Solutions must be prepared in C programming language .To get full credit for each question, you need to provide a brief explanation of your codes and the efficiency analysis with comments.

1) Devise an algorithm that takes an nxn adjacency matrix of a directed graph as input, and determines whether the directed graph contains a simple cycle of odd length or not. The running time of your algorithm should be O(n+m) where n is the number of vertices and m is the number of edges contained in the graph. Also, all the inputs given to the algorithm are assumed to be legitimate, i.e. the users are assumed to provide only nxn matrices that consist of only 0s and 1s as inputs.

2) *a)* Devise an O(nlogn)-time algorithm that takes a sequence S of n integers $a_1, \ldots, a_n$ and a separate integer x as input, and outputs 'YES' if there are two elements in S such that the sum of them is x and 'NO' otherwise. Also, all the inputs given to the algorithm are assumed to be legitimate, i.e. the users are assumed to provide only integers.

 b) Devise an O(n)-time algorithm for the same problem.

3) Devise a dynamic programming algorithm that takes a sequence S of n integers $a_1, \ldots, a_n$ as input, and outputs the longest and strictly increasing subsequence. Note that the numbers in the subsequence don't need to be successive. Also, all the inputs given to the algorithm are assumed to be legitimate, i.e. the users are assumed to provide only integers.

4) Devise a dynamic programming algorithm that takes three bit strings $X = x_1 \ldots x_n$, $Y = y_1 \ldots y_m$, and $Z = z_1 \ldots z_{n+m}$, and output 'YES' if Z is an interleaving of X and Y, and 'NO' otherwise. Note that the string Z is called an interleaving of X and Y if Z can be obtained by interleaving the bits in X and Y in a way that it maintains the left-to-right order of the bits in X and Y. For example, if $X = 101$ and $Y = 01$, then $Z_1 = 10011$ is an interleaving of X and Y, but $Z_2 = 11010$ is not. Also, all the inputs given to the algorithm are assumed to be legitimate, i.e. the users are assumed to provide only bit strings.