Relations

Methods: Logic, Part 3a

Michael Franke

Topics covered

(i) tuples

(ii) Cartesian products

(iii) relations

(iv) properties of relations

Tuples

Order-sensitive collections

Sets

- order of elements is irrelevant
  $\{a, b\} = \{b, a\}$
- elements cannot reoccur
  $\{a, a\} = \{a\}$

Tuples

- order of elements is relevant
  $\langle a, b\rangle \neq \langle b, a\rangle$
- elements can reoccur
  $\langle a, a\rangle \neq \langle a\rangle$

Tuples

Terminology

An *n-tuple* is a tuple with n elements (in order).
For $n = 1$, we conventionally define: $\langle x \rangle = x$.
For small $n \geq 1$, there are special words:

$n = 2$ ordered pair

$n = 3$ triple

$n = 4$ quadruple

$n = 5$ quintuple

...

Cartesian product

The Cartesian product of two sets X and Y is a set of pairs:

$$X \times Y = \{\langle x, y\rangle \mid x \in X \text{ and } y \in Y\}$$

The Cartesian product of n sets is a set of n-tuples:

$$X_1 \times X_2, \ldots, \times X_n = \{\langle x_a, x_2, \ldots, x_n\rangle \mid x_i \in X_i \text{ for all } 1 \leq i \leq n\}$$

Examples

$$\begin{aligned} X &= \{a, b\} \\ Y &= \{c, d\} \\ X \times X &= \{\langle a, a\rangle, \langle a, b\rangle, \langle b, a\rangle, \langle b, b\rangle\} \\ X \times Y &= \{\langle a, c\rangle, \langle a, d\rangle, \langle b, c\rangle, \langle b, d\rangle\} \\ Y \times X &= \{\langle c, a\rangle, \langle d, a\rangle, \langle c, b\rangle, \langle d, b\rangle\} \end{aligned}$$

Relations

Definition

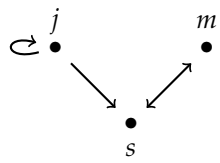
Any subset of an n-place Cartesian product is called an n-ary relation.

Example

$P = \{j, m, s\}$ — a set of people

$L \subseteq P \times P$ — binary relation encoding who loves whom:

$$L = \{\langle x, y\rangle \in P \times P \mid x \text{ loves } y\}$$
$$= \{\langle j, j\rangle, \langle j, s\rangle, \langle m, s\rangle, \langle s, m\rangle\}$$



Relations

Terminology & notation

- if $\langle x, y \rangle \in R$, we can also use
  - prefix notation: Rxy
  - infix notation: Rxy
  - postfix notation: xyR

[used in this course; except for math stuff like $1 \leq 2$]

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