Kvanttilaskenta, kevät 2015 – Viikko 4

Rodion "rodde" Efremov

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edx Problem 1

False. By definition of reversibility we should have x at the output of R_f.

edx Problem 2

False. The quantum circuit should modify each α_x.

edx Problem 3

True. Straight from the slides.

edx Problem 4

$$
\begin{aligned}
H^{\otimes 2} \frac{1}{\sqrt{2}} |00\rangle &= \frac{1}{\sqrt{2}} |{+}{+}\rangle \\
&= \frac{1}{\sqrt{2}} \left(\frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle \right) \left(\frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle \right) \\
&= \frac{1}{2\sqrt{2}} \Bigg(|00\rangle + |01\rangle + |10\rangle + |11\rangle \Bigg).
\end{aligned}
$$

Also

$$
\begin{aligned}
H^{\otimes 2} \frac{1}{\sqrt{2}} |11\rangle &= \frac{1}{\sqrt{2}} |{-}{-}\rangle \\
&= \frac{1}{\sqrt{2}} \left(\frac{1}{\sqrt{2}} |0\rangle - \frac{1}{\sqrt{2}} |1\rangle \right) \left(\frac{1}{\sqrt{2}} |0\rangle - \frac{1}{\sqrt{2}} |1\rangle \right) \\
&= \frac{1}{2\sqrt{2}} \Bigg(|00\rangle - |01\rangle - |10\rangle + |11\rangle \Bigg),
\end{aligned}
$$

so

$$\begin{aligned}
H^{\otimes 2}\psi &= H^{\otimes 2}\left(\frac{1}{\sqrt{2}}\left|00\right\rangle + \frac{1}{\sqrt{2}}\left|11\right\rangle\right) \\
&= \frac{1}{2\sqrt{2}}\left(2\left|00\right\rangle + 2\left|11\right\rangle\right) \\
&= \frac{1}{\sqrt{2}}\left(\left|00\right\rangle + \left|11\right\rangle\right) \\
&= \psi.
\end{aligned}$$

edx Problem 5

Let

$$\left|\psi\right\rangle = \frac{1}{\sqrt{2}}(\left|01\right\rangle + \left|10\right\rangle).$$

$$\begin{aligned}
H^{\otimes 2}\frac{1}{\sqrt{2}}\left|01\right\rangle &= \frac{1}{\sqrt{2}}\left|+-\right\rangle \\
&= \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\left|0\right\rangle + \frac{1}{\sqrt{2}}\left|1\right\rangle\right)\left(\frac{1}{\sqrt{2}}\left|0\right\rangle - \frac{1}{\sqrt{2}}\left|1\right\rangle\right) \\
&= \frac{1}{2\sqrt{2}}\left(\left|00\right\rangle - \left|01\right\rangle + \left|10\right\rangle - \left|11\right\rangle\right).
\end{aligned}$$

$$\begin{aligned}
H^{\otimes 2}\frac{1}{\sqrt{2}}\left|10\right\rangle &= \frac{1}{\sqrt{2}}\left|-+\right\rangle \\
&= \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\left|0\right\rangle - \frac{1}{\sqrt{2}}\left|1\right\rangle\right)\left(\frac{1}{\sqrt{2}}\left|0\right\rangle + \frac{1}{\sqrt{2}}\left|1\right\rangle\right) \\
&= \frac{1}{2\sqrt{2}}\left(\left|00\right\rangle + \left|01\right\rangle - \left|10\right\rangle - \left|11\right\rangle\right).
\end{aligned}$$

Now we see that $H^{\otimes 2}\left|\psi\right\rangle$ is

$$\begin{aligned}
\frac{1}{2\sqrt{2}}\left(\left|00\right\rangle - \left|01\right\rangle + \left|10\right\rangle - \left|11\right\rangle\right) + \frac{1}{2\sqrt{2}}\left(\left|00\right\rangle + \left|01\right\rangle - \left|10\right\rangle - \left|11\right\rangle\right) &= \frac{1}{2\sqrt{2}}(2\left|00\right\rangle - 2\left|11\right\rangle) \\
&= \frac{1}{\sqrt{2}}(\left|00\right\rangle - \left|11\right\rangle).
\end{aligned}$$

edx Problem 6

Yes. Both may yield $\left|00\right\rangle$ with probability $\frac{1}{2}$ and $\left|11\right\rangle$ with probability $\frac{1}{2}$.

edx Problem 7

Apply circuit A and then D.

edx Problem 8

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

edx Problem 9

Suppose

$$H^{\otimes 3} |\psi\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle).$$

Since $H^{\otimes 3}$ is reversible, if we apply it again to $H^{\otimes 3} |\psi\rangle$, we will obtain $|\psi\rangle$. Let us calculate that ket by ket:

$$\begin{aligned} H^{\otimes 3} \frac{1}{\sqrt{2}} |000\rangle &= \frac{1}{\sqrt{2}} |+++\rangle \\ &= \frac{1}{4}\Bigg(|0\rangle + |1\rangle \Bigg)^3 \\ &= \frac{1}{4}\Bigg(|0\rangle + |1\rangle \Bigg)\Bigg(|00\rangle + |01\rangle + |10\rangle + |11\rangle \Bigg) \\ &= \frac{1}{4}\Bigg(|000\rangle + |001\rangle + |010\rangle + |011\rangle + |100\rangle + |101\rangle + |110\rangle + |111\rangle \Bigg). \end{aligned}$$

$$\begin{aligned} H^{\otimes 3} \frac{1}{\sqrt{2}} |111\rangle &= \frac{1}{\sqrt{2}} |---\rangle \\ &= \frac{1}{4}\Bigg(|0\rangle - |1\rangle \Bigg)^3 \\ &= \frac{1}{4}\Bigg(|0\rangle - |1\rangle \Bigg)\Bigg(|00\rangle - |01\rangle - |10\rangle + |11\rangle \Bigg) \\ &= \frac{1}{4}\Bigg(|000\rangle - |001\rangle - |010\rangle + |011\rangle - |100\rangle + |101\rangle + |110\rangle - |111\rangle \Bigg). \end{aligned}$$

Now

$$\begin{aligned}|\psi\rangle &= H^{\otimes 3}H^{\otimes 3}|\psi\rangle \\ &= H^{\otimes 3}\frac{1}{\sqrt{2}}|000\rangle + H^{\otimes 3}\frac{1}{\sqrt{2}}|11\rangle \\ &= \frac{1}{4}(2|000\rangle + 2|011\rangle + 2|101\rangle + 2|110\rangle) \\ &= \frac{1}{2}(|000\rangle + |011\rangle + |101\rangle + |110\rangle).\end{aligned}$$

edx Problem 10

(a)

$$\frac{1}{2^{n-1}}.$$

(b) We see a uniformly random string $y \in \{0,1\}^n$.

edx Problem 11

$$\frac{1}{\sqrt{2^n}}\sum_{x\in\{0,1\}^n}(-1)^{x\cdot f(x)}|x\rangle|f(x)\rangle.$$