Cone Adaptation

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1 Basic model

1.1 Differential equations

The conversion from photon absorption to neuron current can be fully characterized by differential equations as

$$\frac{dOpsin(t)}{dt} = -\sigma Opsin(t) + R^*(t) \tag{1}$$

$$\frac{dPDE(t)}{dt} = Opsin(t) - \phi PDE(t) + \eta \tag{2}$$

$$\frac{dcGMP}{dt} = S(t) - PDE(t)cGMP(t) \tag{3}$$

$$\frac{dCa(t)}{dt} = qI(t) - \beta Ca(t) \tag{4}$$

$$\frac{dCa_{slow}(t)}{dt} = \beta_{slow}(Ca_{slow}(t) - Ca(t)) \tag{5}$$

$$S(t) = \frac{S_{max}}{1 + (Ca(t)/K_{Gc})^n} \tag{6}$$

$$I(t) = \frac{kcGMP(t)^h}{1 + (Ca_{slow}/Ca_{dark})} \tag{7}$$

Here, R^* is the isomerization rate per cone per second.

1.2 Dark current

In dark, the differential equations can be simplified as

$$\begin{aligned}
&Opsin_{dark} = 0\\
&PDE_{dark} = \eta/\phi\\
&S_{dark} = (\eta/\phi)cGMP_{dark}\\
&q = \frac{2\beta Ca_{dark}}{kcGMP^h_{dark}}\\
&Ca_{slow}(t) = Ca(t) = Ca_{dark}\\
&S_{max} = (\eta/\phi)cGMP_{dark}(1 + (Ca_{dark}/K_{Gc})^n)\\
&I_{dark} = kcGMP^h_{dark}/2
\end{aligned}$$

Thus, S_{max} and q can be computed by other parameters as

$$
\begin{aligned}
q &= \frac{2\beta Ca_{dark}}{kcGMP_{dark}^h} \\
S_{max} &= (\eta/\phi)cGMP_{dark}(1+(Ca_{dark}/K_{Gc})^n)
\end{aligned}
$$

1.3 Steady state response

If the input stimulus is static, i.e. $R^*(t) = r$ for all t, we could simplify equation (1) to (7) and compute the steady state response as

$$
\begin{aligned}
Opsin_r &= r/\sigma \\
PDE_r &= \frac{r/\sigma + \eta}{\phi} \\
S_r &= \frac{r/\sigma + \eta}{\phi} cGMP_r \\
qI_r &= \beta Ca_r \\
Ca_{slow} &= Ca_r \\
S_r &= \frac{S_{max}}{1+(Ca_r/K_{Gc})^n} \\
I_r &= \frac{kcGMP_r^h}{1+(Ca_r/Ca_{dark})}
\end{aligned}
$$

Thus, steady state response current follows

$$
\begin{aligned}
I_r &= \frac{kcGMP_r^h}{1+(Ca_r/Ca_{dark})} \\
&= \frac{k(S_r \frac{\phi}{r+\eta})^h}{1+\frac{qI_r}{\beta Ca_{dark}}} \\
&= \frac{k\phi^h \beta Ca_{dark}}{(r/\sigma+\eta)^h} \frac{S_r^h}{\beta Ca_{dark} + qI_r} \\
&= \frac{k\phi^h \beta Ca_{dark} S_{max}^h}{(r/\sigma+\eta)^h} \frac{1}{(\beta Ca_{dark} + qI_r)(1+(\frac{qI_r}{\beta K_{Gc}})^n)^h}
\end{aligned}
$$

1.4 Compare with existing model

Felice A. Dunn, et al. reported that steady state response follows that [1]

$$
I_r \propto \frac{1}{1+(45000/r)^{0.7}}
$$

The comparison of two models (normalized) are shown in figure 1 below.

[1] Felice A. Dunn, et al. Light adaptation in cone vision involves switching between receptor and post-receptor sites, doi:10.1038/nature06150

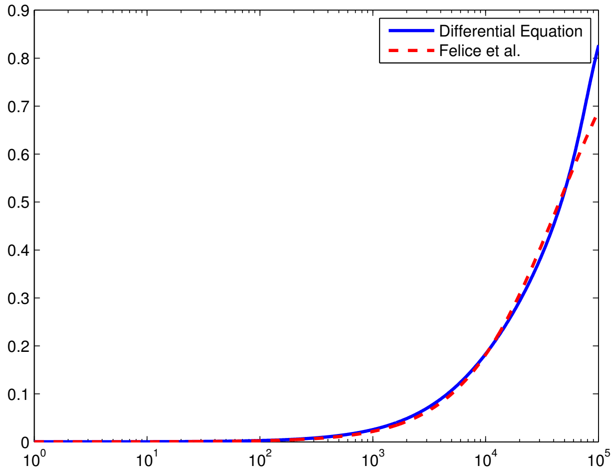

Figure 1: Steady state response from differential equations (Blue) and Felice et al. model (Red) for isomerization rate between 10 to 100000

2 Questions

In ppt slides, there is a σ before *Opsin* in equation (2).