Kvanttilaskenta, kevät 2015 – Viikko 2

Rodion "rodde" Efremov

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edx Problem 1

Let

$$U = \begin{pmatrix} -2i & 5i \\ 5 & 1-i \end{pmatrix}.$$

Now

$$U^\dagger = \begin{pmatrix} 2i & 5 \\ -5i & 1+i \end{pmatrix}.$$

edx Problem 2

All unitary matrices are self-inverse.

edx Problem 3

We are given a U that maps $|0\rangle$ to $\frac{1}{\sqrt{2}}|0\rangle + \frac{1+i}{2}|1\rangle$ and $|1\rangle$ to $\frac{1-i}{2}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$. So we have that

$$\begin{aligned} U|0\rangle &= \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ &= \begin{pmatrix} a \\ c \end{pmatrix} \\ &= \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1+i}{2} \end{pmatrix}. \end{aligned}$$

Also, we have that

$$\begin{aligned} U|1\rangle &= \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ &= \begin{pmatrix} b \\ d \end{pmatrix} \\ &= \begin{pmatrix} 1-i \\ -\frac{1}{\sqrt{2}} \end{pmatrix}. \end{aligned}$$

It follows immediately that

$$U = \begin{pmatrix} \frac{1}{\sqrt{2}} & 1-i \\ \frac{1+i}{2} & -\frac{1}{\sqrt{2}} \end{pmatrix}.$$

edx Problem 4

Suppose we have a one-qubit unitary U that maps $|0\rangle$ to $\frac{-3}{5}|0\rangle + \frac{4i}{5}|1\rangle$ and $|+\rangle$ to $\frac{-3-4i}{5\sqrt{2}}|0\rangle + \frac{3+4i}{5\sqrt{2}}|1\rangle$.

We have that

$$\begin{aligned} U|0\rangle &= \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ &= \begin{pmatrix} a \\ c \end{pmatrix} \\ &= \begin{pmatrix} \frac{-3}{5} \\ \frac{4i}{5} \end{pmatrix}. \end{aligned}$$

As $|+\rangle = (\frac{1}{\sqrt{2}} \quad \frac{1}{\sqrt{2}})^T$, we have that

$$\begin{aligned} U|+\rangle &= \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \\ &= \begin{pmatrix} \frac{1}{\sqrt{2}}(a+b) \\ \frac{1}{\sqrt{2}}(c+d) \end{pmatrix} \\ &= \begin{pmatrix} \frac{1}{\sqrt{2}}(-\frac{3}{5}+b) \\ \frac{1}{\sqrt{2}}(\frac{4i}{5}+d) \end{pmatrix} \\ &= \begin{pmatrix} \frac{-3-4i}{5\sqrt{2}} \\ \frac{3+4i}{5\sqrt{2}} \end{pmatrix}. \end{aligned}$$

Now for b we have an equality

$$-\frac{3}{5} + b = \frac{-3-4i}{5},$$

which has solution $b = -\frac{4i}{5}$. For d we have an equality

$$\frac{4i}{5} + d = \frac{3+4i}{5},$$

which has solution $d = \frac{3}{5}$.

So the matrix in question is

$$U = \begin{pmatrix} -\frac{3}{5} & -\frac{4i}{5} \\ \frac{4i}{5} & \frac{3}{5} \end{pmatrix}.$$