Kvanttilaskenta, kevät 2015 – Viikko 2

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edx Problem 1

No, its 0 for $|+\rangle$ and $|-\rangle$ are orthogonal to each other.

edx Problem 2

Yes.

edx Problem 3

Yes, but the implicit multiplication operator is in fact the tensor product $\otimes$.

edx Problem 4

False.

edx Problem 5

False.

edx Problem 6

True.

edx Problem 7

$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$. $|\psi\rangle = \frac{3}{5}|0\rangle - \frac{4}{5}|1\rangle$. Now we wish to compute

$$\begin{aligned}\langle\psi, +\rangle &= \frac{3}{5}\frac{1}{\sqrt{2}} - \frac{4}{5}\frac{1}{\sqrt{2}} \\ &= -\frac{1}{\sqrt{2}}\frac{1}{5} \\ &= -\frac{1}{5\sqrt{2}}.\end{aligned}$$

edx Problem 8

$$-\frac{1}{5\sqrt{2}}|+\rangle + \frac{7}{5\sqrt{2}}|-\rangle .$$

edx Problem 9

Forget the first standard basis measurement as it is not relevant here. At second measurement the chance of getting $|u\rangle$ is $\cos^2\theta$, where θ is the angle between $|u\rangle$ and $|\phi\rangle$. Since

$$\cos\theta = \langle\phi|u\rangle = ab + \sqrt{1-a^2}\sqrt{1-b^2},$$

the desired probability is

$$(ab + \sqrt{1-a^2}\sqrt{1-b^2})^2.$$

edx Problem 10

The third topmost alternative seems suspicious as it mixes sign and bit bases.

edx Problem 11

$$\left(\frac{\sqrt{2}}{\sqrt{3}}|0\rangle + \frac{1}{\sqrt{3}}|1\rangle\right) \otimes \left(\frac{1}{\sqrt{3}}|0\rangle + \frac{\sqrt{2}}{\sqrt{3}}|1\rangle\right) = \frac{\sqrt{2}}{3}|00\rangle + \frac{2}{3}|01\rangle + \frac{1}{3}|10\rangle + \frac{\sqrt{2}}{3}|11\rangle$$

edx Problem 12

$$\begin{aligned}(a|0\rangle + b|1\rangle)(c|0\rangle + d|1\rangle) &= ac|00\rangle + ad|01\rangle + bc|10\rangle + bd|11\rangle \\ &= \frac{1}{2\sqrt{2}}|00\rangle - \frac{1}{2\sqrt{2}}|01\rangle + \frac{\sqrt{3}}{2\sqrt{2}}|10\rangle - \frac{\sqrt{3}}{2\sqrt{2}}|11\rangle\end{aligned}$$

So we have that

$$
\begin{aligned}
ac &= \frac{1}{2\sqrt{2}}, \\
ad &= -\frac{1}{2\sqrt{2}}, \\
bc &= \frac{\sqrt{3}}{2\sqrt{2}}, \\
bd &= -\frac{\sqrt{3}}{2\sqrt{2}}.
\end{aligned}
$$

The way to factorize is to assign $a = \frac{1}{2}$, $b = \frac{\sqrt{3}}{2}$, $c = \frac{1}{\sqrt{2}}$, $d = -\frac{1}{\sqrt{2}}$, so $|a| = \frac{1}{2}$.