# Predicate logic

## Methods: Logic, Part 5

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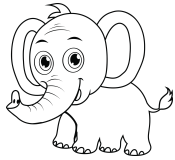
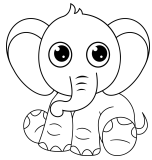
Dumbo 

 Kimbi

| natural language | PropLog | PredLog |
|---|---|---|
| Dumbo is an elephant. | $p$ | $Ed$ |
| Kimbi is an elephant. | $r$ | $Ek$ |
| All elephants are mammals. | ??? | $\forall x\,(Ex \to Mx)$ |
| Dumbo is a mammal. | $q$ | $Md$ |
| Kimbi is a mammal. | $s$ | $Mk$ |

# Elements of the language of predicate logic

- individual constants $a, b, c, \ldots, v$
- predicate letters $A, B, C, D \ldots$
- variables $w, x, y, z$
- parentheses ( )
- sentential connectives $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$
- quantifiers $\exists, \forall$

# Formula of predicate logic

(i) If $A$ is an $n$-ary predicate letter and if $t_1, \ldots, t_n$ are individual constants or variables, then $At_1 \ldots t_n$ is a formula.

(ii) If $\varphi$ is a formula, then so is $\neg\varphi$.

(iii) If $\varphi$ and $\psi$ are formulas, so are:

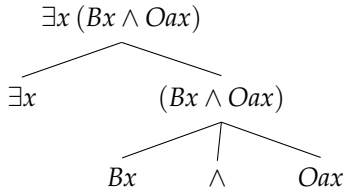
a. $(\varphi \wedge \psi)$ b. $(\varphi \vee \psi)$ c. $(\varphi \rightarrow \psi)$ d. $(\varphi \leftrightarrow \psi)$

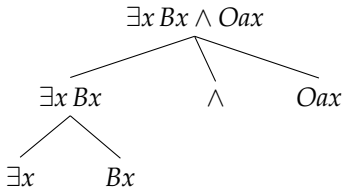
(iv) If $\varphi$ is a formula and if $x$ is a variable, then these are formulas:

a. $\forall x\, \varphi$ [*universal statement*] b. $\exists x\, \varphi$ [*existential statement*]

(v) Anything that cannot be constructed by (i)–(iv) is not a formula.

# Syntactic trees



# Translation

**Domain of quantification**
all human beings

**Translation key**

| | |
|---|---|
| $a$: Alex | $Lxy$: $x$ likes $y$ |
| $b$: Bo | $Px$: $x$ is a pilot |
| $Fx$: $x$ is friendly | $Sxy$: $x$ is a sibling of $y$ |

**Examples**

| | |
|---|---|
| Alex is a pilot. | $Pa$ |
| Bo is an friendly pilot. | $Fb \wedge Pb$ |
| No pilot is friendly. | $\forall x\,(Px \rightarrow \neg Fx)$ |
| Nobody likes pilots. | $\neg\exists x\,\exists y\,(Py \wedge Lxy)$ |
| Bo has a friendly sibling. | $\exists x\,(Sx \wedge Sbx)$ |
| Every pilot has a friendly sibling. | $\forall x\,(Px \rightarrow (\exists y\,(Fy \wedge Sxy)))$ |