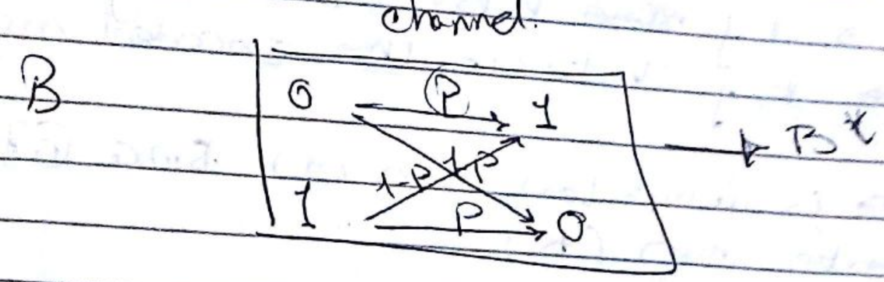
Tutorial 9/1
Channel Coding

After source coding the data is compressed so the Bits used decreases

* The idea now is that the channel has a Probability of error so



$P \Rightarrow Pr\{B^* \neq B\} \Rightarrow$ probability of error

So

* idea of channel coding is to add bits to the original msg with extra bits which carry no Information

for example [Repition code]

$0 \longrightarrow$ | channel coding | $\longrightarrow 000$

$1 \longrightarrow$ | channel coding | $\longrightarrow 111$

& we decode it by Majority Rule

which means if the msg has # of (1's) greater than (# of 0's) so it's (1) & if the msg has # of (0's) greater than (# of 1's) so it's (0)

$010 \longrightarrow$ | Decoder | $\longrightarrow 0$

$110 \longrightarrow$ | Decode | $\longrightarrow 1$ *

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* A Practical Example of More effective is the Hamming Code.

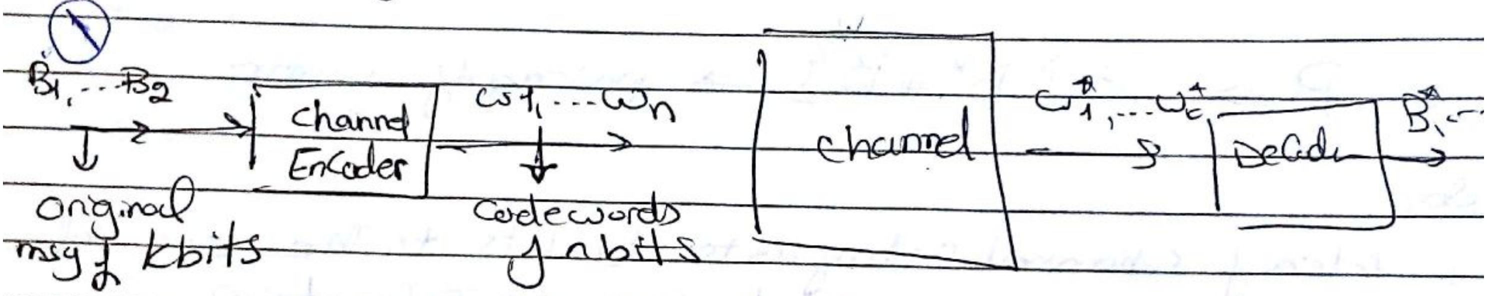
(1) Hamming Code increases the original msg by a certain rate which named

$$\text{Code rate} = \frac{k}{n} < 1$$

where $k \Rightarrow$ # of ~~mess~~ bits in the original msg

$n \Rightarrow$ # of bits in the Encoded msg

⇒ this Code rate is always less than (1) since (n) must be always greater than (k)



* Hamming Code

encoded bits, original bits

(n, k) binary block code

~~The encoder is the multiplication of the~~

It's ~~the~~ a linear code which means that any 2 codewords can be added (XOR) to get another Codeword in the Code

* The ~~code~~ encoded code words are generated by the Multiplication of the Codeword with a generator mtx

$$B \longrightarrow \boxed{\text{Channel Coder}} \longrightarrow \omega = BG.$$

$B \rightarrow [1 \times k]$

$G.$ [2]

$$G = \Big[\underbrace{I}_{k \times k} \;\Big|\; \underbrace{P}_{k \times n-k}\Big]_{k \times n}$$

$P \rightarrow$ coefficient mtx

$$\underbrace{\omega}_{1 \times n} = \underbrace{B}_{1 \times k}\;\underbrace{G}_{k \times n}$$

ω → Encoded Codeword

B → original msg

G → Generation Matrix (2)

[1] Step 1// at the Txer is to have the generation matrix

[2] Step 2// is to Make a code book which contains The encoded msg for each original msg. That can be transmitted

if The msg is of (K bits) so we had (2^k) msg's can be sent & I need to get encoded ~~msg~~ codeword for each ~~coder~~ msg.

$K = 3$ bits $\qquad n = 6$ bits

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Example

$$G = \left[\begin{array}{c:c} 100 & 110 \\ 010 & 011 \\ 001 & 101 \end{array}\right]_{3\times 6}$$

$$\underbrace{\quad}_{I_{3\times 3}} \qquad \underbrace{\quad}_{P_{3\times 3}} \quad 6-3$$

$w = BG$.

Sol

if $B = 001$

$$BG = [001]_{1\times 3} \begin{array}{c} 0 \\ 0 \\ 1 \end{array}\begin{bmatrix} 100110 \\ 010011 \\ 001101 \end{bmatrix}_{3\times 6}$$

$$w = [001101]_{1\times 6}$$

msg's b_i	Code words w_i
000	000000
001	001101
010	010011
011	011110
100	001101
101	101011
110	110101
111	111000

* As usual Multply
R Matrixes is by Multipy & add each Colomn
but Remember

multiply $\Rightarrow$ And $\left.\right\}$ in Binary
Addition $\Rightarrow$ XoR $\quad$ Representation

Andgat

0	0	0
0	1	0
1	0	1
1	1	0

XoR

0	0	0
0	1	1
1	0	1
1	1	0

u Can Make this Sequnce

[0 0 1 0 1]

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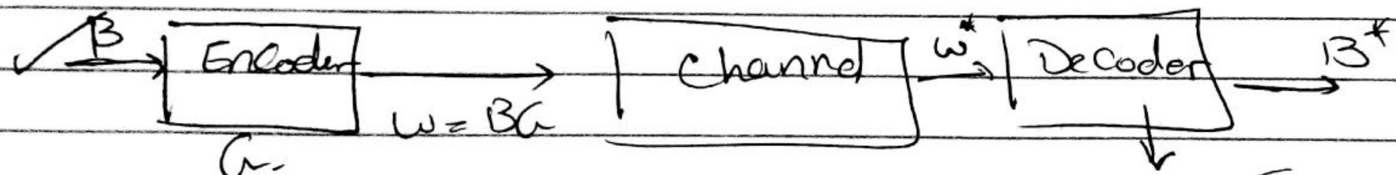
or in an easier way, you just $\boxed{XOR}$ The rows ~~xoring~~ corresponding to The 1's in The msg.

2nd step: Rxer

Second step is The decoder, ~~ys~~, which means to return back to the original msg.



$$B = w^* H^T$$

The Rxer knows The generator matrix with the Txer so He generates Parity check mtx $[H]$

$$H = \left[\begin{array}{c|c} \underset{n-k \times k}{P^T} & \underset{n-k \times n-k}{I} \end{array}\right] \quad n-k \times n$$

$$\underset{(n-k) \text{ bits}}{B^*} = \underset{(1 \times n)}{w^*} \cdot \underset{(n \times n-k)}{H^T}$$

* The idea of Choosing The (H) parity check matrix To be ~~B~~ The transpose of the (G) Matrix is that the Multiplication between the Codewords (w) & the parity check Matrix to be equal to Zero.

* So ~~$B^* = BGH$~~

$$B^* = w^* H^T$$
$$= \underbrace{B G H^T}_{=0}$$

So,, when $Bw^* H$ = Zero ⇒ valid Codeword no error Occurs from channel

$Bw^* H \neq 0$ ⇒ error occured

Now

→ The idea is that by now we know if the received Codeword is in error or not

→ How Can we correct this codeword?

* Some NoTes needed to be Mentioned

① error occured at The Msg if and only if The Noise has (1's)

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Since the noise is the addition of bits (1's) or (0's) to the Signal.
& since adding is the [XORing] so
NOTE that in (XoRing) if you XOR with (1) it's always changed to (1) or (0)

$0 \; 1 \Rightarrow 1$

$1 \; 1 \Rightarrow 0$

0	0	0
0	1	1
1	0	1
1	1	0

while if you (XOR) a bit with (Zero) it's not changed which means no error.

$0 \; 0 \Rightarrow 0$

$1 \; 0 \Rightarrow 1$.

* So if we know the error signal, we know that at the position of (1's) in the error, the original msg at this position will be changed to error.

Example

If we had [Codebook] & the msg was

$B = 101$ $\quad w = 101011 = BG$

$$G = \begin{array}{c} \quad I \qquad P \\ \begin{bmatrix} 1 0 0 & 1 1 0 \\ 0 1 0 & 0 1 1 \\ 0 0 1 & 1 0 1 \end{bmatrix} \end{array}$$

@ Rxer $\quad H = \begin{bmatrix} 1 0 1 & 1 0 0 \\ 1 1 0 & 0 1 0 \\ 0 1 1 & 0 0 1 \end{bmatrix}$

$$\bar{H} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$B^* = w^* H^T$

* if $\omega^* = \omega \Rightarrow$ No error in The channel.

$$B^* = \omega^* H^T$$

$$= 101011 \quad \begin{array}{c} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 1 \end{array} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$B^* = [0\ 0\ 0] \Rightarrow$ valid codeword

while

if $\omega^* \neq \omega \Rightarrow$ error occurs

$\omega = [101011]$ $\quad \omega^* = [101001]$

error

$$B^* = \omega^* H^T = 101001 \quad \begin{array}{c} 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \end{array} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$B^* = [0\ 1\ 0] \Rightarrow$ error occurs

@ This bit

#

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2) Now let's see how can we correct the error?

$$\because \omega^* = (\omega + E) \quad \text{channel error}$$

$$B^* = \omega^* H^T$$
$$= (\omega + E) H^T$$
$$= \underbrace{\omega H^T}_{\text{must be zero}} + \boxed{E H^T}$$

B $G H^T$
zero.

if Error is zero $\Rightarrow$ so B^* will be zero.
while
if Error occurs. $\boxed{E H^T \neq 0}$
so B^* will be not zero which means error occur

3) Idea

* B^* is the result of $\boxed{E H^T}$, so if we can get (E) we can get which digit was (1) so we know which bit in the (ω) was changed

* $E H^T$
$(1 \times n) \longrightarrow$ parity check mtx
same length msg

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2 Now let's see how can we correct the error?

$\because \omega^* = (\omega + E)$ channel error.

$$B^* = \omega^* H^T$$
$$= (\omega + E) H^T$$
$$= \omega H^T + \boxed{E H^T}$$

must be zero

$G H^T$

zero.

if Error is zero $\Rightarrow$ so B^* will be zero.

while

if Error occurs. $(E H^T \neq 0)$

so B^* will be not zero which means error occur

B Idea

* B^* is the result of $[E H^T]$, so if we can get (E) we can get which digit was (1) so we know which bit in the (ω) was changed

* $E H^T$

$\underbrace{(1 \times n)}_{\text{same length of msg}}$ $H^T \rightarrow$ parity check mtx

* We can get all The Combinations of (E) if only 1 bit was in error.

Syndrome (x) B^*	Error pattern (E)
000	000000
110	100000
011	010000
101	001000
100	000100
010	000010
001	000001.

if n=6

* Now Get The result of (EH^T) for all the possible Combinations.

~~00000~~

100000

$$\begin{bmatrix} 110 \\ 011 \\ 101 \\ 100 \\ 010 \\ 001 \end{bmatrix}$$

⇒ this will be easy Because you will just take The row Corresponds to '1')

* So This table will be @ The (Rcer) if He get $\boxed{B^* = \hat{W}H^T}$ is one of These Combinations

So He will get directly The error pattern & get which bit is in error

or The last example $B^* = 010$

So $E = 0000\boxed{1}0$

So $\omega = 101011$ ✓

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The questions

[1] if we had 2 bits in error So we will make all Combinations of only 1 bit in error & Then make all the Combination of 2bit in error ya3ny 2(1's) in The stream.

[2] Now, do all the bits in error Can be Corrected or we have a fixed number?

Solution?

The error Correcting capability of The Code is determined by what is called "Hamming distance".

Hamming distance ⇒ it's The min distance between Any 2 Codewords in The Code

⇒ minimum distance means The number of elements in which they differ

ex// $c_1 = [00101]$ ⇒ $d = 3$ //
$c_2 = [10011]$

So you look @ all The Code book & deside the min distence exsist between a pair of Code words.

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Finally

An (n, k) Linear block code of minimum distance d_{min} can correct up to t error.

$$\boxed{t \leq \left\lfloor \frac{1}{2}(d_{min}-1) \right\rfloor}$$

floor (Integer).

So for previous code $d_{min} = 3$

$$t \leq \frac{3-1}{2} \qquad \boxed{t \leq 1}$$

So $\boxed{t = 1}$

So we can correct only up to 1 bit

So when we make combinations, it was only for 1 bit error.

Final NoTe

The error correction capability will not be effective if one of the codewords is changed into another code word, so we cannot detect that there was an error

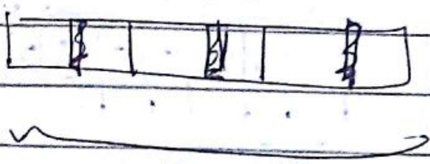
Kind of errors

The channel has 2 Types of errors

Random errors

The probability of error between bits is independent

3 errors



3 Code words

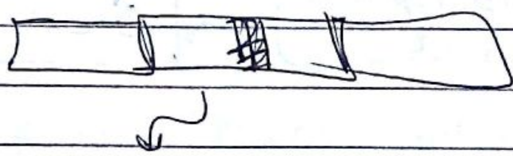
The error will be distributed over Codewords.

So only 1 bit will be in error in The codeword, So we Can Correct Codewords.

Burst errors

There is correlation in error probability for Consecutive bits.

3 Error.



Here incase of burst errors error are packed within The Same Code word.

So,, The decoder will not be able to Correct all The middle Codeword

* Solution.

Interleaving

Interleaving is The idea of distributing Codeword digits before channel So when The burst error occurs at only one Codeword when we rearrange The Codewords again it will be changed To random errors which Can be Solved.

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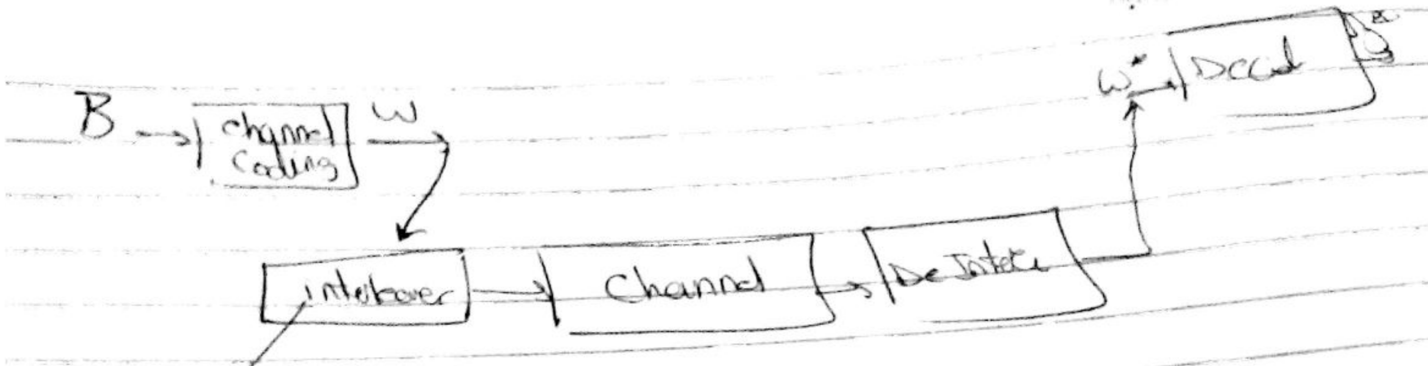

example: For $n=3$

010 → 1/3 repition Cod → 000 111 000

1 2 3 4 5 6 7 8 9

| 1 | 4 | 7 | 2 | 5 | 8 | 3 | 6 | 9 |

Interleaver

010010010

Burst error

010101010

1 4 7 2 5 8 3 6 9

De interleaver

← 010 101 000

2 5 8

010 → 0

101 → 1

100 → 0

So Error only 1 bit in each codeword is Inerro