On the Carrier Frequency Offset Estimation for Frequency Hopping Burst Mode Mobile Radio

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***Abstract*—In this paper, we evaluate the Cramer-Rao bounds (CRB) for the estimation of the carrier frequency offset (CFO) for general QAM modulations with no knowledge of the transmitted sequence (blind operation) in frequency hopping (FH) based short burst mode mobile radios. We investigate the blind CFO estimation problem in FH based mobile system that uses multiple short narrowband bursts in different frequency channels for the transmission of one packet. Joint blind channel and CFO estimation over multiple frequency bursts is considered. In the first part of the paper, we determine the limiting performance by finding the CRB for blind estimation, and compare it to the CRB for known data (genie aided) case. The CRB relates the reliability of CFO estimation to the number of radio frequency bursts (RFB) used, constellation and SNR level, and is an important measure for system design without the need to simulate specific algorithms. It is shown that the linear combination (in MMSE sense) of multiple frequency estimates, each obtained from a single RFB independently, achieves the joint CRB over multiple bursts. In the second part of the paper, some practical blind estimation methods are proposed. They are shown to exhibit close performance to the true CRB.**

I. Introduction

Frequency Hopping (FH) is utilized in many military communication systems in order to provide diversity against fading and reduce the effects of counter measures, i.e., jamming. It is beneficial to spread a message over multiple radio frequency bursts (RFB) to increase the robustness of the system against jamming and fading, but at the same time it is desirable that the length of each RFB is short. However, shorts bursts at each FH aggravate the difficulty of estimating the channel gains and other parameters such as carrier frequency offset (CFO). There is no time for training in such a relatively fast FH system and, the parameters often have to be estimated with no knowledge of the transmitted sequence (blind operation). In this study, we focus on parameter estimation problem in FH based mobile systems that uses multiple short narrowband bursts in different frequency channels for the transmission of one packet.

In many practical systems, carrier frequencies are generated from a master clock. The carrier frequency offset stems from the offset between the transmitter's (Tx) and receiver's (Rx) local oscillators. In a mobile radio environment, Doppler offsets can also be considered. When there is an offset between Tx and Rx clocks and doppler shifts , a CFO, which depends on the hop's carrier frequency, (generated generally from a single frequency reference through frequency multiplication) is observed. Narrowband Rayleigh fading is assumed at each RFB and changes independently from block to block (block fading model [1]).

Knowing that oscillator offsets are slowly varying, an equivalent problem is joint estimation of the channel gains and the offset between the oscillators. This problem at hand is investigated for a FH system with no knowledge of the transmitted data. In order to analyze system performance and find appropriate equipment, it is important to understand the theoretical estimator performance limits. Therefore, analysis is first performed to see estimation performance limits based on the Cramer-Rao lower bound (CRB) for CFO estimation. The performances of the proposed practical estimators are later compared with the analytical bounds.

The CRBs for unmodulated signals (CW) have been evaluated in [2] for AWGN channels and frequently used as an absolute bound. The modified CRB by [3] and some other related works are good approximations for the true CRB at high SNR, but they are significantly loose at low SNR. The true CRB of blind CFO estimation for general QAM modulations is presented in [4] for AWGN channel. Joint channel and frequency offset estimation is studied in [5] for known training sequence case.

This paper is concerned with evaluating the CRB in the estimation of CFO for FH based short burst mobile system for general linear modulations without pilot symbols in Section III. Our work can be seen as a generalization of [4] to the block fading channels including the estimation of multiple complex channel gains of each RFB in addition to the CFO. The CRB found relates the reliability of CFO estimation to the number of radio frequency bursts (RFB) used, constellation and SNR level, and is an important measure for system design without the need to simulate specific algorithms. It is seen that the weighted linear combination (in MMSE sense) of multiple frequency estimates, each obtained from a single RFB independently, yields a new estimate which achieves the true CRB over multiple bursts. In Section IV, two different blind frequency estimators are proposed for FH systems as an adaptation of nonlinearity based approaches in [6], [7].

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II. System Model

We assume a linear modulation (PSK, QAM etc.) and a block fading channel model [8], [9] for the FH system. Under such conditions, the received samples after ideal filtering and sampling at optimum instant can be written as

$$y_k^i = h_i e^{j2\pi v_i k} x_k^i + \eta_k^i \tag{1}$$

$$v_i = f_i T_s \lambda \tag{2}$$

for $k = 0, 1, \cdots, N-1$ and $i = 1, \cdots, B$, where B is the number of RFBs used to transmit one message, and N is the number of symbols in each RFB. f_i is the instantaneous (hopping) frequency of i^{th} RFB. y_k^i, x_k^i, η_k^i are the k^{th} received, transmitted, and noise signals of the i^{th} block respectively. $x_k^i \in \chi$ is chosen from an M-ary alphabet χ: $|\chi| = M$. The average received power $E\{|x_k^i|^2\}$ corresponding to the transmitted signal is set to E_s. The noise term η_k^i is a complex circularly symmetric temporally white Gaussian random variable with zero mean and variance N_o. The fading coefficient h_i is constant for the duration of the block and, independent across blocks. There are a total of B blocks (RFBs) over all transmission. The fading is assumed to be Rayleigh in simulations so that channel power gain $|h_i|^2$ is exponentially distributed with unit mean.

$v_i = f_i T_s \lambda$ is the carrier-frequency offset normalized to $1/T_s$ at the i^{th} block. v_i's stem from the single offset λ between the transmitter's (Tx) and receiver's (Rx) oscillators, and doppler shifts due to the mobility. λ can be thought as a frequency offset per Hertz, then the unit for it is Hz/Hz or ppm (parts per million). Assuming oscillator and doppler offsets are slowly varying, an equivalent problem is the joint estimation of the channel and the offset due to the oscillators and mobility.

For practical applications, burst length N is kept small to limit the effects of counter measures, i.e., jamming, so there is not enough time for training to learn offset and channels. Also, for high values of hopping frequencies f_i's, $v_i = f_i T_s \lambda$ could be so high that even differential modulation techniques can not guarantee reliable communication. Therefore, the only solution is to measure the frequency offset λ accurately with no knowledge of the transmitted sequence x_k^i. Analysis is first performed to see blind estimation performance limits based on the Cramer-Rao bound (CRB).

We will use the convention that $p(a)$ is the probability density function (pdf) for a random variable a. Boldface upper-case letters denote matrices and lower-case letters denote vectors. Scalars are denoted by plain lower-case letters. The superscript $(\cdot)^*$ denotes the complex conjugate for scalars and $(\cdot)^H$ denotes the conjugate transpose for vectors and matrices. $E\{\cdot\}$ stands for the expected value operator. The $(i,j)^{th}$ element of a matrix $\mathbf{A}$ is denoted by $\mathbf{A}(i,j)$.

III. CRB for Blind Frequency Estimation

The frequency offset λ and complex channel gains h_i's are assumed to be non-random parameters to be estimated. We find a true CRB for the joint estimation of frequency offset and channel gains without any information on x_k^i's (blind operation). The CRB on the variances of an unbiased estimator of λ, namely $\hat{\lambda}$, for a sequence of B RFB with N symbols in (1) is evaluated in Appendix A in (23) as

$$CRB\{\hat{\lambda}\} = \sigma_{\lambda_e}^2 = \left(\sum_{i=1}^{B} \frac{1}{\tilde{\sigma}_{\lambda_e}^2(i)} \right)^{-1} \tag{3}$$

where

$$\tilde{\sigma}_{\lambda_e}^2(i) = \left(T_s^2 f_i^2 \left(\frac{N(N-1)}{12} \left[N(4 - 3r_i^2) + (3r_i^2 - 2) \right] \right) E_{\Psi_i}^2 \right)^{-1} \tag{4}$$

and

$$r_i = \sqrt{\frac{|E_{\Phi_i \Psi_i}|^2}{E_{\Phi_i}^2 E_{\Psi_i}^2}} \tag{5}$$

for $i = 1, \ldots, B$. For the evaluation of CRB in (3), the expectations $E_{\Phi_i}^2$, $E_{\Psi_i}^2$ and $E_{\Phi_i \Psi_i}$ are required. They are defined in Appendix A and can be computed by using Monte Carlo simulations or numerical integrations. These values depend on the constellation, channel gains and average SNR, but independent of the value of frequency offset. $\tilde{\sigma}_{\lambda_e}^2(i)$ can be thought as the CRB for λ estimate obtained from i^{th} RFB. The CRB in case of joint estimation is obtained by using $\tilde{\sigma}_{\lambda_e}^2(i)$s as in (3).

Also, it is shown that the true CRB for blind estimation converges to that of known data case as SNR increases in Appendix A. The known data case corresponds to the unmodulated signaling and can be thought as a genie aided operation. Simulation results also verify this, above some critical SNR value and number of RFBs, the CRBs of blind and known data case are almost same. The CRB found is an important measure for system design without the need to simulate specific methods, because it explains the reliability of frequency estimation in terms of the number of RFB, constellation, hopping frequencies and fading levels.

In Appendix B, it is shown that the weighted linear combination (in MMSE sense) of multiple frequency estimates, each obtained from a single RFB independently, produces a new estimate which attains the true CRB over multiple bursts. Thus, there is no need for joint estimation over multiple bursts, instead it is sufficient to get frequency estimates from each RFB independently and linearly combine them to obtain a more reliable estimate. As will be seen, the strongest channels with higher hopping frequency determine the CRB substantially.

IV. Practical Frequency Offset Estimators

There are many practical algorithms for frequency offset estimation in the literature. In [10] and [11], nondata-aided open-loop algorithms were proposed for frequency offset estimation in flat fading channels. Some practical implementations for phase synchronization are introduced in [12]. Performance analysis of a class of nondata-aided frequency offset estimators for flat fading channels is given in [13]. Cyclic stationarity has been exploited for synchronization purposes for linear modulations such as in [14], [15]. However, it is seen that the cyclostationarity based approaches yield unsatisfactory results with short bursts, since their performances significantly depend on the reliable estimation of the cyclic spectrum. However, it is not straightforward to extend the algorithms mentioned above to short burst FH systems.

In this section, some practical blind CFO estimation methods are proposed based on the nonlinear least-squares (NLS) estimation. The proposed methods can be seen as extensions of blind feedforward estimators in [7] and [16] to the case of FH systems. The NLS estimators in [7] and [16] represent a generalized form of frequency offset estimators which remove the effect of modulation by using some non-linearity (NL). This type of estimators was originally proposed by Viterbi and Viterbi (V&V) as a blind carrier phase estimators for M-PSK signaling [6], [17] and, the V&V algorithm is adapted to 16-QAM in [18]. In [7] and [16], NL used is optimized according to SNR level and constellation.

We adapt V&V type of estimators in [7], considering the effect of different channel gains and hopping frequency values at different RFBs, to our problem. Previous works consider only CFO estimation for single flat channel, but in case of FH, each RFB undergoes different fading states and the estimator has to take the strength of fading into account. We propose two different CFO estimators. The first one is a NLS type estimator using multiple blocks (RFB) and considering different instantaneous SNR levels of each RFB. We call this method *NLS estimator over multiple bursts*. However, as it is rather complex to implement, we also consider other solutions.

The second estimation method is based on the linear combination (in MMSE sense) of multiple frequency estimates each obtained from a single RFB independently. We call this estimator *ad hoc estimator*. V&V type estimation can be used in each RFB separately in this case, then the linear combination (depending on the SNR levels and used frequency values in each RFB) of each estimates produce operation close to the CRB at high SNR.

A. NLS estimator over multiple bursts

The equivalent discrete time channel in (1) can be written as

$$y_k^i = h_i e^{j2\pi v_i k} x_k^i + \eta_k^i = \rho_i(k) e^{j\psi_i(k)} \tag{6}$$

where $k = 0, 1, \cdots, N-1$, $i = 1, \cdots, B$, and $v_i = f_i T_s \lambda$. Applying non-linearity (NL) to each RFB as in [7] for a single burst, one gets

$$z_k^i = \mathcal{F}_i(\rho_i(k)) e^{jH\psi_i(k)},\ i = 1, \dots, B \tag{7}$$

Function $\mathcal{F}_i()$ and a scalar H are used to remove modulation for general QAM classes in [16]. Considering $h_i = |h_i| e^{j\theta_i}$, it can be shown by using the similar approach in [7] and [16] that

$$\begin{aligned} z_k^i &= E\{z_k^i\} + d_k^i = C_i e^{jH(\theta_i + 2\pi v_i k)} + d_k^i \\ &= a_i e^{jH(2\pi v_i k)} + d_k^i,\ i = 1, \dots, B \end{aligned} \tag{8}$$

where $C_i = \left|E\{z_k^i\}\right|$ is a real-valued constant independent of k, $a_i = C_i e^{jH\theta_i}$, and $d_k^i = z_k^i - E\{z_k^i\}$ is the disturbance. d_k^i is shown to be WSS and

$$E\{d_k^i (d_l^j)^*\} = \sigma_{d_i}^2 \delta_{ij}(k - l) \tag{9}$$

and $\sigma_{d_i}^2$ can be found for a given constellation, SNR level and non-linearity ($\mathcal{F}_i$) at each RFB. In [7], the noise variance after NL was found for a given constellation and SNR in a single burst. In our problem, we have multiple RFB and different fading levels. Thus, instantaneous SNRs are different at each RFB and, so do the noise variances after NL. $SNR_i = \frac{E_s}{N_o}|h_i|^2$ at the i^{th} RFB and has to be estimated for proper frequency offset estimation. It can be approximated as

$$SNR_i \approx \left(\frac{1}{N} \sum_{k=0}^{N-1} |y_k^i|^2 - 1 \right) \tag{10}$$

One can optimize NL (the function $\mathcal{F}_i$) and H at each RFB, depending on SNR_i and for a given constellation χ, to produce frequency estimates with minimum variance and $\sigma_{d_i}^2$ by following a similar procedure as in [16]. However, this is beyond the scope of this paper. Defining, $\mathbf{z_i} = [z_0^i, \cdots, z_{N-1}^i]^T$, $\mathbf{b_i} = [1, e^{2\pi H v_i}, \dots, e^{2\pi H (N-1) v_i}]^T$, $\mathbf{d_i} = [d_0^i, \cdots, d_{N-1}^i]^T$, and $\mathbf{a} = [a_1, \cdots, a_B]^T$ we can write an equivalent model after NL as

$$\mathbf{z_i} = a_i \mathbf{b_i} + \mathbf{d_i},\ i = 1, \dots, B \tag{11}$$

The joint ML estimate for a_i's and λ can be written by assuming disturbance process d_k^i's are Gaussian. The estimated CFO is actually the weighted nonlinear least square (NLS) estimates such that,

$$\{\hat{\mathbf{a}}, \hat{\lambda}\} = \underset{\{\mathbf{a}, \lambda\}}{\operatorname{argmin}} \sum_{i=1}^{B} \frac{||\mathbf{z_i} - a_i \mathbf{b_i}||^2}{\sigma_{d_i}^2} \tag{12}$$

For a known λ, one can estimate a_i as $\hat{a_i} = \frac{\mathbf{b_i}^H \mathbf{z_i}}{||\mathbf{b_i}||^2}$ ($||\mathbf{b_i}||^2 = N$), then putting this estimate in (12), one gets

$$\left\{ \begin{array}{c} \hat{\lambda} = \underset{\lambda}{\operatorname{argmax}} \sum_{i=1}^{B} \frac{1}{\sigma_{d_i}^2} \frac{|\mathbf{b_i}^H \mathbf{z_i}|^2}{N} \\ \hat{a_i} = \frac{\mathbf{b_i}^H \mathbf{z_i}}{N},\ i = 1, \dots, B \end{array} \right\} \tag{13}$$

B. Ad Hoc Frequency Estimator

Using the results obtained in Appendix B, one may consider to obtain frequency estimates at each burst independently and linearly combine them. If the hypothetical estimator guarantees estimates with variance equal to the true CRB for single burst, then the final estimate, obtained by the filtering of each independent single burst estimates, achieves the true CRB found in Appendix A. This yields a practical implementation of frequency offset estimators for FH systems. In this Ad Hoc mode, one does not need to consider all FH bands as in the case of NLS over multiple bursts, instead independent frequency offset estimates can be obtained for different hopping bands, and then the final estimate can be constructed by simple weighting according to the used RFBs as in (27) in Appendix B. The weights are shown to be inversely proportional with the variance of each single burst estimate and at high SNR ($\frac{E_s}{N_o}$), filter weights can be approximated as (from Appendix B),

$$w_i \approx \frac{f_i^2 SNR_i}{\sum_{i=1}^{B} f_i^2 SNR_i} \tag{14}$$

where $SNR_i = \frac{E_s}{N_o}|h_i|^2$ (it can be found as in (10)). This means that the RFBs with higher channel gains and hopping frequencies (f_i) contribute to the final estimate substantially. This is reasonable since it is easy to observe frequency offset at higher f_is and the estimates from RFB with high SNR are more reliable. We use the above filtering as an ad hoc estimator in simulations.

V. Simulation Results

In this section, we present the results of Monte Carlo simulations related to CRB and the proposed algorithms. We assume that hopping frequencies vary for each packet transmission, thus $f_i T_s$ is a random variable and taken as uniformly distributed between 10^3 and 10^4. This correspond to for example $T_s = 10\ \mu\ sec$ and FH between 100 MHz and 1 GHz operationally. The fading is assumed to be Rayleigh and independent from burst to burst with unity power. Although our analyses are valid for general QAM modulations, only 8-PSK is used here. V&V type NL is used at each RFB, simply 8^{th} power is used for the simulation of proposed algorithms. There are a total of 10000 Monte Carlo channel realizations for h_i and f_i, $i = 1, \cdots, B$. The block length (N) is chosen as 25.

In Fig. 1, the mean value of the square-root of CRB in (3), averaged over different h_i and f_i realizations, is plotted for different average SNR and number of RFB (B) values. As seen in the figure, CRB for the blind case converges to that for the known data case after certain SNR values for $B > 1$. We can say that $B = 1$ is not sufficient to obtain reliable estimates up to 30 dB. This is due to the lack of diversity order (or degrees of freedom). If the channel fades, gross estimation error occur which affect the mean value in Fig. 1. One gains approximately 15 dB in average by making transmission over 2 independent bursts.

In Fig. 2, proposed algorithms in Section IV are simulated for $B = 16$. For Ad Hoc estimators, in order to eliminate the effects of RFBs with weak channel gain leading to gross error at the final estimate, we set the filter coefficients found by (14) to zero if SNR_i is below the $\text{median}\left\{\{SNR_i\}_{i=1}^{B}\right\}$. NLS type estimator and forward backward linear predictor (FBLP) [19] with polynomial order 5 are used to get single burst estimates respectively for the Ad Hoc Estimator. The proposed methods are shown to attain CRB at high SNR values above 20 dB. The performance gap between Ad-Hoc methods and the true CRB at high SNR is due to the use of suboptimal estimators in single burst. Their performance at low SNR can be further improved by varying the NL at each RFB depending on channel gains, and using optimal filtering in (32).

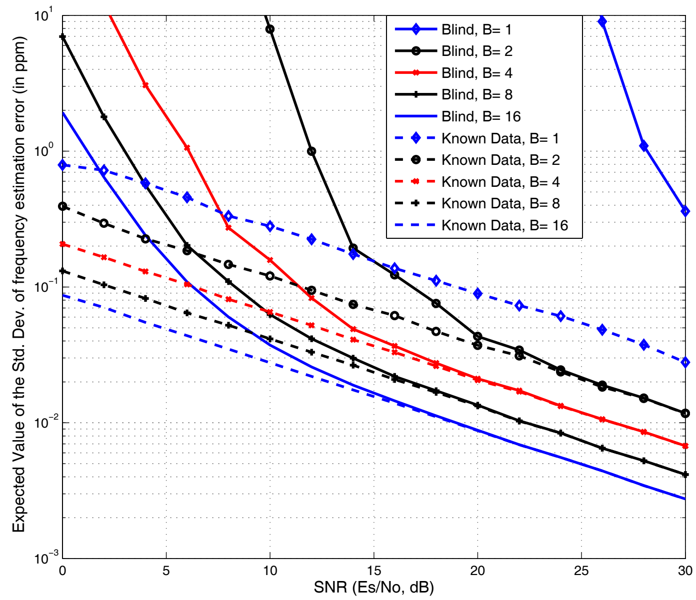

Fig. 1. True CRBs (in ppm) for blind and genie aided operations for 8-PSK and different number of RFBs, $B = \{1, 2, 4, 8, 16\}$, $N = 25$, and $f_i T_s$ is uniform between 10^3 and 10^4

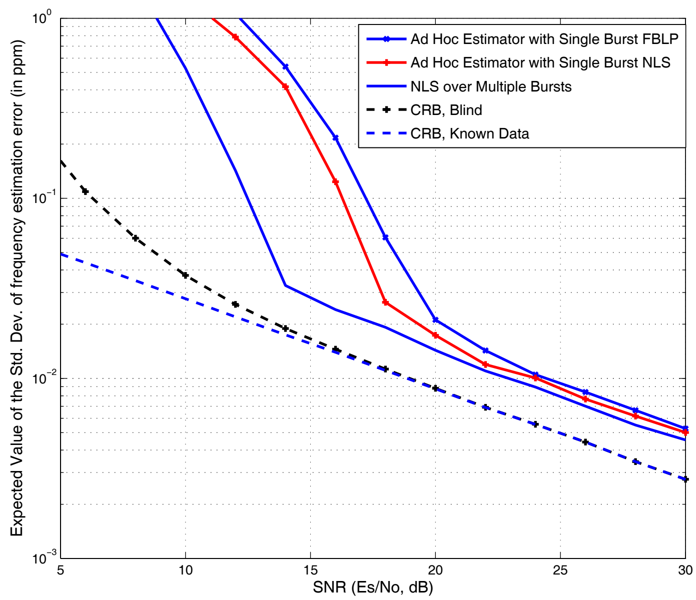

Fig. 2. Standard deviations for the frequency estimate of different methods for 8-PSK. $B = 16$, $N = 25$, and $f_i T_s$ is uniform between 10^3 and 10^4. 8^{th} power NL is used.

VI. Conclusions

In this paper, the true CRB for the blind frequency offset estimation in FH burst mode mobile radio is obtained. This CRB shows how the number of RFBs used, hopping frequencies, channel gains, and constellations affect the performance and can be seen as an important tool for FH system design. Moreover, it is seen that the linear combination (in MMSE sense) of multiple frequency estimates, each obtained from a single RFB independently, also attains the true CRB. In addition, practical frequency estimators, operating close to the

CRB for the FH system, are suggested. For future studies, multipath channels can also be incorporated and the robustness of the proposed schemes will be investigated under different scenarios.

APPENDIX A

In this appendix, we highlight the major steps leading to the CRB in the blind estimation of λ. To begin with, some variables to be used in the analysis are defined as

$$\begin{aligned} \mathbf{y_i} &= [y_0^i, \cdots, y_{N-1}^i]^T, \\ \mathbf{Y} &= [\mathbf{y_1}, \cdots, \mathbf{y_B}], \\ \mathbf{h} &= [h_1, \cdots, h_B]^T. \end{aligned} \tag{15}$$

where $\mathbf{x_i}$ and $\mathbf{X}$ are defined similarly. The Fisher information matrix [20] can be found for the estimation of complex unknown parameters $\mathbf{h}$ and λ. For the received samples from different frequency bursts in (1), one can write the logarithm of conditional pdf required for CRB evaluation as in

$$\ln p(\mathbf{Y}|\mathbf{h}, \lambda) = \sum_{i=1}^{B} \sum_{k=0}^{N-1} \Lambda_i(y_k^i) - NB\ln(M) \tag{16}$$

where

$$\Lambda_i(y_k^i) = \ln\left\{ \sum_{x_k^i \in \chi} \frac{1}{\pi N_o} \exp\left\{ -\frac{1}{N_o} |y_k^i - h_i e^{j2\pi v_i k} x_k^i|^2 \right\} \right\} \tag{17}$$

The following derivatives of the log likelihood function in (16) are required for the evaluation of the $(B+1)\times(B+1)$ Fisher Information matrix $\mathbf{J}$ used to find CRB (See the top of the next page). Let us define $E_y\{|\Phi_m(y_k^m)|^2\} = E_{\Phi_m}^2$, $E_y\{|\Psi_m(y_k^m)|^2\} = E_{\Psi_m}^2$ and $E_y\{\Phi_m(y_k^m)\Psi_m(y_k^m)^*\} = E_{\Phi_m\Psi_m}$ for $m = 1, \ldots, B$ which are independent of the symbol time index, k. These expectations are important metrics in CRB evaluation. Recalling $y_k^i = h^i e^{j2\pi v_i k} a_k^i + \eta_k^i$, the expectations above are taken over both a_k^i and η_k^i such that

$$E_{\Phi_i}^2 = \frac{1}{M} \sum_{a_k^i \in \chi} E_{\eta_k^i} \left\{ |\Phi_i(h^i e^{j2\pi v_i k} a_k^i + \eta_k^i)|^2 \right\} \tag{22}$$

and similar for $E_{\Psi_i}^2$ and $E_{\Phi_i\Psi_i}$. By using the Sherman-Morrison formula for matrix inversions, one can find a Cramer-Rao lower bound for the variance of λ estimate such that

$$\begin{aligned} \sigma_{\lambda_e}^2 &= \mathbf{J}^{-1}(B+1, B+1) \\ &= \left(\sum_{i=1}^{B} \frac{1}{\tilde{\sigma}_{\lambda_e}^2(i)} \right)^{-1} \end{aligned} \tag{23}$$

where $\tilde{\sigma}_{\lambda_e}^2(i)$ is defined in (4) for $i = 1, \ldots, B$. It can be easily seen that the CRB is independent of the value of frequency offset λ, since the expectations are invariant to the rotations of η_k^i's.

For the genie aided case, we can write the Fisher matrix by using the similar procedure as in the blind case, but in this case , data symbols x_k^i's are known. After some algebraic manipulations, a CRB for the variance of λ estimate can be obtained as

$$CRB\{\hat{\lambda}\} = \sigma_{\lambda_e}^2 = \left(\sum_{i=1}^{B} \frac{1}{\tilde{\sigma}_{\lambda_e}^2(i)} \right)^{-1} \tag{24}$$

where $\tilde{\sigma}_{\lambda_e}^2(i) = \left[T_s^2(2\pi)^2 \left(2A_i^3 - \frac{(A_i^2)^2}{A_i^1} \right) f_i^2 |h_i|^2 \right]^{-1}$, and A_i^1, A_i^2 and A_i^3 are defined as,

$$A_i^1 = \frac{1}{N_o} \sum_{k=0}^{N-1} |x_k^i|^2, \; A_i^2 = \frac{1}{N_o} \left(\sum_{k=0}^{N-1} k |x_k^i|^2 \right) \tag{25}$$

$$A_i^3 = \frac{1}{N_o} \left(\sum_{k=0}^{N-1} k^2 |x_k^i|^2 \right) \tag{26}$$

for $i = 1, \ldots, B$. It is interesting to note that two CRBs in blind and known data mode have similar expressions except the definition of $\tilde{\sigma}_{\lambda_e}^2(i)$. The details of the appendix can be found in the journal version of this paper.

As can be seen from (24), strongest channels with higher hopping frequencies among the other RFBs determine the variance of the frequency offset estimate substantially. It can be shown that as SNR goes to ∞, the correlation coefficients r_i in (5) goes to $\sqrt{\frac{1}{2}}$ for $i = 1, \cdots, B$, and the true CRB for blind estimation in (23) goes to the that of the known data case or unmodulated case in (24).

APPENDIX B

In this appendix, the best linear combination (in MMSE sense) of multiple frequency estimates, each obtained from a single RFB independently, is found. One may wonder the variance of $\hat{\lambda}$ in this case. The CRB of this *Ad Hoc* scheme can be found by assuming that the hypothetical estimator operates and achieves the CRB of single burst at each RFB. Then, these independent estimates from each RFB, namely $\hat{\lambda}_i$, with variance $\tilde{\sigma}_{\lambda_e}^2(i)$ can be linearly combined in MMSE sense to yield a final unbiased estimate as

$$\hat{\lambda} = \sum_{i=1}^{B} w_i \hat{\lambda}_i = \mathbf{w}^T \mathbf{a}, \quad \sum_{i=1}^{B} w_i = 1, \tag{27}$$

where $\mathbf{w} = [w_1, \ldots, w_B]^T$ and $\mathbf{a} = [\hat{\lambda}_1, \ldots, \hat{\lambda}_B]^T$

$$\mathbf{R}_{\hat{\lambda}} = E\{\mathbf{a}\mathbf{a}^T\} \tag{28}$$

$$E\{\hat{\lambda}_i \hat{\lambda}_j\} = \lambda^2 + \tilde{\sigma}_{\lambda_e}^2(i)\delta_{ij}, \; i, j = 1, \ldots, B. \tag{29}$$

The parameter λ is the desired but unknown offset value. Defining $\mathbf{D} = \mathrm{diag}\{\tilde{\sigma}_{\lambda_e}^2(1), \ldots, \tilde{\sigma}_{\lambda_e}^2(B)\}$, all ones vector $\underline{\mathbf{1}} = [1, \ldots, 1]_{B\times 1}^T$, and $\mathbf{p} = E\{\mathbf{a}\lambda\} = \lambda^2 \underline{\mathbf{1}}$ is the cross correlation vector.

The Wiener filter (MMSE) solution for $\mathbf{w}$ can be written without considering the constraint $\sum_{i=1}^{B} w_i = 1$ (for unbiased final estimate) as

$$\begin{aligned} \mathbf{w_{mmse}} &= \mathbf{R}_{\hat{\lambda}}^{-1}\mathbf{p} = \left(\underline{\mathbf{1}}\underline{\mathbf{1}}^T \lambda^2 + \mathbf{D} \right)^{-1} \lambda^2 \underline{\mathbf{1}} \\ &= \left(\mathbf{D}^{-1} - \frac{\lambda^2 \mathbf{D}^{-1} \underline{\mathbf{1}}\underline{\mathbf{1}}^T \mathbf{D}^{-1}}{1 + \lambda^2 \underline{\mathbf{1}}^T \mathbf{D}^{-1} \underline{\mathbf{1}}} \right) \lambda^2 \underline{\mathbf{1}} \\ &= \alpha \left(\mathbf{D}^{-1} \underline{\mathbf{1}} \right) \end{aligned} \tag{30}$$

$$\frac{\partial \ln p(\mathbf{Y}|\mathbf{h},\lambda)}{\partial h_i} = \sum_{l=1}^{B}\sum_{k=0}^{N-1}\frac{\partial \Lambda_i(y_k^l)}{\partial h_i} = \sum_{k=0}^{N-1}\Phi_i(y_k^i),\ i=1,\cdots,B \tag{18}$$

where

$$\Phi_i(y_k^i) = \frac{\sum_{x_k^i\in\chi}\left\{\frac{1}{N_o}((y_k^i)^* e^{j2\pi v_i k}x_k^i - |x_k^i|^2(h_i)^*)\exp(-\frac{1}{N_o}|y_k^i - h_i e^{j2\pi v_i k}x_k^i|^2)\right\}}{\sum_{x_k^i\in\chi}\exp\left\{-\frac{1}{N_o}|y_k^i - h_i e^{j2\pi v_i k}x_k^i|^2\right\}} \tag{19}$$

and

$$\frac{\partial \ln p(\mathbf{Y}|\mathbf{h},\lambda)}{\partial \lambda} = \sum_{i=1}^{B}\sum_{k=0}^{N-1}\frac{\partial \Lambda_i(y_k^i)}{\partial \lambda} = T_s\sum_{i=1}^{B} f_i\sum_{k=0}^{N-1}k\Psi_i(y_k^i) \tag{20}$$

where

$$\Psi_i(y_k^i) = \frac{\sum_{x_k^i\in\chi}\left\{\frac{1}{N_o}4\pi\Re\{j(y_k^i)^* h_i e^{j2\pi v_i k}x_k^i\}\exp(-\frac{1}{N_o}|y_k^i - h_i e^{j2\pi v_i k}x_k^i|^2)\right\}}{\sum_{x_k^i\in\chi}\exp\left\{-\frac{1}{N_o}|y_k^i - h_i e^{j2\pi v_i k}x_k^i|^2\right\}} \tag{21}$$

$\alpha = \frac{\lambda^2}{1+\lambda^2\underline{\mathbf{1}}^T\mathbf{D}^{-1}\underline{\mathbf{1}}}$ is only a scalar and so, one can change it in order to satisfy the constraint: $\sum_{i=1}^{B} w_i = 1$. Then, the optimal filter coefficients can be written by normalizing the filter in (30) as,

$$\mathbf{w} = \frac{\mathbf{D}^{-1}\underline{\mathbf{1}}}{\underline{\mathbf{1}}^T\mathbf{D}^{-1}\underline{\mathbf{1}}} \tag{31}$$

or more explicitly,

$$w_i = \frac{1}{\tilde{\sigma}^2_{\lambda_e}(i)}\left(\sum_{i=1}^{B}\frac{1}{\tilde{\sigma}^2_{\lambda_e}(i)}\right)^{-1} \tag{32}$$

The mean square error (MSE) of the final λ estimate (obtained from the linear combination of each single burst estimates) can be calculated by using (31) as,

$$\begin{aligned} MSE &= E\{|\lambda - \mathbf{w}^T\mathbf{a}|^2\} = \lambda^2 - 2\mathbf{w}^T\mathbf{p} + \mathbf{w}^T\mathbf{R}_{\hat{\lambda}}\mathbf{w} \\ &= \frac{1}{\underline{\mathbf{1}}^T\mathbf{D}^{-1}\underline{\mathbf{1}}} = \left(\sum_{i=1}^{B}\frac{1}{\tilde{\sigma}^2_{\lambda_e}(i)}\right)^{-1} \end{aligned} \tag{33}$$

which is independent of λ and more interestingly coincides with the true CRB as expressed by the RHS of (23) and (24). It is observed that this scheme achieves the joint CRB over multiple bursts.

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