Blind estimation of multi-CFO for distributed MIMO-OFDM system in frequency-selective fading channels

Yingtao Niu
Nanjing Telecommunication Technology Institute
Nanjing, China
niuyingtao78@hotmail.com

Qun Chen
College of Math and Physics
Nanjing University of Information Science and Technology
Nanjing, China
chenqun98@163.com

Jianzhong Chen
Nanjing Telecommunication Technology Institute
Nanjing, China
cjz63@hotmail.com

Abstract—**A novel blind multiple carrier frequency offset (multi-CFO) estimation algorithm is developed for distributed multi-input multi-output orthogonal frequency division multiplexing (MIMO-OFDM) systems over frequency-selective fading channels. In the algorithm, different multiple CFOs are estimated by cyclostationarity (CS) of the received signals and least square (LS) criterion. Theory analysis shows that the blind algorithm adapts to synchronization under any distributed stationary noise. Meanwhile, it has no special limitation on the number of transmit and receive antennas. Simulation results verify that the proposed algorithm maintains stability at low signal-to-noise ratio (SNR). It can also achieve excellent performance in single-input single-output (SISO) OFDM systems.**

Keywords- multi-CFO; cyclostationarity; LS; MIMO-OFDM

I. INTRODUCTION

Recently, multi-input multi-output orthogonal frequency division multiplexing (MIMO-OFDM) has gained increased interest because of its high data rate and its ability to combat fading[1]. However, the performance of the MIMO-OFDM system critically depends on the accurate estimation of the carrier frequency offset (CFO). MIMO systems may be categorized into the centralized and the distributed system. For the centralized system, the antennas of transmitter and receiver are placed near to each other which make the same CFO exists between different transmit and receive antenna. Compared with the centralized MIMO system, the distributed system can achieve a higher diversity gain, which is a promising solution for future MIMO system. However, the transmitters and the receivers in a distributed system are sparsely placed. The oscillator mismatch and Doppler shift are often different between each pair of receive and transmit antenna, which make different CFOs exist between each pair of transmit and receive antenna and the carrier synchronization of distributed system much more complex than that of centralized system and traditional single-input single-output (SISO) system. Hence, effective carrier synchronization algorithms are more important to distributed MIMO-OFDM system.

Some carrier synchronization schemes have been proposed for MIMO-OFDM systems, most of which rely on periodically transmitted pilots or training sequences[2, 3]. However, data-aided schemes lower the spectral efficiency for the existence of pilots or training sequences. Furthermore, in some noncooperative (e.g., tactical) links, blind estimators are the only option. Research on blind schemes has been motivated because of the above-mentioned reasons. At present, the literature of blind algorithm on SISO/MIMO-OFDM carrier synchronization is relatively scarce. An algorithm exploited the rotational invariance of OFDM signal is proposed in [4] for SISO-OFDM which can estimate one CFO within bandwidth of OFDM. A method based on kurtosis is proposed in [5] for SISO/MIMO-OFDM that can achieve satisfactory performance within one subcarrier space at high signal-to-noise ratio (SNR). Another important class of blind CFO estimator exploits the second-order cyclostationarity (CS) of the received signal [6, 7]. However, these blind carrier synchronization algorithms can estimate only one CFO that is unsuitable for distributed MIMO-OFDM system with multi-CFO.

In this paper, the authors have proposed a blind multi-CFO estimation algorithm for the distributed MIMO-OFDM system in frequency-selective fading channels. The cyclic correlation of received signal is computed initially, and then least square (LS) criterion is used to estimate all the CFOs of each pair of transmit and receive antenna. This paper is organized as follows. In Section Ⅱ, the MIMO-OFDM signal model is presented, and the authors' assumptions and the problem statement are provided. Section Ⅲ introduces the proposed algorithm in detail. Section Ⅳ presents computer simulation results. Finally, in Section Ⅴ, some concluding remarks are provided.

II. SYSTEM MODEL

Consider a distributed MIMO-OFDM system with P transmit antennas and Q receive antennas. The discrete-time baseband equivalent signal of pth transmit antenna is given by

$$s_p(n) = \sum_{k=0}^{N-1} \sum_{l=-\infty}^{\infty} c_{p,k,i} g_{tr}(n - lM) e^{j(2\pi/N)k(n-lM)}, \ \ p = 1,2,...,P \quad (1)$$

where $c_{p,k,l}$ is the complex information symbol, N is the number of subcarrier, $M = N + L_{GI}$ is the OFDM symbol length, and L_{GI} is the guard region. $g_{tr}(n)$ denotes the transmitter pulse shaping filter, which is used to suppress out-of-band radiation. The frequency-selective fading channel in the time domain between pth transmit antenna and qth receive antenna is modeled as a finite impulse response (FIR) filter with L order such as $h_{pq} = \sum_{l_1=0}^{L-1} \alpha_{pq}(l_1)\delta(n-l_1)$, where $\alpha_{pq}(l_1)$ is the l_1th channel tap gain. If the CFOs between each transmit and receive antenna are different and if perfect symbol timing synchronization is achieved, the qth receive antenna's discrete-time signal is given by

$$r_q'(n) = \sum_{p=1}^{P} e^{j2\pi n\theta_{pq}} \sum_{l_1=0}^{L-1} \alpha_{pq}(l_1) s_p(n-l_1) + v(n) \quad (2)$$

The relative CFO θ_{pq} between pth transmit antenna and qth receive antenna is defined as $\theta_{pq} = \Delta f_{pq} T_s$, where Δf_{pq} is absolute CFO and T_s is sample period. $v(n)$ is additive noise. After passing through the matched filter $g_{re}(n)$, the received signal can be expressed as $r_q(n) = r_q'(n) * g_{re}(n)$ (* denotes convolution). Thus, the received signal that passed through $g_{re}(n)$ can be rewritten as

$$\begin{aligned} r_q(n) = \sum_{p=1}^{P} e^{j2\pi n\theta_{pq}} \sum_{l_1=0}^{L-1} \alpha_{pq}(l_1) \sum_{k=0}^{N-1} \sum_{l=-\infty}^{\infty} c_{p,k,l}\, g(n-lM-l_1) \\ \bullet e^{j(2\pi/N)k(n-lM-l_1)} + v(n),\ q = 1,2,...,Q \end{aligned} \quad (3)$$

where $g(n) = g_{tr}(n) * g_{re}(n)$. In (1), (2), and (3), the following assumptions are used: (a) The complex information symbols $c_{p,k,l}$ for transmission is zero-mean independent and identically distributed (i.i.d.) sequence with variance σ_c^2. (b) The channel state information is quasi-static and known. The l_1th tap gain of channel $\alpha_{pq}(l_1)$ is i.i.d. complex Gaussian random variable and $\sum_{l_1=0}^{L} Var(\alpha_{pq}(l_1)) = 1$, where $Var(\bullet)$ is the variance operator. (c) $v(n)$ is wide-sense stationary complex process, independent of $c_{p,k,l}$.

The objective of this paper is to derive estimation of all $P \times Q$ CFOs $\{\theta_{pq}, p \in [1,P], q \in [1,Q]\}$ from consecutive samples of the received signal $r_q(n)$ in nondata-aided scenario.

III. Proposed Algorithm

The time-varying correlation of a general nonstationary process $r_q(n)$ is defined as $R_q(n,\tau) = E\{r_q(n) r_q^*(n-\tau)\}$, where τ is an integer lag. From (3), $R_q(n,\tau)$ can be written as

$$\begin{aligned} R_q(n,\tau) = \sigma_c^2 \sum_{p=1}^{P} e^{j2\pi\tau\theta_{pq}} \sum_{l_1=0}^{L-1}\sum_{l_2=0}^{L-1} \alpha_{pq}(l_1)\alpha_{pq}^*(l_2) \sum_{k=0}^{N-1} e^{j(2\pi/N)k(\tau+l_2-l_1)} \\ \bullet \sum_{l=-\infty}^{\infty} g(n-lM-l_1) g(n-\tau-lM-l_2) + m_v(\tau) \end{aligned} \quad (4)$$

where $m_v(\tau)$ is given by $m_v(\tau) = E\{v(n)v^*(n-\tau)\}$. One can easily prove $R_q(n,\tau) = R_q(n+lM,\tau)$. Therefore, $r_q(n)$ is a CS signal with period M. From (4), the second order cyclic correlation of the qth receive antenna's signal turns out to be

$$\begin{aligned} M_q(k,\tau) &= \frac{1}{M}\sum_{n=0}^{M-1} R_q(n,\tau) e^{-j(2\pi/M)kn} \\ &= \frac{\sigma_c^2}{M}\sum_{p=1}^{P} e^{j2\pi\tau\theta_{pq}} \sum_{l_1=0}^{L-1}\sum_{l_2=0}^{L-1} \alpha_{pq}(l_1)\alpha_{pq}^*(l_2)\Gamma_N(\tau+l_2-l_1) G(k,0) e^{-j(2\pi/M)kl_1} \\ &\quad + m_v(\tau)\delta(k) \\ &= \sum_{p=1}^{P} B_{pq}(k,\tau) e^{j2\pi\tau\theta_{pq}} + m_v(\tau)\delta(k) \end{aligned} \quad (5)$$

where

$$B_{pq}(k,\tau) = \frac{\sigma_c^2}{M}\sum_{l_1=0}^{L-1}\sum_{l_2=0}^{L-1} \alpha_{pq}(l_1)\alpha_{pq}^*(l_2)\Gamma_N(\tau+l_2-l_1) G(k,0) e^{-j(2\pi/M)kl_1}$$

$$G(k,0) = \sum_{n=-\infty}^{\infty} g(n)g(n) e^{-j(2\pi/M)kn}$$

$$\Gamma_N(\tau+l_2-l_1) = \sum_{m=0}^{N-1} e^{j(2\pi/N)m(\tau+l_2-l_1)} = N\delta[\tau-(l_1-l_2)]$$

where $\delta(k)$ is impulse function. When $\tau = (l_1 - l_2) \in [0, L-1]$, $\Gamma_N(\tau+l_2-l_1) \neq 0$. If the shaping pulse filter $g(n)$ in (1) is a raised cosine impulse, then it is known that $G(k,\tau)$ will be nonzero only for $k = 0, \pm 1$ [6]. From (5), CFOs is separated from the complex information symbols. The stationary noise term $m_v(\tau)\delta(k)$ affects cyclic correlation at $k = 0$. If $k = 1$ or $k = -1$ (nonzero cyclic frequency) is considered and $k = 0$ is excluded in estimator, the effect of noise will be suppressed completely in theory. In fact, suppressing the effect of additive stationary noise is the main advantage of the signal cyclostationarity in nonzero cyclic frequency [8].Therefore, (k,τ) that meets $M_q(k,\tau) \neq 0$ can be written as

$$\begin{aligned} I &\triangleq \{(k,\tau) | M_q(k,\tau) \neq 0,\ m_v(\tau)\delta(k) = 0\} \\ &= \{(k,\tau) | B_{pq}(k,\tau) \neq 0,\ \delta(k) = 0\} \\ &= \{k = \pm 1; \tau \in [0, L-1]\} \end{aligned}$$

Now the problem has transformed into the method of obtaining the estimated value $\hat{\theta}_q = (\hat{\theta}_{1q}, \hat{\theta}_{2q}, ..., \hat{\theta}_{Pq})$ from the observed value of $\{\hat{M}_q(k,\tau), B_{pq}(k,\tau) | (k,\tau) \in I\}$, which can be solved as an optimization problem by LS criterion as follows

$$\hat{\theta}_q = \arg\min \sum_{\tau=0}^{L-1} \left\{ \sum_{p=1}^{P} B_{pq}(k,\tau) \mathrm{e}^{j2\pi\tau\theta_{pq}} - \hat{M}_q(k,\tau) \right\}^2 \quad (6)$$

This is a nonlinear LS problem. In this paper, the Gauss-Newton iterative algorithm (see Appendix) can be applied to solve the problem[9], which is advantageous for computation and for the fast convergence rate. As it is a mature optimization algorithm and there are existing programs to perform the algorithm, the solution is not described in detail in this paper.

In simulation, we can use the function "lsqnonlin" of MATLAB for solution of the problem. The proposed algorithm is blind algorithm. Therefore, the channel state information should also be estimated by blind algorithm such as [10]. If the index q of receive antenna from 1 to Q is chosen, all the CFOs in (3) can be solved. It is obvious that the proposed algorithm is independent of the number of receive antennas and the transmit antenna number $P \le L$. This algorithm fits not only the raised cosine pulse filter but also the other shaping pulse filters.

Compared with traditional CS approach, the key idea behind the proposed CFO estimator is to extract the CFO-term from the received signal and eliminate the effect of complex information symbol and stationary noise by CS, and then solve multi-CFO by the LS criterion. The traditional CS algorithm only makes use of a certain autocorrelation time delay τ for cyclic correlation. Hence, it can only estimate one CFO. The proposed algorithm exploits all cyclic correlations corresponding to $\tau = 0 \sim L-1$ which can estimate multi-CFO. In practice, $M_q(k,\tau)$ can be estimated from a finite sample data record $r_q(n)$ with the T_L symbol number.

$$\hat{M}_q(k,\tau) = \frac{1}{(T_L M - \tau)} \sum_{n=\tau}^{T_L M-1} r_q(n) r_q^*(n-\tau) e^{-j(2\pi/M)kn} \qquad (7)$$

But computational efficiency of (7) is quite low, which require LMT_L complex addition and $2LMT_L$ complex multiplication. If the method based on fast Fourier transform (FFT)[11] is used, only $LMT_L + L\log_2 M$ complex addition and $LM + (L/2)\log_2 M$ complex multiplication are required.

IV. SIMULATION RESULTS

In this section, simulation results are provided to demonstrate the performance of the proposed estimator. $g(n)$ is a raised cosine pulse with a rolloff factor of 0.9. The complex information symbols are i.i.d. 16-QAM symbols with $\sigma_c^2 = 1$. $v(n)$ is additive white Gaussian noise (AWGN) with variance of σ_v^2. The SNR of the received signal is defined as $\mathrm{SNR} = \sigma_c^2 / \sigma_v^2$. The channel tap gain $\alpha_{pq}(l_1)$ is modeled as assumption (b) (see Section 2, System Model). Additionally, $Var(\alpha_{pq}(l_1))$ decay exponentially with delay l_1. The Gauss-Newton iterative algorithm is performed by MATLAB function "lsqnonlin". The initial value of CFO in the iterative algorithm is assumed as zero. The authors chose $k = 1$ in (5) and the OFDM symbol number $T_L = 200$. To evaluate the performance of multi-CFO estimator, the average mean square error (MSE) of qth receive antenna is defined as

$$\mathrm{MSE} = \frac{1}{P} \sum_{p=1}^{P} \mathrm{MSE}(\theta_{pq})$$

where $\mathrm{MSE}(\theta_{pq})$ is the MSE of estimated value $\hat{\theta}_{pq}$. In our simulation, the *1*st receive antenna's MSE is investigated. All results are obtained by averaging over 3000 independent Monte Carlo trials.

A. *The proposed estimator's performance versus SNR under different subcarrier numbers*

Two transmit antennas and two receive antennas are assumed to be used and CFOs from the transmit antennas to *1*st receive antenna is $\theta_{11} = 0.19$, $\theta_{21} = 0.10$. The channel order is $L = 7$. The MSE performance is simulated at subcarrier number $N = 32, 64, 128$, with SNR ranging from 0 to 30 *dB*.

According to (5), if $k \ne 0$, the influence of noise $m_v(\tau)\delta(k)$ will be suppressed. From Fig. 1, it can be observed that the estimator is insensitive to the stationary noise. In comparison, performance of kurtosis-based approach[5] and traditional ESPRIT approach[4] falls obviously with the decrease in SNR. The other disadvantages of the kurtosis-based approach and the traditional ESPRIT approach are one CFO estimation. However, it has to be noted that the performance of the proposed algorithm does not improve with increasing SNR. Therefore, the proposed algorithm fits the course carrier synchronization at low SNR($\le 5dB$), whereas the kurtosis-based method fits the fine carrier synchronization at high SNR. Furthermore, it can be inferred that the use of more subcarriers leads to better performance of estimator, which is attributed to the fact that more the number of subcarriers, more is the sample data in fixed number symbol. By [8], the variance of cyclic correlation is inversely proportional to the sample number for estimation. Therefore, the proposed algorithm fits the system with more subcarriers.

B. *The proposed estimator's performance versus SNR under different order of frequency selective channel*

It is assumed that two transmit antennas and two receive antennas are used and CFOs from the transmit antennas to *1*st receive antenna is $\theta_{11} = 0.19$, $\theta_{21} = 0.10$. The subcarrier number is $N = 64$. The MSE performance is simulated for the order of channel $L = 5, 7, 9$ from 0 to 30 *dB*.

From Fig. 2, it can be found that the order of channel have considerable effect on the MSE performance of the estimator. The higher the channel order, better is the performance of the system. In short, the proposed algorithm makes use of the

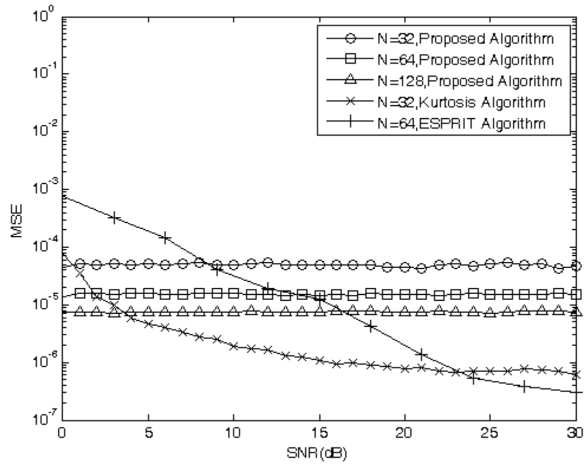

Figure 1. Performance of the proposed estimator versus SNR under different subcarrier numbers

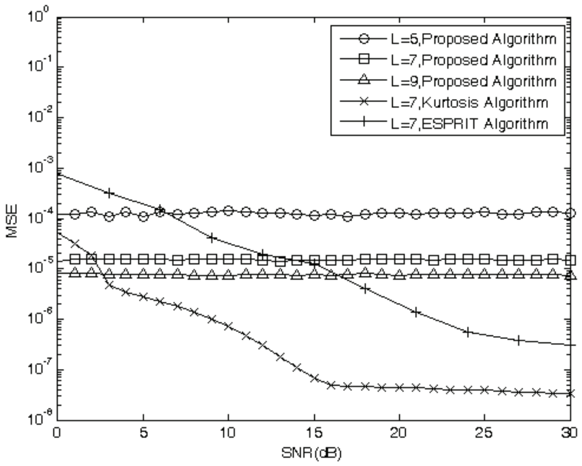

Figure 2. Performance of the proposed estimator versus SNR under different orders of frequency-selective channel

diversity of the multi-path in frequency-selective channel.

C. The proposed estimator's performance versus SNR for different number of CFO estimation

One, two, and three transmit antennas and the same number of receive antennas with $N=128$ and $L=9$ are assumed, respectively. CFOs from the transmit antennas to *1*st receive antenna are as follows respectively: $\{\theta_{11}=0.10\}$, $\{\theta_{11}=0.19, \theta_{21}=0.10\}$ and $\{\theta_{11}=0.19,\ \theta_{21}=0.10, \theta_{31}=0.05\}$.

Fig. 3 shows that the performance of MSE becomes worse with the increase of transmit antennas for the fixed subcarrier number and order of channel. To obtain better performance, the proposed algorithm would be used in the channel in which the order is much greater than the number of CFOs.

Furthermore, the proposed algorithm can also be applied to SISO-OFDM system. From Fig. 3, one can observe that MSE performance of the proposed algorithm is very advantageous over conventional CS algorithm[7] in frequency-selective

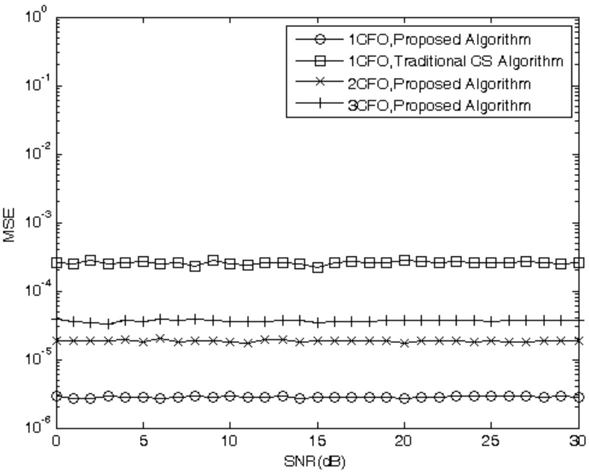

Figure 3. Performance of the proposed estimator versus SNR for different numbers of CFO estimation

fading channel for one CFO estimate. The reason for this advantage is that the proposed algorithm exploits the diversity of multipath in frequency selective channel.

V. Conclusion

In this paper, the authors have proposed a novel blind multi-CFO estimation algorithm for distributed MIMO-OFDM systems in frequency-selective channels. The algorithm exploits the cyclic correlation in nonzero cyclic frequency for eliminating the effect of the complex information symbols and stationary noise. Then LS criterion is used to estimate different multi-CFO between each pair of receive and transmit antenna. It does not require pilot symbols or training sequences, thus saving valuable bandwidth. The simulation results show that the proposed method achieves excellent performance when the order of channel is much greater than the number of CFO. It has no special limitation for the number of receive and transmit antennas. Furthermore, it is robust to any distributed stationary noise because of the second order CS. It can also be used for SISO-OFDM system and is very advantageous over the conventional CS algorithm. However it requires more sample data because of cyclic statistic. Therefore, the proposed algorithm is suitable for continuous-transmission MIMO-OFDM systems at low SNR.

Appendix

The solution of nonlinear LS problem is to find a vector $\mathbf{x}\in\mathbf{R}^n$ which satisfies

$$f(\mathbf{x})=\min_{\mathbf{z}\in\mathbf{R}^n} f(\mathbf{z}),\ f(\mathbf{x})=\frac{1}{2}\sum_{i=1}^{m} r_i^2(\mathbf{x})=\frac{1}{2}\|\mathbf{r}(\mathbf{x})\|_2^2, m\ge n \qquad (8)$$

where $r_i(\mathbf{x})$ is a nonlinear function which is defined in $\mathbf{R}^n$ and $\mathbf{r}(\mathbf{x})=(r_1(\mathbf{x}),r_2(\mathbf{x}),...,r_m(\mathbf{x}))^T$. If this problem is considered as a solution of overdetermined nonlinear equation system $\mathbf{r}(\mathbf{x})=0$, we can approach $\mathbf{r}(\mathbf{x})$ by giving the linear model of the neighborhood of point $\mathbf{x}_c$.

$$\tilde{\mathbf{r}}_c(\mathbf{x})=\mathbf{r}(\mathbf{x}_c)+J(\mathbf{x}_c)(\mathbf{x}-\mathbf{x}_c) \qquad (9)$$

where $J(\mathbf{x}_c)$ is the Jacobi matrix of $\mathbf{r}(\mathbf{x}_c)$ .Therefore approximate solution of (8) can be educed by the linear LS problem

$$\min_{\mathbf{x}}\|\mathbf{r}(\mathbf{x}_c)+J(\mathbf{x}_c)(\mathbf{x}-\mathbf{x}_c)\|_2 \qquad (10)$$

Then Gauss-Newton algorithm is obtained. Only one order derivative is used in this algorithm. If the approximate vector of $\mathbf{r}(\mathbf{x})$ is computed by (10) directly, the basic Gauss-Newton algorithm is obtained. If $\mathbf{x}^{(k)}$ is the approximate solution, the solution $\mathbf{p}_k$ of LS problem

$$\min\|\mathbf{r}(\mathbf{x}^{(k)})+J(\mathbf{x}^{(k)})\mathbf{p}_k\|_2, \mathbf{p}_k\in\mathbf{R}^n \qquad (11)$$

can be used as revised vector, where $J(\mathbf{x}^{(k)})$ is the Jacobi matrix of $\mathbf{r}(\mathbf{x}^{(k)})$. Therefore, the new approximate solution can be written as $\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \mathbf{p}_k$. This LS problem can be solved by QR decomposition of $J(\mathbf{x}^{(k)})$. The advantage of the algorithm is that the solution of the LS problem can be given only by one iteration. Furthermore, the local convergence's rate of the algorithm is very fast for the general nonlinear problem and the almost consistent problem. But the convergence's rate may be slow for the strong nonlinear problem. The Gauss-Newton algorithms also include damped Gauss-Newton algorithm, trust region algorithm, etc[9].

REFERENCES

[1] G. L. Stüber, J. R. Barry, S. W. Mclaughlin, Y. G. Li, M. A. Ingram, and T. G. Pratt, "Broadband MIMO-OFDM wireless communications," *Proceedings of the IEEE,* vol. 92, pp. 271-294, February 2004.

[2] T. Aoki and M. Sandell, "Pilot design for frequency offset tracking in MIMO-OFDM systems," in *PIMRC'06*, Helsinki, Finland, 2006, pp. 1-5.

[3] Y. Li and H. Minn, "Consistent Pilot Designs For Frequency Offset Estimation in MIMO-OFDM Systems," in *ICASSP 2007*, Honolulu, USA, 2007, pp. 249-252.

[4] U. Tureli, H. Liu, and M. D. Zoltowski, "OFDM blind carrier offset estimation: ESPRIT," *IEEE Transactions on Communications,* vol. 48, pp. 1459-1461, September 2000.

[5] Y. Yao and G. B. Giannakis, "Blind carrier frequency offset estimation in SISO, MIMO, and multiuser OFDM systems," *IEEE Transactions on Communications,* vol. 53, pp. 173-183, January 2005.

[6] F. Gini and G. B. Giannakis, "Frequency offset and symbol timing recovery in flat-fading channels: a cyclostationary approach," *IEEE Transactions on Communications,* vol. 46, pp. 400-411, March 1998.

[7] H. Bölcskei, "Blind estimation of symbol timing and carrier frequency offset in wireless OFDM systems," *IEEE Transactions on Communications,* vol. 49, pp. 988-999, June 2001.

[8] Y. Wang, L. Jin, M. Yao, and Q. Yin, "Research on performance of suppressing noise by cyclic auto-correlation function," *Journal of Data Acquisition and Processing (China),* vol. 14, pp. 148-152, June 1999.

[9] M. Wei, *Theory and computation of generalized least square problems*. Beijing: Science Press, 2006.

[10] I. Bradaric, A. P. Petropulu, and K. I. Diamantaras, "On blind identifiability of FIR-MIMO with cyclostationary inputs using second order statistics," *IEEE Transactions on Signal Processing,* vol. 51, pp. 434-441, February 2003.

[11] M. Hua, "The application of the 2-order cyclostationarity of signals in Wireless mobile communication," in *Dept. Elect. Eng.* Ph. D. Hefei: Univ. STC., 2006.