Kvanttilaskenta, kevät 2015 – Viikko 6

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February 17, 2015

edx Problem 1

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle = \begin{pmatrix}\alpha\\ \beta\end{pmatrix}.$$

$$|-\rangle = \begin{pmatrix}\frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{2}}\end{pmatrix}.$$

$$CNOT = \begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{pmatrix}.$$

Now

$$\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{pmatrix}\begin{pmatrix}\alpha\\ \beta\\ \frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{2}}\end{pmatrix} = \begin{pmatrix}\alpha\\ \beta\\ -\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{pmatrix}.$$

(a) The resulting state is not entangled. (b) The probability in question is 1. (c) The state of the first qubit does not change.

edx Problem 2

Is the "phase inversion" unitary its own inverse? Answer: Yes.

edx Problem 3

Is the "inversion about the mean" unitary its own inverse? Answer: Yes.

edx Problem 4

edx Problem 5

edx Problem 6

Now, consider the case where $\frac{N}{4}$ elements are marked instead of just one. If we run one iteration of Grover's algorithm and measure, what is the probability that we see a marked element?

It appears that the probability in question is $\binom{N}{4}^{-1}$.

edx Problem 7

Which of the following observables correspond to a standard basis measurements Answer: I.

edx Problem 8

Which of the following observables correspond to a sign basis measurement? Answer: H.

edx Problem 9

Suppose we measure a qubit $\alpha |0\rangle + \beta |1\rangle$ with respect to the observable $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. What is the outcome of the measurement? Answer: (1) It is 0 with probability $|\alpha|^2$ with the new state $|0\rangle$, and 1 with probability $|\beta|^2$ with the new state $|1\rangle$.

edx Problem 10

edx Problem 11

edx Problem 12

The Hamiltonian is

$$\begin{pmatrix} 1 & 4 \\ 4 & 1 \end{pmatrix}.$$

The states of definite energy are

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \end{pmatrix},$$

and the corresponding energies are 5 and -3.

edx Problem 13