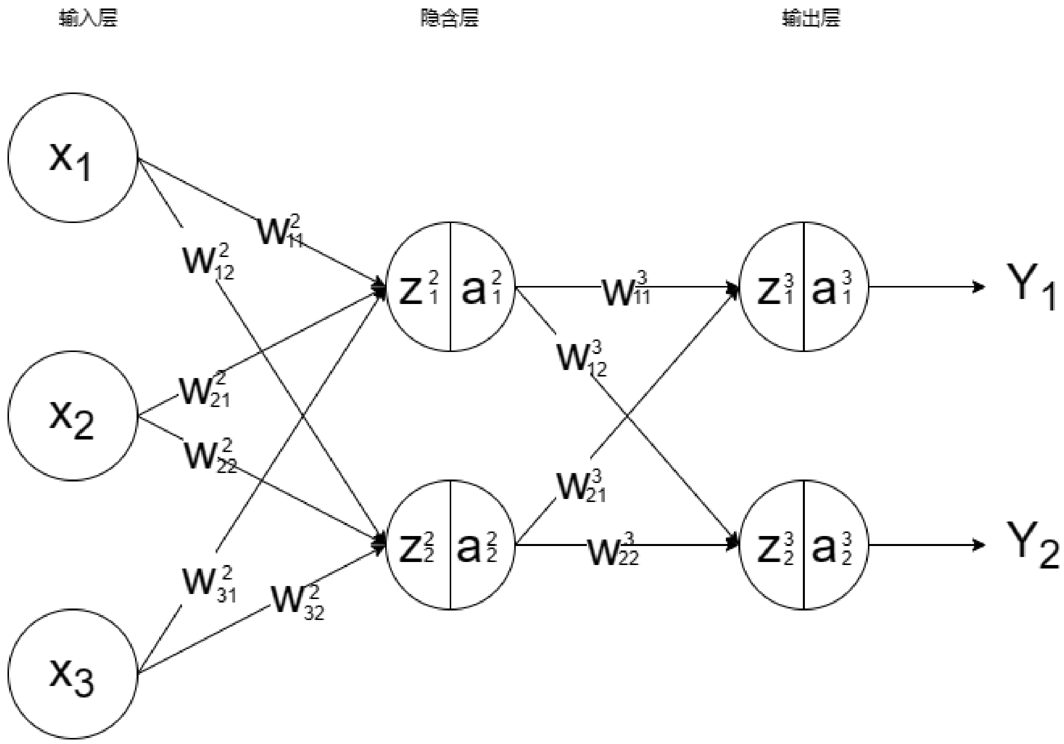
目录

BP 算法模型总览：

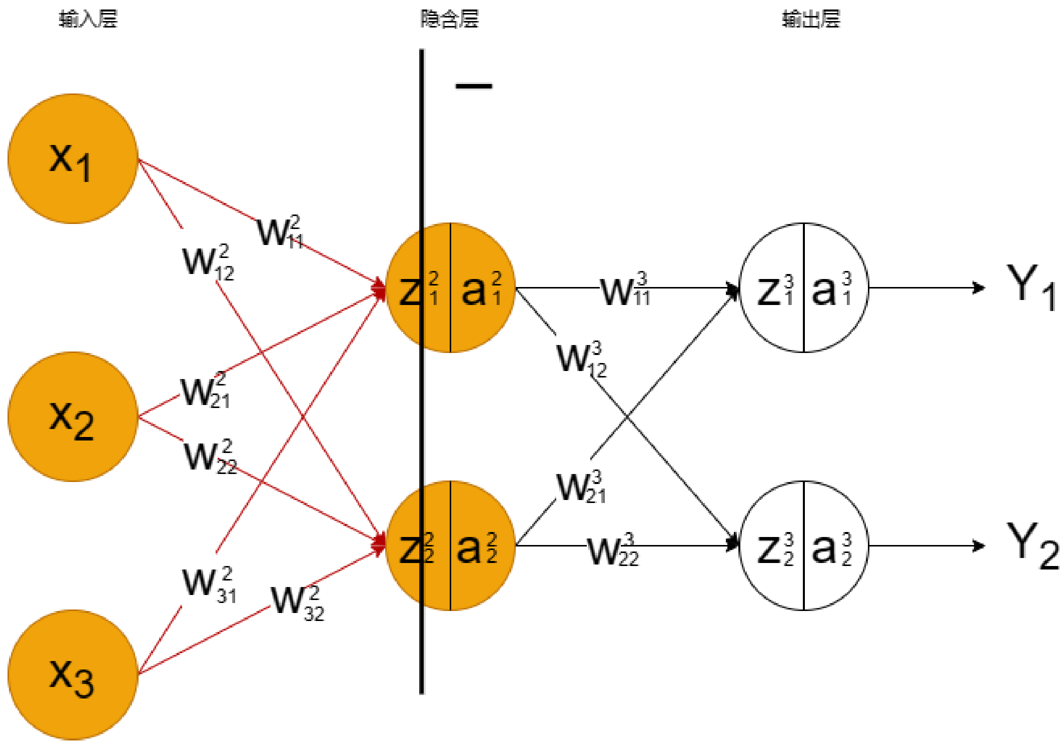


$$E = \frac{1}{2}\left[\left(Y_1 - \widehat{Y_1}\right)^2 + \left(Y_2 - \widehat{Y_2}\right)^2\right]$$

$$a = f(z) = \frac{1}{1 + e^{-z}}$$

正向传播:

(1)输入层 -> 隐含层



$$z_1^2 = x_1 \cdot w_{11}^2 + x_2 \cdot w_{21}^2 + x_3 \cdot w_{31}^2$$

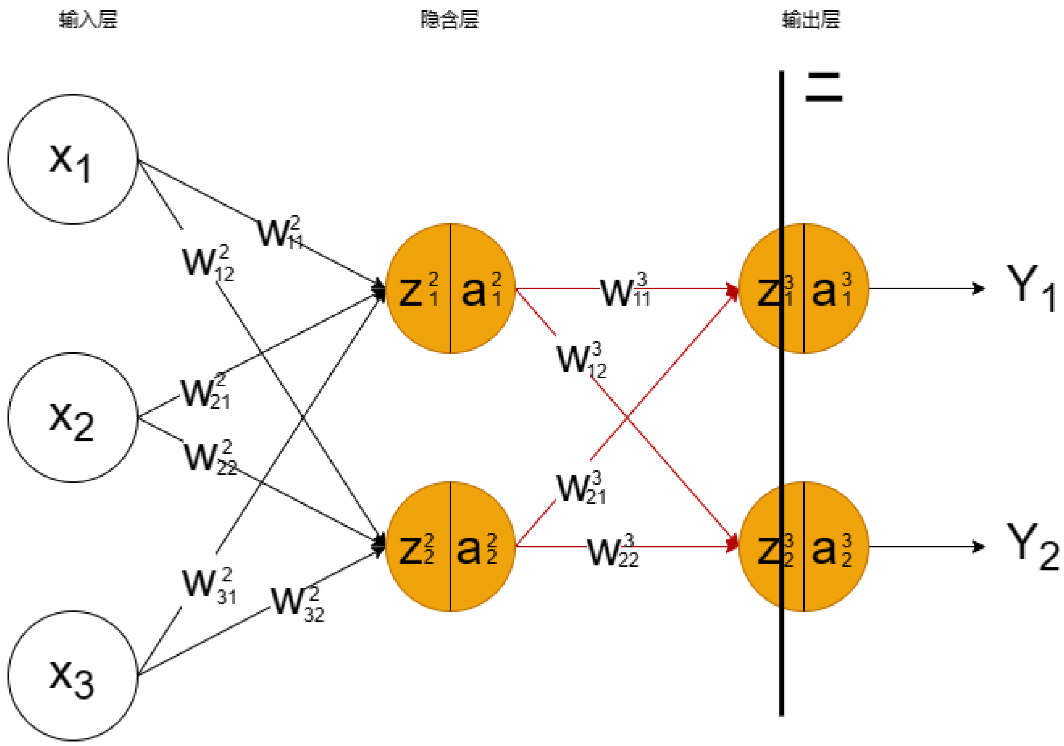
$$z_2^2 = x_1 \cdot w_{12}^2 + x_2 \cdot w_{22}^2 + x_3 \cdot w_{32}^2$$

隐含层内部转化:

$$a_1^2 = f(z_1^2) = \frac{1}{1 + e^{-z_1^2}}$$

$$a_2^2 = f(z_2^2) = \frac{1}{1 + e^{-z_2^2}}$$

(2)隐含层 -> 输出层



$$z_1^3 = a_1^2 \cdot w_{11}^3 + a_2^2 \cdot w_{21}^3$$

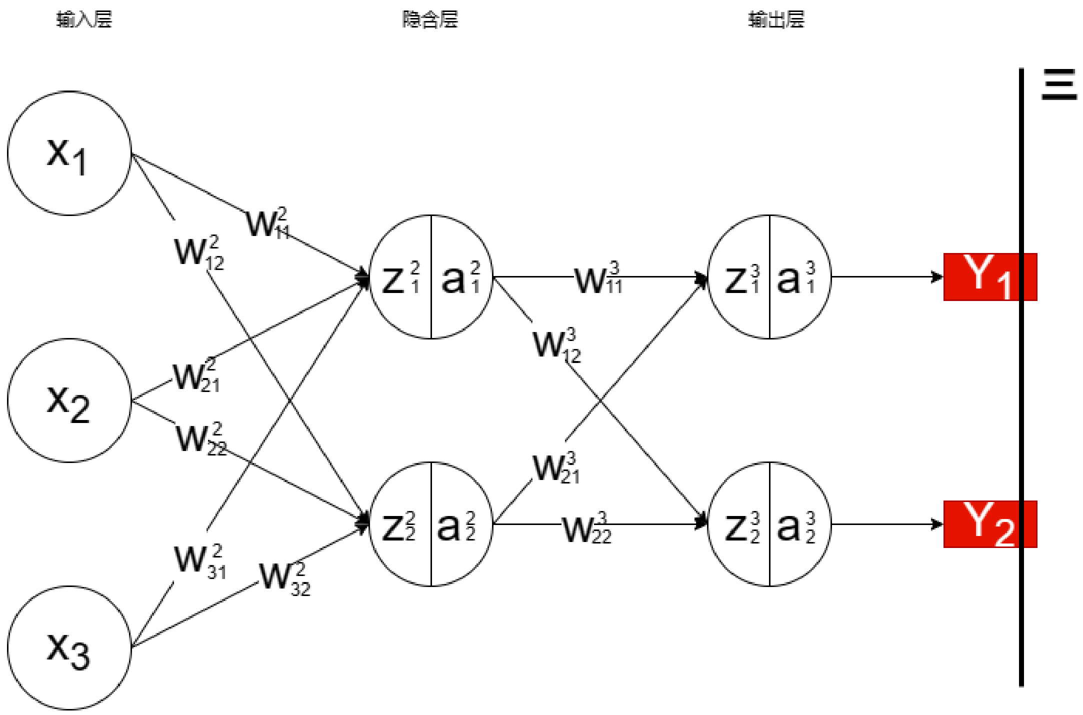
$$z_2^3 = a_1^2 \cdot w_{12}^3 + a_2^2 \cdot w_{22}^3$$

输出层内部转化:

$$a_1^3 = f(z_1^3) = \frac{1}{1+e^{-z_1^3}}$$

$$a_2^3 = f(z_2^3) = \frac{1}{1+e^{-z_2^3}}$$

(3)输出



$$Y_1 = a_1^3 = \frac{1}{1 + e^{-z_1^3}}$$

$$Y_2 = a_2^3 = \frac{1}{1 + e^{-z_2^3}}$$

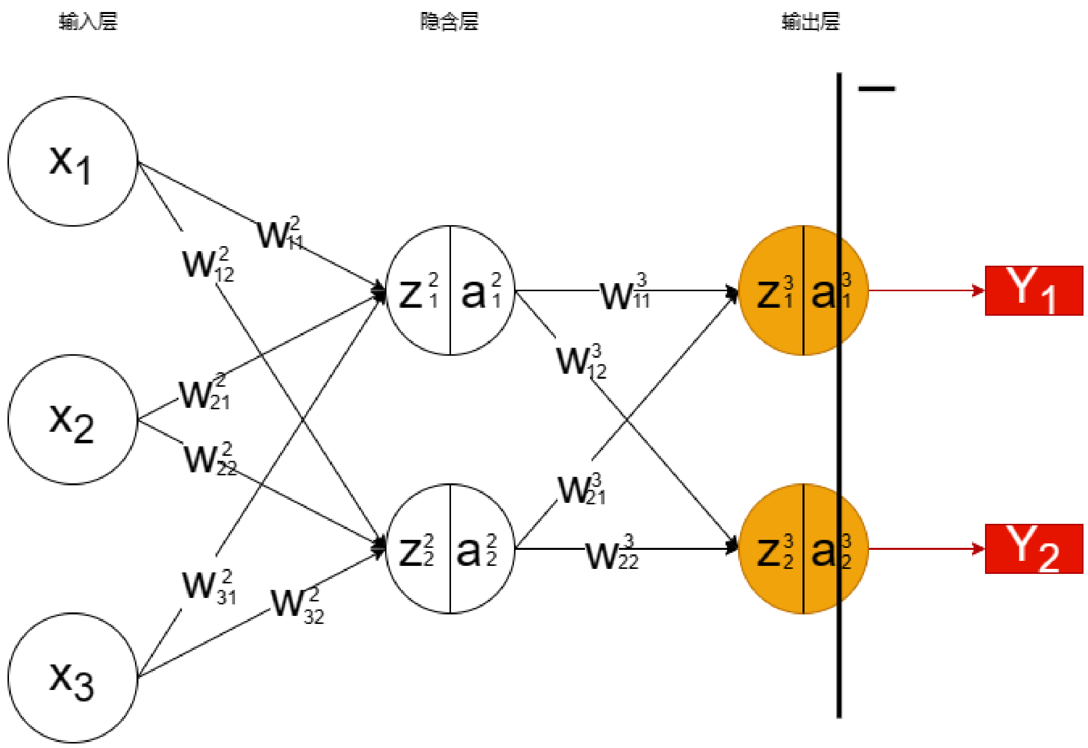
反向传播:（从后向前，所以是反向）

从 E 开始：

$$E = \frac{1}{2}\left[\left(Y_1 - \widehat{Y_1}\right)^2 + \left(Y_2 - \widehat{Y_2}\right)^2\right]$$

其中，（$\widehat{\boldsymbol{Y_1}}$ ， $\widehat{\boldsymbol{Y_2}}$ ） 为已知的标准答案 或 称为 label 标签，

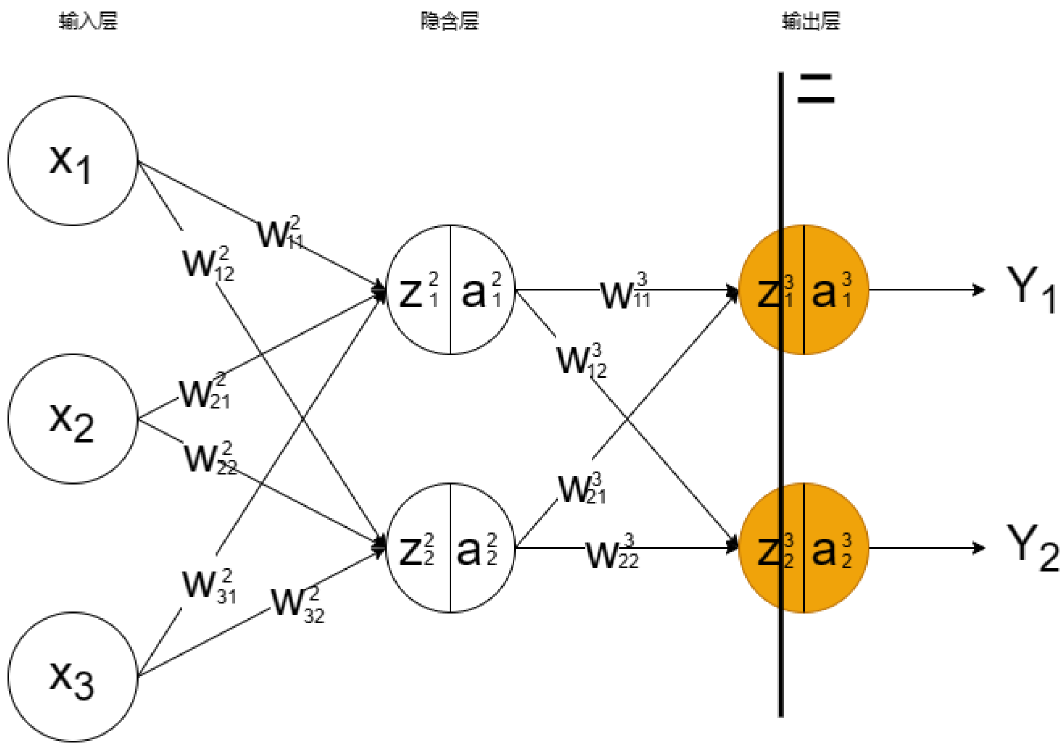
一、求 E 对$\boldsymbol{Y_1}, \boldsymbol{Y_2}$ 或 $\boldsymbol{a_1^3}, \boldsymbol{a_2^3}$的偏导：



$$\frac{\partial E}{\partial Y_1} = \frac{\partial E}{\partial a_1^3} = Y_1 - \widehat{Y_1}$$

$$\frac{\partial E}{\partial Y_2} = \frac{\partial E}{\partial a_2^3} = Y_2 - \widehat{Y_2}$$

二、求 E 对Z_1^3, Z_2^3的偏导：



$$\frac{\partial E}{\partial Z_1^3} = \frac{\partial E}{\partial a_1^3} \cdot \frac{\partial a_1^3}{\partial Z_1^3}$$

由上可知:

$$\frac{\partial E}{\partial a_1^3} = Y_1 - \widehat{Y_1}$$

由条件知:

$$a_1^3 = \frac{1}{1 + e^{-z_1^3}}$$

则:

$$\frac{\partial a_1^3}{\partial Z_1^3} = \frac{e^{-z_1^3}}{\left(1 + e^{-z_1^3}\right)^2} = a_1^3 - (a_1^3)^2$$

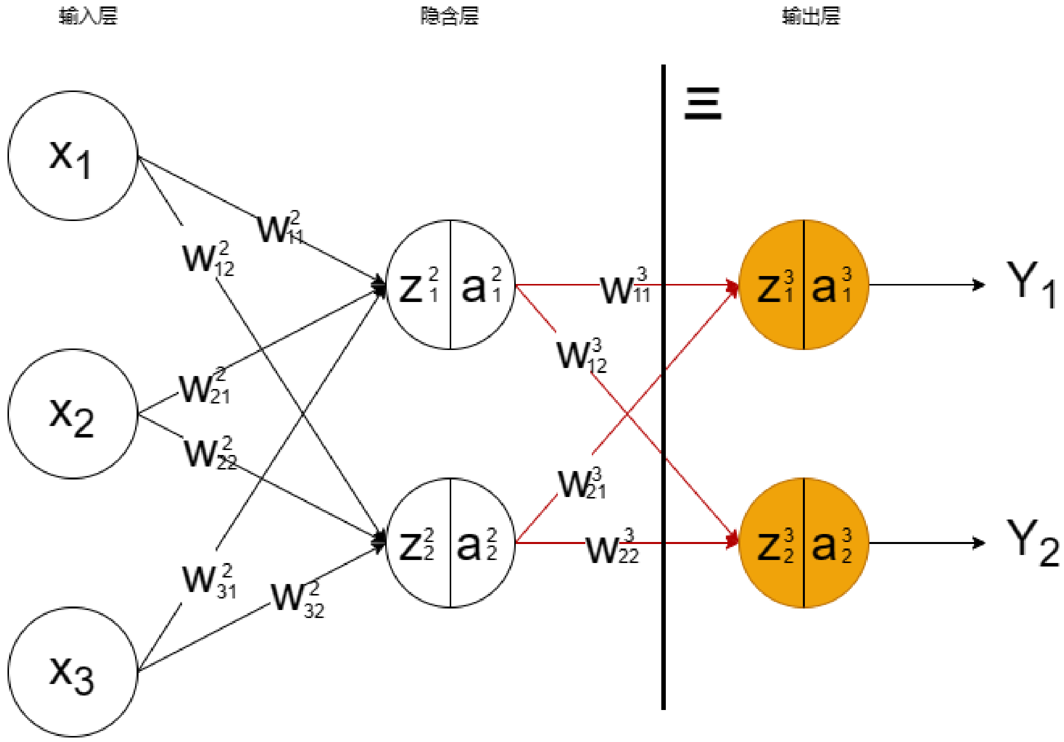
求解得:

$$\frac{\partial E}{\partial Z_1^3} = \left(Y_1 - \widehat{Y_1}\right) \cdot (a_1^3 - (a_1^3)^2)$$

同理可得:

$$\frac{\partial E}{\partial Z_2^3} = \left(Y_2 - \widehat{Y_2}\right) \cdot (a_2^3 - (a_2^3)^2)$$

三、求 E 对$\boldsymbol{w^3_{11}}$, $\boldsymbol{w^3_{21}}$, $\boldsymbol{w^3_{12}}$, $\boldsymbol{w^3_{22}}$的偏导



$$\frac{\partial E}{\partial w^3_{11}} = \frac{\partial E}{\partial Z^3_1} \cdot \frac{\partial Z^3_1}{\partial w^3_{11}}$$

由上可知:

$$\frac{\partial E}{\partial Z^3_1} = \left(Y_1 - \widehat{Y}_1\right) \cdot \left(a^3_1 - (a^3_1)^2\right)$$

由于:

$$z^3_1 = a^2_1 \cdot w^3_{11} + a^2_2 \cdot w^3_{21}$$

$$z^3_2 = a^2_1 \cdot w^3_{12} + a^2_2 \cdot w^3_{22}$$

则:

$$\frac{\partial Z_1^3}{\partial w_{11}^3} = a_1^2$$

得:

$$\frac{\partial E}{\partial w_{11}^3} = \left(Y_1 - \widehat{Y_1}\right) \cdot (a_1^3 - (a_1^3)^2) \cdot a_1^2$$

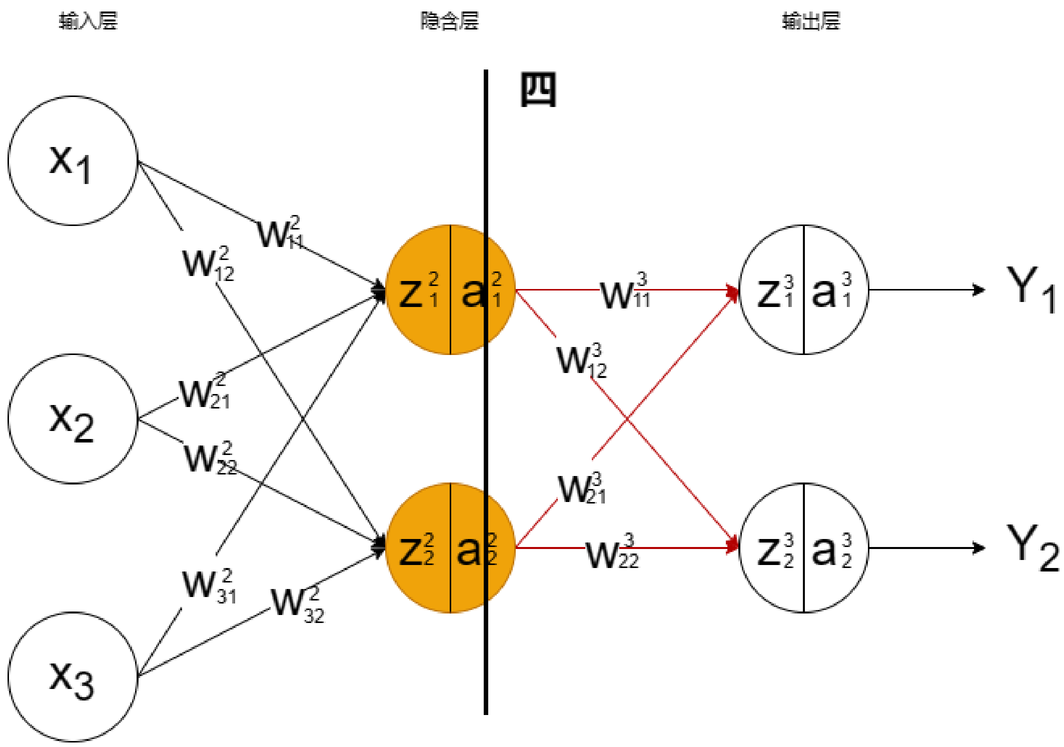
同理可得:

$$\frac{\partial E}{\partial w_{21}^3} = \left(Y_1 - \widehat{Y_1}\right) \cdot (a_1^3 - (a_1^3)^2) \cdot a_2^2$$

$$\frac{\partial E}{\partial w_{12}^3} = \left(Y_2 - \widehat{Y_2}\right) \cdot (a_2^3 - (a_2^3)^2) \cdot a_1^2$$

$$\frac{\partial E}{\partial w_{22}^3} = \left(Y_2 - \widehat{Y_2}\right) \cdot (a_2^3 - (a_2^3)^2) \cdot a_2^2$$

四、求 E 对$\boldsymbol{a_1^2}, \boldsymbol{a_2^2}$的偏导



$$\frac{\partial E}{\partial a_1^2} = \frac{\partial E}{\partial z_1^3} \cdot \frac{\partial z_1^3}{\partial a_1^2} + \frac{\partial E}{\partial z_2^3} \cdot \frac{\partial z_2^3}{\partial a_1^2}$$

由上可知:

$$\frac{\partial E}{\partial Z_1^3} = \left(Y_1 - \widehat{Y_1}\right) \cdot \left(a_1^3 - (a_1^3)^2\right)$$

$$\frac{\partial E}{\partial Z_2^3} = \left(Y_2 - \widehat{Y_2}\right) \cdot \left(a_2^3 - (a_2^3)^2\right)$$

已知:

$$z_1^3 = a_1^2 \cdot w_{11}^3 + a_2^2 \cdot w_{21}^3$$

$$z_2^3 = a_1^2 \cdot w_{12}^3 + a_2^2 \cdot w_{22}^3$$

求导得:

$$\frac{\partial z_1^3}{\partial a_1^2} = w_{11}^3$$

$$\frac{\partial z_2^3}{\partial a_1^2} = w_{12}^3$$

求解得:

$$\frac{\partial E}{\partial a_1^2} = \left(Y_1 - \widehat{Y_1}\right) \cdot (a_1^3 - (a_1^3)^2) \cdot w_{11}^3 +$$

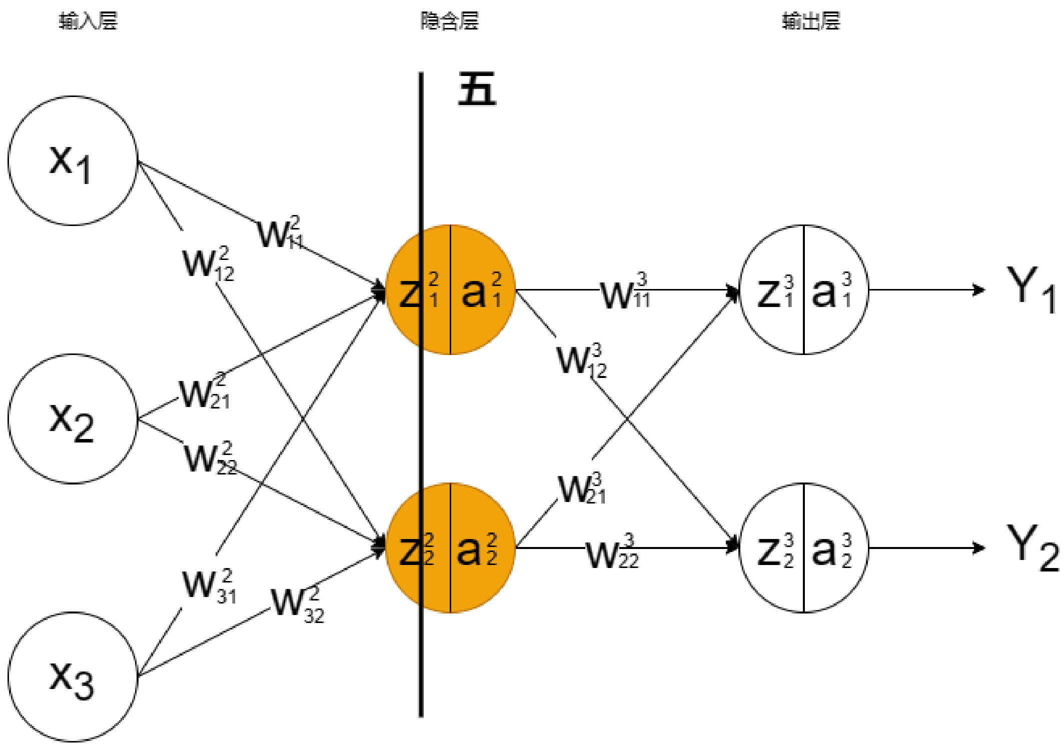
$$\left(Y_2 - \widehat{Y_2}\right) \cdot (a_2^3 - (a_2^3)^2) \cdot w_{12}^3$$

同理可得:

$$\frac{\partial E}{\partial a_2^2} = \left(Y_1 - \widehat{Y_1}\right) \cdot (a_1^3 - (a_1^3)^2) \cdot w_{21}^3 +$$

$$\left(Y_2 - \widehat{Y_2}\right) \cdot (a_2^3 - (a_2^3)^2) \cdot w_{22}^3$$

五、求 E 对z_1^2, z_2^2的偏导



$$\frac{\partial E}{\partial z_1^2} = \frac{\partial E}{\partial a_1^2} \cdot \frac{\partial a_1^2}{\partial z_1^2}$$

由上可知:

$$\frac{\partial E}{\partial a_1^2} = \left(Y_1 - \widehat{Y_1}\right) \cdot (a_1^3 - (a_1^3)^2) \cdot w_{11}^3 +$$

$$\left(Y_2 - \widehat{Y_2}\right) \cdot (a_2^3 - (a_2^3)^2) \cdot w_{12}^3$$

已知:

$$a_1^2 = \frac{1}{1 + e^{-z_1^2}}$$

求导:

$$\frac{\partial a_1^2}{\partial z_1^2} = \frac{e^{-z_1^2}}{\left(1+e^{-z_1^2}\right)^2} = a_1^2 - (a_1^2)^2$$

求解得:

$$\frac{\partial E}{\partial z_1^2} = \left[\left(Y_1 - \widehat{Y_1}\right)\cdot(a_1^3 - (a_1^3)^2)\cdot w_{11}^3 + \left(Y_2 - \widehat{Y_2}\right)\cdot(a_2^3 - (a_2^3)^2)\cdot w_{12}^3\right]$$

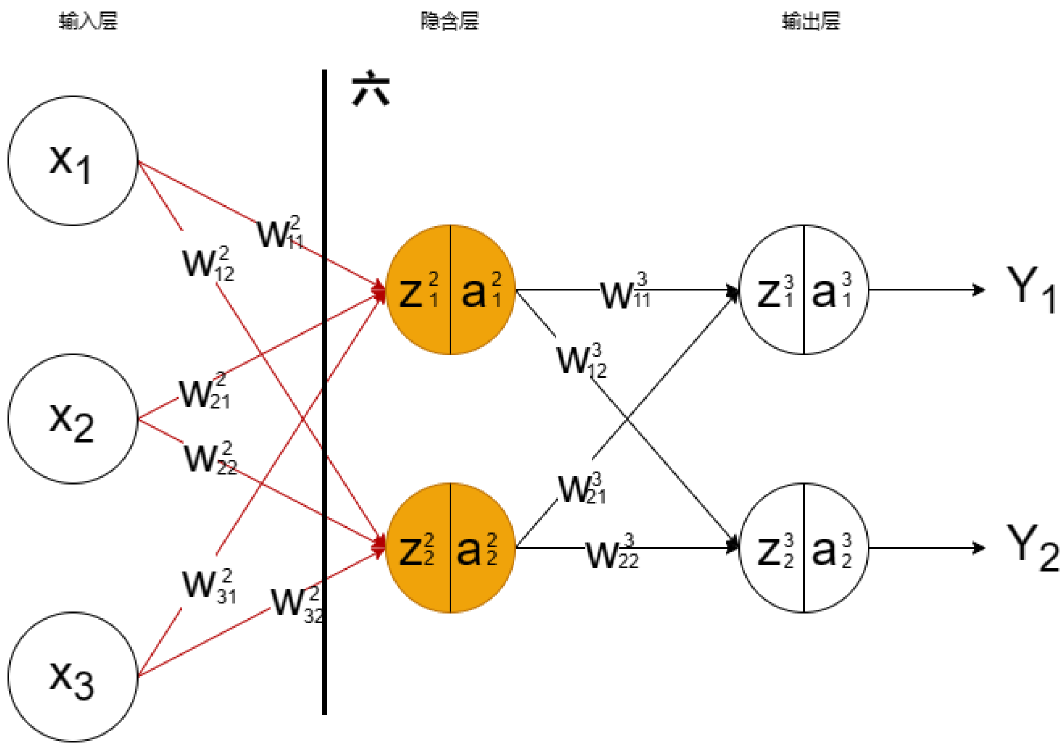
$$\cdot (a_1^2 - (a_1^2)^2)$$

同理可得:

$$\frac{\partial E}{\partial z_2^2} = \left[\left(Y_1 - \widehat{Y_1}\right)\cdot(a_1^3 - (a_1^3)^2)\cdot w_{21}^3 + \left(Y_2 - \widehat{Y_2}\right)\cdot(a_2^3 - (a_2^3)^2)\cdot w_{22}^3\right]$$

$$\cdot (a_2^2 - (a_2^2)^2)$$

六、求 E 对$\boldsymbol{w_{11}^2}, \boldsymbol{w_{21}^2}, \boldsymbol{w_{31}^2}, \boldsymbol{w_{12}^2}, \boldsymbol{w_{22}^2}, \boldsymbol{w_{32}^2}$的偏导



$$\frac{\partial E}{\partial w_{11}^2}=\frac{\partial E}{\partial z_1^2}\cdot\frac{\partial z_1^2}{\partial w_{11}^2}$$

由上可知:

$$\frac{\partial E}{\partial z_1^2}=\left[\left(Y_1-\widehat{Y}_1\right)\cdot\left(a_1^3-(a_1^3)^2\right)\cdot w_{11}^3+\left(Y_2-\widehat{Y}_2\right)\cdot\left(a_2^3-(a_2^3)^2\right)\cdot w_{12}^3\right]$$

$$\cdot\left(a_1^2-(a_1^2)^2\right)$$

已知:

$$z_1^2=x_1\cdot w_{11}^2+x_2\cdot w_{21}^2+x_3\cdot w_{31}^2$$

求导得:

$$\frac{\partial z_1^2}{\partial w_{11}^2} = x_1$$

求解得:

$$\frac{\partial E}{\partial w_{11}^2} = \left[\left(Y_1 - \widehat{Y_1}\right) \cdot (a_1^3 - (a_1^3)^2) \cdot w_{11}^3 + \left(Y_2 - \widehat{Y_2}\right) \cdot (a_2^3 - (a_2^3)^2) \cdot w_{12}^3\right]$$

$$\cdot (a_1^2 - (a_1^2)^2) \cdot x_1$$

同理可得:

$$\frac{\partial E}{\partial w_{21}^2} = \left[\left(Y_1 - \widehat{Y_1}\right) \cdot (a_1^3 - (a_1^3)^2) \cdot w_{11}^3 + \left(Y_2 - \widehat{Y_2}\right) \cdot (a_2^3 - (a_2^3)^2) \cdot w_{12}^3\right]$$

$$\cdot (a_1^2 - (a_1^2)^2) \cdot x_2$$

$$\frac{\partial E}{\partial w_{31}^2} = \left[\left(Y_1 - \widehat{Y_1}\right) \cdot (a_1^3 - (a_1^3)^2) \cdot w_{11}^3 + \left(Y_2 - \widehat{Y_2}\right) \cdot (a_2^3 - (a_2^3)^2) \cdot w_{12}^3\right]$$

$$\cdot (a_1^2 - (a_1^2)^2) \cdot x_3$$

$$\frac{\partial E}{\partial w_{12}^2} = \left[\left(Y_1 - \widehat{Y_1}\right) \cdot (a_1^3 - (a_1^3)^2) \cdot w_{21}^3 + \left(Y_2 - \widehat{Y_2}\right) \cdot (a_2^3 - (a_2^3)^2) \cdot w_{22}^3\right]$$

$$\cdot (a_2^2 - (a_2^2)^2) \cdot x_1$$

$$\frac{\partial E}{\partial w_{22}^2} = \left[\left(Y_1 - \widehat{Y_1}\right) \cdot (a_1^3 - (a_1^3)^2) \cdot w_{21}^3 + \left(Y_2 - \widehat{Y_2}\right) \cdot (a_2^3 - (a_2^3)^2) \cdot w_{22}^3\right]$$

$$\cdot (a_2^2 - (a_2^2)^2) \cdot x_2$$

$$\frac{\partial E}{\partial w_{32}^2} = \left[\left(Y_1 - \widehat{Y_1}\right) \cdot (a_1^3 - (a_1^3)^2) \cdot w_{21}^3 + \left(Y_2 - \widehat{Y_2}\right) \cdot (a_2^3 - (a_2^3)^2) \cdot w_{22}^3\right]$$

$$\cdot (a_2^2 - (a_2^2)^2) \cdot x_3$$

Ps:

关于 sigmoid 函数求导

$$f(z) = \frac{1}{1 + e^{-z}}$$

变形:

$$f(z) = (1 + e^{-z})^{-1}$$

$$\frac{\partial y}{\partial z} = -1 \cdot (1 + e^{-z})^{-2} \cdot e^{-z} \cdot (-1)$$

$$\frac{\partial y}{\partial z} = (1 + e^{-z})^{-2} \cdot e^{-z}$$

得:

$$\frac{\partial y}{\partial z} = \frac{e^{-z}}{(1 + e^{-z})^2} = y - y^2$$