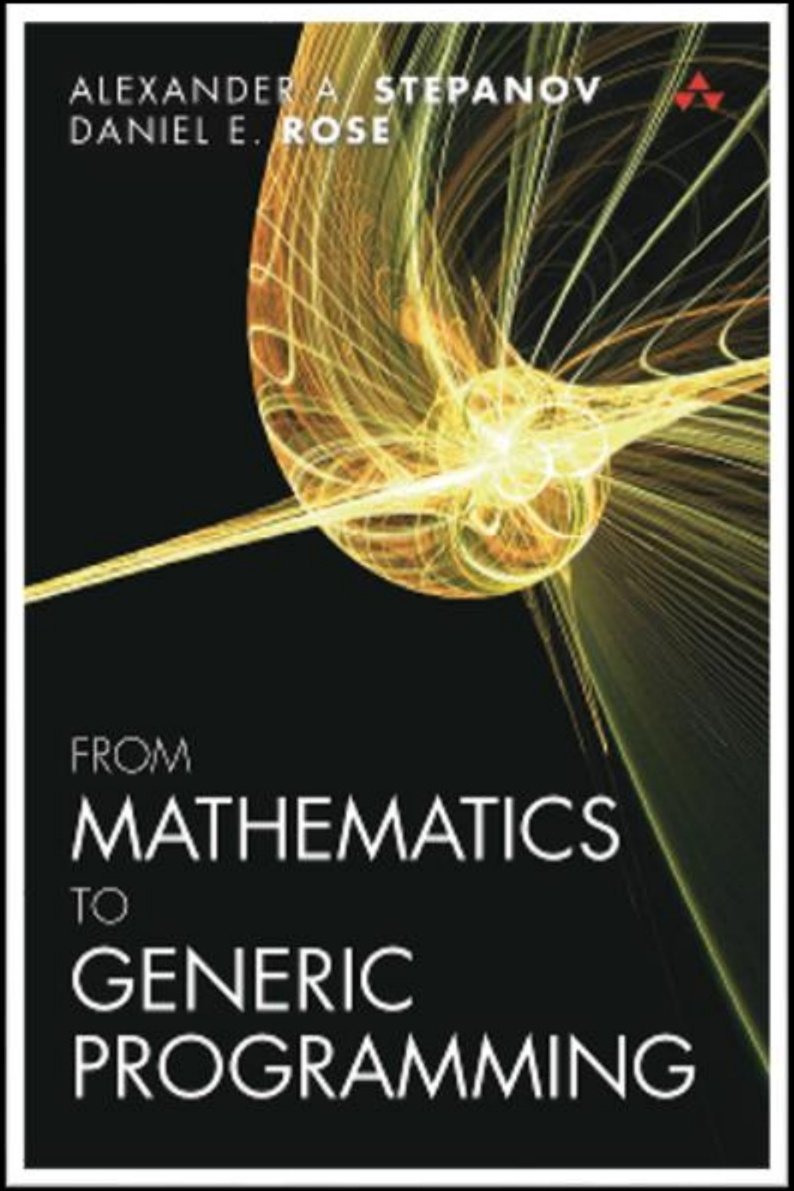
# Meetup

CoC Berlin Code of Conduct

DISCORD

Discord Link: https://discord.gg/nxwbTHd
Github Repo: https://github.com/codereport/FM2GP-2025
code_report: Twitter | BlueSky | Mastodon
CoC: https://berlincodeofconduct.org/




# From Mathematics to Generic Programming

## Chapter 6 & 8

1. What This Book Is About
2. The First Algorithm
3. Ancient Greek Number Theory
4. Euclid’s Algorithm
5. The Emergence of Modern Number Theory
6. Abstraction in Mathematics
7. Deriving a Generic Algorithm
8. More Algebraic Structures
9. Organizing Mathematical Knowledge
10. Fundamental Programming Concepts
11. Permutation Algorithms
12. Extensions of GCD
13. A Real-World Application

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| STRUCTURE | OPERATIONS | ELEMENTS | AXIOMS |
|---|---|---|---|
| *semigroup* | $x \circ y$ | | $x \circ (y \circ z) = (x \circ y) \circ z$ |
| Example: positive integers under addition | | | |
| *monoid* | $x \circ y$ | $e$ | $x \circ (y \circ z) = (x \circ y) \circ z$ |
| | | | $x \circ e = e \circ x = x$ |
| Example: strings under concatenation | | | |
| *group* | $x \circ y$ | $e$ | $x \circ (y \circ z) = (x \circ y) \circ z$ |
| | $x^{-1}$ | | $x \circ e = e \circ x = x$ |
| | | | $x \circ x^{-1} = x^{-1} \circ x = e$ |
| Example: invertible matrices under multiplication | | | |

| STRUCTURE | OPERATIONS | ELEMENTS | AXIOMS |
|---|---|---|---|
| *abelian group* | $x \circ y$ | $e$ | $x \circ (y \circ z) = (x \circ y) \circ z$ |
| | $x^{-1}$ | | $x \circ e = e \circ x = x$ |
| | | | $x \circ x^{-1} = x^{-1} \circ x = e$ |
| | | | $x \circ y = y \circ x$ |
| Example: two-dimensional vectors under addition | | | |
| *semiring* | $x + y$ | $0_R$ | $x + (y + z) = (x + y) + z$ |
| | $xy$ | $1_R$ | $x + 0 = 0 + x = x$ |
| | | | $x + y = y + x$ |
| | | | $x(yz) = (xy)z$ |
| | | | $1 \neq 0$ |
| | | | $1x = x1 = x$ |
| | | | $0x = x0 = 0$ |
| | | | $x(y + z) = xy + xz$ |
| | | | $(y + z)x = yx + zx$ |
| Example: natural numbers | | | |
| *ring* | $x + y$ | $0_R$ | $x + (y + z) = (x + y) + z$ |
| | $-x$ | $1_R$ | $x + 0 = 0 + x = x$ |
| | $xy$ | | $x + -x = -x + x = 0$ |
| | | | $x + y = y + x$ |
| | | | $x(yz) = (xy)z$ |
| | | | $1 \neq 0$ |
| | | | $1x = x1 = x$ |
| | | | $0x = x0 = 0$ |
| | | | $x(y + z) = xy + xz$ |
| | | | $(y + z)x = yx + zx$ |
| Example: integers | | | |

As we did before, we can also define some other structures more concisely in terms of others:

| STRUCTURE | DEFINITION |
|---|---|
| *integral domain* | A commutative ring that has no zero divisors (elements other than 0 whose product is 0) |
| *Euclidean domain* | An integral domain that has quotient and remainder operations and a norm that decreases when remainder is computed |
| *field* | An integral domain where every nonzero element is invertible (Example: rational numbers) |
| *prime field* | A field that does not have a proper subfield |
| *module* | Consists of a primary set that is an additive group $G$ and a secondary set of coefficients that is a ring $R$, with distributive multiplication of coefficients over elements of $G$ |
| *vector space* | A module where the ring $R$ is also a field |

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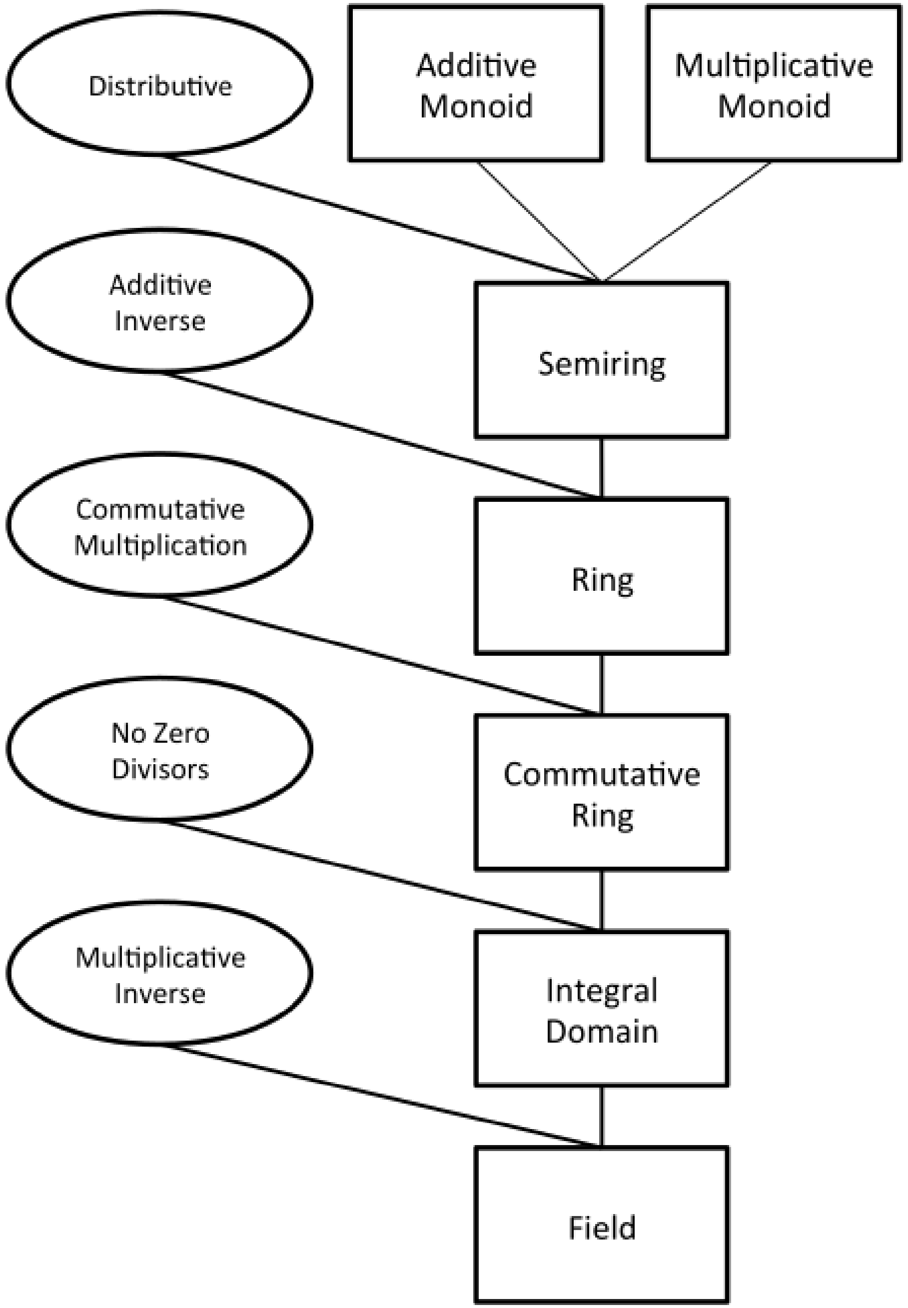
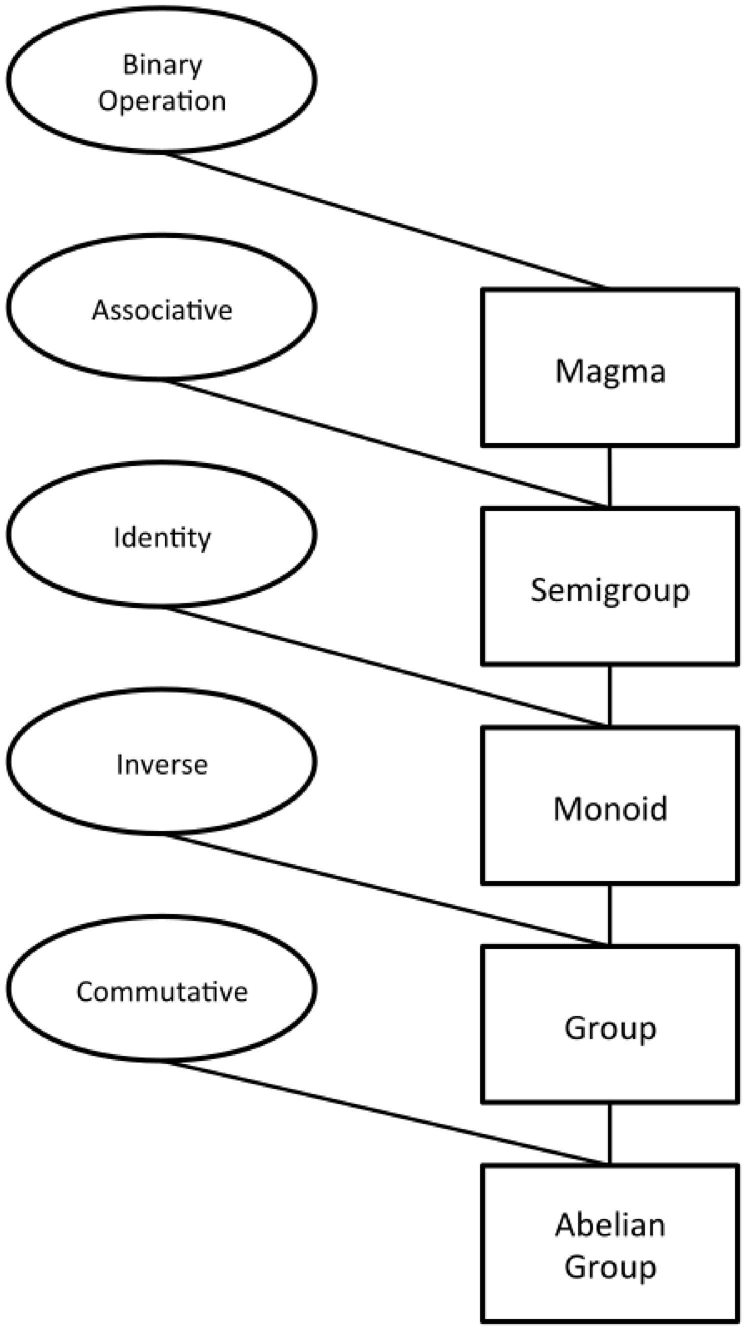
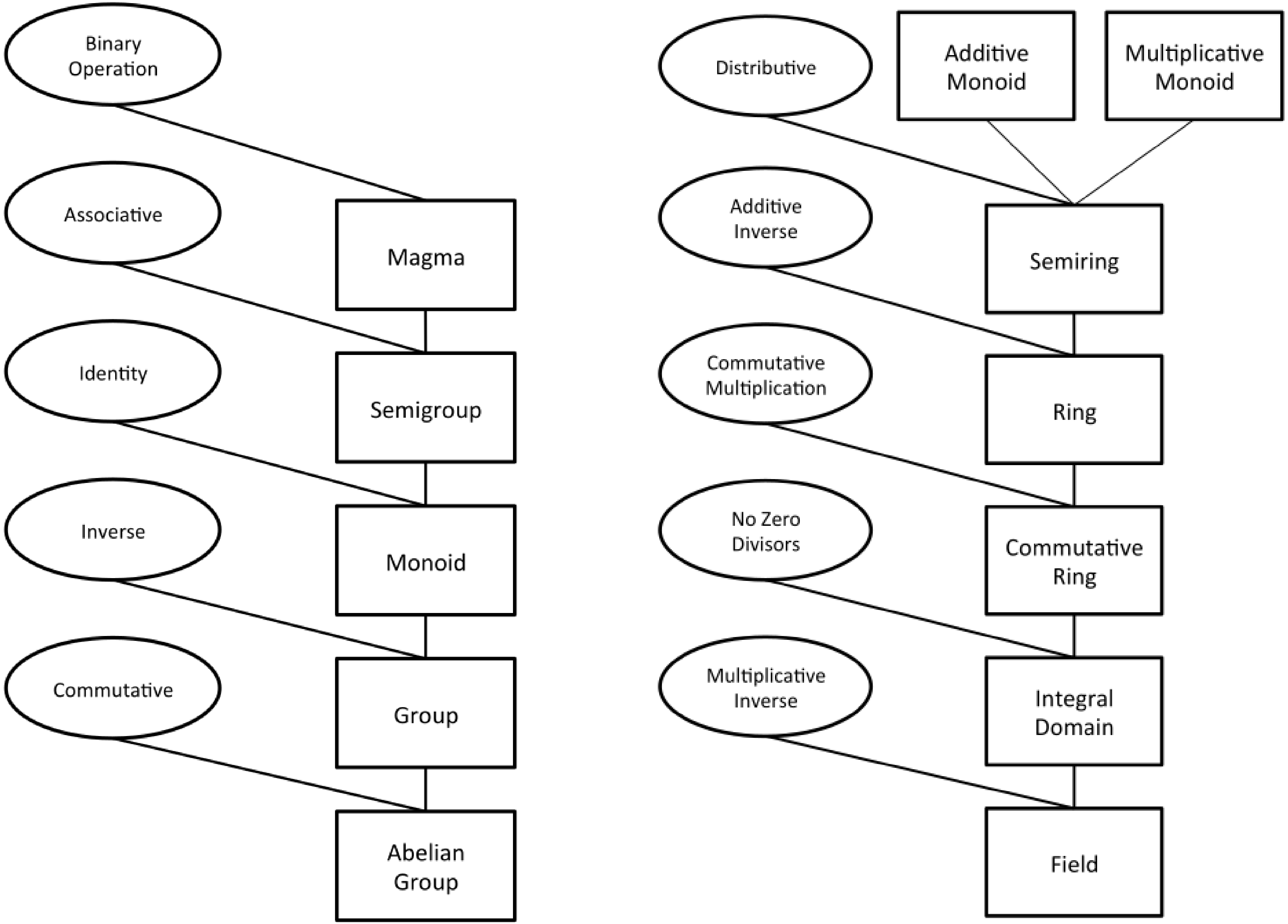


Distributive
Additive Monoid
Multiplicative Monoid
Additive Inverse
Semiring
Commutative Multiplication
Ring
No Zero Divisors
Commutative Ring
Multiplicative Inverse
Integral Domain
Field

| STRUCTURE | DEFINITION |
| --- | --- |
| *additive semigroup* | semigroup where operation is $+$ and (by convention) commutes |
| *additive monoid* | additive semigroup with identity element 0 |
| *subgroup* | group that is a subset of another group |
| *cyclic group* | group where all elements can be obtained by raising (at least) one element to different powers |


Binary Operation
Associative
Identity
Inverse
Commutative
Magma
Semigroup
Monoid
Group
Abelian Group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# discussion

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