr_f - tcr_f\right)\right) + \sum_{p\in P}\sum_{f\in F}\alpha_{pfr}(cfuel \\ + c_{co_2}rc)wfuel_{pf} - \sum_{f\in F}\alpha_{pfr}\pi_f^1 - \sum_{a\in A'}\beta_{par}\pi_{ag}^2 - \pi_p^3 \end{aligned} \tag{38}$$

4.3. Solving pricing

The column generation process is to iteratively solve the pricing problem, finding routes with negative reduced cost to add to the restricted linear master problem (RLMP). Since there are $|P|$ aircraft, we solve an equal number of subproblems at each iteration. We continue the iterations until all the found routes have a positive reduced cost. At the same time, we solve the RLMP optimally. The subproblems are formulated as a particular instance of shortest path problems on a connected network that a modified label-setting algorithm can solve.

Quite a few studies use labelling algorithms for flight recovery problems (Eggenberg et al. 2010, Sinclair et al. 2016, Liang et al. 2018). Drawing inspiration from them, we developed a modified label-setting algorithm based on dynamic programming to solve multiple subproblems. In our proposed modified label-setting algorithm, we first consider a directed network$G(V,E)$. For any aircraft$p$, nodes i and j can be directly connected if the arrival airport of flight i and the departure airport of flight j are identical. Similarly, nodes 0 and i can be directly connected if the start airport of the aircraft p is the same as that of the departure airport of the flight i.

The state $S = (i, rt, e, Nv, De, Cr, fu)$ represents a partial path that arrives at the node i with the current landing airport e and is available to the next node at a timert. Nv is the set of flights that are covered. De is the set of departure delay times of the covered flights and Cr is a set of the duration of the flights represented by the visited nodes. fu is the changes in fuel consumption. Furthermore, c denotes the cost of the state$S = (i, rt, e, Nv, De, Cr, fu)$, and $\bar{c}$ indicates the reduced cost. When an aircraft visits the next flight j via arc (i,j) from the current state$S = (i, rt, e, Nv, De, Cr, fu)$, we deduce the new state$S' = (j, rt', e', Nv', De', Cr', fu')$. Moreover, c' is the cost of the new stateS', and $\bar{c}'$ is the reduced cost. We propose the extending algorithm with the following two conditions to solve the subproblem. Some additional decision variables are introduced as follows:

Decision variables.

De_f Departure delay times of the flight$f$;
Cr_f Actual duration of flight$f$;
fu_{pf} Changes in fuel consumption of aircraft p executing the flight$f$.

Condition (1). Consider the first condition $rt \leqslant dt_j$ with no delays on the flight$j$. The aircraft does not need to accelerate to reduce the flight time, and hence there is no changes in fuel consumption. In this case, the relationships between the state $S = (i, rt, e, Nv, De, Cr, fu)$ and the new state $S' = (j, rt', e', Nv', De', Cr', fu')$ are as follows:

1. $rt' = at_j + gt_p$;
2. $e' = a, \forall a \in A, aa_{fa} = 1$;
3. $Nv' = Nv \cup \{j\}$;
4. $De_j' = 0,\ De' = De \cup \left\{De_j'\right\}$;
5. $Cr_j' = Tcr_j,\ Cr' = Cr \cup \left\{Cr_j'\right\}$;
6. $fu' = fu$;
7. $c' = c + SC_{pj}$;
8. $\bar{c}' = \bar{c} + SC_{pj} - \pi_j^1$.

Condition (2). We consider the second condition, i.e.,$rt > dt_j$. Let pre be the visited flight node before node$i$. If $i = 0$ and$at_i + gt_p \leqslant dt_j + cruc_j + T_j^{up}$, or $i \neq 0$ and$at_{pre} + gt_p + Tcrl_i \leqslant dt_j + cruc_j + T_j^{up}$, the following linear program for aircraft p and route r needs to be solved to determine the variables of the new state.

LP (PR).

$$Min \sum_{k\in Nv\cup\{j\}} g_k(De_k - Tcr_k + Cr_k) + \sum_{k\in Nv\cup\{j\}} (cfuel + c_{co_2}rc)fu_{pk} \tag{39}$$

$$s.t.\ at_k + De_k - Tcr_k + Cr_k + gt_p - dt_l - De_l \leqslant 0,\ \forall k, l \in Nv\cup\{j\}, k < l \tag{40}$$

$$De_k - Tcr_k + Cr_k \leqslant T_k^{up},\ \forall k \in Nv\cup\{j\} \tag{41}$$

$$dt_k + De_k \geqslant st_p,\ \forall k \in Nv\cup\{j\} \tag{42}$$

$$at_k + De_k - Tcr_k + Cr_k \leqslant et_p,\ \forall k \in Nv\cup\{j\} \tag{43}$$

$$Cr_k \geqslant Tcrl_k,\ \forall k \in Nv\cup\{j\} \tag{44}$$

$$Cr_k \leqslant Tcr_k,\ \forall k \in Nv\cup\{j\} \tag{45}$$

$$De_k - Tcr_k + Cr_k \geqslant 0,\ \forall k \in Nv\cup\{j\} \tag{46}$$

$$\sum_{t\in Tin_k, t\leqslant cruc_k} fuely_{pkt} = 1,\ \forall k \in Nv\cup\{j\}, ism_k \leqslant 0 \tag{47}$$

$$fu_{pk} = \sum_{t\in Tin_k, t\leqslant cruc_k} fv_{pkt}fuely_{pkt},\ \forall k \in Nv\cup\{j\}, ism_k \leqslant 0 \tag{48}$$

$$Tcr_k - Cr_k \leqslant \sum_{t\in Tin_k, t\leqslant cruc_k} bound_{pkt}fuely_{pkt},\ \forall k \in Nv\cup\{j\}, ism_k \leqslant 0 \tag{49}$$

$$Tcr_k - Cr_k > \sum_{t\in Tin_k, 1\leqslant t\leqslant cruc_k} bound_{pk(t-1)}fuely_{pkt},\ \forall k \in Nv\cup\{j\}, ism_k \leqslant 0 \tag{50}$$

$$fuely_{pkt} \in \{0,1\},\ \forall k \in Nv\cup\{j\}, \forall t \in Tin_k, t \leqslant cruc_k \tag{51}$$

The objective function (39) minimizes the costs associated with flight delays and additional fuel consumption and carbon emission costs. Constraint (40) stipulates the time required between two adjacent flight nodes. Constraint (41) ensures that the arrival delay of a flight does not exceed the maximum delay. Constraints (42) and (43) ensure that all the flights are completed within the available time of the aircraft. Constraints (44) - (46) enforce the lower and upper limits of any flight duration. Constraints (47) - (50) linearize the discrete function between the reduced flight duration and the changes in fuel consumption.

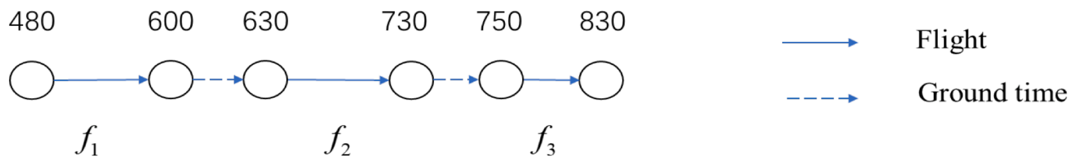

Fig. 4. Example for LP (PR).

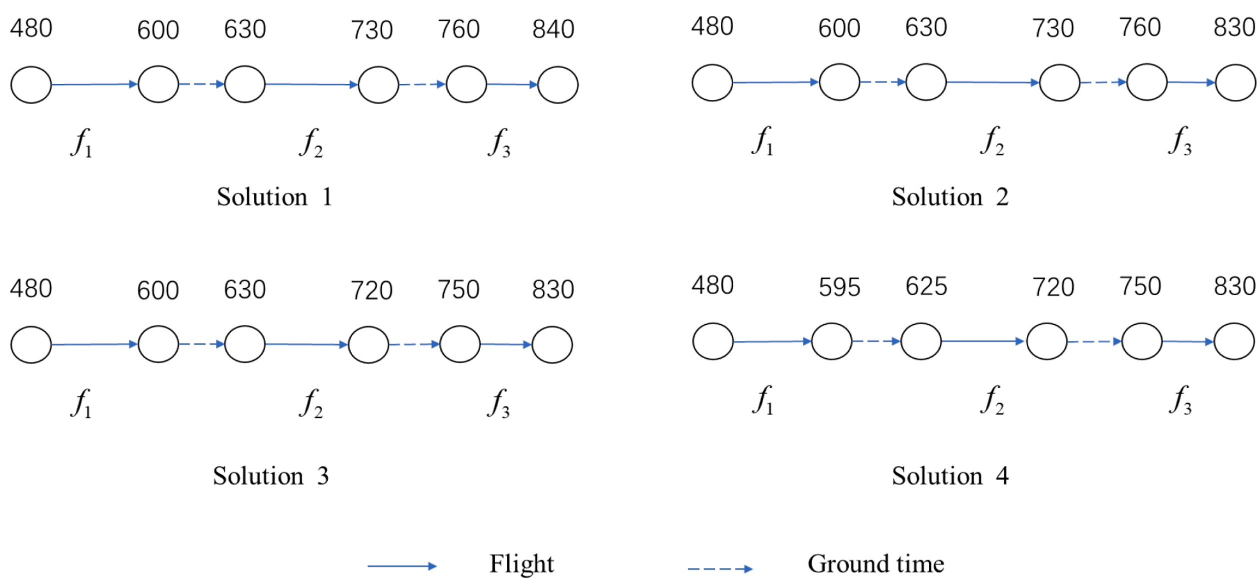

Fig. 5. Four feasible solutions for the example.

Constraint (51) indicates the binary nature of the variables.

Next, an example is used to explain why LP (PR) needs to be solved, as shown in Fig. 4. The minimum ground time between flightsf_1, f_2 and f_3 is 30 min. The departure times of flight 1 and flight 2 have been partially delayed. After the label visit flights 1 and 2 in turn, the time between flights 2 and 3 is less than the minimum ground time.

Let us suppose the flight duration cannot be reduced. In such a scenario, the only solution is to delay the departure time of flight 3 by 10 min, thereby delaying its arrival by 10 min, as shown in Solution 1 in Fig. 5. This is similar to the traditional shortest path problem. However, if the duration of flights 1, 2, and 3 can be reduced, there may be multiple feasible solutions, of which only four are listed in Fig. 5. Delay flight 3 departure time and reduce flight 3 duration, as shown in Solution 2. Reduce the flight duration by 10 min so that flight 3 will not be delayed, as shown in Solution 3, and reduce the duration of flight 1 and flight 2, as shown in Solution 4. This involves how to assign those 10 min to the delay of flight 3 or the reduced flight duration of flights 1, 2, and 3 to minimize the total cost. Therefore, we obtain the optimal path by solving LP (PR).

By solving the LP(PR), we can obtain the values ofDe_k, Cr_k and fu_{pk} for the flight node $k \in Nv \cup \{j\}$ and the objective value obj_{pr} of LP (PR). The relationships between the state $S = (i, rt, e, Nv, De, Cr, fu)$ and the new state $S^{'} = (j, rt^{'}, e^{'}, Nv^{'}, De^{'}, Cr^{'}, fu^{'})$ are as follows:

1. $rt^{'} = at_j + gt_p + De_j - Tcr_j + Cr_j$;
2. $e^{'} = a, \forall a \in A, aa_{fa} = 1$;
3. $Nv^{'} = Nv \cup \{j\}$;
4. $De^{'} = \{De_k | k \in Nv \cup \{j\} \}$;
5. $Cr^{'} = \{Cr_k | k \in Nv \cup \{j\} \}$;
6. $fu^{'} = \sum_{k \in Nv \cup \{j\}} fu_{pk}$;
7. $c^{'} = obj_{pr} + \sum_{k \in Nv^{'}} SC_{pk}$;
8. $\bar{c}^{'} = c^{'} - \sum_{k \in Nv^{'}} \pi_k^1$.

Let $L = (i, rt, e, Nv, De, Cr, fu, c, \bar{c})$ be a label to store the state. The extending algorithm first generates labels from a dummy source node. Each time, unexpanded labels with minimal reduced costs are selected to visit a new flight node to generate a new label, until all the labels are expanded. The relationships between the new label $L^{'} = (j, rt^{'}, e^{'}, Nv^{'}, De^{'}, Cr^{'}, fu^{'}, c^{'}, \bar{c}^{'})$ and the original label $L = (i, rt, e, Nv, De, Cr, fu, c, \bar{c})$ refer to the states $S^{'}$ and S as mentioned before.

After deriving the new labels, the dominance rules are used to discard the labels that cannot bear better solutions. The dominance rules are as follows:

Label $L^{'}$ dominates L if.

1. $j = i$;
2. $\bar{c}^{'} < \bar{c}$;
3. $rt^{'} \leqslant rt$.

Rule 1 guarantees that two labels both rooted at the same node. Rule 2 confirms that the negative reduced cost of $L^{'}$ is smaller than$L$. Rule 3 means that the available time to the next node in the label $L^{'}$ is not greater than that in the label$L$.

After a label has been expanded, the reduced cost will be updated with π_{ag}^2 andπ_p^3. Then if the reduced cost is negative, the label will be stored and waiting to be selected into the column pool.

Algorithm 1 presents the modified label-setting algorithm. The modified label-setting algorithm extends labels from the dummy source node and generates all the non-dominated labels with time complexity$O(n^2)$, where n is the number of nodes.

Algorithm 1 The modified label-setting algorithm

Input:$p, \pi_f^1, \pi_{ag}^2, \pi_p^3, i, j \in \{0, 1, ..., |F| + 1\}$,etc.
Output:$\mho_l$;
Initialization:

(continued on next page)

(*continued*)

Algorithm 1 The modified label-setting algorithm

$\mho \leftarrow \varnothing$, $\mho$ is the set of labels
$\mho_l \leftarrow \varnothing$
for $i = 1$ to $|F|$ **do**
Initialize the value of attributes in L using the$(0, i)$.
$\mho \leftarrow \mho \cup \{L\}$
end for

for (L in $\mho$)
Select a label with the minimum negative reduced cost $\bar{c}$ that has not been visited.
The label belongs to node$i$. Let pre be the visited flight node before node$i$.
for ($j = 1$ to $|F|$) **do**
if($e = da_j, rt \leqslant dt_j$)
Create a new label L_1 using L and(i, j).
Initialize the value of attributes inL_1.
if ($e = da_j, rt > dt_j$)
if ($i = 0, at_i + gt_p \leqslant dt_j + cruc_j + T_j^{up}$, or $i \neq 0, at_{pre} + gt_p + Tcrl_i \leqslant dt_j + cruc_j + T_j^{up}$)
Create a new label L_2 using L and(i, j).
Solve LP (PR) and initializeL_2.
end if
end if
Check if L_1 or L_2 is dominated. The dominated labels are discarded.
if L_1 or L_2 is not dominated
$\mho \leftarrow \mho \cup \{L_1\}$ or$\mho \leftarrow \mho \cup \{L_2\}$.
end if
if L_s is dominated by L_1 orL_2, where $L_s \in \mho$
$\mho \leftarrow \mho \setminus \{L_s\}$
end if
end for
update $\bar{c}$ withπ_{ag}^2, π_p^3.
if ($\bar{c} < 0$)
$\mho_l \leftarrow \mho_l \cup \{L_l\}$.
end if
end for
return $\mho_l$

4.4. Initializing columns

The approach employed to initialize columns significantly influences the computational time of the column generation algorithm. We propose three ways to generate the initial columns of the RLMP - the heuristic insertion algorithm, the basic route generation algorithm, and the shortest path algorithm. The heuristic insertion algorithm is a greedy procedure. In this, we sort different flights chronologically based on their departure times and sequentially assign them to aircraft without violating feasibility. During the assignment process, maintenance tasks carry higher priority. The basic route generation algorithm assigns feasible single flight routes for different aircraft and eventually reconciles the flights to obtain viable solutions. Lastly, while generating columns using the shortest path algorithm, we use the modified label-setting algorithm to solve the subproblem and set the corresponding dual variables to 0. The disadvantage of this method is that it may take a relatively long time.

4.5. Accelerating strategies

4.5.1. Limiting the number of flights performed by aircraft

Let nf_p be the number of flights performed by aircraft p in the original plan and$NF = \max\left\{nf_p, p \in P\right\}$. When we solve the subproblem, we set an upper limit on the number of flights executed by the aircraft toNF. The number of flights in the column obtained by this method cannot exceedNF. Columns with more than NF flights and negative reduced cost are ignored. If the above method does not result in a column with negative reduced cost, we remove the upper limit on the number of flights and solve it again to obtain the exact solution to the subproblem.

4.5.2. Adding multiple negative reduced-cost columns

Adding multiple negative reduced-cost columns is a commonly used strategy to improve column generation (Barnhart and Cohn, 2004). Each time the subproblems are solved, the modified label-setting algorithm reserves three negative reduced-cost columns and adds them to the RLMP. Only negative reduced-cost columns are reserved, and the three most negative are selected to be added to the RLMP.

4.5.3. Algorithm parallelization

The subproblems can be solved in parallel as they are independent of each other (Liang et al., 2018). Each time the subproblems are solved, the modified label-setting algorithm will run $|P|$ times, where $|P|$ represents the number of aircraft. This process is independent of each other and can be run in parallel. Therefore, we parallelize the subproblem solving process to accelerate the convergence.

4.6. Branch-and-price algorithm

The branch-and-price algorithm is a combination of column generation and branch-and-bound algorithm. The column generation algorithm obtains the optimal solution of the LMP through constant iterations between the RLMP and the subproblem. If the solution of the LMP is fractional, use the branching strategy to obtain an integer solution. If the solution is an integer, the obtained solution is optimal.

If the solution is fractional, we use a best-first search strategy to explore the branch-and-bound tree and branch on $\sum_{r \in R} \alpha_{pfr} x_{pr}$ closest to 0.5. Two child nodes are created by fixing $\sum_{r \in R} \alpha_{pfr} x_{pr} = 1$ that means the flight f must be performed by aircraft$p$, and $\sum_{r \in R} \alpha_{pfr} x_{pr} = 0$ that means the flight f is prohibited from being performed by aircraft$p$. If the flight f must be performed by aircraft$p$, any edge (p', f) where $p' \neq p$ will be deleted, and columns related to those edges the will be removed. If the flight f is prohibited from being performed by aircraft$p$, edge (p, f) will be deleted, and columns related to the edge (p, f) will be removed.

5. Computational experiments

5.1. Experimental setup

The sponsor of this consultancy project provided us with scenarios/instances based on the actual operational data of an airline. While we used the CPLEX 12.10 version to solve the RLMP, we coded the proposed column generation approach in C++. We conducted all the experiments on a laptop with 16 GB RAM, a 2.4 GHz Intel i5-9300H CPU and a 64-bit Windows 10 operating system. Based on the computer configuration, 8 threads were used for parallelization. We set a computational time limit of 1 h (3600 s) for both approaches. Table 1 summarizes the characteristics of the nine scenarios. The continuous closure time of the airport is 200 to 600 min. From a practitioner's point of view, scenarios 1, 2 and 3 are small-sized, scenarios 4, 5 and 6 are medium-sized, and scenarios 7, 8 and 9 are enormous.

The recovery horizon is within one day. The delay cost is related to passengers. The swap cost is a matrix that includes the penalty cost when the number of seats in the aircraft cannot cover all passengers when the swap occurs. Airlines usually provide the cost parameters such as delay

Table 1
Characteristics of the scenarios.

Scenario	No. of flights	No. of fleets	No. of maintenances	No. of aircrafts	No. of airports	No. of closed airports
1	11	1	1	2	5	0
2	20	1	1	5	11	1
3	42	2	2	10	21	1
4	63	3	3	14	27	2
5	72	3	3	16	28	2
6	94	3	4	22	31	2
7	121	4	6	27	38	2
8	147	5	6	33	41	3
9	172	5	7	38	45	3

Table 2
Computational results.

Scenario	CPLEX		CG				GAP
	CP obj.	CP time (s)	LP obj.	IP obj.	CG iterations	CG time (s)	
1	1387.26	2.15	1387.26	1387.26	10	0.89	0.00 %
2	19703.40	4.53	19703.40	19703.40	8	1.28	0.00 %
3	20360.12	74.49	20360.12	20360.12	13	4.15	0.00 %
4	80745.42	694.82	80745.42	80745.42	12	7.45	0.00 %
5	52446.68	1653.66	52446.68	52446.68	13	9.70	0.00 %
6	–	–	81251.98	81251.98	28	125.08	–
7	–	–	144601.54	144601.54	23	227.06	–
8	–	–	143318.22	143318.22	48	1005.53	–
9	–	–	150319.99	150319.99	63	2375.70	–
Average Gap	–	–	–	–	–	–	0.00 %

cost per minute per passenger, swap cost for different flights, cancellation cost per flight, changes in fuel consumption for different flight durations, etc. These values vary based on the airline and the geographies they operate. Owing to the non-disclosure agreement (NDA) signed with the airline, we desist from presenting the actual values of these parameters here. However, in line with the inputs received from practitioners, we have chosen to assign the following approximate values to different parameters:

- g_f: Passenger delay cost per unit time – \$ 5–15 per minute
- cl_f: Flight cancellation cost – \$ 8,000–10,000
- SC_{pf}: Swap cost – \$ 8–200 for swappable aircraft and ∞ for non-swappable aircraft
- $W1$: Penalties for violation of aircraft assignment constraints – \$ 10,000,000
- $cfuel$: Fuel price – \$ 0.6 per Kilogram
- rc: Carbon dioxide emission constant (amount of CO2 produced for every Kilogram of fuel burn) – 3.15
- c_{co_2}: Unit carbon emission cost – \$ 0.02 per Kilogram
- gt_p: Minimum turnaround time between two consecutive flights – 30–40 min
- T_f^{up}: Maximum acceptable arrive delay time for any flight and maintenance – 180 min by default
- Percentage of the Maximum flight duration allowed to be reduced for any flight – 10 % – 15 %

Airport closures directly affect flight availability. Here, the data is preprocessed to reflect airport closures directly on flights to reduce model complexity. For example, an airport is closed from 8:00 am to 12:00 am. If the original scheduled departure or arrival time of a flight is within the airport closing time window, the departure time of the flight will be postponed. If the resulting delay exceeds the default maximum allowable arrival delay time, then the flight will be cancelled directly. Otherwise, the flight will be held and scheduled, and the maximum allowable arrival delay time T_f^{up} is modified accordingly.

Table 3
Share of each cost component in the total costs.

Scenario	Cancellations costs	Delays costs	Fuel & carbon emission costs	Swap costs
1	0.00	1034.90	297.36	55.00
2	18105.00	1561.80	36.60	0.00
3	9160.00	8686.80	744.22	1769.10
4	74040.00	5765.40	587.82	352.20
5	45895.00	3878.40	353.78	2319.50
6	65060.00	11675.40	1190.88	3325.70
7	131125.00	8869.40	1475.24	3131.90
8	131125.00	7453.80	1220.12	3519.30
9	131315.00	10887.90	2021.49	6095.60

Table 4
Run time of different accelerating strategies.

Scenario	Basic	Limiting flight number	Multiple columns	Parallelization	All
1	1.75	1.71	1.25	1.96	0.89
2	2.45	2.25	1.34	1.56	1.28
3	15.03	10.54	4.52	5.29	4.15
4	26.65	23.98	14.16	11.39	7.45
5	51.48	29.66	19.78	21.79	9.70
6	577.53	516.92	324.28	237.06	125.08
7	1302.67	1240.10	744.42	796.31	227.06
8	–	–	2479.98	2397.41	1005.53
9	–	–	–	–	2375.70

5.2. Computational results

Table 2 lists the computational results obtained. The column “CP obj.” is the objective function value of the MIQP model as reported by CPLEX, with “CP time (s)” indicating the run time. Columns “LP obj.” and “IP obj.” represent the solution of RLMP and integer solution of the proposed column generation algorithm, respectively. Columns “CG iterations” and “CG time (s)” report the number of iterations of the subproblem and the run time of the column generation algorithm, respectively. Lastly, the column “GAP” reports the optimality gap, where GAP can be calculated with$(\text{IP obj.} - \text{CP obj.})/\text{CP obj.} \times 100\backslash\%$.

The results show that CPLEX converges to optimality in the stipulated time for only 5 out of 9 scenarios. Scenarios 6–9 cannot be solved with MIQP by CPLEX since the model definition phase exceeds the computer’s memory. On the contrary, the proposed column generation approach takes a tiny fraction of the CPLEX’s computational time and converges for all nine scenarios in less than 1 h. We can observe that the “LP obj” and “IP obj” values are identical for all nine scenarios indicating zero optimality gap, which means that there is no branch-and-bound process. These results establish the strength and validity of the proposed column generation approach and reflect its suitability for handling real-life operational scenarios. Table 3 provides flight cancellations costs, flight delays costs, changes in fuel consumption & carbon emission costs and swap costs of each scenario. Evidently, in most scenarios, the cancellation costs dominate the rest.

5.3. Sensitivity analysis

5.3.1. Different accelerating strategies

In Section 4.5, we describe three accelerating strategies to expedite the convergence of the column generation approach. We test them thoroughly to assess their relevance with a computational time limit of 3600 s. In Table 4, the column “Basic” implies the absence of any acceleration strategy. Similarly, “Limiting flight number” means deploying the acceleration strategy described in Section 4.5.1. “Multiple columns”

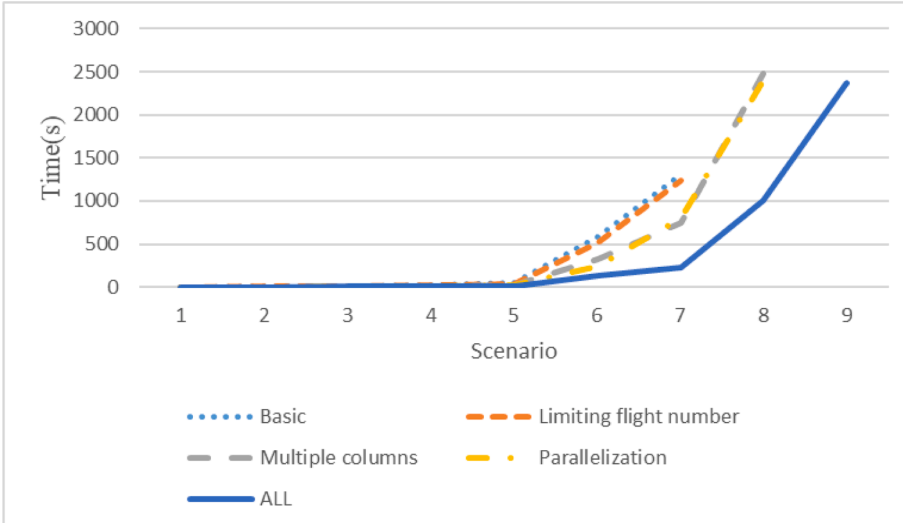

Fig. 6. Run time of different accelerating strategies.

Table 5
Results of different numbers of dividing points.

Scenario	5		21		Time gap	Obj. gap
	Obj.	Time (s)	Obj.	Time (s)		
1	1822.61	0.67	1387.26	0.89	32.84 %	–23.89 %
2	19706.99	0.81	19703.40	1.28	58.02 %	−0.02 %
3	20423.19	3.75	20360.12	4.15	10.67 %	−0.31 %
4	80756.33	4.89	80745.42	7.45	52.35 %	−0.01 %
5	52471.38	12.34	52446.68	9.70	−21.39 %	−0.05 %
6	81315.42	88.78	81251.98	125.08	40.89 %	−0.08 %
7	144637.66	698.26	144601.54	227.06	−67.48 %	−0.02 %
8	143346.22	2144.20	143318.22	1005.53	−53.10 %	−0.02 %
9	150380.28	2208.81	150319.99	2375.70	7.56 %	−0.04 %
Average Gap	–	–	–	–	6.71 %	−2.72 %

Table 6
Results without and with aircraft assignment constraints.

Scenario	without aircraft assignment		with aircraft assignment			with aircraft assignment (80 %)		
	Obj.	Time (s)	Obj.	Time (s)	Obj. gap	Obj.	Time (s)	Obj. gap
1	1387.26	0.89	1387.26	0.89	0.00 %	1387.26	1.47	0.00 %
2	13208.14	1.02	19703.40	1.28	49.18 %	19703.40	1.11	49.18 %
3	20360.12	4.43	20360.12	4.15	0.00 %	20360.12	3.23	0.00 %
4	53387.08	6.27	80745.42	7.45	51.25 %	53387.08	7.05	0.00 %
5	34085.34	6.36	52446.68	9.70	53.87 %	52446.68	9.98	53.87 %
6	66536.00	137.54	81251.98	125.08	22.12 %	72077.68	125.16	8.33 %
7	91204.99	356.88	144601.54	227.06	58.55 %	117827.61	322.08	29.19 %
8	88054.39	819.12	143318.22	1005.53	62.76 %	107539.56	997.72	22.13 %
9	106022.15	2651.81	150319.99	2375.70	41.78 %	107704.39	3075.68	1.59 %
Average Gap	–	–	–	–	37.72 %	–	–	18.25 %

refer to deploying the acceleration strategy as described in Section 4.5.2, and "Parallelization" indicates the acceleration strategy as explained in Section 4.5.3. Lastly, as the name suggests, "All" indicates the use of all three acceleration strategies. The run time of the different accelerating strategies is shown in Fig. 6.

The results show that all the acceleration strategies contribute to the quicker convergence of the proposed approach. Among the three, the effect of multiple columns strategy is similar to that of parallelization the subproblems, while the strategy of limiting the flight number has the least impact.

5.3.2. Effect of the discretization of the fuel consumption function

To study the effect of the number of discrete points in the fuel consumption function, we solve all the nine scenarios with 5 and 21 dividing points separately. A different number of dividing points implies varying temporal discretization. We intend to study if this parameter affects the convergence time and solution quality. We tabulate the results of improved column generation in Table 5.

The results show that higher granularity results in less costly solutions. In all scenarios, when there are more dividing points, the cost of solution decreases to varying degrees. In terms of solution time, more dividing points are more suitable for solving large-scale scenario. Therefore, using fuel consumption functions with higher granularity helps to obtain a better solution.

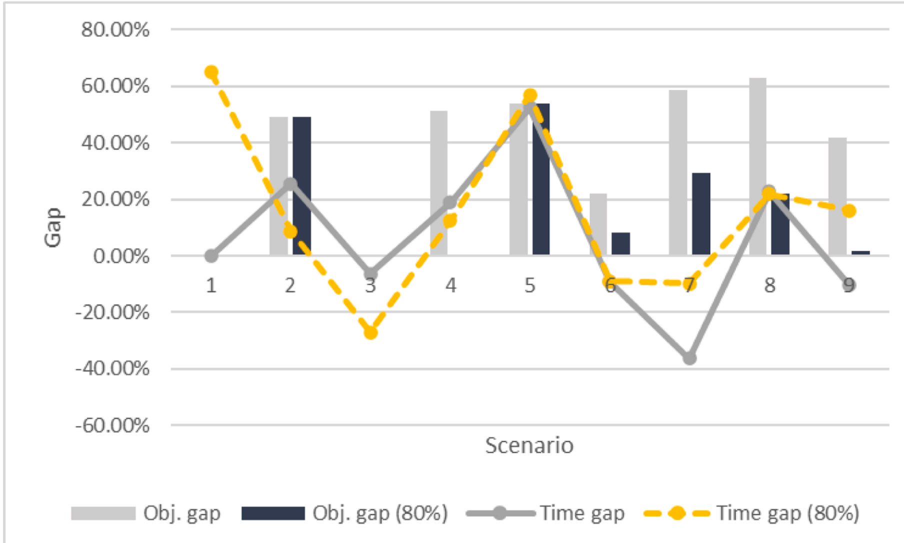

Fig. 7. Obj. gap and time gap with and without aircraft assignment constraints.

Table 7
Results of Scenario 6 in various types of costs.

Count index	without aircraft assignment	with aircraft assignment	with aircraft assignment (80 %)
No. of cancellations	5	7	6
Cancellation costs	45955.00	65060.00	55765.00
No. of swaps	22	19	20
Swap costs	3846.80	3325.70	3497.50
No. of delays	19	16	16
Delay costs	15101.30	11675.40	11624.30
Fuel & carbon emission costs	1632.90	1190.88	1190.88

5.3.3. *Effect of aircraft assignment constraints*

We solve two versions of the master problem – one with aircraft assignment constraints and another without them. We intend to investigate the effect on the computational time and the total costs using the improved column generation. We also conducted experiments where the aircraft assignment constraints were not fully satisfied, requiring about 80 % of the number of aircraft of each type required at each airport. The results are as shown in Table 6.

For all the scenarios, by enforcing aircraft assignment constraints, the results show will lead to increased costs. The reason why the cost of Scenario 1 and 3 did not increase, from the experimental results, is that there are no flight cancellations in the recovery plan or all cancellations are round-trip flights, so it does not affect the number of aircraft at each airport after the recovery period. The test results in Table 6 show that by enforcing aircraft assignment constraints, the average total costs increase by almost 37.72 %, compared to 18.25 % for an 80 % satisfaction rate. This situation presents a trade-off for practitioners. The obj. gap and time gap with and without aircraft assignment constraints are shown in Fig. 7.

When we do not consider the aircraft assignment constraints, we carry the risk of having a lesser number of aircraft required for future flights. For instance, at the end of the recovery plan, if there are three aircraft of the same type at Airport A but the next day's flight plan requires five aircraft of this type, then there will be no aircraft to perform some flights. Airlines usually respond to this situation by modifying the schedule, cancelling some flights, or sometimes ferrying idle aircraft

Table 9
Recovery costs with and without flight duration change.

Scenario	without flight duration change		with flight duration change		GAP
	Obj.	Time (s)	Obj.	Time (s)	
1	2186.20	0.61	1387.26	0.89	−36.54 %
2	19758.00	0.39	19703.40	1.28	−0.28 %
3	21501.10	1.87	20360.12	4.15	−5.31 %
4	195461.70	3.27	80745.42	7.45	−58.69 %
5	53031.50	5.25	52446.68	9.70	−1.10 %
6	116201.00	52.03	81251.98	125.08	−30.08 %
7	225372.70	90.29	144601.54	227.06	−35.84 %
8	223731.10	241.45	143318.22	1005.53	−35.94 %
9	175719.60	882.62	150319.99	2375.70	−14.45 %
Average Gap	–	–	–	–	−24.25 %

Table 8
Experimental results for smaller cancellation cost.

Scenario	without aircraft assignment		with aircraft assignment			with aircraft assignment (80 %)		
	Obj.	Time (s)	Obj.	Time (s)	Obj. gap	Obj.	Time (s)	Obj. gap
1	1387.26	1.42	1387.26	1.20	0.00 %	1387.26	1.17	0.00 %
2	6845.14	1.10	7029.90	1.06	2.70 %	7029.90	1.08	2.70 %
3	12723.63	2.53	12723.63	2.75	0.00 %	12723.63	3.02	0.00 %
4	21033.08	6.53	28917.42	8.23	37.49 %	21033.08	7.81	0.00 %
5	14653.34	5.61	20320.18	8.87	38.67 %	20320.18	9.03	38.67 %
6	31959.58	103.45	35709.98	87.11	11.73 %	33042.18	104.86	3.39 %
7	39140.91	304.40	52621.44	226.72	34.44 %	45682.11	230.22	16.71 %
8	36002.39	780.53	51324.62	790.17	42.56 %	41900.56	854.90	16.38 %
9	46974.39	1742.14	58165.59	1986.81	23.82 %	48710.40	1898.15	3.70 %
Average Gap	–	–	–	–	21.27 %	–	–	9.06 %

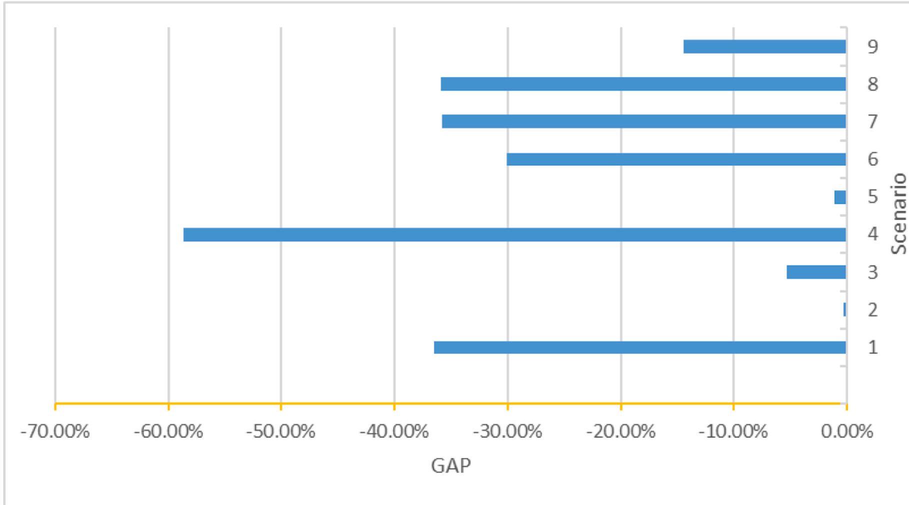

Fig. 8. Gap in recovery costs with and without flight duration change.

from other airports to run original follow-on flights. In essence, the airlines' response defers part of the recovery costs to the future and affects the normal operation of future flights. It makes sense to pay more for the current recovery costs when airlines believe that the normal operation of flights after the recovery period is very important. A compromise might be to compare expected future losses with current additional recovery costs and make logical decisions on a case-by-case basis, such as setting partially satisfied aircraft assignment constraints. As the experimental results show, the current recovery costs can be reduced as much as possible on the basis of guaranteeing the operation of some important flights after the recovery period by setting partially satisfied aircraft assignment constraints.

For a more detailed presentation, taking Scenario 6 as an example, the changes in various types of costs are given in Table 7. It can be seen from Table 7 that the cost increase with considering aircraft assignment is mainly caused by the flight cancellation costs, but considering aircraft assignment guarantees the normal execution of subsequent flights after the end of the recovery period.

Next, we set a smaller flight cancellation cost of 0.3 times the original value. Experiments were carried out using improved column generation without considering the aircraft assignment, considering the aircraft assignment, and considering the incompletely satisfied aircraft assignment (about 80 %). The experimental results are shown in Table 8. The recovery costs considering the aircraft assignment constraints increased by about 21.27 % on average, while the recovery costs considering the incompletely satisfied aircraft assignment increased by about 9.06 % on average. Therefore, when the flight cancellation cost is relatively small, considering the aircraft assignment constraints has a smaller impact on the total recovery costs, which means that the airlines can pay a small price to obtain the normal operation of the flight after the recovery period.

Table 10
Results of the experiments with different fuel prices.

Scenario	fuel price = 0.4		fuel price = 1.0	
	Obj.	Time (s)	Obj.	Time (s)
1	1291.99	1.07	1544.41	1.38
2	19687.99	0.87	19720.85	0.89
3	20077.36	3.56	20677.46	3.70
4	80554.11	6.67	81071.52	6.98
5	52317.71	10.92	52599.34	8.92
6	80784.58	115.93	81844.99	134.49
7	144118.09	287.76	145201.29	327.84
8	142929.31	1299.09	143806.05	977.50
9	149644.38	2306.09	151412.77	1572.34

5.3.4. *Effect of varying the flight duration*

To quantify the savings in the recovery costs by incorporating the recovery option of varying the flight duration, we conduct experiments with and without it using the improved column generation. The results are tabulated in Table 9, where GAP is the change in recovery costs.

As expected, incorporating the recovery option of varying the flight duration helps in saving a significant amount of recovery costs, about 24 % on average. The gap in recovery costs with and without flight duration change is shown in Fig. 8. Changing the flight duration helps execute some flights that would have had to be cancelled otherwise. The only drawback with considering this recovery option is the additional computational time required to solve the problem.

5.3.5. *Analysis of different fuel prices*

Aircraft fuel prices are volatile, so the fuel prices were set to different values for experiments. The fuel price is set to 0.4 and 1.0, respectively, compared to the solution with a fuel price of 0.6. The results of the experiments using the improved column generation are shown in Table 10.

Fig. 9 shows the difference between the solutions with fuel prices of 0.4 and 1.0 and the solution with fuel price of 0.6, which is used as a benchmark. Combining Table 10 and Fig. 9, it can be seen that when the fuel price increases, the total recovery costs will increase, and vice versa. Therefore, the recovery option of changing the flight duration can save more costs when the fuel price is low.

5.3.6. *Analysis of different aircraft assignment requirements*

In this subsection, we report another aircraft assignment constraints, i.e., for designated airports, the number of aircraft after the recovery plan should be the same as that after the end of the original plan, represented by Constraint 2. In contrast, the aircraft assignment constraints considered above are represented by Constraint 1. The modification of the mathematical model for Constraint 2 is shown in Appendix A. We conducted experiments and showed the results of improved column generation in Table 11. Using Constraint 2 implies more stringent aircraft assignment, and Scenario 5 cannot obtain a solution that satisfies Constraint 2 and therefore suffers a penalty cost. Overall, the implementation of more demanding aircraft assignment may lead to higher costs, as some flights may have to be cancelled to meet the demanding assignment requirements. This requires practitioners to

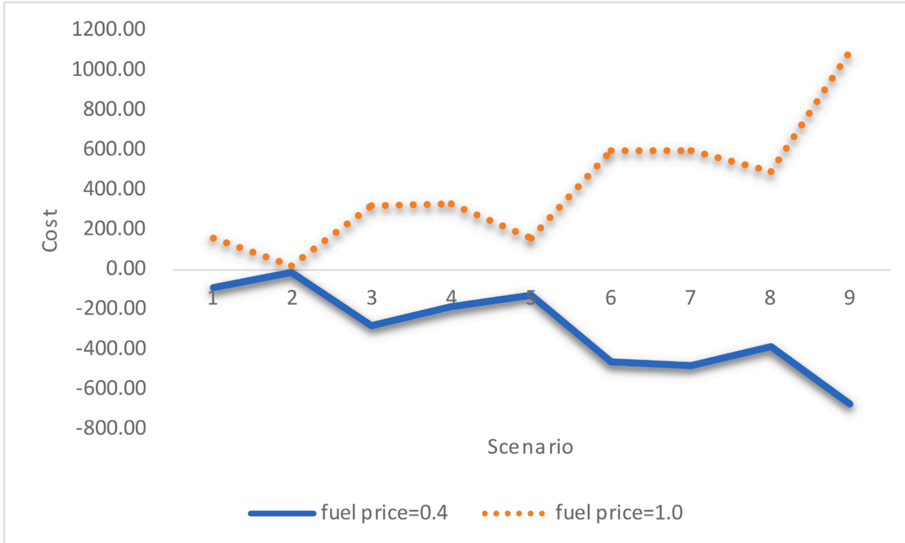

Fig. 9. Cost difference between different fuel prices.

Table 11
Results of different aircraft assignment requirements.

Scenario	Constraint 1		Constraint 2		GAP
	Obj.	Time (s)	Obj.	Time (s)	
1	1387.26	0.89	1387.26	1.01	0.00 %
2	19703.40	1.28	19703.40	1.28	0.00 %
3	20360.12	4.15	29477.92	4.65	44.78 %
4	80745.42	7.45	80745.42	7.71	0.00 %
5	52446.68	9.70	1061408.31	8.59	1923.79 %
6	81251.98	125.08	107180.14	140.62	31.91 %
7	144601.54	227.06	180642.28	313.19	24.92 %
8	143318.22	1005.53	179446.66	846.67	25.21 %
9	150319.99	2375.70	204347.92	1875.14	35.94 %
Average Gap	–	–	–	–	231.84 %

weigh the current cost, the expected future cost, and other factors such as policies in actual operation.

6. Conclusions and scope for future work

This paper distinguishes itself from the rest of the studies in the context of disrupted flight recovery problems by considering two critical real-life characteristics: a) A new implementation of the recovery option by changing flight duration. We use a piecewise function to describe the discrete functional relationship between the reduced flight duration and changes in fuel consumption and b) aircraft assignment constraints - enough aircraft should be assigned to airports at the end of the recovery period. Based on a series of experiments with real data, we have come to the following conclusions:

- The proposed MIQP model is suitable for solving small-scale scenarios with commercial solvers such as CPLEX but not for solving large-scale scenarios. In contrast, the proposed improved column generation algorithm can solve large-scale scenarios in a reasonable time. The experimental results show that the three strategies we adopted to accelerate the column generation process are very effective in facilitating the faster convergence of the algorithm.
- The recovery option of changing flight duration can save an average of 24 % of recovery costs. The proposed method can generate high-quality solutions in real time and make it suitable for real-life deployment.
- Our results suggest that enforcing aircraft assignment constraints may result in higher current recovery costs. However, implementing aircraft assignment constraints ensure the normal operation of subsequent flights and prevents spillover of the recovery costs/actions into the future.

We can extend this work in multiple ways. For example, the early departure of flights when all the passengers have boarded is one of the possible recovery options. Developing heuristics/*meta*-heuristics for large-scale scenarios that can generate good quality solutions quickly can be taken up further. On the basis of the research in this paper, the simultaneous aircraft and crew recovery problem can be further studied. Otherwise, it would be interesting to study the problem in a bi-objective context with total recovery costs as the first objective and penalty cost for violating aircraft assignment as the second objective.

CRediT authorship contribution statement

Jie Li: Methodology, Investigation, Data curation, Writing – original draft. **Kunpeng Li:** Software, Data curation, Writing – original draft. **Qiannan Tian:** Methodology, Data curation, Writing – review & editing. **P.N. Ram Kumar:** Validation, Writing – review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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Appendix A

Here the MIQP model and the master problem model are modified to accommodate Constraint 2 in subsection 5.3.6. The first is the modification of MIQP for Constraint 2. Let *QALL* indicates the total difference in the number of aircraft that does not meet the aircraft assignment constraints. The objective function (1) becomes.

$$Min \sum_{f\in F} cl_f\left(1-\sum_{p\in P}\sum_{i\in Index} z_{pfi}\right)+\sum_{f\in F}\left(g_f\left(d_f-\left(Tcr_f-tcr_f\right)\right)\sum_{p\in P}\sum_{i\in Index} z_{pfi}\right)+\sum_{p\in P}\sum_{f\in F, ism_f\leqslant 0}(cfuel+c_{co_2}rc)wfuel_{pf} +\sum_{p\in P}\sum_{f\in F}\left(SC_{pf}\sum_{i\in Index} z_{pfi}\right)+W1\cdot QALL \tag{52}$$

Constraint (4) becomes.

$$\sum_{p\in P, ft_p=g}\sum_{f\in F}\left(aa_{fa}\sum_{i\in Index} z_{pfi}\right)-\sum_{p\in P, ft_p=g}\sum_{f\in F}\left(da_{fa}\sum_{i\in Index} z_{pfi}\right)-\left(h_{ag}-eh_{ag}\right)=QA_{ag},\ \forall a\in A^{'}, g\in G \tag{53}$$

Constraint (5) becomes.

$$QALL=\sum_{a\in A^{'}}\sum_{g\in G}\left|QA_{ag}\right| \tag{54}$$

Constraint (54) can be linearized as follows by introducing the variable $QA^{'}_{ag}$.

$$QALL=\sum_{a\in A^{'}}\sum_{g\in G} QA^{'}_{ag} \tag{55}$$

$$QA_{ag}\leqslant QA^{'}_{ag}, \forall a\in A^{'}, g\in G \tag{56}$$

$$-QA_{ag}\leqslant QA^{'}_{ag}, \forall a\in A^{'}, g\in G \tag{57}$$

For the master problem formulation, let $q^{'}_{ag}$ be the surplus that the recovery plan does not meet the aircraft assignment constraints. The objective function (28) becomes.

$$Min\sum_{r\in R} c_{pr}x_{pr}+\sum_{f\in F} cl_f y_f+W1\cdot\sum_{a\in A^{'}}\sum_{g\in G}\left(q_{ag}+q^{'}_{ag}\right) \tag{58}$$

Constraint (30) becomes.

$$\sum_{p\in P, ft_p=g}\sum_{r\in R}\beta_{par}x_{pr}+q_{ag}-q^{'}_{ag}=h_{ag},\ \forall a\in A^{'}, g\in G \tag{59}$$

And add the following constraint.

$$q^{'}_{ag}\geqslant 0, \forall a\in A^{'}, g\in G \tag{60}$$

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