



# Integrated recovery of aircraft and passengers after airline operation disruption based on a GRASP algorithm



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## ABSTRACT

This paper considers the integrated recovery of both aircraft routing and passengers. A mathematical model is proposed based on both the flight connection network and the passenger reassignment relationship. A heuristic based on a GRASP algorithm is adopted to solve the problem. A passenger reassignment solution is demonstrated to be optimal in each iteration for a special case. The effectiveness of the heuristic is illustrated through experiments based on synthetic and real-world datasets. It is shown that the integrated recovery of flights and passengers can decrease both the recovery cost and the number of disrupted passengers.

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## 1. Introduction

The airline industry plays a key role in the global transportation system. When scheduling flights, passenger travel demand is clearly a key consideration for commercial airlines. In an increasingly competitive market, it is becoming increasingly important for airlines to efficiently utilize their resources, such as aircraft and crew members (AhmadBeygi et al., 2008). Thus airlines often spend a great amount of time and effort planning flight schedules several months in advance. The dispatchers at an airline's operations control center are in charge of daily flight schedules, which include the routing of both aircraft and crew members during the day.

Despite the efforts airlines put into scheduling, one of the major challenges that the airline industry currently faces is disruption to regular flight operations. Unforeseen events, such as severe weather, the late arrival of crew members, and aircraft maintenance, can occur and lead to delayed or cancelled flights, stranded passengers, and even airport closures. If not properly and promptly addressed, such disruptions will affect crew connections and passenger itineraries and may significantly damage an airline's profitability and reputation. Airlines in the USA lost \$40.7 billion to flight delays in 2007, according to a May 22, 2008, report from the Joint Economic Committee from the House and Senate. Therefore, it is essential for an airline to develop contingency plans in response to disruptions to remain competitive and meet passenger demand.

In the literature, Teodorovic and Guberinic (1984) and Teodorovic and Stojkovic (1990, 1995) pioneered the study of airline schedule recovery. However, many of the methods used for aircraft recovery are highly similar to those used for aircraft assignment in flight scheduling. For example, Jarrah et al. (1993), Rakshit et al. (1996), Mathaisel (1996),

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Yan and Yang (1996), and Cao and Kanafani (1997a,b) proposed minimum cost network flow models for the aircraft recovery problem and then solved them using network flow algorithms. Yan and Yang (1996) introduced a classical time-space network flow model to describe the aircraft rerouting problem and used four models to gradually combine flight arcs, grounded arcs, protection arcs and ferrying into a single model. Bard et al. (2001) presented a time-band network for the aircraft rerouting problem in response to grounding and delays. Eggenberg et al. (2007) solved the aircraft recovery problem with a column generation algorithm based on a time-band network. Argüello et al. (1997), Løve et al. (2005), Andersson (2006), and Liu et al. (2006, 2008) tried to obtain the near-optimal feasible solution to the aircraft recovery problem using meta-heuristics.

In addition to the recovery of aircraft, researchers have studied passenger recovery. Bratu and Barnhart (2006) proposed two models, a Disrupted Passenger Metric model (DPM) and a Passenger Delay Metric model (PDM), which are based on a time-band network represented by Bard et al. (2001). In DPM, only approximate delay costs were considered along with passenger disruption costs whereas in PDM, delay costs were more accurately computed by explicitly modelling passenger disruptions and recovery options. Three different scenarios with different levels of disruptions were analyzed with only DPM because PDM is not suitable for real-world operations.

Nevertheless, integrating the recovery of multiple resources is a difficult task due to the size of the resultant problem, and few attempts have been recorded in the literature. Lettovsky (1997) was one of the first to propose the integrated recovery of three resources: aircraft, crew members and passengers. However, only a portion of this model was implemented. Jafari and Zegordi (2010) presented a model that simultaneously recovers aircraft and passengers. The model considers the length of recovery periods, the use of aircraft rotations, and passenger itineraries. However, with its high computational complexity, the method can only handle small-scale disruptions with 13 aircraft for swapping within an airline.

Bisaillon et al. (2011) developed a large neighborhood search heuristic for the airline recovery problem by combining aircraft routing and passenger reassignment with the objective of minimizing operating costs and impacts on passengers. The core of the heuristic consists of a construction phase, a repair phase and an improvement phase. In each iteration, the optimization of aircraft routes is given higher priority, and then passenger itineraries are reassigned based on optimized aircraft routes. Sinclair et al. (2014) improved the heuristic by adding some additional steps in each phase. Their algorithm found 17 best solutions for 22 instances within a 10-min time frame. However, the computation results of the above two studies were only used for the 2009 ROADEF Challenge instead of for a real-life situation.

Arikan et al. (2013) derived a mathematical model for the joint problem by superimposing the passenger itinerary network onto the aircraft network. Cruise speed control was considered when trying to balance fuel consumption costs and delay propagation. The problem was formulated as a mixed-integer nonlinear program and solved using CPLEX. Several simultaneous disruptions were optimized on a four-hub network of a major U.S. airline in less than a minute on average. Similarly, Chan et al. (2013) proposed a formulation that integrates the recovery of aircraft and passengers with the objective of minimizing the sum of passenger delay cost and airline operation cost. In their model, airline operation cost included the cost of late flight arrivals and an inconvenience cost due to the transshipment of passengers to other airlines. However, methods to solve the model were not offered.

Hu et al. (2011) established an integer programming model based on a time-band network. Solutions for single-fleet aircraft recovery were obtained by first solving the linear programming relaxation in Lingo solver and then applying a rounding heuristic to obtain an integer feasible solution. The data used for the computation was only associated with 12 aircraft. Later Hu et al. (2015) extended the model to multi-fleet aircraft routing and passenger transiting optimization. The integer model was shown to be NP-hard, and the solution was obtained via CPLEX Solver directly. Moreover, both studies assumed that all passenger itineraries are comprised of a single flight leg.

In this paper, we propose a new approach to the integrated recovery problem of both aircraft and passengers. The goal is to find the optimal trade-off between passenger delay cost, passenger reassignment cost and the cost of refunding tickets. This approach ensures the consideration of passengers' travel demand during post-disruption recovery. We develop an integer programming model based on a flight connection network and passenger reassignment relationship. A heuristic algorithm is designed based on the GRASP algorithm. In the heuristic, for any newly available aircraft routings, passengers whose original itineraries are disrupted will be reassigned to other available itineraries with the same destinations, and the passenger reassignment solution is then shown to be optimal for a special case. For random levels of disruption based on empirical data, simulation results indicate that our approach is able to reduce recovery cost and passenger disruption. This suggests that our model can aid decision support so that passenger disruptions can be reduced along with controlling operating costs and recovering flight schedules.

The remainder of the paper is organized as follows. Section 2 provides a formal description of the problem, including the statement and the mathematical formulation. Section 3 introduces the heuristic in details, including passenger reassignment, initial feasible solution construction and improvement. Section 4 presents the computational results. The paper concludes in Section 5.

## 2. Formal problem description

### 2.1. Problem statement

When disruptions occur, the common first step for the operational control center is to collect information about flight schedules, aircraft routing, maintenance schedules and crew status. Based on such information, a recovery period is

defined as a target to promote fast recovery by the end of the recovery period, all flights should be operating as originally scheduled. During the recovery period, different recovery options can be used to control flight delay propagation and to recover from the disruption as quickly as possible, such as aircraft swapping, flight delays, flight cancellation and passenger reassignment. In practice, aircraft swapping between the same aircraft type has higher priority over swapping between different types, as each aircraft type has different characteristics that limit cross-assignments, especially the seating capacities. Substitutions are allowed between certain types in accordance with airline policies and aircraft configurations. Intuitively, aircraft with a larger capacity can substitute for smaller ones but not vice versa. For flight delays, the delay time of the flight segments will be computed according to the available time for the aircraft covering them or the arrival time of the earlier flight in the new aircraft routing. If no other recovery method (such as aircraft swapping or flight delays) is available for a disrupted aircraft within a reasonable time window, the only available decision is to cancel the flight and rebook its passengers on subsequent flights or even flights by other airlines. Passenger reassignment means that passengers whose original itineraries are disrupted will be reassigned to other available itineraries with the same destination. While other recovery method, like ferrying aircraft (transporting an empty aircraft between airports), is not considered in this paper, because in practice, ferrying aircraft is an option of last resort and is to be avoided when at all possible.

This paper considers the integrated recovery problem of aircraft and passengers (IRPAP). The goal is to reschedule disrupted flights as soon as possible and minimize the disruption of passengers by swapping aircraft, flight delays, flight cancellation or passenger reassignment. The impact of disruptions is measured by the sum of passenger delay cost, passenger reassignment cost and passenger refund cost. In a real-world flight rescheduling situation, when the staffs in AOCC (Aviation Operations Control Center) search for attractive routing alternatives, they do not consider modifying the route of an aircraft that requires maintenance service during the immediate recovery period. Additionally they do not consider explicitly the crew requirements to candidate routings. When an aircraft routing is selected, it is passed to the crew coordinators for a feasibility review and is deemed acceptable as long as appropriate crew members can be found to fly the aircraft. Therefore, to be consistent with airlines' real-world actions, aircraft with maintenance scheduled during the recovery period will not be considered for aircraft route swapping and modifications to aircraft routes are not limited by crew availability.

For a passenger itinerary, if one flight of the itinerary is cancelled, or the connection time between adjacent flights of the itinerary is not sufficient, then the itinerary is defined to be disrupted. In practice, there are three options for accommodating passengers whose itineraries are disrupted: (1) reassign them to other itineraries associated with the same airline and the same destination (2) negotiate with other carriers to fly passengers to their original destinations but on flights operated by other airlines; or (3) refund tickets or provide passengers with overnight accommodations and itineraries for the next day. Usually passengers have the choice between the second and the third options because the two options have similar costs to an airline. Because the problem we address is limited to a single airline, the second and third options are combined in our analysis. We define the passengers of the second and third options as those who are refunded. Passengers on a delayed flight will receive compensation, which corresponds to the passenger delay cost. The cost of passenger reassignment between different itineraries is associated with the difference in the arrival time between the two itineraries. Passengers who are refunded will incur both a monetary cost and a reputation cost to the airline.

In addition to the aforementioned assumptions, the following flow balance and operational constraints must also be taken into account when rerouting aircraft: (i) Each flight can be covered by only one aircraft or be cancelled; (ii) the new departure time for a flight should not be earlier than its planned departure time prior to the disruption; (iii) the delay time for each flight should not be greater than a pre-determined threshold; (iv) subsequent departures for each flight should be scheduled only after completing the previous flight, with a minimum turnaround time added. In other words, aircraft flow should be continuous; (v) an airport's arrival curfew time should be observed; (vi) at the end of the recovery period, a sufficient number of aircraft of different types should be positioned at each airport; (vii) when assigning passengers from a disrupted itinerary to other available itineraries, the first flight of the new itineraries that accept passengers must not depart earlier than that of the original itinerary and (viii) the number of passengers on each flight cannot be greater than the seating capacity of the aircraft assigned to this flight.

## 2.2. Mathematical formulation

An integer programming formulation, based on aircraft routings, is constructed to describe the problem more accurately. Aircraft routings satisfy the constraints of continuous aircraft flow and airport curfew time. This model is a set partitioning problem with side constraints, which means that it allocates limited resources (aircraft) to activities (flight segments).

**Index**

$e$	Fleet index
$r$	Aircraft route index
$p$	Aircraft index
$f$	Flight index
$s$	Airport index

**Sets**

$E$	Set of fleet types
$F$	Set of flights
$R$	Set of aircraft routes
$S$	Set of airports
$P$	Set of aircraft
$I$	Set of passenger itineraries
$P(e)$	Set of aircraft that belong to fleet $e$ ; $e \in E$
$R(f)$	Set of aircraft routes covering flight $f$ ; $f \in F$
$R(s)$	Set of aircraft routes end in the airport station $s$ ; $s \in S$ , $e \in E$
$F(i)$	Set of flights covered by the itinerary $i$ ; $i \in I$

**Parameters**

$d_{rjp}$	Delay cost per passenger in flight $f$ covered by route $r$ and aircraft $p$ ; $r \in R$ , $f \in F$ , $p \in P$
$g_i^{jrp}$	Cost per passenger reassigned from itinerary $i$ to itinerary $j$ whose last flight is covered by route $r$ and aircraft $p$ ; $i \in I$ , $j \in Q(i)$
$c_i$	Cost of each passenger who was initially in itinerary $i$ but is refunded eventually; $f \in F$
$m_{rjp}$	Delay time of flight $f$ covered by route $r$ and aircraft $p$ ; $r \in R(f, p)$ , $f \in F$ , $p \in P$
$h_s^e$	Number of aircraft belonging to fleet $e$ that should terminate at airport $s$ ; $s \in S$ , $e \in E$
$M$	The maximum amount of time a flight can be delayed
$N_f$	Number of passengers originally in flight $f$ ; $f \in F$
$N_i$	Number of passengers originally in itinerary $i$ ; $i \in I$
$N_e$	Number of seats in aircraft fleet $e$ ; $e \in E$
$v$	The minimum amount of time for a passenger to make a connection between two joint flights in one itinerary
$n_i$	Number of flights in the itinerary $i$ ; $i \in I$
$f_{il}$	The flight indexed $l$ in the itinerary $i$ ; $l \in \{1, 2, \dots, n_i\}$ , $i \in I$
$q_{ik}$	The scheduled connection time between the flight $f_{ik}$ and the flight $f_{i,k+1}$ ; $k \in \{1, 2, \dots, n_i\}$ , $i \in I$

**Decision variables**

$x_{rp}$	=1 if route $r$ covered by aircraft $p$ is considered in the solution, =0 otherwise; $r \in R$ , $p \in P$
$z_i$	=1 if itinerary $i$ is disrupted, =0 otherwise; $i \in I$
$t_i^{jrp}$	Number of passengers reassigned from itinerary $i$ to itinerary $j$ whose last flight is covered by the route $r$ and plane $p$ due to the disruption of itinerary $i$ ; $i \in I$ , $j \in Q(i)$
$y_i$	Number of passengers who were initially in itinerary $i$ but are eventually refunded; $i \in I$

$$\min c(x_{rp}, t_i^{jrp}, y_i) = \sum_{f \in F} \sum_{p \in P} \sum_{r \in R(f)} d_{rjp} N_f x_{rp} + \sum_{p \in P} \sum_{r \in R} \sum_{i \in I} \sum_{j \in I} g_i^{jrp} t_i^{jrp} + \sum_{i \in I} c_i y_i \quad (1)$$

s.t.

$$\sum_{p \in P} \sum_{r \in R(f)} x_{rp} \leq 1 \quad \forall f \in F \quad (2)$$

$$\sum_{p \in P(e)} \sum_{r \in R(s)} x_{rp} = h_s^e \quad \forall s \in S, \quad e \in E \quad (3)$$

$$\sum_{r \in R} x_{rp} \leq 1 \quad \forall p \in P \quad (4)$$

$$\sum_{p \in P} \sum_{r \in R(f)} m_{rjp} x_{rp} \leq M \quad \forall f \in F \quad (5)$$

$$\sum_{p \in P} \sum_{r \in R} \sum_{j \in I} t_i^{jrp} + y_i = N_i z_i \quad \forall i \in I \quad (6)$$

$$\sum_{p \in P} \sum_{r \in R} \sum_{j \in I} t_j^{irp} \leq (N_e - N_f) \sum_{p \in P(e)} \sum_{r \in R(f)} x_{rp} \quad \forall f \in F(i), \quad e \in E, \quad i \in I \quad (7)$$

$$z_i \geq 1 - \sum_{p \in P} \sum_{r \in R(f)} x_{rp} \quad \forall f \in F(i), \quad i \in I \quad (8)$$

$$\sum_{p \in P} \sum_{r \in R(f_{i,k+1})} m_{rf_{i,k+1}p} x_{rp} - \sum_{p \in P} \sum_{r \in R(f_{i,k})} m_{rf_{i,k}p} x_{rp} \geq (v - q_{ik})(1 - z_i) - Mz_i \quad \forall k \in \{1, \dots, n_i\}, \quad i \in I \quad (9)$$

$$x_{rp} = \{0, 1\} \quad \forall r \in R, \quad p \in P \quad (10)$$

$$z_i = \{0, 1\} \quad \forall i \in I \quad (11)$$

$$y_i = \{0, 1, 2, \dots\} \quad \forall i \in I \quad (12)$$

$$t_i^{jp} = \{0, 1, 2, \dots\} \quad \forall r \in R, \quad p \in P, \quad i, j \in I \quad (13)$$

In the model, the objective function (1) minimizes the sum of delay cost, reassignment cost and refund cost of passengers. Constraint (2) ensures that each flight is either cancelled or reassigned to one aircraft route. Constraint (3) ensures the balance of aircraft at each airport station by the end of the recovery period so that flight schedules for the next period/day can then begin as scheduled. Constraint (4) stipulates that each aircraft can at most be assigned to one aircraft route. Constraint (5) states that the delay time of each flight cannot exceed the maximum delay time limits. Constraint (6) implements the requirement of passenger flow balance by considering rerouting and ticket refunding. In other words, if itinerary  $i$  is disrupted, then all passengers should be reassigned to other available itineraries or be refunded, otherwise passengers should remain in the original itinerary. Constraint (7) states that for each flight, the total number of passengers in it must not exceed the capacity of the aircraft covering it. Constraints (8) and (9) define the disruption of itineraries, which means that if one flight of this is cancelled or the connection between adjacent flights in the itinerary is disrupted, then we define the itinerary as disrupted. Constraints (10)–(13) define the decision variables as being integer or binary.

IRPAP can be modelled accurately with the approach listed in Eqs. (1)–(13), but it is computationally very difficult to solve. The difficulty is due to the fact that for each aircraft  $p$ , all possible routes that it can be covered by  $p$  must be generated. This is an extremely large search spaces when the number of aircraft is large and the recovery period is long. In an attempt to generate feasible plans for aircraft routings and passenger reassignment, we develop a heuristic based on a greedy randomized adaptive search procedure (GRASP) to address this problem. As a practical heuristic, the problem size can be reduced by limiting the consideration to only a part of the routes for all aircraft, including the directly disrupted aircraft and available aircraft for swapping.

### 3. Solution method

A heuristic algorithm for combinatorial optimization problems, GRASP was described initially in [Feo and Resende \(1989\)](#). There are two phases within each GRASP iteration: the first intelligently constructs an initial solution via an adaptive randomized greedy function; the second applies a local search procedure to the constructed solution in the hope of finding an improvement. The concept behind the first phase is similar to the semi-greedy heuristic in [Hart and Shogan \(1987\)](#). Therefore, GRASP is a meta-heuristic combining a semi-greedy heuristic and a local search heuristic.

When considering aircraft groundings and delays, the initial solution obtained by simply delaying or temporally cancelling those flights to which affected aircraft were assigned may be feasible or near feasible. The advantage of using such an initial solution is that it is exactly or close to the original aircraft routing planned by the airline. Furthermore, it reflects the purpose of disruption management—decrease the derivation from the original flight schedule.

The GRASP that we used in this research incorporates the basic components of GRASP. The initial solution is that the flights assigned to delayed aircraft will be delayed successively as long as the airport curfew constraints are not violated. Then flights assigned to grounded aircraft will be inserted into other available aircraft routings in the descending order of cancellation cost. Finally all passengers will be reassigned using a passenger reassignment algorithm. A semi-greedy approach is added into the local search heuristic of the second phrase. The main algorithm (MA) of GRASP in this paper is represented as follows.

- Step 1. One initial feasible solution  $(x_0, t_0, y_0)$  is obtained, set  $index = 0$ , and a positive integer  $Maxindex$  is given.
- Step 2. Provide the neighborhood of the initial solution  $(x_0, t_0, y_0)$  and define it as  $N(x_0, t_0, y_0)$ . Find several solutions from  $N(x_0, t_0, y_0)$  and put them into  $rcl$  in an ascending order of solution costs and denote as  $(x'_1, t'_1, y'_1)$ ,  $(x'_2, t'_2, y'_2), \dots, (x'_n, t'_n, y'_n)$  successively, which satisfy  $c(x'_1, t'_1, y'_1) < c(x_0, t_0, y_0)$ .
- Step 3. Choose one solution  $(x'_i, t'_i, y'_i)$  randomly from  $rcl$ , let  $(x_0, t_0, y_0) = (x'_i, t'_i, y'_i)$  and proceed to Step 2; otherwise proceed to Step 4.
- Step 4. If  $index = Maxindex$ , then proceed to Step 5, otherwise let  $index = index + 1$  and proceed to Step 2.
- Step 5. The heuristic stops and output  $(x_0, t_0, y_0)$ .

From the analysis above, we need to address two issues: (1) how to find a suitable passenger reassignment for aircraft routings in each iteration; (2) how to define the neighborhood  $N(x_0, t_0, y_0)$  of the feasible solution. We describe our approaches in the following subsections.

#### 3.1. Optimal reassignment of passengers

Each feasible solution is composed of feasible aircraft routings, cancelled flights and appropriate passenger reassignment. In this section, a passenger reassignment algorithm is designed to obtain the suitable passenger reassignment solution based

on new aircraft routings in each iteration. In the algorithm, passenger itineraries are first classified and then treated in descending order of the cancellation cost of each itinerary. Then, passengers of each itinerary are reassigned in order by solving a minimal cost path flow problem repeatedly.

Let  $PI$  be the set of disrupted passenger itineraries that can be classified. It can be classified by departure airports and arrival airports, and each  $i \in PI$  has the same origin and the same destination. All components in  $PI$  are treated in the descending order of the cancellation cost of each itinerary. For a given itinerary  $i \in PI$ , the available paths are found from the origin of itinerary  $i$  to its destination within the recovery period, and then the available path network, denoted by  $N_i = (V, s, s', t, t', A, b, c)$ , is constructed. Each path is composed of new scheduled flights that can accommodate the passengers reassigned in the feasible aircraft routings. In the path network,  $V$  represents an airport station at a given point in time,  $s$  and  $s'$  represent the origin of itinerary  $i$ , and  $t$  and  $t'$  represent the destination of itinerary  $i$ .  $A = \{(s, s')\} \cup \{(t', t)\} \cup A_1 \cup A_2 \cup A_3 \cup A_4 \cup \{(s', t')\}$ .  $\{(s, s')\}$  and  $A_1 = \{(s, v) | v \in V\}$  represent the initiated reassignment of passengers of disrupted itinerary  $i$ .  $\{(t', t)\}$  and  $A_4 = \{(v, t) | v \in V\}$  represent the completed reassignment of passengers of disrupted itinerary  $i$ . Each arc  $(u, v) \in A_2$  represents a flight for which the actual departure time is not earlier than planned departure time of itinerary  $i$ . Each arc  $(u, v) \in A_3$  represents a possible connection of passengers between two flights if the two flights satisfy constraints (8) and (9), where  $u$  and  $v$  refer to the same airport station.  $\{(s', t')\}$ , a direct arc from  $s'$  to  $t'$ , represents the reassignment of passengers in itinerary  $i$  to other airlines or the refund of their tickets. There are two labels in each arc. The first label (denoted as  $b$ ) is cost per passenger in the new itinerary flow – it is a function of the total delay at the destination with respects to planned itinerary  $i$ . The second label (denoted as  $c$ ) refers to the capacity of each arc in the new itinerary. For a given arc  $(u, v) \in \{(s, s')\} \cup \{(t', t)\} \cup A_1 \cup A_3 \cup A_4$ ,  $b(u, v) = 0$ ,  $c(u, v) = n_i$ , where  $n_i$  represents the number of passengers in itinerary  $i$  and  $b(s', t') = c_i$ ,  $c(s', t') = n_i$ . For a given flight arc  $(u, v) \in A_2$ , which denotes flight  $f$ , if the itinerary of  $n_f$  passengers in flight  $f$  is disrupted, then the capacity of the flight arc  $c(u, v) = N_e - Np_f + n_f - res$  and  $b(u, v) = delay_i$ , where  $res$  represents the number of passengers who have been reassigned to itineraries that contained flight  $f$  and  $delay_i$  represents the total delay time at the destination of the new itinerary with respect to planned itinerary  $i$ .

Fig. 1 illustrates the available path graph  $N_i$  for a specified itinerary between PEK and SHA. Each arc has two labels, including the cost and the capacity. There are three possible paths from PEK to SHA: PEK–SHA, PEK–CKG–SHA,  $s' \rightarrow t'$ . The first two paths represent passengers who are reassigned to new itineraries within the same airline. There is also one special path from  $s'$  to  $t'$  with the arc label of (1040, 96) in which the passengers are refunded. Among the 96 passengers, 24 passengers would be assigned to path PEK–SHA, 24 passengers would be assigned to path PEK–CKG–SHA, and the other 48 passengers are ultimately refunded.

The passenger reassignment algorithm (PRA) is described as follows.

Input:  $PI = \emptyset$ ,  $I$ , new aircraft routings  $R'$ , itinerary transiting relationship  $T$ ;

- Step 1. For each  $i \in I$ , if it violates the constraints (8) and (9), then  $PI = PI \cup \{i\}$ ,  $I = I \setminus \{i\}$ .
- Step 2. Sort the elements of  $PI$  in descending order of  $c_i$ , if  $I \neq \emptyset$ , then return to Step 1, otherwise proceed to Step 3.
- Step 3. Choose one  $i$  from  $PI$ , evaluate its  $n_i$ , and construct  $N_i = (V, s, s', t, t', A, b, c)$ ,  $PI = PI \setminus \{i\}$ .
- Step 4. Let  $val\ fl = 0$  as the initial flow from  $s$  to  $t$ .
- Step 5. If  $val\ fl = n_i$  and  $PI \neq \emptyset$ , then  $T = T \cup \{fl\}$  and return to Step 3; if  $val\ fl = n_i$  and  $PI = \emptyset$  then proceed to Step 8; if  $val\ fl < n_i$  then proceed to Step 6.
- Step 6. Construct an incremental network  $N(fl)$  and find a minimum cost path, denoted by  $M$ , from  $s$  to  $t$ ; proceed to Step 7.

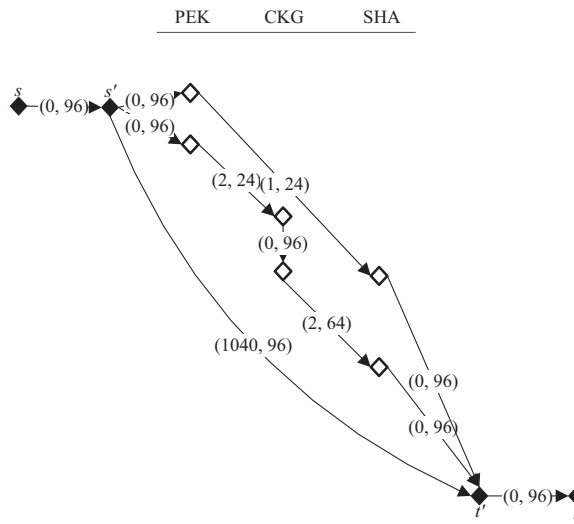


Fig. 1. An example of path network  $N_i$  for itineraries from PEK to SHA.

Step 7. Let  $C(M)$  denote the minimum arc capacity of path  $M$ , and let  $\theta = \min\{C(P), n_i - val\}$ ; add  $\theta$  to  $fl$  along  $M$  in network  $N$ , and obtain a new flow  $fl$ ; return to Step 5.

Step 8. Output  $T$ , and exit the algorithm.

From the definition of the capacity of flight arc  $b$  and the algorithm PRA, we know that there may exist two itineraries  $i, j \in PI$ , which satisfy  $N_i$  and  $N_j$  contain the same flight  $f$ . If passenger itineraries have been reassigned in the network  $N_i$ , then the seat capacity of flight  $f$  has changed. Therefore, the order for reassigning the itineraries has an important impact on the effect of passenger reassignment.

If each path in  $N_i, \forall i \in PI$  is composed of only one flight, that is if the departure airport and arrival airport of the flight are the origin and destination of the itinerary, respectively, then there is no flight covered by any two itineraries simultaneously in  $N_i$ . As  $\forall i, j \in PI$ , the origin or the destination of  $i$  and  $j$  are different, so there is no flight covered by any two networks  $N_i$  and  $N_j$ . The reassignment of disrupted itinerary  $i$  has no impact on the construction of  $N_j$ . Let  $dep(f)$  and  $arr(f)$  represent the departure airport and arrival airport of flight  $f$ ,  $ori(i)$  and  $des(i)$  represent the origin and destination of itinerary  $i \in PI$ . Then we have the following theorem.

**Theorem 1.** For any flight arc (denoted as  $f$ ) in  $N_i, \forall i \in PI$ , if  $dep(f) = ori(i)$  and  $arr(f) = des(i)$ , then PRA can obtain the optimal reassignment of all disrupted passengers for each solution with several feasible aircraft routings.

**Proof.** First, we need to prove that for  $\forall i, j \in PI$ ,  $N_i$  and  $N_j$  are mutually independent; that is, there is no existing flight  $f \in N_i \cap N_j$ .

If  $\exists f \in N_i \cap N_j$ , then  $dep(f) = ori(i)$  and  $arr(f) = des(i)$  &  $dep(f) = ori(j)$  and  $arr(f) = des(j) \Rightarrow ori(i) = ori(j)$  and  $des(i) = des(j)$ .

However, according to the classification principle of  $PI$ , itinerary  $i$  and itinerary  $j$  should not have the same origin and destination. So the initial assumption must be false.

Next, we need to prove that for  $\forall i \in PI$ , the optimal solution can be obtained for network  $N_i$ .

In the passenger reassignment algorithm, Steps 4–7 correspond to the minimum cost path algorithm. From the work of Tomizawa (1972), we know that the minimum cost flow can be obtained by the minimum cost path algorithm.  $\square$

Theorem 1 is more concerned on theoretical analysis, and it suggests that PRA can get an optimal solution for a special case, even though it is relatively restrictive. In real-world situations, not all itineraries can be finished via only a flight. Therefore, the approximately optimal solution of passenger reassignment can generally be derived from PRA.

### 3.2. Neighborhood search definition

Before describing the neighborhood, we introduce four notations.

$Pair = \{S_1, S_2\}$	Route pair, which can be defined as two aircraft routes, $S_1, S_2 \in R$ , or as one aircraft route and one cancellation route, $S_1 \in RC, S_2 \in R$
$Cap_i$	Capacity of aircraft $p_i$ ; $p_i \in P$
$dep(f)$	Departure airport of flight $f$
$arr(f)$	Arrival airport of flight $f$

There are three types of methods to generate neighborhood solutions: *insert*, *cross*, and *cancel*.

An *Insert* and *cross* can only be performed on route pairs. A flight originally covered by a smaller capacity aircraft can be inserted into a route for a larger capacity aircraft, while the reverse is not true. A *Cross* can only be performed between two aircraft routes with the same fleet type. A *Cancel* is only performed on individual aircraft routes.

Let  $S_1 = \{f_{1,1}, f_{1,2}, \dots, f_{1,n}\}$  and  $S_2 = \{f_{2,1}, f_{2,2}, \dots, f_{2,m}\}$ .

(1) *insert*: if flight string  $S_2$  satisfies  $S_2 \in R$ , and  $Cap_1 \leq Cap_2$ , then seek a flight string  $S'_1 = \{f_{1,u}, f_{1,u+1}, \dots, f_{1,v}\} \subseteq S_1$ , there are four methods to insert  $S'_1$  into  $S_2$ : ① *head-insert*; ② *mid-insert*; ③ *tail-insert*; ④ *tail-del-insert*.

① *head-insert*: if  $dep(f_{1,u}) = arr(f_{1,v}) = dep(f_{2,1})$ , then let  $S_2 = S_2 \cup S'_1$  and  $S_1 = S_1/S'_1$ , which satisfy

$$S_1 = \{f_{1,1}, \dots, f_{1,u-1}, f_{1,v+1}, \dots, f_{1,n}\}$$

$$S_2 = \{f_{1,u}, \dots, f_{1,v}, f_{2,1}, \dots, f_{2,m}\}$$

② *mid-insert*: if  $\exists 1 < j < m$  satisfy  $dep(f_{1,u}) = arr(f_{1,v}) = dep(f_{2,j})$ , then let  $S_2 = S_2 \cup S'_1$  and  $S_1 = S_1/S'_1$ , which satisfy

$$S_1 = \{f_{1,1}, \dots, f_{1,u-1}, f_{1,v+1}, \dots, f_{1,n}\}$$

$$S_2 = \{f_{2,1}, \dots, f_{2,j-1}, f_{1,u}, \dots, f_{1,v}, f_{2,j}, \dots, f_{2,m}\}$$

③ *tail-insert*: if  $dep(f_{1,u}) = arr(f_{1,v}) = arr(f_{2,m})$ , then let  $S_2 = S_2 \cup S'_1$  and  $S_1 = S_1/S'_1$ , which satisfy

$$S_1 = \{f_{1,1}, \dots, f_{1,u-1}, f_{1,v+1}, \dots, f_{1,n}\}$$

$$S_2 = \{f_{2,1}, \dots, f_{2,m}, f_{1,u}, \dots, f_{1,v}\}$$



- ④ *tail-del-insert*: if  $S_1 \in R$ ,  $v = n$  and  $dep(f_{1,u}) = arr(f_{2,m})$ , then let  $S_2 = S_2 \cup S_1$  and  $S_1 = S_1/S_1$ , which satisfy
- $$S_1 = \{f_{1,1}, \dots, f_{1,u-1}\}$$
- $$S_2 = \{f_{2,1}, \dots, f_{2,m}, f_{1,u}, \dots, f_{1,v}\}$$
- (2) *Cross*: if  $S_2 \in R$ ,  $S_1 \in R$  and  $Cap_1 = Cap_2$ , then seek two flight strings  $S'_1 = \{f_{1,u}, f_{1,u+1}, \dots, f_{1,v}\} \subseteq S_1$  and  $S'_2 = \{f_{2,j}, f_{2,j+1}, \dots, f_{2,i}\} \subseteq S_2$ . There are two methods for implementing a cross between two routes: ① *mid-cross*; ② *tail-cross*.
- ① *mid-cross*: if  $dep(f_{1,u}) = arr(f_{2,j})$  and  $arr(f_{1,u}) = arr(f_{2,i})$ , then let  $S_2 = S_2/S'_2 \cup S'_1$  and  $S_1 = S_1/S'_1 \cup S'_2$ , which satisfy
- $$S_1 = \{f_{1,1}, \dots, f_{1,u-1}, f_{2,j}, \dots, f_{2,i}, f_{1,u+1}, \dots, f_{1,v}\}$$
- $$S_2 = \{f_{2,1}, \dots, f_{2,j-1}, f_{1,u}, \dots, f_{1,v}, f_{2,i+1}, \dots, f_{2,m}\}$$
- ② *tail-cross*: if  $S_2 \in R$ ,  $S_1 \in R$  and  $dep(f_{1,u}) = arr(f_{2,j})$ , then let  $S_2 = S_2/S'_2 \cup S'_1$  and  $S_1 = S_1/S'_1 \cup S'_2$ , which satisfy
- $$S_1 = \{f_{1,1}, \dots, f_{1,u-1}, f_{2,j}, \dots, f_{2,m}\}$$
- $$S_2 = \{f_{2,1}, \dots, f_{2,j-1}, f_{1,u}, \dots, f_{1,v}\}$$
- (3) *Cancel*: If  $S_1 \in R$ , then seek a flight circle  $fc = \{f_{1,u}, f_{1,u+1}, \dots, f_{1,v}\} \subseteq S_1$ ; let  $S_1 = S_1/fc$  and  $RC = RC \cup fc$ .

Examples of the three methods for generating neighborhood solution are given in Table 1 and Fig. 2. Table 1 lists two sample routes, where Route 1 includes three flights and Route 2 has four flights. Route 1 begins with flight 11 from PEK (Beijing) to ZHA (Zhanjiang, Jiangsu) and ends with flight 13 from PEK (Beijing) to NGB (Ningbo, Zhejiang). Route 2 originates in PEK (Beijing) with flight 21 and terminates in PEK (Beijing) after the completion of flight 24. Fig. 2a illustrates these routes. They will be used to illustrate the neighborhood construction operations (in Fig. 2b–h).

The first method to generate neighborhood solutions is *insert*. An example of a *head-insert* involves removing flights 11 and 12 from Route 1 and placing them in front of Route 2. The results for Route 1 (13) and Route 2 (11–12–21–22–23–24) are shown in Fig. 2b. An example of a *mid-insert* is removing flights 11 and 12 from Route 1 and relocating them after flight 22 in Route 2. This leads to Route 1 (13) and Route 2 (21–22–11–12–23–24) as depicted in Fig. 2c. An example of a *tail-insert* is removing flights 11 and 12 from Route 1 and replacing them at the end of Route 2. The resulting route pair (see Fig. 2d) is Route 1 (13) and Route 2 (21–22–23–24–11–12). Note that all of the three insert methods require Route 1 and Route 2 to be covered by aircraft of the same type and that they preserve the origination and termination airports for each route in the route pair. For example, Route 1 remains at HEF and Route 2 remains at PEK. The fourth insert method, called a *tail-del-insert*, requires the two routes to both be aircraft routes with the same fleet types. For example, flight 13 is removed from Route 1 and placed at the end of Route 2. The results in Route 1 (11–12) and in Route 2 (21–22–23–24–13) are shown in Fig. 2e. If Route 1 is cancelled, the removed flights must be a circuit.

The first way of using the *Cross* method to generate neighborhood solution is a *mid-cross*. For example, flights 11 and 12 in Route 1 can be exchanged for flights 21 and 22 in Route 2. As shown in Fig. 2g, this provides Route 1 (21–22–13) and Route 2 (11–12–23–24). The second type of *Cross* is a *tail-cross*. Exchanging flight 13 with flights 23 and 24 is an example of this type of operation, which leads to Route 1 (11–12–23–24) and Route 2 (21–22–13). A *tail-cross* can only be applied between two aircraft routes. If one of routes is cancelled, this type of operation will violate aircraft balance.

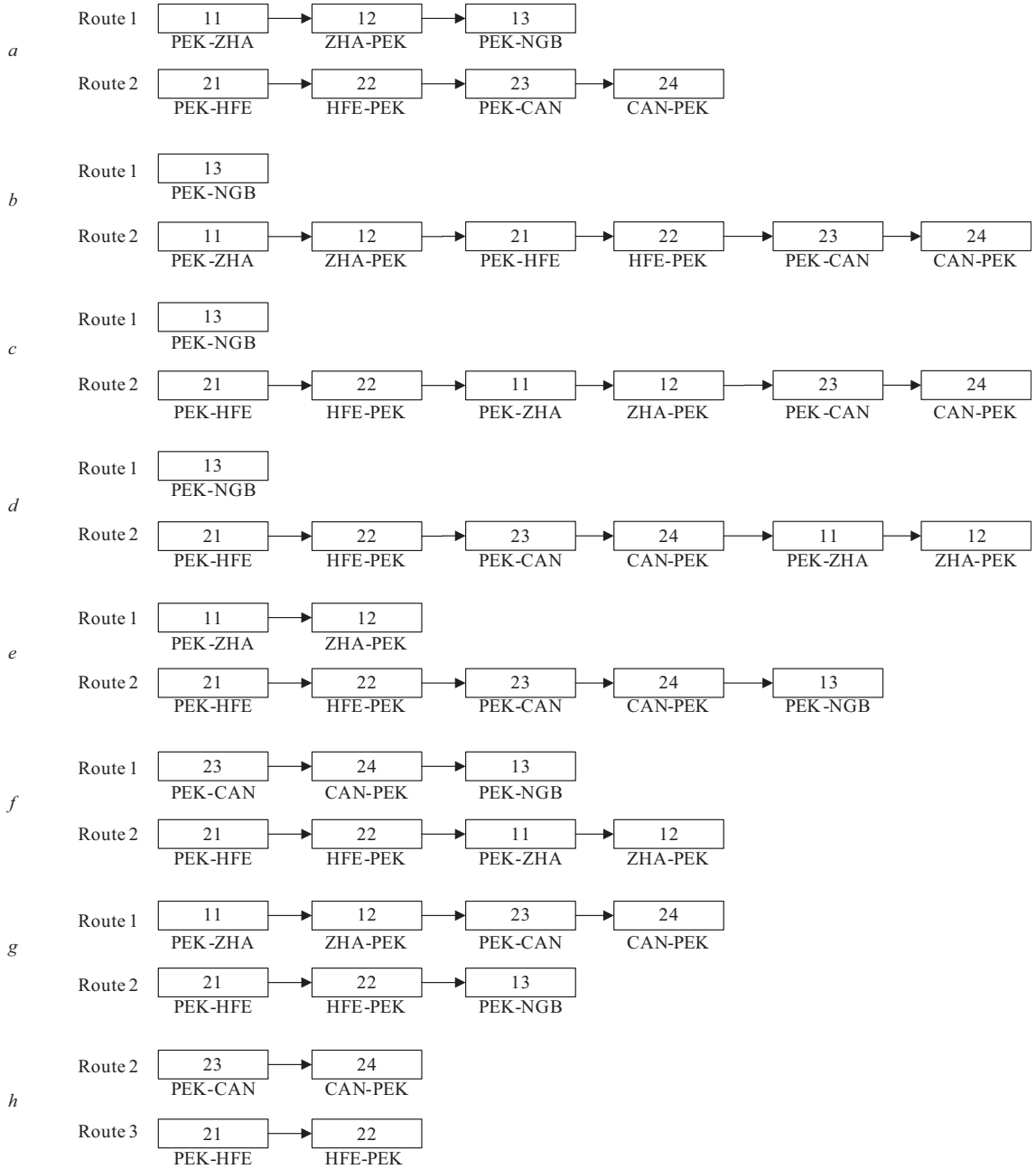
The operations *insert* and *cross* are performed on a route pair. The *cancel* operation is performed only on one aircraft route. This operation simply removes a flight circuit from the aircraft route and creates a new route in response to a cancellation in the aircraft route. The motivation for this operation is to allow for the cancellation of a circuit of flights without having to find a cancelled route in which to fit it. The result is the cancellation of the circuit and the reduction of flights in the route from which the circuit is removed. To illustrate this, we create empty cancellation route 3 and place flights 21 and 22 in it. The result of this operation gives (Fig. 2h) Route 2 (23–24) and route 3 (21–22). Only circuits are considered for the *cancel* operation, so that the remaining aircraft route remains feasible with respect to flow continuity and aircraft balance.

Table 2 presents all feasible neighbors for the sample routes. Both Route 1 and Route 2 are assumed to be aircraft routes with the same fleet types. If the aircraft fleet types between Route 1 and Route 2 are different, for example, if the aircraft capacity of Route 1 is smaller than that of Route 2, then solutions 5–19 would be infeasible. Because solutions 5 and 6 violate the constraints of aircraft balance, across operation can only occur between routes with the same aircraft fleet types. Similarly, if the aircraft capacity of Route 1 is larger than that of Route 2, then solutions 5, 6, 16, 17, 18, 19 would be infeasible because they cause Route 1 to violate the constraints of aircraft balance. All of these feasible neighborhood solutions satisfy both the maximum time and curfew time limitations.

**Table 1**  
Routing pair sample.

Route	Flight	Departure airport	Arrival airport
1	11	PEK	ZHA
	12	ZHA	PEK
	13	PEK	NGB
2	21	PEK	HFE
	22	HFE	PEK
	23	PEK	CAN
	24	CAN	PEK





**Fig. 2.** Sample of routing pairs and neighbors.

Another issue that we need to address is how to determine the capacity of *rcl*. Generally speaking, there are two methods: one is quantity regulation, and the other is quality regulation. Quantity regulation means that given a constant *m*, the top *m* optimal solutions are chosen from the neighborhood solutions and placed into *rcl*. Quality regulation gives a parameter  $0 \leq \alpha \leq 1$ , and a solution value in  $[c^{min}, c^{min} + \alpha(c^{max} - c^{min})]$  will be placed into *rcl*, where  $c^{min}$  and  $c^{max}$  are the maximum and minimum objective value, respectively, from the neighborhood solutions in each iteration. As  $c^{min}$  can hardly be evaluated until the global optimal solution is found, our research chooses quantity regulation to determine the capacity of *rcl*.

#### 4. Computational result

To explain and evaluate our method, two types of examples are generated. First, a small example with 5 aircraft is used to illustrate how our method works. Then, real-world empirical data for a Boeing 737 fleet from a major Chinese airline are used to test the method. The fleet consists of 87 aircraft covering approximately 340 domestic flights in daily operation.

**Table 2**  
Feasible neighborhood routes for the sample routing pair.

No.	Route 1	Route 2
1	11–12–13	21–22–23–24
2	13	11–12–21–22–23–24
3	13	21–22–11–12–23–24
4	13	21–22–23–24–11–12
5	11–12	21–22–23–24–13
6	NULL	21–22–23–24–11–12–13
7	21–22–11–12–13	23–24
8	11–12–21–22–13	23–24
9	23–24–11–12–13	21–22
10	11–12–23–24–13	21–22
11	21–22–23–24–11–12–13	NULL
12	11–12–21–22–23–24–13	NULL
13	21–22–13	11–12–23–24
14	23–24–13	21–22–11–12
15	21–22–23–24–13	11–12
16	11–12–21–22–23–24	13
17	23–24	21–22–11–12–13
18	11–12–23–24	21–22–13
19	21–22–23–24	11–12–13

All computations were performed on a DELL INSPIRON N4110 workstation with Intel i5-2410 CPU and 2.3 GB of RAM. The GRASP algorithm was coded in C++. Solution times are limited to 600 s.

When the airline experiences disruption in their schedule, dispatchers follow a series of rules to aid the recovery. In the order of descending priority, flights with very important persons (VIP) are rescheduled first, followed by international flights that are not carrying any VIPs, and finally, the remaining flights ordered by the affected number of passengers. To calculate the actual solution costs, we wrote a heuristic that implements this scheme. For the scenarios tested, we only consider domestic flights with no VIPs on board. In the heuristic, domestic flights in disrupted aircraft are prioritized in the descending order of the number of passengers aboard and are assigned to available aircraft one by one. Finally, flights that still cannot be accommodated are cancelled. The passengers on disrupted itineraries are transferred to other itineraries whenever possible. Finally, the remaining passengers will be refunded finally.

Furthermore, we validate that reassigning aircraft and passengers simultaneously is better than addressing aircraft and passengers separately, which is common in most airlines. We present another benchmark method called the separate recovery method (SRM), which specifically focuses on aircraft recovery and passenger reassignment in the sequential order.

**Table 3**  
Information from a simple example.

plane_ID	flight_ID	Departure airport	Arrival airport	Scheduled departure time	Scheduled arrival time	Refunding cost per passenger
1	11	PEK	XFN	9:00:00 AM	10:44:00 AM	1040
	12	XFN	PEK	11:25:00 AM	1:02:00 PM	970
	13	PEK	SHE	5:43:00 PM	6:43:00 PM	600
	14	SHE	PEK	7:40:00 PM	8:50:00 PM	710
2	21	PEK	HET	9:58:00 AM	10:48:00 AM	480
	22	HET	PEK	11:42:00 AM	12:35:00 PM	530
	23	PEK	HET	7:55:00 PM	8:45:00 PM	500
	24	HET	PEK	9:32:00 PM	10:22:00 PM	490
3	31	PEK	WEH	7:13:00 AM	8:14:00 AM	610
	32	WEH	PEK	10:16:00 AM	11:25:00 AM	690
	33	PEK	HLH	3:44:00 PM	5:16:00 PM	920
	34	HLH	PEK	6:34:00 PM	8:12:00 PM	980
4	41	HET	PEK	7:47:00 AM	8:31:00 AM	440
	42	PEK	CHG	10:45:00 AM	11:26:00 AM	410
	43	CHG	PEK	12:20:00 PM	1:05:00 PM	450
	44	PEK	YNT	7:47:00 PM	8:41:00 PM	540
	45	YNT	PEK	9:27:00 PM	10:27:00 PM	600
5	51	PEK	TGO	7:46:00 AM	8:50:00 AM	640
	52	TGO	PEK	10:07:00 AM	11:18:00 AM	710
	53	PEK	YNZ	1:41:00 PM	3:02:00 PM	810
	54	YNZ	PEK	4:13:00 PM	5:58:00 PM	1050
	55	PEK	HET	9:30:00 PM	10:20:00 PM	450

#### 4.1. Experiment with a sample problem

A sample problem (see Table 3) for 5 aircraft is introduced to evaluate the performance of our heuristics against the benchmark SRM. To make a fair comparison, we set up our experiments as follows. The disruption scenario is that aircraft 2 and 3 are both grounded, and aircraft 1 and 4 must be delayed for 2 h due to maintenance. Minimum turnaround time is assumed to be 40 min, the maximum delay time limitation is 4 h, and the recovery period varies from 7:00 AM to 12:00 AM.

Fig. 3 shows the aircraft routing solution derived from the airline heuristic, and we know that flights 23, 24, 33 and 34 are all cancelled. Aircraft routings from SRM and our heuristic are displayed in Figs. 4 and 5, respectively. From Figs. 4 and 5, we know that flight 55 and 24 are both cancelled.

Several itineraries have been disrupted due to the aircraft rerouting in Figs. 3–5. Take one disrupted itinerary HLH–PEK–HET, which includes flights 34 and 55, with 48 passengers as an example. For the aircraft routing solution in Fig. 3, the path network for HLH–PEK–HET is shown in Fig. 6. As a result, all passengers in itinerary HLH–PEK–HET are refunded. For the aircraft routing solution in Fig. 4, the new actual departure time of flight 23 is 9:02 PM, which is earlier than the scheduled departure time of flight 55. Thus no available itinerary satisfies the connection time of the passengers. The path network for HLH–PEK–HET is shown in Fig. 6 as well and all 24 passengers are ultimately refunded. For the aircraft routing solution in Fig. 5, let us again take disrupted itinerary HLH–PEK–HET, which includes flights 34 and 55, as the example. As flights 34 and 23 are reassigned to aircraft 1, and the new actual departure time of flight 23 is 9:30 PM, the connection time of the

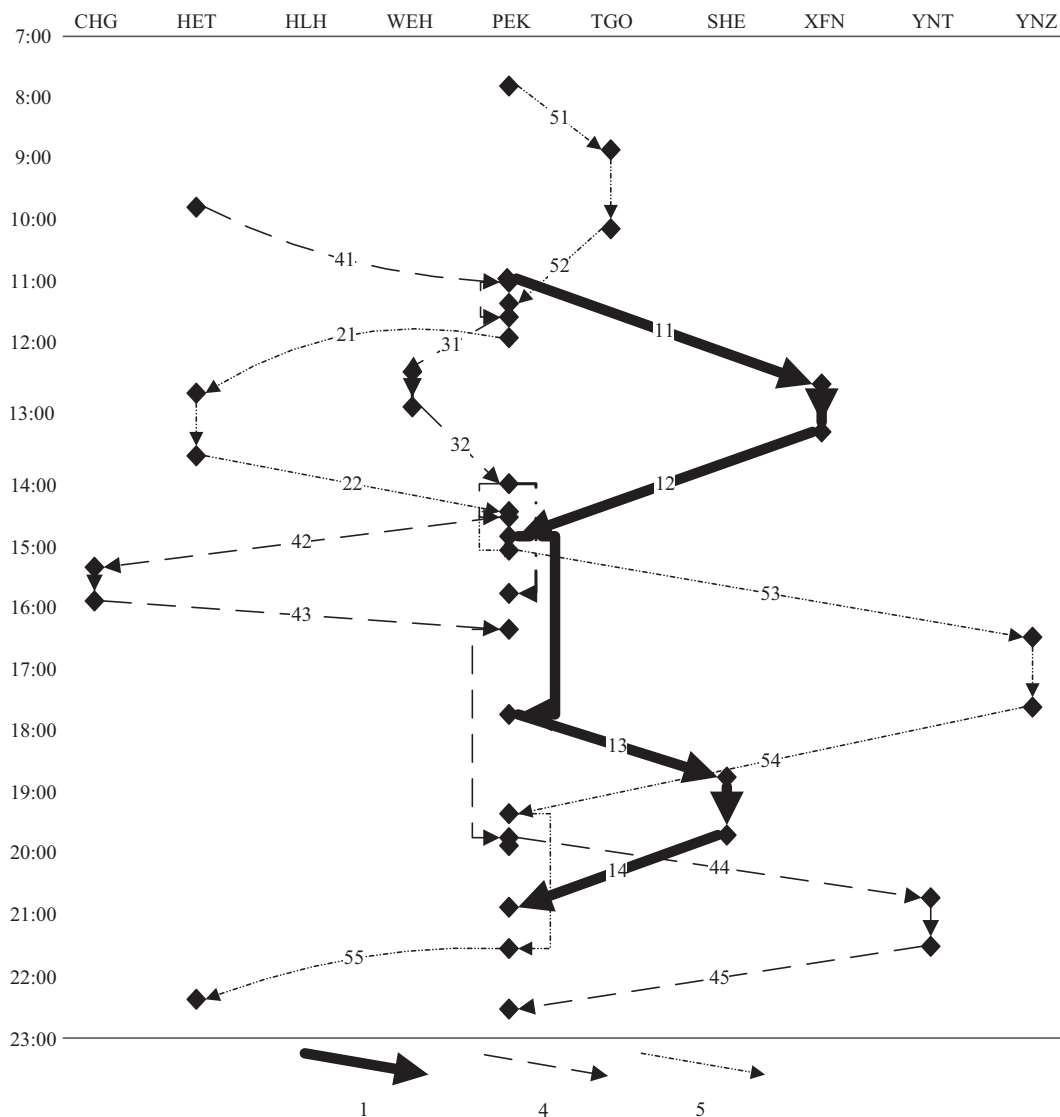


Fig. 3. Aircraft routing solution derived from airline heuristic.

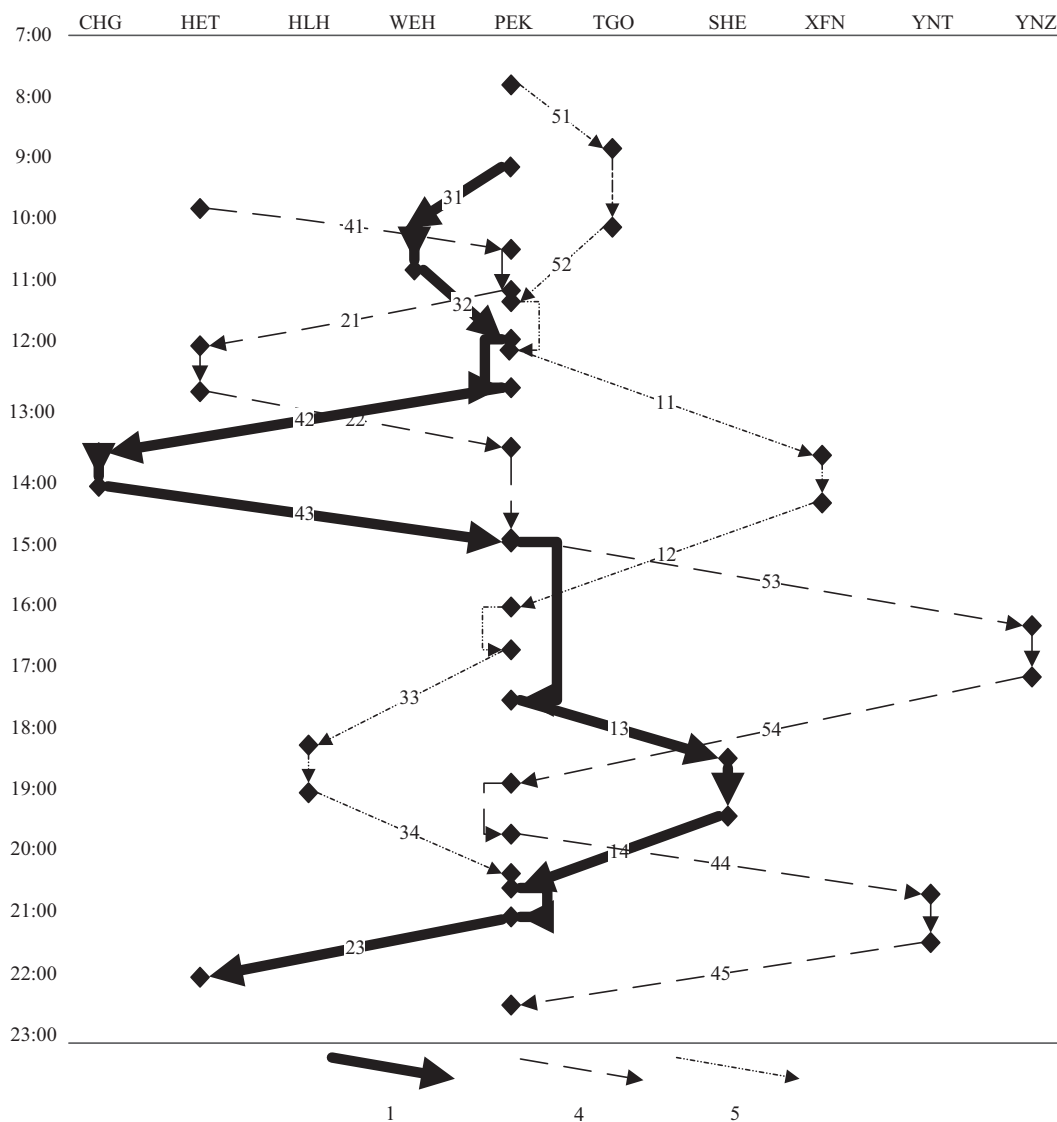


Fig. 4. Aircraft routing solution derived from SRM.

passenger reassignment is now sufficient. The path network for HLH–PEK–HET is shown in Fig. 7. A total of 24 passengers are reassigned to a new itinerary consisting of flights 34 and 23, and another 24 passengers are ultimately refunded.

Table 4 compares different methods, where the first column presents the original solution to cancel all disrupted flights without any recovery effort. The second column describes the actual solution, which reflects the cost that the airline would have incurred if their rules were followed for cancelling and rerouting passengers when faced with a disruption. The third column shows the solution from SRM, and the last column lists the solution from our heuristic.

We compare the three solutions using various metrics, including the number of delayed flights in the solution (“#delayed fls”), the number of cancelled flights in the solution (“#cancelled fls”), the total delay time of all flights in the solution (“total delay t”), the number of passengers reassigned to other available itineraries in the solution (“#reassigned p”), the number of passengers from disrupted itineraries who are refunded (“#refunded p”), flight delay cost (“delay cost”), passenger reassignment cost (“reassigned cost”), refund cost (“refund cost”), and total cost (“total cost”) of the solution.

As we can observe, the comparison in Table 4 shows that both SRM and our heuristic generate solutions with fewer cancelled flights, fewer passengers refunded, and lower cost compared the original solution and the actual solution.

More importantly, by considering passenger reassignment during aircraft recovery instead of after aircraft recovery, our heuristic can lower the number of passengers who are refunded and thus reduce the total cost incurred after the disruption. For the aircraft routing shown in Fig. 4, even if flight 23 can be delayed until 9:30 PM to wait for passengers from flight 55 and can thereby finish the itinerary HLH–PEK–HET, the delay cost of the solution from SRM would increase and total cost of SRM would still be higher than that of our heuristic.

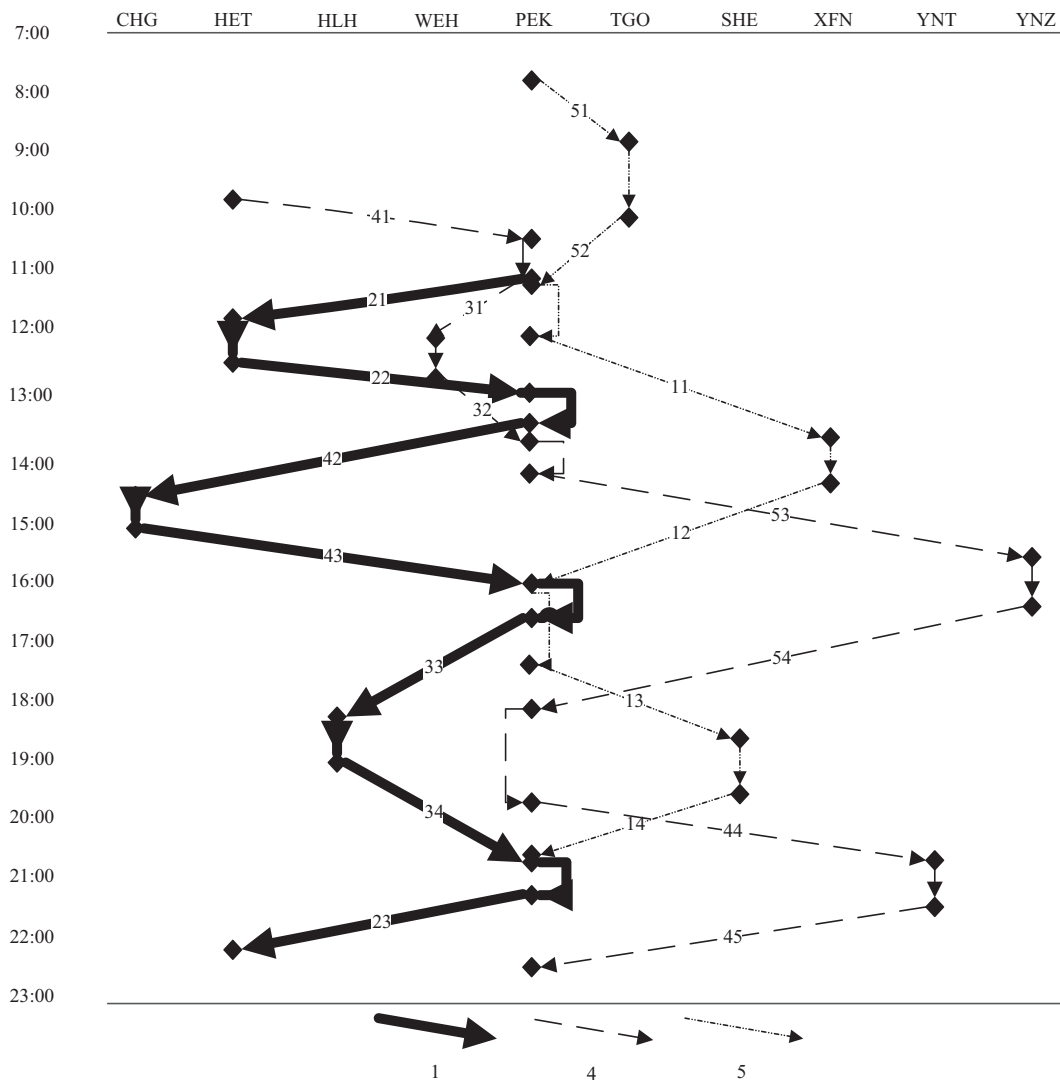


Fig. 5. Aircraft routing solution derived from our heuristic.

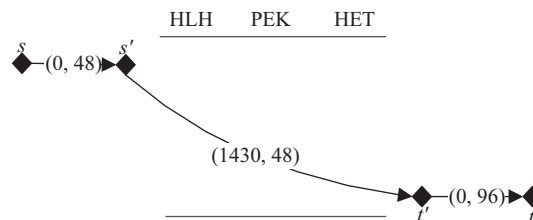


Fig. 6. Path network for HLH-PEK-HET based on the aircraft routing in Figs. 3 and 4.

#### 4.2. Experiments based on real-world data

To further evaluate the utility of our heuristic, we also test it on real-world empirical data provided by a major Chinese airline. The data include flight schedule (e.g., flight ID, flight departure and arrival times, departure and arrival airports) and aircraft data (e.g., aircraft ID, aircraft type and aircraft capacity). Examples of our aircraft data are shown in Table 5. The data-set is divided into multiple instances, with each corresponding to a randomly generated disruption scenario. The number of aircraft in the Boeing 737 fleet ranges from 5 to 87. The sub-fleet set is 3, and the flight set ranges from 22 to 340. The maximum number of airports covered in one instance is 95.

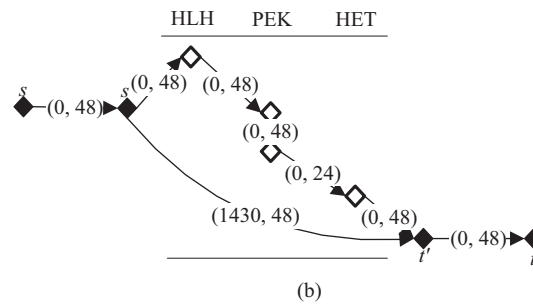


Fig. 7. Path network for HLH-PEK-HET based on the aircraft routing in Fig. 5.

Table 4

Comparison of solutions from different methods for the example.

	Original solution	Airline heuristic	SRM	Our heuristic
#delayed fls	5	11	13	14
#cancelled fls	8	4	2	2
total delay t (min)	23,820	93,600	95,400	97,680
#reassigned p	–	24	0	24
#refunded p	5200	360	192	168
delay cost	3811	8985	9060	8824
reassigned cost	–	346	0	0
refund cost	499,200	312,640	137,280	102,960
total cost	503,011	321,971	146,340	111,784

In addition to the empirical data, we use synthetic data on passengers, including the number of passengers on each flight and the cost for each passenger refunded.

Based on the original schedule, disruption scenarios (instances) were randomly generated as follows.

- 2–10 aircraft are grounded throughout the day.
- 2–5 aircraft are delayed for a period of time ranging from 1 h to 4 h.
- A 40-min minimal turnaround time is assumed at all airports.
- Delay cost for each passenger is CNY 0.1/min.
- The number of the passengers in each planned aircraft is 80% of its capacity.
- Reassignment cost for each passenger is determined according to the difference between the actual arrival time of the new itinerary and the arrival time of the scheduled itinerary. The rate is assumed to be CNY 0.15/min.
- The cost for each passenger refunded refers to the average ticket price of the itinerary.
- The recovery period is one day, that is, from 7:00 AM to 12:00 AM.

Sub-fleet substitution feasibility and aircraft sub-fleet capacities are listed in Table 6, where “y” indicates that the row fleet can substitute the column fleet to cover a flight as scheduled; otherwise, an “n” appears.

A total of 57 instances are tested in this paper. The procedure was restarted a total of five times per instance, and each instance could obtain a satisfactory solution within 600 s. A feasible solution for each instance could even be obtained within 30 s. Some instances with a slight disruption needed no more than 1 s to approach the optimal solution. Table 7 compares the average solution cost for the 57 instances tested. The solution from our heuristic offers a great improvement over the other three solution types. The comparison between SRM and our heuristic reveals that our heuristic can achieve 10.7% fewer passengers refunded as well as almost 10% lower total cost. The comparison between the airline heuristic and our heuristic

Table 5

Examples of aircraft data used in our study.

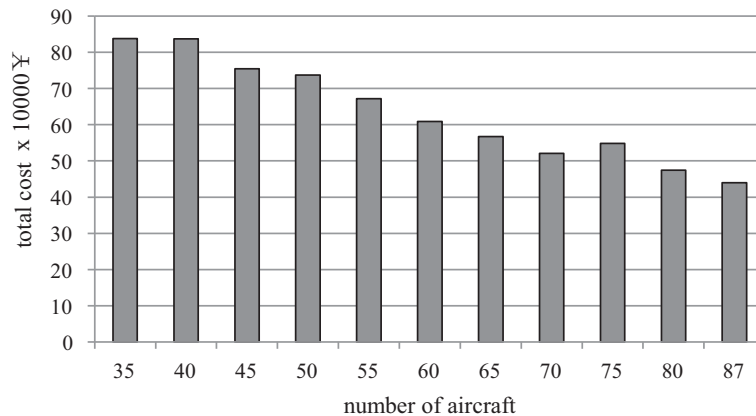
Plane_ID	Type	Available time	Curfew time	Capacity
B2580	737-300	2013-6-2 AM 07:45:00	2013-6-3 AM 12:00:00	120
B2587	737-300	2013-6-2 AM 09:00:00	2013-6-3 AM 12:00:00	120
B2588	737-300	2013-6-2 AM 07:15:00	2013-6-3 AM 12:00:00	120
B2627	737-300	2013-6-2 PM 07:47:00	2013-6-3 AM 12:00:00	120
B2630	737-300	2013-6-2 AM 07:46:00	2013-6-3 AM 12:00:00	120
B5196	73D	2013-6-2 AM 10:10:00	2013-6-3 AM 12:00:00	160
B5197	73D	2013-6-2 PM 07:49:00	2013-6-3 AM 12:00:00	160
...	...	...	...	...

**Table 6**  
Sub-fleet substitutions and capacities.

Sub-fleet	733	73D	738
Capacities	120	160	160
733	–	n	n
73D	y	–	y
738	y	y	–

**Table 7**  
Comparison of average solution cost.

	Original solution	Airline heuristic	SRM	Our heuristic
#delayed fls	6	29	45	48
#cancelled fls	21	18	6	7
total delay t (min)	33,368	1,641,089	1,591,185	1,590,927
#reassigned p	–	382	107	170
#refunded p	2542	1853	730	656
delay cost	5531	8286	11,746	11,385
reassigned cost	–	27,931	11,301	12,193
refund cost	2,641,136	1,686,690	647,328	578,182
total cost	2,646,667	1,722,907	668,374	601,760



**Fig. 8.** Comparison of total cost with different numbers of aircraft for swapping.

shows sharp differences. The average number of disrupted passengers derived from the airline heuristic is much greater than that provided by our heuristic. In addition, solution values from our heuristic dominate in the distribution of delay, re-assigned and refunded costs with a reduction of approximately 65%.

Of course, reading too much into the comparisons would be a mistake because the airline heuristic lacks any of the intelligence that one would find in approaches such as tabu search or simulated annealing. Nevertheless, the results demonstrate the degree to which the use of meta-heuristics can improve what has been commonly used by airlines. In addition to the reason stated above, another reason that the airline heuristic performs so poorly is that it ranks flights mostly by the number of passengers on board the disrupted aircraft. By contrast, our heuristic provides a system-wide view embodied in our heuristic, which implicitly trades off the three cost components to arrive at a satisfactory solution within the constraints of the airline.

Fig. 8 illustrates the relationship between the numbers of aircraft available for swapping and the average solution cost derived from our heuristic. The “total cost” decreases significantly with an increase in the number of swapping opportunities. In the experiments, we also find that the complexity of the problem greatly depends on the number of swapping opportunities. For minimum swapping opportunities, all test instances are solved near optimally in fewer than 100 s (approximately 49 s on the average). Therefore, when disruptions occur, suitable numbers of aircraft are chosen for swapping according to the scale of the disruption, which will promote an efficient recovery.

## 5. Conclusions

In this paper, we propose a new approach to solve the integrated aircraft and passenger recovery problem after disruptions to normal operations. Our approach leverages both the GRASP algorithm and passenger reassignment. An initial feasible solution is first obtained by inserting the affected aircraft into the pool of available aircraft and implementing a passenger



reassignment algorithm. Then, neighborhood solutions are derived from an incumbent by comparing two routes and passenger reassignment successively. Three neighbor generation methods, *insert*, *cross*, and *cancel*, are applied to route pairs and single aircraft routes to enumerate feasible neighbor aircraft routing solutions. The best of the neighbor solutions are stored on a restricted candidate list from which one is selected at random and becomes the new incumbent. This procedure is repeated until one of several stopping criteria is met. In the passenger reassignment algorithm (PRA), we found that no matter how many flight legs are included in a passenger's original itinerary, if the new itinerary is composed of one flight leg, the optimal reassignment for the passenger can be obtained from PRA.

Through a sample example and some computational experiments for different disruption scenarios, we show that our integrated recovery of aircraft and passengers can achieve lower recovery cost and fewer passengers who are refunded compared with similar methods that recover aircraft and passenger itineraries separately. At the same time, we found that when applying the airline's heuristic to construct recovery schedules, the costs were often an order of magnitude higher than those obtained with our heuristic.

There are several directions for future work. First, while we have considered the post-disruption recoveries of both aircraft and passengers, we make assumptions on crew members' availability. Thus an extended model that integrated the recovery of aircraft, passengers and crew members would be desirable in practice. Second, integrated recovery is a classical large scale practical problem, so developing more efficient combinatorial algorithms, such as parallel meta-heuristics, can also be used to reduce the search time to find an acceptable solution matching the AOCC response time.

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