Relations between & operations on sets

Methods: Logic, Part 1b

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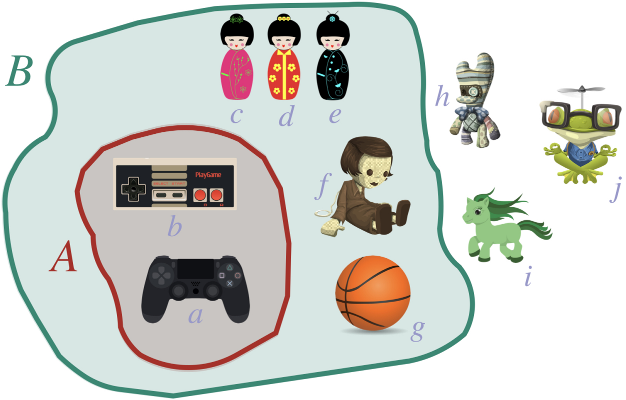
Content covered

- relations between sets:
  - (proper) subset (proper) superset
- operations on sets:
  - power set
  - logical operations:
    - intersection
    - union
    - difference
    - complement

Subsets

For any two sets X and Y, X is a **subset** of Y if all elements of X are also elements of Y.
If X is a subset of Y, we write: $X \subseteq Y$.
If X is not a subset of Y, we write: $X \nsubseteq Y$.

Example 1: $A \subseteq B$



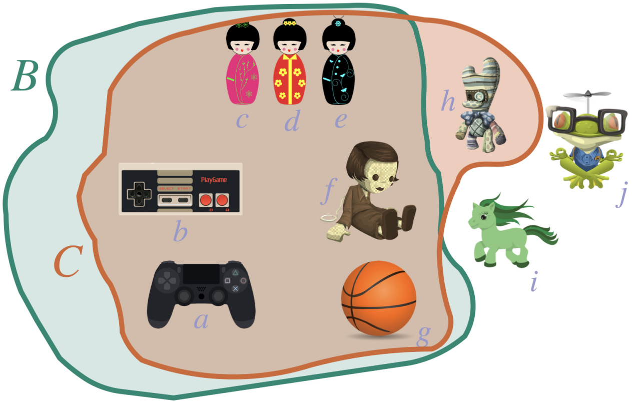
Subsets

For any two sets X and Y, X is a **subset** of Y if all elements of X are also elements of Y.

If X is a subset of Y, we write: $X \subseteq Y$.

If X is not a subset of Y, we write: $X \nsubseteq Y$.

Example 2: $C \nsubseteq B$



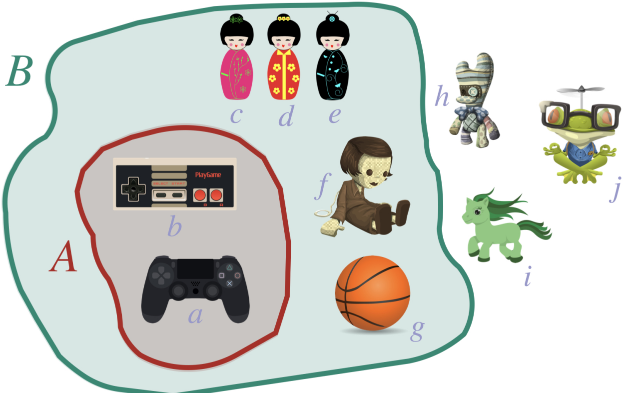
Proper subsets

For any two sets X and Y, X is a **proper subset** of Y if all elements of X are also elements of Y and there is at least one element $y \in Y$ such that $y \notin X$.

If X is a proper subset of Y, we write: $X \subset Y$.

If X is not a proper subset of Y, we write: $X \not\subset Y$.

Example 3: $A \subset B$



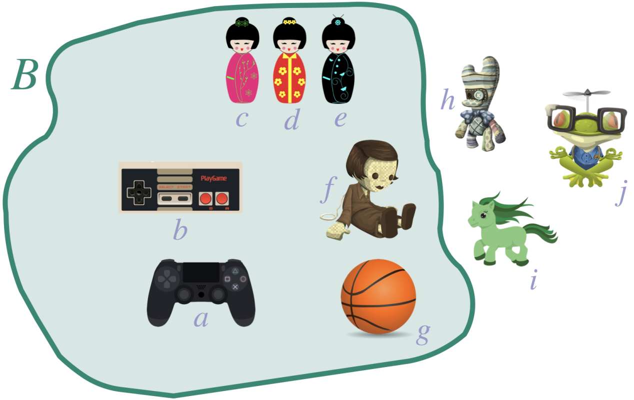
Proper subsets

For any two sets X and Y, X is a **proper subset** of Y if all elements of X are also elements of Y and there is at least one element $y \in Y$ such that $y \notin Y$.

If X is a proper subset of Y, we write: $X \subset Y$.

If X is not a proper subset of Y, we write: $X \not\subset Y$.

Example 4: $B \not\subset B$ **(but** $B \subseteq B$**)**



Superset and proper supersets

If X is a (proper) subset of Y, then Y is a **(proper) superset** of X.
If Y is a superset of X, we write: $X \subseteq Y$ or $Y \supseteq X$.

Power set

The **power set** $\mathcal{P}(X)$ of X is the set of all subsets of X:

$$\mathcal{P}(X) = \{Y \mid Y \subseteq X\}$$

Example 5:

$$X = \{a, b\}$$

$$\mathcal{P}(X) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$$

Logical operations on sets

$$X \cap Y = \{z \mid z \in X \text{ and } z \in Y\} \qquad \text{[intersection]}$$

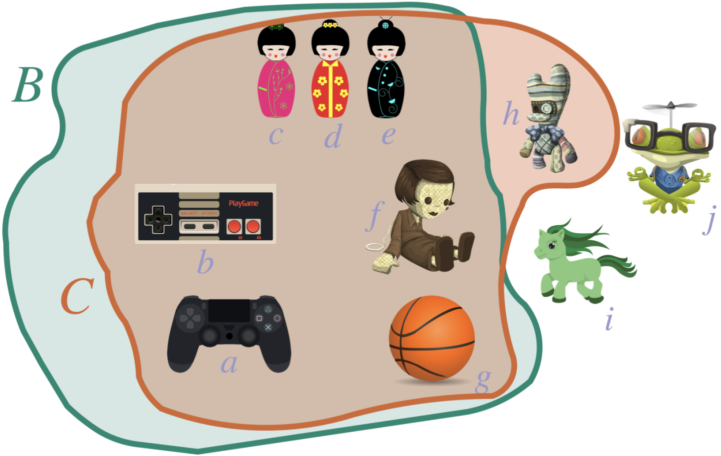
$$X \cup Y = \{z \mid z \in X \text{ or } z \in Y\} \qquad \text{[union]}$$

$$X \setminus Y = \{z \mid z \in X \text{ and } z \notin Y\} \qquad \text{[difference]}$$

$$\overline{X} = \{z \in U \mid z \notin X\} \qquad \text{[complement]}$$

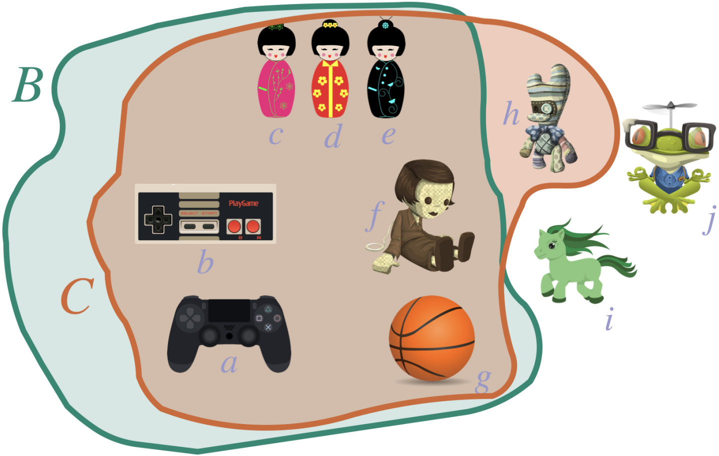
Example: Intersection

Example 6: $B \cap C = \{a, b, \ldots, g\}$



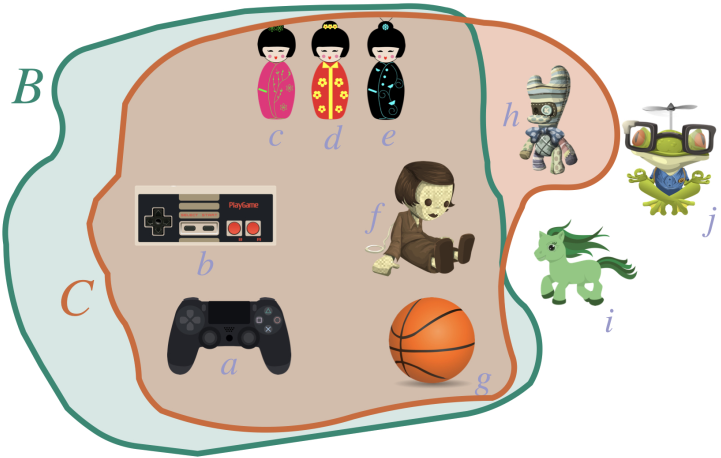
Example: Union

Example 7: $B \cup C = \{a, b, \ldots, g, h\}$



Example: Difference

Example 8: $C \setminus B = \{h\}$ **and** $B \setminus C = \emptyset$



Example: Complement

Example 9: $\overline{B} = \{h, i, j\}$ **and** $\overline{C} = \{i, j\}$