Sets

Some definitions about sets

- Sets and subsets:

  set : group of elements

  subset : a set wich contains sets

- Empty set: $\emptyset$

- Union: $A \cup B$ is equal to A or B

- Intersection: $A \cap B$ is equal to A and B

- Complement

- Disjoint set : sets that do not have any overlap

- Partition: If we have a set A which can be describe by the disjoint union of sets $A_1, A_2, \dots, A_n$

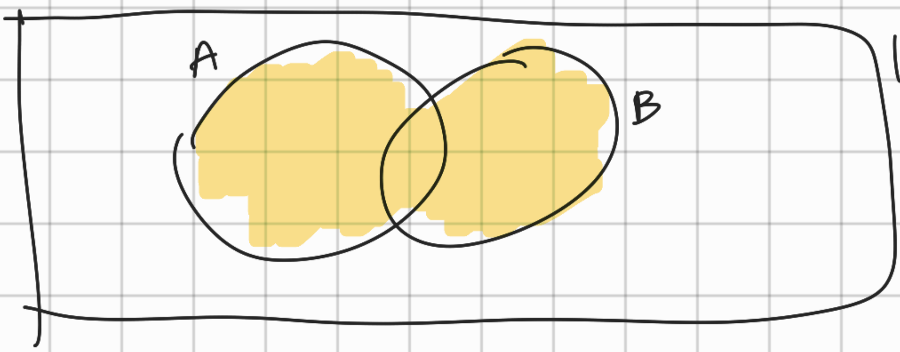
  $$A = \bigcup_{i=1}^{n} A_i,$$

  then, $\{A_i\}_{i=1}^{n}$ is a partition of A.

Example 1.1.

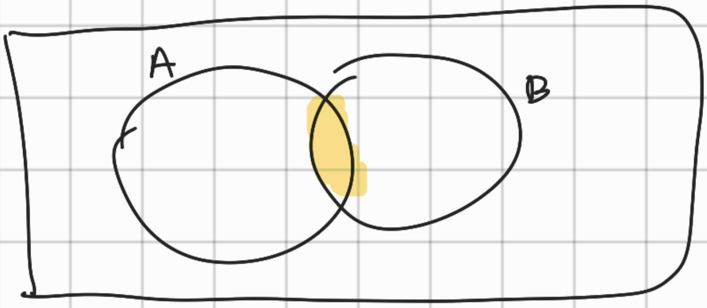
$A = \{$All stat majors at Havard$\}$

$B = \{$All student that live in dorm at Havard$\}$

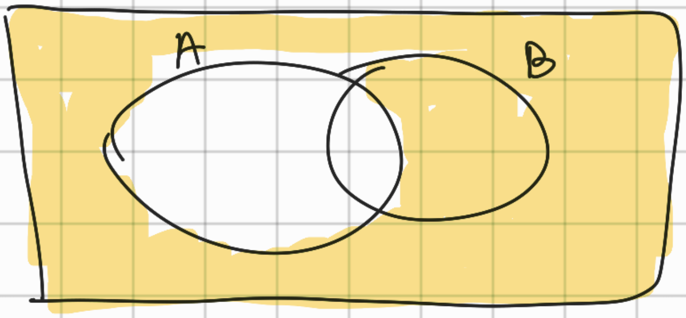
$A \cup B =$ 

Venn Diagram

$A \cap B =$



$A^c =$



Naive Probability

Concept 1.1 (Naive definition of Probability):

$$P(\text{Event}) = \frac{\#\text{ favorable outcomes}}{\#\text{ outcomes}}$$

Concept 1.2 (Multiplication Rule):

In a process where the first step has n_1 choices, the second step has n_2 choices, all the way up to the r^{th} step that has n_r choices, the total number of possible outcomes is

$$n_1 \cdot n_2 \cdot \ldots \cdot n_r .$$

Concept 1.3 (Factorial):

Suppose we want to count the number of ways to order n elements without replacement. To solve this problem we use the factorial definition:

$$n! = n(n-1)(n-2) \ldots 3 \cdot 2 \cdot 1$$

Note that "$n!$" is the number of ways to order n elements (or the number of permutations of n elements) without replacement.

Concept 1.4 (Binomial Coefficient):

$$\binom{n}{x} = \frac{n!}{(n-x)!\,x!}$$

"$\binom{n}{x}$" gives the number of ways that "x" objects can be pick from a population of "n" objects (or the number of combinations possible for a set with "x" elements).

Note: $$\binom{n}{x} = \frac{n!}{\underbrace{(n-x)!}_{\text{overcounting term}}\ \underbrace{x!}_{\text{order term}}}$$

number of possible combinations of n elements

The order does not matter

Sampling table

Number of ways to select k objects from n objects when...

	Order matters	order doesn't matter
with replacement	n^k	$\binom{n+k-1}{k}$
without replacement	$\frac{n!}{(n-k)!}$	$\binom{n}{k}$

n: refers to the population, k: refers to the group.

Usually orders matters when we are working with permutations. Whe we work with combinations the order doesn't matter.

- Permutation: order matter ;
- Combination: order doesn't matter.