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Corroborating the Role of Magnetic Fields and Turbulence in Star Formation via Theoretical Model

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To all those who seek eternally

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1 Introduction

1.1 Motivation

Since ancient times, we have been fascinated with the heavenly bodies, especially stars and their working and significance in the universe. Star formation is one of the most important concepts in astrophysics. The fundamental process of star formation is at the nexus of galaxy formation, planetary science and stellar evolution. Even though our view of it has considerably changed in the past decades due to precise observations and theoretical progress yet there are still numerous open and unsolved problems in the area of star formation.

An important question is the role which magnetic fields play in the process of star formation. Shu et al. (1987) and McKee and Ostriker (2007) have extensively reviewed this field.
There are two major classes of star formation theories which differ by the role played by magnetic field. According to some theories, magnetic fields play a major role in setting both initial mass function of stars and timesecales for star formation (such as Shu et al. (1987), Basu and Ciolek (2004)). The **strong field models**(such as Mouschovias (1991), Mouschovias and Ciolek (1999)) consists of magnetic fields controlling the formation and evolution of the star forming molecular clouds, with ambipolar diffusion driving core formation and their gravitational collapse to form protostars. While some theories downplay the significance of role of magnetic field in star formation and state it is supersonic turbulence instead that controls star formation. There, it is theorized that star formation is a consequence of the collapse of mildly supersonic or transonic to subsonic molecular cores emerging out of supersonic turbulent flows in the interstellar medium, these are **weak field models** (such as Mac Low and Klessen (2004) and McKee and Ostriker (2007), Padoan and Nordlund (1999)). And in other theories of star formation, with some recent simulations and theoretical developments both magnetic fields and turbulence seem to play significant roles. (such as Nakamura and Li (2008, 2011), Tilley and Pudritz (2007), Kudoh and Basu (2008), Vázquez-Semadeni et al. (2011)). Some weak field models, have evolved into including more than just hydrodynamic turbulence, and have considered magnetohydrodynamic turbulence(*as we would discuss in the Background Literature Review*) too. Still, it is an open problem as to what are the main quantitative processes which influence star formation rates majorly and how.

1.2 Aims and Objectives

Star-forming molecular clouds are endowed with magnetic fields which are said to be inherited and condensed out from the interstellar medium (see Shore (2003)). These fields in turn influence a molecular cloud's morphology and evolution (see Crutcher (2012)). However, it's still an open problem as to what are the main quantitative outcomes set by magnetic fields in the process of star formation. In view of approaching this question, we would be studying the effect of magnetic field in a molecular cloud which leads to star formation with the help of simulations using our developed mathematical model. Our objective in this project is to develop a theoretical model using magnetohydrodynamics(MHD) equations and some previous observational work to investigate links between magnetic field, turbulence and star formation rates.

2 Background & Literature Review

2.1 Ideal Magnetohydrodynamics(MHD) Equations

For firmly establishing the background, and for usefulness of readers, a short derivation and explanation of the ideal MHD equations is given here (follows from Hennebelle and Inutsuka (2019) and Spruit (2013)) We will begin with the ideal MHD, which amongst other approximations assume non-relativistic limit, i.e. in other words,the fluid velocities are much smaller than speed of light c. The MHD equations assume that fluids are like perfect conductors. Their evolution is described by the Maxwell equations, which are given as follows **in CGS units**:

$$\nabla \cdot B = 0 \tag{1}$$

$$\nabla \cdot E = 4\pi\rho_e \tag{2}$$

$$c\nabla \times E = -\frac{\partial B}{\partial t} \tag{3}$$

$$c\nabla \times B = 4\pi j + \frac{\partial E}{\partial t} \tag{4}$$

ρ_e is the fluid charge and j is the current density. The charge conservation equation links these two together:

$$\frac{\partial \rho_e}{\partial t} + \nabla \cdot j = 0 \tag{5}$$

As we know, in a perfect conductor at rest, electric field E_R vanishes, but it's different when the conductor is in motion. The fields in the frame of observer, E and B, and the rest fields E_R and B_R are related to each other with the help of Lorentz transformation (see Landau and Lifshitz (1984), Spruit (2013)). Taking Lorentz force F and F_R, we get:

$$F = q\,(E + \frac{v}{c} \times B) \tag{6}$$

$$F_R = qE_R \tag{7}$$

As force doesn't depend on reference frame: $F = F_R$ and so

$$E_R = E + \frac{v}{c} \times B \tag{8}$$

$$B_R = B \tag{9}$$

As we have assumed a perfect conductor, so $E_R = 0$, and

$$E = -\frac{v}{c} \times B \tag{10}$$

By Equation (3) and Equation (10), we get

$$\frac{\partial B}{\partial t} = \nabla \times (v \times B) \tag{11}$$

The Lorentz force per volume would be expressed as

$$f_L = \rho_e E + \frac{1}{c} j \times B \tag{12}$$

In non-relativistic limit, displacement current in eqn (4) can be neglected, so we get

$$\nabla \times B = 4\frac{\pi}{c} j \tag{13}$$

As, we have assumed local electroneutrality, so $\rho_e = 0$ and

$$f_L = \frac{(\nabla \times B) \times B}{4\pi} \tag{14}$$

And, so by the above work, we get the **standard ideal MHD equations**

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0 \tag{15}$$

$$\rho \left[\frac{\partial v}{\partial t} + (v.\nabla) v \right] = -\nabla P + \frac{(\nabla \times B) \times B}{4\pi} \tag{16}$$

$$\rho \left[\frac{\partial e}{\partial t} + (v.\nabla) e \right] = -P(\nabla \cdot v) - \rho \mathcal{L} \tag{17}$$

$$\frac{\partial B}{\partial t} = \nabla \times (v \times B) \tag{18}$$

where $\mathcal{L}$ is the net loss function and describes the radiative heating and cooling of the gas. This can be complemented by a state equation to close this system of equations. A perfect gas is a good assumption for ISM, $P = (\gamma - 1)\rho_\epsilon$ where γ is the adiabatic index of the gas.

Equation (18) is also known as MHD **induction equation** and Equation (15) is known as MHD **continuity equation**

By Equation (12) and Equation (14), the Lorentz force acting per unit volume on a current carrying fluid is given by:

$$f_L = \frac{1}{c} j \times B = \frac{1}{4\pi} (\nabla \times B) \times B \tag{19}$$

As we see, here f_L is expressed as the force per unit volume exerted by a magnetic field on a conducting fluid that's electrically neutral.

2.2 Alfvén's Theorem

In a perfectly conducting fluid, the magnetic flux is a property of the loop i.e. magnetic flux through any closed loop moving with the fluid is conserved with time.

$$\frac{d\Phi_B}{dt} = 0 \tag{20}$$

Also, known as flux-freezing theorem. The 'frozen-in' nature of magnetic fields is of crucial importance in astrophysics(see example in 4.3.1 and 4.3.2 in Davidson (2001)) , where R_m i.e. Magnetic Reynold's number(*will be defined in next section 2.3*), is usually very high.
The equivalence of Alfvén's theorem i.e. Equation (20) with the ideal MHD induction equation i.e. Equation (18) is derived on (Pg 40 of Spruit (2013))

We would be giving a short derivation of Alfven's theorem using the previous equations:
By Equation (10) i.e.ideal MHD Ohm's Law, $E = -(v/c) \times B$
Consider the magnetic flux Φ through a contour, C, which is co-moving with the plasma

$$\Phi_B = \int_S B\,.dS \tag{21}$$

where S is some surface which spans C. The time derivative of magnetic flux Φ is made up of two parts, one partial time derivative due to time variation of B over surface S and the other part is due to motion of C

$$\left(\frac{\partial \Phi_B}{\partial t}\right)_1 = \int_S \frac{\partial B}{\partial t}\,.dS \tag{22}$$

Using Faraday Maxwell's equation i.e. Equation (3)

$$\left(\frac{\partial \Phi_B}{\partial t}\right)_1 = -\int_S (c\boldsymbol{\nabla} \times E)\,.dS \tag{23}$$

Second partial time derivative of flux due to motion of C. If dl is an element of C then $V \times dl$ is the area swept out by dl per unit time. So, the flux crossing this area is $B.(v \times dl)$

$$\left(\frac{\partial \Phi_B}{\partial t}\right)_2 = \int_C B.(v \times dl) = \int_C (B \times v).dl \tag{24}$$

Using Stokes theorem, we get

$$\left(\frac{\partial \Phi_B}{\partial t}\right)_2 = \int_S \boldsymbol{\nabla} \times (B \times v)\,.dS = -\int_S \boldsymbol{\nabla} \times (v \times B)\,.dS \tag{25}$$

Hence, the total time derivative of flux

$$\frac{d\Phi_B}{dt} = -\int_S \boldsymbol{\nabla} \times (cE + v \times B)\,.dS \tag{26}$$

Applying MHD Ohm's Law, Equation (10), we get $d\Phi_B/dt = 0$

Hence, proved

2.3 Coupling Navier Stokes Equations with MHD Equations

the referred equation numbers might be wrong in some places, I will correct them

2.3.1 Basic Navier Stokes Equations

Take f as the force acting on continuum, ϕ is it's potential, and ρ is the density of fluid. Let v be the flow velocity with components (v_x, v_y, v_z) in Cartesian Coordinates and p be pressure and τ as deviatoric stress tensor of order 2
The short derivations and presentation below of the Navier Stokes equations of continuity and momentum follow from (Ch 5:The Differential Equations of Flow of Themelis (1995), Anderson (1992) , *especially the list of equations for viscous and inviscid flow on 2.8.1 and 2.8.2 of the book Anderson (1992)*)

Equation of Continuity

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot (\rho \vec{v}) = 0 \quad \text{or} \quad \frac{\partial \rho}{\partial t} + \rho \boldsymbol{\nabla} \cdot \vec{v} = 0 \tag{27}$$

In 3D cartesian form it is

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho v_x)}{\partial x} + \frac{\partial (\rho v_y)}{\partial y} + \frac{\partial (\rho v_z)}{\partial z} = 0 \tag{28}$$

For steady state conditions, there is no mass accumulation and the equation of continuity becomes

$$\frac{\partial (\rho v_x)}{\partial x} + \frac{\partial (\rho v_y)}{\partial y} + \frac{\partial (\rho v_z)}{\partial z} = 0 \tag{29}$$

For an incompressible fluid, it has negligible variation in density of fluid, so we get

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0 \tag{30}$$

Equation of Momentum

$$\frac{\partial (\rho \vec{v})}{\partial t} + \boldsymbol{\nabla} \cdot (\rho \vec{v}\vec{v}) = -\boldsymbol{\nabla} p - \boldsymbol{\nabla} \cdot \tau + \rho \vec{f} \tag{31}$$

Here f is not force but the acceleration of any force acting such as gravity.
In 3D cartesian form, we can write it as three momentum equations:

$$\rho \left[\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x)}{\partial z} \right] = \rho f_x - \frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} \tag{32}$$

$$\rho \left[\frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y)}{\partial z} \right] = \rho f_y - \frac{\partial p}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} \tag{33}$$

$$\rho \left[\frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z)}{\partial z} \right] = \rho f_z - \frac{\partial p}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z} \tag{34}$$

In index notation, we express the viscous stress tensor

$$\tau_{ij} = 2\mu\, S_{ij} + \lambda S_{mm}\, \delta_{ij} \tag{35}$$

where S_{ij} is the strain rate tensor. μ and λ are the constants to relate viscous stress with strain rate μ is the molecular viscosity coefficient and λ is the bulk viscosity coefficient.
By Stoke's hypothesis, $\lambda = -(2/3)\mu$ which is frequently used, but not yet confirmed. (see 2.6 of Anderson (1992))
$S_{xx} = (1/2)\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_x}{\partial x}\right)$ and $\bar{\bar{\delta}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ In expanded form, we can write viscous tensor as

$$\tau_{xx} = 2\mu S_{xx} + \lambda \left[\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right] \delta_{mm} = 2\mu \frac{\partial v_x}{\partial x} + \lambda(\boldsymbol{\nabla} \cdot \vec{v}) \tag{36}$$

As, for incompressible fluids, $\boldsymbol{\nabla} \cdot \vec{v} = 0$, so

$$\tau_{xx} = 2\mu \frac{\partial v_x}{\partial x}\, , \tau_{yy} = 2\mu \frac{\partial v_y}{\partial y}\, , \tau_{zz} = 2\mu \frac{\partial v_z}{\partial z} \tag{37}$$

We do the following using similar steps,

$$\tau_{xy} = \tau_{yx} = \mu \left[\frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y}\right] \tag{38}$$

$$\tau_{zy} = \tau_{yz} = \mu \left[\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z}\right] \tag{39}$$

$$\tau_{zx} = \tau_{xz} = \mu \left[\frac{\partial v_z}{\partial y} + \frac{\partial v_y}{\partial z}\right] \tag{40}$$

Therefore, the set of Equations (32-34) in expanded form for an incompressible fluid can be written as:

$$\rho \left[\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z}\right] = \rho f_x - \frac{\partial p}{\partial x} + \mu \left[\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right] \tag{41}$$

$$\rho \left[\frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z}\right] = \rho f_y - \frac{\partial p}{\partial y} + \mu \left[\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right] \tag{42}$$

$$\rho \left[\frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z}\right] = \rho f_z - \frac{\partial p}{\partial z} + \mu \left[\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right] \tag{43}$$

Energy Equation (Conservation Form)

$$\begin{aligned} \frac{\partial pe}{\partial t} &+ \boldsymbol{\nabla} \cdot (\rho e \vec{v}) \\ &= p\dot{q} + \boldsymbol{\nabla} \cdot (k \boldsymbol{\nabla} T) - p(\boldsymbol{\nabla} \cdot \vec{v}) \\ &+ \lambda(\boldsymbol{\nabla} \cdot \vec{v})^2 + 2\mu \left[\boldsymbol{\nabla} \cdot \vec{v}^2 - \sum_{i}^{x,y,z} \sum_{i \neq j\;\; j}^{x,y,z} \frac{\partial v_i}{\partial i} \frac{\partial v_j}{\partial j}\right] \\ &+ (\boldsymbol{\nabla} \times \vec{v})^2 + 2 \sum_{i}^{x,y,z} \sum_{i \neq j\;\; j}^{x,y,z} \frac{\partial v_i}{\partial i} \frac{\partial v_j}{\partial j} \end{aligned} \tag{44}$$

where $\dot{q}$ is the rate of volumetric heat addition per unit mass, e is the internal energy and k is the thermal conductivity.(see 2.7 in Anderson (1992) for detailed discussion on energy equation)

The conservation form of energy equation in terms of total energy i.e. $(e + (v^2/2))$, is :

$$\frac{\partial}{\partial t}\left[\rho(e+\frac{v^2}{2})\right] + \boldsymbol{\nabla}\cdot\left[\rho\left(e+\frac{v^2}{2}\vec{v}\right)\right] = p\dot{q} + \boldsymbol{\nabla}\cdot(k\boldsymbol{\nabla}T) - \boldsymbol{\nabla}\cdot(vp) + \sum_{j}^{x,y,z}\sum_{k}^{x,y,z}\frac{\partial(v_j\tau_{jk})}{\partial k} \tag{45}$$

Note that there are many other possible forms of the energy equation; for e.g., the equation can be written in terms of enthalpy h, or total enthalpy i.e. $(h + v^2/2)$. We will not be deriving or mentioning them here. (to see that in detail, please refer these Anderson (2010), Liepmann and Roshko (2013), Anderson (2003))

But the given equations of momentum and energy conservation are too complicated and big, taking into consideration a lot of aspects which are not necessary especially in our study in astrophysics. (Choudhuri (1998) in Chapter 3) gives a simplified version of these two equations by making reasonable assumptions, these simple equations would prove useful for us later on in developing the concept of gas cloud stability and collapse. The final equations are as follows in order, momentum equation and then energy equation:

$$\frac{\partial v}{\partial t} + (v.\boldsymbol{\nabla})v = -\frac{1}{\rho}\boldsymbol{\nabla}p + f + \frac{\mu}{\rho}\boldsymbol{\nabla}^2 v$$

$$\rho\left(\frac{\partial e}{\partial t} + v.\boldsymbol{\nabla}e\right) - \boldsymbol{\nabla}\cdot(k\boldsymbol{\nabla}T) + p\boldsymbol{\nabla}\cdot v = 0$$

2.3.2 Coupling Continuity equation and Induction MHD equation

In many common astrophysical applications, viscosity can be neglected. Here, we restrict our attention to inviscid flow (*i.e. by definition a flow where the dissipative, transport phenomena of viscosity, mass diffusion and thermal conductivity are neglected*), as extension to a viscous fluid can be done similarly as to how we did in the continuity equation and momentum equation in Section 2.3.1 Gravity is an important external force, and so it is given as: (follows from Spruit (2013))

$$f_g = \rho g = -\rho\boldsymbol{\nabla}\phi \tag{46}$$

where g is the acceleration due to gravity, ϕ is it's potential

Using Equation (19), we can get an Equation of motion which follows from ordinary fluid's momentum equation(non-conservation)

$$\rho\frac{dv}{dt} = -\boldsymbol{\nabla}p + \frac{1}{4\pi}(\boldsymbol{\nabla}\times B)\times B + \rho g \tag{47}$$

here d/dt is the Lagrangian time-derivative,

$$\frac{d}{dt} = \frac{\partial}{\partial t} + v\cdot\boldsymbol{\nabla} \tag{48}$$

MHD induction equation does for the magnetic field what the continuity equation (Equation (27)) does for the mass density, but there are important differences because of the divergence-free vector nature of the field.
The Equation of motion (47) and MHD induction equation (18) together determine the two vectors B and v. Compared with ordinary fluid mechanics, there is a new vector field, whose evolution is expressed by MHD induction equation and an additional force appears in equation of motion.

These two equations can be solved together, and reduces MHD in a way that it behaves more like a fluid than EM now. In the Ideal MHD equations (Equations (15) - (18)), three of the four Maxwell equations are accounted, but Equation (2) i.e. Gauss's Law is unneeded now as it has been overcome by the fact that electric field in MHD follows directly from a frame transformation (Equation (10)).

One form of combining MHD induction equation (Equation (27)) and equation of continuity, is **Walén's Equation** which describes the change of the ratio of magnetic flux to mass density with variation of fluid velocity along a field line

$$\frac{d}{dt}\left(\frac{B}{\rho}\right) = \left(\frac{B}{\rho}.\boldsymbol{\nabla}\right) v \tag{49}$$

For a better look and interpretation of the ideal MHD equations, we can rewrite Lorentz force (from Equation (19)) in a different way(see Pg 13-14 of Spruit (2013) lecture for more discussion) now after coupling induction MHD equation and continuity equation

$$f_L = \frac{1}{4\pi}(\boldsymbol{\nabla} \times B) \times B = -\frac{1}{8\pi}\boldsymbol{\nabla} B^2 + \frac{1}{4\pi}(B.\boldsymbol{\nabla})B \tag{50}$$

The first term on the right is the gradient of what is known as the **magnetic pressure** $B^2/8\pi$ (in SI system, it would be $B^2/2\mu$)

The second term describes a force due to the variation of magnetic field strength in the direction of the field that is known as **magnetic curvature force**

The term $(1/4\pi)(B.\boldsymbol{\nabla})B$ (in SI system, it would be $(B.\boldsymbol{\nabla})B/\mu$ represents **magnetic tension** as B^2/μ). The effects of tension in a magnetic field manifest themselves more indirectly, through the curvature of field lines. (for more detail and discussion on same, please see 1.3.2 and 1.3.3 i.e. Pg 14-16 in Spruit (2013))

2.4 Magnetic Diffusivity and Magnetic Reynold's Number

Assume (without validation for an instant) that current is proportional to electric field in the fluid frame: $j' = \sigma_c E'$ i.e. linear Ohm's Law. Here, σ_c is electrical conductivity, not charge density σ. In the case of non-relativistic limit, as shown in Section 2.1, we take $B = B'$, $j = j' = c/(4\pi)\nabla \times B$, so, we have

$$\frac{j}{\sigma_c} = E' = (E + \frac{v}{c} \times B) \tag{51}$$

By Equation (46)**(the one just above)** and basic Maxwell's induction equation i.e. Equation (3), we get

$$\frac{\partial B}{\partial t} = \nabla \times [v \times B - \eta \nabla \times B] \tag{52}$$

where $\eta = \dfrac{c^2}{4\pi\sigma_c} = \dfrac{1}{\mu\,\sigma_c}$ i.e. **magnetic diffusivity**

here μ is the permeability of free space (don't be confused by SI and CGS system here, we are working on CGS here, but sometimes expression in SI also given). Suppose η is constant in space, then

$$\partial_t B = \nabla \times (v \times B) + \eta \nabla^2 B \tag{53}$$

Note that when $\eta \neq 0$ i.e. in the presence of diffusion, Alfvén's theorem (here Equation (20)) fails to hold, as magnetic field lines about the loop co-moving with the fluid are not conserved with time.(refer to Pg 41 of Spruit (2013) for more detail)

The influence of the second term that represents diffusion in Equation (51)**the previous to previous equation** can be quantified using dimensional analysis.
Let L and V be characteristic values for the length scale and velocity of the problem. Non-dimensionalising **this eqn which we talk about** Equation (51), we get

$$\frac{\partial B^*}{\partial t^*} = \nabla^* \times (v^* \times B^*) + \frac{1}{R_m} \nabla^{*2} B^* \tag{54}$$

where $R_m = \dfrac{LV}{\eta} = LV\mu\sigma_c$ i.e. **Magnetic Reynold's number**

Magnetic Reynold's number is useful to the relative importance between the advection term and diffusion term for the relative strengths of advection and diffusion. If $R_m \gg 1$, then advection is more important (ideal MHD limit)
If $R_m \ll 1$, then diffusion is more important In most astrophysical problems, $R_m \ggg 1$ is very large due to large dimensions of astrophysical systems. If the astrophysical system doesn't have a solid boundary, diffusion term and viscosity can be neglected and ideal MHD incorporated except in subvolumes where small scale lengths develop such as in current sheets. However, in one major astrophysics topic i.e. theory of accretion disks, although the Reynold's number is high, still viscosity and diffusion plays an important role, and cannot be neglected. (see 5.7 in Choudhuri (1998))

An ionized plasma can have a magnetic diffusivity i.e. η of the order known as *Spitzer value* when dominant non-ideal effect in electrical resistance due to Coulomb interactions of electrons with ions and neutrals

$$\eta \sim 10^{12}\, T^{-3/2}\, cm^2 s^{-1} \tag{55}$$

with temperature T in Kelvin (see Pg 71 of Spruit (2013))

2.5 Hydromagnetic Waves

It has been known that, a magnetic fluid i.e. compressible, supports three types of waves, out of which, only one is there which resembles sound waves from classical hydrodynamics. This one wave would be our focus here, as some of it's properties.

The derivation below follows from (14.5 of Choudhuri (1998) and 1.8 in Spruit (2013)).
Consider a simple case, perturbing uniform fluid (initially at rest $v = 0$) with a homogenous magnetic field B, neglecting dissipative effects such as viscosity, electrical resistivity and heat conduction. Take the initial magnetic field B (here in Cartesian coordinates) along the z direction:

$$B = B_0\,\hat{z} \tag{56}$$

here, B_0 is a non-negative constant. So, y and x coordinates are equivalent, then we can ignore one of them, take x, by restricting our attention to perturbations δs which are independent of x :

$$\delta_x\,\delta s = 0 \tag{57}$$

Let δB, $\delta\rho$ and δp be little perturbations in magnetic field, density and pressure. Taking $B + \delta B$ as magnetic field, $\rho + \delta\rho$ as the density, and $p + \delta p$ as the pressure. We can write the linear equations of motion and induction as:

$$\rho\frac{\partial v}{\partial t} = \boldsymbol{\nabla}\delta p + \frac{1}{4\pi}(\boldsymbol{\nabla}\times\delta B)\times B \text{ (follows from Equation (47))} \tag{58}$$

$$\frac{\partial \delta B}{\partial t} = \boldsymbol{\nabla}\times(v\times B) \text{ (follows from MHD Induction Equation (18))} \tag{59}$$

Equation of continuity in our case would be (here ρ is constant), using Equation (27))

$$\frac{\partial\delta\,\rho}{\partial t} + \rho v = 0 \tag{60}$$

As we are neglecting heat conduction, assume adiabatic conditions, so

$$\delta p = \left(\frac{\partial p}{\partial\rho}\right)_{ad}\delta\rho = {c_s}^2\,\delta\rho \tag{61}$$

here the derivative is taken at constant entropy, c_s is the speed with which acoustic waves travel in the absence of the magnetic fields, and ${c_s}^2 = (\gamma p/\rho)$ where $\gamma = (c_p/c_v)$ i.e. ratio of specific heats for an ideal gas.

The components of Equation (58) and (59) are:

$$\rho\frac{\partial v_x}{\partial t} = -\frac{\partial\delta p}{\partial x} + \frac{B}{4\pi}\left(\frac{\partial\delta B_x}{\partial z} - \frac{\partial\delta B_z}{\partial x}\right), \quad \rho\frac{\partial v_z}{\partial t} = -\frac{\partial\delta p}{\partial z} \tag{62}$$

$$\rho\frac{\partial v_y}{\partial t} = \frac{B}{4\pi}\frac{\partial\delta B_y}{\partial z} \tag{63}$$

$$\frac{\partial\delta B_x}{\partial t} = B\frac{\partial v_x}{\partial z}, \quad \frac{\partial\delta B_z}{\partial t} = -B\frac{\partial v_x}{\partial x} \tag{64}$$

$$\frac{\partial\delta B_y}{\partial t} = B\delta v_y z \tag{65}$$

Upon solving Equations (58) - (65), one set of solutions we get, is:
$v_x = B_z = \delta B_x = \delta B_z = \delta p = \delta\rho = 0$, here δB_y and v_y would be determined by Equation (63) and (65). These can together be merged to form the wave equation

$$\left(\frac{\partial^2}{\partial t^2} - {v_A}^2 \frac{\partial^2}{\partial z^2}\right)(\delta B_y, v_y) = 0 \tag{66}$$

where v_A is known as **Alfvén's velocity** and the amplitudes of the Equation (66) are related as

$$\frac{\delta|B_y|}{B} = \frac{|v_y|}{v_A} \tag{67}$$

These solutions is known as Alfvén's waves. As we see, wave equation of it includes components in only y and z coordinates, which implies, this wave propagates only in $y - z$ plane.
As the ideal medium is assumed to be homogenous and time-independent, the occuring perturbation can be decomposed as planar waves which can be represented in the usual way in terms of a complex amplitude.
An arbitrary quantity varies with space and time as:

$$q = q_0 \exp\left[i(\omega t - kx)\right] \tag{68}$$

Here time and spatial representation derivatives are replaced by $i\omega$ and $-ik$ respectively, where ω is angular velocity, $q_0 \in \mathbb{C}$ is a constant and $k \in \mathbb{R}$ is the direction of propagation of the wave.
By Equation (68) and (66), we get the basic dispersion relation for Alfvén's waves

$$\omega^2 = {k_z}^2 {v_A}^2 \tag{69}$$

By which, we get

$$\omega = v_A k_z \cos\theta \tag{70}$$

where θ is the angle between magnetic field and propagation vector, here k_z. Note that group velocity i.e. the direction of propagation of wave energy is

$$\frac{\partial\omega}{\partial k} = v_A \hat{z} \tag{71}$$

Since Equation (70) only involves k_z, we can say that Alfvén waves propagate along the magnetic field with frequency depending on just the wave number component along B and displacement being always in transverse direction

As we mentioned just after Equation (50) about magnetic field having a tension i.e. $B^2/4\pi$ associated with it along the field lines. Ideally, a magnetic field in a plasma can be thought to be a stretched string. The magnetic tension tries to oppose any distortion caused by a transverse perturbation. In this case, so take a transverse Alfvén wave moving along the field lines, velocity of such a wave can be given similar to how we express the velocity of a wave moving along a stretched string i.e. $\sqrt{\text{tension/ density}}$, and so we get an expression of Alfvén's velocity v_A (see 14.5 in Choudhuri (1998))

$$v_A = \sqrt{\frac{B^2/4\pi}{\rho}} = \frac{B}{\sqrt{4\pi\rho}} \tag{72}$$

For a more detailed and rigorous derivation of Alfvén's velocity, see Chen (2016), and for further discussion on Alfvén waves, see the former and Pg 29 onwards of Spruit (2013).

2.6 Further Developments upon MHD covered

2.6.1 Sonic, Alfvénic Mach Numbers and Plasma β

Remember Equation (47), ignore any external forces acting like gravity, and so we get

$$\rho \frac{dv}{dt} = -\boldsymbol{\nabla} p + \frac{1}{4\pi}(\boldsymbol{\nabla} \times B) \times B \tag{73}$$

Take a characteristic length scale l and time scale $\mathcal{T}$, v_0 for velocity and B_0 for field strength. For the ideal case of transmission of sound wave, assume a compressible medium. For the sake of simplicity take the isothermal equation of state $p = \mathcal{R}\,\rho\,T$ where T is the temperature and $\mathcal{R}$ is the gas constant. Let all dimensionless variables be denoted with an asterix * (follows from Pg 18-19 of Spruit (2013))

$$t = \mathcal{T}\,t^* \ , \ \ \boldsymbol{\nabla} = \frac{\boldsymbol{\nabla}^*}{l} \ , \ \ v = v_0\,v^* \ , \ \ B = B_0\,B^* \tag{74}$$

Let c_i be the isothermal sound speed

$$c_i{}^2 = \frac{p}{\rho} = \mathcal{R}T \tag{75}$$

Multiplying equation of motion by (l/ρ).Similar to non-dimensionalisation, we did to get Equation (54), we proceed as follows:

$$v_0\,\frac{l}{\mathcal{T}}\,\frac{d}{dt^*}v^* = -c_i{}^2\,\boldsymbol{\nabla}^*\ln\rho + v_A{}^2\,(\boldsymbol{\nabla}^* \times B^*) \times B^* \tag{76}$$

The characteristic value for phenomenon occurring at the time scale $\mathcal{T}$ is simply $v_0 = l/\mathcal{T}$. To get a zero-dimension form of the equation of motion, we divide Equation (76) by $c_i{}^2$

$$\mathcal{M}^2 \frac{d}{dt^*}v^* = -\boldsymbol{\nabla}^*\ln\rho + \frac{2}{\beta}(\boldsymbol{\nabla}^* \times B^*) \times B^* \tag{77}$$

here $\mathcal{M}_s$ is the **Mach number** of the flow i.e. given by

$$\mathcal{M}_s = \frac{v_o}{c_i} \tag{78}$$

and β is the **plasma-β** i.e. ratio of gas pressure to magnetic pressure given by

$$\beta = \frac{c_i{}^2}{v_A{}^2} = \frac{8\pi p}{B_0{}^2} \tag{79}$$

Similar, to how we have defined $\mathcal{M}$, we define **Alfvénic Mach Number** $\mathcal{M}_A$ i.e.

$$\mathcal{M}_A = \frac{v_0}{v_A} \tag{80}$$

As l, v_0, $\mathcal{T}$ and B_0 are taken as representative values in the beginning, so the starred quantities(with asterix) defined in Equation (74), are all of order unity. The dimensionless parameters $\mathcal{M}$.

2.6.2 Cases of β and $\mathcal{M}_s$ to consider

We would consider here some cases of M_s and β which are important to pay attention to (the discussion below follows from Spruit (2013) and please refer 1.4 in it for more detail)

- $\underline{\mathcal{M}_s \ll 1}$: a highly subsonic flow. In this case, the LHS of Equation (77) can be neglected, and β becomes highly important in determining the character of the problem.

- $\underline{\beta \gg 1}$: by definition of β, in this case, gas pressure $\gg$ magnetic pressure that implies gas pressure $\gg$ magnetic energy density which further implies that the second term in the RHS of Equation (77) is too small. So, the first term in RHS must be small as well, as $\boldsymbol{\nabla}^* \ln \rho \ll 1$, meaning that acting magnetic forces result in extremely small changes in density.In the absence of external field, we can make an approximation for constant density in such kind of high β environments.

- $\underline{\beta \ll 1}$: by definition of β, gas pressure $\ll$ magnetic pressure, so the second term in RHS of Equation (77) must be large, but due to the presence of logarithm in the first term, it won't be able to balance it out. Therefore, in case of, low β, low $\mathcal{M}_s$, for magnetic forces to be little, we must have the following condition

$$(\boldsymbol{\nabla} \times B^*) \times B^* \sim \mathcal{O}(\beta) \ll 1 \tag{81}$$

  for $\beta \to 0$, we have two solutions:

  - vanishing current i.e. $\boldsymbol{\nabla} \times B = 0$: In this case, the field is known as *potential field* as it has a scalar potential, say ϕ_m such that $B = -\boldsymbol{\nabla}\phi_m$ with $\boldsymbol{\nabla} \cdot B = 0 \implies \boldsymbol{\nabla}^2\phi_m = 0$
  - $\boldsymbol{\nabla} \times B \parallel B$: $(\boldsymbol{\nabla} \times B) \times B = 0$. In general, this case describes *force free fields*. (see 1.5 in Spruit (2013), Kulsrud (2020), Roberts (1987) for more detailed discussion on force-free and potential fields)

- Significant $\mathcal{M}_s$ and large β : In this case, the second term in RHS of Equation (77) can be neglected, by which we get a classical hydrodynamics scenario with magnetic field playing a passive role.

- Small β but field neither force-free nor potential field : In such a special case, balance can only be attained when $\mathcal{M}_s$ is of the order of $1/\beta \gg 1$ which implies existence of supersonic flows with velocities $v_0 \sim v_A$. In some theories of star forming molecular clouds such as weak-field models mentioned before (for eg. Mac Low and Klessen (2004) , McKee and Ostriker (2007), Padoan and Nordlund (1999)) , the magnetic fields are approximated to be in such a regime.

- $\underline{\beta \approx 1}$: This case is sometimes known as *equipartition* i.e. approximate equality of thermal and magnetic energy densities (follows from definition of β). In some situations, equipartitions also refers to a state where kinetic energy density of flow is comparable to magnetic energy density i.e.

$$\frac{1}{2}\rho v^2 \approx \frac{B^2}{8\pi} \implies v^2 \approx \frac{B^2}{4\pi\rho} \implies v \approx v_A \tag{82}$$

2.7 Virial theorem and Beyond

Before trying to understand how magnetic fields play a role in regulating the collapse and fragmentation of molecular clouds, we must begin with some basic physical considerations.
Virial theorem is quite an important theorem as by using it, we can analyze the relative importance of magnetic fields with respect to self-gravity, turbulent ram pressure and thermal pressure of molecular clouds.

In Eulerian form, virial theorem can be written like as below i.e. Equation (83) for a fixed volume. This builds up from the Lagrangian form of virial theorem for a fixed mass. (Chandrasekhar and Fermi (1953a,b), Mestel and Spitzer (1956))
Take a fixed control volume V, containing fluid with density ρ, having velocity v, it's magnetic field as B and gravitational potential as ϕ, is given by (McKee and Zweibel (1992)) as follows:

$$\frac{1}{2}\ddot{I} = 2(\mathcal{T} - \mathcal{T}_0) + (\mathcal{B} - \mathcal{B}_0) + \mathcal{W} - \frac{1}{2}\frac{d}{dt}\int_{\delta V} (\rho v r^2)\, dS \tag{83}$$

(follows from the explanation in Krumholz and Federrath (2019), McKee and Ostriker (2007)) here,
$\ddot{I}$ is the second derivative of the generalized moment of inertia of the mass within V
$\mathcal{T} = \frac{1}{2}\int (3P + \rho v^2)\, dV$ is the total translational thermal plus kinetic energy.

$\mathcal{B} = \frac{1}{8\pi}\int B^2 dV$ is the total magnetic energy

$\mathcal{W} = -\int \rho r \cdot \nabla\phi\, dV$ is the gravitational potential energy
$\mathcal{T}_0$ and $\mathcal{B}_0$ are fluid and magnetic stresses respectively about the surface of V

The RHS of the equation describes how the forces acting on V cause the material inside the volume to accelerating inbound or outbond direction. The last term in the expression there i.e. the time-derivate of the mass flux about a surface δV of volume V, expresses the change in inertia, not due to the forces but because of the bulk flow of mass across the boundary, within the control volume.
Take ratios of force terms on RHS of Equation (21) in order to yield zero-dimension ratios gives numbers which describe their relative importance. Furthermore, take the ratio of magnetic to two parts of kinetic term, we get (from Krumholz and Federrath (2019))

$$\frac{\mathcal{B}}{(3/2)\int P\, dV} \sim \frac{B^2/8\pi}{P} \sim \beta^{-1} \tag{84}$$

$$\frac{\mathcal{B}}{(1/2)\int \rho v^2\, dV} \sim \frac{B^2/8\pi}{\rho v^2} \sim (\frac{v_A}{v})^2 \sim \mathcal{M}_A^{-2} \tag{85}$$

$$\text{where} \qquad v_A = \frac{B}{\sqrt{4\pi\rho}} \quad \text{i.e. } \textbf{Alfvén velocity} \tag{86}$$

here, β is the **plasma parameter(or efficiency parameter)** defined in Equation (79) and $\mathcal{M}_A$ is **Alfvén Mach number**, defined in Equation (80).

Assuming that the external fields are negligible compared to the volume's self gravity, so $\mathcal{W} \sim -GM^2/R$
For a uniform spherical volume, assuming the condition of negligibility other force fields compared to self gravity, $\mathcal{W} = -\frac{3}{5}\frac{GM^2}{R}$ would be the gravitational energy of a uniform sphere with an extra factor a to account for deviations in shape and density distribution. (see Fissel and BLAST-Pol Collaboration (2012))
So, for an arbitrary volume V, take the ratio of magnetic and gravitational terms, we get(follows from Krumholz and Federrath (2019))

$$\frac{\mathcal{B}}{\mathcal{W}} \sim \frac{(B^2R^3)/8\pi}{GM^2/R} \tag{87}$$

here M is the mass contained in volume and $R = V^{1/3}$ is the characteristic size. In ideal MHD, the magnetic flux through the volume is conserved by Alfvén's Theorem (**add equation no of Alfven's theorem**), so the ratio can be re-written in terms of magnetic flux as $\Phi_B \sim BR^2$ and so

$$\frac{\mathcal{B}}{\mathcal{W}} \sim \frac{\Phi_B^2}{GM^2} \sim \mu_\Phi^2 \tag{88}$$

$$\text{where} \quad M_\Phi = \frac{1}{2\pi}\frac{\Phi_B}{\sqrt{G}} \quad \text{i.e. } \textbf{magnetic critical mass} \tag{89}$$

M_Φ is the magnetic critical mass which is the maximum mass that can be supported from collapse by a specified magnetic flux or gravitational collapse(Mouschovias and Spitzer (1976))
Note that the exact coefficient in M_Φ depends weakly on the configuration of the mass; the value $1/2\pi$ that we have adopted in Equation (28) is by assuming an infinite thin sheet (Nakano and Nakamura (1978))
The relative importance of magnetic field in supporting a cloud against gravitational collapse is usually quantified by the mass to flux ratio i.e. $\mu_\Phi = M/M_\Phi$
For a cold cloud in equilibrium,(see Fissel and BLAST-Pol Collaboration (2012))

$$M = c_\Phi \frac{\Phi_B}{G^{1/2}} \tag{90}$$

If we take the simplest case for a uniform poloidal field through a spherical cloud $\Phi_B = \pi R^2 B$
In the Equation (28), c_Φ is a constant of order unity which depends on the distribution of magnetic fields and density within the cloud. A cold cloud with a constant mass to flux ratio poloidal field has $c_\Phi \approx 0.17$ (Tomisaka et al. (1988))
If $\mu_\Phi > 1$, the cloud is called *supercritical* and the magnetic field cannot prevent gravitational collapse of a cloud on it's own. However, if $\mu_\Phi < 1$ then the cloud is said to be *subcritical*, and gravitational collapse of this cloud is not possible. Based on classical star formation theories(such as Shu et al. (1987)), for a subcritical core to be able to collapse, either the mass must change (such as from flows along field lines), or μ_Φ must change via **ambipolar diffusion**(which results from magnetic field lines in dense regions not being perfectly frozen to the gas).
Also known as ion-neutral drift, it's a non-ideal mechanism operating on large-scales that allows redistribution of Φ_B in weakly-ionized plasmas due to weak ion-neutral coupling. This is characterized by the ambipolar diffusion Reynold's number R_{AD}(Zweibel and Brandenburg (1997), Li et al. (2006, 2008) i.e. comparable to the ordinary hydrodynamical R_e. Here R_{AD} gives the measure of the ratio of size-scale of turbulent flow to size scale where viscous dissipation occurs.

An important point to note, dimensionless ratios here, $\mathcal{M}_A$, β, and μ_Φ do not include surface fluid stress term T_0, surface magnetic stress B_0 and bulk flow term $(1/2)\, d/dt \int_{\delta V} (\rho v r^2)\, dS$. By some simulations(Dib et al. (2007)), these defined dimensionless quantities can make order unity contribution to RHS of Equation (21). We have omitted them also, because observationally measuring these quantities is more difficult than the volumetric terms. Still, we must know that, the relative importance of magnetic forces might be altered if we could properly include these surface magnetic terms.

Points to note : We have implicitly assumed μ_Φ to be constant, which is only true when the flux is conserved, that holds for ideal MHD, but non-ideal effects do play a major role at some point in the star formation process, as evidenced by the fact that magnetic fields of young stars are more weaker than in the situation where all the magnetic flux which threads a typical $\sim 1 M_\odot$ interstellar cloud were to be trapped into the star when it collapses(such as e.g. Paleologou and Mouschovias (1983)).
At largest scales, non-ideal mechanism(ambipolar diffusion) operates. As per (McKee et al. (2010)), observed molecular clumps are seen to have $R_{AD} \approx 20$ that places them near but not exactly close to the ideal MHD limit (*i.e.* $R_{AD} \rightarrow \infty$0. So, we would be assuming ideal MHD here. Although, we to have to keep in mind, that if non-ideal effect are added where relevant, then the model might be more realistic and accurate.

2.8 Stability and Collapse of Gas Cloud

2.8.1 Jeans Instability and Criterion

Consider an infinite, uniform, homogenous gas cloud at rest with density ρ, pressure p and temperature T, both of which are constant everywhere. Note that this state is not a well-defined equilibrium.

(The derivation and explanation below follows from chapter 7 of Choudhuri (1998), chapter 26 of Kippenhahn et al. (2012), chapter 8 of Dyson and Williams (1997) and Höfner (2010)).
We would be investigating under which circumstances, such a configuration can become undtable due to self-gravity. The gas cloud is described by the Navier Stokes equation of continuity and motion.(*here we would be neglecting viscosity*)

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot (\rho v) = 0 \tag{91}$$

$$\frac{\partial v}{\partial t} + (v.\boldsymbol{\nabla})v = -\frac{1}{\rho}\boldsymbol{\nabla} p - \boldsymbol{\nabla}\Phi \tag{92}$$

By Poisson's equation,

$$\boldsymbol{\nabla}^2\Phi = 4\pi G\rho \tag{93}$$

where G is the Newtonian gravitational constant, k_B is the Boltzmann constant, μ is the mean molecular weight and m is the mass of gas particles
Assume the gas is isothermal, and so we have the equation of state

$$p = \frac{\mathcal{R}}{\mu}\rho T = \frac{(k_B/m)}{\mu}\rho T = {c_s}^2\rho \tag{94}$$

This assumption is justified as energy exchange by radiation is usually quite efficient for interstellar matter which means the time scales for thermal adjustment are short compared to the dynamical processes being studied here.
Take constant ρ_0 as initial density, p_0 as the initial pressure for the gas. For equilibrium, let $\rho = \rho_0 =$ constant , $T = T_0 =$ constant, and $v_0 = 0$. ϕ_0 is determined by $\boldsymbol{\nabla}^2\Phi_0 = 4\pi G\rho_0$ and by boundary conditions at infinity Here, we would consider a small perturbation, as below:

$$\rho = \rho_0 + \rho_1 \;\; , \;\; p = p_0 + p_1 \;\; , \;\; \Phi = \Phi_0 + \Phi_1 \;\; , \;\; v = v_1 \tag{95}$$

Here, the functions with subscript 1 depend on space and time.
The perturbation of equation of continuity gives:

$$\frac{\partial \rho_1}{\partial t} + \rho_0 \boldsymbol{\nabla} \cdot v_1 = 0 \tag{96}$$

Linearizing Equation (95) and substituting in (92) and (94), assuming the perturbations are isothermal(c_s *is not perturbed*) and ignoring non-linear quantities, we get

$$\rho_0\frac{\partial v_1}{\partial t} = -{c_s}^2\boldsymbol{\nabla}\rho_1 - \rho_0\boldsymbol{\nabla}\Phi_1 \tag{97}$$

$$\boldsymbol{\nabla}^2\Phi_1 = 4\pi G\rho_1 \tag{98}$$

We have now three equations (96), (97) and (98), satisfied by the three perturbation variables ρ_1, v_1 and Φ_1. This is a system of linear homogeneous system of differential equations with constant coefficients. We therefore can assume that solutions exist with the space and time dependence proportional to $\exp[i(kx + \omega t)]$ such that:

$$\frac{\partial}{\partial x} = ik \ , \ \frac{\partial}{\partial y} = \frac{\partial}{\partial z} = 0 \ , \ \frac{\partial}{\partial t} = i\omega \tag{99}$$

and we obtain

$$\omega v_1 + \frac{k{c_s}^2}{\rho_0}\rho_1 + k\Phi_1 = 0 \tag{100}$$
$$k\rho_0 v_1 + \omega\rho_1 = 0 \tag{101}$$
$$4\pi G\rho_1 + k^2\Phi_1 = 0 \tag{102}$$

The set of equations above (100)- (102) can only have non-trivial solutions if

$$\begin{vmatrix} \omega & (k{c_s}^2/\rho_0) & k \\ k\rho_0 & \omega & 0 \\ 0 & 4\pi G & k^2 \end{vmatrix} = 0 \tag{103}$$

i.e. if

$$\omega^2 = k^2{c_s}^2 - 4\pi G\rho_0 \tag{104}$$

where k is a non-vanishing wave number. There exist two different cases:

- if k is sufficiently large then $k^2{c_s}^2 - 4\pi G\rho_0 > 0$ and $\omega \in \mathbb{R}$. The perturbation varies periodically in time and equilibrium is stable with respect to perturbations of such short wavelengths.
  In the case, of limit $k \to \infty$ in Equation (104), we get $\omega^2 = k^2{c_s}^2$ which corresponds to isothermal sound waves.
- if $k^2{c_s}^2 - 4\pi G\rho_0 < 0$ then ω is of the form $i\xi$ where $\xi \in \mathbb{R}$. So, perturbations $\sim \exp(\xi t)$ which grow exponentially with time i.e. unstable equilibrium.

The intermediate between these two cases corresponds the critical wave number

$$k_J = \left(\frac{4\pi G\rho_0}{{c_s}^2}\right)^{(1/2)} \tag{105}$$

or to the critical wavelength

$$\lambda_J = \frac{2\pi}{k_J} = \left(\frac{\pi}{G\rho_0}\right)^{(1/2)} c_s \tag{106}$$

Therefore, a perturbation with a wave number $k < k_J$ (or a wavelength $\lambda > \lambda_J$) is unstable.

The condition for instability $\lambda > \lambda_J$ is known as **Jeans Criterion**.
In the other words, if the size of perturbation is larger than critical wavelength (λ_J), then the enhanced self-gravity can overpower the excess pressure so that the perturbation grows. The corresponding critical which is often referred as **Jeans Mass** is given by (using Equation (94))

$$M_J = \frac{4}{3}\pi{\lambda_J}^3\rho_0 = \frac{4}{3}\pi^{5/2}\left(\frac{k_B T_0}{Gm}\right)^{3/2}\frac{1}{\rho^{1/2}} \tag{107}$$

So, $M_J \propto {T_0}^{3/2}\,{\rho_0}^{-1/2}$ If the perturbation in a uniform gas cloud with temperature T_0 and density ρ_0, involves $M > M_J$ then it can be said that gas in the perturbed region would keep contracting due to the enhanced self gravity eventually fragmenting into pieces due to Jean's instability.

A cloud which exceeds Jeans Mass collapses and undergoes *fragmentation*. The collapse is assumed to be isothermal more than adiabatic. As, cloud falls together and fragments of it become unstable and collapse faster than the cloud as a whole, Jeans mass becomes smaller than the mass of the parent collapsing cloud. This process goes on until the collapse remains isothermal. (Note: The concept of Jeans Mass is derived for an equilibrium state, but for estimation purposes at the order-of-magnitudem we can use it). As the process gets more complex, strictly following hydrodynamics and thermodynamics becomes difficult as symmetry ceases to exist due to the random behaviour, but it is estimated that when thermal adjustment time becomes comparable with the free-fall time, the collapses stops to be isothermal and tends to the adiabatic state, leading not all subfragments to fall together, terminating fragmentation. A detailed estimate, considering radiation processes was given by Hoyle (1953) and another by Rees (1976) who gave an estimate of the mass limit of fragmentation without specifying the detailed radiation process. To see more detail upon how this fragmentation eventually leads to star formation, see Chapter 27 of Kippenhahn et al. (2012), Dyson and Williams (1997), Stahler and Palla (2004), Krumholz et al. (2011))

2.8.2 Free-Fall Time for Homogeneous Cloud Collapse

In the previous subsection 2.8.1, we had made a lot of assumptions to simplify our study of Jeans instability for an interstellar gas. In order to make the investigation more realistic, we need an appropriate model consisting of an isothermal sphere, containing ideal gas, with a finite radius imbedded in a medium of some pressure. However, as the scope of this topic goes beyond our need at the moment, we wouldn't be covering it in this background literature review. See 26.2 of Kippenhahn et al. (2012) for explicit development of this investigation.

Jeans Criterion follows from an ordinary first-order perturbation theory by which we obtain conditions under which perturbations of equilibrium stange tend to develop exponentially. But this does not give enough information about a fully developed collapse and the final product. In order to know that, the next step taken is to study this process through non-linear perturbations. Here, we begin with the simplified case i.e. of spherically symmetrical cloud. The derivation and explanation below follows from (chapter 27 of Kippenhahn et al. (2012), chapter 8 of Dyson and Williams (1997) and Höfner (2010))

Consider a spherically symmetrical, homogeneous collapsing cloud having mass M, radius R, assuming free-fall i.e. the forces due to pressure gradients would be neglected. As, the gravitational force for sphere $\sim -GM\ R^2$, pressure term in the equation of motion can be approximated by

$$\left|\frac{1}{\rho}\frac{\partial P}{\partial R}\right| \approx \frac{P}{\rho R} \approx \frac{\mathcal{R}T}{\mu R} \tag{108}$$

The ratio of gravitation to pressure term $\propto M/(RT)$, which during isothermal collapse increases as R decreases. (*M is const.*) As we know, the gravitational attraction from the centre of the sphere would be Gm/r^2 where m is the mass within the sphere of radius r. Since, we are neglecting pressure, the sphere collapses in free-fall, by the equation of motion

$$\ddot{r} = -\frac{Gm}{r^2} \tag{109}$$

here, we can take $m = (4/3)\pi {r_0}^3 \rho_0$ where r_0 and ρ_0 are the initial values of radius and mass of sphere at the beginning of collapse, let $\rho_0 =$ const. Multiplying, Equation (109) by $\dot{r}$ and integrating it, we get

$$\frac{1}{2}\dot{r}^2 = \frac{4\pi {r_0}^3}{3r} G\rho_0 + C \tag{110}$$

Take the integration const C such that $dotr = 0$ at the beginning, when $r = r_0$, so we obtain

$$\frac{\dot{r}}{r_0} = \pm\sqrt{\frac{8\pi G}{3}\rho_0 \left(\frac{r_0}{r} - 1\right)} \tag{111}$$

for $r \in \mathbb{R}$, $r < r_0$ i.e. only the negative sign of the RHS of this equation gives relevant solutions. Take a variable θ defined as

$$\cos^2\theta = \frac{r}{r_0} \tag{112}$$

So,

$$\frac{\dot{r}}{r_0} = -2\dot{\theta}\cos\theta\sin\theta \ , \ \ \frac{r_0}{\dot{r}} - 1 = \frac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta \tag{113}$$

Using Equation (111)

$$2\dot{\theta}\cos^2\theta = \sqrt{\frac{8\pi G\rho_0}{3}} \tag{114}$$

By basic trigonometry, we can derive the following identity

$$2\dot{\theta}\cos^2\theta = \frac{d}{dt}\left(\theta + \frac{1}{2}\sin 2\theta\right) \tag{115}$$

Using this identity, in our equation, we get

$$\theta + \frac{1}{2}\sin 2\theta = t\sqrt{\frac{8\pi G\rho_0}{3}} \tag{116}$$

where the integration const C is chosen such that the beginning of the collapse (i.e. when $r = r_0$ or $\theta = 0$) coincides with $t = 0$. Note that, r_0 is absent in the equation (116). So, the solution $\theta(t)$ can be used for all mass shells. Similarly, r/r_0 and $\dot{r}/r_0$ at a given time t in Equation (113) is also valid for all mass shell. This implies that the sphere undergoes a *homologous contraction*. So, the relative density variation doesn't depend on the r_0 and the sphere at all. The time it would take to reach the centre ($r = 0$ or $\theta = \pi/2$) is known as **free-fall time**

$$t_{\text{ff}} = \sqrt{\frac{3\pi}{32\, G\rho_0}} \tag{117}$$

which follows from Equation (116) and is applicable to all mass shells. Keep in mind that, before the centre is reached, the pressure will become relevant as the gas becomes opaque and T increases. In that situation, the free-fall approximation has to be abandoned and finally the collapse would terminate. (see 27.2 - 27.5 of Kippenhahn et al. (2012) for more detail)

The sound travel time across the cloud is given by (using equation (94) too)

$$t_s = \frac{R}{c_s} = R\sqrt{\frac{\rho}{p}} = R\sqrt{\frac{\mu}{\mathcal{R}T}} \tag{118}$$

When $t_s < t_{\text{ff}}$, pressure forces overcome gravity temporarily, returning the system to stable equilibrium. While, when $t_s > t_{\text{ff}}$ gravity is able to overcome pressure forces leading to gravitational collapse of cloud. So, a condition of collapse

$$t_s \geq t_{\text{ff}} \tag{119}$$

Using Equation (119), we can get the condition of critical Jeans Mass again and Jeans Instability.

The view of star formation presented here is extremely simplified, neglecting important effects such as magnetic field, cloud rotation and turbulence. Collapse of cloud increases the magnetic field strength and so the magnetic pressure. This acts in a way similar to thermal pressure, to oppose the collapse . Any collapsing cloud having have zero angular momentum is highly unlikely . In the absence of a braking torque, cloud's rotational velocity increases because angular momentum is conserved during gravitational collapse. If the magnetic field is connected to surrounding gas, the necessary braking torque maybe produced. These are crucial points to keep in mind when dealing with star formation. (see 8.1.4 in Dyson and Williams (1997)).
As we move on, the star formation processes would become more realistic than it is now, as we would be taking both magnetic fields and turbulence driving parameter(*described in next section*) into consideration as well.

2.9 Playing with Turbulence and Magnetic Fields

2.9.1 Introduction

Turbulence can be found to play a dominant role almost everywhere throughout the Milky Way, especially in galactic processes like star formation (Ferrière (2001), Elmegreen and Scalo (2004), McKee and Ostriker (2007) , Padoan et al. (2014)). The energy source of this turbulence has been extremely debatable for a long time, many processes have been considered to be responsible for interstellar turbulence (Elmegreen and Scalo (2004),Mac Low and Klessen (2004), Federrath et al. (2016)) such as galaxy mergers (Renaud et al. (2014)), gravitational potential of spiral arms (Falceta-Gonçalves et al. (2014)), baroclinicity and differential galactic rotation (Del Sordo and Brandenburg (2011)), supernova explosions (Hennebelle and Iffrig (2014), Padoan et al. (2016)), stellar feedback (Lee et al. (2012)), gravitational collapse and accretion (Klessen and Hennebelle (2010) , Krumholz and Burkert (2010), Federrath et al. (2011), Robertson and Goldreich (2012), Krumholz and Burkhart (2016)), and MHD instabilities (Tamburro et al. (2009)).
All these relatively different driving phenomenon mentioned above, differ not only because of their varying spatial scale of activity but also in the way they drive turbulence.(Herron et al. (2016)). Being such an immense topic in astrophysics, our discussion of turbulence is just a brief review and would be taking help from variety of exhaustive reviews on the subject such as Frisch and Kolmogorov (1995), Biskamp (2003), Falgarone and Passot (2003), Elmegreen and Scalo (2004), Schekochihin and Cowley (2007)

Based on the previous background review, the dynamic nature of MHD flow can be characterized by two non-dimensional parameters fully

$$R_e{}^{-1} = \frac{\mu}{Lv} \ \ , \ \ R_m{}^{-1} = \frac{\eta}{Lv} \tag{120}$$

where R_e is the kinetic Reynold's number(of ordinary hydrodynamics) with viscosity μ and R_m is the Magnetic reynold's number with η as the magnetic diffusivity. In the case, R_e and R_m become sufficiently larger than unity, the MHD flow undergoes a transition from the laminar state (*stationary stream-line topology*) to turbulence in which the fluid motion seems to become erratic and unpredictable. (Falgarone and Passot (2003))

The study of MHD turbulence is broadly categorized by considering kinetic energy $E^K = (1/2)\int_V dV\, v^2$ of the flow and the magnetic energy $E^M = (1/2)\int_V dV\, b^2$ where V is the spatial volume of the system. When $E^K \gg E^M$ (*condition for dynamo problem*i.e. amplification of magnetic field by plasma turbulence) , the magnetic field is passively advected by the fluid and when $E^K \ll E^M$, the strong magnetic field forces fluid motion into the state of quasi-two dimensionality (e.g. - encountered in solar corona or terrestrial laboratory experiments with magnetic plasma confinement). To study, the intrinsic nature of the non-linear interaction between the turbulent fields v and B, we look at the turbulence with $E^K \sim E^M$ and a magnetic Prandtl number $\mathrm{Pr_m} = \mu/\eta$ i.e. of the order of unity. (Falgarone and Passot (2003)).
Although our understanding about the working of turbulence is surprising little but there have been many simple models and tools that describe it, some of those which would be mentioned here.

2.9.2 Velocity Statistics

Velocity fluctuates in space and time in a turbulent medium, making the statistical approach a reasonable way to proceed with the study of such fluctuations. Astrophysics employs a lot of statistical tools, but here we would be discussing some of the simpler ones, and make two assumptions which simplify our work that are assuming homogeniety and isotropic nature of turbulence. In a real molecular cloud, both of these assumptions aren't strictly true as huge magnetic fields provide a preferred direction, so the isotropic condition is an ideal case. But as we move on, we can build up from these and relax those assumptions. (the explanation and derivation follows from Krumholz (2015))

Take $v(x)$ as the velocity at position x within volume V. The autocorrelation function which describes how it varies with position is given by

$$A(r) = \frac{1}{V}\int v(x)\cdot v(x+r) \equiv \langle\, v(x)\cdot v(x+r)\,\rangle \tag{121}$$

where the angle brackets denote the average over all positions x. $A(0) = \langle |v|^2\rangle$ is the mean square velocity in the fluid. If the velocity is taken to be isotropic, then we see, $A(r)$ doesn't depend on direction, but only on $r = |r|$. So, by $A(r)$, we can tell the relative difference between the velocities at points separated by some distance r. This can be thought of in a better way in Fourier space than in a real space, so the autocorrelation resembles a Fourier transform. The Fourier transform of the velocity field is

$$v(k) = \frac{1}{(2\pi)^{3/2}}\int v(x)\, e^{-ikx}\, dx \tag{122}$$

The power spectrum is defined as

$$\Psi(k) = |\tilde{v}(k)|^2 \tag{123}$$

As turbulence is taken to be isotropic, the power spectrum depends only on the magnitude of the wave number, $k = |k|$, and not on it's direction, so it is usual to define the power per unit radius in k-space

$$P(k) = 4\pi k^2 \Psi(k) \tag{124}$$

where this is the total power integrated over some shell from k to $k + dk$ in k-space. By Parseval's theorem, we have

$$\int P(k)\, dk = \int |\tilde{v}(k)|^2\, dk = \int v(x)^2\, dx \tag{125}$$

i.e. the integral of power spectral density over all wavenumbers which is equal to the integral of the square of the velocity over all space. For an incompressible flow, the integral of power spectrum gives the kinetic energy per unit mass in the flow. By Weiner-Khinchin theorem , $P(k)$ is just the Fourier transform of the autocorrelation function (Hecht (2002))

$$\Psi(k) = \frac{1}{(2\pi)^{3/2}}\int A(r)\, e^{-ikr}\, dr \tag{126}$$

The power spectrum at a wavenumber k tells the total power there is in the motions at that wavenumber, on that characteristic length scale. It is also another way to look at the spatial scaling of turbulence, describing the power in the turbulent motions in terms of the wavenumber $k = 2\pi/\lambda$. The power spectrum also gives us information about the variation of velocity dispersion measured over a region of characteristic size.

Consider a volume of size l and measure the velocity dispersion within it. Furthermore, suppose the power spectrum is described the power law $P(k) \propto k^{-n}$ then the kinetic energy per unit mass within the region is upto factors of order unity,

$$KE = \sigma_v(l)^2 \tag{127}$$

another way of expressing KE is in terms of power spectrum, integrating over the modes that are small enough to fit within the volume under consideration

$$KE \sim \int_{2\pi/l}^{\infty} P(k)dk \propto l^{n-1} \tag{128}$$

Using which, we get

$$\sigma_v = c_s \left(\frac{l}{l_s}\right)^{(n-1)/2} \tag{129}$$

where the relationship is normalized by defining sonic scale l_s as the size of a region within which the velocity dispersion is equal to the thermal sound speed of gas i.e. c_s. Also, if $P(k) \propto k^{-n}$, then

$$v(l) \propto \sigma_v(l) \propto \Delta v(l) \propto l^q \tag{130}$$

with $q = (n-3)/2$.
If the system is considered spatially symmetric with each dimension $\approx l$, then the one-dimensional velocity dispersion along a given line-of-sight can be related to the 3D velocity dispersion by $\sigma = \sigma_v(l)/\sqrt{3}$

2.9.3 Brief Review of Some Models

If the turbulent system is closed with appropriate periodic and boundary conditions, a number of invariants are considered in the ideal limit $\mu = \eta = 0$ (Woltjer (1958)). For incompressible, three dimensional MHD, these irregular ideal invariants are given as follow with their decay rates at finite viscosity and magnetic diffusivity. (explanation follows from Falgarone and Passot (2003))
Total Energy

$$E = E^K + E^M = \frac{1}{2}\int_V dV\,(v^2 + b^2)\ ,\ \ \dot{E} = -\int_V dV\,(\mu\omega^2 + \eta j^2) \tag{131}$$

Magnetic Helicity

$$H^M = \frac{1}{2}\int_V dV(a)\ ,\ \ \dot{H}^M = -\eta\int_V dV\, j\cdot B \tag{132}$$

Cross helicity

$$H^C = \frac{1}{2}\int_V dV(v\cdot b)\ ,\ \ \dot{H}^C = \frac{\mu+\eta}{2}\int_V dV\,\omega\cdot j \tag{133}$$

These three ideal variants are quite important in characterizing the macroscopic properties of MHD turbulence. The magnetic helicity H^M gives a measure for linkage and twist of magnetic field lines(Moffatt (1969)) while the cross helicity H^C gives the overall relationship between magnetic and velocity field. (Biskamp (2003)). The turbulent fields v and b are here assumed to be spatially homogeneous in their respective statistical properties. However, this assumption is seen in real to be only approximately valid in constricted small spatial regions. Due to statistical homogeneity alongwith quasi-ergodocity hypothesis, we can replace ensemble averages by spatial time averages by neglecting the influence of distant boundaries. Periodic conditions are then applied at the surface outside of volume V which contains the turbulent flow.

One of the most famous and considered kinda model of turbulence for subsonic, hydrodynamic turbulence is the K41 theory given in Kolmogorov (1941)a, Kolmogorov (1941)b (*the first paper was written in Russian 1941, it's English version came as Kolmogorov (1991)*), Frisch and Kolmogorov (1995). However, it doesn't include magnetic field. Real interstellar clouds aren't actually subsonic and hydrodynamic (Krumholz (2015)).

It regards at its basis a turbulent velocity field which is the superposition of structures(which he calls eddies) that are characterized by a spatial scale $\boldsymbol{l}$ and and associated velocity field increment $\delta v_l = [v(r+l) - v(r)] \cdot \boldsymbol{l}/l$ As we are additionally assuming isotropy, the field increments depend solely on l, by which we can define the characteristic eddy velocity

$$v_l = \sqrt{\langle \delta {v_l}^2 \rangle} \tag{134}$$

Using the K41 picture, we can differentiate three spatial scale ranges:

- the energy containing scales which drive flow
- dissipation range at smallest scales, where dissipative effects dominate
- the inertial range where non-linear interaction which influence the driving, dissipation and govern the dynamics are negligible

(the explanation follows from Falgarone and Passot (2003) As is experimentally observed that structure functios of order p , $S^v_p(l) = \langle \delta v^p_l \rangle$ follows power-laws in l with constant $p-$ dependent exponents ζ_p. As K41 theory builds up, it predicts values for scaling these exponents in the limiting case of infinite R_e. Structure functions are statistical moments of the two-point probability distribution of respective turbulent field, and so their scaling exponents describes the statistical properties of turbulence. For e.g. S^v_2 is directly to linked to one-dimensional energy spectrum while the p-dependence of the ζ_p explains the intermittency of flow structures. The treatment of K41 in the sense of aymptotic statistical variance properties of real-turbulence is given in detail in Frisch and Kolmogorov (1995). Also, an important realization of Kolmogorov was that in the spatial hierarchy of eddies, the larger ones transfer a fraction of their energy(i.e. resulting energy flux) towards smaller counterparts (Richardson and Lynch (2007)), the size of interacting eddies differing only slightly i.e. the transfer proceeds in small local steps in spatial Fourier space. These *cascades* are usually considered to be cause by non-linearities in Navier-Stokes and MHD equations. The energy flux is estimated by order of magnitude as ${v_l}^2/\tau_l$ with the non-linear eddy turnover time $\tau_l = l/v_l$

In a fully-developed turbulence, where we include all the assumptions explained above, the non-energy flux in the inertial-range equals the energy dissipation ϱ which gives by order of magnitude

$$\varrho \sim \frac{{v_l}^2}{\tau_l} = \frac{{v_l}^3}{l} \implies v_l \sim (\varrho l)^{1/3} \implies S^v_3(l) = -\frac{4}{5}\varrho l \tag{135}$$

The Fourier transform of S^v_2 scales as the one-dimensional energy spectrum defined as $E_1(k_1) = (1/2)\int dk_2 \int dk_3\, \boldsymbol{v(k)} \cdot \boldsymbol{v^*(k)}$ with $k_i = (2\pi/l_i)$ Here, since turbulent is taken to be isotropic, so the angle-integrated energy spectrum

$$E(k) = \int_0^{4\pi} d\Omega\, E(k)|_{|k|=k} \tag{136}$$

defining $E(\boldsymbol{k}) = (1/2)|\boldsymbol{v}(\boldsymbol{k})|^2$, The real space exponent ζ_2 of $S_2^v(\boldsymbol{l})$ corresponds to an inertial range Fourier-space scaling $\sim k^{-1+\zeta_2}$ which yields the experimentally well supported **Kolmogorov spectrum**

$$E(k) = C_k\, \varrho^{2/3}\, k^{-5/3} \tag{137}$$

with the Kolmogorov constant $C_k \approx 1.6$ The **Kolmogorov disspation length** l_{K41} gives an order of magnitude estimate of the spatial scales where energy dissipation $\sim v_l^2\mu/l^2$ begins to dominate the non-linear energy dynamics $\sim v_l^2/\tau_l$ that marks the beginning of dissipation range. With the approximations above,

$$\frac{\mu}{l^2} \sim {\tau_l}^{-1} = \frac{v_l}{l} \sim \frac{(\varrho l)^{1/3}}{l} \implies \boldsymbol{l}_{\mathsf{K41}} = \left(\frac{\boldsymbol{\mu^3}}{\boldsymbol{\varrho}}\right)^{1/4} \tag{138}$$

(the explanation follows from McKee and Ostriker (2007)) Important point to note that, because velocities $v(l) \sim {}_v(l) \sim \Delta v()$ in molecular clouds are in general not small compared to c_s, for sufficiently large l, so one cannot expect Kolmogorov theory to apply. Even, some portion of energy at a given scale is directly dissipated via shocks rather than by *Richardson cascading*, there are a network of intersecting shocks in the limit of zero pressure, also known as *Burger's turbulence* (Frisch and Bec (2001)). Since the power spectrum corresponding to a velocity discontinuity in one-dimension has $P(k) \propto k^2$, an isotropic system of shocks in three dimensions would also yield power-law scalings for the velocity correlations, with $n = 4$ and $q = 1/2$. Note that these correlations can be expressed in a power-law form even if there's not any case of conservative inertial cascade

Turbulence in a magnetized system is inherently different from unmagnetized case because of the additional wave families, non-linear couplings and additional diffusive processes (including resistive and ion-neutral drift terms) involved. In the case, magnetic field B is strong, the Alfvén velocity v_A satisfies $v_A \gg v(l)$, here a directionality is introduced such that correlations of the flow variables depend differently on $r_\parallel$, $r_\perp$, $k_\parallel$ and $k_\perp$, that are the displacement and wavevector components parallel and perpendicular to $\hat{B}$

The IK i.e. Iroshnikov-Kraichnan phenomenology for MHD turbulence is just a modification of K41 approach while introducing a different model for non-linear flux. (see Iroshnikov (1964), Kraichnan (1965) for more detail on IK picture).
The K41 and IK models are spatially isotropic in a way that turbulent structures are characterized by a single length scale l. However, in the presence of the magnetic field B, the characteristic interaction time of Alfvén wave like distortions of extent λ along the field lines $tau_l \sim \lambda/b_0$ is way shorter than non-linear turnover time $\tau_l \sim l/z_l$ of their components of extent l and amplitude z_l $\perp B$. Furthermore, non-linear energy flux shows a clear anisotropy.(Shebalin et al. (1983), Grappin (1986)) (explanation follows from Falgarone and Passot (2003)).
Against this background, for *incompressible* MHD, Goldreich and Sridhar developed a semi-phenomenological GS95 model (Sridhar and Goldreich (1994), Goldreich and Sridhar (1995); Goldreich and Sridhar (1997)), where they introduced the idea of a critically-balanced anisotropic cascade, in which the non-linear mixing time $\perp B$ and propagation time $\parallel B$ remain comparable for wavepackets at all scales, so $v_A k_\parallel \sim v(k_\perp, k_\parallel) k_\perp$. Interactions between oppositely-directed Alfvén

wavepackets travelling along B cannot change their parallel wavenumbers $k_{\parallel} = l \cdot \hat{B}$, so that energy transfers produced by these collisions involve primarily $k_{\perp}$ i.e. cascade though spatial scales $l_{\perp} = 2\pi/k_{\perp}$ with $v(l_{\perp})^3/l_{\perp} \sim$ constant. Fusing critical balance with a perpendicular cascade yields anisoptropic power spectra (larger in $k_{\perp}$ direction); at a given level of power, the theory predicts $k_{\parallel} \propto {k_{\perp}}^{2/3}$ and magnetic fields and velocities have the same power spectra.

But if see the case of strong compressibility ($c_s \ll v$ and moderate or strong magnetic fields ($c_s \ll v_A \lesssim v$), which usually applies within molecular clouds, there is as of yet, no simple conceptual theory, to characterize the energy transfer between scales and to describe the spatial correlations in the velocity and magnetic fields. On universal scale, the flow can be dominated by large-scale (magnetized) shocks which directly transfer energy from macroscopic to microscopic degrees of freedom. Even if velocity differences are insufficient to induce sufficient shocks, for trans-sonic motions compressibility implies strong coupling among all MHD wave families. However, within a sufficiently small sub-volume of a cloud (*away from shock interfaces*), velocity differences may be sufficienty subsonic that the incompressible MHD limit and the Alfvénic cascade, such as in GS model, hold approximately in a local region.

Intermittency effects (i.e. one of the crucial properties of turbulence is unfortunately not considered in classical models of turbulence. The K41 and IK models, which implicitly assume spatial uniformity of energy dissipation ϱ give the scaling exponents ζ_p of structure functions of order p as $\zeta_p^{\mathrm{K41}} = p/3$ and $\zeta_p^{\mathrm{IK}} = p/4$. There is however, seen to deviations of high-order structure function exponents from the value $p/3$ and in non-Gaussian tales of velocity increment probability-distribution functions (PDFs) (Sreenivasan and Antonia (1997), Lis et al. (1996)) in the turbulent solar wind (Burlaga (1991)) as well as direct numerical simulations(DNS) of MHD turbulence (Politano, H. et al. (1998), Biskamp and Müller (1999); Biskamp and Müller (2000)). The *anomalous scaling* is attributed to departure from self-similarity of turbulent fields in the inertial range, the effect of which is linked to the spatial distribution of the turbulent structures responsible for energy dissipation by Kolmogorov's refined similarity hypothesis(Kolmogorov (1962))that introduces the local energy dissipation in a sphere of radius l, ϱ_l. Instead of spatial homoegeneous distribution of energy dissipation, the turbulent system is interspersed with small regions of intense dissipation, resulting in a pronounced spatial intermittency.

Some models which account for intermittency in prediciting correlation functions for incompressible, unmagnetized turbulence are (She and Leveque (1994),Dubrulle (1994). Boldyrev (2002) proposed for a model for the compressible MHD case, but in that the direct dissipation of large-scales modes in shocks was omitted. Research on formal turbulence theory is still quite an active field (see Elmegreen and Scalo (2004) for a review of the recent theoretical literature in this area) yet a comprehensive framework is still to be seen. For a review of recent numerical simulations validating some of the above models such as the SL picture, and other developments in the theory of turbulence see McKee and Ostriker (2007). For review on extensions of incompressible theory, see Falgarone and Passot (2003)

2.9.4 Further on Interstellar Turbulence

All turbulent systems seem to share one common thing i.e. a large Reynold's number. The properties of the flows on all scales depend on R_e and R_m. Flows with $R_e < 100$ are laminar, chaotic structures develop gradually as R_e increases, and those with $R_e \sim 10^3$ are considerably less chaotic than those with $R_e \sim 10^7$. Observations of star forming clouds and accretion disks are very chaoticwith $R_e > 10^8$ and $R_m > 10^{16}$. (Falgarone and Passot (2003))
As mentioned in the previous section, turbulence plays a critical role in molecular cloud support and star formation and the issue of the time scale of turbulent decay is quite an important subject of discussion. The decay of MHD turbulence poses problems in understanding things such as turbulent motions seen within molecular clouds without star formation(Myers (1999)) and rates of star formation(McKee (1999)). Earlier studies have attributed the rapid decay of turbulence to compressibility effects (Low (1999)). GS95 predicts and numerical simulations e.g. CLV02a, confirm that MHD turbulence decays rapidly even in the incompressible limit. The effect of imbalance (i.e. flux of wave packets travelling in one direction is significantly larger than those travelling in opposite direction) (Matthaeus et al. (1998), Ting et al. (1986), Ghosh et al. (1988), Biskamp (2002), Maron and Goldreich (2001)) on the turbulence decay time scale. In the ISM, many energy sources are localized, both in space and time. For e.g., in terms of energy injection, stellar outflows are essentially point energy sources. With these localized energy sources, it is natural that interstellar turbulence be typically imbalanced. (follows from Falgarone and Passot (2003), see more for detail on interstellar turbulence)

2.9.5 Observation of Magnetic Fields

Observation of magnetic fields is critical to elevating our understanding about various processes in the galactic formation in which magnetic fields have been shown to play a part either through simulations, theoretical models or through observations.Here, we would be giving an extremely brief review, taking points from some sources which give exhaustive reviews in this field (Falgarone and Passot (2003), McKee and Ostriker (2007)). As we know, the stars are weakly magnetized, while the interstellar medium is strongly magnetized. The strength and morphology of uniform (B_u) and random (B_r) components of magnetic fields in Galactic Scales has been area of much intensive research for a long time, since the discovery of the interstellar polarization of starlight.

The role of magnetic fields in star formation process has been the subject of many theoretical and observational investigations(Beck (2001)). Recent reviews include those of McKee (1999), Mouschovias and Ciolek (1999), Shu et al. (1999)). An important question is whether or not magnetic energy density is comparable to other energy densities, such as gravitational, thermal and turbulent. If that's true, then magnetic fields must play a significant role in cloud evolution and star formation while if that's false, then their role can be either secondary or even unimportant. There are two ways which this question can be structured, one is by asking "What is the magnetic field morphology?" and the other is "What is magnetic field strength"? (see Magnetic field observations in Falgarone and Passot (2003) for more detail)

In order to verify the theoretical models, and observe the presence of presence of magnetic fields and their strength and influence, observational techniques are used.(McKee and Ostriker (2007)) Two primary observational methods to measure the magnetic field strengths in dense ISM are Zeeman effect (measures the line of sight component, B_{los} and the Chandrashekhar-Fermi (CF) method (Chandrasekhar and Fermi (1953a)) that measures the field component in the plane of sky, B_{pos}, by comparing the fluctuation in the direction of B_{pos} with those in the velocity field(see reviews by Crutcher (2005), Heiles and Crutcher (2005)). The morphology of the field which is needed for CF method is measured from dust polarization and from linear polarization of spectral lines.(Goldreich and Kylafis (1981)) In the diffuse ISM, magnetic fields are also obtained by Faraday rotation and synchroton observations, with results consistent with Zeeman observations (see Crutcher et al. (2003) for more detail on observational techniques, for development in brief, see (McKee and Ostriker (2007))

2.9.6 Turbulence Driving Parameter

Star formation depends on the driving of molecular cloud turbulence, and differences in the driving can produce an order of magnitude difference in the star formation rate. The turbulent driving is characterized by the parameter , with $= 0$ for compressive, curl-free driving (e.g. accretion or supernova explosions), and $= 1$ for solenoidal, divergence-free driving (e.g. Galactic shear)

3 Development of Mathematical Model

4 Computational Analysis of Model

Table 1: **Star-Formation Properties of 10 SFC-GMC Complexes from Milky Way (Sorted by Luminosities)**

SFC No.	Cloud no.	v (km s^{-1})	σ_v (km s^{-1})	α_{vir}	$\tau_{\rm ff}$	R (pc)	$\epsilon_{\rm ff,\ br}$	M_g ($M_\odot$)	Σ_g ($M_\odot$pc^{-2})
227	1	-75.34	10.80	9.58e+00	9.32	66.84	9.14e-01	9.47e+05	6.31e+01
228	2	-55	7.65	9.58e-01	7.41	119.13	5.77e-02	8.49e+06	1.61e+02
68	3	98.41	5.34	6.90e-01	4.66	61.64	5.69e-02	2.97e+06	2.23e+02
111	4	-3.21	3.48	2.83e-01	5.26	70.60	4.79e-02	3.51e+06	2.18e+02
274	5	-1.41	9.37	2.82e+00	5.61	64.29	6.86e-02	2.33e+06	1.51e+02
2	6	15.44	3.30	6.98e-01	5.22	42.39	1.71e-01	7.69e+05	1.13e+02
249	7	-95.96	6.88	3.54e+00	7.97	59.93	1.99e-01	9.33e+05	7.36e+01
110	8	-1.04	3.75	3.49e-01	5.62	73.23	3.90e-02	3.43e+06	1.88e+02
72	9	102.25	5.59	1.55e+00	5.73	52.90	1.03e-01	1.24e+06	8.71e+01
191	10	-50.97	5.53	7.67e-01	4.45	57.79	3.80e-02	2.69e+06	2.34e+02

(table extracted from Table 2, 3 of Lee et al. (2016))

Note that the order of SFC no's appear shuffled because here they have adopted mass-weighted distances of matched GMCs for each SFC here. The physical radius R is defined as $d\tan(R_{\rm ang})$. The gas surface density $\Sigma_g = M_g/(d\tan(\pi R_{\rm max}R_{\rm min})^2$ while the virial parameter $\alpha_{vir} = 5{\sigma_v}^2R/GM_g$ and $\epsilon_{\rm ff,\ br}$ is as defined in the paper Lee et al. (2016)

Due to the heavy size of images, we are currently adding them in a separate doc, which you can find out from the right, it's named "graphs.tex", the page number of that doc would be set accordingly and after downloading the two pdfs., we would just merge them

5 Discussion and Conclusion

6 Future Scope

6.1 Introduction

The Paper talks about the development and the process of MHD and SFR in the terms of field lines. But the future scope is the secondary approach to understand the MHD just after the formation of the star, and recalculating the possible outcomes from the coronal mass ejection of the star. The Literature Reviews (such as Farhang et al. (2018) Vassiliadis et al. (1998) Inoue (2016)) in this context boldly claim about the tubular approach of solving flares and resolving the parameters that influences the MHD of the Stars During the studies of the MHD Equations, we thought of what other parameters might influence the MHD and among them the Flares and Solar Coronal effects tops the listing and although many terminal papers has been published individually on solar corona. But the factor of including MHD Influences to understand the models in this point of view is a scope of future development of this field.

6.2 Methodology

On Monitoring the behaviours of the SFR-β Graphs and Analysis of the previous approach models in order to solve this statement, it has been observed a vivid approach of understanding solar flares in terms of tubular flow and planar flow. Thus, One-Dimensional Streak Line Flow has been chosen as the elementary analysis of the fluid particular behaviour of the flares to contribute into Coronal Effects and ultimately resulting into MHD Contribution.

6.3 Declaration

The team holds a small part of progress on this approach and is open to access if claimed. The team eventually wants to meet the minute criteria to study the basics of the origin of MHD affected due to the turbulence caused by the interpretation of Coronal Effects and Solar flare. It is also evident that we are open to work with the other teams who claims to work on this field!

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