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Unit-2

Ordinary differential Equations of Higher Order

- Solutions of Differential Equations:- A solution of differential equation is a relation, free from derivative b/w the variable which satisfies the given below/.

#) General Solutions:- General Solution or complete solution of a differential equation is the solution in which number of arbitrary constants is equal to order of differential equation.

Thus $\boxed{y = C_1 \cos x + C_2 \sin x}$, (then the order of given differential will be two), is general solution of $\frac{d^2y}{dx^2} + y = 0$

#) Particular Solutions:- A particular solution of a differential equation is the solution which is obtained from general solution by giving particular values to arbitrary constants.

#) Linear Differential Equations:- A Linear Differential Equation is said to be linear if the dependent variable and its derivative occur only in the first degree and are not multiple together. For ex: $\frac{dy}{dx} + y = x$ order 1 degree 1

The general form of linear differential equations of first order is $\boxed{\frac{dy}{dx} + py = Q}$ where P & Q are functions of x only or constants.

#) Solutions of Linear Differential Eq^n of first order:-

$$\boxed{y \times I.F = \int Q \times I.F \, dx + c}$$

where, Integrating factor $(I.F) = e^{\int P\,dx}$

and c is arbitary constant of integration.

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① Solve the eq^n $\frac{dy}{dx} + \frac{3x^2}{1+x^3} y = \frac{\sin^2 x}{1+x^3}$

Sol:- $I.F = e^{\int P dx} \Rightarrow e^{\int \frac{3x^2}{1+x^3} dx}$

$\Rightarrow 1+x^3 = t$

$3x^2 dx = dt$

$\Rightarrow e^{\int \frac{dt}{t}} \Rightarrow e^{\log t} \Rightarrow t$

$I.F \Rightarrow (1+x^3)$

$$y(1+x^3) = \int \frac{\sin^2 x}{\cancel{(1+x^3)}} \cancel{(1+x^3)} \, dx$$

$$y(1+x^3) = \int \frac{1-\cos 2x}{2} \, dx$$

$$= \frac{1}{2} \int (1-\cos 2x) dx$$

$$\boxed{y(1+x^3) = \frac{1}{2}\left(x - \frac{\sin 2x}{2}\right) + C}$$

② Solve the eq^n $\frac{dy}{dx} + \frac{3}{x} dx = \frac{1}{x^4}$

$I.F = e^{\int P dx} = e^{3\int \frac{1}{x} dx} = e^{3\log x}$

$= e^{\log x^3}$

$= x^3$

$$y x^3 = \int \frac{1}{\cancel{x^4}} \times \cancel{x^3} \, dx$$

$$\boxed{y x^3 = \log x + C}$$

③ Solve $\cos^2 x \frac{dy}{dx} + y = \tan x$

$$\frac{dy}{dx} + \sec^2 x \, y = \tan x \sec^2 x$$

$$\frac{dy}{dx} + P y = Q$$

$$I.F = e^{\int P\,dx} = e^{\int \sec^2 x\,dx} = e^{\tan x}$$

$$y\,e^{\tan x} = \int \tan x \sec^2 x\, e^{\tan x}\,dx$$

$\tan x = t$

$\sec^2 x\,dx = dt$

$$y\,e^{\tan x} = \int t e^t\,dt$$
$$= t e^t - \int 1\cdot e^t\,dt$$
$$= t e^t - e^t + c$$

$$y\,e^{\tan x} = \tan x\, e^{\tan x} - e^{\tan x} + c$$

$$y\,\cancel{e^{\tan x}} = \cancel{e^{\tan x}}\left(\tan x - 1 + \frac{c}{e^{\tan x}}\right)$$

$$\boxed{y = \tan x - 1 + c_1}$$ c & c_1 are arbitrary constant

④ Solve the eqn $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1$

$$\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}} = \frac{dy}{dx}$$

$$\frac{dy}{dx} + \frac{y}{\sqrt{x}} = \frac{e^{-2\sqrt{x}}}{\sqrt{x}}$$

$$I.F = e^{\int P\,dx} = e^{\int \frac{1}{\sqrt{x}}dx} = e^{2\sqrt{x}}$$

$$y\,e^{2\sqrt{x}} = \int \frac{\cancel{e^{-2\sqrt{x}}}}{\sqrt{x}} \times \cancel{e^{2\sqrt{x}}}\,dx$$

$$y\,e^{2\sqrt{x}} = \int \frac{1}{\sqrt{x}}\,dx$$

$$\boxed{y\,e^{2\sqrt{x}} = 2\sqrt{x} + c}$$

⑤ Solve the eqn $(1+y^2)\,dx = (\tan^{-1}y - x)\,dy$

$$\frac{1+y^2}{\tan^{-1}y - x} = \frac{dy}{dx}$$

$$\cancel{\frac{1+y^2}{\tan^{-1}y-x}} \qquad \frac{dx}{dy} = \frac{\tan^{-1}y - x}{1+y^2}$$

$$\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1}y}{1+y^2}$$

$$I.f = e^{\int P\,dx} = e^{\int \frac{1}{1+y^2}dy} = e^{\tan^{-1}y}$$

$$x\,e^{\tan^{-1}y} = \int \frac{\tan^{-1}y}{1+y^2} e^{\tan^{-1}y}\,dy$$

$\tan^{-1}y = t$

$\frac{1}{1+y^2}dy = dt$

$$= \int t\cdot e^t\,dt$$

$$= t\cdot e^t - \int e^t$$

$$= t\cdot e^t - e^t$$

$(x = \tan^{-1}y - 1) + c$

$$x\,e^{\tan^{-1}y} = \tan^{-1}y\; e^{\tan^{-1}y} - e^{\tan^{-1}y}$$

⑥ Solve the eq^n $(y\log y)\,dx + (x - \log y)\,dy = 0$

$$(x - \log y)\,dy = (-y\log y)\,dx$$

$$\frac{dy}{dx} = \frac{-y\log y}{x - \log y}$$

$$\frac{dx}{dy} = \frac{x}{-y\log y} + \frac{\cancel{\log y}}{y\,\cancel{\log y}}$$

$$\frac{dx}{dy} = \frac{-x}{y\log y} + \frac{1}{y}$$

$$\frac{dx}{dy} + \frac{x}{y\log y} = \frac{1}{y}$$

$$I.F = e^{\int P\,dy} = e^{\int \frac{1}{y\log y}dy} = e^{\int \frac{1}{t}dt}$$

$$= e^{\log t} = t$$

$$= \log y$$

$$\frac{dx}{dy} = \frac{1}{y}\left(\frac{-x}{\log y} + 1\right)$$

$$\frac{dx}{dy} = \frac{1}{y}\left(\frac{-x + \log y}{\log y}\right)$$

$$\frac{dy}{dx} = \frac{y\log y}{\log y - x}$$

$$\frac{dy}{dx} = \frac{y\,\cancel{\log y}}{\cancel{\log y}\left(1 - \frac{x}{\log y}\right)}$$

$$\frac{dy}{dx} = \frac{y}{1 - \frac{x}{\log y}} - 0$$

$\log y = t$

$\frac{1}{y}dy = dt$

$\log y = t$

$\frac{1}{y}dy = dt$

$$x\log y = \int \frac{1}{y}\log y\,dy + c$$

$$= \int t\,dt + c$$

$$x\log y = \frac{t^2}{2} + c$$

$$x\log y = \frac{(\log y)^2}{2} + c$$

*) Differential Equations:- An equation involving dependent variable, independent variable and the differential cofficents of the dependent variable with respect to independent variable is known as differential Equation such as; $\frac{dy}{dx} = \cot x$, $\frac{d^2y}{dx^2} + \frac{dy}{dx} + 1 = \sin x$

$$\sqrt{1 + \left(\frac{dy}{dx}\right)^3} = 0$$

#) Ordinary Differential Equation:- A Differential Equation which envoles only one independent variable is called ordinary Differential Equations. for ex:- $\frac{dy}{dx} = \cot x$

$\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = e^x$

#) Partial Differential Equation:- A differential which involves more than one independent varaible is called paratial differential Equation. for example:-

$$x\frac{dz}{dx} + y\frac{dz}{dy} = z$$

* Order of Differential Equations:- The order of differential Equation is the order of highest order derivative occuring in the differential Equation

for Example:-

Order of $\frac{d^2y}{dx^2} + \sqrt{1 + \left(\frac{dy}{dx}\right)^3} = 0$ is 2

Order of $\frac{dy}{dx} = \cot x$ is 1

Order of $f = \frac{\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{3/2}}{\frac{d^2y}{dx^2}}$ is 2

* **Degree of differential Equations:** The degree of differential Equation is the degree of highest order derivative present in the differential Equation when it is free from radical sign and function powers.

For example:

Degree of $\left(\frac{d^2y}{dx^2}\right)^2 = 1+\left(\frac{dy}{dx}\right)^3$, is 2

Degree of $\frac{dy}{dx} = \cot x$, is 1

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① Find degree of $\frac{d^2y}{dx^2} + \sqrt{1+\left(\frac{dy}{dx}\right)^3} = 0$

$$\frac{d^2y}{dx^2} = -\sqrt{1+\left(\frac{dy}{dx}\right)^3}$$

Squaring both side

$$\left(\frac{d^2y}{dx^2}\right)^2 = 1+\left(\frac{dy}{dx}\right)^3$$

Degree is 2 //

② Find degree of $\rho = \frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2}}{\frac{d^2y}{dx^2}}$

$$\rho\frac{d^2y}{dx^2} = \left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2}$$ Squaring

$$\rho^2\left(\frac{d^2y}{dx^2}\right)^2 = \left\{1+\left(\frac{dy}{dx}\right)^2\right\}^3$$

Degree is 2

Order is 2