Sets, elements & cardinality

Methods: Logic, Part 1a

Michael Franke

Content covered

- basics notions of naïve set theory:
  - universe
  - element
  - set
  - membership
  - cardinality
- ways of describing or defining sets

Universe

All the stuff we care about

Set

A collection of elements

Set

A collection of elements

Set

A collection of elements

Notation

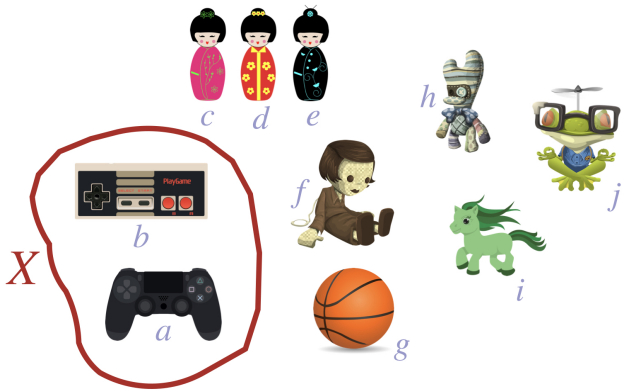
$X = \{a, b\}$

$a \in X$

$c \notin X$

Convention

$\{a, b\} = \{b, a\}$



Ways of describing or defining sets

1. by listing elements
2. by characteristic property
3. by recursive definition

Ways of describing or defining sets

1. **by listing elements**
2. by characteristic property
3. by recursive definition

$X = \{0, 1, 2, 3, \dots\}$

$Y = \{5, 7, 9, 11, \dots, 21\}$

Ways of describing or defining sets

1. by listing elements
2. **by characteristic property**
3. by recursive definition

$$X = \{x \in U \mid x \text{ is a game controler}\}$$
$$= \{a, b\}$$

$$Y = \{x \in X \mid x \text{ is retro}\}$$
$$= \{b\}$$

Ways of describing or defining sets

1. by listing elements
2. by characteristic property
3. **by recursive definition**

Definition:
$\mathfrak{L}$ is a set of strings (symbols), such that

1. **anchor:** all natural numbers $\{0, 1, 2, \dots\}$ are part of $\mathfrak{L}$
2. **step:** if $x, y \in \mathfrak{L}$, then so are the strings "(x + y)" and "(x * y)"
3. **exhaustion:** nothing else is in $\mathfrak{L}$

Examples:

$$5 \in \mathfrak{L}$$
$$3.5 \notin \mathfrak{L}$$
$$((4 * 2) + 3) \in \mathfrak{L}$$
$$(3 + 2 * 3) * 3 \notin \mathfrak{L}$$

Cardinality

Number of elements of a set

$$X = \{a, b, c, d\}$$
$$|X| = 4 \qquad \text{[finite set]}$$

$$Y = \{0, 1, 2, 3, \dots\}$$
$$|Y| = \infty \qquad \text{[infinite set]}$$