Computing debt cuts leading to global zero-equity - example

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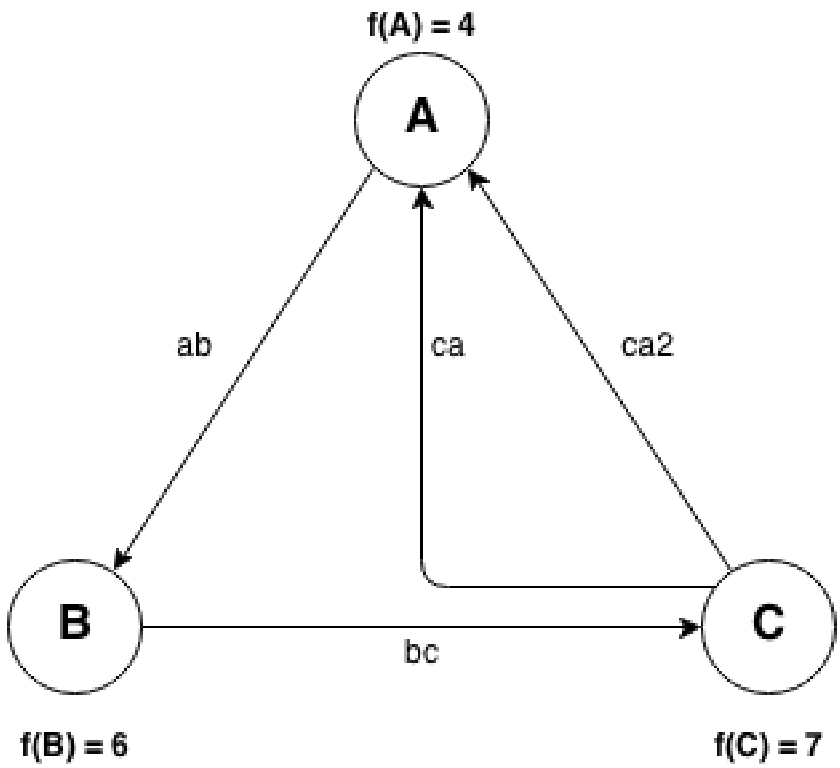

ab $(3, 0.14, \infty, 1)$,

bc $(2, 0.11, 4, 1.4)$,

ca $(2.5, 0.05, 6, 0.5)$,

ca2 $(1, 0.27, \infty, 1.7)$.

Equilibrium equation for node A:

$$\begin{aligned}
\Xi(ab)e^{0.14(11.7-6)} - \Xi(ca)\Big(1+\frac{0.05}{6}\Big)^{\lfloor 6(11.7-4)\rfloor} - \Xi(ca2)e^{0.27(11.7-4)} &= \\
\underline{2.221\,\Xi(ab) - 1.464\,\Xi(ca) - 7.996\,\Xi(ca2)} &= \\
\mathfrak{C}_{T_G}(3e^{0.14(6-1)}, 0.14, \infty, 6) - {} & \\
\mathfrak{C}_{T_G}(2.5\Big(1+\frac{0.05}{6}\Big)^{\lfloor 6(4-0.5)\rfloor}, 0.05, 6, 4) - {} & \\
\mathfrak{C}_{T_G}(e^{0.27(4-1.7)}, 0.27, \infty, 4) &= \\
\mathfrak{C}_{T_G}(6.041, 0.14, \infty, 6) - \mathfrak{C}_{T_G}(2.975, 0.05, 6, 4) - \mathfrak{C}_{T_G}(1.861, 0.27, \infty, 4) &= \\
13.417 - 4.358 - 14.881 = \underline{-5.822}. &
\end{aligned}$$

Equilibrium equation for node B:

$$\begin{aligned}
\Xi(bc)\Big(1+\frac{0.11}{4}\Big)^{\lfloor 4(11.7-7)\rfloor} - \Xi(ab)e^{0.14(11.7-6)} &= \\
\underline{1.629\,\Xi(bc) - 2.221\,\Xi(ab)} &= \\
\mathfrak{C}_{T_G}(2\Big(1+\frac{0.11}{4}\Big)^{\lfloor 4(7-1.4)\rfloor}, 0.11, 4, 7) - \mathfrak{C}_{T_G}(3e^{0.14(6-1)}, 0.14, \infty, 6) &= \\
\mathfrak{C}_{T_G}(3.632, 0.11, 4, 7) - \mathfrak{C}_{T_G}(6.041, 0.14, \infty, 6) &= \\
3.632\Big(1+\frac{0.11}{4}\Big)^{\lfloor 4(11.7-7)\rfloor} - 6.041e^{0.14(11.7-6)} &= \\
5.918 - 13.417 &= \\
\underline{-7.499} &
\end{aligned}$$

Equilibrium equation for node C:

$$\begin{aligned}
\Xi(ca)\Big(1+\frac{0.05}{6}\Big)^{\lfloor 6(11.7-4)\rfloor} + \Xi(ca2)e^{0.27(11.7-4)} - \Xi(bc)\Big(1+\frac{0.11}{4}\Big)^{\lfloor 4(11.7-7)\rfloor} &= \\
\underline{1.464\,\Xi(ca) + 7.996\,\Xi(ca2) - 1.629\,\Xi(bc)} &= \\
\mathfrak{C}_{T_G}(2.5\Big(1+\frac{0.05}{6}\Big)^{\lfloor 6(4-0.5)\rfloor}, 0.05, 6, 4) + \mathfrak{C}_{T_G}(e^{0.27(4-1.7)}, 0.27, \infty, 4) - {} & \\
\mathfrak{C}_{T_G}(2\Big(1+\frac{0.11}{4}\Big)^{4(7-1.4)}, 0.11, 4, 7) &= \\
\mathfrak{C}_{T_G}(2.976, 0.05, 6, 4) + \mathfrak{C}_{T_G}(1.861, 0.27, \infty, 4) - \mathfrak{C}_{T_G}(3.633, 0.11, 4, 7) &= \\
4.359 + 14.881 - 5.920 &= \\
\underline{13.32} &
\end{aligned}$$

Next, the matrix:

$$\begin{bmatrix} 2.221 & 0 & -1.464 & -7.996 & -5.822 \\ -2.221 & 1.629 & 0 & 0 & -7.499 \\ 0 & -1.629 & 1.464 & 7.996 & 13.32 \end{bmatrix}.$$

Add the first row to the second one:

$$\begin{bmatrix} 2.221 & 0 & -1.464 & -7.996 & -5.822 \\ 0 & 1.629 & -1.464 & -7.996 & -13.32 \\ 0 & -1.629 & 1.464 & 7.996 & 13.32 \end{bmatrix}.$$

Now add the second row to the third:

$$\begin{bmatrix} 2.221 & 0 & -1.464 & -7.996 & -5.822 \\ 0 & 1.629 & -1.464 & -7.996 & -13.32 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Divide the 1st row by 2.221:

$$\begin{bmatrix} 1 & 0 & -0.659 & -3.600 & -2.621 \\ 0 & 1.629 & -1.464 & -7.996 & -13.32 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Divide the 2nd row by 1.629:

$$\begin{bmatrix} 1 & 0 & -0.659 & -3.600 & -2.621 \\ 0 & 1 & -0.899 & -4.908 & -8.177 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Now, $\Xi(ca), \Xi(ca2)$ are the independent variables, and

$$\begin{aligned} \Xi(ab) &= -2.621 + 0.659\,\Xi(ca) + 3.600\,\Xi(ca2) \\ \Xi(bc) &= -8.177 + 0.899\,\Xi(ca) + 4.908\,\Xi(ca2). \end{aligned}$$

Also,

$$\begin{aligned} &0 \le \Xi(ca) \le 2.5\Big(1 + \frac{0.05}{6}\Big)^{\lfloor 6(4-0.5) \rfloor} && = 2.976 \\ &0 \le \Xi(ca2) \le e^{0.27(4-1.7)} && = 1.861 \\ &0 \le \Xi(ab) \le 3e^{0.14(6-1)} && = 6.041 \\ &0 \le \Xi(bc) \le 2\Big(1 + \frac{0.11}{4}\Big)^{\lfloor 4(7-1.4) \rfloor} && = 3.535 \end{aligned}$$

The last two inequalities are equivalent to

$$\begin{aligned} &0 \le -2.621 + 0.659\Xi(ca) + 3.600\Xi(ca2) && \le 6.041 \\ &0 \le -8.177 + 0.899\Xi(ca) + 4.908\Xi(ca2) && \le 3.535 \end{aligned}$$

Now, we obtain:

$$\begin{aligned} 2.621 \le 0.659\Xi(ca) + 3.600\Xi(ca2) \le 8.662 \\ 8.177 \le 0.899\Xi(ca) + 4.908\Xi(ca2) \le 11.712 \end{aligned}$$

The objective function is

$$\Xi(ab) + \Xi(bc) + \Xi(ca) + \Xi(ca2) = 2.558\,\Xi(ca) + 9.508\,\Xi(ca2).$$

The optimal solution is $\Xi(ca) = 0$, $\Xi(ca2) = 1.665$ so the cuts are:

$$\begin{aligned}
\Xi(ab) &= 3.373 \\
\Xi(bc) &= 0 \\
\Xi(ca) &= 0 \\
\Xi(ca2) &= 1.665
\end{aligned}$$