

Unit-1 Advanced Data Structures

Hashing.

What is Hashing?

It is basically a method for storing and retrieving data from database in $O(1)$ time.

Terminologies of Hashing

- *Search Keys*: a key on the basis of which data is stored and can be retrieved.
- *Hash Table*: It is a data structure which stores the search keys and it provides a methodology to store data in a proper way and it is similar to an array.
- *Hash Functions*: There are different methods of **Hash Functions**, some of them are:
 1. $K \bmod 10$
 2. $K \bmod n$
 3. Mid Square Method
 4. Folding Method

Example:

Search Keys: (24, 52, 91, 67, 48, 83) *Hash Function*: $K \bmod 10$ *Hash Table*:

0	
1	91
2	52
3	83
4	24
5	
6	
7	67
8	48
9	

Data is inserted in the table by using the **Hash Function $K \bmod 10$** where K is the **Search Key**.

Collision Resolution Techniques

What is Collision? Collision is when 2 search keys are to be placed in the same **Hash Table Column** according to the **Hash Function** used is known as **Collision**.

Example:

Hash Function to be used is $K \bmod 6$ **Search Keys** are (24, 19, 32, 44)

Hash Table Created:

0	24
1	19
2	32
3	
4	
5	

Now, inserting the **Keys** in the Hash Table according to the **Hash Function** which is $K \bmod 6$. Therefore, $24 \bmod 6$, remainder = 0, $19 \bmod 6$, remainder = 1, $32 \bmod 6$, remainder = 2, $44 \bmod 6$, remainder = 2

Now, there is a **collision** so there are different methods to resolve collision like

- Chaining(Open Hashing)
- Open Addressing(Closed Hashing)
 1. Linear Probing
 2. Quadratic Probing
 3. Double Hashing

Chaining(Open Hashing): In Chaining we utilise some extra space by adding a **linked list** to the column which is under **collision** without fully utilising the given space.

Load Factor: It is the ratio of the number of search keys to the number of columns in the **Hash Table**.

$$\alpha = \frac{\text{Number of Search Keys}}{\text{Number of Columns}}$$

Where, α = Load Factor
 Number of Search Keys = Number of Search Keys
 Number of Columns = Number of Columns in the Hash Table

Advantages of Chaining:

- Deletion is easy to perform.
- Insertion is performed in $O(1)$ time.(constant time)

Disadvantages of Chaining:

- Searching worst case is $O(n)$ time.
- Extra space is required for the linked list.

Open Addressing(Closed Hashing): In this method we first utilise the space provided to us before using any extra space.

Linear Probing: In this method we use the **Hash Function** to find the **Hash Table Column** and if there is a **collision** then we move to the next column and check if it is empty or not, if it is empty then we insert the **Search Key** there otherwise we move to the next column and so on.

Formula for Linear Probing:

$$R(K, i) = [H(K) + i] \bmod n$$

Where, $R(K, i)$ = Hash Function $H(K)$ = Search Key i = Number of times we have moved to the next column also know as **prob number/collision number** n = Number of Columns in the Hash Table

Primary Clustering: It is a situation in which the **Search Keys** are inserted in the **Hash Table** in a way that they form a cluster.

Secondary Clustering: When two or more keys are competing for the same slot in the **Hash Table** then it is known as **Secondary Clustering**.

Advantages of Linear Probing:

- No Extra Space is required.

Disadvantages of Linear Probing:

- Search time is $O(n)$ time in worst case.
- Deletion is difficult to perform.
- Primary Clustering occurs.
- Secondary Clustering occurs.

Imp Question Hashing

Question: The keys **1, 3, 12, 4, 25, 6, 18, 20, 8** are inserted into empty hash table of length 10 using open addressing with hash function $h(i) = i^2 \bmod 10$ and linear probing. What is the resultant hash table and find the maximum probe value?

Answer: **$i = 1, 3, 12, 4, 25, 6, 18, 20, 8$**

Calculating the hash values: $1^2 \bmod 10 = 1$ $3^2 \bmod 10 = 9$ $12^2 \bmod 10 = 4$ $4^2 \bmod 10 = 6$ $25^2 \bmod 10 = 5$ $6^2 \bmod 10 = 6$ $18^2 \bmod 10 = 4$ $20^2 \bmod 10 = 0$ $8^2 \bmod 10 = 4$

$\therefore h(i) = 1, 9, 4, 6, 5, 6, 4, 0, 4$

Hash Table:

0	20
1	1
2	8
3	
4	12

5	25
6	4
7	6
8	18
9	3

The maximum probe value is 9 for the key 8.

Quadratic Probing: In this method we use the **Hash Function** to find the **Hash Table Column** and if there is a **collision** then we move to the next column and check if it is empty or not, if it is empty then we insert the **Search Key** there otherwise we move to the next column and so on but the difference is that we move to the next column by using the formula i^2 where i is the number of times we have moved to the next column.

Formula:

$$R(K, i) = [H(K) + i^2] \bmod n$$

Where, $R(K, i)$ = Hash Function $H(K)$ = Search Key i = Number of times we have moved to the next column also know as **prob number/collision number** n = Number of Columns in the Hash Table

Advantages of Quadratic Probing:

- No Extra Space is required.
- No Primary Clustering occurs.

Disadvantages of Quadratic Probing:

- Search time is $O(n)$ time in worst case.
- Secondary Clustering occurs.
- No gaurantee of finding an empty slot.

Double Hashing: In this method we use the **Hash Function** to find the **Hash Table Column** and if there is a **collision** then we move to the next column and check if it is empty or not, if it is empty then we insert the **Search Key** there otherwise we move to the next column and so on but the difference is that we move to the next column by using the formula $i * \text{Hash Function}$ where i is the number of times we have moved to the next column.

Formula:

$$R(K, i) = [H(K) + i * H'(K)] \bmod n$$

Where, $R(K, i)$ = Hash Function $H(K)$ = Search Key i = Number of times we have moved to the next column also know as **prob number/collision number** $H'(K)$ = Second Hash Function n = Number of Columns in the Hash Table

Advantages of Double Hashing:

- No extra space is required.

- No Primary Clustering occurs.
- No Secondary Clustering occurs.

Disadvantages of Double Hashing:

- Search time is $O(n)$ time in worst case.

Example:

$$H(k) = k \bmod 11 \quad H'(k) = 8 - (k \bmod 8) \quad [H(k) + i * H'(k)] \bmod 11$$

Keys = 20, 34, 45, 70, 56

Calculating hash values for the first hash function:

$$20 \bmod 11 = 9 \quad 34 \bmod 11 = 1 \quad 45 \bmod 11 = 1 \quad 70 \bmod 11 = 4 \quad 56 \bmod 11 = 1$$

Calculating hash values for the second hash function:

$$\begin{aligned} 8 - (20 \bmod 8) &= 8 - 4 = 4 & 8 - (34 \bmod 8) &= 8 - 2 = 6 & 8 - (45 \bmod 8) &= 8 - 5 = 3 & 8 - (70 \bmod 8) &= 8 - 6 = 2 & 8 - (56 \bmod 8) &= 8 - 0 = 8 \end{aligned}$$

Hash Table:

0	
1	34
2	
3	56
4	45
5	
6	70
7	
8	
9	20
10	