Kvanttilaskenta, kevät 2015 – Viikko 4

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edx Problem 1

False. By definition of reversibility we should have x at the output of R_f.

edx Problem 2

False. The quantum circuit should modify each α_x.

edx Problem 3

True. Straight from the slides.

edx Problem 4

$$
\begin{aligned}
H^{\otimes 2}\frac{1}{\sqrt{2}}|00\rangle &= \frac{1}{\sqrt{2}}|{+}{+}\rangle \\
&= \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right)\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right) \\
&= \frac{1}{2\sqrt{2}}\Bigg(|00\rangle + |01\rangle + |10\rangle + |11\rangle\Bigg).
\end{aligned}
$$

Also

$$
\begin{aligned}
H^{\otimes 2}\frac{1}{\sqrt{2}}|11\rangle &= \frac{1}{\sqrt{2}}|{-}{-}\rangle \\
&= \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right)\left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right) \\
&= \frac{1}{2\sqrt{2}}\Bigg(|00\rangle - |01\rangle - |10\rangle + |11\rangle\Bigg),
\end{aligned}
$$

so

$$\begin{aligned}
H^{\otimes 2}\psi &= H^{\otimes 2}\left(\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle\right) \\
&= \frac{1}{2\sqrt{2}}\left(2|00\rangle + 2|11\rangle\right) \\
&= \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right) \\
&= \psi.
\end{aligned}$$

edx Problem 5

Let

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle).$$

$$\begin{aligned}
H^{\otimes 2}\frac{1}{\sqrt{2}}|01\rangle &= \frac{1}{\sqrt{2}}|+-\rangle \\
&= \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right)\left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right) \\
&= \frac{1}{2\sqrt{2}}\left(|00\rangle - |01\rangle + |10\rangle - |11\rangle\right).
\end{aligned}$$

$$\begin{aligned}
H^{\otimes 2}\frac{1}{\sqrt{2}}|10\rangle &= \frac{1}{\sqrt{2}}|-+\rangle \\
&= \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right)\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right) \\
&= \frac{1}{2\sqrt{2}}\left(|00\rangle + |01\rangle - |10\rangle - |11\rangle\right).
\end{aligned}$$

Now we see that $H^{\otimes 2}|\psi\rangle$ is

$$\begin{aligned}
\frac{1}{2\sqrt{2}}\left(|00\rangle - |01\rangle + |10\rangle - |11\rangle\right) + \frac{1}{2\sqrt{2}}\left(|00\rangle + |01\rangle - |10\rangle - |11\rangle\right) &= \frac{1}{2\sqrt{2}}(2|00\rangle - 2|11\rangle) \\
&= \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle).
\end{aligned}$$

edx Problem 6

Yes. Both may yield $|00\rangle$ with probability $\frac{1}{2}$ and $|11\rangle$ with probability $\frac{1}{2}$.