Digital Image Processing

Intensity Transformations and Spatial Filtering

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Overview

- **Basic Concepts**
- **Intensity transformation and spatial filtering**
- **Basic intensity transformation functions**
- **Piecewise linear transformation functions**
- **Histogram processing**

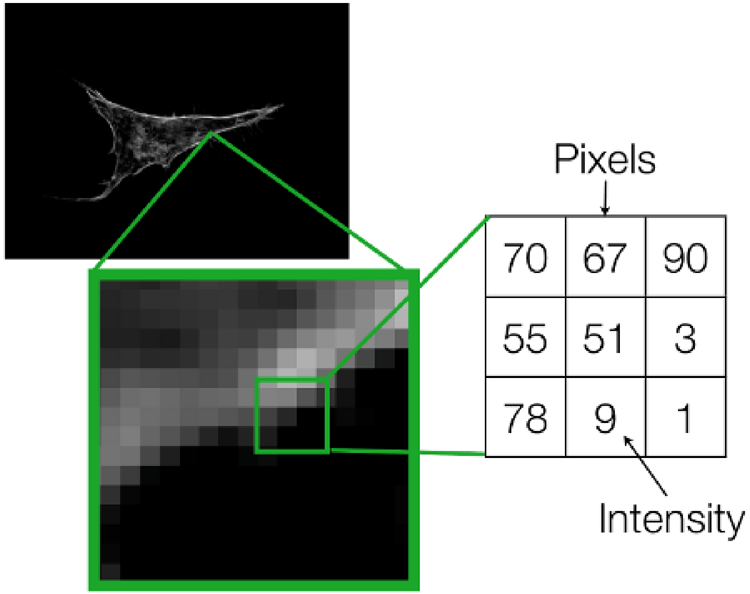
Basic Concepts

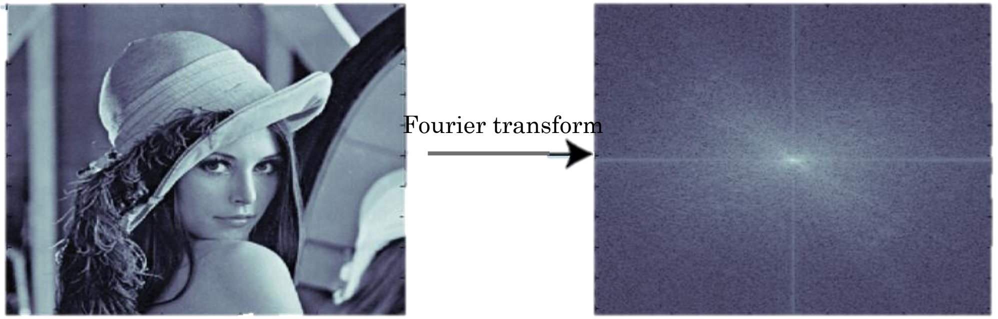
Spatial Domain

- Image plan itself, direct manipulation of pixels in an image.

Transform Domain

- Process the transform coefficients, not directly process the intensity values of the image plane
- E.g. In frequency domain operations are performed on the Fourier transform of an image.





Spatial Domain

Frequency Domain

Basic Concepts

Spatial Domain Process

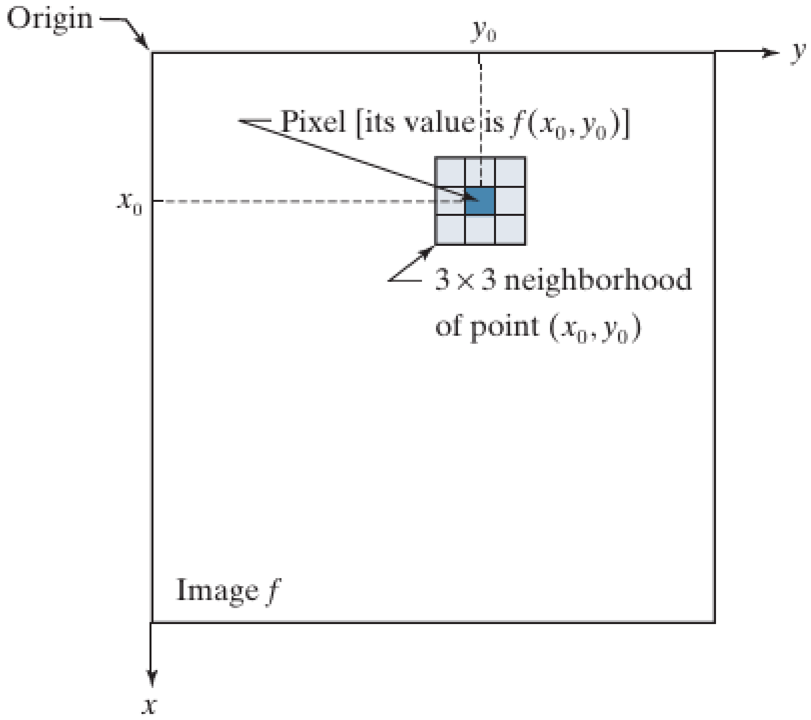
$$g(x, y) = T[f(x, y)]$$

$f(x, y)$: input image

$g(x, y)$: output image

T : An operator on f defined over a neighborhood of point (x, y)

A 3 x 3 neighborhood about a point (x_0 , y_0) in an image. The neighborhood is moved from pixel to pixel in the image to generate an output image.



Basic Concepts

Types of operations in spatial domain

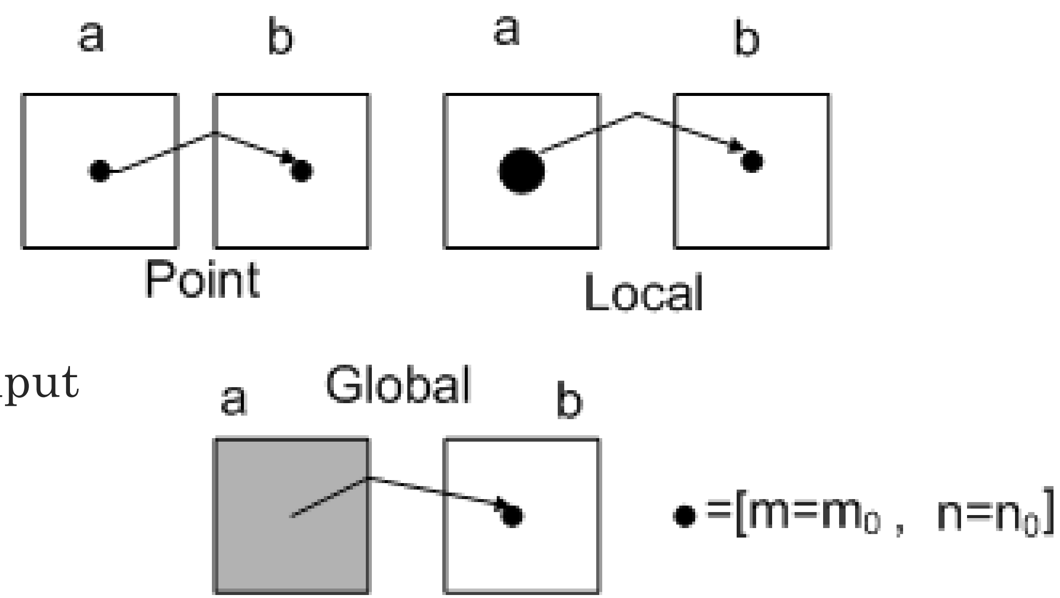
Point/pixel Operations

- Output value at specific coordinates (x, y) is dependent only on the input value at (x, y)
- In this case the neighborhood is 1x1

$$s = T(r)$$

Local Operations

- The output value at (x, y) is dependent on the input values in the neighborhood of (x, y)



Global Operations

- The output value at (x, y) is dependent on all the values in the input image

Basic Concepts

Linear vs Nonlinear Operations

$$H[f(x,y)] = g(x,y)$$

- Given two arbitrary constants, a and b, and two arbitrary images f(x, y) and f(x,y),
- H is said to be a linear operator if

$$H[af_1(x,y) + bf(x,y)] = aH[f_1(x,y)] + bH[f_2(x,y)]$$

→ Additivity

$$H[af_1(x,y) + bf_2(x,y)] = \mathrm{a}\,\mathrm{g}_1(\mathrm{x},\mathrm{y}) + \mathrm{b}\,\mathrm{g}_2(\mathrm{x},\mathrm{y})$$

→ Homogeneity

- An operator that fails to satisfy these properties is said to be nonlinear.

Examples
Linear => sum operator
Nonlinear => max operator

Intensity Transformation and Spatial Filtering

Intensity Transformations

- Intensity transformations operate on single pixels of an image
- E.g. Contrast manipulation, image thresholding

Before Contrast Enhancement After Contrast Enhancement

Spatial Filtering

- Performs operations on the neighbor hood of every pixel in an image
- E.g. image smoothing and sharpening

Before After

Image Enhancement

- Process an image to make the result more suitable than the original image for a specific application
- Image enhancement is subjective (problem oriented)
- Intensity transformation and spatial filtering techniques are often used for image enhancement

Basic Intensity Transformation Functions

Image Negatives

$$s = L - 1 - r$$

Applications

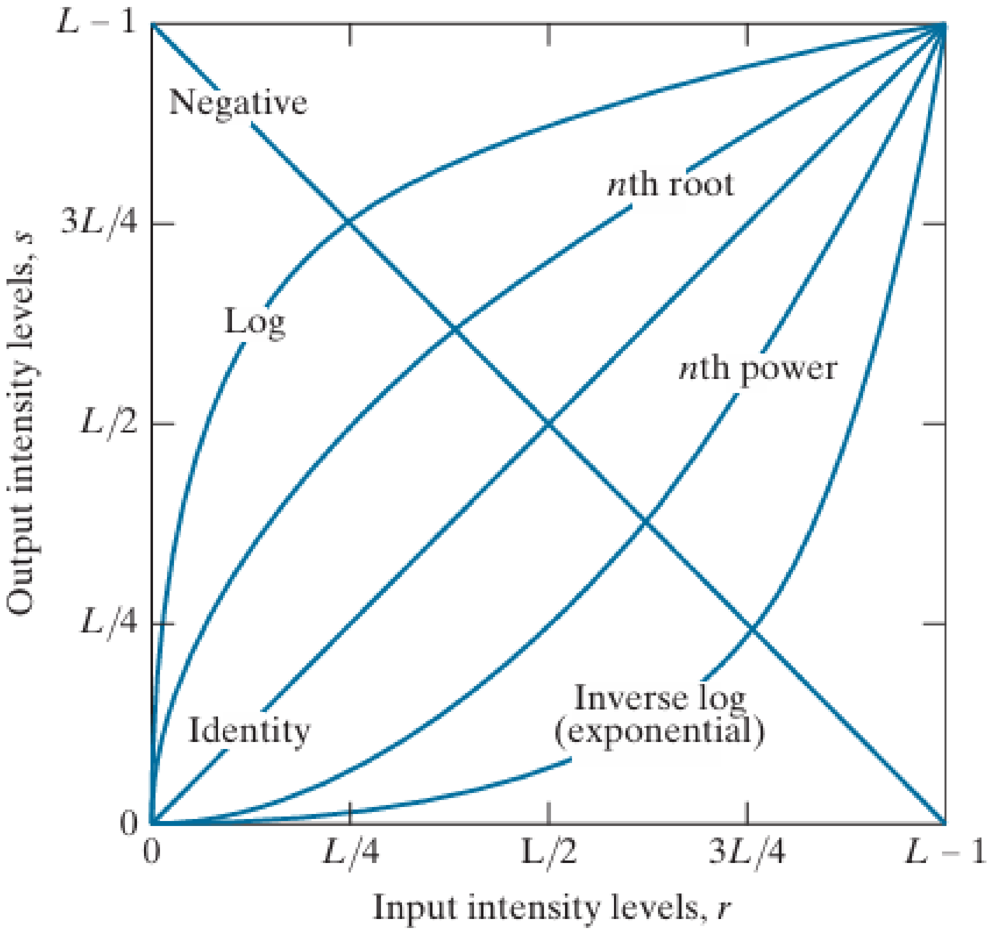
- Enhancing white or gray detail embedded in dark regions.

L = 256

0	50	200
60	128	30
186	255	40

$\longrightarrow$

255	205	55
195	127	225
69	0	215

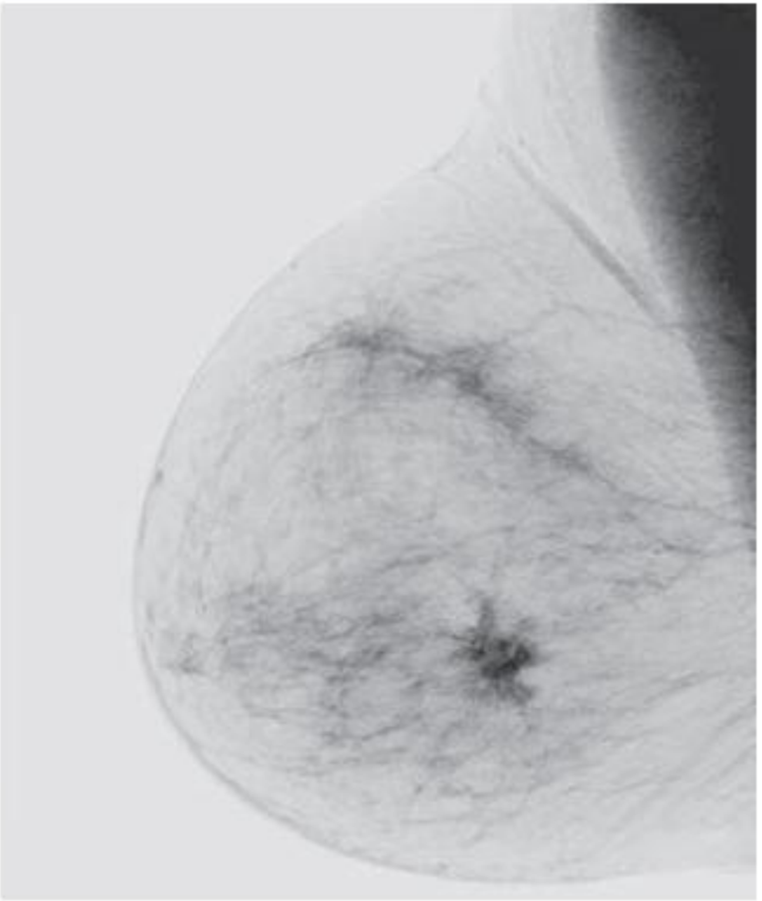


Some basic intensity transformation functions

Basic Intensity Transformation Functions

Image Negatives

A digital mammogram



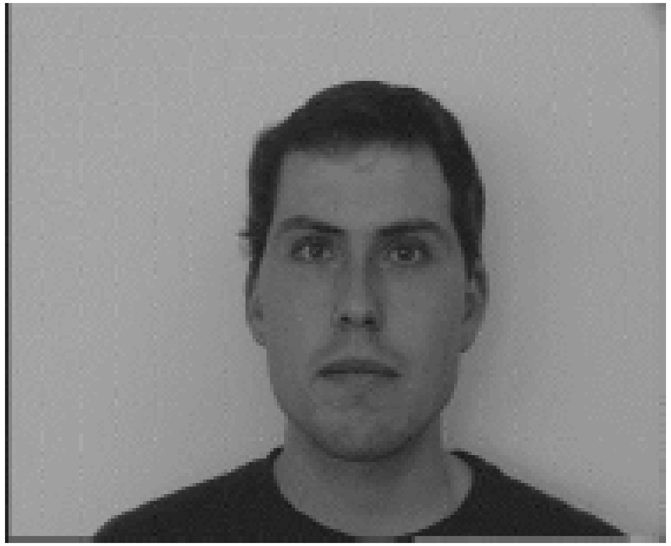
Negative image obtained using image negatives

Basic Intensity Transformation Functions

Image Scaling

$$s = T(r) = a.r$$

Original image



$f(x, y)$

Scaled image

$a \cdot f(x, y)$

Basic Intensity Transformation Functions

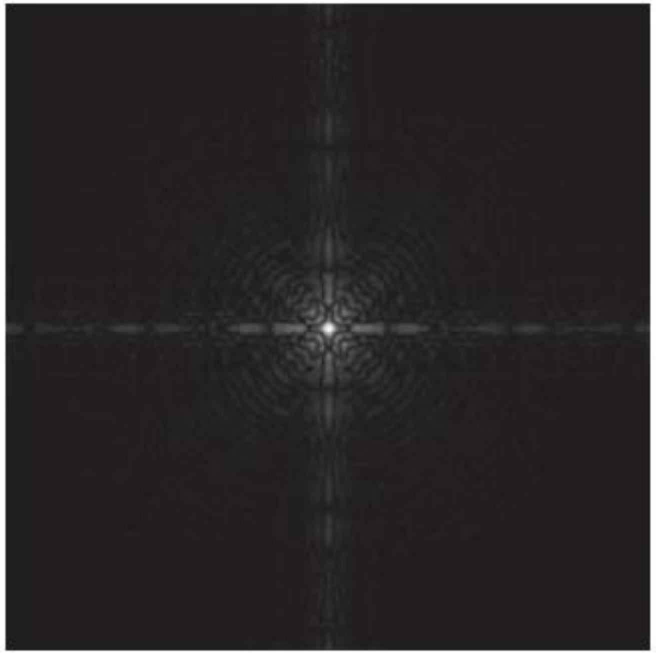
Log Transformations

$$s = c \log(1 + r)$$

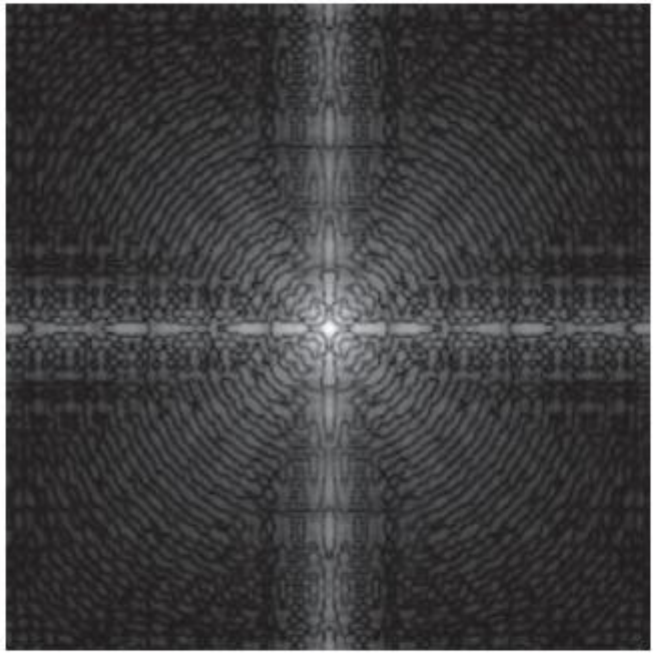
Used to expand the values of dark pixels in an image, while compressing the higher-level values.

Applications

- This transformation is suitable for the case when the dynamic range of a processed image far exceeds the capability of the display device (e.g. display of the Fourier spectrum of an image)
- Also called "dynamic-range compression / expansion"



Fourier spectrum displayed as a grayscale image



Result of applying the log transformation with c=1

Basic Intensity Transformation Functions

Power-law (Gamma) Transformations

$$s = c\, r^{\gamma}$$

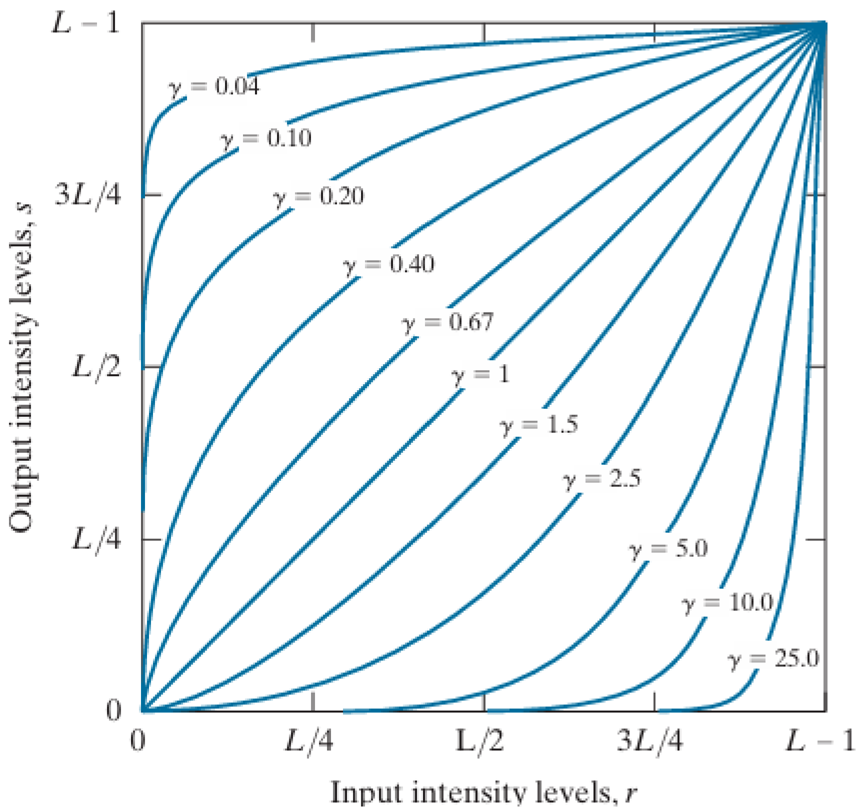
For $\gamma > 1$: Expand values of dark pixels, compress values of brighter pixels

For $\gamma > 1$: Compresses values of dark pixels, expand values of brighter pixels

Applications

- The response of many devices used for image capture, printing, and display obey a power law

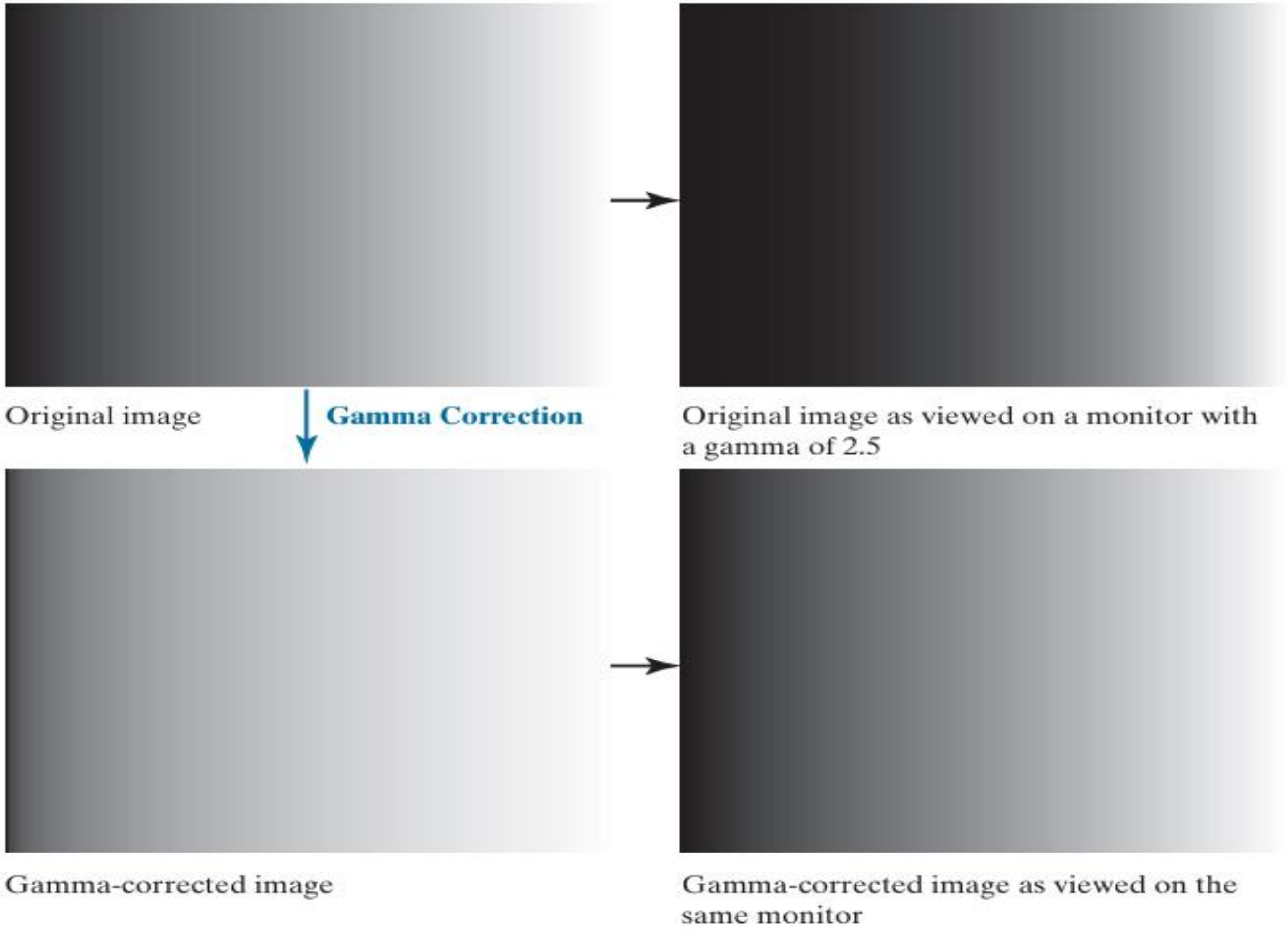
The process used to correct these power-law response phenomena is called gamma correction or gamma encoding.



Plots of the gamma equation for various values of gamma (c =1 in all cases)

Basic Intensity Transformation Functions

Power-law (Gamma) Transformations



Response of CRT to Linear wedge

Output of monitor

Basic Intensity Transformation Functions

Power-law (Gamma) Transformations

MRI image of fractured human spine

Result of applying power-law transformation

c = 1, $\gamma = 0.6$

Result of applying power-law transformation

c = 1, $\gamma = 0.4$

Result of applying power-law transformation

c = 1, $\gamma = 0.3$

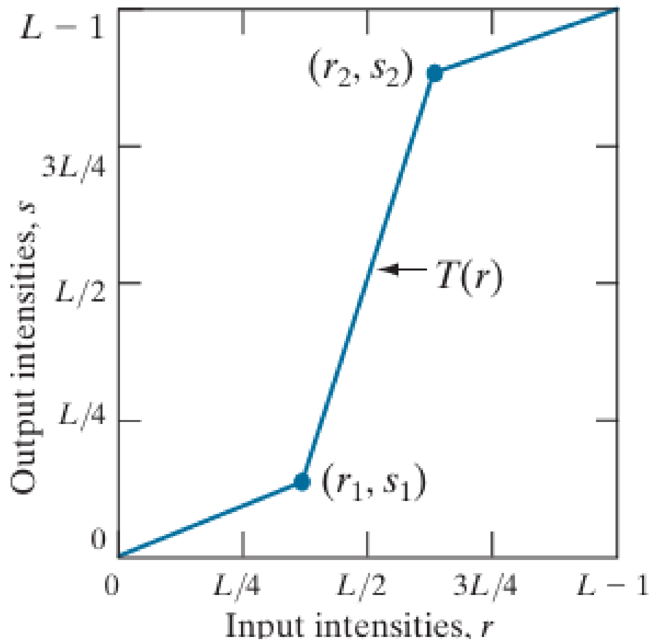
Piecewise Linear Transformations

Contrast Stretching

Increasing the dynamic range of the gray levels for low contrast images.

Low-contrast images can result from

- Poor illumination,
- Lack of dynamic range in the imaging sensor, or
- Wrong setting of a lens aperture during image acquisition

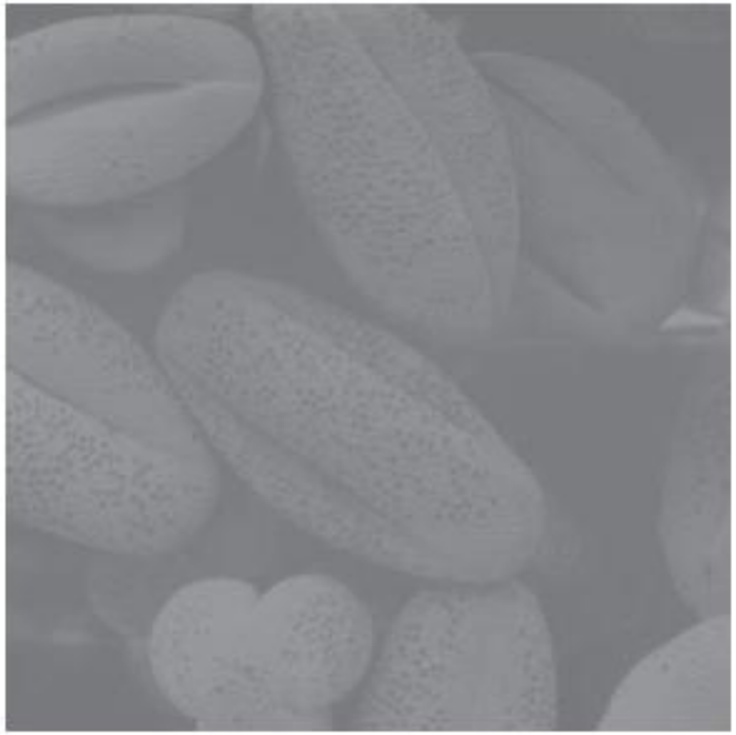


$$s = T(r) = \begin{cases} a_1\, r, & 0 \le r < r \\ a_2(r - r_1) + s_1, & r_1 \le r < r_2 \\ a_3(r - r_2) + s_2, & r_2 \le r \le (L-1) \end{cases}$$

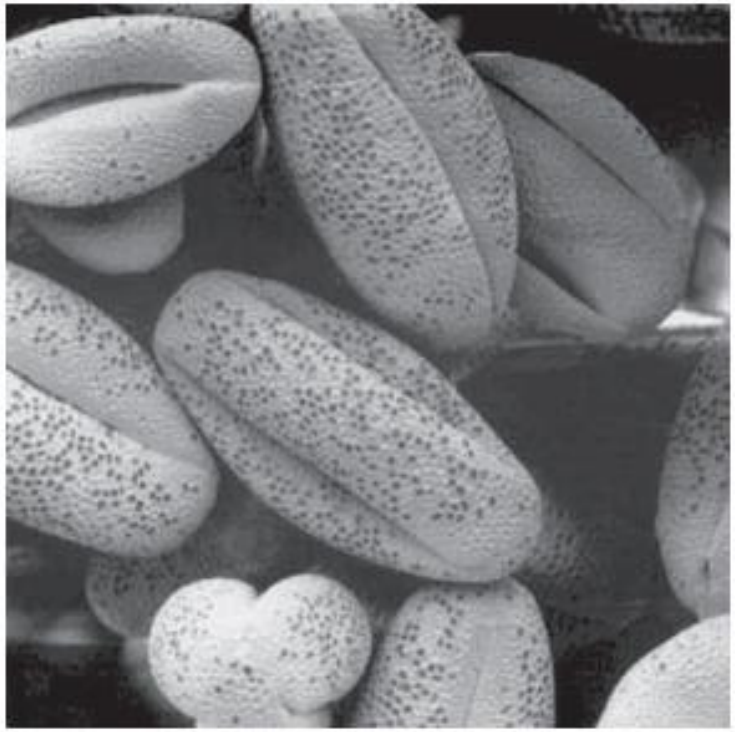
$$s1 = T(r1)$$
$$s2 = T(r2)$$

Piecewise Linear Transformations

Contrast Stretching



Original Image

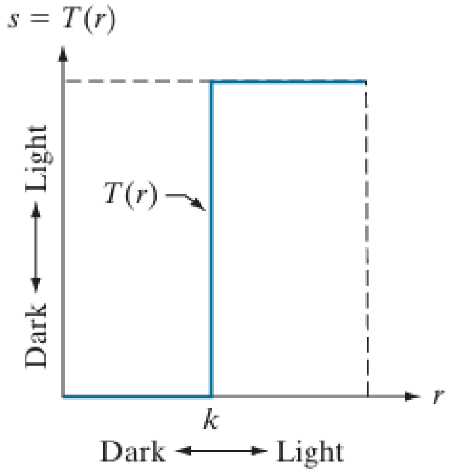


Result of contrast stretching

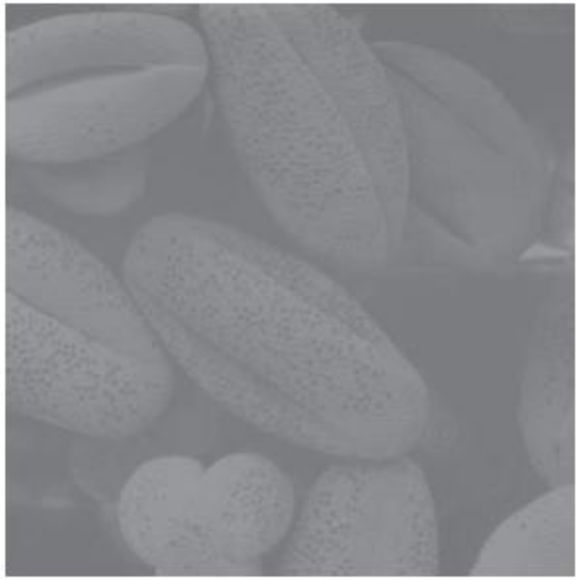
Piecewise Linear Transformations

Thresholding

A technique to convert a grayscale image into a binary image by setting pixels below a threshold to black (0) and those above the threshold to white (255).



$$s = T(r) = \begin{cases} 0, & 0 \le r < k \\ 255, & k \le r \le (L-1) \end{cases}$$



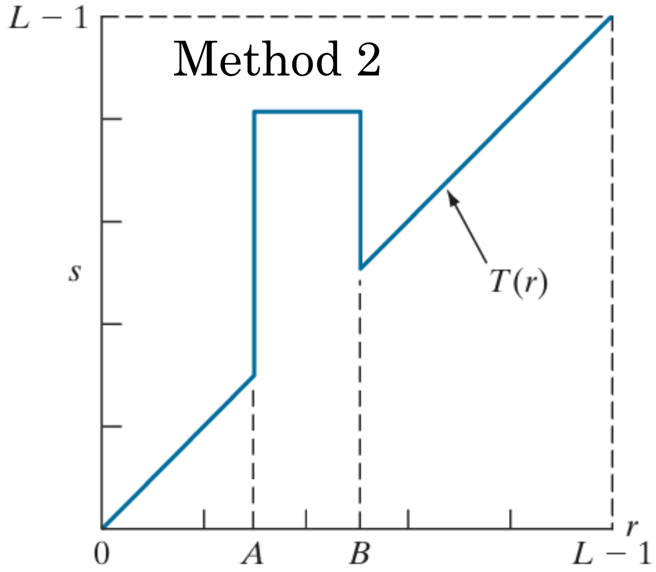
Piecewise Linear Transformations

Intensity-Level Slicing

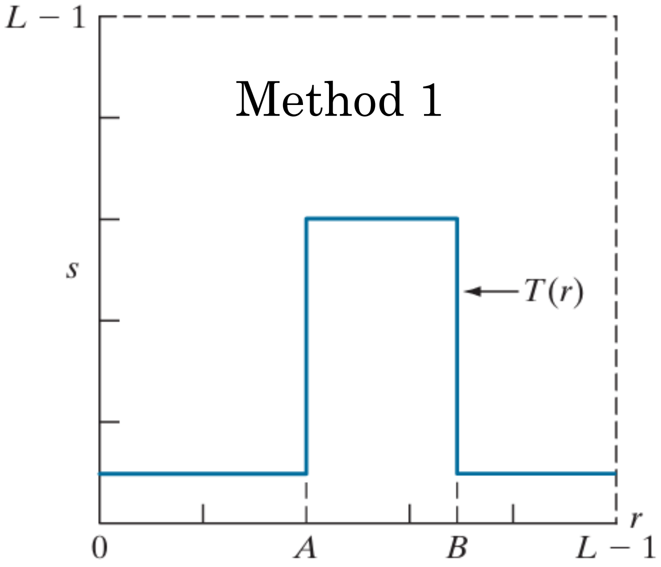
- Highlighting a specific range of intensities in an image often is of interest.

Applications

- enhancing features in satellite imagery, such as masses of water, and enhancing flaws in X-ray images



Highlights range [A ,B] and reduces all other intensities to a lower level.



Highlights range [A , B] and leaves other intensities unchanged.

Piecewise Linear Transformations

Intensity-Level Slicing

Original Image

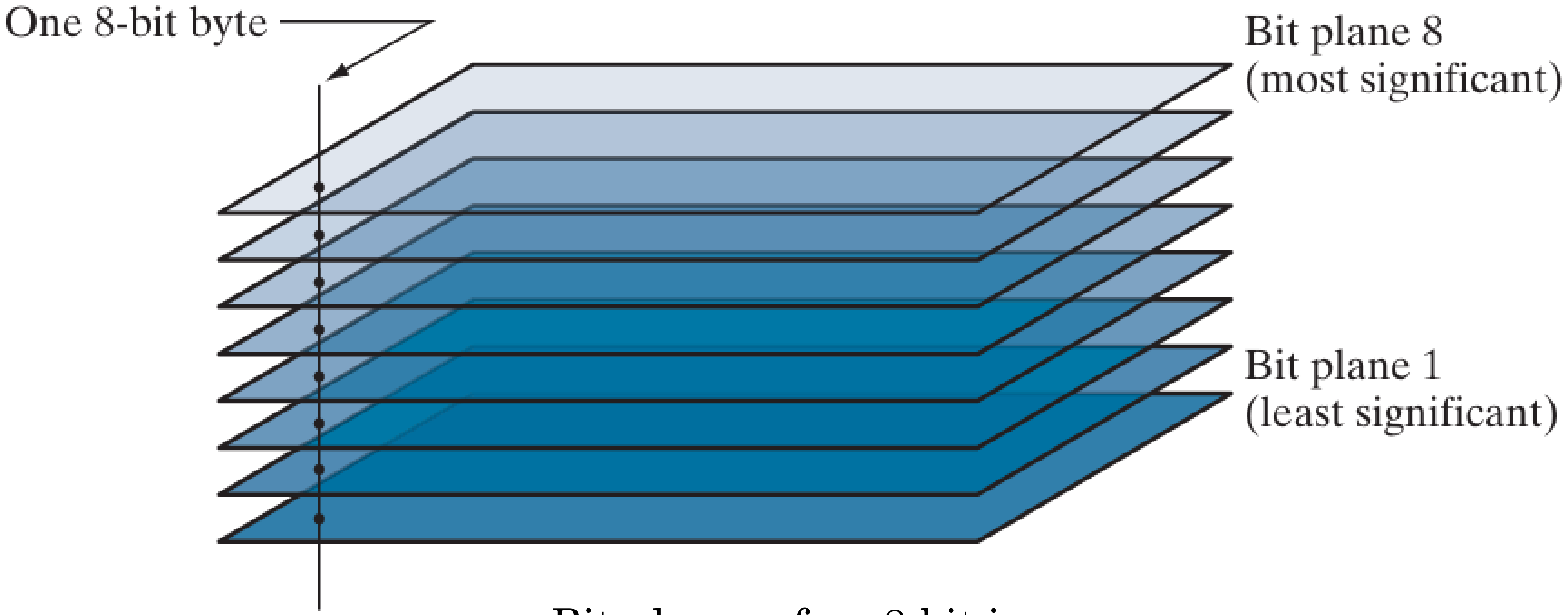
Method 1 result

Method 2 result

Piecewise Linear Transformations

Bit-Plane Slicing

- Highlight the contribution made to total image appearance by specific bits.



Bit-planes of an 8-bit image

Piecewise Linear Transformations

Bit Plane Slicing for 8-bit Grayscale Image

Binary (decimal)

MSB → LSB

0 ↓ 255

8^{th} bit	7^{th} bit	6^{th} bit	5^{th} bit	4^{th} bit	3^{rd} bit	2^{nd} bit	1^{st} bit
0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)
:	:	:	:	:	0 (1)	0 (1)	1 (1)
:	:	0 (31)	0 (15)	0 (7)	0 (2)	1 (2)	0 (2)
:	0 (63)	1 (32)	1 (16)	1 (8)	0 (3)	1 (3)	1 (3)
:	1 (64)	:	:	:	1 (4)	0 (4)	0 (4)
:	:	1 (63)	1 (31)	1 (15)	1 (5)	0 (5)	1 (5)
:	:	0 (64)	0 (32)	0 (16)	1 (6)	1 (6)	0 (6)
0 (127)	1 (127)	:	:	:	1 (7)	1 (7)	1 (7)
1 (128)	0 (128)	0 (95)	0 (47)	0 (23)	0 (8)	0 (8)	0 (8)
:	:	1 (96)	1 (48)	1 (24)	0 (9)	0 (9)	1 (9)
:	:	:	:	:	0 (10)	1 (10)	0 (10)
:	0 (191)	1 (127)	1 (63)	1 (31)	0 (11)	1 (11)	1 (11)
:	1 (192)	:	:	:	:	:	:
:	:	:	:	:	:	:	:
:	:	:	:	:	:	:	:
1 (255)	1 (255)	1 (255)	1 (255)	1 (255)	1 (255)	1 (255)	1 (255)

Piecewise Linear Transformations

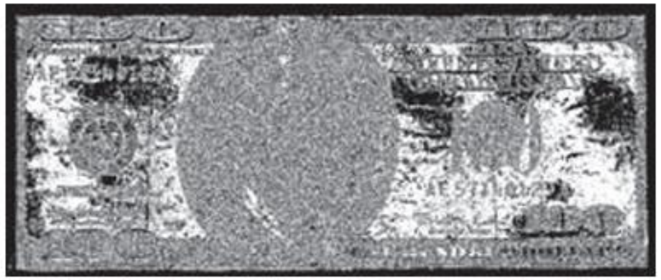
Bit-Plane Slicing

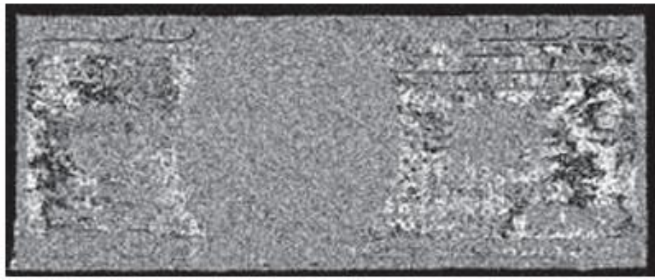
Original Image

Bit-8 plane


Bit-7 plane


Bit-6 plane


Bit-5 plane


Bit-4 plane


Bit-3 plane


Bit-2 plane


Bit-1 plane

Histogram Processing

Histogram

A histogram shows how frequently each intensity value occurs in an image.

Unnormalized Histogram

$$h(r_k) = n_k \qquad for\ k = 0, 1, 2, \ldots\ldots.., L-1$$

- $r_k \rightarrow\ k^{th}$intensity value
- $n_k \rightarrow$ number of pixels in image with intensity r_k

Normalized Histogram

$$p(r_k) = \frac{h(r_k)}{MN} = \frac{n_k}{MN}$$

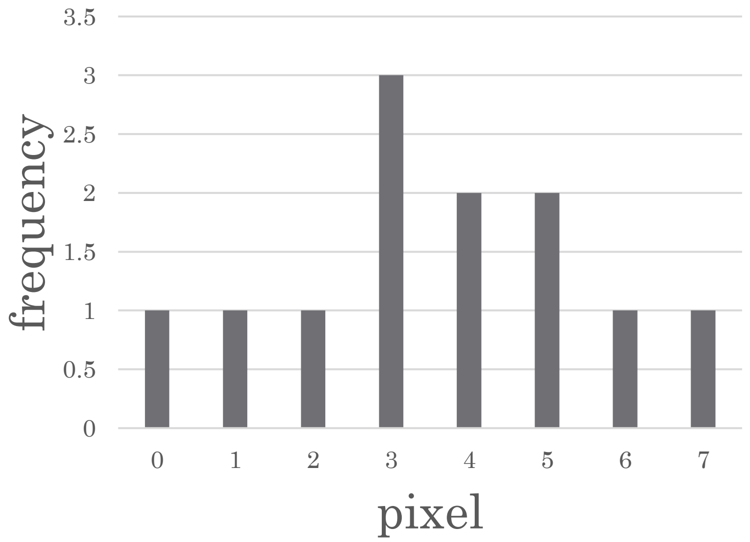
- n_k →Number of pixels in the image of size M x N with intensity r_k
- The sum of $p(r_k)$ for all values of k is always 1

image

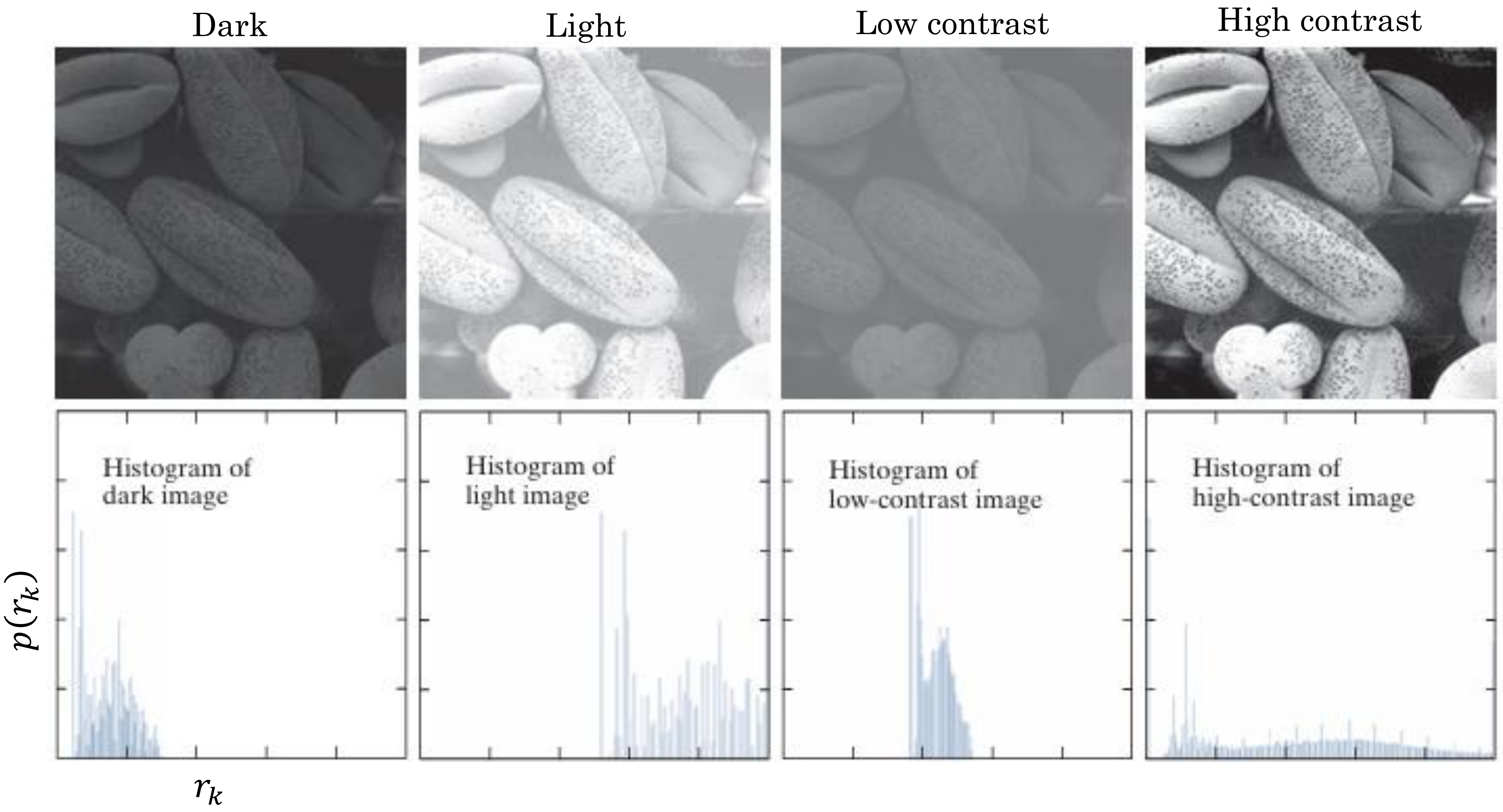
3	2	5
3	1	3
4	5	0
6	7	4

M × N

Pixel (r_k)	r_0	r_1	r_2	r_3	r_4	r_5	r_6	r_7
Frequency	1	1	1	3	2	2	1	1



Histogram Processing



Histogram Processing

Histogram Equalization

Adjust the contrast of an image by modifying the intensity distribution of the histogram

Histogram Equalization steps

1. Compute the histogram of the image.

$$h(r_k) = n_k \qquad for\ k = 0, 1, 2, \dots \dots.., L-1$$

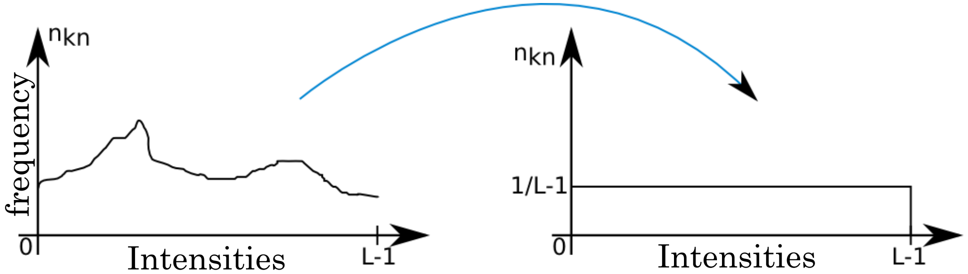
2. Normalize the histogram to get the probability distribution.

$$p(r_k) = \frac{h(r_k)}{MN} = \frac{n_k}{MN}$$

3. Calculate the cumulative distribution function (CDF).

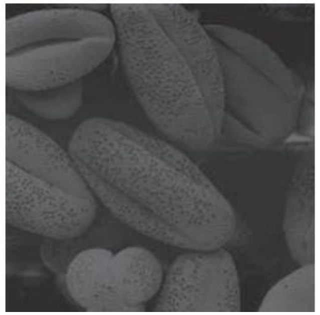
$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j) \qquad k = 0, 1, 2, \dots.., L-1$$

4. Use the CDF to map the old pixel values to new ones for equalized distribution.

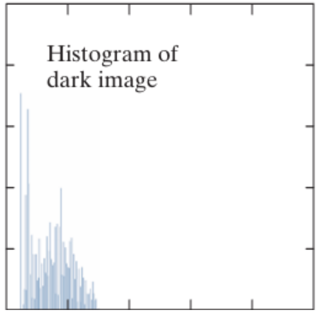


Original Histogram

Equalized Histogram

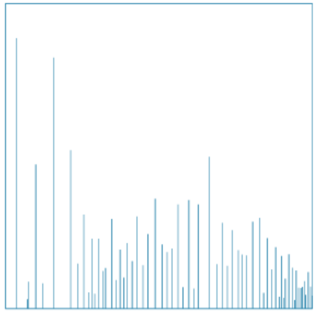


Dark image



Dark image histogram

Histogram-equalized image



Equalized histogram

Histogram Processing

Histogram Equalization Example

Suppose a 3-bit image (L=8) of size 64 x 64, pixels (MN = 4096)

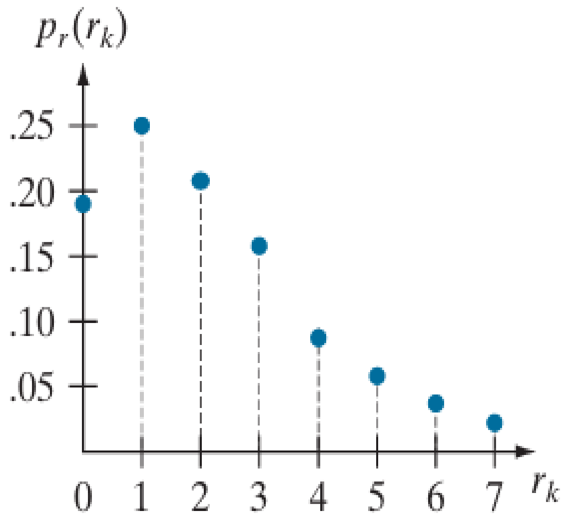
- r_k: Intensity levels
- n_k: Number of pixels at intensity r_k
- Cdf : Comulative Distribution Function

r_k	n_k	$p(r_k) = n_k/MN$	cdf	7 * cdf	Round
$r_0 = 0$	790	0.19	0.19	1.33	1
$r_1 = 1$	1023	0.25	0.44	3.08	3
$r_2 = 2$	850	0.21	0.65	4.55	5
$r_3 = 3$	656	0.16	0.81	5.67	6
$r_4 = 4$	329	0.08	0.89	6.23	6
$r_5 = 5$	245	0.06	0.95	6.65	7
$r_6 = 6$	122	0.03	0.98	6.86	7
$r_7 = 7$	81	0.02	1	7.00	7

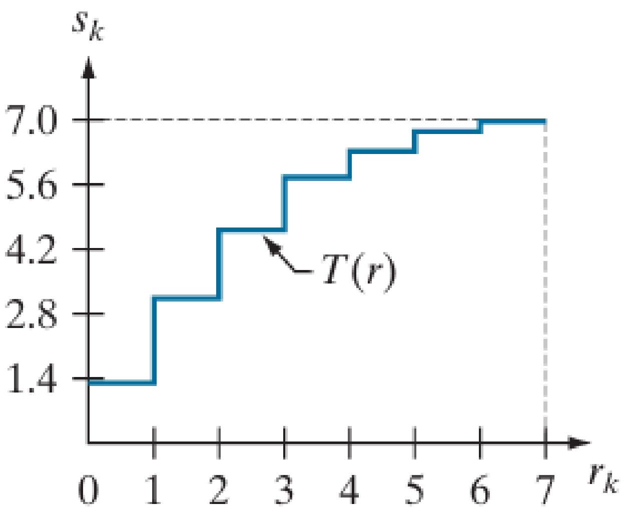
These are the values of the equalized histogram

Histogram Processing

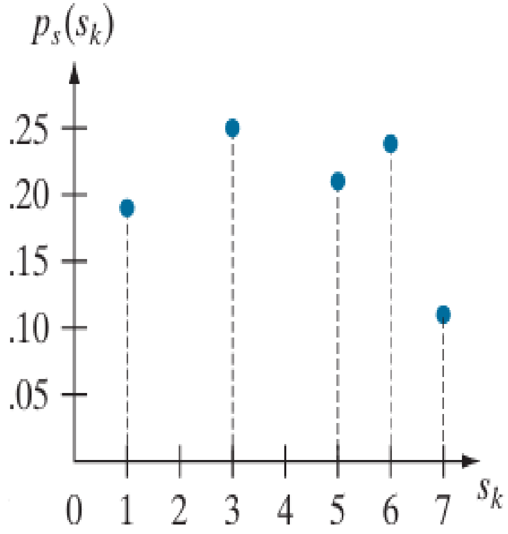
Histogram Equalization Example



Original histogram



Transformation function



Equalized histogram

Thank You