Predicate Logic

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Formulas of predicate logic; atomic sentences, predicate letters &
individual constants; quantifier scope and binding; predicate-logical
meaning of natural language sentences; semantics of predicate logic;

1 Motivation

2 Formulas of predicate logic

2.1 Basic ingredients of predicate-logical formulas

The formulas of PREDLOG consist of a number of building blocks.

- individual constants $a, b, c, \ldots, v$

- predicate letters $A, B, C, D \ldots$

- variables w, x, y, z

- quantifiers $\exists, \forall$

- brackets $(\ )$

- sentential connectives (like PROPLOG) $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$

Individual constants are denoted by lower-case Roman letters ($a, b, c, \ldots, v$)
up to v.[1] Individual constants are like proper names: they refer to exactly one
individual.[2]

Predicate letters are denoted with upper-case Roman letters ($A, B, C, D \ldots$).
Predicate letters will be used to denote relations. Each predicate letter has a
unique *arity*, i.e., the number of elements that the relevant relation requires.
For example, the predicate letter L may stand for a two-place relations such
as "x loves y". The arity of L would therefore be 2. We expect L to have two
arguments, so that *Lab*, *Lax* or *Lxy* would be well-formed expressions (see
below), while *Labc* or *La* would not be.

Variables are denoted by lower-case Roman letters (w, x, y, z), starting
from w. Variables are only interpretable in the *scope* of a quantifier, a tech-
nical concept we will introduce later. As a first intuitive guide, think of vari-
ables as similar to pronouns which are used to refer to an unnamed individual
introduced by a quantifying expression like in these examples:

For every boy it holds that *he* ...   [*he* = some boy]
There is a boy for which it holds that *he* ...   [*he* = some boy]

Quantifiers are special functional elements of the language of PREDLOG.
The quantifier $\exists$ is the *existential quantifier*. It is read as "there is" or "there

[1] If need be, we can also use additional
indices like a_1, a_2 etc. This also holds for
variables and predicate letters.

[2] Individuals in the sense of predicate
logic need not be humans or animals. An
individual is any kind of entity that can have
properties or stand in some kind of relation
to any other property. For example, constant
m may denote Michael's copy of *Moby Dick*.

exists." The quantifier ∀ is the *universal quantifier*. It is read as "for all." For example, the formula ∃*x*(*Bx* ∧ *Ix*) could mean that there is an individual which has the property denoted by *B* (e.g., it is a book) and the property denoted by *I* (e.g., it is interesting). This formula would express that there is at least one interesting book.

3   *Quantifier scope & binding*