Introduction to Machine Learning, Fall 2014 - Exercise session IV

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Problem 1 (6 points)

(a)

We seek to compute

$$P(Y \mid \mathbf{X}) = \frac{P(\mathbf{X} \mid Y)P(Y)}{P(\mathbf{X})},$$

where

$$P(\mathbf{X}) = \sum_{i=0}^{2} P(\mathbf{X} \mid Y = i)P(Y = i).$$

(0, 0)

If $\mathbf{X} = (0, 0)$,

$$\begin{aligned} P((0,0)) &= \sum_{i=0}^{2} P((0,0) \mid Y = i)P(Y = i) \\ &= 0.2 \times 0.4 + 0.6 \times 0.3 + 0.1 \times 0.3 \\ &= 0.08 + 0.18 + 0.03 \\ &= 0.29. \end{aligned}$$

$$\begin{aligned} P(Y = 0 \mid (0,0)) &= \frac{P((0,0) \mid Y = 0)P(Y = 0)}{P((0,0))} \\ &= \frac{0.2 \times 0.4}{0.29} \\ &= \frac{0.08}{0.29} \\ &\approx 0.276, \end{aligned}$$

$$
\begin{aligned}
P(Y=1\,|(0,0)) &= \frac{P((0,0)\,|\,Y=1)P(Y=1)}{P((0,0))} \\
&= \frac{0.6\times 0.3}{0.29} \\
&= \frac{0.18}{0.29} \\
&\approx 0.621,
\end{aligned}
$$

$$
\begin{aligned}
P(Y=2\,|(0,0)) &= \frac{P((0,0)\,|\,Y=2)P(Y=2)}{P((0,0))} \\
&= \frac{0.1\times 0.3}{0.29} \\
&= \frac{0.03}{0.29} \\
&\approx 0.103.
\end{aligned}
$$

(0, 1)

If $\mathbf{X} = (0,1)$,

$$
\begin{aligned}
P((0,1)) &= \sum_{i=0}^{2} P((0,1)\,|\,Y=i)P(Y=i) \\
&= 0.4\times 0.4 + 0.1\times 0.3 + 0.3\times 0.3 \\
&= 0.16 + 0.03 + 0.09 \\
&= 0.28.
\end{aligned}
$$

$$
\begin{aligned}
P(Y=0\,|(0,1)) &= \frac{P((0,1)\,|\,Y=0)P(Y=0)}{0.28} \\
&= \frac{0.4\times 0.4}{0.28} \\
&= \frac{0.16}{0.28} \\
&\approx 0.571,
\end{aligned}
$$

$$
\begin{aligned}
P(Y=1\,|(0,1)) &= \frac{P((0,1)\,|\,Y=1)P(Y=1)}{0.28} \\
&= \frac{0.1\times 0.3}{0.28} \\
&= \frac{0.03}{0.28} \\
&\approx 0.107,
\end{aligned}
$$

$$
\begin{aligned}
P(Y=2\,|(0,1)) &= \frac{P((0,1)\,|\,Y=2)P(Y=2)}{0.28} \\
&= \frac{0.3\times 0.3}{0.28} \\
&= \frac{0.09}{0.28} \\
&\approx 0.321.
\end{aligned}
$$

(0, 2)

If $\mathbf{X} = (0, 2)$,

$$
\begin{aligned}
P((0,2)) &= \sum_{i=0}^{2} P((0,2)\,|\,Y=i)P(Y=i) \\
&= 0.0\times 0.4 + 0.1\times 0.3 + 0.2\times 0.3 \\
&= 0.03 + 0.06 \\
&= 0.09,
\end{aligned}
$$

$$
\begin{aligned}
P(Y=0\,|(0,2)) &= \frac{P((0,2)\,|\,Y=0)P(Y=0)}{0.09} \\
&= \frac{0.0\times 0.4}{0.09} \\
&= 0,
\end{aligned}
$$

$$
\begin{aligned}
P(Y=1\,|(0,2)) &= \frac{P((0,2)\,|\,Y=1)P(Y=1)}{0.09} \\
&= \frac{0.1\times 0.3}{0.09} \\
&= \frac{0.03}{0.09} \\
&\approx 0.333,
\end{aligned}
$$

$$
\begin{aligned}
P(Y=2\,|(0,2)) &= \frac{P((0,2)\,|\,Y=2)P(Y=2)}{0.09} \\
&= \frac{0.2\times 0.3}{0.09} \\
&= \frac{0.06}{0.09} \\
&\approx 0.666.
\end{aligned}
$$

(1, 0)

If $\mathbf{X} = (1, 0)$,

$$\begin{aligned} P((1,0)) &= \sum_{i=0}^{2} P((1,0) \,|\, Y = i) P(Y = i) \\ &= 0.1 \times 0.4 + 0.1 \times 0.3 + 0.4 \times 0.3 \\ &= 0.04 + 0.03 + 0.12 \\ &= 0.19, \end{aligned}$$

$$\begin{aligned} P(Y = 0 \,|(1,0)) &= \frac{P((1,0) \,|\, Y = 0) P(Y = 0)}{0.19} \\ &= \frac{0.1 \times 0.4}{0.19} \\ &= \frac{0.04}{0.19} \\ &\approx 0.211, \end{aligned}$$

$$\begin{aligned} P(Y = 1 \,|(1,0)) &= \frac{P((1,0) \,|\, Y = 1) P(Y = 1)}{0.19} \\ &= \frac{0.1 \times 0.3}{0.19} \\ &= \frac{0.03}{0.19} \\ &\approx 0.158, \end{aligned}$$

$$\begin{aligned} P(Y = 2 \,|(1,0)) &= \frac{P((1,0) \,|\, Y = 2) P(Y = 2)}{0.19} \\ &= \frac{0.4 \times 0.3}{0.19} \\ &= \frac{0.12}{0.19} \\ &\approx 0.632. \end{aligned}$$

(1, 1)

If $\mathbf{X} = (1, 1)$,

$$\begin{aligned} P((1,1)) &= \sum_{i=0}^{2} P((1,1) \,|\, Y = i) P(Y = i) \\ &= 0.2 \times 0.4 + 0.1 \times 0.3 + 0.0 \times 0.3 \\ &= 0.08 + 0.03 \\ &= 0.11, \end{aligned}$$

$$
\begin{aligned}
P(Y=0\,|(1,1)) &= \frac{P((1,1)\,|\,Y=0)P(Y=0)}{0.11} \\
&= \frac{0.2\times 0.4}{0.11} \\
&= \frac{0.08}{0.11} \\
&\approx 0.727,
\end{aligned}
$$

$$
\begin{aligned}
P(Y=1\,|(1,1)) &= \frac{P((1,1)\,|\,Y=1)P(Y=1)}{0.11} \\
&= \frac{0.1\times 0.3}{0.11} \\
&= \frac{0.03}{0.11} \\
&\approx 0.273,
\end{aligned}
$$

$$
\begin{aligned}
P(Y=2\,|(1,1)) &= \frac{P((1,1)\,|\,Y=2)P(Y=2)}{0.11} \\
&= \frac{0.0\times 0.3}{0.11} \\
&= 0.0.
\end{aligned}
$$

(1, 2)

If $\mathbf{X} = (1, 2)$,

$$
\begin{aligned}
P((1,2)) &= \sum_{i=0}^{2} P((1,2)\,|\,Y=i)P(Y=i) \\
&= 0.1\times 0.4 + 0.0\times 0.3 + 0.0\times 0.3 \\
&= 0.04,
\end{aligned}
$$

$$
\begin{aligned}
P(Y=0\,|(1,2)) &= \frac{P((1,2)\,|\,Y=0)P(Y=0)}{0.04} \\
&= \frac{0.1\times 0.4}{0.04} \\
&= 1.0,
\end{aligned}
$$

$$
\begin{aligned}
P(Y=1\,|(1,2)) &= \frac{P((1,2)\,|\,Y=1)P(Y=1)}{0.04} \\
&= \frac{0.0\times 0.3}{0.11} \\
&= 0.0,
\end{aligned}
$$

$$
\begin{aligned}
P(Y=2\,|(1,2)) &= \frac{P((1,2)\,|\,Y=2)P(Y=2)}{0.04} \\
&= \frac{0.0\times 0.3}{0.11} \\
&= 0.0.
\end{aligned}
$$

Summary

	$Y=0$	$Y=1$	$Y=2$
$(0,0)$	0.276	0.621	0.103
$(0,1)$	0.571	0.107	0.321
$(0,2)$	0.0	0.333	0.667
$(1,0)$	0.211	0.158	0.631
$(1,1)$	0.727	0.273	0.0
$(1,2)$	1.0	0.0	0.0

The optimal Bayes classifier is

$$
Y(\mathbf{X}) = \begin{cases} 0 & \text{if } \mathbf{X} \in \{(0,1),(1,1),(1,2)\} \\ 1 & \text{if } \mathbf{X} = (0,0) \\ 2 & \text{if } \mathbf{X} \in \{(0,2),(1,0)\}. \end{cases}
$$

(b)

The error rate is

$$
\sum_{x_1=0}^{1}\sum_{x_2=0}^{2} P(x_1,x_2)E(x_1,x_2),
$$

where $E(x_1,x_2)$ is the error probability at point (x_1,x_2), which is defined as the sum of two least probabilities.

$\mathbf{X}$	$E(\mathbf{X})$	$P(\mathbf{X})$	$E(\mathbf{X})P(\mathbf{X})$
$(0,0)$	0.379	0.29	0.110
$(0,1)$	0.428	0.28	0.120
$(0,2)$	0.333	0.09	0.030
$(1,0)$	0.369	0.19	0.070
$(1,1)$	0.273	0.11	0.030
$(1,2)$	0	0.04	0

So the error rate is $0.110 + 0.120 + 0.030 + 0.070 + 0.030 = 0.36$.

(c)

For naïve Bayes classifier in this context, we have

$$
P(\mathbf{X}\,|\,Y=y) = \prod_{i=1}^{2} P(X_i\,|\,Y=y) = P(X_1\,|\,Y=y)P(X_2\,|\,Y=y),
$$

so we seek to compute

$$P(Y = y \,|\, \mathbf{X}) = \frac{P(\mathbf{X} \,|\, Y = y)P(Y = y)}{P(\mathbf{X})} = \frac{P(X_1 \,|\, Y = y)P(X_2 \,|\, Y = y)P(Y = y)}{P(\mathbf{X})},$$

where

$$\begin{aligned} P(\mathbf{X}) &= \sum_{i=0}^{2} P(\mathbf{X} \,|\, Y = i)P(Y = i) \\ &= \sum_{i=0}^{2} P(X_1 \,|\, Y = i)P(X_2 \,|\, Y = i)P(Y = i). \end{aligned}$$

$(0, 0)$

If $\mathbf{X} = (0, 0)$, and

$$\begin{aligned} P(\mathbf{X}) &= \sum_{i=0}^{2} P(X_1 \,|\, Y = i)P(X_2 \,|\, Y = i)P(Y = i) \\ &= (0.2 + 0.4 + 0.0) \times (0.2 + 0.1) \times 0.4 \\ &+ (0.6 + 0.1 + 0.1) \times (0.6 + 0.1) \times 0.3 \\ &+ (0.1 + 0.3 + 0.2) \times (0.1 + 0.4) \times 0.3 \\ &= 0.6 \times 0.3 \times 0.4 + 0.8 \times 0.7 \times 0.3 + 0.6 \times 0.5 \times 0.3 \\ &= 0.072 + 0.168 + 0.09 \\ &= 0.33. \end{aligned}$$

$$\begin{aligned} P(Y = 0 \,|\, (0, 0)) &= \frac{P(X_1 \,|\, Y = 0)P(X_2 \,|\, Y = 0)P(Y = 0)}{0.33} \\ &= \frac{(0.2 + 0.4 + 0.0) \times (0.2 + 0.1) \times 0.4}{0.33} \\ &= \frac{0.6 \times 0.3 \times 0.4}{0.33} \\ &= \frac{0.072}{0.33} \\ &\approx 0.218, \end{aligned}$$

$$\begin{aligned} P(Y = 1 \,|\, (0, 0)) &= \frac{P(X_1 \,|\, Y = 1)P(X_2 \,|\, Y = 1)P(Y = 1)}{0.33} \\ &= \frac{(0.6 + 0.1 + 0.1) \times (0.6 + 0.1) \times 0.3}{0.33} \\ &= \frac{0.8 \times 0.7 \times 0.3}{0.33} \\ &= \frac{0.168}{0.33} \\ &\approx 0.509, \end{aligned}$$

$$
\begin{aligned}
P(Y=2\,|\,(0,0)) &= \frac{P(X_1\,|\,Y=2)P(X_2\,|\,Y=2)P(Y=2)}{0.33} \\
&= \frac{(0.1+0.3+0.2)\times(0.1+0.4)\times 0.3}{0.33} \\
&= \frac{0.6\times 0.5\times 0.3}{0.33} \\
&= \frac{0.09}{0.33} \\
&\approx 0.273.
\end{aligned}
$$

$(0,1)$

If $\mathbf{X}=(0,1)$, and

$$
\begin{aligned}
P(\mathbf{X}) &= \sum_{i=0}^{2} P(X_1\,|\,Y=i)P(X_2\,|\,Y=i)P(Y=i) \\
&= (0.2+0.4+0.0)\times(0.4+0.2)\times 0.4 \\
&+ (0.6+0.1+0.1)\times(0.1+0.1)\times 0.3 \\
&+ (0.1+0.3+0.2)\times(0.3+0.0)\times 0.3 \\
&= 0.6\times 0.6\times 0.4 + 0.8\times 0.2\times 0.3 + 0.6\times 0.3\times 0.3 \\
&= 0.144 + 0.048 + 0.054 \\
&= 0.246.
\end{aligned}
$$

$$
\begin{aligned}
P(Y=0\,|\,(0,1)) &= \frac{P(X_1=0\,|\,Y=0)P(X_2=1\,|\,Y=0)P(Y=0)}{0.246} \\
&= \frac{(0.2+0.4+0.0)\times(0.4+0.2)\times 0.4}{0.246} \\
&= \frac{0.6\times 0.6\times 0.4}{0.246} \\
&= \frac{0.144}{0.246} \\
&\approx 0.585,
\end{aligned}
$$

$$
\begin{aligned}
P(Y=1\,|\,(0,1)) &= \frac{P(X_1=0\,|\,Y=1)P(X_2=1\,|\,Y=1)P(Y=1)}{0.246} \\
&= \frac{(0.6+0.1+0.1)\times(0.1+0.1)\times 0.3}{0.246} \\
&= \frac{0.8\times 0.2\times 0.3}{0.246} \\
&= \frac{0.048}{0.246} \\
&\approx 0.195,
\end{aligned}
$$

$$
\begin{aligned}
P(Y=2\,|\,(0,1)) &= \frac{P(X_1=0\,|\,Y=2)P(X_2=1\,|\,Y=2)P(Y=2)}{0.246} \\
&= \frac{(0.1+0.3+0.2)\times(0.3+0.0)\times 0.3}{0.246} \\
&= \frac{0.6\times 0.3\times 0.3}{0.246} \\
&= \frac{0.054}{0.246} \\
&\approx 0.219.
\end{aligned}
$$

$(0,2)$

If $\mathbf{X}=(0,2)$, and

$$
\begin{aligned}
P(\mathbf{X}) &= \sum_{i=0}^{2} P(X_1\,|\,Y=i)P(X_2\,|\,Y=i)P(Y=i) \\
&= (0.2+0.4+0.0)\times(0.0+0.1)\times 0.4 \\
&+ (0.6+0.1+0.1)\times(0.1+0.0)\times 0.3 \\
&+ (0.1+0.3+0.2)\times(0.2+0.0)\times 0.3 \\
&= 0.6\times 0.1\times 0.4 + 0.8\times 0.1\times 0.3 + 0.6\times 0.2\times 0.3 \\
&= 0.024+0.024+0.036 \\
&= 0.084.
\end{aligned}
$$

$$
\begin{aligned}
P(Y=0\,|\,(0,2)) &= \frac{P(X_1=0\,|\,Y=0)P(X_2=2\,|\,Y=0)P(Y=0)}{0.084} \\
&= \frac{(0.2+0.4+0.0)\times(0.0+0.1)\times 0.4}{0.084} \\
&= \frac{0.6\times 0.1\times 0.4}{0.084} \\
&= \frac{0.024}{0.084} \\
&\approx 0.286,
\end{aligned}
$$

$$
\begin{aligned}
P(Y=1\,|\,(0,2)) &= \frac{P(X_1=0\,|\,Y=1)P(X_2=2\,|\,Y=1)P(Y=1)}{0.084} \\
&= \frac{(0.6+0.1+0.1)\times(0.1+0.0)\times 0.3}{0.084} \\
&= \frac{0.8\times 0.1\times 0.3}{0.084} \\
&= \frac{0.024}{0.084} \\
&\approx 0.286,
\end{aligned}
$$

$$
\begin{aligned}
P(Y=2\,|\,(0,2)) &= \frac{P(X_1=0\,|\,Y=2)P(X_2=2\,|\,Y=2)P(Y=2)}{0.084} \\
&= \frac{(0.1+0.3+0.2)\times(0.2+0.0)\times 0.3}{0.084} \\
&= \frac{0.6\times 0.2\times 0.3}{0.084} \\
&= \frac{0.036}{0.084} \\
&\approx 0.429.
\end{aligned}
$$

$(1,0)$

If $\mathbf{X}=(1,0)$, and

$$
\begin{aligned}
P(\mathbf{X}) &= \sum_{i=0}^{2} P(X_1\,|\,Y=i)P(X_2\,|\,Y=i)P(Y=i) \\
&= (0.1+0.2+0.1)\times(0.2+0.1)\times 0.4 \\
&+ (0.1+0.1+0.0)\times(0.6+0.1)\times 0.3 \\
&+ (0.4+0.0+0.0)\times(0.1+0.4)\times 0.3 \\
&= 0.4\times 0.3\times 0.4 + 0.2\times 0.7\times 0.3 + 0.4\times 0.5\times 0.3 \\
&= 0.048+0.042+0.06 \\
&= 0.15.
\end{aligned}
$$

$$
\begin{aligned}
P(Y=0\,|\,(1,0)) &= \frac{P(X_1=1\,|\,Y=0)P(X_2=0\,|\,Y=0)P(Y=0)}{0.15} \\
&= \frac{(0.1+0.2+0.1)\times(0.2+0.1)\times 0.4}{0.15} \\
&= \frac{0.4\times 0.3\times 0.4}{0.15} \\
&= \frac{0.048}{0.15} \\
&= 0.32,
\end{aligned}
$$

$$
\begin{aligned}
P(Y=1\,|\,(1,0)) &= \frac{P(X_1=1\,|\,Y=1)P(X_2=0\,|\,Y=1)P(Y=1)}{0.15} \\
&= \frac{(0.1+0.1+0.0)\times(0.6+0.1)\times 0.3}{0.15} \\
&= \frac{0.2\times 0.7\times 0.3}{0.15} \\
&= \frac{0.042}{0.15} \\
&= 0.28,
\end{aligned}
$$

$$
\begin{aligned}
P(Y=2\,|\,(1,0)) &= \frac{P(X_1=1\,|\,Y=2)P(X_2=0\,|\,Y=2)P(Y=2)}{0.15} \\
&= \frac{(0.4+0.0+0.0)\times(0.1+0.4)\times 0.3}{0.15} \\
&= \frac{0.4\times 0.5\times 0.3}{0.15} \\
&= \frac{0.06}{0.15} \\
&= 0.4.
\end{aligned}
$$

$(1,1)$

If $\mathbf{X}=(1,1)$, and

$$
\begin{aligned}
P(\mathbf{X}) &= \sum_{i=0}^{2} P(X_1\,|\,Y=i)P(X_2\,|\,Y=i)P(Y=i) \\
&= (0.1+0.2+0.1)\times(0.4+0.2)\times 0.4 \\
&+ (0.1+0.1+0.0)\times(0.1+0.1)\times 0.3 \\
&+ (0.4+0.0+0.0)\times(0.3+0.0)\times 0.3 \\
&= 0.4\times 0.6\times 0.4 + 0.2\times 0.2\times 0.3 + 0.4\times 0.3\times 0.3 \\
&= 0.096 + 0.012 + 0.036 \\
&= 0.144.
\end{aligned}
$$

$$
\begin{aligned}
P(Y=0\,|\,(1,1)) &= \frac{P(X_1=1\,|\,Y=0)P(X_2=1\,|\,Y=0)P(Y=0)}{0.144} \\
&= \frac{(0.1+0.2+0.1)\times(0.4+0.2)\times 0.4}{0.144} \\
&= \frac{0.4\times 0.6\times 0.4}{0.144} \\
&= \frac{0.096}{0.144} \\
&\approx 0.667,
\end{aligned}
$$

$$
\begin{aligned}
P(Y=1\,|\,(1,1)) &= \frac{P(X_1=1\,|\,Y=1)P(X_2=1\,|\,Y=1)P(Y=1)}{0.144} \\
&= \frac{(0.1+0.1+0.0)\times(0.1+0.1)\times 0.3}{0.144} \\
&= \frac{0.2\times 0.2\times 0.3}{0.144} \\
&= \frac{0.012}{0.144} \\
&\approx 0.083,
\end{aligned}
$$

$$
\begin{aligned}
P(Y=2\,|\,(1,1)) &= \frac{P(X_1=1\,|\,Y=2)P(X_2=1\,|\,Y=2)P(Y=2)}{0.144} \\
&= \frac{(0.4+0.0+0.0)\times(0.3+0.0)\times 0.3}{0.144} \\
&= \frac{0.4\times 0.3\times 0.3}{0.144} \\
&= \frac{0.036}{0.144} \\
&= 0.25.
\end{aligned}
$$

$(1,2)$

If $\mathbf{X}=(1,2)$, and

$$
\begin{aligned}
P(\mathbf{X}) &= \sum_{i=0}^{2} P(X_1\,|\,Y=i)P(X_2\,|\,Y=i)P(Y=i) \\
&= (0.1+0.2+0.1)\times(0.0+0.1)\times 0.4 \\
&+ (0.1+0.1+0.0)\times(0.1+0.0)\times 0.3 \\
&+ (0.4+0.0+0.0)\times(0.2+0.0)\times 0.3 \\
&= 0.4\times 0.1\times 0.4 + 0.2\times 0.1\times 0.3 + 0.4\times 0.2\times 0.3 \\
&= 0.016 + 0.006 + 0.024 \\
&= 0.046.
\end{aligned}
$$

$$
\begin{aligned}
P(Y=0\,|\,(1,2)) &= \frac{P(X_1=1\,|\,Y=0)P(X_2=2\,|\,Y=0)P(Y=0)}{0.046} \\
&= \frac{(0.1+0.2+0.1)\times(0.0+0.1)\times 0.4}{0.046} \\
&= \frac{0.4\times 0.1\times 0.4}{0.046} \\
&= \frac{0.016}{0.046} \\
&\approx 0.348,
\end{aligned}
$$

$$
\begin{aligned}
P(Y=1\,|\,(1,1)) &= \frac{P(X_1=1\,|\,Y=1)P(X_2=2\,|\,Y=1)P(Y=1)}{0.046} \\
&= \frac{(0.1+0.1+0.0)\times(0.1+0.0)\times 0.3}{0.046} \\
&= \frac{0.2\times 0.1\times 0.3}{0.046} \\
&= \frac{0.006}{0.046} \\
&\approx 0.130,
\end{aligned}
$$

$$
\begin{aligned}
P(Y=2\,|\,(1,1)) &= \frac{P(X_1=1\,|\,Y=2)P(X_2=2\,|\,Y=2)P(Y=2)}{0.046} \\
&= \frac{(0.4+0.0+0.0)\times(0.2+0.0)\times 0.3}{0.046} \\
&= \frac{0.4\times 0.2\times 0.3}{0.046} \\
&= \frac{0.024}{0.046} \\
&= 0.522.
\end{aligned}
$$

Summary

	$Y=0$	$Y=1$	$Y=2$
$(0,0)$	0.218	0.509	0.273
$(0,1)$	0.585	0.195	0.219
$(0,2)$	0.286	0.286	0.429
$(1,0)$	0.32	0.28	0.4
$(1,1)$	0.667	0.083	0.25
$(1,2)$	0.348	0.130	0.522

The Naïve Bayes classifier is

$$
Y(\mathbf{X}) = \begin{cases} 0 & \text{if } \mathbf{X} \in \{(0,1),(1,1)\} \\ 1 & \text{if } \mathbf{X} = (0,0) \\ 2 & \text{if } \mathbf{X} \in \{(0,2),(1,0),(1,2)\}. \end{cases}
$$

(d)

$\mathbf{X}$	$E(\mathbf{X})$	$P(\mathbf{X})$	$E(\mathbf{X})P(\mathbf{X})$
$(0,0)$	0.491	0.330	0.162
$(0,1)$	0.415	0.246	0.102
$(0,2)$	0.571	0.084	0.048
$(1,0)$	0.600	0.150	0.09
$(1,1)$	0.333	0.144	0.048
$(1,2)$	0.478	0.046	0.022

So the error rate is 0.162+0.102+0.048+ $0.090 + 0.048 + 0.022 = 0.472$.

(e)

The error rate of naïve Bayes classifier is 31% higher than that of optimal Bayes classifier. And no, I really had to calculate all the stuff to "guess".

Problem 2 (3 points)

Problem 3 (15 points)