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Simultaneous Carrier Frequency Offset Estimation for Multi-point Transmission in OFDM Systems

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Abstract—**Orthogonal frequency division multiplexing (OFDM) combining with the coordinated multi-point (CoMP) transmission technique has been proposed to improve performance of the receivers located at the cell border. However, the inevitable carrier frequency offset (CFO) will destroy the orthogonality between subcarriers and induce strong inter-carrier interference (ICI) in OFDM systems. In a multi-point transmission system, the impact of CFO is more severe because of the mismatch of carrier frequencies among multiple transmitters. To relieve the performance degradation, CFO estimation and compensation is essential. For simultaneous estimation of multiple CFOs, the performance of conventional CFO estimation schemes is significantly degraded by the mutual interference among the signals coming from multiple transmitters. In this work, we propose a maximum likelihood (ML)-based estimator for simultaneous estimation of multiple CFOs, and design the training sequences that are robust to the variation in CFOs. According to the simulations, our scheme can eliminate the mutual interference effectively, approaching the mutual interference-free performance.**

Keywords—Orthogonal frequency division multiplexing (OFDM); coordinated multi-point (CoMP); carrier frequency offset (CFO).

I. Introduction

Carrier frequency offset (CFO), which is generally caused by oscillator mismatch or Doppler effects between the transmitter and the receiver, is one of the major technical problems of the orthogonal frequency division multiplexing (OFDM) technology. To maintain orthogonality between subcarriers, CFO estimation and compensation at the receiver is essential and crucial. The conventional CFO estimators for OFDM can be classified into three categories, including pilot tone-aided (PTA)-based [1]-[8], cyclic prefix-based (CPB) [1], and training symbols-based (TSB) [3], [4], [9], [12].

The coordinated multi-point (CoMP) transmission technique has been proposed for OFDM systems to enhance the receiving performance of the users located at the cell border. The principle of CoMP is using multiple base stations (BSs) or relay stations (RSs) to simultaneously transmit the same information to the user equipments (UEs) at the cell border. As a result, multiple signals coming from different BSs/RSs in same band can be received to improve the receiving performance. In the CoMP-OFDM systems, the multiple transmitters must be tightly time-synchronized in order to prevent mutual interference. However, it is generally true that different transmitters have different CFOs. Hence, the orthogonality between subcarriers collapses at the receiver and the performance is severely degraded.

This work was supported in part by the National Science Council, Taiwan, R.O.C., under Grant NSC 100-2219-E-007-010.

Under the scenario applying CoMP transmission, an UE receiver must estimate multiple CFOs corresponding to multiple receiving signals for compensation. However, different signals interfere with each other at the receiver, and thus the mutual interference significantly degrades the CFO estimation performance. This also implies that the conventional CFO estimators, which aim at the single CFO estimation, are not suitable for CoMP-OFDM systems. The goal of this work is to propose an effective approach that can estimate multiple CFOs simultaneously under the interference of multiple received signals. We propose a maximum likelihood (ML)-based multi-CFO estimator for CoMP-OFDM systems. Based on the Zadoff-Chu sequences, we design a scheme for finding the optimal set of training sequences that minimizes the mutual interference and are robust to the variation in multiple CFOs. The designed training sequences are referred to as the robust orthogonal training sequences (ROTS). By using the proposed ROTS, we propose the robust multi-CFO estimation (RMCE) scheme that can estimate multi-CFO by using only one symbol.

II. Preliminaries

A. System Model

The OFDM system is assumed to comprise N subcarriers. By using CoMP transmission, the signal received at a desired UE is the combination of the signals from the K BSs corresponding to CoMP transmission. The propagation channel from the k-th BS to the UE is assumed to be an L_k-tap multipath fading channel with the l-th path channel coefficient $h_{k,l}$ assumed to be complex-Gaussian distributed and time-invariant during an OFDM symbol interval. Let the time-domain discrete signal transmitted by the k-th BS be denoted as $x_k[i]$, where i is the time-sample index for $-N_{cp} \le i \le N-1$, and N_{cp} be the cyclic prefix (CP) length. Then, we have the time-domain discrete signal for multi-CFO estimation expressed as

$$y[i] = \sum_{k=1}^{K} \exp\left(\frac{j2\pi i w_k}{N}\right) \sum_{l=0}^{L_k-1} h_{k,l} x_k[i-l-\mu_k] + v[i], \qquad (1)$$

for $0 \le i \le N-1$, where $-0.5 < w_k < 0.5$ is the residual CFO, normalized to the carrier spacing, corresponding to the signal from the k-th BS; μ_k is the integer-valued timing error of the signal from the k-th BS; and $v[i]$ is the zero-mean additive white Gaussian noise (AWGN) with variance σ_v^2.

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Let the channel length plus the timing error be restricted within a range (0, Δ), i.e., $\Delta \geq L_k + \mu_k$ for $1 \leq k \leq K$, where Δ is smaller than N_{cp}. After removing the CP, the received N-sample vector can be written as

$$\mathbf{y} = \left[y[0], \cdots, y[N-1]\right]^T = \sum\nolimits_{k=1}^{K} \mathbf{\Phi}_k \mathbf{A}_k \mathbf{h}_k + \mathbf{v}, \tag{2}$$

where $\mathbf{v} = \left[v[0], v[1], \cdots, v[N-1]\right]^T$ is the AWGN vector;

$$\mathbf{\Phi}_k = \text{diag}\left[\exp\left(j\phi_k[0]\right), \cdots, \exp\left(j\phi_k[N-1]\right)\right], \tag{3}$$

with $\phi_k[i] = 2\pi w_k i/N + \varphi_k$ and φ_k denoting the phase of the first time sample; the channel vector

$$\mathbf{h}_k = \left[\mathbf{0}^T_{\mu_k \times 1}, h_k[0], h_k[1], \cdots, h_k[L_k - 1], \mathbf{0}^T_{(\Delta-\mu_k-L_k)\times 1}\right]^T \tag{4}$$

with $\mathbf{0}_{i\times 1}$ denoting an all-zero vector of length i; and

$$\mathbf{A}_k = \begin{bmatrix} x_k[0] & x_k[N-1] & \cdots & x_k[N-\Delta+1] \\ x_k[1] & x_k[0] & \cdots & x_k[N-\Delta+2] \\ \vdots & \vdots & \ddots & \vdots \\ x_k[N-1] & x_k[N-2] & \cdots & x_k[N-\Delta] \end{bmatrix}_{N\times\Delta} \tag{5}$$

is an $N\times\Delta$ matrix. For simplification, we can rewrite (2) as

$$\mathbf{y} = \mathbf{P}\mathbf{h} + \mathbf{v}, \tag{6}$$

where $\mathbf{P} = [\mathbf{\Phi}_1\mathbf{A}_1, \cdots, \mathbf{\Phi}_K\mathbf{A}_K]_{N\times K\Delta}$ and $\mathbf{h} = [\mathbf{h}_1^T, \cdots, \mathbf{h}_K^T]^T$.

B. *Maximum-Likelihood Estimation Approach*

Based on the TSB approach, the transmitted signals from all BSs for CFO estimation are known at the UE. The logarithm of the conditional probability density function (PDF) of $\mathbf{y}$, given $\mathbf{w}$ and $\mathbf{h}$, can be written as

$$\begin{aligned} \ln p(\mathbf{y} \mid \mathbf{w}, \mathbf{h}) &= -\frac{N}{2}\ln(2\pi\sigma_v^2) - \frac{1}{2\sigma_v^2}[\mathbf{y} - \mathbf{P}\mathbf{h}]^H[\mathbf{y} - \mathbf{P}\mathbf{h}] \\ &= C_0 - C_1[\mathbf{y} - \mathbf{P}\mathbf{h}]^H[\mathbf{y} - \mathbf{P}\mathbf{h}], \end{aligned} \tag{7}$$

where $C_0 = -N\ln(2\pi\sigma_v^2)/2$, $C_1 = 1/2\sigma_v^2$, and the superscript H denotes the conjugate transposition. By taking differentiation at the log-likelihood function with respect to $\mathbf{h}$ and then setting it to zero, the channel estimator is obtained as follows [10]:

$$\hat{\mathbf{h}} = (\mathbf{P}^H\mathbf{P})^{-1}\mathbf{P}^H\mathbf{y}. \tag{8}$$

Substituting (8) into (7), we obtain

$$\ln p(\mathbf{y} \mid \mathbf{w}) = C_0 - C_1\left[\mathbf{y} - \mathbf{P}(\mathbf{P}^H\mathbf{P})^{-1}\mathbf{P}^H\mathbf{y}\right]^H\left[\mathbf{y} - \mathbf{P}(\mathbf{P}^H\mathbf{P})^{-1}\mathbf{P}^H\mathbf{y}\right]. \tag{9}$$

As a result, the ML estimator of $\mathbf{w}$ is obtained by

$$\begin{aligned} \hat{\mathbf{w}} &= \arg\max_{\mathbf{w}}\{\ln p(\mathbf{y} \mid \mathbf{w})\} \\ &= \arg\max_{\mathbf{w}}\left\{C_0 - C_1\left[\mathbf{y} - \mathbf{P}(\mathbf{P}^H\mathbf{P})^{-1}\mathbf{P}^H\mathbf{y}\right]^H\left[\mathbf{y} - \mathbf{P}(\mathbf{P}^H\mathbf{P})^{-1}\mathbf{P}^H\mathbf{y}\right]\right\}. \end{aligned} \tag{10}$$

By expanding (10) and ignoring the terms irrelevant to $\mathbf{w}$, including C_0, C_1 and $|\mathbf{y}|^2$, the ML estimator of $\mathbf{w}$ becomes

$$\hat{\mathbf{w}} = \arg\max_{\mathbf{w}}\left\{\mathbf{y}^H\mathbf{P}\left(\mathbf{P}^H\mathbf{P}\right)^{-1}\mathbf{P}^H\mathbf{y}\right\}. \tag{11}$$

III. Robust Multi-CFO Estimation (RMCE) Scheme

A. *Complexity Reduction*

The multi-CFOs estimator shown in (11) involves multiple rounds of matrix inversion, and thus the computational complexity is too high to be realized in real-time applications. However, if we can make (by using the well-designed training sequences) the matrix $\mathbf{P}^H\mathbf{P}$ to be approximated to $c\mathbf{I}$, where c is an arbitrary constant and $\mathbf{I}$ is the identity matrix, the computational complexity can be reduced significantly. The matrix $\mathbf{P}^H\mathbf{P}$ can be represented by K^2 sub-matrices as

$$\mathbf{P}^H\mathbf{P} = \begin{bmatrix} \mathbf{B}^{(1,1)} & \cdots & \mathbf{B}^{(1,K)} \\ \vdots & \ddots & \vdots \\ \mathbf{B}^{(K,1)} & \cdots & \mathbf{B}^{(K,K)} \end{bmatrix}_{K\Delta\times K\Delta}. \tag{12}$$

The sub-matrix $\mathbf{B}^{(m,n)}$, for $1 \leq m, n \leq K$, is given by

$$\mathbf{B}^{(m,n)} = (\mathbf{\Phi}_m\mathbf{A}_m)^H\mathbf{\Phi}_n\mathbf{A}_n, \tag{13}$$

with the elements

$$\left[\mathbf{B}^{(m,n)}\right]_{l,g} = \sum_{i=0}^{N-1} x_m^*\left[\langle N+1-l+i\rangle_N\right]x_n\left[\langle N+1-g+i\rangle_N\right]e^{-j\left(\frac{2\pi i\theta_{m,n}}{N}+\varphi_n-\varphi_m\right)}, \tag{14}$$

where $\theta_{m,n} = (w_m - w_n)$ and $\langle\cdot\rangle_N$ is the modulo-N operator.

Note that the off-diagonal entries in $\mathbf{P}^H\mathbf{P}$ can be regarded as the interference from the same BS on different propagation paths or from other BSs. To assure that the signal used for multi-CFOs estimation is mutual-interference free, we should make all the off-diagonal entries to be zero. Hence, for constant-amplitude training sequences, the entries in $\mathbf{P}^H\mathbf{P}$ must be

$$\left[\mathbf{B}^{(m,n)}\right]_{l,g} = \begin{cases} N, & \forall m = n \text{ and } l = g \\ 0, & \text{otherwise.} \end{cases} \tag{15}$$

This implies that the time-domain training sequences must have the cross-correlation property

$$\begin{aligned} &\sum_{i=0}^{N-1} x_m^*\left[\langle N+1-l+i\rangle_N\right]x_n\left[\langle N+1-g+i\rangle_N\right]e^{j\left(\frac{2\pi i\theta_{m,n}}{N}+\varphi_n-\varphi_m\right)} \\ &= N\delta(m-n)\delta(l-g). \end{aligned} \tag{16}$$

As a result, under the achievement of $\mathbf{P}^H\mathbf{P} \approx N\mathbf{I}$, the ML estimator of $\mathbf{w}$ can be simplified as

$$\hat{\mathbf{w}} = \arg\max_{\mathbf{w}}\left\{\mathbf{y}^H\mathbf{P}\mathbf{P}^H\mathbf{y}/N\right\}. \tag{17}$$

By neglecting the factor N and expanding (17), we have

$$\hat{\mathbf{w}} = \arg\max_{\mathbf{w}}\left\{\mathbf{y}^H\mathbf{\Phi}_1\mathbf{A}_1\mathbf{A}_1^H\mathbf{\Phi}_1^H\mathbf{y} + \cdots + \mathbf{y}^H\mathbf{\Phi}_K\mathbf{A}_K\mathbf{A}_K^H\mathbf{\Phi}_K^H\mathbf{y}\right\}. \tag{18}$$

According to (18), the joint estimation problem can be separated into K independent estimation sub-problems, each of which solves the estimation of the CFO corresponding to a BS.

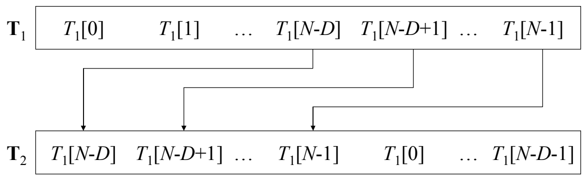

Fig. 1 The relation between two training sequences.

The estimator of w_k for the signal from the k-th BS is given by

$$\hat{w}_k = \arg\max_{w_k} \left\{ \mathbf{y}^H \mathbf{\Phi}_k \mathbf{A}_k \mathbf{A}_k^H \mathbf{\Phi}_k^H \mathbf{y} \right\} \tag{19}$$

and

$$\hat{\mathbf{h}}_k = (\mathbf{A}_k^H \hat{\mathbf{\Phi}}_k^H \mathbf{y})^T / N, \tag{20}$$

where $\hat{\mathbf{\Phi}}_k$ is the matrix by substituting $\hat{w}_k$ into $\mathbf{\Phi}_k$.

By applying the orthogonal sequences as the time-domain training sequences, there will be no mutual interference when there is no CFO between each BS and the UE. However, when there are CFOs, mutual interference becomes inevitable and the estimation performance is degraded. Therefore, our goal is to design a set of time-domain training sequences, which can minimize the mutual interference and are robust to the variation in multiple CFOs.

B. Robust Orthogonal Training Sequences (ROTS)

In this work, we use the Zadoff-Chu sequences [11] as the bases of training sequences. Assuming that the sequence length N is even, a Zadoff-Chu sequence $\mathbf{Z} = [Z[0], \cdots, Z[N-1]]^T$ is defined by

$$Z[i] = \exp(jM\pi i^2 / N), \text{ for } i = 0, \cdots, N-1, \tag{21}$$

where M is an integer relatively prime to N. Then, we can generate a set of orthogonal training sequences for multiple BSs by circular shifting in time samples. Denoting the employed training sequences as $\mathbf{T}_k = [T_k[0], \cdots, T_k[N-1]]^T$, for $1 \le k \le K$, we define the training sequence used at the k-th BS as

$$\mathbf{T}_k = \mathbf{Z}\langle D_k \rangle = [Z[N-D_k], \cdots, Z[N-D_k-1]]^T, \tag{22}$$

where $\mathbf{Z}\langle D_k \rangle$ is a circular-shifting version of $\mathbf{Z}$ by right shifting D_k samples. Without loss of generality, we assume that the first training sequence is equal to the original Zadoff-Chu sequence, i.e., $\mathbf{T}_1 = \mathbf{Z}$ and $D_1 = 0$.

By using the training sequences, we have the following two propositions regarding to the mutual interference from the same BS on different propagation paths or from other BSs.

Proposition 1: By applying the training sequences, the signals coming from the same BS on different propagation taps are still mutually orthogonal for a non-zero CFO under a multipath fading channel.

Proposition 2: By applying the training sequences, the signals coming from two different BSs cannot maintain the mutual orthogonality when the two corresponding CFOs are different.

The proof is skipped here because of limited available space.

Based on *Proposition 1*, we have $(\mathbf{\Phi}_m\mathbf{A}_m)^H\mathbf{\Phi}_m\mathbf{A}_m = N\mathbf{I}$ and $\mathbf{B}^{(m,m)} = N\mathbf{I}$ for $1 \le m \le K$. Furthermore, based on *Proposition 2*, we also find that the mutual interference between the m-th and n-th BSs corresponds to the entry in the sub-matrix $\mathbf{B}^{(m,n)}$, excepting for the effect of the channel vectors $\mathbf{h}_m$ and $\mathbf{h}_n$. As a result, in general, $\mathbf{B}^{(m,n)} \ne \mathbf{0}$, for $m \ne n$. Hence, we conclude that the difference in the CFOs of different BSs collapses the orthogonality among training sequences and the prerequisite $\mathbf{P}^H\mathbf{P} = c\mathbf{I}$ cannot be exactly achieved anymore. Moreover, in the design of ROTS for multi-CFO estimate, we only need to focus on the minimization of the mutual interference coming from different BSs. Thus, we modified the criterion for searching the set of training sequences as follows:

$$\{\mathbf{T}_1, \mathbf{T}_2, \cdots, \mathbf{T}_K\} = \arg\min_{\{\mathbf{T}_1', \mathbf{T}_2', \cdots, \mathbf{T}_K'\}} \left\| \left(N \times \left(\mathbf{P}^H\mathbf{P}\right)^{-1} - \mathbf{I} \right) \right\|_F, \tag{23}$$

where $\mathbf{S} = \{\mathbf{T}_1, \ldots, \mathbf{T}_K\}$ are the set of optimal training sequences, and $\|\mathbf{U}\|_F$ denotes the Frobenius norm of a complex matrix $\mathbf{U}$ given by

$$\|\mathbf{U}\|_F = \sqrt{\mathrm{tr}\left(\mathbf{U}^H\mathbf{U}\right)}. \tag{24}$$

The optimal training sequence set $\mathbf{S}$ can be possibly found by exhaustive search. To efficiently find $\mathbf{S}$, we discuss the results under the scenarios with different numbers of involved BSs.

C. ROTS for the Scenario with Two BSs

Considering the scenario with two BSs, i.e. $K = 2$, $\mathbf{T}_1$ is set to be $\mathbf{Z}$ and $\mathbf{T}_2$ is a D_2-sample circular shifted version of $\mathbf{T}_1$, i.e., $T_2[i] = T_1[\langle i - D_2 \rangle_N]$, as depicted in Fig. 1. Based on (12), (13) and the fact that $(\mathbf{\Phi}_m\mathbf{A}_m)^H\mathbf{\Phi}_m\mathbf{A}_m = N\mathbf{I}$, $\mathbf{P}^H\mathbf{P}$ for $K = 2$ can be written as

$$\mathbf{P}^H\mathbf{P} = \begin{bmatrix} N \times \mathbf{I} & \mathbf{A}_1^H \mathbf{\Phi}_1^H \mathbf{\Phi}_2 \mathbf{A}_2 \\ \mathbf{A}_2^H \mathbf{\Phi}_2^H \mathbf{\Phi}_1 \mathbf{A}_1 & N \times \mathbf{I} \end{bmatrix}. \tag{25}$$

If all the entries in $\mathbf{A}_1^H \mathbf{\Phi}_1^H \mathbf{\Phi}_2 \mathbf{A}_2$ and $\mathbf{A}_2^H \mathbf{\Phi}_2^H \mathbf{\Phi}_1 \mathbf{A}_1$ are small enough, we can achieve the approximation $\mathbf{P}^H\mathbf{P} \approx c\mathbf{I}$. Note that $\mathbf{A}_2^H \mathbf{\Phi}_2^H \mathbf{\Phi}_1 \mathbf{A}_1$ is complex symmetric to $\mathbf{A}_1^H \mathbf{\Phi}_1^H \mathbf{\Phi}_2 \mathbf{A}_2$, and thus we only need to focus on $\mathbf{A}_1^H \mathbf{\Phi}_1^H \mathbf{\Phi}_2 \mathbf{A}_2$. Substituting (25) into (23), the criterion for $K = 2$ can be written as

$$\mathbf{S} = \{\mathbf{T}_1, \mathbf{T}_2\} = \arg\min_{\{\mathbf{T}_1', \mathbf{T}_2'\}} \left\| \mathbf{A}_1^H \mathbf{\Phi}_1^H \mathbf{\Phi}_2 \mathbf{A}_2 \right\|_F. \tag{26}$$

After some manipulation, we have

$$\left[\mathbf{A}_1^H \mathbf{\Phi}_1^H \mathbf{\Phi}_2 \mathbf{A}_2 \right]_{l,g} = e^{j\varpi} \sum_{i=0}^{N-1} e^{-j2\pi i\left[\theta_{1,2} + M(D_2+g-l)\right]/N}. \tag{27}$$

where $\varpi = (\varphi_2 - \varphi_1) - \pi M\left[(1-l)^2 - (1-D_2-g)^2\right]/N$. Then, substituting (27) into (26), we obtain

$$\mathbf{S} = \{\mathbf{T}_1, \mathbf{T}_2\} = \arg\min_{M, D_2} \sqrt{\sum_{l=1}^{\Delta} \sum_{g=1}^{\Delta} \left| \sum_{i=0}^{N-1} e^{-j2\pi i\left[\theta_{1,2} + M(D_2+g-l)\right]/N} \right|^2}, \tag{28}$$

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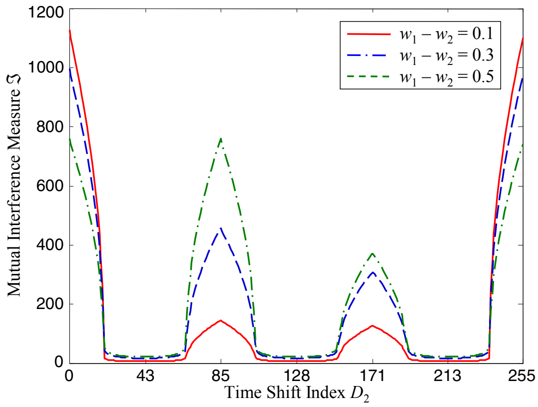

Fig. 2 The mutual interference measure $\Im$ versus the time shift index D_2 for $M = 3$, $N = 256$ and $\Delta = 20$.

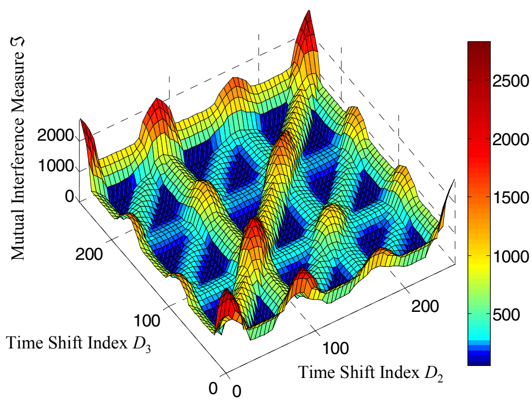

Fig. 3 The mutual interference measure $\Im$ versus the time shift indices (D_2, D_3) for $M = 3$, $N = 256$ and $\Delta = 20$, with $w_1 = 0.1$, $w_2 = -0.3$ and $w_3 = 0.5$.

TABLE 1 The optimal training sequence index/indices that achieve the minimum value of $\Im$ for $M = 3$ and $N = 256$.

$K = 2$ (D_2)	$K = 3$ (D_2, D_3)
43, 128, 213	(28, 57), (28, 142), (28, 228), (114, 57), (114, 142), (114, 228), (199, 57), (199, 142), (199, 228)

where $|\exp(j\varpi)| = 1$ that can be ignored. Note that the determination of the optimal training sequence set $\mathbf{S} = \{\mathbf{T}_1, \mathbf{T}_2\}$ relies only on the parameters M and D_2. For simplification, we define $\lambda_{l,g} = \left|\sum_{i=0}^{N-1} \exp\left(-j2\pi i\left[\theta_{1,2} + M(D_2 + g - l)\right]/N\right)\right|$ which can be represented by using a two variable function

$$Q(M,q) \equiv \left|\sum_{i=0}^{N-1} \exp\left(-j2\pi i\left[\theta_{1,2} + Mq\right]/N\right)\right|, \tag{29}$$

where q is an integer. By combining the terms with the same value of q into a term, we define the *interference metric* as

$$\Im(M,D_2) = \sqrt{\sum_{l=1}^{\Delta}\sum_{g=1}^{\Delta}\lambda_{l,g}^2} = \sqrt{\sum_{s=-\Delta+1}^{\Delta-1}\left(\Delta - |s|\right)Q^2(M,D_2+s)}\cdot \tag{30}$$

Consequently, given M, we have the optimal value of D_2 as

$$D_2^{(opt)} = \arg\min_{D_2} \Im(M,D_2)\cdot \tag{31}$$

Since $\theta_{1,2}$ is a random variable unknown to the receiver, the selection of $D_2^{(opt)}$ must be robust to the variation of $\theta_{1,2}$. Based on the assumption that $-0.5 < w_k < 0.5$, we have the value of $\theta_{1,2}$ in the range $-1 < \theta_{1,2} < 1$. Based on our investigation and derivation, we conclude that the optimal value of D_2 is to make

$$\left\langle MD_2^{(opt)}\right\rangle_N \approx N/2\cdot \tag{32}$$

Note that there are M possible values of D_2 that achieve (32).

Fig. 2 shows the values of the interference metric $\Im(M, D_2)$ versus the time shift index D_2 for $M = 3$, $N = 256$, $\Delta = 20$, $K = 2$ and different values of $\theta_{1,2}$. As we expected, a small value of $\Im(M, D_2)$ can be obtained when $\left\langle MD_2\right\rangle_N \approx N/2$, regardless of the variation of $\theta_{1,2}$. Correspondingly, the M possible values of D_2 that achieve (32) are 43, 128 and 213, as shown in Table 1.

D. ROTS for the Scenario with More than Two BSs

It is noted that the results for $K = 2$ are not the feasible solution for the scenario with more than two BSs. For the example with $M = 3$ and $N = 256$, if $\mathbf{T}_1 = \mathbf{Z}$ and $\mathbf{T}_2 = \mathbf{Z}\langle 43\rangle$ are selected, $\mathbf{T}_3 = \mathbf{Z}\langle 128\rangle$ or $\mathbf{T}_3' = \mathbf{Z}\langle 213\rangle$ is not a feasible training sequence for the 3-rd BS. Although these two sequences have very low cross-correlation with $\mathbf{T}_1$, they have very high cross-correlation with $\mathbf{T}_2$. Hence, in the scenario with more than two BSs, the optimal set of training sequences must be jointly considered. By setting $\mathbf{T}_1 = \mathbf{Z}$, the optimal relative circular-shift set $\mathbf{D} = \{D_2,\ldots, D_K\}$, corresponding to $\mathbf{T}_2,\ldots, \mathbf{T}_K$, is determined by

$$\mathbf{D}^{(opt)} = \left\{D_2^{(opt)}, \cdots, D_K^{(opt)}\right\} = \arg\min_{\mathbf{D}} \Im(M,\mathbf{D})\cdot \tag{33}$$

where the interference metric of the total mutual interference is defined as

$$\Im(M,\mathbf{D}) = \sum_{m=1}^{K}\sum_{\substack{n=1\\ n\neq m}}^{K}\left\|(\mathbf{\Phi}_m\mathbf{A}_m)^H\mathbf{\Phi}_n\mathbf{A}_n\right\|_F\cdot \tag{34}$$

Based on the same arguments for the scenario with two BSs, we conclude that the optimal set $\mathbf{D}^{(opt)}$ must satisfy

$$\left\langle MD_k^{(opt)}\right\rangle_N \approx (k-1)N/K,\ \text{for } 2 \le k \le K. \tag{35}$$

Fig. 3 shows the values of the interference metric $\Im(M, \mathbf{D})$ versus the time shift indices (D_2, D_3) for $M = 3$, $N = 256$, $\Delta = 20$ and $K = 3$, with the CFOs $w_1 = 0.1$, $w_2 = -0.3$ and $w_3 = 0.5$. Accordingly, there are 18 positions achieving the minimum value of $\Im$. Without considering the ordering, there are 9 pairs of ($D_2^{(opt)}$, $D_3^{(opt)}$) that minimize the mutual interference, as shown in Table 1.

IV. Simulation Results

In the simulations, the total number of subcarriers is $N = 256$, and the length of CP is $N/4$. The channel model is assumed to be a 3-tap multi-path Rayleigh fading channel with exponentially decayed power-delay profile, and the channel gains remain unchanged during an OFDM symbol interval. The training sequences used in the BSs are the proposed ROTS. The estimation performance, represented by the mean square error (MSE), of the proposed RMCE scheme is presented, and

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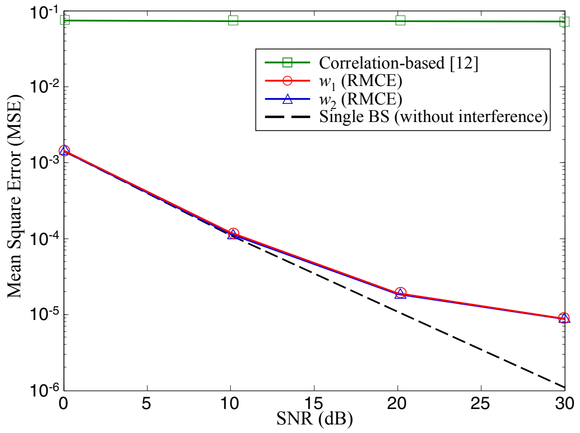

Fig. 4 Estimation performance under the scenario with two BSs.

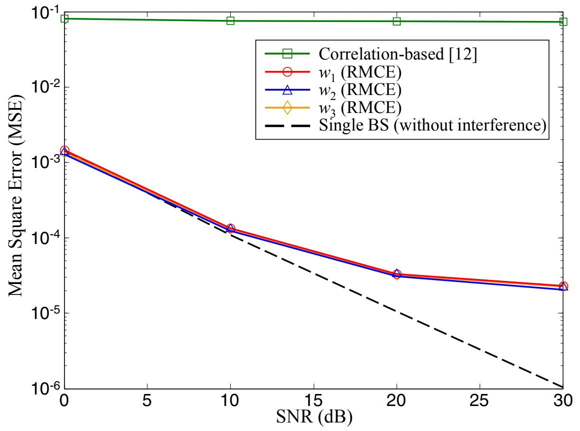

Fig. 5 Estimation performance under the scenario with three BSs.

that of the conventional correlation-based scheme using pseudo-noise (PN) sequences [12] is shown for comparison. The results are obtained by averaging over 30000 runs of simulation, where the CFO values are generated randomly in (-0.5, +0.5). We also show the performance of the scenario with only one BS, i.e., the mutual interference-free case, as the baseline, since it is the best achievable performance. Note that the estimation performance is based on the signal received in an OFDM symbol and all the K CFOs are estimated simultaneously.

Fig. 4 shows the MSE performance versus the signal-to-noise ratio (SNR) under the scenario with two BSs, i.e., $K = 2$. The applied ROTS is obtained based on the parameters $M = 3$ and $N = 256$, and the chosen parameter is $D_2 = 43$. It is found that the performance of the correlation-based scheme is severely degraded by the mutual interference. For the proposed scheme, although the signal-to-interference ratio (SIR) is 0 dB in the scenario with two BSs, the estimation performance still approaches the baseline, implying that the mutual interference is eliminated effectively.

Fig. 5 shows the MSE performance under the scenario with three BSs, i.e., $K = 3$. The chosen parameters for ROTS are $D_2 = 28$ and $D_3 = 57$. Note that the SIR is -3 dB in the scenario with three BSs. The proposed RMCE scheme still performs very well. In the normal operation region, i.e., SNR ≤ 15 dB, the estimation performance is still very close to the baseline. As a result, the estimation performance is almost not impacted by the mutual interference.

V. Conclusion

In this work, we have proposed an effective, low-complexity estimation schemes, RMCE, for simultaneous multi-CFO estimation in CoMP-OFDM systems over multi-path fading channels. We propose the design of training sequences, ROTS, that are robust to the variation in multiple CFOs. The well-designed training sequences lead to very low mutual interference between the signals transmitted by different BSs. As a result, the RMCE scheme achieves very good estimation performance under the interference coming from other multi-point transmission BSs. The proposed ROTS can be applied to the scenarios with different numbers of BSs joining the CoMP transmission. However, if more BSs join the CoMP transmission, the estimation performance will be degraded in the very high SNR region.

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