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- Linear Programming Problem -

Linear Programming is a mathematical technique which involves the allocation of limited resources in an optimal manner on the basis of a given criteria of optimality.

→ LP deals with the optimization (maximisation or minimization) of a function of variables known as objective function.

→ It is subject to set of linear inequalities &/or, linear equalities known as constraints.

Matrix formed object of a LPP :-

The LPP can be written as max/min : $z = cx$ (objective function; c → cost/value profit)

subject to constraint : $Ax (\leq, =, \geq) b$ and $x \geq 0$ (restraints)

Let $x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}_{n \times 1}$ be the decision vector.

then, $c = (c_1, c_2, \dots c_n)$ is a cost/value vector

& $(A)_{m \times n} = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$ is coeff. of constrains.

& $b = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}$ is called requirement/availability or resource vector.

General form of LPP :-

The LPP can be written as max/min : $z = c_1x_1 + c_2x_2 + \dots + c_nx_n$

& subject to constraint : $A = a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \; (\leq, =, \geq) \; b_1$

$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \; (\leq, =, \geq) \; b_2$

$\vdots$

$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \ (\leq, =, \geq)\ b_m$

$\&\ x_1, x_2 \cdots x_n \geq 0$

Mathematical Formulation of LPP :-

1. Identify the decision variables to be considered.
2. Formulate/construct the objective funcⁿ to be optimised
3. Formulate the constraints of the problem.
4. Add the non-negative ~~constra~~ restriction.

16.02.23

Q. A company management judge 2 products p1, p2. Each product is ~~proceed~~ processed in 2 machine M_1, M_2. While p1 requires 2 min on M_1, & 4 min on M_2, while p2 " 3 " on ", & 3 " " ". The machines M_1 & M_2 are available for 1000 & 1500 min respectively. The profit on each product p1 & p2 is ₹2 & ₹4 respectively to work. The company must mfg. 80 p1's & 100 p2's, but not more than 120 p1's. Formulate the problem as an LPP.

	P1 (x_1)	P2 (x_2)	Availability $(\leq)$
M1	2 min	3 min	1000 min
M2	4 min	3 min	1500 min
Profit (c)	2 RS	4 RS	

Let the mfg. produced x_1 & x_2 no. of p_1 & p_2 respectively.

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Max $Z_{max} = 2x_1 + 4x_2$ (objective func.)

The total time required for machine M_1 for both the product is : $2x_1 + 3x_2$

But, ∵ the availability time for M_1 is 1000 min

∴ $2x_1 + 3x_2 \leq 1000$ (1st constraint)

The total time required for machine M_2 for both the product is : $4x_1 + 3x_2$

∵ the availability time for M_2 is 1500 min.

∴ $4x_1 + 3x_2 \leq 1500$ (2nd constraint)

Given, for p_1, $80 \leq x_1 \leq 120$

& for p_2, $x_2 \geq 100$

~~Since, the company mfg. 80 p's & "~~

Since, it is impossible to produce (-ve) no. of products p1 & p2, so, $x_1 \geq 0$ & $x_2 \geq 0$

∴ The LPP is given by :

Max $z = 2x_1 + 4x_2$

subject to :

$2x_1 + 3x_2 \leq 1000$

$4x_1 + 3x_2 \leq 1500$

$80 \leq x_1 \leq 120$

$x_2 \geq 100$

&

$x_1, x_2 \geq 0$

Q2. A doctor recommends 2 foods f1 & f2 to a patient for a diet, which includes atleast 1000 units of vitamins, 850 units of protein & 700 units of fat. Each unit of

food f1 contains 15 units of vitamins, 18 units of protein & 12 units of fat, while each unit of food f2 contains 22 ~~15~~ units of vitamin, 15 units of protein & 16 units of fat. The cost of f1 & f2 is ₹5 & ₹8 respectively.
Formulate the LPP to ~~obtain~~ ~~to~~ minimize the cost for a diet.

Ans.

	f1 (x_1)	f2 (x_2)	Atleast ($\geq$)
Vitamin	15	22	1000
Protein	18	15	850
Fat	12	16	700
Cost	~~₹~~5 Rs.	8	

Let the doctor recommends x_1 & x_2 units of food f1 & f2 respectively.

Min $z = 5x_1 + 8x_2$

The ~~required~~ total vitamin ~~for~~ from both the foods is: $15x_1 + 22x_2$

$\because$ It is recommended to take atleast 1000 units of vitamins.

$$\therefore \quad 15x_1 + 22x_2 \geq 1000$$

The total protein from both the foods is: $18x_1 + 15x_2$

$\because$ It is recommended to take atleast 850 units of protein. $\therefore$ $18x_1 + 15x_2 \geq 850$

The total fat from both the foods is: $12x_1 + 16x_2$

$\because$ It is recommended to take atleast 700 units of fat $\therefore$ $12x_1 + 16x_2 \geq 700$

$\because$ It is impossible to have ~~+ve~~ -ve amounts of vitamins, protein & fat in f1 & f2. $\therefore$ $x_1 \geq 0$ & $x_2 \geq 0$

The LPP is given by.

$$\text{Min } Z = 5x_1 + 8x_2$$

subject to :

$$15x_1 + 22x_2 \geq 1000$$
$$18x_1 + 15x_2 \geq 850$$
$$12x_1 + 16x_2 \geq 700$$
$$\& \; x_1, x_2 \geq 0$$

* Solution of a LPP :-

Any specification or values for the decision variable is called a solution.

* ~~Visible so~~ Feasible solution :-

A feasible sol^n is a sol^n for which all the constrains are satisfied.

Note

Any solution to a LPP which satisfies the non-negative restrictions of the LPP is called its feasible solution.

* Infeasible solution :-

An infeasible sol^n is a sol^n for which atleast one contraint is violated.

* Feasible Region :-

A region in which all the constraints are satisfied simultaneously is called a feasible region.

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* Optimal Solution

Any feasible solⁿ which optimises (maximise/minimise) the objective funcⁿ is called ~~an~~ its optimal solⁿ.

* Solution of a LPP -

1) Graphical Method (easiest)
2) Simplex Method (more than 2 variables)

1) Solution by Graphical Method -

→ Convert the inequality constraint to equation.
→ Plot each equation on the graph.
→ Determine the feasible region.
→ Find the coordinates of the corner points of the feasible region.
→ Find the optimal solⁿ.
→ ~~Solve the~~

Q. Solve the LPP by Graphical Method :-

Max $Z = 3x_1 + 5x_2$

s.t. $x_1 \leq 4$

$x_2 \leq 6$

$3x_1 + 2x_2 \leq 18$

and $x_1, x_2 \geq 0$ → due to this the feasible region ~~graph~~ will be in 1st quadrant always

Ans. Let us convert the inequalities as eqⁿ.

$x_1 = 4$, the intercept point is $(4,0)$

$x_2 = 6$, " " " $(0,6)$

$3x_1 + 2x_2 = 18$ " " " are $(6,0)$ and $(0,9)$

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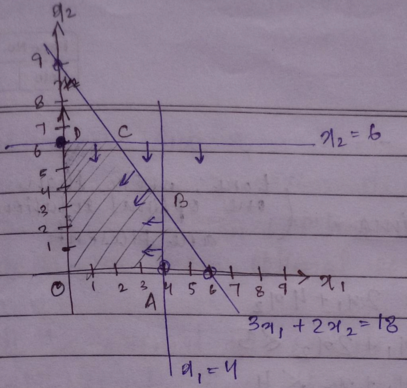

Rough
$3x_1 \leq 18$
$2x_2 \leq 18$

OABCD is the feasible region
we have, O (0,0)
A (4,0)
D (0,6)

The co-ordinate of point B is given by
$x_1 = 4$
at $3x_1 + 2x_2 = 18$
$\Rightarrow x_2 = \frac{18-12}{2} = 3$.

$\therefore$ B (4,3)

Co-ord of pt. C is obtained from,
$x_2 = 6$
at $3x_1 + 2x_2 = 18$
$\Rightarrow x_1 = \frac{18-12}{3} = 2$

$\therefore$ C (2,6)

Points	$z = 3x_1 + 5x_2$	Remarks
O(0,0)	z = 0	
A(4,0)	z = 12	
B(4,3)	z = 27	
C(2,6)	z = 36	Maximum value occurs. (the optimal solution)
D(0,6)	z = 30	

all these are feasible solutions

$\therefore$ Solⁿ of LPP: $x_1 = 2$, $x_2 = 6$
& Max $z = 36$

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Exceptional Cases -

~~Alte:~~

Alternative Optima - (here, more than one optimal solutions are present)

Q. Solve Max $z = 2x_1 + 4x_2$

s.t. $x_1 + 2x_2 \leq 5$

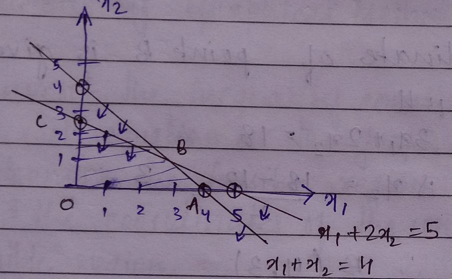
$x_1 + x_2 \leq 4$

& $x_1, x_2 \geq 0$

Convert the inequalities as eqn

$x_1 + 2x_2 = 5$, intercept point : $(5,0)$ & $(0, 5/2)$

$x_1 + x_2 = 4$, " " : $(4,0)$ & $(0,4)$



Rough

$x_1 \leq 5$

$x_2 \leq 5$

$x_1 \leq 4$

OABC is the feasible region

where, O $(0,0)$

A $(4,0)$

~~B~~ C $(0, 2.5)$

The co-ord of point B ~~is~~, can be obtained from,

$$x_1 + 2x_2 = 5$$
$$-x_1 + x_2 = 4$$
$$x_2 = 1$$
$$\therefore x_1 = 3$$
$$\therefore B(3,1)$$

Points	$Z = 2x_1 + 4x_2$		Remark
O $(0,0)$	$Z = 0$		
A $(4,0)$	$Z = 8$		
B $(3,1)$	$Z = 10$	max	} ~~Another Optim~~ any point b/w B, C are feasible solⁿ
C $(0, 2.5)$	$Z = 10$	max	

The solutions are $x_1 = 3, x_2 = 1$ & Max $Z = 10$;
& $x_1 = 0$, $x_2 = 2.5$ & Max$Z = 10$

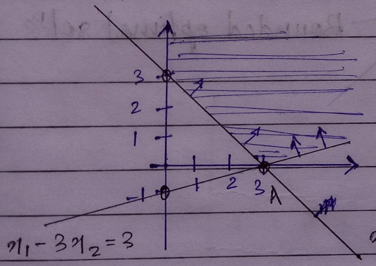
Q. How do you know an LPP has optimal solⁿ in graphical method?

If the optimal solⁿ occurs at more than one solⁿ point, then the LPP has an alternative optimal solⁿ.

- Unbounded solution -

Q. Max $Z = 2x_1 + x_2$
s.t. $x_1 + x_2 \geq 3$
$x_1 - 3x_2 \leq 3$
& $x_1, x_2 \geq 0$

Convert the inequalities as eqⁿ
$x_1 + x_2 = 3$, intercept point: $(3,0)$, $(0,3)$
$x_1 - 3x_2 = 3$, " " : $(3,0)$, $(0,-1)$



Here, LPP has an unbounded solution.

Rough
$x_1 \leq 3$
$-3x_2 \geq 3$
$x_2 \geq -1$

→ If the question were about (min z), then the solⁿ would've existed, i.e. $A(3,0)$.

21.2.23

★ Here, the feasible region lies in an unbounded space, so, the LPP has an unbounded solution.

Note: In some LPP, the values of the variables may increase indefinitely without violating any of the constraints, i.e. the solⁿ space is unbounded. As a result, the objective value may increase (Maximisation problem/case) or decrease (Minimization case) indefinitely then, the sol^n is called unbounded sol^n.

- Unbounded sol^n space but bounded optimal sol^n -

Q. Solve Min $Z = x_1 + 2x_2$

s.t. $x_1 + x_2 \geq 3$

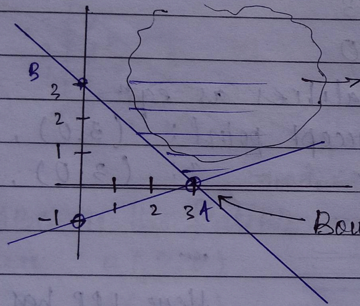
$x_1 - 3x_2 \leq 3$

& $x_1, x_2 \geq 0$

Convert the inequalities as eq^n.

$x_1 + x_2 = 3$, intercept points are $(3,0), (0,3)$

$x_1 - 3x_2 = 3$, " " " $(3,0), (0,-1)$



Points	$Z = x_1 + 2x_2$
A(3,0)	Z = 3
B(0,3)	Z = 6

the sol^n is $x_1 = 3$ & $x_2 = 0$ & Min $z = 3$

- No feasible sol^n or Infeasible sol^n -

Q. Solve the LPP, Max $z = 2x_1 + 5x_2$

s.t. $x_1 + x_2 \leq 1$

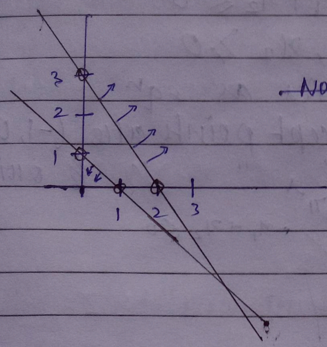
$3x_1 + 2x_2 \geq 6$

& $x_1, x_2 \geq 0$

Convert the inequalities as eq^n

$x_1 + x_2 = 1$, intercept points are $(1,0), (0,1)$

$3x_1 + 2x_2 = 6$, " " " $(2,0), (0,3)$



~~No common~~ common one

Since, there is no, feasible region ~~among the~~ ~~in the~~ among the constraints. ∴ LPP has no feasible sol^n.

Q. Solve the LPP, Max $Z = 2x_1 + x_2$

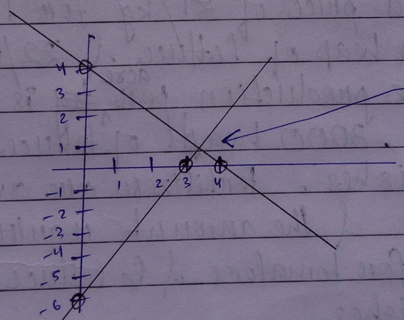
s.t. $x_1 + x_2 = 4$

$2x_1 - x_2 = 6$

& $x_1, x_2 \geq 0$

$x_1 + x_2 = 4$, intercept points are $(4,0), (0,4)$

$2x_1 - x_2 = 6$, " " " $(3,0), (0,-6)$



The intersection point is the feasible region.

Q. ~~An LPP with~~

- An LPP with redundant constraint -

The redundant constraint does not form boundary of feasible region, but it has impact on the sol^n of the problem & removal of which does not change the sol^n.

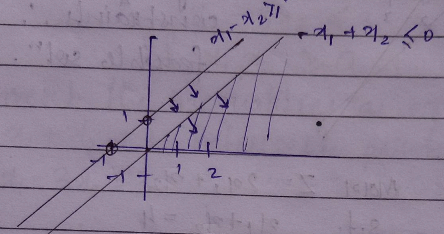
Q. Find the redundant constraint of the given LPP

$$\text{Max } z = 3x_1 + 5x_2$$
$$\text{s.t. } x_1 - x_2 \geq -1$$
$$-x_1 + x_2 \leq 0$$
$$\& \; x_1, x_2 \geq 0$$

Convert the inequalities as eq^n

$x_1 - x_2 = -1$, intercept points are $(-1,0), (0,1)$

$-x_1 + x_2 = 0$, " " " & origin



→ Here, the constraints $x_1 - x_2 \geq -1$ is a redundant constraint.

Q. A farmer has a 100 acre farm ~~she~~. He can sell all tomatoes, lettuce & radishes & can get a price of ₹1/kg for tomatoes & ₹0.75 for a heap of lettuce & ₹2/kg for radishes. The average production per acre ~~kg~~ is 2000 kg of tomatoes, 3000 heaps of lettuce & 1000 kg of radishes. Fertilizers are available for ₹0.5 /kg & the amount required per acre is 100 kg each for tomatoes & lettuce and 50kg for radishes.

Labour required for sowing, cultivating & harvesting per acre is 5 men/day for radishes & tomatoes & 6 men/day for lettuce. A total of 400 men/day of labourer are available at ₹20/men/day.

Formulate the LPP to maximise, the farmer's total profit.

Ans. ~~Let~~ Let the farmer can grow, x_1, x_2 & x_3 acre of farm for tomatoes, lettuce & radishes respectively.

The total sale of the farmer:

$$(1)\,2000x_1 + (0.75)\,3000x_2 + (2)(1000)\,x_3$$
$$= 2000x_1 + 2250x_2 + 2000x_3$$

Total expenditure in fertilizer:

$$0.5(100x_1 + 100x_2 + 50x_3)$$
$$= 50x_1 + 50x_2 + 25x_3$$

Total expenditure in labour:

$$20(5x_1 + 6x_2 + 5x_3) = 100x_1 + 120x_2 + 100x_3$$

Profit is: (total selling − expenditure in fertilizer − expenditure in labour)

$$= (2000x_1 + 2250x_2 + 2000x_3) - (50x_1 + 50x_2 + 25x_3) - (100x_1 + 120x_2 + 100x_3)$$

Objective func^n is $\text{Max } Z = (1850x_1 + 2080x_2 + 1875x_3)$

Since, the farmer has 100 acre of farm

$$\therefore\ x_1 + x_2 + x_3 \leq 100$$

Total labour required for tomatoes, lettuce, & radishes is $(5x_1 + 6x_2 + 5x_3)$, $\because$ the availability ~~of~~ of labours is 400.

$\therefore 5x_1 + 6x_2 + 5x_3 \leq 400$

Since it is impossible to produce (-ve) acre of land for producing tomatoes, lettuce, raddishes,

$\therefore x_1, x_2, x_3 \geq 0$

So, the LPP is given by Max Z =

$$\text{Max } Z = 1850x_1 + 2080x_2 + 1875x_3$$

s.t.

$$x_1 + x_2 + x_3 \leq 100$$

$$5x_1 + 6x_2 + 5x_3 \leq 400$$

$$\& \; x_1, x_2, x_3 \geq 0$$

22/2/23 - Basic Solution - ^nC_m

~~Giving a sy~~

~~Gi~~

Given a system of 'm' linear equations with 'n' variables provided that $m < n$, any solution that is obtained by solving m variables keeping the remaining ($n-m$) variables zero, is called basic soln.

→ One of the basic soln is :

$$X_1 = \{x_1, x_2, \ldots, x_m, \underbrace{0, 0, \ldots, 0}_{n-m \text{ zeroes}}\}$$

$$X_2 = \{0, x_2, x_3, \ldots, x_m, \underbrace{0, 0, \ldots 0}_{n-m-1}\}$$

Here, $x_1, x_2, \ldots, x_m$ are known as basic variables & $0, 0, \ldots, 0$ are known as non-basic.

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→ If $(A)_{m\times n}$, then the no. of basic solⁿ will be atmost ${}^{n}C_m = \frac{n!}{m!(n-m)!}$

- Basic Feasible Solution -

If all the basic variables are non (-ve), then the basic solⁿ is known as Basic Feasible Solⁿ (BFS)

Non-degenerate BFS -

If none of the basic variables are zero, then, the BFS is known as, non-degenerate BFS.

Degenerate BFS -

A BFS ~~is~~ is known as degenerate, if one or more basic variables are zero.

Q. Find the basic solution of the following equation.

$$x_1 + 2x_2 + x_3 = 4 \qquad m=3,\ n=2$$
$$2x_1 + x_2 + 5x_3 = 5 \qquad {}^{3}C_2 = 3 \text{ solutions.}$$

$AX = b$

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 5 \end{pmatrix}, \ X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \ b = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$$

~~Let, the submatrices be~~ Let B_1, B_2, B_3 be 3 submatrices,

$$B_1 = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}, \ B_2 = \begin{pmatrix} 2 & 1 \\ 1 & 5 \end{pmatrix}, \ B_3 = \begin{pmatrix} 1 & 1 \\ 2 & 5 \end{pmatrix}$$

∴ 1st solⁿ : $B_1 x_1 = b$

$$\Rightarrow \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 4 \\ 5 \end{pmatrix}$$

$$= -\frac{1}{3} \begin{pmatrix} 1 & -2 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ 5 \end{pmatrix} = -\frac{1}{3} \Big(4 \cdot$$

$X_1 = (2, 1, 0)$

→ It is basic solution (BS)
→ It is BFS (Both solⁿ are +ve)
→ It is Non-degenerate BFS (no basic variable is zero)

$B_2 X_2 = b$

$$\Rightarrow \begin{pmatrix} x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 1 & 5 \end{pmatrix}^{-1} \begin{pmatrix} 4 \\ 5 \end{pmatrix}$$

$|A| = (5 \times 2 - 1 \times 1)$

$$= \frac{1}{9} \begin{pmatrix} 5 & -1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 4 \\ 5 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{5}{3} \\ \frac{2}{3} \end{pmatrix}$$

$$X_2 = \left(0, \frac{5}{3}, \frac{2}{3}\right)$$

→ It is BS, BFS, & Non-degenerate BFS

$B_3 X_3 = b \Rightarrow X_3 = B_3^{-1} b$

$$\Rightarrow \begin{pmatrix} x_1 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 2 & 5 \end{pmatrix}^{-1} \begin{pmatrix} 4 \\ 5 \end{pmatrix}$$

$$= \frac{1}{3} \begin{pmatrix} 5 & -1 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ 5 \end{pmatrix}$$

$$= \begin{pmatrix} 5 \\ -1 \end{pmatrix}$$

$$X_3 = (5, 0, -1)$$

→ It is BS, Non-BFS, Non degenerate BS

Q. (i) $x_1 + x_2 + x_3 = 8$

(ii) $2x_1 + x_2 + x_4 = 10$

Here, $n = 4$

$m = 2$

$\therefore$ 6 solutions $= {}^4C_2$

$Ax = b$

$$A = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{pmatrix}, \quad x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}, \quad b = \begin{pmatrix} 8 \\ 10 \end{pmatrix}$$

submatrices, $B_1 = \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}, B_2 = \begin{pmatrix} 1 & 1 \\ 2 & 0 \end{pmatrix}, B_3 = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$

$B_4 = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, B_5 = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, B_6 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

1st solution: $B_1 X_1 = b$

$$\Rightarrow \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 8 \\ 10 \end{pmatrix} \Rightarrow \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 8 \\ 10 \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 8 \\ 10 \end{pmatrix}$$

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- Standard form of LPP -

(i) All constraints are expressed as equations.
(ii) RHS of each constraint is non-negative.
(iii) All variables are non-negative.

→ Minimization of a func^n Z is equivalent to maximization, if $\text{Min } Z = -\text{Max}(-Z)$

for min z
e.g. $\text{Min } Z = \underset{\text{pay}}{x_1} + \underset{\text{pay}}{x_2} - \underset{\text{gain}}{x_3}$

convert to Max Z by adding '-' →
$\therefore -x_1 - x_2 + x_3$

→ If a variable x_1 is unrestricted in sign, then it can be expressed as 2 non negative variables, i.e.

$$x_1 = x_1' - x_1''$$
$$\& \; x_1' \geq 0, \; x_1'' \geq 0$$

23.02.23 **Slack variable**

If a constant is of type $\leq$, to make it equality, we add a non-negative variable which is called slack variable.

e.g. $x_1 + 2x_2 + x_3 \leq 5$
$x_1 + 2x_2 + x_3 + x_4 = 4$
Here, $x_4 \geq 0$ is a slack variable.

Surplus variable

If a constant is of type $\geq$, to make it equality, we ~~subs~~ subtract, a non-negative variable called surplus variable.

e.g. $x_1 + x_2 + 3x_3 \geq 60$

$\Rightarrow x_1 + x_2 + 3x_3 - x_4 = 60$

Hence, $x_4 \geq 0$ is a surplus variable.

Note- The cost ~~or~~ profit value on the slack & surplus variables are ~~zero~~ always zero.

Q. *Use simplex method to solve the LPP,

$\text{Max } z = 3x_1 + 2x_2$

s.t. $x_1 + x_2 \leq 4$

$x_1 - x_2 \leq 2$

& $x_1, x_2 \geq 0$.

The standard form is given by

$\text{Max } Z = 3x_1 + 2x_2 + 0x_3 + 0x_4$

s.t. $x_1 + x_2 + x_3 = 4$

$x_1 - x_2 + x_4 = 2$

& $x_1, x_2, x_3, x_4 \geq 0$

cost value ← C_j

Basic variable

		C_j → 3	2	0	0		minimum ratio
C_B	B.V.	x_1	x_2	x_3	x_4	sol^n	
0	x_3	1	1	1	0	4	4/1
0	x_4	$\boxed{1}$	-1	0	1	2	2/1
	Z_j	0	0	0	0		
	$Z_j - C_j$	-3	-2	0	0		

cost value of B.V.

$Z = C_B x$

pivot

always an identity matrix

→ pivot row (min)

→ $z = 0$

pivot column (most negative)

B → feasible

C → optimality

→ In maximization problem, if all $z_j - c_j \geq 0$, then optimum occurs.

→ In minimization problem, if all $z_j - c_j \leq 0$, then optimum occurs.

→ The value of $Z_z - C_j$ of basic variables are always zero.

→ If the table is not optimum, select the most ~~—~~ negative as a pivot column or as a ~~f~~ entering variable.

→ For a leaving variable, find min. ratio of $\left(\frac{\text{solution}}{\text{pivot col. element}}\right)$

24/02/23

→ We will introduce x_1 in place of x_4 in Basic Column.

→

C_B	C_j	3	2	0	0		
C_B	BV	x_1	x_2	x_3	x_4	solⁿ	min ratio
0	x_3	0	$\boxed{2}$	1	−1	2	$2/2 = 1$ ⟷
3	x_1	1	−1	0	1	2	–
	Z_j	3	−3	0	3	$Z = 6$	
	$Z_j - C_j$	0	−5	0	3		
			↑				

new cell value = (old value) − (new value) (corresponding element in the pivot column)

Note

$\rightarrow \because Z_2 - C_2 = -5$, which is (-ve), so, it is not optimal solⁿ

Note → If in the pivot column, the elements are -ve or zero, we will not consider for min. result.
(-ve will yeild infeasible
0 " " ∞)

	C_j	3	2	0	0	
C_B	BV	x_1	x_2	x_3	x_4	solⁿ
2	x_2	0	1	1/2	-1/2	1
3	x_1	1	0	1/2	1/2	3
	Z_j	3	2	5/2	1/2	Z = 11
	$Z_j - C_j$	0	0	5/2	1/2	

$\rightarrow \because$ all $Z_j - C_j \geq 0$ & it is a maximization problem, so, optimum occurs here.

→ ~~of optimal solⁿ is~~
The optimal is, $x_1 = 3$
$x_2 = 1$
Max $Z = 11$

Q. Solve the LPP by using simplex method.

Max $Z = 2x_1 - x_2 + x_3$

s.t. $3x_1 + x_2 + x_3 \leq 60$

$x_1 - x_2 + 2x_3 \leq 10$

$-x_1 - x_2 + x_3 \geq -20$

& $x_1, x_2, x_3 \geq 0$

The LPP can be written as

Max $Z = 2x_1 - x_2 + x_3$

s.t. $3x_1 + x_2 + x_3 \leq 60$

$x_1 - x_2 + 2x_3 \leq 10$

$x_1 + x_2 - x_3 \leq 20$

& $x_1, x_2, x_3 \geq 0$

The standard from is given by

~~Max Z = 3x, - x~~

$$\text{Max } Z = 2x_1 - x_2 + x_3$$

s.t. $3x_1 + x_2 + x_3 + x_4 = 60$

$x_1 - x_2 + 2x_3 + x_5 = 10$

$x_1 + x_2 - x_3 + x_6 = 20$

& $x_i \geq 0$, $i = 1$ to 6

~~Objective function~~

	C_j	2	-1	1	0	0	0		
C_B	BV	x_1	x_2	x_3	x_4	x_5	x_6	soln	min ratio
0	x_4	3	1	1	1	0	0	60	60/3 = 20
0	x_5	$\boxed{1}$	-1	2	0	1	0	10	10/1 = 10 →
0	x_6	1	1	-1	0	0	1	20	20/1 = 20
	Z_j	0	0	0	0	0	0	z = 0	
	$Z_j - C_j$	-2	1	-1	0	0	0		
		↑							

→ Not optimal

0 - 1 + 3

-3

	C_j	2	-1	1	0	0	0		
C_B	BV	x_1	x_2	x_3	x_4	x_5	x_6	soln	min ratio
0	x_4	0	4	-5	1	-3	0	30	30/4 = 7.5
2	x_1	1	-1	2	0	1	0	10	← (-ve not considered
0	x_6	0	$\boxed{2}$	-3	0	-1	1	10	10/2 = 5 →
	Z_j	2	-2	4	0	2	0		
	$Z_j - C_j$	0	-1	3	0	2	0		
			↑						

→ Not optimal

-1-2(1)

-3

0

4-2(-1) 1-

20-10

0-1(1-2(1)

	C_j	x_1
C_B	BV	0
0	x_4	0
2	x_1	1
-1	x_2	0
	Z_j	2
	$Z_j - C_j$	0

	C_j	2
C_B	BV	x_1
0	x_4	2/3
1	x_3	-2/3
-1	x_2	-1
	Z_j	1/3
	$Z_j - C_j$	-5/3
		↑

	C_j	2
C_B	BV	x_1
2	x_1	1
1	x_3	0
-1	x_2	0
	Z_j	2
	$Z_j - C_j$	0

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Q. Solve the following LPP using simplex method.

$$\text{Min } Z = x_1 - 3x_2 + 2x_3$$
$$\text{s.t. } 3x_1 - x_2 + 3x_3 \le 7$$
$$-2x_1 + 4x_2 \le 12$$
$$-4x_1 + 3x_2 + 8x_3 \le 10$$
$$\& \; x_1, x_2, x_3 \ge 0$$

~~The LPP can be written as.~~
The standared form is given by

$$\text{Min } Z = x_1 - 3x_2 + 2x_3 + 0x_4 + 0x_5 + 0x_6$$
$$\text{s.t. } 3x_1 - x_2 + 3x_3 + x_4 = 7$$
$$-2x_1 + 4x_2 + x_5 = 12$$
$$-4x_1 + 3x_2 + 8x_3 + x_6 = 10$$
$$\& \; x_i \ge 0, \; i = (1,2,3,4,5,6)$$

	C_j	1	-3	2	0	0	0		
C_B	BV	x_1	x_2	x_3	x_4	x_5	x_6	solⁿ	min ratio
0	x_4	3	-1	3	1	0	0	7	–
0	x_5	-2	$\boxed{4}$	0	0	1	0	12	$12/4 = 3 \rightarrow$
0	x_6	-4	3	8	0	0	1	10	$10/3 = 3.33$
	Z_j	0	0	0	0	0	0	Z = 0	
	$Z_j - C_j$	-1	3 ↑	-2	0	0	0		

→ In minimisation problem, for optimum value, all $Z_j - C_j \le 0$,
Here $Z_2 - C_2 > 0$, which is not optimum

if new value = 0, col^n remains same

	C_j	1	-3	2	0	0	0		
C_B	BV	x_1	x_2	x_3	x_4	x_5	x_6	sol^n	min ratio
0	x_4	5/2	0	3	1	1/4	0	10	$10 \times \frac{2}{5} = 4$ →
-3	x_2^*	-1/2	1	0	0	1/4	0	3	-
0	x_6	-5/2	0	8	0	-3/4	1	1	-
	Z_j	3/2	-3	0	0	-3/4	0	z = -9	
	$Z_j - C_j$	1/2	0	-2	0	-3/4	0		

↑

	C_j	1	-3	2	0	0	0	
C_B	BV	x_1	x_2	x_3	x_4	x_5	x_6	sol^n
1	x_1	1	0	6/5	2/5	1/10	0	4
-3	x_2	0	1	3/5	1/5	3/10	0	5
0	x_6	0	0	11	1	-1/2	1	11
	Z_j	1	-3	-3/5	-1/5	-8/10	0	z = -11
	$Z_j - C_j$	0	0	-13/5	-1/5	-8/5	0	

∵ all $Z_j - C_j \leq 0$ & it is a minimization problem. So optimum occurs here.

The optimal value is $x_1 = 4$, $x_2 = 5$, $x_3 = 0$, $x_4 = 0$, $x_5 = 0$, $x_6 = 11$

Min $Z = -11$

$\frac{-3}{4}$

- <u>Unbounded sol^n in simplex method</u> -

Max $Z = -x_1 + 4x_2$

s.t. $x_1 - x_2 \leq 1$

$-2x_1 + x_2 \leq 2$

& x_1 & $x_2 \geq 0$

The standard form is given by,

Max $Z = -x_1 + 4x_2$

s.t. $x_1 - x_2 + x_3 = 1$

$-2x_1 + x_2 + x_4 = 2$

& $x_1, x_2, x_3 \geq 0$

C_B	BV	x_1	x_2	x_3	x_4	sol^n	min ratio
	C_j	1	4	0	0		
0	x_3	1	1	0	1	1	–
0	x_4	-2	$\boxed{1}$	0	1	2	2 →
	Z_j	0	0	0	0	Z = 0	
	$Z_j - C_j$	1	-4 ↑	0	0		

$\because Z_2 - C_2 = -4$ ∴ It is not optimum.

C_B	BV	x_1	x_2	x_3	x_4	sol^n	min ratio
	C_j	-1	4	0	0		
0	x_3	-1	0	1	1	3	–
4	x_2	-2	1	0	1	2	–
	Z_j	-8	4	0	4	Z = 8	
	$Z_j - C_j$	-7	0	0	4		

1+1(2

→ Since all the elements in pivot column are negative, so we won't get leaving variable.

→ so the LPP has an unbounded soln

Q. How do you know a LPP has an unbounded sol^n in simplex method.

If all the elements of the pivot column are (-ve) or 0 then we can't find min ratio for leaving variable. & the LPP has an unbounded sol^n.

1Q. Max $Z = 3x_1 + 2x_2 + 5x_3$

s.t. $x_1 + 2x_2 + x_3 \le 430$

$3x_1 + 2x_3 \le 460$

$x_1 + 4x_2 \le 420$

& $x_1, x_2, x_3 \ge 0$

2Q. Min $Z = x_1 + x_2 + 3x_3$

s.t. $3x_1 + 2x_2 + x_3 \leq 3$

$2x_1 + x_2 + 2x_3 \leq 2$

& $x_1, x_2, x_3 \geq 0$

Ans: The standard form is given by -

$$\text{Max } Z = 3x_1 + 2x_2 + 5x_3 + 0x_4 + 0x_5 + 0x_6$$

s.t. $x_1 + 2x_2 + x_3 + x_4 = 430$

$3x_1 + 2x_3 + x_5 = 460$

$x_1 + 4x_2 + x_6 = 420$

& $x_i \geq 0$, $i = 1, 2, \dots, 6$

	C_j	3	2	5	0	0	0		
C_B	BV	x_1	x_2	x_3	x_4	x_5	x_6	sol^n	min ratio
0	x_4	1	2	1	1	0	0	430	$430/1 = 430$
0	x_5	3	0	2	0	1	0	460	$460/2 = \cancel{153.33}$ 230 $\rightarrow$
0	x_6	1	4	0	0	0	1	420	$420/0 = \cancel{420}$ ∞
	Z_j	0	0	0	0	0	0	$Z = 0$	
	$Z_j - C_j$	-3	-2	-5	0	0	0		

$\uparrow$

	C_j	3	2	5	0	0	0		
C_B	BV	x_1	x_2	x_3	x_4	x_5	x_6	sol^n	min ratio
0	x_4	-1/2	2	0	1	-1	0	200	$\cancel{200(\ )} = \cancel{400}$ $\frac{200}{2} = 100$ $\leftarrow$
5	x_3	3/2	0	1	0	1/2	0	230	$\frac{230 \times 2}{0} = \infty = \cancel{153.33}$
0	x_6	1	4	0	0	0	1	420	$420/4 = \cancel{420}$ 105
	Z_j	15/2	0	5	0	5/2	0	$Z = 1150$	
	$Z_j - C_j$	9/2	-2	0	0	5/2	0		

$\uparrow$

	C_j	3	2	5	0	0	0		min ratio
C_B	BV	x_1	x_2	x_3	x_4	x_5	x_6	sol^n	
2	x_2	$-1/4$	1	0	$1/2$	$-1/2$	0	100	$\frac{100 \times 4}{-1} = -400$
5	x_3	$3/2$	0	1	0	$1/2$	0	230	$\frac{230 \times 2}{3} = 153.33$
0	x_6	2	0	0	-2	2	1	20	$20/2 = 10$
	Z_j	7	2	5	1	$\frac{3}{2}$	0	Z = 1350	
	$Z_j - C_j$	-4	0	0	-1	$-3/2$	0		

	C_j	3	2	5	0	0	0		
C_B	BV	x_1	x_2	x_3	x_4	x_5	x_6	sol^n	min ratio
3	x_1	1	-4	0	-2	2	0	-400	$-400 \times \frac{1}{-4} = 100 \rightarrow$
5	x_3	0	0	1	0	$1/2$	0	230	∞
0	x_6	0	0	0	-2	2	1	20	∞
	Z_j	3	-12	5	-6	$17/2$	0	-50	
	$Z_j - C_j$	0	-14	0	-6	$17/2$	0		

~~Wrong!~~

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Q. Solve the following LPP by using simplex method.

$$\text{Max } Z = 4x_1 + 6x_2$$

s.t. $2x_1 + 3x_2 \le 6$

$x_1 - x_2 \le 1$

& $x_1, x_2 \ge 0$

Further, determine the alternative ~~optimum~~ optimal solⁿ if it exists.

RHS is +ve. ∴ Go for standard form.

The standard form is :

$$\text{Max } Z = 4x_1 + 6x_2 + 0x_3 + 0x_4$$

s.t. $2x_1 + 3x_2 + x_3 = 6$

$x_1 - x_2 + x_4 = 1$

& $x_i \ge 0$, $i = 1, 2, 3, 4$

Initial Table

	C_j	4	6	0	0		
C_B	BV	x_1	x_2	x_3	x_4	solⁿ	min ratio
0	x_3	2	3	1	0	6	6/3 = 2 →
0	x_4	1	−1	0	1	1	1/. −
	Z_j	0	0	0	0	Z = 0	
	$Z_j - C_j$	−4	−6	0	0		

↑

Optimal Table

	C_j	4	6	0	0		
C_B	BV	x_1	x_2	x_3	x_4	solⁿ	min ratio
6	x_2	2/3	1	1/3	0	2	2/(2/3) = 3
0	x_4	5/3	0	1/3	1	3	3/(5/3) = 9/5 →
	Z_j	4	6	2	0	Z = 12	
	$Z_j - C_j$	0	0	2	0		

↑

$1 - \frac{2}{3} \times 0 1$

$0 - \frac{1}{3} \times$

All $Z_j - C_j \ge 0$, so, it is optimum

$x_1 = 0$

$x_2 = 2$

Max $Z = 12$

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We know that, $Z_j - C_j$ of all the BV are always zero. But in this optimal table, $Z_1 - C_1 = 0$ though x_1 is Non-Basic variable. That indicates the LPP has an alternative optimal sol^n.

To get an alternative optimal sol^n, we can take x_1 as entering variable.

Alternative sol^n :

	C_j	4	6	0	0	
C_B	BV	x_1	x_2	x_3	x_4	sol^n
6	x_2	0	1	$1/5$	$-2/5$	$4/5$
4	x_1	1	0	$\frac{1}{5}$	$3/5$	$9/5$
	Z_j	4	6	2	0	$z = 12$
	$Z_j - C_j$	0	0	2	0	

$\frac{1}{3} - \frac{1}{5}\left(\frac{2}{3}\right)$

$0 - \frac{3}{5}\left(\frac{2}{3}\right)$

$2 - \frac{9}{5}\left(\frac{2}{3}\right)$

$\frac{6}{5} + \frac{4}{3}$

$-\frac{12}{5} +$

Sol^n is : $x_1 = 9/5$
$x_2 = 4/5$
Max $Z = 12$

Q. How do you know if LPP has an alternative optimal sol^n in simplex method?

If $Z_j - C_j$ of a non-BV is zero, that indicates that the LPP has an alternative sol^n.

- Degeneracy in Simplex Method -

Q. Max $Z = 6x_1 + 2x_2$

s.t. $2x_1 + x_2 \leq 4$

$4x_1 - 3x_2 \leq 8$

$x_1 + 6x_2 \leq 5$

& $x_1, x_2 \geq 0$

Standard form :

$$\text{Max } Z = 6x_1 + 2x_2 + 0x_3 + 0x_4 + 0x_5$$

$$\text{s.t. } 2x_1 + x_2 + x_3 = 4$$

$$4x_1 - 3x_2 + x_4 = 8$$

$$x_1 + 6x_2 + x_5 = 5$$

$$\& \; x_i \geq 0, \; i = 1,2,3,4,5$$

	C_j	6	2	0	0	0		
C_B	BV	x_1	x_2	x_3	x_4	x_5	sol^n	min ratio
0	x_3	2	1	1	0	0	4	$4/2 = 2$
0	x_4	4	-3	0	1	0	8	$8/4 = 2 \rightarrow$
0	x_5	1	6	0	0	1	5	$5/1 = 5$
	Z_j	0	0	0	0	0	$Z = 0$	
	$Z_j - C_j$	-6	-2	0	0	0		
		↑						

case of degeneracy better take lower one: x_4

	C_j	6	2	0	0	0		
C_B	BV	x_1	x_2	x_3	x_4	x_5	sol^n	min ratio
0	x_3	0	5/2	1	-1/2	0	0	$0/5/2 = 0 \rightarrow$
6	x_1	1	-3/4	0	1/4	0	2	-
0	x_5	0	27/4	0	-1/4	1	3	$\frac{3 \times 4}{27} = 4/9$
	Z_j	6	-9/2	0	3/2	0	$Z = 12$	
	$Z_j - C_j$	0	-13/2	0	3/2	0		
			↑					

$\frac{5}{2} \times \frac{2}{1}$

	C_j	6	2	0	0	0		
C_B	BV	x_1	x_2	x_3	x_4	x_5	sol^n	min ratio
2	x_2	0	1	2/5	-1/5	0	0	$0 \leftrightarrow$
6	x_1	1	0	-2/5	9/20	0	2	~~4/5~~ -
0	x_5	0	0	0	-1/4	1	3	∞
	Z_j	6	2	-8/5	23/10	0	$Z = 12$	
	$Z_j - C_j$	0	0	-8/5	23/10	0		
				↑				

$\frac{4}{5} - \frac{12}{5}$

$\frac{-8}{5}$

$\frac{-2}{5} + \frac{54}{20}$

$\frac{-2}{5} + \frac{1}{2}$

	C_j	6	2	0	0	0	
C_B	BV	x_1	x_2	x_3	x_4	x_5	Sol^n
0	x_3	0	5/2	1	-1/2	0	0
6	x_1	1	1	0	1/4	0	2
0	x_5	0	0	0	-1/4	1	3
	Z_j	6	6	0	3/2	0	z=12
	Z_j-C_j	0	4	0	3/2	0	

Sol^n is: ~~x_2=0~~

$x_1 = 2$

$x_2 =$

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Q. How do you get degeneracy in simplex method?
In simplex method, if a tie occurs in the min. ratio, then it's a case of degeneracy.

Tricks -
→ If the new value = 0, colⁿ remains same.
→ If any element in the pivot colⁿ is zero, & next table, the corresponding row is unchanged.

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- Artificial Variable -
A non (-ve) variable which is introduced in the constant to get an initial basic variable (an identity matrix is called Artificial variable)

- Constraints are of "$\geq$" or "="
The constraints which are of $\geq$, that can be solved by - (i) Big 'M' method
(ii) Two-phase method

(i) Big 'M' method:
→ In Big 'M' method, the cost value of artificial variable is '-M' for maximization problem or "+M" or "M" " minimization ",
where "M" is the largest (+ve) number.

Q. Solve, a LPP, Max $Z = 4x_1 + 5x_2 - 3x_3$

$$\text{s.t.} \quad x_1 + x_2 + x_3 = 10$$
$$x_1 - x_2 \geq 1$$
$$2x_1 + 3x_2 + x_3 \leq 40$$
$$\& \; x_1, x_2, x_3 \geq 0$$

by big 'M' method.

The standard form is given by.

Max $Z = 4x_1 + 5x_2 - 3x_3$ → art. var.

s.t. $x_1 + x_2 + x_3 + x_4 = 10$

$x_1 - x_2 - x_5 + x_6 = 1$

x_5 → surplus var.

x_6 → art var. to have I matrix

$2x_1 + 3x_2 + x_3 + x_7 = 40$

& $x_i \geq 0$ & $i = 1 \cdots 7$

Here, $x_4, x_6 \geq 0$ are artificial variable.

$x_5 \geq 0$ is the surplus variable

$x_7 \geq 0$ is the slack.

Max $Z = 4x_1 + 5x_2 - 3x_3 + (-M)x_4 + 0 \cdot x_5 + (-M)x_6 + 0 \cdot x_7$

	C_j	4	5	-3	-M	0	-M	0		
C_B	BV	x_1	x_2	x_3	x_4	x_5	x_6	x_7	sol^n	min ratio
-M	x_4	1	1	1	1	0	0	0	10	10
-M	x_6	1	-1	0	0	-1	1	0	1	1 →
0	x_7	2	3	1	0	0	0	1	40	20
	Z_j	-2M	0	-M	-M	M	-M	0	Z = -11M	
	$Z_j - C_j$	-2M -4	-5	-M +3	0	M	0	0		
		↑								

→ We will introduce x_1 in place of x_6 in the basis column.

	C_j	4	5	-3	-M	0	-M	0		
C_B	BV	x_1	x_2	x_3	x_4	x_5	~~x_6~~	x_7	sol^n	min ratio
-M	x_4	0	2	1	1	1	~~0~~	0	9	9/2 = 4.5 →
4	x_1	1	-1	0	0	-1	~~1~~	0	1	
0	x_7	0	5	1	0	2	~~0~~	1	38	38/5 = 7.6
	Z_j	4	-2M -4	-M	-M	-M -4		0	Z = -9M + 4	
	$Z_j - C_j$	0	-2M -9	-M +3	0	-M -4		0		
			↑							

→ Since, x_6 is an artificial variable & already left from the basis column, so, we won't consider x_6 further in any tables.

	C_j	4	5	-3	-M	0	0	
C_B	BV	x_1	x_2	x_3	~~x_4~~	x_5	x_7	sol^n
5	x_2	0	1	$1/2$		$1/2$	0	$9/2$
4	x_1	1	0	$\frac{1}{2}$		$-1/2$	0	$11/2$
0	x_7	0	0	$-\frac{3}{2}$		$-1/2$	1	$31/2$
	Z_j	4	5	$9/2$		$1/2$	0	$Z_j = 89/2$
	$Z_j - C_j$	0	0	$\frac{15}{2}$		$1/2$	0	

$2 - \frac{1}{2} \times 5$

$38 - \frac{9}{2} \times 5$

→ ∵ x_4 is an artificial variable & already left from the basis column, so we won't consider x_4 further.

→ ∵ All $Z_j - C_j \geq 0$ & it is maximization problem, so optimum occur here.

The optimal sol^n is:

$x_1 = 11/2$

$x_2 = 9/2$

$x_3 = 0$

1. Solve LPP by using Big 'M' method.

$\text{Max } Z = -x_1 + x_2$

s.t. $x_1 + x_2 \leq 1$

$2x_1 + 3x_2 \geq 6$

& $x_1, x_2 \geq 0$

Standard form is given by:

$\text{Max } Z = -x_1 + x_2 + 0 \cdot x_3 + 0x_4 + (-M)x_5$

s.t. $x_1 + x_2 + x_3 = 1$

$2x_1 + 3x_2 - x_4 + x_5 = 6$

& $x_1, x_2, x_3, x_4, x_5 \geq 0$

	C_j	-1	1	0	0	-M		min ratio
C_B	BV	x_1	x_2	x_3	x_4	x_5	sol^n	
0	x_3	1	1	1	0	0	1	$1/1 = 1$ →
-M	x_5	2	3	0	-1	1	6	$6/3 = 2$
	Z_j	-2M	+3M	0	M	-M	Z = -6M	
	$Z_j - C_j$	-2M	-3M	0	M	0		
		+1	-1					

↑

-3-1
-2+1

	C_j	-1	1	0	0	-M	
C_B	BV	x_1	x_2	x_3	x_4	x_5	sol^n
1	x_2	1	1	~~1~~	0	0	1
-M	x_5	-1	0	~~-3~~	-1	1	3
	Z_j	1+M	1	~~1+3M~~	M	-M	Z = 1-3M
	$Z_j - C_j$	2+M	0	~~1+3M~~	M	0	

2-1(3)
0-1(3)
6-1(3)

x_3 cannot be striked. Only artificial variables can be.

$x_1 = 0$
$x_2 = 1$
$Z = 1-3M$

3/3/23 The LPP has an infeasible solution because, the artificial variable x_5 is present in basis column with +ve value.

~~Assume Hard~~

Q. How do you find, LPP has an infeasible sol^n?
A. If in the optimal table, the artificial variable appears in basis column on a +ve level, then the LPP has infeasible sol^n or pseudooptimal sol^n.

Q. Solve Min $Z = 4x_1 + 8x_2 + 3x_3$

s.t. $x_1 + x_2 \geq 2$

$2x_1 + x_3 \geq 5$

& $x_1, x_2, x_3 \geq 0$

RHS is +ve — min ∴ +M

∴ The standard form is given by:

$\text{Min } Z = 4x_1 + 8x_2 + 3x_3 - 0 \cdot x_4 + (M)x_5 + 0 \cdot x_6 + Mx_7$

s.t. $x_1 + x_2 - x_4 + x_5 = 2$

$2x_1 + x_3 - x_6 + x_7 = 5$

& $x_i \geq 0$, $i = 1, 2, \ldots, 7$

$\boxed{\text{* we can ignore } x_5, x_7 \text{ b/c we get I matrix from } x_2, x_3}$

$x_4, x_6 \geq 0$ are surplus variable

& $x_5, x_7 \geq 0$ are artificial variable

	C_j	4	8	3	0	M	0	M		
C_B	BV	x_1	x_2	x_3	x_4	x_5	x_6	x_7	sol^n	min ratio
M	x_5	1	1	0	-1	1	0	0	2	$2/1 = 2 \rightarrow$
M	x_7	2	0	1	0	0	-1	1	5	$5/2 = 2.5$
	Z_j	3M	M	M	-M	M	-M	M	Z = 7M	
	$Z_j - C_j$	3M-4	M-8	M-3	-M	0	-M	0		

↑

	C_j	4	8	3	0	M	0	M		
C_B	BV	x_1	x_2	x_3	x_4	~~x_5~~	x_6	x_7	sol^n	min ratio
4	x_1	1	1	0	-1	~~1~~	0	0	2	~~2/-1 = -2~~ −
M	x_7	0	-2	1	2		-1	1	1	$1/2 = 0.5 \rightarrow$
	Z_j	4	4-2M	M	-4+2M		-M	M	8+M = Z	
	$Z_j - C_j$	0	-2M-4	M-3	2M-4		-M	0		

↑

	C_j	4	8	3	0	M	0	M	
C_B	BV	x_1	x_2	x_3	x_4	~~x_5~~	x_6	~~x_7~~	sol^n
4	x_1	1	0	1/2	0		-1/2		5/2
0	x_4	0	-1	1/2	1		-1/2		1/2
	Z_j	4	0	2	0		-2		Z=10
	$Z_j - C_j$	0	-8	-1	0		-2		