CHAPTER

08

Electromagnetic waves

We have learnt that an electric current produces magnetic field and a magnetic field changing with time gives rise to an electric field. This brought together the phenomena of electricity and magnetism into a coherent and unified theory. After this discovery, Maxwell in 1865, pointed out that a change in either electric or magnetic field with time produces the other field. From this Maxwell concluded that variation of electric and magnetic field vectors perpendicular to each other leads to the production of electromagnetic disturbances in space. These disturbances show properties of waves and can travel in space even without any material medium. These waves are called **electromagnetic waves**.

Maxwell also concluded that electromagnetic waves are transverse in nature and that light is an electromagnetic wave. In this chapter, we will study characteristics of electromagnetic waves and electromagnetic spectrum followed by the concept of displacement current and Maxwell's equations.

1) Displacement current

While applying the Ampere's circuital law to find magnetic field at a point outside a capacitor connected to a time varying current, Maxwell noticed an inconsistency in Ampere's law. He then suggested the existence of an additional current and called it **displacement current**.

"Displacement current is a current which is produced due to the rate of change of electric flux with respect to time". It is given by

$$\boxed{I_d = \varepsilon_0 \frac{d}{dt}(\phi_E)}$$

For the relation of displacement current, consider a parallel plate capacitor which is being charged as shown in the figure.

According to Maxwell, contradiction in Ampere's law is because of some missing term. This missing term must be such that one gets the same magnetic fields at point P, no matter what surface is used.

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This missing term must be related with changing electric field which passes through area A_1 between the plates of capacitor shown in the figure.

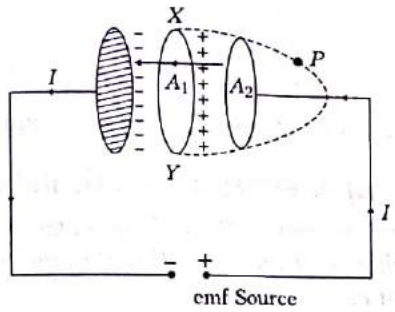


Fig. 8.1 A parallel plate capacitor with an emf source

If at an instant charge on the plates of capacitor is Q, area of plate is A then electric field between the plates is uniform, which is given by

$$E = \frac{Q}{A\varepsilon_0} \quad ...(i)$$

(This field is perpendicular to area A_1)

Electric field outside the plates is zero.

The electric flux through area A_1

$$\phi_E = E \times A_1 = \frac{Q}{A\varepsilon_0} \times A$$

($A = A_1$ because flux will pass through area A)

$$\therefore \quad \phi_E = \frac{Q}{\varepsilon_0} \quad ...(ii)$$

If charge on the capacitor plate changes with time then

$$I = \frac{dQ}{dt}$$

So, $$\frac{d\phi_E}{dt} = \frac{1}{\varepsilon_0}\frac{dQ}{dt} \Rightarrow \frac{d\phi_E}{dt} = \frac{1}{\varepsilon_0} \times \frac{dQ}{dt}$$

This implies that

$$\frac{dQ}{dt} = \varepsilon_0 \frac{d\phi_E}{dt} \Rightarrow I_d = \varepsilon_0 \frac{d\phi_E}{dt}$$

This current I_d (the missing term in the Ampere's law) passes through the surface A_1 and is known as Maxwell displacement current.

Hence, Ampere's circuital law $\left(\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I\right)$ was modified to,

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 (I_c + I_d)$$

$$\boxed{\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \left(I_c + \varepsilon_0 \frac{d\phi_E}{dt}\right)}$$

It is called **modified Ampere's circuital law** or **Ampere Maxwell's circuital law.**

where, I_c = conduction current,

$I_d = \varepsilon_0 \frac{d\phi_E}{dt}$ = displacement current and

ϕ_E = flux of the electric field through the area bounded by closed curve.

The following inferences can be drawn from the above discussion as given below

(i) The conduction and displacement currents are individually discontinuous but the currents together possess the property of continuity through any closed electric circuit.

(ii) The displacement current is precisely equal to the conduction current when both are present in different parts of the circuit.

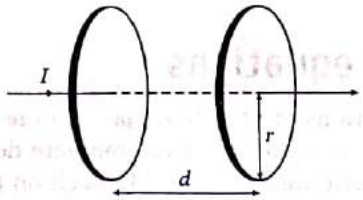
(iii) The displacement current arises due to rate time of change of electric flux (or electric field) between the two plates of the capacitor.

(iv) Just as the conduction current, the displacement current is also the source of varying magnetic field.

Example 8.1 *Figure shows a capacitor made of two circular plates each of radius 12 cm and separated by 5.0 cm. The capacitor is being charged by an external source (not shown in the figure). The charging current is constant and equal to 0.15A.*

(i) Obtain the displacement current across the plates.

(ii) Is Kirchhoff's first rule (junction rule) valid at each plate of the capacitor? Explain.



Sol. (i) As displacement current across the plates,

$$I_d = \frac{d}{dt}(\varepsilon_0 \phi_E) = \varepsilon_0 \frac{d\phi_E}{dt}$$

where, ϕ_E = electric flux across the loop $= \int_{loop} \mathbf{E} \cdot d\mathbf{S}$

$$\phi_E = EA = \frac{q}{\varepsilon_0}$$

$$\therefore \quad I_d = \varepsilon_0 \frac{d}{dt}\left(\frac{q}{\varepsilon_0}\right) = \frac{\varepsilon_0}{\varepsilon_0} \cdot \frac{dq}{dt} = I = 0.15 \text{ A}$$

i.e., displacement current is equal to conduction current.

(ii) Yes, Kirchhoff's first rule is valid at each plate of the capacitor. Net current is equal to the sum of the conduction current and the displacement current.

Example 8.2 *A parallel plate capacitor consists of two circular plates of radius $R = 0.1$ m. They are separated by a short distance. If electric field between the capacitor plates changes as $\frac{dE}{dt} = 6 \times 10^{13} \frac{V}{m \times s}$. Find displacement current between the plates.*

Sol. Given, area of plates, $A = \pi R^2 = 3.14 \times (0.1)^2 m^2$

and $\frac{dE}{dt} = 6 \times 10^{13} \frac{V}{m \times s}$

Displacement current between the plates,

$$I_d = \varepsilon_0 \frac{d\phi}{dt}$$

$$I_d = \varepsilon_0 A\left(\frac{dE}{dt}\right) \qquad \left[\because \frac{d\phi}{dt} = A\frac{dE}{dt}\right]$$

$$= 8.85 \times 10^{-12} \times 3.14 \times (0.1)^2 \times 6 \times 10^{13}$$

$$\Rightarrow \quad I_d = 16.67 \text{ A}$$

Example 8.3 *Consider a parallel plate capacitor is charged by a constant current I, with plate area A and separation between the plates d. Consider a plane surface of area A/4 parallel to the plates and drawn symmetrically between the plates. Find the displacement current through this area.*

Sol. Charge on the capacitor plates at time t is given by,

$$q = It$$

Electric field between the plates of the capacitor at this instant is,

$$E = \frac{q}{A\varepsilon_0} = \frac{It}{A\varepsilon_0}$$

So, electric flux through the given area,

$$\phi_E = \left(\frac{A}{4}\right) \cdot E = \frac{It}{4\varepsilon_0}$$

Therefore, displacement current,

$$I_d = \varepsilon_0 \frac{d\phi_E}{dt} = \varepsilon_0 \frac{d}{dt}\left(\frac{It}{4\varepsilon_0}\right) = \frac{I}{4}$$

Maxwell's equations

Maxwell's equations are the basic laws of electricity and magnetism. These equations give complete description of all electromagnetic interactions. Maxwell on the basis of his equations, predicted the existence of electromagnetic waves. These equations are as fundamental to electromagnetic phenomenon as Newton's law are for the study of mechanical phenomenon.

There are four Maxwell's equations which are explained as given below

1. Gauss's law of electrostatics

This law states that the total electric flux through any closed surface is always equal to $\frac{1}{\varepsilon_0}$ *times the net charge enclosed by that surface.* This law relates electric field with a charge distribution. It is given by

$$\boxed{\oint \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0}}$$

This equation is called **Maxwell's first equation.**

2. Gauss's law of magnetism

This law states that the net magnetic flux through any closed surface is always zero.

It is given by

$$\boxed{\oint \mathbf{B} \cdot d\mathbf{S} = 0}$$

This equation is called **Maxwell's second equation.**

3. Faraday's law of electromagnetic induction

The line integral of electric field along a closed path is equal to the rate of change of magnetic flux through the surface bounded by that closed path.

It is given by

$$\boxed{\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\phi_B}{dt}}$$

This equation is called **Maxwell's third equation.**

4. Ampere-Maxwell's circuital law

This law states that *the line integral of the magnetic field along a closed path is equal to* μ_0 *times the total current (i.e., sum of conduction current and displacement current) threading the surface bounded by that closed path.* It is given by

$$\boxed{\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0\left(I_c + \varepsilon_0 \frac{d\phi_E}{dt}\right)}$$

This equation is called **Maxwell's fourth equation.**

Example 8.4 *A parallel plate capacitor of area 40 cm^2 and plate separation 3.0 mm is charged initially to 80 μC. Due to radioactive source nearby, the medium between the plates gets slightly conducting and the plate loses charge initially at the rate of 2×10^{-7} Cs^{-1}. What is the magnitude and direction of displacement current? What is the magnetic field between the plates?*

Sol. Due to leaking, there is a flow of positive charge from the positive plate to the negative plate (or the flow of negative charge in the reverse direction). Thus the conduction current within the plates is from the positive plate to the negative plate. Now the displacement current is

$$I_D = \varepsilon_0 \frac{d\phi_E}{dt} = \varepsilon_0 A \frac{dE}{dt} = \varepsilon_0 A \frac{d}{dt}\left(\frac{q}{\varepsilon_0 A}\right) = \varepsilon_0 A \frac{1}{\varepsilon_0 A}\frac{dq}{dt}$$

or

$$I_D = \frac{dq}{dt} = 2 \times 10^{-7} \text{ Cs}^{-1}$$

The direction of displacement current is opposite to that of electric field E and hence opposite to the conduction current. But its magnitude is same as that of the conduction current. The net current between the plates is zero.

Using Ampere's law (with I_D there replaced by $I = I_C + I_D = 0$),

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I + \mu_0(I_C + I_D) = \mu_0 \times 0 = 0$$

So, the magnetic field within the plates is zero at all points.

Check point 8.1

1. According to Maxwell's hypothesis, a changing electric field gives rise to
   (a) an emf (b) electric current
   (c) magnetic field (d) pressure radiant
2. Maxwell's in his famous equation of electromagnetism introduced the concept of
   (a) AC current (b) DC current
   (c) displacement current (d) impedance
3. Dimensions of $\varepsilon_0 \frac{d\phi_E}{dt}$ are same as that of
   (a) charge (b) potential
   (c) capacitance (d) current
4. The law which states that the variations of electric field causes magnetic field is
   (a) Faraday's law
   (b) Biot-Savart law
   (c) modified Ampere's law
   (d) Lenz's law
5. The conduction current in ideal case through a circuit is zero when charge on capacitor is
   (a) zero
   (b) maximum
   (c) any transient value
   (d) depends on capacitor used
6. According a Ampere-Maxwell's circuital law,
   (a) $\oint \mathbf{E}\cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$ (b) $\oint \mathbf{E}\cdot d\mathbf{l} = -\frac{d\phi_E}{dt}$
   (c) $\oint \mathbf{B}\cdot d\mathbf{S} = 0$ (d) $\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0\left(I_c + \varepsilon_0 \frac{d\phi_E}{dt}\right)$
7. What is the displacement current between the square plate of side 1 cm of a capacitor, if electric field between the plates is changing at the rate of 3×10^6 $Vm^{-1}s^{-1}$?
   (a) 2.7×10^{-9} A (b) 3.2×10^{-5}A
   (c) 4.2×10^{-6}A (d) 4.0×10^{-5}A
8. A capacitor has been charged by a DC source. What are the magnitude of conduction and displacement current when it is fully charged?
   (a) $I_d = I_c = 1$ (b) $I_d = I_c = 0$
   (c) $I_d = 1, I_c = 0$ (d) $I_d = 0, I_c = 1$
9. A parallel plate capacitor of plate separation 2 mm is connected in an electric circuit having source voltage 400 V. What is the value of the displacement current for 10^{-6} s, if the plate area is 60 cm^2?
   (a) 1.062×10^{-2}A (b) 2.062×10^{-2}A
   (c) 3.062×10^{-2}A (d) 5.062×10^{-2}A
10. Maxwell's equations describe the fundamental laws of
   (a) Only electricity (b) Only magnetism
   (c) Only mechanics (d) Both (a) and (b)

2 Electromagnetic waves

Electromagnetic waves (or EM waves) are the waves which are composed of changing electric field **E** and magnetic field **B**, sinusoidally and propagating through space such that, the two fields are perpendicular to each other and perpendicular to the direction of wave propagation.

Sources of electromagnetic waves

These waves are produced by accelerating charged particles which can be related with following physical phenomena.

(i) A charge oscillating harmonically is a source of EM waves of same frequency.

A simple L-C oscillator and energy source as shown in figure can produce EM waves of desired frequency $\left(\nu = \frac{1}{2\pi\sqrt{LC}}\right)$.

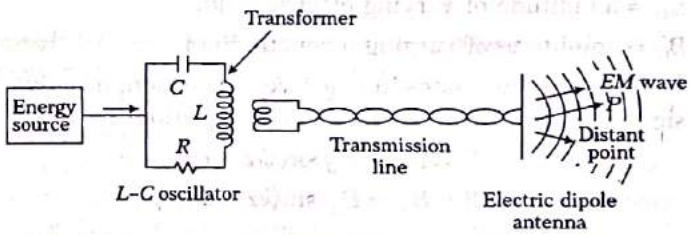


Fig. 8.2 An arrangement for generating electromagnetic wave

L-C oscillator produces a sinusoidal current in the antenna, which generates the wave. P is a distant point at which a detector can monitor the wave travelling past it.

(ii) Electromagnetic waves are produced when high speed electron enters into target of high atomic weight.

(iii) Electromagnetic waves are produced during de-excitation of nucleus in radioactivity.

(iv) An electron in a stationary orbit orbiting around its nucleus does not emit EM waves. But during transistion from higher energy orbit to lower energy orbit, it emits EM waves.

Graphical representation of propagation electromagnetic waves

During propagation of EM waves through a medium, electric field (**E**) and magnetic field **B** vary sinusoidally. Electric field and magnetic field become maximum at same position and time.

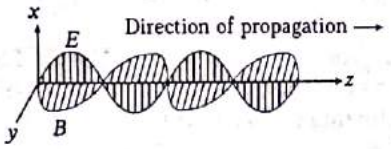


Fig. 8.3 A plane EM wave travelling along z-axis

Also **E** and **B** are perpendicular to each other as well as to the direction of propagation of the wave (**E** × **B**). In the given figure, electric field is along x-direction, magnetic field is along y-direction and the wave propagates along z-direction.

Therefore, $\mathbf{E}(x) \perp \mathbf{B}(y) \perp z$ (direction of propagation of the wave z).

Equations of electromagnetic waves

The EM wave propagating in the positive z-direction may be represented by following equation

Here, $E = E_x = E_0 \sin(kz - \omega t)$

$B = B_y = B_0 \sin(kz - \omega t)$

where, $k = 2\pi/\lambda$ (λ = wavelength),

$\omega = 2\pi\nu$ (ν = frequency),

E_0 = amplitude of varying electric field,

B_0 = amplitude of varying magnetic field

If the wave propagates in negative z-direction, negative sign is replaced by positive sign and equations become

$$E = E_x = E_0 \sin(kz + \omega t)$$

$$B = B_y = B_0 \sin(kz + \omega t)$$

Characteristic of electromagnetic waves

Characteristic of electromagnetic waves are given below

(i) These waves do not require any material medium for propagation.

(ii) These waves travel in free space with the speed of light (3.0×10^8 m/s).

(iii) Electromagnetic waves are transverse in nature.

(iv) The oscillating electric and magnetic field vectors of an EM wave are oriented along mutually perpendicular directions and are in phase.

(v) These waves do not deflected by electric field as well as magnetic field.

(vi) EM waves are polarised.

(vii) EM waves carry energy and momentum.

(viii) EM waves exert pressure called radiation pressure.

(ix) Electric field vector of EM waves is responsible for its optical effects.

Physical quantities associated with EM waves

Important physical quantities associated with EM waves are listed as below

(i) Speed

In free space it's speed, $c = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}} = \dfrac{E_0}{B_0} = 3 \times 10^8$ m/s

In medium, $v = \dfrac{1}{\sqrt{\mu\varepsilon}}$,

where, μ_0 = absolute permeability

ε_0 = absolute permittivity

μ = permeability of medium

ε = permittivity of medium

(ii) Energy

The energy in EM waves is divided equally between the electric and magnetic fields.

Energy density (energy per unit volume) of electric field, $u_e = \dfrac{1}{2}\varepsilon_0 E^2$, energy density of magnetic field, $u_m = \dfrac{1}{2}\dfrac{B^2}{\mu_0}$

The total energy per unit volume is

$$u = u_e + u_m = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\frac{B^2}{u_0} = \varepsilon_0 E^2 = \frac{B^2}{\mu_0}$$

Also, average energy density,

$$u_{av} = \frac{1}{4}\varepsilon_0 E_0^2 + \frac{1}{4}\frac{B_0^2}{\mu_0} = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{B_0^2}{2\mu_0}$$

(iii) Intensity

The energy crossing per unit area per unit time perpendicular to the direction of propagation of EM wave is called intensity of wave. *i.e.*,

$$I = \frac{\text{total EM energy } (u)}{\text{surface area}(A) \times \text{time}(t)}$$

$$= \frac{\text{total energy density} \times \text{volume}}{\text{surface area} \times \text{time}}$$

$$I = u_{av} \times c = \frac{1}{2}\varepsilon_0 E_0^2 c = \frac{1}{2}\frac{B_0^2}{\mu_0} \cdot c \text{ Wm}^{-2}$$

Note **Intensity due to a point source** Consider a point source S, emanating electromagnetic waves (light) in all directions.

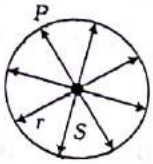


Fig. 8.4 A point source emitting light in all directions

If power of the source is P, then intensity of light at any point P is

$$I = \frac{u}{A \times t} = \frac{P}{A} = \left(\frac{P}{4\pi r^2}\right) \text{W/m}^2$$

In the same wave, intensity of a line source, $I \propto 1/r$ and for a plane source intensity is independent of r.

(iv) Momentum

EM waves also carry momentum. If a portion of EM wave of energy u propagating with speed c, then linear momentum $= \frac{\text{energy } (u)}{\text{speed } (c)}$.

If wave is incident on a completely absorbing surface, then momentum delivered $p = \frac{u}{c}$. If wave is incident on a totally reflecting surface, then momentum delivered $p = \frac{2u}{c}$.

(v) Poynting vector

The rate of flow of energy in an electromagnetic wave is described by the vector **S**, called the **poynting vector,** which is defined by the expression,

$$\mathbf{S} = \frac{1}{\mu_0}(\mathbf{E} \times \mathbf{B})$$

Its magnitude S is related to the rate at which energy is transported by a wave across a unit area at any instant.

Since, **E** and **B** are mutually perpendicular.

Hence, $|\mathbf{E} \times \mathbf{B}| = EB$

Thus, $S = \frac{EB}{\mu_0}$

The SI unit of poynting vector **S** is $\text{Js}^{-1}\text{-m}^2$ or Wm^{-2}.

Note The direction of the poynting vector **S** of an EM wave at any point gives the waves direction of travel and the direction of energy transport at that point.

(vi) Radiation pressure

It the momentum imparted per second per unit area, on which the light falls.

For a perfectly reflecting surface $P_r = \frac{2S}{c}$; S = poynting vector; c = speed of light

For a perfectly absorbing surface, $P_a = \frac{S}{c}$.

Example 8.4 *An L- C resonant circuit contains a 100 pF capacitor and a 400 μH inductor. It is set into oscillation coupled to an antenna. Find the frequency of the radiated electromagnetic waves.*

Sol. Given, $C = 100 \text{ pF} = 100 \times 10^{-12} \text{ F}$, $L = 400 \text{ μH}$

$$L = 400 \times 10^{-6} \text{ H}$$

Frequency of radiated EM wave is

$$\nu = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2 \times 3.14\sqrt{400 \times 10^{-6} \times 100 \times 10^{-12}}}$$

$$= 0.0796 \times 10^7 \text{Hz}$$

Example 8.5 *Electromagnetic waves travel in a medium with a speed of $2 \times 10^8 \text{ ms}^{-1}$. The relative magnetic permeability of the medium is 1. Find the relative electrical permittivity.*

Sol. Given, $v = 2 \times 10^8 \text{ms}^{-1}$ and $\mu_r = 1$

The speed of electromagnetic waves in a medium is given by

$$v = \frac{1}{\sqrt{\mu\varepsilon}} \quad \text{...(i)}$$

where, μ and ε are absolute permeability and absolute permittivity of the medium respectively.

Now, $\mu = \mu_0\mu_r$ and $\varepsilon = \varepsilon_0\varepsilon_r$

∴ Eq. (i) becomes, $v = \frac{1}{\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}} = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \times \frac{1}{\sqrt{\mu_r\varepsilon_r}}$

$$\Rightarrow \quad v = \frac{c}{\sqrt{\mu_r\varepsilon_r}} \qquad \left[\because c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}\right]$$

On squaring both sides, we get $\varepsilon_r = \frac{c^2}{v^2\mu_r} = \frac{(3 \times 10^8)^2}{(2 \times 10^8)^2 \times 1} = 2.25$

Example 8.6 *An electromagnetic wave of frequency 40 MHz travels in free space in the x-direction.*

(i) Determine the wavelength of the wave.

(ii) At some point and at some instant, the electric field has its maximum value of 750 NC^{-1} and is along the Y-axis. Calculate the magnitude and direction of the magnetic field at this position and time.

Sol (i) Wavelength of the wave, $\lambda = \frac{c}{\nu} = \frac{3.0 \times 10^8}{40 \times 10^6} = 7.5 \text{ m}$

(ii) Given, maximum value of electric field, $E_0 = 750 \text{ NC}^{-1}$

∴ Magnetic field, $B_0 = \frac{E_0}{c} = \frac{750}{3 \times 10^8} = 2.5 \times 10^{-6} \text{ T}$

Since, **E** and **B** are mutually perpendicular and they both are perpendicular to the propagation of wave. Thus, we concluded that magnetic field is in negative z-direction.

Example 8.7 *Find the amplitude of electric and magnetic fields in a parallel beam of light of intensity 4.0 Wm^{-2}.*

Sol. The intensity of plane electromagnetic wave is,

$$I = \frac{1}{2}\varepsilon_0 E_0^2 c$$

$\therefore$ Amplitude of electric field, $E_0 = \sqrt{\dfrac{2I}{\varepsilon_0 c}}$

$$= \sqrt{\frac{2\times 4}{8.86\times 10^{-12}\times 3\times 10^{8}}} = 54.87\ \text{NC}^{-1}$$

Further, amplitude of magnetic field,

$$B_0 = \frac{E_0}{c} = \frac{54.87}{3.0\times 10^{8}}\ \text{T} = 1.83\times 10^{-7}\ \text{T}$$

Example 8.8 *The magnetic field in a plane electromagnetic wave is given by*

$$B_y = (2\times 10^{-7}\ T)\ sin\ (0.5\times 10^{3} x + 1.5\times 10^{11}\ t)$$

(i) What is the wavelength and frequency of the wave?

(ii) Write an expression for the electric field.

Sol. (i) Comparing the given equation with the standard equation of electromagnetic wave,

$$B = B_0 \sin(kx + \omega t)$$

We have, $k = \dfrac{2\pi}{\lambda} = 0.5\times 10^{3}$

$\therefore$ $\lambda = \dfrac{2\pi}{0.5\times 10^{3}}\ \text{m} = 1.26\ \text{cm}$

$\omega = 2\pi\nu = 1.5\times 10^{11}$

$\therefore$ $\nu = \dfrac{1.5\times 10^{11}}{2\pi}\ \text{Hz} = 2.38\times 10^{10}\ \text{Hz}$

(ii) Given, $B_0 = 2\times 10^{-7}\ \text{T}$

$\therefore$ $E_0 = B_0 c = 2\times 10^{-7}\times 3\times 10^{8} = 60\ \text{NC}^{-1}$

The electric field is perpendicular to both, the magnetic field and the direction of propagation of wave. Thus,

$$E_z = (60\ \text{NC}^{-1})\sin(0.5\times 10^{3} x + 1.5\times 10^{11} t)$$

Example 8.9 *The magnetic field in a plane electromagnetic wave is given by*

$B = (200\ \mu T)\sin\left[(4.0\times 10^{15}\ s^{-1})\left(t - \dfrac{x}{c}\right)\right]$. *Find the average energy density corresponding to the electric field.*

Sol. Average energy density corresponding to electric and magnetic field is equal which is given by

$$u_{av} = \frac{1}{4}\varepsilon_0 E_0^2 = \frac{B_0^2}{4\mu_0}$$

Substituting the values,

$$u_{av} = \frac{(200\times 10^{-6})^2}{4\,(4\pi\times 10^{-7})} = 0.008\ \text{Jm}^{-3}$$

Example 8.10 *The electric field of an electromagnetic wave is given by*

$$E = (50\ NC^{-1})\ sin\ \omega\,(t - x/c)$$

Find the energy contained in a cylinder of cross-section $10\ cm^2$ *and length 50 cm along the X-axis.*

Sol. The average value of energy density (energy/volume) is given by

$$u_{av} = \frac{1}{2}\varepsilon_0 E_0^2$$

Total volume of the cylinder, $V = A\cdot l$

$\therefore$ Total energy contained in the cylinder,

$$u = (u_{av})(V) = \left(\frac{1}{2}\varepsilon_0 E_0^2\right)(Al)$$

Substituting the values, we have

$$u = \frac{1}{2}\times(8.86\times 10^{-12})(50)^2(10\times 10^{-4})(50\times 10^{-2})\ \text{J}$$

$$= 5.5\times 10^{-12}\ \text{J}$$

Example 8.11 *In a region of free space, the electric field at some instant of time is,* $E = (80\hat{\mathbf{i}} + 32\hat{\mathbf{j}} - 64\hat{\mathbf{k}})\ \text{Vm}^{-1}$ *and the magnetic field is* $B = (0.2\ \hat{\mathbf{i}} + 0.08\ \hat{\mathbf{j}} + 0.29\hat{\mathbf{k}})\ \mu T$.

(i) Show that these two fields are perpendicular to each other.

(ii) Determine the poynting vector for these fields.

Sol. (i) **E** will be perpendicular to **B**, if $\mathbf{E}\cdot\mathbf{B} = 0$

Now,

$$\mathbf{E}\cdot\mathbf{B} = (80\hat{\mathbf{i}} + 32\hat{\mathbf{j}} - 64\hat{\mathbf{k}})\cdot(0.2\hat{\mathbf{i}} + 0.08\hat{\mathbf{j}} + 0.29\hat{\mathbf{k}})$$

$$= (80\times 0.2 + 32\times 0.08 - 64\times 0.29) = 0$$

Thus, we can say that $\mathbf{E}\perp\mathbf{B}$.

(ii) The poynting vector, $\mathbf{S} = \dfrac{1}{\mu_0}(\mathbf{E}\times\mathbf{B})$

$$= \frac{1}{4\pi\times 10^{-7}}\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 80 & 32 & -64 \\ 0.2 & 0.08 & 0.29 \end{vmatrix}$$

$$= [11.52\hat{\mathbf{i}} - 28.8\hat{\mathbf{j}}]\times 10^{6}\ \text{Wm}^{-2}$$

Example 8.12

(i) Find the energy stored in a 90 cm length of a laser beam operating at 6 mW.

(ii) Find the amplitude of electric field in a parallel beam of light of intensity 17.7 W/m^2.

Sol. (i) The time taken by wave to move a distance 90 cm,

$$t = \frac{90\times 10^{-2}}{3\times 10^{8}} = 3\times 10^{-9}\ \text{s}$$

Energy contained in 90 cm length,

$$u = Pt = 6\times 10^{-3}\times 3\times 10^{-9} = 18\times 10^{-12}\ \text{J}$$

(ii) Intensity of light, $I = \dfrac{1}{2}\varepsilon_0 E_0^2 c$

$$\Rightarrow\ 17.7 = \frac{1}{2}(8.85\times 10^{-12})E_0^2\times 3\times 10^{8}$$

$$\Rightarrow\ E_0^2 = \frac{4}{3}\times 10^{4}$$

Therefore, the amplitude of the electric field in the parallel beam, $E_0 = \dfrac{2}{3}\times 10^{2}\ \text{V/m}$

Example 8.13 *Light with an energy flux of* $18\ Wcm^{-2}$ *falls on a non-reflecting surface at normal incidence. If the surface has an area of* $20\ cm^2$, *then find the average force exerted on the surface during a span of 30 min.*

Sol. Total energy falling on the surface,

$$u = (18\ \text{Wcm}^{-2})(30\times 60\ \text{s})\ (20\ \text{cm}^2) = 6.48\times 10^5\ \text{J}$$

Therefore, the total momentum delivered is,

$$p = \frac{u}{c} = \frac{6.48\times 10^5}{3.0\times 10^8}\ \text{kg ms}^{-1} = 2.16\times 10^{-3}\ \text{kgms}^{-1}$$

The average force exerted on the surface is,

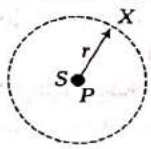
$$F = \frac{p}{t} = \frac{2.16\times 10^{-3}}{30\times 60}\quad [\because F\cdot t = \text{change in momentum}]$$

$$= 1.2\times 10^{-6}\ \text{N}$$

Note If the surface is a perfect reflector, then the change in momentum will be twice the above value, so the force will be twice or 2.4×10^{-6} N.

Example 8.14 *Evaluate the amplitude of electric and magnetic field produced by the radiation coming from a 20π W bulb at distance of 2m. Assume that the efficiency of the bulb 20% and it is a point source.*

Sol. Consider the situation shown below,



Intensity at distance r from a point source (bulb), $I = \dfrac{P}{4\pi r^2}$

Efficiency, $\eta = \dfrac{\text{Output}}{\text{Input}} = \dfrac{P}{P'}$

$$\Rightarrow \quad P = \eta P' = \left(\frac{20}{100}\right)(20\pi) = 4\pi\text{W}$$

$$I = \frac{P}{4\pi r^2} = \frac{4\pi}{4\pi(2)^2} = \frac{1\ \text{W}}{4\ \text{m}^2} = 0.25\ \text{W/m}^2$$

Also, intensity of EM wave is given by,

$$I = \frac{1}{2}\varepsilon_0 E_0^2 c$$

$$\Rightarrow \quad \frac{1}{4} = \frac{1}{2}\varepsilon_0 E_0^2 \times 3\times 10^8$$

$$\Rightarrow \quad E_0^2 = \frac{10^{-8}}{6\varepsilon_0} = \frac{10^{-8}}{6\times 8.85\times 10^{-12}} = \frac{10^4}{6\times 8.85}$$

$\therefore$ Amplitude of electric field, $E_0 = \dfrac{100}{\sqrt{6\times 8.85}} = 13.73\ \text{V/m}$

$\therefore$ Amplitude of magnetic field, $B_0 = \dfrac{E_0}{c} = \dfrac{13.73}{3\times 10^8} = 4.57\times 10^{-8}\ \text{T}$

3) Electromagnetic spectrum

The orderly arrangement of EM waves in increasing or decreasing order of wavelength λ or frequency ν is called **electromagnetic spectrum**. The range varies from 10^{-12} m to 10^4 m *i.e.*, from γ-rays to radiowaves as shown in the figure given below.

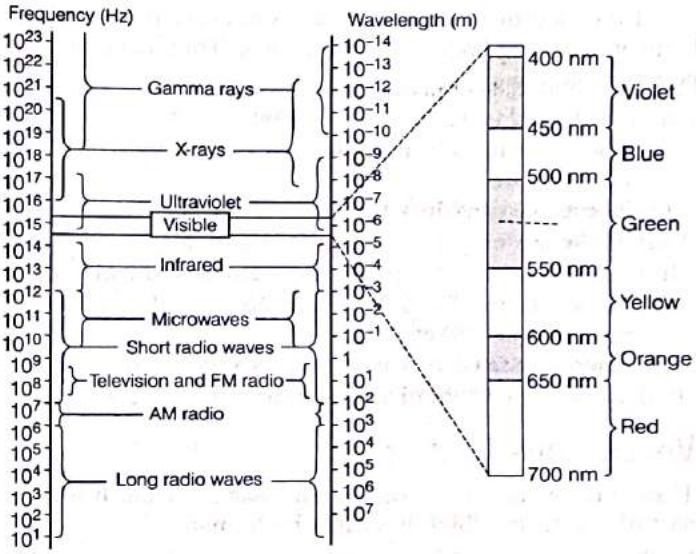


Fig. 8.4 Electromagnetic spectrum

The wavelength ranges, frequency ranges and uses of the various regions of electromagnetic spectrum are summarised below.

Radio waves

Radio waves were first predicted by James Clerk Maxwell. Radiowaves are produced due to oscillating charge particles between plates of capacitor in L-C circuit.

Uses of radiowaves are given below

(i) These are used in AM (Amplitude Modulation) from 530 kHz to 1710 kHz. These are also used in ground wave propagation.

(ii) These are used in TV waves ranging from 54 MHz to 890 MHz.

(iii) These are used in FM (Frequency Modulation) ranging from 88 MHz to 108 MHz.

(iv) UHF (Ultra High Frequency) waves are used in cellular phones.

Microwaves

These waves were also predicted by James Clerk Maxwell. Microwaves are called **short wavelength radio waves** which are produced by vacuum tubes.

Uses of microwaves are given below

(i) These are used in RADAR systems for aircraft navigation.

(ii) These are used in microwave oven for cooking purpose.

(iii) These are used in the study of atomic and molecular structures.

(iv) These are used to measure the speed of vehicle, speed of cricket ball, etc.

Infrared waves

These waves were discovered by Herschell. These waves are also called **heat waves**. These waves are produced by heat radiating bodies and molecules. They have high penetration power.

Uses of infrared waves are given below

(i) These are used in heat therapy to heat muscular pain or sprain.
(ii) These are used in satellite for army purpose.
(iii) These are used in weather forecasting.
(iv) These are used for producing dehydrated fruits.
(v) These are used in solar water heater, solar cells TV remote and cooker.
(vi) These are used in TV remote as signal carrier.
(vii) These are used in night vision or infrared photography.

Visible rays

These rays were discovered by Sir Issac Newton. It is that part of spectrum which is visible by human eye.

Visible rays are used by the optical organs of humans and animals for *three primary purposes given below*

(i) To see things, avoid bumping from them and escape danger.
(ii) To find stuff to eat.
(iii) To find other living things with which to consort, so as to prolong the species.

Ultraviolet rays

These rays were discovered by Ritter in 1801. UV-rays are produced by special lamps and very hot bodies. The sun is an important source of UV-rays but fortunately absorbed by ozone layer.

Uses of ultraviolet rays are given below

(i) These are used in burglar alarm.
(ii) These are used in checking mineral sample.
(iii) These are used to study molecular structure and in detection of invisible writing.
(iv) These are used to kill germs in minerals.
(v) These are used to sterilise surgical instruments.

X-rays

These rays were discovered by German professor Roentgen.

Uses of X-rays are given below

(i) These are used in surgery to detect the fracture, diseased organs, stones in the body, etc.
(ii) These are used in engineering to detect fault, crack on bridges, testing of welds.
(iii) These are used at metro station to detect metal or explosive material.
(iv) These are used in scientific research.

Gamma (γ) rays

γ-rays were discovered by Rutherford. They travel with the speed of light and having high penetration power.

Uses of gamma (γ) rays are given below

(i) These are used to produce nuclear reaction.
(ii) These are used in radio therapy for the treatment of tumour and cancer.
(iii) These are used in food industry to kill pathogenic microorganism.
(iv) These are used to provide valuable information about the structure of atomic nucleus.

Production and detection of different types of electromagnetic waves

Type	Wavelength range	Frequency range	Production	Detection
Radio wave	> 0.1 m	10^4 to 10^9 Hz	Rapid acceleration and decelerations of electrons in aerials	Receiver's aerials
Microwave	0.1 m to 1 mm	10^9 to 10^{11} Hz	Klystron valve or magnetron valve	Point contact diodes
Infrared wave	1 mm to 700 nm	10^{11} to 10^{14} Hz	Vibration of atoms and molecules	Thermopiles bolometer, infrared photographic film
Visible rays	700 nm to 400 nm	4×10^{14} to 8×10^{14} Hz	Electrons in atoms emit light when they move from higher energy level to a lower energy level	The eye, photocells, photographic film
Ultraviolet rays	400 nm to 1 nm	10^{14} to 10^{17} Hz	Inner shell electrons in atoms moving from higher energy level to a lower energy level	Photocells, photographic film
X-rays	1 nm to 10^{-3} nm	10^{16} to 10^{21}Hz	X-ray tubes or inner shell electrons	Photographic film, Geiger tubes, ionisation chamber
Gamma rays	$<10^{-3}$nm	10^{18} to 10^{22} Hz	Radioactive decay of the nucleus	Photographic film, Geiger tubes, ionisation chamber

Check point 8.2

1. A L-C resonant circuit contains a 200 pF capacitor and a 100 μH inductor. It is set into oscillation coupled to an antenna. The wavelength of the radiated electromagnetic waves is
   (a) 377 mm (b) 266 m
   (c) 377 cm (d) 3.77 cm

2. The velocity of electromagnetic wave is parallel to
   (a) $\mathbf{B} \times \mathbf{E}$ (b) $\mathbf{E} \times \mathbf{B}$
   (c) $\mathbf{E}$ (d) $\mathbf{B}$

3. An electromagnetic wave, going through vacuum is described by $E = E_0 \sin(kx - \omega t)$. Which of the following is independent of wavelength?
   (a) k (b) ω
   (c) k/ω (d) $k\omega$

4. Which of the following statements is incorrect about electromagnetic waves?
   (a) These are transverse in nature
   (b) These are produced by accelerating charges
   (c) They travel with the same speed in all media
   (d) They travel in free space with the speed of light

5. In electromagnetic wave, the phase difference between electric and magnetic field vectors **E** and **B** is
   (a) zero (b) $\pi/2$
   (c) π (d) $\pi/4$

6. The oscillating electric and magnetic field vectors of an electromagnetic wave are oriented along
   (a) the same direction but differ in phase by 90°
   (b) the same direction and are in phase
   (c) mutually perpendicular directions and are in phase
   (d) mutually perpendicular directions and differ in phase by 90°

7. Which of the following relation is correct?
   (a) $\sqrt{\varepsilon_0}E_0 = \sqrt{\mu_0}B_0$ (b) $\sqrt{\mu_0\varepsilon_0} = B_0/E_0$
   (c) $E_0 = \sqrt{\mu_0\varepsilon_0}B_0$ (d) $\sqrt{\mu_0}E_0 = \sqrt{\varepsilon_0}B_0$

8. In an apparatus, the electric field was found to oscillate with amplitude of 18 Vm^{-1}. The magnitude of the oscillating magnetic field will be
   (a) 4×10^{-6} T (b) 6×10^{-8} T
   (c) 9×10^{-9} T (d) 11×10^{-11} T

9. The rms value of the electric field of the light coming from the sun is 720 NC^{-1}. The average total energy density of the electromagnetic wave is
   (a) 6.37×10^{-9} J m^{-3} (b) 81.35×10^{-12} J m^{-3}
   (c) 3.5×10^{-3} J m^{-3} (d) 4.58×10^{-6} J m^{-3}

10. The electric field of a plane electromagnetic wave varies with time of amplitude 2 Vm^{-1} propagating along Z-axis. The average density of the magnetic field is (in J m^{-3})
    (a) 13.29×10^{-12} (b) 8.86×10^{-12}
    (c) 17.72×10^{-12} (d) 4.43×10^{-12}

11. Radiations of intensity 0.5 Wm^{-2} are striking a metal plate. The pressure on the plate is
    (a) 0.166×10^{-8} Nm^{-2} (b) 0.332×10^{-8} Nm^{-2}
    (c) 0.111×10^{-8} Nm^{-2} (d) 0.083×10^{-8} Nm^{-2}

12. In an electromagnetic wave, the electric and magnetising fields are 100 Vm^{-1} and 0.265 Am^{-1}. The maximum energy flow is
    (a) 26.5 Wm^{-2} (b) 36.5 Wm^{-2}
    (c) 46.7 Wm^{-2} (d) None of these

13. Which of the following waves have the maximum wavelength?
    (a) X-rays (b) IR-rays
    (c) UV-rays (d) Radio waves

14. Which of the following electromagnetic waves are used in weather forecasting?
    (a) Infrared waves (b) Radio waves
    (c) Microwaves (d) Visible rays

15. Electromagnetic radiation of highest frequency is
    (a) infrared radiations (b) visible radiation
    (c) radio waves (d) γ-rays

Chapter exercises

A Taking it together

(Assorted questions of the chapter for advanced level practice)

1. Wave which cannot travel in vacuum is
(a) X-rays (b) infrasonic
(c) ultraviolet (d) radio waves

2. Energy of electromagnetic waves is due to their
(a) wavelength
(b) frequency
(c) electric and magnetic fields
(d) None of the above

3. Which of the following represents an infrared wavelength?
(a) 10^{-4} cm (b) 10^{-5} cm
(c) 10^{-6} cm (d) 10^{-7} cm

4. Which of the following is of shortest wavelength?
(a) X-rays (b) γ-rays
(c) Microwaves (d) Radio waves

5. The frequency of visible light is of the order of
(a) 10^{14} Hz (b) 10^{10} Hz
(c) 10^{6} Hz (d) 10^{4} Hz

6. The structure of solids is investigated by using
(a) cosmic rays (b) X-rays
(c) γ-rays (d) infrared radiations

7. Energy stored in electromagnetic oscillations is in the form of
(a) electrical energy (b) magnetic energy
(c) Both (a) and (b) (d) None of these

8. The range of wavelength of the visible light is
(a) 10 Å to 100 Å
(b) 4000 Å to 8000 Å
(c) 8000 Å to 10000 Å
(d) 10000 Å to 15000 Å

9. Which of the following electromagnetic waves have the longest wavelength?
(a) Heat waves (b) Light waves
(c) Radio waves (d) Microwaves

10. Radio waves diffract around building although light waves do not. The reason is that radio waves
(a) travel with speed larger than c
(b) have longer wavelength than light
(c) carry news
(d) are not electromagnetic waves

11. Frequency of wave is 6×10^{15} Hz. The wave is
(a) radiowave (b) microwave
(c) X-ray (d) None of these

12. The frequency 1057 MHz of radiation arising from two close energy levels in hydrogen belongs to
(a) radio waves (b) infrared waves
(c) microwaves (d) γ-rays

13. The part of the spectrum of the electromagnetic radiation used to cook food is
(a) ultraviolet rays (b) cosmic rays
(c) X-rays (d) microwaves

14. Which of the following shows green house effect?
(a) Ultraviolet rays (b) Infrared rays
(c) X-rays (d) None of these

15. Which force in nature exits everywhere?
(a) Nuclear force (b) Electromagnetic force
(c) Weak force (d) Gravitation

16. Which of the following waves are used in RADAR systems for aircraft navigation?
(a) X-rays (b) Infrared rays
(c) Ultraviolet rays (d) Microwaves

17. In an electromagnetic wave, the average energy density associated with electric field is
(a) $CV^2/2$ (b) $Q^2/2C$ (c) $\varepsilon_0^2/2E_0$ (d) $\varepsilon_0 E^2/2$

18. In an electromagnetic wave, the average energy density associated with magnetic field is
(a) $Li_0^2/2$ (b) $B^2/2\mu_0$
(c) $\mu_0 B^2/2$ (d) $\mu_0/2B^2$

19. Total energy density of electromagnetic waves in vacuum is given by the relation

(a) $\frac{1}{2}\cdot\frac{E^2}{\varepsilon_0}+\frac{B^2}{2\mu_0}$ (b) $\frac{1}{2}\varepsilon_0 E^2+\frac{1}{2}\mu_0 B^2$

(c) $\frac{E^2+B^2}{c}$ (d) $\frac{1}{2}\varepsilon_0 E^2+\frac{B^2}{2\mu_0}$

20. In a plane electromagnetic wave, the electric field oscillates sinusoidally at a frequency of 2×10^{10} Hz and amplitude 48 Vm^{-1}. The wavelength of the wave is

(a) 24×10^{-10} m (b) 1.5×10^{-2} m

(c) 4.16×10^{8} m (d) 3×10^{8} m

21. Consider the following statements regarding a beam of LASER.

(I) It is highly monochromatic.

(II) It is an electromagnetic radiation from a source.

(III) It travels with the speed of light.

Which of the statements given above are correct?

(a) Both I and II

(b) Both II and III

(c) Both I and III

(d) I, II and III

22. For a medium with permittivity ε and permeability μ, the velocity of light is given by

(a) $\sqrt{\mu/\varepsilon}$ (b) $\sqrt{\mu\varepsilon}$

(c) $1/\sqrt{\mu\varepsilon}$ (d) $\sqrt{\varepsilon/\mu}$

23. The speed of electromagnetic wave in vacuum depends upon the source of radiation

(a) increases as we move from γ-rays to radio waves

(b) decreases as we move from γ-rays to radio waves

(c) is same for all of them

(d) None of the above

24. Velocity of electromagnetic waves in vacuum is given by

(a) $\sqrt{\mu_0\varepsilon_0}$ (b) $\sqrt{\frac{\mu_0}{\varepsilon_0}}$

(c) $\sqrt{\frac{\varepsilon_0}{\mu_0}}$ (d) $\frac{1}{\sqrt{\mu_0\varepsilon_0}}$

25. Which one of the following is the property of a monochromatic, plane electromagnetic waves in free space?

(a) Electric and magnetic fields have a phase difference of $\pi/2$

(b) The energy contribution of both electric and magnetic fields are equal

(c) The direction of propagation is in the direction of $\mathbf{B}\times\mathbf{E}$

(d) The pressure exerted by the wave is the product of its speed and energy density

(e) The speed of the wave is B/E

26. The essential distinction between X-rays and γ-rays is that

(a) γ-rays have smaller wavelength than X-rays

(b) γ-rays emanate from nucleus while X-rays emanate from outer part of the atom

(c) γ-rays have greater ionising power than X-rays

(d) γ-rays are more penetrating than X-rays

27. An EM wave radiates outwards from a dipole antenna, with E_0 as the amplitude of its electric field vector. The electric field E_0 which transports significant energy from the source falls off as **[NCERT Exemplar]**

(a) $\frac{1}{r^3}$ (b) $\frac{1}{r^2}$

(c) $\frac{1}{r}$ (d) remains constant

28. The sun delivers 10^4 Wm^{-2} of electromagnetic flux to the earth's surface. The total power that is incident on a roof of dimensions $(10\times 10)\,m^2$ will be

(a) 10^4 W (b) 10^5 W

(c) 10^6 W (d) 10^7 W

29. If **E** and **B** represent electric and magnetic field vectors of the electromagnetic wave, then the direction of propagation of electromagnetic wave is along **[NCERT Exemplar]**

(a) **E** (b) **B** (c) $\mathbf{B}\times\mathbf{E}$ (d) $\mathbf{E}\times\mathbf{B}$

30. If ε_0 and μ_0 are respectively, the electric permittivity and the magnetic permeability of free space, ε and μ the corresponding quantities in a medium, the refractive index of the medium is

(a) $\sqrt{\frac{\mu\varepsilon}{\mu_0\varepsilon_0}}$ (b) $\frac{\mu\varepsilon}{\mu_0\varepsilon_0}$

(c) $\sqrt{\frac{\mu_0\varepsilon_0}{\mu\varepsilon}}$ (d) $\sqrt{\frac{\mu\mu_0}{\varepsilon\varepsilon_0}}$

31. Which is the correct expression of velocity of light?

(a) $\frac{1}{\sqrt{\varepsilon_0\mu_0}}$ (b) $\frac{E_0}{B_0}$

(c) $\frac{c}{\mu}$ (d) All of these

32. Which of the following statement is false for the properties of electromagnetic waves?

(a) Both electric and magnetic field vectors attain the maxima and minima at the same place and same time

(b) The energy in electromagnetic wave is divided equally between electric and magnetic vectors

(c) Both electric and magnetic field vectors are parallel to each other and perpendicular to the direction of propagation of wave

(d) These waves do not require any material medium for propagation

33. The average electric field of electromagnetic waves in certain region of free space is 9×10^{-4} NC^{-1}. Then, the average magnetic field in the same region is of the order of

(a) 27×10^{-4} T (b) 3×10^{-12} T
(c) $\left(\frac{1}{3}\right)\times10^{-12}$ T (d) 3×10^{12} T

34. The electric and the magnetic field, associated with an electromagnetic wave, propagating along the Z-axis, can be represented by

(a) $[E = E_0\,\hat{\mathbf{k}}, B = B_0\hat{\mathbf{i}}]$ (b) $[E = E_0\,\hat{\mathbf{j}}, B = B_0\,\hat{\mathbf{j}}]$
(c) $[E = E_0\,\hat{\mathbf{j}}, B = B_0\,\hat{\mathbf{k}}]$ (d) $[E = E_0\,\hat{\mathbf{i}}, B = B_0\,\hat{\mathbf{j}}]$

35. The magnetic field amplitude of an electromagnetic wave is 2×10^{-7}T. Its electric field amplitude, if the wave is travelling in free space is

(a) 6 V/m (b) 60 V/m
(c) 10/6 V/m (d) None of these

36. A parallel plate capacitor is charged to 60μC. Due to a radioactive source, the plate losses charge at the rate of 1.8×10^{-8} Cs^{-1}. The magnitude of displacement current is

(a) 1.8×10^{-8}Cs^{-1} (b) 3.6×10^{-8}Cs^{-1}
(c) 4.1×10^{-11}Cs^{-1} (d) 5.7×10^{-12}Cs^{-1}

37. The pressure exerted by an electromagnetic wave of intensity I (Wm^{-2}) on a non-reflecting surface is (c is the velocity of light)

(a) Ic (b) Ic^2
(c) I/c (d) I/c^2

38. An L-C resonant circuit contains a 200 pF capacitor and a 200 μH inductor. It is set into oscillation coupled to an antenna. The wavelength of the radiated electromagnetic waves is

(a) 377mm (b) 377 m
(c) 377 cm (d) 3.77 cm

39. Radiation of intensity 1W/m^2 are striking on a metal plate. The pressure on the plate is

(a) 0.66×10^{-8} N/m^2 (b) 0.25 N/m^2
(c) 2 N/m^2 (d) 4 N/m^2

40. A linearly polarised electromagnetic wave given as $\mathbf{E} = E_0\hat{\mathbf{i}}\cos\,(kz-\omega t)$ is incident normally on a perfectly reflecting infinite wall at $z = a$. Assuming that the material of the wall is optically inactive, the reflected wave will be given as **[NCERT Exemplar]**

(a) $\mathbf{E}_r = E_0\hat{\mathbf{i}}(kz-\omega t)$
(b) $\mathbf{E}_r = E_0\hat{\mathbf{i}}\cos(kz+\omega t)$
(c) $\mathbf{E}_r = -E_0\hat{\mathbf{i}}\cos(kz+\omega t)$
(d) $\mathbf{E}_r = E_0\hat{\mathbf{i}}\sin(kz-\omega t)$

41. The magnetic field in a plane electromagnetic wave is given by $B_y = 2\times10^{-7}\sin(0.5\times10^3 x + 1.5\times10^{11}t)$.

This electromagnetic wave is

(a) a visible light (b) an infrared wave
(c) a microwave (d) a radio wave

42. An electromagnetic wave going through vacuum is described by $E = E_0\sin(kx-\omega t)$, $B = B_0\sin(kx-\omega t)$. Which of the following equations is true?

(a) $E_0\,k = B_0\,\omega$ (b) $E_0\,\omega = B_0\,k$
(c) $E_0\,B_0 = \omega k$ (d) None of these

43. A plane electromagnetic wave propagating in the x-direction has wavelength of 6.0 mm. The electric field is in the y-direction and its maximum magnitude of 33 Vm^{-1}. The equation for the electric field as function of x and t is

(a) $11\sin\pi(t-x/c)$ (b) $33\sin\pi\times10^{11}(t-x/c)$
(c) $33\sin\pi(t-x/c)$ (d) $11\sin\pi\times10^{11}(t-x/c)$

44. An electromagnetic wave travels in vacuum along z-direction with $\mathbf{E} = (E_1\hat{\mathbf{i}} + E_2\hat{\mathbf{j}})\cos(kz-\omega t)$. Choose the correct options from the following **[NCERT Exemplar]**

(a) The associated magnetic field is given as
$$\mathbf{B} = \frac{1}{c}(E_1\hat{\mathbf{i}} - E_2\hat{\mathbf{j}})\cos(kz-\omega t)$$
(b) The associated magnetic field is given as
$$\mathbf{B} = \frac{1}{c}(E_1\hat{\mathbf{i}} - E_2\hat{\mathbf{j}})\cos(kz-\omega t)$$
(c) The given electromagnetic field is circularly polarised
(d) The given electromagnetic wave is plane polarised

45. A plane electromagnetic wave is propagating along the z-direction. If the electric field component of this wave is in the direction $(\hat{\mathbf{i}}+\hat{\mathbf{j}})$, then which of the following is the direction of the magnetic field component.

(a) $(-\hat{\mathbf{i}}+\hat{\mathbf{j}})$ (b) $(\hat{\mathbf{i}}-\hat{\mathbf{j}})$ (c) $(-\hat{\mathbf{i}}-\hat{\mathbf{j}})$ (d) $(\hat{\mathbf{i}}+\hat{\mathbf{j}})$

46. The ratio of contributions made by the electric field and magnetic field components to the intensity of an EM wave is **[NCERT Exemplar]**

(a) $c:1$ (b) $c^2:1$ (c) $1:1$ (d) $\sqrt{c}:1$

47. A perfectly reflecting mirror has an area of 1 cm^2. Light energy is allowed to fall on it for 1 h at the rate of 10 W cm^{-2}. The force that acts on the mirror is

(a) 3.35×10^{-8}N (b) 6.7×10^{-8}N
(c) 1.34×10^{-7}N (d) 2.4×10^{-4}N

48. One requires 11 eV of energy to dissociate a carbon monoxide molecule into carbon and oxygen atoms. The minimum frequency of the appropriate electromagnetic radiation to achieve the dissociation lies in **[NCERT Exemplar]**

(a) visible region (b) infrared region
(c) ultraviolet region (d) microwave region

49. The electric field intensity produced by the radiations coming from 100 W bulb at a 3 m distance is E. The electric field intensity produced by the radiations coming from 50 W bulb at the same distance is **[NCERT Exemplar]**

(a) $\frac{E}{2}$ (b) $2E$ (c) $\frac{E}{\sqrt{2}}$ (d) $\sqrt{2}E$

50. Light with an energy flux of 20 W/cm^2 falls on a non-reflecting surface at normal incidence. If the surface has an area of 30 cm^2, then the total momentum delivered (for complete absorption) during 30 min is **[NCERT Exemplar]**

(a) 36×10^{-5} kg-m/s (b) 36×10^{-4} kg-m/s
(c) 108×10^{4} kg-m/s (d) 1.08×10^{7} kg-m/s

51. Magnitude of the electric and magnetic fields in an electromagnetic wave radiated by a 200 W bulb at a distance 2 m from it is assuming efficiency of bulb is 5% and it behaves like a point source.

(a) 12.27 N/C, 4.09×10^{-8} T (b) 10 N/C, 10^{-5} T
(c) 11.7 N/C, 5×10^{-6} T (d) 5 N/C, 10^{-4} T

B) Medical entrance special format questions

Assertion and reason

Directions (Q. Nos. 1-5) *These questions consist of two statements each linked as Assertion and Reason. While answering these questions you are required to choose any one of the following five responses :*

(a) If both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are true but Reason is not correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If Assertion is false but Reason is true.
(e) If both Assertion and Reason are false.

1. **Assertion** Microwaves are used in microwaves oven for cooking purpose.

Reason The frequency of microwaves is from 10^{14} to 10^{17} Hz.

2. **Assertion** In electromagnetic wave, the direction of variations of electric and magnetic field are perpendicular to each other and also perpendicular to the direction of wave propagation.

Reason Electromagnetic waves are transverse in nature.

3. **Assertion** Ultraviolet radiations of higher frequency wave are dangerous to human being.

Reason Ultraviolet radiation are absorbed by the atmosphere.

4. **Assertion** The frequency of the electromagnetic wave naturally equals the frequency of oscillation of the charge.

Reason The energy associated with the propagating wave comes at the expense of the energy of the source.

5. **Assertion** Like light radiation, thermal radiation are also electromagnetic radiation.

Reason The thermal radiations require no medium for propagation.

Match the columns

1. Match Column I with Column II and select the correct option from the choices given below the columns.

	Column I		Column II
(A)	$\oint_S \mathbf{E}\cdot d\mathbf{S} = q/\varepsilon_0$	(p)	Faraday's law of EMI
(B)	$\oint_S \mathbf{B}\cdot d\mathbf{S} = 0$	(q)	Gauss's law in magnetism
(C)	$\oint \mathbf{B}\cdot d\mathbf{l} = -\frac{d\phi_B}{dt}$	(r)	Maxwell-Ampere's circuital law
(D)	$\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0\left[I_c + \varepsilon_0 \frac{d\phi}{dt}\right]$	(s)	Gauss's law in electrostatics

2. Match Column I (electromagnetic wave type) with Column II (its application) and select the correct option from the choices given below the columns.

	Column I		Column II
(A)	Infrared waves	(p)	to treat muscular strain
(B)	Radio waves	(q)	for broadcasting
(C)	X-rays	(r)	to detect of fracture of bones
(D)	Ultraviolet	(s)	absorbed by the ozone layer of the atmosphere

C. Medical entrances' gallery

(Collection of questions asked in NEET & various medical entrance exams)

1. Out of the following options which one can be used to produce a propagating electromagnetic wave? **[NEET 2016]**
(a) A stationary charge
(b) A chargeless particle
(c) An accelerating charge
(d) A charge moving at constant velocity

2. The frequencies of X-rays, γ-rays and ultraviolet rays are respectively p, q and r, then **[Guj. CET 2015]**
(a) $p>q, q>r$ (b) $p<q, q>r$ (c) $p<q, q<r$ (d) $p>q, q<r$

3. The electric vector vibration of an electromagnetic wave is given by $E = (50\ \text{NC}^{-1}) \sin \omega\left(t - \frac{x}{c}\right)$.
The intensity of the wave is **[UP CPMT 2015]**
(a) 2.3 Wm^{-2} (b) 4.3 Wm^{-2}
(c) 3.3 Wm^{-2} (d) 1.8 Wm^{-2}

4. Which radiations are used in treatment of muscles ache? **[Manipal 2015]**
(a) Ultraviolet (b) Infrared
(c) Microwave (d) X-rays

5. In case of linearly polarised light, the magnitude of the electric field vector **[Manipal 2015]**
(a) does not change with time
(b) varies periodically with time
(c) is parallel to the direction of propagation
(d) increases and decreases linearly with time

6. A leaf which contains only green pigments is illuminated by a laser light of wavelength 0.6328 μm. It would appear to **[Manipal 2015]**
(a) black (b) green (c) brown (d) red

7. The ratio of amplitude of magnetic field to the amplitude of electric field for an electromagnetic wave propagating in vacuum is equal to **[CBSE AIPMT 2014]**
(a) the speed of light in vacuum
(b) reciprocal of speed of light in vacuum
(c) the ratio of magnetic permeability to the electric susceptibility of vacuum
(d) unity

8. In electromagnetic wave, according to Maxwell, changing electric field gives **[MHT CET 2014]**
(a) stationary magnetic field (b) conduction current
(c) eddy current (d) displacement current

9. The speed of light in an isotropic medium depends on **[Kerala CEE 2014]**
(a) the nature of the source
(b) its direction of propagation
(c) its intensity
(d) None of the above

10. The energy of gamma (γ) ray photon is E_γ and that of an X-ray photon is E_X. If the visible light photon has an energy of E_v, then we can say that **[WB JEE 2014]**
(a) $E_X > E_\gamma > E_v$ (b) $E_\gamma > E_v > E_X$
(c) $E_\gamma > E_X > E_v$ (d) $E_X > E_v > E_\gamma$

11. The speed of electromagnetic waves in vacuum is equal to **[UK PMT 2014]**
(a) $\mu_0\varepsilon_0$ (b) $\sqrt{\mu_0\varepsilon_0}$
(c) $\frac{1}{\sqrt{\mu_0\varepsilon_0}}$ (d) $\frac{1}{\mu_0\varepsilon_0}$

12. A plane electromagnetic wave of frequency 20 MHz travels through a space along x-direction. If the electric field vector at a certain point in space is 6 Vm^{-1}, then what is the magnetic field vector at that point? **[KCET 2014]**
(a) 2×10^{-8}T (b) $\frac{1}{2} \times 10^{-8}$T
(c) 2 T (d) $\frac{1}{2}$ T

13. In electromagnetic spectrum, the frequencies of γ-rays and X-rays and ultraviolet rays are denoted by n_1, n_2 and n_3 respectively, then **[MHT CET 2014]**
(a) $n_1 > n_2 > n_3$ (b) $n_1 < n_2 < n_3$
(c) $n_1 > n_2 < n_3$ (d) $n_1 < n_2 > n_3$

14. The electromagnetic waves detected using a thermopile and used in physical therapy are **[Kerala CEE 2014]**
(a) gamma radiations (b) X-rays
(c) ultraviolet radiations (d) infrared radiations
(e) microwave radiations

15. The wavelength of X-rays is in the range **[Guj. CET 2014]**
(a) 0.01 Å to 1 Å (b) 0.001 nm to 1 nm
(c) 0.001 μm to 1 μm (d) 0.001 cm to 1 cm

16. The wavelength of the short radio waves, microwaves, ultraviolet waves are λ_1, λ_2 and λ_3, respectively. Arrange them in decreasing order. **[Guj. CET 2014]**
(a) $\lambda_1, \lambda_2, \lambda_3$ (b) $\lambda_1, \lambda_3, \lambda_2$
(c) $\lambda_3, \lambda_2, \lambda_1$ (d) $\lambda_2, \lambda_1, \lambda_3$

17. The transverse nature of electromagnetic waves is proved by which of the following? **[J & K CET 2013]**
(a) Interference phenomena (b) Diffraction phenomena
(c) Dispersion phenomena (d) Polarisation phenomena

18. Which component of electromagnetic spectrum have maximum wavelength? **[J & K CET 2013]**
(a) Radio waves (b) Visible spectrum
(c) Gamma rays (d) X-rays

19. The condition under which a microwave oven heats up a food item containing water molecules most efficiently is [NEET 2013]
(a) the frequency of the microwave must match the resonant frequency of the water molecules
(b) the frequency of the microwave has no relation with natural frequency of water molecules
(c) microwave are heat waves, so always produce heating
(d) infrared waves produce heating in a microwave oven

20. If c is the velocity of light in free space, the time taken by light to travel a distance x in a medium refractive index μ is [MP PMT 2013]
(a) $\frac{x}{c}$ (b) $\frac{\mu x}{c}$ (c) $\frac{x}{\mu c}$ (d) $\frac{c}{\mu x}$

21. The electric field associated with an electromagnetic wave in vacuum is given by $\mathbf{E} = \hat{\mathbf{i}}\, 40 \cos(kz - 6 \times 10^8 t)$, where E, z and t are in volt m^{-1}, metre and second, respectively. The value of wave vector k is [CBSE-AIPMT (Screening) 2012]
(a) 2 m^{-1} (b) 0.5 m^{-1}
(c) 6 m^{-1} (d) 3 m^{-1}

22. The wave function (in SI unit) for a light wave is given as $\psi(x,t) = 10^3 \sin \pi(3 \times 10^6 x - 9 \times 10^{14} t)$. The frequency of the wave is equal to [AMU 2012]
(a) 4.5×10^{14} Hz (b) 3.5×10^{14} Hz
(c) 3.5×10^{10} Hz (d) 2.5×10^{10} Hz

Answers

Check point 8.1

1. (c) 2. (c) 3. (d) 4. (c) 5. (b) 6. (d) 7. (a) 8. (b) 9. (a) 10. (d)

Check point 8.2

1. (b) 2. (b) 3. (c) 4. (c) 5. (a) 6. (c) 7. (b) 8. (b) 9. (d) 10. (b)
11. (b) 12. (a) 13. (d) 14. (a) 15. (d)

A. Taking it together

1. (b) 2. (c) 3. (a) 4. (b) 5. (a) 6. (b) 7. (c) 8. (b) 9. (c) 10. (b)
11. (d) 12. (a) 13. (d) 14. (b) 15. (b) 16. (d) 17. (d) 18. (b) 19. (d) 20. (b)
21. (d) 22. (c) 23. (c) 24. (d) 25. (b) 26. (b) 27. (c) 28. (c) 29. (d) 30. (a)
31. (d) 32. (c) 33. (b) 34. (d) 35. (b) 36. (a) 37. (c) 38. (b) 39. (a) 40. (b)
41. (c) 42. (a) 43. (b) 44. (d) 45. (a) 46. (c) 47. (b) 48. (c) 49. (c) 50. (b)
51. (a)

B. Medical entrance special format questions

Assertion and reason

1. (c) 2. (a) 3. (b) 4. (b) 5. (b)

Match the columns

1. A → s, B → q, C → p, D → r 2. A → p, B → q, C → r, D → s

C. Madical entrances' gallery

1. (c) 2. (b) 3. (c) 4. (b) 5. (b) 6. (a) 7. (b) 8. (d) 9. (d) 10. (c)
11. (c) 12. (a) 13. (a) 14. (d) 15. (b) 16. (a) 17. (d) 18. (a) 19. (a) 20. (b)
21. (a) 22. (a)

Hints and explanations

Check point 8.1

2. (*c*) Maxwell in his famous equation of electromagnetism introduced the concept of displacement current.

3. (*d*) We have displacement current

$$I_d = \varepsilon_0 \frac{d\phi_E}{dt}$$

So, dimensions of $\varepsilon_0 \frac{d\phi_E}{dt}$ are same as that of current.

4. (*c*) Modified Ampere's law states that the variations of electric field causes magnetic field.

5. (*b*) When capacitor is fully charged, no current flows through the circuit. The potential of the plate became constant and there is no change in the electric field between the plates. So, both conduction current in the circuit and displacement current between the plate became zero.

6. (*d*) Ampere-Maxwell's circuital law is given by

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \left(I_c + \varepsilon_0 \frac{d\phi_E}{dt} \right)$$

7. (*a*) Displacement current,

$$I_d = \varepsilon_0 A \frac{dE}{dt}$$

We have, $\varepsilon_0 = 8.85 \times 10^{-12} C^2 N^{-1} m^{-2}$

Area, $A = 1\ cm^2 = 10^{-4} m^2$ and $\frac{dE}{dt} = 3 \times 10^6 Vm^{-1}s^{-1}$

$\Rightarrow \quad I_d = 8.85 \times 10^{-12} \times 10^{-4} \times 3 \times 10^6$

$\therefore$ Displacement current, $I_d = 2.7 \times 10^{-9} A$

8. (*b*) Displacement current, $I_d = \varepsilon_0 \frac{d\phi}{dt} = \varepsilon_0 \frac{d(q/\varepsilon_0)}{dt} = 0$

Conduction current, $I_c = c\frac{dv}{dt} = 0$ [as voltage becomes constant when the capacitor became fully charged]

9. (a) Displacement current, $I_d = \varepsilon_0 \frac{d\phi_E}{dt}$

$$= \varepsilon_0 \frac{EA}{t} = \frac{\varepsilon_0 (V/d) \times A}{t} = \frac{\varepsilon_0 VA}{td}$$

$$= \frac{8.85 \times 10^{-12} \times 400 \times (60 \times 10^{-4})}{10^{-6} \times (2 \times 10^{-3})}$$

$$= 1.062 \times 10^{-2} A$$

10. (*d*) Maxwell's equations describe the fundamental law of electricity and magnetism both.

Check point 8.2

1. (*b*) Frequency of *L-C* oscillations is

$$\nu = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2 \times 3.14\sqrt{100 \times 10^{-6} \times 200 \times 10^{-12}}}$$

$$= 1.126 \times 10^6 \text{ Hz}$$

So, wavelength, $\lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{1.126 \times 10^6} = 266 \text{ m}$

3. (*c*) The angular wave number, $k = \frac{2\pi}{\lambda}$, where λ is the wavelength. The angular frequency is $\omega = 2\pi\nu$

Ratio, $\frac{k}{\omega} = \frac{2\pi/\lambda}{2\pi\nu} = \frac{1}{\nu\lambda} = \frac{1}{c} = \text{constant}$

7. (*b*) As, $\frac{E_0}{B_0} = c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \Rightarrow \sqrt{\mu_0 \varepsilon_0} = \frac{B_0}{E_0}$

Here, c = speed of light

8. (*b*) As, $c = \frac{E_0}{B_0}$

$$\therefore B_0 = \frac{E_0}{c} = \frac{18}{3 \times 10^8} = 6 \times 10^{-8} \text{ T}$$

9. (*d*) Average energy density,

$$u_{av} = \frac{1}{2}\varepsilon_0 E_0^2$$

$$= \frac{1}{2}\varepsilon_0 (\sqrt{2}E_{rms})^2 = \varepsilon_0 E_{rms}^2$$

$$= 8.85 \times 10^{-12} \times (720)^2$$

$$= 4.58 \times 10^{-6} \text{ Jm}^{-3}$$

10. (*b*) Amplitude of electric field and magnetic field are related by the relations, $\frac{E_0}{B_0} = c$

Average energy density of the magnetic field is

$$u_B = \frac{1}{4}\frac{B_0^2}{\mu_0} = \frac{1}{4}\frac{E_0^2}{\mu_0 c} \qquad \left(\because B_0 = \frac{E_0}{c}\right)$$

$$= \frac{1}{4}\varepsilon_0 E_0^2 \qquad \left(\because c = \frac{1}{\sqrt{\mu_0 E_0}}\right)$$

$$= \frac{1}{4} \times 8.854 \times 10^{-12} \times 2^2$$

$$= 8.854 \times 10^{-12} \text{ Jm}^{-3}$$

$$= 8.86 \times 10^{-12} \text{ Jm}^{-3}$$

11. (*b*) As metal is reflecting surfaces for reflecting surface radiation pressure,

$$P_r = \frac{2S}{c} = \frac{2\times 0.5}{3\times 10^8} = 0.332\times 10^{-8}\ \text{Nm}^{-2}$$

12. (*a*) Here, $E_0 = 100\ \text{Vm}^{-1}$,

$B_0 = 0.265\ \text{Am}^{-1}$

$\therefore$ Maximum rate of energy flow, $S = \frac{E_0 \times B_0}{\mu_0}$

$= 100\times 0.265 = 26.5\ \text{Wm}^{-2}$

A) Taking it together

1. (*b*) Infrasonic wave is not an electromagnetic wave.

10. (*b*) Radio waves diffract around building although light waves do not. The reason is that radio waves have longer wavelength than light.

14. (*b*) Infrared radiations are reflected by lower clouds and keeps the earth warm. Hence shows green house effect.

16. (*d*) $\lambda_{\text{radio waves}} > \lambda_{\text{IR rays}} > \lambda_{\text{UV rays}} > \lambda_{\text{X-rays}}$

19. (*d*) The energy in EM waves is divided equally between the electric and magnetic fields.

The total energy per unit volume is $u = u_e + u_m$

$$= \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0}$$

20. (*b*) $\because$ Speed, $c = \nu\lambda$

$\therefore$ wavelength, $\lambda = \frac{c}{\nu} = \frac{3\times 10^8}{2\times 10^{10}} = 1.5\times 10^{-2}$ m

22. (*c*) For a medium with permittivity ε and permeability μ, velocity of light is given by

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

23. (*c*) Speed of electromagnetic waves in vacuum $= \frac{1}{\sqrt{\mu_0\varepsilon_0}}$

24. (*d*) The velocity of electromagnetic waves in free space (vacuum) is given by $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = 3\times 10^8\ \text{ms}^{-1}$

25. (*b*) The energy in EM waves is divided equally between the electric and magnetic fields.

26. (*b*) The wavelength of the γ-rays is shorter. However, the main distinguishing feature is the nature of emission.

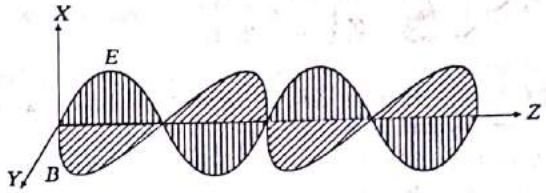
27. (*c*) From a dipole antenna, the electromagnetic waves are radiated outwards.

The amplitude of electric field vector (E_0) which transports significant energy from the source falls off inversely as the distance (r) from the antenna *i.e.*, $E_0 \propto \frac{1}{r}$.

28. (*c*) Total power $= 10^4 \times (10\times 10) = 10^6$ W

29. (*d*) The direction of propagation of electromagnetic wave is perpendicular to both electric field vector **E** and magnetic field vector **B**, *i.e.* in the direction of **E** × **B**.

This can be seen by the diagram given below



Here, electromagnetic wave is along the z-direction which is given by the cross product of **E** and **B**.

30. (*a*) Refractive index, $\eta = \frac{c}{v} = \frac{\frac{1}{\sqrt{\mu_0\varepsilon_0}}}{\frac{1}{\sqrt{\mu\varepsilon}}} = \sqrt{\frac{\mu\varepsilon}{\mu_0\varepsilon_0}}$

31. (*d*) The velocity of electromagnetic waves in free space is given by

$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = \frac{E_0}{B_0} = 3\times 10^8\ \text{ms}^{-1}$$

Moreover, velocity of light in a medium with refractive index μ is, $v = \frac{c}{\mu}$.

32. (*c*) The time varying electric and magnetic fields are mutually perpendicular to each other and also perpendicular to the direction of propagation of the wave.

33. (*b*) Given, $E = 9\times 10^{-4}\ \text{NC}^{-1}$

We know that, $c = \frac{E}{B} \Rightarrow B = \frac{9\times 10^{-4}}{3\times 10^8} = 3\times 10^{-12}$ T

35. (*b*) Speed of light, $c = \frac{E_0}{B_0} \Rightarrow E_0 = B_0 c$

Given, $B_0 = 2\times 10^{-7}$ T, $c = 3\times 10^8$ m/s

$\therefore \quad E_0 = 2\times 10^{-7}\times 3\times 10^8 = 60$ T m/s $= 60$ V/m

36. (*a*) Displacement current is given by

$$I_D = \frac{dq}{dt} = 1.8\times 10^{-8}\ \text{Cs}^{-1}$$

37. (*c*) When a surface intercepts electromagnetic radiation, a force and a pressure are exerted on the surface. As the surface is non-reflecting, so it is completely absorbed and in such case the force is

$$F = \frac{IA}{c}$$

The radiation pressure is the force per unit area,

$$P = \frac{F}{A} = \frac{I}{c}$$

38. (*b*) Frequency of L-C oscillations is

$$\nu = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\times 3.14\sqrt{200\times 10^{-6}\times 200\times 10^{-12}}}$$

$$= 0.0796\times 10^7\ \text{Hz}$$

So, wavelength, $\lambda = \frac{c}{\nu} = \frac{3\times 10^8}{0.0796\times 10^7} = 377$ m

39. (*a*) As metal is reflecting surfaces and for reflecting surface radiation pressure, $p_r = \frac{2S}{c} = \frac{2\times 1}{3\times 10^8} = 0.667\times 10^{-8}\text{Nm}^{-2}$

40. (*b*) When a wave is reflected from denser medium, then the type of wave does not change but only its phase changes by 180° or π radian.

Thus, for the reflected wave $\hat{\mathbf{z}} = -\hat{\mathbf{z}}$, $\hat{\mathbf{i}} = -\hat{\mathbf{i}}$ and additional phase of π in the incident wave.

41. (*c*) We have, $B_y = 2\times 10^{-7}\sin(0.5\times 10^3 x + 1.5\times 10^{11} t)$

Comparing with the standard equation, we get

$$B_y = B_0 \sin(kx + \omega t)$$
$$k = 0.5\times 10^3$$
$$\Rightarrow \quad \lambda = \frac{2\pi}{0.5\times 10^3} = 0.01256 \qquad \left(\because \lambda = \frac{2\pi}{k}\right)$$

The wavelength range of microwaves is 10^{-3} to 0.3. The wavelength of this wave lies between 10^{-3} to 0.3, so the equation represents a microwave.

42. (*a*) From $k = \frac{2\pi}{\lambda}$ and $\omega = 2\pi\nu$

We get, $\frac{k}{\omega} = \frac{\frac{2\pi}{\lambda}}{2\pi\nu} = \frac{\nu}{\lambda} = \frac{1}{c}$

As, $\frac{E_0}{B_0} = c \Rightarrow \frac{k}{\omega} = \frac{B_0}{E_0}$

$$E_0 k = B_0 \omega$$

43. (*b*) Angular frequency, $\omega = 2\pi\nu = \frac{2\pi c}{\lambda}$ $\quad [\because \nu = c/\lambda]$

$$= \frac{2\pi\times 3\times 10^8}{6\times 10^{-3}} = \pi\times 10^{11}\,\text{rads}^{-1}$$

The equation for the electric field , along Y-axis in the electromagnetic wave is

$$E_y = E_0 \sin\omega\left(t - \frac{x}{c}\right) = 33\sin\pi\times 10^{11}(t - x/c)$$

44. (*d*) Here, in electromagnetic wave, the electric field vector is given as, $\mathbf{E} = (E_1\hat{\mathbf{i}} + E_2\hat{\mathbf{j}})\cos(kz - \omega t)$

In electromagnetic wave, the associated magnetic field vector,

$$\mathbf{B} = \frac{\mathbf{E}}{c} = \frac{E_1\hat{\mathbf{i}} + E_2\hat{\mathbf{j}}}{c}\cos(kz - \omega t)$$

Also, **E** and **B** are perpendicular to each other and the propagation of electromagnetic wave is perpendicular to **E** as well as **B**, so the given electromagnetic wave is plane polarised.

45. (*a*) The magnetic field component is perpendicular to the direction of propagation and the direction of electric field. Using vector algebra $\mathbf{E}\times\mathbf{B}$ should be in z-direction.

$$\therefore \quad (\hat{\mathbf{i}} + \hat{\mathbf{j}})\times(-\hat{\mathbf{i}} + \hat{\mathbf{j}}) = 2\hat{\mathbf{k}}$$

46. (*c*) Intensity in terms of electric field, $u_{av} = \frac{1}{2}\varepsilon_0 E_0^2$

Intensity in terms of magnetic field, $u_{av} = \frac{1}{2}\frac{B_0^2}{\mu_0}$

Now, taking the intensity in terms of electric field.

$$(U_{av})_{\text{electric field}} = \frac{1}{2}\varepsilon_0 E_0^2$$
$$\Rightarrow \quad = \frac{1}{2}\varepsilon_0 (cB_0)^2 \qquad [\because E_0 = cB_0]$$
$$= \frac{1}{2}\varepsilon_0 \times c^2 B_0^2$$

But, $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$

$$\therefore \quad (u_{av})_{\text{electric field}} = \frac{1}{2}\varepsilon_0 \times \frac{1}{\mu_0\varepsilon_0} B_0^2 = \frac{1}{2}\frac{B_0^2}{\mu_0}$$
$$= (u_{av})_{\text{magnetic field}}$$

Thus, the energy in electromagnetic wave is divided equally between electric field vector and magnetic field vector. Therefore, the ratio of contributions by the electric field and magnetic field components to the intensity of an electromagnetic wave is 1 : 1.

47. (*b*) Let, E = energy falling on the surface per second = 10J

Momentum of photons, $p = \frac{h}{\lambda} = \frac{h}{(c/\nu)} = \frac{h\nu}{c} = \frac{E}{c}$

On reflection, change in momentum per second = force

$$= 2p = \frac{2E}{c} = \frac{2\times 10}{3\times 10^8} = 6.7\times 10^{-8}\text{N}$$

48. (*c*) Given, energy required to dissociate a carbon monoxide molecule into carbon and oxygen atoms $E = 11\text{eV}$

We know that, $E = h\nu$

where $h = 6.62\times 10^{-34}$J-s, ν = frequency

$\Rightarrow \quad 11\text{ eV} = h\nu,$

$$\Rightarrow \quad \nu = \frac{11\times 1.6\times 10^{-19}}{h}\text{ J} = \frac{11\times 1.6\times 10^{-19}}{6.62\times 10^{-34}}\text{ J}$$
$$= 2.65\times 10^{15}\text{ Hz}$$

This frequency radiation belongs to ultraviolet region.

49. (*c*) We know that, $E_0 \propto \sqrt{P_{av}}$

$$\therefore \quad \frac{(E_0)_1}{(E_0)_2} = \sqrt{\frac{(P_{av})_1}{(P_{av})_2}}$$
$$\Rightarrow \quad \frac{E}{(E_0)_2} = \sqrt{\frac{100}{50}}$$
$$(E_0)_2 = E/\sqrt{2}$$

50. (*b*) Given, energy flux, $\phi = 20\text{ W/cm}^2$

Area, $A = 30\text{ cm}^2$

Time, $t = 30\text{ min} = 30\times 60\text{ s}$

Now, total energy falling on the surface in time t is, $u = \phi A t = 20\times 30\times(30\times 60)$ J

Momentum of the incident light $= \frac{U}{c}$

$$= \frac{20\times 30\times(30\times 60)}{3\times 10^8} = 36\times 10^{-4}\text{ kg-m/s}$$

Momentum of the reflected light = 0

$\therefore$ Momentum delivered to the surface

$$= 36\times 10^{-4} - 0 = 36\times 10^{-4}\text{ kg-m/s}$$

51. (a) Effective power of the bulb, $P = \frac{5}{100} \times 200 = 10\text{ W}$

Intensity at distance r is given by

$$I = \frac{P}{4\pi r^2} = \frac{10}{4 \times 3.14 \times 2^2}\text{ W/m}^2 = \frac{10}{16 \times 3.14}\text{ W/m}^2$$

$$= 0.199\text{ W/m}^2 = 0.2\text{ W/m}^2$$

$$I = \frac{1}{2}\varepsilon_0 E_0^2 c$$

$$E_0^2 = \frac{2I}{c \times \varepsilon_0}$$

$$\Rightarrow \quad E_0 = \sqrt{\frac{2I}{c \times \varepsilon_0}}$$

$$\Rightarrow \quad E_0 = \sqrt{\frac{2 \times 0.2}{3 \times 10^8 \times 8.85 \times 10^{-12}}} \Rightarrow E_0 = 12.27\text{ N/C}$$

$$\Rightarrow \quad B_0 = \frac{E_0}{c} = \frac{12.27}{3 \times 10^8} = \frac{12.27}{3} \times 10^{-8}\text{ T} = 4.09 \times 10^{-8}\text{ T}$$

B) Medical entrance special format question

Assertion and Reason

1. (c) The frequency range of microwave is from 10^9 to 10^{11} Hz.

2. (a) As per Maxwell's equations, electric and magnetic fields in an electromagnetic wave are perpendicular each other and to the direction of wave propagation. Hence, electromagnetic waves are transverse in nature.

3. (b) The wavelength of these waves ranges between 4000Å to 100 Å that is smaller wavelength and higher frequency. They are absorbed by atmosphere and convert oxygen into ozone. They cause skin diseases and they are harmful to eye and cause permanent blindness.

4. (b) The oscillating electric and magnetic fields, thus regenerate each other, as the wave propagates through the space.

   The frequency of the electromagnetic wave naturally equals the frequency of oscillation of the charge. The energy associated with the propagating wave comes at the expense of the energy of the source, the accelerated charge.

5. (b) Light radiations and thermal radiations both belong to electromagnetic spectrum. Light radiation belongs to visible region while thermal radiation belongs to infrared region of EM spectrum.

   Also, EM radiations require no medium for propagation.

C) Medical entrances' gallery

1. (c) Electric charge at rest has electric field in the region around it, but no magnetic field. Whereas, a moving charge produces both the electric and magnetic fields. If a charge is moving with a constant velocity, then the electric and magnetic fields will not change with time, hence no EM wave will be produced. But, if the charge is moving with a non-zero acceleration, then both the electric and magnetic field will change with space and time, it then produces EM wave.

2. (b) *Increasing order of wavelengths as given below*

   γ-rays < X-rays < ultraviolet rays

   Frequency order,

   γ-ray > X-rays > ultraviolet rays

   $\Rightarrow \quad q > p > r$

   Thus, $q > p$

   $q > r$

   $p > r$

3. (c) The intensity is, $I = \frac{1}{2}\varepsilon_0 E_0^2 c$

   $$= \frac{1}{2}(8.85 \times 10^{-12}\text{N-m}^2\text{C}^{-2}) \times (50\text{ NC}^{-1})^2 \times 3 \times 10^8\text{ ms}^{-1}$$

   $$= 3.3\text{ Wm}^{-2}$$

4. (b) Infrared rays are used in physical therapy.

5. (b) The magnitude of electric field vector varies periodically with time because it is in the form of electromagnetic wave.

6. (a) Laser light is of red colour which is not reflected by green, pigment of the leaf and so it appears black.

7. (b) The ratio of amplitude of magnetic field to the amplitude of electric field for an electromagnetic wave propagating in vacuum is equal to the reciprocal of speed of light in vacuum.

   i.e., $\frac{B_0}{E_0} = \frac{1}{c}$

8. (d) According to Maxwell, time varying electric field produced displacement current.

   $$I_D = \varepsilon_0 \frac{d\phi_E}{dt}$$

9. (d) The speed of light in an isotropic medium depends on its wavelength, $v = \frac{c}{\lambda}$

10. (c) $E_\gamma \geq 100$ keV, $E_X = 100$ eV to 100 keV.

    So, we can say that $E_\gamma > E_X > E_v$.

11. (c) According to Maxwell, it is found that the electromagnetic waves travel in free space (or vacuum) with a speed is given by

    i.e., $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8\text{ ms}^{-1}$

    where, μ_0 and ε_0 are absolute permeability and permittivity of the free space, respectively.

    where, $\mu_0 = 4\pi \times 10^{-7}\text{Wb A}^{-1}\text{m}^{-1}$

    $\varepsilon_0 = 8.85 \times 10^{-12}\text{ C}^2\text{N}^{-1}\text{m}^{-2}$

12. (a) The magnetic field, $B = \frac{E}{c}$, where $c = 3 \times 10^8\text{ ms}^{-1}$

    $$\Rightarrow \quad B = \frac{6}{3 \times 10^8} = 2 \times 10^{-8}\text{ T}$$

13. (a) Frequency range of ultraviolet rays $\simeq 10^{16} - 8 \times 10^{14}\ (n_3)$

    Frequency range of X-rays $\simeq 3 \times 10^{19} - 10^{16}\ (n_2)$

    Frequency range of γ-rays $\simeq 5 \times 10^{20} - 3 \times 10^{19}\ (n_1)$

    $\therefore \quad n_1 > n_2 > n_3$

14. (*d*) The electromagnetic waves detected using a thermopile and used in physical therapy are infrared radiations for treating muscular strain.

15. (*b*) X-rays wavelength range $= 1\times10^{-11}$m to 1×10^{-8}m
$= 0.001$nm to 1 nm

16. (*a*) We know that

(i) Short radio waves *i.e.*, wavelength range
$= 1\times10^{-1}$ m to 1×10^{4} m

(ii) Microwaves wavelength range
$= 1\times10^{-3}$ m to 3×10^{-1} m

(iii) Ultraviolet waves wavelength range
$= 1\times10^{-8}$ m to 4×10^{-8} m

So, decreasing order is $\lambda_1, \lambda_2, \lambda_3$.

17. (*d*) Phenomena of restricting the vibration of light in a particular direction is called polarisation, it helps to prove the transverse nature of electromagnetic waves.

18. (*a*) Radio waves have maximum wavelength in the range of 0.1 m to 10^4 m.

20. (b) Velocity of light in a medium $v = \frac{c}{\mu}$

We knows time taken $t = \frac{x}{v}$

$$\Rightarrow \quad t = \frac{x}{c/\mu}$$

$$\Rightarrow \quad t = \frac{\mu x}{c}$$

21. (*a*) Electromagnetic wave equation,

$$E = E_0 \cos(kz - \omega t) \quad \text{...(i)}$$

Speed of electromagnetic wave, $v = \frac{\omega}{k}$

Given, equation,

$$\mathbf{E} = \hat{\mathbf{i}}\,40 \cos(kz - 6\times10^8 t) \quad \text{...(ii)}$$

On comparing Eqs. (i) and (ii), we get

$$\omega = 6\times10^8 \text{ and } E_0 = 40\hat{\mathbf{i}}$$

Here, wave factor, $k = \frac{\omega}{v} = \frac{6\times10^8}{3\times10^8} = 2\text{ m}^{-1}$

22. (*a*) The wave function

$$\psi(x, t) = 10^3 \sin\pi\,(3\times10^8 x - 9\times10^4 t)$$

Here,

Angular frequency $\omega = 9\times10^{14}\pi$

$$\omega = 2\pi\nu$$

$$\nu = \frac{\omega}{2\pi} = \frac{9\times10^{14}\pi}{2\pi} = 4.5\times10^{14}\text{ Hz}$$

CHAPTER 09

Ray optics

Light is a form of energy which produces sensation of vision to our eyes and makes objects visible. The branch of physics which deals with the study of nature of light, its properties, effects and propagation is known as optics. The subject of optics can be divided into two mains branches—ray optics and wave optics.

(i) **Ray or geometrical optics** This branch of optics deals with propagation of light in terms of rays which are valid, if sizes of obstacles are large in comparison with wavelength of light (in nm range). Ray optics concerns itself with image formation, reflection, refraction and dispersion of light by geometrical methods.

(ii) **Wave or physical optics** This branch of optics deals with wave phenomenon like interference, diffraction and polarisation.

In this chapter, we are going to learn ray optics step by step followed by light sources as well as properties of light.

Inside

1) Reflection of light
- Reflection by a plane mirror

2) Spherical mirrors
- Sign convention
- Image formation by spherical mirrors
- Mirror formula
- Magnification
- Power of mirror
- Uses of spherical mirrors

3) Refraction of light
- Refractive index (μ or n)
- Image due to refraction at a plane surface
- Refraction through a glass slab

4) Refraction at spherical surfaces
- Lenses
- Law of formation of image by lens
- Image formation by lens

5) Prism
- Dispersion of light by a prism
- Combination of prisms

6) Optical instruments

7) Defects of vision

Light sources

We obtain light from different types of natural sources like sun, stars etc, and from artificial sources (man made) like-candle, electric bulb, etc. *On the basis of this light source can be categorised in two types as follows*

(i) **Luminous sources** These sources can produce their own light *e.g.*, sun, stars, electric bulb, etc.

(ii) **Non-luminous sources** These sources cannot produce their own light but returns the light coming from a luminous source. *e.g.*, moon, glass, etc.

Properties of light

Following are the properties of light

(i) Light is itself invisible but in its presence we can see objects due to scattering of light.

(ii) Light is an electromagnetic wave, requires no medium for its propagation, its speed in vacuum is 3×10^8 m/s.

(iii) The wavelength of visible light is 400 nm to 700 nm.

(iv) Light can be considered to propagate from one point to another, along a straight line joining them. The path is called a **ray of light** and a bundle of such rays constitutes a **beam of light**.

(v) Light can pass through transparent mediums like glass, air, etc. It cannot pass through non-transparent mediums like wood, iron, etc.

(vi) Speed of light is different in different mediums.

(vii) Light can reflect from polished surfaces.

(viii) When light travels from one medium to another medium, it deviates from the path.

1 Reflection of light

When light is incident on a polished smooth surface, then most of the incident light returns to the same medium. This phenomenon of returning of light after striking a smooth polished surface is called **reflection of light**.

After reflection, velocity, wavelength and frequency of light remains same but intensity decreases.

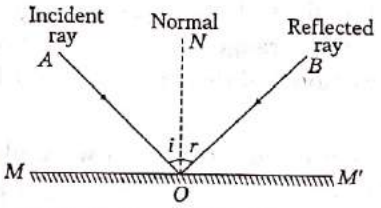


Fig. 9.1 Reflection of light (plane surface)

Some important terms related with reflection of light

(i) **Reflecting surface** The surface, after striking which, light reflects is called reflecting surface (MM').

(ii) **Incident ray** The incident ray (AO) on the reflecting surface is called incident ray.

(iii) **Reflected ray** The reflected ray (OB) after striking the reflecting surface is called reflected ray.

(iv) **Point of incidence** It is the point (O) on the reflecting surface at which incident light strikes.

(v) **Normal** It is the normal (ON) to the surface drawn from point of incidence (O) on the reflecting surface.

(vi) **Angle of incidence** It is the angle (i) between incident ray (AO) and the normal (ON). In the given figure, $\angle AON = i$.

(vii) **Angle of reflection** It is the angle (r) between reflected ray (OB) and the normal (ON). In the given figure, $\angle NOB = r$.

Laws of reflection

There are two laws for reflection of light from a smooth reflecting surface.

(i) **First law** The angle of incidence is equal to the angle of reflection, *i.e.*, $\angle i = \angle r$.

(ii) **Second law** The incident ray, the reflected ray and the normal at the point of incidence all lie in the same plane.

These laws are valid for plane as well as curved surfaces as shown in the figure below.

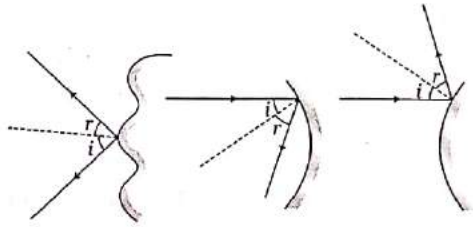
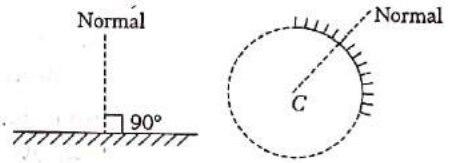


Fig. 9.2

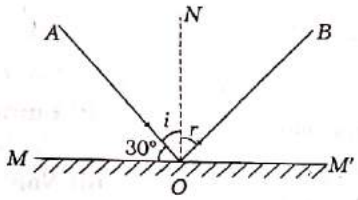
Note

1. In geometrical optics angles are measured with normal.
2. Also in geometrical optics, we deal with plane and spherical surfaces. In case of spherical surface, normal to surface will pass through centre of sphere.



Example 9.1 *A ray of light is incident at an angle of 30° with the plane of a plane mirror. Find the angle of reflection.*

Sol. Consider the situation shown in the figure. Angle made by the incident ray with the plane of the mirror MM' is $\angle MOA = 30°$. Therefore, angle of incidence,



$$\angle AON = i = \angle NOM - \angle AOM = 90° - 30° = 60°$$

Also, angle of reflection, $\angle BON = r = i = 60°$

Image

Light rays emerging from an object (or a point) after reflection or refraction meets or appears to meet at a point is called image of object (or first point).

Images are of two types

(i) Real image

When light rays emerging from a point object really meets at a point, the later point is called real image of the first point. These types of images can be obtained on a screen.

(ii) Virtual image

When light rays emerging from a point object appears to meet at a point, the later point is called virtual image of the first point. These types of images cannot be obtained on a screen.

Note If the incident rays diverge from a point object, the object is real object.

If rays are converging towards a mirror, the point where these rays would meet if there was no mirror is virtual object.

Mirror

There are two types of mirror

(i) **Plane mirror** The mirror whose reflecting surface is plane is called plane mirror. It's one side is silvered and reflection take place from other side as shown in the figure.

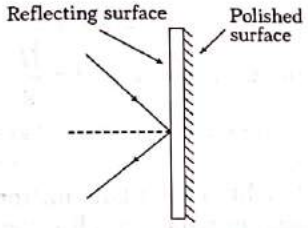


Fig. 9.3 Plane mirror

(ii) **Spherical mirror** It is the part of a spherical transparent medium. One of the curved surfaces is polished and reflection takes place from other side.

Spherical mirrors are of two types

(a) **Concave mirror** In this mirror, outer curved surface is polished and reflection takes place from inner curved surface as shown in figure.

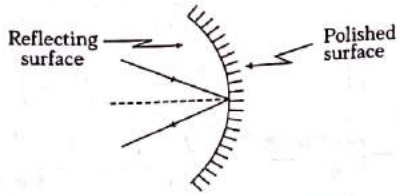


Fig. 9.4 Concave mirror

(b) **Convex mirror** In this mirror, inner curved, surface is polished and reflection takes place from outer curved surface as shown in figure.

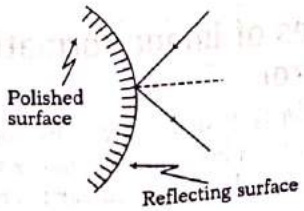


Fig. 9.5 Convex mirror

Reflection by a plane mirror

Characteristics of a plane mirror are given below

Properties of image formed by plane mirror

The image formed by a plane mirror is **virtual, erect,** of **same size** and **at the same distance from the mirror** as the distance of object from mirror. The ray diagram of the image of a point object and of an extended object is as shown below.

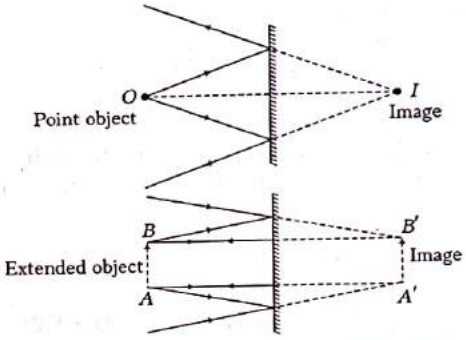


Fig. 9.6 Formation of images by a plane mirror

When a convergent beam is incident on the plane mirror, the object (O) is virtual as shown in figure, in this case image (I) formed by the plane mirror is real.

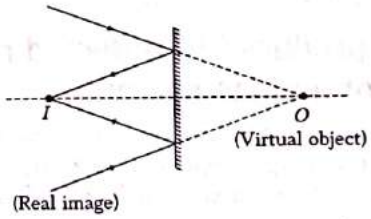
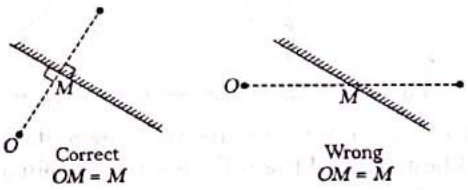


Fig. 9.7 Formation of image by plane mirror of a virtual object

Note To find the location of image of an object from an inclined plane mirror, you have to see the perpendicular distance of the object from the mirror.



Deviation produced by a plane mirror

The angle between incident ray and the ray extended in the direction of incident ray, is called deviation produced by plane mirror.

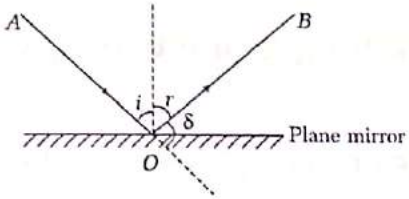


Fig. 9.9 Deviation produced by plane mirror by single reflection

Deviation produced by plane mirror is given by

$$\delta = 180° - (i + r)$$
$$= 180° - 2i \qquad (\because i = r)$$
$$= 180° - 2r$$

The deviation produced by reflection at two mirrors inclined to each other at angle θ is

$$\delta = 360° - 2\theta$$

It is independent of angle of incidence.

(i) If two plane mirrors are inclined to each other at 90°, then the ray reflected at two mirrors becomes parallel to incident ray.

As, $\delta = 360° - 2\theta$

Here, $\theta = 90°$

$\therefore$ $\delta = 360 - 2 \times 90 = 180°$

i.e., the reflected ray is parallel to incident ray.

(ii) If two plane mirrors are inclined to each other at 60° and a ray of light is incident on one mirror parallel to other, then after reflection from two mirrors, the reflected ray becomes parallel to the first mirror.

Rotation produced in reflected ray due to rotation of a plane mirror

(i) For a given fixed incident ray, if the plane mirror is rotated through an angle θ in the plane of incidence, the reflected ray turns through an angle 2θ.

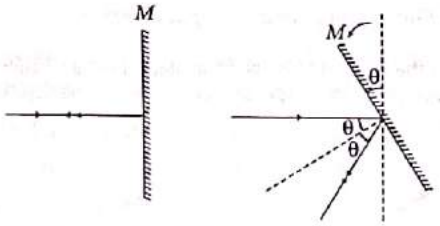


Fig. 9.10 Rotating plane mirror by angle θ

(ii) If a plane mirror is rotated in its own plane, the incident ray and the reflected ray remain at same positions.

Field of view of an object for a plane mirror

The field of view is the region between the extreme reflected rays and depends on the location of the object in front of the mirror.

Two important results based on the field of view are given below

(i) The minimum height of a plane mirror to see object's full height in it is $H/2$, where H is the height of object. But the mirror should be placed in a fixed position as shown in Fig. 9.11.

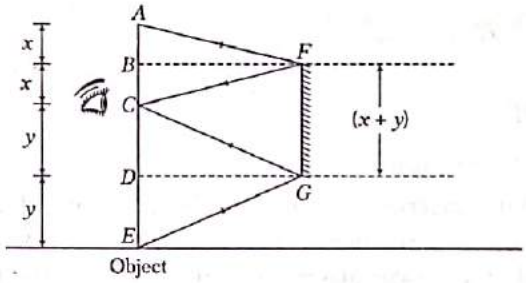


Fig. 9.11 Minimum height of plane mirror to see full height

Height of the object $AE = 2(x + y) = H$

Height of the mirror $= (x + y) = \dfrac{H}{2}$

Note The mirror can be placed anywhere between the centre line BF (of AC) and DG (of CE).

(ii) Minimum height of the plane mirror fixed on the wall of a room in which an observer at the centre of the room can see the full image of the wall behind him, is one third the height of wall.

The minimum height of mirror in this case should be $H/3$, where H is the height of wall. The ray diagram in this case is drawn in Fig. 9.12.

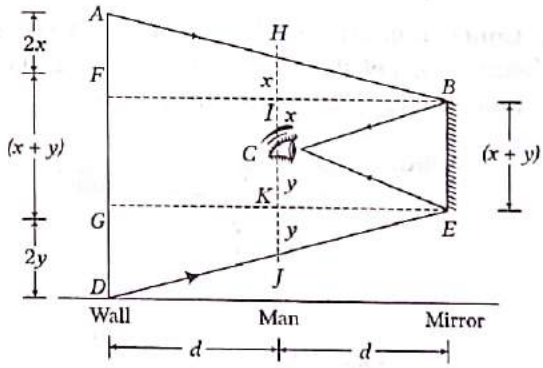


Fig. 9.12 Minimum height of plane mirror for the wall of the room

Height of the wall is 3 $(x + y)$ while that of the mirror is $(x + y)$.

Special cases of image formation through a plane mirror

(i) If the object is displaced by a distance a towards or away from the plane mirror, image will also be displaced by a distance a towards or away from the mirror.

(ii) If an object approaches or recedes from the plane mirror with velocity v, then the image also moves in the same manner from the mirror with velocity v. But the velocity of the image with respect to object will be $2v$.

(iii) If plane mirror moves a distance x towards or away from the object, the image will move a distance $2x$ towards or away from the mirror/object.

(iv) If both plane mirror and object are moved by a distance x, each in opposite directions, the image will be displaced by a distance $3x$ in the direction of the displacement of the mirror.

(v) If a luminous object is placed in front of a thick glass mirror, multiple images are formed due to multiple reflections. The second image formed by the first reflection by the polished surface is much brighter than the others. The intensity of the other images rapidly fade away.

(vi) If an object is placed between two mirrors facing each other at an angle θ the number of images is given by $\frac{360°}{\theta} = N$ and actual number of images are n, where

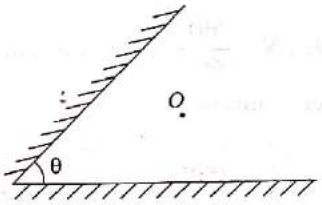


Fig. 9.13

(a) $n = N - 1$, if N is even integer.

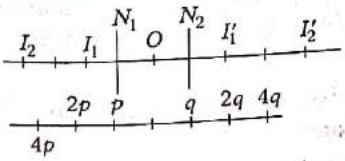
(b) $n = N$, if N is odd integer and object is not on the bisector of mirrors

(c) $n = N - 1$, if N is odd integer and object is on the bisector of mirrors and

(d) If $\frac{360}{\theta} = N$ is a fraction, the number of images will be equal to its lower boundary of integral part.

Note

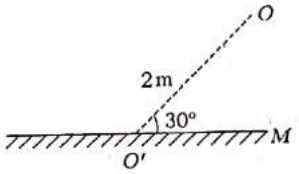
1. If the mirrors are placed parallel to each other like
Number of images = ∞ and distance between nth images,



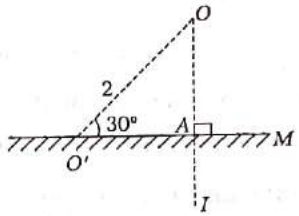
$$I_m I_m' = 2mp + 2nq \quad \text{or} \quad I_n I_n' = 2n(p+q)$$

(ii) The number of images formed by two mutually perpendicular ($\theta = 90°$) mirrors will be 3. All these three images will lie on a circle with centre at C, the point of intersection of mirrors M_1 and M_2 and whose radius is equal to distance between C and the object O.

Example 9.2 *A point object O is at an angle of 30° from the plane mirror M, as shown in figure. If OO′ = 2 m, then find the location of image.*



Sol. To find the location of image, we have to draw a perpendicular on the mirror M from point O as shown in figure.

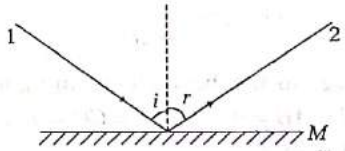


Now from the triangle $O'AO$, we have

$$\sin 30° = \frac{OA}{2} \Rightarrow OA = 2 \times \sin 30° = 1 \text{ m}$$

$\therefore$ Distance of the image from the mirror is $AI = OA = 1$ m.

Example 9.3 *Consider a ray of light is incident on a plane mirror M. If deviation produced by the mirror in the incident light ray is 120° then, find i and r.*



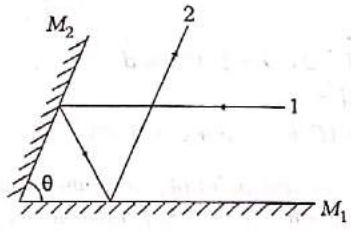
Sol. Deviation produced by the mirror is,

$$\delta = 180° - 2i = 120° \quad \text{(Given)}$$

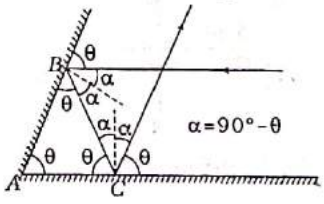
$$\Rightarrow \quad 2i = 60° \Rightarrow i = 30°$$

$$\therefore \quad r = i = 30°$$

Example 9.4 *Two plane mirrors M_1 and M_2 are inclined at angle θ as shown. A ray of light 1, which is parallel to M_1 strikes M_2 and after two reflections, the ray 2 becomes parallel to M_2. Find the angle θ.*



Sol. Different angles are as shown in figure. In triangle ABC,

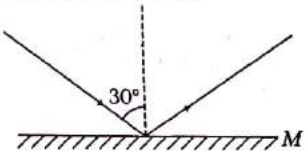


From ΔABC,

$$\theta + \theta + \theta = 180°$$

$$\therefore \quad \theta = 60°$$

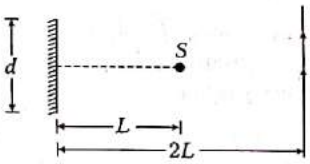
Example 9.5 *A ray of light is incident on a plane mirror at an angle of 30° as shown in figure. Now the mirror is rotated through 30° in anti-clockwise direction. Find the angle and sense of rotation of the reflected ray.*



Sol. Angle rotated by the plane mirror, $\theta = 30°$, anti-clockwise.

$\therefore$ Angle rotate by reflected ray, $\theta_r = 2\theta = 60°$, anti-clockwise.

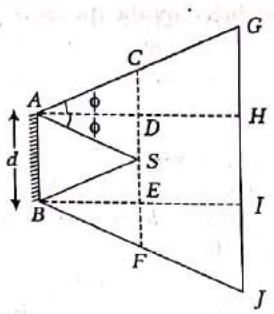
Example 9.6 *A point source of light S, placed at a distance L in front of the centre of a mirror of width d, hangs vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror at a distance 2L from it as shown. Find the greatest distance over which he can see the image of the light source in the mirror.*



Sol. The ray diagram will be as shown in figure given below.

$$HI = AB = d \Rightarrow DS = CD = d/2$$

Since, $\quad AH = 2AD$

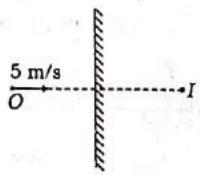


$\therefore \quad GH = 2\,CD = 2\,(d/2) = d$

Similarly, $\quad IJ = d$

$\therefore \quad GJ = GH + HI + IJ = d + d + d = 3d$

Example 9.7 *Consider a point object moving towards a stationary plane mirror with velocity 5 m/s. Find velocity of image w.r.t.*



(i) the mirror *(ii) the object*

Sol. The object is moving towards right with a velocity of $u = 5$ m/s

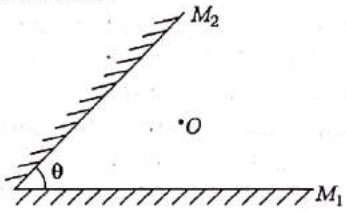
(i) Velocity of image w.r.t. the mirror,

$$\mathbf{v}_{IM} = -\mathbf{u} = 5 \text{ m/s, towards left.}$$

(ii) Velocity of image w.r.t. the object,

$$\mathbf{v}_{IO} = -2\mathbf{u} = 2 \times 5 \text{ m/s} = 10 \text{ m/s, towards left.}$$

Example 9.8 *Consider two plane mirrors inclined at an angle θ as shown in figure. Find the number of images of object O formed these mirrors when*



(i) $\theta = 60°$

(ii) $\theta = 72°$ *and the object is at 40° from* M_1

(iii) $\theta = 72°$ *and the object is at 36° from* M_1

(iv) $\theta = 80°$

Sol. Two mirrors M_1 and M_2 are inclined at angle θ.

Let, $N = \dfrac{360°}{\theta}$

(i) For $\theta = 60°$, $N = \dfrac{360°}{60°} = 6 =$ even integer

$\therefore$ Number of images formed

$$n = N - 1 = 6 - 1 = 5$$

(ii) For $\theta = 72°$, $N = \dfrac{360°}{72°} = 5 =$ odd integer and the object is not on the bisector,

$\therefore$ Number of images formed, $n = N = 5$

(iii) For $\theta = 72°$, $N = \dfrac{360°}{72°} = 5 =$ odd integer also bisector is at 36° not at 30°.

$\therefore$ Number of images formed

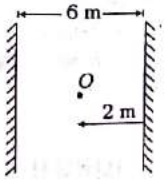
$$n = N - 1 = 5 - 1 = 4$$

(iv) For $\theta = 80°$, $N = \dfrac{360°}{80°} = 4.5 =$ fraction number

$\therefore$ Number of images formed,

$$n = \text{integral part} = 4$$

Example 9.9 *Two plane mirrors are placed parallel to each other at a separation of 6m as shown in figure. Find the (i) number of images of the object O. (ii) separation between 5th image.*



Sol. Given, mirrors are placed parallel to each other. Therefore

(i) Number of images formed $= \infty$

(ii) Distance between 5th images, $I_5I_5' = 2 \times 5(x + y)$

where, $x = 6 - 2 = 4$ m $\Rightarrow$ $y = 2$ m

$\Rightarrow \quad I_5I_5' = 10(4 + 2) = 60$ m

Check point 9.1

1. Light falls on a plane reflecting surface. For what angle of incidence is the reflected ray normal to the incident ray.
(a) 60° (b) 45° (c) 90° (d) 30°

2. Which of the following is correct for the image formed by a plane mirror?
(a) Always real (b) Always virtual
(c) Virtual and laterally inverted (d) Real and laterally inverted

3. A plane mirror reflects a pencil of light to form a real image. Then the pencil of light incident on the mirror is
(a) parallel (b) convergent
(c) divergent (d) None of these

4. A plane mirror produces a magnification of
(a) −1 (b) +1
(c) zero (d) Between 0 and $+\infty$

5. A ray of light is incident on a plane mirror at an angle of 30°. The deviation produced in the ray is
(a) 30° (b) 60°
(c) 90° (d) 120°

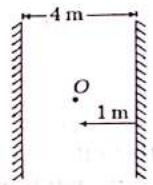
6. If the reflected ray is rotated by an angle of 4θ in clockwise direction then the mirror was rotated by
(a) 2θ in anti-clockwise direction
(b) 4θ in anti-clockwise direction
(c) 2θ in clockwise direction
(d) 4θ in clockwise direction

7. A man is 180 cm tall and his eyes are 10 cm below the top of his head. In order to see this entire height right from top to head, he uses a plane mirror kept at a distance of 1 m from him. The minimum length of the plane mirror is
(a) 180 cm (b) 90 cm
(c) 85 cm (d) 170 cm

8. An object is moving towards a stationary plane mirror with a speed of 2 m/s. Velocity of the image w.r.t. the object is
(a) 2 m/s towards right (b) 4 m/s towards right
(c) 2 m/s towards left (d) 4 m/s towards left

9. To get three images of a single object, one should have two plane mirrors at an angle of
(a) 30° (b) 60° (c) 90° (d) 150°

10. An object (O) is placed between two parallel plane mirrors as shown in figure. Distance between 4th image is



(a) 16 m (b) 32 m (c) 8 m (d) 64 m

2 Spherical mirrors

Spherical mirror is a mirror whose reflecting surface is a part of a hollow sphere.

Spherical mirrors are of two types

(i) Concave mirror

A spherical mirror whose reflecting surface is towards the centre of the sphere is called concave spherical mirror.

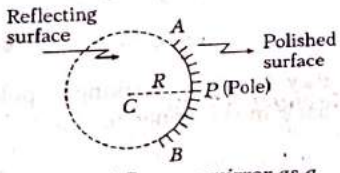


Fig.9.14 Concave mirror as a part of hollow sphere

(ii) Convex Mirror

A spherical mirror whose reflecting surface is away from the centre of the sphere is called convex mirror.

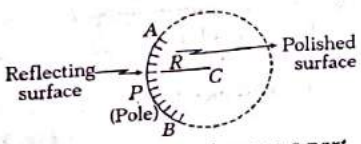


Fig. 9.15 Convex mirror as a part of hollow sphere

Terms related to spherical mirror

In case of these thin spherical mirrors important terms and their definitions are as follows

(i) **Pole** It is the mid-point (P) of the mirror.

(ii) **Centre of curvature** It is the centre C of the sphere of which the mirror is a part.

(iii) **Radius of curvature** Distance between pole and centre of curvature is called radius of curvature diagram the distance CP is radius of curvature.

(iv) **Principal axis** The line joining the pole P and centre of curvature C when produced is called principal axis.

(v) **Aperture** Effective diameter of light reflecting area of the mirror is called aperture (AB).

(vi) **Principal focus or focus** Principal focus is a point on the principal axis of the mirror at which the light rays coming parallel to principal axis actually meet after reflection (or appears to meet).

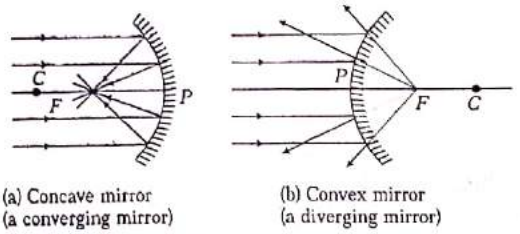


Fig. 9.16 Principal focus of (a) concave mirror (b) convex mirror

Focus of concave mirror is real, while focus of convex mirror is virtual.

(vii) **Focal length** The distance between pole and focus of a spherical mirror is called its focal length. It is represented by f. Focal length is independent of medium. *i.e.*, If a spherical mirror is placed in water or any other mediums, its focal length remains unchanged.

Note Focal length of a plane mirror is infinity and hence its power is zero.

(vii) Focal length of a mirror is equal to half the radius of curvature of the mirror.

i.e., $f = \frac{R}{2}$.

Sign convention

To derive the relevant formulae for reflection by spherical mirrors (concave or convex), we must first adopt a sign convention for measuring distances.

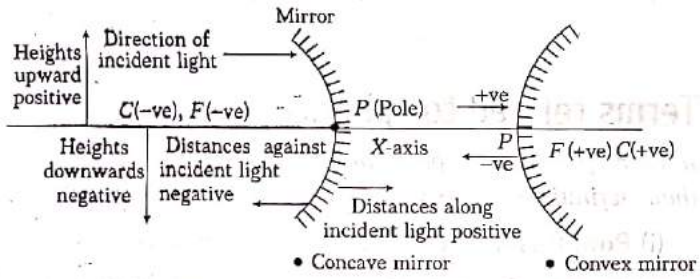


Fig. 9.17 Cartesian sign convention

According to the Cartesian sign convention,

(i) All the distances are measured from pole (P) of the mirrors.

(ii) The principle axis of the mirror is taken as X-axis and the pole as origin.

(iii) Distances measured in the direction of the incident light are taken as positive and opposite to the direction of incident light as negative.

(iv) The heights measured upwards with respect to X-axis and normal to the principal axis of the mirror are taken as positive and the heights measured downwards are taken as negative.

Rules for image formation in spherical mirrors

In ray optics, to locate the image of an object, tracing of a ray as it reflects is very important. *Following four types of rays are used for image formation.*

Ray 1. A ray through the centre of curvature which strikes the mirror normally and is reflected back along the same path.

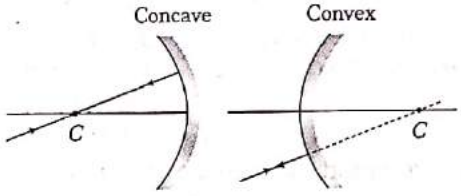


Fig. 9.18

Ray 2. A ray parallel to principal axis after reflection either actually passes through the principal focus F or appears to diverge from it.

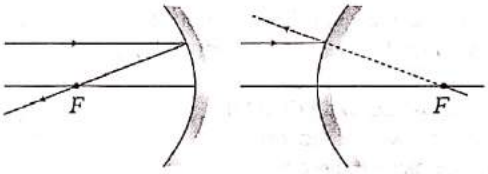


Fig. 9.19

Ray 3. A ray passing through the principal focus F or a ray which appears to converge at F is reflected parallel to the principal axis.

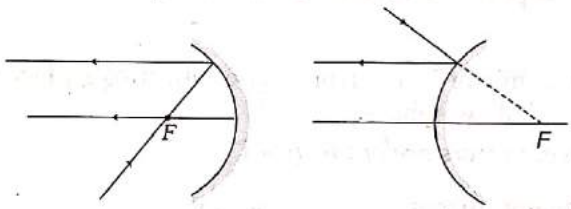


Fig. 9.20

Ray 4. A ray striking at pole P is reflected symmetrically back in the opposite side.

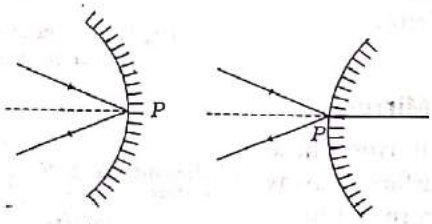


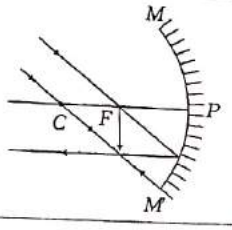
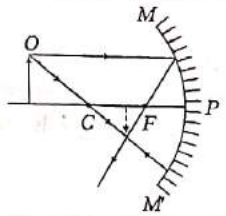
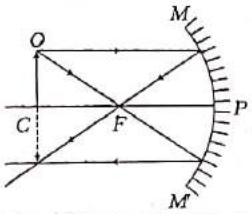
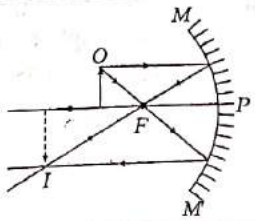
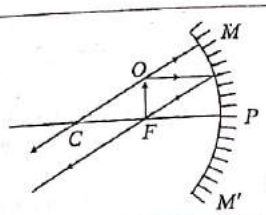
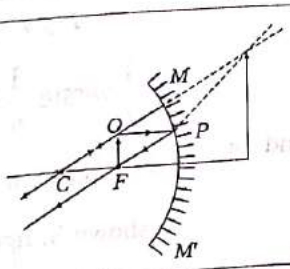
Fig. 9.21

Image formation by spherical mirrors

1. Image formation of concave mirror

In case of a concave mirror, the image is erect and virtual when the object is placed between F and P. In all other positions of object, the image is real and inverted as shown in the table given below.

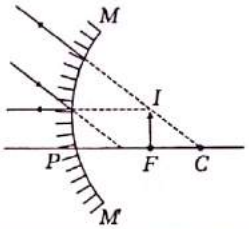
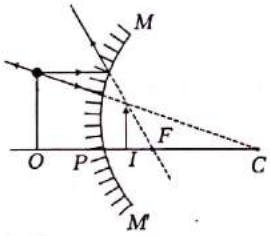
Table Image formation by concave mirror

S.No.	Position of object	Ray diagram	Properties of image
1.	At infinity		Real, inverted, very small at F
2.	Between infinity and C		Real, inverted, diminished between F and C
3.	At C		Real, inverted, equal in size at C
4.	Between F and C		Real, inverted and very large between $2F$ and infinity
5.	At F		Real, inverted, very large at infinity
6.	Between F and P		Virtual, erect, large in size behind the mirror

2. Image formation by convex mirror

Image formed by convex mirror is always virtual, erect and diminished no matter where the object is. All the images formed by this mirror will be between pole and focus as shown in the table given below.

Table. Image formation by convex mirror

S.No.	Position of Object	Ray Diagram	Details of Image
1.	At infinity		Virtual, erect, very small in size at F
2.	In front of mirror		Virtual, erect, diminished between P and F

Mirror formula

The relation among object distance (u), image distance (v) and focal length (f) of a mirror (concave or convex) can be established as

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

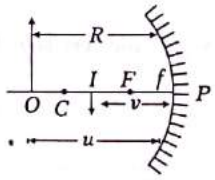


Fig. 9.22 Image and object distances in a concave mirror

Important informations related to mirror formula

(i) For real extended object, if the image is erect and virtual, then for,

Concave mirror : $m > 1$,

Plane mirror : $m = 1$ and

Convex mirror : $m < 1$

If the image is real, then the mirror is concave and for the object

Beyond C : $m < -1$

At C : $m = -1$ and Between C and F : $m > -1$

(ii) Newton's formula

In case of spherical mirrors, if object distance x_1 and image distance x_2 are measured from focus instead of pole, $u = (f + x_1)$ and $v = (f + x_2)$. The mirror formula, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ reduces to, $\frac{1}{f+x_2}+\frac{1}{f+x_1}=\frac{1}{f}$

which on simplification gives,

$$x_1x_2 = f^2$$

This formula is called **Newton's formula.**

(iii) *v versus u* graph for a plane mirror

The focal length and the radius of curvature are infinite for a plane mirror.

$\therefore$ From mirror formula $\frac{1}{v}+\frac{1}{u}=0$

$\Rightarrow$ $v = -u$

Thus ***v versus u* graph for a plane mirror is a straight line with slope equal to – 1** as shown in figure.

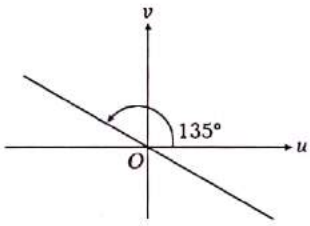


Fig. 9.23 v versus u graph of a plane mirror

(iv) *v versus u* graph for a spherical mirror

For a spherical mirror v *versus* u graph is a rectangular hyperbola as shown in figure.

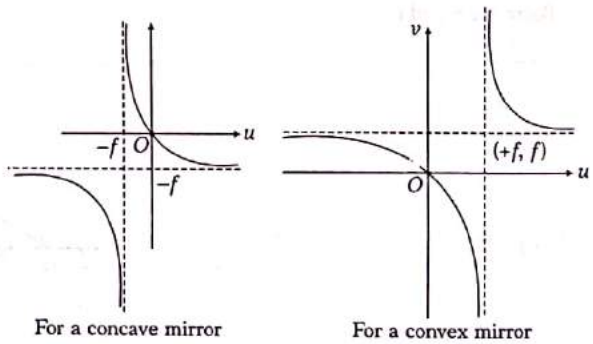


Fig. 9.24 v versus u graph for a spherical mirror

(v) $\frac{1}{v}$ *versus* $\frac{1}{u}$ graph for a spherical mirror

For a spherical mirror $\frac{1}{v}$ *versus* $\frac{1}{u}$ graph is a straight line as shown in figure.

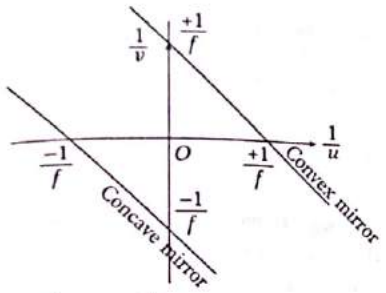


Fig. 9.25 $\frac{1}{v}$ *versus* $\frac{1}{u}$ *graph for a spherical mirror*

Note Coordinates of object if given by x_0, y_0, then coordinates of image is

$$(x_1, y_1) = \left(\frac{fx_0}{x_0 - f}, \frac{fy_0}{f - x_0}\right)$$

Magnification

The ratio of height of image formed by the mirror and height of object is called magnification produced by the mirror.

The lateral, transverse or linear magnification (m)

When an object is placed perpendicular to principal axis then it is defined as the lateral, transverse or linear magnification which is given as,

$$\boxed{m = \frac{\text{Height of image}}{\text{Height of object}}} \quad \text{...(i)}$$

Also, $$m = -\frac{v}{u} = \frac{f}{f-u} = \frac{f-v}{f}$$

Axial or longitudinal magnification (m_L)

When an object is placed along principal axis

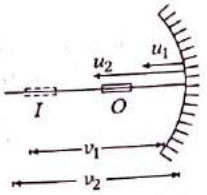


Fig. 9.26 An object placed in front of concave mirror

$$m_L = \frac{-\text{ Length of image}}{\text{Length of object}} = -\frac{(v_2 - v_1)}{u_2 - u_1} = -\frac{\Delta v}{\Delta u}$$

If length of the object is small,

$$m_L = \frac{dv}{du} = \frac{1}{v} + \frac{1}{u} + \frac{1}{f}$$

By diffraction

$$\frac{1}{v^2}\frac{dv}{du} - \frac{1}{u^2} = 0$$

or $$\frac{dv}{du} = -\left(\frac{v^2}{u^2}\right) \Rightarrow m_L = -m^2$$

Here, du = object length, dv = image length

Also, when object is moving then we can write

$$\frac{\text{Velocity of image } (v_I)}{\text{Velocity of object } (v_O)} = -\left(\frac{v}{u}\right)^2 = -\left(\frac{f}{u-f}\right)^2$$

(a) When object moves from $-\infty$ to F, the real image moves from F to $-\infty$ and when the object is placed between $-\infty$ to C, then

$$|v_I| < |v_o|$$

But, when the object is placed between C to F, then

$$|v_I| < |v_o|$$

(b) When the object moves from F to P, the virtual image moves from t_∞ to P, then

$$|v_I| > |v_o|$$

(c) When object is either at C or P, then

$$|v_2| = |v_o|$$

Areal magnification (m_a)

It is given by ratio of area of image (A_I) and area of object (A_O)

$$\therefore\ m_a = \frac{A_I}{A_O} = \left(\frac{v}{u}\right)^2 = \left(\frac{f}{f-u}\right)^2 = \left(\frac{f-v}{f}\right)^2$$

Note We have derived $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ and $m = -\frac{v}{u}$ for special cases of the position of object, but the same result can be derived for other cases also.

Example 9.10 *An object is 30.0 cm from a spherical mirror, along the central axis. The absolute value of lateral magnification is 1/2. The image produced is inverted. What is the focal length of the mirror?*

Sol. Image is inverted, so it is real, u and v both are negative.

Magnification is $\frac{1}{2}$, therefore, $v = \frac{u}{2}$.

Given, $u = -30$ cm and $v = -15$ cm

Using the mirror formula,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

we have, $$\frac{1}{f} = \frac{1}{-15} - \frac{1}{30} = -\frac{1}{10}$$

$\therefore$ Focal length $f = -10$ cm

Example 9.11 *A 4.5 cm needle is placed 12 cm away from a convex mirror of focal length 15 cm. Give the location of the image and the magnification. Describe what happens as the needle is moved farther from the mirror?*

Sol. Using the relation,

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}, \text{ we get}$$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{15} - \left(\frac{1}{-12}\right)$$

$$\therefore \quad v = \frac{20}{3} \text{ cm} = +6.67 \text{ cm} \approx 6.7 \text{ cm}$$

The positive sign shows that the image is formed behind the mirror.

Using the relation, magnification $m = -\frac{v}{u}$, we get

$$m = -\frac{(20/3)}{-12} = +\frac{5}{9}$$

Hence, the image is virtual, erect and diminished.

As the needle is moved farther from the mirror, the image moves towards the focus (upto the focus only) and gets smaller and smaller in size.

Example 9.12 *A small candle 2.5 cm in size is placed 27 cm in front of a concave mirror of radius of curvature 36 cm. At what distance from the mirror should a screen be placed in order to obtain a sharp image? Describe the nature and size of the image.*

Sol. (a) Using the relation, $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{-18} - \frac{1}{(-27)}$$

$$\therefore \quad v = -54 \text{ cm}$$

The negative sign shows that the image is formed in front of the mirror, *i.e.*, on the side of the object itself. Thus, the screen must be placed at a distance of 54 cm in front of the mirror.

(b) Magnification, $m = -\frac{v}{u} = \frac{I}{O}$

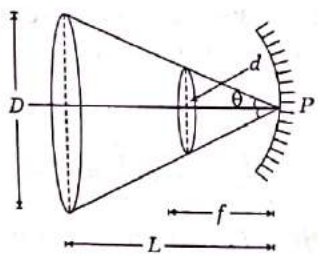
we get, $\frac{I}{2.5} = -\frac{(-54)}{(-27)}$

or $I = (2.5) \times (-2) = -5$ cm

Thus clearly the image is real, inverted and magnified.

Example 9.13 *Determine the diameter of the image formed by a spherical concave mirror of focal length 8 m. The diameter of the moon is 3450 km and the distance between the earth and the moon is approx 4×10^5 km.*

Sol.



The image will be formed at focus because object is at large distance.

The angle subtended by diameter of moon at pole,

$\tan\theta = \frac{D}{L}$, since θ is small, $\tan\theta = \frac{D}{L}$ radian

By similar triangles,

$$\theta = \frac{d}{f} = \frac{D}{L}$$

Diameter of image of moon,

$$d = f\theta = f\frac{D}{L} = 8 \times \frac{3450}{4 \times 10^5} = 0.069 \text{ m} = 6.9 \text{ cm}$$

Example 9.14 *Find the distance of object from a concave mirror of focal length 10 cm, so that image size is four times the size of the object.*

Sol. Concave mirror can form real as well as virtual image. Here nature of image is not given in the question. So, we will consider two possible cases.

***Case I* (When image is real)** Real image is formed on the same side of the object, *i.e.*, u, v and f all are negative. So let,

$$u = -x$$

then, $v = -4x$ as $\left|\frac{v}{u}\right| = |m| = 4$ and $f = -10$ cm

Substituting in, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

We have $\frac{1}{-4x} - \frac{1}{x} = \frac{1}{-10}$ or $\frac{5}{4x} = \frac{1}{10}$

$$\therefore \quad x = 12.5 \text{ cm}$$

Note $|x| > |f|$ and we know that in case of a concave mirror, image is real when object lies beyond F.

***Case II* (When image is virtual)** In case of a mirror image is virtual when it is formed behind the mirror, *i.e.*, u and f are negative while v is positive. So let,

$$u = -y$$

then, $v = +4y$ and $f = -10$ cm

Substituting in, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

we have, $\frac{1}{4y} - \frac{1}{y} = \frac{1}{-10}$

or $\frac{3}{4y} = \frac{1}{10}$ or $y = 7.5$ cm

Note Here $|y| < |f|$ as we know that image is virtual when the object lies between F and P.

Example 9.15 *A concave mirror has a radius of curvature of 24 cm. How far is an object from the mirror, if an image is formed that is*

(i) virtual and 3.0 times the size of the object,

(ii) real and 3.0 times the size of the object and

(iii) real and 1/3 the size of the object?

Sol. Given, $R = -24$ cm (concave mirror)

Hence, $f = \frac{R}{2} = -12$ cm

(i) Image is virtual and 3 times larger. Hence, u is negative and v is positive. Simultaneously, $|v| = 3|u|$. So let,

$$u = -x$$

then, $v = +3x$

Substituting in the mirror formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, we have

$$\frac{1}{3x} - \frac{1}{x} = \frac{1}{-12}$$

$$\therefore \quad x = 8 \text{ cm}$$

Therefore, object distance is 8 cm.

(ii) Image is real and three times larger. Hence, u and v both are negative and $|v| = 3|u|$. So let,

$$u = -x$$

then, $$v = -3x$$

Substituting in mirror formula, we have

$$\frac{1}{-3x} - \frac{1}{x} = -\frac{1}{12} \quad \text{or} \quad x = 16 \text{ cm}$$

∴ Object distance should be 16 cm.

(iii) Image is real and $\frac{1}{3}$rd the size of object. Hence, both u and v are negative and $|v| = \frac{|u|}{3}$. So let,

$$u = -x$$

then, $$v = -\frac{x}{3}$$

Substituting in the mirror formula, we have

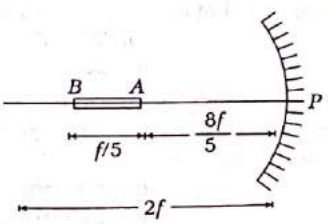
$$-\frac{3}{x} - \frac{1}{x} = -\frac{1}{12}$$

∴ $$x = 48 \text{ cm}$$

∴ Object distance should be 48 cm.

Example 9.16 *A thin stick of length f/5 is placed along the principal axis of a concave mirror of focal length f such that its image which is real and elongated just touches the stick. What is the magnification?*

Sol.



One end of image coincides with one end of stick *i.e.*, either A or B is centre of curvature.

Since image is elongated *i.e.*, point B is centre of curvature. Image of B is formed on B itself.

Image of A

$$u = -(2f - f/5) = \frac{-8f}{3}; \quad f = -f, v = ?$$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \quad \Rightarrow \quad \frac{1}{v} + \frac{1}{-8f/5} = \frac{1}{-f}$$

$$\frac{1}{v} = -\frac{1}{f} + \frac{5}{8f} = \frac{-8+5}{8f} = \frac{-3}{8f} \Rightarrow v = -\frac{8f}{3}$$

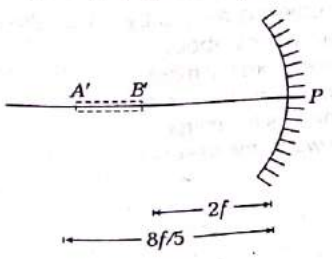


Image length, $A'B' = \frac{8f}{5} - 2f = -2f/5$

Longitudinal magnification,

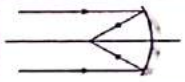
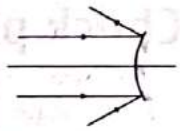
$$m_L = \frac{-A'B'}{AB} = \frac{2f/5}{f/5} = 2$$

Power of a mirror

It is the ability of a mirror to deviate the path of rays striking it.

$$P = \frac{-1}{f(\text{metre})} = \frac{-100}{f(\text{m})} \text{ Diopter (D)}$$

Following table gives, the sign of P and f for different type of mirrors.

Nature of lens/mirror	Focal length (f)	Power $P_M = -1/f$	Converging /diverging	Ray diagram
Concave mirror	−ve	+ve	converging	
Convex mirror	+ve	−ve	diverging	

Example 9.17 *A convex mirror is formed from a spherical surface of radius 20 cm. Find power of the mirror.*

Sol. Given, radius of curvature, $R = 20$ cm

∴ Focal length of the mirror,

$$f = \frac{R}{2} = \frac{20}{2} = 10 \text{ cm} = 0.1 \text{ m}$$

∴ Power of the mirror, $P = \frac{1}{f} = \frac{1}{0.1} = +10 \text{ D}$

Example 9.18 *An object is 40 cm from a spherical mirror, along the central axis. The absolute value of lateral magnification is 1/2. The image produced is inverted. What is the power of the mirror?*

Sol. Image is inverted, so it is real u and v both are negative.

Magnification is $\frac{1}{2}$, therefore, $v = \frac{u}{2}$.

Given, $u = -40$ cm and $v = -20$ cm

Using the mirror formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

We have, $$\frac{1}{f} = \frac{1}{-20} - \frac{1}{40} = \frac{-2-1}{40} = \frac{-3}{40}$$

$$\Rightarrow \quad f = -\frac{40}{3} \text{ cm} = -\frac{40}{3} \times \frac{1}{100} \text{ m} = -\frac{2}{15} \text{ m}$$

∴ Power of the mirror, $P = -\frac{1}{f} = \frac{-1}{-\frac{2}{15}} = \frac{15}{2} = +7.5 \text{ D}$

Uses of spherical mirrors

Uses of spherical mirrors (concave and convex mirrors) are given below.

Concave mirrors

(i) Concave mirror is used as reflectors in search light, cinema projectors and headlights of vehicles, etc.

(ii) Concave mirror is also used as face looking or sharing mirror because it forms erect and magnified image.

(iii) Concave mirrors are used in dish antennas.

(iv) Concave mirrors are used by dentists and ENT specialists, in solar cookers, etc.

Convex mirrors

(i) Convex mirrors are used as a reflector in street lamps to diverge the light over a large area.

(ii) Convex mirrors are used as rear view mirror (or driver's mirror) in vehicles, because it has a wider field of view than a plane mirror as shown in figure.

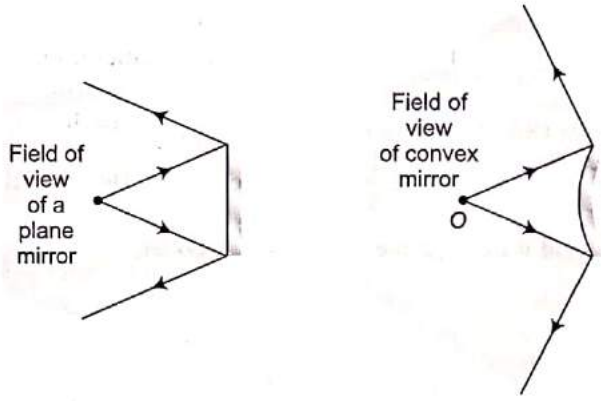


Fig. 9.27

Check point 9.2

1. A concave mirror of focal length f (in air) is immersed in water ($\mu = 4/3$). The focal length of the mirror in water will be

(a) f (b) $\frac{4}{3}f$

(c) $\frac{3}{4}f$ (d) $\frac{7}{3}f$

2. An object is placed 40 cm from a concave mirror of focal length 20 cm. The image formed is

(a) real, inverted and same in size
(b) real, inverted and smaller in size
(c) virtual, erect and larger in size
(d) virtual, erect and smaller in size

3. A point object is placed at a distance of 30 cm from a convex mirror of focal length 30 cm. The image will form at

(a) infinity (b) pole
(c) focus (d) 15 cm behind the mirror

4. An object is placed at a distance of 30 cm from a concave mirror and its real image is formed at a distance of 30 cm from the mirror. The focal length of the mirror is

(a) 15 cm (b) 45 cm
(c) 30 cm (d) 20 cm

5. A convex mirror of focal length f forms an image which is $1/n$ times the object. The distance of the object from the mirror is

(a) $(n-1)f$ (b) $\left[\frac{n-1}{n}\right]f$

(c) $\left[\frac{n+1}{n}\right]f$ (d) $(n+1)f$

6. The focal length of a concave mirror is 50 cm. Where an object be placed, so that its image is two times and inverted

(a) 75 cm (b) 60 cm
(c) 125 cm (d) 50 cm

7. An object of size 7.5 cm is placed in front of a convex mirror of radius of curvature 25 cm at a distance of 40 cm.The size of the image should be

(a) 2.3 cm (b) 1.78 cm
(c) 1 cm (d) 0.8 cm

8. The image formed by a convex mirror of focal length 30 cm is a quarter of the size of the object. The distance of the object from the mirror is

(a) 30 cm (b) 90 cm
(c) 120 cm (d) 60 cm

9. A concave mirror gives an image three times as large as the object placed at a distance of 20 cm from it. For the image to be real, the focal length should be

(a) 10 cm (b) 15 cm
(c) 20 cm (d) 30 cm

10. A point object is placed at a distance of 10 cm and its real image is formed at a distance of 20 cm from a concave mirror. If the object is moved by 0.1 cm towards the mirror, the image will shift by about

(a) 0.4 cm away from the mirror
(b) 0.8 cm away from the mirror
(c) 0.4 cm towards the mirror
(d) 0.8 cm towards the mirror

3) Refraction of light

When a light passes from one medium to another medium, it deviates from its original path. The phenomenon of change in path of light at the boundary of separation of two medium, as it goes from one medium to another medium called **refraction.**

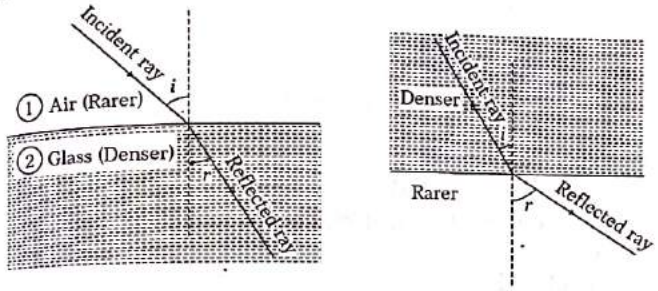


Fig.9.28 Refraction of light

- When ray of light goes from a rarer medium to a denser medium, it bends towards the normal.
- When a ray of light goes from a denser medium to a rarer medium, it bends away from the normal.

Laws of refraction

There are two laws of refraction as follows

(i) For two particular media, the ratio of the sine of the angle of incidence to the sine of the angle of refraction is constant, *i.e.*,

$$\frac{\sin i}{\sin r} = \text{constant} = \frac{\mu_2}{\mu_1} \quad \text{...(i)}$$

This is known as **Snell's law.**

(ii) The incident ray, the reflected ray and the refracted ray all lie in the same plane.

Note The angle of deviation on refraction is $\delta = |i - r|$, If the incident ray is normal, refraction takes place without bending. In this case $i = r = 0$, *i.e.*, incident ray passes undeviated.

Refractive index (μ or n)

The constant ratio $\frac{\sin i}{\sin r}$ is called the relative **refractive index**, for light passing from the first to the second medium.

Thus,

$$\mu = \frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} \quad \text{...(ii)}$$

Absolute refractive Index

When light travels from vacuum (or air) to any transparent medium then refractive index of medium with respect to vacuum (or air) is called it's absolute refractive index *i.e.*,

$$\mu = \frac{\text{speed of light in vacuum}}{\text{speed of light in medium}} = \frac{c}{v}$$

Relative refractive Index

When light travels from medium (1) to medium (2) then refractive index of medium (2) with respect to medium (1) it's called relative refractive index *i.e.*,

$${}_1\mu_2 = \frac{\mu_2}{\mu_1}$$

Refractive index is a dimensionless quantity and having no unit.

Dependence of refractive index

The value of refractive index depends on

(i) The nature of the two media.

(ii) The temperature of the media.

(iii) The wavelength of the light.

(iv) It does not depend on the value of i (however refractive index $= \frac{\sin i}{\sin r}$) for a pair of media.

(v) According to Cauchy's formula,

$$\mu = A + \frac{B}{\lambda^2}$$

where, A and B are constants

$\therefore$ As λ increases, μ decreases.

Important points about refractive index

(i) Refractive index is property of the medium.

(ii) A medium with higher value of μ is called denser medium and with the lower value of μ is called rarer medium.

(iii) The minimum value of refractive index (absolute) is 1 for vacuum. For air, it is 1.003 but practically, we will use $\mu = 1$ for air.

(iv) Maximum value of μ is 2.46 for diamond.

Special points about refraction

(i) We can write Snell's law as,

$$\mu \sin i = \text{constant} \quad \text{...(i)}$$

For two media, $\mu_1 \sin i = \mu_2 \sin r$

or $$\frac{\mu_2}{\mu_1} = \frac{\sin i}{\sin r} = {}_1\mu_2 \quad \text{...(ii)}$$

(ii) From Eq. (i), we can see that $i > r$ if $\mu_2 > \mu_1$, *i.e.*, if a ray of light passes from a rarer to a denser medium it bends towards normal and *vice-versa*.

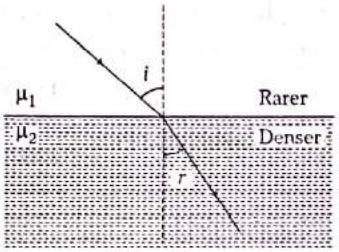


***Fig. 9.29** Light ray travelling from rarer to denser medium*

(iii) Since, ${}_1\mu_2 = \dfrac{\mu_2}{\mu_1}$ and ${}_2\mu_1 = \dfrac{\mu_1}{\mu_2}$

$$\therefore \quad {}_1\mu_2 = \frac{1}{{}_2\mu_1}$$

Similarly, for three mediums,

$${}_1\mu_2 = \frac{\mu_2}{\mu_1}, \quad {}_2\mu_3 = \frac{\mu_3}{\mu_2} \quad \text{and} \quad {}_3\mu_1 = \frac{\mu_1}{\mu_3}$$

$$\therefore \quad {}_1\mu_2 \times {}_2\mu_3 \times {}_3\mu_1 = 1$$

For n mediums,

$${}_1\mu_2 \times {}_2\mu_3 \times {}_3\mu_4 \times \ldots \times (n-1)\mu_n = {}_1\mu_n$$

It is called **reversibility of light**. According to this principal, **when a light ray, after suffering *n* number of reflections and refractions, will has its final path reversed, *i.e.*, it travels back along its entire initial path.**

(iv) Snell's law can also be written as,

$${}_1\mu_2 = \frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} = \frac{\mu_2}{\mu_1} \qquad \ldots\text{(iii)}$$

Here, v_1 is the speed of light in medium 1 and v_2 in medium 2. Similarly, λ_1 and λ_2 are the corresponding wavelengths.

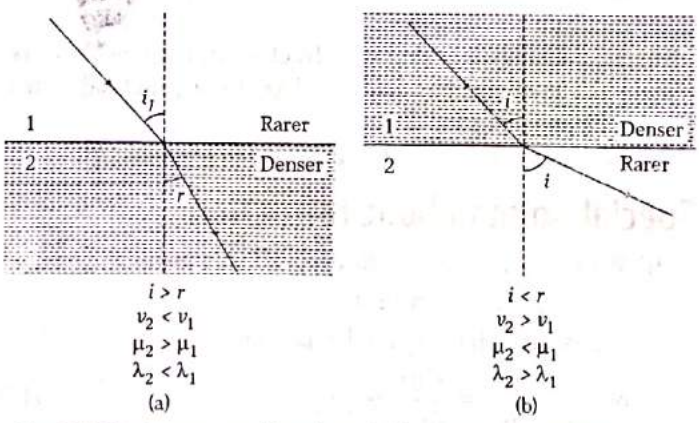


***Fig. 9.30** Light ray travelling from (a) Rarer to denser medium and (b) denser to rarer medium*

(v) As a ray of light moves from medium (1) to medium (2), its wavelength changes but its frequency remains constant.

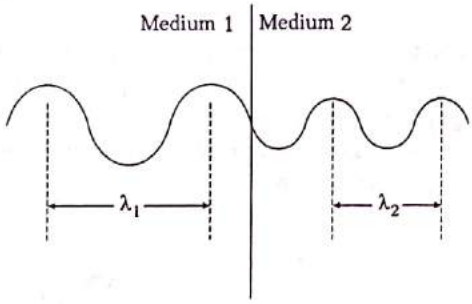


Fig. 9.31

(vi) Experiments show that, if the boundaries of the media are **parallel** the emergent ray *CD*, although laterally displaced. If reaches in the same medium, is parallel to the incident ray, however it is laterally displaced.

We can also directly apply the Snell's law, in medium 1 and 5 *i.e.*,

$$\mu_1 \sin i_1 = \mu_5 \sin i_5$$

$$\Rightarrow \quad i_i = i_5 \text{ if } \mu_1 = \mu_5 \quad (\because \mu \sin i = \text{constant})$$

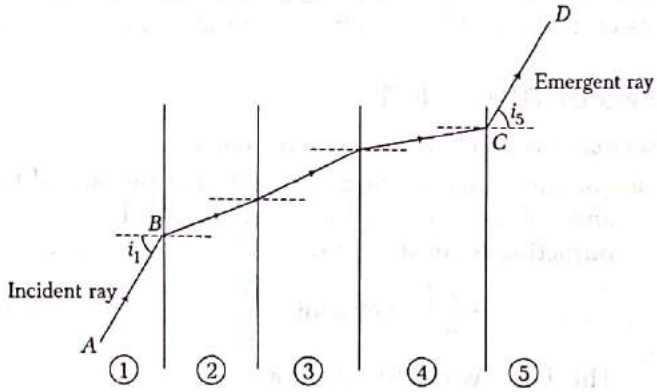


Fig. 9.32

If any of the boundary is not parallel we cannot use this law directly by jumping the intervening media.

Example 9.19 *(i) Find the speed of light of wavelength $\lambda = 780\,nm$ (in air) in a medium of refractive index $\mu = 1.55$.*

(ii) What is the wavelength of this light in the given medium?

Sol. (i) Speed of light, $v = \dfrac{c}{\mu} = \dfrac{3.0 \times 10^8}{1.55}$

$$= 1.94 \times 10^8 \text{ m/s}$$

(ii) Wavelength of light in the given medium,

$$\lambda_{\text{medium}} = \frac{\lambda_{\text{air}}}{\mu} = \frac{780}{1.55} = 503 \text{ nm}$$

Example 9.20 *Light of wavelength 300 nm in medium A enters into medium B through a plane surface. If frequency of light is 5×10^{14} Hz and $v_A / v_B = 4/5$, then find absolute refractive indices of media A and B.*

Sol. Speed, $v_A = \nu\lambda_A = 5 \times 10^{14} \times 300 \times 10^{-9} = 1.5 \times 10^8$ m/s

Absolute refractive index, $\mu_A = \dfrac{c}{v_A} = \dfrac{3 \times 10^8}{1.5 \times 10^8} = 2$

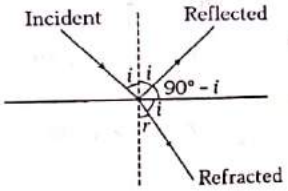
We have, $\mu \propto \dfrac{1}{v}$

$\therefore \quad \dfrac{\mu_A}{\mu_B} = \dfrac{v_B}{v_A} \Rightarrow \dfrac{2}{\mu_B} = \dfrac{5}{4}$

Refractive index, $\mu_B = 0.8 \times 2 = 1.6$

Example 9.21 *A ray of light falls on a glass plate of refractive index $\mu = 1.5$. What is the angle of incidence of the ray, if the angle between the reflected and refracted rays is 90°?*

Sol. In the figure, $r = 90 - i$



From Snell's law,

$$1.5 = \frac{\sin i}{\sin r} = \frac{\sin i}{\sin(90° - i)} = \tan i$$

$\therefore$ Angle of incidence, $i = \tan^{-1}(1.5) = 56.3°$

Example 9.22 *Refractive index of glass with respect to water is (9/8). Refractive index of glass with respect to air is (3/2). Find the refractive index of water with respect to air.*

Sol. Given, ${}_w\mu_g = 9/8$ and ${}_a\mu_g = 3/2$

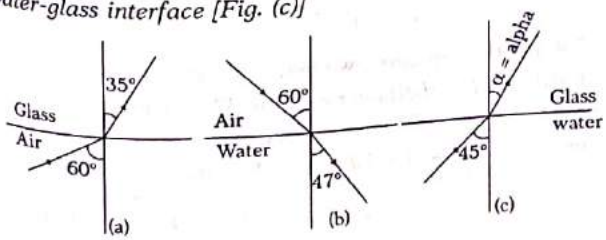
As, ${}_a\mu_g \times {}_g\mu_w \times {}_w\mu_a = 1$

$\therefore \quad \dfrac{1}{{}_w\mu_a} = {}_a\mu_w = {}_a\mu_g \times {}_g\mu_w = \dfrac{{}_a\mu_g}{{}_w\mu_g}$

$\therefore$ Refractive index of water with respect to air,

$${}_a\mu_w = \frac{3/2}{9/8} = \frac{4}{3}$$

Example 9.23 *Figures (a) and (b) show refraction of an incident ray in air at 60° with the normal to a glass-air and water-air interface, respectively. Predict the angle of refraction in glass when the angle of incidence in water is 45° with the normal to a water-glass interface [Fig. (c)]*



Sol. From Fig. (a)

i = angle of incidence = 60°

r = angle of refraction = 35°

$\therefore$ Refractive index of glass with respect to water,

$${}_a\mu_g = \frac{\sin i}{\sin r} = \frac{\sin 60°}{\sin 35°} = \frac{0.8660}{0.5736} = 1.51$$

From Fig. (b), here, $i = 60°$ and $r = 47°$

$\therefore$ Refractive index of water with respect to air,

$${}_a\mu_w = \frac{\sin i}{\sin r} = \frac{\sin 60°}{\sin 47°} = \frac{0.8660}{0.7314} = 1.18$$

$\therefore$ we have ${}_a\mu_g = {}_a\mu_w \times {}_w\mu_g$

or ${}_w\mu_g = \dfrac{{}_a\mu_g}{{}_a\mu_w} = \dfrac{1.51}{1.18} = 1.28$

From Fig. (c), i = angle of incidence = 45°

$\therefore$ Using the relation,

$${}_w\mu_g = \frac{\sin i}{\sin r} \quad \text{or} \quad 1.28 = \frac{\sin 45°}{\sin r}$$

or $\sin r = \dfrac{\sin 45°}{1.28} = \dfrac{1/\sqrt{2}}{1.28}$

$$= \frac{0.707}{1.28} = 0.5525 = \sin 33.54'$$

$\therefore$ Angle of refraction, $r = 33.54°$

Example 9.24 *A light beam passes from medium 1 to medium 2. Show that the emerging beam is parallel to the incident beam.*

Sol. Applying Snell's law at A and B,

$$\mu_1 \sin i_1 = \mu_2 \sin i_2$$

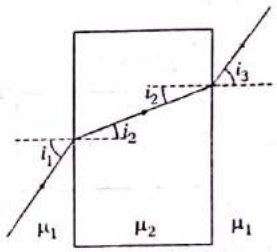


Fig. 24.36

or $\dfrac{\mu_1}{\mu_2} = \dfrac{\sin i_2}{\sin i_1}$ ...(i)

Similarly $\mu_2 \sin i_2 = \mu_1 \sin i_3$

$\therefore \quad \dfrac{\mu_1}{\mu_2} = \dfrac{\sin i_2}{\sin i_3}$ ...(ii)

From Eqs. (i) and (ii),

$$i_3 = i_1$$

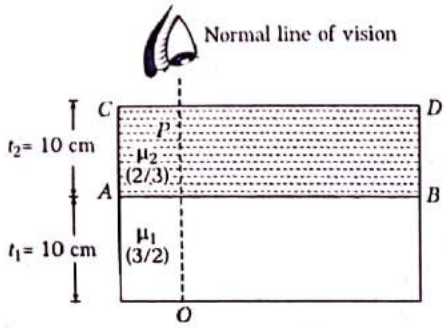
i.e., the emergent ray is parallel to incident ray.

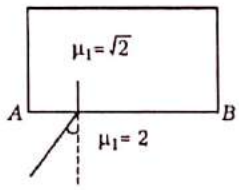
Example 9.25 *A rectangular glass block of thickness 10 cm and refractive index 1.5 is placed over a small coim. A beaker is filled with water of refractive index 2/3 to a height of 10 can and is placed oved a small block.*

(a) Find the apparent position of the object when it is viewed at normal incidence.

(b) If the eye is slowly and moved away from the normal at a certain position, the is found to disapear due to total internal reflection. At what surface does it happen and why?

Sol. Given,





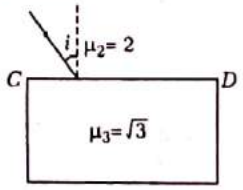
Cricical angle,

$$\mu_2 \sin C = \mu_1 \sin 90°$$

$$2\sin C = \sqrt{2} \Rightarrow \sin C = \frac{1}{\sqrt{2}} = \sin 45°$$

$$C = 45°$$

For T.I.R at AB, $i > C$ *i.e.*, $i > 45°$



Critical angle

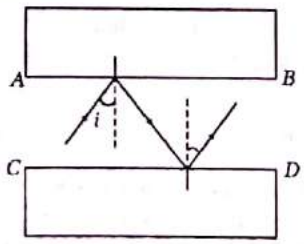
$$\mu_2 \sin C = \mu_3 \sin 90°$$

$$2\sin C = \sqrt{3} \Rightarrow \sin C = \frac{\sqrt{3}}{2} = \sin 60°$$

$$C = 60°$$

For T.I.R. at CD, $i > C$ *i.e.*, $i > 60°$

$$i_{min} = 60°$$



(a) Total shift,

$$OI = t_1(1 - 1/\mu_1) + t_2(1 - 1/\mu_2)$$

$$= 10\left(1 - \frac{1}{3/2}\right) + 10\left(1 - \frac{1}{4/3}\right)$$

$$= \frac{10}{3} + \frac{10}{4}$$

Distance of image from P,

$$PI = PO - OI = (10 + 10) - \left(\frac{10}{3} + \frac{10}{4}\right)$$

$$= 14.17 \text{ cm}$$

(b) The image of coin dispperas when the ray suffers TIR at either at AB or CD

The critical angles for glass-water surface

$$\mu_1 \sin C_1 = \mu_2 \sin 90°$$

$$\frac{3}{2}\sin C_1 = \frac{2}{3}$$

$$\sin C_1 = \frac{4}{9}$$

$$C_1 = \sin^{-1}(4/9)$$

The critical angles for water-air surface

$$\mu_2 \sin C_2 = 1 \times \sin 90°$$

$$\frac{2}{3}\sin C_2 = 1$$

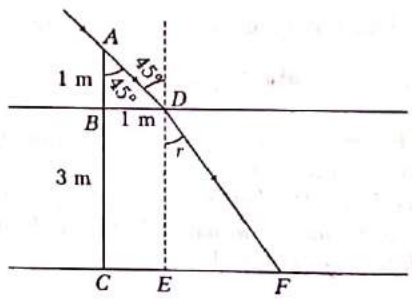
$$\sin C_2 = \frac{3}{2}$$

$C_2 < C_1$ TIR occurs earlier at the water-air surface as eye is moved away from the normal.

Example 9.26 *A pile 4 m high driven into the bottom of a lake is 1 m above the water. Determine the length of the shadow of the pile on the bottom of the lake, if the sun rays make an angle of 45° with the water surface. The refractive index of water is 4/3.*

Sol. From Snell's law,

$$\frac{4}{3} = \frac{\sin 45°}{\sin r}$$



Solving this equation, we get, $r = 32°$

Further, $EF = (DE)\tan r = (3)\tan 32° = 1.88$ m

$\therefore$ Total length of shadow, $L = CF$

or $L = (1 + 1.88)$ m $= 2.88$ m

Image due to refraction at a plane surface

When a light ray passes from one medium to another medium, then it bends at the interface of the two media. Due to this bending or refraction of light, image is formed which creates an illusion of the shifting of the object position.

Following two types of shifts are possible after refraction from a plane surface

Type I *The object (O) is in denser medium (glass, water, etc.) and is seen from rarer medium (air) normally through plane surface.*

In this condition, due to refraction of light, object in the denser medium as seen from rarer medium seems to be raised above as compared to its actual depth below the boundary of denser medium.

In the figure, $\angle AOB$ will be r and $\angle AIB$ is i. For normal incidence, (*i.e.*, small angles of i and r)

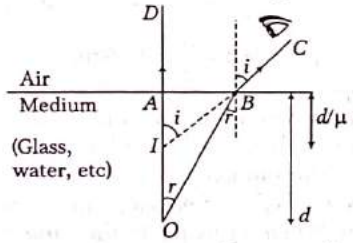


Fig. 9.33 Object is in denser medium

$$\sin i \approx \tan i = \frac{AB}{AI} \quad \text{...(i)}$$

and
$$\sin r \approx \tan r = \frac{AB}{AO} \quad \text{...(ii)}$$

On dividing Eq. (i) by Eq. (ii), we have

$$\frac{\sin i}{\sin r} = \frac{AO}{AI}$$

or
$$\mu = \frac{AO}{AI} \qquad \left(\because \frac{\sin i}{\sin r} = \mu\right)$$

$\therefore$
$$AI = \frac{AO}{\mu} = \frac{d}{\mu}$$

If point O is at a depth of d from a water surface, then the above result is also sometimes written as,

$$AI = d_{\text{apparent}} = \frac{AO\,(= d_{\text{actual}})}{\mu} = \frac{d}{\mu}$$

$$\therefore \text{ Shift} = OI = AO - AI = d\left(1 - \frac{1}{\mu}\right)$$

$$\boxed{\text{Apparent depth} = d\left(1 - \frac{1}{\mu}\right)}$$

where, d is actual depth of the object in the denser medium of refractive index μ.

Lateral magnification

If immiscible liquids of refractive indices μ_1, μ_2 and μ_3 (with $\mu_3 > \mu_2 > \mu_1$) are filled in a vessel. Their depths are d_1, d_2 and d_3, respectively. Then, the apparent depth (for normal incidence) when seen from top of the first liquid will be

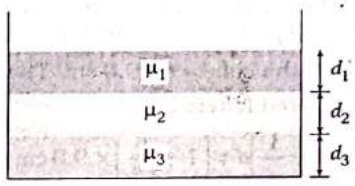


Fig. 9.34

$$d_{\text{app}} = \frac{d_1}{\mu_1} + \frac{d_2}{\mu_2} + \frac{d_3}{\mu_3}$$

Similarly for n liquids,

$$d_{\text{app}} = \frac{d_1}{\mu_1} + \frac{d_2}{\mu_2} + \ldots + \frac{d_n}{\mu_n}$$

Also apparent shift of the bottom upward

$$\text{Apparent shift} = d_1\left(1 - \frac{1}{\mu_1}\right) + d_2\left(1 - \frac{1}{\mu_2}\right) + \ldots + d_n\left(1 - \frac{1}{\mu_n}\right)$$

Type II *The object (O) is in rarer medium (air) and is seen from denser medium (glass, water, etc.), normally through plane surface.*

In this condition, due to refraction of light, object in the rarer medium as seen from denser medium seams to be raised above as compared to its actual height above the boundary of rarer medium. As the ray of light passes from rarer to denser medium, it bends towards the normal and hence the object will appear farther (at I) as shown in figure.

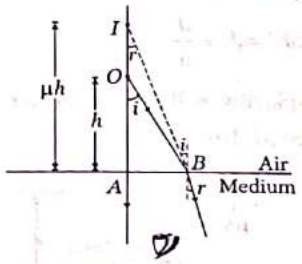


Fig. 9.35 Object is in rarer medium

Proceeding as previous Eqs. (i) and (ii), we get

$$\mu = \frac{\sin i}{\sin r} = \frac{\tan i}{\tan r} = \frac{AB}{AO} \times \left(\frac{AI}{AB}\right)$$

$$\Rightarrow \quad AI = \mu\,(AO) = \mu(h)$$

$\therefore$ Apparent height $(AI) = \mu$ (actual height)

$\therefore$ Shift $= OI = AI - AO = (\mu - 1)h$

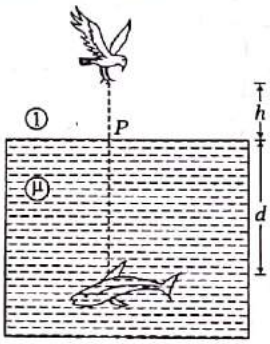
Note In all the above cases, the change in the value of x is μ times whether it is increasing or decreasing. All the relations can be derived for small angles of incidence as done in part (a).

Example 9.27 *A printed page is kept pressed by a glass cube ($\mu = 1.5$) of edge 9.0 cm. By what amount will the printed letters appear to be shifted when viewed from the top?*

Sol. The thickness of the cube $= t = 9.0$ cm. The shift in the position of the printed letters is

$$\Delta d = \left(1 - \frac{1}{\mu}\right)d = \left(1 - \frac{1}{1.5}\right) \times 9.0 \text{ cm} = 3.0 \text{ cm}$$

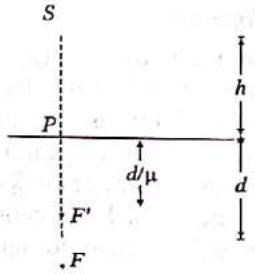
Example 9.28 *Consider the situation as shown in the diagram.*



Find the distance between sparrow and fish as seen by (i) sparrow and (ii) fish.

Sol. (i) For sparrow, fish will appear nearer.

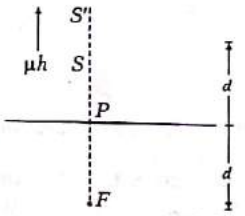
At distance d/μ from P,



$$SF' = h + \frac{d}{\mu}$$

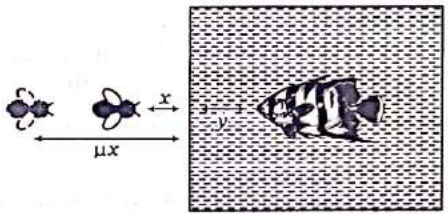
(ii) For fish, sparrow will appear farther

At distance μh from P



$$FS' = d + \mu h$$

Example 9.29 *A fish in an aquarium, approaches the left wall at a rate of 3 ms^{-1}, and observes a fly approaching it at 8 ms^{-1}. If the refractive index of water is (4/3), then find the actual velocity of the fly.*



Sol. For the fish, apparent distance of the fly from the wall of the aquarium is μx. If x is actual distance, then apparent velocity will be $= \dfrac{d(\mu x)}{dt}$

$$(v_{app})_{fly} = \mu\, v_{fly}$$

Now, the fish observes the velocity of the fly to be 8 ms^{-1}.

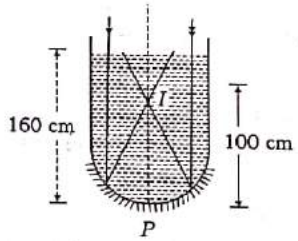
Therefore, apparent relative velocity $= 8$ ms^{-1}

$$\Rightarrow \quad v_{fish} + (v_{app})_{fly} = 8 \text{ ms}^{-1} \Rightarrow 3 + \mu v_{fly} = 8$$

$$\therefore \text{ Velocity of fly } v_{fly} = 5 \times \frac{3}{4} = 3.75 \text{ ms}^{-1}$$

Example 9.30 *A concave mirror of radius of curvature two metre is placed at the bottom of a tank of water. The mirror forms an image of the sun when it is directly overhead. Calculate the distance of the images from the mirror for (i) 160 cm and (ii) 80 cm of water in the tank. (Take, $\mu = 4/3$ for water)*

Sol. (i)

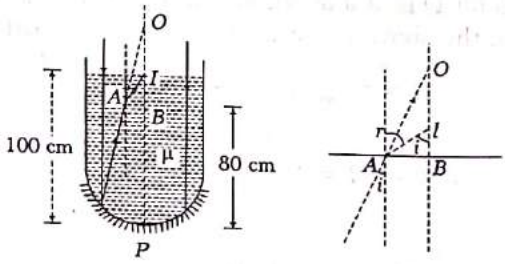


Focal length of mirror, $f = \dfrac{R}{2} = \dfrac{200}{2} = 100$ cm

For parallel rays, image is formed at focus of mirror,

$$PI = 100 \text{ cm}$$

(ii) When depth of water is 80 cm



$$\mu \sin i = 1 \times \sin r$$

As angles i and r are small, $\sin i = \tan i$, $\sin r \simeq \tan r$

$$\mu \tan i = \tan r$$

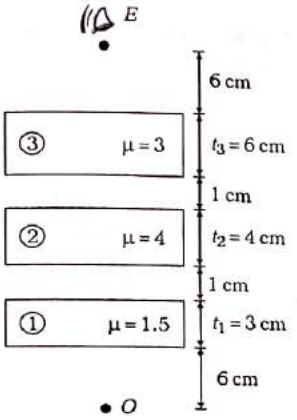
$$\frac{4}{3}\cdot\frac{AB}{OB}=\frac{AB}{BI} \Rightarrow \frac{4}{3}\cdot\frac{1}{20}=\frac{1}{BI}$$

$$BI = 15 \text{ cm} \Rightarrow PI = 80 + BI = 80 + 15 = 95 \text{ cm}$$

Alternatively

$$\mu(BI) = BO \Rightarrow \frac{4}{3}\times BI = 20 \Rightarrow BI = 15 \text{ cm}$$

Example 9.31



Consider the situation shown in figure. Find distance of image of O from eye E.

Sol. Shift by (1); $t_1(1-1/\mu_1) = 3(1-2/3) = 1$ cm

Shift by (2); $t_2(1-1/\mu_2) = 4(1-4) = 3$ cm

Shift by (3); $t_3(1-1/\mu_3) = 6(1-1/3) = 4$ cm

Total shift $= 1 + 3 + 4 = 8$ cm

Image of O is formed at I.

$$OI = 8 \text{ cm} \Rightarrow EO = 6+6+1+4+1+3+6 = 27 \text{ cm}$$

$$EI = EO - OI = 27 - 8 = 19 \text{ cm}$$

Refraction through a glass slab

Normal shift

If a glass slab is placed in the path of a converging or diverging beam of light, then point of convergence or point of divergence appears to be shifted as shown in figure.

Normal shift,

$$OO' = x = \left(1-\frac{1}{\mu}\right)t$$

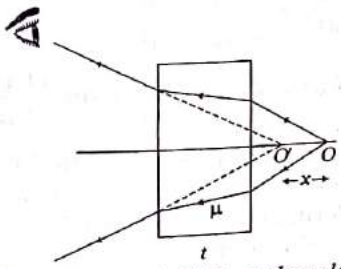


Fig. 9.36 Normal shift by a glass slab

Note Due to slab, if the $\mu_{slab} > \mu_{medium}$, then the shift is along the direction of incident ray and if, $\mu_{slab} > \mu_{medium}$, then the shift will be opposite direction.

Lateral shift

The refracting surfaces of a glass slab are parallel to each other. When a light (ray passes through a glass slab it is refracted twice) at the two parallel faces and finally emerges out parallel to it's incident direction *i.e.*, the ray undergoes no deviation $\delta = 0$.

The angle of emergence (e) is equal to the angle of incidence (i). The lateral shift of the ray is the perpendicular distance between the incident and emergent ray, and it is given by $MN = d = t \sec r \sin(i - r)$.

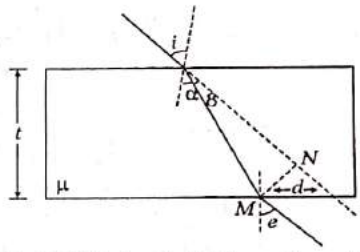


Fig. 9.37 Lateral shift by a glass slab

or
$$d = \left[1 - \frac{\cos i}{\sqrt{\mu^2 - \sin^2 i}}\right] t \sin i$$

For small angle of incidences,

$$d = ti\left(\frac{\mu - 1}{\mu}\right)$$

Note Optical path is defined as distance travelled by light in vacuum in the same time in which it travels a given path length in a medium. Time taken by light ray to pass through the other medium, where x = geometrical path and μx optical path is $\frac{\mu x}{c}$.

Example 9.32 *A small pin fixed on a table top is viewed from above from a distance of 50 cm. By what distance would the pin appear to be raised if it is viewed from the same point through a 15 cm thick glass slab held parallel to the table? Refractive index of glass = 1.5. Does the answer depend on the location of the slab?*

Sol. d = normal shift $= t - \frac{t}{u} = t\left(1 - \frac{1}{\mu}\right)$

$$= 15\left(1 - \frac{1}{1.5}\right) = 15\left(1 - \frac{2}{3}\right)$$

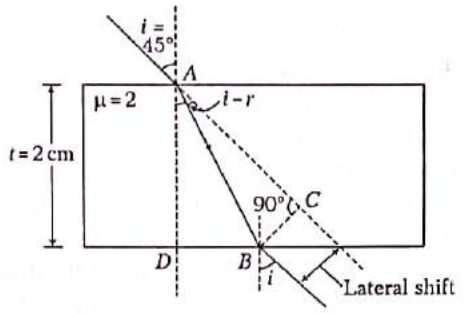
$$= 15 \times \frac{1}{3} = 5 \text{ cm}$$

i.e., the pin appears raised by 5 cm.

No, the answer does not depend upon the location of the slab for small angles of incidence.

Example 9.33 *A light ray is incident at an angle of 45° with the normal to a 4 cm thick plate ($\mu = 2.0$). Find the shift in the path of the light as it emerges out from the plate.*

Sol. Consider the ray diagram as shown below



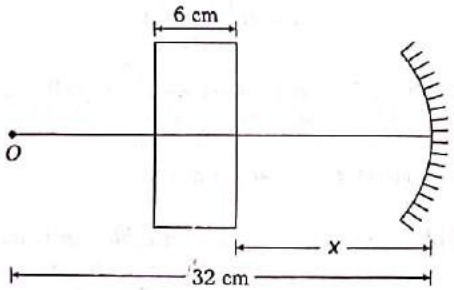
$$\text{Lateral shift, } BC = \frac{t\sin(i-r)}{\cos r} = t\sin i\left[1-\frac{\cos i}{\sqrt{\mu_2-\sin^2 i}}\right]$$

$$= 4\sin 45°\left[1-\frac{\cos 45°}{\sqrt{(2)^2-(\sin^2 45°)}}\right]$$

$$= \frac{4}{\sqrt{2}}\left[\frac{1/\sqrt{2}}{\sqrt{4-1/2}}\right] = 2\sqrt{2}\left(\frac{1}{\sqrt{7}}\right) = 1.06 \text{ cm}$$

Example 9.34 *A point object O is placed in front of a concave mirror of focal length 10 cm. A glass slab of refractive index $\mu = 3/2$ and thickness 6 cm is inserted between object and mirror. Find the position of final image when the distance x shown in figure is*

(i) 5 cm, *(i) 20 cm.*



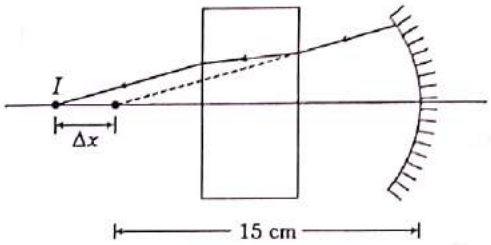
Sol. As we have read in the above article the normal shift produced by a glass slab is,

$$\Delta x = \left(1-\frac{1}{\mu}\right)t = \left(1-\frac{2}{3}\right)(6) = 2 \text{ cm}$$

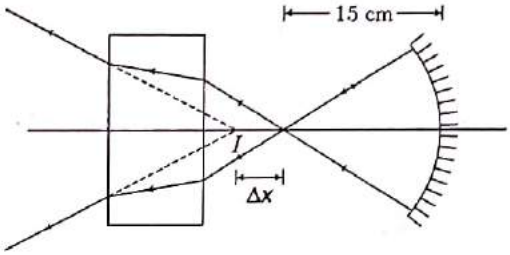
i.e., for the mirror the object is placed at a distance $(32 - \Delta x) = 32 - 2 = 30$ cm from it. Applying mirror formula,

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \quad \text{or} \quad \frac{1}{v}-\frac{1}{30}=-\frac{1}{10} \quad \text{or} \quad v = -15 \text{ cm}$$

(i) **When $x = 5$ cm** The light falls on the slab on its return journey as shown. But the slab will again shift it by a distance, $\Delta x = 2$ cm. Hence, the final **real** image is formed at a distance $(15 + 2) = 17$ cm from the mirror.



(ii) **When $x = 20$ cm** This time also the final image is at a distance 17 cm from the mirror but it is **virtual** as shown.



Critical angle and Total Internal Reflection (TIR)

Whenever a ray of light goes from a denser medium to a rarer medium, it bends away from the normal. As angle of incidence in denser medium increases, angle of refraction also increases in rarer medium.

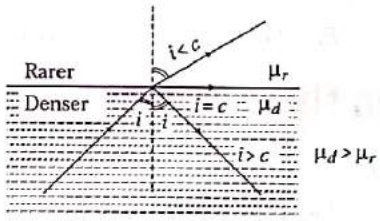


Fig. 9.38 Total internal reflections

The angle of incidence ($\angle i$) in denser medium for which the angle of refraction ($\angle r$) in rarer medium is 90° is called the critical angle ($\angle C$).

$$\Rightarrow \quad \frac{\sin C}{\sin 90°} = \frac{\mu_{\text{rarer}}}{\mu_{\text{denser}}} = \frac{\mu_r}{\mu_d}$$

$$\Rightarrow \quad \sin C = \frac{\mu_r}{\mu_d} \Rightarrow C = \sin^{-1}\left(\frac{\mu_r}{\mu_d}\right)$$

Now, if the angle of incidence ($\angle i$) in the rarer medium is greater than the critical angle ($\angle C$), then the ray instead of suffering refraction is totally reflected back in the same (denser) medium.

This phenomenon is called total internal reflection.
For total internal reflection to take place following set of conditions must be obeyed.

(i) The ray must travel from denser medium to rarer medium.

(ii) The angle of incidence ($\angle i$) must be greater than critical angle ($\angle C$).

When the rarer medium in air,

$$\mu_r = 1 \text{ and } \mu_d = \mu$$

$\therefore$ Critical angle, $C = \sin^{-1}\left(\frac{1}{\mu}\right)$

Applications of total internal reflection

TIR has following applications

(i) Totally reflecting prisms

Prisms are right-angled isosceles triangle which turn, the light ray by 90° or 180°. They are based on the phenomenon of total internal reflection. Refractive index of crown glass is 3/2.

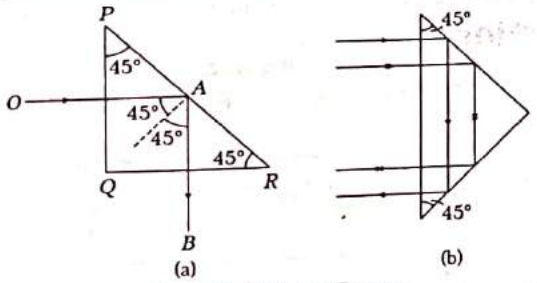


Fig. 9.39 Prism reflectors

Hence, $\theta_C = \sin^{-1}\left(\frac{1}{\mu}\right) = \sin^{-1}\left(\frac{2}{3}\right) \approx 42°$

In totally reflection glass prisms angle of incidence in made 45° (< C).

(ii) Optical fibres

An optical fibre is a thin tube of transparent material that allows light to pass through, without being refracted into the air or another external medium. Optical fibres work on the principle of total internal reflection.

Optical fibres are fabricated with high quality composite glass/quartz fibres. The refractive index of core is higher than that of the cladding. When a signal in the form of light is directed at one end of the fibre at a suitable angle, it undergoes repeated total internal reflection along the length of the fibre and finally comes out from other end. Due to total internal reflection at each stage, no absorption of light takes places *i.e.*, there is no appreciable loss in the intensity of light.

Optical fibres are fabricated in such a way that light reflected at one side of the inner surface strikes the other at an angle larger than critical angle. Even, if fibre is bent, light can easily travel along the length.

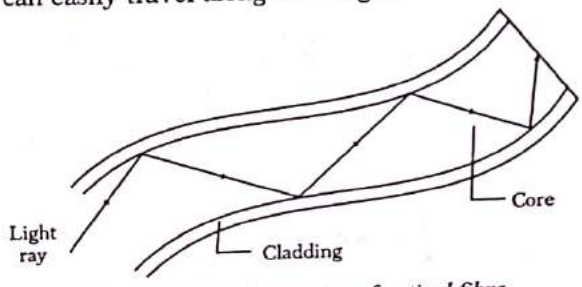


Fig. 9.40 Typical structure of optical fibre

Uses

(*i*) In transmitting electrical and optical signal.

(*ii*) In visual examination of internal organs like esophagus, stomach and intestine.

(*iii*) In decorative lamps.

Note As, we have seen $\theta_C = \sin^{-1}\left(\frac{\mu_R}{\mu_D}\right)$

Suppose, we have two sets of media 1 and 2 and

$$\left(\frac{\mu_R}{\mu_D}\right)_1 < \left(\frac{\mu_R}{\mu_D}\right)_2$$

then, $(\theta_C)_1 < (\theta_C)_2$

So, a ray of light has more chances to have TIR in case 1.

Some examples of total internal reflection

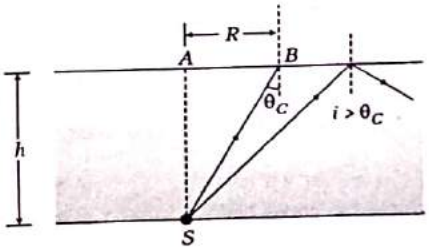
(i) The sparkling of diamond is due to total internal reflection inside it.

(ii) Shining of air bubble in water is also due to total internal reflection. The critical angle of water air is 48.75°.

(iii) The phenomenon of Mirage in deserts and looming in cold region is caused by total internal reflection.

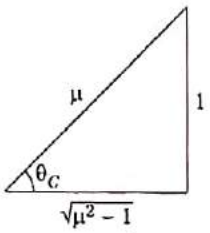
Example 9.35 *An isotropic point source is placed at a depth h below the water surface. A floating opaque disc is placed on the surface of water, so that the bulb is not visible from the surface. What is the minimum radius of the disc? Take refractive index of water = μ.*

Sol. As shown in figure, light from the source will not emerge out of water, if $i > \theta_C$.



Therefore, minimum radius R corresponds to, $i = \theta_C$

In ΔSAB,



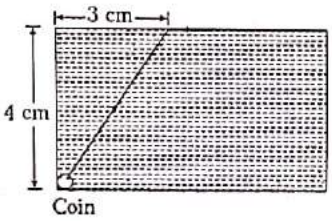
$\sin\theta_C = \frac{1}{\mu}$

$$\frac{R}{h} = \tan\theta_C$$

$$\therefore \quad R = h\tan\theta_C$$

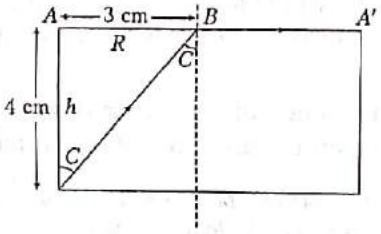
$$\text{or} \quad R = \frac{h}{\sqrt{\mu^2 - 1}}$$

Example 9.36 *A small coin is resting on the bottom of a beaker filled with a liquid. A ray of light from the coin travels upto the surface of the liquid and moves along its surface. Find the speed of light in the liquid.*



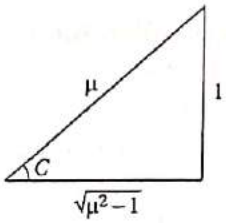
Sol. As shown in figure, a light ray from the coin will not emerge out of liquid if, $i > C$.

Therefore, minimum radius R corresponds to, $i = C$



In ΔSAB, $\frac{R}{h} = \tan C \Rightarrow R = h\tan C \Rightarrow R = \frac{h}{\sqrt{\mu^2 - 1}}$

Given, $R = 3$ cm and $h = 4$ cm



$$\therefore \quad \frac{3}{4} = \frac{1}{\sqrt{\mu^2 - 1}}$$

$$\Rightarrow \quad \mu^2 = \frac{25}{9} \Rightarrow \mu = \frac{5}{3}$$

$$\text{But,} \quad \mu = \frac{C}{v}$$

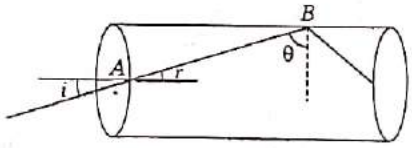
$$\Rightarrow \text{Speed of light, } v = \frac{C}{\mu} = \frac{3 \times 10^8}{5/3} = 1.8 \times 10^8 \text{ ms}^{-1}$$

Example 9.37 *Light enters at an angle of incidence in a transparent rod of refractive index n of the material of the rod. Find the range of n so that the light once entered into it will not leave it through its lateral face, the value of angle of incidence.*

Sol. Let a light ray enters at A and refracted beam is AB. At the lateral face, the angle of incidence is θ. For no refraction at this face, $\theta > C$.

i.e., $\sin\theta > \sin C$

but $\theta + r = 90° \Rightarrow \theta = 90° - r$



$\sin(90° - r) > \sin C$ or $\cos r > \sin C$ ...(i)

By Snell's law, $\mu = \frac{\sin i}{\sin r} \Rightarrow \sin r = \frac{\sin i}{\mu}$

$$\therefore \quad \cos r = \sqrt{1 - \sin^2 r} = \sqrt{\left(1 - \frac{\sin^2 i}{n^2}\right)}$$

From Eq. (i), $\sqrt{1 - \frac{\sin^2 i}{n^2}} > \sin C$

Also, $\sin C = \frac{1}{\mu} \Rightarrow 1 - \frac{\sin^2 i}{n^2} > \frac{1}{\mu^2}$ or $\mu^2 > \sin^2 i + 1$

The maximum value of $\sin i$ is 1.

$$\mu^2 > 2 \text{ or } \mu > \sqrt{2}$$

Check point 9.3

1. A light wave has a frequency of 4×10^{14} Hz and a wavelength of 5×10^{-7} m in a medium. The refractive index of the medium is
(a) 1.5 (b) 1.33 (c) 1.25 (d) 1.75

2. Absolute refractive indices of glass and water are $\frac{3}{2}$ and $\frac{4}{3}$. The ratio of velocities of light in glass and water will be
(a) 4 : 3 (b) 9 : 8 (c) 8 : 9 (d) 3 : 4

3. The refractive index of a certain glass is 1.5 for light whose wavelength in vacuum is 6000 Å. The wavelength of this light when it passes through glass is
(a) 4000 Å (b) 6000 Å
(c) 9000 Å (d) 15000 Å

4. When a ray of light falls on a given plate at an angle of incidence 60°, the reflected and refracted rays are found to be normal to each other. The refractive index of the material of the plate is
(a) $\frac{\sqrt{3}}{2}$ (b) 1.5 (c) 1.732 (d) 2

5. The refractive indices of glass and water with respect to air are 3/2 and 4/3 respectively.The refractive index of glass with respect to water will be
(a) 8/9 (b) 9/8
(c) 7/6 (d) None of these

6. How does refractive index (μ) of a material vary with respect to wavelength (λ)? A and B are constants
(a) $\mu = A + \frac{B}{\lambda^2}$ (b) $\mu = A + B\lambda^2$
(c) $\mu = A + \frac{B}{\lambda}$ (d) $\mu = A + B\lambda$

7. If ${}_i\mu_j$ represents refractive index when a light ray goes from medium i to medium j, then the product ${}_2\mu_1 \times {}_3\mu_2 \times {}_4\mu_3$ is equal to
(a) ${}_3\mu_1$ (b) ${}_3\mu_2$ (c) $\frac{1}{{}_1\mu_4}$ (d) ${}_4\mu_2$

8. μ_1 and μ_2 are the refractive index of two mediums and v_1 and v_2 are the velocity of light in these two mediums respectively. Then, the relation connecting these quantities is
(a) $v_1 = v_2$ (b) $\mu_2 v_1 = \mu_1 v_2$
(c) $\mu_1^2 v_1 = \mu_2^2 v_2$ (d) $\mu_1 v_1 = \mu_2 v_2$

9. When light is refracted into a medium from vacuum
(a) its wavelength and frequency both increase
(b) its wavelength increases but frequency remains unchanged
(c) its wavelength decreases but frequency remains unchanged
(d) its wavelength and frequency both decrease

10. A spot is placed on the bottom of a slab made of transparent material of refractive index 1.5. The spot is viewed vertically from the top when it seems to be raised by 2 cm. Then, the height of the slab is
(a) 10 cm (b) 8 cm (c) 6 cm (d) 4 cm

11. An air bubble inside a glass slab (μ = 1.5) appears 6 cm when viewed from one side and 4 cm when viewed from the opposite side. The thickness of the slab is
(a) 10 cm (b) 6.67 cm (c) 15 cm (d) None of these

12. An under water swimmer is at a depth of 12 m below the surface of water. A bird is at a height of 18 m from the surface of water, directly above his eyes. For the swimmer the bird appears to be at a distance from the surface of water equal to (Refractive index of water is 4/3)
(a) 24 m (b) 12 m (c) 18 m (d) 9 m

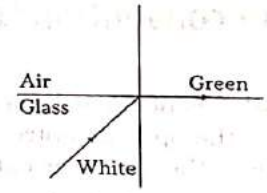
13. A vessel of depth 2 d cm is half filled with a liquid of refractive index μ_1 and the upper half with a liquid of refractive index μ_2. The apparent depth of the vessel seen perpendicularly from about is
(a) $d\left[\frac{\mu_1\mu_2}{\mu_1 + \mu_2}\right]$ (b) $d\left[\frac{1}{\mu_1} + \frac{1}{\mu_2}\right]$
(c) $2d\left[\frac{1}{\mu_1} + \frac{1}{\mu_2}\right]$ (d) $2d\left[\frac{1}{\mu_1\mu_2}\right]$

14. Three immiscible transparent liquids with refractive indices 3/2, 4/3 and 6/5 are arranged one on top of another. The depths of the liquids are 3 cm, 4 cm and 6 cm respectively. The apparent depth of the vessel is
(a) 10 cm (b) 9 cm
(c) 8 cm (d) 7 cm

15. A glass-slab is immersed in water. What will be the critical angle for a light ray at glass-water interface? Where ${}_a n_g = 1.50$, ${}_a n_w = 1.33$ and $\sin^{-1}(0.887) = 62.5$
(a) 48.8° (b) 72.8°
(c) 62.5° (d) 64.5°

16. The wavelength of light in two liquids x and y is 3500 Å and 7000 Å respectively, then the critical angle of x relative to y will be
(a) 60° (b) 45° (c) 30° (d) 15°

17. White light is incident on the interface of glass and air as shown in the figure. If green light is just totally internally reflected, then the emerging ray in air contains



(a) yellow, orange, red (b) violet, indigo, blue
(c) all colours (d) all colours except green

18. The critical angle of a prism is 30°. The velocity of light in the medium is
(a) 1.5×10^8 m/s (b) 3×10^8 m/s
(c) 4.5 m/s (d) None of these

19. A ray of light travelling in a transparent medium falls on a surface separating the medium from air at an angle of incidence of 45°. The ray undergoes total internal reflection. If n is the refractive index of the medium with respect to air, select the possible value of n from the following.
(a) 1.2 (b) 4/3 (c) 1.4 (d) 1.5

20. A glass slab has a critical angle of 30° when placed in air. What will be the critical angle when it is placed in liquid of refractive index 6/5?
(a) 45° (b) 37°
(c) 53° (d) 60°

4 Refraction at spherical surfaces

When two transparent media are separated by a spherical surface, light incident on the surface from one side get refracted into the medium on the other side. Spherical surfaces are of two types as shown in figure.

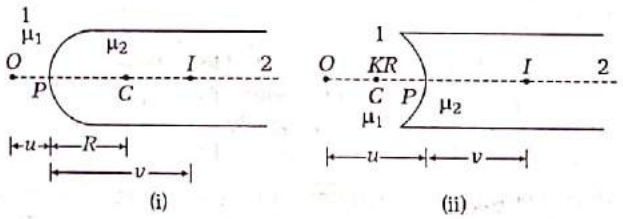


Fig. 9.41 Spherical surface (i) convex (ii) concave

For both surfaces refraction formula is given by

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

where μ_1 = refractive index of first medium,
μ_2 = refractive index of second medium,
R = radius of curvature of spherical surfaces,
u = object distance and
v = image distance

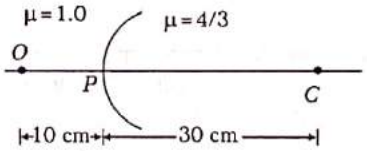
Transverse magnification, $m = \frac{h_I}{h_O} = \frac{\mu_1 v}{\mu_2 u}$

Note For a plane surface, $R = \infty$, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = 0$

Cartesian sign convention for spherical surfaces

(i) The principal axis of the spherical surface is taken as X-axis and the optical centre as origin. Here, the principal axis is the diameter extended.
(ii) The direction of the incident light is taken as the positive direction of X-axis and opposite to it is taken as negative.
(iii) The upward direction is taken as positive and the downward direction as negative.

Example 9.38 *Locate the image of the point object O. The point C is centre of curvature of the spherical surface.*



Sol. Here, $\mu_1 = 1$, $\mu_2 = 4/3$, $u = -10$ cm and $R = 30$ cm

Using the relation, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

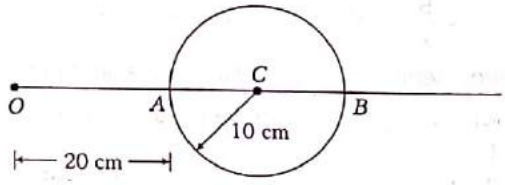
Substituting the values,

$$\Rightarrow \quad \frac{4/3}{v} - \frac{1}{-10} = \frac{4/3 - 1}{30} \Rightarrow \frac{4}{3v} + \frac{1}{10} = \frac{1}{90}$$

$$\Rightarrow \quad \frac{4}{3v} = \frac{1}{90} - \frac{1}{10} = \frac{-8}{90} \Rightarrow v = -15 \text{ cm}$$

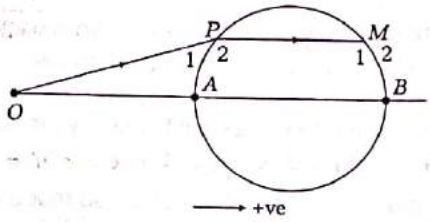
The image is formed 15 cm left of spherical surface and is virtual.

Example 9.39 *A glass sphere of radius R = 10 cm is kept inside water. A point object O is placed at 20 cm from A as shown in figure. Find the position and nature of the image when seen from other side of the sphere. Also draw the ray diagram. Given,* $\mu_g = 3/2$ *and* $\mu_w = 4/3$.



Sol. A ray of light starting from O gets refracted twice. The ray of light is travelling in a direction from left to right. Hence, the distances measured in this direction are taken positive.

Applying, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$, twice with proper signs.



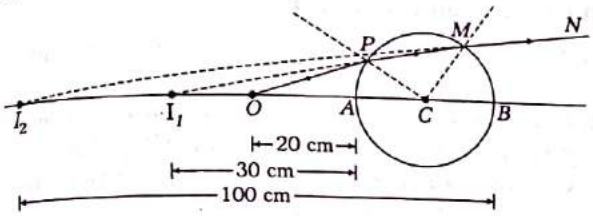
We have, $\frac{3/2}{AI_1} - \frac{4/3}{-20} = \frac{3/2 - 4/3}{10}$ or $AI_1 = -30$ cm

Now, the first image I_1, acts as an object for the second surface, where

$$BI_1 = u = -(30+20) = -50 \text{ cm}$$

$$\therefore \quad \frac{4/3}{BI_2} - \frac{3/2}{-50} = \frac{4/3 - 3/2}{-10}$$

$\therefore BI_2 = -100$ cm *i.e.*, the final image I_2 is **virtual** and is formed at a distance 100 cm (towards left) from B. The ray diagram is as shown.

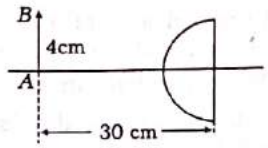


Following points should be noted while drawing the ray diagram.

(i) At P the ray travels from rare to a denser medium. Hence, it will bend towards normal PC. At M, it travels from a denser to a rarer medium, hence, moves away from the normal MC.

(ii) PM ray when extended backwards meets at I_1 and MN ray when extended meets at I_2.

Example 9.40 *A linear object of length 4 cm is placed at 30 cm from the plane surface of hemispherical glass of radius 10 cm. The hemispherical glass is surrounded by water. Find the final position and size of the image.*



Sol. For 1st surface,

$$\mu_1 = \frac{4}{3}, \mu_2 = \frac{3}{2}, u = -20 \text{ cm and } R = +10 \text{ cm}$$

Using, $$\frac{\mu_2}{v} - \frac{\mu_1}{\mu} = \frac{(\mu_2 - \mu_1)}{R}$$

$$\Rightarrow \quad \frac{(3/2)}{v} - \frac{(4/3)}{(-20)} = \frac{(3/2 - 4/3)}{10} \Rightarrow v = -30 \text{ cm}$$

Using, $$\frac{A'B'}{AB} = \frac{\mu_1 v}{\mu_2 u} \Rightarrow \frac{A'B'}{(4 \text{ cm})} = \frac{(4/3)(-30)}{(3/2)(-20)}$$

$$\Rightarrow \quad A'B' = 5.3 \text{ cm}$$

$A'B'$ behaves as the object for plane surface,

$$\mu_1 = \frac{3}{2}, \mu_2 = \frac{4}{3} \text{ and } R = \infty, u' = -40$$

$$\Rightarrow \quad \frac{\mu_1}{v'} = \frac{\mu_2}{u'} \Rightarrow \frac{(4/3)}{v'} = \frac{(3/2)}{(-40)}$$

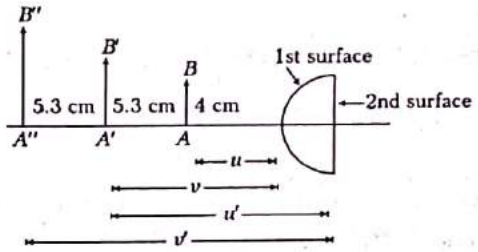
Solving it, we will get, $v' = -35.4$ cm

Now, using, $$\frac{A''B''}{A'B'} = \frac{(\mu_1 v')}{(\mu_2 u')}$$

$$\frac{A''B''}{(5.3)} = \frac{(3/2)(-35.4)}{(4/3)(-40)}$$

$$\Rightarrow \quad A''B'' = 5.3 \text{ cm}$$

The final images in all the above cases are shown in figure.



Lenses

Lens is a transparent medium (material) bounded by two surfaces atleast one of which should be spherical. Refractive index of the medium (material) should also be different from the surroundings.

Lenses are of two types

(i) **Convex or converging lens** In this lens a transparent medium is bounded by two surfaces such that it is thicker in the middle.

It is of fours types (as shown)

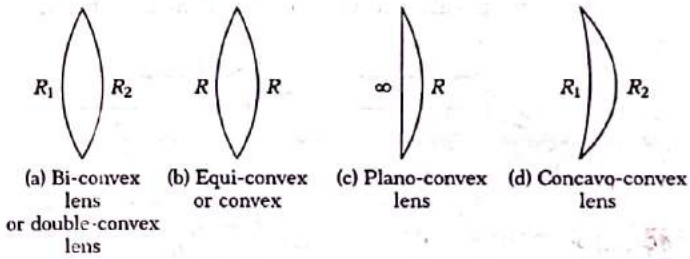


Fig. 9.42 Types of convex lens

(ii) **Concave or diverging lens** In this lens, a transparent medium is bounded by two surfaces such that it is thinner in the middle. It is of four types (as shown)

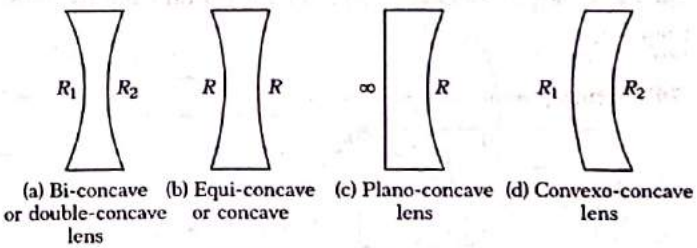


Fig. 9.43 Types of concave lens

Some definitions related to lenses

(i) **Optical centre** The optical centre is a point within or outside the lens, at which incident rays refract without deviation in its path.

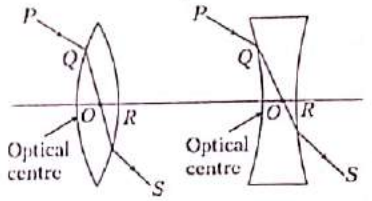


Fig. 9.44 Optical centre

(ii) **Centre of curvature** The centres of the two imaginary spheres of which the lens is a part are called centres of curvature of the lens. A lens has two centres of curvature with respect to its two curved surfaces.

(iii) **Radii of curvature** The radii of the two imaginary spheres of which the lens is a part are called radii of curvature of the lens. A lens has two radii of curvature. These may or may not be equal.

(iv) **Principal axis** The straight line passing through the optical centre of lens is called principal axis of lens.

(v) **Principal focus** Lens has two principal focus.

(a) **First principal focus** It is a point on the principal axis of lens, the rays starting from which (convex lens) or appear to converge at which (concave lens) become parallel to principal axis after refraction.

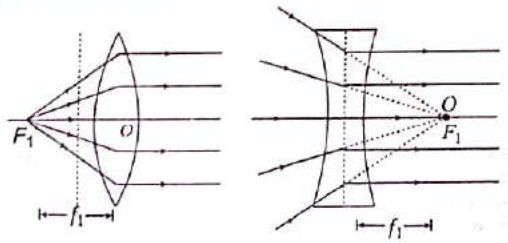


Fig. 9.45 First principal focus

(b) **Second principal focus** It is the point on the principal axis at which the rays coming parallel to the principal axis converge (convex lens) or appear to diverge (concave lens) after refraction from the lens.

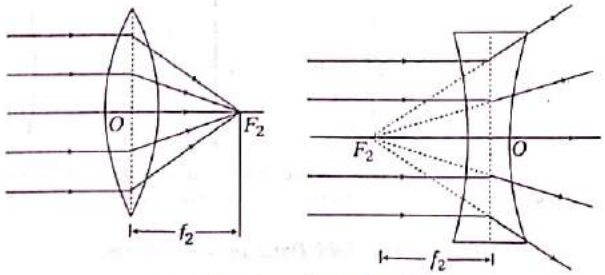


Fig. 9.46 Second principal focus

Both the focus of convex lens are real, while that of concave lens are virtual.

(vi) **Focal length** The distance between focus and optical centre of lens is called focal length of lens.

(vii) **Aperture** The effective diameter of circular boundary of the lens is called aperture.

$$\text{Intensity} \propto (\text{aperture})^2$$

Sign convention for lenses

(i) Sign convention for lenses are same as for spherical surfaces.

(ii) From the Fig. 9.45 and 9.46, we can see that f_1 is negative for a convex lens and positive for a concave lens. But f_2 is positive for convex lens and negative for concave lens.

(iii) $|f_1| = |f_2|$, if the media on the two sides of a thin lens have same refractive index.

(iv) We are mainly concerned with the second focus f_2. Thus, wherever we write the focal length f, it means the second principal focal length. Thus, $f = f_2$ and hence, f is positive for a convex lens and negative for a concave lens.

Law of Formation of Images by Lens

The position and nature of the image by lens, in any case can be obtained either from a ray diagram or by calculation.

(a) Ray diagram

To construct the image of a small object perpendicular to the axis of a lens, two of the following three rays are drawn from the top of the object.

(i) A ray parallel to the principal axis after refraction passes through the principal focus or appears to diverge from it.

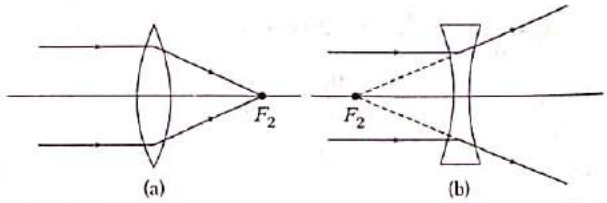


Fig. 9.47 Path of incident ray parallel to principal axis for (a) convex lens (b) concave lens

(ii) A ray through, the optical centre P passes undeviated because the middle of the lens acts like a thin parallel-sided slab.

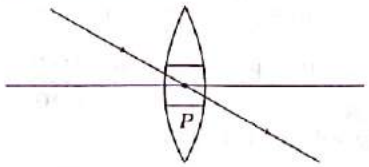


Fig. 9.48 Path of incident ray passing through centre for convex lens

(iii) A ray passing through, the first focus F_1 become parallel to the principal axis after refraction.

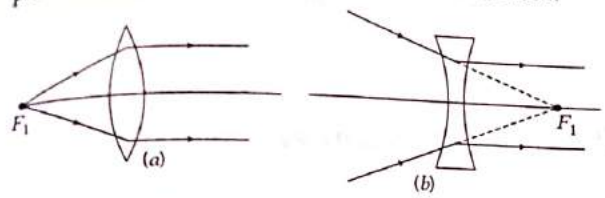


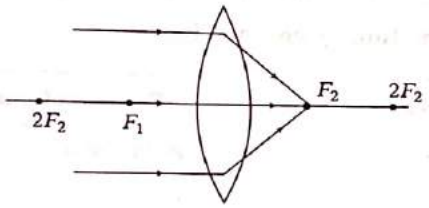
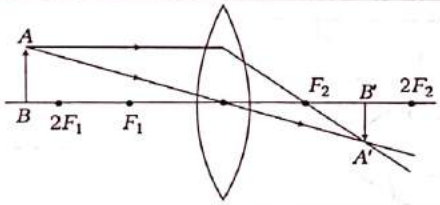
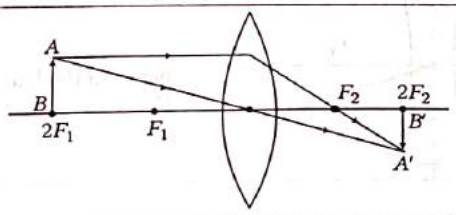
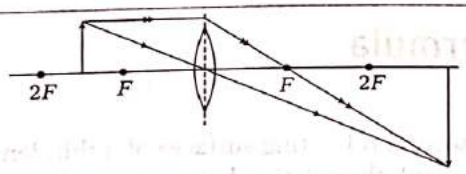
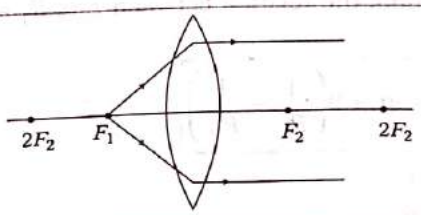
Fig. 9.49 Path incident ray passing through the focus for (a) convex lens (b) concave lens

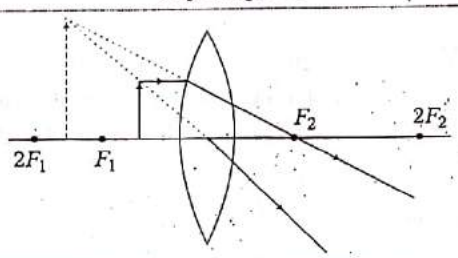
Image formation by lens

Convex lens

The image formed by convex lens depends on the position of object. Formation of image by convex lens for different positions of object is shown in the table given below.

Formation of image by convex lenses

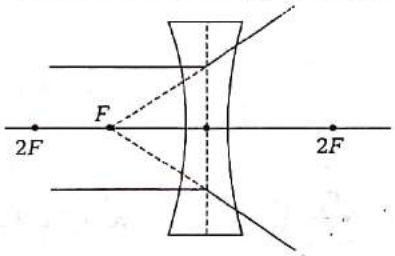
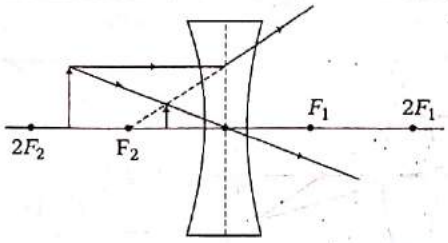
S.No.	Position of object	Ray diagram	Position of image	Nature and size of image
1.	At infinity		At the principal focus (F_2) or in the focal plane	Real, inverted and extremely diminished
2.	Beyond $2F_1$		Between F_2 and $2F_2$	Real, inverted and diminished
3.	At $2F_1$		At $2F_2$	Real, inverted and of same size as the object
4.	Between F_1 and $2F_1$		Beyond $2F_2$	Real, inverted and highly magnified
5.	At F_1		At infinity	Real, inverted and highly magnified

S.No.	Position of object	Ray diagram	Position of image	Nature and size of image
6.	Between F_1 and optical centre		On the same side as the object	Virtual, erect and magnified

Concave lens

The image formed by a concave lens is always virtual, erect and diminished (like a convex mirror). The image formation by concave lens for different positions of object is shown in the table given below.

Image formation by concave lens

S.No.	Position of object	Ray diagram	Position of image	Nature and size of image
1.	At infinity		At the focus	Virtual erect and point size
2.	Anywhere except on the principal axis		Between the lens and F_2	Virtual, erect, diminished

Lens maker's formula and lens formula

Lens maker's formula

If R_1 and R_2 are the radii of curvature of first and second refracting surfaces of a thin lens of focal length f and refractive index μ (with respect to surrounding medium), the relation between f, μ, R_1 and R_2 is known as lens maker's formula and is given by,

$$\boxed{\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

Lens formula

The expression which shows the relation between u, v and f is called lens formula and given by,

$$\boxed{\frac{1}{v} - \frac{1}{u} = \frac{1}{f}}$$

where, u is distance of object from the lens, v is distance of image from the lens, f is focal length of the lens.

Focal length of different lens

Focal lengths of different lenses are given below

(i) **For equibiconvex lens,**

$$R_1 = +R \text{ and } R_2 = -R$$

$$\boxed{f = \frac{R}{2(\mu - 1)}}$$

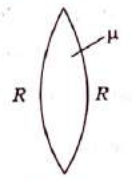


Fig. 9.50 Equibiconvex lens

(ii) **For equibioconcave lens,**

$$R_1 = -R \text{ and } R_2 = +R$$

$$\boxed{f = -\frac{R}{2(\mu - 1)}}$$

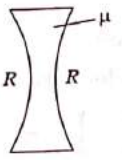


Fig. 9.51 Equibiconcave lens

(iii) **For plano-convex lens** $R_1 = -R$ and $R_2 = \infty$

$$\therefore \quad \boxed{f = \frac{R}{\mu - 1}}$$

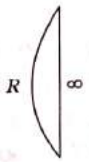


Fig. 9.52 Plano-convex lens

(iv) **For plano-concave lens,** $R_1 = \infty$, $R_2 = R$

$$\Rightarrow \quad \boxed{f = \frac{-R}{(\mu - 1)}}$$

For a lens

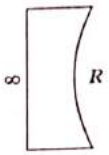


Fig. 9.53 Plane-concave lens

Note **Newton's formula**

In case of thin convex lens, if an object is placed at a distance x_1, from first focus and its image is formed at distance x_2, from the second focus from $x_1 x_2 = f^2$.

If some portion of a lens is covered with black paper full image will be formed by brightness will be reduced.

Lens immersed in a liquid

If a lens (made of glass) of refractive index μ_g is immersed in a liquid of refractive μ_l then its focal length f_l is given by

$$\frac{1}{f_l} = ({}_l\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{...(i)}$$

If f_a is the focal length of lens in air, then

$$\frac{1}{f_a} = ({}_a\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{...(ii)}$$

$$\frac{f_l}{f_a} = \frac{({}_a\mu_g - 1)}{({}_l\mu_g - 1)} = \frac{({}_a\mu_g - 1)}{\left(\frac{{}_a\mu_g}{{}_a\mu_l} - 1\right)}$$

There are three cases based on this

(i) If $\mu_g > \mu_l$, *i.e.*, lens is immersed in a liquid whose refractive index is less than that of material of lens.

Then f_l and f_a are of same sign and $f_l > f_a$ that is the nature of lens remains unchanged, but its focal length increases and hence power of lens decreases.

(ii) If $\mu_g = \mu_l$, *i.e.*, lens is immersed in a liquid whose refractive index is same as that of material of lens.

Then, $f_l = \infty$. It means lens behaves as a plane glass plate and becomes invisible in the medium.

(iii) If $\mu_g < \mu_l$, *i.e.*, lens is immersed in a liquid whose refractive is greater than that of material of lens.

Then f_l and f_a have opposite signs and the nature of lens changes *i.e.*, a convex lens diverges the light rays and concave lens converges the light rays.

Focal length of lens if media are different

Consider a double convex lens of refractive index μ_2. It is surrounded by media of refractive induces μ_1 and μ_3 as shown in figure.

(i) If rays are incident from left, then for refraction at 1.

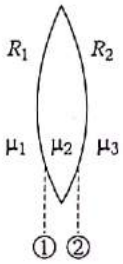


Fig. 9.54 Bi-convex lens with different surrounding refractive index

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1} \quad ...(i)$$

For refraction at (2),

$$\frac{\mu_3}{v'} - \frac{\mu_2}{v} = \frac{\mu_3 - \mu_2}{R_2} \quad ...(ii)$$

On adding Eqs. (i) and (ii), we get

$$\frac{\mu_3}{v'} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1} + \frac{\mu_3 - \mu_2}{R_2}$$

If $u = \infty$, $v' = f$

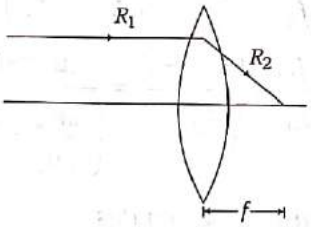


Fig. 9.55 Ray diagram when incident ray comes from left

$$\therefore \quad \boxed{\frac{\mu_3}{f} = \frac{\mu_2 - \mu_1}{R_1} + \frac{\mu_3 - \mu_2}{R_2}}$$

(ii) If rays are incident from right, replace μ_3 by μ_1 and R_2 by R_1.

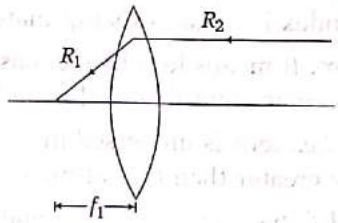
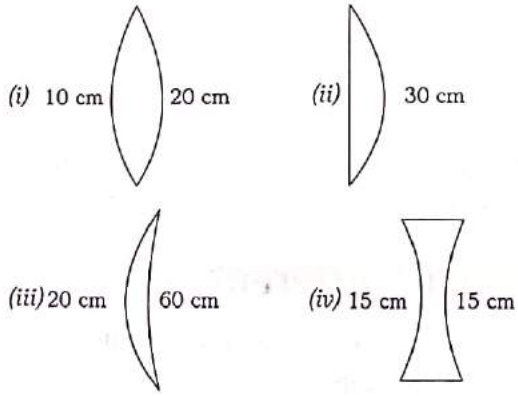


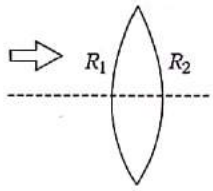
Fig. 9.56 Ray diagram when incident ray comes from right

$$\therefore \quad \boxed{\frac{\mu_1}{f'} = \frac{\mu_2 - \mu_3}{R_2} + \frac{\mu_1 - \mu_2}{R_1}}$$

Example 9.41 *Find focal lengths of lenses made of glass ($\mu = 3/2$) and place in air.*



Sol. (i)



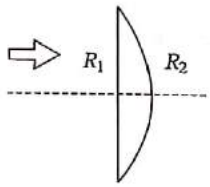
Given, $R_1 = 10$ cm, $R_2 = -20$ cm

On applying Lens maker's formula, we have

$$\frac{1}{f} = (\mu - 1)(1/R_1 - 1/R_2) = \left(\frac{3}{2} - 1\right)\left(\frac{1}{10} - \frac{1}{(-20)}\right) = \left(\frac{1}{2}\right)\left(\frac{3}{20}\right)$$

$\therefore$ Focal length of lens, $f = 40/3$ cm

(ii)



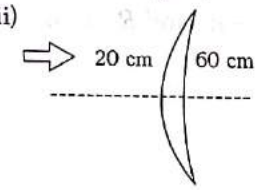
Given, $R_1 = \infty$, $R_2 = 30$ cm

From Lens makerl's formula we have,

$$\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{\infty} - \frac{1}{-30}\right) = \frac{1}{60}$$

$\therefore$ Focal length $f = 60$ cm

(iii)



Given, $R_1 = -20$ cm, $R_2 = 60$ cm

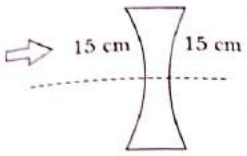
From Lens Maker's formula, we have

$$\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{20} - \frac{1}{60}\right) = \left(\frac{1}{2}\right)\left(\frac{2}{60}\right)$$

$\therefore$ Focal length,

$f = 60$ cm

(iv)



Given, $R_1 = 20$ cm, $R_2 = 20$ cm

From Lens maker's formula, we have

$$\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{-15} - \frac{1}{15}\right) = \left(\frac{1}{2}\right)\left(\frac{-2}{15}\right)$$

$\therefore$ Focal length, $f = -15$ cm

Example 9.42 *Double-convex lenses are to be manufactured from a glass of refractive index 1.55, with both faces of the same radius of curvature. What is the radius of curvature required, if the focal length is to be 20 cm?*

Sol. Using the relation,

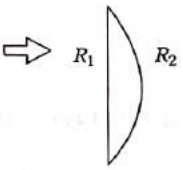
$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right), \text{ we get}$$

$$\frac{1}{20} = (1.55 - 1)\left[\frac{1}{R} - \left(\frac{1}{-R}\right)\right] = 0.55\left[\frac{1}{R} + \frac{1}{R}\right] = 0.55 \times \frac{2}{R}$$

or $$R = 1.10 \times 20 = 22 \text{ cm}$$

Example 9.43 *Find the refractive index of the material of a plano-convex lens, if the radius of curvature of the convex surface is 20 cm and focal length of the lens is 60 cm?*

Sol.



Given, $R_1 = \infty$, $R_2 = 20$ cm, $f = 60$ cm, $\mu = ?$

From Lens maker's formula, we have

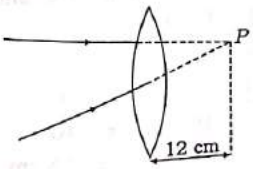
$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \quad \frac{1}{60} = (\mu - 1)\left(\frac{1}{\infty} - \frac{1}{-20}\right) = \frac{\mu - 1}{20}$$

$\therefore$ Refractive index, $\mu = 1 + \frac{1}{3} = \frac{4}{3}$

Example 9.44 *A beam of light converges to a point P. Now a lens is placed on the path of the convergent beam 12 cm apart from P. At what point does the beam converge if the lens is (i) a convex lens of focal length 20 cm, (ii) a concave lens of focal length 16 cm?*

Sol. (i) For convex lens, $f = 20$ cm



Using the formula,

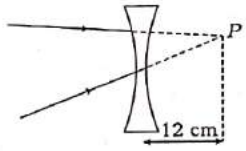
$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}, \text{ we get}$$

$$\frac{1}{v} = \frac{1}{u} + \frac{1}{f} = \frac{1}{12} + \frac{1}{20}$$

$$\therefore \quad v = \frac{60}{8} = 7.5 \text{ cm}$$

i.e., the image is formed on the right of the lens and is real.

(ii) For concave lens, $f = -16$ cm



$\therefore$ Using the relation, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$,

we get $$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{-16} + \frac{1}{12}$$

$$\therefore \quad v = +48 \text{ cm}$$

The image is formed at 48 cm to the right of the lens where the beam would converge and is real.

Example 9.45 *Focal length of a convex lens in air is 10 cm. Find its focal length in water. Given that $\mu_g = 3/2$ and $\mu_w = 4/3$.*

Sol. Using the relations,

$$\frac{1}{f_{air}} = (\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{...(i)}$$

and $$\frac{1}{f_{water}} = \left(\frac{\mu_g}{\mu_w} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{...(ii)}$$

Dividing Eq. (i) by (ii), we get

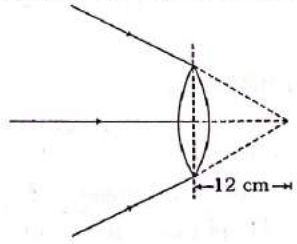
$$\frac{f_{water}}{f_{air}} = \frac{(\mu_g - 1)}{(\mu_g/\mu_w - 1)}$$

Substituting the values, we have

$$f_{water} = \frac{(3/2 - 1)}{\left(\frac{3/2}{4/3} - 1\right)} f_{air} = 4\, f_{air} = 4 \times 10 = 40 \text{ cm}$$

Note Students can remember the result, $f_{water} = 4\, f_{air}$, if $\mu_g = 3/2$ and $\mu_w = 4/3$.

Example 9.46 *If the focal length of the lens is 20 cm, find the distance of the image from the lens in the following figure?*



Sol. According to figure, the point on the right side of the lens of which rays converge will behave as virtual object at the lens.

$\therefore \quad u = +12\text{ cm}, \ f = 20\text{ cm}$

By lens formula, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

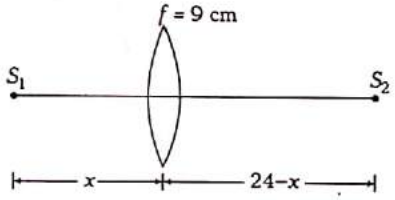
$\therefore \quad \frac{1}{20} = \frac{1}{v} - \frac{1}{12}$

or $\quad \frac{1}{v} = \frac{1}{20} + \frac{1}{12} = \frac{3+5}{60} = \frac{8}{60}$

or $\quad v = \frac{60}{8} = 7.5\text{ cm}$

So, image will be formed on same side of the virtual object at a distance of 7.5 cm from the lens.

Example 9.47 *The distance between two point sources of light is 24 cm. Find out where would you place a converging lens of focal length 9 cm, so that the images of both the sources are formed at the same point.*



Sol. Using the relation, $\frac{1}{v} = \frac{1}{u} = \frac{1}{f}$

For S_1, $\frac{1}{v_1} - \frac{1}{-x} = \frac{1}{9} \Rightarrow \quad \therefore \frac{1}{v_1} = \frac{1}{9} - \frac{1}{x}$ ...(i)

For S_2, $\quad \frac{1}{v_2} - \frac{1}{-(24-x)} = \frac{1}{9}$

$\therefore \quad \frac{1}{v_2} = \frac{1}{9} - \frac{1}{24-x}$ ...(ii)

Since, sign convention for S_1 and S_2 is just opposite. Hence,

$$v_1 = -v_2 \text{ or } \frac{1}{v_1} = -\frac{1}{v_2}$$

$\therefore \quad \frac{1}{9} - \frac{1}{x} = \frac{1}{24-x} - \frac{1}{9}$

Solving this equation we get, $x = 6$ cm. Therefore, the lens should be kept at a distance of 6 cm from either of the object.

Magnification produced by a lens

The ratio of the size of the image to the size of object is called magnification.

Transverse magnification

The lateral, transverse or linear magnification m produced by a lens is defined by,

$$m = \frac{\text{Height of image}}{\text{Height of object}} = \frac{I}{O} = \frac{v}{u}$$

$$m = \frac{f}{f+u} = \frac{f-u}{f}$$

Use sign convention while solving the problems.

Longitudinal magnification (m_L)

When an object is placed along the principal axis then

$$m_L = \frac{\text{Image length}}{\text{Object length}}$$

If length of the object is small,

$$m_L = \frac{dv}{du} = \left(\frac{v}{u}\right)^2 = \left(\frac{f}{u+f}\right)^2 = \left(\frac{f-u}{f}\right)^2$$

where, $du =$ object length, $dv =$ image length

Areal magnification (M_s)

$$m_s = \frac{A_I}{A_O} = m^2 = \left(\frac{f}{f+u}\right)^2$$

(A_I = Area of image and A_O = Area of object)

Example 9.48 *An object of size 3.0 cm is placed at 14 cm from concave lens of focal length 21 cm. Describe the image produced by the lens. What happens if the object is moved further away from the lens?*

Sol. Using the relation, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$, we get

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{-21} + \frac{1}{(-14)}$$

$\therefore \quad v = -8.4$ cm

$\therefore$ The image is virtual, erect and located at 8.4 cm from the lens on the same side as the object.

Also, we know that

$$m = \frac{I}{O} = \frac{v}{u}$$

$\therefore \quad I = \frac{v}{u} \times O = \frac{-8.4}{-14} \times 3 = 1.8$ cm

i.e., the image is of diminished size.

If the object is moved away from the lens, the virtual image moves towards the focus of the lens (but never beyond focus).

Example 9.49 *Find the distance of an object from a convex lens, if image is two times magnified. Focal length of the lens is 10 cm.*

Sol. Convex lens forms both type of images real as well as virtual. Since, nature of the image is not mentioned in the question, we will have to consider both the cases.

When image is real Means v is positive and u is negative with $|v| = 2|u|$. Thus, if

$$u = -x \text{ then, } v = 2x \text{ and } f = 10\text{ cm}$$

Substituting in, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

we have, $\frac{1}{2x} + \frac{1}{x} = \frac{1}{10}$ or $\frac{3}{2x} = \frac{1}{10}$

$\therefore \quad x = 15$ cm

$x = 15$ cm, means object lies between F and $2F$.

When image is virtual Means v and u both are negative. So let, $u = -y$ then $v = -2y$ and $f = 10$ cm

Substituting in, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

we have, $\frac{1}{-2y} + \frac{1}{y} = \frac{1}{10}$ or $\frac{1}{2y} = \frac{1}{10}$

$\therefore$ $y = 5$ cm

$y = 5$ cm, means object lies between F and P.

Displacement method to determine the focal length of a convex lens

The focal length of convex lens can be determined by displacement method. Consider an object and a screen placed at a distance $D (> 4f)$ apart. Let a lens of focal length f be placed between the object and the screen. For two different position of lens two images (I_1 and I_2) of an object are formed at the screen. Let distance between the two positions of the lens is x then the focal length of the lens is given by $f = \frac{D^2 - x^2}{4D} = \frac{x}{m_1 - m_2}$

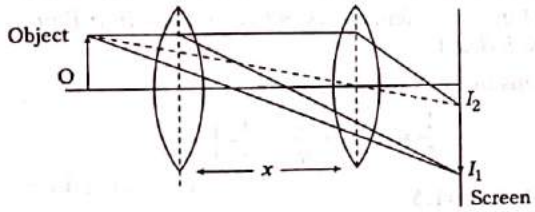


Fig. 9.57 Two convex lenses separated by distance x

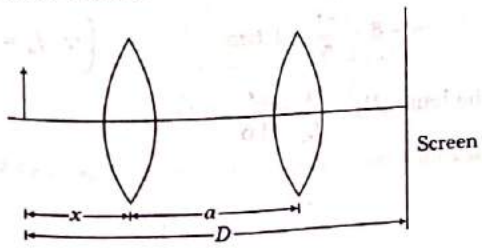
where $m_1 = \frac{I_1}{O}$, $m_2 = \frac{I_2}{O}$, $m_1 m_2 = 1$

Also, size of object $O = \sqrt{I_1 I_2}$

Example 9.50 *An object is placed at a distance of 80 cm from a screen. Where should a convex lens of focal length 15 cm be placed so as to obtain a real image of the object?*

Sol. Given, $D = 80$ cm and $f = 15$ cm

$D > 4f$, there will be two positions of lens for which real image can be obtain.



For lens in position (1), $u = -x$, $v = D - x = 75 - x$, $f = 12$ cm

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{80 - x} - \frac{1}{-x} = \frac{1}{15}$$

$$\frac{80}{(80 - x)x} = \frac{1}{15} \Rightarrow x^2 - 80 + 1200 = 0$$

$$x^2 - 60x - 20x + 1200 = 0$$

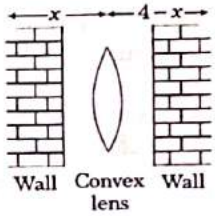
$$\Rightarrow \quad x(x - 60) - 20(x - 60) = 0$$

$$\Rightarrow \quad (x - 20)(x - 60) = 0$$

$$\Rightarrow \quad x = 20 \text{ or } 60 \text{ cm}$$

Example 9.51 *The image of a small electric bulb fixed on the wall of a room is to be obtained on the opposite wall 4m away by means of a large convex lens. Find the maximum possible focal length of the lens required for this purpose.*

Sol. Let a large convex lens is placed between two walls at a distance x from wall on which an electric bulb is fixed.



Using lens formula, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

Putting, $u = x$ and $v = 4 - x$

$$\therefore \quad \frac{1}{f} = \frac{1}{4 - x} - \frac{1}{-x}$$

$$\text{or} \quad \frac{1}{f} = \frac{x + 4 - x}{(4 - x)(x)} \text{ or } \frac{1}{f} = \frac{4}{(4 - x)(x)}$$

$$\text{or} \quad f = \frac{(4 - x)x}{4} \quad \text{...(i)}$$

Now, magnification, $m = \frac{v}{u} = \frac{4 - x}{x}$ or $1 = \frac{4 - x}{x}$

or $x = 4 - x$ or $2x = 4$ or $x = 2$m

From Eq. (i), we get

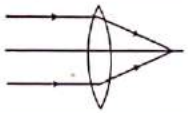
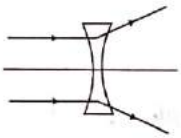
$$f = \frac{(4 - 2)(2)}{4} = \frac{2 \times 2}{4} = 1\text{m}$$

Power of a lens

It is the ability of the lens to deviate the path of rays passing through it. If the lens converges the rays parallel to principal axis, its power is said positive and if it diverges the rays, its power is negative.

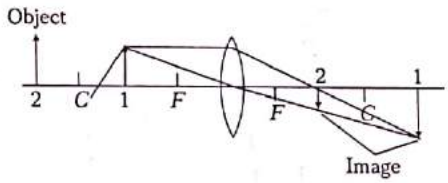
$$P \text{ (in dioptre)} = \frac{1}{f \text{(metre)}}$$

Following table gives the sign of P and f for different type of lenses

Nature of lens/ mirror	Focal length (f)	Power $P_L = \frac{1}{f}$	Converging /diverging	Ray diagram
Convex lens	+ve	+ve	Converging	
Concave lens	−ve	−ve	Diverging	

Example 9.52 *For a given lens, the magnification was found to be twice as large as when the object was 0.15 m distance from it as when the distance was 0.2 m. Find power of the lens.*

Sol. Let as shown 1 and 2 are positions of objects and image in two different situations.



It is given, $\left|\frac{v_1}{u_1}\right| = 2\left|\frac{v_2}{u_2}\right|$

Here, $u_1 = -15$ cm and $u_2 = -20$ cm

$\therefore \quad v_1 = 2v_2 \times \frac{u_1}{u_2} = 2v_2 \times \frac{15}{20} = \frac{3}{2}v_2$

Now, $\quad \frac{1}{f} = \frac{1}{v} - \frac{1}{u} \Rightarrow \frac{1}{f} = \frac{1}{v_1} - \frac{1}{u_1}$ and $\frac{1}{f} = \frac{1}{v_2} - \frac{1}{u_2}$

So, $\quad \frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{v_2} - \frac{1}{u_2}$

$\Rightarrow \quad \frac{2}{3v_2} + \frac{1}{15} = \frac{1}{v_2} + \frac{1}{20} \Rightarrow v_2 = 20$ cm

$$\frac{v_1}{v_1} = 2\frac{v_2}{u_2} = 2 \times \frac{20}{20} = 2$$

$\Rightarrow \quad v_1 = 2u_1 = 2 \times 15 = 30$ cm

$\therefore \quad \frac{1}{f} = \frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{15} + \frac{1}{30} = \frac{3}{30}$

$\Rightarrow \quad f = 10$ cm $= 0.10$ m

$\therefore$ Power of the lens, $P = \frac{1}{f} = \frac{1}{0.1} = +10$D

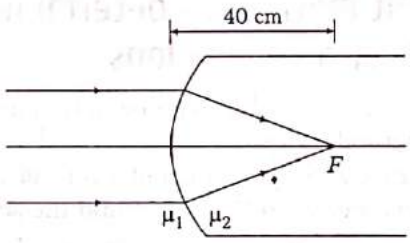
Example 9.53 *A spherical convex surface separates object and image space of refractive index 1.0 and 4/3. If radius of curvature of the surface is 10 cm, find its power.*

Sol. Let us see where does the parallel rays converge (or diverge) on the principal axis. Let us call it the focus and the corresponding length the focal length f.

We have, $\quad \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$$\frac{4/3}{f} - \frac{1.0}{\infty} = \frac{4/3 - 1.0}{+10}$$

or frequency $\quad f = 40$ cm $= 0.4$ m



Since, the rays are converging, its power should be positive. Hence,

P(in dioptre) $= \frac{+1}{f(\text{metre})} = \frac{1}{0.4}$ or $P = 2.5$ D

Example 9.54 *A thin glass (refractive index 1.5) lens has optical power of −5 D in air. Calculate its optical power in a liquid medium with refractive index 1.6.*

Sol. From Lens maker's formula we have,

$$\frac{1}{f_a} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

It is given that, $\mu = 1.5$

$\therefore \quad \frac{1}{f_a} = (1.5 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

and $\quad \frac{1}{f_m} = \left(\frac{\mu_g - \mu_m}{\mu_m}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ ...(i)

$\Rightarrow \quad \frac{1}{f_m} = \left(\frac{1.5}{1.6} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ ...(ii)

Thus, $\quad \frac{f_m}{f_a} = \frac{(1.5 - 1)}{\left(\frac{1.5}{1.6} - 1\right)} = -8$

$\Rightarrow \quad f_m = -8 \times f_a$

$= -8 \times \frac{-1}{5} = 1.6$m $\quad \left(\because f_a = \frac{1}{P} = -\frac{1}{5}\text{m}\right)$

$\therefore$ Power of the lens, $P_m = \frac{\mu}{f_m} = \frac{1.6}{1.6} = 1$ D

Combination of two or more thin lenses in contact

Combinations of lenses in contact are used in many optical instruments to improve their performance.

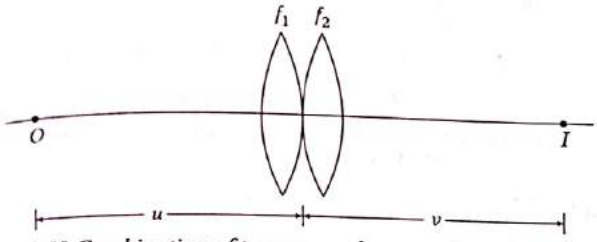


Fig. 9.58 Combination of two convex lenses with zero separation

(i) **When two lens are in contact** Combination will behaves as a lens, which has lesser focal length.

$$\boxed{\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}}$$

Similarly, for more than two lenses in contact, the equivalent focal length is given by the formula,

$$\boxed{\frac{1}{F}=\sum_{i=1}^{n}\frac{1}{f_i}=\frac{1}{f_1}+\frac{1}{f_2}+\ldots}$$

Note Here f_1, f_2 etc., are to be substituted with sign.

Similarly,

(a) Power of combination,

$$\boxed{P=P_1+P_1+\ldots=\sum_{i=1}^{n}P_i}$$

(b) Magnification of combination,

$$\boxed{M=m_1\times m_2\times\ldots=\prod_{i=1}^{n}\text{m}}$$

(ii) **Lenses separated by a distance**

If two lenses of focal lengths f_1 and f_2 are separated by a distance x, then its equivalent focal length

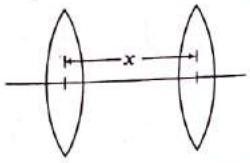


Fig. 9.59 Combination of two convex lenses with x separation

$$\boxed{\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{x}{f_1f_2}}$$

(a) Power of combination,

$$\boxed{P=P_1+P_2-xP_1P_2}$$

(b) Total magnification remains unchanged *i.e.*,

$$\boxed{m=m_1\times m_2}$$

Note Two convex lens behaves as
(i) convex lens, if $(f_1+f_2)>d$
(ii) glass plate, if $(f_1+f_2)=d$
(iii) concave lens, if $(f_1+f_2)<d$

Example 9.55 *A converging lens of focal length 5.0 cm is placed in contact with a diverging lens of focal length 10.0 cm. Find the combined focal length of the system.*

Sol. Here, $f_1=+5.0$ cm and $f_2=-10.0$ cm

Therefore, the combined focal length F is given by

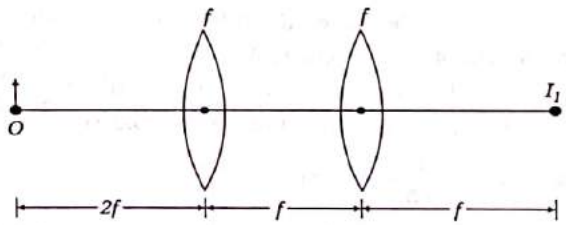
$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{5.0}-\frac{1}{10.0}=+\frac{1}{10.0}$$

$$\therefore \quad F=+10.0 \text{ cm}$$

i.e., the combination behaves as a converging lens of focal length 10.0 cm.

Example 9.56 *Two thin converging lenses are placed on a common axis, so that the centre of one of them coincides with the focus of the other. An object is placed at a distance twice the focal length from the left-hand lens. Where will its image be? What is the lateral magnification? (The focal of each lens is f.)*

Sol.



The image formed by first lens will be at a distance $2f$ with lateral magnification, $m_1=-1$. For the second lens, this image will behave as a virtual object. Using the lens formula,

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

we have,

$$\frac{1}{v}-\frac{1}{f}=\frac{1}{f}$$

$$\therefore \quad v=\frac{f}{2}$$

Magnification, $m_2=\frac{v_2}{u_2}=\frac{(f/2)}{f}=\frac{1}{2}$

Therefore, final image is formed at a distance $f/2$ from the second lens with total lateral magnification,

$$m=m_1\times m_2=(-1)\times\left(\frac{1}{2}\right)=-\frac{1}{2}$$

Example 9.57 *Two equi-convex lenses of focal lengths 30 cm and 70 cm, made of material of refractive index = 1.5, are held in contact coaxially by a rubber band round their edges. A liquid of refractive index 1.3 is introduced in the space between the lenses filling it completely. Find the position of the image of a luminous point object placed on the axis of the combination lens at a distance of 90 cm from it.*

Sol. We have $|R_1| = |R_2| = f_1 = 30$ cm (as $\mu = 1.5$)

Similarly, $|R_3| = |R_4| = f_2 = 70$ cm

The focal length of the liquid lens (in air).

$$\frac{1}{f_3} = (\mu - 1)\left(\frac{1}{R_2} - \frac{1}{R_3}\right) = (1.3 - 1)\left(\frac{1}{-30} - \frac{1}{+70}\right) = -\frac{1}{70}$$

Further, equivalent focal length of the combination,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} = \frac{1}{30} + \frac{1}{70} - \frac{1}{70} = \frac{1}{30}$$

Using the lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{F}$, we have

$$\frac{1}{v} - \frac{1}{-90} = \frac{1}{30} \Rightarrow v = +45 \text{ cm}$$

Thus, image will be formed at a distance of 45 cm from the combination.

Example 9.58 *Two thin equiconvex lenses each of focal length 0.2 m are placed coaxially with their optic centres 0.5m apart. Then find the focal length of the combination.*

Sol. Given, total length of each lens, $f_1 = f_2 = 0.2$ m

Separation between the lenses, $d = 0.5$ m

$\therefore$ Focal length of the combination,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} = \frac{1}{0.2} + \frac{1}{0.2} - \frac{0.5}{(0.2)(0.2)} = -2.5$$

$$\Rightarrow \quad F = -\frac{1}{2.5} = -0.4 \text{ m}$$

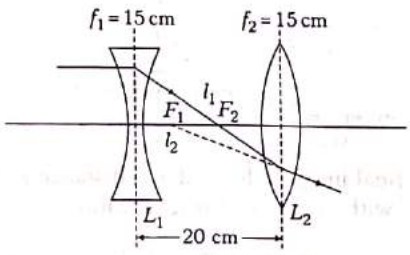
Example 9.59 *Two convex lenses, each of focal length 15 cm, are placed at a separation of 20 cm with their principal axes coinciding.*

(i) Show that a light beam coming parallel to the principal axis diverges as it comes out of the lens system.

(ii) Find the location of the virtual image formed by the lens system of an object placed far away.

(iii) Find the focal length of equivalent lens.

Sol. (i) Consider combination of lenses



(ii) Image by L_1, at distance 15 cm right of it

Image by L_2, I_1 acts object for it

$$\frac{1}{v} - \frac{1}{-5} = \frac{1}{15}$$

$$\Rightarrow \quad \frac{1}{v} = -\frac{2}{15} \Rightarrow v = -7.5 \text{ cm}$$

7.5 cm, left of L_2

(iii) Focal length of two equivalent lens,

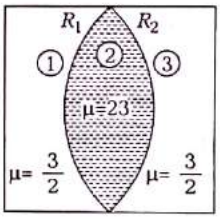
$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} = \frac{1}{15} + \frac{1}{15} - \frac{20}{100} = -\frac{1}{15}$$

$$F = -15 \text{ cm}$$

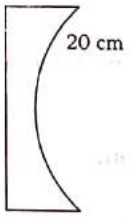
Note That the sign of the focal length is negative and the lens system diverges a parallel beam incident on it.

Example 9.60 *Two plano-concave lenses of glass of refractive 1.5 have radii of curvature of 20 and 30 cm. They are placed in contact with curved surface towards each other and the space between them is filled with a liquid of refractive index 2/3. Find the focal length of the system.*

Sol.

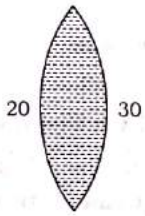


Lens 1



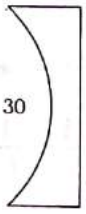
$$\frac{1}{f_1} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{\infty} - \frac{1}{20}\right) = -\frac{1}{40}$$

Lens 2



$$\frac{1}{f_2} = \left(\frac{2}{3} - 1\right)\left(\frac{1}{20} - \frac{1}{30}\right) = \left(\frac{1}{3}\right)\left(\frac{5}{60}\right) = \frac{1}{36}$$

Lens 3



$$\frac{1}{f_3}=\left(\frac{3}{2}-1\right)\left(\frac{1}{-30}-\frac{1}{\infty}\right)=\frac{-1}{60}$$

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}$$

$$=\frac{-1}{40}+\frac{1}{36}-\frac{1}{60}=\frac{-9+10-6}{360}=\frac{-5}{360}$$

$$F=-72\text{ cm}$$

Cutting of lens

(i) If a symmetrical convex lens of focal length f is cut into two parts along its optical axis, then focal length of each part (a plano-convex lens) is $2f$. However, if the two parts are joined as shown in figure, the focal length of combination is again f.

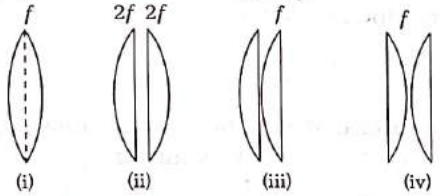


Fig. 9.60 Convex lens cut along its optical axis

(ii) If a symmetrical convex lens of focal length f is cut into two parts along the principal axis, then focal length of each part remains unchanged to f. If these two parts are joined with curved ends on one side focal length of the combination is $f/2$. But on joining two parts in opposite sense the net focal length becomes ∞ (or net power = 0).

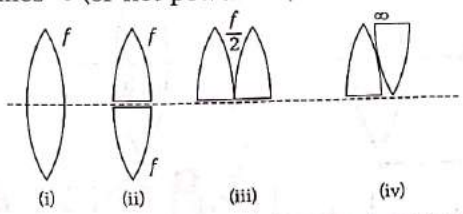


Fig. 9.61 Convex lens cut along its principal axis

Combination of lenses and mirrors

(i) When several lenses or mirror are combined coaxially, image by first serves as an object for second and so on. The net magnification is given by

$$m=m_1\times m_2\times m_3\times\ldots$$

We should keep in mind that direction of incident rays is reversed after every reflection.

(ii) In some cases, object and image are formed at same place after interacting the (lens + mirror) combination. For this, after refraction from lens, rays must retrace their path *i.e.*, must be incident normally on the mirror after refraction from the lens. In other word, we can say that image by lens should be formed at centre of curvature (C) of mirror as shown in figure.

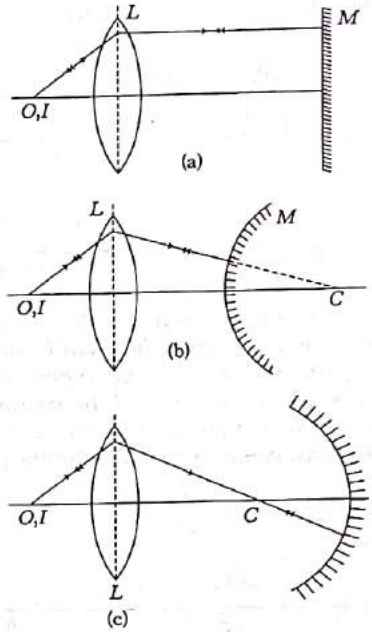
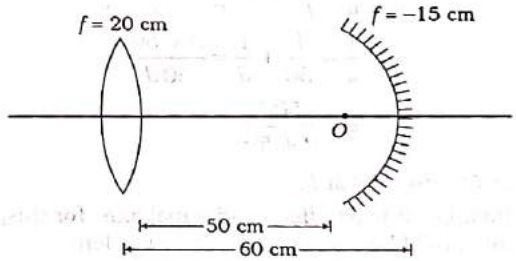


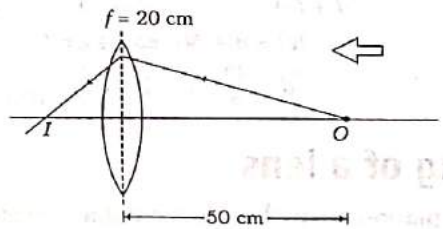
Fig. 9.62 Combinantion of lens and concave mirror

Example 9.61 *A converging lens of focal length 20 cm and a converging mirror of focal length 10 cm are placed 60 cm apart with common principal axis. A point source is placed in between the lens and the mirror at a distance of 50 cm from the lens. Find the locations of the two images formed.*

Sol. Consider the situation,



Directly by lens



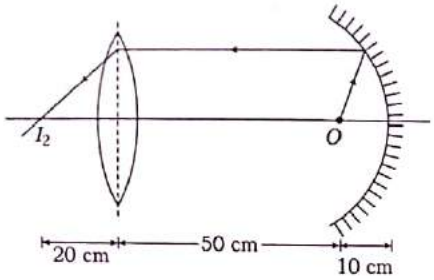
$u=-50$ cm, $f=20$ cm, $v=?$

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}\Rightarrow\frac{1}{v}-\frac{1}{-50}=\frac{1}{20}$$

$$\frac{1}{v}=\frac{1}{20}-\frac{1}{50}=\frac{3}{100}\Rightarrow v=\frac{100}{3}\text{ cm}$$

By mirror and lens By mirror, image is formed at infinite because object is at focus of mirror.

By lens, image is formed at focus of lens.



Example 9.62 *A converging lens and a diverging mirror are placed at a separation of 20 cm. The focal length of the lens is 30 cm and that of the mirror is 50 cm. Where should a point source be placed between the lens and the mirror, so that the light, after getting reflected by the mirror and then getting transmitted by the lens, comes out parallel to the principal axis?*

Sol.

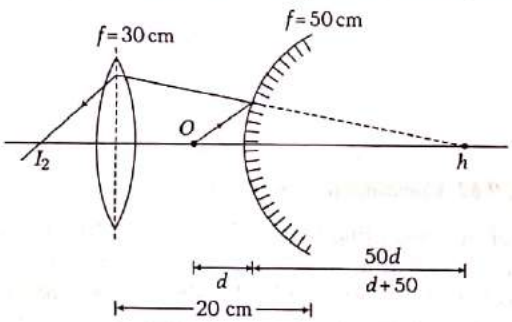


Image by mirror $u = -d, f' = 50$ cm, $v = ?$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f'} \Rightarrow \frac{1}{v} + \frac{1}{-d} = \frac{1}{50}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{50} + \frac{1}{d} = \frac{d+50}{50d}$$

$$v = \frac{50d}{d+50}$$

Image of O is formed at I_1.

Since, final beam is parallel to principal axis, for this, image by mirror should be formed at focus of the lens.

$$\Rightarrow \quad 20 + \frac{50d}{d+50} = 30 \Rightarrow \frac{50d}{d+50} = 10$$

$$\Rightarrow \quad 5d = d + 50 \Rightarrow 4\lambda = 50$$

$$\Rightarrow \quad d = \frac{25}{2} \text{ cm} = 12.5 \text{ cm}$$

Silvering of a lens

(i) Let a plano-convex lens is having a curved surface of radius of curvature R and has refractive index μ. If its plane surface is silvered, it behaves as a concave mirror of focal length. Its focal is,

$$f = \frac{R}{2(\mu - 1)}$$

(ii) If the curved surface of plano-convex lens is silvered, then it behaves as concave mirror of focal length,

$$f = \frac{R}{2\mu}$$

(iii) If one surface of a symmertical double convex lens is silvered, then the lens behaves as a concave mirror of focal length,

$$f = -\frac{R}{2(2\mu - 1)}$$

(iv) If the plane surface of a plano-concave lens is silvered the lens behaves as convex mirror.

Focal length, $f = \dfrac{R}{2(\mu - 1)}$

(v) If the spherical surface of a plano-concave lens is silvered the lens behave convex mirror.

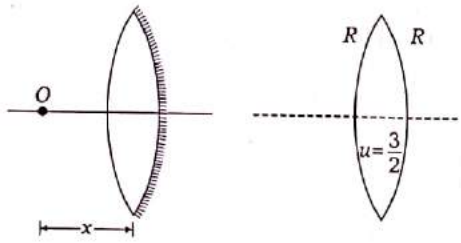
Focal length, $f = \dfrac{R}{2\mu}$

(vi) If one surface of a double concave lens is silvered the lens behave as convex mirror

Focal length, $f = \dfrac{R}{2(2\mu - 1)}$

Example 9.63 *A biconvex thin lens is prepared from glass ($\mu = 1.5$), the two bounding surfaces having equal radii of 30 cm each. One of the surface is silvered from outside to make it reflecting. Where should an object be placed before this lens, so that the image is formed on the object itself?*

Sol.



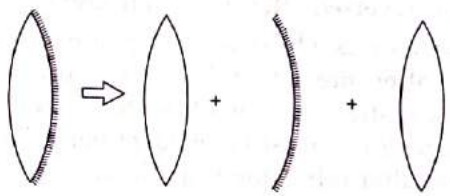
We have, $\dfrac{1}{f} = (\mu - 1)\left[\dfrac{1}{R_1} - \dfrac{1}{R_2}\right]$

$$\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{30} - \frac{1}{30}\right) = \left(\frac{1}{2}\right)\left(\frac{2}{30}\right) \Rightarrow f = 30 \text{ cm}$$

The image by lens should be formed at centre of curvature of mirror.

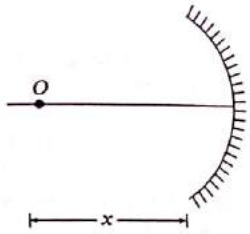
$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{-30} - \frac{1}{-x} = \frac{1}{30} \Rightarrow x = 15.5 \text{ cm}$$

Alternatively

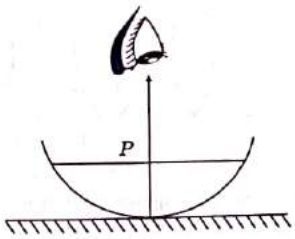


$$\frac{1}{F}=\frac{1}{f_l}+\frac{1}{f_m}+\frac{1}{f_l}=\frac{2}{f_l}+\frac{1}{f_m}$$
$$=\frac{2}{30}+\frac{1}{30/2}=\frac{4}{30}$$
$$F=\frac{30}{4}\text{ cm}$$

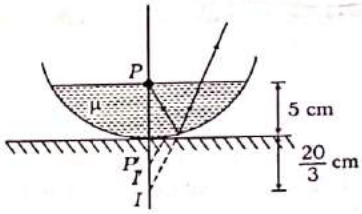
Since, object and image coincides, $u=x=2F=15$ cm



Example 9.65 *A concave mirror of radius 40 cm lies on a horizontal table and water is filled in it upto a height of 5 cm. A small dust particle floats on the water surface at a point P vertically above the point of contact of the mirror with the table. Locate the image of the dust particle as seen from a point directly above it.* $\mu_w=2/3$



Sol.



The dust particle is in water.

Image by concave mirror

$\mu=-5$ cm, $f=-20$ cm, $v=?$

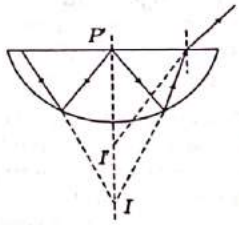
$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}\Rightarrow\frac{1}{v}+\frac{1}{-5}=\frac{1}{-20}$$
$$\Rightarrow\quad\frac{1}{v}=\frac{1}{5}-\frac{1}{20}=\frac{3}{20}\Rightarrow v=\frac{20}{3}\text{ cm}$$

The image of P is formed at I.

Image by refraction After reflection direction of incident rays is reversed. I serves as an object

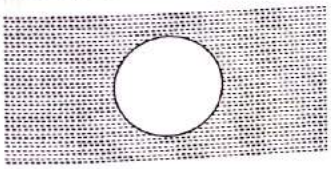
$$\mu_1=\frac{2}{3},\ u=-\left(5+\frac{20}{3}\right)=-\frac{35}{3}\text{ cm}$$
$$\mu_2=1,\ v=?$$
$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=0\Rightarrow\frac{1}{v}-\frac{2/3}{(-35/3)}\Rightarrow v=-\frac{35}{2}=-17.5\text{ cm}$$

Final image is formed at I'. $PI=17.5$ cm



Check point 9.4

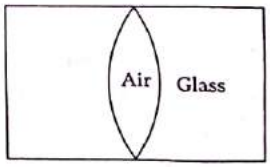
1. An air bubble is contained inside water. It behave as a



(a) concave lens
(b) convex lens
(c) Neither convex nor concave
(d) Cannot say

2. A convex lens has a focal length of 20 cm. It is used to form an image of an object placed 15 cm from lens. The image is
(a) virtual, inverted and enlarged
(b) real, inverted and diminished
(c) real, inverted and enlarged
(d) virtual, erect and enlarged

3. In the figure, an air lens of radii of curvature 10 cm ($R_1=R_2=10$ cm) is cut in a cylinder of glass ($\mu=2/3$). The focal length and the nature of the lens is



(a) 15 cm, concave
(b) 15 cm, convex
(c) ∞, neither concave nor convex
(d) 0, concave

4. A plano-convex lens is made of glass of refractive index 1.5. The radius of curvature of its convex surface is R. Its focal length is
(a) $R/2$ (b) R
(c) $2R$ (d) $1.5R$

5. At what distance from a convex lens of focal length 30 cm an object should be placed, so that the size of image be 1/4th of the object ?
(a) 30 cm (b) 60 cm
(c) 15 cm (d) 150 cm

6. A plano-convex lens is made of flint glass. Its focal length is
(a) inversely proportional to the wavelength of light
(b) longer for red than for blue
(c) longer for blue than for red
(d) the same for all colours

7. Distance of an object from a concave lens of focal length 20 cm is 40 cm. Then linear magnification of the image
(a) $=1$ (b) <1
(c) <1 (d) zero

8. In order to obtain a real image of magnification 2 using converging lens of focal length 20 cm, where should an object be placed
(a) 50 cm (b) 30 cm
(c) –50 cm (d) –30 cm

9. An object is placed at 10 cm from a lens and real image is formed with magnification of 0.5. Then the lens is
(a) concave with focal length of 10/3 cm
(b) convex with focal length of 10/3 cm
(c) concave with focal length of 10 cm
(d) convex with focal length of 10 cm

10. The real image which is exactly equal to the size of an object is to be obtained on a screen with the help of a convex lens of focal length 15 cm. For this, what must be in the distance between the object and screen?
(a) 15 cm (b) 30 cm
(c) 45 cm (d) 60 cm

11. A plano-convex lens of curvature of 30 cm and refractive index 1.5 produces a real image of an object kept 90 cm from it. What is the magnification?
(a) 4 (b) 0.5
(c) 1.5 (d) 2

12. The minimum distance between an object and its real image formed by a convex lens is
(a) $1.5f$ (b) $2f$
(c) $2.5f$ (d) $4f$

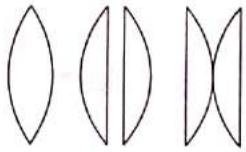
13. A convex lens of refractive index 3/2 has a power of 2.5°. If it is placed in a liquid of refractive index 2, the new power of the lens is
(a) 2.5 D (b) – 2.5 D (c) 1.25 D (d) – 1.25 D

14. Two thin lenses of focal lengths f_1 and f_2 are in contact and co-axial. The combination is equivalent to a single lens of power
(a) $f_1 + f_2$ (b) $\frac{f_1 f_2}{f_1 + f_2}$ (c) $\frac{1}{2}(f_1 + f_2)$ (d) $\frac{f_1 + f_2}{f_1 f_2}$

15. Two thin lenses, one of focal length +60 cm and the other of focal length –20 cm are put in contact. The combined focal length is
(a) +15 cm (b) –15 cm
(c) +30 cm (d) –30 cm

16. A convex lens of focal length 40 cm is in contact with a concave lens of focal length 25 cm. The power of combination is
(a) –1.5 D (b) –6.5 D
(c) +6.5 D (d) +1.5 D

17. Two similar plano-convex lenses are combined together in three different ways as shown in the adjoining figure. The ratio of the focal lengths in three cases will be



(a) 2 : 2 : 1 (b) 1 : 1 : 1
(c) 1 : 2 : 2 (d) 2 : 1 : 1

18. A biconvex lens has a focal length f. It is cut into two parts along a line perpendicular to principal axis. The focal length of each part will be
(a) $f/2$ (b) f (c) $(3/2)f$ (d) $2f$

19. A converging lens is used to form an image on a screen. When the upper half of the lens is covered by an opaque screen, then
(a) half the image will disappear
(b) complete image will disappear
(c) intensity of image will increase
(d) intensity of image will decrease

20. In a plano-convex lens the radius of curvature of the convex lens is 10 cm. If the plane side is polished, then the focal length will be (Refractive index = 1.5)
(a) 20.5 cm (b) 10 cm
(c) 15.5 cm (d) 5 cm

5 Prism

A prism is a portion of a transparent medium generally made of crown or flint glass and bounded by three planes intersect in three parallel straight lines AD, CF and BE as shown in figure.

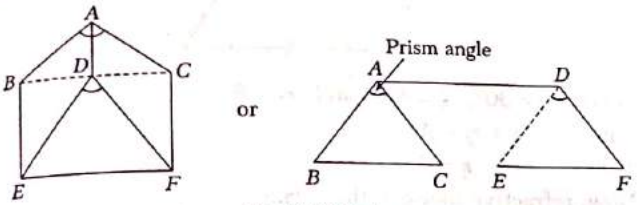


Fig. 9.63 Prism

$ABDE$ and $ACFD$ are two refracting faces as shown in the figure. $\angle A$ is called angle of prism or refracting angle.

Refraction of light through a prism

The figure below shows the passage of light through a triangular prism ABC.

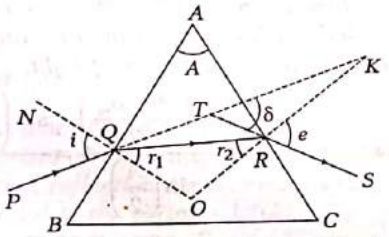


Fig. 9.64 Cross-sectional view of a prism

The angles of incidence and refraction at first face AB are i and r_1.

The angle of refraction at the second face AC is r_2 and the angle of emergence is e.

The angle between the emergent ray RS and incident ray PQ is called angle of deviation (δ).

Here, angle of deviation is given by,

$$\boxed{\delta = (i_1 + i_2) - A} \quad \text{...(i)}$$

where, A is angle of prism and $\angle A = \angle r_1 + \angle r_2$, ... (ii)

or $\delta = (1 + e) - A$ where e is angle of emergence which is equal to i_2.

If angle of incidence and refraction are small, then

$$\delta = (\mu - 1)\, A \quad \text{...(iii)}$$

This expression shows that for a given angle A all rays entering a small angle prism at small angles of incidence suffer the same deviation.

Minimum deviation

The angle of emergence of the ray from the second face equals the angle of incidence of the ray on the first face then deviation produced is minimum.

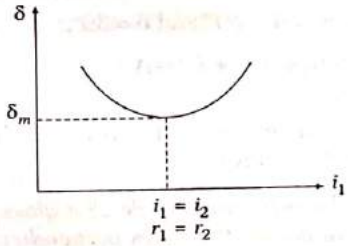


Fig. 9.65 i_1-δ curve

i.e., $\quad i_2 = i_1 = i \quad$ ... (iv)

and $\quad r_1 = r_2 = r \quad$ ... (v)

From Eqs. (ii) and (v), we get

$$r = \frac{A}{2}$$

Further at, $\quad \delta = \delta_m = (i + i) - A$

or $\quad i = \dfrac{A + \delta_m}{2} \Rightarrow \mu = \dfrac{\sin i}{\sin r}$

$$\boxed{\mu = \frac{\sin\left(\dfrac{A + \delta_m}{2}\right)}{\sin\dfrac{A}{2}}} \quad \text{...(vi)}$$

Expression given as Eq. (vi) is also called **prism formula.**

For small angled prism, from Eqs. (iii) or (vi), we get

$$\delta_m = (\mu - 1)A \quad \text{...(x)}$$

Condition of no emergence

For no emergence of light, total internal reflections must takes place at the second surface.

For total internal reflection at second surface

$$r_2 > C$$

So, $\quad A > r_1 + C \quad (\because A = r_1 + r_2)$

Maximum value of $r_1 = C$

So, $A \geq 2C$ for any angle of incidence

If light ray is incident normally on first surface

i.e., $\quad i = 0$, it means $\angle r = 0°$

So in this case, condition of no emergence from second surface is $A > C$

$\therefore \quad \sin A > \sin C$

$$\sin A > \frac{1}{\mu} \Rightarrow \mu > \operatorname{cosec} A$$

Example 9.65 *A ray of light is incident at an angle of 60° on the face of a prism having refracting angle 30°. The ray emerging out of the prism makes an angle 30° with the incident ray. Show that the emergent ray is perpendicular to the face through which, it emerges.*

Sol. Given, $i_1 = 60°, A = 30°$ and $\delta = 30°$.

From the relation, $\delta = (i_1 + i_2) - A$

we have, $i_2 = 0$

This means that the emergent ray is perpendicular to the face through which it emerges.

Example 9.66 *The refracting angle of a glass prism is 30°. A ray is incident onto one of the faces perpendicular to it. Find the angle δ between the incident ray and the ray that leaves the prism. The refractive index of glass is μ =1.5.*

Sol. Given, $A = 30°, \mu = 1.5$ and $i_1 = 0°$

Since, $i_1 = 0°$, therefore, r_1 is also equal to 0°.

Further, since, $r_1 + r_2 = A$

$\therefore \quad r_2 = A = 30°$

Using, $\quad \mu = \dfrac{\sin i_2}{\sin r_2}$

we have, $\quad 1.5 = \dfrac{\sin i_2}{\sin 30°}$

or $\quad \sin i_2 = 1.5 \sin 30° = 1.5 \times \dfrac{1}{2} = 0.75$

$\therefore \quad i_2 = \sin^{-1}(0.75) = 48.6°$

Now, the deviation, $\delta = (i_1 + i_2) - A = (0° + 48.6°) - 30°$

or $\quad \delta = 18.6°$

Example 9.67 *The angle of minimum deviation for a glass prism with $\mu = \sqrt{3}$ equals the refracting angle of the prism. What is the angle of the prism?*

Sol. Given, $\quad A = \delta_m$

Using, $\quad \mu = \dfrac{\sin\left(\dfrac{A + \delta_m}{2}\right)}{\sin\left(\dfrac{A}{2}\right)}$

we have, $\quad \sqrt{3} = \dfrac{\sin\left(\dfrac{A + A}{2}\right)}{\sin\left(\dfrac{A}{2}\right)}$

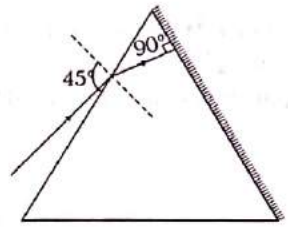
or $\quad \sqrt{3} = \dfrac{\sin A}{\sin\left(\dfrac{A}{2}\right)} = \dfrac{2 \sin \dfrac{A}{2} \cos \dfrac{A}{2}}{\sin\left(\dfrac{A}{2}\right)}$

$\therefore \quad \cos \dfrac{A}{2} = \dfrac{\sqrt{3}}{2}$

$\therefore \quad \dfrac{A}{2} = 30° \text{ or } A = 60°$

Example 9.68 *One face of a prism with a refractive angle of 30° is coated with silver. A ray incident on another face at an angle of 45° is refracted and reflected from the silver coated face and retraces its path. What is the refractive index of the prism?*

Sol.



Given, $A = 30°, \; i_1 = 45°$ and $r_2 = 0$

Since, $\quad r_1 + r_2 = A$

$\therefore \quad r_1 = A = 30°$

Now, refractive index of the prism,

$$\mu = \frac{\sin i_1}{\sin r_1} = \frac{\sin 45°}{\sin 30°} = \frac{1/\sqrt{2}}{1/2} = \sqrt{2}$$

Example 9.69 *A prism is made of glass of unknown refractive index. A parallel beam of light is incident on a face of the prism. The angle of minimum deviation is measured to be 40°. What is the refractive index of the material of the prism? The refracting angle of the prism is 60°. If the prism is placed in water (refractive index 1.33), predict the new angle of minimum deviation of a parallel beam of light.*

Sol. Using the relation, $\mu = \dfrac{\sin\left(\dfrac{A + \delta_m}{2}\right)}{\sin\left(\dfrac{A}{2}\right)} = \dfrac{\sin\left(\dfrac{60° + 40°}{2}\right)}{\sin\left(\dfrac{60°}{2}\right)}$

$$= \frac{\sin 50°}{\sin 30°} = \frac{0.7660}{0.50} = 1.532 \approx 1.53$$

Let δ'_m be the new angle of minimum deviation of a parallel beam of light when the prism is placed in water ${}_a\mu_w = 1.33$, ${}_a\mu_g = 1.532$

$\therefore \quad {}_w\mu_g = \dfrac{{}_a\mu_g}{{}_a\mu_w} = \dfrac{1.532}{1.33} = 1.152$

$${}_w\mu_g = \frac{\sin\left(\dfrac{A + \delta'_m}{2}\right)}{\sin\left(\dfrac{A}{2}\right)}$$

or $\quad 1.152 = \dfrac{\sin\left(\dfrac{60 + \delta'_m}{2}\right)}{\sin \dfrac{60}{2}}$

or $\quad \sin\left(30° + \dfrac{\delta'_m}{2}\right) = 1.152 \times \dfrac{1}{2} = 0.5759 = \sin 35.17°$

or $\quad 30° + \dfrac{\delta'_m}{2} = 35.17° \text{ or } \dfrac{\delta'_m}{2} = 35.17° - 30 = 5.17°$

Angle of minimum deviation, $\delta'_m = 2 \times 5.17° = 10.34° = 10°20'$

Example 9.70 *If the refractive index of a glass prism is $\cot\left(\frac{A}{2}\right)$ and A is angle of prism, then find the angle of minimum deviation.*

Sol. The refractive index (μ) of a prism of angle A and minimum deviation, δ_m is given by

$$\mu = \frac{\sin\left(\frac{4+\delta_m}{2}\right)}{\sin A/2}$$

Given, $\mu = \cos\frac{A}{2}$

$$\therefore \quad \cot\frac{A}{2} = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin(A/2)}$$

$$\Rightarrow \quad \frac{\cos A/2}{\sin A/2} = \frac{\sin\frac{(A+\delta)}{2}}{\sin(A/2)} \Rightarrow \cos\frac{A}{2} = \sin\left(\frac{A+\delta_m}{2}\right)$$

$$\therefore \quad \sin\left(90°-\frac{A}{2}\right) = \sin\left(\frac{A+\delta_m}{2}\right)$$

$$\Rightarrow \quad 90° - \frac{A}{2} = \frac{A+\delta_m}{2} \Rightarrow 180°-A = A+\delta_m$$

$\Rightarrow$ Angle of minimum deviation, $\delta_m = 180°-2A = \pi - 2A$

Example 9.71 *At what angle should a ray of light be incident on the face of a prism of refracting angle 60°, so that it just suffers total internal reflection at the other face? The refractive index of the material of the prism is 1.524.*

Sol. Let C = critical angle for the material of the prism.

$$\therefore \quad \sin C = \frac{1}{\mu} = \frac{1}{1.524} = 0.6562$$

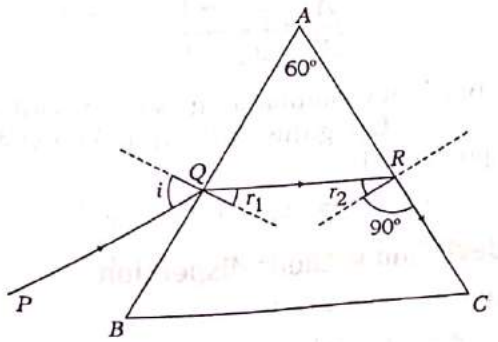
$$\therefore \quad C = \sin^{-1}(0.6562) = 41°$$

We know that for a prism, $A = r_1 + r_2$

Here, $r_2 = C = 41°, A = 60°$

$\therefore \quad 60° = r_1 + 41°$ or $r_1 = 60° - 41° = 19°$

For the face AB, $r_1 = 19°, \mu = 1.524$



$\therefore$ According to Snell's law,

$$\mu = \frac{\sin i}{\sin r_1}$$

or $\quad \sin i = \mu \sin r_1 = 1.524 \sin 19°$

$$= 1.524 \times 0.3256 = 0.4962$$

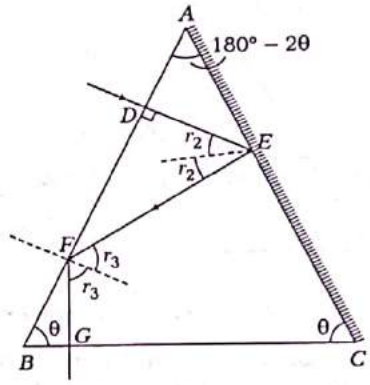
$$i = \sin^{-1}(0.4962) = 29.75°$$

$\therefore$ Angle of incidence, $i = 29.75° \approx 30°$

Example 9.72 *An isosceles glass prism has one of, its faces coated with silver. A ray of light is incident normally on the other face (which is equal to the silvered face). The ray of light is reflected twice on the same sized faces and then emerges through the base of the prism perpendicularly. Find angles of prism.*

Sol. Given, $r_1 = 0$,

$$\therefore \quad r_2 = A = 180° - 2\theta \quad \text{...(i)}$$



$$\angle DFE = 180° - 90° - 2r_2$$

$$= 180° - 90° - 360° + 4\theta$$

$$= 4\theta - 270° \quad \text{...(ii)}$$

$$\therefore \quad r_3 = 90° - \angle DFE = 360° - 4\theta \quad \text{...(iii)}$$

$$\angle BFG = 90° - \theta = 90° - r_3$$

or $\quad r_3 = \theta \quad$ ...(iv)

From Eqs. (iii) and (iv), we have

$$5\theta = 360°$$

$$\therefore \quad \theta = 72° \text{ and } 180° - 2\theta = 36°$$

$\therefore$ Angles of prism are 72°, 72° and 36°.

Dispersion of light by a prism

If a beam of white light, (contains all colours), is sent through the prism, it is separated into a spectrum of colours. The spreading of light into, its colour components is called **dispersion.**

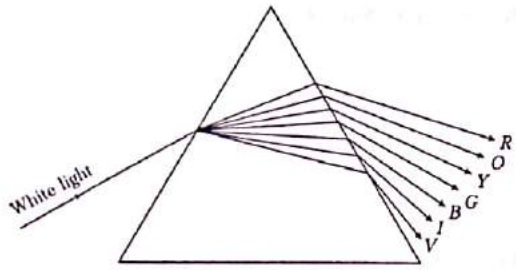


Fig. 9.66 Dispersion of white light

If δ_r, δ_y and δ_v are the deviations for red, yellow and violet components, then average deviation is measured by δ_y as yellow light falls in between red and violet.
So, $\delta_v - \delta_r$ is called **angular dispersion.** The **dispersive power** of a material is defined as the ratio of angular dispersion to the average deviation when a white beam of light is passed through it. It is denoted by ω. As we know,

$$\delta = (\mu - 1)\, A$$

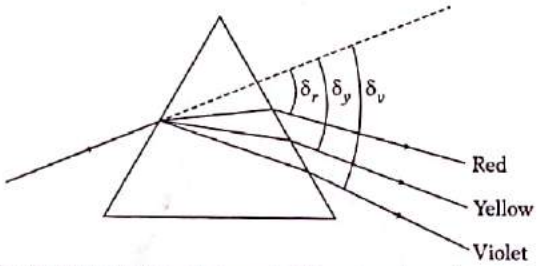


Fig. 9.67 Deviation of rays of different colours in a prism

This equation is valid, when A and i are small. Suppose, a beam of white light is passed through such a prism, the deviation of red, yellow and violet light are given by,

$$\delta_r = (\mu_r - 1)A,$$
$$\delta_y = (\mu_y - 1)A,$$
and $$\delta_v = (\mu_v - 1)\, A$$

The angular dispersion is, $\delta_v - \delta_r = (\mu_v - \mu_r)\, A$ and the average deviation is $\delta_y = (\mu_y - 1)\, A$.

Thus, the dispersive power of the medium is,

$$\boxed{\omega = \frac{\delta_v - \delta_r}{\delta_y} = \frac{\mu_v - \mu_r}{\mu_y - 1}} \quad \text{...(i)}$$

Note

1. Deviation corresponding to yellow coloured light is average or mean deviation, $\mu_y = \frac{\mu_v + \mu_r}{2}$
2. A single prism produces both deviation and dispersion simultaneously.

Example 9.73 *The refractive indices of flint glass for red and violet light are 1.613 and 1.632, respectively. Find the angular dispersion produced by the thin prism of flint glass having refracting angle 4°.*

Sol. Angular dispersion,

$$\theta = (\mu_v - \mu_r)A = (1.632 - 1.613) \times 4 = 0.076°$$

Example 9.74 *Evaluate the dispersive power of flint glass. The refractive indices of flint glass for red, yellow and violet light are 1.613, 1.620 and 1.632, respectively.*

Sol. Dispersive power is given by,

$$\omega = \frac{\mu_v - \mu_r}{\mu_y - 1} = \frac{1.632 - 1.613}{1.620 - 1} = 0.031$$

Combination of prisms

Figure below shows two prisms of refracting angles A and A' and dispersive powers ω and ω' respectively. They are placed in contact in such a way that the two refracting angles are reversed with respect to each other. A ray of light passes through the combination as shown. The deviation produced by the two prisms are,

$$\delta_1 = (\mu - 1)\, A \quad \text{and} \quad \delta_2 = (\mu' - 1)\, A'$$

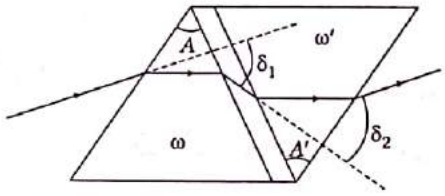


Fig. 9.68 Combination of two prisms

As the two deviations are opposite to each other, the net deviation is,

$$\delta = \delta_1 - \delta_2 = (\mu - 1)A - (\mu' - 1)\, A' \quad \text{...(ii)}$$

Using this equation, the average deviation produced by the combination, if white light is passed is,

$$\delta_y = (\mu_y - 1)A - (\mu'_y - 1)\, A' \quad \text{...(iii)}$$

and the net angular dispersion is,

$$\delta_v - \delta_r = (\mu_v - \mu_r)A - (\mu'_v - \mu'_r)A'$$

But, as $\mu_v - \mu_\gamma = \omega(\mu_y - 1)$ from Eq. (i), we have

$$\delta_v - \delta_r = (\mu_y - 1)\omega A - (\mu'_y - 1)\, \omega'\, A' \quad \text{...(iv)}$$

Dispersion without average deviation

From Eq. (iii), $\delta_\gamma = 0$, if

$$\frac{A}{A'} = \frac{\mu'_y - 1}{\mu_y - 1} \quad \text{...(v)}$$

This is the required condition of dispersion without average deviation. Using this in Eq. (iv), the net angular dispersion produced is :

$$\delta_v - \delta_r = (\mu_y - 1)A\, (\omega - \omega')$$

Average deviation without dispersion

From Eq. (iv), $\delta_v - \delta_r = 0$, if

$$\frac{A}{A'} = \frac{(\mu'_y - 1)\, \omega'}{(\mu_y - 1)\, \omega} = \frac{\mu'_v - \mu'_r}{\mu_v - \mu_r} \quad \text{...(vi)}$$

This is the required condition of average deviation without dispersion. Using the above condition in Eq. (iii), the net average deviation is,

$$\delta_\gamma = (\mu_y - 1)A\left(1 - \frac{\omega}{\omega'}\right)$$

Example 9.75 *White light is passed through a prism of angle 4°. If the refractive indices for red and blue colours are 1.641 are 1.659 respectively, then calculate the angle of deviation between them, also calculate the dispersive power.*

Sol. Angle of deviation,

$$\delta = (\mu_y - 1)A = \left(\frac{\mu_r + \mu_b}{2} - 1\right)4°$$

$$= \left(\frac{1.641 + 1.659}{2} - 1\right) \times 4° = (1.65 - 1) \times 4° = 2.6°$$

Dispersive power,

$$\omega = \frac{\mu_b - \mu_r}{\mu_y - 1} = \frac{1.659 - 1.641}{1.650 - 1} = 0.028$$

Example 9.76 *We have to combine a crown glass prism of angle 4° with a flint glass prism in such a way that the mean ray passes undeviated, find (i) the angle of the flint glass prism needed and (ii) the angular dispersion produced by the combination when white light goes through it. Refractive indices for red, yellow and violet light are 1.514, 1.517 and 1.523 respectively for crown glass and 1.613, 1.620 and 1.632 for flint glass.*

Sol. It is given that, for

Crown glass: $\mu_r = 1.514, \mu_y = 1.517, \mu_v = 1.523, A = 4°$

Flint glass: $\mu_r' = 1.613, \mu_y' = 1.620, \mu_v' = 1.632, A' = ?$

(i) For no deviation $(\mu_y - 1)A = (\mu_y' - 1)A'$

$$\Rightarrow \quad (1.517 - 1) \times 4 = (1.620 - 1)A' \Rightarrow A' = 3.34°$$

(ii) Also, angular dispersion,

$$\theta_0 = \theta_1 - \theta_2$$
$$= |(\mu_v - \mu_r)A - (\mu_v' - \mu_r')A'|$$
$$= |(1.523 - 1.514) \times 4 - (1.632 - 1.613) \times 3.34|$$
$$= 0.027°$$

Example 9.77 *It is given that the dispersive powers of crown and flint glasses are 0.06 and 0.10 respectively. Also the refractive indices for yellow light for these glasses are 1.517 and 1.621, respectively. We have to form an achromatic combination of prism of crown and flint glasses which can produce a deviation of 1° in the yellow ray. Evaluate the refracting angles of the two prisms needed.*

Sol. Crown glass, $\omega = 0.06, \mu_y = 1.517$

Flint glass, $\omega' = 0.10, \mu_y' = 1.621$

For no dispersion,

$$(\mu_v - \mu_r)A = (\mu_v' - \mu_r')A'3$$

$$\Rightarrow \quad \omega(\mu_y - 1)A = \omega'(\mu_y' - 1)A'$$

$$\left[\omega = \frac{\mu_v - \mu_r}{\mu_y - 1}, \omega' = \frac{\mu'_v - \mu_r'}{\mu_y' - 1}\right]$$

$$\Rightarrow \quad 0.06(1.517 - 1)A = 0.10(1.621 - 1)A' \Rightarrow A = 2A'$$

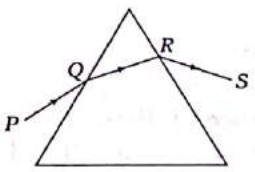
Deviation,

$$\delta_0 = |\mu_y - 1)A - (mu_y' - 1)A'|$$
$$1 = |(1.517 - 1)A - (1.621 - 1)A'|$$
$$= |0.517A - 0.621A'|$$
$$= |0.517 \times 2A' - 0.621A'|$$
$$\Rightarrow \quad 1 = 0.413A' \Rightarrow A' = 2.42°$$

$\Rightarrow$ Angle of prism, $A = 2A' = 4.84°$

Check point 9.5

1. A ray of light is incident at an angle of 60° on one face of a prism of angle 30°. The ray emerging out of the prism makes an angle of 30° with the incident ray. The emergent ray is
   (a) normal to the face through which it emerges
   (b) inclined at 30° to the face through which it emerges
   (c) inclined at 60° to the face through which it emerges
   (d) None of the above
2. When light rays are incident on a prism at an angle of 45°, the minimum deviation is obtained. If refractive index of the material of prism is $\sqrt{2}$, then the angle of prism will be
   (a) 30° (b) 75°
   (c) 90° (d) 60°
3. A ray of light passes through an equilateral glass prism in such a manner that the angle of incidence is equal to the angle of emergence and each of these angles is equal to 3/4 of the angle of the prism. The angle of deviation is
   (a) 45° (b) 39° (c) 20° (d) 30°
4. A ray of light is incident on an equilateral glass prism placed on a horizontal table. For minimum deviation which of the following is true?



   (a) *PQ* is horizontal (b) *QR* is horizontal
   (c) *RS* is horizontal (d) Either *PQ* or *RS* is horizontal

5. In a thin prism of glass (refractive index 1.5), which of the following relations between the angle of minimum deviations δ_m and angle of prism r will be correct?
(a) $\delta_m = r$ (b) $\delta_m = 1.5\,r$
(c) $\delta_m = 2r$ (d) $\delta_m = r/2$

6. The refractive index of a prism for a monochromatic wave is $\sqrt{2}$ and its refracting angle is 60°. For minimum deviation, the angle of incidence will be
(a) 30° (b) 45° (c) 60° (d) 75°

7. The angle of minimum deviation for a prism of refractive index 1.5 is equal to the angle of the prism. The angle of prism is .... (Given, cos 41° = 0.75)
(a) 21° (b) 42° (c) 60° (d) 82°

8. When light of wavelength λ is incident on an equilateral prism kept in, its minimum deviation position, it is found that the angle of deviation equals the angle of the prism itself. The refractive index of the material of the prism for the wavelength λ is, then
(a) $\sqrt{3}$ (b) $\frac{\sqrt{3}}{2}$ (c) 2 (d) $\sqrt{2}$

9. Dispersive power depends upon
(a) the angle of prism
(b) material of prism
(c) deviation produced by prism
(d) height of the prism

10. A thin prism P_1 with angle 6° and made from glass of refractive index 1.54 is combined with another thin prism P_2 of refractive index 1.72 to produce dispersion without deviation. The angle of prism P_2 will be
(a) 5°24′ (b) 4°30′ (c) 6° (d) 8°

6 Optical instruments

Optical instruments (microscope, telescope etc.) are used to assist the eye in viewing an object. We shall discuss these instruments in detail in coming sections followed by some basics of the eye.

The eye

The eye is the organ which gives us the sense of sight.

(i) The eye behave as convex lens of refractive index 1.437.
(ii) Real and inverted image of an object obtained at retina, brain sense it erect.
(iii) Yellow spot is the most sensitive part, the image formed at yellow spot is brighter.
(iv) Blind spot is not sensitive for light.
(v) Eye lens is fixed between ciliary muscles. Its both radius of curvature can be changed by appling pressure on it through ciliary muscles.

Power of accommodation

The ability of eye to see near object as well as far objects is called power of accommodation.

Range of vision

The minimum distance for the clear vision of an object is called **least distance of distinct vision (D).** For normal eye this distance is generally taken to be 25 cm. The range of vision of eye is 25 cm (near point) to ∞ (far point).

Persistence of vision

It is 1/10s *i.e.*, if time interval, between two consecutive light pulse is lesser than 0.1s, eye cannot distinguish them separately.

Visual angle The size of an object as sensed by us is related to the size of the image formed on the retina.

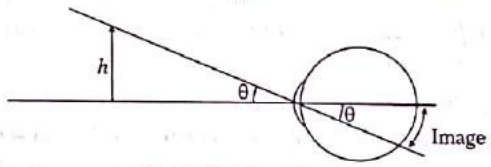


Fig. 9.69 Visual image

The size of the image on the retina is roughly proportional to the angle subtended by the object on the eye. This angle is known as the **visual angle**. Optical instruments are used to increase, this angle artificially in order to improve the clarity.

Note While testing your eye through reading chart, if doctor finds it to 6/2 it implies that you can read a letter from 6m which the normal eye can read from 12m. Thus, 6/6 means normal eye sight.

Magnifying power (*M*)

Magnifying power is the factor by which the image on the retina can be enlarged by using the microscope or telescope.

Magnifying power of a microscope,

$$M = \frac{\text{Visual angle formed by final image}}{\text{Visual angle formed by the object when kept at distance } D}$$

Magnifying power of a telescope,

$$M = \frac{\text{Visual angle formed by final image}}{\text{Visual angle subtended by the object directly when seen from naked eye}}$$

Microscope

It is an optical instrument used to see very small object.

Simple microscope

A simple microscope consists of a single convex lens (converging lens) of small focal length. The image formed by this microscope is erect, virtual enlarged and on same side of object.

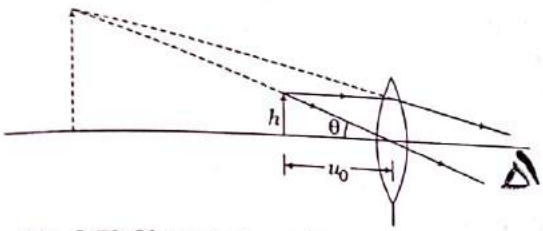


Fig. 9.70 Object in front of a simple microscope

(i) When final image is formed at infinity.

Magnifying power, $M_\infty = \frac{D}{f}$

This is also called magnifying power for normal adjustment.

(ii) When final image is formed at D.

The magnifying power forms at the near point.

$$M_D = 1 + \frac{D}{f}$$

Note

1. That $M_D > M_\infty$, *i.e.*, when final image is formed at 25 cm, angular magnification is increased but eye is most strained. On the other hand, when final image is at infinity, angular magnification is slightly less but eye is relaxed. So, the choice is yours, whether you want to see bigger size with strained eye or smaller size with relaxed eye.
2. That M can be increased by decreasing f, but due to several other aberrations the image becomes too defective at large magnification with a simple microscope. Roughly speaking a magnification upto magnification power upto 4 is trouble free.

Example 9.78 *A 20 D lens is used as a magnifier. Where should the object be placed to obtain maximum angular magnification? (Given, D = 25 cm).*

Sol. Focal length of the lens,

$$f = \frac{1}{20}\text{ m} = \frac{100}{20}\text{ cm} = 5\text{ cm}$$

Maximum angular magnification is obtained when final image is formed at D. Hence,

$$\frac{1}{-25} - \frac{1}{-u_o} = \frac{1}{5} \Rightarrow u_o = 4.17\text{ cm}$$

Example 9.79 *An object is seen through a simple microscope of focal length 20 cm. Find the angular magnification produced, if the image is formed at 30 cm from the lens.*

Sol. Given, $f = +20$ cm and $v = -30$ cm

Using the formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

we have, $\frac{1}{-30} - \frac{1}{-u_o} = \frac{1}{20} \Rightarrow u_o = 12$ cm

The angular magnification,

$$M = \frac{D}{u_o} = \frac{25}{12} = 2.08$$

Compound microscope

Simple microscope has a limited maximum magnification. So, for larger magnification, one can use compound microscope.

It consists of two converging lenses arranged coaxially. The one facing the object is called **objective** and the one close to eye is called **eye-piece**. The objective has a smaller aperture and smaller focal length than those of the eye-piece.

The separation between the objective and the eye-piece (called the length of the microscope L) can be varied by appropriate screws fixed on the panel of microscope.

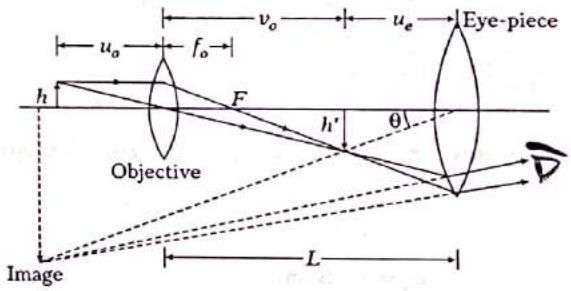


Fig. 9.71 Image formation in a compound microscope

The object is placed beyond first focus of objective, so that an inverted and real image (intermediate image) is formed by the objective. This intermediate image acts as an object for the eye-piece and lies between first focus and pole of eye piece. The final magnified virtual image is formed by the eye-piece.

∴ Magnifying power of the compound microscope,

$$\therefore \quad \boxed{M = \frac{v_o}{u_o}\left(\frac{D}{u_e}\right)}$$

or $M_0 \times M_e$, where M_0 is the magnifying power of objective and M_e is the magnifying power of eye-piece.

(i) When the final image is formed at infinity

∴ Magnifying power, $M_\infty = \frac{v_o}{u_o}\frac{D}{f_e}$

and $$L_\infty = v_o + f_e \quad \left(\because M_\infty = \frac{(L_\infty - f_o - f_e)}{f_o f_e}\right)$$

(ii) When the final image (by eye-piece) is formed at D.

Magnifying power, $M_D = \frac{v_o}{u_o}\left(1+\frac{D}{f_e}\right)$

and $$L_D = v_o + \frac{Df_e}{D+f_e}$$

In general, object is placed very near to the principal focus of objective and f_0 is also very small, then

$$M_D = -\frac{L}{f_o}\left(1+\frac{D}{f_e}\right)$$

Example 9.80 *The separation between the objective and the eye-piece of a compound microscope can be adjusted between 9.8 cm to 11.8 cm. If the focal length of the objective and the eye-piece are 1.0 cm and 6 cm respectively, find the range of the magnifying power, if the image is always needed at 24 cm from the eye.*

Sol. For eye-piece, it is given that

$$f_e = 6 \text{ cm}, \quad v_e = -24 \text{ cm}$$

$$\therefore \quad \frac{1}{-24} - \frac{1}{-u_e} = \frac{1}{6}$$

$$\therefore \quad u_e = 4.8 \text{ cm}$$

When, $L = 9.8$ cm, then $v_o = (9.8 - 4.8)$ cm $= 5.0$ cm

$$\therefore \quad \frac{1}{5.0} - \frac{1}{-u_o} = \frac{1}{1.0}$$

$$\therefore \quad u_o = 1.25 \text{ cm}$$

or Magnifying power, $M = \frac{v_o}{u_o}\frac{D}{u_e} = \left(\frac{5.0}{1.25}\right)\left(\frac{25.0}{4.8}\right) = 20.83$

When, $L = 11.8$ cm, then $v_o = (11.8 - 4.8)$ cm $= 7.0$ cm

$$\therefore \quad \frac{1}{7.0} - \frac{1}{-u_o} = \frac{1}{1.0} \quad \text{or} \quad u_o = 1.17 \text{ cm}$$

$$\therefore \quad M = \frac{v_o}{u_o}\cdot\frac{D}{f_e} = \left(\frac{7.0}{1.17}\right)\left(\frac{25.0}{4.8}\right) = 31.16$$

Therefore, range of magnifying power is from 20.83 to 31.16.

Example 9.81 *A compound microscope has a magnifying power of 100 when the image is formed at infinity. The objective has a focal length of 0.5 cm and the tube length is 6.5 cm. Find the focal length of the eye-piece.*

Sol. When the final image is at infinity,

$$u_e = f_e = \text{tube length} - v_o$$

$$\therefore \quad f_e = 6.5 - v_o \qquad \text{...(i)}$$

Since, $$M_\infty = \frac{v_o}{u_o}\cdot\frac{D}{f_e}$$

$$\therefore \quad 100 = \frac{v_o}{u_o}\cdot\frac{25}{f_e} \quad \text{or} \quad \frac{v_o}{u_o f_e} = 4 \qquad \text{...(ii)}$$

For the objective,

$$\frac{1}{v_o} - \frac{1}{-u_o} = \frac{1}{f_o} = \frac{1}{0.5} \Rightarrow \frac{1}{v_o} + \frac{1}{u_o} = 2 \qquad \text{...(iii)}$$

We have three unknowns v_o, u_o and f_e solving these three equations, we get

$$f_e = 2 \text{ cm}$$

Telescopes

A microscope is used to view the objects placed closed to it. To look at distant objects such as a Star, a planet or a distant tree etc., we use telescopes. *There are three types of refracting telescopes in use*

(i) Astronomical telescope,
(ii) Terrestrial telescope and
(iii) Galilean telescope.

Astronomical telescope

Astronomical telescope are used to see the heavenly bodies. It consists of two converging lenses placed coaxially. The one facing the distant object is called the objective and has a large aperture and large focal length. The other is called the eye-piece, as the eye is placed closed to it.

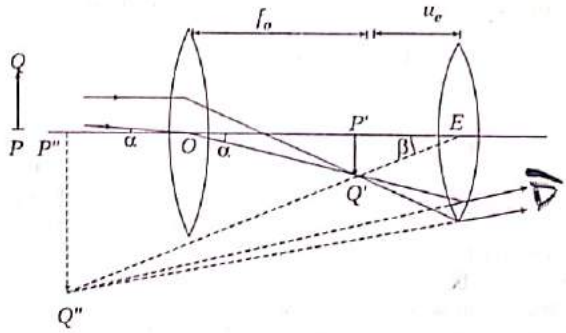


Fig. 9.72 Image formation in an astronomical telescope

The intermediate image formed by objective is real, inverted and small and final image formed by eye-piece inverted and small.

Magnifying power

(i) When object is at infinity

Magnifying power,

$$\boxed{M = \frac{|\beta|}{|\alpha|} = \frac{f_o}{u_e} \quad \text{or} \quad M = \frac{f_o}{u_e}}$$

where, $|\alpha|$ = angle subtended by object on objective and $|\beta|$ = angle subtended by final image at eye-piece.

$$\therefore \quad M_\infty = \frac{f_o}{f_e} \qquad (\because \mu_e = f_e)$$

and $$L_\infty = f_o + f_e$$

(ii) **When the final image is at D**

Magnifying power, $M_D = \frac{f_o}{f_e}\left(1+\frac{f_e}{D}\right)$

and $$L_D = f_o + \frac{Df_e}{D+f_e}$$

Terrestrial telescope

It consists of three converging lens, objective, eye-piece lens and erecting lens. It is used to far off objects on the earth and it's final image is virtual, erect and smaller.

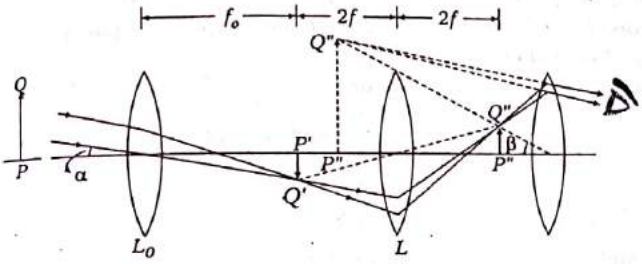


Fig. 9.73 Image formation in a terrestrial telescope

Magnifying power

When image is formed at infinity

$$M_\infty = \frac{f_o}{f_e} \text{ and } L_\infty = f_o + 4f + f_e$$

When image is formed at D

$$M_D = \frac{f_o}{f_e}\left(1+\frac{f_e}{D}\right) \text{ and } L_D = f_o + 4f + \frac{Df_e}{D+f_e}$$

Galilean telescope

In this telescope, a convergent lens is used as the objective and a divergent lens as the eye-piece. The objective lens forms a real and inverted image but the divergent lens comes in between. This intermediate image acts as virtual object for eye-piece. Final image is erect and magnified as shown in figure. The intermediate image is formed at second focus of objective.

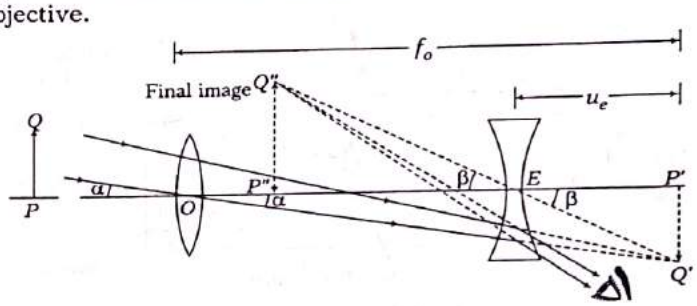


Fig. 9.74 Image formation in a Galilean an telescope

Magnifying power

Magnifying power, $M_\infty = \frac{f_o}{f_e}$ and $L_\infty = f_o - f_e = f_o - u_e$

When the final image is formed at a distance D

Thus, magnifying power, $M_D = \frac{f_o}{f_e}\left(1-\frac{f_e}{D}\right)$

and $$L_D = f_o - \frac{f_e D}{D-f_e}$$

Note

1. In all above formulae of M, we are considering only the magnitude of M.
2. For telescopes, formulae have been derived when the object is at infinity. For the object at some finite distance different formulae will have to be derived.
3. Given below are formulae derived above of M and L in tabular form.

Summary of formula for optical instruments

Name of Optical Instruments	M	L	M_∞	M_D	L_∞	L_D
Simple microscope	$\frac{D}{u_o}$	—	$\frac{D}{f}$	$1+\frac{D}{f}$	—	—
Compound microscope	$\frac{v_o}{u_o}\frac{D}{u_e}$	$v_o + u_e$	$\frac{v_o}{u_o}\frac{D}{f_e}$	$\frac{v_o}{u_o}\left(1+\frac{D}{f_e}\right)$	$v_o + f_e$	$v_o + \frac{Df_e}{D+f_e}$
Astronomical telescope	$\frac{f_o}{u_e}$	$f_o + u_e$	$\frac{f_o}{f_e}$	$\frac{f_o}{f_e}\left(1+\frac{f_e}{D}\right)$	$f_o + f_e$	$f_o + \frac{Df_e}{D+f_e}$
Terrestrial telescope	f_o/u_e	$f_o + 4f + u_e$	$\frac{f_o}{f_e}$	$\frac{f_o}{f_e}\left(1+\frac{f_e}{D}\right)$	$f_o + 4f + f_e$	$f_o + 4f + \frac{Df_e}{D+f_e}$
Galilean telescope	$\frac{f_o}{u_e}$	$f_o - u_e$	$\frac{f_o}{f_e}$	$\frac{f_o}{f_e}\left(1-\frac{f_e}{D}\right)$	$f_o - f_e$	$f_o - \frac{f_e D}{D-f_e}$

Reflecting telescope

Reflecting telescope are based upon the same principle except that the formation of images takes place by reflection instead of by refraction. It consists of concave mirror of large aperture and large focal length (objective). A plane mirror is placed between the concave mirror and its focus. A small convex lens works as eye-piece.

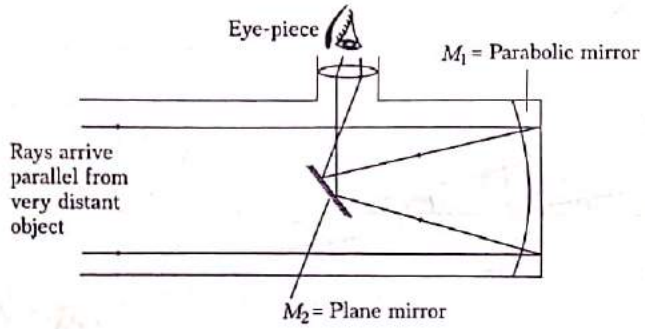


Fig. 9.75 Reflecting telescope

If f_0 is focal length of the concave spherical mirror and f_e the focal length of the eye-piece, the magnifying power of the reflecting telescope is given by

$$m = \frac{f_o}{f_e}$$

Example 9.82 *A small telescope has an objective lens of focal length 140 cm and an eye-piece of focal length 5.0 cm. What is the magnifying power of the telescope for viewing distance objects when*

(i) the telescope is in normal adjustment (i.e., when the final image is at infinity) ?

(ii) the final image is formed at the least distance of distinct vision 25 cm?

Sol. (i) When the telescope is in normal adjustment, the magnifying power is given by

$$M = \frac{f_o}{|f_e|} = +\frac{140}{5} = 28$$

(ii) When the final image is formed at the least distance of distinct vision, then M is given by

$$M = \frac{f_o}{|f_e|}\left(1 + \frac{f_e}{D}\right) = \frac{140}{5}\left(1 + \frac{5.0}{25}\right) = 33.6$$

Example 9.83 *A small telescope has an objective lens of focal length 144 cm and an eye-piece of focal length 6.0 cm. What is the magnifying power of the telescope? What is the separation between the objective and the eye-piece?*

Sol. M = magnifying power of telescope

L = separation between the objective and the eyepiece

In normal adjustment (*i.e.*, when the final image is formed at ∞),

$$M = \frac{f_o}{f_e} = -\frac{144}{6} = -24 \Rightarrow M = 24$$

$$\therefore \quad L = f_o + f_e = 144 + 6 = 150 \text{ cm}$$

Example 9.84 *The eye-piece of an astronomical telescope has a focal length of 10 cm. The telescope is focused for normal vision of distant objects when the tube length is 1.0 m. Find the focal length of the objective and the magnifying power of the telescope.*

Sol. Given, $f_e = 10$ cm, $L_\infty = 1.0$ m $= 100$ cm

Since, $L_\infty = f_o + f_e$

$\therefore$ $f_o = 90.0$ cm

Further, magnification, $M_\infty = \frac{f_o}{f_e} = \frac{90.0}{10.0} = 9.0$

Example 9.85 *An astronomical telescope has an angular magnification of magnitude 5 for distant objects. The separation between the objective and eye-piece is 36 cm and the final image is formed at infinity. Determine the focal length of objective and eye-piece.*

Sol. For final image at infinity,

$$M_\infty = \frac{f_o}{f_e} \text{ and } L_\infty = f_o + f_e$$

$$\therefore \quad 5 = \frac{f_o}{f_e} \quad \text{...(i)}$$

and $$36 = f_o + f_e \quad \text{...(ii)}$$

Solving these two equations, we have

$$f_o = 30 \text{ cm and } f_e = 6 \text{ cm}$$

Example 9.86 *An astronomical telescope is to be designed to have a magnifying power of 50 in normal adjustment. If the length of the tube is 102 cm, find the powers of the objective and the eye-piece.*

Sol. Given, $M_\infty = 50$

$$\therefore \quad \frac{f_o}{f_e} = 50 \quad \text{...(i)}$$

Further, $L_\infty = 102$ cm

$$\therefore \quad f_o + f_e = 102 \text{ cm} \quad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we have

$$f_o = 100 \text{ cm and } f_e = 2 \text{ cm}$$

$\therefore$ Power of object, $P_o = \frac{1}{1.0} = 1$ D

and power of eye-piece, $P_e = \frac{1}{0.02} = 50$ D

Example 9.87 *A telescope has an objective of focal length 50 cm and an eye-piece of focal length 5 cm. The least distance of distinct vision is 25 cm. The telescope is focused for distinct vision on a scale 2 m away from the objective. Calculate (i) magnification produced and (ii) separation between objective and eyepiece.*

Sol. Given, $f_o = 50$ cm and $f_e = 5$ cm

For objective, $\frac{1}{v_o} - \frac{1}{-200} = \frac{1}{50}$

$$\therefore \quad v_o = \frac{200}{3} \text{ cm}$$

$$\Rightarrow \quad m_o = \frac{v_o}{u_o} = \frac{(200/3)}{-200} = -\frac{1}{3}$$

For eye-piece, $\frac{1}{-25} - \frac{1}{u_e} = \frac{1}{5}$

$$\therefore \quad u_e = -\frac{25}{6} \text{ cm and } m_e = \frac{v_e}{u_e} = \frac{-25}{-(25/6)} = 6$$

(i) Magnification, $m = m_o \times m_e = -2$

(ii) Separation between objective and eye-piece,

$$L = v_o + |u_e| = \frac{200}{3} + \frac{25}{6} = \frac{425}{6} = 70.83 \text{ cm}$$

Note Here, object is placed at finite distance from the objective. Hence, formulae derived for angular magnification M cannot be applied directly as they have been derived for the object to be at infinity. Here, it will be difficult to find angular magnification. So, only linear magnification can be obtained.

Example 9.88 *A Galilean telescope is 27 cm long when focused to form an image at infinity. If the objective has a focal length of 30 cm, what is the focal length of the eyepiece?*

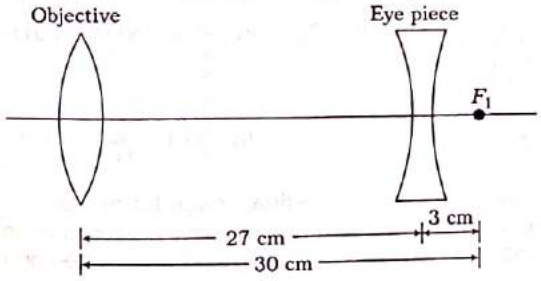
Sol. Given, $f_o = +30$ cm

Length of telescope is given, 27 cm.

Therefore, $u_e = +3$ cm

For the final image at infinity, the intermediate image should lie at first focus of eyepiece of the Galilean telescope.

$\therefore \quad f_e = -3$ cm



Resolving power of a microscope and a telescope

Microscope

The resolving power of a microscope is defined as the reciprocal of the distance between two objects which can be just resolved when seen through the microscope. It depends on the wavelength λ of the light, the refractive index μ of the medium between the object and the objective and the angle θ subtended by a radius of the objective on one of the objects.

Resolving power is reciprocal of resolution limit.

Resolving power, $R = \frac{1}{\Delta d} = \frac{2\mu \sin\theta}{\lambda}$

To increase R, objective and object are immersed in oil.

Telescope

The resolving power of a telescope is defined as the reciprocal of the angular separation between two distant objects which are just resolved by a telescope.

It is given by, $R = \frac{1}{\Delta\theta} = \frac{a}{1.22\lambda}$

Here, a is the diameter of the objective. That is why, telescopes with larger objective aperture are used.

Example 9.89 *Calculate the resolving power of a microscope with cone angle of light falling on the objective equal to 60°. Take, $\lambda = 600$ nm and μ for air = 1.*

Sol. Here, $2\theta = 60°$, $\theta = 30°$, $\lambda = 600 \text{ nm} = 6 \times 10^{-7}$ m, $\mu = 1$

$$\therefore \quad \text{RP} = \frac{1}{d} = \frac{2\mu \sin\theta}{\lambda}$$

$$\text{RP} = \frac{2 \times 1 \times \sin 30°}{600 \times 10^{-9}}$$

$$= \frac{10}{6} \times 10^6 = 1.67 \times 10^6$$

Example 9.90 *Assume that light of wavelength 6000 Å is coming from a star. What is the resolution of a telescope whose objective has diameter of 100 inch?*

Sol. Diameter of objective of telescope,

$D = 100 \text{inch} = 100 \times 2.54 \text{ cm} = 254$ cm

$\lambda = 6000 \text{Å} = 6000 \times 10^{-8} \text{cm} = 6 \times 10^{-5} \text{cm}$

Limit of resolution of telescope,

$$d\theta = \frac{1.22\lambda}{D} = \frac{1.22 \times 6 \times 10^{-5}}{254} = 2.9 \times 10^{-7} \text{ rad}$$

Example 9.91 *The diameter of the pupil of human eye is about 2 mm. Human eye is most sensitive to the wavelength 555 nm. Find the limit of resolution of human eye.*

Sol. Here, $D = 2 \text{mm} = 2 \times 10^{-3}$ m, $\lambda = 555 \text{ nm} = 555 \times 10^{-9}$ m

$$\text{Limit of resolution, } d\theta = \frac{1.22\lambda}{D} = \frac{1.22 \times 555 \times 10^{-9}}{2 \times 10^{-3}}$$

$$= 3.39 \times 10^{-4} \text{ rad}$$

$$d\theta = 3.39 \times 10^{-4} \times \left(\frac{180}{\pi}\right)^0 = 0.0194°$$

$$= 0.0194 \times 60 \text{ min} = 1.2 \text{ min}$$

Check point 9.6

1. For a normal eye, the least distance of distinct vision is
(a) 0.25 m (b) 0.50 m (c) 25 m (d) infinite

2. Image formed on the retina is
(a) real and inverted (b) virtual and erect
(c) real and erect (d) virtual and inverted

3. The focal length of a normal eye lens is about
(a) 1 mm (b) 2 cm (c) 25 cm (d) 1 m

4. An object is placed at a distance u from a simple microscope of focal length f. The angular magnification obtained depends
(a) on f but not on u (b) on u but not on f
(c) on f as well as u (d) neither on f nor on u

5. Magnifying power of a simple microscope is (when final image is formed, at $D = 25$ cm from eye)
(a) $\frac{D}{f}$ (b) $1+\frac{D}{f}$
(c) $1+\frac{f}{D}$ (d) $1-\frac{D}{f}$

6. In a compound microscope, the intermediate image is
(a) virtual, erect and magnified (b) real, erect and magnified
(c) real, inverted and magnified (d) virtual, erect and reduced

7. A compound microscope has two lenses. The magnifying power of one is 5 and the combined magnifying power is 100. The magnifying power of the other lens is
(a) 10 (b) 20 (c) 50 (d) 25

8. The length of the compound microscope is 14 cm. The magnifying power for relaxed eye is 25. If the focal length of eye lens is 5 cm, then the object distance for objective lens will be
(a) 1.8 cm (b) 1.5 cm
(c) 2.1 cm (d) 2.4 cm

9. If the focal length of objective and eye lens are 1.2 cm and 3 cm respectively and the object is put 1.25 cm away from the objective lens and the final image is formed at infinity. The magnifying power of the microscope is
(a) 150 (b) 200
(c) 250 (d) 400

10. The focal length of objective and eye lens of a microscope are 4 cm and 8 cm respectively. If the least distance of distinct vision is 24 cm and object distance is 4.5 cm from the objective lens, then the magnifying power of the microscope will be
(a) 18 (b) 32
(c) 64 (d) 20

11. If the telescope is reversed *i.e.*, seen from the objective side, then
(a) object will appear very small
(b) object will appear very large
(c) there will be no effect on the image formed by the telescope
(d) image will be slightly greater than the earlier one

12. The aperture of a telescope is made large, because to
(a) increase the intensity of image
(b) decrease the intensity of image
(c) have greater magnification
(d) have lesser resolution

13. In an astronomical telescope, the focal length of the objective lens is 100 cm and of eyepiece is 2 cm. The magnifying power of the telescope for the normal eye is
(a) 50 (b) 10 (c) 100 (d) $\frac{1}{50}$

14. The focal lengths of the objective and eye lenses of a telescope are respectively, 200 cm and 5 cm. The maximum magnifying power of the telescope will be
(a) −40 (b) −48 (c) −60 (d) −100

15. The number of lenses in a terrestrial telescope is
(a) two (b) three (c) four (d) six

16. Magnifying power of a Galilean telescope is given by
(a) $\frac{f_o}{f_e}\left(1-\frac{f_e}{D}\right)$ (b) $\frac{f_o}{f_e}\left(1+\frac{f_e}{D}\right)$
(c) $\frac{f_o}{f_e}\left(1+\frac{2f_e}{D}\right)$ (d) $\frac{f_o}{f_e}\left(1-\frac{2f_e}{D}\right)$

17. In Galilean telescope, the final image formed is
(a) real, erect and enlarged (b) virtual, erect and enlarged
(c) real, inverted and enlarged (d) virtual, inverted and enlarged

18. Reflecting telescope consists of
(a) convex mirror of large aperture
(b) concave mirror of large aperture
(c) concave lens of small aperture
(d) None of the above

19. Resolving power of a microscope is given by
(a) $\frac{2\mu\sin\theta}{\lambda^2}$ (b) $\frac{\mu\sin\theta}{\lambda}$
(c) $\frac{2\mu\sin\theta}{\lambda}$ (d) $\frac{2\mu\cos\theta}{\lambda}$

20. The resolving power of telescope whose lens has a diameter of 1.22 m for a wavelength of 5000 Å is
(a) 2×10^5 (b) 2×10^6
(c) 2×10^2 (d) 2×10^4

7 Defects of vision

The common defects of vision are as follows

1. Myopia or short-sightedness or near sightedness

In this defect, distant objects are not clearly visible and eye can see only nearer object. The image of distant object is formed before the retina.

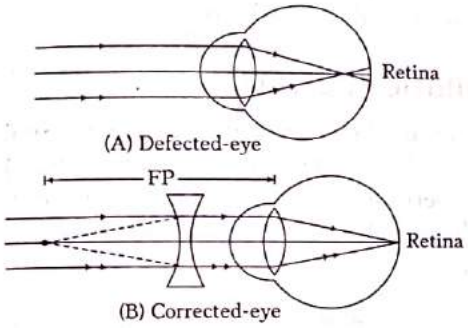


Fig. 9.76 *Myopia defect in eye*

In this defect focal length or radii of curvature of lens reduced or power of lens increases or distance between eye lens and retina increases.

The defect can be remedied by using a concave lens, placed between object and eye (as shown is fig.) so that this lens forms the image of distance object at the far point (F.P.) of the person. Using lens formula, we get

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{-(\text{FP})} - \frac{1}{-(\text{distance of object})} = \frac{1}{f}$$

If object is at infinity then, power of the lens,

$$P = \frac{1}{f} = \frac{1}{-(\text{FP})}$$

2. Farsightedness or longsightedness (hypermetropia or hyperopia)

In this defect, the near objects are not clearly visible and eye can see distance object image of near object is formed behind the retina.

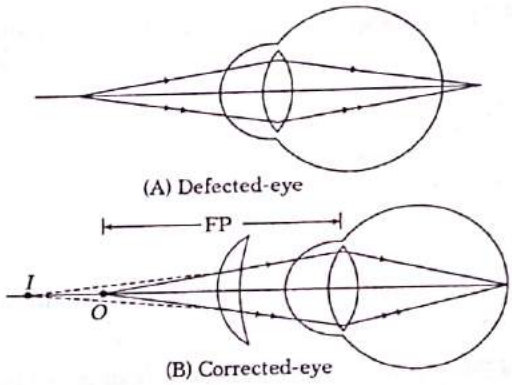


Fig. 9.77 *Hypermetropia defect in eye*

In this defect, focal length or radii of curvature of lens increases or power of lens decreases or distance between eye lens and retina decreases. This defect can be remedied by using a convex lens placed between object and eye (as shown in figure,) so that this lens forms the image of near object at near point (NP) of the person.

Using lens formula, we get

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow -\frac{1}{(\text{NP})} - \frac{1}{-(\text{distance of object})} = \frac{1}{f}$$

Power of the lens, $P = \frac{1}{f}$

3. Astigmatism

In this defect, eye cannot see objects in two orthogonal (perpendicular) directions clearly simultaneously. This defect is remedied by using cylindrical lens.

4. Presbyopia

In this both near and far objects are not clearly visible. This is remedied either by using two separate lenses or by using single spectacle having bifocal lenses.

Example 9.93 *A person who can see things most clearly at a distance of 10 cm, requires spectacles to enable to see clearly things at a distance of 30 cm. What should be the focal length of the spectacles?*

Sol. Since, the person is not able to see distant objects beyond 10 cm. So , he should use concave lens in his spectacles. Concave lens will form the image of object distant 30 cm, at 10 cm it means $u = -30$ cm, $v = -10$ cm.

Using lens formula,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{-10} - \frac{1}{-30} = -\frac{2}{30} \quad \text{or} \quad \frac{1}{f} = \frac{1}{15}$$

So, $f = -15$ cm

Minus sign signifies that lens used in concave.

Example 9.93 *The power of a lens used by short sighted person, is −2D. Find the maximum distance of an object which he can see without spectacles.*

Sol. Power $(P) = \dfrac{1}{\text{Focal length (m)}}$

Given, $P = -2\,\text{D}$

$\therefore$ Focal length $= -\dfrac{1}{2} = 0.5\,\text{m} \Rightarrow F = -50$ cm

Hence, maximum distance of an object, which he can see without spectacles is 50 cm.

Example 9.94 *If a person can sees clearly at a distance of 100 cm, then find the power of lens used to see object at 40 cm.*

Sol. By lens formula,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

Given, $v = -100$ cm and $u = -40$ cm

$$\therefore \quad \frac{1}{f} = \frac{1}{100} - \frac{1}{-40} \Rightarrow \frac{1}{f} = -\frac{1}{100} + \frac{1}{40}$$

$$\Rightarrow \quad \frac{1}{f} = \frac{-1+2.5}{100} = \frac{1.5}{100}$$

Hence, power of lens is

$$P = \frac{100}{f\,(\text{cm})} = 100 \times \frac{1.5}{100} = 1.5\,\text{D}$$

Example 9.95 *A myopic person has been using a spectacles of power 1.0 D for distant vision. During old age he also needs to use separate reading glass of power + 2.0 D. Explain what may have happened?*

Sol. Here, $P = -1\,\text{D}$

$$\therefore \quad f = \frac{1}{P} = \frac{1}{-1}\,\text{m} = -100\,\text{cm}$$

For normal vision, far point it at infinity

$$\therefore \quad u = -\infty$$

Using lens formula, $-\dfrac{1}{u} + \dfrac{1}{v} = \dfrac{1}{f}$ ...(i)

we get, $-\dfrac{1}{-\infty} + \dfrac{1}{v} = -\dfrac{1}{100}$

or $\dfrac{1}{v} = -\dfrac{1}{100}$

$\therefore \quad v = -100$ cm

Thus virtual image of the object at infinity is produced at 100 cm distance using spectacles.

To view objects at distance 25 cm to 100 cm, the person using ability of accommodation of eye which is partially lost in old age for which he needs another spectacles having power.

$$P = +2\,\text{D}$$

$$\therefore \quad f = \frac{1}{P} = \frac{1}{2} = 0.5\,\text{m} = 50\text{ cm} \quad \text{and} \quad v = 25\text{ cm}$$

Using Eq. (i), we get

$$-\frac{1}{u} + \frac{1}{25} = \frac{1}{50}$$

$$\therefore \quad u = 50\text{ cm}$$

$\therefore$ His near point shifts to 50 cm.

Defects of images

Actual image formed by an optical system is usually imperfect. The defects of images are called **aberrations.** The defect may be due to light or optical system. If the defect is due to light, it is called **chromatic aberration,** and if due to optical system, **monochromatic aberration.**

1. Chromatic aberration

The image of an object formed by a lens is usually coloured and blurred. This defect of image is called chromatic aberration. This defect arises due to the fact that focal length of a lens is different for different colours. For a lens,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

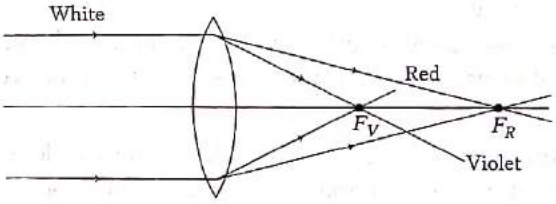


Fig. 9.78 *Chromatic aberration*

As, μ is maximum for violet while minimum for red, violet is focused nearest to the lens while red farthest from it.

The difference between f_R and f_V is a measure of longitudinal chromatic aberration. Thus,

$$\text{LCA} = f_R - f_V = \omega f_Y$$

where, $\omega = \dfrac{\mu_V - \mu_R}{\mu_Y - 1}$

Here, $\mu_Y = \dfrac{\mu_V + \mu_R}{2}$

Condition of achromatism

To get achromatism, we use a pair of two lenses in contact. For two thin lenses in contact we have,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\therefore \quad \frac{-dF}{F^2} = -\frac{df_1}{f_1^2} - \frac{df_2}{f_2^2}$$

The combination will be free from chromatic aberration if,

$$dF = 0$$

$$\therefore \quad \frac{df_1}{f_1^2} + \frac{df_2}{f_2^2} = 0$$

$$\therefore \quad \frac{\omega_1 f_1}{f_1^2} + \frac{\omega_2 f_2}{f_2^2} = 0$$

$$\therefore \quad \frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$$

This is the **condition of achromatism.** From the condition of achromatism, *following conclusions can be drawn*

(i) As ω_1 and ω_2 are positive quantities, f_1 and f_2 should have opposite signs, *i.e.*, if one lens is convex, the other must be concave.

(ii) If $\omega_1 = \omega_2$, means both the lenses are of same material. Then,

$$\frac{1}{f_1} + \frac{1}{f_2} = 0 \quad \text{or} \quad \frac{1}{F} = 0 \text{ or } F = \infty$$

Thus, the combination behaves as a plane glass plate. So, we can conclude that both the lenses should be of different materials or, $\omega_1 \neq \omega_2$

(iii) Dispersive power of crown glass (ω_C) is less than that of flint glass (ω_F).

(iv) If we want the combination to behave as a convergent lens, then convex lens should have lesser focal length or its dispersive power should be more. Thus, convex lens should be made of flint glass and concave lens of crown.

Thus, combination is converging, if convex is made of flint glass and concave of crown. Similarly, for the combination to behave as diverging lens, convex is made of crown glass and concave of flint glass.

2. Monochromatic aberration

This is the defect in image due to optical system. Monochromatic aberration is of many types such as, spherical, coma, distortion, curvature and antigmatism. Here, we shall limit ourselves to spherical aberration only. Spherical aberration arises due to spherical nature of lens (or mirror).

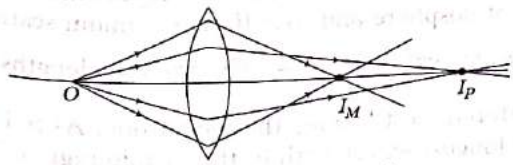


Fig. 9.79 Spherical aberration

The paraxial rays (close to optic axis) get focused at I_P and marginal rays (away from the optic axis) are focused at I_M. Thus, image of a point object O is not a point.

The inability of the lens to form a point image of an axial point object is called spherical aberration. Spherical aberration can never be eliminated but can be minimised by the following methods :

(i) **By using stops** By using stops either paraxial or marginal rays are cut-off.

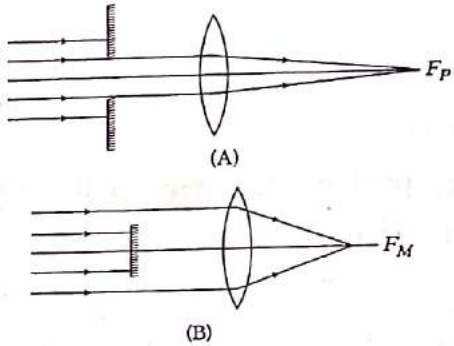


Fig. 9.80 Minimising spherical aberration using stops

(ii) **Using two thin lenses separated by a distance** Two thin lenses separated by a distance, $d = f_2 - f_1$, has the minimum spherical aberration.

(iii) **Using parabolic mirrors** If spherical mirror is replaced by parabolic mirror, spherical aberration is minimised.

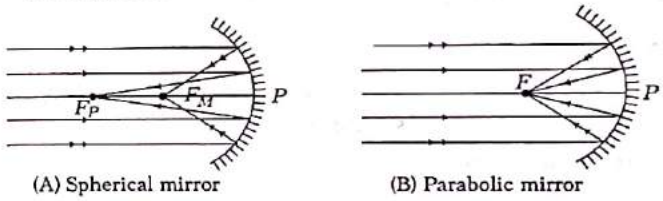


Fig. 9.81 Minimising spherical aberration using mirrors

(iv) **Using lens of large focal** length It has been found that spherical aberration varies inversely as the cube of the focal length. So, if f is large, spherical aberration will be reduced.

(v) **Using plano-convex lens** In case of planoconvex lens spherical aberration is minimised, if its curved surface faces the incident or emergent ray whichever is more parallel.

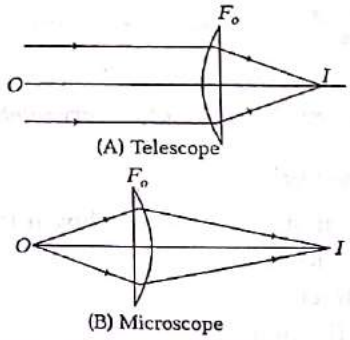


Fig. 9.82 Minimising spherical aberration using plano-convex lens

This is why in telescope, the curved surface faces the object while in microscope curved surface is towards the image.

(vi) **Using crossed lens** For a single lens with object at infinity, spherical aberration is found to be minimum when R_1 and R_2 have the following ratio,

$$\frac{R_1}{R_2} = \frac{2\mu^2 - \mu - 4}{\mu(2\mu + 1)}$$

A lens which satisfies this condition is called a crossed lens.

Some natural phenomena associated with sunlight

The spectrum of colours that we see around us all the time is possible only due to sunlight. The rainbow, blue colour of the sky, white clouds, the red hue at sunrise and sunset, etc., are some of the natural phenomena.

The Rainbow

It is the combined effect of dispersion, refraction and internal reflection of sunlight by spherical water droplets of rain. An observer can see the rainbow only when his back is towards the sun.

Primary Rainbow

It is a result of three steps process shown in the figure.

(i) Refraction with dispersion
(ii) Internal reflection
(iii) Refraction

In primary rainbow, red colour is on the top and violet on the bottom as shown in the figure.

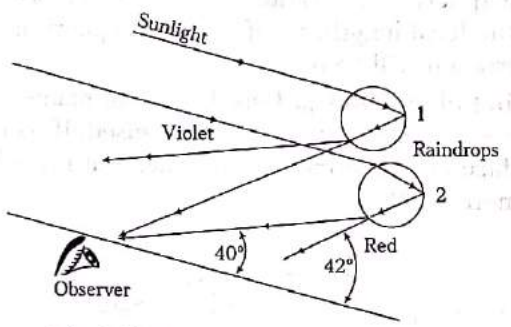


Fig. 9.83 Formation of primary rainbow

Secondary Rainbow

It is a result of four step process as shown in the figure.

(i) Refraction with dispersion
(ii) Internal reflection
(iii) External reflection
(iv) Refraction

It is found that red light emerges at an angle of 50° related to the incoming sunlight and violet light emerges at an angle 53°.

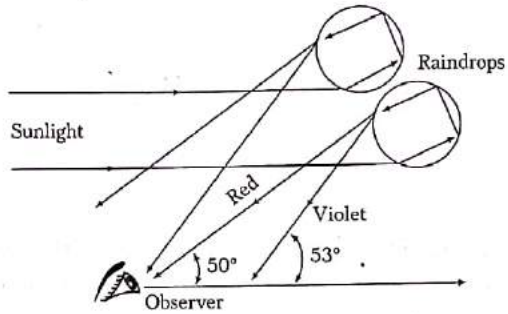


Fig. 9.84 Formation of secondary rainbow

Scattering of light

If the molecules of a medium after absorption of incoming radiations (light) emit them in all directions. This phenomenon is called scattering. If the wavelength of radiation remains unchanged in this process, then the scattering is called elastic otherwise, inelastic.

Rayleigh has shown, theoretically that in case of elastic scattering of light by molecules, the intensity of scattered light depends on both nature of molecules and wavelength of light. According to him,

Intensity of scattered light $\propto \frac{1}{\lambda^4}$

Raman effect is based on inelastic scattering. For this CV. **Raman** was awarded the Noble Prize in 1930.

Scattering helps us in understanding the following:

(i) **Why sky is blue** When white light from the sun enters the earth's atmosphere scattering takes place. As scattering is proportional to $\frac{1}{\lambda^4}$, blue is scattered most. When we look at the sky we receive scattered light which is rich in blue and hence, the sky appears blue.

(ii) **Why sun appears red in the morning and evening** When sun is at the horizon, due to oblique incidence light reaches earth after traversing maximum path in the atmosphere and so suffers maximum scattering. Now, as scattering $\propto \frac{1}{\lambda^4}$, shorter wavelengths are scattered most leaving the longer one. As red light has longest wavelength in the visible region, it is scattered least. This is why sun appears red in the morining and evening. The same reason is why red light is used for danger signals.

Check point 9.7

1. For the myopia defect in eye, it can be removed by
(a) convex lens (b) concave lens
(c) cylindrical lens (d) toric lens

2. A short sighted person can see distinctly only those objects which lie between 10 cm and 100 cm from him. The power of the spectacle lens required to see a distance object is
(a) +0.5 D (b) –1.0 D
(c) –10 D (d) +4.0 D

3. Which of the following statement is correct for hypermetropia?
(a) Near objects are not clearly visible
(b) Distant objects are not clearly visible
(c) Concave lens is used for remedy of hypermetropia
(d) None of the above

4. A person suffering from the defect astigmatism
(a) cannot see any object
(b) cannot see objects in two perpendicular directions simultaneously
(c) cannot see near by objects
(d) cannot see distant objects

5. Astigmatism (for a human eye) can be removed by using
(a) concave lens
(b) convex lens
(c) cylindrical lens
(d) prismatic lens

6. Presbyopia can be removed by using
(a) convex lens (b) concave lens
(c) cylindrical lens (d) bifocal lens

7. The focal length of a converging lens is measured for violet, green and red colours. It is respectively f_v, f_g, f_r. We will find that
(a) $f_v = f_r$ (b) $f_v < f_r$
(c) $f_v > f_r$ (d) $f_g > f_r$

8. A combination is made of two lenses of focal lengths f_1 and f_2 and dispersive powers ω_1 and ω_2 respectively. The combination will be achromatic, if
(a) $\omega_1 = 2\omega_2$ and $f_1 = 2f_2$
(b) $2\omega_1 = \omega_2$ and $f_1 = 2f_2$
(c) $\omega_1 = 2\omega_2$ and $f_1 = -2f_2$
(d) $2\omega_1 = \omega_2$ and $2f_1 = f_2$

9. Rainbow is caused due to
(a) Refraction (b) reflection
(c) dispersion (d) All of these

10. Phenomena associated with scattering is/are
(a) blue colour of the sky
(b) appearence of reddish sun during sunset and sunrise
(c) both (a) and (b)
(d) None of the above

Chapter exercises

A Taking it together

(Assorted questions of the chapter for advanced level practice)

1. White light is passed through a prismm when ..... colour shows maximum deviation.
   (a) red (b) violet
   (c) yellow (d) green

2. When the power of eye lens increases, the defect of vision is produced. The defect is known as
   (a) shortsightedness
   (b) longsightedness
   (c) colourblindness
   (d) None of the above

3. In human eye the focusing is done by
   (a) to and fro movement of eye lens
   (b) to and fro movement of the retina
   (c) change in the convexity of the lens surface
   (d) change in the refractive index of the eye fluids

4. The phenomena involved in the reflection of radiowaves by ionosphere is similar to **[NCERT Exemplar]**
   (a) reflection of light by a plane mirror
   (b) total internal reflection of light in air during a mirage
   (c) dispersion of light by water molecules during the formation of a rainbow
   (d) scattering of light by the particles of air

5. A passenger in an aeroplane **[NCERT Exemplar]**
   (a) should see a rainbow
   (b) may see a primary and a secondary rainbow as concentric circles
   (c) may see a primary and a secondary rainbow as concentric arcs
   (d) should never see a secondary rainbow

6. Myopia is due to
   (a) elongation of eye ball
   (b) irregular change in focal length
   (c) shortening of eye ball
   (d) older age

7. The minimum magnifying power of telescope is M. If the focal length of its eye lens is halved, the magnifying power will become
   (a) $m/2$ (b) $2m$ (c) $3m$ (d) $4m$

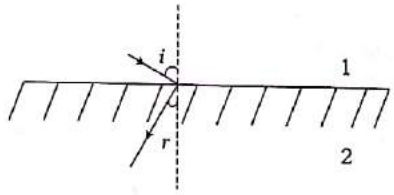
8. When light wave suffers reflection at the interface from air to glass, then the change in phase of the reflected wave is equal to
   (a) zero (b) $\frac{\pi}{2}$ (c) π (d) 2π

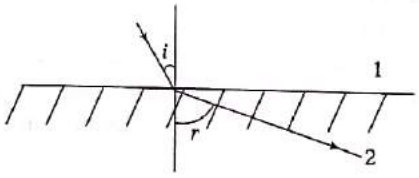
9. A man has height of 6 m. He observes image of 2 m height erect, then mirror used is
   (a) concave (b) convex
   (c) plane (d) None of these

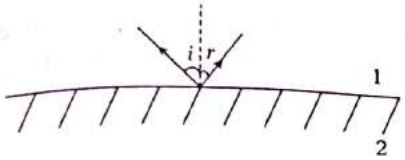
10. The field of view is maximum for
    (a) plane mirror (b) concave mirror
    (c) convex mirror (d) cylindrical mirror

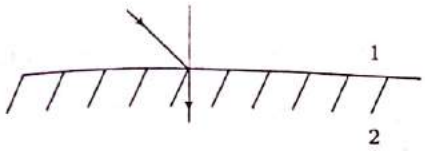
11. A virtual image larger than the object can be obtained by
    (a) concave mirror (b) convex mirror
    (c) plane mirror (d) concave lens

12. There are certain material developed in laboratories which have a negative refractive index figure. A ray incident from air (medium 1) into such a medium (medium 2) shall follow a path given by **[NCERT Exemplar]**

(a)


(b)


(c) 

(d) 

13. Our eyes are most sensitive for which of the following wavelength?
(a) 4500 Å
(b) 5500 Å
(c) 6500 Å
(d) Equally sensitive for all wavelengths of visible spectrum

14. The reason for shining of air bubble in water is
(a) diffraction of light
(b) dispersion of light
(c) scattering of light
(d) total internal reflection of light

15. What will be the colour of sky as seen from the earth, if there were no atmosphere?
(a) Black (b) Blue
(c) White (d) Red

16. A short pulse of white light is incident from air to a glass slab at normal incidence. After travelling through the slab, the first colour to emerge is **[NCERT Exemplar]**
(a) blue (b) green
(c) violet (d) red

17. Which of the following can form a virtual, erect and magnified image?
(a) Plane mirror (b) Concave mirror
(c) Convex mirror (d) Concave lens

18. Rainbow is observed when the sun is
(a) in front of the observer
(b) behind the observer
(c) vertically above the observer
(d) in any of these positions

19. The sky would appear red instead of blue, if
(a) atmospheric particles scatter blue light more than the red light
(b) atmospheric particles scatter all colours equally
(c) atmospheric particles scatter red light more than the blue light
(d) the sun was much hotter

20. A ray of light is incident on the surface of separation of a medium at an angle 45°and is refracted in the medium at an angle 30°. What will be the velocity of light in the medium?
(a) 1.96×10^8 m/s (b) 2.12×10^8 m/s
(c) 2.65×10^8 m/s (d) 1.24×10^8 m/s

21. Which of the following quantities increase when wavelength of light is increased? Consider only the magnitudes.
(a) The power of a converging lens
(b) The focal length of a converging lens
(c) The power of a diverging lens
(d) The focal length of a diverging lens

22. Sun is visible a little before the actual sunrise and until a little after the actual sunset. This is due to
(a) total internal reflection (b) reflection
(c) refraction (d) polarisation

23. In optical fibres, the refractive index of the core is
(a) greater than that of the cladding
(b) equal to that of the cladding
(c) smaller than that of the cladding
(d) independent of that of the cladding

24. An object approaches a convergent of lens from the left of the lens with a uniform speed 5 m/s and stops at the focus. The image **[NCERT Exemplar]**
(a) moves away from the lens with an uniform speed 5 m/s
(b) moves away from the lens with an uniform acceleration
(c) moves away from the lens with a non-uniform acceleration
(d) moves towards the lens with a non-uniform acceleration

25. A point object is placed at the centre of a glass sphere of radius 6 cm and refractive index 1.5. The distance of the virtual image from the surface of the sphere is
(a) 2 cm (b) 4 cm (c) 6 cm (d) 12 cm

26. An object has an image thrice of its original size when kept at 8 cm and 16 cm from a convex lens. Focal length of the lens is
(a) less than 8 cm (b) 8 cm
(c) 16 cm (d) between 8 and 16 cm

27. If there had been one eye of the man, then
(a) image of the object would have been inverted
(b) visible region would have decreased
(c) image would have not been seen three-dimensional
(d) Both (b) and (c)

28. A transparent plastic bag filled with air forms a concave lens. Now, if this bag is completely immersed in water, then it behaves as
(a) divergent lens (b) convergent lens
(c) equilateral prism (d) rectangular slab

29. A normal eye is not able to see objects closer than 25 cm because
(a) the focal length of the eye is 25 cm
(b) the distance of the retina from the eye lens is 25 cm
(c) the eye is not able to increase the focal length beyond a limit
(d) the eye is not able to decrease the focal length beyond a limit

30. A mark at the bottom of a liquid appears to rise by 0.1 m. The depth of the liquid is 1 m. The refractive index of the liquid is

(a) 1.33 (b) 9/10 (c) $\frac{10}{9}$ (d) 1.5

31. Under minimum deviation condition in a prism, if a ray is incident at an angle 30°, then the angle between the emergent ray and the second refracting surface of the prism is

(a) 0° (b) 30°
(c) 45° (d) 60°

32. A prism can have a maximum refracting angle of (C = critical angle for the material of the prism)

(a) 60° (b) C
(c) $2C$ (d) slightly less than 180°

33. You are given four sources of light each one providing a light of a single colour-red, blue, green and yellow. Suppose the angle of refraction for a beam of yellow light corresponding to a particular angle of incidence at the interface of two media is 90°. Which of the following statements is correct, if the source of yellow light is replaced with that of other lights without changing the angle of incidence? **[NCERT Exemplar]**

(a) The beam of red light would undergo total internal reflection
(b) The beam of red light would bend towards normal while it gets refracted through the second medium
(c) The beam of blue light would undergo total internal reflection
(d) The beam of green light would bend away from the normal as it gets refracted through the second medium

34. A beam of light composed of red and green rays is incident obliquely at a point on the face of a rectangular glass slab. When coming out on the opposite parallel face, then the red and green rays emerge from

(a) two points propagating in two different non-parallel directions
(b) two points propagating in two different parallel directions
(c) one point propagating in two different directions
(d) one point propagating in the same direction

35. Object is placed 15 cm from a concave mirror of focal length 10 cm, then the nature of image formed will be

(a) magnified and inverted (b) magnified and erect
(c) small in size and inverted (d) small in size and erect

36. When a lens of refractive index n_1 is placed in a liquid of refractive index n_2, then the lens looks to be disappeared only, if

(a) $n_1 = n_2/2$ (b) $n_1 = 3n_2/2$
(c) $n_1 = n_2$ (d) $n_1 = 5n_2/2$

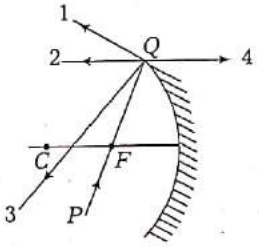
37. If x_1 is the size of the magnified image and x_2 is the size of the diminished image in lens displacement method, then the size of the object is

(a) $\sqrt{x_1x_2}$ (b) x_1x_2 (c) $x_1^2x_2$ (d) $x_1x_2^2$

38. Mark the correct one.

(a) Our eyes can distinguish between real and virtual image
(b) Virtual image can also be taken on screen
(c) If the incident rays are converging at a point, then the object is real
(d) None of the above

39. The direction of ray of light incident on a concave mirror is shown by PQ while directions in which the ray would travel after reflection is shown by four rays marked 1, 2, 3 and 4 (figure). Which of the four rays correctly shows the direction of reflected ray? **[NCERT Exemplar]**



(a) 1 (b) 2 (c) 3 (d) 4

40. Two beams of red and violet colours made to pass separately through a prism (A = 60°). In the minimum deviation position, the angle of refraction inside the prism will be

(a) greater for red colour
(b) equal but not 30° for both the colours
(c) greater for violet colour
(d) 30° for both the colours

41. When sun light is scattered by minute particles of atmosphere, then the intensity of light scattered away is proportional to

(a) (wavelength of light)^4 (b) (frequency of light)^4
(c) (wavelength of light)^2 (d) (frequency of light)^2

42. An object is placed at a distance u from an equiconvex lens such that the distance between the object and its real image is minimum. The focal length of the lens is f. The value of u is

(a) ∞ (b) $1.5f$ (c) $2f$ (d) $4f$

43. If the aperture of a telescope is decreased resolving power will

(a) increases (b) decreases
(c) remain same (d) zero

44. A telescope has an objective of focal length 100 cm and an eye-piece of focal length 5 cm. What is the magnifying power of the telescope when it is in normal adjustment?

(a) 0.2 (b) 2.0 (c) 20 (d) 200

45. Which one of the following is not associated with the total internal reflection?
(a) The mirage formation
(b) Optical fibre communication
(c) The glittering of diamond
(d) Dispersion of light

46. The time required for the light to pass through a glass slab (refractive index = 1.5) of thickness u mm is ($c = 3 \times 10^8$ ms^{-1} speed of light in free space)
(a) 10^{-11} s (b) 2×10^{-11} s
(c) 2×10^{11} s (d) 2×10^{-5} s

47. Electromagnetic radiation of frequency n, wavelength λ, travelling with velocity v in air, enters a glass slab of refractive index μ. The frequency, wavelength and velocity of light in the glass slab will be respectively
(a) $\frac{n}{\mu}, \frac{\lambda}{\mu}, \frac{v}{\mu}$ (b) $n, \frac{\lambda}{\mu}, \frac{v}{\mu}$
(c) $n, \lambda, \frac{v}{\mu}$ (d) $\frac{n}{\mu}, \frac{\lambda}{\mu}, v$

48. We combined a convex lens of focal length f_1 and concave lens of focal length f_2 and their combined focal length was F. The combination of these lenses will behave like a concave lens, if
(a) $f_1 > f_2$ (b) $f_1 < f_2$
(c) $f_1 = f_2$ (d) $f_1 \le f_2$

49. Wavelength of light in vacuum is 5890 Å, then its wavelength in glass ($\mu = 1.5$) will be
(a) 9372 Å (b) 7932 Å
(c) 7548 Å (d) 3927 Å

50. Light travels in two media A and B with speeds 1.8×10^8 ms^{-1} and 2.4×10^8 ms^{-1}, respectively. Then, the critical angle between them is
(a) $\sin^{-1}\left(\frac{2}{3}\right)$ (b) $\tan^{-1}\left(\frac{3}{4}\right)$ (c) $\tan^{-1}\left(\frac{2}{3}\right)$ (d) $\sin^{-1}\left(\frac{3}{4}\right)$

51. The focal length of a thin convex lens for red and blue colours is 100.5 cm and 99.5 cm. The dispersive power of the lens is
(a) 0.01 (b) 0.02 (c) 1.005 (d) 0.995

52. Two mirrors are placed at right angle to each other. A man is standing between them combining his hair. How many images he will see?
(a) 2 (b) 3 (c) 1 (d) zero

53. To get three images of a single object, one should have two plane mirrors at an angle of
(a) 60° (b) 90° (c) 120° (d) 30°

54. Two mirrors are kept at 60° to each other and a body is placed at the middle. The total number of images formed are
(a) six (b) four (c) five (d) three

55. If the focal length of the eye piece of a telescope is doubled, then its magnifying power (m) will be
(a) $2m$ (b) $3m$
(c) $m/2$ (d) $4m$

56. Angular resolving power of human eye is
(a) 3.6×10^3 rad (b) 3.6×10^2 rad
(c) 3.6×10^4 rad (d) 3.6×10^6 rad

57. If the red light is replaced by blue light illuminating the object in a microscope, then the resolving power of the microscope
(a) decreases (b) increases
(c) gets halved (d) remains unchanged

58. A telescope using light having wavelength 5000 Å and using lenses of focal lengths 2.5 cm and 30 cm. If the diameter of the aperture of the objective is 10 cm, then the resolving limit of telescope is
(a) 6.1×10^{-6} rad (b) 5.0×10^{-6} rad
(c) 8.3×10^{-4} rad (d) 7.3×10^{-3} rad

59. An astronomical telescope in normal adjustment receives light from a distance source S, the tube length is now decreased slightly, then
(a) no image will be formed
(b) a virtual image of S will be formed at a finite distance
(c) a large, real image of S will be formed behind the eye piece, far away from it
(d) a small, real image of S will be formed behind the eye-piece closes to it

60. An astronomical telescope has an angular magnification of magnitude 5 for distant objects. The separation between the objective and the eye-piece is 36 cm and the final image is formed at infinity. The focal length f_o of the objective and the focal length f_e of eye-piece are
(a) $f_o = 45$ cm and $f_e = -9$ cm
(b) $f_o = -7.2$ cm and $f_e = 5$ cm
(c) $f_o = 50$ cm and $f_e = 10$ cm
(d) $f_o = 30$ cm and $f_e = 6$ cm

61. The magnification produced by an astronomical telescope for normal adjustment is 10 and the length of the telescope is 1.1 m. The magnification, when the image is formed at least distance of distinct vision is
(a) 6 (b) 14
(c) 16 (d) 18

62. Where should a person stand straight from the pole of a convex mirror of focal length 2.0 m on its axis, so that the image formed become half of his original height?
(a) -2.60 m (b) -4.0 m
(c) -0.5 m (d) -2.0 m

63. The radius of curvature of the curved surface of a plano-convex lens is 20 cm. If the refractive index of the material of the lens be 1.5, then it will **[NCERT Exemplar]**
(a) act as a convex lens only for the objects that lie on its curved side
(b) act as a concave lens for the objects that lie on its curved side
(c) act as a convex lens irrespective of the side on which the object lies
(d) act as a concave lens irrespective of side on which the object lies

64. Wavelength of given light waves in air and in a medium are 6000 Å and 3000 Å, respectively. The critical angle is
(a) $\tan^{-1}\left(\frac{2}{3}\right)$ (b) $\tan^{-1}\left(\frac{3}{2}\right)$ (c) $\sin^{-1}(2)$ (d) $\sin^{-1}\left(\frac{3}{2}\right)$

65. A convex and a concave mirror of radii 10 cm each are placed facing each other and 15 cm apart. An object is placed exactly between them. If the reflection first takes place in concave and then in convex mirror the position of the final image will be
(a) 7 cm behind concave mirror
(b) at the pole of the concave mirror
(c) at the pole of the convex mirror
(d) 6.7 cm in front of concave mirror

66. The focal lengths of the lenses of an astronomical telescope are 50 cm and 5 cm. The length of the telescope when the image is formed at the least distance of distinct vision is
(a) 45 cm (b) 55 cm (c) $\frac{275}{6}$ cm (d) $\frac{325}{6}$ cm

67. An object of 5 cm height is placed 1 m apart from a concave spherical mirror which has a radius of curvature of 20 cm. The size of the image is
(a) 0.11 cm (b) 0.5 cm
(c) 0.55 cm (d) 0.60 cm

68. Two thin lenses of focal lengths 20 cm and 25 cm are placed in contact. The effective power of the combination is
(a) 9 D (b) 2 D (c) 3 D (d) 7 D

69. The nearer point of hypermetropic eye is 40 cm. The lens to be used for its correction should have the power
(a) + 1.5 D (b) − 1.5 D
(c) + 2.5 D (d) + 0.5 D

70. A ray of light passing through a prism of refraction angle 60° has to deviate by at least 30°. Then, refractive index of prism should be
(a) $\leq \sqrt{2}$ (b) $\geq \sqrt{2}$
(c) $\geq \sqrt{3}$ (d) $\leq \sqrt{3}$

71. A liquid is placed in a hollow prism of angle 60°. If angle of the minimum deviation is 30°, then what is the refractive index of the liquid?
(a) $\sqrt{2}$ (b) 1.50 (c) $\frac{4}{3}$ (d) $\frac{5}{4}$

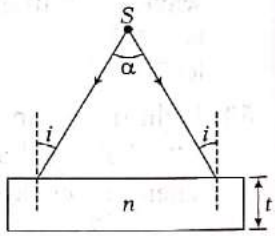
72. When a ray is refracted from one medium to another, then the wavelength changes from 6000 Å to 4000 Å. The critical angle for the interface will be
(a) $\cos^{-1}\left(\frac{2}{3}\right)$ (b) $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$
(c) $\sin^{-1}\left(\frac{2}{3}\right)$ (d) $\cos^{-1}\left(\frac{2}{\sqrt{3}}\right)$

73. In a plano-convex lens, the radius of curvature of convex surface is 10 cm and the focal length of the lens is 30 cm. The refractive index of the material of the lens will be
(a) 1.5 (b) 1.66 (c) 1.33 (d) 3

74. If the refractive index of a material of equilateral prism is $\sqrt{3}$, then angle of minimum deviation of the prism is
(a) 30° (b) 45° (c) 60° (d) 75°

75. A ray of light is incident at 60° on one face of a prism of angle 30° and the emergent ray makes 30° with the incident ray. The refractive index of the prism is
(a) 1.732 (b) 1.414 (c) 1.5 (d) 1.33

76. A diverging beam of light from a point source S having divergence angle α, falls symmetrically on a glass slab as shown in figure. The angles of incidence of the two extreme rays are equal. If the thickness of the glass slab is t and the refractive index n, then the divergence angle of the emergent beam is



(a) zero (b) α
(c) $\sin^{-1}\frac{1}{n}$ (d) $2\sin^{-1}\left(\frac{1}{n}\right)$

77. A convex lens forms an image of an object on a screen 30 cm from the lens. When the lens is moved 90 cm towards the object, then the image is again formed on the screen. Then, the focal length of the lens is
(a) 13 cm (b) 24 cm (c) 33 cm (d) 40 cm

78. If tube length of astronomical telescope is 105 cm and magnifying power is 20 for normal setting, then calculate the focal length of objective
(a) 100 cm (b) 10 cm (c) 20 cm (d) 25 cm

79. A double convex glass lens ($R_1 = R_2 = 10$ cm) having focal length equal to the focal length of a concave mirror. The radius of curvature of the concave mirror is ($\mu_g = 1.5$)

(a) 10 cm (b) 20 cm
(c) 40 cm (d) 15 cm

80. When a thin convex lens is put in contact with a thin concave lens of the same focal length (f), then the resultant combination has focal length equal to

(a) $f/2$ (b) $2f$
(c) 0 (d) ∞

81. If refractive index of glass is 1.50 and of water is 1.33, then critical angle is

(a) $\sin^{-1}\left(\frac{8}{9}\right)$ (b) $\sin^{-1}\left(\frac{2}{3}\right)$
(c) $\cos^{-1}\left(\frac{8}{9}\right)$ (d) None of these

82. The radii of curvature of the two surfaces of a lens are 20 cm and 30 cm and the refractive index of the material of the lens is 1.5. If the lens is concave-convex, then the focal length of the lens is

(a) 24 cm (b) 10 cm
(c) 15 cm (d) 120 cm

83. The power of a biconvex lens is 10 D and the radius of curvature of each surface is 10 cm. Then the refractive index of the material of the lens is

(a) $\frac{3}{2}$ (b) $\frac{4}{3}$
(c) $\frac{9}{8}$ (d) $\frac{5}{3}$

84. An eye specialist prescribes spectacles having a combination of a convex lens of local length 40 cm in contact with a concave lens of focal length 25 cm. The power of this lens combination will be

(a) + 1.5 D (b) – 1.5 D
(c) + 6.67 D (d) – 6.67 D

85. Two lenses are placed in contact with each other and the focal length of combination is 80 cm. If the focal length of one is 20 cm, then the power of the other will be

(a) 1.66 D (b) 4.00 D
(c) –1.00 D (d) –3.75 D

86. A ray of light falls on a denser-rarer boundary from denser side. The critical angle is 45°. The maximum deviation the ray can undergo is

(a) 30° (b) 45°
(c) 90° (d) 120°

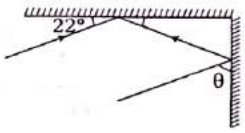
87. A plano-convex lens of refractive index 1.5 and radius of curvature 30 cm is silvered at the curved surface. Now, the lens has been used to form the image of an object. What should be the distance of the object from the lense in order to have a real image of the size of the object?

(a) 20 cm (b) 30 cm
(c) 60 cm (d) 80 cm

88. If light travels a distance x in t_1 sec in air and $10x$ distance in t_2 sec in a medium, the critical angle of the medium, will be

(a) $\tan^{-1}\left(\frac{t_1}{t_2}\right)$ (b) $\sin^{-1}\left(\frac{t_1}{t_2}\right)$
(c) $\sin^{-1}\left(\frac{10t_1}{t_2}\right)$ (d) $\tan^{-1}\left(\frac{10t_1}{t_2}\right)$

89. A ray of light is directed towards a corner reflector as shown. The incident ray makes an angle of 22° with one of the mirrors. *At what angle θ does the ray emerge?*

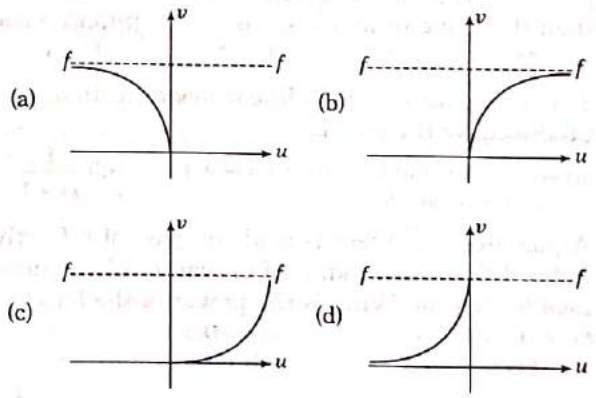


(a) 22° (b) 68°
(c) 44° (d) 46°

90. The focal lengths of the objective and eye lens of a microscope are 1 cm and 5 cm, respectively. If the magnifying power for the relaxed eye is 45, then the length of the tube is

(a) 30 cm (b) 25 cm
(c) 15 cm (d) 12 cm

91. The graph between u and v for a convex mirror is



92. A concave lens of focal length 20 cm placed in contact with a plane mirror acts as a

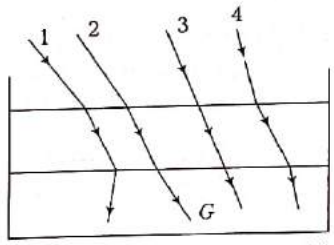
(a) convex mirror of focal length 10 cm
(b) concave mirror of focal length 40 cm
(c) concave mirror of focal length 60 cm
(d) concave mirror of focal length 10 cm

93. A diver at a depth of 12 m in water $\left(\mu = \frac{4}{3}\right)$ sees the sky at a cone of semivertical angle

(a) $\sin^{-1}\left(\frac{4}{3}\right)$ (b) $\tan^{-1}\left(\frac{4}{3}\right)$

(c) $\sin^{-1}\left(\frac{3}{4}\right)$ (d) 90°

94. The optical density of turpentine is higher than that of water while its mass density is lower. Figure shows, a layer of turpentine floating over water in a container. For which one of the four rays incident on turpentine in figure, the path shown is correct? **[NCERT Exemplar]**



(a) 1 (b) 2 (c) 3 (d) 4

95. A plano-convex lens is made of refractive index of 1.6. The focal length of the lens is

(a) 400 cm (b) 200 cm (c) 100 cm (d) 50 cm

96. A plano-convex lens (f = 20 cm) is silvered at plane surface. Now, focal length will be

(a) 20 cm (b) 40 cm (c) 30 cm (d) 10 cm

97. A ray of light incident at an angle θ on a refracting face of a prism emerges from the other face normally. If the angle of the prism is 5° and the prism is made of a material of refractive index 1.5, then the angle of incidence is **[NCERT Exemplar]**

(a) 7.5° (b) 5° (c) 15° (d) 2.5°

98. The divergent lens in m linear magnification produced by the lens is

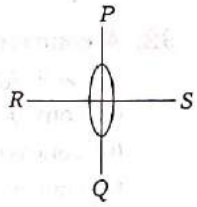
(a) m (b) $\frac{1}{m}$ (c) $m+1$ (d) $\frac{1}{m+1}$

99. A plano-concave lens is made of glass of refractive index 1.5 and the radius of curvature of its curved face is 100 cm. What is the power of the lens?

(a) + 0.5 D (b) – 0.5 D

(c) – 2D (d) + 2D

100. The figure shows and equiconvex lens of focal length f. It the lens is cut along PQ, the focal length of each half will be



(a) $\frac{f}{2}$ (b) f

(c) $2f$ (d) $4f$

101. A glass slab of thickness 8 cm contains the same number of waves as 10 cm long path of water when both are traversed by the same monochromatic light. If the refractive index of water is 4/3, then the refractive index of glass is

(a) $\frac{5}{3}$ (b) $\frac{5}{4}$ (c) $\frac{16}{15}$ (d) $\frac{3}{2}$

102. A microscope is focussed on an ink mark on the top of a table. If we place a glass slab of 3 cm thick on it, how should the microscope be moved to focus the ink spot again? The refractive index of glass is 1.5

(a) 2 cm upwards (b) 2 cm downwards

(c) 1 cm upwards (d) 1 cm downwards

103. The optical path of a monochromatic light is same whether, if it goes through 4.0 cm of glass or 4.5 cm of water. If the refractive index of glass is 1.53, then the refractive index of the water is

(a) 1.30 (b) 1.36 (c) 1.42 (d) 1.46

104. A ray of light travelling in transparent medium of refractive index μ falls, on a surface separating the medium from air at an angle of incidence of 45°. For which of the following value of μ, the ray can undergo total internal reflection?

(a) $\mu = 1.33$ (b) $\mu = 1.41$ (c) $\mu = 1.50$ (d) $\mu = 1.25$

105. A ray of light, travelling in a medium of refractive index μ, is incident at an angle i on a composite transparent plate consisting of three plates of refractive indices μ_1, μ_2 and μ_3. The ray emerges from the composite plate into a medium of refractive index μ_4, at angle x. Then

(a) $\sin x = \sin i$ (b) $\sin x = \frac{\mu}{\mu_4} \sin i$

(c) $\sin x = \frac{\mu_4}{\mu} \sin i$ (d) $\sin x = \frac{\mu_1}{\mu_2}\frac{\mu_3}{\mu_2}\frac{\mu}{\mu_4} \sin i$

106. What should be the angle between two plane mirrors, so that whatever be the angle of incidence, the incident ray and the reflected ray from the two mirrors be parallel to each other?

(a) 60° (b) 90° (c) 120° (d) 45°

107. How much water should be filled in a container of height 21 cm, so that it appears half filled to the observer when viewed from the top of the container ($\mu = 4/3$)

(a) 8 cm (b) 10.5 cm (c) 12 cm (d) 14 cm

108. When a plane mirror is placed horizontally on ground at a distance of 60 m from the base of a tower, then the top of the tower and its image in the mirror subtend an angle of 90° at the corner of the mirror nearer to the base of tower. The height of the tower is

(a) 30 m (b) 60 m (c) 90 m (d) 120 m

109. A prism having refractive index 1.414 and refracting angle 30° has one of the refracting surfaces silvered. A beam of light incident on the other refracting surface will retrace its path, if the angle of incidence is
(a) 0° (b) 30° (c) 60° (d) 45°

110. Two thin lenses have a combined power of +9 D. When they are separated by a distance of 20 cm, then their equivalent power becomes $+\frac{27}{5}$ D. Their individual powers (in dioptre) are
(a) 1, 8 (b) 2, 7 (c) 3,6 (d) 4, 5

111. Correct exposure for a photographic print is 10 s at a distance of one metre from a point source of 20 cd. For an equal fogging of the print placed at a distance of 2 m from a 16 cd source, the necessary time for exposure is
(a) 100 s (b) 25 s (c) 50 s (d) 75 s

112. A ray of light strikes a horizontal plane mirror at an angle of 45°. A second plane mirror is attached at an angle θ with it. If ray after reflection from second mirror runs parallel to the first mirror, then θ is
(a) 45° (b) 60° (c) 67.5° (d) 135°

113. A convex lens of focal length 1.0 m and a concave lens of focal length 0.25 m are 0.75 m apart. A parallel beam of light is incident in the convex lens. The beam emerging after refraction from both lenses is
(a) parallel to principal axis (b) convergent
(c) divergent (d) None of these

114. For an angle of incidence θ on an equilateral prism of refractive index $\sqrt{3}$, the ray refracted is parallel to the base inside the prism. The value of θ is
(a) 30° (b) 45° (c) 60° (d) 75°

115. A car is moving with at a constant speed of 60 km h^{-1} on a straight road. Looking at the rear view mirror, the driver finds that the car following him is at a distance of 100 m and is approaching with a speed of 5 kmh^{-1}. In order to keep track of the car in the rear, the driver begins to glance alternatively at the rear and side mirror of his car after every 2 s till the other car overtakes. If the two cars were maintaining their speeds, which of the following statement (s) is/are correct? **[NCERT Exemplar]**
(a) The speed of the car in the rear is 65 kmh^{-1}
(b) In the side mirror, the car in the rear would appear to approach with a speed of 5 kmh^{-1} to the driver of the leading car
(c) In the rear view mirror, the speed of the approaching car would appear to decrease as the distance between the cars decreases
(d) In the side mirror, the speed of the approaching car would appear to increase as the distance between the cars decreases

116. A thin prism of angle 6° made up of glass of refractive index 1.5 is combined with another prism made up of glass of refractive index 1.75 to produce dispersion without deviation. The angle of second prism is
(a) 7° (b) 9° (c) 4° (d) 5°

117. A double convex lens made of glass (refractive index n =1.5) has both radii of curvature of magnitude 20 cm. Incident light rays parallel to the axis of the lens will converge at a distance L such that
(a) $L = 20$ cm (b) $L = 10$ cm
(c) $L = 40$ cm (d) $L = \frac{20}{3}$ cm

118. When sunlight is scattered by atmospheric atoms and molecules, the amount of scattering of light of wavelength 440 nm is A. The amount of scattering for the light of wavelength 660 nm is approximately
(a) $\frac{4}{9}A$ (b) $2.25A$
(c) $1.5A$ (d) $\frac{A}{6}$

119. A prism, having refractive index $\sqrt{2}$ and refracting angle 30°, has one of the refracting surfaces polished. A beam of light incident on the other refracting surface will trace its path, if the angle of incidence is
(a) 0° (b) 30°
(c) 45° (d) 60°

120. When a ray of light is incident normally on one refracting surface of an equilateral prism, than (Refractive index of the material of the prism = 1.5)
(a) emerging ray is deviated by 30°
(b) emerging ray is deviated by 45°
(c) emerging ray just grazes the second refracting surface
(d) the ray undergoes total internal reflection at the second refracting surface

121. A telescope has a objective of focal length 50 cm and an eye-piece of focal length 5 cm. The least distance of distinct vision is 25 cm. The telescope is focussed for distinct vision on a scale of 200 cm away. The separation between the objective and the eye-piece is
(a) 75 cm (b) 60 cm
(c) 71 cm (d) 74 cm

122. A glass prism has a refractive angle of 90° and a refractive index of 1.5. A ray is incident at an angle of 30°. The ray emerges from an adjacent face at an angle of
(a) 60° (b) 30°
(c) 45° (d) the ray does not emerge

123. An equilateral prism deviates a ray through 45° for the two angles of incidence differing by 20°. The angle of incidence is
(a) 62.5° (b) 42.5°
(c) Both are correct (d) Both are wrong

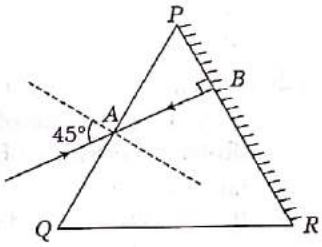
124. The focal length of the objective of a terrestrial telescope is 80 cm and it is adjusted for parallel rays, and its magnifying power is 20. If the focal length of erecting lens is 20 cm, then full length of telescope will be
(a) 84 cm (b) 100 cm
(c) 124 cm (d) 164 cm

125. The length of the tube of a microscope is 10 cm. The focal lengths of the objective and eye lenses are 0.5 cm and 1.0 cm. The magnifying power of the microscope is about
(a) 5 (b) 23 (c) 166 (d) 500

126. The magnifying power of a microscope with an objective of 5 mm focal length is 400. The length of its tube is 20 cm. Then, the focal length of the eye-piece is
(a) 200 cm (b) 160 cm
(c) 2.5 cm (d) 0.1 cm

127. A thin plano-convex lens acts like a concave mirror of radius of curvature 20 cm when its plane surface is silvered. The radius of curvature of the curved surface, if index of refraction of its material is 1.5 will be
(a) 40 cm (b) 30 cm (c) 10 cm (d) 20 cm

128. The face PR of a prism PQR of angle 30° is silvered. A ray is incident on face PQ at an angle of 45° as shown in figure. The refracted ray undergoes reflection on face PR and retraces its path. The refractive index of the prism is



(a) $\sqrt{2}$ (b) $3/\sqrt{2}$
(c) 1.5 (d) 1.33

129. A convex lens has mean focal length of 20 cm. The dispersive power of the material of the lens is 0.02. The longitudinal chromatic aberration for an object at infinity is
(a) 10^3 (b) 0.80 (c) 0.40 (d) 0.20

130. A convex lens of focal length f is placed some, where in between an object and a screen. The distance between object and screen is x. If numerical value of magnification produced by lens is m, then focal length of lens is
(a) $\frac{mx}{(m+1)^2}$ (b) $\frac{mx}{(m-1)^2}$
(c) $\frac{(m+1)^2}{m}x$ (d) $\frac{(m-1)^2}{m}x$

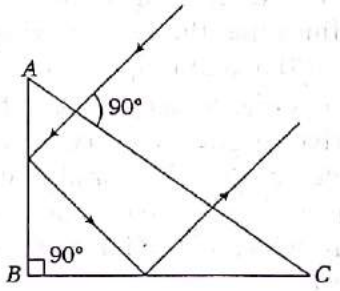
131. A square of side 3 cm is located at a distance 25 cm from a concave mirror of focal length 10 cm. The centre of square is at the axis of the mirror and the plane is normal to axis of mirror. The area enclosed by the image of the square is
(a) 4 cm^2 (b) 6 cm^2
(c) 16 cm^2 (d) 36 cm^2

132. A plano-convex lens has a maximum thickness of 6 cm. When placed on a horizontal table with the curved surface in contact with the table surface, then the apparent depth of the bottom most point of the lens is found to be 4 cm. If the lens is inverted such that the plane face of the lens is in contact with the surface of the table, then the apparent depth of the centre of the plane face is found to be $\frac{17}{3}$ cm. The radius of curvature of the lens is
(a) 68 cm (b) 75 cm
(c) 128 cm (d) 34 cm

133. An object is 20 cm away from a concave mirror with focal length 15 cm. If the object moves with a speed of 5 m/s along the axis, then the speed of the image will be
(a) 45 m/s (b) 27 m/s
(c) 9 m/s (d) 10 m/s

134. The dispersive powers of glasses of lenses used in an achromatic pair are in the ratio 5 : 3. If the focal length of the concave lens is 15 cm, then the nature and focal length of the other lens would be
(a) convex, 9 cm (b) concave, 9 cm
(c) convex, 25 cm (d) concave, 25 cm

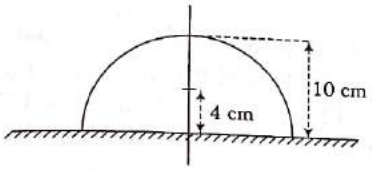
135. A ray falls on a prism ABC ($AB = BC$) and travels as shown in figure. The minimum refractive index of the prism material should be



(a) 4/3 (b) $\sqrt{2}$
(c) 1.5 (d) $\sqrt{3}$

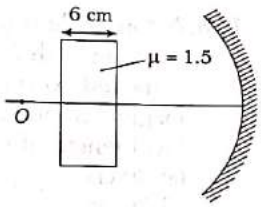
136. If the distances of an object and its virtual image from the focus of a convex lens of focal length f are 1 cm each, then f is
(a) 4 cm (b) $(\sqrt{2}+1)$ cm
(c) $2\sqrt{2}$ cm (d) $(2+\sqrt{2})$ cm

137. A hemispherical paper weight contains a small flower on its axis of symmetry at a distance of 4 cm from its flat surface. Where is the flower appears to an observer when he looks at it along the axis of symmetry from the top? (Index of refraction of glass = 1.5)



(a) 15 cm from top (b) 20 cm from top
(c) 5 cm from top (d) 25 cm from top

138. A slab of glass, of thickness 6 cm and refractive index $\mu = 1.5$ is placed in front of a concave mirror as shown in the figure. If the radius of curvature of the mirror is 40 cm and the reflected image coincides with the object, then the distance of the object from the mirror is



(a) 30 cm (b) 22 cm (c) 42 cm (d) 38 cm

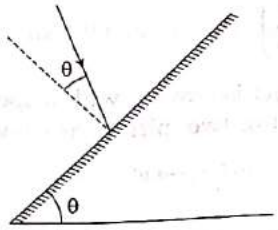
139. An object is kept at a distance of 16 cm from a thin lens and the image formed is real. If the object is kept at a distance of 6 cm from the same lens, then the image formed are equal, then the focal length of the lens will be

(a) 19 cm (b) 17 cm (c) 21 cm (d) 11 cm

140. A short linear object of length b lies along the axis of a concave mirror of focal length f at a distance u from the pole of the mirror, what is the size of image?

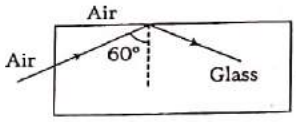
(a) $\left(\frac{f}{u-f}\right)b$ (b) $\left(\frac{f}{u-f}\right)^2 b$ (c) $\left(\frac{f}{u-f}\right)b^2$ (d) $\left(\frac{f}{u-f}\right)$

141. A mirror is inclined at an angle of θ with the horizontal. If a ray of light is incident at an angle of incidence θ, then the reflected ray makes the following angle with horizontal



(a) θ (b) 2θ
(c) θ/2 (d) None of these

142. A light ray going from air is incident (as shown in figure) at one end of a glass fibre (refractive index $\mu = 1.5$) making an incidence angle of 60° on the lateral surface,



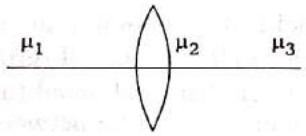
so that it undergoes a total internal reflection. How much time would it take to traverse the straight fibre of length 1 km?

(a) 3.3 μs (b) 6.6 μs
(c) 5.7 μs (d) 3.85 μs

143. A thin convergent glass lens $\mu = 1.5$ has a power of + 5.0 D. When this lens is immersed in a liquid of refractive index μ_l it acts as diverging lens of focal length 100 cm. The value of μ_l should be

(a) $\frac{3}{2}$ (b) $\frac{4}{3}$ (c) $\frac{5}{3}$ (d) 2

144. The figure shows an equiconvex lens. What should be the condition on the refractive indices so that the lens becoming diverging?



(a) $2\mu_3 > \mu_1 - \mu_2$ (b) $2\mu_2 < \mu_1 + \mu_3$
(c) $2\mu_2 > 2\mu_1 - \mu_3$ (d) None of these

145. A ray of light makes an angle of 10° with the horizontal above it and strikes a plane mirror which is inclined at an angle θ to the horizontal. The angle θ for which the reflected ray becomes vertical is

(a) 40° (b) 50°
(c) 80° (d) 100°

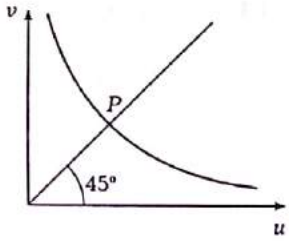
146. In the measurement of the angle of a prism using a spectrometer, the readings of first reflected image are vernier I : 320° 40′ ; vernier II : 140° 30′ and those of the second reflected image are vernier I : 80° 38′; vernier II : 260° 24′. Then, the angle of the prism is

(a) 59° 58′ (b) 59° 56′
(c) 60° 2′ (d) 60° 4′

147. A thin rod of length $f/3$ lies along the axis of a concave mirror of focal length f. One end of its magnified image touches an end of the rod. The length of the image is

(a) f (b) $\frac{1}{2}f$
(c) $2f$ (d) $\frac{1}{4}f$

148. The graph shows part of variation of v with change in u for a concave mirror. Points plotted above the point P on the curve are for values of v



(a) smaller than f
(b) smaller than $2f$
(c) larger than $2f$
(d) larger than f but less than $2f$

149. When an object is at distance x and y from a lens, a real image and a virtual image is formed respectively having same magnification. The focal length of the lens is

(a) $\frac{x+y}{2}$ (b) $x-y$
(c) $\sqrt{xy}$ (d) $x+y$

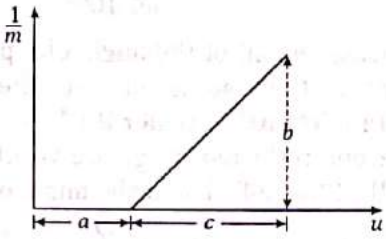
150. A symmetric double convex lens is cut in two equal parts by a plane perpendicular to the principal axis. If the power of the original lens is 4D, then the power of a cut lens will be

(a) 2D (b) 3D
(c) 4D (d) 5D

151. A ray incident at a point as an angle of incidence of 60° enters a glass sphere of refractive index $n = \sqrt{3}$ and is reflected and refracted further at the surface of the sphere. The angle between the reflected and refracted rays at this surface is

(a) 50° (b) 60° (c) 90° (d) 40°

152. The graph shows, how the inverse of magnification $\frac{1}{m}$ produced by a convex thin lens, with variation in object distance u. What was the focal length of the lens used?



(a) $\frac{b}{c}$ (b) $\frac{b}{ca}$ (c) $\frac{bc}{a}$ (d) $\frac{c}{b}$

153. A convex lens of focal length 30 cm forms a real image three times larger than the object on a screen. Object and screen are moved until the image becomes twice the size of the object. If the shift of the object is 6 cm. The shift of screen is

(a) 28 cm (b) 14 cm (c) 18 cm (d) 16 cm

154. An object is placed at 21 cm in front of a concave mirror of radius of a curvature 10 cm. A glass slab of thickness 3 cm and $\mu = 15$ is then placed close to the mirror in the space between the object and the mirror. The position of final image formed is

(a) −3.94 cm (b) 4.3 cm
(c) −4.93 (d) 3.94 cm

155. A thin prism P_1 with angle 4° and made from glass of refractive index 1.54 is combined with another prism P_2 made of glass of refractive index 1.72 to produce dispersion without deviation. The angle of prism P_2 is

(a) 5.33° (b) 4°
(c) 2.6° (d) 3°

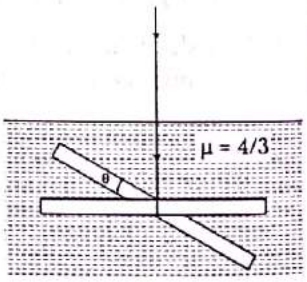
156. A convex lens produces an image of a real object on a screen with a magnification of 1/2. When the lens is moved 30 cm away from the object, the magnification of the image on the screen is 2. The focal length of the lens is

(a) 30 cm (b) 60 cm
(c) 20 cm (d) 15 cm

157. An infinitely long rod lies along with the axis of a concave mirror of focal length f. The near end of the rod is at a distance $u > f$ from the mirror. Its image will have a length

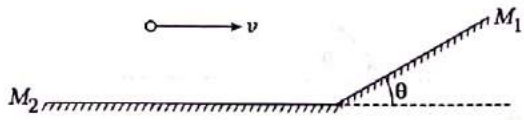
(a) $\frac{f^2}{u-f}$ (b) $\frac{uf}{u-f}$
(c) $\frac{f^2}{u+f}$ (d) $\frac{uf}{u+f}$

158. A plane mirror is placed horizontally inside water ($\mu = 4/3$). A ray falls normally on it. Then mirror is rotated through an angle θ. The minimum value of θ for which ray does not come out of the water surface is



(a) $\pi/4$ (b) $\sin^{-1}\left(\frac{3}{4}\right)$
(c) $\frac{1}{2}\sin^{-1}\left(\frac{3}{4}\right)$ (d) $2\sin^{-1}\left(\frac{3}{4}\right)$

159. A point object is moving with a speed v before an arrangement of two mirrors as shown in figure.



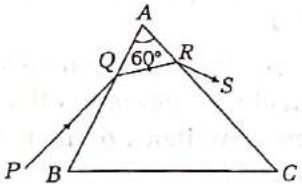
Find the velocity of image in mirror M_1 with respect to image in mirror M_2

(a) $2v \sin(\theta/2)$ (b) $v \sin(\theta/2)$
(c) $2v \cos(\theta/2)$ (d) $v \cos(\theta/2)$

160. If a ray of light in a denser medium strikes a rarer medium at an angle of incidence i, the angles of reflection and refraction are respectively, r and r'. If the reflected and refraction rays are at right angles to each other, the critical angle for the given pair of media is

(a) $\sin^{-1}(\tan r')$ (b) $\sin^{-1}(\tan r)$
(c) $\tan^{-1}(\sin i)$ (d) $\cot^{-1}(\tan i)$

161. A ray PQ incident on the refracting face BA is refracted in the prism BAC as shown in the figure and emerges from the other refracting face AC as RS, such that $AQ = AR$. If the angle of prism $A = 60°$ and the refractive index of the material of prism is $\sqrt{3}$, then the angle of deviation of the ray is



(a) 60° (b) 45°
(c) 30° (d) None of these

162. The xz-plane separates two media A and B with refractive indices μ_1 and μ_2, respectively. A ray of light travels from A and B. Its directions in the two media are given by the unit vectors $\mathbf{r}_A = a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$ and $\mathbf{r}_B = \alpha\hat{\mathbf{i}} + \beta\hat{\mathbf{j}}$ respectively, where $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ are unit vectors in the x and y-directions. Then,

(a) $\mu_1 a = \mu_2 \alpha$ (b) $\mu_1 \alpha = \mu_2 a$
(c) $\mu_1 b = \mu_2 \beta$ (d) None of these

163. A thin lens made of glass of refractive index $\mu = 1.5$ has a focal length equal to 12 cm in air. It is now immersed in water $\left(\mu = \frac{4}{3}\right)$. Its new focal length is

(a) 48 cm (b) 36 cm
(c) 24 cm (d) 12 cm

164. A ray of light is incident on surface of glass slab at an angle 45°. If the lateral shift produced per unit thickness is $\frac{1}{\sqrt{3}}$ m, the angle of refraction produced is

(a) $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (b) $\tan^{-1}\left(1 - \sqrt{\frac{2}{3}}\right)$
(c) $\sin^{-1}\left(1 - \sqrt{\frac{2}{3}}\right)$ (d) $\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{3}-1}}\right)$

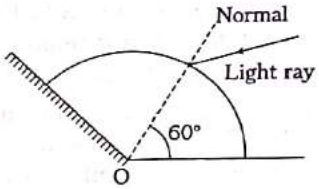
165. In a lake, a fish rising vertically to the surface of water uniformly at the rate of 3 m/s, observes a bird diving vertically towards the water at the rate of 9 m/s. The actual velocity of the dive of the bird is (given, refractive index of water = 4/3)

(a) 3.6 m/s (b) 4.5 m/s
(c) 6.0 m/s (d) 12.0 m/s

166. A circular beam of light (diameter = d) falls on a plane surface of a liquid. The angle of incidence is 45° and refractive index of the liquid is μ. The diameter of the refracted beam is

(a) d (b) $(\mu - 1)d$
(c) $\frac{\sqrt{2\mu^2 - 1}}{d} d$ (d) $\frac{\sqrt{\mu^2 - 1}}{\mu} d$

167. Consider the situation as shown in figure. The point O is the centre. The light ray forms an angle of 60° with the normal.



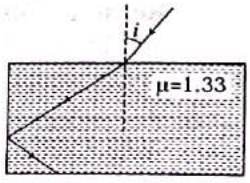
The normal makes an angle 60° with the horizontal and the mirror makes an angle 60° with the normal. The value of refractive index of that spherical portions, so that light ray retraces its path is

(a) $\sqrt{2}$ (b) $\frac{2}{\sqrt{3}}$
(c) $\frac{3}{2}$ (d) $\sqrt{3}$

168. A concave mirror is placed at the bottom of an empty tank with face upwards and axis vertical. When sunlight falls normally on the mirror, then it is focused at distance of 32 cm from the mirror. If the tank filled with water $\left(\mu = \frac{4}{3}\right)$ upto a height of 20 cm, then the sunlight will now get focused at

(a) 16 cm above water level
(b) 9 cm above water level
(c) 24 cm below water level
(d) 9 cm below water level

169. Given a slab with index $n = 1.33$ and incident light striking the top horizontal face at angle i as shown in figure. The maximum value of i for which total internal reflection occurs is



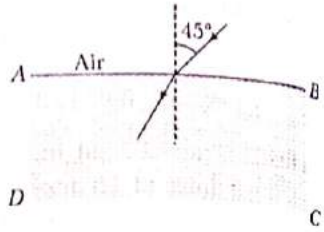
(a) $\sin^{-1}(\sqrt{0.77})$ (b) $\cos^{-1}(\sqrt{0.77})$
(c) $\sin^{-1}(0.77)$ (d) $\sin^{-1}(0.38)$

170. The apparent depth of water in cylindrical water tank of diameter $2R$ cm is reducing at the rate of x cm/min when water is being drained out at a constant rate. The amount of water drained in cc/min is (n_1 = refractive index of air, n_2 = refractive index of water)

(a) $\frac{x\pi R^2 n_1}{n_2}$ (b) $\frac{x\pi R^2 n_2}{n_1}$

(c) $\frac{2\pi R n_1}{n_2}$ (d) $\pi R^2 x$

171. In the figure shown, for an angle of incidence 45°, at the top surface, what is the minimum refractive index needed for total internal reflection at vertical face AD?



(a) $\frac{\sqrt{2}+1}{2}$ (b) $\sqrt{\frac{3}{2}}$ (c) $\sqrt{\frac{1}{2}}$ (d) $\sqrt{2}$

B. Medical entrance special format questions

Assertion and reason

Directions (Q. Nos. 1-20) *These questions consist of two statements each linked as Assertion and Reason. While answering these questions you are required to choose any one of the following five responses.*

(a) If both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are true but Reason is not correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If Assertion is false but Reason is true.
(e) If both Assertion and Reason are false.

1. Assertion The formula connecting u, v and f for a spherical mirror is valid only for mirrors sizes which are very small compared to their radii of curvature.
Reason Laws of reflection are stricity valid for plane surface, but not for large spherical.

2. Assertion Reflected image always travels in opposite direction of object.
Reason Speed of point image formed by reflected rays is always equal to the speed of its point object.

3. Assertion Image formed by a convex mirror can never be real.
Reason Convex mirror is diverging in nature.

4. Assertion There is no dispersion of light, refracted through a rectangular glass slab.
Reason Dispersion of light is the phenomenon of spliting of a beam of white light into its constituent colours.

5. Assertion The images formed by total internal reflection are much brighter than those formed by mirrors or lenses.
Reason There is no loss of intensity in total internal reflection.

6. Assertion In refraction from a plane surface, if object is virtual, then its image will be real.
Reason Plane surface always makes opposite natured image. If object is real, then image is virtual and *vice-versa*.

7. Assertion Shining of diamond is due to the phenomenon of total internal reflection.
Reason Refractive index of diamond is large. Hence, critical angle is small.

8. Assertion Sun appears red during sunrise and sunset due to the phenomenon of refraction.
Reason In refraction of light frequency of light remains unchanged.

9. Assertion A mirror has only one focus, while that of lens has two focii.
Reason Both the focii of a mirror coincide at one point.

10. Assertion Lens formula can be applied only for thin lenses.
Reason For thick lenses one cannot find image position.

11. Assertion The refractive index of diamond is $\sqrt{6}$ and that of liquid is $\sqrt{3}$. If the light travels from diamond to the liquid, it will totally reflected when the angle of incidence is 30°
Reason $\mu = \frac{1}{\sin C}$, where μ is the refractive index of diamond with respect to liquid.

12. Assertion A ray of light travels from first medium to second medium. In first medium, angle of ray with normal is $i_1 = 30°$ and in second medium $i_2 = 60°$. Then, second medium is rarer medium.
Reason Refractive index of first medium with respect to second medium is $\frac{\sin 30°}{\sin 60°}$.

13. **Assertion** When a wave travels from a denser to a rarer medium, its amplitude increases.

Reason In a rarer medium, speed of wave is more.

14. **Assertion** In front of a concave mirror, a point object is placed between focus and centre of curvature. If a glass slab is placed between object and mirror, then image from mirror may become virtual.

Reason Glass slab always makes a virtual image of a real object.

15. **Assertion** If a lens is immersed in a liquid its nature will change *i.e.*, convex will behave concave and *vice-versa*.

Reason If both sides of a lens medium is same, then object can be placed on either side of the lens, image distance remains same.

16. **Assertion** A diverging lens (in air) cannot be made more diverging whatever be the medium we choose to completely immerse the lens.

Reason The minimum refractive index of any medium is 1.

17. **Assertion** Although the surfaces of goggle lens are curved, it does not have any power.

Reason In case of goggles, both the curved surfaces have equal radii of curvature and have centre of curvature of the same side.

18. **Assertion** Focal length of a lens depends on the wavelength of light used.

Reason The more the wavelengths, lesser is the focal length.

19. **Assertion** A hollow lens behaves like a thin glass plate.

Reason Power of this lens becomes zero.

20. **Assertion** If angle of incidence in case of a prism is gradually increased, then deviation produced by prism will first decrease, then increase.

Reason At minimum deviation, $r_1 = \frac{A}{2}$.

Match the columns

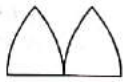
1. Regarding the power of an optical instruments match the following two columns.

	Column I		Column II
(A)	Convex mirror	(p)	Zero
(B)	Concave mirror	(q)	infinite
(C)	Plane mirror	(r)	Positive
(D)	Concave lens	(s)	negative

2. Magnitude of focal lengths of a convex lens, a concave lens, a convex mirror and a concave mirror are 20 cm each. An object is placed at a distance of 30 cm from pole/optical centre of each. Regarding the image match the following two columns.

	Column I		Column II
(A)	Convex mirror	(p)	real
(B)	Concave mirror	(q)	virtual
(C)	Convex lens	(r)	magnified
(D)	Concave lens	(s)	diminished

3. Focal length of a biconvex lens is f. Regarding the focal lengths and match the following two columns.

	Column I		Column II
(A)		(p)	f
(B)		(q)	$2f$
(C)		(r)	$\frac{f}{2}$
(D)		(s)	Zero

4. Match Column I (Phenomenon) with Column II (Principle) and select the correct answer using the codes given below the Column.

	List I (Phenomenon)		List II (Principle)
A.	Blue colour of a sky	(p)	Total internal reflection
B.	Glittering of diamond	(q)	Dispersion of light
C.	Formation of rainbow	(r)	Scattering of light
D.	In the evening when the sun goes down below the horizon, it continues to remain visible for some time	(s)	Refraction of light

5. A thin convex lens has focal length f_1 and a concave lens has focal length f_2. They are kept in contact. Now, match the following two columns.

	Column I		Column II
(A)	If $\lvert f_1 \rvert = \lvert f_2 \rvert$	(p)	Power of system = 0
(B)	If $\lvert f_1 \rvert > \lvert f_2 \rvert$	(q)	Power of system = ∞
(C)	If $\lvert f_1 \rvert < \lvert f_2 \rvert$	(r)	Power of system is positive
		(s)	Power of system is negative

C. Medical entrances' gallery

(Collection of questions asked in NEET & various medical entrance exams)

1. Two identical glass ($\mu_g = 3/2$) equi-convex lenses of focal length f each are kept in contact. The space between the two lenses is filled with water ($\mu_w = 4/3$). The focal length of the combination is **[NEET (II) 2016]**

(a) $f/3$ (b) f (c) $\frac{4f}{3}$ (d) $\frac{3f}{4}$

2. An air bubble in a glass slab with refractive index 1.5 (near normal incidence) is 5 cm deep when viewed from one surface and 3 cm deep when viewed from the opposite face. The thickness (in cm) of the slab is **[NEET (II) 2016]**

(a) 8 (b) 10 (c) 12 (d) 16

3. A person can see clearly objects only when they lie between 50 cm and 400 cm from his eyes. In order to increase the maximum distance of distinct vision to infinity, the type and power of the correcting lens, the person has to use, will be **[NEET (II) 2016]**

(a) convex, + 2.25 D (b) concave, - 0.25 D
(c) concave, - 0.2 D (d) convex, + 0.15 D

4. A astronomical telescope has objective and eye-piece of focal lengths 40 cm, 4 cm, respectively. To view an object 200 cm away from the objective, the lenses must be separated by a distance **[NEET 2016]**

(a) 46.0 cm (b) 50.0 cm (c) 54.0 cm (d) 37.3 cm

5. Match the corresponding entries of Column I with Column II. [Where, m is the magnification produced by the mirror] **[NEET 2016]**

	Column I		Column II
A.	$m = -2$	a.	Convex mirror
B.	$m = -1/2$	b.	Concave mirror
C.	$m = +2$	c.	Real image
D.	$m = +1/2$	d.	Virtual image

(a) A→ a and c, B→ a and d, C→ a and b, D→ c and d
(b) A→ a and d, B→ b and c, C→ b and d, D→ b and c
(c) A→ c and d; B→ b and d; C→ b and c; D→ a and d
(d) A→ b and c; B→ b and c; C→ b and d; D→ a and d

6. The angle of incidence for a ray of light at a refracting surface of a prism is 45°. The angle of prism is 60°. If the ray suffers minimum deviation through the prism, then the angle of minimum deviation and refractive index of the material of the prism respectively, are **[NEET 2016]**

(a) 30°, $\sqrt{2}$ (b) 45°, $\sqrt{2}$ (c) 30°, $\frac{1}{\sqrt{2}}$ (d) 45°, $\frac{1}{\sqrt{2}}$

7. Two identical thin plano-convex glass lenses (refractive index 1.5) each having radius of curvature of 20 cm are placed with their convex surfaces in contact at the centre. The intervening space is filled with oil of refractive index 1.7. The focal length of the combination is **[CBSE AIPMT 2015]**

(a) – 20 cm (b) – 25 cm (c) – 50 cm (d) 50 cm

8. The refracting angle of a prism is A and refractive index of the material of the prism is cot $(A/2)$. The angle of minimum deviation is **[CBSE AIPMT 2015]**

(a) $180° - 3A$ (b) $180° - 2A$ (c) $90° - A$ (d) $180° + 2A$

9. The near point and far point of a person are 40 cm and 250 cm, respectively. Determine the power of the lens he/she should use while reading a book kept at distance 25 cm from the eye. **[AIIMS 2015]**

(a) 2.5 D (b) 5 D (c) 1.5 D (d) 3.5 D

10. An object is seen through a simple microscope of focal length 12 cm. What will be the angular magnification produced, if the image is formed at the near point of the eye which is 25 cm away from it? **[UK PMT 2015]**

(a) 6.08 (b) 3.08 (c) 9.03 (d) 5.09

11. Angle of minimum deviation for a prism of refractive index 1.5 is equal to the angle of prism of given prism. Then, the angle of prism is ..... ($\sin 48°36' = 0.75$) **[Guj. CET 2015]**

(a) 80° (b) 41°24′ (c) 60° (d) 82°48′

12. A ray of light passes from a medium A having refractive index 1.6 to the medium B having refractive index 1.5. The value of critical angle of medium A is .... **[Guj. CET 2015]**

(a) $\sin^{-1}\sqrt{\frac{16}{15}}$ (b) $\sin^{-1}\left(\frac{16}{15}\right)$ (c) $\sin^{-1}\left(\frac{1}{2}\right)$ (d) $\sin^{-1}\left(\frac{15}{16}\right)$

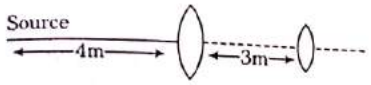
13. The focal lengths of a converging lens are f_v and f_r for violet and red lights, respectively. Which of the following is correct? **[CG PMT 2015]**

(a) $f_v < f_r$
(b) $f_v = f_r$
(c) $f_v > f_r$
(d) It depends on the average refractive index

14. Aperture of human eye is 0.2 cm. The minimum magnifying power of a visual telescope, whose objective has diameter 100 cm, is **[CG PMT 2015]**

(a) 500 (b) 0.002 (c) 0.02 (d) 100

15. An object is located 4 m from the first of two thin converging lenses of focal lengths 2 m and 1 m, respectively. The lenses are separated by 3 m. The final image formed by the second lens is located from the source at a distance of [WB JEE 2015]



(a) 8 m (b) 5.5 m (c) 6 m (d) 6.5 m

16. If $\mu_V = 15230$ and $\mu_R = 15145$, then dispersive power of a crown glass is [UP CPMT 2015]
(a) 0.0164 (b) 0.00701 (c) 0.0132 (d) 0.0320

17. Dispersion of light is caused due to [Manipal 2015]
(a) intensity of light (b) density of medium
(c) wavelength (d) None of these

18. Calculate the focal length of a reading glass of a person, if the distance of distinct vision is 75 cm. [KCET 2015]
(a) 75.2 cm (b) 25.6 cm (c) 100.4 cm (d) 37.5 cm

19. A person wants a real image of his own, 3 times enlarged. Where should he stand in front of a concave mirror of radius of curvature of 30 cm? [KCET 2015]
(a) 90 cm (b) 10 cm (c) 20 cm (d) 30 cm

20. The magnifying power of a convex lens of focal length 10 cm, when the image is formed at the near point is [Kerala CEE 2015]
(a) 6 (b) 5.5 (c) 4 (d) 3.5
(e) 2

21. The velocity of image when object and mirror both are moving towards each other with velocities $4\,ms^{-1}$ and $5\,ms^{-1}$ respectively, is [UP CPMT 2015]
(a) $-14\,ms^{-1}$ (b) $15\,ms^{-1}$
(c) $-9\,ms^{-1}$ (d) $14\,ms^{-1}$

22. The limiting angle of incidence for an optical ray that can be transmitted by an equilateral prism of refractive index $\mu = \sqrt{\frac{7}{3}}$ is given by (angles can be assumed to be small, so that sine of the angle is angle itself) [CG PMT 2015]
(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{2\pi}{3}$ (d) $\frac{\pi}{2}$

23. A ray enters a glass sphere of refractive index $\mu = \sqrt{3}$ at an angle of incidence of 60°, a ray is reflected and refracted at the farther surface of the sphere. The angle between the reflected and refracted rays at this surface is [UP CPMT 2015]
(a) 50° (b) 60° (c) 90° (d) 40°

24. To measure the roughness of the surface of a material, which of the following microscope is preferred for better result output? [Manipal 2015]
(a) Compound microscope (b) Electron microscope
(c) Atomic force microscope (d) None of the above

25. To get three images of single object, one should have two plane mirrors at an angle of [UK PMT 2014]
(a) 60° (b) 90° (c) 120° (d) 30°

26. An object placed at 20 cm in front of a concave mirror produces three times magnified real image. What is the focal length of the concave mirror? [KCET 2014]
(a) 15 cm (b) 6.6 cm (c) 10 cm (d) 7.5 cm

27. A glass slab consists of thin uniform layers of progressively decreasing refractive indices refractive index such that the refractive index of any layer is $\mu - m\Delta\mu$. Here, μ and $\Delta\mu$ denote the refractive index of 0th layer and the difference in refractive index between any two consecutive layers, respectively. The integer $m = 0, 1, 2, 3, \cdots$ denotes the numbers of the successive layers. A ray of light from the 0th layer enters the 1st layer at an angle of incidence of 30°. After undergoing the mth refraction, the ray emerges parallel to the interface. If $\mu = 1.5$ and $\Delta\mu = 0.015$, then the value of m is [WB JEE 2014]
(a) 20 (b) 30 (c) 40 (d) 50

28. In vacuum, to travel distance d, light takes time t and in medium to travel distance $5d$, it takes time T. The critical angle of the medium is [MHT CET 2014]
(a) $\sin^{-1}\left(\frac{5T}{t}\right)$ (b) $\sin^{-1}\left(\frac{5t}{3T}\right)$
(c) $\sin^{-1}\left(\frac{5t}{T}\right)$ (d) $\sin^{-1}\left(\frac{3t}{5T}\right)$

29. The angle of a prism is A. One of its refracting surfaces is silvered. Light rays falling at an angle of incidence $2A$ on the first surface returns back through the same path after suffering reflection at the silvered surface. The refractive index μ, of the prism is [CBSE AIPMT 2014]
(a) $2\sin A$ (b) $2\cos A$ (c) $\frac{1}{2}\cos A$ (d) $\tan A$

30. The equiconvex lens has focal length f. If it is cut perpendicular to the principal axis passing through optical centre, then focal length of each half is [MHT CET 2014]
(a) $\frac{f}{2}$ (b) f (c) $\frac{3f}{2}$ (d) $2f$

31. Two lenses of power 15 D and −3 D are placed in contact. The focal length of the combinations is [Kerala CEE 2014]
(a) 10 cm (b) 15 cm (c) 12 cm (d) 18 cm
(e) 8.33 cm

32. A luminous object is separated from a screen by distance d. A convex lens is placed between the object and the screen such that it forms a distinct image on the screen. The maximum possible focal length of this convex lens is **[WB JEE 2014]**

(a) $4d$ (b) $2d$ (c) $\frac{d}{2}$ (d) $\frac{d}{4}$

33. An object is placed 30 cm away from a convex lens of focal length 10 cm and a sharp image is formed on a screen. Now, a concave lens is placed in contact with the convex lens. The screen now has to be moved by 45 cm to get a sharp image again. The magnitude of focal length of the concave lens is (in cm) **[WB JEE 2014]**

(a) 72 (b) 60 (c) 36 (d) 20

34. A thin glass (refractive index 1.5) lens has optical power of –5D in air. Its optical power in a liquid medium with refractive index 1.6 will be **[UK PMT 2014]**

(a) 1 D (b) –1 D (c) 25 D (d) –25 D

35. A concave lens of focal length f forms an image which is 1/3 times the size of the object. Then, the distance of object from the lens is **[EAMCET 2014]**

(a) $2f$ (b) f (c) $\frac{2}{3}f$ (d) $\frac{3}{2}f$

36. A focal length of a lens is 10 cm. What is power of a lens in dioptre? **[KCET 2014]**

(a) 0.1 D (b) 10 D (c) 15 D (d) 1 D

37. If the focal length of objective lens is increased, then magnifying power of **[CBSE AIPMT 2014]**

(a) microscope will increase but that of telescope decrease
(b) microscope and telescope both will increase
(c) microscope and telescope both will decrease
(d) microscope will decrease but that of telescope will increase

38. Astigmatism is corrected by using **[Kerala CEE 2014]**

(a) cylindrical lens (b) plano-convex lens
(c) plano-concave lens (d) convex lens
(e) concave lens

39. The intermediate image formed by the objective of a compound microscope is **[WB JEE 2014]**

(a) real, inverted and magnified
(b) real, erect and magnified
(c) virtual, erect and magnified
(d) virtual, inverted and magnified

40. An astronomical telescope arranged for normal adjustment has a magnification of 6. If the length of the telescope is 35 cm, then the focal lengths of objective and eye-piece respectively, are **[EAMCET 2014]**

(a) 30 cm, 5 cm (b) 5 cm, 30 cm
(c) 40 cm, 5 cm (d) 30 cm, 6 cm

41. A microscope is having objective of focal length 1 cm and eye-piece of focal length 6 cm. If tube length is 30 cm and image is formed at the least distance of distinct vision, what is the magnification produced by the microscope.(take, D = 25 cm) **[KCET 2014]**

(a) 6 (b) 150 (c) 25 (d) 125

42. Diameter of the objective of a telescope is 200 cm. What is the resolving power of a telescope? Take, wavelength of light = 5000Å. **[KCET 2014]**

(a) 6.56×10^6 (b) 3.28×10^5
(c) 1×10^6 (d) 3.28×10^6

43. A plano-convex lens fits exactly into a plano-concave lens. Their plane surfaces are parallel to each other. If lenses are made of different materials of refractive indices μ_1 and μ_2 and R is the radius of curvature of the curved surface of the lenses, then the focal length of the combination is **[NEET 2013]**

(a) $\frac{R}{2(\mu_1+\mu_2)}$ (b) $\frac{R}{2(\mu_1-\mu_2)}$
(c) $\frac{R}{(\mu_1-\mu_2)}$ (d) $\frac{2R}{(\mu_2-\mu_1)}$

44. For a normal eye, the cornea of eye provides a converging power of 40 D and the least converging power of the eye lens behind the cornea is 20 D. Using this information, the distance between the retina and the cornea-eye lens can be estimated to be **[NEET 2013]**

(a) 5 cm (b) 2.5 cm (c) 1.67 cm (d) 1.5 cm

45. An object is 8 cm high. It is desired to form a real image 4 cm high at 60 cm from the mirror. The type of mirror needed with the focal length is **[J&K CET 2013]**

(a) convex mirror with focal length $f = 40$ cm
(b) convex mirror with focal length $f = 20$ cm
(c) concave mirror with focal length $f = -40$ cm
(d) concave mirror with focal length $f = -20$ cm

46. When an object is placed 40 cm from a diverging lens, its virtual image is formed 20 cm from the lens. The focal length and power of lens are **[J&K CET 2013]**

(a) $F = -20$ cm, $P = -5$ D (b) $F = -40$ cm, $P = -5$ D
(c) $F = -40$ cm, $P = -2.5$ D (d) $F = -20$ cm, $P = -2.5$ D

47. A magnifying glass of focal length 5 cm is used to view an object by a person whose smallest distance of distinct vision is 25 cm. If he holds the glass close to eye, then the magnification is **[J&K CET 2013]**

(a) 5 (b) 6 (c) 2.5 (d) 3

48. A person has a minimum distance of distinct vision as 50 cm. The power of lenses required to read a book at a distance of 25 cm is **[J&K CET 2013]**

(a) 3 D (b) 1 D (c) 2 D (d) 4 D

49. The distance of moon form the earth is 3.8×10^5 km. Supposing that the eye is most sensitive to the light of wavelength 550 nm, the separation of two points on the moon that can be resolved by a 500 cm telescope is **[J&K CET 2013]**

(a) 50 m (b) 55 m (c) 51 m (d) 60 m

50. A small angled prism of refractive index 1.4 is combined with another small angled prism of refractive index 1.6 to produce dispersion without deviation. If the angle of first prism is 6°, then the angle of the second prism is **[EAMCET 2013]**

(a) 8° (b) 6° (c) 4° (d) 2°

51. The magnifying power of the astronomical telescope for normal adjustment is 50. The focal length of the eyepiece is 2 cm. The required length of the telescope for normal adjustment is **[EAMCET 2013]**

(a) 102 cm (b) 100 cm (c) 98 cm (d) 25 cm

52. The speed of light in media M_1 and M_2 are 1.5×10^8 ms^{-1} and 2×10^8 ms^{-1}, respectively. A ray travels from medium M_1 to the medium M_2 with an angle of incidence θ. The ray suffers total internal reflection. Then, the value of the angle of incidence θ is **[CET 2013]**

(a) $>\sin^{-1}\left(\frac{3}{4}\right)$ (b) $<\sin^{-1}\left(\frac{3}{4}\right)$

(c) $=\sin^{-1}\left(\frac{3}{4}\right)$ (d) $\le\sin^{-1}\left(\frac{3}{4}\right)$

53. Radii of curvature of a converging lens are in the ratio 1 : 2. Its focal length is 6 cm and refractive index is 1.5. Then, its radii of curvature are ..., respectively. **[Kerala CET 2013]**

(a) 9 cm and 18 cm (b) 6 cm and 12 cm

(c) 3 cm and 6 cm (d) 4.5 cm and 9 cm

54. The sun light reaches us as white and not as its components because **[CET 2013]**

(a) air medium is dispersive

(b) air medium is non-dispersive

(c) air medium scatter the sunlight

(d) air medium absorbs the sunlight

(e) speed of light depends on wavelength in vacuum

55. The radius of curvature of the convex face of a plano-convex lens is 12 cm and the refractive index of the material of the lens is 1.5. Then, the focal length of the lens is **[MP PMT 2013]**

(a) 6 cm (b) 12 cm (c) 18 cm (d) 24 cm

56. Two thin lenses of focal lengths f_1 and f_2 are placed in contact with each other. Then, the equivalent focal length of the combination will be **[MP PMT 2013]**

(a) f_1+f_2 (b) $\frac{1}{f_1+f_2}$ (c) $\frac{f_1f_2}{f_1+f_2}$ (d) $\frac{f_1+f_2}{f_1f_2}$

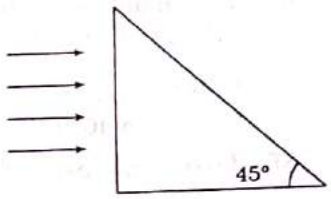
57. The mirrors are inclined at an angle of 50°. The number of images formed for an object placed in between the mirrors is **[MP PMT 2013]**

(a) 5 (b) 6 (c) 7 (d) 8

58. If c is the velocity of light in free space, then the time taken by light to travel a distance x in a medium refractive index μ is **[MP PMT 2013]**

(a) $\frac{x}{c}$ (b) $\frac{\mu x}{c}$ (c) $\frac{x}{\mu c}$ (d) $\frac{c}{\mu x}$

59. A beam of light consisting of red, green and blue colours is incident on a right angled prism as shown in figure. The refractive indices of the material of the prism for above red, green and blue wavelengths are 1.39, 1.44 and 1.47, respectively. The prism will **[UPCPMT 2013]**



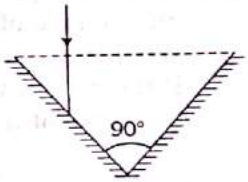
(a) separate part of the blue colour from the red and green

(b) separate part of the red colour from the green and blue colours

(c) separate all the three colours from one another

(d) None of the above

60. A vessel consists of two plane mirrors at right angles as shown in figure. The vessel is filled with water. The total deviation in incident ray is **[UP CPMT 2013]**



(a) 0° (b) 60° (c) 90° (d) 180°

61. For having large magnification power of a compound microscope **[MPPMT 2013]**

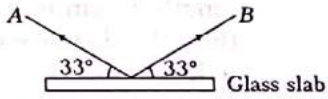
(a) length of the microscope tube must be small

(b) focal lengths of objective lens and eye-piece should be large

(c) focal lengths of objective lens and eye-piece should be small

(d) focal length of eye-piece must be smaller than the focal length of objective lens

62. A beam of light is incident on a glass slab ($\mu = 1.54$) in a direction as shown in the figure. The reflected light is analysed by a polaroid prism. On rotating the polaroid, ($\tan 57° = 1.54$) **[Kerala CET 2013]**



(a) the intensity remains unchanged

(b) the intensity is reduced to zero and remains at zero

(c) the intensity gradually reduced to zero and then again increases

(d) the intensity increase continuously

(e) the intensity increases initially and remains constant afterwards

63. A thin equiconvex lens of refractive index 3/2 and radius of curvature 30 cm is put in water (refractive index = 4/3), its focal length is **[UP CPMT 2012]**
(a) 0.15 m (b) 0.30 m (c) 0.45 m (d) 1.20 m

64. If the focal length of a concave mirror is 50 cm, then where should the object is placed, so that its image is two times and inverted **[BCECE (Mains) 2012]**
(a) 55 cm (b) – 75 cm
(c) 65 cm (d) 85 cm

65. The diameter of the eye ball of a normal eye is about 2.5 cm. The power of the eye lens varies from **[BCECE (Mains) 2012]**
(a) 9 D to 8 D (b) 40 D to 32 D
(c) 44 D to 40 D (d) None of these

66. Two thin lenses when placed in contact, then the power of combination is +10 D. If they are kept 0.25 m apart, then the power reduces to + 6 D. The focal lengths of the lenses (in m) will be **[BCECE (Mains) 2012]**
(a) 0.5 and 0.125
(b) 0.5 and 0.75
(c) 0.125 and 0.75
(d) Both have the same focal length 1.25

67. A ray of light is incident at an angle of incidence i, on one face of a prism of angle A (assumed to be small) and emerges normally from the opposite face. If the refractive index of the prism is μ, then the angle of incidence i, is nearly equal to **[CBSE AIPMT 2012]**
(a) μA (b) $\frac{\mu A}{2}$ (c) A/μ (d) $A/2\mu$

68. A concave mirror of focal length f_1 is placed at a distance d from a convex lens of focal length f_2. A beam of light coming from infinity and falling on this convex lens concave mirror combination returns to infinity. The distance d must be equal **[CBSE AIPMT 2012]**
(a) $f_1 + f_2$ (b) $-f_1 + f_2$ (c) $2f_1 + f_2$ (d) $-2f_1 + f_2$

69. If the image formed by a convex mirror of focal length 30 cm is a quarter of the size of the object, then the distance of the object from the mirror will be **[BHU 2012]**
(a) 30 cm (b) 60 cm (c) 90 cm (d) 120 cm

70. In a compound microscope, the focal length of the objective is 2.5 cm and of eye lens is 5 cm. If an object is placed at 3.75 cm before the objective and the image is formed at the least distance of distinct vision, then the distance between two lenses will be **[BHU 2012]**
(a) 11.67 cm (b) 12 cm
(c) 12.75 cm (d) 13 cm

71. A thin convex lens of refractive index 1.5 has 20 cm focal length in air. If the lens is completely immersed in a liquid of refractive index 1.6, then its focal length will be **[AIIMS 2012]**
(a) - 160 cm (b) - 100 cm
(c) + 10 cm (d) + 100 cm

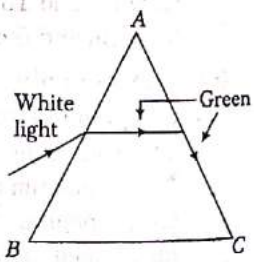
72. **Assertion** The resolving power of a telescope is more, if the diameter of the objective lens is more.

Reason Objective lens of large diameter collects more light. **[AIIMS 2012]**
(a) If both Assertion and Reason are true and Reason is the correct explanation of Assertion
(b) If both Assertion and Reason are true but Reason is not the correct explanation of Assertion
(c) If Assertion is true but Reason is false
(d) If both Assertion and Reason are false

73. A short linear object of length L lies on the axis of a spherical mirror of focal length f at a distance u form the mirror. Its image has an axial length L' equal to **[AMU 2012]**
(a) $L\left[\frac{f}{u-f}\right]^{1/2}$ (b) $L\left[\frac{u+f}{f}\right]^{1/2}$
(c) $L\left[\frac{u-f}{f}\right]^{2}$ (d) $L\left[\frac{f}{u-f}\right]^{2}$

74. An object is placed at a distance of 10 cm from a co-axial combination of two lenses A and B in contact. The combination forms a real image three times the size of the object. If lens B is concave with a focal length of 30 cm. The nature and focal length of lens A is **[AMU 2012]**
(a) convex, 12 cm (b) concave, 12 cm
(c) convex, 6 cm (d) convex, 18 cm

75. White light is incident on face AB of a glass prism. The path of the green component is shown in the figure. If the green light is just totally internally reflected at face AC as shown, then the light emerging from face AC will contain **[AMU 2012]**



(a) yellow, orange and red colours
(b) violet, indigo and blue colours
(c) All colours
(d) All colours except green

76. The focal length of the objective of a terrestrial telescope is 80 cm and it is adjusted for parallel rays, then its power is 20. If the focal length of erecting lens is 20 cm, then full length of the telescope will be **[AFMC 2012]**
(a) 164 cm (b) 124 cm (c) 100 cm (d) 84 cm

Answers

Check point 9.1

1. (b)	2. (c)	3. (b)	4. (b)	5. (d)	6. (a)	7. (b)	8. (d)	9. (c)	10. (b)

Check point 9.2

1. (a)	2. (a)	3. (d)	4. (a)	5. (a)	6. (a)	7. (b)	8. (b)	9. (b)	10. (a)

Check point 9.3

1. (a)	2. (c)	3. (a)	4. (c)	5. (b)	6. (a)	7. (c)	8. (d)	9. (c)	10. (c)
11. (c)	12. (a)	13. (b)	14. (a)	15. (c)	16. (c)	17. (a)	18. (a)	19. (c)	20. (b)

Check point 9.4

1. (b)	2. (d)	3. (a)	4. (c)	5. (d)	6. (d)	7. (b)	8. (d)	9. (b)	10. (b)
11. (d)	12. (d)	13. (d)	14. (d)	15. (d)	16. (a)	17. (b)	18. (d)	19. (d)	20. (b)

Check point 9.5

1. (a)	2. (d)	3. (d)	4. (b)	5. (a)	6. (b)	7. (d)	8. (a)	9. (b)	10. (b)

Check point 9.6

1. (a)	2. (a)	3. (b)	4. (a)	5. (b)	6. (c)	7. (b)	8. (a)	9. (b)	10. (b)
11. (a)	12. (a)	13. (a)	14. (b)	15. (b)	16. (a)	17. (b)	18. (b)	19. (c)	20. (b)

Check point 9.7

1. (b)	2. (b)	3. (a)	4. (b)	5. (c)	6. (d)	7. (c)	8. (c)	9. (d)	10. (c)

A. Taking it together

1. (b)	2. (a)	3. (c)	4. (b)	5. (b)	6. (a)	7. (b)	8. (c)	9. (b)	10. (c)
11. (a)	12. (a)	13. (b)	14. (d)	15. (a)	16. (d)	17. (a)	18. (b)	19. (c)	20. (b)
21. (b,d)	22. (c)	23. (a)	24. (c)	25. (c)	26. (d)	27. (d)	28. (b)	29. (d)	30. (c)
31. (d)	32. (c)	33. (c)	34. (a)	35. (a)	36. (c)	37. (a)	38. (d)	39. (b)	40. (d)
41. (b)	42. (c)	43. (b)	44. (c)	45. (b)	46. (b)	47. (b)	48. (a)	49. (a)	50. (d)
51. (a)	52. (b)	53. (b)	54. (c)	55. (c)	56. (a)	57. (b)	58. (a)	59. (b)	60. (d)
61. (b)	62. (d)	63. (c)	64. (c)	65. (c)	66. (d)	67. (c)	68. (a)	69. (c)	70. (b)
71. (a)	72. (c)	73. (c)	74. (c)	75. (a)	76. (b)	77. (b)	78. (c)	79. (b)	80. (d)
81. (a)	82. (d)	83. (a)	84. (b)	85. (d)	86. (c)	87. (a)	88. (c)	89. (b)	90. (c)
91. (a)	92. (a)	93. (c)	94. (b)	95. (c)	96. (d)	97. (a)	98. (d)	99. (b)	100. (c)
101. (a)	102. (c)	103. (b)	104. (b)	105. (b)	106. (b)	107. (d)	108. (b)	109. (d)	110. (c)
111. (c)	112. (c)	113. (a)	114. (a)	115. (d)	116. (c)	117. (a)	118. (d)	119. (c)	120. (d)
121. (c)	122. (d)	123. (c)	124. (d)	125. (d)	126. (c)	127. (c)	128. (a)	129. (c)	130. (a)
131. (a)	132. (d)	133. (a)	134. (a)	135. (b)	136. (b)	137. (c)	138. (c)	139. (d)	140. (b)
141. (d)	142. (d)	143. (c)	144. (b)	145. (a)	146. (a)	147. (b)	148. (c)	149. (a)	150. (a)
151. (c)	152. (a)	153. (a)	154. (c)	155. (d)	156. (c)	157. (a)	158. (c)	159. (a)	160. (b)
161. (a)	162. (a)	163. (a)	164. (b)	165. (b)	166. (c)	167. (d)	168. (b)	169. (a)	170. (b)
171. (b)									

B. Medical entrance special format question

Assertion and reason

1. (c)	2. (d)	3. (d)	4. (b)	5. (a)	6. (a)	7. (a)	8. (a)	9. (a)	10. (c)
11. (d)	12. (c)	13. (a)	14. (c)	15. (d)	16. (a)	17. (a)	18. (c)	19. (a)	20. (b)

Match the columns

1. (A→ s; B→ r; C→ p; D→ s)
2. (A→ q, s; B→ p, r; C→ p,r; D→ q, s)
3. (A→ q; B→ p; C→ r; D→ p)
4. (A → r; B → p; C → q; D → s)
5. (A→ p; B→ q, s; C→ r)

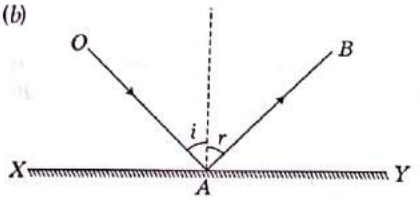
C. Madical entrances' gallery

1. (d)	2. (c)	3. (b)	4. (c)	5. (d)	6. (a)	7. (c)	8. (b)	9. (c)	10. (b)
11. (d)	12. (d)	13. (a)	14. (a)	15. (b)	16. (a)	17. (c)	18. (d)	19. (c)	20. (d)
21. (a)	22. (b)	23. (c)	24. (c)	25. (b)	26. (a)	27. (d)	28. (c)	29. (b)	30. (d)
31. (e)	32. (d)	33. (d)	34. (a)	35. (a)	36. (b)	37. (d)	38. (a)	39. (a)	40. (a)
41. (b)	42. (d)	43. (c)	44. (c)	45. (a)	46. (c)	47. (b)	48. (c)	49. (b)	50. (c)
51. (c)	52. (a)	53. (d)	54. (b)	55. (d)	56. (c)	57. (b)	58. (b)	59. (b)	60. (d)
61. (d)	62. (c)	63. (d)	64. (b)	65. (c)	66. (a)	67. (a)	68. (c)	69. (c)	70. (a)
71. (a)	72. (a)	73. (d)	74. (c)	75. (d)	76. (a)				

Hints and explanations

Check point 9.1

1. (*b*)



Here, XY is a plane reflecting surface.

OA and AB are the incident and reflected rays

i = angle of incidence

r = angle of reflection

As, $AB \perp OA$,

$\therefore$ $i + r = 90°$

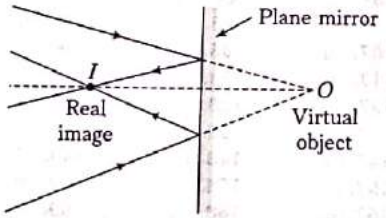
Also, according to laws of reflection,

$$r = i$$

$$\therefore \quad i + i = 90° \Rightarrow 2i = 90°$$

$$i = \frac{90°}{2} = 45°$$

3. (*b*) When convergent beam incident on a plane mirror, then mirror forms real image, for a virtual object.

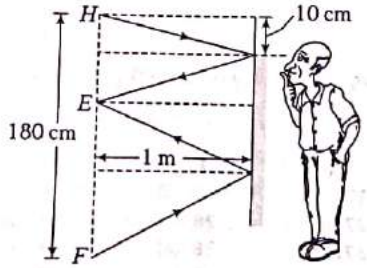


4. (*b*) Size of image formed by a plane mirror is same as that of the object. Hence, its magnification will be 1.

5. (*d*) Deviation by a plane mirror,

$$\delta = 180° - 2\theta = 180° - 60° = 120°$$

7. (*b*) According to ray diagram shown in figure



Length of mirror $= \frac{1}{2}(10 + 170) = 90$ cm

9. (*c*) As number of images, $n = 3$

So, $$n = \frac{360}{\theta} - 1$$

$$3 = \frac{360}{\theta} - 1$$

$$\therefore \quad \theta = 90°$$

10. (b) Distance between 4th images

$$I_u I_u' = 2n(x + y) = 2 \times 4\,[3 + 1] = 32$$

Check point 9.2

1. (*a*) On immersing a mirror in water, focal length of the mirror remains unchanged.

2. (*a*) Image formed is real, inverted and same in size because object is at the centre of curvature of the mirror.

3. (*d*) Given, $u = -30$ cm and $f = 30$ cm

$$\frac{1}{v} + \frac{1}{-30} = \frac{1}{30} \Rightarrow \frac{1}{v} = \frac{1}{30} + \frac{1}{30} = \frac{1}{15}$$

$\therefore$ Distance from the mirror, $v = +15$ cm

4. (*a*) Given, $u = -30$ cm

$$v = -30 \text{ cm}$$

Using relation, $$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\frac{1}{-30} + \frac{1}{-30} = \frac{1}{f}$$

$\therefore$ Focal length of the mirror, $f = -15$ cm

5. (*a*) Magnification, $m = +\frac{1}{n} = -\frac{v}{u}$ s

$$\therefore \quad v = -\frac{u}{n}$$

From mirror formula,

$$\frac{1}{f} = \frac{1}{\left(-\frac{u}{n}\right)} + \frac{1}{u}$$

$\therefore$ The distance of the object from the mirror, $u = -(n-1)f$

6. (*a*) In case of concave mirror, $m = -2$

Also, $$m = \frac{f}{f - u}$$

$$\Rightarrow \quad -2 = \frac{-50}{-50 - u}$$

$\therefore$ Distance of the object from the mirror $u = -75$ cm

7. (b) Magnification,

$$m = \frac{I}{O} = \frac{f}{f-u}$$

$$\therefore \quad \frac{I}{7.5} = \frac{(R/2)}{(R/2)-u} = \frac{(25/2)}{(25/2)-(-40)}$$

$\therefore$ The size of the image, $I = -1.78$ cm

8. (b) Magnification, $m = \frac{f}{f-u}$

$$\Rightarrow \quad \left(+\frac{1}{4}\right) = \frac{+30}{+30-u}$$

$\Rightarrow$ The distance of the object from the mirror, $u = -90$ cm

9. (b) Magnification,

$$m = \frac{f}{f-u} \Rightarrow -3 = \frac{f}{f-(-20)}$$

$\Rightarrow$ The focal length of mirror, $f = -15$ cm

10. (a) From the relation,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

We have, $\frac{-dv}{v^2} - \frac{du}{u^2} = 0$

$$\therefore \quad dv = \frac{v^2}{u^2}(-du) = \left(\frac{20}{10}\right)^2 (0.1) = 0.4 \text{ cm}$$

Check points 9.3

1. (a) Refractive index of the medium,

$$\mu = \frac{c}{v} = \frac{c}{n\lambda} = \frac{3\times10^8}{4\times10^{14}\times5\times10^{-7}} = 1.5$$

2. (c) As, $\mu = \frac{c}{v} \Rightarrow v \propto \frac{1}{\mu}$

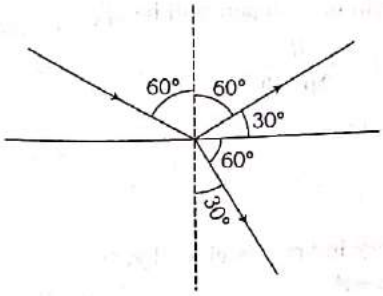
$$\therefore \quad \frac{v_1}{v_2} = \frac{\mu_2}{\mu_1}$$

$\Rightarrow$ Ratio of velocities of light

$$\frac{v_g}{v_w} = \frac{\mu_w}{\mu_g} = \frac{4/3}{3/2} = \frac{8}{9}$$

3. (a) Wavelength of light, $\lambda' = \frac{\lambda}{\mu} = \frac{6000}{1.5} = 4000$ Å

4. (c)



Refractive index of the material,

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin 60°}{\sin 30°} = \sqrt{3} = 1.732$$

5. (b) Refractive index of glass with respect to water,

$${}_w\mu_g = \frac{{}_a\mu_g}{{}_a\mu_w} = \frac{3/2}{4/3} = \frac{9}{8}$$

6. (a) Cauchy's formula, $\left(\mu = A + \frac{B}{\lambda^2}\right)$

7. (c) ${}_2\mu_1 \times {}_3\mu_2 \times {}_4\mu_3 = \frac{\mu_1}{\mu_2} \times \frac{\mu_2}{\mu_3} \times \frac{\mu_3}{\mu_4} = \frac{\mu_1}{\mu_4} = {}_4\mu_1 = \frac{1}{{}_1\mu_4}$

9. (c) When light is refracted into a medium from vacuum its wavelength decreases but frequency remain unchanged.

10. (c) We have, $d_{app} = \frac{d}{\mu}$

or $(d-2) = \frac{d}{1.5}$

$\therefore \quad d = 6$ cm

11. (c) $d_{actual} = (d_{app})\mu$

From one side, $d_1 = 6 \times 1.5 = 9$

From other side, $d_2 = 4 \times 1.5 = 6$

$\therefore \quad d = d_1 + d_2 = 15$ cm

12. (a) Distance from the surface of water,

$$h' = \mu h = \frac{4}{3} \times 18 = 24 \text{ m}$$

13. (b) Apparent depth, $h' = \frac{d_1}{\mu_1} + \frac{d_2}{\mu_2} = \frac{d}{\mu_1} + \frac{d}{\mu_2} = d\left(\frac{1}{\mu_1} + \frac{1}{\mu_2}\right)$

14. (a) As, $d_{app} = \frac{d_1}{\mu_1} + \frac{d_2}{\mu_2} + \frac{d_3}{\mu_3} = \frac{3}{3/2} + \frac{4}{4/3} + \frac{6}{6/5}$

$= 2 + 3 + 5 = 10$ cm

15. (c) The refractive index of glass relative to water is given by,

$${}_wn_g = \frac{{}_an_g}{{}_an_w} = \frac{1.50}{1.33}$$

If C be the critical angle at glass water interface, then

$$\sin C = \frac{1}{{}_wn_g} = \frac{1.33}{1.50} = 0.8867$$

$$C = \sin^{-1}(0.8867) = 62.5$$

16. (c) Critical angle,

$$\sin C = \frac{n_2}{n_1} = \frac{\lambda_1}{\lambda_2} \Rightarrow \sin C = \frac{3500}{7000} = \frac{1}{2}$$

$\therefore \quad C = 30°$

17. (a) Critical angle,

$$\sin C = \frac{1}{\mu}$$

$$\therefore \quad C = \sin^{-1}\left(\frac{1}{\mu}\right)$$

and as μ decreases with increase in λ yellow, orange, red have higher wavelength than green, so μ will be less for these rays, and so critical angle for these rays will be high, hence, if green is just totally internally reflected then yellow, orange and red rays will emerge out.

18. (*a*) We have, $\sin\theta_C = \frac{1}{\mu}$

$\therefore \quad \mu = \frac{1}{\sin 30°} = 2$

The velocity of light, $v = \frac{c}{\mu} = \frac{3.0\times10^8}{2}$

$= 1.5\times10^8$ m/s

19. (*c*) We have, $\sin i > \sin\theta_C$

$\therefore \quad \sin 45° > \frac{1}{n}$ or $\frac{1}{\sqrt{2}} > \frac{1}{n}$

$\therefore \quad n > \sqrt{2}$ or $n > 1.414$

20. (*b*) Refractive index of glass, $\mu_g = \frac{1}{\sin 30°} = 2$

Now, $\sin\theta_C = \frac{6/5}{2} = \frac{3}{5}$

$\therefore \quad \theta_C = 37°$

Check point 9.4

2. (*d*) When object lies between focus and pole of convex lens, image is virtual, erect and enlarged.

3. (*a*) For air lens in glass, focal length of air lens,

$$\frac{1}{f} = ({}_g\mu_a - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= \left(\frac{2}{3} - 1\right)\left[\frac{1}{10} - \left(-\frac{1}{10}\right)\right]$$

$\Rightarrow \quad f = -15$ cm

So, nature of lens is concave type.

4. (*c*) Focal length of plano-convex lens,

$$f = \frac{R}{\mu - 1} = \frac{R}{(15-1)} = 2R$$

5. (*d*) Image is real, because virtual image is enlarged.

$\therefore \quad \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ or $\frac{1}{x/4} + \frac{1}{x} = \frac{1}{30}$ $\quad (\because u = -x)$

$$\frac{5}{x} = \frac{1}{30}$$

$\therefore \quad x = 150$ cm

7. (*b*) From a concave lens image is always smaller in size.

8. (*d*) For real image from converging lens,

$$m = -2$$

$\therefore$ Magnification, $m = \frac{f}{f+u} \Rightarrow -2 = \frac{f}{u+f} = \frac{20}{u+20}$

$\Rightarrow \quad u = -30$ cm

9. (*b*) We have $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \Rightarrow \frac{1}{+5} - \frac{1}{-10} = \frac{1}{f}$ $\quad \left(|v| = \frac{|u|}{2}\right)$

$\therefore$ Focal length, $f = \frac{10}{3}$ cm

10. (*b*) Keep the object at a distance $2f$ from the lens.

11. (*d*) We have, $\frac{1}{v} - \frac{1}{-90} = \frac{1}{f} \Rightarrow (1.5-1)\left(\frac{1}{30}\right) = \frac{1}{60}$

$\therefore \quad v = 180$ cm

$\therefore \quad |m| = \left|\frac{v}{u}\right| = \frac{180}{90} = 2.0$

13. (*d*) We have, $P = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

$$2.5 = \left(\frac{3}{2} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$P = \left(\frac{3/2}{2} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$\therefore \quad P = -1.25$ D

14. (*d*) Focal length of combination,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$F = \frac{f_1 f_2}{f_1 + f_2}$$

Power of lens, $P = \frac{1}{F} = \frac{f_1 + f_2}{f_1 f_2}$

15. (*d*) Focal length of combination,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{60} + \frac{1}{(-20)} \Rightarrow F = -30 \text{ cm}$$

16. (*a*) Focal length of the combination,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

$\Rightarrow \quad \frac{1}{F} = \frac{1}{(+40)} + \frac{1}{(-25)}$

$\Rightarrow \quad F = -\frac{200}{3}$ cm

As, Power, $P = \frac{100}{F} = \frac{100}{(-200/3)} = -1.5$ D

17. (*b*) In each case, there is a combination of two plano-convex lenses placed close to each other. Focal length of combination $\frac{1}{F} = \frac{1}{f} + \frac{1}{f}$ is same in all cases, so ratio of focal length in three cases is 1 : 1 : 1.

18. (*d*) The focal lenght of each part will be $2f$.

20. (*b*) Focal length, $F = \frac{R}{2(\mu - 1)}$

$\Rightarrow F = \frac{10}{2(15-1)} = 10$cm

Check point 9.5

1. (*a*) Deviation of ray in prism is given by,

$$\delta = i + e - A$$

So, $e = \delta + A - i = 30° + 30° - 60° = 0°$

So, emergent ray will be perpendicular to face, or emergent ray will make an angle of 90° with the face through which it emerges.

2. (*d*) Refractive index of prism,

$$\mu = \frac{\sin\frac{A+\delta_m}{2}}{\sin\frac{A}{2}} \Rightarrow i = 45° = \frac{A+\delta_m}{2}$$

So, $$\frac{\sin 45°}{\sin(A/2)} = \sqrt{2}$$

So, $$\frac{1}{2} = \sin\frac{A}{2} \Rightarrow A = 60°$$

3. (*d*) Given, $i = e = \frac{3}{4}A = \frac{3}{4} \times 60 = 45°$

In the position of minimum deviation,

$$2i = A + \delta_m$$

or $$\delta_m = 2i - A$$
$$= 2 \times 45° - 60 = 30°$$

4. (*b*) In minimum deviation position, refracted ray inside an isosceles prism is parallel to the base of the prism.

5. (*a*) Angle of minimum deviation in prism,

$$\delta_m = (\mu - 1)A = (\mu - 1)(2r) \quad (\because r = A/2)$$
$$= (1.5 - 1)(2r) = 0.5 \times 2r = r$$

6. (*b*) Refractive index,

$$\mu = \frac{\sin i}{\sin A/2}$$

$\Rightarrow$ $$\sqrt{2} = \frac{\sin i}{\sin(60°/2)} \text{ or } \sqrt{2} \times \sin 30° = \sin i$$

or $$\sqrt{2} \times \frac{1}{2} = \sin i \Rightarrow i = 45°$$

7. (*d*) Refractive index of prism, $\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

Substituting $A = \delta_m$ and $\mu = 1.5$,

$$1.5 = \frac{\sin A}{\sin\left(\frac{A}{2}\right)}$$

$$\frac{1.5}{2} = \cos\frac{A}{2}$$

$$7.5 = \cos\frac{A}{2}$$

We get, $A = 82°$.

8. (*a*) Refractive index for prism,

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)} = \frac{\sin\left(\frac{60°+60°}{2}\right)}{\sin\left(\frac{60°}{2}\right)} = \sqrt{3}$$

9. (*b*) Dispersive power depends upon material of prism.

10. (*b*) To produce dispersion without deviation,

$$\frac{A'}{A} = \frac{(\mu_y - 1)}{(\mu'_y - 1)} \Rightarrow \frac{A'}{6} = -\frac{(1.54 - 1)}{(1.72 - 1)}$$

$\Rightarrow$ $$A' = -4.5° = 4°30'$$

Check point 9.6

1. (*a*) For normal eye, the least distance of distinct vision is 0.25 m.
2. (*a*) Image formed on the retina is real and inverted.
3. (*b*) The focal length of a normal eye lens is about 2 cm.
4. (*a*) The angular magnification obtained depends on f but not on u.
5. (*b*) When final image is formed at $D = 25$ cm from eye.

In this situation, $v = -D$ from lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f},$$

we have, $$\frac{1}{-D} - \frac{1}{-u} = \frac{1}{f}$$

i.e., $$\frac{D}{u} = 1 + \frac{D}{f}$$

So, Magnifying power $= \frac{D}{u} = \left(1 + \frac{D}{f}\right)$

6. (*c*) In compound microscope, the intermediate image is real, inverted and magnified.

7. (*b*) Magnifying power, $m = m_0 \times m_e$

$$100 = 5 \times m_e$$
$$m_o = 20$$

8. (*a*) Length of compound microscope,

$$L_\infty = v_o + f_e \Rightarrow 14 = v_o + 5$$

$\therefore$ $$v_o = 9 \text{ cm}$$

Magnifying power of microscope for relaxed eye

$$M = \frac{v_o}{u_o} \cdot \frac{D}{f_e} \text{ or } 25 = \frac{9}{u_o} \cdot \frac{25}{5}$$

or distant of object, $u_o = \frac{9}{5} = 1.8$ cm

9. (*b*) When final image is formed at infinity,then magnifying power

$$M_\infty = -\frac{v_o}{u_o} \times \frac{D}{f_e}$$

From, $$\frac{1}{f_o} = \frac{1}{v_o} - \frac{1}{u_o}$$

$$\frac{1}{+1.2} = \frac{1}{v_o} - \frac{1}{(-1.25)}$$

$\Rightarrow$ $$v_o = 30 \text{ cm}$$

$\therefore$ $$|M_\infty| = \frac{30}{1.25} \times \frac{25}{3} = 200$$

10. (*b*) For objective lens,

$$\frac{1}{f_o} = \frac{1}{v_o} - \frac{1}{u_o}$$

or $$\frac{1}{4} = \frac{1}{v_o} - \frac{1}{(-4.5)} \Rightarrow |v_o| = 36 \text{ cm}$$

$\therefore$ Magnifying power, $|M| = \frac{v_o}{u_o}\left(1 + \frac{D}{f_e}\right) = \frac{36}{4.5}\left(1 + \frac{24}{8}\right) = 32$

11. (*a*) If the telescope is reversed *i.e.*, seen from the objective side, then object will appear very small, because in this case magnification becomes reciprocal of initial magnification.

12. (*a*) In a telescope large aperture of objective helps in improving the brightness (image) by gathering more light from distant object.

13. (*a*) Magnification of astronomical telescope for normal eye is,

$$m = -\frac{f_o}{f_e} = -\frac{100}{2} = -50$$

14. (*b*) Magnifying power of astronomical telescope,

$$m = -\frac{f_o}{f_e}\left(1+\frac{f_e}{D}\right) = -\frac{200}{5}\left(1+\frac{5}{25}\right) = -48$$

15. (*b*) In terrestrial telescope, there are three lenses, objective, eye-piece and third is erecting lens, it is placed at $2f$ from intermediate image.

16. (*a*) Magnifying power of a Gallilean telescope is given by,

$$m = \frac{f_o}{f_e}\left(1-\frac{f_e}{D}\right)$$

17. (*b*) If in an astronomical telescope, the convergent eye-piece is replaced by a divergent lens which is placed in such a way that rays from objective are directed towards its focus, final image will be erect, enlarged and virtual, and hence, it is called Gallilean telescope.

18. (*b*) Reflecting telescope consists of concave mirror of large aperture.

19. (*c*) Resolving power of microscope is given by, $\frac{2\mu \sin\theta}{\lambda}$.

20. (*b*) Resolving power of telescope $= \frac{d}{1.22\lambda} = \frac{1.22}{1.22\times 500\times 10^{-10}}$

$= 2\times 10^6$

Check point 9.7

1. (*b*) The defect in the myopia eye is removed by the concave lens.

2. (*b*) Focal length, $f = -d = -100$ cm $= -1$m

$\therefore \quad P = \frac{1}{f} = \frac{1}{-1} = -1\text{D}$

3. (*a*) In case of hypermetropia, eye can see distant objects clearly but nearer objects are not clearly visible.

4. (*b*) A person suffering from the defect astigmatism cannot see object in two perpendicular directions simultaneously.

5. (*c*) Astigmatism can be removed by using cylindrical lens.

6. (*d*) Presbyopia can be removed by using bifocal lens.

8. (*c*) Coordinate of achromatic combination is satisfied, if

$$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$$

9. (*d*) Rainbow is caused due to refraction, reflection and dispersion.

10. (*c*) The phenomena based upon the scattering are
(i) blue colour of the sky
(ii) appearence of reddish sun during sunnset and sunrise.

A Taking it together

1. (*b*) When, white light is passed through a prism violet colour shows maximum deviation.

2. (*a*) In shortsightedness, the focal length of eye lens decreases and so the power of eyelens increases.

4. (*b*) The phenomenon involved in the reflection of radiowaves by ionosphere is similar to total internal reflection of light in air during a mirage *i.e.*, angle of incidence is greater than critical angle.

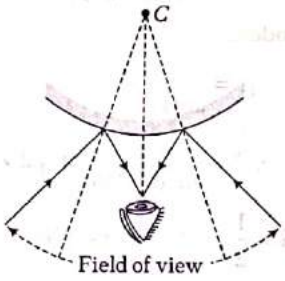
5. (*b*) A passenger in an aeroplane may see a primary and a secondary rainbow like concentric circles.

6. (*a*) In myopia, the focal length or radii of curvature of lens reduced or power of lens increases or distance between eye lens and retina increases.

8. (*c*) From a rarer to denser medium, a phase difference of π occurs.

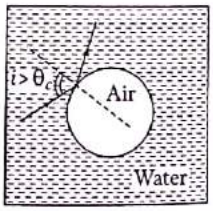
9. (*b*) Erect diminised image is formed by convex mirror.

10. (*c*) In convex mirror, the field of view is increased as compared to plane mirror. This is why it is used as rear view mirror in vehicles.



12. (*a*) The negative refractive index materials are those in which incident ray from air (medium 1) to them refract or bend differently to that of positive refractive index medium.

14. (*d*) An air bubble in water shines due to total internal reflection at its outer surface. Here, light is propagating from denser medium (water) to rarer medium (air) and if $i > \theta_C$ total internal reflection takes place at the surface of the bubble.



15. (*a*) Sky appears blue due to scattering. In absence of atmosphere no scattering will occur. Therefore sky will be seen black.

16. (*d*) Since $v \propto \lambda$, the light of red colour is of highest wavelength and therefore of highest speed. So, after travelling through the slab, the red colour emerge first.

21. (*b,d*) With increase in wavelength, refractive index gets decreased. Therefore, focal length of lens will be increased.

22. (*c*) Sun is visible a little before the actual sunrise and until a little after the actual sunset. This is due to refraction.

23. (*a*) In optical fibres, the refractive index of the core is greater than that of the cladding.

24. (*c*) When an object approaches a convergent lens from the left of the lens with a uniform speed of 5 m/s, then the image moves away from the lens with a non-uniform acceleration.

25. (*c*) All rays coming from object at centre are normal. So, no bending will take place. So, the distance of the virtual image from the surface of sphere is 6cm.

26. (*d*) Magnification, $m = \pm 3$

We have, $m = \dfrac{f}{f+u}$

For vertical image, $3 = \dfrac{f}{f+8}$ ...(i)

For real image, $-3 = \dfrac{f}{f-16}$ ...(ii)

Solving Eqs. (i) and (ii) we get

$$f = 12 \text{ cm}$$

27. (*d*) If there had been one eye of the man, then visible region decreases, so the depth of image will not be seen.

28. (*b*) After completely immersed in water, this bag will behave as convergent lens.

29. (*d*) A normal eye is not able to see object closer than 25 cm because the eye is not able to decrease the focal length beyend a limit.

30. (*c*) Real depth = 1 m

Apparent depth = 1 − 0.1 = 0.9 m

Refractive index, $\mu = \dfrac{\text{Real depth}}{\text{Apparent depth}} = \dfrac{1}{0.9} = \dfrac{10}{9}$

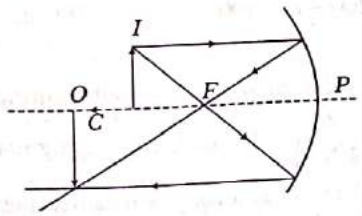
31. (*d*) Under minimum deviation, $i = e = 30°$, so angle between emergent ray and second refracting surface is, $90° - 30° = 60°$.

32. (*c*) For $A > 2C$, the ray of light does not emerge from the opposite fall of the prism.

33. (*c*) According to VIBGYOR, among all given sources of light, the blue light have smallest wavelength. According to Cauchy relationship, smaller the wavelength higher the refractive index and consequently smaller the critical angle. So, corresponding to blue colour, the critical angle is least which facilitates total internal, reflection for the beam of blue light. The beam of green light would also undergo total internal reflection.

34. (*a*) In any medium other than air or vacuum, the velocities of different colours are different.Therefore, both red and green colours are refracted at different angles of refractions. Hence, after emerging from glass slab through opposite parallel face, they appear at two different points and move in two different parallel directions.

35. (*a*) When object is placed between centre of curvature and focus (*i.e.*, $f < u < 2f$), then image will be between $2f$ and ∞, real, inverted, large in size and $m > -1$



36. (*c*) If refractive index of lens n_1 is equal to refractive index of liquid n_2, then lens behaves as a plane glass plate and becames invisible in the medium.

37. (*a*) The size of the object in accordance with the Fresnel's displacement law is given by $\sqrt{x_1 x_2}$.

38. (*d*) Only real image can be taken on screen. Object is real only when incident rays are diverging from it. Our eyes cannot distinguish between real and virtual image.

39. (*b*) The PQ ray of light passes through focus F and incident on the concave mirror, after reflection the ray, should become parallel to the principal axis and shown by ray-2 in the figure.

40. (*d*) Here, angle of prism, $A = 60°$

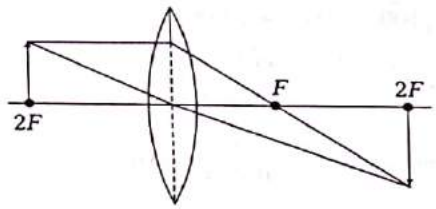
For minimum deviation, $A = 2r$

or $r = \dfrac{A}{2} = \dfrac{60°}{2} = 30°$ for both colours.

41. (*b*) Molecules of a medium after absorbing incoming light radiations, emit them in all directions. This phenomenon is called scattering. According to scientist Rayleigh, the intensity of scattered light $\propto \dfrac{1}{\lambda^4} \propto \nu^4$

where, ν is the frequency.

42. (*c*) For real image, object should be placed at distance $2f$.



43. (*b*) The resolving power of telescope $= \dfrac{a}{1.22\lambda}$

where, a = aperture of telescope

$\therefore$ Resolving power $\propto$ aperture of telescope

If aperture of telescope is decreased, then the resolving power will decrease.

44. (*c*) The magnifying power of the telescope in normal adjustment,

$$m = \frac{f_o}{f_e} = \frac{100}{5} = 20$$

45. (*d*) Dispersion of light is not associated with the total internal reflection.

46. (*b*) We have $n_a c_a = n_g c_g$

$$\frac{n_g}{n_a} = \frac{c_a}{c_g}$$

$$\frac{3}{2} = \frac{3 \times 10^8}{c_g}$$

$$c_g = 2 \times 10^8$$

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$t = \frac{4 \times 10^{-3}}{2 \times 10^8}$$

$$t = 2 \times 10^{-11} \text{ s}$$

47. (*b*) In going from one medium to another frequency remains same but wavelength and velocity decrease with the increase in refractive index *i.e.*, wavelength becomes $\lambda' = \lambda / \mu$ and velocity becomes,

$$v = \frac{c}{\mu}$$

48. (*a*) For combination of one convex and one concave lens

$$\frac{1}{F} = \frac{1}{f_1} - \frac{1}{f_2} \Rightarrow F = \frac{f_1 f_2}{f_2 - f_1}$$

If $f_1 > f_2$, then F will be negative.

49. (*d*) Given, wavelength in vacuum $= 5890$ Å

In a medium of refractive index, the wavelength of light decreases to $\frac{\lambda}{\mu} = \frac{5890}{1.5} = 3927$ Å

50. (*d*) We have,

$$C = \sin^{-1}\left(\frac{v_1}{v_2}\right) = \sin^{-1}\left(\frac{1.8\times10^8}{2.4\times10^8}\right) = \sin^{-1}\left(\frac{3}{4}\right)$$

51. (*a*) We have, $f_y = \sqrt{f_R f_b}$

$$f_y = \sqrt{100.5 \times 99.5} = 99.99$$

Also, $\omega = \frac{f_R - f_b}{f_y} = \frac{100.5 - 99.5}{99.99}$

Dispersive power, $\omega = \frac{1}{99.99} = 0.01$

52. (*b*) The number of images formed depends upon the angle between the mirrors. If two mirrors make an angle θ with each other, then the number of images formed is

$$n = \frac{360°}{\theta} - 1$$

When mirrors are kept mutually perpendicular to each other, then $\theta = 90°$.

$$n = \frac{360}{90} - 1 = 3$$

53. (*b*) Number of images, $n = \frac{360°}{\theta} - 1$

$$3 = \frac{360°}{\theta} - 1$$

$$\Rightarrow \quad \theta = 90°$$

54. (*c*) As, $m = \frac{360}{60°} = 6$

Since, m is an even integer, total number of images will be $(m - 1)$ or 5.

55. (*c*) The magnifying power (m) of a telescope, $m = -\frac{f_o}{f_e}$. Now, if the focal length of the eye-piece (f_e) is doubled, then the new magnification would be equal to $m' = -\frac{f_o}{2f_e} = \frac{m}{2}$.

56. (*a*) The minimum angular separation between two objects, so that they are just resolved is called resolving limit. For eye it is, $1 = \left[\frac{1}{60}\right]^\circ$. Reciprocal of resolving limit is called resolving power (RP),

$$= \frac{1}{\left(\frac{1}{60}\right)^\circ} = \frac{1}{\frac{1}{60} \times \frac{\pi}{180}}$$

$$= \frac{10800}{\pi} = 3.436 \times 10^3 \text{ rad}$$

57. (*b*) Resolving power of microscope $= \frac{2\mu \sin\theta}{\lambda}$

Since, $\lambda_R > \lambda_B$ hence, resolving power increases.

58. (*a*) Resolving limit of telescope

$$= \frac{1.22\lambda}{a} = \frac{1.22 \times (5000 \times 10^{-10})}{0.1} = 6.1 \times 10^{-6} \text{ rad}$$

59. (*b*) In an astronomical telescope,

$$L = f_o + f_e$$

When tube length is decreased, then the image formed by the objective lens will lie between principal focus and optical centre of eye lens instead of lying at the focus of eye lens. Therefore, a virtual image will be formed at a finite distance from the eye lens.

60. (*d*) In this case, $|m| = \frac{f_o}{f_e} = 5$ ...(i)

and length of telescope $= f_o + f_e = 36$ ...(ii)

On solving Eqs. (i) and (ii), we get

$$f_e = 6 \text{ cm}, f_o = 30 \text{ cm}$$

61. (*b*) Magnification, $M = \frac{f_o}{f_e} = 10$ or $f_o = 10 f_e$

Given,

$$f_e + f_o = 1.1 \text{ m}$$

$$f_e + 10 f_e = 1.1 \times 100 \text{ cm}$$

$$11 f_e = 110$$

$$f_e = 10$$

Magnification least distance of distinct vision

$$M_D = \frac{f_o}{f_e}\left(1 + \frac{f_e}{D}\right) = 10\left(1 + \frac{10}{25}\right) = 10\left(\frac{35}{25}\right) = 14$$

62. (*d*) Given, $m = +\frac{1}{2}$, $f = 2$ m $= 200$ cm, $u = ?$

$\therefore$ Magnification, $m = \frac{f}{(f - u)}$

$$\Rightarrow \quad +\frac{1}{2} = \frac{200}{(200 - u)}$$

$$\Rightarrow \quad 200 - u = 400 \Rightarrow u = -200 \text{ cm}$$

$$\Rightarrow \quad u = -2 \text{ m}$$

63. (*c*) Here, $R = 20$ cm, $\mu = 1.5$, on substituting the values in $f = \frac{R}{\mu - 1} = \frac{20}{1.5 - 1} = 40$ cm of converging nature as $f > 0$.

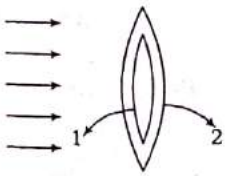
Therefore, lens act as a convex lens irrespective of the side on which the object lies.

64. (*c*) We have, $\frac{1}{\sin C} = \frac{\lambda_1}{\lambda_2}$

$\Rightarrow \frac{1}{\sin C} = \frac{6000}{3000}$

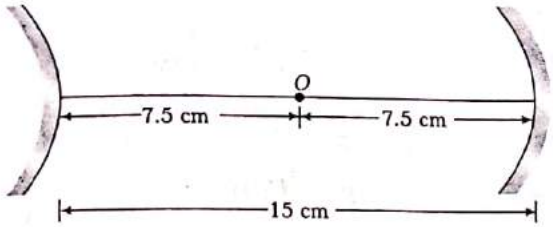
$\Rightarrow \frac{1}{\sin C} = \frac{6}{3} = 2$

$\Rightarrow \sin C = \frac{1}{2}$



$\Rightarrow$ Critical angle between two medium, $C = \sin^{-1}\left(\frac{1}{2}\right)$

65. (*c*) For concave mirror,



Using the relation,

$$\frac{1}{v} - \frac{1}{7.5} = -\frac{1}{5}$$

$\therefore \quad v = -15$ cm

Therefore position of final Image will be at the pole of the convex minor.

66. (*d*) Length of the telescope when final image is formed at least distance of distinct vision is,

$$L = f_o + u_e = f_o + \frac{f_e D}{f_e + D} = 50 + \frac{5 \times 25}{(5+25)} = \frac{325}{6}\text{cm}$$

67. (*c*) We have, $\frac{I}{O} = \frac{f}{(f-u)}$

$\Rightarrow \frac{I}{5} = \frac{-10}{-10-(-100)}$

Size of the image, $I = 0.55$ cm

68. (*a*) We have, the effective power given by,

$$P_{\text{eff}} = P_1 + P_2$$

Also, we know $P = \frac{1}{f}$

Hence, $P_{\text{eff}} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{0.20} + \frac{1}{0.25} = 5 + 4 = 9\text{D}$

69. (*c*) Hypermetropia can be removed by using a convex lens.

Focal length of used lens, $f = +d = +40$ cm

$= +$(defected near point)

$\therefore$ Power of lens $= \frac{100}{f\,(\text{cm})} = \frac{100}{40} = 2.5$ D .

70. (*b*) Given, $A = 60°$ and $\delta_m = 30°$

$\therefore$ Refractive index of prism, $\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\frac{A}{2}} = \frac{\sin 45°}{\sin 30°} = \sqrt{2}$

71. (*a*) Refractive index of the liquid, $\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)} = \sqrt{2}$

72. (*c*) As $_1\mu_2 = \frac{1}{\sin C}$

$\Rightarrow \frac{\mu_2}{\mu_1} = \frac{\lambda_1}{\lambda_2} = \frac{1}{\sin C}$

$\Rightarrow \frac{6000}{4000} = \frac{1}{\sin C}$

$\Rightarrow$ Critical angle, $C = \sin^{-1}\left(\frac{2}{3}\right)$

73. (*c*) We have, $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$$\frac{1}{30} = (\mu - 1)\left(\frac{1}{10}\right)$$

The refractive index of the material of the lens

$$\mu = \frac{4}{3} = 1.33$$

74. (*c*) Refractive index of the material of prism,

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$\Rightarrow \sqrt{3} = \frac{\sin\left(\frac{60° + \delta_m}{2}\right)}{\sin\left(\frac{60°}{2}\right)}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \sin\left(30° + \frac{\delta_m}{2}\right)$$

$\therefore$ Minimum deviation of the prism $\delta_m = 60°$

75. (*a*) As, $A + \delta = i + e$ ...(i)

$\Rightarrow 30° + 30° = 60° + e$

Angle of emergence, $e = 60° - 60° = 0$

$r_1 = A = 30°$

$\therefore$ Refractive index, $\mu = \frac{\sin i}{\sin r} = \frac{\sin 60°}{\sin 30°} = \sqrt{3} = 1.732$

76. (*b*) Since, rays after passing through the glass slab just suffer lateral displacement. So, angle between emergent ray is also α.

77. (*b*) We have, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\because \quad \frac{1}{30} - \frac{1}{(-u)} = \frac{1}{f}$ or $\frac{1}{30} + \frac{1}{u} = \frac{1}{f}$ ...(i)

Similarly, $\frac{1}{120} + \frac{1}{u-90} = \frac{1}{f}$ ...(ii)

On solving Eqs. (i) and (ii), we get

The focal length of the lens, $f = 24$ cm

78. (*c*) Given, $f_o + f_e = 105$

$$f_o + \frac{f_e}{20} = 105$$

$$f_o = 100 \text{ cm}$$

$$f_o + f_e = 105 \text{ cm} \quad \text{and} \quad \frac{f_o}{f_e} = 20 \text{ cm}$$

79. (*b*) We have, $\frac{1}{f} = (1.5 - 1)\left(\frac{1}{10} + \frac{1}{10}\right)$

$\therefore$ Focal length, $f = 10$ cm

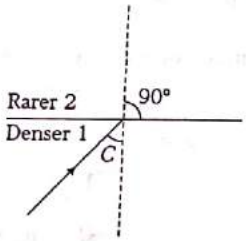
Radius of curvature, $R = 2f = 20$ cm

80. (*d*) Resultant combination of focal length,

$$F_{\text{comb}} = \frac{f_1 f_2}{f_1 + f_2} = \frac{f(-f)}{f + (-f)} = \infty$$

81. (*a*) When a ray of light passes from a denser medium to a rarer medium, then it bends away from the normal at the interface of the two media.

The angle of incidence is measured with respect to the normal at the refractive boundary. It is given by



$$C = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

where, C is critical angle, n_2 is the refractive index of rarer medium and n_1 of the denser medium.

Given, $n_2 = 1.33$, $n_1 = 1.50$

$$C = \sin^{-1}\left(\frac{1.33}{1.50}\right) \Rightarrow C = \sin^{-1}\left(\frac{8}{9}\right)$$

82. (*d*) The focal length of the lens,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{20} - \frac{1}{30}\right) \Rightarrow \frac{1}{f} = 0.5\left(\frac{30 - 20}{600}\right)$$

or $$\frac{1}{f} = \frac{1}{2} \times \frac{10}{600} = \frac{1}{120}$$

or The focal length, $f = 120$ cm

83. (*a*) Power of a biconvex lens is 10 D.

Radius of curvature of each surface = 10 cm.

$$\Rightarrow \quad \frac{1}{R_f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\Rightarrow \quad 10 = (\mu - 1)\left[\frac{100}{10} - \left(-\frac{100}{10}\right)\right]$$

$$\Rightarrow \quad 10 = (\mu - 1)[20]$$

$$\Rightarrow \quad (\mu - 1) = \frac{1}{2}$$

$\Rightarrow$ Refractive index of the material, $\mu = \frac{3}{2}$

84. (*b*) $\therefore$ Power of the lens $= \frac{1}{\text{focal length}}$

Focal length of combination of convex and concave lenses is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

where, f_1 and f_2 be the focal lengths of convex and concave lenses, respectively.

Now, $$\frac{1}{F} = \frac{1}{0.4} + \frac{1}{(-0.25)}$$

$$\frac{1}{F} = -1.5$$

Power, $P = -1.5$ D $\quad \left(\because P = \frac{1}{F}\right)$

85. (*d*) Focal length of the combination,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\Rightarrow \quad \frac{1}{80} = \frac{1}{20} + \frac{1}{f_2}$$

$$\therefore \quad f_2 = -\frac{80}{3} \text{ cm}$$

$\therefore$ Power of second lens,

$$P' = \frac{100}{f_2} = \frac{100}{(-80/3)} = -3.75 \text{ D}$$

86. (*c*) The maximum deviation can be, when it is reflected back at

$$i = \theta_C = 45°$$

$\therefore$ Deviation, $\delta = 180° - 2i = 90°$

87. (*a*) When plano-convex lens is silvered at curved surface, then it becomes concave mirror of focal length,

$$f = \frac{R}{2\mu} = \frac{30}{2 \times 1.5} = 10 \text{ cm}$$

If we have to find real image of same size of the object, then object must be placed at centre of curvature $u = (2f) = 20$cm.

88. (*c*) From the formula, $\sin C = \frac{1}{{}_1\mu_2}$

$$\Rightarrow \quad \sin C = {}_2\mu_2$$

$$\therefore \quad \frac{\mu_1}{\mu_2} = \frac{v_2}{v_1}$$

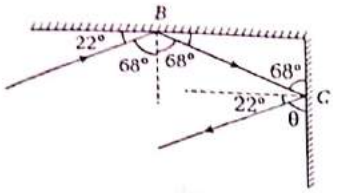
$$\Rightarrow \quad \sin C = \frac{10x/t_1}{x/t_1}$$

$$\sin C = \frac{10t_1}{t_2}$$

$$C = \sin^{-1}\left(\frac{10t_1}{t_2}\right)$$

89. (*b*) From geometry of the figure,

Angle, $\theta = 90 - 22° = 68°$



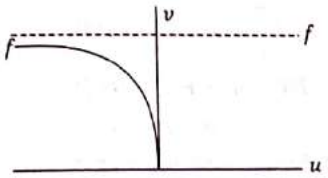
90. (*c*) Magnifying power,

$$m_\infty = \frac{(L_\infty - f_o - f_e)\cdot D}{f_o f_e}$$

$$\Rightarrow \quad 45 = \frac{(L_\infty - 1 - 5)\times 25}{1\times 5}$$

$\Rightarrow$ The length of the tube, $L_\infty = 15$ cm

91. (*a*) u changes from 0 to $-\infty$. Then, v will change from 0 to $+f$.

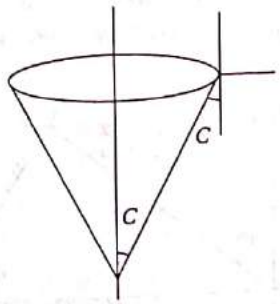


92. (*a*) Power, $P = 2P_L + P_M$

or $\quad -\frac{1}{F} = 2\left(-\frac{1}{0.2}\right) + 0 \quad$ or $\quad F = 10$ cm

On silvering the surface of the lens, it behaves as a mirror. Therefore, the combination behaves as a convex mirror of focal length 10 cm.

93. (*c*) We can represent this diagrammatically as,



Thus, we see that $C = \sin^{-1}\left(\frac{1}{\mu}\right) = \sin^{-1}\left(\frac{3}{4}\right)$

94. (*b*) Here, light ray goes from (optically) rarer medium air to optically denser turpentine, then it bends towards the normal *i.e.*, $i > r$ whereas when it goes from to optically denser medium turpentine to rarer medium water, then it bends away the normal *i.e.*, $i < r$.

95. (*c*) Here, $\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

$$\frac{1}{f} = (1.6 - 1)\left[\frac{1}{60} - \frac{1}{\infty}\right] = \frac{1}{100}$$

$\Rightarrow \quad f = 100$ cm

96. (*d*) The focal length of the plano-convex lens is 20 cm with the plane surface being silvered.

So, the net power of the lens is given by

$$P = \frac{1}{f}\times 2 = \frac{2}{f}$$

Thus, the net focal length is, $\frac{f}{2} = \frac{20}{2} = 10$ cm

97. (*a*) Since, deviation, $\delta = (\mu - 1)A = (1.5 - 1)\times 5° = 2.5°$

By geometry angle of refraction by first surface is 5°.

But $\delta = \theta - r$, so we have $2.5° = \theta - 5°$

On solving, $\theta = 7.5°$

98. (*d*) Given, $u = mf$

$$\therefore \quad \frac{1}{v} - \frac{1}{(-mf)} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{f}\left(1 + \frac{1}{m}\right)$$

$$\Rightarrow \quad -\frac{1}{v} = \frac{m}{m+1} \Rightarrow -\frac{v}{u} = \frac{1}{1+m}$$

99. (*b*) The refractive index is 1.5 and the radius of curvature of the curved face is 100 cm = 1m and of the plane side is ∞, hence

$$P = \frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Power of the lens, $P = (1.5 - 1)\left(\frac{1}{\infty} - \frac{1}{1}\right) = -0.5$ D

100. (*c*) According to lens maker's formula,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R} - \frac{1}{(-R)}\right)$$

$$\Rightarrow \quad \frac{1}{f} = (\mu - 1)\left(\frac{2}{R}\right) \quad \text{...(i)}$$

On cutting, $\quad \frac{1}{f'} = (\mu - 1)\frac{1}{R} - \frac{1}{\infty}$

$$\frac{1}{f'} = (\mu - 1)\left(\frac{1}{R}\right) \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get $f' = 2f$

101. (*a*) $\because$ Refractive index of water with respect to air, ${}_an_w = \frac{4}{3}$

and Refractive index of glass with respect to water, ${}_wn_g = \left(\frac{10}{8}\right)$

$\therefore$ Refractive index of glass with respect to air.

$${}_an_g = {}_an_w \times {}_wn_g = \frac{4}{3}\times\frac{10}{8} = \frac{5}{3}$$

102. (*c*) Given, $n = 1.5$, and $h_o = 3$ cm

Refractive index, $n = \frac{h_o}{h_i} \Rightarrow 1.5 = \frac{3}{h_i}$

Apparent depth, $h_i = \frac{3}{1.5} = 2$ cm

Hence, $3 - 2 = 1$cm upwards.

103. (*b*) As optical paths are equal, hence

$$n_g\, x_g = n_w\, x_w$$

$$n_w = n_g \frac{x_g}{x_w}$$

Refractive index of the water, $n_w = 1.53 \times \frac{4.0}{4.5} = 1.36$

104. (*b*) For total internal reflection,

$$i > C$$

$$\Rightarrow \quad \sin i > \sin C \Rightarrow \sin 45° > \frac{1}{\mu}$$

$$\Rightarrow \quad \mu > \sqrt{2} \Rightarrow \mu > 1.414$$

105. (*b*) From Snell's law,

$$\mu \sin i = \text{constant}$$

$$\mu_4 \sin x = \mu \sin i$$

$$\therefore \quad \sin x = \frac{\mu}{\mu_4} \sin i$$

106. (*b*) As incident ray and reflected ray are parallel to each other it means, deviation,

$$\delta = 180°$$

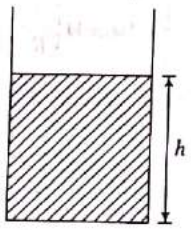
$$\because \quad \delta = 360° - 2\theta$$

$$\therefore \quad 180° = 360° - 2\theta$$

Angle between two plane mirrors

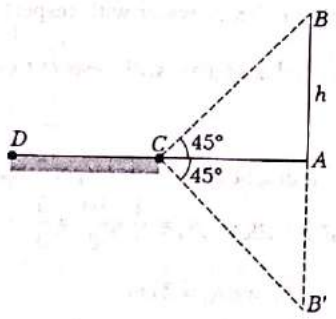
$$\Rightarrow \quad \theta = 90°$$

107. (*d*) According to the question, $\frac{h}{\mu} = \frac{21}{2} = 10.5$



$$\therefore \quad \text{Height, } h = 10.5 \times \frac{4}{3} \Rightarrow h = 14 \text{ cm}$$

108. (*b*) Given, $AC = 60$ m and $\angle BCB' = 90°$



From the figure,

$\therefore$ Angle, $\angle BCA = \frac{\angle BCB'}{2} = 45°$

We have, $\tan 45° = \frac{h}{AC}$

$\Rightarrow AC = h$ and $AB = AC = 60$ m

109. (*d*) At second surface, there is no refraction, so

$$r_2 = 0$$

$$\therefore \quad r_1 = A = 30°$$

From Snell's law,

$$n = \frac{\sin i_1}{\sin r_1}$$

$$\Rightarrow \quad n = \frac{\sin i_1}{\sin 30°}$$

$$\Rightarrow \quad \sin i_1 = \sqrt{2} \times \frac{1}{2} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \sin i_1 = \frac{1}{\sqrt{2}} \Rightarrow i_1 = 45°$$

110. (*c*) Given, $P_1 + P_2 = 9$ ...(i)

We have,

$$P = P_1 + P_2 - dP_1P_2$$

$$\Rightarrow \quad \frac{27}{5} = 9 - \frac{20}{100} \times P_1P_2$$

Again $(P_2 - P_1)^2 = (P_1 + P_2)^2 - 4P_1P_2$

$$\Rightarrow \quad P_2 - P_1 = 3 \quad \text{...(ii)}$$

On solving Eqs. (i) and (ii), we get

$$P_1 = 3 \text{ and } P_2 = 6$$

111. (*c*) For equal fogging of print,

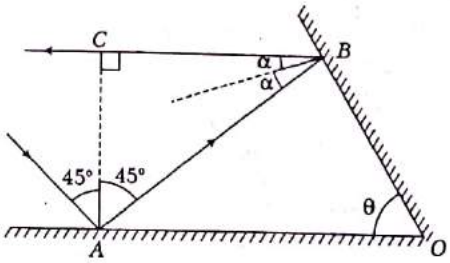
$$I_2 \times t_2 = I_1 \times t_1$$

or $$\frac{L_2}{r_2^2} \times t_2 = \frac{L_1}{r_1^2} \times t_1$$

or $$\frac{16}{4} \times t_2 = \frac{20}{1} \times 10$$

$$\therefore \quad t_2 = 50 \text{ s}$$

112. (*c*) From the figure in ΔABC, $45° + 90° + 2\alpha = 180°$



$$\Rightarrow \quad \text{or } \alpha = 22.5°$$

$$\angle OBA = 90° - \alpha$$

In ΔOAB, $\theta + 45° + 90° - \alpha = 180°$

$$\Rightarrow \quad \theta = 67.5°$$

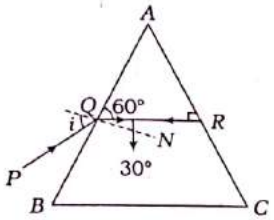
113. (*a*) Power of system $= \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$

$$= \frac{1}{1} + \left(\frac{1}{-0.25}\right) - \frac{0.75}{(1)(-0.25)}$$

$$= 1 - 4 + 3 = -3 + 3 = 0$$

Since, power of the system is zero, therefore the incident parallel beam of light will remain parallel after emerging from the system.

114. (*a*) The ray retraces its path after reflection from the silvered face *AC*. Therefore, $\angle ARQ = 90°$. As is clear from the figure, the angle of reflection *RQN* at face *AB* is 30°.



115. (*d*) The speed of the image of the car would appear to increases as the distance between the cars decreases.

116. (*c*) For dispersion without deviation,

$$\frac{A}{A'} = \frac{(\mu' - 1)}{(\mu - 1)}$$

$$\frac{6°}{A'} = \frac{0.75}{0.50}$$

$$A' = \frac{0.50 \times 6°}{0.75}$$

$$A' = 4°$$

117. (*a*) Here, $n = 1.5$, as per sign convention followed

$$R_1 = +20 \text{ cm and } R_2 = -20 \text{ cm}$$

$$\therefore \quad \frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= (1.5-1)\left[\frac{1}{(+20)} - \frac{1}{(-20)}\right] = 0.5 \times \frac{2}{20} = \frac{1}{20}$$

$$\Rightarrow \quad f = +20 \text{ cm}$$

Incident rays travelling parallel to the axis of lens will converge at its second principal focus. Hence,

$$L = +20 \text{ cm}$$

118. (*d*) By Rayleigh's law of scattering,

$$I \propto \frac{1}{\lambda^4} \quad \text{or} \quad \frac{I_2}{I_1} = \left(\frac{\lambda_1}{\lambda_2}\right)^4$$

$$\frac{I_2}{A} = \left(\frac{440}{660}\right)^4 \quad \text{or} \quad I_2 \cong \frac{A}{6}$$

119. (*c*) Light should fall normally on the silvered face *i.e.*,

$$r_2 = 0$$

$$\therefore \quad r_1 = A = 30°$$

Now, Refractive index, $\mu = \frac{\sin i_1}{\sin r_1}$

or $\sqrt{2} = \frac{\sin i_1}{\sin 30°}$

Angle of incidence, $i_1 = 45°$

120. (*d*) Here, $i_1 = 0 = r_1$

Therefore, $r_2 = A = 60°$

$$\theta_C = \sin^{-1}\left(\frac{1}{1.5}\right) = 41.8°$$

$$\Rightarrow \quad r_2 > \theta_C$$

121. (*c*) For objective,

$$\frac{1}{v_o} - \frac{1}{-200} = \frac{1}{50}$$

$$\therefore \quad v_o = \frac{200}{3} \text{ cm}$$

For eye-piece,

$$\frac{1}{-25} - \frac{1}{(-u_e)} = \frac{1}{5}$$

$$\therefore \quad u_e = \frac{25}{6}$$

Therefore, separation between objective and eye-piece,

$$L = v_o + u_e = \frac{200}{3} + \frac{25}{6} = \frac{425}{6} = 71 \text{cm}$$

122. (*d*) Critical angle, $\theta_C = \sin^{-1}\left(\frac{1}{1.5}\right) = 41.8°$,

Incident angle, $i_1 = 30°$

$$\therefore \quad \sin r_1 = \frac{\sin i_1}{\mu} = \frac{\sin(30°)}{1.5}$$

$$r_1 = 19.5°$$

or Angle of emergence, $r_2 = A - r_1 = 90 - 19.5 = 70.5°$

Since, $r_2 > \theta_C$

$\therefore$ The ray will not emerge from the prism.

123. (*c*) We have, $\delta = i_1 + i_2 - A$

$$45° = (i_1 + i_2) - 60°$$

$$\therefore \quad i_1 + i_2 = 105° \quad \text{...(i)}$$

$$\text{and} \quad i_1 - i_2 = 20° \quad \text{...(ii)}$$

On solving Eqs. (i) and (ii), we get

$$i_1 = 62.5° \text{ and } i_2 = 42.5°$$

124. (*d*) For terrestrial telescope magnification,

$$M = \frac{f_o}{f_e} = \frac{80}{f_e} = 20$$

$$\Rightarrow \quad f_e = 4 \text{ cm}$$

Hence, length of terrestrial telescope

$$= f_o + f_e + 4f$$

$$= 80 + 4 + 4 \times 20$$

$$= 164 \text{ cm}$$

125. (*d*) We assume that final image is formed at infinity, and so

$$M_\infty = \frac{(L_\infty - f_o - f_e)D}{f_o f_e} = \frac{LD}{f_o f_e}$$

$\therefore$ Magnifying power of the microscope, $M_\infty = \frac{10 \times 25}{0.5 \times 1} = 500$

126. (*c*) We assume that final image is formed at infinity and

$$M_\infty = \frac{(L_\infty - f_o - f_e) \cdot D}{f_o f_e} = \frac{LD}{f_o f_e}$$

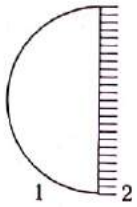
$$\text{or} \quad 400 = \frac{20 \times 25}{0.5 \times f_e}$$

$$\therefore \quad f_e = 2.5 \text{ cm}$$

127. (*c*) We have, $\frac{1}{f} = \frac{2\mu}{R_2} - \frac{2(\mu - 1)}{R_1}$

$$\frac{1}{-10} = \frac{2 \times 1.5}{\infty} - \frac{2(1.5 - 1)}{R}$$

$\therefore$ $R = 10$ cm



128. (*a*) Given, $r_2 = 0$, $r_1 = A = 30°$, and $i_1 = 45°$

$\therefore$ Refractive index, $\mu = \frac{\sin i_1}{\sin r_1} = \frac{\sin 45°}{\sin 30°} = \frac{1/\sqrt{2}}{1/2} = \sqrt{2}$

129. (*c*) The separation between the images formed by extreme wavelengths of visible range is called the longitudinal chromatic given by,

$$f_1 - f_2 = \omega \times f$$

where, ω is dispersive power

Given, $\omega = 0.02$, $f = 20$ cm

$\because$ $f_1 - f_2 = 0.02 \times 20 = 0.40$

130. (*a*) Here, $x = u + v$ ...(i)

Magnification, $m = \frac{f}{f-u} = \frac{f-v}{f}$

Image is real, magnification is negative

$$-m = \frac{f}{f-u} \Rightarrow u = \frac{(m+1)}{m} f$$

From, $-m = \frac{f-v}{f}$

$\Rightarrow$ $v = (m+1)f$

Putting the value of v in Eq. (i), we get

$$x = \frac{(m+1)}{m} f + (m+1)f \Rightarrow f = \frac{mx}{(m+1)^2}$$

131. (*a*) Linear magnification,

$$m = \frac{I}{O} = \frac{f}{u-f} = \frac{10}{25-10}$$

$$m = \frac{10}{15} = \frac{2}{3}$$

Areal magnification $= m^2 = \frac{4}{9}$

Area of image $= \frac{4}{9} \times$ area of object

$= \frac{4}{9} \times (3)^2 = 4$ cm^2

132. (*d*) As, $n = \frac{\text{Real depth}}{\text{Apparent depth}}$

$\Rightarrow$ $n = \frac{6}{4} = \frac{3}{2}$

Also, $\frac{n_1}{u} - \frac{n_2}{v} = \frac{n_1 - n_2}{R} = \frac{1.5}{6} - \frac{4}{17} = \frac{1.5 - 1}{R}$

Radius of curvature, $R = 34$ cm

133. (*a*) We have, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\Rightarrow$ $\frac{1}{v} - \frac{1}{20} = \frac{1}{-15}$

$v = -60$ m

Image speed $= \left(\frac{v^2}{u^2}\right)$ (object speed)

$= \left(\frac{60}{20}\right)^2 (5) = 45$ m/s

134. (*a*) From condition of achromatism,

$$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0 \Rightarrow \frac{\omega_1}{\omega_2} = -\frac{f_1}{f_2} \Rightarrow \frac{5}{3} = \frac{-(-15)}{f_2}$$

Focal length, $f_2 = 9$ cm (convex)

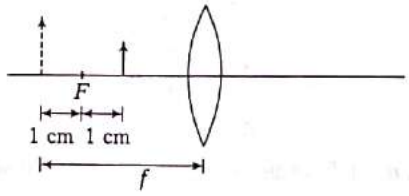
135. (*b*) Angle i at both focus will be 45°. For TIR to take place,

$$i > \theta_C \quad \text{or} \quad \sin i > \sin \theta_C$$

$\therefore$ $\frac{1}{\sqrt{2}} > \frac{1}{\mu}$ or $\mu > \sqrt{2}$

136. (*b*) As, per question,

$$\because \frac{1}{-(f+1)} - \frac{1}{-(f-1)} = \frac{1}{f}$$



On solving, we get $f = (\sqrt{2} + 1)$ cm

137. (*c*) We have, $\frac{\mu_2}{v} = \frac{\mu_1}{u} - \frac{\mu_2 - \mu_1}{R}$

$$\frac{1}{v} = \frac{1.5}{-6} - \frac{1 - 1.5}{-10}$$

$\therefore$ $v = -5$ cm

138. (*c*) Shift from the slab,

$$S = \left(1 - \frac{1}{\mu}\right) t = 2 \text{ cm}$$

Let x be the distance between object and mirror. Then,

$x - 2 = 40$ or $x = 42$ cm

139. (*d*) Lens should be convex. Images should be magnified by focal length 16 cm length should lie between f and $2f$. Because corresponding to 6 cm ($< f$) length image should be magnified.

From the given options, if $f = 11$cm.

Then, 6 cm, $< f$ and 11 cm < 16 cm < 22 cm or $f < 16$ cm $< 2f$

140. (*b*) Using mirror formula,

$$\frac{1}{v} + \frac{1}{(-u)} = \frac{1}{-f}$$

Multiplying with u,

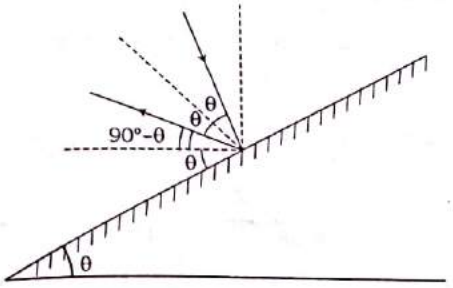
$$\frac{u}{v} = \frac{-u}{f} + 1 = \frac{f-u}{f}$$

$\therefore$ Magnification, $m = \frac{v}{u} = \left(\frac{f}{f-u}\right)$

Now, $|dv| = m^2 |du|$

Size of image $= \left(\frac{f}{f-u}\right)^2 \cdot b$

141. (*d*) We have, $90° - \theta + 2\theta = 90°$



$\therefore \quad \theta = 0°$

Angle of reflected ray with horizontal $= 90° - \theta = 90°$

142. (*d*) When total internal reflection just takes place from lateral surface, $i = C$ *i.e.*, $60° = C$

$$\Rightarrow \quad \sin 60° = \sin C = \frac{1}{\mu} \Rightarrow \mu = \frac{2}{\sqrt{3}}$$

Time taken by light to traverse some distance in a medium,

$$t = \frac{\mu x}{c} = \frac{\frac{2}{\sqrt{3}} \times 10^3}{3 \times 10^8} = 3.85 \mu s$$

143. (*c*) When the lens in air, we have

$$P_a = \frac{1}{f_a} = \frac{\mu_g - \mu_a}{\mu_a}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

When the lens is in liquid, we have

$$P_l = \frac{1}{f_l} = \frac{\mu_g - \mu_l}{\mu_l}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

Given, $\quad P_a = 5, P_l = -1, \mu_a = 1$ and $\mu_g = 1.5$

On solving, we get $\mu_l = \frac{5}{3}$

144. (*b*) We have, $\frac{\mu_2}{v_1} - \frac{\mu_1}{\infty} = \frac{\mu_2 - \mu_1}{R}$ ...(i)

and $\frac{\mu_3}{f} - \frac{\mu_2}{v_1} = \frac{\mu_3 - \mu_2}{-R}$ ...(ii)

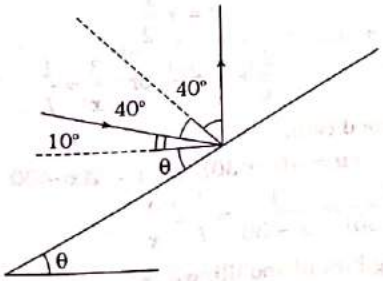
From Eqs. (i) and (ii), we get

$$\frac{\mu_3}{f} = \frac{\mu_2 - \mu_1 - \mu_3 + \mu_2}{R} = \frac{2\mu_2 - (\mu_1 + \mu_3)}{R}$$

Lens will become diverging, if f is negative

i.e., $2\mu_2 < (\mu_1 + \mu_3)$

145. (*a*) If $\theta = 40°$, then ray of light will become vertical after reflection.



146. (*a*) Angle of the prism,

$$A = \frac{\phi}{2}$$

Now, $\quad \phi = \frac{\phi_1 + \phi_2}{2}$

where, $\quad \phi_1 = x_2 - x_1$

$= (360° + 80°38') - (320° 40')$

$= 119° 58'$

and $\quad \phi_2 = y_2 - y_1$

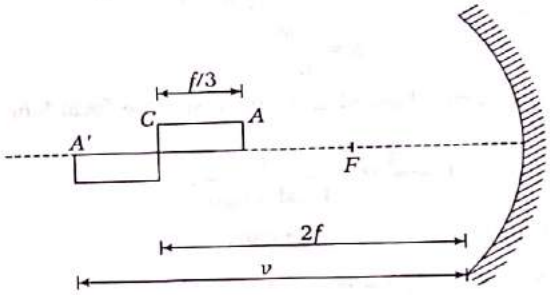
$= 260° 24' - 140° 30' = 119° 54'$

$$\phi = \frac{119° 58' + 119° 54'}{2} = \frac{239° 52'}{2}$$

The angle to prism, $\quad A = \frac{\phi}{2} = \frac{239° 52'}{4} = 59° 58'$

147. (*b*) For end A of the rod,

$$u = 2f - \frac{f}{3} = \frac{5f}{3}$$

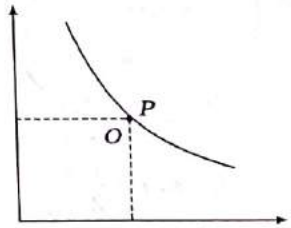


By using, $\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \quad \Rightarrow \quad \frac{1}{-f} = \frac{1}{v} + \frac{1}{(-5f/3)}$

$\therefore \quad v = -\frac{5}{2} f$

So, length of image $= \frac{5}{2} f - 2f = \frac{f}{2}$

148. (*c*) At P, $u = v$ which is possible only when $u = 2f$



If we take another point Q just above P, then $v > 2f$.

149. (*a*) For real image,

$$u = -x$$
$$v = +mx$$

$\therefore \quad \frac{1}{mx} - \frac{1}{-x} = \frac{1}{f}$ ...(i)

For virtual image,

$$u = -y$$

Then, $\quad v = -my$

$$\therefore \quad \frac{1}{-my} - \frac{1}{(-y)} = \frac{1}{f} \quad \text{...(ii)}$$

On solving Eqs. (i) and (ii), we get

$$f = \frac{x+y}{2}$$

150. (*a*) Biconvex lens is cut perpendicularly to the principal axis, it will become a plano-convex lens.

Focal length of biconvex lens,

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = (n-1)\frac{2}{R} \quad (\because R_1 = R, R_2 = -R)$$

$$\Rightarrow \quad f = \frac{R}{2}(n-1) \quad \text{...(i)}$$

For plano-convex lens,

$$\frac{1}{f_1} = (n-1)\left(\frac{1}{R} - \frac{1}{\infty}\right)$$

$$f_1 = \frac{R}{(n-1)} \quad \text{...(ii)}$$

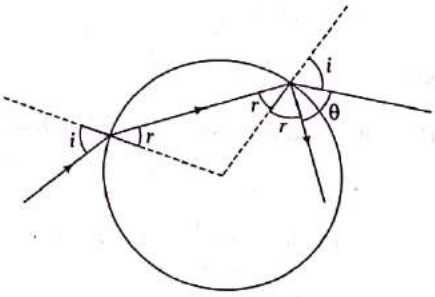
On comparing Eqs. (i) and (ii),we see the focal length become double.

As power of lens, $P \propto \frac{1}{\text{focal length}}$.

Hence, power will become half.

New power $= \frac{4}{2} = 2$ D

151. (*c*) We have, refractive index, $\mu = \frac{\sin i}{\sin r} \Rightarrow \sqrt{3} = \frac{\sin 60°}{\sin r}$



$$\sin r = \frac{1}{2}$$

Refracted angle, $r = 30°$

The angle between the reflected and refracted rays $\theta = 180° - i - r = 90°$

152. (*a*) From, $\frac{1}{v} - \frac{1}{-u} = \frac{1}{+f}$

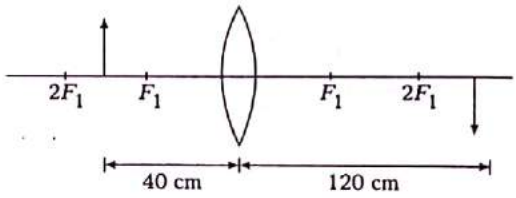
$$v = \frac{uf}{u-f} \text{ or } \frac{v}{u} = m = \frac{f}{u-f}$$

$$\Rightarrow \quad \frac{1}{m} = \left(\frac{1}{f}\right)u - 1$$

Comparing this with equation of straight line, $1/f$ is the slope of line, which is b/c.

153. (*a*) We have, $\frac{1}{3x} - \frac{1}{(-x)} = \frac{1}{+30}$

$$\therefore \quad x = 40 \text{ cm and } 3x = 120 \text{ cm}$$



To decrease the magnification object should be moved towards $2F_1$.

Hence, image will move towards $2F_2$. Let displacement is y. Then,

$$2(40+6) = (120 - y) \quad \left(\because m = \frac{v}{u}\right)$$

$$\therefore \quad y = 28 \text{ cm}$$

154. (*c*) Due to glass slab, increase in path

$$= (\mu - 1)t = (1.5 - 1)3 \text{ cm}$$
$$= 1.5 \text{ cm}$$

$$\therefore \quad u' = -(21 + 1.5) \text{ cm}$$

or $\quad u' = -22.5$ cm

Now, $\quad \frac{1}{-22.5} + \frac{1}{v} = \frac{1}{-5}$

or $\quad \frac{1}{v} = \frac{1}{22.5} - \frac{1}{5}$ or $\frac{1}{v} = \frac{5 - 22.5}{22.5 \times 5}$

or $\quad v = \frac{22.5 \times 5}{17.5}$ or $v = -6.43$ cm

The position of the final image $= -(6.43 - 1.5)$ cm $= -4.93$ cm

155. (*d*) For a thin prism, angle of minimum deviation is given by, $\delta = (\mu - 1)A$

where, μ is refractive index of the prism and A is the angle of prism

For dispersion without deviation,

$$\delta_1 = \delta_2 \Rightarrow (\mu_1 - 1)A_1 = (\mu_2 - 1)A_2$$

$$\Rightarrow \quad A_2 = \frac{(\mu_1 - 1)}{(\mu_2 - 1)}A_1$$

Given, $\mu_1 = 1.54, A_1 = 4°, \mu_2 = 1.72$

$$\Rightarrow \quad A_2 = \frac{(1.54 - 1)}{(1.72 - 1)} \times 4 = 3°$$

156. (*c*) In the first case, let

$$u = -x$$

Then, $\quad v = +\frac{x}{2}$

$$\therefore \quad \frac{2}{x} + \frac{1}{x} = \frac{1}{f} \text{ or } \frac{3}{x} = \frac{1}{f} \quad \text{...(i)}$$

In the second case,

$$u = -(x - 30) \Rightarrow v = 2(x - 30)$$

$$\therefore \quad \frac{1}{2(x-30)} + \frac{1}{x-30} = \frac{1}{f} = \frac{3}{x} \quad \text{...(ii)}$$

On solving Eqs. (i) and (ii), we get

$$f = 20 \text{ cm}$$

157. (*a*) From the relation,

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \quad \text{or} \quad \frac{1}{v}-\frac{1}{u}=\frac{1}{-f}$$

$$\frac{1}{v}=\frac{1}{u}-\frac{1}{f} \quad \text{or} \quad v=\left(\frac{uf}{f-u}\right)$$

Since, $u > f$, v is negative

or $$|v|=\left(\frac{uf}{u-f}\right)>f$$

The end which is at infinity will have its image at focus.

$\therefore$ Length of image,

$$L=|v|-f=\frac{f^2}{u-f}$$

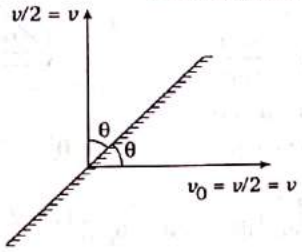
158. (*c*) The reflected ray will rotate by angle 2θ. For TIR to take place at water-air boundary,

$$\sin 2\theta > \sin\theta_C \quad \text{or} \quad \sin 2\theta > \frac{1}{\mu}$$

$$\therefore \quad \theta > \frac{1}{2}\sin^{-1}\left(\frac{3}{4}\right)$$

159. (*a*) Velocity of image, $v_r=\sqrt{v^2+v^2-2v\cdot v\cdot\cos\theta}$

$$=2v\sin(\theta/2)$$



160. (*b*) We know that,

$$\frac{1}{\mu}=\frac{\sin i}{\sin r'}$$

But $$r+r'=90° \quad \text{or} \quad r'=90°-r$$

$$\sin r'=\sin(90°-r')$$

$$=\cos r$$

or $$\sin r'=\cos i$$

$\therefore$ $$\frac{1}{\mu}=\frac{\sin i}{\cos i} \quad \text{or} \quad \frac{1}{\mu}=\tan i$$

or $$\sin i_C=\tan i=\tan r$$

The critical angle for pair of media is,

$$i_C=\sin^{-1}(\tan r)$$

161. (*a*) For a given prism, the angle of deviation depends upon the angle of incidence of the light rays falling on the prism. Taking triangle FQR, we have

$$\delta=\angle FQR+\angle FRQ$$

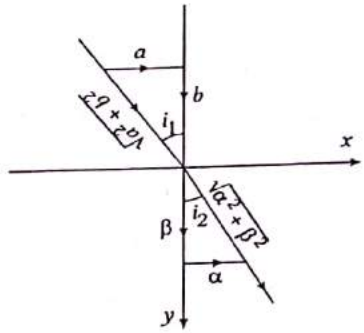
Since, ΔAQR is an equilateral triangle, therefore,

$$\angle FQR=\frac{60°}{2}=30°=\angle FRQ$$

$\therefore$ $$\delta=30°+30°=60°$$

Hence, angle of deviation of the ray is 60°.

162. (a) From the Snell's law, $\mu_1\sin i_1=\mu_2\sin i_2$



$\therefore$ $$\frac{\mu_1 a}{\sqrt{a^2+b^2}}=\frac{\mu_2\alpha}{\sqrt{\alpha^2+\beta^2}}\left(\because \sin\theta=\frac{\text{Height}}{\text{Hypotenuse}}\right)$$

$$\left(\because r_A=a\hat{\mathbf{i}}+b\hat{\mathbf{j}} \text{ and } r_B=\alpha\hat{\mathbf{i}}+\beta\hat{\mathbf{j}} \text{ are unit vectors}\right)$$

$$\sqrt{a^2+b^2}=\sqrt{\alpha^2+\beta^2}=1$$

$\therefore$ $$\mu_1 a=\mu_2\alpha$$

163. (*a*) Focal length in air is given by,

$$\frac{1}{f_a}=({}_a\mu_g-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

The focal length of lens immersed in water is given by,

$$\frac{1}{f_l}=({}_l\mu_g-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

where, R_1, R_2 are radii of curvatures of the two surfaces of lens and ${}_l\mu_g$ is refractive index of glass with respect to liquid.

Also, $${}_l\mu_g=\frac{{}_a\mu_g}{{}_a\mu_l}$$

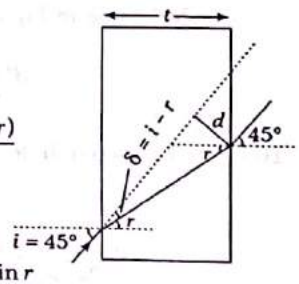
Given, ${}_an_g=1.5$, $f_a=12$ cm and ${}_a\mu_l=4/3$

$\therefore$ $$\frac{f_l}{f_a}=\frac{({}_a\mu_g-1)}{({}_l\mu_g-1)}$$

$$\frac{f_l}{12}=\frac{(1.5-1)}{\left(\frac{1.5}{4/3}-1\right)}=\frac{0.5\times 4}{0.5}$$

$\Rightarrow$ $$f_l=4\times 12=48 \text{ cm}$$

164. (*b*) Here, angle of incidence $i=45°$



$$\frac{\text{Lateral shift } (d)}{\text{Thickness of glass slab } (t)}=\frac{1}{\sqrt{3}}$$

Lateral shift, $$d=\frac{t\sin\delta}{\cos r}=\frac{t\sin(i-r)}{\cos r}$$

$\Rightarrow$ $$\frac{d}{t}=\frac{\sin(i-r)}{\cos r}$$

or $$\frac{d}{t}=\frac{\sin i\cos r-\cos i\sin r}{\cos r}$$

or $$\frac{d}{t}=\frac{\sin 45°\cos r-\cos 45°\sin r}{\cos r}$$

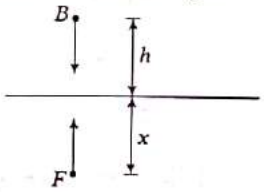
$$=\frac{\cos r-\sin r}{\sqrt{2}\cos r}$$

or $\frac{d}{t} = \frac{1}{\sqrt{2}}(1-\tan r)$ or $\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{2}}(1-\tan r)$

or $\tan r = 1 - \frac{\sqrt{2}}{\sqrt{3}}$

or Angle of refraction, $r = \tan^{-1}\left(1-\frac{\sqrt{2}}{\sqrt{3}}\right)$

165. (*b*) Fish is observer and bird is object.



Apparent distance between F and B at some instant will be,

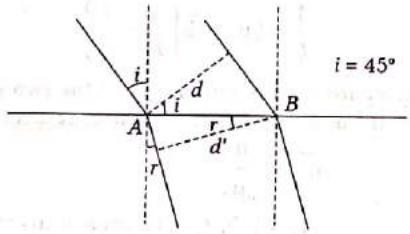
$$y = (x + \mu h)$$

$$\left(-\frac{dy}{dt}\right) = \left(-\frac{dx}{dt}\right) + (\mu)\left(-\frac{dh}{dt}\right)$$

$$9 = 3 + \frac{4}{3}\left(-\frac{dh}{dt}\right)$$

$\therefore$ $-\frac{dh}{dt} = 4.5$ m/s

166. (*c*) From the figure, $AB = \frac{d}{\cos i} = \frac{d'}{\cos r}$



$\therefore$ $d' = \frac{\cos r}{\cos i} d = \sqrt{2}\cos r . d$ ...(i)

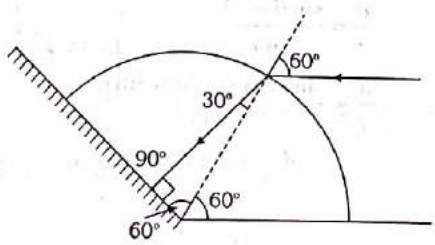
Further, $\sin r = \frac{\sin i}{\mu} = \frac{1}{\sqrt{2}\mu}$

Hence, $\cos r = \sqrt{1-\frac{1}{2\mu^2}} = \frac{\sqrt{2\mu^2-1}}{\sqrt{2}\mu}$

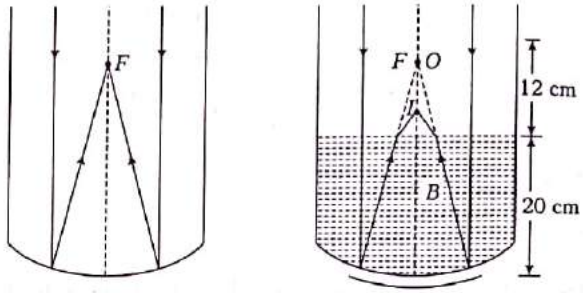
Substituting in Eq. (i), we get

$$d' = \frac{\sqrt{2\mu^2-1}}{\mu} \cdot d$$

167. (*d*) Refractive index, $\mu = \frac{\sin 60°}{\sin 30°} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$



168. (*b*) In first case, when sun is at infinity *i.e.*,



$\therefore$ $u = \infty$

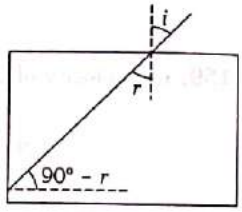
$\Rightarrow$ $\frac{1}{f} = \frac{1}{-32} + \frac{1}{(-\infty)}$

$\therefore$ $f = -32$ cm

When water is filled in that tank upto a height of 20 cm, the image formed by the mirror will act as virtual object O for water surface.

$\therefore$ $BI = BO \times \frac{3}{4} = 12 \times \frac{3}{4} = 9$ cm

169. (*a*) TIR to take place,



$90° - r > \theta_C$

$\Rightarrow$ $r < 90° - \theta_C$

From the relation,

$$\mu = \frac{\sin i}{\sin r}$$

$$\sin i = \mu \sin r$$

Since, for TIR to take place, $r < 90° - \theta_C$

$\therefore$ $i = \sin^{-1}\{\mu \sin r\}$

$i \le \sin^{-1}[\{n \sin(90° - \theta_C)\}]$

or $i \le \sin^{-1}\{\mu \cos\theta_C\}$

or $i \le \sin^{-1}\left\{\mu\sqrt{1-\frac{1}{\mu^2}}\right\}$ $\left(\because \sin\theta_C = \frac{1}{\mu_C}\right)$

or $i \le \sin^{-1}\left\{1.33\sqrt{1-\frac{1}{(1.33)^2}}\right\}$

or $i \le \sin^{-1}\{\sqrt{0.77}\}$

Note For simplyfying the calculation, take $1.33 = \frac{4}{3}$

170. (*b*) As, $y_a = \left(\frac{n_1}{n_2}\right) y$

Here, y = actual depth

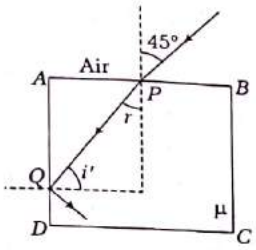
and y_a = apparent depth

$\therefore$ $y = \left(\frac{n_2}{n_1}\right) \cdot y_a \Rightarrow \left(-\frac{dy}{dt}\right) = \frac{n_2}{n_1}\left(-\frac{dy_a}{dt}\right) = \frac{n_2 x}{n_1}$

The amount of water drained in cc/min

$\therefore$ $\frac{dv}{dt} = A \cdot \left(-\frac{dy}{dt}\right) = \frac{\pi R^2 n_2 x}{n_1}$

171. (*b*) At point P,



Refractive index, $\mu = \frac{\sin 45°}{\sin r} \Rightarrow \sin r = \frac{1}{\mu\sqrt{2}}$ ...(i)

At point Q, for total internal reflection,

$$\sin i' = \frac{1}{\mu}$$

From figure, $i' = 90° - r$

$\therefore \sin(90° - r) = \frac{1}{\mu} \Rightarrow \cos r = \frac{1}{\mu}$ ...(ii)

Now, $\cos r = \sqrt{1 - \sin^2 r} = \sqrt{\left(1 - \frac{1}{2\mu^2}\right)} = \sqrt{\left(\frac{2\mu^2 - 1}{2\mu^2}\right)}$ ...(iii)

From Eqs. (ii) and (iii), we get

$$\frac{1}{\mu} = \sqrt{\frac{2\mu^2 - 1}{2\mu^2}}$$

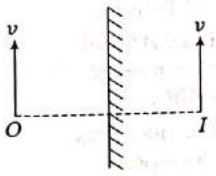
$\Rightarrow$ Refractive index, $\mu = \sqrt{\frac{3}{2}}$

For total internal reflection at vertical face AD.

B) Medical entrance special format questions

Assertion and reason

2. (*d*) In the shown figure, object and image are travelling in same direction.



3. (*d*) For virtual object (u = positive),

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\therefore \quad \frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

If $u < f$, then $v = -$ve *i.e.*, image may be real.

4. (*b*) After refraction at two parallel faces of a glass slab, a ray of light emerges in a direction parallel to the direction of incidence of white light on the slab. As rays of all colours emerge in the same direction (of incidence of white light), hence, there is no dispersion, but only lateral displacement.

5. (*a*) In total internal reflection, 100% of incident light is reflected back into the same medium, and there is no loss of intensity, while in reflection from mirrors and refraction from lenses, there is always same loss of intensity. Therefore, images formed by total internal reflection are much brighter than those formed by mirrors or lenses.

11. (*d*) For total internal reflection the angle of incidence should be greater than the critical angle. As critical angle is 35°. Therefore, total internal reflection is not possible.

12. (*c*) In rarer medium, ray of light moves away from the normal. Further,

$$\mu_1 \sin 30° = \mu_2 \sin 60°$$

$$\therefore \quad \frac{\mu_1}{\mu_2} = {}_2\mu_1 = \frac{\sin 60°}{\sin 30°}$$

13. (*a*) $A_t = \left(\frac{2v_2}{v_1 + v_2}\right) A_i$

If $v_2 > v_1$, then $A_t > A_i$

14. (*c*) Glass slab will shift the object towards mirror. If it comes between pole and focus the image will become virtual.

15. (*d*) Nature of lens will change only, if refractive index of liquid is more than that of material of lens.

16. (*a*) When a diverging lens is immersed in a liquid, then its diverging power will either decrease or it will become converging.

17. (*a*) $\therefore P = \frac{1}{f} = (\mu - 1)\left(\frac{1}{R} - \frac{1}{R}\right) = 0$

18. (*c*) If λ is more, then μ will be less as per Cauchy's formula $\left(\mu = A + \frac{B}{\lambda^2}\right)$. Therefore, focal length will be more.

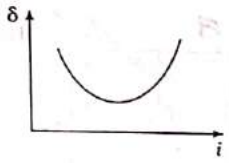
19. (*a*) As, $\frac{1}{f_1} = (\mu - 1)\left(\frac{1}{R} - \frac{1}{R}\right) = 0$

$$\therefore \quad P_1 = 0$$

$$\frac{1}{f_2} = (\mu - 1)\left(\frac{1}{-R} - \frac{1}{(-R)}\right) = 0$$

$$\therefore \quad P_2 = 0$$

20. (*b*) For minimum deviation,



and $r_1 = r_2$

$\because A = r_1 + r_2$

$\therefore r_1 = r_2 = \frac{A}{2}$

Match the columns

2. Convex mirror and concave lens always make virtual and diminished images. In case of concave mirror, object is placed between F and C. In case of convex lens object is placed between F and $2F$. In both the cases image is real and magnified.

5. $\therefore \frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$ or $P = P_1 + P_2$

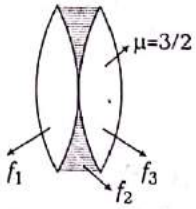
f_1 and P_1 are positive, f_2 and P_2 are negative.

If $|f_1| > |f_2|$, power of system is negative

If $|f_1| < |f_2|$, power of system is positive.

C) Medical entrances' gallery

1. *(d)* Consider the situation shown in figure. Let radius of curvature of lens surfaces is R. The combination is equivalent to three lenses in contact.



$$\therefore \quad \frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} = \frac{2}{f_1} + \frac{1}{f_2} \qquad (\because f_1 = f_3)$$

Now, $\frac{1}{f_1} = \frac{1}{f_3} = (\mu - 1)\left(\frac{2}{R}\right) = \frac{1}{f}$

$$\frac{1}{f_2} = (\mu_w - 1)\left(-\frac{2}{R}\right) = \left(\frac{4}{3} - 1\right)\left(\frac{-2}{R}\right)$$

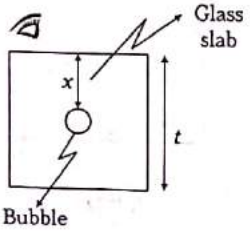
$$= \left(\frac{1}{3}\right)\left(-\frac{2}{R}\right) = \left(\frac{-2}{3}\right)\left(\frac{1}{2(\mu - 1)}\right)\frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{f_2} = \left(-\frac{1}{3}\right)\left(\frac{1}{\frac{3}{2} - 1}\right)\left(\frac{1}{f}\right) = -\frac{2}{3} \times \frac{1}{f}$$

$$\therefore \quad \frac{1}{f_{eq}} = \frac{2}{f} - \frac{2}{3} \times \frac{1}{f} = \frac{6-2}{3f} = \frac{4}{3f}$$

$\Rightarrow$ The focal length of the combination, $f_{eq} = \frac{3f}{4}$

2. *(c)* Let thickness of the given slab is t. According to the question, when viewed from both the surfaces



$$\Rightarrow \quad \frac{x}{\mu} + \frac{t-x}{\mu} = 3 + 5 \Rightarrow \frac{t}{\mu} = 8 \text{ cm}$$

$\Rightarrow$ Thickness of the slab, $t = 8 \times \mu = 8 \times \frac{3}{2} = 12$ cm

3. *(b)* Image distance, $v = 400$ cm $= 4$ m $\Rightarrow u = \infty$

Using lens equation,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{-4} - \frac{1}{\infty} = \frac{1}{f} \Rightarrow f = -4 \text{ m}$$

Now, power of the required lens is,

$$P = \frac{1}{f} = \frac{1}{-4} = -0.25 \text{ D}$$

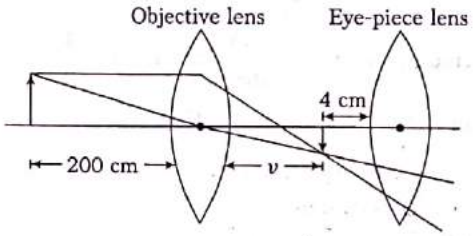
Thus, the person require a concave lens of power -0.25 D.

4. *(c)* According to question,

Focal length of objective lens, $f_o = +40$ cm

Focal length of eye-piece lens, $f_e = 4$ cm

Object distance for objective lens, $u_o = -200$ cm



Applying lens formula for objective lens,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \frac{1}{(-200)} = \frac{1}{40}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{40} - \frac{1}{200} = \frac{5-1}{200} = \frac{4}{200}$$

$$\Rightarrow \quad v = 50 \text{ cm}$$

Image will be form at first focus of eye-piece lens.

So, for normal adjustment distance between objectives and eye-piece lens (length of tube) will be,

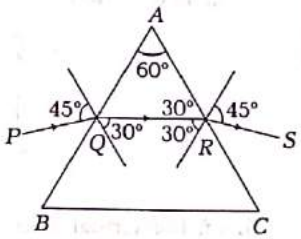
$$v + f_e \Rightarrow 50 + 4 \Rightarrow 54 \text{ cm}$$

5. *(d)* A concave mirror forms real and virtual images, whose magnification can be negative or positive depending upon the position of the object.

If object is placed between focus and pole, then the image obtained will be virtual and its magnification will be positive. In all other cases, concave mirror forms real images whose magnification will be negative.

A convex mirror always forms a virtual image whose magnification will always be positive.

6. *(a)* Consider a ray of light PQ incident at the surface AB and moves along RS, after passing through the prism ABC.



It is given that the incident ray suffers minimum deviation. Therefore, the ray inside the prism must be parallel to the base BC of the prism.

From the geometry of the prism and the ray diagram, then it is clear that,

angle of incidence, $i = 45°$

angle of refraction, $r = r' = 30°$

angle of emergence, $e = 45°$

A = angle of prism $= 60°$

Therefore, minimum deviation suffered by the ray is

$$\delta_{min} = i + e - (r + r') = 90° - 60° = 30°$$

Refractive index of the material,

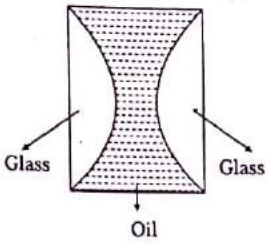
$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\frac{A}{2}}$$

$$\therefore \quad \mu = \frac{\sin\left(\frac{60°+30°}{2}\right)}{\sin\frac{60°}{2}} = \frac{\sin 45°}{\sin 30°} = \frac{1/\sqrt{2}}{1/2} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

7. (*c*) Given, $\mu_g = 1.5$,

$\mu_{oil} = 1.7$

and $R = 20$ cm



From Lens maker's formula for the plano-convex lens,

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

Here, $R_1 = R$ and for plane surface, $R_2 = \infty$

$$\therefore \quad \frac{1}{f_{lens}} = (1.5-1)\left(\frac{1}{R} - 0\right) \Rightarrow \frac{1}{f_{lens}} = \frac{0.5}{R}$$

When the intervening medium is filled with oil, then focal length of the concave lens formed by the oil

$$\frac{1}{f_{concave}} = (1.7-1)\left(-\frac{1}{R} - \frac{1}{R}\right) = -0.7 \times \frac{2}{R} = \frac{-1.4}{R}$$

Here, we have two concave surfaces.

So, $$\frac{1}{f_{eq}} = 2 \times \frac{1}{f_l} + \frac{1}{f_c} = 2 \times \frac{0.5}{R} + \left(\frac{-1.4}{R}\right) = \frac{1}{R} - \frac{1.4}{R} = -\frac{0.4}{R}$$

Focal length of the combination,

$$\therefore \quad f_{eq} = -\frac{R}{0.4} = -\frac{20}{0.4} = -50 \text{ cm}$$

8. (*b*) Refractive index, $\mu = \dfrac{\sin\left(\frac{A+D_m}{2}\right)}{\sin\frac{A}{2}}$

$$\Rightarrow \quad \cot\frac{A}{2} = \frac{\sin\left(\frac{A+D_m}{2}\right)}{\sin\frac{A}{2}}$$

$$\Rightarrow \quad \frac{\cos\frac{A}{2}}{\sin\frac{A}{2}} = \frac{\sin\left(\frac{A+D_m}{2}\right)}{\sin\frac{A}{2}}$$

$$\sin\left(\frac{\pi}{2} - \frac{A}{2}\right) = \sin\left(\frac{A+D_m}{2}\right)$$

$$\Rightarrow \quad \frac{\pi}{2} - \frac{A}{2} = \frac{A}{2} + \frac{D_m}{2}$$

$$\Rightarrow \quad D_m = \pi - 2A$$

Angle of minimum deviation, $D_m = 180° - 2A$

9. (*c*) If the object is placed at a distance 25 cm from the corrected lens, then it should produce the virtual image at 40 cm.

Thus, $u = -25$ cm and $v = -40$ cm

The lens formula is,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \Rightarrow \frac{1}{f} = \frac{-1}{40} + \frac{1}{25}$$

$$\Rightarrow \quad f = \frac{200}{3} \text{ cm} = \frac{2}{3} \text{ m}$$

$\therefore$ Power of the lens, $P = \dfrac{1}{f} = \dfrac{1}{2/3} = \dfrac{3}{2} = +1.5$ D

10. (*b*) Given, focal length of simple microscope = 12 cm

Near point, $D = 25$ cm

From lens formula, $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f} \Rightarrow \dfrac{1}{25} - \dfrac{1}{u} = \dfrac{1}{(-12)}$

$$\Rightarrow \quad \frac{1}{u} = \frac{1}{12} + \frac{1}{25}$$

$$\Rightarrow \quad \frac{1}{u} = \frac{25+12}{300} = \frac{37}{300}$$

$$\therefore \quad u = \frac{300}{37}$$

Now, magnification, $m = \dfrac{v}{u} = \dfrac{25}{\frac{300}{37}} = \dfrac{25 \times 37}{300} \Rightarrow m = 3.08$

11. (*d*) Given, $\mu = 1.5$

As refractive index of a prism, $\mu = \dfrac{\sin\frac{A+\delta}{2}}{\sin\frac{A}{2}}$

where, δ = angle of minimum deviation

and A = refracting angle of prism

As per question, $\delta = A$, then

$$\mu = \frac{\sin\frac{A+A}{2}}{\sin A/2} = \frac{\sin A}{\sin A/2} = \frac{2\sin A/2 \cdot \cos A/2}{\sin A/2} = 2\cos\frac{A}{2}$$

$$\Rightarrow \quad \frac{1.5}{2} = \cos A/2 = \sin(90° - A/2)$$

$$\Rightarrow \quad 90° - A/2 = \sin^{-1}(0.75) = 48°36'$$

$$\Rightarrow \quad 90° - 48°36' = \frac{A}{2} \Rightarrow 41°24' = A/2$$

$$\therefore \quad A = 82°48'$$

12. (*d*) As the law of refraction, we have

$$\frac{\sin i_C}{\sin 90°} = \frac{1.5}{1.6}$$

$$\therefore \quad \sin i_C = \frac{15}{16}$$

Where, i_C = critical angle of medium, $A = \sin^{-1}\left(\dfrac{15}{16}\right)$.

13. (*a*) Let wavelengths of red and violet lights are λ_r and λ_v respectively.

As we know that,

$$\lambda_r > \lambda_v \Rightarrow \mu_v > \mu_r \qquad \left(\because \mu \propto \frac{1}{\lambda}\right)$$

$$\Rightarrow \quad f_r > f_v \qquad \left(\because f \propto \frac{1}{\mu}\right)$$

14. (*a*) Smallest angle that can be resolved by telescope is given by,

$$d\theta = \frac{1.22\lambda}{D}$$

Smallest angle that can be resolved by human eye is given by,

$$d\theta' = \frac{1.22\lambda}{D_e}$$

where, D_e is the aperture of human eye, therefore the telescope must have a minimum magnifying power which must be such that, if an object subtend an angle $d\theta$ at its objective, then the final image produced must subtend an angle $d\theta'$ at the human eye.

$\therefore$ Minimum magnifying power is given by

$$M = \frac{d\theta'}{d\theta} = \frac{D}{D_e}$$

$$\Rightarrow \quad M = \frac{100 \text{ cm}}{0.2 \text{ cm}} = 500$$

[$\because D = 100$ cm and $D_e = 0.2$ cm (given)]

15. (*b*) From the lens formula (for first lens),

$$\frac{1}{f_1} = \frac{1}{v_1} - \frac{1}{u_1} \Rightarrow \frac{1}{2} = \frac{1}{v_1} - \frac{1}{(-4)}$$

$$\Rightarrow \quad \frac{1}{2} + \frac{1}{4} = \frac{1}{v_1} = \frac{3}{4} \quad \text{...(i)}$$

$$\Rightarrow \quad v_1 = \frac{4}{3}, \; u_2 = 3 - \frac{4}{3} = 5/3$$

This image will be treated as the source for second lens, then again from lens formula, we have

$$\frac{1}{f_2} = \frac{1}{v_2} - \frac{1}{u_2} \Rightarrow \frac{1}{1} = \frac{1}{v_2} + 5/3 \quad \text{(by Eq. (i))}$$

$$\Rightarrow \quad 1 - 5/3 = \frac{1}{v_2} \Rightarrow v_2 = -3/2$$

This is the final image distance from 2nd lens. So, the overall distance of image from the primary source (or object).

Then, $d = 4 + 3 - 1.5 = 5.5$

16. (*a*) Here, $\mu_V = 1.5230$ and $\mu_R = 1.5145$

Therefore, mean refractive index,

$$\mu = \frac{1.523 + 1.5145}{2} = 1.5187$$

Now, dispersive power, ω is given by

$$\omega = \frac{\mu_V - \mu_R}{(\mu - 1)} = \frac{1.5230 - 1.5145}{(1.5187 - 1)} = 0.0164$$

17. (*c*) The spliting of white light into its constituent colour is called dispersion of light. In this phenomenon, different colours contained in white emerge from the glass prism in different directions due to their different wavelengths.

18. (*d*) Given, $v = -75$ cm

$u = -25$ cm

We know that, lens formula, we have

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{f} = \frac{-1}{75} + \frac{1}{25}$$

$$\frac{1}{f} = \frac{1}{25} - \frac{1}{75}$$

$$\frac{1}{f} = \frac{3-1}{75}$$

$$\frac{1}{f} = \frac{2}{75}$$

Focal length of a glass, $f = \frac{75}{2} = 37.5$ cm

19. (*c*) Given, $m = -3$, and $R = 30$ cm

$$f = -\frac{R}{2} = -15 \text{ cm}$$

We know that, $m = -\frac{v}{u} \Rightarrow -3 = \frac{-v}{u}$

$$v = 3u$$

Also, $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

$$\frac{1}{-15} = \frac{1}{3u} + \frac{1}{u} \Rightarrow \frac{-1}{15} = \frac{1+3}{3u}$$

$$-3u = 4 \times 15$$

$$\Rightarrow \quad u = \frac{-4 \times 15}{3}$$

Initial position of a person, $u = -20$ cm

20. (*d*) Given, $f = 10$ cm, $D = 25$ cm

We know that,

$$m = 1 + \frac{D}{f} = 1 + \frac{25}{10}$$

$$m = 1 + 2.5 = 3.5$$

21. (*a*) v_{OM} = velocity of object with respect to mirror

$= -v_{IM}$

(Here, v_{IM} = velocity of image with respect to mirror)

$$\Rightarrow \quad v_O - v_M = -(v_I - v_M)$$

$$4\text{ms}^{-1} - (-5\text{ms}^{-1}) = -v_I + (-5 \text{ ms}^{-1})$$

$$v_I = -14 \text{ ms}^{-1}$$

22. (*b*) Given, refractive index of prism, $\mu = \sqrt{7/3}$

and angle of prism, $A = 60°$

Using, $$\mu = \frac{\sin\left(\frac{A + D_m}{2}\right)}{\sin\frac{A}{2}}$$

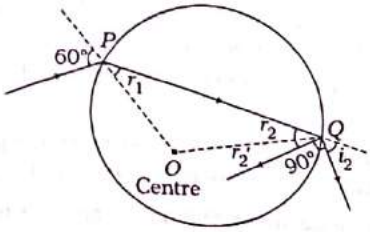
Since, angles can be assumed to be small, so that sine of the angle is angle itself.

i.e., $\sin\left(\frac{A + D_m}{2}\right) \approx \frac{A + D_m}{2}$ and $\sin\frac{A}{2} \approx \frac{A}{2} \Rightarrow \mu = \frac{\frac{A + D_m}{2}}{A/2}$

So, $$\sqrt{\frac{7}{3}} = \frac{(60 + D_m)/2}{60/2} \text{ or } 1.53 = \frac{60 + D_m}{2} \times \frac{1}{30}$$

$$\therefore \quad D_m \approx 30° = \frac{\pi}{6}$$

23. (*c*) The situation can be represented as,



For the refraction at point P,

$$\frac{\sin 60°}{\sin r_1} = \sqrt{3} \Rightarrow r_1 = 30°$$

Since, $r_2 = r_1 \Rightarrow r_2 = 30°$

For refraction at point Q, $\frac{\sin r_2}{\sin i_2} = \frac{1}{\sqrt{3}}$

When, $r_2 = 30°$, then $i_2 = 60°$

$\Rightarrow$ $r_2 = r'_2 = 30°$

$\therefore$ $\alpha = 180° - (r'_2 + i'_2) = 180° - (30° + 60°) = 90°$

24. (*c*) Atomic force microscope is used for measuring the roughness of a material with much better optical diffraction limit.

25. (*b*) Consider the diagram where two plane mirrors are inclined at an angle θ.

Number of images formed is given by,

$$n = \frac{360°}{\theta} - 1 \quad \text{(for odd number of images)}$$

$$\Rightarrow \quad 3 = \frac{360°}{\theta} - 1 \Rightarrow \frac{360°}{\theta} = 4 \Rightarrow \theta = 90°$$

26. (*a*) We know that, linear magnification, $m = \frac{f}{f-u}$

Given, object displaced, $u = -20$ cm

$m = -3$ ($\because$ all real images are inverted)

So, $-3 = \frac{f}{f-(-20)}$

$\Rightarrow$ $-3 = \frac{f}{f+20} \Rightarrow -3f - 60 = f$

$$4f = -60 \Rightarrow f = -\frac{60}{4} = -15 \text{ cm}$$

Since, mirror is concave, $f = -15$ cm

27. (*d*) By Snell's law,

$\mu \sin i = \text{constant}$

$\Rightarrow$ $\mu \sin i = (\mu - m\,\Delta\mu)\sin r$

where, $\mu = 1.5$, $i = 30°$, $r = 90°$, $\Delta\mu = 0.015$

$1.5 \sin 30° = (1.5 - m \times 0.015)\sin 90°$

$$\frac{3}{2} \times \frac{1}{2} = (1.5 - m \times 0.015) \times 1$$

$$\frac{3}{4} - \frac{3}{2} = \frac{-15m}{1000} \Rightarrow 15m = \left(\frac{3}{2} - \frac{3}{4}\right) \times 1000$$

$$15m = \frac{6-3}{4} \times 1000 \Rightarrow m = \frac{3}{4} \times \frac{1000}{15}$$

Magnification, $m = \frac{3000}{60} \Rightarrow m = 50$

28. (*c*) In vacuum, $c = d/t$

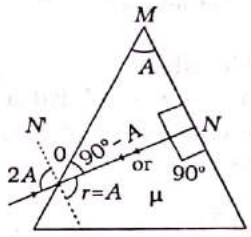
In medium, $v = \frac{5d}{T}$

As refractive index, $\mu = \frac{c}{v} = \frac{d/t}{5d/T} = \frac{T}{5t}$

Also, $\sin C = \frac{1}{\mu}$ ($\because$ C is critical angle)

$\therefore$ $C = \sin^{-1}\left[\frac{5t}{T}\right]$

29. (*b*) For the condition giving in question, ray diagram will be,



From the ray diagram,

$\angle MON = 90° - A$

So, $\angle r = 90° - (90° - A) \Rightarrow \angle r = A$

By Snell's law, $\frac{\sin i}{\sin r} = \mu \Rightarrow \frac{\sin(2A)}{\sin(A)} = \mu$

$$\frac{2\sin A \cos A}{\sin A} = \mu \Rightarrow \mu = 2\cos A$$

30. (*d*) When an equiconvex lens is cut parallel to principal axis, then focal length remains the same and when the lens is cut perpendicular to principal axis, then its focal length becomes twice the original.

31. (*e*) Given, $P_1 = 15\,\text{D}$ and $P_2 = -3\,\text{D}$

$\because$ $P = P_1 + P_2 = 15\,\text{D} - 3\text{D} = 12\,\text{D}$

$\therefore$ $P = \frac{1}{f\,(\text{m})} \Rightarrow P = \frac{100}{f\,(\text{cm})} \Rightarrow 12 = \frac{100}{f}$

Focal length of the combination, $f = \frac{100}{12} = 8.33$ cm

32. (*d*) Focal length of a convex lens by displacement method is given as, $f = \frac{a^2 - b^2}{4a}$.

where, a = distance between the image and object and
b = distance between two positions of lens

Here, $a = d$ and to form a distinct image $b = 0$, so the maximum possible focal length is $d/4$.

33. (*d*) For the first condition, $f_1 = 10$ cm, $u = -30$ cm

then $\frac{1}{f_1} = \frac{1}{v} - \frac{1}{u} \Rightarrow \frac{1}{10} = \frac{1}{v} + \frac{1}{30}$

$$\frac{1}{v} = \frac{1}{10} - \frac{1}{30} \Rightarrow v = \frac{30}{2} = 15$$

For the second condition, when concave lens is placed,

$$v' = (15 + 45) = 60 \text{ cm} \Rightarrow \frac{1}{F} = \frac{1}{v'} - \frac{1}{u}$$

(where, F = focal length of combination)

$\therefore \quad \frac{1}{F} = \frac{1}{60} + \frac{1}{30} \Rightarrow F = \frac{60}{3} = 20$ cm

The magnitude of focal length of concave lens,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} \Rightarrow \frac{1}{20} = \frac{1}{10} + \frac{1}{f_2}$$

$\Rightarrow \quad f_2 = -20$ (negative sign for concave lens)

34. (*a*) As we know that, from Lens maker's formula,

$$\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

As, $\frac{1}{f}$ = power of lens (P)

where, f = focal length

μ_2 and μ_1 = refractive index of 2nd and 1st medium

R_1 and R_2 = radius of curvature of 1st and 2nd curved surface of lens

So, for 1st case, $P_1 = \left(\frac{1.5}{1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ ...(i)

As, $\mu_2 = 1.5$, $\mu_1 = 1$ (air)

where, P_1 = power in initial condition

Now, for 2nd case, $P_2 = \left(\frac{1.5}{1.6} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ ...(ii)

where, P_2 = power in 2nd condition

On dividing Eqs. (i) and (ii), we get

$$\frac{P_1}{P_2} = \frac{\frac{1.5-1}{1}}{\frac{1.5-1.6}{1.6}} = \frac{0.5 \times 1.6}{-0.1} \Rightarrow \frac{-5}{P_2} = \frac{-0.8}{0.1} \quad (\text{as } P_1 = -5\text{D})$$

$$\Rightarrow P_2 = \frac{+5 \times 0.1}{0.8} = \frac{5}{8} = 0.625 \text{ D}$$

So, option (a) is close to 0.625 as $0.625 \approx 1$ D.

35. (*a*) Given, $f = -f \Rightarrow v = \frac{1}{3}u$ (for concave)

According to the lens formula,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} - \frac{1}{f} = \frac{1}{-(1/2)u} + \frac{1}{u} - \frac{1}{f}$$

$$= \frac{-3+1}{u} - \frac{1}{f} = -\frac{2}{u} \Rightarrow u = 2f$$

36. (*b*) Given, $f = 10$ cm and power, $P = ?$

The power of lens, $P = \frac{1}{\text{Focal length (m)}}$

$$P = \frac{1}{f\,(\text{m})} \Rightarrow P = \frac{100}{f\,(\text{cm})} = \frac{100}{10} = 10 \text{ D}$$

37. (*d*) For microscope, $m = \frac{L}{f_o}\frac{D}{f_e} \Rightarrow m \propto \frac{1}{f_o}$

For telescope, $m = \frac{f_o}{f_e}$, $m \propto f_o$

The magnifying power of microscope will decrease but the magnifying power telescope will increase.

38. (*a*) In this defect, eye cannot perpendicular directions clearly *i.e.*, horizontal and vertical lines simultaneously. It is due to imperfect spherical nature of eye lens.

This defect can be removed by using cylindrical lenses. (Torric lenses).

39. (*a*) The intermediate image formed by the objective of a compound microscope is real, inverted and magnified.

40. (*a*) In normal adjustment, the object and the final image both are at infinity and the separation between the objective and the eye-piece is $f_o + f_e$.

Thus, $f_o + f_e = 35$ cm ...(i)

Also, the magnifying power of the telescope in normal adjustment is $m = -\frac{f_o}{f_e} = -6$ (given)

$f_o = 6f_e$ ...(ii)

On solving Eqs. (i) and (ii), we get

$f_o = 30$ cm and $f_e = 5$ cm

41. (*b*) Given, length of tube, $L = 30$ cm, focal length of objective lens, $f_o = 1$cm, focal length of eye-piece, $f_e = 6$ cm, $2D = 25$ cm By compound microscope, $m = \frac{L}{f_o}\left(1 + \frac{D}{f_e}\right)$

$$m = \frac{30}{1}\left(1 + \frac{25}{6}\right) = 30 \times \frac{(6+25)}{6}$$

$$= 5 \times 31 = 155 \text{ cm} \cong 150$$

42. (*d*) Given, wavelength of light,

$$\lambda = 5000\text{Å} = 5000 \times 10^{-10} = 5 \times 10^{-7}\text{m}$$

Diameter of telescope, $D = 200$ cm $= 2$ m

Resolving power of a telescope $= \frac{1}{d\theta} = \frac{D}{1.22\lambda}$

where, $d\theta$ = limit of resolution,

λ = wavelength of light used

and D = diameter of aperture of objective

So, Resolving power $= \frac{2}{1.22 \times 5 \times 10^{-7}} = \frac{2 \times 10^7}{6.1}$

$$= 3.278 \times 10^6 = 3.28 \times 10^6$$

43. (*c*) Focal length of the combination

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \quad ...(i)$$

We have, $\frac{1}{f_1} = (\mu_1 - 1)\left(\frac{1}{\infty} - \frac{1}{-R}\right) = \frac{\mu_1 - 1}{R}$

and $\frac{1}{f_2} = (\mu_2 - 1)\left(\frac{1}{-R} - \frac{1}{\infty}\right) = -\frac{(\mu_2 - 1)}{R}$

Putting these values of $\frac{1}{f_1}$ and $\frac{1}{f_2}$ in Eq. (i)

$$\frac{1}{f} = \frac{(\mu_1 - 1)}{R} - \frac{(\mu_2 - 1)}{R} = \frac{[\mu_1 - 1 - \mu_2 - 1]}{R} = \frac{\mu_1 - \mu_2}{R}$$

$$f = \frac{R}{\mu_1 - \mu_2}$$

44. (*c*) Given, power $(P_1) = 40$ D and power $(P_2) = 20$ D

We have, $P_{eq} = P_1 + P_2 = 40 + 20 = 60$ D

So, $\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2}$

$\Rightarrow$ $f_{eq} = \frac{100}{P_{eq}} = \frac{100}{60} = 1.67$ cm

46. (*c*) We have, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \Rightarrow \frac{1}{f} = \frac{1}{-20} - \left(-\frac{1}{40}\right)$

$= \frac{-2+1}{40} = -\frac{1}{40}$

or $f = -40$ cm

Power of the lens, $P = -\frac{200}{0.40} = -2.5$ D

47. (*b*) Given, $f = 5$ cm, $D = 25$ cm

$\therefore$ Magnification $m = 1 + \frac{D}{f}$

$m = 1 + \frac{25}{5} = 6$

49. (*b*) Resolving power $= \frac{D}{1.22\lambda}$

Given, $D = 3.8 \times 10^5$ km $= 3.8 \times 10^8$ m

$\lambda = 550$ nm $= 550 \times 10^9$ m

$\Rightarrow$ Resolving power $= \frac{3.8 \times 10^8}{1.22 \times 550 \times 10^9} \approx 55$ m

50. (*c*) We know about the thin prism, $\delta_m = (\mu - 1)A$

In given condition, $A_1(\mu_1 - 1) = A_2(\mu_2 - 1)$

Given, $\mu_1 = 1.4$, $\mu_2 = 1.6$ and $A_1 = 6°$

Now, $6(1.4 - 1) = A_2(1.6 - 1)$

$A_2 = \frac{6 \times 0.4}{0.6} \Rightarrow A_2 = \frac{2.4}{0.6}$

$A_2 = 4°$

Hence, the angle of the second prism is 4°.

52. (*a*) For total internal reflection, the angle of incidence of light in denser medium must be greater than the critical angle for the pair of media in contact,

i.e., $i > C$

We have, $\frac{1}{\sin C} = \frac{v_2}{v_1}$

Given, $v_1 = M_1 = 1.5 \times 10^8$ ms^{-1}

$v_2 = M_2 = 2 \times 10^8$ ms^{-1}

Here, $C = \sin^{-1}\left(\frac{M_1}{M_2}\right)$

For total internal reflection, $i > C$

$\therefore$ $i > \sin^{-1}\left(\frac{M_1}{M_2}\right)$

$\Rightarrow$ $i > \sin^{-1}\left(\frac{1.5 \times 10^8}{2 \times 10^8}\right) \Rightarrow i > \sin^{-1}\left(\frac{3}{4}\right)$

53. (*d*) Given, $\frac{R_1}{R_2} = \frac{1}{2}$, $f = 6$ cm and $\mu = 1.5$

Let, $R_1 = R$ and $R_2 = 2R$

We know the focal length of converging lens

by using relation, $\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} + \frac{1}{R_2}\right]$

Substituting all the values, we get

$R = 4.5$ cm

$\therefore$ $R_1 = 4.5$ cm

and $R_2 = 2 \times 4.5 = 9$ cm

54. (*b*) The sun light reaches us as white light and not as its components because air medium is non-dispersive.

55. (*d*) We know that, $\frac{1}{f} = (n - 1)\frac{1}{R}$

Given, $R = 12$ cm and $r = 1.5$

Here, $f = \frac{R}{n-1} = \frac{12}{1.5 - 1} = \frac{12}{0.5} = 24$ cm

56. (*c*) Focal length of combination of two thin lenses,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{f_1 + f_2}{f_1 f_2} \Rightarrow f = \frac{f_1 f_2}{f_1 + f_2}$$

57. (*b*) We know that, the relation for number of images formed for an object placed between the mirror.

$$n = \frac{360°}{\theta} - 1$$

Given, $\theta = 50°$

Now, Number of images, $n = \frac{360}{50°} - 1 = 7.2 - 1 = 6.2 \simeq 6$

58. (*b*) Velocity of light in a medium, $v = \frac{c}{\mu}$

We know that time taken, $t = \frac{x}{v} \Rightarrow t = \frac{x}{c/\mu} \Rightarrow t = \frac{\mu x}{c}$

59. (*b*) The critical angles $\left[C = \sin^{-1}\frac{1}{n}\right]$ for red ($n = 1.39$), green ($n = 1.44$) and blue ($n = 1.47$) lights are 46°, 44° and 43°, respectively.

All colours will strike the hypotenuse face at 45°. Hence, green and blue rays will totally reflected while red rays will get through.

60. (*d*) Angle between the two plane mirrors, $\theta = 90°$

As reflection is independent of medium.

$\therefore$ Deviation produced by the combination of two plane mirrors is, $\delta = 2\pi - 2\theta = 2\pi - 2(\pi/2) = \pi = 180°$

61. (*d*) In order to increase the magnifying power of a compound microscope, the object should have large focal length and eye-piece should have small focal length.

62. (c) Given, $\mu = 1.54$

We know that, $\theta = \tan^{-1}(\mu)$

$\theta = \tan^{-1}(1.54) \Rightarrow \theta = 57°$

On rotating the polaroid the reflected light is fully polarised. Here the intensity gradually reduces to zero and then again increases.

63. (*d*) $\therefore \frac{1}{f}=\left[\frac{\mu_1}{\mu_2}-1\right]\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$,

Given, $R_1 = 0.30$ m,

$R_2 = -0.30$ m, $\mu_1 = 3/2$, $\mu_2 = 4/3$

$$\left[\frac{1}{f}=\left[\frac{3/2}{4/3}-1\right]\left[\frac{1}{0.30}-\left(-\frac{1}{0.30}\right)\right]\right]$$

By putting values, $f = 1.20$ m

64. (*b*) $\therefore$ Magnification, $m=\frac{f}{f-u}$

Here, $m=-2$ and $f=-50$ cm

$\therefore \quad -2=\frac{-50}{-50-u} \Rightarrow 100+2u=-50$

or $\quad u=\frac{-150}{2}=-75$ cm

65. (*c*) An eye see distant objects with full relaxation.

So, $\frac{1}{2.5\times10^{-2}}-\frac{1}{-\infty}=\frac{1}{f}$

or $\quad P=\frac{1}{f}=\frac{1}{2.5\times10^{-2}}=40$ D

An eye see an object at 25 cm with strain

$$\frac{1}{2.5\times10^{-2}}-\frac{1}{25\times10^{-2}}=\frac{1}{f}$$

$\therefore \quad P=\frac{1}{f}=40+4=44$ D

The power of the eye lens various from 44 D to 40 D.

66. (*a*) When lenses are in contact, then power

$$P=\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}$$

$$10=\frac{1}{f_1}+\frac{1}{f_2} \quad \text{...(i)}$$

When they are placed a distance d apart

$$P'=\frac{1}{F'}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1f_2}$$

$$6=\frac{1}{f_1}+\frac{1}{f_2}-\frac{0.25}{f_1f_2} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$f_1f_2=\frac{1}{16} \quad \text{...(iii)}$$

From Eqs. (i) and (iii), we get

$$f_1+f_2=\frac{5}{8} \quad \text{...(iv)}$$

Also, $(f_1-f_2)^2=(f_1+f_2)^2-4f_1f_2=\left(\frac{5}{8}\right)^2-4\times\frac{1}{16}=\frac{9}{64}$

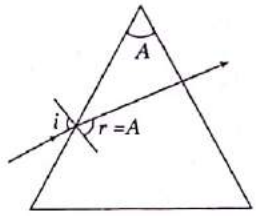
$$f_1-f_2=\frac{3}{8} \quad \text{...(v)}$$

On solving Eqs. (iv) and (v), we get

$f_1=0.5$ m and $f_2=0.125$ m

67. (*a*) Refractive index,

$$\mu=\frac{\sin i}{\sin r} \Rightarrow \mu=\frac{\sin i}{\sin A}$$

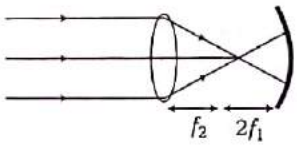


In case of small angle,

$$\mu=\frac{i}{A}$$

Angle of incidence, $\quad i=\mu A$

68. (*c*) $d = 2f_1 + f_2$



69. (*c*) Given, $f = +30$ cm and $m=\frac{1}{4}$

Magnification, $m=\frac{f}{f-u}$

$\therefore \quad \frac{30}{30-u}=\frac{1}{4}$

$30-u=120 \Rightarrow u=-90$ cm

70. (*a*) For objective lens, $\frac{1}{f_o}=\frac{1}{v_o}-\frac{1}{u_o}$

Using proper sign convention,

$$\frac{1}{+2.5}=\frac{1}{v_o}-\frac{1}{(-3.75)} \Rightarrow v_o=7.5 \text{ cm}$$

For eye lens, $\frac{1}{f_e}=\frac{1}{v_e}-\frac{1}{u_e}$

$$\frac{1}{+5}=\frac{1}{(-25)}-\frac{1}{u_e} \Rightarrow u_e=-4.16 \text{ cm}$$

or $\quad |u_e|=4.16$ cm

$\therefore$ Distance between the two lens $=7.5+4.16=11.67$ cm

71. (*a*) From lens formula,

$$\frac{1}{f}=({}_a\mu_g-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$=(1.5-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \quad \text{...(i)}$$

Also, ${}_l\mu_g=\frac{\mu_g}{\mu_l}=\frac{1.5}{1.6}$

$\therefore \quad \frac{1}{f'}=({}_l\mu_g-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

$$\frac{1}{f'}=\left(\frac{1.5}{1.6}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \quad \text{...(ii)}$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{f}{f'} = \frac{\left(\frac{1.5}{1.6}\right)}{(1.5-1)} = -\frac{1}{16 \times 0.5}$$

$$f' = -16 \times 0.5 \times f = 16 \times 0.5 \times 20$$

$$\Rightarrow \quad f' = -160 \text{ cm}$$

72. (*a*) Resolving power of telescope $= \frac{a}{1.22}\lambda$

where, a is the diameter of objective lens and λ is the wavelength of light used. It is obvious that on increasing a more light is collected by objective lens and so, the image formed is more bright. Thus, resolving power of telescope increases.

73. (*d*) From mirror formula,

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \quad \text{...(i)}$$

On differentiating Eq. (i), we obtain

$$0 = \frac{1}{v^2}dv - \frac{1}{v^2}du$$

$$dv = -\left(\frac{v}{u}\right)^2 du \quad \text{...(ii)}$$

Also, from Eq. (i), we get

$$\frac{v}{u} = \frac{f}{u-f} \quad \text{...(iii)}$$

From Eqs. (ii) and (iii), we get

$$dv = -\left(\frac{f}{u-f}\right)^2 L$$

Therefore, size of image is $\left(\frac{f}{u-f}\right)^2 L$

74. (*c*) $\therefore$ Magnification, $m = \frac{v}{-u} = -3$

$$v = 3u = 30 \text{ cm}$$

Focal length of co-axial combination,

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{v} - \frac{1}{u}$$

$$-\frac{1}{30} + \frac{1}{f_2} = \frac{1}{30} + \frac{1}{10} = \frac{4}{30}$$

$$\frac{1}{f_2} = \frac{4}{30} + \frac{1}{30}$$

$$\frac{1}{f_2} = \frac{4+1}{30} = \frac{5}{30}$$

$$f_2 = \frac{30}{5} = 6 \text{ cm}$$

Hence, lens A is convex lens.

75. (*d*) White light is incident on face AB of a glass prism, the green light is just to tally internally reflected at face AC, the light emerging from face AC will contain all colour except green.

76. (*a*) Magnification for parallel rays,

$$m = \frac{f_o}{f_e} \Rightarrow 20 = \frac{80}{f_e} \text{ or } f_e = 4 \text{ cm}$$

If the focal length of erecting lens is 20 cm, then the length of the telescope

$$L_\infty = f_o + 4f + f_e$$

(where, f is the focal length of erecting lens)

$$= 80 + 4 \times 20 + 4$$

$$= 80 + 80 + 4 = 164 \text{ cm}$$

CHAPTER

10 Wave optics

The ray optics uses the geometry of straight lines to explain the macroscopic phenomena like rectilinear propagation of light, reflection of light and refraction of light, etc. However, the microscopic phenomena like interference, diffraction and polarisation could not be accounted by ray optics. To explain these phenomena concept of waves was introduced. The new branch of physics based on the wave concept of light was called wave optics. According to wave theory of light, light is a form of energy which travels through a medium in the form of transverse wave motion. In this chapter, we shall study the various phenomena related to wave nature of light.

Inside

1) Nature of light
- Wavefront
- Huygens' principle of secondary wavelets
- Principle of superposition of waves
- Interference of light wave
- Necessary conditions for interference of light

2) Young's double slit experiment
- Intensity of the fringes
- Lloyd's mirror

3) Diffraction of light
- Diffraction of light due to a single narrow slit
- Width of central maxima
- Fresnel's distance
- Polarisation of light
- Malus's law
- Polarisation of transverse mechanical waves

1) Nature of light

Various theories about the nature of light have been proposed from time to time. *The main two theories are as follows*

1. Newton's corpuscular theory of light

In 1675 AD Newton proposed this theory. According to him,

(i) Light consists of tiny particles called corpuscles which are emitted by a luminous object.

(ii) These corpuscles travels with speed of light in all directions.

(iii) The corpuscles carry energy and momentum with them. When they strike retina of the eye, they produce sensation of vision.

(iv) The corpuscles of different colours are of different sizes. Red coloured corpuscles are larger than blue coloured corpuscles.

(v) This theory could explains the reflection, refraction and rectilinear propagation of light.

(vi) The corpuscular theory could not explain interference, diffraction and polarisation of light.

(vii) Speed of light in denser medium is more than speed of light in a rarer medium according to this theory. Which is incorrect, therefore the Newton's corpuscular theory is wrong.

2. Huygens' wave theory of light

In 1678, a Dutch scientist, Christian Huygens' propounded the wave theory of light. According to him,

(i) Light travels in the form of waves.

(ii) These waves travel in all the directions with the velocity of light.

(iii) The waves of light of different colours have different wavelengths.

(iv) Initially, the light waves were assumed to be longitudinal. But later on while explaining the phenomenon of polarisation the light waves were considered to be transverse.

(v) The whole universe with all matter and space is filled with a luminiferous medium called ether of very low density and very high elasticity.

(vi) Huygen's theory could explain reflection, refraction, interference, diffraction, polarisation but could not explain photoelectric effect and Compton's effect.

(vii) Wave theory introduced the concept of wavefront.

Wavefront

A wavefront is the locus of points (wavelets) having the same phase (a surface of constant phase) of oscillations. A wavelet is the point of disturbance due to propagation of light. A line perpendicular to a wavefront is called a ray.

Types of wavefront

Depending on the shape of source of light, *wavefronts can be of three types which are given below*

(i) Spherical wavefront

When light emerging from a point source Wavefronts are spherical and eccentric with point source at their centre as shown in figure by spheres 1, 2, 3 and 4. Rays are radial as shown in arrows.

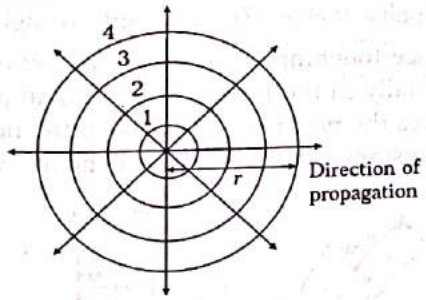


***Fig. 10.1** Spherical wavefront of a point source*

In spherical wavefront,

Amplitude of light waves, $A \propto \frac{1}{r}$

and Intensity of light waves, $I \propto \frac{1}{r^2}$

(ii) Cylindrical wavefront

When the source of light is linear *e.g.*, a straight line source as shown in figure. All the points are equidistant from the source.

Therefore wavefronts 1, 2 and 3 are cylindrical and co-axial with the source as their common axis.

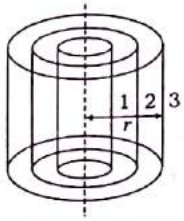


***Fig. 10.2** Cylindrical wavefront of a linear source*

In this case, intensity of light waves, $I \propto \frac{1}{r}$

and amplitude of light waves, $A \propto \frac{1}{\sqrt{r}}$

(iii) Plane wavefront

When light sources are emitting parallel rays, or when the light is coming from a very far-off source
Wavefronts will be planes as shown in figure.

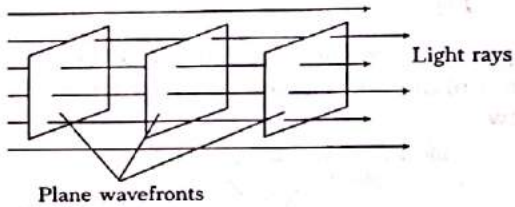


***Fig. 10.3** Plane wavefront of a distant source*

Note wavefronts parallel to xy-plane can be represented by z = constant. Similarly, wavefronts parallel to yz-plane and zx-plane are x = constant and y = constant, respectively.

In this case, intensity of light waves, $I \propto r^0$

and amplitude of light waves, $A \propto r^0$

Shapes of wavefronts of reflected and refracted waves

We have discussed the various types of wavefront. Keeping in mind those shapes, now we are going to see the shapes of wavefronts associated with reflected as well as refracted waves.

(i) Shapes of wavefronts before and after reflection from a plane and curved mirrors are as shown below.

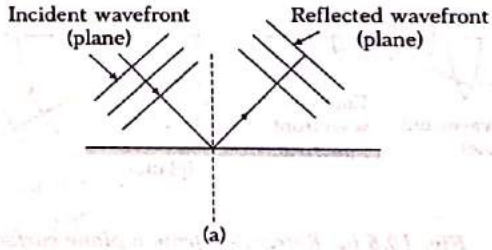
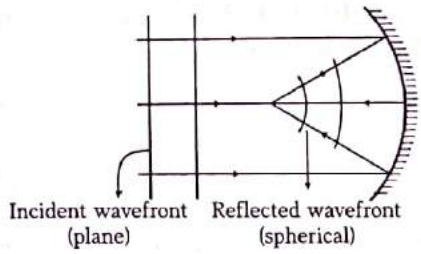
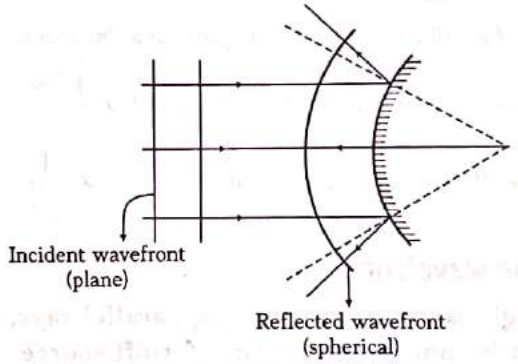


***Fig. 10.4** Reflection from a plane mirror*



(b) Reflection from concave mirror



(c) Reflection from convex mirror

***Fig. 10.5** Reflection from curved mirrors*

(ii) Shapes of wavefronts before and after refraction from a plane and curved surfaces are as shown below.

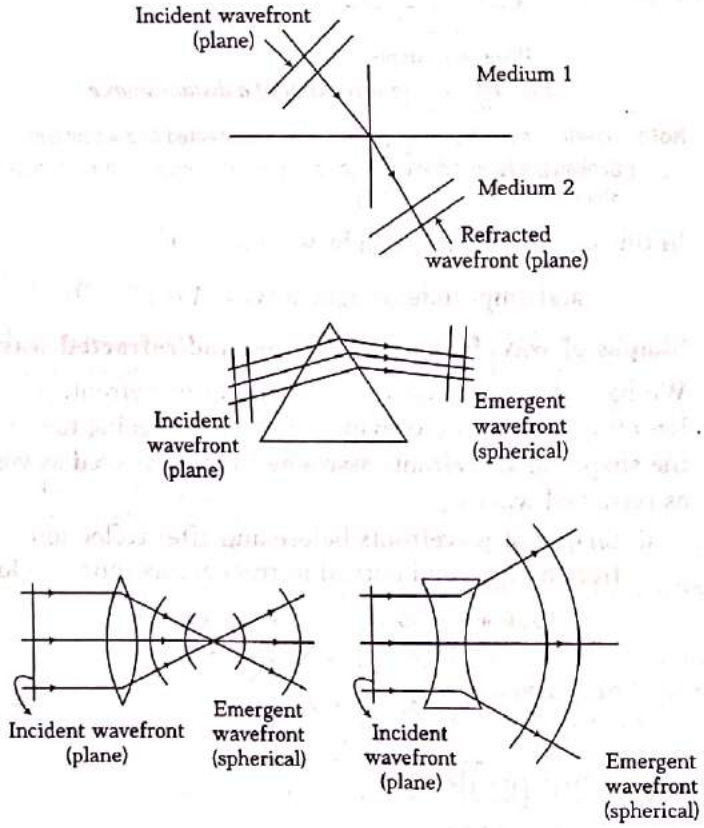
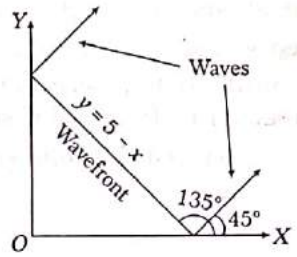


***Fig. 10.6** (a) Refraction from a plane surface*
(b) Refraction from a prism
(c) Refraction through lenses

***Example 10.1** A wavefront is represented by the plane $y = 5 - x$. Find the direction of propagation of wave.*

Sol. Given, wavefront is $y = 5 - x$

This wavefront can be represented by a line as shown in the figure. This wavefront makes an angle of 135° with X-axis. Therefore, the wave must be perpendicular to the wavefront and it will make an angle of 45° with X-axis as shown in figure.



Huygens' principle of secondary wavelets

In 1678, in order to explain the propagation of wave in a medium, Huygens' propounded a principle known as Huygens' principle of secondary wavelets by which at any instant, we can geometrically obtain the position of wavefront. *He made following three assumptions*

(i) Every point on a given wavefront (called primary wavefront) can be regarded as fresh source of new disturbance and sends out its own spherical wavelets called secondary wavelets.

(ii) The secondary wavelets spread in all directions with the velocity of wave (*i.e.*, velocity of light).

(iii) A surface touching these secondary wavelets, tangentially in the forward direction at any instant (Δt) gives the position and shape of the new wavefront at the instant. This is called secondary wavefront.

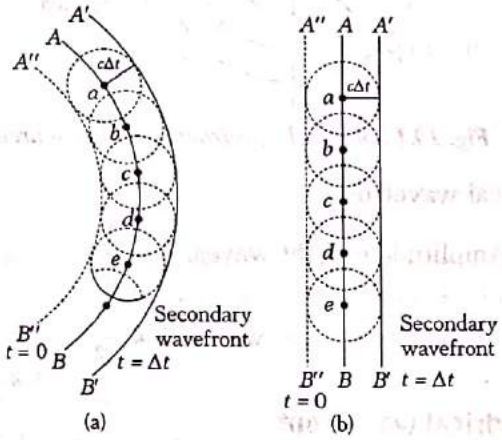


***Fig. 10.7** Formation of secondary wavefront using Huygens' principle*

In Fig. (a), AB is the section of the given spherical wavefront and $A'B'$ is the new wavefront in the forward direction.

In Fig. (b), AB is the section of the given plane wavefront and $A'B'$ is the new wavefront in the forward direction.

Note Huygens' argued that the amplitude of the secondary wavelets is maximum in the forward direction and zero in the backward direction. Hence, the backward secondary wavefront is absent.

Merits

On the basis of Huygens' principle many optical phenomena like interference, diffraction, reflection (or law of refraction) as well refraction (or Snell's law) could be explained.

Demerits

Following are demerits of Huygens' principle

(i) Huygens' principle could not explain, why wavefronts of secondary wavelets are formed in forward direction and not in backward direction?

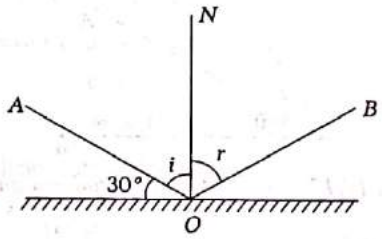
(ii) This theory assumes light waves as longitudinal waves, therefore could not explain polarisation of light.

(iii) Michelson-Morley does not assume the ether particle as a medium in his experiment.

(iv) This principle could not explain electromagnetic nature of light.

Example 10.2 *A plane wavefront incident on a reflecting surface at an angle of 30° with horizontal. Find the angle of reflected wavefront with horizontal.*

Sol. Consider a plane wavefront OA incident at 30° with the horizontal. ON is normal to the reflecting surface. $\angle i$ is angle of incidence and $\angle r$ is angle of reflection. According to law of reflection, $\angle i = \angle r$.



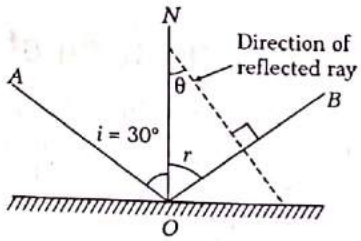
Now, from figure, we have

$$\angle i + 30° = 90° \Rightarrow \angle i = 60° = \angle r$$

Angle with horizontal $= 90° - \angle r = 30°$

Example 10.3 *A plane wavefront incident on a reflecting surface at an angle of 30° with vertical. Find the angle (acute) of reflected wave with vertical.*

Sol. Consider a plane wavefront OA incident at 30° with the vertical. ON is normal to the reflecting surface. $\angle i$ is angle of incidence and $\angle r$ is angle of reflection. According to law of reflection, $\angle i = \angle r = 30°$

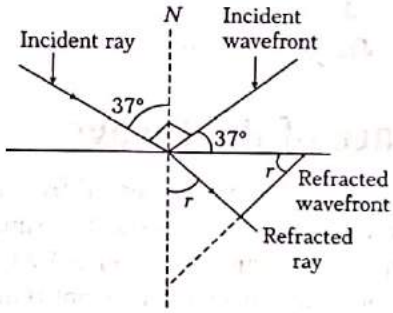


Reflected ray must be perpendicular to the reflected wavefront (OB) as shown in the figure. Therefore, acute angle made by reflected ray with the vertical is $\theta = 90° - r$

$$= 90° - 30° = 60°$$

Example 10.4 *A plane wavefront is incident from air ($\mu = 1$) at an angle of 37° with a horizontal boundary of a refractive medium from air of refractive index $\mu = \frac{3}{2}$. Find the angle of refracted wavefront with the horizontal boundary.*

Sol. It has been given that incident wavefront makes 37° with horizontal. Hence, incident ray makes 37° with normal as the ray is perpendicular to the wavefront.



Now, by Snell's law, $\frac{\sin 37°}{\sin r} = \frac{3}{2}$

$$\Rightarrow \quad \sin r = \frac{2}{3} \times \frac{3}{5} = \frac{2}{5}$$

$\Rightarrow r = \sin^{-1}\left(\frac{2}{5}\right)$ which is same as angle of refractive wavefront with horizontal.

Example 10.5 *A plane wavefront of yellow light with wavelength 0.5 μm in air suffers refraction in a medium in which velocity of light is 2×10^8 ms^{-1}. Then, the wavelength of the light associated with a wavefront in the medium would be*

Sol. For wavelength and speed of light associated with a wavefront, we have

$$\frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2}$$

Here, $\lambda_1 = 0.5\ \mu\text{m}$, $v_1 = 3 \times 10^8\ \text{ms}^{-1}$

and $v_2 = 2 \times 10^8\ \text{ms}^{-1}$

$$\Rightarrow \quad \frac{0.5}{\lambda_2} = \frac{3 \times 10^8}{2 \times 10^8}$$

$\therefore$ The wavelength of the light associated with a wavefront in the medium is $\lambda_2 = 0.33\ \mu\text{m}$

Principle of superposition of waves

According to superposition principle, at a particular point in the medium, the resultant displacement y produced by a number of waves is the vector sum of the displacements produced by each of the waves $(y_1, y_2, \ldots)$.

i.e., $y = y_1 + y_2 + y_3 + y_4 + \ldots$

Clearly, each wave contributes as, if the other wave is not present. The superposition principle which was stated first for mechanical waves is equally applicable to the electromagnetic (light) waves.

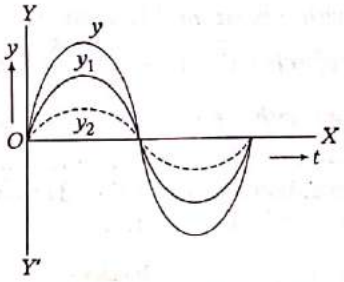


Fig. 10.8 Superposition of waves

Interference of light wave

When two light waves of exactly equal frequency having constant phase difference with respect to time travelled in same direction and superimpose (overlap) with each other, then intensity of resultant wave does not remain uniform in space.

This phenomenon of formation of maximum intensity at some points and minimum intensity at some other points by the two identical light waves travelling in same direction is called the **interference of light.**

Types of interference

According to phase difference in superimposing waves, interference is divided into two categories as follows

(i) Constructive interference

The interference at the points, where the two waves meet in same phase *i.e.*, the intensity of light is maximum, is called the **constructive interference.**

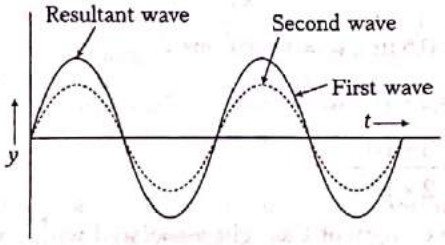


Fig. 10.9 Constructive interference

(ii) Destructive interference

When two waves meet in opposite phase *i.e.*, the intensity of light is minimum is called the **destructive interference.**

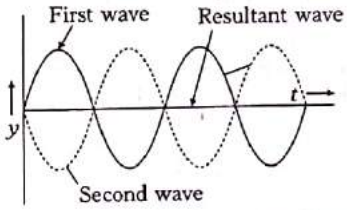


Fig. 10.10 Destructive interference

Note During interference of light redistribution of energy (intensity) takes place but total energy remains conserved.

Expression for resultant intensity in interference of two waves

Let the waves from two sources of light S_1 and S_2, interfere at the point P as shown in figure be represented as

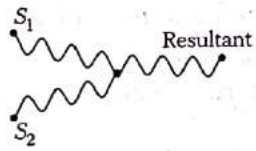


Fig. 10.11 Interference of two waves

$$y_1 = a \sin \omega t \quad \text{and} \quad y_2 = b \sin(\omega t + \phi)$$

where, a and b are the respective amplitudes of the two waves and ϕ is the constant phase angle by which second wave leads the first wave.

Applying superposition principle, the magnitude of the resultant displacement of the waves is

$$y = y_1 + y_2 \quad \text{or} \quad y = a \sin \omega t + b \sin(\omega t + \phi)$$

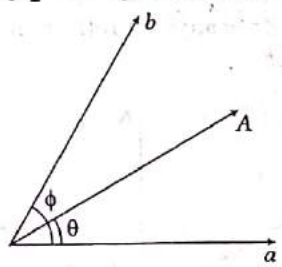


Fig. 10.12 Resultant of amplitudes a and b

Therefore, we get,

$$y = A \sin(\omega t + \theta)$$

where, A is the resultant amplitude and θ is the resultant phase difference as shown in figure.

$$\therefore \quad A = \sqrt{a^2 + b^2 + 2ab \cos \phi}$$

and $$\tan \theta = \frac{b \sin \phi}{a + b \cos \phi}$$

As, intensity is directly proportional to the square of the amplitude of the wave *i.e.*, $I \propto a^2$.

So, for two different cases,

$$I_1 = ka^2, I_2 = kb^2$$

$\therefore$ and $$I = kA^2 = k(a^2 + b^2 + 2ab\cos\phi)$$

$\therefore$ $$\boxed{I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi}$$

Note Relation between path difference, phase difference and time difference

$$\frac{2\pi}{\text{Phase. diff}} = \frac{\lambda}{\text{Path diff.}} = \frac{T}{\text{Time diff.}}$$

For constructive interference

Intensity, I should be maximum, for which

$\cos\phi = \text{maximum} = +1$

$\therefore$ $$\phi = 0, 2\pi, 4\pi, \ldots$$

i.e., phase difference, $\phi = 2n\pi$

where $n = 1, 2, 3, \ldots$

If Δx be the path difference between the interfering waves, then $\Delta x = \frac{\lambda}{2\pi}\phi = \left(\frac{\lambda}{2\pi}\right)(2n\pi) = n\lambda$

$\therefore$ Path difference, $\Delta x = n\lambda$.

where, $n = 1, 2, 3, \ldots$

Hence, *condition for constructive interference at a point is that phase difference between the two waves reaching the point should be zero or an even integral multiple of π. Equivalent path difference between the two waves reaching the point should be zero or an integral multiple of wavelength (λ).*

$\therefore$ Maximum intensity, $$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2} = (\sqrt{I_1} + \sqrt{I_2})^2 = K(a+b)^2$$

For destructive interference

Intensity, I should be minimum, for which

$\cos\phi = \text{minimum} = -1$

$\therefore$ Phase difference, $\phi = \pi, 3\pi, 5\pi, \ldots$

i.e., $$\phi = (2n-1)\pi$$

where, $n = 1, 2, 3, \ldots$

The corresponding path difference between the two waves is

$$\Delta x = \left(\frac{\lambda}{2\pi}\right)\phi = \left(\frac{\lambda}{2\pi}\right)(2n-1)\pi = (2n-1)\frac{\lambda}{2}$$

$\therefore$ Path difference, $\Delta x = (2n-1)\frac{\lambda}{2}$, where $n = 1, 2, 3 \ldots$

Hence, *condition for destructive interference at a point is that, the phase difference between the two waves reaching the point should be an odd integral multiple of π or path difference between the two waves reaching the point should be an odd integral multiple of half the wavelength.*

$\therefore$ Minimum intensity, $$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2} = (\sqrt{I_1} - \sqrt{I_2})^2 = K(a-b)^2$$

Comparison of intensities of maxima and minima

As we know that,

$$I_{max} \propto (a+b)^2 \text{ and } I_{min} \propto (a-b)^2$$

$\therefore$ $$\frac{I_{max}}{I_{min}} = \frac{(a+b)^2}{(a-b)^2} = \frac{\left(\frac{a}{b}+1\right)^2}{\left(\frac{a}{b}-1\right)^2} \Rightarrow \frac{I_{max}}{I_{min}} = \frac{(r+1)^2}{(r-1)^2}$$

where, $r = \frac{a}{b}$ (ratio of amplitudes)

Also, we can write as

$$\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2 = \left(\frac{a+b}{a-b}\right)^2$$

Example 10.6 *Two sources of intensity I and 4I are used in an interference experiment. Find the intensity at a point, where the waves from two sources superimpose with a phase difference of (i) zero, (ii) $\pi/2$, (iii) π and (iv) ratio of maximum and minimum intensity.*

Sol. Given, $I_1 = I$ and $I_2 = 4I$

Intensity, $I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

(i) At $\phi = 0$,

Intensity, $I_R = I + 4I + 2\sqrt{I.4I}\cos 0° = 9I = I_{max}$

(ii) At $\phi = \frac{\pi}{2}$,

Intensity, $I_R = I + 4I + 2\sqrt{I.4I}\cos(\pi/2) = 5I$

(iii) At $\phi = \pi$,

Intensity, $I_R = I + 4I + 2\sqrt{I.4I}\cos(\pi) = I = I_{min}$

(iv) Ratio of maximum and minimum intensity,

$$\frac{I_{max}}{I_{min}} = \frac{9I}{I} = 9:1$$

Example 10.7 *Two sources with intensity I_0 and $4I_0$ respectively, interfere at a point in a medium. Find the ratio of (i) maximum and minimum possible intensities, (ii) ratio of amplitudes.*

Sol. (i) Maximum intensity, $I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$

Here, $I_1 = I_0$ and $I_2 = 4I_0$

$\therefore$ $I_{max} = (\sqrt{I_0} + \sqrt{4I_0})^2 = 9I_0$

and minimum intensity, $I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2 = I_0$

$\Rightarrow$ $$\frac{I_{max}}{I_{min}} = \frac{9I_0}{I_0} = 9:1$$

(ii) Ratio of intensities, $\frac{I_{max}}{I_{min}} = \left(\frac{r+1}{r-1}\right)^2$

where, r = ratio of amplitudes

$\Rightarrow$ $$\frac{9}{1} = \left(\frac{r+1}{r-1}\right)^2 \Rightarrow 3 = \frac{r+1}{r-1}$$

$\Rightarrow$ $$3r - 3 = r + 1 \Rightarrow 2r = 4 \Rightarrow r = 2:1$$

Coherent and incoherent sources

Light sources can be coherent or incoherent which are discussed as follows.

Coherent sources

Two sources of light which continuously emit light waves of same frequency with a zero or constant phase difference between them are called coherent sources.

Incoherent sources

Two sources of light which do not emit light waves with a constant phase difference are called incoherent sources.

Need of coherent sources for the production of interference pattern

When two monochromatic waves of intensity I_1, I_2 and phase difference ϕ meet at a point, then the resultant intensity is given by

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

Here the term $2\sqrt{I_1 I_2}\cos\phi$ is called interference term.

There are two possibilities. (i) If $\cos\phi$ remains constant with time, then the total intensity at any point will be constant. The intensity will be maximum $(\sqrt{I_1} + \sqrt{I_2})^2$ at points, where $\cos\phi$ is 1 and minimum $(\sqrt{I_1} - \sqrt{I_2})^2$ at points, where $\cos\phi = -1$ The sources in this case are coherent.

(ii) If $\cos\phi$ varies continuously with time assumming both positive and negative value, then the average value of $\cos\phi$ will be zero over time interval of measurement. The interference term averages to zero. There will be same intensity $I = I_1 + I_2$ at every point. The two sources in this case are incoherent.

Note In practice, coherent sources are produced either by dividing the wavefront or by dividing the amplitude of the incoming waves.

Example 10.8 *Two incoherent sources of light emitting light of intensity I_0 and $3I_0$ interfere in a medium. Calculate, the resultant intensity at any point.*

Sol. Since, the two sources are incoherent, hence we do not obtain any sustained interference pattern.

We observe an average intensity,

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

Now, $\cos\phi = 0$ in one complete cycle.

$\therefore \quad I = I_1 + I_2$ which is $4I_0$

Requirements to find out coherent sources

(i) Coherent sources of light should be obtained from a single source by some device.

Two coherent sources can be obtained either by

(a) the source and its virtual image (Lloyd's mirror).

(b) the two virtual images of the same source. (Fresnel's biprism)

(c) two real images of the same source. (Young's double slit)

(ii) The two sources should give monochromatic light.

(iii) The path difference between light waves from two sources should be small.

(iv) Coherent sources are produced by

(a) division of wavefront (Young's double slit experiment).

(b) division of amplitude (partial reflection or refraction in thin films).

Necessary conditions for interference of light

In order to obtain a sustained (permanent or stable) and observable interference pattern, the following conditions must be fulfilled.

(i) **Sources must be coherent** In order to produce a stable interference pattern, the individual waves must maintain a **constant phase relationship with one another** *i.e.*, the two interfering sources must emit waves having a constant phase difference between them. If the phase difference between two sources does not remain constant, then the places of maxima and minima shift. In case of mechanical waves, it is possible to keep a constant phase relationship between two different sources. But in case of light, two different light sources cannot be coherent.

(ii) **Same frequency or wavelength** Phase relationship between two waves can be kept constant only when their frequencies are same. Thus we can say that, two coherent sources must have the same frequency.

(iii) **Equality of amplitudes** The amplitudes of two interfering waves should be equal or approximately equal. Otherwise the difference between the intensities of maximas and minimas will be too small and the contrast will be poor. Maximum contrast is, however obtained when $A_1 = A_2$, because then minimum intensity will be zero.

Check point 10.1

1. By corpuscular theory of light, the phenomena which can explained, is
(a) refraction (b) interference
(c) diffraction (d) polarisation

2. According to corpuscular theory of light, the different colours of light are due to
(a) different electromagnetic waves
(b) different force of attraction among the corpuscles
(c) different size of the corpuscles
(d) None of the above

3. Interference proves
(a) transverse nature of light (b) longitudinal nature of light
(c) wave nature of light (d) particle nature of light

4. Two identical light sources S_1 and S_2 emit light of same wavelength λ. These light rays will exhibit interference, if
(a) their phase differences remain constant
(b) their phases are distributed randomly
(c) their light intensities remain constant
(d) their light intensities change randomly

5. Which of the following is conserved when light waves interfere?
(a) Intensity (b) Energy
(c) Amplitude (d) Momentum

6. When interference of light takes place
(a) energy is created in the region of maximum intensity
(b) energy is destroyed in the region of maximum intensity
(c) conservation of energy holds good and energy is redistributed
(d) conservation of energy does not hold good

7. If two waves represented by $y_1 = 4\sin\omega t$ and $y_2 = 3\sin\left(\omega t + \frac{\pi}{3}\right)$ interfere at a point, then the amplitude of the resulting wave will be about
(a) 7 (b) 6 (c) 5 (d) 3.5

8. If the two waves represented by $y_1 = 4\cos\omega t$ and $y_2 = 3\cos(\omega t + \pi/3)$ interfere at a point, then the amplitude of the resulting wave will be about
(a) 7 (b) 5 (c) 6 (d) 3.5

9. Two coherent sources of intensities, I_1 and I_2 produce an interference pattern. The maximum intensity in the interference pattern will be
(a) $I_1 + I_2$ (b) $I_1^2 + I_2^2$
(c) $(I_1 + I_2)^2$ (d) $(\sqrt{I_1} + \sqrt{I_2})^2$

10. What is the path difference in case of destructive interference?
(a) $n\lambda$ (b) $n(\lambda + 1)$ (c) $\frac{(n+1)\lambda}{2}$ (d) $\frac{(2n+1)\lambda}{2}$

11. If the amplitude ratio of two sources producing interference is 3 : 5, then the ratio of intensities at maxima and minima is
(a) 25 : 16 (b) 5 : 3
(c) 16 : 1 (d) 25 : 9

12. Two sources of light of the same frequency produce intensities I and $4I$ at a point P when used individually. If they are used together such that the light from them reach P with a phase difference of $2\pi/3$, then the intensity at P will be
(a) $2I$ (b) $3I$ (c) $4I$ (d) $5I$

13. Two incoherent sources of intensities I and $4I$ superpose, then the resultant intensity is
(a) $5I$ (b) $9I$
(c) $3I$ (d) I

14. For the sustained interference of light, the necessary condition is that the two sources should
(a) have constant phase difference
(b) be narrow
(c) be close to each other
(d) of same amplitude

15. Which of the following is not an essential condition for interference?
(a) The two interfering waves must propagate in almost the same direction
(b) The waves must have the same period and wavelength
(c) The amplitudes of the two waves must be equal
(d) The two interfering beams of light must originate from the same source

2 Young's double slit experiment (YDSE)

One of the first to demonstrate the interference of light was Thomas Young in 1801.

In his experimental arrangement (as shown in Fig. 10.13), monochromatic light (single wavelength) from a narrow vertical slit S falls on two other narrow slits S_1 and S_2 which are very close together and equidistant to S. S_1 and S_2 act as two coherent sources (both being derived from S). The emerging beams spread into the region beyond the slits. Superposition occurs in the shaded area, where the beams overlap. Alternate bright and dark equally spaced vertical bands (interference fringes) can be observed on a screen placed at some distance from the slits.

If either of S_1 or S_2 is covered, then the fringes disappear.

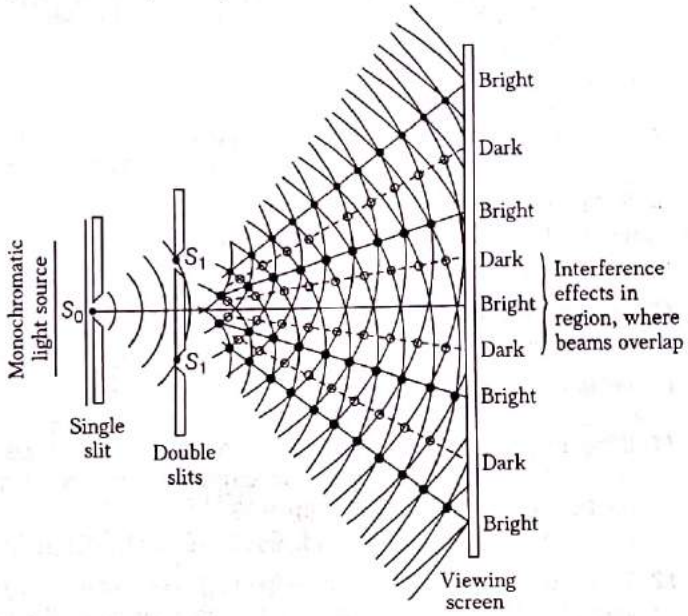


Fig. 10.13 Young's experimental arrangement to produce interference pattern

Path difference

Figure shows the light waves from S_1 and S_2 meeting at an arbitrary point P on the screen.

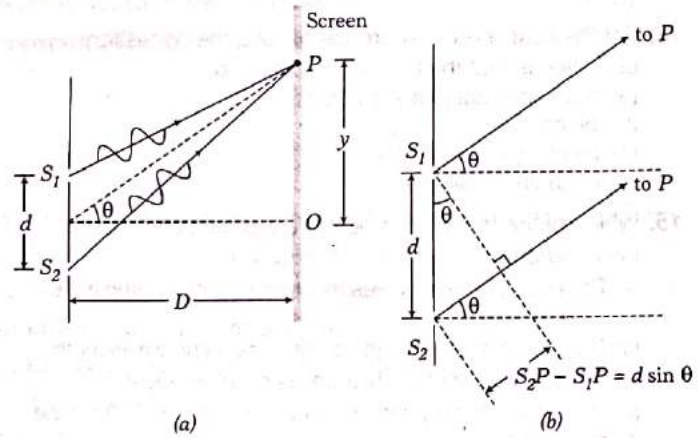


Fig. 10.14 (a) To reach P, the light waves from S_1 and S_2 must travel different distances. (b) The path difference between the two rays is $d \sin\theta$

Since $D >> d$, the two light rays are approximately parallel with a path difference,

$$\Delta x = S_2P - S_1P \approx d \sin\theta$$

In addition to our assumption that $D >> d$, we assume that $d >> \lambda$. These can be valid assumptions because in practice D is often of the order of 1 m, d is a fraction of a millimetre and λ is a fraction of a micrometre for visible light. Under these conditions θ is small, thus we can use the approximation, $\sin\theta \approx \tan\theta = \dfrac{y}{D}$

$\therefore$ Path difference, $\Delta x = \dfrac{y}{D} d$ ...(i)

This is the path difference between two waves meeting at a point on the screen. Due to this path difference some points on the screen are bright and some points are dark. Now, we will discuss the position of these bright, dark fringes and fringe width under following points.

(I) Position of bright fringes

For **maximum** intensity at P,

$$\Delta x = n\lambda \qquad (n = 0, \pm 1, \pm 2, ...)$$

or $$d \sin\theta = n\lambda \qquad (n = 0, \pm 1, \pm 2, ...) \quad ...(ii)$$

The bright fringe corresponding to the integer n is called the nth order (or just nth) bright fringe. The bright fringe for $n = 0$, is known as the central fringe and its centre (point O) is called the central (or zero order) maximum. Higher order bright fringes are situated symmetrically about the central fringe. It is useful to obtain expression for the positions of the bright fringes measured vertical from O.

From Eqs. (i) and (ii), we get

Position of bright fringes,

$$y_{\text{bright}} = \frac{n\lambda}{d} D \quad (n = 0, \pm 1, \pm 2, ...) \quad ...(iii)$$

Example 10.9 *In a YDSE green light of wavelength 500 nm is used. Where will be the second bright fringe be formed for a set up in which separation between slits is 4 mm and the screen is placed 1 m from the slits?*

Sol. Position of second bright fringe, $y_2 = \dfrac{2\lambda D}{d}$

$$= \frac{2 \times 500 \times 10^{-9} \times 1}{4 \times 10^{-3}} = 0.25 \text{ mm}$$

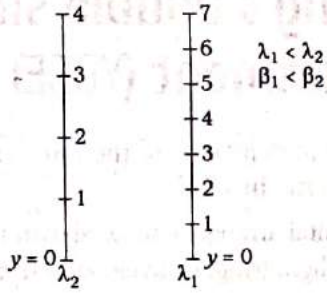
Note While reporting our answer we should report $y_2 = \pm$ 0.25 mm as fringe will be formed both above and below central bright fringe.

Example 10.10 *Bichromatic light is used in YDSE having wavelengths $\lambda_1 = 400$ nm and $\lambda_2 = 700$ nm. Find minimum order of λ_1 which overlaps with λ_2.*

Sol. Let n_1 bright fringe of λ_1 overlaps with n_2 bright fringe of λ_2.

Then,

$$\frac{n_1\lambda_1 D}{d} = \frac{n_2\lambda_2 D}{d}$$



or $$\frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1} = \frac{700}{400} = \frac{7}{4}$$

The ratio $\frac{n_1}{n_2} = \frac{7}{4}$ implies that 7th bright fringe of λ_1 will overlap with 4th bright fringe of λ_2.

Similarly, 14th of λ_1 will overlap with 8th of λ_2 and so on.

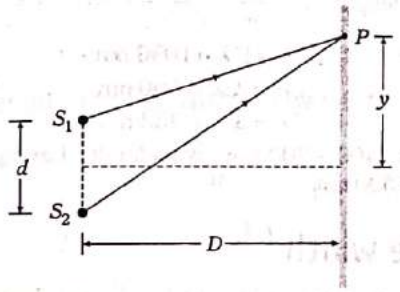
So, the minimum order of λ_1 which overlaps with λ_2 is 7.

Example 10.11 *Young's double slit experiment is carried out using microwaves of wavelength $\lambda = 3$ cm. Distance between the slits is $d = 5$ cm and the distance between the plane of slits and the screen is $D = 100$ cm.*

(i) Find the number of maximas and

(ii) Their positions on the screen.

Sol. (i) The maximum path difference that can be produced = distance between the sources or 5 cm.



Thus, in this case we can have only three maximas, one central maxima and two on its either side for a path difference of $\lambda = 3$ cm.

(ii) For maximum intensity at P,

$$S_2P - S_1P = \lambda$$

or $$\sqrt{(y + d/2)^2 + D^2} - \sqrt{(y - d/2)^2 + D^2} = \lambda$$

Substituting $d = 5$ cm, $D = 100$ cm

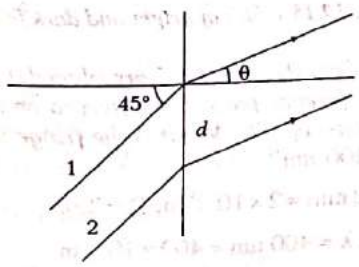
and $\lambda = 3$ cm, we get

$$y = \pm 75 \text{ cm}$$

Thus, the three maximas will be at

$$y = 0 \text{ and } y = \pm 75 \text{ cm}$$

Example 10.12 *Distance between the slits, in YDSE, shown in figure is $d = 20\lambda$, where λ is the wavelength of light used. Find the angle θ, where*



(i) central maxima (where path difference is zero) is obtained.

(ii) third order maxima is obtained.

Sol. Ray 1 has a longer path than that of ray 2 by a distance $d \sin 45°$, before reaching the slits. Afterwards ray 2 has a path longer than ray 1 by a distance $d \sin\theta$. The net path difference is therefore, $d \sin\theta - d \sin 45°$.

(i) Central maximum is obtained, where net path difference is zero,

or $d \sin\theta - d \sin 45° = 0$ or $\theta = 45°$

(ii) Third order maxima is obtained, where net path difference is 3λ,

or $d \sin\theta - d \sin 45° = 3\lambda$

$\therefore$ $$\sin\theta = \sin 45° + \frac{3\lambda}{d}$$

Putting $d = 20\lambda$,

we have $$\sin\theta = \sin 45° + \frac{3\lambda}{20\lambda} \text{ or } \theta \approx 59°$$

(II) Position of dark fringes

In previous point, we obtained the position of bright fringes. In the similar way, we can obtain position of dark fringes as follows.

For minimum intensity at P,

$$\Delta x = (2n - 1)\frac{\lambda}{2} \qquad (n = \pm 1, \pm 2, \ldots)$$

or $$d \sin\theta = (2n - 1)\frac{\lambda}{2} \qquad (n = \pm 1, \pm 2, \ldots) \quad \ldots\text{(iv)}$$

The first minima ($n = \pm 1$) are adjacent to the central maximum on either side. We will obtain expression for position of dark fringes measured vertically from O.

From Eqs. (i) and (iv), we get

Position of dark fringes,

$$y_{\text{dark}} = \frac{(2n - 1)\lambda}{2d} D \quad \ldots\text{(v)}$$
$$(n = \pm 1, \pm 2, \pm 3, \ldots)$$

Example 10.13 *In YDSE, the two slits are separated by 0.1 mm and they are 0.5 m from the screen. The wavelength of light used is 5000 Å. Find the distance between 7th maxima and 11th minima on the screen.*

Sol. Given, $d = 0.1$ mm $= 10^{-4}$ m, $D = 0.5$ m

and $\lambda = 5000$ Å $= 5.0 \times 10^{-7}$ m

The distance between 7th maxima and 11th minima,

$$\Delta y = (y_{11})_{\text{dark}} - (y_7)_{\text{bright}}$$
$$= \frac{(2 \times 11 - 1)\lambda D}{2d} - \frac{7\lambda D}{d}$$

or $$\Delta y = \frac{7\lambda D}{2d} = \frac{7 \times 5.0 \times 10^{-7} \times 0.5}{2 \times 10^{-4}}$$
$$= 8.75 \times 10^{-3} \text{ m}$$
$$= 8.75 \text{ mm}$$

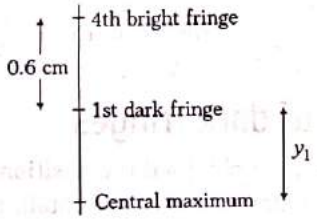
Example 10.14 *In YDSE, the slits are separated by 0.28 mm and the screen is placed 1.4 m away. The distance between the first dark fringe and fourth bright fringe is obtained to be 0.6 cm. Determine the wavelength of the light used in the experiment.*

Sol. Let us first identify the parameters given in the question, $d = 0.28$ mm, $D = 1.4$ m.

Also, separation between first dark fringe and fourth bright fringe is 0.6 cm.

Now, position of 1st dark fringe from central maximum be

$$y_1 = \frac{\lambda D}{2d}$$



Position of 4th bright fringe will be

$$y_2 = \frac{4\lambda D}{d}$$

Separation between them is equal to $\frac{4\lambda D}{d} - \frac{\lambda D}{2d} = 0.6$ cm

$$\Rightarrow \quad \frac{7\lambda D}{2d} = 0.6 \text{ cm}$$

Wavelength of the light,

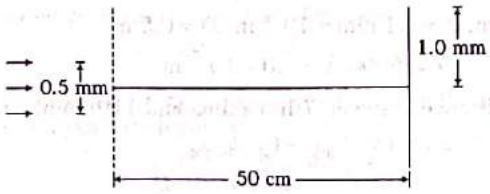
$$\Rightarrow \quad \lambda = \frac{0.6 \times 10^{-2} \times 2 \times 0.28 \times 10^{-3}}{7 \times 1.4} \text{ m}$$

$$= 3.428 \times 10^{-7} \text{ m}$$

Example 10.15 *White coherent light (400 nm – 700 nm) is sent through the slits of a YDSE. ($d = 0.5$ mm, $D = 50$ cm). There is a hole in the screen at a point 1.0 mm away (along the width of the fringes) from the central line*

(i) Which wavelength will be absent in the light coming from the hole?

(ii) Which wavelength(s) will have a strong intensity?



Sol. Given, $d = 0.5$ mm, $D = 50$ cm and $y = 1$ mm

(i) For absent wavelength, use condition of minima,

$$y = (2n+1)\frac{D\lambda}{2d}$$

$$\Rightarrow \quad 10^{-3} = \frac{(2n+1)(0.5)\lambda}{2 \times 0.5 \times 10^{-3}}$$

$$\text{Wavelength, } \lambda = \frac{2 \times 10^{-6}}{(2n+1)} \text{ m} = \frac{2000}{2n+1} \times 10^{-9} \text{ m}$$

$$= \frac{2000}{2n+1} \text{ nm}$$

For $\quad n = 0, \lambda = 2000$ nm

For $\quad n = 1, \lambda = \frac{2000}{3} = 667$ nm

For $\quad n = 2, \lambda = \frac{2000}{5} = 400$ nm

In the range (400 – 700 nm), absent wavelengths are 400 nm, 667 nm.

(ii) For wavelengths of strong intensity, use condition of maxima,

$$y = n\frac{D\lambda}{d} \Rightarrow 10^{-3} = \frac{n(0.5)(\lambda)}{0.5 \times 10^{-3}}$$

$$\text{Wavelength, } \lambda = \frac{10^{-6}}{n} = \frac{1000}{n} \times 10^{-9} \text{ m} = \frac{1000}{n} \text{ nm}$$

For $\quad n = 1, \lambda = 1000$ nm

For $\quad n = 2, \lambda = 500$ nm

For $\quad n = 3, \lambda = 333$ nm

In the range (400 – 700 nm), wavelength having strong intensity is 500 nm.

(III) Fringe width

Distance between two adjacent bright (or dark) fringes is called the fringe width. It is denoted by β.

Thus,

$$\beta = \frac{n\lambda D}{d} - \frac{(n-1)\lambda D}{d} = \frac{\lambda D}{d} \quad \text{or} \quad \boxed{\beta = \frac{\lambda D}{d}} \quad \text{...(vi)}$$

Alternatively, we can show that distance between two successive dark fringes is also $\frac{\lambda D}{d}$.

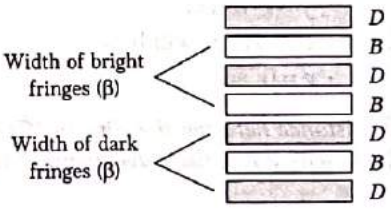


Fig. 10.15 *Width of bright and dark fringes*

Example 10.16 *Two slits in YDSE are placed 2 millimetre from each other. Interference pattern is observed on a screen placed 2 m from the plane of slits. What is the fringe width for a light of wavelength 400 nm?*

Sol. Given, $d = 2$ mm $= 2 \times 10^{-3}$ m, $D = 2$m,

Wavelength, $\lambda = 400$ nm $= 400 \times 10^{-9}$ m

$$\therefore \text{ Fringe width, } \beta = \frac{\lambda D}{d} = \frac{400 \times 10^{-9}}{2 \times 10^{-3}} = 200 \times 10^{-6} \text{m}$$

Example 10.17 *In Young's double slit experiment with monochromatic light, fringes are obtained on a screen placed at some distance from the plane of slits. If the screen is moved by 5 cm towards the slits, then the change in fringe width is 30 μm. If the distance between the slits is 1 mm, then calculate wavelength of the light used.*

Sol. Fringe width, $\beta = \frac{D\lambda}{d}$ ...(i)

When D decreases, then fringe width decreases,

$$\beta - 30 \times 10^{-6} = \frac{(D - 5 \times 10^{-2})\lambda}{d} \quad ...(ii)$$

Subtract Eq. (ii) from Eq. (i), we get

$$30 \times 10^{-6} = \frac{5 \times 10^{-2}\lambda}{d} = \frac{5 \times 10^{-2}\lambda}{1 \times 10^{-3}}$$

wavelength of the light,

$$\lambda = \frac{30 \times 10^{-6} \times 10^{-3}}{5 \times 10^{-2}}$$

$$= 6 \times 10^{-7}\,\text{m} = 600 \times 10^{-9}\,\text{m} = 600\,\text{nm}$$

Important points related with fringe width

As we know that fringe width (β) is the distance between two successive maxima or minima. It is given by

$$\beta = \frac{\lambda D}{d} \quad \text{or} \quad \beta \propto \lambda$$

Two conclusions can be drawn from this relation

(i) If YDSE apparatus is immersed in a liquid of refractive index μ, then wavelength of light and hence fringe width decreases μ times.

(ii) If white light is used in place of a monochromatic light, then coloured fringes are obtained on the screen with red fringes of larger size than that of violet because $\lambda_{red} > \lambda_{violet}$.

But note that centre is still white because path difference there is zero for all colours. Hence, all the wavelengths interfere constructively. At other places light will interfere destructively for those wavelengths for whom path difference is $\lambda/2, 3\lambda/2, ...$, etc, and they will interfere constructively for the wavelengths for whom path difference is $\lambda, 2\lambda, ...$, etc.

Note Shape of fringes on the screen is hyperbolic. But, if the screen is placed at very large distance from slits, then the hyperbola nearly looks straight line in shape.

Example 10.18 *Fringe width in a particular YDSE is measured to be β. What will be the fringe width, if wavelength of the light is doubled, separation between the slits is halved and separation between the screen and slits is tripled?*

Sol. We know that, $\beta = \frac{\lambda D}{d}$

Now, the changed values are $\lambda' = 2\lambda$

$$d' = \frac{d}{2} \Rightarrow D' = 3D$$

The new fringe width will be

$$\beta' = \frac{\lambda' D'}{d'}$$

$$\Rightarrow \quad \beta' = \frac{(2\lambda)(3D)}{\left(\frac{d}{2}\right)} = 12\frac{\lambda D}{d} = 12\beta$$

The fringe width increases to 12 times of the original.

Example 10.19 *Whose fringe width will be larger, the one for red light or the one for yellow light, all other things be the same?*

Sol. We know that, fringe width,

$$\beta = \frac{\lambda D}{d}$$

Now, for same value of D and d, $B \propto \lambda$.

Now, $\lambda_{red} > \lambda_{yellow}$

Hence, $\beta_{red} > \beta_{yellow}$

Hence, fringe width will be greater when red coloured light is used.

Angular width of fringes

Let angular position of nth bright fringe is θ_n and because of its small value $\tan\theta_n \approx \theta_n$

$$\therefore \quad \theta_n = \frac{Y_n}{D} = \frac{nD\lambda/d}{D} = \frac{n\lambda}{d}$$

Similarly, if angular position of $(n+1)$th bright fringe is θ_{n+1}, then

$$\theta_{n+1} = \frac{Y_{n+1}}{D} = \frac{(n+1)\,D\lambda/d}{D} = \frac{(n+1)\lambda}{d}$$

∴ Angular width of a fringe,

$$\theta = \theta_{n+1} - \theta_n = \frac{(n+1)\lambda}{d} - \frac{n\lambda}{d} = \frac{\lambda}{d}$$

Also, $\beta = \frac{D}{d}\lambda$

$$\therefore \quad \boxed{\theta = \frac{\lambda}{d} = \frac{\beta}{D}}$$

It is independent of n *i.e.*, angular width of all fringes are same.

Example 10.20 *Two slits in YDSE are placed 1 mm from each other. Interference pattern is observed on a screen placed 1 m from the plane of slits. What is the angular fringe width for a light of wavelength 400 nm.*

Sol. Given, $d = 1\,\text{mm} = 10^{-3}\,\text{m}$

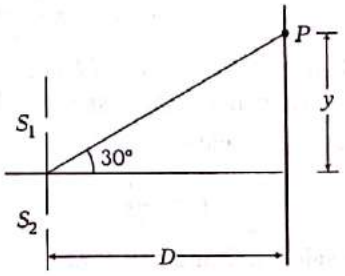
$D = 1\,\text{m}$, $\lambda = 400\,\text{nm} = 400 \times 10^{-9}\,\text{m}$

Angular fringe width,

$$\theta = \frac{\lambda}{d} = \frac{400 \times 10^{-9}}{10^{-3}} = 400 \times 10^{-6}\,\text{rad}$$

Example 10.21 *Two slits are separated by 0.32 mm. A beam of 500 nm strikes the slits producing an interference pattern. Determine the number of maxima observed in the angular range* $-30° < \theta < 30°$.

Sol. Fringe width, $\beta = \frac{\lambda D}{d}$ and $y = \frac{D}{\sqrt{3}}$



Therefore, number of fringe widths in a distance y,

$$n = \frac{y}{\beta} = \frac{d}{\sqrt{3}\lambda}$$

$$= \frac{0.32 \times 10^{-3}}{(\sqrt{3})(500 \times 10^{-9})} = 369.5$$

Therefore, total number of maxima obtained in the angular range $-30° < \theta < 30°$ (including the central one) is

$$N = 2 \times 369 + 1 = 739$$

Example 10.22 *In Young's double slit experiment, separation between slits is 1 mm, distance of screen from slits is 2m. If wavelength of incident light is 500 nm. Determine*

(i) fringe width

(ii) angular fringe width

(iii) distance between 4th bright fringe and 3rd dark fringe.

(iv) if whole arrangement is immersed in water ($\mu_w = 4/3$), *new angular fringe width.*

Sol. Given, $d = 1$ mm, $D = 2$ m, $\lambda = 500$ nm $= 500 \times 10^{-9}$ m

(i) Fringe width,

$$\beta = \frac{D\lambda}{d} = \frac{2 \times 500 \times 10^{-9}}{1 \times 10^{-3}} = 10^{-3} \text{ m} = 1 \text{ mm}$$

(ii) Angular fringe width,

$$\theta_0 = \frac{\lambda}{d} = \frac{500 \times 10^{-9}}{1 \times 10^{-3}} = 5 \times 10^{-4} \text{ rad}$$

(iii) Distance of 4th bright fringe from centre,

$$y_4 = \frac{nD\lambda}{d} = \frac{4D\lambda}{d}$$

Distance of 3rd dark fringe from centre,

$$y_3' = \frac{(2n+1)D\lambda}{d} = \frac{(2 \times 2 + 1)D\lambda}{2d} = \frac{5D\lambda}{2d}$$

Distance between 4th bright fringe and 3rd dark fringe,

$$y_4 - y_3' = \frac{4D\lambda}{d} - \frac{5D\lambda}{2d} = \frac{3D\lambda}{2d}$$

$$= \frac{3}{2}\beta = \frac{3}{2} \times 1 = \frac{3}{2} \text{ mm}$$

(iv) Wavelength in water, $\lambda_w = \frac{\lambda}{\mu_w} = \frac{\lambda}{4/3} = \frac{3}{4}\lambda$

$$(\theta_0)_w = \frac{\theta_0}{\mu_w} = \frac{5 \times 10^{-4}}{4/3} = \frac{15}{4} \times 10^{-4} \text{ rad}$$

Intensity of the fringes

In YDSE (Young's double slit experiment), we are basically interested in finding the resultant intensity at a general point P at a distance y from point O on the screen or at an angle θ from C as shown. Rather we can say we will find intensity I as a function of y or θ.

Since, $d << D$, we can assume that intensity at P due to independent sources S_1 and S_2 are almost equal.

or $$I_1 \approx I_2 = I_0 \text{ (say)}$$

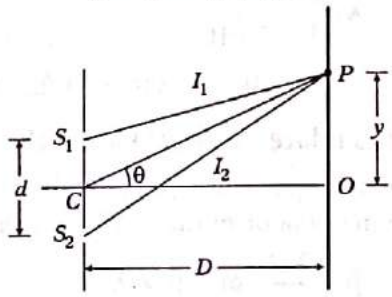


Fig. 10.16 *Intensity in YDSE*

For the two coherent sources S_1 and S_2, the resultant intensity at point P is given by

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

Putting, $$I_1 = I_2 = I_0$$

We have, $$I = I_0 + I_0 + 2\sqrt{I_0 \times I_0}\cos\phi$$

Simplifying the above expression, we get

$$I = 4I_0 \cos^2\frac{\phi}{2}$$

Maximum intensity

From above expression, we can see that intensity is maximum at points, where

$$\cos\frac{\phi}{2} = \pm 1 \quad \text{or} \quad \frac{\phi}{2} = n\pi, n = 0, \pm 1, \pm 2, \ldots$$

or $$\phi = 2n\pi \text{ or } \frac{2\pi}{\lambda}\Delta x = 2n\pi$$

or $\Delta x = n\lambda$, we know that this path difference is for maxima. Thus, intensity of bright points are maximum and given by

$$I_{max} = 4I_0$$

Therefore, we can write $\boldsymbol{I = I_{max}\cos^2\frac{\phi}{2}}$

Minimum intensity

Minimum intensity on the screen is found at points, where

$$\cos\frac{\phi}{2} = 0$$

or
$$\frac{\phi}{2} = \left(n - \frac{1}{2}\right)\pi$$

(where, $n = \pm 1, \pm 2, \pm 3, \ldots$)

or
$$\phi = (2n-1)\,\pi$$

or
$$\frac{2\pi}{\lambda}\,\Delta x = (2n-1)\,\pi$$

$\Rightarrow$
$$\Delta x = (2n-1)\,\frac{\lambda}{2}$$

We know that this path difference corresponds to minima. Thus, intensity of minima are minimum and given by

$$\boldsymbol{I_{min} = 0}$$

Note If both the slits are of equal width, $I_1 \approx I_2 = I_0$ and in that case, $I_{max} = 4I_0$ and $I_{min} = 0$

If the slits are of unequal width, then, $I_1 \neq I_2$ $I_{min} \neq 0$

Distribution of intensity

The distribution of intensity in Young's double slit experiment is shown below.

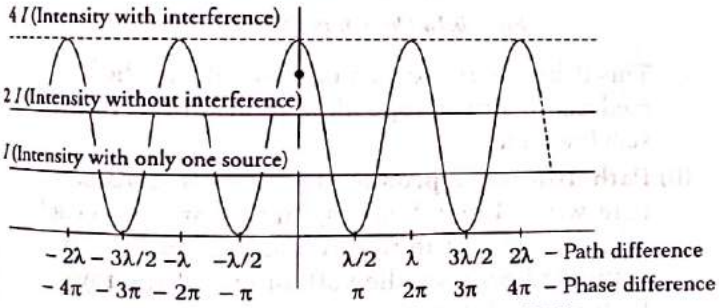


***Fig. 10.17** Distribution of intensity in YDSE*

Example 10.23 *Two coherent sources each emitting light of intensity I_0 interfere, in a medium at a point, where phase difference between them is $\frac{2\pi}{3}$. Then, the resultant intensity at that point would be.*

Sol. Given, phase difference, $\phi = \frac{2\pi}{3}$

and resultant intensity, $I = 4I_0\cos^2\left(\frac{\phi}{2}\right) = 4I_0\cos^2\left(\frac{\phi}{2}\right) = I_0$

Example 10.24 *In a Young's double slit set up using monochromatic light of wavelength λ, the intensity of light at a point, where path difference is 2λ is found to be I_0. What will be the intensity at a point when path difference is $\lambda/3$?*

Sol. We know that path difference is related to the phase difference according to the relation, $\phi = \frac{2\pi x}{\lambda}$

Now, phase difference, where path difference is equal to $\lambda/3$ is equal to $\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{3} = \frac{2\pi}{3}$.

Also, at the point, where path difference $= 2\lambda$

Phase difference will be $\frac{2\pi}{\lambda} \times 2\lambda = 4\pi$

Let individual source intensity be I.

Then,
$$I_0 = 4I\cos^2\frac{4\pi}{2} = 4I$$

Hence,
$$I = \frac{I_0}{4}$$

Now, at point, where $\phi = \frac{2\pi}{3}$

$\Rightarrow$
$$I' = 4I\cos^2\frac{2\pi}{3\times 2}$$

$$I' = 4I\cos^2\left(\frac{\pi}{3}\right) = I = \frac{I_0}{4}$$

Example 10.25 *Maximum intensity in YDSE is I_0. Find the intensity at a point on the screen, where*

(i) the phase difference between the two interfering beams is $\frac{\pi}{3}$.

(ii) the path difference between them is $\frac{\lambda}{4}$.

Sol. (i) We know that, intensity,

$$I = I_{max}\cos^2\left(\frac{\phi}{2}\right)$$

Here, I_{max} is I_0 (*i.e.*, intensity due to independent sources is $I_0/4$). Therefore, at

phase difference, $\phi = \frac{\pi}{3}$ or $\frac{\phi}{2} = \frac{\pi}{6}$

$\therefore$ Intensity, $I = I_0\cos^2\left(\frac{\pi}{6}\right) = \frac{3}{4}I_0$

(ii) Phase difference corresponding to the given path difference, $\Delta x = \frac{\lambda}{4}$ is,

$$\phi = \left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{4}\right) \qquad \left(\because \phi = \frac{2\pi}{\lambda}\Delta x\right)$$

or
$$\frac{\phi}{2} = \frac{\pi}{4}$$

$\therefore$
$$I = I_0\cos^2\left(\frac{\pi}{4}\right) = \frac{I_0}{2}$$

Example 10.26 *In a Young's double slit set up the wavelength of light used is 546 nm. The distance of screen from slits is 1 m. The slit separation is 0.3 mm.*

(i) Compare the intensity at a point P distant 10 mm from the central fringe, where the intensity is I_0.

(ii) Find the number of bright fringes between P and the central fringe.

Sol. Given, $\lambda = 546\text{ nm} = 5.46\times10^{-7}$ m,

$D = 1.0$ m and $d = 0.3$ m $= 0.3\times10^{-3}$m

(i) At a distance, $y = 10$ mm $= 10\times10^{-3}$ m from central fringe, the path difference will be

$$\Delta x = \frac{y\cdot d}{D} = \frac{(10\times10^{-3})(0.3\times10^{-3})}{1.0}$$
$$= 3.0\times10^{-6}\text{ m}$$

The corresponding phase difference between the two beams will be

$$\phi = \frac{2\pi}{\lambda}\cdot\Delta x$$
$$= \left(\frac{2\pi}{5.46\times10^{-7}}\right)(3.0\times10^{-6})\text{ rad}$$
$$\approx 1978°$$
$$\therefore \quad \frac{\phi}{2} = 989°$$
$$\therefore \quad I = I_0\cos^2\frac{\phi}{2} = I_0\cos^2(989) = 3.0\times10^{-4}I_0$$

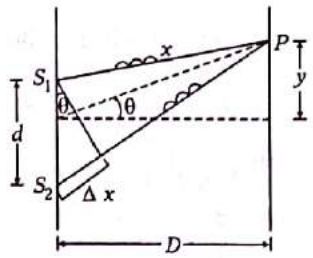
(ii) Fringe width, $\beta = \frac{\lambda D}{d} = \frac{(5.46\times10^{-7})(1.0)}{0.3\times10^{-3}}$ m $= 1.82$ mm

Since, $\frac{y}{\beta} = \frac{10}{1.82} = 5.49$

Therefore, number of bright fringes between P and central fringe will be 5 (excluding the central fringe).

Example 10.27 *Consider an interference arrangement similar to YDSE. Slits S_1 and S_2 are illuminated with coherent microwave sources each of frequency 2 MHz. The sources are synchronised to have zero phase difference. The slits are separated by distance d = 75m. The intensity $I_{(\theta)}$ is measured as a function of θ, where θ is defined as shown in figure.*

If I_0 is maximum intensity, then calculate $I_{(\theta)}$ for (i) $\theta = 0°$, (ii) $\theta = \pi/6$ and (iii) $\theta = \pi/2$.



Sol. Given, $\nu = 2$ MHz $= 10^6$Hz, $d = 75$ m

As, $c = \nu\lambda$

$$\Rightarrow \quad \lambda = \frac{c}{\nu} = \frac{3\times10^8}{2\times10^6} = 150\text{m}$$

Path difference, $\Delta x = d\sin\theta$

Phase difference, $\theta = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda}d\sin\theta = \frac{2\pi}{150}\times75\sin\theta$
$= \pi\sin\theta$

Intensity at point P, $I = I_0\cos^2(0/2) = I_0\cos^2\left(\frac{\pi\sin\theta}{2}\right)$

(i) $\theta = 0, I = I_0$

(ii) $\theta = \frac{\pi}{6} = 30°$

$$I = I_0\cos^2\left(\frac{\pi\sin30°}{2}\right) = I_0\cos^2(\pi/4) = \frac{I_0}{2}$$

(iii) $\theta = \frac{\pi}{2} = 90°, I = 0$

Insertion of transparent slab in YDSE : fringe shift

If a thin transparent plate (or slab) of thickness t and refractive index μ is placed in the path of one of interfering waves in YDSE, then fringe pattern shifts. First we will discuss the optical path, path difference due to the slab and *then shifting in pattern as follows*

(i) **Optical path of a slab** If light travels a distant t in a slab (medium) of refractive index μ in a given time t_0, then in this same time, it travels a distance μt in vacuum.

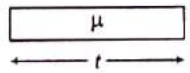


Fig. 10.18 Optical path of a slab

This distance μt is called optical path of the medium in air corresponding to distance t in the slab (medium).

(ii) **Path difference produced by a slab** Consider two light waves 1 and 2 moving in air parallel to each other. If a slab of refractive index μ and thickness t is inserted between the path of one of the rays, then a path difference

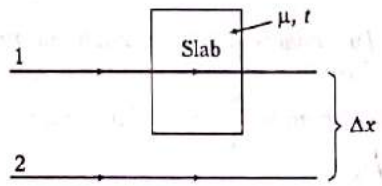


Fig. 10.19 Path difference between rays 1 and 2

$$\Delta x = (\mu - 1)\,t$$

is produced among them.

(iii) **Shifting of fringes** Suppose a glass slab of thickness t and refractive index μ is inserted onto the path of the ray emanating from source S_1, then the whole fringe pattern shifts upwards by a distance $\frac{(\mu-1)\,tD}{d}$. *This can be calculated as follows*

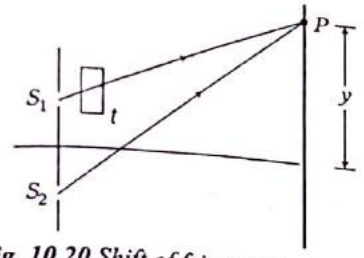


Fig. 10.20 Shift of fringes due to a slab

The net path difference between the two waves reaching at point P is,

$$\Delta x = \Delta x_1 - \Delta x_2 \quad \text{or} \quad \Delta x = \frac{yd}{D} - (\mu - 1)\, t$$

For nth maxima on upper side, $\Delta x = n\lambda$

or $$\frac{yd}{D} - (\mu - 1)\, t = n\,\lambda \;\Rightarrow\; y = \frac{n\lambda D}{d} + \frac{(\mu - 1)\, tD}{d}$$

Earlier it was $\dfrac{n\lambda D}{d}$

$$\therefore \qquad \text{Shift} = \frac{(\mu - 1)\, tD}{d}$$

Following three points are important with regard to above equation

(a) Shift is independent of n, (the order of the fringe) *i.e.*, shift of zero order maximum = shift of 7th order maximum or shift of 5th order maximum = shift of 9th order minimum and so on.

(b) Shift is independent of λ *i.e.*, if white light is used, then shift of red colour fringe = shift of violet colour fringe.

(c) Number of fringes shifted $= \dfrac{\text{Shift}}{\text{Fringe width}}$

$$= \frac{(\mu - 1)\, t\, D/d}{\lambda D/d} = \frac{(\mu - 1)t}{\lambda}$$

These numbers are inversely proportional to λ. This is because shift is same for all colours but fringe width of the colour having smaller value of λ is small, so more number of fringes will shift for this colour.

Example 10.28 *Light travels in a mica sheet of refractive index 1.4 and length 10 cm. Find the optical path equivalent to length of the sheet.*

Sol. Given, length of the sheet, $t = 10$ cm,

refractive index, $\mu = 1.4$

$\therefore$ Optical path $= \mu t = (1.4) \times (10) = 14$ cm

Example 25.29 *Interference fringes are produced by a double slit arrangement and a piece of plane parallel glass of refractive index 1.5 is interposed in one of the interfering beam. If the fringes are displaced through 30 fringe widths for light of wavelength 6×10^{-5} cm, then find the thickness of the plate.*

Sol. Path difference due to the glass slab, $\Delta x = (\mu - 1)t$.

Thirty fringes are displaced due to the slab.

Hence,

$$\Delta x = 30\,\lambda$$

$$\Rightarrow \qquad (\mu - 1)t = 30\,\lambda$$

$$\therefore \qquad t = \frac{30\lambda}{\mu - 1} = \frac{30 \times 6 \times 10^{-5}}{1.5 - 1} = 3.6 \times 10^{-3}\,\text{cm}$$

Example 10.30 *In YDSE using monochromatic light, the fringe pattern shifts by a certain distance on the screen when a mica sheet of refractive index 1.5 and thickness 2 microns is introduced in the path of one of the interfering waves. The mica sheet is then removed and the distance between the plane of slits and the screen is doubled. It is found that the distance between successive maxima (or minima) now is the same as the observed fringe shift upon the introduction of the mica sheet. Calculate the wavelength of the light.*

Sol. Shift $= \dfrac{(\mu - 1)tD}{d}$

New fringe width $= \dfrac{2D\lambda}{d} \;\Rightarrow\; \dfrac{(\mu - 1)tD}{d} = \dfrac{2D\lambda}{d}$

Therefore, wavelength of light,

$$\lambda = \frac{(\mu - 1)t}{2} = \frac{(1.5 - 1) \times 2 \times 10^{-6}}{2}\,\text{m} = 1.0 \times 10^{-6}\,\text{m}$$

Example 10.31 *In YDSE, find the thickness of a glass slab ($\mu = 1.5$) which should be placed before the upper slit S_1, so that the central maximum now lies at a point, where 5th bright fringe was lying earlier (before inserting the slab). Wavelength of light used is 5000 Å.*

Sol. According to the question,

shift = 5 (fringe width)

$$\therefore \qquad \frac{(\mu - 1)\, tD}{d} = \frac{5\lambda D}{d}$$

$\therefore$ Thickness of a glass slab,

$$t = \frac{5\lambda}{\mu - 1} = \frac{25000}{1.5 - 1} = 50000\ \text{Å}$$

Example 10.32 *In a double slit pattern ($\lambda = 6000$ Å), the first order and tenth order maxima fall at 12.50 mm and 14.75 mm from a particular reference point. If λ is changed to 5500 Å, then find the position of zero order and tenth order fringes, other arrangements remaining the same.*

Sol. Distance between 10 fringes is,

$$9\beta = (14.75 - 12.50)\ \text{mm} = 2.25\ \text{mm}$$

$\therefore$ Fringe width, $\beta = 0.25$ mm

As fringe width $\propto \lambda$

When the wavelength is changed from 6000 Å to 5500 Å, then the new fringe width will become,

$$\beta' = \left(\frac{5500}{6000}\right)\beta = \left(\frac{5500}{6000}\right)(0.25)$$

$$\therefore \qquad \beta' = 0.23\ \text{mm}$$

The position of central (or zero order) maxima will remain unchanged. Earlier it was at a position,

$$y_0 = y_1 - \beta$$
$$= 12.50 - 0.25 = 12.25 \text{ mm}$$

The new position of tenth order maxima will be

$$y_{10} = y_0 + 10\beta'$$
$$= 12.25 + (10)(0.23) = 14.55 \text{ mm}$$

Example 10.33 *A double-slit apparatus is immersed in a liquid of refractive index 1.33. It has slit separation of 1 mm and distance between the plane of slits and screen 1.33 m. The slits are illuminated by a parallel beam of light whose wavelength in air is 800 nm.*

(i) Calculate the fringe width.

(ii) One of the slits of the apparatus is covered by a thin glass sheet of refractive index 1.53. Find the smallest thickness of the sheet to bring the adjacent minima on the axis.

Sol. Given, $\mu_m = \frac{4}{3}$, $d = 1\text{mm}$, $D = \frac{4}{3}\text{m}$ and $\lambda = 630$ nm

Wavelength, $\lambda_m = \frac{\lambda}{\mu_m} = \frac{800}{4/3} = 600\text{ nm}$

(i) Fringe width, $\beta = \frac{D\lambda_m}{d} = \frac{\frac{4}{3} \times 600 \times 10^{-9}}{10^{-3}} = 800 \times 10^{-6}\text{m}$

(ii) Let thickness of plate t, $\mu_p = 1.53$

Refractive index, ${}_m\mu_p = \frac{\mu_p}{\mu_m} = \frac{1.53}{4/3}$

Shift due to plate $= \frac{({}_m\mu_p - 1)tD}{d} \Rightarrow \frac{({}_m\mu_p - 1)tD}{d} = \frac{D\lambda_m}{2d}$

Thickness of sheet, $t = \frac{\lambda_m}{2({}_m\mu_p - 1)} = \frac{600 \times 10^{-9}}{2\left(\frac{1.53}{4/3} - 1\right)}$

$$= 2033 \times 10^{-9}\text{m}$$

Example 10.34 *A monochromatic light of $\lambda = 500$ nm is incident on two identical slits separated by a distance of 5×10^{-4} m. The interference pattern is seen on a screen placed at a distance of 1 m from the plane of slits. A thin glass plate of thickness 2.5×10^{-6} m and refractive index $\mu = 1.5$ is placed between one of the slits and the screen. Find the intensity at the centre of the screen, if the intensity is I_0 in the absence of the plate. Also, find the lateral shift of the central maxima and number of fringes crossed through centre.*

Sol. Given, $\lambda = 500$ nm, $d = 5 \times 10^{-4}$ m, $D = 1$ m

$t = 2.5 \times 10^{-6}$m and $\mu = 1.5$

Path difference due to plate,

$$\Delta x = (\mu - 1)t = (1.5 - 1) \times 2.5 \times 10^{-6} = 1.25 \times 10^{-6}\text{ m}$$

Phase difference,

$$\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{500 \times 10^{-9}} \times 0.75 \times 10^{-6} = 5\pi$$

Intensity, $I = I_0 \cos^2(\phi/2) = I_0 \cos^2(2.5\pi) = 0$

Lateral shift,

$$y_0 = \frac{(\mu - 1)tD}{d} = \frac{1.25 \times 10^{-6} \times 1}{5 \times 10^{-4}}$$
$$= 0.25 \times 10^{-2}\text{m} = 2.5\text{ mm}$$

Number of fringes crossing the centre,

$$\frac{y_0}{\beta} = \frac{\frac{(\mu - 1)tD}{d}}{\frac{D\lambda}{d}} = \frac{(\mu - 1)t}{\lambda} = \frac{1.25 \times 10^{-6}}{500 \times 10^{-9}} = 2.5$$

(iii) **Path difference due to insertion of two slabs in YDSE** In the problems of YDSE, our first task is to find path difference. Let us take a typical case. In the figure shown, path of ray 1 is more deviated than path of ray by a distance.

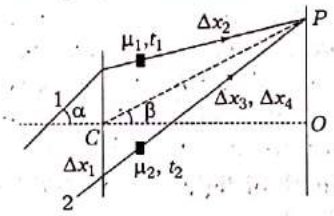


Fig. 10.21 *Insertion of two slabs in a YDSE*

$\Delta x_1 = d \sin\alpha$ and $\Delta x_2 = (\mu_1 - 1)t_1$.

and path of ray 2 is greater than path of ray 1 by a distance.

$$\Delta x_3 = d \sin\beta \text{ and } \Delta x_4 = (\mu_2 - 1)t_2$$

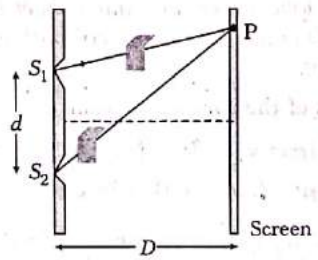
Therefore, net path difference is

$$\Delta x = (\Delta x_1 + \Delta x_2) - (\Delta x_3 + \Delta x_4)$$

Example 10.35 *In a double slit arrangement fringes are produced using light of wavelength 4800Å. One slit is covered by a thin plate of glass of refractive index 1.4 and the other with another glass plate of same thickness but of refractive index 1.7. By doing, so the central bright shifts to original 4th bright fringe from centre. Find the thickness of the glass plate.*

Sol. As we know that shift due to insertion of a slab is

$$\Delta x = \frac{\beta}{\lambda}(\mu - 1)t$$



Shift due to one plate, $\Delta x_1 = \frac{\beta}{\lambda}(\mu_1 - 1)t$

Shift due to another path, $\Delta x_2 = \frac{\beta}{\lambda}(\mu_2 - 1)t$

Net shift, $\Delta x = \Delta x_2 - \Delta x_1 = \frac{\beta}{\lambda}(\mu_2 - \mu_1)t$ ...(i)

Here, it is given that, $\Delta x = 4\beta$ ...(ii)

Hence, $4\beta = \frac{\beta}{\lambda}(\mu_1 - \mu_2)t$

$$\Rightarrow \quad t = \frac{4\lambda}{(\mu_2 - \mu_1)} = \frac{4 \times 4800 \times 10^{-10}}{(1.7 - 1.4)}$$

$$= 6.4 \times 10^{-6}\text{m} = 6.4\,\mu\text{m}$$

Interference in thin films

Interference effects are commonly observed in thin films, such as thin layers of oil on water or the thin surface of a soap bubble. The varied colours observed when white light is incident on such films as the result of the interference of waves reflected from the two surfaces of the film.

Consider a film of uniform thickness t and index of refraction μ, as shown in figure. Let us assume that the light rays travelling in air are nearly normal to the two surfaces of the film. To determine whether the reflected rays interfere constructively or destructively, we first note the following facts.

(i) The wavelength of light in a medium whose refractive index is μ is, $\lambda_\mu = \frac{\lambda}{\mu}$

where, λ is the wavelength of light in vacuum (or air).

(ii) If a wave is reflected from a denser medium, then it undergoes a phase change of 180°. Let us apply these rules to the film shown in figure. The path difference between the two rays 1 and 2 is $2t$ while the phase difference between them is 180°. Hence, condition of **constructive** interference will be

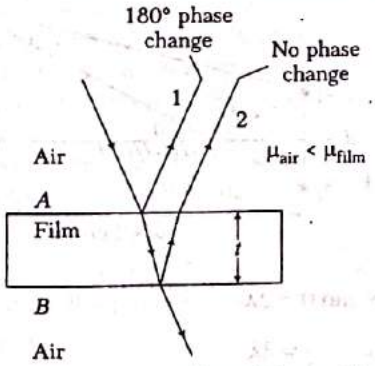


Fig. 10.22 Interference in light reflected from a thin film is due to a combination of rays reflected from the upper and lower surfaces of the film

$$2t = (2n - 1)\frac{\lambda_\mu}{2}$$

where, $n = 1, 2, 3, \ldots$

or, $2\mu t = \left(n - \frac{1}{2}\right)\lambda$ as $\lambda_\mu = \frac{\lambda}{\mu}$

Similarly, condition of destructive interference will be

$$2\mu t = n\lambda, \text{ where } n = 0, 1, 2, \ldots$$

Example 10.36 *Calculate the minimum thickness of a soap bubble film ($\mu = 1.33$) that results in constructive interference in the reflected light, if the film is illuminated with light whose wavelength in free space is $\lambda = 600$ nm.*

Sol. For constructive interference in case of soap film,

$$2\mu t = \left(n - \frac{1}{2}\right)\lambda \qquad \text{(where, } n = 1, 2, 3, \ldots)$$

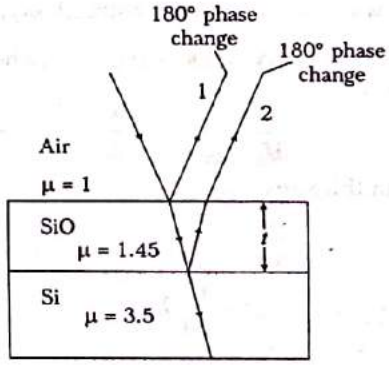
For minimum thickness t, $n = 1$ or $2\mu t = \lambda/2$

The minimum thickness of a soap bubble,

$$t = \frac{\lambda}{4\mu} = \frac{600}{4 \times 1.33} = 112.78 \text{ nm}$$

Example 10.37 *In solar cells, a silicon solar cell ($\mu = 3.5$) is coated with a thin film of silicon monoxide SiO ($\mu = 1.45$) to minimise reflective losses from the surface. Determine the minimum thickness of SiO that produces the least reflection at a wavelength of 550 nm, near the centre of the visible spectrum.*

Sol. The reflected light is a minimum when rays 1 and 2 (shown in figure) meet the condition of destructive interference. Both rays undergo a 180° phase change upon reflection. The net change in phase due to reflection is therefore zero and the condition for a reflection minimum requires a path difference of $\lambda_\mu/2$.



Hence, $2t = \frac{\lambda}{2\mu}$

Thus, the minimum thickness of silicon monoxide

$$t = \frac{\lambda}{4\mu} = \frac{550}{4\,(1.45)} = 94.8 \text{ nm}$$

Note When we observe this film fringes through refracted light waves, then condition for maxima is $2t = n\lambda\mu$ and for maxima,

$2t = (2n - 1)\frac{\lambda\mu}{2}$, where $n = 1, 2, 3, \ldots$

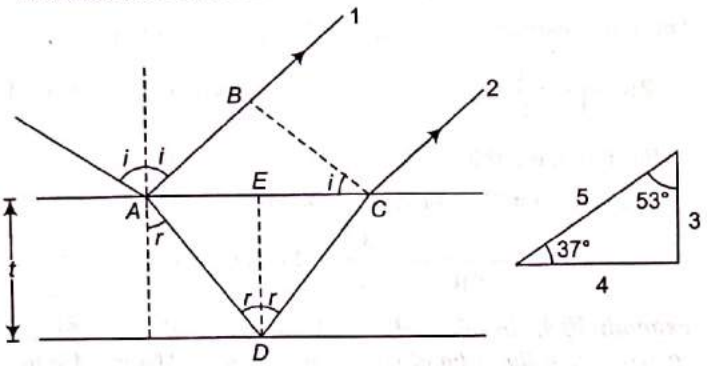
Example 10.38 *A parallel beam of white light falls on a thin film whose refractive index is equal to 4/3. The angle of incidence $i = 53°$. What must be the minimum film thickness, if the reflected light is to be coloured yellow (λ of yellow $= 0.6\,\mu m$) most intensively?* (tan 53° = 4/3)

Sol. From the relation,

$$\mu = \frac{\sin i}{\sin r} \Rightarrow \frac{4}{3} = \frac{\sin 53°}{\sin r}$$

We have, $r \approx 37°$

Interference has to be taken place between rays 1 and 2. They have already a phase difference of π, because ray 1 is reflected from a denser medium.



They have a path difference of

$$\Delta x = 2\mu\,(AD) - AB$$

$$= 2\mu\left(\frac{t}{\cos r}\right) - 2t\,(\tan r)\,(\sin i)$$

$$= 2\left(\frac{4}{3}\right)\left(\frac{5}{3}\right)t - 2(t)\left(\frac{3}{4}\right)\left(\frac{4}{5}\right) = \frac{32}{15}t$$

For the two waves to interfere constructively,

$$\Delta x = (2n-1)\frac{\lambda}{2} \qquad \text{(where, } n = 1, 2, 3, \ldots)$$

or $$\frac{32}{15}t = (2n-1)\frac{\lambda}{2}$$

For minimum thickness, $n = 1$

or $$\frac{32}{15}t = \frac{\lambda}{2}$$

or $$t = \frac{15}{64}\lambda$$

$$= \frac{15 \times 0.6}{64}\,\mu\text{m} = 0.14\ \mu\text{m}$$

Lloyd's mirror

A plane glass plate (acting as a mirror) is illuminated at almost grazing incidence by a light from a slit S_1. A virtual image S_2 of S_1 is formed closed to S_1 by reflection and these two act as coherent sources. The expression giving the fringe width is the same as for the double slit but the fringe system differs in one important respect.

In Lloyd's mirror, if the point P, for example, is such that the path difference $S_2P - S_1P$ is a whole number of wavelengths, the fringe at P is dark not bright. This is due to 180° phase change which occurs when light is reflected from a denser medium.

This is equivalent to adding an extra half wavelength to the path of the reflected wave. At grazing incidence a fringe is formed at O, where the geometrical path difference between the direct and reflected waves is zero and it follows that it will be dark rather than bright.

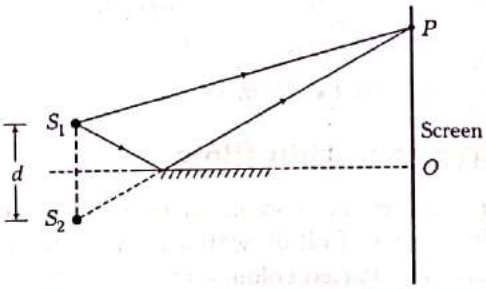


Fig. 10.23 Reflection of light from Lloyd's mirror

Thus, whenever there exists a phase difference of π between the two interfering beams of light, conditions of maximas and minimas are interchanged *i.e.*,

$$\Delta x = n\lambda \qquad \text{(for minimum intensity)}$$

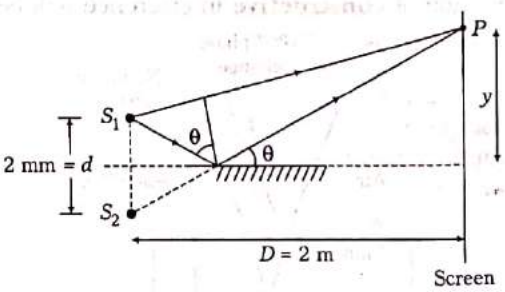
and $$\Delta x = (2n-1)\lambda/2 \text{ (for maximum intensity)}$$

where, $n = 1, 2, 3, \ldots$

Example 10.39 *Light source of wavelength 400 nm used in a Lloyd's mirror, lies 1 mm above the plane mirror. Find the distance of first minima above the mirror. Distance of the screen from the plane of the sources is 2 m.*

Sol. Consider the formation of minimum at point P as shown in figure. Path difference between the rays reaching at point P is

$$\Delta x = \frac{d}{2}\sin\theta = \lambda \qquad \text{(for first minimum)}$$



$\Rightarrow$ $$d \times \tan\theta \approx 2\lambda \qquad (\because \sin\theta \approx \tan\theta)$$

$\Rightarrow$ $$d \times \frac{y}{D} = 2\lambda$$

$\Rightarrow$ $$y = \frac{D}{d}(2\lambda)$$

$\therefore$ Distance of first minima above the mirror,

$$Y = \frac{2}{2 \times 10^{-3}} \times 2 \times 400 \times 10^{-9} = 0.8 \text{ mm}$$

Check point 10.2

1. In Young's double slit experiment, an interference pattern is obtained for $\lambda = 6000$ Å, coming from two coherent sources S_1 and S_2. At certain point P on the screen third dark fringe is formed. Then, the path difference $S_1P - S_2P$ in microns is
(a) 0.75 (b) 1.5
(c) 3.0 (d) 4.5

2. In Young's double slit experiment, when two light waves form third minimum intensity, they have
(a) phase difference of 3π (b) phase difference of $\frac{5\pi}{2}$
(c) path difference of 3λ (d) path difference of $\frac{5\lambda}{2}$

3. Two slits are separated by a distance of 0.5 mm and illuminated with light of $\lambda = 6000$ Å. If the screen is placed 2.5 m from the slits. The distance of the third bright image from the centre will be
(a) 1.5 mm (b) 3 mm (c) 6 mm (d) 9 mm

4. Fringe width decreases with increase in
(a) λ (b) D
(c) d (d) None of these

5. In Young's double slit experiment, the fringe width will remain same, if (D = distance between screen and plane of slits, d = separation between two slits and λ = wavelength of light used)
(a) Both λ and D are doubled
(b) Both d and D are doubled
(c) D is doubled but d is halved
(d) λ is doubled but d is halved

6. Interference was observed in interference chamber when air is present. Now, the chamber is evacuated and if the same light is used, then for the same arrangement
(a) no interference pattern will be obtained
(b) exactly same interference pattern will be obtained with better contrast
(c) the fringe width is slightly decreased
(d) the fringe width is slightly increased

7. Monochromatic green light of wavelength 5×10^{-7} m illuminates a pair of slits 1 mm apart. The separation of bright lines on the interference pattern formed on a screen 2 m away is
(a) 0.25 mm (b) 0.1 mm
(c) 1.0 mm (d) 0.01 mm

8. In double slit experiment, for light of which colour, the fringe width will be minimum
(a) violet (b) red (c) green (d) yellow

9. The Young's double slit experiment is performed with blue and green light of wavelengths 4360 Å and 5460 Å, respectively. If x is the distance of 4th maxima from the central one, then
(a) x(blue) = x(green) (b) x (blue) > x(green)
(c) x(blue) < x(green) (d) $\frac{x\text{ (blue)}}{x\text{ (green)}} = \frac{5460}{4360}$

10. In Young's double slit experiment, green light ($\lambda = 5461$ Å) is used and 60 fringes were seen in the field view. Now, sodium light is used ($\lambda = 5890$ Å), then number of fringes observed are
(a) 40 (b) 60
(c) 50 (d) 55

11. In Young's double slit experiment, if the monochromatic source of light is replaced by white light, then one observes
(a) no interference fringe pattern
(b) coloured fringes
(c) black and white fringes
(d) white central fringe surrounded by few coloured fringes on either side

12. In Young's double slit experiment, the ratio of intensities of bright and dark fringes is 9. This means that
(a) the intensities of individual sources are 5 and 4 units respectively
(b) the intensities of individual sources are 4 and 1 units respectively
(c) the ratio of their amplitudes is 3
(d) the ratio of their amplitudes is 4

13. What happens to the fringe pattern, if in the path of one of the slits a glass plate which absorbs 50% energy is interposed?
(a) The bright fringes become bright and dark fringes become darker
(b) No fringes are observed
(c) The fringe width decreases
(d) None of the above

14. In Young's double slit experiment, instead of taking slits of equal widths, one slit is made twice as wide as the other. Then, in the interference pattern
(a) the intensities of both the maxima and the minima increase
(b) the intensity of maxima increases and the minima has zero intensity
(c) the intensity of maxima decreases and that of the minima increases
(d) the intensity of maxima decreases and the minima has zero intensity

15. Intensity at centre in YDSE is I_0. If one slit is covered, then intensity at centre will be
(a) I_0
(b) $2I_0$
(c) $I_0/4$
(d) $I_0/2$

16. When a thin transparent plate of thickness t and refractive index μ is placed in the path of one of the two interfering waves of light, then the path difference between two waves is
(a) $(\mu + 1)t$ (b) $(\mu - 1)t$
(c) $\frac{(\mu + 1)}{t}$ (d) μt

17. A thin mica sheet of thickness 2×10^{-6} m and refractive index ($\mu = 1.5$) is introduced in the path of the light from upper slit. The wavelength of the wave used is 5000 Å. The central bright maximum will shift
(a) two fringes upward (b) two fringes downward
(c) ten fringes upward (d) None of these

18. In Young's double slit experiment, the aperture screen distance is 2 m. The fringe width is 1 mm. Light of 600 nm is used. If a thin plate of glass ($\mu = 1.5$) of thickness 0.06 mm is placed over one of the slits, then there will be a lateral displacement of the fringes by
(a) 0 cm (b) 5 cm
(c) 10 cm (d) 15 cm

19. Interference fringes were produced in Young's double slit experiment using light of wavelength 5000 Å. When a film of thickness 2.5×10^{-3} cm was placed in front of one of the slits, then the fringe pattern shifted by a distance equal to 20 fringe widths. The refractive index of the material of the film is
(a) 1.25 (b) 1.35
(c) 1.4 (d) 1.5

20. What is the minimum thickness of a thin film ($\mu = 1.2$) that results in constructive interference in the reflected light? If the film is illuminated with light whose wavelength in free space is $\lambda = 500$ nm?
(a) 104 nm (b) 200 nm (c) 300 nm (d) 400 nm

3 Diffraction of light

The phenomenon of bending of light around the corners of an obstacle/aperture of the size of the wavelength of is called diffraction. Other waves, such as sound waves and water waves, also have this property of spreading when passing through apertures or by sharp edges.

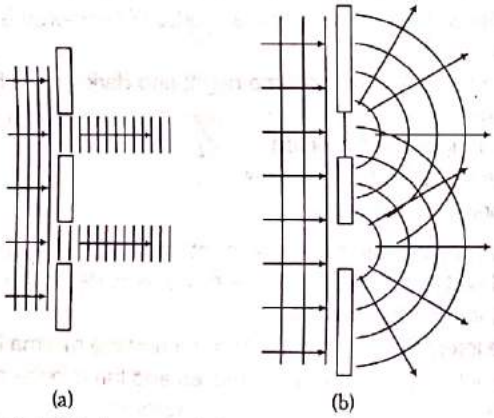


Fig. 10.24 (a) If light waves did not spread out after passing through the slits, no interference would occurs. (b) The light waves from the two slits overlap as they spread out, filling what we expect to be shadowed regions with light and producing interference fringes.

Diffraction pattern

In general, diffraction occurs when waves pass through small openings, around obstacles, or past sharp edges, as shown in figure. When an opaque object is placed between a point source of light and a screen, no sharp boundary exists on the screen between a shadowed region and an illuminated region. The illuminated region above the shadow of the object contains alternating light and dark fringes. Such a display is called a **diffraction pattern.**

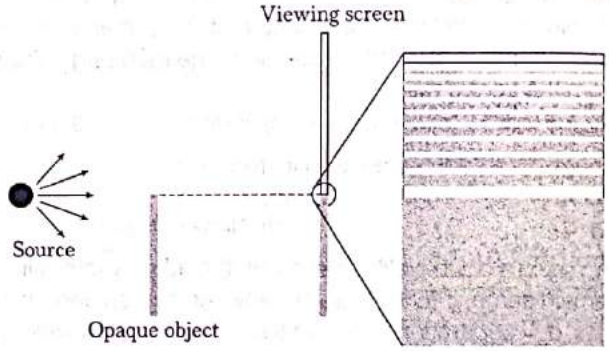


Fig. 10.25 Light from a small source passes by the edge of an opaque object. We might expect no light to appear on the screen below the position of the edge of the object. In reality, light bends around the top edge of the object and enter this region. Because of these effects, a diffraction pattern consisting of bright and dark fringes appears in the region above the edge of the object.

Types of diffraction of light

The phenomenon of diffraction is divided mainly in the following two classes

(i) Fresnel diffraction

If source and screen are at finite distance from the obstacle or aperture, then diffraction observed is known Fresnel diffraction (as shown in the figure).

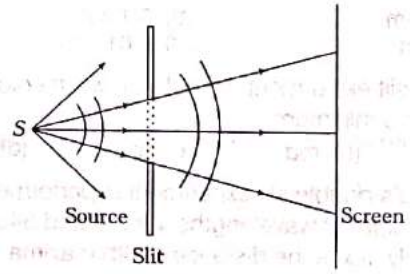


Fig. 10.26 Fresnel diffraction, through a slit

(ii) Fraunhofer diffraction

When all the rays passing through a narrow slit are approximately parallel to one another. This can be achieved experimentally either by placing the screen far from the opening used to create the diffraction or by using a converging lens to focus the rays once they pass through the opening, as shown in figure.

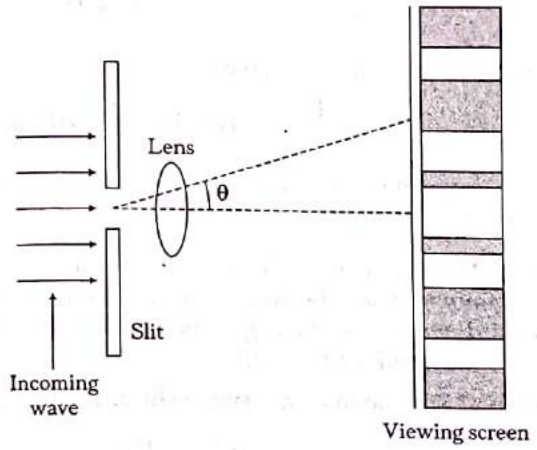


Fig. 10.27 Fraunhofer diffraction pattern of a single slit. The pattern consists of a central bright fringe flanked by much weaker maxima alternating with dark fringes.

The diffraction produced in such an arrangement is called Fraunhofer diffraction.

Fraunhofer Diffraction of light due to a single narrow slit

Suppose a plane wavefront is incident on a slit AB (of width a). Each and every part of the exposed part of the plane wavefront (*i.e.*, every part of the slit) acts as a source of secondary wavelets spreading in all directions. The diffraction is obtained on a screen placed at a large distance. (In practice, this condition is achieved by placing the screen at the focal plane of a converging lens placed just after the slit).

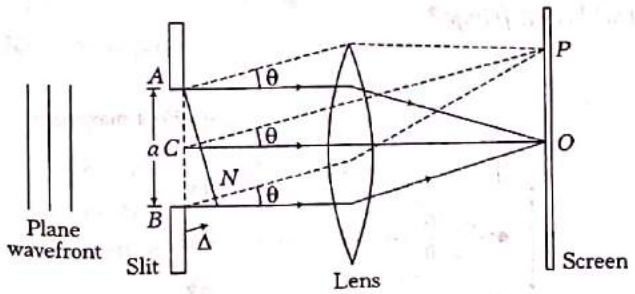


Fig. 10.28 A plane wavefront is incident on the slit AB, which forms a diffraction pattern on the screen

The diffraction pattern consists of a central bright fringe (central maxima) surrounded by dark and bright lines (called secondary minima and maxima).

At point O on the screen, the central maxima is obtained. The wavelets originating from points A and B meets in the same phase at this point, hence at O, intensity is maximum.

Secondary minima For obtaining nth secondary minima at point P on the screen, path difference between the diffracted waves $a\sin\theta = \pm n\lambda$, where $n = 1, 2, 3, \ldots$.

(i) Angular position of nth secondary minima,

$$\sin\theta \approx \theta = \frac{n\lambda}{a}$$

(ii) Distance of nth secondary minima from central maxima,

$$y_n = D \cdot \theta$$

$$\boxed{y_n = \frac{n\lambda D}{a} = \frac{n\lambda f}{a}}$$

where, D = distance between slit and screen
$f \approx D$ = focal length of converging lens.

Secondary maxima For nth secondary maxima at P on the screen.

Path difference $= a\sin\theta = (2n+1)\dfrac{\lambda}{2}$;

where $n = 1, 2, 3, \ldots$

(i) Angular position of nth secondary maxima,

$$\sin\theta \approx \theta \approx \frac{(2n+1)\lambda}{2a}$$

(ii) Distance of nth secondary maxima from central maxima,

$$y_n = D \cdot \theta = \frac{(2n+1)\lambda D}{2a}$$

$$\boxed{y_n = \frac{(2n+1)\lambda f}{2a}}$$

Example 10.40 *A screen is placed 50 cm from a single slit, which is illuminated with 6000 Å light. If distance between first and third minima in the diffraction pattern is 3 mm. What is the width of the slit?*

Sol. Given, $D = 50$ cm and $\lambda = 600$ nm

Distance between first and third minima

$$y_3 - y_1 = \frac{3D\lambda}{a} - \frac{D\lambda}{a} = \frac{2D\lambda}{a}$$

$$\Rightarrow \quad 3 \times 10^{-3} = \frac{2 \times 0.5 \times 600 \times 10^{-9}}{a}$$

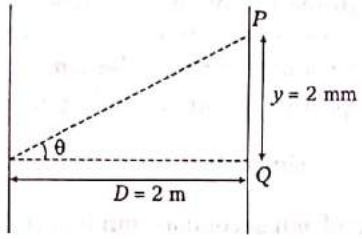
Therefore, width of the slit,

$$a = 200 \times 10^{-6}\,\text{m} = 0.2\,\text{mm}$$

Example 10.41 *A beam of light of wavelength 400 nm falls on a narrow slit and the resulting diffraction pattern is observed on a screen 2 m away from the slit. It is observed that 2nd order minima occurs at a distance of 2 mm from the position of central maxima. Find the width of the slit.*

Sol. Given, $\lambda = 400\text{nm} = 400 \times 10^{-9}\text{m}$,

$D = 2\text{m}$, $y = 2\text{mm} = 2 \times 10^{-3}\text{m}$



Let point P is the point of second order minima.

Now, $\sin\theta = \tan\theta \approx \theta = \dfrac{2 \times 10^{-3}}{2} = 10^{-3}$

Let the slit separation be a.

Now, for second order minima, $\theta = \dfrac{2\lambda}{a} = 10^{-3}$

$$\Rightarrow \quad \frac{2 \times 400 \times 10^{-9}}{a} = 10^{-3}$$

Thus, we obtain $a = 0.8$ mm

Example 10.42 *A beam of light of wavelength 600 nm from a distant source falls on a single slit 2 mm wide and the resulting diffraction pattern is observed on a screen 2m away. What is the distance between the first dark fringes on either side of central bright fringe?*

Sol. Given, $\lambda = 600$ nm, $a = 2 \times 10^{-3}$m and $D = 2$m

Distance of first dark fringe from centre, $y_1 = \dfrac{D\lambda}{a}$

Distance between the first dark fringes on either side of the central bright fringe,

$$2y_1 = \frac{2D\lambda}{a} = \frac{2 \times 2 \times 600 \times 10^{-9}}{2 \times 10^{-3}} = 1200 \times 10^{-6}\text{m}$$

$$= 1.2 \times 10^{-3}\text{m} = 1.2\text{mm}$$

Example 10.43 *A slit of width d is illuminated by red light of wavelength 6500Å. For what value of d will the first maximum fall at angle of diffraction of 30°.*

Sol. For first secondary maximum of the diffraction pattern,

$$d \sin\theta = \frac{3\lambda}{2}$$

$$\therefore \quad d = \frac{3\lambda}{2\sin\theta} = \frac{3 \times 6{,}500 \times 10^{-10}\text{m}}{2 \times \sin 30°} = 1.95 \times 10^{-6}\text{ m}$$

Example 10.44 *A beam of light of wavelength 600 nm from a distant source falls on a single slit 1.0 mm wide and the resulting diffraction pattern is observed on a screen 2 m away. What is the distance between the second bright fringe on either side of the central bright fringe?*

Sol. For the diffraction at a single slit, the position of maxima is given by

$$a \sin\theta = (2n+1)\frac{\lambda}{2}$$

For small value of θ, $\sin\theta \approx \theta = \dfrac{y}{D}$

For second bright fringe, $n = 2$.

$$\therefore \quad a \times \frac{y}{D} = \frac{5\lambda}{2} \text{ or } y = \frac{5\lambda}{2a}D$$

Substituting the values, we have

$$y = \frac{5 \times 2 \times 6 \times 10^{-7}}{2 \times 1 \times 10^{-3}} = 3.0 \times 10^{-3}\text{m} = 3.0\text{mm}$$

Distance between second maxima on either side of central maxima $= 2y = 6.0$ mm.

Example 10.45 *A screen is placed 50 cm from a single slit, which is illuminated with 6000 Å light. If distance between the first and third maxima in the diffraction pattern is 3.0 mm, then what is the width of the slit?*

Sol. Position of nth maxima on a single slit diffraction pattern is given by

$$a \sin\theta = (2n+1)\frac{\lambda}{2}$$

For small value of θ, $\sin\theta \approx \theta = \dfrac{y}{D}$

$$\therefore \quad \frac{y \cdot a}{D} = (2n+1)\frac{\lambda}{2}$$

or

$$y = \frac{(2n+1)\lambda}{2a}D$$

$\therefore$ Distance between third order maxima and first order maxima will be

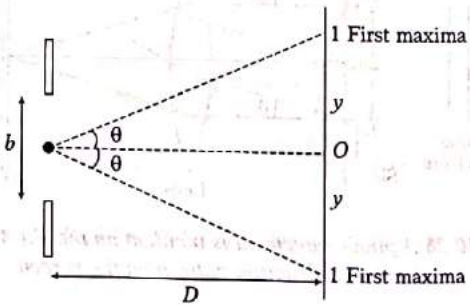
$$\Delta y = y_3 - y_1 = \frac{2 \times (3-1)(\lambda D)}{2a} = \frac{2\lambda D}{a}$$

Substituting the values, we have

$$\Delta y = \frac{(2)(6 \times 10^{-7})(0.5)}{3 \times 10^{-3}} = 2 \times 10^{-4}\text{m} = 0.2\text{ mm}$$

Example 10.46 *A beam of light of wavelength 600 nm from a distance source falls on a single slit 1 mm wide and resulting diffraction pattern is observed on a screen 2 m away. What is the distance between the first bright fringes on either side of the central bright fringe?*

Sol.



In case of diffraction, apart from central maxima the position of the first maxima is given by $\left(a \sin\theta = \frac{3\lambda}{2}\right)$.

$$\Rightarrow \quad \sin\theta = \frac{3\lambda}{2a} \quad \text{...(i)}$$

From geometry of the figure, $\sin\theta = \frac{y}{D}$ ...(ii)

From Eqs. (i) and (ii), we get

$$y = \frac{3\lambda D}{2a} = \frac{3 \times 600 \times 10^{-9}\text{m} \times 2\text{m}}{2 \times 1 \times 10^{-3}\text{m}} = 1.8 \text{ mm}$$

$\therefore$ The distance between the first maxima on either side $= 2y = 3.6$ mm

Example 10.47 *In a single slit diffraction experiment first minimum for $\lambda_1 = 660$ nm coincides with first maxima for wavelength λ_2. Calculate λ_2.*

Sol. Position of minima in diffraction pattern is given by

$$a \sin\theta = n\lambda$$

For first minima of λ_1, we have

$$a \sin\theta_1 = (1)\lambda_1 \quad \text{or} \quad \sin\theta_1 = \frac{\lambda_1}{a} \quad \text{...(i)}$$

The first maxima approximately lies between first and second maxima. For wavelength λ_2 its position will be

$$a \sin\theta_2 = \frac{3}{2}\lambda_2$$

$$\therefore \quad \sin\theta_2 = \frac{3\lambda_2}{2a} \quad \text{...(ii)}$$

The two will coincide, if

$$\theta_1 = \theta_2 \quad \text{or} \quad \sin\theta_1 = \sin\theta_2$$

$$\therefore \quad \frac{\lambda_1}{a} = \frac{3\lambda_2}{2a}$$

or $$\lambda_2 = \frac{2}{3}\lambda_1 = \frac{2}{3} \times 660 \text{ nm} = 440 \text{ nm}$$

Example 10.48 *A lens of focal length 1m forms Fraunhofer diffraction pattern of a single slit of width 0.04 cm in its focal plane. The incident light contains two wavelength λ_1 and λ_2. It is found that the fourth minimum corresponding to λ_1 and the fifth minimum corresponding to λ_2 occur at the same point 0.5 cm from the central maximum. Compute λ_1 and λ_2.*

Sol. The angular position of nth minima,

$$a \sin\theta = n\lambda$$

For λ_1, $a \sin\theta_1 = 4\lambda_1$ ...(i)

For λ_2, $a \sin\theta_2 = 5\lambda_2$ ...(ii)

we have, $\theta_1 = \theta_2 = \theta$

$$a \sin\theta = 4\lambda_1 = 5\lambda_2 \quad \text{...(iii)}$$

$$y = f\theta \Rightarrow \theta = y/f \quad [\because D = f]$$

Separation $y = 0.5$ cm $= 5 \times 10^{-3}$m, focal length, $f = 1$m

$$\theta = \frac{y}{f} = \frac{5 \times 10^{-3}}{1} = 0.005 \text{ rad}$$

$$\text{Wavelength, } \lambda_1 = \frac{a \sin\theta}{4} = \frac{a\theta}{4}$$

$$= \frac{0.04 \times 10^{-2} \times 0.005}{4}$$

$$= 5 \times 10^{-7} \text{ m} = 500 \text{ nm}$$

$$\text{Wavelength, } \lambda_2 = \frac{4\lambda_1}{5} = \frac{4 \times 500}{5} = 400 \text{ nm}$$

Width of central maxima

Diffraction pattern can be graphically shown as below.

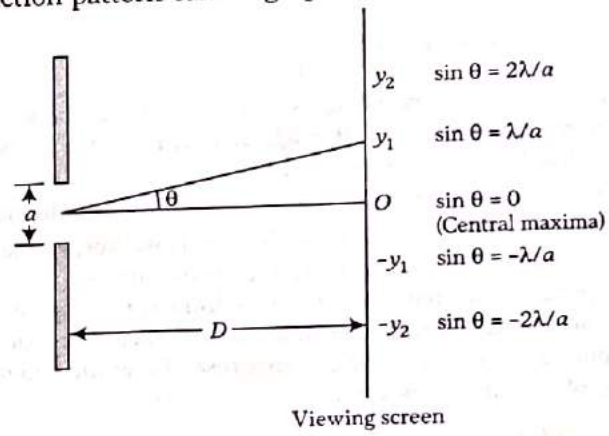


***Fig. 10.29** Fraunhofer diffraction pattern from a single slit of width a. The positions of two minima on each side of the central maximum are labelled*

The point O corresponds to the position of central maxima. And the position $-3\lambda, -2\lambda, -\lambda, \lambda, 2\lambda, 3\lambda, \ldots$ are secondary minima. The above conditions for diffraction maxima and minima are exactly reverse of mathematical conditions for interference maxima and minima. The width of central maximum is the distance between first secondary minimum on either side of the central bright fringe O.

For first secondary minimum,

$$a \sin\theta = \lambda \quad \text{or} \quad \sin\theta = \frac{\lambda}{a} \quad \text{...(i)}$$

If θ is small, $\sin\theta \approx \theta = \frac{y}{D}$ ...(ii)

From Eqs. (i) and (ii), we get

$$\frac{y}{D} = \frac{\lambda}{a}$$

or $$y = \frac{D\lambda}{a}$$

Width of central maximum $= 2y = \frac{2D\lambda}{a}$

As the slit width a increases, width of central maximum decreases.

Angular width of central maxima, $2\theta = \frac{2\lambda}{a}$.

Example 10.49 *Light of wavelength 580 nm is incident on a slit having a width of 0.300 mm. The viewing screen is 2.00 m from the slit. Find the positions of the first dark fringe and the width of the central bright fringe.*

Sol. The two dark fringes that flank the central bright fringe correspond to $n = \pm 1$ in Eq. (i), hence we find that

$$\sin\theta = \pm\frac{\lambda}{a} = \pm\frac{5.80\times 10^{-7}\text{ m}}{0.300\times 10^{-3}\text{ m}} = \pm 1.93\times 10^{-3}$$

From the triangle in figure note that $\tan\theta = y_1/D$. Because θ is very small, we can use the approximation $\sin\theta \approx \tan\theta$, thus $\sin\theta \approx y_1/D$. Therefore, the positions of the first minima measured from the central axis are given by

$$y_1 \approx D\sin\theta = \pm D\frac{\lambda}{a} = \pm 3.87\times 10^{-3}\text{ m}$$

The positive and negative signs correspond to the dark fringes on either side of the central bright fringe. Hence, the width of the central bright fringe is equal to

$2|y_1| = 7.74\times 10^{-3}\text{ m} = 7.74\text{ mm}$. Note that this value is much greater than the width of the slit. However, as the slit width is increased, the diffraction pattern narrows, corresponding to smaller values of θ. In fact, for large values of a, the various maxima and minima are so closely spaced that only a large central bright area resembling the geometric image of the slit is observed.

Example 10.50 *A parallel beam of monochromatic light of wavelength 450 nm passes through a slit of width 0.2 mm. Find the angular divergence in which most of the light is diffracted.*

Sol. Most of the light is diffracted between the two first order minima. These minima occur at angle given by

$$a\sin\theta = \pm n\lambda$$

$$\therefore\quad \sin\theta \approx \theta = \pm\frac{\lambda}{a} = \pm\frac{450\times 10^{-9}}{0.2\times 10^{-3}} = \pm 2.25\times 10^{-3}\text{ rad}$$

$\therefore$ The angular divergence $= 2\theta = 4.5\times 10^{-3}$ rad

Example 10.51 *Angular width of central maximum in the Fraunhoffer diffraction pattern of a slit is measured. The slit is illuminated by light of wavelength 6000 Å. When the slit is illuminated by light of another wavelength, then the angular width decreases by 30%. Calculate the wavelength of this light. The same decrease in the angular width of central maximum is obtained when the original apparatus is immersed in a liquid. Find the refractive index of the liquid.*

Sol. Angular width of central maximum $= \dfrac{2\lambda}{a}$

$$\lambda_1 = 600\text{ nm},\ \theta_1 = \frac{2\lambda_1}{a} \qquad \text{...(i)}$$

$$\theta_2 = \theta_1 \times 0.7 = \frac{2\lambda_2}{a} \qquad \text{...(ii)}$$

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{1}{0.7} = \frac{\lambda_1}{\lambda_2} = \frac{600}{\lambda_2}$$

Wavelength, $\lambda_2 = 420\text{ nm}$

When immersed in liquid, $\lambda_2 = \lambda_1/\mu$

$$\theta_2 = \theta_1 \times 0.7 \qquad \text{...(iii)}$$

$$\Rightarrow \quad \frac{2\lambda_1/\mu}{a} = \frac{2\lambda_1}{a}\times 0.7$$

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{1}{0.7} = \mu$$

Refractive index of the liquid, $\mu = \dfrac{10}{7} = 1.42$

Intensity of single slit diffraction patterns

We can use phasors to determine the light intensity distribution for a single slit diffraction pattern. The proof is beyond our syllabus, we are just writing here the intensity at angle θ.

$$I = I_0\left[\frac{\sin\beta/2}{\beta/2}\right]^2$$

Here, $\dfrac{\beta}{2} = \dfrac{\pi a\sin\theta}{\lambda}$

or $$I = I_0\left[\frac{\sin(\pi a\sin\theta)/\lambda}{(\pi a\sin\theta)/\lambda}\right]^2 \qquad \text{(at angle } \theta\text{)}$$

Here, I_0 is the intensity at $\theta = 0$ (the central maximum).

From this result we see that minima occurs when,

$$\frac{\pi a\sin\theta}{\lambda} = m\pi \text{ or } \sin\theta = m\frac{\lambda}{a},$$

where, $m = \pm 1, \pm 2, \ldots$ (not zero)

This is in agreement with Eq. (i).

Note That $\sin\theta = 0$, corresponds to central maxima while $\dfrac{\pi a\sin\theta}{\lambda} = \pi$, corresponds to first minima.

Intensity distribution curve

The equation of minima gives the values of θ for which the diffraction pattern has zero light intensity *i.e.*, when a dark fringe is formed.

However, it tells us nothing about the variation in light intensity along the screen. The general features of the intensity distribution are shown in figure. A broad central bright fringe is observed; this fringe is flanked by much weaker bright fringes alternating with dark fringes. The various dark fringes occur at the values of θ that satisfy Eq. (i). Each bright fringe peak lies approximately halfway between its bordering dark fringe minima.

Note that the central bright maximum is twice as wide as the secondary maxima.

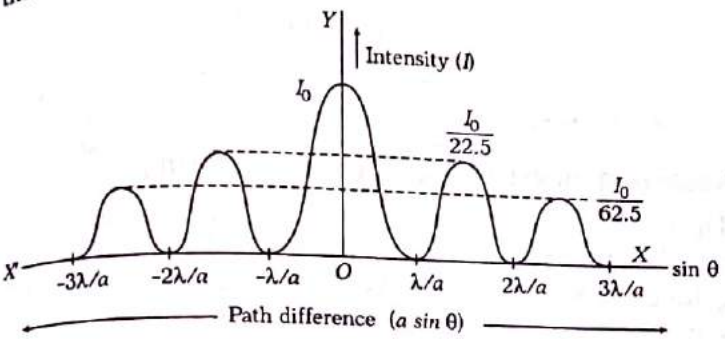
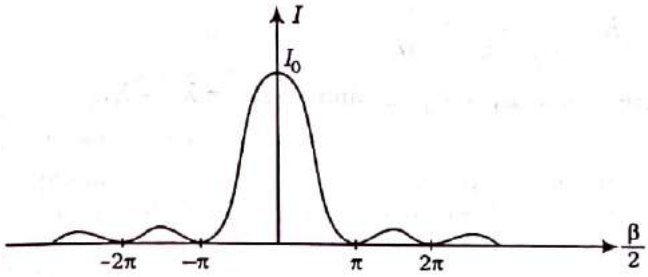


Fig. 10.30 Intensity distribution curve on the screen in diffraction pattern with sin θ

Example 10.52 *Find the ratio of the intensities of the secondary maxima to the intensity of the central maximum for the single-slit Fraunhofer diffraction pattern.*



Sol. To a good approximation, the secondary maxima lie midway between the zero points. From figure, we see that this corresponds to β/2 values of 3π/2, 5π/2, 7π/2, ...

$$\therefore \quad \frac{I_1}{I_0} = \left[\frac{\sin(3\pi/2)}{(3\pi/2)}\right]^2 = \frac{1}{9\pi^2/4} = 0.045$$

$$\frac{I_2}{I_0} = \left[\frac{\sin(5\pi/2)}{5\pi/2}\right]^2 = \frac{1}{25\,\pi^2/4} = 0.016$$

i.e., the first secondary maxima (the ones adjacent to the central maximum) have an intensity of 4.5% that of the central maximum and the next secondary maxima have an intensity of 1.6% that of the central maximum.

Note Secondary maxima does not lie exactly between two minimas.

Fresnel's distance

When a slit or hole of size a is illuminated by a parallel beam, then it is diffracted into an angle of $\approx \frac{\lambda}{a}$.

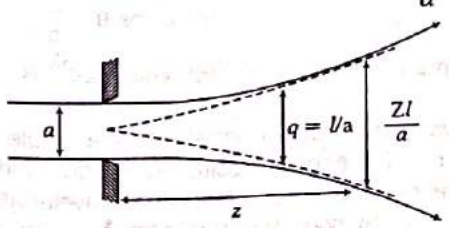


Fig. 25.31 Diffraction of a parallel beam

In travelling a distance Z, size of beam is $Z\lambda / a$.

So, taking $$\frac{Z\lambda}{a} \geq a$$

or $$Z \geq \frac{a^2}{\lambda}$$

Now, distance Z_F is called Fresnel's distance,

$$Z_F = a^2/\lambda$$

Spreading due to diffraction is comfortable upto distance $Z_F/2$ and for distance much greater than Z_F, spreading due to diffraction is prominant.

So, image formation can be explained by ray optics for distance less than Z_F.

Example 10.53 *For what distance is ray optics a good approximation when a plane light wave is incident on a circular aperture of width 2 mm having wavelength 600 nm?*

Sol. Ray optics is a good approximation upto Fresnel distance only.

$$Z_F = \frac{a^2}{\lambda} = \frac{(2\times10^{-3})^2}{(600\times10^{-9})} = 6.7 \text{ m}$$

Hence, the ray optics hold good upto distance of 6.7 m.

Example 10.54 *Estimate how large can be the aperture opening to work with laws of ray optics using a monochromatic light of wavelength 450 nm to a distance of around 20 m.*

Sol. Now, here we are given Fresnel distance equal to 20 m, $\lambda = 450$ nm, we have to estimate value of a.

Putting $Z_F = \frac{a^2}{\lambda}$, $a = \sqrt{Z_F\lambda}$

$$= 3\times10^{-3} \text{ m or } 3 \text{ mm}$$

Difference between interference and diffraction

Interference	Diffraction
1. It is due to superposition of waves from two coherent sources.	1. It is due to superposition of wavelets from same wavefronts.
2. All bright fringes are of same intensity.	2. Intensity decreases with the increase in the order of maxima.
3. The dark fringes are almost black.	3. Dark fringes are not perfectly black.
4. Fringes are of same width $\frac{\lambda D}{d}$.	4. The central maximum is of double the width of the secondary maximum. So, width are not equal.
5. The number of bands is large.	5. The number of bands is small.
6. Bands are equally spaced.	6. Bands are unequally spaced.

Diffraction grating

It consists of large number of equally spaced parallel slits. If light is incident normally on a transmission grating, then the direction of principal maxima is given by

$$d \sin\theta = n\lambda$$

Here, d is the distance between two consecutive slits and is called grating element.

$n = 1, 2, 3, \ldots$ is the order of principal maxima.

Note Diffraction grating is based on combined phenomena of interference and diffraction. It is the results of interference in diffracted waves.

Example 10.55 *A diffraction grating has 1.26×10^4 rulings uniformly spaced over width, $w = 25.4$ mm. It is illuminated at normal incidence by blue light of wavelength 450 nm. At what angles to the central axis do the second order maxima occur.*

Sol. The grating spacing d is

$$d = \frac{w}{N} = \frac{25.4 \times 10^{-3}}{1.26 \times 10^4} = 2.016 \times 10^{-6}\text{m} = 2016 \text{ nm}$$

The second order maxima corresponds to $n = 2$.

For $\lambda = 450$ nm, we have

$$d \sin\theta = 2\lambda \Rightarrow \theta = \sin^{-1}\left(\frac{2\lambda}{d}\right)$$

$$\Rightarrow \quad \theta = \sin^{-1}\left(\frac{2 \times 450 \times 10^{-9}}{2.016 \times 10^{-6}}\right)$$

$\therefore$ Angle, $\theta \approx 27°$

Resolving power of the diffraction grating

The diffraction grating is most useful for measuring wavelengths accurately. Like the prism, the diffraction grating can be used to disperse a spectrum into its wavelength components. Of the two devices, the grating is the more precise, if one wants to distinguish two closely spaced wavelengths.

For two nearly equal wavelengths λ_1 and λ_2 between which a diffraction grating can just barely distinguish, the **resolving power** R of the grating is defined as

$$R = \frac{\lambda}{\lambda_2 - \lambda_1} = \frac{\lambda}{\Delta\lambda}$$

where, $\lambda = (\lambda_1 + \lambda_2)/2$ and $\Delta\lambda = \lambda_2 - \lambda_1$.

Check point 10.3

1. Diffraction of light was discovered by
(a) Young (b) Hertz (c) Girmaldi (d) Malus

2. In Young's double slit experiment, the type of diffraction is
(a) Fresnel (b) Fraunhofer
(c) Both (a) and (b) (d) None of these

3. The bending of beam of light around corners of obstacles is called
(a) reflection (b) diffraction (c) refraction (d) interference

4. To observe diffraction, the size of an obstacle
(a) should be of the same order as wavelength
(b) should be much larger than the wavelength
(c) has no relation to wavelength
(d) should be exactly ($\lambda/2$)

5. Yellow light is used in a single slit diffraction experiment with slit width of 0.6 mm. If yellow light is replaced by X-rays, then the observed pattern will reveal
(a) that the central maximum is narrower
(b) more number of fringes
(c) less number of fringes
(d) no diffraction pattern

6. In Fresnel's class of diffraction, the
(a) obstacle screen distance is finite
(b) the diffracted wavefront is considered as spherical
(c) no convex lens is used to focus the diffraction fringes on the screen
(d) All of the above

7. A slit of width d is illuminated by white light. For red light ($\lambda = 6500$ Å), the first minima is obtained at $\theta = 30°$. Then, the value of d will be
(a) 3250 Å (b) 6.5×10^{-4} mm
(c) 1.24 microns (d) 2.6×10^{-4} cm

8. The light of wavelength 6328 Å is incident on a slit of width 0.2 mm perpendicularly, the angular fringe width will be
(a) 0.36° (b) 0.18° (c) 0.72° (d) 0.09°

9. A diffraction pattern is obtained using a beam of red light. What happens, if the red light is replaced by blue light?
(a) No change
(b) Diffraction bands become narrower and crowded together
(c) Diffraction bands become broader and farther apart
(d) Bands disappear

10. Direction of the first secondary maximum in the Fraunhofer diffraction pattern at a single slit is given by (a is the width of the slit)
(a) $a \sin\theta = \frac{\lambda}{2}$ (b) $a \cos\theta = \frac{3\lambda}{2}$
(c) $a \sin\theta = \lambda$ (d) $a \sin\theta = \frac{3\lambda}{2}$

11. A light wave is incident normally over a single slit of width 24×10^{-5} cm. The angular position of second dark fringe from the central maxima is 30°. What is the wavelength of light?
(a) 6000 Å (b) 5000 Å (c) 3000 Å (d) 1500 Å

12. The first diffraction minima due to a single slit diffraction is at $\theta = 30°$ for a light of wavelength 5000 Å. The width of the slit is
(a) 5×10^{-5} cm (b) 10×10^{-5} cm
(c) 2.5×10^{-5} cm (d) 1.25×10^{5} cm

13. Light of wavelength 6328 Å is incident normally on a slit having a width of 0.2 mm. The width of the central maximum measured from minimum to minimum of diffraction pattern on a screen 9 m away will be nearly
(a) 0.36° (b) 0.18°
(c) 0.72° (d) 0.09°

14. The main difference in the phenomenon of interference and diffraction is that
(a) diffraction is due to interaction of light from the same wavefront, whereas the interference is the interaction of waves from two isolated sources
(b) diffraction is due to interaction of light from the same wavefront, whereas the interference is the interaction of two waves derived from the same source
(c) diffraction is due to interaction of waves derived from the same source, whereas the interference is the bending of light from the same wavefront
(d) diffraction is caused by the reflected waves from a source, whereas interference is caused due to refraction of waves from a source

15. The X-ray cannot be diffracted by means of an ordinary grating due to
(a) large wavelength
(b) high speed
(c) short wavelength
(d) All of the above

Polarisation of light

Light is an electromagnetic wave in which electric and magnetic field vectors vary sinusoidally perpendicular to each other as well as perpendicular to the direction of propagation of wave of light.

"The phenomenon of limiting the vibration of electric field vector in one direction in a plane perpendicular to the direction of propagation of light wave is called polarisation of light."

The plane perpendicular to the plane of oscillation is called plane of polarisation.

Note In ordinary light (sun light, bulb light, etc), the electric field vectors distributed in all directions in a light is called unpolarised light.

Polaroids

These are thin film of ultramicroscopic crystals of quinine iodosulphate with their optic axis parallel to each other and it is used to produce the plane polarised light. It is based on the principle of selective absorption and is more effective than the tourmaline crystal.

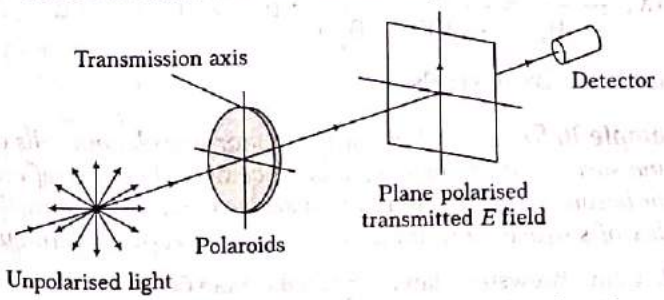


Fig. 10.32 Unpolarised light passes through polaroids

(i) Polaroids allow the light oscillations parallel to the transmission axis, to pass through them.

(ii) The crystal or polaroid on which unpolarised light is incident is called polariser. Crystal or polaroid on which polarised light is incident is called analyser.

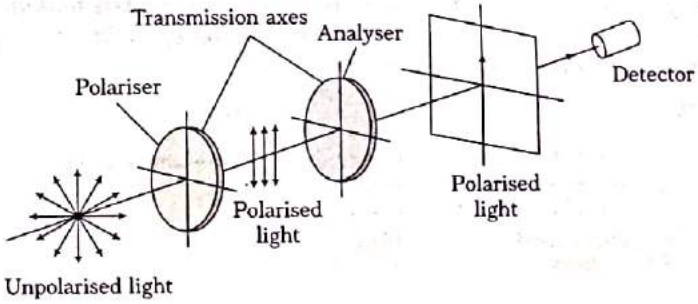


Fig. 10.33 Transmission axis of the polariser and analyser are parallel to each other, so whole of the polarised light passes through analyser

(iii) If the transmission axis of analyser is perpendicular to that of the polariser, then no light passes through the analyser.

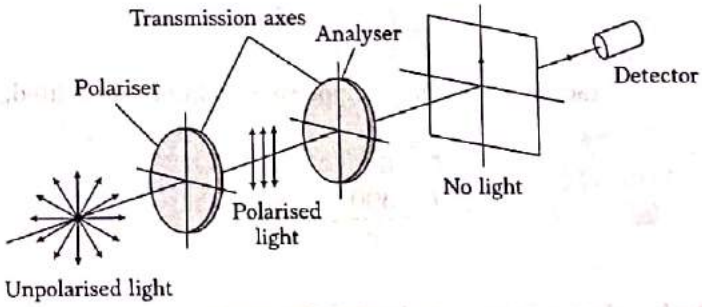


Fig. 10.34 Transmission axis of the analyser is perpendicular to the polariser, hence no light passes through the analyser

Uses of polaroids

Polaroids are used

(i) in sun glass.
(ii) to prepare filters.
(iii) for laboratory purpose.
(iv) in head light of automobiles.
(v) in three-dimensional motion pictures.
(vi) to improve colour constrast in old paintings.

Malus's law

This law states that the intensity of the polarised light transmitted through the analyser varies as the square of the cosine of the angle between the plane of transmission of the analyser and the plane of the polariser.

If I_0 be the intensity of the polarised light incident on the analyser and θ be the angle between the transmission axes of the polariser and the analyser, the intensity of the light transmitted through the analyser is given by $I = I_0 \cos^2\theta$.

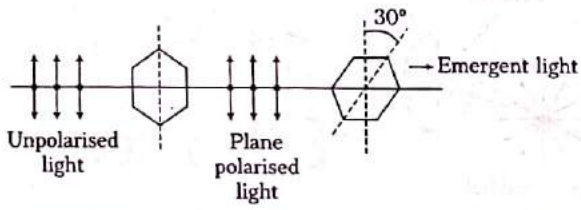
Note If an unpolarised light is converted into plane polarised light (say by passing it through a polaroid), then its intensity becomes half *i.e.*, $I_0 = \frac{I_i}{2}$

So, $$I = \frac{I_i}{2}\cos^2\theta$$

where, I_i = Intensity of unpolarised light.

Example 10.56 *Two polaroids are oriented with their planes perpendicular to incident light and transmission axis making an angle of 30° with each other. What fraction of incident unpolarised light is transmitted?*

Sol.



Suppose intensity of unpolarised light $= I'$

$\therefore$ Intensity of polarised light $= \frac{I'}{2}$

Intensity of emergent light,

$$I = \frac{I'}{2}\cos^2 30°$$

$$= \frac{I'}{2} \times \frac{3}{4} = \frac{3I'}{8}$$

$\therefore$ Fraction of incident unpolarised light of transmitted,

$$\frac{I}{I'} = \frac{3}{8}$$

$$\Rightarrow \quad \frac{I}{I'} = \frac{300}{8} = 37.5\%$$

Polarisation of transverse mechanical waves

If a transverse mechanical wave is passed through a narrow slit, so that the plane of vibration of the wave is parallel to the slit, then the wave passes through the slit with its vibrations being unaffected and the wave is said to be plane polarised.

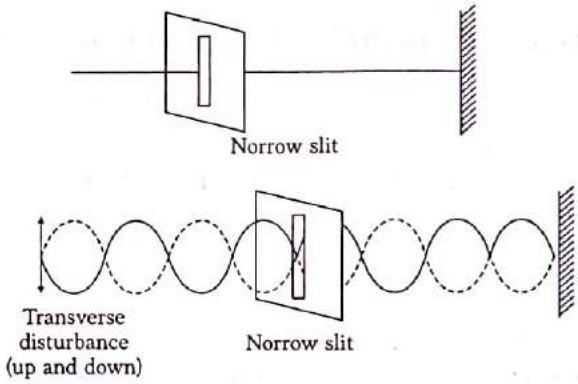


Fig. 10.35 Polarisation of transverse wave

Polarisation by reflection

When a beam of unpolarised light is reflected from a transparent medium (refractive index = μ), then the reflected light is completely plane polarised at a certain angle of incidence (called the angle of polarisation θ_P).

In this case, reflected and refracted waves are perpendicular to each other.

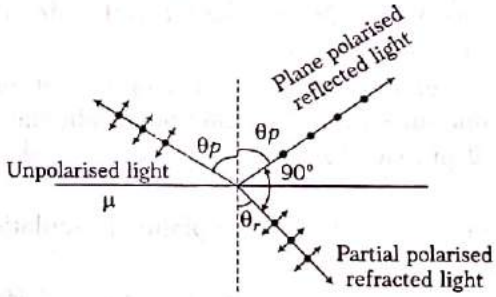


Fig. 10.36 Polarisation of light by reflection

By Snell's law, $\frac{\sin\theta_P}{\sin\theta_r} = \mu$

Now, since reflected ray perpendicular to refracted by

$$90° + \theta_P + \theta_r = 180°$$

$$\Rightarrow \quad \theta_P + \theta_r = 90° \quad \Rightarrow \quad \theta_r = 90° - \theta_P$$

Now, $\frac{\sin\theta_P}{\sin\theta_r} = \frac{\sin\theta_P}{\sin(90° - \theta_P)} = \mu \Rightarrow \tan\theta_P = \mu$

It is called **Brewster's** law.

Example 10.57 *When light of particular wavelength falls on a plane surface at an angle of incidence 60°, then the reflected light becomes completely plane polarised. Find the refractive index of surface material and the angle of refraction through it.*

Sol. Using Brewster's law, $\mu = \tan\theta_P = \tan 60° = \sqrt{3}$

At this condition, $\theta_P + \theta_r = 90°$

$$\therefore \quad \theta_r = 90° - 60°$$

Angle of refraction, $\theta_r = 30°$

Polarisation of light by scattering

When a beam of white light passes through a medium consisting of small particles of size of the order of the wavelength of light (dust, smoke, air molecules), then the light seen in a direction perpendicular to the incident beam appears bluish. This phenomenon is known as 'scattering of light' and the particles causing scattering are called 'scatterers'. The scattering of light is, infact, absorption and re-radiation of light by the scatterers.

The scattering of light can be demonstrated by a simple experiment. A few drops of dilute sulphuric acid is added to a dilute solution of hypo (sodium thiosulphate) prepared in a glass tank.

A precipitate of fine sulphur particles is formed, the particles grow in size with time. Now, a beam of light from a bright source is sent through the tank. The light scattered by the sulphur particles in a direction at right angles to the incident light appears bluish shown in figure below.

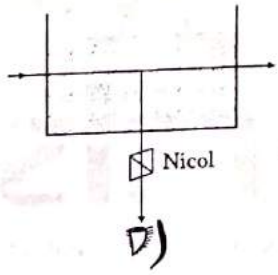


Fig. 10.37 Polarisation of light by sulphur particles

When the scattered bluish light is seen through an analyser (for example, a rotating Nicol) a variation in intensity with minimum intensity zero is found. This shows that the light scattered in a direction perpendicular to the incident light is plane polarised.

Check point 10.4

1. A polariser is used to
(a) reduce intensity of light (b) produce polarised light
(c) increase intensity of light (d) produce unpolarised light

2. Which of the following cannot be polarised?
(a) Radio waves (b) Ultraviolet rays
(c) Infrared rays (d) Ultrasonic waves

3. Light waves can be polarised as they are
(a) transverse (b) of high frequency
(c) longitudinal (d) reflected

4. The transverse nature of light is shown by
(a) interference of light (b) refraction of light
(c) polarisation of light (d) dispersion of light

5. Through which character, we can distinguish the light waves from sound waves
(a) interference (b) refraction
(c) polarisation (d) reflection

6. Which of the following is incorrect?
(a) If the wave is longitudinal, then it must be a mechanical wave
(b) If the wave is mechanical, then it may or may not be transverse wave
(c) Mechanical waves cannot propagate in vacuum
(d) Diffractor helps us to distinguish between sound wave and light wave

7. An optically active compound
(a) rotates the plane polarised light
(b) changes the direction of polarised light
(c) does not allow plane polarised light to pass through
(d) None of the above

8. A polaroid is placed at 45° to an incoming light of intensity I_0. Now, the intensity of light passing through polaroid after polarisation would be
(a) I_0 (b) $I_0/2$ (c) $I_0/4$ (d) zero

9. When the angle of incidence on a material is 60°, then the reflected light is completely polarised. The velocity of the refracted ray inside the material is (in ms^{-1})
(a) 3×10^8 (b) $\left(\frac{3}{\sqrt{2}}\right) \times 10^8$ (c) $\sqrt{3} \times 10^8$ (d) 0.5×10^8

10. The angle of polarisation for any medium is 60°, what will be critical angle for this
(a) $\sin^{-1}\sqrt{3}$ (b) $\tan^{-1}\sqrt{3}$ (c) $\cos^{-1}\sqrt{3}$ (d) $\sin^{-1}\frac{1}{\sqrt{3}}$

11. The angle of incidence at which reflected light is totally polarised for reflection from air to glass (refractive index n) is
(a) $\sin^{-1}(n)$ (b) $\sin^{-1}\left(\frac{1}{n}\right)$ (c) $\tan^{-1}\left(\frac{1}{n}\right)$ (d) $\tan^{-1}(n)$

12. A light has amplitude A and angle between analyser and polariser is 60°. Light is reflected by analyser has amplitude
(a) $A\sqrt{2}$ (b) $A/\sqrt{2}$ (c) $\sqrt{3}A/2$ (d) $A/2$

13. Light is incident on a glass surface at polarising angle of 57.5°. Then, the angle between the incident ray and the refracted ray is
(a) 57.5° (b) 115° (c) 65° (d) 205°

14. When unpolarised light beam is incident from air onto glass ($n = 1.5$) at the polarising angle
(a) reflected beam is polarised 100 percent
(b) reflected and refracted beams are partially polarised
(c) the reason for (a) is that almost all the light is reflected
(d) All of the above

15. The Brewster angle for the glass-air interface is 54.74°. If a ray of light going from air to glass strikes at an angle of incidence 45°, then the angle of refraction is (take, $\tan 54.74° = \sqrt{2}$)
(a) 60° (b) 30° (c) 25° (d) 54.74°

Chapter exercises

A) Taking it together

(Assorted Questions of the chapter for advanced level practice)

1. In a Young's double slit experiment, the source is white light. One of the holes is covered by a red filter and another by a blue filter. In this case, **[NCERT Exemplar]**

(a) there shall be alternate interference patterns of red and blue
(b) there shall be an interference pattern for red distinct from that for blue
(c) there shall be no interference fringes
(d) there shall be an interference pattern for red mixing with one for blue

2. In Young's double slit experiment, the fringe width β. If entire arrangement is placed in a liquid of refractive index n, then the fringe width becomes

(a) $\frac{\beta}{n+1}$ (b) $n\beta$
(c) $\frac{\beta}{n}$ (d) $\frac{\beta}{n-1}$

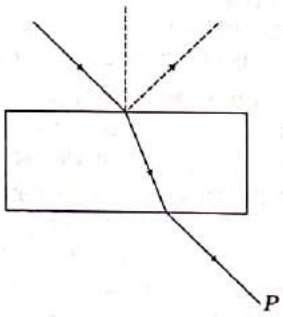
3. Consider the sunlight incident on a slit of width 10^4 Å. The image seen through the slit shall **[NCERT Exemplar]**

(a) be a fine sharp slit white in colour at the centre
(b) a bright slit white at the centre diffusing to zero intensities at the edges
(c) a bright slit white at the centre diffusing to regions of different colours
(d) only be a diffused slit white in colour

4. In Young's double slit experiment, the angular width of a fringe formed on a distant screen is 1°, the wavelength of light used is 6000 Å. The spacing between the slits is

(a) 6×10^{-7}m
(b) 6.8×10^{-5}m
(c) 3.4×10^{-5}m
(d) 3.4×10^{-4}m

5. Consider a light beam incident from air to a glass slab at Brewster's angle as shown in figure. A polaroid is placed in the path of the emergent ray at point P and rotated about an axis passing through the centre and perpendicular to the plane of the polaroid. **[NCERT Exemplar]**



(a) For a particular orientation, there shall be darkness as observed through the polaroid
(b) The intensity of light as seen through the polaroid shall be independent of the rotation
(c) The intensity of light as seen through the polaroid shall go through a minimum but not zero for two orientations of the polaroid
(d) The intensity of light as seen through the polaroid shall go through a minimum for four orientations of the polaroid

6. Two waves of same frequency and same amplitude from two monochromatic sources are allowed to superpose at a certain point. If in one case, the phase difference is 0 and in other case it is $\pi/2$, then the ratio of the intensities in the two cases will be

(a) 1 : 1
(b) 2 : 1
(c) 4 : 1
(d) None of the above

7. What is the minimum thickness of a soap film needed for constructive interference in reflected light, if the light incident on the film is 750 nm? Assume that the index for the film is $n = 1.33$.
(a) 282 nm (b) 70.5 nm
(c) 141 nm (d) 387 nm

8. Monochromatic light from a narrow slit illuminates two parallel narrow slits producing an interference pattern on a screen. The separation between the two slits is now doubled and the distance between the screen and the slits is reduced to half. The fringe width
(a) is doubled (b) becomes four times
(c) becomes one-fourth (d) remains the same

9. Two wavelengths of light of wavelength $\lambda_1 = 4500\,\text{Å}$ and $\lambda_2 = 6000\,\text{Å}$ are sent through a Young's double slit apparatus simultaneously, then
(a) no interference pattern will be formed
(b) the third order bright fringe of λ_1 will coincide with the fourth order bright fringe of λ_2
(c) the third order bright fringe of λ_2 will coincide with the fourth order bright fringe of λ_1
(d) the fringes of wavelength λ_1 will be wider than the fringes of wavelength λ_2

10. How large can be the aperture opening to work with laws of ray optics using a monochromatic light of wavelength 450 nm to a distance of around 20 m?
(a) 6 mm (b) 3 mm
(c) 2 mm (d) 8 mm

11. Two slits separated by a distance of 1 mm, are illuminated with red light of wavelength 6.5×10^{-7} m. The interference fringes are observed on a screen placed 1 m from the slits. The distance between the third dark fringe and the fifth bright fringe is equal to
(a) 0.65 mm (b) 1.63 mm
(c) 3.25 mm (d) 4.88 mm

12. In Young's double slit experiment, 12 fringes are observed to be formed in a certain segment of the screen, when light of wavelength 600 nm is used. If the wavelength of light is changed to 400 nm, number of fringes observed in the same segment of the screen is given by
(a) 12 (b) 18 (c) 24 (d) 30

13. A parallel beam of monochromatic light of wavelength 5000 Å is incident normally on a single narrow slit of width 0.001 mm. The light is focused by a convex lens on a screen placed on the focal plane. The first minimum will be formed for the angle of diffraction equal to
(a) 0° (b) 15°
(c) 30° (d) 60°

14. In Young's double slit experiment, the 10th maximum of wavelength λ_1 is at a distance of y_1 from the central maximum. When the wavelength of the source is changed to λ_2, 5th maximum is at a distance of y_2 from its central maximum. The ratio $\left(\frac{y_1}{y_2}\right)$ is
(a) $\frac{2\lambda_1}{\lambda_2}$ (b) $\frac{2\lambda_2}{\lambda_1}$ (c) $\frac{\lambda_1}{2\lambda_2}$ (d) $\frac{\lambda_2}{2\lambda_1}$

15. It is found that when waves of same intensity from two coherent sources superpose at a certain point, then the resultant intensity is equal to the intensity of one wave only. This means that the phase difference between the two waves at that point is
(a) zero (b) $\frac{\pi}{3}$ (c) $\frac{2\pi}{3}$ (d) π

16. The two slits are 1 mm apart from each other and illuminated with a light of wavelength 5×10^{-7} m. If the distance of the screen is 1m from slits, then the distance between third dark fringe and fifth bright fringe is
(a) 1.5 mm (b) 0.75 mm (c) 1.25 mm (d) 0.625 mm
(e) 2.5 mm

17. What is the minimum thickness of a soap bubble needed for constructive interference in reflected light, if the light incident on the film is 750 nm? Assume that the refractive index for the film is $n = 1.33$.
(a) 282 nm (b) 70.5 nm (c) 141 nm (d) 387 nm

18. A thin mica sheet of thickness 4×10^{-6} m and refractive index ($\mu = 1.5$) is introduced in the path of the light from upper slit. The wavelength of the wave used is 5000 Å. The central bright maximum will shift
(a) 4 fringes upward (b) 2 fringes downward
(c) 10 fringes upward (d) None of these

19. Light of wavelength 589.3 nm is incident normally on two slits of separation 0.1 mm width 0.01 nm. The angular width of the central diffraction maximum at a distance of 1 m from the slits, is
(a) 0.68° (b) 0.34° (c) 2.05° (d) None of these

20. Young's double slit experiment is made in a liquid. The tenth bright fringe in liquid lies on screen where 6th dark fringe lies in vacuum. The refractive index of the liquid is approximately
(a) 1.8 (b) 1.54 (c) 1.67 (d) 1.2

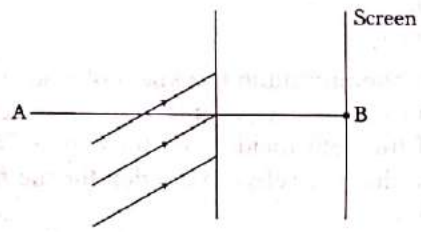
21. Two beams of light having intensities I and $4I$ interferes. The phase difference between the beams is $\pi/2$ at point A and π at point B. Then, the difference between the resultant intensities at A and B is
(a) $2I$ (b) $4I$ (c) $5I$ (d) $9I$

22. In a Young's double slit experiment using red and blue lights of wavelengths 600 nm and 480 nm respectively, the value of n for which the nth red fringe coincides with $(n+1)$th blue fringe is
(a) 5 (b) 4 (c) 3 (d) 2

23. Two beams of light having intensities I and $4I$ interfere to produce a fringe pattern on a screen. The phase difference between the beams is $\pi/2$ at point A and π at point B. Then, the difference between the resultant intensities at A and B is
(a) $2I$ (b) $4I$ (c) $5I$ (d) $7I$

24. In Young's double slit experiment, how many maximas can be obtained on a screen (including the central maximum) on both sides of the central fringes, if $\lambda = 2000$ Å and $d = 7000$ Å?
(a) 12 (b) 7 (c) 18 (d) 4

25. A beam of light parallel to central line AB is incident on the plane of slits. The number of minima obtained on the large screen is n_1. Now, if the beam is tilted by some angle ($\neq 90°$) as shown in figure, then the number of minima obtained is n_2. Then,



(a) $n_1 = n_2$ (b) $n_1 > n_2$
(c) $n_2 > n_1$ (d) n_2 will be zero

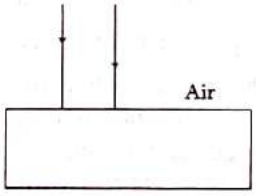
26. Unpolarised light of intensity 32 Wm^{-2} passes through three polarisers such that the transmission axis of the last polariser is crossed with the first. If the intensity of the emerging light is 3 Wm^{-2}. What is the angle between the transmission axes of the first two polarisers ?
(a) 60° (b) 45° (c) 90° (d) 120°

27. In a Young's experiment, one of the slits is covered with a transparent sheet of thickness 3.6×10^{-3} cm due to which position of central fringe shifts to a position originally occupied by 30th bright fringe. The refractive index of the sheet, if $\lambda = 6000$ Å is
(a) 1.5 (b) 1.2 (c) 1.4 (d) 1.6

28. In Young's double slit experiment, the y-coordinates of central maxima and 10th maxima are 2 cm and 5 cm, respectively. When the YDSE apparatus is immersed in a liquid of refractive index 1.5. The corresponding y-coordinates will be
(a) 2 cm, 7.5 cm (b) 3 cm, 6 cm
(c) 2 cm, 4 cm (d) 4/3 cm, 10/3 cm

29. In Young's double slit experiment, wavelength $\lambda = 5000$ Å, the distance between the slits is 0.2 mm and the screen is at 200 cm from the slits. The central maximum is at $x = 0$. The third maximum (taking the central maximum as zeroth maximum) will be at x equal to
(a) 1.67 cm (b) 1.5 cm (c) 0.5 cm (d) 5.0 cm

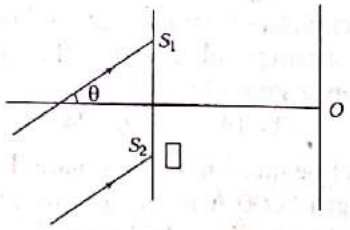
30. A parallel beam of light of intensity I is incident on a glass plate. 25% of light is reflected in any reflection by upper surface and 50% of light is reflected by any reflection from lower surface. Rest is refracted. The ratio of maximum to minimum intensity in interference region of reflected rays is



(a) $\left(\dfrac{\frac{1}{2}+\sqrt{\frac{3}{8}}}{\frac{1}{2}-\sqrt{\frac{3}{8}}}\right)^2$ (b) $\left(\dfrac{\frac{1}{4}+\sqrt{\frac{3}{8}}}{\frac{1}{2}-\sqrt{\frac{3}{8}}}\right)^2$
(c) $\dfrac{5}{8}$ (d) None of these

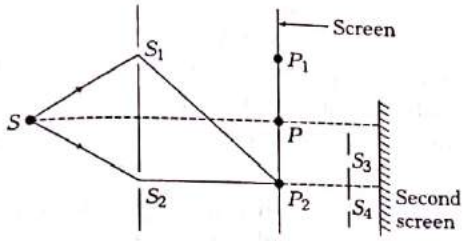
31. In the Young's double slit experiment, the intensities at two points P_1 and P_2 on the screen are respectively I_1 and I_2. If P_1 is located at the centre of a bright fringe and P_2 is located at a distance equal to a quarter of fringe width from P_1, then $\dfrac{I_1}{I_2}$ is
(a) 2 (b) 3
(c) 4 (d) None of these

32. A monochromatic beam of light falls on YDSE apparatus at some angle (say θ) as shown in the figure. A thin sheet of glass is inserted in front of the lower slit S_2. The central bright fringe (path difference = 0) will be obtained



(a) at O
(b) above O
(c) below O
(d) anywhere depending on angle θ, thickness of plate t and refractive index of glass μ

33. Figure shows a standard two slit arrangement with slits S_1, S_2, P_1, P_2 are the two minima points on either side of P (figure).



At P_2 on the screen, there is a hole and behind P_2 is a second 2-slit arrangement with slits S_3, S_4 and a second screen behind them. **[NCERT Exemplar]**

(a) There would be no interference pattern on the second screen but it would be lighted
(b) The second screen would be totally dark
(c) There would be a single bright point on the second screen
(d) There would be a regular two slit pattern on the second screen

34. In Young's double slit experiment, the two slits acts as coherent sources of equal amplitude A and wavelength λ. In another experiment with the same set up the two slits are sources of equal amplitude A and wavelength λ but are incoherent. The ratio of the intensity of light at the mid-point of the screen in the first case to that in the second case is

(a) 4 : 1 (b) 1 : 1 (c) 2 : 1 (d) 1 : 4

35. In the ideal double slit experiment, when a glass-plate (refractive index 1.5) of thickness t is introduced in the path of one of the interfering beams (wavelength λ), the intensity at the position where the central maximum occurred previously remains unchanged. The minimum thickness of the glass-plate is

(a) 2λ (b) $\frac{2\lambda}{3}$ (c) $\frac{\lambda}{3}$ (d) λ

36. In the standard Young's double slit experiment, the intensity on the screen at a point distant 1.25 fringe widths from the central maximum is (assuming slits to be identical)

(a) $\frac{1}{2} I_{max}$ (b) $\frac{1}{4} I_{max}$
(c) $\frac{1}{3} I_{max}$ (d) I_{max}

37. In a Young's double slit experiment, D equals the distance of screen and d is the separation between the slits. The distance of the nearest point to the centre maximum where the intensity is same as that due to a single slit, is equal to

(a) $\frac{D\lambda}{d}$ (b) $\frac{D\lambda}{2d}$ (c) $\frac{D\lambda}{3d}$ (d) $\frac{2D\lambda}{d}$

38. In Young's double slit experiment, white light is used. The separation between the slits is b. The screen is at a distance $d(d >> b)$ from the slits. Some wavelengths are missing exactly in front of one slit. These wavelengths are

(a) $\lambda = \frac{b^2}{2d}$ (b) $\lambda = \frac{2b^2}{d}$
(c) $\lambda = \frac{b^2}{3d}$ (d) $\lambda = \frac{2b^2}{3d}$

39. The intensity of each of the two slits in Young's double slit experiment is I_0. Calculate the minimum separation between the two points on the screen where intensities are $2I_0$ and I_0. Given the fringe width equal to α

(a) $\frac{\alpha}{4}$ (b) $\frac{\alpha}{3}$ (c) $\frac{\alpha}{12}$ (d) None of these

40. One slit of a double slit experiment is covered by a thin glass plate of refractive index 1.4, and the other by a thin glass plate of the refractive index 1.7. The point on the screen where the central maximum fall before the glass plate was inserted, is now occupied by what had been the fifth bright fringe was seen before. Assume the plate have the same thickness t and wavelength of light 480 nm. Then, the value of t is

(a) 2.4 μm (b) 4.8 μm
(c) 8 μm (d) 16 μm

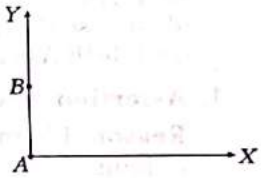
41. An interference is observed due to two coherent sources separated by a distance 5λ along Y-axis, where λ is the wavelength of light. A detector D is moved along the positive X-axis. The number of point on the X-axis excluding the points $x = 0$ and $x = \infty$ at which resultant intensity will be maximum, are

(a) 4 (b) 5 (c) ∞ (d) zero

42. In a Young's double slit experiment, using unequal slit widths, the intensity at a point midway between a bright and dark fringes is $4I$. If one slit is covered by an opaque film, intensity at that point becomes $2I$. If the other is covered instead, then the intensity at that point is

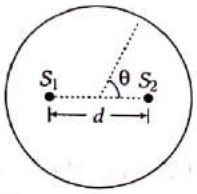
(a) $2I$ (b) $5I$
(c) $(5 + 2\sqrt{2})I$ (d) $(5 + 4\sqrt{2})I$

43. Two coherent light sources A and B are at a distance 3λ from each other (λ = wavelength). The distances from A on the X-axis at which the interference is constructive are



(a) 3λ (b) 8λ
(c) $5\lambda/4$ (d) 8.75λ

44. Two coherent sources separated by distance d are radiating in phase having wavelength λ. A detector moves in a big circle around the two sources in the plane of the two sources. The angular position of $n = 4$ interference maxima is given as



(a) $\sin^{-1}\frac{n\lambda}{d}$ (b) $\cos^{-1}\frac{4\lambda}{d}$

(c) $\tan^{-1}\frac{d}{4\lambda}$ (d) $\cos^{-1}\frac{\lambda}{4d}$

45. Consider a ray of light incident from air onto a slab of glass (refractive index n) of width d, at an angle θ. The phase difference between the ray reflected by the top surface of the glass and the bottom surface is **[NCERT Exemplar]**

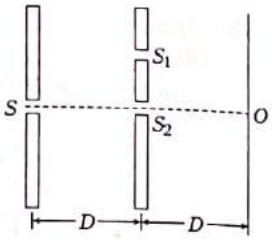
(a) $\frac{4\pi d}{\lambda}\left(1-\frac{1}{n^2}\sin^2\theta\right)^{-1/2}+\pi$

(b) $\frac{4\pi d}{\lambda}\left(1-\frac{1}{n^2}\sin^2\theta\right)^{1/2}$

(c) $\frac{4\pi d}{\lambda}\left(1-\frac{1}{n^2}\sin^2\theta\right)^{1/2}+\frac{\pi}{2}$

(d) $\frac{4\pi d}{\lambda}\left(1-\frac{1}{n^2}\sin^2\theta\right)^{1/2}+2\pi$

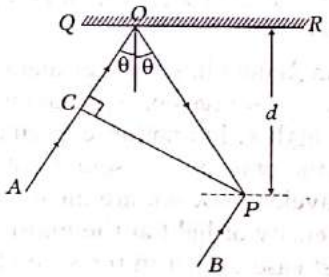
46. Two ideal slits S_1 and S_2 are at a distance d apart and illuminated by light of wavelength λ passing through an ideal source slit S placed on the line through S_2 as shown. The distance between the planes of slits and the source slit is D.



A screen is held at a distance D from the plane of the slits. The minimum value of d for which there is darkness at O is

(a) $\sqrt{\frac{3\lambda D}{2}}$ (b) $\sqrt{\lambda D}$ (c) $\sqrt{\frac{\lambda D}{2}}$ (d) $\sqrt{3\lambda D}$

47. In the below diagram, CP represents a wavefront and AO and BP, the corresponding two rays. Find the condition of θ for constructive interference at P between the rays BP and reflected ray AOP



(a) $\cos\theta = 3\lambda/2d$ (b) $\cos\theta = \lambda/4d$

(c) $\sec\theta - \cos\theta = \lambda/d$ (d) $\sec\theta - \cos\theta = 4\lambda/d$

B) Medical entrance special format questions

Assertion and reason

Directions (Q. Nos. 1-10) *These questions consists of two statements each printed as Assertion and Reason. While answering these questions you are required to choose any one of the following five responses.*

(a) If both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If Assertion is false but Reason is true.
(e) If both Assertion and Reason are false.

1. Assertion Interference shows wave nature of light.

Reason Photoelectric effect proves particle nature of light.

2. Assertion If wavelength is of the order of distance between the slits, then fringe size is large.

Reason Fringe width is given by $\beta = \lambda D/d$.

3. Assertion Two coherent sources transmit waves of equal intensity I_0. Resultant intensity at a point where path difference is $\frac{\lambda}{3}$ is also I_0.

Reason In interference resultant intensity at any point is the average intensity of two individual intensities.

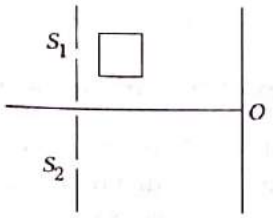
4. Assertion In Young's double slit experiment, ratio $\frac{I_{max}}{I_{min}}$ is infinite.

Reason If width of any one of the slits is slightly increased, then this ratio will decrease.

5. Assertion If a glass slab is placed in front of one of the slits, then fringe width will decrease.

Reason Glass slab will produce an additional path difference.

6. **Assertion** In the YDSE set up shown in figure, a glass slab is inserted in front of the slit S_1 as shown in figure. Then, zero order maxima (where path difference = 0) will lie above point O.



Reason Glass slab will produce an extra path difference in the path of the ray passing through S_1.

7. **Assertion** Fraunhoffer diffraction occurs when all the rays passing through a narrow slit are approximately parallel to one another.

Reason Fraunhofer diffraction pattern can be achieved by placing the screen far from the slit.

8. **Assertion** In Young's double slit experiment, often both the phenomena interference and diffraction are present.

Reason Diffraction results due to superposition of wavelets from different points of the some wavefront.

9. **Assertion** In diffraction phenomenon different maximas have different intensities.

Reason In interference different maximas have same intensities.

10. **Assertion** The unpolarised light and polarised light can be distinguished from each other by using polaroid.

Reason A polaroid is capable of producing plane polarised beam of light.

Matching type questions

1. In Young's double slit experiment, match the following two columns.

Column I	Column II
A. When width of one slit is slightly increased	p. maximum intensity will increase
B. When one slit is closed	q. maximum intensity will decrease
C. When both the sources are made incoherent	r. maximum intensity will remain same
D. When a glass slab is inserted in front of one of the slits	s. fringe pattern will disappear

Note Assume absorption from glass slab to be negligible.

2. In normal YDSE experiment maximum intensity is $4I_0$. In Column I, y-coordinate is given corresponding to centre line. In Column II resultant intensities are given. Match the two columns.

Column I	Column I
A. $y = \frac{\lambda D}{d}$	p. $I = I_0$
B. $y = \frac{\lambda D}{2d}$	q. $I = 2I_0$
C. $y = \frac{\lambda D}{3d}$	r. $I = 4I_0$
D. $y = \frac{\lambda D}{4d}$	s. $I =$ zero

3. In normal YDSE experiment, match the following two columns.

Column I	Column II
A. In YDSE apparatus is immersed in a liquid	p. fringe width will increase
B. When wavelength of light used in increased	q. fringe width will decrease
C. When distance between slits and screen (D) is increased	r. fringe width will remain constant
D. When distance between two slits (d) is increased	s. fringe pattern will disappear

4. Match the following two columns.

Column I	Column II
A. The phenomenon of redistribution of light energy an account of superimposition	p. Coherent sources
B. Emit continous light waves of same wavelength having constant phase difference	q. Interference of light
C. The phenomenon of bending of light around corners	r. Polarization of light
D. The phenomenon of restricting the vibration of ordinary light	s. Diffraction of light

C) Medical entrance gallery

(Collection of questions asked in NEET various medical entrance exams)

1. The interference pattern is obtained with two coherent light sources of intensity ratio n. In the interference pattern, the ratio $\frac{I_{max} - I_{min}}{I_{max} + I_{min}}$ will be **[NEET 2016]**

(a) $\frac{\sqrt{n}}{n+1}$ (b) $\frac{2\sqrt{n}}{n+1}$ (c) $\frac{\sqrt{n}}{(n+1)^2}$ (d) $\frac{2\sqrt{n}}{(n+1)^2}$

2. A linear aperture whose width is 0.02 cm is placed immediately in front of a lens of focal length 60 cm. The aperture is illuminated normally by a parallel beam of wavelength 5×10^{-5} cm. The distance of the first dark band of the diffraction pattern from the centre of the screen is **[NEET 2016]**

(a) 0.10 cm (b) 0.25 cm (c) 0.20 cm (d) 0.15 cm

3. The intensity at the maximum in a Young's double slit experiment is I_0. Distance between two slits is $d = 5\lambda$, where, λ is the wavelength of light used in the experiment. What will be the intensity in front of one of the slits on the screen placed at a distance, $D = 10\ d$? **[NEET 2016]**

(a) $\frac{I_0}{4}$ (b) $\frac{3}{4}I_0$ (c) $\frac{I_0}{2}$ (d) I_0

4. In a diffraction pattern due to a single slit of width a, the first minimum is observed at an angle 30° when light of wavelength 5000 Å is incident on the slit. The first secondary maximum is observed at an angle of **[NEET 2016]**

(a) $\sin^{-1}\left(\frac{2}{3}\right)$ (b) $\sin^{-1}\left(\frac{1}{2}\right)$

(c) $\sin^{-1}\left(\frac{3}{4}\right)$ (d) $\sin^{-1}\left(\frac{1}{4}\right)$

5. Light of wavelength λ is incident on slit of width d. The resulting diffraction pattern is observed on a screen placed at distance D. The linear width of central maximum is equal to width of the slit, then $D = \ldots$. **[Guj. CET 2015]**

(a) $\frac{2\lambda^2}{d}$ (b) $\frac{d^2}{2\lambda}$ (c) $\frac{d}{\lambda}$ (d) $\frac{2\lambda}{d}$

6. A beam of light contains two wavelengths 6500 Å and 5200 Å. They form interference fringes in Young's double slit experiment. What is the least distance (approx) from central maximum, where the bright fringes due to both wavelength coincide? The distance between the slits is 2 mm and the distance of screen is 120 cm. **[CG PMT 2015]**

(a) 0.16 cm (b) 0.32 cm (c) 0.48 cm (d) 1.92 cm

7. Find the right condition(s) for Fraunhofer diffraction due to a single slit. **[WB JEE 2015]**

(a) Source is at infinite distance and the incident beam has converged at the slit
(b) Source is near to the slit and the incident beam is parallel
(c) Source is at infinity and the incident beam is parallel
(d) Source is near to the slit and the incident beam has converged at the slit

8. The parallel beams of monochromatic light of wavelength 4.5×10^{-7} m passes through a long slit with width 0.2×10^{-3} m. The angular divergence in which most of the light is **[UP CPMT 2015]**

(a) 4.5×10^{-3} rad (b) $4.5\ \pi$ rad
(c) 2.25×10^{-3} rad (d) 9×10^{-3} rad

9. The condition for obtaining secondary maxima in the diffraction pattern due to single slit is **[Manipal 2015]**

(a) $a \sin\theta = \frac{n\lambda}{2}$ (b) $a \sin\theta = (2n-1)\lambda/2$
(c) $a \sin\theta = n\lambda$ (d) $a \sin\theta = (2n-1)\lambda$

10. In Young's double slit experiment, when wavelength used is 6000 Å and the screen is 40 cm from the slits, the fringes are 0.012 cm wide. What is the distance between the slits? **[Manipal 2015]**

(a) 0.24 cm (b) 0.2 cm (c) 2.4 cm (d) 0.024 cm

11. The polarising angle of glass is 57°. A ray of light which is incident at this angle will have an angle of refraction as **[KCET 2015]**

(a) 43° (b) 25° (c) 38° (d) 33°

12. To observed diffraction, the size of the obstacle **[KCET 2015]**

(a) should be much larger than the wavelength
(b) has no relation to wavelength
(c) should be of the order of wavelength
(d) should be $\lambda/2$, where λ is the wavelength

13. A plane wave of wavelength 6250 Å is incident normally on a slit of width 2×10^{-2} cm. The width of the principal maximum of diffraction pattern on a screen at a distance of 50 cm will be **[EAMCET 2015]**

(a) 312.5×10^{-3} cm (b) 312.5×10^{-4} cm
(c) 312 cm (d) 312.5×10^{-5} cm

14. For a parallel beam of monochromatic light of wavelength λ diffraction is produced by a single slit whose width a is of the order of the wavelength of the light. If D is the distance of the screen from the slit, the width of the central maxima will be **[CBSE AIPMT 2015]**

(a) $\frac{2D\lambda}{a}$ (b) $\frac{D\lambda}{a}$ (c) $\frac{Da}{\lambda}$ (d) $\frac{2Da}{\lambda}$

15. A beam of light of wavelength 590 nm is focussed by a converging lens of diameter 10 cm at a distance of 20 cm form it. Find the diameter of the disc image formed [UK PMT 2015]
(a) 5.76×10^{-4} cm (b) 7.51×10^{-4} cm
(c) 8.72×10^{-4} cm (d) 9.80×10^{-4} cm

16. A plane polarised light is incident normally on a tourmaline plate. Its **E** vector make an angle of 60° with the optic axis of the plate. Find the percentage difference between initial and final intensities. [Guj.CET 2015]
(a) 50% (b) 25% (c) 75% (d) 90%

17. In a double slit experiment, the two slits are 1 mm apart and the screen is placed 1 m away. A monochromatic light of wavelength 500 nm is used. What will be the width of each slit for obtaining ten maxima of double slit within the central maxima of single slit pattern? [CBSE AIPMT 2015]
(a) 0.2 mm (b) 0.1 mm (c) 0.5 mm (d) 0.02 mm

18. In Young's double slit experiment, the locus of the point P lying in a plane with a constant path difference between two interfering waves is [Kerala CEE 2015]
(a) a hyperbola (b) a straight line
(c) an ellipse (d) a parabola
(e) a circle

19. Wavefront is the locus of all points, where the particles of the medium vibrate with the same [KCET 2014]
(a) phase (b) amplitude (c) frequency (d) period

20. In the Young's double slit experiment, the intensity of light at a point on the screen (where the path difference is λ) is K, (λ being the wavelength of light used). The intensity at a point where the path difference is $\lambda/4$, will be [J&K CET 2014]
(a) K (b) $K/4$ (c) $K/2$ (d) Zero

21. Two coherent sources of intensity ratio α interfere. In interference pattern, $\frac{I_{max} - I_{min}}{I_{max} + I_{min}}$ is equal to [MHT CET 2014]
(a) $\frac{2\alpha}{1+\alpha}$ (b) $\frac{2\sqrt{\alpha}}{1+\alpha}$ (c) $\frac{2\alpha}{1+\sqrt{\alpha}}$ (d) $\frac{1+\alpha}{2\alpha}$

22. Two coherent monochromatic beams of intensities I and $4I$ respectively, are superposed. Then, the maximum and minimum intensities in the resulting pattern are [WB JEE 2014]
(a) $5I$ and $3I$ (b) $9I$ and $3I$ (c) $4I$ and I (d) $9I$ and I

23. Colours appear on a thin soap film and soap bubbles due to the phenomenon of [UK PMT 2014]
(a) interference (b) scattering
(c) diffraction (d) dispersion

24. In Young's double slit experiment, the ratio of maximum and minimum intensities in the fringe system is 9 : 1. The ratio of amplitudes of coherent sources is [UK PMT 2014]
(a) 9 : 1 (b) 3 : 1
(c) 2 : 1 (d) 1 : 1

25. In Young's double slit interference experiment, using two coherent waves of different amplitudes, the intensities ratio between bright and dark fringes is 3. Then, the value of the wave amplitudes ratio that arrive there is [EAMCET 2014]
(a) $\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)$ (b) $\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)$
(c) $\sqrt{3}:1$ (d) $1:\sqrt{3}$

26. The angle of incidence at which reflected light is totally polarised for refraction from air to glass (refractive index μ) is [UK PMT 2014]
(a) $\sin^{-1}\mu$ (b) $\sin^{-1}\left(\frac{1}{\mu}\right)$
(c) $\tan^{-1}\left(\frac{1}{\mu}\right)$ (d) $\tan^{-1}(\mu)$

27. A polarised light of intensity I_0 is passed through another polariser whose pass axis makes an angle of 60 ° with the pass axis of the former. What is the intensity of emergent polarised light from second polariser? [KCET 2014]
(a) $I = I_0$ (b) $I = I_0/6$
(c) $I = I_0/5$ (d) $I_0/4$

28. A fringe width of a certain interference pattern is $\beta = 0.002$ cm. What is the distance of 5th dark fringe centre? [MHT CET 2014]
(a) 1×10^{-2} cm (b) 11×10^{-2} cm
(c) 1.1×10^{-2} cm (d) 3.28×10^{6} cm

29. In Young's double slit experiment, the slits are 2 mm apart and are illuminated by photons of two wavelengths $\lambda_1 = 12000$ Å and $\lambda_2 = 10000$ Å. At what minimum distance from the common central bright fringe on the screen 2 m from the slit will a bright fringe from one interference pattern coincide with a bright fringe from the other? [NEET 2013]
(a) 8 mm (b) 6 mm
(c) 4 mm (d) 3 mm

30. If two slits in Young's experiment are 0.4 mm apart and fringe width on a screen 200 cm away is 2 mm, the wavelength of light illuminating the slits is [J&K CET 2013]
(a) 500 nm (b) 600 nm
(c) 400 nm (d) 300 nm

31. Light of wavelength λ from a point source falls on a small circular obstacle of diameter d. Dark and bright circular rings around a central bright spot are formed on a screen beyond the obstacle. The distance between the screen and obstacle is D. Then, the condition for the formation of rings, is **[EAMCET 2013]**

(a) $\sqrt{\lambda} \approx \frac{d}{4D}$ (b) $\lambda \approx \frac{d^2}{4D}$ (c) $d \approx \frac{\lambda^2}{D}$ (d) $\lambda \approx \frac{D}{4}$

32. Which of the following phenomena support the wave theory of light?

1. Scattering
2. Interference
3. Diffraction
4. Velocity of light in a denser medium is less than the velocity of light in the rare medium

[Karnataka CET 2013]

(a) 1, 2 and 3 (b) 1, 2 and 4
(c) 2, 3 and 4 (d) 1, 3 and 4

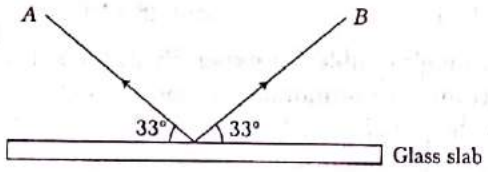
33. λ_1 and λ_2 are used to illuminate the slits. β_1 and β_2 are the corresponding fringe widths. The wavelength λ_1 can produce photoelectric effect when incident on a metal. But the wavelength λ_2 cannot produce photoelectric effect. The correct relation between β_1 and β_2 is **[Karnataka CET 2013]**

(a) $\beta_1 < \beta_2$ (b) $\beta_1 = \beta_2$ (c) $\beta_1 > \beta_2$ (d) $\beta_1 \geq \beta_2$

34. When we close one slit in the Young's double slit experiment, then **[Kerala CET 2013]**

(a) the bandwidth increased
(b) the bandwidth is decreased
(c) the bandwidth remains unchanged
(d) the interference pattern is shifted
(e) the diffraction pattern is observed

35. A beam of light is incident on a glass slab ($\mu = 154$) in a direction as shown in the figure. The reflected light is analysed by a polaroid prism. On rotating the polaroid (take, $\tan 57° = 154$) **[Kerala CET 2013]**



(a) the intensity remains unchanged
(b) the intensity is reduced to zero and remains at zero
(c) the intensity gradually reduced to zero and then again increases
(d) the intensity increases continuously
(e) the intensity increases initially and remains constant afterwards.

36. In Young's double slit experiment, the phase difference between the two waves reaching at the location of the third dark fringe is **[MP PMT 2013]**

(a) π (b) $\frac{3\pi}{2}$ (c) 5π (d) 3π

37. A parallel beam of fast moving electrons is incident normally on a narrow slit. A fluorescent screen is placed at a large distance from the slit. If the speed of the electrons is increased, then which of the following statements is correct? **[NEET 2013]**

(a) Diffraction pattern is not observed on the screen in the case of electrons
(b) The angular width of the central maximum of the diffraction pattern will increase
(c) The angular width of the central maximum will decrease
(d) The angular width of the central maximum will be unaffected

38. White light reflected from a soap film (Refractive Index = 1.5) has a maxima at 600 nm and a minima at 450 nm with no minimum in between. Then, the thickness of the film is **[KCET 2013]**

(a) 1 (b) 2 (c) 3 (d) 4

39. In Young's double slit experiment, the fringe width is 1×10^{-4} m, if the distance between the slit and screen is doubled and the distance between two slits is reduced to half and the wavelength is changed from 6.4×10^{-7} m to 4.0×10^{-7} m, then the value of new fringe width will be **[BCECE (Mains) 2012]**

(a) 2.5×10^{-4} m (b) 2.0×10^{-4} m
(c) 1×10^{-4} m (d) 0.5×10^{-4} m

Directions (Q. Nos. 40-42) *These questions consist of two statements each linked as Assertion and Reason. While answering these questions you are required to choose any one of the following five responses.*

(a) If both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If bot Assertion and Reason are false.

40. Assertion To observe diffraction of light the size of obstacle/aperture should be of the order of 10^{-7} m.

Reason 10^{-7} m is the order of wavelength of visible light. **[AIIMS 2012]**

41. Assertion The pattern and position of fringes always remain same even after the introduction of transparent medium in a path of one of the slit. **[AIIMS 2012]**

Reason The central fringe is bright or dark depends upon the initial phase difference between the two coherence sources.

42. Assertion Corpuscular theory fails in explaining the velocities of light in air and water. **[AIIMS 2012]**

Reason According to corpuscular theory, light should travel faster in denser media than in rarer media.

Answers

Check point 10.1

1. (a)	2. (c)	3. (c)	4. (a)	5. (b)	6. (c)	7. (b)	8. (c)	9. (d)	10. (d)
11. (c)	12. (b)	13. (a)	14. (a)	15. (c)					

Check point 10.2

1. (b)	2. (d)	3. (d)	4. (c)	5. (b)	6. (d)	7. (c)	8. (a)	9. (c)	10. (d)
11. (d)	12. (b)	13. (d)	14. (a)	15. (c)	16. (b)	17. (a)	18. (b)	19. (c)	20. (a)

Check point 10.3

1. (c)	2. (a)	3. (b)	4. (a)	5. (d)	6. (d)	7. (b)	8. (b)	9. (b)	10. (d)
11. (a)	12. (b)	13. (a)	14. (b)	15. (c)					

Check point 10.4

1. (b)	2. (d)	3. (a)	4. (c)	5. (c)	6. (d)	7. (a)	8. (b)	9. (c)	10. (d)
11. (d)	12. (d)	13. (d)	14. (a)	15. (b)					

A. Taking it together

1. (c)	2. (c)	3. (a)	4. (c)	5. (c)	6. (b)	7. (c)	8. (c)	9. (c)	10. (b)
11. (b)	12. (b)	13. (c)	14. (a)	15. (c)	16. (c)	17. (c)	18. (a)	19. (b)	20. (a)
21. (b)	22. (b)	23. (b)	24. (b)	25. (a)	26. (a)	27. (a)	28. (c)	29. (b)	30. (d)
31. (a)	32. (a)	33. (d)	34. (c)	35. (a)	36. (a)	37. (c)	38. (c)	39. (c)	40. (c)
41. (a)	42. (a)	43. (c)	44. (b)	45. (a)	46. (c)	47. (b)			

B. Medical entrance special format questions

Assertion and reason

1. (b)	2. (b)	3. (c)	4. (b)	5. (d)	6. (a)	7. (b)	8. (b)	9. (b)	10. (a)

Match the columns

1. (A → p; B → q, s; C → q; s; D → r)
2. (A → r; B → s; C → p; D → q)
3. (A → q; B → p; C → p; D → q)
4. (A → q; B → p; C → s; D → r)

C. Medical entrances' gallery

1. (b)	2. (d)	3. (c)	4. (c)	5. (b)	6. (a)	7. (c)	8. (a)	9. (b)	10. (b)
11. (d)	12. (c)	13. (a)	14. (a)	15. (a)	16. (c)	17. (a)	18. (a)	19. (a)	20. (c)
21. (b)	22. (d)	23. (a)	24. (c)	25. (c)	26. (d)	27. (d)	28. (c)	29. (b)	30. (c)
31. (b)	32. (c)	33. (a)	34. (e)	35. (c)	36. (b)	37. (b)	38. (c)	39. (a)	40. (a)
41. (b)	42. (a)								

Hints and explanations

Check point 10.1

2. (*c*) According to corpuscular theory of light, the different colours of light are due to different size of the corpuscles.

4. (*a*) For phenomenon of interference, it is necessary that there should be a constant phase difference between two light rays.

7. (*b*) Here phase difference, $\phi = \pi / 3$

So, amplitude of resulting wave

$$A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2 \cos \phi}$$

$$= \sqrt{4^2 + 3^2 + 2 \times 4 \times 3 \cos \pi/3} \approx 6$$

8. (*c*) Given, $\phi = \frac{\pi}{3}$

$$\therefore \quad A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos \phi}$$

$$= \sqrt{16 + 9 + 2(4)(3) \cos 60°} \approx \sqrt{36} = 6$$

9. (*d*) Resultant intensity for two interfering waves is given by

$$I_R = I_1 + I_2 + 2\sqrt{I_1I_2} \cos \phi$$

For maximum, $\cos \phi$ = maximum = 1

i.e., $\phi = 0, \pm 2\pi, \pm 4\pi$

or $\phi = \pm 2\pi n, \quad n = 0, 1, 2, \ldots$

$$\therefore \quad I_{max} = I_1 + I_2 + 2\sqrt{I_1I_2}$$

$$= (\sqrt{I_1} + \sqrt{I_2})^2$$

10. (*d*) For destructive interference,

$\cos \phi$ = min = -1

i.e., $\phi = \pm \pi, \pm 3\pi, \pm 5\pi$

or $\phi = \pm (2n + 1)\pi$

with $n = 0, 1, 2, 3, \ldots$

or $\Delta x = \pm \frac{(2n + 1)\lambda}{2}$

11. (*c*) In interference,

$$I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$$

while $I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$

So, $$\frac{I_{max}}{I_{min}} = \frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2}$$

$$= \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2}$$

$\therefore$ Given $\frac{A_1}{A_2} = \frac{3}{5}$

$$\therefore \quad \frac{I_{max}}{I_{min}} = \frac{(3 + 5)^2}{(3 - 5)^2} = \frac{16}{1}$$

12. (*b*) $I = I_1 + I_2 + 2\sqrt{I_1I_2} \cos \phi$

$$= I + 4I + 2\sqrt{I \times 4I} \times \cos \frac{2\pi}{3}$$

$$= 5I + 4I \times \left(-\frac{1}{2}\right) = 5I - 2I = 3I$$

13. (*a*) $I = I_1 + I_2 = I + 4I = 5I$

14. (*a*) The sources which produce sustained, *i.e.*, observable interference are called coherent sources. In case of interference as ($I = I_1 + I_2 + 2\sqrt{I_1I_2} \cos \phi$), interference will be sustained if the phase difference ϕ at a given point does not vary with time, so phase difference between two sources must be constant.

Check point 10.2

1. (*b*) For dark fringe at P,

$$S_1P - S_2P = \frac{5\lambda}{2}$$

$$= 5 \times \frac{6000}{2} = 15000 \text{ Å} = 1.5 \text{ micron}$$

2. (*d*) For minima, path difference should be an odd multiple of half wavelength.

or $\Delta = (2n - 1)\frac{\lambda}{2}$

For third minima, $n = 3$, $\Delta = (2 \times 3 - 1)\frac{\lambda}{2} = \frac{5\lambda}{2}$

3. (*d*) Distance of nth bright fringe from the centre,

$$y_n = \frac{nD\lambda}{d}$$

So, $$y_3 = \frac{3 \times 6000 \times 10^{-10} \times 2.5}{0.5 \times 10^{-3}}$$

$$= 9 \times 10^{-3} \text{ m} = 9 \text{ mm}$$

4. (*c*) $\beta = \frac{\lambda D}{d}$

5. (*b*) Fringe width, $\beta = \frac{D\lambda}{d}$

If both d and D are doubled, then

$$\beta' = \frac{(2D)\lambda}{2d} = \frac{D\lambda}{d} = \beta$$

6. (*d*) Refractive index is slightly decreased.

7. (*c*) Separation of bright lines or fringe width

$$\beta = \frac{D\lambda}{d} = \frac{5 \times 10^{-7} \times 2}{10^{-3}} = 10^{-3} \text{ m} = 1.0 \text{ mm}$$

8. (*a*) Fringe width in double slit experiment is given by

$$\beta = \frac{D\lambda}{d}$$

It implies that $\beta \propto \lambda$, so in given options λ_v = minimum.

9. (*c*) $\lambda_{blue} < \lambda_{green}$

$\therefore$ $\beta_{blue} < \beta_{green}$

10. (*d*) We have,

$$n_1\lambda_1 = n_2\lambda_2$$

Number of fringes, $n_2 = \frac{n_1\lambda_1}{\lambda_2} = \frac{60 \times 5461 \times 10^{-9}}{5890 \times 10^{-9}} = 55$ fringes

12. (*b*) Given , $\frac{I_{bright}}{I_{dark}} = \frac{I_{max}}{I_{min}} = 9$

or $\left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2 = 9$ or $\frac{a_1 + a_2}{a_1 - a_2} = 3$

$\Rightarrow$ $\frac{a_1}{a_2} = \frac{3+1}{3-1} \Rightarrow \frac{a_1}{a_2} = 2$

So, $\frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} = \frac{4}{1}$

13. (*d*) In normal Young's double slit apparatus,

$$I_1 = I_2 = I_0$$

$\therefore$ $I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2 = 4I_0$

and $I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2 = 0$

When one of the slit is covered by a glass plate.

$$I_1 < I_0, \ I_2 = I_0$$

$\therefore$ $I_{max} < 4I_0$ and $I_{min} > 0$

14. (*a*) In interference between waves of equal amplitudes a, the minimum intensity is zero and the maximum intensity is proportional to $4a^2$. (because slit width $\propto$ amplitude). For waves of unequal amplitudes a and $A(A > a)$, the minimum intensity is non-zero and the maximum intensity is proportional to $(a + A)^2$, which is greater than $4a^2$. So, the intensities of both the maxima and the minima increase.

17. (*a*) On introducing mica sheet, fringe shift $= \frac{\beta}{\lambda}(\mu - 1)t$

where, t is thickness of sheet.

So, shift $= \frac{\beta}{(5000 \times 10^{-10})} \times (1.5 - 1) \times (2 \times 10^{-6}) = 2\beta$

So, central bright maximum will shift two fringes upward.

18. (*b*) When thin plate of glass is introduced, then lateral displacement of fringes

$$= \frac{\beta}{\lambda}(\mu - 1)t$$

$$= \frac{1 \times 10^{-3}}{600 \times 10^{-9}}(1.5 - 1) \times 0.06 \times 10^{-3} = \frac{1}{20} \text{ m} = 5 \text{ cm}$$

19. (*c*) Shift $= \frac{(\mu - 1)tD}{d}$ or $20\left(\frac{\lambda D}{d}\right) = \frac{(\mu - 1)tD}{d}$

$\therefore$ $\mu = 1 + \frac{20\lambda}{t} = 1 + \frac{20(5.0 \times 10^{-7})}{2.5 \times 10^{-5}} = 1.4$

20. (*a*) For constructive interference in case of soap film,

$$2\mu t = \left(n - \frac{1}{2}\right)\lambda \qquad (n - 1, 2, 3, \ldots)$$

For minimum thickness t, $n = 1$ or $2\mu t = \lambda / 2$

The minimum thickness of a soap bubble,

$$t = \frac{\lambda}{4\mu} = \frac{500}{4 \times 12} = 104.17 \approx 104 \text{ nm}$$

Check point 10.3

3. (*b*) Bending of beam of light around corners of obstacles/slit is called diffraction.

6. (*d*) In Fresnel's class of diffraction, the source of light and the screen on which the diffraction pattern is observed are at finite distance from diffracting obstacle. In this no lenses are used and the incident wavefrom is either spherical or cylindrical.

7. (*b*) For first minima, $d \sin\theta = \frac{\lambda}{2}$ or $d = \frac{\lambda}{2\sin\theta}$

8. (*b*) Angular fringe width, $\theta = \frac{\lambda}{d}$

10. (*d*) In single slit, for nth secondary maxima path difference

$$a \sin\theta = (2n + 1)\frac{\lambda}{2}$$

For first secondary maximum, $n = 1$

$\therefore$ $a \sin\theta = \frac{3\lambda}{2}$

11. (*a*) For nth dark fringe, in single slit

$$a \sin\theta = n\lambda$$

For second dark fringe, $a \sin\theta = 2\lambda$

So, $24 \times 10^{-5} \times 10^{-2} \times \sin 30° = 2\lambda$

$\therefore$ $\lambda = 6 \times 10^{-7}\text{m} = 6000 \text{ Å}$

13. (*a*) $d \sin\theta = \lambda \Rightarrow \theta \approx \frac{\lambda}{d}$

$\therefore$ $2\theta = \frac{2\lambda}{d} = \frac{2 \times 6328 \times 10^{-10}}{0.2 \times 10^{-3}} \times \frac{180°}{\pi} = 0.36°$

Check point 10.4

1. (*b*) Polariser produces polarised light.
2. (*d*) Ultrasonic waves are longitudinal waves.
3. (*a*) Only transverse waves can be polarised.
5. (*c*) Polarisation is not shown by sound waves.
6. (*d*) Polarisation help us to distinguish between sound wave and light wave.
7. (*a*) When the plane polarised light passes through certain substance, the plane of polarisation of the light is rotated about the direction of propagation of light through a certain angle.
8. (*b*) $I = I_0 \cos^2\theta = I_0 \cos^2 45° = \frac{I_0}{2}$
9. (*c*) From Brewster's law, $\mu = \tan i_p \Rightarrow \frac{c}{v} = \tan 60° = \sqrt{3}$

$\Rightarrow$ $v = \frac{c}{\sqrt{3}} = \frac{3 \times 10^8}{\sqrt{3}} = \sqrt{3} \times 10^8 \text{ ms}^{-1}$

10. (*d*) By using $\mu = \tan\theta_p$

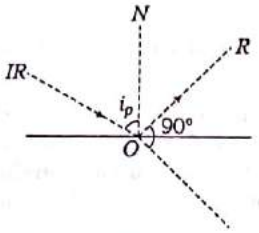
$\Rightarrow \quad \mu = \tan 60° = \sqrt{3}$

Also $\quad C = \sin^{-1}\left(\frac{1}{\mu}\right) \Rightarrow C = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

11. (*d*) $\mu n = \tan\theta_p \Rightarrow \theta_p = \tan^{-1}(n)$

12. (*d*) The amplitude will be $A\cos 60° = A/2$

13. (*d*) Required angle $= 2 \times 57.5 + 90 = 205°$



14. (*a*) According to Brewster's law, when a beam of ordinary light (*i.e.*, unpolarised) is reflected from a transparent medium (like glass), the reflected light is completely plane polarised at certain angle of incidence called the angle of polarisation.

15. (*b*) $\mu = \tan i_p = \tan 54.74° = \sqrt{2}$

$\because \quad \sqrt{2} = \frac{\sin 45°}{\sin r} \Rightarrow \sin r = \frac{1}{2} \Rightarrow r = 30°$

A) Taking it together

1. (*c*) For the interference pattern to be formed on the screen, the sources should be coherent and emits lights of same frequency and wavelength.

In a Young's double slit experiment, when one of the holes is covered by a red filter and another by a blue filter. In this case due to filteration only red and blue lights are present. In YDSE monochromatic light is used for the formation of fringes on the screen. Hence, in this case there shall be no interference fringes.

2. (*c*) We have,

$$\beta = \frac{\lambda D}{d}$$

$\because$ The arrangement is placed in a liquid of refractive index n

$$\therefore \quad \lambda' = \frac{\lambda}{n}$$

Therefore fringe width

$$\beta' = \frac{\lambda D}{dn}$$

or $$\beta' = \frac{\beta}{n}$$

3. (*a*) Given, width of the slit $= 10^4$ Å

$$= 10^4 \times 10^{-10}\ \text{m} = 10^{-6}\ \text{m} = 1\ \mu\text{m}$$

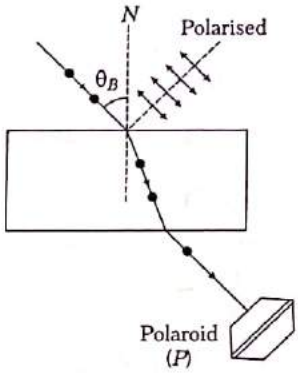
Wavelength of (visible) sunlight varies from 4000 Å to 8000 Å. As the width of slit is comparable to that of wavelength, hence diffraction occurs with maxima at centre. So, at the centre all colours appear *i.e.*, mixing of colours form white patch at the centre.

4. (*c*) $\theta(\text{in radian}) = \frac{\lambda}{d}$

$$\therefore \quad d = \frac{\lambda}{\theta} = \frac{6 \times 10^{-7}}{\pi/180} = 3.4 \times 10^{-5}\text{m}$$

5. (*c*) Consider the diagram the light beam incident from air to the glass slab at Brewster's angle (i_p). The incident ray is unpolarised and is represented by dot (.).

The reflected light is plane polarised represented by arrows. As the emergent ray is unpolarised, hence intensity cannot be zero when passes through polaroid.



6. (*b*) Where phase difference is 0, intensity is maximum.

where, $\phi = \frac{\pi}{2}$ or $\frac{\phi}{2} = \frac{\pi}{4}$,

the intensity will be $I = I_{max}\cos^2\frac{\phi}{2} = \frac{I_{max}}{2}$

$\therefore$ The desired ratio is 2 : 1.

7. (*c*) $\quad 2\mu t = \frac{\lambda}{2}$

$$\therefore \quad t_{min} = \frac{\lambda}{4\mu} = 141\ \text{nm}$$

10. (*b*) Now, here we are given Fresnel distance equal to 20 m $\lambda = 450$ nm, we hae to estimate value of a.

Putting $Z_F = \frac{a^2}{\lambda}$, $a = \sqrt{Z_F\lambda} = 3 \times 10^{-3}$ m or 3 mm

11. (*b*) $\Delta Y = \frac{5\lambda D}{d} - \frac{5\lambda D}{2d} = \frac{5\lambda D}{2d}$

$$= \frac{5 \times 6.5 \times 10^{-7} \times 1}{2 \times 10^{-3}}\ \text{m} = 1.63\ \text{mm}$$

13. (*c*) In single narrow slit, for first minima,

$$d\sin\theta = \lambda \Rightarrow \sin\theta = \frac{\lambda}{d}$$

$$\therefore \quad \theta = \sin^{-1}\left(\frac{5000 \times 10^{-10}}{0.001 \times 10^{-3}}\right) = 30°$$

14. (*a*) We have, $y_1 = 10\ \omega_1 = \frac{10\lambda_1 D}{d}$, $y_2 = 5\omega_2 = \frac{5\lambda_2 D}{d}$

$$\therefore \quad \frac{y_1}{y_2} = \frac{2\lambda_1}{\lambda_2}$$

15. (c) Intensity, $I = 4I_0 \cos^2 \frac{\phi}{2}$

$\because \quad I = I_0$

$\therefore \quad \phi = \frac{2\pi}{3}$

16. (c) Distance of 5th bright fringe from central fringe,

$$x_{5D} = \frac{5\lambda D}{d}$$

Distance of 3rd dark fringe from central fringe,

$$x_{3D} = \frac{(2\times 3 - 1)\times D}{2d} = \frac{5}{2}\frac{\lambda D}{d}$$

Required distance,

$$x_{5D} - x_{3D} = \left(5 - \frac{5}{2}\right)\frac{\lambda D}{2} = \frac{5}{2}\times\frac{5\times 10^{-7}\times 1}{1\times 10^{-3}} = 1.25\,\text{mm}$$

17. (c) We have, $2\mu t = \frac{\lambda}{2}$

$\Rightarrow$ Minimum thickness,

$$t = \frac{\lambda}{4\mu} = \frac{750\times 10^{-9}}{4\times 4/3} = 141\text{ nm}$$

18. (a) On introducing mica sheet, fringe shift $= \frac{\beta}{\lambda}(\mu - 1)t$

where, t is thickness of sheet.

So, shift $= \frac{\beta}{(5000\times 10^{-10})}\times(1.5-1)\times(4\times 10^{-6}) = 4\beta$

So, central bright maximum will shift 4 fringes upward.

19. (b) $\beta = \frac{\lambda D}{d} = \frac{589.3\times 10^{-9}}{0.1\times 10^{-3}}D$

Hence, angular width of the central diffraction.

$$\theta = \frac{\beta}{D} = \frac{589.3\times 10^{-9}}{0.1\times 10^{-3}}\text{ rad}$$

$$= 589.3\times 10^{-5}\times\frac{180}{\pi} = 0.34°$$

20. (a) We have, $5.5\beta = 10\beta'$

$\therefore \quad \frac{\beta}{\beta'} = \frac{10}{5.5} = 1.818 = \mu$

21. (b) $\Delta I = (I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi_1) - (I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi_2)$

$$= 2\sqrt{I_1 I_2}(\cos\phi_1 - \cos\phi_2) = 2\times 2I\left(\cos\frac{\pi}{2} - \cos\pi\right) = 4I$$

22. (b) Let nth red coincides with mth blue, then

$$\frac{n\lambda_R D}{d} = \frac{m\lambda_B D}{d}$$

$$\therefore \quad \frac{n}{m} = \frac{\lambda_B}{\lambda_R} = \frac{480}{600} = \frac{4}{5}$$

i.e., 4th red coincides with 5th blue. Next 8th red will coincide with 10th blue and so on. Thus, $n = 4$

and $\quad m = n + 1 = 5$

23. (b) Resultant intensity, $I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

$I_A = 5I$ and $I_B = I$

$\therefore$ The difference between the intensity, $I_A - I_B = 4I$

24. (b) For maximum (brightness) intensity of the screen, path difference should be integer multiple of wavelength, i.e.,

$$d = \sin\theta = n\lambda$$

$$\sin\theta = \frac{n\lambda}{d} = \frac{n(2000)}{7000} = \frac{n}{3.5}$$

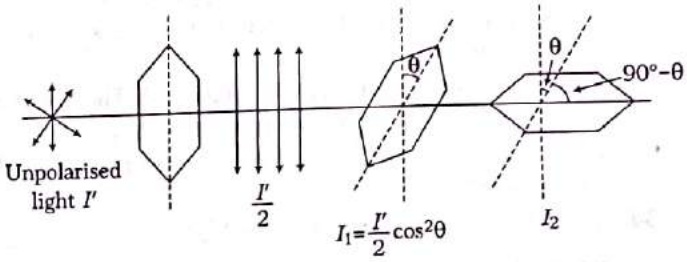
As, $\quad (\sin\theta)_{max} = 1$

So, $\quad n = 0, 1, 2, 3$

Thus, only seven maximas can be obtained on both sides of the screen.

25. (a) Fringe size will not change.

26. (a) $I_2 = I_1\cos^2(90° - \theta) = \frac{I'}{2}\cos^2\theta\sin^2\theta = \frac{I'}{8}(\sin 2\theta)^2$



$$3 = \frac{32}{8}(\sin 2\theta)^2$$

$$\sin 2\theta = \frac{\sqrt{3}}{2} = \sin 60° \text{ or } \sin 120°$$

$$\theta = 30° \text{ or } 60°$$

27. (a) Number of fringes shifted, $N = \frac{(\mu - 1)t}{\lambda}$

$\therefore$ Refractive index of the sheet,

$$\mu = \frac{N\lambda}{t} + 1 = \frac{30\times 6000\times 10^{-10}}{3.6\times 10^{-5}} + 1 = 1.5$$

28. (c) Fringe width, $\beta = \frac{D\lambda}{d}$

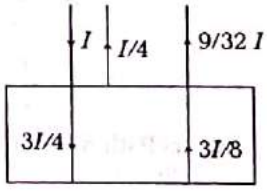
i.e., $\quad \beta \propto \lambda$

So, wavelength λ and hence fringe width β decreases 1.5 times when immersed in liquid. The distance between central maxima and 10th maxima is 3 cm in vacuum. When immersed in liquid it will reduce to 2 cm. Position of central maxima will not change while 10th maxima will be obtained at $y = 4$ cm.

29. (b) Position of third maxima,

$$x_3 = \frac{3\lambda D}{d} = \frac{3(5\times 10^{-5})(200)}{0.02}\text{ cm} = 1.5\text{ cm}$$

30. (d) $I_1 = \frac{I}{4}$ and $I_2 = \frac{9I}{32}$



$\therefore \quad \frac{I_1}{I_2} = \frac{8}{9}$

or $\quad \sqrt{\frac{I_1}{I_2}} = \frac{2\sqrt{2}}{3}$

$$\therefore \quad \frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1/I_2} + 1}{\sqrt{I_1/I_2} - 1}\right)^2 = \left(\frac{2\sqrt{2} + 3}{2\sqrt{2} - 3}\right)^2$$

31. (*a*) According to the question,

$$y = \frac{\omega}{4} = \frac{\lambda D}{4d} \Rightarrow \Delta x = \frac{yd}{D} = \frac{\lambda}{4}$$

Phase difference, $\phi = \frac{2\pi}{\lambda} \cdot \Delta x = \frac{\pi}{2}$

$\therefore \quad \frac{\phi}{2} = \frac{\pi}{4}$

Now, intensity, $I_2 = I_1 \cos^2 \frac{\phi}{2}$ or $\frac{I_1}{I_2} = \frac{1}{\cos^2 \phi/2} = 2$

33. (*d*) According to question, there is a hole at point P_2. From Huygens' principle, wave will propagates from the sources S_1 and S_2. Each point on the screen will acts as secondary sources of wavelets.

Now, there is a hole at point P_2 (minima). The hole will act as a source of fresh light for the slits S_3 and S_4.

Therefore, there will be a regular two slit pattern on the second screen.

34. (*c*) Intensity in the first case, $I_1 = 4I_0$

where, I_0 = intensity due to each source

Intensity in second case, $I_2 = 2I_0$

$\therefore \quad \frac{I_1}{I_2} = \frac{4I_0}{2I_0} = \frac{2}{1}$

35. (*a*) Shift $= \frac{(\mu - 1)tD}{d} = \omega = \frac{\lambda D}{d}$

$\therefore \quad t = 2\lambda \qquad$ (as $\mu = 1.5$)

36. (*a*) Path difference, $\Delta x = \frac{yd}{D} = \left(\frac{1.25\lambda D}{d}\right)\left(\frac{d}{D}\right)$

$= 1.25\lambda = \frac{5}{4}\lambda$

Phase difference, $\phi = \frac{2\pi}{\lambda} \cdot \Delta x$

$= \frac{2\pi}{\lambda} \times \frac{5}{4}\lambda = \frac{5}{2}\pi$

$\therefore \quad \frac{\phi}{2} = \frac{5}{4}\pi = 225°$

$\therefore \quad$ Intensity, $I = I_{max} \cos^2 \frac{\phi}{2} = \frac{1}{2} I_{max}$

37. (*c*) Intensity, $I = 4I_0 \cos^2 \frac{\phi}{2}$

$\because \quad I = I_0$

$\therefore \quad \phi = \frac{2\pi}{3} = \frac{2\pi}{\lambda} \cdot \Delta x = \frac{2\pi}{\lambda}\left(\frac{yd}{D}\right)$

$\therefore \quad y = \frac{\lambda D}{3d}$

38. (*c*) Path difference between the rays reaching in front of slit S_1 is

$$S_1P - S_2P = \sqrt{(b^2 - d^2)} - d$$

For destructive interference at P

$$S_1P - S_2P = \frac{(2n-1)\lambda}{2}$$

i.e., $\sqrt{(b^2 + d^2)} - d = \frac{(2n-1)\lambda}{2}$

$\Rightarrow \quad d\sqrt{\left(1 + \frac{b^2}{d^2}\right)} - d = \frac{(2n-1)\lambda}{2}$

$\Rightarrow \quad d\left(1 + \frac{b^2}{2d^2}\right) - d = \frac{(2n-1)\lambda}{2}$

$\Rightarrow \quad \frac{\beta^2}{2\delta} = \frac{(2\nu - 1)\lambda}{2}$

$\therefore \quad \lambda = \frac{b^2}{(2n-1)d}$

For $\quad n = 1, 2, \ldots,$

$\Rightarrow \quad \lambda = \frac{b^2}{d}, \frac{b^2}{3d} \ldots\ldots$

39. (*c*) $2I_0 = 4I_0 \cos^2 \frac{\phi}{2}$

$\phi = \frac{\pi}{2} = \frac{2\pi}{\lambda} \cdot \Delta x = \frac{2\pi}{\lambda} \cdot \left(\frac{y_1 d}{D}\right)$

$\therefore \quad y_1 = \frac{\lambda D}{4d}$

Further, $I_0 = 4I_0 \cos^2 \frac{\phi}{2}$

$\Rightarrow \quad \phi = \frac{2\pi}{3} = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda}\left(\frac{y_2 d}{D}\right)$

$\therefore \quad y_2 = \frac{\lambda D}{3d}$

Minimum separation, $\Delta y = y_2 - y_1 = \frac{1}{12}\left(\frac{\lambda D}{d}\right) = \frac{\alpha}{12}$

40. (*c*) Five fringes have to be shifted.

$\because \quad 5 = \frac{(\mu_1 - 1)t - (\mu_2 - 1)t}{\lambda} = \frac{(\mu_1 - \mu_2)t}{\lambda}$

$\therefore$ Thickness, $t = \frac{5\lambda}{(\mu_1 - \mu_2)} = \frac{5 \times 480 \times 10^{-9}}{1.7 - 1.4}$

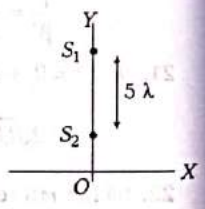
$= 8 \times 10^{-6}$ m

41. (*a*) At $x = 0$, $\Delta x = 5\lambda$

which corresponds to a maximum.

At $x = \infty$, $\Delta x = 0$, this also corresponds to *a* maximum.

$\therefore$ Other possible maxima between $x = 0$ and $x = \infty$ are $\Delta x = \lambda, 2\lambda, 3\lambda, 4\lambda$.



42. (*a*) Location of the point, $y = \frac{\omega}{4} = \frac{\lambda D}{4d}$

$\Rightarrow \quad \Delta x = \frac{yd}{D} = \frac{\lambda}{4}$

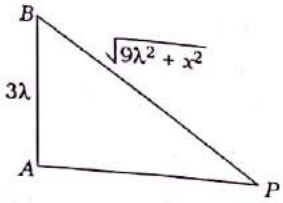
$\Rightarrow \quad \phi = \frac{2\pi}{\lambda} \cdot \Delta x = \frac{\pi}{2}$

$\because \quad I_R = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$

$\therefore \quad 4I = 2I + I_2 + 0$

$\Rightarrow \quad I_2 = 2I$

43. (c) Say P be the point on X-axis, where constructive interference is obtained.



$$BP - AP = 2\lambda \text{ or } \lambda$$

$$\therefore \sqrt{9\lambda^2 + x^2} - x = 2\lambda$$

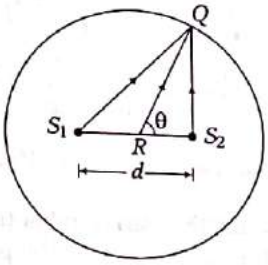
$$\therefore \quad 9\lambda^2 + x^2 = x^2 + 4\lambda^2 + 4x\lambda \Rightarrow x = \frac{5\lambda}{4}$$

Further, $\sqrt{9\lambda^2 + x^2} - x = \lambda$

$$\therefore \quad 9\lambda^2 + x^2 = x^2 + \lambda^2 + 2x\lambda$$

$$\therefore \quad x = 4\lambda$$

44. (b) Path difference at a point Q on the circle is



$$\Delta y = d \cos\theta \quad \text{...(i)}$$

For maxima at Q path difference should be integer multiple of wavelength

$$\Delta y = n\lambda \quad \text{...(ii)}$$

From Eqs. (i) and (ii),

$$n\lambda = d \cos\theta$$

$$\therefore \quad \theta = \cos^{-1}\left(\frac{n\lambda}{d}\right)$$

For $\quad n = 4$

Angular position, $\quad \theta = \cos^{-1}\left(\frac{4\lambda}{d}\right)$

45. (a) Consider the diagram, the ray (P) is incident at an angle θ and gets reflected in the direction P' and refracted in the direction P''. Due to reflection from the glass medium, there is a phase change of π.

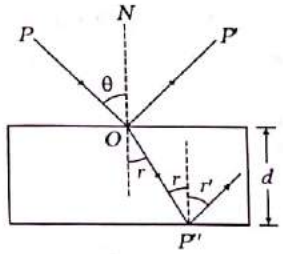
Time taken to travel along OP''

$$\Delta t = \frac{OP''}{v} = \frac{d/\cos r}{c/n} = \frac{nd}{c\cos r}$$

From Snell's law, $n = \frac{\sin\theta}{\sin r} \Rightarrow \sin r = \frac{\sin\theta}{n}$

$$\cos r = \sqrt{1 - \sin^2 r} = \sqrt{1 - \frac{\sin^2\theta}{n^2}}$$

$$\therefore \quad \Delta t = \frac{nd}{c\left(1 - \frac{\sin^2\theta}{n^2}\right)^{1/2}} = \frac{n^2 d}{c}\left(1 - \frac{\sin^2\theta}{n^2}\right)^{-1/2}$$



$$\text{Phase difference} = \Delta\phi = \frac{2\pi}{T} \times \Delta t = \frac{2\pi\, nd}{\lambda}\left(1 - \frac{\sin^2\theta}{n^2}\right)^{-1/2}$$

$$\text{So, net phase difference} = \Delta\phi + \pi = \frac{4\pi d}{\lambda}\left(1 - \frac{1}{n^2}\sin^2\theta\right)^{-1/2} + \pi$$

46. (c) Path difference between the waves reaching at P,

$$\Delta = \Delta_1 + \Delta_2$$

where, Δ_1 = initial path difference

Δ_2 = path difference between the waves after emerging from slits.

Now, $\quad \Delta_1 = SS_1 - SS_2 = \sqrt{(D^2 + d^2)} - D$

and $\quad \Delta_2 = S_1O - S_2O = \sqrt{(D^2 + d^2)} - D$

$$\therefore \quad \Delta = 2\{\sqrt{(D^2 + d^2)} - D\} = 2\{(D^2 + d^2)^{1/2} - D\}$$

$$= 2\left\{\left(D + \frac{d^2}{2D}\right) - D\right\} \text{from binomial expansion}$$

$$= \frac{d^2}{D}$$

For obtaining dark at O, Δ must be equals to $(2n - 1)\frac{\lambda}{2}$

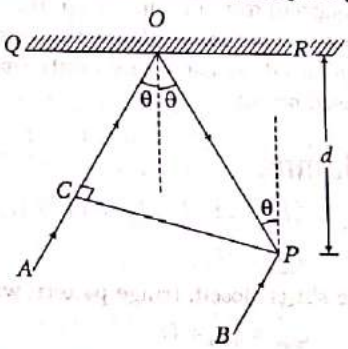
i.e., $\quad \frac{d^2}{D} = (2n - 1)\frac{\lambda}{2}$

$$\therefore \quad d^2 = \frac{(2n - 1)\lambda D}{2} \quad \text{or} \quad d = \sqrt{\frac{(2n - 1)\lambda D}{2}}$$

For minimum distance $n = 1$,

So, $\quad d = \sqrt{\left(\frac{\lambda D}{2}\right)}$

47. (b) Path difference between the two rays is given by



$$\Delta = CO + PO$$

$\because \quad PR = d,$

So, $\quad PO = d \sec\theta$

and $\quad CO = PO \cos 2\theta = d \sec\theta \cos 2\theta$

So, $\Delta = (d \sec\theta + d \sec\theta \cos 2\theta)$

Phase difference between two rays is $\phi = \pi$ (as one ray is reflected one and another is direct).

Now, for constructive interference, path difference should be even multiple of half wavelength.

i.e., $\Delta = \lambda/2, \quad 3\lambda/2, \ldots$

So, $d \sec\theta + d \sec\theta \cos 2\theta = \frac{\lambda}{2}$

or $d \sec\theta (1 + \cos 2\theta) = \frac{\lambda}{2}$

or $d \sec\theta (2\cos^2\theta) = \frac{\lambda}{2}$

$\therefore \quad \cos\theta = \lambda/4d$

B) Medical entrance special format questions

Assertion and reason

3. (*c*) $\phi = \frac{2\pi}{\lambda}\cdot\Delta x = \frac{2\pi}{\lambda}\times\frac{\lambda}{3} = \frac{2\pi}{3}$

and $I = 4I_0 \cos^2\left(\frac{\phi}{2}\right) = 4I_0 \times \frac{1}{4} = I_0$

4. (*b*) $I_{max} > 4I_0$ and $I_{min} = 0$

$\therefore \quad \frac{I_{max}}{I_{min}} = \text{infinite}$

If width of one slit is slightly increased $I_{min} > 0$. Therefore, this ratio will be less than infinite.

5. (*d*) Fringe width does not change by the introduction of glass slab.

6. (*a*) Whole fringe pattern shifts towards that side, where slab is inserted.

10. (*a*) When a polaroid is rotated in the path of unpolarised light, the intensity of light transmitted from polaroid remain undiminished (because unpolarised light contains waves vibrating in all possible planes with equal probability).

However, when the polarised is rotated in path of plane polarised light, its intensity will vary from maximum (when the vibrations of the plane polarised light are parallel to the axis of the polaroid) to minimum (when the direction of the vibrations becomes perpendicular to the axis of the crystal). Thus, using polaroid we can easily verify that whether the light is polarised or not.

Match the columns

1. (a) $I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2 \Rightarrow I_1 = I_0$ and $I_2 > I_0$

$\therefore \quad I_{max} > 4I_0$

(b) When one slit is closed, fringe pattern will disappear.

$I_{max} = I_{min} = I_0$

(c) When both slits are made incoherent, again fringe pattern will disappear.

$I_{max} = I_{min} = I_1 + I_2 = 2I_0$

(d) Glass slab will only shift the fringe pattern.

2. $\Delta x = \frac{yd}{D}, \phi = \frac{2\pi}{\lambda}\cdot\Delta x$ and $I = 4I_0 \cos^2\frac{\phi}{2}$

3. Fringe width, $\beta = \frac{\lambda D}{d}$

In a liquid, λ decreases μ times, therefore ω also decrease μ times.

C) Medical entrances' gallery

1. (*b*) It is given that, $\frac{I_2}{I_1} = n \Rightarrow I_2 = nI_1$

$\therefore$ Ratio of intensities is given by

$$\frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{(\sqrt{I_2} + \sqrt{I_1})^2 - (\sqrt{I_2} - \sqrt{I_1})^2}{(\sqrt{I_1} + \sqrt{I_2})^2 + (\sqrt{I_2} - \sqrt{I_1})^2)}$$

$$= \frac{\left(\sqrt{\frac{I_2}{I_1}} + 1\right)^2 - \left(\sqrt{\frac{I_2}{I_1}} - 1\right)^2}{\left(\sqrt{\frac{I_2}{I_1}} + 1\right)^2 + \left(\sqrt{\frac{I_2}{I_1}} - 1\right)^2}$$

$$= \frac{(\sqrt{n} + 1)^2 - (\sqrt{n} - 1)^2}{(\sqrt{n} + 1)^2 + (\sqrt{n} - 1)^2} = \frac{2\sqrt{n}}{n+1}$$

2. (*d*) Ist minima is formed at a distance $Y = \frac{\lambda D}{a}$.

For the distance of the first dark band of the diffraction pattern from the centre of the screen is given by positon of Ist minima.

i.e., $Y = \frac{\lambda D}{a}$

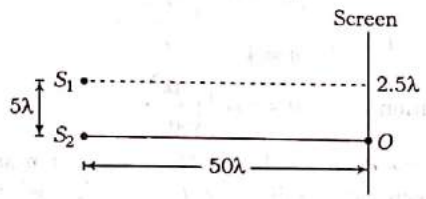
where, λ = wavelength of parallel beams

D = focal length, a = width of linear aperture.

$\Rightarrow \quad Y = \frac{(5\times10^{-5})(0.6)}{0.02\times10^{-2}}$ (given)

$\Rightarrow \quad Y = 0.15$ cm

3. (*c*)



In the above figure, S_1 and S_2 are the two different slits. Given, distance between slits S_1 and S_2, $d = 5\lambda$

distance between screen and slits, $D = 10d = 50\lambda$

Here, λ is the wavelength of light used in the experiment. According to the question, the intensity at maximum in this Young's double slit experiment is I_0.

$\Rightarrow \quad I_{max} = I_0$

$\because$ Path difference $= \frac{dY_n}{D} = \frac{d\times(d/2)}{10d} = \frac{d}{20} = \frac{\lambda}{4}$ $[\because d = 5\lambda]$

A path difference of λ corresponds to phase difference 2π

So, for path difference $\lambda/4$, phase difference

$$\phi = \frac{2\pi}{\lambda}\times\frac{\lambda}{4} = \pi/2 = 90°$$

As we know, $I = I_0 \cos^2 \frac{\phi}{2} \Rightarrow I = I_0 \cos^2 \frac{90°}{2}$

$\Rightarrow \quad I = I_0 \times \left(\frac{1}{\sqrt{2}}\right)^2 \Rightarrow I = \frac{I_0}{2}$

4. (c) As the first minimum is observed at an angle of 30° in a diffraction pattern due to a single slit of width a.

i.e., $n = 1, \theta = 30°$

$\because$ According to Bragg's law of diffraction,

$a \sin\theta = n\lambda \Rightarrow a \sin 30° = (1)\lambda \quad (n = 1)$

$\Rightarrow \quad a = 2\lambda \quad ...(i) \quad \left[\because \sin 30° = \frac{1}{2}\right]$

For Ist secondary maxima,

$\Rightarrow \quad a \sin\theta_1 = \frac{3\lambda}{2} \Rightarrow \sin\theta_1 = \frac{3\lambda}{2a} \quad ...(ii)$

Substitute value of a from Eq. (i) to Eq. (ii), we get

$\sin\theta_1 = \frac{3\lambda}{4\lambda} \Rightarrow \sin\theta_1 = \frac{3}{4} \Rightarrow \theta_1 = \sin^{-1}\left(\frac{3}{4}\right)$

5. (b) In diffraction, fringe width, $\beta = \frac{\lambda D}{d}$

when $\frac{d}{2} = \beta$, then $\frac{d}{2} = \frac{\lambda D}{d} \Rightarrow D = \frac{d^2}{2\lambda}$

6. (a) To find the point of coincidence of bright fringes, we can equate the distance of bright fringes from the central maxima, made by both the wavelengths of light.

Given, distance between the screen and slit,

$D = 120 \text{ cm} = 120 \times 10^{-2} \text{ m}$

Slit width, $d = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}$

$\lambda_1 = 6500 \text{ Å}$ and $\lambda_2 = 5200 \text{ Å}$

Let nth fringe due to $\lambda_2 = 5200$ Å coincide with $(n-1)$th bright fringe due to $\lambda_1 = 6500$ Å

$\therefore \frac{Dn\lambda_2}{d} = \frac{D(n-1)\lambda_1}{d}$ or $n \times 5200 = (n-1)6500$

or $4n = 5n - 5 \Rightarrow n = 5$

So, the least distance required, $y_n = \frac{n\lambda_2 D}{d}$

$= \frac{5 \times 5200 \times 10^{-10} \times 120 \times 10^{-2}}{2 \times 20^{-3}} = 0.16 \times 10^{-2} \text{m} = 0.16 \text{ cm}$

7. (c) The source is at infinity and incident beam is parallel.

8. (a) Most of the light is diffracted between the two first order minima. These minima occur at angles given by $b \sin\theta = \pm\lambda$.

$\Rightarrow \quad \sin\theta = \pm\frac{\lambda}{b} = \pm\frac{4.5 \times 10^{-7} \text{ m}}{0.2 \times 10^{-3} \text{ m}} = \pm 2.25 \times 10^{-3}$

$\Rightarrow \quad \theta \simeq 2.25 \times 10^{-3} \text{ rad}$ (angle is very small)

The angular divergence, $2\theta = 4.5 \times 10^{-3}$ rad

9. (b) $a \sin\theta = (2n-1)\lambda/2$ is the condition for obtaining secondary maxima in the diffraction pattern due to single slit.

10. (b) Fringe width, $\beta = \frac{D\lambda}{d}$

$d = \frac{D\lambda}{\beta} = \frac{6000 \times 10^{-10} \times (40 \times 10^{-2})}{0.012 \times 10^{-2}} = 0.2 \text{ cm}$

11. (d) Given, $\theta_p = 57°$

According to Brewster's law, we have

$\theta_p + r = 90° \Rightarrow r = 90° - \theta_p \Rightarrow r = 90° - 57$

Angle of refraction, $r = 33°$

12. (c) To observed diffraction, the size of the obstacle should be of the order of wavelength. *i.e.*, $\lambda \approx d$.

13. (a) According to the question,

$\lambda = 6250 \text{ Å} = 6250 \times 10^{-10} \times 10^2 \text{ cm} = 6250 \times 10^{-8} \text{ cm}$

Width of the slit, $a = 2 \times 10^{-2}$ cm

The angular width of central maxima, $\beta = 2\theta = 2\frac{\lambda}{a}$

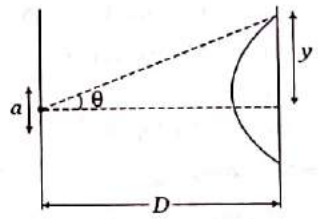
Thus, linear width is

$L = \beta \times D = \frac{2\lambda}{a} \times D = \frac{2 \times 6250 \times 10^{-8}}{2 \times 10^{-2}} \times 50$

$= 6250 \times 10^{-6} \times 50 = 3125 \times 10^{-4}$

$= 312.5 \times 10^{-3}$ cm

14. (a) For the condition of maxima $\sin\theta = \lambda/a$



From the geometry,

$\sin\theta = \theta = \frac{y}{D}$ (for small angle)

So, $\frac{y}{D} = \frac{\lambda}{a} \Rightarrow y = \frac{\lambda D}{a}$

Hence, width of central maxima $= 2y = \frac{2\lambda D}{a}$

15. (a) Given, wavelength, $\lambda = 590 \text{ nm} = 590 \times 10^{-9}$ m

Diameter of lens $= 10 \times 10^{-2}$ m

$\therefore$ Radius of lens $= 5 \times 10^{-2}$ m

Focal length of lens $= 20 \times 10^{-2}$ m

Now, the diameter of image disc

$D = 2R = 2\left(1.22\frac{\lambda D}{b}\right) = \left(\frac{122 \times 590 \times 10^{-9} \times 20 \times 10^{-2}}{5 \times 10^{-2}}\right) \times 2$

$= 5.76 \times 10^{-4}$ cm

16. (c) According to law of Malus, when a beam of completely plane polarised light is incident on an analyser, the resultant intensity of light (I) transmitted from an analyser varies directly as the square of cosine angle between plane of analyser and polariser.

Let initial intensity is I and final intensity is I', then

$I' = I\cos^2 60 = I\left(\frac{1}{2}\right)^2 = \frac{I}{4}$

Now, % difference between initial and final intensities,

$= \frac{I - (I/4)}{I} \times 100 = \frac{3I}{4} \times \frac{1}{I} \times 100 = \frac{3}{4} \times 100 = 75\%$

17. (*a*) Given $d = 1\,\text{mm} = 1\times10^{-3}\text{m}$, $D = 1\text{m}$

$$\lambda = 500\text{ nm} = 5\times10^{-7}\text{ m}$$

As width of central maxima = width of 10 maxima

$$\therefore\ \frac{2D\lambda}{a} = 10\left(\frac{\lambda D}{d}\right) \Rightarrow a = \frac{d}{5} = \frac{10^{-3}}{5} = 0.2\times10^{-3}\text{m}$$

$$a = 0.2\text{ mm}$$

18. (*a*) In Young's double slit experiment, the locus of the point P lying in a plane with a constant path difference between the two interfering waves is a hyperbola.

19. (*a*) On the wavefront, all the points are in the same phases.

20. (*c*) For net intensity, $I' = 4I_0\cos^2\frac{\phi}{2}$

***Case* I** $\quad K = 4I_0\cos^2\left(\frac{2\pi}{2}\right) \qquad \left(\because\ \phi = \frac{2\pi}{\lambda}\times\lambda\right)$

$$K = 4I_0\cos^2(\pi) \Rightarrow K = 4I_0 \qquad \text{...(i)}$$

***Case* II** $\quad K' = 4I_0\cos^2\left(\frac{\pi/2}{2}\right) \qquad \left(\because\ \phi = \frac{2\pi}{\lambda}\times\frac{\lambda}{4}\right)$

$$K' = 4I_0\cos^2(\pi/4) \Rightarrow K' = 2I_0 \qquad \text{...(ii)}$$

Comparing Eqs. (i) and (ii), we get, $K' = K/2$

21. (*b*) $\frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{(a_1+a_2)^2 - (a_1-a_2)^2}{(a_1+a_2)^2 + (a_1-a_2)^2}$

$[\because I_{max} = (a_1+a_2)^2, I_{min} = (a_1-a_2)^2$, where a = amplitude$]$

$$= \frac{4a_1a_2}{2(a_1^2+a_2^2)} = \frac{2a_1a_2}{(a_1^2+a_2^2)}$$

Now, dividing the numerator and denominator by a_1a_2, we get

$$\frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{2}{\left(\frac{a_1}{a_2} + \frac{a_2}{a_1}\right)} \qquad \left(\because \frac{I_1}{I_2} = \alpha \Rightarrow \frac{a_1}{a_2} = \sqrt{\alpha}\right)$$

$$\Rightarrow \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{2}{\left(\sqrt{\alpha} + \frac{1}{\sqrt{\alpha}}\right)} = \frac{2\sqrt{\alpha}}{(\alpha+1)}$$

22. (*d*) We know that,

The maximum intensity, $I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$ ...(i)

The minimum intensity, $I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$ ...(ii)

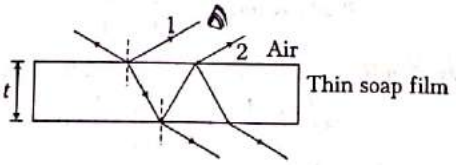
So, the ratio of the maximum and minimum intensities is

$$\frac{I_{max}}{I_{min}} = \frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2} = \frac{(\sqrt{4I} + \sqrt{I})^2}{(\sqrt{4I} - \sqrt{I})^2}$$

$$\Rightarrow \frac{I_{max}}{I_{min}} = \left(\frac{3\sqrt{I}}{\sqrt{I}}\right)^2 = \frac{9}{1} = 9:1$$

Hence, the maximum and minimum intensities in the resulting pattern are $9I$ and I.

23. (*a*) The diagram below shows a thin soap film and a soap bubble.



The rays 1 and 2 produce the phenomenon of interference when they meet. The path difference between the rays are proportional to thickness of film,

i.e., path difference $(\Delta x) \propto t$

As, thickness t varies so Δx varies, maximas and minimas appear for different wavelengths in incident white light. When white light is to used to the path difference (phase difference) will satisfy the condition of maxima for certain wavelengths and condition of minima for other wavelengths. This gives coloured appearance of the film.

24. (*c*) $\frac{I_{max}}{I_{min}} = \left(\frac{a_{max}}{a_{min}}\right)^2 = \left(\frac{a_1+a_2}{a_1-a_2}\right)^2$

According to given question, $\frac{I_{max}}{I_{min}} = \frac{9}{1}$

$$\Rightarrow \quad 9 = \left(\frac{a_1/a_2+1}{a_1/a_2-1}\right)^2 \Rightarrow (3)^2 = \left(\frac{a_1/a_2+1}{a_1/a_2-1}\right)^2$$

$$\Rightarrow \quad 3 = \frac{a_1/a_2+1}{a_1/a_2-1} \Rightarrow 3\left(\frac{a_1}{a_2}\right) - 3 = \frac{a_1}{a_2} + 1$$

$$\Rightarrow \quad 2\left(\frac{a_1}{a_2}\right) = 4 \Rightarrow \frac{a_1}{a_2} = \frac{4}{2} = \frac{2}{1} \Rightarrow a_1 : a_2 = 2:1$$

25. (*c*) According to given question,

$$I_1 : I_2 = 3:1,\quad a_1 : a_2 = ?$$

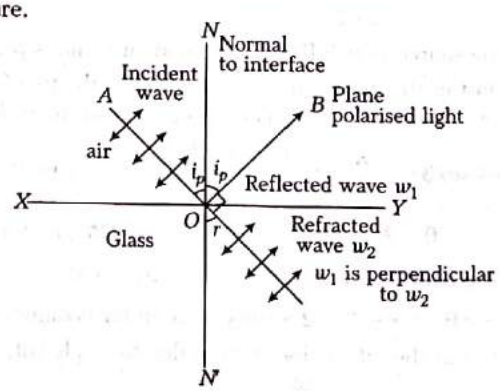
The intensity of light due to an illuminated slit is directly proportional to the width of the slit, if w_1 and w_2 be the width of two silts and I_1 and I_2 are the intensities of light on the screen due to the respective slits, then

$$\frac{w_1}{w_2} = \frac{I_1}{I_2} = \left(\frac{a_1}{a_2}\right)^2$$

where, a_1 and a_2 are the amplitudes of the corresponding light wave.

Here, $\frac{w_1}{w_2} = \frac{3}{1} \Rightarrow \left(\frac{a_1}{a_2}\right)^2 = \frac{3}{1}$ or $a_1 : a_2 = \sqrt{3}:1$

26. (d) Suppose, i_p is the Brewster's angle, when unpolarised light is incident at an polarising angle, the reflected component along OB and refracted component along OC in the following figure.



Now, $\quad \angle BOY + \angle YOC = 90°$

$$(90° - i_p) + (90° - r) = 90°$$

Here, r is the angle of refraction.

i.e. $90° - i_p = r$

According to Snell's law, $\mu = \frac{\sin i}{\sin r}$

When $i = i_p, r = 90° - i_p$

Then, $\mu = \frac{\sin i_p}{\sin(90° - i_p)}$

$= \frac{\sin i_p}{\cos i_p} = \tan i_p$

$\therefore$ $i_p = \tan^{-1}(\mu)$

where, i_p is polarised angle.

27. (*d*) By Malus's law, $I = I_0 \cos^2\theta$ (I = Intensity of emergent polarised light, I_0 = Intensity passed through polariser) where, $\theta = 60°, I = ?$

$\Rightarrow$ $I = I_0 \times \cos^2 60° = I_0 \times \left(\frac{1}{2}\right)^2 = \frac{I_0}{4}$

28. (*c*) The distance of 5th dark fringe from centre is given by

$$x_n = (2n+1)\frac{\lambda D}{2d}$$

As, $n = 4/5$ for 5th dark fringe

So, $x_5 = \frac{11}{2}\frac{\lambda D}{d} = \frac{11}{2}\beta$ $\left(\because \frac{\lambda D}{d} = \beta\right)$

So, $= 11/2 \times 0.002 = 0.011$ cm $= 1.1 \times 10^{-2}$ cm

29. (*b*) Given $\lambda_1 = 12000$ Å and $\lambda_2 = 10000$ Å, $D = 2$ cm and $d = 2$ mm $= 2 \times 10^{-3}$ cm.

We have $\frac{\lambda_1}{\lambda_2} = \frac{12000}{10000} = \frac{6}{5} = \frac{n_2}{n_1}$

as $x = \frac{n_1\lambda_1 D}{d} = \frac{5 \times 12000 \times 10^{-10} \times 2}{2 \times 10^{-3}}$

$= 5 \times 1.2 \times 10^4 \times 10^{-10} \times 10^3 = 6$ mm

30. (*c*) Fringe width $\beta = \frac{D\lambda}{d}$

Given, $d = 0.4$ mm $= 0.04$ cm, $D = 200$ cm

$\beta = 2$ mm $= 0.2$ cm

$\therefore$ $0.2 = \frac{200 \times \lambda}{0.04}$ $\Rightarrow$ $\lambda = 400$ nm

31. (*b*) Here the condition for the formation of rings is

$$\lambda \approx \frac{d^2}{4D}$$

32. (*c*) Wave theory of light explains the phenomenon of interference, diffraction and velocity of light in a denser medium or rarer medium but this theory fails to explain the scattering of light.

33. (*a*) Fringe width $\beta \propto \lambda$. As given that λ_1 produces photo electric effect, so $\lambda_1 < \lambda_2$.

Hence, $\beta_1 < \beta_2$

34. (*e*) When we close one slit in the Young's double slit experiment, then the single slit diffraction pattern is observed.

35. (*c*) Given $\mu = 1.54$

We know that $\theta = \tan^{-1}(\mu) \Rightarrow \theta = \tan^{-1}(1.54) \Rightarrow \theta = 57°$

On rotating the polaroid, the reflected light is fully polarised. Here, the intensity gradually reduces to zero and then again increases.

36. (*b*) We know that, phase difference in YDSE experiment, $\phi = \frac{n\pi}{2}$

Here, $n = 3 \Rightarrow \phi = \frac{3\pi}{2}$

38. (*c*) We know for maxima

$$2\mu t \cos r = \frac{(2n+1)\lambda_1}{2} \quad ...(i)$$

For minima $2\mu t \cos r = (n+1)\lambda_2$ ...(ii)

Given here, $\lambda_1 = 600$ nm and $\lambda_2 = 450$ nm

Dividing Eq. (i) by (ii) and substituting values of λ_1 and λ_2, we get

$$\frac{(2n+1)\lambda_1}{2(n+1)\lambda_2} = 1$$

$$90m + 90 = 60n + 60$$

$$30n = 30 \Rightarrow n = 1$$

Using one of the equation, we get

In given condition $2\mu t = (2n+1)\frac{\lambda}{2}$

$$2 \times 1.5 \times t = (2 \times 1 + 1)\frac{600}{2}$$

$$3t = \frac{3 \times 600}{2}$$

$\Rightarrow$ $t = 300$ nm $= 3 \times 10^{-6}$ m $= 3$ units

39. (*a*) Fringe width, $\beta = \frac{D\lambda}{d}$

According to question, $\frac{\beta_2}{\beta_1} = \frac{\lambda_2 D_2 d_1}{\lambda_1 D_1 d_2}$

Here, $D_2 = 2D_1$

$d_2 = \frac{d_1}{2}$

$\Rightarrow$ $\frac{\beta_2}{\beta_1} = \frac{\lambda_2(2D_1) \times d_1}{\lambda_1(D_1) \times \left(\frac{d_1}{2}\right)} = 4\frac{\lambda_2}{\lambda_1} = \frac{4 \times 4 \times 10^{-7}}{6.4 \times 10^{-7}}$

$\Rightarrow$ $\beta_2 = \frac{4 \times 4 \times 10^{-7} \times 1 \times 10^{-4}}{6.4 \times 10^{-7}} = 2.5 \times 10^{-4}$ m

40. (*a*) For diffraction to occur, the size of an obstacle/aperture is comparable to the wavelength of light wave.

41. (*b*) If a transparent medium of thickness t and refractive index μ is introduced in the path of one of the slits, then effective path in air is increased by an amount $(\mu - 1)t$ due to introduction of plate. Therefore, the zeroth fringe shifts to a new position where the two optical paths are equal. In such case fringe width remains unchanged.

42. (*a*) According to Newton's corpuscular theory of light, the light should travel faster in denser media than in rarer media. It is contrary to present theory of light which explains that light travels faster in air (rarer) than in water (denser).

CHAPTER

11

Dual nature of radiation and matter

We know that, light have the properties of electromagnetic waves. Various phenomena like interference, diffraction, polarization etc, demonstrated by light were explained by considering the light as waves. But some other phenomena like photoelectric effect etc, could not be explained by the wave theory of light. To explain photoelectric effect, Einstein used Planck's quantum theory, according to which light consists of the packets of energy. Each packet of energy is called **photon** or **quantum** of light. These packets of energy travel in a straight line with the speed of light. Thus, the particle nature of light was established.

Hence, it was concluded that light is of dual nature, as some phenomena were explained by wave theory of light and some by particle nature of light.

Similar to light, matter also has dual nature, *e.g.*, electron is a matter particle as it contains mass and charge but fast moving electrons, *i.e.*, cathode rays show the properties of waves. Thus, matter also exhibits dual nature.

In this chapter, we will study the dual nature of radiation and matter followed by electron emission, photoelectric effect and matter waves.

Inside

1) Photoelectric effect
   - Experimental study of photoelectric effect
2) Laws of photoelectric emission
   - Einstein's explanation of photoelectric effect : Einstein's photoelectric equation
3) Planck's quantum theory (Particle nature of light : the photon)
4) Photocell
5) Compton effect
6) Dual nature of radiation
   - Matter waves : de-Broglie waves
7) Davisson and Germer experiment (Experimental verification of waves matter)
8) Electron microscope

Electron emission

In metals, the electrons in the outer shells (valence electrons) are loosely bound to the atoms, hence they are free to move easily within the metal surface but cannot leave the metal surface. Such electrons are called free electrons.

These free electrons can be emitted from the metals, if they have sufficient energy to overcome the attractive pull of metal surface.

The phenomenon of emission of electrons from the surface of a metal is called **electron emission.**

The minimum required energy for the electron emission from the metal surface can be supplied to the free electrons by any of the following physical processes

(i) **Thermionic emission** The energy to the free electrons can be given by heating the metal. This process of emission of electrons is known as thermionic emission. The electrons so emitted are known as **thermions.**

(ii) **Field emission** When a conductor is put under strong electric field ($\approx 10^8$ V/m), the free electrons on it experience an electric force in the opposite direction of field. Beyond a certain limit, electrons start coming out of the metal surface. Emission of electrons from a metal surface by this method is called the field emission.

Note Field emission is also known as **cold emission** or **cold cathode emission.** One of the example of cold emission is spark plug.

(iii) **Secondary emission** Emission of electrons from a metal surface by the bombardment of high speed electrons or other particles is known as secondary emission. The electrons so emitted are called secondary electrons.

(iv) **Photoelectric emission** It is the phenomenon of emission of electrons from the surface of metal when light radiations of suitable frequency fall on it. Here, the energy to free electrons is supplied by light photons.

Work function

To pull out electrons from the surface of the metal, a certain minimum amount of energy is required. This minimum energy required by the electrons is called the work function of the metal. It is denoted by ϕ_0.

It is measured in eV (electron volt). 1 *eV is defined as the energy gained by an electron when it has been accelerated by a potential difference of one volt* (1 *V*), so that 1 eV = 1.602×10^{-19}J.

Cesium is regarded as the best photosensitive material as it has lowest work function (ϕ_0 =2.14 eV), whereas work function for platinum is highest (ϕ_0 = 5.65 eV).

Note Work function depends on the properties of the metal and the nature of its surface. It decreases with increase in temperature.

1 Photoelectric effect

The phenomenon of emission of electrons from the surface of metal, when radiations of suitable frequency fall on it, is known as photoelectric effect.

The emitted electrons (as shown in Fig. 11.1) are called **photoelectrons** and the current, so produced is called **photoelectric current.**

Alkali metals like lithium, sodium, potassium, cesium etc, show photoelectric effect with visible light, whereas the metals like zinc, cadmium, magnesium etc are sensitive only to ultraviolet light.

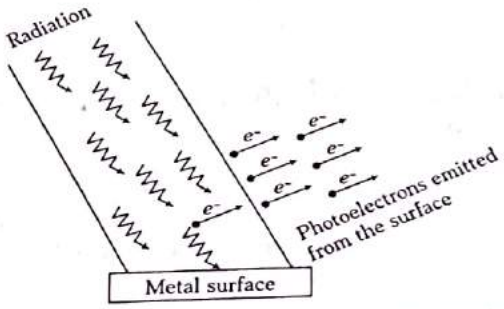


Fig. 11.1 Emission of photoelectrons from the metal surface

The phenomenon of photoelectric effect was discovered by Heinrich Hertz in 1887 while detailed investigation on photoelectric emission was done by Wilhelm hallwachs and Philipp Lenard.

Experimental study of photoelectric effect

An extensive study of photoelectric effect was made by Lenard and RA Millikan. Fig. 11.2 shows the experimental arrangement for the study of photoelectric effect.

It consists of an evacuated glass or quartz tube which encloses a photosensitive plate *C* (called emitter) and a metal plate *A* (called collector). A transparent quartz window is sealed onto the glass tube which permits ultraviolet radiation to pass through it and fall on photosensitive plate *C*. The electrons are emitted by the plate *C* and are collected by the plate *A*. When the collector plate *A* is positive with respect to the emitter plate *C*, the electrons are attracted to it and hence photoelectron current is constituted.

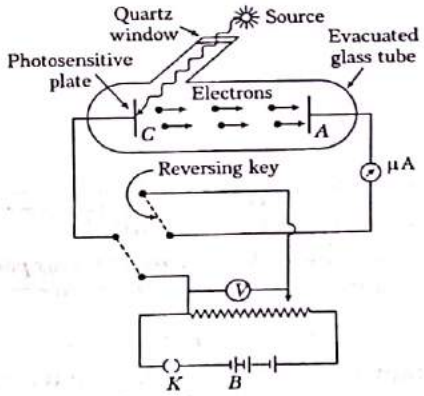


Fig. 11.2 Experimental arrangement for the study of photoelectric effect

The experimental arrangement given above is used to study the variation of photoelectric current with intensity of radiation, frequency of radiation and the potential difference between the plates A and C.

These variations are described below

Effect of intensity of light on photocurrent

For a fixed frequency of incident radiation and accelerating potential, the photoelectric current increases linearly with increase in intensity of incident light.

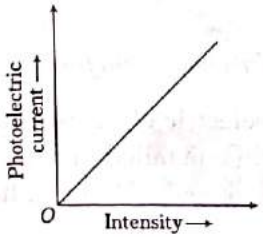


***Fig. 11.3** Variation of photoelectric current with intensity of light*

As the photoelectric current is directly proportional to the number of photoelectrons emitted per second. So for constant frequency of incident radiation, the number of photoelectrons emitted per second is directly proportional to the intensity of the incident radiations.

Intensity $\propto$ Photoelectric current

$$\propto \text{number of photoelectrons emitted/second} \propto \frac{1}{(\text{distance})^2}$$

Effect of potential on photoelectric current

For a fixed frequency and intensity of incident light, photoelectric current increases with increase in potential applied to the collector, as shown in the graph.

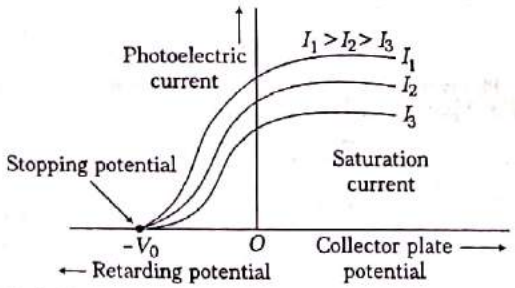


***Fig. 11.4** Variation of photoelectric current versus potential for different intensities but constant frequency*

From the above graph, we observe that

(i) After a certain value of accelerating potential, when all photoelectrons reach the plate A, the photocurrent does not increase. On increasing the value of accelerating potential, this maximum value of photoelectric current is called **saturation current.**

(ii) When the potential is decreased, the current decreases but does not become zero at zero potential. This shows that even in the absence of accelerating potential, few photoelectrons manage to reach the plate A on their own due to their kinetic energy.

(iii) For a particular frequency of incident radiation, the minimum negative (retarding) potential V_0 given to plate A for which the photoelectric current becomes zero and this is called **cut-off potential** or **stopping potential.**

In this condition, no electron (even the most energetic electron) reaches to plate A, *i.e.*, the stopping potential is sufficient to repel even the most energetic photoelectron with maximum kinetic energy K_{max}.

Hence, maximum kinetic energy of emitted electrons is given as,

$$\boxed{K_{max} = eV_0 = \frac{1}{2}mv_{max}^2}$$

where, m is the mass of photoelectron and v_{max} is the maximum velocity of emitted photoelectron.

Note For the radiation of a given frequency and material of plate C, the value of stopping potential V_0 is independent of the intensity of the incident radiation. It means, that the maximum kinetic energy of emitted photoelectron depends on the light source and the emitter plate material but is independent of intensity of incident radiation.

***Example 11.1** The photoelectric cut-off voltage in a certain experiment is 1.5 V. What is the maximum kinetic energy of photoelectrons emitted?*

Sol. Maximum kinetic energy of photoelectrons $K_{max} = eV_0$

where, $e = 1.6 \times 10^{-19}$ C, $V_0 = 1.5$ V

$$\therefore \quad K_{max} = 1.6 \times 10^{-19} \times 1.5 = 2.4 \times 10^{-19} \text{ J}$$

***Example 11.2** In an experimental study of photoelectric effect, the stopping potential for a metallic surface is 5.68 V. What will be the maximum velocity of the electrons?*

Sol. Stopping potential, $V_0 = 5.68$ V

$$m_e = 9.1 \times 10^{-31} \text{ kg}$$

Maximum KE, $K_{max} \Rightarrow \frac{1}{2}mv_{max}^2 = eV_0$

$$\Rightarrow v_{max} = \sqrt{\frac{2eV_0}{m}} = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 5.68}{9.1 \times 10^{-31}}} = 1.414 \times 10^6 \text{ ms}^{-1}$$

Effect of frequency of incident radiation on stopping potential

If we take radiations of different frequencies but of same intensity. For each radiation, we study the variation of photoelectric current against the potential difference between the plates as shown in the graph.

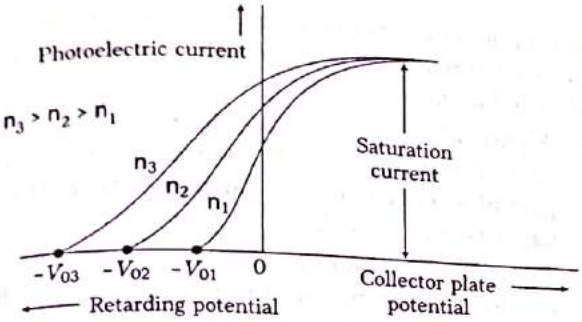


Fig. 11.5 Variation of photoelectric current versus potential for different frequencies but constant intensity of incident radiation

From the above graph, we observe that

(i) The value of stopping potential is different for radiation of different frequencies but value of saturation current (for given intensity) remains same for all incident radiations as intensity remains constant.

(ii) The value of stopping potential is more negative for radiation of higher incident frequency. That means the energy of the emitted electrons depends on the frequency of incident radiations. Greater the frequency of incident radiation, greater is the maximum kinetic energy of photoelectrons, consequently greater retarding potential or stopping potential is required to stop them completely.

(iii) The value of saturation current depends on the intensity of incident radiation but is independent of the frequency of the incident radiation.

Stopping potential *vs* frequency graph

If we plot a graph between stopping potential and the frequency of the incident radiation for two different metals *A* and *B*, we get the graph as shown in the figure.

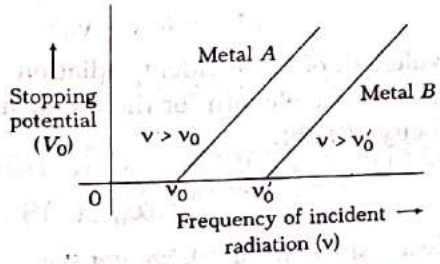


Fig. 11.6 Graph of stopping potential versus frequency of incident light

From the above graph, we observe that

(i) The stopping potential V_0 varies linearly with the frequency of incident radiation for a given photosensitive material.

(ii) There exists a certain minimum cut-off frequency ν_0 for which the stopping potential is zero. This minimum frequency is called **threshold frequency**.

It can be defined as,

"The minimum frequency of light which can emit photoelectrons from a material is called **threshold frequency** or **cut-off frequency** of that material". It is a characteristic property of material.

For a frequency lower than cut-off frequency, no photoelectric emission is possible even if the intensity is large. If frequency of incident radiation is more than the threshold frequency, the photoelectric emission starts instantaneously without any apparent time lag (-10^{-9} s or less), even when the incident radiation has low intensity.

2 Laws of photoelectric emission

On the basis of the experimental results on photoelectric effect, Lenard and Millikan gave *the following laws of photoelectric emission*

(i) For a given photosensitive material and frequency of incident radiation, the photoelectric current or number of photoelectrons ejected per second is directly proportional to the intensity of the incident light.

(ii) For a given material and frequency of incident radiation, saturation current is found to be proportional to the intensity of incident radiation, whereas the stopping potential is independent of its intensity.

(iii) For a given material, there exists a certain minimum frequency of the incident radiation below which no emissions of photoelectrons take place. This frequency is called **threshold frequency**.

Above the threshold frequency, the maximum kinetic energy of the emitted photoelectrons or equivalent stopping potential is independent of the intensity of the incident light but depends only upon the frequency (or wavelength) of the incident light.

(iv) The photoelectric emission is an instantaneous process. The time lag between the incidence of radiations and emission of photoelectrons is very small, less than 10^{-9} s, even when incident radiation is made exceedingly dim.

Failure of wave theory in explaining photoelectric effect

The wave theory of light given by Huygen could not explain the photoelectric emission due to the following main reasons

(i) According to the wave picture of light, the free electrons at the surface of the metal absorb the radiant energy continuously.

The greater the intensity of radiation, the greater should be the energy absorbed by each electron. The maximum kinetic energy of the photoelectrons on the surface is then expected to increase with increase in intensity.

But according to experimental facts, the maximum kinetic energy of ejected photoelectrons is independent of intensity of incident radiation.

(ii) According to wave theory of light, no matter what the frequency of radiation is, a sufficiently intense beam of radiation should be able to impart enough energy to the electrons, so that they exceed the minimum energy needed to escape from metal surface.

A threshold frequency, therefore should not exist which contradicts the experimental fact that no photoelectric emission takes place below that threshold frequency, no matter whatsoever may be its intensity.

(iii) According to the wave theory of light, the absorption of energy by electron takes place continuously over the entire wavefront of the radiation. Since a large number of electrons absorb energy, the energy absorbed per electron per unit time turns out to be small.

Hence, it will take hours or more for a single electron to come out of the metal which contradicts the experimental fact that photoelectron emission is instantaneous ($\sim 10^{-9}$ s).

Einstein's explanation of photoelectric Effect : Einstein's photoelectric equation

In 1905, Albert Einstein explained the various laws of photoelectric emission on the basis of Planck's quantum theory. According to Planck's quantum theory, light radiations consist of tiny packets of energy called **quanta**. One quantum of light radiation is called a **photon** which travels with the speed of light.

The energy of a photon is given by

$$\boxed{E = h\nu}$$

where, h is Planck's constant and ν is the frequency of light radiation.

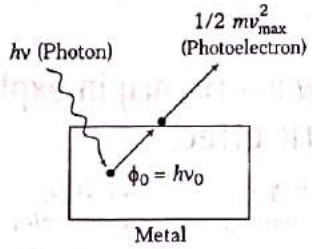


Fig. 11.7 Emission of photoelectron by a metal when a photon is absorbed by it

When a photon of energy $h\nu$ falls on a metal surface, the energy of the photon is absorbed by the electron and is used in following two ways

(i) A part of energy is utilised to overcome the surface barrier and come out of the metal surface. This part of energy is called **work function**. It is expressed as $\phi_0 = h\nu_0$.

(ii) The remaining part of the energy is used in giving a velocity v to the emitted photoelectron. This is equal to the maximum kinetic energy of the photoelectrons $\left(\frac{1}{2}mv^2_{max}\right)$, where m is the mass of the photoelectron.

According to the law of conservation of energy,

$$h\nu = \phi_0 + \frac{1}{2}mv^2_{max} = h\nu_0 + \frac{1}{2}mv^2_{max}$$

$$\therefore \quad \frac{1}{2}mv^2_{max} = K_{max} = h(\nu - \nu_0) = h\nu - \phi_0$$

$$\therefore \quad \boxed{K_{max} = h\nu - \phi_0}$$

This equation is called **Einstein's photoelectric equation.**

$$\Rightarrow \quad K_{max} = E - W \qquad [\because W = \phi_0]$$

Relation between stopping potential (V_0) and frequency (ν) of incident light

Maximum kinetic energy is given by

$$K_{max} = h(\nu - \nu_0)$$

If stopping potential is V_0, then

$$K_{max} = eV_0$$

$$\therefore \quad eV_0 = h(\nu - \nu_0) \qquad \text{...(i)}$$

If λ = wavelength of the incident radiation

λ_0 = threshold wavelength for the metal surface

c = velocity of light.

Then, $\nu = \frac{c}{\lambda}$ and $\nu_0 = \frac{c}{\lambda_0}$

Putting these values in Eq. (i), we get

$$eV_0 = h\left(\frac{c}{\lambda} - \frac{c}{\lambda_0}\right) \Rightarrow eV_0 = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) \qquad \text{...(ii)}$$

Slope of the graph between V_0 and ν_0

Consider the equations

$$eV_0 = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) \Rightarrow V_0 = \frac{h}{e}\left(\frac{c}{\lambda} - \frac{c}{\lambda_0}\right)$$

$$\boxed{V_0 = \frac{h}{e}\nu - \frac{h\nu_0}{e}} \qquad \text{...(iii)}$$

From the Eq. (iii) it is clear that graph between V_0 and ν_0 is a straight line. The slope of this line is given by $\tan\theta = \frac{h}{e}$, also intercept on V_0-axis is given by $\frac{h\nu_0}{e}$, as shown in the graph

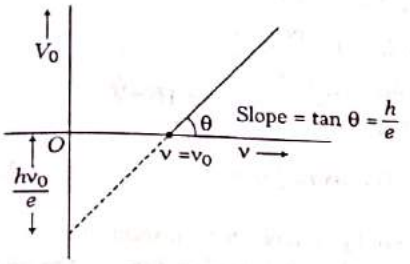


Fig. 11.8 Graph of V_0 versus ν

In terms of wavelength, we can write

$$V_0 = \frac{hc}{\lambda e} - \frac{hc}{\lambda_0 e} \quad \text{...(iv)}$$

From the Eq. (iv), it is clear that graph between V_0 and $\frac{1}{\lambda}$ is also a straight line. The slope of this line is given by $\tan\theta = \frac{hc}{e}$, also intercept on V_0-axis is given by $\frac{hc}{\lambda_0 e}$, as shown in the graph.

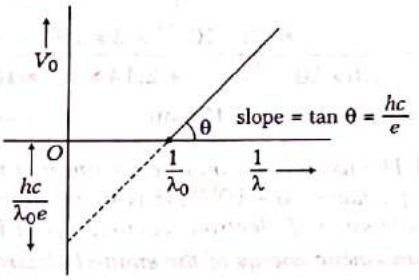


Fig. 11.9 Graph of V_0 versus $1/\lambda$

Explanation of photoelectric effect from Einstein's equation

Einstein explained all the observations of photoelectric effect using Einstein's photoelectric equation.

These are as follow

(i) From photoelectric equation it is clear that K_{max} is independent of intensity of radiation but depends linearly on ν. Einstein explained that single electron absorbs energy of a single photon.

(ii) Photoelectric emission is possible only if $h\nu > \phi_0$.

or $h\nu > h\nu_0$ $(\because \phi_0 = h\nu_0)$

$\Rightarrow$ $\nu > \nu_0$

where ν_0 is threshold frequency. If $\nu < \nu_0$, then kinetic energy of electron comes out to be negative, which is not possible, so no photoelectric emission is possible below ν_0, even if the incident radiation of high intensity and long duration falls on the surface.

(iii) Intensity of radiation (I) is proportional to the number of energy quanta per unit area per unit time (n). Higher the value of n, higher the number of electrons emitted (for $\nu > \nu_0$).

That is the reason (for $\nu > \nu_0$)

Photocurrent ∝ Intensity

(iv) The absorption of photon by electron is instantaneous process, hence emission of electrons is instantaneous.

Note Practically only one electron is ejected by 1 million photons.

$$\text{Quantum efficiency} = \frac{\text{Number of electrons ejected / sec}}{\text{Number of photons incident / sec}}$$

$$\Rightarrow \quad \boxed{\eta = \frac{n'}{n} \times 100\%}$$

Example 11.3 *The work function for a certain metal is 4.2 eV. Will this metal give photoelectric emission for incident radiation of wavelength 330 nm?*

Sol. Here, W = Work function of certain metal = 4.2 eV

$$= 4.2 \times 1.6 \times 10^{-19}\ \text{J} = 6.72 \times 10^{-19}\ \text{J}$$

$[\because 1\ \text{eV} = 1.6 \times 10^{-19}\ \text{J}]$

If E be the energy of the photon of incident light, then

$$E = h\nu = \frac{hc}{\lambda} = \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{330 \times 10^{-9}}\ \text{J} = 6.01 \times 10^{-19}\ \text{J}$$

The photoelectric emission is possible only if $E = h\nu > \phi$ where ϕ is work function of metal. Here $E < \phi$, so this metal will not give photoelectric emission for incident radiation of wavelength 330 nm.

Example 11.4 *The threshold frequency for a certain metal is 3.3×10^{14} Hz. If light of frequency 8.2×10^{14} Hz is incident on the metal, predict the cut-off voltage for the photoelectric emission.*

Sol. Using the relation, $h\nu - h\nu_0 = eV_0$,

We get $V_0 = h\left(\frac{\nu - \nu_0}{e}\right)$

Here, $\nu_0 = 3.3 \times 10^{14}$ Hz, $\nu = 8.2 \times 10^{14}$ Hz

$$= 6.62 \times 10^{-34} \times \left(\frac{8.2 \times 10^{14} - 3.3 \times 10^{14}}{1.6 \times 10^{-19}}\right) = 2.03\ \text{V}$$

Example 11.5 *In an experiment on photoelectric effect, the slope of the cut-off voltage versus frequency of incident light is found to be 4.12×10^{-15} Vs. Calculate the value of Planck's constant.*

Sol. We know, $eV_0 = h\nu - W$

$\therefore$ $V_0 = \left(\frac{h}{e}\right)\nu - \frac{W}{e}$

$\therefore$ Slope $= \frac{h}{e}$ or $4.12 \times 10^{-15} = \frac{h}{e}$

$$\therefore \quad h = 4.12\times 10^{-15}\times e$$
$$= 4.12\times 10^{-15}\times 1.6\times 10^{-19}\ \text{Js}^{-1}$$
$$= 6.592\times 10^{-34}\ \text{Js}^{-1}$$

Example 11.6 *Light of frequency 7.21×10^{14} Hz is incident on a metal surface. Electrons with a maximum speed of 6.0×10^{5} m/s are ejected from the surface. What is the threshold frequency for photoemission of electrons?*

Sol. Using Einstein's photoelectric equation, $h\nu - h\nu_0 = \frac{1}{2}mv_{max}^2$,

We get $h\nu_0 = h\nu - \frac{1}{2}mv_{max}^2$

$$\therefore \quad \nu_0 = \nu - \frac{mv_{max}^2}{2h}$$

Here, $\nu = 7.21\times 10^{14}$ Hz

$v_{max} = 6\times 10^5$ m/s

m = mass of electron $= 9.1\times 10^{-31}$ kg

$\therefore$ Threshold frequency, ν_0

$$= 7.21\times 10^{14} - \frac{9.1\times 10^{-31}\times (6.0\times 10^5)^2}{2\times 6.62\times 10^{-34}}$$
$$= 7.21\times 10^{14} - 2.47\times 10^{14}$$
$$= 4.74\times 10^{14}\ \text{Hz}$$

Example 11.7 *The photoelectric work function of potassium is 2.3 eV. If light having a wavelength of 2800 Å falls on potassium, find*

(a) the kinetic energy in electron volts of the most energetic electrons ejected.

(b) the stopping potential in volts.

Sol. Given, $W = 2.3$ eV, $\lambda = 2800$ Å

$$\therefore\ E\,(\text{in eV}) = \frac{hc}{\lambda(\text{in Å})} = \frac{12375}{\lambda\,(\text{in Å})} = \frac{12375}{2800} = 4.4\ \text{eV}$$

(a) As $K_{max} = E - W = (4.4 - 2.3)$ eV $= 2.1$ eV

(b) Using the relation, $K_{max} = eV_0$

$\therefore \quad 2.1\ \text{eV} = eV_0$ or $V_0 = 2.1$ V

Example 11.8 *Light of wavelength 488 nm is produced by an argon laser which is used in the photoelectric effect. When light from this spectral line is incident on the emitter, the stopping (cut-off) potential of photoelectrons is 0.38 V. Find the work function of the material from which the cathode is made.*

Sol. Energy of incident photon,

$$E = \frac{hc}{\lambda} = \frac{6.62\times 10^{-34}\times 3\times 10^8}{488\times 10^{-9}}$$
$$= 4.07\times 10^{-19}\ \text{J}$$

Using formula, $eV_0 = E - W$,

We get $W = E - eV_0$

where $E = h\nu$

W = Work function of the material

Here, $E = 4.08\times 10^{-19}$ J

$e = 1.6\times 10^{-19}$ V

$V_0 = 0.38$ V

$$\therefore \quad W = 4.08\times 10^{-19}\ \text{J} - (1.6\times 10^{-19}\times 0.38)$$
$$= (4.7 - 0.608)\times 10^{-19}\ \text{J}$$
$$= 3.62\times 10^{-19}\ \text{J}$$
$$= \frac{3.462\times 10^{-19}}{1.6\times 10^{-19}}\ \text{eV} = 2.16\ \text{eV}$$

Example 11.9 *The work function of caesium is 2.14 eV. Calculate*

(i) the threshold frequency for caesium and

(ii) the wavelength of the incident light, if the photocurrent is brought to zero by a stopping potential of 0.60 V.

Given, $h = 6.63\times 10^{-34}$ Js.

Sol. Here, $V_0 = 0.60$ V

$$\phi_0 = 2.14\ \text{eV} = 2.14\times 1.6\times 10^{-19}\ \text{J}$$

(i) Threshold frequency, $\nu_0 = \frac{\phi_0}{h}$ $\quad [\because \phi_0 = h\nu_0]$

$$= \frac{2.14\times 1.6\times 10^{-19}}{6.63\times 10^{-34}} = 5.16\times 10^{14}\ \text{Hz}$$

(ii) As, $eV_0 = \frac{hc}{\lambda} - \phi_0$ or $\lambda = \frac{hc}{(eV_0 + \phi_0)}$

$$= \frac{6.63\times 10^{-34}\times 3\times 10^8}{(1.6\times 10^{-19}\times 0.6 + 2.14\times 1.6\times 10^{-19})}$$
$$\approx 454\times 10^{-9}\text{m} = 454\ \text{nm}$$

Example 11.10. *The work function of caesium metal is 2.14 eV. When light of frequency 6×10^{14} Hz is incident on the metal surface, photoemission of electrons occurs. What is the*

(a) maximum kinetic energy of the emitted electrons?

(b) stopping potential, and

(c) maximum speed of the emitted photoelectrons?

Sol. (a) Using Einstein's photoelectric equation,

$$E_{max} = h\nu - \phi_0$$
$$= 6.62\times 6\times 10^{-20} - 2.14\times 1.6\times 10^{-19}$$
$$= 39.72\times 10^{-20} - 34.24\times 10^{-20}\ \text{J}$$
$$= 5.48\times 10^{-20}\ \text{J}$$
$$= \frac{0.548\times 10^{-19}}{1.6\times 10^{-19}}\ \text{eV} \quad [\because 1\ \text{eV} = 1.6\times 10^{-19}\ \text{J}]$$
$$= 0.342\ \text{eV}$$

(b) As, $eV_0 = E_{max}$

or $V_0 = \frac{E_{max}}{e}$

$$= \frac{0.342\times 1.6\times 10^{-19}\ \text{J}}{1.6\times 10^{-19}\ \text{C}}$$
$$= 0.342\ \text{JC}^{-1} = 0.342\ \text{V}$$

(c) As we know that, $\frac{1}{2}mv_{max}^2 = E_{max}$

$$\therefore \quad v_{max} = \sqrt{\frac{2E_{max}}{m}} = \sqrt{\frac{2\times 5.48\times 10^{-20}}{9.1\times 10^{-31}}} = 3.45\times 10^5 \text{ ms}^{-1}$$

Example 11.11 *When light of wavelength 4000 Å is incident on a barium emitter, emitted photoelectrons in a transverse magnetic field strength $B = 5.2\times 10^{-6}$ T moves in a circular path. Find the radius of the circle. Given work function of barium is 2.5 eV and mass of electron is 9.1×10^{-31} kg, $e = 1.6\times 10^{-19}$ C, $h = 6.6\times 10^{-34}$ Js.*

Sol. Using the relation, $K_{max} = E - \phi_0$

$$\frac{1}{2}mv_{max}^2 = \frac{hc}{\lambda} - \phi_0 \Rightarrow v_{max}^2 = \frac{2}{m}\left[\frac{hc}{\lambda} - \phi_0\right]$$

$$= \frac{2}{9.1\times 10^{-31}}\left[\frac{(6.6\times 10^{-34})\times(3\times 10^8)}{4\times 10^{-7}} - 2.5\times 1.6\times 10^{-19}\right]$$

$$= 20.88\times 10^{10}$$

$$\therefore v_{max} = 4.57\times 10^5 \text{ m/s}$$

Radius of circular path described by charged particle in a magnetic field is given by

$$Bev_{max} = \frac{mv_{max}^2}{r}$$

$$\Rightarrow r = \frac{mv_{max}}{Be} = \frac{(9.1\times 10^{-31})\times(4.57\times 10^5)}{5.2\times 10^{-6}\times 1.6\times 10^{-19}} = \frac{1}{2}\text{ m}$$

Example 11.12 *Light of wavelength 2475 Å is incident on barium. Photoelectrons emitted describe a circle of radius 100 cm by a magnetic field of flux density $\frac{1}{\sqrt{17}}\times 10^{-5}$ tesla. Find the work function of the barium.* $\left(\text{Given } \frac{e}{m} = 1.7\times 10^{11}\right)$

Sol. Radius of circular path described by a charged particle in a magnetic field B is given by

$$r = \sqrt{\frac{2mK}{qB}} \Rightarrow K = \frac{q^2B^2r^2}{2m} = \left(\frac{e}{m}\right)\frac{eB^2r^2}{2}$$

where, K = kinetic energy of a particle

q = charge on a particle

r = radius of circular path

$$= \frac{1}{2}\times 1.7\times 10^{11}\times 1.6\times 10^{-19}\times\left(\frac{1}{\sqrt{17}}\times 10^{-5}\right)\times(1)^2$$

$$= 8\times 10^{-20}\text{ J} = 0.5\text{ eV}$$

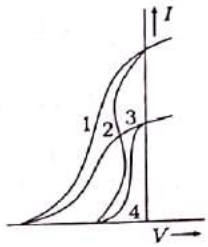
Using the relation,

$$E = \phi + K_{max} \Rightarrow \phi = E - K_{max}$$

where, $E = h\nu = \frac{hc}{\lambda} = \frac{12375}{\lambda(\text{in Å})}$

$$= \left(\frac{12375}{2475}\right)\text{eV} - 0.5\text{ eV} = 4.5\text{ eV}$$

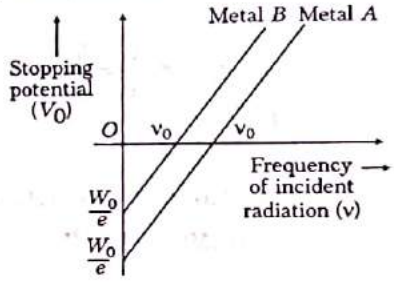
Example 11.13 *The given graph shows the variation of photoelectric current I versus applied voltage V for two different photosensitive materials and for two different intensities of the incident radiations. Identify the pairs of curves that correspond to different materials but same intensity of incident radiation.*



Sol. Curves 1 and 2 correspond to similar materials, while curves 3 and 4 represents different materials, since the value of stopping potential for the pair of curves (1 and 2) and (3 and 4) are the same. For given frequency of the incident radiation, the stopping potential is independent of its intensity.

So, the pairs of curves (1 and 3) and (2 and 4) correspond to different materials but same intensity of incident radiation.

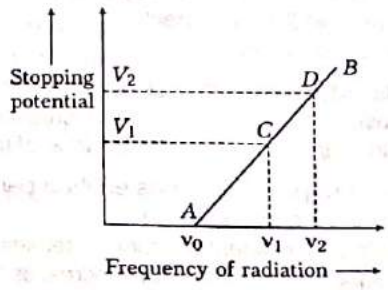
Example 11.14 *The graph shows the variation of stopping potential with frequency of incident radiation for two photosensitive metals A and B. Which one of the two has higher value of work function? Justify your answer.*



Sol. Metal A has higher value of work function because the slope of both materials are constant and the intercept of the line depends on work function.

Example 11.15 *Draw a graph to show the variation of stopping potential with frequency of radiation incident on a metal plate. How can the value of Planck's constant be determined from this graph?*

Sol. The variation of stopping potential with the frequency of radiation, incident on a metal plate is a straight line AB as shown in the figure.



Take two points C and D on the graph. The corresponding frequency of radiation is ν_1, ν_2 and stopping potential is V_1, V_2.

Then, $$eV_1 = h\nu_1 - \phi_0$$

and $$eV_2 = h\nu_2 - \phi_0$$

$\therefore \quad e(V_2 - V_1) = h(\nu_2 - \nu_1)$

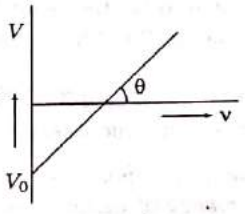
or $$h = \frac{e(V_2 - V_1)}{\nu_2 - \nu_1}$$

Thus, Planck's constant can be determined.

Example 11.16 *In an experiment on photoelectric effect, the slope of the cut-off voltage versus frequency of incident light is found to be 4.12×10^{-15} V-s. Calculate the value of Planck's constant.*

Sol. Given, slope of graph, $\tan\theta = 4.12 \times 10^{-15}$ V-s

Charge on electron, $e = 1.6 \times 10^{-19}$ C



For slope of graph, $\tan\theta = \dfrac{V}{\nu}$

We know that,

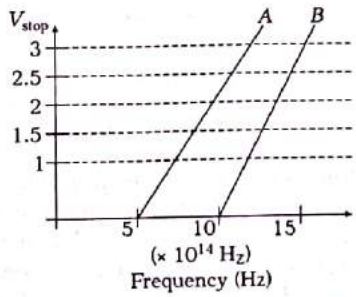
$$h\nu = eV \text{ or } \frac{V}{\nu} = \frac{h}{e}$$

$$\therefore \quad \frac{h}{e} = 4.12 \times 10^{-15}$$

$$h = 1.6 \times 10^{-19} \times 4.12 \times 10^{-15}$$

$$= 6.592 \times 10^{-34} \text{Js}$$

Example 11.17 *A student performs an experiment on photoelectric effect, using two materials A and B. A plot of V_{stop} versus ν is given in the figure.*



(*i*) *Which material A or B has a higher work function?*

(*ii*) *Given the electric charge of an electron = 1.6×10^{-19} C, find the value of h obtained from the experiment for both A and B. Comment on whether it is consistent with the Einstein's theory.*

Sol. (*i*) Given, threshold frequency of A is given by

$\nu_{OA} = 5 \times 10^4$ Hz and

For B, $\nu_{OB} = 10 \times 10^{14}$ Hz

We know that work function,

$$\phi = h\nu_0 \text{ or } \phi_0 \propto \nu_0 \Rightarrow \phi_0 \propto \nu_0$$

So, $$\frac{\phi_{OA}}{\phi_{OB}} = \frac{5 \times 10^{14}}{10 \times 10^{14}} < 1 \Rightarrow \phi_{OA} < \phi_{OB}$$

Thus, work function B is higher than A.

(*ii*) For metal A, slope $= \dfrac{h}{e} = \dfrac{2}{(10-5)10^{14}}$

or $$h = \frac{2 \times e}{5 \times 10^{14}} = \frac{2 \times 1.6 \times 10^{-19}}{5 \times 10^{14}}$$

$$= 6.4 \times 10^{-34} \text{ Js}$$

For metal B, slope $= \dfrac{h}{e} = \dfrac{2.5}{(15-10)10^{14}}$

or $$h = \frac{2.5 \times e}{5 \times 10^{14}} = \frac{2.5 \times 1.6 \times 10^{-19}}{5 \times 10^{14}}$$

$$= 8 \times 10^{-34} \text{ Js}$$

Since the value of h from experiment for metals A and B is different. Hence, experiment is not consistent with theory.

Check point 11.1

1. Which of the following is correct?
(a) Hertz discovered the phenomenon of photoelectric emission
(b) Lenards made detailed study of photoelectric effect.
(c) Both (a) and (b) are correct
(d) None of the above

2. When intensity of light increases, then photoelectric current
(a) increases (b) decreases
(c) remains same (d) None of these

3. The number of photoelectrons emitted per second from a metal surface increases when
(a) the energy of incident photons increases
(b) the frequency of incident light increases
(c) the wavelength of the incident light increases
(d) the intensity of the incident light increases

4. When stopping potential is applied in an experiment on photoelectric effect, no photocurrent is observed. This means that
(a) the emission of photoelectrons is stopped
(b) the photoelectrons are emitted but are reabsorbed by the emitter metal
(c) the photoelectrons are accumulated near the collector plate
(d) the photoelectrons are dispersed from the sides of the apparatus.

5. If the work function of potassium is 2 eV, then its photoelectric threshold wavelength is
(a) 310 nm (b) 620 nm (c) 6200 nm (d) 3100 nm

6. Maximum kinetic energy of emitted electron from the surface of a metal is K. When frequency of incident light is doubled, then maximum kinetic energy of emitted electrons is K' then
(a) $K' > 2K$ (b) $K' < 2K$
(c) $K' = 2K$ (d) Data is insufficient

7. If the frequency of light in a photoelectric experiment is doubled, then the stopping potential will
(a) be doubled (b) be halved
(c) become more than double (d) become less than double

8. When frequency increases, slope of stopping potential *versus* frequency graph
(a) increases (b) decreases
(c) remains same (d) slope is always zero

9. The work function of aluminium is 4.2 eV. If two photons, each of energy 3.5 eV strike an electron of aluminium, then emission of electrons will be
(a) possible
(b) not possible
(c) data is incomplete
(d) depend upon the area of the surface

10. Photoelectric effect supports quantum nature of light because
(a) the energy of released electron is discrete
(b) the maximum kinetic energy of photoelectrons depends only on the frequency of light and not on its intensity
(c) even when the metal surface is faintly illuminated, the photoelectrons leave the surface immediately
(d) electric charge of the photoelectrons is quantised

3 Planck's quantum theory (Particle nature of light : the photon)

In 1990, Max Planck proposed his quantum theory of radiation. According to this theory, the energy of an electromagnetic wave is not continuously distributed over the wavefront, instead of an electromagnetic wave travels in the form of discrete packets or bundles of energy called quanta. One quantum of light radiation is called a **photon**, which travels with the speed of light. Photoelectric effect thus gave evidence that light consists of packets of energy.

Energy of a photon is given by

$$E = h\nu = \frac{hc}{\lambda}$$

where, h ($\simeq 6.63 \times 10^{-34}$ Js) is the Planck's constant, ν is the frequency of radiation or photon, c is the speed of light and λ is the wavelength of photon. The energy E is expressed in electron volt (eV).

The momentum of photon is given by

$$\boxed{p = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda} \text{ kg-ms}^{-1}}$$

Characteristic properties of photons

Different characteristic properties of photons are given below

(i) In interaction of radiation with matter, radiation behaves as if it is made up of particles called **photons**.

(ii) A photon in vacuum travels with a speed of light, c (*i.e.*, 3×10^8 ms^{-1}) in straight line.

(iii) It has zero rest mass, *i.e.*, the photon cannot exist at rest.

(iv) According to the theory of relativity, the mass m of a particle moving with velocity v, comparable with the velocity of light c is given by

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\Rightarrow \quad \boxed{m_0 = m\sqrt{1 - \frac{v^2}{c^2}}}$$

where, m_0 is the mass of the particle at rest.

As, a photon moves with the speed of light, *i.e.*, $v = c$, hence, $m_0 = 0$. So, rest mass of photon is zero.

(v) The inertial mass of a photon is given by

$$m = \frac{E}{c^2} = \frac{h}{c\lambda} = \frac{h\nu}{c^2}$$

(vi) Irrespective of the intensity of radiation, all the photons of a particular frequency ν or wavelength λ have the same energy

$$E\left(= h\nu = \frac{hc}{\lambda}\right) \text{ and momentum } p\left(= \frac{h\nu}{c} = \frac{h}{\lambda}\right).$$

(vii) Energy of a photon depends upon frequency of the photon, so the energy of the photon does not change when photon travels from one medium to another.

(viii) Wavelength of the photon changes in different media, so velocity of a photon is different in different media.

(ix) Photons are not deflected by electric and magnetic fields. This shows that photons are electrically neutral.

(x) Total energy and total momentum are conserved in photon-particle collision. However, the number of photons may not be conserved in collision. As, the photon may be absorbed or a new photon may be created in a collision.

Example 11.18 *What is the energy, in joule, associated with a photon of wavelength 4000 Å?*

Sol. Energy of a photon, $E = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{4 \times 10^{-7}}$

$= 4.95 \times 10^{-19}$ J

Example 11.19 *The momentum of photon of electromagnetic radiation is 3.3×10^{-29} kg-ms^{-1}. Find out the frequency and wavelength of the wave associated with it.*

Sol. (i) $h = 6.63 \times 10^{-34}$ Js, $c = 3 \times 10^8$ ms^{-1}

$p = 3.3 \times 10^{-29}$ kg-ms^{-1}

Momentum, $p = \frac{h\nu}{c}$

$\Rightarrow \quad \nu = \frac{pc}{h} = \frac{3.3 \times 10^{-29} \times 3 \times 10^8}{6.63 \times 10^{-34}}$

$= 1.5 \times 10^{13}$ Hz

(ii) $\lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{1.5 \times 10^{13}} = 2 \times 10^{-5}$ m

Example 11.20 *The wavelength of a photon is 1.4 Å. It collides with an electron. The energy of the scattered electron is 4.26×10^{-16} J. Find the wavelength of photon after collision. Given $h = 6.63 \times 10^{-34}$ Js.*

Sol. Initial wavelength of photon λ_1.

Final wavelength of photon λ_2.

$\therefore 4.26 \times 10^{-16} = \frac{hc}{\lambda_1} - \frac{hc}{\lambda_2}$

$\frac{hc}{\lambda_2} = \frac{hc}{\lambda_1} - 4.26 \times 10^{-16}$

$\frac{6.63 \times 10^{-34} \times 3 \times 10^8}{\lambda_2} = 14.21 \times 10^{-16} - 4.26 \times 10^{-16}$

$\lambda_2 = 2 \times 10^{-10}$ m $= 2.0$ Å

Example 11.21 *Find the ratio of momentum of an electron and an α-particle, which are accelerated from rest by a potential difference of 100 V.*

Sol. Momentum, $p = mv$

and $\quad v = \sqrt{\frac{2qv}{m}}$

$p = \sqrt{2qmv}$

$\Rightarrow \quad p \propto \sqrt{qm}$

$\Rightarrow \quad \frac{p_e}{p_\alpha} = \sqrt{\frac{e \times m_e}{2e \times m_\alpha}} = \sqrt{\frac{m_e}{2m_\alpha}}$

Intensity of light

A moving photon carries momentum with it. Therefore, pressure and intensity are also associated with photons.

Intensity or energy flux is defined as the energy crossing per unit area normally per second.

Consider an area A through which light (photons) of total energy E and intensity I is passing normally as shown in the figure.

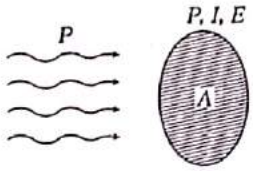


***Fig. 11.10** Photon of energy E passing normally through as area A*

If power of the source is P, then we can write

Total energy of photons, $E = Nh\nu = N\left(\frac{hc}{\lambda}\right)$

$\therefore$ Power of source, $P = \frac{E}{t} = \frac{Nh\nu}{t} = nh\nu = \frac{nhc}{\lambda} \quad \left[\because n = \frac{N}{t}\right]$

Also we can write

$$\boxed{n = P/E = \frac{P}{h\nu} = \frac{P\lambda}{hc}}$$

$\Rightarrow$ Intensity, $\boxed{I = \frac{P}{A} = \frac{E}{tA} = \frac{nh\nu}{A} = n\frac{hc}{A\lambda}}$

where, A = area of normal to incident light

N = number of photons

n = number of photons/s

At a distance r from a point source of power P intensity is given by $I = \frac{P}{4\pi r^2} \Rightarrow I \propto \frac{1}{r^2}$. For a line source, $I \propto \frac{P}{2\pi rl}$

$\Rightarrow \quad I \propto \frac{1}{r}$.

Example 11.22 *The eye can detect 5×10^4 photons per square metre per second of green light ($\lambda = 5000$ Å) while the ear can detect 10^{-13} (W/m^2). Find the factor by which the eye is more sensitive as a power detector than the ear.*

Sol. As, $E = \frac{hc}{\lambda(\text{Å})}$

$\Rightarrow \quad E = \frac{12375}{5000} = 2.475$ eV $= 4 \times 10^{-19}$ J

I_{Eye} = (Photon flux) × (Energy of photon)

$= (5 \times 10^4) \times (4 \times 10^{-19}) = 2 \times 10^{-14}$ W/m^2

Lesser the intensity = More sensitive is the detector

$\therefore \quad \frac{S_{Eye}}{S_{Ear}} = \frac{I_{Ear}}{I_{Eye}} = \frac{10^{-13}}{2 \times 10^{-14}} = 5$

Example 11.23 *The intensity of sunlight reaching the surface of the earth is 1.388×10^3 W/m^2. How many photons (nearly) per square metre are incident on the earth per second? Assume that the photons in the sunlight have an average wavelength of 550 nm.*

Sol. Intensity = Energy of photons reaching the earth per second per unit area

$$I = 1.388\times 10^3 \text{ W/m}^2$$
$$= 1.388\times 10^3 \text{ Js}^{-1}\text{ m}^{-2}$$

Let n = number of photons incident on earth per sec per m^2

and E' = energy of each photon $= \dfrac{hc}{\lambda}$

If E be the energy of all photons reaching the surface of earth per sec per unit area, then

$$E = nE' = \frac{nhc}{\lambda}$$

$$\therefore \quad \frac{nhc}{\lambda} = I$$

or
$$n = \frac{I}{\left(\dfrac{hc}{\lambda}\right)} = \frac{1.388\times 10^3}{hc}\times\lambda$$
$$= \frac{1.388\times 10^3\times 550\times 10^{-9}}{6.62\times 10^{-34}\times 3\times 10^8}$$
$$= 3.85\times 10^{21}$$

Example 11.24 *A 100 W sodium lamp radiates energy uniformly in all directions. The lamp is located at the centre of a large sphere that absorbs all the sodium light which is incident on it. The wavelength of the sodium light is 589 nm. (a) What is the energy per photon associated with the sodium light? (b) At what rate are the photons delivered to the sphere?*

Sol. (a) Using the formula, $E = \dfrac{hc}{\lambda}$, we get

$$E = \frac{6.62\times 10^{-34}\times 3\times 10^8}{589\times 10^{-9}}$$
$$= 3.38\times 10^{-19}\text{ J}$$
$$= \frac{3.38\times 10^{-19}}{1.6\times 10^{-19}} = 2.1\text{ eV}$$

(b) Let n = rate at which the photons are delivered to the sphere.

$$n = \frac{\text{Energy radiated per second}}{\text{Energy of each photon}} = \frac{P}{E}$$
$$= \frac{100}{3.38\times 10^{-19}} = 2.96\times 10^{20}\text{ photons/s}$$

Example 11.25 *(a) Find the momentum of a photon of light of wavelength 660 nm.*

(b) Calculate the number of photons emitted per second by a 66 W source of monochromatic light of wavelength 660 nm.

(c) How many photons per second does a one watt bulb emit if its efficiency is 20% and the wavelength of light emitted is 400 nm?

(d) Monochromatic light of wavelength 200 nm is incident normally on a surface of area 2 cm^2. If the intensity of light is 100 W/m^2; calculate the rate at which photons strike the surface.

Sol. (a) Momentum of a photon, $p = \dfrac{h}{\lambda} = \dfrac{6.6\times 10^{-34}}{660\times 10^{-9}}$

$$= 1.00\times 10^{-27}\text{ kg-m/s}$$

(b) Power of the source, $P = \dfrac{nhc}{\lambda}$

$$\therefore \quad n = \frac{P\lambda}{hc} = \frac{66\times 660\times 10^{-9}}{6.6\times 10^{-34}\times 3\times 10^8}$$
$$= 2.2\times 10^{20}$$

(c) Efficiency $= \dfrac{\text{Output power}}{\text{Input power}}$

$$\Rightarrow \quad \frac{20}{100} = \frac{P}{1} \Rightarrow P = \frac{1}{5}\text{ W}$$

$\therefore$ Number of photons emitted,

$$n = \frac{P\lambda}{hc} = \frac{1}{5}\times\frac{400\times 10^{-9}}{6.6\times 10^{-34}\times 3\times 10^8}$$
$$= 0.04\times 10^{19} = 4\times 10^{17}$$

(d) Rate at which photons strike the surface

$$n = \frac{P\lambda}{hc} = \frac{IA\lambda}{hc} = \frac{100\times 10^{-3}\times 2\times 10^{-4}\times 200\times 10^{-9}}{6.6\times 10^{-34}\times 3\times 10^8}$$
$$= 0.2\times 10^{14} = 2\times 10^{13}\text{ per second}$$

Example 11.26 *A small plate of metal (work function = 1.17 eV) is placed at a distance of 2 m from a monochromatic light source of wavelength 4.8×10^{-7} m and power 1.0 W. The light falls normally on the plate. Find the number of photons striking the metal plate per square meter per second. If a constant uniform magnetic field of strength 10^{-4} T is applied parallel to the metal surface, find the radius of the largest circular path followed by the emitted photoelectrons.*

Sol. Intensity of light at distance r from point source,

$$I = \frac{P}{4\pi r^2} = n\frac{hc}{\lambda}$$

$$n = \frac{I\lambda}{hc} = \frac{P\lambda}{4\pi r^2 hc} = \frac{1\times 4.8\times 10^{-7}}{4\pi(2)^2\times 6.6\times 10^{-34}\times 3\times 10^8}$$
$$= 4.8\times 10^{16}$$

$$E = \frac{1242}{\lambda(nm)}\text{ eV} = \frac{1242}{480} = 2.59\text{ eV}$$

$$E = \phi + K_{max} \Rightarrow 2.59 = 1.17 + K_{max} \Rightarrow K_{max} = 1.42\text{ eV}$$

$$R = \frac{\sqrt{2mK_{max}}}{eB}$$
$$= \frac{\sqrt{2\times 9.1\times 10^{-31}\times 1.42\times 1.6\times 10^{-19}}}{1.6\times 10^{-19}\times 10^{-4}} = 4\times 10^{-2}\text{ m}$$

Force and pressure exerted by incident light

When light is incident on a surface then it exerts a force (or pressure) on the surface. Let light of intensity I is incident normally on a surface of area A as shown in the figure.

We will now calculate the expression for force and pressure being exerted on perfectly reflecting and perfectly absorbing surfaces.

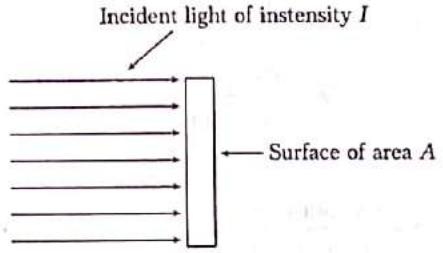


Fig. 11.11 Light is incident normally on a surface

Force and pressure exerted on perfectly reflecting surface

Suppose N photons are incident on a perfectly reflected surface in time t as shown in the figure.

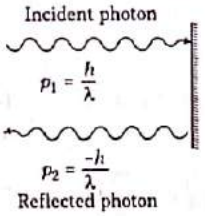


Fig. 11.12 Perfectly reflecting surface

Momentum of photons before striking the surface, $p_1 = \dfrac{Nh}{\lambda}$

Momentum of photons after striking the surface, $p_2 = \dfrac{-Nh}{\lambda}$

Change in momentum of photons,

$$= p_2 - p_1 = -\frac{Nh}{\lambda} - \frac{Nh}{\lambda} = -\frac{2Nh}{\lambda}$$

But change in momentum of surface $= \Delta p = \dfrac{2Nh}{\lambda}$

Therefore, force on the surface

$$F = \frac{\Delta p}{\Delta t} = \frac{2Nh}{\lambda t} = n\left(\frac{2h}{\lambda}\right) \qquad \left[\because n = \frac{N}{t}\right]$$

As power, $\quad P = n\dfrac{hc}{\lambda}$

$$\therefore \qquad n = \frac{P\lambda}{hc} \Rightarrow F = \left(\frac{2h}{\lambda}\right) \times \left(\frac{P\lambda}{hc}\right) = \frac{2P}{c}$$

$$\therefore \quad \boxed{\text{Pressure on the surface} = \frac{F}{A} = \frac{2P}{cA} = \frac{2I}{c}} \qquad \left[\because I = \frac{P}{A}\right]$$

where, c is speed of light.

Special case

When a beam of light is incident at angle ϕ on perfectly reflecting surface

$$F = \frac{2P}{c}\cos\phi = n\left(\frac{2h}{\lambda}\right)\cos\phi$$

$$\boxed{\text{Pressure} = \frac{F}{A} = \frac{2P}{cA}\cos\phi = \frac{2I\cos\phi}{c}}$$

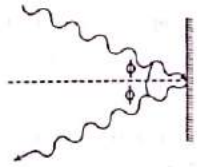


Fig. 11.13 Incident beam making angle 'ϕ' on perfectly reflecting surface.

Force and pressure on perfectly absorbing surface

When the N photons are incident on a perfectly absorbing surface, then in time t.

Force exerted on the surface,

$$F = \frac{\Delta p}{\Delta t} = \frac{\dfrac{Nh}{\lambda} - 0}{t} = \frac{nh}{\lambda} \qquad \left[\because n = \frac{N}{t}\right]$$

Power, $P = \dfrac{nhc}{\lambda} \Rightarrow n = \dfrac{P\lambda}{hc}$

$$\therefore \qquad F = \left(\frac{P\lambda}{hc}\right)\left(\frac{h}{\lambda}\right) = \frac{P}{c}$$

$$\boxed{\text{Pressure on the surface} = \frac{F}{A} = \frac{P}{Ac} = \frac{I}{c}}$$

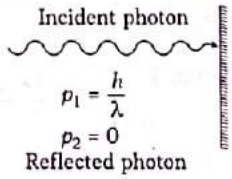


Fig. 11.14 Perfectly absorbing surface

Example 11.27 *(a) A parallel beam of monochromatic light of wavelength 500 nm is incident normally on a perfectly reflecting surface. The power through any cross section of the beam is 5 W. Find*

(i) the number of photons absorbed per second by the surface and

(ii) the force exerted by the light beam on the surface.

(b) Radiation of wavelength 200 nm, propagating in the form of a parallel beam, fall normally on a plane metallic surface. The intensity of the beam is 10 mW/m^2 and its cross-sectional area is 2.0 mm^2. Find the pressure exerted by the radiation on the metallic surface, if the radiation is completely absorbed.

(c) *A parallel beam of monochromatic light of wavelength 500 nm is incident on a totally reflecting plane mirror. The angle of incidence is 30° with the plane of the mirror and the number of photons striking the mirror per second is 2.0×10^{19}. Compute the force exerted by the light beam on the mirror.*

(d) *A beam of white light is incident normally on a plane surface absorbing 60% of the light and reflecting the rest. If the incident beam carries 5 W of power, calculate the force exerted by it on the surface.*

Sol. (a) (i) Power, $P = n\dfrac{hc}{\lambda}$

$\therefore$ Number of photons,

$$n = \frac{P\lambda}{hc} = \frac{5 \times 500 \times 10^{-9}}{6.6 \times 10^{-34} \times 3 \times 10^{8}} = 1.26 \times 10^{19}$$

(ii) Force on the surface

$$F = \frac{2P}{c} = \frac{10}{3 \times 10^{8}} = 3.3 \times 10^{-8}\ \text{N}$$

(b) Intensity of the radiation,

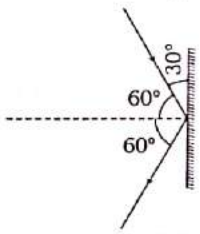
$$I = \frac{P}{A} = \frac{5 \times 10^{-3}}{1 \times 10^{-6}} = 5 \times 10^{3}\ \text{W/m}^2$$

Pressure exerted by the radiation,

$$p = \frac{I}{c} = \frac{10 \times 10^{3}}{3 \times 10^{8}} = 3.3 \times 10^{-5}\ \text{N/m}^2$$

(c) Consider the situation shown in the figure given below. Given that angle of the incident ray with the normal

$$\phi = 90° - 30° = 60°$$



Force exerted by the light beam on the mirror,

$$F = \frac{2P\cos\phi}{c} = \frac{2P\cos 60°}{c}$$

$$= \frac{2 \times n\dfrac{hc}{\lambda}\cos 60°}{c}$$

$$= \frac{2 \times 10^{19} \times 6.6 \times 10^{-34}}{500 \times 10^{-9}}$$

$$= 2.64 \times 10^{-8}\ \text{N}$$

(d) As we know that, $F = 2P/c$, if light is completely reflected $F = P/c$, if light is completely absorbed.

Here, in this problem, 60% light is absorbed and 40% is reflected.

$$F = \frac{60}{100} \times \frac{P}{c} + \frac{40}{100} \times \frac{2P}{c} = 1.4\frac{P}{c}$$

$$= \frac{1.4 \times 5}{3 \times 10^{8}} = 2.33 \times 10^{-8}\ \text{N}$$

4 Photocell

It is a device which converts light energy into electrical energy. It is also called an **electric eye.**

It works on the principle of photoelectric emission.

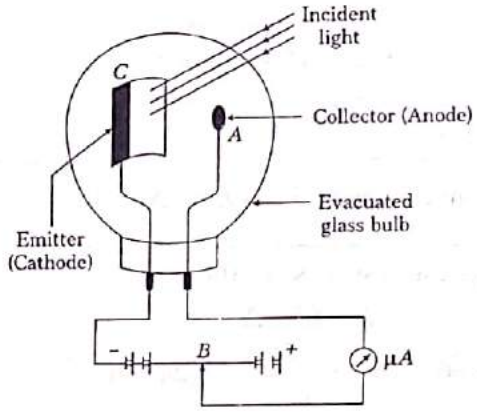


Fig. 11.15 Photocell

Construction

A photocell consists of a semi-cylindrical photosensitive metal plate C (emitter) and a wire loop A (collector) supported in an evacuated glass or quartz bulb. When light of suitable wavelength falls on the emitter C, photoelectrons are emitted. These photoelectrons are drawn to collector by electric field between cathode and anode.

Applications of photocell

Some applications of photocell are as below

(i) Used in television camera for telecasting scenes and in photo telegraphy.
(ii) Used in counting devices.
(iii) Used in burglar alarm and fire alarm.
(iv) To measure the temperature of stars.
(v) Automatic switching of street lights.
(vi) Used as photoelectric sensors.
(vii) Used for the determination of Planck's constant.

5 Compton effect

The scattering of a photon by an electron is called **compton effect**. The scattered photon will have less energy (more wavelength) than that of incident photon (less wavelength). The energy lost by the photon is taken by electron as kinetic energy. Hence, energy and

momentum remains conserved. The schematic representation of compton scattering experiment is shown in the figure.

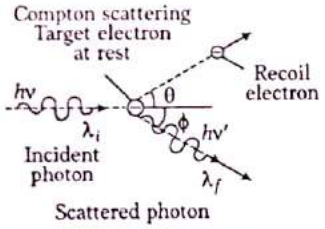


Fig. 11.16 Compton scattering

The change in wavelength due to Compton effect is called Compton shift.

$$\therefore \quad \boxed{\text{Compton shift, } \lambda_f - \lambda_i = \Delta\lambda = \frac{h}{m_0 c}(1-\cos\phi)}$$

Here, m_0 is the rest mass of the electron.

(a) If $\phi = 0°, \Delta\lambda = 0$

(b) If $\phi = 90°$, $\Delta\lambda = \dfrac{h}{m_0 c} = 0.24$ nm

(c) If $\phi = 180°$, $\Delta\lambda = \dfrac{2h}{m_0 c} = 0.48$ nm

(called Compton wavelength)

Compton shift depends only on the scattering angle. It does not depend on the incident wavelength or on the scattering material.

Note In Compton effect only a part of energy of incident photon is absorbed by the electron but in photoelectric effect, the energy of incident photons is completely absorbed by the electron.

Example 11.28 *X-rays of wavelength 10 pm are scattered from a target. (a) Find the wavelength of the X-rays scattered through 45°. (b) Find the maximum wavelength present in the scattered X-rays. (c) Find the maximum kinetic energy of the recoil electrons.*

Sol. When X-rays are scattered from a target, the phenomenon is known as compton scattering.

(a) The change in wavelength is given by

$$\Delta\lambda = \lambda' - \lambda = \lambda c\,(1 - \cos\phi)$$

where, λ_c = Compton wavelength $= \dfrac{h}{mc}$

For electron, $\lambda_c = 2.426 \times 10^{-12}$ m $= 2.426$ pm

λ' = Wavelength of the X-rays scattered throught angle 45°

$\therefore \ \lambda' = \lambda + \lambda_c(1 - \cos 45°)$

$= 10.0 \text{ pm} + 0.293\,\lambda_c = 10.7$ pm

(b) $\lambda' - \lambda = \Delta\lambda$ is maximum when $(1 - \cos\phi) = 2$

$\Rightarrow \ \lambda' = \lambda + 2\lambda_c = 10.0 \text{ pm} + 4.9 \text{ pm} = 14.9$ pm

(c) The maximum recoil kinetic energy is equal to the difference between the energy of the incident and scattered photon, so

$$\text{KE}_{\max} = h(\nu - \nu') = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)$$

where, λ' = maximum wavelength present in the scattered X-rays

$$\therefore \text{KE}_{\max} = \frac{(6.626 \times 10^{-34})(3 \times 10^8)}{10^{-12}\text{m/pm}}\left(\frac{1}{10.0 \text{ pm}} - \frac{1}{14.9 \text{ pm}}\right)$$

$$= 6.54 \times 10^{-15} \text{ J} = 40.8 \text{ keV}$$

Check point 11.2

1. Light consists of packets of energy that are associated with particles called
(a) photon
(b) electron
(c) light cannot be treated as particle
(d) All of the above

2. The rest mass of photon is
(a) zero (b) ∞
(c) between 0 and ∞ (d) equal to that of an electron

3. A photon in motion has a mass
(a) $c/h\nu$ (b) ν/h (c) $h\nu$ (d) $h\nu/c^2$

4. The momentum of a photon is 2×10^{-16} g-cms^{-1}. Its energy is
(a) 0.61×10^{-26} erg (b) 2.0×10^{-26} erg
(c) 6×10^{-8} erg (d) 6×10^{-8} erg

5. The momentum of the photon of wavelength 5000 Å will be
(a) 1.3×10^{-27} kg-ms^{-1} (b) 1.3×10^{-28} kg-ms^{-1}
(c) 4×10^{-29} kg-ms^{-1} (d) 4×10^{-18} kg-ms^{-1}

6. Which of one is correct?
(a) $E^2 = p^2c^2$ (b) $E^2 = p^2c$ (c) $E^2 = pc^2$ (d) $E^2 = p^2/c^2$

7. Two photons having
(a) equal wavelengths have equal linear momenta
(b) equal energies have equal linear momenta
(c) equal frequencies have equal linear momenta
(d) All of the above

8. The number of photons of wavelength 540 nm emitted per second by an electric bulb of power 100 W is (taking $h = 6 \times 10^{-34}$ Js)
(a) 100 (b) 1000 (c) 3×10^{20} (d) 3×10^{18}

9. Light of intensity 10^4 W/m^2 is falling on a perfectly reflecting surface. Pressure on the surface is
(a) 6.67×10^{-5} N/m^2 (b) 4.5×10^{-4} N/m^2
(c) 3×10^{-3} N/m^2 (d) None of these

10. Light of intensity I_0, exerts a pressure of 4.5×10^{-5} N/m^2 on a perfectly absorbing surface. Value of I_0 is
(a) 1.4×10^2 W/m^2 (b) 1.4×10^4 W/m^2
(c) 1.4×10^{-4} W/m (d) None of these

6 Dual nature of radiation

With the experimental verification, it was proved that light has dual nature, in some phenomena it behaves like wave and in some phenomena it behaves like particle depending upon the dimensions of object with which the particle interacts.

In 1924, French physicist Lewis de-Broglie suggested like radiation, matter too should have dual nature, *i.e.*, the particles like electrons, protons, neutrons, etc, can have particle as well as wave nature.

Matter waves : de-Broglie waves

According to de-Broglie, a wave is associated with moving material particle which controls the particle in every respect. The wave associated with moving particle is called **matter waves** or **de-Broglie wave** and it propagates in the form of wave packets with group velocity. Wavelength associated with de-Broglie wave is called **de-Broglie wavelength** and is given by

$$\boxed{\lambda = \frac{h}{mv} = \frac{h}{p}}$$

where, m and v are the mass and velocity of the particle respectively and p is momentum of particle and h is Planck's constant.

According to Planck's quantum theory, the energy of the photon is given by

$$E = h\nu = \frac{hc}{\lambda} \quad \text{...(i)}$$

According to Einstein's theory, the energy of the photon is given by

$$E = mc^2 \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\boxed{\lambda = \frac{h}{mc} = \frac{h}{p}}$$

where, $p = mc$ is momentum of a photon.

Calculation of de-Broglie wavelength of an accelerating electron

Let an electron of charge e having mass m be accelerated from rest through a potential difference V, then

Gain in kinetic energy of electron is given by

$$K = \frac{1}{2} m_e v^2$$

Work done on electron $= eV$

$$\therefore \quad \frac{1}{2} m_e v^2 = eV \Rightarrow v = \sqrt{\frac{2eV}{m_e}}$$

Momentum is given by $p = mv = \sqrt{2eV m_e}$

The wavelength associated with moving electron is given by

$$\boxed{\lambda = \frac{h}{\sqrt{2eVm_e}}}$$

Substituting the value of h, e and m_e in above equation,

we get $$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19}\ \text{V}}}$$

$$\boxed{\lambda = \frac{12.27}{\sqrt{V}} \text{Å}}$$

Similarly, de-Broglie wavelength for other charged particles can be given by $\lambda = \frac{h}{p}$

where, $p = \sqrt{2m(KE)} = \sqrt{2mqV}$

(i) **For proton** $\lambda = \frac{h}{\sqrt{2m_p q}}$

On putting the values of h, m_p and q, we get

$$\lambda = \frac{0.286}{\sqrt{V}} \text{Å}$$

(ii) **For deuteron** $\lambda = \frac{0.202}{\sqrt{V}}$ Å

(iii) **For α-particle** $\lambda = \frac{0.101}{\sqrt{V}}$ Å

de-Broglie wavelength associated with uncharged particles

For neutron de-Broglie wavelength is given as

$$\lambda_{\text{Neutron}} = \frac{0.286 \times 10^{-10}}{\sqrt{E\ (\text{in eV})}}\ \text{m} = \frac{0.286}{\sqrt{E\ (\text{in eV})}} \text{Å}$$

Energy of thermal neutrons or gas molecules at ordinary temperature is given by

$$E = kT \Rightarrow \boxed{\lambda = \frac{h}{\sqrt{3mkT}}}$$

where, T = Absolute temperature

k = Boltzmann's constant $= 1.38 \times 10^{-23}$ joule/kelvin

Also, $$\boxed{\lambda_{\text{Thermal neutrons}} = \frac{25.17}{\sqrt{T}} \text{Å}}$$

Characteristics of matter waves

Characteristics of matter waves are given below

(i) Matter waves are not electromagnetic waves in nature.

(ii) Matter waves are non-mechanical waves, *i.e.*, they can travel in vacuum.

(iii) Matter waves are independent of charge, *i.e.*, they are associated with every moving particle (whether charged or uncharged).

(iv) Observation of matter waves is possible only when the de-Broglie wavelength is of the order of size of the particle (*i.e.*, the waves are diffracted).

(v) The phase velocity of the matter waves can be greater than the speed of the light.

(vi) The number of de-Broglie waves associated with nth orbital electron is n.

Example 11.29 *A particle of mass 1 mg has the same wavelength as an electron moving with a velocity of $3\times10^6\,ms^{-1}$. What is the velocity of the particle?*

Sol. de-Broglie wavelength, $\lambda=\dfrac{h}{mv}$

As both particle and electron are having same wavelength therefore their momentum will be equal

$$m_p v_p = m_e v_e$$

$$\Rightarrow \quad v_p = \frac{m_e v_e}{m_p} = \frac{9.1\times10^{-31}\times3\times10^6}{10^{-6}}$$

$$\Rightarrow \quad v_p = 2.7\times10^{-18}\ \text{ms}^{-1}$$

Example 11.30 *Find out the energy that should be added to an electron in terms of initial energy to reduce its de-Broglie wavelengths from 10^{-10} m to 0.5×10^{-10} m.*

Sol. de-Broglie wavelength is given by

$$\lambda=\frac{h}{\sqrt{2mE}} \Rightarrow \lambda\propto\frac{1}{\sqrt{E}}$$

$$\frac{\lambda_1}{\lambda_2}=\sqrt{\frac{E_2}{E_1}} \Rightarrow \frac{10^{-10}}{0.5\times10^{-10}}=\sqrt{\frac{E_1}{E_2}}$$

$$\Rightarrow \quad E_2 = 4E_1$$

Hence, added energy $=E_2-E_1=3E_1$

Example 11.31 *When the momentum of a proton is changed by an amount p_0, the corresponding change in the de-Broglie wavelength is found to be 0.25%. Find the original momentum of the photon.*

Sol. As $\lambda\propto\dfrac{1}{p} \Rightarrow \dfrac{\Delta p}{p}=-\dfrac{\Delta\lambda}{\lambda}$

$$\Rightarrow \quad \frac{p_0}{p}=\frac{0.25}{100} \Rightarrow p=400p_0$$

Example 11.32 *The kinetic energy of an electron is 5 eV. Calculate the de-Broglie wavelength associated with it ($h=6.6\times10^{-34}$ Js, $m_e=9.1\times10^{-31}$ kg)*

Sol. de-Broglie wavelength associated with the electron,

$$\lambda=\frac{h}{\sqrt{2m\,(\text{KE})}}$$

$$=\frac{6.6\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times5\times1.6\times10^{-19}}}$$

$$=5.469\times10^{-10}\text{m}=5.47\text{Å}$$

Example 11.33 *What is the*

(a) momentum,

(b) speed and

(c) de-Broglie wavelength of an electron with kinetic energy of 120 eV?

Sol. (a) Using the relation, $E=\dfrac{p^2}{2m}$,

We get $\quad p=\sqrt{2mE}$

$$=\sqrt{2\times9.1\times10^{-31}\times120\times1.6\times10^{-19}}$$

$$=5.91\times10^{-24}\ \text{kg-ms}^{-1}$$

(b) Speed, $v=\dfrac{p}{m}=\dfrac{5.91\times10^{-24}}{9.1\times10^{-31}}=6.5\times10^6\ \text{ms}^{-1}$

(c) de-Broglie wavelength is given by

$$\lambda=\frac{h}{p}=\frac{6.62\times10^{-34}}{5.91\times10^{-24}}=1.12\times10^{-10}\ \text{m}$$

Example 11.34 *What is the de-Broglie wavelength of a nitrogen molecule in air at 300 K? Assume that the molecule is moving with the root-mean-square speed of molecules at this temperature. (Atomic mass of nitrogen = 14.0076 u)*

Sol. Let v = rms speed of N_2 molecule at 300 K.

or $\quad v=\sqrt{\dfrac{3kT}{m}}$

where, k = Boltzmann constant $=1.38\times10^{-23}\ \text{JK}^{-1}$

or $\quad v=\sqrt{\dfrac{3\times1.38\times10^{-23}\times300}{28.0152\times1.67\times10^{-27}}}=5.15\times10^2\ \text{ms}^{-1}$

Using the formula, $\lambda=\dfrac{h}{mv}$, we get

$$\lambda=\frac{h}{mv}=\frac{6.62\times10^{-34}}{28.0152\times1.67\times10^{-27}\times5.15\times10^2}$$

$$=2.75\times10^{-11}\ \text{m}$$

Example 11.35 *The wavelength of light from the spectral emission line of sodium is 589 nm. Find the kinetic energy at which*

(a) an electron, and

(b) a neutron, would have the same de-Broglie wavelength.

Sol. (a) Let E_1 = KE of electron

Using the relation, $\lambda=\dfrac{h}{\sqrt{2m_eE}}$, we get

$$\lambda^2 = \frac{h^2}{2m_e E}$$

$$\therefore \quad E_1 = \frac{h^2}{2m_e\lambda^2} = \frac{(6.62\times 10^{-34})^2}{2\times 9.1\times 10^{-31}\times (589\times 10^{-9})^2}$$

$$= 6.94\times 10^{-25}\ \text{J}$$

$$= \frac{6.94\times 10^{-25}}{1.6\times 10^{-19}}\ \text{eV} = 4.34\times 10^{-6}\ \text{eV}$$

(b) $E_2 = \text{KE of neutron}$

$$E_2 = \frac{h^2}{2m_n\lambda^2} = \frac{(6.62\times 10^{-34})^2}{2\times 1.67\times 10^{-27}\times (589\times 10^{-9})^2}$$

$$= 3.782\times 10^{-28}\ \text{J}$$

$$= \frac{3.782\times 10^{-28}}{1.6\times 10^{-19}}\ \text{eV} = 2.36\times 10^{-9}\ \text{eV}$$

Example 11.36

(a) For what kinetic energy of a neutron will the associated de-Broglie wavelength be 1.40×10^{-10} m?

(b) Also find the de-Broglie wavelength of a neutron, in thermal equilibrium with matter, having an average kinetic energy of $\frac{3}{2}kT$ at 300 K.

Sol. (a) Using the formula, $\lambda = \frac{h}{\sqrt{2mE}}$, we get

$$\lambda^2 = \frac{h^2}{2mE}$$

or

$$E = \frac{h^2}{2m\lambda^2}$$

$$= \frac{(6.62\times 10^{-34})^2}{2\times 1.67\times 10^{-27}\times (1.40\times 10^{-10})^2}$$

$$= 6.69\times 10^{-21}\ \text{J}$$

$$= \frac{6.69\times 10^{-21}}{1.6\times 10^{-19}}$$

$$= 4.184\times 10^{-2}\ \text{eV}$$

(b) Here, T = absolute temperature = 300 K

k = Boltzmann constant

$= 1.38\times 10^{-23}\ \text{JK}^{-1}$

E = kinetic energy of neutron

$$= \frac{3}{2}kT = \frac{3}{2}\times 1.38\times 10^{-23}\times 300$$

$$= 6.21\times 10^{-21}\ \text{J}$$

Using the relation,

$$\lambda = \frac{h}{\sqrt{2mE}}, \text{ we get}$$

$$\lambda = \frac{6.62\times 10^{-34}}{\sqrt{2\times 1.67\times 10^{-27}\times 6.21\times 10^{-21}}}$$

$$= 1.455\times 10^{-10}\ \text{m} = 1.455\ \text{Å}$$

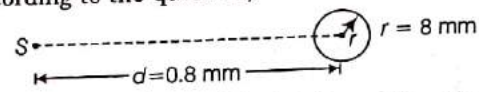
Example 11.37 *In a photoelectric effect set up, a point source of light of power 3.2×10^{-3} W emits mono-energetic photons of energy 5.0 eV. The source is located at a distance of 0.8 m from the centre of a stationary metallic sphere of work function 3.0 eV and of radius 8 min. The efficiency of photoelectron emission is one for every 10^6 incident photons. Assume that the sphere is isolated and initially neutral and that photoelectrons are instantly swept away after emission.*

(a) Find the ratio of the wavelength of incident light to the de-Broglie wavelength of the fastest photoelectron emitted.

(b) It is observed that the photoelectron emission stops at a certain time t after the light source is switched on. Why?

(c) Evaluate the time t.

Sol. According to the question,



(a) Using the relation, $E = \phi + K_{max} \Rightarrow 5 = 3 + K_{max}$

$K_{max} = 2$ eV

de-Broglie wavelength is given as

$$\lambda_d = \frac{h}{p} = \frac{h}{\sqrt{2mK_{max}}} \quad \text{...(i)}$$

and incident wavelength is given as

$$\lambda = \frac{hc}{E} \quad \text{...(ii)}$$

Dividing Eq. (i) by Eq. (ii),

$$\frac{\lambda}{\lambda_d} = \frac{c\sqrt{2mK_{max}}}{E}$$

Substituting the values,

$$= \frac{3\times 10^8\sqrt{2\times 9.1\times 10^{-31}\times 2\times 1.6\times 10^{-19}}}{5\times 1.6\times 10^{-19}}$$

$$= \frac{3\times 10^8\times 2}{5}\sqrt{\frac{9.1\times 10^{-31}}{1.6\times 10^{-19}}}$$

$$= 1.2\times 10^8\times \frac{1}{4}\times 10^{-6}\sqrt{91} = 286.18$$

(b) When electrons are ejected from sphere, it becomes positively charged and thus potential increases. When potential of sphere becomes equal to stopping potential, emission of electrons stopped.

(c) Let charge on sphere be q when its potential equals stopping potential $V = V_s$

As we know, $\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r} = V_s$

Substituting the waves,

$$\Rightarrow \quad \frac{9\times 10^9 q}{8\times 10^{-3}} = 2 \quad \Rightarrow \quad q = \frac{16}{9}\times 10^{-12}\text{C} \quad \text{...(iii)}$$

Also, charge is given by $q = n_0 et$ ...(iv)

Comparing Eqs. (iii) and (iv), we get

$$\frac{16}{9}\times 10^{-12} = 10^5\times 1.6\times 10^{-19}\, t$$

$$\Rightarrow \quad t = \frac{10}{9}\times 100 = \frac{100}{9} = 111\ \text{s}$$

7 Davisson and Germer experiment (Experimental verification of waves matter)

The wave nature of the material particles as predicted by de-Broglie wave was confirmed by **Davisson** and **Germer** (1927) in United States and by **GP Thomson** (1928) in **Scotland.**

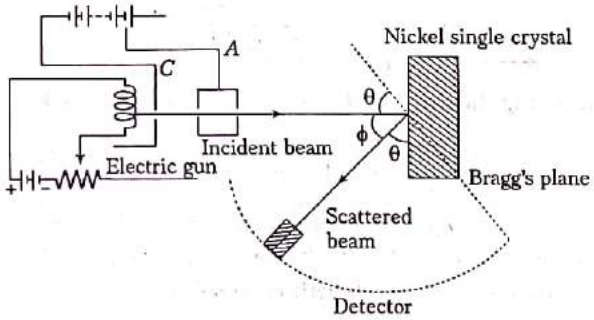


Fig. 11.17 Davisson-Germer electron diffraction arrangement

Experimental arrangement used by Davisson and Germer is as shown in the figure. Electrons from hot tungsten cathode (C) are accelerated by a potential difference V between the cathode and anode (A). A narrow hole in the anode renders the electrons into a fine beam of electrons and allows it to strike the nickel crystal.

The electrons are scattered in all directions by the atoms in the crystal. The intensity of the electron beam scattered in a given direction is found with the help of a detector. By rotating the detector, the intensity of the scattered beam can be measured for different values of ϕ, the angle between incident and the scattered direction of electron beam.

Davisson and Germer found that the intensity of scattered beam of electrons was not the same but different at different angles of scattering.

Intensity of scattered beam of electrons was found to be maximum when angle of scattering is 50° and accelerating potential is 54 V.

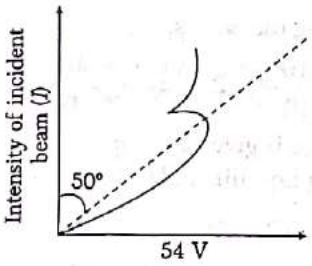


Fig. 11.18

Here, $\theta + \phi + \theta = 180°$, *i.e.*, $\theta = 65°$ $[\because \phi = 50°]$

For Ni crystal, lattice spacing $d = 0.91$ Å

For first principle maxima, $n = 1$

According to Bragg's equation for diffraction, $2d \sin\theta = n\lambda$ gives

$$\lambda = 1.65 \text{ Å}$$

According to de-Broglie hypothesis,

$$\lambda = \frac{12.27}{\sqrt{V}} \text{Å} = \frac{12.27}{\sqrt{54}} = 0.167 \text{ nm}$$

$\therefore$ de-Broglie wavelength of moving electron at $V = 54$ V is 1.67 Å which is in close agreement with 1.65 Å.

This proves the existence of de-Broglie waves for slow moving electrons.

Example 11.38 *In Davisson and Germer experiment, electron beam of wavelength 1.5 Å is incident on a crystal of lattice spacing 3 Å. Find the incident angle for first maxima.*

Sol. It is given that $d = 3$ Å, $\lambda = 1.5$ Å, $n = 1$

For the crystal we can write

$$2d \sin\theta = n\lambda$$

$$\Rightarrow \quad 2 \times 3 \times \sin\theta = 1 \times 1.5$$

$$\Rightarrow \quad \sin\theta = \frac{1}{4} \Rightarrow \theta = 14.4°$$

$\therefore$ Angle of incidence of electron beams, $\theta = 14.4°$

Example 11.39 *In Davisson and Germer experiment, second maxima is obtained for an electron beam of wavelength 2.5 Å and incident at an angle of 30°. Evaluate the lattice spacing of the crystal.*

Sol. It is given that wavelength of electron beam, $\lambda = 2.5$ Å

Angle of incidence, $\theta = 30°$

$n = 2$ (for second maxima), $d = ?$

As we know that, $2d \sin\theta = n\lambda$

$$\Rightarrow \quad 2d \sin 30° = 2 \times 2.5 \text{ Å}$$

$$\Rightarrow \quad 2 \times d \times \frac{1}{2} = 2 \times \frac{5}{2} \Rightarrow d = 5 \text{ Å}$$

$\therefore$ Lattice spacing, $d = 5$ Å.

8 Electron microscope

Electron microscope is an important application of de-Broglie wave used to study very minute objects like viruses, microbes etc. Like light radiations, electron beams behave as waves but with much smaller wavelength. It uses electric and magnetic field to concentrate electron beam. The magnifying power of a microscope is inversely related with the wavelength of radiations used. Here, the

electrons are focussed with the help of electric and magnetic lenses. Also, the viewing screen used should be fluorescent so that image produced should be visible. An electron microscope can have a very high magnification of $\approx 10^5$.

Example 11.40 *An electron microscope uses electrons accelerated by a voltage of 50 kV. Determine the de-Broglie wavelength associated with the electrons. If other factors (such as numerical aperture etc.) are taken to be roughly the same, how does the resolving power of an electron microscope compare with that of an optical microscope which uses yellow light?*

Sol. Here, $V = 50\text{ kV} = 5 \times 10^4\text{ V}$, $m_e = 9.11 \times 10^{-31}\text{ kg}$

KE of an electron,

$$K = 50\text{ eV} = 1.6 \times 10^{-19} \times 5 \times 10^4\text{ J} = 8 \times 10^{-15}\text{ J}$$

$\therefore$ de-Broglie wavelength of electrons is

$$\lambda = \frac{h}{\sqrt{2mK}} = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.11 \times 10^{-31} \times 8 \times 10^{-15}}}\text{ m}$$

$$= \frac{6.63 \times 10^{-11}}{12.07}\text{ m} = 5.5 \times 10^{-12}\text{ m}$$

Wavelength of yellow light, $\lambda_y = 5.9 \times 10^{-7}$ m

Resolving power of a microscope $\propto 1/\lambda$

$$\therefore \frac{\text{Resolving power of electron microscope}}{\text{Resolving power of optical microscope}} = \frac{\lambda_y}{\lambda}$$

$$= \frac{5.9 \times 10^{-7}}{5.5 \times 10^{-12}} \equiv 10^5$$

Thus, the resolving power of an electron microscope is about 10^5 times greater than that of an optical microscope.

Check point 11.3

1. The idea of matter waves was given by
(a) Davisson and Germer (b) de-Broglie
(c) Einstein (d) Planck

2. The de-Broglie wavelength associated with the particle of mass m moving with velocity v is
(a) h/mv (b) mv/h (c) mh/v (d) m/hv

3. The wavelength of the matter wave is independent of
(a) mass (b) velocity (c) momentum (d) charge

4. If the velocity of an electron is doubled, its de-Broglie frequency will
(a) be halved (b) remain same
(c) be doubled (d) become four times

5. Proton and α-particles have the same de-Broglie wavelength. Which quantity will remain same for both of them.
(a) Charge (b) Energy (c) Speed (d) Momentum

6. A photon, an electron and a very small unknown particle have the same wavelength. The one with the most energy
(a) is the photon
(b) is the electron
(c) is the particle nucleus
(d) depends upon the wavelength and the properties of the particle

7. If λ_p and λ_e denote the de-Broglie wavelength of proton and electron after they are accelerated from rest through the same potential difference, then
(a) $\lambda_e = \lambda_p$ (b) $\lambda_e < \lambda_p$ (c) $\lambda_e > \lambda_p$ (d) $\lambda_e = \lambda_p/2$

8. The de-Broglie wavelength of a body of mass m and kinetic energy E is given by
(a) $\frac{h}{\sqrt{2mE}}$ (b) $\frac{h}{2mE}$
(c) $\frac{\sqrt{2mE}}{h}$ (d) $\frac{h}{mE}$

9. When the kinetic energy of an electron is increased, the wavelength of the associated wave will
(a) increase
(b) decrease
(c) not depend on the kinetic energy
(d) None of the above

10. de-Broglie wavelength (λ) of a charged particle accelerated from rest by potential difference V is related as
(a) $\lambda \propto V$ (b) $\lambda \propto \sqrt{V}$ (c) $\lambda \propto \frac{1}{\sqrt{V}}$ (d) $\lambda \propto \frac{1}{V}$

11. The de-Broglie wavelength of a neutron at 27°C is λ. What will be its wavelength at 927°C?
(a) $\frac{\lambda}{2}$ (b) $\frac{\lambda}{3}$ (c) $\frac{\lambda}{4}$ (d) $\frac{\lambda}{9}$

12. The de-Broglie wavelength associated with an electron, accelerated through a potential difference of 100 V is
(a) 0.12 nm (b) 0.15 nm (c) 0.20 nm (d) 0.25 nm

13. A particle having zero rest mass and non-zero energy and momentum must travel with a speed
(a) equal to c, the speed of light in vacuum
(b) tending to infinity
(c) greater than c
(d) less than c

14. Matter waves
(a) are longitudinal waves
(b) are electromagnetic waves in nature
(c) always travel with the speed of light
(d) show diffraction

15. In Davisson and Germer experiment, intensity of scattered beam of electron was found to be maximum when angle of scattering is
(a) 50° (b) 45° (c) 30° (d) 90°

Chapter exercises

A) Taking it together

(Assorted Questions of the Chapter for Advanced Level Practice)

1. Photoelectric effect was successfully explained first by
(a) Planck (b) Hallwach
(c) Hertz (d) Einstein

2. A non-monochromatic light is used in an experiment on photoelectric effect. The stopping potential is
(a) related to the mean wavelength
(b) related to the longest wavelength
(c) related to the shortest wavelength
(d) not related to the wavelength

3. The de-Broglie wavelength λ of an electron accelerated through a potential V (in volt) is
(a) $\frac{1.227}{\sqrt{V}}$ nm (b) $\frac{0.1227}{\sqrt{V}}$ nm
(c) $\frac{0.01227}{\sqrt{V}}$ nm (d) $\frac{0.1227}{\sqrt{V}}$ Å

4. Let p and E denote the linear momentum and energy of a photon. If the wavelength is decreased, then
(a) Both p and E increase
(b) p increases and E decreases
(c) p decreases and E increases
(d) Both p and E decrease

5. The maximum kinetic energy of the photoelectrons varies
(a) inversely with the intensity and is independent of the frequency of the incident radiation
(b) inversely with the frequency and is independent of the intensity of the incident radiation
(c) linearly with the frequency and the intensity of the incident radiation
(d) linearly with the frequency and is independent of the intensity of the incident radiation

6. If the particles listed below all have the same kinetic energy, which one would possess the shortest de-Broglie wavelength?
(a) Deuteron (b) α-particle
(c) Proton (d) Electron

7. If K be the kinetic energy and m the mass of a moving particle, then the de-Broglie wavelength of the particle is
(a) $\lambda = \frac{h}{\sqrt{mK}}$ (b) $\lambda = \frac{2h}{\sqrt{mK}}$
(c) $\lambda = \frac{h}{2\sqrt{mK}}$ (d) $\lambda = \frac{h}{\sqrt{2mK}}$

8. Photoelectric emission occurs only when the incident light has more than a certain minimum
(a) wavelength (b) intensity
(c) frequency (d) power

9. The momentum of a photon is 3.3×10^{-29} kg-ms^{-1}. Its frequency will be
(a) 3×10^3 Hz (b) 6×10^3 Hz
(c) 7.5×10^{12} Hz (d) 1.5×10^{13} Hz

10. An electron and proton are accelerated through the same potential difference. The ratio of their de-Broglie wavelength will be
(a) $(m_p/m_e)^{1/2}$ (b) m_e/m_p
(c) m_p/m_e (d) 1

11. If the de-Broglie wavelengths for a proton and for α-particle are equal, then the ratio of their velocities will be in the ratio
(a) 4 : 1 (b) 2 : 1
(c) 1 : 2 (d) 1 : 4

12. The approximate wavelength of a photon of energy 2.48 eV is
(a) 500 Å (b) 5000 Å
(c) 2000 Å (d) 1000 Å

13. The de-Broglie wavelength of an electron having 80 eV of energy is nearly (1eV $= 1.6 \times 10^{-19}$ J, Mass of electron $= 9.1 \times 10^{-31}$kg, Planck's constant $= 6.6 \times 10^{-34}$ Js)
(a) 140 Å (b) 0.14 Å
(c) 14 Å (d) 1.4 Å

14. The energy of incident photons corresponding to maximum wavelength of visible light is
(a) 3.2 eV (b) 7 eV (c) 1.75 eV (d) 1 eV

15. The momentum of a photon is 6.6×10^{-29} kg- ms^{-1}. The frequency of the radiation (in Hz) is
(a) 3×10^{13} (b) 3×10^{8} (c) 3×10^{15} (d) 3×10^{6}

16. If the energy of the photon is increased by a factor of 4, then its momentum
(a) does not change
(b) decreases by a factor of 4
(c) increases by a factor of 4
(d) increases by a factor of 2

17. The number of photons of wavelength 540 nm emitted per second by an electric bulb of power 100 W is approximately (taking $h = 6\times 10^{-34}$ Js)
(a) 4×10^{16} (b) 4×10^{10}
(c) 3×10^{20} (d) 3×10^{18}

18. The work function for Al, K and Pt is 4.28 eV, 2.30 eV and 5.65 eV respectively. Their respective threshold frequencies would be
(a) Pt > Al > K (b) Al > Pt > K
(c) K > Al > Pt (d) Al > K > Pt

19. An electron of mass m_e and a proton of mass m_p are moving with the same speed. The ratio of their de-Broglie wavelengths λ_e/λ_p is
(a) 1 (b) 1836 (c) $\frac{1}{1836}$ (d) 918

20. The frequency of a photon having energy 100 eV is (Take $h = 6.62\times 10^{-34}$ Js; 1 eV = 1.6×10^{-19} J)
(a) 2.417×10^{-16} Hz (b) 2.417×10^{16} Hz
(c) 2.417×10^{17} Hz (d) 10.54×10^{17} Hz

21. The de-Broglie wavelength of neutrons in thermal equilibrium is
(a) $\frac{30.8}{\sqrt{T}}$ Å (b) $\frac{3.08}{\sqrt{T}}$ Å
(c) $\frac{0.308}{\sqrt{T}}$ Å (d) $\frac{0.0308}{\sqrt{T}}$ Å

22. Find the maximum wavelength of light that can cause photoelectric effect in Lithium (work function = 2.5 eV)
(a) 497 nm (b) 397 nm
(c) 749 nm (d) 237 nm

23. Photon and electron are given energy (10^{-2} J). Wavelengths associated with photon and electron are λ_{ph} and λ_{el}, then correct statement will be
(a) $\lambda_{ph} > \lambda_{el}$ (b) $\lambda_{ph} < \lambda_{el}$
(c) $\lambda_{ph} = \lambda_{el}$ (d) $\frac{\lambda_{el}}{\lambda_{ph}} = c$

24. A radio transmitter radiates 1 kW power at a wavelength 198.6 m. How many photons does it emit per second?
(a) 10^{10} (b) 10^{20} (c) 10^{30} (d) 10^{40}

25. According to Einstein's photoelectric equation, the graph of KE of the photoelectron emitted from the metal *versus* the frequency of the incident radiation gives a straight line graph, whose slope
(a) depends on the intensity of the incident radiation
(b) depends on the nature of the metal and also on the intensity of incident radiation
(c) is same for all metals and independent of the intensity of the incident radiation
(d) depends on the nature of the metal

26. Sodium and copper have work functions 2.3 eV and 4.5 eV respectively. Then, the ratio of their threshold wavelengths is nearest to
(a) 1 : 2 (b) 2 : 1 (c) 1 : 4 (d) 4 : 1

27. Monochromatic light of frequency f is incident on emitter having threshold frequency f_0. The kinetic energy of ejected electron will be
(a) hf (b) $h(f - f_0)$
(c) hf_0 (d) $h(f + f_0)$

28. The work function for aluminium surface is 4.2 eV. The cut-off wavelength for the photoelectric effect is
(a) 2955 Å (b) 4200 Å (c) 2000 Å (d) 1000 Å

29. Light of wavelength 3500 Å is incident on two metals *A* and *B*. *A* of work function 4.2 eV and *B* of work function 1.19 eV. The photoelectron will be emitted by
(a) metal *A*
(b) metal *B*
(c) Both *A* and *B*
(d) Neither metal *A* nor metal *B*

30. The retarding potential necessary to stop the emission of photoelectron, when a target material of work function 1.24 eV is irradiated with light of wavelength 4.36×10^{-7} m is
(a) 0.36 V (b) 1.60 V (c) 2.84 V (d) 4.08 V

31. Light of frequency $4\nu_0$ is incident on the metal of threshold frequency ν_0. The maximum kinetic energy of the emitted photoelectron is
(a) $3h\nu_0$ (b) $2h\nu_0$ (c) $\frac{3}{2}h\nu_0$ (d) $\frac{1}{2}h\nu_0$

32. The surface of some material is radiated, by waves of $\lambda_1 = 3.5\times 10^{-7}$ m and $\lambda_2 = 5.4\times 10^{-7}$ m, respectively. The ratio of the stopping potential in two cases is 2 : 1. The work function of the material is
(a) 0.86 eV (b) 1.345 eV
(c) 1.05 eV (d) 2.20 eV

33. The masses of two particles having same kinetic energies are in the ratio 1 : 2. Then, their de-Broglie wavelengths are in the ratio
(a) 2 : 1 (b) 1 : 2
(c) $\sqrt{2}$: 1 (d) $\sqrt{3}$: 1

34. Let n_r and n_b be respectively the number of photons emitted by red bulb and blue bulb of equal power in given time
(a) $n_r = n_b$ (b) $n_r < n_b$
(c) $n_r > n_b$ (d) None of these

35. When light of wavelength 300 nm falls on a photoelectric emitter, photoelectrons are liberated. For another emitter, light of wavelength 600 nm is sufficient for liberating photoelectrons. The ratio of the work function of the two emitters is
(a) 1 : 2 (b) 2 : 1
(c) 4 : 1 (d) 1 : 4

36. The work functions of metals A and B are in the ratio 1 : 2. If light of frequencies f and $2f$ are incident on metal surfaces of A and B respectively, the ratio of the maximum kinetic energies of photoelectron emitted is (f is greater than threshold frequency of A, $2f$ is greater than threshold frequency of B)
(a) 1 : 1 (b) 1 : 2
(c) 1 : 3 (d) 1 : 4

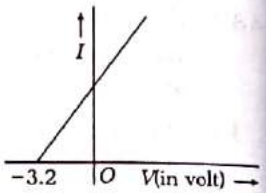
37. The de-Broglie wavelength of an electron in a metal at 27°C is
(a) 3.5×10^{-9} m (b) 4.2×10^{-9} m
(c) 5.6×10^{-9} m (d) 6.2×10^{-9} m

38. A proton accelerated through a potential difference of 100 V, has de-Broglie wavelength λ_0. The de-Broglie wavelength of an α-particle, accelerated through 800 V is
(a) $\frac{\lambda_0}{\sqrt{2}}$ (b) $\frac{\lambda_0}{2}$ (c) $\frac{\lambda_0}{4}$ (d) $\frac{\lambda_0}{8}$

39. An electron and a photon possess the same de-Broglie wavelength. If E_e and E_p respectively are the energies of electron and photon and v and c are their respective velocities, then $\frac{E_e}{E_p}$ is equal to
(a) $\frac{v}{c}$ (b) $\frac{v}{2c}$ (c) $\frac{v}{3c}$ (d) $\frac{v}{4c}$

40. Calcium plate has maximum possible radiation of wavelength λ of 400 nm to eject electrons. Its work function is
(a) 2.3 eV (b) 3.1 eV
(c) 4.5 eV (d) None of these

41. In a photoelectric experiment, the relation between applied potential difference between cathode and anode (V) and the photoelectric current (I) was found to be shown in graph below. If Planck's constant $h = 6.6 \times 10^{-34}$ Js, the frequency of incident radiation would be nearly (in s^{-1})



(a) 0.436×10^{18} (b) 0.436×10^{17}
(c) 0.775×10^{15} (d) 0.775×10^{16}

42. Maximum KE of a photoelectron is E when the wavelength of incident light is λ. If energy becomes four times when wavelength is reduced to one-third, then work function of the metal is
(a) $\frac{3hc}{\lambda}$ (b) $\frac{hc}{3\lambda}$ (c) $\frac{hc}{\lambda}$ (d) $\frac{hc}{2\lambda}$

43. In a photoelectric experiment, the stopping potential V_s is plotted against the frequency ν of incident light. The resulting curve is a straight line which makes an angle θ with the ν-axis. The $\tan\theta$ will be equal to (ϕ = work function of surface)
(a) h/e (b) e/h (c) $-\phi/e$ (d) eh/ϕ

44. The work function of a metal is 1.6×10^{-19} J. When the metal surface is illuminated by the light of wavelength 6400 Å, then the maximum kinetic energy of emitted photoelectrons will be
(Planck's constant $h = 6.4 \times 10^{-34}$ Js)
(a) 14×10^{-19} J (b) 2.8×10^{-19} J
(c) 1.4×10^{-19} J (d) 1.4×10^{-19} eV

45. The work function for tungsten and sodium are 4.5 eV and 2.3 eV, respectively. If the threshold wavelength λ for sodium is 5600 Å, then the value of λ for tungsten is
(a) 5893 Å (b) 10683 Å
(c) 2862 Å (d) 528 Å

46. In a photoelectric experiment for 4000 Å incident radiation, the potential difference to stop the ejection is 2 V. If the incident light is changed to 3000 Å, then the potential required to stop the ejection of electrons will be
(a) 2 V (b) less than 2 V
(c) zero (d) greater than 2 V

47. When radiation is incident on photoelectron emitter, the stopping potential is found to be 9 V. If e/m for the electron is 1.8×10^{11} C kg^{-1}, the maximum velocity of the ejected electrons is
(a) 6×10^5 ms^{-1} (b) 8×10^5 ms^{-1}
(c) 1.8×10^6 ms^{-1} (d) 1.8×10^5 ms^{-1}

48. An electron of mass m when accelerated through a potential difference V has de-Broglie wavelength λ. The de-Broglie wavelength associated with a proton of mass M accelerated through the same potential difference will be

(a) $\lambda\frac{m}{M}$ (b) $\lambda\sqrt{\frac{m}{M}}$

(c) $\lambda\frac{M}{m}$ (d) $\lambda\sqrt{\frac{M}{m}}$

49. A photocell is illuminated by a point source of light, which is placed at a distance d from the cell. If the distance becomes $d/2$, then number of electrons emitted per second will

(a) remain the same (b) become four times

(c) become two times (d) become one-fourth

50. Threshold frequency for a metal is 10^{15} Hz. Light of $\lambda = 4000$ Å falls on its surface. Which of the following statements is correct?

(a) No photoelectric emission takes place

(b) Photoelectrons come out with zero speed

(c) Photoelectrons come out with 10^3 ms^{-1} speed

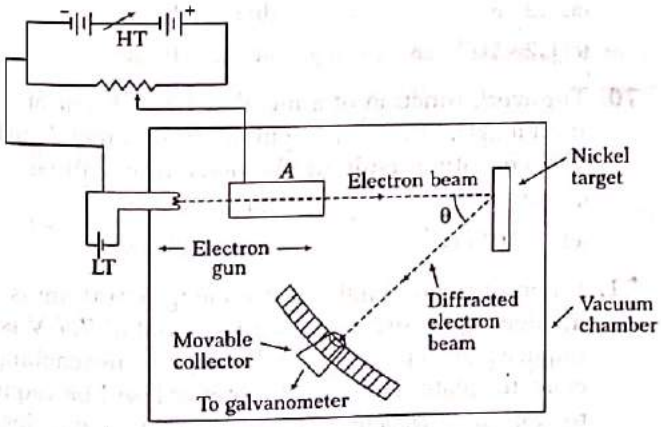
(d) Photoelectrons come out with 10^5 ms^{-1} speed

51. The threshold frequency of the metal of the cathode in a photoelectric cell is 1×10^{15} Hz. When a certain beam of light is incident on the cathode, it is found that a stopping potential 4.144 V is required to reduce the current to zero. The frequency of the incident radiation is

(a) 2.5×10^{15} Hz (b) 2×10^{15} Hz

(c) 4.144×10^{15} Hz (d) 3×10^{16} Hz

52. Consider the figure given below. Suppose the voltage applied to A is increased. The diffracted beam will have the maxima at a value of θ that [NCERT Exemplar]



(a) will be larger than the earlier value

(b) will be the same as the earlier value

(c) will be less than the earlier value

(d) will depend on the target

53. If maximum velocity with which an electron can be emitted from photocell is 4×10^6 cm/s, the stopping potential is (mass of electron $= 9\times 10^{-31}$ kg)

(a) 30 V

(b) 45 V

(c) 59 V

(d) Information is insufficient

54. An electron is moving with an initial velocity $\mathbf{v} = v_0\hat{\mathbf{i}}$ and is in a magnetic field $\mathbf{B} = B_0\hat{\mathbf{j}}$. Then, its de-Broglie wavelength will be [NCERT Exemplar]

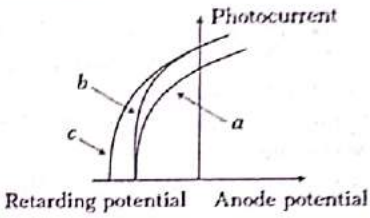
(a) remain constant

(b) increase with time

(c) decrease with time

(d) increase and decrease periodically

55. The figure shows a plot of photocurrent *versus* anode potential for a photosensitive surface for three different radiations. Which one of the following is a correct statement?



(a) Curves a and b represent incident radiations of different frequencies and different intensities

(b) Curves a and b represent incident radiations of same frequency but of different intensities

(c) Curves b and c represent incident radiations of different frequencies and different intensities

(d) Curves b and c represent incident radiations of same frequency having same intensity

56. Threshold wavelength for a metal is 5200 Å. The photoelectron will be ejected, if it is irradiated by light from

(a) 50 W infrared lamp (b) 1 W infrared lamp

(c) 50 W ultraviolet lamp (d) 0.5 W infrared lamp

57. When a monochromatic point source of light is at a distance r from a photoelectric cell, the cut-off voltage is V and the saturation current is I. If the same source is placed at a distance $3r$ away from the photoelectric cell, then

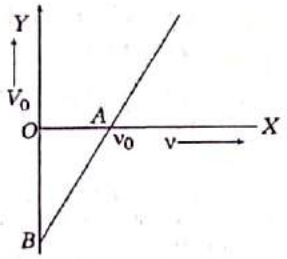
(a) the saturation current will not change

(b) the saturation current will change to $I/9$

(c) the stopping potential will not change

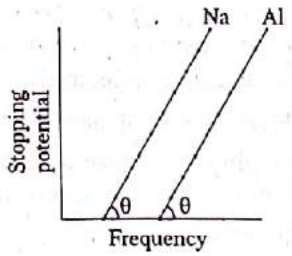
(d) the stopping potential will increase to $3V$

58. In an experiment on photoelectric effect the frequency ν of the incident light is plotted against the stopping potential V_0. The work function of the photoelectric surface is given by (e is electronic charge)



(a) $OB \times e$ in eV (b) OB in volt
(c) OA in eV (d) The slope of the line AB

59. From the figure describing photoelectric effect, we may infer correctly that



(a) Na and Al both have the same threshold frequency
(b) Maximum kinetic energy for both the metals depend linearly on the frequency
(c) The stopping potentials are different for Na and Al for the same change in frequency
(d) Al is better photosensitive material than Na

60. Two electrons are moving with the same speed v. One electron enters a region of uniform electric field while the other enters a region of uniform magnetic field. Then after some time if the de-Broglie wavelengths of the two are λ_1 and λ_2, then
(a) $\lambda_1 = \lambda_2$ (b) $\lambda_1 > \lambda_2$
(c) $\lambda_1 < \lambda_2$ (d) $\lambda_1 > \lambda_2$ or $\lambda_1 < \lambda_2$

61. The de-Broglie wavelength associated with an electron moving with a velocity 0.5c and rest mass $= 9.1 \times 10^{-31}$kg is
(a) 4.2×10^{-12} m (b) 3.6×10^{-12} m
(c) 5.0×10^{-12}m (d) 4.2×10^{-10}m

62. An X-ray beam of wavelength 1.0 Å is incident on a crystal of lattice spacing 2.8 Å. The value of Bragg angle for first order of diffraction is
(a) 17.8° (b) 10.3°
(c) 30° (d) 45°

63. The work functions for metals A, B and C are respectively 1.92 eV, 2.0 eV and 5 eV. According to Einstein's equation, the metal(s) which will emit photoelectrons for a radiation of wavelength 4100 Å is/are
(a) A only (b) A and B
(c) All of these (d) None of these

64. The maximum wavelength of radiation that can produces photoelectric effect in a certain metal is 200 nm. The maximum kinetic energy acquired by electron due to radiation of wavelength 100 nm will be
(a) 12.4 eV (b) 6.2 eV (c) 100 eV (d) 200 eV

65. A particle of mass M at rest decays into two particles of masses m_1 and m_2, having non-zero velocities. The ratio of the de-Broglie wavelengths of the particles, λ_1/λ_2 is
(a) m_1/m_2 (b) m_2/m_1
(c) 1 : 1 (d) $\sqrt{m_2}/\sqrt{m_1}$

66. A particle is dropped from a height H. The de-Broglie wavelength of the particle as a function of height is proportional to **[NCERT Exemplar]**
(a) H (b) $H^{1/2}$ (c) H^0 (d) $H^{-1/2}$

67. The photoelectric threshold wavelength for silver is λ_0. The energy of the electron ejected from the surface of silver by an incident wavelength λ ($\lambda < \lambda_0$) will be **[NCERT Exemplar]**
(a) $hc(\lambda_0 - \lambda)$ (b) $\dfrac{hc}{\lambda_0 - \lambda}$
(c) $\dfrac{h}{c}\left(\dfrac{\lambda_0 - \lambda}{\lambda\lambda_0}\right)$ (d) $hc\left(\dfrac{\lambda_0 - \lambda}{\lambda\lambda_0}\right)$

68. If the momentum of an electron is changed by Δp, then the de-Broglie wavelength associated with it changes by 0.50%. The initial momentum of electron will be
(a) $\dfrac{\Delta p}{200}$ (b) $\dfrac{\Delta p}{199}$
(c) $199\,\Delta p$ (d) $400\,\Delta p$

69. The wavelength of a photon needed to remove a proton from a nucleus which is bound to the nucleus with 1 MeV energy is nearly **[NCERT Exemplar]**
(a) 1.2 nm (b) 1.2×10^{-3} nm
(c) 1.2×10^{-6} nm (d) 1.2×10 nm

70. The work function of a metal is 1 eV. Light of wavelength 3000 Å is incident on this metal surface. The velocity of emitted photoelectrons will be
(a) 10 ms^{-1} (b) 1×10^3 ms^{-1}
(c) 1×10^4 ms^{-1} (d) 1×10^6 ms^{-1}

71. When ultraviolet light of wavelength 100 nm is incident upon silver plate, a potential of 7.7 V is required to stop the photoelectrons from reaching the collector plate. How much potential will be required to stop the photoelectrons when light of wavelength 200 nm is incident upon silver?
(a) 1.5 V (b) 3.85 V
(c) 2.35 V (d) 15.4 V

72. When 1 cm thick surface is illuminated with light of wavelength λ, the stopping potential is V. When the same surface is illuminated by light of wavelength 2λ, the stopping potential is $V/3$. Threshold wavelength for metallic surface is
(a) $4\lambda/3$ (b) 4λ (c) 6λ (d) $8\lambda/3$

73. If 5% of the energy supplied to a bulb is irradiated as visible light, how many quanta are emitted per second by a 100 W lamp? (Assume wavelength of visible light as 5.6×10^{-5} cm)
(a) 1.4×10^{19} (b) 3×10^{3}
(c) 1.4×10^{-19} (d) 3×10^{4}

74. Consider a beam of electrons (each electron with energy E_0) incident on a metal surface kept in an evacuated chamber. Then, [NCERT Exemplar]
(a) no electrons will be emitted as only photons can emit electrons
(b) electrons can be emitted but all with an energy, E_0
(c) electrons can be emitted with any energy, with a maximum of $E_0 - \phi$ (ϕ is the work function)
(d) electrons can be emitted with any energy, with a maximum of E_0

75. Minimum light intensity that can be perceived by normal human eye is about 10^{-10} Wb m^{-2}. What is the minimum number of photons of wavelength 660 nm that must enter the pupil in 1 s for one to see the object? Area of cross-section of the pupil is 10^{-4} m^2.
(a) 3.3×10^{2} (b) 3.3×10^{3}
(c) 3.3×10^{4} (d) 3.3×10^{5}

76. A proton, a neutron, an electron and an α-particle have same energy. Then, their de-Broglie wavelengths compare as [NCERT Exemplar]
(a) $\lambda_p = \lambda_n > \lambda_e > \lambda_\alpha$ (b) $\lambda_\alpha < \lambda_p = \lambda_n < \lambda_e$
(c) $\lambda_e < \lambda_p = \lambda_n > \lambda_\alpha$ (d) $\lambda_e = \lambda_p = \lambda_n = \lambda_\alpha$

77. A photon of energy E ejects a photoelectron from a metal surface whose work function is W_0. If electron having maximum kinetic energy enters into a uniform magnetic field of induction B in a direction perpendicular to the field and describes a circular path of radius r, then the radius r is given by, (in the usual notation)
(a) $\frac{\sqrt{2m(E-W_0)}}{eB}$ (b) $\sqrt{2m(E-W_0)\,eB}$
(c) $\frac{\sqrt{2m(E-W_0)}}{mB}$ (d) None of these

78. A photosensitive metallic surface has work function ϕ. If photon of energy 3ϕ fall on this surface, the electron comes out with a maximum velocity of 6×10^6 ms^{-1}. When the photon energy is increased to 9ϕ, there maximum velocity of photoelectron will be
(a) 12×10^6 m/s (b) 6×10^6 m/s
(c) 3×10^6 m/s (d) 24×10^6 m/s

79. Radiations of two photon having energies twice and five times the work function of metal are incident successively on the metal surface. The ratio of the maximum velocity of the photoelectrons emitted in the two cases will be
(a) 1 : 1 (b) 1 : 2
(c) 1 : 3 (d) 1 : 4

80. Work function of nickel is 5.01 eV. When ultraviolet radiation of wavelength 2000 Å is incident on the surface of nickel, electrons are emitted. What will be the maximum velocity of emitted electrons?
(a) 3×10^8 ms^{-1}
(b) 6.46×10^5 ms^{-1}
(c) 10.36×10^5 ms^{-1}
(d) 8.54×10^6 ms^{-1}

81. When radiation of wavelength λ is incident on a metallic surface, the stopping potential is 4.8 V. If the same surface is illuminated with radiation of double the wavelength, then the stopping potential becomes 1.6 V. Then the threshold wavelength for the surface is
(a) 2λ (b) 4λ
(c) 6λ (d) 8λ

82. A photon has the same wavelength as the de-Broglie wavelength of electron. Given that c = speed of light and v = speed of electron. Which of the following relation is correct? [Here E_e = Kinetic energy of electron, E_{ph} = energy of photon, p_e = momentum of electron and p_{ph} = momentum of photon]
(a) $\frac{E_e}{E_{ph}} = \frac{2c}{v}$ (b) $\frac{E_e}{E_{ph}} = \frac{v}{2c}$
(c) $\frac{p_e}{p_{ph}} = \frac{2c}{v}$ (d) $\frac{p_e}{p_{ph}} = \frac{c}{2v}$

83. We wish to see inside an atom. Assuming the atom to have a diameter of 100 pm [1 picometer (pm) = 10^{-12} m], this means that one must be able to resolve a width of, say 10 pm. If an electron microscope is used, the minimum electron energy required is about
(a) 1.5 keV (b) 15 keV
(c) 150 keV (d) 1.5 MeV

84. X-rays of wavelength 10.0 pm are scattered from a target. The wavelength of the X-rays scattered through 30° is
(a) 10.32 pm (b) 12 pm
(c) 10 nm (d) 10 Å

85. A 1100 W light bulb is placed at the centre of a spherical chamber of radius 20 cm. Assume that 60% the energy supplied to the bulb is converted into light and that the surface of the chamber is perfectly absorbing. The force and pressure exerted by the light on the surface of chamber is
(a) 3.6×10^{-6} N, 4.4×10^{-6} N/m^2
(b) 7.3×10^{-6} N, 8.8×10^{-6} N/m^2
(c) 3.6×10^{-6} N, 8.8×10^{-6} N/m^2
(d) 7.3×10^{-6} N, 4.4×10^{-6} N/m^2

86. A parallel beam of monochromatic light of wavelength 663 nm is incident on a totally reflecting plane mirror. The angle of incidence is 60° and the number of photons striking the mirror per second is 1.0×10^{19}. The value of the force exerted by it on the surface is
(a) 10^{-6} N (b) 10^{-7} N (c) 10^{-8} N (d) 10^{-9} N

87. A parallel beam of monochromatic light of wavelength 550 nm falls normally on a plane metallic surface which is perfectly reflecting. The power of the beam is 5 mW and its cross-sectional area is 1.0 mm^2. Find the force and pressure exerted by the radiation on the metallic surface.
(a) 1.65×10^{-11} N, 1.65×10^{-5} N/m^2
(b) 3.3×10^{-11} N, 3.3×10^{-5} N/m^2
(c) 1.65×10^{-11} N, 3.3×10^{-5} N/m^2
(d) 3.3×10^{-11} N, 1.65×10^{-5} N/m^2

88. In a photoemissive cell with exciting wavelength λ, the fastest electron has speed v. If the exciting wavelength is changed to $\frac{\lambda}{4}$, the speed of the fastest emitted electron will be
(a) $2v$ (b) $\frac{v}{2}$ (c) $< 2v$ (d) $> 2v$

89. Light of wavelength λ, strikes a photoelectric surface and electrons are ejected with an energy E. If E is to be increased to exactly twice its original value, the wavelength changes to λ', where
(a) λ' is less than $\lambda/2$
(b) λ' is greater than $\lambda/2$
(c) λ' is greater than $\lambda/2$ but less than λ
(d) λ' is exactly equal to $\lambda/2$

90. In a photocell, with exciting wavelength λ, the faster electron has speed v. If the exciting wavelength is changed to $3\lambda/4$, the speed of the fastest electron will be
(a) $v\left(\frac{3}{4}\right)^{1/2}$ (b) $v\left(\frac{4}{3}\right)^{1/2}$
(c) less than $v\left(\frac{4}{3}\right)^{1/2}$ (d) greater than $v\left(\frac{4}{3}\right)^{1/2}$

91. Light of wavelength 330 nm falling on a piece of metal ejects electrons with sufficient energy which requires voltage V_0 to prevent them from reaching a collector. In the same setup, light of wavelength 220 nm, ejects electrons which require twice the voltage V_0 to stop them in reaching a collector. The numerical value of voltage V_0 is
(a) $\frac{16}{15}$ V (b) $\frac{15}{16}$ V (c) $\frac{15}{8}$ V (d) $\frac{8}{15}$

92. An electron (mass m) with an initial velocity $\mathbf{v} = v_0\hat{\mathbf{i}}$ $(v_0 > 0)$ is in an electric field $\mathbf{E} = -E_0\hat{\mathbf{i}}$ (E_0 = constant > 0). Its de-Broglie wavelength at time t is given by **[NCERT Exemplar]**
(a) $\dfrac{\lambda_0}{\left(1 + \frac{eE_0}{m}\frac{t}{v_0}\right)}$ (b) $\lambda_0\left(1 + \frac{eE_0 t}{mv_0}\right)$
(c) λ_0 (d) $\lambda_0 t$

93. Light of wavelength 0.6 μm from a sodium lamp falls on a photocell and causes the emission of photoelectrons for which the stopping potential is 0.5 V. With light of wavelength 0.4 μm from a sodium lamp, stopping potential is 1.5 V. With this data, the value of h/e is
(a) 4×10^{-19} Vs (b) 0.25×10^{15} Vs
(c) 4×10^{-15} Vs (d) 4×10^{-8} VsV

94. An electron (mass m) with an initial velocity $\mathbf{v} = v_0\hat{\mathbf{i}}$ is in an electric field $\mathbf{E} = E_0\hat{\mathbf{j}}$. If $\lambda_0 = h/mv_0$, its de-Broglie wavelength at time t is given by **[NCERT Exemplar]**
(a) λ_0 (b) $\lambda_0\sqrt{1 + \frac{e^2E_0^2t^2}{m^2v_0^2}}$
(c) $\dfrac{\lambda_0}{\sqrt{1 + \frac{e^2E_0^2t^2}{m^2v_0^2}}}$ (d) $\dfrac{\lambda_0}{\left(1 + \frac{e^2E_0^2t^2}{m^2v_0^2}\right)}$

95. When photons of energy 4.25 eV strike the surface of a metal A, the ejected photoelectrons have maximum kinetic energy T_A expressed in eV and de-Broglie wavelength λ_A. The maximum kinetic energy of photoelectron liberated from another metal B by photons of energy 4.70 eV is $T_B = (T_A - 1.50)$ eV. If the de-Broglie wavelength of these photoelectrons is $\lambda_B = 2\lambda_A$, then choose the wrong option.
(a) the work function of A is 2.25 eV
(b) the work function of B is 4.20 eV
(c) $T_A = 2.00$ eV
(d) $T_B = 2.75$ eV

B. Medical entrance special format questions

Assertion and reason

Directions (Q. Nos. 1-7) *These questions consist of two statements each linked as Assertion and Reason. While answering these questions you are required to choose any one of the following five responses :*

(a) If both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are true but Reason is not correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If Assertion is false but Reason is true.
(e) If both Assertion and Reason are false.

1. **Assertion** Electromagnetic theory of light failed to explain photoelectric effect.

   Reason Photoelectric effect can only be explained by assuming particle nature of light.

2. **Assertion** Photoelectric emission is an instantaneous process.

   Reason There is very small time tag ($\sim 10^{-9}$ s) between incidence of light and emission of photoelectron.

3. **Assertion** If distance of the point source is increased from the photoelectric plate, then stopping potential will remain unchanged.

   Reason Saturation current will decrease.

4. **Assertion** If red light is just able to emit photoelectrons from a metal surface, then violet light cannot emit the photoelectrons from that metal.

   Reason Energy of violet light photons is more than the red light photons.

5. **Assertion** When wavelength of incident radiation in a photoelectric experiment is decreased, then the stopping potential will decrease.

   Reason With decrease in wavelength energy of incident light increases.

6. **Assertion** In photoelectric effect, if the intensity of light is doubled, then maximum kinetic energy of photoelectrons will become double.

   Reason The maximum kinetic energy of emitted photoelectrons depends on nature of material of the plate.

7. **Assertion** If wavelength of incident photons is halved, then maximum kinetic energy of photoelectrons will become two times.

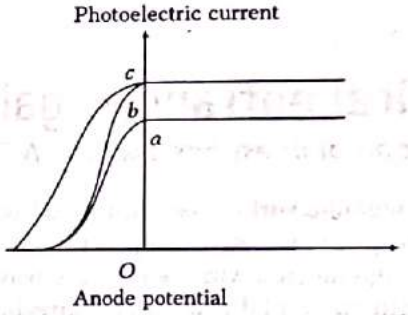
   Reason Energy of photon $\propto 1/\lambda$.

8. **Assertion** A particle at rest breaks into two particles of different masses. They fly off in different directions. Their de-Broglie wavelengths will be different.

   Reason Their speed will be different.

Match the columns

1. The adjoining figure shows the variation of photocurrent with anode potential for a photosensitive surface for three different radiations. Let I_a, I_b and I_c be the intensities and f_a, f_b and f_c be the frequencies for the curves a, b and c respectively.



Now match the Column I with Column II.

Column I	Column II
A. $\frac{f_a}{f_b}$	p. 1
B. $\frac{I_a}{I_b}$	q. < 1
C. $\frac{I_b}{I_c}$	r. > 1
D. $\frac{V_a}{V_c}$	

2. The light rays having wavelength 690 nm are falling on a metal surface of work function 1.2 eV. Now match the Column I with Column II.

Column I		Column II
A. Stopping potential to be applied to stop the emitting electrons.	p.	1.8 eV
B. Energy of incident photons	q.	1 V
C. Kinetic energy of emitted electrons	r.	0.6 eV

3. Light of wavelength 248 nm is incident on a metal surface of threshold frequency 1.0×10^{15} Hz. Match the Column I with Column II.

Column I		Column II
A. Energy of incident photons	p.	4.1 eV
B. Work function of the metal	q.	0.9 eV
C. Maximum kinetic energy of emitted electrons.	r.	6.6 eV
	s.	5 eV

4. Light rays of intensity 3.3×10^{-3} W/m^2 and wavelength 6000 Å falls normally on a photosensitive surface of area 2 cm^2 and work function 2.0 eV. Assuming that there is no light loss due to reflection. Match the results given in Column II for the physical quantities given in Column I.

Column I	Column II
A. Energy (in J) of incident photon	(p) 2×10^{12}
B. Work function (in J) of surface	(q) 3.3×10^{-19}
C. Emission of photoelectrons is	(r) 3.2×10^{-19}
D. Number of photoelectrons emitted per second is	
	(s) Possible

C. Medical entrances' gallery

(Collection of questions asked in NEET & various medical entrance exams)

1. When a metallic surface is illuminated with radiation of wavelength λ, the stopping potential is V. If the same surface is illuminated with radiation of wavelength 2λ, the stopping potential is $V/4$. The threshold wavelength for the metallic surface is **[NEET 2016]**

(a) 5λ (b) $\frac{5}{2}\lambda$ (c) 3λ (d) 4λ

2. An electron of mass m and a photon have same energy E. The ratio of de-Broglie wavelengths associated with them is **[NEET 2016]**

(a) $\left(\frac{E}{2m}\right)^{\frac{1}{2}}$ (b) $c(2mE)^{\frac{1}{2}}$

(c) $\frac{1}{c}\left(\frac{2m}{E}\right)^{\frac{1}{2}}$ (d) $\frac{1}{c}\left(\frac{E}{2m}\right)^{\frac{1}{2}}$

(c being velocity of light)

3. Photons with energy 5 eV are incident on a cathode C in a photoelectric cell. The maximum energy of emitted photoelectrons is 2 eV. When photons of energy 6 eV are incident on C, no photoelectrons will reach the anode A, if the stopping potential of A relative to C is **[NEET 2016 (II)]**

(a) + 3 V (b) + 4 V (c) − 1 V (d) − 3 V

4. Electrons of mass m with de-Broglie wavelength λ fall on the target in an X-ray tube. The cut-off wavelength (λ_0) of the emitted X-ray is **[NEET 2016 (II)]**

(a) $\lambda_0 = \frac{2mc\lambda^2}{h}$ (b) $\lambda_0 = \frac{2h}{mc}$

(c) $\lambda_0 = \frac{2m^2c^2\lambda^3}{h^2}$ (d) $\lambda_0 = \lambda$

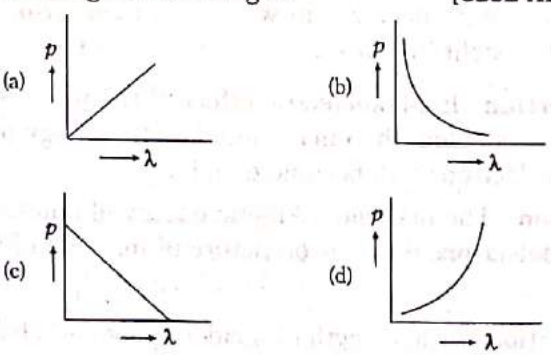
5. A radiation of energy 'E' falls normally on a perfectly reflecting surface. The momentum transferred to the surface is (c = velocity of light) **[CBSE AIPMT 2015]**

(a) $\frac{E}{c}$ (b) $\frac{2E}{c}$ (c) $\frac{2E}{c^2}$ (d) $\frac{E}{c^2}$

6. A certain metallic surface is illuminated with monochromatic light of wavelength λ. The stopping potential for photoelectric current for this light is $3V_0$. If the same surface is illuminated with light of wavelength 2λ, the stopping potential is V_0. The threshold wavelength for this surface for photoelectric effect is **[CBSE AIPMT 2015]**

(a) 6λ (b) 4λ (c) $\frac{\lambda}{4}$ (d) $\frac{\lambda}{6}$

7. Which of the following figures represent the variation of particle momentum and the associated de-Broglie wavelength? **[CBSE AIPMT 2015]**



8. When a light beam of frequency ν is incident on a metal surface, photoelectrons are emitted. If these electrons describe a circle of radius r in a magnetic field of strength B, then the work function of the metal is **[EAMCET 2015]**

(a) $h\nu + \frac{Ber}{2m_e}$ (b) $h\nu - \frac{(Ber)^2}{2m_e}$

(c) $h\nu + \frac{(Ber)^2}{2m_e}$ (d) $h\nu - \frac{Ber}{2m_e}$

9. The work function of metals is in the range of 2 eV to 5 eV. Find which of the following wavelength of light cannot be used for photoelectric effect? (Consider, Planck constant = 4×10^{-15} eVs, velocity of light = $3 \times 10^8 ms^{-1}$) **[WB JEE 2015]**

(a) 510 nm (b) 650 nm (c) 400 nm (d) 570 nm

10. Consider two particles of different masses. In which of the following situations the heavier of the two particles will have smaller de-Broglie wavelength? **[WB JEE 2015]**

(a) Both have a free fall through the same height
(b) Both move with the same kinetic energy
(c) Both move with the same linear momentum
(d) Both move with the same speed

11. The de-Broglie wavelength associated with an electron accelerated by a potential of 64 V is **[Manipal 2015]**

(a) 0.153 nm (b) 0.613 nm (c) 1.22 nm (d) 0.302 nm

12. The electrons are emitted in the photoelectric effect from a metal surface **[Manipal 2015]**

(a) only if the temperature of the surface is high
(b) with a maximum velocity proportional to the frequency of the incident radiation
(c) only if the frequency of the incident radiation is above a certain threshold value
(d) at a rate that is independent of the nature of the metal

13. In a laser tube, all the photons **[Manipal 2015]**

(a) have same energy (b) have same wavelength
(c) move in same direction (d) move with same speed

14. If the kinetic energy of the particle is increased to 16 times its previous value, the percentage change in the de-Broglie wavelength of the particle is **[CBSE AIPMT 2014]**

(a) 25 (b) 75 (c) 60 (d) 50

15. What is the de-Broglie wavelength of the electron accelerated through a potential difference of 100 V? **[KCET 2014]**

(a) 12.27 Å (b) 1.227 Å
(c) 0.1227 Å (d) 0.001227 Å

16. When the energy of the incident radiation is increased by 20%, the kinetic energy of the photoelectrons emitted from a metal surface increased from 0.5 eV to 0.8 eV. The work function of the metal is **[UP CPMT 2014]**

(a) 0.65 eV (b) 1 eV (c) 1.3 eV (d) 1.5 eV

17. Light of wavelength λ_A and λ_B falls on two identical metal plates A and B respectively. The maximum kinetic energy of photoelectrons is K_A and K_B respectively, then which one of the following relations is true? ($\lambda_A = 2\lambda_B$) **[MHT CET 2014]**

(a) $K_A < \frac{K_B}{2}$ (b) $2K_A = K_B$ (c) $K_A = 2K_B$ (d) $K_A > 2K_B$

18. The maximum kinetic energy of the photoelectrons depends only on **[KCET 2014]**

(a) charge (b) frequency
(c) incident angle (d) pressure

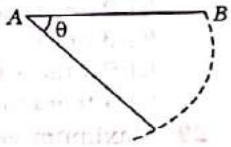
19. A light of wavelength 5000 Å falls on a sensitive plate with photoelectric work function 1.90 eV. Kinetic energy of the emitted photoelectrons will be (given, $h = 6.62 \times 10^{-34}$ Js) **[UP CPMT 2104]**

(a) 0.1 eV (b) 2 eV (c) 0.58 eV (d) 1.581 eV

20. When monochromatic light falls on a photosensitive material, the number of photoelectron emitted per second is n and their maximum kinetic energy is K_{max}. If the intensity of incident light is doubled, then **[EAMCET 2014]**

(a) n is doubled but K_{max} remains same
(b) K_{max} is double but n remains same
(c) Both n and K_{max} are doubled
(d) Both n and K_{max} are halved

21. Find the correct statement(s) about photoelectric effect. **[WB JEE 2014]**



(a) There is no significant time delay between the absorption of a suitable radiation and the emission of electrons.
(b) Einstein analysis gives a threshold frequency above which no electron can be emitted.
(c) The maximum kinetic energy of the emitted photoelectrons is proportional to the frequency of incident radiation.
(d) The maximum kinetic energy of electrons does not depend on the intensity of radiation.

22. If the wavelength of incident light falling on a photosensitive material decreases, then **[Kerala CEE 2014]**

(a) photoelectric current increases
(b) stopping potential decreases
(c) stopping potential remains constant
(d) stopping potential increases
(e) None of the above

23. The de-Broglie wavelength of an electron is the same as that of a 50 keV X-ray photon. The ratio of the energy of the photon to the kinetic energy of the electron is (the energy equivalent of electron mass of 0.5 MeV) **[WB JEE 2014]**

(a) 1 : 50 (b) 1 : 20 (c) 20 : 1 (d) 50 : 1

24. For photoelectric emission from certain metal, the cut-off frequency is ν. If radiation of frequency 2ν

impinges on the metal plate, the maximum possible velocity of the emitted electron will be (m is the electron mass) **[NEET 2013]**

(a) $\sqrt{\frac{h\nu}{(2m)}}$ (b) $\sqrt{\frac{h\nu}{m}}$ (c) $\sqrt{\frac{2h\nu}{m}}$ (d) $2\sqrt{\frac{h\nu}{m}}$

25. The wavelength λ_e of an electron and λ_p of a photon of same energy E are related by **[NEET 2013]**

(a) $\lambda_p \propto \lambda_e^2$ (b) $\lambda_p \propto \lambda_e$ (c) $\lambda_p \propto \sqrt{\lambda_e}$ (d) $\lambda_p \propto \frac{1}{\sqrt{\lambda_e}}$

26. Einstein's photoelectric equation is **[J & K CET 2013]**

(a) $E_{max} = h\nu - \phi$ (b) $E = mc^2$

(c) $E^2 = p^2c^2 + m_0^2c^4$ (d) $E = \left(\frac{1}{2}\right)mv^2$

27. The Davisson-Germer experiment is the direct evidence of **[J & K CET 2013]**

(a) particle nature of electrons
(b) wave nature of electrons
(c) wave nature of light
(d) particle nature of light

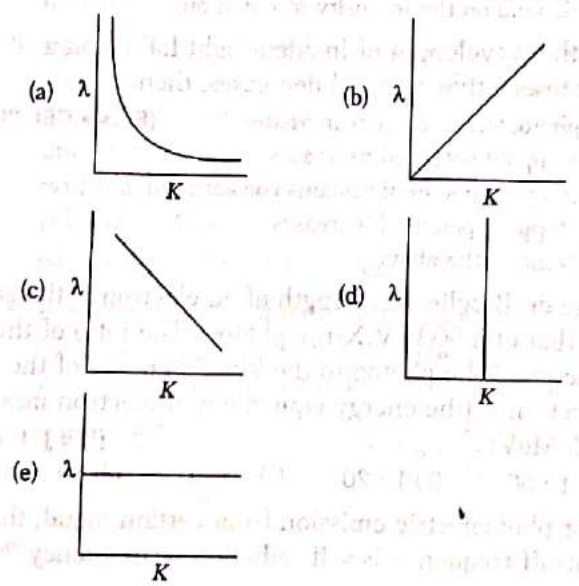
28. The additional energy that should be given to an electron to reduce its de-Broglie wavelength from 1 nm to 0.5 nm is **[KCET 2013]**

(a) 2 times the initial kinetic energy
(b) 3 times the initial kinetic energy
(c) 0.5 times the initial kinetic energy
(d) 4 times the initial kinetic energy

29. Maximum velocity of the photoelectron emitted by a metal is 1.8×10^6 ms^{-1}. Take the value of specific charge of the electron is 1.8×10^{11} C kg^{-1}. Then, the stopping potential in volt is **[KCET 2013]**

(a) 1 (b) 3 (c) 9 (d) 6

30. Identify the graph depicting the variation of the de-Broglie wavelength λ of an electron with its kinetic energy K **[Kerala CEE 2013]**



31. Photocells convert **[Kerala CEE 2013]**

(a) heat energy into electrical energy
(b) light energy into mechanical energy
(c) thermal energy into mechanical energy
(d) light energy into electrical energy
(e) electrical energy into light energy

32. Maximum kinetic energy of electrons emitted in photoelectric effect increases when **[MP PMT 2013]**

(a) intensity of light is increased
(b) light source is brought nearer the metal
(c) frequency of light is decreased
(d) wavelength of light is decreased

33. In photoelectric effect, the photoelectric current **[UP CPMT 2013]**

(a) depends both on intensity and frequency of incident beam
(b) does not depend on frequency but depends only on intensity of incident beam
(c) increases when frequency of incident beam increases
(d) decreases when frequency of incident beam increases

34. A 200 W sodium street lamp emits yellow light of wavelength 0.6 μm. Assuming it to be 25% efficient in converting electrical energy to light, the number of photons of yellow light it emits per second is **[CBSE-AIPMT 2012]**

(a) 1.5×10^{20} (b) 6×10^{18} (c) 62×10^{20} (d) 3×10^{19}

35. An α-particle moves in a circular path of radius 0.83 cm in the presence of a magnetic field of 0.25 Wb m^{-2}. The de-Broglie wavelength associated with the particle will be **[CBSE-AIPMT 2012]**

(a) 1 Å (b) 0.1 Å (c) 10 Å (d) 0.01 Å

36. What should be the velocity of an electron so that its momentum becomes equal to that of a photon of wavelength 5200 Å? **[Manipal 2012]**

(a) 700 ms^{-1} (b) 1000 ms^{-1} (c) 1400 ms^{-1} (d) 2800 ms^{-1}

37. The de-Broglie wavelength of an electron moving with a velocity 1.5×10^8 ms^{-1} is equal to that of a photon. The ratio of the kinetic energy of the electron to the energy of the photon is **[AMU 2012]**

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) 2 (d) 4

38. The threshold frequency for a certain photosensitive metal is ν_0. When it is illuminated by light of frequency $\nu = 2\nu_0$, the maximum velocity of photoelectrons is v_0. What will be the maximum velocity of the photoelectrons when the same metal is illuminated by light of frequency $\nu = 5\nu_0$? **[AMU 2012]**

(a) $\sqrt{2}\,v_0$ (b) $2v_0$ (c) $2\sqrt{2}\,v_0$ (d) $4v_0$

39. The name of ions knocked out from the hot surface is **[AFMC 2012]**

(a) nuclei (b) neutrons (c) electrons (d) protons

Answers

Check point 11.1

1. (c)	2. (a)	3. (d)	4. (b)	5. (b)	6. (a)	7. (c)	8. (c)	9. (b)	10. (b)

Check point 11.2

1. (a)	2. (a)	3. (d)	4. (c)	5. (a)	6. (a)	7. (d)	8. (c)	9. (a)	10. (b)

Check point 11.3

1. (b)	2. (a)	3. (d)	4. (c)	5. (d)	6. (a)	7. (c)	8. (a)	9. (b)	10. (c)
11. (a)	12. (a)	13. (a)	14. (d)	15. (a)					

A. Taking it together

1. (a)	2. (c)	3. (a)	4. (a)	5. (d)	6. (b)	7. (d)	8. (c)	9. (d)	10. (a)
11. (a)	12. (b)	13. (d)	14. (c)	15. (a)	16. (c)	17. (c)	18. (a)	19. (b)	20. (b)
21. (a)	22. (a)	23. (a)	24. (c)	25. (c)	26. (b)	27. (b)	28. (a)	29. (b)	30. (b)
31. (a)	32. (c)	33. (c)	34. (c)	35. (b)	36. (b)	37. (d)	38. (d)	39. (b)	40. (b)
41. (b)	42. (b)	43. (a)	44. (c)	45. (c)	46. (d)	47. (c)	48. (b)	49. (d)	50. (a)
51. (b)	52. (c)	53. (b)	54. (a)	55. (b)	56. (c)	57. (b)	58. (a)	59. (b)	60. (d)
61. (a)	62. (b)	63. (b)	64. (b)	65. (c)	66. (d)	67. (d)	68. (c)	69. (b)	70. (b)
71. (a)	72. (b)	73. (a)	74. (d)	75. (c)	76. (b)	77. (a)	78. (b)	79. (b)	80. (b)
81. (b)	82. (b)	83. (b)	84. (a)	85. (a)	86. (c)	87. (b)	88. (d)	89. (c)	90 (b)
91. (c)	92. (a)	93. (c)	94. (c)	95. (d)					

B. Medical entrance special format questions

Assertion and reason

1. (a)	2. (a)	3. (b)	4. (d)	5. (d)	6. (d)	7. (d)	8. (d)

Match the columns

1. (A → p; B → q; C → p; D → q)
2. (A → r; B → p; C → r)
3. (A → s; B → p; C → q)
4. (A → q; B → r; C → s; D → p, s)

C. Medical entrances' gallery

1. (c)	2. (d)	3. (d)	4. (a)	5. (b)	6. (b)	7. (b)	8. (b)	9. (b)	10. (d)
11. (a)	12. (c)	13. (d)	14. (b)	15. (b)	16. (b)	17. (a)	18. (b)	19. (c)	20. (a)
21. (a)	22. (d)	23. (c)	24. (c)	25. (a)	26. (a)	27. (b)	28. (b)	29. (c)	30. (a)
31. (d)	32. (c)	33. (b)	34. (a)	35. (d)	36. (d)	37. (a)	38. (b)	39 (c)	

Hints and explanations

Check point 11.1

5. (*b*) $\phi_0 = h\nu = \frac{hc}{\lambda_0}$, where $\phi_0 = 2\,\text{eV}$

$\therefore \lambda\,(\text{in Å}) = \frac{12375}{2} \simeq 6200\,\text{Å} = 620\,\text{nm}$

6. (*a*) As, $K_{max} = E - W$

$\Rightarrow \quad K = E - W \quad$ ...(i)

$\Rightarrow \quad K' = 2E - W$

$= 2(E) - (E - K) \quad$ [Using (i)]

$= E + K = K + W + K = W + 2K$

$\Rightarrow \quad K' > 2K \quad [\because W > 0]$

7. (*c*) As, $eV_0 = h\nu - W$

or $\quad V_0 = \frac{h}{e}\nu - \frac{W}{e}$

When frequency is doubled,

$$V_0' = \frac{2h\nu}{e} - \frac{W}{e} = 2\left(\frac{h\nu}{e} - \frac{W}{e}\right) + \frac{W}{e} = 2V_0 + \frac{W}{e}$$

or $\quad V_0' > 2V_0$

9. (*b*) For photoemission to take place, $h\nu > \phi_0$.

Here, $E < \phi_0$, so it is not possible.

Check point 11.2

4. (*c*) Momentum, $p = \frac{E}{c}$

$\Rightarrow \quad E = pc = 2 \times 10^{-16} \times (3 \times 10^8) = 6 \times 10^{-8}$ erg

5. (*a*) Momentum,

$$p = \frac{h}{\lambda} = \frac{6.6 \times 10^{-34}}{(5000 \times 10^{-10})} = 1.3 \times 10^{-27}\ \text{kg-ms}^{-1}$$

6. (*a*) Momentum, $p = \frac{E}{c}$

$\therefore \quad E = pc \Rightarrow E^2 = p^2c^2$

8. (*c*) Power, $P = \frac{nhc}{\lambda}$

$\therefore \quad 100 = \frac{n \times 6 \times 10^{-34} \times 3 \times 10^8}{540 \times 10^{-9}} \Rightarrow n = 3 \times 10^{20}$

9. (*a*) Pressure $= \frac{2I}{c}$ (For perfectly reflecting surface)

where, c is speed of light.

$= \frac{2 \times 10^4}{3 \times 10^8} = \frac{2}{3} \times 10^{-4}\ \text{N/m}^2$

$= 0.667 \times 10^{-4} = 6.67 \times 10^{-5}\ \text{N/m}^2$

10. (*b*) Pressure $= \frac{I_0}{c}$ (For a perfectly absorbing surface)

$\Rightarrow \quad 4.5 \times 10^{-5} = \frac{I_0}{3 \times 10^8}$

$\Rightarrow \quad I_0 = 13.5 \times 10^3 = 1.35 \times 10^4 \simeq 1.4 \times 10^4\ \text{W/m}^2$

Check point 11.3

4. (*c*) As, $\lambda = \frac{h}{mv}$

where, v = velocity of an electron

λ = de-Broglie wavelength

Also $\lambda \propto 1/\nu$ (de-Broglie frequency)

When v is doubled, λ gets halved and ν (de-Broglie frequency) gets doubled.

5. (*d*) As, $\lambda = \frac{h}{p} \Rightarrow p_p = p_\alpha$

6. (*a*) All the three have same wavelength, therefore same momentum say p.

Energy of photon $E = pc$. Therefore, energy of photon is maximum.

7. (*c*) $\lambda = \frac{h}{\sqrt{2qVm}}$ or $\lambda \propto \frac{1}{\sqrt{m}}$

$m_e < m_p$

$\therefore \quad \lambda_e > \lambda_p$

9. (*b*) Wavelength, $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$

$\therefore \quad \lambda \propto \frac{1}{\sqrt{E}} \quad$ (h and m are constants)

So, when the KE of an electron is increased, the wavelength of the associated wave will decrease.

11. (*a*) As, $\lambda \propto \frac{1}{\sqrt{T}} \Rightarrow \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{T_2}{T_1}}$

Here, $T_2 = 927°\text{C} = (927 + 273)\text{K}$

$T_1 = 27°\text{C} = (27 + 273)\text{K}$

$\lambda_1 = \lambda$

$\therefore \quad \frac{\lambda}{\lambda_2} = \sqrt{\frac{1200}{300}} \Rightarrow \lambda_2 = \frac{\lambda}{2}$

12. (*a*) The de-Broglie wavelength is $\lambda = \frac{1227}{\sqrt{V}}$ nm

Here, $V = 100$ volt

$\lambda = \frac{1.227}{\sqrt{100}}\ \text{nm} = 0.123\ \text{nm}$

A Taking it together

3. (a) The de-Broglie wavelength,

$$\lambda = \frac{1.227}{\sqrt{V}} \text{ nm}$$

7. (d) de-Broglie wavelength of the particle,

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}$$

8. (c) By the concept of threshold minimum frequency needed for photoelectric emission.

$$\frac{1}{2}mv^2 = h(\nu - \nu_0)$$

So, for possible emission, $\nu \geq \nu_0$

9. (d) Momentum, $p = \frac{E}{c} = \frac{h\nu}{c}$

$$\therefore \quad \nu = \frac{pc}{h} = \frac{3.3 \times 10^{-29} \times 3 \times 10^8}{6.6 \times 10^{-34}}$$

$$= 1.5 \times 10^{13} \text{ Hz}$$

10. (a) As, $\lambda = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2qVm}}$

So, $\lambda \propto \frac{1}{\sqrt{m}}$

$$\therefore \quad \frac{\lambda_e}{\lambda_p} = \left(\frac{m_p}{m_e}\right)^{1/2}$$

11. (a) As, $\lambda = \frac{h}{p} = \frac{h}{mv}$

So, $\frac{h}{m_1 v_1} = \frac{h}{m_2 v_2}$

$$\therefore \quad \frac{v_1}{v_2} = \frac{m_2}{m_1} = \frac{4}{1} \qquad [\because m_\alpha = 4m_p]$$

12. (b) Energy (in eV) $= \frac{12375}{\lambda(\text{Å})}$

$$\Rightarrow \quad \lambda = \frac{12375}{2.48} = 4989.8 \text{ Å} \approx 5000 \text{ Å}$$

13. (d) As, $\lambda = \frac{h}{\sqrt{2mE}} = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 80 \times 1.6 \times 10^{-19}}}$

$$= 1.4 \text{ Å}$$

14. (c) ΔE (in eV) $= \frac{12375}{\lambda \text{ (in Å)}}$

$$= \frac{12375}{7000} = 1.75 \text{ eV}$$

15. (a) As, $p = \frac{h\nu}{c}$

$$\therefore \quad \nu = \frac{pc}{h} = \frac{(6.6 \times 10^{-29})(3 \times 10^8)}{(6.6 \times 10^{-34})}$$

$$= 3 \times 10^{13} \text{ Hz}$$

16. (c) Momentum, $p = \frac{h}{\lambda}$

or $\quad E = h\nu = \frac{hc}{\lambda} = pc$

So, if the energy of photon is increased by a factor of 4, then its momentum will also increase by a factor of 4.

17. (c) Number of photons emitted,

$$n = \frac{\text{Power}}{\text{Energy of one photon}} = \frac{P\lambda}{hc}$$

$$= \frac{100 \times 540 \times 10^{-9}}{(6 \times 10^{-34} \times 3 \times 10^8)} = 3 \times 10^{20}$$

18. (a) Work function, $W_0 = h\nu_0$

where, ν_0 = threshold frequency

So, $\quad W \propto \nu_0$

Hence, Pt > Al > K

19. (b) de-Broglie wavelength,

$$\lambda = \frac{h}{mv}$$

For electron,

$$\lambda_e = \frac{h}{m_e v}$$

For photon,

$$\lambda_p = \frac{h}{m_p v}$$

$$\therefore \quad \frac{\lambda_e}{\lambda_p} = \frac{\frac{h}{m_e v}}{\frac{h}{m_p v}}$$

$$\frac{\lambda_e}{\lambda_p} = \frac{m_p}{m_e}$$

$$\frac{\lambda_e}{\lambda_p} = \frac{1.67 \times 10^{-27}}{9.1 \times 10^{-31}} = 0.18 \times 10^4 = 1836$$

20. (b) Given, $E = 100 \text{ eV} = 100 \times 1.6 \times 10^{-19}$ J

and $\quad h = 6.662 \times 10^{-34}$ Js

$$E = h\nu$$

or $\quad \nu = \frac{E}{h} = \frac{100 \times 1.6 \times 10^{-19}}{6.662 \times 10^{-34}}$

$$= 0.24 \times 10^{17}$$

$$= 2.417 \times 10^{16} \text{ Hz}$$

21. (a) de-Broglie wavelength, $\lambda = \frac{h}{\sqrt{2mkT}}$

$$= \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 1.67 \times 10^{-27} \times 1.38 \times 10^{-23} T}}$$

$$= \frac{6.62 \times 10^{-34}}{2.15 \times 10^{-25} \sqrt{T}} \text{ m}$$

$$= \frac{3.079}{\sqrt{T}} \times 10^{-9} \text{ m} = \frac{30.79}{\sqrt{T}} \text{ Å} \approx \frac{30.8}{\sqrt{T}} \text{ Å}$$

22. (*a*) Work function of lithium, $\phi_0 = 2.5$ eV

$$\lambda = \frac{hc}{\phi_0}$$

$$= \frac{(4.14 \times 10^{-15}\ \text{eV-s}) \times (3 \times 10^8\ \text{m/s})}{2.5\ \text{eV}}$$

$$= 497\ \text{nm}$$

23. (*a*) Wavelength of photon will be greater than that of electron because mass of photon is less than that of electron

$$\Rightarrow \quad \lambda_{ph} > \lambda_{el} \qquad \left[\because \lambda \propto \frac{1}{\sqrt{m}}\right]$$

24. (*c*) Number of photons, $n = \frac{p}{E} = \frac{p}{h\nu} = \frac{p\lambda}{hc}$

$$= \frac{1000 \times 198.6}{6.6 \times 10^{-34} \times 3 \times 10^8} \approx 10^{30}$$

25. (*c*) According to Einstein's photoelectric equation,

$$KE_{max} = h\nu - \phi_0$$

Comparing with the equation of straight line

$$y = mx + c$$

We get, slope of graph = h (same for all metals)

26. (*b*) Work function, $W_0 = h\nu_0$

For sodium, $\quad W_{01} = h\nu_{01} = \frac{hc}{\lambda_0}$

$$\Rightarrow \quad \frac{2.3\ \text{eV}}{h} = \nu_{01} = \frac{c}{\lambda_{01}}$$

For copper, $\quad W_{02} = h\nu_{02}$

$$\Rightarrow \quad \frac{4.5\ \text{eV}}{h} = \nu_{02} = \frac{c}{\lambda_{02}}$$

$$\Rightarrow \quad \frac{\lambda_{01}}{\lambda_{02}} = \frac{ch}{2.3} \times \frac{4.5}{ch} = \frac{2}{1}$$

$$\Rightarrow \quad \lambda_{01} : \lambda_{02} = 2 : 1$$

28. (*a*) If λ_0 is threshold wavelength, then work function

$$\phi_0 = \frac{hc}{\lambda_0}$$

$$\Rightarrow \quad \lambda_0 = \frac{hc}{\phi_0} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{4.2 \times 1.6 \times 10^{-19}}$$

$$\approx 2955\ \text{Å}$$

29. (*b*) $E = \frac{12375}{\lambda(\text{Å})} = \frac{12375}{3500}\ \text{eV} = 3.53\ \text{eV}$

Since, $E > W_B$ but $E < W_A$, photoelectrons will be emitted by metal B.

30. (*b*) $\quad E = \frac{12375}{\lambda\ (\text{in Å})} = \frac{12375}{4360} = 2.84\ \text{eV}$

$\therefore \quad K_{max} = E - W = 1.6$ eV

or the retarding potential is 1.6 V.

32. (*c*) $E_1 = \frac{12375}{3500}\ \text{eV} = 3.54\ \text{eV}$

$$E_2 = \frac{12375}{5400}\ \text{eV} = 2.29\ \text{eV}$$

Now, $\quad \frac{2}{1} = \frac{3.54 - W}{2.29 - W}$

Solving this equation we get, $W = 1.05$ eV.

33. (*c*) $\frac{K_1}{K_2} = \frac{1}{2}$ (Given)

We know, $\quad \lambda = \frac{h}{\sqrt{2mE}} \Rightarrow \lambda \propto \frac{1}{\sqrt{E}}$

$$\Rightarrow \quad \frac{\lambda_1}{\lambda_2} = \frac{\sqrt{E_2}}{\sqrt{E_1}} \Rightarrow \frac{\lambda_1^2}{\lambda_2^2} = \frac{E_2}{E_1} = \frac{K_2}{K_1}$$

$$\Rightarrow \quad \frac{\lambda_1^2}{\lambda_2^2} = \frac{2}{1} \Rightarrow \lambda_1 : \lambda_2 = \sqrt{2} : 1$$

34. (*c*) $p = n\frac{hc}{\lambda} \Rightarrow n \propto \lambda$

$$\lambda_r > \lambda_b$$

$$\therefore \quad n_r > n_b$$

35. (*b*) Work function, $\phi_0 = \frac{hc}{\lambda_0}$

$$\Rightarrow \quad \phi_0 \propto \frac{1}{\lambda_0} \Rightarrow \frac{(\phi_0)_1}{(\phi_0)_2} = \frac{(\lambda_0)_2}{(\lambda_0)_1} = \frac{600}{300} = 2 : 1$$

36. (*b*) $\quad K_1 = hf - W \qquad (\because W_1 = W,\ f_1 = f)$

$$\Rightarrow \quad K_2 = 2hf - 2W \qquad (\because W_2 = 2W,\ f_2 = 2f)$$

$$\therefore \quad \frac{K_1}{K_2} = \frac{1}{2}$$

37. (*d*) Mass of an electron, $m = 9.11 \times 10^{-31}$ kg

$$T = 27 + 273 = 300\ \text{K}$$

de-Broglie wavelength of electron is $\lambda = \frac{h}{\sqrt{3mKT}}$

$$= \frac{6.6 \times 10^{-34}}{\sqrt{3 \times 9.11 \times 10^{-31} \times 1.38 \times 10^{-23} \times 300}}\ \text{m}$$

$$= 6.2 \times 10^{-9}\ \text{m}$$

38. (*d*) As, $\lambda = \frac{h}{\sqrt{2qVm}} \propto \frac{1}{\sqrt{qVm}}$

$$\therefore \quad \frac{\lambda_\alpha}{\lambda_p} = \frac{\sqrt{q_p V_p m_p}}{\sqrt{q_\alpha V_\alpha m_\alpha}} = \sqrt{\frac{1 \times 100 \times 1}{2 \times 800 \times 4}} = \frac{1}{8}$$

$$\therefore \quad \lambda_\alpha = \frac{\lambda_p}{8} = \frac{\lambda_0}{8}$$

39. (*b*) de-Broglie wavelength, $\lambda = \frac{h}{\sqrt{2mE_e}} = \frac{hc}{E_p}$

or $\quad 2mE_e = \frac{E_p^2}{c^2} \qquad$ ...(i)

But $E_e = \frac{1}{2}mv^2$ or $m = \frac{2E_e}{v^2}$

From Eq. (i), we get

$$2\left[\frac{2E_e}{v^2}\right]E_e = \frac{E_p^2}{c^2} \quad \text{or} \quad \frac{4E_e^2}{v^2} = \frac{E_p^2}{c^2}$$

or $\quad \frac{E_e^2}{E_p^2} = \frac{v^2}{4c^2} \quad$ or $\quad \frac{E_e}{E_p} = \frac{v}{2c}$

40. (*b*) The minimum energy required for the emission of photoelectron from a metal surface is called the work function ϕ_0 of that metal. If ν_0 is threshold frequency, then

$$\phi_0 = h\nu_0$$

But frequency $(\nu_0) = \dfrac{\text{velocity } (c)}{\text{wavelength } (\lambda)}$

$$\therefore \quad \phi_0 = \frac{hc}{\lambda}$$

Given, $h = 6.6 \times 10^{-34}$ Js, $c = 3 \times 10^8$ m/s,

$\lambda = 400$ nm $= 400 \times 10^{-9}$ m

$$\therefore \quad \phi_0 = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{400 \times 10^{-9}} \text{ J}$$

$$\phi_0 = 4.95 \times 10^{-19} \text{ J}$$

$$\phi_0 = \frac{4.95 \times 10^{-19}}{16 \times 10^{-19}} = 3.1 \text{ eV}$$

42. (*b*) As,

$$E = \frac{hc}{\lambda} - \phi_0 \quad \text{...(i)}$$

$$4E = \frac{hc}{\lambda/3} - \phi_0 \quad \text{...(ii)}$$

Solving these equations we get, $\phi_0 = \dfrac{hc}{3\lambda}$

43. (*a*) $eV_s = (h\nu) - \phi$

$$\therefore \quad V_s = \left(\frac{h}{e}\right)\nu - \frac{\phi}{e}$$

Comparing with equation of straight line $y = mx + c$

Slope $(m) = \tan\theta = \dfrac{h}{e}$

44. (*c*) Maximum kinetic energy,

$$K_{max} = \frac{hc}{\lambda} - \phi_0 = \frac{6.4 \times 10^{-34} \times 3 \times 10^8}{6400 \times 10^{-10}} - 1.6 \times 10^{-19}$$

$$= 1.4 \times 10^{-19} \text{ J}$$

45. (*c*) Work function, $\phi_0 = \dfrac{hc}{\lambda_0}$

$$\therefore \quad \frac{(\phi_0)_T}{(\phi_0)_{Na}} = \frac{\lambda_{Na}}{\lambda_T}$$

or

$$\lambda_T = \frac{\lambda_{Na} \times (\phi_0)_{Na}}{(\phi_0)_T} = \frac{5600 \times 2.3}{4.5} = 2862 \text{ Å}$$

46. (*d*) By decreasing the wavelength of incident light energy of incident light will increase.

47. (*c*) As, $\dfrac{1}{2}mv_{max}^2 = eV_0$

$$\therefore \quad v_{max} = \sqrt{\frac{2eV_0}{m}} = \sqrt{2 \times 1.8 \times 10^{11} \times 9}$$

$$= 1.8 \times 10^6 \text{ ms}^{-1}$$

48. (*b*) As,

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$\Rightarrow \quad \lambda \propto \frac{1}{\sqrt{m}}$$

$$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{m_2}{m_1}}$$

So,

$$\frac{\lambda}{\lambda_2} = \sqrt{\frac{M}{m}}$$

$$\therefore \quad \lambda_2 = \lambda\sqrt{\left(\frac{m}{M}\right)}$$

49. (*b*) Intensity of incident light $\propto$ No. of electrons emitted per sec

$$\propto \frac{1}{(\text{distance})^2} = 4 \text{ times}$$

50. (*a*) Frequency of light of wavelength ($\lambda = 4000$ Å) is

$$\nu = \frac{c}{\lambda}$$

$$\therefore \quad \nu = \frac{3 \times 10^8}{4000 \times 10^{-10}} = 0.75 \times 10^{15}$$

This frequency is less than the given threshold frequency. Hence, no photoelectric emission takes place.

51. (*b*) Stopping potential, $V_0 = \dfrac{h}{e}(\nu - \nu_0)$

or

$$(\nu - \nu_0) = \frac{V_0 \times e}{h} = \frac{4.144 \times 16 \times 10^{-19}}{6.63 \times 10^{-34}}$$

$$\nu - 1 \times 10^{15} = 10^{15}$$

$$\text{Frequency} = 10^{15} + 1 \times 10^{15} = 2 \times 10^{15} \text{ Hz}$$

52. (*c*) In Davisson-Germer experiment, the de-Broglie wavelength associated with electron is

$$\lambda = \frac{12.27}{\sqrt{V}} \text{ Å} \quad \text{...(i)}$$

where, V is the applied voltage.

If there is a maxima of the diffracted electrons at an angle θ, then

$$2d \sin\theta = \lambda \quad \text{...(ii)}$$

From Eq. (i), we note that if V is inversely proportional to the wavelength λ.

i.e., V will increase with the decrease in the λ.

From Eq. (ii), we note that wavelength λ is directly proportional to $\sin\theta$ and hence, θ.

So, with the decrease in λ, θ will also decrease.

Thus, when the voltage applied to A is increased. The diffracted beam will have the maxima at a value of θ that will be less than the earlier value.

53. (*b*) Given, $v_{max} = 4 \times 10^8$ cm/s $= 4 \times 10^6$ m/s

$$\therefore \quad K_{max} = \frac{1}{2}mv_{max}^2 = \frac{1}{2} \times 9 \times 10^{-31} \times (4 \times 10^6)^2$$

$$= 7.2 \times 10^{-18} \text{ J} = 45 \text{ eV}$$

Hence, stopping potential $|V_0| = \dfrac{K_{max}}{e} = \dfrac{45 \text{ eV}}{e} = 45$ V

54. (*a*) Given, $\mathbf{v} = v_0\hat{\mathbf{i}}$ and $\mathbf{B} = B_0\hat{\mathbf{j}}$

Force on moving electron due to magnetic field is

$$\mathbf{F} = -e(\mathbf{v} \times \mathbf{B}) = -e[v_0\hat{\mathbf{i}} \times B_0\hat{\mathbf{j}}] \Rightarrow = -ev_0B_0\hat{\mathbf{k}}$$

As this force is perpendicular to **v** and **B**, so the magnitude of **v** will not change, *i.e.*, momentum ($= mv$) will remain constant in magnitude.

Hence, de-Broglie wavelength $\lambda = \frac{h}{mv}$ remains constant.

56. (*c*) Wavelength $\propto \frac{1}{\text{Energy}}$ Hence, wavelength should be smaller than threshold wavelength.

57. (*b*) When the distance is increased, frequency of incident light and hence, the stopping potential does not change. But the intensity and hence, the saturation current decreases nine times. [$\because$ Intensity of incident radiation $\propto \frac{1}{(\text{distance})^2}$]

58. (*a*) According to Einstein's equation,

$$h\nu = \phi_0 + K_{max}$$

where, ϕ_0 is the work function.

$$\Rightarrow \quad h\nu = \phi_0 + eV_0$$

$$\Rightarrow \quad V_0 = \frac{h\nu}{e} - \frac{\phi_0}{e}$$

Comparing with equation, $y = mx + c$

$$\phi_0 = -(\text{intercept}) \times e = OB \times e \text{ in } eV$$

59. (*b*) Stopping potential equals to maximum kinetic energy since, stopping potential is varying linearly with the frequency. Therefore, maximum KE for both the metals also vary nearly with frequency.

60. (*d*) The electrons may be accelerated or retarded.

61. (*a*) Mass of electron in motion is

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{m_0}{\sqrt{1-\frac{(0.5c)^2}{c^2}}} = \frac{m_0}{\sqrt{0.75}}$$

$$\therefore \quad \lambda = \frac{h}{mv} = \frac{6.6\times10^{-34}\times\sqrt{0.75}}{m_0 v}$$

$$= \frac{6.6\times10^{-34}\times\sqrt{0.75}}{9.1\times10^{-31}\times15\times10^{8}} = 4.2\times10^{-12} \text{ m}$$

62. (*b*) Here, $\lambda = 10$ Å, $d = 2.8$ Å, $n = 1$

Bragg's equation is $2d\sin\theta = n\lambda$

$$\therefore \quad 2\times2.8\times\sin\theta = 1\times10$$

$$\sin\theta = 0.1786$$

$$\theta = \sin^{-1}(0.1786) \simeq 10.3$$

63. (*b*) Work function for wavelength of 4100 Å is

$$W = \frac{hc}{\lambda}$$

$$= \frac{6.62\times10^{-34}\times3\times10^{8}}{4100\times10^{-10}} = 4.8\times10^{-19} \text{ J}$$

$$= \frac{4.8\times10^{-19}}{16\times10^{-19}} \text{ eV} = 3 \text{ eV}$$

Now, we have

$W_A = 1.92$ eV, $W_B = 2.0$ eV, $W_C = 5$ eV

Since, $W_A < W$

and $W_B < W$, hence A and B will emit photoelectrons.

64. (*b*) Maximum kinetic energy,

$$K_{max}(\text{in eV}) = \frac{hc}{e}\left[\frac{1}{\lambda} - \frac{1}{\lambda_0}\right]$$

$$= 12375\left[\frac{1}{1000} - \frac{1}{2000}\right] = 6.2 \text{ eV}$$

65. (*c*) By law of conservation of momentum,

$$0 = mv_1 + m_2 v_2$$

$$m_1 v_1 = -m_2 v_2$$

Negative sign indicates that both the particles are moving in opposite directions. Now, de-Broglie wavelengths,

$$\lambda_1 = \frac{h}{m_1 v_1} \text{ and } \lambda_2 = \frac{h}{m_2 v_2}$$

$$\therefore \quad \frac{\lambda_1}{\lambda_2} = \frac{m_2 v_2}{m_1 v_1} = 1$$

66. (*d*) Velocity of a body falling from a height H is given by

$$v = \sqrt{2gH}$$

We know that de-Broglie wavelength

$$\lambda = \frac{h}{mv} = \frac{h}{m\sqrt{2gH}} \Rightarrow = \frac{h}{m\sqrt{2g}\sqrt{H}}$$

Here, $\frac{h}{m\sqrt{2g}}$ is a constant ϕ (say K).

So, $$\lambda = K\frac{1}{\sqrt{H}} \Rightarrow \lambda \propto \frac{1}{\sqrt{H}}$$

$$\Rightarrow \quad \lambda \propto H^{-1/2}$$

67. (*d*) We know that $E = W + \text{KE}$

$$\text{KE} = E - W = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$$

$$= hc\left[\frac{1}{\lambda} - \frac{1}{\lambda_0}\right] = hc\left[\frac{\lambda_0 - \lambda}{\lambda\lambda_0}\right]$$

68. (*c*) de-Broglie wavelength, $\lambda = \frac{h}{p}$

$$\Rightarrow \quad \lambda - \frac{0.5\lambda}{100} = \frac{h}{p + \Delta p}$$

$$\Rightarrow \quad \frac{199\lambda}{200} = \frac{h}{p + \Delta p} = \frac{199h}{200p}$$

$$\Rightarrow \quad p + \Delta p = \frac{200}{199}p$$

$$\Rightarrow \quad p = 199\,\Delta p$$

69. (*b*) Given in the question,

Energy of a photon, $E = 1\text{MeV} = 10^6$ eV

Now, $hc = 1243$ eVnm

Now, $E = \frac{hc}{\lambda}$

$$\Rightarrow \quad \lambda = \frac{hc}{E} = \frac{1243 \text{ eVnm}}{10^6 \text{ eV}}$$

$$= 1.24\times10^{-3} \text{ nm}$$

71. (*a*) $\lambda_1 = \frac{12375}{E_1\,(\text{eV})}\,\text{Å} = 1000\,\text{Å}$

$\therefore \quad E_1 = 12.375\,\text{eV}$

Similarly, $\frac{12375}{\lambda_2(\text{Å})}\,\text{eV} = \frac{12375}{2000} = 6.1875\,\text{eV}$

Now, $E_1 - W_0 = eV_s$

and $E_2 - W_0 = eV_s$

Hence, $12.375 - W_0 = 7.7\,eV_s$

and $6.1875 - W_0 = eV_s$

Solving, we get $V_s = 1.5\,\text{V}$

72. (*b*) According to the question,

$$eV = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) \quad \text{...(i)}$$

and
$$\frac{eV}{3} = hc\left(\frac{1}{2\lambda} - \frac{1}{\lambda_0}\right) \quad \text{...(ii)}$$

Dividing Eq. (i) by Eq. (ii), we get

$$3 = \frac{\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)}{\left(\frac{1}{2\lambda} - \frac{1}{\lambda_0}\right)}$$

or
$$3\left(\frac{1}{2\lambda} - \frac{1}{\lambda_0}\right) = \frac{1}{\lambda} - \frac{1}{\lambda_0}$$

or
$$\frac{3}{2\lambda} - \frac{1}{\lambda} = \frac{3}{\lambda_0} - \frac{1}{\lambda_0}$$

or
$$\frac{1}{2\lambda} = \frac{2}{\lambda_0}$$

Threshold wavelength for metallic surface,

$$\lambda_0 = 4\,\lambda$$

73. (*a*) Energy radiated as visible light $= \frac{5}{100} \times 100 = 5\,\text{J/s}$

Let n be the number of photons emitted per second. Then,

$$nh\nu = E = 5$$

$$\therefore \quad n = \frac{5\lambda}{hc} = \frac{5 \times 5.6 \times 10^{-7}}{(6.62 \times 10^{-34})\,(3 \times 10^8)}$$

$$= 1.4 \times 10^{19}$$

74. (*d*) When a beam of electrons of energy E_0 is incident on a metal surface kept in an evacuated chamber, electrons can be emitted with maximum energy E_0 (due to elastic collision) and with any energy less than E_0, when a part of incident energy of electron is used in liberating the electrons from the surface of metal.

75. (*c*) Intensity, $I = 10^{-10}\,\text{Wbm}^{-2} = 10^{-10}\,\text{Js}^{-1}\,\text{m}^{-2}$. Let the number of photons required be n.

Then, $I = \frac{nh\nu}{A} = \frac{nh\nu}{10^{-4}} = 10^{-10}$

Hence, $n = 10^{-10} \times 10^{-4}/h\nu = 10^{-14}\,\frac{\lambda}{hc}$

$$= \frac{10^{-14} \times 660 \times 10^{-9}}{6.6 \times 10^{-34} \times 3 \times 10^8} = 3.3 \times 10^4$$

76. (*b*) We know that the relation between λ and K is given by

$$\lambda = \frac{h}{\sqrt{2mK}}$$

Here, for the given value of energy K, $\frac{h}{\sqrt{2K}}$ is a constant.

Thus, $\lambda \propto \frac{1}{\sqrt{m}}$

$\therefore \quad \lambda_p : \lambda_n : \lambda_e : \lambda_\alpha$

$$\Rightarrow \quad = \frac{1}{\sqrt{m_p}} : \frac{1}{\sqrt{m_n}} : \frac{1}{\sqrt{m_e}} : \frac{1}{\sqrt{m_\alpha}}$$

Since, $m_p = m_n$, hence $\lambda_p = \lambda_n$

As, $m_\alpha > m_p$, therefore $\lambda_\alpha < \lambda_p$

As, $m_e < m_n$, therefore $\lambda_e > \lambda_n$

Hence, $\lambda_\alpha < \lambda_p = \lambda_n < \lambda_e$

77. (*a*) As $\frac{mv^2}{r} = qvB \Rightarrow \frac{mv}{qB} = r$

Also, $mv = P \Rightarrow P = \sqrt{2mk}$

$$\Rightarrow \quad r = \frac{\sqrt{2Km}}{Bq} = \frac{\sqrt{2(E - W_0)\,m}}{eB}$$

78. (*b*) The maximum velocity of photoelectron will remain same, *i.e.*, 6×10^8 m/s, because it does not depend on work function.

79. (*b*) As, $E - W_0 = \frac{1}{2}mv^2$

First case, $2W_0 - W_0 = \frac{1}{2}mv_1^2$ ...(i)

Second case, $5W_0 - W_0 = \frac{1}{2}mv_2^2$ ...(ii)

Therefore, from Eqs. (i) and (ii),

$$\frac{2W_0 - W_0}{5W_0 - W_0} = \frac{v_1^2}{v_2^2} \quad \text{or} \quad \frac{v_1}{v_2} = \frac{1}{2}$$

80. (*b*) Energy corresponding to 2000 Å

$$E = \frac{12375}{\lambda} = \frac{12375}{2000}\,\text{eV} = 6.2\,\text{eV}$$

Maximum kinetic energy,

$$K = h\nu - h\nu_0$$
$$K = E - W$$
$$= (6.2 - 5.01)\,\text{eV}$$
$$= 1.19\,\text{eV}$$

Now, $k_{max} = \frac{1}{2}mv_{max}^2$

$$\Rightarrow \frac{1}{2} \times 9.1 \times 10^{-31} \times v_{max}^2 = 1.19 \times 1.6 \times 10^{-19}$$

or
$$v_{max}^2 = \frac{1.19 \times 1.6 \times 10^{-19} \times 2}{9.1 \times 10^{-31}}$$

or
$$v_{max}^2 = 0.418 \times 10^{12} = 41.8 \times 10^{10}$$

or
$$v_{max} = 6.46 \times 10^5\,\text{m/s}$$

81. (*b*) From equation,

$$eV_0 = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$$

$$4.8 = \frac{hc}{e}\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) \quad \text{...(i)}$$

and $$1.6 = \frac{hc}{e}\left(\frac{1}{2\lambda} - \frac{1}{\lambda_0}\right) \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)}{\left(\frac{1}{2\lambda} - \frac{1}{\lambda_0}\right)} = \frac{4.8}{1.6} \Rightarrow \lambda_0 = 4\lambda$$

82. (*b*) Given, $\lambda_{ph} = \lambda_e$

$\therefore$ $$\frac{h}{p_{ph}} = \frac{h}{p_e}$$

$\therefore$ $$p_{ph} = p_e \quad \therefore \quad \frac{E_{ph}}{c} = \frac{2E_e}{v}$$

$\therefore$ $$\frac{E_e}{E_{ph}} = \frac{v}{2c}$$

83. (*b*) As, $\lambda = \frac{h}{\sqrt{2mK}}$

or $$\lambda^2 = \frac{h^2}{2mK}$$

$\therefore$ $$K = \frac{h^2}{2m\lambda^2} = \frac{(6.6\times10^{-34})^2}{2\times9.1\times10^{-31}\times(10\times10^{-12})^2}\text{ J}$$

$$= 15.1\text{ keV}$$

84. (*a*) The change in wavelength is given by

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0c}(1-\cos\phi)$$

where, λ' = wavelength of the X-rays scattered

and $\lambda_c = \frac{h}{m_0c}$ = compton wavelength = 2.426×10^{-12} m

$\Rightarrow$ $$\lambda' = \lambda + \lambda_c(1-\cos 30°)$$

$$= 10\text{ pm} + 0.134\,\lambda_c = 10.32\text{ pm}$$

85. (*a*) Consider a spherical chamber of radius 20 cm. Bulb is placed at point *s* as shown in figure.

Intensity of light at surface,

$$I = \frac{0.6P}{4\pi r^2}$$

Force, exerted on the surface, $F = \frac{P}{c}$

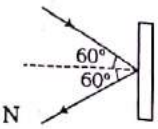
where P is power of bulb.

$\therefore$ $$F = \frac{1100}{3\times10^8} = 3.6\times10^{-6}\text{ N}$$

Pressure exerted by light, $P = \frac{I}{c} = \frac{0.6P}{4\pi r^2 c} = \frac{0.6\times1100}{4\pi(0.2)^2\times3\times10^8}$

$$= 4.4\times10^{-6}\text{ N/m}^2$$

86. (*c*) Force totally reflecting surface is,



$$F = \frac{2P\cos 60°}{c}$$

$$= \frac{2\times\frac{nhc}{\lambda}\cos 60°}{c} = \frac{10^{19}\times6.6\times10^{-34}}{663\times10^{-9}} = 10^{-8}\text{ N}$$

87. (*b*) As, $I = \frac{P}{A} = \frac{5\times10^{-3}}{1\times10^{-6}} = 5\times10^3\text{ W/m}^2$

Force on the surface, $F = \frac{2P}{c} = \frac{2\times5\times10^{-3}}{3\times10^8} = 3.3\times10^{-11}\text{ N}$

Pressure on the surface, $= \frac{2I}{c} = \frac{2\times5\times10^{-3}}{3\times10^8} = 3.3\times10^{-5}\text{ N/m}^2$

88. (*d*) As, $\frac{1}{2}mv_{max}^2 = \frac{hc}{\lambda} - \phi_0$

$$v_{max} = \sqrt{\frac{2}{m}\left(\frac{hc}{\lambda} - \phi_0\right)} = v$$

$$v'_{max} = \sqrt{\frac{2}{m}\left(\frac{hc}{\lambda/4} - \phi_0\right)}$$

$$= 2\sqrt{\frac{2}{m}\left(\frac{hc}{\lambda} - \phi_0\right) + \frac{6}{m}\phi_0}$$

$$> 2v$$

89. (*c*) Energy of photoelectron

$$E = \frac{hc}{\lambda} - \phi_0 \Rightarrow \frac{hc}{\lambda} = E + \phi_0 \quad \text{...(i)}$$

where ϕ_0 is the work function for the metal surface (constant). When $E' = 2E$, then

$$2E = \frac{hc}{\lambda'} - \phi_0$$

$\Rightarrow$ $$\frac{hc}{\lambda'} = 2E + \phi_0 \quad \text{...(ii)}$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{\lambda'}{\lambda} = \frac{E + \phi_0}{2E + \phi_0}$$

$$\frac{\lambda'}{\lambda} = \frac{(E + \phi_0)}{2\left(E + \frac{\phi_0}{2}\right)}$$

$\therefore$ $$\frac{\lambda'}{\lambda} > \frac{1}{2} \text{ or } \lambda' > \frac{\lambda}{2}$$

$\therefore$ $$\lambda > \lambda' > \frac{\lambda}{2}$$

90. (*b*) According to Einstein's photoelectric equation,

$$\frac{hc}{\lambda_1} = W_0 + \frac{1}{2}mv_1^2$$

and $$\frac{hc}{\lambda_2} = W_0 + \frac{1}{2}mv_2^2$$

These expressions show that

$$v^2 \propto \left(\frac{1}{\lambda}\right)$$

$$\therefore \quad \frac{v_1}{v_2} = \sqrt{\frac{\left(\frac{1}{\lambda_1}\right)}{\left(\frac{1}{\lambda_2}\right)}} = \sqrt{\frac{\lambda_2}{\lambda_1}} = \sqrt{\frac{3\lambda/4}{\lambda}}$$

$$\therefore \quad v_2 = v\left(\frac{4}{3}\right)^{1/2}$$

91. (*c*) Let W be the work function of metal. Then,

$$eV_0 = \frac{hc}{330 \times 10^{-9}} - W \quad \text{...(i)}$$

$$e(2V_0) = \frac{hc}{220 \times 10^{-9}} - W \quad \text{...(ii)}$$

Solving these two equations, we get

$$V_0 = \frac{10^9 \times h \times c}{110 \times e \times 6}$$

$$= \frac{10^9 \times 6.6 \times 10^{-34} \times 3 \times 10^8}{110 \times 1.6 \times 10^{-19} \times 6} = \frac{15}{8}\text{ V}$$

92. (*a*) Initial de-Broglie wavelength of electron,

$$\lambda_0 = \frac{h}{mv_0} \quad \text{...(i)}$$

Force on electron in electric field,

$$\mathbf{F} = -e\mathbf{E} = -e[-E_0\hat{\mathbf{i}}] = eE_0\hat{\mathbf{i}}$$

Acceleration of electron, $a = \frac{\mathbf{F}}{m} = \frac{eE_0\hat{\mathbf{i}}}{m}$

Velocity of electron after time t,

$$v = v_0\hat{\mathbf{i}} + \left(\frac{eE_0\hat{\mathbf{i}}}{m}\right)t = \left(v_0 + \frac{eE_0}{m}t\right)\hat{\mathbf{i}} = v_0\left(1 + \frac{eE_0}{mv_0}t\right)\hat{\mathbf{i}}$$

de-Broglie wavelength associated with electron at time t is

$$\lambda = \frac{h}{mv}$$

$$\Rightarrow \quad = \frac{h}{m\left[v_0\left(1 + \frac{eE_0}{mv_0}t\right)\right]} = \frac{\lambda_0}{\left[1 + \frac{eE_0}{mv_0}t\right]} \quad \left[\because \lambda_0 = \frac{h}{mv_0}\right]$$

93. (*c*) As, $eV = \frac{hc}{\lambda} - W$

$$\therefore \quad V = \left(\frac{h}{e}\right)\cdot\frac{c}{\lambda} - \frac{W}{e}$$

$$V_1 = \left(\frac{h}{e}\right)\frac{c}{\lambda_1} - \frac{W}{e} \quad \text{...(i)}$$

$$V_2 = \left(\frac{h}{e}\right)\frac{c}{\lambda_2} - \frac{W}{e} \quad \text{...(ii)}$$

Solving these two equations, we get

$$\frac{h}{e} = \frac{\lambda_1\lambda_2(V_1 - V_2)}{c(\lambda_2 - \lambda_1)}$$

$$= \frac{(0.6 \times 0.4 \times 10^{-12})(1.0)}{(3 \times 10^8)(0.2 \times 10^{-6})} = 4 \times 10^{-15}\text{ Vs}$$

94. (*c*) Initial de-Broglie wavelength of electron,

$$\lambda_0 = \frac{h}{mv_0}$$

Force on electron in electric field,

$$\mathbf{F} = -e\mathbf{E} = -eE_0\hat{\mathbf{j}}$$

Acceleration of electron, $\mathbf{a} = \frac{\mathbf{F}}{m} = \frac{eE_0\hat{\mathbf{j}}}{m}$

It is acting along negative Y-axis.

The initial velocity of electron along X-axis, $\mathbf{v}_{x_0} = v_0\hat{\mathbf{i}}$. Initial velocity of electron along Y-axis, $\mathbf{v}_{y_0} = 0$.

Velocity of electron after time t along X-axis, $\mathbf{v}_x = v_0\hat{\mathbf{i}}$

($\because$ there is no electron along X-axis.)

Velocity of electron after time t along Y-axis,

$$\mathbf{v}_y = 0 + \left(-\frac{eE_0}{m}\hat{\mathbf{j}}\right)t = -\frac{eE_0}{m}t\hat{\mathbf{j}}$$

Magnitude of velocity of electron after time t is

$$\mathbf{v} = \sqrt{v_x^2 + v_y^2} = \sqrt{v_0^2 + \left(\frac{-eE_0}{m}t\right)^2}$$

$$\Rightarrow \quad = v_0\sqrt{1 + \frac{e^2E_0^2t^2}{m^2v_0^2}}$$

de-Broglie wavelength, $\lambda' = \frac{h}{mv}$

$$\Rightarrow \quad = \frac{h}{mv_0\sqrt{1 + e^2E_0^2t^2/(m^2v_0^2)}} = \frac{\lambda_0}{\sqrt{1 + e^2E_0^2t^2/m^2v_0^2}}$$

95. (*d*) We know, $K_{max} = E - W$

$$\therefore \quad T_A = 4.25 - W_A \quad \text{...(i)}$$

$$T_B = (T_A - 1.50) = 4.70 - W_B \quad \text{...(ii)}$$

From these two equations, we have

$$W_B - W_A = 1.95\text{ eV} \quad \text{...(iii)}$$

de-Broglie wavelength is given by

$$\lambda = \frac{h}{\sqrt{2Km}} \text{ or } \lambda \propto \frac{1}{\sqrt{K}}$$

$$\therefore \quad \frac{\lambda_B}{\lambda_A} = \sqrt{\frac{K_A}{K_B}} \Rightarrow 2 = \sqrt{\frac{T_A}{T_A - 1.5}}$$

$$\Rightarrow \quad T_A = 2\text{ eV}, W_A = 2.25\text{ eV}, W_B = 4.20\text{ eV}$$

and $\quad T_B = 0.5\text{ eV}$

B) Medical entrances special format questions

Assertion and reason

1. (*a*) The electromagnetic theory of light failed to explain photoelectric effect. It cannot be explained by assuming wave nature of radiation. If wave nature of radiation is assumed, then it will take quite a long time (about one year) for electron to come out of the metal surface.

2. (*a*) There is a very small time lag between the instants, the photon of light falls on the metal surface and instantly a photoelectron is emitted from the metal surface.

3. (*b*) Stopping potential is dependent on frequency, but, independent of the intensity. So, increasing the distance, affects the intensity as.

$$\text{Intensity} \propto \frac{1}{(\text{distance})^2}$$

So, the stopping potential will not change. But name of saturation current depends on the intensity of incident radiation, so, more the distance, the intensity will decrease. Hence, the saturation current will also decrease.

5. (*d*) Decreasing the λ of incident photon means energy of incident light is increased, so E_k increases and hence, stopping potential increases.

6. (*d*) The maximum kinetic energy of emitted photoelectrons depends on the radiation source and nature of material of plate but is independent of the intensity.

So, it will remain unchanged.

7. (*d*) $K = \frac{hc}{\lambda} - W$

$$K' = \frac{hc}{\lambda/2} - W = 2\left(\frac{hc}{\lambda} - W\right) + W = 2K + W$$

i.e., $K' > 2K$

8. (*d*) $\lambda = \frac{h}{p}$

Their momenta are same. Hence their wavelengths are same.

Match the columns

1. (A → p, B → q, C → p, D → q)

In the figure, stopping potential $V_a = V_b$ but V_a or $V_b < V_c$. Hence, we conclude that $f_a = f_b$ but f_a or $f_b < f_c$ (since stopping potential increases with increase in frequency or incident radiation). Moreover $I_a < I_b$ but $I_b = I_c$.

2. (A → r, B → p, C → r)

From Einstein's photoelectric equation, we have

$$E_k = h\nu - W$$

where, $h\nu$ is energy of photon absorbed by the electron in the metal, W is the work function of the metal.

Also, the potential at which the photoelectric current becomes zero, is called stopping potential or cut-off potential. Since, the stopping potential stops the maximum energy electrons, it is a measure of the maximum kinetic energy of the electrons.

Given, $h\nu = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{690 \times 10^{-9}} = 1.8 \text{ eV}$

$$W = 1.2 \text{ eV}$$

$$\therefore \quad E_k = 1.8 - 1.2 = 0.6 \text{ eV}$$

Thus, stopping potential, $V_0 = \frac{E_k}{e} = \frac{0.6 \times 1.6 \times 10^{-19}}{1.6 \times 10^{19}} = 0.6 \text{ V}$

3. (A → s, B → p, C → q)

Given, $\lambda = 248 \text{ nm} = 248 \times 10^{-9} \text{ m}$

$$\therefore \quad E = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{248 \times 10^{-9}}$$

$$= 5 \text{ eV}$$

$$\nu_0 = 1.0 \times 10^{15} \text{ Hz}$$

$$\therefore \quad W = h\nu_0 = \frac{6.626 \times 10^{-34} \times 10 \times 10^{15}}{1.6 \times 10^{-19}} \text{ eV}$$

$$= 4.141 \text{ eV} \approx 4.1 \text{ eV}$$

From Einstein's photoelectric equation,

$$\text{KE}_{max} = h\nu - h\nu_0$$

$$\Rightarrow \quad \text{KE}_{max} = 5 - \frac{6.6 \times 10^{-34} \times 1.6 \times 10^{15}}{1.0 \times 10^{-19}}$$

$$= 5 - 4.1 = 0.9 \text{ eV}$$

4. (A → q; B → r; C → s; D → p)

Energy of incident photon $= \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{6000 \times 10^{-10}}$

$$= 3.3 \times 10^{-19} \text{ J}$$

Work function of surface $= 2 \text{ eV} = 2 \times 1.6 \times 10^{-19}$

$$= 3.2 \times 10^{-19} \text{ J}$$

Energy of incident photon is more than the work function of the surface. Therefore, emission of photoelectrons is possible. Number of photoelectrons emitted = Number of incident photons per second on 2 cm^2 (= 2×10^{-4} m^2) area

$$n = \frac{\text{Intensity of incident radiations} \times \text{surface area}}{\text{Energy of one photon}}$$

$$n = \frac{2 \times 3.3 \times 10^{-3} \times 10^{-4}}{3.3 \times 10^{-19}} = 2 \times 10^{12} \text{ per second}$$

C) Medical entrances' gallery

1. (*c*) In Ist case, when a metallic surface is illuminated with radiation of wavelength λ, the stopping potential is V. So, photoelectric equation can be written as

$$eV = \frac{hc}{\lambda} - \frac{hc}{\lambda_0} \quad \text{...(i)}$$

In IInd case, when the same surface is illuminated with radiation of wavelength 2λ, the stopping potential is $\frac{V}{4}$. So, photoelectric equation can be written as

$$\frac{eV}{4} = \frac{hc}{2\lambda} - \frac{hc}{\lambda_0}$$

$$\Rightarrow \quad eV = \frac{4hc}{2\lambda} - \frac{4hc}{\lambda_0} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\Rightarrow \quad \frac{hc}{\lambda} - \frac{hc}{\lambda_0} = \frac{4hc}{2\lambda} - \frac{4hc}{\lambda_0}$$

$$\Rightarrow \quad \frac{1}{\lambda} - \frac{1}{\lambda_0} = \frac{2}{\lambda} - \frac{4}{\lambda_0} \Rightarrow \lambda_0 = 3\lambda$$

2. (*d*) Since, it is given that electron has mass m.
de-Broglie wavelength for an electron will be given as

$$\lambda_e = \frac{h}{p} \quad ...(i)$$

where, h = Planck's constant
p = linear momentum of electron

As kinetic energy of electron, $E = \frac{p^2}{2m}$

$$\Rightarrow \quad p = \sqrt{2mE} \quad ...(ii)$$

From Eqs. (i) and (ii), we get

$$\lambda_e = \frac{h}{\sqrt{2mE}} \quad ...(iii)$$

Energy of a photon can be given as

$$E = h\nu$$

$$\Rightarrow \quad E = \frac{hc}{\lambda_p} \Rightarrow \lambda_p = \frac{hc}{E} \quad ...(iv)$$

Hence, λ_p = de-Broglie wavelength of photon.
Now, divide Eq. (iii) by Eq. (iv), we get

$$\frac{\lambda_e}{\lambda_p} = \frac{h}{\sqrt{2mE}} \cdot \frac{E}{hc}$$

$$\Rightarrow \quad \frac{\lambda_e}{\lambda_p} = \frac{1}{c} \cdot \sqrt{\frac{E}{2m}}$$

3. (*d*) Here, we will use Einstein's photoelectric equation.
We know that,

$$E = (\text{KE})_{max} + \text{Work function } (\phi)$$

where, $\phi = h\nu_0$, $E = h\nu$

$$(\text{KE})_{max} = \frac{1}{2}mv_0^2$$

$$\Rightarrow \quad (\text{KE})_{max} = h\nu - \phi$$

$$\Rightarrow \quad 2\text{eV} = 5 \text{ eV} - \phi \quad \text{(given)}$$

$$\Rightarrow \quad \phi = 3 \text{ eV}$$

Thus, $V_{cathode} - V_{anode} = 3\text{ V}$

$$\Rightarrow \quad V_{anode} - V_{cathode} = -3\text{ V}$$

4. (*a*) Given, mass of electrons = m
and de-Broglie wavelength = λ

So, kinetic energy of electron $= \frac{p^2}{2m} = \frac{\left(\frac{h}{\lambda}\right)^2}{2m} = \frac{h^2}{2m\lambda^2}$

Now, maximum energy of photon can be given by

$$E = \frac{hc}{\lambda_0} = \frac{h^2}{2m\lambda^2}$$

$$\Rightarrow \quad \lambda_0 = \frac{hc \times 2\lambda^2 . m}{h^2} = \frac{2mc\lambda^2}{h}$$

5. (*b*) The radiation energy is given by $E = \frac{hc}{\lambda}$

Initial momentum of the radiation is $\mathbf{p}_i = \frac{h}{\lambda} = \frac{E}{c}$

The reflected momentum is $\mathbf{p}_r = -\frac{h}{\lambda} = -\frac{E}{c}$

So, the change in momentum of light is

$$\Delta\mathbf{p}_{light} = \mathbf{p}_r - \mathbf{p}_i = -\frac{2E}{c}$$

Thus, the momentum transferred to the surface is

$$\Delta\mathbf{p}_{light} = \frac{2E}{c}$$

6. (*b*) From photoelectric equation,

$$h\nu = W + eV_0 \quad \text{(where, } W \text{ = work function)}$$

So $\quad \frac{hc}{\lambda} = W + 3eV_0 \quad ...(i)$

Also, $\quad \frac{hc}{2\lambda} = W + eV_0 \Rightarrow \frac{hc}{\lambda} = 2W + 2eV_0 \quad ...(ii)$

Subtracting Eq. (i) from Eq. (ii), we get

$$0 = W - eV_0 \Rightarrow W = eV_0$$

From Eq. (i), $\quad \frac{hc}{\lambda} = eV_0 + 3eV_0 = 4eV_0$

The threshold wavelength is given by

$$\lambda_{th} = \frac{hc}{W} = \frac{4eV_0\lambda}{eV_0} = 4\lambda$$

7. (*b*) The de-Broglie wavelength is given by

$$\lambda = \frac{h}{p} \Rightarrow p\lambda = h$$

This equation is in the form of $yx = c$, which is the equation of a rectangular hyperbola. Hence, the graph given in option (b) is the correct one.

8. (*b*) When charged particle is moving on a circular path in a uniform magnetic field B, then

$$\frac{mv^2}{r} = qvB \Rightarrow \frac{mv}{r} = qB \Rightarrow v = \frac{qBr}{m}$$

Thus, kinetic energy $= \frac{1}{2}mv^2 = \frac{1}{2}m \times \left(\frac{qBr}{m}\right)^2 = \frac{1}{2m}(qBr)^2$

Now for photoelectric effect, Einstein's equation is

$$h\nu = W + (\text{KE})$$

where, $\quad$ W = Work function

$$\Rightarrow \quad h\nu - \text{KE} = W$$

$$\Rightarrow \quad W = h\nu - \frac{(qBr)^2}{2m}$$

Here, charged particle is a photoelectron denoted by e, so

$$W = h\nu - \frac{(eBr)^2}{2m_e}$$

9. (*b*) Given that, $2 \text{ eV} \le \phi \le 5 \text{ eV}$

Wavelength corresponding to minimum and maximum values of work function are

$$\lambda_{max} = \frac{hc}{E} = \frac{4 \times 10^{-15} \times 3 \times 10^8}{2}$$

$$= 6 \times 10^{-7}\text{m} = 600 \text{ nm}$$

and $\lambda_{min} = \frac{hc}{E} = \frac{4 \times 10^{-15} \times 3 \times 10^8}{5}$

$= 2.4 \times 10^{-7}\text{m} = 240 \text{ nm}$

Clearly, wavelength range would be

$240 \le \lambda \le 600$

Thus, wavelength 650 nm is not suitable for photoelectric effect.

10. (*d*) The de-Broglie equation is

$$\lambda = \frac{h}{mv} \Rightarrow m = \frac{h}{\lambda v} \Rightarrow m \propto \frac{1}{\lambda}, \text{ if } \frac{h}{v} = \text{constant}$$

Again, $\frac{h}{v}$ will be constant only when v is constant.

Thus, in present case the de-Broglie wavelength of heavier particle will be smaller, when the velocities of two particles will be same.

11. (*a*) The de-Broglie wavelength of electron associated with voltage is given by

$$\lambda = \frac{1.227}{\sqrt{V}} \text{ nm} = \frac{1.227}{\sqrt{64}} \text{ nm} = 0.153 \text{ nm}$$

14. (*b*) For de-Broglie wavelength,

$$\lambda_1 = \frac{h}{p} = \frac{h}{\sqrt{2mK}} \quad \text{...(i)}$$

$$\lambda_2 = \frac{h}{\sqrt{2m\,16K}} = \frac{h}{4\sqrt{2mK}} = \frac{\lambda_1}{4} \quad \text{...(ii)}$$

$\lambda_2 = 25\%$ of λ_1

There is 75% change in the wavelength.

15. (*b*) de-Broglie wavelength of an electron is given by

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2meV}} = \frac{12.27}{\sqrt{V}} \text{ Å}$$

where, h = Planck's constant

m = mass of electron

e = electronic charge and

V = potential difference with which electron is accelerated.

$$\lambda = \frac{12.27}{\sqrt{100}} = \frac{12.27}{10} = 1.227 \text{ Å}$$

16. (*b*) From Einstein's photoelectric equation,

$(KE)_{max} = h\nu - \phi_0$

For first condition, $0.5 = E - \phi_0$ ...(i)

For second condition,

$$0.8 = \left(E + \frac{E \times 20}{100}\right) - \phi_0$$

$\Rightarrow \quad 0.8 = 1.2E - \phi_0$ ...(ii)

From Eqs. (i) and (ii), we get

$$0.3 = 0.2E \Rightarrow E = \frac{3}{2} = 1.5 \text{ eV}$$

From Eq. (i), $\phi_0 = E - 0.5 = 1.5 - 0.5 = 1$ eV

17. (*a*) According to Einstein's photoelectric equation,

$$K_A = \frac{hc}{\lambda_A} - \phi \Rightarrow 2K_A = \frac{2hc}{\lambda_A} - 2\phi$$

or $\quad 2K_A + \phi = \frac{2hc}{\lambda_A} - \phi$

Wavelength of metal B is half of metal A, therefore

$$K_B = \frac{2hc}{\lambda_A} - \phi$$

$\Rightarrow \quad 2K_A + \phi = K_B \Rightarrow K_A < \frac{K_B}{2}$

18. (*b*) Above the threshold frequency, the maximum kinetic energy of the emitted photoelectrons depends on the frequency of the incident light, but is independent of the intensity of the incident light.

19. (*c*) From photoelectric equation, $E_K = E - W$

where, E_K is kinetic energy of emitted photoelectrons, W is the work function and E is the energy supplied.

$$E = h\nu = \frac{hc}{\lambda}$$

where, h is Planck's constant, c is the speed of light and λ is the wavelength.

$\therefore \quad E = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{5000 \times 10^{-10}}$

$= 3.96 \times 10^{-19}$ J

Also, $\quad 1 \text{ eV} = 1.6 \times 10^{-19}$ J

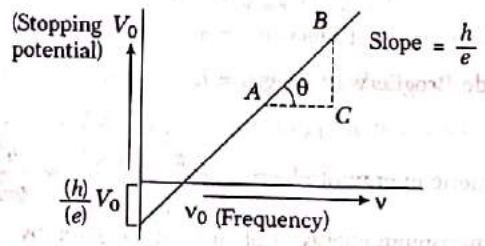
$\therefore \quad E = \frac{3.96 \times 10^{-19}}{1.6 \times 10^{-19}} = 2.48$ eV

Hence, $\quad E_K = 2.48 - 1.90 = 0.58$ eV

20. (*a*) We know that, $\quad K_{max} = eV_0$

where, e is the charge in the electron. Since, at the stopping potential the photoelectric current cannot be obtained by increasing the intensity of the incident light, it is clear that the stopping potential or the maximum kinetic energy of the photoelectrons, does not depend upon the intensity of the incident light. If the intensity of light be doubled, then the number of photoelectrons emitted per second is doubled. So, n is doubled but K_{max} remains same.

22. (*d*) If the wavelength of incident light falling on a photosensitive meterial decreases, then stopping potential increases.



$$V_0 = \frac{h}{e}(\nu - \nu_0) \Rightarrow V_0 = \frac{h}{e}\left(\frac{c}{\lambda} - \frac{c}{\lambda_0}\right) \Rightarrow V_0 \propto \frac{1}{\lambda}$$

23. (*c*) de-Broglie wavelength, $\lambda = \frac{h}{\sqrt{2mK}}$

The kinetic energy of the electron,

$$K_{electron} = \frac{h^2}{2m \cdot \lambda^2} \quad \text{...(i)}$$

where, h = Planck's constant, λ = wavelength

The photon energy,

$$E_{\text{photon}} = \frac{ch}{\lambda} \quad \text{...(ii)}$$

From Eqs. (ii) and (i), we get

$$\frac{E_{\text{photon}}}{K_{\text{electron}}} = \frac{ch/\lambda}{h^2/2m\cdot\lambda^2} = \frac{hc\cdot\lambda^2 2m}{h^2\cdot\lambda}$$

where, $m = 0.5 \text{ MeV} = 5\times10^5 \text{ eV}$

$$\frac{h}{\lambda} = 50\times10^3 \text{ eV} = \frac{2m\lambda c}{h}$$

$$= \frac{2m}{h/\lambda c} = \frac{2\times5\times10^5}{50\times10^3} = 20 \qquad \left(\because m = \frac{h}{c\lambda}\right)$$

$$= 20:1$$

24. (*c*) As $\frac{1}{2}mv_{\max}^2 = h\nu \Rightarrow v_{\max}^2 = \frac{2h\nu}{m}$

$$\therefore \quad v_{\max} = \sqrt{\frac{2h\nu}{m}}$$

25. (*a*) We know, for electron,

$$\lambda_e = \frac{h}{\sqrt{2m_eE}} \Rightarrow \lambda_e \propto \frac{1}{\sqrt{E}} \quad \text{...(i)}$$

For photon,

$$\lambda_p = \frac{hc}{E}$$

$$\Rightarrow \quad \lambda_p \propto \frac{1}{E} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\lambda_p \propto \lambda_e^2$$

27. (*b*) Davisson-Germer experiment verified the wave nature of electrons using Ni crystal.

28. (*b*) $\lambda = \frac{h}{\sqrt{2mK}} \Rightarrow \lambda \propto \frac{1}{\sqrt{K}}$

$$\therefore \quad \frac{\lambda_1^2}{\lambda_2^2} = \frac{K_2}{K_1} \Rightarrow \frac{1}{(0.5)^2} = \frac{K_2}{K_1} \Rightarrow K_2 = 4K$$

Additional energy required $= K_2 - K_1 = 4K - K = 3K$

29. (*c*) Given, $v = 1.8\times10^6 \text{ ms}^{-1}$

$$\frac{e}{m} = 1.8\times10^{11} \text{ Ckg}^{-1}$$

We have, $eV_0 = \frac{1}{2}mv^2 \Rightarrow V_0\frac{e}{m} = \frac{v^2}{2}$

$$\Rightarrow \quad V_0\times1.8\times10^{11} = \frac{1.8\times1.8\times(10^6)^2}{2} \Rightarrow V_0 = 9\text{V}$$

30. (*a*) de-Broglie wavelength is given by

$$\lambda = \frac{h}{p} \quad \text{and} \quad \frac{h}{p} \propto \frac{1}{\sqrt{K}}$$

So, $\lambda \propto \frac{1}{\sqrt{K}}, \lambda\sqrt{K} = \text{constant}$

Here, the graph between λ and K will be a curved line.

32. (*c*) Einstein postulated that the energy of light depends on the frequency, whereas in classical theory, the energy depends only on the intensity and is independent of the frequency.

34. (*a*) Efficient power, $P = \frac{N}{t}\times\frac{hc}{\lambda} = 200\times0.25$

$$\frac{N}{t} = 50\times\frac{\lambda}{hc} = \frac{50\times0.6\times10^{-6}}{6.6\times10^{-34}\times3\times10^8}$$

$$= 1.5\times10^{20} \text{ per second}$$

35. (*d*) We know

$$R = \frac{mv}{qB} \text{ and } \lambda = \frac{h}{mv}$$

$$\lambda = \frac{h}{qBR} = \frac{6.6\times10^{-34}}{1.6\times10^{-19}\times0.83\times10^{-2}\times0.25} = 0.01\text{Å}$$

36. (*c*) Momentum, $p = mv = \frac{h}{\lambda}$ or $v = \frac{h}{m\lambda}$

$$\therefore \quad v = \frac{6.62\times10^{-34}}{9.1\times10^{-31}\times5.2\times10^{-7}}$$

$$\Rightarrow \quad v = \frac{6.62\times10^4}{9.1\times5.1} \Rightarrow v \cong 1400 \text{ m/s}$$

37. (*a*) $K_e = \frac{1}{2}mv^2$ and $\lambda = \frac{h}{mv}$

$$K_e = \frac{1}{2}\left(\frac{h}{\lambda v}\right)v^2 = \frac{vh}{2\lambda}, K_p = \frac{hc}{\lambda}$$

$$\therefore \quad \frac{K_e}{K_p} = \frac{v}{2c} = \frac{1.5\times10^8}{2\times3\times10^8} = \frac{1}{4}$$

CHAPTER

12 Atoms

Atoms are the basic units of matter and defining structure of elements. In 1898, JJ Thomson for the first time proposed the physical structure of atom and named it as 'Plum-Pudding model'. Then, Rutherford performed an experiment on α-particle scattering in early 90's to investigate the atomic structure. But his work was rejected on the basis of classical theory. His shortcomings was rectified by Neil Bohr through his atomic model.

In this chapter, we will study Bohr's model, energy levels, hydrogen spectrum and X-rays.

1 Rutherford α-particle scattering experiment and discovery of nucleus

Rutherford with his two assistants, Geiger and Marsden did an experiment in 1911, in which they directed a narrow beam of α-particle onto gold foil of about 10^{-8} m thick.

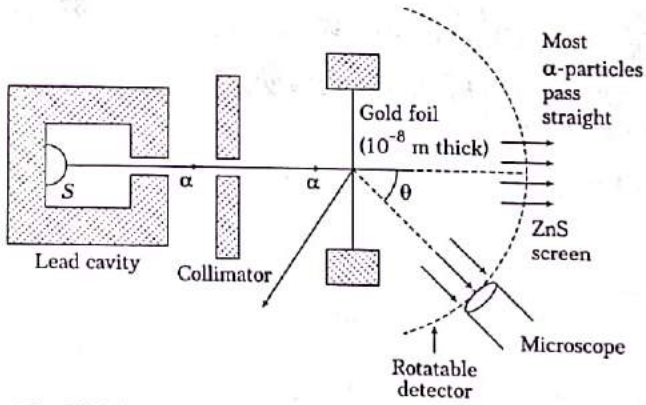


Fig. 12.1 Experimental arrangement for Rutherford's theory

α-particle were emitted by some radioactive material (polonium) kept inside a thick lead box. A very fine beam of α-particles pass through a small hole in the lead screen. This well collimated beam was then allowed to fall on a thin gold foil. While passing through the gold foil, α-particles scattered through different angles. A zinc sulphide screen was placed on the other side of the gold foil. This screen was movable, so as to receive the α-particles, scattered from the gold foil at angles varying from 0° to 180°. When an α-particle strikes the screen, then it produces a flash of light which were observed by the microscope.

Inside

1) Rutherford α-particle scattering experiment and discovery of nucleus
2) Bohr's atomic model
3) Energy of electron in *n*th orbit
   - Energy of atom
4) Hydrogen spectrum or the line spectra of hydrogen atom
5) X-rays
   - Bragg's law

Observations

Rutherford made following observations from the experiment

(i) Most of the α-particles pass through the foil straight away undeflected.

(ii) Some of them are deflected through small angles.

(iii) Few α-particles (in 8000) are deflected through the angle more than 90°.

(iv) A very few α-particles return back *i.e.*, deflected by 180°. The α-particles reverse their motion without actually touching the gold foil.

(v) The scattering is proportional to the target thickness for thin targets.

(vi) The number of α-particles scattered per unit area $N(\theta)$ at scattering angle θ varies inversely as $\sin^4(\theta/2)$.

$$N(\theta) \propto \frac{1}{\sin^4(\theta/2)}$$

This number $[N(\theta)]$ can be represented by following graph.

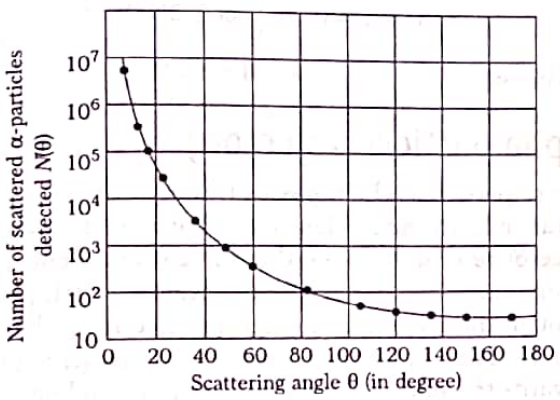


***Fig. 12.2** Number of α-particle scattered versus scattering angle (θ)*

Example 12.1 *In Rutherford's α-particle experiment with thin gold foil, 8100 scattered α-particles per unit area per minute were observed at an angle of 60°. Find the number of scattered α-particles per unit area per minute at an angle of 120°.*

Sol. It is given that $\theta_1 = 60°$, $\theta_2 = 120°$, $N_1 = 8100$, $N_2 = ?$

Number of α-particles scattered at an angle θ,

$$N \propto \frac{1}{\sin^4\left(\frac{\theta}{2}\right)} \Rightarrow \frac{N_1}{N_2} = \frac{\left(\sin^4\frac{\theta_2}{2}\right)}{\left(\sin^4\frac{\theta_1}{2}\right)}$$

$$\Rightarrow \frac{8100}{N_2} = \frac{\left(\sin^4\frac{120°}{2}\right)}{\left(\sin^4\frac{60°}{2}\right)} \Rightarrow \frac{8100}{N_2} = \frac{\sin^4 60°}{\sin^4 30°}$$

$$\Rightarrow N_2 = \frac{8100 \times \frac{1}{16}}{\frac{9}{16}} = 900$$

Rutherford's atomic model

The essential features of Rutherford's model of the atom or planetary model of the atom are as follows

(i) Most of the mass (at least 99.95%) and all of the charge of an atom is concentrated in a very small region called atomic nucleus.

(ii) Nucleus is positively charged and its size is of the order of 10^{-15} m ≈ 1 fermi. The nucleus occupies only about 10^{-12} of the total volume of the atom or less.

(iii) In an atom, there is maximum empty space and the electrons revolve around the nucleus in the same way as the planets revolve around the sun (*i.e.*, in elliptical orbits)

Note Electrons are light weight as compared to α-particles to deflect them, whereas when α-particle come in contact with nucleus, which is very massive, due to Coulomb's force of repulsion, the path of the particles on deflection is hyperbolic but the nucleus remains at rest.

Distance of closest approach

The minimum distance from the nucleus upto which the α-particle approaches, is called the distance of closest approach (r_0). At this distance (when particle deflect by 180°) whole of the KE of α-particle converts into electrostatic potential energy.

i.e., $$\frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{(Ze)\,2e}{r_0}$$

[∵ charge on α-particle is $+2e$ and charge on nucleus is Ze, where Z is atomic number]

$$\Rightarrow \quad r_0 = \frac{1}{4\pi\varepsilon_0}\frac{2Ze^2}{\left[\frac{1}{2}mv^2\right]} = \frac{4kZe^2}{mv^2} \qquad \left(\because k = \frac{1}{4\pi\varepsilon_0}\right)$$

where, m is mass and v is velocity of α-particle

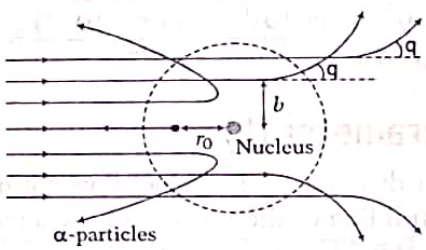


***Fig. 12.3** Distance of closest approach (r_0)*

Example 12.2 *An α-particle of energy 5 MeV is scattered through 180° by a fixed uranium nucleus. Determine the distance of the closest approach.*

Sol. According to law of conservation of energy,

kinetic energy of α-particle = potential energy of α-particle at distance of closest approach

i.e., $$\frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$$

Given, atomic number of uranium, $Z = 92$,

Charge of electron $e = 1.6 \times 10^{-19}$C,

Charge of α-particle, $q_1 = 2e$,

Charge of uranium nucleus,

$$q_2 = 92e$$

$$\therefore \quad 5 \text{ MeV} = \frac{9 \times 10^9 \times (2e) \times (92e)}{r} \quad \left(\because \frac{1}{2}mv^2 = 5 \text{ MeV}\right)$$

$$\Rightarrow \quad r = \frac{9 \times 10^9 \times 2 \times 92 \times (1.6 \times 10^{-19})^2}{5 \times 10^6 \times 1.6 \times 10^{-19}}$$

$$(\because 1 \text{ MeV} = 1.6 \times 10^{-19} \text{J})$$

$$\therefore \quad r = 5.3 \times 10^{-14} \text{ m}$$

$$= 5.3 \times 10^{-12} \text{ cm}$$

Example 12.3 *Atomic number of silver metal is 47. Calculate the speed at which a beam of protons has to be fired at a sheet of silver foil, if the protons were able to approach to within 2.5×10^{-14}m of the silver nucleus?*

Sol. The distance between protons and silver nucleus is $r = 2.5 \times 10^{-14}$m

Mass of proton is, $m = 1.67 \times 10^{-27}$ kg

Charge of protons is, $q_1 = 1.6 \times 10^{-19}$C

Atomic number of silver, $Z = 47$

Charge of silver nucleus, $q_2 = Ze = (47)(1.6 \times 10^{-19}\text{C})$

By energy conservation at a distance r the kinetic energy of proton must be equal to the potential energy of proton at that point. Therefore, we get

$$K = U \Rightarrow \frac{1}{2}mv^2 = \frac{kq_1q_2}{r}$$

Therefore, $$v = \sqrt{\frac{2kq_1q_2}{mr}}$$

$$= \sqrt{\frac{2 \times (9 \times 10^9)(1.6 \times 10^{-19})(47)(1.6 \times 10^{-19})}{(1.67 \times 10^{-27})(2.5 \times 10^{-14})}} = 2.28 \times 10^7 \text{ ms}^{-1}$$

Impact parameter (*b*)

Perpendicular distance of the velocity vector of α-particle from the central line of the nucleus of the atom is called impact parameter.

Mathematically, it is expressed as

$$\boxed{b = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Ze^2 \cot\frac{\theta}{2}}{\text{KE}}}$$

Thus, smaller the impact parameter (*b*), greater the scattering angle (θ) and kinetic energy of α-particle.

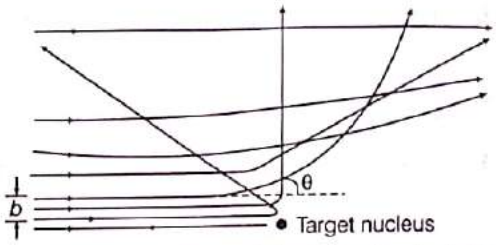


Fig. 12.4 *Trajectory of α-particles in the Coulomb field of a target nucleus. The impact parameter b and scattering angle θ are also depicted.*

Example 12.4 *In Rutherford scattering experiment, what will be the correct angle for α scattering for an impact parameter, $b = 0$?*

Sol. We know that impact parameter, $b \propto \cot\frac{\theta}{2} \Rightarrow$

$$b = K\cot\frac{\theta}{2}$$

where, K is a constant

Here, if, $b = 0$.

$$K\cot\theta/2 = 0 \Rightarrow \cot\theta/2 = 0 \Rightarrow \frac{\theta}{2} = \frac{\pi}{2}$$

Hence, $$\theta = \pi = 180°$$

Alpha particle trajectory

The α-particles which passes through the atom at a large distance from the nucleus experience a small electrostatic force of repulsion due to the nucleus and hence, undergo a very small deflection the α-particles which passes through the atom at a smaller distance from the nucleus suffer a large deflection. The α-particles which travel towards the nucleus directly, slow down and ultimately comes to rest and then after being deflected through 180° retrace their path.

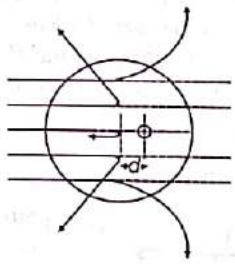


Fig 12.5 *Trajectory of α-particles close to an atom*

Electron orbits

The Rutherford nuclear model of the atom pictures the atom as an electrically neutral sphere consisting of a very small, massive and positively charged nucleus at the centre surrounded by the revolving electrons in their respective dynamically stable orbits. The electrostatic force of attraction F_e between the revolving electrons and the nucleus provides the requisite centripetal force (

F_c) to keep them in their orbits. Thus, for a dynamically stable orbit in a H-atom,

$$F_c = F_e \Rightarrow \frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{r^2} \quad (\because Z = 1)$$

Thus, the relation between the orbit radius and the electron velocity is $r = \dfrac{e^2}{4\pi\varepsilon_0 mv^2}$

The kinetic energy (K) and electrostatic potential energy (U) of the electron in H-atom are

$$K = \frac{1}{2}mv^2 = \frac{e^2}{8\pi\varepsilon_0 r} \quad \left(\because mv^2 = \frac{e^2}{4\pi\varepsilon_0 r}\right)$$

and $$U = -\frac{e^2}{4\pi\varepsilon_0 r}$$

(the negative sign in U signifies that the electrostatic force is in the $-r$ direction or attractive in nature.) Thus, the total mechanical energy E of the electron in a H-atom is

$$E = K + U = \frac{e^2}{8\pi\varepsilon_0 r} - \frac{e^2}{4\pi\varepsilon_0 r} = -\frac{e^2}{8\pi\varepsilon_0 r}$$

The total energy of the electron is negative. This implies the fact that the electron is bound to the nucleus. If E were positive, an electron will not follow a closed orbit around the nucleus and it would leave the atom.

Example 12.5 *It is found experimentally that 13.6 eV energy is required to separate a hydrogen atom into a proton and an electron. Calculate the velocity of the electron in hydrogen atom.*

Sol. Total energy of the electron in hydrogen atom is
$-13.6\text{ eV} = -13.6\times 1.6\times 10^{-19}\text{ J} = -2.2\times 10^{-18}\text{ J}$

Total energy of the electron in hydrogen atom is

$$E = -\frac{e^2}{8\pi\varepsilon_0 r}$$

$$\therefore \quad -\frac{e^2}{8\pi\varepsilon_0 r} = -2.2\times 10^{-18}$$

Orbital radius, $r = -\dfrac{e^2}{8\pi\varepsilon_0 E}$

$$\Rightarrow \quad r = \frac{(9\times 10^9\text{ Nm}^2\text{ C}^2)(1.6\times 10^{-19}\text{ C})^2}{(2)(-2.2\times 10^{-18}\text{ J})}$$

$$\Rightarrow \quad r = 5.3\times 10^{-11}\text{ m}$$

The velocity of the revolving electron is given by

$$r = \frac{e^2}{4\pi\varepsilon_0 mv^2}$$

$$\Rightarrow \quad v = \frac{e}{\sqrt{4\pi\varepsilon_0 mr}}$$

Putting, $m = 9.1\times 10^{-31}$ kg and $r = 5.3\times 10^{-11}$ m

We have $$v = \frac{(1.6\times 10^{-19})}{\sqrt{4\pi\varepsilon_0\times 9.1\times 10^{-31}\times 5.3\times 10^{-11}}}$$

$$v = 2.2\times 10^6\text{ m/s}$$

Drawbacks of rutherford model

Rutherford's model suffers two major drawbacks

(i) Regarding stability of atom

Electrons revolving around the nucleus have centripetal acceleration. According to classical electromagnetic theory, the electrons must radiate energy in the form of electromagnetic wave.

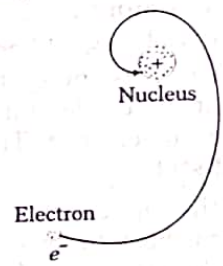


Fig. 12.6 *An atomic electron should, classically, spiral rapidly into the nucleus as it radiates energy due to its acceleration.*

Due to this continuous loss of energy of the electrons, the radii of their orbits should be continuously decreasing and ultimately the electrons should fall in the nucleus (as shown in Fig. 12.6). Thus, atom cannot remain stable.

(ii) Regarding explanation of line spectrum

Due to continuous decrease in radii of electron's orbit, the frequency of revolution of electron will also change. According to classical theory of electromagnetism, frequency of EM wave emitted by electron is equal to frequency of revolution of electron.

So, due to continuous change in frequency of revolution of electron, it will radiate EM waves of all frequencies *i.e.*, the spectrum of these waves will be continuous in nature. But, this is not the case, experimentally we get line spectrum. Rutherford model was unable to explain line spectrum.

2 Bohr's atomic model

Bohr proposed a model for hydrogen atom which is applicable for some lighter atoms in which a single electron revolves around a stationary nucleus of positive charge Ze called hydrogen like atom, *e.g.*, ${}_1H^1$, ${}_1H^2$, He^+, Li^{++}, etc.

Bohr's model is based on the following three postulates

(i) Bohr postulated that an electron in an atom can move around nucleus in certain circular '**stable orbits**' without emitting radiations.

(ii) Electrons can revolve only in those orbits in which their angular momentum is an integral multiple of $h/2\pi$, where h is Planck's universal constant. If the mass of the electron be m and it is revolving with velocity v in an orbit of radius r, then its angular momentum will be mvr.

According to Bohr's postulate, we have

angular momentum, $mvr = \frac{nh}{2\pi}$

where, n is an integer (n=1, 2, 3, ...) and is called the **principal quantum number** of the orbit. This equation is called 'Bohr's quantisation condition'.

(iii) When the atom receives energy from outside, then one (or more) of its outer electrons leaves its orbit and goes to some higher orbit. This state of the atom is called **excited state**.

The electron in the higher orbit stays only for 10^{-8} second and returns back to lower orbit. While returning back, the electron radiates energy in the form of electromagnetic waves.

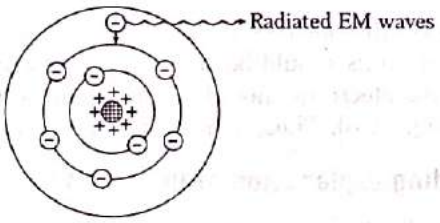


Fig. 12.7 Electron falls down to lower orbit and emits energy

If the energy of electron in the higher orbit be E_2 and that in the lower orbit be E_1, then the frequency ν of the radiated waves is given by

$$h\nu = E_2 - E_1 \quad \text{or} \quad \nu = \frac{E_2 - E_1}{h}$$

This equation is called **Bohr's frequency condition.**

Example 12.6 *Compute the angular momentum in 4th orbit, if L is the angular momentum of the electron in the 2nd orbit of hydrogen atom.*

Sol. We know that, the angular momentum of the electron of hydrogen atom,

$$L_n = \frac{nh}{2\pi} \quad \text{...(i)}$$

But according to question, the angular momentum in 2nd orbit is L (n = 2).

So, $$L = \frac{2h}{2\pi} = \frac{h}{\pi} \quad \text{...(ii)}$$

Hence, the angular momentum in 4th orbit is

$$L_4' = \frac{4h}{2\pi} = \frac{2h}{\pi} = 2L \quad \text{[from Eq. (ii)]}$$

Mathematical analysis of Bohr's atomic model of hydrogen like atoms

A hydrogen like atom consists of a tiny positively charged nucleus and an electron revolving in a stable circular orbit around the nucleus. Let e, m and v be respectively, the charge, mass and velocity of the electron and r the radius of the orbit. The positive charge of the nucleus is Ze, where Z is the atomic number (in case of hydrogen atom, Z = 1). As the centripetal force is provided by the electrostatic force of attraction, we have

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{(Ze)\times e}{r^2} \quad \text{or} \quad \boxed{mv^2 = \frac{Ze^2}{4\pi\varepsilon_0 r}} \quad \text{...(i)}$$

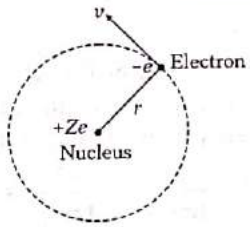


Fig. 12.7 An electron is moving around a nucleus of hydrogen like atoms

From the first postulate, the angular momentum of the electron is,

$$\boxed{mvr = n\frac{h}{2\pi}} \quad \text{...(ii)}$$

Radius of electron in *n*th orbit

Squaring Eq. (ii) and dividing by Eq. (i), we get

$$\boxed{r_n = n^2\frac{h^2\varepsilon_0}{\pi m Ze^2}} \quad (\because r = r_n) \quad \text{...(iii)}$$

This is the equation for the radii of the permitted orbits. According to this equation, $r_n \propto n^2$.

Since, n = 1, 2, 3, ... so on, it follows that the radii of the permitted orbits increase in the ratio 1 : 4 : 9 : 16 ... so on, from the first orbit.

Bohr's radius

The radius of the first orbit (n = 1) of H-atom (Z = 1) will be

$$\boxed{r_1 = \frac{h^2\varepsilon_0}{\pi m e^2}} \quad \text{...(iv)}$$

Substituting the value of constants, r_1 = 0.53 Å. This is called Bohr's radius. From Eqs. (iii) and (iv), we get Radius of nth orbit is,

$$r_n = 0.53\frac{n^2}{Z}\text{Å} = \frac{n^2}{Z}r_1.$$

Variations of r_n

Using the relation, $r_n = \frac{n^2}{Z} r_1$, *we can plot the following graphs*

r_n versus Z	r_n versus n	r_n versus n^2

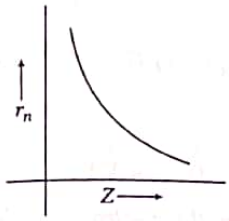
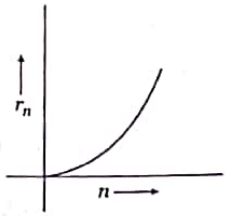
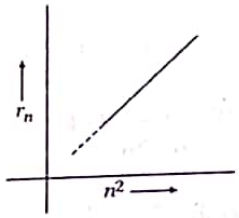

		

Fig. 12.8 r_n versus Z graph	*Fig. 12.9 r_n versus n graph*	*Fig. 12.10 r_n versus n^2 graph*
Here, $r_n \propto 1/Z$	Here, $r_n \propto \sqrt{n}$	Here, $r_n \propto n^2$
Nature of graph will be rectangular hyperbola.	Nature of graph will be parabola.	Nature of graph will be straight line.

Example 12.7 *Using known values for hydrogen atom, calculate radius of third orbit for Li^{2+}.*

Sol. As, $Z = 3$ for Li^{+2}

Further we know that, $r_n = \frac{n^2}{Z} r_1$

Substituting, $n = 3$, $Z = 3$ and $r_1 = 0.529\,\text{Å}$

We have, $r_3 = \frac{(3)^2}{(3)}(0.529)\,\text{Å} = 1.587\,\text{Å}$

Speed of electron in *n*th orbit

From Eq. (ii), we have $v_n = n\frac{h}{2\pi mr}$

Putting the value of r from Eq. (iii), we get

$$v_n = \frac{Ze^2}{2h\varepsilon_0}\frac{1}{n} \quad \text{...(v)}$$

where, principal quantum number,

$$n = 1, 2, 3, \ldots, \text{so on.}$$

Thus, $v_n \propto \frac{1}{n}$

This shows that the velocity of electron is maximum in the lowest orbit ($n = 1$) and goes on decreasing in higher orbits.

The velocity of electron in the first orbit ($n = 1$) of H-atom ($Z = 1$) is

$$v_1 = \frac{e^2}{2h\varepsilon_0} = \frac{c}{137} = 2.2 \times 10^6 \text{ ms}^{-1} \quad \text{...(vi)}$$

Thus, from Eqs. (v) and (vi), we get

Velocity of electron in nth orbit is,

$$v_n = 2.2 \times 10^6 \frac{Z}{n} \text{ms}^{-1} = \frac{Z}{n} v_1$$

Variations of v_n

Using the relation, $v_n = \frac{Z}{n} v_1$, *we can plot the following graph*

v_n versus Z	v_n versus n

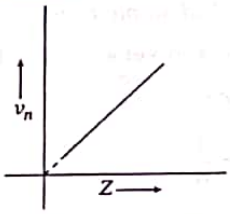
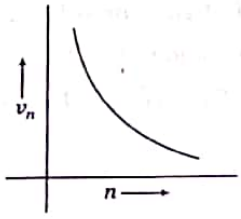

	

Fig. 12.11 v_n versus Z graph	*Fig. 12.12 v_n versus n graph*
Here, $v_n \propto Z$	Here, $v_n \propto 1/n$
Nature of graph will be straight line.	Nature of graph will be rectangular hyperbola.

Example 12.8 *Using known values for hydrogen atom, calculate speed of electron in fourth orbit of He^+.*

Sol. As, $Z = 2$ for He^+. Also we know that, $v_n = \frac{Z}{n} v_1$

Substituting $n = 4$, $Z = 2$ and $v_1 = 2.19 \times 10^6 \text{ ms}^{-1}$, we get

$$v_4 = \left(\frac{2}{4}\right)(2.19 \times 10^6) \text{ ms}^{-1}$$

$$= 1.095 \times 10^6 \text{ ms}^{-1}$$

Time period and frequency of revolution electron in *n*th orbit

Number of revolutions completed per second by the electron in a stationary orbit, around the nucleus is known as frequency.

It is represented by ν.

From $\quad v_n = r\omega_n = r_n(2\pi\nu_n) \qquad (\because \omega_n = 2\pi\nu_n)$

$\therefore \quad \nu_n = \dfrac{v_n}{2\pi r_n}$

Putting the values of v_n and r_n in above equation, we get

$$\nu_n = \frac{1}{2\pi r_n}\frac{Ze^2}{2h\varepsilon_0}\frac{1}{n} \Rightarrow \nu_n = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Ze^2}{nhr}$$

$$\Rightarrow \quad \nu_n = \frac{kZe^2}{nhr} \qquad \left(\because k = \frac{1}{4\pi\varepsilon_0}\right)$$

∴ Time period of revolution of electron in nth orbit,

$$T_n = \frac{1}{\nu_n} = \frac{nhr}{kZe^2}$$

Since, $\quad r_n \propto \dfrac{n^2}{Z}$

$\therefore \quad T_n \propto \dfrac{n^3}{Z^2}$ and $\nu_n \propto \dfrac{Z^2}{n^3}$

Also, $\quad \omega_n = 2\pi\nu_n \Rightarrow \omega_n \propto \dfrac{Z^2}{n^3}$

Example 12.9 *Find the ratio of product of velocity and time period of electron orbiting in 2nd and 3nd stable orbits.*

Sol. We know that, time period $T_n \propto n^3$ and velocity $v_n \propto 1/n$.

$\Rightarrow$ Time period $(T_n) \times$ velocity $(v_n) \propto n^2$

$$\therefore \quad \frac{T_2 v_2}{T_3 v_3} = \frac{2^2}{3^2} = \frac{4}{9}$$

Example 12.10 *In the Bohr model of hydrogen atom, the electron is pictured to rotate in a circular orbit of radius 5×10^{-11} m, at a speed $2.2 \times 10^6 ms^{-1}$. Calculate the current associated with electron motion?*

Sol. Frequency of revolution of electron can be written as,

$$f = \frac{v}{2\pi r} = \frac{2.2 \times 10^6}{2\pi(5 \times 10^{-11})} = 7.1 \times 10^{15}\ \text{Hz}$$

∴Current associated, $I = q/t = qf = (1.6 \times 10^{-19})(7 \times 10^{15}) \qquad \left(\because \dfrac{1}{t} = f\right)$

$$= 11.2 \times 10^{-4}\text{A} = 1.12\ \text{mA}$$

Example 12.11 *In a hydrogen atom, the electron is making $6.6 \times 10^{15} revs^{-1}$ around the nucleus in an orbit of radius 0.528 Å. Find the magnetic moment ($A - m^2$) associated with the motion of the electron.*

Sol. According to the question,

angular frequency $\omega = 6.6 \times 10^{15}\ \text{revs}^{-1}$

For 1 revolution frequency, $\omega = 6.6 \times 10^{15} \times 2\pi$

Radius $r = 0.528$ Å and Charge $e = 1.6 \times 10^{-19}$ C

Current associated with the motion of electron,

$$I = \frac{q}{t} = qf = \frac{e\omega}{2\pi} = \frac{1.6 \times 10^{-19} \times 6.6 \times 10^{15} \times 2\pi}{2\pi} \qquad (\because \omega = 2\pi\nu)$$

$$= 1.6 \times 10^{-19} \times 6.6 \times 10^{15} = 6.6 \times 1.6 \times 10^{-4}$$

$$= 1.056 \times 10^{-3}\ \text{A}$$

Magnetic moment, $M = IA = 1.056 \times 10^{-3} \times \pi \times r^2$

$$= 3.14 \times 1.056 \times 10^{-3} \times (0.53\ \text{Å})^2$$

$$= 9.314 \times 10^{-24}\ \text{A-m}^2 \approx 1 \times 10^{-23}\ \text{A-m}^2$$

Check point 12.1

1. The radius of an atom is of the order of
(a) 10^{-8}m (b) 10^{-9} m (c) 10^{-11}m (d) 10^{-10} m

2. According to classical theory of Rutherford's model, the path of electron will be
(a) parabolic (b) hyperbolic
(c) circular (d) elliptical

3. An alpha nucleus of energy $\frac{1}{2}mv^2$ bombards a heavy nuclear target of charge Ze. Then, the distance of closest approach for the alpha nucleus will be proportional to
(a) v^2 (b) $1/m$ (c) $1/v^4$ (d) $1/Ze$

4. The number of α-particles scattered per unit area $N(\theta)$ at scattering angle θ varies inversely as
(a) $\cos^4\left(\frac{\theta}{2}\right)$ (b) $\sin^4\left(\frac{\theta}{2}\right)$ (c) $\tan^4\left(\frac{\theta}{2}\right)$ (d) $\cot^4\left(\frac{\theta}{2}\right)$

5. In Rutherford scattering experiment for scattering angle of 180°, what will be the value of impact parameter?
(a) 0 (b) ∞
(c) 1 (d) Data is insufficient

6. The concept of stationary orbits was proposed by
(a) Neil Bohr (b) JJ Thomson (c) Rutherford (d) I Newton

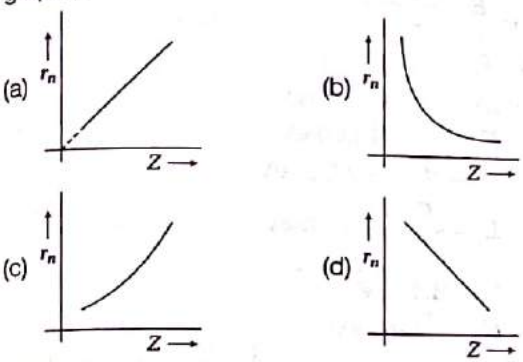
7. Bohr's atom model assumes
(a) the nucleus is of infinite mass and is at rest
(b) electrons in a quantised orbit will not radiate energy
(c) mass of electron remains constant
(d) All the above conditions

8. The angular momentum (L) of an electron moving in a stable orbit around nucleus is
(a) half integral multiple of $\frac{h}{2\pi}$ (b) integral multiple of h
(c) integral multiple of $\frac{h}{2\pi}$ (d) half integral multiple of h

9. In Bohr's model, the atomic radius of the first orbit is r_0, then the radius of the third orbit is
(a) $r_0/9$ (b) r_0
(c) $9r_0$ (d) $3r_0$

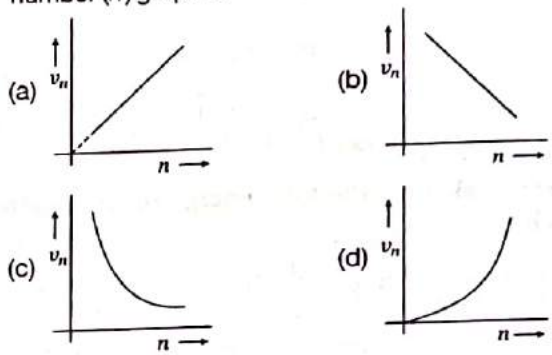
10. In which of the following systems will the radius of the first orbit ($n = 1$) be minimum?
(a) Hydrogen atom
(b) Deuterium atom
(c) Singly ionised helium
(d) Doubly ionised lithium

11. Radius (r_n) of electron in nth orbit *versus* atomic number (Z) graph is



12. The ratio of the speed of the electrons in the ground state of hydrogen to the speed of light in vacuum is
(a) 1/2 (b) 2/137 (c) 1/137 (d) 1/237

13. Speed (v_n) of electron in nth orbit *versus* principal quantum number (n) graph is



14. The orbital frequency of an electron in the hydrogen atom is proportional to
(a) n^3 (b) n^{-3} (c) n (d) n^0

15. Product of velocity and time period of electron orbiting in nth stable orbit is proportional to
(a) n^3 (b) $\frac{1}{n}$ (c) n (d) n^2

3 Energy of electron in *n*th orbit

The energy E of an electron in an orbit is the sum of kinetic and potential energies.

1. Kinetic energy

Electrons possess kinetic energy because of its motion. Closer orbits have greater kinetic energy than outer ones.

$$\text{KE}_n = \frac{1}{2}mv_n^2$$

Substituting the values of v_n from Eq. (v),

i.e., $$v_n = \frac{Ze^2}{2\pi\varepsilon_0}\left(\frac{1}{n}\right)$$

we get $$\text{KE}_n = \frac{mZ^2e^4}{8\varepsilon_0^2 n^2 h^2}$$

or $$\text{KE}_n = \frac{mZ^2e^4}{8\varepsilon_0^2 h^2}\left(\frac{1}{n^2}\right)$$

In simplified form, in terms of Rydberg's constant

$$\boxed{\text{KE}_n = \frac{RhcZ^2}{n^2}}$$

where, $$R = \frac{me^4}{8\varepsilon_0^2 ch^3} = \text{Rydberg's constant}$$

$$= 1.09 \times 10^7 \text{ per m}$$

2. Potential energy

The potential energy of the electron in an orbit is due to the electrostatic attraction by nucleus.

$$U_n = \frac{1}{4\pi\varepsilon_0}\frac{(Ze)(-e)}{r_n}$$

$$= -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r_n}$$

Substituting the value of r_n from Eq. (iii)

i.e., $$r_n = \frac{n^2h^2\varepsilon_0}{\pi mZe^2}$$

we get $$U_n = \frac{-mZ^2e^4}{4\varepsilon_0^2 n^2 h^2} \quad \text{...(viii)}$$

In simplified form, in terms of Rydberg's constant

$$\boxed{U_n = -\frac{2RhcZ^2}{n^2}}$$

3. Total energy

The total energy of the electron is therefore

$$E_n = \text{KE}_n + U_n = \frac{Ze^2}{8\pi\varepsilon_0 r_n} - \frac{Ze^2}{4\pi\varepsilon_0 r_n} = -\frac{Ze^2}{8\pi\varepsilon_0 r_n}$$

Substituting for r_n from Eq. (iii), we get

$$E_n = \frac{-mZ^2e^4}{8\varepsilon_0^2 h^2}\left(\frac{1}{n^2}\right) \quad \text{...(ix)}$$

This is the expression for the total energy of the electron in the nth orbit.

or $$\boxed{E_n = \frac{-Rhc}{n^2}}$$

Also, from the above equations, we can write

$$\text{PE} = -2\text{KE}$$

$$\Rightarrow \quad U_n = -2\text{KE}_n$$

$$\Rightarrow \quad |U_n| = 2|\text{KE}_n|$$

$$\therefore \quad E_n = \text{KE}_n - 2\text{KE}_n = -\text{KE}_n$$

Some general points related to energy of electron

(i) $Rhc = 1\,\text{Rydberg} = 13.6$ eV.

(ii) For hydrogen atom $Z = 1$, substituting the standard values, we get $E_n = \frac{-13.6}{n^2}$ eV.

(iii) Negative energy of the electron shows that the electron is bound to the nucleus and is not free to leave it.

(iv) For $n = 1$, $E_1 = -13.6$ eV. Substituting $n = 2, 3, 4, \ldots\ldots$ etc, we get energies of electron in different orbits.

$$E_2 = -3.40 \text{ eV}, E_3 = -1.51 \text{ eV} \ldots E_\infty = 0.$$

Example 12.12 *The ground state energy of hydrogen atom is -13.6 eV. What is the kinetic and potential energies of the electron in this state?*

Sol. As we know, kinetic energy = – total energy

$$= -(-13.6)\text{ eV} = 13.6 \text{ eV}$$

and potential energy = + 2 (total energy)

$$= 2(-13.6)\text{ eV} = -27.2 \text{ eV}$$

Example 12.13 *The total energy of electron in the ground state of hydrogen atom is – 13.6 eV. Find the kinetic energy of an electron in the first excited state.*

Sol. The energy of electron in hydrogen atom when it revolves in nth orbit is,

$$E = \frac{-13.6}{n^2} \text{ eV}$$

In the ground state, $n = 1$

$$\therefore \quad E = \frac{-13.6}{1^2} = -13.6 \text{ eV}$$

For $n = 2$, $$E = \frac{-13.6}{2^2} = -3.4 \text{ eV}$$

So, kinetic energy of electron in the first excited state *i.e.*, for $n = 2$ is, kinetic energy = – (total energy),

$$K = -E = -(-3.4) = 3.4 \text{ eV}$$

Example 12.14 *Find the kinetic energy, potential energy and total energy of electron in first and second orbit of hydrogen atom, if potential energy in first orbit is taken to be zero.*

Sol. The energy of electron in hydrogen atom when it revolves in nth orbit is,

$$E = \frac{-13.6}{n^2} \text{ eV}$$

For $n = 1$, $E_1 = -13.60$ eV

Kinetic energy = – (Total energy)

$$K_1 = -E_1 = 13.60 \text{ eV}$$

$$U_1 = 2E_1 = -27.20 \text{ eV}$$

$$E_2 = \frac{E_1}{(2)^2} = -3.40 \text{ eV}$$

$$K_2 = 3.40 \text{ eV}$$

and $$U_2 = -6.80 \text{ eV}$$

Now, $U_1 = 0$ *i.e.*, potential energy has been increased by 27.20 eV. So, we will increase U and E in all energy states by 27.20 eV while kinetic energy will remain unchanged. Changed values in tabular form are as under.

Orbit	K (eV)	U(eV)	E(eV)
First	13.60	0	13.60
Second	3.40	20.40	23.80

Energy of atom

According to Bohr's theory, in a given stationary orbit, an electron has a definite value of energy. Energy of a hydrogen or hydrogen like atom is given by this definite value of energy of the electron in the nth orbit. Therefore, energy of the nth orbit of the hydrogen like atom is

$$E_n = -13.6\frac{Z^2}{n^2} \text{ eV}$$

where, Z = atomic number of the atom.

The electrons in the atom can jump from one orbit to another when the atom emits or absorbs energy.

During absorption of energy electron jumps from lower to higher orbit. Similarly, during emission of energy an electron jumps from higher to lower orbit. When electron jumps from higher to lower or lower to higher orbits respectively, it emits or absorb a photon and the energy of a photon is equal to the difference of energies of two orbits.

Let an electron of the atom jumps from n_1 to n_2 ($n_2 > n_1$)

$$\therefore E_{n_2} - E_{n_1} = -13.6\frac{Z^2}{n_2^2} - \left(-13.6\frac{Z^2}{n_1^2}\right)$$

$$\Rightarrow \quad \Delta E = 13.6Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Energy of absorbed photon,

$$\Delta E = h\nu = \frac{hc}{\lambda}$$

where, ν = frequency of absorbed photon
λ = wavelength of absorbed photon
c = speed of light

Also, we can write

$$\Delta E = h\nu = \frac{hc}{\lambda} = E_{n_2} - E_{n_1} = \frac{Z^2me^4}{8\varepsilon_0^2h^2}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\Rightarrow \quad \frac{1}{\lambda} = \frac{Z^2me^4}{8h^3\varepsilon_0^2c}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\Rightarrow \quad \boxed{\frac{1}{\lambda} = Z^2R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)}$$

where, R is Rydberg's constant.

Note The reciprocal of the wavelength is called wave number.

$$\Rightarrow \quad \bar{\nu} = 1/\lambda$$

Energy levels for hydrogen atom

Energy of nth state of an atom is, $E_n = -13.6\dfrac{Z^2}{n^2}$ eV ...(i)

For hydrogen, $Z = 1; E_n = -\dfrac{13.6}{n^2}$ eV

The energy of an atom is least, when its electron is revolving in an orbit closest to the nucleus *i.e.*, for which $n = 1$.

$\therefore E_1 = -13.6$ eV, this is the energy when the electron revolve in smallest allowed orbit.

For $n = 2$, $E_2 = \dfrac{-13.6}{2^2} = -3.4$ eV, etc.

For $n = 3, 4, \ldots$, so on the absolute value of energy E is smaller, so the energy is progressively larger in outer orbits as shown in the figure.

The lowest state of an atom is called the **ground state**, this state has lowest energy. The energy of this state for H-atom is -13.6 eV.

Therefore, the minimum energy required to free the electron form the ground state of the hydrogen atom is 13.6 eV, it is called **ionisation energy** of hydrogen atom.

At room temperature, most of the hydrogen atoms are in groud state. When a hydrogen atom receives energy by processes such as electron collisions, the atom may acquire sufficient energy to raise the electron to higher energy states. The atom is then said to be in an **excited state.**

From Eq. (i), for $n = 2$, the energy E_2 is -3.40 eV. It means that the energy required to excite an electron in hydrogen atom to its first excited state, is an energy equal to $E_2 - E_1 = -3.40$ eV $-(-13.6)$ eV $= 10.2$ eV.

Similarly, $E_3 = -1.51$ eV and $E_3 - E_1 = 12.09$ eV, or to excited the hydrogen atom from its ground state ($n = 1$) to second excited state ($n = 3$), 12.09 eV energy is required, and so on.

From these excited states the electron can then fall back to a state of lower energy, emitting a photon in the process. Thus, as the excitation of hydrogen atom increases (that is as n increases) the value of minimum energy required to free the electron from the excited atom decreases.

The energy level diagram for the stationary state of a hydrogen atom, computed from Eq. (i), is given in Fig. 12.13. The principal quantum number n labels the stationary states in the ascending order of energy. In this diagram, the highest energy state corresponds to $n = \infty$ in Eq. (i) and has an energy of 0 eV. This is the energy of the atom when the electron is completely removed ($r = \infty$) from the nucleus and is at rest.

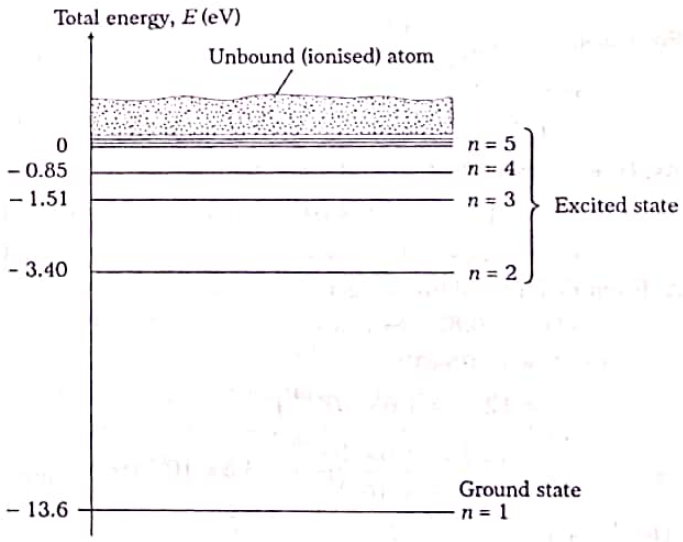


Fig. 12.13 The energy level diagram for the hydrogen atom.

Example 12.15 *A difference of 2.3 eV separates two energy levels in an atom. What is the frequency of radiation emitted when the atom make a transition from the upper level to the lower level?*

Sol. Here, difference of energy,

$$\Delta E = 2.3 \text{ eV} = 2.3 \times 1.6 \times 10^{-19} \text{ J}$$

and Planck's constant, $h = 6.62\times 10^{-34}$ J-s

$[\because 1\text{ eV} = 1.6\times 10^{-19}\text{ J}]$

Using the relation, $h\nu = \Delta E = E_2 - E_1$

or $$\nu = \frac{\Delta E}{h} = \frac{2.3\times 1.6\times 10^{-19}}{6.62\times 10^{-34}}$$

$$= 0.5559\times 10^{15}\text{ Hz} \approx 5.6\times 10^{14}\text{ Hz}$$

Example 12.16 *Determine the wavelength of the radiation required to excite the electron in Li^{++} from the first to the third Bohr orbit.*

Sol. Given, for Li^{++}, $Z = 3$ and as the excitation is from first to third Bohr orbit, so $n_1 = 1$, $n_2 = 3$

Using the relation,

$$\frac{1}{\lambda} = Z^2R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = (3)^2R\left(\frac{1}{1^2} - \frac{1}{3^3}\right) = 8R$$

$$\Rightarrow \text{Wavelength, } \lambda = \frac{1}{8R} = \frac{1}{8\times 1.097\times 10^7} = 0.114\times 10^{-7}$$

$$= 11.4\text{ nm}$$

Example 12.17 *A hydrogen atom initially in the ground level absorbs a photon, which excites it to the n = 4 level. Determine the wavelength and frequency of photon.*

Sol. If $E = h\nu$ be the energy absorbed, then

we know, $h\nu = \Delta E \Rightarrow h\nu = E_4 - E_1$

Now, using the relation, $E_n = \dfrac{-13.6}{n^2}$

For $n = 4$, $$E_4 = \frac{-13.6}{4^2}$$

$$= \frac{-13.6}{16} = -0.85\text{ eV} \quad \text{...(i)}$$

As, H-atom initially at ground level, for $n = 1$,

$[\because 1\text{eV} = 1.6\times 10^{-19}\text{ J and } h = 6.62\times 10^{-34}\text{ J}]$

$$E_1 = -13.6\text{ eV} \quad \text{...(ii)}$$

$\therefore$ From Eqs. (i) and (ii), we get

$$h\nu = -0.85 - (-13.6)$$

$$= 12.75\text{ eV}$$

$$= 12.75\times 1.6\times 10^{-19}\text{ J}$$

$$\nu = \frac{12.75\times 1.6\times 10^{-19}}{6.62\times 10^{-34}} = 3.8\times 10^{15}\text{ Hz}$$

Using the relation, $c = \nu\lambda$,

we get $$\lambda = \frac{c}{\nu} = \frac{3\times 10^8}{3.8\times 10^{15}} = 974.02\times 10^{-10}\text{ m} \approx 974\text{ Å}$$

Example 12.18 *A hydrogen like atom of atomic number Z is in an excited state of quantum number 2n. It can emit a maximum energy photon of 204 eV. If it makes a transition to quantum state n, a photon of energy 40.8 eV is emitted. Find n, Z and the ground state energy (in eV) for this atom. Also, calculate the minimum energy (in eV) that can be emitted by this atom during de-excitation. Ground state energy of hydrogen atom is 13.6 eV.*

Sol. Given, $E_{2n} - E_1 = 204$ eV

Using the relation,

$$\Delta E = -13.6Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

We get, $$(13.6)Z^2\left(1 - \frac{1}{4n^2}\right) = 204 \quad \text{...(i)}$$

$$E_{2n} - E_n = 40.8\text{ eV}$$

$\therefore$ $$13.6\,Z^2\left(\frac{1}{n^2} - \frac{1}{4n^2}\right) = 40.8 \quad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get, $n = 2$ and $Z = 4$

We know that, in ground state ($n = 1$), $E = -13.6Z^2$eV

So, $$E_1 = (-13.6)\,Z^2\text{ eV}$$

$$= (-13.6)\,(4)^2\text{ eV} = -217.6\text{ eV}$$

During de-excitation, minimum energy emitted is,

$$E_{\min} = E_{2n} - E_{2n-1} = E_4 - E_3$$

$$= \frac{-217.6}{4^2} - \left(\frac{-217.6}{3^2}\right) = 10.58\text{ eV}$$

Example 12.19 *A moving hydrogen atom makes a head on collision with a stationary hydrogen atom. Before collision both atoms are in ground state and after collision they move together. What is the minimum value of the kinetic energy of the moving hydrogen atom, such that one of the atoms reaches one of the excitation state?*

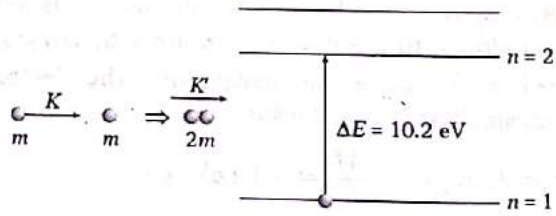
Sol. Let K be the kinetic energy of the moving hydrogen atom and K', the kinetic energy of combined mass after collision.

From conservation of linear momentum,

$$p = p' \text{ or } \sqrt{2Km} = \sqrt{2K'(2m)} \text{ or } K = 2K' \quad \text{...(i)}$$

From conservation of energy,

$$K = K' + \Delta E \quad \text{...(ii)}$$



Solving Eqs. (i) and (ii), we get

$$\Delta E = \frac{K}{2}$$

Using the relation,

$$\Delta E = -13.6Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

For $n_1 = 1$ and $n_2 = 2$, we get

$$\Delta E = -13.6(1)^2\left(\frac{1}{(1)^2} - \frac{1}{2^2}\right) = 10.2\text{eV} \quad (\because Z = 1)$$

So, minimum value of ΔE for hydrogen atom is 10.2 eV.

or $$\Delta E \geq 10.2 \text{ eV}$$

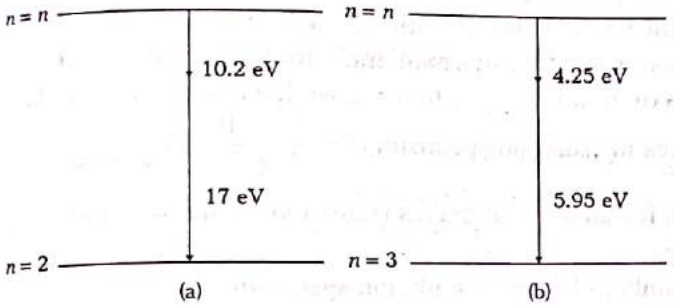
$\therefore$ $$\frac{K}{2} \geq 10.2$$

$\therefore$ $$K \geq 20.4 \text{ eV}$$

Therefore, the minimum kinetic energy of moving hydrogen is 20.4 eV.

Example 12.20 *A hydrogen like atom (atomic number Z) is in a higher excited state of quantum number n. It can emit two photons of energies 10.20 eV and 17.00 eV. When make transition to the first excited state. But emits photons of energies 4.25 eV and 5.95 eV, if make transition to second excited state. Determine Z and n.*

Sol. According to the question,



We know, energy of electron in nth orbit is given by

$$\Delta E = 13.6Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

***Case* I**

Energy emitted by photon, $\Delta E = 10.20 + 17$

$$= 27.2 \text{ eV}$$

So, $$10.2 + 17 = 13.6Z^2\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \quad (\because n_1 = 2, n_2 = n)$$

$$27.2 = 13.6Z^2\left(\frac{1}{4} - \frac{1}{n^2}\right) \quad \text{...(i)}$$

***Case* II**

Energy emitted by photons, $\Delta E = 4.25 + 5.95 = 10.2$ eV

$$4.25 \times 5.95 = 13.6Z^2\left(\frac{1}{3^2} - \frac{1}{n^2}\right) \quad (\because n_1 = 3, n_2 = n)$$

$$10.2 = 13.6Z^2\left(\frac{1}{9} - \frac{1}{n^2}\right) \quad \text{...(ii)}$$

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{27.2}{10.2} = \frac{1/4 - 1/n^2}{1/9 - 1/n^2}$$

$$\Rightarrow \quad \frac{27.2}{9} - \frac{27.2}{n^2} = \frac{10.2}{4} - \frac{10.2}{n^2}$$

$$\Rightarrow \quad \frac{17.2}{n^2} = \frac{27.9}{9} - \frac{10.2}{4}$$

$$\Rightarrow \quad n^2 = 36 \Rightarrow n = 6$$

Substituting the value of n in Eq. (i), we get

$$27.2 = 13.6Z^2\left(\frac{1}{4} - \frac{1}{n^2}\right) = 13.6Z^2\left(\frac{1}{4} - \frac{1}{6^2}\right)$$

$$= 13.6Z^2\left(\frac{1}{4} - \frac{1}{36}\right) = 13.6Z^2\left(\frac{9-1}{36}\right) = 13.6Z^2 \times \frac{2}{9}$$

$$Z^2 = 9 \Rightarrow Z = 3$$

Excitation energy and excitation potential

The energy required to take an atom from its lower state to higher state is known as excitation energy. For example 10.2 eV is excitation energy from $n = 1$ to $n = 2$ for H-atom. Also, 12.1 eV is excitation energy from $n = 1$ to 3. Similarly, the potential through which an electron should be accelerated to gain higher state is called excitation potential. For H-atom from $n = 1$ to $n = 2$ and $n = 1$ to 3 excitation potentials are 10.2 V and 12.1 V, respectively.

Ionisation energy and ionisation potential

The energy required to ionise an atom is called ionisation energy. It is the energy required to make the electron jump from the present orbit to the infinite orbit.

$$E_{\text{ionisation}} = E_\infty - E_n = 0 - \left[-13.6\frac{Z^2}{n^2}\right]$$

$$= +\frac{13.6Z^2}{n^2} \text{ eV}$$

Similarly, the potential difference through which an electron should be accelerated to get ionisation energy is called ionisation potential.

$$\therefore \text{Ionisation potential} = \frac{E_{\text{ionisation}}}{e} = \frac{13.6Z^2}{n^2} \text{ V}$$

Binding energy

The energy required to separate the constituent of a nucleus to infinity is called binding energy. The binding energy of hydrogen atom in ground state is therefore

$$\text{BE} = -(E_1) = -(-13.6 \text{ eV}) = 13.6 \text{ eV}.$$

Example 12.21 *A doubly ionised lithium atom is hydrogen like with atomic number 3. Find the wavelength of the radiation required to excite the electron in Li^{2+} from the first to the third Bohr orbit. The ionisation energy of the hydrogen atom is 13.6 eV.*

Sol. We know, $E_n' = -\frac{Z^2}{n^2}(13.6 \text{ eV})$

By putting $Z = 3$ (given), we have

$$E_n = -\frac{122.4}{n^2} \text{ eV}$$

As, excitation is from 1st to 3rd Bohr orbit
i.e., $n_1 = 1, n_2 = 3$.

$$\Rightarrow \quad E_1 = -\frac{122.4}{(1)^2} = -122.4 \text{ eV}$$

and $$E_3 = -\frac{122.4}{(3)^2} = -13.6 \text{ eV}$$

$$\therefore \quad \Delta E = E_3 - E_1 = 108.8 \text{ eV}$$

The corresponding wavelength is,

$$\lambda = \frac{12375}{\Delta E \text{ (in eV)}} \text{ Å} = \frac{12375}{108.8} \text{ Å} = 113.74 \text{ Å}$$

Absorption and emission spectrum

An atom can be excited by incident photon or by collision. If an atom absorbs a photon, it goes to higher orbits/state and remains, there for a time of 10^{-8} second and returns back to ground state by different ways.

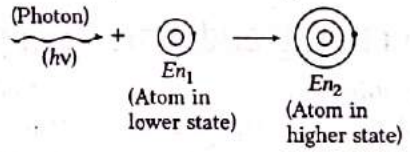


Fig. 12.14 Absorption of photon by an atom

From the above diagram, it is clear that

$$En_1 + h\nu = En_2$$

Let us now consider an example in which a photon of energy 12.09 eV is incident on hydrogen atom in ground state.

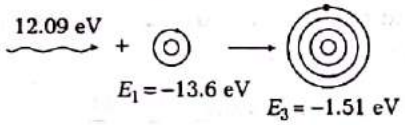


Fig. 12.15 Absorption of photon of energy 12.09 eV by an atom

Energy of the atom after absorption

$$= 12.09 + (-13.6) = -1.51 \text{ eV}$$
$$= E_3$$

After absorbing photon the atom goes to state $n = 3$, hence in this absorption spectrum we have single line as shown below

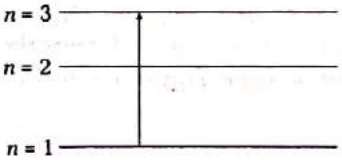


Fig. 12.16 Absorption spectrum

Now, in this state $n = 3$, the atom remains for the time 10^{-8}s and then returns to ground state by emitting photon(s). This atom can come directly to $n = 1$ by emitting one photon or by emitting two photons from $n \to 2$, $2 \to 1$. Therefore, possible transitions are

$3 \to 2$, $2 \to 1$, $3 \to 1$, as shown in the figure

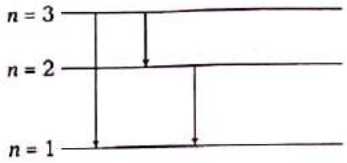


Fig. 12.17 Emission spectrum

From the figure it is clear that, number of emission spectrum equal to two. Out of three transitions, frequency of emitted photon will be maximum corresponding to $3 \to 1$ (since, $\Delta E = h\nu$) therefore wavelength will be minimum. In the transition $3 \to 2$, frequency of emitted photon will be minimum and hence wavelength will be maximum. For $n = 3$ to $n = 1$, we have seen that number of lines in emission spectrum $N = \frac{3(3-1)}{2} = 3$

So, for an atom to makes transition from $n = n_1$ to $n = n_2$, then

Number of lines in emission spectrum,

$$N = \frac{(n_1 - n_2)(n_1 - n_2 + 1)}{2}$$

Putting $n_1 = n$ and $n_2 = 1$, this reduces to $N = \frac{n(n-1)}{2}$

Note Emission/absorption of photons only takes place when the energy of the photon is exactly equal to the energy level difference.

Example 12.22 *Monochromatic radiation of wavelength λ is incident on a hydrogen sample in ground state. Hydrogen atoms absorb a fraction of light and subsequently emit radiation of ten different wavelengths. Find the value of λ.*

Sol. From nth energy state a total of $\frac{n(n-1)}{2}$ lines are emitted.

$$\therefore \quad \frac{n(n-1)}{2} = 10 \quad \text{or} \quad n = 5$$

Further, $$E_n = \frac{E_1}{n^2}$$

$$\therefore \quad E_5 = \frac{-13.6}{(5)^2} = -0.544 \text{ eV}$$

The energy needed to take a hydrogen atom from its ground state to $n = 5$ is,

$$\Delta E = E_5 - E_1$$
$$= -0.54 - (-13.6)$$
$$= 13.06 \text{ eV}$$

The corresponding wavelength of light,

$$\lambda = \frac{12375}{\Delta E} \text{ Å} = \frac{12375}{13.06} = 947.5 \text{ Å}$$

Example 12.23 *Ionisation potential of hydrogen atom is 13.6 V. Hydrogen atoms in the ground state are excited by monochromatic radiation of photon energy is 12.1 eV. According to Bohr's theory, find the possible spectral lines emitted by hydrogen.*

Sol. Ionisation energy corresponding to ionisation potential

$$= -13.6 \text{ eV}$$

Photon energy incident $= 12.1$ eV

So, the energy of electron in excited state

$$= -13.6 + 12.1 = -1.5 \text{ eV}$$

i.e., $$E_n = -\frac{13.6}{n^2} \text{ eV}$$

$$-1.5 = \frac{-13.6}{n^2} \Rightarrow n^2 = \frac{-13.6}{-1.5} = 9 \Rightarrow n = 3$$

i.e., the energy of electron in excited state corresponds to third orbit.

The possible spectral lines are when electron jumps from orbit 3rd to 2nd, 3rd to 1st and 2nd to 1st. Thus, three spectral lines are emitted.

Example 12.24 *An electron is moving in an orbit of a hydrogen atom from which there can be a maximum of six transition. An electron is moving in an orbit of another hydrogen atom from which there can be a maximum of three transition. Find ratio of the velocities of the electron in these two orbits.*

Sol. Number of spectral lines obtained due to transition of electrons from nth orbit to lower orbit is given by

$$N = \frac{n(n-1)}{2}$$

Case I $$6 = \frac{n_1(n_1 - 1)}{2}$$

$$\Rightarrow \quad n_1^2 - n_1 - 12 = 0 \Rightarrow (n_1 - 4)(n_1 + 3) = 0 \Rightarrow n_1 = 4$$

because n_1 cannot be negative *i.e.,* -3.

Case II $$3 = \frac{n_2(n_2 - 1)}{2}$$

$$\Rightarrow \quad n_2^2 - n_2 - 6 = 0 \Rightarrow (n_2 + 2)(n_2 - 3) = 0 \Rightarrow n_2 = 3$$

because n_2 cannot be negative *i.e.,* -2.

Velocity of electron in hydrogen atom in nth orbit,

$$v_n \propto \frac{1}{n} \Rightarrow \frac{v_n}{v_n'} = \frac{n_2}{n_1} \Rightarrow \frac{v_6}{v_3} = \frac{3}{4}$$

4 Hydrogen spectrum or the line spectra of hydrogen atom

Hydrogen spectrum consists of discrete bright lines in a dark background and it is specifically known as **hydrogen emission spectrum.**

There is one more type of hydrogen spectrum that exists where we get dark lines on the bright background, it is known as **absorption spectrum.**

Balmer find an empirical formula by the observation of a small part of this spectrum and it is represented by

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right), \text{ where } n = 3, 4, 5, \ldots$$

where, R is a constant called **Rydberg constant** and its value is $1.097 \times 10^7 \text{ m}^{-1}$.

So, $$\lambda = 1.522 \times 10^6 \text{ m}^{-1} = 656.3 \text{ nm for } n = 3$$

Other series of spectra for hydrogen were subsequently discovered and known by the name of their discoverers.

The lines of Balmer series are found in the visible part of the spectrum. Other series were found in the invisible parts of the spectrum. e.g. Lyman series in the ultraviolet region and Paschen, Brackett and Pfund in the infrared region.

The wavelengths of line in these series can be expressed by the following formulae.

(i) For Lyman series

$$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right), \text{ where } n = 2, 3, 4, \ldots$$

(ii) For Balmer series

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right), \text{ where } n = 3, 4, 5, \ldots$$

(iii) For Paschen series

$$\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right), \text{ where } n = 4, 5, 6, \ldots$$

(iv) For Brackett series

$$\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n^2}\right), \text{ where } n = 5, 6, 7, \ldots$$

(v) For Pfund series

$$\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n^2}\right), \text{ where } n = 6, 7, 8, \ldots$$

Explanation The different series of hydrogen spectrum can be explained by Bohr's theory. According to Bohr's theory, if the ionised state of hydrogen atom be taken as zero energy level (at ∞) then the energies of the different energy levels of the atom can be expressed by the following formula.

$$E_n = -\frac{Rhc}{n^2}, \text{ where } n = 1, 2, 3, \ldots$$

where, R is Rydberg's constant and h is Planck's constant. The integer n is called quantum number.

The energy levels of hydrogen atom are shown in the figure.

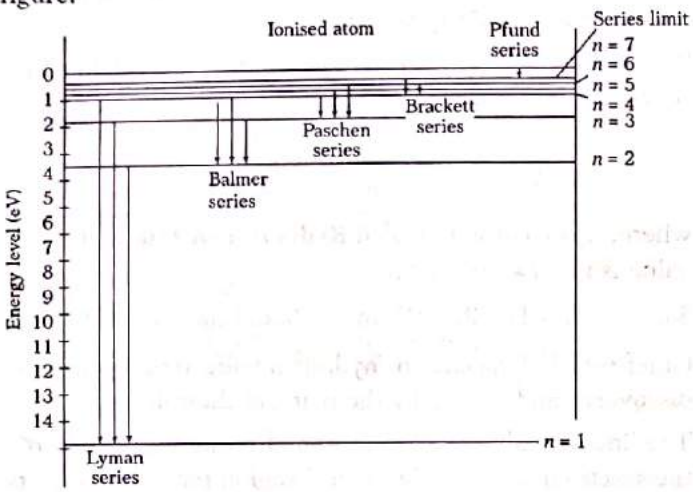


Fig. 12.18 Emission spectrum of H-atom

When the atom gets energy from outside, its electron goes from the lower energy level to some higher energy level. But it returns from there, within 10^{-8} s, to the lower energy level directly or through other lower energy levels. While returning back, the atom emits photons.

Series limit

The transitions corresponding to maximum energy or smallest wavelength of a given series of hydrogen spectrum is called series limit.

To calculate wavelength corresponding to series limit, we use $n_2 = \infty$.

For Lyman $\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{\infty}\right) \Rightarrow \lambda = \frac{1}{R}$

For Balmer $\frac{1}{\lambda} = \frac{R}{4}$

In general series limit $\frac{1}{\lambda} = \frac{R}{n^2}$

where, $n = 1, 2, 3, 4, \ldots$

Note In Balmer series, first line is also called H_α line, second line as H_β line and so on.

Example 12.25 *A 12.5 eV electron beam is used to bombard gaseous hydrogen at room temperature. What series of wavelengths will be emitted?*

Sol. Energy of electron beam, $E = 12.5$ eV

Planck's constant, $h = 6.62 \times 10^{-34}$ J

So, using the relation, $E = \frac{hc}{\lambda}$, we get

$$\lambda = \frac{hc}{E} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{12.5 \times 1.6 \times 10^{-19}}$$
$$= 0.993 \times 10^{-7} \text{ m}$$
$$= 993 \times 10^{-10} \text{ m} = 993 \text{ Å}$$

This wavelength falls in the range of Lyman series (912 Å to 1216 Å), thus we conclude that Lyman series is emitted.

Example 12.26 *In H-atom, a transition takes place from $n = 3$ to $n = 2$ orbit. Calculate the wavelength of the emitted photon, will the photon be visible? To which spectral series will this photon belong? (Take, $R = 1.097 \times 10^7 m^{-1}$)*

Sol. The wavelength of the emitted photon is given by

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

When the transition takes place from $n = 3$ to $n = 2$, then

$$\frac{1}{\lambda} = (1.097 \times 10^7)\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$
$$= 1.097 \times 10^7 \times \frac{5}{36}$$
$$\therefore \quad \lambda = \frac{36}{1.097 \times 10^7 \times 5} = 6.563 \times 10^{-7} \text{m}$$
$$= 6563 \text{ Å}$$

λ falls in the visible (red) part of the spectrum, hence the photon will be visible. This photon is the first member of the Balmer series.

Example 12.27 *What is the shortest wavelength present in the Paschen series of spectral lines?*

Sol. For Paschen series, $n_1 = 3$, $n_2 = \infty$ for shortest wavelength. If λ_{min}, be the minimum wavelength in this series of spectral lines, then

$$\frac{1}{\lambda_{min}} = R\left(\frac{1}{3^2} - \frac{1}{\infty^2}\right) = \frac{R}{9}$$

or $$\lambda_{min} = \frac{9}{R} = \frac{9}{1.097 \times 10^7} \text{ m}$$
$$= 8204.2 \times 10^{-10} \text{ m}$$

Example 12.28 *Find the largest and shortest wavelengths in the Brackett series for hydrogen. In what region of the electromagnetic spectrum does each series lie?*

Sol. The transition equation for Brackett series is given by

$$\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n^2}\right), \; n = 2, 3, \ldots$$

The largest wavelength is corresponding to $n = 5$.

$$\therefore \quad \frac{1}{\lambda_{max}} = 1.097 \times 10^7\left(\frac{1}{16} - \frac{1}{25}\right)$$
$$= 0.0246 \times 10^7 \text{ m}$$
$$\therefore \quad \lambda_{max} = 40.650 \times 10^{-7} \text{m} = 40650 \text{ Å}$$

The shortest wavelength corresponds to $n = \infty$.

$$\therefore \quad \frac{1}{\lambda_{min}} = 1.097 \times 10^7\left(\frac{1}{4^2} - \frac{1}{\infty}\right)$$

or $$\lambda_{min} = 14.58 \times 10^{-7} \text{ m} = 14585 \text{ Å}$$

Both of these wavelengths lie in infrared region of electromagnetic spectrum.

Example 12.29 *If the series limit wavelength of the Lyman series for hydrogen atom is 912 Å, then find the series limit wavelength for the Balmer series for the hydrogen atom.*

Sol. For series limit of Balmer series, we can write,

$$n_2 = 2, n_1 = \infty$$

$$\Rightarrow \quad \frac{1}{\lambda} = R\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right) = R\left[\frac{1}{(2)^2} - \frac{1}{(\infty)^2}\right] = \frac{R}{4}$$

$$\therefore \quad \lambda = \frac{4}{R} = \frac{4}{10967800} = 4 \times 912 \times 10^{-10}\text{m}$$

$$= 4 \times 912 \text{ Å} = 3648 \text{ Å}$$

Example 12.30 *Calculate (i) the wavelength and (ii) the frequency of the H_β line of the Balmer series for hydrogen.*

Sol. (i) H_β line of Balmer series corresponds to the transition from $n = 4$ to $n = 2$ level. Using Eq. (xiii), the corresponding wavelength for H_β line is,

$$\frac{1}{\lambda} = (1.097 \times 10^7)\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = 0.2056 \times 10^7$$

$$\therefore \quad \lambda = 4.9 \times 10^{-7} \text{ m}$$

(ii) Further, $f = \frac{c}{\lambda} = \frac{3 \times 10^8}{4.9 \times 10^{-7}} = 6.12 \times 10^{14}$ Hz

Example 12.31 *The wavelength of the first line of Lyman series for hydrogen is identical to that of the second line of Balmer series for some hydrogen like ion Y. Calculate energies of the first three levels of Y.*

Sol. Wavelength of the first line of Lyman series for hydrogen atom ($Z = 1$) will be given by the equation,

$$\frac{1}{\lambda_1} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3R}{4} \quad \text{...(i)}$$

The wavelength of second Balmer line for hydrogen like ion Y is

$$\frac{1}{\lambda_2} = RZ^2\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \frac{3RZ^2}{16} \quad \text{...(ii)}$$

Given that, $\lambda_1 = \lambda_2$ or $\frac{1}{\lambda_1} = \frac{1}{\lambda_2}$

i.e., $\frac{3R}{4} = \frac{3RZ^2}{16}$

$\therefore \quad Z = 2$

i.e., Y ion is He^+. The energies of first four levels of Y are,

$$E_1 = -(13.6)\, Z^2 = -54.4 \text{ eV}$$

$$E_2 = \frac{E_1}{(2)^2} = -13.6 \text{ eV}$$

and $E_3 = \frac{E_1}{(3)^2} = -6.04$ eV

Example 12.32 *Hydrogen atom in its ground state is excited by a radiation of $\lambda = 990.7$ Å. How many different wavelength are possible in the resulting emission spectrum? Find the longest wavelength amongst these.*

Sol. Wavelength of radiation, $\lambda = 990.7$ Å

$= 99$ nm

As we know, $\Delta E = \frac{1242 \text{ eV}}{\lambda(\text{nm})} = \frac{1242}{99} = 12.45$ eV

Using the relation, $\Delta E = 13.6 Z^2\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$

($\because n = 1, Z = 1$ because H-atom is in ground state)

$$12.45 = 13.6(1)^2\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$$

$$\Rightarrow \quad n = 3$$

Number of wavelengths in resulting spectrum,

$$\frac{n(n-1)}{2} = \frac{3(3-1)}{2} = 3$$

and we also know,

$$\lambda \propto \frac{1}{\Delta E}$$

So, for ΔE to be minimum, it will be in the transition $3 \rightarrow 2$.

Using the relation, $\Delta E = 13.6\, Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$$= 13.6.(1)^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 13.6 \times \frac{5}{36}$$

$$= 1.88 \text{ eV}$$

So, the wavelength, $\lambda = \frac{1242}{1.88} = 660.63$ nm.

Limitations of Bohr's model

The limitation of Bohr's Model are as follows

(i) This model is applicable only to a single electron atoms/ions.

(ii) It does not explain the fine structure of spectral lines. Application of external magnetic field (Zeeman effect) and external electric field (Stark effect) to radiating atoms could not be explained.

(iii) Bohr's assumption regarding stationary orbits is against Heisenberg uncertainty principle.

(iv) No explanation is given regarding principle of quantisation of angular momentum.

(v) This models does not explain why orbits of electrons are taken as circular whereas elliptical orbits are also possible.

de-Broglie explanation of Bohr's second postulate of quantisation

According to de-Broglie, a stationary orbit is that which contains an integral number of de-Broglie standing waves associated with the revolving electron.

For an electron revolving in nth circular orbit of radius r_n,

Total distance covered = circumference of the orbit = $2\pi r_n$

$\therefore$ For the permissible orbit, $2\pi r_n = n\lambda$

According to de-Broglie wevelength, $\lambda = \dfrac{h}{mv_n}$

where, v_n is speed of electron revolving in nth orbit

$$\therefore \quad 2\pi r_n = \frac{nh}{mv_n} \text{ or } mv_n r_n = \frac{nh}{2\pi} = n(h/2\pi)$$

i.e., angular momentum of electron revolving in nth orbit must be an integral multiple of $h/2\pi$, which is the quantum condition proposed by Bohr's in second postulate.

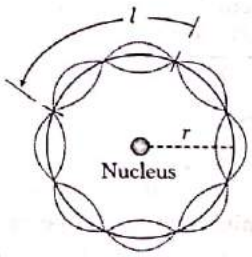


Fig. 12.19 *A standing wave is shown on a circular orbit*

Example 12.33 *An electron of a hydrogen like atom is in excited state. If total energy of the electron is –4.6 eV, then evaluate*

(i) the kinetic energy and

(ii) the de-Broglie wavelength of the electron.

Sol. Given, total energy of electron, $E = -4.6$ eV

(i) Kinetic energy of electron, $K = -$(Total energy E,)

$$\Rightarrow \quad = -E = -(-4.6) = 4.6 \text{ eV}$$

(ii) de-Broglie wavelength,

$$\lambda_d = \frac{h}{\sqrt{2mK}} = \frac{6.6\times 10^{-34}}{\sqrt{2\times 9.1\times 10^{-31}\times 4.6\times 1.6\times 10^{-19}}}$$

$$= 0.57\times 10^{-9} \text{ m} = 0.57 \text{ nm}$$

Example 12.34 *The wavelength of light from the spectral emission line of sodium is 589 nm. Find the kinetic energy at which*

(i) an electron, and

(ii) a neutron, would have the same de-Broglie wavelength.

Sol. Given Planck's constant, $h = 6.62\times 10^{-34}$ J,

mass of electron, $m_e = 9.1\times 10^{-31}$ kg

Mass of neutron, $m_n = 1.67\times 10^{-27}$ kg

(i) Let E_1 = KE of electron

Using the relation, $\lambda = \dfrac{h}{\sqrt{2m_e E}}$, we get

$$\Rightarrow \quad \lambda^2 = \frac{h^2}{2m_e E}$$

$$\therefore \quad E_1 = \frac{h^2}{2m_e\lambda^2} = \frac{(6.62\times 10^{-34})^2}{2\times 9.1\times 10^{-31}\times (589\times 10^{-9})^2}$$

$$= 6.94\times 10^{-25} \text{ J}$$

$$= \frac{6.94\times 10^{-25}}{1.6\times 10^{-19}} \text{ eV} \qquad [\because 1\text{eV} = 1.6\times 10^{-19}\text{J}]$$

$$= 4.34\times 10^{-6} \text{ eV}$$

(ii) E_2 = KE of neutron

$$\Rightarrow \quad E_2 = \frac{h^2}{2m_n\lambda^2} = \frac{(6.62\times 10^{-34})^2}{2\times 1.67\times 10^{-27}\times (589\times 10^{-9})^2}$$

$$= 3.782\times 10^{-28} \text{ J}$$

$$= \frac{3.782\times 10^{-28}}{1.6\times 10^{-19}} \text{ eV} = 2.36\times 10^{-9} \text{ eV}$$

Example 12.35 *Let us assume that the de-Broglie wave associated with an electron forms a standing wave between the atoms arranged in a one-dimensional array with nodes at each of the atomic sites. It is found that one such standing wave is formed, if the distance between the atoms of the array is 3Å. A similar standing wave is again formed, if d is increased to 3.5Å but not for any intermediate value of d. Find the energy of the electrons in electron volts and the least value of d for which the standing wave of the type described above can form.*

Sol. For standing waves, the separation between consecutive nodes or antinodes.

$$d = n\frac{\lambda}{2} \qquad \text{...(i)}$$

$$n = 1, 2, \ldots$$

For next node,

$$d' = (n+1)\frac{\lambda}{2} \qquad \text{...(ii)}$$

$$d' - d = \frac{\lambda}{2} \Rightarrow 3.5 - 3 = \frac{\lambda}{2} \Rightarrow \lambda = 1\text{Å}$$

de-Broglie wavelength, $\lambda = \dfrac{h}{\sqrt{2mK}}$

$$\Rightarrow \quad K = \frac{h^2}{2m\lambda^2} = \frac{(6.6\times 10^{-34})^2}{2(9.1\times 10^{-31})\times (10^{-10})^2} = 2.415\times 10^{-17}\text{J}$$

$$= \frac{2.415\times 10^{-17}}{1.6\times 10^{-19}} = 150 \text{ eV}$$

For minimum value of d, $n = 1$

$$\therefore \quad d = n\frac{\lambda}{2} = \frac{\lambda}{2} = \frac{1}{2} = 0.5\text{Å}$$

Example 12.36 *Hydrogen gas in the atomic state is excited to an energy level such that the electrostatic potential energy becomes –3.02 eV. Now, the photoelectric plate hoving W = 4.6 eV is exposed to the emission spectra of this gas. Assuming all the transition to be possible, find the minimum de-Broglie wavelength of ejected photoelectrons.*

Sol. Potential energy, $U = -3.02$ eV

We know,

$$\text{Total energy, } E = \frac{\text{Potential energy } (U)}{2} = \frac{-3.02}{2} = -1.511 \text{ eV}$$

Using the relation, $E_n = \dfrac{-13.6}{n^2}$ eV

$$\Rightarrow \quad -1.511 = \frac{-13.6}{n^2} \Rightarrow n^2 = 9 \Rightarrow n = 3$$

Three transitions are possible *i.e.*, $3 \to 2, 2 \to 1, 3 \to 1$. The emission will take place for those transition in which $\Delta E > 1$.

Further, using the relation, $\Delta E = 13.6 Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

For $3 \to 2$,

$$\Delta E = -13.6\left(\frac{1}{3^2} - \frac{1}{2^2}\right) = -13.6 \times 5/36 = +1.88 \text{ eV}$$

For $3 \to 1$,

$$\Delta E = -13.6\left(\frac{1}{3^2} - \frac{1}{1^2}\right) = +13.6 \times \frac{8}{9} = 12.08 \text{ eV}$$

Minimum KE of ejected electron is possible in $3 \to 1$.

So, according to the relation, $\Delta E = \phi + K_{max}$

where, ϕ = work function $12.08 = 4.6 + K_{max}$

$$\Rightarrow \quad K_{max} = 7.48 \text{ eV}$$

$\therefore$ Maximum de-Broglie wavelength $\lambda_d = \frac{h}{\sqrt{2mK_{max}}}$

$$= \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 7.48 \times 1.6 \times 10^{-19}}}$$

$$= 0.45 \text{ nm}$$

Check point 12.2

1. Kinetic energy of electron in nth orbit is given by
(a) $\frac{Rhc}{2n^2}$ (b) $\frac{2Rhc}{n}$ (c) $\frac{Rhc}{n}$ (d) $\frac{Rhc}{n^2}$

2. The ground state energy of hydrogen atom is –13.6 eV. The kinetic energy of the electron in this state is
(a) 1.85 eV (b) 13.6 eV (c) 6.8 eV (d) 3.4 eV

3. Potential energy (PE_n) and kinetic energy (KE_n) of electron in nth orbit are related as
(a) $PE_n = KE_n$ (b) $PE_n = -2KE_n$
(c) $PE_n = 2KE_n$ (d) $PE_n = -KE_n$

4. The ground state energy of hydrogen atom is –13.6 eV. What is the potential energy of the electron in this state?
(a) 0 eV (b) –27.2 eV (c) 1 eV (d) 2 eV

5. In the nth orbit, the energy of an electron, $E_n = -\frac{13.6}{n^2}$ eV for hydrogen atom. The energy required to take the electron from first orbit to second orbit will be
(a) 10.2 eV (b) 12.1 eV
(c) 13.6 eV (d) 3.4 eV

6. In hydrogen atom, if the difference in the energy of the electron in $n = 2$ and $n = 3$ orbits is E, the ionisation energy of hydrogen atom is
(a) $13.2E$ (b) $7.2E$ (c) $5.6E$ (d) $3.2E$

7. Ionisation potential (IP) and ionisation energy (IE) are related as
(a) IP = (IE)e (b) $IP = \frac{IE}{e}$ (c) $IE = \frac{IP}{e^2}$ (d) $IP = \frac{IE}{e^2}$

8. Atomic hydrogen is excited from the ground state to the nth state. The number of lines in the emission spectrum will be
(a) $\frac{n(n+1)}{2}$ (b) $\frac{n(n-1)}{2}$ (c) $\frac{(n-1)^2}{2}$ (d) $\frac{(n+1)^2}{2}$

9. Which one of the series of hydrogen spectrum is in the visible region?
(a) Lyman series (b) Balmer series
(c) Paschen series (d) Brackett series

10. In which of the following systems will the wavelength corresponding to $n = 2$ to $n = 1$ be minimum?
(a) Hydrogen atom (b) Deuterium atom
(c) Singly ionised helium (d) Doubly ionised lithium

11. The ratio of the largest to shortest wavelengths in Balmer series of hydrogen spectra is
(a) $\frac{25}{9}$ (b) $\frac{17}{6}$ (c) $\frac{9}{5}$ (d) $\frac{5}{4}$

12. In which of the following transitions will the wavelength be minimum?
(a) $n = 5$ to $n = 4$ (b) $n = 4$ to $n = 3$
(c) $n = 3$ to $n = 2$ (d) $n = 2$ to $n = 1$

13. Wavelength corresponding to series limit is
(a) $\frac{1}{\lambda} = \frac{2R}{n}$ (b) $\frac{1}{\lambda} = \frac{R^2}{n^2}$ (c) $\frac{1}{\lambda} = \frac{R}{n^2}$ (d) $\frac{1}{\lambda} = \frac{R}{n}$

14. For the Bohr's first orbit of circumference $2\pi r$, the de-Broglie wavelength of revolving electron will be
(a) $2\pi r$ (b) πr (c) $\frac{1}{2\pi r}$ (d) $\frac{1}{4\pi r}$

15. Which of the following is incorrect regarding limitations of Bohr's model?
(a) This model is applicable only to single electron system
(b) It does not explain fine structure of spectral lines
(c) It's assumption regarding stationary orbits supports Heisenberg uncertainty principle
(d) None of the above

5 X-rays

In 1895, **Wilhelm Roentgen** found that a highly penetrating radiation of unknown nature is produced when fast moving electrons strike a target of high atomic number and high melting point.

These radiations were given a name X-rays as their nature was unknown (in mathematics an unknown quantity is normally designated by X).

Later it was discovered that these are high energy photons (or electromagnetic waves).

Hence, X-rays are a form of electromagnetic radiation with wavelengths ranging from 0.1 Å to 100 Å.

However, the boundaries of this category are not sharp. The shorter wavelength end overlaps gamma rays and the longer wavelength end overlaps ultraviolet rays *i.e.*, wavelengths are in the order of $\lambda_{\gamma\text{-rays}} < \lambda_{\text{X-rays}} < \lambda_{\text{UV-rays}}$

Production of X-rays

Figure shows a diagram of a highly evacuated glass tube called filament, X-ray tube or the **collidge tube**.

A cathode (a plate connected to negative terminal of a battery) is heated by a tungsten filament through which an electric current is passed, supplies electrons by thermionic emission.

The filament is coated with oxides of barium or strontium to have an emission of electrons even at low temperature.

The high potential difference V maintained between the cathode and a metallic target accelerate the electrons toward the target.

The target is a material of high atomic weight, high melting point and high thermal conductivity made of tungsten or molybdenum and is embedded in a copper block.

The face of the target is at an angle relative to the electron beam, and the X-rays that leave the target pass through the side of the tube.

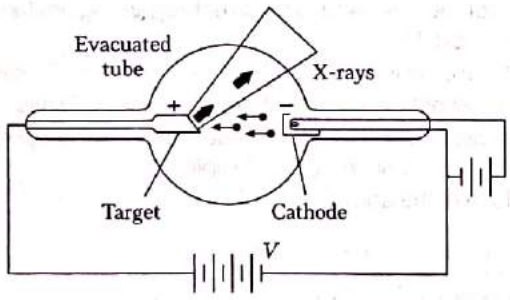


Fig. 12.20 An X-ray tube

Kinetic energy of electron is converted into energy of photon during the production of X-rays. The higher the accelerating voltage V, the electrons and the shorter the wavelengths of the X-rays and hence greater will be its penetrating power, due to higher energy.

Properties of X-rays

Properties of X-rays are given below

(i) X-rays are electromagnetic waves with wavelength range 0.1 Å-100 Å.
(ii) X-rays are invisible.
(iii) They travel in straight line with speed of light.
(iv) They show all important properties of light rays such as reflection, refraction, interference, diffraction and polarisation, etc.
(v) X-rays are not deflected by electric and magnetic fields.
(vi) These show continuous spectrum.
(vii) X-rays carry no charge.
(viii) Ionisation power of the X-rays is measured in Roentgen.
(ix) They do not pass through heavy metals and bones.
(x) They effect the photographic plates.
(xi) Lead is best absorber of X-rays.
(xii) Long exposure to X-rays is injurious for human body.
(xiii) The wavelength of X-rays is very small as compared to the wavelength of light.

Classification of X-rays

X-rays are of two types, continuous and characteristic. While the former depends only on the accelerating voltage V, the later depends on the target used.

(i) Continuous X-rays

When accelerated electrons enters into the target material, it loses its KE and finally is brought to rest. Inside the material, may after several collisions. A part of this lost KE is converted into a photon, remaining part goes into heating the target. The energy of photon formed may be anything between 0 and eV, depending on energy lost. Wavelength is minimum, when energy loss is maximum.

$$\lambda_{\min} = \frac{hc}{eV}$$

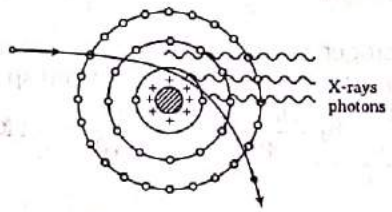


Fig. 12.21 Emission of X-ray photons forming continuous X-ray spectrum

The continuous X-rays (or bremsstrahlung X-rays) produced at a given accelerating potential V vary in wavelength, but none has a wavelength shorter than a certain value λ_{min}. This minimum wavelength corresponds to the maximum energy of the X-rays which in turn is equal to the maximum kinetic energy qV or eV of the striking electrons. Thus,

$$\frac{hc}{\lambda_{min}} = eV \quad \text{or} \quad \lambda_{min} = \frac{hc}{eV}$$

After substituting values of h, c and e, we obtain the following simple formula for λ_{min}.

$$\lambda_{min}\ (\text{in Å}) = \frac{12375}{V} \qquad ...(i)$$

If V is increased, then λ_{min} decreases. This wavelength is also known as the **cut-off wavelength** or the **threshold wavelength**.

Intensity-wavelength graph

Following figure shows the intensity of X-ray variation with different wavelength for various accelerating voltages applied to X-ray tube. From the graph, it is clear that the wavelengths at which the intensity is maximum depends on the accelerating voltage, being shorter for higher voltage and vice-versa.

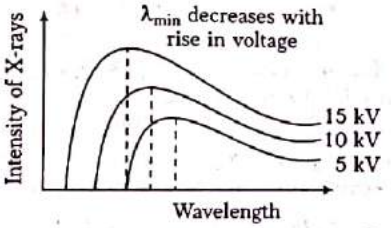


***Fig. 12.22** Variation of intensity of X-rays with wavelength*

Example 12.37 *Find the cut-off wavelength for the continuous X-rays coming from an X-ray tube operating at 40 kV.*

Sol. Cut-off wavelength λ_{min} is given by

$$\lambda_{min}\ (\text{in Å}) = \frac{12375}{V(\text{in volts})} = \frac{12375}{40 \times 10^3} = 0.31\ \text{Å}$$

Example 12.38 *An X-ray tube is operated at 30 kV. If a particular electron loses 10% of its kinetic energy to emit an X-ray photon during the collision. Find the wavelength and maximum frequency associated with this photon.*

Sol. Kinetic energy of electron = 30 keV

According to the question, 10% of this energy is converted to photon which is given by

$$\frac{10}{100} \times 30 = 3\ \text{kV}$$

$$\therefore \quad \lambda_{min}(\text{in Å}) = \frac{12375}{V(\text{in volt})} = \frac{12375}{3 \times 10^3} = 4.12\ \text{Å}$$

Also, $$\nu_{max} = \frac{c}{\lambda_{min}} = \frac{3 \times 10^8}{(4.12 \times 10^{-10})} = 7.2 \times 10^{17}\ \text{Hz}$$

Example 12.39 *The operating voltage in an X-ray tube is increased to 3 times the original value when the short wavelength limits shifts by 25 nm. Find the original value of the operating voltage.*

Sol. As we know, $\frac{hc}{\lambda} = eV \Rightarrow \lambda \propto \frac{1}{V}$

$$\therefore \quad \frac{\lambda_1}{\lambda_2} = \frac{V_2}{V_1}$$

According to the question,

$$\frac{\lambda}{\lambda - 25} = \frac{3V}{V}$$

$$\Rightarrow \quad \lambda = 3\lambda - 3 \times 25$$

$$\Rightarrow \quad \lambda = 37.5\ \text{nm} = 375\ \text{Å}$$

$$\therefore \quad E(eV) = \frac{12375}{\lambda\ (\text{in Å})} = \frac{12375}{375} = 33\ \text{eV}$$

$$\Rightarrow \quad eV_1 = 33\ \text{eV}$$

$$\Rightarrow \quad V_1 = 33\ \text{V}$$

Example 12.40 *The current flowing through the X-ray tube which is operating at 25 kV is 1mA. Calculate the number of electrons hitting the target per second. Also, find the energy falling on the target per second.*

Sol. Current flowing in X-ray tube = 1 mA

Potential difference = 25 kV

Current (I) is given by $I = ne$

where, n = number of electrons hitting the target per second.

$$\therefore \quad n = \frac{I}{e} = \frac{1 \times 10^{-3}}{1.6 \times 10^{-19}} = 6.25 \times 10^{15}$$

Now, the energy falling on the target per second is equal to the power consumed by the X-ray tube *i.e.*, $P = VI$

Here, $V = 25\ \text{kV} = 25 \times 10^3\ \text{V}$

$$I = 1\ \text{mA} = 1 \times 10^{-3}\ \text{A}$$

$$\therefore \quad P = (25 \times 10^3)(1 \times 10^{-3}) = 25\ \text{W}$$

(ii) Characteristic X-rays

Some of the fast moving electrons with high velocity penetrate the surface atoms of the target material and knock out the electrons even from the inner most shells of atom. Now, a vacancy is created at that place.

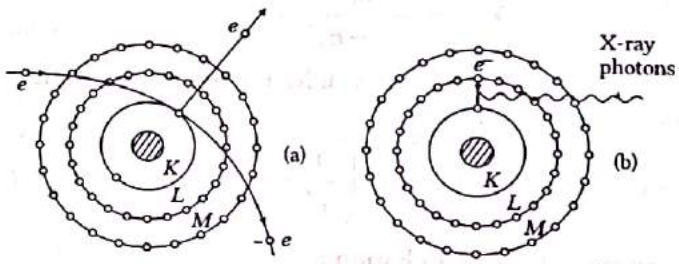


***Fig. 12.23** (a) Electron knocked out from inner shell (b) Emission of X-ray photons forming characteristic X-ray spectrum*

Electrons from higher shell jumps to fill the created vacancy. When the electron jumps from a higher energy level E_2 to lower energy orbit E_1, it radiates energy $(E_2 - E_1)$. Thus, this energy difference is radiated in the form of X-rays of very small but definite wavelength which depends upon target material. The X-ray spectrum consists of sharp lines and is called **characteristic X-ray spectrum.**

***K, L, M* ... series** If the electron striking the target ejects an electron from the *K*-shell of the target atom, a vacancy is created in the *K*-shell. Immediately an electron from one of the outer shell, jumps to *K*-shell emitting an X-ray photon of energy equal to the energy difference between the two shells.

The X-ray photons emitted due to the jump of electron from the *L, M, N* shell to the *K*-shells gives $K_\alpha, K_\beta, K_\gamma$ lines respectively of the *K*-series of the spectrum.

If the electron striking the target ejects an electron from the *L*-shell of the target atom, an electron from the *M, N*, ... shells jumps to the *L*-shell, so that X-ray photons of lesser energy are emitted. In a similar way, the formation of *M*-series, *N*-series, etc, can be explained.

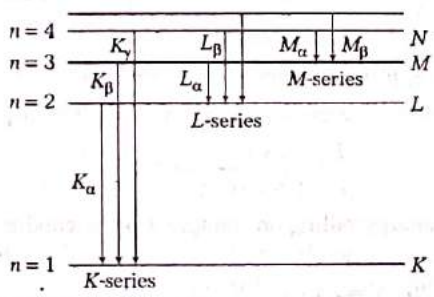


Fig. 12.24 *Energy level diagram for K, L and M-series*

From the above energy level diagram we have

$$E_K > E_L > E_M > E_N \ldots$$

Where $E_K, E_L, E_M, E_N, \ldots$ are magnitudes of energies of different levels.

Now, we can write, $\lambda_{K_\alpha} = \dfrac{hc}{E_K - E_L}, \lambda_{K_\beta} = \dfrac{hc}{E_K - E_M}$

$$\lambda_{L_\alpha} = \frac{hc}{E_L - E_M}, \text{ etc.}$$

Also, the characteristics wavelength depends upon the target material as below

$$\frac{1}{\lambda} = R\,(Z-b)^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

and energy of X-ray radiation as

$$\Delta E = Rhc(Z-b)^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\Rightarrow \quad \nu = Rc\,(Z-b)^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

where, Z = atomic number of the target, b = constant

Note X-rays of high frequency or low wavelength are known as hard X-rays and X-rays of low frequency or high wavelength are known as soft X-rays.

Intensity-wavelength graph

At certain sharply defined wavelengths, the intensity of X-rays is very large as marked $K_\alpha, K_\beta, \ldots$, as shown in figure.

These X-rays are known as characteristic X-rays. At other wavelengths the intensity varies gradually and these X-rays are called continuous X-rays.

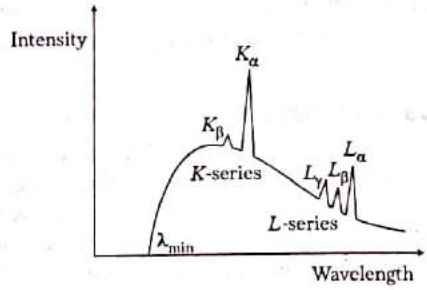


Fig. 12.25 *X-ray spectrum*

Example 12.41 *The wavelength of K_α X-ray for an element is 21.3 pm. It takes 12.5 keV to knock out an electron from the L-shell of the atom of the element. What should be the minimum accelerating voltage across an X-ray tube having the element as target which allows production of K_α X-ray?*

Sol. Since, $E_K - E_L = \dfrac{hc}{\lambda_{K_\alpha}}$

It is given that $\lambda_{K_\alpha} = 21.3 \text{ pm} = 0.0213 \text{ nm}$

Also, $\quad E(\text{eV}) = \dfrac{1242}{\lambda(\text{nm})} = \dfrac{1242}{0.0213} = 58.3 \text{ keV}$

$\Rightarrow \quad E_K - E_L = E$

$\Rightarrow \quad E_K - 12.5 = 58.3$

$\Rightarrow \quad E_K = 70.8 \text{ keV}$

$\therefore$ Acceleration potential = 70.8 kV

Example 12.42 *The wavelength of K_α-line in copper is 1.54 Å. Find the ionisation energy of K-electron in copper (in joule).*

Sol. For K_α-line we can write,

$$\frac{1}{\lambda_\alpha} = R(Z-1)^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = R(Z-1)^2 \times \frac{3}{4} \Rightarrow R(Z-1)^2 = \frac{4}{3\lambda_\alpha}$$

Ionisation energy is given by

$$E = Rhc\,(Z-1)^2\left(\frac{1}{1^2} - \frac{1}{\infty}\right) = hc\,\frac{4}{3\lambda_\alpha}$$

$$= \frac{6.6\times 10^{-34} \times 3\times 10^8 \times 4}{3\times 1.54\times 10^{-10}} = 1.7\times 10^{-15}\text{ J}$$

Example 12.43 *The wavelength of K_α X-ray line for an element is 0.42 Å. Find the wavelength of K_β line emitted by the same element.*

Sol. The characteristic wavelength for particular element is given by $\frac{1}{\lambda} = R(Z-b)^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

For K_α line,

$$\frac{1}{\lambda_\alpha} = (Z-b)^2 R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = (Z-b)^2 R\left(\frac{3}{4}\right)$$

For K_β line,

$$\frac{1}{\lambda} = (Z-b)^2 R\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = (Z-b)^2 R\left(\frac{8}{9}\right)$$

$$\Rightarrow \quad \frac{\lambda_\beta}{\lambda_\alpha} = \frac{(3/4)}{(8/9)} = \frac{27}{32}$$

It is given that λ_α is 0.42 Å.

$$\therefore \quad \lambda_\beta = 0.42 \times \frac{27}{32} = 0.35\,\text{Å}$$

Example 12.44 *The energy of an element with a vacancy in K-shell is 35.2 keV, in L-shell is 5.25 keV and in M-shell is 0.55 keV higher than the energy of the atom with no vacancy. Find the frequency of K_α, K_β and L_α X-rays of that element.*

Sol. It is given that,

$$E_K = 35.2 \text{ keV}, E_L = 5.25 \text{ keV}, E_M = 0.55 \text{ keV}$$

$$\Rightarrow \quad E_K - E_L = \frac{hc}{\lambda_{K_\alpha}} = h\nu_{K_\alpha}$$

$$\Rightarrow \quad (35.2 - 5.25) \times 10^3 \times 1.6 \times 10^{-19} = 6.6 \times 10^{-34}\, \nu_{K_\alpha}$$

$$\Rightarrow \quad \nu_{K_\alpha} = \frac{29.95 \times 10^3 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}} = 7.2 \times 10^{18} \text{ Hz}$$

Similarly, $E_K - E_M = h\nu_{K_\beta}$

$$\Rightarrow \quad \nu_{K_\beta} = \frac{(35.2 - 0.55) \times 10^3 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}} = 8.4 \times 10^{18} \text{ Hz}$$

Also, $E_L - E_M = h\nu_{k_\alpha}$

$$\Rightarrow \quad \nu_{K_\alpha} = \frac{(5.25 - 0.55) \times 10^3 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}} = 1.1 \times 10^{18} \text{ Hz}$$

Moseley's law for characteristic spectrum

Although multielectron atoms cannot be analysed with the Bohr model. Henery GJ Moseley in 1914 made an effort towards this. Moseley measured the frequencies of characteristic X-rays emitted from a large number of elements and plotted the square root of the frequency $\sqrt{f}$ against the atomic number Z of the element. He discovered that the plot is very close to a straight line. He concluded that the spectra of different elements are very similar and with increasing atomic number, the spectral lines merely shift towards higher frequencies.

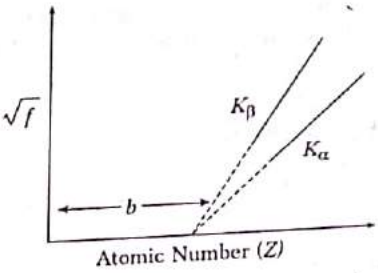


Fig. 12.26 Frequency versus atomic number graph

He also gave the following relation, $\sqrt{f} = a(Z-b)$

where, f = frequency of emitted line
Z = atomic number of target
a = proportionality constant
b = screening constant
$(Z-b)$ = effective atomic number.

For K_α X-rays, when an electron from the L shell move a transition to the vacant K shell, effective charge at centre $= (Z-1)\,e \Rightarrow b = 1$

For K_α transition, $\Delta E = h\nu = Rhc\,(Z-b)^2\left[\frac{1}{1^2} - \frac{1}{2^2}\right]$

$$\sqrt{\nu} = \sqrt{\frac{3RC}{4}}\,(Z-b)$$

From Moseley's law

$$\sqrt{\nu} = a\,(Z-b) \Rightarrow a = \sqrt{\frac{3RC}{4}},\ b = 1$$

Here, a and b does not depend on the nature of target. *Different values of a and b are as follows:*

$b = 1$	for	K-series
$b = 7.4$	for	L-series
$b = 19.2$	for	M-series

Moseley's law supported Bohr's theory. It experimentally determined the atomic number (Z) of elements.

If the graph was plotted between $\sqrt{f}$ and the atomic weight, then straight line was not obtained. So, he concluded that the elements in the periodic table should be arranged in the increasing order of atomic number and not of atomic weight. When these changes were made, the anomalies of the Mendeleev's table disappeared.

Example 12.45 *Use Moseley's law with $b = 1$ to find the frequency of the K_α X-rays of La ($Z = 57$), if the frequency of the K_α X-rays of Cu ($Z = 29$) is known to be 1.88×10^{18} Hz.*

Sol. Using the equation, $\sqrt{f} = a\,(Z-b)$ (for K-series, $b = 1$)

$$\Rightarrow \quad \frac{f_{La}}{f_{Cu}} = \left(\frac{Z_{La} - 1}{Z_{Cu} - 1}\right)^2 \quad \text{or} \quad f_{La} = f_{Cu}\left(\frac{Z_{La} - 1}{Z_{Cu} - 1}\right)^2$$

$$= 1.88 \times 10^{18}\left(\frac{57-1}{29-1}\right)^2 = 7.52 \times 10^{18} \text{ Hz}$$

Example 12.46 *The K_α X-rays of aluminium ($Z = 13$) and copper ($Z = 29$) have wavelength 887 pm and 140 pm, respectively. Use Moseley's law $\sqrt{f} = a(Z-b)$ to find the wavelength of the K_α X-ray of iron ($Z = 26$).*

Sol. According of Moseley's law,

$$\sqrt{f} = a(Z-b) \Rightarrow \sqrt{\frac{c}{\lambda}} = a(Z-b)$$

For aluminium, $\sqrt{\frac{c}{887}} = a(13-b)$ ...(i)

For copper, $\sqrt{\frac{c}{140}} = a(29-b)$ ...(ii)

On dividing Eq. (ii) by Eq. (i), we have,

$$\sqrt{\frac{887}{140}} = \frac{29-b}{13-b} \Rightarrow 2.5(13-b) = 29-b \Rightarrow b = 2.3$$

$$\therefore \sqrt{\frac{c}{140}} = a(29-b) \Rightarrow \sqrt{\frac{3\times10^8}{140\times10^{-12}}} = a(29-2.3)$$

$$\Rightarrow 1.46\times10^9 = a(26.7) \Rightarrow a = 0.055\times10^9 \approx 5\times10^7$$

Now, for iron we can write as

$$\sqrt{\frac{c}{\lambda}} = a(Z-b) = 5\times10^7\,(26-2.3) = 118.5\times10^7$$

$$\Rightarrow \lambda = \frac{3\times10^8}{(118.5\times10^7)^2} = 2.13\times10^{-10} = 2\,\text{Å}$$

Absorption of X-rays

When a beam of X-rays of intensity I_0 passes through a thickness t of any material, they get absorbed as shown in the figure.

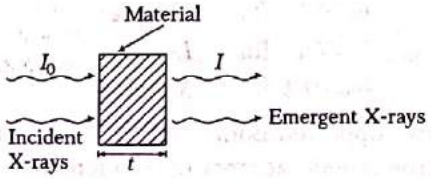


Fig. 12.27 *Absorption of X-rays when incident on any material*

Thus, the intensity of the emergent X-rays is given by

$$I = I_0 e^{-\mu t}$$

where, I_0 = intensity of incident X-rays

I = intensity of emergent X-rays

μ = absorption coefficient of the material

Example 12.47 *If the intensity of an X-ray becomes $\frac{I_0}{3}$ from I_0 after travelling 3.0 cm inside a target, then find its intensity after travelling 9 cm.*

Sol. Given that, $I_1 = \frac{I_0}{3}$ at $t = 3.0$ cm, $I_2 = ?$ at $t = 9$ cm

Since, intensity of X-ray after travelling a distance t becomes

$$I = I_0 e^{-\mu t} \Rightarrow I_1 = \frac{I_0}{3} = I_0 e^{-\mu\times3.0} \Rightarrow e^{-\mu\times3.0} = \frac{1}{3}$$

Again, we have $I_2 = I_0 e^{-\mu\times9} = I_0\,(e^{-\mu\times3.0})^2$

$$\Rightarrow \quad I_2 = I_0\left(\frac{1}{3}\right)^2 = \frac{I_0}{9}$$

Bragg's law

When X-rays are incident on a crystal these rays may be diffracted by suitable diffracting centres as shown in the figure. And when the path difference is equal to n (a whole number) times of wavelength, a constructive interference will occur. To obtain a maximum intensity we have

$$\boxed{\text{Path difference, } 2d\sin\theta = n\lambda}$$

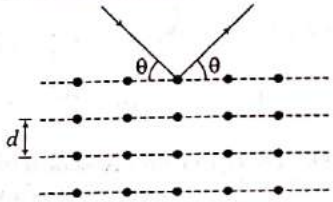


Fig. 12.28 *X-rays incident on a crystal*

where, $n = 1, 2, 3, \ldots$

θ = angle of incidence, d = inter-atomic distance

and λ = wavelength of X-rays

Example 12.48 *The distance between two successive atomic planes of a calcite crystal is 0.3 nm. Find the minimum angle for bragg scattering of 0.3 Å X-rays.*

Sol. To obtain maximum intensity, we must have

$$2d\sin\theta = n\lambda$$

Given, $\lambda = 0.3\text{Å} = 0.3\times10^{-10}$ m, $d = 0.3$ nm $= 0.3\times10^{-9}$ m

For θ to be minimum, $n = 1$

$$\Rightarrow \quad \sin\theta = \frac{n\lambda}{2d} = \frac{1\times0.3\times10^{-10}}{2\times0.3\times10^{-9}} = 0.05 \text{ rad}$$

$$\Rightarrow \quad \sin\theta \approx \theta = \frac{0.05\times180}{\pi} = 2.86°$$

Uses of X-rays

Uses of X-rays are given below

(i) X-rays are used for medical imaging.

(ii) X-rays are used in treating cancer.

(iii) They are useful for determining crystal structure by X-ray crystallography.

(iv) X-rays are useful for airport security.

(v) In art, the change occurring in old oil paintings can be examined by X-rays.

(vi) X-rays are used in laboratories for materials characterisation.

Check point 12.3

1. X-rays were discovered by
(a) Becquerel (b) Roentgen (c) Marie Curie (d) Von Laue

2. Which of the following wavelength falls in X-ray region?
(a) 10000 Å (b) 1000 Å
(c) 1 Å (d) 10^{-2} Å

3. X-rays are in nature similar to
(a) beta rays (b) gamma rays
(c) de-Broglie waves (d) cathode rays

4. The nature of X-rays spectrum is
(a) continuous (b) line
(c) continuous and line (d) None of these

5. The kinetic energy of electrons that strike the target is increased, then the cut-off wavelength of continuous X-ray spectrum
(a) increases (b) decreases
(c) no change (d) cannot be said

6. If V be the accelerating voltage, then the maximum frequency of continuous X-rays is given by
(a) $\frac{eh}{V}$ (b) $\frac{hV}{e}$ (c) $\frac{eV}{h}$ (d) $\frac{h}{eV}$

7. An X-ray tube is operated at 50 kV. The minimum wavelength produced is
(a) 0.5 Å (b) 0.75 Å
(c) 0.25 Å (d) 1 Å

8. X-rays are being produced in a tube operating at 10^5 V. The velocity of X-rays produced is
(a) 3×10^8 ms^{-1} (b) 2.8×10^8 ms^{-1}
(c) 3.1×10^8 ms^{-1} (d) 3×10^{10} ms^{-1}

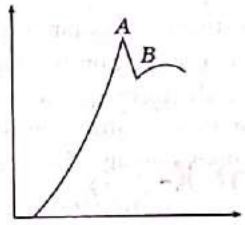
9. The energy of a photon of characteristic X-ray from a coolidge tube comes from
(a) the kinetic energy of the striking electron
(b) the kinetic energy of the free electrons of the target
(c) the kinetic energy of the ions of the target
(d) an atomic transition in the target

10. In X-ray spectrum, transition of an electron from an outer shell to an inner shell gives a characteristics X-ray spectral line. If we consider the spectral line K_β, L_β and M_α, then
(a) K_β and L_β have a common inner shell
(b) K_β and L_β have a common outer shell
(c) L_β and M_α have a common outer shell
(d) L_β and M_α have a common inner shell

11. In X-ray spectrum wavelength λ of line K_α depends on atomic number Z as
(a) $\lambda \propto Z^2$ (b) $\lambda \propto (Z-1)^2$
(c) $\lambda \propto \frac{1}{(Z-1)}$ (d) $\lambda \propto \frac{1}{(Z-1)^2}$

12. For characteristic X-ray, choose the correct option.
(a) $E(K_\gamma) < E(K_\beta) < E(K_\alpha)$ (b) $E(K_\alpha) < E(L_\alpha) < E(M_\alpha)$
(c) $\lambda(K_\gamma) < \lambda(K_\beta) < \lambda(K_\alpha)$ (d) $\lambda(M_\alpha) < \lambda(L_\alpha) < \lambda(K_\alpha)$

13. Sharp peak point A represents



(a) characteristic X-rays (b) continuous X-rays
(c) Bremsstrahlung (d) discontinuous spectrum

14. Moseley's law relates the frequencies of line X-rays with the following characteristics of the target element
(a) its density (b) its atomic weight
(c) its atomic number
(d) interplaner spacing of the atomic planes

15. Bragg's law for X-rays is
(a) $d \sin\theta = 2n\lambda$ (b) $2d \sin\theta = n\lambda$
(c) $n \sin\theta = 2\lambda d$ (d) None of these

Chapter exercises

A) Taking it together

(Assorted questions of the chapter for advanced level practice)

1. The penetrating power of X-rays increases with the increase in its
(a) velocity (b) intensity
(c) frequency (d) wavelength

2. X-rays are not used for radar purpose because
(a) they are not reflected by the target
(b) they are not electromagnetic waves
(c) they are completely absorbed by the air
(d) they sometimes damage the target

3. The X-ray beam coming from an X-ray tube will be
(a) monochromatic
(b) having all wavelengths larger than a certain minimum wavelength
(c) having all wavelengths smaller than a certain maximum wavelength
(d) having all wavelengths between a minimum and a maximum wavelength

4. Consider a photon of continuous X-ray coming from a coolidge tube. Its energy comes from
(a) the kinetic energy of the striking electron
(b) the kinetic energy of the free electrons of the target
(c) the kinetic energy of the ions of the target
(d) an atomic transition in the target

5. Hydrogen atom does not emit X-rays because
(a) its energy levels are too close to each other
(b) its energy levels are too far apart
(c) it has a very small mass
(d) it has a single electron

6. Moseley's law for characteristic X-rays is $\sqrt{\nu} = a(Z - b)$. In this
(a) Both a and b are independent of the material
(b) a is independent but b depends on the material
(c) b is independent but a depends on the material
(d) Both a and b depends on the material

7. A set of atoms in an excited state decays **[NCERT Exemplar]**
(a) in general to any of the states with lower energy
(b) into a lower state only when excited by an external electric field
(c) all together simultaneously into a lower state
(d) to emit photons only when they collide

8. X-ray beam can be deflected by
(a) magnetic field (b) electric field
(c) Both (a) and (b) (d) None of these

9. In an X-rays tube, the intensity of the emitted X-rays beam is increased by
(a) increasing the filament current
(b) decreasing the filament current
(c) increasing the target potential
(d) decreasing the target potential

10. If the current in the circuit for heating the filament is increased, the cut-off wavelength
(a) will increase
(b) will decrease
(c) will remain unchanged
(d) may increase or decrease

11. X-rays are produced by jumping of
(a) electrons from lower to higher energy orbit of atom
(b) electrons from higher to lower energy orbit of atom
(c) protons from lower to higher energy orbit of nucleus
(d) protons from higher to lower energy orbit of nucleus

12. The difference in angular momentum associated with the electron in the two successive orbits of hydrogen atom is
(a) $\frac{h}{\pi}$ (b) $\frac{h}{2\pi}$
(c) $\frac{h}{2}$ (d) $(n-1)\frac{h}{2\pi}$

13. Which of the following is not correct in Bohr model of hydrogen atom?
(a) The radius of nth orbit is proportional to n^2
(b) The total energy of electron in nth orbit is proportional to n
(c) The angular momentum of an electron in an orbit is an integral multiple of $h/2\pi$
(d) The magnitude of the potential energy of an electron in any orbit is greater than its kinetic energy

14. Which of the following parameters are the same for all hydrogen like atoms and ions in their ground states?
(a) Radius of the orbit
(b) Speed of the electrons
(c) Energy of the atom
(d) Orbital angular momentum of the electron

15. The second line in Paschen series is obtained when the electron makes transition from
(a) fourth orbit to third orbit
(b) seventh orbit to third orbit
(c) sixth orbit to third orbit
(d) fifth orbit to third orbit

16. An electron moves in a circular orbit at a distance from a proton with kinetic energy E. To escape to infinity, the minimum energy which must be supplied to the electron is
(a) E (b) $2E$ (c) $0.5E$ (d) $E\sqrt{2}$

17. Molybdenum is used as a target element for production of X-rays because it is
(a) light and can easily deflect electrons
(b) light and can absorb electrons
(c) a heavy element with a high melting point
(d) an element having high thermal conductivity

18. An X-ray has a wavelength of 0.010 Å. Its momentum is
(a) 2.126×10^{-23} kg ms^{-1}
(b) 6.626×10^{-22} kg ms^{-1}
(c) 3.456×10^{-25} kg ms^{-1}
(d) 3.313×10^{-22} kg ms^{-1}

19. X-rays of $\lambda = 1$ Å have frequency
(a) 3×10^{8} Hz (b) 3×10^{18} Hz
(c) 3×10^{10} Hz (d) 3×10^{15} Hz

20. The simple Bohr model cannot be directly applied to calculate the energy levels of an atom with many electrons. This is because [NCERT Exemplar]
(a) of the electrons not being subject to a central force
(b) of the electrons colliding with each other
(c) of screening effects
(d) the force between the nucleus and an electron will no longer be given by Coulomb's law

21. The velocity of an electron in the first orbit of H-atom is v. The velocity of an electron in the 2nd orbit of He^+ is
(a) $2v$ (b) v
(c) $v/2$ (d) $v/4$

22. A X-ray tube operates at an accelerating potential of 20 kV. Which of the following wavelengths will be absent in the continuous spectrum of X-rays?
(a) 12 pm (b) 75 pm
(c) 65 pm (d) 95 pm

23. A metal block is exposed to beams of X-ray of different wavelengths 2Å, 4Å, 6Å and 8Å. X-rays of which wavelength penetrate most?
(a) 2 Å (b) 4 Å (c) 6 Å (d) 8 Å

24. O_2 molecule consists of two oxygen atoms. In the molecule, nuclear force between the nuclei of the two atoms [NCERT Exemplar]
(a) is not important because nuclear forces are short-ranged
(b) is as important as electrostatic force for binding the two atoms
(c) cancels the repulsive electrostatic force between the nuclei
(d) is not important because oxygen nucleus have equal number of neutrons and protons

25. The patient is asked to drink $BaSO_4$ for examining the stomach by X-rays because X-rays are
(a) reflected by heavy atoms
(b) refracted by heavy atoms
(c) less absorbed by heavy atoms
(d) more absorbed by heavy atoms

26. The orbital angular momentum of electron in the n_1th shell of element of atomic number Z_1 is L_1 and the same in the n_2th shell of element of atomic number Z_2 is L_2. If $L_2 > L_1$, then
(a) $n_2 > n_1$ (b) $Z_2 > Z_1$
(c) $n_2 Z_2 > n_1 Z_1$ (d) Both (a) and (b)

27. For production of characteristic K_β X-rays, the electron transition is
(a) $n = 2$ to $n = 1$ (b) $n = 3$ to $n = 2$
(c) $n = 3$ to $n = 1$ (d) $n = 4$ to $n = 2$

28. Hydrogen atom is excited from ground state to another state with principal quantum number equal to 4. Then, the number of spectral lines in the emission spectra will be
(a) 2 (b) 3
(c) 5 (d) 6

29. Consider the following two statements A and B and identify the correct choice in the given answer.
(A) The characteristic X-ray spectrum depends on the nature of the material of the target.
(B) The short wavelength limit of continuous X-ray spectrum varies inversely on the potential difference applied to the X-ray tube.
(a) A is true and B is false
(b) A is false and B is true
(c) Both A and B are true
(d) Both A and B are false

30. In a hypothetical Bohr hydrogen, the mass of the electron is doubled. The energy E_0 and radius r_0 of the first orbit will be (a_0 is the Bohr radius)
(a) $E_0 = -27.2$ eV, $r_0 = a_0/2$
(b) $E_0 = -27.2$ eV, $r_0 = a_0$
(c) $E_0 = -13.6$ eV $r_0 = a_0/2$
(d) $E_0 = -13.6$ eV, $r_0 = a_0$

31. Ionisation energy of a hydrogen-like ion A is greater than that of another hydrogen-like ion B. Let r, u, E and L represent the radius of the orbit, speed of the electron, energy of the atom and orbital angular momentum of the electron, respectively. In ground state
(a) $r_A > r_B$ (b) $u_A > u_B$ (c) $E_A < E_B$ (d) $L_A > L_B$

32. If ω is the speed of electron in the nth orbit hydrogen atom, then
(a) $\omega \propto n^{1/2}$ (b) $\omega \propto \frac{1}{n}$
(c) $\omega \propto \frac{1}{n^2}$ (d) $\omega \propto \frac{1}{n^3}$

33. Hydrogen atom emits blue light when it jumps from $n = 4$ energy level to the $n = 2$ level. Which colour of light would the atom emit when it changes from the $n = 5$ level to the $n = 2$ level?
(a) Red (b) Yellow (c) Green (d) Violet

34. The potential difference between the cathode and the target electrode in a coolidge tube is 24.75 kV. The minimum wavelength of the emitted X-rays is
(a) 0.1 Å (b) 0.5 Å (c) 1 Å (d) 5 Å

35. In Rutherford scattering experiment, what will be the ratio of impact parameter for scattering angles $\theta_1 = 90°$ and $\theta_2 = 120°$.
(a) 1 (b) $\sqrt{2}$ (c) 2 (d) $\sqrt{3}$

36. Taking the Bohr radius as $a_0 = 53$ pm, the radius of Li^{++} ion in its ground state, on the basis of Bohr's model, will be about **[NCERT Exemplar]**
(a) 53 pm (b) 27 pm
(c) 18 pm (d) 13 pm

37. An electron jumps from the 4th orbit to 2nd orbit of the hydrogen atom. Given, Rydberg's constant $R = 10^5\ cm^{-1}$, the frequency in hertz of emitted radiation will be
(a) $\frac{3}{16} \times 10^5$ (b) $\frac{3}{16} \times 10^{15}$
(c) $\frac{9}{16} \times 10^{15}$ (d) $\frac{3}{4} \times 10^{15}$

38. When an electron in hydrogen atom is excited, from its 4th to 5th stationary orbit, the change in angular momentum of electron is (Planck's constant, $h = 6.6 \times 10^{-34}$ J/s)
(a) 4.16×10^{-34} J-s (b) 3.32×10^{-34} J-s
(c) 1.05×10^{-34} J-s (d) None of these

39. The magnetic moment (μ) of a revolving electron around the nucleus varies with principal quantum number n as
(a) $\mu \propto n$ (b) $\mu \propto 1/n$
(c) $\mu \propto n^2$ (d) $\mu \propto 1/n^2$

40. The electron in a hydrogen atom makes a transition from $n = n_1$ to $n = n_2$ state. The time period of the electron in the initial state (n_1) is eight times that in the final state (n_2). The possible values of n_1 and n_2 are
(a) $n_1 = 8, n_2 = 1$ (b) $n_1 = 4, n_2 = 2$
(c) $n_1 = 2, n_2 = 4$ (d) $n_1 = 1, n_2 = 8$

41. Two elements A and B with atomic numbers Z_A and Z_B are used to produce characteristic X-rays with frequencies ν_A and ν_B, respectively. If $Z_A : Z_B = 1:2$, then $\nu_A : \nu_B$ will be
(a) $1:\sqrt{2}$ (b) 1 : 8 (c) 4 : 1 (d) 1 : 4

42. An electron revolves round a nucleus of charge Ze. In order to excite the electron from the state $n = 2$ to $n = 3$, the energy required is 47.2 eV. Z is equal to
(a) 3 (b) 4 (c) 5 (d) 2

43. As per Bohr model, the minimum energy (in eV) required to remove an electron from the ground state of doubly ionised Li atom is
(a) 1.51 (b) 13.6 (c) 40.8 (d) 122.4

44. Excitation energy of a hydrogen like ion in its first excitation state is 40.8 eV. Energy needed to remove the electron from the ion in ground state is
(a) 54.4 eV (b) 13.6 eV (c) 40.8 eV (d) 27.2 eV

45. Ionisation potential of hydrogen atom is 13.6 V. Hydrogen atoms in the ground state are excited by monochromatic radiation of photon energy 12.1 eV. The spectral lines emitted by hydrogen atoms according to Bohr's theory will be
(a) One (b) Two
(c) Three (d) Four

46. An electron makes a transition from orbit $n = 4$ to the orbit $n = 2$ of a hydrogen atom. The wave number of the emitted radiations (R = Rydberg's constant) will be
(a) $\frac{16}{3R}$ (b) $\frac{2R}{16}$ (c) $\frac{3R}{16}$ (d) $\frac{4R}{16}$

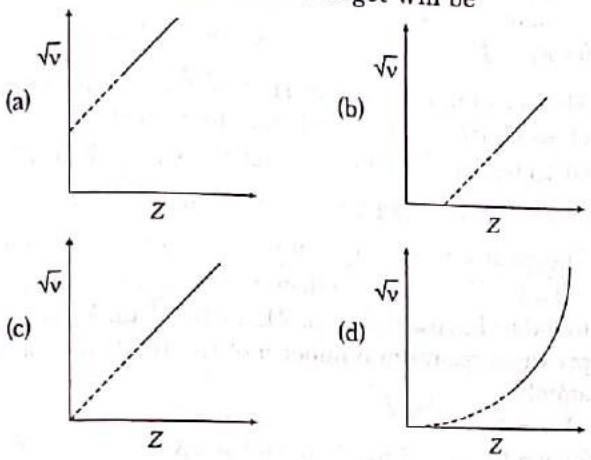
47. The K_α and K_β lines of characteristic X-ray spectrum of molybdenum are 0.76 Å and 0.64 Å, respectively. The wavelength of L_α line is
(a) 1.4 Å (b) 2.4 Å
(c) 4.1 Å (d) 3.6 Å

48. The longest wavelength that can be analysed by a sodium chloride crystal of spacing $d = 2.82$ Å in the second order is
(a) 2.82 Å (b) 5.64 Å
(c) 8.46 Å (d) 11.28 Å

49. The wavelength of K_α, X-rays for lead isotopes Pb^{208}, Pb^{206}, Pb^{204} are λ_1, λ_2 and λ_3, respectively. Then

(a) $\lambda_1 = \lambda_2 > \lambda_3$
(b) $\lambda_1 > \lambda_2 > \lambda_3$
(c) $\lambda_1 < \lambda_2 < \lambda_3$
(d) $\lambda_2 = \sqrt{\lambda_1 \lambda_3}$

50. The graph between the square root of the frequency of a specific line of characteristic spectrum of X-rays and the atomic number of the target will be



51. The ratio between acceleration of the electron in singly ionised helium atom and doubly ionised lithium atom (both in ground state) is

(a) $\frac{4}{9}$
(b) $\frac{27}{8}$
(c) $\frac{8}{27}$
(d) $\frac{9}{4}$

52. Magnetic moment of an electron in nth orbit of hydrogen atom is

(a) $\frac{neh}{\pi m}$
(b) $\frac{neh}{4\pi m}$
(c) $\frac{meh}{2\pi n}$
(d) $\frac{meh}{4\pi n}$

(m = mass of electron, h = Planck's constant)

53. A sphere of work function, $\phi = 4.6$ eV is suspended in a vacuum chamber by an insulating thread. Radiation of wavelength, $\lambda = 0.2$ μm stricke on the sphere. The maximum electric potential of the sphere will be

(a) 4.6 V
(b) 6.2 V
(c) 1.6 V
(d) 3.2 V

54. The wavelength of radiation emitted is λ_0 when an electron jumps from the third to the second orbit of hydrogen atom. For the electron jump from the fourth to the second orbit of the hydrogen atom, the wavelength of radiation emitted will be

(a) $\frac{16}{25}\lambda_0$
(b) $\frac{20}{27}\lambda_0$
(c) $\frac{27}{20}\lambda_0$
(d) $\frac{25}{16}\lambda_0$

55. Hard X-rays for the study of fractures in bones should have a minimum wavelength of 10^{-11} m. The accelerating voltage for electrons in X-ray machine should be

(a) <124.2 kV
(b) >124.2 kV
(c) Between 60 kV and 70 kV
(d) equal to 100 kV

56. An electron with kinetic energy 5 eV is incident on a H-atom in its ground state. The collision

(a) must be elastic
(b) may be partially elastic
(c) must be completely inelastic
(d) may be completely inelastic

57. When an electron jumps from the orbit $n = 2$ to $n = 4$, then wavelength of the radiations absorbed will be (R is Rydberg's constant)

(a) $\frac{16}{3R}$
(b) $\frac{16}{5R}$
(c) $\frac{5R}{16}$
(d) $\frac{3R}{16}$

58. The minimum energy required to ionise an atom is the energy equal to

(a) add one electron to the gaseous state of atom
(b) excite the atom from its ground state to its first excited state
(c) remove one outermost electron from the gaseous state of atom
(d) remove one innermost electron from the gaseous state of atom

59. If ν_1 is the frequency of the series limit of Lyman series, ν_2 is the frequency of the first line of Lyman series and ν_3 is the frequency of the series limit of the Balmer series. Then,

(a) $\nu_1 - \nu_2 = \nu_3$
(b) $\nu_1 = \nu_2 - \nu_3$
(c) $\frac{1}{\nu_2} = \frac{1}{\nu_1} + \frac{1}{\nu_3}$
(d) $\frac{1}{\nu_1} = \frac{1}{\nu_2} + \frac{1}{\nu_3}$

60. The shortest wavelength of the Brackett series of hydrogen like atom (atomic number = Z) is the same as the shortest wavelength of the Balmer series of hydrogen atom. The value of Z is

(a) 3
(b) 4
(c) 5
(d) 2

61. If an α-particle of mass m, charged q and velocity v is incident on a nucleus charge Q and mass m, then the distance of closest approach is

(a) $\frac{Qq}{4\pi\varepsilon_0 m^2}$
(b) $\frac{Qq}{2\pi\varepsilon_0 m v^2}$
(c) $\frac{Qq\, m v^2}{2}$
(d) $\frac{Qq}{m v^2}$

62. The energy of an electron in excited hydrogen atom is -3.4 eV. Then, according to Bohr's theory, the angular momentum of the electron is

(a) 2.1×10^{-34} J-s
(b) 3×10^{-34} J-s
(c) 2×10^{-34} J-s
(d) 0.5×10^{-34} J-s

63. The shortest wavelength which can be obtained in hydrogen spectrum is ($R = 10^7\ m^{-1}$)
(a) 1000 Å (b) 800 Å (c) 1300 Å (d) 2100 Å

64. The wavelength of the first spectral line of sodium 5896 Å. The first excitation potential of sodium atom will be (Planck's constant, $h = 6.63 \times 10^{-34}$ J-s)
(a) 4.2 V (b) 3.5 V
(c) 2.1 V (d) None of these

65. The shortest wavelength in Lyman series is 91.2 nm. The longest wavelength of the series is
(a) 121.6 nm (b) 182.4 nm (c) 243.4 nm (d) 364.8 nm

66. Energy of 24.6 eV is required to remove one of the electron from a neutral helium atom. The energy (in eV) required to remove both the electrons from a neutral helium atom is
(a) 38.2 (b) 49.2 (c) 51.8 (d) 79.0

67. If the radius of first Bohr's orbit is x, then de-Broglie wavelength of electron in 3rd orbit is nearly
(a) $2\pi x$ (b) $6\pi x$ (c) $9x$ (d) $\frac{x}{3}$

68. An excited hydrogen atom emits a photon of wavelength λ in returning to the ground state. The quantum number n of the excited state is given by (R = Rydberg constant)
(a) $\sqrt{\lambda R(\lambda R - 1)}$ (b) $\sqrt{\frac{\lambda R}{(\lambda R - 1)}}$
(c) $\sqrt{\frac{(\lambda R - 1)}{\lambda R}}$ (d) $\sqrt{\frac{1}{\lambda R(\lambda R - 1)}}$

69. An alpha nucleus of energy $\frac{1}{2}mv^2$ bombards a heavy nuclear target of charge Ze. Then, the distance of closest approach for the alpha nucleus will be proportional to
(a) $1/m$ (b) $1/v^4$ (c) $1/Ze$ (d) v^2

70. If the frequency of K_α X-ray emitted from the element with atomic number 31 is f, then the frequency of K_α X-ray emitted from the element with atomic number 51 would be
(a) $\frac{5f}{3}$ (b) $\frac{51f}{31}$ (c) $\frac{9f}{25}$ (d) $\frac{25f}{9}$

71. According to Moseley's law the ratio of the slope of graph between $\sqrt{f}$ and Z for K_β and K_α is
(a) $\sqrt{\frac{32}{27}}$ (b) $\sqrt{\frac{27}{32}}$ (c) $\sqrt{\frac{5}{36}}$ (d) $\sqrt{\frac{36}{5}}$

72. The ratio of speed of the electron in the first Bohr orbit of hydrogen and the speed of light is equal to (where e, h and c have their usual meanings)
(a) $\frac{2\pi hc}{e^2}$ (b) $\frac{e^2c}{2\pi h}$ (c) $\frac{e^2h}{2\pi c}$ (d) $\frac{e^2}{2\varepsilon_0 hc}$

73. If scattering particles are 56 for 90° angle, then at an angle 60, it will be
(a) 224 (b) 256
(c) 98 (d) 108

74. de-Broglie wavelength of an electron in the nth Bohr orbit of hydrogen atom is λ_n and the angular momentum is J_n, then
(a) $J_n \propto \lambda_n$ (b) $\lambda_n \propto \frac{1}{J_n}$
(c) $\lambda_n \propto J_n^2$ (d) None of these

75. The recoil momentum of H-atom due to the transition of an electron from $n = 4$ state to $n = 1$ state is
(a) 13.6×10^{-19} kg ms^{-1} (b) 6.8×10^{-27} kg ms^{-1}
(c) 12.75×10^{-24} kg ms^{-1} (d) None of these

76. The radius of hydrogen atom in its ground state is 5.3×10^{-11} m. After collision with an electron it is found to have a radius of 21.2×10^{-11} m. What is the principal quantum number n of the final state of the atom?
(a) $n = 4$ (b) $n = 2$
(c) $n = 16$ (d) $n = 3$

77. The acceleration of electron in the first orbit of hydrogen atom is
(a) $\frac{4\pi^2 m}{h^3}$ (b) $\frac{h^2}{4\pi^2 mr}$ (c) $\frac{h^2}{4\pi^2 m^2 r^3}$ (d) $\frac{m^2h^2}{4\pi^2 r^3}$

78. An α-particle accelerated through V volt is fired towards a nucleus. Its distance of closest approach is r. If a proton accelerated through the same potential is fired towards the same nucleus, the distance of closest approach of proton will be
(a) r (b) $2r$ (c) $\frac{r}{2}$ (d) $\frac{r}{4}$

79. In a Rutherford scattering experiment, when a projectile of charge Z_1 and mass M_1 approaches a target nucleus of charge Z_2 and mass M_2, the distance to closest approach is r_0. The energy of the projectile is
(a) directly proportional to $M_1 \times M_2$
(b) directly proportional to $Z_1 Z_2$
(c) directly proportional to Z_1
(d) directly proportional to mass M_1

80. In the Bohr's model of a hydrogen atom, the centripetal force is furnished by the Coulomb attraction between the proton and the electron. If a_0 is the radius of the ground state orbit, m is the mass, e is the charge on electron and ε_0 is the permittivity of free space, the speed of the electron is
(a) $\frac{e}{\sqrt{\varepsilon_0 a_0 m}}$ (b) zero
(c) $\frac{e}{\sqrt{4\pi\varepsilon_0 a_0 m}}$ (d) $\frac{\sqrt{4\,\varepsilon_0 a_0 m}}{e}$

81. The first member of the Balmer's series of the hydrogen has a wavelength λ, the wavelength of the second member of its series is

(a) $\frac{27}{20}\lambda$ (b) $\frac{20}{27}\lambda$ (c) $\frac{27}{10}\lambda$ (d) None of these

82. The diagram shows the path of four α-particles of the same energy being scattered by the nucleus of an atom simultaneously. Which of these are/is not physically possible

(a) Both 3 and 4 (b) Both 2 and 3
(c) Both 1 and 4 (d) Only 4

83. In a hydrogen atom, the binding energy of the electron in the ground state is E_1. Then the frequency of revolution of the electron in the nth orbit is

(a) $\frac{2E_1}{n^3h}$ (b) $\frac{2E_1n^3}{h}$ (c) $\sqrt{\frac{2mE_1}{n^3h}}$ (d) $\frac{E_1n^2}{h}$

84. The ratio of the wavelength for $2 \rightarrow 1$ transition in Li^{++}, He^+ and H is

(a) 1:2:3 (b) 1:4:9 (c) 4:9:36 (d) 3:2:1

85. The electron in a hydrogen atom makes transition from M shell to L-shell. The ratio of magnitude of initial to final acceleration of the electron is

(a) 9 : 4 (b) 81 : 16
(c) 4 : 9 (d) 16 : 81

86. If the wavelength of the first line of the Balmer series of hydrogen atom is 6561 Å, the wavelength of the second line of the series should be

(a) 13122 Å (b) 3280 Å
(c) 4860 Å (d) 2187 Å

87. If the wavelength of first member of Balmer series of hydrogen spectrum is 6563 Å, then the wavelength of second member of Balmer series will be

(a) 1215 Å
(b) 4861 Å
(c) 6050 Å
(d) data given is insufficient to calculate the value

88. The wavelength of K_α line for an element of atomic number 43 is λ. then the wavelength of K_α line for an element of atomic number 29 is

(a) $\frac{43}{29}\lambda$ (b) $\frac{42}{28}\lambda$ (c) $\frac{9}{4}\lambda$ (d) $\frac{4}{9}\lambda$

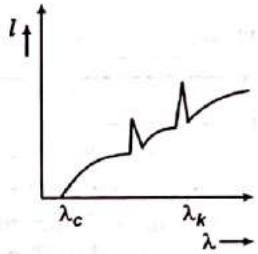
89. For the ground state, the electron in the H-atom has an angular momentum $= h$, according to the simple Bohr model. Angular momentum is a vector and hence there will be infinitely many orbits with the vector pointing in all possible directions. In actuality, this is not true, **[NCERT Exemplar]**

(a) because Bohr model gives incorrect values of angular momentum
(b) because only one of these would have a minimum energy
(c) angular momentum must be in the direction of spin of electron
(d) because electrons go around only in horizontal orbits

90. If λ_{Cu} is the wavelength of K_α X-ray line of copper (atomic number 29) and λ_{Mo} is the wavelength of the K_α X-ray line of molybdenum (atomic number 42), then the ratio $\lambda_{Cu}/\lambda_{Mo}$ is close to

(a) 1.99 (b) 2.14
(c) 0.50 (d) 0.48

91. The intensity of X-rays from a coolidge tube is plotted against wavelength as shown in the figure. The minimum wavelength found is λ_c and the wavelength of the K_α line is λ_k. As the accelerating voltage is increased



(a) $(\lambda_k - \lambda_c)$ increases (b) $(\lambda_k - \lambda_c)$ decreases
(c) λ_k increases (d) λ_k decreases

92. Which element has a K_α line of wavelength 1.785 Å?

(a) Copper (b) Cobalt
(c) Sodium (d) Aluminium

93. A H-atom moving with speed v makes a head on collision with a H-atom at rest. Both atoms are in ground state. The minimum value of velocity v for which one of the atom may excite is ($m_H = 1.67 \times 10^{-27}$ kg)

(a) 6.25×10^4 ms^{-1} (b) 8×10^4 ms^{-1}
(c) 7.25×10^4 ms^{-1} (d) 13.6×10^4 ms^{-1}

94. The wavelength of the first line of Lyman series for hydrogen atom is equal to that of the second line of Balmer series for a hydrogen like ion. The atomic number Z of hydrogen like ion is

(a) 4 (b) 1
(c) 2 (d) 3

95. In an inelastic collision an electron excites a hydrogen atom from its ground state to a M-shell state. A second electron collides instantaneously with the excited hydrogen atom in the M-shell state and ionizes it. At least how much energy the second electron transfers to the atom in the M-shell state?

(a) + 3.4 eV (b) + 1.51 eV
(c) – 3.4 eV (d) – 1.51 eV

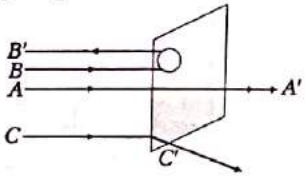
96. A hydrogen-like atom emits radiations of frequency 2.7×10^{15} Hz when it makes a transition from $n = 2$ to $n = 1$. The frequency emitted in a transition from $n = 3$ to $n = 1$ will be

(a) 1.8×10^{15} Hz (b) 3.2×10^{15} Hz
(c) 4.7×10^{5} Hz (d) 6.9×10^{15} Hz

97. Two H-atoms in the ground state collide inelastically. The maximum amount by which their combined kinetic energy is reduced is **[NCERT Exemplar]**

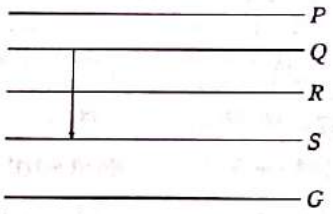
(a) 10.20 eV (b) 20.40 eV
(c) 13.6 eV (d) 27.2 eV

98. A beam of fast moving alpha particles were directed towards a thin film of gold. The parts A', B' and C' of the transmitted and reflected beams corresponding to the incident parts A, B and C of the beam, are shown in the adjoining diagram. The number of alpha particles in



(a) B' will be minimum and in C' maximum
(b) A' will be maximum and in B' minimum
(c) A' will be minimum and in B' maximum
(d) C' will be minimum and in B' maximum

99. Figure shows the energy levels P, Q, R, S and G of an atom where G is the ground state. A red line in the emission spectrum of the atom can be obtained by an energy level change from Q to S. A blue line can be obtained by following energy level change



(a) P to Q (b) Q to P
(c) R to S (d) R to G

100. The binding energy of a H-atom, considering an electron moving around a fixed nuclei (proton), is

$$B = -\frac{me^4}{8n^2\varepsilon_0^2h^2} \quad (m = \text{electron mass})$$

If one decides to work in a frame of reference where the electron is at rest, the proton would be moving around it. By similar arguments, the binding energy would be

$$B = -\frac{Me^4}{8n^2\varepsilon_0^2h^2} \quad (M = \text{proton mass})$$

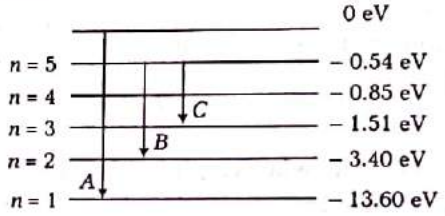
This last expression is not correct, because **[NCERT Exemplar]**

(a) n would not be integral
(b) Bohr-quantisation applies only two electron
(c) the frame in which the electron is at rest is not inertial
(d) the motion of the proton would not be in circular orbits, even approximately.

101. If the intensity of an X-ray becomes $\frac{I_0}{2}$ from I_0 after travelling 2.0 cm inside a target, then its intensity after travelling a distance of 4 cm will be

(a) $\frac{I_0}{2}$ (b) $\frac{I_0}{4}$ (c) $2I_0$ (d) I_0

102. In figure the energy levels of the hydrogen atom have been shown along with some transitions marking A, B, C. The transitions A, B, and C respectively,. represents



(a) the first member of the Lyman series, third member of Balmer series and second member of Paschen series
(b) the ionisation potential of H, second member of Balmer series and third member of Paschen series
(c) the series limit of Lyman series, second member of Balmer series and second member of Paschen series
(d) the series limit of Lyman series, third member of Balmer series and second member of Paschen series

103. The ionisation energy of the electron in the hydrogen atom in its ground state is 13.6 eV. The atoms are excited to higher energy levels to emits radiations of 6 wavelengths. Maximum wavelength of emitted radiation corresponds to the transition between

(a) $n = 3$ and $n = 2$ states (b) $n = 3$ and $n = 1$ state
(c) $n = 2$ and $n = 1$ state (d) $n = 4$ and $n = 3$ states

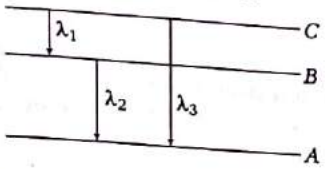
104. If an electron is revolving around the hydrogen nucleus at a distance of 0.1 nm, what should be its speed?

(a) 2.188×10^6 ms^{-1} (b) 1.094×10^6 ms^{-1}
(c) 4.376×10^6 ms^{-1} (d) 1.60×10^6 ms^{-1}

105. Imagine an atom made up of a proton and a hypothetical particle of double the mass of the electron but having the same charge as the electron. Apply the Bohr atomic model and consider all possible transitions of this hypothetical particle to the first excited level. The longest wavelength of photon in the Balmer series has wavelength λ (given in terms of the Rydberg constant R for hydrogen atom) equal to

(a) $\frac{9}{(5R)}$ (b) $\frac{36}{(5R)}$ (c) $\frac{18}{(5R)}$ (d) $\frac{4}{(5R)}$

106. Energy levels A, B and C of a certain atom corresponding to increasing values of energy *i.e.*, $E_A < E_B < E_C$. If λ_1, λ_2 and λ_3 are the wavelengths of radiations corresponding to the transitions C to B, B to A and C to A respectively, which of the following statements is correct?



(a) $\lambda_3 = \lambda_1 + \lambda_2$ (b) $\lambda_3 = \dfrac{\lambda_1\lambda_2}{\lambda_1 + \lambda_2}$

(c) $\lambda_1 + \lambda_2 + \lambda_3 = 0$ (d) $\lambda_3^2 = \lambda_1^2 + \lambda_2^2$

107. The electron in a hydrogen atom makes a transition $n_1 \rightarrow n_2$, where n_1 and n_2 are the principal quantum numbers of two states. Assume the Bohr model to be valid. The time period of the electron in the initial state is eight times of the final state. The possible values of n_1 and n_2 are

(a) $n_1 = 4, n_2 = 2$ (b) $n_1 = 8, n_2 = 2$

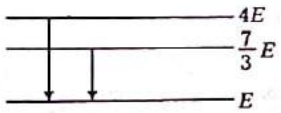
(c) $n_1 = 8, n_2 = 1$ (d) $n_1 = 6, n_2 = 2$

108. If elements with principal quantum number $n > 4$ were not allowed in nature, then the number of possible elements would be

(a) 32 (b) 60

(c) 18 (d) 4

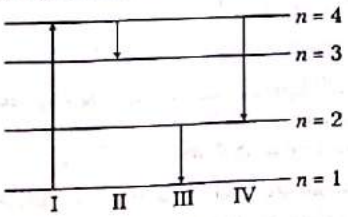
109. The following diagram indicates the energy levels of a certain atom when the system moves from $4E$ level to E. A photon of wavelength λ_1 is emitted. The wavelength of photon produced during its transition from $7/3\,E$ level to E is λ_2. The ratio λ_1/λ_2 will be



(a) $\dfrac{4}{9}$ (b) $\dfrac{4}{8}$

(c) $\dfrac{3}{8}$ (d) $\dfrac{5}{9}$

110. The diagram shows the energy levels for an electron in a certain atom. Which transition shown represents the emission of a photon with the most energy?



(a) III (b) IV (c) I (d) II

111. When the voltage applied to an X-ray tube is increased from $V_1 = 10$ kV to $V_2 = 20$ kV, the wavelength difference between the K_α line and the short wavelength limit of the continuous X-ray spectrum increases by a factor 3. The atomic number of the element of which the tube anticathode is made will be

(a) 62 (b) 56 (c) 45 (d) 29

112. The ionisation potential of hydrogen atom is -13.6 eV. An electron in the ground state of a hydrogen atom absorbs a photon of energy 12.75 eV. How many different spectral lines can one expect when the electron make a downward transition

(a) 1 (b) 4

(c) 2 (d) 6

113. The distance of closest approach of an α-particle fired towards a nucleus with momentum p, is r. If the momentum of the α-particle is $2p$, then the corresponding distance of closest approach is

(a) $r/2$ (b) $2r$

(c) $4r$ (d) $r/4$

114. The first excited state of hydrogen atom is 10.2 eV above its ground state. The temperature is needed to excite hydrogen atoms to first excited level, is

(a) 7.9×10^4 K (b) 3.5×10^4 K

(c) 5.8×10^4 K (d) 14×10^4 K

115. Hydrogen ($_2H^1$), deuterium ($_1H^2$), singly ionised helium $(_2He^4)^+$ and doubly ionised lithium $(_3Li^6)^{++}$ all have one electron around the nucleus. Consider an electron transition from $n = 2$ to $n = 1$. If the wave lengths of emitted radiation are λ_1, λ_2, λ_3 and λ_4 respectively, then approximately which one of the following is correct?

(a) $4\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$ (b) $\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$

(c) $\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$ (d) $\lambda_1 = 2\lambda_2 = 3\lambda_3 = 4\lambda_4$

116. Suppose an electron is attracted towards the origin by a force $\dfrac{k}{r}$, where k is a constant and r is the distance of the electron from the origin. By applying Bohr model to this system, the radius of the nth orbital of the electron is found to be r_n and the kinetic energy of the electron to be T_n. Then, which of the following is true?

(a) T_n independent of n, $r_n \propto n$ (b) $T_n \propto \dfrac{1}{n}, r_n \propto n$

(c) $T_n \propto \dfrac{1}{n}, r_n \propto n^2$ (d) $T_n \propto \dfrac{1}{n^2}, r_n \propto n^2$

117. A hydrogen like atom of atomic number Z is in an excited state of quantum number $2n$. It can emit a maximum energy photon of 204 eV. If it makes a transition to quantum state n, then a photon of energy 40.8 eV is emitted. The value of n will be

(a) 1 (b) 2 (c) 3 (d) 4

B) Medical entrance special format questions

Assertion and reason

Directions (Q. Nos. 1-10) *These questions consist of two statements each linked as Assertion and Reason. While answering of these questions you are required to choose any one of the following five responses.*

(a) If both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are true but Reason is not correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If Assertion is false but Reason is true.
(e) If both Assertion and Reason are false.

1. **Assertion** Angular momentum of single electron in any orbit of hydrogen type atoms is independent of the atomic number of the element.

   Reason In ground state angular momentum is minimum.

2. **Assertion** The radius of second orbit of hydrogen atom is 2.1Å.

   Reason Radius of nth orbit of hydrogen atom $r_n \propto n^2$.

3. **Assertion** Energy E_1 is required to remove first electron from helium atom and energy E_2 is to required to remove the second electron. Then $E_1 < E_2$.

   Reason Ionisation energy of single electron of He^+ is 54.4 eV.

4. **Assertion** The spectral series 'Balmer' of the hydrogen atom lies in the visible region of the electromagnetic spectrum.

   Reason Wavelength of light in the visible region lies is the range of 400 nm to 700 nm.

5. **Assertion** It is essential that all the lines available in the emission spectrum will also be available in the absorption spectrum.

   Reason The spectrum of hydrogen atom is only absorption spectrum.

6. **Assertion** Second orbit circumference of hydrogen atom is two times the de-Broglie wavelength of electrons in that orbit.

   Reason de-Broglie wavelength of electron in ground state is minimum.

7. **Assertion** By increasing the accelerating voltage in coolidge tube wavelength of characteristic X-rays does not change.

   Reason Cut-off wavelength is inversely proportional to the acceleration voltage.

8. **Assertion** Energy of characteristic X-rays is more than the energy of continuous X-rays.

   Reason Characteristic X-rays are produced due to transition of electrons from higher energy states to lower energy states.

9. **Assertion** Wavelength of characteristic X-rays is given by

$$\frac{1}{\lambda} \propto \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

   in the transition from n_2 to n_1. In the above relation proportionality constant does not depend upon the target material.

   Reason Continuous X-rays are target independent.

10. **Assertion** Wavelength of characteristic X-rays is given by

$$\frac{1}{\lambda} \propto \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

   in transition from n_2 to n_1. In the above relation proportionality constant is series dependent. For different series (K-series, L-series, etc.) value of this constant will be different.

   Reason For L-series value of this constant is less than the value for K-series.

Match the columns

1. In Column I physical quantities corresponding to hydrogen and hydrogen like atoms are given. In Column II, powers of principal quantum number n are given on which those physical quantities depend. Match the two columns.

	Column I		Column II
A.	Angular velocity of circular motion of electron	(p)	-4
B.	Centripetal acceleration in circular motion of electron	(q)	1
C.	Angular momentum of electron	(r)	-3
D.	Moment of inertia of electron about centroidal axis	(s)	4

2. Ground state energy of hydrogen atom is E_0. Match the following two columns.

	Column I		Column II
A.	Electrostatic potential energy in ground state of hydrogen atom	(p)	E_0
B.	Total energy in first excited state of He^+ ion	(q)	$-E_0$
C.	Kinetic energy of electron in first excited state of He^+ ion	(r)	$2E_0$
D.	Kinetic energy of electron in ground state of hydrogen atom	(s)	$-2E_0$

3. Wavelength of X-ray photon is a and momentum of X-ray photon is b.
Now, match the following two columns.

	Column I		Column II
A.	Wavelength of γ-ray photon	(p)	$> a$
B.	Momentum of γ-ray photon	(q)	$< a$
C.	Wavelength of UV ray photon	(r)	$> b$
D.	Momentum of UV ray photon	(s)	$< b$

4. Match the Column :

	Column I		Column II
A.	$\frac{Z^3}{n^5}$	(p)	Angular speed
B.	$\frac{Z^2}{n^2}$	(q)	Magnetic field at the centre due to revolution of electron
C.	$\frac{Z^2}{n^3}$	(r)	Potential energy of an electron in nth orbit
D.	$\frac{Z}{n}$	(s)	Speed of electron in nth orbit

Medical entrances' gallery

(Collection of questions asked in NEET & various medical entrance exams)

1. Given the value of Rydberg constant is $10^7\ m^{-1}$, the wave number of the last line of the Balmer series in hydrogen spectrum will be **[NEET 2016]**
(a) $0.5 \times 10^7\ m^{-1}$ (b) $0.25 \times 10^7\ m^{-1}$
(c) $2.5 \times 10^7\ m^{-1}$ (d) $0.025 \times 10^4 m^{-1}$

2. When an α-particle of mass m moving with velocity v bombards on a heavy nucleus of charge Ze, its distance of closest approach from the nucleus depends on m as **[NEET 2016]**
(a) $\frac{1}{\sqrt{m}}$ (b) $\frac{1}{m^2}$
(c) m (d) $\frac{1}{m}$

3. Electrons of mass m with de-Broglie wavelength λ fall on the target in an X-ray tube. The cut-off wavelength (λ_0) of the emitted X-ray is **[NEET 2016]**
(a) $\lambda_0 = \frac{2mc\lambda^2}{h}$ (b) $\lambda_0 = \frac{2h}{mc}$
(c) $\lambda_0 = \frac{2m^2c^2\lambda^3}{h^2}$ (d) $\lambda_0 = \lambda$

4. If an electron in a hydrogen atom jumps from the 3rd orbit to the 2nd orbit, it emits a photon of wavelength λ. When it jumps from the 4th orbit to the 3rd orbit, the corresponding wavelength of the photon will be **[NEET 2016]**
(a) $\frac{16}{25}\lambda$ (b) $\frac{9}{16}\lambda$ (c) $\frac{20}{7}\lambda$ (d) $\frac{20}{13}\lambda$

5. Number of spectral lines in hydrogen atom is **[Guj. CET 2015]**
(a) 8 (b) 6 (c) 15 (d) ∞

6. The excitation potential of hydrogen atom in the first excited state is **[CG PMT 2015]**
(a) 13.6 V (b) –13.6 V (c) 10.2 V (d) –10 V

7. Frequency of K_β X-ray of a material, frequency of K_αX-ray and frequency of L_α X-ray are related as **[UP CPMT 2015]**
(a) $\nu_{K_\beta} = \nu_{K_\alpha} + \nu_{L_\alpha}$ (b) $\nu_{K_\beta} = \nu_{K_\alpha} - \nu_{L_\alpha}$
(c) $\nu_{K_\beta} = \frac{\nu_{K_\alpha}}{\nu_{L_\alpha}}$ (d) $\nu_{K_\beta}^2 = \frac{\nu_{K_\alpha} \cdot \nu_{L_\alpha}}{2}$

8. The magnetic moment (μ) of a revolving electron around the nucleus varies with principal quantum number n as **[Manipal 2015]**
(a) $\mu \propto 1/n$ (b) $\mu \propto 1/n^2$
(c) $\mu \propto n$ (d) $\mu \propto n^2$

9. The ionisation energy of the electron in the hydrogen. Atom in its ground state is 13.6 eV. The atoms are excited to higher energy levels to emit radiations of 6 wavelengths.
Maximum wavelength of emitted radiation corresponds to the transition between **[Manipal 2015]**
(a) $n = 3$ to $n = 1$ states
(b) $n = 4$ to $n = 3$ states
(c) $n = 3$ to $n = 2$ states
(d) $n = 2$ to $n = 1$ states

10. A photon of wavelength 300nm interacts with a stationary hydrogen atom in ground state. During the interaction, whole energy of the photon is transferred to the electron of the atom. State which possibility is correct. (Consider, Planck's constant $= 4 \times 10^{-15}$eVs, velocity of light $= 3 \times 10^8 ms^{-1}$, ionisation energy of hydrogen = 13.6 eV) **[WB JEE 2015]**
(a) Electron will be knocked out of the atom
(b) Electron will go to any excited state of the atom
(c) Electron will go only to first excited state of the atom
(d) Electron will keep orbiting in the ground state of the atom

11. Consider 3rd orbit of He^+ (Helium), using non-relativistic approach, the speed of electron in this orbit will be (given $K = 9 \times 10^9$ constant, $Z = 2$ and h (Planck's constant) $= 6.6 \times 10^{-34}$ J-s) **[CBSE AIPMT 2015]**

(a) $2.92 \times 10^6 ms^{-1}$ (b) $1.46 \times 10^6\ ms^{-1}$
(c) $0.73 \times 10^6\ ms^{-1}$ (d) $3 \times 10^8\ ms^{-1}$

12. What is the wavelength of light for the least energetic photon emitted in the Lyman series of the hydrogen spectrum. (Take, hc = 1240 eV-nm) **[KCET 2015]**

(a) 122 nm (b) 82 nm (c) 150 nm (d) 102 nm

13. The wavelength of k_α, X-rays produced by an X-ray tube 0.76 Å. The atomic number of the anode material of the tube is **[CG PMT 2015]**

(a) 20 (b) 60 (c) 41 (d) 80

14. The de-Broglie wavelength of an electron in 4th orbit is (where, r = radius of 1st orbit) **[MHT CET 2014]**

(a) $2\pi r$ (b) $4\pi r$ (c) $8\pi r$ (d) $16\pi r$

15. The de-Broglie wavelength of an electron is the same as that of a 50 keV X-ray photon. The ratio of the energy of the photon to the kinetic energy of the electron is (the energy equivalent of electron mass of 0.5 MeV) **[WB JEE 2014]**

(a) 1 : 50 (b) 1 : 20 (c) 20 : 1 (d) 50 : 1

16. The ionisation energy of hydrogen is 13.6 eV. The energy of the photon released when an electron jumps from the first excited state ($n = 2$) to the ground state of hydrogen atom is **[WB JEE 2014]**

(a) 3.4 eV (b) 4.53 eV (c) 10.2 eV (d) 13.6 eV

17. Hydrogen atom in ground state is excited by a monochromatic radiation of λ = 975 Å. Number of spectral lines in the resulting spectrum emitted will be **[CBSE AIPMT 2014]**

(a) 3 (b) 2 (c) 6 (d) 10

18. If, an electron in hydrogen atom jumps from an orbit of level $n = 3$ to an orbit of level $n = 2$, emitted radiation has a frequency (R = Rydberg's constant, c = velocity of light) **[MHT CET 2014]**

(a) $\frac{3Rc}{27}$ (b) $\frac{Rc}{25}$ (c) $\frac{8Rc}{9}$ (d) $\frac{5Rc}{36}$

19. Ratio of longest wavelengths corresponding to Lyman and Balmer series in hydrogen spectrum is **[NEET 2013]**

(a) $\frac{5}{27}$ (b) $\frac{3}{23}$ (c) $\frac{7}{29}$ (d) $\frac{9}{31}$

20. The Rutherford scattering experiment proves that an atom consists of **[J & K CET 2013]**

(a) a sphere of positive charge in which electrons are embedded like seeds of water-melon
(b) a sphere of negative charge in which protons are embedded like seeds of water-melon
(c) a sphere of electron cloud in which the positive charge is placed at the centre of the sphere
(d) a sphere of neutral charge

21. According to Bohr model of hydrogen atom, only those orbits are permissible which satisfy the condition **[J & K CET 2013]**

(a) $mv = nh$ (b) $\frac{mv^2}{r} = n\left(\frac{h}{2\pi}\right)$
(c) $mvr = n\left(\frac{h}{2\pi}\right)$ (d) $mvr^2 = n\left(\frac{h}{2\pi}\right)$

22. In Moseley's law $\sqrt{\nu} = a(z - b)$, the values of the screening constant for K-series and L-series of X-rays are respectively **[EAMCET 2013]**

(a) 1, 6.4 (b) 1, 4 (c) 4, 6 (d) 2, 4

23. The K_α X-ray of molybdenum has a wavelength of 71×10^{-12} m. If the energy of a molybdenum atom with K-electron removed is 23.32 keV, then the energy of molybdenum atom when an L-electron removed is ($hc = 12.42 \times 10^{-7}$ eV) **[EAMCET 2013]**

(a) 17.5 keV (b) 40.82 keV
(c) 23.32 keV (d) 5.82 keV

24. Pick out the correct statement from the following **[Karnataka CET 2013]**

(a) Mercury vapour lamp produces line emission spectrum
(b) Oil flame produces line emission spectrum
(c) Band spectrum helps us to study molecular structure
(d) Sunlight spectrum is an example for line absorption spectrum

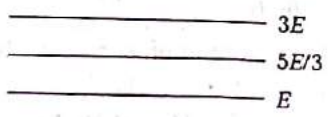
25. Light emitted during the de excitation of electron from $n = 3$ to $n = 2$, when incident on a metal, photoelectrons are just emitted from that metal. In which of the following de excitations photoelectric effect is not possible? **[Karnataka CET 2013]**

(a) From $n = 2$ to $n = 1$ (b) From $n = 3$ to $n = 1$
(c) From $n = 5$ to $n = 2$ (d) From $n = 4$ to $n = 3$

26. The ionisation energy of an electron in the ground state of helium atom is 24.6 eV. The energy required to remove both the electron is **[Karnataka CET 2013]**

(a) 51.8 eV (b) 79 eV (c) 38.2 eV (d) 49.2 eV

27. The figure shows the energy level of certain atom. When the electron de excites from $3E$ to E, an electromagentic wave of wavelength λ is emitted. What is the wavelength of the electromagnetic wave emitted when the electron de excites from $\frac{5E}{3}$ to E? **[Karnataka CET 2013]**

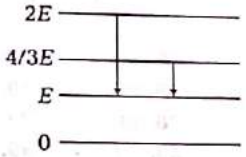


(a) 3λ (b) 2λ (c) 5λ (d) $\frac{3\lambda}{5}$

28. Spectrum of X-rays is [MP PMT 2013]
(a) continuous (b) linear
(c) continuous and linear (d) band

29. Which series of hydrogen spectrum corresponds to ultraviolet region? [MP PMT 2013]
(a) Balmer series (b) Brackett series
(c) Paschen series (d) Lyman series

30. The energy levels of a certain atom are represented in adjoining figure. During the transition from $2E$ to E level, a photon of wavelength λ is emitted. The wavelength of photon produced during transition from $\frac{4}{3}E$ level to E will be [UP CPMT 2013]



(a) 3λ (b) $\lambda/3$ (c) $3\lambda/3$ (d) $4\lambda/3$

31. As per Bohr model, the minimum energy (in eV) required to remove an electron from the ground state of doubly ionised Li-atom ($Z = 3$) is [UP CPMT 2013]
(a) 1.51 (b) 28.7 (c) 53.9 (d) 122.4

32. If the series limit of Lyman series for hydrogen atom is equal to the series limit of Balmer series for a hydrogen like atom, then atomic number of this hydrogen like atom is [BCECE Mains 2012]
(a) 1 (b) 2 (c) 3 (d) 4

33. Electron in hydrogen atom first jumps from third excited state to second excited state and then from second excited to the first excited state. The ratio of the wavelengths $\lambda_1 : \lambda_2$ emitted in the two cases is [CBSE AIPMT 2012]
(a) 7/5 (b) 27/20 (c) 27/5 (d) 20/7

34. An electron of a stationary hydrogen atom passes from the fifth energy level to the ground level. The velocity that the atom acquired as a result of photon emission will be [CBSE AIPMT 2012]
(m is the mass of the electron, R is Rydberg constant and h is Planck's constant)
(a) $\frac{24\,hR}{25\,m}$ (b) $\frac{25\,hR}{24\,m}$ (c) $\frac{25\,m}{24\,hR}$ (d) $\frac{24\,m}{25\,hR}$

Direction (Q. No. 35 and 36) *In each of the following question a statement of Assertion is given following by a corresponding statement of Reason just below it. Of the statement mark the correct answer as*

(a) If both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If both Assertion and Reason are false.

35. **Assertion** Balmer series lies in the visible region of electromagnetic spectrum. [AIIMS 2012]

Reason $\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$, where $n = 3, 4, 5, \ldots$.

36. **Assertion** A beam of charged particles is employed in the treatment of cancer. [AIIMS 2012]

Reason Charged particles on passing through a material medium loss their energy by causing ionisation of the atoms along their path.

37. In Bohr model of hydrogen atom, the force on the electron depends on the principal quantum number (n) as [BHU 2012]
(a) independent of n (b) $F \propto \frac{1}{n^5}$
(c) $F \propto \frac{1}{n^4}$ (d) $F \propto \frac{1}{n^3}$

38. The wavelength of K_α line in copper is 1.5 Å. The ionisation energy of K electron in copper is [Manipal 2012]
(a) 11.2×10^{-17} J (b) 12.9×10^{-16} J
(c) 1.7×10^{-15} J (d) 10×10^{-16} J

39. The wavelength of first line of Balmer series is 6563 Å. The wavelength of first line of Lyman series will be [Manipal 2012]
(a) 1215.4 Å (b) 2500 Å
(c) 7500 Å (d) 600 Å

40. The electron of a hydrogen atom revolves round the proton in a circular nth orbit of radius $r_n = \varepsilon_0 n^2 h^2 / (\pi m e^2)$ with a speed $v_n = \frac{e^2}{2\varepsilon_0 nh}$. The current due to the circulating charge is proportional to [AMU 2012]
(a) e^2 (b) e^3
(c) e^5 (d) e^6

41. If 10000 V is applied across an X-ray tube, what will be the ratio of de-Broglie wavelength of the incident electrons to the shortest wavelength X-ray produced? $\left(\frac{e}{m} \text{ for electron} = 1.8 \times 10^{11}\,\text{C kg}^{-1}\right)$ [JCECE 2012]
(a) 0.1 (b) 0.2
(c) 0.3 (d) 1.0

42. Consider a hydrogen like atom whose energy in nth excited state is given by $E_n = -\frac{13.6Z^2}{n^2}$, when this excited atom makes a transition from excited state to ground state, most energetic photons have energy $E_{max} = 52.224$ eV and least energetic photons have energy $E_{min} = 1.224$ eV. The atomic number of atom is [JCECE 2012]
(a) 2 (b) 4
(c) 5 (d) None of these

Answers

Check point 12.1

1. (d)	2. (b)	3. (b)	4. (b)	5. (a)	6. (a)	7. (b)	8. (c)	9. (c)	10. (d)
11. (b)	12. (c)	13. (c)	14. (b)	15. (d)					

Check point 12.2

1. (d)	2. (b)	3. (b)	4. (b)	5. (a)	6. (b)	7. (b)	8. (b)	9. (b)	10. (d)
11. (c)	12. (d)	13. (c)	14. (a)	15. (c)					

Check point 12.3

1. (b)	2. (c)	3. (b)	4. (c)	5. (b)	6. (c)	7. (c)	8. (a)	9. (d)	10. (c)
11. (d)	12. (c)	13. (a)	14. (c)	15. (b)					

A. Taking it together

1. (c)	2. (a)	3. (d)	4. (a)	5. (a)	6. (a)	7. (a)	8. (d)	9. (a)	10. (c)
11. (b)	12. (b)	13. (b)	14. (d)	15. (d)	16. (a)	17. (d)	18. (b)	19. (b)	20. (a)
21. (b)	22. (a)	23. (a)	24. (a)	25. (d)	26. (a)	27. (c)	28. (d)	29. (c)	30. (a)
31. (b)	32. (d)	33. (d)	34. (b)	35. (d)	36. (c)	37. (c)	38. (c)	39. (a)	40. (b)
41. (d)	42. (c)	43. (d)	44. (a)	45. (c)	46. (c)	47. (c)	48. (a)	49. (d)	50. (b)
51. (c)	52. (b)	53. (a)	54. (b)	55. (a)	56. (a)	57. (a)	58. (c)	59. (a)	60. (d)
61. (b)	62. (a)	63. (a)	64. (c)	65. (a)	66. (d)	67. (b)	68. (b)	69. (a)	70. (d)
71. (a)	72. (d)	73. (a)	74. (a)	75. (b)	76. (b)	77. (c)	78. (a)	79. (b)	80. (c)
81. (b)	82. (d)	83. (a)	84. (c)	85. (d)	86. (c)	87. (b)	88. (c)	89. (a)	90. (b)
91. (a)	92. (b)	93. (a)	94. (c)	95. (b)	96. (b)	97. (a)	98. (b)	99. (d)	100. (c)
101. (b)	102. (d)	103. (d)	104. (d)	105. (c)	106. (b)	107. (a)	108. (b)	109. (a)	110. (a)
111. (d)	112. (d)	113. (d)	114. (a)	115. (c)	116. (a)	117. (b)			

B. Medical entrance special format questions

Assertion and reason

1. (b)	2. (b)	3. (b)	4. (b)	5. (e)	6. (b)	7. (b)	8. (d)	9. (b)	10. (c)

Match the columns

1. (A → r, B → p, C → q, D → s)
2. (A → r, B → p, C → q, D → q)
3. (A → q, B → r, C → p, D → s)
4. (A → s, B → p, C → q, D → s)

C. Medical entrances' gallery

1. (b)	2. (d)	3. (a)	4. (c)	5. (d)	6. (c)	7. (a)	8. (c)	9. (b)	10. (d)
11. (b)	12. (a)	13. (c)	14. (c)	15. (c)	16. (c)	17. (a)	18. (d)	19. (a)	20. (c)
21. (c)	22. (a)	23. (d)	24. (a,c,d)	25. (d)	26. (b)	27. (a)	28. (a)	29. (d)	30. (a)
31. (d)	32. (b)	33. (d)	34. (a)	35. (a)	36. (b)	37. (c)	38. (b)	39. (a)	40. (c)
41. (a)	42. (a)								

Hints and explanations

Check point 12.1

3. (*b*) For the distance of closest approach, we can write,

$$\frac{1}{2}mv^2 = \frac{K \times (Ze)(Ze)}{r_0} \Rightarrow r_0 \propto \frac{1}{m}$$

5. (*a*) We know that impact parameter

$$b = K \cot \frac{\theta}{2}$$

Given, $\theta = 180°$

$\therefore$ $b = K \times \cot 90° = 0$

7. (*b*) Bohr's atom model assumes that electron is in a quantised orbit will not radiate energy.

9. (*c*) $r \propto n^2$

10. (*d*) $r_n \propto \frac{n^2}{Z}$ or $r_n \propto \frac{1}{Z}$

12. (*c*) $v_n = \frac{Ze^2}{2h\varepsilon_0}\frac{1}{n}$

The velocity of electron in the first orbit ($n = 1$) of H-atom ($Z = 1$) is

$$v_1 = \frac{e^2}{2h\varepsilon_0} = \frac{c}{137}$$

$$\therefore \quad \frac{v_1}{c} = \frac{1}{137}$$

14. (*b*) Time period of orbital motion electron, $T = \frac{4\varepsilon_0^2 n^3 h^3}{mZ^2e^4}$

$$\Rightarrow \quad T \propto n^3 \Rightarrow \frac{1}{\text{Frequency } (f)} \propto n^3$$

$$\Rightarrow \quad f \propto n^{-3}$$

15. (*d*) We know that, time period $\propto n^3$ and velocity $\propto \frac{1}{n}$

$\Rightarrow$ Time period $\times$ velocity $\propto n^2$

Check point 12.2

2. (*b*) Kinetic energy of electron in ground state = – (Total energy in this state)

$$= -(-13.6)\text{eV} = 13.6\text{eV}$$

4. (*b*) Potential energy of electron in ground state = 2 × total energy in this state = $2 \times (-13.6 \text{ eV}) = -27.2$ eV

5. (*a*) For $n = 2$, using, $E_n = \frac{-13.6\text{eV}}{n^2}$, $E_2 = -\frac{13.6}{(2)^2} = -3.4$ eV

For $n = 1$, $E_1 = -13.6$ eV

$\therefore$ $E_{1\to 2} = -3.4 - (-13.6) = 10.2$ eV

6. (*b*) $E_3 - E_2 = E$ or $E = -13.6\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = E_1\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$$\Rightarrow \quad \frac{E_1}{9} - \frac{E_1}{4} = E \quad \text{or} \quad E_1 = -7.2E$$

$\therefore$ Ionisation energy of hydrogen atom is $7.2E$.

10. (*d*) Wavelength is minimum for which Z is maximum.

11. (*c*) $\lambda \propto \frac{1}{\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)}$

$$\frac{\lambda_{\text{longest}}}{\lambda_{\text{shortest}}} = \frac{\left(\frac{1}{4} - \frac{1}{\infty}\right)}{\left(\frac{1}{4} - \frac{1}{9}\right)} = \frac{(1/4)}{(5/36)} = \frac{9}{5}$$

12. (*d*) The wavelength will be minimum for which $\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$ is maximum.

14. (*a*) According to Bohr's first postulate,

$$mvr = \frac{nh}{2\pi}$$

$$\therefore \quad 2\pi r = n\left(\frac{h}{mv}\right) = n\lambda$$

For $n = 1, \lambda = 2\pi r$

Check point 12.3

1. (*b*) X-rays were discovered by Roentgen.

2. (*c*) X-rays are electromagnetic waves of wavelength ranging from 0.1 to 100 Å.

3. (*b*) Both X-rays and gamma rays are electromegnetic in nature.

4. (*c*) In X-ray tube, when high speed electrons strike the target they penetrate the target. They loss their kinetic energy and come to rest inside the metal. The electron before finally being stopped makes several collisions with the atoms in the target. At each collision one of the following two types of X-rays get formed (i) continuous X-rays (ii) characteristic X-rays.

6. (*c*) Energy, $E = eV = h\nu_{max}$

$$\therefore \quad \nu_{max} = \frac{eV}{h}$$

7. (*c*) Minimum wavelength produced

$$\lambda_{min} = \frac{12375}{50 \times 10^3} \text{ Å} = 0.247 = 0.25 \text{ Å}$$

8. (*a*) X-rays travel in straight line with speed of light *i.e.*, 3×10^8 m/s.

10. (*b*) If L_β and M_α have a common outer shell, in X-rays spectrum transition of an electron from an outer shell to inner shell gives a characteristics X-rays spectral line.

11. (*d*) In X-ray spectrum wavelength λ of K_α line is given by

$$\frac{1}{\lambda_{K_\alpha}} = \frac{3}{4}(Z-1)^2$$

$$\therefore \quad \lambda_{K_\alpha} \propto \frac{1}{(Z-1)^2}$$

12. (*c*) As, $E(K_\gamma) > E(K_\beta) > E(K_\alpha)$

$$\Rightarrow \quad \lambda(K_\gamma) < \lambda(K_\beta) < \lambda(K_\alpha) \qquad \left[\because \lambda \propto \frac{1}{E}\right]$$

14. (*c*) According to Moseley's law.

$$\nu \propto (Z-b)^2 \Rightarrow \nu = a(Z-b)^2$$

Z = atomic number of element and (a, b are constants).

15. (*b*) Bragg's law for diffraction of X-rays is

$$2d \sin\theta = n\lambda.$$

A) Taking it together

2. (*a*) When X-rays fall on a target, they are absorbed by the target, and they are not reflected by the target, so they are not used for radar purposes.

7. (*a*) A set of atoms in an excited state decays in general to any of the states with lower energy.

8. (*d*) X-rays are electromagnetic waves in nature and they are not deflected by electric and magnetic fields.

16. (*a*) If kinetic energy is E, then total energy will be $-E$ *i.e.*, energy E must be supplied to the electron.

18. (*b*) $p = \frac{h}{\lambda} = \frac{6.6\times10^{-34}}{0.01\times10^{-10}} = 6.626\times10^{-22}$ kg ms^{-1}.

20. (*a*) The simple Bohr model cannot be directly applied to calculate the energy levels of an atom with many electrons. This is because of the electrons not being subject to a central force.

21. (*b*) $v \propto \frac{Z}{n}$, since Z and n both have become twice, velocity of electron in second orbit of He^+ will be v.

23. (*a*) Penetrating power is greater for lower wavelength.

24. (*a*) In the molecules, nuclear force between the nuclei of the two atoms is not important because nuclear forces are short-ranged.

25. (*d*) $BaSO_4$ is heavy element and it absorbs X-rays.

26. (*a*) $L = \frac{nh}{2\pi}$, it does not depend on Z.

28. (*d*) If $n = 4$, then

$$\text{lines} = \frac{n(n-1)}{2} = 6$$

30. (*a*) $E \propto m$ and $r \propto \frac{1}{m}$

i.e., energy will become two times and radius will become half.

32. (*d*) $L_n = \frac{nh}{2\pi} \Rightarrow I\omega = \frac{nh}{2\pi} \Rightarrow (mr^2)\,\omega = \frac{nh}{2\pi}$

As $\quad r \propto n^2$

$$\therefore \quad \omega \propto \frac{1}{n^3}$$

35. (*d*) Impact parameter, $b \propto \cot\frac{\theta}{2}$

36. (*c*) The atomic number of lithium is 3, therefore, the radius of Li^{++} ion in its ground state, on the basis of Bohr's model, will be about $\frac{1}{3}$ times to that of Bohr radius.

Therefore, the radius of lithium ion is near $\frac{53}{3} \approx 18$ pm

38. (*c*) Change in the angular momentum,

$$\Delta L = L_2 - L_1 = \frac{n_2 h}{2\pi} - \frac{n_1 h}{2\pi}$$

$$\Delta L = \frac{h}{2\pi}(n_2 - n_1) = \frac{6.6\times10^{-34}}{2\times3.14}(5-4) = 1.05\times10^{-34}\ \text{J-s}$$

39. (*a*) Magnetic moment μ of a revolving electron is given by

$$\mu = \frac{e}{2m}\mathbf{L} = \frac{e}{2m}\cdot\frac{nh}{2\pi} = \left(\frac{eh}{2m}\right)\cdot\frac{n}{2\pi} = \frac{n}{2\pi}\mu_B$$

Here, $\quad \mu_B = \frac{eh}{2m}$ is Bohr magneton

$$\therefore \quad \mu \propto n$$

40. (*b*) In a hydrogen atom the time period is given by $T \propto n^3$

$$\frac{T_1}{T_2} = \left(\frac{n_1}{n_2}\right)^3 \Rightarrow \frac{8}{1} = \left(\frac{n_1}{n_2}\right)^3 \Rightarrow \frac{n_1}{n_2} = \frac{2}{1}$$

Thus, the values must be $n_1 = 4$ and $n_2 = 2$.

41. (*d*) According to Moseley's law, $\sqrt{\nu} \propto Z$

$$\therefore \quad \frac{\nu_A}{\nu_B} = \left(\frac{Z_A}{Z_B}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

42. (*c*) $Z^2\left[\frac{13.6}{4} - \frac{13.6}{9}\right] = 47.2$

$$\therefore \quad Z = 5$$

43. (*d*) $E \propto Z^2$

$\therefore$ Energy required = $(3)^2\,(13.6)$ eV = 122.4 eV

44. (*a*) Excitation energy, $\Delta E = E_2 - E_1 = 13.6\,Z^2\left[\frac{1}{1^2} - \frac{1}{2^2}\right]$

$$40.8 = 13.6\,Z^2 \times \frac{3}{4}$$

$$\therefore \quad Z = 2$$

So, required energy to remove the electron from ground state

$$= +\frac{13.6\,Z^2}{(1)^2} = 13.6\,(2)^2 = 54.4\ \text{eV}$$

45. (*c*) Final energy of electron $= -13.6 + 12.1 = -1.51$ eV

This energy corresponds to third level *i.e.*, $n = 3$

Hence, number of spectral lines emitted

$$= \frac{n(n-1)}{2} = \frac{3(3-1)}{2} = 3$$

46. (*c*) Wave number is reciprocal of wavelength,

$$\therefore \quad \frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$\nu = \frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{4^2}\right]$$

$$= R\left[\frac{1}{4} - \frac{1}{16}\right] = \frac{3R}{16}$$

47. (*c*) $E_2 - E_1 = \frac{12375}{0.76}$ eV

$$E_3 - E_1 = \frac{12375}{0.64} \text{ eV} \quad \text{...(ii)}$$

$$\therefore \quad E_3 - E_2 = \frac{12375}{0.64} - \frac{12375}{0.76}$$

or $\lambda_{L_\alpha} = \frac{12375}{E_3 - E_2} = 4.1$ Å

48. (*a*) Use Bragg's X-ray diffraction law,

$$n\lambda = 2d \sin\theta$$

$$\therefore \quad \lambda = \frac{2d \sin\theta}{n}$$

For longest wavelength take $\sin\theta = 1$

$$\therefore \quad \lambda = \frac{2 \times 2.82 (\text{Å})}{2} \quad [\because n = 2 \text{ for second order}]$$

$$= 2.82 \text{ Å}$$

49. (*d*) Since, atomic numbers are same, $\lambda_1 = \lambda_2 = \lambda_3$

or $\lambda_2 = \sqrt{\lambda_1 \lambda_3}$

50. (*b*) According to Moseley's law,

$$\nu \propto (Z-b)^2 \quad \text{or} \quad \sqrt{\nu} \propto (Z-b)$$

So, option (b) is correct.

51. (*c*) $a = \frac{v^2}{r}$ or $a \propto \frac{(Z^2)}{(1/Z)}$

$$\therefore \quad a \propto Z^3$$

For singly ionised helium atom, $Z = 2$

and doubly ionised lithium atom, $Z = 3$

$$\Rightarrow \quad \frac{a_{He}}{a_{Li}} = \frac{(2)^3}{(3)^3} = \frac{8}{27}.$$

52. (*b*) $\frac{M}{L} = \frac{q}{2m} = \frac{e}{2m}$

$$\therefore \quad M = \left(\frac{e}{2m}\right) L = \left(\frac{e}{2m}\right)\left(\frac{nh}{2\pi}\right) = \left(\frac{neh}{4\pi m}\right)$$

53. (*c*) $E_{Photon} = \frac{12400}{\lambda(\text{Å})} = \frac{12400}{2000} = 6.2$ eV

$$KE_{max} = E_{Photon} - \phi = 1.6 \text{ eV}$$

Maximum potential of sphere $= 1.6$ eV

54. (*b*) For $n = 3$ to $n = 2$ transition

$$\frac{1}{\lambda_1} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = \frac{5R}{36}$$

For $n = 4$ to $n = 2$ transition

$$\frac{1}{\lambda_2} = R\left[\frac{1}{2^2} - \frac{1}{4^2}\right] = \frac{3R}{16}$$

$$\therefore \quad \frac{\lambda_2}{\lambda_1} = \frac{20}{27}$$

$$\therefore \quad \lambda_2 = \frac{20}{27}\lambda_1 = \frac{20}{27}\lambda_0$$

55. (*a*) $V_{max} = \frac{12400 \times 10^{-10}}{10^{-11}} = 124\text{kV} \Rightarrow V < 124\text{kV}$

56. (*a*) In hydrogen atom $E_2 - E_1 = 10.2$ eV

Since, $5 \text{ eV} < 10.2 \text{ eV}$

The electron cannot excite the hydrogen atom. The collision must be therefore elastic.

57. (*a*) Wavelength is given by

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \Rightarrow \frac{1}{\lambda} = R\left[\frac{1}{(2)^2} - \frac{1}{(4)^2}\right]$$

$$\Rightarrow \quad \frac{1}{\lambda} = R\left(\frac{1}{4} - \frac{1}{16}\right) \Rightarrow \frac{1}{\lambda} = R\left(\frac{4-1}{16}\right)$$

$$\Rightarrow \quad \frac{1}{\lambda} = \frac{3R}{16} \Rightarrow \lambda = \frac{16}{3R}$$

58. (*c*) The minimum energy required to ionise an atom is equal to the energy to excite an electron in the atom from its ground state corresponding to $n = 1$ to its ionisation level corresponding to $n = \infty$. If the atom is given an amount of energy equal to the ionisation energy, the electron is just able to become a free electron and the atom is about to be ionised.

When this occurs, one outermost electron from the atom is just to be removed from the atom. Much higher energy is necessary to free an innermost electron from the atom because this will occur only after the outermost electrons are all removed.

59. (*a*) We know that frequency, $\nu = Rc\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

This gives, $\nu_1 = Rc\left(1 - \frac{1}{\infty}\right) = Rc \Rightarrow \nu_2 = Rc\left(1 - \frac{1}{4}\right) = \frac{3}{4}Rc$

$$\nu_3 = Rc\left(\frac{1}{4} - \frac{1}{\infty}\right) = \frac{Rc}{4} \Rightarrow \nu_1 - \nu_2 = \nu_3$$

60. (*d*) According to the question,

$$(\lambda_{shortest}) \text{ Brackett} = (\lambda_{shortest}) \text{ Balmer}$$

$$\Rightarrow \quad \frac{1}{(\lambda_{shortest}) \text{ Brackett}} = \frac{1}{(\lambda_{shortest}) \text{ Balmer}}$$

$$\Rightarrow \quad Z^2 R\left(\frac{1}{4^2}\right) = R\left(\frac{1}{4}\right) \Rightarrow Z^2 = 4 \Rightarrow Z = 2$$

61. (*b*) According to the question,

For the distance of closest approach kinetic energy will totally be converted to potential energy.

Hence, $\frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r_0} \Rightarrow r_0 = \frac{Qq}{2\pi\varepsilon_0 mv^2}$

62. (*a*) According to the question,

Energy, $E = -3.4$ eV

This corresponds to $n = 2$.

So, the angular momentum of the electron, $mvr = \frac{nh}{2\pi}$

$$L = mvr = \frac{2 \times 6.626 \times 10^{-34}}{2 \times 3.14}$$

$$= \frac{6.626 \times 10^{-34}}{3.14} = 2.1 \times 10^{-34} \text{ J-s}$$

63. (*a*) The wavelength can be given as

$$\frac{1}{\lambda} = Z^2 R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \quad \text{...(i)}$$

For λ to be shortest $\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$ should be maximum

So, $n_2 = \infty$ and $n_1 = 1$

Now, from Eq. (i), we get

$$\frac{1}{\lambda} = R\left(\frac{1}{1}\right) \Rightarrow \lambda = \frac{1}{R} = \frac{1}{10^7} \text{ m} \simeq 1000 \text{ Å}$$

64. (*c*) The energy of first excitation of sodium is $E = h\nu = \frac{hc}{\lambda}$

$$\therefore \quad E = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{5896 \times 10^{-10}} \text{ J}$$

$$E = 3.37 \times 10^{-19} \text{J}$$

Also, since 1.6×10^{-19} J = 1 eV

$$\therefore \quad E = \frac{3.37 \times 10^{-19}}{1.6 \times 10^{-19}} \Rightarrow E = 2.1 \text{ eV}$$

Hence, corresponding first excitation potential is 2.1 V.

65. (*a*) The wavelength (λ) of lines is given by

$$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$$

For Lyman series, the shortest wavelength is for $n = \infty$ and longest is for $n = 2$

$$\therefore \quad \frac{1}{\lambda_S} = R\left(\frac{1}{1^2}\right) \quad \text{...(i)}$$

$$\frac{1}{\lambda_L} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3}{4}R \quad \text{...(ii)}$$

On dividing Eq. (ii) by Eq. (i), we get

$$\frac{\lambda_L}{\lambda_S} = \frac{4}{3}$$

Given, $\lambda_S = 91.2 \text{ nm} \Rightarrow \lambda_L = 91.2 \times \frac{4}{3} = 121.6 \text{ nm}$

66. (*d*) After remaining one electron from helium atom it will become hydrogen like atom.

$$\therefore \quad E = 24.6 + (13.6)(2)^2 = 79 \text{ eV}$$

67. (*b*) $r_3 = (3)^2 \cdot r_1 = 9x$

Now, $3\lambda_3 = (2\pi r_3) = 18\pi x$

$\therefore \quad \lambda_3 = 6\pi x$

68. (*b*) $\frac{1}{\lambda} = R\left(1 - \frac{1}{n^2}\right)$

$$\therefore \quad \frac{R}{n^2} = R - \frac{1}{\lambda} = \frac{\lambda R - 1}{\lambda}$$

$$\therefore \quad n = \sqrt{\frac{\lambda R}{\lambda R - 1}}$$

69. (*a*) Distance of closet approach, $r_0 = \frac{Ze^2}{2mv^2\pi\varepsilon_0}$.

$$\Rightarrow \quad r_0 \propto \frac{1}{m}$$

70. (*d*) $f \propto (Z-1)^2$ for K-series

$$\frac{f'}{f} = \frac{(51-1)^2}{(31-1)^2} = \frac{25}{9}$$

$$\therefore \quad f' = \frac{25}{9} f$$

71. (*a*) $\sqrt{f} \propto (Z-1)$

$$\text{slope} \propto \sqrt{\frac{1}{n_1^2} - \frac{1}{n_2^2}}$$

$$\therefore \quad \frac{(\text{slope})\, K_\beta}{(\text{slope})\, K_\alpha} = \frac{\sqrt{1 - \frac{1}{9}}}{\sqrt{1 - \frac{1}{4}}} = \sqrt{\frac{8}{9} \times \frac{4}{3}} = \sqrt{\frac{32}{27}}$$

72. (*d*) Speed of electron in first orbit of hydrogen atom is given by

$$v = \frac{e^2}{2\varepsilon_0 h}$$

$$\therefore \quad \frac{v}{c} = \frac{e^2}{2\varepsilon_0 hc}$$

73. (*a*) According to scattering formula,

$$N \propto \frac{1}{\sin^4(\theta/2)} \Rightarrow \frac{N_2}{N_1} = \left[\frac{\sin(\theta_1/2)}{\sin(\theta_2/2)}\right]^4$$

$$\Rightarrow \quad \frac{N_2}{N_1} = \left[\frac{\sin\frac{90°}{2}}{\sin\frac{60°}{2}}\right]^4 = \left[\frac{\sin 45°}{\sin 30°}\right]^4$$

$$\Rightarrow \quad N_2 = (\sqrt{2})^4 \times N_1 = 4 \times 56 = 224$$

74. (*a*) $\lambda = \frac{h}{p} = \frac{h}{mv}$

or $\lambda \propto \frac{1}{v}$

$$\therefore \quad \lambda_n \propto n \Rightarrow J_n = n\frac{h}{2\pi} \text{ i.e., } J_n \propto n$$

Hence, $\lambda_n \propto J_n$

75. (*b*) $\Delta E_{4-1} = \left(13.6 - \frac{13.6}{16}\right) \text{ eV} = 12.75 \text{ eV}$

Momentum of hydrogen atom = Momentum of photon

$$= \frac{E}{c} = \frac{12.75 \times 1.6 \times 10^{-19}}{3.0 \times 10^8} = 6.8 \times 10^{-27} \text{ kg ms}^{-1}$$

76. (*b*) $r \propto n^2$ *i.e.*, $\frac{r_f}{r_i} = \left(\frac{n_f}{n_i}\right)^2$

$$\Rightarrow \quad \frac{21.2\times10^{-11}}{5.3\times10^{-11}} = \left(\frac{n}{1}\right)^2 \Rightarrow n^2 = 4 \Rightarrow n = 2$$

77. (*c*) $mvr = \frac{h}{2\pi}$ (in first orbit)

$$\therefore \quad v = \frac{h}{2\pi mr} \Rightarrow a = \frac{v^2}{r} = \frac{h^2}{4\pi^2 m^2 r^3}$$

78. (*a*) Decrease in kinetic energy = increase in potential energy

$$\therefore \quad qV = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q\cdot q}{r}$$

$$\therefore \quad r = \frac{Q}{(4\pi\varepsilon_0)V}$$

i.e., r is independent of q.

79. (*b*) The energy of the projectile will be equivalent to potential energy of the charge system

$$E = \frac{1}{4\pi\varepsilon_0}\frac{(Z_1e)(Z_2e)}{r_0} \Rightarrow \text{Energy, } E \propto Z_1Z_2$$

81. (*b*) The wavelength of series for n is given by

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

where, R is Rydberg's constant.

For Balmer series, $n = 3$ gives the first member of series and $n = 4$ gives the second member of series. Hence,

$$\frac{1}{\lambda_1} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = R\left(\frac{5}{36}\right) \quad \text{...(i)}$$

$$\Rightarrow \quad \frac{1}{\lambda_2} = R\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = R\left(\frac{12}{16\times4}\right) = \frac{3R}{16} \quad \text{...(ii)}$$

$$\Rightarrow \quad \frac{\lambda_2}{\lambda_1} = \frac{16}{3}\times\frac{5}{36} = \frac{20}{27}$$

Given, $\quad \lambda_1 = \lambda \Rightarrow \lambda_2 = \frac{20}{27}\lambda$

82. (*d*) α-particles cannot be attracted by the nucleus.

83. (*a*) $E_1 = \frac{me^4}{8\varepsilon_0^2 h^2}$

$$f_n = \frac{v_n}{2\pi r_n} = \frac{(e^2/2\varepsilon_0 nh)}{(2\pi)\varepsilon_0 n^2h^2/\pi me^2} = \frac{me^4}{4\varepsilon_0^2 n^3h^3} = \frac{8E_1\varepsilon_0^2h^2}{4\varepsilon_0^2n^3h^3} = \frac{2E_1}{n^3h}$$

84. (*c*) $\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \Rightarrow \lambda \propto \frac{1}{Z^2}$

$$\lambda_{Li^{++}} : \lambda_{He^+} : \lambda_H = 4:9:36$$

85. (*d*) $a = \omega^2 r$

As $\quad \omega \propto \frac{1}{n^3}$ and $r \propto n^2$

$$a \propto \frac{1}{n^4}$$

$$\Rightarrow \quad \frac{a_M}{a_L} = \left(\frac{2}{3}\right)^4 = \frac{16}{81}$$

86. (*c*) Wavelength of spectral line in Balmer series is given by

$$\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{n^2}\right]$$

For first line of Balmer series, $n = 3$

$$\therefore \quad \frac{1}{\lambda_1} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = \frac{5R}{36}$$

For second line of Balmer series $n = 4$

$$\frac{1}{\lambda_2} = R\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \frac{3R}{16}$$

$$\therefore \quad \frac{\lambda_2}{\lambda_1} = \frac{20}{27}$$

$$\therefore \quad \lambda_1 = \frac{20}{27}\times6561 = 4860 \text{ Å}$$

87. (*b*) $\lambda \propto \frac{1}{\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)}$

$$\therefore \quad \frac{\lambda}{6563} = \frac{(1/4 - 1/9)}{(1/4 - 1/16)} = \frac{(5/36)}{(12/64)}$$

$$\therefore \quad \lambda = 4861 \text{ Å}$$

88. (*c*) For K_α line, $\lambda_{K_\alpha} \propto \frac{1}{(Z-1)^2}$

So, $\quad \frac{\lambda_2}{\lambda_1} = \left(\frac{Z_1 - 1}{Z_2 - 1}\right)^2$

$$\Rightarrow \quad \frac{\lambda_2}{\lambda_1} = \left(\frac{43-1}{29-1}\right)^2 = \left(\frac{42}{28}\right)^2 \Rightarrow \lambda_2 = \frac{9}{4}\lambda$$

89. (*a*) In the simple Bohr model, only the magnitude of angular momentum is kept equal to some integral multiple of $\frac{h}{2\pi}$, where, h is Planck's constant and thus, the Bohr model gives incorrect values of angular momentum.

90. (*b*) $\frac{\lambda_{Cu}}{\lambda_{Mo}} = \left(\frac{Z_{Mo} - 1}{Z_{Cu} - 1}\right)^2 = \frac{41\times41}{28\times28} = \frac{1681}{784} = 2.144$

91. (*a*) Wavelength λ_k is independent of the accelerating voltage (V), while the minimum wavelength λ_c is inversely proportional to V. Therefore, as V increases λ_k remains unchanged whereas λ_c decreases or $\lambda_k - \lambda_c$ will increase.

92. (*b*) We know that, $\frac{1}{\lambda_{k_\alpha}} = R(Z-1)^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$

$$(Z-1)^2 = \frac{4}{3}\times\frac{1}{\lambda_{k_\alpha}R} = 680$$

$$(Z-1) = 26$$

$$Z = 27 \rightarrow \text{cobalt}$$

93. (*a*) Minimum 10.2 eV energy is required to excite a hydrogen atom. In perfectly inelastic collision maximum energy is lost.

$$\therefore \quad \text{energy loss} = \frac{1}{2}(2m)\left(\frac{v}{2}\right)^2 = 10.2\times1.6\times10^{-19}$$

$$\frac{1}{4}mv^2 = 1.632 \times 10^{-18}$$

$$\therefore \quad \text{velocity } v = \sqrt{\frac{1.632 \times 4 \times 10^{-18}}{1.67 \times 10^{-27}}} = 6.25 \times 10^4 \text{ ms}^{-1}$$

94. (*c*) Lyman series of H-atom, we can write

$$\frac{hc}{\lambda} = Rhc\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

where, symbols have their usual meaning.

and for second line of Balmer series of H-like ion

$$\frac{hc}{\lambda} = Z^2Rhc\left(\frac{1}{2^2} - \frac{1}{4^2}\right)$$

Therefore, $\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = Z^2\left(\frac{1}{4} - \frac{1}{16}\right)$

$$\left(1 - \frac{1}{4}\right) = Z^2\left(\frac{1}{4} - \frac{1}{16}\right) \Rightarrow Z = 2.19 \simeq 2$$

95. (*b*) The ground state of hydrogen ($n = 1$) is represented by K, the first excited state ($n = 2$) is represented by L, the second excited state ($n = 3$) is represent by M. So, the energy transferred by the second electron can be given by

$$E = -13.6\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = -13.6\left(\frac{1}{3^2} - \frac{1}{\infty^2}\right) \simeq -1.51 \text{ eV}$$

96. (*b*) According to Bohr's theory, the wavelength of the radiation

$$\frac{1}{\lambda} = Z^2R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \Rightarrow \frac{\nu}{c} = Z^2R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\nu = cZ^2R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \Rightarrow 2.7 \times 10^{15} = cZ^2R\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

$$\nu = cZ^2R\left(\frac{1}{1^2} - \frac{1}{3^2}\right) \Rightarrow \nu = \frac{32}{27} \times 2.7 \times 10^{15}$$

$$= 3.2 \times 10^{15} \text{ Hz}$$

97. (*a*) The total energy associated with the two H-atoms in the ground state collide in elastically $= 2 \times (13.6 \text{ eV}) = 27.2$ eV. The maximum amount by which their combined kinetic energy is reduced when any one of them goes into first excited state after the inelastic collision.

The total energy associated with the two H-atoms after the collision

$$= \left(\frac{13.6}{2^2}\right) + (13.6) = 17.0 \text{ eV}$$

Therefore, maximum loss of their combined kinetic energy

$$= 27.2 - 17.0 = 10.2 \text{ eV}$$

98. (*b*) Because atom is hollow and whole mass of atom is concentrated in a small centre called nucleus. So, A' will be maximum while B' minimum.

99. (*d*) If E is the energy radiated in transition, then from figure

$$E_{R \to G} > E_{Q \to S} > E_{R \to S} > E_{Q \to R} > E_{P \to Q}$$

For obtaining blue line energy radiated should be more. Hence, correct option is (d).

100. (*c*) When one decides to work in a frame of reference where the electron is at rest, the given expression is not true as it forms the non- inertial frame of reference.

101. (*b*) Given that, $I_1 = \frac{I_0}{2}$ at $t = 2.0$ cm, $I_2 = ?$ at $t = 4$ cm

Since, intensity of X-ray after travelling a distance t becomes

$$I = I_0e^{-\mu t}$$

$$\Rightarrow \quad I_1 = \frac{I_0}{2} I_0e^{-\mu} \times 2.0 \Rightarrow e^{-\mu \times 2.0} = \frac{1}{2}$$

Again, we have

$$I_2 = I_0e^{-\mu \times 4} = I_0 (e^{-\mu \times 2.0})^2$$

$$\Rightarrow \quad I_2 = I_0\left(\frac{1}{3}\right)^2 = \frac{I_0}{4}$$

102. (*d*) The transition marked as A represents series limit of Lyman series, B represents third member of Balmer series and C represents second member of Paschen series.

103. (*d*) We know that, the wavelength of emitted radiation is given by

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = R \times K \text{ where, } K = \frac{1}{n_1^2} - \frac{1}{n_2^2}$$

For λ_{max}, K should be minimum.

Which corresponds to $n_1 = 3$, $n_2 = 4$. So, transition takes place between $n = 4$ and $n = 3$ states.

104. (*d*) Given, radius, $r = 0.1$ nm

We know that, $r_n = \frac{n^2}{Z} r_0$

For $n = 1$, $r_0 = 0.53$ Å $= 0.53 \times 10^{-10}$ m

$$= 5.3 \times 10^{-11} = 5.3 \times 10^{-2} \times 10^{-9} = 0.053 \text{ nm}$$

According to the question,

$$r_n = 0.1 \text{ nm} = \frac{n^2}{1} \times 0.053 \text{ nm}$$

$$\Rightarrow \quad n^2 = \frac{0.1}{0.053} = \frac{100}{53} \Rightarrow n = \frac{10}{\sqrt{53}}$$

We know that, $mvr = \frac{nh}{2\pi} \Rightarrow v = \frac{nh}{2\pi mr}$

By putting the values above equation gives, $v = 1.60 \times 10^6$ m/s

105. (*c*) $E \propto m$

$$\therefore \quad \lambda \propto \frac{1}{m}$$

In case of hydrogen atom

$$\frac{1}{\lambda} = R\left(\frac{1}{4} - \frac{1}{9}\right) \text{ or } \lambda = \frac{36}{5R}$$

For this atom,

$$\lambda' = \frac{\lambda}{2} = \frac{18}{5R}$$

106. (*b*) Let energy corresponding to state A, B and C be E_A, E_B and E_C.

So, from figure

$$(E_C - E_B) + (E_B - E_A) = (E_C - E_A)$$

or $$\frac{hc}{\lambda_1} + \frac{hc}{\lambda_2} = \frac{hc}{\lambda_3} \Rightarrow \lambda_3 = \frac{\lambda_1\lambda_2}{\lambda_1 + \lambda_2}$$

107. (*a*) $T = \frac{2\pi r}{v}$

$\therefore \quad T \propto \frac{r}{v}$ or $T \propto \frac{(n^2)}{(1/n)}$ or $T \propto n^3$

$T_i = 8T_f$

$\therefore \quad n_i = 2n_f$

108. (*b*) If all the elements having $n > 4$ are removed the number of elements that will be present in the periodic table are calculated as

$n = 1$ represents K shell and the number of elements having K shell = 2 [in accordance with $2n^2$]

$n = 2$ represents L shell and the number of elements having L shell = 8

$n = 3$ represents M shell and the number of elements having M shell = 18

$n = 4$ represents N shell and the number of elements having N shell = 32

So, the total number of elements having $n < 5$ are

$$2 + 8 + 18 + 32 = 60$$

109. (*a*) For transition from $4E$ to E,

$$(4E - E) = \frac{hc}{\lambda_1} \Rightarrow \lambda_1 = \frac{hc}{3E} \quad \text{...(i)}$$

For transition from $\frac{7}{3}E$ to E

$$\left(\frac{7}{3}E - E\right) = \frac{hc}{\lambda_2} \Rightarrow \lambda_2 = \frac{3hc}{4E} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get, $\frac{\lambda_1}{\lambda_2} = \frac{4}{9}$

110. (*a*) Energy of emitted radiation, $E = Rhc\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$$\Rightarrow R_{(4 \to 3)} = Rhc\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = Rhc\left(\frac{7}{9 \times 16}\right) = 0.05\,Rhc$$

$$E_{(4 \to 2)} = Rhc\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = Rhc\left(\frac{3}{16}\right) = 0.2Rhc$$

$$E_{(2 \to 1)} = Rhc\left[\frac{1}{(1)^2} - \frac{1}{(2)^2}\right] = Rhc\left(\frac{3}{4}\right) = 0.75Rhc$$

and $E_{(1 \to 4)} = Rhc\left[\frac{1}{(4)^2} - \frac{1}{(1)^2}\right] = -\frac{8}{9}Rhc = -0.9Rhc$

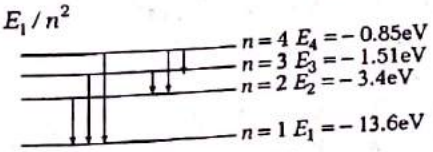
Thus, transition III gives largest amount of energy.

111. (*d*) $\lambda_\alpha - \lambda'_m = 3(\lambda_\alpha - \lambda_m)$

$$\therefore \frac{12375}{10.2(Z-1)^2} - \frac{12375}{20 \times 10^3} = 3\left[\frac{12375}{10.2(Z-1)^2} - \frac{12375}{10 \times 10^3}\right]$$

Solving, we get $Z = 29$

112. (*d*) $E = E_1 / n^2$



Energy levels of H-atom

Energy used for excitation is 12.75 eV

i.e., $(-13.6 + 12.75)$ eV $= -0.85$eV

The photons of energy 12.75 eV can excite the fourth level of H-atom. Therefore, six lines will be emitted.

$\left(n\frac{(n-1)}{2}\text{ lines}\right)$.

113. (*d*) At the distance of closed approach, r

$$K = \frac{1}{4\pi\varepsilon_0}\frac{(2e)(Ze)}{r} \Rightarrow r = \frac{2Ze^2}{4\pi\varepsilon_0 K}$$

where, Ze = charge of the nucleus

$2e$ = charge of the alpha particle

K = kinetic energy of the alpha particle

$$\because \quad K = \frac{p^2}{2m}$$

where, p is the momentum of the α-particle and m is the mass of the electron

$$\therefore \quad r = \frac{2Ze^2 2m}{4\pi\varepsilon_0 p^2} \text{ or } r \propto \frac{1}{p^2}$$

$$\frac{r'}{r} = \left(\frac{p}{p'}\right)^2 = \left(\frac{p}{2p}\right)^2 = \frac{1}{4} \Rightarrow r' = \frac{r}{4}$$

114. (*a*) According to kinetic interpretation of temperature,

$$\text{KE} = \left(\frac{1}{2}mv^2\right) = \frac{3}{2}kT$$

$$\Rightarrow 10.2 \times 1.6 \times 10^{-19} = \frac{3}{2} \times (1.38 \times 10^{-23})T \Rightarrow T = 7.9 \times 10^4 \text{ K}$$

115. (*c*) $\frac{1}{\lambda} = RZ^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$

$$\frac{1}{\lambda_1} = R(1)^2(3/4), \frac{1}{\lambda_2} = R(1)^2(3/4)$$

$$\frac{1}{\lambda_3} = R(2)^2(3/4), \frac{1}{\lambda_4} = R3^2(3/4)$$

$$\frac{1}{\lambda_1} = \frac{1}{4\lambda_3} = \frac{1}{9\lambda_4} = \frac{1}{\lambda_2}$$

So, option (c) is correct.

116. (*a*) According to Bohr theory, $mvr = n\frac{h}{2\pi} \Rightarrow v = \frac{nh}{2\pi mr}$

and $\quad \frac{mv^2}{r} \propto \frac{k}{r} \Rightarrow \frac{m}{r}\left(\frac{n^2h^2}{4\pi^2m^2r^2}\right) \propto \frac{k}{r}$

$\Rightarrow \quad r_n \propto n$

Kinetic energy of the electron,

$$T = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{n^2h^2}{4\pi^2m^2r^2}\right) \Rightarrow T_n \propto \frac{n^2}{r^2}$$

But as $r \propto n$, therefore $T \propto n^0$

117. (*b*) Let ground state energy (in eV) be E_1.

Then, from the given condition,

$$E_{2n} - E_1 = 204\text{eV or } \frac{E_1}{4n^2} - E_1 = 204\text{ eV}$$

$$\Rightarrow E_1\left(\frac{1}{4n^2} - 1\right) = 204\text{ eV} \quad \text{...(i)}$$

and $E_{2n} - E_n = 40.8\text{eV}$

$$\Rightarrow \quad \frac{E_1}{4n^2} - \frac{E_1}{n^2} = E_1\left(-\frac{3}{4n^2}\right) = 40.8\text{eV} \quad \text{...(ii)}$$

Form Eqs. (i) and (ii), we get

$$\frac{1-\frac{1}{4n^2}}{3/4n^2} = 5 \Rightarrow n = 2$$

B) Medical entrance special format questions

Assertion and reason

1. (*b*) $L = n\left(\frac{h}{2\pi}\right)$ *i.e.*, $L \propto n$. In ground state, $n = 1$,

so L = minimum.

2. (*b*) Radius of Bohr's orbit $r \propto n^2$

$$\Rightarrow \quad \frac{r_1}{r_2} = \left(\frac{n_1}{n_2}\right)^2 \Rightarrow = \frac{5.29\times10^{-11}}{r_2} = \left(\frac{1}{2}\right)^2$$

$$\Rightarrow \quad r_2 = 5.29\times10^{-11}\times4 = 21.16\times10^{-11}\text{ m}$$

3. (*b*) $E_H = 13.6$ eV (ionisation energy)

$\therefore E_{He^+} = 13.6(Z)^2 = (13.6)(Z)^2 = 54.4$ eV

5. (*e*) **For Assertion** It is not essential that all the lines available in the emission spectrum will also be available in the absorption spectrum.

For Reason The spectrum of hydrogen atom may not be absorption spectrum.

6. (*b*) $mv_2r_2 = 2\left(\frac{h}{2\pi}\right)$

$$\therefore 2\pi r_2 = 2\left(\frac{h}{mv_2}\right) = 2\lambda_2$$

Further, $\lambda = \frac{h}{p}$

Speed of momentum is maximum in ground state. Hence, λ is minimum.

9. (*b*) Proportionality constant $\propto (Z-b)^2$.

10. (*c*) This proportionality constant $\propto (Z-b)^2$.

For *K*-series : $b = 1$

For *L*-series : $b = 7.4$

Match the columns

1. (A) $\omega = \frac{v}{r} \propto \frac{(1/n)}{(n^2)} \propto n^{-3}$

(B) $a = r\omega^2 \propto (n^2)(n^{-6}) \propto n^{-4}$

(C) $L = n\frac{h}{2\pi}$ or $L \propto n$

(D) $I = mr^2$ or $I \propto r^2 \propto (n^2)^2 \propto n^4$

2. As $E \propto \frac{Z^2}{n^2}$

$\therefore \quad U = 2E, K = -E$

3. Momentum is given by $p = \frac{h}{\lambda}$ or $p \propto \frac{1}{\lambda}$

Here, wavelength of X-ray photon = *a*

momentum of X-ray photon = *b*

$\lambda_{\gamma\text{-rays}} < \lambda_{\text{X-rays}} < \lambda_{\text{UV-rays}}$

4. A-q, B-r, C-p, D-s

$$\omega = 2\pi n = 2\pi\frac{v}{2\pi r} = \frac{v}{r} \Rightarrow v \propto \frac{Z}{n}, r \propto \frac{n^2}{Z}, E_n \propto \frac{Z^2}{n^2}$$

$$\omega \propto \frac{Z^2}{n^3}, B_{\text{centre}} = \mu_0\frac{eV}{\frac{2\pi r}{2r}} = \frac{\mu_0 eV}{4\pi r^2} \propto \frac{Z^3}{n^5}$$

C) Medical entrances' gallery

1. (*b*) Given, Rydberg constant, $R = 10^7\text{ m}^{-1}$

$\because$ For last line in Balmer series, $n_2 = \infty$, $n_1 = 2$

As we know that,

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \Rightarrow \frac{1}{\lambda} = 10^7\left(\frac{1}{2^2} - \frac{1}{\infty}\right)$$

$$\bar{v} = \frac{1}{\lambda} = \frac{10^7}{4} = 0.25\times10^7\text{ m}^{-1}$$

2. (*d*) When an α-particle of mass *m* moving with velocity *v* bombards on a heavy nucleus of charge *Ze*, then there will be no loss of energy, so initial kinetic energy of α-particle = potential energy of α-particle at closest approach.

$$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{2Ze^2}{4\pi\varepsilon_0 r_0} \Rightarrow r_0 \propto \frac{1}{m}$$

3. (*a*) Cut-off wavelength occurs when incoming electron losses its complete energy in collision. This energy appears in the form of X-rays.

Given, mass of electrons = *m*

de-Broglie wavelength = λ

So, kinetic energy of electron $= \frac{p^2}{2m} = \frac{\left(\frac{h}{\lambda}\right)^2}{2m} = \frac{h^2}{2m\lambda^2}$

Now, maximum energy of photon can be given by

$$E = \frac{hc}{\lambda_0} = \frac{h^2}{2m\lambda^2} \Rightarrow \lambda_0 = \frac{hc\times2\lambda^2\cdot m}{h^2} = \frac{2mc\lambda^2}{h}$$

4. (*c*) Excess energy of electron appears as photon.

From Rydberg's formula,

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5R}{36}$$

$$\frac{1}{\lambda'} = R\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = \frac{7R}{144}$$

$$\frac{1}{\lambda} / \frac{1}{\lambda'} = \frac{5R}{36} \div \frac{7R}{144}$$

$$\Rightarrow \quad \frac{\lambda'}{\lambda} = \frac{5R}{36}\times\frac{144}{7R} = \frac{20}{7} \Rightarrow \lambda' = \frac{20}{7}\lambda$$

5. (*d*) Number of spectral lines in hydrogen atom is ∞.

6. (*c*) Energy of electron in ground state of hydrogen atom,

$$E_1 = -13.6 \text{ eV} \qquad \left[\because E_n = \frac{-13.6}{n^2} \text{ eV}\right]$$

$\therefore$ Energy of electron in first excited state of hydrogen atom,

$$E_2 = \frac{-13.6}{2^2} = -3.4 \text{ eV}$$

Hence, the excitation energy of hydrogen atom in first excited state,

$$E = E_2 - E_1 = -3.4 - (-13.6) = 10.2 \text{ eV}$$

Therefore, excitation potential is 10.2 V.

7. (*a*) As energy of K_α X-rays is

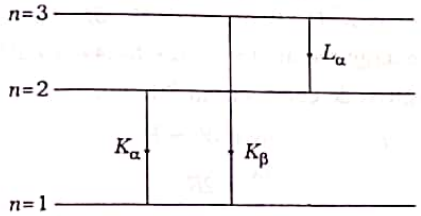
$$E_{K_\alpha} = E_{K_\beta} - E_L \qquad \text{...(i)}$$

$$E_{K_\beta} = E_{K_\alpha} + E_{L_\alpha} \qquad \text{...(ii)}$$

On adding Eq. (i) and Eq. (ii)

$\therefore$ $$h\nu_{K_\beta} = h\nu_{K_\alpha} + h\nu_{L_\alpha} \qquad [E = h\nu]$$

or $$\nu_{K_\beta} = \nu_{K_\alpha} + \nu_{L_\alpha}$$



8. (*c*) Orbital magnetic moment of an electron

$$\mu_1 = \frac{neh}{4\pi m_e}$$

$\therefore$ $$\mu_1 \propto n$$

9. (*b*) Number of wavelengths $= \dfrac{n(n-1)}{2}$

where, n = number of orbit from which transition takes place

$\therefore$ $$6 = \frac{n(n-1)}{2} \Rightarrow n = 4$$

$\therefore$ The wavelength of emitted radiations will be maximum for transition $n = 4$ to $n = 3$.

10. (*d*) The energy of the photon,

$$E = \frac{hc}{\lambda} = \frac{4 \times 10^{-15} \text{eVs} \times 3 \times 10^8 \text{ms}^{-1}}{300 \times 10^{-9} \text{m}} = 4\text{eV}$$

The ionisation energy is 13.6 eV which is greater than energy of photon, so atom cannot come into excited state and will remain in ground state.

11. (*b*) Energy of electron in the 3rd orbit of He^+ is

$$E_3 = -13.6 \times \frac{Z^2}{n^2} \text{ eV} = -13.6 \times \frac{4}{3^2} \text{ eV}$$

$$= -13.6 \times \frac{4}{9} \times 1.6 \times 10^{-19} \text{ J}$$

From Bohr's model, $E_3 = -KE_3 = -\dfrac{1}{2} m_e v^2$

$$\Rightarrow \frac{1}{2} \times 9.1 \times 10^{-31} \times v^2 = -13.6 \times \frac{4}{9} \times 1.6 \times 10^{-19}$$

$$\Rightarrow v^2 = \frac{13.6 \times 4 \times 16 \times 10^{-19} \times 2}{9.1 \times 10^{-31} \times 9}$$

$$\Rightarrow v^2 = 2.12 \times 10^{12} \Rightarrow v = 1.46 \times 10^6 \text{ms}^{-1}$$

12. (*a*) Given, $hc = 1240$ eV

For least energetic photon, $E = E_2 - E_1 = 10.2$ eV

We know that, $E = \dfrac{hc}{\lambda}$; $\lambda = \dfrac{hc}{E}$; $\lambda = \dfrac{1240}{10.2}$

$$\lambda = 121.56 \approx 122 \text{ nm}$$

13. (*c*) Given, wavelength, $\lambda_{K_\alpha} = 0.76 \text{ Å} = 0.76 \times 10^{-10}$ m

Moseley's empirical formula for K_α X-rays when adapted to Bohr model is given by

$$h\nu_{K_\alpha} = \frac{3}{4} \times 13.6 (Z-1)^2 \text{ eV}$$

or $$\frac{hc}{\lambda_{K_\alpha}} = \frac{3}{4} \times 13.6 \times 1.6 \times 10^{-19} (Z-1)^2 \text{ J}$$

$$[\because 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}]$$

$$\Rightarrow \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{0.76 \times 10^{-10}} = 16.08 \times 10^{-19} (Z-1)^2$$

$$[\because h = 6.626 \times 10^{-34} \text{ J-s and } c = 3 \times 10^8 \text{ms}^{-1}]$$

$$\Rightarrow 26.15 \times 10^{-16} = 16.08 \times 10^{-19} (Z-1)^2$$

$$\Rightarrow 1.63 \times 10^3 = (Z-1)^2 \Rightarrow 16.3 \times 10^2 = (Z-1)^2$$

$$\Rightarrow Z - 1 = 40.4$$

$\therefore$ $$Z = 40.4 + 1 \approx 41$$

14. (*c*) de-Broglie wavelength, $\lambda = \dfrac{h}{p}$ ...(i)

According to Bohr's quantisation condition,

$$mvr_n = \frac{nh}{2\pi} \Rightarrow pr_n = \frac{nh}{2\pi} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\lambda = \frac{h \times 2\pi r_n}{nh} \Rightarrow \lambda = \frac{2\pi r_n}{n}$$

For fourth orbit ($n = 4$) $\Rightarrow$ $\lambda = \dfrac{2\pi r_4}{4}$ ...(iii)

Moreover, $r \propto n^2 \Rightarrow \dfrac{r_1}{r_4} = \dfrac{(1)^2}{(4)^2} \Rightarrow r_4 = 16r$

Substituting the value of r_4 in Eq. (iii), we get

$$\lambda = \frac{2\pi [16 r_1]}{4} = 8\pi r_1 = 8\pi r$$

15. (*c*) de-Broglie wavelength, $\lambda = \dfrac{h}{\sqrt{2mK}}$

The kinetic energy of the electron,

$$K_{\text{electron}} = \frac{h^2}{2m \cdot \lambda^2} \qquad \text{...(i)}$$

where, h = Planck's constant, λ = wavelength

The photon energy, $E_{\text{photon}} = \dfrac{ch}{\lambda}$ ...(ii)

From Eqs. (ii) and (i), we get

$$\frac{E_{\text{photon}}}{K_{\text{electron}}} = \frac{ch/\lambda}{h^2/2m \cdot \lambda^2} = \frac{hc \cdot \lambda^2 2m}{h^2 \cdot \lambda} = \frac{2m\lambda c}{h} \qquad \left[\lambda = \frac{hc}{\nu}\right]$$

$$= \frac{2m}{h/\lambda c} = \frac{2 \times 5 \times 10^5}{50 \times 10^3} = 20 = 20:1 \qquad \left[\because m = \frac{h}{c\lambda}\right]$$

16. (*c*) The energy required to removed electron in the first excited state($n = 2$)

The energy of the photon $= Rhc\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$

$$\Rightarrow \quad 13.6\left(\frac{1}{1} - \frac{1}{4}\right) = 13.6\left(\frac{4-1}{4}\right)$$

$$= 13.6 \times \frac{3}{4} = \frac{40.8}{4} = 10.2 \text{ eV}$$

17. (*a*) Energy provided to the ground state electron

$$= \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{975 \times 10^{-10}} = \frac{6.6 \times 3}{975} \times 10^{-16}$$

$$= 0.020 \times 10^{-16} = 2 \times 10^{-18} \text{ J} = \frac{20 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV}$$

$$= \frac{20}{1.6} = 12.5 \text{ eV}$$

It means the electron is jump to $n = 3$ from $n = 1$

The number of possible spectral lines from $n = 3$ to $n = 1$ is 3

i.e., $(n = 3) \longrightarrow (n = 1)$

$(n = 3) \longrightarrow (n = 2)$

and $(n = 2) \longrightarrow (n = 1)$

18. (*d*) When an electron jumps from orbit n_i to orbit n_f ($n_i > n_f$), the frequency of emitted photon is given by

$$\nu = cR\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \Rightarrow \nu = cR\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$\nu = cR\left(\frac{5}{36}\right) \Rightarrow \nu = \frac{5cR}{36}$$

19. (*a*) $\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

For longest wavelength of Lyman series $n_1 = 1$ and $n_2 = 2$

For longest wavelength of Balmer series $n_1 = 2$ and $n_2 = 3$

$$\text{So } \frac{\lambda_{\text{Lyman}}}{\lambda_{\text{Balmer}}} = \frac{5}{27}$$

20. (*c*) Rutherford's model states that an atom is a sphere of diameter about 10^{-10} m with whole of its positive charge concentrated at the centre.

21. (*c*) According to Bohr's model, $mvr = \frac{nh}{2\pi}$.

22. (*a*) In Moseley's law $\sqrt{\nu} = a(Z - b)$, the values of the screening constant for K-series and L-series of X-rays are respectively, 1 and 6.4.

23. (*d*) Given, $E_k = 23.32 \text{ keV} = 23.32 \times 1.6 \times 10^{-19}$ eV

$hc = 12.42 \times 10^{-7}$ eV

$\lambda_{k\alpha} = 71 \times 10^{-12}$ m

Now, $E_L - E_K = \frac{hc}{h_{k\alpha}}$

$$E_L - 23.32 \times 1.6 \times 10^{-19} = \frac{12.42 \times 10^{-7}}{71 \times 10^{-12}}$$

$$\Rightarrow \quad E_L = 5.82 \text{ keV}$$

24. (*a,c,d*) The line emission spectra is produced by excited substance in atomic state, as by mercury vapour lamp. Line absorption spectra is produced by e.m. radiations, that has passed through a medium hence, sunlight spectrum is a line absorption spectrum. The Band spectrum is used to study the molecular structure. Here, *a*, *c* and *d* are the correct statements.

25. (*d*) The photoelectric effect is possible when light of sufficient energy incident on metal surface. When electron de excites form $n = 3$ to $n = 2$, the light produces photoelectric effect. So for the given options, the photoelectric effect is not possible for that level in which energy of emitted light is less than that for $n = 3$ to $n = 2$.

$$(h\nu)_{4-3} < (h\nu)_{3-2}$$

Hence, photoelectric effect is not possible.

26. (*b*) Ionisation energy in ground state = 24.6 eV

Energy required to remove 2^n electron from He^{2+}

$$= Z^2 (13.6) \text{ eV} = (2)^2 (13.6) = 54.4 \text{ eV}$$

So, total energy required = 24.6 + 54.4 = 79 eV

27. (*a*) As electron de excites from $3E \rightarrow E$

$\therefore \quad h\nu = 3E - E$

or $\quad \frac{hc}{\lambda} = 2E \quad$ ...(i)

When electron de excites from $\frac{5E}{3} \rightarrow E$

$$\frac{hc}{\lambda'} = \frac{5E}{3} - E = \frac{2E}{3} \quad \text{...(ii)}$$

Substituting the value of E from Eq. (i) in above equation

$$\frac{hc}{\lambda'} = \frac{2}{3}\left(\frac{hc}{\lambda_2}\right) \Rightarrow \lambda' = 3\lambda$$

28. (*a*) Continuous spectrum of X-rays consist of radiations of all possible X-ray wavelengths within a certain range starting from λ_{min} onwards.

30. (*a*) When transition takes place from higher energy level to lower energy level, the energy difference

$$\Delta E = h\nu = \frac{hc}{\lambda}$$

$\therefore \quad \lambda = \frac{hc}{\Delta E}$ So, $\frac{\lambda_2}{\lambda_1} = \frac{\Delta E_1}{\Delta E_2}$

When $\quad \Delta E_1 = 2E - E = E$; $\lambda_1 = \lambda$

and for $\quad \Delta E_2 = \frac{4}{3}E - E = \frac{E}{3}$

$$\frac{\lambda_2}{\lambda} = \frac{E/E}{3} = 3 \Rightarrow \lambda_2 = 3\lambda$$

31. (*d*) Minimum energy, $E = 13.6 Z^2$ eV $= 13.6 (3)^2 = 122.4$ eV

32. (*b*) By using, $\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

For hydrogen atom, $\frac{1}{(\lambda_{min})_H} = R\left[\frac{1}{1^2} - \frac{1}{\infty}\right] = R$

$$\frac{1}{(\lambda_{min})_H} = R \text{ or } (\lambda_{min})_H = \frac{1}{R} \quad \text{...(i)}$$

For another atom,

$$\frac{1}{(\lambda_{min})_{atom}} = RZ^2\left(\frac{1}{2^2} - \frac{1}{\infty}\right) = \frac{RZ^2}{4}$$

$$\Rightarrow \quad (\lambda_{min})_{atom} = \frac{4}{RZ^2} \quad ...(ii)$$

From Eqs. (i) and (ii), we get, $\frac{1}{R} = \frac{4}{RZ^2} \Rightarrow Z = 2$

33. (*d*) Here, for wavelength λ_1

$n_1 = 4$ and $n_2 = 3$

and for λ_2, $n_1 = 3$ and $n_2 = 2$

We have $\frac{hc}{\lambda} = -13.6\left[\frac{1}{n_2^2} - \frac{1}{n_1^2}\right]$ So, for λ_1

$$\Rightarrow \quad \frac{hc}{\lambda_1} = -13.6\left[\frac{1}{(4)^2} - \frac{1}{(3)^2}\right]$$

$$\Rightarrow \quad \frac{hc}{\lambda_1} = 13.6\left[\frac{7}{144}\right] \quad ...(i)$$

Similarly, for λ_2

$$\Rightarrow \quad \frac{hc}{\lambda_2} = -13.6\left[\frac{1}{(3)^2} - \frac{1}{(2)^2}\right]$$

$$\Rightarrow \quad \frac{hc}{\lambda_2} = 13.6\left[\frac{5}{36}\right] \quad ...(ii)$$

Hence, from Eqs. (i) and (ii), we get

$$\frac{\lambda_1}{\lambda_2} = \frac{20}{7}$$

34. (*a*) Here, $E_5 - E_1 = \frac{hc}{\lambda}$

and $\quad \frac{Rhc}{25} - Rhc = \frac{hc}{\lambda} \Rightarrow \frac{24}{25}R = \frac{1}{\lambda}$

But $\quad p = \frac{h}{\lambda}$ and $v = \frac{h}{m\lambda} = \frac{24}{25}\frac{Rh}{m}$

35. (*a*) The wavelength in Balmer series is given by

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right), n = 3, 4, 5$$

$$\frac{1}{\lambda_{max}} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$\Rightarrow \quad \lambda_{max} = \frac{36}{5R} = \frac{36}{5 \times 1.097 \times 10^7} = 6563\text{Å}$$

and $\quad \frac{1}{\lambda_{min}} = R\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right)$

$$\Rightarrow \quad \lambda_{max} = \frac{4}{R} = \frac{4}{1.097 \times 10^7} = 3646\text{Å}$$

The wavelength 6563Å and 3646Å lie in visible region. Therefore, Balmer series lies in visible region.

36. (*b*) A radiation consists of a beam of charged particles. When radiation is used for cancer treatment, then on falling upon the cancerous tissues, it destroys the, cancer cells.

37. (*c*) We have, $F \propto \frac{v^2}{r}$

Also, $\quad v \propto \frac{1}{n}$ and $r \propto n^2 \Rightarrow F \propto \frac{1}{n^4}$

38. (*b*) Required ionisation energy, $E = \frac{hc}{\lambda}$

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{1.5 \times 10^{-10}} = 12.9 \times 10^{-16} \text{ J}$$

39. (*a*) $\frac{\lambda_{Lyman}}{\lambda_{Balmer}} = \frac{\left(\frac{1}{2^2} - \frac{1}{3^2}\right)}{\left(\frac{1}{1^2} - \frac{1}{2^2}\right)} = \frac{5}{27} \quad \left[\because \frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)\right]$

$$\Rightarrow \quad \lambda_{Lyman} = \frac{5}{27} \times \lambda_{Balmer} = \frac{5}{27} \times 6563 = 1215.4 \text{ Å}$$

40. (*c*) Time period of electron, $T = \frac{2\pi r}{v} = \frac{\frac{2\pi \times \varepsilon_0 n^2h^2}{\pi me^2}}{\frac{e^2}{2\varepsilon_0 nh}}$

$$T = \frac{4\varepsilon_0^2 n^3 h^3}{me^4}$$

Current, $\quad i = \frac{e}{T} = \frac{e}{\frac{4\varepsilon_0^2 n^3 h^3}{me^4}} \Rightarrow i = \frac{me^5}{4\varepsilon_0^2 n^3 h^3}$ i.e., $i \propto e^5$

41. (*a*) For incident electron, $\frac{1}{2}mv^2 = eV$

or $\quad p^2 = 2meV$

$\therefore$ de-Broglie wavelength, $\lambda_1 = \frac{h}{p} = \frac{h}{\sqrt{2meV}}$

Shortest X-rays wavelength, $\lambda_2 = \frac{hc}{eV}$

$$\therefore \quad \frac{\lambda_1}{\lambda_2} = \frac{1}{c}\sqrt{\left(\frac{V}{2}\right)\left(\frac{e}{m}\right)}$$

$$= \frac{\sqrt{\frac{10^4}{2} \times 1.8 \times 10^{11}}}{3 \times 10^8} = 0.1$$

42. (*a*) Maximum energy is liberated for transition $E_n \rightarrow 1$ and minimum energy for $E_n \rightarrow E_{n-1}$

Hence, $\quad \frac{E_1}{n^2} - E_1 = 52.224 \text{ eV} \quad ...(i)$

and $\quad \frac{E_1}{n^2} - \frac{E_1}{(n-1)^2} = 1.224 \text{ eV} \quad ...(ii)$

Solving Eqs. (i) and (ii), we get

$$E_1 = -54.4 \text{eV and } n = 5$$

But $\quad E_1 = -\frac{13.6Z^2}{1^2}$

$$\therefore \quad -54.4 = \frac{13.6}{1^2}Z^2 \Rightarrow Z = 2$$

CHAPTER

13 Nuclei

In the last chapter, we learnt that atomic nucleus was discovered in the year 1911 by Rutherford and his associates from the study of large angle scattering of alpha particles from the thin metal foils. In this chapter, we shall study the constituents of the nucleus and how they are held together. We shall also study different properties of nuclei such as size, mass, density and stability of nuclei and associated phenomena such as radioactivity, nuclear fission and nuclear fusion.

1 Nucleus

As per the Rutherford's experiment, it was concluded that, the entire positive charge and mass of an atom was present at its centre known as **nucleus.**

Composition of nucleus

Nucleus is composed of two types of particles

(i) Protons-positively charged particles

(ii) Neutrons, neutral particles

Mass of neutron is slightly greater than the mass of the proton.

$$\text{Mass of proton, } m_p = 16726231 \times 10^{-37}\text{kg}$$

$$\text{Mass of neutron, } m_n = 16749286 \times 10^{-37}\text{kg}$$

$$m_p \simeq 1840\, m_e, \text{ where } m_e = \text{mass of electron}$$

Atomic number

Atomic number of an element is the number of proton present inside the nucleus of a neutral atom. It is also equal to the number of electrons revolving in various orbits around the nucleus of the atom. It is represented by Z.

Atomic number (Z) = Number of proton = Number of electron.

Mass number

The total number of neutrons and protons, *i.e.*, number of nucleous present inside the nucleus of an element is called its mass number A. Number of neutrons is represented by N.

So, **Mass number (A) = $Z + N$**

Inside

1) Nucleus
- Isotopes, isobars and isotones

2) Mass energy relation
- Binding energy of nucleus
- Nuclear forces
- Nuclear stability
- Nuclear reaction

3) Nuclear energy

4) Radioactivity

It is represented by $_Z$(chemical symbol)^A or $_ZX^A$.

Total charge of the nucleus is equal to the total charge on all Z protons in it = Ze, where $e = 1.6 \times 10^{-19}$ C.

Size of nucleus

From various experiments, it was found that the volume of the nucleus is directly proportional to the number of nucleons (mass number) constituting the nucleus.

If R is the radius of the nucleus having mass number A, then

$$V = \frac{4}{3}\pi R^3 \propto A \Rightarrow R \propto A^{1/3}$$

$$\Rightarrow \quad \boldsymbol{R = R_0 A^{1/3}}$$

where, $R_0 = 1.2 \times 10^{-15}$ m, is the range of nuclear size.

It is also known as **nuclear unit radius.**

Owing to the small size of the nucleus, fermi (fm) is found to be a convenient unit of length in Nuclear Physics.

It is defined as,

$$1 \text{ fermi (fm)} = 10^{-15} \text{ m}$$

Example 13.1 *What is the nuclear radius of ^{125}Fe, if that of ^{27}Al is 6.4 fermi?*

Sol. Radius of the nucleus is,

$$R = R_0 A^{1/3}$$

$$\frac{R_{Fe}}{R_{Al}} = \left(\frac{A_{Fe}}{A_{Al}}\right)^{1/3} = \left(\frac{125}{27}\right)^{1/3}$$

$$\text{Radius, } R_{Fe} = \frac{5}{3}R_{Al} = \frac{5}{3} \times 6.4 = 10 \text{ fm}$$

Nuclear density

Density of nuclear matter is the ratio of mass of nucleus and its volume.

If m is the average mass of a nucleon ($\simeq 1.67 \times 10^{-27}$ kg) and A is the mass number of element, then the mass of nucleus = mA. If R is the nuclear radius, then

$$\text{Volume of nucleus} = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi(R_0 A^{1/3})^3 = \frac{4}{3}\pi R_0^3 A$$

$$\text{As, density of nuclear matter} = \frac{\text{mass of nucleus}}{\text{volume of nucleus}}$$

$$\therefore \quad \rho = \frac{mA}{\frac{4}{3}\pi R_0^3 A} \quad \text{or} \quad \boxed{\rho = \frac{3m}{4\pi R_0^3}}$$

Thus, the density of nucleus is a constant, independent of A, for all nuclei. Different nuclei are likes drop of liquid of constant density. The density of nuclear matter is approximately 2.3×10^{17} kgm^{-3}. This density is very large as compared to ordinary matter.

Note Number of nucleon per unit volume $= \dfrac{\text{Density}}{\text{Mass of one nucleon}}$

$$= \frac{10^{17}}{1.67 \times 10^{-27}} \text{ Nu/m}^3$$

$$\simeq 10^{44} \text{ Nu/m}^3$$

Nuclide

A nuclide is a specific nucleus of an atom, which is characterised by its atomic number Z and mass number A.

Example 13.2 *Assuming that protons and neutrons have equal masses, calculate how many times nuclear matter is denser than water. Take mass of nucleon $= 1.67 \times 10^{-27}$ kg and $R_0 = 1.2 \times 10^{-15}$ m.*

Sol. Density of nucleus (of water)

$$\rho = \frac{3m}{4\pi R_0^3} = \frac{3 \times 1.67 \times 10^{-27}}{4 \times \frac{22}{7} \times (1.2 \times 10^{-15})^3}$$

$$= \frac{7 \times 3 \times 1.67 \times 10^{18}}{88 \times 1.2 \times 1.2 \times 1.2} = 2.307 \times 10^{17} \text{ kg/m}^3$$

Density of water $\rho' = 10^3$ kg/m^3

$$\therefore \quad \frac{\rho}{\rho'} = \frac{2.307 \times 10^{17}}{10^3} = 2.307 \times 10^{14}$$

Nuclear spin

Protons and neutrons inside a nucleus moves in a well defined quantum states and thus they have orbital angular momentum. But, both of them also have internal spin angular momentum.

The total angular momentum of the nucleus is the resultant of all the spin and orbital angular momentum of the individual nucleons. This total angular momentum of the nucleus is known as nuclear spin of that nucleus.

Note All nuclei with a given Z and N are all together taken as **nuclide**.
e.g., all $^{32}_{16}S$ together taken as one nuclide.

Isotopes, isobars and isotones

Isotopes

The atoms of same elements having the same number of protons but different number of neutrons are called isotopes. In other words, isotopes have same value of atomic number (Z) but different value of mass number (A). Almost every element has isotopes. Because of the same atomic number, isotopes of an element have the same place in the periodic table.

The isotopes of some elements are

$$_1H^1, {}_1H^2, {}_1H^3;\ {}_8O^{16}, {}_8O^{17}, {}_8O^{18};\ {}_2He^3, {}_2He^4$$

In nature, the isotopes of chlorine ($_{17}Cl^{35}$ and $_{17}Cl^{37}$) are found in the ratio 75.4% and 24.6%. When chlorine is prepared in laboratory, its atomic mass is found to be,

$$M = (35 \times 0.754) + (37 \times 0.246) = 35.5$$

Note Since, the isotopes have the same atomic number, they have the same chemical properties. Their physical properties are different as they have different mass numbers. Two isotopes, thus cannot be separated by chemical method, but they can be separated from the physical methods.

Example 13.3 *Two stable isotopes of $_3Li^6$ and $_3Li^7$ have respective abundances of 7.5% and 92.5%. Those isotopes have masses 6.01512 u and 7.01600 u respectively. Find the atomic weight of lithium.*

Sol. Atomic weight = Weighted average of the isotopes

$$= \frac{6.01512 \times 7.5 + 7.01600 \times 92.5}{(7.5 + 92.5)}$$

$$= \frac{45.1134 + 648.98}{100} = 6.941 \text{ u}$$

Isobars

The atoms of different elements having the same mass number (A) but different atomic number (Z) are called isobars. They have different places in periodic table. Their chemical as well as physical properties are different. $_1H^3$ and $_2He^3$, $_8O^{17}$ and $_9F^{17}$ are examples of isobars.

Isotones

The atoms of different elements having equal number of neutrons ($A - Z$) but different number of protons are called isotones.

$_3Li^7$ and $_4Be^8$, $_1H^3$ and $_2He^4$ are examples of isotones.

Example 13.4 *Select the pairs of isotones from the following nuclei. $_{12}Mg^{24}$, $_1H^3$, $_2He^4$, $_{11}Na^{23}$.*

Sol. Isotones have same number of neutrons, $N = A - Z$

(i) $_1H^3$ and $_2He^4$

Number of neutrons $= 3 - 1$ or $4 - 2 = 2$

(i) $_{12}Mg^{24}$ and $_{11}Na^{23}$

Number of neutrons $= 24 - 12$ or $23 - 11 = 12$

Note Nuclei having same mass number A but with proton number (Z) and neutron number ($A-Z$) interchanged are called **mirror nuclei**.

e.g., $_1H^3$ and $_2H^3$, $_3Li^7$ and $_4Be^7$.

Check point 13.1

1. The mass number of a nucleus is
(a) always less than its atomic number
(b) always more than its atomic number
(c) always equal to its atomic number
(d) sometimes more than and sometimes equal to its atomic number

2. Outside a nucleus
(a) neutron is stable
(b) proton and neutron both are stable
(c) neutron is unstable
(d) Neither neutron nor proton is stable

3. Nucleus of an atom whose mass number is 24 consists of
(a) 11 electrons, 11 protons and 13 neutrons
(b) 11 electrons, 13 protons and 11 neutrons
(c) 11 protons and 13 protons
(d) 11 protons and 13 electrons

4. For a nucleus to be stable, the correct relation between neutron number N and proton number Z for light nuclei is
(a) $N > Z$ (b) $N = Z$
(c) $N < Z$ (d) $N > Z$

5. In helium nucleus, there are
(a) 2 protons and 2 electrons
(b) 2 neutrons, 12 protons and 2 electrons
(c) 2 protons and 2 neutrons
(d) 2 positrons and 2 protons

6. In $_{88}Ra^{226}$ nucleus, there are
(a) 138 protons and 88 neutrons
(b) 138 neutrons and 88 protons
(c) 226 neutrons and 88 electrons
(d) 226 neutrons and 138 electrons

7. As compared ^{12}C atom, ^{14}C atom are
(a) Two extra protons and two extra electrons
(b) Two extra protons but no extra electrons
(c) Two extra neutrons and no extra electrons
(d) Two extra neutrons and two extra electrons

8. The radius of a nucleus of a mass number A is directly proportional to
(a) A^3 (b) A
(c) $A^{2/3}$ (d) $A^{1/3}$

9. The radius of $_{29}Cu^{64}$ nucleus in fermi is (given, $R_0 = 1.2 \times 10^{-15}$ m)
(a) 4.8 (b) 1.2 (c) 7.7 (d) 9.6

10. The three stable isotopes of neon $_{10}Ne^{20}$, $_{10}Ne^{21}$ and $_{10}Ne^{22}$ have respective abundances of 90.51%, 0.27% and 9.22%. The atomic masses of the three isotopes are 19.99 u, 20.99 u and 21.99 u respectively. The average atomic mass of neon will be
(a) 20.17 u (b) 30.17 u
(c) 20.98 u (d) 30.98 u

2) Mass energy relation

According to Einstein, mass and energy are interconvertible. He predicted that, if the energy of a body changes by an amount E, its mass changes by an amount m given by the equation, $E = \Delta mc^2$

where, c is the speed of light. Everyday examples of energy gain are too small to produce detectable changes of mass. But in nuclear physics this plays an important role. Mass appears as energy and the two can be regarded as equivalent.

Mass defect of nucleus

It has been found that the rest mass of the nucleus of a stable atom is always less than the sum of the masses of its constituents nucleons (protons and neutrons) in the free state. The difference between the sum of the masses of the nucleons constituting a nucleus and the rest-mass of the nucleus is called mass defect (Δm).

Mass defect Δm = (mass of protons + mass of neutrons) − mass of the nucleus

Let us consider an atom of element ${}_ZX^A$.

Its atomic number is Z and mass number is A. Thus, its nucleus has Z protons and $(A - Z)$ neutrons. If m_p is the mass of a proton, m_n is the mass of a neutrons and m_N the mass of the nucleus, then the mass defect is,

$$\Delta m = Zm_p + (A - Z)m_n - m_N$$

Atomic mass unit or amu

In nuclear physics mass is measured in unified atomic mass units abbreviated amu or u as mass of an atom is very small.

(i) The amu is defined as $\frac{1}{12}$ th mass of a ${}_6C^{12}$ atom.

(ii) 1 amu (or 1 u) $= 1.66 \times 10^{-27}$ kg.

(iii) It can be shown using relation ($\Delta E = \Delta mc^2$) that 1 u mass has energy 931.5 MeV

For 1 u, $\Delta m = 1.66 \times 10^{-27}$ kg

$$c = 3 \times 10^8 \text{ ms}^{-1}$$

$$\Delta E = \Delta mc^2 = 1.66 \times 10^{-27} (3 \times 10^8)^2$$

$$= 1.494 \times 10^{-10} \text{ J}$$

$$= \frac{1.49 \times 10^{-10}}{1.6 \times 10^{-19}} \text{ eV}$$

$$= 931.5 \text{ MeV}$$

$\therefore$ **1 u = 931.5 meV**

Example 13.5 *Calculate the mass defect of Helium* $[{}_2He^4]$.

Mass of proton $= 1.007276$ u,

Mass of neutron $= 1.008665$ u,

Mass of ${}_2He^4 = 4.001506$ u

Sol. Mass defect = Mass of nucleons − Mass of nucleus

[Here, A(mass number) = 4

Z(atomic number) = 2

$\Rightarrow$ Number of protons = 2

Number of neutrons $= A - Z = 4 - 2 = 2$

= Mass of 2 protons + Mass of 2 neutrons − Mass of nucleus

$= [2 \times 1.007276 + 2 \times 1.008665 - 4.001506] = 0.030376$ u

Example 13.6 *Calculate the energy required to remove the least tightly bound neutron in* ${}_{20}Ca^{40}$. *Given that*

Mass of ${}_{20}Ca^{40} = 39.962589$ *amu*

Mass of ${}_{20}Ca^{39} = 38.970691$ *amu*

Mass of neutron $= 1.008665$

Sol. Here in order to remove a neutron, energy has to be supplied.

Mass defect, $\Delta m = M({}^{39}_{20}\text{Ca}) + M({}^{1}_{0}n) - M({}^{40}_{20}\text{Ca})$

$= 38.970691 + 1.008665 - 39.962589$

$= 0.016767$ amu

Equivalent energy $= 0.06767 \times 931$ ($\because$ 1 amu = 931 MeV)

$= 15.6$ MeV

Example 13.7 *A neutron breaks into a proton and electron. Calculate the energy produced in this reaction in* $m_e = 9 \times 10^{-31}$ *kg,* $m_p = 1.6725 \times 10^{-27}$ *kg,* $m_n = 1.6747 \times 10^{-27}$ *kg,* $c = 3 \times 10^8$ *m/s.*

Sol. We have ${}^1_0n \rightarrow {}^1_1p + {}^0_{-1}e$

Mass defect, $\Delta m = m_n - m_p - m_e$

$= [1.6747 \times 10^{-27} - 1.6725 \times 10^{-27} - 0.0009 \times 10^{-27}]$

$= 1.3 \times 10^{-30}$ kg

Energy produced $= \Delta mc^2 = 1.3 \times 10^{-30} \times (3 \times 10^8)^2$

$= 11.7 \times 10^{-14}$ J $= \frac{11.7 \times 10^{-14}}{1.6 \times 10^{-13}} = 0.73$ MeV

($\because$ 1 MeV $= 1.6 \times 10^{-13}$ J)

Packing fraction

$$= \frac{\text{Actual isotopic mass} - \text{Mass number}}{\text{Mass number}} = \frac{m - A}{A}$$

It may be positive, negative and zero.

Packing fraction of ${}^{40}C\left(\frac{n}{P} = 1\right)$ is zero.

If isotopic mass is less than mass number, packing fraction is negative, indicating some has been converted into energy, called BE, are move stable.

In general, lower the packing fraction, greater is the BE per nucleon, and hence greater is the stability.

Binding energy of nucleus

The binding energy (E_b) of a nucleus is defined as the minimum energy required to separate its nucleons and place them at rest at infinite distance apart.

or

The binding energy of a nucleus may be defined as the energy equivalent to the mass defect of the nucleus.

If Δm is mass defect, then according to Einstein's mass energy relation.

Binding energy $= \Delta mc^2 = [\{Zm_p + (A - Z)m_n\} - m_N] \cdot c^2$

This binding energy is expressed in joule if Δm is measured in kg.

If Δm is measured in amu, then binding energy

$$E_b = \Delta m \text{ amu} = [\{Zm_p + (A - Z)m_n\} - m_N] \text{ amu}$$

$$\boldsymbol{E_b = \Delta m \times 931 \text{ MeV}}$$

Note Binding energy can also be written as,

$E_b = [Zm_H + (A - Z)m_N - m_A]c^2$

where m_H is the mass of H-atom and m_A the atomic mass. By using the masses of H-atoms rather than protons, masses of the electrons in the atom cancel out. We do this because it is atomic masses that are measured directly by mass spectrometer. A slight error made by doing so but that is negligible.

Example 13.8 *Find the binding energy of an α-particle from the following data.*

Mass of the helium nucleus $m\,({}_2He^4) = 4.001265u$

Mass of proton $m_p = 1.007277\ u$

Mass of neutron $m_n = 1.008666\ u$

(Take, $1\ u = 931.4813$ *MeV)*

Sol. α-particle consists of 2 protons and 2 neutrons

Mass defect, $\Delta m = (Zm_p + Nm_n - m({}^4_2\text{He})$

$= 2 \times 1.007277 + 2 \times 1.008666 - 4.001265$

$= 0.3062$ u

Binding energy $= \Delta m \times 931.4813$ MeV

$= 0.03062 \times 931.48 = 28.52$ MeV

Binding energy per nucleon

The average energy required to release a nucleon from the nucleus is called binding energy per nucleon.

Binding energy per nucleon

$$= \frac{\text{Total binding energy}}{\text{Mass number (i.e. total number of nucleons)}}$$

$$= \frac{\Delta m \times 931 \text{MeV}}{A \text{ nucleon}}$$

$$\Rightarrow \boxed{E_{bn} = \frac{E_b}{A}}$$

Binding energy per nucleon ∝ stability of nucleus.

Example 13.9 *Binding energy of* ${}_{17}Cl^{35}$ *and* ${}_{15}P^{31}$ *are 287.67 MeV and 262.48 MeV. Which of the two nuclei is more stable?*

Sol. Binding energy per nucleon of ${}_{17}\text{Cl}^{35} = \dfrac{\text{Binding energy}}{\text{Mass number}}$

$$= \frac{287.67}{35} = 8.22 \text{ MeV}$$

Binding energy per nucleon of ${}_{15}\text{P}^{31} = \dfrac{262.48}{31} = 8.47 \text{MeV}$

BE of ${}_{15}\text{P}^{31}$ is more than BE of ${}_{17}\text{Cl}^{35}$.

Hence, ${}_{15}\text{P}^{31}$ is more stable.

Example 13.10 *If the binding energy per nucleon of deuteron is 1.115 MeV, find its mass defect in atomic mass unit.*

Sol. Binding energy per nucleon is $E_{bn} = \dfrac{E_b}{A}$

For deuteron, atomic mass, $A = 2$

Substituting the values in above equation,

$$\therefore \quad 1.115 \text{ MeV} = \frac{E_b}{2}$$

$$E_b = 2 \times 1.115 \Rightarrow E_b = \Delta mc^2 \Rightarrow \Delta m = \frac{E_b}{c^2}$$

Mass defect, $\Delta m = \dfrac{2 \times 1.115}{931.5}$ u $= 0.0024$ u

Binding energy curve

It is the graph between binding energy per nucleon and total number of nucleons (*i.e.*, mass number *A*).

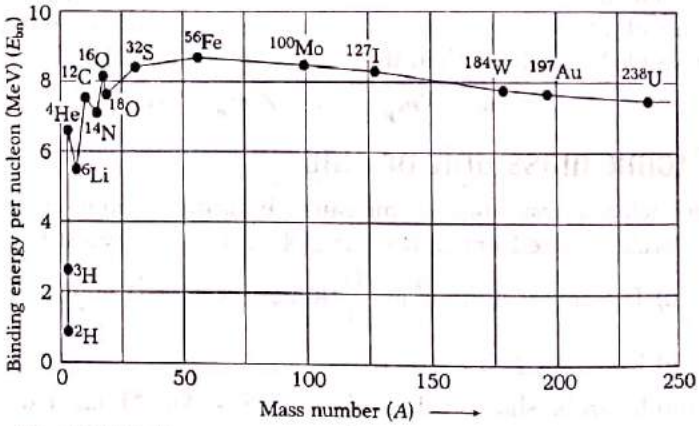


Fig. 13.1 *Binding energy per nucleon as a function of mass number*

Some features of this curve are given below

(i) The curve has almost a flat maximum roughly from $A = 50$ to $A = 80$ corresponding to an average BE/nucleon of about 8.5 MeV. So, the nuclei having mass number between 50 to 80 are most stable. Iron (Fe^{56}) ($A = 56$), having a BE/nucleon of about 8.8 MeV has maximum stability.

(ii) For nuclei having mass number more than 80, the average binding energy per nucleon decreases slowly and drop to about 7.6 MeV for Uranium ($A = 238$). The lower value of binding energy per nucleon fails

to overcome the repulsion among protons in nuclei having large number of protons. *i.e.*, the nuclei of heavier atoms, beyond $_{83}Bi^{209}$ are radioactive.

(iii) For nuclei having mass number below 20, it decreases sharply. For example, for heavy hydrogen (H^2), it is only about 1.1 MeV. This shows that the nuclei having mass number below 20 are comparatively less stable.

(iv) Below $A = 50$, the curve does not fall continuously, but has subsidiary peaks at $_8O^{16}$, $_6C^{12}$ and $_2He^4$. This shows that these (even-even) nuclei are more stable than their immediate neighbours.

Conclusions from the graph

(i) The curve shows that very heavy and very light nuclei have a lower average BE/nucleon than the nuclei of intermediate masses. So, if a very heavy nucleus (such as U^{238}) be split into two lighter nuclei near the flat maximum of the curve, the BE/nucleon will increase. Hence, energy will be released in the process. This method of releasing nuclear energy by breaking up a heavy nucleus into two lighter nuclei of comparable masses is called **nuclear fission.** It is the basis of nuclear bombs and nuclear reactors.

(ii) Alternatively, if two, or more, very light nuclei (such as H^2) can be combined to form a heavier nucleus (such as He^4), the BE/nucleon will again increase and now by a much greater amount than in the fission process. This will result in much larger release of energy. This method of releasing nuclear energy is called **nuclear fusion.** It occurs inside sun and other stars and is the source of their energy.

Nuclear forces

As we know that nucleus consists of positively charged protons and neutral particle neutrons. The protons due to their positive charge repel each other, so there must be a strong forces inside the nucleus which overcome the repulsive force of protons and bind them together inside the nucleus, this force is called nuclear force.

Therefore, the forces that holds the nucleons together inside the nucleus of an atom are called nuclear forces.

Some important properties of nuclear forces are given below

(i) Nuclear forces are charge independent.

(ii) Nuclear forces are short range forces. These do not exist at large distances greater than 10^{-15}m.

(iii) These are attractive force and cause stability of the nucleus.

(iv) Nuclear forces are non-central force.

(v) Nuclear forces are the strongest forces in nature.

(vi) Nuclear forces depends on the direction of the spins of the nucleons. The force is stronger if the spin of nucleons is parallel and is weak if it is anti-parallel.

(vii) Electrical and gravitational forces follows inverse square law (*i.e.*, $F \propto 1/r^2$), but nuclear forces are complicated in nature and does not follows it.

Yukawa's Meson theory of nuclear forces

A Japanese scientist Yukawa in 1935 suggested that the nuclear forces are 'exchange forces' which are produced by the exchange of new particles called π mesons between nucleons. These particles were later on actually discovered in cosmic radiation.

There are three types of π mesons, π^+, π^- and π^0. There is a continuous exchange of π-mesons between protons and neutrons due to which they continue to be converted into one another.

$$p \rightarrow \pi^+ + n \text{ and } n \rightarrow p + \pi^-$$

The forces between a pair of neutrons or a pair of protons are the result of the exchange of neutral meson (π^0) between them *i.e.*,

$$p \rightarrow p' + \pi^0 \text{ and } n \rightarrow n' + \pi^0$$

Thus exchange of π meson between nucleons keeps the nucleons bound together. It is responsible for the 'nuclear force'.

Nuclear Stability

The stability of a nucleus is determined by the value of its binding energy per nucleon. Higher the binding energy of nucleon, more stable is the nucleus. The stability of nucleus is also determined by its neutron to proton ratio. A plot of number of neutrons and number of protons is shown in the figure below.

In the figure, the solid line shows the nuclei with equal number of protons and neutron ($N = Z$). Only light nuclei are on this line, *i.e.* they are stable if they contain approximately same number of protons and neutrons.

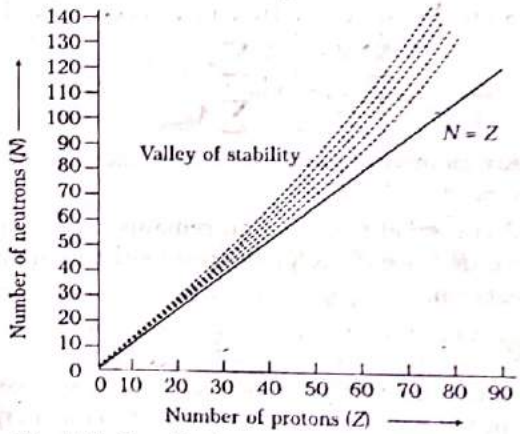


***Fig. 13.2** Graphical representation of number of neutrons versus number of protons*

Heavy nuclei are stable only when they have more neutrons than protons.

The long narrow region shown in the figure, which contains the cluster of short lines representing stable nuclei is referred to as the valley of stability.

The nuclide $^{209}_{83}Bi$ is the heaviest stable nucleus.

Experimental study shows that the stable nuclei contain even number of protons or neutrons or both.

Magic numbers

We know that the electrons in an atom are grouped in 'shells' and 'sub-shells'. Atoms with 2, 10, 18, 36, 54 and 86 electrons have all of their shells completely filled. Such atoms are unusually stable and chemically inert. A similar situation exists with nuclei also. Nuclei having 2, 8, 20, 28, 50, 82 and 126 nucleons of the same kind (either protons or neutrons) are more stable than nuclei of neighbouring mass numbers. These numbers are called as 'magic numbers'.

Nuclear reaction

The process by which the identity of a nucleus is changed when it is bombarded by an energetic particle is called nuclear reaction. *The general expression for the nuclear reaction is a follow*

$$\underset{\text{(Parent nucleus)}}{X} + \underset{\text{(Incident particle)}}{a} \longrightarrow \underset{\text{(Compound nucleus)}}{C}$$

$$\longrightarrow \underset{\text{(Product nucleus)}}{Y} + \underset{\text{(Daughter nucleus)}}{b} + \underset{\text{energy}}{Q}$$

Here, X and a are the reactants, Y and b are the products and Q is the kinetic energy released in the nuclear reaction.

Conservation laws obeyed by nuclear reaction are

(i) Mass number and electric charge is conserved

i.e., $\sum Z_{\text{initial}} = \sum Z_{\text{final}}$;

$\sum A_{\text{initial}} = \sum A_{\text{final}}$

(ii) Linear momentum and angular momentum are conserved.

(iii) Total energy of the reaction remains conserved, where in, term Q, helps to balance the total energy of reaction.

Q-value

Q-value means the difference between the rest mass energy of initial constituents and the rest mass energy of final constituents of a nuclear reaction.

Q-value of nuclear reaction

As considered, $X + a \longrightarrow Y + b + Q$

where, $Q = U_i - U_f$

Here, U_i = Rest mass energy of initial constituents

U_f = Rest mass energy of final constituents

$= (M_X + M_a - M_Y - M_b)\cdot c^2$

If, $Q = +\text{ve}$ = reaction is exothermic

and $Q = -\text{ve}$ = reaction is endothermic.

e.g.,

For α-decay, $_ZX^A \rightarrow {}_{Z-2}Y^{A-4} + {}_2He^4$

So, Q value will be,

$$Q = [m({}_ZX^A) - m({}_{Z-2}Y^{A-4}) - m({}_2He^4)].c^2$$

which represents the kinetic energy available to the products.

For β^-decay, $_ZX^A \rightarrow {}_{Z+1}Y^A + e^- + \bar{\nu}$

$$U_i = [m({}_ZX^A) - Zm_e]\cdot c^2$$

$$U_f = [m({}_{Z+1}Y^A) - (Z+1)m_e + m_e]\cdot c^2$$

$\Rightarrow$ Kinetic energy available to the products,

$$Q = U_i - U_f = [m({}_ZX^A) - m({}_{Z+1}Y^A)].c^2$$

For β^+-decay, $_ZX^A \rightarrow {}_{Z-1}Y^A + e^+ + \nu$

$$U_i = [m({}_ZX^A) - Zm_e]\cdot c^2$$

$$U_f = [m({}_{Z-1}Y^A) - (Z-1)m_e + m_e]\cdot c^2$$

$\Rightarrow$ Kinetic energy available to the products,

$$Q = [m({}_ZX^A) - m({}_{Z-1}Y^A) - 2m_e]\cdot c^2$$

For electron capture, $_ZX^A \rightarrow {}_{Z-1}Y^A$

$$Q = [m({}_ZX^A) - m({}_{Z-1}Y^A)]\cdot c^2$$

Example 13.11 *A deuteron strikes $_7N^{14}$ nucleus with the subsequent emission of an α-particle. Find the atomic number, mass number and chemical name of the element so produced.*

Sol. The nuclear reaction as per the given data is given as,

$$_7N^{14} + {}_1H^2 \longrightarrow {}_ZX^A + {}_2He^4 \quad (\alpha\text{-particle})$$

According to laws of conservation

(i) For charge,

$$\sum Z_{\text{initial}} = \sum Z_{\text{final}}$$

$\Rightarrow$ $7 + 1 = Z + 2 \Rightarrow Z = 6$

(ii) For mass number,

$$\sum A_{\text{initial}} = \sum A_{\text{final}}$$

$\Rightarrow$ $14 + 2 = A + 4 \Rightarrow A = 12$

$\Rightarrow$ $_6X^{12} \longrightarrow$ the element is carbon $_6C^{12}$.

3 Nuclear energy

The energy released during nuclear reaction is called nuclear energy. Generally, there is loss of mass in nuclear reaction, which results (according to Einstein's equation) into release of energy.

Consequently, if nuclei with less total binding energy transform to nuclei with greater binding energy, there will be a net energy release. This happens when a heavy nucleus decays into two or more intermediate mass fragments (fission) or when light nuclei fuse into a heavier nucleus (fusion).

In a nuclear reaction, the energy released is of the order of MeV. For same quantity of matter, nuclear sources produces a million times more energy than a chemical source.

Two distinct ways of obtaining energy from nucleus are as

(i) Nuclear fission (ii) Nuclear fusion

Nuclear fission

The process of splitting of a heavy nucleus into two lighter nuclei of comparable masses (after bombardment with a energetic particle) with liberation of energy is called nuclear fission.

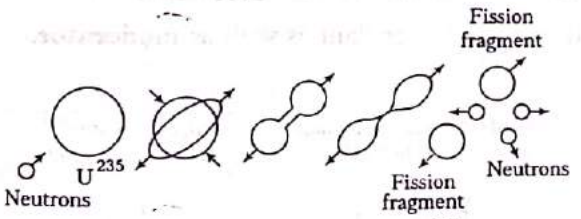


Fig. 13.3 Nuclear fission in U^{235}

Fission reaction resulting from the absorption of neutron is known as induced fission.

One of the common example of fission is when uranium isotope $_{92}U^{235}$ bombarded with a neutron breaks into two intermediate mass nuclear fragments.

$$_{92}U^{235} + {}_0n^1 \rightarrow \underset{\text{Unstable nucleus}}{_{92}U^{236}} \rightarrow {}_{56}Ba^{144} + {}_{36}Kr^{89} + 3\,{}_0n^1 + Q$$

This is one of the type into which the fission of $_{92}U^{235}$ takes place, there can be other combinations also.

In general, fission nuclei contains excess of neutrons, therefore are highly unstable. Hence, undergo beta decay until they reaches a stable end product.

The energy released in U^{235} fission is about 200 MeV or 0.8 MeV per nucleon and on an average 2.5 neutrons are liberated. These neutrons are called **fast neutrons** and their energy is about 2 MeV (each). These fast neutrons can escape from the reaction. So, in order to proceed the reaction they are needed to slowed down. Fission of U^{235} occurs by slow neutrons only of energy about 1 eV or even by thermal neutrons of energy about 0.025 eV.

Note The mass of the compound nucleus must be greater than the sum of masses of fission products.

The $\frac{\text{binding energy}}{A}$ of compound nucleus must be less than that of the fission products. Hence, mass difference *i.e.*, mass defect, is converted into energy ($E = mc^2$). Thus, tremendous amount of energy is released, first appears as kinetic energy of the fragments and neutrons, then is transferred to the surrounding appearing as heat.

Chain reaction

In nuclear fission many neutrons are produced along with the release of large energy. Under favourable conditions these neutrons can cause further fission of other nuclei, producing large number of neutrons.

Thus, a chain of nuclear fissions is established which continues until the whole of the uranium is consumed.

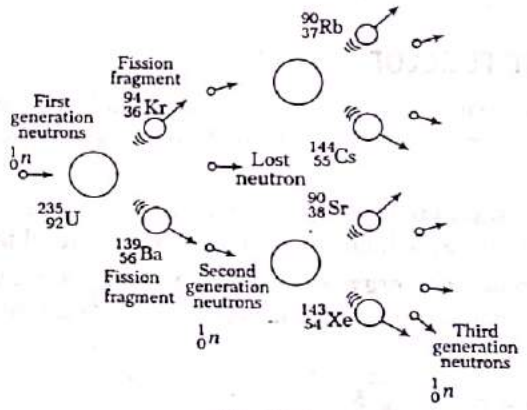


Fig. 13.4

In the chain reaction, the number of nuclei undergoing fission increases very fast.

So, the energy produced takes a tremendous magnitude very soon.

Important points related to chain reaction

A chain reaction once started will remain steady, accelerate or retard depends upon neutron reproduction factor or multiplication factor (R) which is defined as

$$R = \frac{\text{Rate of production of neutrons}}{\text{Rate of loss of neutrons}}$$

If $R = 1$, then chain reaction will be steady. The size of the fissionable sample used is said to be the critical size and it's mass, the critical mass.

If $R > 1$, then chain reaction grows faster. Size of the material is super critical.

If $R < 1$, then chain reaction comes to hault. Size of the material is said as sub-critical.

There are two types of chain reaction

(i) **Controlled chain reaction** The reaction which can be controlled by absorbing suitable number of neutrons at each stage of the reaction, so that all neutrons are absorbed except one, which is available for further fission. This type of reaction is called controlled chain reaction. The rate of this reaction is slow and the energy liberated is always less than the explosive energy. The reproduction factor is, $R = 1$. Nuclear reactor works on this principle.

(ii) **Uncontrolled chain reaction** The reaction which cannot be controlled externally, rather there are more than one neutron taking part in reaction, then the reaction will accelerate at such a rapid rate that the whole material will explode within few micro-seconds, liberating tremendous amount of energy. The reproduction factor is $R > 1$. Atom bombs works on this principle.

Nuclear reactor

A nuclear reactor is a device in which nuclear fission can be carried out through a sustained and controlled chain reaction.

It is also called an atomic pile. It is thus a source of controlled energy which is utilised for many useful purposes.

Nuclear reactor energy is used to generate steam, which operates on a turbine and turns a electric generator.

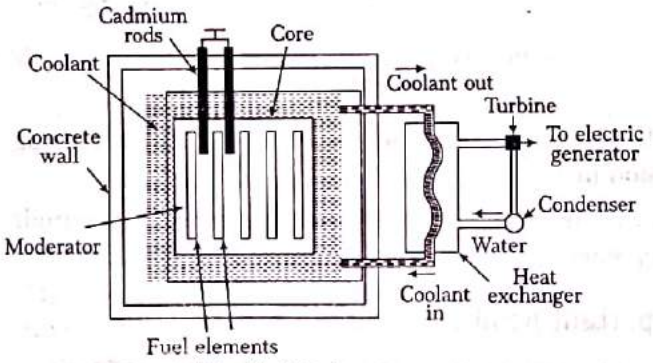


Fig. 13.5 Nuclear reactor

Construction

The key components of the nuclear reactor are as follows

(i) **Fissionable material (Fuel)** The fissionable material used in the reactor is called the fuel of the reactor. Uranium isotope (U^{235}), thorium isotope (Th^{232}) and plutonium isotopes (Pu^{239}, Pu^{240} and Pu^{241}) are the most commonly used fuel in the reactor.

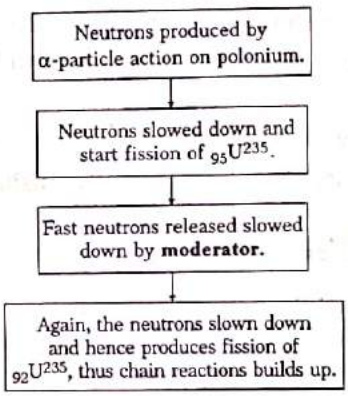
(ii) **Moderator** Moderator is used to slow down the fast moving neutrons. Most commonly used moderators are graphite and heavy water (D_2O). Heavy water is the best moderator.

(iii) **Control rods** Cadmium rods are inserted into the core of the reactor because they can absorb the neutrons. The neutrons available for fission are controlled by moving the cadmium rods in or out of the core of the reactor.

(iv) **Coolant** It is a liquid used to remove heat from nuclear reactor core and transfer it to electrical generator and environment. Ordinary water under high pressure is used as coolant.

(v) **Shielding** It is the protective covering made of concrete wall to protect from harmful radiations.

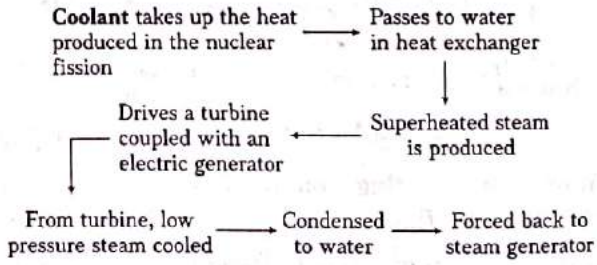
(vi) **Working**



By adjusting **control rods**, the chain reaction can be controlled.

As **water** is used as coolant as well as **moderator.**

Hence,



Uses of nuclear reactor

(i) In electric power generation.

(ii) To produce radioactive isotopes for their use in medical, science, agriculture and industry.

(iii) In manufacturing of Pu^{239} which is used in atom bomb.

Note **Breeder reactor** The reactors which can produce fuel more than they use *i.e.*, produce more fissile material than it consumes. *e.g.*, $^{238}_{92}U$ isotope, when bombarded with fast neutrons, transformations occurs like

$$_{92}U^{238} + {}_0n^1 \longrightarrow {}_{92}U^{239} \xrightarrow{-\beta^-} {}_{93}Np^{239} \xrightarrow[-\beta]{} {}_{94}Pu^{239}$$

Thus, breeder reactor produces more fissionable nuclei *i.e.*, more fuel in the form of Pu^{239} from non-fissile uranium.

Atom bomb

In the atomic bomb an increasing uncontrolled chain reaction occurs in a very short time when two pieces of Uranium-235 are rapidly brought together to form a mass greater than the critical size.

Example 13.12 *Neon-23 decays in the following way*

$$_{10}Ne^{23} \longrightarrow {}_{11}Na^{23} + {}_{-1}^{0}e + \bar{\nu}$$

Find the minimum and maximum kinetic energy that the beta particle ($_{-1}^{0}e$) can have. The atomic masses of $_{10}Ne^{23}$ and $_{11}Na^{23}$ are 22.9945 u and 22.9898 u, respectively.

Sol. Here, atomic masses are given (not the nuclear masses), but still we can use them for calculating the mass defect because mass of electrons get cancelled both sides. Thus,

mass defect, $\Delta m = m(_{10}Ne^{23}) - m(_{11}Na^{23})$

$$= (22.9945 - 22.9898) = 0.0047 \text{ u}$$

Kinetic energy is given by $Q = \Delta m \cdot c^2$

$\therefore \quad Q = (0.0047\text{u})\,(931.5 \text{ MeV/u}) = 4.4 \text{ MeV}$

Hence, the energy of beta particles can range from 0 to 4.4 MeV.

Example 13.13 *Find the amount of energy produced in joules due to fission of 1 g of $_{92}U^{235}$ assuming that 0.1% of mass is transformed into energy. Atomic mass of $_{92}U^{235} = 235$ amu, Avogadro number $N_A = 6.023 \times 10^{23}$. Given that the energy released per fission is 200 MeV.*

Sol. It is given that 0.1% of 1gm of $_{92}U^{235}$ is being used. So,

mass used, $m = \dfrac{0.1}{100} \times 1 = 10^{-3}$ g

Number of fissions $= \dfrac{\text{Given mass}}{\text{Atomic mass}} \times N_A = \dfrac{10^{-3}}{235} \times 6.023 \times 10^{23}$

Energy produced, E = Number of fissions × Energy released per fission

$$= \frac{6.023 \times 10^{20}}{235} \times 200 \times 1.6 \times 10^{-13} = 8.2 \times 10^{7} \text{ J}$$

$(\because 1 \text{ MeV} = 1.6 \times 10^{-13} \text{ J})$

Example 13.14 *Find the amount of energy released when 1 atom of Uranium $_{92}U^{235}$ (235.0439 amu) undergoes fission by slow neutron (1.0087 amu) and is splitted into Krypton $_{36}Kr^{92}$ (91.8973 amu) and Barium $_{56}B^{141}$ (140.9139 amu) assuming no energy is lost. Hence, find the energy in kWh, when 1 g of it undergoes fission.*

Sol. Fission equation, $_{92}U^{235} + {}_{0}n^{235} \rightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3({}_{0}n^{1})$

Mass defect,

$$\Delta m = [m(_{92}U^{235}) + m_n] - [m(_{66}Ba^{141}) + m(_{36}Kr^{92}) + 3m_n]$$

$$= [(235.0439 + 1.0087) - (140.9139 + 91.8973 + 3 \times 1.0087)]$$

$$= 0.2135 \text{ amu}$$

Equivalent energy, $E = \Delta mc^2 = \Delta m \times 931 \text{ MeV}$

$$= 0.2153 \times 931 = 200 \text{ MeV}$$

Number of atoms/fission in 1 g of atom is

$$n = \frac{N_A}{A} = \frac{1}{235} \times 6.023 \times 10^{23} = 2.56 \times 10^{21}$$

Energy released in fission of 1 g of $_{92}^{235}U$

$$\Rightarrow \quad n \times E = 2.56 \times 10^{21} \times 200 \text{ MeV}$$

$$= 2.56 \times 10^{21} \times 200 \times 1.6 \times 10^{-13} \text{ J}$$

$(\because 1 \text{ MeV} = 1.6 \times 10^{-13} \text{ J})$

$$= \frac{2.56 \times 10^{21} \times 200 \times 1.6 \times 10^{-13}}{3.6 \times 10^{6}} \text{ kWh}$$

$$= 2.28 \times 10^{4} \text{ kWh}$$

Example 13.15 *In a neutron induced fission of $_{92}U^{235}$ nucleus, usable energy of 185 MeV is released. If a $_{92}U^{235}$ reactor is continuously operating it at a power level of 100 MW how long will it take for 1 kg of uranium to be consumed in this reactor?*

Sol. Total number of nuclei in 1 kg of uranium,

n = (Number of moles) × (Avogadro's number)

$$= \left(\frac{1000}{235}\right)(6.02 \times 10^{23}) = 2.56 \times 10^{24}$$

By the fission of one nucleus, 185 MeV energy is released. Hence total energy released by the fission of 1 kg of uranium is,

$$E = (2.56 \times 10^{24})\,(185 \times 1.6 \times 10^{-13}) \text{ J}$$

$$= 7.58 \times 10^{12} \text{ J}$$

Power of the power plant is given 100 MW or 10^8 J/s. Therefore total time upto which the power plant can be run by 1 kg uranium is, $t = \dfrac{7.58 \times 10^{13}}{10^{8}} = 7.58 \times 10^{5}$ s

Nuclear fusion

When two light nuclei combine to form a heavier nucleus, then the process is called nuclear fusion.

In order to fuse two nuclei, they must come together in the range of nuclear force and to do this, they must overcome electrical repulsion of their positive charges. Hence, the kinetic energy required for fusion depends on the Coulomb barrier height ≈ 400 keV for two protons. This energy is only achieved at high temperature.

Hence, for fusion, high pressure ≈ 10^{16} atm and high temperature of the order of 10^7 K to 10^8 K is required and so the reaction is called **thermonuclear reaction.** *e.g.*, two deuterons (heavy-hydrogen nuclei) can be fused to form a triton (tritium) according to following reaction.

$$_{1}H^{2} + {}_{1}H^{2} \rightarrow {}_{1}H^{3} + {}_{1}H^{1} + 4.0 \text{ MeV (energy)}$$

The triton so formed can further fuse with a third **deuteron** to form an α-particle.

$$_{1}H^{3} + {}_{1}H^{2} \rightarrow {}_{2}He^{4} + {}_{0}n^{1} + 17.6 \text{ MeV (energy)}$$

The net result of the two reactions is the fusion of three deuterons and the formation of α-particle ($_2He^4$), a neutron $_0n^1$ and a proton ($_1H^1$). The total energy released is 21.6 MeV.

Solar energy is due to fusion in the interior of the sun which has a temperature of 1.5×10^7 K ($<< 3\times10^8$ K). The fusion in star and sun involves protons whose energies are much above the average energy.

The fusion reaction in the sun is a multi-step process in which hydrogen is fused into helium, which is follows as

$$_1H^1 + {}_1H^1 \longrightarrow {}_1H^2 + e^+ + \nu + 0.42\text{ MeV} \quad ...(i)$$

$$e^+ + e^- \longrightarrow \gamma + \gamma + 1.02\text{ MeV} \quad ...(ii)$$

$$_1H^2 + {}_1H^1 \longrightarrow {}_2He^3 + \gamma + 5.49\text{ MeV} \quad ...(iii)$$

$$_2He^3 + {}_2He^3 \longrightarrow {}_2He^4 + {}_1H^1 + {}_1H^1 + 12.86\text{ MeV} \quad ...(iv)$$

First three reactions occurs twice to react to fourth reaction.

$\Rightarrow$ 2(i) + 2(ii) + 2(iii) + (iv)

$$\Rightarrow \quad 4\,{}_1H^1 + 2e^- \longrightarrow {}_2He^4 + 2\nu + 6\gamma + 26.7\text{ MeV}$$

Thus, four hydrogen atoms combine to form an $_2He^4$ atom with a release of 26.7MeV of energy.

As the hydrogen in the core gets depleted and becomes helium, the core start to cool. The star begins to collapse under its own gravity, which decreases the temperature of the core.

Note Hydrogen bomb is due to nuclear fusion.

Controlled thermonuclear fusion

The aim of thermonuclear fusion is to generate steady power by heating the nuclear fuel to the temperature in the range of 10^8K. At these temperature, the fuel is a mixture of positive ions and electrons (plasma). The challenge is to confine this plasma, since no container can stand such a high temperature. Several countries around the world including India are developing techniques for this.

Note If a controlled fusion reaction can be achieved an almost unlimited supply of energy will become available from deuterium in the water of oceans.

Example 13.16 *The binding energy per nucleon of deuterium and helium atom is 1.1 MeV and 7.0 MeV, respectively. If two deuterium nuclei fuse to form helium atom. Find the energy released.*

Sol. Given, fusion reaction, $_1H^2 + {}_1H^2 \longrightarrow {}_2He^4$ + energy

Binding energy of a ($_1H^2$) deuterium nuclei

$$= 2\times1.1 = 2.2\text{ MeV}$$

Total binding energy of two deuterium nuclei

$$= 2.2\times2 = 4.4\text{ MeV}$$

Binding energy of a ($_2He^4$) nuclei $= 4\times7 = 28$ MeV

So, energy released in fusion = Final BE − Initial BE

$$= 28 - 4.4 = 23.6\text{ MeV}$$

Example 13.17 *In the fusion reaction $_1H^2 + {}_1H^2 \rightarrow {}_1He^3 + {}_0n^1$, the masses of deuteron, helium and neutron expressed in amu are 2.015, 3.017 and 1.009, respectively. If 1 kg of deuterium undergoes complete fusion, then find the amount of total energy released.*

1 amu = 931.5 MeV/c^2.

Sol. Given, fusion reaction, $_1H^2 + {}_1H^2 \rightarrow {}_2He^3 + {}_0n^1$

Mass defect, $\Delta m = 2\times m(_1H^2) - [m(_2He^4) + M_n]$

$$= [(2.015\times2) - (3.017 + 1.009)]$$

$$= 4\times10^{-3}\text{ amu}$$

Equivalent energy, $E = \Delta mc^2 = \Delta m \times 931.5\text{ MeV}/c^2$

$$= 4\times10^{-3}\times931.5 = 3.726\text{ MeV}$$

Number of atoms in 1 kg of $_1H^2$

$$= \frac{\text{Given mass}}{\text{Atomic mass}} \times \text{Avogadro number}$$

$$= \frac{1000}{2}\times6.023\times10^{23}$$

In one reaction, two atoms of $_1^2H$ are used.

So, total energy released

$$= \frac{1}{2}\left(\frac{1000}{2}\times6.023\times10^{23}\right)\times3.726\times1.6\times10^{-13} \simeq 9\times10^{13}\text{ J}$$

Example 13.18 *In a nuclear reactor U^{235} undergoes fission releasing energy 100 MeV. The reactor has 20% efficiency and the power produces is 2000 MW. If the reactor is to function for 5 yr, find the total mass of uranium required.*

Sol. Given, *output power*, $P_o = 2000\text{ MW} = 2\times10^9$W

$$\text{Efficiency, } \eta = \frac{\text{Output power } P_o}{\text{Input power } P_i}$$

$$\Rightarrow \quad \frac{20}{100} = \frac{2\times10^9}{P_i} \Rightarrow P_i = 10^{10}\text{ W}$$

Consider the mass of uranium be m.

Number of fissions = Number of atoms

$$= \frac{\text{Given mass}}{\text{Atomic mass}} \times \text{Avogadro number}$$

$$= \frac{m}{A}\times N_A = \frac{m}{235}\times6.023\times10^{23}$$

Energy produced, E = Number of fissions × Energy per fission

$$= \frac{m}{235}\times6.023\times10^{23}\times200\text{ MeV}$$

$$= \frac{m}{235}\times6.023\times10^{23}\times100\times1.6\times10^{-13}\text{ J}$$

$$(\because 1\text{ MeV} = 1.6\times10^{-13}\text{ J})$$

$$= m\times0.041\times10^{12}\text{ J} = m\times4.1\times10^{10}\text{ J}$$

$$\text{Power} = \frac{\text{Energy}}{\text{Time}} \Rightarrow 10^{10} = \frac{m\times4.1\times10^{10}}{5\times365\times24\times3600}$$

$$m = 3.84\times10^7\text{ g} = 38.4\times10^3\text{ kg}$$

Check point 13.2

1. 1 g of hydrogen is converted into 0.993 g of helium in a thermonuclear reaction. The energy released is
(a) 63×10^7 J (b) 63×10^{10} J (c) 63×10^{14} J (d) 63×10^{20} J

2. If an electron and a positron annihilitate, then the energy released is
(a) 3.2×10^{-13} J (b) 1.6×10^{-13} J
(c) 4.8×10^{-13} J (d) 6.4×10^{-13} J

3. The mass defect in a particular nuclear reaction is 0.3 g. The amount of energy liberated in kilowatt hours is (velocity of light = 3×10^8 ms^{-1})
(a) 1.5×10^6 (b) 2.5×10^6
(c) 3×10^6 (d) 7.5×10^6

4. If m, m_n and m_p are the masses of $_ZX^A$ nucleus, neutron and proton respectively, then
(a) $m < (A - Z)_{mn} + Z_{mp}$ (b) $m = (A - Z)_{mn} + Z_{mp}$
(c) $m = (A - Z)_{mp} + Z_{mn}$ (d) $m > (A - Z)_{mn} + Z_{mp}$

5. The masses of neutron and proton are 1.0087 amu and 1.0073 amu, respectively. If the neutrons and protons combine to form a helium nucleus (alpha particles) of mass 4.0015 amu the binding energy of the helium nucleus will be (1 amu = 931 MeV)
(a) 28.4 MeV (b) 20.8 MeV (c) 27.3 MeV (d) 14.2 MeV

6. In a fission reaction $^{236}_{92}U \rightarrow {}^{117}X + {}^{117}Y + n + n$, the binding energy per nucleon of X and Y is 8.5 MeV, whereas of ^{236}U is 7.6 MeV. The total energy liberated will be about
(a) 200 keV (b) 2 MeV (c) 200 MeV (d) 2000 MeV

7. Fission of nuclei is possible because the binding energy per nucleon in them
(a) increases with mass number at high mass numbers
(b) decreases with mass number at high mass numbers
(c) increases with mass number at low mass number
(d) decreases with mass number at low mass number

8. A chain reaction is continuous due to
(a) large mass defect
(b) large energy
(c) production of more neutrons in fission
(d) None of the above

9. The principle of controlled chain reaction is used in
(a) atomic energy reactor (b) atom bomb
(c) the core of sun (d) artificial radioactivity

10. Which of these is a fusion reaction?
(a) $^3H_1 + {}^2H_1 \rightarrow {}^4He_1 + {}^1n_0$
(b) $^{238}U_{92} + {}^{206}Pb_{82} + 8(^4He_2) + 6(^0B_1)$
(c) $^{12}C_7 \rightarrow {}^{12}C_6 + \beta + \gamma$
(d) None of the above

11. Solar energy is mainly caused due to
(a) fission of uranium present in the sun
(b) fusion of protons during synthesis of heavier elements
(c) gravitational contraction
(d) burning of hydrogen in the oxygen

12. Nuclear fusion is common to the pair
(a) thermonuclear reactor, uranium based nuclear reactor
(b) energy production in sun, uranium based nuclear reactor
(c) energy production in sun, hydrogen bomb
(d) disintegration of heavy nuclei, hydrogen bomb

13. Fusion reaction takes place at high temperature because
(a) molecules break up at high temperature
(b) nuclei break up at high temperature
(c) atoms get ionised at high temperature
(d) kinetic energy is high enough to over come the coulomb repulsion between nuclei

14. Heavy water is used as moderator in a nuclear reactor. The function of the moderator is
(a) to control the energy released in the reactor
(b) to absorb neutrons and stop chain reaction
(c) to cool the reactor factor
(d) to slow down the neutron to thermal energies

15. Pick out the correct statement from the following
(a) Energy released per unit mass of the reactant is less in case of fusion reaction
(b) Packing fraction may be positive or may be negative
(c) Pu^{239} is not suitable for a fission reaction
(d) For stable nucleus, the specific binding energy is low

4 Radioactivity

Radioactivity was discovered by Henry Becquerel in uranium salt in the year 1896.

The phenomenon of spontaneous emission of radiation by nucleus of some elements is known as radioactivity. The elements which show this phenomenon are called radioactive elements.

Atoms are radioactive if their nuclei are unstable and spontaneously (and randomly) emit various (α, β and/or γ) radiations and decay into stable nucleus.

When naturally occuring nuclei are unstable, we call the phenomena natural radioactivity. When some nuclei are made radioactive, nuclei in laboratory, then phenomena is known as **artificial radioactive.**

Some examples of radioactive substances are uranium, radium, thorium, polonium, neptunium, etc.

Generally, three types of emission are emitted from radioactive nuclei, which α, β and γ radiations.

Properties of radioactive radiations

Properties of α-particles

α-particle carries +2*e* charge and mass equal 4 times that of proton *i.e.*, is equal to doubly ionised helium atom ($_2He^4$).

Some important properties of α-particles are as follows

(i) α-particles are deflected in electric and magnetic fields. They are positively charged.

(ii) The velocity of α-particles vary from 0.01-0.1 times *c* (velocity of light).

(iii) α-particles have low penetrating power.

(iv) α-particles have high ionisation power. Their ionising power is 100 times greater than that of β-rays and 10000 times greater than that of γ-rays.

(v) α-particles produce fluorescence in substances like zinc sulphide and barium platinocyanide. They produce scintillation on fluorescent screen.

(vi) α-particles feebly affect photographic plate. They also produce heating effect, when stopped and can cause incurable burns on human body.

(vii) All α-particles coming from a source have same energy.

Properties of β-particles

(i) β-particles are deflected by electric and magnetic fields. They are negatively charged. Their deflection is much larger than the deflection of α-particles. This shows that β-particles are much lighter than the α-particles.

(ii) The velocity of β-particles vary from 0.01 to 0.99 of the velocity of light.

(iii) The penetrating power of β-particles is about 100 times larger than the penetrating power of α-particles. They can pass through 1mm thick sheet of aluminium.

(iv) β-particles ionise gases, but their ionising power is much smaller, only (1/100)th of the ionising power of α-particles. As β-particles cannot produce ionisation continuously, their tracks in cloud chamber do not appear to be continuous.

(v) β-particles produce fluorescence in calcium tungstate, barium platinocyanide and zinc sulphide.

(vi) β-particles affect photographic plate more than do the α-particles.

Properties of γ-rays

(i) γ-rays are not deflected by electric and magnetic fields. This indicates that they have no charge.

(ii) γ-rays travel with the speed of light (3×10^8 ms^{-1}).

(iii) The penetrating power of γ-rays is much more than that of α and β-particles. They can pass through 30 cm thick iron sheet.

(iv) γ-rays ionise gases, but their ionisation power is very small compared to that of α and β-particles.

(v) γ-rays produce fluorescence.

(vi) γ-rays affect photographic plate more than do β-particles.

(vii) γ-rays are diffracted by crystals in the same way as X-rays.

Note No radioactive substance emits both α and β-particles simultaneously. γ-rays are emitted along with both α and β-particles.

Comparison of the properties of α-particle, β-particle and γ-rays

S.No.	Property	α-particle	β-particle	γ-rays
1.	Nature	Helium nucleus	Fast moving electrons	Electromagnetic waves
2.	Charge	+ 2*e*	−*e*	zero
3.	Rest mass	6.67×10^{-27}kg	9.1×10^{-31} kg	zero
4.	Speed	1.4×10^7 to 2.2×10^7 ms^{-1}	1 to 99% of *c* $= 3 \times 10^8$ ms^{-1}	$c = 3 \times 10^8$ ms^{-1}
5.	Ionising power	10^4	10^2	1
6.	Penetrating power	1	10^2	10^4

Radioactive decay

A radioactive nucleus (parent nucleus), decays by particle emission into another nucleus (daughter nucleus), this phenomenon is called **radioactive decay.**

In nature, three types of radioactive decay occurs

1. Alpha decay

An alpha particle is a helium nucleus. Thus a nucleus emitting an alpha particle loses two protons and two neutrons. Therefore, the atomic number *Z* decreases by 2, the mass number *A* decreases by 4 and the neutron number *N* decreases by 2. The decay can be written as,

$$_ZX^A = {}_{Z-2}Y^{A-4} + {}_2He^4$$

where, X is the parent nucleus and Y is the daughter nucleus. As examples ${}_{92}U^{238}$ and ${}_{88}Ra^{226}$ are both alpha emitters and decay according to,

$$ {}_{92}U^{238} \longrightarrow {}_{90}Th^{234} + {}_{2}He^{4} $$

$$ {}_{88}Ra^{226} \longrightarrow {}_{86}Rn^{222} + {}_{2}He^{4} $$

As a general rule, in any decay sum of mass numbers A and atomic numbers Z must be the same on both sides.

Alpha decay occurs with nuclei that are very heavy, hence, unstable. So, emitting alpha particle, mass number decreases and it moves towards stability.

2. Beta decay

Beta decay involves the emission of electrons or positron. A positron is a form of antimatter, which has a charge equal to $+e$ and a mass equal to that of electron. The electrons and positron emitted in β-decay do not exist inside the nucleus. They are created at the time of emission, just as photons are created when an atom makes a transition from higher to a lower energy state.

There are three different types of β-decay

(i) In β^- **decay** a neutron in the nucleus is transformed into a proton, an electron and an antineutrino.

An anti-neutrino is represented by symbol $\bar{\nu}$. It has zero rest mass, is chargeless and its spin quantum number is $\pm\frac{1}{2}$.

$$ n \longrightarrow p + e^- + \bar{\nu} $$

Thus a parent nucleus with atomic number Z and mass number A decays by β^- emission into a daughter nuclei with atomic number $Z+1$ and the same mass number A.

$$ {}_{Z}X^{A} \xrightarrow{\beta^-} {}_{Z+1}Y^{A} $$

β^- decay occurs in nuclei that have too many neutrons *i.e.*, N/Z ratio is too large for stability. An example of β^- decay is the decay of carbon 14 into nitrogen,

$$ {}_{6}C^{14} \longrightarrow {}_{7}N^{14} + e^- + \bar{\nu} $$

(ii) In β^+ **decay**, a proton changes into a neutron with the emission of a positron and a neutrino.

Neutrino and anti-neutrino are anti-particles to each other.

$$ p \longrightarrow n + e^+ + \nu $$

Positron (e^+) emission from a nucleus decreases the atomic number Z by 1 while keeping the same mass number A.

$$ {}_{Z}X^{A} \xrightarrow{\beta^+} {}_{Z-1}Y^{A} $$

β^+ decay occurs in nuclei that have too many protons *i.e.*, N/Z ratio is too small for stability.

β^+ decay can occur only when the neutral atomic mass of the original atom (${}_{Z}X^{A}$) is at least two electron mass ($2m_e$) larger than that of the final atom (${}_{Z-1}Y^{A}$).

$$ {}^{13}_{7}N \longrightarrow {}^{13}_{6}C + e^+ + \nu $$

Example 13.19 *How many α and β^--particles will be emitted when ${}_{90}Th^{232}$ changes to ${}_{83}Bi^{212}$?*

Sol. ${}_{90}Th^{232} \longrightarrow {}_{83}Bi^{212}$

Decrease in mass number $= 232 - 212 = 20$

Number of α-particles emitted due to the above decrease in mass number $= \frac{20}{4} = 5$

Expected decrease in atomic number due to emission of 5α-particles $= 5 \times 2 = 10$

Expected atomic number of the nucleus formed $= 90 - 10 = 80$

But the atomic number of nucleus formed $= 83$

Increase in atomic number $= 83 - 80 = 3$

Number of β^--particles emitted $= 3$.

Thus, 5 α-particles and $3\beta^-$-particles are emitted.

(iii) **Electron capture** is competitive with positron emission since both processes lead to the same nuclear transformation.

This occurs when a parent nucleus captures one of its own orbital electrons and emits a neutrino.

$$ {}_{Z}X^{A} + e^{-1} \longrightarrow {}_{Z-1}Y^{A} + \nu $$

In most cases, it is a K-shell electron that is captured, and for this reason the process is referred to as K-capture. One example is the capture of an electron by ${}_{4}Be^{7}$

$$ {}_{4}Be^{7} + e^- \longrightarrow {}_{3}Li^{7} + \nu $$

Note

1. Electron capture occurs in those nuclides on which $\frac{N}{Z}$ ratio is too small for stability but β^+ emission is not energetically possible.
2. After an electron-capture, a vacancy is created in the atomic shell and hence X-rays are emitted.

3. Gamma decay

In ordinary physical and chemical transformation the nucleus always remains in its ground state. When a nucleus is placed in an excited state, by a radioactive transformation, it can decay to the ground state by emission of one or more photons called gamma rays and this process is called gamma decay.

A gamma ray is emitted when an α or a β decay results in a daughter nucleus in a an excited state.

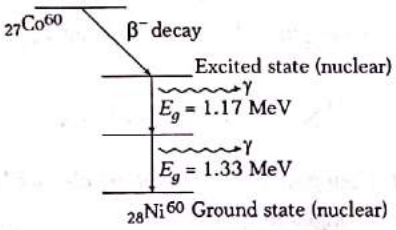


Fig. 13.6 *β-decay of $_{27}Co^{60}$ followed by 2 γ decay*

Note In both α and β-decay, the value of Z of a nucleus changes and nucleus of one element becomes the nucleus of a different element, whereas in γ-decay, the element does not change, the nucleus merely goes from an excited state to a less excited state.

Example 13.20 *A radioactive nucleus undergoes a series of decay according to scheme.*

$$A \xrightarrow{\alpha} A_1 \xrightarrow{\beta} A_2 \xrightarrow{\alpha} A_3 \xrightarrow{\gamma} A_4$$

If the mass number and atomic number of A are 180 and 72 respectively, then what are these numbers for A_4?

Sol. In α- decay, $^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$

Here, $_{72}A^{180} \xrightarrow{\alpha} {}_{70}A_1^{176} + {}^{4}_{2}He$

In β^-decay, $^{A}_{Z}X \longrightarrow {}^{A}_{Z+1}Y + e^- + \bar{\nu}$

Here, $_{70}A_1^{176} \xrightarrow{\beta} {}_{71}A_2^{176} + e^- + \bar{\nu}$

Again, $_{71}A_2^{176} \xrightarrow{\alpha} {}_{69}A_3^{172} + {}^{4}_{2}He$

In γ-decay, $^{A}_{Z}X \longrightarrow {}^{A}_{Z}X + \gamma$

$$_{69}A_3^{172} \longrightarrow {}_{69}A_4^{172} + \gamma$$

So, we have,

$$_{72}A^{180} \xrightarrow{\alpha} {}_{70}A_1^{176} \xrightarrow{\beta} {}_{71}A_2^{176} \xrightarrow{\alpha} {}_{69}A_3^{172} \xrightarrow{\gamma} {}_{69}A_4^{172}$$

Pair production and pair annihilation

Pair production

When an energetic γ-ray photon falls on a heavy substance, it is absorbed by some nucleus of the substance and its energy gives rise to the production of an electron and a positron.

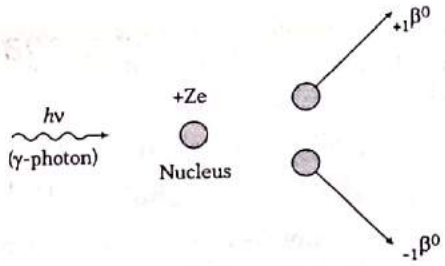


Fig.13.7

This phenomenon is called **pair production** and may be represented by the equation

$$h\nu \longrightarrow {}_{+1}\beta^0 + {}_{-1}\beta^0$$

From the equation, $E = mc^2$, the least energy of photon required for pair production will be,

$$E = 2(9.1\times10^{-31}\,\text{kg})(3.0\times10^{8}\ \text{ms}^{-1})^2$$

$$= 1.64\times10^{-13}\,\text{J}$$

$$= \frac{1.64\times10^{-13}}{1.6\times10^{-13}}\ \text{MeV}$$

$$= 1.02\ \text{MeV}$$

Hence, for pair production, it is essential that the energy of γ-photon must be at least 1.02 MeV.

Note

1. If the energy of γ-photon is less than 1.02 MeV, it would cause photoelectric effect or Compton effect on striking the matter (no pair-production).
2. If the energy of γ-photon is more than 1.02 MeV, then electron and positron are produced and the energy in excess of 1.02 MeV is obtained as the kinetic energy of these particles. This kinetic energy is not distributed equally between these two particles, but the positron gets more than half. This is because (positively charged) nucleus accelerates the escaping (positively charged) positron due to repulsion, but retards the electron due to attraction.

Pair annihilation

The converse phenomenon of pair production is called pair annihilation.

When an electron and a positron come very close to each other, they annihilate each other by combining together and two γ-photons are produced.

This phenomenon can be represented by the following equation.

$$\underset{\text{(Positron)}}{_{+1}\beta^0} + \underset{\text{(Electron)}}{_{-1}\beta^0} \longrightarrow \underset{\text{(γ-photons)}}{h\nu + h\nu}$$

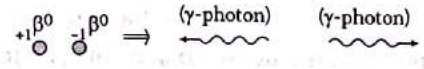


Fig. 13.6 *Pair annihilation*

Note That two photons (not one) are produced in pair-annihilation to conserve both the energy and the momentum.

Rutherford and Soddy's law regarding radioactive decay

According to this law, the rate of decay of radioactive nuclei at any instant is proportional to the number of nuclei present at that instant.

Let N be the number of nuclei present in a radioactive substance at any instant t. Let dN be the number of nuclei that disintegrates in a short interval dt.

Then, the rate of disintegration $\frac{-dN}{dt}$ is proportional to N

i.e., $$-\frac{dN}{dt} \propto N \text{ or } \boxed{-\frac{dN}{dt} = \lambda N}$$

where, λ is a constant for the given substance and is called **decay constant** (or disintegration constant or radioactive constant or transformation constant) and the –ve sign indicates that radioactive nuclei are decreasing with time. For a given element, the value of λ is constant but for different elements, it is different. From the above equation, we have $\frac{dN}{N} = -\lambda\, dt$

Integrating above equation, we get

$$\int_0^N dN/N = -\lambda \int_0^t dt$$

$$\log_e N = -\lambda t + C \quad \text{...(i)}$$

where, C is the integration constant. To determine C, we apply the initial conditions. Suppose, there were N_0 nuclei in the beginning *i.e.*, $N = N_0$ at $t = 0$.

Then,

$$\log_e N_0 = C$$

Substituting this value of C in Eq.(i), we have

$$\log_e N = -\lambda t + \log_e N_0$$

$$\log_e N - \log_e N_0 = -\lambda t$$

or $$\log_e \frac{N}{N_0} = -\lambda t$$

$\Rightarrow$ $$\frac{N}{N_0} = e^{-\lambda t}$$

$\Rightarrow$ $$\boxed{N = N_0 e^{-\lambda t}} \quad \text{...(ii)}$$

where, N_0 and N are numbers of nuclei in a radioactive substance at time $t = 0$ and after time t, respectively. According to this equation, the decay of a radioactive substance is exponential *i.e.*, the decay is rapid in the beginning and then its rate decreases continuously. It means that a radioactive substance will take infinite time in decaying completely.

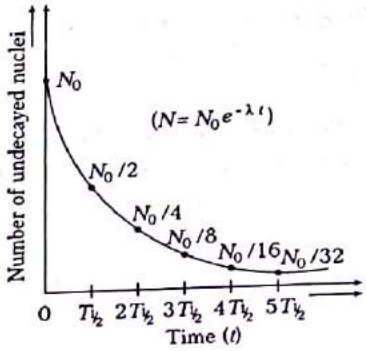


Fig. 13.8. *Exponential decay of a radioactive substance*

Note Number of nuclei decayed after time t,

$$N_1 = N_0 - N = N_0 - N_0 e^{-\lambda t} = N_0 (1 - e^{-\lambda t})$$

The corresponding graph is as shown in figure.

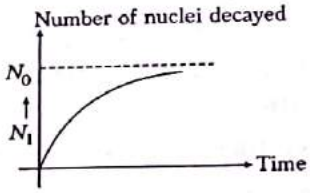


Fig. 13.9

Note Probability of a nucleus to survive in time t is,

$$P(\text{survival}) = \frac{N}{N_0} = e^{-\lambda t}$$

Probability of a nucleus to disintegrate in time t is,

$$P(\text{disintegration}) = 1 - P(\text{survival}) = 1 - e^{-\lambda t}$$

Example 13.21 *At time $t = 0$, number of nuclei of a radioactive substance are 100. At $t = 1$ s these numbers become 90. Find the number of nuclei at $t = 2$ s.*

Sol. In 1 second 90% of the nuclei have remained undecayed, so in another 1 second 90% of 90 *i.e.*, 81 nuclei will remain undecayed.

Activity of radioactive substance

The rate of decay of a radioactive substance is called the activity (R) of the substance.

i.e., $$R = \frac{-dN}{dt}$$

Also, according to Rutherford and Soddy law,

$$\frac{-dN}{dt} \propto N \text{ or } -\frac{dN}{dt} = \lambda N$$

Thus, $$R = -\frac{dN}{dt} \text{ or } R = \lambda N$$

or $$R = \lambda N_0 e^{-\lambda t}$$

or $$\boxed{R = R_0 e^{-\lambda t}} \quad \text{...(iv)}$$

where, $R_0 = \lambda N_0$ is the activity of the radioactive substance at time $t = 0$. The activity *versus* time graph is shown in Fig. 13.10.

Thus, the number of nuclei and hence the activity of the radioactive substance decrease exponentially with time.

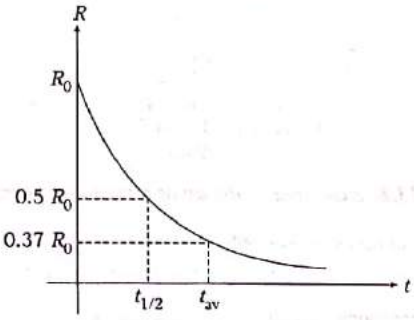


Fig. 13.10

Units of activity The SI unit for the decay rate is the becquerel (Bq), but the curie (Ci) and rutherford (Rd) are often used in practice.

$$1 \text{ Bq} = 1 \text{decay/s}, \quad 1 \text{ Ci} = 3.7 \times 10^{10} \text{ Bq}$$

and $$1 \text{Rd} = 10^6 \text{ Bq}$$

Note Activity per unit mass or per unit volume is known as **specific activity**.

Half-life

The time interval in which the mass of a radioactive substance, the number of its nuclei, is reduced to half its initial value is called 'half-life' of that substance. It is represented by $T_{1/2}$.

From the equation, $N = N_0 e^{-\lambda t}$

At half-life, $t = T_{1/2}$, $N = \frac{N_0}{2}$

$$\therefore \quad \frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}} \Rightarrow \frac{1}{2} = e^{-\lambda T_{1/2}}$$

$$\Rightarrow \quad e^{\lambda T_{1/2}} = 2$$

Taking log on both sides, we get

$$\lambda T_{1/2} = \log_e 2 \Rightarrow T_{1/2} = \frac{\log_e 2}{\lambda}$$

$$= \frac{2.303 \log_{10} 2}{\lambda} = \frac{0.3010 \times 2.303}{\lambda}$$

$[\because \log_{10} 2 = 0.3010]$

$$\boxed{T_{1/2} = \frac{0.6931}{\lambda}}$$

After n half-life, the number of nuclei left undecayed is given by $t = nt_{1/2}$, $N = N_0 \left(\frac{1}{2}\right)^n$ and $R = R_0 \left(\frac{1}{2}\right)^n$.

Note The half-life rule is applicable to any fraction, for example,

$$N_0 \xrightarrow{t_{1/3}} \frac{N_0}{3} \xrightarrow{t_{1/3}} \frac{N_0}{3^2} \xrightarrow{t_{1/3}} \frac{N_0}{3^3} \longrightarrow$$

Example 13.22 *There is a stream of neutrons with a kinetic energy of 0.0327 eV. If the half-life of neutrons be 700 s, what fraction of neutrons will decay before they travel a distance of 10 m? Mass of neutron equal to 1.675×10^{-27} kg.*

Sol. Kinetic energy of neutrons is given to be 0.0327 eV. From the relation, $K = \frac{1}{2} mv^2$, we can find the speed of the neutrons.

$$\therefore \quad K = \frac{1}{2} mv^2$$

or $$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2 \times 0.0327 \times 1.6 \times 10^{-19}}{1.675 \times 10^{-27}}}$$

$$= 2.5 \times 10^3 \text{ ms}^{-1}$$

Time taken by the neutrons to travel a distance of 10 m with this speed, $t = \frac{10}{2.5 \times 10^3} = 4.0 \times 10^{-3}$ s

Fraction of neutrons decayed in the given time interval,

$$= \frac{N_0 (1 - e^{-\lambda t})}{N_0} = (1 - e^{-\lambda t})$$

where, $t_{1/2} = 700$ s (given)

$$t_{1/2} \text{ (half-life)} = \frac{0.693}{\lambda}; \ (\lambda = \text{decay constant})$$

$$\Rightarrow \quad \lambda = 0.693 / t_{1/2} = \frac{0.693}{700}$$

$$= 1 - e^{-\left(\frac{0.693}{700}\right)(4.0 \times 10^{-3})} = 3.96 \times 10^{-6}$$

Example 13.23 *The disintegration rate of a certain radioactive sample at any instant is 4750 disintegrations per minute. Five minutes later the rate becomes 2700 per minute. Calculate*

(i) decay constant and *(ii) half-life of the sample.*

Sol. (i) From the relation,

$$R = R_0 e^{-\lambda t} \quad \text{(here, } R = \text{activity of sample)}$$

Substituting the values, we have, $2700 = 4750 e^{-5\lambda}$

$$\therefore \quad 5\lambda = \ln\left(\frac{4750}{2700}\right) = 0.56$$

$$\therefore \quad \text{Wavelength, } \lambda = 0.113 \text{ min}^{-1}$$

(ii) Half-life of the sample, $t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{0.113} = 6.132$ min

Example 13.24 *At time $t = 0$, activity of a radioactive substance is 1600 Bq, at $t = 8$ s activity remains 100 Bq. Find the activity at $t = 2$ s.*

Sol. Activity of the sample after n half life is $R = R_0 \left(\frac{1}{2}\right)^n$

Given, $R = \frac{R_0}{16}$

$$\therefore \quad \frac{R_0}{16} = R_0\left(\frac{1}{2}\right)^n$$

or number of half-lives, $n = 4$

Four half-lives are equivalent to 8 s. Hence, 2 s is equal to one half-life. So in one half-life activity will remain half of 1600 Bq, *i.e.* 800 Bq.

Example 13.25 *A ^7Li target is bombarded with a proton beam current of 10^{-4} A for 1 hour to produce ^7Be of activity 2.8×10^8 disintegrations per second. Assuming that one ^7Be radioactive nuclei is produced by bombarding 1000 protons, determine its half-life.*

Sol. Charge, $q = It = 10^{-4} \times 1 \times 3600 = 0.36$ C

$$\text{Number of protons} = \frac{q}{e} = \frac{0.36}{1.6 \times 10^{-19}} = 2.25 \times 10^{18}$$

$$\text{Number of } ^7\text{Be nuclei} = \frac{2.25 \times 10^{18}}{1000} = 2.25 \times 10^{15}$$

$$\text{Activity, } R = \frac{\ln 2}{T_{1/2}} N, \text{ where } R = 2.8 \times 10^8 \text{ dps (given)}$$

$$\text{Half-life of substance, } T_{1/2} = \frac{N \ln 2}{R} = \frac{2.25 \times 10^{15} \times 0.693}{2.8 \times 10^8}$$

$$= 0.556 \times 10^7 \text{ s}$$

Example 13.26 *Determine the amount of $_{84}Po^{210}$ (polonium) necessary to provide a source of α-particles of 5 millicurie strength, if half-life of polonium is 138 days. (Given, 1 curie = 3.7×10^{10} disintegrations/s)*

Sol. From the relation, $R = \lambda N$, we have

$$N = \frac{R}{\lambda} \qquad \text{(here, } R = \text{activity of sample)}$$

where, $R = 5 \times 10^{-3} \times 3.7 \times 10^{10}$ dps

$$t_{1/2} = 138 \text{ days} = 138 \times 24 \times 3600 \text{ s}$$

$$\therefore \quad N = \frac{(5 \times 10^{-3})(3.7 \times 10^{10})}{(0.693/138 \times 24 \times 3600)} \qquad \left(\because \lambda = \frac{0.693}{t_{1/2}}\right)$$

$$= 3.18 \times 10^{15}$$

$$\text{Number of moles required, } n = \frac{N}{N_A} = \frac{3.18 \times 10^{15}}{6.02 \times 10^{23}} = 5.28 \times 10^{-9}$$

where, N_A = Avogadro number.

∴ Amount of polonium required for the desired activity,

$$m = \text{number of moles} \times \text{given mass} = (5.28 \times 10^{-9})(210) \text{ g}$$

$$= 1.11 \times 10^{-6} \text{ g}$$

Example 13.27

(i) *Calculate the activity of two mg sample $_{28}Sr^{90}$ whose half-life period is 28 yr.*

(ii) *An experiment is performed to determine the half-life of a radioactive substance which emits one beta particle for each decay process. Observations show that an average of 8.4 β-particles are emitted each second by 2.5 mg of the substance. The atomic weight of the substance is 230. Calculate the half-life of the substance.*

Sol. (i) Number of atoms, $N = \dfrac{\text{Mass}}{\text{Atomic weight}} \times N_A$;

(where N_A is Avogadro number)

$$= \frac{2 \times 10^{-3}}{90} \times 6.023 \times 10^{23}$$

$$= 13.38 \times 10^{18}$$

$$\text{Decay constant, } \lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{28} \text{ /yr}$$

$$\text{Acitivity, } R = \lambda N = \frac{0.693}{28} \times 13.38 \times 10^{18}$$

$$= 0.331 \times 10^{18} \text{/yr}$$

$$= \frac{0.331 \times 10^{18}}{365 \times 24 \times 3600} = 10^{10} \text{ dps}$$

(ii) Half-life of the substance,

$$N = \frac{2.5 \times 10^{-3}}{230} \times 6.023 \times 10^{23} = 6.5 \times 10^{18}$$

$$\text{Activity, } R = 8.4/\text{s} = \lambda N = \frac{\ln 2}{T_{1/2}} N \Rightarrow T_{1/2} = \frac{\ln 2}{A} N$$

Half-life of the substance,

$$T_{1/2} = \frac{0.693 \times 6.5 \times 10^{18}}{8.4}$$

$$= 0.54 \times 10^{18} \text{ s} = 1.7 \times 10^{10} \text{ yr}$$

Example 13.28 *In an experiment on two radioactive isotopes of an element (which do not decay into each other), their number of atoms ratio at a given instant was found to be 3. The rapidly decaying isotope has larger mass and an activity of 1.0 μCi initially. The half-lives of the two isotopes are known to be 12 hours and 16 hours. What would be the activity of each isotope and their number of atoms ratio after two days?*

Sol. We know, $t = nt_{1/2}$,

Here, $t = 2$ days (given)

And after n half-life, number of nuclei left undecayed is,

$$N = N_0\left(\frac{1}{2}\right)^n.$$

For first isotope,

$$24 \times 2 = n_1 \times 12 \Rightarrow n_1 = \frac{48}{12}$$

Half-lives of first isotopes,

$$N_A = (N_A)_0\left(\frac{1}{2}\right)^{n_1} = (N_A)_0\left(\frac{1}{2}\right)^{\frac{48}{12}} = (N_A)_0\left(\frac{1}{2}\right)^4$$

For second isotope,

$$24 \times 2 = n_2 \times 16 \Rightarrow n_2 = \frac{48}{16}$$

Half-lives of second isotope,

$$N_B = (N_B)_0\left(\frac{1}{2}\right)^{n_2} = (N_B)_0\left(\frac{1}{2}\right)^{\frac{48}{16}} = (N_B)_0\left(\frac{1}{2}\right)^3$$

∴ Ratio of half-lives of two isotope,

$$\frac{N_A}{N_B}=\left(\frac{N_A}{N_B}\right)_0\left(\frac{1}{2}\right)=\frac{3}{2}$$

Average or mean life of radioactive substance

The average of the lives of all the nuclei in a radioactive substance is called the 'mean life' or 'average life' of that substance. It is denoted by τ.

$$t_{av}=\tau=\frac{\int_0^\infty t\lambda N_0 e^{-\lambda t}dt}{N_0}$$

It can be shown that the mean life of a radioactive substance is equal to the reciprocal decay constant.

i.e., $$\tau=\frac{1}{\lambda}$$

Also, $\lambda=\frac{0.6931}{T_{1/2}}$, where $T_{1/2}$ is the half-life of the substance.

$$\therefore \quad \tau=\frac{T_{1/2}}{0.6931}=1.443\,T_{1/2}$$

Thus, mean life is longer than half-life. The reason is that the last few nuclei of the radioactive substance have very long lives.

Example 13.29 *At a given instant there are 25% undecayed rodioactive nuclei in a sample. After 10 s the number of undecayed nuclei reduces to 12.5%. Calculate the mean life of the nuclei.*

Sol. The number of undecayed nuclei reduces from 25% to 12.5% in 10 s. It means that the half-life of the sample is 10 s.

i.e., $T_{1/2}=10$ s

But $T_{1/2}=0.6931/\lambda$, where λ is decay

Thus, $$\lambda=\frac{0.6931}{T_{1/2}}=\frac{0.6931}{10\text{ s}}$$

The mean life of the sample,

$$\tau=1/\lambda=\frac{10}{0.6931}=14.435\text{ s}$$

Example 13.30 *1 g of a radioactive substance disintegrates at the rate of 3.7×10^{10} dps. The atomic mass of the substance is 226. Calculate its mean life.*

Sol. We know, number of nuclei present, $N=\frac{m}{M}N_A$

where, m = given mass, M = atomic mass

N_A = Avogadro number

Activity, $R=\lambda N=\frac{1}{\tau}\frac{mN_A}{M}$

where, τ = mean life.

Mean life of substance, $$\tau=\frac{mN_A}{MR}=\frac{1\times6.023\times10^{23}}{226\times3.7\times10^{10}}\text{ s}$$
$$=7.2\times10^{10}\text{ s}$$

Successive disintegration

Let A be a radioactive nuclei (decay constant λ_1) disintegrates onto another radioactive nuclei B (decay constant λ_2), then

$$A\xrightarrow{\lambda_1}B\xrightarrow{\lambda_2}X\text{ (stable)}$$

At $t=0$, $N_A=N_0$; $N_B=0$; $N_X=0$

At $t=t$, $N_A=N$; $N_B=N'$; $N_X=N''$

$$\frac{dN}{dt}=-\lambda_1 N$$

$$\frac{dN'}{dt}=\lambda_1 N-\lambda_2 N'$$

$$\frac{dN''}{dt}=-\lambda_2 N''$$

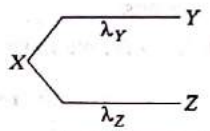
(i) If $(t_{1/2})_1 >> (t_{1/2})_2$; $\lambda_1 << \lambda_2$

$\lambda_1 N=\lambda_2 N'$ (Secular equilibrium)

(ii) If $(t_{1/2})_1 > (t_{1/2})_2$; $\lambda_1 < \lambda_2$

$\lambda_1 N=(\lambda_2-\lambda_1)N'$ (Transient equilibrium)

Note Parallel radioactive disintegration



$$\boxed{\lambda_X=\lambda_Y+\lambda_Z}$$

where, λ_X = decay constant of X

λ_Y = decay constant of Y

λ_Z = decay constant of Z

Example 13.31 *In the chemical analysis of a rock the mass ratio of two radioactive isotopes is found to be 100 : 1. The mean lives of the two isotopes are 4×10^9 yr and 2×10^9 yr, respectively. If it is assumed that at the time of formation the atoms of both the isotopes were in equal proportion, calculate the age of the rock. Ratio of the atomic weights of the two isotopes is 1.02 : 1.*

Sol. At the time of observation ($t=t$),

$$\frac{m_1}{m_2}=\frac{100}{1}\quad\text{(given)}$$

Further it is given that, $\frac{A_1}{A_2}=\frac{1.02}{1}$

Number of atoms, $N=\frac{m}{A}$

$$\therefore \quad \frac{N_1}{N_2}=\frac{m_1}{m_2}\times\frac{A_2}{A_1}=\frac{100}{1.02}\quad\text{...(i)}$$

Let N_0 be the number of atoms of both the isotopes at the time of formation, then

$$\frac{N_1}{N_2} = \frac{N_0 e^{-\lambda_1 t}}{N_0 e^{-\lambda_2 t}} = e^{(\lambda_2 - \lambda_1)t} \quad ...(ii)$$

Equating Eqs. (i) and (ii), we have

$$e^{(\lambda_2 - \lambda_1)t} = \frac{100}{1.02}$$

or $$(\lambda_2 - \lambda_1)t = \ln(100) - \ln(1.02)$$

$$\therefore \quad t = \frac{\ln(100) - \ln(1.02)}{\left(\frac{1}{2\times 10^9} - \frac{1}{4\times 10^9}\right)} \quad (\because \text{Mean life, } \tau = 1/\lambda)$$

Substituting the values, we have, $t = 1.834 \times 10^{10}$ yr

Example 13.32 *Uranium ores on the earth at the present time typically have a composition consisting of 99.3% of the isotope $_{92}U^{238}$ and 0.7% of the isotope $_{92}U^{235}$. The half lives of these isotopes are 4.47×10^9 yr and 7.04×10^8 yr, respectively. If these isotopes were equally abundant when the earth was formed, estimate the age of the earth.*

Sol. Let N_0 be number of atoms of each isotope at the time of formation of the earth ($t = 0$) and N_1 and N_2 the number of atoms at present ($t = t$). Then, according to law of radioactivity,

$$N_1 = N_0 e^{-\lambda_1 t} \quad ...(i)$$

and $$N_2 = N_0 e^{-\lambda_2 t} \quad ...(ii)$$

$$\therefore \quad \frac{N_1}{N_2} = e^{(\lambda_2 - \lambda_1)t} \quad ...(iii)$$

Further it is given that, $$\frac{N_1}{N_2} = \frac{99.3}{0.7} \quad ...(iv)$$

Equating Eqs. (iii) and (iv) and taking log on both sides, we have

$$(\lambda_2 - \lambda_1)t = \ln\left(\frac{99.3}{0.7}\right)$$

$$\therefore \quad t = \left(\frac{1}{\lambda_2 - \lambda_1}\right)\ln\left(\frac{99.3}{0.7}\right)$$

where λ_1 and λ_2 are the decay constant.

Substituting the values, we have

$$t = \frac{1}{\frac{0.693}{7.04\times 10^8} - \frac{0.693}{4.47\times 10^9}} \ln\left(\frac{99.3}{0.7}\right)$$

$$\left(\because t_{1/2} = \frac{0.693}{\lambda} \Rightarrow \lambda = \frac{0.693}{t_{1/2}}\right)$$

or age of the earth, $t = 5.97\times 10^9$ yr

Example 13.33 *The nuclei of two radioactive isotopes of same substance A^{236} and A^{234} are present in the ratio of 4 : 1 in an ore obtained from some other palnet. Their half lives are 30 min and 60 min, respectively. Both isotopes are alpha emitters and the activity of the isotope with half-life 30 min is 10^6 dps. Calculate after how much time their activities will become identical. Also, calculate the time required to bring the ratio of their atoms to 1:1.*

Sol. We have, $\left(\frac{N_1}{N_2}\right)_0 = \frac{4}{1}$

Half-life of two isotopes are,

$$(T_{1/2})_1 = 30 \text{ min} \quad \text{and } (T_{1/2})_2 = 60 \text{ min}$$

Activity of first isotope is, $(R_1)_0 = 10^6$ dps

We know, Activity of radioactive substance is, $R_0 = \lambda N_0$ and decay constant is, $\lambda = \frac{0.693}{t_{1/2}}$, where $t_{1/2}$ = half-life.

$$\therefore \quad \frac{(R_1)_0}{(R_2)_0} = \frac{\lambda_1 (N_1)_0}{\lambda_2 (N_2)_0} = \frac{(T_{1/2})_2}{(T_{1/2})_1}\left(\frac{N_1}{N_2}\right)_0$$

$$\frac{10^6}{(R_2)_0} = \frac{60}{30} \times \frac{4}{1} \Rightarrow (R_2)_0 = \frac{1}{8} \times 10^6 \text{ dps}$$

Let their activities become equal after time t,

$$\Rightarrow \quad (R_1 = R_2)$$

We know, after n half-life, $R = R_0\left(\frac{1}{2}\right)^n$,

$$\Rightarrow \quad (R_1)_0\left(\frac{1}{2}\right)^{n_1} = (R_2)_0\left(\frac{1}{2}\right)^{n_2} \text{ and } t = nT_{1/2}$$

We have, $n_1 = \frac{t}{(T_{1/2})_1} = \frac{t}{30}$ and $n_2 = \frac{t}{(T_{1/2})_2} = \frac{t}{60}$

$$\therefore \quad 10^6\left(\frac{1}{2}\right)^{\frac{t}{30}} = \frac{1}{8} \times 10^6\left(\frac{1}{2}\right)^{\frac{t}{60}}$$

$$\left(\frac{1}{2}\right)^{\frac{t}{30} - \frac{t}{60}} = \frac{1}{8} = \left(\frac{1}{2}\right)^3 \Rightarrow \left(\frac{1}{2}\right)^{\frac{t}{60}} = \left(\frac{1}{2}\right)^3 \Rightarrow \frac{t}{60} = 3$$

$\Rightarrow$ Time, $t = 180$ min

According to question, when $N_1 = N_2$

After n half-life, number of nuclei left undecayed is,

$$N = N_0\left(\frac{1}{2}\right)^n,$$

$$(N_1)_0\left(\frac{1}{2}\right)^{t/30} = (N_2)_0\left(\frac{1}{2}\right)^{t/60}$$

or $$\left(\frac{1}{2}\right)^{t/60} = \left(\frac{N_2}{N_1}\right)_0 = \frac{1}{4} = \left(\frac{1}{2}\right)^2$$

$$t/60 = \frac{1}{2} \Rightarrow \text{time, } t = 120 \text{ min}$$

Example 13.34 *The mean lives of a radioactive substance are 1620 years and 405 years for α-emission and β-emission, respectively. Find out the time during which three fourths of a sample will decay if it is decaying both by α-emission and β-emission simultaneously.*

Sol. For α- emission and β-emission respectively, according to parallel disintegration $\lambda = \lambda_\alpha + \lambda_\beta$

$$= \frac{1}{T_\alpha} + \frac{1}{T_\beta} = \frac{1}{1620} + \frac{1}{405} = \frac{5}{1620}\,\text{yr}^{-1}$$

$$\left(\because \text{ Mean life} = \frac{1}{\text{Decay constant}}\right)$$

If 3/4th sample will decay *i.e.*, remaining 1/4th

After n half-life,

$$N = N_0\left(\frac{1}{2}\right)^n \Rightarrow \frac{N_0}{4} = N_0\left(\frac{1}{2}\right)^n \Rightarrow n = 2$$

Time during 3/4th sample will decay, $t = nT_{1/2} = n\dfrac{\ln 2}{\lambda}$

$$= 2 \times \frac{0.693}{5/1620} = 449 \text{ yr}$$

Example 13.35 *U^{238} is found to be in secular equilibrium with Ra^{226} in its ore. If chemical analysis shows 1 nuclei of Ra^{226} per 3.6×10^6 nuclei of U^{238}, find the half-life of U^{238}. Given the half-life is 1500 years.*

Sol. According to secular equilibrium,

$$\lambda_U N_U = \lambda_{Ra} N_{Ra}$$

where, λ_U = decay constant of uranium

λ_{Ra} = decay constant of Ra

N_U = number of nuclei of U

N_{Ra} = number of nuclei of Ra

We know,

$$\text{Decay constant } (\lambda) = \frac{\ln 2}{\text{Half time } (t_{1/2})}$$

$$\Rightarrow \quad \frac{\ln 2}{(t_{1/2})_U} N_U = \frac{\ln 2}{(t_{1/2})_{Ra}} N_{Ra}$$

$$\Rightarrow \quad \frac{3.6 \times 10^6}{(t_{1/2})_U} = \frac{1}{1500}$$

$$\Rightarrow \quad (t_{1/2})_U = 3.6 \times 10^6 \times 1500$$

$$= 5400 \times 10^6 = 5.4 \times 10^9 \text{ yr}$$

Example 13.36 *A radioactive element decays by β -emission. A detector records n-beta particles in 2 s and in next 2 s it records 0.45 n beta particles. Find mean life.*

Sol. Let N_0 specifies initial number of nuclei at time $t = 0$,

where, N = number of undecayed nuclei in time t.

According to the law of radioactivity, $N = N_0 e^{-\lambda t}$

The number of decayed nuclei in time t,

$$N' = N_0 - N = N_0(1 - e^{-\lambda t}) = n \quad \text{...(i)}$$

The number of undecayed nuclei in next time t,

$$N_0' = Ne^{-\lambda t} = N_0 e^{-\lambda t} . e^{-\lambda t} = N_0 e^{-2\lambda t}$$

Number of decayed nuclei in next time t,

$$N'' = N - N_0'$$

$$= N_0 e^{-\lambda t} - N_0 e^{-2\lambda t}$$

$$= N_0 e^{-\lambda t}(1 - e^{-\lambda t}) \quad \text{...(ii)}$$

$$= 0.45\,n$$

On dividing Eq. (ii) by Eq. (i), we get

$$e^{-\lambda t} = 0.45 = \frac{9}{20}$$

$$\Rightarrow \quad e^{\lambda t} = \frac{20}{9} \Rightarrow \lambda t = \ln\left(\frac{20}{9}\right)$$

As, time = 2 s (given)

$$\Rightarrow \quad 2 \times \lambda = \ln\left(\frac{20}{9}\right) \Rightarrow \lambda = \frac{1}{2}\ln\left(\frac{20}{9}\right)$$

So, mean life, $\tau = \dfrac{1}{\lambda} = \dfrac{2}{\ln\left(\dfrac{20}{9}\right)}$

Example 13.37 *A small quantity of a solution containing ^{24}Na radio nuclide (half-life 15 h) of activity 1.0 micro-curie is injected into the blood of a person. A sample of the blood of volume 1 cm^3 taken after 5 hours shows an activity of 360 disintegration per minute. Determine the total volume of blood in the body of the person, Assume that the radioactive mixed uniformly in the blood of the person.*

Sol. Half-life, $t_{1/2} = 15$ h

Initial activity, $R_0 = 1\,\mu\text{Ci} = 10^{-6} \times 3.7 \times 10^{10}$

$$= 3.7 \times 10^4 \text{ dps}$$

At $t = 5$h, activity of sample, $R = \dfrac{360}{60}$dps,

volume of blood, $V = 1\text{cm}^3$

Let initial volume of blood $= V_0$ cm^3

According to specific activity,

$$\frac{R}{V} = \frac{R_0}{V_0} e^{-\lambda t} = \frac{R_0}{V_0} e^{\left(\frac{0.693}{t_{1/2}} t\right)} \quad \left(\because \lambda = \frac{0.693}{t_{1/2}}\right)$$

$$\frac{360/60}{1} = \frac{3.7 \times 10^4}{V_0} e^{\left(\frac{0.693}{15} \times 5\right)}$$

$$6 = \frac{3.7 \times 10^4}{V_0 \times e^{0.231}}$$

Volume of the blood, $V_0 = 4894.72 \text{ cm}^3 = 4.894$ L

Example 13.38 *A 25 g piece of charcoal is found in some ruins of an ancient city. The sample shows a C^{14} activity of 250 decay/minutes. How long has the tree this charcoal came from been dead? Given, the half-life of C^{14} is 5730 years. The ratio of C^{14} and C^{12} in the living sample is 1.3×10^{-12}.*

Sol. The decay constant λ of ^{14}C will be

$$\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{(5730)(365 \times 24 \times 60 \times 60)}$$

$$= 3.84 \times 10^{-12} \text{ s}^{-1}$$

The number of C^{14} nuclei can be calculated in two steps. First, the number of C^{12} nuclei in 25.0 g of carbon is,

$$N(^{12}C) = \frac{m}{M} N_A = (6.02 \times 10^{23})\left(\frac{25}{12}\right) = 1.25 \times 10^{24}$$

The ratio of C^{14} to C^{12} in the live sample was 1.3×10^{-12}, we see that number of C^{14} nuclei in 25.0 g before decay was,

$$N_0(C^{14}) = (1.3 \times 10^{-12})(1.25 \times 10^{24}) = 1.6 \times 10^{12}$$

Hence, the initial activity of the sample was,

$$R_0 = \lambda N_0$$
$$= (3.84 \times 10^{-12})(1.6 \times 10^{12})$$
$$= 6.14 \text{ decay/s} = 370 \text{ decay/min}$$

Activity at the time of observation is given as, 250 decay/min.

From the equation, $R = R_0 e^{-\lambda t}$

We have, $\lambda t = \ln\left(\frac{R_0}{R}\right)$ or $t = \frac{1}{\lambda}\ln\left(\frac{R_0}{R}\right)$

Time, $t = \frac{1}{3.84 \times 10^{-12}} \ln\left(\frac{370}{250}\right)$

$$= 1.0 \times 10^{11} \text{ s} = 3200 \text{ yr}$$

Applications of radioactivity

Applications of radioactivity are given below

(i) Na-24 is used for testing blood circulation.
(ii) Radio mercury-203 is used for detecting brain tumor.
(iii) Phosphorous-31 is used for skin diseases.
(iv) C^{14} is used for determining age of archaeological sample.

Check point 13.3

1. Alpha rays emitted from a radioactive substance are
(a) negatively charged particles
(b) ionised hydrogen nuclei
(c) doubly ionised helium atom
(d) uncharged particles having the mass equal to proton

2. Pick out the correct statement from the following.
(a) α-particles are not deflected by electric and magnetic field.
(b) β-particle has less penetrating power than α-particles.
(c) γ-rays has the least ionising power than α and β-particles.
(d) γ-rays has the least penetrating power than α and β-rays.

3. In the reaction, identify X, $_7N^{14} + \alpha \longrightarrow {}_8X^{17} + {}_1P^1$
(a) An oxygen nucleus with mass 17
(b) An oxygen nucleus with mass 16
(c) A nitrogen nucleus with mass 17
(d) A nitrogen nucleus with mass 16

4. β-decay means emission of electrons from
(a) innermost electron orbit (b) a stable nucleus
(c) outermost electrons orbit (d) radioactive nucleus

5. During a negative beta decay
(a) an atomic electron is ejected
(b) an electron which is already present with in the nucleus is ejected
(c) a neutron in the nucleus decays emitting on electron
(d) a part of the binding energy is converted into electron

6. A radioactive nucleus $_{92}X^{235}$ decays to $_{91}Y^{231}$. Which of the following particles are emitted?
(a) One alpha and one electron
(b) Two deuterons and one positron
(c) One alpha and one proton
(d) One proton and four neutrons

7. The half-life of a radioactive element which has only 1/32 of its original mass left after a lapse of 60 days is
(a) 12 days (b) 32 days (c) 60 days (d) 64 days

8. A radioactive substance emits n beta particles in the first 2 seconds and 0.5 n beta particles in the next 2 seconds. The mean life of the sample is
(a) 4 s (b) 2 s
(c) $\frac{2}{(\ln 2)}$ s (d) 2 (ln 2) s

9. The half-life ($T_{1/2}$) and the disintegration constant (λ) of a radioactive substance are related as
(a) $\lambda T_{1/2} = 1$ (b) $\lambda T_{1/2} = 0.693$
(c) $\frac{T_{1/2}}{\lambda} = 0.693$ (d) $\frac{\lambda}{T_{1/2}} = 0.693$

10. Half-life of radioactive element depend upon
(a) amount of element present (b) temperature
(c) pressure (d) nature of element

11. The half-life of polonium is 140 days. After how many days, 16 g polonium will be to 1 g (or 15 g will decay)
(a) 700 days (b) 280 days
(c) 560 days (d) 420 days

12. Certain radioactive substance reduces to 25% of its value in 16 days. Its half-life is
(a) 32 days (b) 8 days
(c) 64 days (d) 28 days

13. C^{14} has half-life 5700 years. At the end of 11400 years, the actual amount left is
(a) 0.5 of original amount (b) 0.25 of original amount
(c) 0.125 of original amount (d) 0.0625 of original amount

14. If the half-life of a radioactive sample is 10 hours its mean life is
(a) 14.4 h (b) 7.2 h (c) 20 h (d) 6.93 h

15. Mean life of a radioactive sample is 100 second. Then, its half-life is
(a) 0.693 (b) 1
(c) 10^{-4} (d) 1.155

Chapter exercises

A) Taking it together

(Assorted questions of the chapter for advanced level practice)

1. Following process is known as $h\nu \rightarrow e^{+} + e^{-}$
(a) Pair production (b) Photoelectric effect
(c) Compton effect (d) Zeeman effect

2. The operation of a nuclear reactor is said to be critical, if the multiplication factor (k) has a value
(a) 1 (b) 1.5
(c) 2.1 (d) 2.5

3. Which of the following isotopes is used for the treatment of cancer
(a) K^{40} (b) Co^{60}
(c) Sr^{90} (d) I^{131}

4. Identify the correct statement given below.
(a) Alpha rays have the highest penetrating power
(b) Beta rays have the highest penetrating power
(c) Gamma rays have the highest penetrating power
(d) All three kinds of rays have the same penetrating power

5. When a nucleus in an atom undergoes a radioactive decay, then the electronic energy levels of the atom **[NCERT Exemplar]**
(a) do not change for any type of radioactivity
(b) change for α and β-radioactivity but not for γ-radioactivity
(c) change for α-radioactivity but not for others
(d) change for β-radioactivity but not for others

6. Heavy stable nuclei have more neutrons than protons. This is because of the fact that **[NCERT Exemplar]**
(a) neutrons are heavier than protons
(b) electrostatic force between protons are repulsive
(c) neutrons decay into protons through beta decay
(d) nuclear forces between neutrons are weaker than that between protons

7. In one average life
(a) half the active nuclei decay
(b) less than half the active nuclei decay
(c) more than half the active nuclei decay
(d) All the nuclei decay

8. Which of the following is a wrong description of binding energy in a nucleus?
(a) It is the energy required to break a nucleus into its constituent nucleons
(b) It is the energy made available when free nucleons combine to form a nucleus
(c) It is the sum of the rest mass energies of its nucleons minus the rest mass energy of the nucleus
(d) None of the above

9. As compared to ^{12}C-atom, ^{14}C-atom has
(a) two extra protons and two extra electrons
(b) two extra protons but no extra electron
(c) two extra neutrons and no extra electron
(d) two extra neutrons and two extra electrons

10. When the number of nucleons in nuclei increase, then the binding energy per nucleon
(a) increases continuously with mass number
(b) decreases continuously with mass number
(c) remains constant with mass number
(d) first increases and then decreases with increase of mass number

11. The binding energy per nucleon is maximum in the case of
(a) $^{4}_{2}He$ (b) $^{56}_{26}Fe$ (c) $^{141}_{56}Ba$ (d) $^{235}_{92}U$

12. Which of the following process will result into fission reaction?
(a) $_{92}U^{235}$ is bombarded with fast moving neutrons
(b) $_{92}U^{235}$ is bombarded with thermal neutrons
(c) U^{238} is bombarded with slow moving neutrons
(d) U^{235} being unstable breaks into smaller fragments

13. What fissionable material was used in the bomb dropped at Nagasaki (Japan) in the year 1945?
(a) Uranium (b) Neptunium
(c) Berkelium (d) Plutonium

14. The principle of controlled chain reaction is used in
(a) atomic energy reactor (b) atom bomb
(c) the core of sun (d) artificial radioactivity

15. In an atomic bomb, the energy is released due to
(a) chain reaction of neutrons and $_{92}U^{235}$
(b) chain reaction of neutrons and $_{92}U^{238}$
(c) chain reaction of neutrons and $_{92}U^{240}$
(d) chain reaction of neutrons and $_{92}U^{236}$

16. Boron rods are used in nuclear reactors because
(a) boron can absorb neutrons
(b) strength is given to the plant
(c) boron speeds up neutrons
(d) the reactors look good

17. A nuclear reaction along with the masses of the particle taking part in it is as follows

$$\underset{1.002\ \text{amu}}{A} + \underset{1.004\ \text{amu}}{B} \rightarrow \underset{1.001\ \text{amu}}{C} + \underset{1.003\ \text{amu}}{D} + Q\,\text{MeV}$$

The energy Q liberated in the reaction is
(a) 1.234 MeV (b) 0.931 MeV
(c) 0.465 MeV (d) 1.862 MeV

18. The curve of binding energy per nucleon as a function of atomic mass number has a sharp peak for helium nucleus.
This implies that helium
(a) can easily be broken up
(b) is very stable
(c) can be used as fissionable material
(d) is radioactive

19. In the given nuclear reaction A, B, C, D and E represents

$$_{92}U^{238} \xrightarrow{\alpha} {}_{B}Th^{A} \xrightarrow{\beta} {}_{D}Pa^{C} \xrightarrow{E} {}_{92}U^{234}$$

(a) $A = 234, B = 90, C = 234, D = 91$ and $E = \beta$
(b) $A = 234, B = 90, C = 238, D = 94$ and $E = \alpha$
(c) $A = 238, B = 93, C = 234, D = 91$ and $E = \beta$
(d) $A = 234, B = 90, C = 234, D = 93$ and $E = \alpha$

20. An atom of mass number 15 and atomic number 7 captures an α-particle and then emits a proton. The mass number and atomic number of the resulting product will respectively be
(a) 14 and 2 (b) 15 and 3
(c) 16 and 4 (d) 18 and 8

21. The binding energies of the nuclei A and B are E_a and E_b, respectively. Three atoms of the element B fuse to give one atom of element A and an energy Q is released. Then, E_a, E_b and Q are related as
(a) $E_a - 3E_b = Q$ (b) $3E_b - E_a = Q$
(c) $E_a + 3E_b = Q$ (d) $E_b + 3E_a = Q$

22. The total number of α and β particles emitted in the nuclear reaction

$$^{238}_{92}U \longrightarrow {}^{214}_{82}Pb \text{ is}$$

(a) 6 (b) 7
(c) 8 (d) 10

23. Suppose we consider a large number of containers each containing initially 10000 atoms of a radioactive material with a half-life of 1 yr. After 1 yr, **[NCERT Exemplar]**
(a) All the containers will have 5000 atoms of the material
(b) All the containers will contain the same number of atoms of the material but that number will only be approximately 5000
(c) The containers will in general have different numbers of the atoms of the material but their average will be close to 5000
(d) None of the containers can have more than 5000 atoms

24. 90% of a radioactive sample is left undecayed after time t has elapsed. What percentage of the initial sample will decay in a total time $2t$?
(a) 20% (b) 19%
(c) 40% (d) 38%

25. M_x and M_y denote the atomic masses of the parent and the daughter nuclei respectively, in radioactive decay. The Q-value for a β^- decay is Q_1 and that for a β^+ decay is Q_2. If m_e denotes the mass of an electron, then which of the following statements is correct? **[NCERT Exemplar]**
(a) $Q_1 = (M_x - M_y)c^2$ and $Q_2 = [M_x - M_y - 2m_e]c^2$
(b) $Q_1 = (M_x - M_y)c^2$ and $Q_2 = (M_x - M_y)c^2$
(c) $Q_1 = (M_x - M_y - 2m_e)c^2$ and $Q_2 = (M_x - M_y + 2c_e)c^2$
(d) $Q_1 = (M_x - M_y + 2m_e)c^2$ and $Q_2 = (M_x - m_y + 2m_e)c^2$

26. Half-life of a radioactive substance is 20 min. The time between 20% and 80% decay will be
(a) 20 min (b) 40 min
(c) 30 min (d) 25 min

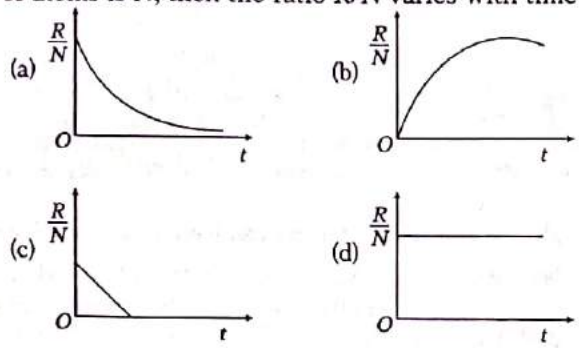
27. A freshly prepared radioactive source of half-life 2 h emits radiation of intensity which is 64 times the permissible safe level. The minimum time after which it would be possible to work safely with this source is
(a) 128 h (b) 24 h
(c) 6 h (d) 12 h

28. The decay constant of a radioactive substance is λ. Its half-life and mean life respectively, are
(a) $\frac{1}{\lambda}$ and $\log_e 2$ (b) $\frac{\log_e 2}{\lambda}$ and $\frac{1}{\lambda}$
(c) $2\log_e 2$ and $\frac{1}{\lambda}$ (d) $\frac{1}{\lambda}$ and $\frac{1}{\lambda}$

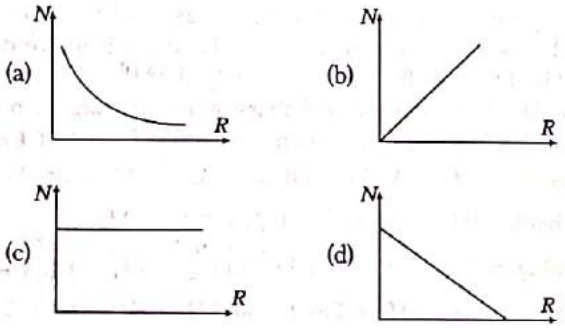
29. A radioactive substance disintegrates 1/64 of initial value in 60 s. The half-life of this substance is
(a) 5 s (b) 10 s (c) 30 s (d) 20 s

30. In the reaction $^2_1H + {}^3_1H \rightarrow {}^4_2He + {}^1n_0$, if the binding energies of ^2_1H, ^3_1H and ^4_2He are respectively, a, b and c (in MeV), then the energy (in MeV) released in this reaction is
(a) $c + a - b$ (b) $c - a - b$
(c) $a + b + c$ (d) $a + b - c$

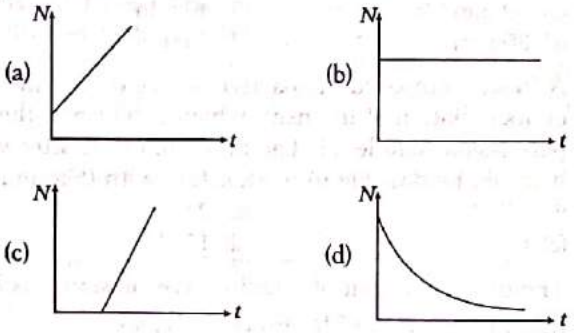
31. A radioactive sample has N_0 active atoms at $t = 0$. If the rate of disintegration at any time is R and the number of atoms is N, then the ratio R/N varies with time as



32. The plot of the number (N) of decayed atoms *versus* activity (R) of a radioactive substance is



33. The graph between the instantaneous concentration (N) of a radioactive element and time (t) is



34. Two radioactive materials X_1 and X_2 have decay constants 10λ and λ, respectively. If initially they have the same number of nuclei, then the ratio of the number of nuclei of X_1 to that of X_2 will be $1/e$ after a time

(a) $\left(\frac{1}{10\lambda}\right)$ (b) $\frac{1}{(11\lambda)}$

(c) $\frac{11}{(10\lambda)}$ (d) $\frac{1}{(9\lambda)}$

35. Two radioactive nuclei A and B have disintegration constants λ_A and λ_B and initially N_A and N_B number of nuclei of them are taken, then the time after which their undisintegrated nuclei are same is

(a) $\frac{\lambda_A\lambda_B}{(\lambda_A - \lambda_B)}\ln\left(\frac{N_B}{N_A}\right)$ (b) $\frac{1}{(\lambda_A + \lambda_B)}\ln\left(\frac{N_B}{N_A}\right)$

(c) $\frac{1}{(\lambda_B - \lambda_A)}\ln\left(\frac{N_B}{N_A}\right)$ (d) $\frac{1}{(\lambda_A - \lambda_B)}\ln\left(\frac{N_B}{N_A}\right)$

36. What is the binding energy per nucleon of ${}_6C^{12}$ nucleus?

Given, mass of C^{12} $(m_c) = 12.000$ u

Mass of proton $(m_p) = 1.0078$ u

Mass of neutron $(m_n) = 1.0087$ u

and 1 amu $= 931.4 \frac{\text{MeV}}{c^2}$

(a) 5.26 MeV (b) 6.2 MeV (c) 4.65 MeV (d) 7.68 MeV

37. A radioactive isotope X with a half-life of 1.37×10^9 yr decays to Y which is stable. A sample of rock from the moon was found to contain both the elements X and Y which were in the ratio of 1 : 7. The age of the rock is

(a) 1.96×10^8 yr (b) 3.85×10^9 yr

(c) 4.11×10^9 yr (d) 9.59×10^9 yr

38. If the energy released in the fission of one nucleus is 200 MeV, then the number of nuclei required per second in a power plant of 16 kW will be

(a) 0.5×10^{14} (b) 0.5×10^{22}

(c) 5×10^{12} (d) 5×10^{14}

39. A common example of β-decay is

$${}_{15}P^{32} \rightarrow {}_{16}P^{32} + x + y$$

Then x and y stand for

(a) electron and neutrino

(b) positron and neutrino

(c) electron and antineutrino

(d) positron and antineutrino

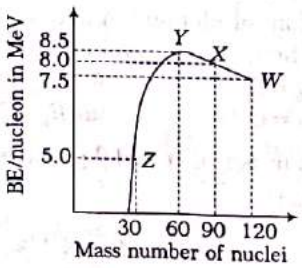
40. A nucleus with $Z = 92$ emits the following in a sequence:

$\alpha, \beta^-, \beta^-, \alpha, \alpha, \alpha, \alpha, \alpha, \beta^-, \beta^-, \alpha, \beta^+, \beta^+, \alpha$. The Z of the resulting nucleus is

(a) 74 (b) 76

(c) 78 (d) 82

41. Binding energy per nucleon *versus* mass number curve for nuclei is shown in the figure W, X, Y and Z are four nuclei indicated on the curved. The process that would release energy is



(a) $Y \rightarrow 2Z$ (b) $W \rightarrow X + Y$

(c) $W \rightarrow 2Y$ (d) $X \rightarrow Y + Z$

42. Tritium is an isotope of hydrogen whose nucleus triton contains 2 neutrons and 1 proton. Free neutrons decay into $p + e^- + \bar{\nu}$. If one of the neutrons in triton decays, then it would transform into He^3 nucleus. This does not happen. This is because **[NCERT Exemplar]**
(a) triton energy is less than that of a He^3 nucleus
(b) the electron created in the beta decay process cannot remain in the nucleus
(c) Both the neutrons in triton have to decay simultaneously resulting in a nucleus with 3 protons, which is not a He^3 nucleus
(d) Free neutrons decay due to external perturbations which is absent in triton nucleus

43. The gravitational force between a H-atom and another particle of mass **m** will be given by Newton's law $F = G\frac{M \cdot m}{r^2}$, where r is in m and **[NCERT Exemplar]**
(a) $M = m_{proton} + m_{electron}$
(b) $M = m_{proton} + m_{electron} - \frac{B}{c^2}$ ($B = 13.6$ eV)
(c) M is not relate to the mass of the hydrogen atom
(d) $M = m_{proton} + m_{electron} - \frac{|V|}{c^2}$ ($|V|$ = magnitude of the potential energy of electron in the H-atom)

44. In a sample of radioactive material, what fraction of the initial number of active nuclei will remain undisintegrated after half of the half-life of the sample?
(a) $\sqrt{2} - 1$ (b) $1/\sqrt{2}$ (c) $1/2\sqrt{2}$ (d) 1/4

45. In a radioactive sample, the fraction of initial number of radioactive nuclei, which remains undecayed after n mean lives is
(a) $\frac{1}{e^n}$ (b) e^n (c) $1 - \frac{1}{e^n}$ (d) $\left(\frac{1}{e-1}\right)^n$

46. At any instant, the ratio of the amount of radioactive substance is 2 : 1. If their half-lives be respectively, 12 h and 16 h then after 2 days, what will be the ratio of the substances?
(a) 1 : 1 (b) 2 : 1 (c) 1 : 2 (d) 1 : 4

47. A radioactive isotope X with half-life 1.5×10^9 yr decays into a stable nucleus Y. A rock sample contains both elements X and Y in the ratio 1: 15. The age of the rock is
(a) 3.0×10^9 yr (b) 4.5×10^9 yr
(c) 6.0×10^9 yr (d) 9.0×10^9 yr

48. The half-life of a sample of a radioactive substance is 1 h. If 8×10^{10} atoms are present at $t = 0$, then the number of atoms decayed in the duration $t = 2$ h to $t = 4$ h will be
(a) 2×10^{10} (b) 1.5×10^{10}
(c) zero (d) infinity

49. Half-life of a radioactive substance is 20 min. Difference between points of time when it is 33% disintegrated and 67% disintegrated is approximately
(a) 10 min (b) 20 min (c) 30 min (d) 40 min

50. A and B are two radioactive substances whose half-lives are 1 and 2 yr, respectively. Initially 10 g of A and 1 g of B is taken. The time (approximate) after which they will have same quantity remaining is
(a) 6.62 yr (b) 5 yr (c) 3.2 yr (d) 7 yr

51. The decay constant of a radioactive element is defined as the reciprocal of the time interval after which the number of atoms of the radioactive element falls to nearly
(a) 50% of its original number
(b) 36.8% of its original number
(c) 63.2% of its original number
(d) 75% of its original number

52. The decay constant of radioactive element is $1.5 \times 10^{-9} s^{-1}$. Its mean-life (in second) will be
(a) 1.5×10^9 (b) 4.62×10^8
(c) 6.67×10^8 (d) 10.35×10^8

53. A radioactive material has mean lives of 1620 yr and 520 yr for α and β-emission, respectively. The material decays by simultaneous α and β-emission. The time in which 1/4th of the material remains intact is
(a) 4675 yr (b) 720 yr (c) 545 yr (d) 324 yr

54. A radioactive nucleus is being produced at a constant rate α per second. Its decay constant is λ. If N_0 are the number of nuclei at time $t = 0$, then maximum number of nuclei possible are
(a) $\frac{\alpha}{\lambda}$ (b) $N_0 + \frac{\alpha}{\lambda}$ (c) N_0 (d) $\frac{\lambda}{\alpha} + N_0$

55. After three 'average life times' of a radioactive sample, the amount of substance remains x % of the original amount. Then, approximate value of x is
(a) 14 (b) 37 (c) 7 (d) 5

56. An archaeologist analyses the wood in a prehistoric structure and finds that C^{14} (half-life = 5700yr) to C^{12} ratio is only one-third of that found in the cells buried plants. The age of the wood is about
(a) 5700 yr (b) 2850 yr (c) 11400 yr (d) 22800 yr

57. A radioactive element A of decay constant λ_A decays into another radioactive element B of decay constant λ_B. Initially the number of active nuclei of A was N_0 and B was absent in the sample. The maximum number of active nuclei of B is found at $t = 2 \ln 2 / \lambda_A$. The maximum number of active nuclei of B is
(a) $\frac{N_0}{4}$ (b) $\frac{\lambda_A}{\lambda_B} N_0 e^{-\lambda_B t}$
(c) $\frac{\lambda_A}{\lambda_B} \frac{N_0}{4}$ (d) None of these

58. Two identical samples (same material and same amount) P and Q of a radioactive substance having mean life T are observed to have activities R_P and R_Q respectively, at the time of observation. If P is older than Q, then the difference in their age is

(a) $T\ln\left(\frac{R_p}{R_q}\right)$ (b) $T\ln\left(\frac{R_q}{R_p}\right)$

(c) $T\left(\frac{R_p}{R_q}\right)$ (d) $T\left(\frac{R_p}{R_q}\right)$

59. Energy of 24.6 eV is required to remove one of the electron from a neutral helium atom. The energy (in eV) required to remove both the electrons from a neutral helium atom is

(a) 38.2 (b) 49.2 (c) 51.8 (d) 79.0

60. The activity of a sample of radioactive material is R_1 at time t_1 and R_2 at time $t_2\,(t_2 > t_1)$. Its mean life is T. Then,

(a) $R_1t_1 = R_2t_2$ (b) $\frac{R_1 - R_2}{t_2 - t_1} = \text{constant}$

(c) $R_2 = R_1e^{(t_1 - t_2)/T}$ (d) $R_2 = R_1e^{(t_1/t_2)/T}$

61. A heavy nucleus at rest breaks into two fragments which fly off with velocities in the ratio 8 : 1. The ratio of radii of the fragments is

(a) 1 : 2 (b) 1 : 4 (c) 4 : 1 (d) 2 : 1

62. For a substance, the average life for α-emission is 1620 yr and for β-emission is 405 yr. After how much time the $\frac{1}{4}$ of the material remains after α and β-emission?

(a) 1500 yr (b) 300 yr

(c) 449 yr (d) 810 yr

63. Two radioactive nuclei P and Q, in a given sample decay into a stable nucleus R. At time $t = 0$, number of P species are $4N_0$ and that of Q are N_0. Half-life of P (for conversion to R) is 1 min, where as that of Q is 2 min. Initially there are no nuclei of R present in the sample. When number of nuclei of P and Q are equal, then the number of nuclei of R present in the sample would be

(a) $\frac{5N_0}{2}$ (b) $2N_0$ (c) $3N_0$ (d) $\frac{9N_0}{2}$

64. The half-life of a radioactive substance is 40 yr. How long will take to reduce to one-fourth of its original amount and what is the value of decay constant?

(a) 40 yr, 0.9173/yr (b) 90 yr, 9.017/yr

(c) 80 yr, 0.0173/yr (d) None of these

65. Starting with a sample of pure ^{66}Cu, $\frac{7}{8}$ of it decays into Zn in 15 min. The corresponding half-life is

(a) 5 min (b) $7\frac{1}{2}$ min (c) 10 min (d) 15 min

66. A stationary radioactive nucleus of mass 210 units disintegrates into an alpha particle of mass 4 units and residual nucleus of mass 206 units. If the kinetic energy of the alpha particle is E, then the kinetic energy of the residual nucleus is

(a) $\left(\frac{2}{105}\right)E$ (b) $\left(\frac{2}{103}\right)E$

(c) $\left(\frac{103}{105}\right)E$ (d) $\left(\frac{103}{2}\right)E$

67. A bone containing 200 g carbon-14 has a β-decay rate of 375 decay/min. Calculate the time that has elapsed, since the death of the living one. Given the rate of decay for the living organism = 15 decay per min per gram of carbon and half-life of carbon-14 = 5730 yr.

(a) 22920 yr (b) 11460 yr

(c) 17190 yr (d) None of these

68. Assuming that about 200 MeV energy is released per fission of $_{92}U^{235}$ nuclei. What would be the mass of U^{235} consumed per day in the fission of reactor of power 1 MW approximately?

(a) 10 kg (b) 1 kg

(c) 1 g (d) 10 g

69. The activity of a radioactive sample is measured as N_0 counts per minute at $t = 0$ and $\frac{N_0}{e}$ counts per minute at $t = 5$ min. The time (in minutes) at which the activity reduces to half its value is

(a) $5\log_e 2$ (b) $\log_e \frac{2}{5}$

(c) $\frac{5}{\log_e 2}$ (d) $5\log_{10} 2$

70. In the nuclear fusion reaction $^2_1H + ^3_1H \rightarrow ^4_2He + ^1_0n$, given that the repulsive potential energy between the two nuclei is -7.7×10^{-14} J. The temperature at which the gases must be heated to initiate the reaction is nearly, [Boltzmann's constant, $k = 1.38 \times 10^{-23}$ J/K]

(a) 10^9K (b) 10^7K

(c) 10^5K (d) 10^3K

71. The half-life of radioactive radon in 3.8 days. The time at the end of which $\frac{1}{20}$ th of the radon sample will remain undecayed is (given, $\log_{10} e = 0.4343$)

(a) 3.8 days (b) 16.5 days

(c) 33 days (d) 76 days

72. The half-life of radioactive polonium (Po) is 138.6 days. For ten lakh polonium atoms, the number of disintegrations in 24 h is

(a) 2000 (b) 3000

(c) 4000 (d) 5000

73. If half-life of a substance is 3.8 days and its quantity is 10.38 g. Then, substance quantity remaining left after 19 days will be
(a) 0.151 g (b) 0.32 g (c) 1.51 g (d) 0.16 g

74. Two radioactive substance A and B have decay constants 5λ and λ, respectively. At $t = 0$ they have the same number of nuclei. The ratio of number of nuclei of A to those of B will be $\left(\frac{1}{e}\right)^2$ after a time interval.
(a) $\frac{1}{4\lambda}$ (b) 4λ (c) 2λ (d) $\frac{1}{2\lambda}$

75. A star initially has 10^{40} deuterons. It produces energy *via* the processes ${}^2_1H + {}^2_1H \rightarrow {}^3_1H + p$ and ${}^2_1H + {}^3_1H \rightarrow {}^4_2He + n$. where, the masses of the nuclei are
$m({}^2H) = 2.014$ amu, $m(p) = 1.007$ amu,
$m(n) = 1.008$ amu and $m({}^4He) = 4.001$ amu, if the average power radiated by the star is 10^{16} W. The deuteron supply of the star is exhausted in a time of the order of
(a) 10^{18} s (b) 10^{28} s
(c) 10^{12} s (d) 10^{16} s

76. The sun radiates energy in all directions. The average radiation received on the earth surface from the sun $1.4\ kW/m^2$. The average earth-sun distance 1.5×10^{11} m. The mass lost by the sun per day is (1 day = 86400 s)
(a) 4.4×10^9 kg (b) 7.6×10^{14} kg
(c) 3.8×10^{12} kg (d) 3.8×10^{14} kg

77. If 10% of a radioactive material decays in 5 days, then the amount of original material left after 20 days is approximately
(a) 60% (b) 65% (c) 70% (d) 75%

78. A small quantity of solution containing Na^{24} radio nuclide of activity 1 microcurie is injected into the blood of a person. A sample of the blood of volume 1 cm^3 taken after 5 h shows an activity of 296 disintegration per minute. What will be the total value of the blood in the body of the person? Assume that the radioactive solution mixes uniformly in the blood of the person. (Take, 1 curie $= 3.7 \times 10^{10}$ disintegration per second and $e^{-\lambda t} = 0.7927$, where λ = disintegration constant)
(a) 5.94 L (b) 2 L
(c) 317 L (d) 1 L

B) Medical entrance special format questions

Assertion and reason

Directions (Q. Nos. 1-9) *These questions consist of two statements each linked as Assertion and Reason. While answering of these questions you are required to choose any one of the following five responses.*

(a) If both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are true but Reason is not correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If Assertion is false but Reason is true.
(e) If both Assertion and Reason are false.

1. **Assertion** Radioactive nuclei emit β^{-1} particles.
Reason Electrons exist inside the nucleus.

2. **Assertion** Density of all the nuclei is same.
Reason Radius of nucleus is directly proportional to the cube root of mass number.

3. **Assertion** Ionisation power of α-particles is maximum while penetration power of γ-rays is maximum.
Reason α-particles are heaviest particles, while γ-rays have maximum energy.

4. **Assertion** The binding energy per nucleon, for nuclei with atomic mass number $A > 100$, decreases with A
Reason The nuclear forces are weak for heavier nuclei.

5. **Assertion** In any nuclear process energy is released, if total binding energy of daughter nuclei is more than the total binding energy of parent nuclei.
Reason Binding energy per nucleon is of the order of MeV.

6. **Assertion** In one half-life of a radioactive substance more number of nuclei are decayed than in one average life.
Reason Average life = Half- life/ln (2)

7. **Assertion** Two different radioactive substances have initially same number of nuclei. Their decay constants are λ_1 and λ_2 $(< \lambda_1)$. Then, initially first radioactive substance decays at faster rate.
Reason Half-life of first radioactive substance is less.

8. **Assertion** Positive value of packing fraction implies a large value of binding energy.
Reason The ratio of the difference between the atomic mass of nucleus and the mass number of the nucleus to that of atomic mass is called packing fraction.

9. **Assertion** If high pressure is applied on a radioactive substance rate of radioactivity will increase.
Reason Radioactivity is a nuclear process.

Match the columns

1. From a radioactive substance x numbers of α-particles and y numbers of β-particles are emitted. As a result atomic number decreases by n and mass number by m. Then, match the following two columns.

	Column I		Column II
(A)	$x = 1$ and $y = 2$	(p)	$n = 2, m = 0$
(B)	$x = 2$ and $y = 1$	(q)	$n = 0, m = 4$
(C)	$x = 0$ and $y = 2$	(r)	$n = 2, m = 4$
(D)	$x = 1$ and $y = 0$	(s)	None

2. Match the statement in Column I to the appropriate statement(s) from Column II.

	Column I		Column II
(A)	Nuclear fusion	(p)	Converts some matter into energy
(B)	Nuclear fission	(q)	Generally possible for nuclei with low atomic number
(C)	β-decay	(r)	Generally possible for nuclei with higher atomic number
(D)	Exothermic nuclear reaction	(s)	Essentially proceeds by weak nuclear forces

3. Match the statement in Column I to the appropriate statement(s) from Column II.

	Column I		Column II
(A)	Stability of nucleus decided by	(p)	−ve
(B)	Four radioactive substance spontaneously decays because its	(q)	Binding energy per nucleon is minimum
(C)	For the stable orbit or bound orbit, total energy	(r)	Neutron-proton ratio
(D)	Stopping potential	(s)	Packing fraction

4. Match Column I of the nuclear processes with Column II containing parent nucleus and one of the end products of each process.

	Column I		Column II
(A)	Alpha decay	(p)	${}^{15}_{8}O \rightarrow {}^{15}_{7}N + \ldots$
(B)	β^+ decay	(q)	$U^{238} \rightarrow U^{234} + \ldots$
(C)	Fission	(r)	${}^{185}_{83}Bi \rightarrow {}^{184}_{82}Pb + \ldots$
(D)	Proton emission	(s)	${}^{239}_{94}Pb \rightarrow {}^{140}_{57}La + \ldots$

Medical entrances' gallery

(Collection of questions asked in NEET & various medical entrance exams)

1. The half-life of a radioactive substance is 30 min. The time (in minutes) taken between 40% decay and 85% decay of the same radioactive substance is **[NEET 2016]**
(a) 15 (b) 30 (c) 45 (d) 60

2. The kinetic energy of α-particle emitted in the α-decay of ${}_{88}Ra^{226}$ is [given, mass number of Ra = 222 u] **[UP CPMT 2015]**
(a) 5.201 MeV (b) 3.301 MeV
(c) 6.023 MeV (d) 4.871 MeV

3. If radius of the ${}^{27}_{13}Al$ nucleus is taken to be R_{Al}, then the radius of ${}^{125}_{53}Te$ nucleus is nearly **[CBSE AIPMT 2015]**
(a) $\left(\frac{53}{13}\right)^{1/3} R_{Al}$ (b) $\frac{5}{3} R_{Al}$ (c) $\frac{3}{5} R_{Al}$ (d) $\left(\frac{13}{53}\right)^{1/3} R_{Al}$

4. Two spherical nuclei have mass numbers 216 and 64 with their radii R_1 and R_2, respectively. The ratio, $\frac{R_1}{R_2}$ is equal to **[AIIMS 2015]**
(a) 3 : 2 (b) 1 : 3
(c) 1 : 2 (d) 2 : 3

5. In a nuclear reactor, the number of U^{235} nuclei undergoing fissions per second is 4×10^{20}. If the energy released per fission is 250 MeV, then the total energy released in 10 h is (1 eV = 1.6×10^{-19} J) **[EAMCET 2015]**
(a) 576×10^6 J (b) 576×10^{12} J
(c) 576×10^{15} J (d) 576×10^{18} J

6. Nuclear fusion is not found in **[Kerala CEE 2015]**
(a) thermonuclear reactor (b) hydrogen bomb
(c) energy production in sun (d) atom bomb
(e) energy production in stars

7. The approximate ratio of nuclear mass densities of ${}^{197}_{79}Au$ and ${}^{107}_{47}Ag$ nuclei is **[Kerala CEE 2015]**
(a) 197 : 107 (b) 47 : 79 (c) 79 : 47 (d) 1 : 1
(e) 107 : 197

8. A radioactive element X disintegrates successively
$$X \xrightarrow{\beta^-} X_1 \xrightarrow{\alpha} X_2 \xrightarrow{\beta^-} X_3 \xrightarrow{\alpha} X_4$$
If atomic number and atomic mass number of X are respectively, 72 and 180, then what are the corresponding values for X_4? **[Guj. CET 2015]**
(a) 69, 172 (b) 69, 176 (c) 71, 176 (d) 70, 172

9. The energy released by the fission of one uranium atom is 200MeV. The number of fission per second required to produce 6.4W power is [Guj. CET 2015]
(a) 2×10^{11} (b) 10^{11} (c) 10^{10} (d) 2×10^{10}

10. By the successive disintegration of $_{92}U^{238}$, the final product obtained is $_{82}Pb^{206}$, then how many number of α and β-particles are emitted? [Guj. CET 2015]
(a) 6 and 8 (b) 8 and 6
(c) 12 and 6 (d) 8 and 12

11. If 20% of a radioactive element decays in 10 days, then amount of original material left after 30 days is (approx) [UP CPMT 2015]
(a) 78 % (b) 62 %
(c) 51 % (d) 48 %

12. A nucleus at rest splits into two nuclear parts having radii in the ratio 1 : 2. Their velocities are in the ratio [KCET 2015]
(a) 4 : 1 (b) 8 : 1
(c) 2 : 1 (d) 6 : 1

13. What is the energy released by fission of 1 g of U^{235}? (Assume 200 MeV energy is liberated on fission of 1 nucleus) [CG PMT 2015]
(a) 2.26×10^4 kWh (b) 1.2×10^4 kWh
(c) 4.5×10^2 kWh (d) 2.5×10^2 kWh

14. The half-life of radium is 1620 yr and its atomic weight is 226 kg per kilo mol. The number of atoms that will decay from its 1g sample per second will be (take, Avogadro's number, $N_A = 6.023\times10^{23}$ atom/kilo mol) [Manipal 2015]
(a) 3.61×10^{10} (b) 3.11×10^{15}
(c) 3.6×10^{12} (d) 3.11×10^{15}

15. The relationship between decay constant λ and half-life T of a radioactive substance is [UP PMT 2015]
(a) $\lambda=\frac{\log_{10}2}{T}$ (b) $\lambda=\frac{\log_e 2}{T}$
(c) $\lambda=\frac{\log_2 10}{T}$ (d) $\lambda=\frac{\log_2 e}{T}$

16. The binding energy per nucleon of ^7_3Li and ^4_2He nuclei are 5.60 MeV and 7.06 MeV, respectively. In the nuclear reaction $^7_3Li+^1_1H\rightarrow^4_2He+^4_2He+Q$, the value of energy Q released is [CBSE AIPMT 2014]
(a) 19.6 MeV (b) −2.4 MeV
(c) 8.4 MeV (d) 17.3 MeV

17. For the stability of any nucleus, [KCET 2014]
(a) binding energy per nucleon will be more
(b) binding energy per nucleon will be less
(c) number of electrons will be more
(d) None of the above

18. A uranium nucleus $_{92}U^{238}$ emits an α-particle and a β-particle in succession. The atomic number and mass number of the final nucleus will be [Kerala CEE 2014]
(a) 90, 233 (b) 90, 238 (c) 91, 238 (d) 93, 238
(e) 91, 234

19. A radio isotope X with a half-life 1.4×10^8 yr decays of Y which is stable. A sample of the rock from a cave was found to contain X and Y in the ratio 1 : 7. The age of the rock is [CBSE AIPMT 2014]
(a) $1.96\times\times10^9$ yr (b) 3.92×10^9 yr
(c) 4.20×10^9 yr (d) 8.40×10^9 yr

20. After 300 days, the activity of a radioactive sample is 5000 dps (disintegration per second). The activity becomes 2500 dps after another 150 days. The initial activity of the sample (in dps) is [Kerala CEE 2014]
(a) 20000 (b) 10000 (c) 7000 (d) 25000
(e) 15000

21. For the radioactive nuclei that undergo either α or β decay, which one of the following cannot occur? [WB JEE 2014]
(a) Isobar of original nucleus is produced
(b) Isotope of the original nucleus is produced
(c) Nuclei with higher atomic number than that of the original nucleus is produced
(d) Nuclei with lower atomic number than that of the original nucleus is produced

22. In a given reaction,
$$_ZX^A \rightarrow {}_{Z+1}Y^A \rightarrow {}_{Z-1}K^{A-4} \rightarrow {}_{Z-1}K^{A-4}$$
The radioactive radiations are emitted in the sequence of [UK PMT 2014]
(a) α, β, γ (b) γ, α, β
(c) β, α, γ (d) γ, β, α

23. The control rods used in a nuclear reactor can be made up of [Kerala CEE 2014]
(a) graphite (b) cadmium
(c) uranium (d) barium
(e) lead

24. The fusion reaction in the sun is a multi-step process in which the [Kerala CEE 2014]
(a) helium is burned into deuterons
(b) helium is burned into hydrogen
(c) deuteron is burned into hydrogen
(d) hydrogen is burned into helium
(e) helium is burned into neutrons

25. The nuclear fusion reaction between deuterium and tritium takes place [EAMCET 2014]
(a) at low temperature and low pressure
(b) at very high temperature and very high pressure
(c) when the temperature is near absolute zero
(d) at ordinary temperature and pressure

26. The half-life of a radioactive isotope X is 20 yr. It decays to another element Y which is stable. The two elements X and Y were found to be in the ratio 1 : 7 in a sample of a given rock. The age of the rock is estimated to be **[NEET 2013]**
(a) 40 yr (b) 60 yr (c) 80 yr (d) 100 yr

27. A certain mass of hydrogen is changed to helium by the process of fusion. The mass defect in fusion reaction is 0.02866 u. The energy liberated per fusion is (given 1 u = 931 MeV) **[NEET 2013, EAMCET 2013]**
(a) 2.67 MeV (b) 26.7 MeV
(c) 6.675 MeV (d) 13.35 MeV

28. A U^{235} reactor generates power at a rate of P producing 2×10^{18} fission per second. The energy released per fission is 185 MeV. The value of P is **[EAMCET 2013]**
(a) 59.2 MW (b) 370×10^{18} MW
(c) 0.59 MW (d) 370 MW

29. The purpose of using heavy water in nuclear reactor is **[EAMCET 2013]**
(a) to increase the energy released in nuclear fission
(b) to cool the reactor to room temperature
(c) to make the dynamo blades to work well
(d) to decrease the energy of fast neutrons to the thermal energy

30. The ratio of volumes of nuclei (assumed to be in spherical shape) with respective mass numbers 8 and 64 is **[Kerala CET 2013]**
(a) 0.5 (b) 2 (c) 0.125 (d) 0.25
(e) 4

31. Radioactivity of a sample at T_1 time is R_1 and at time T_2 is R_2. If half-life of sample is T, then in time $(T_2 - T_1)$, the number of decayed atoms is proportional to **[MP PMT 2013]**
(a) $R_1T_2 - R_2T_1$ (b) $(R_1 - R_2)T$
(c) $\frac{(R_1 - R_2)}{T}$ (d) $R_1 - R_2$

32. Which pair is isotonic? **[MP PMT 2013]**
(a) ${}_7N^{13}, {}_7N^{14}$ (b) ${}_6C^{14}, {}_6C^{12}$
(c) ${}_6C^{14}, {}_7N^{14}$ (d) ${}_7N^{13}, {}_6C^{12}$

33. The phenomenon of radioactivity **[UP CPMT 2013]**
(a) increases on applied pressure
(b) is exothermic change which increases or decreases with temperature
(c) is nuclear process which does not depend on external forces
(d) None of the above

34. An atom bomb weighing 1 kg explodes releasing 9×10^{13} J of energy. What percentage of mass is converted into energy? **[UP CPMT 2013]**
(a) 0.1% (b) 1% (c) 2% (d) 10%

35. Pick out the correct statements from the following
I. Electron emission during β-decay is always accompanied by neutrons.
II. Nuclear force is charge independent.
III. Fusion is the chief source of stellar energy.
[Karnataka CET 2013]
(a) Both I and II are correct (b) Both I and III are correct
(c) Only I is correct (d) Both II and III are correct

36. A nucleus ${}_ZX^A$ emits an α-particle with velocity v. The recoil speed of the daughter nucleus is **[Karnataka CET 2013]**
(a) $\frac{A-4}{4v}$ (b) $\frac{4v}{A-4}$ (c) v (d) $\frac{v}{4}$

37. A radioactive substance emits 100 beta particles in the first 2 s and 50 beta particles in the next 2s. The mean life of the sample is **[Karnataka CET 2013]**
(a) 4 s (b) 2 s
(c) $\frac{2}{0.693}$ s (d) 2×0.693 s

38. When alpha particle captures and electron, then it becomes a **[Kerala CET 2013]**
(a) helium atom (b) hydrogen atom
(c) helium ion (d) hydrogen ion
(e) lithium ion

39. If a nucleus ${}_ZX^A$ emits 9 α and 5 β-particles, then the ratio of total protons and neutrons in the final nucleus is **[BCECE Mains 2012]**
(a) $\frac{(Z-13)}{(A-36)}$ (b) $\frac{(Z-13)}{(A-Z-13)}$
(c) $\frac{(Z-18)}{(A-36)}$ (d) $\frac{(Z-13)}{(A-Z-23)}$

40. If the half-life of a material is 10 yr, then in what time, it becomes $\frac{1}{4}$th part of the initial amount **[BCECE Mains 2012]**
(a) 15 yr (b) 20 yr
(c) 25 yr (d) 30 yr

41. A mixture consists of two radioactive materials A_1 and A_2 with half-lives of 20 s and 10 s, respectively. Initially the mixture has 40 g of A_1 and 160 g of A_2. The amount of the two in the mixture will become equal after **[CBSE AIPMT 2012]**
(a) 60 s (b) 80 s
(c) 20 s (d) 40 s

42. If $t_{1/2}$ is the half-life of a substance, then $t_{3/4}$ is the time in which **[CBSE AIPMT 2012]**
(a) substance decays 1/2
(b) substance decays (3/4)th
(c) substance remains 1/2
(d) substance remains (3/4)th

43. The half-life of radioactive element is 600 yr. The fraction of sample that would remain after 3000 yr is [AIIMS 2012, Manipal 2012]
(a) 1/2 (b) 1/16
(c) 1/8 (d) 1/32

44. What is the Q-value of the reaction?
$$^{1}H + {}^{7}Li \longrightarrow {}^{4}He + {}^{4}He$$
The atomic masses of ^{1}H, ^{4}He and ^{7}Li are 1.0078254u, 4.0026034u and 7.016004u, respectively. [Manipal 2012]
(a) 17.35 MeV (b) 18.06 MeV
(c) 177.35 MeV (d) 170.35 MeV

45. A nucleus of mass number 220, initially at rest, emits an α-particle. If the Q-value of the reaction is 5.5 MeV, then the energy of the emitted α-particle will be [AMU 2012]
(a) 4.8 MeV (b) 5.4 MeV
(c) 5.5 MeV (d) 6.8 MeV

46. A radioactive substance emits n beta particles in the first 2 s and 0.5 n beta particles in the next 2 s. The mean life of the sample is [AMU 2012]
(a) 4 s (b) 2 s
(c) $\frac{2}{(\ln 2)}$ s (d) 2 (ln 2) s

Answers

Check point 13.1

1. (d)	2. (c)	3. (a)	4. (b)	5. (c)	6. (b)	7. (c)	8. (d)	9. (a)	10. (a)

Check point 13.2

1. (b)	2. (b)	3. (d)	4. (a)	5. (a)	6. (c)	7. (b)	8. (c)	9. (a)	10. (a)
11. (b)	12. (c)	13. (d)	14. (d)	15. (b)					

Check point 13.3

1. (c)	2. (c)	3. (a)	4. (d)	5. (a)	6. (a)	7. (a)	8. (c)	9. (b)	10. (d)
11. (c)	12. (b)	13. (b)	14. (a)	15. (d)					

A. Taking it together

1. (a)	2. (a)	3. (b)	4. (c)	5. (b)	6. (b)	7. (c)	8. (d)	9. (c)	10. (d)
11. (b)	12. (b)	13. (d)	14. (a)	15. (a)	16. (a)	17. (d)	18. (b)	19. (a)	20. (d)
21. (a)	22. (c)	23. (c)	24. (b)	25. (a)	26. (b)	27. (d)	28. (b)	29. (b)	30. (b)
31. (d)	32. (d)	33. (d)	34. (d)	35. (c)	36. (d)	37. (c)	38. (d)	39. (c)	40. (c)
41. (c)	42. (a)	43. (b)	44. (b)	45. (a)	46. (a)	47. (c)	48. (b)	49. (b)	50. (a)
51. (b)	52. (c)	53. (c)	54. (a)	55. (d)	56. (c)	57. (c)	58. (b)	59. (d)	60. (c)
61. (a)	62. (c)	63. (d)	64. (c)	65. (a)	66. (b)	67. (c)	68. (c)	69. (a)	70. (a)
71. (b)	72. (d)	73. (b)	74. (d)	75. (c)	76. (d)	77. (b)	78. (a)		

B. Medical entrance special format questions

Assertion and reason

1. (c)	2. (a)	3. (a)	4. (c)	5. (b)	6. (d)	7. (a)	8. (d)	9. (d)

Match the columns

1. (A→q; B→s; C→p; D→r)
2. (A→p, q; B→p, r; C→s; D→p, q, r)
3. (A→r, s; B→q; C→p; D→p)
4. (A→q; B→p; C→s; D→r)

C. Medical entrances' gallery

1. (d)	2. (d)	3. (b)	4. (a)	5. (b)	6. (d)	7. (d)	8. (d)	9. (a)	10. (b)
11. (c)	12. (b)	13. (a)	14. (a)	15. (b)	16. (d)	17. (a)	18. (e)	19. (c)	20. (a)
21. (b)	22. (c)	23. (b)	24. (d)	25. (b)	26. (b)	27. (c)	28. (a)	29. (d)	30. (c)
31. (b)	32. (d)	33. (c)	34. (a)	35. (d)	36. (b)	37. (c)	38. (c)	39. (d)	40. (b)
41. (d)	42. (b)	43. (d)	44. (a)	45. (b)	46. (c)				

Hints and explanations

Check point 13.1

1. (*d*) The mass number of a nucleus is more than its atomic number, but in hydrogen, atomic number and mass number are equal.

2. (*c*) Outside a nucleus, neutron is unstable.

3. (*a*) Nucleus of an atom whose atomic mass is 24 consists of 11 protons and 13 electrons.

4. (*b*) For stability in case of lighter nuclei $\frac{N}{Z} = 1$ and for heavier nuclei $\frac{N}{Z} > 1$.

5. (*c*) Helium nucleus = ${}_2He^4$

Number of protons = $Z = 2$

Number of neutrons = $A - Z = 2$

6. (*b*) ${}_ZX^A = {}_{88}Ra^{226}$

Number of proton = $Z = 88$

Number of neutrons = $A - Z = 226 - 88 = 138$

7. (*c*) For ${}_6C^{12}$

Proton = 6, electron = 6 and neutron = 6

For ${}_6C^{14}$

Proton = 6, electron = 6 and neutron = 8

8. (*d*) Radius of nucleus, $R = R_0 A^{1/3}$.

where, R_0 is a constant and A is the mass number.

$$R_{Cu} = (1.2\times10^{-15})(64)^{1/3}$$
$$= (1.2\times10^{-15})\times(4^3)^{1/3}$$
$$= 4.8\times10^{-15}\text{ m} = 4.8\text{ fm}$$

10. (*a*) The mass of three isotopes are 19.99 u, 20.99 u and 21.99 u.

Their relative abundances are 90.51%, 0.27% and 9.22%

∴ Average atomic mass of neon

$$m = \frac{90.51\times19.99\times0.27\times20.99 + 9.22\times21.99}{(90.51 + 0.27 + 9.22)}$$
$$= \frac{2017.7}{100} = 20.17\text{ u}$$

Check point 13.2

1. (*b*) Mass defect, $\Delta m = 1 - 0.993 = 0.007$ g

∴ Energy released, $E = (\Delta m)c^2$

$$= (0.007\times10^{-3})\times(3\times10^8)^2 = 63\times10^{10}\text{ J}$$

2. (*b*) Mass of electron = Mass of positron

$$= 9.1\times10^{-31}\text{ kg}$$

According to positron annihilation,

$${}_+\beta^0 + {}_-\beta^0 \rightarrow h\nu + h\nu$$

Energy released, $E = (2m)\cdot c^2$

$$= 2\times9.1\times10^{-31}\times(3\times10^8)^2 = 1.6\times10^{-13}\text{J}$$

3. (*d*) $E = \Delta mc^2$; $\Delta m = 0.3\text{ g} = 0.3\times10^{-3}$ kg (given)

$\Rightarrow$ Amount of energy $E = \frac{0.3}{1000}\times(3\times10^8)^2 = 2.7\times10^{13}$ J

$$= \frac{2.7\times10^{13}}{3.6\times10^6} = 7.5\times10^6\text{ kWh}$$

$(\because 1\text{ kWh} = 3.6\times10^6\text{ J})$

4. (*a*) The mass of nucleus formed is always less than the sum of the masses, the constituent proton and neutrons

i.e., $m < (A - Z)_{mn} + Z_{mp}$

5. (*a*) BE = $\Delta mc^2 = \Delta m\, 931$MeV

$[Zm_p + (A - Z)m_n - m_N]\times931$

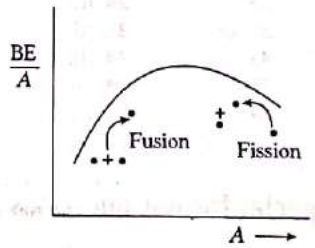
$= [2(1.0087 + 1.0073) - 4.0015]\times931$

$= 28.4$ MeV

6. (*c*) Energy released = Products energy − Reactant energy

$$= 8.5\times234 - 7.6\times236$$
$$= 195.4\text{ MeV} \approx 200\text{ MeV}$$

7. (*b*)



8. (*c*) Chain reaction occurs due to the production of more neutrons.

9. (*a*) In nuclear reactor, nuclear fission can be carried out through sustained and controlled chain reaction.

10. (*a*) In fusion reaction, two lighter nuclei combine to form a heavy nuclei.

11. (*b*) Solar energy is main caused due to fusion of proton during synthesis of heavier elements.

12. (*c*) The energy released in sun and hydrogen bomb are due to nuclear fusion.

13. (*d*) Fusion reaction takes place at high temperature because kinetic energy is high enough to overcome the Coulomb repulsion between nuclei.

14. (*d*) Heavy water is used as moderator in a nuclear reactor. The function of the moderator is to slow down the neutrons of thermal energies.

15. (*b*) Packing fraction may be positive, negative and zero.

Check point 13.3

1. (*c*) Alpha rays emitted from a radioactive substance are doubly ionised helium atom.

2. (*c*) γ-rays, ionise gases but their ionisation power is very small compared to that of α and β-particles.

4. (*d*) β-decay means emission of electron from radioactive nucleus.

5. (*a*) Negative β-decay is expressed by the equation,
$n \longrightarrow p^+ + e^- + \nu^-$.

6. (*a*) ${}_{92}X^{235} \xrightarrow{\alpha} {}_{90}X^{231} \xrightarrow{\beta^-} {}_{91}Y^{231}$.

Therefore, one alpha and one electron are emitted.

7. (*a*) We know, after n half-life, undecayed nuclei left are,

$N = N_0\left(\frac{1}{2}\right)^n$; $t = n\,t_{1/2}$

We have, $\frac{N_0}{32} = N_0\left(\frac{1}{2}\right)^{60/T} \Rightarrow 5 = \frac{60}{T}$

$\Rightarrow$ Half-life of a radioactive element, $T = 12$ days

8. (*c*) The half-life of radioactive substance is $T_{1/2} = 2$ s

Mean life, $\tau = \frac{T_{1/2}}{\ln 2} = \frac{2}{\ln 2}$ s

9. (*b*) We have, $N = N_0 e^{-\lambda}$

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}} \Rightarrow 2 = e^{\lambda T_{1/2}}$$

By taking log both the side,

$$\log 2 = \lambda T_{1/2}$$
$$\lambda T_{1/2} = 0.693$$

10. (*d*) Half-life of a substance does not depends upon amount, temperature and pressure. It depends upon the nature of the substance.

11. (*c*) We have, $\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/140}$ $\left[\because N = N_0\left(\frac{1}{2}\right)^n \text{ and } t = nt_{1/2}\right]$

$$\frac{1}{16} = \left(\frac{1}{2}\right)^{t/140} \Rightarrow \frac{t}{140} = 4 \quad (t_{1/2} = 140 \text{ days})$$

$\Rightarrow$ $t = 560$ days

12. (*b*) We have, $N = N_0\left(\frac{1}{2}\right)^{t/T} \Rightarrow \frac{N_0}{4} = N_0\left(\frac{1}{2}\right)^{16/T} \Rightarrow T = 8$ days

13. (*b*) $N = N_0 \times \left(\frac{1}{2}\right)^{11400/5700}$ $(\because T_{1/2} = 5700 \text{ yr and } T = 11400 \text{ yr})$

$$= N_0\left(\frac{1}{2}\right)^2 = 0.25N_0$$

14. (*a*) Mean life $= \frac{\text{Half-life}}{0.6931} = \frac{10}{0.6931} = 14.4$ h

15. (*d*) Mean life, $T = 1/\lambda = 100$ s

Half life $= \frac{0.693}{\lambda} = \frac{0.693 \times 100}{60} = 1.155$ min

A) Taking it together

1. (*a*) When an energetic γ-ray photon falls on a heavy substance, it absorbed by same nucleus of the substance and an electron and positron are emitted. This phenomenon is called pair production.

5. (*b*) β-particle carries one unit of negative charge, an α-particle carries 2 units of positive charge and γ-particle carries no charge, therefore electronic energy levels of the atom changes for α and β-decay but not for γ-decay.

6. (*b*) Stable heavy nuclei have more neutrons than protons. This is because electrostatic force between protons is repulsive which may reduce stability.

7. (*c*) The average life of a radioactive element is 1.44 times, the half life of the element.

8. (*d*) BE $= \Delta mc^2$

15. (*a*) To sustain a chain reaction in atom bomb one of the neutrons emitted in the fission of ${}^{235}U$ must be captured by another ${}^{235}U$ nucleus and cause it to fission.

17. (*d*) $Q = (1.002 + 1.004 - 1.001 - 1.003)\,(931.5)$ MeV
$= 1.863$ MeV

18. (*b*) The elements high on the BE *versus* mass number plot are very tightly bound and hence are stable. And the elements those are lower on this plot, are less tightly bound and hence are unstable.
Since, helium nucleus shown a peak on this plot, so it is very stable.

19. (*a*) ${}_{92}U^{238} \xrightarrow{\alpha} {}_{90}Th^{234} \xrightarrow{\beta} {}_{91}Pa^{234} \xrightarrow{-1\beta^0} {}_{92}U^{234}$

So, $A = 234, B = 90, C = 234, D = 91$ and $E = \beta$

20. (*d*) ${}_7X^{15} + \alpha$-particle $\longrightarrow {}_9Y^{19} - {}_1H^1 \longrightarrow {}_8Z^{18}$

21. (*a*) Three atoms of $B \longrightarrow$ one atom of A

$\therefore \quad Q = E_a - 3E_b$

22. (*c*) ${}^{238}_{92}U \longrightarrow {}^{214}_{82}Pb + 6\,{}^4_2He + 2\,{}^0_{-1}e$

23. (*c*) Radioactivity is a process due to which a radioactive material spontaneously decays. In half-life (t=1yr) of the material on the average half, the number of atoms will decay. Therefore, the containers will in general have different number of atoms of the material but their average will be approx. 5000.

24. (*b*) $100 \xrightarrow{t} 90\% \xrightarrow{t} 81\%$
81% is undecayed. Therefore, 19% is decayed.

25. (*a*) Let the nucleus is ${}_zX^A$. β^+ decay is represented as

$${}_zX^A \rightarrow {}_{z-1}Y^A + {}_{+1}e^0 + n + Q_2$$

$$\therefore \quad Q_2 = [m_n({}_zX^A) - m_n({}_{z-1}y^A) - m_e]\,c^2$$
$$= [m_n({}_zX^A) + zm_e - m_n({}_{z-1}Y^A) - (z-1)\,m_e - 2m_e]\,c^2$$
$$= [m({}_zX^A) - m({}_{Z-1}Y^A) - 2m_e]\,c^2 = (M_x - M_y - 2m_e)\,c^2$$

β^-decay is represented as

$$= {}_zX^A \rightarrow {}_{z+1}A^Y + {}_{-1}e^0 + \bar{\nu} + \alpha_1$$

$$Q_1 = [m_n({}_zX^A) - m_n({}_{z+1}Y^A) - m_e]c^2$$

$$= [m_n({}_zX^A) + zm_e - m_n({}_{z+1}Y^A) - (z+1)me]c^2$$

$$= [m({}_zX^A) - m({}_{z-1}Y^A)]c^2 = (M_x - M_y)c^2$$

26. (*b*) Here, half-life of radioactive substance, $T = 20$ min

We have, $N = N_0\left(\frac{1}{2}\right)^{t/T}$

So, for 20% decay, $\frac{N}{N_0} = \frac{80}{100} = \left(\frac{1}{2}\right)^{t_1/20}$ ...(i)

For 80% decay, $\frac{N}{N_0} = \frac{20}{100} = \left(\frac{1}{2}\right)^{t_2/20}$ ...(ii)

On dividing Eq. (ii) by Eq. (i), we get

$$\left(\frac{1}{2}\right)^{\left(\frac{t_2 - t_1}{20}\right)} = \frac{1}{4}$$

or $$\left(\frac{1}{2}\right)^{\left(\frac{t_2 - t_1}{20}\right)} = \left(\frac{1}{2}\right)^2$$

$\therefore$ The time between 20% and 80% decay $t_2 - t_1 = 40$ min

27. (*d*) After n number of half-life, $N = N_0\left(\frac{1}{2}\right)^n$

$$1 = 64\left(\frac{1}{2}\right)^n \Rightarrow \left(\frac{1}{2}\right)^6 = \left(\frac{1}{2}\right)^n$$

$n = 6$ *i.e.*, 6 half-lives

$\therefore$ Minimum time, $t = 6 \times 2 = 12$ h

29. (*b*) We have, $\frac{1}{64} = \left(\frac{1}{2}\right)^n$

$n = 6$

6 half-lives are equal to 60 s.

$\therefore$ 1 half-life = 10 s

30. (*b*) During fusion, BE of daughter nucleus is always greater than the total energy of the parent nuclei, so energy released

$$= c - (a + b) = c - a - b$$

31. (*d*) Rate, $R = \lambda N$

$\therefore$ $\frac{R}{N} = \lambda$ (Constant)

So, graph between R/N and t be a straight line parallel to the time axis.

32. (*d*) We have, $N = N_0e^{-\lambda t}$ and $R = R_0e^{-\lambda t} = \lambda N_0e^{-\lambda t}$

$\therefore$ $N_{\text{decayed}} = N_0 - N = N_0 - N_0e^{-\lambda t} = N_0 - \frac{R}{\lambda}$

This is equation of straight line with negative slope.

33. (*d*) By using $N = Ne^{-\lambda t}$

It shows that N decreases exponentially with time.

34. (*d*) We have, $\frac{1}{e} = \frac{N_0e^{-10\lambda t}}{N_0e^{-\lambda t}} = e^{-9\lambda t}$ or $t = \frac{1}{9\lambda}$ $\left(\text{given} \frac{N_1}{N_2} = \frac{1}{e}\right)$

35. (*c*) We have, $N_Ae^{-\lambda_A t} = N_Be^{-\lambda_B t}$

$\therefore$ $e^{(\lambda_B - \lambda_A)t} = \frac{N_B}{N_A}$

$\therefore$ Time, $t = \frac{1}{\lambda_B - \lambda_A}\ln\left(\frac{N_B}{N_A}\right)$

36. (*d*) Binding energy per nucleon

$$= \frac{[(m_pZ + m_n(A - Z)] - M] \times 931}{\text{Mass number } A}$$

$$= \left(\frac{6 \times 1.0078 + 6 \times 1.0087 - 12}{12}\right) \times 931.4$$

$= 7.68$ MeV

37. (*c*) If in the rock there is no Y element, then the time taken by element X to reduce to $\frac{1}{8}$th the initial value will be equal to

$$\frac{1}{8} = \left(\frac{1}{2}\right)^n \text{ or } n = 3$$

Therefore, from the beginning three half-life time is spent. Hence, the age of the rock is

$$= 3 \times 1.37 \times 10^9 = 4.11 \times 10^9 \text{ yr}$$

38. (*d*) Energy released in the fission of one nucleus = 200 MeV

$$= 200 \times 10^6 \times 1.6 \times 10^{-19} = 3.2 \times 10^{-11} \text{ J}$$

$$P = 16 \text{ kW} = 16 \times 10^3 \text{ Watt}$$

Now, number of nuclei required per second

$$n = \frac{P}{E} = \frac{16 \times 10^3}{3.2 \times 10^{11}} = 5 \times 10^{14}$$

39. (*c*) In β^- decay a neutron in the nucleus is transformed into a proton, an electron and an anti-neutrino

$$n \rightarrow P + \bar{e} + \bar{\nu}$$

So, x and y, stand for electron and anti-neutrino.

40. (*c*) $Z_{\text{resulting nucleus}} = 92 - 8 \times 2 + 4 \times 1 - 2 \times 1 = 78$

41. (*c*) Energy is released in a process when total binding energy (BE) of the nucleus is increased or we can say when total BE of products is more than the reactants. By calculation we can see that only in case of option (c), this happens.

Given, $W \rightarrow 2Y$

BE of reactants $= 120 \times 7.5 = 900$ MeV

and BE of products $= 90 \times 8 + 60 \times 8.5 = 1230$ MeV

i.e., BE of products > BE of reactants.

42. (*a*) Tritium (${}_1H^3$) contains 1 proton and 2 neutrons. A neutron decays as, $n \rightarrow p + e^- + \bar{\nu}$, the nucleus may have 2 protons and one neutron *i.e.*, tritium will transform into a He^3 (2 protons and 1 neutron).

Triton energy is less than that of a He^3 nucleus *i.e.*, transformation is not allowed energetically.

43. (*b*) Given, $F = \frac{GMm}{r^2}$; where

M = effective mass of hydrogen atom

= mass of electron + mass of proton $- (B^2/c^2)$

where, B is BE of hydrogen atom = 13.6 eV

44. (*b*) We have, $\frac{N}{N_0} = e^{-\lambda t} = e^{-\left(\frac{\ln 2}{t_{1/2}}\right)\left(\frac{t_{1/2}}{2}\right)} = e^{-\left(\frac{\ln 2}{2}\right)} = \frac{1}{\sqrt{2}}$

$\left(\because \lambda = \frac{\ln 2}{t_{1/2}}; t = nt_{1/2}, \text{ here } n = \frac{1}{2}\right)$

45. (*a*) $\frac{N}{N_0} = e^{-\lambda t} = e^{-\lambda\left(\frac{n}{\lambda}\right)} = \frac{1}{e^n}$ $\left(\because \text{ mean life, } t = \frac{1}{\lambda}\right)$

46. (*a*) Number of half-lives in two days for substances 1 and 2 respectively are $n_1 = \frac{2 \times 24}{12} = 4$ and $n_2 = \frac{2 \times 24}{1.6} = 3$

By using $N = N_0\left(\frac{1}{2}\right)^n$

$$\frac{N_1}{N_2} = \frac{(N_0)_1}{(N_0)_2} \times \frac{\left(\frac{1}{2}\right)^{n_1}}{\left(\frac{1}{2}\right)^{n_2}}$$

Ratio, $\frac{N_1}{N_2} = \frac{2}{1} \times \frac{\left(\frac{1}{2}\right)^4}{\left(\frac{1}{2}\right)^3} = \frac{1}{1}$

47. (*c*) Initially whole isotope was X $[\because X : Y = 1 : 15]$

i.e., $N_0 = 1 + 15 = 16$ and after decaying into Y

$\Rightarrow$ We have, $N = N_0 e^{-\lambda t} \Rightarrow 1 = 16 e^{-\frac{0.693}{1.5 \times 10^9} \times t}$

$\Rightarrow$ $\ln\left(\frac{1}{16}\right) = e^{-\frac{\ln 2}{t/2}}$

$\Rightarrow$ The age of the rock, $t = 6.00 \times 10^9$ yr

So, to convert 16 to 1, a four half-lives is required.

48. (*b*) As, $N = N_0\left(\frac{1}{2}\right)^{t/T}$

Number of atoms at $t = 2$ h,

$$N_1 = 8 \times 10^{10}\left(\frac{1}{2}\right)^{\frac{2}{1}} = 2 \times 10^{10}$$

Number of atoms at $t = 4$ h,

$$N_2 = 8 \times 10^{10}\left(\frac{1}{2}\right)^{\frac{4}{1}} = \frac{1}{2} \times 10^{10}$$

$\therefore$ Number of atoms decayed in given duration

$$= \left(2 - \frac{1}{2}\right) \times 10^{10} = 1.5 \times 10^{10}$$

49. (*b*) Decay constant, $\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{20} = 0.03465$

Now, time of decay, $t = \frac{2.303}{\lambda}\log\frac{N_0}{N}$

$\Rightarrow$ $t_1 = \frac{2.303}{0.03465}\log\frac{100}{67} = 11.6$ min

and $t_2 = \frac{2.303}{0.03465}\log\frac{100}{33} = 32$ min

So, time difference between points of time

$$= t_1 - t_2 = 32 - 11.6 = 20.4 \text{ min} \approx 20 \text{ min}$$

50. (*a*) We have, $N = N_0\left(\frac{1}{2}\right)^{t/T}$

$\therefore$ $N_A = 10\left(\frac{1}{2}\right)^{t/1} \Rightarrow N_B = 1\left(\frac{1}{2}\right)^{t/2}$

Given, $N_A = N_B$

So, $10\left(\frac{1}{2}\right)^t = \left(\frac{1}{2}\right)^{t/2}$

So, $10 = \left(\frac{1}{2}\right)^{-t/2}$ or $10 = 2^{t/2}$

$\therefore$ $\log_{10} 10 = \frac{t}{2}\log_{10} 2$

$1 = \frac{t}{2} \times 0.3010$

$\therefore$ Time, $t = 6.62$ yr

51. (*b*) Number of atoms remains undecayed, $N = N_0 e^{-\lambda t}$

As, $\lambda = \frac{1}{t}$ (given)

$\Rightarrow$ $N = N_0 e^{-\lambda(t/\lambda)} = \frac{N_0}{e} = 36.8\%$ of N_0

53. (*c*) $\lambda = (\lambda_1 + \lambda_2) = \frac{1}{1620} + \frac{1}{520} = 2.54 \times 10^{-3}$ yr^{-1}

$\therefore$ $t_{1/2} = \frac{\ln(2)}{\lambda} = 272.8$ yr

(1/4) th of the material remains intact after 2 half-lives.

54. (*a*) Maximum number of nuclei will be present when rate of decay = rate of formation

$$\lambda N = \alpha$$

Maximum number of nuclei, $N = \alpha/\lambda$

55. (*d*) The value of x

$$x = \frac{N}{N_0} \times 100 = \left\{e^{-\lambda\left(\frac{3}{\lambda}\right)}\right\} \times 100 = 5\%$$

56. (*c*) $\frac{N_0}{N} = \frac{1}{4} = \left(\frac{1}{2}\right)^{t/5700}$

$\therefore$ $\left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^{t/5700}$

$\therefore$ $\frac{t}{5700} = 2$

$\therefore$ The age of the wood, $t = 11400$ yr

57. (*c*) We have, $\lambda_B N_B = \lambda_A N_A$

$\therefore$ $N_B = \frac{\lambda_A}{\lambda_B} \cdot N_A$

The given time is equivalent to two half-lives of A. Hence,

$N_A = \frac{N_0}{4}$ $\left(\because N = N_0\left(\frac{1}{2}\right)^2\right)$

$\therefore$ $N_B = \frac{\lambda_A}{\lambda_B}\left(\frac{N_0}{4}\right)$

58. (*b*) Activity, $R = R_0 e^{-\lambda t}$

$\because \quad \frac{R_p}{R_q} = \frac{R_0 e^{-\lambda t_1}}{R_0 e^{-\lambda t_2}}$

$\therefore \quad e^{\lambda(t_1 - t_2)} = \frac{R_q}{R_p}$

$\therefore \quad \frac{t_1 - t_2}{T} = \ln \frac{R_q}{R_p} \qquad \left(\because \lambda = \frac{1}{T}\right)$

$\therefore$ The difference image, $(t_1 - t_2) = T \ln\left(\frac{R_q}{R_p}\right)$

59. (*d*) After remaining one electron from helium atom it will become hydrogen like atom.

$\therefore \quad E = 24.6 + (13.6)(2)^2 = 79$ eV

60. (*c*) Activity, $R = R_0 e^{-\lambda t}$; $\therefore R_1 = R_0 e^{-t/T}$

$\Rightarrow$ Again, $R_2 = R_0 e^{-t_2/T}$

$\therefore R_2 = R_1 e^{t_1/T} \cdot e^{-t_2/T} \Rightarrow R_2 = R_1 e^{\frac{(t_1 - t_2)}{T}}$

61. (*a*) By conservation of momentum, $m_1 v_1 = m_2 v_2$

$\Rightarrow \quad \frac{v_1}{v_2} = \frac{8}{1} = \frac{m_2}{m_1} \qquad$ ...(i)

Also, from $r \propto A^{1/3}$,

$\therefore \quad \frac{r_1}{r_2} = \left(\frac{A_1}{A_2}\right)^{1/3} = \left(\frac{1}{8}\right)^{1/3} = \frac{1}{2}$

62. (*c*) $\lambda_\alpha = \frac{1}{1620}$ per year and $\lambda_\beta = \frac{1}{405}$ per year and it is given that the fraction of the remained activity $\frac{R_1}{R_2} = \frac{1}{4}$

Total decay constant,

$$\lambda = \lambda_\alpha + \lambda_\beta = \frac{1}{1620} + \frac{1}{405} = \frac{1}{324} \text{ per yr}$$

We know that, $R = R_0 e^{-\lambda t} \Rightarrow t = \frac{1}{\lambda} \log \frac{R_0}{R}$

$\Rightarrow \quad t = \frac{1}{\lambda} \log_e 4 = \frac{2}{\lambda} \log_e 2 = 324 \times 2 \times 0.693 = 449$ yr

63. (*d*) Initially $P \to 4N_0$; $Q \to N_0$

Half-life, $T_P = 1$ min; $T_Q = 2$ min

Let after time t, number of nuclei of P and Q are equal

i.e., $\quad \frac{4N_0}{2^{t/1}} = \frac{N_0}{2^{t/2}} \qquad \left(\because N = \frac{N_0}{2^{t/T_{1/2}}}\right)$

or $\quad \frac{4}{2^{t/2}} = 1$ or $t = 4$ min

So, $\quad$ at $t = 4$ min

$N_P = \frac{(4N_0)}{2^{4/1}} = \frac{N_0}{4}$

and at $\quad t = 4$ min, $N_Q = \frac{N_0}{2^{4/2}} = \frac{N_0}{4}$

So, number of nuclei of R in the sample

$$= \left(4N_0 - \frac{N_0}{4}\right) + \left(N_0 - \frac{N_0}{4}\right) = \frac{9N_0}{2}$$

64. (*c*) To reduce one-fourth it takes time, $t = 2(T_{1/2}) = 2 \times 40 = 80$ yr

So, decay constant, $\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{40} = 0.0173$ yr

65. (*a*) According to the question, (7/8) part decays *i.e.*, remaining part is (1/8).

We have, $N = N_0 \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}} \Rightarrow \frac{1}{8} = \left(\frac{1}{2}\right)^{\frac{15}{T_{1/2}}}$

$\Rightarrow$ Half-life $T_{1/2} = 5$ min

66. (*b*) Momentum remains constant,

$$K = \frac{p^2}{2m} \text{ or } K \propto \frac{1}{m} \Rightarrow \frac{K_n}{K_\alpha} = \frac{m_\alpha}{m_n} = \frac{4}{206} = \frac{2}{103}$$

$\therefore$ Kinetic energy of the residual nucleus

$$K_n = \left(\frac{2}{103}\right) K_\alpha = \left(\frac{2}{103}\right) E$$

67. (*c*) $R_i = (15 \times 200) = 3000$ decay/min

$R = 375$ decay/min $= \frac{R_i}{8}$

$\therefore$ Time, $t = 3$ (half-lives) $= 3 \times 5730 = 17190$ yr

68. (*c*) As, $n = \frac{m}{A} N_A \Rightarrow E = \frac{m}{A} N_A \times 200 \times 1.6 \times 10^{-13}$ J

Power $= \frac{E}{t} = \frac{m N_A \times 3.2 \times 10^{-11}}{235 \times 24 \times 3600} = 10^6$

$$\text{Mass} = \frac{1 \times 10^6 \times 24 \times 3600 \times 235}{200 \times 1.6 \times 10^{-13} \times 6.02 \times 10^{23}} \approx 1 \text{ g}$$

69. (*a*) We have, $N = N_0 e^{-\lambda t} \Rightarrow \frac{N_0}{e} = N_0 e^{-\lambda(5)} \Rightarrow \lambda = \frac{1}{5}$

Now, $\quad \frac{N_0}{2} = N_0 e^{-\lambda(t)} \Rightarrow t = \frac{1}{\lambda} \ln 2 = 5 \ln 2$

70. (*a*) Kinetic energy of the molecules of a gas at a temperature T is $\frac{3}{2} kT$.

$\therefore$ To initiate the reaction $\frac{3}{2} kT = 7.7 \times 10^{-14}$ J

$\Rightarrow \quad \frac{3}{2} \times 1.38 \times 10^{-23} T = 7.7 \times 10^{-14}$

$\Rightarrow$ Temperature, $T = 3.7 \times 10^9$ K

71. (*b*) By the formula $N = N_0 e^{-\lambda t}$

Given, $\quad \frac{N}{N_0} = \frac{1}{20}$ and $\lambda = \frac{0.6931}{3.8} \Rightarrow 20 = e^{\frac{0.6931 \times t}{3.8}}$

$(\because t_{1/2} = 3.8$ days)

On taking log both sides, we get

or $\quad \log 20 = \frac{0.6931 \times t}{3.8} \log_{10} e$

or $\quad 1.3010 = \frac{0.6931 \times t \times 0.4343}{3.8} \Rightarrow t = 16.5$ days

72. (*d*) We have, $n = \frac{24}{24 \times 138.6} = \frac{1}{138.6}$

Now, $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^{1/138.6}$

$$\Rightarrow \quad N = 1000000\left(\frac{1}{2}\right)^{1/138.6} = 995011$$

So, number of disintegration

$$= 1000000 - 995011 = 4989 \approx 5000$$

73. (*b*) Number of half-lives, $n = \frac{19}{3.8} = 5$

Now, $$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$$

$$\Rightarrow \quad \frac{N}{10.38} = \left(\frac{1}{2}\right)^5 \Rightarrow N = 10.38 \times \left(\frac{1}{2}\right)^5 = 0.32 \text{ g}$$

74. (*d*) Number of nuclei remained after time t can be written as

$$N = N_0 e^{-\lambda t}$$

where, N_0 is initial number of nuclei of both the substances

$$N_1 = N_0 e^{-5\lambda t} \quad \text{...(i)}$$

and $$N_2 = N_0 e^{-\lambda t} \quad \text{...(ii)}$$

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{N_1}{N_2} = e^{(-5\lambda + \lambda)t} = e^{-4\lambda t} = \frac{1}{e^{4\lambda t}}$$

But, we have given

$$\frac{N_1}{N_2} = \left(\frac{1}{e}\right)^2 = \frac{1}{e^2}$$

Comparing the powers, we get

$$2 = 4\lambda t \Rightarrow t = \frac{2}{4\lambda} = \frac{1}{2\lambda}$$

75. (*c*) Net process is, $3\,{}_1H^2 \rightarrow {}_2He^4 + p + n$

Mass defect, $\Delta m = (3 \times 2.014) - 1.007 - 1.008 - 4.001$

$$= 0.026 \text{ amu}$$

Energy released by the fusion of three deuterons

$$= \Delta m \times 931.48 = 0.026 \times 931.48 \text{ MeV}$$
$$= 0.026 \times 931.48 \times 1.6 \times 10^{-13} \text{ J} = 3.87 \times 10^{-12} \text{ J}$$

$$\therefore \text{ Total energy released} = \frac{10^{40}}{3} \times 3.87 \times 10^{-12} \text{ J} = 1.29 \times 10^{28} \text{ J}$$

$$\text{Total time} = \frac{1.29 \times 10^{28}}{10^{16}} = 1.29 \times 10^{12} \text{ s} \left(\because \text{ power} = \frac{\text{energy}}{\text{time}}\right)$$

76. (*d*) Energy radiated $= 14 \text{ kW/m}^2$

$$= 1.4 \text{ kJ/sm}^2 = \frac{1.4 \text{ kJ}}{\frac{1}{86400} \text{ day m}^2} = \frac{1.4 \times 86400 \text{ kJ}}{\text{day m}^2}$$

Total energy radiated/day

$$E = \frac{4\pi \times (1.5 \times 10^{11})^2 \times 1.4 \times 86400}{1} \frac{\text{kJ}}{\text{day}} = E$$

$\therefore$ We have, $E = mc^2$

$$\Rightarrow \quad m = \frac{E}{c^2} = \frac{4\pi (1.5 \times 10^{11})^2 \times 1.4 \times 86400}{(3 \times 10^8)^2} \times 10^3$$

$$= 3.8 \times 10^{14} \text{ kg}$$

77. (*b*) We have $N = N_0 e^{-\lambda t}$

$$\therefore \quad 0.9N_0 = N_0 e^{-\lambda \times 5} \Rightarrow 5\lambda = \log_e \frac{1}{0.9} \quad \text{...(i)}$$

and $$xN_0 = N_0 e^{-\lambda \times 20} \Rightarrow 20\lambda = \log_e\left(\frac{1}{x}\right) \quad \text{...(ii)}$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{1}{4} = \frac{\log_e(1/0.9)}{\log_e(1/x)} = \frac{\log_{10}(1/0.9)}{\log_{10}(1/x)} = \frac{\log_{10} 0.9}{\log_{10} x}$$

$$\Rightarrow \quad \log_{10} x = 4\log_{10} 0.9 \quad \Rightarrow \quad x = 0.658 = 65.8\% \simeq 65\%$$

78. (*a*) R_0 = Initial activity = 1 microcurie = 3.7×10^4 dps

r = Activity in 1 cm^3 of blood at $t = 5$ h

$$= \frac{296}{60} \text{ dps} = 4.93 \text{ dps}$$

R = Activity of whole blood at time $t = 5$ h

Total volume should be $V = \frac{R}{r} = \frac{R_0 e^{-\lambda t}}{r}$

$$= \frac{3.7 \times 10^4 \times 0.7927}{4.93} = 5.94 \times 10^3 \text{cm}^3 = 5.94 \text{ L}$$

B) Medical entrance special format questions

Assertion and reason

1. (*c*) The β^- particle (*e*) is emitted from the nucleus in same radioactive transformation. So electrons do not exist in the nucleus but they result in same nuclear transformation.

6. (*d*) Average life is more. Hence, more nuclei decay in one average life.

7. (*a*) $t_{1/2} = \frac{0.693}{\lambda} \Rightarrow t_{1/2} \propto 1/\lambda \Rightarrow \lambda \propto \frac{1}{t_{1/2}}$

8. (*d*) Mass defect per nucleon is called the packing fraction,

i.e., $$P = \frac{\Delta m}{A} = \frac{m \sim A}{A}$$

Positive value of P does not mean that Δm is large. Since binding energy $E = (\Delta m)c^2$

9. (*d*) Rate of a nuclear process cannot be altered by altering pressure or temperature.

Match the columns

1. By the emission of an α-particle, atomic number decreases by 2 and mass number by 4. But by the emission of a β-particle, atomic number increases by 1 while mass number remains unchanged.

3. (i) Stability of nucleus is decided by neutron-proton ratio, packing fraction and binding energy per nucleon.

(ii) For radioactive substance binding energy per nucleon is minimum. So, they are unstable.

(iii) For bound orbit, total energy is always negative.

(iv) Stopping potential is the particular negative potential when no electron reaches the plate.

4. In α-decay mass number decreases by 4 and atomic number decreases by 2.

In β^+ decay mass number remains unchanged while atomic number decreases by 1.

In fission, parent nucleus break into almost two equal fragments.

In proton emission both mass number and atomic number decreases by 1.

C) Medical entrances' gallery

1. (*d*) Half-life of a radioactive substance is,

$$T_{1/2} \propto \log\left(\frac{N_0}{N}\right)$$

Given, $N_1 = 0.6N_0$ ($\because$ 40% decay)

$N_2 = 0.15N_0$ ($\because$ 85% decay)

Putting these in the formula,

$$\frac{N_2}{N_1} = \frac{0.15N_0}{0.6N_0} = \frac{1}{4} = \left(\frac{1}{2}\right)^2$$

So, two half-life periods has passed.

Thus, time taken $= 2 \times t_{1/2} = 2 \times 30 = 60$ min

2. (*d*) The nuclear should be ${}_{88}Ra^{226} \longrightarrow {}_{86}Rn^{222} + \alpha$

Energy corresponding mass defect,

$$Q = \begin{bmatrix} \text{(mass number of Ra)} \\ -\text{(mass number of Rn)} \\ -m\,(4He) \end{bmatrix} U \times 931.5 \text{ MeV} = 4.871 \text{ MeV}$$

3. (*b*) Radius of the nucleus is given by

$$R = R_0 A^{1/3} \Rightarrow R \propto A^{1/3}$$

$$\therefore \quad \frac{R_{Al}}{R_{Te}} = \left(\frac{A_{Al}}{A_{Te}}\right)^{1/3} = \left(\frac{27}{125}\right)^{1/3} = \frac{3}{5}$$

$$R_{Te} = \frac{5}{3} R_{Al}$$

4. (*a*) Radius of nuclei having mass number A is determined as

$$R = R_0 A^{1/3} \quad \text{(where, } R_0 = \text{constant)}$$

where, $R_0 = 1.2 \times 10^{-15}$m

This implies, $\frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3} = \left(\frac{216}{64}\right)^{1/3} = \frac{6}{4}$

$\Rightarrow \quad R_1 : R_2 = 3 : 2$

5. (*b*) The fission per second $= 4 \times 10^{20}$

So, the energy released per second

$$= 4 \times 10^{20} \times 250 \text{ MeV}$$
$$= 4 \times 10^{20} \times 250 \times 10^6 \times 1.6 \times 10^{-19} \text{ J}$$

Therefore, energy released in 10 h $= 36 \times 10^3$ s is,

$$E = 36 \times 10^3 \times 4 \times 10^{20} \times 250 \times 10^6 \times 1.6 \times 10^{-19} \text{ J}$$
$$= 36 \times 4 \times 250 \times 1.6 \times 10^{10} \text{ J} = 57600 \times 10^{10}$$
$$= 576 \times 10^{12} \text{ J}$$

6. (*d*) Nuclear fission is not found in atom bomb because it works on the nuclear fission.

7. (*d*) Nuclear mass density is independent of mass number, therefore ratio of nuclear mass densities of Au^{197} and Ag^{107} is 1 : 1.

8. (*d*) The disintegration can be shown as below.

$${}_{72}X^{180} \xrightarrow[-2\alpha]{-2\bar{\beta}} X_4$$

Loss in atomic number $= 72 + 2 - 4 = 70$

Loss in mass number $= 180 - 8 = 172$

So, final nucleus is ${}_{70}X_4^{172}$.

9. (*a*) Let there is n number of fission per second produces a power of 6.4 W, then

$$n \times 200 \times 10^6 \times 16 \times 10^{-19} \text{ Js}^{-1} = 6.4 \text{ Js}^{-1}$$

$$\therefore \quad n = \frac{6.4}{200 \times 10^{-13} \times 1.6} = \frac{4}{2 \times 10^{-11}} = 2 \times 10^{11}$$

10. (*b*) The number of α-particles, n_1

$$= \frac{\text{change in mass number}}{4} = \frac{238 - 206}{4} \Rightarrow n_1 = \frac{32}{4} = 8$$

Now, number of β-particles, $n_2 = 82 - (92 - 2n_1)$

$$= 82 - (92 - 2 \times 8) = 82 - 76 = 6$$

11. (*c*) Amount of element left undecayed after 10 days

$$= 1 - \frac{20}{100} = \frac{8}{10}$$

Now, $N = \frac{8}{10} N_0$ and $t = 10$ days

We have, $N = N_0 e^{-\lambda t}$

$$\frac{8}{10} N_0 = N_0 e^{-\lambda t} \Rightarrow e^{\lambda t} = \frac{10}{8} \Rightarrow \lambda = \frac{\log_e (10/8)}{10}$$

$\therefore \quad \lambda = 0.022$

If $T_{1/2}$ is half-life period

$$T_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{0.022} = 31.5 \text{ days}$$

Again, $N = N_0 \left(\frac{1}{2}\right)^n = N_0 \left(\frac{1}{2}\right)^{\frac{31.5}{30}}$

$\Rightarrow \quad$ % of $N / N_0 = 51.2\%$

12. (*b*) Given, $\frac{R_1}{R_2} = \frac{1}{2}$

According to conservation of linear momentum, we have

$$m v_1 = m_2 v_2$$

$$\Rightarrow \quad \frac{v_1}{v_2} = \frac{m_2}{m_1} = \left(\frac{R_2}{R_1}\right)^3 \quad (\because m \propto A \propto R^3)$$

i.e., $\frac{v_1}{v_2} = \left(\frac{2}{1}\right)^3 = \frac{8}{1}$

13. (*a*) Energy released in one fission of ${}^{235}_{92}U$ nucleus = 200 MeV

Mass of uranium = 1g

We know that, 235 g of ${}^{235}U$ has 6.023×10^{23} atoms or nuclei.

$\therefore$ Energy released in fission of 1 g of U^{235},

$$E = \frac{6.023 \times 10^{23} \times 1 \times 200}{235} = 5.1 \times 10^{23} \text{ MeV}$$
$$= 5.1 \times 10^{23} \times 1.6 \times 10^{-13} \text{ J}$$
$$= \frac{5.1 \times 10^{23} \times 1.6 \times 10^{-13}}{3.6 \times 10^6} \text{ kWh} = 2.26 \times 10^4 \text{ kWh}$$

14. (*a*) We have, $\frac{dN}{dt} = \lambda N$

and $\lambda = \frac{0.6931}{T_{1/2}} = \frac{0.6931}{1620 \times 365 \times 24 \times 60 \times 60}$

$$N = \frac{6.023 \times 10^{23}}{226}$$

$$\therefore \frac{dN}{dt} = \frac{0.6931 \times 6.023 \times 10^{23}}{1620 \times 365 \times 24 \times 60 \times 60 \times 226} = 3.61 \times 10^{10}$$

15. (*b*) The relation between decay constant λ and half-life T can be given as

$$T = \frac{0.693}{\lambda}$$

$$\therefore \quad T = \frac{\log_e 2}{\lambda}$$

16. (*d*) The binding energy for ${}_1H^1$ is around zero and also not given in the question, so we can ignore it

$$Q = 2\,(4 \times 7.06) - 7 \times (5.60) = (8 \times 7.06) - (7 \times 5.60)$$
$$= 56.48 - 39.2 = 17.28 \text{ MeV} \simeq 17.3 \text{ MeV}$$

17. (*a*) The binding energy of a nucleus is the energy required to take its nucleus away from one another. It is generally expressed as binding energy per nucleon. It is a measure of the stability of the nucleus. Higher the binding energy per nucleon, more stable is the nucleus.

18. (*e*) Simply, we can analyse from the equation,

$${}_{92}U^{238} \xrightarrow{-\alpha} {}_{90}U^{234} \xrightarrow{-\beta} {}_{91}Y^{234}$$

19. (*c*) According to the question, ratio of $X : Y$ is given $= 1 : 7$

$$\Rightarrow \quad \frac{m_X}{m_Y} = \frac{1}{7} \Rightarrow 7m_X = m_Y$$

Let the initial total mass is m.

$$\Rightarrow \quad m_X + m_Y = m \Rightarrow \frac{m_Y}{7} + m_Y = m$$

$$\Rightarrow \quad \frac{8m_Y}{7} = m \Rightarrow m_Y = \frac{7}{8}m$$

Only $\frac{1}{8}$ part remains left undisintegrated.

Number of half-lives involve are three as

$$1 \xrightarrow{T_{1/2}} \frac{1}{2} \xrightarrow{T_{1/2}} \frac{1}{4} \xrightarrow{T_{1/2}} \frac{1}{8}$$

So, time taken to become $\frac{1}{8}$ unstable part

$$= 3 \times T_{1/2} = 3 \times 1.4 \times 10^9 = 4.20 \times 10^9 \text{ yr}$$

20. (*a*) As, activity droped from 5000 dps to 2500 dps in 150 days means $T_{1/2} = 150$ days

$\Rightarrow$ 300 days $= 2T_{1/2}$

So, initial activity $= 2 \times 5000 \times 2 = 20000$ dps

22. (*c*) Suppose a given nucleus is ${}_ZX^A$.

In α-decay, the mass number of the product nucleus is four less than that of decaying nucleus, while the atomic number decreases by two.

$${}_ZX^A \xrightarrow{\alpha\text{-decay}} {}_{Z-2}X^{A-4} + {}_2He^4\ (\alpha)$$

In β-decay, the mass number of product nucleus remains same but atomic number increases or decreases by one.

$${}^A_ZX \xrightarrow{\beta^- \text{ decay}} {}^A_{Z+1}Y + {}^0_{-1}e + \nu$$

or

$${}^A_ZX \xrightarrow{\beta^+ \text{ decay}} {}^A_{Z-1}Y + {}^0_{+1}e + \nu$$

Gamma decay is the phenomenon of emission of gamma rays photon from a radioactive nucleus.

$${}_ZX^A \longrightarrow {}_ZX^A + \gamma$$

Hence, the given reaction will have sequence as follows

$${}_ZX^A \xrightarrow{\beta^- \text{ emission}} {}_{Z+1}Y \xrightarrow{\alpha\text{-emission}}$$

$${}_{Z-1}K^{A-Y} \xrightarrow{\gamma\text{-emission}} {}_{Z-1}K^{A-\gamma}$$

23. (*b*) The control rods used in a nuclear reactor can be made up of cadmium because control rods absorbs fast moving neutrons.

24. (*d*) The fusion reaction in the sun is a multi-step process in which the hydrogen is burned into helium. Because in fusion, lighter nuclei combine to form heavier nucleus.

25. (*b*) In particle, the nuclear fusion is a very difficult process. Since, the nuclei to be fused are positively charged, they would repel one another strongly. Hence, they must be brought very close together not only by high pressure but also with high kinetic energy ($\simeq 0.1$ MeV). For this, a temperature of the other of 10^8 K is required. Such high temperature are available in the sun and stars. On the earth, they may be produced by exploding a nuclear fission bomb.

26. (*b*) Ratio of X and $Y = 1 : 7$

$$\frac{m_X}{m_Y} = \frac{1}{7} \qquad m_Y = 7m_X$$

Let the initial total mass $= m$

$$\Rightarrow \quad m_X + m_Y = m$$

$$\Rightarrow \quad \frac{m_Y}{7} + m_Y = m$$

$$\frac{8m_Y}{7} = m \Rightarrow m_Y = \frac{7}{8}m$$

Only $\left(\frac{1}{8}\right)$th part remains undisintegrated, number of half-lives involve are three as

$$1 \xrightarrow{T_{1/2}} \frac{1}{2} \xrightarrow{T_{1/2}} \frac{1}{4} \xrightarrow{T_{1/2}} \frac{1}{8}$$

So, total time taken to become $\frac{1}{8}$ unstable part

$$= 3T_{1/2} = 3 \times 20 \text{ yr} = 60 \text{ yr}$$

27. (*c*) $4H^1_1 \longrightarrow 2He^4 + 2\beta^+ + 2\gamma$

Here, $\Delta m = 0.02866$u

$\therefore$ Energy liberated $= \Delta m \times 931$MeV.

$$= 0.02866 \times 931 = 26.7 \text{ MeV}$$

$\therefore$ Energy liberated per fusion (*i.e.*, $n = 4$) $= \frac{26.7}{4}$ MeV

$$= 6.675 \text{ MeV}$$

28. (*a*) Power, $P = \frac{nE}{t}$

Given, $n = 2 \times 10^{18}$ fission per second

$E = 185$ MeV

Here, $P = \frac{2\times10^{18}\times185\times10^{6}\text{ eV}}{1}$

$\Rightarrow \quad = 2\times10^{18}\times185\times10^{6}\times16\times10^{-19} = 59.2\text{ MW}$

29. (*d*) The purpose of using heavy water in nuclear reactor is to decrease the energy of fast neutrons to thermal energy.

30. (*c*) Nucleus of mass number A has a radius, $R = R_0A^{1/3}$

where, $R_0 = 1.2\times10^{-15}\text{m}$

$\therefore \quad \frac{R_8}{R_{64}} = \left[\frac{8}{64}\right]^{1/3} = \left[\frac{1}{8}\right]^{1/3}$

$\therefore$ Ratio of volume $= \frac{1}{8} = 0.125$

31. (*b*) Let $R_1 = N_1\lambda$ and $R_2 = N_2\lambda$

Mean life, $T = \frac{1}{\lambda}$

$$R_1 - R_2 = (N_1 - N_2)\lambda = (N_1 - N_2)\frac{1}{T}$$

So, number of atoms disintegrated in $(T_2 - T_1)$ is

$$= N_1 - N_2 = (R_1 - R_2)T$$

32. (*d*) Since, $N = 13 - 7 = 6$, $C = 12 - 6 = 6$

33. (*c*) Phenomenon of radioactivity is a nuclear process which does not depend on external forces.

34. (*a*) From $E = (\Delta m)c^2$

$$\Delta m = \frac{E}{c^2} = \frac{9\times10^{13}}{(3\times10^8)^2} = 10^{-3}\text{ kg}$$

$\therefore \quad \frac{\Delta m}{m}\times100 = \frac{10^{-3}}{1}\times100 = 0.1\ \%$

35. (*d*) We know that nuclear forces are charge independent and the chief source of stellar energy is fussion.

36. (*b*) According to the question,

$${}_ZX^A \longrightarrow {}_{Z-2}Y^{A-4} + 2\,\text{He}^4$$

Let the recoil speed of daughter nucleus is v'. By the law of conservation of momentum, we have

Initial momentum = Final momentum

$$0 = (A-4)v' + 4v$$

or $\quad (A-4)v' = -4v$ or $v' = -\frac{4v}{(A-4)}$

37. (*c*) Given, in 1st two seconds 100 β particles and in next two seconds 50 β particles are emitted.

$\therefore \quad T_{1/2} = 2\text{ s}$

Hence, mean life, $T = \frac{t_{1/2}}{0.693} = \frac{2}{0.693}$

39. (*d*) The given situation can be shown as

$${}_ZX^A \xrightarrow{9\alpha} {}_{Z-18}X^{A-36} \xrightarrow{5\beta} {}_{Z-13}X^{A-36}$$

$\therefore$ Number of protons, $P = (Z-13)$ and number of neutrons,

$$N = (A-36) - (Z-13) = (A-Z-23)$$

$\therefore \quad \frac{P}{N} = \frac{(Z-13)}{(A-Z-23)}$

40. (*b*) We know that, $N = N_0e^{\frac{-t}{T_{1/2}}}$

$$\frac{1}{4} = e^{-t/10} \Rightarrow \left(\frac{1}{2}\right)^2 \Rightarrow t = 20\text{ yr}$$

41. (*d*) Given, $\quad N_A = N_B$

$$N_0^A\left(\frac{1}{2}\right)^{t/20} = N_0^B\left(\frac{1}{2}\right)^{t/10}$$

$$40\left(\frac{1}{2}\right)^{t/20} = 160\left(\frac{1}{2}\right)^{t/10} \Rightarrow t = 40\text{ s}$$

42. (*b*) $t_{1/2}$ is given as the half-life of the substance *i.e.*, substance decays half, so in $t_{3/4}$ time in which substance decays $\frac{3}{4}$th.

43. (*d*) We know that, $n = \frac{t}{T}$

where, T = half-life

Given, $\quad t = 3000\text{ yr}$

$T = 600\text{ yr}$

$n = \frac{3000}{600} = 5$

Then, $\quad \frac{N}{N_0} = \left(\frac{1}{2}\right)^n$

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^5 = \frac{1}{32}$$

44. (*a*) The total mass of the initial particles,

$$m_i = 1.007825 + 7.016004 = 8.023829\text{ u}$$

and the total mass of final particles,

$$m_f = 2\times4.002603 = 8.005206\text{ u}$$

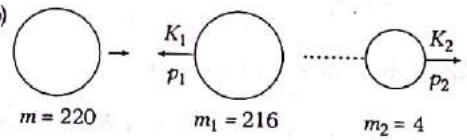
Difference between initial and final mass of particles,

$$\Delta m = m_i - m_f = 8.023829 - 8.005206 = 0.018623\text{u}$$

The Q-value is given by

$$Q = (\Delta m)c^2 = 0.018623\times931.5 = 17.35\text{ MeV}$$

45. (*b*)



Q-value of the reaction is 5.5 MeV

i.e., $\quad K_1 + K_2 = 5.5\text{ MeV}$ ...(i)

By conservation of linear momentum,

$$p_1 = p_2 \Rightarrow \sqrt{2(216)K_1} = \sqrt{2(4)K_2}$$

$$K_2 = 54K_1 \quad \text{...(ii)}$$

On solving Eqs. (i) and (ii), we have

The energy of emitted α-particle, $K_2 = 5.4\text{ MeV}$

46. (*c*) We have, $\lambda = \frac{0.693}{T_{1/2}}$

Mean life, $\tau = \frac{1}{\lambda} = \frac{t_{1/2}}{0.693} = \frac{2}{2.303\log_2} = \frac{2}{\ln 2}\text{ s}$

CHAPTER 14

Solids and semiconductor devices

Solid is a state of matter which has a definite shape and a definite volume. Most of the solids can be classified in three types as per their electrical conductivity—conductors, insulators and semiconductors.

Conductors are those through which electric charge can flow easily, while insulators are those through which electric charge is difficult to flow. There are, however, certain solids whose electrical conductivity is intermediate between conductors and insulators. They are called semiconductors. Silicon and germanium are a few examples of semiconductors.

In this chapter, we will discuss basic concepts of semiconductor physics and some semiconductor devices like junction diodes (two electrode device) and bi-polar junction transistor (three electrode device). Then, we will explore the basics of logic gates.

Classification of conductors, semiconductors and insulators

On the basis of the relative values of electrical conductivity (σ) or resistivity ($\rho = 1/\sigma$), *the solids are broadly classified as*

(i) **Conductors or Metals** They possess very low resistivity (or high conductivity).

$$\rho \sim 10^{-2} - 10^{-8}\ \Omega\text{m}$$

$$\sigma \sim 10^{2} - 10^{8}\ \text{S m}^{-1}$$

(ii) **Semiconductors** They have resistivity or conductivity intermediate to metals and insulators.

$$\rho \sim 10^{-5} - 10^{6}\ \Omega\text{m}$$

$$\sigma \sim 10^{5} - 10^{-6}\ \text{Sm}^{-1}$$

(iii) **Insulators** They have high resistivity (or low conductivity).

$$\rho \sim 10^{11} - 10^{19}\ \Omega\text{m}$$

$$\sigma \sim 10^{-11} - 10^{-19}\ \text{Sm}^{-1}$$

The values of ρ and σ given above are indicative of magnitude and could well go outside the ranges as well.

Inside

1) Energy bands in solids
2) Types of semiconductors
3) *p-n* junction diode
4) Junction transistors
5) Analog and digital circuits
6) Logic gates

1 Energy bands in solids

According to Bohr model, the energy of electrons in an isolated atom is decided by the orbit in which it revolves. When two atoms comes close to each other to form solid, their outer orbits of electrons come very close even overlap. Thus, the motion of electron in a solid is different from that in isolated atom. In a crystal each electron has a unique position, *i.e.*, no two electrons can have the same position. So each electron will have a different energy level, there different energy levels with continuous energy variation are called **energy bands.**

Valency band and conduction band

The energy band which includes the energy levels of valence electrons is called **valence band** and the energy band above it is called **conduction band.**

In isolated condition, *i.e.* with no external energy, all the valence electrons will reside in valence band. Normally conduction band is empty but when it overlaps on valence band, electrons can move freely into it.

Energy band gap

If there is some gap between the conduction band and the valence band then electrons in the valence band will remain bound and no free electrons will be available in conduction band.

The energy band gap is also defined as the difference between the highest valence band energy (E_V) and lowest conduction band energy (E_C),

i.e., $$E_g = E_C - E_V$$

The energy gap for different materials is different.

Note If λ is the wavelength of radiation used in shifting, the electron from valence band to conduction band, then energy band gap (E_g) is

$$E_g = h\nu = hc/\lambda$$

where, h is called Planck's constant and c is the velocity of light.

Forbidden bands or forbidden energy gap

The various energy bands in a solid may or may not overlap depending upon the structure of solid. If they do not overlap, then the intervals between them represent energies which the electrons in the solid cannot have. These intervals are called forbidden bands (as shown in the figure). It is also called forbidden energy gap because free electrons cannot exist in this gap.

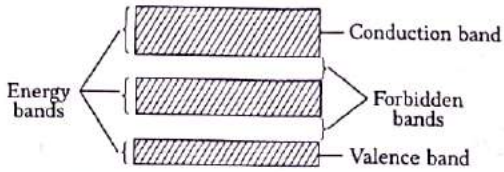


Fig. 14.1 Forbidden bands in a solid

If however, the adjacent energy bands in a solid overlap, the electrons have a continuous distribution of allowed energies as shown in the figure.

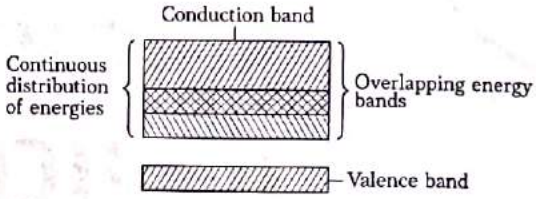


Fig. 14.2 Overlapping of energy bands

Note As temperature increases, forbidden energy gap decreases.

Fermi energy

It is the maximum possible energy possessed by free electrons of a material at absolute zero temperature (*i.e.*, 0 K). The value of fermi energy is different for different materials.

Example 14.1 *The maximum wavelength at which solid begin to absorb energy is 10000 Å. Calculate the energy gap of a solid (in eV).*

Sol. The energy band gap is given by $E_g = h\nu = \frac{hc}{\lambda}$

where, h = Planck's constant, c = speed of light and λ = wavelength at which solid absorbs energy

On putting the values of h, c and λ, we get

$$E_g = \frac{(6.626 \times 10^{-34}\ \text{Js})(3 \times 10^8\ \text{m/s})}{(10000 \times 10^{-10}\ \text{m})}$$

$$= 1.98 \times 10^{-19}\ \text{J} = \frac{1.98 \times 10^{-19}}{1.6 \times 10^{-19}}\ \text{eV} = 1.24\ \text{eV}$$

Energy band formation in sodium

The electronic configuration of sodium atom which has 11 electrons is $1s^2$, $2s^2$, $2p^6$ and $3s^1$. The levels $1s$, $2s$ and $2p$ are completely filled. The level $3s$ is half filled and the levels above $3s$ are empty.

Consider a group of N sodium atoms all in ground state separated from each other by large distances such as in sodium vapour. There are total $11N$ electrons. Each atom has two energy states in $1s$ energy level. So, there are $2N$ identical energy states labelled as $1s$ and all of them are filled from $2N$ electrons. Similarly, energy level $2p$ has $6N$ identical energy states which are also completely filled. In $3s$ energy levels, N of the $2N$ states are filled by the electrons and the remaining N states are empty.

These ideas are shown in the table given below.

Energy level	Total available energy states	Total occupied states
1s	2N	2N
2s	2N	2N
2p	6N	6N
3s	2N	N
3p	6N	0

In the above discussion, we have assumed that N sodium atoms are widely spread and hence the electrons of one atom do not interact with others.

As a result energy states of different states (*e.g.*, 1s) are identical. When atoms are drawn closer to one another electron of one atom starts interacting with the electrons of the neighbouring atoms of the same energy states.

For example, 1s electrons of one atom interact with 1s electrons of the other.

Due to interaction of electrons the energy states are not identical, but a sort of energy band is formed.

Thus, the collection of these closely spaced energy levels are called an **energy band**.

These bands are shown in figure given below.

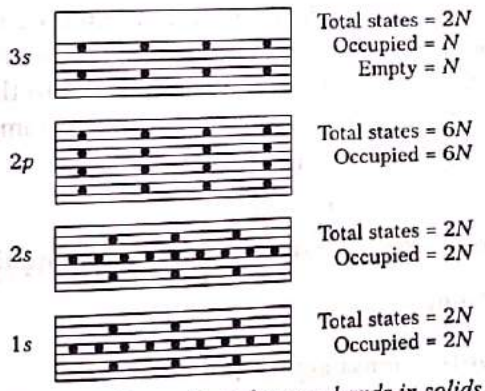


Fig. 14.3 Formation of energy bands in solids

Classification of solids on the basis of energy bands

Depending on whether the energy band gap is zero, large or small, the solids may be classified into conductors, insulators and semiconductors, *as explained below*

1. Conductors (Metals)

In case of metals either the conduction band is partially filled and valence band is partially empty, or conduction band and valence band overlap.

In case of overlaping electrons from valence band can easily move into conduction band, thus large number of electrons available for conduction. In case valence band is empty, electrons from its lower level can move to higher level making conduction possible.

This is the reason why resistance of metals is low or the conductivity is high.

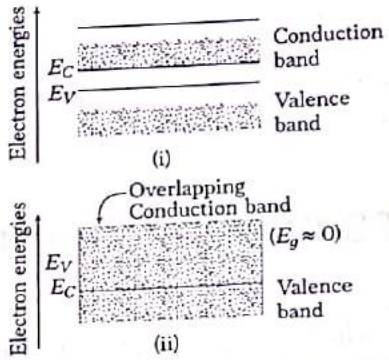


Fig. 14.4 Energy band gap for conductors

2. Insulators

In insulators, the valence band is completely filled whereas the conduction band is completely empty.

As there is no electron in conduction band so no electrical conduction is possible. The energy gap between conduction band and valence band is so large ($E_g > 3$ eV) that no electron in valence band can be provided so much energy from any external source that it can jump this energy gap.

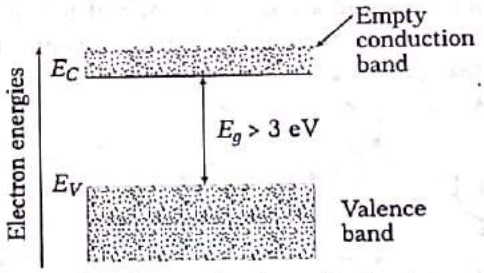


Fig. 14.5 Energy band gap for insulators

3. Semiconductors

The energy band structure of a semiconductor is shown in figure. It is similar to that of an insulator but with a comparatively small energy gap ($E_g < 3$eV). At absolute zero temperature, the conduction band of semiconductors is totally empty and valence band is completely filled.

Therefore, they are insulators at low temperatures.

However, at room temperature, some electrons in the valence band acquire thermal energy greater than energy band gap and jump over to the conduction band where

they are free to move under the influence of even a small electric field and acquire small conductivity.

Hence the resistance of semiconductor is not as high as that of insulators.

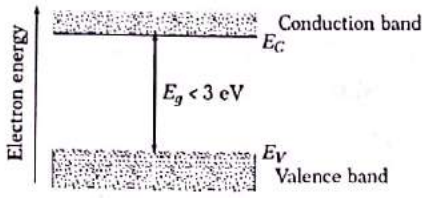


Fig. 14.6 Energy band gap for semiconductors

2 Types of Semiconductors

As discussed above, in semiconductors the conduction band and the valence band are separated by a relatively small energy gap. For silicon, this gap is 1.1 eV and for germanium it is 0.7 eV. However at 0 K, they behave as an insulator.

On the basis of purity, semiconductors are divided in two group

1. Intrinsic semiconductors

A pure (free from impurity) semiconductor which has a valency 4 is called an intrinsic semiconductor. Pure germanium, silicon in their natural state are intrinsic semiconductors.

Si and Ge both have 4 valence electrons. In crystaline structure each Si or Ge atom share each of its four valence electrons with four nearest neighbour atoms and forms covalent bond.

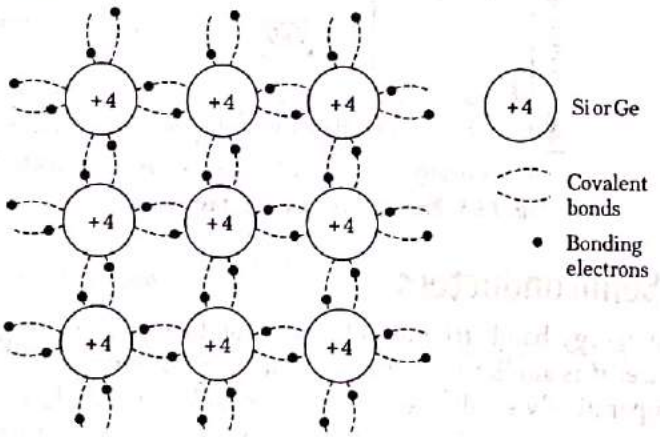


Fig. 14.7 Schematic two-dimensional representation of Si or Ge structure showing covalent bonds at low temperature (all bonds intact). +4 symbol indicates inner cores of Si or Ge.

At temperature close to zero, all shared electrons are tightly bound and so no free electrons are available to conduct electricity through the crystal.

At room temperature, however a few of the covalent bonds are broken due to thermal agitation and thus some of the valence electrons become free. Thus, we can say that a valence electrons is shifted to conduction band leaving a vacancy of electron in valence band. This vacancy is known as **hole** and carries a positive charge e.

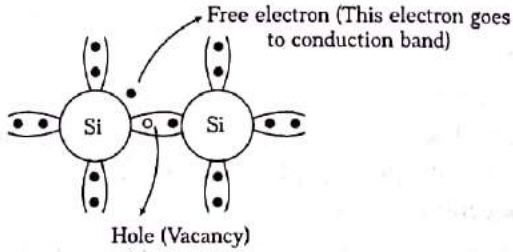


Fig. 14.8 Generation of hole due to thermal agitation

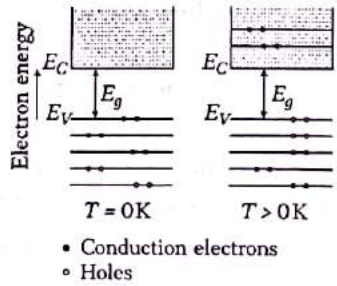


Fig. 14.9 Energy band diagram for an intrinsic semiconductor at $T = 0$ K and $T > 0$ K

In intrinsic (or pure) semiconductors, the number of free electrons per unit volume (n_e) is equal to the number of holes per unit volume (n_h) and it is also equal to the number of intrinsic charge carriers per unit volume (n_i).

Mathematically, we have the relation

$$n_e = n_h = n_i$$

Here, (n_i) is also called intrinsic carrier concentration.

At equilibrium, $\boxed{n_e n_h = n_i^2}$

This is known as **mass-action law.**

Under the action of an electric field, holes move towards negative potential giving hole current, I_h and electron towards positive potential giving electron current I_e. The total current is the sum of the electron current I_e and the hole current I_h.

i.e., $$I = I_e + I_h$$

Example 14.3 *In an intrinsic (pure) semiconductor, the number of conduction electrons is 7×10^{19} per cubic metre. Find the total number of current carriers (electrons and holes) in the same semiconductor of size 1 cm × 1 cm × 1mm.*

Sol. In an intrinsic semiconductor, $n_e = n_h$

where, n_e = number of conduction electrons,

and n_h = number of holes per unit volume

Given, $n_e = 7 \times 10^{19}$ per m^3

$\therefore$ $n_h = n_e = 7 \times 10^{19}$ per m^3

So, total current carrier density

$$n_e + n_h = 7 \times 10^{19} + 7 \times 10^{19} = 14 \times 10^{19} \text{ per } m^3$$

Now, total number of current carrier

$$= \text{Number density} \times \text{volume}$$
$$= (14 \times 10^{19} \text{ per } m^3) \times (10^{-2} m \times 10^{-2} m \times 10^{-3} m$$
$$= 1.4 \times 10^{13}$$

Example 14.4 *Pure Si at 300 K has equal electron (n_e) and hole (n_h) concentration of 1.5×10^{16} m^{-3}. By some means hole concentration increases to 3×10^{22} m^{-3}. Now, calculate n_e in the Si.*

Sol. Given, $n_i = 1.5 \times 10^{16}$ m^{-3}, $n_h = 3 \times 10^{22}$ m^{-3}

For the semiconductor in thermal equilibrium $n_e n_h = n_i^2$ (Law of mass action)

$$\Rightarrow \quad n_e = \frac{n_i^2}{h_h} = \frac{(1.5 \times 10^{16})^2}{3 \times 10^{22}} = 7.5 \times 10^9 m^{-3}$$

Example 14.4 *The number of conduction electrons (in per cubic metre) present in a pure semiconductor is 6.5×10^{19}. Calculate the number of holes in a sample having dimensions 1 cm × 1 cm × 2 mm ?*

Sol. Here, number of conduction electrons (n_e) = 6.5×10^{19} m^{-3}

Volume of the sample = 1 cm × 1 cm × 2 mm = 2×10^{-7} m^3

Number of holes in the sample (n_h) = Number of electrons in the sample (n_e)

$$= n_e \times V$$
$$= 6.5 \times 10^{19} \times 2 \times 10^{-7}$$
$$= 1.3 \times 10^{13}$$

2. Extrinsic semiconductors

The conductivity of an intrinsic semiconductor is very poor (unless the temperature is very high). At ordinary temperature, only one covalent bond breaks in 10^9 atoms of Ge. It means that only 1 atom in 10^9 atoms is available for conduction. Conductivity of an intrinsic (pure) semiconductor is significantly increased, if some pentavalent or trivalent impurity is mixed with it. Such impure semiconductors are called **extrinsic or doped semiconductors.**

Note When some desirable impurity is added to intrinsic semiconductors deliberately then this process is called doping and the impurity atoms are called dopants.

There are two types of dopants used in doping the tetravalent Si or Ge.

(i) Trivalent (valency 3) atoms: *e.g.*, Indium (In), Boron (B), Aluminium (Al) etc.

(ii) Pentavalent (valency 5) atoms: *e.g.*, Arsenic (As), Antimony (Sb) phosphorus (P) etc.

Depending upon the nature of impurity added in intrinsic semiconductor, the extrinsic semiconductors are of two types.

(i) *p*-type (ii) *n*-type

p-type semiconductors

It is obtained when a trivalent impurity (*e.g.*, boron, aluminium, gallium or indium) is added to a germanium or silicon crystal. The impurity atom has one valence electron less than Si or Ge, therefore its three valence electrons form covalent bonds with neighbouring three Ge (or Si) atoms while the fourth valence electron of Ge (or Si) is not able to form the bond.

So, the bond between the fourth neighbouring Ge (or Si) and the trivalent impurity has a vacancy or hole as shown in Fig. 14.10(a). This vacancy can be filled by an electron in outer orbit in the atom, (it may jump to fill the vacancy) leaving a vacancy or hole at its own site. With this electron the trivalent impurity atom becomes negatively charged. So the dopant atom of *p*-type semiconductor can be considered as core of the negative charge along with its associated hole.

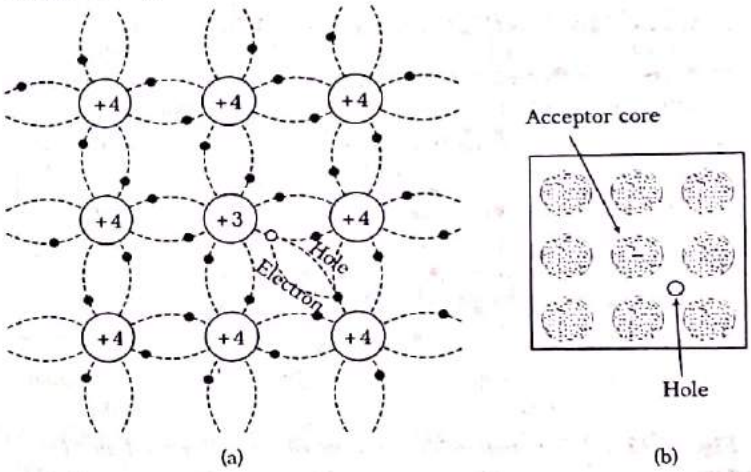


Fig. 14.10 *(a) Trivalent acceptor atom (In Al, B etc) doped in tetravalent Si or Ge lattice giving p-type semiconductor. (b) Commonly used schematic representation of p-type material which shows only the fixed core of the substituent acceptor with one effective additional negative charge and its associated hole.*

The trivalent impurity atoms are called **acceptor atoms** because they create holes which accept electrons.

Following points are worthnoting regarding p-type semiconductors

(a) Holes are the majority charge carriers and electrons are minority charge carriers in case of *p*-type semiconductors or number of holes are much greater than the number of electrons.

$$n_h >> n_e ; i_h > i_e$$

(b) p-type semiconductor is electrically neutral.
(c) p-type semiconductor can be shown as
(d) Conductivity $\sigma = n_h \mu_h e$

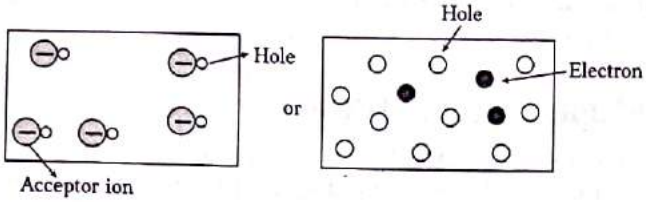


Fig. 14.11 Representation of p-type semiconductor

n-type semiconductors

It is obtained when a pentavalent impurity atom (*e.g.*, antimony, phosphorus or arsenic) is added to a Ge (or Si) crystal. Here the impurity atom has one valence electron more than Si or Ge, therefore four of the five valence electrons of the impurity atom form covalent bonds with four neighbouring Ge (or Si) atoms and the fifth valence electron becomes free to move inside the crystal lattice. Thus, by doping pentavalent impurity, number of free electrons increases.

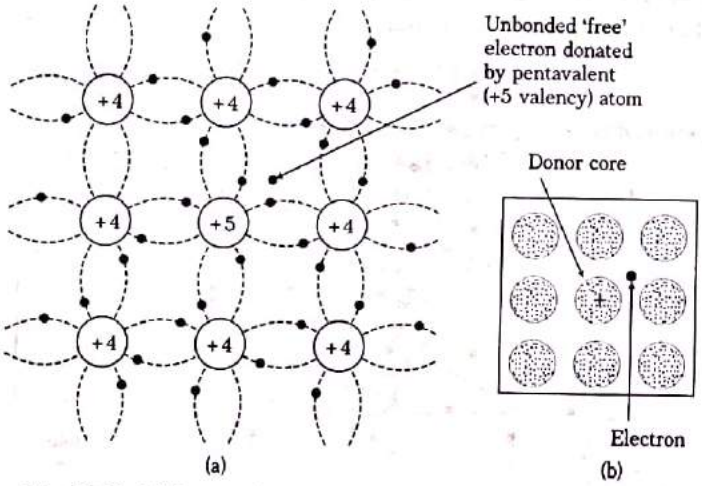


Fig. 14.12 (a) Pentavalent donor atom (As, SB, P, etc.) doped for tetravalent Si or Ge giving n-type semiconductor, and (b) Commonly used schematic representation of n-type material which shows only the fixed cores of the substituent donors with one additional effective positive charge and its associated extra electron.

The impurity (pentavalent) atoms are called **donor atoms** because they donate conduction electrons to the crystal. The number of electrons made available for conduction (by dopant atoms) depend strongly upon the dopant level and is independent of increase in temperature.

Following points are worthnoting regarding n-type semiconductors

(a) Electrons are the majority charge carriers and holes are minority charge carriers or number of electrons are much greater than the number of holes

$$n_e >> n_h; i_e >> i_h$$

(b) n-type semiconductor is also electrically neutral.
(c) n-type semiconductor can be shown as
(d) Conductivity $\sigma = n_e \mu_e e$

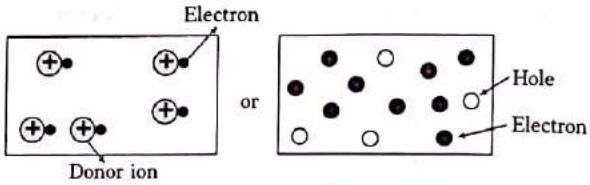


Fig. 14.13 Representation of n-type semiconductor

Energy band in extrinsic semiconductors

Energy band structure of semiconductor is effected due to doping. In extrinsic semiconductors, additional energy states due to donor impurities (E_D) and acceptor impurities (E_A) also exist. In the energy band diagram of n-type semiconductor the donor energy level E_D is slightly below the bottom E_C of conduction band and the electrons from this level move into conduction band with very small supply of energy. So the conduction band have most of the electrons coming from donor impurities.

In p-type semiconductors, the acceptor energy level E_A is slightly above the top energy level E_V of the valence band. With very small supply of energy an electron from the valence band can jump to the level E_A and ionise the acceptor negatively. At room temperature most of the acceptor atoms get ionised leaving hole in valence bond. Thus, at room temperature the density of holes in valence band is mainly due to impurity.

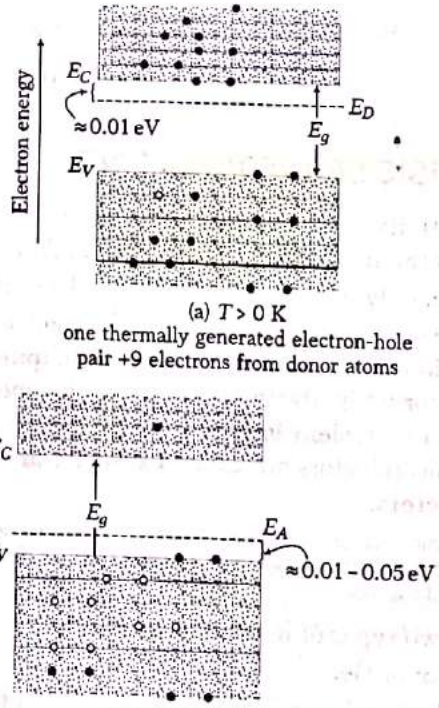


Fig. 14.14 Energy bands of (a) n-type semiconductor at $T > 0$ K, (b) p-type semiconductor at $T > 0$ K

Electrical conduction through semiconductors

When a battery is connected across a semiconductor (whether intrinsic or extrinsic) a potential difference is developed across its ends. Due to the potential difference an electric field is produced inside the semiconductors. A current (although very small) starts flowing through the semiconductor. This current may be due to the motion of (i) free electrons (i_e) and (ii) holes (i_h). Electrons move in opposite direction of electric field while holes move in the same direction.

The motion of holes towards right (in the figure) take place because electrons from right hand side come to fill this hole creating a new hole in their own position. Thus, we can say that holes are moving from left to right. Thus, current in a semiconductor can be written as,

$$i = i_e + i_h$$

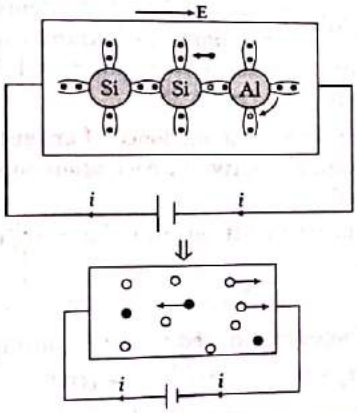


Fig. 14.15 Conduction through semiconductors

But it should be noted that mobility of holes is less than the mobility of electrons.

Conductivity of semiconductors is given

$$\boxed{\sigma = n_e e\mu_e + n_h e\mu_h}$$

where n_e and n_h are densities of conduction electrons and holes respectively and μ_e and μ_h are their respective mobilities.

where, $\mu_e = \dfrac{V_e}{E}, \mu_n = \dfrac{V_h}{e}$

Effect of temperature on conductivity of semiconductor

The conductivity of a semiconductor increases (or the resistivity decreases) with rise in temperature. This is because, as the temperature rises, more and more of the covalent bonds in the crystal lattice break, thus creating more and more charge carriers (electron-hole pairs). This results in increasing conductivity.

The temperature dependence of charge-carrier density is given by

$$n_i \propto e^{-E_g/2kT}$$

where n_i is number of electron-hole pair, E_g is the energy gap of the semiconductor and k is Boltzmann's constant. Thus, the charge-carrier density and hence the conductivity of the semiconductor increases exponentially with rise in temperature.

Example 14.5 *A p-type semiconductor has acceptor level 50 meV above the valence band. Find the maximum wavelength of light that can create a hole.*

Sol. To create a hole, an electron of valence band has to be excited into one of the acceptor levels which are 50 meV above the valence band. Hence a minimum of 50 meV energy is needed to create a hole.

As $$E_{min} = \frac{hc}{\lambda_{max}}$$

$$\therefore \quad \lambda_{max} = \frac{hc}{E_{min}} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{50 \times 1.6 \times 10^{-22}}$$

$$= 2.47 \times 10^{-5} \text{ m}$$

Example 14.6 *A Ge specimen is doped with Al. The concentration of acceptor atoms is ~ 10^{21} atoms/m^3. Given that the intrinsic concentration of electron hole pairs is ~ 10^{19}/m^3, then find the concentration of electrons in the specimen.*

Sol. According to law of mass action, $n_i^2 = n_h n_e$

Here, n_i = intrinsic concentration of electron hole pair $= 10^{19}/\text{m}^3$

n_h = concentration of acceptor atoms $= 10^{21}$ atoms/m^3

$$\therefore \quad n_e = \frac{n_i^2}{n_h} = \frac{(10^{19})^2}{(10^{21})} = 10^{17}/\text{m}^3$$

Example 14.7 *A semiconductor has equal electron atom hole concentration of 6×10^8 m^{-3}. On doping with certain impurity, electron concentration increases to 9×10^{12} m^{-3}.*

(i) Identify the new semiconductor obtained after doping.

(ii) Calculate the new hole concentration. (iii) How does the energy gap vary with doping?

Sol. (i) As the electron concentration increases after doping, so the new semiconductor obtained is of n-type.

(ii) As $n_e n_h = n_i^2$ (Law of mass action)

Here, $n_i = 6 \times 10^8 \text{ m}^{-3}, n_e = 9 \times 10^{12} \text{ m}^{-3}$

$$\therefore \quad n_h = \frac{n_i^2}{n_e} = \frac{(6 \times 10^8)^2}{9 \times 10^{12}}$$

$$= 4 \times 10^4 \text{ m}^{-3}$$

(iii) The energy gap decreases with doping.

Example 14.8 *Suppose a pure Si crystal has 5×10^{28} atoms m^{-3}. It is doped by 1 ppm concentration of pentavalent As. Calculate the number of electrons and holes. Given that $n_i = 1.5 \times 10^{16}\ m^{-3}$.*

Sol. Here, $n_i = 1.5 \times 10^{16}\ \text{m}^{-3}$

Doping concentration of pentavalent As atoms

$= 1$ ppm $= 1$ part per million

$\therefore$ Number density of pentavalent as atoms,

$$N_D = \frac{5 \times 10^{28}}{10^6} = 5 \times 10^{22}\ \text{atom m}^{-3}$$

Now, the thermally generated electrons ($n_i \propto 10^{16}\ \text{m}^{-3}$) are negligibly small as compared to those produced by doping, so

$$n_e \simeq N_D = 5 \times 10^{22}\ \text{m}^{-3}$$

Also, $n_e n_h = n_i^2$ (Law of mass action)

$$\therefore \quad n_h = \frac{n_i^2}{n_e} = \frac{1.5 \times 10^{16} \times 1.5 \times 10^{16}}{5 \times 10^{22}} = 4.5 \times 10^9\ \text{m}^{-3}$$

Example 14.9 *What will be conductivity of pure silicon crystal at 300K temperature. If electron hole pairs per cm^3 is 1.072×10^{10} at this temperature and mobilities are $\mu_h = 1350\ cm^2$ /volt sec and $\mu_e = 480\ cm^2$ /volt second.*

Sol. Conductivity of pure silicon

$$\sigma = n_i e \mu_e + n_i e \mu_h = n_i e\ (\mu_e + \mu_h)$$

where n_i is the total number of charge carrier, μ_e and μ_h are the mobility of electron and holes respectively.

Here, $n_i = 1.072 \times 10^{10}$ per cm^3

$\mu_h = 1350\ \text{cm}^2/\text{volt sec}$

$\mu_c = 480\ \text{cm}^2/\text{volt sec}$

$$\Rightarrow \quad \sigma = (1.072 \times 10^{10})(1.6 \times 10^{-19})(1350 + 480)$$

$$= 3.14 \times 10^{-6}\ \text{mho/cm}$$

Example 14.10 *An intrinsic germanium semiconductor is to be made n-type semiconductor of conductivity 6 mho/cm. Calculate the number density of donor atoms required, if the mobility of electrons in n-type semiconductor is 3850 cm^2/volt sec.*

Sol. In n-type semiconductor, $n_e >> n_h$

Let number density of donor atoms, $N_0 = n_e$

Conductivity of semiconductor is given by

$$\sigma = n_e e \mu_e$$

On putting the values, we get

$$6 \times 10^2 = n_e \times 1.6 \times 10^{-19} \times 3850 \times 10^{-4}$$

$$\Rightarrow \quad n_e = 9.7 \times 10^{21}/\text{m}^3$$

Example 14.11 *The concentration of hole-electron pairs in pure germanium at $T = 300\ K$ is 7×10^{15} per cubic metre. Antimony is doped into germanium a proportion of 1 atom 10^7 Ge atoms. Assuming that half of the impurity atoms contribute electron in the conduction band, calculate the factor by which the number of charge carriers increases due to doping the number of germanium atoms per cubic metre is 5×10^{28}.*

Sol. In pure semiconductor electron-hole pair $= 7 \times 10^{15}\text{m}^{-3}$

Total charge carrier $n_{\text{total initial}} = n_h + n_e = 14 \times 10^{15}$ after doping donor impurity $N_D = \dfrac{5 \times 10^{28}}{10^7} = 5 \times 10^{21}$

and $$n_e = \frac{N_D}{2} = 2.5 \times 10^{21}$$

So, $$n_{\text{final}} = n_h + n_e$$

$$\Rightarrow \quad n_{\text{final}} \approx n_e \approx 2.5 \times 10^{21} \qquad (\because n_e >> n_h)$$

$$\text{Factor} = \frac{n_{\text{final}} - n_{\text{initial}}}{n_{\text{initial}}}$$

$$= \frac{2.5 \times 10^{21} - 14 \times 10^{15}}{14 \times 10^{15}}$$

$$\approx \frac{2.5 \times 10^{21}}{14 \times 10^{15}} = 1.8 \times 10^5$$

Example 14.12 *The number of silicon atoms per m^3 is 5×10^{28}. This is doped simultaneously with 5×10^{22} atoms per m^3 of arsenic and 5×10^{20} per m^3 atoms of indium. Calculate the number of electrons and holes. Given that $n_i = 1.5 \times 10^{16}/m^3$. Is the material n-type or p-type.*

Sol. We know that for each atom doped of arsenic one free electron is received. Similarly, for each atom doped of indium a vacancy is created.

So, the number of free electrons introduced by pentavalent impurity added,

$$n_e = N_{As} = 5 \times 10^{22}\ \text{m}^3 \qquad \text{...(i)}$$

The number of holes introduced by trivalent impurity added.

Now, $$n_e - n_h = 5 \times 10^{22} - 5 \times 10^{20}$$

$$= 4.95 \times 10^{22} \qquad \text{...(ii)}$$

So, $$(n_e + n_h)^2 = (n_e - n_h)^2 + 4 n_e n_h$$

$$n_e + n_h = \sqrt{(4.95 \times 10^{22})^2 + 4(1.5 \times 10^{16})^2}$$

$$(\because n_e n_h = n_i^2) \quad \text{...(iii)}$$

Adding Eqs. (iii) and (ii),

$$2n_e = 4.95 \times 10^{22} + \sqrt{(4.95 \times 10^{22})^2 + 4(1.5 \times 10^{16})^2}$$

$$n_e = \frac{1}{2}[4.95 \times 10^{22} + \sqrt{(4.95 \times 10^{22})^2}]$$

$$= 4.95 \times 10^{22}/\text{m}^3$$

Now $$n_i^2 = n_h \times n_e$$

or $$n_h = \frac{n_i^2}{n_e} = \frac{(1.5 \times 10^{16})^2}{4.95 \times 10^{22}} = 4.54 \times 10^9\ /\ \text{m}^3$$

As number of electrons $n_e (= 4.95 \times 10^{22})$ is greater than number of holes $n_h (= 4.5 \times 10^9)$.

So, the material is n-type semiconductor.

Example 14.13 *Find the resistivity of a sample in which 10^{19} atoms of phosphorous are added per m^3. Take the resistivity of pure silicon as 3000 ohm-metre and the mobilities of electrons and holes as 0.15 m^2/V-s and 0.030 m^2/V-s respectively.*

Sol. The conductivity of a pure silicon is given by

$$\sigma_i = n_i e(\mu_e + \mu_h)$$

Also, the resistivity is reciprocal of conductivity.

$$\therefore \quad \rho_i = \frac{1}{\sigma_i} = \frac{1}{n_i e(\mu_e + \mu_h)}$$

$$\Rightarrow \quad n_i = \frac{1}{\rho_i e(\mu_e + \mu_h)}$$

$$= \frac{1}{3000 \times 1.6 \times 10^{-19}(0.15 + 0.030)}$$

$$= 1.157 \times 10^{16}/m^3$$

When 10^{19} atoms of phosphorus (donor atoms) are added per m^3, we have

$$n_e >> n_i \quad \text{or} \quad n_e >> n_h$$

$$\Rightarrow \quad n_e = 10^{19}$$

$$\therefore \quad \rho = \frac{1}{n_e e \mu_e} = \frac{1}{10^{19} \times 1.6 \times 10^{-19} \times 0.15}$$

$$= 4.17 \text{ ohm m}$$

Example 14.14 *A pure germanium plate of area 3.5×10^{-4} m^2 and of thickness 1.5×10^{-3} m is connected across a battery of potential 5 V. Find the amount of current produced at room temperature in the germanium sample. Also find the amount of heat generated in the plate in 120 second. Given that the concentration of carriers in germanium at room temperature is 1.6×10^6 per cubic metre. The mobilities of electrons and holes are 0.4 $m^2V^{-1}s^{-1}$ and 0.2 m^2 V^{-1} s^{-1} respectively.*

Sol. Conductivity of semiconductor is given by

$$\sigma = n_e e \mu_e + n_h e \mu_h$$

Since semiconductor is intrinsic

$$\Rightarrow \quad n_e = n_h = n_i \text{ or } \sigma = n_i e(\mu_e + \mu_h)$$

$$= 1.6 \times 10^6 \times 1.6 \times 10^{-19}(0.4 + 0.2)$$

$$= 1.536 \times 10^{-13} \text{ ohm}^{-1}\text{m}^{-1}$$

Current flowing, $I = jA$

where, j = current density $= \sigma E = \sigma\left(\frac{V}{d}\right)$

$$= 1.536 \times 10^{-13}\left(\frac{5}{1.5 \times 10^{-3}}\right)(3.5 \times 10^{-4}) = 1.79 \times 10^{-13} \text{ A}$$

Heat produced, $H = VIt = 5 \times 1.79 \times 10^{-13} \times 120$

$$= 10.74 \times 10^{-11} \text{ J}$$

Example 14.15 *The energy gap of pure Si is 1.12 eV. The mobilities of electrons and holes are respectively 0.140 $m^2V^{-1}s^{-1}$ and 0.050 $m^2V^{-1}s^{-1}$. They are independent of temperature. The intrinsic carrier concentration is given by*

$$n_i = n_0 e^{-E_g/2kT}$$

where n_0 is a constant, E_g the gap width and k the Boltzmann's constant whose value is 1.38×10^{-23} JK^{-1}. Find the ratio of the electrical conductivities of Si at 400 K and 200 K.

Sol. Conductivity of semiconductor is given by

$$\sigma = n_e e \mu_e + n_h e \mu_h$$

Here, $n_e = n_h = n_i$ (pure semiconductor)

$$\sigma = n_i e(\mu_e + \mu_h) = e(\mu_e + \mu_h) n_i$$

$$= e(\mu_e + \mu_h) n_0 e^{-E_g/2kT}$$

$$\frac{\sigma_{400}}{\sigma_{200}} = \frac{e^{\frac{-E_g}{2k \times 400}}}{e^{\frac{-E_g}{2k \times 200}}} = e^{E_g/800k}$$

$$= e^{1.12 \times 1.6 \times 10^{-19}/800 \times 1.38 \times 10^{23}} = e^{16.23}$$

Check point 14.1

1. The expected energy of the electrons at absolute zero is called
   (a) fermi energy
   (b) emission energy
   (c) work function
   (d) potential energy
2. The conduction band in a solid is partially filled at 0 K. The solid sample is a
   (a) conductor (b) semiconductor
   (c) insulator (d) None of these
3. The valence band and conduction band of a solid overlap at low temperature, the solid may be
   (a) a metal (b) a semiconductor
   (c) an insulator (d) None of these
4. In an insulator, forbidden energy gap between the valence band and conduction band is of the order of
   (a) 1 MeV (b) 0-1 MeV
   (c) 1 eV (d) 5 eV
5. The forbidden energy band gap in conductors semiconductor and insulators are E_{g_1}, E_{g_2} and E_{g_3} respectively. The relations among them is
   (a) $E_{g_1} = E_{g_2} = E_{g_3}$ (b) $E_{g_1} < E_{g_2} < E_{g_3}$
   (c) $E_{g_1} > E_{g_2} > E_{g_3}$ (d) $E_{g_1} < E_{g_2} > E_{g_3}$
6. Doping of intrinsic semiconductor is done
   (a) to neutralise charge carries
   (b) to increase the concentration of majority charge carriers
   (c) to make it neutral before disposal
   (d) to carry out further purification

7. The majority charge carries in p-type semiconductor are
(a) electrons (b) protons
(c) holes (d) neutrons

8. A p-type semiconductor can be obtained by adding
(a) arsenic to pure silicon
(b) gallium to pure silicon
(c) antimony to pure germanium
(d) phosphorous to pure germanium

9. To obtain p-type Si semiconductor we need to dope pure Si with
(a) aluminium (b) phosphorous
(c) oxygen (d) germanium

10. In p-type semiconductor the majority and minority charge carries are respectively.
(a) Protons and electrons
(b) Electrons and protons
(c) Electrons and holes
(d) Holes and electrons

11. The charge on a hole is equal to the charge of
(a) zero (b) proton
(c) neutron (d) electron

12. If n_e and n_h represent the number of free electrons and holes respectively in a semiconducting material, then for n-type semiconducting material.
(a) $n_e << n_h$ (b) $n_e >> n_h$
(c) $n_e = n_h$ (d) $n_e = n_h = 0$

13. Which statement is correct?
(a) n-type germanium is negatively charged and p-type germanium is positively charged.
(b) Both n-type and p-type germanium are neutral.
(c) n-type germanium is positively charged and p-type germanium is negatively charged.
(d) Both n-type and p-type germanium are negatively charged.

14. When a semiconductor is heated, its resistance
(a) decreases
(b) increases
(c) remains unchanged
(d) Nothing is definite

15. Which of the following has negative temperature coefficient of resistance?
(a) Copper (b) Aluminium
(c) Iron (d) Germanium

3 *p-n* junction

A p-type or n-type silicon crystal can be made by adding appropriate impurity as discussed above. Separately p- and n-type semiconductors have limited use. When the two types are combined, a number of new characteristic appears, which make the combination a very useful device. So, when a p-type semiconductor is suitably joined to an n-type semiconductor, then resulting arrangement is called p-n junction.

A p-n junction is formed either when a p-type semiconductor is grown in n-type semiconductor or n-type is grown on p-type semiconductor as given below.

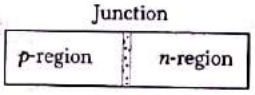


Fig. 14.16 A p-n junction

Formation of depletion region in *p-n* junction

In an n-type semiconductor, the concentration of electrons is more than concentration of holes. Similarly, in a p-type semiconductor, the concentration of holes is more than that of concentration of electrons. During formation of p-n junction and due to the concentration gradient across p and n-sides, holes diffuse from p-side to n-side ($p \rightarrow n$) and electrons diffuse from n-side to p-side ($n \rightarrow p$).

The diffused charge carriers combine with their counterparts in the immediate vicinity of the junction and neutralise each other.

Thus, near the junction positive charge is built on n-side and negative charge on p-side as shown in the figure.

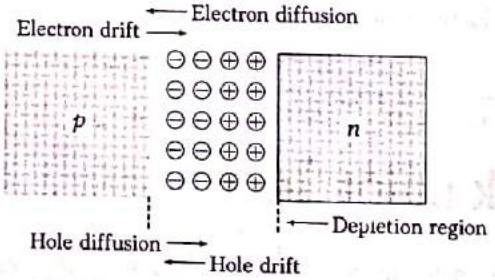


Fig. 14.17 p-n junction formation process

This set up potential difference across the junction and an internal electric field E_i directed from n-side to p-side. The equilibrium is established when the field E_i becomes strong enough to stop further diffusion of the majority charge carriers (however it helps the minority charge carriers to diffuse across the junction).

The region on either side of the junction which becomes depleted (free) from the mobile charge carriers is called depletion region or **depletion layer.** The width of depletion region is of the order of 10^{-6} m.

The potential difference developed across the depletion region is called the potential barrier. Potential barrier depends on dopant concentration in the semiconductor and temperature of the junction.

On the average the potential barrier in p-n junction is ~ 0.5 V and is given by $E_B = \frac{V}{d} \Rightarrow \boxed{V = E_B\, d}$

where E_B = barrier electric field

d = width of depletion region

For Ge, $V_B = 0.3$ V and for silicon $V_B = 0.7$ V

Given below are the graphs of potential versus distance, charge density versus distance and electric field versus distance across the depletion region.

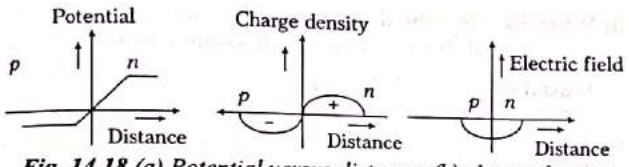
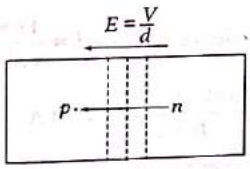


***Fig. 14.18** (a) Potential versus distance, (b) charge density versus distance, (c) Electric field versus distance across the depletion region*

***Example 14.16** A potential barrier of 0.5 V exists across a p-n junction (i) If the depletion region is 5×10^{-7} m wide. What is the intensity of the electric field in the region (ii) An electron with speed 5×10^5 ms^{-1} approaches the p-n junction from the n-side, with what speed will it enter the p-side?*

Sol. (i) Width of depletion layer, $d = 5 \times 10^{-7}$ m

$$\text{Electric field, } E_B = \frac{V}{d} = \frac{0.5\text{ V}}{5 \times 10^{-7}} = 10^6 \text{ volt m}^{-1}$$



(ii) According to work-energy theorem,

$$\frac{1}{2} m_e v_i^2 = eV + \frac{1}{2} m_e v_f^2$$

$$\text{Speed, } v_f = \sqrt{\frac{m_e v_i^2 - 2\,\text{eV}}{m_e}}$$

$$= \sqrt{\frac{9 \times 10^{-31} \times (5 \times 10^5)^2 - 2 \times 1.6 \times 10^{-19}}{9 \times 10^{-31}}}$$

$$= 2.7 \times 10^5 \text{ ms}^{-1}$$

Current flow across a *p-n* junction

1. Diffusion current

Because of concentration difference, holes try to diffuse from the p-side to the n-side and electrons from n-side to p-side at the p-n junction. However, only those holes/ electrons cross the junction which have high kinetic energy. The diffusion give rise to a current from p-side to n-side called **diffusion current.**

2. Drift current

Because of thermal collisions electron-hole pairs are created at every part of a p-n junction. However, if an electron-hole pair is created in the depletion region, the electron is pushed by the electric field towards the n-side and the hole towards the p-side. This gives rise to a current from n-side to p-side called the **drift current.**

Thus, $I_{df} \longrightarrow$ from p-side to n-side

$I_{dr} \longrightarrow$ from n-side to p-side

In steady state, $I_{df} = I_{dr}$ or $I_{net} = 0$

Semiconductor diode or *p-n* junction diode

A semiconductor diode [Fig. (a)] is basically a p-n junction with metallic contacts provided at the ends for the application of an external voltage. It is a two terminal device. A p-n junction diode is represented by the symbol as shown in Fig. (b).

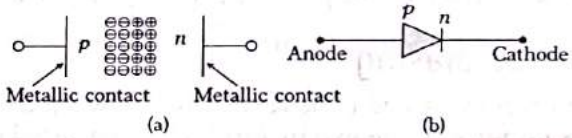


***Fig. 14.19** (a) Semiconductor diode, (b) Symbol for p-n junction diode*

The direction of arrow indicates the conventional direction of current.

Biasing of junction diode

Biasing is the method of connecting external battery or emf source to a p-n junction diode. The junction diode can be connected to an external battery in two ways, called forward biasing and reverse biasing of the junction.

1. Forward biasing

A junction is said to be forward biased when the positive terminal of the external battery is connected to the p-side and negative terminal to the n-side of the diode.

Flow of current in forward biasing

In this situation, the forward voltage opposes the potential barrier, due to which the potential barrier decreases and hence width of depletion layer decreases. Under the effect of external electric field, holes in the p-region and electrons in n-region, both move towards the junction.

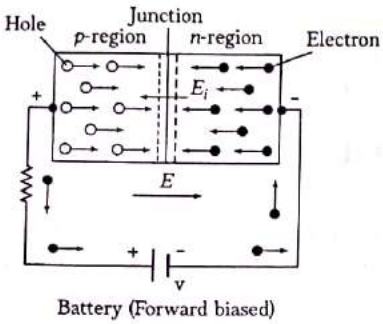


Fig. 14.20 Forward biased p-n junction diode

These holes and electrons mutually combine just near the junction and cease to exist. For each electron-hole combination, a covalent bond breaks up in the p-region near the positive terminal of the battery.

Of the hole and electron so produced, the hole moves towards the junction, while the electron enters the positive terminal of the battery through the connecting wire.

Just at this moment, an electron is released from the negative terminal of the battery which enters the n-region to replace the electron lost by combining with a hole at the junction.

Thus, a current called **forward current**, is constituted by the motion of majority charge carriers across the junction. In forward bias, the junction diode offers low resistance (ideally zero). Also, $I_{df} > I_{dr}$ or I_{net} is from p-side to n-side.

2. Reverse biasing

A junction diode is said to be reverse biased when the positive terminal of the external battery is connected to the n-side and negative terminal to the p-side of the diode.

Flow of current in reverse biasing

In this situation, the reverse voltage supports the potential barrier, due to which the potential barrier increases and hence width of depletion layer increases.

Under the effect of external electric field, holes in the p-region and electrons in the n-region are pushed away from the junction *i.e.*, they cannot be combined at the junction.

So, there is almost no flow of current due to majority charge carriers.

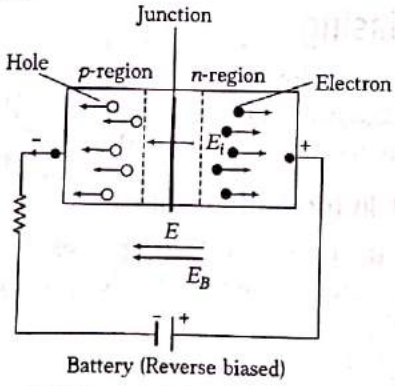
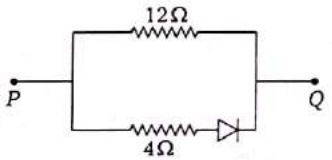


Fig. 14.21 Reverse biased p-n junction diode

However, a very small current due to minority charge carriers, flows across the junction. This current is called **reverse current.** Here $I_{dr} > I_{df}$ or I_{net} is from n-side to p-side.

Note The p-n junction allows a much larger current flow in forward biasing than in reverse biasing. This is crudely, the basis of the action of a p-n junction as a rectifier.

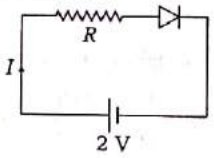
Example 14.17 *Find the net resistance of the network shown in the figure between the points P and Q if. (i)$V_P > V_Q$, (ii)$V_P < V_Q$*



Sol. (i) When $V_P > V_Q$, the diode is forward biased. Hence, the resistance of diode will be taken as zero. So the net resistance is $\frac{4 \times 12}{4+12}\Omega = 3\Omega$

(ii) When $V_P < V_Q$, the diode is reverse biased. Hence, there will be no current in the diode branch. So, the net resistance is $12\,\Omega$.

Example 14.18 *The diode used in the circuit shown in the figure has a constant voltage drop of 0.5 V at all current and a maximum power rating of 200 mW. What should be the value of the resistor R, connected in series with the diode, for obtaining maximum current?*

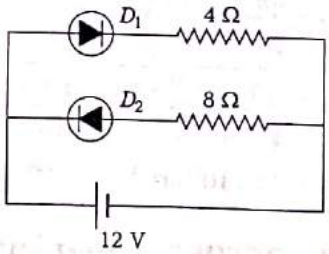


Sol. Current through diode (or circuit), $I = \frac{\text{Power}}{\text{Voltage}}$

$$\therefore \quad I = \frac{200 \times 10^{-3}\text{W}}{0.5\text{ V}} = 0.4\text{ A}$$

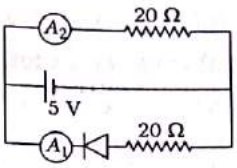
and resistance $R = \frac{\text{Net voltage}}{\text{Current}} = \frac{2-0.5}{0.4} = 3.75\Omega$

Example 14.19 *Find current passing through 4 Ω and 8 Ω resistance in the circuit shown in figure.*

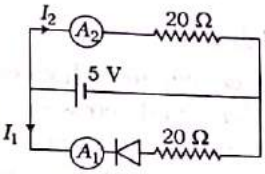


Sol. In the given circuit, diode D_1 is forward biased and D_2 is reverse biased. Hence, D_1 will conduct but D_2 will not. Therefore, current through 8 Ω resistance will be zero while through 4 Ω resistance will be, $\frac{12}{4} = 3$ A.

Example 14.20 *Consider the circuit diagram shown below. Two ammeters A_1 and A_2 are connected across a diode and resistor respectively. Find the amount of current flowing through these two ammeters. Neglect the resistances of the meters.*



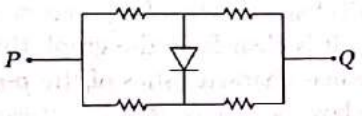
Sol. Let the current I_1 flows across a diode and I_2 flows across a resistor as shown below



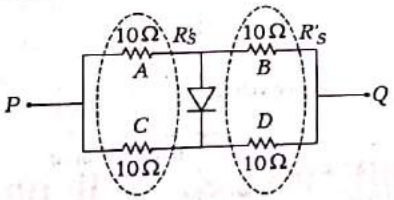
Diode is in reverse biased condition, so it will not conduct. Hence, ammeter A_1 will not show any reading. Now, the value of I_2 will be

$$I_2 = \frac{V}{R} = \frac{5}{20} = 0.25 \text{ A}$$

Example 14.21 *Find the net resistance between two points P and Q. If the value of each resistances shown in the figure is 10 Ω.*



Sol. The given circuit is in the form of a wheatstone bridge,



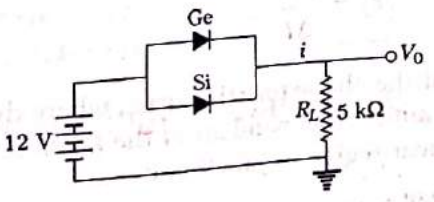
i.e.
$$\frac{A}{B} = \frac{C}{D}$$

Hence no current will from through diode, *i.e.*, it will not offer any resistance. Hence, net resistance will be

$$R_N = \frac{R_S \times R'_S}{R_S + R'_S}$$

where R_S and $R'_S = (10+10)\Omega = 20\Omega \Rightarrow R_N = 10\Omega$

Example 14.22 *(a) Calculate the value of V_o and i if the silicon and germanium diode start conducting at 0.7 V and 0.3 V, respectively.*



(b) *If the Ge diode connection is now reversed, what will be the new values of V_0 and i?*

Sol. (a) Ge diode will start conducting before the silicon diode does so. The effective forward voltage across Ge diode is (12 – 0.3) V =11.7 V. This will appear as the output voltage across the load *i.e.*,

$$V_o = 11.7 \text{ volt}$$

The current through R_L,

$$i = \frac{11.7}{5 \times 10^3} \text{ A} = 2.34 \text{ mA}$$

(b) On reversing the connection of Ge diode, it will be reverse biased and conduct no current. Only Si diode will conduct. Therefore,

$$V_o = (12 - 0.7)\text{V} = 11.3 \text{ volt}$$

and current $i = \dfrac{11.3}{5 \times 10^3}$ A

$$= 2.26 \text{ mA}$$

Voltage-current characteristic curve of a *p-n* junction diode

The voltage-current characteristic curves of a *p-n* junction diode are graphs showing the change in current flowing through the junction with change in a applied voltage, when the junction is forward-biased and reverse-biased.

1. Forward biased characteristics

When the diode is forward biased *i.e.*, *p*-side is kept at higher potential, the current in the diode changes with the voltage applied across the diode.

A graph is plotted between voltage and current. The curve so obtained is the forward characteristic of the diode. At the start when applied voltage is low, the current through diode is almost zero. It is because of potential barrier, which opposes the applied voltage. Till the applied voltage exceeds the potential barrier, the current increases very slowly with increase in applied voltage (*OA* portion of the graph).

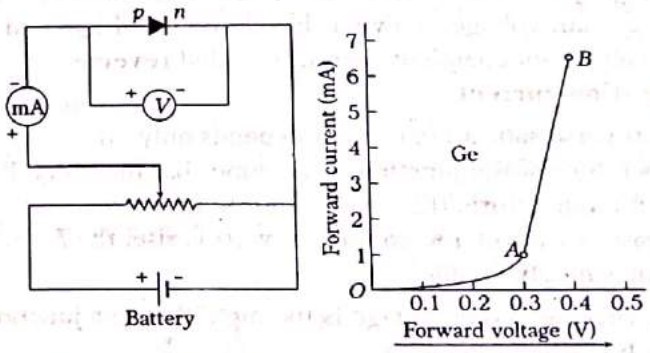


Fig. 14.22 Forward biased characteristics of a diode

After this voltage, the diode current increases rapidly (*AB* portion of the graph), even for very small increase in the diode voltage. In this situation the diode behaves like a conductor. The forward voltage beyond which the current through the function starts increasing rapidly with voltage is called the **threshold voltage or cut-off voltage** or **knee voltage.**

If line *AB* is extended back, it cuts the voltage axis at potential barrier voltage.

The value of the cut in voltage is about 0.2 V for a germanium diode and 0.7 V for a silicon diode.

2. Reverse biased characteristics

The circuit diagram for studying reverse biased characteristics is shown in the figure.

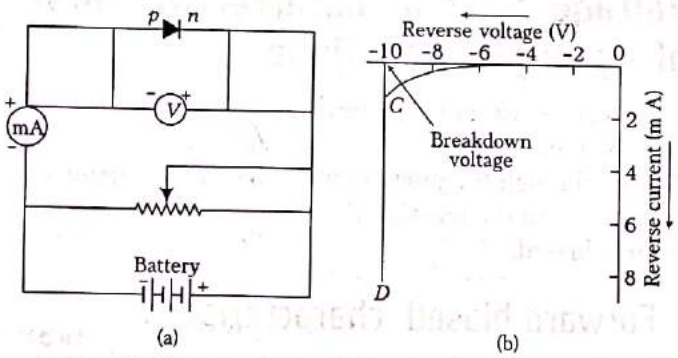


Fig. 14.23 Reverse biased characteristics of a diode

In reverse biased, the applied voltage supports the flow of minority charge carriers across the junction. So, a very small current flows across the junction due to minority charge carriers. Motion of minority charge carriers is also supported by internal potential barrier, so all the minority carriers cross over the junction.

Therefore, the small reverse current remains almost constant over a sufficiently long range of reverse bias, increasing very little with increasing voltage (*OC* portion of the graph). This reverse current is voltage independent upto certain voltage known as **breakdown voltage** and this voltage independent current is called **reverse saturation current.**

This reverse saturation current depends only on temperature of the junction. If is found that for every 1° rise in temperature, the reverse saturation current increases by about 7%. So for every 10°C rise, the I_0 becomes nearly double.

If the reverse biased voltage is too high, then *p-n* junction diode breaks.

It is of following two types.

(a) Zener breakdown

When reverse bias voltage is increased the electric field across the junction also increases. At some stage the electric field becomes so high that it breaks the covalent bonds creating electron-hole pairs. Thus a large number of carriers are generated. So, this causes a large current to flow. This mechanism is known as **Zener breakdown.**

(b) Avalanche breakdown

At high reverse voltage, due to high electric field, the minority charge carriers, while crossing the junction acquire very high velocities. These by collision breaks down the covalent bonds generating more carriers. A chain reaction is established giving rise to high current. This mechanism is called **Avalanche breakdown.**

Note Zener breakdown occurs in a highly doped *p-n* junction whereas Avalanche breakdown occurs in a lightly doped *p-n* junction.

Dynamic Resistance

The complete *VI* characteristic of a junction diode is shown in figure. It is clear from the graph that both the forward reverse bias characteristics of the *p-n* junction do not obey Ohm's law. So, the resistance offered by junction diode will depend on the applied voltage.

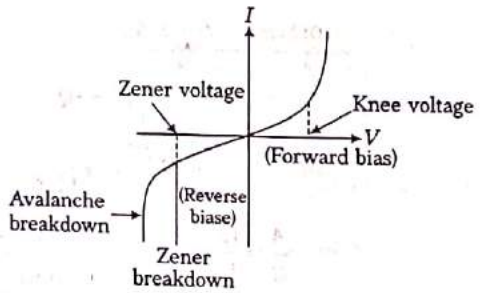


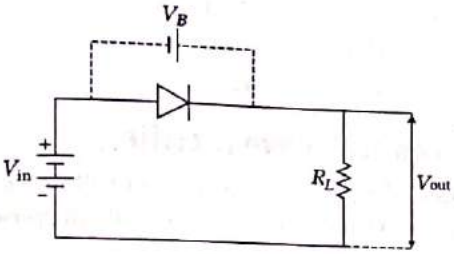
Fig. 14.24 Complete VI characteristic of a junction diode

Here, we will take the dynamic resistance which is defined as the ratio of small charge in voltage to the small change in current produced. It is also called dynamic resistance of the junction diode and is denoted by r_d.

Thus, $$r_d = \frac{\Delta V}{\Delta I}$$

The region of the characteristic curve, where dynamic resistance is almost independent of the applied voltage is called the linear region of junction diode.

Example 14.23 *A silicon diode is connected to a load resistance R_L as shown in the figure. Let $V_{in} = 10$ V and $R_L = 10\,k\Omega$.*
(a) If barrier voltage, $V_B = 0.7$ V then calculate
(i) output voltage across R_L
(ii) current in diode and
(iii) forward resistance.



(b) If diode is assumed ideal, then what will be
(i) output voltage and
(ii) output current in diode?

Sol. (a) (i) As, $V_{out} = V_{in} - V_B$
where V_B = Barrier voltage
$\therefore V_{out} = 10 - 0.7 = 9.3$ V

(ii) As diode is in forward biased state, so it will conduct

$$\therefore I = \frac{V_{out}}{R_L} = \frac{9.3}{10 \times 10^3} = 0.9 \text{ mA}$$

(iii) $r_f = \dfrac{V_B}{I} = \dfrac{0.7}{0.9 \times 10^{-3}} = 777\Omega$

(b) For ideal diode, $r_f = 0, V_B = 0$
(i) $V_{out} = V_{in} = 10$ V
(ii) $I = \dfrac{V_{out}}{R_L} = \dfrac{10}{10 \times 10^3} = 1$ mA

p-n Junction diode as a rectifier

A rectifier is a device which converts an alternating current (or voltage) into a direct (or unidirectional) current (or voltage).

Principle A *p-n* junction diode can work as an excellent rectifier because it permits current in one direction only. It offers a low resistance for the current to flow when it is forward biased, but a very high resistance when reverse biased. Thus, it allows current to flow through it in only one direction and acts as a rectifier.

The junction diode can be used either as an half wave rectifier or as a full-wave rectifier.

1. *p-n* junction diode as half-wave rectifier

A simple rectifier circuit called the half-wave rectifier, using only one diode is shown in figure. The AC voltage to be rectified is connected to the primary coil of a step-down transformer. Secondary coil is connected to the diode through resistors R_L across which, output is obtained.

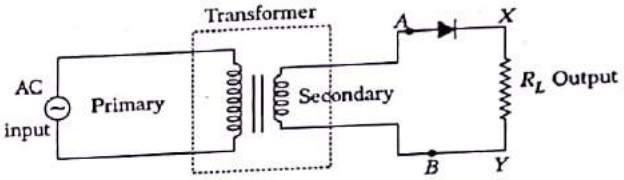


Fig. 14.25 Circuit diagram of a half-wave rectifier

Working

During positive half cycle of the input AC, (*i.e.*, $V_A < 0$) the *p-n* junction is forward biased. Thus, the resistance in *p-n* junction becomes low and current flows. Hence, we get output in the load.

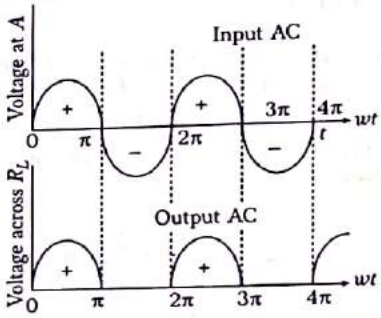


Fig. 14.26 Input and output waveform

During negative half cycle of the input AC, (*i.e.*, $V_A < 0$) the *p-n* junction is reverse biased. Thus, the resistance of *p-n* junction is high and current does not flow. Hence, no output is in the load.

Important points related to half wave rectifier

(i) **Average output** in one cycle

$$I_{DC} = \frac{I_0}{\pi} \text{ and } V_{DC} = \frac{V_0}{\pi}; I_0 = \frac{V_0}{r_f + R_L},$$

$$V_{DC} = \frac{V_0}{\pi\left(1 + \dfrac{r_f}{R_L}\right)} \quad (r_f = \text{forward biased resistance})$$

(ii) **rms output** $I_{rms} = \dfrac{I_0}{2}, V_{rms} = \dfrac{V_0}{2}$

(iii) The ratio of the effective alternating component of the output voltage or current to the DC component is known as **ripple factor.**

$$r = \frac{I_{AC}}{I_{DC}} = \left[\left(\frac{I_{rms}}{I_{DC}}\right)^2 - 1\right]^{1/2} = 1.21$$

The smaller the ripple factor, more effective will be rectifier.

(iv) **Peak inverse voltage** (PIV) The maximum reverse biased voltage that can be applied before commencement of Zener region is called the peak inverse voltage. When diode is not conducting PIV across it $= V_0$

(v) **Efficiency** It is given by

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{40.6\%}{1 + r_f / R_L}$$

If $R_L >> r_f$, then $\eta = 40.6\%$. If $R_L = r_f$, then $\eta = 20.3\%$

(vi) The ratio of i_{rms} and i_{DC} is known as form factor.

$$\text{Form factor} = \frac{I_{rms}}{I_{DC}} = \frac{\pi}{2} = 1.57$$

(vii) The ripple frequency (ω) for half wave rectifier is same as that of AC.

Example 14.24 *The applied input AC to a half wave rectifier is 60 W. The DC output is 20 W. Find the rectification efficiency. Also calculate the value of power efficiency.*

Sol. Rectification efficiency is the ratio of DC output power to the AC input power, *i.e.*,

$$\text{Rectification efficiency } \eta = \frac{\text{DC output power}}{\text{AC input power}}$$

Here, DC output power = 20 W

AC input power = 60 W

$$\therefore \text{ Rectification efficiency, } \eta = \frac{20}{60} \times 100 = 33.3\%$$

Also, power efficiency is given by

$$\text{PE} = \frac{\text{DC output power}}{\text{AC input power for half cycle}} \times 100\,\% = \frac{20}{30} \times 100\,\% = 66.67\%$$

Example 14.25 *A semiconductor diode is used as a half rectifier having internal resistance 200 Ω. The voltage applied is given by $V = 50 \sin \omega t$ volt and load resistance is 650 Ω. Calculate (i) maximum output current (ii) DC output current (iii) DC output power and (iv) DC output voltage.*

Sol. Here, $V = (50 \sin \omega t)$ volt

Comparing it with general equation $V = V_0 \sin\omega t$, we get

$$V_0 = 50 \text{ volt}, R_L = 650\,\Omega, \; r_d = 200\,\Omega$$

(i) Maximum output current, $I_0 = \dfrac{V_0}{r_d + R_L}$

$$= \frac{50}{(200 + 650)} = 58 \text{ mA}$$

(ii) DC output current, $I_{DC} = \dfrac{I_0}{\pi} = 18.5$ mA

(iii) DC output power $= I_{DC}^2 \cdot R_L = 0.22$ W

(iv) DC output voltage $= I_{DC} \cdot R_L = 12.02$ W

2. *p-n* junction diode as full wave rectifier

The full wave rectifiers are of two types

(i) Centre tap full wave rectifier

(ii) Full wave bridge rectifier.

These are described in detail below

(i) Centre tap full wave rectifier

Figure shows a circuit which is used in full wave rectification. Two diodes are used for this purpose.

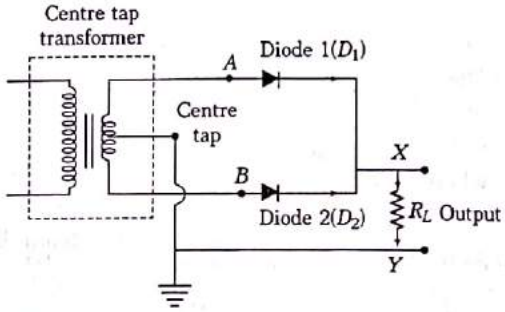


Fig. 14.27 *Circuit diagram of full wave rectifier*

Working

During the positive half cycle of the input AC, the diode D_1 is forward biased and the diode D_2 is reverse biased. The forward current flows through diode D_1.

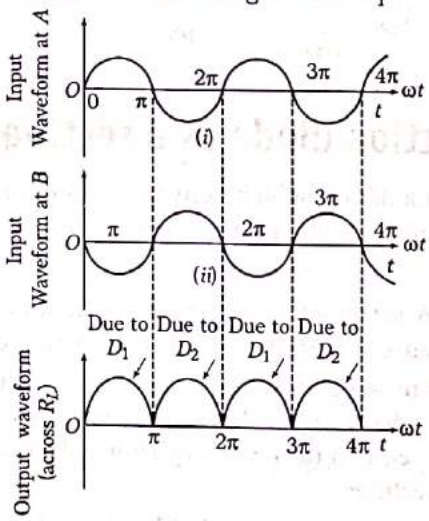


Fig. 14.28 *Input and output waveform of a fullwave rectifier*

During the negative half cycle, the diode D_1 is reverse biased and diode D_2 is forward biased. Thus, current flows through diode D_2. Thus, we find that during both the halves, current flows in the same direction.

(ii) Full wave bridge rectifier

Another full-wave rectifier called the bridge rectifier uses four diodes as shown in figure.

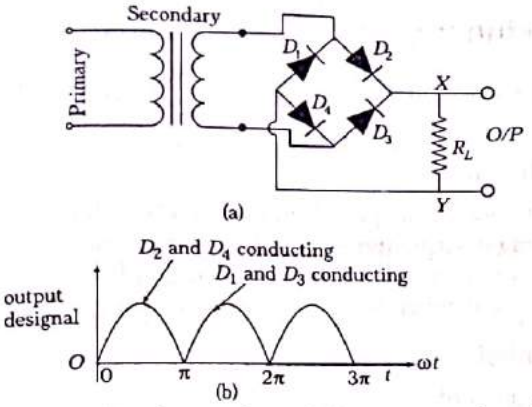


Fig. 14.29 (a) Bridge rectifier and (b) output waveforms for a bridge rectifier

For one-half cycle diodes D_1 and D_3 are forward biased and D_2 and D_4 are reverse biased. So, D_1 and D_3 conduct but D_2 and D_4 don't. Current through R_L flows from X to Y. In another half cycle D_2 and D_4 are forward biased and D_1 and D_3 are reverse biased. So, in this half cycle D_2 and D_4 conduct but D_1 and D_3 do not. Current again flows from X to Y through R_L. Thus, we see that current through R_L always flows in one direction from X to Y.

Important points related to full wave rectifier

(i) Output voltage is obtained across the load resistance R_i. It is not constant but pulsating in nature.

(ii) **Average output**

$$V_{av} = \frac{2V_0}{\pi}, I_{av} = \frac{2I_0}{\pi}, V_{DC} = \frac{2V_0}{\pi\left(1+\frac{r_f}{R_L}\right)}$$

(iii) **rms output** $V_{rms} = \frac{V_0}{\sqrt{2}}, I_{rms} = \frac{I_0}{\sqrt{2}}$

(iv) **Ripple factor** $r = 0.48 = 48\%$

(v) **Ripple frequency** The ripple frequency of full wave rectifier

$= 2 \times$ (Frequency of input AC)

(vi) **Peak inverse voltage (PIV)** Its value is $2V_0$

(vii) **Efficiency** $\eta_{\%} = \frac{812}{1+\frac{r_f}{R_L}}$ for $r_f << R_L$, $\eta = 812\%$

(viii) **Form factor,** $f = \frac{I_{rms}}{I_{DC}} = \frac{\pi}{2\sqrt{2}}$

Note The output signal frequency in full wave rectification is double that of input signal frequency but in half wave rectification, the input and output signal frequency is same.

Example 14.26 *In a full-wave rectifier circuit operating from 100 Hz mains frequency, what is the fundamental frequency in the ripple?*

Sol. In full-wave rectification, output signal (ripple) frequency is double that of input frequency. So, output frequency is 200 Hz.

Example 14.27 *In a centre tap full wave rectifier, the value of the load resistance is $2k\Omega$. The voltage applied across the half the secondary winding is given by $V = 220 \sin 314t$. Assume that the each diode has a forward bias dynamic resistance of $20\,\Omega$. Calculate (i) the peak value of current (ii) the DC value of current (iii) the rms value of current.*

Sol. Comparing $V = 220 \sin 314\,t$ with general equation

$$V = V_0 \sin \omega t,$$

we get

$$V_0 = 220\text{V and } \omega = 314$$

Also, it is given that $R_L = 2\text{k}\Omega$, $r_d = 20\,\Omega$

(i) Peak value of current,

$$I_0 = \frac{V_0}{(r_d + R_L)} = \frac{220}{20+2000} \approx 109\text{ mA}$$

(ii) DC value of current,

$$I_{DC} = \frac{2I_0}{\pi} = \frac{2\times 109}{314} = 69.4\text{ mA}$$

(iii) rms value of current, $I_{rms} = \frac{I_0}{\sqrt{2}} = 0.707 \times 109$

$$= 77.06\text{ mA}$$

Special types of *p-n* junction diode

The junction diodes are of many types

1. Light emitting diode (LED)

It is an important light source used in optical communication and it converts biasing electrical energy into light energy. It is a forward-biased *p-n* junction which emits light spontaneously.

(i) Symbol

It is represented by the symbol shown. In actual, it has two leads, the shorter lead represents *n*-or cathode side while the longer lead represents *p*-or anode side.

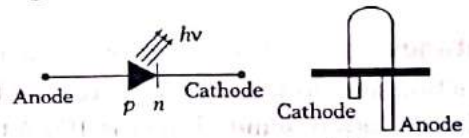


Fig. 14.30 LED symbol

(ii) Working

When *p-n* junction is forward biased, electrons and holes moves towards opposite sides of junction through it.

Therefore, there are excess minority carriers on the either side of the junction boundary, which recombines with majority carriers near the junction.

On recombination of electron and hole, the energy is given out in the form of heat and light.

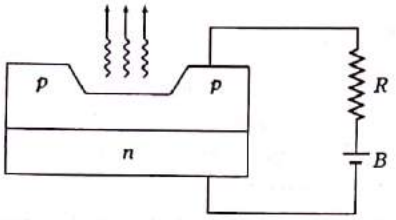


Fig. 14.31 A forward biased LED

(iii) *V-I* characteristics

V-I characteristics of LED are given below, which is similar to that of a simple junction diode. But the threshold voltages are much higher and slightly different for each colour. The reverse breakdown voltages of LEDs are very low. The colour of light emitted by a given LED, depends on its band gap energy. The photon emitted by an LED is of energy equal to or slightly less than the band gap energy. Forward current conducted by the junction determines the intensity of light emitted by LED.

A low voltage DC supply is required to operate an LED.

Current drawn by LED's is of the order of milliampere. So, in practice, a resistor of suitable value is joined in series with the LED to limit the current upto the safe value required.

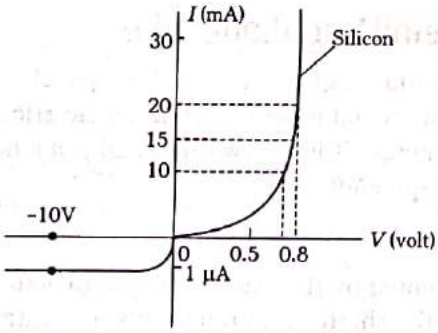


Fig. 14.32 I-V characteristics of LED

(iv) Advantages

(*a*) Fast action and no warm up time required.

(*b*) The bandwidth of emitted light is 100 Å to 500 Å. So, its nearly (not exactly) monochromatic.

(*c*) Long-life and ruggedness.

(*d*) Low operational voltage and less power consumed.

(*e*) Fast on-off switching capability.

Example 14.28 *A voltage drop of 2 V occurs across a light emitting diode (LED) and a current of 10 μA is passed through it when it is operated with a 6 V battery having a limiting resistor R. What is the value of R?*

Sol. Current, $I = \frac{V}{R}$ or $R = \frac{V}{I}$

Resistance of limiting resistor, $R = \frac{(6-2)\text{V}}{10 \times 10^{-6}\text{A}} = 400\,\text{k}\Omega$

2. Photodiode

A photodiode is a special type of junction diode used for detecting optical signals. It is a reverse biased *p-n* junction made from a photosensitive material. In photodiode, current carriers are generated by photons through photo excitation.

(i) Symbol

The symbol of a photodiode is

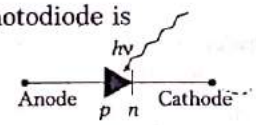


Fig. 14.33 symbol of a photodiode

(ii) Construction

A photodiode is fabricated with a transparent cover to allow light to fall on the is diode and is operated under reverse bias.

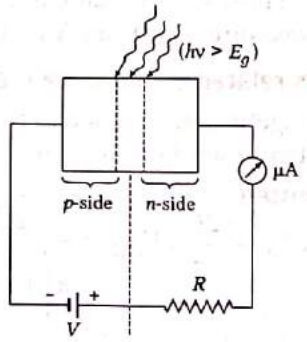


Fig. 14.34 A reverse biased photodiode illuminated with light

(iii) Working

When the photodiode is illuminated with light (photons), having energy greater than the energy gap of the semiconductor, then electron hole pairs are generated due to the absorption of photons. These charge carriers contribute to the reverse current. They increase the conductivity of the semiconductor.

Larger the intensity of the incident light, larger will be the conductivity of the semiconductor. The value of reverse saturation current increases with the increase in intensity of incident light.

Hence, a measurement of the change in the reverse saturation current on illumination can give the values of light intensity.

(iv) V-I Characteristics

Its V-I characteristics are shown in the figure, given below. We observe from the figure that current in photodiode changes with the change in light intensity (I) when reverse bias is applied.

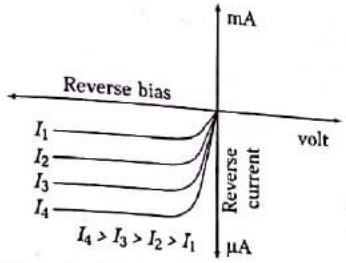


Fig. 14.35 V-I characteristics of photodiode at different intensites

(v) Uses

The p-n photodiode can operate at frequencies of the order of 1MHz, in contrast to the slow bulk-type photo conductor which is limited to a frequency range from 100 to 5000 Hz. Hence, the photodiodes are extensively used in high-speed reading of computer punched cards, light-detection systems, light-operated switches, counting of objects interrupting a light-beam etc.

Example 14.29 *A p-n photodiode is made of a material with a band gap of 1.5 eV. What is the minimum wavelength of radiation that can be absorbed by the material?*

Sol Use energy $E = h\nu = \dfrac{hc}{\lambda}$

The minimum wavelength of radiation

$$\lambda = \frac{hc}{E} = \frac{(6.4\times10^{-34}\ \text{J-s})\times(3\times10^{8}\ \text{ms}^{-1})}{1.5\times1.6\times10^{-19}\ \text{J}}$$

$$= 8.3\times10^{-7}\ \text{m} = 830\ \text{nm}$$

$$= \frac{1.5\times\ 1.6\times\ 10^{-19}\text{J}}{6.6\times\ 10^{-34}\text{J-s}\times3\times10^{8}\ \text{ms}^{-2}} \approx 1.2\times10^{6}\ \text{Hz}$$

3. Solar cell

Solar cell is a p-n junction diode, which converts solar energy into electrical energy. It is based on photovoltaic effect.

(i) Symbol

Its symbol is

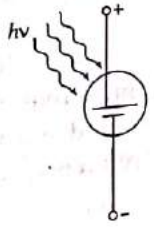


Fig. 14.36 A symbol of a solar cell

(ii) Construction

It consists of a silicon or gallium-arsenide p-n junction diode packed in a can with glass window on the top. Here, a thin layer of p-type is grown on a n-type semiconductor. The top of the p-layer has few finger electrodes for the light to reach it. The bottom of the n-layer is provided with a current collecting electrode.

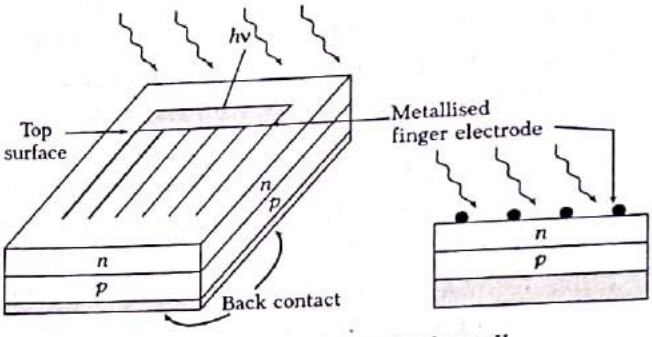


Fig. 14.37 pn junction of solar cell

(iii) Working

When photons of light (of energy $h\nu > E_g$) falls at the junction, electron hole pair are generated near the junction and they move in opposite directions due to junction field. They will be collected at the two sides of the junction, giving rise to a photovoltage between the top and bottom metal electrodes. The top metal contact acts as positive electrode and bottom metal contact acts as negative electrode. When an external load is connected across metal electrodes, a photo current flows.

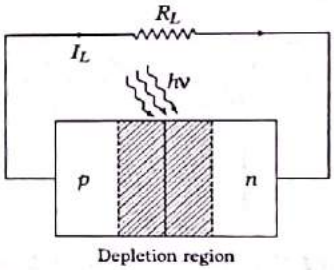


Fig. 14.38 Flow of photocurrent in a solar cell

(iv) I-V characteristics

The I-V characteristics of solar cell are shown in the figure. We can see in the figure, that it is drawn in the fourth quadrant of the coordinate axes, because a solar cell does not draw current but supplies the same to the load.

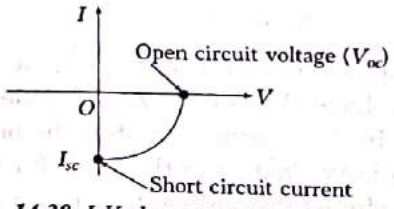


Fig. 14.39 I-V characteristic of a solar cell

(v) Uses

Solar cells are used in space vehicles for charging batteries in day time and then taking electrical power from them during night.

4. Zener diode

Zener diode is a reverse biased heavily doped *p-n* junction diode. It is operated in breakdown region.

Zener diode is designed to operate in the reverse breakdown voltage continuously without being damaged.

This can be achieved by changing the thickness of the depletion layer to which the voltage is applied. Current through the Zener diode is controlled by an external resistance. Zener diode is represented by the symbol.

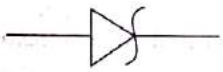


Fig. 14.40 Symbol of Zener diode

(i) *V-I* Characteristics

The *V-I* characteristics of Zener diode is shown below and we observe that when the applied reverse voltage (V) reaches the breakdown voltage (V_Z) of the Zener diode, there is a large change in the current.

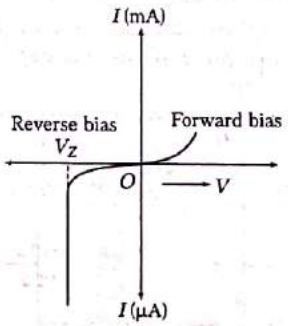


Fig. 14.41 VI characteristics of a Zener diode

But after the breakdown voltage V_Z, a large change in the current can be produced by almost insignificant change in the reverse bias voltage.

(ii) Zener diode as a voltage regulator

This is the most important application of a Zener diode.

(a) Principle

When the applied reverse voltage (V) reaches the breakdown voltage (V_Z) of the Zener diode, there is a large change in the current. So, after the breakdown voltage V_Z, a large change in the current can be produced by almost insignificant change in the reverse bias voltage *i.e.*, Zener voltage remains constant even though the current through the Zener diode varies over a wide range.

Zener diode is joined in reverse bias to the fluctuating. DC input voltage through a resistance R.

The constant output voltage is taken across a load resistance connected in parallel with Zener diode.

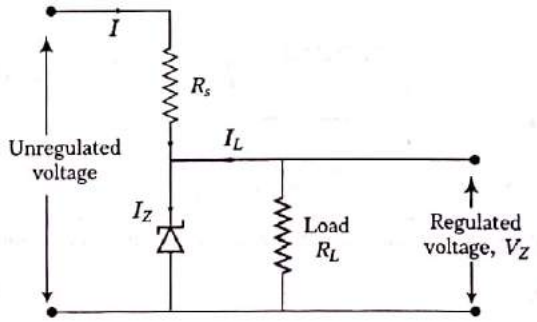


Fig. 14.42 Circuit diagram of Zener diode as voltage regulator

(b) Working

Here, when input DC voltage increases beyond a certain limit, the current through the circuit rises sharply, causing a sufficient increase in the voltage drop across the resistor R_s. Thus, the voltage across the Zener diode remains constant and also the output voltage remains constant at V_Z. When the input DC voltage decreases, the current through the circuit goes down sharply causing sufficient decrease in the voltage drop across the resistance. Thus, the voltage across the Zener diode remains constant and also the output voltage across R_L remains constant at V_Z.

Hence, the output voltage remains constant in both conditions. Figure shows the graph of output voltage V_0 *versus* input voltage V_{in} of a Zener diode. Clearly, the output voltage remains constant after the reverse breakdown voltage V_Z.

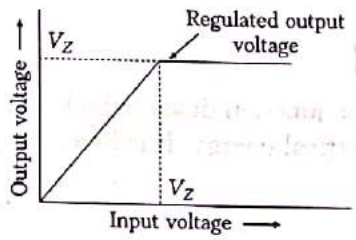
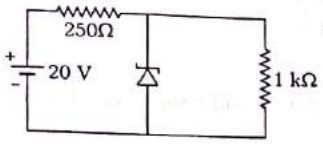


Fig. 14.43 Graph of V_0 versus V_{in} for a Zener diode

(iii) Uses

Though zener diode finds numerous applications in electronics but mainly it is used as a voltage stabilizer (voltage regulator) and as a fixed "reference voltage" for calibrating voltmeters.

Example 14.30 *The breakdown voltage of a Zener diode is 12 V. It is used in a voltage regulator circuit shown in figure. Find the current through the diode.*



Sol. Current through 250 Ω resistor is

$$I_1 = \frac{\text{Net voltage}}{\text{Resistance}} = \frac{(20-12)}{250\,\Omega} = 32 \times 10^{-3}\text{A} = 32\text{ mA}$$

Current through 1 kΩ resistor is $I_2 = \frac{12\text{ V}}{1\text{ k}\Omega}$

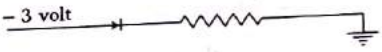
$$= 12 \times 10^{-3}\text{A} = 12\text{ mA}$$

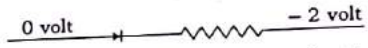
So, current through the Zener diode is $I_1 - I_2 = 32\text{ mA} - 12\text{ mA}$

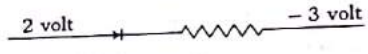
$$= 20\text{ mA}$$

Check point 14.2

1. In the depletion region of a unbiased *pn*-junction diode, there are
   (a) only electrons (b) only holes
   (c) Both electrons and holes (d) only fixed ions
2. The cause of the potential barrier in a *p-n* diode is
   (a) depletion of positive charges near the junction
   (b) concentration of positive charges near the junction
   (c) depletion of negative charge near the junction
   (d) concentration of positive and negative charges near the junction.
3. Barrier potential of a *p-n* junction diode does not depend on
   (a) temperature (b) forward bias
   (c) doping density (d) diode design
4. What control the conduction of *p-n*-junction?
   (a) Majority carries (b) Minority carries
   (c) Holes (d) Electron
5. In the forward bias arrangement of a *pn*-junction diode,
   (a) the *n*-end is connected to the positive terminal of the battery
   (b) the *p*-end is connected to the positive terminal of the battery
   (c) the direction of current is from *n*-end to *p*-end in the diode
   (d) the *p*-end is connected to the negative terminal of battery
6. In forwards bias, the width of potential barrier in a *p-n* junction diode
   (a) increases
   (b) decreases
   (c) remains content
   (d) first increases then decreases
7. On adjusting the *p-n* junction diode in forward biased
   (a) depletion layer increases (b) resistance increases
   (c) Both decreases (d) None of these
8. If the two ends *p* and *n* of a *p-n* diode junction are joined by a wire, then
   (a) there will not be a steady current in the circuit
   (b) there will not be a steady current from *n* side to *p* side
   (c) there will be a steady current from *p* side to *n* side
   (d) there may not be a current depending upon the resistance of the connecting wire
9. The reverse biasing in a *p-n* junction diode
   (a) decreases the width of potential barrier
   (b) increases the width of potential barrier
   (c) increases the number of minority charge carriers
   (d) increases the number of majority change carriers
10. A reverse-biased diode is
    (a)

    (b) 

    (c) 

    (d) 

11. On increasing the reverse bias to a large value in a *p-n* junction diode, current
    (a) increases slowly (b) remains fixed
    (c) suddenly increases (d) decreases slowly
12. In *p-n* junction avalanche current flows in circuit when biasing is
    (a) forward (b) reverse
    (c) zero (d) excess
13. The *p-n* junction diode is used as
    (a) an amplifier (b) a rectifier
    (c) an oscillator (d) a modulator
14. In a *p-n* junction photocell, the value of photo electromotive force produced by monochromatic light is proportional to
    (a) the voltage applied at the *p-n* junction
    (b) the barrier voltage at the *p-n* junction
    (c) the intensity of the light falling on the cell
    (d) the frequency of the light falling on the cell
15. What is the order of the reverse saturation current before breakdown in a Zener diode
    (a) ampere
    (b) milli-ampere
    (c) it depends on the applied voltage
    (d) micro-ampere

4 Junction transistors

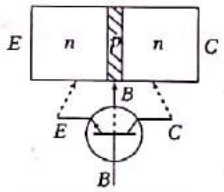
A junction transistor is a three terminal device which is formed by sandwiching a thin wafer of one type of semiconductors between two layers of another type. The *n-p-n* transistor has a *p*-type wafer between two *n*-type layers. Similarly the *p-n-p* transistor has a *n*-type wafer between two *p*-type layers.

Type of junction transistors

There are two types of junction transistors

n-p-n transistor

A *n-p-n* transistor is formed by sandwiching a thin layer of *p*-type semiconductor between two *n*-type semiconductors as shown in figure. Arrow indicates the direction of conventional current. In *n-p-n* transistor, it will always point outwards, *i.e.*, towards the emitter.

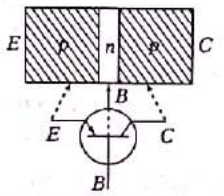


n-p-n transistor

***Fig. 14.44** n-p-n transistor and its circuit symbol*

p-n-p transistor

A *p-n-p* transistor is formed by sandwiching a thin layer of *n*-type semiconductor between two *p*-type semiconductors as shown in figure. The arrowhead in the symbol points inwards *i.e.*, towards the base.



p-n-p transistor

***Fig. 14.45** p-n-p transistor and its circuit symbol*

Terminals of a transistor

A transistor is basically a three terminal device. Terminals come out from the emitter, base and the collector for external connections.

Emitter (*E*)

It is heavily doped and of moderate size as it emits the majority charge carriers into the base by which current flows in the transistor.

Base (*B*)

The base region is very lightly doped and thin. Most of the charge carriers emitted by the emitter pass through it to the collector.

Collector (*C*)

The doping level of collector is neither very high nor very low, however, it lies between the doping levels of emitter and base. The size of the collector region is larger than the other two regions because it collects the charge carriers from the base.

Important points related to transistor

(i) In normal operation of a transistor, the emitter base junction is always forward biased and collector-base junction is reverse biased.

(ii) The arrow on the emitter-base line shows the direction of current between emitter and base.

(iii) In an *n-p-n* transistor, for example, there are a large number of conduction electrons in the emitter and a large number of holes in the base.

(iv) If the junction is forward biased the electrons will diffuse from emitter to the base and holes will diffuse from the base to the emitter.

(v) The direction of electric current at this junction is therefore from the base to the emitter.

(vi) In the connecting wires, electrons are always responsible for current flow, but inside the transistor holes are mainly responsible for current flow in *p-n-p* transistor and electrons in case of *n-p-n* transistor.

Biasing of transistor

Biasing of a transistor is defined as the process of applying external voltages or potential difference to it.

The biasing of the two *p-n* junction transistor, *i.e.* emitter junction and collector junction depends on the four modes, *i.e.*, Active mode, saturation mode, cut-off mode and inverse mode.

Active mode is also known as linear mode operation. A transistor is mostly used in the active region of operation, *i.e.*, emitter base junction is forward biased and collector base junction is reverse biased.

In **saturation mode**, maximum collector current flows and transistor acts as a closed switch from collector to emitter terminal. In **cut-off mode**, the circuit behaves as open switch where only leakage current flows.

The emitter and collector are interchanged in **inverse mode.**

Different modes of operation of a transistor is given below.

Operating mode	Emitter base bias	Collector base bias
Active	Forward	Reverse
Saturation	Forward	Forward
Cut off	Reverse	Reverse
Inverse	Reverse	Forward

Working of a transistor

1. *p-n-p* transistor

From figure, it is clear that emitter-base junction is forward biased and collector base junction is reverse biased.

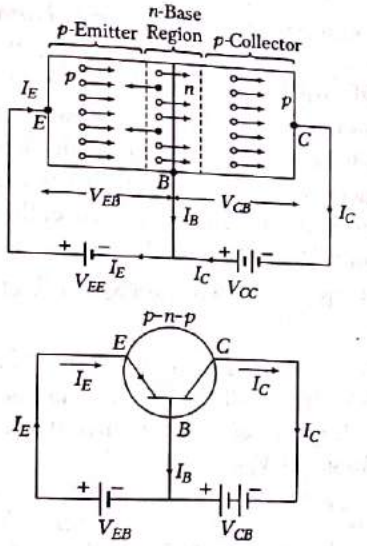


***Fig. 14.46** Action of p-n-p transistor and its biasing*

The resistance of emitter-base junction is very low. So, the voltage of V_{EB} is quite small (*i.e.*, 1.5 V) where as the resistance of collector-base junction is very high, so the voltage of V_{CB} is very high (*i.e.*, 45 V). The forward bias of the emitter-base circuit repels the holes of emitter towards the base and electrons of base towards the emitter. As the base is very thin and lightly doped, most of the holes (> 95%) entering it pass on to collector while a very few of them (< 5%) recombine with the electrons of the base region.

As soon as a hole combines with an electron, an electron from the negative terminal of the battery V_{EB} enters the base. This sets up a small base current I_B.

Each hole entering the collector region combines with an electron from the negative terminal of the battery V_{CB} and gets neutralised. This creates collector current I_C. Both the base current I_B and collector current I_C combine to form emitter current I_E.

$$\therefore \quad I_E = I_B + I_C$$

Thus inside the *p-n-p* transistor, the current conduction is due to holes while electrons are the charge carriers in the external circuit.

2. *n-p-n* transistor

In this transistor, the emitter-base junction is forward biased and its resistance is very low. So, the voltage of V_{EB} is quite small.

The collector-base junction is reverse biased. The resistance of this junction is very high. So, the voltage of V_{CB} is quite large (≈ 45V).

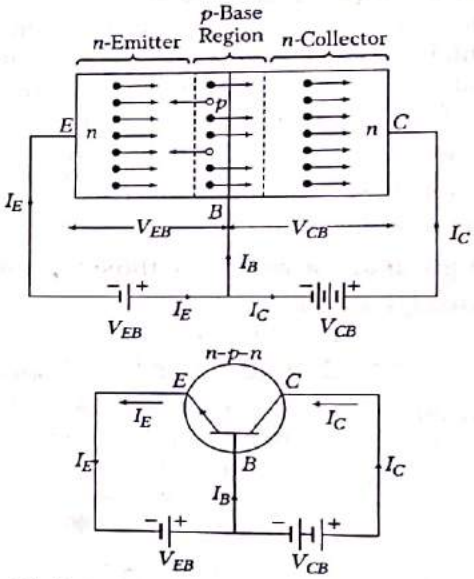


***Fig. 14.47** Action of n-p-n transistor and its biasing*

The forward bias of the emitter-base circuit repels the electrons of emitter towards the base, setting up emitter current I_E. As the base is very thin and lightly doped, a very few electrons (< 5%) from the emitter combine with the holes of base, giving rise to base current I_B and the remaining electrons (> 95%) are pulled by the collector which is at high positive potential. The electrons are finally collected by the positive terminal of battery V_{CB}, giving rise to collector current I_C.

As soon as an electron from the emitter combines with a hole in the base region, an electron leaves the negative terminal of the battery V_{EB} and at the same time the positive terminal of battery V_{EB} receives an electron from the base. This sets a base current I_B. Similarly, corresponding to each electron that goes from collector to positive terminal of V_{CB}, an electron enters the emitter from negative terminal of V_{CB}.

Hence, Emitter current = Base current + Collector current

or $$I_E = I_B + I_C \qquad (I_B << I_C)$$

Here, I_B is a small fraction of I_C depending on the shape of transistor, thickness of base, doping levels, bias voltages, etc.

Example 14.31 *In n-p-n transistor circuit, the collector current is 10 mA. If 95% of the electrons emitted reach the collector, what is the base current?*

Sol. $I_C = 95\%\, I_E \Rightarrow I_C = 0.95\, I_E \Rightarrow I_E = \dfrac{I_C}{0.95} = \dfrac{100}{95} \times 10\,\text{mA}$

$= 10.53$ mA

Now, $I_E = I_C + I_B$

$\therefore$ Base current $I_B = I_E - I_C = 10.53 - 10 = 0.53$ mA

Transistor circuit configurations

A transistor is a three terminal device. One terminal out of the three serves as a reference point for the entire circuit. This terminal should be common to the input and output circuits and is connected to ground. So, a transistor can be used in the following three configurations:

1. Common-Emitter (CE) Configuration
2. Common-Base (CB) Configuration
3. Common-Collector (CC) Configuration

The figure given below shows the three types of circuit arrangements for an *n-p-n* transistor.

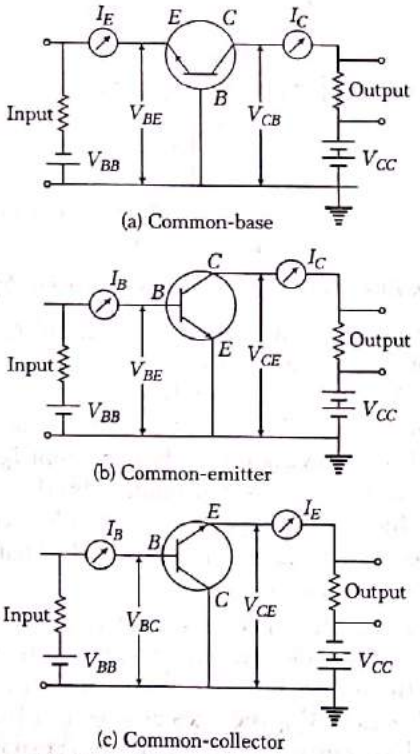


Fig. 14.48 *Configurations for n-p-n transistor*

Common emitter characteristics

The graphs drawn between voltages and currents, when emitter of a transistor is common to input and output circuits are known as CE characteristics of a transistor.

The circuit diagram for studying CE characteristics of a *n-p-n* transistor is given below.

Here, base-emitter circuit is forward biased with battery V_{BB} and emitter-collector circuit is reverse biased with battery V_{CC}.

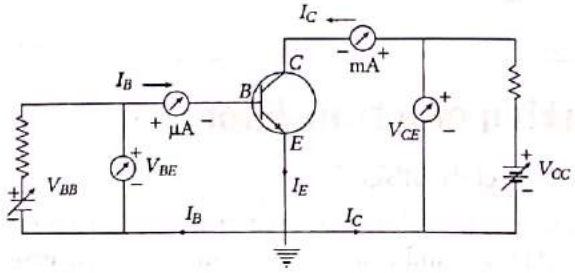


Fig. 14.49

Three types of characteristic curves are obtained.

1. Emitter or input characteristics

It is a graphical relation between the base-emitter voltage (V_{BE}) and the base-emitter current (I_B) by keeping collector-emitter-voltage (V_{CE}) constant is called input characteristics of the transistor. Adjust collector-emitter voltage at a suitable high value V_{CE} (say = +10 V).

It is necessary so as to make the base-collector junction reverse biased.

Now, with the help of rheostat, gradually increase the value of base-emitter voltage V_{BE} in small steps and note the corresponding values of base current I_B. Similarly, take the readings for $V_{CE} = 4$ V.

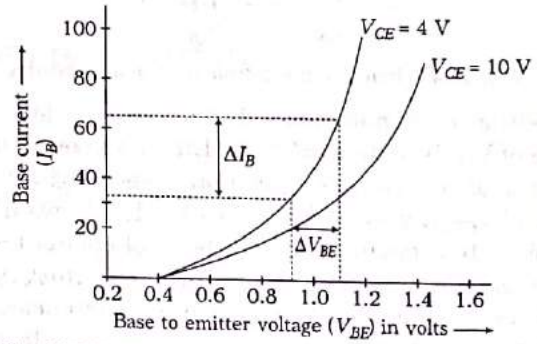


Fig. 14.50 *Input characteristics of a CE n-p-n transistor*

Observations from the graph

From the graph, it can be concluded that

(i) For a given collector voltage, the base current increases rapidly with increase in emitter-base voltage. It means that input resistance is very small.

(iii) For a higher negative collector voltage, the base current increases more rapidly with the collector voltage.

Input resistance

It is defined as the ratio of small change in base-emitter voltage (ΔV_{BE}) to the resulting change in the base current (ΔI_B) at constant collector-emitter voltage (V_{CE}). It is reciprocal of slope of I_B-V_{BE} curve.

$$\text{Input resistance, } R_i = \left(\frac{\Delta V_{BE}}{\Delta I_B}\right)_{V_{CE}=\text{constant}}$$

2. Collector or output characteristics

A graphical relation between the collector-emitter voltage (V_{CE}) and collector current (I_C) by keeping base current constant is called **output characteristics** of the transistor. To study the output characteristics of transistor, we keep value of base current I_B fixed (say at 10 μA) with the help of V_{BE}. Now, gradually change the value of V_{CE} and note the values of collector current I_C. Plot I_C-V_{CE} graph. Repeat the process for different constant values of I_B. The value of V_{CE} upto which the I_C changes with V_{CE} is called knee voltage. The transistors are operated in the region above knee voltage. The output characteristics are shown as,

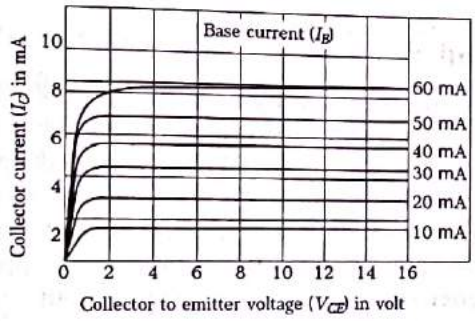


Fig. 14.51 Output characteristics of a CE n-p-n transistor

Observation from the graph

From the graph, it can be concluded

(i) From a given value of base current, the collector current is not zero even when collector voltage is zero.

(ii) For a given base current, there is a rapid increase in collector current for a increase in low negative collector voltage. It is the region of low collector resistance, transistor is never operated in this region.

(iii) For a given base current, the collector current become saturated for a certain collector voltage. Beyond which there is no increase in collector current for further increase in negative collector voltage. It is the region of high collector resistance.

Output resistance

From the output characteristics, we define output resistance of transistor as the ratio of change in collector-emitter voltage to the resulting change in collector current at constant base current. Thus,

$$\text{Output resistance, } r_0 = \left(\frac{\Delta V_{CE}}{\Delta I_C}\right)_{I_B=\text{constant}}$$

$$= \text{Reciprocal of slope of } I_C\text{-}V_{CE} \text{ curve}$$

3. Transfer Characteristics

The graph between the collector current (I_C) and the base current (I_B) at constant values of collector-emitter voltage (V_{CE}).

The transfer characteristic of a transistor is almost a straight line.

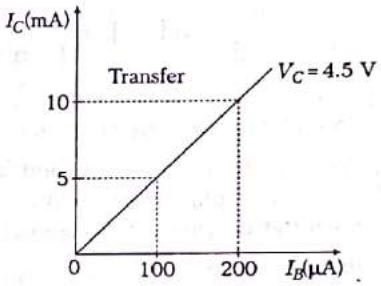


Fig. 14.52 Transfer characteristics of CE n-p-n transistor

Current amplification factor

As we have already studied the current amplification factor (β) of a transistor in *CE* configuration is given by

$$\beta_{AC} = \left(\frac{\Delta I_C}{\Delta I_B}\right)_{V_{CE}=\text{constant}}$$

Its value is very large ($\beta_{AC} >> 1$).

The direct ratio of I_C and I_B gives the DC current gain (β_{dc}) of the transistor.

$$\beta_{DC} = \frac{I_C}{I_B}$$

Note Since I_C increases with I_B almost linearly and $I_C = 0$ when $I_B = 0$, therefore, the values of both β_{AC} and β_{DC} are nearly equal.

Current amplification factors for different configurations of a transistor

There are two types of current amplification factors in a transistor

1. Common base current amplification factor

It is the ratio of small change in the collector current to the small change in the emitter current at constant collector base voltage. It is denoted by α and is given by

$$\boxed{\alpha = \left[\frac{\Delta I_C}{\Delta I_E}\right]_{V_{CB}=\text{constant}}}$$

2. Common emitter current amplification factor

It is the ratio of small change in the collector current to the small change in the base current at constant collector-emitter voltage. It is denoted by β and is given by

$$\beta = \left[\frac{\Delta I_C}{\Delta I_B}\right]_{V_{CE}=\text{constant}}$$

Relation between α and β

$$\alpha = \frac{\beta}{1+\beta} \quad \text{and} \quad \beta = \frac{\alpha}{1-\alpha}$$

Note α and β are independent of current, if the emitter-base junction is forward biased and the collector-base junction is reverse biased.

Example 14.32 *The current gain for common base amplifier is 0.98. Find the current amplification factor, if the transistor is used in common emitter configuration.*

Sol. It is given that $\alpha = 0.98$

As we know, $\beta = \dfrac{\alpha}{1-\alpha}$

where α = current amplification factor for common base amplifier

β = current amplification for common emitter amplifier

$$\Rightarrow \quad \beta = \frac{0.98}{(1-0.98)} = 49$$

Example 14.33 *The current factor of a transistor in a common base arrangement in 0.98. Find the change in collector current corresponding to a change of 5.0 mA in emitter current. What would be the cnange in base current?*

Sol. Given, $\alpha = 0.98$ and $\Delta i_e = 5.0$ mA

From the definition of, $\alpha = \dfrac{\Delta i_c}{\Delta i_e}$

Change in collector current,

$$\Delta i_c = (\alpha)(\Delta i_e) = (0.98)(5.0)\text{ mA} = 4.9\text{ mA}$$

Further, change in base current,

$$\Delta i_b = \Delta i_e - \Delta i_c = 0.1\text{ mA}$$

Example 14.34 *The current amplification factor for common emitter a arrangement is 59. If the emitter current is 5 mA. Find the value of collector current.*

Sol. Current amplification factor for common emitter amplifier (β) = 59

Emitter current, $(I_E) = 5$ mA

As $\quad \alpha = \dfrac{\beta}{1+\beta}$ Also $\alpha = \dfrac{I_C}{I_E}$

$$\therefore \quad \frac{I_C}{I_E} = \frac{\beta}{1+\beta} \Rightarrow I_C = \left(\frac{\beta}{1+\beta}\right) I_E$$

$$I_C = \left(\frac{59}{60}\right) \times 5 = 4.92\text{ mA}$$

Example 14.35 *In an n-p-n transistor (in common emitter mode) 10^{10} electrons enter the emitter in 10^{-6} s. Only 2% of the electrons are lots in the base. Calculate the base current and the current amplification factor. Charge on electron is 1.6×10^{-19} C.*

Sol. Current $\quad i_e = \dfrac{q}{t} = \dfrac{10^{10} \times 1.6 \times 10^{-19}}{10^{-6}} = 1.6\text{ mA}$

Base current, $i_b = 2\%$ of $i_e = \left(\dfrac{2}{100}\right) \times 1.6\text{ mA} = 0.032\text{ mA}$

$\therefore \quad i_c = i_e - i_b = 1.568$ mA

Therefore current amplifications factor,

$$\beta = \frac{i_c}{i_b} = \frac{1.568}{0.032} = 49$$

Transistor as an amplifier

An amplifier is a device which is used for increasing the amplitude of input signal. A transistor can be used for amplifying a weak signal. For a transistor to be operated as amplifier, three different basic circuits can be used. These are common bases, common emitter and common collector.

Principle

In a transistor, there are two *p-n* junctions one is forward biased (low resistance) and other is reverse biased (high resistance). The weak input signal is applied across the forward biased junction and output signal is taken across the reverse biased junction. Since, the input and output currents are almost equal, the output signal appears with a much higher voltage, because of high output resistance.

The emitter is always forward biased whereas the collector is always reverse biased no matter which configuration has to be used.

Common emitter amplifier using a *n-p-n* transistor

The circuit diagram for *n-p-n* transistor as an amplifier is shown in the figure given below:

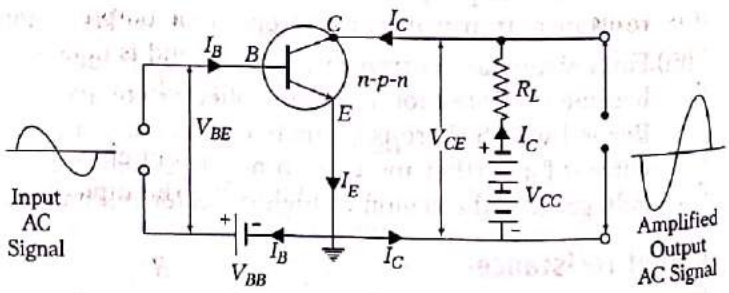


Fig. 14.53 *Circuit diagram of transistor as an amplifier*

The emitter-base circuit is forward biased by a low voltage battery V_{BB}, that means the resistance of input circuit is small. The base emitter voltage is given by $V_{BE} = V_{BB} - I_B R_B$

The collector-emitter circuit is reverse biased by a high voltage battery V_{CC}, that means the resistance of output circuit is high. R_L is a load resistance connected in collector-emitter circuit. The weak input AC signal is applied across the base-emitter circuit and amplified output signal is obtained across collector-emitter circuit. When no AC voltage is applied to the input circuit, we have

$$I_E = I_B + I_C \quad \text{...(i)}$$

Due to collector current I_C, the voltage drop across load resistance (R_L) is $I_C R_L$. Therefore, the collector-emitter voltage V_{CE} is given by

$$V_{CE} = V_{CC} - I_C R_L \quad \text{...(ii)}$$

When the input AC voltage signal is applied across base-emitter circuit, it changes base-emitter voltage and hence, emitter current I_E changes which in turn changes the collector current I_C. So, the collector-emitter voltage V_{CE} varies in accordance with Eq. (ii). This variation in V_{CE} appears as an amplified output.

Note The output voltage signal is 180° out of phase with the input voltage signal in the common emitter amplifier.

Gains in common-emitter amplifier

DC current gain

It is defined as the ratio of the collector current to the base current and is denoted by β_{DC}. Thus,

$$\beta_{DC} = \left(\frac{I_C}{I_B}\right)_{V_{CE}=\text{constant}}$$

AC current gain

It is defined as the ratio of the small change in the collector current to the small change in the base current at a constant collector-to-emitter voltage and is denoted by β_{AC}.

Thus, $$\beta_{AC} = \left(\frac{\Delta I_C}{\Delta I_B}\right)_{V_{CE}=\text{constant}}$$

Note The value of current gain β is from 15 to 50 which is much greater than α.

AC voltage gain

It is defined as the ratio of the change in the output voltage to the change in the input voltage and is denoted by A_V. Suppose, on applying an AC input voltage signal, the input base current changes by ΔI_B and correspondingly to the output collector current changes by ΔI_C. If R_{in} and R_{out} are the resistances of the input and the output circuits respectively, then

$$A_V = \frac{\Delta I_C \times R_{out}}{\Delta I_B \times R_{in}} = \frac{\Delta I_C}{\Delta I_B} \times \frac{R_{out}}{R_{in}}$$

Note The voltage gain in common-emitter amplifier is larger compared to that in common base amplifier.

Now, $\Delta I_C / \Delta I_B$ is the AC current gain β_{AC} and R_{out}/R_{in} is the resistance gain $A_V = \beta_{AC} \times$ Resistance gain

i.e., $\boxed{\text{Voltage gain} = \text{Current gain} \times \text{Resistance gain}}$

AC power gain

It is defined as the ratio of the change in the output power to the change in the input power. Since,

$$\text{Power} = \text{Current} \times \text{Voltage}$$

We have, AC power gain = AC current gain × AC voltage gain

$$= \beta_{AC} \times A_V = \beta_{AC} \times (\beta_{AC} \times \text{Resistance gain})$$

$$= \beta_{AC}^2 \times \text{Resistance gain}$$

Note As $\beta_{AC}^2 >> \alpha_{AC}^2$, the AC power gain of a common emitter amplifier is much larger than that of a common base amplifier.

Transconductance (g_m)

It is defined as the ratio of the change in the collector current to the change in the base-to-emitter voltage at constant collector-to-emitter voltage and is denoted by g_m.

Thus, $$g_m = \left(\frac{\Delta I_C}{\Delta V_{BE}}\right)_{V_{CE}}$$

We can express it in terms of β_{AC},

$$g_m = \frac{\Delta I_C}{\Delta I_B} \times \frac{\Delta I_B}{\Delta V_i} \qquad [\because V_{BE} = V_i]$$

Now, $$\frac{\Delta I_C}{\Delta I_B} = \beta_{AC}$$

and $$\frac{\Delta V_i}{\Delta I_B} = R_B \qquad \text{[a resistance of the input circuit]}$$

$$\therefore \quad g_m = \frac{\beta_{AC}}{R_{in}}$$

The unit if g_m is Ω^{-1} (ohm^{-1}) or S (siemen).

Note Common emitter amplifier using *n-p-n* transistor is same as common emitter amplifier using *p-n-p* transistor.

Important points in common base amplifier

(i) The output voltage signal is in phase with the input voltage signal.

(ii) The common base amplifier is used to amplify high (radio)-frequency signals and to match a very low source impedence (~20 Ω) to a high load impedence (~100 kΩ).

Example 14.36 *In a transistor connected in common emitter mode $R_0 = 4\ k\Omega$, $R_i = 1\ k\Omega$, $i_c = 1\ mA$ and $i_b = 20\ \mu A$. Find the voltage gain.*

Sol. Current gain, $\beta = \frac{i_c}{i_b} = \frac{1 \times 10^{-3}}{20 \times 10^{-6}} = 50$

$\therefore$ Voltage gain, $A_V = \beta\left(\frac{R_0}{R_i}\right) = (50)\left(\frac{4}{1}\right) = 200$

Example 14.37 *A load of 3 $k\Omega$ is connected in the collector branch of an amplifier circuit using a transistor in common emitter mode. The current gain $\beta = 40$. The input resistance of the transistor is 0.40 $k\Omega$. If the input current is changed by 40 μ A.*

(a) Calculate the change in output voltage.

(b) Also find the change in the input voltage.

(c) Calculate the power gain.

Sol. Given, Load resistance, $R_L = 3\,k\Omega = 3 \times 10^3\,\Omega$, $\beta = 40$

$r_i = 0.40\,k\Omega = 0.4 \times 10^3\,\Omega$

Base current $\Delta I_B = 40\,\mu A = 40 \times 10^{-6}$ A

(a) Output voltage

$V_0 = \Delta I_C R_L = \beta \Delta I_B R_L = 40 \times 40 \times 10^{-6} \times 3 \times 10^3 = 4.8$ V

(b) $\Delta V_{BE} = \Delta I_B r_i = 40 \times 10^{-6} \times 0.4 \times 10^3 = 16 \times 10^{-3}$ V

(c) Power gain, $A_P = \frac{\beta^2 R_L}{r_i}$

$= (40)^2\left(\frac{3 \times 10^3}{0.4 \times 10^3}\right) = 12000$

Example 14.38 *A transistor is used in common emitter mode in an amplifier circuit. When a signal of 40 mV is added to the base-emitter voltage, the base current changes by 40 μA and the collector current changes by 4 mA. The load resistance is 5 $k\Omega$. Calculate*

(a) β,

(b) the input resistance R_i,

(c) the transconductance g_m and

(d) the voltage gain.

Sol. Given, $\Delta i_b = 40\,\mu A$ and $\Delta i_c = 4$ mA

(a) $\beta = \frac{\Delta i_c}{\Delta i_b} = \frac{4 \times 10^{-3}}{40 \times 10^{-6}} = 100$

(b) $\Delta V_i = (\Delta i_b) \times R_i$

$\therefore\ R_i = \frac{\Delta V_i}{\Delta i_b} = \frac{40 \times 10^{-3}}{40 \times 10^{-6}} = 1000\,\Omega = 1\,k\Omega$

(c) Transconductance $g_m = \frac{\Delta i_c}{\Delta V_i} = \frac{4 \times 10^{-3}}{40 \times 10^{-3}} = 0.1\,\Omega^{-1}$

(d) Voltage gain, $A_V = \beta\left(\frac{R_{out}}{R_{in}}\right) = (100)\left(\frac{5}{1}\right) = 500$

Example 14.39 *A transistor is used in common-emitter mode in an amplifier circuit. When a signal of 40 mV is added to the base-emitter voltage, the base current changes by 40 μA and the collector current changes by 4 mA. The load resistance is 5 $k\Omega$. Calculate*

(a) the factor β *(b) the input resistance R_{BE}*

(c) the transconductance and *(d) the voltage gain*

Sol. Given, $V_i = \Delta V_{BE} = 40\,mV = 40 \times 10^{-3}$ V

$\Delta I_B = 40\,\mu A = 40 \times 10^{-6}$ A

$\Delta I_C = 4\,mA = 4 \times 10^{-3}$ A

and $R_L = 5\,k\Omega = 5 \times 10^3\,\Omega$

(a) Factor $\beta = \frac{\Delta I_C}{\Delta I_B} = \frac{4 \times 10^{-3}}{40 \times 10^{-6}} = 100$

(b) Input resistance, $r_i = \frac{\Delta V_{BE}}{\Delta I_B} = \frac{40 \times 10^{-3}}{40 \times 10^{-6}} = 1000\,\Omega$

(c) Transconductance, $g_m = \frac{\Delta I_C}{\Delta V_{BE}} = \frac{4 \times 10^{-3}}{40 \times 10^{-3}} = 0.1$mho

(d) Voltage gain, $A_V = \beta\frac{R_L}{r_i} = 100 \times \frac{5 \times 10^3}{10^3} = 500$

Example 14.40 *In a common emitter amplifier, the load resistance of the output circuit is 500 times the resistance of the input circuit. If $\alpha = 0.98$, then find the voltage gain and power gain.*

Sol. Given, $\alpha = 0.98$ and $\frac{R_{out}}{R_{in}} = 5000$, $\beta = \frac{\alpha}{1-\alpha} = \frac{0.98}{1-0.98} = 49$

(i) Voltage gain $= (\beta)\left(\frac{R_{out}}{R_{in}}\right) = (49)(500) = 24500$

(ii) Power gain $= (\beta^2)\left(\frac{R_{out}}{R_{in}}\right) = (49)^2(500) = 1200500$

Example 14.41 *A transistor is connected in common-emitter (CE) configuration. The collector supply is 8 V and the voltage drop across a resistor of 800 Ω in the collector circuit is 0.5 V. If the current gain factor α is 0.96, find the base current.*

Sol. Given, $V_{CC} = 8\,V$, $V_0 = \Delta I_C R_L = 0.5\,V$, $R_L = 800\,\Omega$

and $\alpha = 0.96$

As $\beta = \frac{\alpha}{1-\alpha} = \frac{0.96}{1-0.96} = 24$

We have $V_0 = \Delta I_C R_L$

$\therefore\ 0.5 = \Delta I_C \times 800$

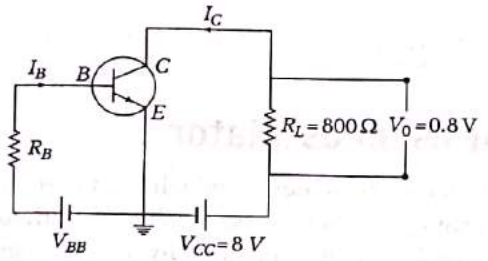
Collector current, $\Delta I_C = \frac{0.5}{800}$ A

Current amplification factor, $\beta = \frac{\Delta I_C}{\Delta I_B}$

Base current, $\Delta I_B = \frac{\Delta I_C}{\beta} = \frac{0.5/800}{24} = 26 \times 10^{-6}\,A = 26\,\mu A$

Example 14.42 *An n-p-n transistor is connected in common-emitter configuration in which collector supply is 8 V and the voltage drop across the load resistance of 800 Ω connected in the collector circuit is 0.8 V. If current gain factor is (25/26), determine the collector-emitter voltage and base current. If the internal resistance of the transistor is 200 Ω, calculate the voltage gain and power gain.*

Sol.



We have, $V_0 = \Delta I_C R_L$; $0.8 = \Delta I_C \times 800 \Rightarrow \Delta I_C = 10^{-3}$A

$$V_{CC} = V_{CE} + \Delta I_C R_L \Rightarrow 8 = V_{CE} + 10^{-3} \times 800$$

and collector emitter voltage, $V_{CE} = 8 - 0.8 = 7.2$ V

Current gain factor, $\alpha = \frac{25}{26}$, $\beta = \frac{\alpha}{1-\alpha} = \frac{25/26}{1-25/26} = 25$

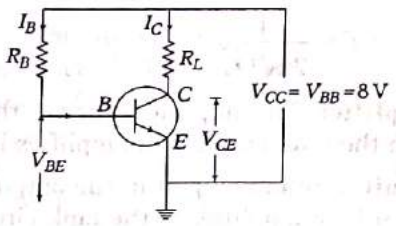
Base current, $\Delta I_B = \frac{\Delta I_C}{\beta} = \frac{10^{-3}}{25} = 4 \times 10^{-5}\text{A} = 40\,\mu\text{A}$

Voltage gain, $A_V = \beta \frac{R_L}{r_i} = 25 \times \frac{800}{200} = 100$

Power gain, $A_P = \beta^2 \frac{R_L}{r_i} = \beta A_V = 25 \times 100 = 2500$

Example 14.43 *An n-p-n transistor in a common-emitter mode is used as a simple voltage amplifier with a collector current of 4 mA. The terminal of 8 V battery is connected to the collector through a load resistance R_L and to the base through a resistance R_B. The collector-emitter voltage $V_{CE} = 4$ V, base-emitter voltage $V_{BE} = 0.6$ V and base current amplification factor $\beta_{DC} = 100$. Calculate the values of R_L and R_B.*

Sol.



Given, $V_{CE} = 4$ V, $V_{BE} = 0.6$ V, $\beta = 100$

and $\Delta I_C = 4\text{mA} = 4 \times 10^{-3}$A

we have, $V_{CE} + \Delta I_C R_L = V_{CC}$

$$4 \times 10^{-3} R_L = 8 - 4 = 4$$

Load resistance, $R_L = 1000\,\Omega$

Base current, $I_B = I_C / \beta = \frac{4 \times 10^{-3}}{100} = 4 \times 10^{-5}$A

We have $V_{CC} = I_B R_B + V_{BE}$

$$8 = I_B R_B + 0.6$$

$\therefore$ $I_B R_B = 7.4$ V

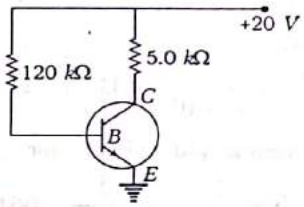
$$4 \times 10^{-5} R_B = 7.4$$

Resistance through the base

$$R_B = \frac{7.4}{4} \times 10^5 = 1.85 \times 10^5\,\Omega$$

$$= 185\,\text{k}\Omega$$

Example 14.44 *In the following circuit, the value of β is 200. Find I_B, V_{CE}, V_{BE} and V_{BC}, when $I_C = 2.5$ mA. Find whether transistor is in active, cut off or saturation state.*



Sol. $\beta = \frac{I_C}{I_B} \Rightarrow I_B = \frac{I_C}{\beta} = \frac{2.5}{200} = 0.0125$ mA

Applying Kirchhoff's law to base emitter loop,

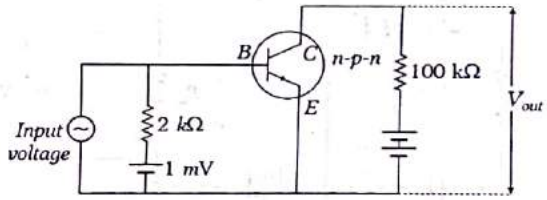
$$V_{CE} = V_C - I_C R_C = 20 - (2.5 \times 10^{-3}) \times (5 \times 10^3) = 7.5 \text{ V}$$

$$V_{BE} = 20 - I_B R_B = 20 - (0.0125 \times 10^{-3}) \times (120 \times 10^3)$$
$$= 18.5 \text{ V}$$

$\therefore$ $V_{BC} = (V_{BE} - V_{CE}) = (18.5 - 7.5) = 11$ V

The transistor is in active state because *EB* region is forward biased and *CB* region is reverse biased.

Example 14.45 *In the following common emitter configuration an n-p-n transistor with current gain β = 95 is used. What is the output voltage of the amplifier?*



Sol. Input current, $I_{in} = \frac{V_{in}}{R_{in}} = \frac{1\text{ mV}}{2\text{k}\Omega} = 0.5 \times 10^{-6}$ A

$\because$ $\beta = \frac{I_{out}}{I_{in}}$

$\therefore$ Output current,

$$I_{out} = \beta \cdot I_{in}$$
$$= 95 \times 0.5 \times 10^{-6}\text{ A}$$
$$= 0.47 \times 10^{-4}\text{ A}$$

$\therefore$ Output voltage,

$$V_{out} = I_{out} \cdot R_{out}$$
$$= (0.47 \times 10^{-4}\text{ A})(100\text{k}\Omega) = 4.7\text{mV}$$

Example 14.46 *A p-n-p transistor is used in common-emitter mode in an amplifier circuit. A change of 45 μA in the base current brings a change of 3 mA in collector current and 0.05 V in base-emitter voltage.*
Find the

(a) input resistance r_i and

(b) the base current amplification factor (β).

(c) If a load of 7kΩ is used, then also find the voltage gain of the amplifier.

Sol. Given,

$$\Delta I_B = 45\,\mu A = 45\times10^{-6}A,\ \Delta I_C = 3mA = 3\times10^{-3}A$$

$$\Delta V_{BE} = 0.05V,\ R_L = 7k\Omega = 7\times10^3\Omega$$

(a) Input resistance

$$r_i = \frac{\Delta V_{BE}}{\Delta I_B} = \frac{0.05}{45\times10^{-6}} = 1.1\times10^3\,\Omega$$

(b) Base current amplification factor

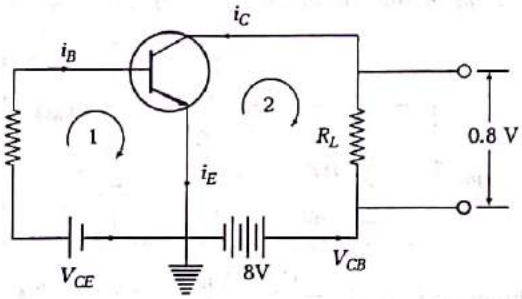
$$\beta = \frac{\Delta I_C}{\Delta I_B} = \frac{3\times10^{-3}}{45\times10^{-6}} = 66.67$$

(c) Voltage gain of the amplifier

$$A_V = \beta\frac{R_L}{r_i} = \frac{66.67\times7\times10^3}{1.1\times10^3} = 424.27$$

Example 14.47 *An n-p-n transistor is connected in common emitter configuration in which collector supply is 8 V and the voltage drop across the load resistance of 800 Ω connected in the collector circuit is 0.8 V V. If current amplification factor is, 25, determine collector-emitter voltage and base current. If the internal resistance of the transistor is 200 Ω, calculate the voltage gain and the power gain.*

Sol. The corresponding circuit is shown in figure.



Voltage across $R_L = i_c R_L = 0.8$ V (given)

$$\therefore \quad i_c = \frac{0.8}{R_L} = \frac{0.8}{800}\,A = 1\ mA$$

Further it is given that, $\beta = 25 = \frac{i_c}{i_b}$

$$\therefore \quad i_b = \frac{i_c}{25} = 40\,\mu A$$

Collector-Emitter Voltage (V_{CE})

Applying Kirchhoff's second law in loop 2, we have

$$V_{CE} = V_{CC} - i_c R_L = (8 - 0.8)\ V = 7.2\ V$$

Voltage gain (A_V)

Voltage gain, $A_V = \beta\left(\frac{R_{out}}{R_{in}}\right)$ or $A_V = 25\left(\frac{800}{200}\right) = 100$

Power gain

$$\text{Power gain} = \beta^2\left(\frac{R_{out}}{R_{in}}\right) = (25)^2\left(\frac{800}{200}\right) = 2500$$

Note Kirchhoff's laws can be applied in a transistor circuit in the similar manner as is done in normal circuits.

Transistor as an oscillator

An oscillator is an electronic device which produces electric oscillation of constant frequency and amplitude, without any externally applied input signal. It converts DC energy obtained from a battery into AC energy.

1. Principle of an oscillator

Figure shows the block diagram of an oscillator. An oscillator may be regarded as amplifier which provides its own input signal.

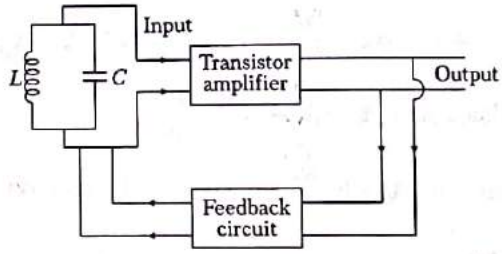


Fig. 14.54 *Principle for an oscillator*

Essential parts of a transistor oscillator are

(i) **Tank Circuit**
It is a parallel combination of an inductance L and a capacitance C. The frequency of electric oscillations in the tank circuit is

$$f = \frac{1}{2\pi\sqrt{LC}}$$

(ii) **Transistor Amplifier** The amplifier receives the oscillations from the tank circuit and amplifies it.

(iii) **Feedback Circuit** It returns a part of the output power of the transistor amplifier to the tank circuit and produces undamped oscillations. This process is called positive feedback.

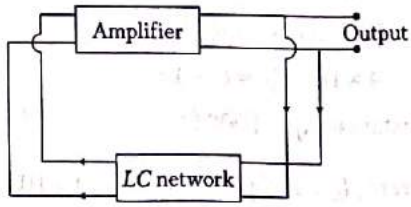


Fig. 14.55 Feedback circuit

2. Construction

A basic circuit using a common-emitter *n-p-n* transistor as an oscillator is given below. A tank circuit (*L-C* circuit) is connected in the emitter-base circuit. A small coil *L* called **feedback or tickler** coil is connected in the emitter-collector circuit. The coil *L* is inductively coupled with the coil *L* of the tank circuit.

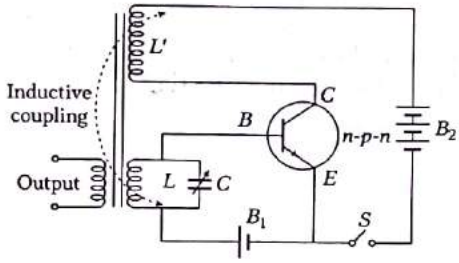


Fig. 14.56 Transistor as an oscillator

3. Working

When the switch *S* is closed, electrical oscillations are developed in the tank circuit. The circuit amplifies these oscillations. A part of the amplified signal in the collector circuit is feedback in the base circuit by the coupling between *L* and *L'*. This cycle repeats again and again to give electric oscillations of constant amplitude and of constant frequency

$$f = \frac{1}{2\pi\sqrt{LC}}$$

The oscillations of the desired frequency can be obtained by changing the value of capacitance *C* of the variable capacitor. The oscillations can be transferred to an external circuit by mutual induction in coil connected in that circuit.

4. Need for positive feedback

The oscillations are damped due to the presence of some inherent electrical resistance in the circuit. Hence, the amplitude of oscillations decreases rapidly and ultimately stops. To have oscillations of constant amplitude, a positive or negative feedback is necessary from the output circuit to the input circuit so that the losses in the circuit can be compensated.

Note

1. Gain of the amplifier with positive feedback is given by

$$A = \frac{V_o}{V_i} \text{ (Gain without feedback)}$$

2. $A = \dfrac{V_0}{V_i + mV_0}$ (Gain with feedback)

where, m = feedback fraction

Transistor as a switch

Transistor has an ability to turn things ON and OFF. So, the most common use of transistor in an electronic circuit is as simple switches.

A transistor can be used as a switch because its collector current is directly controlled by the base current.

To understand it properly, letus consider a basic circuit using a common-emitter *n-p-n* transistor as a switch which is given below.

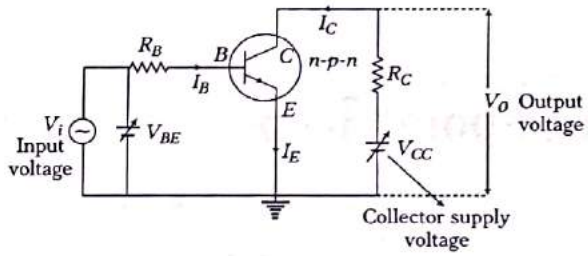


Fig. 14.57 Common-emitter *n-p-n* transistor as a switch

Apply Kirchhoff's voltage rule to the input and output sides of this circuit, we get

$$V_i = I_B \cdot R_B + V_{BE} \quad (V_i = \text{DC input voltage})$$

and

$$V_o = V_{CC} - I_C \cdot R_C \quad (V_i = \text{DC output voltage})$$

In case of silicon transistor, if $V_i < 0.6$ V, I_B will be zero, hence, I_C will be zero and transistor will operate in **cut-off state,** and $V_0 = V_{CC}$.

When $V_i > 0.6$ V, some I_B flows, so some I_C flows and transistor will now operate in **active state** and hence output V_0 decreases as the term $I_C R_C$ increases. With increase in V_i, the I_C increases almost linearly and hence V_0 decreases linearly till its value becomes less than about 1 volt.

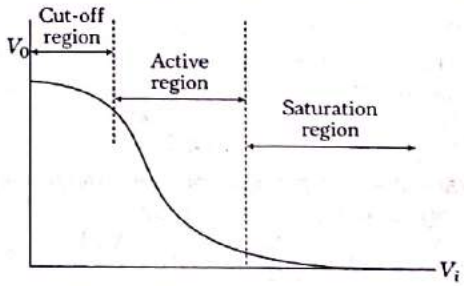


Fig. 14.58 Transfer characteristic

Beyond this, the change becomes non-linear and transistor now goes into the saturation state. When V_i is further increased, V_0 is found to decrease towards zero (however, it may never become to zero).

Now the operation of transistor as a switch can be understood in the following way. When V_i is low (in

cut-off region), V_0 is high. If V_i is high (in saturation region), then V_0 is low, almost zero.

If we define low and high states as below and above certain voltage levels corresponding to cut-off and saturation of the transistor, then we can say that a low input switches the transistor off and high input, switches it on. Alternatively, we can say that a low input, to the transistor gives a high output and a high input gives a low output. The switching circuits are designed in such a way that the transistor does not remain in active state.

Effect of temperature on the working of transistor

A transistor is a temperature-sensitive device. In the transistor, conduction is due to the movement of charge carriers, *i.e.*, electrons and holes. When temperature of the transistor increases, many covalent bonds may break up, resulting in the formation of more electrons and holes. Due to which current will increase in the transistor and this current gives rise to the production of more heat energy. The excess heat so produced causes complete breakdown of the transistor.

Check point 14.3

1. In a *p-n-p* transistor the base is the *n*-region. Its width relative to the *p*- region is
(a) smaller (b) larger
(c) same (d) not related

2. The heavily and lightly doped region of a bipolar junction transistor are respectively.
(a) base and emitter (b) base and collector
(c) collector and base (d) emitter and base

3. The concentrations of impurities in a transistor are
(a) equal for the emitter, base and collector region
(b) least for the emitter region
(c) largest for the emitter region
(d) largest for the base region

4. An *n-p-n* transistor conducts when
(a) both collector and emitter are positive with respect to the base
(b) collector is positive and emitter is negative with respect to the base
(c) collector is positive and emitter is at same potential as the base
(d) both collector and emitter are negative with respect to the base.

5. In the *CB* mode of a transistor when the collector voltage is changed by 0.5 V. The collector current changes by 0.05 mA. The output resistance will be
(a) 10 kΩ (b) 20 kΩ
(c) 5 kΩ (d) 2.5 kΩ

6. For a transistor, in a common emitter arrangement, the alternating current gain β is given by
(a) $\beta = \left(\frac{\Delta I_C}{\Delta I_B}\right)_{V_C}$ (b) $\beta = \left(\frac{\Delta I_B}{\Delta I_C}\right)_{V_C}$ (c) $\beta = \left(\frac{\Delta I_C}{\Delta I_E}\right)_{V_C}$ (d) $\beta = \left(\frac{\Delta I_E}{\Delta I_C}\right)_{V_C}$

7. While a collector to emitter voltage is constant in a transistor, the collector current changes by 8.2 mA when the emitter current changes by 8.3 mA. The value of forward current ratio *B* is
(a) 82 (b) 83 (c) 8.2 (d) 8.3

8. In a common base mode of a transistor, the collector current is 5.488 mA for an emitter current of 5.60 mA. The value of the base current amplification factor (β) will be
(a) 50 (b) 51 (c) 48 (d) 49

9. Consider an *n-p-n* transistor amplifier in common-emitter configuration. The current gain of the transistor is 100. If the collector current changes by 1mA. What will be the change in emitter current.
(a) 1.1 mA (b) 1.01 mA
(c) 0.01 mA (d) 10 mA

10. When *p-n-p* transistor is used as an amplifier
(a) electrons move from base to emitter
(b) holes move from emitter to base
(c) electrons move from base to emitter
(d) holes move from base to emitter

11. When *n-p-n* transistor is used as an amplifier
(a) electrons move from base to collector
(b) holes move from emitter to base
(c) electron move from collector to base
(d) holes move from base to emitter

12. Which of the following is used to produce radio waves of constant amplitude?
(a) oscillator (b) FET (c) rectifier (d) amplifier

13. An amplifier has a voltage gain $A_V = 1000$. The voltage gain in dB is
(a) 30 dB (b) 60 dB (c) 3 dB (d) 20 dB

14. An oscillator is nothing but an amplifier with
(a) positive feedback (b) large gain
(c) no feedback (d) negative feedback

15. A common-emitter amplifier gives an output of 3 V for an input of 0.01V. If β of the transistor is 100 and the input resistance is 1kΩ; then the collector resistance is
(a) 1 kΩ (b) 3 kΩ
(c) 30 kΩ (d) 30 Ω

5 Analog and digital circuits

All electronic circuits are broadly divided into two categories

1. Analog circuits

A continuous time-varying voltage or current signal is called analog signal.

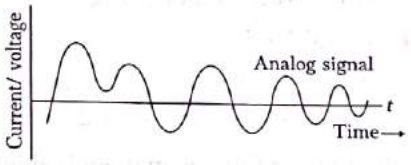


***Fig. 14.59** : Analog signal*

The electronic circuits which process analog signals are called analog circuits.

The devices like amplifiers, radio, television, oscillators etc. make use of analog signals.

2. Digital circuits

A signal that can take only two discrete values of current or voltage is called digital signals.

A digital signal can take only two values, *i.e.*, 0 and 1 which are marked as low and high values respectively.

In the square wave shown in Fig 14.60, a signal of 0 V represents binary 0 and a signal of 5V represents binary 1.

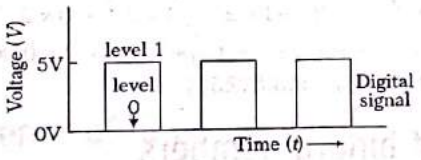


***Fig. 14.60** Digital signal*

The electrical circuits which process digital signals are called digital circuits. *e.g.*, Pocket calculators, burglar alarms, modern computers etc.

Advantages of digital circuits

(i) It has high accuracy and precision.
(ii) It is easier to design.
(iii) Information can be stored easily.
(iv) Digital circuits are less affected by the noise.
(v) They typically uses less bandwidth.
(vi) It enables long distance transmission.

Binary System

There are number of questions which have only two answers Yes or No. A statement can be either True or False. A switch can be either ON or OFF. These values may be represented by two symbols 0 and 1. In a number system in which we have only two digits is called a **binary system.**

In binary system usually we write 1 for positive response (*e.g.*, when a switch is ON) and 0 for negative (when switch is OFF).

Decimal and binary number systems

1. Decimal number system

In decimal number system, ten digits are used, *i.e.*, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. The base of the decimal number system is 10.

The place values are found by raising the base 10 to the power of the place.

Also, powers are numbered to the left of the decimal point starting with 0, and to the right of the decimal point starting with −1.

e.g., 2876 = 2000 + 800 + 70 + 6

MSD LSD

$$= 2 \times 10^3 + 8 \times 10^2 + 7 \times 10^1 + 6 \times 10^0$$

LSD = Least significant digit

MSD = Most significant digit

2. Binary number system

In binary number system, only two digits 0 and 1 are used. In binary system, the base is 2. The two binary digits 0 and 1 are called 'bits' and a group of bits is known as a byte.

In a binary number, the place value of each bit corresponds to some power of 2. For example,

$$1101.011 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + 0 \times 2^{-1} + 1 \times 2^{-2} + 1 \times 2^{-3}$$

$$= 8 + 4 + 0 + 1 + 0 + 0.25 + 0.125$$

$$= 13.375$$

3. Conversion of a decimal number into its equivalent binary number

A decimal number can be converted into binary number by using divide by 2 rule. We go on dividing the given decimal number by 2, until the quotient is zero and write down the remainder after each division. These remainders when taken in reverse order form the required binary number. Let us convert 23 into its binary equivalent.

Table

Divide by 2	23	Remainder
Divide by 2	11	1 LSD
Divide by 2	5	1
Divide by 2	2	1
Divide by 2	1	0
	0	1 MSD

The remainders in reverse order are 10111. Hence, the binary equivalent of 23 is 10111. Symbolically we write,

$$(23)_{10} = (10111)_2$$

The conversion of a decimal fraction into binary fraction is performed by multiply-by-2 rule. Let us convert 0.125 into its binary equivalent.

Table

	Binary
$0.125 \times 2 = 0.250$	0
$0.250 \times 2 = 0.500$	0
$0.500 \times 2 = 1.000$	1
$0.000 \times 2 = 0.000$	0

The recovered binaries taken from top to bottom form the required binary fraction. The binary equivalent of 0.125 is therefore written 0.0010.

Note If a decimal number contains both the integral and the fractional part, such as 23.125, then after determining the binary equivalent of each part separately, the two parts are combined. Thus, binary equivalent of 23.125 is 10111.0010.

Example 14.57 *Convert the decimal number* 10.625 *into its binary equivalent.*

Sol. Integral part is 10. It can be converted into its binary part by using divide by 2 rule.

Table

Divide by 2	10	Remainder
Divide by 2	5	0
Divide by 2	2	1
Divide by 2	1	0
	0	1

Thus, $(10)_{10} = (1010)_2$

Fractional part is 0.625. It can be converted into its binary part by using multiply-by-2 rule.

Table 28

	Binary
$0.625 \times 2 = 1.250$	1
$0.250 \times 2 = 0.500$	0
$0.500 \times 2 = 1.000$	1
$0.000 \times 2 = 0.000$	0

Thus, $(0.625)_{10} = 0.1010$

$\therefore$ $(10.625)_{10} = (1010.1010)_2$

4. Conversion of a binary number into its equivalent decimal number

In binary number system, base is 2. Therefore, the least significant digit in the binary number is the coefficient of 2 with power zero. As we move towards the left side of LSD, the power of 2 goes on increasing.

For example $(\underset{\text{MSD}}{\underset{\downarrow}{1}}00110\ \underset{\text{LSD}}{\underset{\downarrow}{1}})_2$

$$= 1 \times 2^6 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^0 = 77$$

Note that, while finding the decimal equivalent of a fractional binary number, the multipliers will be $2^{-1}, 2^{-2}, 2^{-3} \ldots$ starting from MSD, till LSD is reached.

For example $(0.\underset{\text{MSD}}{\underset{\downarrow}{1}}01\ \underset{\text{LSD}}{\underset{\downarrow}{1}})$

$$= 1 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3} + 1 \times 2^{-4}$$

$$= \frac{1}{2} + 0 + \frac{1}{8} + \frac{1}{16} = 0.6875$$

$$(0.1011)_2 = (0.6875)_{10}$$

Example 14.58 *Convert the binary number* 101011.1101 *into equivalent decimal number.*

Sol. $101011.1101 = 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 0 \times 2^2$

$$+ 1 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2} + 0 \times 2^{-3} + 1 \times 2^{-4}$$

$$= 32 + 0 + 8 + 0 + 2 + 1 + 0.5 + 0.25 + 0 + 0.0625 = 43.8125$$

$\therefore (101011.1101)_2 = (43.8125)_{10}$

5. Sum of binary numbers

The sum of digits 0 and 1 with themselves and with each other are given by the following laws.

(i) $0 + 0 = 0$ (ii) $1 + 0 = 1$

(iii) $0 + 1 = 1$ (iv) $1 + 1 = 10$

Now, let us take few examples of different types. Through these examples we can easily understand the method of adding two binary numbers.

Example 14.59 *Add the binary numbers* 101 *and* 11.

Sol. We first write these numbers in columns as below.

$$\begin{array}{r} 1\,1\,1 \\ +\,1\,1 \\ \hline 1\,0\,0\,0 \end{array}$$

Rule Starting from right, in the first column we get $1 + 1 = 10$. The 0 is written below this column and 1 is carried in the next column. In the second column,

$1 + (0 + 1) = 1 + 1 = 10$. From the value 0 is written below this column and 1 is carried in the third column. Now in the third column $1 + 1 = 10$ is obtained. Thus, $101 + 11 = 1000$

Example 14.60 *Add the binary numbers* 101 *and* 110.

Sol. Rewriting the numbers in columns as below.

$$\begin{array}{r} 101 \\ +110 \\ \hline 1011 \end{array}$$

The sum is in accordance with the four laws given in the beginning. Thus, $101 + 110 = 1011$

Example 14.61 *Add the binary numbers* 1101 *and* 1110.

Sol. Writing the numbers in columns we have,

$$\begin{array}{r} 1101 \\ +1110 \\ \hline 11011 \end{array}$$

Here, in the last column we had $1 + (1 + 1)$ which we wrote 11. This is because, $1 + (1 + 1) = 1 + 10 = 10$

$$\begin{array}{r} +1 \\ \hline 11 \end{array}$$

Example 14.62 *Add the binary numbers* 111, 101 *and* 110.

Sol. For adding more than two binary numbers the above method is repeated again and again. If we want to add 111, 101 and 110. First we will add 101 and 101 by the method discussed above, then add 110 with the sum of 101 and 101.

$$\begin{array}{r} 111 \\ 101 \\ \hline 1100 \end{array}$$

Now add 1100 and 110

$$\begin{array}{r} 1100 \\ +110 \\ \hline 10010 \end{array}$$

6. Difference of binary numbers

The four laws of difference (in binary system) are as below :

(i) $0 - 0 = 0$ (ii) $1 - 0 = 1$ (iii) $1 - 1 = 0$ (iv) $10 - 1 = 1$

Now, let us take an example in support of the above four laws.

Example 14.63 *Subtract the binary number* 100 *from the binary number* 101.

Sol. Let us write the given numbers in columns as below,

$$\begin{array}{r} 101 \\ -100 \\ \hline 001 \end{array}$$

Thus, $101 - 100 = 001$ or simply 1.

Example 14.64 *Subtract the binary number* 1110 *from the binary number* 11011.

Sol. Rewriting the given numbers in columns as below.

$$\begin{array}{r} 11011 \\ -1110 \\ \hline 1101 \end{array}$$

In this example, first and second column were subtracted in accordance with the four laws given in the starting. In the third column, for subtracting 1 from 0 we carry 1 from the fourth column. Thus, $10 - 1 = 1$ is obtained. In the fourth column, for subtracting 1 from 0 (which remains 0 after giving 1 to the third column) we carry 1 from the fifth column. Thus $10 - 1 = 1$ is obtained.

Thus, $11011 - 1110 = 1101$

6 Logic gates

Logic gates are the building blocks of digital circuit that makes use of diodes and transistors to perform switching function. It is used for performing a particular logical operations. In other words,

A gate is a digital circuit that follows certain logical relationship between the input and output voltages.

A logic gate has one output but one or more inputs. There are three basic gates named as OR gate, AND gate and NOT gate.

Each basic logic gate is indicated by a logic symbol and its function is defined and described either by a truth table or by a boolean expression.

1. Truth table

It is a table that shows all possible input combinations and the corresponding output combinations for a logic gate. To understand the concept of truth table, let us take an example. A bulb is connected to an AC source *via* two switches S_1 and S_2.

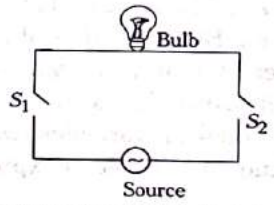


Fig. 14.61 *A bulb is connected via two switches*

In binary system, we will write 0, if the switch (or bulb) is off and write 1 if it is on. Further, let us write

A for state of switch S_1

B for state of switch S_2

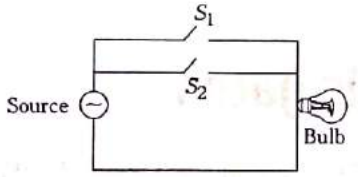
and C for state of the bulb.

Now, let us make a table (called truth table).

Switch S_1	Switch S_2	Bulb	A	B	C
Off	On	Off	0	1	0
On	Off	Off	1	0	0
Off	Off	Off	0	0	0
On	On	On	1	1	1

Here, A and B represents the input combination and C represents the output for this particular circuit. This table is called truth table and is used to define or decribe the function of a particular logic gate. Now, let us discuss one example based on this.

Example 14.48 *Make a truth table corresponding to the circuit shown in figure.*



Sol. Here, the two switches can be operated in different combinations. Note that the two switches are connected in parallel. So, if one switch is OFF, the bulb will still glow. We will write 0, if the switch is OFF and write 1 if it is ON. Further, A stands for state of switch S_1 and B stands for state of switch S_2 and C for state of bulb. Now, let us make a truth table based on this.

Switch S_1	Switch S_2	Bulb	A	B	C
On	Off	On	1	0	1
Off	On	On	0	1	1
Off	Off	Off	0	0	0
On	On	On	1	1	1

2. Boolean expression

Boolean algebra is a different kind of algebra in which only two states of variables (0 and 1) called boolean variables are allowed. Hence, to describe the functioning of a logic gate in terms of an equation, we use boolean expression. It is defined as the expression showing the combination of two Boolean variables resulting into a new Boolean variable known as Boolean expression.

Some useful postulates and laws of boolean algebra

(i) **Boolean postulates**

(a) $0 + A = A$ (b) $1 + A = 1$

(c) $0 \cdot A = 0$ (d) $1 \cdot A = A$

(e) $A + \overline{A} = 1$

(ii) **Identity law**

(a) $A + A = A$ (b) $A \cdot A = A$

(iii) **Negation law** $\overline{\overline{A}} = A$

(iv) **De-Morgan's theorem** It states that the complement of the whole sum is equal to the product of individual complements and *vice-versa*, *i.e.*,

(a) $\overline{A + B} = \overline{A} \cdot \overline{B}$ (b) $\overline{A \cdot B} = \overline{A} + \overline{B}$

De-Morgan's theorem also states that,

(a) $\overline{\overline{A} + \overline{B}} = \overline{\overline{A}} \cdot \overline{\overline{B}} = A \cdot B$

(b) $\overline{\overline{A} \cdot \overline{B}} = \overline{\overline{A}} + \overline{\overline{B}} = A + B$

(v) **Commutative laws**

(a) $A + B = B + A$ (b) $A \cdot B = B \cdot A$

(vi) **Associative laws**

(a) $A + (B + C) = (A + B) + C$

(b) $A \cdot (B \cdot C) = (A \cdot B) \cdot C$

(vii) **Distributive laws**

(a) $A \cdot (B + C) = A \cdot B + A \cdot C$

(b) $(A + B) \cdot (A + C) = A + BC$

(viii) **Absorption laws**

(a) $A + A \cdot B = A$ (b) $A \cdot (A + B) = A$

(c) $A \cdot (A + \overline{A}) = A$ (d) $\overline{A} \cdot (A + B) = \overline{A} \cdot B$

(ix) **Boolean identities**

(a) $A + \overline{A}B = A + B$

(b) $A(\overline{A} + B) = AB$

(c) $A + BC = (A + B)(A + C)$

(d) $(\overline{A} + B) \cdot (A + C) = \overline{A}C + AB$

Logic system

Let us now study the three types of logic gates in detail.

1. OR gate

An OR gate has two or more inputs and one output. In this gate, if any of the input or all the inputs are 1 then output is 1.

Let the two input of OR gate be A and B respectively and its output be Y.

Or Operation Consider the circuit given below.

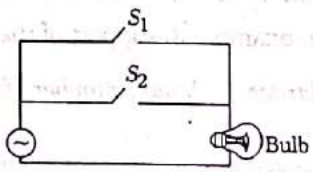


Fig. 14.62 *Circuit for OR operation*

The two switches are connected in parallel. So, when S_1 OR S_2 is closed, the bulb is ON, *i.e.*, it glows. So, it represents OR operation.

Boolean expression

$$Y = A + B$$

Logic symbol

Fig. 14.63 Logic symbol of OR gate

Truth table

Input		Output
A	B	$Y = A + B$
0	0	0
0	1	1
1	0	1
1	1	1

OR Gate using diodes

As shown in figure a two input OR gate can be realised by using two ideal diodes D_1 and D_2 and a resistor R. The negative terminal of the battery is grounded (*i.e.*, it is at zero volt) and corresponds to the 0 state, and the positive terminal (which is at say 5V) corresponds to the 1 state.

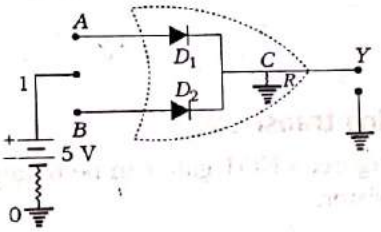


Fig. 14.64 Realisation of OR gate using two p-n junction diodes

The following four cases are possible

(a) **When $A = 0$ and $B = 0$.** Both the diodes are connected to earth (0V). They do not conduct. Output across R is zero, *i.e.*, $Y = 0$.

(b) **When $A = 0$ and $B = 1$.** D_1 is connected to earth. It does not conduct. D_2 is connected to 5V, it gets forward biased and conducts. Voltage drop across D_2 is zero and the full voltage of 5V appears across R. So $Y = 1$.

(c) **When $A = 1$ and $B = 0$.** D_1 gets forward biased and D_2 does not conduct. Voltage drop across R is again 5V. So $Y = 1$.

(d) **When $A = 1$ and $B = 1$.** Both D_1 and D_2 get forward biased and conduct current. But D_1 and D_2 are in parallel. Voltage drop across R is still 5V. So $Y = 1$.

Hence, the circuit shown in figure above satisfies the truth-table of OR gate.

(vi) Input and output waveforms for an OR gate

The output of an OR gate is high when either A or B or both the inputs are high. Therefore the output waveform Y will be as shown in Fig. 14.65.

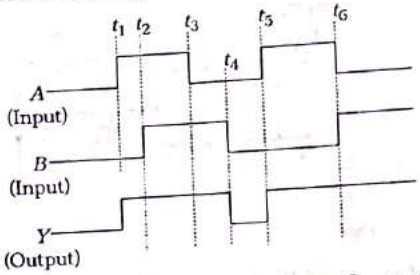


Fig. 14.65 Input and output waveforms for an OR gate

2. AND gate

It has two or more inputs and one output. In this gate, output is 1 only when all the inputs are 1. Let the inputs of AND gate be A and B respectively and its output be Y.

AND operation

Consider the circuit given below

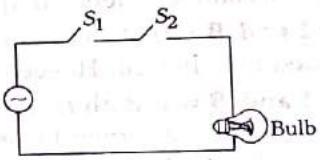


Fig. 14.66 Circuit for AND operation

Now, the two switches are connected in series. If any one switch is open, the current will now flow and hence bulb will not glow. So, when S_1 AND S_2 is closed, bulb is on, *i.e.*, it glow. So, it represents AND operation.

Boolean expression

$$Y = A \cdot B$$

Logic symbol

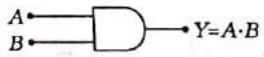


Fig. 14.67 Logic symbol of AND gate

Truth table

Input		Output
A	B	$Y = A \cdot B$
0	0	0
0	1	0
1	0	0
1	1	1

AND gate using diodes

As shown in figure two input AND gate can be realised by using two ideal junction diodes D_1 and D_2. Here the resistance R is kept permanently connected to the +ve terminal of 5V battery.

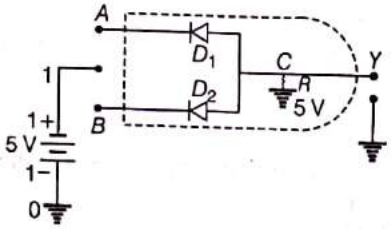


Fig. 14.68 Realisation of an AND Gate

The following four cases are possible:

(a) **When $A = 0$ and $B = 0$.** The input terminals A and B are earthed (0V). The two diodes get forward biased and conduct current. But both diodes are shorted. The point Y also gets earthed through the shorted diodes. Hence output $Y = 0$.

(b) **When $A = 0$ and $B = 1$.** Diode D_1 is forward biased but shorted. Diode D_2 is not forward biased and hence does not conduct. Hence output $Y = 0$.

(c) **When $A = 1$ and $B = 0$.** D_1 does not conduct D_2 is forward biased but shorted. Hence output $Y = 0$.

(d) **When $A = 1$ and $B = 1$.** Both D_1 and D_2 do not conduct as they are not forward biased. The output voltage is equal to the battery voltage of 5V. Hence, $Y = 1$ Hence, the circuit shown in figure above satisfies the truth table of AND gate.

Input and output waveform for an AND gate

The output of an AND gate is high when both the inputs are high, otherwise the output is low. Therefore, the output waveform Y for AND gate will be as shown in Fig 14.68

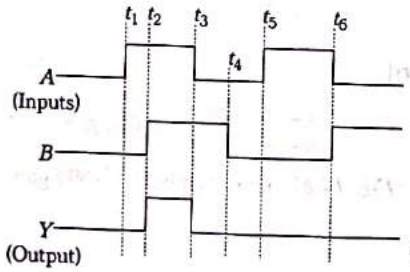


Fig. 14.69 Input and output waveform of an AND gate

3. NOT gate

It has only one input and one output.

It gives an inverted version of its input, *i.e.*, if input is 1, then output is 0 and *vice-versa*. Let the input of NOT gate be A and its output be Y.

NOT operation

Consider the circuit given below

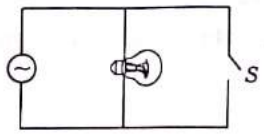


Fig. 14.70 Circuit for NOT operation

Here, only one switch is used and it is connected in parallel with bulb. So, when S is open, bulb is ON, i.e., it glows. It represents inverted or NOT operation.

Boolean expression

$$Y = \overline{A}$$

Logic symbol

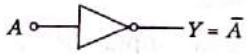


Fig. 14.71 Logic symbol NOT gate

Truth table

Input	Output
A	$Y = \overline{A}$
0	1
1	0

NOT gate using transistor

As shown in figure, a NOT gate can be obtained by using an n-p-n transistor.

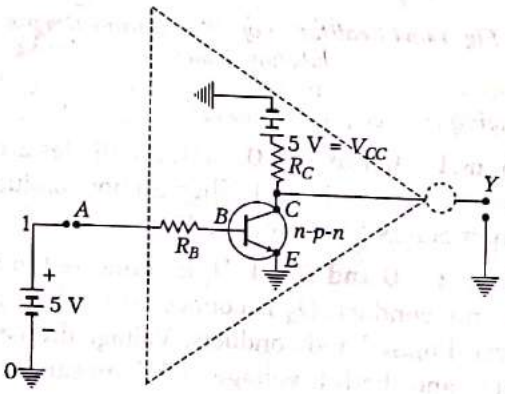


Fig. 14.72 Realisation of NOT gate

The base resistor R_B and the collector resistance R_C are so chosen that when a voltage of 5V is applied at the base of the transistor, a large collector current flow, the voltage at Y drops and the base-collector junction is forward biased.

The following two cases are possible :

(i) **When input $A = 0$.** The base of the transistor is earthed, the base-emitter junction is not forward biased and the collector-base junction is reverse biased. Hence the base current and the collector current are both zero. The transistor is in the cut-off

mode. No voltage drop occurs across R_C. The voltage at Y is 5 V. Hence output $Y = 1$

(ii) **When input $A = 1$.** The input terminal A is at 5 V, both emitter and collector are forward biased. A large collector current flows. The transistor is in the saturation mode. The voltage drop across R_C is almost 5 V. Hence, output $Y = 0$. Thus, the circuit of given figure satisfies the truth table of a NOT gate.

Input and output waveforms for NOT gate

The output of a NOT gate is high when the input is low and *vice-versa*. Therefore, the output waveform Y for NOT gate is as shown in Fig 14.73

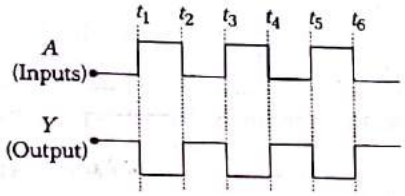


Fig. 14.73: Input and waveforms for NOT gate

Combination of logic gates

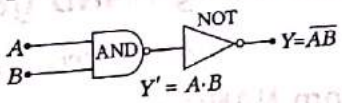
Various combinations of three basic gates (*i.e.*, OR, AND and NOT) give rise to complicated digital circuits. Let us now discuss a few combinations of these basic gates using their symbols.

1. NAND gate

It has two or more inputs and one output. This is an AND gate followed by a NOT gate. In this gate, if all the inputs are 1, then output will be 0. Let the two inputs of NAND gate be A and B respectively and its output be Y.

Boolean expression

$$Y = \overline{A \cdot B} \quad \text{or} \quad Y = \overline{AB}$$



Logic symbol

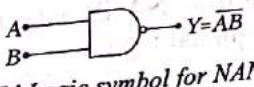


Fig. 14.74 Logic symbol for NAND Gate

Truth table

Input			Output
A	B	$Y' = A \cdot B$	$Y = \overline{AB}$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

Input and output waveforms for NAND gate

The output waveform for NAND gate will be as shown in Fig. 14.75.

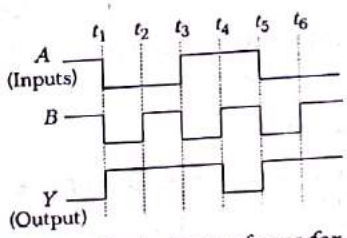


Fig. 14.75 Input and output waveforms for NAND gate

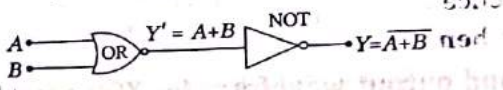
2. NOR gate

It has two or more inputs and one output. When a NOT operation is applied after OR gate, then it is called NOR gate.

In this gate, if all the inputs are 0, then only output will be 1 otherwise it is 0. Let the two inputs be A and B respectively and its output be Y.

Boolean expression

$$Y = \overline{A + B}$$



Logic symbol

Fig. 14.76 : Logic symbol of NOR gate

Truth table

Input			Output
A	B	$Y' = A + B$	$Y = \overline{A + B}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

Input and output waveforms for NOR gate

The output waveform for NOR gate will be as shown in Fig. 14.77

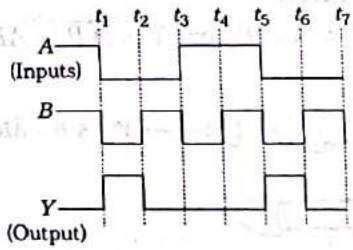


Fig. 14.77 Input and output waveforms for NOR gate

3. XOR gate

It is also called the exclusive OR function. It is a function of two logical variables A and B which evaluates to 1 if one of two variables is 0 and the other is 1. The function is zero, if both the variables are 0 or 1.

Boolean expression

$$Y = A \text{ XOR } B \text{ or } Y = A \oplus B \text{ or } Y = \overline{A}B + A\overline{B}$$

Logic symbol

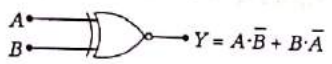


Fig. 14.78 Logic symbol of XOR gate

Truth table

Input						Output
A	B	$\overline{A}$	$\overline{B}$	$A \cdot \overline{B}$	$\overline{A} \cdot B$	$A = A \cdot \overline{B} + \overline{A} \cdot B$
0	0	1	1	0	0	0
0	1	1	0	0	1	1
1	0	0	1	1	0	1
1	1	0	0	0	0	0

Input and output waveform for XOR gate

The output waveform for XOR gate will be as shown in Fig. 14.79.

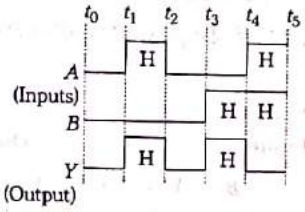


Fig. 14.79 Input and output waveform for XOR gate

4. XNOR gate

It is also called exclusive NOR function. It is a function of two logical variables A and B which evaluates to 0, if one of two variable is 0 and the other is 1. The function is one, if both the variables are 0 or 1.

Boolean expression

$$Y = A \odot B \quad \text{or} \quad Y = \overline{A}\,\overline{B} + AB$$

Logic symbol

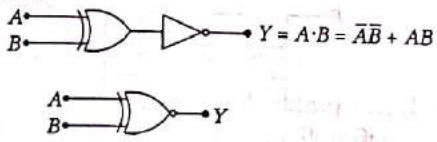


Fig. 14.80 : Logic symbol of XNOR gate

Truth table

Input		Output
A	B	$Y = \overline{A}\,\overline{B} + AB$
0	0	1
0	1	0
1	0	0
1	1	1

Input and output waveform for XNOR gate

The output waveform Y for XNOR gate is as shown in Fig. 14.81

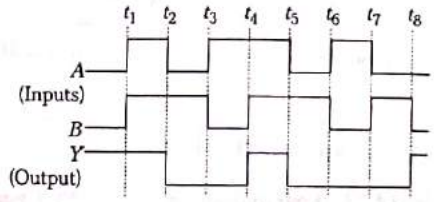


Fig. 14.81 Input and output waveform for XNOR gate

NAND and NOR gates as digital building blocks

The repeated use of the OR, AND or the NOT gates alone cannot given a different gate. But the repeated use of the NAND or the NOR gates alone can give all basic gates like OR, AND and NOT gates. Hence, the NAND and the NOR gates are also called **universal gates**. In digital circuits, these gates serve as digital building blocks. Now, let us derive the basic logic gates one by one from NAND and NOR gates.

1. Logic gates using NAND gate

Using NAND gate we can realise different gates as follows

NOT gate from NAND gate

To obtain NOT gate from NAND gate, the two inputs of the NAND gate are joined together.

Boolean expression

The Boolean expression for a NAND gate is given by

$$Y = \overline{A \cdot B}$$

When $B = A$,

$$Y = \overline{A \cdot A}$$

$$= \overline{A} \qquad [\because A \cdot A = A]$$

It is the Boolean expression for NOT gate.

Logic symbol

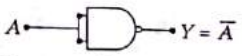


Fig. 14.82 Realisation of NOT gate using NAND gate

Truth table

Input	Output
$A = B$	Y
0	1
1	0

AND gate from NAND gate

To obtain AND gate form NAND gate, one NOT gate (made from NAND gate) is used. The output of the NAND gate is given to the input of the NOT gate. The logic circuit so obtained will work as AND gate.

Boolean expression

The output of the NAND gate is given by

$$Y' = \overline{AB}$$

This output will form the input of the NOT gate.

$\therefore$ Final output is given by

$$Y = \overline{\overline{AB}}$$

$$= A \cdot B \quad \text{(Boolean expression for AND gate)}$$

Logic symbol

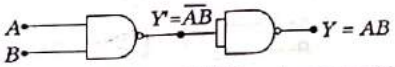


Fig. 14.83 Realisation of AND gate using NAND gate

Truth table

A	B	Y'	Y
0	0	1	0
0	1	1	0
1	0	1	0
1	1	0	1

OR gate from NAND gate

To obtain OR gate from NAND gate, two NOT gates are used. The outputs of two NOT gate (obtain from NAND gate) is given to the inputs of the NAND gate. The logic circuit so obtained will work as OR gate.

Boolean expression

The Boolean expression for a NAND gate having $\overline{A}$ and $\overline{B}$ as its two input is

$$Y = \overline{(\overline{A} \cdot \overline{B})}$$

$$= \overline{\overline{A}} + \overline{\overline{B}} \quad \text{(Applying De Morgan's theorem)}$$

$$= A + B \quad \text{(Boolean expression for OR gate)}$$

Logic symbol

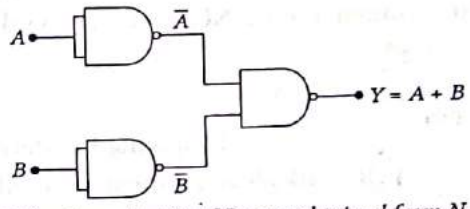


Fig. 14.84 Logic symbol for OR gate obtained from NAND gate

Truth table

Input				Output
A	B	$\overline{A}$	$\overline{B}$	Y
0	0	1	1	0
0	1	1	0	1
1	0	0	1	1
1	1	0	0	1

2. Logic gates using NOR gate

Using NOR gate we can realise different gates as follows

NOT gate from NOR gate

To obtain NOT gate from NOR gate, two inputs of NOR gate are joined together.

Boolean expression

The Boolean expression for NOR gate is given by

$$Y = \overline{A + B}$$

Putting $B = A, Y = \overline{A + A} = \overline{A}$ $\quad (\because A + A = A)$

This is same as Boolean expression for NOT gate.

Logic symbol

Fig. 14.85 Realisation of NOT gate using NOR gate

Truth table

A	Y
0	1
1	0

AND gate from NOR gate

To obtain AND gate from NOR gate, two NOT gates (made from NOR gates are connected to a NOR gate. The logic circuit so obtained will work as AND gate.

Boolean expression

The Boolean expression for a NOR gate having $\overline{A}$ and $\overline{B}$ as two inputs is given by

$$Y = \overline{(\overline{A} + \overline{B})} = \overline{\overline{A}} \cdot \overline{\overline{B}}$$

(Applying De Morgan's theorem)

$$= A \cdot B \quad \text{(Boolean expression for AND gate)}$$

Logic Symbol

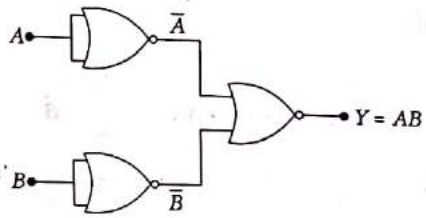


Fig. 14.86 Realisation of AND gate using NOR gate

Truth table

A	B	$\overline{A}$	$\overline{B}$	Y
0	0	1	1	0
1	0	0	1	0
0	1	1	0	0
1	1	0	0	1

OR gate from NOR gate

To obtain OR gate from NOR gate. We connect NOR gate to the NOT gate (made from NOR gate. The logic circuit obtained will work as OR gate.

Boolean expression

The output of the NOR gate is given by $Y' = \overline{A + B}$

The output of the NOR gate forms the input of the NOT gate.

∴ The final output is given by

$$Y = \overline{Y'} = \overline{\overline{A + B}}$$

$$= A + B \quad \text{(Boolean expression for OR gate)}$$

Logic symbol

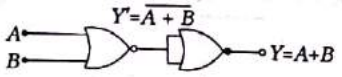
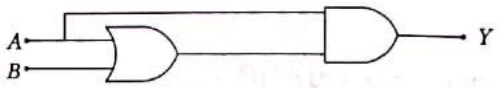


Fig. 14.87 Realisation of OR gate using NOR gate

Truth table

A	B	Y'	Y
0	0	1	0
1	0	0	1
0	1	0	1
1	1	0	1

Example 14.49 *Draw the truth table for the function Y of A and B for the following logic gate.*



Sol. The output X in terms of the input A and B can be written as,

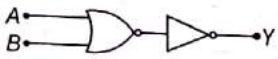
$$X = A \cdot (A + B).$$

Let us make the truth table corresponding to this function.

Truth table

Input		Output	
A	B	A + B	X = A · (A + B)
0	0	0	0
0	1	1	0
1	0	1	1
1	1	1	1

Example 14.50 *Write the truth table for the following circuit. Name the equivalent gate that this circuit represents.*



Sol. The given combination consists of NOR gate and NOT gate, so equivalent gate is OR gate.

Truth table

A	B	Y = A + B
0	0	0
0	1	1
1	0	1
1	1	1

From the truth table, it is clear that the output is 1 only when atleast one of the inputs is at the high state (i.e., 1).

Example 14.51 *Let $X = A \cdot \overline{BC}$. Evaluate X for*

(a) $A = 1, B = 0, C = 1,$

(b) $A = B = C = 1$ *and*

(c) $A = B = C = 0.$

Sol. (a) When, $A = 1, B = 0$ and $C = 1$

$$BC = 0$$

$$\therefore \quad \overline{BC} = 1 \text{ or } A\overline{BC} = 1$$

(b) When, $A = B = C = 1$

Then, $BC = 1$ or $\overline{BC} = 0 \quad \therefore \quad A\overline{BC} = 0$

(c) When, $A = B = C = 0$

Then, $BC = 0$

$\therefore \quad \overline{BC} = 1$ or $A\overline{BC} = 0$

Example 14.52 *Write the truth table for the logical function $D = (A \text{ OR } B) \text{ AND } B$.*

Sol. A OR B is a logical function, say it is equal to X, *i.e.*,

$$X = A \text{ OR } B$$

Now, $$Y = X \text{ AND } B$$

The corresponding truth table is as under.

Truth Table

A	B	X = A OR B	Y = (A OR B) AND B
1	0	1	0
0	1	1	1
0	0	0	0
1	1	1	1

Note The given function can also be written as, $Y = (A + B) \cdot B$

Example 14.53 *Write down truth tables for*

(a) $X = \overline{A} \cdot \overline{B} + \overline{A} \cdot B$ (b) $X = (A + \overline{B}) + \overline{A \cdot B}$

Sol. (a)

A	B	$\overline{A}$	$\overline{B}$	$\overline{A} \cdot \overline{B}$	$\overline{A} \cdot B$	$X = \overline{A} \cdot \overline{B} + \overline{A} \cdot B$
0	0	1	1	1	0	1
0	1	1	0	0	1	1
1	0	0	1	0	0	0
1	1	0	0	0	0	0

(b)

A	B	$\overline{B}$	$A + \overline{B}$	$A \cdot B$	$\overline{A \cdot B}$	$X = (A + B) + (\overline{A \cdot B})$
0	0	1	1	0	1	1
0	1	0	0	0	1	1
1	0	1	1	0	1	1
1	1	0	1	1	0	1

Example 14.54 *Let $X = A\overline{BC} + B\overline{CA} + C\overline{AB}$. Find the output Y if following inputs are given*

(a) $A = 1, B = 0, C = 1$ (b) $A = B = C = 1$

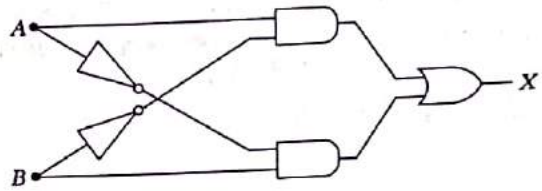
(c) $A = B = C = 0$

Sol.

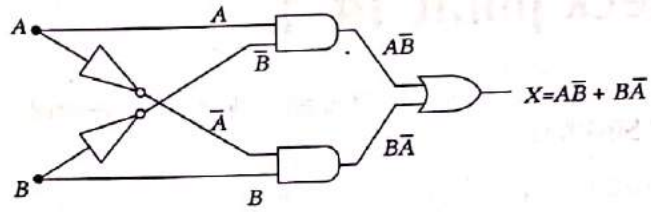
A	B	C	$\overline{AB}$	$\overline{BC}$	$\overline{CA}$	$A\overline{BC}$	$B\overline{CA}$	$C\overline{AB}$	Y
0	0	0	1	1	1	0	0	0	0
1	1	1	0	0	0	0	0	0	0
1	0	1	1	1	0	1	0	1	1

(a) Y = 1 (b) Y = 0 (c) Y = 0

Example 14.55 *Write the truth table for the circuit given in figure.*



Sol. The circuit can be redrawn as,



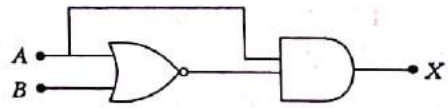
The corresponding truth table is shown below.

Table 14.17

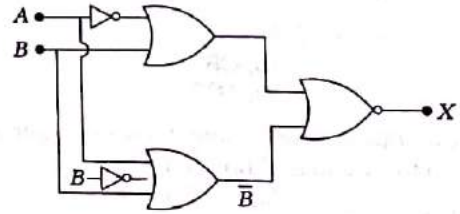
A	B	$\overline{A}$	$\overline{B}$	$A\overline{B}$	$B\overline{A}$	$A\overline{B} + B\overline{A} = X$
0	0	1	1	0	0	0
0	1	1	0	0	1	1
1	0	0	1	1	0	1
1	1	0	0	0	0	0

Example 14.56 *Construct truth table for the function X of A and B as shown in the figure.*

(a)

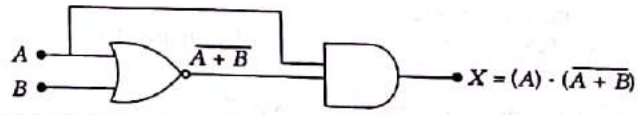


(b)



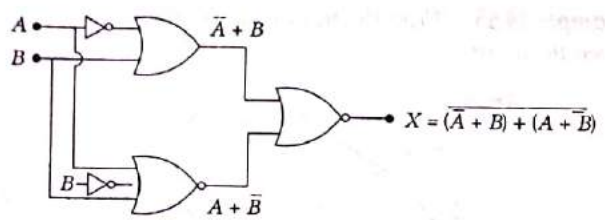
Sol. First write the Boolean expression on the figure and then construct truth table.

(a)



A	B	$\overline{A + B}$	$X = A \cdot (\overline{A + B})$
0	0	1	0
0	1	0	0
1	0	0	0
1	1	0	0

(b)



A	B	$\bar{A}$	$\bar{B}$	$\bar{A} + B$	$A + \bar{B}$	$X = \overline{(\bar{A} + B) + (A + \bar{B})}$
0	0	1	1	1	1	0
0	1	1	0	1	0	0
1	0	0	1	0	1	0
1	1	0	0	1	1	0

Check point 14.4

1. The binary number 10111 is equivalent to the decimal number
(a) 19 (b) 31
(c) 23 (d) 22

2. Sum of the two binary numbers $(100010)_2$ and $(11011)_2$ is
(a) $(111101)_2$ (b) $(111111)_2$
(c) $(101111)_2$ (d) $(111001)_2$

3. The output of OR gate is 1
(a) if both inputs are zero
(b) if either or both inputs are 1
(c) only if both input are 1
(d) if either input is zero

4. If A and B are two inputs in AND gate, then AND gate has an output of 1 when the values of A and B are
(a) $A = 0, B = 0$ (b) $A = 1, B = 1$
(c) $A = 1, B = 0$ (d) $A = 0, B = 1$

5. A gate has the following truth table

P	1	1	0	0
Q	1	0	1	0
R	1	0	0	0

The gate is
(a) NOR (b) OR
(c) NAND (d) AND

6. If A and B are two inputs in AND gate, then AND gate has an output of 0 when the value of A and B are
(a) $A = 0, B = 0$ (b) $A = 0, B = 1$
(c) $A = 1, B = 0$ (d) All are correct

7. A gate in which all the inputs must be low to get a high output is called
(a) NAND gate (b) An inverter
(c) NOR gate (d) AND gate

8. The output of a NAND gate is 0
(a) if both inputs are 0
(b) if one input is 0 and the other input is 1
(c) if both inputs are 1
(d) either if both inputs are 1 or if one of the inputs is 1 and the other 0

9. The truth table given below is for

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

(A and B are the inputs, Y is the output)
(a) NOR (b) AND (c) XOR (d) NAND

10. The boolean equation of NOR gate is
(a) $C = A + B$ (b) $C = \overline{A + B}$
(c) $C = A \cdot B$ (d) $C = \overline{A \cdot B}$

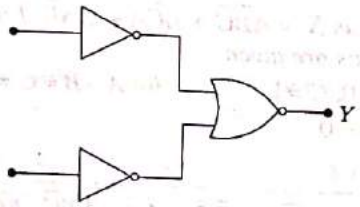
11. Any digital circuit can be realised by repetitive use of only
(a) NOT gates (b) OR gates (c) AND gates (d) NOR gates

12. The logic gates behind NOR gate is that it gives
(a) high output when both the inputs are low
(b) low output when both the inputs are low
(c) high output when both the inputs are high
(d) None of the above

13. The logic gates having output 1 for the inputs of 1 and 0 are
(a) AND and OR (b) OR and NOR
(c) NAND and NOR (d) NAND and OR

14. If the two inputs of a NAND gate are shorted, the gate is equivalent to
(a) XOR (b) OR (c) NOR (d) NOT

15. Which logic gate is represented by the following combination of logic gates



(a) OR (b) NAND (c) AND (d) NOR

Chapter exercises

A. Taking it together

(Assorted questions of the chapter for advanced level practice)

1. Number of electrons in the valence shell of a semiconductor is
(a) 1 (b) 2 (c) 3 (d) 4

2. Hole is **[NCERT Exemplar]**
(a) an anti-particle of electron
(b) a vacancy created when an electron leaves a covalent bond
(c) absence of free electrons
(d) an artificially created particle

3. A piece of copper and the other of germanium are cooled from the room temperature to 80 K, then which of the following would be a correct statement?
(a) Resistance of each increases
(b) Resistance of each decreases
(c) Resistance of copper increases while that of germanium decreases
(d) Resistance of copper decreases while that of germanium increases

4. The valence band and conduction band of a solid, overlap at low temperature, the solid may be
(a) a metal (b) a semiconductor
(c) an insulator (d) None of these

5. In an insulator, the forbidden energy gap between the valence band and conduction band is of the order of
(a) 1 MeV (b) 0.1 MeV (c) 1 eV (d) 5 eV

6. In a semiconductor, the separation between conduction band and valence band is of the order of
(a) 100 eV (b) 10 eV (c) 1 eV (d) zero

7. In a good conductor, the energy gap between the conduction band and the valence band is
(a) infinite (b) wide (c) narrow (d) zero

8. In semiconductors, at room temperature the
(a) valence band is partially empty and the conduction band is partially filled
(b) valence band is completely filled and the conduction band is partially filled
(c) valence band is completely filled
(d) conduction band is completely empty

9. A piece of semiconductor is connected in series in an electric circuit. On increasing the temperature, the current in the circuit will
(a) decrease (b) remain unchanged
(c) increase (d) stop flowing

10. For transistor action, which of the following statements are correct?
(a) Base, emitter and collector regions should have similar size and doping concentrations
(b) The base region must be very thin and lightly doped
(c) The emitter junction is forward biased and collector junction is reverse biased
(d) Both the emitter junction as well as the collector junction are forward biased

11. The forbidden energy gap in the energy bands of germanium at room temperature is about
(a) 1.1 eV (b) 0.1 eV
(c) 0.67 eV (d) 6.7 eV

12. The energy band gap of Si is
(a) 0.70 eV
(b) 1.1 eV
(c) between 0.70 eV to 1.1 eV
(d) 5 eV

13. What enables Ge to behave as semiconductor even though all electrons in the valence band form covalent bonds? It is due to the small width of
(a) valence band (b) conduction band
(c) forbidden energy gap (d) None of these

14. E_g for silicon is 1.12 eV and that for germanium is 0.72 eV. Therefore, it can be concluded that
(a) more number of electron hole pairs will be generated in silicon than in germanium at room temperature
(b) less number of electron hole pairs will be generated in silicon than in germanium at room temperature
(c) equal number of electron hole pairs will be generated in both at lower temperatures
(d) equal number of electron hole pairs will be generated in both at higher temperatures

15. When the electrical conductivity of a semiconductor is due to the breaking of its covalent bonds, then the semiconductor is said to be
(a) donor (b) acceptor
(c) intrinsic (d) extrinsic

16. Let n_p and n_e be the numbers of holes and conduction electrons in an intrinsic semiconductor
(a) $n_p > n_e$ (b) $n_p = n_e$
(c) $n_p < n_e$ (d) $n_p \neq n_e$

17. The majority charge carriers in p-type semiconductor are
(a) electrons (b) protons
(c) holes (d) neutrons

18. A p-n junction has a thickness of the order of
(a) 1 cm (b) 1 mm
(c) 10^{-6} m (d) 10^{-12} cm

19. A p-type semiconductor can be obtained by adding
(a) arsenic to pure silicon
(b) gallium to pure silicon
(c) antimony to pure germanium
(d) phosphorous to pure germanium

20. p-type semiconductor is formed when
A. As impurity is mixed in Si
B. Al impurity is mixed in Si
C. B impurity is mixed in Ge
D. P impurity is mixed in Ge
(a) Both A and C
(b) Both A and D
(c) Both B and C
(d) Both B and D

21. At room temperature, a p-type semiconductor has
(a) large number of holes and few electrons
(b) large number of free electrons and few holes
(c) equal number of free electrons and holes
(d) no electrons or holes

22. In an n-type silicon, which of the following statement is true?
(a) Electrons are majority carriers and trivalent atoms are the dopants
(b) Electrons are minority carriers and pentavalent atoms are the dopants
(c) Holes are minority carriers and pentavalent atoms are the dopants
(d) Holes are majority carriers and trivalent atoms are the dopants

23. Which of the following statements is not true?
(a) The resistance of intrinsic semiconductor decreases with increase of temperature
(b) Doping pure Si with trivalent impurities give p-type semiconductors
(c) The majority carriers in n-type semiconductors are holes
(d) A p-n junction can act as a semiconductor diode

24. n-type semiconductor is formed
(a) when a germanium crystal is doped with an impurity containing three valence electrons
(b) from pure germanium
(c) from pure silicon
(d) when a germanium crystal are doped with an impurity containing five valence of electrons

25. An n-type semiconductor is
(a) negatively charged
(b) positively charged
(c) neutral
(d) negatively or positively charged depending upon the amount of impurity

26. Electrical conductivity of intrinsic and p-type semiconductor increases with increase in
(a) pressure (b) volume (c) density (d) temperature

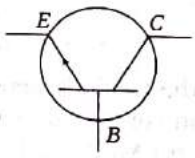
27. In an insulator, the forbidden energy gap between the valence band and conduction band is of the order of
(a) 1 MeV (b) 0.1 MeV (c) 1 eV (d) 5 eV

28. The valence of the impurity atom that is to be added to germanium crystal, so as to make it a n-type semiconductor, is
(a) 6 (b) 5 (c) 4 (d) 3

29. An n-type and a p-type silicon can be obtained by doping pure silicon with
(a) sodium and magnesium respectively
(b) phosphorus and boron respectively
(c) boron and phosphorus respectively
(d) indium and sodium respectively

30. The potential barrier for silicon diode is approximately
(a) 0.2 V (b) 0.6 V (c) 1.1 V (d) 1.4 V

31. The symbol given in figure represents



(a) n-p-n transistor
(b) p-n-p transistor
(c) forward biased p-n junction diode
(d) reverse biased n-p junction diode

32. The part of a transistor which is heavily doped to produce a large number of majority carriers, is
(a) base (b) emitter
(c) collector (d) None of these

33. In comparison to half wave rectifier, full wave rectifier gives lower
(a) efficiency (b) average DC
(c) average output voltage (d) None of these

34. Zener diode is used as
(a) half wave rectifier (b) full wave rectifier
(c) AC voltage stabiliser (d) DC voltage stabiliser

35. Serious drawback of the semiconductor device is that they
(a) cannot be used with high voltage
(b) pollute the environment
(c) are costly
(d) do not last for long time

36. Least doped region in a transistor is
(a) either emitter or collector
(b) base
(c) emitter
(d) collector

37. Intrinsic semiconductor is electrically neutral. Extrinsic semiconductor having large number of current carriers would be
(a) positively charged
(b) negatively charged
(c) positively charged or negatively charged depending upon the type of impurity that has been added
(d) electrically neutral

38. In a semiconducting material, the mobilities of electrons and holes are μ_e and μ_h, respectively. Which of the following is true?
(a) $\mu_e > \mu_h$ (b) $\mu_e < \mu_h$
(c) $\mu_e = \mu_h$ (d) $\mu_e < 0, \mu_h > 0$

39. When a forward bias is applied to *p-n* junction, then it
(a) raises the potential barrier
(b) reduces the majority carrier current to zero
(c) lowers the potential barrier
(d) None of the above

40. The conductivity of a semiconductor increases with increase in temperature, because **[NCERT Exemplar]**
(a) number density of free current carries increases
(b) relaxation time increases
(c) both number density of carries and relaxation time increase
(d) number density of carries increases, relaxation time decreases but effect of decrease in relaxation time is much less than increase in number density

41. The drift current in a *p-n* junction is
(a) from the *n*-side to the *p*-side
(b) from the *p*-side to the *n*-side
(c) from the *n*-side to the *p*-side, if the junction is forward biased and in the opposite direction, if it is reverse biased
(d) from the *p*-side to the *n*-side, if the junction is forward biased and in the opposite direction, if it is reverse biased

42. The diffusion current in a *p-n* junction is
(a) from the *n*-side to the *p*-side
(b) from the *p*-side to the *n*-side
(c) from the *n*-side to the *p*-side, if the junction is forward biased and in the opposite direction, if it is reverse biased
(d) from the *p*-side to the *n*-side, if the junction is forward biased and in the opposite direction, if it is reverse biased

43. Diffusion current in a *p-n* junction is greater than the drift current in magnitude,
(a) if the junction is forward biased
(b) if the junction is reverse biased
(c) if the junction is unbiased
(d) None of the above

44. In the forward bias arrangement of a *p-n*-junction diode, the
(a) *n*-end is connected to the positive terminal of the battery
(b) *p*-end is connected to the positive terminal of the battery
(c) direction of current is from *n*-end to *p*-end in the diode
(d) *p*-end is connected to the negative terminal of battery

45. Potential barrier developed in a junction diode opposes
(a) minority carrier in both regions only
(b) majority carrier only
(c) electrons in *n*-region
(d) holes in *p*-region

46. If n_e and v_d be the number of electrons and drift velocity in a semiconductor. When the temperature is increased
(a) n_e increases and v_d decreases
(b) n_e decreases and v_d increases
(c) Both n_e and v_d increases
(d) Both n_e and v_d decreases

47. The dominant mechanisms for motion of charge carriers in forward and reverse biased silicon *p-n* junctions are
(a) drift in forward bias, diffusion in reverse bias
(b) diffusion in forward bias, drift in reverse bias
(c) diffusion in both forward and reverse bias
(d) drift in both forward and reverse bias

48. Which of the statement is true for *p*-type semiconductors?
(a) Electrons are majority carriers and trivalent atoms are the dopants
(b) Electrons are minority carriers and pentavalent atoms are the dopants
(c) Holes are minority carriers and pentavalent atoms are the dopants
(d) Holes are majority carriers and trivalent atoms are the dopants

49. The reverse biasing in a *p-n* junction diode
(a) decreases the potential barrier
(b) increases the potential barrier
(c) increases the number of minority charge carriers
(d) increases the number of majority charge carriers

50. In p-n junction, avalanche current flows in circuit when biasing is
(a) forward (b) reverse (c) zero (d) excess

51. In an unbiased p-n junction, holes diffuse from the p-region to n-region because
(a) free electrons in the n-region attract them
(b) they move across the junction by the potential difference
(c) hole concentration in p-region is more as compared to n-region
(d) All of the above

52. In a transistor
(a) both the emitter and the collector are equally doped
(b) base is more heavily doped than the collector
(c) collector is more heavily doped than the emitter
(d) the base is made very thin and is lightly doped

53. A transistor has three impurity regions. All the three regions have different doping levels. In order of increasing doping level, the regions are
(a) emitter, base and collector
(b) collector, base and emitter
(c) base, emitter and collector
(d) base, collector and emitter

54. n-p-n transistor is more useful than a p-n-p transistor because
(a) n-p-n transistor offers less resistance to the flow of current
(b) charge carriers in n-p-n move more easily than the charge carriers in p-n-p
(c) p-n-p transistor is very costly compared to an n-p-n transistor
(d) None of the above

55. In case of n-p-n transistor the collector current is always less than the emitter current because
(a) collector side is reverse biased and emitter side forward biased
(b) a few electrons are lost in the base and only the remaining ones reach the collector
(c) collector side is forward biased and emitter side is reverse biased
(d) collector being reverse biased attracts less electrons

56. In a p-n-p transistor with normal bias
(a) only holes cross the collector junction
(b) only majority carriers cross the collector junction
(c) the collector junction has a low resistance
(d) the emitter-base junction is forward biased and the collector-base junction is reverse biased

57. The transistors provide good power amplification when they are used in
(a) common collector configuration
(b) common emitter configuration
(c) common base configuration
(d) None of the above

58. The working of a triode valve and transistor are similar. A transistor is a current operated device while a triode valve is a
(a) current operated device (b) voltage operated device
(c) charge operated device (d) magnetically operated device

59. When the p-end of the p-n junction is connected to the negative terminal of the battery and the n-end to the positive terminal of the battery, then the p-n junction behaves like
(a) a conductor (b) an insulator
(c) a super conductor (d) a semiconductor

60. Select the correct statement.
(a) In a full wave rectifier, two diodes work alternately
(b) In a full wave rectifier, two diodes work simultaneously
(c) the efficiency of full wave and half wave rectifiers is same
(d) The full wave rectifier is bi-directional

61. Temperature coefficient of resistance of semiconductor is
(a) zero (b) constant
(c) positive (d) negative

62. A semiconducting device is connected in a series circuit with a battery and a resistance. A current is found to pass through the circuit. If the polarity of the battery is reversed, then the current drops to almost zero, the device may be
(a) extrinsic semiconductor (b) intrinsic semiconductor
(c) p-n junction (d) p-n-p transistor

63. GaAs (with a band gap = 1.5 eV) as an LED can emit
(a) blue light (b) green light
(c) UV rays (d) X-rays (e) infrared rays

64. If no external voltage is applied across p-n junction, then there would be
(a) no electric field across the junction
(b) an electric field pointing from n-type to p-type side across the junction
(c) an electric field pointing from p-type to n-type side across the junction
(d) a temporary electric field during formation of p-n junction that would subsequently disappear

65. Carbon, silicon and germanium have four valence electrons each. These are characterised by valence and conduction bands separated by energy band gap respectively equal to $(E_g)_C$, $(E_g)_{Si}$ and $(E_g)_{Ge}$. Which of the following statements is true?
(a) $(E_g)_{Si} < (E_g)_{Ge} < (E_g)_C$ (b) $(E_g)_C < (E_g)_{Ge} < (E_g)_{Si}$
(c) $(E_g)_C < (E_g)_{Si} < (E_g)_{Ge}$ (d) $(E_g)_{Ge} < (E_g)_{Si} < (E_g)_C$

66. When a diode is heavily doped, then
(a) the Zener voltage will be low
(b) the avalanche voltage will be high
(c) the depletion region will be thin
(d) the leakage current will be low

67. For a transistor amplifier, the voltage gain
(a) remains constant for all frequencies
(b) is high at high and low frequencies and constant in the middle frequency range
(c) is low at high and low frequencies and constant at mid frequencies
(d) None of the above

68. The electrical circuit used to get smooth DC output from a rectifier circuit is called
(a) oscillator (b) filter
(c) amplifier (d) logic gates

69. To use a transistor as an amplifier
(a) Both junctions are forward biased
(b) Both junctions are reverse biased
(c) the emitter base junction is forward biased and the collector base junction is reverse biased
(d) no biasing voltages are required

70. In the case of constants α and β of a transistor
(a) $\alpha = \beta$ (b) $\beta < 1, \alpha > 1$ (c) $\alpha\beta = 1$ (d) $\beta > 1, \alpha < 1$

71. The base of transistor is made very thin compared to emitter and collector. By doing this which one of the following is achieved?
(a) High emitter current (b) Low power gain
(c) Low emitter current (d) High power gain

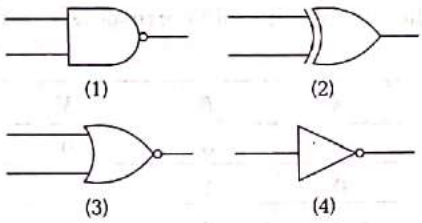
72. In an *n-p-n* transistor, the collector current is 10 mA. If 90% of the electrons emitted reach the collector, then the emitter current will be about
(a) 9 mA (b) 11 mA (c) 1 mA (d) 0.1 mA

73. What is β value for transistor whose $\alpha = 0.98$?
(a) 29 (b) 38 (c) 49 (d) 56

74. In a transistor α is related to β by the relation
(a) $\beta = \frac{\alpha + 1}{\alpha}$ (b) $\beta = \frac{\alpha - 1}{\alpha}$
(c) $\beta = \frac{\alpha}{1 - \alpha}$ (d) $\beta = \frac{\alpha}{\alpha + 1}$

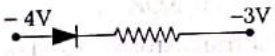
75. The maximum efficiency of full wave rectifier is
(a) 100% (b) 25.20%
(c) 40.2% (d) 81.2%

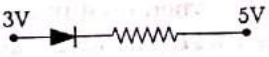
76. Given below are symbols for some logic gates.

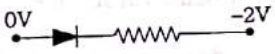


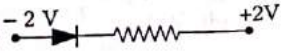
The XOR gate and NOR gates respectively are
(a) 1 and 2 (b) 2 and 3 (c) 3 and 4 (d) 1 and 4

77. The forward bias diode is

(a) 

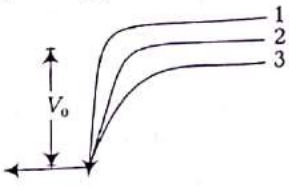
(b) 

(c) 

(d) 

78. In the Boolean algebra which of the following is not equal to A?
(a) $A \cdot A$ (b) $A + A$ (c) $\overline{A} \cdot A$ (d) $\overline{\overline{A} + \overline{A}}$

79. In figure given below V_0 is the potential barrier across a *p-n* junction, when no battery is connected across the junction **[NCERT Exemplar]**

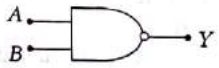


(a) 1 and 3 both correspond to forward bias of junction
(b) 3 corresponds to forward bias of junction and 1 corresponds to reverse bias of junction
(c) 1 corresponds to forward bias and 3 corresponds to reverse bias of junction
(d) 3 and 1 both correspond to reverse bias of junction

80. If a full wave rectifier circuit is operating in 50 Hz mains, then the fundamental frequency in the ripple will be
(a) 50 Hz (b) 70.7 Hz (c) 100 Hz (d) 25 Hz

81. With an AC input from 50 Hz power line, the ripple frequency is
(a) 50 Hz in the DC output of half wave as well as full wave rectifier
(b) 100 Hz in the DC output of half wave as well as full wave rectifier
(c) 50 Hz in the DC output of half wave and 100 Hz in DC output of full wave rectifier
(d) 100 Hz in the DC output of half wave and 50 Hz in the DC output of full wave rectifier

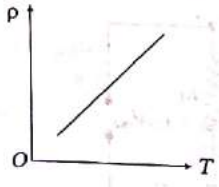
82. The given symbol represents

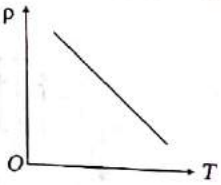


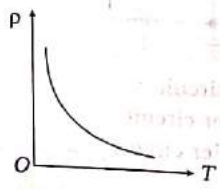
(a) NAND gate (b) OR gate
(c) AND gate (d) NOR gate

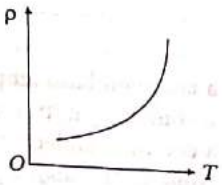
83. The Boolean expression of NOR gate is
(a) $C = A + B$ (b) $C = \overline{A + B}$
(c) $C = A \cdot B$ (d) $C = \overline{A \cdot B}$

84. The temperature (T) dependence on resistivity (ρ) of a semiconductor is represented by

(a) 

(b) 

(c) 

(d) 

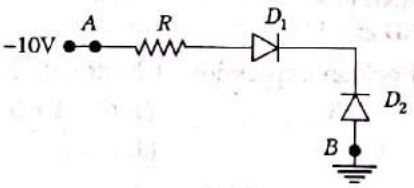
85. Under which of the following conditions does an avalanche breakdown in a semiconductor diode occur?
(a) When potential barrier is reduced to zero
(b) When reverse bias exceeds a certain value
(c) When forward bias exceeds a certain value
(d) When forward current exceeds a certain value

86. The width of depletion region in a *p-n* junction diode is 500 nm and an intense electric field of 6×10^5 Vm^{-1} is also found to exist. The height of potential barrier is
(a) 0.30 V (b) 0.40 V (c) 3 V (d) 4 V

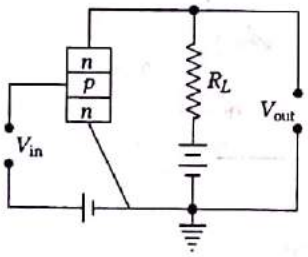
87. Mobilities of electrons and holes in a sample of intrinsic germanium at room temperature are 0.36 m^2 /Vs and 0.17 m^2 /Vs. The electron and hole densities are each equal to 2.5×10^{19} m^3. The electrical conductivity of germanium is
(a) 0.47 Sm^{-1} (b) 5.18 Sm^{-1}
(c) 2.12 Sm^{-1} (d) 1.09 Sm^{-1}

88. In a semiconductor, the concentration of electrons is 8×10^{14} / cm^3 and that of the holes is 5×10^{12} cm^3. The semiconductor is
(a) *p*-type (b) *n*-type
(c) Intrinsic (d) *p-n-p*-type

89. In figure given below, assuming the diodes to be ideal **[NCERT Exemplar]**
(a) D_1 is forward biased and D_2 is reverse biased and hence current flows from *A* to *B*
(b) D_2 is forward biased and D_1 is reverse biased and hence no current flows from *B* to *A* and *vice-versa*
(c) D_1 and D_2 are both forward biased and hence current flows from *A* to *B*
(d) D_1 and D_2 are both reverse biased and hence no current flows from *A* to *B* and *vice-versa*



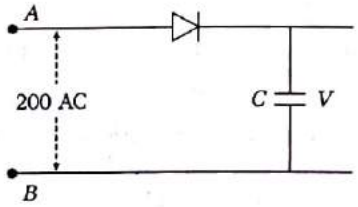
90. An *n-p-n* transistor circuit is arranged as shown in figure. It is



(a) a common base amplifier circuit
(b) a common emitter amplifier circuit
(c) a common collector amplifier circuit
(d) None of the above

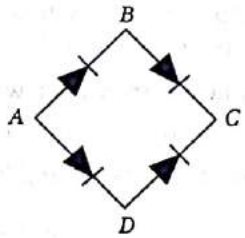
91. A Si specimen is made into *p*-type semiconductor by doping on an average one indium atom per 6×10^7 silicon atoms. If the number density of atoms in Si be 6×10^{28} m^{-3}, then what is the indium atom per cm^3?
(a) 10^{12} (b) 10^{15} (c) 10^{18} (d) 10^{20}

92. A 220 V AC supply is connected between points *A* and *B* (figure). What will be the potential difference *V* across the capacitor? **[NCERT Exemplar]**



(a) 220V (b) 110 V (c) 0 V (d) $220\sqrt{2}$V

93. In the diagram, the input is across the terminals *A* and *C* and the output is across *B* and *D*. Then, the output is



(a) zero (b) same as the input
(c) full wave rectifier (d) half wave rectifier

94. When forward bias is applied to a *p-n* junction, then what happens to the potential barrier V_B and the width of charge depleted region *x* ?
(a) V_B increases, *x* decreases (b) V_B decreases, *x* increases
(c) V_B increases, *x* increases (d) V_B decreases, *x* decreases

95. The following truth table corresponds to the logic gate :

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

(a) NAND (b) OR
(c) AND (d) XOR

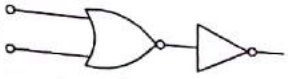
96. The transfer ratio β of a transistor is 50. The input resistance of the transistor when used in the common emitter configuration is 1 kΩ. The peak value of the collector AC current for a peak value of AC input voltage of 0.01 V is
(a) 100 μA (b) 0.01 μA
(c) 0.25 μA (d) 500 μA

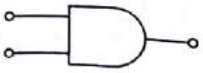
97. The voltage gain of an amplifier with 9% negative feedback is 10. The voltage gain without feedback will be

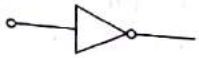
(a) 90 (b) 10 (c) 1.25 (d) 100

98. Which represents NAND gate?

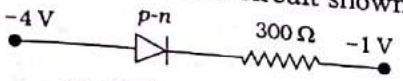
(a)

(b) 

(c) 

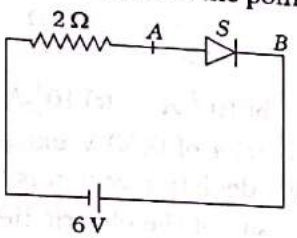
(d) 

99. What is the current in the circuit shown below?



(a) Zero (b) 10^{-2} A (c) 1 A (d) 0.10 A

100. The diode shown in the circuit is a silicon diode. The potential difference between the points A and B will be



(a) 6 V (b) 0.6 V (c) 0.7 V (d) zero

101. The value of DC voltage in half wave rectifier in converting AC voltage $V = 100\sin(314t)$ into DC is

(a) 100 volt (b) 50 volt
(c) 30.3 volt (d) 0

102. Below we give four entries for the truth table of two point input OR gate. Which two are wrong?

	A	B	Y
i	1	0	1
ii	1	1	0
iii	0	0	1
iv	0	1	1

(a) i, ii (b) ii, iii (c) iii, iv (d) iv, i

103. Input signal to a common emitter amplifier having a voltage gain of 1000 is given by $V_i = (0.004\text{ V})\sin(\omega t + \pi/2)$. The corresponding output signal is

(a) $(40\text{ V})\sin(\omega t + \pi/2)$ (b) $(0.004\text{ V})\cos(\omega t + \pi/2)$
(c) $(4\text{ V})\cos(\omega t - \pi/2)$ (d) $(4\text{ V})\sin(\omega t - \pi/2)$

104. In a common base transistor circuit, the current gain is 0.98. On changing the emitter current by 5 mA, the change in collector current is

(a) 0.196 mA (b) 2.45 mA (c) 4.9 mA (d) 5.1 mA

105. The forbidden energy gap in Ge is 0.72 eV. Given, $hc = 124000$ eV-Å. The maximum wavelength of radiation that will generate electron hole pair is

(a) 172220 Å (b) 172.2 Å
(c) 17222 Å (d) 1722 Å

106. When the emitter current of a transistor is changed by 1 mA, then its collector current changes by 0.990 mA. In the common base circuit current gain for the transistor is

(a) 0.099 (b) 1.01 (c) 1.001 (d) 0.990

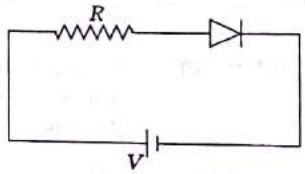
107. What will be the input of A and B for the Boolean expression $\overline{(A+B)\cdot(A\cdot B)} = 1$?

(a) 0, 0 (b) 0, 1
(c) 1, 0 (d) 1, 1

108. In a transistor, the current amplification factor α is 0.9. The transistor is connected in common base configuration. The change in collector current when emitter current changes by 4 mA is

(a) 4 mA (b) 12 mA (c) 24 mA (d) 3.6 mA

109. For the given circuit of p-n junction diode, which of the following statement is correct?



(a) In forward biasing the voltage across R is V
(b) In forward biasing the voltage across R is $2V$
(c) In reverse biasing the voltage across R is V
(d) In reverse biasing the voltage across R is $2V$

110. A p-type semiconductor has acceptor level 57 MeV above the valence band. The maximum wavelength of light required to create hole is

(a) 57 Å (b) 57×10^{-3} Å
(c) 217100 Å (d) 11.61×10^{-33} Å

111. The given truth table is of

A	X
0	1
1	0

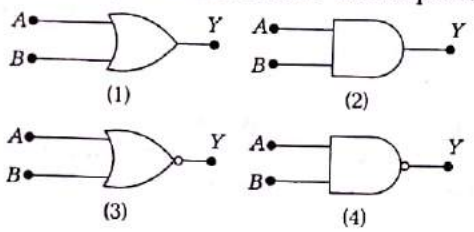
(a) OR gate (b) AND gate
(c) NOT gate (d) None of the above

112. The below truth table corresponds to

A	B	Y
0	0	1
1	0	1
0	1	1
1	1	0

(a) NAND gate (b) XOR gate
(c) OR gate (d) NOR gate

113. Given below are four logic gate symbols (figure). Those for OR, NOR and NAND are respectively

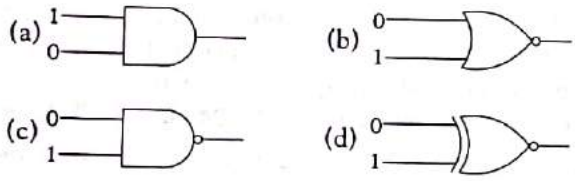


(a) 1, 4, 3 (b) 4, 1, 2 (c) 1, 3, 4 (d) 4, 2, 1

114. The decimal equivalent of the binary number $(11010.101)_2$ is

(a) 9.625 (b) 25.265 (c) 26.625 (d) 26.265

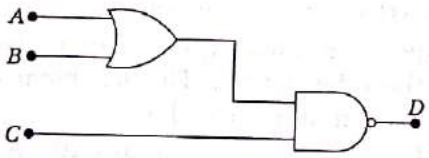
115. Which of the following gates will have an output of 1?



116. When the inputs of two input logic gate are 0 and 0, then the output is 1. When the inputs are 1 and 0, then the output is zero. The logic gate is of the type

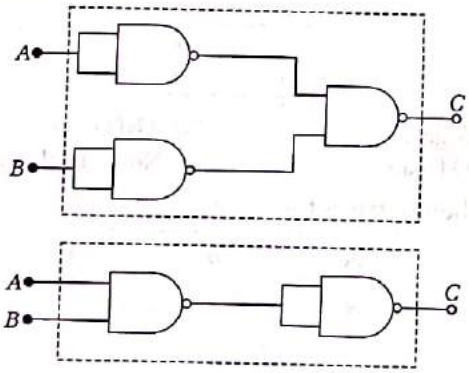
(a) XOR (b) NAND
(c) NOR (d) OR

117. For the given combination of gates, if the logic states of inputs A, B and C are as follows $A = B = C = 0$ and $A = B = 1$, $C = 0$, then the logic states of output D are



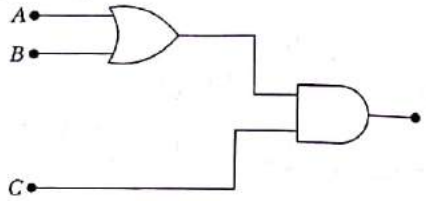
(a) 0, 0 (b) 0, 1 (c) 1,0 (d) 1, 1

118. The combination of NAND gates shown here are equivalent to an



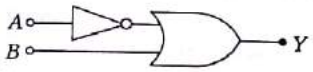
(a) OR gate and an AND gate respectively
(b) AND gate and a NOT gate respectively
(c) AND gate and an OR gate respectively
(d) OR gate and a NOT gate respectively

119. To get an output 1 from the circuit shown in the figure, the input must be



(a) $A = 0, B = 1, C = 0$ (b) $A = 1, B = 0, C = 0$
(c) $A = 1, B = 0, C = 1$ (d) $A = 1, B = 1, C = 0$

120. The Boolean expression for the circuit given in the figure is



(a) $Y = A + \overline{B}$ (b) $Y = \overline{A + B}$
(c) $Y = \overline{A} + B$ (d) $Y = A + B$

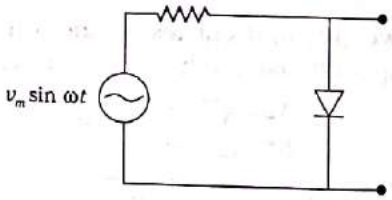
121. In the circuit given below, the value of the current is

(a) zero (b) 10^{-2} A (c) 10^{2} A (d) 10^{-3} A

122. A potential barrier of 0.50 V exists across a p-n junction. If the depletion region is 5.0×10^{-7} m wide, then the intensity of the electric field in this region is

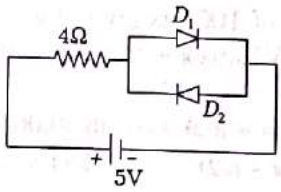
(a) 1×10^{6} Vm^{-1} (b) 1×10^{5} Vm^{-1}
(c) 2×10^{5} Vm^{-1} (d) 2×10^{6} Vm^{-1}

123. The output of the given circuit in figure given below, **[NCERT Exemplar]**



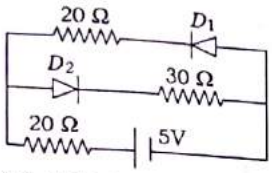
(a) would be zero at all times
(b) would be like a half wave rectifier with positive cycles in output
(c) would be like a half wave rectifier with negative cycles in output
(d) would be like that of a full wave rectifier

124. In the network shown,



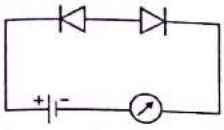
(a) the potential difference across D_2 is 5 V
(b) current through resistor equals 1.25 A
(c) current through diode D_1 is 1.25 A
(d) current through diode D_2 is 1.25 A

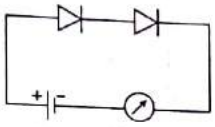
125. The current in the circuit will be

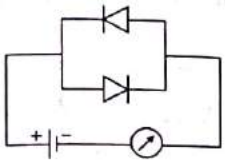


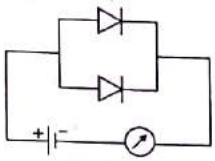
(a) 5/40 A (b) 5/50 A (c) 5/10 A (d) 5/20 A

126. Which circuit will not show current in ammeter?

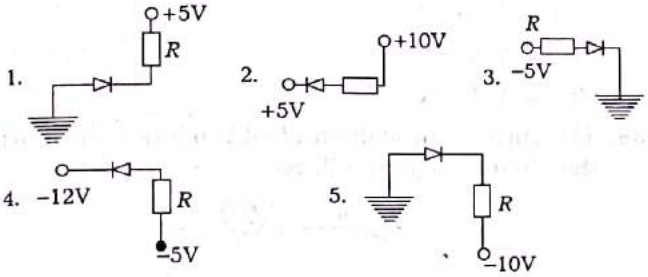
(a)


(b)


(c)


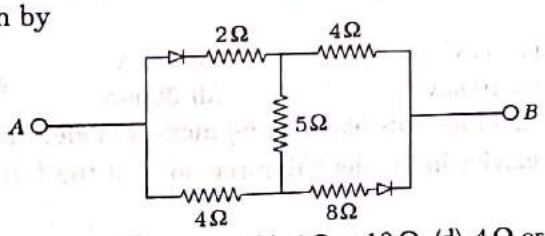
(d)


127. In the given figure, which of the diodes are forward biased?



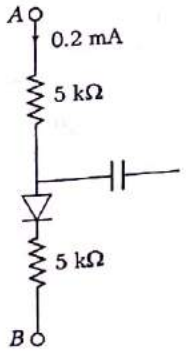
(a) 1, 2, 3 (b) 2, 4, 5 (c) 1, 3, 4 (d) 2, 3, 4

128. The equivalent resistance of the circuit across AB is given by



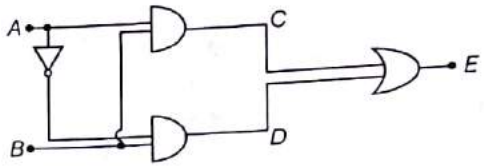
(a) 4 Ω (b) 13 Ω (c) 4 Ω or 13 Ω (d) 4 Ω or zero

129. In the circuit shown in figure given below, if the diode forward voltage drop is 0.3 V, the voltage difference between A and B is [NCERT Exemplar]



(a) 1.3 V (b) 2.3 V
(c) 0 (d) 0.5 V

130. Truth table for the given circuit is [NCERT Exemplar]



(a)

A	B	E
0	0	1
0	1	0
1	0	1
1	1	0

(b)

A	B	E
0	0	1
0	1	0
1	0	0
1	1	0

(c)

A	B	E
0	0	0
0	1	1
1	0	0
1	1	1

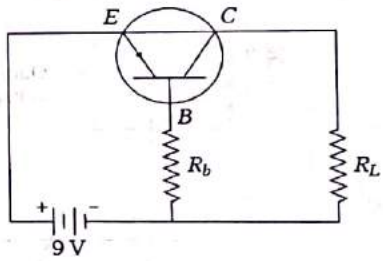
(d)

A	B	E
0	0	0
0	1	1
1	0	1
1	1	0

131. The input resistance of a common-emitter amplifier is 2kΩ and AC current gain is 20. If the load resistor used is 5kΩ, then calculate the transconductance of the transistor used

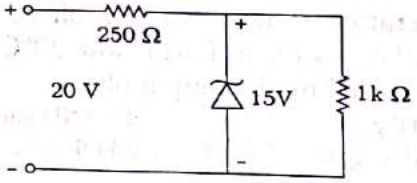
(a) 0.01 Ω^{-1} (b) 0.03 Ω^{-1}
(c) 0.04 Ω^{-1} (d) 0.07 Ω^{-1}

132. In a transistor circuit shown here the base current is 35 μA. The value of the resistor R_b is



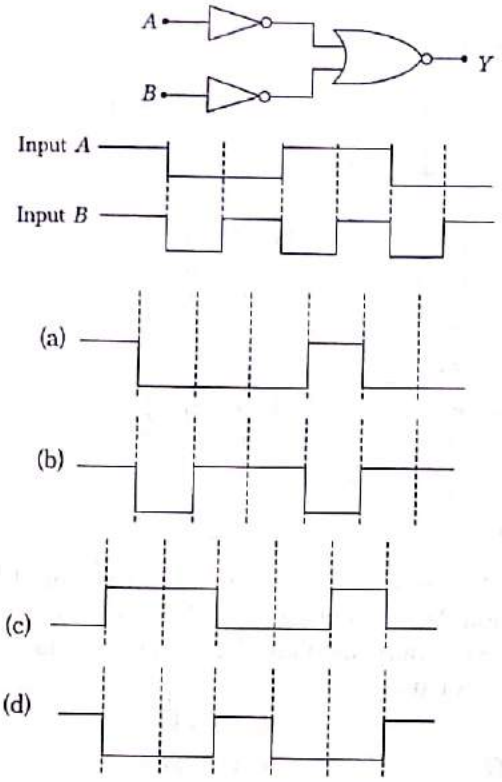
(a) 123.5 kΩ
(b) 257 kΩ
(c) 380.05 kΩ
(d) None of these

133. A Zener diode, having breakdown voltage equal to 15 V, is used in a voltage regulator circuit shown in figure. The current through the diode is

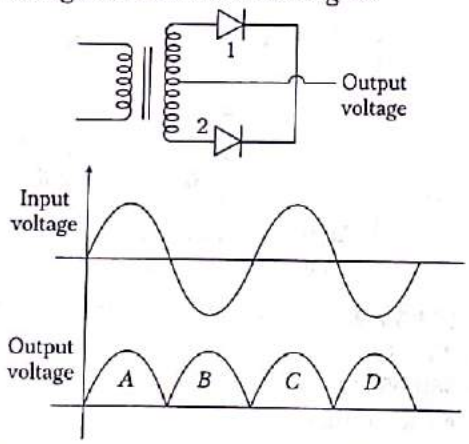


(a) 20 mA (b) 5 mA
(c) 10 mA (d) 15 mA

134. The logic circuit shown below has the input waveforms A and B as shown. Pick out the correct output waveform.



135. A full wave rectifier circuit along with the input and output voltages is shown in the figure



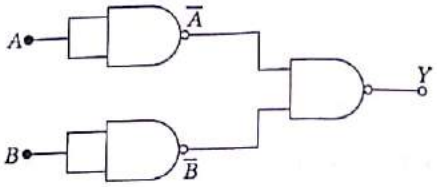
The contribution to output voltage from D_2 is

(a) A, C (b) B, D (c) B, C (d) A, D

136. A researcher wants an alarm to sound when the temperature of air in his controlled research chamber rises above 40°C or falls below 20°C. The alarm can be triggered by the output of a

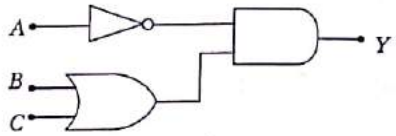
(a) NOT gate (b) AND gate
(c) NAND gate (d) OR gate

137. The combination of the gates shown in the figure below produces



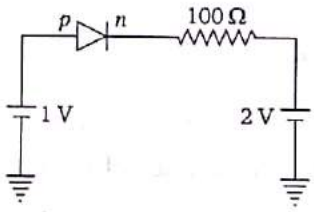
(a) NOR gate
(b) OR gate
(c) AND gate
(d) XOR gate

138. The Boolean expression for the circuit given in figure is



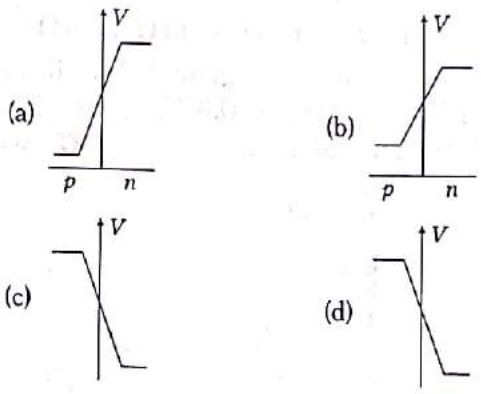
(a) $Y = \overline{A} \cdot B + C$
(b) $Y = \overline{A} \cdot (\overline{B} + \overline{C})$
(c) $Y = \overline{A} \cdot (B + \overline{C})$
(d) $Y = \overline{A} \cdot (B + C)$

139. The current through an ideal p-n junction shown in the circuit diagram will be

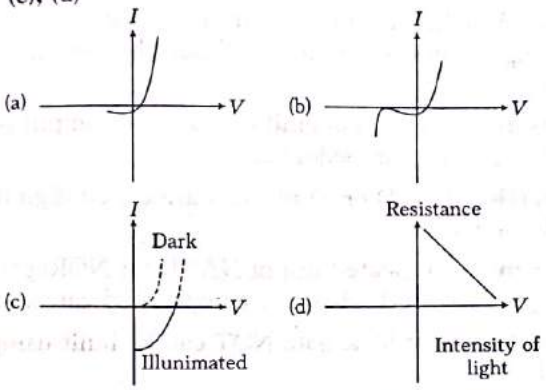


(a) zero (b) 1 mA
(c) 10 mA (d) 30 mA

140. In a forwards biased p-n-junction diode, the potential barrier in the depletion region is of the form

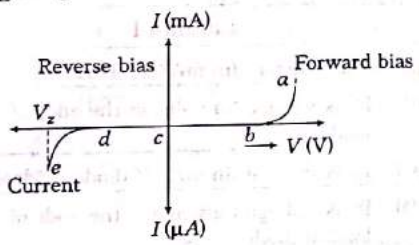


141. Identify the semiconductor devices whose characteristics are given below, in the order (a), (b), (c), (d)



(a) Zener diode, Simple diode, Light dependent resistance, Solar cell
(b) Solar cell, Light dependent resistance, Zener diode , Simple diode
(c) Zener diode, Solar cell, Simple diode, Light dependent resistance
(d) Simple diode, Zener diode, Solar cell, Light dependent resistance

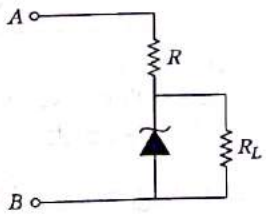
142. The graph given below represents the I-V characteristics of a Zener diode. Which part of the characteristics curve is most relevant for its operation as a voltage regulator



(a) ab (b) bc (c) cd (d) de

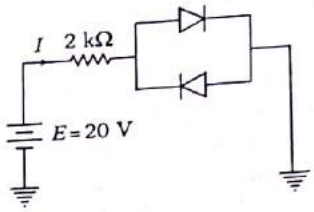
143. A diode having potential difference 0.5 V across its junction which does not depend on current, is connected in series with resistance of 20 Ω across source. If 0.1 A passes through resistance, then what is the voltage of the source?
(a) 1.5 V (b) 2.0 V (c) 2.5 V (d) 5 V

144. If the voltage between the terminals A and B is 17 V and Zener breakdown voltage is 9 V, then the potential across R is



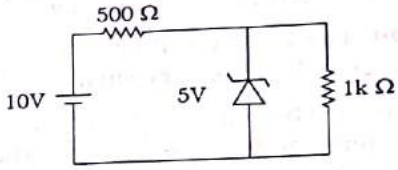
(a) 6 V
(b) 8 V
(c) 9 V
(d) 17 V

145. Assuming the diodes to be of silicon with forward resistance zero, the current I in the following circuit is



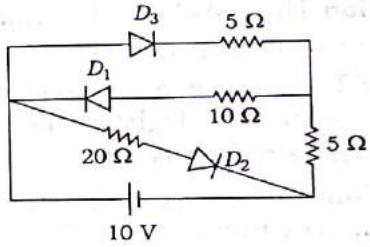
(a) 0
(b) 9.65 mA
(c) 10 mA
(d) 10.36 mA

146. In the following circuit, the current flowing through 1 kΩ resistor is



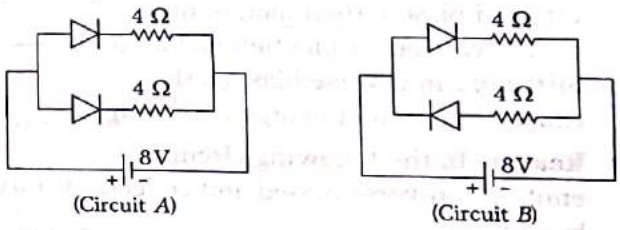
(a) 0 mA (b) 5 mA
(c) 10 mA (d) 15 mA

147. In the given circuit



The current through the battery is
(a) 0.5 A
(b) 1 A
(c) 1.5 A
(d) 2 A

148. Currents flowing in each of the circuits A and B respectively are



(a) 1 A, 2 A (b) 2 A, 1 A
(c) 4 A, 2 A (d) 2 A, 4 A

B. Medical entrance special format questions

Assertion and reason

Directions (Q. Nos. 1-10) *These questions consists of two statements each printed as Assertion and Reason. While answering these questions you are required to choose any one of the following five responses.*

(a) If both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If Assertion is false but Reason is true.
(e) If both Assertion and Reason are false.

1. **Assertion** If the temperature of a semiconductor is increased, then its resistance decreases.

   Reason The energy gap between conduction band and valence band is very small.

2. **Assertion** The number of electrons in a *p*-type silicon semiconductor is less than the number of electrons in a pure silicon semiconductor at room temperature.

   Reason It is due to law of mass action.

3. **Assertion** The resistivity of a semiconductor increases with temperature.

   Reason The atoms of a semiconductor vibrate with larger amplitudes of higher temperatures thereby increasing its resistivity.

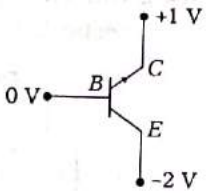
4. **Assertion** A rectifier converts as alternating current into a direct current.

   Reason A *p-n* junction diode can work as a rectifier.

5. **Assertion** Zener diode works on a principle of breakdown voltage.

   Reason Current increases suddenly after breakdown voltage.

6. **Assertion** When *p-n* junction is forward biased, then motion of charge carriers at junction is due to diffusion. In reverse biasing, the cause of motion of charge is drifting.



   Reason In the following circuit emitter is reverse biased and collector is forward biased.

7. **Assertion** A transistor amplifier in common emitter configuration has a low input impedence.

   Reason The base to emitter region is forward biased.

8. **Assertion** In a common emitter transistor amplifier, the input current is much less than the output current.

   Reason The common emitter transistor amplifier has very high input impedence.

9. **Assertion** NAND or NOR gates are called digital building blocks.

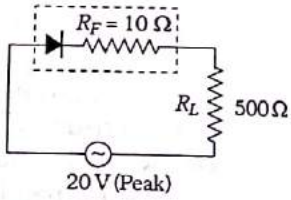
   Reason The repeated use of NAND (or NOR) gates can produce all the basic or complicated gates.

10. **Assertion** The logic gate NOT can be built using diode.

    Reason The output voltage and the input voltage of the diode have 180° phase difference.

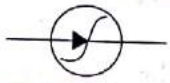
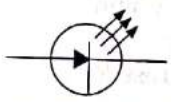
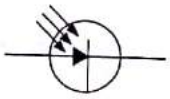
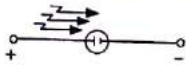
Match the columns

1. In the circuit shown the barrier voltage of diode is 0.7 V. Match the physical quantities given in Column I to the results given in Column II



	Column I		Column II
(A)	Peak current (in mA) in diode	(p)	37.8
(B)	Peak voltage (in volts) at the ends of load	(q)	40.0
(C)	Peak current (in mA), if diode is ideal	(r)	20.0
(D)	Peak voltage (in volts) at the ends of load, if diode is ideal	(s)	18.9
		(t)	Zero

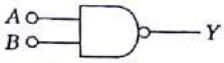
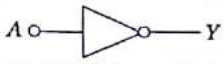
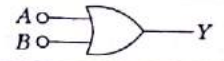
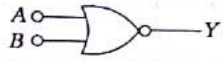
2. Match the symbol of diode given in Column I to the result given in Column II.

	Column I		Column II
(A)		(p)	Light emitting diode
(B)		(q)	Zener diode
(C)		(r)	Photo diode
(D)		(s)	Solar cells

3. Match the boolean expression of gate given in Column I to the gate given in Column II.

Column I		Column II	
(A)	$Y = A + B$	(p)	NAND Gate
(B)	$Y = A \cdot B$	(q)	NOT Gate
(C)	$Y = \overline{A}$	(r)	AND Gate
(D)	$Y = \overline{A \cdot B}$	(s)	OR Gate

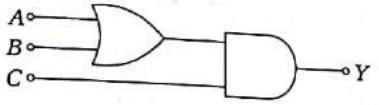
4. Match the symbol of gate given in Column I to the gate given in Column II.

Column I			Column II
(A)		(p)	NOT
(B)		(q)	OR
(C)		(r)	NAND
(D)		(s)	NOR

C. Medical entrances' gallery

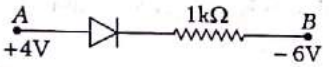
(Collection of questions asked in NEET & various medical entrance exams)

1. To get output 1 for the following circuit, the correct choice for the input is **[NEET 2016]**



(a) $A = 0, B = 0, C = 0$ (b) $A = 1, B = 1, C = 0$
(c) $A = 1, B = 0, C = 1$ (d) $A = 0, B = 1, C = 0$

2. Consider the junction diode as ideal. The value of current flowing through AB is **[NEET 2016]**



(a) 10^{-2} A (b) 10^{-1} A (c) 10^{-3} A (d) 0 A

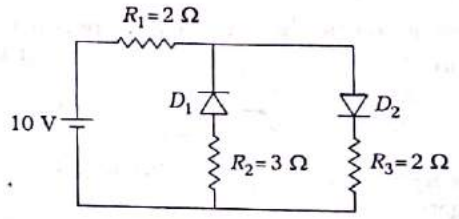
3. A n-p-n transistor is connected in common emitter configuration in a given amplifier. A load resistance of 800 Ω is connected in the collector circuit and the voltage drop across it is 0.8V. If the current amplification factor is 0.96 and the input resistance of the circuits is 192 Ω, then the voltage gain and the power gain of the amplifier will respectively be **[NEET 2016]**

(a) 3.69, 3.84 (b) 4, 4
(c) 4, 3.69 (d) 4, 3.84

4. For CE transistor amplifier, the audio signal voltage across the collector resistance of 2 kΩ is 4V. If the current amplification factor of the transistor is 100 and the base resistance is 1 kΩ, then the input signal voltage is **[NEET (II) 2016]**

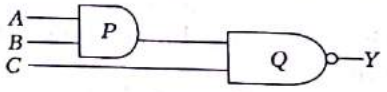
(a) 10 mV (b) 20 mV
(c) 30 mV (d) 15 mV

5. The given circuit has two ideal diodes connected as shown in the figure below. The current flowing through the resistance R_1 will be **[NEET (II) 2016]**



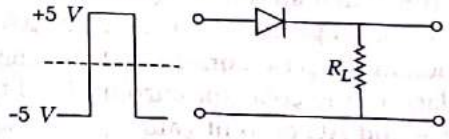
(a) 2.5 A (b) 10.0 A (c) 1.43 A (d) 3.13 A

6. What is the output Y in the following circuit, when all the three inputs A, B, C are first 0 an then 1 ?

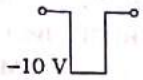


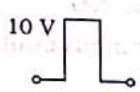
(a) 0, 1 (b) 0, 0
(c) 1, 0 (d) 1, 1 **[NEET (II) 2016]**

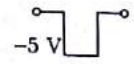
7. If in a p-n junction, a square input signal of 10 V is applied as shown,



then the output across R_L will be **[CG PMT 2015]**

(a) 

(b) 

(c) 

(d)

8. The Boolean expression $P + \overline{P}Q$, where P and Q are the inputs of the logic circuit, represents **[AIIMS 2015]**

(a) AND gate (b) NAND gate
(c) NOT gate (d) OR gate

9. When an input signal 1 is applied to a NOT gate, then its output is **[UK PMT 2015]**
(a) 1 (b) 0
(c) either 0 or 1 (d) any positive value

10. The impurity atoms with which pure silicon may be doped to make it a p-type semiconductor are those of **[UK PMT 2015]**
(a) phosphorous (b) antimony
(c) boron (d) iron

11. Let n_p and n_e be the number of holes and conduction electrons respectively, in an intrinsic semiconductor, then **[UK PMT 2015]**
(a) $n_p > n_e$ (b) $n_p = n_e$
(c) $n_p < n_e$ (d) None of these

12. Zener diode is used for **[UK PMT 2015]**
(a) rectification (b) amplification
(c) filtration (d) stabilisation

13. In a transistor, the value of β is 100, then the value of α will be **[UK PMT 2015]**
(a) 0.01 (b) 0.1 (c) 0.99 (d) 1

14. The arrangement shown in figure performs the logic function of **[UK PMT 2015]**

(a) AND gate (b) NAND gate
(c) OR gate (d) XOR gate

15. A semiconductor is having electron and hole mobilities μ_n and μ_p, respectively.
If its intrinsic carrier density is n_i, then what will be the value of hole concentration P for which the conductivity will be minimum at a given temperature? **[AIIMS 2015]**
(a) $n_i\sqrt{\frac{\mu_n}{\mu_p}}$ (b) $n_h\sqrt{\frac{\mu_n}{\mu_p}}$ (c) $n_i\sqrt{\frac{\mu_p}{\mu_n}}$ (d) $n_h\sqrt{\frac{\mu_p}{\mu_n}}$

16. A change of 0.04 V takes place between the base and the emitter when an input signal is connected to the CE transistor amplifier. As a result, 20 μA change take place in the base current and a change of 2mA takes place in the collector current. Find the input resistance and AC current gain. **[Guj. CET 2015]**
(a) 1 kΩ, 100 (b) 2 kΩ, 100
(c) 2 kΩ, 200 (d) 1 kΩ, 200

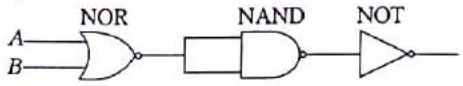
17. In a common emitter transistor amplifier, the voltage gain is **[UK PMT 2015]**
(a) $\alpha\frac{R_{AC}}{R_{in}}$ (b) $\alpha\frac{R_{in}}{R_{AC}}$ (c) $\beta\frac{R_{AC}}{R_{in}}$ (d) $\beta\frac{R_{in}}{R_{AC}}$

18. In a n-p-n transistor about 10^{10} electrons enter the emitter in 2μs, when it is connected to a battery. Then, $I_E = \ldots$ μA. **[Guj. CET 2015]**
(a) 400 (b) 200 (c) 800 (d) 1600

19. Which gate can be obtained by shorting both the input terminals of a NOR gate? **[Guj. CET 2015]**
(a) NOT (b) OR (c) AND (d) NAND

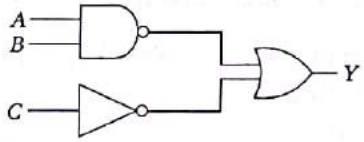
20. Pure Si at 300 K has equal electrons (n_e) and hole (n_h) concentrations of $1.5\times10^{+16}\,m^{-3}$. Dopping by indium, n_h increases to $3\times10^{22}\,m^{-3}$. What is n_e in doped Si? **[CG PMT 2015]**
(a) $2.5\times10^9\,m^{-3}$ (b) $5.5\times10^9\,m^{-3}$
(c) $7.5\times10^9\,m^{-3}$ (d) $6\times10^7\,m^{-3}$

21. The equivalent circuit is



The circuit represents **[Manipal 2015]**
(a) NAND gate (b) OR gate
(c) AND gate (d) NOR gate

22. The inputs to the digital circuit are as shown below. The output Y is **[WB JEE 2015]**



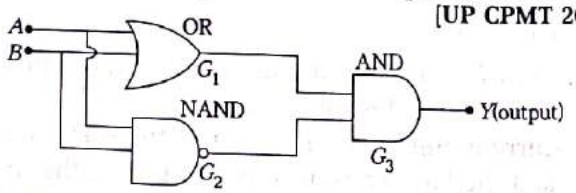
(a) $A+B+\overline{C}$ (b) $(A+B)\overline{C}$
(c) $\overline{A}+\overline{B}+\overline{C}$ (d) $\overline{A}+\overline{B}+C$

23. The p-type semiconductor is **[UP CPMT 2015]**
(a) positively charged
(b) negatively charged
(c) electrically neutral
(d) uncharged at 0 K or $-273\,°C$ but charged at temperature higher than 0 K

24. The current gain of a transistor in common base configuration is 0.96. Corresponding to the change in emitter current 10 mA, the change in base current would be **[UP CPMT 2015]**
(a) 0.03 mA (b) 0.2 mA (c) 0.4 mA (d) 0.6 mA

25. The liquid-crystal phase of a matter is called **[Manipal 2015]**
(a) mesophase (b) gel
(c) crystalline liquid (d) None of these

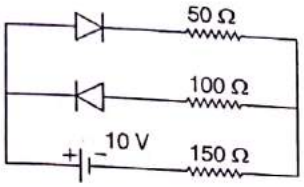
26. The following logic gate circuit in equivalent to an **[UP CPMT 2015]**



(a) NAND (b) OR (c) XOR (d) NOT

27. A common emitter amplifier is designed with n-p-n transistor ($\alpha = 0.99$). The input impedence is 1kΩ and load is 10 kΩ. The voltage gain will be **[Manipal 2015]**
(a) 9900 (b) 99 (c) 9.9 (d) 990

28. Assume that each diode as shown in the figure has a forward bias resistance of 50 Ω and an infinite reverse bias resistance. The current through the resistance 150 Ω is [WB JEE 2015]



(a) 0.66 A (b) 0.05 A
(c) zero (d) 0.04 A

29. The input characteristics of a transistor in CE mode is the graph obtained by plotting [KCET 2015]
(a) I_B against I_C at constant V_{CE}
(b) I_B against V_{BE} at constant V_{CE}
(c) I_B against I_C at constant V_{BE}
(d) I_B against V_{CE} at constant V_{BE}

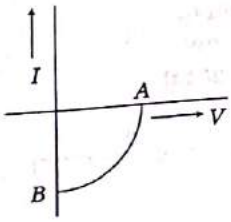
30. In a semiconducting material, the mobilities of electrons and holes are μ_e and μ_h, respectively. Which of the following is true? [Manipal 2015]
(a) $\mu_e < \mu_h$ (b) $\mu_e = \mu_h$
(c) $\mu_e < 0, \mu_h > 0$ (d) $\mu_e > \mu_h$

31. The barrier potential of a p-n junction depends on
I. type of semiconductor material
II. amount of doping
III. temperature
Which one of the following is correct? [CBSE AIPMT 2014]
(a) Both I and II (b) Only II
(c) Both II and III (d) All of these

32. The conductivity in the intrinsic semiconductor does not depend on [CG PMT 2015]
(a) band gap (b) temperature
(c) fermi energy (d) effective mass of charge carriers

33. An LED is constructed from a p-n junction based on a certain semi-conducting material whose energy gap is 1.9 eV. Then, the wavelength of the emitted light is [KCET 2015]
(a) 6.5×10^{-7} m (b) 2.9×10^{-9} m
(c) 9.1×10^{-5} m (d) 1.6×10^{-8} m

34. The given graph represents V-I characteristic for a semiconductor device. [CBSE AIPMT 2014]



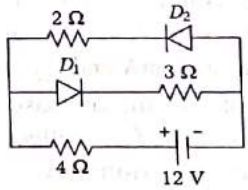
Which of the following statement is correct?
(a) It is V-I characteristic for solar cell, where point A represents open circuit voltage and point B short circuit current
(b) It is for a solar cell and points A and B represent open circuit voltage and current, respectively
(c) It is for a photodiode and points A and B represent open circuit voltage and current, respectively
(d) It is for a LED and points A and B represents open circuit voltage and short circuit current, respectively

35. In insulators (CB is Conduction Band and VB is Valence Band) [MHT CET 2014]
(a) VB is partially filled with electrons
(b) CB is partially filled with electrons
(c) CB is empty and VB is filled with electrons
(d) CB is filled with electrons and VB is empty

36. Identify the wrong statement. [Kerala CEE 2014]
(a) In conductors, the valence and conduction bands overlap
(b) Substances with energy gap of the order of 10 eV are insulators
(c) The resistivity of semiconductors is lower than metals
(d) The conductivity of metals is high
(e) The resistivity of a semiconductor is lower than that of an insulator

37. If the band gap between valence band and conduction band in a material is 5.0 eV, then the material is [WB JEE 2014]
(a) semiconductor (b) good conductor
(c) superconductor (d) insulator

38. The circuit has two oppositely connected ideal diodes in parallel. What is the current flowing in the circuit? [KCET 2015]

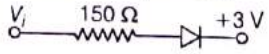


(a) 2.31 A (b) 1.71 A (c) 1.33 A (d) 2 A

39. In n-type semiconductor, electrons are majority charge carriers but it does not show any negative charge. The reason is [KCET 2014]
(a) electrons are stationary
(b) electrons neutralise with holes
(c) mobility of electrons is extremely small
(d) atom is electrically neutral

40. Identify the wrong statement with reference to solar cell. [Kerala CEE 2014]
(a) It is a p-n junction diode with no external bias
(b) It uses materials of high optical absorption
(c) It uses materials with band gap of 5 eV
(d) It converts light energy into electrical energy
(e) It uses materials such as GaAs, Si

41. In the circuit shown below, assume the diode to be ideal. When V_i increases from 2 V to 6 V, then the change in the current is (in mA) **[WB JEE 2014]**



(a) zero (b) 20 (c) 80/3 (d) 40

42. Zener diode is used for **[UK PMT 2014, Haryana PMT, CG PMT 2010]**
(a) amplification
(b) rectification
(c) voltage regulation
(d) produce oscillation in an oscillator

43. In common base circuit of a transistor, current amplification factor is 0.95. Calculate the emitter current, if base current is 0.2 mA. **[MHT CET 2014]**
(a) 2 mA (b) 4 mA (c) 6 mA (d) 8 mA

44. A tuned amplifier circuit is used to generate a carrier frequency of 2 MHz for the amplitude modulation. The value of $\sqrt{LC}$ is **[KCET 2014]**
(a) $\frac{1}{2\pi \times 10^6}$ (b) $\frac{1}{2 \times 10^6}$ (c) $\frac{1}{3\pi \times 10^6}$ (d) $\frac{1}{4\pi \times 10^6}$

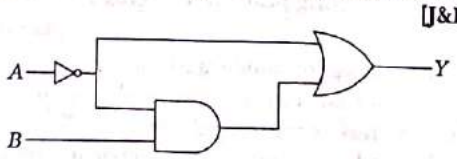
45. If α (current gain) of transistor is 0.98, then what is the value of β (current gain) of the transistor? **[KCET 2014]**
(a) 0.49 (b) 49 (c) 4.9 (d) 5

46. In a transistor output characteristics commonly used in common emitter configuration, the base current I_B, the collector current I_C and the collector-emitter voltage V_{CE} have values of the following orders of magnitude in the active region **[WB JEE 2014]**
(a) I_B and I_C both are in μA and V_{CE} in V
(b) I_B is in μA, I_C is in mA and V_{CE} in V
(c) I_B is in mA, I_C is in μA and V_{CE} in mV
(d) I_B is in mA, I_C is in mA and V_{CE} in mV

47. For the action of a Common Base (CB) transistor, (E = emitter, B = base, C = collector) the required CB, EB junction bias conditions are **[EAMCET 2014]**
(a) Both EB and CB junction forward bias
(b) Both EB and CB junction reverse bias
(c) EB junction forward bias, CB junction reverse bias
(d) EB junction reverse bias, CB junction forward bias

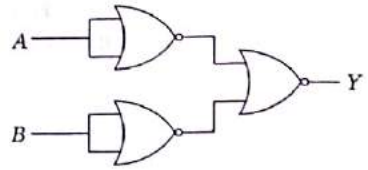
48. The minimum number of NAND gates used to construct an OR gate is **[Kerala CEE 2014]**
(a) 4 (b) 6 (c) 5 (d) 3 (e) 2

49. The output Y of the logic circuit given below is **[J&K CET 2014]**



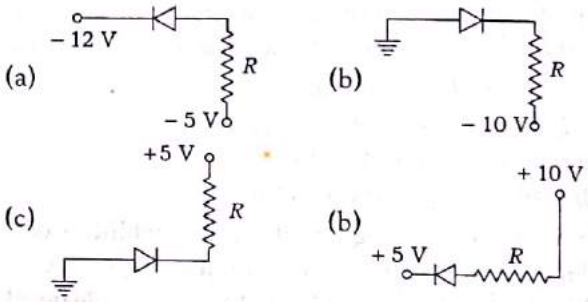
(a) $\overline{A} + B$ (b) $\overline{A}$ (c) $\overline{(\overline{A} + B)} \cdot \overline{A}$ (d) $\overline{(\overline{A} + B)} \cdot A$

50. For the given digital circuit, identify the logic gate it



[KCET 2014]
(a) OR gate
(b) NOR gate
(c) NAND gate
(d) AND gate

51. Of the diodes shown in the following diagrams, which one is reverse biased. **[UK PMT 2014]**



52. The truth tables of logic gates G_1, G_2, G_3 and G_4 are given here. Identify them correctly. **[EAMCET 2014]**

G_1

Inputs		Output
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

G_2

Inputs		Output
A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

G_3

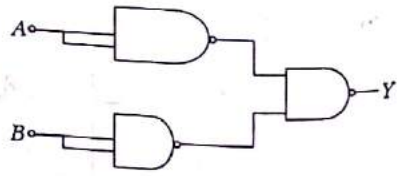
Inputs		Output
A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

G_4

Inputs		Output
A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

(a) G_1-OR, G_2-AND, G_3-NOR, G_4-NAND
(b) G_1-OR, G_2-NOR, G_3-AND, G_4-NAND
(c) G_1-AND, G_2-OR, G_3-NAND, G_4-NOR
(d) G_1-OR, G_2-NOR, G_3-NAND, G_4-AND

53. The combination of gates shown below yields **[UK PMT 2014]**



(a) NAND gate
(b) NOR gate
(c) XOR gate
(d) OR gate

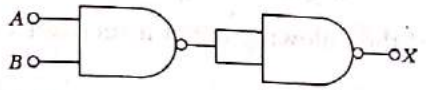
54. In a n-type semiconductor, which of the following statement is true? **[NEET 2013]**

(a) Electrons are majority carriers and trivalent atoms are dopants
(b) Electrons are minority carriers and pentavalent atoms are dopants
(c) Holes are minority carriers and pentavalent atoms are dopants
(d) Holes are majority carriers and trivalent atoms are dopants

55. In a common emitter (CE) amplifier having a voltage gain G, the transistor used has transconductance 0.03 mho and current gain 25. If the above transistor is replaced with another one with transconductance 0.02 mho and current gain 20, then the voltage gain will become **[NEET 2013]**

(a) $\frac{2}{3}G$ (b) $1.5\,G$ (c) $\frac{1}{3}G$ (d) $\frac{5}{4}G$

56. The output (X) of the logic circuit shown in figure will be



[NEET 2013]

(a) $X = \bar{A} \cdot \bar{B}$ (b) $X = \overline{A \cdot B}$
(c) $X = A \cdot B$ (d) $X = \overline{A + B}$

57. The necessary condition in making of a junction transistor (E-emitter, B-base and C-collector) **[EAMCET 2013]**

(a) E and B are lightly doped and C is heavily doped
(b) E is heavily doped, B is thin and lightly doped and C is moderately doped
(c) E and C are lightly doped and B is thick and heavily doped
(d) E and B are heavily doped and C is lightly doped

58. A p-n-p transistor is used in common emitter mode in an amplifier circuit. When base current is changed by an amount ΔI_B, then the collector current changes by 4 mA. If the current amplification factor is 60, then the value of ΔI_B is **[EAMCET 2013]**

(a) 15 μA (b) 240 mA (c) 66.6 μA (d) 60 μA

59. In a ionised gas, the mobile charge carriers are **[Kerala CET 2013]**

(a) electrons
(b) both positive and negative ions
(c) electrons and positively charged ions
(d) positive ions only
(e) negative ions only

60. Which one of the following statements is not correct in case of a semiconductor? **[BCECE (Mains) 2012]**

(a) Temperature coefficient of resistance is negative
(b) Doping increases conductivity
(c) At absolute zero temperature, it behaves like an insulator
(d) Resistivity is in between that of a conductor and insulator

61. In p-type semiconductor, the acceptor level lies **[Kerala CET 2013]**

(a) near the conduction band
(b) halfway between conduction and valence bands
(c) within conduction band
(d) near the valence band
(e) within the valence band

62. If the feedback voltage is increased in a negative feedback amplifier, then **[Kerala CET 2013]**

(a) both gain and distortion decrease
(b) the distortion increase
(c) the gain decrease and distortion increase
(d) the gain increase
(e) both gain and distortion increase

63. When a p-n junction is reverse biased, then the current through the junction is mainly due to **[MP PMT 2013]**

(a) Only diffusion of charges
(b) Only drift of charges
(c) Both drift and diffusion of charges
(d) Neither drift nor diffusion of charges

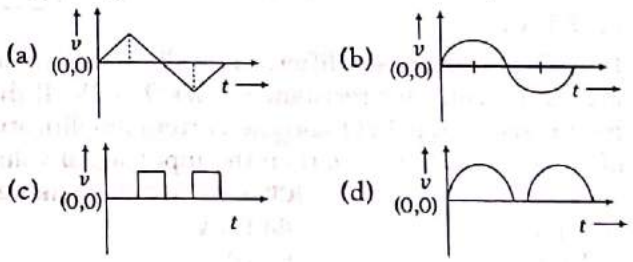
64. What is the value of $\bar{A} + A$ in the Boolean algebra? **[UP CPMT 2013]**

(a) A (b) 0 (c) 1 (d) $\bar{A}$

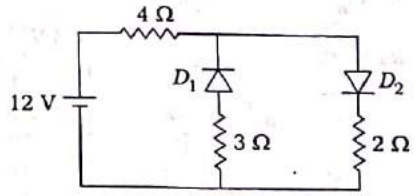
65. The depletion layer in the p-n junction region is caused by **[UP CPMT 2013]**

(a) drift of holes
(b) drift of electrons
(c) diffusion of carriers
(d) migration of impurity ions

66. Out of the following curves, which one represents a digital signal? **[UP CPMT 2013]**



67. The circuit has two oppositely connected ideal diodes in parallel as shown in figure. What is the current flowing in the circuit ? **[UP CPMT 2013]**

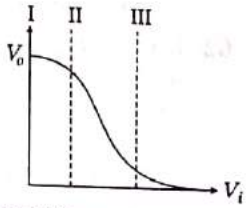


(a) 1.33 A (b) 1.71 A
(c) 2.00 A (d) 2.31 A

68. The output of an AND gate is connected to both the inputs of a NOR gate, then this circuit will act as a **[Kerala CET 2013]**

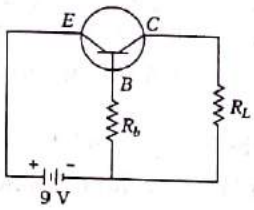
(a) OR gate (b) NOR gate
(c) AND gate (d) NOT gate
(e) NAND gate

69. Transfer characteristic [output voltage (V_o) *versus* input voltage (V_i)] for a base biased transistor in CE configuration is as shown in the figure. For using transistor as a switch, it is used **[BCECE 2012; CBSE AIPMT (Screening) 2012]**



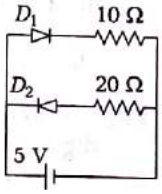
(a) Only in region III (b) Both in region I and III
(c) Only in region II (d) Only in region I

70. In a transistor circuit shown in figure, if the base current is 35 μA, then the value of resistor R_b is **[BCECE (Mains) 2012]**



(a) 257 kΩ (b) 123.5 kΩ
(c) 380.05 kΩ (d) None of these

71. Find the current in the circuit. **[CBSE AIPMT (Screening) 2012]**

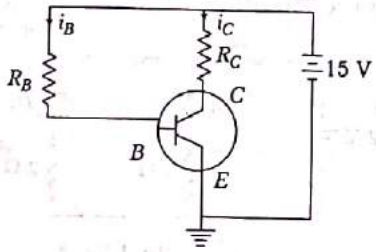


(a) 0.75 A
(b) Zero
(c) 0.25 A
(d) 0.5 A

72. In a CE transistor amplifier, the audio signal voltage across the collector resistance of 2 kΩ is 2V. If the base resistance is 1 kΩ and the current amplification of the transistor is 100, then the input signal voltage is **[CBSE AIPMT (Screening) 2012]**

(a) 0.1 V (b) 1.0 V
(c) 1 mV (d) 10 mV

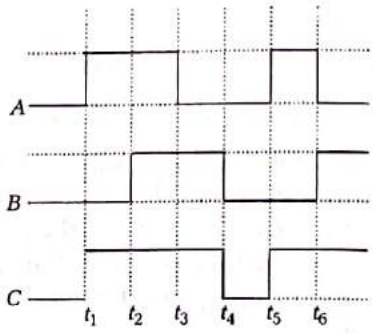
73. If for the following common emitter circuit, $\beta = 100$, $V_{CE} = 7$ V, V_{BE} is negligible and $R_C = 2$ kΩ, then $i_B = ?$



[BCECE (Mains) 2012]

(a) 0.04 mA (b) 0.03 mA
(c) 0.02 mA (d) 0.01 mA

74. The figure shows a logic circuit with two inputs A and B and the output C. The voltage wave forms across A, B and C are as given. The logic circuit gate is **[CBSE AIPMT (Screening) 2012]**

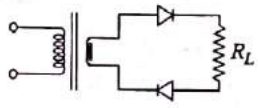


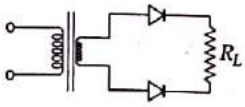
(a) OR gate (b) NOR gate (c) AND gate (d) NAND gate

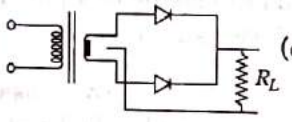
75. In intrinsic semiconductor at room temperature number of electrons and holes are **[Manipal 2012]**

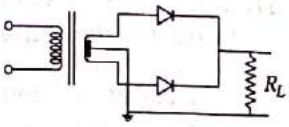
(a) equal (b) zero (c) unequal (d) infinite

76. Which of the following is not a rectifier circuit? **[AMU 2012]**

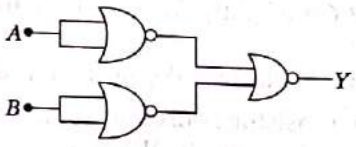
(a)


(b)


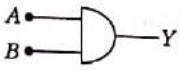
(c)


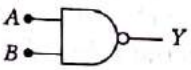
(d)


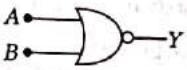
77. The following network of gates **[AMU 2012]**

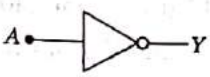


is equivalent to

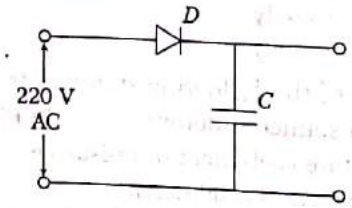
(a)


(b)


(c)


(d)


78. A diode is connected to 220 V (rms) AC in series with a capacitor as shown in figure. The voltage across the capacitor is **[JCECE 2012]**



(a) 220 V (b) 110 V (c) 311.1 V (d) $\frac{220}{\sqrt{2}}$ V

Answers

Check point 14.1

1. (a)	2. (a)	3. (a)	4. (d)	5. (b)	6. (b)	7. (c)	8. (b)	9. (a)	10. (d)
11. (b)	12. (b)	13. (b)	14. (a)	15. (d)					

Check point 14.2

1. (d)	2. (d)	3. (d)	4. (a)	5. (b)	6. (b)	7. (c)	8. (a)	9. (b)	10. (a)
11. (c)	12. (b)	13. (b)	14. (c)	15. (d)					

Check point 14.3

1. (a)	2. (d)	3. (c)	4. (b)	5. (a)	6. (a)	7. (a)	8. (d)	9. (b)	10. (b)
11. (a)	12. (a)	13. (a)	14. (a)	15. (b)					

Check point 14.4

1. (c)	2. (a)	3. (b)	4. (b)	5. (d)	6. (d)	7. (b)	8. (c)	9. (d)	10. (b)
11. (d)	12. (a)	13. (d)	14. (d)	15. (c)					

A. Taking it together

1. (d)	2. (b)	3. (d)	4. (a)	5. (d)	6. (c)	7. (d)	8. (a)	9. (c)	10. (b,c)
11. (c)	12. (b)	13. (c)	14. (b)	15. (c)	16. (b)	17. (c)	18. (c)	19. (b)	20. (c)
21. (a)	22. (c)	23. (c)	24. (d)	25. (c)	26. (d)	27. (c)	28. (b)	29. (b)	30. (b)
31. (a)	32. (b)	33. (d)	34. (c)	35. (a)	36. (b)	37. (d)	38. (a)	39. (c)	40. (d)
41. (a)	42. (b)	43. (a)	44. (b)	45. (b)	46. (a)	47. (b)	48. (d)	49. (b)	50. (b)
51. (c)	52. (d)	53. (d)	54. (b)	55. (b)	56. (b)	57. (b)	58. (b)	59. (b)	60. (a)
61. (d)	62. (c)	63. (c)	64. (b)	65. (d)	66. (c)	67. (c)	68. (b)	69. (c)	70. (d)
71. (d)	72. (b)	73. (c)	74. (c)	75. (d)	76. (b)	77. (c)	78. (c)	79. (b)	80. (c)
81. (c)	82. (a)	83. (b)	84. (c)	85. (b)	86. (a)	87. (c)	88. (b)	89. (b)	90 (b)
91. (b)	92. (d)	93. (a)	94. (d)	95. (b)	96. (d)	97. (d)	98. (a)	99. (a)	100. (a)
101. (c)	102. (b)	103. (d)	104. (c)	105. (c)	106. (d)	107. (a)	108. (d)	109. (a)	110. (c)
111. (c)	112. (a)	113. (c)	114. (c)	115. (c)	116. (c)	117. (d)	118. (a)	119. (c)	120. (c)
121. (b)	122. (a)	123. (c)	124. (b,c)	125. (b)	126. (a)	127. (b)	128. (c)	129. (b)	130. (c)
131. (a)	132. (b)	133. (b)	134. (a)	135. (b)	136. (d)	137. (b)	138. (d)	139. (a)	140. (b)
141. (d)	142. (c)	143. (c)	144. (b)	145. (c)	146. (b)	147. (c)	148. (c)		

B. Medical entrance special format questions

Assertion and reason

1. (a)	2. (a)	3. (e)	4. (b)	5. (b)	6. (b)	7. (a)	8. (c)	9. (a)	10. (e)

Match the columns

1. (A → p, B → s, C → q, D → r)
2. (A → q, B → p, C → r, D → s)
3. (A → s, B → r, C → q, D → p)
4. (A → r, B → p, C → q, D → s)

C. Medical entrances' gallery

1. (c)	2. (a)	3. (d)	4. (b)	5. (a)	6. (c)	7. (d)	8. (d)	9. (b)	10. (c)
11. (b)	12. (d)	13. (c)	14. (b)	15. (a)	16. (b)	17. (c)	18. (c)	19. (a)	20. (c)
21. (d)	22. (c)	23. (c)	24. (c)	25. (a)	26. (c)	27. (d)	28. (b)	29. (b)	30. (d)
31. (d)	32. (c)	33. (a)	34. (a)	35. (c)	36. (c)	37. (d)	38. (b)	39 (d)	40. (c)
41. (b)	42. (c)	43. (b)	44. (d)	45. (b)	46. (b)	47. (c)	48. (d)	49. (b)	50. (d)
51. (c)	52. (a)	53. (d)	54. (c)	55. (a)	56. (c)	57. (b)	58. (c)	59. (c)	60. (d)
61. (d)	62. (a)	63. (b)	64. (c)	65. (c)	66. (c)	67. (c)	68. (e)	69. (b)	70. (a)
71. (d)	72. (d)	73. (a)	74. (a)	75. (a)	76. (a)	77. (a)	78. (c)		

Hints and explanations

Check point 14.1

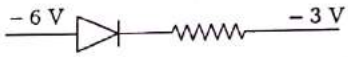
1. (*a*) The expected energy of the electrons at absolute zero is called fermi energy.
2. (*a*) The conduction band in a solid is partially filled at 0 K. the solid sample is a conductor.
3. (*a*) In conductor, valence band and conduction band overlaps.
4. (*d*) In insulators, the forbidden energy gap is largest and it is of the order of 6 eV.
5. (*b*) In insulators, the forbidden energy gap is very larger, in case of semiconductor it is moderate and in conductors the energy gap is zero.
6. (*b*) Doping of intrinsic semiconductor is done to increase concentration of majority charge carriers.
7. (*c*) In *p*-type semiconductor, holes are the majority charge carries.
8. (*b*) Gallium to pure silicon.
9. (*a*) Aluminium is trivalent impurity which is used to obtain *p*-type semiconductor.
10. (*d*) In *p*-type semiconductor, holes are majority charge carries and electrons are minority charge carries.
11. (*b*) The charge on hole is positive which is equivalent to charge on proton.
12. (*b*) For *n*-type semiconducting material number of free electrons is very greater than number of holes *i.e.*, $n_e >> n_h$.
14. (*a*) When a semiconductor is heated, its resistance decreases.
15. (*d*) Temperature coefficient of semiconductor is negative.

Check point 14.2

1. (*d*) In the depletion region of a unbiased *p-n*-junction diode there are only fixed ions.
2. (*d*) The cause of the potential barrier in a *p-n* diode is concentration of positive and negative charge near the junction.
4. (*a*) Majority of carriers controls the conduction of *p-n* junction.
5. (*b*) In the forward bias arrangement of a *p-n*-junction diode the *p*-end is connected to the positive terminal of the battery.
6. (*b*) In forward biasing, width of depletion layer decreases.
8. (*a*) There will not be a steady current in the circuit. If the two ends *p* and *n* of a *p-n* diode are joined by a circle.
9. (*b*) In reverse biasing, width of the depletion layer increases.
10. (*a*) As, $-6\,V < -3\,V$

    i.e., *p* is at low potential than *n*

    ∴ Diode is reverse biased



11. (*c*) When a large reverse voltage is applied across a *p-n* junction diode a huge current flows in the reverse direction suddenly. This is called breakdown of *p-n* junction diode.
12. (*b*) At a particular reverse voltage in *p-n* junction, a huge current flows in reverse direction known as avalanche current.
13. (*b*) It is used to convert AC into DC (rectifier).
14. (*c*) When a light falls on the junction, new hole electron pairs are created. No. of produced electron hole pairs depend upon No. of photons. So photo emf or current is proportional to intensity of light.

Check point 14.3

1. (*a*) The base is always thin.
2. (*d*) In a bipolar junction transistor, emitter is heavily doped, base is lightly doped and collector is moderately doped.
4. (*b*) A *n-p-n* transistor conducts when collector is positive and emitter is negative with respect to the base.
5. (*a*) Here $\Delta V_C = 0.5V$, $\Delta I_C = 0.05\,mA$

   Output resistance is given by

$$R_{out} = \frac{\Delta V_C}{\Delta I_C} = \frac{0.5}{0.05 \times 10^{-3}} = 10^4\ \Omega = 10\,k\Omega$$

6. (*a*) The alternating current gain β is given by $\beta = \left(\frac{\Delta I_C}{\Delta I_B}\right)_{VC}$
7. (*a*) $\beta = \left(\frac{\Delta I_C}{\Delta i_B}\right)_{V_{CE}} = \frac{8.2}{(8.3 - 8.2)} = 82$
8. (*d*) $\alpha = \frac{i_C}{i_E} = \frac{5.488}{5.6} = 0.98$; $\beta = \frac{\alpha}{1-\alpha} = \frac{0.98}{1-0.98} = 49$
9. (*b*) Current gain, $\beta = \frac{\Delta I_C}{\Delta I_B} \Rightarrow \Delta I_B = \frac{1 \times 10^{-3}}{100}$

   $= 10^{-5}\,A = 0.01\,mA$

   By using $\Delta i_E = \Delta i_B + \Delta i_C$; $\Delta i_C = 0.01 + 1 = 1.01$ mA
10. (*b*) In *p-n-p* transistor, emitter-base is forward-biased hence, holes move from emitter to base.

11. (*a*) When *n-p-n* transistor is used as an amplifier majority charge carrier *i.e.*, electrons of *n*-type emitter move from emitter to base end and then base to collector.

12. (*a*) Oscillator can produce radio waves of constant amplitude.

13. (*a*) Voltage gain $= 10\log_{10} 1000$ dB $= 30$ dB

14. (*a*) In oscillator a portion of the output power is returned back to the input in phase with the starting power. This process is termed as positive feedback.

15. (*b*) $\frac{V_{out}}{V_{in}} = \frac{I_{out}}{I_{in}} \times \frac{R_{out}}{R_{in}}$

$$\Rightarrow R_{out} = \frac{3}{0.01} \times 1000 \times \frac{1}{100} = 3\text{k}\Omega$$

Check point 14.4

1. (*c*) $10111 = 1\times 2^4 + 0\times 2^3 + 1\times 2^2 + 1\times 2^1 + 1\times 2^0$
$= 16 + 0 + 4 + 2 + 1 = 23$

2. (*a*)

$$\begin{array}{r} 100010 \\ +\ \ 11011 \\ \hline 111101 \\ \hline \end{array}$$

$\therefore\ (100010)_2 + (11011)_2 = (111101)_2$

3. (*b*) The output of OR gate is $Y = A + B$.

4. (*b*) For AND gate, if output is 1, than both inputs must be 1.

5. (*d*) The boolean expression for AND gate is $R = P.Q$

$R = P.Q \Rightarrow 1.1 = 1$
$1.0 = 0;\ 0.1 = 0$ and $0.0 = 0$

6. (*d*) For AND gate, if output is 0, then one or both the inputs must be 0.

7. (*b*) A gate in which all the inputs must be low to get a high output is called on inverter.

8. (*c*) If inputs are A and B, then output for NAND gate is $Y = \overline{AB}$.
If $A = B = 1, Y = \overline{1.1} = \overline{1} = 0$

9. (*d*) The given truth table is for NAND gate.

10. (*b*) The boolean expression for NOR gate is $\overline{Y} = \overline{A + B}$
i.e., if $A = B = 0$ (low) $\Rightarrow Y = \overline{0 + 0} = \overline{0} = 1$ (High)

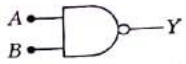
11. (*d*) Any digital circuit can be realised by repetitive use of NOR gate or NAND gate. NOR gates or NAND gates are called universal gates because by using these gates we can realise other baisc gates like OR, AND and NOT gates.

13. (*d*)

Input		Output			
		OR	AND	NOR	NAND
0	0	0	0	1	1
0	1	1	0	0	1
1	0	1	0	0	1
1	1	1	1	0	0

From the above truth tables it is clear that the logic gates giving output '1' for the inputs 1 and 0 are NAND and OR gates.

14. (*d*) NAND gate

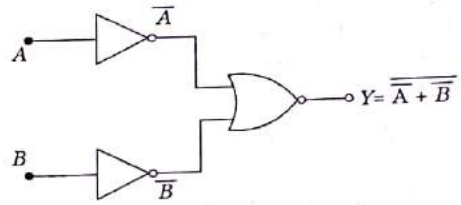


$Y = \overline{A \cdot B}$

When the two inputs are shorted

$Y = \overline{A}$ = which is equivalent to a NOT gate.

15. (*c*)



Hence, output $Y = \overline{\overline{A} + \overline{B}}$

According to De-morgan's theorem

$$Y = \overline{\overline{A} + \overline{B}} = \overline{\overline{A}} \cdot \overline{\overline{B}} = A \cdot B$$

This is the output equation of AND gate.

A. Taking it together

2. (*b*) The concept of hole describes the lack of an electron at a position where one could exist in an atom or atomic lattice. If an electron is excited into a higher state, it leaves a hole in its old state.

Thus, hole can be defined as a vacancy created when an electron leaves a covalent bond.

3. (*d*) Resistance of conductor (Cu) decreases with decrease in temperature while that of semiconductor (Ge) increases with decrease in temperature.

5. (*d*) A solid with completely filled valence band is an insulator, if the energy gap between the valence band and the empty conduction band is larger that is about 5 eV.

6. (*c*) In semiconductors, forbidden energy gap is of the order of 1 eV.

7. (*d*) In a good conductor, the conduction band and valence band overlap each other.

8. (*a*) In semiconductors, at a room temperature, the valence band is partially empty and the conduction band is partially filled.

9. (*c*) On increasing the temperature, the current in the circuit will increase because with rise in temperature. resistance of semiconductor decreases, hence overall resistance of the circuit decreases, which in turn increases the current in the circuit.

10. (*b, c*) The base region must be very thin and lightly doped. The emitter function is forward biased and collector junction is reverse biased.

12. (*b*) The energy band gap of Si is 1.1 eV.

15. (*c*) When the electrical conductivity of a semiconductor is due to the breaking of its covalent bonds, then the semiconductor is said to be intrinsic.

17. (*c*) The majority charge carriers in *p*-type semiconductor are holes, while electrons are minority charge carriers.

18. (*c*) The thickness of *p-n* junction has an order of 10^{-6} m.

19. (*b*) *p*-type semiconductor can be made by replacing a silicon atom in the crystal lattice with a gallium (a trivalent impurity).

20. (*c*) For making *p*-type semiconductor, we add trivalent impurity. So, B impurity is mixed in Ge and Al impurity is mixed in Si.

21. (*a*) At room temperature, a *p*-type semiconductor has large number of holes and few electrons, as holes are majority charge carriers.

22. (*c*) Holes are minority carriers and pentavalent atoms are dopants.

28. (*b*) Pentavalent impurity atom is to be added to Ge crystal so as to make a *n*-type semiconductor.

36. (*b*) In a transistor, emitter is heavily doped, base is least doped while collector is moderately doped.

37. (*d*) Extrinsic semiconductors are also electrically neutral.

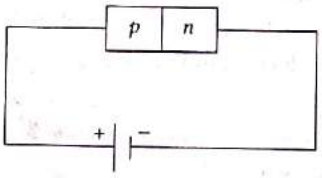
38. (*a*) Mobility of electrons are higher than that of holes because electrons need less energy to move.

39. (*c*) Lower the potential barrier.

40. (*d*) The conductivity of a semiconductor increases with increase in temperature, because the number density of current carries increases, relaxation time decreases but effect of decrease in relaxation is much less than increase in number density.

43. (*a*) At junction, a potential barrier/depletion layer is formed, with *n*-side at higher potential and *p*-side at lower potenital. Therefore, there is an electric field at the junction directed from *n*-side to *p*-side.

44. (*b*) In the forward bias arrangement of a *p-n* junction diode, the *p*-end is connected to the positive terminal of the battery.



46. (*a*) As, $i = neAv_d$

So, $v_d = \dfrac{i}{neA}$

So, when temperature is increased, n_e increases and v_d decreases.

47. (*b*) In forward biasing, the diffusion current increases and drift current remains constant, so net current is due to diffusion. In reverse biasing diffusion is not possible, so net current (very small) is due to the drift.

48. (*d*) Holes are majority charge carriers and trivalent atoms are dopants.

49. (*b*) In reverse biasing of *p-n* junction diode, the applied reverse voltage establishes an electric field which acts in the same direction as the electric field in the potential barrier. Therefore, the height of potential barrier is increased.

50. (*b*) At a specific reverse voltage in *p-n* junction, a huge current flows in reverse direction known as Avalanche current.

51. (*c*) Hole concentration in *p*-region is more as compared to *n*-region.

56. (*b*) In forward bias, the applied forward potential is opposite to potential barrier of depletion region, therefore potential barrier is decreased.

61. (*d*) Resistance of semiconductor decreases with increase in temperature.

65. (*d*) $(E_g)_{Ge} < (E_g)_{Si} < (E_g)_C$

$[\because (E_g)_{Ge} = 0.7\text{ eV}, (E_g)_{Si} = 1.1\text{ eV}, (E_g)_C = 5.5\text{ eV}]$

67. (*c*) A transistor amplifier, the voltage gain, is low at high and low frequencies and constant at mid frequencies.

68. (*b*) From rectifier we get pulsating DC, which consists of AC ripples, to remove these unwanted ripples we use filter circuits.

70. (*d*) α is current gain in case of common base amplifier while β is current gain in case of common emitter amplifier. α is always less than 1 and β is greater than 1.

72. (*b*) Collector current = 10 mA

$\therefore \quad 10\% = \dfrac{10}{90} \times 100 = 11.11$

or emitter current is 11.11 mA.

73. (*c*) Given, $\alpha = 0.98$

$\therefore \quad \beta = \dfrac{\alpha}{1-\alpha} = \dfrac{0.98}{1-0.98} = 49$

75. (*d*) For full wave rectifier $\eta = \dfrac{81.2}{1+\dfrac{r_f}{R_L}}$

$\Rightarrow \quad \eta_{max} = 81.2\% \qquad (r_j << R_L)$

77. (*c*) In forward biasing of a diode, *p*-side is at higher potential.

78. (*c*) $A \cdot A = A$

$A + A = A \Rightarrow \overline{\overline{A+A}} = \overline{\overline{A}} \cdot \overline{\overline{A}} = A$

$\overline{A} \cdot A = 0$

79. (*b*) When *p-n* junction is forward biased, it opposes the potential barrier junction, when *p-n* junction is reverse biased, it supports the potential barrier junction, resulting increase in potential barrier across the junction.

80. (*c*) For full wave rectifier, the fundamental frequency in ripple is twice that of input frequency.

83. (*b*) A NOR gate is combination of NOT and OR gate. The boolean expression of NOR gate is $C = \overline{A+B}$

84. (*c*) With the rise in temperature, resistivity of semiconductors decreases exponentially.

86. (*a*) Given, $E = 6 \times 10^5 \text{ Vm}^{-1}$

$$d = 500 \text{ nm} = 500 \times 10^{-9} \text{m}$$

$$V = ?$$

$\therefore$ Potential barrier, $V = Ed = (6 \times 10^5)(500 \times 10^{-9})$

$$= 0.30 \text{ V}$$

87. (*c*) The electrical conductivity of germanium,

$$\sigma = \frac{1}{\rho} = e(\mu_e n_e + \mu_h n_n)$$

$$= (1.6 \times 10^{-19})[0.36 + 0.17](2.5 \times 10^{19})$$

$$= 2.12 \text{ Sm}^{-1}$$

88. (*b*) As $n_e >> n_h$

So, it is a *n*-type semiconductor.

89. (*b*) In the given circuit *p*-side of *p-n* function D_1 is connected to lower voltage and *n*-side of D_1 to higher voltage.

Thus *D* is reverse biased.

The *p* -side of *p-n* junction D_2 is at higher potential and *n*-side of D_2 is at lower potential. Therefore D_2 is forward biased.

Hence, current flows through the junction *B* to *A*

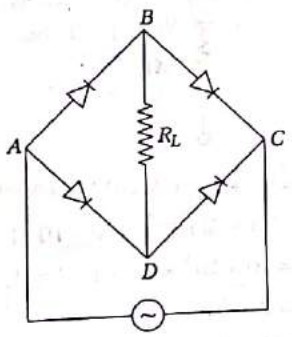
91. (*b*) In 1 cm^3 of Si, there will be total $\frac{6 \times 10^{28}}{10^6}$

$= 6 \times 10^{22}$ atoms per 6×10^7 silicon atoms there is one indium atom. Therefore, total number of indium atoms will be

$$\frac{6 \times 10^{22}}{6 \times 10^7} = 10^{15}$$

92. (*d*) As *p-n* junction conducts during positive half cycle only, the diode connected here will work is positive half cycle. Potential difference across C = peak voltage of the given AC voltage $= V_0 = V_{rms}\sqrt{2} = 220\sqrt{2}$ V.

93. (*a*) During the first half cycle when $V_a > V_c$, all the four diodes are forward biased. Hence, no current will flow through R_L. During second half cycle when $V_c > V_a$, all the four diodes are reverse biased. Again no current will flow through R_L.



94. (*d*) In forward biasing, both potential barrier V_B and the width of charge depleted region *x* decreases.

96. (*d*) Given, $\beta = \frac{\Delta i_c}{\Delta i_b} = 50$, $R_i = 1\text{k}\Omega = 10^3\,\Omega$, $\Delta V_i = 0.01\text{V}$

$$\therefore \quad \Delta i_{in} = \Delta i_b = \frac{\Delta V_i}{R_i} = \frac{0.01}{10^3} = 10^{-5} \text{ A}$$

$$\therefore \quad \Delta i_c = \beta\, \Delta i_b = 50 \times 10^{-5} \text{ A} = 500\ \mu\text{A}$$

97. (*d*) $A = \frac{V_0}{V_i + mV_0}$

Here, $V_0 = A(V_i - 0.09V_0) \Rightarrow \frac{V_0}{V_i} = 10 \therefore A = 100$

98. (*a*) In figure (a), first gate is AND gate and second gate is NOT gate. So, it is NAND gate.

99. (*a*) Here, diode is in reverse biased condition. In reverse biasing, it acts as open an circuit, hence no current flows.

100. (*a*) In the given circuit, diode is in reverse biasing, so it acts as open circuit. Hence, potential difference between *A* and *B* is 6 V.

103. (*d*) There is a phase difference of 180 in case of CE amplifier.

Also, $A_V = \frac{V_0}{V_i} \therefore V_0 = 4\text{V}$

Hence, $V_0 = (4\text{ V})\sin(\omega t - \pi/2)$

104. (*c*) $\alpha = \frac{\Delta i_c}{\Delta i_e}$

$\therefore \quad \Delta i_c = \alpha\, \Delta i_e = (0.98)(5) = 4.9 \text{ mA}$

105. (*c*) Energy gap, $E_g = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E_g} = \frac{12400\text{eVÅ}}{0.72\text{eV}} = 17222\text{ Å}$

106. (*d*) Given, $\Delta i_c = 0.990$ mA

$$\therefore \quad \alpha = \frac{\Delta i_c}{\Delta i_e} = \frac{0.990}{1} = 0.990$$

107. (*a*) The given Boolean expression can be written as

$$Y = (\overline{A+B}) \cdot (\overline{AB})$$

$$= (\overline{A} \cdot \overline{B}) \cdot (\overline{A} + \overline{B}) = (\overline{A} \cdot \overline{A}) \cdot \overline{B} + \overline{A}(\overline{B} \cdot \overline{B})$$

$$= \overline{A} \cdot \overline{B} + \overline{A} \cdot \overline{B} = \overline{A} \cdot \overline{B}$$

So, the truth table is

A	B	Y
0	0	1
1	0	0
0	1	0
1	1	0

109. (*a*) In case of forward biasing, resistance of *p-n* junction diode is zero. So, whole voltage appears across the resistance.

110. (*c*) $\lambda \text{ (in Å)} = \frac{12375}{E \text{ (in eV)}} = \frac{12375}{57 \times 10^{-3}}\text{ Å} \approx 217100\text{ Å}$

111. (*c*) In the truth table, output $X = \overline{A}$. So, this truth table is of NOT gate.

114. (*c*) $1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 + 1 \times 2^{-1}$

$$+ 0 \times 2^{-2} + 1 \times 2^{-3} = 26.625$$

115. (*c*) For option (c), it is a NAND gate, its output $= \overline{0.1} = \overline{0} = 1$

117. (*d*) The output D for the given combination,

$$D = \overline{(A+B)\cdot C} = \overline{(A+B)} + \overline{C}$$

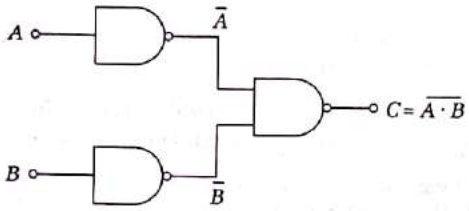
If $A = B = C = 0$, then

$$D = \overline{(0+0)} + \overline{0} = \overline{0} + \overline{0} = 1 + 1 = 1$$

If $A = B = 1$, $C = 0$, then

$$D = \overline{(1+1)} + \overline{0} = \overline{1} + \overline{0} = 0 + 1 = 1$$

118. (*a*) For upper part,

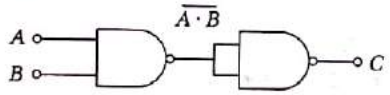


Here, $C = \overline{\overline{A}\cdot\overline{B}} = \overline{\overline{A}} + \overline{\overline{B}}$

$= A + B$ [from De-Morgan's theorem]

Here, output C is equivalent to OR gate.

For lower part,



$$C = \overline{\overline{AB}\cdot\overline{AB}} = \overline{\overline{AB}} + \overline{\overline{AB}} = AB + AB = AB$$

In this case output C is equivalent to AND gate.

119. (*c*) The boolean expression for the given combination is output

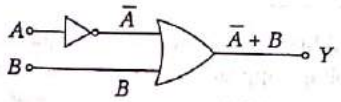
$$Y = (A+B)\cdot C$$

The truth table is

A	B	C	$Y = (A+B)\,C$
0	0	0	0
1	0	0	0
0	1	0	0
0	0	1	0
1	1	0	0
0	1	1	1
1	0	1	1
1	1	1	1

Hence, $A = 1$, $B = 0$ and $C = 1$

120. (*c*)



121. (*b*) Diode is in forward biased condition. So, current flow

$$i = \frac{(4-1)}{300} = 10^{-2}\ \text{A}$$

122. (*a*) Intensity of electric field,

$$E = \frac{V}{d} = \frac{0.50}{5\times10^{-7}} = 1.0\times10^{6}\ \text{Vm}^{-1}$$

123. (*c*) Due to forward biased during positive half cycle of input AC voltage, the resistance of p-n junction is low. The current in the circuit is maximum. In this situation, a maximum potential difference will appear across resistance connected in series of circuit. This result into zero output voltage across p-n junction.

Due to reverse biase during negative half cycle of AC voltage, the p-n junction is reverse biased. The resistance of p-n junction becomes high which will be more than resistance in series. That is why, there will be voltage across p-n junction with negative cycle in output.

124. (*b,c*) D_1 is forward biased and D_2 reverse biased.

Therefore, current through the resistance and D_1 will be equal and which is equal to $\frac{5}{4} = 1.25$ A

125. (*b*) D_1 is reverse biased. Therefore, no current will flow through 20 Ω resistance.

∴ 30 Ω and 20 Ω resistance will be in series. Total current will be 5/50 A.

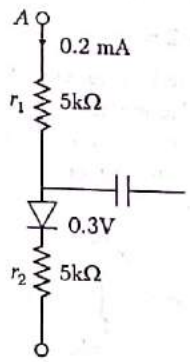
126. (*a*) First diode is in reverse biasing it acts as open circuit, hence no current flows.

127. (*b*) For forward biasing, p-side is at higher potential and n-side is at lower potential, so figures 2, 4 and 5 are forward biased.

128. (*c*) If $V_A > V_B$, then both the junction diodes are forward biased and the given circuit diagram becomes a balanced Wheatstone bridge. The equivalent resistance in this case becomes 4 Ω.

If $V_A < V_B$, then the diodes are reverse biased. In that case 4 Ω, 5 Ω and 4 Ω are in series.

129. (*b*) Consider the fig. (b) given here, suppose the potential difference between A and B. $r_1 = 5\,\text{k}\Omega$ and $r_2 = 5\,\text{k}\Omega$ are resistance in series connection.



Then, $V - 0.3 = [(r_1 + r_2)\,10^3] \times (0.2\times10^{-3})]$ [$\because V = ir$]

$= [(5+5)\,10^3] \times (0.2\times10^{-3})$

$= 10\times10^3 \times 0.2\times10^{-3} = 2$

$\Rightarrow$ $V = 2 + 0.3 = 2.3$ V

130. (*c*) Here, $C = AB$ and $D = \overline{A}.B$

$$E = C + D = (AB) + (\overline{A}\,.\,B)$$

Explanation *The truth table of this arrangement of gates can be given by*

A	B	$\overline{A}$	$C = A.B$	$d = \overline{A}.B$	$E = (C + D)$
0	0	1	0	0	0
0	1	1	0	1	1
1	0	0	0	0	0
1	1	0	1	0	1

131. (*a*) $g_m = \dfrac{A_V}{R_L}$

where, A_V = voltage gain $= \beta_{ac} \times$ resistance gain

and R_L = load resistance

$\therefore \quad A_V = 20 \times \dfrac{5}{2} = 50$

$\therefore \quad g_m = \dfrac{50}{5 \times 10^3} = 0.01\Omega^{-1}$

132. (*b*) Given, $V = 9\text{V}$

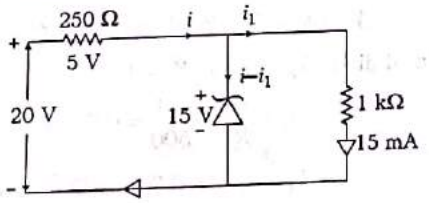
$i_b = 35\ \mu\text{A} = 35 \times 10^{-6}\text{A}$

$R_b = ?$

We know that, $V = i_b R_b$

$\Rightarrow \quad R_b = \dfrac{V}{i_b} = \dfrac{9}{35 \times 10^{-6}} = 257\ \text{k}\Omega$

133. (*b*) Voltage across Zener diode is constant



$i_{1\text{k}\Omega} = \dfrac{15\ \text{volt}}{1\text{k}\Omega} = 15\ \text{mA}$

$i_{250\Omega} = \dfrac{(20-15)\text{V}}{250\,\Omega} = \dfrac{5\text{V}}{250\Omega} = \dfrac{20}{1000}\text{A} = 20\ \text{mA}$

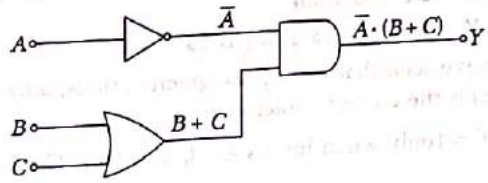
$\therefore \quad i_{\text{Zener diode}} = (20 - 15) = 5\ \text{mA}$

135. (*b*) In the positive half cycle of input *ac* signal diode D_1 is forward biased and D_2 is reverse biased so in the output voltage signal, A and C are due to D_1. In negative half cycle of input AC signal D_2 conducts hence output signals B and D are due to D_2.

137. (*b*) The Boolean expression of given combination of gates is written as $Y = \overline{\overline{A} \cdot \overline{B}} = \overline{\overline{A}} + \overline{\overline{B}} = A + B$

So, it produces OR gate.

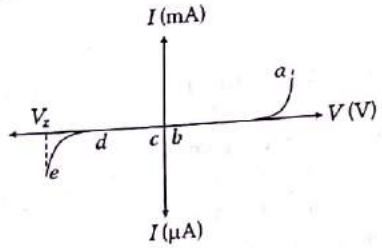
138. (*d*)



139. (*a*) The diode is in reverse biasing, so no current flows through it.

140. (*b*) Potential across the *p-n* junction varies symmetrically linear, having *p* side negative and *n* side positive.

142. (*c*)



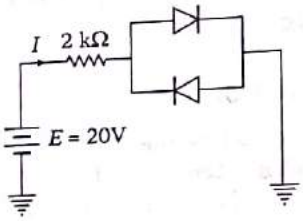
If the reverse bias is greater than the V_z there is breakdown condition. In breakdown region *i.e.*, $V_i > V_z$, for a log range of load, (R_L).

143. (*c*) Voltage of the source,

$V' = V_D + iR = 0.5 + 0.1 \times 20 = 2.5\ \text{V}$

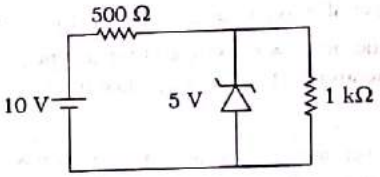
144. (*b*) The potential across $R = 17\,\text{V} - 9\,\text{V} = 8\,\text{V}$

145. (*c*) Total resistance in the circuit $= 2\,\text{k}\Omega \Rightarrow E = 20\,\text{V}$



$\therefore$ The current $= \dfrac{20}{2000\,\Omega} = 10\ \text{mA}$

146. (*b*)



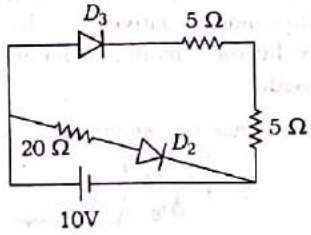
In the given circuit, the Zener diode is used as a voltage regulating device.

Hence, the voltage across 1 kΩ is 5 V.

Current flowing through 1 kΩ resistor is

$I = \dfrac{5\,\text{V}}{1 \times 10^3\,\Omega} = 5 \times 10^{-3}\text{A} = 5\ \text{mA}$

147. (*c*) In the given circuit, diode D_1 is reverse biased, so it will not conduct. Diodes D_2 and D_3 are forward biased, so they will conduct. The corresponding equivalent circuit is a shown in the figure.



The equivalent resistance of the circuit is

$$R_{eq} = \frac{(5+5)\times 20}{(5+5)+20} = \frac{10\times 20}{10+20} = \frac{200}{30} = \frac{20}{3}\Omega$$

Current through the battery, $I = \dfrac{10\text{ V}}{\frac{20}{3}\Omega} = 1.5\text{ A}$

148. (*c*) In circuit *A*, both the (*p-n*) junction diodes act as forward biased. Hence current flows in circuit *A*,

Total resistance *R* is given by

$$\frac{1}{R} = \frac{1}{4} + \frac{1}{4} \Rightarrow \frac{1}{R} = \frac{2}{4} \Rightarrow R = 2\,\Omega$$

According to Ohm's law

$$V = I_A R$$

$$\Rightarrow \quad 8 = I_A \times 2 \Rightarrow I_A = 4\text{ A}$$

In circuit *B*, lower *p-n* junction diode is reverse biased. Hence, no current will flow but upper diode is forward biased, so current can flow through it.

$V = I_B R \Rightarrow 8 = I_B \times 4 \Rightarrow I_B = 2\text{ A}$.

B. Medical entrance special format questions

Assertion and reason

1. (*a*) In semiconductors, the energy gap between conduction band and valence band is small (≈ 1 eV). Due to temperature rise, electron in the valence band gain thermal energy and may jump across the small energy gap, (to the conduction band). Thus, conductivity increases and hence resistance decreases.

2. (*a*) According to law of mass action, $n_i^2 = n_e n_h$. In intrinsic semiconductors $n_i = n_e = n_h$ and for *p*-type semiconductor n_e would be less than n_i, since n_h is necessarily more than n_i.

3. (*e*) Resistivity of a semiconductor decreases with the temperature. The atoms of a semiconductor vibrate with larger amplitudes at higher temperature thereby increasing its conductivity not resistivity.

5. (*b*) When the reverse voltage across the Zener diode is equal to or more than the breakdown voltage, then the reverse current increases sharply.

6. (*b*) In forward biasing of *p-n* junction current flows due to diffusion of majority charge carriers. While in reverse biasing current flows due to drifting of minority charge carriers.

The circuit given in the reason is a *p-n-p* transistor having emitter more negative w.r.t. base, so it is reverse biased and collector is more positive *w.r.t.* base, so it is forward biased.

7. (*a*) Input impedence of common emitter configuration

$$= \left|\frac{\Delta V_{BE}}{\Delta i_B}\right|_{V_{CE}=\text{constant}}$$

where, ΔV_{BE} = voltage across base and emitter (base emitter region is forward biased)

Δi_B = base current which is order of few microampere.

Thus, input impedence of common emitter is low.

8. (*c*) In common emitter transistor amplifier, current gain $\beta > 1$, so output current > input current, hence assertion is correct. Also, input circuit has low resistance due to forward biasing of emitter base junction, hence reason is false.

9. (*a*) These gates are called digital building blocks because using these gates only (either NAND or NOR) we can compile all other gates also (like OR, AND, NOT, XOR)

10. (*e*) In diode, the output is in same phase with the input, therefore it cannot be used to built NOT gate.

Match the columns

1. Diode gets forwardly biased during positive half cycle of alternating source and current flows. Let the barrier voltage of diode is V_B and its forward resistance, be R_F and load resistance be R_L, then peak voltage of applied signal is

$$V_0 = V_B + i_0 (R_F + R_L)$$

$$\therefore \quad i_0 = \frac{V_0 - V_B}{R_F + R_L}$$

$$= \frac{20 - 0.7}{10 + 500} = 37.8\times 10^{-3}\text{ A} = 37.8\text{ mA}$$

Peak voltage at the ends of the load R_L,

$$V_L = i_0 \times R_L = 37.8\times 10^{-3}\times 500 = 18.9\text{ V}$$

For ideal diode, $V_B = 0$ and $R_F = 0$

$$\therefore \quad i_0 = \frac{V_0}{R_L} = \frac{20}{500} = 0.04\text{ A} = 40\text{ mA}$$

and $\quad V_L = i_0 R_L = 0.04\times 500 = 20\text{ V}$

C. Medical entrances' gallery

1. (*c*) Consider the given figure.

The resultant boolean expression of the above logic circuit will be

$$Y = (A + B)\cdot C$$

Now, let us try with inputs *A*, *B* and *C* given in the options and lets see, which one of them will give output 1 at *Y*.

If $A = 0, B = 0, C = 0$, then

$$Y = (0+0)\cdot 0 \Rightarrow Y = 0$$

If $A = 1, B = 1, C = 0$, then

$$Y = (1+1)\cdot 0 \Rightarrow Y = 1\cdot 0 \Rightarrow Y = 0$$

If $A = 1, B = 0, C = 1$, then

$$Y = (1+0)\cdot 1 \Rightarrow Y = 1\cdot 1 \Rightarrow \boxed{Y = 1}$$

If $A = 0, B = 1, C = 0$, then

$$Y = (0+1)\cdot 0 \Rightarrow Y = 1\cdot 0 \Rightarrow Y = 0$$

So, we have seen that among the given options, only option (c) is the correct choice, i.e.

Output $Y = 1$ only when inputs $A = 1, B = 0$ and $C = 1$.

2. (*a*) Let us assume that current through the diode is I.
From the given condition,

$$\because \quad I = \frac{V_A - V_B}{R} = \frac{4-(-6)}{1\,\text{k}\Omega} = \frac{10}{1\times 10^3} = 10^{-2}\,\text{A}$$

3. (*d*) Given, resistance across load, $R_L = 800\,\Omega$
Voltage drop across load, $V_L = 0.8$ V
Input resistance of circuit, $R_i = 192\,\Omega$

Collector current is given by $I_C = \frac{V_L}{R_L} = \frac{0.8}{800} = \frac{8}{8000} = 1\text{mA}$

$$\therefore \text{ Current amplification} = \frac{\text{Output current}}{\text{Input current}}$$

$$= \frac{I_C}{I_B} = 0.96 \quad \Rightarrow \quad I_B = \frac{1\,\text{mA}}{0.96}$$

$$\therefore \text{ Voltage gain, } A_V = \frac{V_L}{V_{\text{in}}} = \frac{V_L}{I_B R_i} = \frac{0.8\times 0.96}{10^{-3}\times 192} = 4$$

$$\Rightarrow \qquad A_V = 4$$

$$\text{and power gain, } A_P = \frac{I_C^2 R_L}{I_B^2 R_i} = \left(\frac{I_C}{I_B}\right)^2 \cdot \frac{R_L}{R_i} = (0.96)^2 \times \frac{800}{192}$$

$$A_P = 3.84$$

4. (*b*) Given, collector resistance $= R_{\text{out}} = 2\,\text{k}\Omega$
Current amplification factor, $\beta = 100$
Base resistance, $R_{\text{in}} = 1\text{k}\Omega$
Output signal voltage $= 4$V
Putting all the values in given equation, we get

$$A_V = \beta\frac{R_{\text{out}}}{R_{\text{in}}} = 100\times\frac{2\,\text{k}\Omega}{1\text{k}\Omega} \Rightarrow A_V = 200$$

$$\text{Now,} \quad A_V = \frac{(V_{\text{out}})_{\text{AC}}}{(V_{\text{in}})_{\text{AC}}} = 200 \Rightarrow (V_{\text{in}})_{\text{AC}} = \frac{4}{200} = 20\,\text{mV}$$

5. (*a*) We know that a diode only conducts in forward biased condition. In the given circuit, the diode D_1 will be in reverse bias, so it will block the current and diode D_2 will be in forward bias, so it will pass the current

$$i = \frac{V}{R_1 + R_3} = \frac{10}{2+2} = 2.5\text{A}$$

6. (*c*) Output of the given circuit is given by

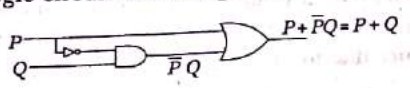
$$y = \overline{(AB)(C)}$$

When $\quad A = B = C = 0,\ Y_1 = \overline{(0)(0)(0)} = \overline{0} = 1$

When $\quad A = B = C = 1,\ Y_2 = \overline{(1)(1)} = 0$

7. (*d*) As the diode is forward biased, so it takes positive value. Hence, option (d) is correct.

8. (*d*) The logic circuit can be figured as



Clearly, the gate is an OR gate

9. (*b*) When signal 1 is applied to a NOT gate, then its output will be opposite to that of the input.

10. (*c*) To make p-type semiconductor, silicon may doped with a trivalent impurity such as boron, etc.

11. (*b*) For an intrinsic or pure semiconductor, the number densities of electron and hole are the same, i.e.

$$n_p = n_e$$

12. (*d*) The common application of Zener diode is stabilisation or voltage regulation.

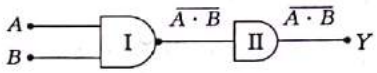
13. (*c*) We know that the relation between α and β is given as

$$\beta = \frac{\alpha}{1-\alpha} \Rightarrow 100 = \frac{\alpha}{1-\alpha}$$

$$\Rightarrow \qquad 100 - 100\,\alpha = \alpha$$

$$\therefore \qquad \alpha = \frac{100}{101} = 0.99$$

14. (*b*) Given arrangement is



Gate I $\rightarrow$ NAND gate

Gate II $\rightarrow$ AND gate

Above arrangement represents NAND gate.

15. (*a*) The overall conductivity of a semiconductor is

$$\sigma = n_e\, e\, \mu_e + n_p\, e\, \mu_p$$

$$\sigma = e\,[\mu_e\, n_e + n_p\, \mu_p]$$

Also, $n_e\, n_p = n_i^2$ [for an intrinsic semiconductor]

$$\Rightarrow \qquad n_e = \frac{n_i^2}{n_p}$$

$$\text{Now,} \qquad \sigma = e\left[\frac{n_i^2}{n_p}\mu_n + \mu_p n_p\right]$$

$$\text{On differentiating, } \frac{d\sigma}{dn_p} = e\left[\frac{n_i^2}{n_p^2}\mu_n + \mu_p\right] = 0$$

$$\Rightarrow \qquad \mu_p = \frac{n_i^2}{n_p^2}\mu_n, \Rightarrow n_p = n_i\sqrt{\frac{\mu_n}{\mu_p}}$$

16. (*b*) Given, $\quad \Delta I_B = 20\ \mu\text{A} = 20\times 10^{-6}\text{A}$

$$\Delta I_C = 2\ \text{mA} = 2\times 10^{-3}\text{A}$$

and $\quad \Delta V_{BE} = 0.04$ V

$$\therefore \quad \Delta V_{BE} = R_{BE}\times\Delta I_B$$

$$\Rightarrow \qquad 0.04 = R_{BE}\times 20\times 10^{-6}\,\Omega \Rightarrow R_{BE} = 2\,\text{k}\Omega$$

$$\text{Now, current gain} = \frac{\Delta I_C}{\Delta I_B} = \frac{2\times 10^{-3}\,\Omega}{20\times 10^{-6}\,\Omega}$$

$$= \frac{1}{10}\times 10^3 = 100$$

17. (*c*) In a common emitter transistor amplifier, the voltage gain is $\beta \frac{R_{AC}}{R_{in}}$.

18. (*c*) The current, $I = \frac{\Delta Q}{\Delta t} = \frac{10^{10} \times 1.6 \times 10^{-19}}{2 \times 10^{-6}}$

$\Rightarrow \quad I = 10^{10} \times 10^{-19} \times 10^{6} \times 0.8 = 0.8 \times 10^{-3}$

$= 800 \times 10^{-6}\ \text{A} = 800\ \mu\text{A}$

19. (*a*) The output of OR gate is given by $A + B = Y$

The output of NOR gate

$$\overline{A+B} = Y$$

When, $A = B$, then $\overline{A+A} = \overline{A}$ = NOT gate

20. (*c*) Given, $n_e = n_h = 1.5 \times 10^{16}\ \text{m}^{-3}$

$\therefore \quad n_e \cdot n_h = n_i^2 \Rightarrow n_i^2 = (1.5 \times 10^{16})^2$

After dopping by indium

$$n_h = 3 \times 10^{22}\ \text{m}^{-3}$$

$\therefore \quad n_e = \frac{n_i^2}{n_h} = \frac{(1.5 \times 10^{16})^2}{3 \times 10^{22}} = 0.75 \times 10^{10}\ \text{m}^{-3}$

$\Rightarrow \quad n_e = 7.5 \times 10^{9}\ \text{m}^{-3}$

21. (*d*) Output of NOR gate, $y' = \overline{A+B}$

Output of NAND gate,

$$y'' = \overline{(A+B) \cdot (A+B)} = \overline{(A+B)} \cdot \overline{(A+B)}$$

$$= (A+B)(A+B) = A + B$$

Output of NOT gate,

$$y''' = \overline{A+B}$$

$\therefore$ The boolean expression is for NOR gate.

22. (*c*) The output of digital circuit can be given as $Y = \overline{AB} + \overline{C}$

It will be same as $Y = \overline{A} + \overline{B} + \overline{C}$

23. (*c*) Each and every extrinsic semiconductor is electrically neutral whether it is *p*-type or *n*-type.

24. (*c*) We know that, $\alpha = \frac{\Delta I_C}{\Delta I_E}$

$\Rightarrow \quad \Delta I_C = \alpha\ \Delta I_E = 0.96 \times 10 = 9.6\ \text{mA}$

The change in base current,

$$\Delta I_B = \Delta I_E - \Delta I_C$$

$$= 10\ \text{mA} - 9.6\ \text{mA} = 0.4\ \text{mA}$$

25. (*a*) The liquid-crystal phase of a liquid is called mesophase.

26. (*c*) The output of $G_1 = A + B$

Output of $G_2 = \overline{A \cdot B}$

and output of $G_3 = (A+B)\,\overline{(A \cdot B)} = A\overline{B} + B\overline{A}$

This output is same as the output of an XOR gate.

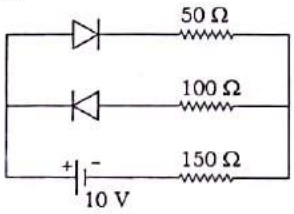
27. (*d*) Voltage gain = $\beta \times$ Resistance gain

$$\beta = \frac{\alpha}{1-\alpha} = \frac{0.99}{(1-0.99)} = 99$$

$$\text{Resistance gain} = \frac{10 \times 10^3}{10^3} = 10$$

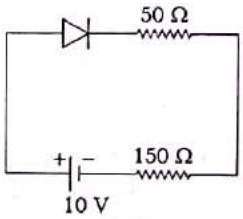
$$\text{Voltage gain} = 99 \times 10 = 990$$

28. (*b*) The circuit is



As the lower diode attached to 100 Ω resistance is in reversed biased, so it is non-conducting.

Now, the circuit can be redrawn as,

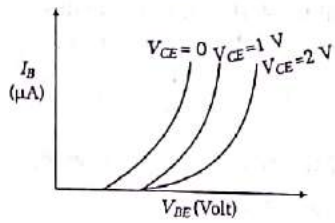


$\therefore$ Current through the circuit, $I = \frac{V}{R} = \frac{10}{200} = 0.05\ \text{A}$

29. (*b*) Input characteristics curve is drawn between base current I_B and emitter base voltage V_{EB} at constant collector emitter voltage V_{CE}.

Dynamic input resistance,

$$R_i = \left(\frac{\Delta V_{BE}}{\Delta I_B}\right)_{V_{CE} = \text{constant}}$$



30. (*d*) The mobility of e^- is greater than holes because e^- acquire less energy to move.

31. (d) Barrier potential should depend on all the three point given. Barrier potential depends on the material used to make *p-n* junction diode (whether it is Si or Ge). It should also depends on amount of doping due to which the number of majority carriers will change. It should also depend on temperature due to number of minority carriers will change.

32. (*c*) The conductivity of the intrinsic semicondutor is independent of fermi energy. It depends upon band gap, temperature as well as effective mass of charge carriers.

33. (*a*) Given, energy gap, $E_g = 1.9 \text{ eV} = 1.9 \times 1.6 \times 10^{-19}$ V

We know that, $E_g = \frac{hc}{\lambda}, \lambda = \frac{hc}{E_g}$

Wavelength of the emitted light,

$$\lambda = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1.9 \times 1.6 \times 10^{-19}}$$

$$\lambda = 6.5 \times 10^{-7} \text{m}$$

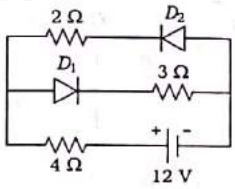
34. (*a*) *V-I* characteristic of a solar cell is shown in the question, where *A* represents open circuit voltage and *B* shows short circuit current.

35. (*c*) In insulators conduction band is empty and valence band is filled with electrons.

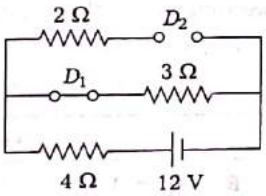
36. (*c*) The resistivity of semiconductors is lower than metals,this is not correct.

37. (*d*) The band gap of 5 eV corresponds to that of an insulator.

38. (*b*)



In the given circuit, D_1 is forward biased and D_2 is reverse biased. So, D_1 works like a closed switch and D_2 works like an open circuit. So, circuit becomes



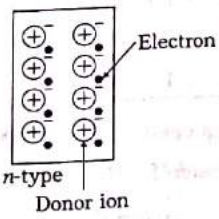
Now, current in the circuit,

$$I = \frac{E}{R_S} = \frac{12}{4+3}$$

$$\Rightarrow \quad I = \frac{12}{7}$$

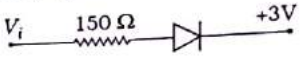
$$\Rightarrow \quad I = 1.71 \text{ A}$$

39. (*d*) The *n*-type semiconductor region has (negative) electrons as majority charge carriers and an equal number of fixed positively charged donor ions. Again, the material as a whole is neutral. That is a reason that atom is electrically neutral.



40. (*c*) The wrong statement with reference to a solar cell is that, it uses materials with band gap of 5 eV. Because for solar cell, band gap < 3 eV.

41. (*b*) Initial current, $I_{in} = 0$

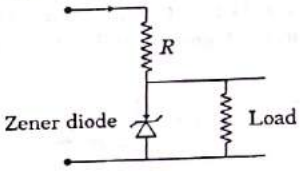


Final current, $I_f = \frac{3}{150} = 0.02\text{A}$

So, change in current, $\Delta I = I_f - I_{in}$

$$= 0.02 - 0 = 0.02\text{A} = 20 \text{ mA}$$

42. (c) When the applied reverse voltage (*V*) reaches the breakdown voltage (V_Z) of the Zener diode, then there is a large change in the current. So, after the breakdown voltage V_Z, a large change in the current can be produced by almost in significant change in the reverse bias voltage.



As Zener voltage remains constant even though the current through the Zener diode varies over a wide range, hence it is used to obtain regulated voltage output as shown in the above diagram.

43. (b) Given, $\alpha = 0.95, I_b = 0.2$ mA

As, $\alpha = \frac{I_c}{I_e} = \frac{I_e - I_b}{I_e} = 1 - \frac{I_b}{I_e}$

$$\Rightarrow \quad \frac{I_b}{I_e} = 1 - \alpha = 1 - 0.95 = 0.05$$

$$\Rightarrow \quad I_e = \frac{I_b}{0.05} = \frac{0.2}{0.05} \Rightarrow I_e = 4 \text{ mA}$$

44. (*d*) For transistor as an oscillator, $f = \frac{1}{2\pi\sqrt{LC}}$

where, $f = 2 \text{ MHz} = 2 \times 10^6$ Hz and $\sqrt{LC} = ?$

$$\Rightarrow \sqrt{LC} = \frac{1}{2\pi f} = \frac{1}{2\pi \times 2 \times 10^6} = \frac{1}{4\pi \times 10^6}$$

45. (*b*) α and β are the AC current gains of a transistor in the common-base configuration and in the common-emitter configuration respectively and are related as

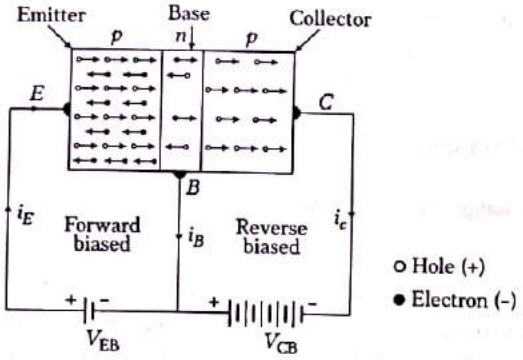
$$\beta = \frac{\alpha}{1-\alpha} = \frac{0.98}{1-0.98} \qquad [\because \alpha = 0.98]$$

$$= \frac{0.98}{0.02} = \frac{98}{2} = 49$$

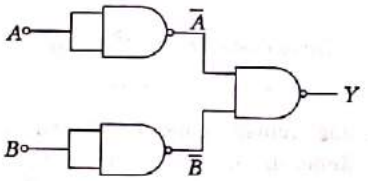
46. (*b*) As I_B is in μA, I_C is in mA and V_{CE} in V.

47. (*c*) A common base circuit of a *p-n-p* transistor is shown in figure. The emitter base (*p-n*) junction on the left is given a small forward bias (fraction of a volt) by an emitter base battery V_{EB}, while the base collector (*n-p*) junction is given a

large reverse bias (a few volt) by another battery V_{CB}.
So, the required solution is emitter base junction forward bias and collector base junction reverse bias.

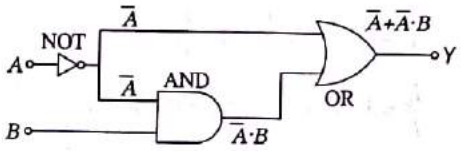


48. (*d*) If the two inputs of a NAND gate are joined together. It works as a NOT gate. Now, if the inputs A and B are inverted by using two NOT gates (obtained from two NAND gates) and the resulting output $\overline{A}$ and $\overline{B}$ are fed to a third NAND gates.



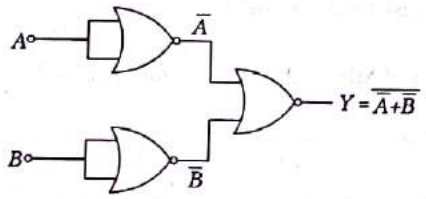
So, three NAND gates are required to make an OR gate.

49. (*b*) By absorption law, output $Y = \overline{A} + \overline{A} \cdot B$



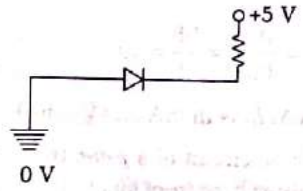
or $\quad Y = \overline{A} \cdot B + \overline{A} = \overline{A} \cdot (B + 1) = \overline{A} \cdot 1 = \overline{A}$

50. (*d*) According to the logical relationship, $\overline{\overline{A} + \overline{B}} = \overline{\overline{A}} \cdot \overline{\overline{B}} = A \cdot B$



So, it is AND gate.

51. (*c*) A diode is symbolically shown in the adjacent diagram. The diode is said to be forward biased when, $V_A > V_B$.



The diode is said to be reverse biased when, $V_A < V_B$

In the option (b), $5V > 0V$
Hence, $\quad V_B > V_A$
So, the diode is reverse biased.

52. (*a*) The OR gate is a device that has two (or more) input variables A and B and one output variable Y and follows the boolean expression, $A + B = Y$, read as 'A OR B equals Y'.
Truth table for given gate (G_1) combination is

Input		Output
A	B	$Y = A + B$
0	0	0
0	1	1
1	0	1
1	1	1

Similarly, the AND gate is also has two inputs and one output and follows boolean expression $A \cdot B = Y$, read as A AND B equals Y. Truth table for given gate (G_2) combination is

Input		Output
A	B	$Y = A \cdot B$
0	0	0
0	1	0
1	0	0
1	1	1

The NOR gate is a combination of OR and NOT gates.
The boolean expression for the NOR gate is

$$\overline{A + B} = Y.$$

Truth table for given gate (G_3) combination is

Input		Output	
A	B	$Y' = A + B$	$Y = \overline{A + B} = \overline{Y'}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

or

Input		Output
A	B	$Y = \overline{A + B}$
0	0	1
0	1	0
1	0	0
1	1	0

The NAND gate is a combination of AND and NOT gates.
The boolean expression for the NAND gate is

$$\overline{A \cdot B} = Y$$

Truth table for given gate (G_4) combination is

Input		Output	
A	B	$Y' = A \cdot B$	$Y = \overline{A \cdot B} = \overline{Y}'$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

or

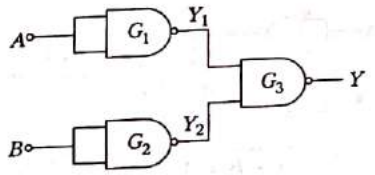
Input		Output
A	B	$Y = \overline{A \cdot B}$
0	0	1
0	1	1
1	0	1
1	1	0

So, the required solution is

$A \rightarrow$ OR gate, $B \rightarrow$ AND gate, $C \rightarrow$ NOR gate and

$D \rightarrow$ NAND gate.

53. (*d*) The combination of three NAND gates are shown in the diagram.



Y_1 = Output due to NAND gate $1 = \overline{A \cdot A} = \overline{A}$

Y_2 = Output due to the NAND gate $2 = \overline{B \cdot B} = \overline{B}$

Output due to NAND gate 3

$$Y = \overline{Y_1 \cdot Y_2} = \overline{Y_1} + \overline{Y_2} = \overline{\overline{A}} + \overline{\overline{B}} = A + B$$

This is OR gate. Hence, the combination is equivalent to an OR gate.

Truth table for given gate combination is

Input		Output		
A	B	Y_1	Y_2	Y
1	1	0	0	1
1	0	0	1	1
0	1	1	0	1
0	0	1	1	0

54. (*c*) The *n*-type semiconductor can be produced by doping an impurity atom of valence 5, i.e. pentavalent atoms. i.e. Phosphorus.

55. (*a*) As, $G = \left(\frac{\beta}{R_i}\right) R_L \Rightarrow G = g_m R_L$ $\left[\because g_m = \frac{\beta}{R_i}\right]$

$\Rightarrow$ $G \propto g_m$

$\therefore$ $\frac{G_2}{G_1} = \frac{g_{m_1}}{g_{m_2}} \Rightarrow G_2 = \frac{0.02}{0.03} \times G$

$\therefore$ Voltage gain, $G_2 = \frac{2}{3} G$

56. (*c*) $X = \overline{\overline{AB}} = A \cdot B$ (*i.e.*, AND gate)

If the output X of NAND gate is connected to the input of NOT gate (made from NAND gate by joining two inputs) from the given figure, then we get back an AND gate.

57. (*b*) For making a junction transistor, the emitter E is heavily doped, the base B is thin and lightly doped and the collector C is moderately doped semiconductor materials and with together.

58. (*c*) $\beta = \left(\frac{\Delta I_C}{\Delta I_B}\right)_{V_{CE} = \text{constant}}$

$$60 = \frac{4 \times 10^{-3}}{\Delta I_B} \Rightarrow \Delta I_B = \frac{4 \times 10^{-3}}{60} = 66.6 \mu A$$

59. (*c*) In an ionised gas, the mobile charge carriers are electron and positively charged ions.

60. (*d*) Resistivity of a semiconductor is $10^5 - 10^{\circ} \Omega$-m while that of insulator is $10^8 \Omega$ - m and conductor is $10^{-2} - 10^{-8} \Omega$-m.

61. (*d*) In *p*-type semiconductor, the acceptor level lies near the valence band.

62. (*a*) If the feedback voltage is increased in negative feedback amplifier, then both gain and distortion decreases.

63. (*b*) When the *p-n* junction is reverse biased, then the current across the junction is mainly due to drifting of minority carriers.

64. (*c*) $\overline{A} + A$ should always be equal to 1. As addition of inputs like 1 to 0 or 0 to 1 always gives 1.

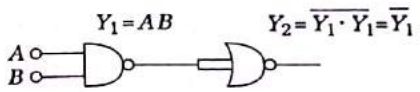
65. (*c*) The depletion is the *p-n* junction region is caused due to the diffusion of carriers on either side of the junction.

66. (*c*) A digital signal has only two values of voltage variation with time. Its value is either zero or maximum.

67. (*c*) The diode D_1 is reversed biased and D_2 is forward biased. The resistance of D_1 becomes infinite and of D_2 is zero. Therefore, current in the circuit,

$$I = \frac{\text{emf}}{\text{total resistance}} = \frac{12}{4 + 2} = 2 \text{ A}$$

68. (*e*)



Truth table for the circuit

A	B	$Y_1 = AB$	$Y_2 = \overline{Y_1}$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

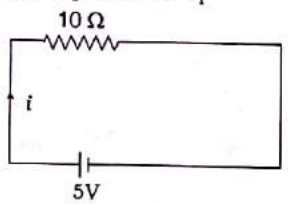
It represents a NAND gate.

69. (*b*) For using transistor as a switch, it is used in cut-off state and saturation state only.

70. (*a*) By using, $V_b = i_b R_b$

$$\Rightarrow \quad R_b = \frac{V_b}{i_b} = \frac{9}{35 \times 10^{-6}} = 257\text{k}\Omega$$

71. (*d*) Here in circuit D_1 is forward bias and D_2 is reverse bias. Current will flow only in diode D_1



The current supplied by the battery, $i = \frac{5}{10} = 0.5$ A

72. (*d*) Current amplification factor,

$$\beta = \frac{\Delta I_C}{\Delta I_B}$$

Collector resistance,

$$\Delta I_C = \frac{2V}{2 \times 10^3 \Omega} = 1 \times 10^{-3}\text{A}$$

Base current,

$$\Delta I_B = \frac{V_B}{R_B} = \frac{V_B}{1 \times 10^3} = V_B \times 10^{-3}$$

Given, $\beta = 100$

Now, $100 = \frac{10^{-3}}{V_B \times 10^{-3}}$

$$\Rightarrow \quad V_B = \frac{1}{100}\text{ V} = 10\text{ mV}$$

73. (*a*) As $V = V_{CE} + i_C R_C$

$$\Rightarrow \quad 15 = 7 + i_C \times 2 \times 10^3$$

$$\Rightarrow \quad i_C = \frac{8}{2 \times 10^3} = 4\text{ mA}$$

$$\therefore \quad \beta = \frac{i_C}{i_B}$$

or $$i_B = \frac{i_C}{\beta} = \frac{4}{100} = 0.04\text{ mA}$$

74. (*a*) From the given waveforms, the following truth table can be made

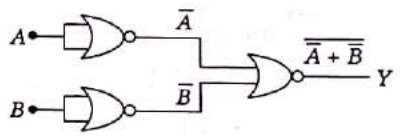
Input		Output
A	B	C
0	0	0
1	0	1
1	1	1
0	1	1

This truth table is similar to OR gate. So, logic circuit gate is OR gate.

Note The logic of the OR gate is, if any of the input is one, then output is one.

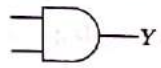
75. (*a*) At room temperature, due to thermal vibrations, the few bonds of intrinsic semiconductor are broken and producing equal number of electrons and holes in the semiconductor.

77. (*a*)



$$\overline{\overline{A} + \overline{B}} = \overline{\overline{A \cdot B}} = A \cdot B$$

This Boolean expression represents AND gate *i.e.*,



78. (*c*) As, $V_{rms} = \frac{V_0}{\sqrt{2}}$

$$\Rightarrow \quad 220 = \frac{V_0}{\sqrt{2}}$$

$$\Rightarrow \quad V_0 = 311.1\text{ V}$$