

## A simplified carrier frequency offset estimation algorithm for burst digital transmission

Xin Man <sup>a</sup>, Haitao Zhai <sup>b</sup> and Eryang Zhang <sup>c</sup>

School of Electronic Science and Engineering,  
 National University of Defense Technology, Changsha 410073, P. R. China

<sup>a</sup>manxin09@163.com, <sup>b</sup>zht25@163.com, <sup>c</sup>ey\_zhang@163.com

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**Abstract.** In this letter, we present a carrier frequency offset estimation method for burst digital transmission, by introducing a step factor into an earlier estimation algorithm to reduce the computational complexity, with little loss of the estimation accuracy. Performance simulations show that this method can separate the estimation range and accuracy, avoiding the weakness of reaching large estimation range at the expense of estimation accuracy.

### Introduction

Synchronization is crucial for digital transmission systems to estimate a number of parameters such as the carrier frequency offset, phase offset, timing error, etc. Most of the synchronizers work in data-aided (DA) or non-data-aided (NDA) modes [1, 2, 3, 4]. For burst transmission, DA synchronizing algorithms are usually employed to get good performance with short preambles.

Concerning the problem of carrier frequency offset estimation for burst digital transmission, the Fitz algorithm [5] works effectively in DA mode, and however, it possesses the drawback of increasing the estimation range at the cost of estimation accuracy. Although the Mengali&Morelli (M&M) algorithm [6] resolves this problem effectively, it is too complex to implement.

We propose a simplified method based on the M&M algorithm for frequency offset estimation, which has some advantages over the previous methods. Firstly, it introduces a step factor into the M&M algorithm to reduce the computational complexity with little loss of estimation accuracy and variable acquisition range. Secondly, it separates the estimation range and the estimation accuracy, and avoids the weakness of reaching high estimation accuracy at the expense of estimation range. In a word, the proposed method is more flexible and applicable than both the M&M and the Fitz algorithm.

### System model

Assuming negligible filter distortion, sampling at the symbol rate, and perfect timing recovery in the digital receiver, the complex samples at the output of the matched filter can be expressed as:

$$x(k) = c_k e^{j(2\pi k \Delta f T + \Delta \theta)} + n_k \quad (1)$$

where  $c_k$  is unit-amplitude symbol,  $n_k$  is the complex additional gaussian white noise which is independent in phase and quadrature component samples, and have identical variance  $N_0/2$ .  $\Delta f T$  represents the residual carrier frequency offset, normalized to the symbol rate (NFO) and  $\Delta \theta$  accounts for the phase offset between local oscillators within the transmitter and the receiver.  $\Delta f T$  and  $\Delta \theta$  are assumed to be deterministic and constant during a burst transmission.

Equation (1) shows that  $x(k)$  depends on  $c_k$ , which is exactly known by the receivers in DA mode. We can remove the dependence on  $c_k$  by multiplying  $x(k)$  by  $c_k^*$ , which gets

$$z(k) = e^{j(2\pi k \Delta f T + \Delta \theta)} + n_k c_k^* \quad (2)$$

where  $c_k^*$  is the conjugation of  $c_k$  and  $n_k c_k^*$  is statistically equivalent to  $n_k$ .

From the observation of a few consecutive samples  $\{z(k), 0 < k < L_0 - 1\}$ , the M&M algorithm is derived as the following equations.

$$\Delta \hat{f} = \frac{1}{2\pi T} \sum_{m=1}^N w(m) \times [T(m, 1)]_{2\pi} \quad (3)$$

$$w(m) = \frac{3[(L_0 - m)(L_0 - m + 1) - N(L_0 - N)]}{N(4N^2 - 6NL_0 + 3L_0^2 - 1)} \quad (4)$$

$$T(m, j) = \arg\{R(m)\} - \arg\{R(m - j)\} \quad (5)$$

$$R(m) = \frac{1}{L_0 - m} \sum_{k=m}^{L_0-1} z(k) z^*(k - m), \quad 1 \leq m \leq N \quad (6)$$

where  $[\cdot]_{2\pi}$  denotes the modulo  $2\pi$  operation,  $L_0$  means the number of sequence symbols and  $N$  should normally satisfy that  $N < L_0$ .

### The simplified carrier frequency offset estimation method

The original M&M algorithm is too complex to implement. To reduce the computational complexity, we introduce a step factor into (3) and get the simplified algorithm.

$$\Delta \hat{f} = \frac{1}{2\pi T} \sum_{m \in M} w(m) \times [T(m, d)]_{2\pi} \quad (7)$$

where  $M = \{1, 1+d, 1+2d, \dots, N\}$  and  $d$  is the step factor.

It is easy to see that equation (7) is a generalization version of (3) and we name it the S-M&M algorithm in that it is simpler than the original one. We will see in Fig. 1 that the complexity of this new method falls at the cost of its estimation range.

Fig. 1 illustrates the estimation mean,  $E[\Delta \hat{f}T]$  versus  $\Delta fT$ . The ideal line is indicated as a reference. We assume the modulation scheme is QPSK, the estimation interval is of  $L_0 = 256$  symbols and we set the parameter  $N$  to be 128.

It is apparent that the estimation range of the S-M&M algorithm is approximately  $\Delta fT \in (-1/2d, 1/2d)$  and it is controlled by the step factor  $d$ .

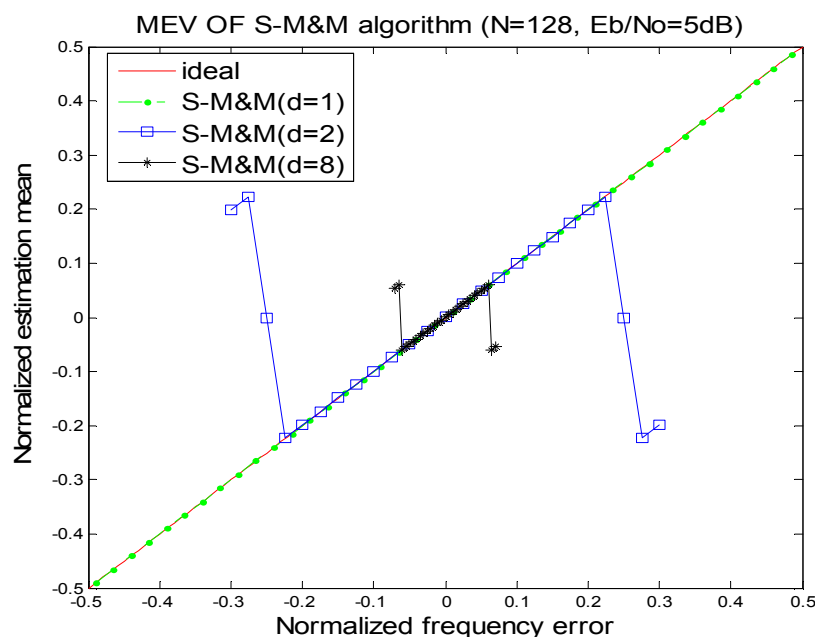


Fig. 1. The estimation mean of the S-M&M algorithm

In next section, we will discuss how the performance of this new algorithm compares with the M&M and the Fitz algorithm. Before proceeding we will first introduce the Fitz algorithm, which reads

$$\Delta \hat{f} = \frac{1}{\pi T N (N+1)} \sum_{m=1}^N \arg \{R(m)\} \quad (8)$$

With (8), the estimation range of NFO  $\Delta f T$  is less than  $1/2N$ .

### Computer simulations

We make some computer simulations to compare the performance of the S-M&M algorithm with other algorithms. We still assume the modulation scheme is QPSK and  $L_0 = 256$ .

Fig. 2 compares normalized variance performance for different algorithms. We assume  $N$  equals 128 and the normalized frequency offset  $|\Delta f T| < 1/2N$ . The MCRB is also shown as a baseline.

From the figure we can easily get the following two points. 1) The estimation accuracy of the S-M&M algorithm appears to have little loss as the step factor  $d$  increases. 2) For the same  $L_0$  and  $N$ , the Fitz algorithm excels a little in the estimation accuracy over the S-M&M algorithm; however, the estimation range of the S-M&M algorithm is about  $N/d$  times of that of the Fitz algorithm.

Fig. 3 gives out the variance performance comparison of the Fitz and the S-M&M algorithm when they have the same estimation range. We set  $N = 2$  for the Fitz algorithm and  $N = 128$ ,  $d = 2$  for the S-M&M algorithm. The NFO is assumed to be less than 0.25. From the figure we could easily know that the estimation accuracy of the S-M&M algorithm is much better than that of the Fitz algorithm for a given estimation range.

The analysis is as follows. The Fitz algorithm controls the estimation range and accuracy by the same parameter  $N$ , and the estimation range may be increased at the expense of worse accuracy, or the estimation accuracy can be improved at the cost of reduced range. However, it is not the case for the S-M&M algorithm, which controls the estimation accuracy and range by parameter  $N$  and  $d$  separately. In a word, the S-M&M algorithm is more flexible than the Fitz algorithm and usually has higher estimation accuracy for the cases of large frequency offset.

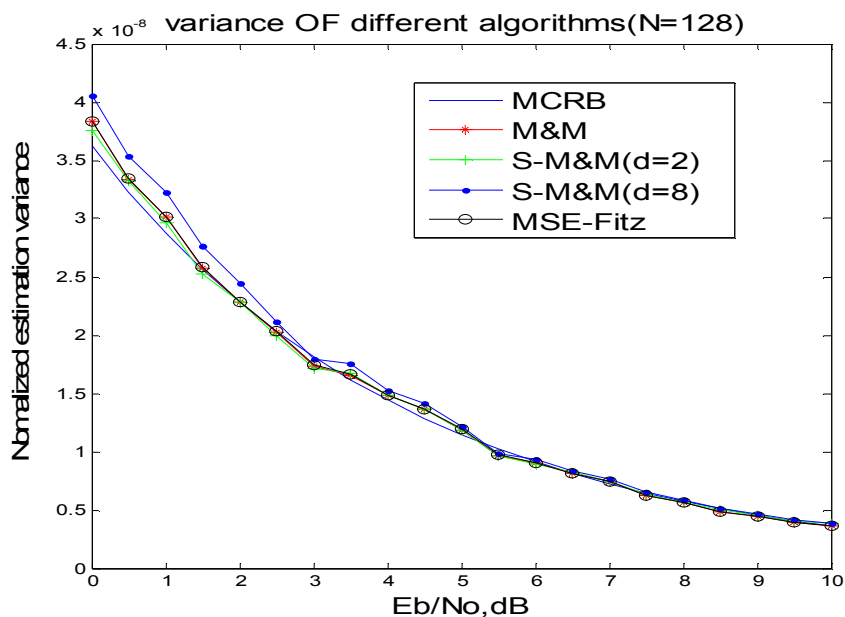


Fig. 2. The estimation variance of different algorithms

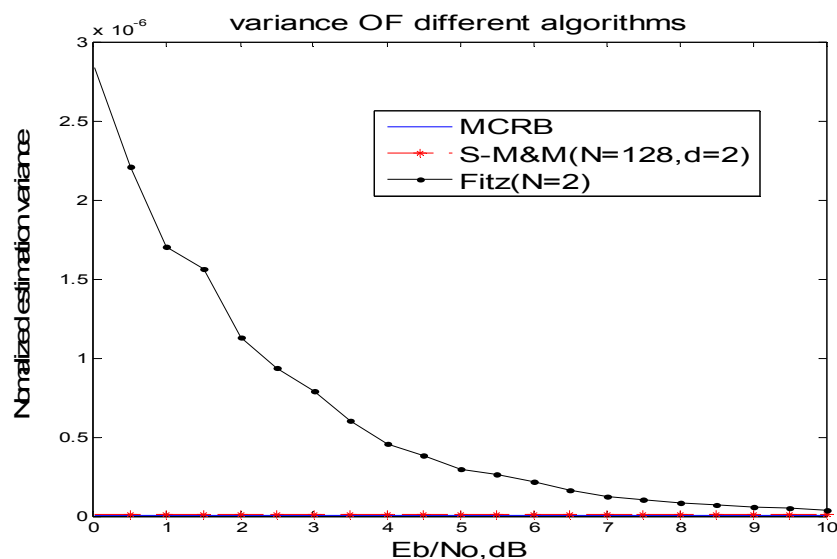


Fig. 3. The variance comparison with the Fitz algorithm

## Conclusions

We have proposed a simplified carrier frequency estimation algorithm for burst transmission, which we named the S-M&M algorithm. When the Fitz algorithm encounters the contradiction of estimation range and accuracy, this new method resolves it as the M&M algorithm does. So taking the estimation range and estimation accuracy into account synthetically, we can draw a conclusion that the S-M&M method outperforms the Fitz algorithm. As a generalization of the M&M algorithm, the new method is more flexible and we have the choice of reducing the computational complexity according to a given frequency offset range, with little degradation of the estimation accuracy compared to the M&M algorithm.

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