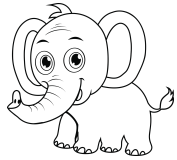
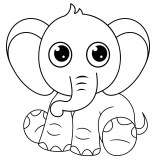
Predicate logic

Methods: Logic, Part 5

Michael Franke



Dumbo



Kimbi

natural language	PropLog	PredLog
Dumbo is an elephant.	p	Ed
Kimbi is an elephant.	q	Ek
All elephants are mammals.	???	$\forall x\,(Ex \to Mx)$
Dumbo is a mammal.	r	Md
Kimbi is a mammal.	s	Mk

Elements of the language of predicate logic

- individual constants $a, b, c, \ldots, v$
- predicate letters $A, B, C, D \ldots$
- variables w, x, y, z
- parentheses ( )
- sentential connectives $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$
- quantifiers $\exists, \forall$

Formula of predicate logic

(i) If A is an n-ary predicate letter and if $t_1, \ldots, t_n$ are individual constants or variables, then $At_1 \ldots t_n$ is a formula.

(ii) If φ is a formula, then so is $\neg\varphi$.

(iii) If φ and ψ are formulas, so are:

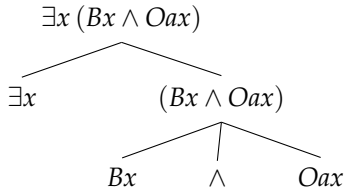
a. $(\varphi \wedge \psi)$ b. $(\varphi \vee \psi)$ c. $(\varphi \rightarrow \psi)$ d. $(\varphi \leftrightarrow \psi)$

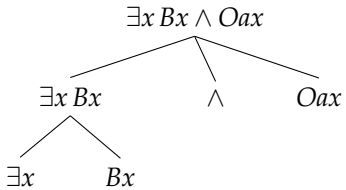
(iv) If φ is a formula and if x is a variable, then these are formulas:

a. $\forall x\, \varphi$ [*universal statement*] b. $\exists x\, \varphi$ [*existential statement*]

(v) Anything that cannot be constructed by (i)–(iv) is not a formula.

Syntactic trees





Translation

Domain of quantification
all human beings

Translation key

a: Alex	Lxy: x likes y
b: Bo	Px: x is a pilot
Fx: x is friendly	Sxy: x & y are siblings

Examples

Alex is a pilot.	Pa
Bo is a friendly pilot.	$Fb \wedge Pb$
No pilot is friendly.	$\forall x\,(Px \rightarrow \neg Fx)$
Nobody likes pilots.	$\neg\exists x\,\exists y\,(Py \wedge Lxy)$
Bo has a friendly sibling.	$\exists x\,(Fx \wedge Sbx)$
Every pilot has a friendly sibling.	$\forall x\,(Px \rightarrow (\exists y\,(Fy \wedge Sxy)))$