Variational Quantum Algorithms for Solving Vehicle Routing Problem

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***Abstract*—Vehicle Routing Problem (VRP) is considered one of the most challenging problems with application in several domains, including transportation and logistics distribution. VRP is known to be NP-Hard problem. Several algorithms have been proposed to solve VRP in polynomial time. However, these algorithms are inefficient if the VRP instance increases. Recently, researchers investigated how quantum computing can be used to solve VRP. In particular, two promising variational quantum algorithms have been studied, i.e., Variational Quantum Eigensolver (VQE) and Quantum Approximate Optimization Algorithm (QAOA). In this paper, we implement both algorithms on IBM Qiskit and compare their evaluation in solving several instances of VRP. We observe that current Noisy-Intermediate Scale Quantum (NISQ) devices cannot solve VRP instances beyond 5 nodes and 2 vehicles. Furthermore, we observe that classical optimizers still provide better results. However, we believe that with the rapid advancement of quantum computing manufacturing, VQE and QAOA can provide better performance as compared to classical computing.**

***Index Terms*—Quantum computing, VRP, VQE, QAOA**

I. Introduction

The Vehicle Routing Problem (VRP) is classified as one of the most challenging problems in the field of scheduling and optimization [1]. VRP caught the attention of researchers because of the potential of significant reduction of cost pertaining to transportation and logistics distribution [2], [3]. VRP is being used in several applications including supply chain management [4] and last-mile delivery services [5].

The objective of VRP is to obtain the optimal route for a fleet of vehicles to serve set of customers. Several variations of VRP have been proposed where one or two constraints are introduced, for example time [6], capacity [7], and distance constraints [8].Several algorithms for solving VRP problems have been proposed [9]. However, classical algorithms for solving VRP are often computationally intensive and are inefficient when the size of the problem increases. Recently, researchers started to investigate the feasibility of using Quantum Computing in solving VRP [10]–[13].

Quantum computing is an emerging computing paradigm that exploits quantum physics principles, i.e., superposition and entanglement, to manipulate qubits (the quantum equivalent to bits). There are several general purpose quantum algorithms that were investigated to solve VRP on quantum computers, including Variational Quantum Eigensolver (VQE) [11], [14] and Quantum Approximate Optimization Algorithms (QAOA) [10], [15].

The objective of this paper is to assess the capability of the state-of-the-art quantum computers in solving VRP. In particular, we implement two variational quantum algorithms, namely, VQE and QAOA, using IBM Qiskit [16] and compare their performance in solving several instances of VRP. We systematically varied the size of the VRP instances, the problem configuration, and the associated constraints to incrementally increase the complexity of the VRP instances. For every instance, we solved it using classical approach (i.e., optimization solvers) and quantum approach (i.e., VQE and QAOA). We compare the performance of both approaches using time complexity. Furthermore, we provide discussion on some of the current limitations of the existing Noisy Intermediate-Scale Quantum (NISQ) hardware and algorithms with respect to VRP applications.

The rest of this paper is organized as follows. In Section II, we provide brief background about VRP and Quantum Computing. In Section III, we describe the implementation and, in Section IV, we present the results. We conclude in Section V.

II. Background

A. Vehicle Routing Problem

VRP is a combinatorial problem that computes the best routes from a starting point (depot) to a number of destinations (clients) and back to the depot for a given a number of available vehicles. VRP is often modeled as a graph, on which the best routes are obtained. Formally, let n be the number of clients (indexed as $1, ..., n$), and K be the number of available vehicles. Let $x_{ij} = \{0, 1\}$ be the binary decision variable which, if it is 1, activates the segment of route from node i to node j. Node 0 is, by convention, the depot. In a fully connected graph, there are $n(n+1)$ binary decision variables.

If two nodes i and j have a direct link from i to$j$,we write $i \sim j$. We also denote with $\delta(i)^+$ the set of nodes to which i has a link, i.e., $j \in \delta(i)^+$ if and only if $i \sim j$. Similarly, we denote with $\delta(i)^-$ the set of nodes which are connected to i, in the sense that $j \in \delta(i)^-$ if and only if $j \sim i$. The VRP can formulated as the following.

$$f = \min_{\{x_{ij}\}_{i \sim j} \in \{0,1\} \sum_{i \sim j} w_{ij} x_{ij}} \tag{1}$$

$$s.t.$$

$$\sum_{j \in \delta(i)^+} x_{ij} = 1 \quad \forall i \in \{1, ..., n\} \tag{2}$$

$$\sum_{j \in \delta(i)^-} x_{ij} = 1 \quad \forall i \in \{1, ..., n\} \tag{3}$$

$$\sum_{i \in \delta(0)^+} x_{0i} = K \tag{4}$$

$$\sum_{j \in \delta(0)^+} x_{j0} = K \tag{5}$$

The cost function (Equation 1) is linear in the cost functions and weighs the different arches based on a positive weight $w_{ij} > 0$, which models the distance between node i and node j. Equations 2 and 3 enforce that from and to every client, only one link is allowed, while equations 4 and 5 enforce that from and to the depot, exactly K links are allowed.

1) VRP Variants: There exist several variants of the VRP that are formulated based on the nature of the transported goods, the quality of required service and the features of the vehicles and the customers. An example of such variants is the Capacitated Vehicle Routing Problem with Time Windows (CVRPTW) which includes constraints that bound both the capacities of the vehicles and the time windows of delivery [17].

2) Time Complexity of VRP: It was proven that VRP is non-deterministic polynomial-time hardness (NP-hard) [1].As a result, there is no exact polynomial algorithm to solve such problem especially with increasing the parameters' values (nodes/customers) or adding more variables (depth) to the problem.

B. Quantum Computing

Quantum computing emerged from the use of quantum physics laws in computing. Quantum computing is based on four postulates: 1) the state of the qubit, 2) linear combination of qubits, 3) probabilistic outcome of measurements, and 4) time evolution of a system through quantum gates. Physical implications of these postulates have appeared in terms of quantum superposition and quantum entanglement, which are the major principles in quantum computing. A quantum bit (Qubit) is the fundamental unit of computation that can take two possible states: 0 and 1 (denoted as as $|0\rangle$ and $|1\rangle$, respectively). A qubit can also take both states simultaneously, i.e., a qubit can be in a superposition state. In general, a qubit in a superposition state can be denoted as $|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$, where α and β are two complex numbers such that $|\alpha|^2 + |\beta|^2 = 1$. When a qubit is measured, the qubit state collapses into one of the states, i.e., $|0\rangle$ or $|1\rangle$, with probabilities α^2 and β^2, respectively. Operations on qubits are performed using quantum gates that transform the qubits state over time. An n-qubit quantum gate is represented using $2^n \times 2^n$ unitary matrices (U) such that $UU^\dagger = U^\dagger U = I$. Tensor product operation can be used to combine qubits and form larger quantum states. For example, for two qubits $|\psi\rangle$ and $|\phi\rangle$, their tensor product is denoted by $|\psi\rangle \otimes |\phi\rangle$ (or $|\psi\phi\rangle$, for short) and results in a combination of four possible states, i.e., $|\psi\phi\rangle = \alpha |00\rangle + \beta + |01\rangle + \gamma |10\rangle + \delta |11\rangle$ such that $|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$.

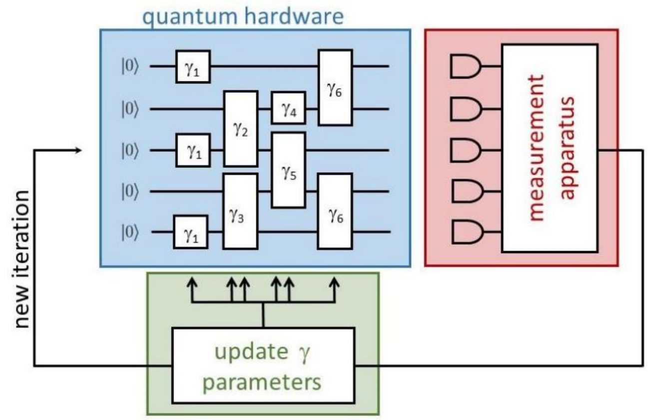

Fig. 1. Overall Procedure of VQE

C. Variational Quantum Algorithms

The Variational Quantum Algorithms (VQA) is a type of hybrid quantum classical algorithm which is best fit for NISQ devices. The hybrid approach include training a closed loop optimization between classical and quantum computers. Famous applications of it are in chemistry optimization and machine learning [18]. In general, VQA contains four steps. First, a cost function is defined to encode the solution of the domain-specific problem. Then, a quantum circuit is executed to prepare an Ansatz (a guessed solution to the problem). The Ansatz is then optimized using the hybrid classical-quantum loop to solve the optimization problem. In the final step, VQAs use classical optimizers to better optimize the parameters. Although this approach is widely used in current NISQ devices, challenges include trainability and accuracy. Additionally, the efficient optimization of the quantum circuit for these variational parameters is still an area of research. We explain two widely-used VQAs in the following sections.

1) VQE: VQE is a hybrid classical and quantum algorithm which uses variational principle to determine the Hamiltonian's ground state energy. The VQE has mainly two steps: 1) preparing Ansatz quantum state $|\Psi(\theta)\rangle$ and 2) measure the expectation value $\langle\Psi(\theta)| H |\Psi(\theta)\rangle$. This variational principle ensures that expectation value is greater that the smallest eigenvalue of H. VQE iterates between the quantum part and classical part until the expected values convergence. Figure 1 illustrates the overall procedure of VQE. Main challenges with VQE is the dramatical impact on performance for selecting the parameterized state (Ansatz). Additionally, another area of

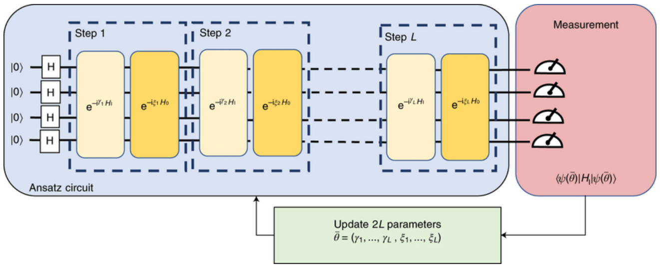

Fig. 2. Overall Structure of QAOA.

Problem	VQE	QAOA	Classical
Nodes=3 Vehicles=2	20.01	20.01	20.01
Nodes=4 Vehicles=2	24.34	23.86	23.86
Nodes=5 Vehicles=2	11713.48	98.26	98.26

TABLE I
COMPARISON OF VRP SOLUTIONS OBTAINED BY VQE, QAOA, AND CLASSICAL OPTIMIZER.

focus is the possibility of getting a local minimum instead of global.

2) QAOA: QAOA is considered the most broadly studied technique to solve combinatorial optimization problems in NISQ devices. This is due to the performance of the algorithm and its wide applications which made it of a great interest to the quantum research community. Similar to VQE, the QAOA is a general method to get approximate solutions for combinatorial optimization. The starting point of the QAOA is to specify cost and mixer Hamiltonians, H_C and H_M, that encode the ground state and the solution to the problem, respectively. Then, two variational circuits are constructed to compute $e^{-i\gamma H_C}$ and $-i\alpha H_C$. Finally, the algorithm ends up by sampling from the circuit to get an approximate solution of the desired optimization problem. Figure 2 illustrate the structure of QAOA.

III. Experiments and Results

The objective of this paper is to evaluate and compare two quantum algorithms, namely, VQE and QAOA, in solving VRP. In particular, we aim to evaluate the maximum instance of VRP that current state-of-the-art NISQ device can solve. Therefore, we attempt to find solutions for the general (n, k) VRP problems, where n is number of nodes (cities) and k is number of vehicles. We observed that the largest instances that NISQ devices can run is (5,2) Then, we assess quantum algorithms and compare them to the classical approach and list findings and recommendations.

For implementation, we have used IBM Qiskis platform, which provides an open-source software development kit (SDK) for developers to work with quantum computers at the three levels of circuits, pulses, and algorithms. Figure 3 demonstrates the layers and modules in IBM Qiskit.

Figure 4 illustrates the three (n, k) VRP scenarios for (3,2), (4,2) and (5,2) and shows the optimal solutions obtained by

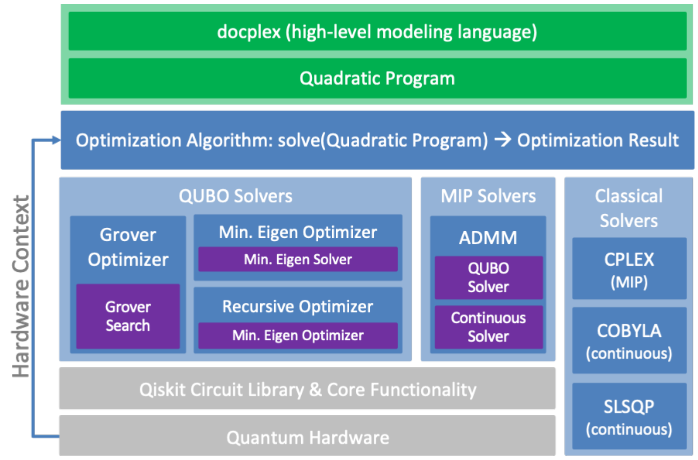

Fig. 3. Architecture of Qiskit.

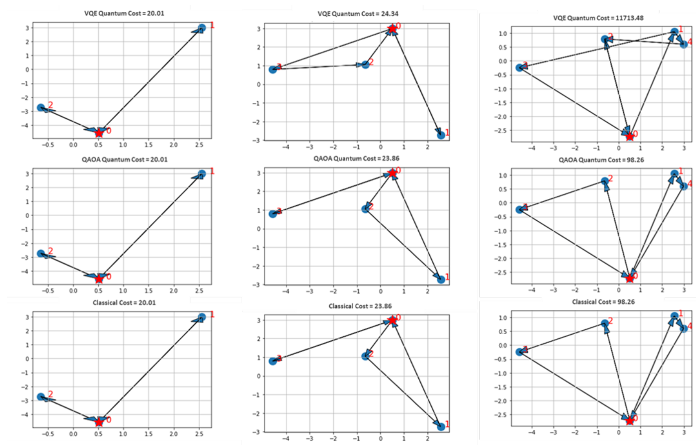

Fig. 4. Comparison of VRP solutions obtained by VQE, QAOA, and classical optimizer.

VQE, QAOA, and CPLEX classical optimizer. Table I lists the of our experiments. We observe that large VRP instances, current quantum computers can't execute quantum algorithms (VQE QAOA). Additionally, QAOA was found to perform better than VQE when increasing the size of VRP variables. Also, finding optimal solution for both algorithms depends on choosing appropriate values of steps (Iteration) and Ansatz. This leads to the importance of having further studies to enhance current quantum algorithms in order to cater for different problem sizes.

IV. Conclusion

Quantum technology, even with Noisy Intermediate-Scale Quantum (NISQ), provides a promising journey for solving combinatorial problems such as Vehicle Routing Problem (VRP). However, both algorithms can currently cater only for small scale of the VRP. As stated in many reports, a lot of development in hardware and software is needed. The current limitation of number of qubits and error correction prevent scaling up the usability of the hardware for VRP cases. Also, current algorithms including VQE and QAOA need to be enhanced to better represent the VRP and cater for more complicated cases. When those issues (hardware and software) are resolved, it is expected that quantum solutions will be globally adopted.

REFERENCES

[1] P. Toth and D. Vigo, *The vehicle routing problem*. SIAM, 2002.

[2] G. D. Konstantakopoulos, S. P. Gayialis, and E. P. Kechagias, "Vehicle routing problem and related algorithms for logistics distribution: A literature review and classification," *Operational research*, vol. 22, no. 3, pp. 2033–2062, 2022.

[3] R. Moghdani, K. Salimifard, E. Demir, and A. Benyettou, "The green vehicle routing problem: A systematic literature review," *Journal of Cleaner Production*, vol. 279, p. 123691, 2021.

[4] L. Ren, X. Fan, J. Cui, Z. Shen, Y. Lv, and G. Xiong, "A multi-agent reinforcement learning method with route recorders for vehicle routing in supply chain management," *IEEE Transactions on Intelligent Transportation Systems*, 2022.

[5] Y. Ancele, M. H. Hà, C. Lersteau, D. B. Matellini, and T. T. Nguyen, "Toward a more flexible vrp with pickup and delivery allowing consolidations," *Transportation Research Part C: Emerging Technologies*, vol. 128, p. 103077, 2021.

[6] R. Baldacci, A. Mingozzi, and R. Roberti, "Recent exact algorithms for solving the vehicle routing problem under capacity and time window constraints," *European Journal of Operational Research*, vol. 218, no. 1, pp. 1–6, 2012.

[7] M. Noorizadegan and B. Chen, "Vehicle routing with probabilistic capacity constraints," *European Journal of Operational Research*, vol. 270, no. 2, pp. 544–555, 2018.

[8] V. Nagarajan and R. Ravi, "Approximation algorithms for distance constrained vehicle routing problems," *Networks*, vol. 59, no. 2, pp. 209–214, 2012.

[9] R. Elshaer and H. Awad, "A taxonomic review of metaheuristic algorithms for solving the vehicle routing problem and its variants," *Computers & Industrial Engineering*, vol. 140, p. 106242, 2020.

[10] U. Azad, B. K. Behera, E. A. Ahmed, P. K. Panigrahi, and A. Farouk, "Solving vehicle routing problem using quantum approximate optimization algorithm," *IEEE Transactions on Intelligent Transportation Systems*, 2022.

[11] N. Mohanty, B. K. Behera, and C. Ferrie, "Analysis of the vehicle routing problem sovled via hybrid quantum algorithms in presence of noisy channels," *arXiv preprint arXiv:2205.07630*, 2022.

[12] S. J. Weinberg, F. Sanches, T. Ide, K. Kamiya, and R. Correll, "Supply chain logistics with quantum and classical annealing algorithms," *arXiv preprint arXiv:2205.04435*, 2022.

[13] W. Ma, X. Zhang, Y. Xu, and F. Gao, "Loading is the key: A novel genetic quantum algorithm for sdvrp," in *2021 IEEE Congress on Evolutionary Computation (CEC)*. IEEE, 2021, pp. 1817–1825.

[14] P. Atchade-Adelomou, "Quantum algorithms for solving hard constrained optimisation problems," *arXiv preprint arXiv:2202.13125*, 2022.

[15] D. Fitzek, T. Ghandriz, L. Laine, M. Granath, and A. F. Kockum, "Applying quantum approximate optimization to the heterogeneous vehicle routing problem," *arXiv preprint arXiv:2110.06799*, 2021.

[16] "Ibm quantum." https://quantum-computing.ibm.com/, accessed: 2021.

[17] C. Natalia, V. Triyanti, G. Setiawan, and M. Haryanto, "Completion of capacitated vehicle routing problem (cvrp) and capacitated vehicle routing problem with time windows (cvrptw) using bee algorithm approach to optimize waste picking transportation problem," *Journal of Modern Manufacturing Systems and Technology*, vol. 5, no. 2, pp. 69–77, 2021.

[18] M. Cerezo, A. Arrasmith, R. Babbush, S. C. Benjamin, S. Endo, K. Fujii, J. R. McClean, K. Mitarai, X. Yuan, L. Cincio *et al.*, "Variational quantum algorithms," *Nature Reviews Physics*, vol. 3, no. 9, pp. 625–644, 2021.