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Kernel Estimation for a Non-Parametric Cointegrating Regression Model

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Outline

Introduction

Question

How do we model this relationship?

Solution?

Ordinary Least Squares Regression

Solution?

Ordinary Least Squares Regression

$$y_t = \beta_0 + \beta_1 x_t + \epsilon_t$$

Results of performing an OLS regression

	Estimate	Std. Error	t value	p value	
β_0	1.595	0.526	3.028	0.0028	**
β_1	1.044	0.065	-16.2	$<$ 2e16	***

Multiple R-squared: 0.5698, Adjusted R-squared: 0.5676

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no

no

They were independent random walks

Why OLS doesn't hold with nonstationary processes

$$y_t - \beta_0 + \beta_1 x_t = \epsilon_t$$

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Why OLS doesn't hold with nonstationary processes

nonstationary

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Why OLS doesn't hold with nonstationary processes

stationary

$$y_t - \beta_0 + \beta_1 x_t = \epsilon_t$$

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Why OLS doesn't hold with nonstationary processes

nonstationary $\neq$ stationary

$$y_t - \beta_0 + \beta_1 x_t = \epsilon_t$$

however . . .

What if . . .

$$y_t - \beta_0 + \beta_1 x_t = \epsilon_t$$

Shortcomings

- Nonlinearity in economic processes

Shortcomings

- Nonlinearity in economic processes

Example

The Cobb Douglas production function

$$y = \phi x_1^{\alpha} x_2^{\beta}$$

where y is the total production, x_1 is labour input and x_2 is capital input.

Shortcomings

- Nonlinearity in economic processes
- Relationship unchanged over an extended period of time

Nonlinear cointegration model

$$y_t = f(x_t) + \text{error}$$

- y_t, x_t nonstationary
- f nonlinear
- stationary error

Question

What is f?

Parametric vs Non-Parametric

Parametric

Parametric vs Non-Parametric

Parametric

- Misspecification

Parametric vs Non-Parametric

Parametric

- Misspecification

Non-Parametric

Parametric vs Non-Parametric

Parametric

- Misspecification

Non-Parametric

- Let the data speak for itself

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Nonlinear + Nonstationary

	Stationary	Nonstationary
Linear	–	–
Nonlinear	–	✓

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Difficulties

Nonstationary + Nonparametric = ?

Difficulties

Nonstationary + Nonparametric = wandering +

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Difficulties

Nonstationary + Nonparametric = wandering + local behaviour

Difficulties

Nonstationary + Nonparametric = Difficult

Difficulties

Nonstationary + Nonparametric = Reduced rate of convergence

Difficulties

Nonstationary + Nonparametric = New techniques required

Local Time

Definition (Local Time)

The local time up to time t at x is

$$L(t,x) = \lim_{\epsilon \downarrow 0} \frac{1}{2\epsilon} \int_0^t I(|B_s - a|)ds$$

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Local Time

Definition (Local Time)

The local time up to time t at x is

$$L(t,x) = \lim_{\epsilon \downarrow 0} \frac{1}{2\epsilon} \int_0^t I(|B_s - a|)ds$$

$L(t,x)$ is the amount of time that the path of B_t spends at x up to time t

Kernel Regression

the Nadaray-Watson Kernel Estimator:

$$\hat{f}(x) = \frac{\sum_{t=1}^{n} y_t K_h(x_t - x)}{\sum_{t=1}^{n} K_h(x_t - x)}$$

where

$$K_h(x) = \frac{1}{h} K(x/h)$$

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Note

h controls the size of the neighbourhood, and the rate at which K_h decays

The Equation

$$y_t = f(x_t) + u_t$$

- $x_t = \sum_{j=1}^{t} \epsilon_j, \quad \epsilon_j \sim iid(0,1)$
- Nonparametric estimator of f: NW Kernel Estimator.
- $u_t = ?$

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Previous work

- Wang, Phillips (2009): error process as a martingale difference sequence

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- We consider the error process as a linear process

Linear Process

Definition (Linear Process)

Let u_t be defined by

$$u_t = \sum_{j=0}^{\infty} \phi_j \epsilon_{t-j}, \quad t \geq 1$$

where $\{\epsilon_i\}$ is a sequence of iid random variables with mean 0 and variance 1.

Linear Process

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Let u_t be defined by

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- linear aggregation of random shocks

Methodology

$$y_t = f(x_t) + u_t$$

Recall the kernel estimator:

$$\hat{f}(x) = \frac{\sum_{t=1}^{n} y_t K_h(x_t - x)}{\sum_{t=1}^{n} K_h(x_t - x)}$$

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Methodology

$$\hat{f}(x) - f(x) = \frac{\sum_{t=1}^{n} u_t K_h(x_t - x)}{\sum_{t=1}^{n} K_h(x_t - x)} + \frac{\sum_{t=1}^{n} (f(x_t) - f(x)) K_h(x_t - x)}{\sum_{t=1}^{n} K_h(x_t - x)}$$

Methodology

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