# Probability theory: random variables

## Methods: Logic, Part 6

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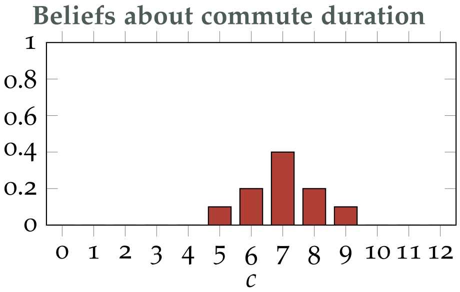
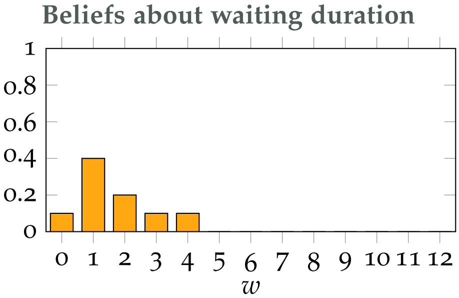



## Random variable notation

$$P(T \leq 15) \approx 0.48$$

where $T = C + W + C$

# Random variable

A **random variable** is a function $X : \Omega \to \mathbb{R}$

Notation:

- $P(X = x) = P(\{\omega \in \Omega \mid X(\omega) = x\})$
- $P(X \leq x) = P(\{\omega \in \Omega \mid X(\omega) \leq x\})$
- $P(C + W + C \leq 15) = P(\{\omega \in \Omega \mid C_1(\omega) + W(\omega) + C_2(\omega) \leq 15\})$

# Example

- flip coin with bias $\theta$ twice (independent outcomes)
- set of elementary outcomes:

$$\Omega_{\text{2flips}} = \{\langle \text{heads}, \text{heads} \rangle, \langle \text{heads}, \text{tails} \rangle, \langle \text{tails}, \text{heads} \rangle, \langle \text{tails}, \text{tails} \rangle\}$$

- random variable:

$$X^{\theta}_{\text{2flips}}(\langle \text{tails}, \text{tails} \rangle) = 0 \qquad X^{\theta}_{\text{2flips}}(\langle \text{heads}, \text{tails} \rangle) = 1$$

$$X^{\theta}_{\text{2flips}}(\langle \text{tails}, \text{heads} \rangle) = 1 \qquad X^{\theta}_{\text{2flips}}(\langle \text{heads}, \text{heads} \rangle) = 2$$

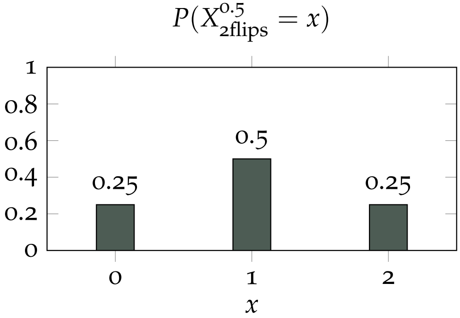
- Probabilities:

$$P(X^{\theta}_{\text{2flips}} = 0) = (1 - \theta)^2$$

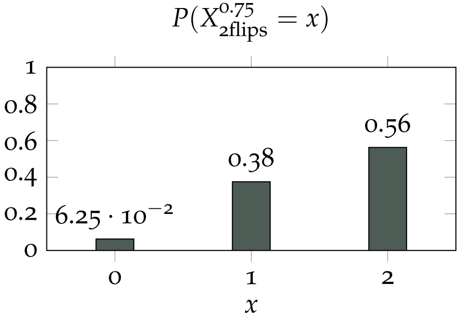
$$P(X^{\theta}_{\text{2flips}} = 1) = (1 - \theta)\theta$$

$$P(X^{\theta}_{\text{2flips}} = 2) = \theta^2$$

# Example



$P(X^{0.5}_{2\text{flips}} = x)$
1
0.8
0.6
0.4
0.2
0
0.25
0.5
0.25
0
1
2
$x$

$P(X^{0.75}_{2\text{flips}} = x)$
1
0.8
0.6
0.4
0.2
0
$6.25 \cdot 10^{-2}$
0.38
0.56
0
1
2
$x$

# Expected value

**Expected value of a random variable**

$$\mathbb{E}_X = \sum_x x \cdot P(X = x)$$

e.g., $\mathbb{E}_T \approx 15.67$

**Expected value of a function**

$$\mathbb{E}_X f = \sum_x f(x) \cdot P(X = x)$$

e.g., with $f(x) = x^2$, we have $\mathbb{E}_T f \approx 250$