

Digital Image Processing

Intensity Transformations and Spatial Filtering

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Overview

- **Basic Concepts**
- **Intensity transformation and spatial filtering**
- **Basic intensity transformation functions**
- **Piecewise linear transformation functions**
- **Histogram processing**

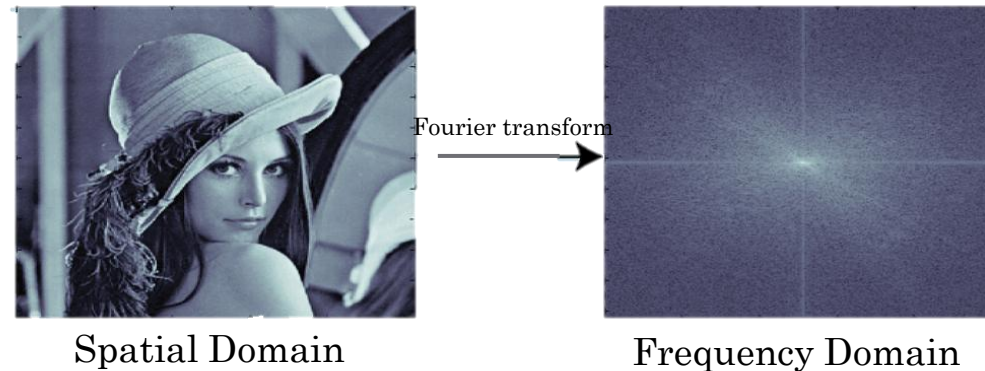
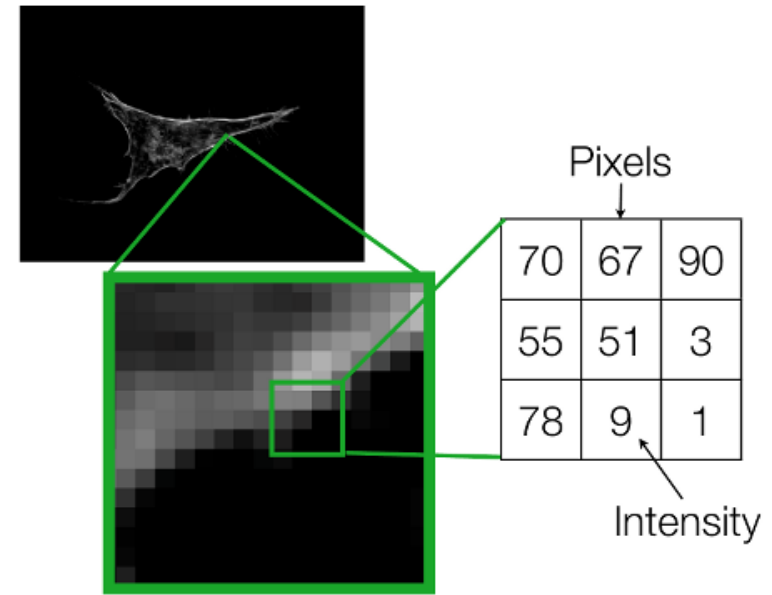
Basic Concepts

Spatial Domain

- Image plan itself, direct manipulation of pixels in an image.

Transform Domain

- Process the transform coefficients, not directly process the intensity values of the image plane
- E.g. In frequency domain operations are performed on the Fourier transform of an image.



Basic Concepts

Spatial Domain Process

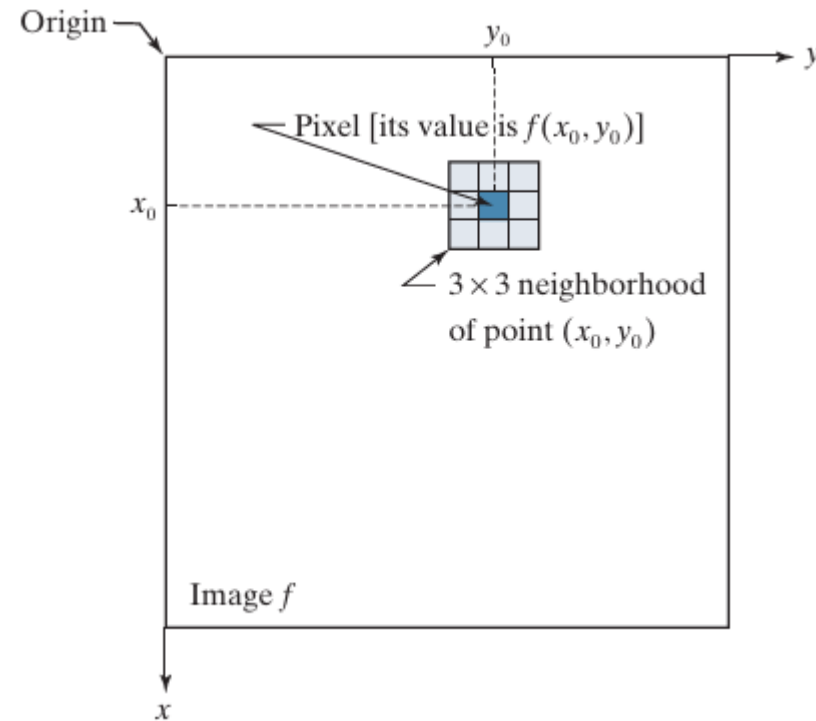
$$g(x, y) = T[f(x, y)]$$

$f(x, y)$: input image

$g(x, y)$: output image

T : An operator on f defined over a neighborhood of point (x, y)

A 3×3 neighborhood about a point (x_0, y_0) in an image. The neighborhood is moved from pixel to pixel in the image to generate an output image.



Basic Concepts

Types of operations in spatial domain

Point/pixel Operations

- Output value at specific coordinates (x, y) is dependent only on the input value at (x, y)
- In this case the neighborhood is 1x1

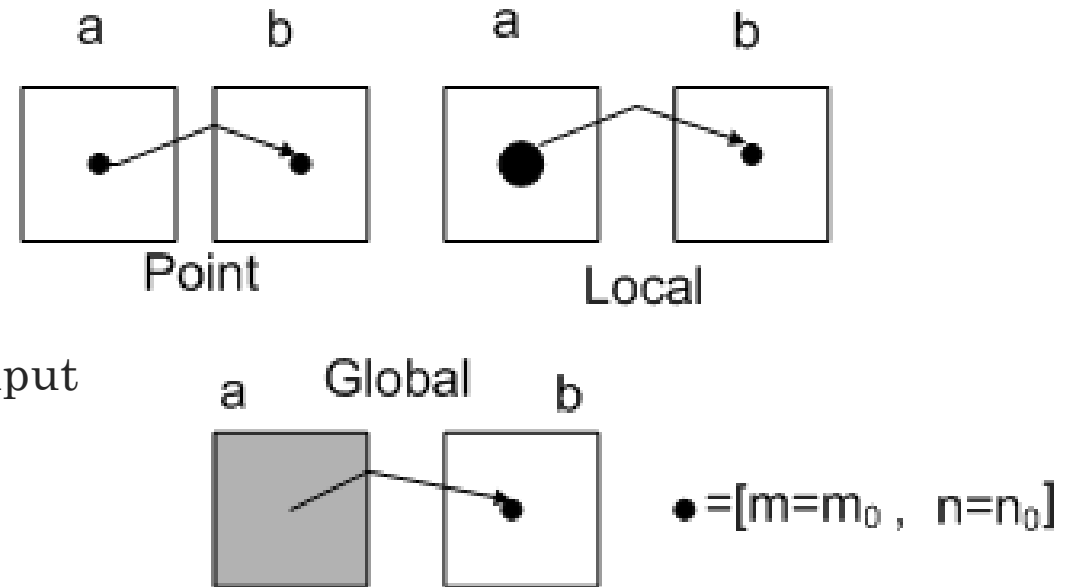
$$s = T(r)$$

Local Operations

- The output value at (x, y) is dependent on the input values in the neighborhood of (x, y)

Global Operations

- The output value at (x, y) is dependent on all the values in the input image



Basic Concepts

Linear vs Nonlinear Operations

$$H[f(x, y)] = g(x, y)$$

- Given two arbitrary constants, a and b , and two arbitrary images $f_1(x, y)$ and $f_2(x, y)$,
- H is said to be a linear operator if

$$H[af_1(x, y) + bf_2(x, y)] = aH[f_1(x, y)] + bH[f_2(x, y)]$$

→ Additivity

$$H[af_1(x, y) + bf_2(x, y)] = a g_1(x, y) + b g_2(x, y)$$

→ Homogeneity

- An operator that fails to satisfy these properties is said to be nonlinear.

Examples

Linear \Rightarrow sum operator

Nonlinear \Rightarrow max operator

Intensity Transformation and Spatial Filtering

Intensity Transformations

- Intensity transformations operate on single pixels of an image
- E.g. Contrast manipulation, image thresholding

Spatial Filtering

- Performs operations on the neighborhood of every pixel in an image
- E.g. image smoothing and sharpening

Image Enhancement

- Process an image to make the result more suitable than the original image for a specific application
- Image enhancement is subjective (problem oriented)
- Intensity transformation and spatial filtering techniques are often used for image enhancement



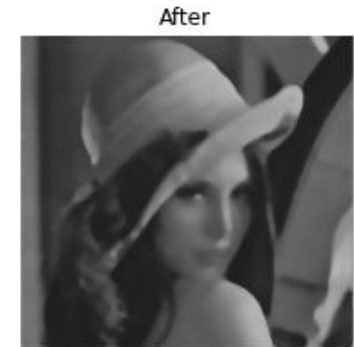
Before Contrast Enhancement



After Contrast Enhancement



Before



After

Basic Intensity Transformation Functions

Image Negatives

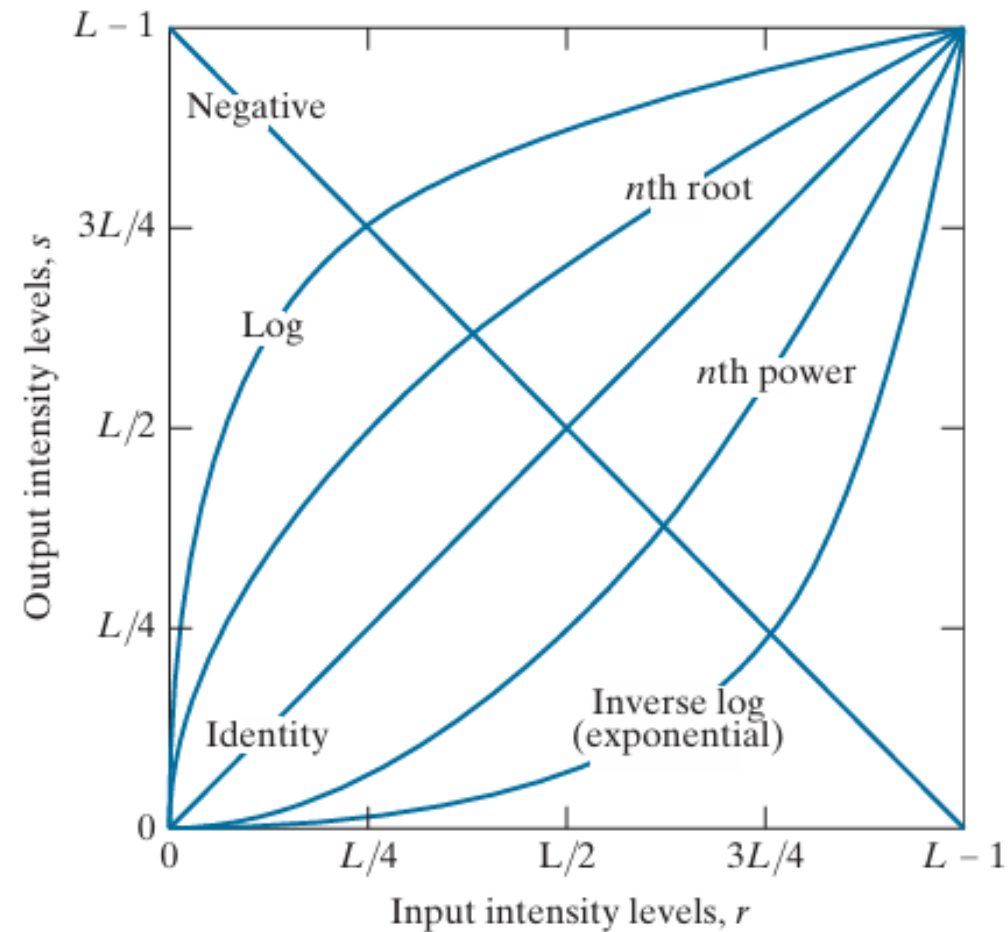
$$s = L - 1 - r$$

Applications

- Enhancing white or gray detail embedded in dark regions.

$$L = 256$$

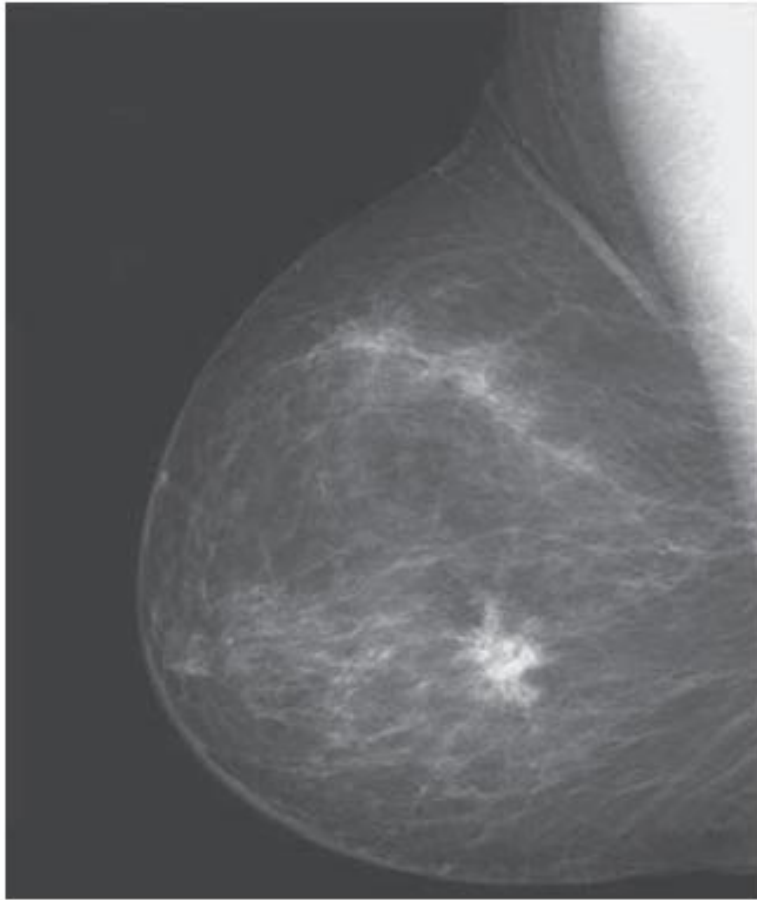
0	50	200	→	255	205	55
60	128	30		195	127	225
186	255	40		69	0	215



Some basic intensity transformation functions

Basic Intensity Transformation Functions

Image Negatives



A digital mammogram



Negative image obtained using image negatives

Basic Intensity Transformation Functions

Image Scaling

$$s = T(r) = a \cdot r$$

Original image



$f(x,y)$

Scaled image



$a \cdot f(x,y)$

Basic Intensity Transformation Functions

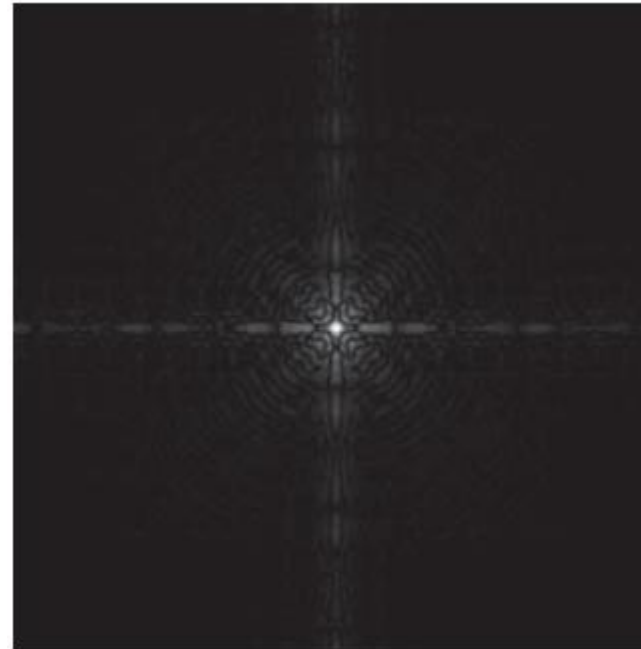
Log Transformations

$$s = c \log(1 + r)$$

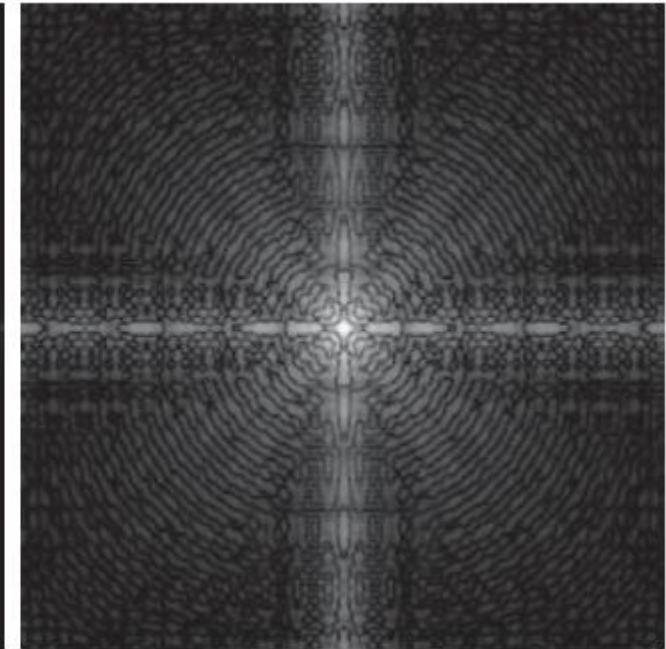
Used to expand the values of dark pixels in an image, while compressing the higher-level values.

Applications

- This transformation is suitable for the case when the dynamic range of a processed image far exceeds the capability of the display device (e.g. display of the Fourier spectrum of an image)
- Also called “[dynamic-range compression / expansion](#)”



Fourier spectrum displayed as a grayscale image



Result of applying the log transformation with $c=1$

Basic Intensity Transformation Functions

Power-law (Gamma) Transformations

$$s = c r^\gamma$$

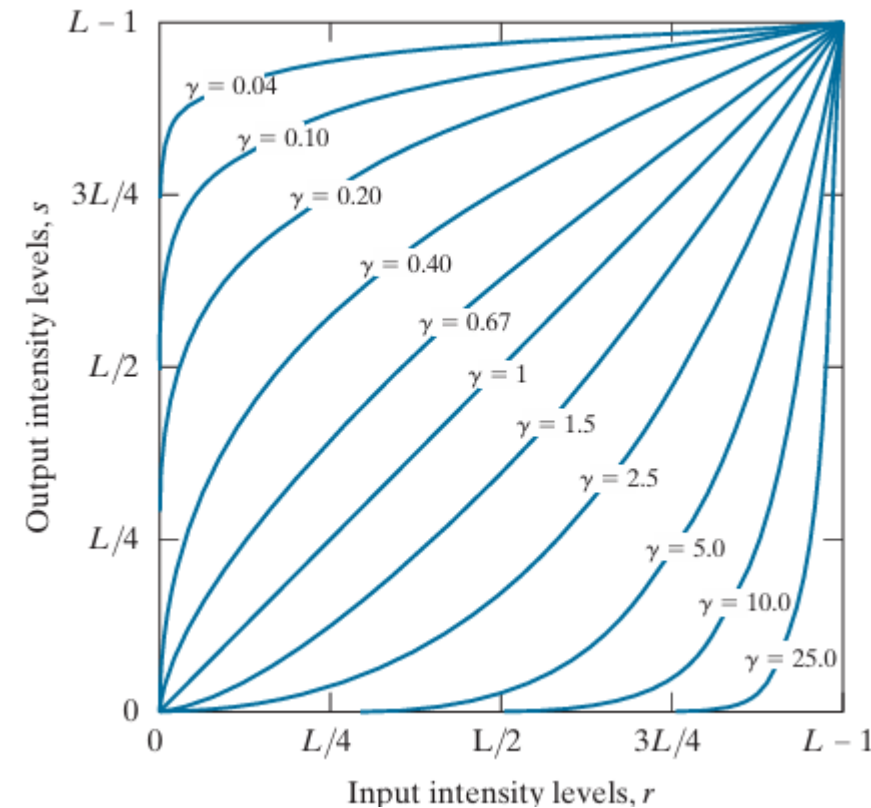
For $\gamma > 1$: Expand values of dark pixels, compress values of brighter pixels

For $\gamma < 1$: Compresses values of dark pixels, expand values of brighter pixels

Applications

- The response of many devices used for image capture, printing, and display obey a power law

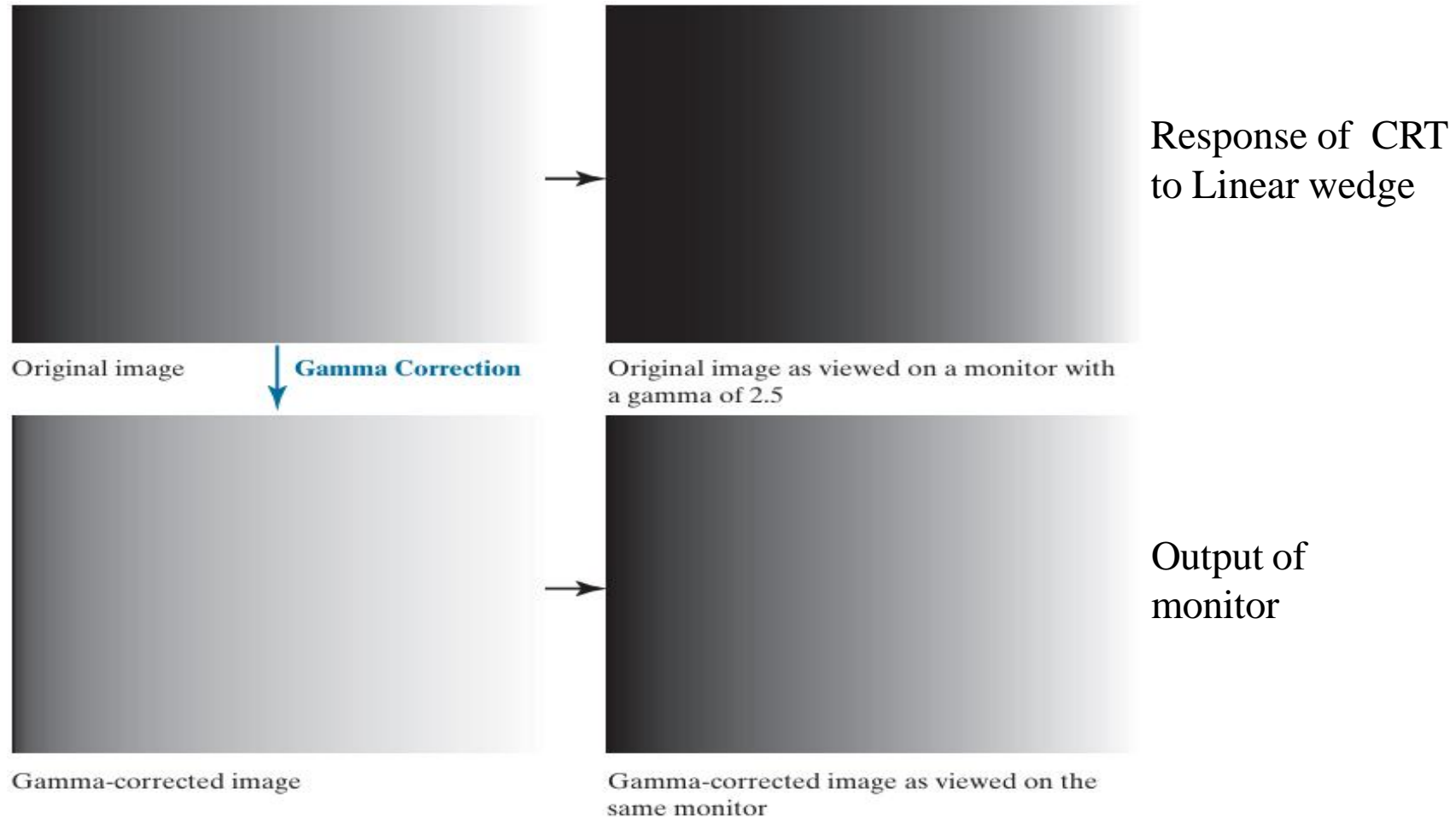
The process used to correct these power-law response phenomena is called gamma correction or gamma encoding.



Plots of the gamma equation for various values of gamma ($c = 1$ in all cases)

Basic Intensity Transformation Functions

Power-law (Gamma) Transformations



Basic Intensity Transformation Functions

Power-law (Gamma) Transformations



MRI image of
fractured
human spine



Result of applying
power-law
transformation

$$c = 1, \gamma = 0.6$$



Result of applying
power-law
transformation

$$c = 1, \gamma = 0.4$$



Result of applying
power-law
transformation

$$c = 1, \gamma = 0.3$$

Piecewise Linear Transformations

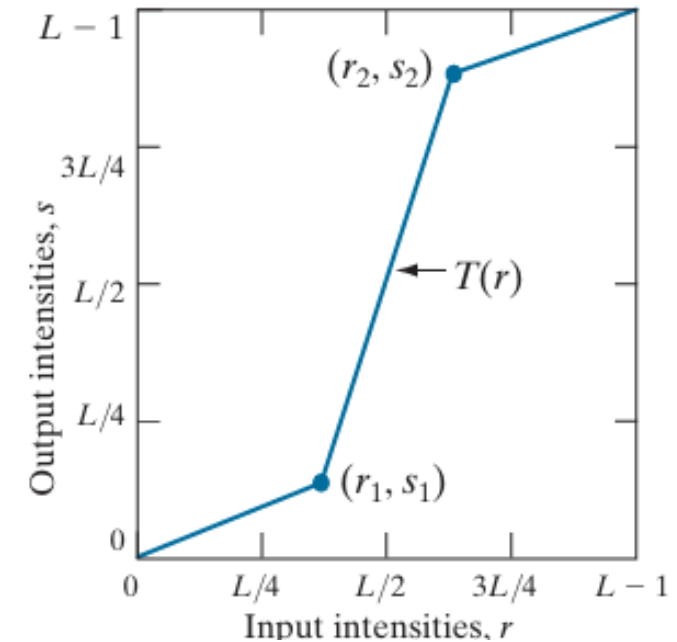
Contrast Stretching

Increasing the dynamic range of the gray levels for low contrast images.

Low-contrast images can result from

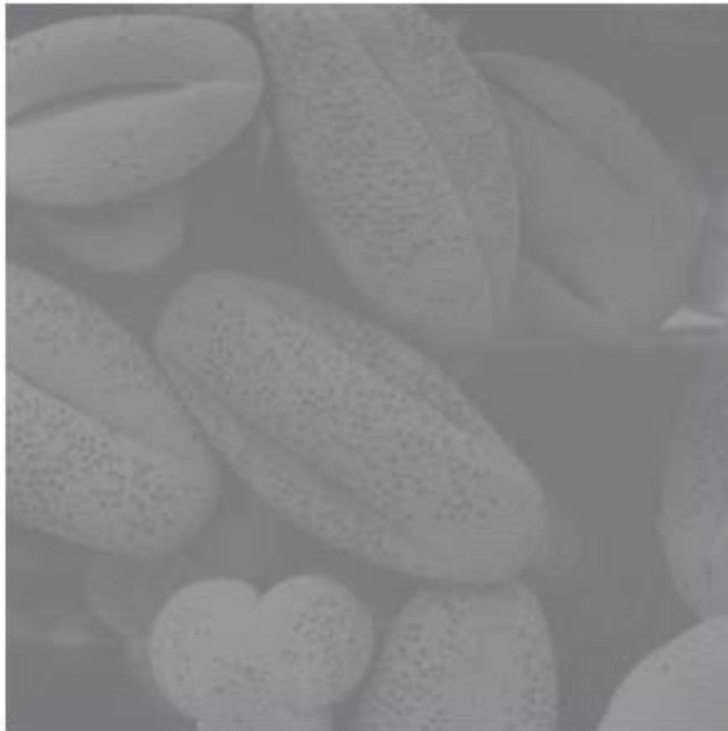
- Poor illumination,
- Lack of dynamic range in the imaging sensor, or
- Wrong setting of a lens aperture during image acquisition

$$s = T(r) = \begin{cases} a_1 r, & 0 \leq r < r_1 \\ a_2(r - r_1) + s_1, & r_1 \leq r < r_2 \\ a_3(r - r_2) + s_2, & r_2 \leq r \leq (L - 1) \end{cases}$$
$$s_1 = T(r_1)$$
$$s_2 = T(r_2)$$



Piecewise Linear Transformations

Contrast Stretching



Original Image



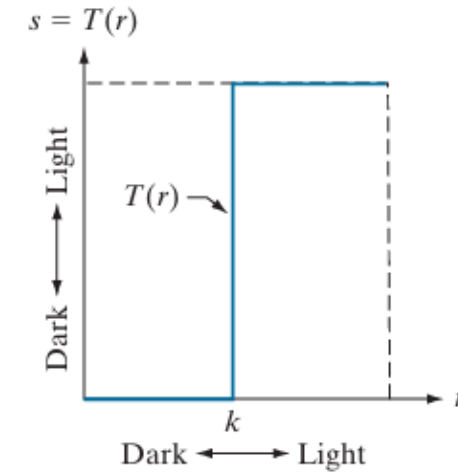
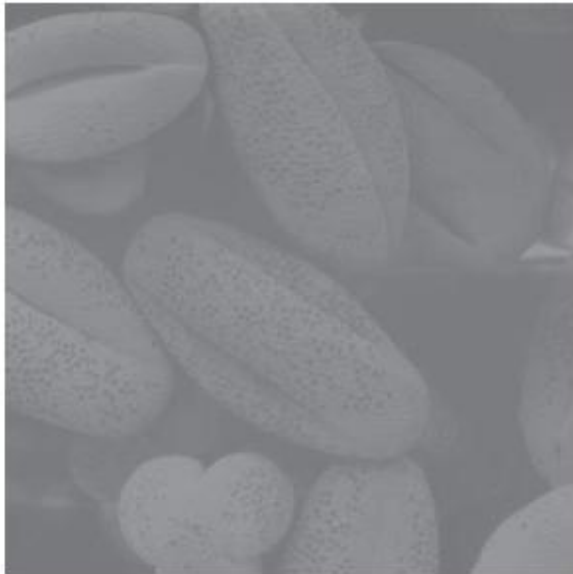
Result of contrast stretching

Piecewise Linear Transformations

Thresholding

A technique to convert a grayscale image into a binary image by setting pixels below a threshold to black (0) and those above the threshold to white (255).

$$s = T(r) = \begin{cases} 0, & 0 \leq r < k \\ 255, & k \leq r \leq (L - 1) \end{cases}$$



Piecewise Linear Transformations

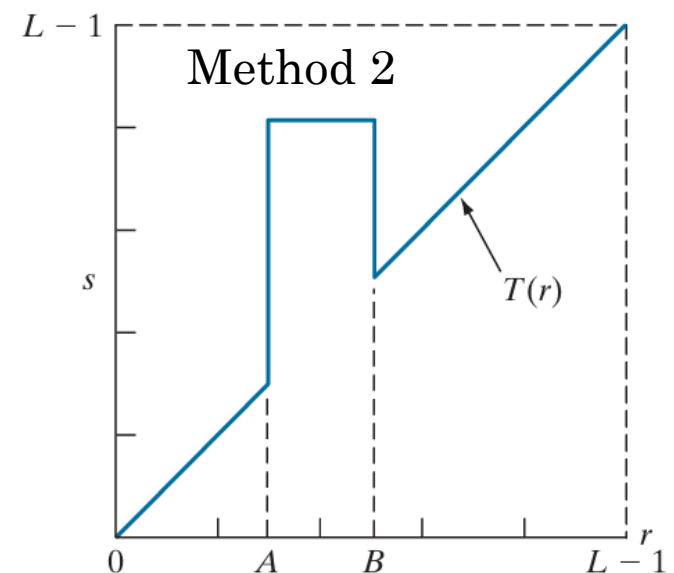
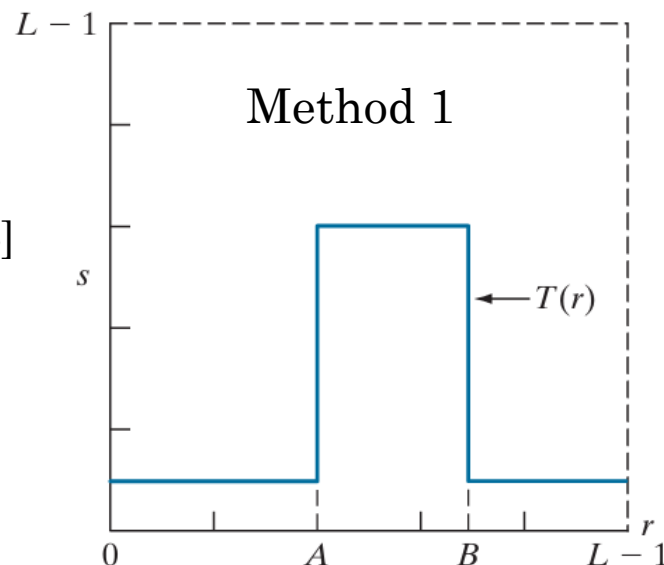
Intensity-Level Slicing

- Highlighting a specific range of intensities in an image often is of interest.

Applications

- enhancing features in satellite imagery, such as masses of water, and enhancing flaws in X-ray images

Highlights range $[A, B]$ and leaves other intensities unchanged.



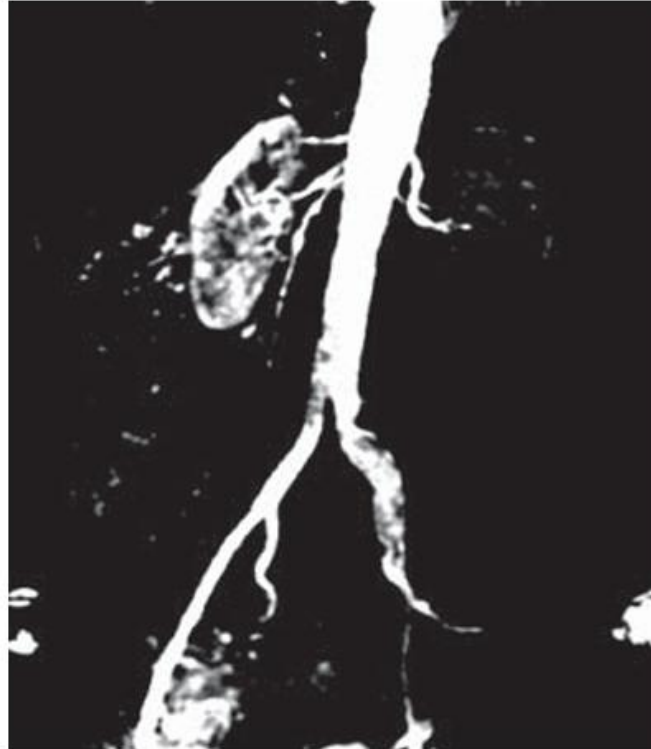
Highlights range $[A, B]$ and reduces all other intensities to a lower level.

Piecewise Linear Transformations

Intensity-Level Slicing



Original Image



Method 1 result

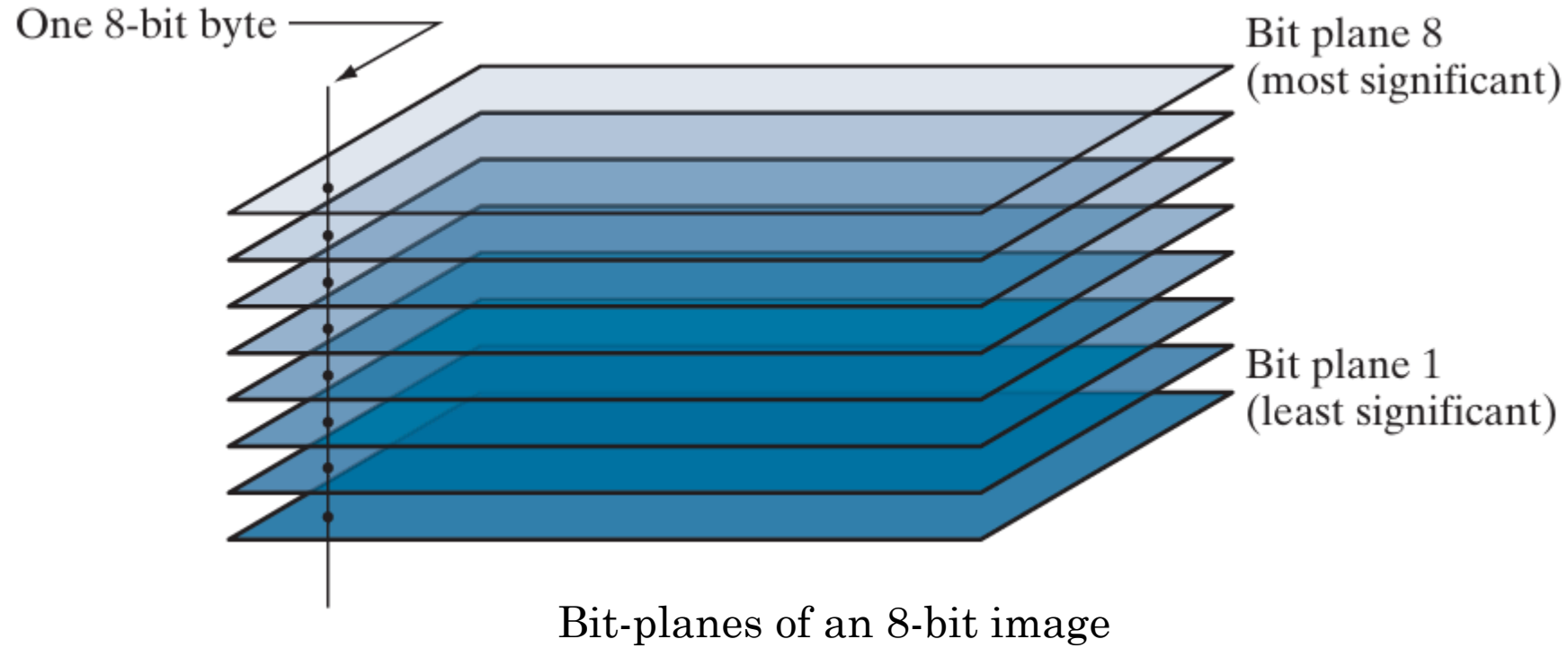


Method 2 result

Piecewise Linear Transformations

Bit-Plane Slicing

- Highlight the contribution made to total image appearance by specific bits.



Piecewise Linear Transformations

Bit Plane Slicing for 8-bit Grayscale Image

MSB Binary (decimal) → LSB

	8 th bit	7 th bit	6 th bit	5 th bit	4 th bit	3 rd bit	2 nd bit	1 st bit
0	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)
	:	:	:	:	:	0 (1)	0 (1)	1 (1)
	:	:	0 (31)	0 (15)	0 (7)	0 (2)	1 (2)	0 (2)
	:	0 (63)	1 (32)	1 (16)	1 (8)	0 (3)	1 (3)	1 (3)
	:	1 (64)	:	:	:	1 (4)	0 (4)	0 (4)
	:	:	1 (63)	1 (31)	1 (15)	1 (5)	0 (5)	1 (5)
	:	:	0 (64)	0 (32)	0 (16)	1 (6)	1 (6)	0 (6)
	0 (127)	1 (127)	:	:	:	1 (7)	1 (7)	1 (7)
	1 (128)	0 (128)	0 (95)	0 (47)	0 (23)	0 (8)	0 (8)	0 (8)
	:	:	1 (96)	1 (48)	1 (24)	0 (9)	0 (9)	1 (9)
	:	:	:	:	:	0 (10)	1 (10)	0 (10)
	:	0 (191)	1 (127)	1 (63)	1 (31)	0 (11)	1 (11)	1 (11)
	:	1 (192)	:	:	:	:	:	:
	:	:	:	:	:	:	:	:
	:	:	:	:	:	:	:	:
255	1 (255)	1 (255)	1 (255)	1 (255)	1 (255)	1 (255)	1 (255)	1 (255)

Piecewise Linear Transformations

Bit-Plane Slicing



Original Image



Bit-8 plane



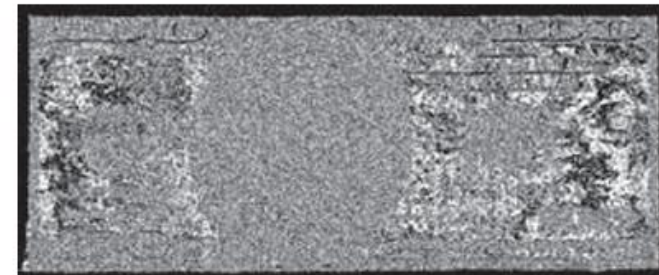
Bit-7 plane



Bit-6 plane



Bit-5 plane



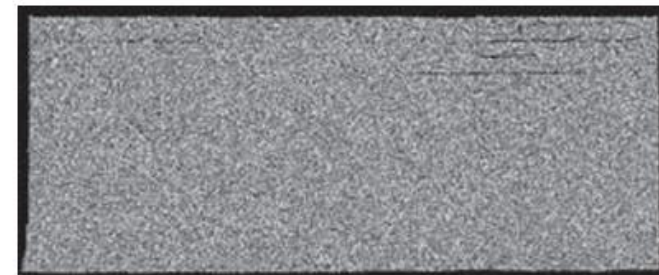
Bit-4 plane



Bit-3 plane



Bit-2 plane



Bit-1 plane

Histogram Processing

Histogram

Unnormalized histogram of digital image is defined as

$$h(r_k) = n_k \quad \text{for } k = 0, 1, 2, \dots, L - 1$$

$r_k \rightarrow k^{th}$ intensity value

$n_k \rightarrow$ number of pixels in image with intensity r_k

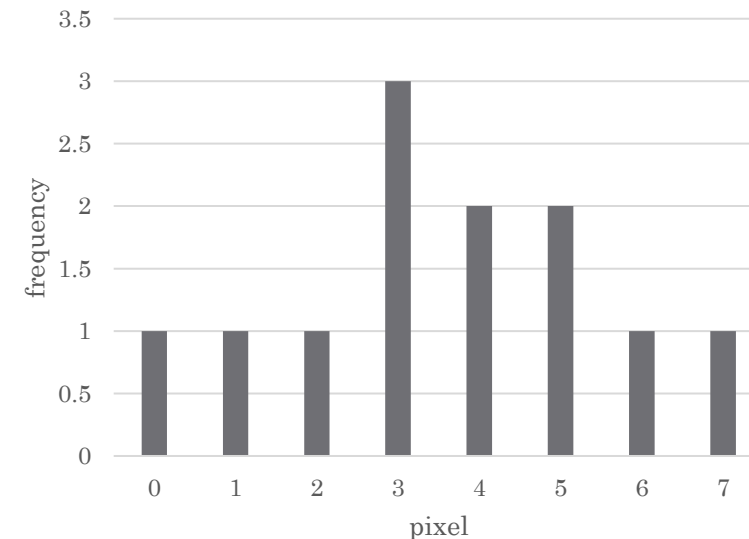
Normalized Histogram

$$p(r_k) = \frac{h(r_k)}{MN} = \frac{n_k}{MN}$$

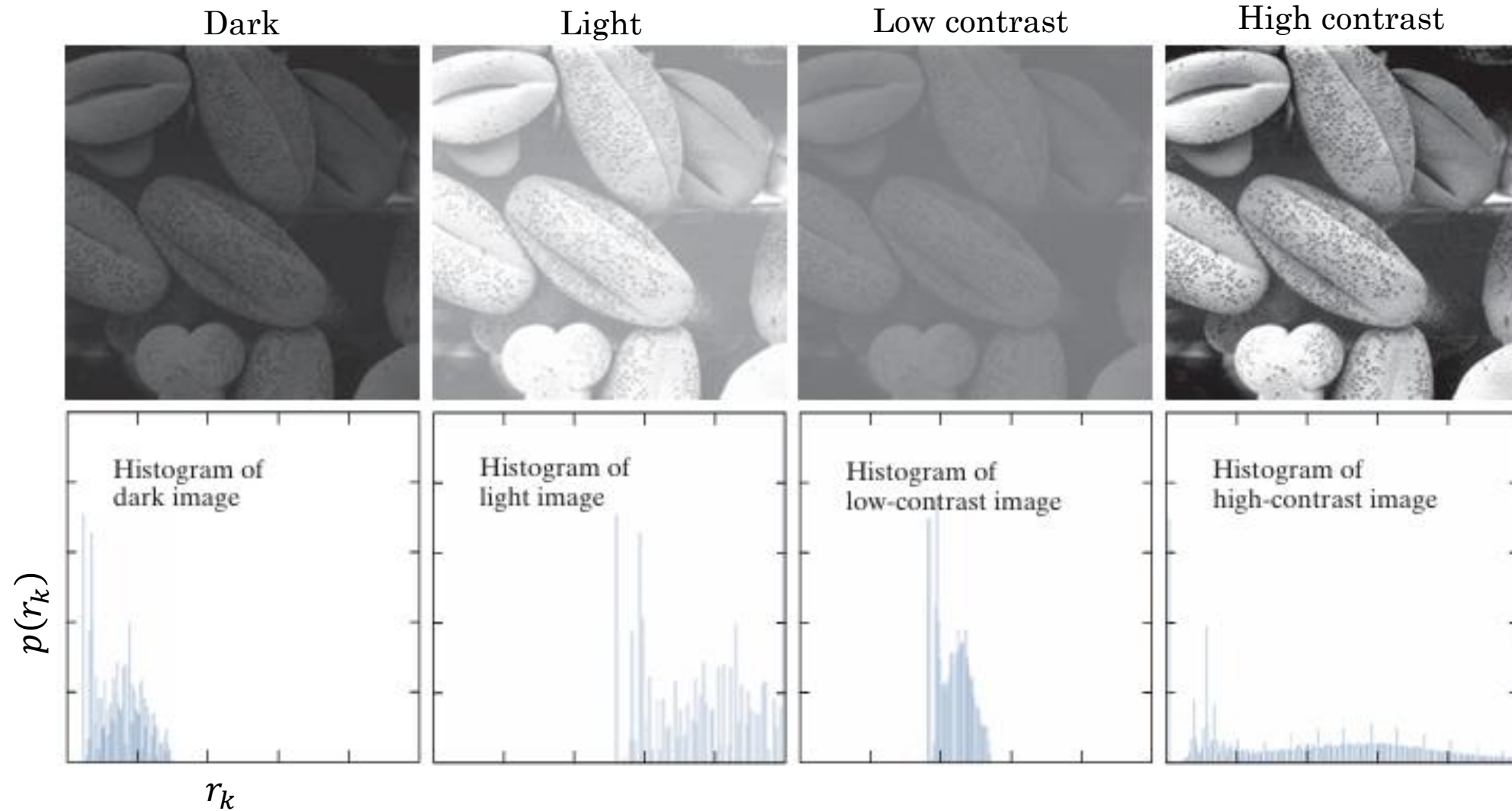
- $n_k \rightarrow$ Number of pixels in the image of size $M \times N$ with intensity r_k
- The sum of $p(r_k)$ for all values of k is always 1

image

3	2	5
3	1	3
4	5	0
6	7	4



Histogram Processing



Histogram Processing

Histogram Equalization

Histogram equalization is the process of uniformly distributing the image histogram over the entire intensity axis by choosing a proper intensity transformation function.

Equalized Histogram

$$p(r_k) = \frac{h(r_k)}{MN} = \frac{n_k}{MN}$$

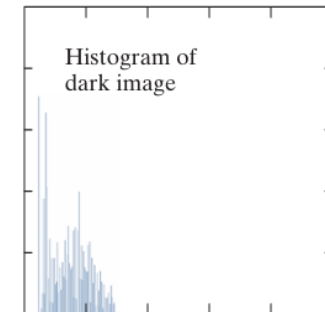
Transformation function

$$s_k = T(r_k) = (L - 1) \sum_{j=0}^k p_r(r_j) \quad k = 0, 1, 2, \dots, L - 1$$

L -> #intensity levels in the image



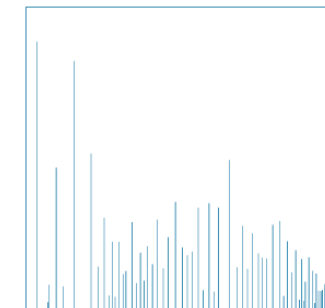
Dark image



Dark image histogram



Histogram-equalized image



Equalized histogram


Histogram Processing

Histogram Equalization Example

Suppose a 3-bit image ($L=8$) of size 64×64 , pixels ($MN = 4096$)

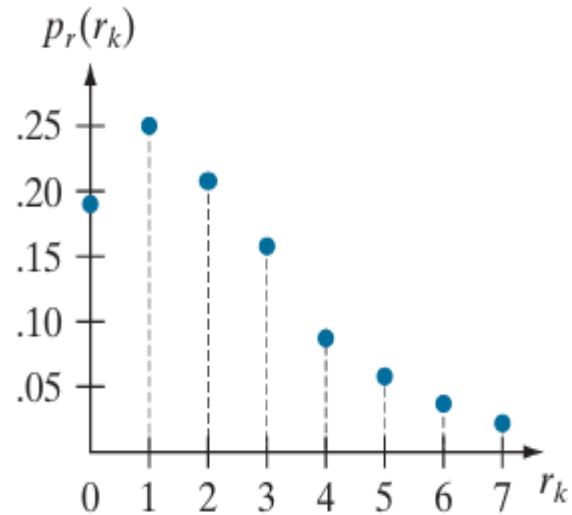
r_k	n_k	$p(r_k) = n_k/MN$	cdf	$7 * \text{cdf}$	Round
$r_0 = 0$	790	0.19	0.19	1.33	1
$r_1 = 1$	1023	0.25	0.44	3.08	3
$r_2 = 2$	850	0.21	0.65	4.55	5
$r_3 = 3$	656	0.16	0.81	5.67	6
$r_4 = 4$	329	0.08	0.89	6.23	6
$r_5 = 5$	245	0.06	0.95	6.65	7
$r_6 = 6$	122	0.03	0.98	6.86	7
$r_7 = 7$	81	0.02	1	7.00	7

These are the values of the
equalized histogram

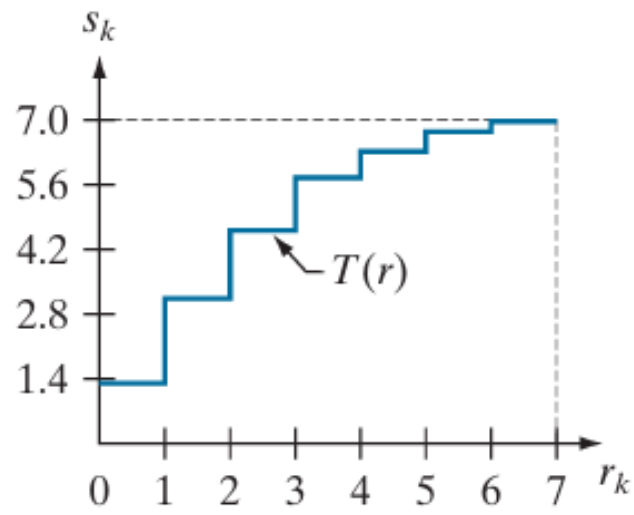


Histogram Processing

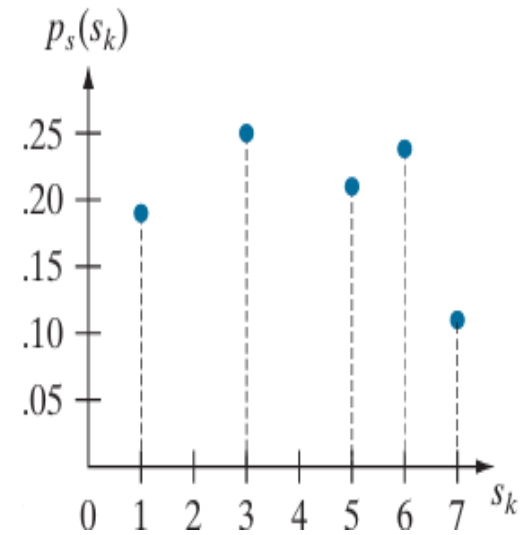
Histogram Equalization Example



Original histogram



Transformation function



Equalized histogram

Thank You