Kvanttilaskenta, kevät 2015 – Viikko 2

Rodion "rodde" Efremov

January 21, 2015

edx Problem 1

No, its 0 for $|+\rangle$ and $|-\rangle$ are orthogonal to each other.

edx Problem 2

Yes.

edx Problem 3

Yes, but the implicit multiplication operator is in fact the tensor product $\otimes$.

edx Problem 4

False.

edx Problem 5

False.

edx Problem 6

True.

edx Problem 7

$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$. $|\psi\rangle = \frac{3}{5}|0\rangle - \frac{4}{5}|1\rangle$. Now we wish to compute

$$\begin{aligned}\langle \psi, +\rangle &= \frac{3}{5}\frac{1}{\sqrt{2}} - \frac{4}{5}\frac{1}{\sqrt{2}} \\ &= -\frac{1}{\sqrt{2}}\frac{1}{5} \\ &= -\frac{1}{5\sqrt{2}}.\end{aligned}$$

edx Problem 8

$$-\frac{1}{5\sqrt{2}}|+\rangle + \frac{7}{5\sqrt{2}}|-\rangle .$$

edx Problem 9

Forget the first standard basis measurement as it is not relevant here. At second measurement the chance of getting $|u\rangle$ is $\cos^2\theta$, where θ is the angle between $|u\rangle$ and $|\phi\rangle$. Since

$$\cos\theta = \langle\phi|u\rangle = ab + \sqrt{1-a^2}\sqrt{1-b^2},$$

the desired probability is

$$(ab + \sqrt{1-a^2}\sqrt{1-b^2})^2.$$

edx Problem 10

The third topmost alternative seems suspicious as it mixes sign and bit bases.

edx Problem 11

$$\left(\frac{\sqrt{2}}{\sqrt{3}}|0\rangle + \frac{1}{\sqrt{3}}|1\rangle\right) \otimes \left(\frac{1}{\sqrt{3}}|0\rangle + \frac{\sqrt{2}}{\sqrt{3}}|1\rangle\right) = \frac{\sqrt{2}}{3}|00\rangle + \frac{2}{3}|01\rangle + \frac{1}{3}|10\rangle + \frac{\sqrt{2}}{3}|11\rangle$$

edx Problem 12

$$\begin{aligned}(a|0\rangle + b|1\rangle)(c|0\rangle + d|1\rangle) &= ac|00\rangle + ad|01\rangle + bc|10\rangle + bd|11\rangle \\ &= \frac{1}{2\sqrt{2}}|00\rangle - \frac{1}{2\sqrt{2}}|01\rangle + \frac{\sqrt{3}}{2\sqrt{2}}|10\rangle - \frac{\sqrt{3}}{2\sqrt{2}}|11\rangle\end{aligned}$$

So we have that

$$\begin{aligned}
ac &= \frac{1}{2\sqrt{2}}, \\
ad &= -\frac{1}{2\sqrt{2}}, \\
bc &= \frac{\sqrt{3}}{2\sqrt{2}}, \\
bd &= -\frac{\sqrt{3}}{2\sqrt{2}}.
\end{aligned}$$

The way to factorize is to assign $a = \frac{1}{2}$, $b = \frac{\sqrt{3}}{2}$, $c = \frac{1}{\sqrt{2}}$, $d = -\frac{1}{\sqrt{2}}$, so $|a| = \frac{1}{2}$.

edx Problem 13

(a)

The probability is $\frac{1}{2}$.

(b)

$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$.

edx Problem 14

$$\left(\frac{4}{5}\right)^2 + \left(-\frac{2}{5}\right)^2 = \frac{16}{25} + \frac{4}{25} = \frac{20}{25}.$$

edx Problem 15

$|0+\rangle, |0-\rangle, |1+\rangle, |1-\rangle$.

edx Problem 16

There si no way to entangle two qubits by a partial measurement.

edx Problem 17

As

$$|0\rangle = \frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle, |1\rangle = \frac{1}{\sqrt{2}}|+\rangle - \frac{1}{\sqrt{2}}|-\rangle,$$

we have that

$$\begin{aligned}
|\psi\rangle &= \frac{1}{\sqrt{2}}|0\rangle + \frac{e^{i\theta}}{\sqrt{2}}|1\rangle \\
&= \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle\right) + \frac{e^{i\theta}}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}|+\rangle - \frac{1}{\sqrt{2}}|-\rangle\right) \\
&= \frac{1}{2}|+\rangle + \frac{1}{2}|-\rangle + \frac{e^{i\theta}}{2}|+\rangle - \frac{e^{i\theta}}{2}|-\rangle \\
&= \frac{1+e^{i\theta}}{2}|+\rangle + \frac{1-e^{i\theta}}{2}|-\rangle .
\end{aligned}$$

edx Problem 18

$$\frac{1+\cos\theta}{2}.$$

QCE 3.1

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$. $X = |0\rangle\langle 1| + |1\rangle\langle 0|$. $Y = -i|0\rangle\langle 1| + i|1\rangle\langle 0|$. Now

$$\begin{aligned}
X|\psi\rangle &= (|0\rangle\langle 1| + |1\rangle\langle 0|)(\alpha|0\rangle + \beta|1\rangle) \\
&= \alpha(|0\rangle\langle 1| + |1\rangle\langle 0|)|0\rangle + \beta(|0\rangle\langle 1| + |1\rangle\langle 0|)|1\rangle \\
&= \alpha(|0\rangle\langle 1|0\rangle + |1\rangle\langle 0|0\rangle) + \beta(|0\rangle\langle 1|1\rangle + |1\rangle\langle 0|1\rangle) \\
&= \alpha|0\rangle \times 0 + \alpha|1\rangle \times 1 + \beta|0\rangle \times 1 + \beta|1\rangle \times 0 \\
&= \alpha|1\rangle + \beta|0\rangle .
\end{aligned}$$

$$\begin{aligned}
Y|\psi\rangle &= (-i|0\rangle\langle 1| + i|1\rangle\langle 0|)(\alpha|0\rangle + \beta|1\rangle) \\
&= \alpha(-i|0\rangle\langle 1| + i|1\rangle\langle 0|)|0\rangle + \beta(-i|0\rangle\langle 1| + i|1\rangle\langle 0|)|1\rangle \\
&= \alpha(-i|0\rangle\langle 1|0\rangle + i|1\rangle\langle 0|0\rangle) + \beta(-i|0\rangle\langle 1|1\rangle + i|1\rangle\langle 0|1\rangle) \\
&= \alpha i|1\rangle + \beta(-i)|0\rangle \\
&= \alpha i|1\rangle - \beta i|0\rangle .
\end{aligned}$$

QCE 3.2

Suppose we have a qubit

$$\begin{pmatrix}\alpha \\ \beta\end{pmatrix}.$$

Now

$$\begin{pmatrix}0 & 1 \\ 1 & 0\end{pmatrix}\begin{pmatrix}\alpha \\ \beta\end{pmatrix} = \begin{pmatrix}0\times\alpha + 1\times\beta \\ 1\times\alpha + 0\times\beta\end{pmatrix} = \begin{pmatrix}\beta \\ \alpha\end{pmatrix}.$$

QCE 3.3

$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}, |-\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}.$$

Now

$$\begin{aligned}
X|+\rangle &= X\frac{|0\rangle + |1\rangle}{\sqrt{2}} \\
&= X\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \\
&= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \\
&= \begin{pmatrix} 1 \times \frac{1}{\sqrt{2}} + 0 \times \frac{1}{\sqrt{2}} \\ 0 \times \frac{1}{\sqrt{2}} - 1\frac{1}{\sqrt{2}} \end{pmatrix} \\
&= \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} \\
&= \frac{|0\rangle - |1\rangle}{\sqrt{2}} \\
&= |-\rangle.
\end{aligned}$$

Also

$$\begin{aligned}
X|-\rangle &= X\frac{|0\rangle - |1\rangle}{\sqrt{2}} \\
&= X\begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} \\
&= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} \\
&= \begin{pmatrix} 1 \times \frac{1}{\sqrt{2}} + 0 \times -\frac{1}{\sqrt{2}} \\ 0 \times \frac{1}{\sqrt{2}} - 1\left(-\frac{1}{\sqrt{2}}\right) \end{pmatrix} \\
&= \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \\
&= \frac{|0\rangle + |1\rangle}{\sqrt{2}} \\
&= |+\rangle.
\end{aligned}$$