

Introduction to Machine Learning, Fall 2014 - Exercise session IV

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November 19, 2014

Problem 1 (6 points)

We seek to compute

$$P(Y | \mathbf{X}) = \frac{P(\mathbf{X} | Y)P(Y)}{P(\mathbf{X})},$$

where

$$P(\mathbf{X}) = \sum_{i=0}^2 P(\mathbf{X} | Y = i)P(Y = i).$$

(0, 0)

If $\mathbf{X} = (0, 0)$,

$$\begin{aligned} P(Y = 0 | (0, 0)) &= \frac{P((0, 0) | Y = 0)P(Y = 0)}{\sum_{i=0}^2 P((0, 0) | Y = i)P(Y = i)} \\ &= \frac{0.2 \times 0.4}{0.2 \times 0.4 + 0.6 \times 0.3 + 0.1 \times 0.3} \\ &= \frac{0.08}{0.08 + 0.18 + 0.03} \\ &= \frac{0.08}{0.29} \\ &\approx 0.276, \end{aligned}$$

$$\begin{aligned}
P(Y = 1 | (0, 0)) &= \frac{P((0, 0) | Y = 1)P(Y = 1)}{\sum_{i=0}^2 P((0, 0) | Y = i)P(Y = i)} \\
&= \frac{0.6 \times 0.3}{0.2 \times 0.4 + 0.6 \times 0.3 + 0.1 \times 0.3} \\
&= \frac{0.18}{0.08 + 0.18 + 0.03} \\
&= \frac{0.18}{0.29} \\
&\approx 0.621,
\end{aligned}$$

$$\begin{aligned}
P(Y = 2 | (0, 0)) &= \frac{P((0, 0) | Y = 2)P(Y = 2)}{\sum_{i=0}^2 P((0, 0) | Y = i)P(Y = i)} \\
&= \frac{0.1 \times 0.3}{0.2 \times 0.4 + 0.6 \times 0.3 + 0.1 \times 0.3} \\
&= \frac{0.03}{0.08 + 0.18 + 0.03} \\
&= \frac{0.03}{0.29} \\
&\approx 0.103.
\end{aligned}$$

(0, 1)

If $\mathbf{X} = (0, 1)$,

$$\begin{aligned}
P(Y = 0 | (0, 1)) &= \frac{P((0, 1) | Y = 0)P(Y = 0)}{\sum_{i=0}^2 P((0, 1) | Y = i)P(Y = i)} \\
&= \frac{0.4 \times 0.4}{0.2 \times 0.4 + 0.6 \times 0.3 + 0.1 \times 0.3} \\
&= \frac{0.08}{0.08 + 0.18 + 0.03} \\
&= \frac{0.08}{0.29} \\
&\approx 0.276,
\end{aligned}$$

$$\begin{aligned}
P(Y = 1 | (0, 1)) &= \frac{P((0, 1) | Y = 1)P(Y = 1)}{\sum_{i=0}^2 P((0, 1) | Y = i)P(Y = i)} \\
&= \frac{0.1 \times 0.3}{0.2 \times 0.4 + 0.6 \times 0.3 + 0.1 \times 0.3} \\
&= \frac{0.03}{0.08 + 0.18 + 0.03} \\
&= \frac{0.03}{0.29} \\
&\approx 0.103,
\end{aligned}$$

$$\begin{aligned}
P(Y = 2 | (0, 1)) &= \frac{P((0, 1) | Y = 2)P(Y = 2)}{\sum_{i=0}^2 P((0, 1) | Y = i)P(Y = i)} \\
&= \frac{0.3 \times 0.3}{0.2 \times 0.4 + 0.6 \times 0.3 + 0.1 \times 0.3} \\
&= \frac{0.03}{0.08 + 0.18 + 0.03} \\
&= \frac{0.03}{0.29} \\
&\approx 0.103.
\end{aligned}$$

Problem 2 (3 points)

Problem 3 (15 points)