Optimizing Aircraft Routings in response to Groundings and Delays

Article *in* IIE Transactions · October 2001

DOI: 10.1023/A:1010987008497

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IIE Transactions (2001) **33**, 931–947 Vol. 33, No. 10

Optimizing aircraft routings in response to groundings and delays

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Received January 1999 and accepted July 2000

This paper presents the time-band optimization model for reconstructing aircraft routings in response to groundings and delays experienced over the course of the day. Whenever the schedule is disrupted, the immediate objective of the airlines is to minimize the cost of reassigning aircraft to flights taking into account available resources and other system constraints. Associated costs are measured by flight delays and cancellations. The time-band model is constructed by transforming the routing problem into a time-based network in which the time horizon is discretized. The resulting formulation is an integral minimum cost network flow problem with side constraints. Conditions for which an exact solution to the model represents an optimal solution for the original problem are stated. The transformation procedure is polynomial with respect to the number of airports and flights in the schedule. Computational experience shows that the underlying network structure of the transformed problem often leads to integral solutions when solved with a standard linear programming code. Empirical results using Continental Airline data demonstrate that the solutions obtained are either provably optimal or no more than a few percentage points from the lower bound.

1. Introduction

Airlines spend a great deal of effort developing flight schedules for each of their fleets. A flight schedule consists of the originating city, departure time, destination, and arrival time for flights that the airline intends to serve. Due to the seasonality of passenger travel, these schedules are created every 2 to 3 months. In the short term, aircraft are typically assigned to cover specific flights every day for a 1 week time horizon. The ordered sequence of flights to which an aircraft is assigned is called an *aircraft route*. A collection of aircraft routes that can be used to service scheduled flights is defined as an *aircraft routing*. The great expense of purchasing and maintaining aircraft motivates airlines to maximize their utilization by creating aircraft routings with as little embedded idle time as possible. Thus when an aircraft is unexpectedly grounded or delayed, an airline's ability to meet its flight schedule is compromised.

Unplanned aircraft shortages and resulting flight schedule disruptions are an unavoidable occurrence in the daily operations of an airline. Aircraft are grounded or temporarily delayed when equipment failures make flying unsafe, when severe weather closes an airport, or when the required flight crews are unavailable. Flights that are grounded or delayed jeopardize their assigned routes. Real-time decisions must be made to minimize lost revenues, passenger inconvenience, and operational costs by reassigning available aircraft and canceling or delaying flights.

In extreme cases where severe weather closes airports in a region, aircraft are prohibited from taking off or landing at the affected airports. This results in massive flight cancellations whose effects propagate through the system causing missed connections and other disruptions at upstream and downstream airports. Accordingly, an airline must be able to modify its aircraft routing in order to minimize the effects of extraordinary irregular events like severe weather. The airline must also be able to get back on schedule as quickly as possible after the irregular events have passed and the entire system becomes operational. For example, if a hub like Cleveland with about 250 daily departures is closed due to a blizzard, and if each departing flight averages \$15 000 in revenue, then, assuming a 50% recapture of

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passengers, about $937 500 at the hub airport alone could be saved if the airlines can get back on schedule in half a day instead of a full day. The results would be even more dramatic if we considered the revenue that could be saved by re-routing aircraft at open airports.

Due to the complexity of this problem, operations personnel are unable in practice to evaluate alternate solutions with respect to these objectives and are content merely to find feasible solutions. In this paper we present a time-band optimization model for generating cost-effective aircraft routings in response to schedule disruptions. While the idea of discretizing the time axis to deal with the continuous nature of flow is common in many areas (e.g., see Jarrah *et al.* (1993) for an application to mail flows) there have been limited applications in the airline industry. One of the few was reported by Daskin and Panayotopoulos (1989) who addressed the problem of assigning aircraft to routes that originate and terminate at the same hub. They used Lagrangian relaxation in conjunction with a heuristic to find upper and lower bounds.

With regard to irregular operations, Teodorovic and Guberinic (1984) developed a branch and bound procedure for minimizing total passenger delay. Subsequently, Teodorovic and Stojkovic (1990) gave a lexicographic dynamic programming scheme to minimize the number of canceled flights and the total passenger delay. Neither of these papers considered flight delay and cancellation costs. Their chief concern was for total passenger delay and the number of flight cancellations.

Jarrah *et al.* (1993) introduced two complementary minimum cost network flow models. The first addresses flight delays and the second flight cancellations. Yu (1996), and Yu and Luo (1997) also provided a network representation of the problem and from it derived an optimization model that appears to be NP-hard. No solution techniques were proposed. Their ideas served as a foundation for the work presented by Argüello (1997) where more formal optimization models are constructed and heuristic procedures are introduced.

A problem related to the one we investigate here arises from the Ground Delay Program (GDP) imposed by the FAA at airports experiencing inclement weather. For the GDP, a reduced number of flight arrivals and departures are mandated for a specific period of time. The designation of flight delays and cancellations are determined locally with respect to the airport where the ground delay program is imposed. Yu and Luo (1997) considered a variety of objectives for this problem as well as scenarios where resources may and may not be reassigned to other scheduled flights. They presented efficient algorithms to solve several simple cases, proved the NP-hardness of some difficult cases, and provided heuristics. Vasquez-Marquez (1991) considered the options available for shifting flight arrivals and gave a traveling salesman problem formulation (although it appears that only an assignment problem must be solved in practice). The model was implemented as part of a decision support system for American Airlines. Vranas *et al.* (1994) looked at the GDP from a system-wide prospective considering multiple airports and down-the-line effects. They developed several integer programming models for minimizing overall ground and airborne delay costs subject to reduced capacity at the network airports. They also proposed a rounding heuristic to find feasible solutions.

This paper presents the time-band model for solving the aircraft routing problem in response to schedule disruptions. This model is constructed by transforming the problem into a time-based network whose time horizon is discretized. Conditions for the exactness of the model are provided as well as solution methods. In the next section, we give a formal definition of the problem under consideration. The optimization model is presented in Section 3. The complexity of the transformation procedure and the structure of the resulting formulation are described in Section 4. In Section 5, the data and empirical results are presented. Conclusions are offered in Section 6.

2. Problem definition

Currently, there are no comprehensive automated systems in use at commercial airlines that resolve the irregular operations routing problem. When aircraft are grounded or delayed, it is typical for operations managers to query databases containing flight schedule, aircraft routing, aircraft maintenance, and crew schedule information. Armed with these data, they find simple routing alternatives that put the airline back on schedule. Each fleet is considered separately. The guiding rule is to find the simplest alternative with the least schedule disruption. The most attractive candidates are those that contain the least number of canceled flights. The routing alternative that minimizes disruptions is the one that incurs the least cost in terms of passenger inconvenience or lost flight revenues. A routing alternative that gets the airline back on schedule is one that positions aircraft at the end of the recovery period so that the original flight schedule can then be resumed.

When operations managers search for attractive routing alternatives, they do not consider modifying the route of an aircraft that requires maintenance service during the immediate recovery period. They also do not consider explicitly the crew requirements to staff candidate routings. When a routing is selected, it is passed to the crew coordinators for a feasibility review and is deemed acceptable only if appropriate personnel can be found to fly the aircraft.

2.1. *Constraints*

The problem that this paper addresses closely reflects the problem that must be solved in practice. The objective is to minimize the flight cancellation and delay costs associated with a recovery aircraft routing in response to groundings and delays. Flight cancellation costs are defined as the profit lost by not operating scheduled flights, assuming a certain amount of passenger recapture. Delay costs are the direct costs incurred due to flight delays, and may include overtime pay and additional fuel expenses.

All aircraft routings considered must be feasible with respect to the following constraints: (i) every flight in each aircraft route must depart from the station where the immediately preceding flight arrived; (ii) a minimum turnaround time must be observed between each flight arrival and subsequent departure; (iii) the recovery period extends to the end of the current day; (iv) aircraft must be positioned at the end of the recovery period so that the flight schedule can be resumed the next day; (v) station departure curfew restrictions must be observed; and (vi) no aircraft scheduled for maintenance during the recovery period can have its original route altered.

To evaluate aircraft routings, cancellation and delay costs must be given with the flight schedule. The cost of any alternate schedule is composed of the cost of each aircraft route and the sum of the cancellation costs for each canceled flight. The cost of an aircraft route is determined by observing the delay that flights will incur in the route and summing the costs for each of those delays. The first constraint above implies that the flow must be continuous and that no ferrying – transporting empty aircraft to another location – is permitted. In practice, ferrying aircraft is an option of last resort and is to be avoided when at all possible. The third constraint is somewhat arbitrary because we would like to recover as soon as possible. If we cannot get back on schedule by the end of the day, it is an easy matter to extend the recovery period to the following day or beyond. The fourth constraint is enforced by requiring specific quantities of aircraft to be located at individual airports by the end of the recovery period. This constraint is defined as aircraft balance. In keeping with airline practice, aircraft with maintenance scheduled during the recovery period will not be considered for route modifications. In addition, only single fleets will be considered, and crew availability will not affect the generation of alternate routings.

2.2. *Mathematical models*

Argüello (1997) introduced a resource assignment model and a multicommodity flow model to represent the irregular operations problem. The former is a path formulation whose objective is to find the least cost assignment of aircraft resources to flight sequences. The latter is based on a network representation of the problem in which the least cost flow of aircraft and cancellation commodities is determined.

The resource assignment model captures the details of the problem in the most concise and intuitive manner, and is similar to the set partitioning formulation used in vehicle routing (Desrochers *et al.*, 1992). This model treats aircraft as resources which are assigned to feasible routes and is extremely difficult to solve because the number of feasible aircraft routes is exponential with respect to the number of flights thus making enumeration impractical. In addition, this model is a general integer program for which there does not appear to be any special structure that can be exploited. Column generation techniques may be useful in generating feasible aircraft routings which can then be evaluated, but it is not clear that such methods will be efficient in practice.

The multicommodity flow model is generated from a network composed of a series of bipartite networks, each representing activity at a station. The individual bipartite networks consist of flight departure nodes, flight arrival nodes, commodity (aircraft) source nodes, and aircraft sink nodes. Figure 1 illustrates a two commodity network at one station. Nodes s_1 and g_1 are source nodes for aircraft and cancellation flows, respectively. The source and arrival nodes have arcs directed into the flight and sink nodes. The stations are connected by flight arcs directed from the flight nodes to the corresponding arrival nodes. Flow from a specific aircraft source node to a particular station sink node represents an aircraft route to which an aircraft may be assigned. Flights on a cancellation flow are canceled. The network can be constructed efficiently because the number of nodes required is linear with respect to the overall number of flights, aircraft, and stations while the number of arcs required is quadratic with respect to the same parameters. Flow costs throughout the network are determined by assigning cost functions on the flight arcs. Cancellation flow through a flight arc incurs the corresponding flight's cancellation cost. Aircraft flow through a flight arc is assigned the cost of servicing the corresponding flight at its specific position in the flow. Note that the cost for aircraft flow is not constant but depends on the path leading up to the flight arc. There is no cost for flows along other arcs.

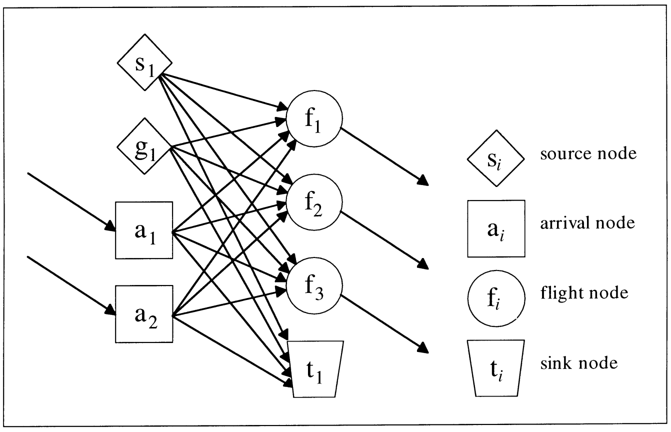

Fig. 1. Two-commodity network for one station.

The resulting mathematical model is a two-commodity minimum cost binary flow network with homologous arcs, an NP-hard problem (Carey and Johnson, 1979). The difficulty in solving this model, is that accurate delay costs cannot be placed *a-priori* on any of the flight arcs because these costs are a function of the flight's position in a specific aircraft route. This suggests a dynamic programming approach where routes are explicitly enumerated from one station to the next. We have not pursued this idea beyond a basic dynamic programming formulation.

Both the resource assignment model and the multicommodity flow model provide insight into the difficulty of the irregular operations problem. Although the resource assignment model captures the essence of the problem, the formulation eludes efficient solution techniques. The multicommodity flow model provides a first cut at a network representation but does not appear to be readily solvable. In the next section we present a time-based network that allows for the computation of realistic arc costs and the development of good and, in most cases, exact lower bounds to the original problem.

3. Time-band model

In order to develop a workable model that can be used for managing irregular operations, we transform the problem into a time-based network. The network is then used to develop a mathematical representation that captures delay and cancellation costs for each flight. The motivation for this approach is to exploit the underlying network structure associated with aircraft routings. The resulting mathematical program resembles a single commodity integral minimum cost flow problem with side constraints.

The idea is to partition the recovery period into time bands or discrete intervals. Station activity is then aggregated during each of the resulting time intervals. This aggregation allows us to model the flight connections to which an aircraft may be assigned and to approximate the costs associated with the possible connections. The output of the transformation is a network positioned on a two-dimensional plane in which one axis represents time and the other space or station location. A node in the transformation network represents the possible activity at a station during a specific segment of time. Because the network is time based, nodes are placed according to the segment of time they represent. Arcs directed into a node represent the arrival of an aircraft during a specific time segment for a scheduled flight. Arcs originating at a node represent specific flights that an available aircraft at the node may service.

All flights scheduled to depart from a station during the recovery period are made available to each node in the corresponding station. Therefore, if five flights are scheduled to depart Cleveland during the problem time horizon, then each node representing a segment of time at Cleveland will have five flight arcs originating from it. This allows all permutations of flight connections to be examined. Because each node represents a specific time interval, a delay cost may be calculated and associated with each flight arc. The strength of this network is that aircraft available at each node may be assigned to any of the flights scheduled to depart that node's station. Thus all possible aircraft routes may be considered for the set of available aircraft, and flight delay costs may be associated with each connection made at a node in the network.

Arc costs associated with flight delays are calculated by using the earliest arrival into a node as the available time for all aircraft at that node. The earliest available time is used as a reference point from which flights may depart. Flight delays are the difference of the scheduled departure time and the node's earliest available time. This convention underestimates delay costs because it links flight departure times to the first available aircraft at the node. An aircraft that becomes available later in the node's time segment will not be able to service a delayed flight as early as the first aircraft into the node. Thus the later arriving aircraft will incur a greater delay cost than earlier arriving aircraft. This implies that the delay costs placed on the flight arcs may underestimate the actual delay costs. However, these costs are exact if all arrivals into a node are at the earliest possible time. If all flight delay costs are exact at every node, then the network and resulting mathematical representation are exact.

3.1. *Transformation network*

The transformation network consists of two types of nodes and two types of arcs:

Station-time nodes:	nodes representing aggregate activity for a specific time segment at a station.
Station-sink nodes:	nodes representing the end of the recovery period at stations.
Flight arcs:	arcs representing specific scheduled flights.
Termination arcs:	arcs originating at station-time nodes and terminating at station-sink nodes; the purpose of these arcs is to allow for the termination of an aircraft route at the destination of the last flight in the route.

In the construction, a two-dimensional time-space network is developed. The time dimension is used for the placement of the station-time nodes. The space dimension separates the stations. For example, if a time axis is placed vertically along the network, then each column in the network represents a station. This is important because the placement of nodes in the network is dependent on the time segment and station each node represents. This allows each station to be represented by a column of station-time nodes and exactly one station-sink node. The number of station-time nodes for a station depends on the discretization of the recovery period for that station. Thus the time bands need not be uniform throughout the station space. Each station's time horizon may be discretized uniquely if desired.

The sample schedule presented in Table 1 will be used to illustrate the transformation procedure. This schedule assigns three aircraft to service 12 flights over four stations. The scheduled route for aircraft 1 originates and terminates in Boise (BOI). The routes for aircraft 2 and 3 originate and terminate in Seattle (SEA) and Spokane (GEG), respectively. Figure 2 depicts a network representation of the schedule. The network consists of 30-minute uniform time bands at each station and enforces a 40-minute minimum turnaround time. The nodes in the figure are marked by the time that the corresponding flight is available to depart rather than the actual departure time. For example, flight 11 arrives in Seattle at 1520 and connects to flight 12 which is available for departure at 1600 but is not scheduled to depart until 1605.

Let the transformation network also have 30-minute uniform time bands at each station and require a minimum turnaround time of 40 minutes. It is initialized by the placement of the station-time nodes at each time band. This results in a two-dimensional network consisting of nodes only. Assuming a midnight departure curfew time and no arrival curfew, the nodes placed at 2400 are the station-sink nodes. Initialization of the network continues with the marking of the appropriate station-time nodes with the earliest time at which an aircraft can become available. Assume that the current time is 1:30 p.m. and it is known that aircraft 2, located in Seattle, will be unavailable for the rest of the day. If we

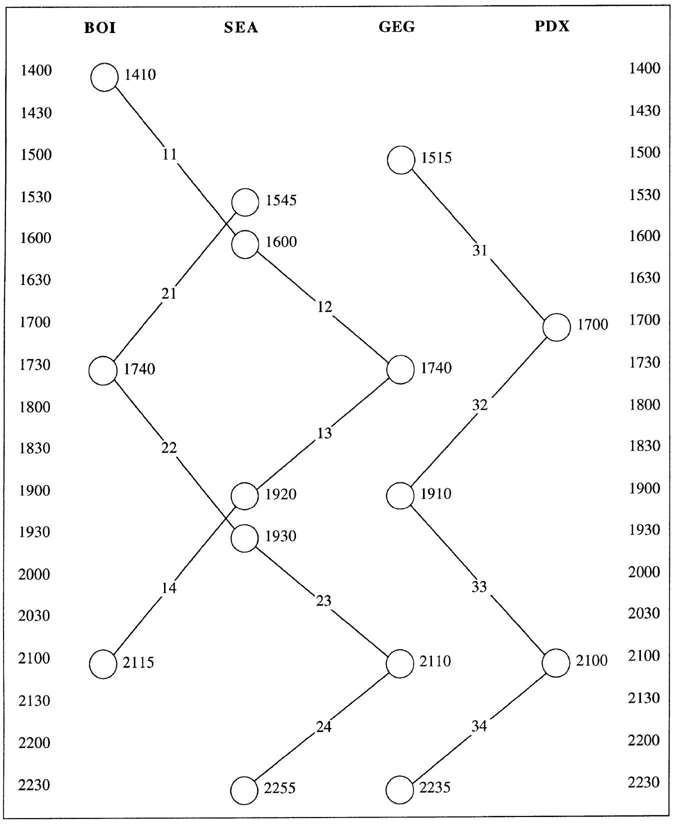

Fig. 2. Network representation of sample problem.

assume that at the beginning of the recovery period aircraft are available at their initial scheduled flight's departure time, then station-time node BOI:1400-1429 is marked with time 1410 to represent the availability of aircraft 1 (see corresponding node in Fig. 2). Similarly, station-time node GEG:1500-1529 is marked with time 1515 for aircraft 3. Initialization is completed by sorting the marked station-time nodes in non-decreasing order with respect to starting segment time. Thus for the sample problem BOI:1400-1429 is placed before GEG:1500-1529.

Table 1. Sample problem flight schedule

Aircraft id	*Flight id*	*Origin*	*Destination*	*Departure*	*Arrival*	*Duration*	*Cancellation cost*
Aircraft 1	11	BOI	SEA	1410	1520	1:10	7350
	12	SEA	GEG	1605	1700	0:55	10 231
	13	GEG	SEA	1740	1840	1:00	7434
	14	SEA	BOI	1920	2035	1:15	14 191
Aircraft 2	21	SEA	BOI	1545	1700	1:15	11 189
	22	BOI	SEA	1740	1850	1:10	12 985
	23	SEA	GEG	1930	2030	1:00	11 491
	24	GEG	SEA	2115	2215	1:00	9581
Aircraft 3	31	GEG	PDX	1515	1620	1:05	9996
	32	PDX	GEG	1730	1830	1:00	15 180
	33	GEG	PDX	1910	2020	1:10	17 375
	34	PDX	GEG	2100	2155	0:55	15 624

Flight arcs are placed in the network to allow all possible feasible flight connections in an aircraft route. For example, in the sample problem, all flight arcs representing flights 11 and 22 originate from a station-time node for BOI. The station-time node's marked time determines each flight arc's departure time. If the marked time for a station-time node is earlier than a flight's scheduled departure time, then the scheduled departure time is used as the actual departure time for the flight arc; otherwise, the marked time is used. This is important because it provides a method for approximating the delay in a flight arc. The delay is the positive difference of the flight arc's departure time and the corresponding scheduled departure time. To insure that the minimum turnaround time constraint is enforced, the designated turnaround time is added to the expected flight duration to determine the length of a flight arc. For example, the flight arc for flight 11 from BOI:1400-1429 has a departure time of 1410 and length equal to the 70-minute flight duration plus the 40-minute turnaround time. This arc connects BOI:1400-1429 with SEA:1600-1629. However, BOI:1730-1759 connects with SEA:1930-1959 along the arc corresponding to flight 22. This is because the departure time for this arc is 1740 and its length is 110 minutes.

The flight arcs are placed in the network iteratively. Let Λ be the list of station-time nodes ordered according to their marked earliest arrival time. Thus after initialization for the sample problem, $\Lambda = \{$BOI:1400-1429, GEG:1500-1529$\}$. Each iteration begins by removing the first node λ in Λ. For each of the scheduled departing flights from the station corresponding to λ, place a flight arc from λ to the appropriate destination node as described in the preceding paragraph. If the destination node is a marked station-time node, mark it with the earlier of the marked time and the flight arc's destination (departure + duration + turnaround) time. If the destination node is an unmarked station-time node, mark it with the flight arc's destination time and insert it into Λ. The delay cost for each flight arc is calculated according to the arc's delay. This process is repeated until flight arcs originate from all marked station-time nodes. Unmarked station-time nodes may be removed from the network. Termination arcs are then placed from each station-time node to the corresponding station's station-sink node.

The pseudocode for the network transformation procedure is given in Fig. 3 (a and b). The resulting network for the sample problem is displayed in Fig. 4 and contains 25 nodes and 87 arcs. All of the nodes are station-time nodes except 5, 11, 19 and 25 which are station-sink nodes. The arcs are made up of 66 flight arcs and 21 termination arcs. By construction, all station-time nodes associated with a particular airport have the same number of emanating arcs. This implies that some arcs represent delayed flights. For example, one of the two arcs from node 2 to node 8 represents flight 11 delayed until the arrival of flight 21 at 1700. The second arc represents the original flight 22 which is also represented by the arc from node 1 to node 8. If arc (1,8) is flow, it means that either flight 11 was cancelled and that the aircraft assigned to that flight was used for flight 22 or a second aircraft was available at node 1.

Table 2 contains the non-zero delay costs for the arcs in the transformation network assuming a \$20 per minute delay cost function. This is a conservative estimated (see Rakshit *et al.*, 1996).

3.2. *Mathematical formulation*

At first glance the time-band network might suggest that all we need to do is solve a simple integral minimum cost flow problem. This is not the case because the network does not provide for the cancellation of flights nor does it guarantee that all flights are assigned to a unique aircraft route or are canceled. These requirements lead to the mathematical formulation given below.

Indices

i, j = node indices;
k = flight index.

Sets

F = set of flights;
$G(i)$ = set of flights originating at station-time node i;
$H(k,i)$ = set of destination nodes for flight k originating at station-time node i;
I = set of station-time nodes;
J = set of station-sink nodes;
$L(i)$ = set of flights terminating at node $i \in J$;
$M(k,i)$ = set of origination station-time nodes for flight k terminating at node $i \in J$;
$P(k)$ = set of station-time nodes from which flight k originates;
$Q(i)$ = set of station-time nodes terminating at station-sink node $i \in J$.

Parameters

a_i = number of aircraft that become available at station-time node i at time zero;
c_k = cost of canceling flight k;
d_{ij}^k = delay cost of flight k from station-time node i to node j;
h_i = number of aircraft required to terminate at station-sink node $i \in J$.

Variables

x_{ij}^k = amount of aircraft flow for flight k from station-time node i to node j;
y_k = cancellation indicator for flight k;
z_i = number of aircraft flow from station-time node i to station-sink node at same station.

```
(a)
Procedure Initialization

input: Ψ = original flight schedule, Π = station recovery period discretization plan
output: Γ = time-space network, Λ = ordered list of marked station-time nodes

begin
for each station do                                   /* create nodes */
    begin
    for each time segment in recovery period do
        begin
        place station-time node λ under station column at beginning of time segment
        mark(λ) = 9999
        end
    end
for each available aircraft do                        /* mark aircraft supply nodes */
    begin
    locate appropriate station-time node λ
    mark(λ) = min {mark(λ), aircraft available time}
    end
let Λ = { }
for each station-time node λ do                       /* build list of marked station-time nodes */
    begin
    if mark(λ) < 9999 then place node in Λ
    end
sort Λ
end
```

Fig. 3. (a) Network transformation pseudocode, part 1, and (b) part 2.

Mathematical formulation

$$\text{minimize} \quad \sum_{k\in F}\sum_{i\in P(k)}\sum_{j\in H(k,i)} d_{ij}^k x_{ij}^k + \sum_{k\in F} c_k y_k, \tag{1a}$$

subject to (flight cover)

$$\sum_{i\in P(k)}\sum_{j\in H(k,i)} x_{ij}^k + y_k = 1 \quad \forall k \in F, \tag{1b}$$

(station-time node flow)

$$\sum_{k\in G(i)}\sum_{j\in H(k,i)} x_{ij}^k + z_i - \sum_{k\in L(i)}\sum_{j\in M(k,i)} x_{ji}^k = a_i \quad \forall i \in I, \tag{1c}$$

(station-sink node flow)

$$\sum_{k\in L(i)}\sum_{j\in M(k,i)} x_{ji}^k + \sum_{j\in Q(i)} z_j = h_i \quad \forall i \in J, \tag{1d}$$

(binary aircraft flow)

$$x_{ij}^k \in \{0,1\} \quad \forall k \in F,\ i \in I,\ j \in H(k,i), \tag{1e}$$

(binary cancellation flow)

$$y_k \in \{0,1\} \quad \forall k \in F, \tag{1f}$$

(integration termination arc flow)

$$z_i \in Z_+ = \{0,1,2,\ldots\} \quad \forall i \in I. \tag{1g}$$

The objective function (1a) minimizes the delay cost of flights in aircraft routes and cancellation costs for canceled flights. Constraint (1b) enforces the requirement that all flights must be in an aircraft route or canceled. The station-time node flow constraint (1c) enforces aircraft utilization while the station-sink node flow constraint (1d) enforces aircraft balance. Both of these constraints represent conservation of flow. If there are gate restrictions in a particular time band, this would be taken into account by adjusting the parameter a_i. The binary aircraft and cancellation flow constraints (1e) and (1f) merely stipulate that a flight may not be partially serviced or canceled. Constraint (1g) precludes the fractional termination of aircraft routes.

3.3. *Solution methodology*

Using the transformation network and the resulting mathematical formulation, a solution methodology can be specified as follows:

Step 1. (Initialization) Input original flight schedule and aircraft routing. Identify the grounded and delayed aircraft. Set time-band lengths for each station.

(b)

Procedure Transformation

```
input: Ψ, Γ with nodes only, Λ, Ωφ(t) = delay cost function for flight φ delayed by t time units
output: completed Γ, ω(φ,λ,ν) = delay cost of flight φ from node λ to node ν

begin
let σ = minimum turnaround time
while Λ not empty do
    begin
    remove first node λ in Λ
    let t = mark(λ)
    let s = station represented by λ
    for each departing flight scheduled from s do
        begin
        φ = flight id
        ŝ = destination station for flight φ
        δ = expected duration for flight φ
        η = max{t, scheduled departure time for flight φ}
        γ = η + δ + σ
        ν = node corresponding to station ŝ and time γ
        place flight arc Γ in from λ to ν
        ω(φ,λ,ν) = Ωφ(max{0, η + δ - scheduled arrival time for flight φ})
        if ν ∉ Λ then
            begin
            Λ = Λ ∪ {ν}
            resort Λ
            end
        mark(ν) = min{mark(ν), γ}
        end
    end
for each station-time node λ do
    begin
    if mark(λ) = 9999 then remove λ from Γ
    end
for each station-time node λ do
    begin
    place termination arc in Γ from λ to corresponding station-sink node
    end
end
```

Fig. 3. (*Continued*)

Step 2. (Network transformation) Transform the problem into the time-band network.

Step 3. (Mathematical representation) Create the mathematical program (1) from the time-band network.

Step 4. (Relaxed solution) Solve the relaxed linear program associated with (1). Set the optimal objective function value as the lower bound. If the solution is integral, go to Step 6.

Step 5. (Integer solution) Call mixed-integer solver to generate integer solution for (1).

Step 6. (Create schedule) Following the flows from the station-time nodes where aircraft are available to the station-sink nodes indicated in the integral solution, create the aircraft routes for the recovery schedule. Calculate the cost of the resulting schedule. Check the schedule for feasibility. If not feasible, reduce time-band lengths and go back to Step 2.

Step 7. (Assessment of results) Output schedule, accompanying cost, and lower bound. Compare deviation of schedule cost with lower bound.

The quality of the solution provided by this procedure can be improved arbitrarily by decreasing time-band lengths until an acceptable deviation from the lower bound is achieved. One approach is to initially set coarse time-band lengths, an optimistic deviation from the lower bound, and a moderate CPU limit. The time-band lengths could then be reduced systematically until the acceptable deviation is achieved or the CPU limit is exceeded.

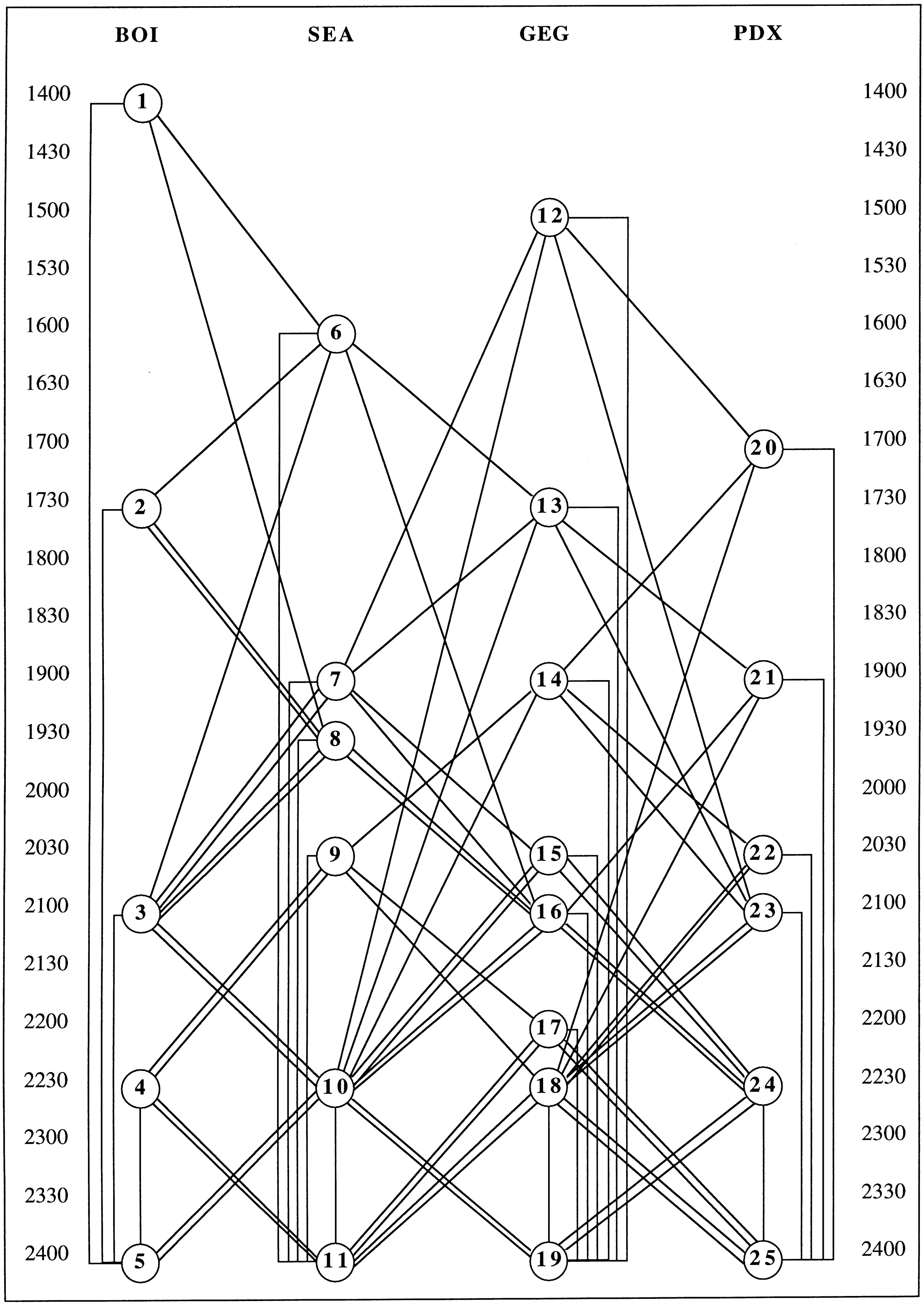

Fig. 4. Time-band transformation network for sample problem.

Table 2. Non-zero flight arc delay costs for sample problem

Flight id, ϕ	*Origin node*, λ	*Destination node*, v	*Arc delay cost*, $d(\phi, \lambda, v)$
11	2	8	4500
11	3	10	8500
11	4	11	10 300
12	7	15	3900
12	8	16	4100
12	9	17	5700
12	10	19	7800
13	14	9	1800
13	15	10	3900
13	16	10	4100
13	17	11	5700
13	18	11	5800
14	8	3	200
14	9	4	1800
14	10	5	3900
21	6	2	300
21	7	3	4300
21	8	3	4500
21	9	4	6100
21	10	5	8200
22	2	8	300
22	3	10	4300
22	4	11	6100
23	9	18	1600
23	10	19	3700
24	17	11	1400
24	18	11	1500
31	13	21	2900
31	14	22	4700
31	15	24	6800
31	16	24	7000
31	17	25	8600
31	18	25	8700
32	21	16	2300
32	22	18	4100
32	23	18	4200
32	24	19	6200
33	15	24	2100
33	16	24	2300
33	17	25	3900
33	18	25	4000
34	24	19	2000

It should be mentioned that the feasibility checking involved in Step 6 is with respect to curfews. The discretization procedure and corresponding mathematical representation guarantee feasibility with respect to all constraints except curfews. Nevertheless, the use of the earliest arrival convention in the transformation network may lead to differences between computed and actual arrival times at particular station-time nodes. These differences affect the later arriving aircraft and have downstream consequences. When a schedule is constructed from the solution of (1), some aircraft flows may not satisfy the curfew constraints. This situation can arise if the time-band network is not exact; however, it becomes more unlikely as the time-band lengths are decreased.

4. Complexity and exactness

One of the strengths of the time-band model is that it can be constructed efficiently. The transformation initialization procedure creates the nodes, and marks and sorts those where aircraft are available. The number of nodes in the transformation network is pseudo-linear with respect to the number of stations. Because the number of station-time nodes at each station is dependent on the number of time bands at the station, it can be stated that the number of nodes in the network is $O(mt)$ where t is the maximum number of time bands used for any station and m is the number of stations. Because t is arbitrary, the number of nodes is only pseudo-linear with respect to the number of stations. At most, the initial marking of nodes can be done in $O(r(m + t))$ time where r is the number of available aircraft. This comes from a linear search of stations and then time segments which must be executed r times to find the appropriate nodes to be marked. Ordering the marked nodes can be done in $O(r\log r)$ time. The initialization is dominated by creation of the nodes.

The transformation procedure itself consists of placing flight arcs, removing unmarked nodes, and placing termination arcs. If n is the number of flights in the schedule, then the number of flight arcs in the network is $O(mnt)$, again only pseudo-polynomial with respect to m and n. The effort required to place one flight arc is composed of the effort to find the appropriate destination node and to place a previously unmarked node into the ordered list of marked nodes. Searching for a node can be done in $O(m + t)$ time. Placing an element into the ordered list can be done in $O(\log(mt))$ in the worst case with a list consisting of all nodes. The first part of the actual transformation requires $O(mnt(m + \mathrm{t} + \log(mt)))$ time which is dominated by $O(m^2nt)$. The removal of unmarked nodes and the placement of termination arcs can be executed by simply inspecting all nodes and searching for the corresponding station-sink nodes, thereby requiring $O(mt^2)$ effort. The entire transformation is then dominated by the placement of the flight arcs which requires $O(m^2nt)$ effort and is pseudo-polynomial with respect to the number of stations and flights in the schedule. Thus careful control of the discretization of the recovery period will allow for an efficient transformation of the problem to the time-band network.

Given the real-time nature of the problem, it is imperative that the time-band network be solved quickly. The model itself is a general integer program. However, its underlying minimum cost network flow structure allows for investigation of efficient solution methods. It is possible, though not guaranteed, that solving this model first by finding a minimum cost flow solution that ignores the flight cover constraint and using this solution as an initial basis for a linear programming solution method will produce feasible solutions. This would have the advantage of generating feasible solutions in polynomial time with respect to the size of the network. Our experimental results reported in the next section confirm this.

Because the time-band model approximates delay costs, it is important to recognize those instances when the model is exact. This occurs when all arrivals into each station-time node occur near-simultaneously at the marked time. Delays are underestimated for arrivals that take place after the marked time so the corresponding delay costs are understated. A decrease in time-band length on average reduces the size and likelihood of error between the computed and actual arrival times, and hence improves the accuracy of the model. Nevertheless, one must be careful in increasing the number of time bands to achieve greater accuracy because this number has a direct effect on the effort required to transform and solve the original problem.

5. Computational experience

The time-band scheme was implemented in ANSI C and run on a SUN Sparcstation 10. CPLEX was used to solve the linear programs resulting from the relaxation of the integrality constraints (1f) and (1g). When fractional solutions arose, the mixed-integer component of CPLEX was called to find the optimum.

Experimentation with the options available in CPLEX for solving the various subproblems resulted in the selection of the hybrid network optimization – dual simplex algorithm for solving the linear programs and the network simplex algorithm and hybrid network optimization – dual simplex algorithm for solving the mixed-integer programs, respectively. The details are discussed by Argüello (1997). The algorithms selected were observed to solve the time-band problems most efficiently with respect to time, and for the linear programs, they also provided the greatest percentage of integral solutions.

The sample problem introduced previously required less than 0.1 CPU seconds to solve. The results are presented in Table 3. The solution to the linear program resulting from the relaxation of the integrality constraints (1f) and (1g) produced an optimal objective value of \$22 165 which is a lower bound. The solution was integral and feasible. After adjusting arrival and departure times to match the real flows, the resultant schedule had an actual value of \$23 265. This was proved to be optimal through enumeration. Thus the lower bound was not tight, and the 30-minute time-band lengths used did not result in an exact representation of the problem. If enumeration had not been used to verify the optimality of the schedule, we could only state that it was within 5% of the optimum.

For full testing, the time-band model was applied to a flight schedule obtained from Continental Airlines for their 737-100 fleet. This consisted of 162 flights serviced by 27 aircraft over a network of 30 stations in the continental United States.

The cancellation costs associated with the 737-100 flight schedule represent lost profit as determined by a survey of actual fares for the flights, assuming 75% capacity, 50% passenger recapture, and 10% profit margin. A uniform delay cost of \$20 per minute was assessed for all delayed flights. A 35-minute turnaround time was observed at all stations and a departure curfew of midnight local time was assumed.

A total of 427 instances were tested. Included in these were all 27 cases where one aircraft is grounded. The remaining cases consisted of 100 random instances of two, three, four, and five grounded aircraft each. It was

Table 3. Time-band solution for sample problem; total cost = \$23 265

Aircraft id	*Flight id*	*Origin*	*Destination*	*Departure*	*Arrival*	*Delay cost*	*Cancellation cost*
Aircraft 1	11	BOI	SEA	1410	1520		–
	21	SEA	BOI	1600	1715	300	–
	22	BOI	SEA	1755	1905	300	–
	23	SEA	GEG	1945	2045	300	–
	24	GEG	SEA	2125	2225	200	–
	14	SEA	BOI	2305	0020	4500	–
Cancel	12	SEA	GEG	–	–	–	10 231
	13	GEG	SEA	–	–	–	7434
Aircraft 3	31	GEG	PDX	1515	1620	–	–
	32	PDX	GEG	1730	1830	–	–
	33	GEG	PDX	1910	2020	–	–
	34	PDX	GEG	2100	2155	–	–

assumed that the grounded aircraft are identified before the start of the day and that the recovery period extends to the end of the day. Uniform time-band lengths of 5, 15, and 30 minutes were tested as well as the combination of 15- and 30-minute time bands at spoke stations, and 5- and 15-minute time bands at hubs (CLE and EWR). Thus five time-band models, with 5-, 5/15-, 15-, 15/30-, and 30-minute time-band lengths were investigated. The 5/15 and 15/30 combinations were tested to observe the benefits of modeling a subset of stations with smaller time-band lengths. The hub stations were selected as the subset based on the hypothesis that smaller time-band lengths at stations with greater activity would increase the accuracy of the model.

Tables 4 through 8 present the results of the application of each of the models to the 427 problem instances. The columns labeled "1," "2," "3," "4," and "5" represent results for the corresponding number of grounded aircraft. For example, results for the 100 random instances with two grounded aircraft are listed under the column labeled "2." The "All" column represents the results for all of the 427 instances tested.

The tables are divided into four sections. The first section presents lower bound, time-band, and cancellation averages. The "Lower bound" row gives the average lower bound value as obtained by solving the relaxed time-band linear program. The "Time band" row presents the average of the actual schedule cost corresponding to the solution of (1). The "Cancel cost" is the average cost of simply canceling all flights to which grounded aircraft are assigned. The schedule resulting from canceling grounded aircraft flights is often infeasible with respect to aircraft balance, yet it provides a suitable value with which the lower bound and time-band solution may be compared. It should be noted that in every instance, the schedule corresponding to the integral solution was feasible. Thus it was never necessary to loop through Steps 2 through 6 of the algorithm.

The second section of the tables gives the quantity of MIPs solved and the average CPU time in seconds required to perform specific operations. The "Transformation time" row indicates the average time required to transform a problem instance to the time-band network. The "LP time" is the average time required to solve the relaxed linear program. The "MIP time" is the average time required to solve problem (1) to optimality. This was necessary whenever the linear programs terminated with fractional solutions. The "Schedule construction" is the

Table 4. Results for uniform 5-minute time bands

	Grounded aircraft					
	1	*2*	*3*	*4*	*5*	*All*
Lower bound	13 586	26 084	39 448	51 678	64 823	43 490
Time band solution	14 169	26 763	40 143	52 316	65 526	44 162
Cancel cost	14 688	29 228	44 482	58 439	73 399	49 066
MIPs solved	1	3	7	4	2	17
Transformation time (Sec)	27.67	27.70	27.58	27.45	27.11	27.47
LP time (Sec)	91.21	121.17	137.21	150.89	159.50	138.97
MIP time (Sec)	129.00	138.76	208.25	232.18	584.62	241.24
Schedule construction (Sec)	0.04	0.04	0.04	0.04	0.03	0.04
Nodes	3422	3412	3400	3389	3375	3396
Flight arcs	21 321	21 242	21 148	21 065	20 965	21 119
Cancellation arcs	162	162	162	162	162	162
Absorption arcs	3392	3382	3370	3359	3345	3366
Rows	3584	3574	3562	3551	3537	3558
Columns	24 875	24 786	24 680	24 585	24 472	24 646
Constraint matrix density (%)	0.08	0.08	0.08	0.08	0.08	0.08
Optimal	0	0	0	0	0	0
1%	0	0	15	32	51	98
2%	0	35	68	92	99	294
3%	12	67	91	99	100	369
4%	16	91	97	100	100	404
5%	17	96	100	100	100	413
10%	27	100	100	100	100	427
15%	27	100	100	100	100	427
20%	27	100	100	100	100	427
25%	27	100	100	100	100	427

Table 5. Results for 15-minute time bands at spoke stations and 5-minute time bands at hubs

	Grounded aircraft					
	1	*2*	*3*	*4*	*5*	*All*
Lower bound	13 151	25 542	38 862	51 060	64 212	42 910
Time band solution	14 594	27 299	40 593	52 709	65 922	44 605
Cancel cost	14 688	29 228	44 482	58 439	73 399	49 066
MIPs solved	0	1	4	3	6	14
Transformation time (Sec)	5.90	5.88	5.87	5.94	5.97	4.91
LP time (Sec)	32.54	39.82	44.56	50.44	58.70	47.38
MIP time (Sec)	0.00	223.12	157.92	238.91	172.42	186.15
Schedule construction (Sec)	0.03	0.03	0.03	0.03	0.04	0.03
Nodes	1461	1458	1456	1452	1447	1454
Flight arcs	13 470	13 429	13 404	13 359	13 298	13 379
Cancellation arcs	162	162	162	162	162	162
Absorption arcs	1431	1428	1426	1422	1417	1424
Rows	1623	1620	1618	1614	1609	1616
Columns	15 063	15 018	14 992	14 943	14 877	14 964
Constraint matrix density (%)	0.18	0.18	0.18	0.18	0.18	0.18
Optimal	0	0	0	0	0	0
1%	0	0	0	0	1	1
2%	0	1	6	7	23	37
3%	0	5	18	49	68	140
4%	0	13	48	78	90	229
5%	1	28	63	91	99	282
10%	15	90	100	100	100	405
15%	19	100	100	100	100	419
20%	27	100	100	100	100	427
25%	27	100	100	100	100	427

average time required to construct a schedule corresponding to the integral solution obtained from the LP or MIP.

The third section presents the attributes of the time-band network and resulting mathematical program. The nodes and arcs refer to the network and the rows and columns are for the mathematical representation. The "Constraint matrix density" refers to the density of non-zero entries in the constraint matrix. It should be noted that the number of rows is the sum of the number of nodes and the number of flights, the number of columns is the sum of the flight, cancellation, and absorption arcs, and the number of constraint matrix entries equals (3 × flight arcs) + cancellation arcs + (2 × absorption arcs).

The final section in the tables gauges the quality of the schedules obtained with respect to the computed lower bound. Each entry in this section lists the number of instances for which the generated schedule cost matches the lower bound or is within a fixed percentage of the lower bound. For example, Table 4 shows that of the 100 instances with five grounded aircraft, the schedule corresponding to the time-band solution was within 1% of the lower bound 51 times and all schedules were within 3% of the lower bound. Similarly, the schedules generated for every one of the 427 instances were always within 10% of the lower bound.

Comparison of the data in Tables 4 through 8 leads to some fundamental and not surprising observations. As the time-band lengths are reduced from 30 to 5 minutes, the relative lower bound value increases, the relative time-band solution value decreases, the computation time increases, the size of the network and mathematical model grows, and the gap between the lower bound and schedule value decreases. Figures 5 through 7 illustrate the relationship between the time-band lengths and the solution value, CPU time, and number of aircraft grounded. In these figures, tb$x$ refers to the solution time bands of length x minutes. As the time-band lengths are reduced we observe a linear improvement in solution value but an exponential increase in running time. Figure 7 also indicates that increases in the number of grounded aircraft have a linear effect on the running time for the time-band model, regardless of the length of the time bands. It should be noted that the CPU time used for Figs. 6 and 7 is the average time required to transform the problem instance to the time-band network, develop the representative mathematical program, solve the mathematical program, and construct the schedule from

Table 6. Results for uniform 15-minute time bands

	Grounded aircraft					
	1	*2*	*3*	*4*	*5*	*All*
Lower bound	12 852	25 080	38 483	50 667	63 869	42 522
Time band solution	14 975	28 076	41 489	54 005	67 167	45 616
Cancel cost	14 688	29 228	44 482	58 439	73 399	49 066
MIPs solved	0	1	3	1	1	6
Transformation time (Sec)	4.21	4.22	4.20	4.19	4.17	4.20
LP time (Sec)	14.53	17.86	19.61	21.14	23.03	20.04
MIP time (Sec)	0.00	56.38	20.00	67.20	75.10	43.11
Schedule construction (Sec)	0.03	0.03	0.03	0.03	0.03	0.03
Nodes	1271	1268	1268	1264	1261	1266
Flight arcs	7908	7891	7885	7857	7834	7869
Cancellation arcs	162	162	162	162	162	162
Absorption arcs	1241	1238	1238	1234	1231	1236
Rows	1433	1430	1430	1426	1423	1428
Columns	9311	9291	9285	9253	9226	9267
Constraint matrix density (%)	0.20	0.20	0.20	0.20	0.20	0.20
Optimal	0	0	0	0	0	0
1%	0	0	0	0	0	0
2%	0	0	2	1	1	4
3%	1	3	5	1	4	14
4%	1	3	8	2	13	27
5%	3	8	10	7	44	72
10%	8	23	86	99	100	316
15%	14	81	100	100	100	395
20%	15	100	100	100	100	415
25%	18	100	100	100	100	418

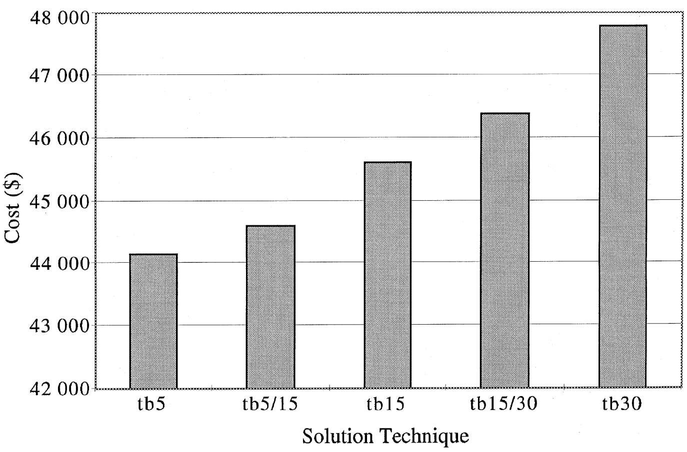

Fig. 5. Comparison of average solution value for varying time-band lengths.

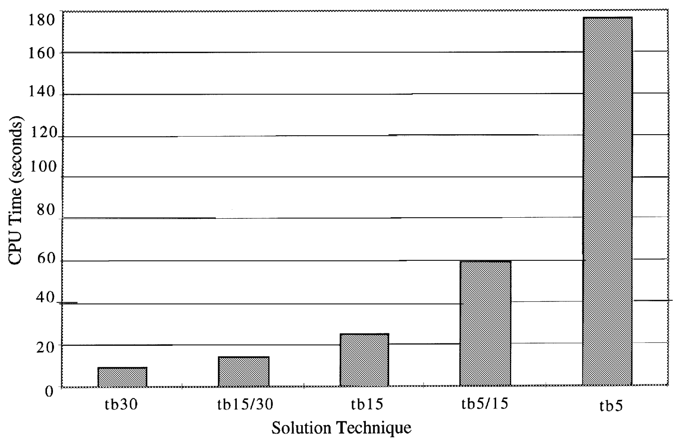

Fig. 6. Comparison of average CPU time for varying time-band lengths.

the solution generated. This is equivalent to the average overall processing time required to set up and solve a problem instance.

In general, shorter time-band lengths produce tighter lower bounds and lower cost schedules; however, this may not be true for a particular instance due to the inexactness of the model. To see this, let us focus on a single station-time node and recall that as a consequence of the earliest available time convention delay costs will be underestimated for flights that do not depart at the marked time. Consider a 10-minute time-band model and assume that a flight arrives at the end of one of the time bands. Its delay

Table 7. Results for 30-minute time bands at spoke stations and 15-minute time bands at hubs

	Grounded aircraft					
	1	*2*	*3*	*4*	*5*	*All*
Lower bound	12 378	24 279	37 466	49 428	62 524	41 461
Time band solution	15 349	28 694	42 205	54 833	68 097	46 364
Cancel cost	14 688	29 228	44 482	58 439	73 399	49 066
MIPs solved	0	1	1	4	4	10
Transformation time (Sec)	1.85	1.85	1.84	1.83	1.83	1.84
LP time (Sec)	8.92	9.66	11.35	11.90	13.02	11.32
MIP time (Sec)	0.00	36.37	32.58	32.49	42.12	36.74
Schedule construction (Sec)	0.03	0.03	0.03	0.03	0.03	0.03
Nodes	766	764	764	762	760	763
Flight arcs	5965	5952	5946	5925	5907	5934
Cancellation arcs	162	162	162	162	162	162
Absorption arcs	736	734	734	732	730	733
Rows	928	926	926	924	922	925
Columns	6863	6849	6842	6818	6799	6829
Constraint matrix density (%)	0.31	0.31	0.31	0.31	0.31	0.31
Optimal	0	0	0	0	0	0
1%	0	0	0	0	0	0
2%	0	0	0	0	0	0
3%	0	1	0	0	0	1
4%	0	3	3	0	0	6
5%	0	3	5	0	0	8
10%	6	11	15	33	77	142
15%	8	20	75	98	100	301
20%	11	59	98	100	100	368
25%	13	87	100	100	100	400

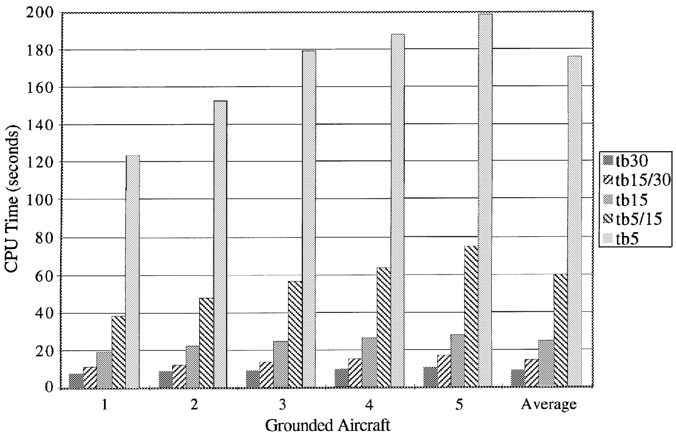

Fig. 7. Comparison of average CPU time for varying time-band lengths.

cost will likely be underestimated because the model assumes that it arrives at the marked time which is usually at the beginning of the time bands. Now consider a 30-minute time-band model for the same problem and assume that the flight at issue arrives at the beginning of one of the time bands. In this case, its delay cost will not be underestimated so the results will be more accurate. If all other flights incur the same costs, the solution obtained from the longer time-band model will be superior.

It should be noted that even at a relatively coarse time-band length of 30 minutes, the discretization procedure generates solutions that are measurably better to those obtained by simply canceling the grounded flights. In addition, they are always feasible unlike the latter. This point is particularly important because it is not obvious that most of the cancellation solutions are infeasible with respect to balance. Solutions generated in practice are likely to have costs higher than the simple cancellation solution because of the adjustments made to recover aircraft balance. Thus in about 10 seconds of processing time, the 30-minute time-band model produces high quality feasible solutions. This is a significant advance over current practice.

6. Summary and conclusions

The time-band model presented in this paper provides a methodology for generating provably good solutions in response to schedule disruptions. The transformation

Table 8. Results for uniform 30-minute time bands

	Grounded aircraft					
	1	*2*	*3*	*4*	*5*	*All*
Lower bound	12 167	23 528	36 386	47 987	60 933	40 309
Time band solution	15 264	29 074	43 427	56 825	70 629	47 793
Cancel cost	14 688	29 228	44 482	58 439	73 399	49 066
MIPs solved	0	1	0	0	1	2
Transformation time (Sec)	1.48	1.50	1.49	1.49	1.48	1.49
LP time (Sec)	5.76	6.61	7.35	7.82	8.54	7.46
MIP time (Sec)	0.00	25.93	0.00	0.00	25.38	25.66
Schedule construction (Sec)	0.03	0.03	0.03	0.03	0.03	0.03
Nodes	711	709	709	707	705	708
Flight arcs	4411	4401	4399	4384	4369	4390
Cancellation arcs	162	162	162	162	162	162
Absorption arcs	681	679	679	677	675	678
Rows	873	871	871	869	867	870
Columns	5254	5242	5240	5222	5206	5229
Constraint matrix density (%)	0.32	0.32	0.32	0.32	0.32	0.32
Optimal	0	0	0	0	0	0
1%	0	0	0	0	0	0
2%	0	0	0	0	0	0
3%	0	2	1	0	0	3
4%	0	3	2	0	0	5
5%	0	3	5	0	0	8
10%	6	12	13	3	3	37
15%	11	19	28	25	34	117
20%	14	36	48	59	93	250
25%	15	52	77	92	99	335

network can be constructed in pseudo-polynomial time with respect to the number of flights and stations in the original schedule. The resulting problem is a variation of an integral minimum cost network flow problem. The complicating factor in the transformation is the time-band lengths which can be controlled arbitrarily. The network flow problem is solved as a linear program, or integer program if necessary, to obtain a lower bound as well as a solution to the original problem. The cost of the corresponding schedule is then compared to the lower bound to gauge its quality.

The application of the proposed scheme using data provided by Continental Airlines has produced very promising results. In particular, the use of 5-minute time bands leads to solutions that are provably within 5% of optimal for 97% of the instances and within 10% of optimal for every instance tested. On average, these are obtained in about 3 CPU minutes which is more than sufficient for real-time control. Because the lower bounds generated are not necessarily tight, it is likely that many of these solutions are optimal. The use of longer time bands results in a linear erosion of the lower bound and solution quality but requires significantly less processing time. The computational effort grows linearly with the number of groundings and exponentially with the level of discretization. Thus a major schedule disruption resulting in station closures and aircraft groundings is manageable by carefully controlling the time-band length.

Acknowledgements

This work was partially supported by the Texas Higher Education Coordinating Board under grant ARP-003.

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Biographies

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Contributed by the Applied Optimization Department