In [2]:

```
import numpy as np
import matplotlib.pyplot as plt
import random

# Define true function
def f(x):
    return (1 + x) * np.sin(x) + np.cos(2 * x)

# Generate training data
x_vals = np.linspace(-3, 3, 1000)
y_vals = f(x_vals)
training_data = [(np.array([[x]]), np.array([[y]])) for x, y in zip(x_vals, y_vals)]

# Activation functions
def sigmoid(z):
    return 1.0 / (1.0 + np.exp(-z))

def sigmoid_prime(z):
    return sigmoid(z) * (1 - sigmoid(z))

def tanh(z):
    return np.tanh(z)

def tanh_prime(z):
    return 1 - np.tanh(z)**2

# Original Network Class
class Network:
    def __init__(self, sizes):
        self.num_layers = len(sizes)
        self.sizes = sizes
        self.biases = [np.random.randn(y, 1) for y in sizes[1:]]
        self.weights = [np.random.randn(y, x) for x, y in zip(sizes[:-1], sizes[1:])]

    def feedforward(self, a):
        for b, w in zip(self.biases, self.weights):
            a = sigmoid(np.dot(w, a) + b)
        return a

    def SGD(self, training_data, epochs, mini_batch_size, eta, test_data=None):
        training_data = list(training_data)
        n = len(training_data)
        for j in range(epochs):
            random.shuffle(training_data)
            mini_batches = [training_data[k:k + mini_batch_size] for k in range(0, n, mini_ba
            for mini_batch in mini_batches:
                self.update_mini_batch(mini_batch, eta)
            print(f"Epoch {j + 1} complete")

    def update_mini_batch(self, mini_batch, eta):
        nabla_b = [np.zeros(b.shape) for b in self.biases]
        nabla_w = [np.zeros(w.shape) for w in self.weights]
        for x, y in mini_batch:
            delta_nabla_b, delta_nabla_w = self.backprop(x, y)
            nabla_b = [nb + dnb for nb, dnb in zip(nabla_b, delta_nabla_b)]
            nabla_w = [nw + dnw for nw, dnw in zip(nabla_w, delta_nabla_w)]
        self.weights = [w - eta * nw / len(mini_batch) for w, nw in zip(self.weights, nabla_w
        self.biases = [b - eta * nb / len(mini_batch) for b, nb in zip(self.biases, nabla_b)]

    def backprop(self, x, y):
        nabla_b = [np.zeros(b.shape) for b in self.biases]
        nabla_w = [np.zeros(w.shape) for w in self.weights]
        activation = x
        activations = [x]
        zs = []
        for b, w in zip(self.biases, self.weights):
            z = np.dot(w, activation) + b
```

```python
            zs.append(z)
            activation = sigmoid(z)
            activations.append(activation)
        delta = self.cost_derivative(activations[-1], y) * sigmoid_prime(zs[-1])
        nabla_b[-1] = delta
        nabla_w[-1] = np.dot(delta, activations[-2].T)
        for l in range(2, self.num_layers):
            z = zs[-l]
            sp = sigmoid_prime(z)
            delta = np.dot(self.weights[-l + 1].T, delta) * sp
            nabla_b[-l] = delta
            nabla_w[-l] = np.dot(delta, activations[-l - 1].T)
        return nabla_b, nabla_w

    def cost_derivative(self, output_activations, y):
        return output_activations - y

# Train and plot for part (b)
net = Network([1, 20, 1])
net.SGD(training_data, epochs=300, mini_batch_size=10, eta=0.5)
predicted_vals = [net.feedforward(np.array([[x]])).flatten()[0] for x in x_vals]

import matplotlib.pyplot as plt
plt.figure(figsize=(10, 5))
plt.plot(x_vals, y_vals, label="True f(x)", linewidth=2)
plt.plot(x_vals, predicted_vals, '--', label="NN Output", linewidth=2)
plt.legend()
plt.grid(True)
plt.title("Neural Network Approximation of f(x)")
plt.xlabel("x")
plt.ylabel("f(x)")
plt.show()
```

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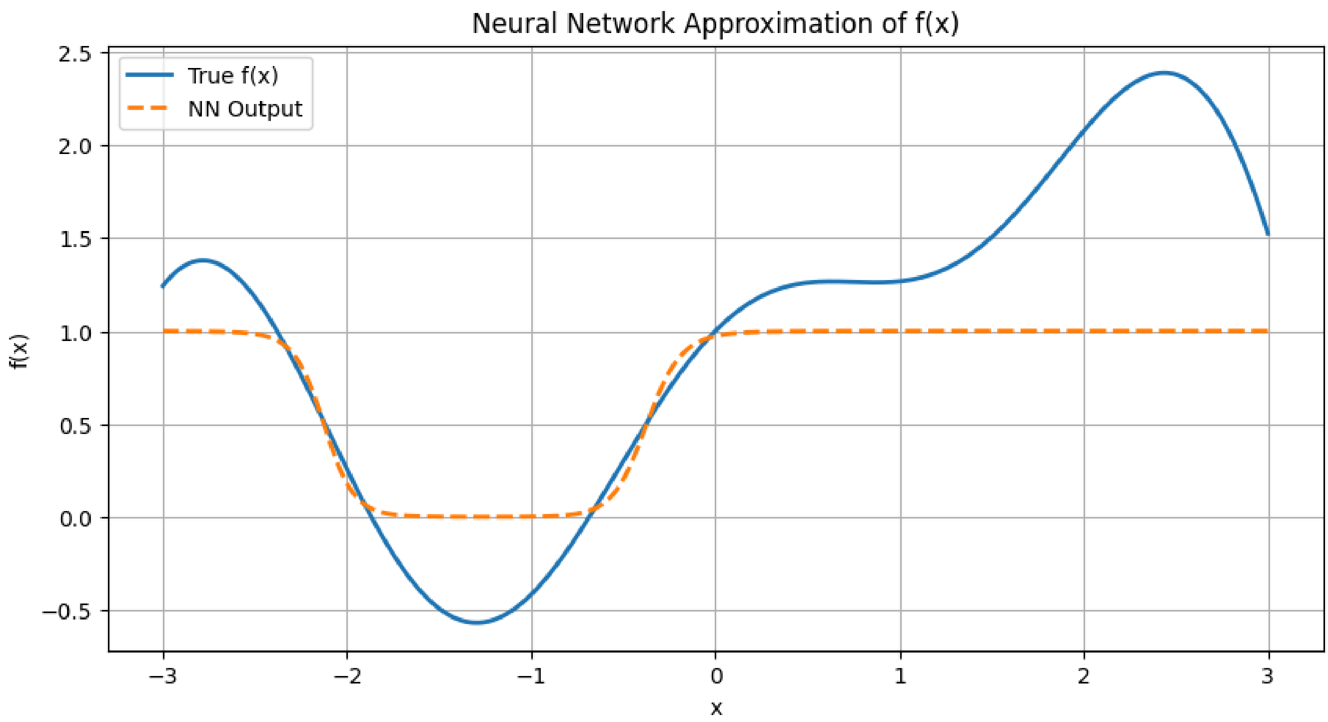

Architecture Details: Input Layer: 1 neuron — represents the single input x∈R. No bias or activation is applied to the input layer. Hidden Layer: 20 neurons Each neuron: Receives input from the single input neuron. Applies a sigmoid activation function to introduce non-linearity. Biases are used for each neuron in this layer. Output Layer: 1 neuron Collects input from all 20 neurons in the hidden layer. Applies the sigmoid activation function to produce the final output

(b) Training and Evaluation of the Neural Network

In [3]:
```
net = Network([1, 20, 1])
net.SGD(training_data, epochs=300, mini_batch_size=10, eta=0.5)
```

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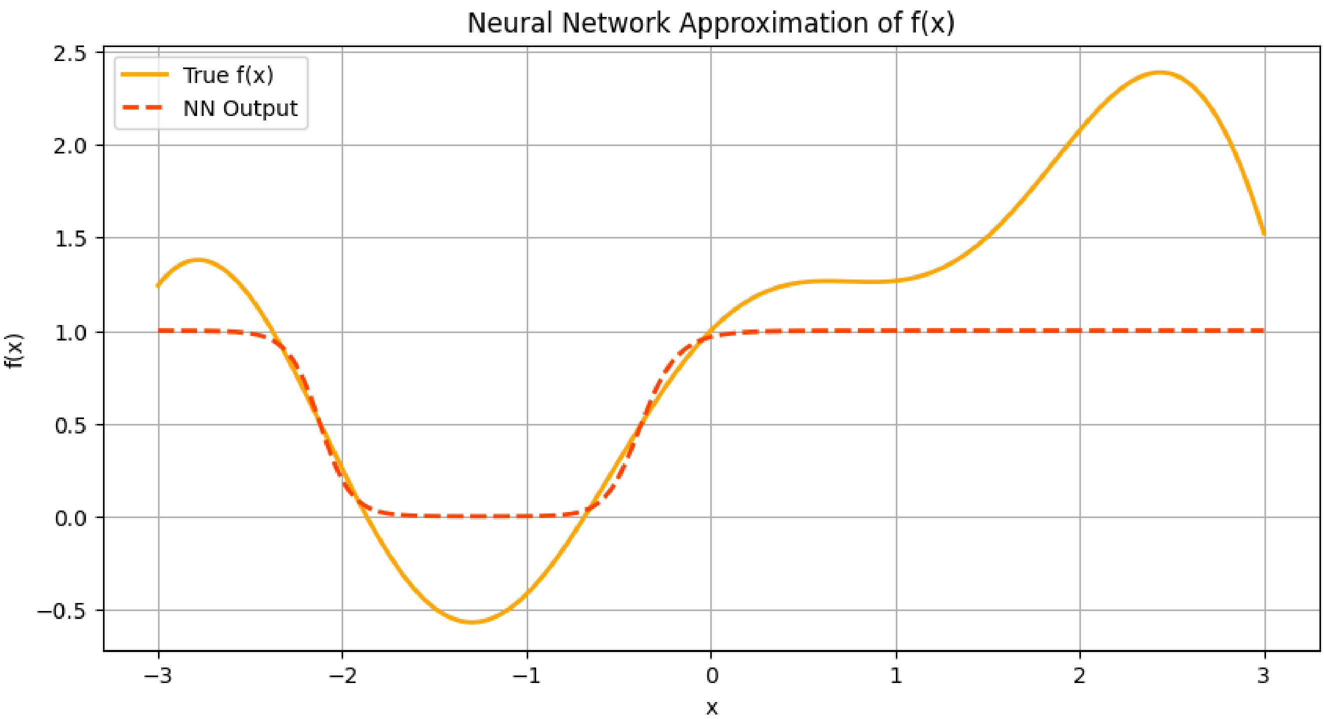
```

In [4]:

```
predicted_vals = [net.feedforward(np.array([[x]])).flatten()[0] for x in x_vals]

plt.figure(figsize=(10, 5))
plt.plot(x_vals, y_vals, label="True f(x)", linewidth=2, color='orange')
plt.plot(x_vals, predicted_vals, '--', label="NN Output", linewidth=2, color='orangered')
plt.legend()
plt.grid(True)
plt.title("Neural Network Approximation of f(x)")
plt.xlabel("x")
plt.ylabel("f(x)")
plt.show()
```



(c) Create a modified version of the Network class (call it ModNetwork) to obtain an architecture more suited to the approximation task you should perform. Clearly describe in at most a couple of sentences the modifications you per￾formed and why they should work

In [9]:

```
# Define tanh and its derivative for use in ModNetwork
def tanh(z):
    return np.tanh(z)
```

```python
def tanh_prime(z):
    return 1 - np.tanh(z) ** 2

# Modified version of Network class: ModNetwork
class ModNetwork:
    def __init__(self, sizes):
        self.num_layers = len(sizes)
        self.sizes = sizes
        self.biases = [np.random.randn(y, 1) for y in sizes[1:]]
        self.weights = [np.random.randn(y, x) for x, y in zip(sizes[:-1], sizes[1:])]

    def feedforward(self, a):
        for i, (b, w) in enumerate(zip(self.biases, self.weights)):
            z = np.dot(w, a) + b
            if i < self.num_layers - 2:  # Hidden layers: tanh
                a = tanh(z)
            else:  # Output layer: linear
                a = z
        return a

    def SGD(self, training_data, epochs, mini_batch_size, eta):
        training_data = list(training_data)
        n = len(training_data)
        for j in range(epochs):
            random.shuffle(training_data)
            mini_batches = [training_data[k:k + mini_batch_size] for k in range(0, n, mini_ba
            for mini_batch in mini_batches:
                self.update_mini_batch(mini_batch, eta)
            print(f"Epoch {j + 1} complete")

    def update_mini_batch(self, mini_batch, eta):
        nabla_b = [np.zeros(b.shape) for b in self.biases]
        nabla_w = [np.zeros(w.shape) for w in self.weights]
        for x, y in mini_batch:
            delta_nabla_b, delta_nabla_w = self.backprop(x, y)
            nabla_b = [nb + dnb for nb, dnb in zip(nabla_b, delta_nabla_b)]
            nabla_w = [nw + dnw for nw, dnw in zip(nabla_w, delta_nabla_w)]
        self.weights = [w - eta * nw / len(mini_batch) for w, nw in zip(self.weights, nabla_w
        self.biases = [b - eta * nb / len(mini_batch) for b, nb in zip(self.biases, nabla_b)]

    def backprop(self, x, y):
        nabla_b = [np.zeros(b.shape) for b in self.biases]
        nabla_w = [np.zeros(w.shape) for w in self.weights]
        activation = x
        activations = [x]
        zs = []
        for i, (b, w) in enumerate(zip(self.biases, self.weights)):
            z = np.dot(w, activation) + b
            zs.append(z)
            if i < self.num_layers - 2:
                activation = tanh(z)
            else:
                activation = z  # Linear output
            activations.append(activation)
        delta = (activations[-1] - y)  # MSE derivative for linear output
        nabla_b[-1] = delta
        nabla_w[-1] = np.dot(delta, activations[-2].T)
        for l in range(2, self.num_layers):
            z = zs[-l]
            sp = tanh_prime(z)
            delta = np.dot(self.weights[-l + 1].T, delta) * sp
            nabla_b[-l] = delta
            nabla_w[-l] = np.dot(delta, activations[-l - 1].T)
        return nabla_b, nabla_w
```

Modified Network: ModNetwork

In [10]:
```
modnet = ModNetwork([1, 40, 20, 1])  # or [1, 20, 1] for part (d)
modnet.SGD(training_data, epochs=300, mini_batch_size=10, eta=0.5)
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```
<ipython-input-9-652b2e16000e>:65: RuntimeWarning: invalid value encountered in multiply
  delta = np.dot(self.weights[-l + 1].T, delta) * sp
```

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(d) Create a new network using the new class as “modnet=ModNetwork([1,20,1])” and describe its architecture.

In [11]:

```python
# Create ModNetwork instance with architecture [1, 20, 1] as required in part (d)
modnet_d = ModNetwork([1, 20, 1])

# Train the model using the same training data and settings
modnet_d.SGD(training_data, epochs=300, mini_batch_size=10, eta=0.5)

# Evaluate the model output
predicted_mod_d = [modnet_d.feedforward(np.array([[x]])).flatten()[0] for x in x_vals]

# Plot the model output vs true function
plt.figure(figsize=(10, 5))
plt.plot(x_vals, y_vals, label="True f(x)", linewidth=2, color='orange')
plt.plot(x_vals, predicted_mod_d, '--', label="ModNetwork [1, 20, 1]", linewidth=2, color='bl
plt.legend()
plt.grid(True)
plt.title("ModNetwork [1, 20, 1] Approximation of f(x)")
plt.xlabel("x")
plt.ylabel("f(x)")
plt.show()
```

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<ipython-input-9-652b2e16000e>:65: RuntimeWarning: invalid value encountered in multiply
  delta = np.dot(self.weights[-l + 1].T, delta) * sp
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