

Relations between & operations on sets

Methods: Logic, Part 1b

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Content covered

- relations between sets:
 - (proper) subset (proper) superset
- operations on sets:
 - power set
 - logical operations:
 - intersection
 - union
 - difference
 - complement

Subsets

For any two sets X and Y , X is a **subset** of Y if all elements of X are also elements of Y .

If X is a subset of Y , we write: $X \subseteq Y$.

If X is not a subset of Y , we write: $X \not\subseteq Y$.

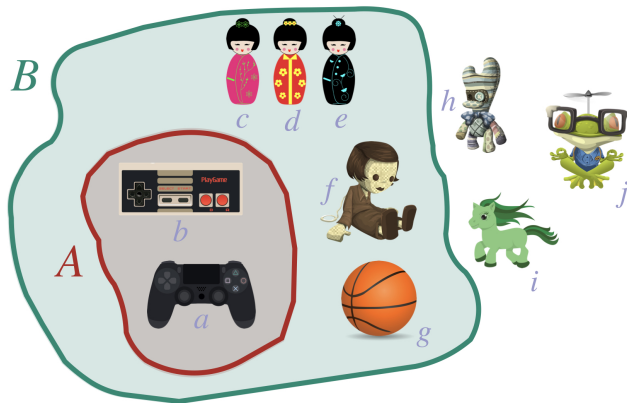
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Example 1: $A \subseteq B$



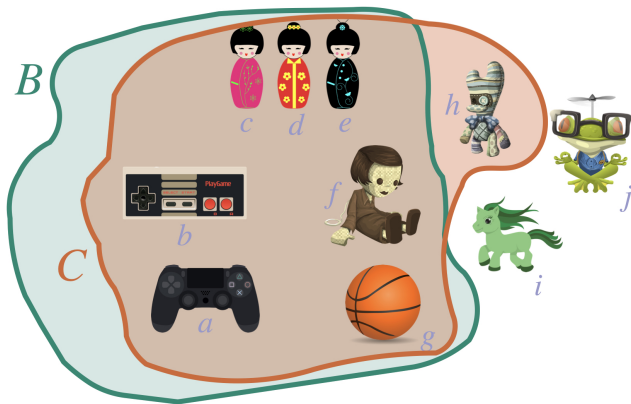
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Example 2: $C \not\subseteq B$



Proper subsets

For any two sets X and Y , X is a **proper subset** of Y if all elements of X are also elements of Y and there is at least one element $y \in Y$ such that $y \notin X$.

If X is a proper subset of Y , we write: $X \subset Y$.

If X is not a proper subset of Y , we write: $X \not\subset Y$.

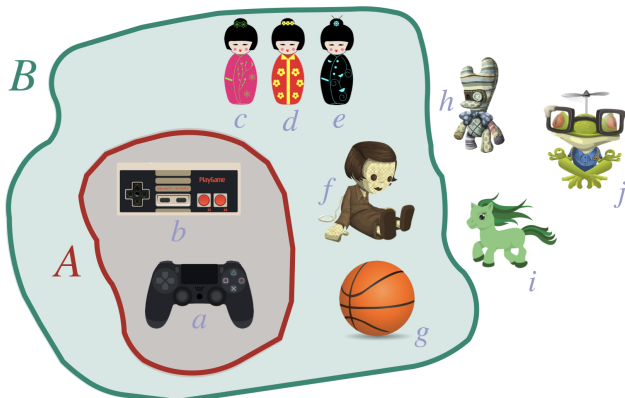
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Example 3: $A \subset B$



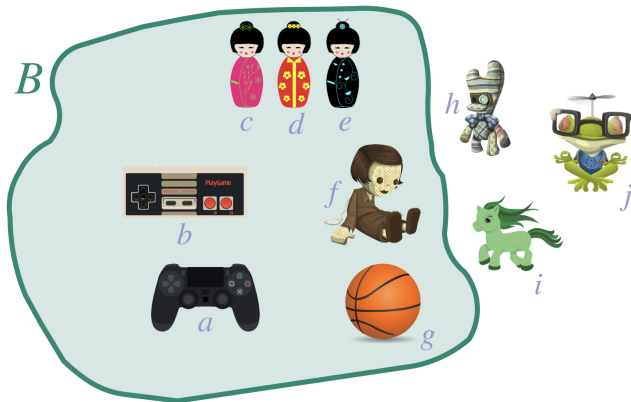
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For any two sets X and Y , X is a **proper subset** of Y if all elements of X are also elements of Y and there is at least one element $y \in Y$ such that $y \notin X$.

If X is a proper subset of Y , we write: $X \subset Y$.

If X is not a proper subset of Y , we write: $X \not\subset Y$.

Example 4: $B \not\subset B$ (but $B \subseteq B$)



Superset and proper supersets

If X is a (proper) subset of Y , then Y is a **(proper) superset** of X .

If Y is a superset of X , we write: $X \subseteq Y$ or $Y \supseteq X$.

Power set

The **power set** $\mathcal{P}(X)$ of X is the set of all subsets of X :

$$\mathcal{P}(X) = \{Y \mid Y \subseteq X\}$$

Example 5:

$$X = \{a, b\}$$

$$\mathcal{P}(X) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$$

Logical operations on sets

$$X \cap Y = \{z \mid z \in X \text{ and } z \in Y\}$$

[intersection]

$$X \cup Y = \{z \mid z \in X \text{ or } z \in Y\}$$

[union]

$$X \setminus Y = \{z \mid z \in X \text{ and } z \notin Y\}$$

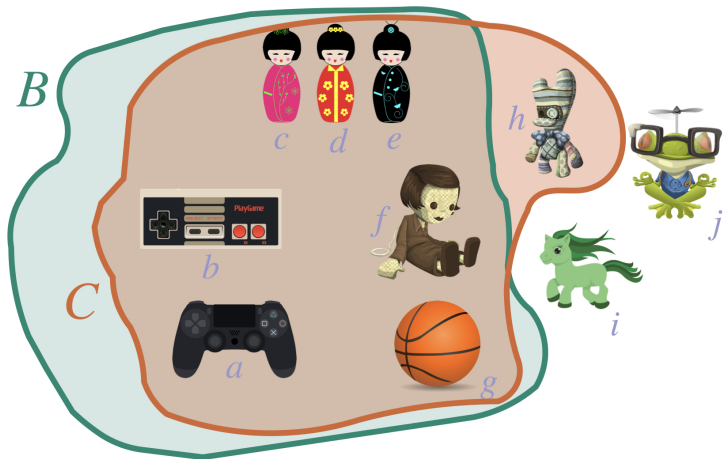
[difference]

$$\overline{X} = \{z \in U \mid z \notin X\}$$

[complement]

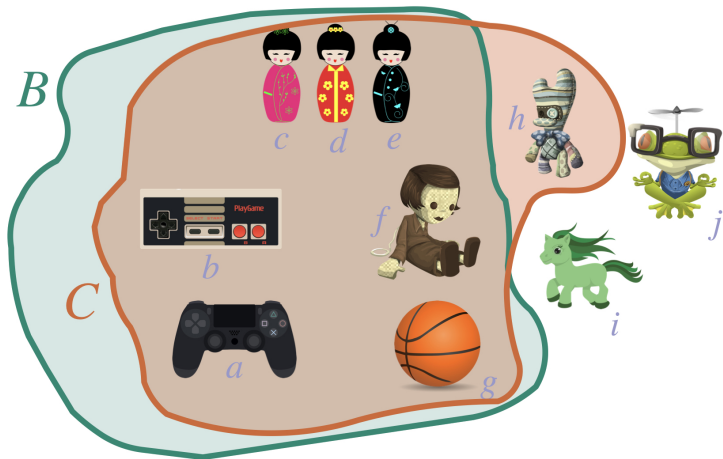
Example: Intersection

Example 6: $B \cap C = \{a, b, \dots, g\}$



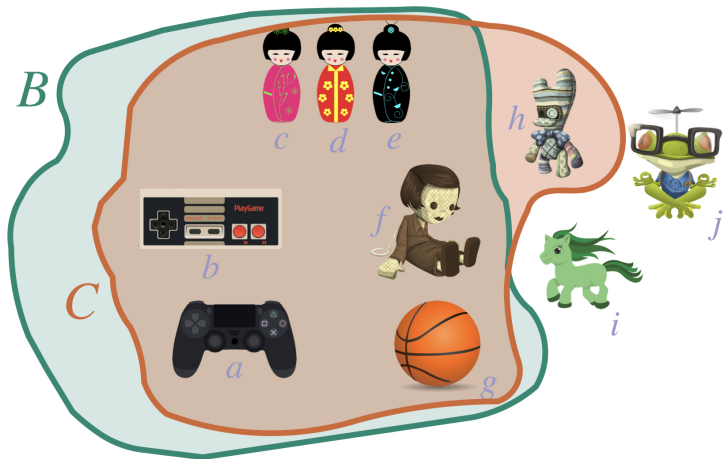
Example: Union

Example 7: $B \cup C = \{a, b, \dots, g, h\}$



Example: Difference

Example 8: $C \setminus B = \{h\}$ and $B \setminus C = \emptyset$



Example: Complement

Example 9: $\overline{B} = \{h, i, j\}$ and $\overline{C} = \{i, j\}$

