

# Assignment 2

## STAT40850 Bayesian Analysis (online)

2024/2025

AUTHOR

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## Instructions

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- Hand-in date: **Wednesday 5th March 2025 at 11:59pm.**
- You should submit it to the Assignment 2 object in Brightspace.
- You should submit a pdf file and corresponding source file (e.g., rmd or qmd file) containing your answers to the questions.
- You may submit it multiple times before the deadline, but only the last version will be marked.

## Dataset

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The **foodexp** dataset is a sample of **38 observations** gathered from a European rural community on **3 variables**:

- **food**: daily household **food expenditure in euro**;
- **income**: household **daily income in euro**;
- **persons**: number of **persons living in household**.

To load the dataset:

```
load(url("https://acaimo.github.io/teaching/data/foodexp.RData"))
```

We would like to understand the association of the household **income** with household **expenditure for food**.

## Questions

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1. **Define a Bayesian linear model** and **justify the selection of appropriate prior parameters** based on reasonable prior beliefs. Then, visualize at least 1,000 prior predictions of the regression line  $\mu = \alpha + \beta x$ .

[20]

2. Implement your Bayesian model using Stan and summarise the results obtained. Generate new data  $\tilde{y}$  from the posterior predictive distribution. Interpret on the results obtained.

[15]

3. Estimate, a priori and a posteriori, the 95% interval and the median of the expected daily household food expenditure for food ( $\mu$ ) given an household daily income of 50 euro ( $x = 50$ ) and interpret the results obtained.

[20]

4. Visualise the 80% posterior prediction interval for the estimated model.

[10]

5. Visualise the posterior predictive distribution  $p(\tilde{y} | y)$  for an household daily income corresponding to 72 euro ( $x = 72$ ).

[15]

6. Estimate the prior and posterior predictive probability of observing  $\tilde{y} > 25$  for an household daily income corresponding to 68 euro ( $x = 68$ ). Briefly compare the differences in the obtained probabilities.

[20]