

第四章 正弦交流电路

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4—1 有一正弦电压 $u = 311\sin(100\pi t + \frac{\pi}{3})$ V, 试求 (1) 角频率 ω 、频率 f 、周期 T 、有效值 U 和初相位 φ_u ; (2) $t = 0$ 和 $t = 0.1$ s 时电压的瞬时值; (3) 画出电压的波形图。

(答案: (1) 314 rad/s ; 50 Hz ; 0.02 s ; 220 V ; $\varphi_u = \frac{\pi}{3} \text{ rad}$

(2) $u_{(t=0)} = u_{(t=0.1)} = 269.3 \text{ V}$)

① $\omega = 100\pi = 314 \text{ rad/s}$

② $f = \frac{\omega}{2\pi} = 50 \text{ Hz}$

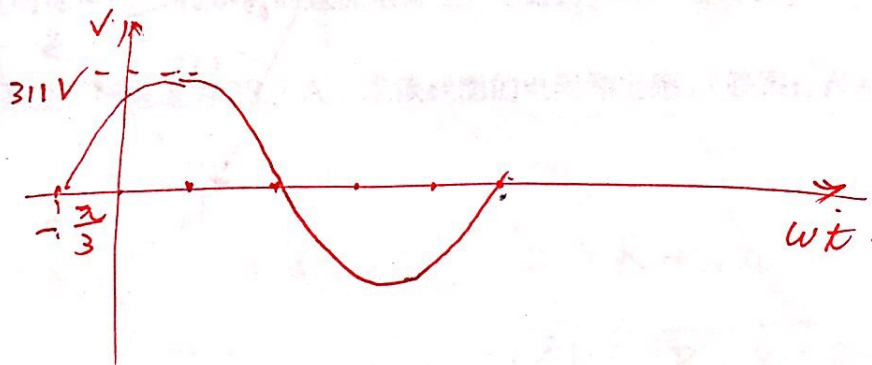
$T = \frac{1}{f} = 0.02 \text{ s}$

$U = \frac{311}{\sqrt{2}} = 220 \text{ V}$

$\varphi_u = \frac{\pi}{3} \text{ rad}$

(2) $u_{t=0} = 311 \sin \frac{\pi}{3}$

$u_{t=0.1} = 311 \sin(100\pi \times 0.1 + \frac{\pi}{3}) = 311 \sin \frac{7\pi}{3}$
 $= 269.3 \text{ V}$



① 波形图表示. 相对相角, 绝对相角

② 相量的画法和表示. \dot{U} 有效值 U , 瞬时值 u

③ $\frac{U}{I} = |Z| = \sqrt{R^2 + X^2}$

④ 瞬时值不带直接参考运算

4—2 已知 $i_1 = 10\sin(314t + 30^\circ)$ A, $i_2 = 10\sin(314t - 60^\circ)$ A, $i = i_1 + i_2$ 。试用相量法求 i , 并画出三个电流的相量图。(答案: $i = 10\sqrt{2}\sin(314t - 15^\circ)$ A)

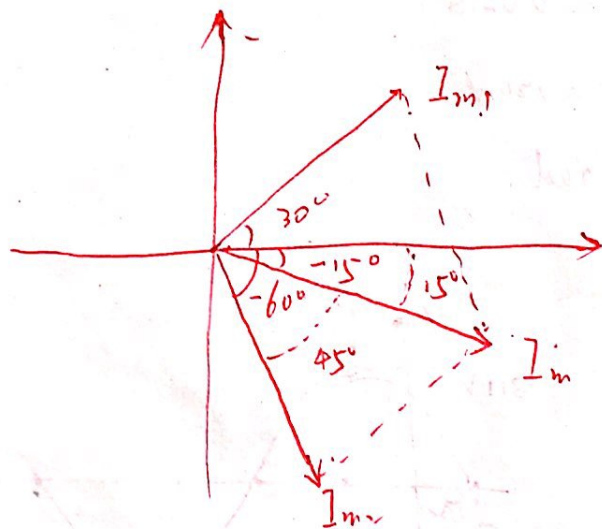
$$\dot{I}_{m1} = 10 \angle 30^\circ = \cancel{10 \cos 30^\circ} + j \underset{\checkmark}{10 \sin 30^\circ} = 5\sqrt{3} + 5j$$

$$\dot{I}_{m2} = 10 \angle -60^\circ = \cancel{10 \cos 60^\circ} + j \underset{\checkmark}{10 \sin 60^\circ} = 5 - 5\sqrt{3}j$$

$$\dot{I}_m = \dot{I}_{m1} + \dot{I}_{m2} = 13.66 - 3.66j$$

$$= 14.14 \angle -15^\circ$$

$$i = 14.14 \sin(314t - 15^\circ) \text{ A}$$



试用

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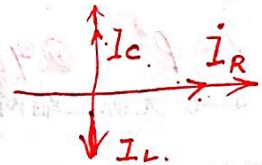
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 4—3 电压 $u = 220\sqrt{2} \sin 314t$ V, 分别作用在 (1) $R = 100 \Omega$; (2) $L = 0.5$ H;

 (3) $C = 10 \mu\text{F}$ 的元件上。试求 i_R 、 i_L 、 i_C , 并画出相量图。(答案: $i_R = 2.2\sqrt{2} \sin 314t$ A;

 $i_L = 1.4\sqrt{2} \sin(314t - 90^\circ)$ A; $i_C = 0.69\sqrt{2} \sin(314t + 90^\circ)$ A)

$$\begin{aligned}
 (1) \dot{I}_R &= \frac{\dot{U}}{R} = \frac{220\angle 0^\circ}{100} = 2.2\angle 0^\circ \text{ A} \quad \dot{U} = 220\angle 0^\circ \quad jX_L = j\omega L = 314 \times 0.5 \angle 90^\circ \\
 \dot{I}_L &= \frac{\dot{U}}{jX_L} = \frac{220\angle 0^\circ}{j314 \times 0.5} = 1.4\angle -90^\circ \text{ A} \quad \dot{I}_L = 1.4\sqrt{2} \sin(314t - 90^\circ) \text{ V} \\
 -jX_C &= -j \frac{1}{314 \times 10 \times 10^{-6}} = -j \frac{1}{314 \times 10^{-5}} \angle -90^\circ \\
 \dot{I}_C &= \frac{\dot{U}}{-jX_C} = \frac{220}{-j \frac{1}{314 \times 10^{-5}}} \angle 90^\circ = 0.69\angle 90^\circ \text{ A} \quad \dot{I}_C = 0.69\sqrt{2} \sin(314t + 90^\circ) \text{ A}
 \end{aligned}$$


 4—4 一个电感线圈接在 $U = 120$ V 的直流电源上, 电流为 20 A, 若接在 $f = 50$ Hz, $U = 220$ V 的交流电源上, 则电流为 28.2 A, 求该线圈的电阻和电感。(答案: $R = 6 \Omega$;

 $L = 15.9$ mH)

$$R = \frac{U}{I} = \frac{120}{20} = 6 \Omega$$

$$Z = R + jX_L$$

$$\omega L = \frac{U}{I} = \frac{220}{28.2}$$

$$|Z| = \sqrt{R^2 + X_L^2}$$

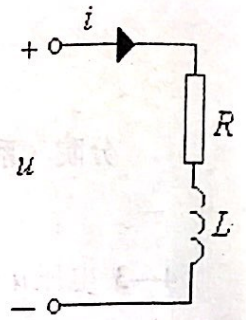
$$L = \frac{U}{I \cdot 2\pi f} = \frac{220}{28.2 \cdot 6.28 \times 50} = \frac{8792}{1000} = 8.792 \text{ mH}$$

$$\begin{cases} \frac{U}{I} = |Z| = \sqrt{R^2 + X_L^2} \\ X_L = \omega 2\pi f \cdot L \end{cases}$$

$$\Rightarrow \quad \text{Resulting in } L = 15.9 \text{ mH}$$

$$L = 15.9 \text{ mH}$$

4-5 如图所示电路中 $R = 4 \Omega$ ，频率 $f = 50 \text{ Hz}$ 时，电路的功率 $P = 16 \text{ W}$ ，功率因数 $\lambda = 0.8$ ，求 $f = 25 \text{ Hz}$ 时电路的电流



题 4-5 电路图

I' ，有功功率 P' ，无功功率 Q' 和视在功率 S' 。（答案：

$I' = 2.34 \text{ A}$ ； $P' = 21.9 \text{ W}$ ； $Q' = 8.21 \text{ Var}$ ； $S' = 23.4 \text{ VA}$ ）

$$P = I^2 R \Rightarrow I = \sqrt{\frac{P}{R}} = 2 \text{ A}$$

$$P = \frac{U^2}{R} \Rightarrow U_R = 8 \text{ V}$$

$$\cos \varphi = 0.8 \Rightarrow \varphi = 37^\circ$$

$$P = UI \cos \varphi \Rightarrow U = 10 \text{ V}$$

$$I' = \frac{U}{|Z|} = \frac{10}{\sqrt{4^2 + 1.5^2}} = 2.34 \text{ A}$$

$$\frac{X_L}{R} = \tan 37^\circ \Rightarrow X_L = 3 \Omega$$

$$2\pi \cdot 50 \cdot L = 3 \Rightarrow X_L = 2\pi \cdot 25 \cdot L = 1.5 \Omega$$

$$\varphi' = \arctan \frac{1.5}{4} = 20.5^\circ$$

$$\cos \varphi' = 0.94$$

$$\therefore P' = UI' \cos \varphi' = 21.9 \text{ W}$$

$$Q' = UI' \sin \varphi' = 8.21 \text{ Var}$$

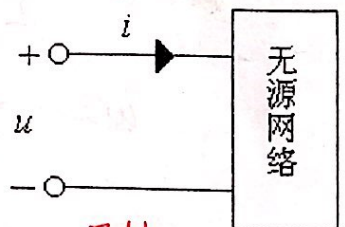
$$S = UI = 23.4 \text{ VA}$$

4-6 无源二端网络如图所示，输入端电压 $u = 220\sqrt{2} \sin(314t + 20^\circ) \text{ V}$ ，电流

$i = 4.4\sqrt{2} \sin(314t - 33^\circ) \text{ A}$ ，求该二端网络的等效电路（两个元件串联）和元件参数值；

并求二端网络的功率因数及输入的有功功率和无功功率。（答案： $R = 30 \Omega$ ；

$L = 0.127 \text{ H}$ ； $\cos \varphi = 0.6$ ； $P = 580.8 \text{ W}$ ； $Q = 773 \text{ Var}$ ）



题 4-6 电路图

$$\dot{U} = 220 \angle 20^\circ$$

$$\dot{I} = 4.4 \angle -33^\circ$$

$$Z = \frac{\dot{U}}{\dot{I}} = \frac{220}{4.4} \angle 53^\circ = 50 + 40j \Rightarrow R = 30 \Omega$$

$$\omega L = 40 \Omega$$

$$\lambda = \cos \varphi = \cos 53^\circ = 0.6$$

$$L = \frac{40}{2\pi \cdot 314} = 0.127 \text{ H}$$

$$S_R = UI = 220 \cdot 4.4 = 968 \text{ VA}$$

$$P = P_R \cos \varphi = 580.8 \text{ W}$$

$$Q = P_R \sin \varphi = 773 \text{ Var}$$

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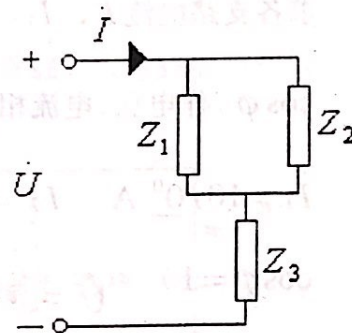
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4—7 在图示电路中, 电源电压 $\dot{U} = 220 \angle 0^\circ \text{ V}$, 阻抗

$$Z_1 = 4.4 + j2.65 \Omega, \quad Z_2 = 21.69 - j12.6 \Omega,$$

$Z_3 = 1.5 + j2.6 \Omega$ 求电路的有功功率, 无功功率和视在功率。

(答案: $P = 5.39 \text{ kW}$; $Q = 4 \text{ kVar}$; $S = 6.71 \text{ kVA}$)



$$Z = Z_1 // Z_2 + Z_3 = \frac{Z_1 Z_2}{Z_1 + Z_2} + Z_3 = 5.78 + j4.31 = 7.15 \angle 36.7^\circ \quad \text{题 4-7 电路图}$$

$$\dot{I} = \frac{\dot{U}}{Z} = \frac{220 \angle 0^\circ}{7.15 \angle 36.7^\circ} = 30.77 \angle -36.7^\circ$$

$$P = UI \cos \varphi = 220 \cdot 30.77 \cos 36.7^\circ \approx 5.42 \text{ kW}$$

$$Q = UI \sin \varphi = 220 \cdot 30.77 \sin 36.7^\circ \approx 4.0 \text{ kVar}$$

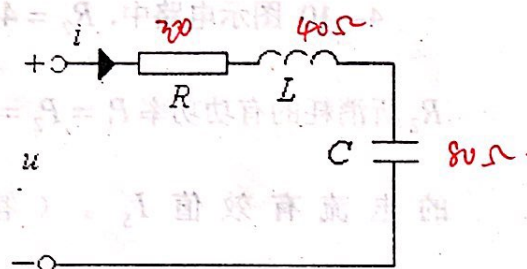
$$S = UI = 6.71 \text{ kVA}$$

4—8 图示电路中, $u = 100\sqrt{2} \sin 314t \text{ V}$,

$R = 30 \Omega$, $X_L = 40 \Omega$, $X_C = 80 \Omega$ 。(1) 求电流 i 及

功率因数 λ ; (2) 若 u 的有效值不变, 调节其频率使电路

谐振, 求谐振时的电流 I_0 及谐振频率 f_0 。(答案: (1)



$$i = 2\sqrt{2} \sin(314t + 53.1^\circ) \text{ A}; \quad \lambda = 0.6 \quad (2)$$

$$I_0 = 3.33 \text{ A}; \quad f_0 = 70.6 \text{ Hz}$$

$$\lambda = \cos \varphi = 0.6$$

$$X_L = 314L = 40 \Omega \Rightarrow L =$$

$$X_C = \frac{1}{314C} = 80 \Omega \Rightarrow C =$$

$$\dot{U} = 100 \angle 0^\circ$$

$$Z = 30 + j(40 - 80) = 30 - j40 = 50 \angle -53^\circ$$

$$\dot{I} = \frac{\dot{U}}{Z} = 2 \angle 53^\circ \Rightarrow i$$

$$2\pi f_0 = \frac{1}{\sqrt{LC}}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = 70.6 \text{ Hz}$$

谐振时, $X_L = X_C$ 是纯阻 $Z = R = 30 \Omega$

4—9 在图示电路中, 已知:

$$Z_1 = 12 + j16 \Omega, Z_2 = 10 - j20 \Omega, \dot{U} = 120 + j160 \text{ V},$$

求各支路电流 \dot{I} 、 \dot{I}_1 、 \dot{I}_2 , 总有功功率 P 及总功率因数

$\cos \varphi$, 作电压、电流相量图。(答案: $\dot{I} = 10 / 53.13^\circ \text{ A}$

$$\dot{I}_1 = 10 / 0^\circ \text{ A} \quad \dot{I}_2 = 8.94 / 116.56^\circ \text{ A} \quad P = 2000 \text{ W}$$

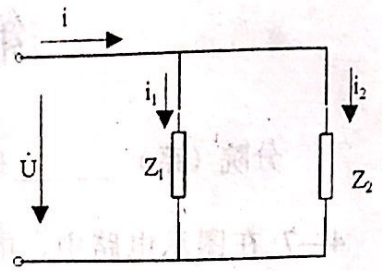
$$\cos \varphi = 1) \quad \dot{U} = 200 / 53.13^\circ \quad Z_1 = 20 / 53.13^\circ \quad Z_2 = 22.36 / 63.43^\circ$$

$$Z_1 = |Z_1| \angle \varphi_1, \quad \dot{I}_1 = \frac{\dot{U}}{Z_1} = 10 / 0^\circ \quad Z = Z_1 // Z_2 = 12 \angle 0^\circ$$

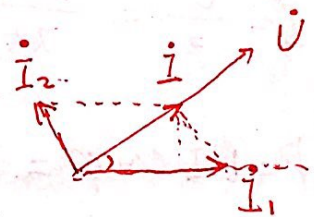
$$Z_2 = |Z_2| \angle \varphi_2, \quad \dot{I}_2 = \frac{\dot{U}}{Z_2} = 8.94 / 116.56^\circ$$

$$\dot{I} = \dot{I}_1 + \dot{I}_2 = 10 / 53.13^\circ \quad \dot{U} \text{ 与 } \dot{I} \text{ 同相位 } \varphi = 0 \quad \cos \varphi = 1$$

$$P = UI \cos \varphi = UI = 200 \times 10 = 2000 \text{ W}$$



习题 4-9 电路图

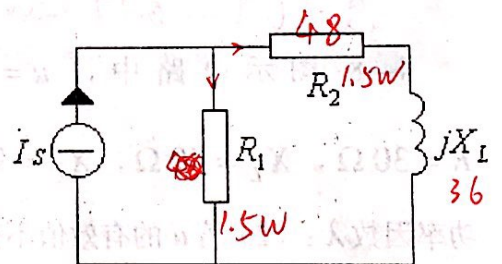


4—10 图示电路中, $R_2 = 48 \Omega$, $X_L = 36 \Omega$, R_1 、

R_2 所消耗的有功功率 $P_1 = P_2 = 1.5 \text{ W}$, 求 R_1 及电流源

的电流有效值 I_s 。(答案: $R_1 = 75 \Omega$;

$$I_s = 0.303 \text{ A})$$



或求电压有效值

题 4—10 电路图

$$① \text{ 支路 2 中 } Z = R_2 + jX_L = 48 + j36 \quad \angle \varphi = 60^\circ$$

$$P_2 = I^2 R_2 \Rightarrow I_2 = \sqrt{\frac{P_2}{R_2}} = 0.177 \text{ A} \quad \therefore \lambda = \cos 37^\circ = 0.8$$

$$② \text{ 令 } \dot{U} = U \angle 0^\circ \quad \text{由 } \begin{cases} UI_2 \times 0.8 = 1.5 \text{ W} \\ UI_1 = 1.5 \text{ W} \end{cases} \quad \text{得 } I_1 = 0.8 I_2 = 0.8 \times 0.177 = 0.142$$

$$\text{由 } I_1^2 R_1 = 1.5 \text{ W} \quad \text{得 } R_1 = 74.8 \Omega$$

$$③ \dot{I}_s = \dot{I}_1 + \dot{I}_2$$

$$= 0.142 \angle 0^\circ + 0.177 \angle -37^\circ \quad I_s = |\dot{I}_s| = 0.303 \text{ A}$$

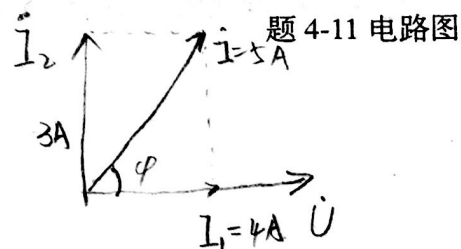
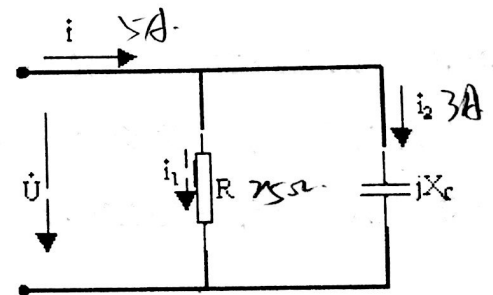
$$= 0.253 - j0.107$$

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4—11 在图示电路中, 电流有效值 $I = 5\text{ A}$, $I_2 = 3\text{ A}$, $R = 25\ \Omega$, 求电路的阻抗 $|Z|$

为多少? (答案: $|Z| = 20\ \Omega$)



解. 由向量关系可得

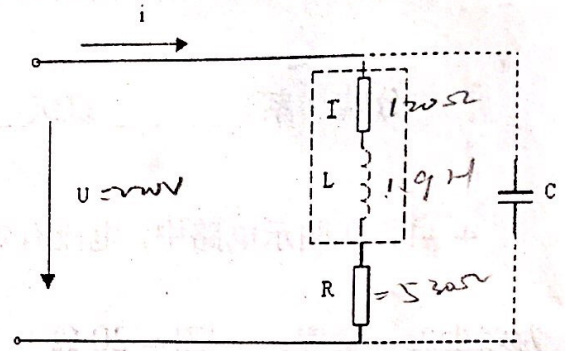
$$\varphi = \arcsin \frac{3}{5}$$

$$I_1 = I \cos \varphi = 4\text{ A}$$

$$U = I_1 R = 4 \cdot 25 = 100\text{ V}$$

$$|Z| = \frac{U}{I} = \frac{100}{5} = 20\ \Omega$$

4—12 日光灯电路如图所示，灯管电阻 $R=530\ \Omega$ ，镇流器电阻 $r=120\ \Omega$ ，电感 $L=1.9\text{ H}$ ，接在 220 V ， 50 Hz 交流电源上，求电路电流；灯管电压；镇流器电压； P 、 Q 、 S 及 $\cos\varphi_1$ ，要把电路功率因数提高到 $\cos\varphi=0.85$ ，问在日光灯两端应并多大电容？（答案：



习题 4—12 电路图

$I=0.25\text{ A}$ $U_R=132.16\text{ V}$ $U_{rL}=152.14\text{ V}$
 $P=40.63\text{ W}$ $Q=37.29\text{ Var}$ $S=55.15\text{ VA}$ $\cos\varphi_1=0.74$ $C=0.8\ \mu\text{F}$)

$$Z = r + R + j\omega L = 120 + 530 + j2\pi \cdot 50 \cdot 1.9 = 650 + j596.6 \quad |Z| = 882\ \Omega$$

$$\varphi = \arctan \frac{596.6}{650} = 42.5^\circ$$

$$\cos\varphi = 0.74$$

$$I = \frac{220}{|Z|} = 0.25\text{ A}$$

$$U_R = I \cdot R = 0.25 \cdot 530 = 132.5\text{ V}$$

$$U_{rL} = I \cdot |Z_L| = 0.25 \cdot \sqrt{120^2 + 596.6^2} = 152.14\text{ V}$$

$$S = U \cdot I = 220 \cdot 0.25 = 55\text{ VA}$$

$$Q = UI \sin\varphi = 37.2\text{ Var}$$

$$P = UI \cos\varphi = 40.7\text{ W}$$

$$\cos\varphi_1 = 0.85 \quad \tan\varphi_1 = 0.62 \quad \tan\varphi = 0.92$$

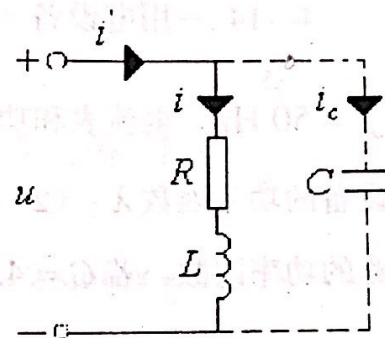
$$C = \frac{P}{\omega U^2} (\tan\varphi - \tan\varphi_1)$$

$$= \frac{40.7}{2\pi \cdot 50 \cdot (220)^2} (0.92 - 0.62) = 0.8 \times 10^{-6}\text{ F} = 0.8\ \mu\text{F}$$

4—13 在图示 R 、 L 串联电路中，已知 $i = 2.82\sqrt{2} \sin 314t$ A， $R = 60 \Omega$ ， $L = 0.255$ H，求 (1)

若在电路两端并联 $C = 11.3 \mu\text{F}$ 的电容，电源供出电流的有效值变化了多少？(2) 并联电容后的功率因数。(答案：(1)

$\Delta I = 0.71$ A； (2) $\lambda' = 0.8$)



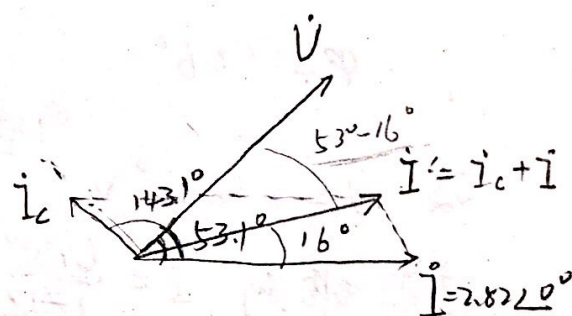
题 4—13 电路图

解： $\omega = 314$ $Z = 60 + j\omega L = 60 + j80$

$$\varphi = \arctan \frac{80}{60} = 53.1^\circ$$

$$U = \frac{I \cdot R}{\cos \varphi} = \frac{282 \cdot 60}{\cos 53.1} = 282 \text{ V}$$

$$\dot{U} = U \angle 53.1^\circ = 282 \angle 53.1^\circ$$



$$\dot{i}_c = \frac{U}{X_c} \angle 90^\circ = \frac{282 \angle 53.1^\circ}{314 \times 11.3 \times 10^{-6} \angle 90^\circ} = 1 \angle 143.1^\circ$$

$\varphi' = \dot{i}'$ 和 \dot{U} 的夹角

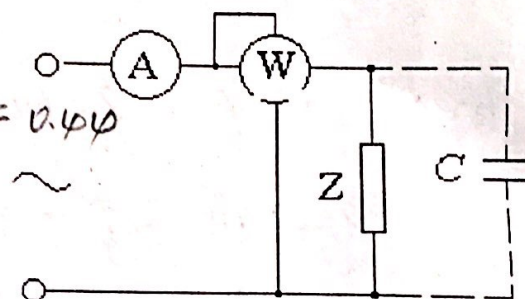
$$\begin{aligned} \dot{I}' &= \dot{I} + \dot{I}_c = 2.82 \angle 0^\circ + 1 \angle 143.1^\circ \\ &= 2.82 + (-0.8 + j0.6) \\ &= 2.02 + j0.6 \\ &= 2.11 \angle 16.5^\circ \end{aligned}$$

$$\therefore \Delta I = 2.82 - 2.11 = 0.71 \text{ A}$$

$$\lambda' = \cos(53.1 - 16.5) = 0.8$$

4—14 一用电设备（电感性负载）接于 220 V 的交流电源上，如图所示，电源频率 $f = 50 \text{ Hz}$ ，电流表和功率表测得的电流 $I = 0.41 \text{ A}$ ，功率 $P = 40 \text{ W}$ 。试求（1）该电器设备的功率因数 λ ；（2）因该电器设备是电感性负载，故可用并联电容器 C 来提高整个电路的功率因数。若 $C = 4.75 \mu\text{F}$ ，电流表的读数和整个电路的功率因数为多少？（答案：

（1） $\lambda = 0.4435$ ；（2） $I' = 0.186 \text{ A}$ ； $\lambda' = 0.977$ ）



题 4—14 电路图

$$\because P = UI \cos \varphi \Rightarrow \lambda = \cos \varphi = \frac{P}{UI} = \frac{40}{220 \times 0.41} = 0.4435$$

$$\therefore \varphi = 63.6^\circ$$

$$\text{设 } \dot{U} = 220 \angle 0^\circ \text{ V}$$

$$\text{并联电容前 } \dot{I} = \frac{U \angle 0^\circ}{|Z| \angle \varphi} = 0.41 \angle -63.6^\circ$$

$$\dot{I}_C = \frac{\dot{U}}{-jX_C} = \frac{220 \angle 0^\circ}{\frac{1}{2\pi \cdot 50 \cdot 4.75 \times 10^{-6}} \angle 90^\circ} = 0.328 \angle 90^\circ$$

$$\dot{I}' = \dot{I} + \dot{I}_C = 0.186 \angle -12.3^\circ \text{ A}$$

$$\left\{ \begin{aligned} \lambda' &= \frac{P}{UI'} = \frac{40 \text{ W}}{220 \cdot 0.186} = 0.977 \end{aligned} \right.$$

$$\left\{ \begin{aligned} \text{或 } \lambda' &= \cos -12.3^\circ = 0.977 \end{aligned} \right.$$

\therefore 并联电容后电流表的读数为 0.186 A

整个电路功率因数为 0.977