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CHAPTER

01 Electric charges and fields

1 Electric charges

Electric charge can be defined as an intrinsic property of elementary particles of matter which give rise to electric force between various objects. It is represented by q. The SI unit of electric charge is coulomb (C). A proton has positive charge ($+e$) and an electron has negative charge ($-e$), where $e = 1.6 \times 10^{-19}$ C.

Important points regarding electric charge

Regarding charge the following points are worthnoting

(i) Like charges repel each other and unlike charges attract each other.

(ii) The property which differentiates the two kinds of charges is called the 'polarity' of charge. If an object possesses an electric charge, then it is said to be electrified or charged. When its net charge is zero, then it is said to be neutral (just like neutron).

(iii) Charge is a scalar quantity and can be of two types positive or negative. When some electrons are removed from the atom, it acquires a positive charge and when some electrons are added to the atom, it acquires a negative charge.

(iv) It can be transferred from one body to another.

(v) Charge is invariant, *i.e.*, it does not depend on the velocity of charged particle.

(vi) A charged particle at rest produces electric field. A charged particle in an unaccelerated motion produces both electric and magnetic fields but does not radiate energy. But an accelerated charged particle not only produces an electric and magnetic fields but also radiates energy in the form of electromagnetic waves.

(vii) 1 coulomb $= 3 \times 10^9$ esu $= \frac{1}{10}$ emu of charge. Where esu is electrostatic unit of charge. Its CGS unit is stat coulomb.

(viii) The dimensional formula of charge $[q] = [AT]$.

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Properties of electric charges

If the sizes of charged bodies are very small as compared to the distance between them, we treat them as 'point charges'. In addition to being positive or negative, *the charges have the following properties*

Additivity of charges

Additivity of charges is a property by virtue of which total charge of a system is obtained simply by adding algebraically all the charges present anywhere on the system.

If a system contains two point charges q_1 and q_2, then the total charge of the system is obtained simply by adding algebraically q_1 and q_2, *i.e.*, charges add up like real numbers. Proper signs have to be used while adding the charges in a system.

$$q_{\text{net}} = q_1 + q_2$$

Note Charge has magnitude but no direction.

Example 1.1 *What is the total charge of a system containing five charges +1, +2, −3, +4 and −5 in some arbitrary unit?*

Sol. As charges are additive in nature, *i.e.*, the total charge of a system is the algebraic sum of all the individual charges located at different points inside the system, *i.e.*,

$$q_{\text{net}} = q_1 + q_2 + q_3 + q_4 + q_5$$

$\therefore$ Total charge $= +1 + 2 - 3 + 4 - 5 = -1$ in the same unit.

Charge is conserved

The total charge of an isolated system is always conserved. It is not possible to create or destroy net charge carried by any isolated system. It can only be transferred from one body to another body. Pair production and pair annihilation are two examples of conservation of charge.

Quantisation of charge

The charge on any body can be expressed as the integral multiple of basic unit of charge, *i.e.*, charge on one electron (e). This phenomenon is called **quantisation of electric charge**. It can be written as $q = \pm ne$.

where, $n = 1, 2, 3, \cdots$ is any integer, positive or negative and e is the basic unit of charge.

Charge is said to be quantised because it can have only discrete values rather than any arbitrary value, i.e. free particle has no charge at all or a charge of $+10\,e$ or $-6\,e$ but not a free particle with a charge of, say $3.57\,e$.

Note

1. Just like charge other physical quantities like energy, angular momentum and mass are also quantised. The quantum of energy is $h\nu$ and that of angular momentum is $\frac{h}{2\pi}$. Quantum of mass is yet not known.
2. The protons and neutrons are combination of other entities called quarks, which have charges $\frac{1}{3}e$ and $\pm\frac{2}{3}e$. However, isolated quarks have not been observed, so quantum of charge is still e.

Example 1.2 *How many electrons are there in one coulomb of negative charge?*

Sol. The negative charge is due to the presence of excess electrons. Because an electron has a charge whose magnitude is $e = 1.6 \times 10^{-19}$ C, the number of electrons is equal to the charge q divided by the charge e on each electron.

Therefore, the number n of electrons is

$$n = \frac{q}{e} = \frac{1.0}{1.6 \times 10^{-19}}$$

$$= 6.25 \times 10^{18} \text{ electrons}$$

Example 1.3 *A sphere of lead of mass 10 g has net charge -2.5×10^{-9} C.*

(i) Find the number of excess electrons on the sphere.

(i) How many excess electrons are per lead atom? Atomic number of lead is 82 and its atomic mass is 207 g/mol.

Sol. (i) The charge of an electron

$$= -16 \times 10^{-19}\text{C}$$

Net charge on sphere $= -2.5 \times 10^{-9}$C

So, the number of excess electrons

$$= \frac{-2.5 \times 10^{-9}\text{C}}{-1.6 \times 10^{-19}\text{C}} = 1.5 \times 10^{10} \text{ electrons}$$

(ii) Atomic number of lead is 82.

Atomic mass of lead is 207 g/mol.

$\therefore$ 10 g of lead will have $\frac{10 \text{ g}}{207 \text{ g/mol}} \times 6.02 \times 10^{23}$ atoms/mol

$$= 2.91 \times 10^{22} \text{ atoms}$$

$\therefore$ The number of excess electrons per atom

$$= \frac{1.56 \times 10^{10}}{2.91 \times 10^{22}}$$

$$= 5.36 \times 10^{-13} \text{ electrons}$$

Conductors and insulators

The quantisation of electric charge is the property by virtue of which all free charges are integral multiple of a basic unit of charge represented by C.

Conduction The substances through which electric charges can flow easily are called conductors.

Metals, human body and animal bodies, graphite acids, etc, are conductor.

Insulators The substances through which electric charges cannot flow easily are called insulators.

Most of the non-metals like glass, diamond, porcelain, plastic, nylon, wood, mica, etc, are insulators.

Methods of Charging

Mainly there are three methods of charging a body these are given below

1. Charging by rubbing

When two bodies are rubbed together, some electrons from one body pass onto the other body. The positive and negative charges appears in equal amount simultaneously due to the transfer of electrons. The body that donates the electrons becomes positively charged while that which receives electrons becomes negatively charged. *e.g.*, When a glass rod is rubbed with a silk cloth, the glass rod acquires some positive charge by losing electrons and the silk cloth acquires negative charge by the same amount by gaining electrons as shown in figure.

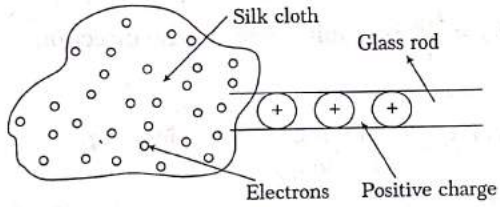


***Fig. 1.1** A glass rod rubbed with a silk cloth*

Ebonite on rubbing with wool becomes negatively charged making the wool positively charged.

2. Charging by contact or conduction

Charging by conduction Take two conductors, one charged and other uncharged. Bring the conductors in contact with each other. The charge (whether – ve or + ve) under its own repulsion will spread over both the conductors. Thus, the conductors will be charged with the same sign. This is called as charging by conduction (through contact).

3. Charging by induction

By electrostatic induction If a charged body is brought near an uncharged body, then one side of neutral body (closer to charged body) becomes oppositely charged while the other side becomes similarly charged as shown in figure.

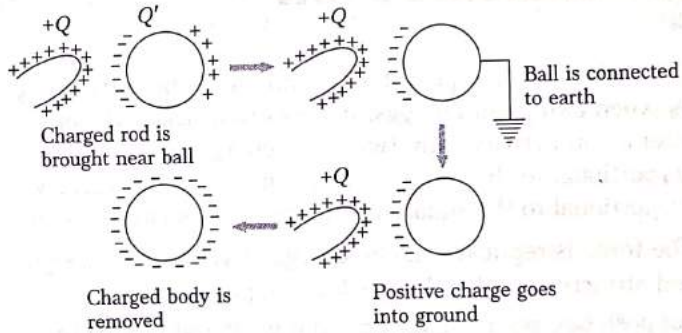


***Fig. 1.2** Charging by induction*

In this process, charging is done without actual contact of bodies.

Check point 1.1

1. Which of the following is correct regarding electric charge?
(i) If a body is having positive charge, *i.e.*, shortage of electrons
(ii) If a body is having negative charge *i.e.*, excess of electrons
(iii) Minimum possible charge = $\pm 1.6 \times 10^{-19}$C
(iv) Charge is quantised, *i.e.*, $Q = \pm ne$, where $n = 1, 2, 3, 4, \ldots$
(a) Both (i) and (ii) (b) Both (ii) and (iii)
(c) (i), (ii), (iii) (d) All of these

2. One metallic sphere *A* is given positive charge whereas another identical metallic sphere *B* of exactly same mass as of *A* is given equal amount of negative charge. Then,
(a) mass of *A* and mass of *B* still remain equal
(b) mass of *A* increases
(c) mass of *B* decreases
(d) mass of *B* increases

3. When 10^{14} electrons are removed from a neutral metal sphere, then the charge on the sphere becomes
(a) 16 μC (b) –16 μC (c) 32 μC (d) –32 μC

4. A conductor has 14.4×10^{-19}C positive charge. The conductor has (charge on electron = 1.6×10^{-19}C)
(a) 9 electrons in excess
(b) 27 electrons in short
(c) 27 electrons in excess
(d) 9 electrons in short

5. Charge on α-particle is
(a) 4.8×10^{-19} C (b) 1.6×10^{-19} C
(c) 3.2×10^{-19}C (d) 6.4×10^{-19} C

6. A body has –80 micro coulomb of charge. Number of additional electrons in it will be
(a) 8×10^{-5} (b) 80×10^{-17}
(c) 5×10^{14} (d) 1.28×10^{-17}

7. When a glass rod is rubbed with silk, it
(a) gains electrons from silk
(b) gives electrons to silk
(c) gains protons from silk
(d) gives protons to silk

8. A comb runs through one's dry hair attracts small bits of paper. This is due to
(a) comb is a good conductor
(b) paper is a good conductor
(c) the atoms in the paper get polarised by the charged comb
(d) the comb possesses magnetic properties

9. When a body is earth connected, electrons from the earth flow into the body. This means the body is
(a) uncharged (b) charged positively
(c) charged negatively (d) an insulator

10. Consider a neutral conducting sphere. A positive point charge is placed outside the sphere. The net charge on the sphere is then
(a) Negative and distributed uniformly over the surface of the sphere
(b) Negative and appears only at the point on the sphere closest to the point charge
(c) Negative and distributed non-uniformly over the entire surface of the sphere
(d) zero

2 Coulomb's law

Coulomb's law is a quantitative statement about the force between two point charges. It states that "the force of interaction between any two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them".

The force is repulsive, if the charges have the same sign and attractive, if the charges have opposite signs.

Suppose two point charges q_1 and q_2 are separated in vacuum by a distance r, then force between two charges is given by

Fig. 1.3 Two charges separated by distance r

$$F_e = \frac{K|q_1 q_2|}{r^2}$$

The constant K is usually put as $K = \frac{1}{4\pi\varepsilon_0}$, where ε_0 is called the permittivity of free space and has the value $\varepsilon_0 = 8.854 \times 10^{-12} C^2/N\text{-}m^2$. For all practical purposes we will take $\frac{1}{4\pi\varepsilon_0} \simeq 9 \times 10^9 N\text{-}m^2/C^2$.

Coulomb's law in vector form

Consider two point of charges q_1 and q_2 placed in vacuum distance r from each other they repel each other.

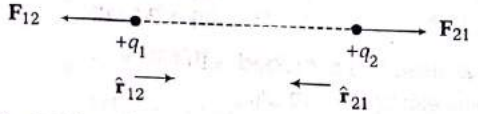


Fig. 1.4 Repulsive Coulombian forces for $q_1 q_2 > 0$

In vector form, Coulomb's law may be expressed as

$$\mathbf{F}_{12} = \text{Force on charge } q_1 \text{ due to } q_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2} \hat{\mathbf{r}}_{12}$$

where, $\hat{\mathbf{r}}_{12} = \frac{\mathbf{r}_{12}}{r}$ is a unit vector in the direction from q_1 to q_2.

Similarly, $\mathbf{F}_{21}$ = force on charge q_2 due to q_1

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2} \hat{\mathbf{r}}_{21}$$

where, $\hat{\mathbf{r}}_{21} = \frac{\mathbf{r}_{21}}{r}$ is a unit vector in the direction from q_2 to q_1.

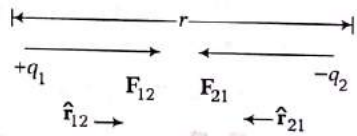


Fig. 1.5 Attractive Coulombian forces for $q_1 q_2 < 0$

Important points related to Coulomb's law

Important points related to Coulomb's law are given below

(i) The electric force is an action reaction pair, *i.e.*, the two charges exert equal and opposite forces on each other. Thus, Coulomb's law obeys Newton's third law. $\mathbf{F}_{12} = -\mathbf{F}_{21}$

(ii) The electric force is conservative in nature.

(iii) Coulomb's law as we have stated above can be used for point charges in vacuum. When a dielectric medium is completely filled in between charges, rearrangement of the charges inside the dielectric medium takes place and the force between the same two charges decreases by a factor of K (dielectric constant).

$$F_e' = \frac{F_e}{K} = \frac{1}{4\pi\varepsilon_0 K r} \cdot \frac{q_1 q_2}{r^2} = \frac{1}{4\pi\varepsilon} \cdot \frac{q_1 q_2}{r^2} \quad \text{(in medium)}$$

Here, $\varepsilon = \varepsilon_0 K$ is called permittivity of the medium.

(iv) The electric force is much stronger than gravitational force, $F_e >> F_g$, as value of $\frac{1}{4\pi\varepsilon_0} = 9\times10^9$ N-m^2/C^2.

Which is much more than the value of gravitational constant, $G = 6.67\times10^{-11}$ N-m^2 kg^{-2}.

Example 1.4 *A proton and an electron are placed 1.6 cm apart in free space. Find the magnitude and nature of electrostatic force between them.*

Sol. Charge on electron $= -1.6\times10^{-19}$C

Charge on proton $= 1.6\times10^{-19}$C

Using Coulomb's law,

$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$, where r is the distance between proton and electron.

$$= \frac{9\times10^9\times(1.6\times10^{-19})(-1.6\times10^{-19})}{(1.6\times10^{-2})^2} = -9\times10^{-25}\text{ N}$$

Negative sign indicates that the force is attractive in nature.

Example 1.5 *The electrostatic force on a small sphere of charge 0.4 μC due to another small sphere of charge −0.8 μC in air is 0.2 N. (i) What is the distance between the two spheres? (ii) What is the force on the second sphere due to the first?*

Sol. (i) Using the relation, $F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r^2}$, we get

$$r^2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{F}$$

Here, $q_1 = 0.4\,\mu\text{C} = 0.4\times10^{-6}$C,

$q_2 = 0.8\,\mu\text{C} = 0.8\times10^{-6}$C

and $F = 0.2$ N

$$\Rightarrow \quad r^2 = 9\times10^9\times\frac{0.4\times10^{-6}\times(0.8\times10^{-6})}{0.2}$$

$$= 144\times10^{-4}\text{m}^2$$

Therefore, distance between the two spheres,

$$r = 12\times10^{-2}\text{m} = 0.12\text{ m}$$

(ii) Electrostatic forces always, appear in pairs and follows Newton's 3rd law of motion.

$\therefore |\mathbf{F}_{21}|$ = Force on q_2 due to $q_1 = 0.2$ N and is attractive in nature.

Note In the coulomb's expression for finding force between two charges, do not use sign of charge because this is magnitude of force.

Example 1.6 *Nucleus $_{92}U^{238}$ emits α-particle ($_2He^4$). α-particle has atomic number 2 and mass number 4. At any instant α-particle is at distance of 9×10^{-15} m from the centre of nucleus of uranium. What is the force on α-particle at this instant?*

$$_{92}U^{238} \longrightarrow {}_2He^4 + {}_{90}Th^{234}$$

Sol. Force, $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$

$_{92}U^{238}$ has charge $92e$. When α-particle is emitted, charge on residual nucleus is $92e - 2e = 90e$

$\therefore \quad q_1 = 90e,\ q_2 = 2e$, and $r = 9\times10^{-15}$m

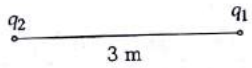
$$\therefore \quad F = 9\times10^9\times\frac{(90e)(2e)}{(9\times10^{-15})^2}$$

Force on α-particle,

$$F = \frac{9\times10^9\times90\times(1.6\times10^{-19})^2\times2}{(9\times10^{-15})^2} = 512\text{ N}$$

Example 1.7 *Two identical spheres having positive charges are placed 3m apart repel each other with a force 8×10^{-3} N. Now, charges are connected by a metallic wire, they begin to repel each other with a force of 9×10^{-3} N. Find initial charges on the spheres.*

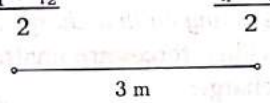
Sol. Let charges are q_1 and q_2 placed 3 m apart from each other.



$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} \qquad \text{[Coulomb's law]}$$

$$\Rightarrow \quad 8\times10^{-3} = \frac{9\times10^9 q_1 q_2}{(3)^2} \Rightarrow q_1 q_2 = 8\times10^{-12}$$

When identical spheres are touched or connected by a metallic wire, charge on each sphere will be $\left(\frac{q_1+q_2}{2}\right)$



According to given situation in the question,

$$\Rightarrow \quad 9\times10^{-3} = \frac{9\times10^9\left(\frac{q_1+q_2}{2}\right)^2}{(3)^2}$$

$$\Rightarrow \quad \left(\frac{q_1+q_2}{2}\right)^2 = 9\times10^{-12}$$

$$\Rightarrow \quad q_1 + q_2 = 6\times10^{-6} \qquad \text{...(i)}$$

$$\Rightarrow \quad (q_1 - q_2)^2 = (q_1+q_2)^2 - 4q_1 q_2$$

$$= (6\times10^{-6})^2 - 4\times8\times10^{-12}$$

$$= 4\times10^{-12}$$

$$\Rightarrow \quad q_1 - q_2 = 2\times10^{-6} \qquad \text{...(ii)}$$

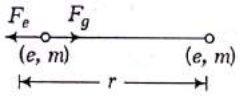
Solving Eqs. (i) and (ii), we get

$$q_1 = 4\times10^{-6}\text{C},\ q_2 = 2\times10^{-6}\text{C}$$

Therefore, initial charges on the sphere are 4 μC and 2 μC.

Example 1.8 *Two protons are placed at some separation in vacuum. Find the ratio of electric and gravitational force acting between them.*

Sol. Let the distance between two protons having charge $+e$ and mass m is placed at a distance r from each other as shown in figure.



$$\therefore \quad F_e = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} \text{ (Coulomb's law)} = \frac{1}{4\pi\,\varepsilon_0}\frac{e \times e}{r^2}$$

$$F_g = G\frac{m_1 m_2}{r^2} \text{ (Gravitation law)} = G\frac{m \times m}{r^2}$$

where, F_e and F_g are electric and gravitational force respectively, on putting the values, we get

$$\frac{F_e}{F_g} = \frac{\dfrac{1}{4\pi\varepsilon_0}\dfrac{e}{r^2}}{\dfrac{Gm^2}{r^2}} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{m^2}\cdot\frac{1}{G}$$

$$= \frac{(9\times10^9)(1.6\times10^{-19})^2}{(1.67\times10^{-27})^2}\times\frac{1}{(6.67\times10^{-11})}$$

$$= 1.2\times10^{36} \simeq 10^{36}$$

$$F_e >> F_g$$

Force between multiple charges (Superposition principle)

According to the principle of superposition, *"total force on a given charge due to number of charges is the vector sum of individual forces acting on that charge due to all the charges"*. The individual forces are unaffected due to the presence of other charges.

Suppose a system contains n point charges $q_1, q_2, \ldots, q_n$. Then, by the principle of superposition, the force on q_1, due to all the other charges is given by

$$\mathbf{F}_1 = \mathbf{F}_{12} + \mathbf{F}_{13} + \ldots\ldots + \mathbf{F}_{1n}$$

$$\mathbf{F}_1 = \frac{q_1}{4\pi\varepsilon_0}\left[\frac{q_2\hat{\mathbf{r}}_{12}}{r_{12}^2} + \frac{q_3\hat{\mathbf{r}}_{13}}{r_{13}^2} + \ldots\ldots\frac{q_n\hat{\mathbf{r}}_{1n}}{r_{1n}^2}\right]$$

$$= \frac{q_1}{4\pi\varepsilon_0}\sum_{i=2}^{n}\frac{q_{1i}\,\hat{\mathbf{r}}_{1i}}{\mathbf{r}_{1i}^2}$$

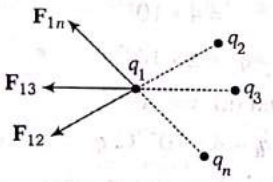


Fig. 1.5 *Force between charges $q_1, q_2, q_3, \ldots, q_n$*

Example 1.9 *Equal charges each of 20 μ are placed at $x = 0$ 2, 4, 8, 16 cm on X-axis. Find the force experienced by the charge at $x = 2$ cm.*

Sol. Force on charge at $x = 2$ cm due to charge at $x = 0$ cm and $x = 4$ cm, are equal and opposite they cancel.

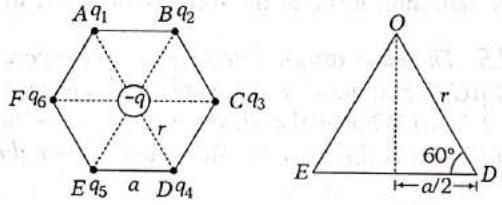
Net force on charge at $x = 2$ cm is resultant of repulsive forces due to two charges at $x = 8$ cm and $x = 16$ cm.

$$\therefore \quad F = \frac{q_1 \times q_2}{4\pi\varepsilon_0}\left[\frac{1}{(0.08-0.02)^2} + \frac{1}{(0.16-0.02)^2}\right]$$

$$= 9\times10^9[20\times10^{-6}]^2\left[\frac{1}{(0.06)^2} + \frac{1}{(0.14)^2}\right] = 1.2\times10^3 \text{ N}$$

Example 1.10 *Five point charges each of value $+q$ are placed on five vertices of a regular hexagon of side a metre. What is the magnitude of the force on a point charge of value $-q$ coulomb placed at the centre of the hexagon?*

Sol. Let the centre of the hexagon be O. When the centre is joined with the vertices of a hexagon, then six triangles are formed. Consider ΔODE



$$\frac{a/2}{r} = \cos 60° = \frac{1}{2}$$

$$\therefore \quad a = r$$

$$q_1 = q_2 = \ldots q_5 = q$$

Net force on $-q$ is due to q_3 because forces due to q_1 and due to q_4 are equal and opposite, so cancel each other. Similarly, forces due to q_2 and q_5 also cancel each other. Hence, the net force on $-q$ is

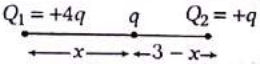
$$F = \frac{1}{4\pi\,\varepsilon_0}\cdot\frac{(q)(q)}{r^2} \qquad \text{[towards } q_3\text{]}$$

or

$$F = \frac{1}{4\pi\,\varepsilon_0}\cdot\frac{q^2}{a^2}$$

Example 1.11 *Two fixed charges $+4q$ and $+q$ are at a distance 3 m apart. At what point between the charges, a third charge $+q$ must be placed to keep it in equilibrium?*

Sol. Remember, if Q_1 and Q_2 are of same nature (means both positive or both negative), then the third charge should be put between (not necessarily at mid-point) Q_1 and Q_2 on the straight line joining them. But, if Q_1 and Q_2 are of opposite nature, then the third charge will be put outside and close to that charge which is lesser in magnitude.



Here Q_1 and Q_2 are of same nature of third charge, q will be kept in between at a distance x from Q_1 (as shown in figure). Hence, q will be at a distance $(3-x)$ from Q_2. Since, q is in equilibrium, so net force on it must be zero. The forces applied by Q_1 and Q_2 on q are in opposite direction, so just balance their magnitude.

Force on q by $Q_1 = \frac{KQ_1q}{x^2}$ and that by $Q_2 = \frac{KQ_2q}{(3-x)^2}$

Now, $\frac{KQ_1q}{x^2} = \frac{KQ_2q}{(3-x)^2}$ or $\frac{Q_1}{x^2} = \frac{Q_2}{(3-x)^2}$ or $\frac{4}{x^2} = \frac{1}{(3-x)^2}$

Take the square root, $\frac{2}{x} = \frac{1}{(3-x)}$

or $6-2x = x$ (after cross multiplication) or, $x = 2$ m. So, q will be placed at a distance 2 m from Q_1 and at 1 m from Q_2.

Note If $q_1q_2 > 0$

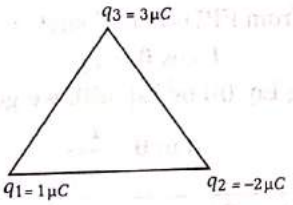
$$x_1 = \frac{r_0}{\sqrt{\frac{q_2}{q_1}} + 1}$$ [x_1 is distance from q_1 between q_1 and q_2]

If $q_1q_2 < 0$

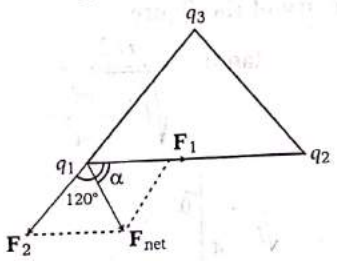
$$x_1 = \frac{r_0}{\sqrt{\frac{q_2}{q_1}} - 1}$$ [x_1 in this case is not between the charges.]

Example 1.12 *Three charges $q_1 = 1\,\mu C$, $q_2 = -2\,\mu C$ and $q_3 = 3\,\mu C$ are placed on the vertices of an equilateral triangle of side 1.0 m. Find the net electric force acting on charge q_1.*

How to proceed *Charge q_2 will attract charge q_1 (along the line joining them) and charge q_3 will repel charge q_1. Therefore, two forces will act on q_1, one due to q_2 and another due to q_3. Since, the force is a vector quantity both of these forces (say $\mathbf{F}_1$ and $\mathbf{F}_2$) will be added by vector method. Following are two methods of their addition.*



Sol. **Method 1.** In the figure,



$$|\mathbf{F}_1| = F_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{r^2}$$

= magnitude of force between q_1 and q_2

Here, $q_1 = 1\,\mu C = 1\times10^{-6}$ C

$q_2 = 2\,\mu C = 2\times10^{-6}$ C

$$\Rightarrow \quad F_1 = \frac{(9.0\times10^9)(1.0\times10^{-6})(2.0\times10^{-6})}{(1.0)^2} = 1.8\times10^{-2}\text{ N}$$

Similarly,

$$|\mathbf{F}_2| = F_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_3}{r^2}$$

where, $q_3 = 3\mu C = 3\times10^{-6}$ C

= magnitude of force between q_1 and q_2

$$\Rightarrow \quad F_2 = \frac{(9.0\times10^9)(1.0\times10^{-6})(3.0\times10^{-6})}{(1.0)^2}$$

$$= 2.7\times10^{-2}\text{ N}$$

Now, net force, $|\mathbf{F}_{net}| = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 120°}$

$$= \left(\sqrt{(1.8)^2 + (2.7)^2 + 2(1.8)(2.7)\left(-\frac{1}{2}\right)}\right)\times10^{-2}\text{ N}$$

$$= 2.38\times10^{-2}\text{ N}$$

and $\tan\alpha = \frac{F_2\sin 120°}{F_1 + F_2\cos 120°}$

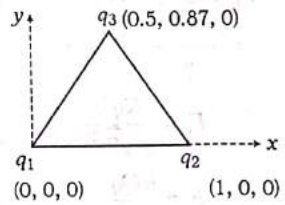
$$= \frac{(2.7\times10^{-2})(0.87)}{(1.8\times10^{-2}) + (2.7\times10^{-2})\left(-\frac{1}{2}\right)}$$

or $\alpha = 79.2°$

Thus, the net force on charge q_1 is 2.38×10^{-2} N at an angle $\alpha = 79.2°$ with a line joining q_1 and q_2 as shown in the figure.

Method 2. In this method let us assume a coordinate axes with q_1 at origin as shown in figure. The coordinates of q_1, q_2 and q_3 in this coordinate system are (0, 0, 0), (1 m, 0, 0) and (0.5 m, 0.87 m, 0), respectively. Now,

$\mathbf{F}_1$ = force on q_1 due to charge q_2



$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{|\mathbf{r}_1 - \mathbf{r}_2|^3}(\mathbf{r}_1 - \mathbf{r}_2)$$

$$= \frac{(9.0\times10^9)(1.0\times10^{-6})(-2.0\times10^{-6})}{(1.0)^3}$$

$$\times[(0-1)\hat{\mathbf{i}} + (0-0)\hat{\mathbf{j}} + (0-0)\hat{\mathbf{k}}]$$

$$= (1.8\times10^{-2}\,\hat{\mathbf{i}})\text{ N}$$

and $\mathbf{F}_2$ = force on q_1 due to charge q_3

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_3}{|\mathbf{r}_1 - \mathbf{r}_3|^3}(\mathbf{r}_1 - \mathbf{r}_3)$$

$$= \frac{(9.0\times 10^9)(1.0\times 10^{-6})(3.0\times 10^{-6})}{(1.0)^3}$$

$$\times[(0-0.5)\,\hat{\mathbf{i}} + (0-0.87)\,\hat{\mathbf{j}} + (0-0)\,\hat{\mathbf{k}}]$$

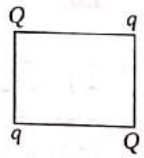
$$= (-1.35\,\hat{\mathbf{i}} - 2.349\,\hat{\mathbf{j}})\times 10^{-2}\,\text{N}$$

Therefore, net force on q_1 is

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2$$

$$= (0.45\,\hat{\mathbf{i}} - 2.349\,\hat{\mathbf{j}})\times 10^{-2}\,\text{N}$$

Example 1.13 *Four charges Q, q, Q and q are kept at the four corners of a square as shown below. What is the relation between Q and q, so that the net force on a charge q is zero?*

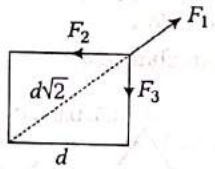


Sol. Here, both the q will have same sign either positive or negative. Similarly, both the Q will have same sign. Let us make the force on upper right corner q equal to zero.

Lower q will apply a repelling force F_1 on upper q because both the charges have same sign. To balance this force both Q must apply attractive forces $\mathbf{F}_2$ and $\mathbf{F}_3$ of equal magnitude, hence, Q and q will have opposite signs. Now, the resultant of $\mathbf{F}_2$ and $\mathbf{F}_3$ will be $F\sqrt{2}$ (Parallelogram law of vector addition), if $|\mathbf{F}_2| = |\mathbf{F}_3| = F$. Also note that $F\sqrt{2}$ will be exactly and opposite to F_1.

So, $$F_1 = F\sqrt{2}$$

From, Coulomb's law,

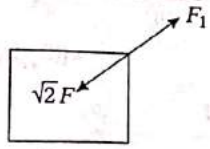


$$F_1 = \frac{Kq^2}{(d\sqrt{2})^2}$$

and $$F = \frac{KQq}{d^2}$$

$\because$ $$F_1 = \sqrt{2}F$$

$\therefore$ $$\frac{q^2}{(d\sqrt{2})^2} = \frac{\sqrt{2}Qq}{d^2}$$



$\therefore$ $$Q = \frac{q}{2\sqrt{2}}$$

But as we said Q and q have opposite sign, so $q = -2\sqrt{2}Q$.

Applications of electric force (Coulomb's law)

1. For solving problems related to string

Let us consider two identical balls, having mass m and charge q. These are suspended from a common point by two insulating strings each of length L as shown in figure.

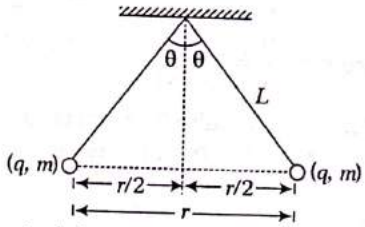


Fig. 1.6 *Two identical charged balls are suspended from same point*

The balls repel and come into equilibrium at separation r. The ball is in equilibrium under the following forces.

(i) Weight of the ball, $w = mg$

(ii) Electric force, $F_e = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q\cdot q}{r^2} = \dfrac{q^2}{4\pi\varepsilon_0 r^2}$ ...(i)

(iii) **Tension in the string** For this, let us draw FBD of a ball under consideration.

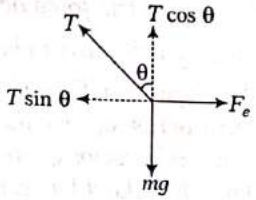


Fig. 1.7 *FBD of one of the balls*

It is clear from FBD that, $T\sin\theta = F_e$ ...(ii)

$T\cos\theta = mg$ ...(iii)

On dividing Eq. (ii) by Eq. (iii), we get

$\Rightarrow$ $$\tan\theta = \frac{F_e}{mg}$$ ...(iv)

or $$T = \sqrt{(T\sin\theta)^2 + (T\cos\theta)^2} = \sqrt{F_e^2 + (mg)^2}$$ ...(v)

From geometry of the figure,

$$\tan\theta = \frac{r/2}{\sqrt{L^2 - \frac{r^2}{4}}}$$

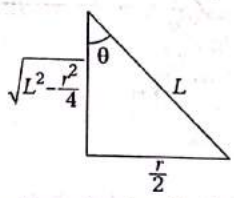


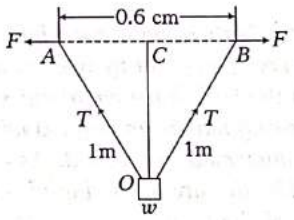
Fig. 1.8 *Formation of a triangle through r/2 and L*

If θ is small, $\tan\theta \simeq \sin\theta = \dfrac{r/2}{L}$

Now, let us solve some problems related to this to make our concepts more clear.

Example 1.14 *Two identical helium filled balloons A and B fastened to a weight of 5 g by threads floats in equilibrium as shown in figure. Calculate the charge on each balloons, assuming that they carry equal charges.*

Sol. In figure, A and B are two balloons attached to weight w by string $OA = OB = 1$ m



$AB = 0.6$ m, C is the centre of AB

$\therefore \quad AC = BC = 0.3$ m

$$OC = \sqrt{OA^2 - AC^2} = \sqrt{(1)^2 - (0.3)^2} = 0.954$$

Force of repulsion between the balloons,

$$F = \frac{q \times q}{4\pi\varepsilon_0 (AB)^2}$$

As the balloons float in equilibrium.

$\therefore$ Using triangle law of vectors, we get

$$\frac{F}{AC} = \frac{w}{OC}$$

where, w = upthrust acting on each balloon.

As $W = w + w$

$$\Rightarrow \quad w = \frac{W}{2}$$

$$\therefore \quad F = \frac{AC}{OC} \times \frac{W}{2}$$

$$\Rightarrow \quad \frac{q \times q}{4\pi\varepsilon_0 (AB)^2} = \frac{AC}{OC} \times \frac{mg}{2}$$

$$\Rightarrow \quad q^2 = 4\pi\varepsilon_0 (AB)^2 \times \frac{AC}{OC} \times \frac{mg}{2}$$

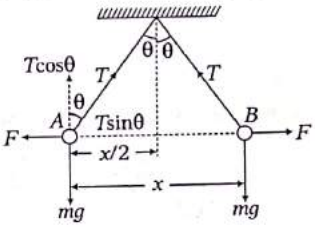
$$= \frac{(0.6)^2 \times (0.3) \times (5 \times 10^{-3} \times 9.8)}{9 \times 10^9 \times 0.954 \times 2}$$

$$\Rightarrow \quad q^2 = \frac{0.36 \times 0.30 \times 49}{9 \times 0.954 \times 2} \times 10^{-7}\text{C}$$

$$\Rightarrow \quad q = \pm 5.6 \times 10^{-7}\ \text{C}$$

Example 1.15 *Two identical pith balls, each of mass m and charge q are suspended from a common point by two strings of equal length l. Find charge q in terms of given quantities, if angle between the strings is 2θ in equilibrium.*

Sol. Let x be the separation between balls in equilibrium. According to the question, following figure can be drawn.



Each ball is in equilibrium under the action of the following forces

(a) Weight of the ball, mg

(b) Repulsive electric force, $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q^2}{x^2}$

(c) Tension in the string, T

Resolving T in the horizontal and vertical direction, since ball is in equilibrium,

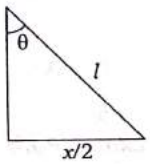
$$T\sin\theta = F \quad \text{...(i)}$$

$$T\cos\theta = mg \quad \text{...(ii)}$$

By Eqs. (i) and (ii), we get

$$\tan\theta = \frac{F}{mg} = \frac{\dfrac{1}{4\pi\varepsilon_0}\dfrac{q^2}{x^2}}{mg} \quad \text{...(iii)}$$

Form figure, $\dfrac{x}{2} = l\sin\theta \Rightarrow x = 2l\sin\theta$



Put in Eq. (iii), we get

Also, $\quad q^2 = 4\pi\varepsilon_0 mgx^2 \tan\theta$

$$= 4\pi\varepsilon_0 mg(2l\sin\theta)^2 \tan\theta$$

$$q = [16\pi\varepsilon_0\, mgl^2 \sin^2\theta \tan\theta]^{1/2}$$

Example 1.16 *Two identical balls, each having a charge q and mass m, are suspended from a common point by two insulating strings each of length L. The balls are held at a separation x and then released. Find*

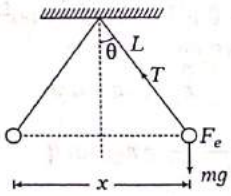
(i) the electric force on each ball.

(ii) the component of the resultant force on a ball along and perpendicular to string.

(iii) the tension in the string.

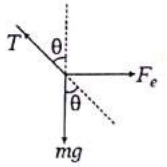
(iv) the acceleration of one of the balls. Consider the situation only for the instant just after the release.

Sol. Let x be the separation between the balls. According to question, the following figures can be drawn

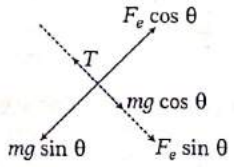


(i) Electric force between balls, $F_e = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q^2}{x^2}$

(ii)



Resolving forces along and perpendicular to string.



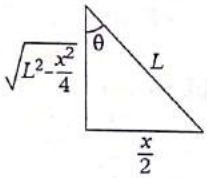
Resultant force on ball along the string,

$T - (mg\cos\theta + F_e \sin\theta) = 0$ [the string is unstretchable]

Similarly, force perpendicular to the string

$= |F_e \cos\theta - mg\sin\theta|$

θ can be obtained from geometry.



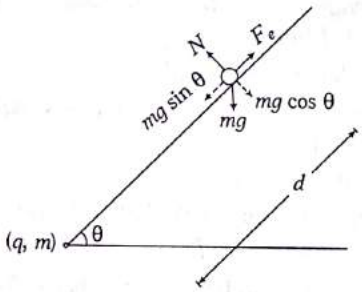
(iii) Since the net force on the ball along string is zero, hence

$$T = mg\cos\theta + F_e \sin\theta$$

(iv) Acceleration of ball, $a = \dfrac{|F_e \cos\theta - mg\sin\theta|}{m}$

Example 1.17 *A particle A having charge q and mass m is placed at the bottom of a smooth inclined plane of inclination θ. Where should a block B, having same charge and mass, be placed on the incline, so that it may remain in equilibrium?*

Sol. The following figure can be drawn in accordance with the question,



Let block B be placed at distance d. The block B is in equilibrium. $mg\cos\theta$ is balanced by normal reaction and $mg\sin\theta$ by repulsive electric force.

$$F_e = mg\sin\theta$$

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\frac{q^2}{d^2} = mg\sin\theta$$

$$\Rightarrow \quad d = q\sqrt{\frac{1}{4\pi\varepsilon_0 mg\sin\theta}}$$

2. Lami's theorem

In few problems of electrostatics Lami's theorem is very useful. According to this theorem, *if three concurrent forces F_1, F_2 and F_3 as shown in figure are in equilibrium* or if $F_1 + F_2 + F_3 = 0$, then

$$\boxed{\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}}$$

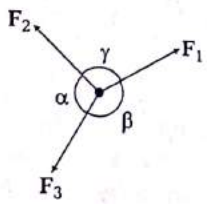


***Fig. 1.9** Three forces passing through a point*

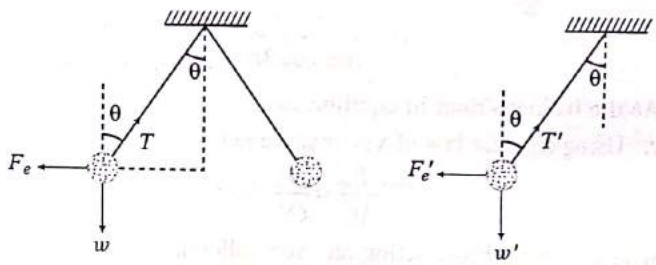
Example 1.18 *Two identical balls each having a density ρ are suspended from a common point by two insulating strings of equal length. Both the balls have equal mass and charge. In equilibrium each string makes an angle θ with vertical. Now, both the balls are immersed in a liquid. As a result the angle θ does not change. The density of the liquid is σ. Find the dielectric constant of the liquid.*

Sol. Each ball is in equilibrium under the following three forces

(i) tension (ii) electric force and

(iii) weight

So, by applying Lami's theorem,



In vacuum In liquid

In the liquid, $F_e' = \dfrac{F_e}{K}$

where, K = dielectric constant of liquid

and $w' = w -$ upthrust

Applying Lami's theorem in vacuum,

$$\frac{w}{\sin(90° + \theta)} = \frac{F_e}{\sin(180° - \theta)} \quad \text{or} \quad \frac{w}{\cos\theta} = \frac{F_e}{\sin\theta} \qquad \text{...(i)}$$

Similarly in liquid, $\dfrac{w'}{\cos\theta} = \dfrac{F_e'}{\sin\theta}$ ...(ii)

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{w}{w'} = \frac{F_e}{F_e'} \quad \text{or} \quad K = \frac{w}{w - \text{upthrust}} \qquad \left(\text{as } \frac{F_e}{F_e'} = K\right)$$

$$= \frac{V\rho g}{V\rho g - V\sigma g} \qquad [V = \text{volume of ball}]$$

or $K = \dfrac{\rho}{\rho - \sigma}$

Check point 1.2

1. There are two charges + 1μC and +5μC. The ratio of the forces acting on them will be
(a) 1 : 5 (b) 1 : 1
(c) 5 : 1 (d) 1 : 25

2. The force between two charges 0.06 m apart is 5 N. If each charge is moved towards the other by 0.04 m, then the force between them will become
(a) 7.20 N (b) 11.25 N
(c) 22.50 N (d) 45.00 N

3. The value of electric permittivity of free space is
(a) $9 \times 10^9 NC^2/m^2$
(b) $8.85 \times 10^{-12} N\text{-}m^2/C^2s$
(c) $8.85 \times 10^{-12} C^2/N\text{-}m^2$
(d) $9 \times 10^9 C^2/N\text{-}m^2$

4. Two charges each equal to 2μC are 0.5 m apart. If both of them exist inside vacuum, then the force between them is
(a) 1.89 N (b) 2.44 N
(c) 0.144 N (d) 3.144 N

5. The charges on two spheres are +7μC and −5 μC, respectively. They experience a force F. If each of them is given an additional charge of −2μC, then the new force attraction will be
(a) F (b) $F/2$
(c) $F/\sqrt{3}$ (d) $2F$

6. Two charges of equal magnitudes and at a distance r exert a force F on each other. If the charges are halved and distance between them is doubled, then the new force acting on each charge is
(a) $F/8$ (c) $F/4$
(c) $4F$ (d) $F/16$

7. A metallic sphere having no net charge is placed near a finite metal plate carrying a positive charge. The electric force on the sphere will be
(a) towards the plate
(b) away from the plate
(c) parallel to the plate
(d) zero

8. Two charged spheres separated at a distance d exert a force F on each other. If they are immersed in a liquid of dielectric constant 2, then what is the force (if all conditions are same)
(a) $\frac{F}{2}$ (b) F
(c) $2F$ (d) $4F$

9. Two charges placed in air repel each other by a force of 10^{-4}N. When oil is introduced between the charges, then the force becomes 2.5×10^{-5}N.
The dielectric constant of oil is
(a) 2.5 (b) 0.25
(c) 2.0 (d) 4.0

10. Two point charges placed at a certain distance r in air exert a force F on each other. Then, the distance r' at which these charges will exert the same force in a medium of dielectric constant K is given by
(a) r (b) r/K
(c) $r/\sqrt{K}$ (d) $r\sqrt{K}$

11. F_g and F_e represent gravitational and electrostatic force respectively between electrons situated at a distance 10 cm. The ratio of F_g/F_e is of the order of
(a) 10^{42} (b) 10^{-21}
(c) 10^{24} (d) 10^{-43}

12. Two protons are a distance of 1×10^{-10}cm from each other. The forces acting on them are
(a) nuclear force and gravitational force
(b) nuclear force and coulomb force
(c) coulomb force and gravitational force
(d) nuclear, coulomb and gravitational force

13. Two particles of equal mass m and charge q are placed at a distance of 16 cm. They do not experience any force. The value of $\frac{q}{m}$ is
(a) 1 (b) $\sqrt{\frac{\pi\varepsilon_0}{G}}$
(c) $\sqrt{\frac{G}{4\pi\varepsilon_0}}$ (d) $\sqrt{4\pi\varepsilon_0 G}$

14. A charge q_1 exerts some force on a second charge q_2. If a third charge q_3 is brought near q_2, then the force exerted by q_1 on q_2
(a) decreases
(b) increases
(c) remains the same
(d) increases, if q_3 is of same sign as q_1 and decreases, if q_3 is of opposite sign as q_1

15. Electric charges of 1μC, −1μC and 2μC are placed in air at the corners A, B and C respectively of an equilateral triangle ABC having length of each side 10 cm. The resultant force on the charge at C is
(a) 0.9 N (b) 1.8 N
(c) 2.7 N (d) 3.6 N

3 Electric field

The region surrounding a charge or distribution of charge in which its electrical effects can be observed or experienced is called the **electric field** of the charge or distribution of charge.

Electric field at a point can be defined in terms of either a vector function **E** called **electric field strength** or a scalar function V called **electric potential.**

Electric field strength

The electric field strength (often called electric field) at a point in an electric field is defined as the electrostatic force $\mathbf{F}_e$ per unit positive charge. Thus, if the electrostatic force experienced by a small test charge q_0 is $\mathbf{F}_e$, then field strength at that point is defined as

$$\boxed{\mathbf{E} = \lim_{q_0 \to 0} \frac{\mathbf{F}_e}{q_0}}$$

The electric field is a vector quantity and its direction is the same as the direction of the force $\mathbf{F}_e$ on a positive test charge. The SI unit of electric field is N/C. The dimensions for **E** is $[MLT^{-3}A^{-1}]$. For a positive charge the electric field will be directed radially outwards from the charge. On the other hand, if the source charge is negative, the electric field vector at each point is directed radially inwards.

Note Suppose there is an electric field strength **E** at some point in an electric field, then the electrostatic force acting on a charge $+q$ is qE in the direction of **E**, while on the charge $-q$ it is qE in the opposite direction of **E**.

Example 1.19 *An electric field of 10^5 N/C points due west at a certain spot. What are the magnitude and direction of the force that acts on a charge of $+2\ \mu C$ and $-5\ \mu C$ at this spot?*

Sol. Electrostatic force, $F = qE$

Force on $+2\ \mu C = qE$

where, E is the electric field of magnitude 10^5 N/C

$= (2 \times 10^{-6})(10^5) = 0.2$ N (due west)

Force on $-5\ \mu C = (5 \times 10^{-6})(10^5)$

$= 0.5$ N (due east)

Example 1.20 *Calculate the magnitude of an electric field which can just suspend a deuteron of mass 3.2×10^{-27} kg freely in air.*

Sol. Upward force on the deuteron due to electric field qE is equal to weight mg of deuteron $qE = mg$

$$\therefore \quad E = \frac{mg}{q} = \frac{3.2 \times 10^{-27} \times 9.8}{1.6 \times 10^{-19}} = 19.6 \times 10^{-8}\ \text{NC}^{-1}$$

Electric field due to a point charge

The electric field produced by a point charge q can be obtained in general terms from Coulomb's law. The magnitude of the force exerted by the charge q on a test charge q_0 is

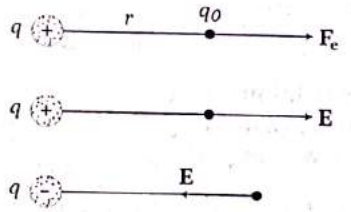


Fig. 1.10 Direction of electric field due to positive and negative charges

$$F_e = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qq_0}{r^2}$$

Therefore, the intensity of the electric field at this point is given by $E = \dfrac{F}{q_0}$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}}$$

Note Suppose a charge q is placed at a point whose position vector is $\mathbf{r}_q$ and we want to find the electric field at a point P whose position vector is $\mathbf{r}_p$, then in vector form the electric field is given by

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{|\mathbf{r}_p - \mathbf{r}_q|^3} (\mathbf{r}_p - \mathbf{r}_q)$$

Here, $\mathbf{r}_p = x_p\hat{\mathbf{i}} + y_p\hat{\mathbf{j}} + z_p\hat{\mathbf{k}}$

and $\mathbf{r}_q = x_q\hat{\mathbf{i}} + y_q\hat{\mathbf{j}} + z_q\hat{\mathbf{k}}$

Example 1.21 *Find the electric field strength due to a point charge of $5\ \mu C$ at a distance of 80 cm from the charge.*

Sol. Given, $q = 5\ \mu C = 5 \times 10^{-6}$ C

$r = 80$ cm $= 80 \times 10^{-2}$ m.

Electric field strength,

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$

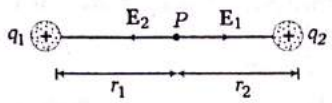
$$\Rightarrow \quad E = 9 \times 10^9 \times \frac{5 \times 10^{-6}}{(80 \times 10^{-2})^2}$$

$$\Rightarrow \quad E = 7.0 \times 10^4\ \text{N/C}$$

Example 1.22 *Two positive point charges $q_1 = 16\ \mu C$ and $q_2 = 4\ \mu C$, are separated in vacuum by a distance of 3.0 m. Find the point on the line between the charges, where the net electric field is zero.*

Sol. Between the charges, the two field contributions have opposite directions, and the net electric field is zero at a point (say P), where the magnitudes of $\mathbf{E}_1$ and $\mathbf{E}_2$ are equal. However, since, $q_2 < q_1$, point P must be closer to q_2, in order

that the field of the smaller charge can balance the field of the larger charge.



At P, $\quad E_1 = E_2$

or $\quad \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1}{r_1^2} = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{q_2}{r_2^2}$

$\therefore \quad \dfrac{r_1}{r_2} = \sqrt{\dfrac{q_1}{q_2}} = \sqrt{\dfrac{16}{4}} = 2 \qquad$ ...(i)

Also, $\quad r_1 + r_2 = 3.0$ m $\qquad$ ...(ii)

Solving these equations, we get

$$r_1 = 2 \text{ m} \quad \text{and} \quad r_2 = 1 \text{ m}$$

Thus, the point P is at a distance of 2 m from q_1 and 1 m from q_2.

Example 1.23 *A charge $q = 1\ \mu C$ is placed at point (1 m, 2 m, 4 m). Find the electric field at point P (0 m, – 4 m, 3 m).*

Sol. Here, $\quad \mathbf{r}_q = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$

and $\quad \mathbf{r}_p = -4\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

$\therefore \quad \mathbf{r}_p - \mathbf{r}_q = -\hat{\mathbf{i}} - 6\hat{\mathbf{j}} - \hat{\mathbf{k}}$

or $\quad |\mathbf{r}_p - \mathbf{r}_q| = \sqrt{(-1)^2 + (-6)^2 + (-1)^2} = \sqrt{38}$ m

Now, electric field, $\mathbf{E} = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{q}{|\mathbf{r}_p - \mathbf{r}_q|^3}(\mathbf{r}_p - \mathbf{r}_q)$

Substituting the values, we have

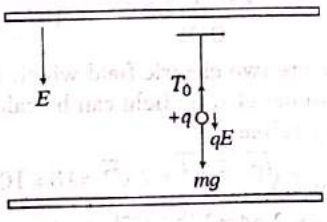
$$\mathbf{E} = \frac{(9.0\times10^9)(1.0\times10^{-6})}{(38)^{3/2}}(-\hat{\mathbf{i}} - 6\hat{\mathbf{j}} - \hat{\mathbf{k}})$$

$$\mathbf{E} = (-38.42\ \hat{\mathbf{i}} - 230.52\ \hat{\mathbf{j}} - 38.42\ \hat{\mathbf{k}})\text{N/C}$$

Example 1.24 *A ball having charge q and mass m is suspended from a string of length L between two parallel plates where a vertical electric field E is established. Find the time period of simple pendulum if electric field is directed (i) downward and (ii) upward.*

Sol. (i) When electric field is downward

For simple pendulum,



Time period, $T = 2\pi\sqrt{\dfrac{L}{a}}$

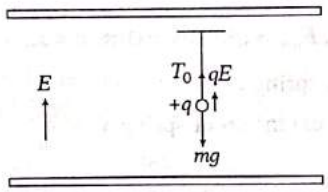
where, a effective acceleration.

$$a = \frac{\text{net external force on the ball}}{\text{mass of the ball}}$$

$$\Rightarrow \quad a = \frac{mg + qE}{m} = g + \frac{qE}{m}$$

$\therefore$ Time period, $T = 2\pi\sqrt{\dfrac{L}{g + \dfrac{qE}{m}}}$

(ii) When electric field is upward



Effective acceleration, $a = \dfrac{mg - qE}{m} = g - \dfrac{qE}{m}$

Time period, $\quad T = 2\pi\sqrt{\dfrac{L}{g - \dfrac{qE}{m}}}$

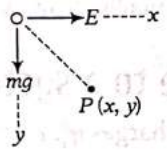
Example 1.25 *A ball of mass m having a charge q is released from rest in a region where a horizontal electric field E exists.*

(i) Find the resultant force acting on the ball.

(ii) Find the trajectory followed by the ball.

Sol. The forces acting on the ball are weight of the ball in vertically downward direction and the electric force in the horizontal direction.

(i) Resultant force, $F = \sqrt{(mg)^2 + (qE)^2}$.



(ii) Let the ball be at point P after time t.

As, $\quad F = ma = qE$

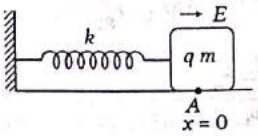
$\therefore \quad a = \dfrac{qE}{m}$

$$\left.\begin{aligned} x\text{-direction, } x &= \frac{1}{2}\frac{qE}{m}t^2 \\ y\text{-direction, } y &= \frac{1}{2}gt^2 \end{aligned}\right\} \text{[from second law of Newton]}$$

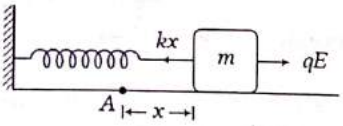
$$\frac{y}{x} = \frac{g}{qE/m} \Rightarrow y = \frac{mgx}{qE}$$

Trajectory is straight line.

Example 1.26 *A block of mass m having charge q is attached to a spring constant k. This arrangement is placed in uniform electric field E on smooth horizontal surface as shown in the figure. Initially, spring in unstretched. Find the extension of spring in equilibrium position and maximum extension of spring.*



Sol. Due to electric force qE acting on the block, spring extends. Let at some instant extension of spring be x.



Net force on block, $F_{net} = qE - kx$

In equilibrium, $F_{net} = qE - kx = 0 \Rightarrow x = x_0 = \frac{qE}{k}$

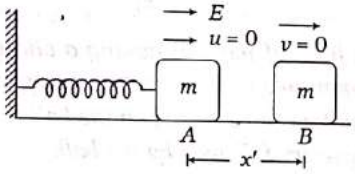
where, k is the spring constant.

In equilibrium extension of spring $x_0 = \frac{qE}{k}$

Maximum extension $= 2x_0 = \frac{2qE}{k}$

Method 2

Let the body is displaced from position A to position B. Let the maximum extension produced be x'. Then, from conservation law,



$$\frac{1}{2}kx'^2 = qEx' \Rightarrow x' = \frac{2qE}{k}$$

In fact, the block is executing SHM of time period,

$$T = 2\pi\sqrt{\frac{m}{k}}, \text{ amplitude of oscillation} = \frac{qE}{k}.$$

Electric field due to a system of charges

Consider a system of charges $q_1, q_2, \ldots, q_n$ with position vectors $\mathbf{r}_{p_1}, \mathbf{r}_{p_2}, \mathbf{r}_{p_3}, \ldots, \mathbf{r}_{p_n}$ relative to some origin O as shown in the figure.

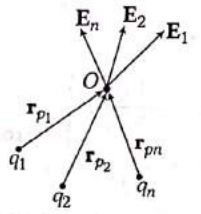


Fig. 1.11 Electric field due to system of charges $q_1, q_2, \ldots, q_n$

By the principle of superposition, the electric field $\mathbf{E}$ at point P due to system of charges will be given by

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \ldots \mathbf{E}_n$$

where, $\mathbf{E}_1, \mathbf{E}_2, \ldots \mathbf{E}_n$ be the electric field at P due to charges $q_1, q_2, \ldots, q_n$ respectively.

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{p_1}^2}\hat{\mathbf{r}}_{p_1} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{p_2}^2}\hat{\mathbf{r}}_{p_2} + \ldots \frac{1}{4\pi\varepsilon_0}\frac{q_n}{r_{p_n}^2}\hat{\mathbf{r}}_{p_n}$$

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_{p_i}^2}\hat{\mathbf{r}}_{p_i}$$

Example 1.27 *An infinite number of charges each equal to q are placed along X-axis at $x = 1$, $x = 2$, $x = 4$, $x = 8$ and so on. Find the electric field at the point $x = 0$ due to this set up of charges.*

Sol. At the point $x = 0$, the electric field due to all the charges are in the same negative x-direction and hence get added up.

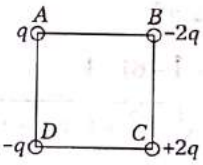
$$E = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{1^2} + \frac{q}{2^2} + \frac{q}{4^2} + \frac{q}{8^2} + \ldots\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \ldots\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{1}{1 - 1/4} = \frac{q}{3\pi\varepsilon_0}$$

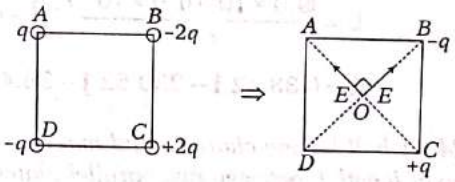
This electric field is along negative X-axis.

Example 1.28 *Four charges are placed at the corners of a square of side 10 cm as shown in figure having. If q is 1 μC, then what will be electric field intensity at the centre of the square?*



Sol. Side of square, $a = 0.1$m, Magnitude of charge, $q = 1\times10^{-6}$ C

Half of diagonal of the square $= \frac{a}{\sqrt{2}} = \frac{0.1}{\sqrt{2}}$



Electric field due to charge q,

$$E = \frac{9\times10^9 \times 1\times10^{-6}}{\left(\frac{0.1}{\sqrt{2}}\right)^2}$$

$$= \frac{9\times10^3 \times 2}{0.01} = 18\times10^5 \text{ NC}^{-1}$$

At centre there are two electric field which are perpendicular to each other, so net electric field can be calculated using superposition principle.

$$\therefore \quad E_{net} = \sqrt{E^2 + E^2} = E\sqrt{2} = 18\times10^5 \times \sqrt{2}$$

$$= 2.54\times10^6 \text{ NC}^{-1}$$

Electric field lines

An **electric field line** is an imaginary line or curve drawn through a region of space so that its tangent at any point is in the direction of the electric field vector at that point.

Electric field lines are introduced by Michael Faraday to visualise electric field

The electric field lines have the following properties

(i) The tangent to a line at any point gives the direction of **E** at that point.

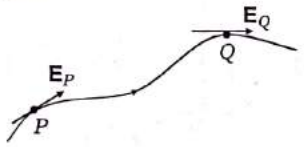


Fig. 1.12 Electric field at points P and Q

In the given figure, electric points P and Q are along the tangents (E_P and E_Q). This is also the path on which a positive test charge will tend to move if free to do so.

(ii) Electric field lines always begin from a positive charge and end on a negative charge and do not start or stop in midspace.

(iii) The number of lines leaving a positive charge or entering a negative charge is proportional to the magnitude of the charge. For example if 100 lines are drawn leaving a + 4 μC charge then 75 lines would have to end on a -3 μC charge.

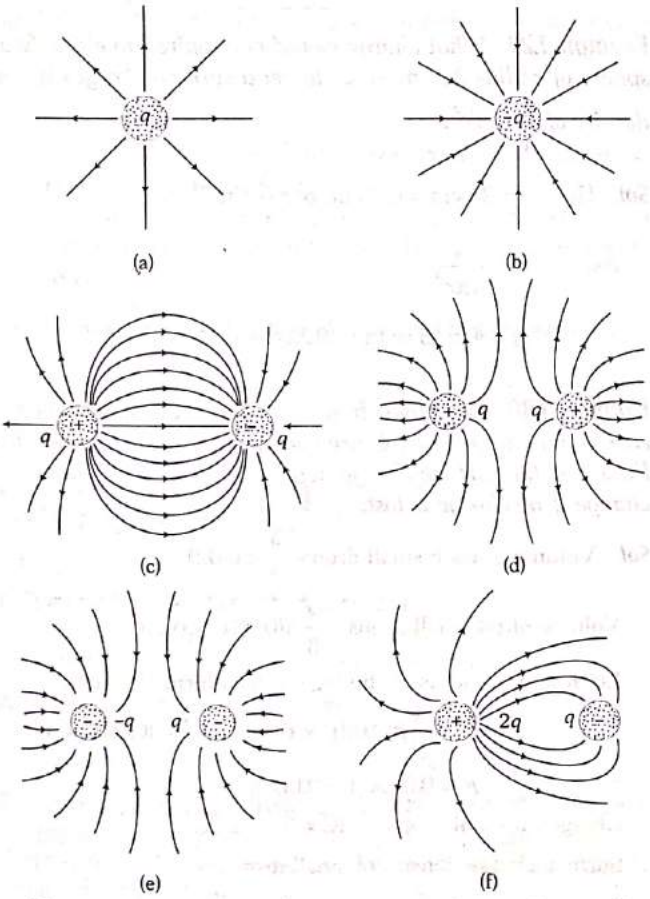


Fig. 1.12 Electric field lines associated with a single as well as combination of charges

(iv) Two lines can never intersect. If it happens, then two tangents can be drawn at their point of intersection *i.e.*, intensity at that point will have two directions which is not possible.

(v) In a uniform field, the field lines are straight, parallel and uniformly spaced.

(vi) The electric field lines can never form closed loops as a line can never start and end on the same charge.

(vii) Electric field lines also give us an indication of the equipotential surface (surface which has the same potential)

(viii) Electric field lines always flow from higher potential to lower potential.

(ix) In a region, where there is no electric field, lines are absent. This is why inside a conductor (where, electric field is zero) there cannot be any electric field line.

(x) Electric field lines of force ends or starts normally from the surface of a conductor.

(xi) The relative closeness of the electric lines of force in different regions of space indicates the relative strength of the electric field in different regions. In regions, where lines of force are closer, the electric field is stronger, whereas in regions, where line of force are further apart the field is weaker.

Therefore, in the given figure $|E_A| > |E_B|$.

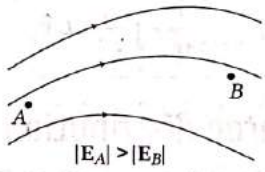


Fig. 1.13 Electric field strength at points A and B

Continuous charge distribution

In most of the cases, we deal with charges having magnitude greater than the charge of an electron. For this, we can imagine that the charge is spread in a region in a continuous manner. Such a charge distribution is known as continuous charge distribution.

Consider a point charge q_0 lying near a region of continuous charge distribution which is made up of large number of small charges dq as shown in figure.

According to Coulomb's law, the force on a point charge q_0 due to small charge dq is

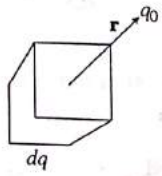


Fig. 1.14 Force on a point charge q_0 due to a continuous charge distribution

$$\mathbf{F} = \frac{q_0}{4\pi\varepsilon_0}\int \frac{dq}{r^2}\hat{\mathbf{r}} \quad \text{...(i)}$$

where, $\hat{\mathbf{r}} = \frac{\mathbf{r}}{r}$

There are three types of continuous charge distribution

1. Line charge distribution (λ)

It is a charge distribution along a one-dimensional curve or line, L in space as shown in figure.

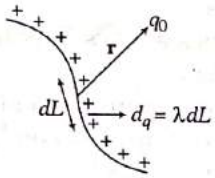


***Fig. 1.15** Line charge distribution*

The charge contained per unit length of the line at any point is called linear charge density. It is denoted by λ.

i.e., $\lambda = \frac{dq}{dL}$

Its SI unit is Cm^{-1}.

Electric field due to the line charge distribution at the location of charge q_0 is

$$\boxed{\mathbf{E}_L = \frac{1}{4\pi\varepsilon_0}\int_L \frac{\lambda}{r^2} dL\hat{\mathbf{r}}}$$

2. Surface charge distribution (σ)

It is a charge distribution spread over a two-dimensional surface S in space as shown in figure.

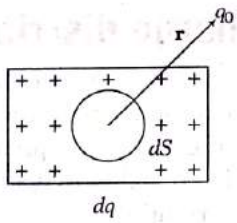


***Fig. 1.16** Surface charge distribution*

The charge contained per unit area at any point is called surface charge density. It is denoted by σ, *i.e.*,

Surface charge density, $\sigma = \frac{dq}{dS}$

Its SI unit is Cm^{-2}.

Electric field due to the surface charge distribution at the location of charge q_0 is

$$\boxed{\mathbf{E}_S = \frac{1}{4\pi\varepsilon_0}\int_S \frac{\sigma}{r^2} dS\hat{\mathbf{r}}}$$

3. Volume charge distribution (ρ)

It is a charge distribution spread over a three-dimensional volume or region V of space as shown in figure.

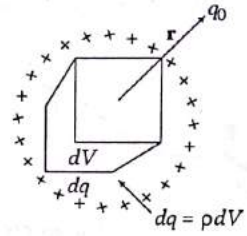


***Fig. 1.17** Volume charge distribution*

The charge contained per unit volume at any point is called volume charge density. It is denoted by ρ, *i.e.*,

$$\rho = \frac{dq}{dV}$$

Its SI unit is Coulomb per cubic metre (Cm^{-3}).

Electric field due to the volume charge distribution at the location of charge q_0 is

$$\boxed{\mathbf{E}_V = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho}{r^2} dV\hat{\mathbf{r}}}$$

Example 1.29 *What charge would be required to electrify a sphere of radius 25 cm, so as to get a surface charge density of $\frac{3}{\pi}$ Cm^{-2}?*

Sol. Here, $r = 25$ cm $= 0.25$ m, $\sigma = \frac{3}{\pi}$ Cm^{-2}

As, $\sigma = \frac{q}{4\pi r^2}$

$\therefore$ $q = 4\pi r^2\sigma = 4\pi \times (0.25)^2 \times \frac{3}{\pi}$ C $= 0.75$ C

Example 1.30 *Sixty four drops of radius 0.02 m and each carrying a charge of 5 μC are combined to form a bigger drop. Find how the surface charge density of electrification will change if no charge is lost.*

Sol. Volume of each small drop $= \frac{4}{3}\pi(0.02)^3$ m^3

Volume of 64 small drops $= \frac{4}{3}\pi(0.02)^3 \times 64$ m^3

Let R be the radius of the bigger drop formed. Then,

$$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi(0.02)^3 \times 64 \quad \text{or} \quad R^3 = (0.02)^3 \times 4^3$$

$\therefore$ $R = 0.02 \times 4 = 0.08$ m

Charge on small drop $= 5\,\mu C = 5 \times 10^{-6}$ C

Surface charge density of small drop,

$$\sigma_1 = \frac{q}{4\pi r^2} = \frac{5 \times 10^{-6}}{4\pi(0.02)^2} Cm^{-2}$$

Surface charge density of bigger drop,

$$\sigma_2 = \frac{5\times10^{-6}\times 64}{4\pi(0.08)^2}\ \text{Cm}^{-2}$$

$$\therefore \quad \frac{\sigma_1}{\sigma_2} = \frac{5\times10^{-6}}{4\pi(0.02)^2}\times\frac{4\pi(0.08)^2}{5\times10^{-6}\times 64} = \frac{1}{4} = 1:4$$

Electric field of a ring of charge

Electric field at distance x from the centre of uniformly charged ring of total charge q on its axis is given by

$$\boxed{E_x = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{qx}{(x^2+R^2)^{3/2}}}$$

Direction of this electric field is along the axis and away from the ring in case of positively charged ring and towards the ring in case of negatively charged ring.

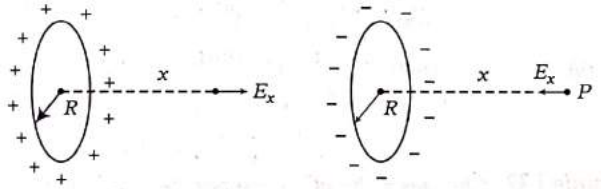


Fig. 1.18 Electric field of positively and negatively charged rings

Special cases

From the above expression, we can see that

(i) $E_x = 0$ at $x = 0$ *i.e.*, **field is zero at the centre of the ring.** This would occur because charges on opposite sides of the ring would push in opposite directions on a test charge at the centre, and the forces would add to zero.

(ii) $E_x = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{x^2}$ for $x \gg R$, *i.e.*, when the point P is much farther from the ring, its field is the same as that of a point charge. To an observer far from the ring, the ring would appear like a point, and the electric field reflects this.

(iii) E_x will be maximum where $\frac{dE_x}{dx} = 0$. Differentiating E_x w.r.t. x and putting it equal to zero, we get $x = \frac{R}{\sqrt{2}}$ and E_{max} comes out to be, $\frac{2}{\sqrt[3]{3}}\left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R^2}\right)$.

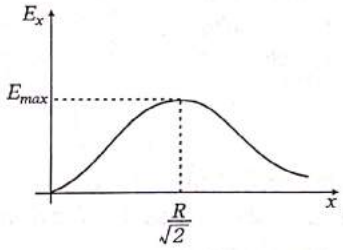


Fig. 1.19 Variation of electric field on the axis of a ring

Example 1.31 *A charge of 4×10^{-9} C is distributed uniformly over the circumference of a conducting ring of radius 0.3 m. Calculate the field intensity at a point on the axis of the ring at 0.4 m from its centre also at the centre.*

Sol. Given, $q = 0.4\times10^{-9}$ C, $a = 0.3$ m

$r = 0.4$ m

Electric field intensity at 0.4 m from its centre,

$$E = \frac{qx}{4\pi\varepsilon_0(R^2+x^2)^{3/2}}$$

$$= \frac{9\times10^9\times4\times10^{-9}\times0.4}{(0.4^2+0.3^2)^{3/2}}$$

$$E = \frac{14.4}{(0.5)^2} = 115.2\ \text{N/C}$$

At the centre of the ring, $x = 0$

$\therefore$ Electric field intensity, $E = 0$

Check point 1.3

1. The unit of intensity of electric field is
(a) newton/coulomb (b) joule/coulomb
(c) volt-metre (d) newton/metre

2. Which of the following is deflected by electric field?
(a) X-rays (b) γ-rays
(c) Neutrons (d) α-particles

3. A charged particle of mass 5×10^{-5} kg is held stationary in space by placing it in an electric field of strength 10^7NC^{-1} directed vertically downwards. The charge on the particle is
(a) $-20\times10^{-5}\,\mu$C (b) $-5\times10^{-5}\,\mu$C
(c) $5\times10^{-5}\,\mu$C (d) $20\times10^{-5}\,\mu$C

4. Electric field strength due to a point charge of 5μC at a distance 80 cm from the charge is
(a) 8×10^4 NC^{-1} (b) 7×10^4 NC^{-1}
(c) 5×10^4 NC^{-1} (d) 4×10^4 NC^{-1}

5. The electric field due to a charge at a distance of 3 m from it is 500 NC^{-1}. The magnitude of the charge is

$$\left[\frac{1}{4\pi\varepsilon_0} = 9\times10^9\ \text{N-m}^2/\text{C}^2\right]$$

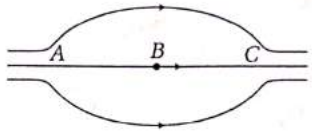
(a) 2.5 μC (b) 2.0 μC
(c) 1.0 μC (d) 0.5 μC

6. Two charges +5 μC and +10 μC are placed 20 cm apart. The net electric field at the mid-point between the two charges is
(a) 4.5×10^6 NC^{-1} directed towards +5 μC
(b) 4.5×10^6 NC^{-1} directed towards +10 μC
(c) 13.5×10^6 NC^{-1} directed towards +5 μC
(d) 13.5×10^6 NC^{-1} directed towards +10 μC

7. Two point charges $+8q$ and $-2q$ are located at $x = 0$ and $x = L$, respectively. The location of a point on the X-axis at which the net electric field due to these two point charges is zero, is
(a) $8L$ (b) $4L$
(c) $2L$ (d) $L/4$

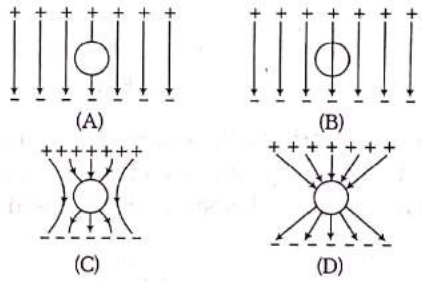
8. A cube of side b has a charge q at each of its vertices. The electric field due to this charge distribution at the centre of this cube will be $\frac{1}{4\pi\varepsilon_0}$ times
(a) q/b^2 (b) $q/2b^2$
(c) $32q/b^2$ (d) zero

9. The figure shows some of the electric field lines corresponding to an electric field. The figure suggests



(a) $E_A > E_B > E_C$ (b) $E_A = E_B = E_C$
(c) $E_A = E_C > E_B$ (d) $E_A = E_C < E_B$

10. An uncharged sphere of metal is placed in between two charged plates as shown. The lines of force look like



(a) A (b) B (c) C (d) D

4 Electric dipole

A pair of equal and opposite point charges, that are separated by a short distance is known as **electric dipole.** Electric dipole occurs in nature in a variety of situations *e.g.*, in HF, H_2O, HCl etc, the centre of positive charges does not fall exactly over the centre of negative charges. Such molecules are electric dipoles.

Dipole moment

The product of magnitude of one charge and the distance between the charges is called the magnitude of the electric dipole moment p. Suppose the charges of dipole are $-q$ and $+q$ and the small distance between them is $2a$.
Then, the magnitude of the electric dipole moment is given by

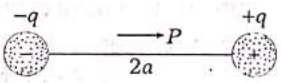


Fig. 1.20 Electric dipole

$$p = (2a)\, q$$

The electric dipole moment is a vector **p** whose direction is along the line joining the two charges pointing from the negative charges to the positive charges.
Its SI unit is Coulomb-metre.

Example 1.32 *Charges ± 20 nC are separated by 5 mm. Calculate the magnitude and direction of dipole moment.*

Sol. Given, $q_1 = q_2 = \pm 20$ nC $= \pm 20 \times 10^{-9}$ C

Distance $= 2a = 5$ mm $= 5 \times 10^{-3}$ m

Dipole moment, $p = q(2a) = 20 \times 10^{-9} \times 5 \times 10^{-3}$

$= 10^{-10}$ cm

The direction of **p** is from negative charge to positive charge.

Example 1.33 *A system has two charges, $q_A = 2.5 \times 10^{-7}$ C and $q_B = -2.5 \times 10^{-7}$ C located at points A(0,0, −15 cm) and B(0, 0, +15 cm) respectively. What is the electric dipole moment of the system?*

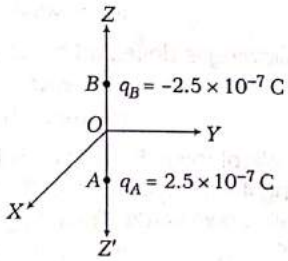
Sol. Electric dipole moment,

p = magnitude of either charge × dipole length

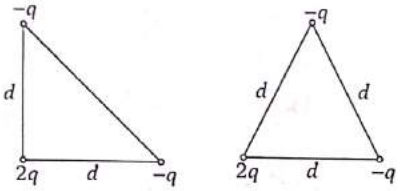
$= q_A \times AB = 2.5 \times 10^{-7} \times 0.30$

$= 7.5 \times 10^{-8}$ C-m

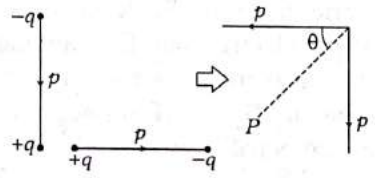
The electric dipole moment is directed from B to A, *i.e.*, from negative charge to positive charge.



Example 1.34 *Three charges are placed as shown. Find dipole moment of the arrangements.*



Sol. Here, two dipoles are formed. These are shown in diagram below.



Resultant dipole moment,

$$P = \sqrt{2}p = \sqrt{2}qd,\ \theta = 45°$$

The field of an electric dipole or dipole field

The electric field produced by an electric dipole is called a **dipole field**. The total charge of the electric dipole is zero but dipole field is not zero. It can be found out from Coulomb's law and the superposition principle. We will find electric field of an electric dipole at two points as shown below.

1. Electric field at an axial point of an electric dipole

Let us calculate electric field at the point P at a distance r from the centre of the dipole on the axial line of the dipole on the side of the charge q as shown in figure.

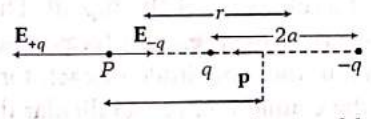


Fig. 1.21 *A point P is at an axial point of the dipole*

$$\mathbf{E}_{-q} = \frac{-q}{4\pi\varepsilon_0 (r+a)^2}$$

$$\mathbf{E}_{+q} = \frac{q}{4\pi\varepsilon_0 (r-a)^2}$$

The total field at P is, $\mathbf{E} = \mathbf{E}_{+q} + \mathbf{E}_{-q}$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{4ar}{(r^2-a^2)^2} \qquad [\because 2aq = p]$$

$$\therefore \qquad \mathbf{E} = \frac{2(2aq)r}{4\pi\varepsilon_0 (r^2-a^2)^2} = \frac{2r\,\mathbf{p}}{4\pi\varepsilon_0 (r^2-a^2)^2}$$

For short dipole, *i.e.*, for $r >> a$, $\boxed{\mathbf{E} = \frac{2\mathbf{p}}{4\pi\varepsilon_0 r^3}}$

Direction of $\mathbf{E}$ is same as $\mathbf{p}$.

2. Electric field at an equatorial point of an electric dipole

The magnitude of the electric fields due to the two charges $+q$ and $-q$ are given by

$E_{+q} = \dfrac{q}{4\pi\varepsilon_0 (r^2+a^2)}$ and $E_{-q} = \dfrac{q}{4\pi\varepsilon_0 (r^2+a^2)}$ and they are equal. The directions of E_{+q} and E_{-q} are as shown in the figure.

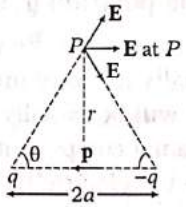


Fig. 1.22 *A point P is at an equatorial point of the dipole*

The components normal to the dipole axis cancel away. The components along the dipole axis add up. The total electric field $\mathbf{E}$ at P is opposite to dipole moment vector $\mathbf{p}$. So, we have $\mathbf{E} = -[(E_{+q} + E_{-q})\cos\theta]$

$$= \frac{-2q}{4\pi\varepsilon_0 (r^2+a^2)} \cdot \frac{a}{(r^2+a^2)^{\frac{1}{2}}} \left(\because \cos\theta \frac{a}{(r^2+a^2)^{\frac{1}{2}}}\right)$$

$$\Rightarrow \mathbf{E} = \frac{-2aq}{4\pi\varepsilon_0 (r^2+a^2)^{3/2}} = \frac{-\mathbf{p}}{4\pi\varepsilon_0 (r^2+a^2)^{3/2}} \quad [\because 2aq = p]$$

For short dipole, $r >> a$

$$\therefore \quad \boxed{\mathbf{E} = \frac{-\mathbf{p}}{4\pi\varepsilon_0 r^3}}$$

Note The electric field due to short dipole at large distance ($r >> a$) is proportional to $\frac{1}{r^3}$. If we take the limit, when the dipole size $2a$ approaches zero, the charge q approaches infinity in such a way that the product, $p = q \times 2a$ is finite. Such a dipole is referred to as a point dipole (ideal dipole).

Example 1.35 *Two opposite charges each of magnitude $2\mu C$ are 1 cm apart. Find electric field at a distance of 5 cm from the mid-point on axial line of the dipole. Also, find the field on equatorial line at the same distance from mid-point.*

Sol. Electric field (E) on axial line is given by $\dfrac{2pr}{4\pi\varepsilon_0(r^2-a^2)^2}$

where, p is dipole moment = either charge × dipole length

Thus, $p = q \cdot l = (2\times10^{-6})\times(0.01)$

Also, $r = 5\times10^{-2}$ m

$$\therefore \quad E = \frac{9\times10^9\times2(2\times10^{-6}\times10^{-2})\times5\times10^{-2}}{[(5\times10^{-2})^2-(0.5\times10^{-2})^2]^2}$$

$$= 2.93\times10^6\ \text{NC}^{-1}$$

Similarly, electric field (E) on equatorial line is given by

$$= \frac{p}{4\pi\varepsilon_0(r^2+a^2)^{3/2}}$$

The symbols have the same meaning as above,

$$E = \frac{9\times10^9\times(2\times10^{-6}\times10^{-2})}{[(5\times10^{-2})^2+(0.5\times10^{-2})^2]^{3/2}}$$

$$E = 1.46\times10^6\ \text{NC}^{-1}$$

3. Electric field at the position (r, θ)

Due to the positive charge of the dipole electric field at point P will be in radially outward direction and due to the negative charge it will be radially inward. Now, we have considered the radial component (E_r) and transverse component (E_θ) of the net electric field (E) as shown in figure.

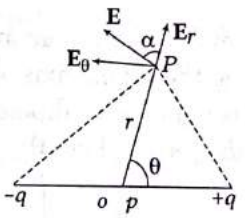


Fig. 1.23 Radial and transverse component of the electric field E of the dipole at point P(r, θ)

$$\therefore \quad E_r = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2p\cos\theta}{r^3} \quad \text{and} \quad E_\theta = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p\sin\theta}{r^3}$$

$\therefore$ Net electric field at point P is $E = \sqrt{E_r^2+E_\theta^2}$

$$\Rightarrow \quad \boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}\sqrt{1+3\cos^2\theta}}$$

Distance of the electric field,

$$\tan\alpha = \frac{E_\theta}{E_r} = \frac{1}{2}\tan\theta$$

Example 1.36 *What is the magnitude of electric field intensity due to a dipole of moment 2×10^{-8} C-m at a point distance 1 m from the centre of dipole, when line joining the point to the centre of dipole makes an angle of 60° with dipole axis?*

Sol. Given, $p = 2\times10^{-8}$, $r = 1$ m, $\theta = 60°$

$\therefore$ Electric field intensity,

$$E = \frac{p}{4\pi\varepsilon_0 r^3}\sqrt{3\cos^2\theta+1}$$

$$= \frac{2\times10^{-8}\times9\times10^9}{(1)^3}\times\sqrt{3(\cos60°)^2+1}$$

$$= 238.1\ \text{N/C}$$

Force on dipole

Suppose an electric dipole of dipole moment $|\mathbf{p}| = 2aq$ is placed in a uniform electric field $\mathbf{E}$ at an angle θ. Here, θ is the angle between $\mathbf{p}$ and $\mathbf{E}$. A force $\mathbf{F}_1 = q\mathbf{E}$ will act on positive charge and $\mathbf{F}_2 = -q\mathbf{E}$ on negative charge. Since, $\mathbf{F}_1$ and $\mathbf{F}_2$ are equal in magnitude but opposite in direction.

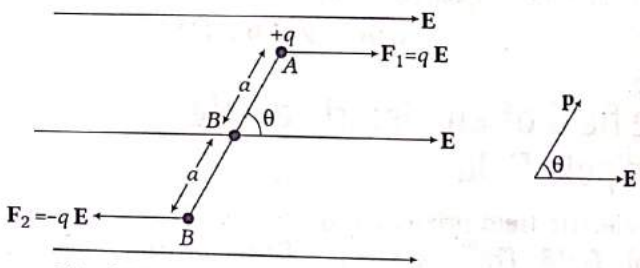


Fig. 1.24 Electric dipole in a uniform electric field

Hence, $\quad \mathbf{F}_1 + \mathbf{F}_2 = 0$

or $\quad \mathbf{F}_{net} = 0$

Thus, net force on a dipole in uniform electric field is zero. While in a non-uniform electric field, it may or may not be zero.

Torque on an electric dipole

The two equal and opposite forces shown in the diagram above act at different points of the dipole. They form a couple which exerts a **torque**. This torque has a magnitude equal to the magnitude of each force multiplied by the arm of the couple *i.e.*, perpendicular distance between the two anti-parallel forces.

Magnitude of torque $= qE\times2a\sin\theta$

$$= 2qaE\sin\theta$$

$$\boxed{\tau = pE\sin\theta} \qquad [\because p = (2a)q]$$

or $\quad \boxed{\tau = \mathbf{p}\times\mathbf{E}}$

Thus, the magnitude of torque is $\tau = pE\sin\theta$. The direction of torque is perpendicular to the plane of paper inwards. Further this torque is zero at $\theta = 0°$ or $\theta = 180°$, *i.e.*, when the dipole is parallel or anti-parallel to $\mathbf{E}$ and maximum at $\theta = 90°$.

Thus, variation of τ with θ is as shown in graph below.

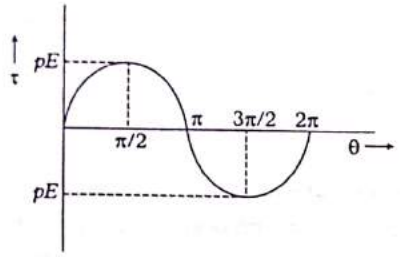


Fig. 1.25 Variation of τ with θ

Example 1.37 *An electric dipole with dipole moment 4×10^{-9} C-m is aligned at 30° with the direction of a uniform electric field of magnitude 5×10^4 NC^{-1}. Calculate the magnitude of the torque acting on the dipole.*

Sol. Using the formula, $\tau = pE \sin\theta$

Here, dipole moment, $p = 4 \times 10^{-9}$ C-m

and $E = 5 \times 10^4$ NC^{-1}

Angle between E and p, $\theta = 30°$, substituting the values, we get

$$\tau = 4 \times 10^{-9} \times 5 \times 10^4 \times \sin 30°$$

$$= 20 \times 10^{-5} \times \frac{1}{2}$$

$$= 10^{-4} \text{ N-m}$$

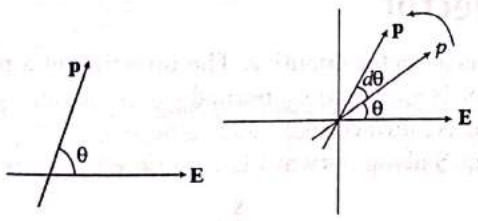
Work done in rotating a dipole in a uniform electric field

When an electric dipole is placed in a uniform electric field **E**, [Fig. (a)] a torque, $\tau = \mathbf{p} \times \mathbf{E}$ acts on it. If we rotate the dipole through a small angle $d\theta$ as shown in Fig. (b) the work done by the torque is

$$dW = \tau d\theta$$

$$\Rightarrow \quad dW = -pE \sin\theta \, d\theta$$

The work is negative as the rotation $d\theta$ is opposite to the torque.



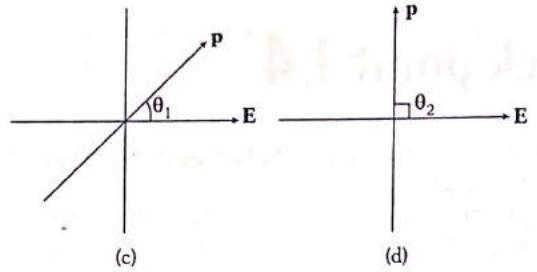


Fig. 1.26 Dipole at different angles with electric field

Total work done by external forces in rotating a dipole from, $\theta = \theta_1$ to $\theta = \theta_2$ [Fig. (c) and (d)] will be given by

$$W = \int_{\theta_1}^{\theta_2} pE \sin\theta d\theta$$

$$W_{\text{external forces}} = pE (\cos\theta_1 - \cos\theta_2)$$

and work done by electric forces,

$$W_{\text{electric force}} = -W_{\text{external force}} = pE (\cos\theta_2 - \cos\theta_1)$$

Taking $\theta_1 = \theta$ and $\theta_2 = 90°$

We have,

$$W_{\text{electric dipole}} = \mathbf{p} \cdot \mathbf{E} (\cos 90° - \cos\theta) = -pE\cos\theta$$

$$= -\mathbf{p} \cdot \mathbf{E}$$

Note If dipole is placed in non-uniform electric field, then magnitude and direction of electric field is different at every point and it will experience both net force and net torque.

Example 1.38 *An electric dipole of dipole moment $p = 5 \times 10^{-18}$ C-m lying along uniform electric field $E = 4 \times 10^4$ NC^{-1}. Calculate the work done is rotating the dipole by 60°.*

Sol. It is given that electric dipole moment, $p = 5 \times 10^{-18}$ C-m

Electric field strength, $E = 4 \times 10^4$ NC^{-1}

When the electric dipole is placed in an electric field **E**, a torque $\tau = \mathbf{p} \times \mathbf{E}$ acts on it. This torque tries to rotate the dipole through an angle θ.

If the dipole is rotated from an angle θ_1 to θ_2, then work done by external force is given by

$$W = pE (\cos\theta_1 - \cos\theta_2) \quad \text{...(i)}$$

Putting $\theta_1 = 0°$, $\theta_2 = 60°$ in the Eq. (i), we get

$$W = pE (\cos 0° - \cos 60°)$$

$$= pE (1 - 1/2) = \frac{pE}{2}$$

$$= \frac{5 \times 10^{-18} \times 4 \times 10^4}{2} = 10^{-13} \text{ J}$$

$$\Rightarrow \quad W = 0.1 \times 10^{-12} \text{ J} = 0.1 \text{ pJ}$$

Check point 1.4

1. The electric dipole moment of an electron and a proton 4.3 nm apart, is
(a) 6.8×10^{-28} C-m (b) 2.56×10^{-29} C^2/m
(c) 3.72×10^{-14} C/m (d) 11×10^{-46} C^2/m

2. If E_a be the electric field strength of a short dipole at a point on its axial line and E_e that on the equatorial line at the same distance, then
(a) $E_e = 2E_a$ (b) $E_a = 2E_e$ (c) $E_a = E_e$ (d) None of these

3. Electric field at a far away distance r on the axis of a dipole is E_0. What is the electric field at a distance $2r$ on perpendicular to bisector?
(a) $\frac{E_0}{16}$ (b) $-\frac{E_0}{16}$ (c) $\frac{E_0}{8}$ (d) $-\frac{E_0}{8}$

4. The electric field due to an electric dipole at a distance r from its centre in axial position is E. If the dipole is rotated through an angle of 90° about its perpendicular axis, then the magnitude of electric field at the same point will be
(a) E (b) $E/4$ (c) $E/2$ (d) $2E$

5. The torque acting on a dipole of moment **p** in an electric field **E** is
(a) $\mathbf{p} \cdot \mathbf{E}$ (b) $\mathbf{p} \times \mathbf{E}$ (d) zero (d) $\mathbf{E} \times \mathbf{p}$

6. When an electric dipole **p** is placed in a uniform electric field **E**, then at what angle between **p** and **E** the value of torque will be maximum?
(a) 90° (b) 0° (c) 180° (d) 45°

7. An electric dipole is kept in non-uniform electric field. It experiences
(a) a force and a torque (b) a force but not a torque
(c) a torque but not a force (d) Neither a force nor a torque

8. An electric dipole of moment **p** is placed normal to the lines of force of electric intensity **E**, then the work done in deflecting it through an angle of 180° is
(a) pE (b) $+2pE$
(c) $-2pE$ (d) zero

9. A molecule with a dipole moment p is placed in an electric field of strength E. Initially, the dipole is aligned parallel to the field. If the dipole is to be rotated to be anti-parallel to the field, then the work required to be done by an external agency is
(a) $-2pE$ (b) $-pE$
(c) pE (d) $2pE$

10. Two opposite and equal charges 4×10^{-8}C when placed 2×10^{-2} cm away from a dipole. If this dipole is placed in an external electric field 4×10^{8} NC^{-1}, then the value of maximum torque and the work done in rotating it through 180° will be
(a) 64×10^{-4} N-m and 64×10^{-4} J
(b) 32×10^{-4} N-m and 32×10^{-4} J
(c) 64×10^{-4} N-m and 32×10^{-4} J
(d) 32×10^{-4} N-m and 64×10^{-4} J

5 Electric flux

Electric flux over an area in an electric field is a measure of the number of field lines crossing a surface. It is denoted by ϕ_E. Let **E** be electric field at the location of the surface element $d\mathbf{S}$. The electric flux through the entire surface is given by

$$\phi_E = \int_S \mathbf{E} \cdot d\mathbf{S} \Rightarrow \phi_E = \int_S E\, dS \cos\theta = E \cos\theta \int_S dS$$

Here, θ is smaller angle between **E** and d **S**.

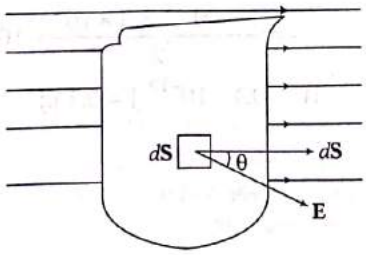


Fig. 1.27 Electric flux over an area dS

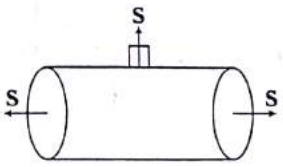
For a closed surface, θ is the angle between **E** and outward normal to the area element.

Electric flux is a scalar quantity having SI unit V-m or $N\text{-}m^2\ C^{-1}$.

Note As an electric flux can also be defined as the flow of the field lines through a surface when field lines leave or flow out of a closed surface, ϕ_E is positive, when they enter or flow into the surface, ϕ_E is negative.

Area vector

The area is a vector quantity. The direction of a planar area vector is specified by normal to the plane, *e.g.*, in case of closed surface like cube, sphere, etc. Direction of area vector **S** along outward is considered to be positive.



(a) Arrowhead gives direction of area

In case of open surface, any normal direction can be considered positive.

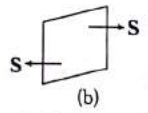


Fig. 1.28 (a) and (b)

The flux of electric field passing through an area is the dot product of electric field vector and area vector

$$\phi = \mathbf{E} \cdot \mathbf{S} = ES\cos\theta$$

Example 1.39 *The electric field in a region is given by* $\mathbf{E} = a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$. *Here, a and b are constants. Find the net flux passing through a square area of side l parallel to yz-plane.*

Sol. A square area of side l parallel to yz-plane in vector form can be written as, $\mathbf{S} = l^2\hat{\mathbf{i}}$

Given, $\mathbf{E} = a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$

$\therefore$ Electric flux passing through the given area will be

$$\phi_E = \mathbf{E} \cdot \mathbf{S} = (a\hat{\mathbf{i}} + b\hat{\mathbf{j}}) \cdot (l^2\hat{\mathbf{i}}) = al^2$$

Example 1.40 *A rectangular surface of sides 10 cm and 15 cm is placed inside a uniform electric field of 25 NC^{-1}, such that normal to the surface makes an angle of 60° with the direction of electric field. Find the flux of electric field through the rectangular surface.*

Sol. Here, E = electric field = 25 NC^{-1}

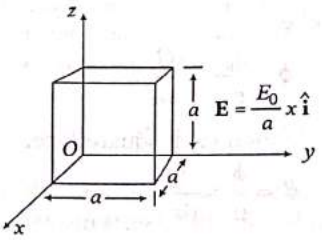
S = surface area of rectangle = $l \times b = 10 \times 15$ cm

Flux, $\phi = ES\cos\theta = (25NC^{-1})(0.15 \times 0.10)\ m^2(\cos 60°)$

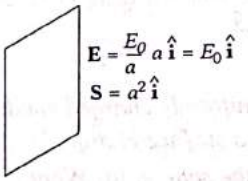
$= 0.1875\ N\text{-}m^2C^{-1}$

Example 1.41 *The electric field in a region is given by* $\mathbf{E} = \frac{E_0}{a}x\hat{\mathbf{i}}$. *Find the electric flux passing through a cubical volume bounded by the surfaces $x = 0$, $x = a$, $y = 0$, $y = a$, $z = 0$ and $z = a$.*

Sol.



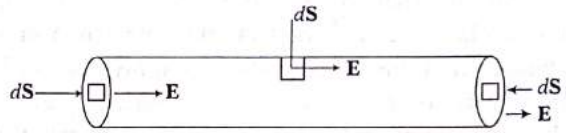
On four faces, electric field and area are perpendicular, hence there will be no flux. One face is at origin, *i.e.*, $x = 0$, $E = 0$ hence, there will be no flux. On the sixth face, $x = a$



$$\phi = \mathbf{E} \cdot \mathbf{S} = E_0\hat{\mathbf{i}} \cdot a^2\hat{\mathbf{i}} = E_0 a^2$$

Example 1.42 *A cylinder is placed in a uniform electric field* $\mathbf{E}$ *with axis parallel to the field. Find the total electric flux through the cylinder.*

Sol.



Flux through the entire cylinder,

$$\phi_E = \underbrace{\int \mathbf{E} \cdot d\mathbf{S}}_{\text{Right plane face}} + \underbrace{\int \mathbf{E} \cdot d\mathbf{S}}_{\text{Left plane face}} + \underbrace{\int \mathbf{E} \cdot d\mathbf{S}}_{\text{curved surface}}$$

$$\Rightarrow \phi_E = \int \mathbf{E} \cdot d\mathbf{S}\cos 180° + \int \mathbf{E} \cdot d\mathbf{S}\cos 0° + \int \mathbf{E} \cdot d\mathbf{S}\cos 90°$$

$$= -\mathbf{E}\int d\mathbf{S} + \mathbf{E}\int d\mathbf{S} + 0$$

$$= -\mathbf{E} \times \pi r^2 + \mathbf{E} \times \pi r^2 = 0$$

Gauss's law

According to Gauss's law, "*the net electric flux through any closed surface is equal to the net charge enclosed by it divided by* ε_0". Mathematically, it can be written as

$$\phi_E = \oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$$

Gauss's theorem in simplified form can be written as under

$$ES = \frac{q_{in}}{\varepsilon_0} \quad \text{...(i)}$$

but this form of Gauss's law is applicable only under the following two conditions

(i) the electric field at every point on the surface is either perpendicular or tangential.

(ii) magnitude of electric field at every point where it is perpendicular to the surface has a constant value (say E).

Here, S is the area, where electric field is perpendicular to the surface.

Important points regarding Gauss's law in electrostatics

Important points regarding Gauss's law in electrostatics are given below

(i) Gauss's law is true for any closed surface, no matter what its shape or size.

(ii) The term q on the right side of Gauss's law includes the sum of all charges enclosed by surface. The charges may be located anywhere inside the surface.

(iii) In the situation, when the surface is so taken that there are some charges inside and some outside, the electric field (whose flux appears on the left side of

Gauss's law) is due to all the charges, both inside and outside S. However, the term q on the right side of Gauss's law represents only the total charge inside S.

(iv) The surface that we choose for the application of Gauss's law is called the Gaussian surface. You may choose any Gaussian surface and apply Gauss's law. However, do not let the Gaussian surface pass through any discrete charge, because electric field is not well defined at the location of discrete charge. However, the Gaussian surface can pass through a continuous charge distribution.

(v) Gauss's law is often useful towards a much easier calculation of the electrostatic field, when the system has some symmetry. This is achieved by the choice of a suitable Gaussian surface.

(vi) Finally, Gauss's law is based on the inverse square dependence on distance as taken in the Coulomb's law. Any violation of Gauss's law will indicate departure from the inverse square law.

Special cases

Some points related to calculation of electric flux in different cases

(i) If surface contains number of charges, as shown in figure, then q_{in} can be calculated as

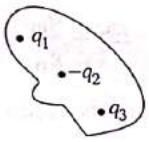


Fig. 1.29 Charges enclosed by a surface

$$q_{in} = q_1 - q_2 + q_3$$

$$\therefore \quad \phi = \frac{q_{in}}{\varepsilon_0} = \frac{q_1 - q_2 + q_3}{\varepsilon_0}$$

(ii) If a charge q is placed at the centre of a cube, then the flux passing through cube, $\phi = \frac{q_{in}}{\varepsilon_0} = \frac{q}{\varepsilon_0}$

The flux passing through each face of cube,

$$\phi' = \frac{\phi}{6} = \frac{q}{6\varepsilon_0} \qquad \text{(by symmetry)}$$

(iii) To calculate flux passing through open surface, first make surface close in such a manner that point charge comes at the centre and then apply symmetry concept, *e.g.*,

(a) A charge q is placed at the centre of an imaginary hemispherical surface.

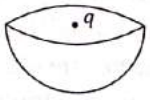


Fig. 1.30 Charge at the centre of a hemisphere

First, make the surface close by placing another hemisphere.

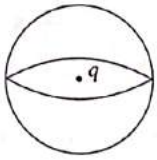


Fig. 1.31 Assume another symmetrical hemisphere to form a sphere

Flux through sphere, $\phi = \frac{q_{in}}{\varepsilon_0} = \frac{q}{\varepsilon_0}$

By symmetry, flux through hemisphere,

$$\phi' = \frac{\phi}{2} = \frac{q}{2\varepsilon_0}$$

(b) A charge Q is placed at a distance $a/2$ above the centre of a horizontal, square of edge a.

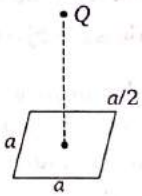


Fig. 1.32 Charge at a distance a/2 from square

First, make the surface close by placing five square faces ($a \times a$), so that a cube is formed and charge Q is at centre of a cube.

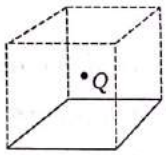


Fig. 1.33 a cube

Flux through the cube,

$$\phi = \frac{q_{in}}{\varepsilon_0} = \frac{Q}{\varepsilon_0}$$

By symmetry, flux through each square face,

$$\phi' = \frac{\phi}{6} = \frac{Q}{6\varepsilon_0}$$

Note In case of closed symmetrical body with charge q at its centre, the electric flux linked with each half will be $\frac{\phi_E}{2} = \frac{q}{2\varepsilon_0}$. If the symmetrical closed body has n identical faces with point charge at its centre, flux linked with each face will be $\frac{\phi_E}{n} = \frac{q}{n\varepsilon_0}$.

Example 1.43 *An uniformly charged conducting sphere of 2.4 m diameter has a surface charge density of 80.0 $\mu C/m^2$. (i) Find the charge on the sphere. (ii) What is the total electric flux leaving the surface of the sphere?*

Sol. (i) Using the relation, $\sigma = \frac{q}{4\pi R^2}$, we get

$$q = 4\pi R^2 \times \sigma = 4 \times \frac{22}{7} \times (12)^2 \times 80 \times 10^{-6} = 1.45 \times 10^{-3}\,\text{C}$$

(ii) Using Gauss's theorem, we get

$$\phi = \frac{q}{\varepsilon_0} = \frac{1.45 \times 10^{-3}}{8.854 \times 10^{-12}} = 1.64 \times 10^8\ \text{N-m}^2\text{C}^{-1}$$

Example 1.44 *A point charge causes an electric flux of -1.0×10^3 N-m^{2}C^{-1} to pass through a spherical Gaussian surface of 10.0 cm radius centred on the charge. (i) If the radius of the Gaussian surface were doubled, how much flux would pass through the surface? (ii) What is the value of the point charge?*

Sol. (i) According to Gauss's law, the electric flux through a Gaussian surface depends upon the charge enclosed inside the surface and not upon its size. Thus, the electric flux will remain unchanged

i.e., -1.0×10^3 N-m^{2}C^{-1}.

(ii) Using the formula,

$$\phi = \frac{q}{\varepsilon_0} \text{ (Gauss's, theorem), we get}$$

$$\Rightarrow \quad q = \varepsilon_0 \phi = 8.854 \times 10^{-12} \times (-1.0 \times 10^3)$$

$$= -8.854 \times 10^{-9}\,\text{C} = -8.8\,\text{nC}$$

Example 1.45 *A point charge q is placed at the centre of a cube. What is the flux linked.*

(i) with all the faces of the cube

(ii) with each face of the cube

(iii) if charge is not at the centre, then what will be the answers of parts (i) and (ii)?

Sol. (i) According to Gauss's law, $\phi_{\text{total}} = \frac{q_{\text{in}}}{\varepsilon_0} = \frac{q}{\varepsilon_0}$.

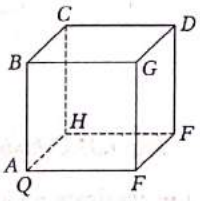
(ii) The cube is a symmetrical body with 6 faces and the point charge is at its centre, so electric flux linked with each face will be

$$\phi_{\text{each face}} = \frac{\phi_{\text{total}}}{6} = \frac{q}{6\varepsilon_0}$$

(iii) If charge is not at the centre, the answer of part (i) will remain same while that of part (ii) will change.

Example 1.46 *A point charge Q is placed at one corner of a cube. Find flux passing through a cube.*

Sol. First, make the surface close by placing three identical cubes at three sides of given cube and four cubes above. Now, charge comes at the centre of 8 cubes. The flux passing through each cube will be 1/8th of the flux Q/ε_0. Hence, flux passing through given cube $= Q/8\varepsilon_0$.



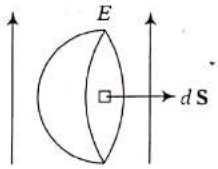
Example 1.47 *A hemispherical body of radius R is placed in a uniform electric field E. What is the flux linked with the curved surface, if the field is*

(i) parallel to the base *(ii) perpendicular to the base*

Sol. We know, flux passing through closed surface,

$$\phi = \oint \mathbf{E} \cdot d\mathbf{S} = \frac{q_{\text{in}}}{\varepsilon_0}$$

(i)



Charge inside hemisphere, $q_{\text{in}} = 0$

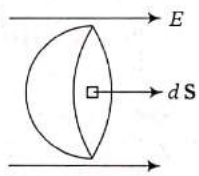
i.e., $\oint \mathbf{E} \cdot d\mathbf{S} = 0$

$$\Rightarrow \quad \phi_{\text{curved}} + \phi_{\text{plane}} = 0$$

$$\Rightarrow \quad \phi_{\text{curved}} + ES\cos 90° = 0$$

$$\Rightarrow \quad \phi_{\text{curved}} = 0$$

(ii)



Also, $\phi_{\text{curved}} + \phi_{\text{plane}} = 0$

$$\Rightarrow \quad \phi_{\text{curved}} + ES\cos 0° = 0$$

$$\Rightarrow \quad \phi_{\text{curved}} + E\pi R^2 = 0$$

$$\Rightarrow \quad \phi_{\text{curved}} = -E\pi R^2$$

Applications of Gauss's law

To calculate electric field by Gauss's theorem, we will draw a Gaussian surface (either sphere or cylinder, according to the situation) in such a way that electric field is perpendicular at each point of surface and its magnitude is same at every point and then apply Gauss's law. Let us start with some simple cases.

1. Electric field due to an infinitely long straight uniformly charged wire

Consider a long line charge with a linear charge density (charge per unit length), λ. To calculate the electric field at a point, located at a distance r from the line charge. We construct a Gaussian surface, a cylinder of any arbitrary length l of radius r and its axis coinciding with the axis of the line charge. This cylinder have three surfaces. One is curved surface and the two plane parallel surfaces.

Field lines at plane parallel surfaces are tangential (so flux passing through these surfaces is zero). The magnitude of

electric field is having the same magnitude (say E) at curved surface and simultaneously the electric field is perpendicular at every point of this surface.

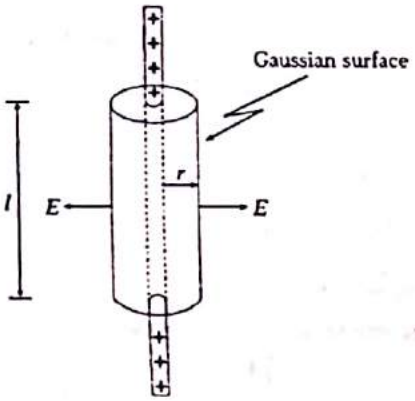


Fig. 1.34 Cylindrical Gaussian surface around a line charge

Hence, we can apply the Gauss's law as, $E\,S = \dfrac{q_{in}}{\varepsilon_0}$

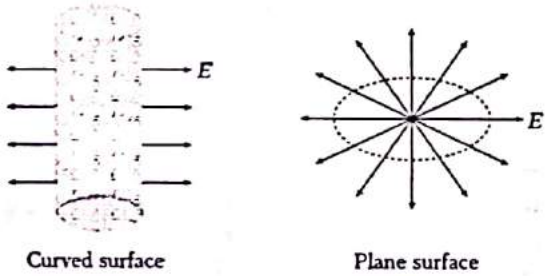


Fig. 1.35 Electric flux through different surfaces

Here, S = area of curved surface = $(2\pi rl)$
and q_{in} = net charge enclosing this cylinder = λl

$\therefore \quad E\,(2\pi rl) = \dfrac{\lambda l}{\varepsilon_0}$

$\therefore \quad \boxed{E = \dfrac{\lambda}{2\pi\varepsilon_0 r}}$

i.e., $\quad E \propto \dfrac{1}{r}$

or E-r graph is a rectangular hyperbola as shown in figure.

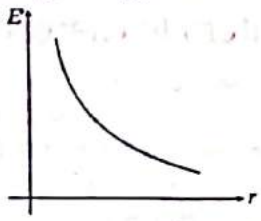


Fig. 1.36 E-r graph for a long charged wire

Example 1.48 *An infinite line charge produces a field of 9×10^4 NC^{-1} at a distance of 2 cm. Calculate the linear charge density.*

Sol. As we know electric field due to an infinite line charge is given by the relation,

$$E = \frac{1}{2\pi\varepsilon_0}\cdot\frac{\lambda}{r}$$

where, λ is linear charge density, r is the distance of a point, where an electric field is produced from the line charge.

or $\quad \lambda = 2\pi\varepsilon_0 rE$

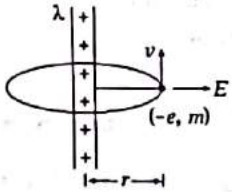
Here, $E = 9\times10^4\ \text{NC}^{-1}$, $r = 2\ \text{cm} = 0.02\ \text{m}$

$\therefore$ Linear charge density,

$$\lambda = \frac{1}{9\times10^9}\times\frac{0.02\times9\times10^4}{2} = 10^{-7}\ \text{Cm}^{-1}$$

Example 1.49 *A long cylindrical wire carries a positive charge of linear density λ. An electron $(-e, m)$ revolves around it in a circular path under the influence of the attractive electrostatic force. Find the speed of the electron.*

Sol. Electric field at perpendicular distance r,



$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

The electric force on electron, $F = eE$

To move in circular path, necessary centripetal force is provided by electric force.

$$F_c = \frac{mv^2}{r} = eE = \frac{e\lambda}{2\pi\varepsilon_0 r}$$

$\therefore$ Speed of the electron, $\quad v = \sqrt{\dfrac{e\lambda}{2\pi\varepsilon_0 m}},\ v \propto r^0$

2. Electric field due to a plane sheet of charge

Consider a flat thin sheet, infinite in size with constant surface charge density σ (charge per unit area).

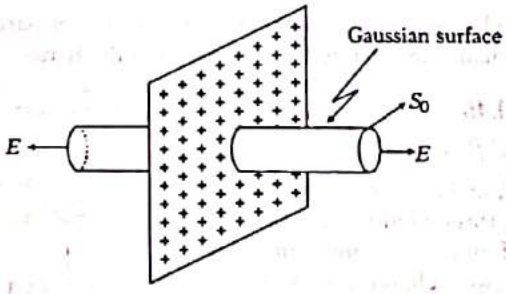


Fig. 1.37 Cylindrical Gaussian surface for a plane charged sheet

Let us draw a Gaussian surface (a cylinder) with one end on one side and other end on the other side and of cross-

sectional area S_0. Field lines will be tangential to the curved surface, so flux passing through this surface is zero. At plane surfaces, electric field has same magnitude and perpendicular to surface. Hence, using

$$ES = \frac{q_{in}}{\varepsilon_0}$$

$$\therefore \quad E(2S_0) = \frac{(\sigma)(S_0)}{\varepsilon_0}$$

$$\therefore \quad \boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

Thus, we see that the **magnitude of the field due to plane sheet of charge is independent of the distance from the sheet.**

Important point

Suppose two plane sheets having charge densities $+\sigma$ and $-\sigma$ are placed at some separation.

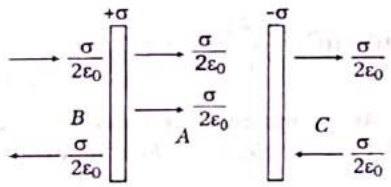


Fig. 1.38 Plane charged sheets placed close to each other

Electric field at A,

$$E_A = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

Electric field at B,

$$E_B = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} \Rightarrow E_B = 0$$

Similarly, $\quad E_C = 0$

If two plane sheets having opposite charges are kept near to each other, then electric field exists only between them.

Example 1.50 *A large plane sheet of charge having surface charge density 5×10^{-6} Cm^{-2} lies in xy-plane. Find the electric flux through a circular area of radius 0.1 m. If the normal to the circular area makes an angle of 60° with the Z-axis.*

Sol. Given, $\sigma = 5 \times 10^{-6}$ C/m^2, $r = 0.1$ m, $\theta = 60°$

$$\phi = ES\cos\theta = \left(\frac{\sigma}{2\varepsilon_0}\right)\pi r^2 \cos\theta$$

$$= \frac{5 \times 10^{-6}}{2 \times 8.85 \times 10^{-12}} \times \frac{22}{7} \times (0.1)^2 \cos 60°$$

$$\phi = 4.44 \times 10^3 \text{ N-m}^2/\text{C}$$

Example 1.51 *Two large, thin metal plates are parallel and close to each other. On their inner faces, the plates have surface charge densities of opposite signs and of magnitude 1.77×10^{-11} coulomb per square metre. What is electric field*

(i) to the left of the plates?

(ii) to the right of the plates?

(iii) in between the plates?

Sol. (i) Fields due to both the plates will be equal and opposite, so net field will be zero.

(ii) Field outside the plates will be equal and opposite, so net field will be zero.

(iii) In between the plates, the fields due to both the plates will be adding up, so net field will be $\frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$ from positive to negative plate,

$$\therefore \quad E = \frac{\sigma}{\varepsilon_0} = \frac{1.77 \times 10^{-11}}{8.85 \times 10^{-12}} = 2 \text{ N/C}$$

3. Electric field near a charged conducting surface

When a charge is given to a conducting plate, it distributes, itself over the entire outer surface of the plate. The surface charge density σ is uniform and is the same on both surfaces, if plate is of uniform thickness and of infinite size. This is similar to the previous one the only difference is that this time charges are on both sides. Hence, applying,

$$ES = \frac{q_{in}}{\varepsilon_0}$$

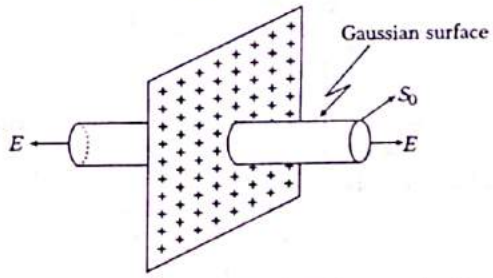


Fig. 1.39 Cylindrical Gaussian surface for a conducting surface

Here, $\quad S = 2S_0$

and $\quad q_{in} = (\sigma)(2S_0)$

$$\therefore \quad E(2S_0) = \frac{(\sigma)(2S_0)}{\varepsilon_0}$$

$$\therefore \quad \boxed{E = \frac{\sigma}{\varepsilon_0}}$$

The electric field near a charged conducting surface of any shape is σ/ε_0 and perpendicular to surface.

4. Electric field due to a uniformly charged thin spherical shell of solid conducting sphere

Let O be the centre and R the radius of a thin, isolated spherical shell carrying a charge $+q$ which uniformly distributed on the surface. We have to determine electric field intensity due to this shell at points outside the shell, on the surface of the shell and inside the shell.

At external point

We can construct a Gaussian surface (a sphere) of radius $r > R$. At all points of this sphere the magnitude of electric field is the same and its direction is perpendicular to the surface. Thus, we can apply Gauss's theorem,

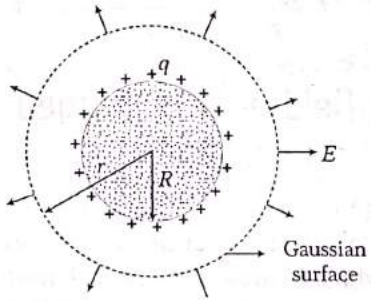


Fig. 1.40 Spherical Gaussian surface around a spherical shell

$$E\,S = \frac{q_{in}}{\varepsilon_0}$$

or
$$E\,(4\pi r^2) = \frac{q}{\varepsilon_0}$$

$\therefore$
$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}}$$

Hence, the electric field at any external point is the same as, if the total charge is concentrated at centre.

At the surface of sphere, $r = R$

$\therefore$
$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R^2}}$$

At an internal point

In this case, the Gaussian surface encloses no charge

i.e., $\phi = E(4\pi r^2) = 0$

$$E_{\text{inside}} = 0$$

The electric field intensity is zero everywhere inside the charged shell.

The variation of electric field (E) with the distance from the centre (r) is as shown in figure.

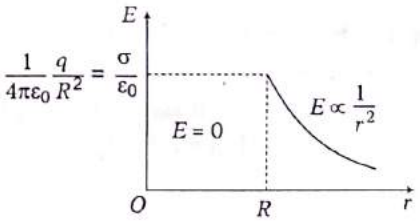


Fig. 1.41 E-r graph for a spherical shell

Note

1. At the surface graph is discontinuous.
2. $E_{\text{surface}} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R^2} = \frac{q/4\pi R^2}{\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$

Example 1.52 *A thin spherical shell of metal has a radius of 0.25 m and carries charge of 0.2 micro coulomb. Calculate the electric intensity at 3.0 m from the centre of the shell.*

Sol. The intensity at an external point at distant r from the centre of the shell is given by

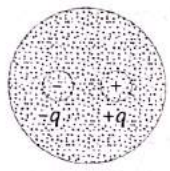
$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$$

Here, $r = 3.0$ m

$$\therefore \quad E = (9.0\times10^9)\times\frac{2\times10^{-7}}{(3.0)^2} = 200\text{ N/C}$$

Example 1.53 *An electric dipole is placed at the centre of a spherical shell. Find the electric field at an external point of the shell.*

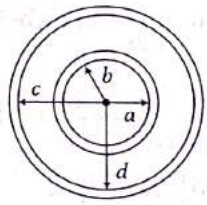
Sol. Net charge inside the sphere, $q_{in} = 0$. Therefore, according to Gauss's law net flux passing through the sphere is zero. Therefore, electric field at an external point will be zero.



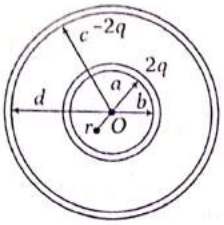
Example 1.54 *A small conducting spherical shell with inner radius a and outer radius b is concentric with a larger conducting spherical shell with inner radius c and outer radius d. The inner shell has total charge +2q and the outer shell has charge + 4q.*

(i) *What is the total charge on the (a) inner surface of the small shell, (b) outer surface of the small shell (c) inner surface of the large shell, (d) outer surface of the large shell?*

(ii) *Calculate the electric field in terms of q and the distance r from the common centre of two shells for (a) $r < a$, (b) $a < r < b$, (c) $b < r < c$, (d) $c < r < d$, (e) $r > d$*



Sol. Charge distribution,



(i) Total charge on inner shell $= 2q$
Total charge on outer shell $= 4q$
Charge distribution
(a) Charge on inner surface on inner shell $= 0$
(b) Charge on outer surface of inner shell $= 2q$
(c) Charge on inner surface of outer shell $= -2q$
(facing surface have equal and opposite charges)
(d) Charge on outer surface of outer shell $= 6q$
(total charge on outer shell is $4q$)

(ii) To calculate electric field, draw a sphere with centre O through that point, where electric field is required. Assume charge to be concentrated at centre and apply formula of point charge.
(a) $r < a$, enclosed charge $= 0$, $E = 0$
(b) $a < r < b$, enclosed charge $= 0$, $E = 0$ or electric field inside conductor $= 0$
(c) $b < r < c$, enclosed charge $= 2q$, $E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2q}{r^2}$
(d) $c < r < d$, enclosed charge $= 0$, $E = 0$, or electric field inside conductor $= 0$
(e) $r > d$, enclosed charge $= 6q$, $E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{6q}{r^2}$

5. Electric field due to a solid sphere of charge

Suppose positive charge q is uniformly distributed throughout the volume of a non-conducting solid sphere of radius R.

At an internal point

For finding the electric field at a distance $r\,(< R)$ from the centre let us choose as our Gaussian surface a sphere of radius r, concentric with the charge distribution. From symmetry, the magnitude E of electric field has the same value at every point on the Gaussian surface, and the direction of **E** is radial at every point on the surface. So, applying Gauss's law, we have

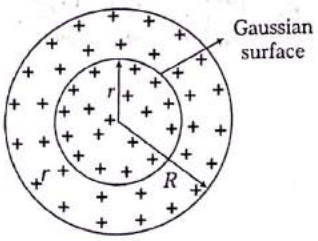


Fig. 1.42 Spherical Gaussian surface inside a solid sphere

$$ES = \frac{q_{in}}{\varepsilon_0} \quad \text{...(i)}$$

Here, $\quad S = 4\pi r^2$ and $q_{in} = (\rho)\left(\frac{4}{3}\pi r^3\right)$

Here, ρ = charge per unit volume $= \frac{q}{(4/3)\,\pi R^3}$

Substituting these values in Eq. (i), we have

$$\boxed{E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R^3} \cdot r} \quad \text{or} \quad E \propto r$$

At the centre, $\quad r = 0$, so $\quad \boldsymbol{E = 0}$

At surface of the sphere, $r = R$, so $\quad \boldsymbol{E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R^2}}$

At an external point

To find the electric field outside the charged sphere, we use a spherical Gaussian surface of radius $r\,(> R)$. This surface encloses the entire charged sphere, so $q_{in} = q$, and Gauss's law gives,

$$E\,(4\pi r^2) = \frac{q}{\varepsilon_0} \quad \text{or} \quad E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \quad \text{or} \quad E \propto \frac{1}{r^2}$$

Thus, for a uniformly charged solid sphere, we have the following formulae for magnitude of electric field.

$$E_{inside} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R^3} \cdot r$$

$$\Rightarrow \quad E_{surface} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R^2}$$

and $$E_{outside} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$

Thus, the electric field at any external point is the same as, if the total charge is concentrated at centre.

The variation of electric field (E) with the distance from the centre of the sphere (r) is shown in figure.

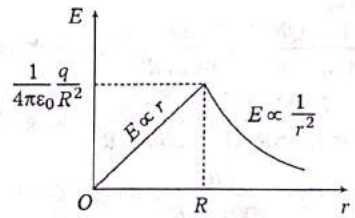


Fig. 1.43 E-r graph for a solid sphere

Note If we set $r = R$ in either of the two expressions for E (outside and inside the sphere), we get the same result $E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R^2}$

this is because E is continuous function of r in this case. By contrast, for the charged conducting sphere the magnitude of electric field is discontinuous at $r = R$ (it jumps from $E = 0$ to $E = \sigma/\varepsilon_0$).

Example 1.55 *At a point 20 cm from the centre of a uniformly charged dielectric sphere of radius 10 cm, the electric field 100 V/m. Find the electric field at 3 cm from the centre of the sphere.*

Sol. Electric field outside of the sphere, $E_{out} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

Electric field inside the dielectric sphere, $E_{in} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R^3}x$

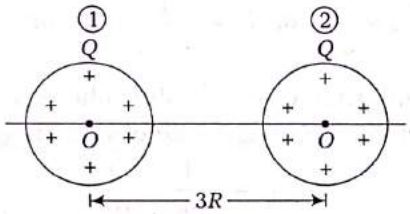
$$\therefore \quad E_{in} = E_{out} \times \frac{r^2 x}{R^3}$$

$$\Rightarrow \quad E = 100 \times \frac{3 \times (20)^2}{10^3} = 120 \text{ V/m}$$

Example 1.56 *Two non-conducting spheres of radius R have charge Q uniformly distributed. The centres of spheres are $x = 0$ and $x = 3R$. Find the magnitude and direction of the net electric field on the X-axis,*

(i) $x = 0$ (ii) $x = \frac{R}{2}$ (iii) $x = \frac{3R}{2}$ (iv) $x = 4R$

Sol. At O, origin ($x = 0$)



Electric field inside sphere at distance r from centre

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qr}{R^3}$$

On the surface or outside, whole charge is assumed to be concentrated at centre.

(i) At $x = 0$, $E_1 = 0$, $E_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{(3R)^2}$, towards left

$$E_{net} = E_2 = \frac{Q}{36\pi\varepsilon_0 R^2}, \text{ along } -X\text{-axis}$$

(ii) At $x = R/2$,

$$E_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{QR/2}{R^3} = \frac{Q}{8\pi\varepsilon_0 R^2}, \text{ along } +X\text{-axis}$$

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{(5R/2)^2} = \frac{Q}{25\pi\varepsilon_0 R^2}, \text{ along } -X\text{-axis}$$

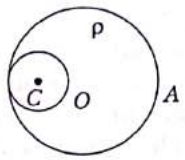
$$E_{net} = E_1 - E_2 = \frac{17Q}{200\pi\varepsilon_0 R^2}, \text{ along } +X\text{-axis}$$

(iii) At $x = 3R/2$, $E_{net} = 0$

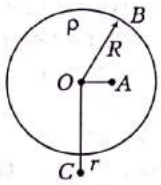
(iv) At $x = 4R$, $E_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{(4R)^2}$, $E_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}$

$$E_{net} = E_1 + E_2 = \frac{17Q}{64\pi\varepsilon_0 R^2}, \text{ along } +X\text{-axis}$$

Example 1.57 *A non-conducting sphere of radius R has a spherical cavity of radius R/2 as shown. The solid part of the sphere has a uniform volume charge density ρ. Find the magnitude and direction of electric field at point (a) O and (b) A*

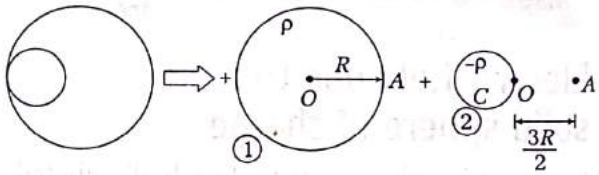


Sol. For a non-conducting sphere of radius R having volume charge density ρ.



(i) $r < R$ (inside), $E = \frac{\rho r}{3\varepsilon_0}$ (ii) $r > R$ (outside), $E = \frac{\rho R^3}{3\varepsilon_0 r^2}$

(iii) $r = R$ (surface), $E = \frac{\rho R}{3\varepsilon_0}$



(a) At O, $E_1 = 0$, $E_2 = \frac{\rho R/2}{3\varepsilon_0} = \frac{\rho R}{6\varepsilon_0}$, towards left

$$\Rightarrow \quad E_0 = \frac{\rho R}{6\varepsilon_0}, \text{ towards left}$$

(b) At A, $E_1 = \frac{\rho R}{3\varepsilon_0}$, towards right

$$E_2 = \frac{\rho (R/2)^3}{3\varepsilon_0 (3R/2)^2} = \frac{\rho R}{54\varepsilon_0}, \text{ towards left}$$

$$\therefore E_A = E_1 - E_2 = \frac{\rho R}{\varepsilon_0}\left[\frac{1}{3} - \frac{1}{54}\right] = \frac{17\rho R}{54\varepsilon_0}, \text{ towards right}$$

Check point 1.5

1. The SI unit of electric flux is
(a) weber
(b) newton coulomb^{-1}
(c) volt × metre
(d) joule coulomb^{-1}

2. A surface $S = 10\,\hat{\mathbf{j}}$ is kept in an electric field $\mathbf{E} = 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 7\hat{\mathbf{k}}$. How much electric flux will come out through this surface?
(a) 40 unit
(b) 50 unit
(c) 30 unit
(d) 20 unit

3. A cube of side a is placed in a uniform electric field $\mathbf{E} = E_0\hat{\mathbf{i}} + E_0\hat{\mathbf{j}} + E_0\hat{\mathbf{k}}$. Total electric flux passing through the cube would be
(a) $E_0 a^2$
(b) $2E_0 a^2$
(c) $6E_0 a^2$
(d) None of these

4. Flux coming out from a positive unit charge placed in air, is
(a) ε_0
(b) ε_0^{-1}
(c) $(4\pi\varepsilon_0)^{-1}$
(d) $4\pi\varepsilon_0$

5. If the electric flux entering and leaving an enclosed surface respectively, is ϕ_1 and ϕ_2, then the electric charge inside the surface will be
(a) $(\phi_1 + \phi_2)\varepsilon_0$
(b) $(\phi_2 - \phi_1)\varepsilon_0$
(c) $(\phi_1 + \phi_2)/\varepsilon_0$
(d) $(\phi_2 - \phi_1)/\varepsilon_0$

6. Charge of 2 C is placed at the centre of a cube. What is the electric flux passing through one face?
(a) $\frac{1}{(3\varepsilon_0)}$
(b) $\left(\frac{1}{4}\right)\varepsilon_0$
(c) $\frac{2}{\varepsilon_0}$
(d) $\frac{3}{\varepsilon_0}$

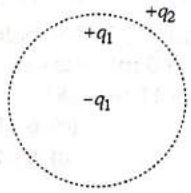
7. For a given surface, the Gauss's law is stated as $\int \mathbf{E}\cdot d\mathbf{S} = 0$. From this, we can conclude that
(a) E is necessarily zero on the surface
(b) E is perpendicular to the surface at every point
(c) The total flux through the surface is zero
(d) The flux is only going out of the surface

8. The total electric flux emanating from a closed surface enclosing an α-particle (e = electronic charge) is
(a) $\frac{2e}{\varepsilon_0}$
(b) $\frac{e}{\varepsilon_0}$
(c) $e\,\varepsilon_0$
(d) $\frac{\varepsilon_0 e}{4}$

9. The inward and outward electric flux for a closed surface in units of N-m^{2}C^{-1} are respectively, 8×10^3 and 4×10^3. Then, the total charge inside the surface is [where, ε_0 = permittivity constant]
(a) 4×10^3 C
(b) -4×10^3 C
(c) $\frac{(-4\times10^3)}{\varepsilon}$ C
(d) $-4\times10^3\,\varepsilon_0$ C

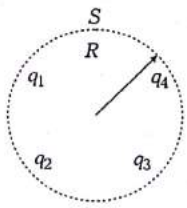
10. If the flux of the electric field through a closed surface is zero, then
(i) the electric field must be zero everywhere on the surface
(ii) the electric field may be zero everywhere on the surface
(iii) the charge inside the surface must be zero
(iv) the charge in the vicinity of the surfaces must be zero
(a) (i), (ii)
(b) (ii), (iii)
(c) (ii), (iv)
(d) (i), (iii)

11. Consider the charge configuration and spherical Gaussian surface as shown in the figure. When calculating the flux of the electric field over the spherical surface the electric field will be due to



(a) q_2
(b) only the positive charges
(c) all the charges
(d) $+q_1$ and $-q_1$

12. q_1, q_2, q_3 and q_4 are point charges located at points as shown in the figure and S is a spherical Gaussian surface of radius R. Which of the following is true according to the Gauss's law?



(a) $\oint_S (\mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3)\cdot d\mathbf{A} = \frac{q_1 + q_2 + q_3}{2\varepsilon_0}$
(b) $\oint_S (\mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3)\cdot d\mathbf{A} = \frac{(q_1 + q_2 + q_3)}{\varepsilon_0}$
(c) $\oint_S (\mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3)\cdot d\mathbf{A} = \frac{(q_1 + q_2 + q_3 + q_4)}{\varepsilon_0}$
(d) None of the above

13. The Gaussian surface for calculating the electric field due to a charge distribution is
(a) any surface near the charge distribution
(b) always a spherical surface
(c) a symmetrical closed surface containing the charge distribution, at every point of which electric field has a single fixed value
(d) None of the given options

14. Gauss's law should be invalid, if
(a) there were magnetic monopoles
(b) the inverse square law were not exactly true
(c) the velocity of light were not a universal constant
(d) None of the above

15. Two parallel infinite line charges $+\lambda$ and $-\lambda$ are placed with a separation distance R in free space. The net electric field exactly mid-way between the two line charges is
(a) zero
(b) $\frac{2\lambda}{\pi \varepsilon_0 R}$
(c) $\frac{\lambda}{\pi \varepsilon_0 R}$
(d) $\frac{1}{2\pi \varepsilon_0 R}$

16. An infinite line charge produces a field of 18×10^9 N/C at 0.02 m. The linear charge density is
(a) 2×10^{-7} C/m
(b) 10^{-8} C/ m
(c) 10^7 C/m
(d) 10^{-4} C/m

17. A charge of 17.7×10^{-4} C is distributed uniformly over a large sheet of area 200 m^2. The electric field intensity at a distance 20 cm from it in air will be
(a) 5×10^5 N/C
(b) 6×10^5 N/C
(c) 7×10^5 N/C
(d) 8×10^5 N/C

18. From what distance should a 100 eV electron be fired towards a large metal plate having a surface charge of -2.0×10^{-6} Cm^{-2}, so that it just fails to strike the plate?
(a) 0.50 mm
(b) 0.44 m
(c) 0.60 mm
(d) 0.77 mm

19. If the electric field near the earth's surface be 300 V/m directed downwards, then the surface, density of charge on earth's surface is
(a) 3.0×10^{-9} C/m^2
(b) 5.0×10^{-9} C/m^2
(c) 2.6×10^{-9} C/m^2
(d) 7.0×10^{-9} C/m^2

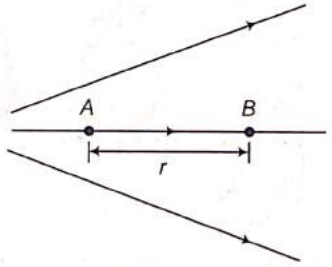
20. A thin spherical shell of metal has a radius of 0.25 m and carries a charge of 0.2 mC. The electric intensity at a point for outside the shell will be
(a) 2.88×10^4 N/C
(b) 3.4×10^4 N/C
(c) 3.25×10^4 N/C
(d) 3.88×10^4 N/C

Chapter exercises

A Taking it together

(Assorted questions of the chapter for advanced level practice)

1. Figure shows the electric lines of force emerging from a charged body. If the electric field at A and B are E_A and E_B respectively and if the distance between A and B is r, then



(a) $E_A > E_B$ (b) $E_A < E_B$ (c) $E_A = \frac{E_B}{r}$ (d) $E_A = \frac{E_B}{r^2}$

2. The unit of physical quantity obtained by the line integral of electric field is
(a) NC^{-1} (b) Vm^{-1}
(c) JC^{-1} (d) $C^2N^{-1}m^{-2}$

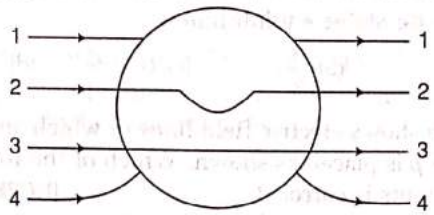
3. Two point charges +2 C and + 6 C repel each other with a force of 12 N. If a charge of – 4 C is given to each of these charges, the force now is
(a) 4 N (repulsive) (b) 4 N (attractive)
(c) 12 N (attractive) (d) 8 N (repulsive)

4. Three equal charges are placed on the three corners of a square. If the force between q_1 and q_2 is F_{12} and that between q_1 and q_3 is F_{13}, then the ratio of magnitudes (F_{12}/F_{13}) is
(a) 1/2 (b) 2
(c) $1/\sqrt{2}$ (d) $\sqrt{2}$

5. The insulation property of air breaks down at $E = 3 \times 10^6$ V/m. The maximum charge that can be given to a sphere of diameter 5 m is approximately (in coulombs)
(a) 2×10^{-2} (b) 2×10^{-3}
(c) 2×10^{-4} (d) 2×10^{-5}

6. The electric field near a conducting surface having a uniform surface charge density σ is given by
(a) $\frac{\sigma}{\varepsilon_0}$ and is parallel to the surface
(b) $\frac{2\sigma}{\varepsilon_0}$ and is parallel to the surface
(c) $\frac{\sigma}{\varepsilon_0}$ and is normal to the surface
(d) $\frac{2\sigma}{\varepsilon_0}$ and is normal to the surface

7. A metallic solid sphere is placed in a uniform electric field. The lines of force follow the path(s) shown in figure as



(a) 1 (b) 2
(c) 3 (d) 4

8. A conductor has been given a charge -3×10^{-7} C by transferring electron. Mass increase (in kg) of the conductor and the number of electrons added to the conductor are respectively
(a) 2×10^{-16} and 2×10^{31}
(b) 5×10^{-31} and 5×10^{19}
(c) 3×10^{-19} and 9×10^{16}
(d) 2×10^{-18} and 2×10^{12}

9. Two point charges of 20 μC and 80μC are 10 cm apart where will the electric field strength be zero on the line joining the charges from 20 μC charge
(a) 0.1 m (b) 0.04 m
(c) 0.033 m (d) 0.33 m

10. For a dipole $q = 2 \times 10^{-6}$ C and $d = 0.01$m. Calculate the maximum torque for this dipole, if $E = 5 \times 10^5 \text{NC}^{-1}$

(a) 1×10^{-3} N-m^{-1} (b) 10×10^{-3} N-m^{-1}
(c) 10×10^{-3} N-m (d) 1×10^2 N-m^2

11. What is the magnitude of a point charge due to which the electric field 30 cm away has the magnitude 2 N/C $[1/4\pi\varepsilon_0 = 9 \times 10^9 \text{N-m}^2/\text{C}^2]$

(a) 2×10^{-11} C (b) 3×10^{-11} C (c) 5×10^{-11} C (d) 9×10^{-11} C

12. A dipole of electric dipole moment p is placed in a uniform electric field of strength E. If θ is the angle between positive directions of p and E, then the potential energy of the electric dipole is largest when θ is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) π (d) zero
(e) $\frac{2}{3}\pi$

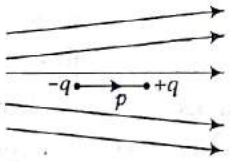
13. The ratio of electrostatic and gravitational forces acting between electron and proton separated by a distance 5×10^{-11}m, will be (charge on electron $= 1.6 \times 10^{-19}$C, mass of electron $= 9.1 \times 10^{-31}$ kg, mass of proton $= 1.6 \times 10^{-27}$ kg, $G = 6.7 \times 10^{-11}$ N-m^2/kg^2)

(a) 2.36×10^{39} (b) 2.36×10^{40}
(c) 2.34×10^{41} (d) 2.34×10^{42}

14. An electric dipole is placed in an uniform electric field with the dipole axis making an angle θ with the direction of the electric field. The orientation of the dipole for stable equilibrium is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) 0 (d) $\frac{\pi}{2}$

15. Figure shows electric field lines in which an electric dipole p is placed as shown. Which of the following statements is correct? **[NCERT Exemplar]**



(a) The dipole will not experience any force
(b) The dipole will experience a force towards right
(c) The dipole will experience a force towards left
(d) The dipole will experience a force upwards

16. Two similar small spheres having $+q$ and $-q$ charge are kept at a certain distance. F force acts between the two. If in the middle of two spheres, another similar small sphere having $+q$ charge is kept, then it will experience a force in magnitude and direction as

(a) zero having no direction
(b) $8F$ towards $+q$ charge
(c) $8F$ towards $-q$ charge
(d) $4F$ towards $+q$ charge

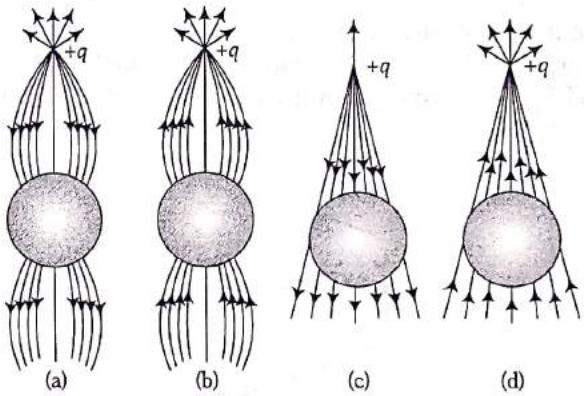
17. Two small conducting spheres of equal radius have charges $+10\ \mu$C and $-20\ \mu$C respectively and placed at a distance R from each other. They experience force F_1. If they are brought in contact and separated to the same distance, they experience force F_2. The ratio of F_1 to F_2 is

(a) 1 : 8 (b) −8 : 1 (c) 1 : 2 (d) −2 : 1

18. A charge Q is placed at each of the two opposite corners of a square. A charge q is placed at each of the other two corners. If the resultant force on Q is zero, then

(a) $Q = \sqrt{2}q$ (b) $Q = -\sqrt{2}q$
(c) $Q = 2\sqrt{2}q$ (d) $Q = -2\sqrt{2}q$

19. A positive point charge is brought near an isolated conducting sphere as shown in figure. The electric field is best given by **[NCERT Exemplar]**



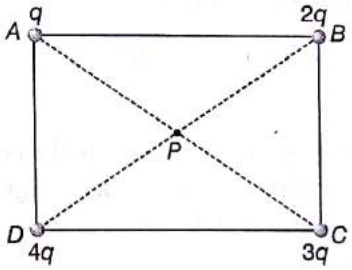
20. A charge q is lying at mid-point of the line joining the two similar charges Q. The system will be in equilibrium, if the value of q is

(a) $Q/2$ (b) $-Q/2$ (c) $Q/4$ (d) $-Q/4$

21. The centres of two identical small conducting spheres are 1 m apart. They carry charges of opposite kind and attract each other with a force F. When they are connected by a conducting thin wire they repel each other with a force $F/3$. What is the ratio of magnitude of charges carried by the spheres initially?

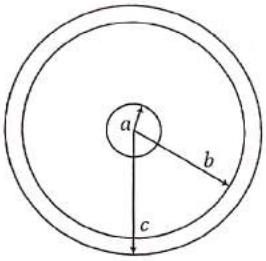
(a) 1 : 1 (b) 2 : 1
(c) 3 : 1 (d) 4 : 1

22. q, $2q$, $3q$ and $4q$ charges are placed at the four corners A, B, C and D of a square. The field at the centre P of the square has the direction along



(a) AB (b) CB
(c) AC (d) BD

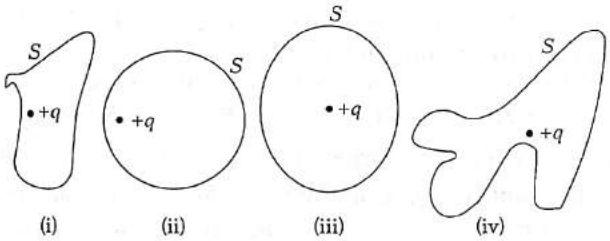
23. A solid conducting sphere of radius a has a net positive charge $2Q$. A conducting spherical shell of inner radius b and outer radius c is concentric with the solid sphere and has a net charge $-Q$. The surface charge density on the inner and outer surfaces of the spherical shell will be



(a) $-\frac{2Q}{4\pi b^2}, \frac{Q}{4\pi c^2}$ (b) $-\frac{Q}{4\pi b^2}, \frac{Q}{4\pi c^2}$

(c) $0, \frac{Q}{4\pi c^2}$ (d) None of these

24. The electric flux through the surface **[NCERT Exemplar]**



(a) in Fig. (iv) is the largest
(b) in Fig. (iii) is the least
(c) in Fig. (ii) is same as Fig. (iii) but is smaller than Fig. (iv)
(d) is the same for all the figures

25. Two point charges q and $2q$ are placed some distance apart. If the electric field at the location of q be E, then that at the location of $2q$ will be

(a) $3E$ (b) $E/2$
(c) E (d) None of these

26. A ball with charge $-50e$ is placed at the centre of a hollow spherical shell has a net charge of $-50e$. What is the charge on the shell's outer surface?

(a) $-50e$ (b) Zero (c) $-100e$ (d) $+100e$

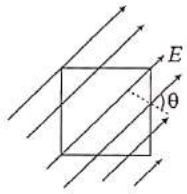
27. A mass $m = 20$ g has a charge $q = 3.0$ mC. It moves with a velocity of 20 ms^{-1} and enters a region of electric field of 80 NC^{-1} in the same direction as the velocity of the mass. The velocity of the mass after 3 s in this region is

(a) 80 ms^{-1} (b) 56 ms^{-1} (c) 44 ms^{-1} (d) 40 ms^{-1}

28. Two parallel metal plates having charges $+Q$ and $-Q$ face each other at a certain distance between them. If the plate are now dipped in kerosene oil tank, the electric field between the plates will

(a) became zero (b) increase
(c) decrease (d) remain same

29. A square surface of side L metre in the plane of the paper is placed in a uniform electric field E (volt/m) acting along the same place at an angle θ with the horizontal side of the square as shown in figure. The electric flux linked to the surface in unit of V-m, is

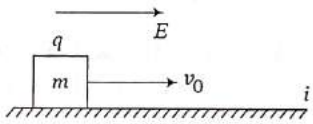


(a) EL^2 (b) $EL^2 \cos\theta$
(c) $EL^2 \sin\theta$ (d) zero

30. Three concentric metallic spherical shells of radii R, $2R$, $3R$ are given charges Q_1 Q_2 Q_3, respectively. It is found that the surface charge densities on the outer surfaces of the shells are equal. Then, the ratio of the charges given to the shells $Q_1 : Q_2 : Q_3$ is

(a) 1 : 2 : 3 (b) 1 : 3 : 5
(c) 1 : 4 : 9 (d) 1 : 8 : 18

31. A charged block is projected on a rough horizontal surface with speed v_0. The value of coefficient of friction if the kinetic energy of the block remains constant is



(a) $\frac{qE}{mg}$ (b) $\frac{qE}{m}$
(c) qE (d) None of these

32. Electric charges $q, q, -2q$ are placed at the corners of an equilateral triangle ABC of side l. The magnitude of electric dipole moment of the system is

(a) ql (b) $2ql$
(c) $\sqrt{3}ql$ (d) $4ql$

33. A point charge $+q$ is placed at a distance d from an isolated conducting plane. The field at a point P on the other side of the plane is **[NCERT Exemplar]**

(a) directed perpendicular to the plane and away from the plane
(b) directed perpendicular to the plane but towards the plane
(c) directed radially away from the point charge
(d) directed radially towards the point charge

34. Under the influence of the Coulomb field of charge $+Q$, a charge $-q$ is moving around it in an elliptical orbit. Find out the correct statement(s).

(a) The angular momentum of the charge $-q$ is constant
(b) The linear momentum of the charge $-q$ is constant
(c) The angular velocity of the charge $-q$ is constant
(d) The linear speed of the charge $-q$ is constant

35. Two identical conducting spheres carrying different charges attract each other with a force F when placed in air medium at a distance d apart. The spheres are brought into contact and then taken to their original positions. Now, the two spheres repel each other with a force whole magnitude is equal to that of the initial attractive force. The ratio between initial charges on the spheres is

(a) $-(3+\sqrt{8})$ only

(b) $-3+\sqrt{8}$ only

(c) $-(3+\sqrt{8})$ or $(-3+\sqrt{8})$

(d) $+\sqrt{3}$

(e) $-\sqrt{8}$

36. The electric field at a distance $\frac{3R}{2}$ from the centre of a charged conducting spherical shell of radius R is E. The electric field at a distance $\frac{R}{2}$ from the centre of the sphere is

(a) zero (b) E

(c) $E/2$ (d) $E/3$

37. Under the action of a given coulombic force the acceleration of an electron is 2.5×10^{22} ms^{-2}. Then, the magnitude of the acceleration of a proton under the action of same force is nearly

(a) 1.6×10^{-19} ms^{-2} (b) 9.1×10^{31} ms^{-2}

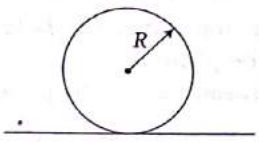
(c) 1.5×10^{19} ms^{-2} (d) 1.6×10^{27} ms^{-2}

38. A drop of 10^{-6} kg water carries 10^{-6} C charge. What electric field should be applied to balance its weight (assume $g=10$ ms^{-2})

(a) 10 V/m upward (b) 10 V/m downward

(c) 0.1 V/m downward (d) 0.1 V/m upward

39. If linear charge density of a wire as shown in the figure is λ



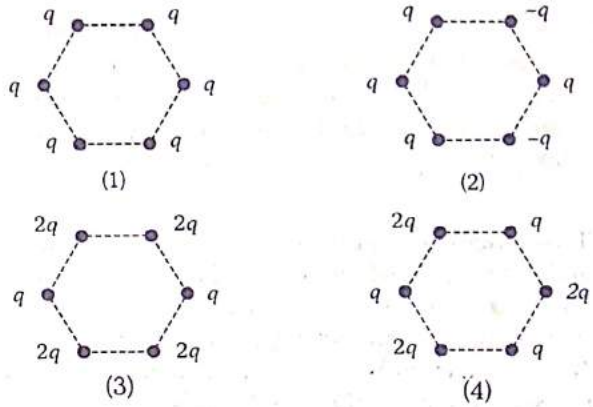
(a) potential at the centre is $\frac{\lambda}{2\varepsilon_0}$

(b) electric field at the centre of the loop is $\frac{\lambda}{2\pi\varepsilon_0 R}$

(c) electric field at the centre of the loop is $\frac{\lambda}{2\pi\varepsilon_0 R}+\frac{\lambda}{2\varepsilon_0 R}$

(d) None of the above

40. Figures below show regular hexagons, with charges at the vertices. In which of the following cases the electric field at the centre is not zero?



(a) 1 (b) 2 (c) 3 (d) 4

41. Electric field intensity at a point in between two parallel sheets with like charges of same surface charge densities (σ) is

(a) $\frac{\sigma}{2\varepsilon_0}$ (b) $\frac{\sigma}{\varepsilon_0}$ (c) zero (d) $\frac{2\sigma}{\varepsilon_0}$

42. The electric charges are distributed in a small volume. The flux of the electric field through a spherical surface of radius 10 cm surrounding the total charge is 20 V-m. The flux over a concentric sphere of radius 20 cm will be

(a) 20 Vm (b) 10 Vm (c) 40 Vm (d) 5 Vm

43. Two charges of charge -4 μC and $+4$ μC are placed of the point $A(1, 0, 4)$ and $B(2, -1, 5)$ located in an electric field $\mathbf{E}=0.20\hat{\mathbf{i}}$ V/C-m. Then, torque acting on the dipole will be

(a) 2.31×10^{-4} N/m (b) 7.98×10^{-4} C/m

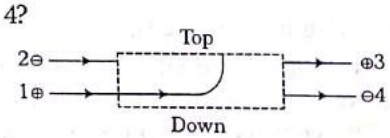
(c) 7.11×10^{-4} C/m (d) 7.04×10^{-4} C/m

44. An infinite line charge produces a field of 7.182×10^{8} N/C at distance of 2 cm. The linear charge density is

(a) 7.27×10^{-4} C/m (b) 7.98×10^{-4} C/m

(c) 7.11×10^{-4} C/m (d) 7.04×10^{-4} C/m

45. The figure shows the path of a positively charged particle 1 through a rectangular region of uniform electric field as shown in the figure. What is the direction of electric field and the direction of particles 2, 3 and 4?



(a) Top, down, top, down

(b) Top, down, down, top

(c) Down, top, top, down

(d) Down, top, down, down

46. The electric intensity due to a dipole of length 10 cm and having a charge of 500 μC, at a point on the axis at a distance 20 cm from one of the charges in air, is

(a) 6.25×10^{7} N/C (b) 9.28×10^{7} N/C

(c) 13.1×11^{10} N/C (c) 20.5×10^{7} N/C

47. Two electric dipoles of moment p and $64p$ are placed in opposite direction on a line at a distance of 25 cm. The electric field will be zero at point between the dipoles whose distance from the dipole of moment p is

(a) 5 cm (b) $\frac{25}{9}$ cm (c) 10 cm (d) $\frac{4}{13}$ cm

48. Two spherical conductors B and C having equal radii and carrying equal charges in them repel each other with a force F. When kept apart at same distance. A third spherical conductor having same radius as that of B but uncharged is brought in contact with B, then brought in contact with C and finally removed away from both. The new force of repulsion between B and C is

(a) $F/4$ (b) $3F/4$
(c) $F/8$ (d) $3F/8$

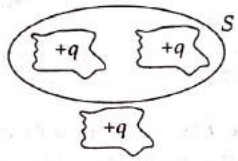
49. A sample of HCl gas is placed in an electric field of 3×10^4 NC^{-1}. The dipole moment of each HCl molecule is 6×10^{-30} C-m. The maximum torque that can act on a molecule is

(a) 2×10^{-34} C^{2}N^{-1}m (b) 2×10^{-34} N-m
(c) 18×10^{-26} N-m (d) 0.5×10^{34} C^{-2}N^{-1}m^{-1}

50. A water molecule has an electric dipole moment 6.4×10^{-30} cm when it is in vapour state. The distance in metre between the centre of positive and negative charge of the molecule is

(a) 4×10^{-10} (b) 4×10^{-11}
(c) 4×10^{-12} (d) 4×10^{-13}

51. Figure shown below is a distribution of charges. The flux of electric field due to these charges through the surface S is



(a) $3q/\varepsilon_0$ (b) $2q/\varepsilon_0$
(c) q/ε_0 (d) zero

52. There exists an electric field of 1 NC^{-1} along Y direction. The flux passing through the square of 1 m placed in xy-plane inside the electric field is

(a) 1.0 Nm^{2}C^{-1} (b) 10.0 Nm^{2}C^{-1}
(c) 2.0 Nm^{2}C^{-1} (d) zero

53. Two positive ions, each carrying a charge q, are separated by a distance d. If F is the force of repulsion between the ions, then the number of electrons missing from each ion will be (e being the charge on an electron)

(a) $\frac{4\pi\varepsilon_0 F d^2}{e}$ (b) $\sqrt{\frac{4\pi\varepsilon_0 F e^2}{d^2}}$ (c) $\sqrt{\frac{4\pi\varepsilon_0 F d^2}{e^2}}$ (d) $\frac{4\pi\varepsilon_0 F d^2}{e^2}$

54. 'All charge on a conductor must reside on its outer surface'. This statement is true

(a) in all cases
(b) for spherical conductors only (Both solid and hollow)
(c) for hollow spherical conductors only
(d) for conductors which do not have any sharp points or corners

55. Two charges $+4e$ and $+e$ are at a distance apart. At what distance, a charge q must be placed from charge $+e$, so that it is in equilibrium?

(a) $x/2$ (b) $2x/3$ (c) $x/3$ (d) $x/4$

56. Among two discs A and B, first has radius 10 cm and charge 10^{-6} C and second has radius 30 cm and charge 10^{-5} C. When they are touched, charges on both are, q_A and q_B respectively, will be

(a) $q_A = 2.75\,\mu$C, $q_B = 3.15\,\mu$C
(b) $q_A = 1.09\,\mu$C, $q_B = 1.53\,\mu$C
(c) $q_A = q_B = 5.5\,\mu$C
(d) None of the above

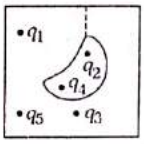
57. Two charges are at a distance d apart. If a copper plate of thickness $d/2$ is kept between them, then effective force will be

(a) $\frac{F}{2}$ (b) zero (c) $2F$ (d) $\sqrt{2F}$

58. Two point charges $-q$ and $+\frac{q}{2}$ are situated at the origin and at the point $(a, 0, 0)$ respectively. The point along the X-axis where the electric field vanishes is

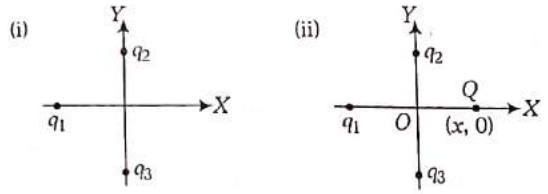
(a) $x = \frac{a}{\sqrt{2}}$ (b) $x = \sqrt{2}a$ (c) $x = \frac{\sqrt{2}a}{\sqrt{2}-1}$ (d) $x = \frac{\sqrt{2}a}{\sqrt{2}+1}$

59. Five charges q_1, q_2, q_3, q_4, and q_5 are fixed at their positions as shown in figure, S is a Gaussian surface. The Gauss's law is given by $\int_S E \cdot dS = \frac{q}{\varepsilon_0}$. Which of the following statements is correct? **[NCERT Exemplar]**



(a) **E** on the LHS of the above equation will have a contribution from q_1, q_5 and q_1, q_5 and q_3 while q on the RHS will have a contribution from q_2 and q_4 only
(b) **E** on the LHS of the above equation will have a contribution from all charges while q on the RHS will have a contribution from q_2 and q_4 only
(c) **E** on the LHS of the above equation will have a contribution from all charges while q on the RHS will have a contribution from q_1, q_3 and q_5 only
(d) Both **E** on the LHS and q on the RHS will have contribution from q_2 and q_4 only

60. In figure two positive charges q_2 and q_3 fixed along the Y-axis, exert a net electric force in the + x-direction on a charge q_1 fixed along the X-axis. If a positive charge Q is added at $(x, 0)$, then the force on q_1 **[NCERT Exemplar]**



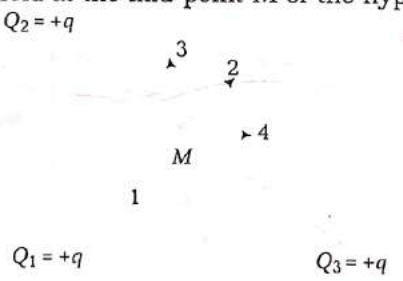
(a) shall increase along the positive X-axis.
(b) shall decrease along the positive X-axis.
(c) shall point along the negative X-axis.
(d) shall increase but the direction changes because of the intersection of Q with q_2 and q_3

61. A hemisphere is uniformly charged positively. The electric field at a point on a diameter away from the centre is directed **[NCERT Exemplar]**
(a) perpendicular to the diameter
(b) parallel to the diameter
(c) at an angle tilted towards the diameter
(d) at an angle tilted away from the diameter

62. A ring of radius R is uniformly charged. Linear charge density is λ. An imaginary sphere of radius R is drawn with its centre on circumference of ring. Total electric flux passing through the sphere would be

(a) $\frac{2\pi R\lambda}{\varepsilon_0}$ (b) $\frac{\pi R\lambda}{\varepsilon_0}$
(c) zero (d) None of these

63. Three point charges as shown are placed at the vertices of an isosceles right angled triangle. Which of the numbered vectors coincides in direction with the electric field at the mid-point M of the hypotenuse?

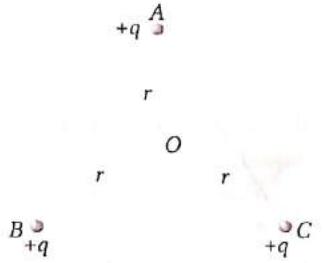


(a) 1 (b) 2 (c) 3 (d) 4

64. Two identical charged spheres of material density ρ, suspended from the same point by inextensible strings of equal length make an angle θ between the strings. When suspended in a liquid of density σ the angle θ remains the same. The dielectric constant K of the liquid is

(a) $\frac{\rho}{\rho - \sigma}$ (b) $\frac{\rho - \sigma}{\rho}$
(c) $\frac{\rho}{\rho + \sigma}$ (d) $\frac{\rho + \sigma}{\rho}$

65. ABC is an equilateral triangle. Charges $+q$ are placed at each corner. The electric intensity at O, the centroid of the triangle, O will be



(a) $\frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$ (b) $\frac{3}{4\pi\varepsilon_0} \cdot \frac{q}{r}$
(c) zero (d) $\frac{1}{4\pi\varepsilon_0} \cdot \frac{3q}{r^2}$

66. A wire of linear charge density λ passes through a cuboid of length l, breadth b and height h in such a manner that flux through the cuboid is maximum. The position of wire is now changed, so that the flux through the cuboid is minimum. $l > b > h$, then the ratio of maximum flux to minimum flux will be

(a) $\frac{\sqrt{l^2 + b^2 + h^2}}{h}$ (b) $\frac{\sqrt{l^2 + b^2}}{h}$
(c) $\frac{h}{\sqrt{l^2 + b^2}}$ (d) $\frac{l}{\sqrt{l^2 + b^2 + h^2}}$

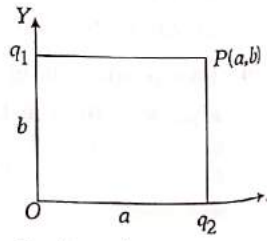
67. Equal charges q are placed at the four corners A, B, C and D of a square of length a. The magnitude of the force on the charge at B will be

(a) $\frac{3q^2}{4\pi\varepsilon_0 a^2}$ (b) $\frac{q^2}{4\pi\varepsilon_0 a^2}$
(c) $\left(\frac{1 + 2\sqrt{2}}{2}\right)\frac{q^2}{4\pi\varepsilon_0 a^2}$ (d) $\left(2 + \frac{1}{\sqrt{2}}\right)\frac{q^2}{4\pi\varepsilon_0 a^2}$

68. A small element l is cut from a circular ring of radius a and λ charge per unit length. The net electric field at the centre of ring is

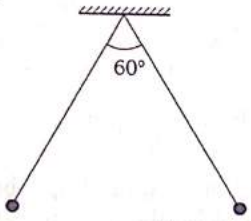
(a) zero (b) $\frac{-\lambda l}{4\pi\varepsilon_0 a^2}$
(c) infinity (d) $\frac{\lambda}{4\pi\varepsilon_0 l}$

69. Two point charges $q_1 = 2\,\mu C$ and $q_2 = 1\,\mu C$ are placed at distances $b = 1$ cm and $a = 2$ cm from the origin of the Y and X-axis as shown in figure. The electric field vector at point $P(a, b)$ will subtend an angle θ with the X-axis given by



(a) $\tan\theta = 1$ (b) $\tan\theta = 2$
(c) $\tan\theta = 3$ (d) $\tan\theta = 4$

70. Two small spherical balls each carrying a charge $Q = 10\mu C$ (10 micro-coulomb) are suspended by two insulating threads of equal lengths 1 m each, from a point fixed in the ceiling. It is found that in equilibrium threads are separated by an angle 60° between them, as shown in the figure. What is the tension in the threads (Given, $\frac{1}{(4\pi\varepsilon_0)} = 9 \times 10^9$ Nm/C^2)



(a) 18 N (b) 1.8 N
(c) 0.18 N (d) None of these

71. An infinite number of charges, each of charge 1 μC, are placed on the X-axis with coordinates $x = 1, 2, 4, 8, \ldots, \infty$. If a charge of 1 C is kept at the origin, then what is the net force acting on 1C charge
(a) 9000 N (b) 12000 N (c) 24000 N (d) 36000 N

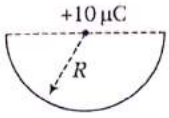
72. An electron moving with the speed $5 \times 10^6 ms^{-1}$ is shooted parallel to the electric field of intensity 1×10^3 N/C. Field is responsible for the retardation of motion of electron. Now, evaluate the distance travelled by the electron before coming to rest for an instant (mass of electron $= 9 \times 10^{-31}$ kg charge $= 1.6 \times 10^{-19}$ C)
(a) 7m (b) 0.7 mm (c) 7 cm (d) 0.7 cm

73. An electric dipole coincides on Z-axis and its mid-point is on origin of the coordinates system. The electric field at an axial point at a distance z from origin is $\mathbf{E}_{(z)}$ and electric field at an equatorial point at a distance y from origin is $\mathbf{E}_{(y)}$. Here, $z = y >> a$, so $\left|\frac{\mathbf{E}_{(z)}}{\mathbf{E}_{(y)}}\right| = \ldots$
(a) 1 (b) 4 (c) 3 (d) 2

74. Three point charges $+q$, $-2q$ and $+q$ are placed at points $(x = 0, y = a, z = 0)$, $(x = 0, y = 0\ z = 0)$ and $(x = a, y = 0, z = 0)$, respectively. The magnitude and direction of the electric dipole moment vector of this charge assembly are
(a) $\sqrt{2}qa$ along $+y$-direction
(b) $\sqrt{2}qa$ along the line joining points $(x = 0, y = 0, z = 0)$ and $(x = a, y = a, z = 0)$
(c) qa along the line joining points $(x = 0, y = 0, z = 0)$ and $(x = a, y = a, z = 0)$
(d) $\sqrt{2}qa$ along $+x$-direction

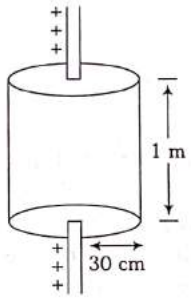
75. A cylinder of radius R and length L is placed in a uniform electric field E parallel to the cylinder axis. The total flux for the surface of the cylinder is given by
(a) $2\pi R^2 E$ (b) $\pi R^2 / E$ (c) $(\pi R^2 / \pi R) / E$ (d) zero

76. A charge 10μC is placed at the centre of a hemisphere of radius $R = 10$ cm as shown. The electric flux through the hemisphere (in MKS units) is



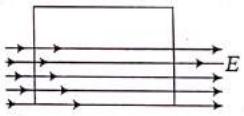
(a) 20×10^5 (b) 10×10^5
(c) 6×10^5 (d) 2×10^5

77. Electric charge is uniformly distributed along a long straight wire of radius 1 mm. The charge per centimetre length of the wire is Q coulomb. Another cylindrical surface of radius 50 cm and length 1m symmetrically encloses the wire as shown in the figure. The total electric flux passing through the cylindrical surface is



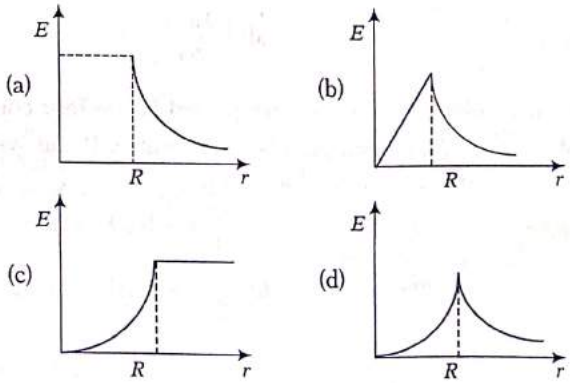
(a) $\frac{Q}{\varepsilon_0}$ (b) $\frac{100Q}{\varepsilon_0}$ (c) $\frac{10Q}{(\pi\varepsilon_0)}$ (d) $\frac{100Q}{(\pi\varepsilon_0)}$

78. A square surface of side L metres is in the plane of the paper. A uniform electric field $\mathbf{E}$ (volt/m), also in the plane of the paper, is limited only to the lower half of the square surface, (see figure). The electric flux in SI units associated with the surface is

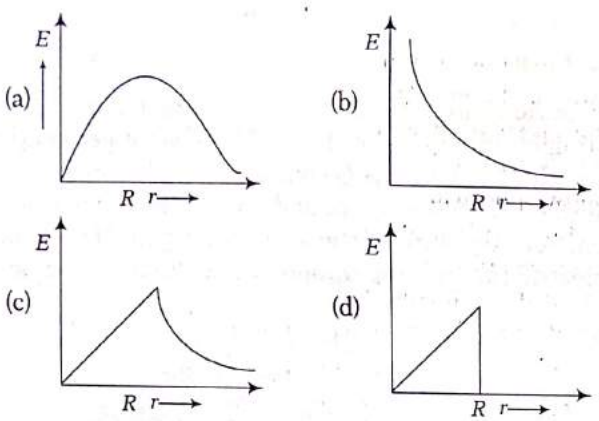


(a) zero (b) EL^2
(c) $EL^2/(2\varepsilon_0)$ (d) $EL^2/2$

79. Which of the following graphs shows the variation of electric field E due to a hollow spherical conductor of radius R as a function of distance from the centre of the sphere?



80. In a uniformly charged sphere of total charge Q and radius R, the electric field E is plotted as function of distance from the centre. The graph which would correspond to the above will be



81. An electric dipole is situated in an electric field of uniform intensity E whose dipole moment is p and moment of inertia is I. If the dipole is displaced slightly from the equilibrium position, then the angular frequency of its oscillations is

(a) $\left(\frac{pE}{I}\right)^{1/2}$ (b) $\left(\frac{pE}{I}\right)^{3/2}$

(c) $\left(\frac{I}{pE}\right)^{1/2}$ (d) $\left(\frac{p}{IE}\right)^{1/2}$

82. Two point charges $(+Q)$ and $(-2Q)$ are fixed on the X-axis at positions a and $2a$ from origin, respectively. At what positions on the axis, the resultant electric field is zero

(a) Only $x = \sqrt{2}a$ (b) Only $x = -\sqrt{2}a$

(c) Both $x = \pm\sqrt{2}a$ (d) Only $x = \frac{3a}{2}$

83. Charge q_2 of mass m revolves around a stationary charge q_1 in a circular orbit of radius r. The orbital periodic time of q_2 would be

(a) $\left[\frac{4\pi^2 mr^3}{kq_1q_2}\right]^{1/2}$ (b) $\left[\frac{kq_1q_2}{4\pi^2 mr^3}\right]^{1/2}$

(c) $\left[\frac{4\pi^2 mr^4}{kq_1q_2}\right]^{1/2}$ (d) $\left[\frac{4\pi^2 mr^2}{kq_1q_2}\right]^{1/2}$

84. Four charges equal to $-Q$ are placed at the four corners of a square and a charge q is at its centre. If the system is in equilibrium, the value of q is

(a) $\frac{-Q}{4}(1+2\sqrt{2})$ (b) $\frac{Q}{4}(1+2\sqrt{2})$

(c) $\frac{-Q}{2}(1+2\sqrt{2})$ (d) $\frac{Q}{2}(1+2\sqrt{2})$

85. The distance between the two charges 25 μC and 36 μC is 11 cm. At what point on the line joining the two, the intensity will be zero?

(a) At a distance of 5 cm from 25 μC
(b) At a distance of 5 cm from 36 μC
(c) At a distance of 4 cm from 25 μC
(d) At a distance of 4 cm from 36 μC

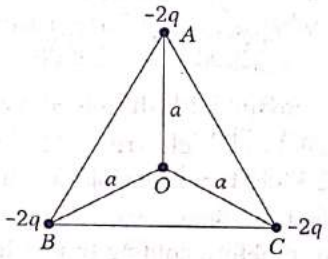
86. The electric field at a point due to an electric dipole, on an axis inclined at an angle $\theta(<90°)$ to the dipole axis, is perpendicular to the dipole axis, if the angle θ is

(a) $\tan^{-1}(2)$ (b) $\tan^{-1}\left(\frac{1}{2}\right)$ (c) $\tan^{-1}(\sqrt{2})$ (d) $\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

87. If 10^{10} electrons are acquired by a body every second, the time required for the body to get a total charge of C will be

(a) 2h (b) 2 days (c) 2 yr (d) 20 yr

88. ABC is an equilateral triangle. Charges $-2q$ are placed at each corner. The electric intensity at O will be



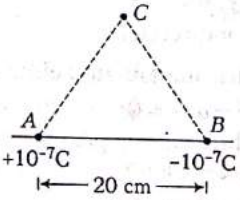
(a) $\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

(c) zero (d) $\frac{1}{4\pi\varepsilon_0}\frac{3q}{r^2}$

89. Two equally charged, identical metal spheres A and B repel each other with a force F. The spheres are kept fixed with a distance r between them. A third identical, but uncharged sphere C is brought in contact with A and then placed at the mid-point of the line joining A and B. The magnitude of the net electric force on C is

(a) F (b) $F/4$ (c) $F/2$ (d) $4F$

90. Two point charges $+10^{-7}$C and -10^{-7}C are placed at A and B 20 cm apart as shown in the figure. Calculate the electric field at C, 20 cm apart from both A and B.



(a) 1.5×10^{-5} N C^{-1} (b) 2.2×10^{4} NC^{-1}

(c) 3.5×10^{6} NC^{-1} (d) 3.0×10^{5} NC^{-1}

91. Infinite charges of magnitude q each are lying at $x = 1, 2, 4, 8, \ldots$ metre on X-axis. The value of intensity of electric field at point $x = 0$ due to these charges will be

(a) $12 \times 10^9\ q\ \text{NC}^{-1}$ (b) zero

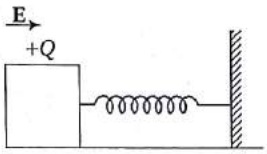
(c) $6 \times 10^9\ q\ \text{NC}^{-1}$ (d) $4 \times 10^9\ q\ \text{NC}^{-1}$

92. Two copper balls, each weighing 10 g, are kept in air 10 cm apart. If one electron from every 10^6 atoms is transferred from one ball to the other, then the coulomb force between them is (atomic weight of copper is 63.5)

(a) 2.0×10^{10} N (b) 2.0×10^4 N

(c) 2.0×10^8 N (d) 2.0×10^6 N

93. A wooden block performs SHM on a frictionless surface with frequency ν_0. The block carries a charge $+Q$ on its surface. If now a uniform electric field **E** is switched on as shown, then SHM of the block will be



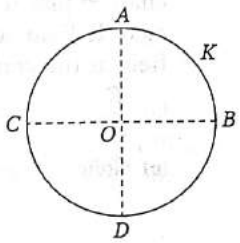
(a) of the same frequency and with shifted mean position

(b) of the same frequency and with the same mean position

(c) of changed frequency and with shifted mean position

(d) of changed frequency and with the same mean position

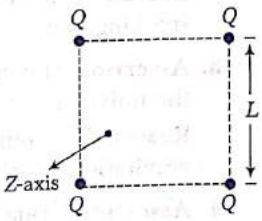
94. A thin conducting ring of radius R is given a charge $+Q$. The electric field at the centre O of the ring due to the charge on the part AKB of the ring is E. The electric field at the centre due to the charge on the part $ACDB$ of the ring is



(a) E along KO (b) $3E$ along OK

(c) $3E$ along KO (d) E along OK

95. Four point +ve charges of same magnitude (Q) are placed at four corners of a rigid square frame as shown in figure. The plane of the frame is perpendicular to Z-axis. If a negative point charge is placed at a distance z away from the above frame ($z << L$), then



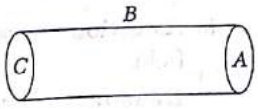
(a) negative charge oscillates along the Z-axis

(b) it moves away from the frame

(c) it moves slowly towards the frame and stays in the plane of the frame

(d) it passes through the frame only once

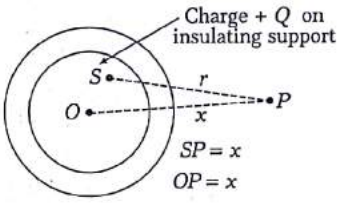
96. A hollow cylinder has a charge q coulomb within it. If ϕ is the electric flux in units of volt-metre associated with the curved surface B, then the flux linked with the plane surface A in units of volt-metre will be



(a) $\frac{1}{2}\left(\frac{q}{\varepsilon_0} - \phi\right)$ (b) $\frac{q}{2\varepsilon_0}$

(c) $\frac{\phi}{3}$ (d) $\frac{q}{\varepsilon_0} - \phi$

97. The adjacent diagram shows a charge $+Q$ held on an insulating support S and enclosed by a hollow spherical conductor. O represents the centre of the spherical conductor and P is a point such that $OP = x$ and $SP = r$. The electric field at point P will be



(a) $\frac{Q}{4\pi\varepsilon_0 x^2}$ (b) $\frac{Q}{4\pi\varepsilon_0 r^2}$

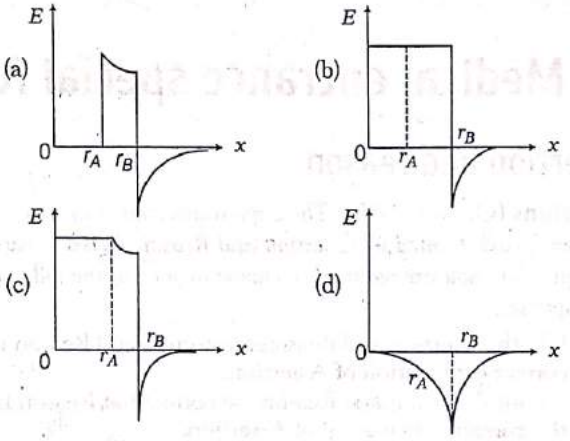
(c) 0 (d) None of these

98. An infinitely long thin straight wire has uniform linear charge density of $\frac{1}{3}\ \text{cm}^{-1}$. Then, the magnitude of the electric intensity at a point 18 cm away is (given, $\varepsilon_0 = 8.8 \times 10^{-12}\ \text{C}^2\text{Nm}^{-2}$)

(a) $0.33 \times 10^{11}\ \text{NC}^{-1}$ (b) $3 \times 10^{11}\ \text{NC}^{-1}$

(c) $0.66 \times 10^{11}\ \text{NC}^{-1}$ (d) $1.32 \times 10^{11}\ \text{NC}^{-1}$

99. Two concentric conducting thin spherical shells A and B having radii r_A and r_B ($r_B > r_A$) are charged to Q_A and $-Q_B$ ($|Q_B| > |Q_A|$). The electrical field along a line, (passing through the centre) is

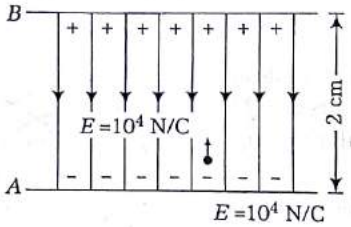


100. Two identical charged spheres suspended from a common point by two massless strings of length l are initially a distance $d (d << l)$ apart because of their mutual repulsion. The charge begins to leak from both the spheres at a constant rate. As a result the charges approach each other with a velocity v. Then, as a function of distance x between them

(a) $v \propto x^{-1/2}$ (b) $v \propto x^{-1}$

(c) $v \propto x^{1/2}$ (d) $v \propto x$

101. An electron is released from the bottom plate A as shown in the figure ($E = 10^4$ N/C). The velocity of the electron when it reaches plate B will be nearly equal to

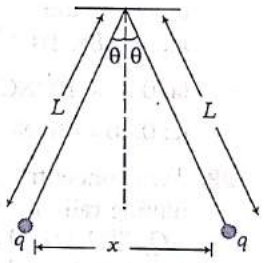


(a) 0.85×10^7 ms^{-1} (b) 1.0×10^7 ms^{-1}
(c) 1.25×10^7 ms^{-1} (d) 1.65×10^7 ms^{-1}

102. Charge q is uniformly distributed over a thin half ring of radius R. The electric field at the centre of the ring is

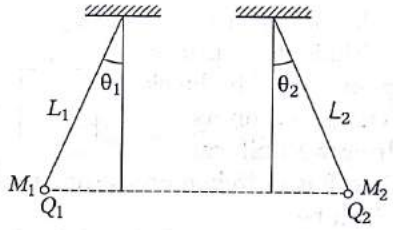
(a) $\frac{q}{2\pi^2\varepsilon_0 R^2}$ (b) $\frac{q}{4\pi^2\varepsilon_0 R^2}$
(c) $\frac{q}{4\pi\varepsilon_0 R^2}$ (d) $\frac{q}{2\pi\varepsilon_0 R^2}$

103. In the given figure, two tiny conducting balls of identical mass m and identical charge q hang from non-conducting threads of equal length L. Assume that θ is so small that $\tan\theta \simeq \sin\theta$, then for equilibrium x is equal to



(a) $\left(\frac{q^2L}{2\pi\varepsilon_0 mg}\right)^{1/3}$ (b) $\left(\frac{qL^2}{2\pi\varepsilon_0 mg}\right)^{1/3}$
(c) $\left(\frac{q^2L^2}{4\pi\varepsilon_0 mg}\right)^{1/3}$ (d) $\left(\frac{q^2L}{4\pi\varepsilon_0 mg}\right)^{1/3}$

104. Two small spheres of masses M_1 and M_2 are suspended by weightless insulating threads of lengths L_1 and L_2. The spheres carry charges Q_1 and Q_2, respectively. The spheres are suspended such that they are in level with one another and the threads are inclined to the vertical at angles of θ_1 and θ_2 as shown. Which one of the following conditions is essential, if $\theta_1 = \theta_2$?



(a) $M_1 \neq M_2$ but $Q_1 = Q_2$ (b) $M_1 = M_2$
(c) $Q_1 = Q_2$ (d) $L_1 = L_2$

105. At what distance along the central axis of a uniformly charged plastic disc of radius R is the magnitude of the electric field equal to one-half the magnitude of the field at the centre of the surface of the disc?

(a) $\frac{R}{\sqrt{2}}$ (b) $\frac{R}{\sqrt{3}}$ (c) $\sqrt{2}R$ (d) $\sqrt{3}R$
(e) None of these

B. Medical entrance special format questions

Assertion and reason

Directions (Q. Nos. 1-15) *These questions consist of two statements each printed as Assertion and Reason. While answering these questions you are required to choose anyone of the following five responses.*

(a) If both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If Assertion is false but Reason is true.
(e) If both Assertion and Reason are false.

1. Assertion Mass of ion is slightly differed from its element.

Reason Ion is formed, when some electrons are removed or added, so mass changes.

2. Assertion When charges are shared between any two bodies, then no charge is really lost but some loss of energy does occur.

Reason Some energy disappears in the form of heat, sparking, etc.

3. Assertion The coulomb force is the dominating force in the universe.

Reason The coulomb force is weaker than the gravitational force.

4. Assertion Due to two point charges electric field cannot be zero at some point simultaneously.

Reason Field is a vector quantity.

5. Assertion At the centre of the line joining two equal and opposite charges, $E = 0$.

Reason At the centre of the line joining two equal and similar charge, $E \neq 0$.

6. Assertion A point charge produces a uniform electric field.

Reason Due to a point charge, electric lines of forces are parallel and equidistant.

7. **Assertion** A point charge is brought in an electric field. The field at a nearby point will increase, whatever be the nature of the charge.
Reason The electric field is independent of the nature of charge.

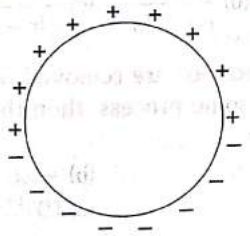
8. **Assertion** On going away from a point charge or a small electric dipole, electric field decreases at the same rate in both the cases.
Reason Electric field is inversely proportional to square of distance from the charge or an electric dipole.

9. **Assertion** Electric lines of force cross each other.
Reason Electric field at a point superimposes to give one resultant electric field.

10. **Assertion** The surface charge densities of two spherical conductors of different radii are equal. Then, the electric field intensities near their surface are also equal.
Reason Surface charge density is equal to charge per unit area.

11. **Assertion** Two identical balls are charged by q. They are suspended from a common point by two insulating threads l each. In equilibrium, the angle between the tension in the threads is 180°. (Ignore gravity).
Reason In equilibrium tension in the springs is
$$T = \frac{1}{4\pi\varepsilon_0}\frac{q \cdot q}{l^2}$$

12. **Assertion** Half of the ring is uniformly positively charged and other half uniformly negatively charged. Then, electric field is zero at centre.



Reason At the centre of uniformly charged ring, electric field is zero.

13. **Assertion** If a dipole is enclosed by a surface, then according to Gauss's law, electric flux linked with it will be zero.
Reason The charge enclosed by a surface is zero.

14. **Assertion** In a region, where uniform electric field exists, the net charge within volume of any size is zero.
Reason The electric flux within any closed surface in region of uniform electric field is zero.

15. **Assertion** With the help of Gauss's theorem we can find electric field at any point.
Reason Gauss's theorem can be applied for any type of charge distribution.

Match the columns

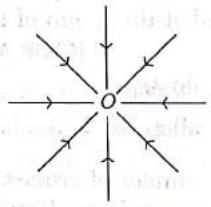
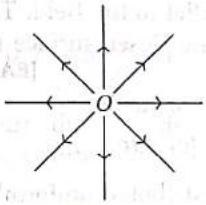
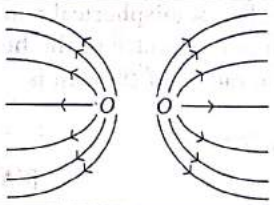
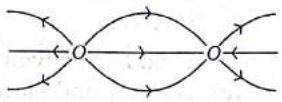
1. Match the following two columns.

	Column I		Column II
A.	Electric charge	p.	$[M^{-1}L^{-3}T^4A^2]$
B.	Electric field strength	q.	$[MLT^{-3}A^{-1}]$
C.	Absolute permittivity	r.	$[MT^{-3}A^{-1}]$
D.	Electric dipole	s.	None

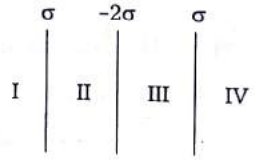
2. Two charges $+q$ and $-q$ are placed at distance r. Match the following two columns when distance between them is changed to r'.

	Column I		Column II
A.	$r' = 2r$	p.	Force between them will become half
B.	$r' = \frac{r}{2}$	q.	Force between them will become $\frac{1}{4}$ th
C.	$r' = 4r$	r.	Force between them will become four times
D.	$r' = \frac{r}{4}$	s.	None

3. Match the field lines given in Column I with the charge configuration due to which field lines exist in Column II.

Column I		Column II
A.		p. A pair of equal and opposite charges
B.		q. A pair of positive chargs
C.		r. A single positive charge
D.		s. A single negative charge

4. Four metallic plates are charged as shown in figure. Now, match the following two columns.



	Column I		Column II
A.	Electric field in region-I	p.	$\frac{\sigma}{\varepsilon_0}$
B.	Electric field in region-II	q.	$\frac{2\sigma}{\varepsilon_0}$
C.	Electric field in region-III	r.	$\frac{\sigma}{2\varepsilon_0}$
D.	Electric field in region-IV	s.	zero

5. Match the following two columns.

	Column I		Column II
A.	Electric field outside the uniformly charged non-conducting sphere	p.	zero
B.	Electric field inside the conducting sphere	q.	$E \propto r^0$
C.	Electric field due to infinite thin plane sheet of charge	r.	$E \propto \frac{1}{r^2}$
D.	Electric field at centre of hemispherical charged body	s.	None

C. Medical entrances' gallery

(Collection of questions asked in NEET & various medical entrance exams)

1. An electric dipole is placed at an angle of 30° with an electric field intensity 2×10^5 N/C. It experiences a torque equal to 4 N-m. The charge on the dipole, if the dipole length is 2 cm is **[NEET 2016]**
(a) 8 mC (b) 2 mC
(c) 5 mC (d) 7 μC

2. The electric field in a certain region is acting radially outward and is given by $E = Ar$. A charge contained in a sphere of radius a centred at the origin of the field will be given by **[CBSE AIPMT 2015]**
(a) $4\pi\varepsilon_0 Aa^2$ (b) $A\varepsilon_0 a^2$
(c) $4\pi\varepsilon_0 Aa^3$ (d) $\varepsilon_0 Aa^3$

3. A Gaussian surface in the cylinder of cross-section πa^2 and length L is immersed in a uniform electric field **E** with the cylinder axis parallel to the field. The flux ϕ of the electric field through the closed surface is **[EAMCET 2015]**
(a) $2\pi a^2\mathbf{E}$ (b) $\pi a^2 \mathbf{E}L$
(c) $\pi a^2(2+L)\mathbf{E}$ (d) zero

4. A total charge of 5 μC is distributed uniformly on the surface of the thin walled semispherical cup. If the electric field strength at the centre of the hemisphere is 9×10^8 NC^{-1}, then the radius of the cup is
$\left(\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ N-m}^2\text{C}^{-2}\right)$ **[EAMCET 2015]**
(a) 5 mm (b) 10 mm
(c) 5 cm (d) 10 cm

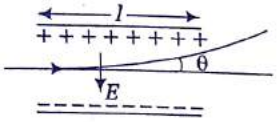
5. Two small spherical shells A and B are given positive charges of 9 C and 4 C respectively and placed such that their centres are separated by 10 m. If P is a point in between them, where the electric field intensity is zero, then the distance of the point P from the centre of A is **[Kerala CEE 2015]**
(a) 5 m (b) 6 m (c) 7 m (d) 8 m
(d) 4 m

6. A point charge q is situated at a distance r on axis from one end of a thin conducting rod of length L having a charge Q [uniformly distributed along its length]. The magnitude of electric force between the two is **[Guj. CET 2015]**
(a) $\frac{KQq}{r^2}$ (b) $\frac{2KQ}{r(r+L)}$ (c) $\frac{KQq}{r(r-L)}$ (d) $\frac{KQq}{r(r+L)}$

7. When 10^{19} electrons are removed from a neutral metal plate through some process, then the charge on it becomes **[Guj. CET 2015]**
(a) + 1.6 C (b) – 1.6 C
(c) 10^{19} C (d) 10^{-19} C

8. A charge Q is uniformly distributed over a large plastic plate. The electric field at point P close to centre of plate is 10 Vm^{-1}. If the plastic plate is replaced by copper plate of the same geometrical dimension and carrying the same charge Q, then the electric field at that point will be **[CG PMT 2015]**
(a) zero (b) 5 Vm^{-1}
(c) 10 Vm^{-1} (d) 20 Vm^{-1}

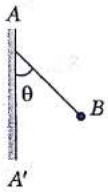
9. A uniform electric field is created between two parallel charged plates as shown below. An electron enters the field symmetrically between the plates with a speed of v_0. The length of each plate is l. Find the angle of deviation of path of the electron as it comes out of the field. **[CG PMT 2015]**



(a) $\theta = \tan^{-1}\dfrac{El}{mv_0^2}$ (b) $\theta = \tan^{-1}\left(\dfrac{eEl}{mv_0^2}\right)$

(c) $\theta = \tan^{-1}\left(\dfrac{eEl}{mv_0}\right)$ (d) $\theta = \tan^{-1}\left(\dfrac{eE}{mv_0^2}\right)$

10. The line AA' is on charged infinite conducting plane which is perpendicular to the plane of the paper. The plane has a surface density of charge σ and B is ball of mass m with a like charge of magnitude q. B is connected by string from a point on the line AA'. The tangent of angle (θ) formed between the line AA' and the string is **[WB JEE 2015]**



(a) $\dfrac{q\sigma}{2\varepsilon_0 mg}$ (b) $\dfrac{q\sigma}{4\pi\varepsilon_0 mg}$

(c) $\dfrac{q\sigma}{2\pi\varepsilon_0 mg}$ (d) $\dfrac{q\sigma}{\varepsilon_0 mg}$

11. The angle between the dipole moment and electric field at any point on the equatorial plane is **[KCET 2015]**

(a) 180° (b) 0° (c) 45° (d) 90°

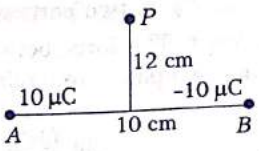
12. Pick out the statement which is incorrect? **[KCET 2015]**

(a) A negative test charge experiences a force opposite to the direction of the field
(b) The tangent drawn to a line of force represents the direction of electric field
(c) Field lines never intersect
(d) The electric field lines form closed loop

13. An electron of mass M_e, initially at rest, moves through a certain distance in a uniform electric field in time t_1. A proton of mass M_p also initially at rest, takes time t_2 to move through an equal distance in this uniform electric field. Neglecting the effect of gravity, the ratio t_2 / t_1 is nearly equal to **[AIIMS 2015]**

(a) 1 (b) $\sqrt{\dfrac{M_p}{M_e}}$ (c) $\sqrt{\dfrac{M_e}{M_p}}$ (d) 1836

14. Two charges of 10 μC and −10 μC are placed at points A and B separated by a distance of 10 cm. Find the electric field at a point P on the perpendicular bisector of AB at a distance of 12 cm from its middle point. **[UK PMT 2015]**



(a) 16.4×10^6 NC^{-1} (b) 28.4×10^6 NC^{-1}

(c) 8.2×10^6 NC^{-1} (d) 4.1×10^6 NC^{-1}

15. If the electric field lines is flowing along axis of a cylinder, then the flux of this field through the cylindrical surface with the axis parallel to the field is [where, r = radius of cylinder] **[UP CPMT 2015]**

(a) $\dfrac{\sigma}{2\pi r\varepsilon_0}$ (b) $\dfrac{\sigma}{\varepsilon_0}$ (c) zero (d) $\dfrac{\sigma}{2\,\varepsilon_0}$

16. An inclined plane of length 5.60 m making an angle of 45° with the horizontal is placed in a uniform electric field $E = 100\,\text{Vm}^{-1}$. A particle of mass 1 kg and charge 10^{-2} C is allowed to slide down from rest position from maximum height of slope. If the coefficient of friction is 0.1, then the time taken by the particle to reach the bottom is **[Guj. CET 2015]**

(a) 1 s (b) 1.41 s

(c) 2 s (d) None of these

17. Two charged spheres separated at a distance d exert a force F on each other. If they are immersed in a liquid of dielectric constant $K = 2$, then the force (if all conditions are same) is **[UK PMT 2014]**

(a) $F/2$ (b) F (c) $2F$ (d) $4F$

18. If a charge on the body is 1 nC, then how many electrons are present on the body? **[KCET 2014]**

(a) 1.6×10^{19} (b) 6.25×10^9

(c) 6.25×10^{27} (d) 6.25×10^{28}.

19. Electric field at a point of distance r from a uniformly charged wire of infinite length having linear charge density λ is directly proportional to **[Kerala CEE 2014]**

(a) r^{-1} (b) r (c) r^2 (d) r^{-2}

(e) None of these

20. Two equal and opposite charges of masses m_1 and m_2 are accelerated in a uniform electric field through the same distance. What is the ratio of their accelerations, if their ratio of masses is $\dfrac{m_1}{m_2} = 0.5$? **[KCET 2014]**

(a) $\dfrac{a_1}{a_2} = 0.5$ (b) $\dfrac{a_1}{a_2} = 1$ (c) $\dfrac{a_1}{a_2} = 2$ (d) $\dfrac{a_1}{a_2} = 3$

21. An electric dipole of dipole moment **p** is placed in a uniform external electric field **E**. Then, the **[Kerala CEE 2014]**

(a) torque experienced by the dipole is **E** × **p**
(b) torque is zero, if **p** is perpendicular to **E**
(c) torque is maximum, if **p** is perpendicular to **E**
(d) potential energy is maximum, if **p** is parallel to **E**
(e) potential energy is maximum, if **p** is perpendicular to **E**

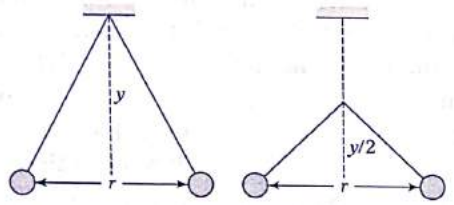
22. An electric dipole placed in a non-uniform electric field experiences **[UK PMT 2014]**

(a) Both a torque and a net force
(b) Only a force but no torque
(c) Only a torque but no net force
(d) No torque and no net force

23. What is the nature of Gaussian surface involved in Gauss's law of electrostatics? **[KCET 2014]**

(a) Scalar (b) Electrical
(c) Magnetic (d) Vector

24. Two pith balls carrying equal charges are suspended from a common point by strings of equal length, the equilibrium separation between them is r. Now, the strings are rigidly clamped at half the height. The equilibrium separation between the balls now becomes **[NEET 2013]**



(a) $\left(\frac{1}{\sqrt{2}}\right)^2$ (b) $\left(\frac{r}{\sqrt[3]{2}}\right)$ (c) $\left(\frac{2r}{\sqrt{3}}\right)$ (d) $\left(\frac{2r}{3}\right)$

25. An electric charge does not have which of the following properties? **[J&K CET 2013]**

(a) Total charge conservation
(b) Quantisation of charge
(c) Two types of charge
(d) Circular line of force

26. The net electric force on a charge of $+3\,\mu C$ at the mid-point on the line joining two charges of magnitude $+2\,\mu C$ and $-2\,\mu C$ separated by the distance of 6 mm, is **[J&K CET 2013]**

(a) 6000 N (b) 500 N
(c) 60 N (d) zero

27. The force of repulsion between two electrons at a certain distance is F. The force between two protons separated by the same distance is $(m_p = 1836 m_e)$ **[Kearala CET 2013]**

(a) $2F$ (b) F (c) $1836F$ (d) $\frac{F}{1836}$
(e) $\frac{F}{4}$

28. A point charge q is placed at a distance $x/2$ directly above the centre of a cube of side x. The flux through the cube is **[MP PMT 2013]**

(a) q/ε_0 (b) $q/2\varepsilon_0$ (c) $\frac{q}{4\varepsilon_0}$ (d) q/ε_0

29. Electrical force is acting between two charges kept in vacuum. A copper plate is placed between the charges, the force now is **[MP PMT 2013]**

(a) more (b) less but not zero
(c) remains unchanged (d) zero

30. Equal charge q each are placed at the vertices A and B of an equilateral triangle ABC of side a. The magnitude of electric intensity at the point C is **[UP CPMT 2012]**

(a) $\frac{q}{4\pi\varepsilon_0 a^2}$ (b) $\frac{\sqrt{2}\,q}{4\pi\varepsilon_0 a^2}$
(c) $\frac{\sqrt{3}\,q}{4\pi\varepsilon_0 a^2}$ (d) $\frac{2q}{4\pi\varepsilon_0 a^2}$

31. If two charges $+4e$ and $+e$ are at a distance x apart, then at what distance charge q must be placed from $+e$, so that it is in equilibrium? **[BCECE (Mains) 2012]**

(a) $\frac{x}{2}$ (b) $\frac{x}{3}$ (c) $\frac{x}{6}$ (d) $\frac{2x}{3}$

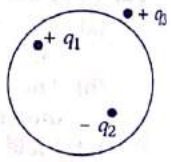
32. If a mass of 20 g having charge 3.0 mC moving with velocity 20 ms^{-1} enters a region of electric field of 80 NC^{-1} in the same direction as the velocity of mass, then the velocity of mass after 3 s in the region will be **[BCECE 2012]**

(a) 40 ms^{-1} (b) 44 ms^{-1} (c) 56 ms^{-1} (d) 80 ms^{-1}

33. A rod lies along the X-axis with one end at the origin and the other at $x \to \infty$. It carries a uniform charge $\lambda\,Cm^{-1}$. The electric field at the point $x = -a$ on the axis will be **[AMU 2012]**

(a) $\mathbf{E} = \frac{\lambda}{4\pi\varepsilon_0 a}(-\hat{\mathbf{i}})$ (b) $\mathbf{E} = \frac{\lambda}{4\pi\varepsilon_0 a}(\hat{\mathbf{i}})$
(c) $\mathbf{E} = \frac{\lambda}{2\pi\varepsilon_0 a}(-\hat{\mathbf{i}})$ (d) $\mathbf{E} = \frac{\lambda}{2\pi\varepsilon_0 a}(\hat{\mathbf{i}})$

34. The given figure shows a spherical Gaussian surface and a charge distribution. When calculating the flux of electric field through the Gaussian surface, then the electric field will be due to **[AMU 2012]**



(a) $+q_3$ alone (b) $+q_1$ and $+q_3$
(c) $+q_1$, $+q_3$ and $-q_2$ (d) $+q_1$ and $-q_2$

35. What is the flux through a cube of side a, if a point charge of q is a one of its corner? **[AIPMT 2012]**

(a) $\frac{2q}{\varepsilon_0}$ (b) $\frac{q}{8\varepsilon_0}$
(c) $\frac{q}{\varepsilon_0}$ (d) $\frac{q}{2\varepsilon_0} 6a^2$

36. If the electric field is given by $(5\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 9\hat{\mathbf{k}})$, then the electric flux through a surface of area 20 unit lying in the yz- plane will be **[AFMC 2012]**

(a) 100 unit (b) 80 unit (c) 180 unit (d) 20 unit

37. Charge Q is divided into two parts which are then kept some distance apart. The force between them will be maximum, if the two parts are having the charge **[AFMC 2012]**

(a) $\frac{Q}{2}$ each (b) $\frac{Q}{4}$ and $\frac{3Q}{4}$
(c) $\frac{Q}{3}$ and $\frac{2Q}{3}$ (d) None of these

Answers

Check point 1.1

1. (d)	2. (d)	3. (a)	4. (d)	5. (c)	6. (c)	7. (b)	8. (c)	9. (a)	10. (d)

Check point 1.2

1. (b)	2. (b)	3. (c)	4. (c)	5. (a)	6. (d)	7. (a)	8. (a)	9. (d)	10. (c)
11. (d)	12. (c)	13. (d)	14. (c)	15. (b)					

Check point 1.3

1. (a)	2. (d)	3. (b)	4. (b)	5. (d)	6. (a)	7. (c)	8. (d)	9. (c)	10. (c)

Check point 1.4

1. (a)	2. (b)	3. (d)	4. (c)	5. (b)	6. (a)	7. (a)	8. (d)	9. (d)	10. (d)

Check point 1.5

1. (c)	2. (a)	3. (d)	4. (b)	5. (b)	6. (a)	7. (c)	8. (a)	9. (d)	10. (b)
11. (c)	12. (d)	13. (c)	14. (b)	15. (b)	16. (a)	17. (a)	18. (b)	19. (c)	20. (a)

A. Taking it together

1. (a)	2. (c)	3. (b)	4. (b)	5. (b)	6. (c)	7. (d)	8. (d)	9. (c)	10. (c)
11. (a)	12. (c)	13. (a)	14. (c)	15. (c)	16. (c)	17. (b)	18. (d)	19. (a)	20. (d)
21. (c)	22. (b)	23. (a)	24. (d)	25. (d)	26. (d)	27. (b)	28. (c)	29. (d)	30. (b)
31. (a)	32. (c)	33. (a)	34. (a)	35. (c)	36. (a)	37. (c)	38. (a)	39. (b)	40. (b)
41. (c)	42. (a)	43. (b)	44. (b)	45. (a)	46. (a)	47. (a)	48. (d)	49. (c)	50. (b)
51. (b)	52. (d)	53. (c)	54. (a)	55. (c)	56. (c)	57. (b)	58. (c)	59. (b)	60. (a)
61. (a)	62. (d)	63. (b)	64. (a)	65. (c)	66. (a)	67. (c)	68. (b)	69. (b)	70. (b)
71. (b)	72. (c)	73. (d)	74. (b)	75. (d)	76. (c)	77. (b)	78. (a)	79. (a)	80. (c)
81. (a)	82. (b)	83. (a)	84. (b)	85. (a)	86. (c)	87. (d)	88. (c)	89. (a)	90. (b)
91. (a)	92. (c)	93. (a)	94. (d)	95. (a)	96. (a)	97. (a)	98. (a)	99. (a)	100. (a)
101. (a)	102. (a)	103. (a)	104. (b)	105. (b)					

B. Medical entrance special format questions

Assertion and Reason

1. (a)	2. (b)	3. (e)	4. (b)	5. (e)	6. (e)	7. (e)	8. (e)	9. (d)	10. (b)
11. (c)	12. (d)	13. (a)	14. (a)	15. (c)					

Match the Columns

1. (A→s; B→q; C→p; D→s)
2. (A→q; B→r; C→s; D→s;)
3. (A→s; B→r; C→q; D→p)
4. (A→s; B→p; C→p; D→s)
5. (A→r; B→p; C→q; D→q)

C. Medical entrances' gallery

1. (b)	2. (c)	3. (d)	4. (a)	5. (b)	6. (d)	7. (a)	8. (c)	9. (b)	10. (a)
11. (a)	12. (d)	13. (b)	14. (d)	15. (c)	16. (b)	17. (a)	18. (b)	19. (a)	20. (c)
21. (c)	22. (a)	23. (d)	24. (b)	25. (d)	26. (d)	27. (b)	28. (d)	29. (b)	30. (c)
31. (b)	32. (c)	33. (c)	34. (c)	35. (b)	36. (a)	37. (a)			

Hints and explanations

Check point 1.1

2. (*d*) Negative charge means excess of electron which increases the mass of sphere *B*.

3. (*a*) Charge on the sphere, $q = ne$

$$= 10^{14} \times 1.6 \times 10^{-19}$$

$$\therefore \quad q = 1.6 \times 10^{-5}\text{C} = 16\mu\text{C}$$

4. (*d*) As conductor has positive charge. So, there is a deficiency of electrons.

$$\therefore \text{ Number of electrons} = \frac{14.4 \times 10^{-19}}{1.6 \times 10^{-19}} = 9$$

5. (*c*) Charge on α-particle, $q = ne$

$$\therefore \quad q = +2e = 2 \times 1.6 \times 10^{-19}$$

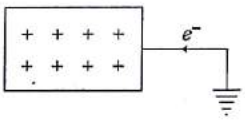
$$= 3.2 \times 10^{-19}\text{C}$$

6. (*c*) As, $q = Ne$ or $N = \frac{q}{e}$

$$\therefore \text{Number of electrons, } N = \frac{80 \times 10^{-6}}{1.6 \times 10^{-19}} = 5 \times 10^{14}$$

7. (*b*) When we rub glass rod with silk, excess electrons are transferred from glass to silk. So, glass rod becomes positive and silk becomes negative.

9. (*a*) When positively charged body connected to earth, then electrons flow from earth to body and body becomes neutral.



Check point 1.2

1. (*b*) Force, $F_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(1 \times 10^{-6})(5 \times 10^{-6})}{r^2}$

[if distance between them is r]

Also, force, $F_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(5 \times 10^{-6})(1 \times 10^{-6})}{r^2}$

$$\therefore \quad \frac{F_1}{F_2} = \frac{1}{1}$$

But direction of F_1 and F_2 are different.

2. (*b*) According to Coulomb's law,

$$F \propto \frac{1}{r^2} \Rightarrow \frac{F_1}{F_2} = \left(\frac{r_2}{r_1}\right)^2$$

$$\therefore \quad \frac{5}{F_2} = \left(\frac{0.04}{0.06}\right)^2$$

$\therefore$ Force between two charges, $F_2 = 11.25$ N

4. (*c*) Force, $F = 9 \times 10^9 \cdot \frac{q^2}{r^2}$

$$\Rightarrow \quad F = 9 \times 10^9 \cdot \frac{(2 \times 10^{-6})^2}{(0.5)^2} = 0.144 \text{ N}$$

5. (*a*) $F = \frac{1}{4\pi\varepsilon_0} \frac{(+7 \times 10^{-6})(-5 \times 10^{-6})}{r^2} = -\frac{1}{4\pi\varepsilon_0} \frac{35 \times 10^{12}}{r^2}$N

$$F = \frac{1}{4\pi\varepsilon_0} \frac{(+5 \times 10^{-6})(-7 \times 10^{-6})}{r^2} = -\frac{1}{4\pi\varepsilon_0} \frac{35 \times 10^{12}}{r^2}\text{N}$$

6. (*d*) $F = K \cdot \frac{q^2}{r^2}$. If q is halved, r is doubled, then

$$\Rightarrow \quad F' = K \frac{(q/2)^2}{(2r)^2}$$

$$\Rightarrow \quad F' = K \frac{q^2}{16r^2}$$

$$\Rightarrow \quad F' = \frac{F}{16}$$

The new force acting on each charge is $\frac{F}{16}$.

8. (*a*) Dielectric constant, $K = \frac{F_{\text{in medium}}}{F_{\text{in air}}}$

So, $\quad F' = \frac{F}{2}$

9. (*d*) By using, $K = \frac{F_a}{F_m} \Rightarrow K = \frac{10^{-4}}{2.5 \times 10^{-5}} = 4$

10. (*c*) We have, $F = F'$ or $\frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r'^2 K}$

$$\Rightarrow \quad r' = \frac{r}{\sqrt{K}}$$

11. (*d*) Gravitational force, $F_g = \frac{G(m_e)(m_e)}{r^2}$

Also, electrostatic force, $F_e = \frac{1}{4\pi\varepsilon_0} \frac{(e)(e)}{r^2}$

$$\therefore \quad \frac{F_g}{F_e} = \frac{G(m_e)^2}{\left(\frac{1}{4\pi\varepsilon_0}\right)e^2} = \frac{6.67\times10^{-11}\times(9.1\times10^{-31})^2}{9\times10^9\times(1.6\times10^{-19})^2}$$

$$= 2.39\times10^{-43}$$

So, ratio of F_g/F_e is of order 10^{-43}.

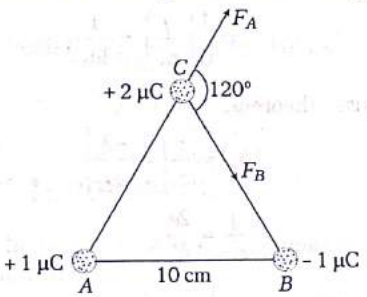
13. (*d*) They will not experience any force, if $|\mathbf{F}_G| = |\mathbf{F}_e|$

$$\Rightarrow \quad \frac{Gm^2}{r^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{r^2}$$

$$\Rightarrow \quad \frac{q^2}{m^2} = 4\pi\varepsilon_0 G$$

$$\Rightarrow \quad q/m = \sqrt{4\pi\varepsilon_0 G}$$

15. (*b*) Let F_A = Force on charge at C due to charge at A



$$\therefore \quad F_A = 9\times10^9\times\frac{10^{-6}\times2\times10^{-6}}{(10\times10^{-2})^2} = 1.8\text{ N}$$

Similarly, F_B = Force on point C due to charge at B

$$= 9\times10^9\times\frac{10^{-6}\times2\times10^{-6}}{(10\times10^{-2})^2} = 1.8\text{ N}$$

$\therefore$ Net force on C,

$$F_{net} = \sqrt{(F_A)^2 + (F_B)^2 + 2F_AF_B\cos120°}$$

$$= \sqrt{(1.8)^2 + (1.8)^2 + 2(1.8)(1.8)(-1/2)} = 1.8\text{ N}$$

Check point 1.3

1. (*a*) Electric field = Force per unit charge = $\frac{F}{q_0}$.

The unit of intensity of electric field is newton/coulomb.

2. (*d*) α-particles are charged particles, so they are deflected by electric field.

3. (*b*) $QE = mg \Rightarrow Q = \frac{mg}{E} = \frac{5\times10^{-5}\times10}{10^7} = 5\times10^{-5}\,\mu\text{C}$

Since, electric field is acting downward, so for balance charge must be negative.

4. (*b*) Electric field, $E = 9\times10^9\cdot\frac{Q}{r^2}$

$$= 9\times10^9\times\frac{5\times10^{-6}}{(0.8)^2} = 7\times10^4\text{ N/C}$$

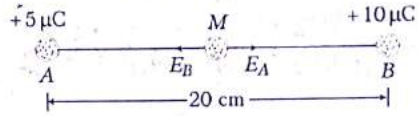
5. (*d*) Electric field,

$$E = 9\times10^9\times\frac{Q}{r^2} \Rightarrow 500 = 9\times10^9\times\frac{Q}{(3)^2}$$

$$\Rightarrow \quad Q = 0.5\,\mu\text{C}$$

6. (*a*) E_A = Electric field at mid-point M due to $+5\,\mu$C charge

$$= 9\times10^9\times\frac{5\times10^{-6}}{(0.1)^2} = 45\times10^5\text{ N/C}$$



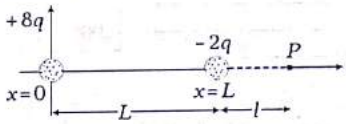
E_B = Electric field at M due to $+10\,\mu$C charge

$$= 9\times10^9\times\frac{10\times10^{-6}}{(0.1)^2} = 90\times10^5\text{ N/C}$$

Net electric field at $M = |\mathbf{E}_B| - |\mathbf{E}_A| = 45\times10^5$ N/C

$= 4.5\times10^6$ N/C, in the direction of E_B, *i.e.*, towards $+5\,\mu$C charge.

7. (*c*) The net field will be zero at a point outside the charges and near the charge which is smaller in magnitude.

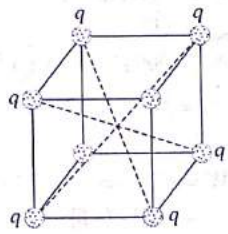


Suppose electric field is zero at P as shown in the figure.

Hence, at P, $K\frac{8q}{(L+l)^2} = \frac{K\cdot(2q)}{l^2} \Rightarrow l = L$.

So, distance of P from origin is, $L + L = 2L$.

8. (*d*) Due to symmetry of charges, electric field intensity is equal and opposite due to charges. So, they cancel out. Hence, net electric field due to charge distribution at centre of cube is zero.



9. (*c*) At point A and C, electric field lines are dense and equally spaced, so $E_A = E_C$.

While at B, they are far apart.

$$\therefore \quad E_A = E_C > E_B$$

10. (*c*) Electric lines of force never intersect the conductor. They are perpendicular and slightly curved near the surface of conductor.

Check point 1.4

1. (*a*) Dipole moment, $p = q\times r = 1.6\times10^{-19}\times4.3\times10^{-9}$

$$= 6.8\times10^{-28}\text{ c-m}$$

2. (*b*) On equatorial line electric field is given by

$$E_{equat} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2p}{r^3}$$

On axial line, $$E_{axial} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{r^3}$$

$\therefore \quad E_{axial} = 2E_{equat}$

But, since there directions are opposite.

Hence, $\quad E_a = 2E_e$

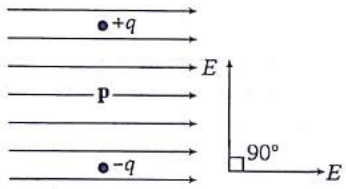
3. (*d*) Electric field, $E_{axis} = \dfrac{2kP}{r^3}$ [along P]

Electric field, $\quad E_{\perp} = \dfrac{kP}{(2r)^3}$ [opposite to P]

4. (*c*) If dipole is rotated through an angle of 90° about it's perpendicular axis, then given point comes on equatorial line. So, field becomes half of previous value, *i.e.*, $E/2$.

8. (*d*) Here, $\quad \theta_1 = 90°$

$\theta_2 = 90° + 180° = 270°$



$$\therefore \text{ Work done} = \int_{\theta_1 = 90°}^{\theta_2 = 270°} pE \sin\theta \, d\theta$$

$$= [-pE \cos\theta]_{90°}^{270°} = 0$$

9. (*d*) Work done in rotating the dipole,

$$W = pE(\cos\theta_1 - \cos\theta_2)$$
$$= pE(\cos 0° - \cos 180°)$$
$$= pE[1-(-1)] = 2pE$$

10. (*d*) Maximum torque is given by

$$\tau_{max} = pE \qquad [\because \sin 90° = 1]$$
$$= (2ql)E = (4\times10^{-8} \times 2\times10^{-4})\times 4\times10^{8}$$
$$= 32\times10^{-4} \text{ N-m}$$

If $\theta = 180°$, then

work done, $\quad W = pE(1-\cos 180°)$

$$= pE[1-(-1)]$$
$$W = 2pE$$
$$= 2\times32\times10^{-4}$$
$$= 64\times10^{-4}\text{J}$$

Check point 1.5

1. (*c*) Electric flux, $\phi = \mathbf{E}\cdot\mathbf{A}$

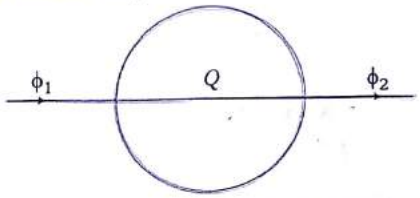
$$\therefore \text{ SI unit of electric flux} = \frac{\text{newton}}{\text{coulomb}} \times \text{metre}^2$$
$$= \frac{(\text{newton-metre})\,\text{metre}}{\text{coulomb}}$$
$$= \frac{\text{joule}\times\text{metre}}{\text{coulomb}} = \text{volt-metre}$$

2. (*a*) Electric flux through the surface,

$$\phi = \mathbf{E}\cdot\mathbf{S} = (2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 7\hat{\mathbf{k}})\cdot(10\hat{\mathbf{j}}) = 40 \text{ unit}$$

3. (*d*) Net electric flux from a closed surface in uniform electric field is always zero.

5. (*b*) From Gauss's law,



$$\text{Net flux} = \frac{\text{Total charge enclosed}}{\varepsilon_0}$$
$$= \frac{1}{\varepsilon_0}\times Q$$
$$\therefore \quad Q = \varepsilon_0(\phi_2 - \phi_1)$$

6. (*a*) Flux from one face $= \dfrac{1}{6}$ (total flux)

$$= \frac{1}{6}\left(\frac{q}{\varepsilon_0}\right) = \frac{1}{3\varepsilon_0} \qquad [\because q = 2\text{C}]$$

8. (*a*) By Gauss's theorem,

$$|\phi_E| = \left|\frac{q}{\varepsilon_0}\right| = \left|\frac{-2e}{\varepsilon_0}\right|$$
$$\frac{q}{\varepsilon_0} = \frac{2e}{\varepsilon_0}$$

9. (*d*) By Gauss's law, $\phi = \dfrac{1}{\varepsilon_0}[Q_{enclosed}]$

$$\Rightarrow \quad Q_{enclosed} = \phi\varepsilon_0 = [-8\times10^3 + 4\times10^3]\varepsilon_0$$
$$= -4\times10^3\varepsilon_0 \text{ C}$$

12. (*d*) By using $\int \mathbf{E}\cdot d\mathbf{A} = \int (\mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 + \mathbf{E}_4)\cdot dA$

$$= \frac{1}{\varepsilon_0}[Q_{enclosed}] = \frac{(q_1 + q_2 + q_3 + q_4)}{\varepsilon_0}$$

15. (*b*) According to Gauss's theorem,

$$\phi = \frac{q}{\varepsilon_0}$$

or $\quad E\cdot 2\pi R\times l = \dfrac{\lambda}{\varepsilon_0}$

$$\Rightarrow \quad E = \frac{1}{2\pi}\frac{\lambda}{\varepsilon_0(R/2)}\times 2 \qquad [\because l = 1]$$
$$= \frac{2\lambda}{\pi\varepsilon_0 R}$$

16. (*a*) Linear charge density, $E = \dfrac{\lambda}{2\pi\varepsilon_0 r} = \dfrac{2\lambda}{4\pi\varepsilon_0 r}$

or $\quad \lambda = \dfrac{E\times4\pi\varepsilon_0 r}{2} = 18\times10^4 \times \dfrac{1}{9\times10^9}\times\dfrac{0.02}{2}$

$$= 2\times10^{-7} \text{ C/m}$$

17. (*a*) Surface charge density, $\sigma = \dfrac{q}{A} = \dfrac{17.7\times10^{-4}\text{C}}{200\text{m}^2}$

The electric field outside the sheet is given by

$$E = \frac{\sigma}{2\varepsilon_0} = \frac{8.85\times10^{-6}}{2\times8.85\times10^{-6}} = 5\times10^5 \text{ N/C}$$

18. (*b*) Electric field, $E = \sigma/\varepsilon_0$

$$F = Ee = \frac{\sigma e}{\varepsilon_0} = \frac{(2.0\times10^{-6})(1.6\times10^{-19})}{8.85\times10^{-12}}$$

The work done by the electron against this force in travelling a distance x metre,

$$W = Fx = \frac{(2.0)\times(10^{-6})(1.6\times10^{-19})}{(8.85\times10^{-12})}$$

$$K = 100 \text{ eV} = 16\times10^{-19}\times100 \text{ J}$$

$$\therefore \quad \frac{(2.0\times10^{-6})(16\times10^{19})x}{8.85\times10^{-12}} = 100\times(16\times10^{-19})$$

$$x = 0.4425 \text{ mm}$$

19. (*c*) We have, $E = \sigma/\varepsilon_0$

$$\sigma = E\varepsilon_0 = 300\times(8.85\times10^{-12}) = 2.65\times10^{-9} \text{ C/m}^2$$

20. (*a*) Electric field, $E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R^2} = (9\times10^9)\times\frac{0.2\times10^{-6}}{(0.25)^2}$

$$= 2.88\times10^4 \text{ N/C}$$

A) Taking it together

1. (*a*) Electric lines are closer at A. So, strength of electric field at A is more than B.

3. (*b*) Force, $F = k\frac{q_1q_2}{r^2} \Rightarrow 12 = k\frac{2\times6}{r^2}$ ...(i)

$\therefore$ A charge of -4C is given to each of these charges

$\therefore$ $q_1 = 2$ and $q_2 = -2$

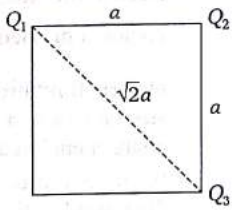
and $F' = k\frac{(+2)(-2)}{r^2}$ ...(ii)

On dividing Eq. (ii) by Eq. (i), we get

$$\frac{F'}{12} = \frac{-4}{12}$$

$$F' = 4 \text{ N (attractive)}$$

4. (*b*) Force, $F_{12} = \frac{k(q^2)}{a^2}$



Force, $F_{13} = \frac{k(q^2)}{(\sqrt{2}a)^2}$

$\therefore$ Ratio, $\frac{F_{12}}{F_{13}} = \frac{2}{1}$

5. (*b*) Electric field on the surface of a conducting sphere is

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$$

$\therefore$ Charge, $q = Er^2\cdot4\pi\varepsilon_0$

$$= \frac{3\times10^6\times(2.5)^2}{9\times10^9} = 2.0831\times10^{-3} \text{ C}$$

7. (*d*) Electric line of force are perpendicular to the surface of a conductor. Inside the sphere, no lines are present.

8. (*d*) Here, $q = -3\times10^{-7}$C

Number of electrons transferred to the conductor is,

$$n = \frac{q}{e} = \frac{-3\times10^{-7}\text{C}}{-1.6\times10^{-19}\text{C}} \approx 2\times10^{12}$$

Mass of one electron, $m_e = 9.1\times10^{-31}$ kg

Mass increase of the conductor $= m_e\times n$

$$= 9.1\times10^{-31}\times2\times10^{12}$$

$$= 18.2\times10^{-19} \text{ kg}$$

$$\approx 2\times10^{-18} \text{ kg}$$

9. (*c*) Let neutral point be obtained as a distance x from 20 μC charge. Hence at neutral point

$$\frac{1}{4\pi\varepsilon_0}\frac{20}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{80}{(10-x)^2}$$

$$\frac{20}{x^2} = \frac{80}{(10-x)^2}$$

$$x = +0.033 \text{ m}$$

10. (*c*) Torque, $\tau_{max} = pE = q(2l)E = 2\times10^{-6}\times0.01\times5\times10^5$

$$= 10\times10^{-3} \text{ N-m}$$

11. (*a*) Electric field due to a point charge, $E = \frac{q}{4\pi\varepsilon_0 r^2}$.

$$\therefore \quad q = E\times4\pi\varepsilon_0 r^2 = 2\times\frac{1}{9\times10^9}\times\left(\frac{30}{100}\right)^2 = 2\times10^{-11} \text{ C}$$

12. (*c*) The potential energy of an electric dipole in a uniform electric field is

$$U = -\mathbf{p}\cdot\mathbf{E}$$

$$U = -pE\cos\theta$$

For U to be maximum,

$$\cos\theta = -1 \Rightarrow \theta = \pi$$

13. (*a*) Gravitational force, $F_G = \frac{Gm_em_p}{r^2}$

$$F_G = \frac{6.7\times10^{-11}\times9.1\times10^{-31}\times1.6\times10^{-27}}{(5\times10^{-11})^2}$$

$$= 3.9\times10^{-47}\text{N}$$

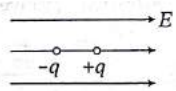
Electrostatic force, $F_e = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$

$$F_e = \frac{9\times10^9\times1.6\times10^{-19}\times1.6\times10^{-19}}{(5\times10^{-11})^2}$$

$$= 9.22\times10^{-8} \text{ N}$$

So, $\frac{F_e}{F_G} = \frac{9.22\times10^{-8}}{3.9\times10^{-47}} = 2.36\times10^{39}$

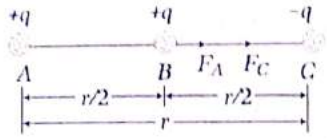
14. (*c*) For stable equilibrium, the angle θ should be 0°.



15. (*c*) The space between the electric field lines is increasing, here from left to right and its characteristics states that, strength of electric field decreases with the increase in the space between electric field lines. As a result, force on charges also decreases from left to right.

Thus, the force on charge $-q$ is greater than force on charge $+q$ in turn dipole will experience a force towards left.

16. (*c*) Situation is shown in figure.



Force between A and C,

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{r^2}$$

When sphere B is kept at the mid-point of line joining A and C, then net force on B is

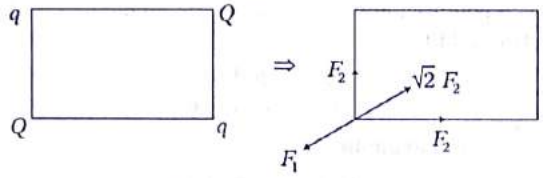
$$F_{net} = F_A + F_C = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{(r/2)^2} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{(r/2)^2}$$

$$= 8 \cdot \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{r^2} = 8F$$

17. (*b*) We have, $q_1' = q_2' = \left(\frac{q_1 + q_2}{2}\right) = \left(\frac{10 - 20}{2}\right) = -5\,\mu C$

$$\therefore \quad \frac{F_1}{F_2} = \frac{q_1 q_2}{q_1' q_2'} = \frac{(10)(-20)}{(-5) \times (-5)} = -\frac{8}{1}$$

18. (*d*) Q and q should be of opposite signs.



$$\sqrt{2}F_2 = F_1$$

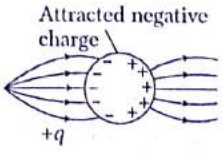
$$\therefore \quad \sqrt{2}\left[\frac{1}{4\pi\varepsilon_0} \cdot \frac{Q(q)}{a^2}\right] = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q \cdot Q}{(\sqrt{2}a)^2},$$

Charge, $Q = -2\sqrt{2}q$

19. (*a*) When a positive point charge is brought near an isolated conducting sphere without touching the sphere, then the free electrons in the sphere are attracted towards the positive charge. This leaves an excess of positive charge on the rear (right) surface of sphere.

Both kinds of charges are bound in the metal sphere and cannot escape. They, therefore reside on the surface.

Thus, the left surface of sphere has an excess of negative charge and the right surface of sphere has an excess of positive charge as given in the figure below



Electric field lines start from positive charge and end at negative charge (in this case from point positive charge to negative charge created inside the sphere).

Also, electric field line emerges from a positive charge, in case of single charge and ends at infinity.

Here, all these conditions are fulfilled in figure.

20. (*d*) For the system to be in equilibrium, net force on charge Q should be zero. So, Q and q should be unlike in nature.

$$\therefore \quad \frac{k \cdot Qq}{r^2} = \frac{kQ \cdot Q}{(2r)^2}$$

or charge, $q = \frac{-Q}{4}$

21. (*c*) From Coulomb's law,

$$q_1 q_2 = F \qquad [\because r = 1\,m] \quad ...(i)$$

$$\left(\frac{q_1 - q_2}{2}\right)\left(\frac{q_1 - q_2}{2}\right) = \frac{F}{3} \qquad ...(ii)$$

Solving Eqs. (i) and (ii), we get

$$\frac{q_1}{q_2} = \frac{3}{1}$$

22. (*b*) $\mathbf{E}_q + \mathbf{E}_{3q}$ is along PA

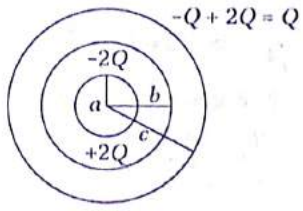
$\mathbf{E}_{2q} + \mathbf{E}_{4q}$ is along PB

$\therefore$ $\mathbf{E}_{net}$ is along CB.

23. (*a*) Surface charge density $(\sigma) = \frac{\text{Charge}}{\text{Surface area}}$

So, surface charge density on the inner surface

$$\sigma_{inner} = \frac{-2Q}{4\pi b^2}$$



and surface charge density $\sigma_{outer} = \frac{Q}{4\pi c^2}$

24. (*d*) Gauss's law of electrostatic states that, the total of the electric flux through a closed surface is equal to the charge enclosed divided by the permittivity, *i.e.*, $\phi = \frac{Q}{\varepsilon_0}$. Thus, electric flux through a surface does not depend on the shape, size or area of a surface but it depends on the number of charges enclosed by the surface.

So, here in this question, all the figures have same electric flux as all of them has single positive charge.

25. (*d*) The electric field at $2q$ will be half of what it is at q *i.e.*, $-\frac{\mathbf{E}}{2}$.

26. (*d*) The net charge on the outer surface is

$$-(-50e - 50e) = 100e$$

27. (*b*) When charged particle enters in a uniform electric field, then force on charged particle,

$$F = qE$$

Also, $F = ma$

$\therefore$ $ma = qE$

or acceleration, $a = \frac{qE}{m} = \frac{3 \times 10^{-3} \times 80}{20 \times 10^{-3}} = 12\ ms^{-2}$

So, from equations of motion,

$$v = u + at$$
$$= 20 + 12 \times 3 = 56 \text{ ms}^{-1}$$

28. (*c*) Electrical field in vacuum, $E_0 = \frac{\sigma}{\varepsilon_0}$

In medium, $E = \frac{\sigma}{\varepsilon_0 K}$

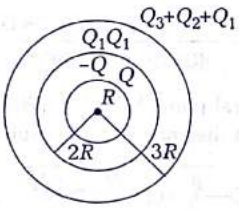
If $K > 1$, then $E_2 < E_0$.

i.e., if the plates are dipped in kerosene oil tank, the electric field between the plates will decrease.

29. (*d*) Flux of electric field E through any area A is defined as $\phi = EA\cos\theta$ or $\phi = \mathbf{E}\cdot\mathbf{A} = 0$ as the lines are parallel to the surface.

30. (*b*) On the outer surfaces of the shell surface charge densities are equal

$$\frac{Q_3 + Q_2 + Q_1}{4\pi(3R)^2} = \frac{Q_2 + Q_1}{4\pi(2R)^2} = \frac{Q_1}{4\pi R^2}$$



$$\frac{Q_3 + Q_2 + Q_1}{9} = \frac{Q_2 + Q_1}{4}$$

and $$\frac{Q_2 + Q_1}{4} = \frac{Q_1}{1}$$

$$Q_2 = 3Q_1$$

$$\therefore \quad \frac{Q_3 + Q_2 + Q_1}{3} = \frac{Q_1}{1}$$

$$Q_3 = 5Q_1$$

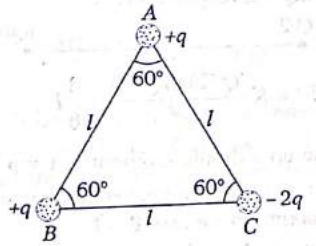
Hence, $Q_1 : Q_2 : Q_3 = 1 : 3 : 5$

31. (*a*) Force on a charged particle, $qE = \mu mg$

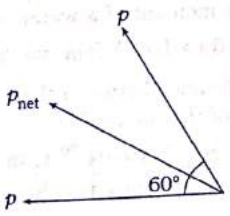
where, μ is coefficient of friction

$$\therefore \quad \mu = \frac{qE}{mg}$$

32. (*c*)



The direction of dipole moment is shown in figure.



Net electric dipole moment,

$$p_{net} = \sqrt{p^2 + p^2 + 2pp\cos 60°} = \sqrt{3}p$$
$$= \sqrt{3}\,ql \qquad [\because p = ql]$$

33. (*a*) When a point positive charge brought near an isolated conducting plane, then some negative charge developes on the surface of the plane towards the charge and an equal positive charge developes on opposite side to the plane. This process is called charging by induction.

34. (*a*) Torque about Q of charge $-q$ is zero, so angular momentum of the charge $-q$ is constant but distance between charges is changing, so force is changing, so speed and velocity are changing.

35. (*c*) $F_1 = \frac{kQ_1Q_2}{d^2}$ and $F_2 = \frac{k\left(\frac{Q_1 - Q_2}{2}\right)^2}{d^2}$

According to question, $F_1 = F_2$

$$Q_1Q_2 = \frac{(Q_1 - Q_2)^2}{4}$$

$$\Rightarrow \quad 4Q_1Q_2 = Q_1^2 + Q_2^2 - 2Q_1Q_2$$
$$0 = Q_1^2 + Q_2^2 - 6Q_1Q_2$$
$$\Rightarrow \quad \frac{Q_1}{Q_2} = -3 \pm \sqrt{8}$$

36. (*a*) Electric field inside shell is zero.

37. (*c*) The acceleration of the electron due to given coulombic force, F is $a_e = \frac{F}{m_e}$ ...(i)

where, m_e is the mass of the electron.

The acceleration of the proton due to same force F is

$$a_p = \frac{F}{m_p} \quad \text{...(ii)}$$

where, m_p is the mass of the proton.

On dividing Eq. (ii) by Eq. (i), we get $\frac{a_p}{a_e} = \frac{m_e}{m_p}$

$$a_p = \frac{a_e m_e}{m_p} = \frac{(2.5 \times 10^{22}\text{ ms}^{-2})(9.1 \times 10^{-31}\text{ kg})}{(1.67 \times 10^{-27}\text{kg})}$$
$$= 13.6 \times 10^{18}\text{ ms}^{-2} \approx 1.5 \times 10^{19}\text{ ms}^{-2}$$

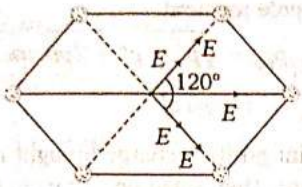
38. (*a*) By using, $QE = mg$

$\Rightarrow E = \frac{mg}{Q} = \frac{10^{-6} \times 10}{10^{-6}} = 10$ V/m, upward because charge is positive.

39. (*b*) Electric field at centre due to circular portion is zero.

So, the electric field at the centre of loop is $\frac{\lambda}{2\pi\varepsilon_0 R}$.

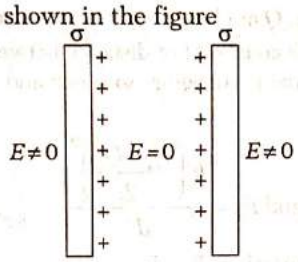
40. (*b*) In Figs. (1), (3) and (4), net electric field is zero because electric field at a point due to positive charge acts away from the charge and due to negative charge, it acts towards the charge so for Fig. (2), net electric field is not zero.



Here, net electric field in Fig. (2) is

$$= \sqrt{(2E)^2 + (2E)^2 + (2E)(2E)\cdot 2\cos 120°} = 2E$$

41. (*c*) Situation is shown in the figure



Electric field between sheets $= \frac{2}{2\varepsilon_0}(\sigma - \sigma) = 0$

42. (*a*) According to Gauss's law, total flux coming out of a closed surface enclosing charge q is given by $\phi = \int \mathbf{E}\cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$.

From this expression, it is clear that total flux linked with a closed surface only depends on the enclosed charge and independent of the shape and size of the surface.

$$\phi = \int \mathbf{E}\cdot d\mathbf{S} = \frac{q}{\varepsilon_0} = 20\text{ V-m} \qquad \text{[given]}$$

This $\frac{q}{\varepsilon_0}$ is constant as long as the enclosed charge is constant.

The flux over a concentric sphere of radius 20 cm = 20 V-m.

43. (*b*) Given, $A(1, 0, 4)$ and $B(2, -1, 5)$

$\therefore \quad 2\mathbf{a} = \mathbf{AB} = [(2-1)\hat{\mathbf{i}} + (-1-0)\hat{\mathbf{j}} + (5-4), \hat{\mathbf{k}}]$

$2\mathbf{a} = [\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}]$

Torque, $\tau = \mathbf{p}\times\mathbf{E} = 2(2\mathbf{a})\times\mathbf{E}$

$\tau = 4\times10^{-6}(\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}})\times 20\hat{\mathbf{i}}$

$\tau = 8\times10^{-5}(\hat{\mathbf{k}} + \hat{\mathbf{j}})$

Magnitude of torque,

$\tau = 8\times10^{-5}\sqrt{1^2 + 1^2}$

$\tau = 1.131\times10^{-4}$ N-m

44. (*b*) Given, $E = 7.18\times10^8$ N/C, $r = 2$ cm $= 2\times10^{-2}$

Electric field is given by

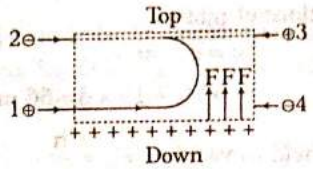
$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

$$\lambda = 2\pi\varepsilon_0 rE$$

$$\lambda = \frac{4\pi\varepsilon_0 rE}{2} = \frac{2\times10^{-2}\times7.182\times10^8}{2\times9\times10^9}$$

$$= 7.98\times10^4 \text{ C/m}$$

45. (*a*) The figure shows, the path of a + ve charged particle (1) through a rectangular region of uniform electric field.

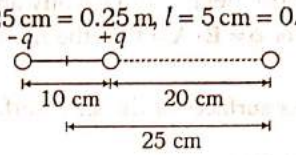


Since, + ve charged particle moves as a parabolic path in electric field, it means the direction of electric field is upward. The direction of particle (2) which is – ve is downward.

The direction of deflection of particle (3) which is + ve is upward and direction of deflection of particle (4) is downward.

46. (*a*) Given, $p = (500\times10^{-6})\times(10\times10^{-2}) = 5\times10^{-5}$ C-m

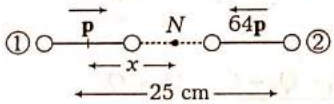
$r = 25$ cm $= 0.25$ m, $l = 5$ cm $= 0.05$ m



Electric field intensity, $E = 9\times10^9\cdot\frac{2pr}{(r^2 - l^2)^2}$

$$E = \frac{9\times10^9\times2\times5\times10^{-5}\times0.25}{\{(0.25)^2 - (0.05)^2\}^2} = 6.25\times10^7 \text{ N/C}$$

47. (*a*) Suppose neutral point N lies at a distance x from dipole of moment p or at a distance $(25 - x)$ from dipole of 64 p.



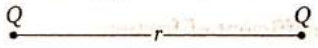
At point N electric field due to dipole ①|

= Electric field due to dipole ②|

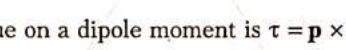
$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\cdot\frac{2p}{x^3} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2(64p)}{(25-x)^3}$$

$$\Rightarrow \quad \frac{1}{x^3} = \frac{64}{(25-x)^3} \Rightarrow x = 5 \text{ cm}$$

48. (*d*) Initially force, $F = k\frac{Q^2}{r^2}$



Finally when a third spherical conductor cames in contact alternately with B and C then removed, so charges on B and C are $Q/2$ and $3Q/4$, respectively.



Now, force, $F' = K\frac{(Q/2)(3Q/4)}{r^2} = \frac{3}{8}F$

49. (*c*) The torque on a dipole moment is $\tau = \mathbf{p}\times\mathbf{E}$. The maximum value is when $\mathbf{p}$ and $\mathbf{E}$ are perpendicular to each other, so that $pE\sin\theta$ is maximum *i.e.*, $\sin\theta = 1$.

$$\tau = (3\times10^4 \text{ NC}^{-1})(6\times10^{-30} \text{ C-m}) = 18\times10^{-26} \text{ N-m}$$

50. (*b*) Electric dipole moment of a water molecule,

$$p = 6.4\times10^{-30} \text{ C-m} \Rightarrow p = qd$$

where, d is the distance between the centre of positive and negative charge of the molecule

$$\Rightarrow \quad d = \frac{p}{q} = \frac{6.4\times10^{-30} \text{ C-m}}{1.6\times10^{-19} \text{ C}} = 4\times10^{-11} \text{ m}$$

51. (*b*) Flux of electric field, $\phi = \frac{1}{\varepsilon_0} \times Q_{encl} = \frac{1}{\varepsilon_0}(2q)$

52. (*d*) The flux passing through the square of 1 m placed in xy-plane inside the electric field is zero because by Gauss's theorem, we can say, closed circuit is not formed.

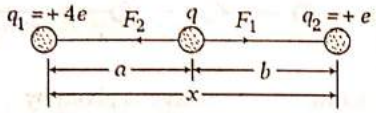
53. (*c*) Two positive ions each carrying a charge q are kept at a distance d, then it is found that force of repulsion between them is $F = \frac{kqq}{d^2} = \frac{1}{4\pi\varepsilon_0}\frac{qq}{d^2}$

where, $q = ne$

$\therefore$ $F = \frac{1}{4\pi\varepsilon_0}\frac{n^2e^2}{d^2}$

Number of electron, $n = \sqrt{\frac{4\pi\varepsilon_0 Fd^2}{e^2}}$

55. (*c*) For equilibrium of charge q, forces on charge q due to charges q_1 and q_2 should be equal.



$\therefore$ $F_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(4e)(q)}{a^2}$

and $F_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(e)(q)}{b^2}$

Also, $x = a + b$

$\therefore$ $\frac{1}{4\pi\varepsilon_0}\frac{(4e)(q)}{a^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(e)q}{(x-a)^2}$

or $\frac{4}{a^2} = \frac{1}{(x-a)^2}$ or $\frac{2}{a} = \frac{1}{x-a}$ or $2x - 2a = a$

$\therefore$ $a = \frac{2x}{3} \Rightarrow b = x/3$

56. (*c*) When touched, charge flows from higher potential to lower potential till it equalises on the two discs.

Given, $q_1 = 10^{-6}\text{C}$, $q_2 = 10^{-5}\text{C}$

$\therefore$ $q = \frac{q_1 + q_2}{2}$

$= \frac{10^{-6} + 10^{-5}}{2} = 5.5\ \mu\text{C}$

57. (*b*) From Coulomb's law, electric force between two charges is directly proportional to product of charges and inversely proportional to square of distance between them. *i.e.*,

$$F = K\frac{q_1q_2}{d^2}$$

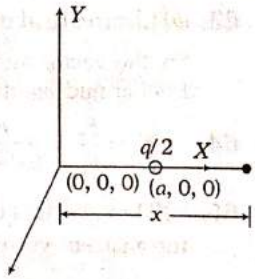
where, $K = \frac{1}{4\pi\varepsilon_0}$ = proportionality constant

If a medium is placed between the charges, then

$$F' = \frac{1}{4\pi\varepsilon_0 K}\frac{q_1q_2}{d^2}$$

Since, medium placed between the charge is a metallic plate, So for it $K = \infty$. Hence, $F' = 0$ (zero)

58. (*c*) Suppose the field vanishes at a distance x, we have



$$\frac{Kq}{x^2} = \frac{Kq/2}{(x-a)^2}$$

or $2(x-a)^2 = x^2$ or $\sqrt{2}(x-a) = x$

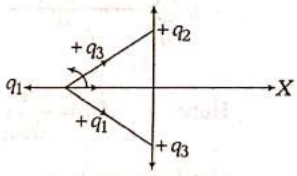
or $(\sqrt{2}-1)x = \sqrt{2}a$

or $x = \frac{\sqrt{2}a}{\sqrt{2}-1}$

59. (*b*) According to Gauss's law, the term q on the right side of the equation $\int_S \mathbf{E}\cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$ includes the sum of all charges enclosed by the surface.

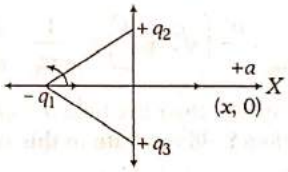
The charges may be located anywhere inside the surface, if the surface is so chosen that there are some charges inside and some outside, then the electric field on the left side of equation is due to all the charges, both inside and outside S. So, $\mathbf{E}$ on LHS of the above equation will have a contribution from all charges while q on the RHS will have a contribution from q_2 and q_4 only.

60. (*a*) The net force on q_1 by q_2 and q_3 is along the $+x$-direction, so nature of force between q_1, q_2 and q_1, q_3 is attractive. This can be represent by the figure given alongside.



The attractive force between these charges states that q_1 is a negative charge (since, q_2 and q_3 are positive).

Thus, nature of force between q_1 and newly introduced charge Q (positive) is attractive and net force on q_1 by q_2, q_3 and Q are along the same direction as given in the diagram below.



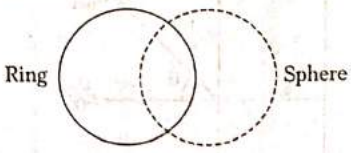
The figure given above clearly shows that the force on q_1 shall increase along the positive X-axis due to the positive charge Q.

61. (*a*) When the point is situated, at a point on diameter away from the centre of hemisphere charged uniformly positively, then the electric field is perpendicular to the diameter. The component of electric intensity parallel to the diameter cancel out.

62. (*d*) As imaginary sphere of radius R is drawn with its centre on circumference of ring.

So, $q_{in} < \pi R\lambda$.

where, λ is linear charge density



$\therefore$ $\phi < \frac{\pi R\lambda}{\varepsilon_0}$

63. (*b*) Electric field due to Q_2 and Q_3 cancel each other.

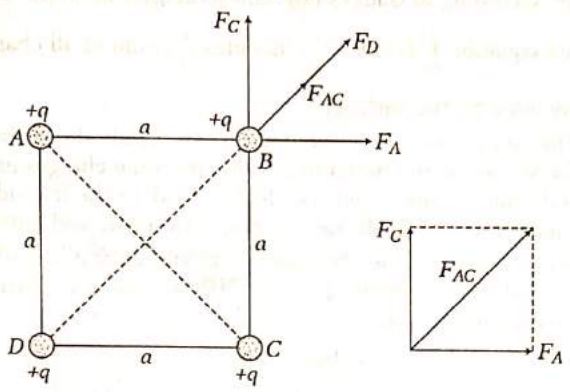
So, the vector numbered 2 coincides in direction with electric field at mid-point M of the hypotenuse.

64. (*a*) $K_r = \frac{F_a}{F_m} = \frac{mg}{(m - m_1)g} = \frac{\rho V}{(\rho - \sigma) V} = \frac{\rho}{(\rho - \sigma)}$

65. (*c*) Three vectors of equal magnitude are inclined at 120° with the adjacent vector. So, net electric intensity will be zero.

66. (*a*) $\frac{\sqrt{l^2 + b^2 + h^2}}{h}$.

67. (*c*)



Here, $F_A = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{a^2} \Rightarrow F_C = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{a^2}$

Net force on B, $F_{\text{net}} = F_{AC} + F_D$

$$= \sqrt{F_A^2 + F_C^2} + F_D$$

$$= \sqrt{\left\{\left(\frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2}\right)^2 + \left(\frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2}\right)^2\right\}} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{(a\sqrt{2})^2}$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{a^2}\left(\sqrt{2} + \frac{1}{2}\right) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{a^2}\left(\frac{1 + 2\sqrt{2}}{2}\right)$$

68. (*b*) If ring is complete, then net field at centre is zero. If small portion is cut, then field opposite to this is not cancelled out.

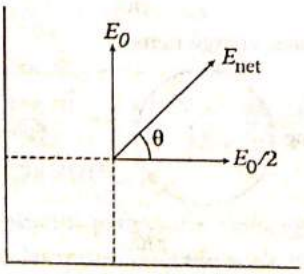
Charge of small portion $= \lambda \cdot l$

$\therefore$ Electric field due to this portion $= \frac{\lambda l}{4\pi\varepsilon_0 a^2}$

Let E_R is electric field of remaining portion.

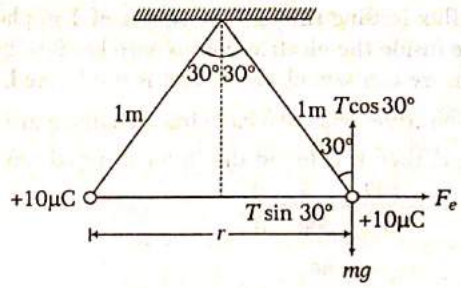
$\therefore \quad E_R + \frac{\lambda l}{4\pi\varepsilon_0 a^2} = 0 \Rightarrow E_R = \frac{-\lambda l}{4\pi\varepsilon_0 a^2}$

69. (*b*) $E \propto \frac{q}{r^2} \Rightarrow \tan\theta = \frac{E_0}{E_0/2} = 2$



70. (*b*) In the following figure, in equilibrium,

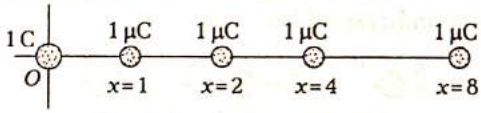
$F_e = T \sin 30°, r = 1\text{m}$



$$\Rightarrow 9 \times 10^9 \cdot \frac{Q^2}{r^2} = T \times \frac{1}{2} \Rightarrow 9 \times 10^9 \cdot \frac{(10 \times 10^{-6})^2}{1^2} = T \times \frac{1}{2}$$

$\Rightarrow$ Tension in the threads $T = 1.8$ N

71. (*b*) The schematic diagram of distribution of charges on X-axis is shown in figure below.



Total force acting on 1 C charge is given by

$$F = \frac{1}{4\pi\varepsilon_0}\left[\frac{1 \times 1 \times 10^{-6}}{(1)^2} + \frac{1 \times 1 \times 10^{-6}}{(2)^2} + \frac{1 \times 1 \times 10^{-6}}{(4)^2} + \frac{1 \times 1 \times 10^{-6}}{(8)^2} + \ldots \infty\right]$$

$$= \frac{10^{-6}}{4\pi\varepsilon_0}\left(\frac{1}{1} + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \ldots \infty\right) = 9 \times 10^9 \times 10^{-6}\left(\frac{1}{1 - (1/4)}\right)$$

$$= 9 \times 10^9 \times 10^{-6} \times \frac{4}{3} = 9 \times \frac{4}{3} \times 10^3 = 12000 \text{ N}$$

72. (*c*) Electric force, $qE = ma \Rightarrow a = \frac{qE}{m}$

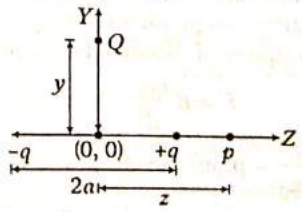
$\therefore \quad a = \frac{1.6 \times 10^{-19} \times 1 \times 10^3}{9 \times 10^{-31}} = \frac{1.6}{9} \times 10^{15}$

$u = 5 \times 10^6$ and $v = 0$

$\therefore$ From $v^2 = u^2 - 2as \Rightarrow s = \frac{v^2}{2a}$

$\therefore$ Distance, $s = \frac{(5 \times 10^6)^2 \times 9}{2 \times 1.6 \times 10^{15}} = 7$ cm (approx.)

73. (*d*) The magnitude of electric field at a axial point P at a distance z from the origin is given by



$$|\mathbf{E}_{(z)}| = \frac{4qaz}{4\pi\varepsilon_0 (z^2 - a^2)^2} = \frac{2pz}{4\pi\varepsilon_0 (z^2 - a^2)^2}$$

where, $p = 2qa$ is the electric dipole moment

For $z >> a$, $|\mathbf{E}_{(z)}| = \frac{2p}{4\pi\varepsilon_0 z^3}$

The magnitude of electric field at a equatorial point Q at a distance y from the origin is given by

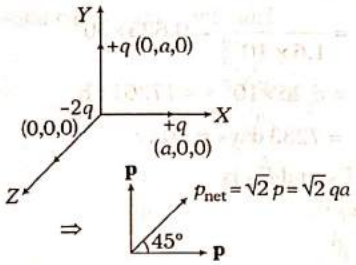
$$|\mathbf{E}_{(y)}| = \frac{1}{4\pi\varepsilon_0}\frac{2qa}{(y^2+a^2)^{3/2}} = \frac{p}{4\pi\varepsilon_0(y^2+a^2)^{3/2}}$$

For $y >> a$, $|\mathbf{E}_{(y)}| = \dfrac{p}{4\pi\varepsilon_0 y^3}$

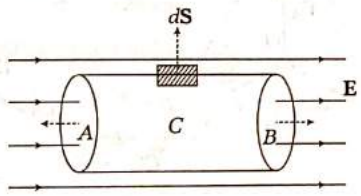
For $z = y >> a$,

$\therefore \quad \dfrac{|\mathbf{E}_{(z)}|}{|\mathbf{E}_{(y)}|} = 2$

74. (*b*)



75. (*d*) Flux through surface, A $\phi_A = E \times \pi R^2$ and $\phi_B = -E \times \pi R^2$



Flux through curved surface, $C = \int \mathbf{E}\cdot d\,\mathbf{S} = \int EdS\cos 90° = 0$

$\therefore$ Total flux through cylinder $= \phi_A + \phi_B + \phi_C = 0$

76. (*c*) According to Gauss's theorem, electric flux through the sphere $= \dfrac{q}{\varepsilon_0}$.

$\therefore$ Electric flux through the hemisphere $= \dfrac{1}{2}\dfrac{q}{\varepsilon_0}$

$$= \frac{10\times10^{-6}}{2\times8.854\times10^{-12}} = 0.56\times10^6\ \text{N-m}^2\text{C}^{-1}$$

$$\approx 0.6\times10^6\ \text{N-m}^2\text{C}^{-1} = 6\times10^5\ \text{N-m}^2\text{C}^{-1}.$$

77. (*b*) Charge enclosed by cylindrical surface (length 100 cm) is $Q_{encl} = 100Q$. By applying Gauss's law,

$$\phi = \frac{1}{\varepsilon_0}(Q_{encl}) = \frac{1}{\varepsilon_0}(100\,Q)$$

78. (*a*) Electric flux, $\phi_E = \int \mathbf{E}\cdot d\mathbf{S}$

$$= \int E\,dS\cos\theta = \int E\,dS\cos 90° = 0$$

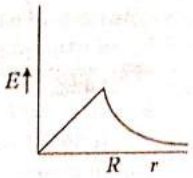
The lines are parallel to the surface.

79. (*a*) Electric field due to a hollow spherical conductor is governed by following equation $E = 0$, for $r < R$ ...(i)

and $E = \dfrac{Q}{4\pi\varepsilon_0 r^2}$, for $r \geq R$ ...(ii)

i.e., inside the conductor field will be zero and outside the conductor will vary according to $E \propto \dfrac{1}{r^2}$.

80. (*c*) The field increases linearly from zero inside the sphere (from $r = 0$ to $r = R$) become maximum at the surface of the sphere and decrease rapidly with distance $\left(\propto \dfrac{1}{r^2}\right)$ outside the sphere. So, the graph will be



81. (*a*) When dipole is given a small angular displacement θ about it's equilibrium position, then the restoring torque will be

$$\tau = -pE\sin\theta = -pE\,\theta \qquad (\text{as } \sin\theta = \theta)$$

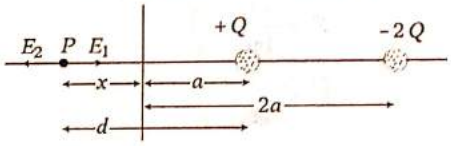
or $\quad I\dfrac{d^2\theta}{dt^2} = -pE\,\theta \qquad \left(\text{as } \tau = I\alpha = I\dfrac{d^2\theta}{dt^2}\right)$

or $\quad \dfrac{d^2\theta}{dt^2} = -\omega^2\theta$ with $\omega^2 = \dfrac{pE}{I} \Rightarrow \omega = \sqrt{\dfrac{pE}{I}}$

82. (*b*) Suppose electric field is zero at a point P lies at a distance d from the charge $+Q$.

At P, $\quad \dfrac{KQ}{d^2} = \dfrac{K(2Q)}{(a+d)^2}$

$\Rightarrow \quad \dfrac{1}{d^2} = \dfrac{2}{(a+d)^2} \Rightarrow d = \dfrac{a}{(\sqrt{2}-1)}$



Since, $d > a$ *i.e.*, point P must lies on negative X-axis as shown at a distance x from origin,

hence $x = d - a$

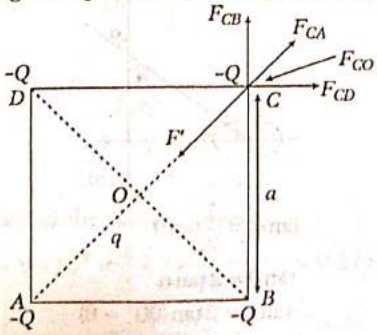
$= \dfrac{a}{(\sqrt{2}-1)} - a = \sqrt{2}\,a$. Actually, P lies on negative X-axis, so $x = -\sqrt{2}a$

83. (*a*) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1q_2}{r^2} = mr\,\omega^2 = \dfrac{4\pi^2 mr}{T^2} \qquad \left(\because \omega = \dfrac{2\pi}{T}\right)$

$$T^2 = \frac{(4\pi\varepsilon_0)r^2(4\pi^2 mr)}{q_1q_2}$$

Time period, $T = \left[\dfrac{4\pi^2 mr^3}{kq_1q_2}\right]^{1/2}$

84. (*b*) The system is in equilibrium means the force experienced by each charge is zero. It is clear that charge placed at centre would be in equilibrium for any value of q, so we are considering the equilibrium of charge placed at any corner.



$$F_{CD} + F_{CA}\cos 45^\circ + F_{CD}\cos 45^\circ = 0$$

$$\Rightarrow \frac{1}{4\pi\varepsilon_0}\cdot\frac{(-Q)(-Q)}{a^2} + \frac{1}{4\pi\varepsilon_0}\frac{(-Q)(-Q)}{(\sqrt{2}a)^2}\times\frac{1}{\sqrt{2}} + \frac{1}{4\pi\varepsilon_0}\frac{(-Q)q}{(\sqrt{2}a/2)^2}\times\frac{1}{\sqrt{2}} = 0$$

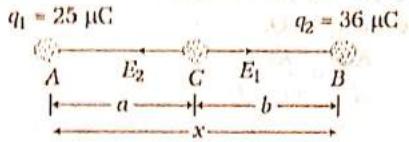
$$\Rightarrow \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{a^2} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{2a^2}\cdot\frac{1}{\sqrt{2}} - \frac{1}{4\pi\varepsilon_0}\cdot\frac{2Qq}{a^2}\times\frac{1}{\sqrt{2}} = 0$$

$$\Rightarrow Q + \frac{Q}{2\sqrt{2}} - \sqrt{2}q = 0 \Rightarrow \quad 2\sqrt{2}Q + Q - 4q = 0$$

$$4q = (2\sqrt{2}+1)Q$$

$$q = \frac{Q}{4}(2\sqrt{2}+1)$$

85. (*a*) Let electric field is zero at point O in the figure.



$\therefore \quad E_1 = E_2$

$\therefore \quad E_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1}{a^2}$

$E_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_2}{b^2}$

Also, $\quad x = a + b$

or $\quad 11 = a + b$

$\therefore \quad b = 11 - a$

Now, $\quad \frac{1}{4\pi\varepsilon_0}\frac{q_1}{a^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_2}{(11-a)^2}$

$\therefore \quad \frac{q_1}{q_2} = \frac{a^2}{(11-a)^2}$

or $\quad \sqrt{\frac{q_1}{q_2}} = \frac{a}{11-a}$ or $\sqrt{\frac{25}{36}} = \frac{a}{11-a}$

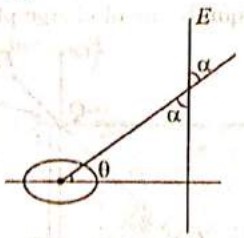
or $\quad \frac{5}{6} = \frac{a}{11-a}$

or $\quad 6a = 55 - 5a$

$\therefore \quad a = 5\,\text{cm}$

So, intensity will be zero at a distance of 5 cm from 25 μC.

86. (*c*) Here, $\alpha + \theta = 90^\circ$



$\tan\alpha = \frac{1}{2}\tan\theta$

or $\quad \tan\theta = 2\tan\alpha$

or $\quad \tan\theta = 2\tan(90^\circ - \theta)$

or $\quad \tan^2\theta = 2$ or $\tan\theta = \sqrt{2}$

$\theta = \tan^{-1}(\sqrt{2})$

87. (*d*) 1 electron has a charge of 1.6×10^{-19} C.

10^{10} electrons would have a charge of

$$q = ne = 1.6\times10^{-19}\times10^{10}$$

$$= 1.6\times10^{-9}\,\text{C}$$

Thus, in 1s charge accumulated $= 1.6\times10^{-9}$ C

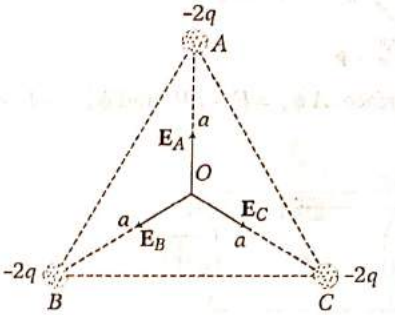
So, time taken to accumulate 1 C

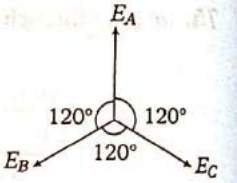
$$= \frac{1}{1.6\times10^{-9}} = 0.625\times10^{9}$$

$$= 6.25\times10^{8}\ \text{s} = 173611\ \text{h}$$

$$= 7233\ \text{days} \approx 20\ \text{yr}$$

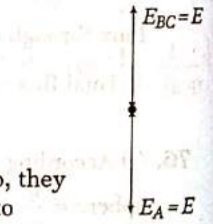
88. (*c*) The resultant of E_B and E_C is





$$= \sqrt{E^2 + E^2 + 2E\cdot E\cos 120^\circ}$$

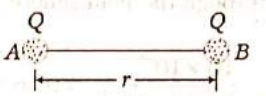
$$= \sqrt{2E^2 + 2E^2\cdot\left(-\frac{1}{2}\right)} = E$$



Now, situation is shown in figure.

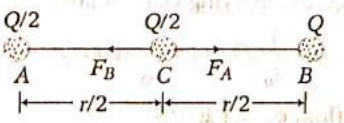
Here, E_A and E_{BC} are equal and opposite so, they cancel out. So, resultant electric field due to E_A, E_B and E_C are zero.

89. (*a*) ***Case* I**



$$F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{r^2} \quad \ldots\text{(i)}$$

***Case* II**



When sphere C is touched to A, then equal charge $Q/2$ distributes on A and C.

$$\therefore \quad F_A = \frac{1}{4\pi\varepsilon_0}\frac{(Q/2)^2}{(r/2)^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{r^2}$$

$$F_B = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(Q)(Q/2)}{(r/2)^2} = 2\cdot\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{r^2}$$

$\therefore$ Net force on C, $F_{net} = F_B - F_A = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{r^2} = F$

90. (*b*) The electric field at C due to charge $+10^{7}$C at A is

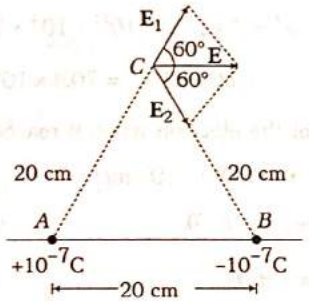
$$\mathbf{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{10^{-7}}{(0.2)^2} \text{ along } AC$$

The electric field at C due to charge -10^{-7}C at B is

$$\mathbf{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{10^{-7}}{(0.2)^2} \text{ along } CB$$

As $\quad |\mathbf{E}_1| = |\mathbf{E}_2|$

By symmetry, the vertical components will cancel out and horizontal components will add.



$\therefore$ The resultant electric field at C is

$$E = 2E_1\cos 60° = 2\times\frac{1}{4\pi\varepsilon_0}\frac{10^{-7}}{(0.2)^2}\times\frac{1}{2}$$

$$= \frac{9\times10^{9}\times10^{-7}}{(0.2)^2} = 2.2\times10^{4}\text{ N/C}$$

91. (*a*) Net field at origin, $E = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{4^2}+\ldots\infty\right]$

$$= \frac{q}{4\pi\varepsilon_0}\left[1+\frac{1}{4}+\frac{1}{16}+\ldots+\infty\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{1-(1/4)}\right]$$

$$= 12\times10^{9}q\text{ N/C}$$

92. (*c*) Number of electrons,

$$n = \frac{6\times10^{23}}{63.5}\times10\times\frac{1}{10^6} = \frac{6\times10^{18}}{63.5}$$

As $\quad q = ne$

$$q = \frac{6\times10^{18}\times1.6\times10^{-19}}{63.5}$$

or $\quad q = 1.5\times10^{-2}$ C

$$F = \frac{9\times10^{9}\times1.5\times10^{-2}\times1.5\times10^{-2}}{\left(\frac{10}{100}\right)^2}$$

$$= 2.0\times10^{8}\text{ N}$$

93. (*a*) The frequency will be same, $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ but due to the constant qE force, the equilibrium position gets shifted by $\frac{qE}{K}$ in forward direction. So option (a) is correct.

94. (*d*) Electric field at the centre of charged circular ring is zero. Hence, electric field at O due to the part $ACDB$ is equal in magnitude and opposite in direction that due to the part AKB.

95. (*a*) The negative charge oscillates, the resultant force acts as a restoring force and proportional to displacement. When it reaches the plane xy, then the resultant force is zero and the mass moves down due to inertia. Thus, oscillation is set.

96. (*a*) Total flux, $\phi_{\text{total}} = \phi_A + \phi_B + \phi_C = \frac{q}{\varepsilon_0}$

$\because \quad \phi_B = \phi$ and $\phi_A = \phi_C = \phi'$ [assumed]

$\therefore \quad 2\phi' + \phi = \frac{q}{\varepsilon_0}$

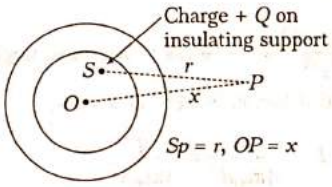
The flux through the plane surface A, $\phi' = \frac{1}{2}\left(\frac{q}{\varepsilon_0} - \phi\right)$

97. (*a*) According to Gauss's theorem,

$$\oint \mathbf{E}\cdot d\mathbf{S} = \frac{1}{\varepsilon_0}Q_{\text{enclosed}}$$

$$E\cdot 4\pi x^2 = \frac{Q}{\varepsilon_0}$$

Electric field of the point P, $E = \frac{Q}{4\pi\varepsilon_0 x^2}$



$Sp = r$, $OP = x$

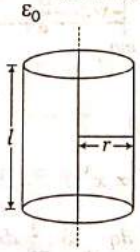
98. (*a*) Charge density of long wire,

$$\lambda = \frac{1}{3}\text{ C-m}$$

From Gauss's theorem,

$$\oint \mathbf{E}\cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$$

$$E\oint dS = \frac{q}{\varepsilon_0} \text{ or } E2\pi rl = \frac{q}{\varepsilon_0}$$



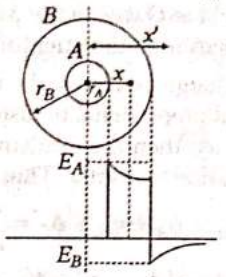
$$\Rightarrow \quad E = \frac{q}{2\pi\varepsilon_0 rl} = \frac{q/l}{2\pi\varepsilon_0 r}$$

$$\Rightarrow \quad \frac{\lambda\times2}{2\pi\varepsilon_0 r\times2} = \frac{\lambda\times2}{4\pi\varepsilon_0 r}$$

The magnitude of the electric intensity,

$$E = 9\times10^{9}\times\frac{1}{3}\times2\times\frac{1}{18\times10^{-2}} = 0.33\times10^{11}\text{ NC}^{-1}$$

99. (*a*) Inside the shell A, electric field $E_{in} = 0$



At the surface of shell A,

$$E_A = \frac{KQ_A}{r_A^2} \longrightarrow \text{[a fixed positive value]}$$

Between the shell A and B, at a distance x from the common centre,

$$E = \frac{K \cdot Q_A}{x^2} \longrightarrow \text{[as } x \text{ increases, } E \text{ decreases]}$$

At the surface of shell B,

$$E_B = \frac{K(Q_A - Q_B)}{r_B^2} \longrightarrow \text{[a fixed negative value because } |Q_A| < |Q_B|]$$

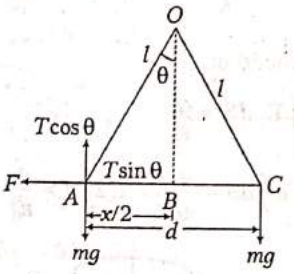
Outside the both shell, at a distance x' from the common centre,

$$E_{out} = \frac{K(Q_A - Q_B)}{x'^2} \longrightarrow \text{[as } x' \text{ increase negative value of } E_{out} \text{ decrease and it becomes zero at } x = \infty]$$

100. (*a*) $T \sin\theta = F = \dfrac{q^2}{4\pi\varepsilon_0 d^2} = \dfrac{q}{4\pi\varepsilon_0 x^2}$

and $T \cos\theta = mg$

$$\therefore \quad \tan\theta = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{x^2 mg}$$



or $\dfrac{x}{2l} = \dfrac{q^2}{4\pi\varepsilon_0 x^2 mg}$

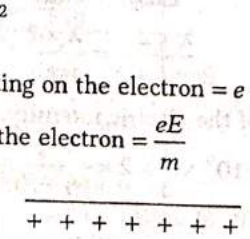
or $\dfrac{x}{2l} \propto \dfrac{q^2}{x^2}$, or $q^2 \propto x^3$

$$\Rightarrow \quad q \propto x^{\frac{3}{2}} \Rightarrow \frac{dq}{dt} \propto \frac{3}{2} x^{\frac{1}{2}} \frac{dx}{dt}$$

$$v \propto x^{-\frac{1}{2}} \qquad \left(\frac{dq}{dt} = \text{constant}\right)$$

101. (*a*) The force acting on the electron $= e \cdot E$.

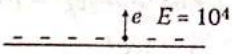
Acceleration of the electron $= \dfrac{eE}{m}$



$$u = 0, v = ?$$

$$\therefore \quad v^2 - u^2 = 2as$$

$$s = 2 \times 10^{-2}\text{m}$$



$$\therefore \quad v^2 = 2\left(\frac{e}{m}\right) E \times 2 \times 10^{-2} \text{ m}$$

$$\frac{e}{m} = 1.76 \times 10^{11} \text{ C/kg}$$

$$\therefore \quad v^2 = 2 \times 1.76 \times 10^{11} \times 10^4 \times 2 \times 10^{-2}$$

$$= 7.04 \times 10^{13} = 70.4 \times 10^{12}$$

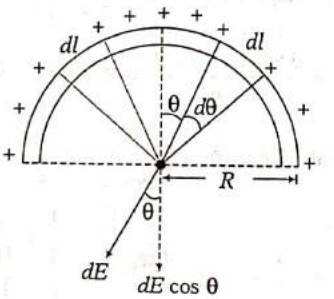
The velocity of the electron when it reaches plate B,

$$v \approx 0.85 \times 10^7 \text{ m/s}$$

102. (*a*) From figure, $dl = R\, d\theta$,

Charge on $dl = \lambda R\, d\theta$ $\left(\because \lambda = \dfrac{q}{\pi R}\right)$

Electric field at centre due to dl is $dE = K\dfrac{\lambda R d\theta}{R^2}$.



We need to consider only the component $dE \cos\theta$, as the component $dE \sin\theta$ will cancel out because of the field at C due to the symmetrical element dl'.

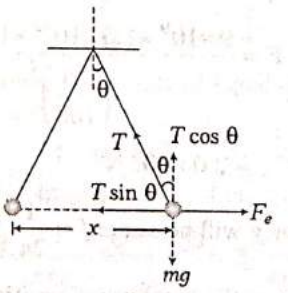
Total field at centre $= 2\int_0^{\pi/2} dE \cos\theta$

$$= \frac{2K\lambda}{R}\int_0^{\pi/2} \cos\theta\, d\theta = \frac{2K\lambda}{R} = \frac{q}{2\pi^2\varepsilon_0 R^2}$$

103. (*a*) In equilibrium,

$$F_e = T \sin\theta \quad \text{...(i)}$$

$$mg = T \cos\theta \quad \text{...(ii)}$$



$$\tan\theta = \frac{F_e}{mg} = \frac{q^2}{4\pi\varepsilon_0 x^2 \times mg}$$

Also, $\tan\theta \approx \sin\theta = \frac{x/2}{L}$

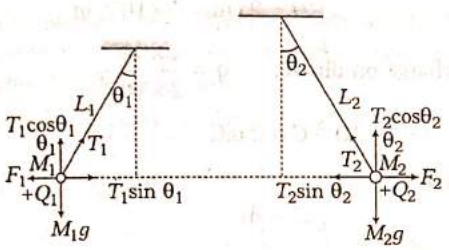
Hence, $\frac{x}{2L} = \frac{q^2}{4\pi\varepsilon_0 x^2 \times mg}$

$\Rightarrow$ $x^3 = \frac{2q^2L}{4\pi\varepsilon_0\, mg} \Rightarrow x = \left(\frac{q^2L}{2\pi\varepsilon_0\, mg}\right)^{1/3}$

104. (*b*) The three forces acting on each sphere are

(i) Tension (ii) Weight

(iii) Electrostatic force of repulsion



For sphere 1, in equilibrium, from figure,

$T_1\cos\theta_1 = M_1g$ and $T_1\sin\theta_1 = F_1$

$\therefore$ $\tan\theta_1 = \frac{F_1}{M_1g}$

For sphere 2, in equilibrium, from figure,

$T_2\cos\theta_2 = M_2g_1$ and $T_2\sin\theta_2 = F_2$

$\therefore$ $\tan\theta_2 = \frac{F_2}{M_2g}$

Force of repulsion between two charges are same.

$\therefore$ $F_1 = F_2$

$\theta_1 = \theta_2$ only, if $\frac{F_1}{M_1g} = \frac{F_2}{M_2g}$

But $F_1 = F_2$, then $M_1 = M_2$

105. (*b*) At a point on the axis of uniformly charged disc at a distance x above the centre of the disc, the magnitude of the electric field is,

$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{x^2+R^2}}\right]$$

$\Rightarrow$ $E_c = \frac{\sigma}{2\varepsilon_0}$, such that $\frac{E}{E_c} = \frac{1}{2}$

Then, $1 - \frac{x}{\sqrt{x^2+R^2}} = \frac{1}{2}$

or $\frac{x}{\sqrt{x^2+R^2}} = \frac{1}{2}$

On squaring both sides and multiplying by $x^2 + R^2$ to obtain

$$x^2 = \frac{x^2}{4} + \frac{R^2}{4}$$

Thus, $x^2 = \frac{R^2}{3} \Rightarrow x = \frac{R}{\sqrt{3}}$

B. Medical entrance special format questions

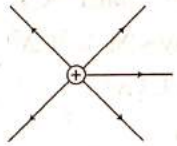
Assertion and reason

2. (*b*) Charge is always conserved but energy is lost in the form of heat.

3. (*e*) Gravitational force is the dominating force in nature. It is the weakest force. Also, Coulomb's force >> gravitational force.

5. (*e*) At the centre of the line joining two equal and opposite charge, $E \neq 0$. For similar charge, $E = 0$.

6. (*e*) Lines are not parallel and equidistant. Therefore, field is non-uniform.



7. (*e*) Electric field at the nearby point will be resultant of existing field and field due to the charge brought. It may increase or decrease, if the charge is positive or negative depending on the position of the point with respect to the charge brought.

8. (*e*) The rate of decrease of electric field is different in the two cases. In case of a point charge, it decreases as $1/r^2$ but in the case of electric dipole, it decreases more rapidly, as $E \propto 1/r^3$.

9. (*d*) If electric lines of force cross each other, then the electric field at the point of intersection will have two direction simultaneously which is not possible physically.

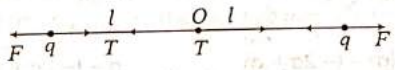
10. (*b*) As $\sigma_1 = \sigma_2$ [given]

$\therefore \frac{q_1}{4\pi r_1^2} = \frac{q_2}{4\pi r_2^2}$, or $\frac{q_1}{q_2} = \frac{r_1^2}{r_2^2}$ [let r_1 and r_2 be two different radii]

Then, the ratio of electric field intensities near the surface of spherical conductor,

$$\frac{E_1}{E_2} = \frac{q_1}{4\pi\varepsilon_0 r_1^2} \times \frac{4\pi\varepsilon_0 r_2^2}{q_2} = \frac{q_1}{q_2} \times \frac{r_2^2}{r_1^2} = 1 \text{ i.e., } E_1 = E_2$$

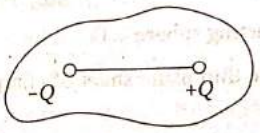
11. (*c*) In equilibrium, figure can be drawn as



$$T = \frac{1}{4\pi\varepsilon_0}\frac{q \cdot q}{(2l)^2}$$

12. (*d*) For a given ring, electric field acts in downward direction. At the centre of uniformly charged ring electric field is zero.

13. (*a*) If a dipole is enclosed by a surface as shown in figure,



then $Q_{enclosed} = 0$

$\therefore$ $\phi = 0$ [from Gauss's law]

14. (*a*) Flux, $\phi_{net} = \frac{q_{in}}{\varepsilon_0}$

If a closed body, is placed in an electric field (either uniform on non-uniform) total flux linked with it will be zero.

i.e., $\phi_{net} = 0$

$\Rightarrow$ $q_{in} = 0$

15. (*c*) With the help of Gauss's theorem, we can find electric flux at any point and it is true that Gauss's theorem can be applied for any symmetric of charge distribution.

Match the columns

1. Electric charge = [AT]

Electric field strength = $[MLT^{-3}A^{-1}]$

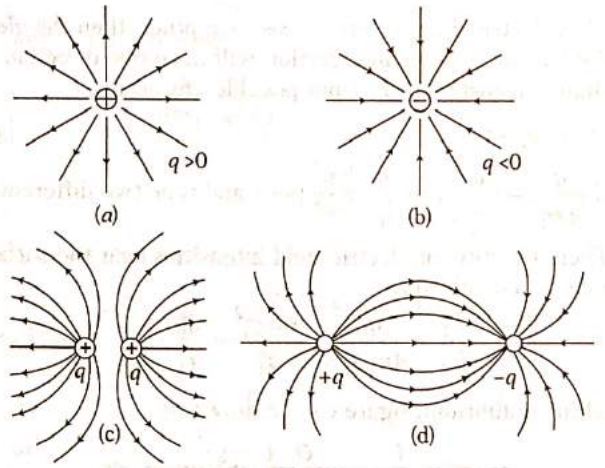
Absolute permittivity = $[M^{-1}L^{-3}T^4A^2]$

Electric dipole = $[M^0L^1T^1A^1]$

2. From Coulomb's law,

$$F = \frac{k\,q_1q_2}{r^2} \text{ or } F \propto \frac{1}{r^2}$$

3. Figure shows the field lines around some simple charge configurations. The field lines are in three-dimensional space curves, though the figure shows them only in a plane. The field lines of a single positive charge are radially outward while those of a single negative charge are radially inward. The field lines around a system of two positive charges (q, q) give a pictorial description of their mutual repulsion.



4. (c) $E_I = \frac{(\sigma) - (-2\sigma + \sigma)}{2\varepsilon_0} = 0$; $E_{II} = \frac{\sigma - (-2\sigma + \sigma)}{2\varepsilon_0} = \frac{\sigma}{E_0}$

$E_{III} = \frac{(\sigma - 2\sigma) - \sigma}{2\varepsilon_0} = -\frac{\sigma}{E_0}$; $E_{IV} = \frac{(\sigma - 2\sigma + \sigma)}{2\varepsilon_0} = 0$

5. E_{out} for non-conducting sphere $= \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$ or $E \propto \frac{1}{r^2}$.

E_{inside} for conducting sphere = 0.

E due to infinite thin plane sheet of charge $= \frac{\sigma}{2\varepsilon_0}$ or $(E \propto r^0)$

E due to hemispherical charged body $= \frac{\sigma}{4\varepsilon_0}$ or $E \propto r^0$

C. Medical entrance gallery

1. (*b*) Torque on an electric dipole in an electric field,

$$\tau = \mathbf{p} \times \mathbf{E}$$

$\Rightarrow$ $|\tau| = pE \sin\theta$

where, θ is angle between **E** and **p**

$\Rightarrow$ $4 = p \times 2 \times 10^5 \times \sin 30°$

$\Rightarrow$ $p = 4 \times 10^{-5}$

$\therefore$ $q2l = 4 \times 10^{-5}$

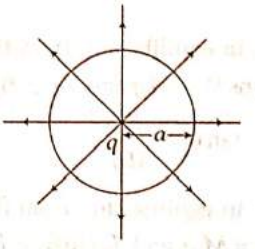
where, $2l = 2\text{ cm} = 2 \times 10^{-2}$ m

$\therefore$ The charge on dipole, $q = \frac{4 \times 10^{-5}}{2 \times 10^{-2}}$

$\Rightarrow$ 2×10^{-3} C = 2 mC

2. (*c*) Given,

$$E = Ar \quad \text{...(i)}$$



Here, $r = a$

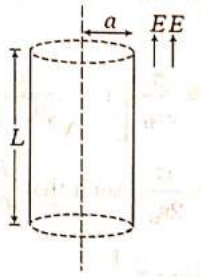
$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{a^2}$$

From Eq. (i), we get

$$\frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{a^2} = Aa$$

$\Rightarrow$ A charge at the origin of the field, $q = 4\pi\varepsilon_0 Aa^3$

3. (*d*) Gaussian surface is given below,



Net flux crossing through surface of cylinder is given by

ϕ = Flux through upper disc + Flux through lower disc + Flux through curved surface

$= E(\pi a^2) + (-E\pi a^2) + 0$

$= \pi a^2 E - \pi a^2 E = 0$

4. (*a*) The field at centre of hemispherical cup is given by

$$E = \frac{\sigma}{4\varepsilon_0} \text{ and } \sigma = \frac{Q}{S}$$

$$\therefore \quad E = \frac{Q}{S \times 4\varepsilon_0}$$

On putting values, we get

$$\Rightarrow \quad 9\times10^8 = \frac{Q}{2\pi R^2 \times 4\varepsilon_0} \Rightarrow 9\times10^8 = \frac{5\times10^{-6}}{4\pi\varepsilon_0 \times 2R^2}$$

$$\Rightarrow \quad 9\times10^8 = \frac{5\times10^{-6}\times9\times10^9}{2R^2}$$

$$\Rightarrow \quad 2R^2 = 5\times10^{-6}\times10 = 5\times10^{-5}$$

$$R = \sqrt{\frac{5}{2}\times10^{-5}} = \sqrt{2.5\times10^{-5}}$$

$$\Rightarrow \quad R = \sqrt{25\times10^{-6}} = 5\times10^{-3}\text{ m} = 5\text{ mm}$$

5. (*b*) At the point P electric field intensity is zero.
So, the electric field intensity on both charges are same.
Let, radius for q_1 charge be r.
So, radius for q_2 charge is $(10-r)$.

$$\frac{1}{4\pi\varepsilon_0}\frac{q_1}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{(10-r)^2}$$

$$\Rightarrow \quad \frac{q_1}{r^2} = \frac{q_2}{(10-r)^2} \Rightarrow \frac{9}{r^2} = \frac{4}{(10-r)^2} \quad (\because q_1 = 9\text{C}, q_2 = 4\text{C})$$

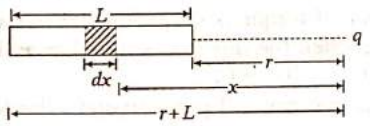
$$\Rightarrow \quad \left(\frac{3}{r}\right)^2 = \left(\frac{2}{10-r}\right)^2 \Rightarrow \frac{3}{r} = \frac{2}{10-r}$$

$$\Rightarrow \quad 3(10-r) = 2r \Rightarrow 30 - 3r = 2r$$

$$\Rightarrow \quad 5r = 30$$

$$\Rightarrow \quad r = 6\text{m}$$

6. (*d*) As a point charge q is situated at a distance r on axis from one end of a thin conducting rod of length L having charge Q as shown in figure.
Total charge $= Q$



Consider an elementary charge dq having length dx at a distance x from the charge q, then

$$dq = \rho dx = \frac{Q}{L}dx$$

Now, force between dq and q

$$dF = \frac{1}{4\pi\varepsilon_0}\frac{q\,dq}{x^2} = K\frac{q\,Q\,dx}{L\,x^2}$$

$$\therefore \text{ Total force, } F = \int dF = \frac{KQq}{L}\int_r^{r+L} x^{-2}dx$$

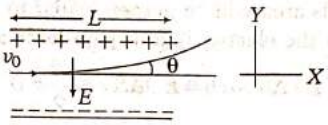
$$= \frac{KQq}{L}\left[-\frac{1}{x}\right]_r^{r+L} = \frac{-KQq}{L}\left[\frac{1}{r+L} - \frac{1}{r}\right]$$

$$= \frac{-KQq}{L}\left[\frac{r-r-L}{(r+L)r}\right] = \frac{KQq}{r(r+L)}$$

7. (*a*) As 10^{19} is removed from the neutral metal.
So, loss of charge $= 1.6\times10^{-19}\times10^{19} = 1.6$ C
So, charge on metal $= +1.6$ C

8. (*c*) Electric field at centre of plastic sheet is the same as at close to the centre of the copper plate.

9. (*b*) Initial velocity of the electron along x-direction,



$$u_x = v_0$$

As applied electric field is vertical,

$$v_x = u_x = v_0 \Rightarrow u_y = 0 \text{ and } u_y = u_y + a_y t$$

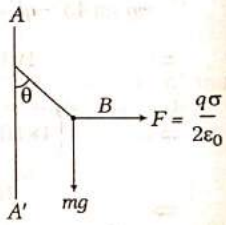
or $$v_y = 0 + \frac{eE}{m}\times\frac{L}{v_0} \qquad [\because L = v_0 t]$$

or $$v_y = \frac{eEL}{mv_0}$$

We can write, $\tan\theta = \dfrac{v_y}{v_x}$

$$\Rightarrow \quad \tan\theta = \frac{eEL}{mv_0}\times\frac{1}{v_0} = \frac{eEL}{mv_0^2} \quad \Rightarrow \theta = \tan^{-1}\left(\frac{eEL}{mv_0^2}\right)$$

10. (*a*) The diagram is as follows



The electric field due to charged infinite conducting sheet is

$$E = \frac{\sigma}{2\varepsilon_0}$$

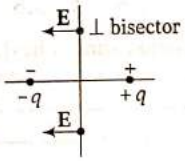
Now, force (electric force) on the charged ball is **F**

$$= qE = \frac{q\sigma}{2\varepsilon_0}$$

The resultant of electric force and mg balance the tension produced in the string.

So, $\tan\theta = \dfrac{F_e}{mg} = \dfrac{q\sigma/2\varepsilon_0}{mg} = \dfrac{q\sigma}{2\varepsilon_0 mg}$

11. (*a*) The angle between the dipole moment and electric field at any point on the equatorial plane is 180°.



12. (*d*) Electric field lines does not form closed loop as line can never start and end on the same charge. So, the option (d) is incorrect.

13. (*b*) $s = ut + \frac{1}{2}at^2 = \frac{1}{2}at^2$ or $t = \sqrt{\frac{2s}{a}}$

As S is same, $\quad t \propto \dfrac{1}{\sqrt{a}} \Rightarrow \dfrac{t_2}{t_1} = \sqrt{\dfrac{a_1}{a_2}} = \sqrt{\dfrac{F_e/M_e}{F_p/M_p}} = \sqrt{\dfrac{M_p}{M_e}}$

14. (*d*) At equatorial, electric field,

$$E = \frac{9\times10^9\times2ql}{(r^2+l^2)^{3/2}}$$

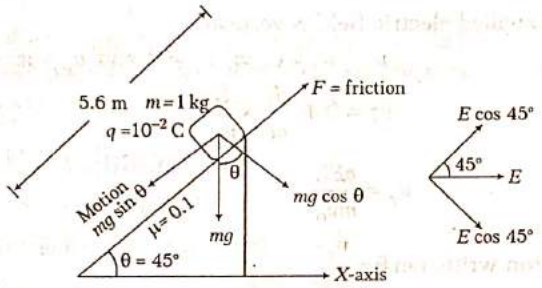
$$E = \frac{9\times10^9\times2\times5\times10^{-2}\times10\times10^{-6}}{[(0.12)^2+(0.05)^2]^{3/2}}$$

Electric field, $E = 4.1\times10^6$ N/C

15. (*c*) Assuming the small area ΔS on the cylindrical surface. The normal to this area will be perpendicular to the axis of the cylinder, but the electric field is parallel to axis.

Hence, $\Delta\phi = \mathbf{E}\cdot\Delta\mathbf{S}\cos\theta = E\cdot\Delta S\cos\frac{\pi}{2} = 0$

16. (*b*) The figure of the above situation is shown below



The electric field, $E = 100\,\text{V m}^{-1}$

For the downward motion of the particle of mass 1kg.

$mg\sin 45° - qE\cos 45° - \mu\,(mg\cos 45° + qE\sin 45°) = ma$

$$\Rightarrow \quad 1\times10\times\frac{1}{\sqrt{2}} - \frac{1}{100}\times100\times\frac{1}{\sqrt{2}} - \frac{1}{10}$$

$$\Rightarrow \quad \left(1\times10\times\frac{1}{\sqrt{2}} + \frac{1}{100}\times100\times\frac{1}{\sqrt{2}}\right) = 1\times a$$

$$\Rightarrow \quad \frac{10}{\sqrt{2}} - \frac{1}{\sqrt{2}} - \frac{1}{10}\left(\frac{10}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = a$$

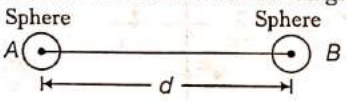
$$\Rightarrow \quad \frac{10}{\sqrt{2}} - \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} - \frac{1}{10\sqrt{2}} = a$$

$$\Rightarrow \quad a = \frac{79}{10\sqrt{2}} = \frac{7.9}{\sqrt{2}}$$

$$\Rightarrow \quad a = 5.6\,\text{ms}^{-2}$$

Now, time taken by the particle to cover 5.6 m distance along the incline plane is $t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2\times5.6}{5.6}} = \sqrt{2} = 1.41$ s

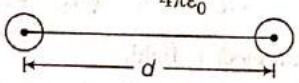
17. (*a*) Consider the situation shown in the diagram.



Let charges on sphere A and B are q_1 and q_2, respectively. Force between the charges placed in air by Coulomb's law (spheres can be assumed as point charges placed at their centres)

$$F_{AB} = K_1\frac{q_1q_2}{d^2} \quad \text{...(i)}$$

where, $K_1 = \frac{1}{4\pi\varepsilon_0}$



When the spheres are immersed in a liquid, force between the charges is $F'_{AB} = \frac{q_1q_2}{4\pi\varepsilon_r d^2} = \frac{q_1q_2}{4\pi\varepsilon_0 K}\times\frac{1}{d^2}$

$$= \frac{1}{(4\pi\varepsilon_0)\times2}\times\frac{q_1q_2}{d^2} = \frac{K_1}{2}\times\frac{q_1q_2}{d^2} \quad \text{...(ii)}$$

where, K is dielectric constant of the liquid $= 2$

From Eqs. (i) and (ii), we get $\frac{F_{AB}}{F'_{AB}} = \frac{K_1}{K_1/2} = 2$

According to question, $F_{AB} = F \Rightarrow \frac{F}{F'_{AB}} = 2 \Rightarrow F'_{AB} = \frac{F}{2}$

18. (*b*) Given, charge on the body,

$$q = 1\text{nC} = 1\times1\times10^{-9}\text{ C} \quad (\because 1\text{ nC} = 1\times10^{-9}\text{C})$$

Charge on the electron $e = 1.6\times10^{-19}$

From the property of quantisation of charge, $q = ne$

$\Rightarrow$ Number of charges, $n = \frac{q}{e} = \frac{1\times10^{-9}}{1.6\times10^{-19}}$

$$= 0.625\times10^{-9}\times10^{19} = 6.25\times10^{9}$$

19. (*a*) We know that, electric field at distance r from an infinitely long line charge is given by

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} \Rightarrow \lambda = 2\pi\varepsilon_0\cdot rE \text{ or } E\propto r^{-1}$$

20. (*c*) We know that,

$$F_1 = q_1E = m_1a_1 \quad \text{...(i)}$$

$$F_2 = q_2E = m_2a_2 \quad \text{...(ii)}$$

(force experienced by charge in electric field)

As $\quad q_1 = q_2 \Rightarrow F_1 = F_2$

From Eqs. (i) and (ii), we get

$$\frac{a_1}{a_2} = \frac{F_1/m_1}{F_2/m_2} = \frac{F\cdot m_2}{F\cdot m_1}$$

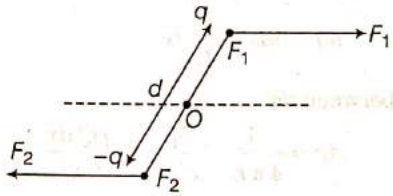
$$\Rightarrow \quad \frac{a_1}{a_2} = \frac{m_2}{m_1} = \frac{1}{0.5} = \frac{10}{5} = 2 \quad \left(\because \frac{m_1}{m_2} = 0.5\right)$$

21. (*c*) Torque on dipole form, $\tau = p\times E\sin\theta$

The direction of torque is perpendicular to the plane of paper inward *i.e.*, when the dipole is parallel or anti-parallel to **E** and maximum at $\theta = 90°$.

So, torque is maximum if **p** is perpendicular to **E**.

22. (*a*) Consider the diagram, where an electric dipole is placed in non-uniform electric field.



Electric field at the sight of q is E_1

and electric field at the sight of $-q$ is E_2

Net force on the dipole, $|E_1| \neq |E_2|$.

F_1 = force on the charge $q = qE_1$.

F_2 = force on the charge $-q = -qE_2$.

Net force on the dipole,

$$F = F_1 + F_2 = q(E_1 - E_2)$$

$$\therefore \quad |E_1| \neq |E_2| \Rightarrow |F| \neq 0$$

τ_1 = torque on the dipole due to $E_1 = p\times E_1$ [clockwise]

τ_2 = torque on the dipole due to $E_2 = p\times E_1$ [clockwise]

where, $p = \frac{1}{2}qd$ is dipole moment of the dipole,

$$\tau_{net} = \tau_1 + \tau_2 = p\times E_1 + p\times E_2 \neq 0$$

Hence, both torque (τ_{net}) and force (F) act on the dipole.

23. (*d*) The Gauss's law in electrostatics gives a relation between electric flux through any closed hypothetical surface (called a Gaussian surface) and the charge enclosed by the surface. So, the nature of Gaussian surface is vector.

24. (*b*) $\tan\theta = \frac{F}{mg} \Rightarrow \frac{r/2}{y} = \frac{kq^2}{r^2 mg} \Rightarrow y \propto r^3$

Therefore, $\left(\frac{r'}{r}\right)^3 = \frac{y/2}{y} \Rightarrow r' = r\left(\frac{1}{2}\right)^{1/3}$

25. (*d*) Electric lines of force forms open loops while magnetic lines of force forms closed loops.

26. (*d*) From Coulomb's law, force due to same charge is repulsive and that due to opposite is attractive.

$$F = K\left[\frac{2\times 3}{3^2} - \frac{2\times 3}{3^2}\right] = 0$$

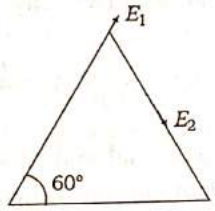
27. (*b*) In same medium, the force between two protons separated the force will F, as it is independent of the masses but depends on the charge of the particle.

28. (*d*) We know that total flux is through all its six faces is

$$\phi_T = \frac{q}{\varepsilon_0}$$

But electric flux through one face of cube, $\phi_T = \frac{q}{6\varepsilon_0}$.

30. (*c*)

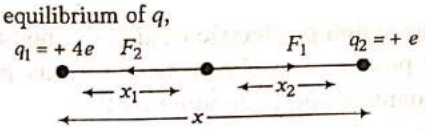


Here, $E_1 = E_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{a^2} \Rightarrow E' = \sqrt{E_1^2 + E_2^2 + 2E_1E_2\cos\theta}$

Given, $\theta = 60°$

$$E' = \sqrt{3E^2} \Rightarrow E' = \frac{q\sqrt{3}}{4\pi\varepsilon_0 a^2}$$

31. (*b*) For equilibrium of q,



$$|F_1| = |F_2|$$

$$\frac{1}{4\pi\varepsilon_0}\frac{qq_1}{x_1^2} = \frac{1}{4\pi\varepsilon_0}\frac{qq_2}{x_2^2}$$

$$x_2 = \frac{x}{\sqrt{\frac{q_1}{q_2}} + 1} = \frac{x}{\sqrt{\frac{4e}{e}} + 1} = \frac{x}{3}$$

32. (*c*) As the mass is moving in the electric field, then

$$ma = qE$$

$$\Rightarrow \quad a = \frac{qE}{m} = \frac{3\times 10^{-3}}{20\times 10^{-3}} = 12\ \text{ms}^{-2}$$

By using, $v = u + at$

$$v = 20 + 12\times 3 = 56\ \text{ms}^{-1}$$

33. (*c*) The electric field at the point, $x = -a$,

$$\mathbf{E}_{out} = \frac{\lambda}{2\pi\varepsilon_0 a}(-\hat{\mathbf{i}})$$

34. (*c*) The electric field is due to all charges present whether inside or outside the given surface. So option (c) is correct.

35. (*b*) Charge enclosed $= q/8$

Therefore, flux $\phi = \frac{q_{enclosed}}{\varepsilon_0}$

$$\phi = \frac{q}{8\varepsilon_0}$$

36. (*a*) Electric flux is equal to the product of an area element and the perpendicular component of **E**. As the surface is lying in yz-plane,

$\therefore \quad \mathbf{E}\cdot d\mathbf{A} = \phi = (5)(20) = 100$ unit

37. (*a*) $F = \frac{Kq(Q-q)}{r^2}$ ...(i)

On differentiating of Eq. (i) w.r.t. q, we get

$$\frac{dF}{dq} = \frac{d}{dq}\left[\frac{Kq}{r^2}(Q-q)\right]$$

or $$\frac{dF}{dq} = \frac{K}{r^2}\frac{d}{dq}[Qq - q^2]$$

or $$\frac{dF}{dq} = \frac{K}{r^2}[Q - 2q] \quad \text{...(ii)}$$

But we know that, when force is maximum, then, $\frac{dF}{dq} = 0$

Then, from Eq. (ii), we have

$$\frac{K}{r^2}[Q - 2q] = 0 \quad \text{or} \quad Q - 2q = 0$$

or $$Q = 2q \quad \text{or} \quad q = \frac{Q}{2}$$

So, the other part $= Q - \frac{Q}{2} = \frac{Q}{2}$

Hence, the each part have the same charge $\frac{Q}{2}$.

CHAPTER

02

Electrostatic potential and capacitance

Inside

1) Electric potential
- Electrostatic potential due to a point charge
- Electrostatic potential due to a system of charge
- Electric potential due to a continuous charge distribution
- Electric potential due to electric dipole
- Relation between electric field and potential

2) Equipotential surfaces
- Variation of electric potential on the axis of a charged ring
- Potential due to spherical charge
- Electric potential due to a group of concentric shells

3) Electric potential energy
- Potential energy of a system of charges
- Equilibrium of dipole

4) Electrostatics of conductors
- Dielectrics and polarisation
- Capacitors and capacitance
- Parallel plate capacitor

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- Special method to solve combination of capacitors
- Kirchhoff's law for capacitor circuits

The electric field around a charge can be described into two ways by an electric field (**E**) and by electrostatic or electric potential (V).

The electric field (**E**) is a vector quantity and we have already discussed about electric field in detail in the previous chapter.

In this chapter, we will study about the electric potential and how these quantities are interrelated to each other. We will also study about capacitor, *i.e.*, a device used for storing electric energy.

1 Electric potential

The electric potential at any point in the region of electric field is defined as the amount of work done in bringing a unit positive test charge from infinity to that point along any arbitrary path. It is a scalar quantity and is denoted by V.

$$\boxed{\text{Electric potential } (V) = \frac{\text{Work done } (W)}{\text{Charge } (q_0)}}$$

According to the nature of charge, electric potential can be positive (due to positive charge), negative (due to negative charge) or zero.

Unit and dimensional formula of electric potential

Its SI unit is volt (V) and 1 volt $= \dfrac{1 \text{ joule}}{1 \text{ coulomb}}$

and CGS unit is stat volt (esu), where 1 volt $= \dfrac{1}{300}$ stat volt

Its dimensional formula is $[ML^2T^{-3}A^{-1}]$.

Example 2.1 *How much work will be done in bringing a charge of 400 μC from infinity to some point P in the region of electric field? Given that the electric potential at point P is 20V.*

Sol. As electric potential at any point can be written as

$$\text{Electric potential } (V) = \frac{\text{work done } (W)}{\text{charge } (q_0)} \quad \text{... (i)}$$

Given, $V_P = 20\text{V}$ and $q_0 = 400\ \mu\text{C} = 400 \times 10^{-6}\text{C}$

On putting the values in Eq. (i), we get

$$20 = \frac{W}{400 \times 10^{-6}}$$

$$\therefore \quad W = 20 \times 400 \times 10^{-6} = 8 \times 10^{-3}\text{J}$$

Example 2.2 *Find the work done by some external force in moving a charge $q = 2\ \mu C$ from infinity to a point, where electric potential is 10^4 V.*

Sol. Given, charge $q = 2\ \mu\text{C}$ and electric potential $V = 10^4$ V

Using the relation,

$$W = qV$$

We have, $$W = (2 \times 10^{-6})(10^4) = 2 \times 10^{-2}\ \text{J}$$

Electric potential difference

The electric potential difference between two points A and B is equal to the work done by the external force in moving a unit positive charge against the electrostatic force from point B to A along any path between these two points.

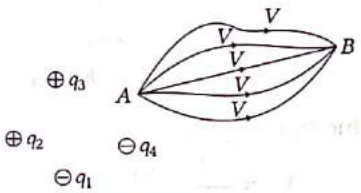


Fig 2.1 Electric potential between points A and B

If V_A and V_B be the electric potential at point A and B respectively, then $\Delta V = V_A - V_B$

or $$\boxed{\Delta V = \frac{W_{AB}}{q}}$$

The SI unit of potential difference is volt (V).

The dimensional formula for electric potential difference is given by $[ML^2T^{-3}A^{-1}]$.

Note Following three formulae are very useful in the problems related to work done in electric field.

$$(W_{a-b})_{\text{electric force}} = q_0 (V_a - V_b)$$
$$(W_{a-b})_{\text{external force}} = q_0 (V_b - V_a) = -(W_{a-b})_{\text{electric force}}$$
$$(W_{\infty - a})_{\text{external force}} = q_0 V_a$$

Here, q_0, V_a and V_b are to be substituted with sign.

Example 2.3 *The electric potential at point A is 20 V and at B is – 40 V. Find the work done by an external force and electrostatic force in moving an electron slowly from B to A.*

Sol. Here, the test charge is an electron, *i.e.*,

$$q_0 = -1.6 \times 10^{-19}\ \text{C}$$
$$V_A = 20\ \text{V}$$
and $$V_B = -40\ \text{V}$$

Work done by external force

$$(W_{B-A})_{\text{external force}} = q_0 (V_A - V_B)$$
$$= (-1.6 \times 10^{-19})[(20) - (-40)]$$
$$= -9.6 \times 10^{-18}\ \text{J}$$

Work done by electric force

$$(W_{B-A})_{\text{electric force}} = -(W_{B-A})_{\text{external force}}$$
$$= -(-9.6 \times 10^{-18}\ \text{J})$$
$$= 9.6 \times 10^{-18}\ \text{J}$$

Note We have covered this in theory but do not add after example here, we can see that the electron (a negative charge) moves from B (lower potential) to A (higher potential) and the work done by electric force is positive. Therefore, we may conclude that whenever a negative charge moves from a lower potential to higher potential work done by the electric force is positive or when a positive charge moves from lower potential to higher potential the work done by the electric force is negative.

Electrostatic potential due to a point charge

Let P be the point at a distance r from the origin O at which the electric potential due to charge $+q$ is required.

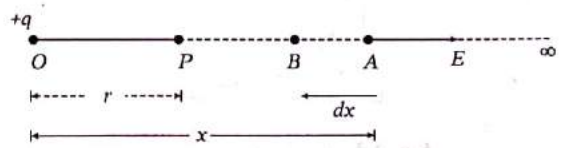


Fig. 2.2 Electrostatic potential at point due to a charge

The electric potential at a point P is the amount of work done in carrying a unit positive charge from ∞ to point P.

Suppose a test charge q_0 is placed at point A at distance x from O.

The electrostatic force acting on charge q_0 is given by

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_0}{x^2}, \quad \text{along } OA \quad \text{...(i)}$$

Small work done in moving the charge through a distance dx from A to B is given by

$$dW = \mathbf{F} \cdot d\mathbf{x} = F dx \cos 180° = -F dx$$
$$[\because \cos 180° = -1]$$

Total work done in moving a positive test charge from ∞ to the point P is given by

$$W = \int_{\infty}^{r} -F dx = \int_{\infty}^{r} -\frac{1}{4\pi\varepsilon_0}\frac{qq_0}{x^2}\,dx$$

$$= -\frac{qq_0}{4\pi\varepsilon_0}\int_{\infty}^{r} x^{-2}dx = -\frac{qq_0}{4\pi\varepsilon_0}\left[\frac{-1}{x}\right]_{\infty}^{r}$$

$$= \frac{qq_0}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{1}{\infty}\right] \qquad \left(\because \int x^{-2}\,dx = -\frac{1}{x}\right)$$

$$W = \frac{qq_0}{4\pi\varepsilon_0 r} \qquad \text{...(iii)}$$

From the definition of electric potential,

$$\therefore \qquad \boxed{V = \frac{W}{q_0} = \frac{q}{4\pi\varepsilon_0 r}} \qquad \text{...(iv)}$$

A positively charged particle produces a positive electric potential. A negatively charged particle produces a negative electric potential.

Here, we assume that electrostatic potential is zero at infinity. Eq. (iv) shows that at equal distances from a point charge q, value of V is same.

Hence, electrostatic potential due to a single charge is spherically symmetric.

Figure given below shows the variation of electrostatic potential with distance, *i.e.*, $V \propto \frac{1}{r}$.

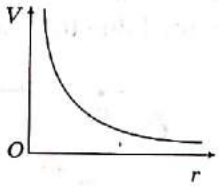


Fig. 2.3 *Variation of v w.r.t. r*

Example 2.4 *Determine the potential at a point 0.50 m (i) from a + 20 μC point charge (ii) from a −20 μC point charge.*

Sol. The potential due to a point charge is given by $V = \frac{kq}{r}$.

(i) At a distance of 0.50 m from a positive 20 μC charge, the potential,

$$V = \frac{kq}{r} = (9.0 \times 10^9\ \text{N-m}^2/\text{C}^2)\left(\frac{20 \times 10^{-6}\ \text{C}}{0.50\ \text{m}}\right)$$

$$= 3.6 \times 10^5\ \text{V}$$

(ii) For the negative charge,

$$V = (9.0 \times 10^9\ \text{N-m}^2/\text{C}^2)\left(\frac{-20 \times 10^{-6}\ \text{C}}{0.50\ \text{m}}\right)$$

$$= -3.6 \times 10^5\ \text{V}$$

Example 2.5 *What is the electrostatic potential at the surfac a silver nucleus of diameter 12.4 fermi? Atomic number (Z) silver is 47.*

Sol. Given, radius of silver nucleus, $r = \frac{12.4}{2} = 6.2$ fm

$= 6.2 \times 10^{-15}$ m

[1 fermi $= 10^{-15}$

and $\qquad Z = 47$

$\therefore \qquad$ Charge, $q = Ze = 47 \times 1.6 \times 10^{-19}$ C

$\therefore$ Electrostatic potential,

$$V = \frac{q}{4\pi\varepsilon_0 r} = \frac{9 \times 10^9 \times 47 \times 1.6 \times 10^{-19}}{6.2 \times 10^{-15}}$$

$$= 1.09 \times 10^7\ \text{V}$$

Electrostatic potential due to a system of charges

Let there be a number of point charges $q_1, q_2, q_3, \ldots, q_n$ a distances $r_1, r_2, r_3, \ldots, r_n$ respectively from the point P, where electric potential is to be calculated.

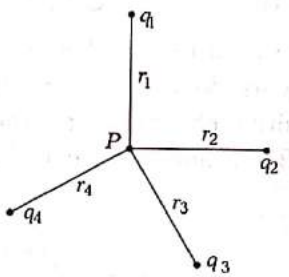


Fig. 2.4 *A system of charges*

Potential at P due to charge q_1,

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{1P}}$$

Similarly,

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{2P}}$$

$$V_3 = \frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_{3P}}$$

$$\vdots \qquad \vdots$$

$$V_n = \frac{1}{4\pi\varepsilon_0}\frac{q_n}{r_{nP}}$$

Using superposition principle, we obtain resultant potential at P due to total charge configuration as the algebraic sum of the potentials due to individual charges.

$$V = V_1 + V_2 + V_3 + \cdots + V_n$$

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_{1P}} + \frac{q_2}{r_{2P}} + \frac{q_3}{r_{3P}} + \cdots + \frac{q_n}{r_{nP}}\right)$$

$$\Rightarrow \quad \boxed{V = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_{iP}}}$$

The net potential at a point due to multiple charges is equal to the algebraic sum of the potentials due to individual charges at that particular point.

Mathematically, it is expressed as

$$V_{net} = \sum_{i=1}^{n} V_i$$

Note If $r_1, r_2, r_3, \ldots, r_n$ are position vectors of the charges $q_1, q_2, q_3, \ldots, q_n$ respectively, then electrostatic potential at point P whose position vector is $\mathbf{r}_0$ would be

$$V = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{|\mathbf{r}_0 - \mathbf{r}_i|}$$

Example 2.6 *Three point charges $q_1 = 1\ \mu C$, $q_2 = -2\ \mu C$ and $q_3 = 3\ \mu C$ are placed at (1 m, 0, 0), (0, 2m, 0) and (0, 0, 3 m) respectively. Find the electric potential at origin.*

Sol. The net electric potential at origin is,

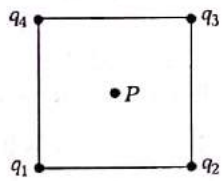
$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3}\right]$$

Substituting the values, we have

$$V = (9.0 \times 10^9)\left(\frac{1}{1.0} - \frac{2}{2.0} + \frac{3}{3.0}\right) \times 10^{-6}$$

$$= 9.0 \times 10^3\ \text{V}$$

Example 2.7 *In the given figure, there are four point charges placed at the vertices of a square of side, $a = 1.4$ m. If $q_1 = +18\ nC$, $q_2 = -24\ nC$, $q_3 = +35\ nC$ and $q_4 = +16\ nC$, then find the electric potential at the centre P of the square. Assume the potential to be zero at infinity.*



Sol. The distance of the point P from each charge is

$$r = \frac{a}{\sqrt{2}} = \frac{1.4\ \text{m}}{1.4} = 1\ \text{m}$$

Potential, $V = V_1 + V_2 + V_3 + V_4$

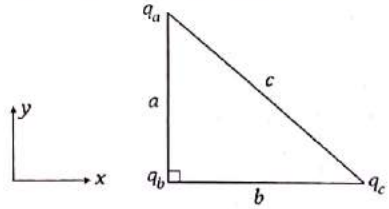
$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \frac{q_4}{r_4}\right]$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 + q_2 + q_3 + q_4}{4}$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{(18 - 24 + 35 + 16) \times 10^{-9}\ \text{C}}{1\ \text{m}}$$

$$= (9 \times 10^9) \times (45 \times 10^{-9})\ \text{V} = 405\ \text{V}$$

Example 2.8 *Suppose that three point charges, q_a, q_b and q_c are arranged at the vertices of a right-angled triangle, as shown. What is the absolute electric potential at the position of the third charge, if $q_a = -6.0\ \mu C$, $q_b = +4.0\ \mu C$, $q_c = +2.0\ \mu C$, $a = 4.0$ m and $b = 3.0$ m?*



Sol. The electric potential at P (the position of the third charge) due to the presence of the first charge is

$$V_a = k_e\frac{q_a}{c} = (9 \times 10^9)\frac{(-6 \times 10^{-6})}{\sqrt{4^2 + 3^2}} = -1.08 \times 10^4\ \text{V}$$

Likewise, the electric potential due to the presence of the second charge is

$$V_b = k_e\frac{q_b}{b} = (9 \times 10^9)\frac{(4 \times 10^{-6})}{(3)} = 1.20 \times 10^4\ \text{V}$$

The net potential of the third charge V_c is simply the algebraic sum of the potentials due to the others two charges taken in isolation. Thus,

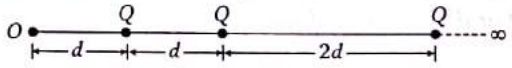
$$V_c = V_a + V_b = 1.20 \times 10^3\ \text{V}$$

Example 2.9

(i) The charges each Q are placed, at $x = d, 2d, 4d, \ldots, \infty$. Find electric potential at origin O.

(ii) A charge + Q is placed at each of the points $x = d$, $x = 3d$, $x = 5d, \ldots, \infty$ on the X-axis, and a charge $-Q$ is placed at each of the points, $x = 2d$, $x = 4d$, $x = 6d, \ldots, \infty$. Find the electric potential at the origin O.

Sol. (i)



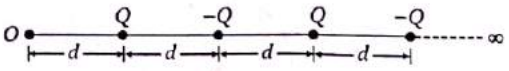
Electric potential,

$$V_O = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{d} + \frac{Q}{2d} + \frac{Q}{4d} + \ldots\infty\right]$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{d}\left[1 + \frac{1}{2} + \frac{1}{4} + \ldots\infty\right]$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{d}\left[\frac{1}{1 - 1/2}\right] = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Q}{d}$$

(ii)



Electric potential,

$$V_O = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{d} - \frac{Q}{2d} + \frac{Q}{3d} - \frac{Q}{4d} + \ldots\infty\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{d}\left[1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots\infty\right]$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{d}\log_e 2$$

Electric potential due to a continuous charge distribution

We can imagine that a continuous charge distribution consists of a number of small charge elements located at position $\mathbf{r}_i$. If $\mathbf{r}$ is the position vector of point P, then the electric potential at point P due to the continuous charge distribution can be written as

$$V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{|\mathbf{r}-\mathbf{r}_i|}$$

When the charge is distributed continuously in a volume V, $dq = \rho\, dV$, where ρ is volume charge density. The potential at point P due to the **volume charge distribution** will be

$$V_V = \frac{1}{4\pi\varepsilon_0}\int_V \frac{\int \rho\, dV}{|\mathbf{r}-\mathbf{r}_i|}$$

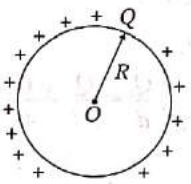
When the charge is distributed continuously over an area S, $dq = \sigma dS$, where σ is **surface charge density.**

$$\therefore \qquad V_S = \frac{1}{4\pi\varepsilon_0}\int_S \frac{\sigma dV}{|\mathbf{r}-\mathbf{r}_i|}$$

When the charge is distributed uniformly along a line L, $dq = \lambda dL$, where λ is **line charge density.**

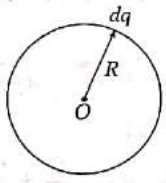
$$\therefore \qquad V_L = \frac{1}{4\pi\varepsilon_0}\int_L \frac{\lambda\, dL}{|\mathbf{r}-\mathbf{r}_i|}$$

Example 2.10 *A charge Q is distributed uniformly on a ring of radius R as shown in the following diagrams. Find the electric potential at the centre O of the ring.*



Charge on whole ring

Sol. Consider a small charge dq. Electric potential at O due to dq,



$$dV = \frac{1}{4\pi\varepsilon_0}\cdot\frac{dq}{R}$$

$$V_0 = \Sigma\, dV = \frac{1}{4\pi\varepsilon_0 R}\Sigma dq = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R}$$

Electric potential due to electric dipole

We determine the potential due to an electric dipole.

Let AB be an electric dipole of length $2a$ and let P be any point, where $OP = r$.

Let θ be the angle between r and the dipole axis.

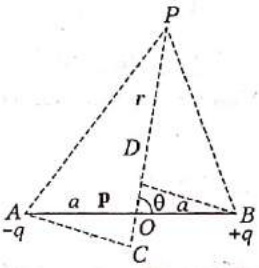


Fig. 2.5

$$AB = 2a,\ AO = OB = a,\ OP = r$$

In ΔOAC, $\qquad \cos\theta = \dfrac{OC}{OA} = \dfrac{OC}{a}$

$\therefore \qquad OC = a\cos\theta$

Also, $\qquad OD = a\cos\theta$

If $r >> a$, $PA = PC = OP + OC = r + a\cos\theta$

$$PB = PD = OP - OD = r - a\cos\theta$$

V is the potential due to electric dipole,

$$V = \left(\frac{1}{4\pi\varepsilon_0}\right)\left[\frac{q}{PB} - \frac{q}{PA}\right]$$

$$V = \left(\frac{1}{4\pi\varepsilon_0}\right)q\left[\frac{1}{(r - a\cos\theta)} - \frac{1}{(r + a\cos\theta)}\right]$$

$$V = \left(\frac{1}{4\pi\varepsilon_0}\right)\left[\frac{2a\,q\cos\theta}{r^2 - a^2\cos\theta}\right]$$

$$V = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{p\cos\theta}{(r^2 - a^2\cos^2\theta)}$$

If $r >> a$, $a^2\cos^2\theta$ can be neglected in comparison to r^2

Then, the resultant potential at point P,

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}}$$

where, p is dipole moment.

Special Cases

(i) When the point P lies on the axial line of the dipole on the side of positive charge, $\theta = 0° \Rightarrow \cos\theta = 1°$

$$\therefore \qquad V = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{p}{(r^2 - a^2)}$$

If r is very large as compared to $2l$, $r >> 2a$, then a^2 can be neglected in comparison to a^2, then potential at the point P,

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{r^2}$$

(ii) When P lies on other side,

$$\theta = 180° \Rightarrow \cos\theta = -1$$

$$\therefore \quad V = -\left(\frac{1}{4\pi\varepsilon_0}\right)\frac{p}{(r^2 - a^2)}$$

(iii) When the point P lies on equatorial line, $\theta = 90°$

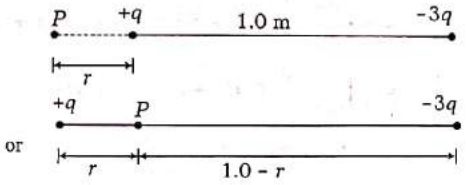
$$\Rightarrow \quad \cos\theta = 0$$

$$\therefore \quad V = 0$$

(iv) In general, $V = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{\mathbf{p}\cdot\mathbf{r}}{r^3}$, for a short dipole, at a point whose position vector with respect to dipole is $\mathbf{r}$.

Example 2.11 *Find out the points on the line joining two charges + q and − 3q (kept at a distance of 1.0 m), where electric potential is zero.*

Sol. Let P be the point on the axis either to the left or to the right of charge $+q$ at a distance r where potential is zero. Hence,



$$V_P = \frac{q}{4\pi\varepsilon_0 r} - \frac{3q}{4\pi\varepsilon_0(1+r)} = 0$$

Solving this, we get $r = 0.5$ m

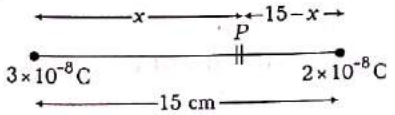
Further, $$V_P = \frac{q}{4\pi\varepsilon_0 r} - \frac{3q}{4\pi\varepsilon_0(1-r)} = 0$$

which gives, $r = 0.25$ m

Thus, the potential will be zero at point P on the axis which is either 0.5 m to the left or 0.25 m to the right of charge $+q$.

Example 2.12 *Two charges 3×10^{-8} C and 2×10^{-8} C are located 15 cm apart. At what point on the line joining the two charges in the electric potential zero? Take the potential at infinity to be zero.*

Sol. Let us take the origin O at the location of the positive charge. The line joining the two charges is taken to be the X-axis; the negative charge is taken to be on the right side of the origin.



Let P be the required point on the X-axis, where the potential is zero. If x is the x-coordinate of P, obviously x must be positive. (There is no possibility of potentials due to the two charges adding up to zero for $x < 0$.) If x lies between O and A, we have

$$\frac{1}{4\pi\varepsilon_0}\left[\frac{3\times10^{-8}}{x\times10^{-2}} - \frac{2\times10^{-8}}{(15-x)\times10^{-2}}\right] = 0$$

where, x is in cm, *i.e.*,

$$\frac{3}{x} - \frac{2}{15-x} = 0$$

which gives, $x = 9$ cm

If x lies on the extended line OA, the required condition is

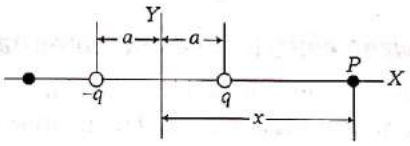
$$\frac{3}{x} - \frac{2}{x-15} = 0$$

which gives,

$$x = 45 \text{ cm}$$

Thus, electric potential is zero at 9 cm and 45 cm away from the positive charge on the side of the negative charge. Note that the formula for potential used in the calculation required choosing potential to be zero at infinity.

Example 2.13 *An electric dipole consists of two charges of equal magnitude and opposite sign separately by a distance 2a as shown in figure. The dipole is along the X-axis and is centred at the origin.*



(i) Calculate the electric potential at point P.

(ii) Calculate V at a point far from the dipole.

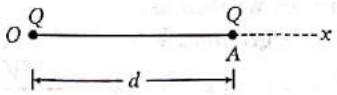
Sol. (i) For a point in figure,

$$V = k_e \Sigma \frac{q_i}{r_i} = k_e\left(\frac{q}{x-a} - \frac{q}{x+a}\right) = \frac{2k_e qa}{x^2 - a^2}$$

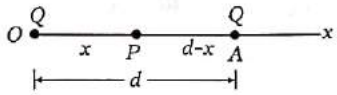
(ii) If point P is far from the dipole, such that $x >> a$, then a^2 can be neglected in the terms, $x^2 - a^2$ and V becomes

$$V = \frac{2k_e qa}{x^2} \qquad [x >> a]$$

Example 2.14 *Sketch the variation of electric potential on the x-axis with respect to x for $x = -\infty$ to $x = +\infty$ in the following cases.*



Sol.



Electric potential at P,

$$V_P = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{x} + \frac{Q}{d-x}\right] = \frac{Q}{4\pi\varepsilon_0} \cdot \frac{d}{x(d-x)}$$

V_P is minimum, if $x(d-x)$ is maximum.

$x(d-x)$ is maximum, if $x = d - x \Rightarrow x = d/2$.

At mid-point of OA, electric potential is minimum.

At $O: x \to 0, V \to \infty$

At $A: x \to d, V \to \infty$

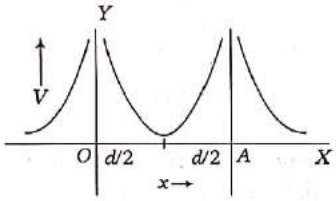
From O to A, electric potential decreases reaching to minimum value and then increases.

Left of O: At distance x from O,

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{x} + \frac{Q}{d+x}\right]$$

$x \to 0, \ V \to \infty$

$x \to -\infty, V \to 0$



Right of A: At distance x from O,

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{x} + \frac{Q}{x-d}\right]$$

$x \to d, \ V \to \infty$

$x \to \infty, \ V \to 0$

Relation between electric field and potential

Let us first consider the case when electric potential V is known and we want to calculate $\mathbf{E}$. The relation is as under,

In case of cartesian coordinates

$$\mathbf{E} = E_x\hat{\mathbf{i}} + E_y\hat{\mathbf{j}} + E_z\hat{\mathbf{k}}$$

Here, $E_x = -\dfrac{\partial V}{\partial x} = -$ (partial derivative of V w.r.t. x)

$E_y = -\dfrac{\partial V}{\partial y} = -$ (partial derivative of V w.r.t. y)

$E_z = -\dfrac{\partial V}{\partial z} = -$ (partial derivative of V w.r.t. z)

$$\therefore \quad \mathbf{E} = -\left[\frac{\partial V}{\partial x}\hat{\mathbf{i}} + \frac{\partial V}{\partial y}\hat{\mathbf{j}} + \frac{\partial V}{\partial z}\hat{\mathbf{k}}\right]$$

This is also sometimes written as,

$$\mathbf{E} = -\text{ gradient } V$$

$$= -\text{ grad } V = -\nabla V = -\frac{dV}{dr}$$

$$\boxed{\mathbf{E} = -\frac{dV}{dr}}$$

Then, negative sign shows that the potential decreases in the direction of electric field.

Potential gradient is a vector quantity.

Example 2.15 *The electric potential in a region is represented*

$$V = 2x + 3y - z$$

Obtain expression for electric field strength.

Sol. $$\mathbf{E} = -\left[\frac{\partial V}{\partial x}\hat{\mathbf{i}} + \frac{\partial V}{\partial y}\hat{\mathbf{j}} + \frac{\partial V}{\partial z}\hat{\mathbf{k}}\right]$$

Here, $$\frac{\partial V}{\partial x} = \frac{\partial}{\partial x}(2x + 3y - z) = 2$$

$$\frac{\partial V}{\partial y} = \frac{\partial}{\partial y}(2x + 3y - z) = 3$$

$$\frac{\partial V}{\partial z} = \frac{\partial}{\partial z}(2x + 3y - z) = -1$$

Electric field, $\mathbf{E} = -2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$

Example 2.16 *Find V_{ab} in an electric field*

$$\mathbf{E} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})\frac{N}{C}$$

where, $\mathbf{r}_a = (\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}})\,m$

and $\mathbf{r}_b = (2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}})\,m$

Sol. Here, the given field is uniform (constant). So using,

$$dV = -\mathbf{E}\cdot d\mathbf{r}$$

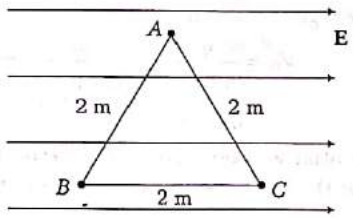
or $$V_{ab} = V_a - V_b = -\int_b^a \mathbf{E}\cdot d\mathbf{r}$$

$$= -\int_{(2,1,-2)}^{(1,-2,1)} (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})\cdot(dx\,\hat{\mathbf{i}} + dy\,\hat{\mathbf{j}} + d$$

$$= -\int_{(2,1,-2)}^{(1,-2,1)} (2\,dx + 3\,dy + 4\,dz)$$

$$= -[2x + 3y + 4z]_{(2,1,-2)}^{(1,-2,1)} = -1 \text{ V}$$

Example 2.17 *In uniform electric field, $\mathbf{E} = 10\ NC^{-1}$ as shou in figure, find*



(i) $V_A - V_B$ *(ii)* $V_B - V_C$

Sol. (i) $V_B > V_A$, so $V_A - V_B$ will be negative.

Further, $d_{AB} = 2\cos 60° = 1$ m

$\therefore \quad V_A - V_B = -Ed_{AB}$

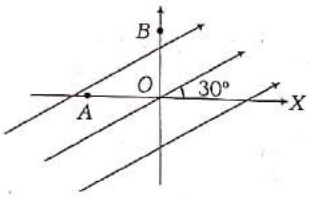
$= (-10)(1) = -10$ V

(ii) $V_B > V_C$, so $V_B - V_C$ will be positive.

Further, $d_{BC} = 2.0$ m

$\therefore \quad V_B - V_C = (10)(2) = 20$ V

Example 2.18 *A uniform electric field of $100\ Vm^{-1}$ is directed at 30° with the positive X-axis as shown in figure. Find the potential difference, V_{BA} if OA = 2 m and OB = 4 m.*



Sol. This problem can be solved by both the methods as discussed below.

Method 1 Electric field in vector form can be written as

$$\mathbf{E} = (100 \cos 30°\ \hat{\mathbf{i}} + 100 \sin 30°\ \hat{\mathbf{j}})\ \text{Vm}^{-1}$$

$$= (50\sqrt{3}\ \hat{\mathbf{i}} + 50\hat{\mathbf{j}})\ \text{Vm}^{-1}$$

$$A \equiv (-2\ \text{m}, 0, 0)$$

and $B \equiv (0, 4\ \text{m}, 0)$

$$\therefore\ V_{BA} = V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{r}$$

$$= -\int_{(-2\ \text{m}, 0, 0)}^{(0, 4\ \text{m}, 0)} (50\sqrt{3}\ \hat{\mathbf{i}} + 50\ \hat{\mathbf{j}}) \cdot (dx\ \hat{\mathbf{i}} + dy\ \hat{\mathbf{j}} + dz\ \hat{\mathbf{k}})$$

$$= -[50\sqrt{3}\ x + 50\ y]_{(-2\ \text{m}, 0, 0)}^{(0, 4\ \text{m}, 0)} = -100\ (2 + \sqrt{3})\ \text{V}$$

Method 2 We can also use, $V = Ed$

With the view that, $V_A > V_B$ or $V_B - V_A$ will be negative.

Here, $d_{AB} = OA \cos 30° + OB \sin 30°$

$$= 2 \times \frac{\sqrt{3}}{2} + 4 \times \frac{1}{2} = (\sqrt{3} + 2)$$

$$\therefore\ V_B - V_A = -Ed_{AB} = -100\ (2 + \sqrt{3})$$

Example 2.19 *Two points A and B are 2 cm apart and a uniform electric field E acts along the straight line AB directed from A to B with $E = 200\ NC^{-1}$. A particle of charge $+10^{-6}$ C is taken from A to B along AB. Calculate*

(i) the force on the charge.

(ii) the potential difference, $V_A - V_B$ and

(iii) the work done on the charge by **E**.

Sol. (i) Electrostatic force on the charge,

$$F = qE = (10^{-6})\ (200) = 2 \times 10^{-4}\ \text{N}$$

(ii) In uniform electric field,

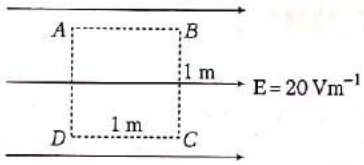
$$V = Ed$$

or potential difference, $V_A - V_B = 200 \times 2 \times 10^{-2} = 4\ \text{V}$

(iii) $W = Fs \cos \theta = (2 \times 10^{-4})\ (2 \times 10^{-2}) \cos 0°$

$$= 4 \times 10^{-6}\ \text{J}$$

Example 2.20 *In the uniform electric field shown in figure, find*



(i) $V_A - V_D$ *(ii)* $V_A - V_C$

(iii) $V_B - V_D$ *(iv)* $V_C - V_D$

Sol. Using the relation, $V = Ed$ (in uniform **E**)

where, V = PD between the two points

E = magnitude of **E** and

d = projection of line (joining two points) along **E**.

(i) $V_A - V_D = 0$ as $d = 0$

(ii) $V_A - V_C = +(20)\ (1) = 20\ \text{V}$

(iii) $V_B - V_D = -(20)\ (1) = -20\ \text{V}$

(iv) $V_C - V_D = -(20)\ (1) = -20\ \text{V}$

Example 2.21 *An electric field* $\mathbf{E} = (20\hat{\mathbf{i}} + 30\hat{\mathbf{j}})$ *newton/coulomb exists in the space. If the potential at the origin is taken to be zero, find the potential at (2m, 2m).*

Sol. We have, $E = \dfrac{-dV}{dr}$ so, it can be written in vector form as

$$dV = -\mathbf{E} \cdot d\mathbf{r}$$

Note, we can write **E** as $E_x\hat{\mathbf{i}} + E_y\hat{\mathbf{j}} + E_z\hat{\mathbf{k}}$

and $d\mathbf{r} = dx\hat{\mathbf{i}} + dy\hat{\mathbf{j}} + dz\hat{\mathbf{k}}$

Therefore, $\mathbf{E} \cdot d\mathbf{r} = E_x \cdot dx + E_y \cdot dy + E_z \cdot dz$

In the given question, the z-component of E or the point is not given. So, we can write

$$\mathbf{E} \cdot d\mathbf{r} = E_x \cdot dx + E_y \cdot dy$$

Now, $dV = -\mathbf{E} \cdot d\mathbf{r}$

$$= -(20\hat{\mathbf{i}} + 30\hat{\mathbf{j}}) \cdot (dx\hat{\mathbf{i}} + dy\hat{\mathbf{j}})$$

or $dV = -20\ dx - 30\ dy$

Now, we will have to integrate it within limits. Given, $V = 0$ when $x = 0$ and $y = 0$ (lower limit) and we have to calculate V. When $x = 2$ and $y = 2$ (upper limit)

Therefore, $\int_0^V dV = -20 \cdot \int_{x=0}^{x=2} dx - 30 \int_{y=0}^{y=2} dy$

or $[V]_0^V = -20\ [x]_0^2 - 30[y]_0^2$

or $V - 0 = -20\ (2 - 0) - 30(2 - 0)$

or Potential, $V = -40 - 60 = -100\ \text{V}$

Check point 2.1

1. Find the work done by some external force in moving a charge $q = 4\,\mu C$ from infinity to a point, where electric potential is 10^4 V.
(a) 4×10^{-2} J (b) 2×10^{-2} J (c) 8×10^{-2} J (d) 1×10^{-2} J

2. Equal charges are given to two spheres of different radii. The potential will
(a) be more one the smaller sphere
(b) be more on the bigger sphere
(c) be equal on both the spheres
(d) depend on the nature of the materials of the spheres

3. The electric potential at a point in free space due to a charge Q coulomb is $Q \times 10^{11}$ volts. The electric field at the point is
(a) $4\pi\varepsilon_0 Q \times 10^{20}$ Vm^{-1} (b) $12\pi\varepsilon_0 Q \times 10^{22}$ Vm^{-1}
(c) $4\pi\varepsilon_0 Q \times 10^{22}$ Vm^{-1} (d) $12\pi\varepsilon_0 Q \times 10^{20}$ Vm^{-1}

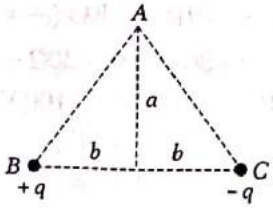
4. In an hydrogen atom, the electron revolves around the nucleus in an orbit of radius 0.53×10^{-10} m. Then, the electrical potential produced by the nucleus at the position of the electron is
(a) −13.6 V (b) − 27.2 V
(c) 27.2 V (d) 13.6 V

5. Three charges $2q, -q, -q$ are located at the vertices of an equilateral triangle. At the centre of the triangle,
(a) the field is zero but potential is non-zero
(b) the field is non-zero but potential is zero
(c) Both field and potential are zero
(d) Both field and potential are non-zero

6. In a region of constant potential,
(a) the electric field is uniform
(b) the electric field is zero
(c) there can be no charge inside the region
(d) the electric field shall necessarily change if a charge is placed outside the region

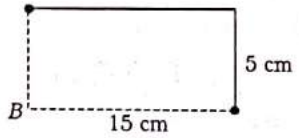
7. The work done in bringing a 20 C charge from point A to point B for distance 0.2 m is 2 J. The potential difference between the two points will be (in volt)
(a) 0.2 (b) 8 (c) 0.1 (d) 0.4

8. As shown in the figure, charges $+q$ and $-q$ are placed at the vertices B and C of an isosceles triangle. The potential at the vertex A is



(a) $\frac{1}{4\pi\varepsilon_0} \cdot \frac{2q}{\sqrt{a^2 + b^2}}$ (b) zero
(c) $\frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{\sqrt{a^2 + b^2}}$ (d) $\frac{1}{4\pi\varepsilon_0} \cdot \frac{(-q)}{\sqrt{a^2 + b^2}}$

9. In the rectangle shown below, the two corners have charge $q_1 = -5\,\mu C$ and $q_2 = +2.0\,\mu C$. The work done in moving a charge $+3.0\,\mu C$ from B to A is



(a) 2.8 J (b) 3.5 J (c) 4.5 J (d) 5.5 J

10. Two point charges $-q$ and $+q$ are located at points $(0, 0, -a)$ and $(0, 0, a)$, respectively. The potential at a point $(0, 0, z)$, where $z > a$ is
(a) $\frac{qa}{4\pi_0 z^2}$ (b) $\frac{q}{4\pi\varepsilon_0 a}$
(c) $\frac{2qa}{4\pi\varepsilon_0 (z^2 - a^2)}$ (d) $\frac{2qa}{4\pi\varepsilon_0 (z^2 + a^2)}$

11. Two plates are 2 cm apart a potential difference of 10 V is applied between them, the electric field between the plates is
(a) 20 NC^{-1} (b) 500 NC^{-1}
(c) 5 NC^{-1} (d) 250 NC^{-1}h

12. At a certain distance from a point charge the electric field i 500 Vm^{-1} and the potential is 3000 V. What is this distance
(a) 6 m (b) 12 m
(c) 36 m (d) 144 m

13. Two charges of $4\,\mu C$ each are placed at the corners A and of an equilateral triangle of side length 0.2 m in air. The electric potential at C is $\left[\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ N-m}^2\text{C}^{-2}\right]$
(a) 9×10^4 V (b) 18×10^4 V
(c) 36×10^4 V (d) 36×10^{-4} V

14. The electric potential V at any point $O(x, y, z$ all in metres) space is given by $V = 4x^2$ V. The electric field at the point $(1\ m, 0, 2m)$ in volt metre^{-1} is
(a) 8 along negative X-axis (b) 8 along positive X-axis
(c) 16 along negative X-axis (d) 16 along positive Z-axis

15. The electric potential V is given as a function of distance x (metre) by $V = (5x^2 + 10x - 9)$ V. Value of electric field at $x = 1$ is
(a) − 20 Vm^{-1} (b) 6 Vm^{-1} (c) 11 Vm^{-1} (d) − 23 Vm^{-1}

2 Equipotential surfaces

Any surface over which the electric potential is same everywhere is called an **equipotential surface.** For a single charge q, the potential is given by $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$.

This shows that if r is constant, then V will be constant. Thus, equipotential surfaces of a single point charge are concentric spherical surfaces centred at the charge.

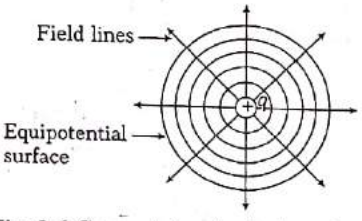


Fig. 2.6 Concentric spherical surfaces having charge q at the centre

Properties of a equipotential surfaces

(i) Potential difference between any two points in an equipotential surface is zero.

(ii) Two equipotential surfaces can never intersect each other. If two equipotential surfaces intersect each other, then at the point of intersection of these will be two directions of electric field which is impossible.

(iii) As the work done by electric force is zero when a test charge is moved along the equipotential surface, it follows that **E** must be perpendicular to the surface at every point so that the electric force $q_0\mathbf{E}$ will always be perpendicular to the displacement of a charge moving on the surface. Thus, field lines and equipotential surfaces are always mutually perpendicular.

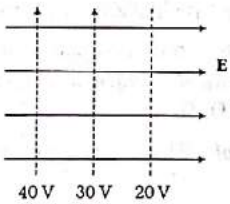


Fig. 2.7

Equipotential surface for charge distribution

(i) The equipotential surfaces are a family of concentric spheres for a point charge or a sphere of charge as shown in the following figure.

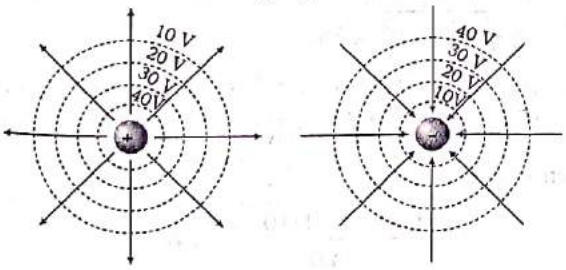


Fig. 2.8 Equipotential surfaces

(ii) When potential is applied between two charged plates, electric field (**E**) is set up between them and this **E** is normal to the equipotential surface as shown below

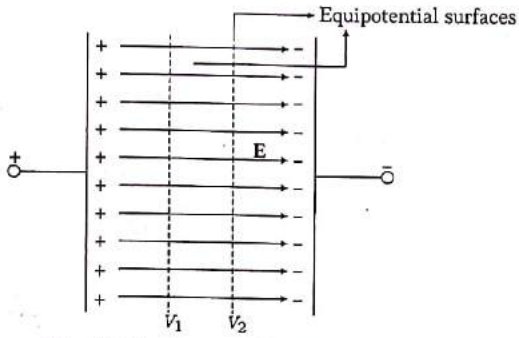


Fig. 2.9 Equipotential surfaces between two charged plates

(iii) The equipotential surfaces are a family of concentric cylinders for a line of charge or cylinder of charge. Equipotential surface for line charge is shown below.

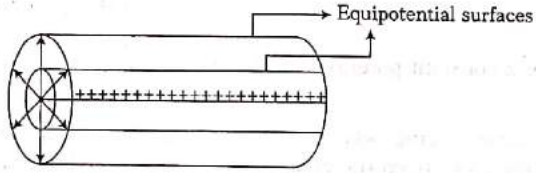


Fig. 2.10 Equipotential surfaces for a line charge distribution

(iv) Equipotential surfaces for two positive and equal charges are shown below.

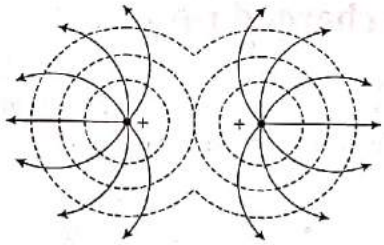


Fig. 2.11 Equipotential surfaces for like charges

(v) Equipotential surfaces for two equal and opposite charges can be drawn as

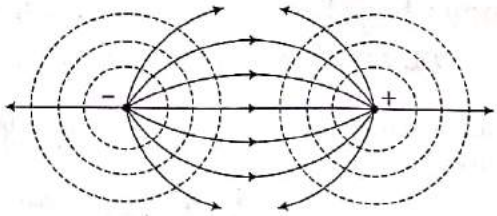


Fig. 2.12 Equipotential surfaces for unlike charges

Note While drawing the equipotential surfaces we should keep in mind the two main points

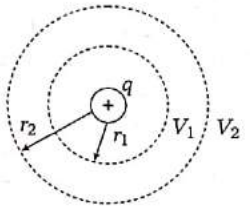
(i) These are perpendicular to field lines at all places.

(ii) Field lines always flow from higher potential to lower potential.

Example 2.22 *Equipotential spheres are drawn round a point charge. As we move away from the charge, will the spacing between two spheres having a constant potential difference decrease, increase or remain constant.*

Sol. We have, $V_1 > V_2$

$$V_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r_1}$$



and $$V_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r_2}$$

Now, $$V_1 - V_2 = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{r_2 - r_1}{r_1 r_2}\right)$$

$\therefore$ $$(r_2 - r_1) = \frac{(4\pi\varepsilon_0)(V_1 - V_2)}{q}(r_1 r_2)$$

For a constant potential difference $(V_1 - V_2)$,

$$r_2 - r_1 \propto r_1 r_2$$

i.e., the spacing between two spheres $(r_2 - r_1)$ increases as we move away from the charge, because the product $r_1 r_2$ will increase.

Variation of electric potential on the axis of a charged ring

Let a charge q is uniformly distributed over the circumference of a ring as shown in Fig. (a) and is non-uniformly distributed in Fig (b).

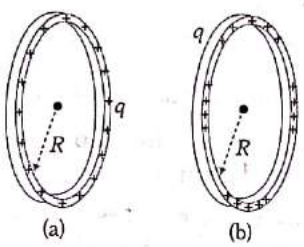


Fig. 2.13 *Variation of electric potential*

The electric potential at the centre of the ring in both the cases is given by

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

where, R is radius of ring.

The electric potential at point P which is at a distance r from the centre of ring is given by

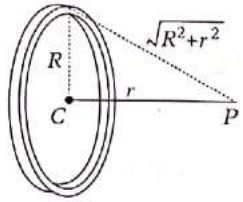


Fig. 2.14 *Electric potential at P at a distance r from centre ring*

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{R^2 + r^2}}$$

From these expressions, we can see that electric potentia is maximum at the centre and decreases as we move aw from the centre on the axis.

This potential varies with the distance r as shown in figure.

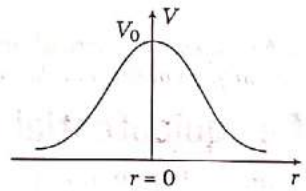
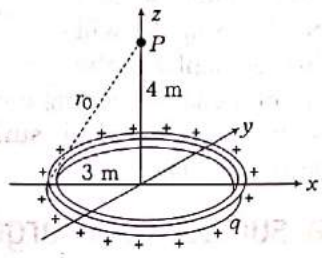


Fig. 2.15 *Variation of potential V with distance r*

In the figure, $$V_0 = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

Example 2.23 *A charge $q = 10\ \mu C$ is distributed uniformly o the circumference of a ring of radius 3 m placed on xy-plane with its centre at origin. Find the electric potential at a point P(0, 0, 4 m).*

Sol. The electric potential at point P would be



$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r_0}$$

Here, r_0 = distance of point P from the circumference of ring

$$= \sqrt{(3)^2 + (4)^2} = 5\text{ m}$$

and $q = 10\ \mu\text{C} = 10^{-5}$ C

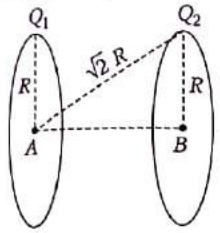
Substituting the values, we have

Electric potential,

$$V = \frac{(9.0 \times 10^9)(10^{-5})}{(5.0)} = 1.8 \times 10^4 \text{ V}$$

Example 2.24 *Two identical thin rings, each of radius R, are coaxially placed at a distance R. If Q_1 and Q_2 are respectively, the charges uniformly spread on the two rings, find the work done in moving a charge q from centre of ring having charge Q_1 to the other ring.*

Sol.



$$V_A = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1}{R} + \frac{Q_2}{\sqrt{2}R}\right]$$

$$V_B = \frac{1}{4\pi\varepsilon_0 R}\left[\frac{Q_2}{R} + \frac{Q_1}{\sqrt{2}R}\right]$$

$$V_B - V_A = \frac{1}{4\pi\varepsilon_0 R}\left[Q_1\left(\frac{1}{\sqrt{2}} - 1\right) + Q_2\left(1 - \frac{1}{\sqrt{2}}\right)\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{(Q_2 - Q_1)(\sqrt{2} - 1)}{\sqrt{2}R}$$

Work done in moving a charge q from A to B,

$$W_{A\to B} = q(V_B - V_A) = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(\sqrt{2} - 1)(Q_2 - Q_1)q}{\sqrt{2}R}$$

Potential due to spherical charge

Electric potential due to charged spherical shell of solid conducting sphere

Let us consider a uniformly charged shell. The radius of shell is R and its total charge is q, which is uniformly distributed over the surface. We can construct a Gaussian surface (a sphere) of radius $r > R$.

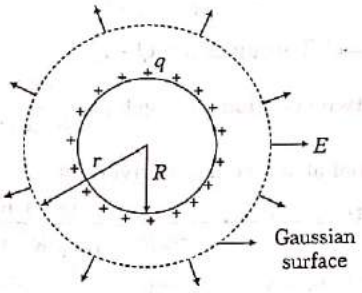


Fig. 2.16

At external points the potential at any point is the same when the whole charge is assumed to be concentrated at the centre. At the surface of the sphere, $r = R$

$\therefore$ Potential, $$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R}$$

The potential is same at all points which is equal to the potential at surface. Thus, we can write,

$$V_{\text{inside}} = V_{\text{surface}} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R}$$

and $$V_{\text{outside}} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r}$$

The potential (V) varies with the distance from the centre (r) as shown in Fig. 2.17.

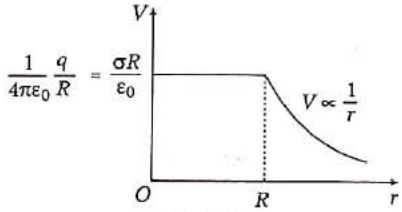


Fig. 2.17

Example 2.25 *A spherical drop of water carrying a charge of 3×10^{-19} C has a potential of 500 V at its surface. What is the radius of the drop? If two drops of the same charge and the same radius combine to form a single spherical drop, what is the potential at the surface of the new drop?*

Sol. The potential V is given by $V = \frac{q}{4\pi\varepsilon_0 r}$

Here, $q = 3 \times 10^{-19}$ C and $V = 500$ V

Volume of one drop is $\frac{4}{3}\pi r^3$.

Total volume of both drops is $\frac{8}{3}\pi r^3$.

Let r' be the radius of the new drop formed, equating the volumes, we have $\frac{4}{3}\pi r'^3 = \frac{8}{3}\pi r^3$. This gives, $r' = 2^{1/3}r$.

Charge on the new drop $= 2q = 6 \times 10^{-19}$ C

New potential, $V = \frac{2q}{4\pi\varepsilon_0 r'}$

$$= \frac{2q}{4\pi\varepsilon_0 2^{1/3}r} = \frac{1000}{2^{1/3}} = 794 \text{ V}$$

Electric potential due to charged spherical shell of non-conducting sphere

For a uniformly charged solid sphere, we have the following formulae for potential,

$$V_{\text{outside}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}; \quad V_{\text{surface}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

$$V_{\text{inside}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}\left[\frac{3}{2} - \frac{1}{2}\frac{r^2}{R^2}\right]$$

At the centre potential will be $V_C = \frac{3}{2}\left[\frac{1}{4\pi\varepsilon_0}\frac{q}{R}\right]$,

which is equal to $\frac{3}{2}$ times the potential at surface. This can be obtained by putting $r = 0$ in the formula of V_{inside}.

The variation of potential (V) with distance from the centre is as shown $\frac{3}{2}\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{R}\right)$

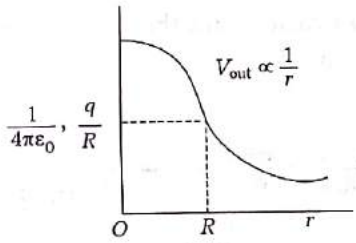


Fig. 2.18

Electric potential due to a group of concentric shells

Consider a pair of two uniformly charged concentric shells having radii a and b ($a < b$) and carrying charges q_1 and q_2, respectively. We will calculate potential at three different points A, B and C. A lies inside the inner shell, B lies in the space between the two shells and C lies outside the outer shell.

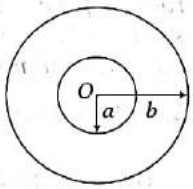


Fig. 2.19

Potential at point C

$OC = r$, where $r > b$.

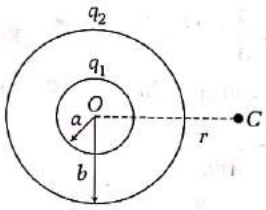


Fig. 2.20

So, potential at C is $V_C = \frac{q_1 + q_2}{4\pi\varepsilon_0 r}$

(due to inner and outer shell)

Potential at point B

Let $OB = r$, where $a < r < b$.

As the point B, lies inside the outer shell,

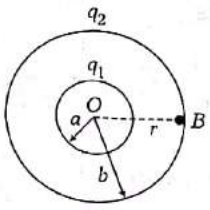


Fig. 2.21

Potential at B due to outer shell is $V_2 = \frac{q_2}{4\pi\varepsilon_0 b}$.

The point B lies outside the inner shell, the potential at B due to inner shell is $V_1 = \frac{q_1}{4\pi\varepsilon_0 r}$.

So, potential at B is $V_B = \frac{q_1}{4\pi\varepsilon_0 r} + \frac{q_2}{4\pi\varepsilon_0 b}$

Potential at point A

Let $OA = r$, such that $r < a$. The point A lies inside the outer shell. So, potential at A, due to outer shell is $V_2 = \frac{q_2}{4\pi\varepsilon_0 b}$.

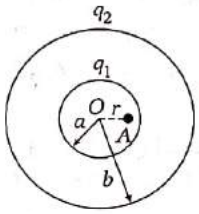


Fig. 2.22

Similarly, potential at A, due to inner shell is $V_1 = \frac{q_1}{4\pi\varepsilon_0 a}$.

So, potential at A is $V_A = \frac{q_1}{4\pi\varepsilon_0 a} + \frac{q_2}{4\pi\varepsilon_0 b}$.

Example 2.26 *A charge Q is distributed over two concentric hollow spheres of radii r and R ($> r$) such that the surface densities are equal. Find the potential at any point inside the smaller sphere.*

Sol. Let q and q' be the charges on the inner and outer sphere.

As surface charge densities are equal.

$$\therefore \quad \frac{q}{4\pi r^2} = \frac{q'}{4\pi R^2}$$

or $$qR^2 = q'r^2$$

Also, $q + q' = Q$. This gives $q = Q - q'$

Solving the two equations, we get $q = \frac{Qr^2}{R^2 + r^2}$, $q' = \frac{QR^2}{R^2 + r^2}$

Now, potential at the centre is given by

$$V_C = \frac{q}{4\pi\varepsilon_0 r} + \frac{q'}{4\pi\varepsilon_0 R} = \frac{Q(r + R)}{4\pi\varepsilon_0 (R^2 + r^2)}$$

This is equal to potential at any point inside the smaller sphere

Motion of charged particle in electric field

Consider a charged particle having charge q and mass m is initially at rest in an electric field of strength E. The particle will experience an electric force which causes its motion. The force experienced by the charged particle is F, where

$$F = qE$$

$\therefore$ Acceleration produced by this force is $a = \frac{F}{m} = \frac{qE}{m}$ ...(i)

Suppose at point A particle is at rest and after some time, t it reaches the point B and attains velocity v.

$\therefore$ $$v = at$$

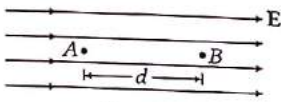


Fig. 2.23

If potential difference between A and B be ΔV and the distance between them is d, then

$$v = \frac{qEt}{m} = \sqrt{\frac{2q\Delta V}{m}} \quad \text{... (ii)}$$

As momentum, $p = mv$

$\therefore$ $$p = m\left(\frac{qEt}{m}\right) \quad \text{[from Eq. (ii)]}$$

$$p = qEt = m \times \sqrt{\frac{2q\Delta V}{m}} = \sqrt{2mq\Delta V}$$

Kinetic energy of a charged particle

Kinetic energy gained by the particle in time t is,

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{qEt}{m}\right)^2 \quad \text{[from Eq. (ii)]}$$

$$= \frac{q^2E^2t^2}{2m} \text{ or } K = \frac{1}{2}m \times \frac{2q\Delta V}{m} = q\Delta V$$

Example 2.27 *A bullet of mass 2 g is moving with a speed of 10 m/s. If the bullet has a charge of 2 μC, through what potential it be accelerated starting from rest, to acquire the same speed?*

Sol. Use the relation, $qV = \frac{1}{2}mv^2$

Here, $q = 2 \times 10^{-6}$ C, $m = 2 \times 10^{-3}$ kg, $v = 10\ \text{ms}^{-1}$

Therefore, potential, $V = \frac{mv^2}{2q} = \frac{2 \times 10^{-3} \times (10)^2}{2 \times 2 \times 10^{-6}} = \frac{10^{-1}}{2 \times 10^{-6}}$

$$= 5 \times 10^4 \text{ V} = 50 \text{ kV}$$

Work done by a charged particle

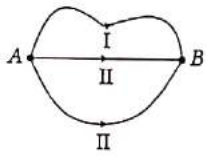
According to work energy theorem, gain in kinetic energy = work done in displacement of charge, *i.e.*, $W = q\Delta V$

where, ΔV = potential difference between the two positions of charge q, $(\Delta V) = \mathbf{E} \cdot \Delta\mathbf{r} = E\Delta r \cos\theta$, where θ is the angle between direction of electric field and direction of motion of charge).

If charge q is given a displacement $\mathbf{r} = (x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}})$ in an electric field $\mathbf{E} = (E_x\hat{\mathbf{i}} + E_y\hat{\mathbf{j}} + E_z\hat{\mathbf{k}})$, then the work done is

$$W = q(\mathbf{E} \cdot \mathbf{r}) = q(E_x x + E_y y + E_z z)$$

Work done in displacing a charge in an electric field is path independent because electric force in field is conservative.



$W_I = W_{II} = W_{III}$

Fig. 2.24

When charged particle enters perpendicularly in an electric field, it describes a parabolic path as shown

(i) **Equation of trajectory** Throughout the motion, particle has uniform velocity along X-axis and horizontal displacement (x) is given by the equation, $x = ut$. Since, the motion of the particle is accelerated along Y-axis.

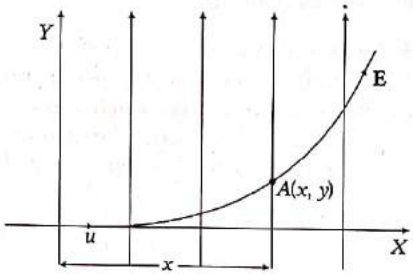


Fig. 2.25

So, $$y = \frac{1}{2}\left(\frac{qE}{m}\right)\left(\frac{x}{u}\right)^2 \quad \left(\because t = \frac{x}{u}\right)$$

This is the equation of a parabola which shows

$$y \propto x^2$$

(ii) **Velocity at any instant** At any instant t, $v_x = u$ and

$v_y = \frac{qEt}{m}$. So, $v = |v| = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + \frac{q^2E^2t^2}{m^2}}$

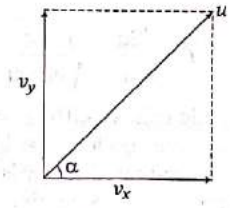


Fig. 2.26

If α is the angle made by v with X-axis, then

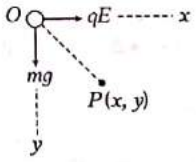
$$\tan\alpha = \frac{v_y}{v_x} = \frac{qEt}{mu}$$

Example 2.28 *A ball of mass m having a charge q is released from rest in a region where a horizontal electric field E exists.*

(i) Find the resultant force acting on the ball.

(ii) Find the trajectory followed by the ball.

Sol. The forces acting on the ball are weight of the ball in vertically downward direction and the electric force in the horizontal direction.



(i) Resultant force, $F = \sqrt{(mg)^2 + (qE)^2}$.

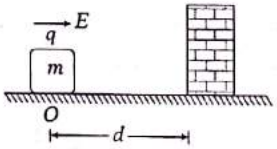
(ii) Let the ball be at point P after time t, then the trajectory followed by ball can be written as

In x-direction, $x = \frac{1}{2}\frac{qE}{m}t^2$

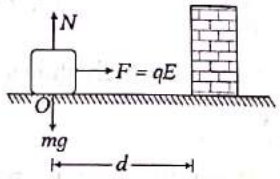
In y-direction, $y = \frac{1}{2}gt^2, \frac{y}{x} = \frac{g}{qe/M}, y = \frac{mgx}{qE}$

Trajectory is a straight line.

Example 2.29 *A block having charge q and mass m is resting on a smooth horizontal surface at a distance d from the wall as shown. Discuss the motion of the block when a uniform electric field E is applied horizontally towards the wall assuming that collision of the block with the wall is completely elastic.*



Sol.



Acceleration of the block can be given as $a = \frac{qE}{m}$.

Time taken by the block to reach at wall $d = \frac{1}{2}at^2$

$$t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{2d}{qE/m}} = \sqrt{\frac{2md}{qE}}$$

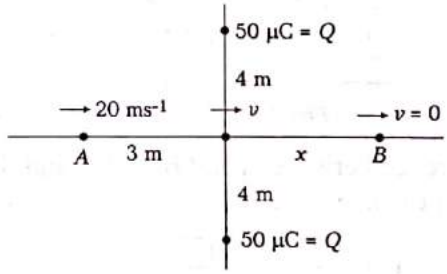
After completely elastic collision, the speed of the block is reversed and then it moves under retardation and comes to rest at O. Again it is accelerated towards the wall and so on. In fact, it is executing oscillatory motion of the time period

$$T = 2t = 2\sqrt{\frac{2md}{qE}}$$

As force qE is constant, the motion of block is not simply harmonic.

Example 2.30 *Two point charges each 50 μC are fixed on Y-axis at y = + 4 m and y = − 4 m. Another charged particle having charge − 50 μC and mass 20 g is moving along the positive X-axis. When it is at x = −3 m, its speed is 20 ms⁻¹. Find the speed of charged particle when it reaches to origin. Also, find distance of charged particle from origin, when its kinetic energy becomes zero.*

Sol.



Let the speed of particle at origin be v. Applying energy conservation between A and O, we get

$$K_A + U_A = K_O + U_O$$

$$K_A + (-Q)V_A = K_O + (-Q)V_O$$

$$\frac{1}{2}mv_A^2 + (-Q)\frac{1}{4\pi\varepsilon_0}\cdot\frac{2Q}{\sqrt{3^2+4^2}} = \frac{1}{2}mv_O^2 + (-Q)\frac{1}{4\pi\varepsilon_0}\cdot\frac{2Q}{4}$$

$$\frac{1}{2}m(v_O^2 - v_A^2) = \frac{2Q^2}{4\pi\varepsilon_0}\left(\frac{1}{4} - \frac{1}{5}\right)$$

$$= 2(50\times10^{-6})^2 \times 9\times10^9 \times \frac{1}{20}$$

$$\frac{1}{2}\times20\times10^{-3}(v_O^2 - 20^2) = 2.25$$

$$\Rightarrow v_O^2 - 400 = 225 \Rightarrow v_O^2 = 625 \Rightarrow v_O = 25\,\text{ms}^{-1}$$

Applying energy conservation between A and B, we have

$$K_A + U_A = K_B + U_B$$

$$K_A + (-Q)V_A = 0 + (-Q)V_B$$

$$\frac{1}{2}mv_A^2 + (-Q)\frac{1}{4\pi\varepsilon_0}\cdot\frac{2Q}{5} = (-Q)\frac{1}{4\pi\varepsilon_0}\cdot\frac{2Q}{\sqrt{4^2+x^2}}$$

$$\frac{1}{2}mv_A^2 = \frac{1}{4\pi\varepsilon_0}2Q^2\left[\frac{1}{5} - \frac{1}{\sqrt{4^2+x^2}}\right]$$

$$\frac{1}{2}\times20\times10^{-3}\times(20)^2 = 9\times10^9\times2\times(50\times10^{-6})^2\left[\frac{1}{5} - \frac{1}{\sqrt{4^2+x^2}}\right]$$

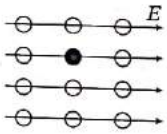
$$4 = 45\left(\frac{1}{5} - \frac{1}{\sqrt{16^2+x^2}}\right) \Rightarrow \frac{1}{\sqrt{16+x^2}} = \frac{1}{5} - \frac{4}{45} = \frac{5}{45} = \frac{1}{9}$$

$$x = \sqrt{65}\text{ m}$$

Check point 2.2

1. The work done to move a charge along an equipotential from A to B
   (a) cannot be defined as $-\int_A^B \mathbf{E}\cdot d\mathbf{l}$
   (b) must be defined as $-\int_A^B \mathbf{E}\cdot d\mathbf{l}$
   (c) is zero
   (d) can have a non-zero value

2. There is a uniform electric field of intensity E which is as shown. How many labelled points have the same electric potential as the fully shaded point?



   (a) 2 (b) 3
   (c) 8 (d) 11

3. The electric potential at the surface of an atomic nucleus ($Z = 50$) of radius 9.0×10^{-3} cm
   (a) 80 V (b) 8×10^6 V
   (c) 9 V (d) 9×10^7 V

4. A hollow metal sphere of radius 5 cm is charged so that the potential on its surface is 10 V. The potential at the centre of the sphere is
   (a) 0 V
   (b) 10 V
   (c) same as at point 5 cm away from the surface
   (d) same as at point 25 cm away from the surface

5. Two charged spheres of radii R_1 and R_2 having equal surface charge density. The ratio of their potential is
   (a) R_1/R_2 (b) R_2/R_1
   (c) $(R_1/R_2)^2$ (d) $(R_2/R_1)^2$

6. The radii of two concentric spherical conducting shells are r_1 and r_2 ($> r_1$). The change on the outer shell is q. The charge on the inner shell which is connected to the earth is
   (a) $q\left(\frac{r_2}{r_1}\right)$ (b) $q^2\left(\frac{r_1}{r_2}\right)$ (c) $-q(r_1/r_2)$ (d) $q^2\left(\frac{r_2}{r_1}\right)$

7. If a charged spherical conductor of radius 10 cm has potential V at a point distant 5 cm from its centre, then the potential at a point distant 15 cm from the centre will be
   (a) $\frac{1}{3}V$ (b) $\frac{2}{3}V$
   (c) $\frac{3}{2}V$ (d) $3V$

8. A hollow conducting sphere of radius R has a charge $(+q)$ on its surface. What is the electric potential within the sphere at a distance $r = R/3$ from its centre
   (a) Zero (b) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$ (c) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$ (d) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$

9. A spherical conductor of radius 2 m is charged to a potential of 120 V. It is now placed inside another hollow spherical conductor of radius 6 m. Calculate the potential to which the bigger sphere would be raised.
   (a) 20 V (b) 60 V
   (c) 80 V (d) 40 V

10. Three concentric spherical shells have radii a, b and $c (a < b < c)$ and have surface charge densities $\sigma, -\sigma$ and σ respectively. If V_A, V_B and V_C denote the potentials of the three shell, ... for $c = a + b$, we have
    (a) $V_C = V_A \neq V_B$ (b) $V_C = V_B \neq V_A$
    (c) $V_C \neq V_B \neq V_A$ (d) $V_C = V_B = V_A$

11. The electrostatic potential of a uniformly charged thin spherical shell of charge Q and radius R at a distance r from the centre is
    (a) $\frac{Q}{4\pi\varepsilon_0 r}$ for points outside and $\frac{Q}{4\pi\varepsilon_0 R}$ for points inside the shell
    (b) $\frac{Q}{4\pi\varepsilon_0 r}$ for both points inside and outside the shell
    (c) zero for points outside and $\frac{Q}{4\pi\varepsilon_0 r}$ for points inside the shell
    (d) zero for both points inside and outside the shell

12. The diameter of a hollow metallic sphere is 60 cm and the sphere carries a charge of 500 μC. The potential at a distance of 100 cm from the centre of the sphere will be
    (a) 6×10^7 V (b) 7×10^6 V
    (c) 4.5×10^6 V (d) 5×10^6 V

13. Obtain the energy in joule acquired by an electron beam when accelerated through a potential difference of 2000 V. How much speed will the electron gain?
    (a) $\frac{8}{3} \times 10^7$ m/s (b) $\frac{7}{3} \times 10^7$ m/s
    (c) $\frac{5}{3} \times 10^7$ m/s (d) $\frac{2}{3} \times 10^7$ m/s

14. A particle A has charge $+16q$ and a particle B has charge $+4q$ with each of them having the same mass m. When allowed to fall from rest through the same electric potential difference, the ratio of their speed $\frac{v_A}{v_B}$ will becomes
    (a) 2 : 1 (b) 1 : 2
    (c) 1 : 4 (d) 4 : 1

15. What potential difference must be applied to produce an electric field that can accelerate an electron to one length the velocity of light?
    (a) 1352 V (b) 2511 V
    (c) 2531 V (d) 3521 V

3 Electric potential energy

Electric potential energy of a system of point charges is defined as the total amount of work done in bringing the different charges to their respective positions from infinitely large mutual separations.

It is represented by U. Thus, electric potential can also be written as potential energy per unit charge, *i.e.*,

$$V = \frac{W}{q} = \frac{U}{q}$$

Electric potential energy is defined only in a conservative field.

Relation between work and energy

If the force **F** is conservative, the work done by **F** can always be expressed in terms of a **potential energy** U. When the particle moves from a point where the potential energy is U_a to a point where it is U_b, the change in potential energy is, $\Delta U = U_b - U_a$.

This is related by the work $W_{a \to b}$ as

$$W_{a \to b} = U_a - U_b = -(U_b - U_a) = -\Delta U \quad \text{...(i)}$$

Here, $W_{a \to b}$ is the work done in displacing the particle from a to b by the **conservative force** (here electrostatic). Moreover, we can see from Eq. (i) that if $W_{a \to b}$ is positive, ΔU is negative and the potential energy decreases. So, whenever the work done by a conservative force is positive, the potential energy of the system decreases and *vice versa*.

That's what happens when a particle is thrown upwards, the work done by gravity is negative, and the potential energy increases.

Electron volt

One electron volt (1eV) is the energy equal to the work done in moving a single elementary charge, e such as the electron or the proton through a potential difference of exactly one volt (1 V).

$$1\,\text{eV} = e\,(1\,\text{V}) = (1.60 \times 10^{-19}\,\text{C})\,(1\,\text{J/C})$$
$$= 1.60 \times 10^{-19}\,\text{J}$$

Example 2.31 *A spherical oil drop, radius 10^{-4} cm has on it at a certain time a total charge of 40 electrons. Calculate the energy that would be required to place an additional electron on the drop.*

Sol. Initial charge on the oil drop $= 40 \times 1.6 \times 10^{-19}$ C

Potential of the oil drop $= \dfrac{q}{4\pi\varepsilon_0 r} = 9 \times 64 \times 10^{-4}$ V

Now, energy required = work done in bringing an electron from infinity to a point of potential.

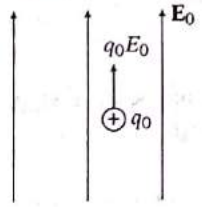
As, $\quad \text{potential} = \dfrac{\text{work}}{\text{charge}}$

$\therefore$ Work = Potential × Charge $= 9 \times 64 \times 10^{-4} \times 1.6 \times 10^{-19}$
$= 921.6 \times 10^{-23}$ J

Example 2.32 *A uniform electric field E_0 is directed along positive y-direction. Find the change in electric potential energy of a positive test charge q_0 when it is displaced in the field from $y_i = a$ to $y_f = 2a$ along the Y-axis.*

Sol. Electrostatic force on the test charge,

$F_e = q_0 E_0$ (along positive y-direction)



$\therefore W_{i-f} = -\Delta U$ or $\Delta U = -W_{i-f} = -[q_0E_0\,(2a - a)] = -q_0E_0a$

Note Here, work done by electrostatic force is positive. Hence, the potential energy is decreasing.

Example 2.33 *Find the change in electric potential energy, ΔU as a charge of 2.20×10^{-6} C, moves from a point A to point B given that the change in electric potential between these points is $\Delta V = V_B - V_A = 24.0$ V.*

Sol. As $\Delta V = \dfrac{\Delta U}{q_0}$, where ΔU is change in potential energy

$\therefore \quad \Delta U = q_0 \Delta V = (2.20 \times 10^{-6}\,\text{C})\,(24.0\,\text{V}) = 5.28 \times 10^{-5}$ J

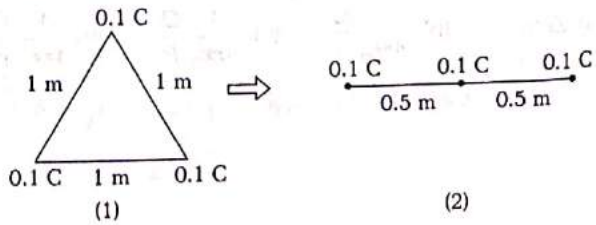
Example 2.34 *A charge is moved in an electric field of a fixed charge distribution from point A to another point B slowly. The work done by external agent in doing so is 100 J. What is the change in potential energy of the charge as it moves from A to B? What is the work done by the electric field of the charge distribution as the charge moves from A to B?*

Sol. Work done, $W_{ext} = \Delta U = U_B - U_A = 100$ J

As, $\mathbf{F}_{ext} = -\mathbf{F}_E$ for the charge to move slowly, so
$W_E = -W_{ext} = -100$ J

Exmple 2.35 *Three point charges of 0.1 C each are placed at the corners of an equilateral triangle with side 1 m. If this system is supplied energy at the rate of 1 kW, how much time will be required to move one of the charges on to the mid-point of the line joining the two others?*

Sol.



$$U_1 = \frac{1}{4\pi\varepsilon_0}\left[\frac{0.1\times 0.1}{1}\times 3\right]$$
$$= 9\times 10^9 \times 3\times 10^{-2} = 27\times 10^7 \text{ J}$$
$$U_2 = \frac{1}{4\pi\varepsilon_0}\left[\frac{0.1\times 0.1}{0.5}\times 2 + \frac{0.1\times 0.1}{1}\right]$$
$$= 9\times 10^9 \times 0.05$$
$$= 45\times 10^7 \text{ J}$$
$$W_{1\to 2} = U_2 - U_1 = 18\times 10^7 \text{ J}$$

Rate at which energy is supplied is given by

$$P = \frac{W}{t}$$
$$t = \frac{W}{P} = \frac{18\times 10^7}{10^3} = 18\times 10^4 \text{ s} = 50 \text{ h}$$

Potential energy of a system of charges

The potential energy for a system of two point charges and for a collection of more than two charges are given below.

1. Potential energy for a system of two point charges

Potential energy of the system of two charges q_1 and q_2 will be work done to bring them from infinity to point A and point B, respectively.

Work done to bring charge q_1 from infinity to point A,

$$W_1 = 0$$

Work done to bring charge q_2 from infinity to point B,

$$W_2 = q_2(V_B - V_A)$$

As,
$$W_2 = q_2 \times \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1}{r}$$

Now, potential energy of the system of charge q_1 and q_2,

$$U = W_1 + W_2$$
$$\boxed{U = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r}}$$

Example 2.36 *In a hydrogen atom, the electron and proton are bound together at a distance of above 0.53Å. Estimate the potential energy of the system in eV, assuming zero potential energy at infinite separation between the electron and the proton.*

Sol. Charge on electron, $q_1 = 1.6\times 10^{-19}$ C

Charge on proton, $q_2 = 1.6\times 10^{-9}$ C

Separation between electron and proton,

$$r = 0.53 \text{ Å} = 0.53\times 10^{-10} \text{ m}$$

The change in energy when electron-proton system is formed is

$$\Delta U = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r}$$
$$= (9\times 10^9)\times\frac{(-1.6\times 10^{-19})(1.6\times 10^{-19})}{0.53\times 10^{-10}} = -4.35\times 10^{-18} \text{ J}$$
$$= -\frac{4.35\times 10^{-18}}{1.6\times 10^{-19}} = -27.2 \text{ eV}$$

At infinite separation, the potential energy is zero, therefore, the energy when the system is formed is

$$U = 0 + \Delta U = -27.2 \text{ eV}$$

Example 2.37 *A point charge $q_1 = -5.8\ \mu C$ is held stationary at the origin. A second point charge $q_2 = +4.3\ \mu C$ moves from the point (0.26 m, 0, 0) to (0.38 m, 0, 0). How much work is done by the electric force on q_2?*

Sol. Work done by the electrostatic forces $= U_i - U_f$

$$= \frac{q_1 q_2}{4\pi\varepsilon_0}\left(\frac{1}{r_i} - \frac{1}{r_f}\right) = \frac{q_1 q_2}{4\pi\varepsilon_0}\left(\frac{r_f - r_i}{r_i r_f}\right)$$
$$= \frac{(-5.8\times 10^{-6})(4.3\times 10^{-6})(9\times 10^9)(0.38-0.26)}{(0.38)(0.26)}$$
$$= -0.272 \text{ J}$$

Example 2.38 *What minimum work must be done by an external force to bring a charge $q = 3.00\ \mu C$ from a great distance away (take $r = \infty$) to a point 0.500 m from a charge $Q = 20.0\ \mu C$?*

Sol. The change in potential energy equal to the (positive of the work required of an external force $W = \Delta U = q(V_b - V_a)$. We get the potentials V_b and V_a using $V = \frac{KQ}{r}$

The work required is equal to the change in potential energy,

$$W = q(V_b - V_a) = q\left(\frac{KQ}{r_b} - \frac{KQ}{r_a}\right)$$

where, $r_b = 0.500$ m and $r_a = \infty$.

Work done,

$$W = (300\times 10^{-6} \text{ C})\frac{(8.99\times 10^9 \text{ N-m}^2/\text{C}^2)(2.00\times 10^{-5} \text{ C})}{(0.500 \text{ m})}$$
$$= 1.08 \text{ J}$$

Example 2.39 *A point charge $q_1 = 9.1\ \mu C$ is held fixed at origin. A second point charge $q_2 = -0.42\ \mu C$ and a mass 3.2×10^{-4} kg is placed on the X-axis, 0.96 m from the origin. The second point charge is released at rest. What is its speed when it is 0.24 m from the origin?*

Sol. From conservation of mechanical energy, we have decrease in gravitational potential energy

= increase in kinetic energy

$$\text{or } \frac{1}{2}mv^2 = U_i - U_f = \frac{q_1 q_2}{4\pi\varepsilon_0}\left(\frac{1}{r_i} - \frac{1}{r_f}\right) = \frac{q_1 q_2}{4\pi\varepsilon_0}\left(\frac{r_f - r_i}{r_i r_f}\right)$$

Speed of the second charge,

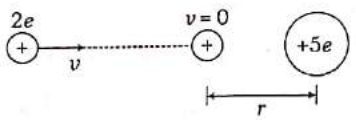
$$v = \sqrt{\frac{q_1 q_2}{2\pi\,\varepsilon_0 m}\left(\frac{r_f - r_i}{r_i r_f}\right)}$$

$$= \sqrt{\frac{(9.1\times10^{-6})(-0.42\times10^{-6})\times 2\times 9\times10^{9}}{3.2\times 10^{-4}}\left(\frac{0.24-0.96}{(0.24)(0.96)}\right)}$$

$$= 26\ \text{ms}^{-1}$$

Example 2.40 *An α-particle with kinetic energy 10 MeV is leading towards a stationary tin nucleus of atomic number 50. Calculate the distance of closest approach.*

Sol. Due to repulsion by the tin nucleus, the kinetic energy of the α-particle gradually decreses at the expense of electrostatic potential energy.



∴ Decrease in kinetic energy = Increase in potential energy

or $$\frac{1}{2}mv^2 = U_f - U_i$$

or $$\frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r} - 0$$

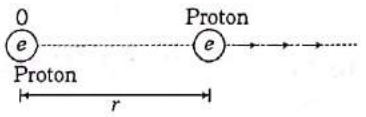
∴ $$r = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(2e)(50e)}{(\text{KE})}$$

Substituting the values, we get

$$\text{Distance, } r = \frac{(9\times 10^9)(2\times 1.6\times 10^{-19})(1.6\times 10^{-19}\times 50)}{10\times 10^6\times 1.6\times 10^{-19}}$$

$$= 14.4\times 10^{-15}\ \text{m}$$

Example 2.41 *A proton is fixed at origin. Another proton is released from rest, from a point at a distance r from origin. Taking charge of portion as e and mass as m, find the speed of the proton (i) at a distnace 2r from origin (ii) at large distance from origin.*



Sol. The proton moves away under electrostatic repulsion.

As there is no external force,

$$W_{\text{ext}} = 0$$

⇒ $$\Delta\text{KE} + \Delta\text{PE} = 0$$

⇒ $$\left(\frac{1}{2}mv^2\right) + (U_f - U_i) = 0$$

(i) We have, $$U_f = \frac{e^2}{4\pi\varepsilon_0(2r)},\ \text{and}\ U_i = \frac{e^2}{4\pi\varepsilon_0(r)}$$

∴ $$\frac{1}{2}mv^2 = \frac{e^2}{4\pi\varepsilon_0(2r)}$$

or speed of the proton, $$v = \sqrt{\frac{e^2}{4\pi\varepsilon_0 rm}}$$

(ii) We have, $$U_f = 0,\ U_i = \frac{e^2}{4\pi\varepsilon_0 r}$$

∴ $$\frac{1}{2}mv^2 = \frac{e^2}{4\pi\varepsilon_0 r}$$

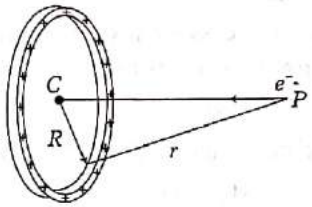
Speed of the proton, $$v = \sqrt{\frac{2e^2}{4\pi\varepsilon_0 rm}}$$

Example 2.42 *A uniformly charged thin ring has radius 10.0 cm and total charge + 12.0 nC. An electron is placed on the ring's axis a distance 25.0 cm from the centre of the ring and is constrained to stay on the axis of the ring. The electron is then released from rest, then.*

(i) describe the subsequent motion of the electron.

(ii) find the speed of the electron when it reaches the centre of the ring.

Sol. (i) The electron will be attracted towards the centre *C* of the ring. At *C* net force is zero, but on reaching *C*, electron has some kinetic energy and due to inertia it crosses *C*, but on the other side it is further attracted towards *C*. Hence, motion of electron is oscillatory about point *C*.



(ii) As the electron approaches *C*, its speed (hence, kinetic energy) increases due to force of attraction towards the centre *C*. This increase in kinetic energy is at the cost of electrostatic potential energy. Thus,

$$\frac{1}{2}mv^2 = U_i - U_f$$

$$= U_P - U_C = (-e)[V_P - V_C] \quad \text{...(i)}$$

Here, *V* is the potential due to ring.

$$V_P = \frac{1}{4\pi\,\varepsilon_0}\cdot\frac{q}{r}\quad (\text{where, } q = \text{charge on ring})$$

$$= \frac{(9\times10^9)(12\times10^{-9})}{(\sqrt{(10)^2+(25)^2})\times10^{-2}} = 401\ \text{V}$$

$$V_C = \frac{1}{4\pi\,\varepsilon_0}\cdot\frac{q}{R}$$

$$= \frac{(9\times10^9)(12\times10^{-9})}{10\times10^{-2}}$$

$$= 1080\ \text{V}$$

Substituting the proper values in Eq. (i), we have

$$\frac{1}{2}\times 9.1\times 10^{-31}\times v^2 = (-1.6\times 10^{-19})(401-1080)$$

$\therefore$ Speed of the electron, $v = 15.45\times 10^6\ \text{ms}^{-1}$

2. Potential energy for a collection of more than two charges

The potential energy of a system of n charges is given by

$$U = \frac{K}{2}\sum_{\substack{i,j\\ i\neq j}}^{n}\frac{q_i q_j}{r_{ij}} \qquad \left(\text{here, } K = \frac{1}{4\pi\varepsilon_0}\right)$$

The factor of 1/2 is applied only with the summation sign because on expanding the summation, each pair is counted twice. It is reqresented by U.

For example electric potential energy of four point charges q_1, q_2, q_3 and q_4 would be given by

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_4q_3}{r_{43}} + \frac{q_4q_2}{r_{42}} + \frac{q_4q_1}{r_{41}} + \frac{q_3q_2}{r_{32}} + \frac{q_3q_1}{r_{31}} + \frac{q_2q_1}{r_{21}}\right] \quad \text{...(ii)}$$

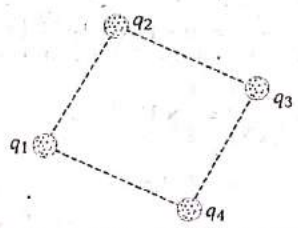


Fig. 2.27

Here, all the charges are to be substituted with sign.

Example 2.43 *Three point charges of 1 C, 2 C and 3 C are placed at the corners of an equilateral triangle of side 1 m. Calculate the work required to move these charges to the corners of a smaller equilateral triangle of side 0.5 m.*

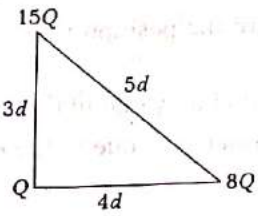
Sol. Work done $= U_f - U_i$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{1}{r_f} - \frac{1}{r_i}\right)[q_3q_2 + q_3q_1 + q_2q_1]$$

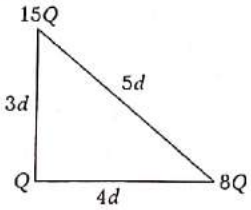
$$= 9\times 10^9\left(\frac{1}{0.5} - \frac{1}{1}\right)[(3)(2) + (3)(1) + (2)(1)]$$

$$= 99\times 10^9\ \text{J}$$

Example 2.44 *Find the electric potential energy of the system of charges.*



Sol. Number of pairs = 3



The electric potential energy for a system of charges is given by

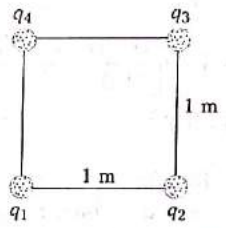
$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_2}{r} + \frac{q_2q_3}{r} + \frac{q_1q_3}{r}\right]$$

On putting the values, we get

$$U = \frac{1}{4\pi\varepsilon_0}\cdot\left[\frac{15Q\cdot Q}{3d} + \frac{8Q\cdot Q}{4d} + \frac{15Q\cdot 8Q}{5d}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{31Q^2}{d}$$

Example 2.45 *Four charges $q_1 = 1\ \mu C$, $q_2 = 2\ \mu C$, $q_3 = -3\ \mu C$ and $q_4 = 4\ \mu C$ are kept on the vertices of a square of side 1 m. Find the electric potential energy of this system of charges.*



Sol. We have,

$$r_{41} = r_{43} = r_{32} = r_{21} = 1\ \text{m}$$

and $$r_{42} = r_{31} = \sqrt{(1)^2 + (1)^2} = \sqrt{2}\ \text{m}$$

Potential energy of a system of three charges,

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_4q_3}{r_{43}} + \frac{q_4q_2}{r_{42}} + \frac{q_4q_1}{r_{41}} + \frac{q_3q_2}{r_{32}} + \frac{q_3q_1}{r_{31}} + \frac{q_2q_1}{r_{21}}\right]$$

$$U = (9.0\times 10^9)(10^{-6})(10^{-6})$$

$$\left[\frac{(4)(-3)}{1} + \frac{(4)(2)}{\sqrt{2}} + \frac{(4)(1)}{1} + \frac{(-3)(2)}{1} + \frac{(-3)(1)}{\sqrt{2}} + \frac{(2)(1)}{1}\right]$$

$$= (9.0\times 10^{-3})\left[-12 + \frac{5}{\sqrt{2}}\right] = -7.62\times 10^{-2}\ \text{J}$$

Note Here, negative sign of U implies that positive work has been done by electrostatic forces in assembling these charges at respective distances from infinity.

Example 2.46 *Two point charges are located on the X-axis, $q_1 = -1\ \mu C$ at $x = 0$ and $q_2 = +1\ \mu C$ at $x = 1$ m.*

(i) Find the work that must be done by an external force to bring a third point charge $q_3 = +1\ \mu C$ from infinity to $x = 2$ m.

(ii) Find the total potential energy of the system of three charges.

Sol. (i) The work that must be done on q_3 by an external force is equal to the difference of potential energy U when the charge is at $x = 2$m and the potential energy when it is at infinity.

$$\therefore \quad W = U_f - U_i = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_3q_2}{(r_{32})_f} + \frac{q_3q_1}{(r_{31})_f} + \frac{q_2q_1}{(r_{21})_f}\right] - \frac{1}{4\pi\varepsilon_0}\left[\frac{q_3q_2}{(r_{32})_i} + \frac{q_3q_1}{(r_{31})_i} + \frac{q_2q_1}{(r_{21})_i}\right]$$

Here, $\quad (r_{21})_i = (r_{21})_f$ and $(r_{32})_i = (r_{31})_i = \infty$

$$\therefore \quad W = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_3q_2}{(r_{32})_f} + \frac{q_3q_1}{(r_{31})_f}\right]$$

Substituting the values, we have

$$W = (9.0\times 10^9)(10^{-12})\left[\frac{(1)(1)}{(1.0)} + \frac{(1)(-1)}{(2.0)}\right]$$

$$= 4.5\times 10^{-3}\ \text{J}$$

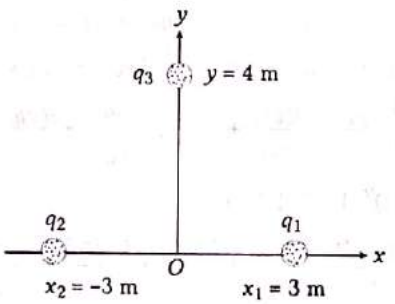
(ii) The total potential energy of the three charges is given by

$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_3q_2}{r_{32}} + \frac{q_3q_1}{r_{31}} + \frac{q_2q_1}{r_{21}}\right)$$

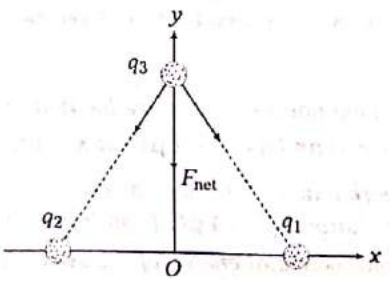
$$= (9.0\times 10^9)\left[\frac{(1)(1)}{(1.0)} + \frac{(1)(-1)}{(2.0)} + \frac{(1)(-1)}{(1.0)}\right](10^{-12})$$

$$= -4.5\times 10^{-3}\ \text{J}$$

Example 2.47 *Two point charges $q_1 = q_2 = 2\ \mu C$ are fixed at $x_1 = +3$ m and $x_2 = -3$ m as shown in figure. A third particle of mass 1 g and charge $q_3 = -4\ \mu C$ are released from rest at $y = 4.0$ m. Find the speed of the particle as it reaches the origin.*



Sol. Here, the charge q_3 is attracted towards q_1 and q_2 both. So, the net force on q_3 is towards origin.



By this force charge is accelerated towards origin, but this acceleration is not constant.

So, to obtain the speed of particle at origin by kinematics, we have to find first the acceleration at same intermediate position and then will have to integrate it with proper limits.

On the other hand, it is easy to use energy conservation principle as the only forces are conservative.

Let v be the speed of particle at origin. From conservation of mechanical energy,

$$U_i + K_i = U_f + K_f$$

$$\text{or} \quad \frac{1}{4\pi\varepsilon_0}\left[\frac{q_3q_2}{(r_{32})_i} + \frac{q_3q_1}{(r_{31})_i} + \frac{q_2q_1}{(r_{21})_i}\right] + 0 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_3q_2}{(r_{32})_f} + \frac{q_3q_1}{(r_{31})_f} + \frac{q_2q_1}{(r_{21})_f}\right] + \frac{1}{2}mv^2$$

Here, $\quad (r_{21})_i = (r_{21})_f$

Substituting the proper values, we have

$$(9.0\times 10^9)\left[\frac{(-4)(2)}{(5.0)} + \frac{(-4)(2)}{(5.0)}\right]\times 10^{-12}$$

$$= (9.0\times 10^9)\left[\frac{(-4)(2)}{(3.0)} + \frac{(-4)(2)}{(3.0)}\right]\times 10^{-12} + \frac{1}{2}\times 10^{-3}\times v^2$$

$$\therefore \quad (9\times 10^{-3})\left(-\frac{16}{5}\right) = (9\times 10^{-3})\left(-\frac{16}{3}\right) + \frac{1}{2}\times 10^{-3}\times v^2$$

$$(9\times 10^{-3})(16)\left(\frac{2}{15}\right) = \frac{1}{2}\times 10^{-3}\times v^2$$

$$\therefore \quad v = 6.2\ \text{ms}^{-1}$$

Potential energy in an external field

(i) Potential energy of a single charge in external field

Potential energy of a single charge q at a point with position vector r in an external field $= q\cdot V(r)$

where $V(r)$ is the potential at a point due to external electric field **E**.

(ii) Potential energy of a system of two charges in an external field

Consider two charges q_1 and q_2 kept in an external field **E** at A and B.

Then, potential energy of a system is given by

$$U = q_1V(r_1) + q_2V(r_2) + \frac{q_1q_2}{4\pi\varepsilon_0 r_{12}}$$

where, r_1 and r_2 are the position vector of point A and B, respectively.

$V(r_1)$ = potential at r_1 due to the external field.

$V(r_2)$ = potential at r_2 due to the external field.

Potential energy of a dipole in a uniform electric field

The work done in rotating the dipole through small angle $d\theta$, then $dW = \tau d\theta$.

$$dW = -pE \sin\theta d\theta$$

Suppose initially dipole is kept in a uniform electric field at angle θ_1. Now, to turn it through an angle θ_2 (with the field). Then, work done

$$\int_{U_1}^{U_2} dW = \int_{\theta_1}^{\theta_2} pE \sin\theta d\theta$$

$$W = -pE[\cos\theta_2 - \cos\theta_1]$$

If $\theta_1 = 0°$ and $\theta_2 = \theta$, *i.e.*, Initially dipole is kept along the field then it turns through θ so work done,

$$W = pE(1 - \cos\theta)$$

Potential energy of dipole is defined as work done in rotating a dipole from a direction perpendicular to the field to the given direction.

If the dipole is rotated by an angle $\theta_1 = 90°$ to $\theta_2 = \theta$, then energy is given by

$$W = U = pE(\cos 90° - \cos\theta)$$
$$= -pE\cos\theta$$
$$= -\mathbf{p}\cdot\mathbf{E}$$

Example 2.48 *When an electric dipole is placed in a uniform electric field making angle θ with electric field, it experiences a torque τ. Calculate the minimum work done in changing the orientation to 2θ.*

Sol. $\tau = pE\sin\theta \Rightarrow \mathbf{pE} = \dfrac{\tau}{\sin\theta}$

$$W = \Delta U = -pE\cos\theta_2 + pE\cos\theta_1$$

$\Rightarrow$ $W = pE[\cos\theta - \cos 2\theta]$

$\Rightarrow$ $W = \dfrac{\tau}{\sin\theta}[\cos\theta - \cos 2\theta]$

Equilibrium of dipole

(i) Work done $\theta = 0°$

$$U = -pE\cos\theta = -pE$$

Thus, the potential energy of a dipole is minimum when its dipole moment is parallel to the external field. This is position of stable equilibrium.

(ii) When $\theta = 180°$

$$U = -pE\cos\theta = +pE$$

Thus, the potential energy of a dipole is maximum when its dipole moment is antiparallel to the external field. This is the position of unstable equilibrium.

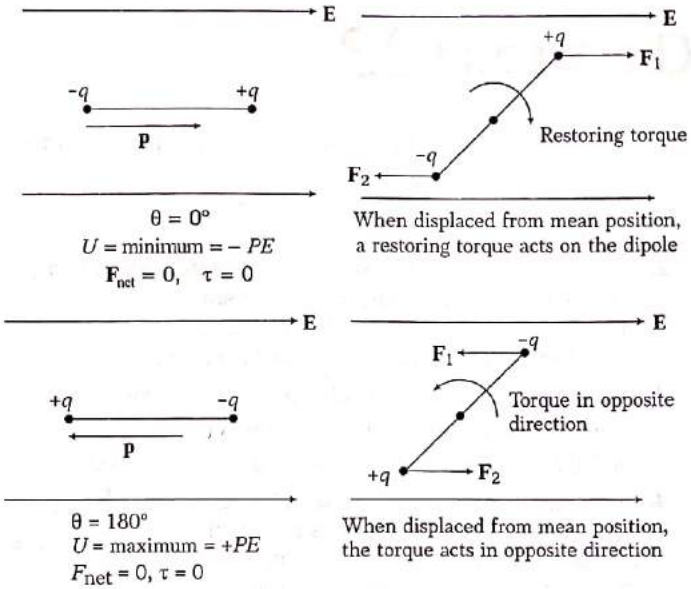


Fig. 2.28

Example 2.49 *Two point-charges $+2e$ and $-2e$ are situated at a distance of 2.4 Å from each other and constitute an electric dipole. This dipole is placed in a uniform electric field of 4.0×10^5 Vm^{-1}. Calculate (i) electric dipole moment, (ii) potential energy of the dipole in equilibrium position, (iii) work done in rotating the dipole through 180° from the equilibrium position.*

Sol. (i) The electric dipole moment is $p = q \times 2l$

where, q is the charge of one end of the dipole and $2l$ is the distance between the two charges.

Here, $q = 2e = 3.2 \times 10^{-19}$ C and $2l = 2.4$ Å

$$= 2.4 \times 10^{-10}\text{m}$$

$\because$ $p = (3.2 \times 10^{-19}) \times (2.4 \times 10^{-10})$

$$= 7.68 \times 10^{-29} \text{ C-m}$$

(ii) The potential energy of a dipole in an electric field E is

$$U = -pE\cos\theta$$

where, θ is the angle between the axis of the dipole and the field. In equilibrium position,

$\theta = 0°$, and so $U_0 - pE$

Here, $p = 7.68 \times 10^{-29}$ C-m and $E = 4.0 \times 10^5$ Vm^{-1}

$\therefore$ $U_0 = -(7.68 \times 10^{-29}$ C-m$) \times (4.0 \times 10^5$ $Vm^{-1})$

$$= -3.07 \times 10^{-23} \text{ J}$$

(iii) The work done (increase in energy) in rotating the dipole through angle θ from the direction of the electric field E is given by

$$W = pE(1 - \cos\theta)$$

If $\theta = 180°$ or $\cos\theta = -1$, then $W = 2pE$

$$= 2 \times (7.68 \times 10^{-29} \text{ C-m} \times (4.0 \times 10^5 \text{ V/m}$$

$$= 6.14 \times 10^{-23} \text{ J}$$

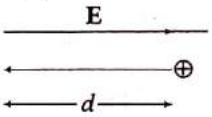
Check point 2.3

1. When a positive charge q is taken from lower potential to a higher potential point, then its potential energy will
(a) decrease (b) increase
(c) remain unchanged (d) become zero

2. When one electron is taken towards the other electron, then the electric potential energy of the system
(a) decrease (b) increases
(c) remains unchanged (d) becomes zero

3. Two positive point charges of $12\,\mu C$ and $8\,\mu C$ are 10 cm apart. The work done in bringing them 4 cm is
(a) 5.8 J (b) 5.8 eV (c) 13 J (d) 13 eV

4. In the figure, a proton moves a distance d in a uniform electric field **E** as shown in the figure. Does the electric field do a positive or negative work on the proton? Does the electric potential energy of the proton increase or decrease



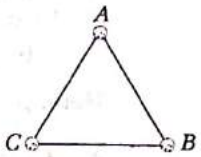
(a) Negative, increase (b) Positive, decrease
(c) Negative, decrease (d) Positive increase

5. The electrostatic potential energy between proton and electron separated by a distance 1 Å is
(a) 13.6 eV (b) 27.2 eV (c) -14.4 eV (d) 1.44 eV

6. Identify the wrong statement.
(a) The electrical potential energy of a system of two protons shall increase if the separation between the two is decreased
(b) The electrical potential energy of a proton-electron system will increase if the separation between the two is decreased
(c) The electrical potential energy of a proton-electron system will increase if the separation between the two is increased
(d) The electrical potential energy of system of two electrons shall increase if the separation between the two is decreased.

7. Two positive point charges of 12 and 5 microcoulombs, are placed 10 cm apart in air. The work needed to bring them 4 cm closer is
(a) 2.4 J (b) 3.6 J (c) 4.8 J (d) 6.0 J

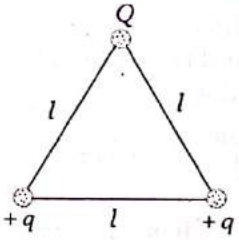
8. Three identical charges each of $2\,\mu C$ are placed at the vertices of a triangle ABC as shown in the figure.



If $AB + AC = 12$ cm and $AB \cdot AC = 32\ cm^2$, the potential energy of the charge at A is
(a) 1.53 J (b) 5.31 J (c) 3.15 J (d) 1.35 J

9. If three charges are placed at the vertices of equilateral triangle of charge q each. What is the net potential energy, if the side of equilateral Δ is l cm.
(a) $\frac{1}{4\pi\varepsilon_0}\frac{q^2}{l}$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{2q^2}{l}$
(c) $\frac{1}{4\pi\varepsilon_0}\frac{3q^2}{l}$ (d) $\frac{1}{4\pi\varepsilon_0}\frac{4q^2}{l}$

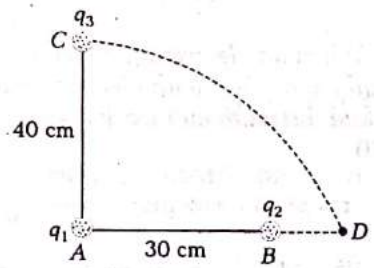
10. Three charges Q, $+q$ and $+q$ are placed at the vertices of an equilateral triangle of side l as shown in the figure. If the net electrostatic energy of the system is zero, then Q is equal to



(a) $\left(-\frac{q}{2}\right)$ (b) $(-q)$ (c) $(+q)$ (d) zero

11. If identical charges $(-q)$ are placed at each corner of a cube of side b, then electric potential energy of charge $(+q)$ which is placed at centre of the cube will be
(a) $\frac{8\sqrt{2}q^2}{4\pi\varepsilon_0 b}$ (b) $\frac{-8\sqrt{2}q^2}{\pi\varepsilon_0 b}$ (c) $\frac{-4\sqrt{2}q^2}{\pi\varepsilon_0 b}$ (d) $\frac{-4q^2}{\sqrt{3}\pi\varepsilon_0 b}$

12. Two charges q_1 and q_2 are placed 30 cm apart as shown in the figure. A third charge q_3 is moved along the arc of a circle of radius 40 cm from C to D. The change in the potential energy of the system is $\frac{q_3}{4\pi\varepsilon_0}k$, where k is



(a) $8q_2$ (b) $8q_1$ (c) $6q_2$ (d) $6q_1$

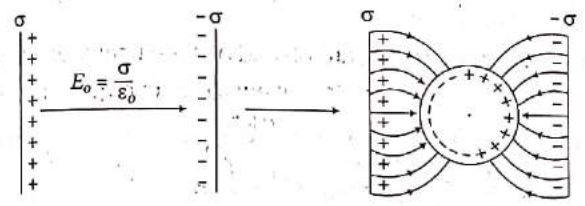
13. For dipole $q = 2\times10^{-6}$ C and $d = 0.01$ m, calculate the maximum torque for this dipole if $E = 5\times10^5$ N/C.
(a) 1×10^{-3} N/m (b) 10×10^{-3} N/m
(c) 10×10^{-3} N/m (d) 1×10^2 N/m

14. A molecule with a dipole moment p is placed in a electric field of strength E. Initially, the dipole is aligned parallel to the field. If the dipole is to be rotated to be anti-parallel to the field, then the work required to be done by an external agency is
(a) $-2pE$ (b) $-pE$
(c) pE (d) $2pE$

15. Three point charges of 1C, 2C and 3C are placed at the corners of an equilateral triangle of side 100 cm. The work done to move these charges to the corners of a similar equilateral triangle of side 50 cm, will be
(a) 9.9×10^{10} J (b) 9.9×10^9 J
(c) 5.2×10^{10} J (d) 5.9×10^9 J

4 Electrostatics of conductors

Whenever a conductor is placed in an external electric field, the free electrons in it experience a force due to it and start moving opposite to the field. This movement makes one side of conductor positively charged and the other as negatively charged. This creates an electric field in the conductor in a direction opposite to external electric field (called induced field). The movement of electrons continues till the net electric field (resultant of external electric field and induced field) becomes zero. This is a state of **electrostatic equilibrium.**



Metallic sphere in electric field

Fig. 2.29

A conductor in electrostatic equilibrium has the following properties

(i) Under static conditions, electric field inside a conductor is zero.

(ii) Just outside a conductor, **E** is normal to its surface. If it were not, then it would have a component along the conductor's surface, that would cause the motion of charges. That contradicts the state of electrostatic equilibrium.

(iii) The whole body of conductor is equipotential, as $E_{inside} = 0$.

(iv) At any point inside that body of the conductor, the electric field due to charges appearing on the surface of conductor is equal and opposite to the external field.

Some phenomena related to electrostatics of conductors are given below.

(i) Electrostatic shielding

Suppose we have a very sensitive electronic instrument that we want to protect from external electric fields that might cause wrong measurements. We surround the instrument with a conducting box or we keep the instrument inside the cavity of a conductor. By doing this charge in the conductor is so distributed that the net electric field inside the cavity becomes zero and the instrument is protected from the external fields. This is called electronic shielding.

It is electrostatic shielding that protects a person from lightning strikes if he is in a car.

Except for spherical surfaces, the charge is not distributed uniformly on the surface of a conductor. At the sharp points or edges, the surface charge density (σ) is very high and hence the electric field becomes very strong. The air around such sharp points may become ionised producing the corner discharge in which the charge jumps from the conductor air because of the dielectric breakdown of air.

(ii) Electrostatic pressure

Electrostatic pressure is defined as the force per unit area on the surface of a conductor due to its own charge. If σ is the surface charge density, then

$$\text{Electrostatic pressure} = \frac{\sigma^2}{2\varepsilon_0} = \frac{1}{2}\varepsilon_0 E^2$$

where, $E = \frac{\sigma}{\varepsilon_0}$ (electric field near conductor)

where, σ = surface charge density

(iii) Effect of metallic slab between two charged plates

Figure shows a metallic slab between two charged plates. The field **E** due to the charged plates is directed towards right and the field **E** due to the induced charge in the slab is directed toward left, and hence the net field inside the slab becomes zero.

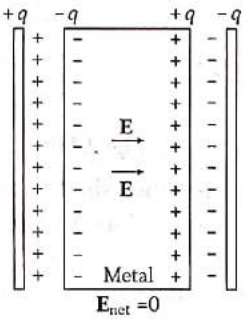


Fig. 2.30

(iv) Earthing a conductor

Earth is a good conductor of electricity. For most practical purpose, its potential is assumed to be zero. A conductor is said to be earthed or grounded whenever it is connected to a earth. In that case, its potential becomes zero.

The charge will flow from conductor to earth or from earth to conductor to make its potential zero. The symbol used for earthing is

Fig. 2.31 A symbol for earthing

Let us discuss some of the examples related to this concept.

(a) Fig. (a) shows a conducting shell with charge q.

Fig. (b) the conducting shell with charge q has been surrounded by another larger conducting shell which is uncharged. Charges $-q$ and $+q$ are induced on its inner and outer surfaces but net charge in the outer shell is still zero.

In the Fig. (c), the outer shell of the Fig. (b) is earthed, The free charge on the outer surface goes to the earth but the inner charge remains bounded to the charge on the inner shell, so that the potential of the outer shell connected to the earth becomes zero.

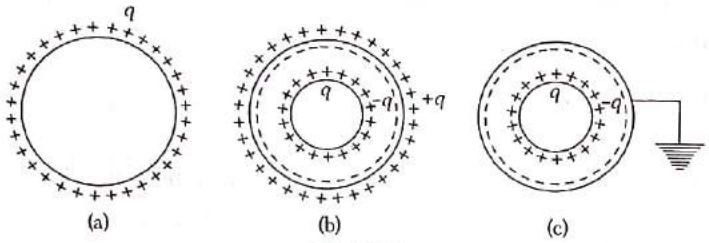


Fig. 2.32

(b) Fig. (a), shows two concentric conducting shells. Some charge q_1 is given to the outer shell. No charge is developed on the inner shell.

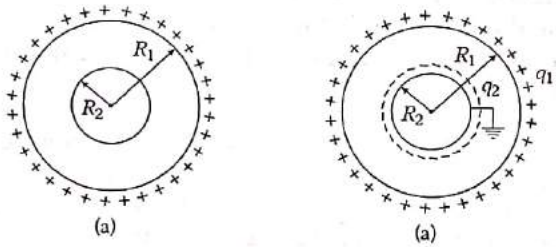


Fig. 2.33

In the Fig. (b), the inner shell is earthed and hence some charge q_2 is developed on it so that its potential becomes zero.

On the surface of the inner shell, the net potential is

$$\frac{1}{4\pi\varepsilon_0}\left(\frac{q_2}{R_1}+\frac{q_1}{R_1}\right)=0$$

or

$$\boxed{q_2=-q_1\frac{R_2}{R_1}}$$

(c) Fig. (a) is an uncharged metallic solid sphere of radius R.

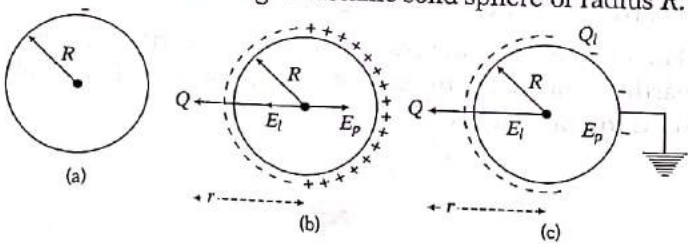


Fig. 2.34

In Fig. (b), a positive point charge Q has been placed at a distance r from the centre of the sphere. The point charge Q exerts force on the electrons in the sphere and hence the free electrons redistribute themselves so that the left half is negatively charged and the right half is positively charged. The charge distribution is non-uniform.

At the centre of the sphere, the field due to the point charge Q is $E_p=\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$, toward right, since the field inside the conductor is zero. The field due the induced charge on the sphere is equal in magnitude of E_p but opposite in direction.

i.e., $\boxed{\mathbf{E}_l=-\mathbf{E}_p}$

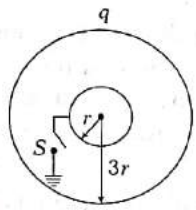
The potential at any point of the sphere = potential at its centre $=\frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$

In the Fig. (c), the sphere is earthed, so $V=0$, hence

$$\frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{r}+\frac{Q_i}{R}\right)=0 \text{ or, } Q_i=\frac{QR}{r}$$

Note that the induced charge Q_i is non-uniformly distribution on the surface of the sphere.

Example 2.50 *Figure shows two conducting thin concentric shells of radii r and 3r. The outer shell carries charge q. Inner shell is neutral. Find the charge that will flow from inner shell to earth after the switch S is closed.*



Sol. Let q' be the charge on inner shell when it is earthed.

$$V_{\text{inner}}=0$$

$$\therefore \quad \frac{1}{4\pi\varepsilon_0}\left[\frac{q'}{r}+\frac{q}{3r}\right]=0$$

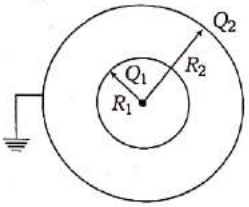
$$\therefore \quad q'=-\frac{q}{3}$$

i.e., $+\frac{q}{3}$ charge will flow from inner shell to earth.

Example 2.51 *A sphere of 4 cm radius is suspended within a hollow sphere of 6 cm radius. The inner sphere is charged to a potential of 3 esu when the outer sphere is earthed, find the charge on the inner sphere.*

Sol. First of all understand that esu means electrostatic unit (CGS unit) and in this system $\frac{1}{4\pi\varepsilon_0}$ is replaced by 1. So,

potential due to Q esu of charge will be Q/R in esu. Now, the diagram for the above question is as follows



Let the inner sphere and outer sphere have charges Q_1 and Q_2 on them, respectively. Their radii being R_1 and R_2 such that ($R_1 < R_2$). Given $R_1 = 4$ cm and $R_2 = 6$ cm. We have two unknowns Q_1 and Q_2. So, we will form two equations by equating to the potential of both the spheres. Consider the outer sphere. It is grounded, so the potential of this sphere is zero.

or $$\frac{Q_1}{R_1} + \frac{Q_2}{R_2} = 0 \quad ...(i)$$

Note, the contribution of Q_2 in potential will be its surface potential but for Q_1 the outer sphere is outside it. So, Q_1 will be treated at the centre.

Hence, from Eq. (i),

$$Q_1 + Q_2 = 0 \Rightarrow Q_2 = -Q_1$$

Now, write the expression for the potential of inner, sphere and make it equal to 3 esu. For the inner sphere both the charges will contribute their respective surface potentials.

Therefore, $\frac{Q_1}{R_1} + \frac{Q_2}{R_2} = 3$

Substituting $Q_2 = -Q_1$, we get

$\frac{Q_1}{R_1} - \frac{Q_2}{R_2} = 3$, therefore, $Q\left[\frac{1}{4} - \frac{1}{6}\right] = 3$ or $Q_1 = 36$ esu

Dielectrics and polarisation

Dielectrics are insulating (non-conducting) materials that can produce electric effect without conduction. Movement of free charges is not possible in a dielectric. So, they behave differently. When a dielectric material is kept in an electric field, then the external field induces dipole moment. Due to which net charges on the surface of the dielectric appears and as a result electric field is produced that opposes the external field. Induced field is less in magnitude than the external field. So, field inside the dielectric gets reduced.

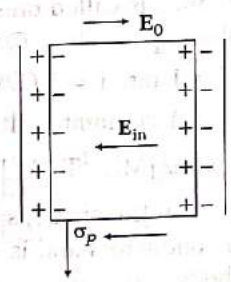


Fig. 2.35 Polarised dielectric slab

$$\mathbf{E} = |\mathbf{E}_0| - |\mathbf{E}_{in}|$$

where, $\mathbf{E}$ = resultant electric field in the dielectric

$\mathbf{E}_0$ = external electric field between two plates

and $\mathbf{E}_{in}$ = electric field inside the dielectric

Types of dielectrics

There are two types of dielectrics

(i) Polar dielectrics

A polar molecule is one in which the centres of positive and negative charges are separated (even when there is no external field). Such molecules have permanent dipole moment, *e.g.*, HCl, H_2O. So, a polar dielectric is one which is having a net dipole moment zero in the absence of electric field due to the random orientation of polar molecules as shown in figure.

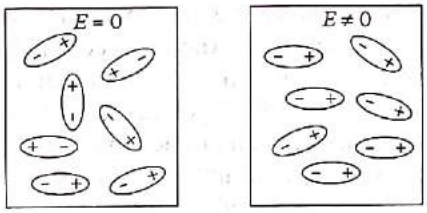


Fig. 2.36 Behaviour of a polar dielectric in external field

In the presence of electric field polar molecules tend to line up in the direction of electric field, and the substance has finite dipole moment.

(ii) Non-polar dielectrics

In a non-polar molecule, the centres of positive and negative charges coincide. The molecule thus has no permanent (or intrinsic) dipole moment, *e.g.*, oxygen (O_2) and hydrogen (H_2) molecules. In an external electric field, the positive and negative charges of a non-polar molecules are displaced in opposite direction. This occurs till the point where external force is balanced by the restoring force (due to internal field in the molecule). Thus, dipole movement is induced in non-polar molecules and dielectric is said to be **polarised**. The induced dipole moments of different molecules add up giving a net dipole moment of the dielectric in the presence of the external field.

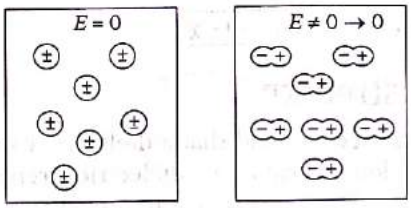


Fig. 2.37 Behaviour of a non-polar dielectric in external field

Dielectric constant (K)

The ratio of the strength of the applied electric field to the strength of the reduced value of the electric field on placing the dielectric between the two charged plates called the dielectric constant of the dielectric medium.

It is also known as **relative permittivity** or **specific inductive capacity** (SIC) and is denoted by K(or ε_r).

Therefore, dielectric constant of a dielectric medium is given by

$$K = \frac{E_0}{E}$$

The value of K is always greater than 1.

Polarisation (P)

It is a process of inducing equal and opposite charges on the two faces of the dielectrics on the application of electric field. In other words, the induced dipole moment developed per unit volume in a dielectric slab on placing it, in an electric field is called polarisation. It is denoted by P. If p is induced dipole moment acquired by an atom of the dielectric and N is the number of atoms per unit volume, then polarisation is given by

$$P = Np$$

The induced dipole moment acquired by the atom is found to be directly proportional to the reduced value of electric field and is given by

$$P = \alpha\varepsilon_0 E_0$$

where, α is constant of proportionality and is called **atomic polarisability.**

Electric susceptibility (χ)

The polarisation density of dielectric slab is directly proportional to the reduced value of the electric field and may be expressed as

$$P = \chi \varepsilon_0 E_0$$

where, χ is a constant of proportionality and is called electric susceptibility of the dielectric slab. It is a dimensionless constant.

Note

1. For vacuum, $\chi = 0$
2. $K = 1 + \chi$

Dielectric strength

The maximum electric field that a dielectric can withstand without breakdown is called its dielectric strength.

Its SI unit is Vm^{-1} and its practical unit is $kV(mm)^{-1}$.

For air it is about 3×10^6 Vm^{-1}.

Capacitors and capacitance

A capacitor is a device which is used to store electric charge or potential energy. It is made up of two isolated conductors carrying equal and opposite charges placed at small separation.

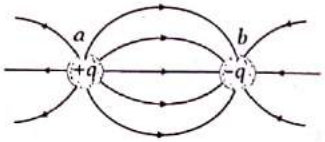


Fig. 2.38

The space between the conductors may be vacuum or an insulating material. Capacitor is represented as

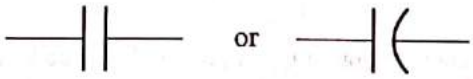


Fig. 2.39

Capacitance of a conductor

When a charge q is given to a conductor, it spreads over the outer surface of the conductor. The whole conductor comes to the same potential (say V). This potential V is directly proportional to the charge q, *i.e.*,

$$V \propto q$$

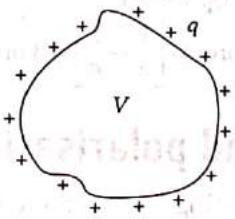


Fig. 2.40

$$q = CV$$

or

$$C = \frac{q}{V}$$

This C is constant of proportionality and is called the **capacitance** of the conductor.

The SI unit of capacitance is called one **farad** (1 F). One farad is equal to coulomb per volt (1 C/V)

$$\therefore \quad 1\ F = 1\ \text{farad} = 1\ C/V$$
$$= 1\ \text{coulomb/volt}$$

Its dimensional formula is $[ML^{-2}T^4A^2]$.

Capacitance C depends on the shape, size and separation of the system of two conductors and is independent of the charge given to the body.

Example 2.52 *A capacitor of 0.75 μF is charged to a voltage of 16 V. What is the magnitude of the charge on each plate of the capacitor?*

Sol. Using $q = CV$, we get

$$q = CV = (0.75 \times 10^{-6}\ \text{F})(16\ \text{V}) = 1.2 \times 10^{-5}\ \text{C}$$

Capacitance of an isolated spherical conductor

When a charge q is given to a spherical conductor of radius R, the potential on it is,

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R}$$

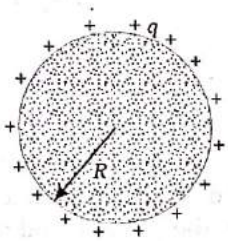


Fig. 2.41

From this expression we find that

$$\frac{q}{V} = 4\pi\varepsilon_0 R = C$$

Thus, capacitance of the spherical conductor is

$$C = 4\pi\varepsilon_0 R$$

From this expression, we can draw the following conclusions

(i) $C \propto R$ or C depends on R only. Which we have already stated that C depends on the dimensions of the conductor. Moreover if two conductors have radii R_1 and R_2, then

$$\frac{C_1}{C_2} = \frac{R_1}{R_2}$$

(ii) Earth is also a spherical conductor of radius $R = 6.4 \times 10^6$ m. The capacity of earth is therefore,

$$C = \left(\frac{1}{9 \times 10^9}\right)(6.4 \times 10^6) \approx 711 \times 10^{-6}\ \text{F}$$

or $C = 711\,\mu\text{F}$

From here, we can see that farad is a large unit. As capacity of such a huge conductor is only 711 μF.

Capacity of a spherical conductor enclosed by an earthed concentric spherical shell

It consists of two concentric conducting spheres of radii a and b ($a < b$). Inner sphere is given charge q while outer sphere is earthed. Potential difference between the spheres is given by

$$V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) \quad \text{...(i)}$$

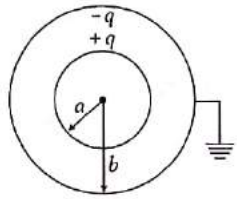


Fig. 2.42

Hence, the capacitance of this system will be

$$C = \frac{q}{V}$$

or $$C = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right) \quad \text{[From Eq. (i)]}$$

Example 2.53 *A sphere of radius 0.03 m is suspended within a hollow sphere of radius 0.05 m. If the inner sphere is charged to a potential of 1500 V and outer sphere is earthed, find the capacitance and the charge on the inner sphere.*

Sol. Here, $a = 0.03$ m, $b = 0.05$ m and $V = 1500$ V

The capacitance of the air-filled spherical capacitor is

$$C = \frac{4\pi\varepsilon_0 ab}{b-a}$$

$$= \frac{0.03 \times 0.05}{9 \times 10^9 \times (0.05 - 0.03)}$$

$$= 8.33 \times 10^{-12}\ \text{F} = 8.33\ \text{pF}$$

Charge, $$q = CV = 8.33 \times 10^{-12} \times 1500$$

$$= 1.25 \times 10^{-8}\ \text{C}$$

Example 2.54 *The thickness of air layer between the two coatings of a spherical capacitor is 2 cm. The capacitor has the same capacitance as the sphere of 1.2 m diameter. Find the radii of its surfaces.*

Sol. Here, $$\frac{4\pi\varepsilon_0 ab}{b-a} = 4\pi\varepsilon_0 R \quad \text{or} \quad \frac{ab}{b-a} = R$$

Now, $b - a = 2$ cm and $R = \frac{1.2}{2}$ m $= 60$ cm

$\therefore$ $$\frac{ab}{2} = 60$$

or $$ab = 120$$

$$(b+a)^2 = (b-a)^2 + 4ab = 2^2 + 4 \times 120 = 484$$

or $$b + a = 22$$

or $$2 + a + a = 22 \quad [\because b - a = 2\ \text{cm}]$$

$\therefore$ $a = 10$ cm and $b = 12$ cm

Cylindrical capacitor

Cylindrical capacitor consists of two co-axial cylinders of radii a and b ($a < b$), inner cylinder is given charge $+q$ while outer cylinder is earthed.

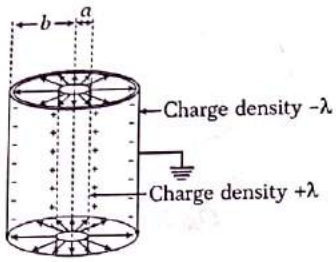


Fig. 2.43

Common length of the cylinders is l, then capacity of a cylindrical capacitor is given by

$$C = \frac{2\pi\varepsilon_0 l}{\log_e (b/a)}$$

Parallel plate capacitor

A parallel plate air capacitor consists of two parallel metallic plates separated by a small distance.

Suppose the area of each plate is A and the separation between the two plates is d. Also assume that the space between the plates contains vacuum. Suppose the charge density on each of these surfaces has a magnitude $\sigma = q/A$.

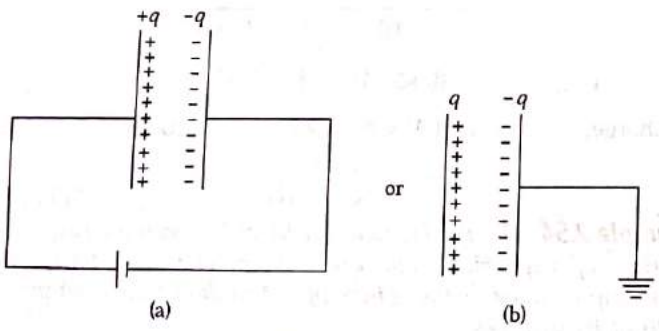


Fig. 2.44

We know that the intensity of electric field at a point between two plane parallel sheets of equal and opposite charges is given by $E = \dfrac{\sigma}{\varepsilon_0}$.

The potential difference between the plates be V volt. Then, the electric field between the plates is given by

$$E = \frac{V}{d}$$

$$\therefore \quad V = Ed = \left(\frac{\sigma}{\varepsilon_0}\right) d = \frac{qd}{A\varepsilon_0}$$

$\therefore$ The capacitance of the parallel plate capacitor is given by

$$C = \frac{q}{V} = \frac{A\varepsilon_0}{d} \quad \text{or} \quad \boxed{C = \frac{\varepsilon_0 A}{d}}$$

The force between plates of capacitor is attractive and can be expressed as

$$F = qE = q\left(\frac{\sigma}{2\varepsilon_0}\right)$$

or

$$\boxed{F = \frac{1}{2}\varepsilon_0 A E^2}$$

If the space between the plates be filled with same dielectric medium of dielectric constant K, then the electric field between the plates is reduced to

$$\boxed{C = \frac{K\varepsilon_0 A}{d} \text{ Farad}}$$

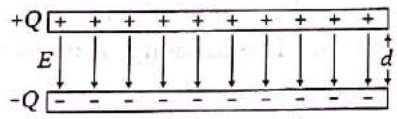
Note

1. Instead of two plates, if there are n similar plates at equal distances from each other and the alternate plates are connected together, then the capacitance of the arrangement is given by

$$C = \frac{(n-1)\,\varepsilon_0 A}{d}$$

2. From the above relation, it is clear that the capacitance depends only on geometrical factors (A and d).

Example 2.55 *A parallel plate capacitor is constructed with plates of area 0.0280 m^2 and separation 0.550 mm. Find the magnitude of the charge on each plate of this capacitor when the potential difference between the plates is 20.1 V.*



Sol. Using the formula,

$$C = \frac{\varepsilon_0 A}{d} = \frac{(8.85 \times 10^{-12}\ \text{C}^2/\text{N-m}^2)(0.0280\ \text{m}^2)}{0.550 \times 10^{-3}\ \text{m}}$$

We obtain the capacitance of the parallel plate capacitor,

$$C = 4.51 \times 10^{-10}\ \text{F}$$

Capacitance of a capacitor partially filled with dielectric

Suppose, a plate of capacitor is partially filled with a dielectric (dielectric constant $= K$) and thickness t ($< d$) is introduced between the plates. Then, the distance between the plates is t in the dielectric and $(d - t)$ in vacuum (or air)

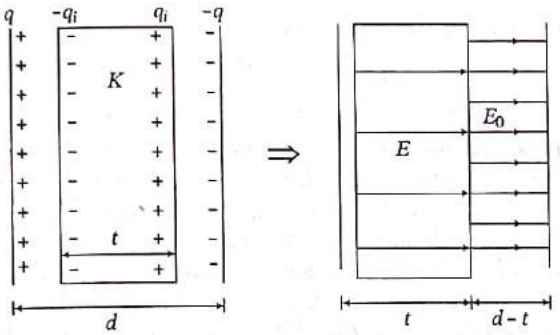


Fig. 2.45

If $E_0 = \dfrac{Q}{\varepsilon_0 A}$ is the electric field in the region, where dielectric is absent, then electric field inside the dielectric will be $E = E_0/K$. The potential difference between the plates of the capacitor is

$$V = Et + E_0(d - t) = \frac{E_0}{K}t + E_0(d - t)$$

$$= E_0\left(d - t + \frac{t}{K}\right) = \frac{q}{A\varepsilon_0}\left(d - t + \frac{t}{K}\right)$$

Now, as per the definition of capacitance,

$$C = \frac{q}{V} = \frac{\varepsilon_0 A}{d - t + \dfrac{t}{K}}$$

or

$$\boxed{C = \frac{\varepsilon_0 A}{d - t + \dfrac{t}{K}}}$$

Special cases

(i) If the slab completely fills the space between the plates, then $t = d$ and therefore

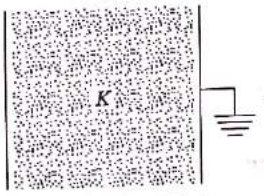


Fig. 2.46

$$C = \frac{\varepsilon_0 A}{d/K} = \frac{K\varepsilon_0 A}{d} \quad \text{or} \quad \frac{C}{C_0} = K$$

where, $C_0 = \dfrac{\varepsilon_0 A}{d}$ is capacitance without the dielectric.

(ii) If a conducting slab ($K = \infty$) is placed between the plates, then

$$C = \frac{\varepsilon_0 A}{d - t + t/\infty} = \frac{\varepsilon_0 A}{d - t}$$

This can also be understood from the following figure

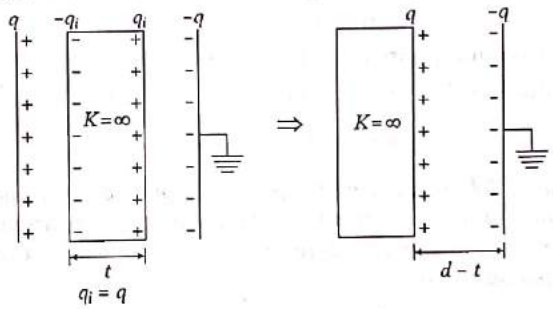


Fig. 2.47

(iii) If the space between the plates is completely filled with a conductor, then $t = d$ and $K = \infty$.

Then, $$C = \frac{\varepsilon_0 A}{d - d + \dfrac{d}{\infty}} = \infty$$

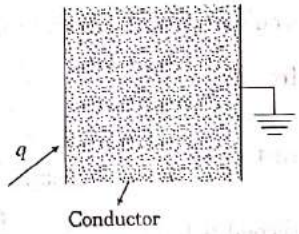


Fig. 2.48

(iv) If more than one dielectric slabs are placed between the capacitor, then

$$C = \frac{\varepsilon_0 A}{d - t_1 - t_2 - \ldots - t_n + \dfrac{t_1}{K_1} + \dfrac{t_2}{K_2} + \ldots + \dfrac{t_n}{K_n}}$$

Example 2.56 *A dielectric slab of thickness 1.0 cm and dielectric constant 5 is placed between the plates of a parallel plate capacitor of plates area 0.01 m^2 and separation 2.0 cm. Calculate the change in capacity on introduction of dielectric. What would be on the change, if the dielectric slabs were conducting?*

Sol. Given, $t = 1.0\text{ cm} = 10^{-2}\text{m}$, $K = 5$, $A = 0.01\text{ m}^2 = 10^{-2}\text{m}^2$

$d = 2\text{ cm} = 2 \times 10^{-2}\text{m}$

Capacity with air in between the plates,

$$C_0 = \frac{\varepsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 10^{-2}}{2 \times 10^{-2}} = 4.425 \times 10^{-12}\text{ F}$$

Capacity with dielectric slab in between the plates,

$$C = \frac{\varepsilon_0 A}{\left[d - \left(1 - \dfrac{1}{K}\right)t\right]} = \frac{8.85 \times 10^{-12} \times 10^{-2}}{2 \times 10^{-2} - 10^{-2}\left(1 - \dfrac{1}{5}\right)}$$

$$= 7.375 \times 10^{-12}\text{ F}$$

Capacity with conducting slab in between the plates,

$$C' = \frac{\varepsilon_0 A}{d-t} = \frac{8.85\times10^{-12}\times10^{-2}}{2\times10^{-2}-1\times10^{-2}} = 8.85\times10^{-12}\text{ F}$$

Increase in capacity on introduction of dielectric

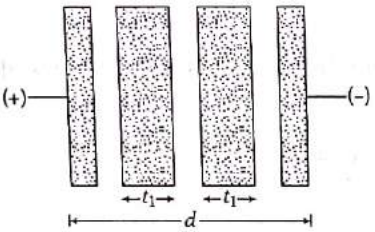
$$= C = C_0 = 7.375\times10^{-12} - 4.425\times10^{-12}$$

$$= 2.95\times10^{-12}\text{ F}$$

Increase in capacity on introduction of conducting slab $= C' - C_0 = 8.85\times10^{-12} - 4.425\times10^{-12}$

$$= 4.425\times10^{-12}\text{ F}$$

Example 2.57 *An air-cored capacitor of plate area A and separation d has a capacity C. Two dielectric slabs are inserted between its plates in two different manners as shown. Calculate the capacitance in it.*



Sol. Let the charges on the plates are Q and $-Q$.

Electric field in free space is $E_0 = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$.

Electric field in first slab is $E_1 = \frac{E_0}{K_1} = \frac{Q}{A\varepsilon_0 K_1}$.

Electric field in second slab is $E_2 = \frac{E_0}{K_2} = \frac{Q}{A\varepsilon_0 K_2}$.

The potential difference between the plates is

$$V = E_0(d - t_1 - t_2) + E_1 t_1 + E_2 t_2$$

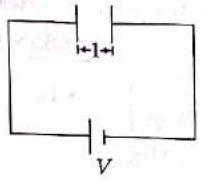
$$\Rightarrow \quad V = E_0\left(d - t_1 - t_2 + \frac{t_1}{K_1} + \frac{t_2}{K_2}\right) \quad \left(\text{as } E_1 = \frac{E_0}{K_1}, E_2 = \frac{E_0}{K_2}\right)$$

$$\therefore \quad V = \frac{Q}{A\varepsilon_0}\left(d - t_1 - t_2 + \frac{t_1}{K_1} + \frac{t_2}{K_2}\right)$$

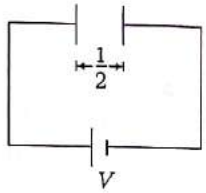
$$\therefore \quad C = \frac{\varepsilon_0 A}{d - t_1 - t_2 + \frac{t_1}{K_1} + \frac{t_2}{K_2}}$$

Example 2.58 *A parallel plate capacitor has plate area A and separation d between the plates. The capacitor is connected to a battery of emf V. (i) Find the charge on the capacitor. (ii) The plate separation is decreased to d/2. Find the extra charge given by the battery to the positive plate.*

Sol. (i) Capacitance, $C = \frac{A\varepsilon_0}{d}$, charge, $Q = CV = \frac{A\varepsilon_0 V}{d}$



(ii) $C' = \frac{A\varepsilon_0}{d/2} = \frac{2A\varepsilon_0}{d} \Rightarrow Q' = C'V = \frac{2A\varepsilon_0 V}{d}$



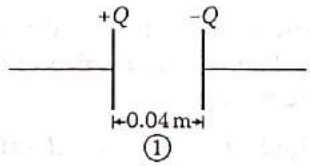
Extra charge given by battery,

$$\Delta Q = Q' - Q = \frac{A\varepsilon_0 V}{d}$$

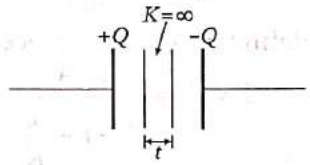
Example 2.59 *The distance between the plates of a parallel plate capacitor is 0.04 m. A field of 5000 V/m is established between the plates and an uncharged metal plate of thickness 0.01 m is inserted into the condenser parallel to its plate. Find the potential difference between the plates (i) before the introduction of the metal plate and (ii) after its introduction. What would be the potential difference, if a plate of dielectric constant K = 2 is introduced in place of metal plate?*

Sol. Potential difference across capacitor,

$$V = Ed = 5000\times0.04 = 200\text{ V}$$



$$C = \frac{A\varepsilon_0}{d} = \frac{A\varepsilon_0}{0.04} \Rightarrow V = 200\text{ V}$$

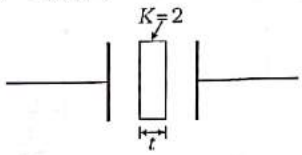


$$C' = \frac{A\varepsilon_0}{d - t + \frac{t}{K}} = \frac{A\varepsilon_0}{0.04 - 0.01 + \frac{0.01}{\infty}} = \frac{A\varepsilon_0}{0.03}$$

Since, charge remains same,

$$Q = CV = C'V' \Rightarrow \frac{A\varepsilon_0}{0.04}\times200 = \frac{A\varepsilon_0}{0.03}V'$$

$\Rightarrow$ Potential, $V' = 150$ V



$$C'' = \frac{A\varepsilon_0}{d - t + \frac{t}{K}} = \frac{A\varepsilon_0}{0.04 - 0.01 + \frac{0.01}{2}} = \frac{A\varepsilon_0}{0.035}$$

$$Q = CV = C''V''$$

$$\frac{A\varepsilon_0}{0.04}\times200 = \frac{A\varepsilon_0}{0.035}V''$$

Potential, $V'' = 175$ V

Effect of dielectric on various parameters

A parallel plate capacitor is charged by a battery which is then disconnected. A dielectric slab is then inserted to fill the space between the plates.

Let q_0, C_0, V_0 and E_0 be the charge, capacitance, potential difference and electric field respectively, before the dielectric slab is inserted.

Now, we will discuss two cases

Case-I Effect of dielectric when the battery is kept disconnected from the capacitor

As $q_0 = C_0V_0, E_0 = \frac{V_0}{d}$.

(i) The charge on the capacitor plates remains q_0.

(ii) The electric field gets reduced

i.e., $E = \frac{E_0}{K}$

(iii) The potential difference gets reduced, *i.e.*, $V = \frac{V_0}{K}$.

(iv) The capacitance increases, *i.e.*, $C = KC_0$.

(v) The energy decreases, *i.e.*, $U = \frac{U_0}{K}$.

Case-II Effect of dielectric when battery remains connected across the capacitor

(i) The charge now becomes Kq_0, *i.e.*, $q = Kq_0$.

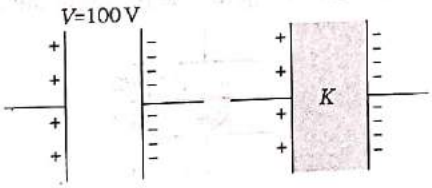
(ii) The electric field remains constant, *i.e.*, $E = E_0$.

(iii) The potential difference remains constant at V_0 *i.e.*, $V = V_0$.

(iv) The capacitance increases from C_0 to C, *i.e.*, $C = KC_0$

(v) The energy increases *i.e.*, $U = KU_0$.

Example 2.60 *An isolated 16 μF parallel plate air capacitor has a potential difference of 100 V. A dielectric slab having relative permittivity (i.e., dielectric constant) = 5 is introduced to fill the space between the two plates completely. Calculate*



(i) the new capacitance of the capacitor.

(ii) the new potential difference between the two plates of capacitor.

Sol. (i) The new capacity of the capacitor, $C = KC_0 = 5 \times 16 = 80\,\mu F$

(ii) Since, the capacitor is isolated, therefore the charge on the capacitor remains the same, thus its capacity is increased an expense of its potential drop.

Therefore, the new potential difference,

$$V = \frac{V_0}{\nu} = \frac{100}{5} = 20\,V$$

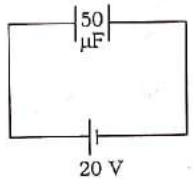
Example 2.61 *A capacitor ($C = 50\,\mu F$) is charged to a potential difference of 20 V. The charging battery is disconnected and the capacitor is connected to another cell of emf 10 V with the positive plate of capacitor joined with the positive terminal of cell.*

(i) Find charge flown through 10 V cell.

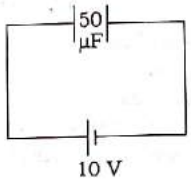
(ii) Is work done by the cell or is it done on the cell? Find its magnitude.

(iii) Find the heat developed in connecting wires.

Sol.



Charge on capacitor, $Q_1 = 50 \times 20 = 1000\,\mu C$



Charge on capacitor, $Q_2 = 50 \times 10 = 500\,\mu C$, $Q_2 < C_1$

(i) Charge flown through cell of emf 10 V,

$$\Delta Q = Q_1 - Q_2 = 500\,\mu C$$

(ii) Since, $Q_2 < Q_1$, work is done on the cell

$W_b = \Delta QV = 500 \times 100 \times 10^{-6} = 5 \times 10^{-3}$ J, W_b is Negative.

(iii) $\Delta U = U_f - U_i = \frac{1}{2}C(V_2^2 - V_1^2) = \frac{1}{2} \times 50 \times 10^{-6}\,(10^2 - 20^2)$

$= -7.5 \times 10^{-3}$ J

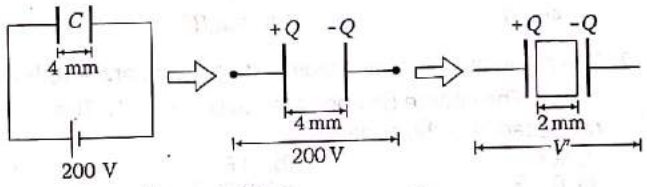
$W_b = \Delta U + H \Rightarrow -5 \times 10^{-3} = -7.5 \times 10^{-3} + H$

$H = 2.5 \times 10^{-3}$ J

where, H = heat developed in connecting wires.

Example 2.62 *A parallel plate capacitor ($C = 50\,\mu F$, $d = 4$ mm) is charged to 200 V and then charging battery is removed. Now, a dielectric slab ($K = 4$) of thickness 2 mm is placed between the plates. Find new potential difference across capacitor.*

Sol.



$$C = \frac{A\varepsilon_0}{d} = \frac{A\varepsilon_0}{4 \times 10^{-3}} = 50 \times 10^{-6}\,F$$

$$C' = \frac{A\varepsilon_0}{d - t + \frac{t}{K}} = \frac{A\varepsilon_0}{\left(4 - 2 + \frac{2}{4}\right) \times 10^{-3}}$$

$$= \frac{A\varepsilon_0}{(5/2) \times 10^{-3}} = \frac{50 \times 10^{-6} \times 4 \times 10^{-3}}{(5/2) \times 10^{-3}} = 80 \times 10^{-6}\,F$$

Since, charge remains same $Q = CV = C'V'$

$$500 \times 200 = 80V'$$

$\Rightarrow$ Potential difference across capacitor $V' = 125$ V

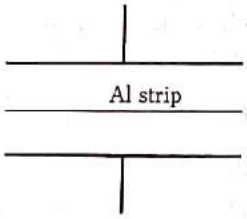
Check point 2.4

1. Identify the false statement.
(a) Electric field is zero inside the conductor and just outside, it is normal to the surface.
(b) Electric field is zero in the cavity of a hollow charged conductor.
(c) A polar dielectric is one in which is having a net dipole moment zero in the absence of electric field.
(d) H_2, N_2, O_2, CO_2 and CH_4 are examples of polar dielectric.

2. Eight drops of mercury of equal radii possessing equal charges combine to form a big drop. Then, the capacitance of bigger drop compared to each individual small drop is
(a) 8 times (b) 4 times (c) 2 times (d) 32 times

3. The capacity of parallel plate condenser depends on
(a) the type of metal used
(b) the thickness of plates
(c) the potential applied across on the plates
(d) the separation between the plates

4. As shown in the figure, a very thin sheet of aluminium is placed in between the plates of the condenser. Then, the capacity



(a) will increase (b) will decrease
(c) remains unchanged (d) may increase or decrease

5. When a lamp is connected in series with capacitor, then
(a) lamp will not glow (b) lamp will burst out
(c) lamp will glow normally (d) None of these

6. The capacity of a spherical conductor is
(a) $\frac{R}{4\pi\varepsilon_0}$ (b) $\frac{4\pi\varepsilon_0}{R}$
(c) $4\pi\varepsilon_0 R$ (d) $4\pi\varepsilon_0 R^2$

7. The potentials of the two plates of capacitor are + 10 V and – 10 V. The charge on one of the plates is 40 C. The capacitance of the capacitor is
(a) 2 F (b) 4 F
(c) 0.5 F (d) 0.25 F

8. The earth has volume V and surface area A then capacitance would be
(a) $4\pi\varepsilon_0 \frac{A}{V}$ (b) $2\pi\varepsilon_0 \frac{A}{V}$
(c) $12\pi\varepsilon_0 \frac{V}{A}$ (d) $12\pi\varepsilon_0 \frac{A}{V}$

9. If the circumference of a sphere is 2 m, then capacitance of sphere in water would be
(a) 2700 pF (b) 2760 pF
(c) 2780 pF (d) 2800 pF

10. The capacitance of a parallel plate capacitor is 12 μF. If the distance between the plates is doubled and area is halved, then new capacitance will be
(a) 8 μF (b) 6 μF
(c) 4 μF (d) 3 μF

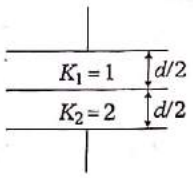
11. A parallel plate condenser has a capacitance 50 μF in air and 110 μF when immersed in an oil. The dielectric constant K of the oil is
(a) 0.45 (b) 0.55
(c) 1.10 (d) 2.20

12. A 500 μF capacitor is charged at a steady rate of 100 μC/s. The potential difference across the capacitor will be 10 V after an interval of
(a) 5 s (b) 25 s
(c) 20 s (d) 50 s

13. There is an air filled 1 pF parallel plate capacitor. When the plate separation is doubled and the space is filled with wax, the capacitance increases to 2 pF. The dielectric constant of wax is
(a) 2 (b) 4
(c) 6 (d) 8

14. A parallel plate capacitor with air between the plates has a capacitance of 9 pF. The separation between its plates is d. The space between the plates is now filled with two dielectrics. One of the dielectrics has dielectric constant $K_1 = 3$ and thickness $d/3$ while the other one has dielectric constant $K_2 = 6$ and thickness $2d/3$. Capacitance of the capacitor is now
(a) 45 pF (b) 40.5 pF
(c) 20.25 pF (d) 1.8 pF

15. Two parallel plate of area A are separated by two different dielectrics as shown in figure. The net capacitance is



(a) $\frac{4\varepsilon_0 A}{3d}$ (b) $\frac{3\varepsilon_0 A}{R}$
(c) $\frac{2\varepsilon_0 A}{d}$ (d) $\frac{\varepsilon_0 A}{d}$

5 Combination of capacitors

1. Series grouping

In a series connection, charge on each capacitor remains same and equals to the main charge supplied by the battery but potential difference across each capacitors is different such that

$$V = V_1 + V_2 + V_3$$

where V_1, V_2, V_3 are the potential difference across capacitor C_1, C_2 and C_3 respectively.

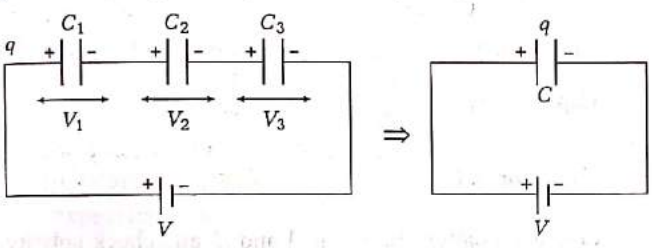


Fig. 2.49

The arrangement shown above is series connection. Here, the equivalent capacitance is

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

Following points are important in case of series combination of capacitors

(i) In series combination potential difference and distributes in the inverse ratio of capacitance, *i.e.*, $V \propto \frac{1}{C}$. and $U \propto \frac{1}{C}$

(ii) If two capacitors having capacitances C_1 and C_2 are connected in series, then

$$C = \frac{C_1 C_2}{C_1 + C_2}$$

$$V_1 = \left(\frac{C_2}{C_1 + C_2}\right)V$$

and $$V_2 = \left(\frac{C_1}{C_1 + C_2}\right)V$$

(iii) If n capacitors of equal capacity C are connected in series, with supply voltage V, then their equivalent capacitance is $\frac{C}{n}$ and potential difference across each capacitor is $V' = \frac{V}{n}$.

(iv) If n identical plates are arranged as shown in figure, then they constitute $(n-1)$ capacitors in series. Each capacitor has capacitance $\frac{\varepsilon_0 A}{d}$

and $$C_{eq} = \frac{\varepsilon_0 A}{(n-1)d}$$

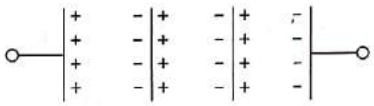


Fig. 2.50

In this situation, except two extreme plates each plate is common to adjacent capacitors.

(v) Below are some of the examples of combination of capacitors arranged in series.

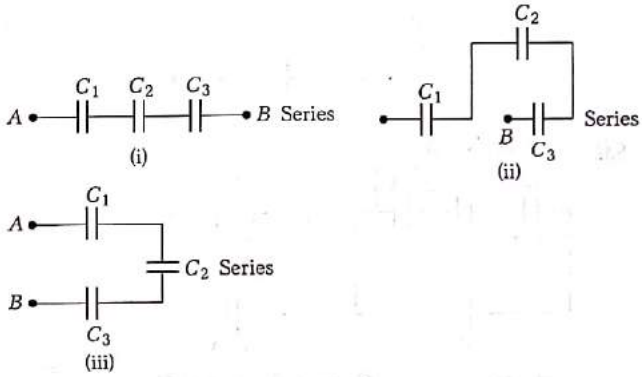
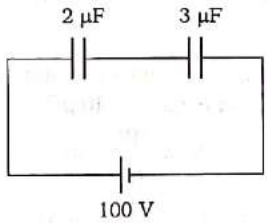
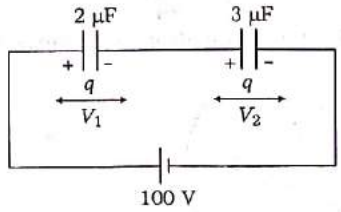


Fig. 2.51

Example 2.63 *In the circuit shown in figure, find*
(i) the equivalent capacitance.
(ii) the charge stored in each capacitor and
(iii) the potential difference across each capacitor.



Sol. (i) The equivalent capacitance



$$C = \frac{C_1 C_2}{C_1 + C_2} \quad \text{or} \quad C = \frac{(2)(3)}{2+3} = 1.2\ \mu F$$

(ii) The charge q, stored in each capacitor is

$$q = CV = (1.2 \times 10^{-6})(100)\ \text{C} = 120\ \mu\text{C}$$

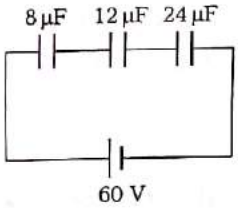
(iii) In series combination, $V \propto \frac{1}{C}$

or $$\frac{V_1}{V_2} = \frac{C_2}{C_1}$$

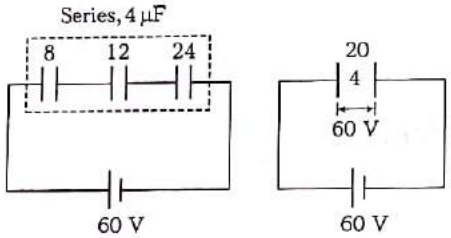
$$\therefore \quad V_1 = \left(\frac{C_2}{C_1 + C_2}\right)V = \left(\frac{3}{2+3}\right)(100) = 60\ \text{V}$$

and $$V_2 = V - V_1 = 100 - 60 = 40\ \text{V}$$

Example 2.64 *Find the potential difference and charge on each capacitor.*

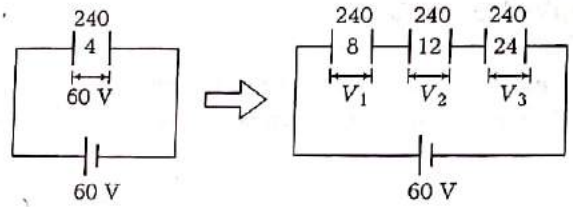


Sol.



Charge on capacitor, $Q = 4 \times 60 = 240\ \mu\text{C}$

Now, the individual potential can be shown as



In series, charge on individual capacitors and equivalent capacitor is same and equal to 240 μC.

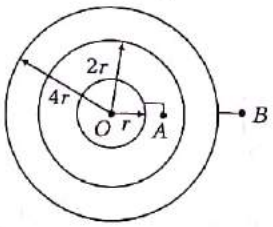
$$V_1 = \frac{240}{8} = 30\ \text{V}$$

$$V_2 = \frac{240}{12} = 20\ \text{V}$$

$$V_3 = \frac{240}{24} = 10\ \text{V}$$

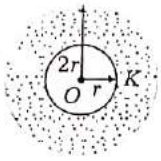
Example 2.65 *Find the equivalent capacitance between A and B.*

(i)



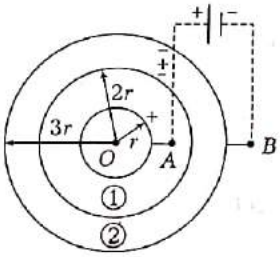
Three conducting concentric shells of radii r, 2r and 4r.

(ii)



An isolated ball-shaped conductor of radius r surrounded by an adjacent concentric layer of dielectric (K) and outer radius 2r.

Sol. (i)



The capacitance of the two capacitors can be calculated as

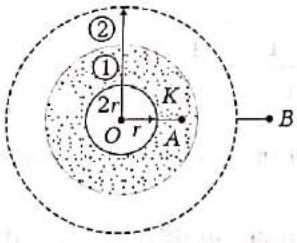
Capacitor 1, $C_1 = \frac{4\pi\varepsilon_0 r \cdot 2r}{(2r - r)} = 8\pi\varepsilon_0 r$

Capacitor 2, $C_2 = \frac{4\pi\varepsilon_0 2r \cdot 3r}{(3r - 2r)} = 24\pi\varepsilon_0 r$

Connect a battery between A and B, and check polarity, +, −, +, −.

C_1 and C_2 are in series $C = \frac{C_1 C_2}{C_1 + C_2} = 6\pi\varepsilon_0 r$

(ii)



Capacitance of 2 capacitors can be calculated as

Capacitor 1

$$C_1 = \frac{4\pi\varepsilon_0 K \cdot r \cdot 2r}{2r - r} = 8\pi\varepsilon_0 Kr$$

Capacitor 2 Isolated sphere of radius $2r$ and outer radius infinite.

$$C_2 = 4\pi\varepsilon_0 2r = 8\pi\varepsilon_0 r$$

C_1 and C_2 in series, as shown in (i)

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{8\pi\varepsilon_0 Kr}{K+1}$$

2. Parallel grouping

In a parallel connection, potential difference across each capacitor remains same and equal to the applied potential difference but charge gets distributed, *i.e.*,

$$q = q_1 + q_2 + q_3$$

where, q_1, q_2 and q_3 the charges across capacitor C_1, C_2 and C_3 respectively q is the net charge flowing across the circuit.

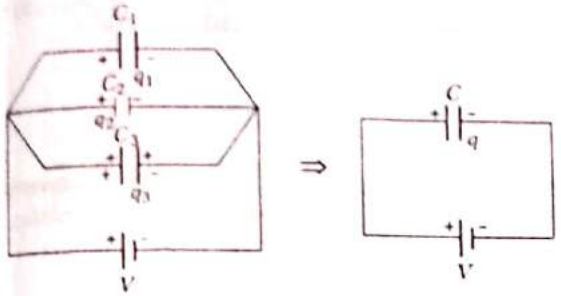


Fig. 2.52

The arrangement shown in figure is called a parallel connection. Here, the equivalent capacitance is equal to

$$C_{eq} = C_1 + C_2 + C_3$$

Following points are important in case of parallel combination of capacitors

(i) In parallel combination, charge energy distributes in the ratio of their capacitance, *i.e.*, $q \propto C$ and $U \propto C$. If two capacitors having capacitance C_1 and C_2 respectively are connected in parallel, then

$$C_{eq} = C_1 + C_2$$

$$q_1 = \left(\frac{C_1}{C_1 + C_2}\right)q \text{ and } q_2 = \left(\frac{C_2}{C_1 + C_2}\right)q$$

(ii) If n identical capacitors are connected in parallel, then equivalent capacitance is $C_{eq} = nC$ and charge on each capacitor is $q' = \frac{q}{n}$.

(iii) If n identical plates are arranged such that even numbered of plates are connected together and odd number of plates are connected together as shown in figure, then $(n-1)$ capacitors are formed and will be in parallel.

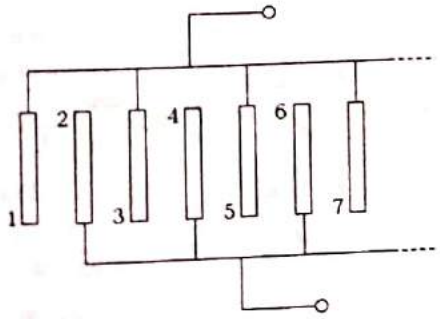


Fig. 2.53

The capacitance of each capacitor is $\frac{\varepsilon_0 A}{d}$ and the equivalent capacitance, $C_{eq} = \frac{(n-1)\varepsilon_0 A}{d}$.

(iv) If there are n identical capacitors, then $\frac{C_P}{C_S} = n^2$

where, C_P is the equivalent capacitance when n capacitors are connected in parallel. Below are some of the examples of combination of capacitors arranged in parallel.

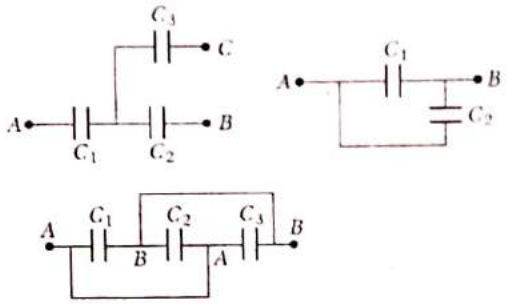
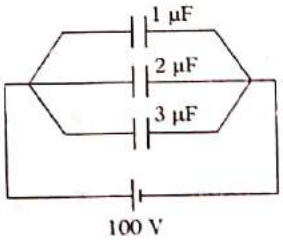


Fig. 2.54

Example 2.66 *In the circuit shown in figure, find*
(i) the equivalent capacitance and
(ii) the charge stored in each capacitor.



Sol. (i) The capacitors are in parallel. Hence, the equivalent capacitance is,

$$C = C_1 + C_2 + C_3 \text{ or } C = (1 + 2 + 3) = 6\ \mu F$$

(ii) Total charge drawn from the battery,

$$q = CV = 6 \times 100\ \mu C = 600\ \mu C$$

This charge will be distributed in the ratio of their capacities. Hence,

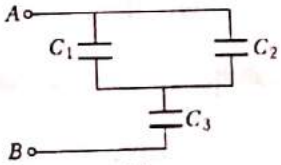
$$q_1 : q_2 : q_3 = C_1 : C_2 : C_3 = 1 : 2 : 3$$

$$\therefore \quad q_1 = \left(\frac{1}{1+2+3}\right) \times 600 = 100\ \mu C$$

$$q_2 = \left(\frac{2}{1+2+3}\right) \times 600 = 200\ \mu C$$

$$\text{and} \quad q_3 = \left(\frac{3}{1+2+3}\right) \times 600 = 300\ \mu C$$

Example 2.67



(a)

(i) Find the equivalent capacitance of the combination shown in the figure (a), when $C_1 = 2.0\ \mu F$, $C_2 = 4.0\ \mu F$ and $C_3 = 3.0\ \mu F$.

(ii) The input terminals A and B in Fig. (a) are connected to a battery of 12 V. Find the potential and the charge of each capacitor.

Sol. (i) C_1 and C_2 are in parallel, hence their equivalent capacitance is

$$C' = C_1 + C_2 = 2.0\,\mu F + 4.0\,\mu F = 6.0\,\mu F$$

Fig. (b) shows the combination of C' and C_3 in series. The final equivalent capacitance, shown in Fig. (c) is given by

$$\frac{1}{C_{eq}} = \frac{1}{C'} + \frac{1}{C_3} = \frac{1}{6.0\,\mu F} + \frac{1}{3.0\,\mu F} = \frac{1}{2.0\,\mu F} \text{ or } C_{eq} = 2.0\,\mu F$$

(ii) To find the charge or potential difference, we retrace the path to the original in Fig. (a).

The charge supplied by the battery is

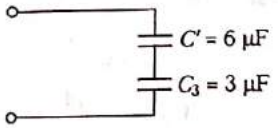
$q = C_{eq}V$, when the inputs are joined to a V-volt battery.

$= (2.0\,\mu F)(12.0\,V) = 24.0\,\mu C$

The charge on each capacitor in series in Fig. (b) is

$$q_3 = q' = q = 24.0\,\mu C$$

So, $$V_3 = \frac{q_3}{C_3} = \frac{24.0\,\mu C}{3.0\,\mu F} = 8\,V$$



(b) C_1 and C_2 are replaced by C'

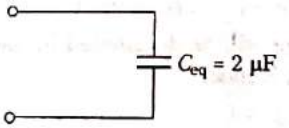
The potential difference across C' in Fig. (b) is

$$V' = \frac{q'}{C'} = \frac{24.0\,\mu C}{6.0\,\mu C} = 4\,V$$

This same potential difference V' appears across C_1 and C_2 in Fig. (a) and hence the charges on them are

$$q_1 = C_1V_1 = (2.0\,\mu F)(4.0\,V) = 8.0\,\mu C$$

and $$q_2 = C_2V_2 = (4.0\,\mu F)(4.0\,V) = 16.0\,\mu C$$

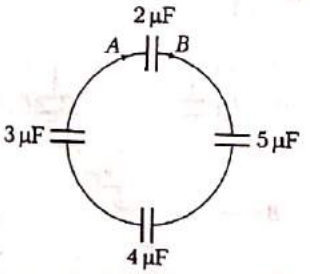


(c) C' and C_3 are replaced by C_{eq}

Thus, we have $V_1 = V_2 = 4.0\,V$, $V_3 = 8\,V$

and $q_1 = 8.0\,\mu C$, $q_2 = 16.0\,\mu C$, $q_3 = 24.0\,\mu C$

Example 2.68 *For the following arrangement, find the equivalent capacitance between A and B.*



Sol. Here, the 3 μF, 4 μF and 5 μF capacitors are in series.

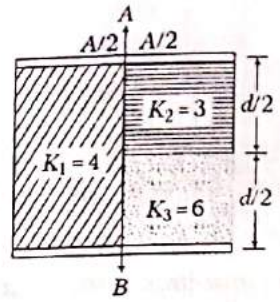
$$\therefore \quad C_{eq} = \left(\frac{1}{3} + \frac{1}{4} + \frac{1}{5}\right)^{-1} = \frac{60}{12 + 20 + 15} = \frac{60}{47}\,\mu F$$

Now, 2 μF and $\frac{60}{47}$ μF are in parallel, so we have

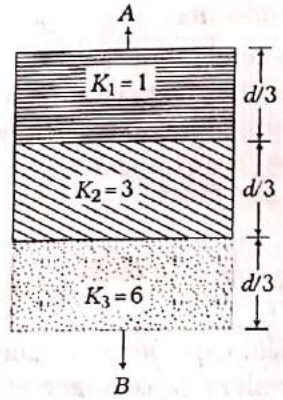
$$C_{eq} = C_1 + C_2 = \frac{154}{47}\,\mu F$$

Example 2.69 *Find the equivalent capacitance between A and B. A : area of each plate; d ; separation between plates*

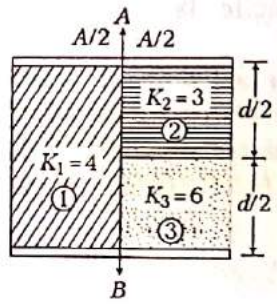
(i)



(ii)



Sol. (i)



Capacitance, $$C_1 = \frac{K_1 \frac{A}{2}\varepsilon_0}{d} = \frac{4\frac{A}{2}\varepsilon_0}{d} = \frac{2A\varepsilon_0}{d} \quad \left(\because C = \frac{KA\varepsilon_0}{d}\right)$$

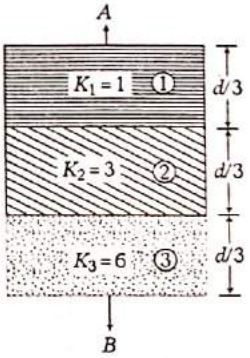
Capacitance, $$C_2 = \frac{K_2 \frac{A}{2}\varepsilon_0}{\frac{d}{2}} = \frac{3A\varepsilon_0}{d}$$

Capacitance, $$C_3 = \frac{K_3 \frac{A}{2}\varepsilon_0}{\frac{d}{2}} = \frac{6A\varepsilon_0}{d}$$

C_2 and C_3 are in series, $$C' = \frac{C_2C_3}{C_2 + C_3} = \frac{2A\varepsilon_0}{d}$$

C' and C_1 are in parallel $= \dfrac{4A\varepsilon_0}{d}$

(ii) Capacitance, $C_1 = \dfrac{K_1 A\varepsilon_0}{d/3} = \dfrac{3A\varepsilon_0}{d}$



Capacitance, $C_2 = \dfrac{K_2 A\varepsilon_0}{d/3} = \dfrac{9A\varepsilon_0}{d}$

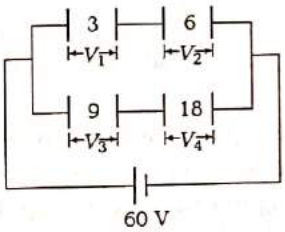
Capacitance, $C_3 = \dfrac{K_3 A\varepsilon_0}{d/3} = \dfrac{18A\varepsilon_0}{d}$

C_1, C_2 and C_3 are in series,

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Equivalent capacitance, $C_{eq} = \dfrac{2A\varepsilon_0}{d}$

Example 2.70 *Find the potential difference and charge on each capacitances. All capacitance in μF.*



Sol. Here, two branches are in parallel. Potential difference across each branch is 60 V. There are two capacitors in series, use direct formulae as explained earlier.

$$V_1 = \left(\frac{6}{3+6}\right) \times 60 = 40\text{ V}$$

$$V_2 = 60 - V_1 = 20\text{ V}$$

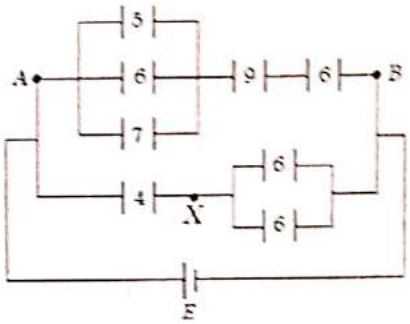
$$V_3 = \left(\frac{18}{9+18}\right) \times 60 = 40\text{ V}$$

$$V_4 = 60 - V_3 = 20\text{ V}$$

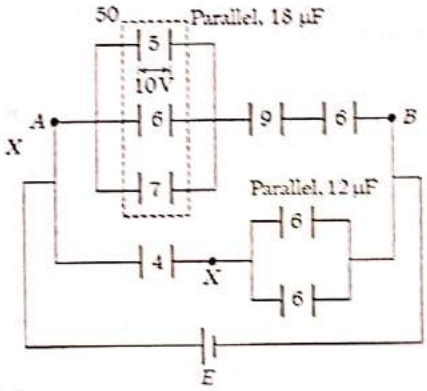
Charge on

$$3\,\mu\text{F}, q_1 = 3 \times 40 = 120\,\mu\text{C}$$
$$6\,\mu\text{F}, q_2 = 6 \times 20 = 120\,\mu\text{C}$$
$$9\,\mu\text{F}, q_3 = 9 \times 40 = 360\,\mu\text{C}$$
$$18\,\mu\text{F}, q_4 = 18 \times 20 = 360\,\mu\text{F}$$

Example 2.71 *If charge on 5 μF capacitor is 50 μC, then find the potential difference on 4 μF and emf of battery.*

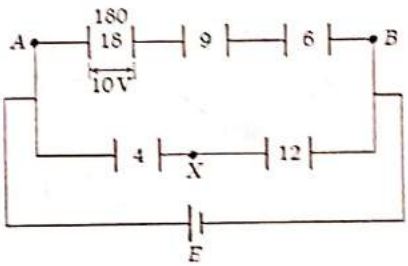


Sol. Since, charge on 5 μF is 50 μC, hence potential difference on it is 10 V. The equivalent circuit can be drawn as

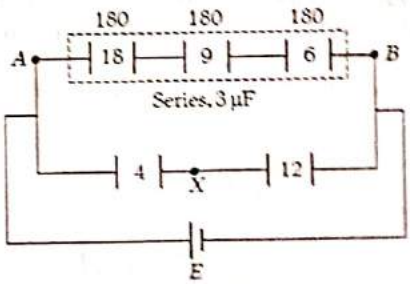


***Step* I** Charge on 5 μF capacitor is 50 μC, hence potential difference across it is 10 V.

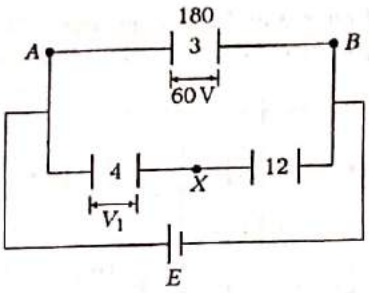
***Step* II**



***Step* III**



Step IV

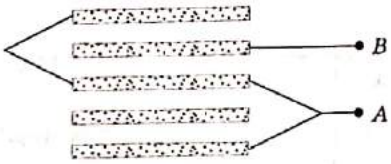


emf of battery = 60 V,

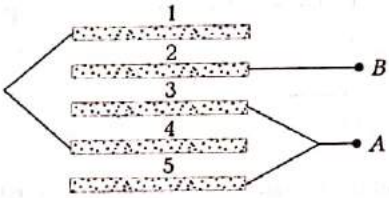
$$V_1 = \left(\frac{12}{4+12}\right) \times 60 = 45\text{ V}$$

Potential difference on 4 μF : 45 V

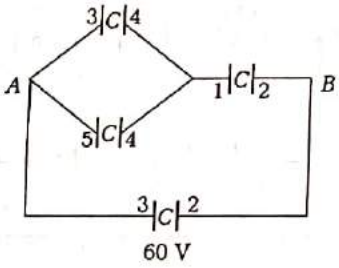
Example 2.72 *In the following figures, area of each plate is A and d is separation between adjacent plates. Find the capacitance of system between points A and B.*



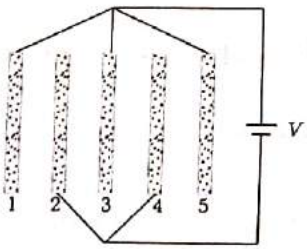
Sol.



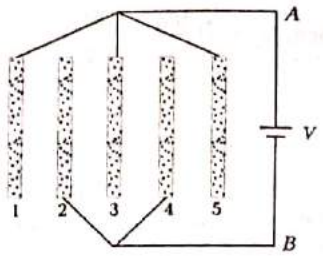
Th equivalent circuit can be drawn as



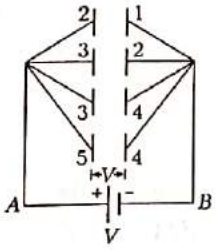
Example 2.73 *Five identical capacitor plates each of area A are arranged such that adjacent plates are at a distance d apart. The plates are connected to a source of emf V as shown. What is the magnitude and nature of charge on plates 1 and 3, respectively?*



Sol.



First, rearrange the plates.



Charge on plate 1, $q = -CV = \frac{-A\varepsilon_0 V}{d}$

Charge on plate 3, $q' = CV + CV = 2CV = \frac{2A\varepsilon_0 V}{d}$

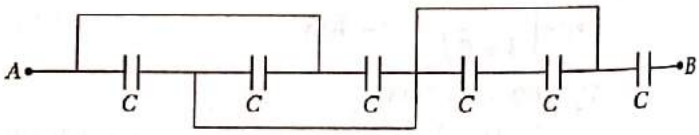
Special method to solve combination of capacitors

1. Method of same potential

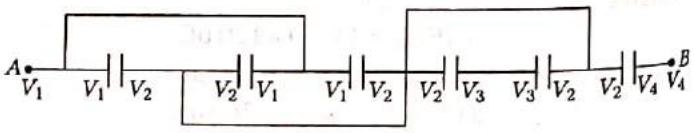
Give any arbitrary potentials (V_1, V_2, ..., etc.) to all terminals of capacitors. But notice that the points connected directly by a conducting wire will have at the same potential.

The capacitors having the same potential difference (PD) are in parallel. Make a table corresponding to the figure. Now corresponding to this table a simplified figure can be formed and from this figure C_{eq} can be calculated.

Example 2.74 *Find equivalent capacitance between points A and B shown in figure.*



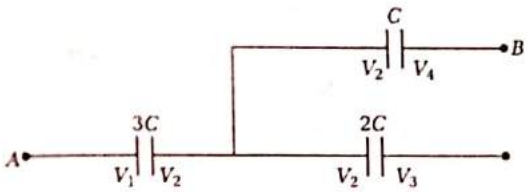
Sol. Three capacitors have potential difference, $V_1 - V_2$. So, they are in parallel. Their equivalent capacitance is 3 C.



Two capacitors have potential difference, $V_2 - V_3$. So, their equivalent capacitance is $2C$ and lastly there is one capacitor across which potential difference is $V_2 - V_4$. So, let us make a table corresponding to this information.

Potential difference	Capacitance
$V_1 - V_2$	$3C$
$V_2 - V_3$	$2C$
$V_2 - V_4$	C

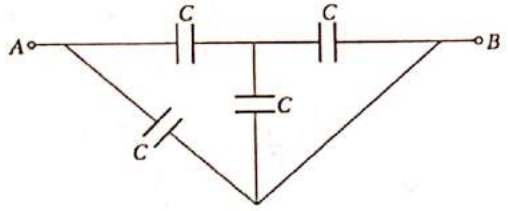
Now corresponding to this table, we make a simple figure as shown in figure.



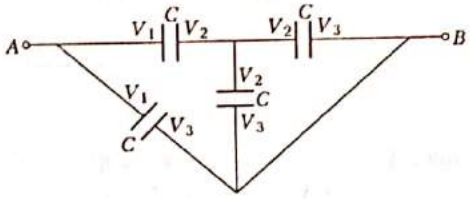
As we have to find the equivalent capacitance between points A and B, across which potential difference is $V_1 - V_4$. From the simplified figure, we can see that the capacitor of capacitance $2C$ is out of the circuit points A and B as shown. Now, $3C$ and C are in series and their equivalent capacitance is

$$C_{eq} = \frac{(3C)(C)}{3C + C} = \frac{3}{4}C$$

Example 2.75 *Find equivalent capacitance between points A and B.*



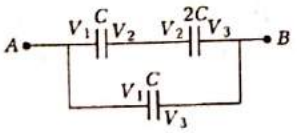
Sol. The circuit can be drawn as



Two capacitors have potential difference $V_2 - V_3$ and equivalent capacitance $2C$. Let us make a corresponding table based on the potential each capacitor is having.

Potential difference	Capacitance
$V_1 - V_2$	C
$V_2 - V_3$	$2C$
$V_1 - V_3$	C

Now corresponding to this table, we make a simple figure as shown below.



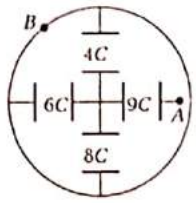
From the figure we can see that C and $2C$ are connected in series.

$$\therefore \quad C' = \frac{(C) + (2C)}{C + 2C} = \frac{2}{3}C$$

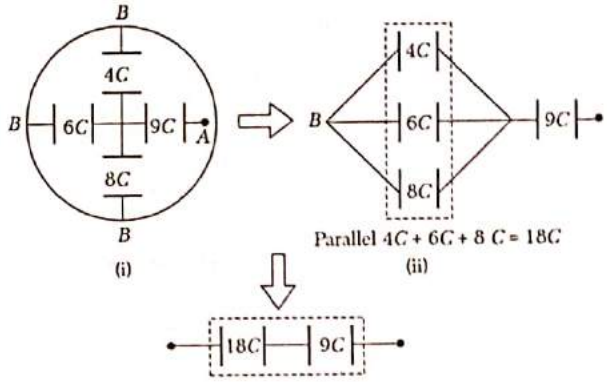
This combination is connected in parallel with C. So, net capacitance of the given circuit is

$$C_{net} = C + \frac{2}{3}C = \frac{5}{3}C$$

Example 2.76 *Find the equivalent capacitance between A and B.*

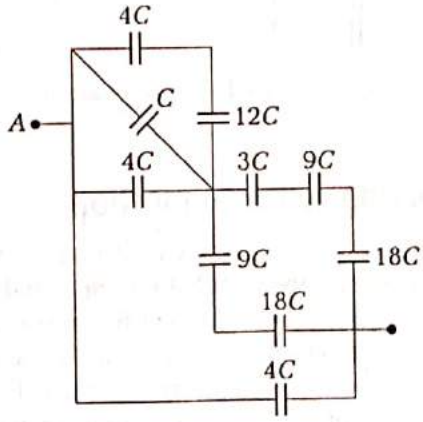


Sol. As we have solved in above examples, we can solve here also using the same technique.



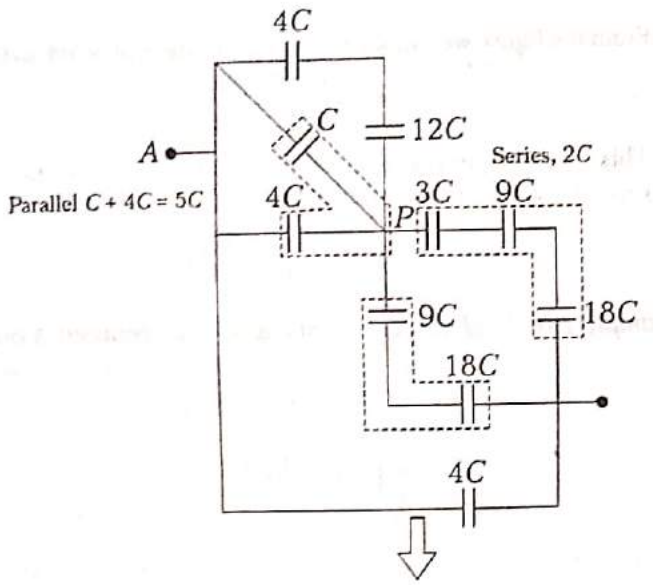
$$\text{Series,} \quad \text{capacitance} = \frac{9C \times 18C}{9C + 18C} = 6C, \; C_{eq} = 6C$$

Example 2.77 *Find the equivalent capacitance between A and B.*

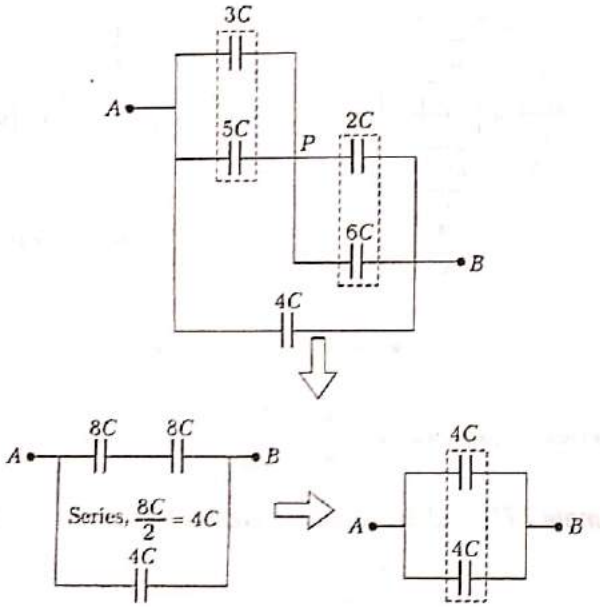


Sol. In series, capacitance $= \dfrac{4C \times 12C}{4C + 12C} = 3C$

In series, $\dfrac{1}{C'} = \dfrac{1}{3C} + \dfrac{1}{9C} + \dfrac{1}{18C} = \dfrac{9}{18C}$, $C' = 2C$



Series, capacitance $= \dfrac{9C \times 18C}{9C + 18C} = 6C$



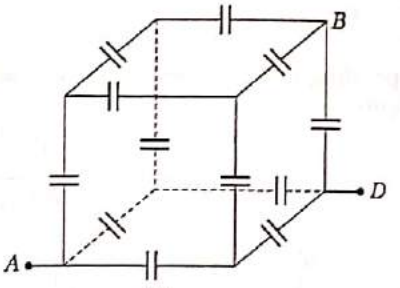
The equivalent capacitance between A and B,

$$C_{eq} = 4C + 4C = 8C$$

2. Connection removal method

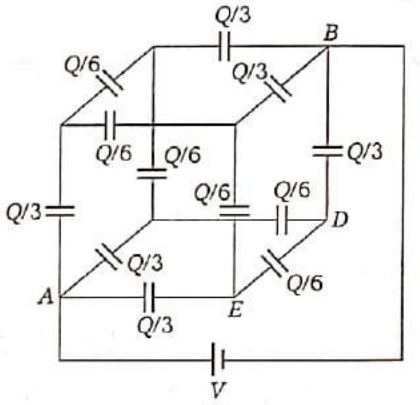
This method is useful when the circuit diagram is symmetric except for the fact that the input and output are reversed. That is the flow of current is a mirror image between input and output above a particular axis. In such cases, some junctions are unnecessarily made. Even if we remove that junction there is no difference in the remaining circuit or current distribution. But after removing the junction, the problem becomes very simple. The following example illustrates the theory.

Example 2.78 *Figure shows a combination of twelve capacitors, each of capacitance C, forming a cube. Find the equivalent capacitance of the combination (i) between the diagonally opposite corners A and B of the cube (ii) between the diagonally opposite corners A and D of a face of the cube.*



(a)

Sol. (i) Suppose the charge supplied by the battery is Q. This will be equally divided on the three capacitors connected to A because on looking from A to B three sides of the cube have identical properties. Hence, each capacitor connected to A has charge $\frac{Q}{3}$. Similarly, each capacitor connected to B also has charge $\frac{Q}{3}$. In the Fig. (b), the charges shown are the charged on the capacitors (*i.e.*, charges on their positive plates)



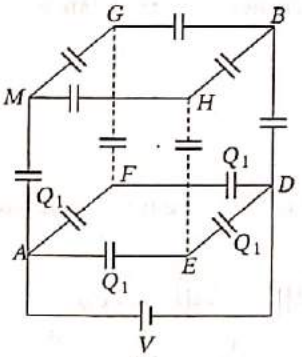
(b)

Now, $V = (V_A - V_E) + (V_E - V_D) + (V_D - V_B)$

$$V = \frac{Q/3}{C} + \frac{Q/6}{C} + \frac{Q/3}{C} = \frac{5Q}{6C}$$

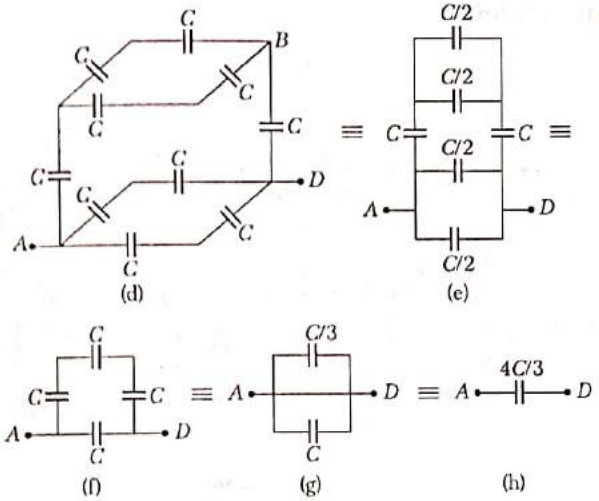
$\therefore$ Equivalent capacitance, $C_{eq} = \dfrac{Q}{V} = \dfrac{6}{5}C$

(ii)

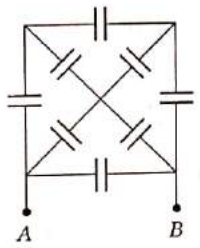


(c)

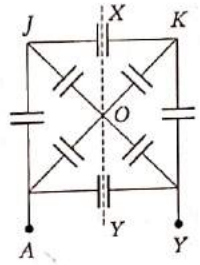
On looking from A to D into the circuit, and from D to A into the circuit, we find symmetry. Hence, the charge on each of the four capacitors of the face $AEDF$ is same (say Q). It means there is no charge on the capacitors between F and G, and between E and H. Hence to find the equivalent capacitance, the combination may be taken without these two capacitors, which has been shown in the figure (d).

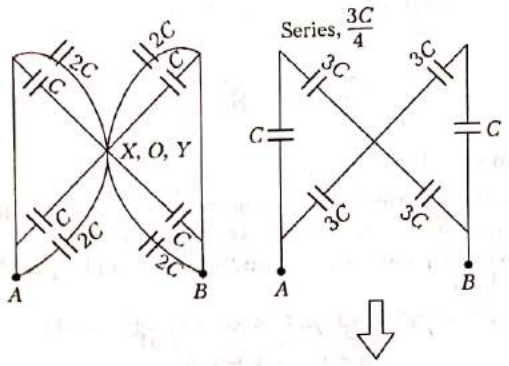


Example 2.79 *Find the equivalent capacitance between A and B. All the capacitors have capacitance C.*

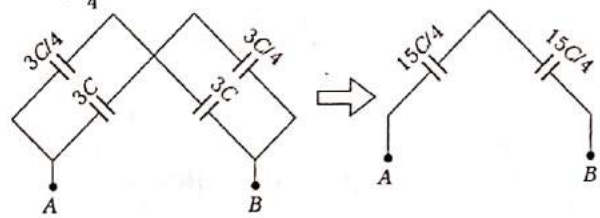


Sol.





Parallel, $\frac{15C}{4}$



Equivalent capacitance between A and B

$$\frac{1}{C_{eq}} = \frac{1}{15\,C/4} + \frac{1}{15/C} \Rightarrow C_{eq} = \frac{15C}{8}$$

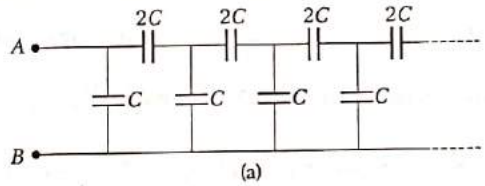
3. Infinite series problems

This consists of an infinite series of identical loops. To find C_{eq} of such a series first we consider by ourself a value (say x) of C_{eq}.

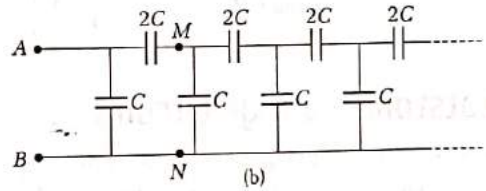
Then we break the chain in such a manner that only one loop is left with us and in place of the remaining portion we connect a capacitor x.

Then we find the C_{eq} and put it equal to x. With this we get a quadratic equation in x. By solving this equation we can find the desired value of x.

Example 2.80 *Find the equivalent capacitance of the infinite ladder shown in the figure (a) between the points A and B.*



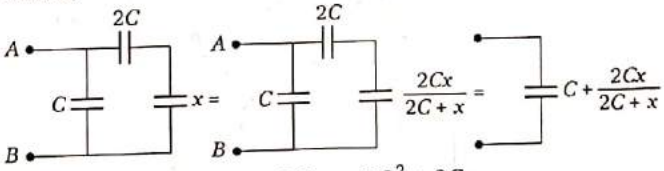
Sol. If we look at the infinite ladder between M and N, the arrangement obtained is exactly same as we have between A and B.



Now, suppose that the equivalent capacitance between A and B is x.

Then the equivalent capacitance between M and N is also x.

Hence,

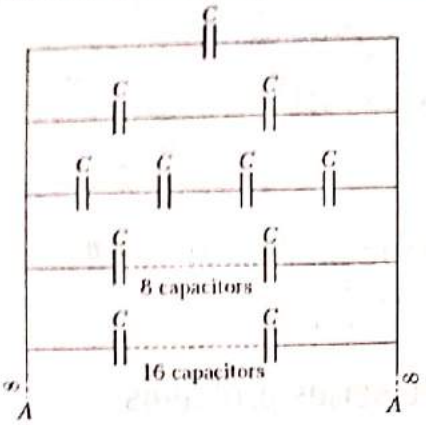


Thus, $$x = C + \frac{2Cx}{2C + x} = \frac{2C^2 + 3Cx}{2C + x}$$

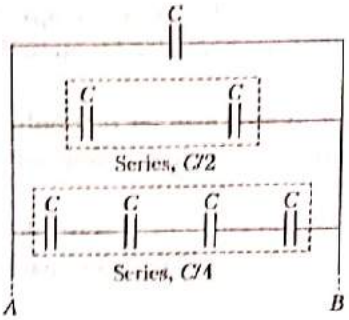
or, $$x^2 - Cx - 2C^2 = 0 \quad \text{or} \quad x = 2C$$

i.e., The equivalent capacitance is $2C$.

Example 2.81 *Find the equivalent capacitance between A and B.*



Sol.



Equivalent capacitance,

In second branch the capacitance are connected in series having net capacitance $\frac{C}{2}$, third branch the capacitance are connected in series having net capacitance $\frac{C}{4}$, ...

$\therefore$ $C, \frac{C}{2}, \frac{C}{4}, \ldots, \infty$ are in parallel, $C_{eq} = C + \frac{C}{2} + \frac{C}{4} + \ldots \infty$

$$= \frac{C\left(1 + \frac{1}{2} + \frac{1}{2^2} + \ldots \infty\right)}{\text{using GP}} = C\left(\frac{1}{1 - \frac{1}{2}}\right) = 2C$$

4. Wheatstone's bridge circuits

Wheatstone's bridge consists of five capacitors, or a number of capacitors which can be reduced to five as shown in the following arrangement.

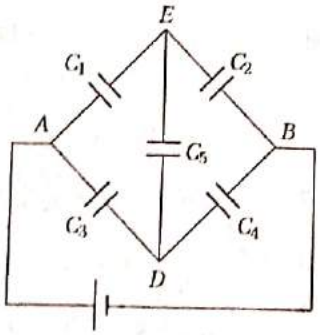


Fig. 2.55

If $\frac{C_1}{C_2} = \frac{C_3}{C_4}$, bridge is said to be balanced and in that case

$$V_E = V_D \quad \text{or} \quad V_E - V_D = 0 \quad \text{or} \quad V_{ED} = 0$$

i.e., no charge is stored in C_5. Hence, it can be removed from the circuit.

Some of the different forms of the Wheatstone bridge is given below.

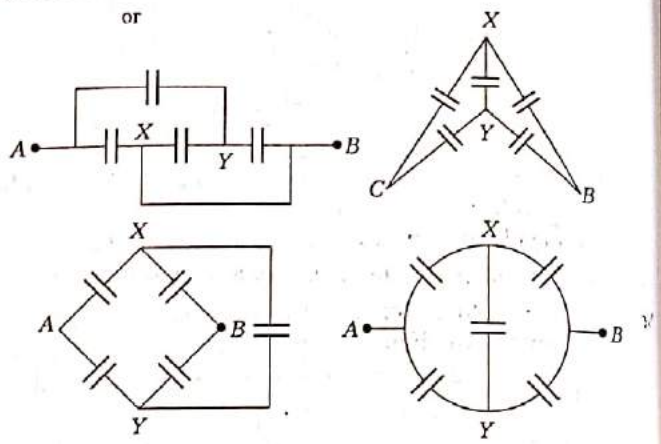
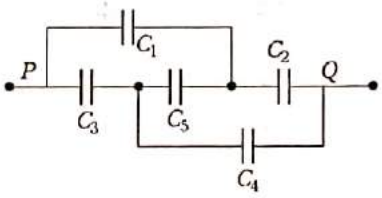


Fig. 2.56

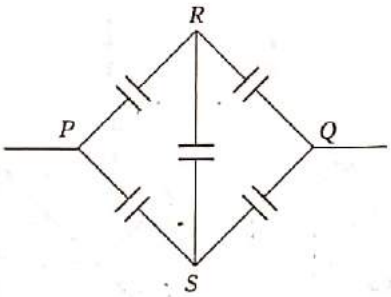
Example 2.82 *In the network of capacitors given ahead. Find the effective capacitance between the points P and Q.*

If $C_1 = C_2 = C_3 = C_4 = 4\,\mu F$

Value is not given C_1 *and* C_2



Sol. The given network consists of two closed capacitor circuits one containing C_1, C_3 and C_5, and the other containing C_2 C_4, C_5. Thus, C_5 is common in both. Hence, the network can be replaced shown in figure which is a Wheatstone bridge arrangement



C_1 and C_2 are in series between the points P and Q similarly C_3 and C_4 are also in series. Suppose the equivalent capacitance of C_1 and C_2 is C' and that of C_3 and C_4 is C''. Then, we have

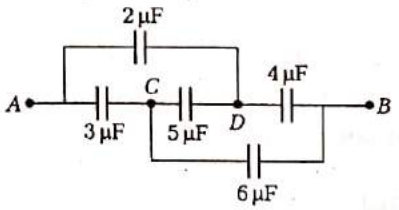
$$C' = \frac{C_1 C_2}{C_1 + C_2} = \frac{4 \times 4}{4 + 4} = 2\,\mu F$$

Similarly, $C'' = 2\,\mu F$

Now, capacitors C' and C'' are in parallel between P and Q. Hence, the equivalent capacitance is given by

$$C = C' + C'' = 2\,\mu F + 2\,\mu F = 4\,\mu F$$

Example 2.83 *(i) Find the equivalent capacitance of the combination between A and B in the figure.*



If the points A and B are maintained at 15V and 0V respectively, then

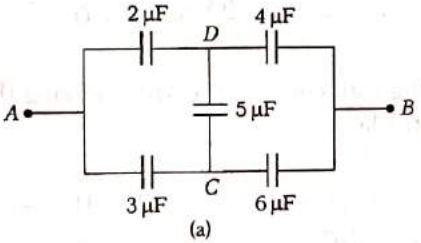
(ii) find the charges on 3 μF, 4 μF and 5 μF capacitors.

(iii) what are the potentials of the points C and D?

Sol. (i) The simplified form of the given combination has been shown in the Fig. (a). This forms Wheatstone' bridge.

Here, $$\frac{C_1}{C_2} = \frac{2\,\mu F}{4\,\mu F} = \frac{1}{2}$$

$$\frac{C_3}{C_4} = \frac{3\,\mu F}{6\,\mu F} = \frac{1}{2}$$



(a)

Thus, $$\frac{C_1}{C_2} = \frac{C_3}{C_4} \quad \text{(balancing condition)}$$

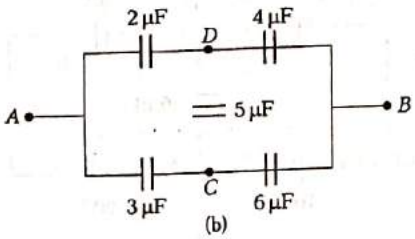
Hence, (a) $$C_{eq} = \frac{C_1C_2}{C_1 + C_2} + \frac{C_3C_4}{C_3 + C_4}$$

$$= \left(\frac{2\times 4}{2+4} + \frac{3\times 6}{3+6}\right)\mu F = \left(\frac{4}{3} + 2\right)\mu F = \frac{10}{3}\,\mu F$$

(ii) Potential difference, $V_A - V_B = 15\text{ V}$

$\therefore$ The charge on $\frac{4}{3}\,\mu F$ capacitor is

$$q_1 = \left(\frac{4}{3}\,\mu F\right)(15\text{ V}) = 20\,\mu C$$



(b)

Hence, the charge on 4 μF capacitor in Fig. (b) is also

$$q_1 = 20\,\mu C$$

The charge on 2 μF capacitor in Fig. (d) is

$$q_2 = (2\,\mu F)(15\text{ V}) = 30\,\mu C$$

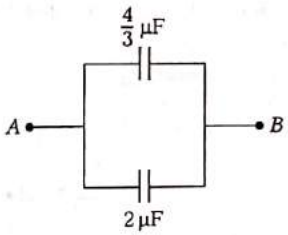
So, the charge on the 3 μF capacitor in Fig. (c) is also

$$q_2 = 30\,\mu C$$

So, the charge on 5 μF capacitor is zero.

Hence, our answers are 30 μC on 3 μF capacitor, 20 μC on 4 μF capacitor and zero on 5 μF capacitor.

(iii) The potential difference across 3 μF capacitor of Fig. (c) is



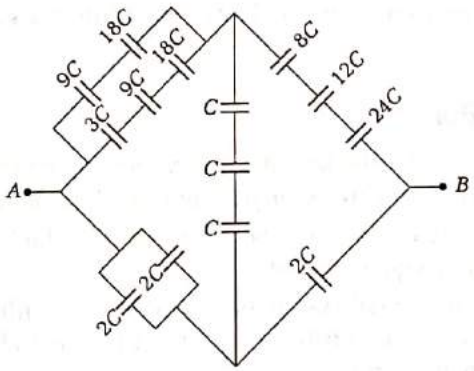
(c)

$$V_A - V_C = \frac{30\,\mu C}{3\,\mu F} = 10\text{ V}$$

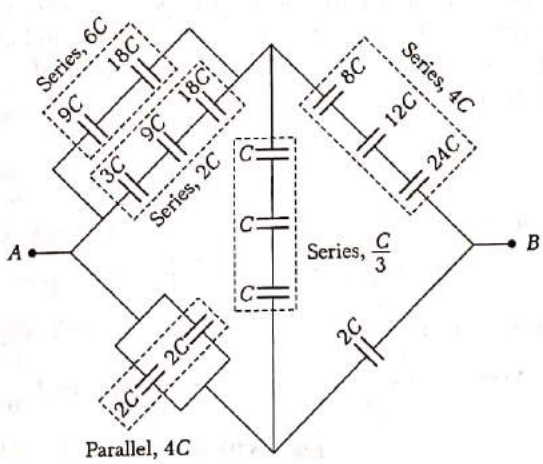
or $$15V - V_C = 10\text{ V}$$

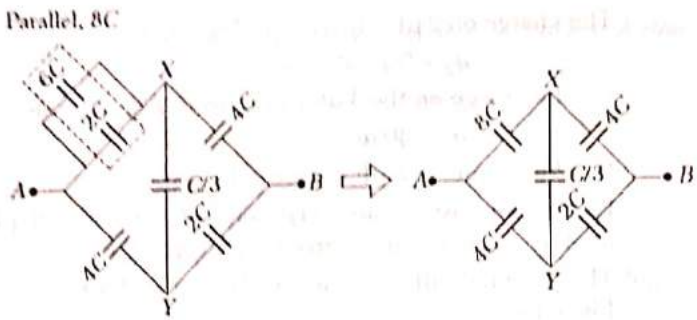
$\Rightarrow$ $$V_C = 5\text{ V}$$

Example 2.84 *Find the equivalent capacitance between A and B.*

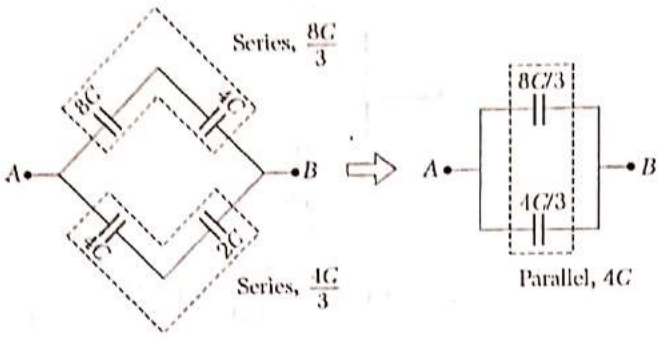


Sol.





Balanced Wheatstone bridge, remove capacitor between X and Y.



The equivalent capacitance between A and $B = 4C$

Kirchhoff's Law for capacitor circuits

Kirchhoff's law can be applied to a circuit containing resistances, capacitors and batteries. *Its two laws are as follows*

First law

This is basically the law of conservation of charge.

Following two points are important regarding the first law

(i) In case of a battery, both terminals of the battery supply equal amount of charge.

(ii) In an isolated system (not connected to either of the terminals of a battery or to the earth) net charge remains constant.

For example, in the figure shown, the positive terminal of the battery supplies a positive charge $q_1 + q_2$. Similarly, the negative terminal supplies a negative charge of magnitude $q_3 + q_4$.

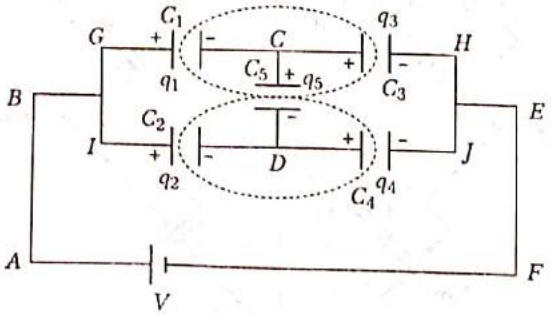


Fig. 2.57

Hence, $$q_1 + q_2 = q_3 + q_4$$

Further, the plates enclosed by the dotted lines form an isolated system, as they are neither connected to a battery terminal nor to the earth. Initially, no charge was present in these plates. Hence, after charging net charge on these plates should be zero. Or

$$q_3 + q_5 - q_1 = 0$$

and $$q_4 - q_2 - q_5 = 0$$

So, these are the three equations which can be obtained from the first law.

Second law

In a capacitor, potential drops when one moves from positive plate to the negative plate by q/C and in a battery it drops by an amount equal to the emf of the battery. Applying second law in loop $ABGHEFA$, we have

$$-\frac{q_1}{C_1} - \frac{q_3}{C_3} + V = 0$$

Similarly, the second law in loop $GCDIG$ gives the equation,

$$-\frac{q_1}{C_1} - \frac{q_5}{C_5} + \frac{q_2}{C_2} = 0$$

Use following sign convention while solving the problems on Kirchhoff's law

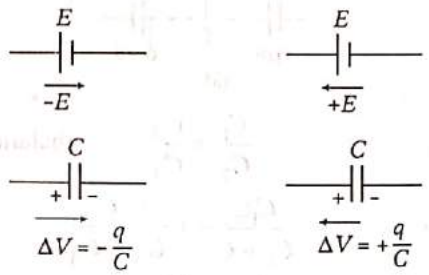
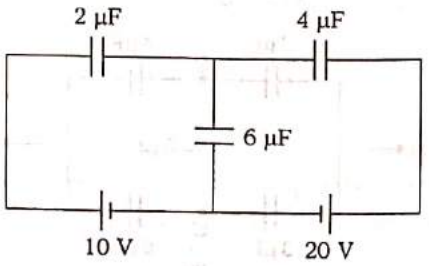


Fig. 2.58

Arrow shows the direction of current or flow of charges.

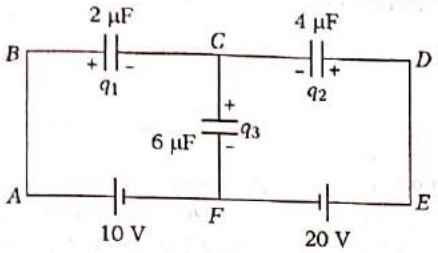
Note When an arrangement of capacitors cannot be simplified by any of the method discussed in the previous section, then we need to apply the Kirchhoff's law to solve the circuit.

Example 2.85 *Find the charges on the three capacitors shown in figure.*



(a)

Sol. Let the charges in three capacitors be as shown in figure.



(b)

Charge supplied by 10 V battery is q_1 and that from 20 V battery is q_2. Then,

$$q_1 + q_2 = q_3 \quad \text{...(i)}$$

This relation can also be obtained in a different manner. The charges on the three plates which are in contact add to zero. Because these plates taken together form an isolated system which can't receive charges from the batteries. Thus,

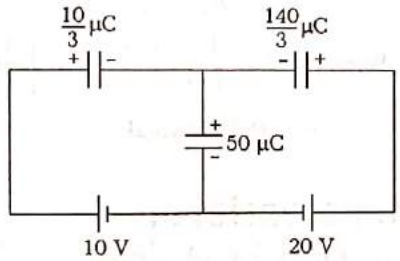
$$q_3 - q_1 - q_2 = 0 \text{ or } q_3 = q_1 + q_2$$

Applying second law in loops *BCFAB* and *CDEFC*, we have

$$-\frac{q_1}{2} - \frac{q_3}{6} + 10 = 0 \text{ or } q_3 + 3q_1 = 60 \quad \text{...(ii)}$$

and $$\frac{q_2}{4} - 20 + \frac{q_3}{6} = 0 \text{ or } 3q_2 + 2q_3 = 240 \quad \text{...(iii)}$$

Solving the above three equations, we have



$$q_1 = \frac{10}{3}\,\mu\text{C}, \quad q_2 = \frac{140}{3}\,\mu\text{C} \text{ and } q_3 = 50\,\mu\text{C}$$

Energy stored in charged capacitor

The total amount of work in charging the capacitor is stored up in the capacitor in the form of electric potential energy. Suppose at time t, a charge q is present on the capacitor and V is the potential of the capacitor. If dq is the amount of charge that is brought against the forces of the field due to the charge already present on the capacitor, the additional work needed will be

$$dW = (dq)\,V = \left(\frac{q}{C}\right)\cdot dq \qquad [\text{as } V = q/C]$$

∴ Total work to charge a capacitor to a charge q_0, can be obtained by integrating additional work from limits 0 to q_0, we get

$$W = \frac{q_0^2}{2C}$$

∴ Energy stored by a charged capacitor,

$$U = W = \frac{q_0^2}{2C} = \frac{1}{2}CV_0^2 = \frac{1}{2}q_0V_0$$

Thus, if a capacitor is given a charge q, then the potential energy stored in it is,

$$\boxed{U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}qV}$$

The above relation shows that the charged capacitor is the electrical analog of a stretched spring whose elastic potential energy is $\frac{1}{2}kx^2$. The charge q is analogous to the elongation x and $\frac{1}{C}$ *i.e.*, the reciprocal of capacitance to the force constant k.

Energy density between the plates

The energy stored per unit volume of space in a capacitor is called energy density.

Charge on either plate of capacitor is

$$Q = \sigma A = \varepsilon_0 EA$$

Energy stored in the capaitor is

$$U = \frac{Q}{2C} = \frac{(\varepsilon_0 EA)^2}{2\cdot\varepsilon_0 A/d} = \frac{1}{2}\varepsilon_0 E^2\cdot Ad$$

Energy density , $$u = \frac{\text{Energy stored}}{\text{Volume of capacitor}} = \frac{U}{Ad}$$

∴ $$\boxed{u = \frac{1}{2}\varepsilon_0 E^2}$$

Total energy stored in series combination or parallel combination of capacitors is equal to the sum of energies stored in individual capacitors, *i.e.*, $U = U_1 + U_2 + U_3 + \ldots$.

Note Change in energy on introducing a dielectric slab.

(i) When a dielectric slab is inserted between the plates of a charged capacitor, with battery connected to its plates. Then, the capacitance becomes K (dielectric constant) times and energy stored in the capacitor becomes KU_0.

(ii) When a dielectric slab is inserted between the plates of a charged capacitor and battery is disconnected. Then, the charge on the plates remains unchanged and energy stored in the capacitor becomes $\frac{U_0}{K}$, *i.e.*, energy decreases.

Example 2.86 *Calculate energy stored in a capacitor of* $5\,\mu F$ *when it is charged to a potential of* 250 *V?*

Sol. Given, $C = 5\,\mu\text{F} = 5\times10^{-6}\,\text{F}, V = 250\,\text{V}$

Energy stored in a capacitor,

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\times(5\times10^{-6})\times(250)^2 = 0.156\,\text{J}$$

Example 2.87 *The plates of a parallel plate capacitor have an area of 90 cm² each and are separated by 2.5 mm. The capacitor is charged to 400 V. How much electrostatic energy is stored in it ? How much when it is filled with a dielectric medium K = 3 and then charged ? If it is first charged as an air capacitor and then filled with the dielectric, then ?*

Sol. Capacitance of the parallel plate capacitor,

$$C_0 = \frac{\varepsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 90 \times 10^{-4}}{2.5 \times 10^{-3}} = 3.19 \times 10^{-11}\,\text{F}$$

The energy stored in the air-capacitor

$$U = \frac{1}{2} C_0 V_0^2 = \frac{1}{2} \times (3.19 \times 10^{-11}) \times (400)^2$$
$$= 2.25 \times 10^{-6}\,\text{J}$$

The capacitance of the dielectric filled capacitor is

$$C = KC_0$$

Energy, $U = \frac{1}{2} CV_0^2 = \frac{1}{2} KC_0 V_0^2$

$$= KU_0 = 3 \times (2.55 \times 10^{-6}) = 7.65 \times 10^{-6}\,\text{J}$$

If the capacitor is first charged end, then filled with the dielectric, then the charge remains constant but the potential difference between the plates decreases. The potential difference is $V = \frac{V_0}{K}$.

The new energy of capacitor,

$$U = \frac{1}{2} CV^2 = \frac{1}{2} (KC_0)(V_0/K)^2$$
$$= \frac{1}{K}\left(\frac{1}{2} C_0 V_0^2\right) = \frac{U_0}{K}$$
$$= \frac{2.55 \times 10^{-6}\,\text{J}}{3} = 8.55 \times 10^{-7}\,\text{J}$$

Example 2.88 *Two capacitors of capacitance $C_1 = 2\,\mu\text{F}$ and $C_2 = 8\,\mu\text{F}$ are connected in series and the resulting combination is connected across 300 V. Calculate the charge, potential difference and energy stored in the capacity separately.*

Sol. If C is the equivalent capacitance,

then $$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{2} + \frac{1}{8} = \frac{5}{8}$$

$$\therefore \quad C = \frac{8}{5} = 1.6\,\mu\text{F}$$

Charge, $q = CV = 1.6 \times 10^{-6} \times 300 = 4.8 \times 10^{-4}$ C

Potential across capacitance, $C_1 = V_1 = \frac{q}{C_1} = 240$ V

Potential across capacitance, $C_2 = V_2 = \frac{q}{C_2} = 60$ V

Energy stored in capacitance,

$$C_1 = \frac{1}{2} C_1 V_1^2 = \frac{1}{2} \times 2 \times 10^{-6} \times (240)^2$$
$$= 5.76 \times 10^{-2}\,\text{J}$$

Energy stored in capacitance, $C_2 = \frac{1}{2} C_2 V_2^2$

$$= \frac{1}{2} \times 8 \times 10^{-6} \times (60)^2$$
$$= 1.44 \times 10^{-2}\,\text{J}$$

Example 2.89 *The capacitance of a variable radio capacitor can be changed from 50 pF to 200 pF by turning the dial from 0° to 180°. With the dial set at 180°, the capacitor is connected to a 400 V battery. After charging, the capacitor is disconnected from the battery and dial is turned at 0°.*

(i) What is the potential difference across the capacitor when dial reads 0°?

(ii) How much work is required to turn the dial, if friction neglected?

Sol. When dial reads 180°, $C = 200$ pF

When dial reads 0°, $C' = 50$ pF

(i) Since, charge remains same,

$$Q = CV = C'V' \Rightarrow 200 \times 400 = 50 \times V$$

The potential difference across the capacitor,

$$V = 1600 \text{ volt}$$

(ii) Charge, $Q = CV = 200 \times 10^{-12} \times 400 = 8 \times 10^{-8}$ C

$$W_{1 \to 2} = U' - U = \frac{Q^2}{2C'} - \frac{Q^2}{2C}$$

$$W = \frac{Q^2}{2}\left(\frac{1}{C'} - \frac{1}{C}\right) = \frac{(8 \times 10^{-8})^2}{2}\left[\frac{1}{50} - \frac{1}{200}\right] \times \frac{1}{10^{-}}$$

$$= 48 \times 10^{-6}\,\text{J} = 48\,\mu\text{J}$$

Common potential or redistribution of charge

When two charged conductors or capacitors are joined together by a conducting wire and having different potentials, then charge flow from capacitor at higher potential to the capacitor at lower potential. This flow of charge continues till their potential becomes equal, this equal potential is called **common potential.**

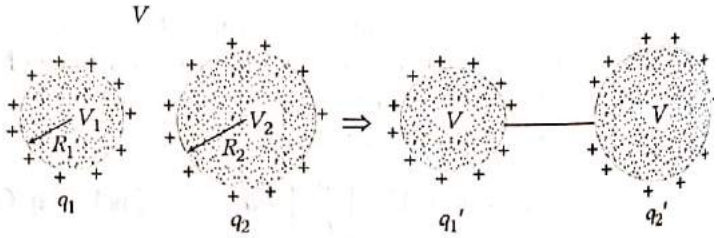


Fig. 2.59

Suppose two conductors of capacities C_1 and C_2 have charges q_1 and q_2 respectively and raised to potential V_1 and V_2 respectively are joined together by a conducting wire, then charge redistributes in these conductors in the ratio of their capacities. Charge redistributes till potential

of both the conductors become equal. Thus, let q_1' and q_2' be the final charges on them, then

$$q_1' = C_1V \text{ and } q_2' = C_2V$$

or
$$\frac{q_1'}{q_2'} = \frac{C_1}{C_2}$$

and if they are spherical conductors, then $\frac{C_1}{C_2} = \frac{R_1}{R_2}$

$$\therefore \quad \frac{q_1'}{q_2'} = \frac{C_1}{C_2} = \frac{R_1}{R_2}$$

where, R_1 and R_2 are radius of capacitors.
Since, the total charge is $(q_1 + q_2)$. Therefore,

$$q_1' = \left(\frac{C_1}{C_1 + C_2}\right)(q_1 + q_2)$$

and
$$q_2' = \left(\frac{C_2}{C_1 + C_2}\right)(q_1 + q_2)$$

Common potential, $V = \frac{\text{Total charge}}{\text{Total capacity}}$

$$\boxed{V = \frac{q_1' + q_2'}{C_1 + C_2} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}}$$

Note If the two capacitors are connected in such a way that their dissimilar plates are connected together, then their common potential is

$$V = \frac{C_1V_1 - C_2V_2}{C_1 + C_2}$$

Loss of energy during redistribution of charge

The charge on the capacitor C_1 was q and after joining it becomes q'. The charge transformed from the capacitor C_1 to the capacitor C_2 is $q - q'$.

Now,
$$q - q' = C_1V - C_1V'$$
$$= C_1\left(V_1 - \frac{C_1V_1 + C_2V_2}{C_1 + C_2}\right)$$

$$\boxed{q - q' = \frac{C_1C_2(V_1 - V_2)}{C_1 + C_2}}$$

This is the quantity of the charge transformed.
The loss of energy due to redistribution of charge is given by

or
$$\boxed{\Delta U = \frac{C_1C_2}{2(C_1 + C_2)}(V_1 - V_2)^2}$$

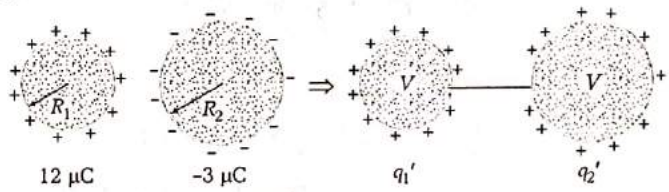
Now as C_1, C_2 and $(V_1 - V_2)^2$ are always positive. $U_i > U_f$ *i.e.*, there is a decrease in energy. Hence, energy is always lost in redistribution of charge. Further,

$$\Delta U = 0, \quad \text{if} \quad V_1 = V_2$$

this is because no flow of charge takes place when both the conductors are at same potential.

Example 2.90 *Two isolated spherical conductors have radii 5 cm and 10 cm respectively. They have charges of 12 μC and - 3 μC. Find the charges after they are connected by a conducting wire. Also find the common potential after redistribution.*

Sol.



Net charge = (12 – 3) μC = 9 μC

charge is distributed in the ratio of their capacities (or radii in case of spherical conductors), *i.e.*,

$$\frac{q_1'}{q_2'} = \frac{R_1}{R_2} = \frac{5}{10} = \frac{1}{2}$$

$$\therefore \quad q_1' = \left(\frac{1}{1+2}\right)(9) = 3\,\mu\text{C} \quad \text{and} \quad q_2' = \left(\frac{2}{1+2}\right)(9) = 6\,\mu\text{C}$$

Common potential,

$$V = \frac{q_1 + q_2}{C_1 + C_2} = \frac{(9 \times 10^{-6})}{4\pi\varepsilon_0(R_1 + R_2)}$$
$$= \frac{(9 \times 10^{-6})(9 \times 10^{9})}{(15 \times 10^{-2})}$$
$$= 5.4 \times 10^5 \text{ V}$$

Example 2.91 *Two parallel plates capacitors A and B having capacitance of 1 μF and 5 μF are charged separately to the same potential of 100 V. Now, the positive plate of A is connected to the negative plate of B and the negative plate of A to the positive plate of B. Find the final charges on each capacitor.*

Sol. As for a capacitor $q = CV$, so initially the charge on each capacitor,

$$q_1 = C_1V_1 = (1 \times 10^{-6}) \times 100 = 100\,\mu\text{C}$$

and
$$q_2 = C_2V_2 = (5 \times 10^{-6}) \times 100 = 500\,\mu\text{C}$$

Now when two capacitors are joined to each other such that positive plate of one is connected with the negative of the other, by conservation of charge

$$q = q_1 + q_2 = |q_1| = |q - q_2| = (500 - 100)\,\mu\text{C} = 400\,\mu\text{C}.$$

So common potential

$$V = \frac{(q_1 + q_2)}{(C_1 + C_2)} = \frac{400 \times 10^{-6}}{(1 + 5) \times 10^{-6}} = \frac{200}{3}\text{ V}$$

and hence after sharing charge on each capacitor

$$q_1' = C_1V = (1 \times 10^{-6}) \times \frac{200}{3} = \frac{200}{3}\,\mu\text{C}$$

$$q_2' = C_2V = (5 \times 10^{-6}) \times \frac{200}{3} = \frac{1000}{3}\,\mu\text{C}$$

Example 2.92 *A capacitor of capacitance 5 μF is charged to potential 20 V and then isolated. Now, an uncharged capacitor is connected in parallel to it. If the charge distributes equally on these capacitors, find total energy stored in capacitors.*

Sol. Let C be unknown capacitance

$$Q_1' = Q_2' \Rightarrow C_1V = C_2V$$
$$5V = CV \Rightarrow C = 5\,\mu\text{F}$$

Common potential,

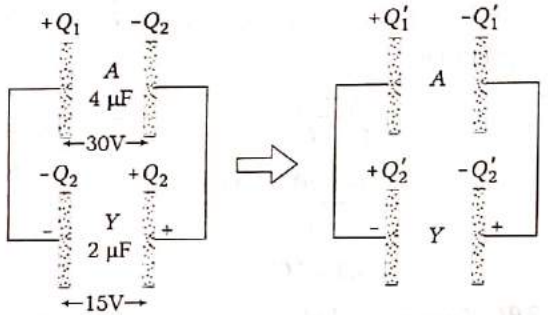
$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{5\times 20 + 5\times 0}{5+5} = 10\text{ V}$$

Total energy stored in capacitors,

$$U_f = \frac{1}{2}(C_1 + C_2)V^2 = \frac{1}{2}(5+5)(10)^2 \times 10^{-6}\text{ J}$$
$$= 500\times 10^{-6} = 500\,\mu\text{J}$$

Example 2.93 *A capacitor A of capacitance 4 μF is charged to 30 V and another capacitor B of capacitance 2 μF is charged to 15 V. Now, the positive plate of A is connected to the negative plate of B and negative plate of A to the positive plate of B. Find the final charge of each capacitor and loss of electrostatic energy in the process.*

Sol.



Here charge on A, $Q_1 = C_1V_1 = 4\times 30 = 120\,\mu\text{C}$

Charge on B, $Q_2 = C_2V_2 = 2\times 15 = 30\,\mu\text{C}$

When positive plate is connected to negative plate, we find net charge as $|Q_1 - Q_2| = 120 - 30 = 90\,\mu\text{C}$

The charge 90 μC will be redistributed in such a manner that capacitors acquire same potential.

Common potential,

$$V = \frac{|C_1V_1 - C_2V_2|}{C_1 + C_2} = \frac{4\times 30 - 2\times 15}{4+2} = 15\text{ V}$$
$$Q_1' = C_1V = 4\times 15 = 60\,\mu\text{C}$$
$$Q_2' = C_2V = 2\times 15 = 30\,\mu\text{C}$$

Loss of energy,

$$\Delta U = \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}(V_1 + V_2)^2 = \frac{1}{2}\cdot\frac{4\times 2}{4+2}(30-15)\times 10^{-6}\text{ J} = 150\,\mu\text{J}$$

Van de Graaff generator

Principle

It is based on the phenomenon of corona discharge and that the charge given to a hollow conductor get transferred to the outer surface to be distributed uniformly.

Use

It is a device which is used for generating high electric potential of the order of ten million volts. Such high electric potentials are needed for accelerating charged particles to very high speeds.

If we place a charge anywhere in a conductor, the charge will move to the outside surface, and the field inside the conductor will be zero.

Theory

Robert Van de Graaff took advantage of this concept in 1931 to build an generator, an apparatus, that produces highly energetic charged particles.

Such particles are useful for microscopic probes of matter such as cancer treatments. Van de Graaff used a device similar in concept to the apparatus shown schematically in figure.

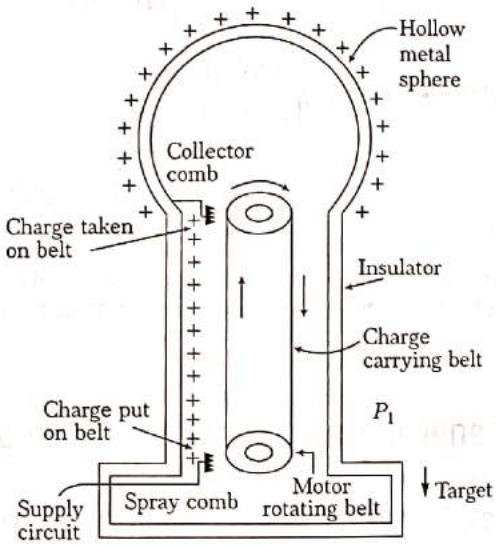


Fig. 2.60

An insulated belt (or chain) continuously bridge charge to the inside of a hollow conductor, which then moves to the outside surface of the conductor. The electric potential on the spherical conducting surface increases as charge flows to its surface $V = \frac{q}{4\pi\varepsilon_0 R}$. An ion source produces charged atoms whose sign in such as to be repelled from the region of high potential and thus accelerated.

Such devices are called Van de Graaff accelerators or Van de Graaff generators, and the beams they or other accelerators produce play an important role in modern technology. For example, such beams are used to make microcircuits.

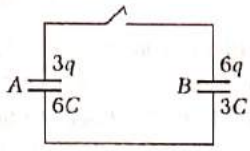
Check point 2.5

1. Three capacitors each of capacitance C and of breakdown voltage V are joined in series. The capacitance and breakdown voltage of the combination will be

(a) $\frac{C}{3}, \frac{V}{3}$ (b) $3C, \frac{V}{3}$

(c) $\frac{C}{3}, 3V$ (d) $3C, 3V$

2. In given circuit when switch S has been closed, then charge on capacitor A and B respectively,



(a) $3q, 6q$ (b) $6q, 3q$

(c) $4.5q, 4.5q$ (b) $5q, 4q$

3. Three condensers each of capacitance 2 F are put in series. The resultant capacitance is

(a) 6 F (b) $\frac{3}{2}$ F

(c) $\frac{2}{3}$ F (d) 5 F

4. Two capacitors of capacitance 2 μF and 3 μF are joined in series. Outer plate of first capacitor is 1000 V and outer plate of second capacitor is earthed (grounded). Now the potential on inner plate of each capacitor will be

(a) 700 V (b) 200 V

(c) 600 V (d) 400 V

5. A series combination of three capacitors of capacities 1 μF, 2 μF and 8 μF is connected to a battery of emf 13 V. The potential difference across the plates of 2 μF capacitor will be

(a) 1 V (b) 8 V

(c) 4 V (d) $\frac{13}{3}$ V

6. A parallel plate capacitor is made by stacking n equally spaced plates connected alternately. If the capacitance between any two plates is C, then the resultant capacitance is

(a) C (b) nC (c) $(n-1)C$ (d) $(n+1)C$

7. Four capacitors of equal capacitance have an equivalent capacitance C_1 when connected in series and an equivalent capacitance C_2 when connected in parallel. The ratio $\frac{C_1}{C_2}$ is

(a) $\frac{1}{4}$ (b) $\frac{1}{16}$ (c) $\frac{1}{8}$ (d) $\frac{1}{12}$

8. Three capacitors of capacitance 3 μF are connected in a circuit. Then, their maximum and minimum capacitances will be

(a) 9 μF, 1 μF (b) 8 μF, 2 μF

(c) 9 μF, 0, μC (d) 3 μF, 2 μF

9. Three capacitors each of capacity 4 μF are to be connected in such a way that the effective capacitance is 6 μF. This can be done by

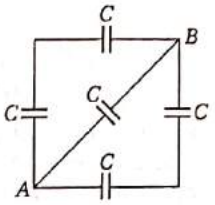
(a) connecting them in parallel

(b) connecting two in series and one in parallel

(c) connecting two in parallel and one in series

(d) connecting all of them in series

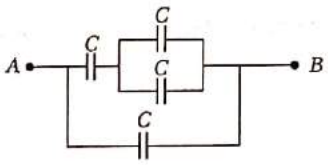
10. In the figure shown, the effective capacitance between the points A and B, if each has capacitance C, is



(a) $2C$ (b) $C/5$

(c) $5C$ (d) $C/2$

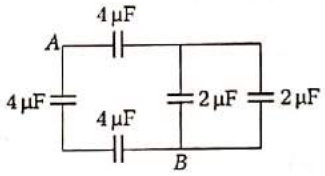
11. Four equal capacitors, each of capacity C, are arranged as shown. The effective capacitance between A and B is



(a) $\frac{5}{8}C$ (b) $\frac{3}{5}C$

(c) $\frac{5}{3}C$ (d) C

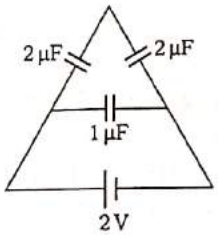
12. In the circuit as shown in the figure, the effective capacitance between A and B is



(a) 3 μF (b) 2 μF

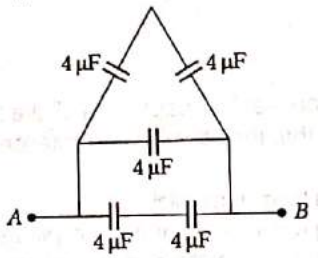
(c) 4 μF (d) 8 μF

13. The charge on anyone of the 2 μF capacitors and 1 μF capacitor will be given respectively (in μC) as



(a) 1, 2 (b) 2, 1 (c) 1, 1 (d) 2,2

14. Equivalent capacitance between A and B is



(a) 8 μF (b) 6 μF
(c) 26 μF (d) $\frac{10}{3}$ μF

15. The energy stored in a capacitor of capacitance 100 μF is 50 J. Its potential difference is
(a) 50 V (b) 500 V
(c) 1000 V (d) 1500 V

16. The potential energy of a charged parallel plate capacitor is U_0. If a slab of dielectric constant K is inserted between the plates, then the new potential energy will be
(a) $\frac{U_0}{K}$ (b) U_0K^2
(c) $\frac{U_0}{K^2}$ (d) U_0^2

17. A series combination of n_1 capacitors, each of value C_1, is charged by a source of potential difference $4V$. When another parallel combination of n_2 capacitors, each of value C_2, is charged by a source of potential difference V, it has the same (total) energy stored in it, as the first combination has. The value of C_2, in terms of C_1 is, then
(a) $\frac{16C_1}{n_1n_2}$ (b) $\frac{2C_1}{n_1n_2}$
(c) $16\frac{n_2}{n_1}C_1$ (d) $2\frac{n_2}{n_1}C_1$

18. If the charge on a capacitor is increased by 2 C, then the energy stored in it increases by 21%. The original charge on the capacitor is
(a) 10 C (b) 20 C
(c) 30 C (d) 40 C

19. A capacitor of capacitance value 1 μF is charged to 30 V and the battery is then disconnected. If it is connected across a 2 μF capacitor, then the energy lost by the system is
(a) 300 μJ (b) 450 μJ (c) 225 μJ (d) 150 μJ
(e) 100 μJ

20. A parallel plate capacitor is charged to a potential difference of 50 volts. It is then discharged through a resistance for 2 seconds and its potential drops by 10 volts. Calculate the fraction of energy stored in the capacitance
(a) 0.14 (b) 0.25
(c) 0.50 (d) 0.64

Chapter exercises

A. Taking it together

(Assorted questions of the chapter for advanced level practice)

1. Angle between equipotential surface and lines of force is
(a) zero (b) 180° (c) 90° (d) 45°

2. From a point charge there is a fixed point A. At that point there is an electric field of 500 V/m and potential of 3000 V. Then, the distance of A from the point charge is
(a) 6 m (b) 12 m
(c) 36 m (d) 144 m

3. A charge of 5 C is given a displacement of 0.5 m. The work done in the process is 10 J. The potential difference between the two points will be
(a) 2 V (b) 0.25 V (c) 1 V (d) 25 V

4. An electron enters in higher potential region V_2 from lower potential region V_1, then its velocity
(a) will increase
(b) will change in direction but not in magnitude
(c) will not change in direction of field
(d) will not change in direction perpendicular to field

5. When the separation between two charges is increased, the electric potential energy of the charges
(a) increases (b) decreases
(c) remains the same (d) may increase or decrease

6. If a positive charge is shifted from a low potential region to a high potential region, then the electric potential energy
(a) increases (b) decreases
(c) remains the same (d) may increase or decrease

7. The work done in carrying a charge of 5 μC from a point A to a point B in an electric field is 10 mJ. Then, potential difference $(V_B - V_A)$ is
(a) + 2 kV (b) – 2 kV
(c) + 200 kV (d) – 200 kV

8. The unit of electric permittivity is
(a) volt/m^2 (b) joule/C (c) farad/m (d) henry/m

9. The capacitance of a capacitor does not depend on
(a) the medium between the plates
(b) the size of the plates
(c) the charges on the plates
(d) the separation between the plates

10. In a charged capacitor, the energy resides in
(a) the positive charges
(b) Both the positive and negative charges
(c) the field between the plates
(d) around the edge of the capacitor plates

11. Which of the following is not true?
(a) For a point charge, the electrostatic potential varies as $1/r$
(b) For a dipole, the potential depends on the position vector and dipole moment vector
(c) The electric dipole potential varies as $1/r$ at large distance
(d) For a point charge, the electrostatic field varies as $1/r^2$

12. The energy stored in a condenser is in the form of
(a) kinetic energy (b) potential energy
(c) elastic energy (d) magnetic energy

13. The potential energy of a charged parallel plate capacitor is U_0. If a slab of dielectric constant K is inserted between the plates then the new potential energy will be
(a) $\frac{U_0}{k}$ (b) U_0k^2 (c) $\frac{U_0}{k^2}$ (d) U_0^2

14. A charge Q is placed at the origin. The electric potential due to this charge at a given point in space is v. The work done by an external force in bringing another charge q from infinity up to the point is
(a) $\frac{V}{q}$ (b) Vq (c) $V + q$ (d) V

15. The force between the plates of a parallel plate capacitor of capacitance C and distance of separation of the plates d with a potential difference V between the plates, is

(a) $\frac{CV^2}{2d}$ (b) $\frac{C^2V^2}{2d^2}$

(c) $\frac{C^2V^2}{d^2}$ (d) $\frac{V^2d}{C}$

16. An electric field of moment p is placed in a uniform electric field E. Then,

(i) the torque on the dipole is $p \times E$

(ii) the potential energy of the system is $p \cdot E$

(iii) the resultant force on the dipole is zero.

(a) (i), (ii) and (iii) are correct
(b) (i) and (iii) are correct and (ii) is wrong
(c) Only (i) is correct
(d) (i) and (ii) are correct and (iii) is wrong
(e) (i), (ii) and (iii) are wrong

17. A positively charged particle is released from rest in an uniform electric field. The electric potential energy of the charge **[NCERT Exemplar]**

(a) remains a constant because the electric field is uniform
(b) increases because the charge moves along the electric field
(c) decreases because the charge moves along the electric field
(d) decreases because the charge moves opposite to the electric field

18. Identify the false statement.

(a) Inside a charged or neutral conductor electrostatic field is zero
(b) The electrostatic field at the surface of the charged conductor must be tangential to the surface at any point
(c) There is no net charge at any point inside the conductor
(d) Electrostatic potential is constant throughout the volume of the conductor
(e) Electric field at the surface of a charged conductor is proportional to the surface charge density

19. Equipotentials at a great distance from a collection of charges whose total sum is not zero are approximately **[NCERT Exemplar]**

(a) spheres
(b) planes
(c) paraboloids
(d) ellipsoids

20. An electron enters in high potential region V_2 from lower potential region V_1, then its velocity

(a) will increase
(b) will change in direction but not in magnitude
(c) No change in direction of field
(d) No change in direction perpendicular to field

21. The capacitance of a metallic sphere is $1\,\mu F$, if its radius is nearly

(a) 9 km (b) 10 m
(c) 1.11 m (d) 1.11 cm

22. The unit of electric field is not equivalent to

(a) N/C (b) J/C
(c) V/m (d) J/C-m

23. The electric potential V is given as a function of distance x (metre) by $V = (5x^2 + 10x - 9)$ V. Value of electric field at $x = 1$ is

(a) -20 V/m (b) 6 V/m
(c) 11 V/m (d) -23 V/m

24. Electric potential at a point x from the centre inside a conducting sphere of radius R and carrying charge Q is

(a) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{x}$

(c) $\frac{1}{4\pi\varepsilon_0}xQ$ (d) zero

25. If a charged spherical conductor of radius 5 cm has potential V at a point distant 5 cm from its centre, then the potential at a point distant 30 cm from the centre will be

(a) $\frac{1}{3}$ V (b) $\frac{1}{6}$ V (c) $\frac{3}{2}$ V (d) 3 V

26. Two plates are at potentials -10 V and $+30$ V. If the separation between the plates be 2 cm. The electric field between them is

(a) 2000 V/m (b) 1000 V/m
(c) 500 V/m (d) 3000 V/m

27. The potential at a point due to an electric dipole will be maximum and minimum when the angles between the axis of the dipole and the line joining the point to the dipole are respectively

(a) 90° and 180° (b) 0° and 90°
(c) 90° and 0° (d) 0° and 180°

28. An electric dipole when placed in a uniform electric field E will have minimum potential energy if the dipole moment makes the following angle with E

(a) π (b) $\pi/2$ (c) zero (d) $3\pi/2$

29. How much kinetic energy will be gained by an α-particle in going from a point at 70 V to another point at 50 V

(a) 40 eV (b) 40 keV (c) 40 MeV (d) 0 eV

30. A charged particle of mass m and charge q is released from rest in an electric field of constant magnitude. The KE of the particle after time t is

(a) $\frac{Eq^2m}{2t^2}$ (b) $\frac{2E^2t^2}{mq}$ (c) $\frac{E^2q^2t^2}{2m}$ (d) $\frac{Eqm}{2t}$

31. Two positive charges 12 μC and 8 μC are 10 cm apart. The work done in bringing them 4 cm closer is
(a) 5.8 J (b) 5.8 eV
(c) 13 J (d) 13 eV

32. The capacitance of the earth, viewed as a spherical conductor of radius 6408 km is
(a) 980 μF (b) 1424 μF
(c) 712 μF (d) 356 μF

33. A thin metal plate P is inserted between the plates of a parallel plate capacitor of capacitance C in such a way that its edges touch the two plates. The capacitance now becomes
(a) $C/2$ (b) $2C$ (c) zero (d) ∞

34. A capacitor of capacity C has charge Q and stored energy is W. If the charge is increased to $2Q$, then the stored energy will be
(a) $2W$ (b) $W/2$ (c) $4W$ (d) $W/4$

35. A 2 μF capacitor is charged to 100 V and then its plates are connected by a conducting wire. The heat produced is
(a) 1 J (b) 0.1 J (c) 0.01 J (d) 0.001 J

36. If there are n capacitors in parallel connected to V volt source, then the energy stored is equal to
(a) nCV^2 (b) $\frac{1}{2}nCV^2$
(c) $\frac{CV^2}{n}$ (d) $\frac{1}{2n}CV^2$

37. A variable condenser is permanently connected to a 100 V battery. If the capacity is changed from 2 μF to 10 μF, then change in energy is equal to
(a) 2×10^{-2} J (b) 2.5×10^{-2} J
(c) 3.5×10^{-2} J (d) 4×10^{-2} J

38. Two condensers of capacity 0.3 μF and 0.6 μF respectively are connected in series. The combination is connected across a potential of 6 V. The ratio of energies stored by the condensers will be
(a) $\frac{1}{2}$ (b) 2 (c) $\frac{1}{4}$ (d) 4

39. A capacitor of capacity C_1, is charged by connecting it across a battery of emf V_0. The battery is then removed and the capacitor is connected in parallel with an uncharged capacitor of capacity C_2. The potential difference across this combination is
(a) $\frac{C_2}{C_1+C_2}V_0$ (b) $\frac{C_1}{C_1+C_2}V_0$
(c) $\frac{C_1+C_2}{C_2}V_0$ (d) $\frac{C_1+C_2}{C_1}V_0$

40. A capacitor is charged by using a battery which is then disconnected. A dielectric slab is then slipped between the plates which results in
(a) reduction of charges on the plates and increase of potential difference across the plates
(b) increase in the potential difference across the plates, reduction in stored energy, but no change in the charge on the plates
(c) decrease in the potential difference across the plates, reduction in stored energy, but no change in the charge on the plates
(d) None of the above

41. A parallel plate air capacitor is charged to a potential difference of V. After disconnecting the battery, distance between the plates of the capacitor is increased using an insulating handle. As a result, the potential difference between the plates
(a) decreases (b) increases
(c) becomes zero (d) does not change

42. Two concentric metallic spherical shells are given equal amount of positive charges. Then,
(a) the outer sphere is always at a higher potential
(b) the inner sphere is always at a higher potential
(c) Both the spheres are at the same potential
(d) no prediction can be made about their potentials unless the actual value of charges and radii are known

43. Dielectric constant of pure water is 81. Its permittivity will be
(a) 7.16×10^{-10} MKS units (b) 8.86×10^{-12} MKS units
(c) 1.02×10^{13} MKS units (d) Cannot be calculated

44. Two spherical conductors each of capacity C are charged to potential V and $-V$. These are then connected by means of a fine wire. The loss of energy is
(a) zero (b) $\frac{1}{2}CV^2$
(c) CV^2 (d) $2\,CV^2$

45. Two spheres A and B of radius 4 cm and 6 cm are given charges of 80 μC and 40 μC, respectively. If they are connected by a fine wire, then the amount of charge flowing from one to the other is
(a) 20 μC from A to B (b) 20 μC from B to A
(c) 32 μC from B to A (d) 32 μC from A to B

46. The electric potential difference between two parallel plates is 2000 V. If the plates are separated by 2 mm, then what is the magnitude of electrostatic force on a charge of 4×10^{-6} C located midway between the plates?
(a) 4 N (b) 6 N
(c) 8 N (d) 1.5×10^{-6} N

47. Two conducting spheres A and B of radii 4 cm and 2 cm carry charges of 18×10^{-8} statcoulomb and 9×10^{-8} statcoulomb respectively of positive electricity. When they are put in electrostatic contact, then the charge will
(a) not flow at all (b) flow from A to B
(c) flow from B to A (d) disappear

48. Two insulated charged spheres of radii R_1 and R_2 having charges Q_1 and Q_2 are respectively, connected to each other. There is
(a) an increase in the energy of the system
(b) no change in the energy of the system
(c) always decrease in energy
(d) a decrease in energy of the system unless $Q_1R_2 = Q_2R_1$

49. A small sphere is charged to a potential of 50 V and a big hollow sphere is charged to a potential of 100 V. Electricity will flow from the smaller sphere to the bigger one when
(a) the smaller one is placed inside the bigger one and connected by a wire
(b) bigger one placed by the side of the smaller one and connected by a wire
(c) Both are correct
(d) Both are wrong

50. Two identical charges are placed at the two corners of an equilateral triangle. The potential energy of the system is U. The work done in bringing an identical charge from infinity to the third vertex is
(a) U (b) $2U$ (c) $3U$ (d) $4U$

51. Four electric charges $+q$, $+q$, $-q$ and $-q$ are placed at the corners of a square of side $2L$. The electric potential at point A, midway between the two charges $+q$ and $+q$, is
(a) $\frac{1}{4\pi\varepsilon_0}\frac{2q}{L}\left(1+\frac{1}{\sqrt{5}}\right)$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{2q}{L}\left(1-\frac{1}{\sqrt{5}}\right)$
(c) zero (d) $\frac{1}{4\pi\varepsilon_0}\frac{2q}{L}(1+\sqrt{5})$

52. A hollow metal sphere of radius 10 cm is charged such that the potential on its surface becomes 80 V. The potential at the centre of the sphere is
(a) 80 V (b) 800 V (c) 8 V (d) zero

53. A parallel plate capacitor has a uniform electric field E (V/m) in the space between the plates. If the distance between the plates is d(m) and area of each plate is $A(\text{m}^2)$, then the energy (joule) stored in the capacitor is
(a) $\frac{1}{2}\varepsilon_0 E^2$ (b) $\varepsilon_0 EAd$
(c) $\frac{1}{2}\varepsilon_0 E^2 Ad$ (d) E^2Ad/ε_0

54. Charges 5 μC and 10 μC are placed 1 m apart. Work done to bring these charges at a distance 0.5 m from each other is ($k = 9 \times 10^9$ SI)
(a) 9×10^4 J (b) 18×10^4 J
(c) 45×10^{-2} J (d) 9×10^{-1} J

55. A particle of mass 2×10^{-3} kg, charge 4×10^{-3} C enters in an electric field of 5 V/m, then its kinetic energy after 10 s is
(a) 0.1 J (b) 1 J (c) 10 J (d) 100 J

56. The ionisation potential of mercury is 10.39 V. How far an electron must travel in an electric field of 1.5×10^6 V/m to gain sufficient energy to ionise mercury?
(a) $\frac{10.39}{1.6 \times 10^{-19}}$ m (b) $\frac{10.39}{2 \times 1.6 \times 10^{-19}}$ m
(c) $10.39 \times 1.6 \times 10^{-19}$ m (d) $\frac{10.39}{1.5 \times 10^6}$ m

57. 0.2 F capacitor is charge to 600 V by a battery, on removing the battery, it is connected with another parallel plate condenser of 1F. The potential decreases to
(a) 100 V (b) 120 V (c) 300 V (d) 600 V

58. Work done in placing a charge of 8×10^{-18} C on a condenser of capacity 100 μF is
(a) 16×10^{-32} J (b) 31×10^{-26} J
(c) 4×10^{-10} J (d) 32×10^{-32} J

59. In a parallel plate capacitor the separation between the plates is 3 mm with air between them. Now a 1 mm thick layer of a material of dielectric constant 2 is introduced between the plates due to which the capacity increases. In order to bring its capacity to the original value the separation between the plates must be made
(a) 1.5 mm (b) 2.5 mm (c) 3.5 mm (d) 4.5 mm

60. The electric potential at any point x, y and z in metres is given by $V = 3x^2$. The electric field at a point (2, 0, 1) is
(a) 12 Vm^{-1} (b) -6 Vm^{-1} (c) 6 Vm^{-1} (d) -12 Vm^{-1}

61. An electron of mass m and charge e is accelerated from rest through a potential difference V in vacuum. The final speed of the electron will be
(a) $V\sqrt{\frac{e}{m}}$ (b) $\sqrt{\frac{eV}{m}}$ (c) $\sqrt{\frac{2eV}{m}}$ (d) $\frac{2eV}{m}$

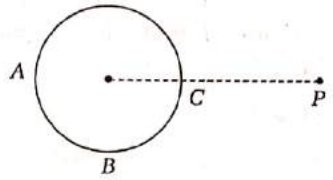
62. If an electron moves from rest from a point at which potential is 50 V to another point at which potential is 70 V, then its kinetic energy in the final state will
(a) 3.2×10^{-10} J (b) 3.2×10^{-18} J
(c) 1 N (d) 1 dyne

63. The work done in bringing a 20 C charge from point A to point B for distance 0.2 m is 2 J. The potential difference between the two points will be (in volt)
(a) 0.2 (b) 8 (c) 0.1 (d) 0.4

64. If 4×10^{20} eV energy is required to move a charge of 0.25 C between two points, then what will be the potential difference between them?
(a) 178 V (b) 256 V
(c) 356 V (d) None of these

65. Kinetic energy of an electron accelerated in a potential difference of 100 V is
(a) 1.6×10^{-17} J (b) 1.6×10^{21} J
(c) 1.6×10^{-29} J (d) 1.6×10^{-34} J

66. A hollow conducting sphere is placed in an electric field produced by a point charge placed at P as shown in figure. Let V_A, V_B and V_C be the potentials at points A, B and C, respectively. Then



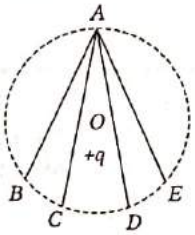
(a) $V_C > V_B$ (b) $V_B > V_C$ (c) $V_A > V_B$ (d) $V_A = V_C$

67. Two unilike charges of magnitude q are separated by a distance $2d$. The potential at a point midway between them is
(a) zero (b) $\frac{1}{4\pi\varepsilon_0}$ (c) $\frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{d}$ (d) $\frac{1}{4\pi\varepsilon_0} \cdot \frac{2q}{d}$

68. Two spheres A and B of radius a and b respectively are at same electric potential. The ratio of the surface charge densities of A and B is
(a) $\frac{a}{b}$ (b) $\frac{b}{a}$ (c) $\frac{a^2}{b^2}$ (d) $\frac{b^2}{a^2}$

69. A capacitor of 2 μF charged to 50V is connected in parallel with another capacitor of 1μF charged to 20V. The common potential and loss of energy will be
(a) 40 V, 300 μ J (b) 50 V, 400 μ J
(c) 40 V, 600 μ J (d) 50 V, 700 μ J

70. In the electric field of a point charge q, a certain charge is carried from point A to B, C, D and E. Then, the work done



(a) is least along the path AB
(b) is least along the path AD
(c) is zero along all the paths AB, AC, AD and AE
(d) is least along AE

71. A uniform electric field having a magnitude E_0 and direction along the positive X-axis exists. If the potential V is zero at $x = 0$, then its value at $X = +x$ will be
(a) $+xE_0$ (b) $-xE_0$
(c) $+x^2E_0$ (d) $-x^2E_0$

72. Two positive point charges of 12 and 5 microcoulombs are placed 10 cm apart in air. The work done needed to bring them 4 cm closer is
(a) 3.6 J (b) 2.6 J
(c) 4.8 J (d) 6.0 J

73. In a uniform electric field a charge of 3 C experiences a force of 3000 N. The potential difference between two points 1 cm apart along the electric lines of force will be
(a) 10 V (b) 100 V
(c) 30 V (d) 300 V

74. A particle A has charge $+q$ and a particle B has charge $+4q$ with each of them having the same mass m. When allowed to fall from rest through the same electric potential difference, the ratio of their speed $\frac{v_A}{v_B}$ will become
(a) 2 : 1 (b) 1 : 2
(c) 1 : 4 (d) 4 : 1

75. Three particles, each having a charge of 10 μC are placed at the corners of an equilateral triangle of side 10 cm. The electrostatic potential energy of the system is (given $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ N-m^2/C^2)
(a) zero (b) infinite
(c) 27 J (d) 100 J

76. A mass $m = 20$ g has a charge $q = 3.0$ mC. It moves with a velocity of 20 m/s and enters a region of electric field of 80 N/C in the same direction as the velocity of the mass. The velocity of the mass after 3 s in this region is
(a) 80 m/s (b) 56 m/s
(c) 44 m/s (d) 40 m/s

77. Four identical charges +50 μC each are placed, one at each corner of a square of side 2 m. How much external energy is required to bring another charge of + 50 μC from infinity to the centre of the square?
(a) 64 J (b) 41 J
(c) 16 J (d) 10 J

78. Two equal charges q are placed at a distance of $2a$ and a third charge $-2q$ is placed at the mid-point. The potential energy of the system is

(a) $\frac{q^2}{8\pi\varepsilon_0 a}$ (b) $\frac{6q^2}{8\pi\varepsilon_0 a}$ (c) $-\frac{7q^2}{8\pi\varepsilon_0 a}$ (d) $-\frac{9q^2}{8\pi\varepsilon_0 a}$

79. An alpha particle is accelerated through a potential difference of 10^6 V. Its kinetic energy will be

(a) 1 MeV (b) 2 MeV (c) 4 MeV (d) 8 MeV

80. The ratio of momenta of an electron and an α-particle which are accelerated from rest by a potential difference of 100 V is

(a) 1 (b) $\sqrt{\frac{2m_e}{m_\alpha}}$ (c) $\sqrt{\frac{m_e}{m_\alpha}}$ (d) $\sqrt{\frac{m_e}{2m_\alpha}}$

81. Two particles of masses m and $2m$ with charges $2q$ and q are placed in a uniform electric field E and allowed to move for same time. Find the ratio of their kinetic energies

(a) 8 : 1 (b) 4 : 1
(c) 2 : 1 (d) 16 : 1

82. A spherical condenser has inner and outer spheres of radii a and b respectively. The space between the two is filled with air. The difference between the capacities of two condensers formed when outer sphere is earthed and when inner sphere is earthed will be

(a) zero (b) $4\pi\varepsilon_0 a$
(c) $4\pi\varepsilon_0 b$ (d) $4\pi\varepsilon_0 a\left(\frac{b}{b-a}\right)$

83. Three charges are placed at the vertices of an equilateral triangle of side 10 cm. Assume $q_1 = 1\,\mu C$, $q_2 = -2\,\mu C$ and $q_3 = 4\,\mu C$. Work done in separating the charges to infinity is

(a) -4.5 J (b) 4.5 J
(c) 45 J (d) None of these

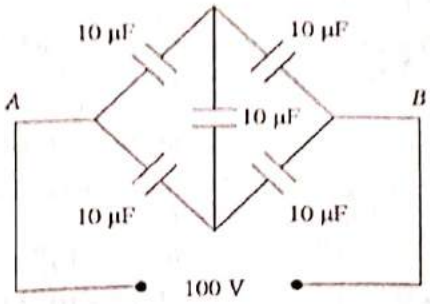
84. At a distance of 1 m from a fixed charge of 1 mC, a particle of mass 2 g and charge $1\,\mu C$ is held stationary. Both the charges are placed on a smooth horizontal surface. If the particle is made free to move, then its speed at a distance of 10 m from the fixed charge will be

(a) $10\,ms^{-1}$ (b) $20\,ms^{-1}$ (c) $60\,ms^{-1}$ (d) $90\,ms^{-1}$

85. A point charge q is surrounded by six identical charges at distance r shown in the figure. How much work is done by the force of electrostatic repulsion, when the point charge at the centre is removed to infinity?

(a) $6q/4\pi\varepsilon_0 r$ (b) $6q^2/4\pi\varepsilon_0 r$
(c) $36q^2/4\pi\varepsilon_0 r$ (d) Zero

86. Five capacitors of 10 μF capacity each are connected to a DC potential of 100 V as shown in the adjoining figure. The equivalent capacitance between the points A and B will be equal to

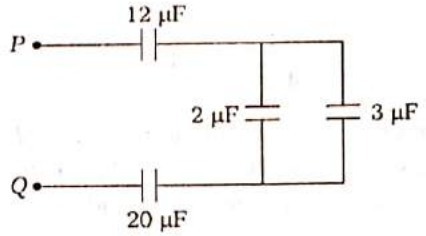


(a) 40 μF (b) 20 μF
(c) 30 μF (d) 10 μF

87. Three capacitors of capacitances 3 μF, 9 μF and 18 are connected once in series and another time in parallel. The ratio of equivalent capacitance in the two cases $\left(\frac{C_S}{C_P}\right)$ will be

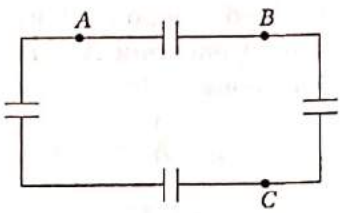
(a) 1:15 (b) 15:1
(c) 1:1 (d) 1:3

88. In the circuit diagram shown in the adjoining figure the resultant capacitance between points P and Q



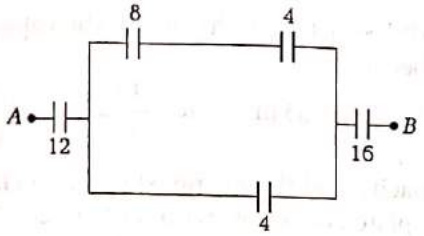
(a) 47 μF (b) 3 μF
(c) 60 μF (d) 10 μF

89. Four capacitors each of capacity 3 μF are connected as shown in the adjoining figure. The ratio of equivalent capacitance between A and B and between A and C will be



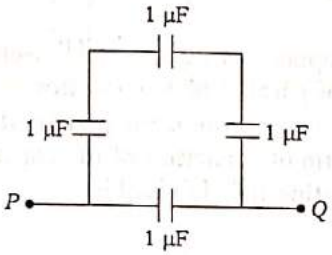
(a) 4 : 3 (b) 3 : 4
(c) 2 : 3 (d) 3 : 2

90. What is the equivalent capacitance between A and B in the given figure (all are in farad)?



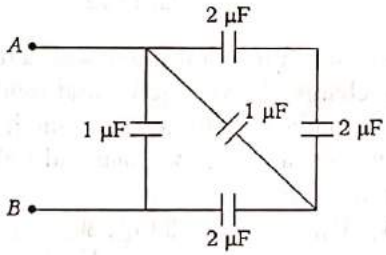
(a) $\frac{13}{18}$ F (b) $\frac{48}{13}$ F (c) $\frac{1}{31}$ F (d) $\frac{240}{71}$ F

91. Four capacitors are connected as shown. The equivalent capacitance between the points P and Q is



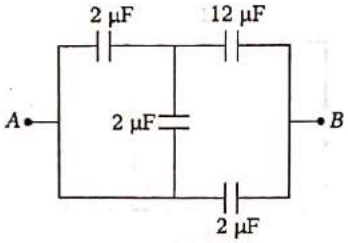
(a) 4 μF (b) $\frac{1}{4}$ μF
(c) $\frac{3}{4}$ μF (d) $\frac{4}{3}$ μF

92. The total capacity of the system of capacitors shown in the adjoining figure between the points A and B is



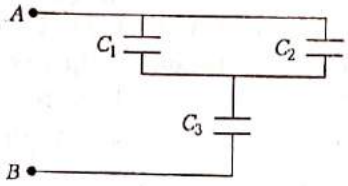
(a) 1 μF (b) 2 μF
(c) 3 μF (d) 4 μF

93. Four capacitors are connected in a circuit as shown in the figure. The effective capacitance in μF between points A and B will be



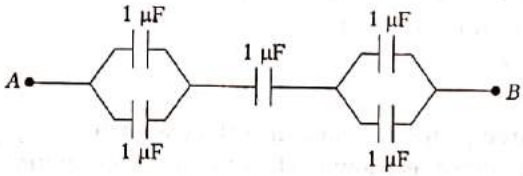
(a) $\frac{28}{9}$ (b) 4 (c) 5 (d) 18

94. In the given network capacitance, $C_1 = 10$ μF, $C_2 = 5$ μF and $C_3 = 4$ μF. What is the resultant capacitance between A and B (approximately)



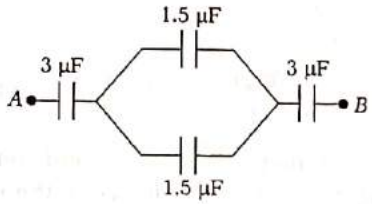
(a) 2.2 μF (b) 3.2 μF (c) 1.2 μF (d) 4.7 μF

95. The equivalent capacitance between points A and B is



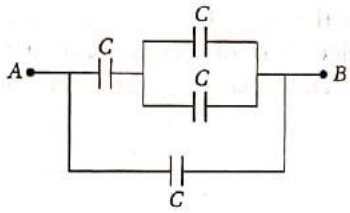
(a) 2 μF (b) 3 μF (c) 5 μF (d) 0.5 μF

96. The capacitance between the points A and B in the given circuit will be



(a) 1 μF (b) 2 μF (c) 3 μF (d) 4 μF

97. Four equal capacitors, each of capacity C, are arranged as shown. The effective capacitance between points A and B is



(a) $\frac{5}{8}C$ (b) $\frac{3}{5}C$ (c) $\frac{5}{3}C$ (d) C

98. There are 7 identical capacitors. The equivalent capacitance when they are connected in series is C. The equivalent capacitance when they are connected in parallel is
(a) $C/49$ (b) $C/7$ (c) $7C$ (d) $49C$

99. The capacitance of a parallel plate capacitor is 16 μF. When a glass slab is placed between the plates, the potential difference reduces to 1/8th of the original value. What is dielectric constant of glass?
(a) 4 (b) 8 (c) 16 (d) 32

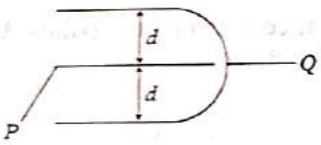
100. A parallel plate condenser with air between the plates possesses the capacity of 10^{-12} F. Now, the plates are removed apart, so that the separation is twice the original value. The space between the plates is filled with a material of dielectric constant 4.0. Then new value of the capacity is (in farad)

(a) 4×10^{-12} (b) 3×10^{-12}
(c) 2×10^{-12} (d) 0.5×10^{-12}

101. Three condensers each of capacity C microfarad are connected in series. An exactly similar set is connected in parallel to the first one. The effective capacity of the combination is 4 μF. Then, the value of C in microfarad is

(a) 8 (b) 6
(c) 4 (d) 2

102. Three plates of common surface area A are connected as shown. The effective capacitance between points P and Q will be

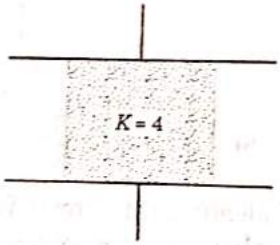


(a) $\frac{\varepsilon_0 A}{d}$ (b) $\frac{3\varepsilon_0 A}{d}$ (c) $\frac{3}{2}\frac{\varepsilon_0 A}{d}$ (d) $\frac{2\varepsilon_0 A}{d}$

103. Eight drops of mercury of equal radii combine to form a big drop. The capacitance of the bigger drop as compared to each smaller drop is

(a) 2 times (b) 8 times (c) 4 times (d) 16 times

104. Consider a parallel plate capacitor of capacity 10 μF with air filled in the gap between the plates. Now, one-half of the space between the plates is filled with a dielectric of dielectric constant 4, as shown in the figure. The capacity of the capacitor changes to



(a) 25 μF (b) 20 μF (c) 40 μF (d) 5 μF

105. A capacitor of capacity C is connected with a battery of potential V. The distance between its plates is reduced to half, assuming that the battery remains the same. Then, the new energy given by the battery will be

(a) $CV^2/4$ (b) $CV^2/2$
(c) $3CV^2/4$ (d) CV^2

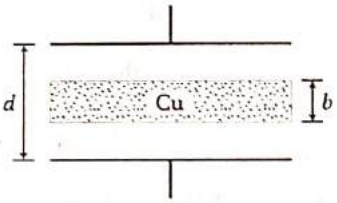
106. A parallel plate capacitor has plate separation d and capacitance 25 μF. If a metallic foil of thickness $\frac{2}{7}d$ is introduced between the plates, the capacitance would become

(a) 25 μF (b) 35 μF (c) $\frac{125}{7}$ μF (d) $\frac{175}{2}$ μF

107. The capacity and the energy stored in a charged parallel plate condenser with air between its plates are respectively, C_0 and W_0. If the air is replaced by glass (dielectric constant = 5) between the plates, the capacity of the plates and the energy stored in it will respectively be

(a) $5C_0, 5W_0$ (b) $5C_0, \frac{W_0}{5}$ (c) $\frac{C_0}{5}, 5W_0$ (d) $\frac{C_0}{5}, \frac{W_0}{5}$

108. A slab of copper of thickness b is inserted in between the plates of parallel plate capacitor as shown in figure. The separation of the plate is d. If $b = d/2$, then the ratio of capacities of the capacitor after and before inserting the slab will be

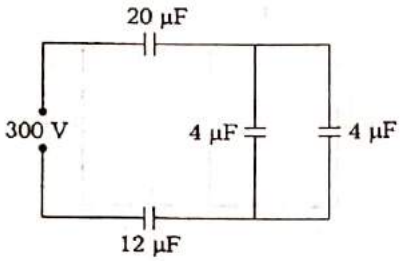


(a) $\sqrt{2}:1$ (b) $2:1$
(c) $1:1$ (d) $1:\sqrt{2}$

109. A charged capacitor when filled with a dielectric $K = 3$ has charge Q_0, voltage V_0 and field E_0. If the dielectric is replaced with another one having $K = 9$ the new values of charge voltage and field will be respectively

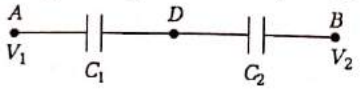
(a) $3Q_0, 3V_0, 3E_0$ (b) $Q_0, 3V_0, 3E_0$
(c) $Q_0, \frac{V_0}{3}, 3E_0$ (d) $Q_0, \frac{V_0}{3}, \frac{E_0}{3}$

110. In the adjoining figure, four capacitors are shown with their respective capacities and the potential difference is applied. The charge and the potential difference across the 4 μF capacitor will be



(a) 600 μC; 150 V (b) 300 μC; 75 V
(c) 800 μC; 200 V (d) 580 μC; 145 V

111. Two condensers C_1 and C_2 in a circuit are joined as shown in figure. The potential of point A is V_1 and that of B is V_2. The potential of point D will be



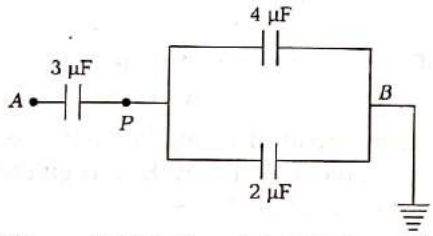
(a) $\frac{1}{2}(V_1+V_2)$ (b) $\frac{C_2V_1+C_1V_2}{C_1+C_2}$

(c) $\frac{C_1V_1+C_2V_2}{C_1+C_2}$ (d) $\frac{C_2V_1-C_1V_2}{C_1+C_2}$

112. Three capacitors of 2 μF, 3 μF and 6 μF are joined in series and the combination is charged by means of a 24 V battery. The potential difference between the plates of the 6 μF capacitor is

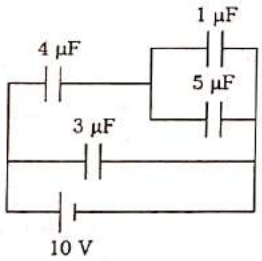
(a) 4 V (b) 6 V (c) 8 V (d) 10 V

113. In the figure a potential of + 1200 V is given to point A and point B is earthed, what is the potential at the point P?



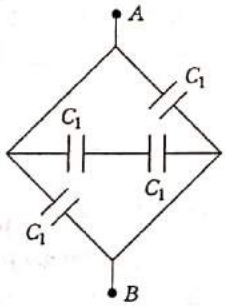
(a) 100 V (b) 200 V (c) 400 V (d) 800 V

114. The charge on 4 μF capacitor in the given circuit is (in μC)



(a) 12 (b) 24 (c) 36 (d) 32

115. Four identical capacitors are connected as shown in diagram. When a battery of 6 V is connected between A and B, then the charge stored is found to be 1.5 μC. The value of C_1 is



(a) 2.5 μF (b) 15 μF (c) 1.5 μF (d) 0.1 μF

116. A dielectric slab of thickness d is inserted in a parallel plate capacitor whose negative plate is at $x = 0$ and positive plate is at $x = 3d$. The slab is equidistant from the plates. The capacitor is given some charge. As one goes from 0 to $3d$

(a) the magnitude of the electric field remains the same
(b) the direction of the electric field remains the same
(c) the electric potential increases continuously
(d) the electric potential increases at first, then decreases and again increases

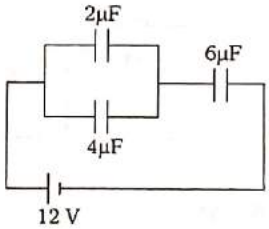
117. A 2 μF condenser is charged upto 200 V and then battery is removed. On combining this with another uncharged condenser in parallel, the potential differences between two plates are found to be 40 V. The capacity of second condenser is

(a) 2 μF (b) 4 μF
(c) 8 μF (d) 16 μF

118. Consider two conductors. One of them has a capacity of 2 units and the capacity of the other is unknown. They are charged until their potentials are 4 and 5 units respectively. The two conductors are now connected by a wire when their common potential is found to be 4.6 units. Then the unknown capacity has the value (in the same units as above)

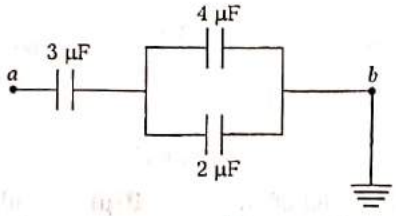
(a) 6 (b) 5
(c) 4 (d) 3

119. Two capacitors 2 μF and 4 μF are connected in parallel. A third capacitor of 6 μF capacity is connected in series. The combination is connected across a 12 V battery. The voltage across a 2 μF capacitor is



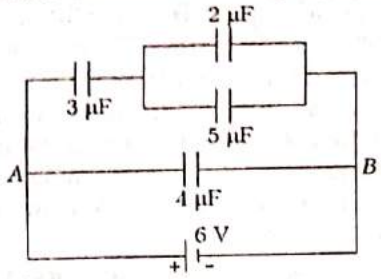
(a) 2 V (b) 6 V
(c) 8 V (d) 1 V

120. In the given circuit, if point b is connected to earth and a potential of 1200 V is given to a point a, the charge on 4 μF capacitor is



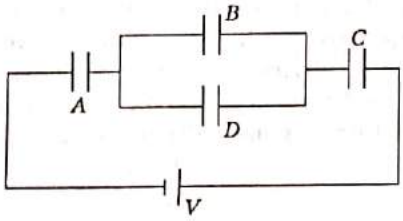
(a) 800 μC (b) 1600 μC
(c) 2400 μC (d) 3000 μC

121. A circuit is shown in the given figure. Find out the charge on the condenser having capacity 5 μF.



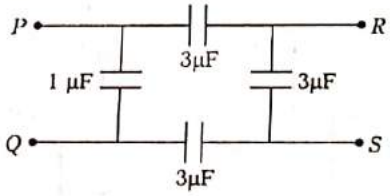
(a) 4.5 μC (b) 9 μC (c) 7 μC (d) 30 μC

122. A potential of $V = 3000$ V is applied to a combination of four initially uncharged capacitors as shown in the figure. Capacitors A, B, C and D have capacitances $C_A = 6.0\ \mu F$, $C_B = 5.2\ \mu F$, $C_C = 1.5\ \mu F$ and $C_D = 3.8\ \mu F$, respectively. If the battery is disconnected, then potential difference across capacitor B is (approximately)



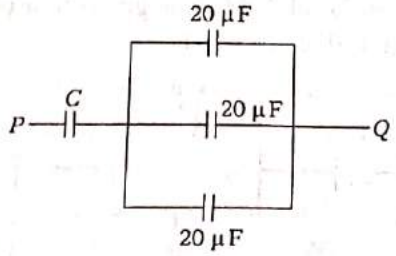
(a) 3000 V (b) zero (c) 530 V (d) 350 V

123. Four capacitors are arranged as shown. All are initially uncharged. A 30 V battery is placed across terminal PQ to charge the capacitors and is then removed. The voltage across the terminals RS is then (in volt)



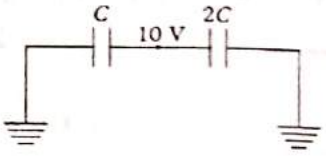
(a) 10 (b) 20 (c) 30 (d) 40

124. If the equivalent capacitance between points P and Q of the combination of the capacitors show in figure below is 30 μF, the capacitor C is



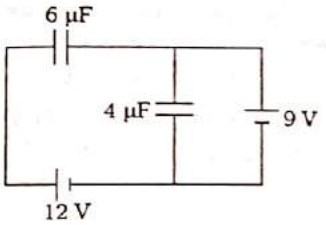
(a) 60 μF (b) 30 μF (c) 10 μF (d) 5 μF

125. In the circuit shown in figure $C = 6$ μF. The charge stored in the capacitor of capacity C is



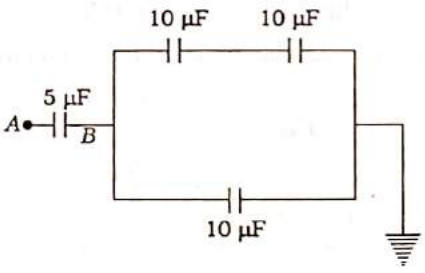
(a) zero (b) 90 μC
(c) 40 μC (d) 60 μC

126. In the circuit shown in figure. Charge stored in 6 μ capacitor will be



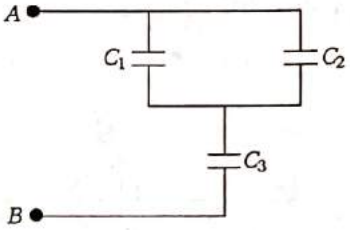
(a) 18 μC (b) 54 μC
(c) 36 μC (d) 72 μC

127. In the given circuit, if point C is connected to the earth and a potential of + 2000 V is given to the point A, the potential at B is



(a) 1500 V (b) 1000 V
(c) 500 V (d) 400 V

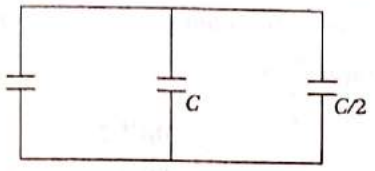
128. In the given network capacitance $C_1 = 10\ \mu F$ $C_2 = 5\mu F$ and $C_3 = 4\mu F$. What is the resultant capacitance between A and B?



(a) 2.2 μF (b) 3.2 μF
(c) 1.2 μF (d) 4.7 μF

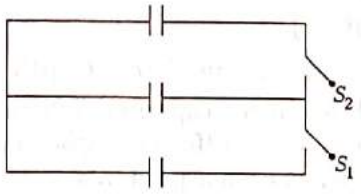
129. Two condensers one of capacity C and the other capacity $\frac{C}{2}$, are connected to a V volt battery as shown.

The work done in charging fully both the condensers is



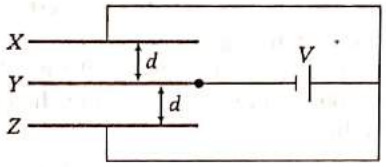
(a) $2CV^2$ (b) $1/4CV^2$ (c) $3/4CV^2$ (d) $\frac{1}{2}CV^2$

130. In the circuit shown here $C_1 = 6\mu F$, $C_2 = 3\mu F$ and battery $B = 20$ V. The switch S, is first closed. It is then opened and afterwards S_2 is closed. What is the charge finally on C_2?



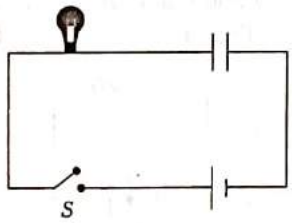
(a) 120 μC (b) 80 μC (c) 40 μC (d) 20 μC

131. Consider the arrangement of three plates X, Y and Z each of area A and separation d. The energy stored when the plates are fully charged is



(a) $\frac{\varepsilon_0 A V^2}{2d}$ (b) $\frac{\varepsilon_0 A V^2}{d}$ (c) $\frac{2\varepsilon_0 A V^2}{d}$ (d) $\frac{3\varepsilon_0 A V^2}{2d}$

132. A light bulb, a capacitor and a battery are connected together as shown here, with switch S initially open. When the switch S is closed, then which one of the following is true



(a) the bulb will light up for an instant when the capacitor starts charging
(b) the bulb will light up when the capacitor is fully charged
(c) the bulb will not light up at all
(d) the bulb will light up and go off at regular intervals

133. Point charges $+4q$, $-q$ and $+4q$ are kept on the X-axis at points $x = 0$, $x = a$ and $x = 2a$, respectively. Then,

(a) Only $-q$ is in stable equilibrium
(b) None of the charges are in equilibrium
(c) All the charges are in unstable equilibrium
(d) All the charges are in stable equilibrium

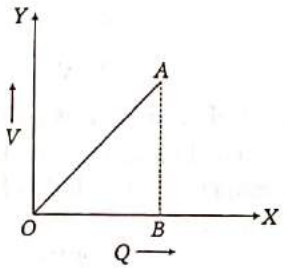
134. Two spherical conductors of radii 4 cm and 5 cm are charged to the same potential. If σ_1 and σ_2 be respective value of surface density of charge on both the conductors, then the ratio of σ_1/σ_2 will be

(a) $\frac{16}{25}$ (b) $\frac{25}{10}$ (c) $\frac{4}{5}$ (d) $\frac{5}{4}$

135. A hollow charged metal sphere has radius r. If the potential difference between its surface and a point at a distance $3r$ from the centre is V, then electric field intensity at a distance $3r$ is

(a) $\frac{V}{2r}$ (b) $\frac{V}{3r}$ (c) $\frac{V}{6r}$ (d) $\frac{V}{4r}$

136. Charge Q on a capacitor varies with voltage V as shown in the figure, where Q is taken along the X-axis and V along the Y-axis. The area of triangle OAB represents

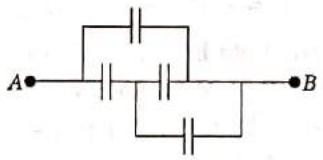


(a) capacitance
(b) capacitive reactance
(c) electric field between the plates
(d) energy stored in the capacitor

137. How many 1μF capacitors must be connected in parallel to store a charge of 1 C with a potential of 110 V across the capacitors?

(a) 990 (b) 900 (c) 9090 (d) 909

138. In the figure below, the capacitance of each capacitor is 3 μF. The effective capacitance between points A and B is



(a) $\frac{3}{4}\mu F$ (b) 3 μF
(c) 6 μF (d) 5 μF

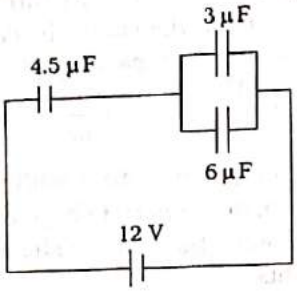
139. A 500 μF capacitor is charged at the steady rate of 100 μC/ s. How long will it take to raise the potential difference between the plates of the capacitor to 10 V?

(a) 5 s (b) 10 s (c) 50 s (d) 100 s

140. A ball of mass 1 g and charge 10^{-8} C moves from a point A, where potential is 600 V to the point B where potential is zero. Velocity of the ball at the point B is 20 cms^{-1}. The velocity of the ball at the point A will be

(a) 22.8 cms^{-1} (b) 228 cms^{-1}
(c) 16.8 ms^{-1} (d) 168 ms^{-1}

141. In the circuit shown in the figure, the potential difference across the 4.5 μF capacitor is



(a) $\frac{8}{3}$ V (b) 4V
(c) 6V (d) 8V

142. Three capacitors of capacitances 1 μF, 2 μF and 4μF are connected first in a series combination, and then in parallel combination. The ratio of their equivalent capacitances will be

(a) 2:49 (b) 49:2
(c) 4:49 (d) 49:4

143. An electron moving with the speed 5×10^6 per/s is shooted parallel to the electric field of intensity 1×10^3 N/C. Field is responsible for the retardation of motion of electron. Now evaluate the distance travelled by the electron before coming to rest for an instant (mass of $e = 9 \times 10^{-31}$ kg, charge $= 1.6 \times 10^{-19}$ C)

(a) 7 m (b) 0.7 mm
(c) 7 cm (d) 0.7 cm

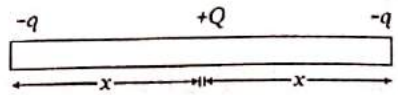
144. The electric potential at a point (x, y, z) is given by

$$V = -x^2y - xz^2 + 4$$

The electric field **E** at that point is

(a) $\mathbf{E} = \hat{\mathbf{i}}(2xy + z^3) + \hat{\mathbf{j}}x^2 + \hat{\mathbf{k}}3xz^2$
(b) $\mathbf{E} = \hat{\mathbf{i}}2xy + \hat{\mathbf{j}}(x^2 + y^2) + \hat{\mathbf{k}}(3xz - y^2)$
(c) $\mathbf{E} = \hat{\mathbf{i}}z^3 + \hat{\mathbf{j}}xyz + \hat{\mathbf{k}}z^2$
(d) $\mathbf{E} = \hat{\mathbf{i}}(2xy - z^3) + \hat{\mathbf{j}}xy^2 + \hat{\mathbf{k}}3z^2x$

145. Three charges $-q, +Q$ and $-q$ are placed in a straight line as shown.



If the total potential energy of the system is zero, then the ratio $\frac{q}{Q}$ is

(a) 2 (b) 5.5
(c) 4 (d) 1.5

146. The mutual electrostatic potential energy between two protons which are at a distance of 9×10^{-15} m, in $_{92}U^{235}$ nucleus is

(a) 1.56×10^{-14} J
(b) 5.5×10^{-14} J
(c) 2.56×10^{-14} J
(d) 4.56×10^{-14} J

147. Three capacitor of capacitance C (μF) are connected in parallel to which a capacitor of capacitance C is connected in series. Effective capacitance is 3.75, then capacity of each capacitor is

(a) 4 μF (b) 5 μF
(c) 6 μF (d) 8 μF

148. Figure shows some equipotential lines distributed in space. A charged object is moved from point A to point B. **[NCERT Exempla**

(a) The work done in Fig. (i) is the greatest
(b) The work done in Fig. (ii) is least
(c) The work done is the same in Fig. (i), Fig.(ii) and Fig. (iii)
(d) The work done in Fig. (iii) is greater than Fig. (ii) but equal to that in

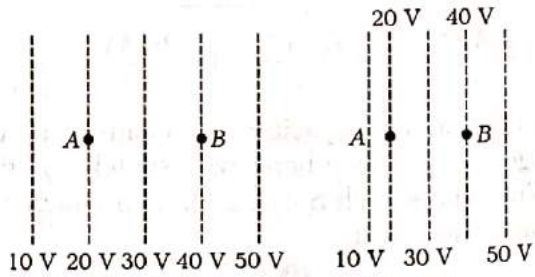


Fig. (i) Fig. (ii)

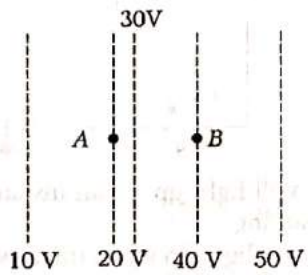


Fig. (iii)

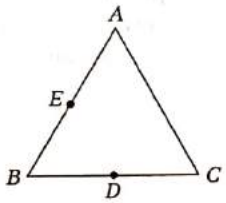
149. The electrostatic potential on the surface of a charged conducting sphere is 100V. Two statements are made in this regard S_1 at any point inside the sphere, electric intensity is zero. S_2 at any point inside the sphere, the electrostatic potential is 100 V. Which of the following is a correct statement? **[NCERT Exemplar]**

(a) S_1 is true but S_2 is false
(b) Both S_1 and S_2 are false
(c) S_1 is true, S_2 is also true and S_1 is the cause of S_2
(d) S_1 is true, S_2 is also true but the statements are independent

150. Two conducting spheres of radii 3 cm and 1 cm are separated by a distance of 10 cm in free space. If the spheres are charged to same potential of 10 V each, then the force of repulsion between them is

(a) $\left(\frac{1}{3}\right)\times 10^{-9}$ N (b) $\left(\frac{2}{9}\right)\times 10^{-9}$ N
(c) $\left(\frac{1}{9}\right)\times 10^{-9}$ N (d) $\left(\frac{4}{9}\right)\times 10^{-9}$ N
(e) $\left(\frac{2}{3}\right)\times 10^{-9}$ N

151. Three charges, each $+q$, are placed at the corners of an isosceles triangle ABC of sides BC and AC, $2a$. D and E are the mid-points of BC and CA. The work done in taking a charge Q from D to E is



(a) zero (b) $\frac{3qQ}{4\pi\varepsilon_0 a}$ (c) $\frac{3qQ}{8\pi\varepsilon_0 a}$ (d) $\frac{qQ}{4\pi\varepsilon_0 a}$

152. An electric charge 10^{-3} μC is placed at the origin (0, 0) of xy-coordinate system. Two points A and B are situated at $(\sqrt{2}, \sqrt{2})$ and (2, 0), respectively. The potential difference between the points A and B will be

(a) 9 V (b) zero (c) 2 V (d) 3.5 V

153. Two identical thin rings each of radius 10 cm carrying charges 10 C and 5 C are coaxially placed at a distance 10 cm apart. The work done in moving a charge q from the centre of the first ring to that of the second is

(a) $\frac{q}{8\pi\varepsilon_0}\cdot\left[\frac{\sqrt{2}+1}{\sqrt{2}}\right]$ (b) $\frac{q}{8\pi\varepsilon_0}\left[\frac{\sqrt{2}-1}{\sqrt{2}}\right]$
(c) $\frac{q}{4\pi\varepsilon_0}\cdot\left[\frac{\sqrt{2}+1}{\sqrt{2}}\right]$ (d) $\frac{q}{4\pi\varepsilon_0}\cdot\left[\frac{\sqrt{2}-1}{\sqrt{2}}\right]$

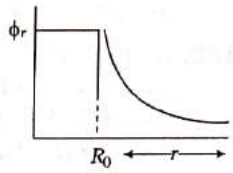
154. Two equal charges q of opposite sign separated by a distance $2a$ constitute an electric dipole of dipole moment p. If P is a point at a distance r from the centre of the dipole and the line joining the centre of the dipole to this point makes an angle θ with the axis of the dipole, then the potential at P is given by $(f >> 2a)$ (where $p = 2qa$).

(a) $V = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$ (b) $V = \frac{p\cos\theta}{4\pi\varepsilon_0 r}$
(c) $V = \frac{p\sin\theta}{4\pi\varepsilon_0 r}$ (d) $V = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$

155. The electrostatic potential ϕ_r of a spherical symmetrical system kept at origin, is shown in the adjacent figure, and given as

$$\phi_r = \frac{q}{4\pi\varepsilon_0 r}\ (r \geq R_0)$$

$$\phi_r = \frac{q}{4\pi\varepsilon_0 R_0}\ (r \leq R_0)$$



Which of the following option is incorrect?

(a) For spherical region $r \leq R_0$ total electrostatic energy stored is zero
(b) With in $r = 2R_0$, total charge is $q/2$
(c) There will be no charge anywhere except at $r = R$
(d) Electric field is discontinuous at $r = R_0$

156. An electric dipole of moment p placed in a uniform electric field E has minimum potential energy when the angle between p and E is

(a) zero (b) $\frac{\pi}{2}$ (c) π (d) $\frac{3\pi}{2}$

157. Electric charges of + 10 μC, + 5 μC, − 3 μC and + 8 μC are placed at the corners of a square of side $\sqrt{2}$ m, the potential at the centre of the square is

(a) 1.8 V (b) 1.8×10^6 V
(c) 1.8×10^5 V (d) 1.8×10^4 V

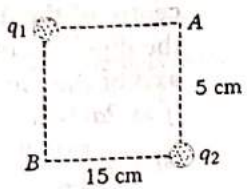
158. The displacement of a charge Q in the electric field $\mathbf{E} = e_1\hat{\mathbf{i}} + e_2\hat{\mathbf{j}} + e_3\hat{\mathbf{k}}$ is $\mathbf{r} = a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$. The work done is

(a) $Q(ae_1 + be_2)$
(b) $Q\sqrt{(ae_1)^2 + (be_2)^2}$
(c) $Q(e_1 + e_2)\sqrt{a^2 + b^2}$
(d) $Q(\sqrt{e_1^2 + e_2^2})(a + b)$

159. Two electric charges 12 μC and − 6 μC are placed 20 cm apart in air. There will be a point P on the line joining these charges and outside the region between them, at which the electric potential is zero. The distance of P from − 6 μC charge is

(a) 0.10 m (b) 0.15 m
(c) 0.20 m (d) 0.25 m

160. In the rectangle, shown below, the two corners have charges $q_1 = -5\,\mu C$ and $q_2 = +2.0\,\mu C$. The work done in moving a charge $+3.0\,\mu C$ from B to A is (take $1/4\,\pi\varepsilon_0 = 10^{10}$ N-m^2/C^2)



(a) 2.8 J (b) 3.5 J
(c) 4.5 J (d) 5.5 J

161. Electric potential at any point is $V = -5x + 3y + \sqrt{15}z$, then the magnitude of the electric field is

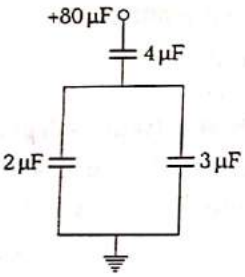
(a) $3\sqrt{2}$ (b) $4\sqrt{2}$ (c) $5\sqrt{2}$ (d) 7

162. A thin spherical conducting shell of radius R has a charge q. Another charge Q is placed at the centre of the shell. The electrostatic potential at a point P a distance $R/2$ from the centre of the shell is

(a) $\frac{(q+Q)}{4\pi\varepsilon_0}\frac{2}{R}$ (b) $\frac{2Q}{4\pi\varepsilon_0 R}$

(c) $\frac{2Q}{4\pi\varepsilon_0 R} - \frac{2q}{4\pi\varepsilon_0 R}$ (d) $\frac{2Q}{4\pi\varepsilon_0 R} + \frac{q}{4\pi\varepsilon_0 R}$

163. In the given circuit, a charge of $+80\,\mu C$ is given to the upper plate of the $4\,\mu F$ capacitor. Then, in the steady state, the charge on the upper plate of the $3\,\mu F$ capacitor is



(a) $+32\,\mu C$ (b) $+40\,\mu C$
(c) $+48\,\mu C$ (d) $+80\,\mu C$

164. Capacitance of a parallel plate capacitor becomes 4/3 times its original value, if a dielectric slab of thickness $t = d/2$ is inserted between the plates (d is the separation between the plates). The dielectric constant of the slab is

(a) 8 (b) 4
(c) 6 (d) 2

165. Point charge $q_1 = 2\,\mu C$ and $q_2 = -1\,\mu C$ are kept at points $x = 0$ and $x = 6$ respectively. Electrical potential will be zero at points

(a) $x = 2$ and $x = 9$
(b) $x = 1$ and $x = 5$
(c) $x = 4$ and $x = 12$
(d) $x = -2$ and $x = 2$

166. Eight small drops, each of radius r and having same charge q are combined to form a big drop. The ratio between the potentials of the bigger drop and the smaller drop is

(a) 8 : 1 (b) 4 : 1
(c) 2 : 1 (d) 1 : 8

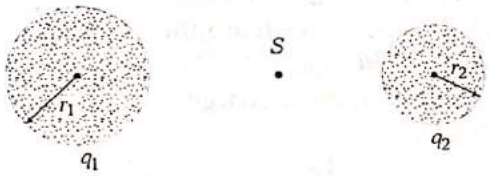
167. Eight oil drops of same size are charged to a potenti of 50 V each. These oil drops are merged into one single large drop. What will be the potential of the large drop?

(a) 50 V (b) 100 V
(c) 200 V (d) 400 V

168. Three charges Q, $(+q)$ and $(+q)$ are placed at the vertices of an equilateral triangle. If the net electrostatic energy of the system is zero, then Q is equal to

(a) $(-q/2)$ (b) $-q$
(c) $+q$ (d) zero

169. Consider a system composed of two metallic sphere of radii r_1 and r_2 connected by a thin wire and switc S as shown in the figure. Initially S is in open position, and the spheres carry charges q_1 and q_2 respectively. If the switch is closed, the potential of the system is



(a) $\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_1 r_2}$ (b) $\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1+q_2}{r_1+r_2}\right)$

(c) $\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1}+\frac{q_2}{r_2}\right)$ (d) $\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1+q_2}{\sqrt{r_1 r_2}}\right)$

170. Separation between the plates of a parallel plate capacitor is d and the area of each plate is A. Wher a slab of material of dielectric constant K and thickness $t (t < d)$ is introduced between the plates, capacitance becomes

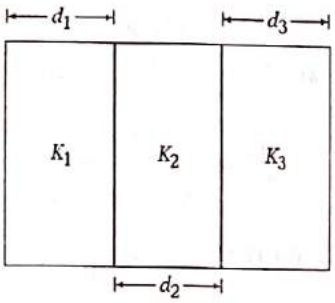
(a) $\frac{\varepsilon_0 A}{d + t\left(1 - \frac{1}{K}\right)}$ (b) $\frac{\varepsilon_0 A}{d + t\left(1 + \frac{1}{K}\right)}$

(c) $\frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{K}\right)}$ (d) $\frac{\varepsilon_0 A}{d - t\left(1 + \frac{1}{K}\right)}$

171. The distance between the circular plates of a parall plate condenser 40 mm in diameter, in order to hav same capacity as a sphere of radius 1 m is

(a) 0.01 mm (b) 0.1 mm
(c) 1.0 mm (d) 10 mm

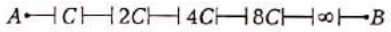
172. The expression for the capacity of the capacitor formed by compound dielectric placed between the plates of a parallel plate capacitor as shown in figure, will be (area of plate = A)



(a) $\dfrac{\varepsilon_0 A}{\left(\dfrac{d_1}{K_1}+\dfrac{d_2}{K_2}+\dfrac{d_3}{K_3}\right)}$ (b) $\dfrac{\varepsilon_0 A}{\left(\dfrac{d_1+d_2+d_3}{K_1+K_2+K_3}\right)}$

(c) $\dfrac{\varepsilon_0 A(K_1K_2K_3)}{d_1d_2d_3}$ (d) $\varepsilon_0\left(\dfrac{AK_1}{d_1}+\dfrac{AK_2}{d_2}+\dfrac{AK_3}{d_3}\right)$

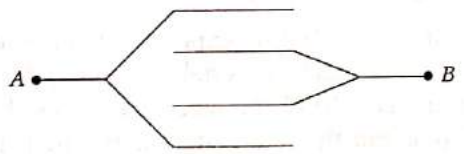
173. The equivalent capacitance between A and B will be



(a) $2C$ (b) $\dfrac{C}{2}$

(c) $3C$ (d) $\dfrac{2}{C}$

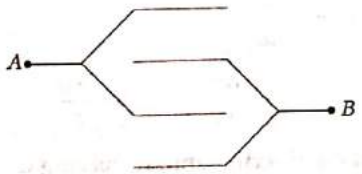
174. Four plates of equal area A are separated by equal distances d and are arranged as shown in the figure. The equivalent capacity is



(a) $\dfrac{2\varepsilon_0 A}{d}$ (b) $\dfrac{3\varepsilon_0 A}{d}$

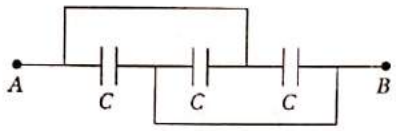
(c) $\dfrac{4\varepsilon_0 A}{d}$ (d) $\dfrac{\varepsilon_0 A}{d}$

175. Four plates of the same area of cross-section are joined as shown in the figure. The distance between each plate is d. The equivalent capacity across A and B will be



(a) $\dfrac{2\varepsilon_0 A}{d}$ (b) $\dfrac{3\varepsilon_0 A}{d}$ (c) $\dfrac{3\varepsilon_0 A}{2d}$ (d) $\dfrac{\varepsilon_0 A}{d}$

176. Three equal capacitors, each with capacitance C are connected as shown in figure. Then the equivalent capacitance between point A and B is

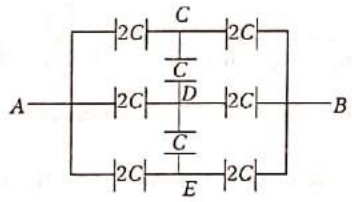


(a) C (b) $3C$ (c) $\dfrac{C}{3}$ (d) $\dfrac{3C}{2}$

177. If a slab of insulating material 4×10^{-3} m thick is introduced between the plates of a parallel plate capacitor, the separation between plates has to be increased by 3.5×10^{-3} m to restore the capacity to original value. The dielectric constant of the material will be

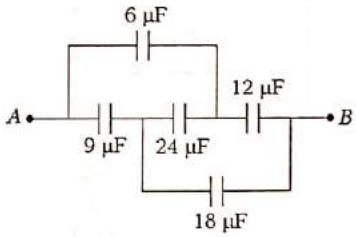
(a) 6 (b) 8 (c) 10 (d) 12

178. In this figure the equivalent capacitance between A and B will be



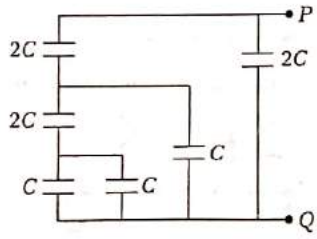
(a) $\dfrac{C}{2}$ (b) $\dfrac{C}{3}$ (c) $3C$ (d) $2C$

179. In the connections shown in the adjoining figure, the equivalent capacity between points A and B will be



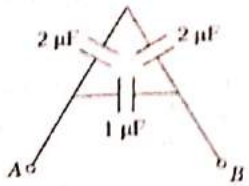
(a) 10.8 μF (b) 69 μF (c) 15 μF (d) 10 μF

180. The resultant capacitance of given circuit between points P and Q is



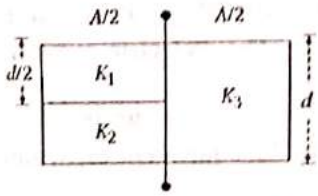
(a) $3C$ (b) $2C$ (c) C (d) $\dfrac{C}{3}$

181. What is the effective capacitance between points A and B in the given figure?



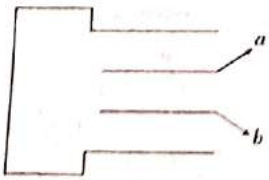
(a) 1 μF (b) 2 μF
(c) 1.5 μF (d) 2.5 μF

182. In the figure a capacitor is filled with dielectrics. The resultant capacitance is



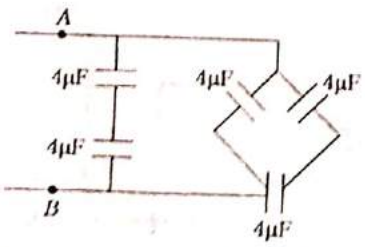
(a) $\frac{2\varepsilon_0 A}{d}\left[\frac{1}{K_1}+\frac{1}{K_2}+\frac{1}{K_3}\right]$ (b) $\frac{\varepsilon_0 A}{d}\left[\frac{1}{K_1}+\frac{1}{K_2}+\frac{1}{K_3}\right]$
(c) $\frac{2\varepsilon_0 A}{d}[K_1+K_2+K_3]$ (d) None of these

183. Four metallic plates each with a surface area of one side A are placed at a distance d from each other. The plates are connected as shown in the circuit diagram. Then the capacitance of the system between points a and b is



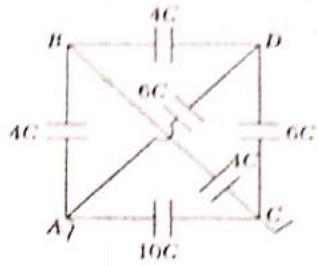
(a) $\frac{3\varepsilon_0 A}{d}$ (b) $\frac{2\varepsilon_0 A}{d}$
(c) $\frac{2\varepsilon_0 A}{3d}$ (d) $\frac{3\varepsilon_0 A}{2d}$

184. The equivalent capacitance between points A and B in the circuit will be



(a) 10/3 μF (b) 4 μF
(c) 6 μF (d) 8 μF

185. The equivalent capacitance between the points A and C is given by



(a) $\frac{10}{3}C$ (b) $15C$ (c) $\frac{3}{10}C$ (d) $20C$

186. Potential difference between two points $(V_A - V_B)$ in an electric field $\mathbf{E} = (2\hat{\mathbf{i}} - 4\hat{\mathbf{j}})$ N/C, where $A = (0, 0)$ and $B = (3\text{ m}, 4\text{ m})$ is

(a) 10 V (b) −10 V (c) 16 V (d) −16 V

187. A and B are two thin concentric hollow conductors having radii a and $2a$ and charge $2Q$ and Q, respectively. If potential of outer sphere is 5 V, then potential of inner sphere is

(a) 20 V (b) 10 V (c) $\frac{25}{3}$ V (d) $\frac{50}{3}$ V

188. A spherical conductor of radius 2 m is charged to a potential of 120 V. It is now placed inside another hollow spherical conductor of radius 6 m. Calculate the potential of bigger sphere, if the smaller sphere is made to touch the bigger sphere

(a) 120 V (b) 60 V (c) 80 V (d) 40 V

189. In Millikan's oil drop experiment, an oil drop carrying a charge Q is held stationary by a potential difference 2400 V between the plates. To keep a drop of half the radius stationary, the potential difference had to be made 600 V. What is the charge on the second drop

(a) $\frac{Q}{4}$ (b) $\frac{Q}{2}$ (c) Q (d) $\frac{3Q}{2}$

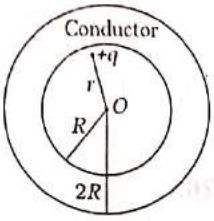
190. There are four concentric shells A, B, C and D of radii a, $2a$, $3a$ and $4a$, respectively. Shells B and D are given charges $+q$ and $-q$, respectively. Shell C is now earthed. The potential difference $V_A - V_C$ is $\left(\text{take, } K = \frac{1}{4\pi\varepsilon_0}\right)$

(a) $\frac{Kq}{2a}$ (b) $\frac{Kq}{3a}$ (c) $\frac{Kq}{4a}$ (d) $\frac{Kq}{6a}$

191. A solid conducting sphere having a charge Q is surrounded by an uncharged concentric conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of the hollow shell be V.

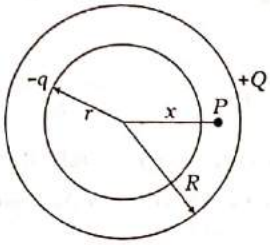
If the shell is now given a charge of $-3Q$, then the new potential difference between the same two surface is
(a) V (b) $2V$ (c) $4V$ (d) $-2V$

192. A point charge q is placed at a distance r from the centre O of an uncharged spherical shell of inner radius R and outer radius $2R$. The distance $r < R$. The electric potential at the centre of the shell will be



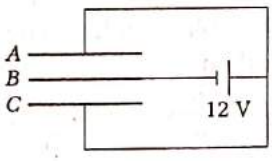
(a) $\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r}-\frac{1}{2R}\right)$ (b) $\frac{q}{4\pi\varepsilon_0 r}$
(c) $\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r}+\frac{1}{2R}\right)$ (d) $\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r}-\frac{1}{R}\right)$

193. A hollow sphere of radius r is put inside another hollow sphere of radius R. The charges on the two are $+Q$ and $-q$ as shown in the figure. A point P is located at a distance x from the common centre such that $r < x < R$. The potential at the point P is



(a) $\frac{1}{4\pi\varepsilon_0}\left(\frac{Q-q}{x}\right)$ (b) $\frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R}-\frac{q}{r}\right)$
(c) $\frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R}-\frac{q}{x}\right)$ (d) $\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r}-\frac{Q}{x}\right)$

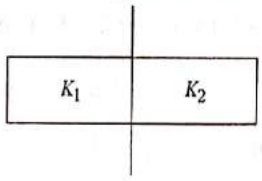
194. Three plates A, B, C each of area 50 cm^2 have separation 3 mm between A and B and 3 mm between B and C. The energy stored when the plates are fully charged is



(a) 1.6×10^{-9} J (b) 2.1×10^{-9} J
(c) 5×10^{-9} J (d) 7×10^{-9} J

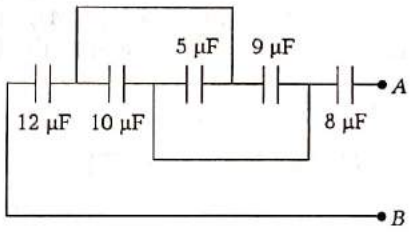
195. A parallel plate capacitor with air as medium between the plates has a capacitance of 10 μF. The area of capacitor is divided into two equal halves and filled with two media as shown in the figure having dielectric constant $K_1 = 2$ and $K_2 = 4$.

The capacitance of the system will now be



(a) 10 μF (b) 20 μF (c) 30 μF (d) 40 μF

196. The capacities and connection of five capacitors are shown in the adjoining figure. The potential difference between the points A and B is 60 V. Then, the equivalent capacity between A and B and the charge on 5 μF capacitance will be respectively



(a) 44 μF, 300 μC (b) 16 μF, 150 μC
(c) 15 μF, 200 μC (d) 4 μF, 50 μC

197. A charge $+Q$ is uniformly distributed over a thin ring of the radius R. The velocity of an electron at the moment when it passes through the centre O of the ring, if the electron was initially at far away on the axis of the ring is (m = mass of electron, $K = \frac{1}{4\pi\varepsilon_0}$)

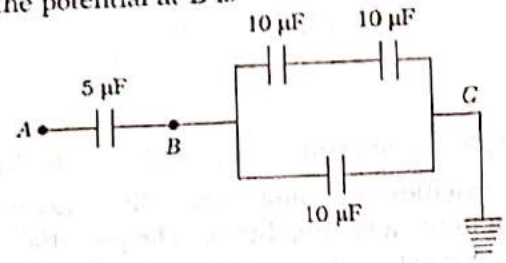
(a) $\sqrt{\left(\frac{2KQe}{mR}\right)}$ (b) $\sqrt{\left(\frac{KQe}{m}\right)}$ (c) $\sqrt{\left(\frac{Kme}{QR}\right)}$ (d) $\sqrt{\left(\frac{KQe}{mR}\right)}$

198. A parallel plate capacitor of capacitance C is connected to a battery and is charged to a potential difference V. Another capacitor of capacitance $2C$ is connected to another battery and is charged to potential difference $2V$. The charging batteries are now disconnected and the capacitors are connected in parallel to each other in such a way that the positive terminal of one is connected to the negative terminal of the other. The final energy of the configuration is

(a) zero (b) $\frac{25CV^2}{6}$ (c) $\frac{3CV^2}{2}$ (d) $\frac{9CV^2}{2}$

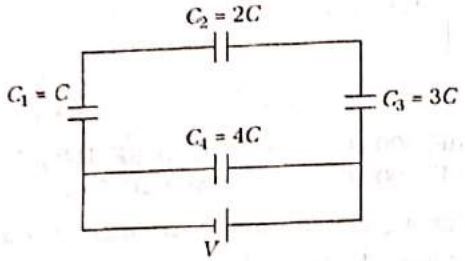
199. Condenser A has a capacity of 15 μF when it is filled with a medium of dielectric constant 15. Another condenser B has a capacity of 1 μF with air between the plates. Both are charged separately by a battery of 100 V. After charging, both are connected in parallel without the battery and the dielectric medium being removed. The common potential now is
(a) 400 V (b) 800 V (c) 1200 V (d) 1600 V

200. In the given circuit if point C is connected to the earth and a potential of +2000 V is given to the point A, the potential at B is



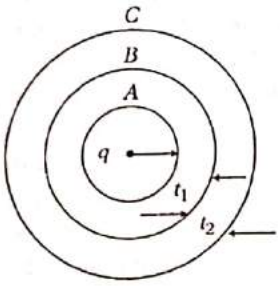
(a) 1500 V (b) 1000 V (c) 500 V (d) 400 V

201. A network of four capacitors of capacity equal to $C_1 = C, C_2 = 2C, C_3 = 3C$ and $C_4 = 4C$ are connected to in a battery as shown in the figure. The ratio of the charges on C_2 and C_4 is



(a) $\frac{22}{3}$ (b) $\frac{3}{22}$
(c) $\frac{7}{4}$ (d) $\frac{4}{7}$

202. Figure shows three spherical and equipotential surfaces A, B and C round a point charge q. The potential difference $V_A - V_B = V_B - V_C$. If t_1 and t_2 be the distances between them, then

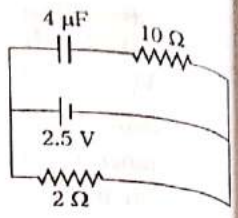


(a) $t_1 = t_1$ (b) $t_1 > t_2$
(c) $t_1 < t_2$ (d) $t_1 \le t_2$

203. A charged oil drop of mass 2.5×10^{-7} kg is in space between the two plates, each of area 2×10^{-2} m^2 of a parallel plate capacitor. When the upper plate has a charge of 5×10^{-7} C and the lower plate has an equal negative charge, then the oil remains stationary. The charge of the oil drop is (take, $g = 10$ m/s^2)

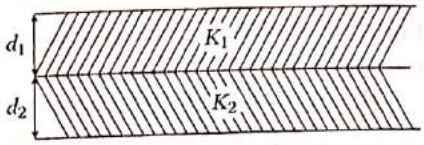
(a) 9×10^{-1} C (b) 9×10^{-6} C
(c) 8.85×10^{-13} C (d) 1.8×10^{-14} C

204. A capacitor of 4 μF is connected as shown in the circuit. The internal resistance of the battery is 0.5 Ω. The amount of charge on the capacitor plates will be **[NCERT Exemplar]**



(a) 0 (b) 4 μ C (c) 16 μC (d) 8 μC

205. A parallel plate capacitor is made of two dielectric blocks in series. One of the blocks has thickness d_1 and dielectric constant K_1 and the other has thickness d_2 and dielectric constant K_2 as shown in figure. This arrangement can be thought as a dielectric slab of thickness d $(= d_1 + d_2)$ and effective dielectric constant K. The K is **[NCERT Exemplar]**



(a) $\frac{K_1 d_1 + K_2 d_2}{d_1 + d_2}$ (b) $\frac{K_1 d_1 + K_2 d_2}{K_1 + K_2}$
(c) $\frac{K_1 K_2 (d_1 + d_2)}{(K_1 d_2 + K_2 d_1)}$ (d) $\frac{2K_1 K_2}{K_1 + K_2}$

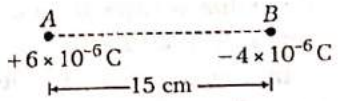
206. A number of condensers, each of the capacitance 1 μF and each one of which gets punctured if a potential difference just exceeding 500 V is applied, are provided. An arrangement suitable for giving capacitance of 2 μF across which 3000 V may be applied requires at least

(a) 6 component capacitors (b) 12 component capacitors
(c) 72 component capacitors (d) 2 component capacitors

207. A series combination of n_1 capacitors, each of value C_1, is charged by a source of potential difference 4V. When another parallel combination of n_2 capacitors, each of value C_2, is charged by a source of potential difference V, it has the same (total) energy stored in it, as the first combination has. The value of C_2, in terms of C_1 is, then

(a) $\frac{2C_1}{n_1 n_2}$ (b) $16\frac{n_1}{n_2}C_1$ (c) $2\frac{n_2}{n_1}C_1$ (d) $\frac{16C_1}{n_1 n_2}$

208. Two charges + 6 μC and − 4 μC are placed 15 cm apart as shown. At what distances from A to its right, the electrostatic potential is zero (distances in cm)



(a) 4, 9, 60 (b) 9, 45, infinity
(c) 20, 45, infinity (d) 9, 15, 45

209. Assume that an electric field $\mathbf{E} = 30x^2\hat{\mathbf{i}}$ exists in space. Then, the potential difference $V_A - V_O$, where V_O is the potential at the origin and V_A is the potential at $x = 2$ m is

(a) 120 V (b) – 120 V (c) – 80 V (d) 80 V

210. An electron initially at rest falls a distance of 1.5 cm in a uniform electric field of magnitude 2×10^4 N/C. The time taken by the electron to fall this distance is

(a) 1.3×10^2 s (b) 2.1×10^{-2} s

(c) 1.6×10^{-10} s (d) 2.9×10^{-9} s

211. A long, hollow conducting cylinder is kept coaxially inside another long, hollow conducting cylinder of larger radius. Both the cylinders are initially electrically neutral.

(a) A potential difference appears between the two cylinders when a charge density is given to the inner cylinder

(b) A potential difference appears between the two cylinders when a charge density is given to the outer cylinder

(c) No potential difference appears betwen the two cylinders when a uniform line charge is kept along the axis of the cylinders

(d) No potential difference appears between the two cylinders when same charge density is given to both the cylinders

212. The potential at a point x (measured in μm) due to some charges situated on the X-axis is given by $V(x) = \dfrac{20}{(x^2 - 4)}$ volts. The electric field E at $x = 4$ μm is given by

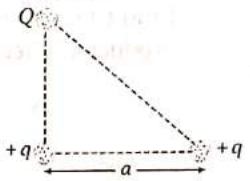
(a) 5/3 volt/μm and in the negative x-direction

(b) 5/3 volt/μm and in the positive x-direction

(c) 10/9 volt/μm and in the negative x-direction

(d) 10/9 volt/μm and in the positive x-direction

213. Three charge Q, $+q$ and $+q$ are placed at the vertices of a right angled isosceles triangle as shown. The net electrostatic energy of the configuration is zero, if Q is equal to



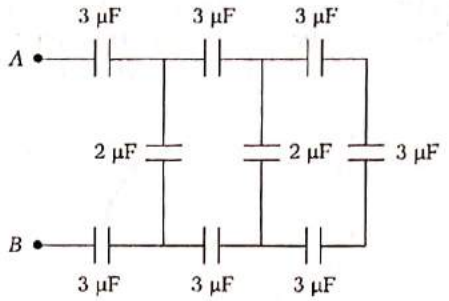
(a) $\dfrac{-q}{1+\sqrt{2}}$ (b) $\dfrac{-2q}{2+\sqrt{2}}$ (c) $-2q$ (d) $+q$

214. Four point charges each $+q$ is placed on the circumference of a circle of diameter $2d$ in such a way that they form a square. The potential at the centre is

(a) 0 (b) $\dfrac{4q}{d}$

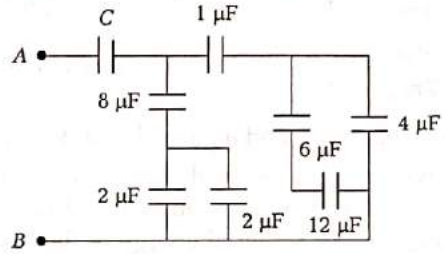
(c) $\dfrac{4d}{q}$ (d) $\dfrac{q}{4d}$

215. The resultant capacitance between A and B in the following figure is equal to



(a) 1 μF (b) 3 μF (c) 2 μF (d) 1.5 μF

216. In the following circuit, the resultant capacitance between A and B is 1 μF. The value of C is



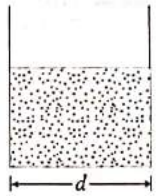
(a) $\dfrac{32}{11}$ μF (b) $\dfrac{11}{32}$ μF (c) $\dfrac{23}{32}$ μF (d) $\dfrac{32}{23}$ μF

217. A small conducting sphere of radius r is lying concentrically inside a bigger hollow conducting sphere of radius R. The bigger and smaller spheres are charged with Q and q $(Q > q)$ and are insulated from each other. The potential difference between the spheres will be

(a) $\dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{q}{r} - \dfrac{Q}{R}\right)$ (b) $\dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{Q}{R} + \dfrac{q}{r}\right)$

(c) $\dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{q}{r} - \dfrac{q}{R}\right)$ (d) $\dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{q}{R} - \dfrac{Q}{r}\right)$

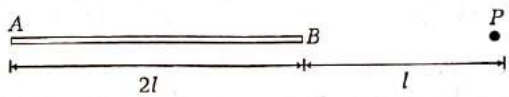
218. A parallel plate air capacitor has a capacitance C. When it is half filled with a dielectric of dielectric constant 5, then the percentage increase in the capacitance will be



(a) 400% (b) 66.6%

(c) 33.3% (d) 200%

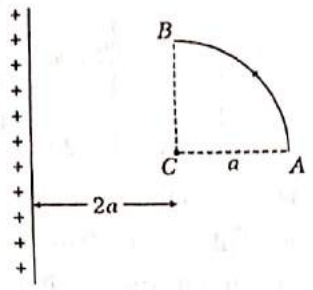
219. Charge Q is uniformly distributed on a dielectric rod AB of length $2l$. The potential at P shown in the figure is equal to



(a) $\dfrac{Q}{4\pi\varepsilon_0(2l)}$ (b) $\dfrac{Q}{4\pi\varepsilon_0(l)}\ln 2$

(c) $\dfrac{Q}{4\pi\varepsilon_0(2l)}\ln 3$ (d) None of these

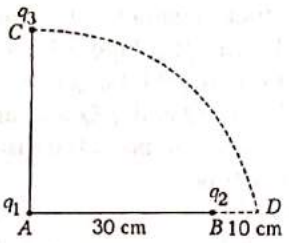
220. The arc AB with the centre C and the infinitely long wire having linear charge density λ are lying in the same plane. The minimum amount of work to be done to move a point charge q_0 from point A to B through a circular path AB of radius a is equal to



(a) $\frac{q_0\lambda}{4\pi\varepsilon_0}\ln\left(\frac{2}{3}\right)$ (b) $\frac{q_0\lambda}{2\pi\varepsilon_0}\ln\left(\frac{3}{2}\right)$

(c) $\frac{q_0\lambda}{2\pi\varepsilon_0}\ln\left(\frac{2}{3}\right)$ (d) None of these

221. Two charges q_1 and q_2 are placed 30 cm apart, as shown in figure. A third charge q_3 is moved along the arc of a circle of radius 40 cm from C to D. The change in the potential energy of the system is $\frac{q_3}{4\pi\varepsilon_0}K$, where K is



(a) $8q_2$ (b) $8q_1$

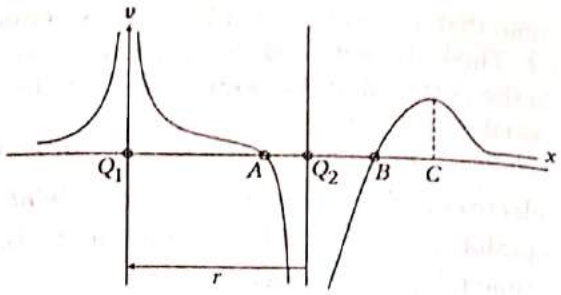
(c) $6q_2$ (d) $6q_1$

222. Three identical metallic uncharged spheres A, B and C of radius a are kept on the corners of an equilateral triangle of side $d(d >> a)$. A fourth sphere (radius a) which has charge Q touches A and is then removed to a position for away. B is earthed and then the earth connection is removed. C is then earthed. The charge on C is

(a) $\frac{Qa}{2d}\left(\frac{2d-a}{2d}\right)$ (b) $\frac{Qa}{2d}\left(\frac{2d-a}{d}\right)$

(c) $\frac{Qa}{2d}\left(\frac{a-d}{d}\right)$ (d) $\frac{2Qa}{d}\left(\frac{d-a}{2d}\right)$

223. The curve represents the distribution of potential along the straight line joining the two charges Q_1 and Q_2 (separated by a distance r), then which is of the following statements are correct?



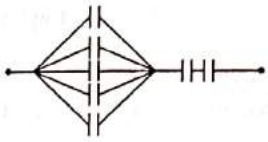
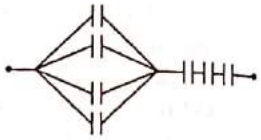
1. $|Q_1| > |Q_2|$
2. Q_1 is positive in nature
3. A and B are equilibrium points
4. C is a point of unstable equilibrium

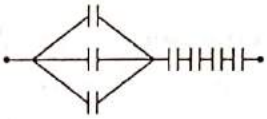
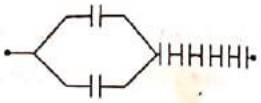
Which of the above statements are correct?

(a) 1 and 2 (b) 1, 2 and 3

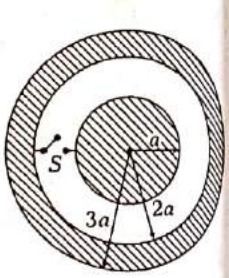
(c) 1, 2 and 4 (d) 1, 2, 3 and 4

224. Seven capacitors each of capacity 2 μF are to be so connected to have a total capacity (10/11) μF. Which will be the necessary figure as shown?

(a)  (b) 

(c)  (d) 

225. A solid conducting sphere of radius a having a charge q is surrounded by a concentric conducting spherical shell of inner radius $2a$ and outer radius $3a$ as shown in figure. Find the amount of heat produced when switch is closed



$$K = \frac{1}{4\pi\varepsilon_0}$$

(a) $\frac{Kq^2}{2a}$ (b) $\frac{Kq^2}{3a}$

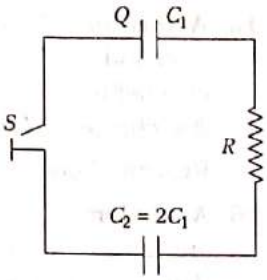
(c) $\frac{Kq^2}{4a}$ (d) $\frac{Kq^2}{6a}$

226. Three identical charges are placed at corners of an equilateral triangle of side l. If force between any two charges is F, then the work required to double the dimensions of triangle is

(a) $-3Fl$ (b) $3Fl$

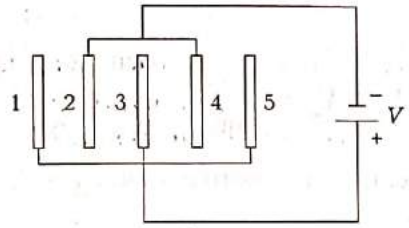
(c) $\left(-\frac{3}{2}\right)Fl$ (d) $\left(\frac{3}{2}\right)Fl$

227. Two capacitors C_1 and $C_2 = 2C_1$ are connected in a circuit with a switch between them as shown in the figure. Initially the switch is open and C_1 holds charge Q. The switch is closed. At steady state, the charge on each capacitor will be



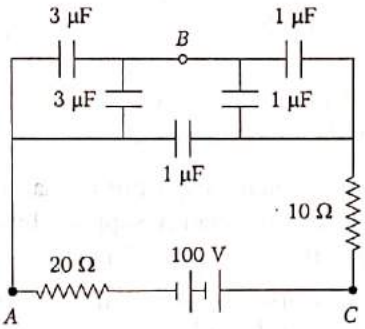
(a) $Q, 2Q$ (b) $Q/3, 2Q/3$
(c) $3Q/2, 3Q$ (d) $2Q/3, 4Q/3$

228. Five identical plates each of area A are joined as shown in the figure. The distance between the plates is d. The plates are connected to a potential difference of V volt. The charge on plates 1 and 4 will be



(a) $-\frac{\varepsilon_0 AV}{d}, \frac{2\varepsilon_0 AV}{d}$ (b) $\frac{\varepsilon_0 AV}{d}, \frac{2\varepsilon_0 AV}{d}$
(c) $\frac{\varepsilon_0 AV}{d}, \frac{-2\varepsilon_0 AV}{d}$ (d) $\frac{-\varepsilon_0 AV}{d}, \frac{-2\varepsilon_0 AV}{d}$

229. In the figure shown, what is the potential difference between the points A and B and between B and C respectively in steady state?



(a) $V_{AB} = V_{BC} = 100$ V
(b) $V_{AB} = 75$ V, $V_{BC} = 25$ V
(c) $V_{AB} = 25$ V, $V_{BC} = 75$ V
(d) $V_{AB} = V_{BC} = 50$ V

B. Medical entrance special format questions

Assertion and reason

Directions (Q. Nos. 1-16) *These questions consist of two statements each linked as Assertion and Reason. While answering these questions you are required to choose any one of the following five responses*

(a) If both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are true but Reason is not correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If Assertion is false but Reason is true.
(e) If both Assertion and Reason are false.

1. **Assertion** Due to two point charges electric field and electric potential can't be zero at some point simultaneously.

   Reason Field is a vector quantity and potential a scalar quantity.

2. **Assertion** Conductor having equal positive charge and volume, must also have same potential.

   Reason Potential depends only an charge and volume of conductor.

3. **Assertion** Circuit containing capacitors should be handled cautiously even when there is no current.

   Reason The capacitors are very delicate and so quickly breakdown.

4. **Assertion** When two positive point charges move away from each other, then their electrostatic potential energy decreases.

   Reason Change in potential energy between two points is equal to the work done by electrostatic forces.

5. **Assertion** When two charged spheres are connected to each other by a thin conducting wire, charge flow from bigger sphere to smaller sphere, if initial charges on them are same.

   Reason Electrostatic potential energy will be lost in redistribution of charges.

6. **Assertion** For a non-uniformly charged thin circular ring with net charge zero, the electric field at any point on axis of the ring is zero.

   Reason For a non-uniformly charged thin circular ring with net charge zero, the electric potential at each point on axis of the ring is zero.

7. Assertion Five charges $+q$ each are placed at five vertices of a regular pentagon. A sixth charge $-Q$ is placed at the centre of pentagon. Then, net electrostatic force on $-Q$ is zero.

Reason Net electrostatic potential at the centre is zero.

8. Assertion A charged capacitor is disconnected from a battery. Now, if its plate are separated further, the potential energy will fall.

Reason Energy stored in a capacitor is equal to the work done in charging it.

9. Assertion When a capacitor is charged by a battery, half of the energy supplied by the battery is stored in the capacitor and rest half is lost.

Reason If resistance in the circuit is zero, then there will be no loss of energy.

10. Assertion When a metallic plate is partially inserted between the plates of a capacitor its capacity increases.

Reason If conductivity of conducting plate is more, then increase in capacity will be more.

11. Assertion If the distance between parallel plates of a capacitor is halved and dielectric constant is made three times, then the capacitance becomes 6 times.

Reason Capacity of the capacitor depends upon the nature of the material between the plates.

12. Assertion Spherical shell A in the shown figure has a charge $-q$ and shell B has charge $+q$. When these two are connected by a thin conducting wire, then whole of the electrostatic potential energy stored between the shell is lost.

Reason Whole of the inner charge will transfer to outer shell B.

13. Assertion When a dielectric is placed in an electric field, the electric field with in the dielectric as well as. Potential difference across the capacitor plates are reduced by a factor K (battery is not connected).

Reason The dielectric constant is the ratio of the absolute permittivity of the dielectric to the permittivity of the free space.

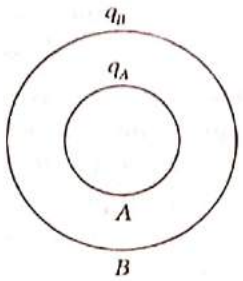
14. Assertion No work is done in moving a charge along equatorial line.

Reason The electric potential is everywhere zero on the equatorial line of a dipole.

15. Assertion A and B are two conducting spheres of same radius. A being solid and B hollow. Both are charged to the same potential. Then, charge on A = charge on B.

Reason Potentials on both are same.

16. Assertion Two concentric spherical shells A and B are charged by charges q_A and q_B. If q_A is negative, then $V_A - V_B$ is also negative.



Reason $V_A - V_B = \left(\frac{q_A}{4\pi\varepsilon_0}\right)\left(\frac{1}{R_A} - \frac{1}{r_B}\right)$

Therefore, if q_A is negative, then $V_A - V_B$ is also negative.

Match the columns

1. Two charges $+q$ and $-q$ are placed at distance r. Match the following two columns when distance between them is changed to r'.

Column I		Column II
A. $r' = 2r$	(p)	Potential energy will become half
B. $r' = \frac{r}{2}$	(q)	Force between them will become $\frac{1}{4}$th
C. $r' = 4r$	(r)	Potential energy will become four times
D. $r' = \frac{r}{4}$	(s)	None

2. A capacitor is connected with a battery. With battery remains connected some changes are done in capacitor/battery, which are given in Column I. Corresponding to it match the two columns.

Column I		Column II
A. Distance between capacitor plates is halved	(p)	Capacity of capacitor will become two times
B. A metallic slab is half filled between the plates	(q)	Charge stored in capacitor will become two times
C. A dielectric slab of dielectric constant $K = 2$ is completely filled between the plates	(r)	Energy stored in capacitor will become two times
	(s)	Capacitance become infinite

3. A parallel plate capacitor is charged by a battery which is then disconnected. A dielectric slab is then inserted to fill the space between the plates. Match the changes that could occur with Column II.

Column I	Column II
A. Charge on the capacitor plates	(p) Decrease by a factor of K
B. Intensity of electric field	(q) Increase by a factor of K
C. Energy stored	(r) Remains same
D. Capacitance	(s) None

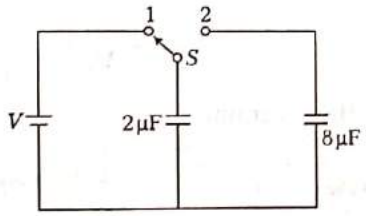
4. The area of parallel plates of an air-filled capacitor is $0.20\ m^2$ and the distance between them is 0.01 m. The potential difference across the plates is 3000 V. When a 0.01 m thick dielectric sheet is introduced between the plates, then the potential difference decreases to 1000 V. Now, match the two columns (all in SI units).

Column I	Column II
A. Capacitance of air-filled capacitor	(p) 5.31×10^{-7}
B. Charge on each plate	(q) 5.31×10^{-10}
C. Dielectric constant of the material	(r) 3
D. Capacitance after the dielectric sheet is introduced	(s) 1.77×10^{-10}

C. Medical entrances' gallery

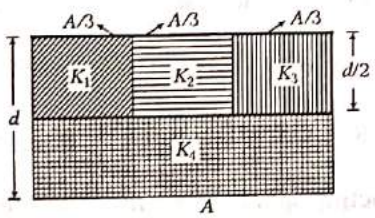
(Collection of questions asked in NEET & various medical entrance exams)

1. A capacitor of 2μF is charged as shown in the figure. When the switch S is turned to position 2, the percentage of its stored energy dissipated is **[NEET 2016]**



(a) 20% (b) 75% (c) 80% (d) 0%

2. A parallel plate capacitor of area A, plate separation d and capacitance C is filled with four dielectric materials having dielectric constants K_1, K_2, K_3 and K_4 as shown in the figure below. If a single dielectric material is to be used to have the same capacitance C in this capacitor, then its dielectric constant K is given by **[NEET 2016]**



(a) $K = K_1 + K_2 + K_3 + 3K_4$

(b) $K = \frac{2}{3}(K_1 + K_2 + K_3) + 2K_4$

(c) $\frac{2}{K} = \frac{3}{K_1 + K_2 + K_3} + \frac{1}{K_4}$

(d) $\frac{1}{K} = \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} + \frac{3}{2K_4}$

3. A parallel plate air capacitor of capacitance C is connected to a cell of emf V and then disconnected from it. A dielectric slab of dielectric constant K, which can just fill the air gap of the capacitor, is now inserted in it. Which of the following is incorrect? **[CBSE AIPMT 2015]**

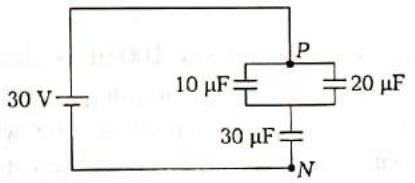
(a) The potential difference between the plates decreases K times

(b) The energy stored in the capacitor decreases K times

(c) The change in energy stored is $\frac{1}{2}CV^2\left(\frac{1}{K}-1\right)$

(d) The charge on the capacitor is not conserved

4. Calculate the charge on equivalent capacitance of the combination shown in figure between the points P and N. **[UK PMT 2015]**



(a) 450 μC (b) 225 μC (c) 350 μC (d) 900 μC

5. An isolated sphere has a capacitance of 50 pF. What would be the radius of the sphere? **[UK PMT 2015]**

(a) 90 cm (b) 45 cm
(c) 11.50 cm (d) 5.75 cm

6. A parallel plate capacitor has 91 plates, all are identical and arranged with same spacing between them. If the capacitance between adjacent plates is 3 pF. What will be the resultant capacitance? **[EAMCET 2015]**

(a) 273 pF (b) 30 pF
(c) 94 pF (d) 270 pF

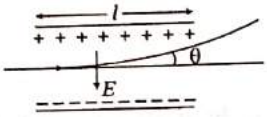
7. A particle of mass 1.96×10^{-15} kg is kept in equilibrium between two horizontal metal plates having potential difference of 400 V separated apart by 0.02 m. Then, the charge on the particle is (e = electronic charge) **[Kerala CEE 2015]**

(a) $3e$ (b) $6e$ (c) $2e$ (d) $5e$
(e) $4e$

8. The distance of the closest approach of an alpha particle fired at a nucleus with kinetic energy K is r_0. The distance of the closest approach when the α particle is fired at the same nucleus with kinetic energy $2K$ will be **[Guj. CET 2015]**

(a) $4r_0$ (b) $\frac{r_0}{2}$ (c) $\frac{r_0}{4}$ (d) $2r_0$

9. A uniform electric field is created between two parallel charged plates as shown below. An electron enters the field symmetrically between the plates with a speed of v_0. The length of each plate is l. Find the angle of deviation of path of the electron as it comes out of the field. **[CG PMT 2015]**



(a) $\theta = \tan^{-1}\frac{El}{mv_0^2}$ (b) $\theta = \tan^{-1}\left(\frac{eEl}{mv_0^2}\right)$

(c) $\theta = \tan^{-1}\left(\frac{eEl}{mv_0}\right)$ (d) $\theta = \tan^{-1}\left(\frac{eE}{mv_0^2}\right)$

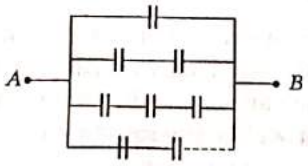
10. A battery charges a parallel plate capacitor separated by distance (d) so that the energy (V_0) is stored in the system. A slab of dielectric constant (K) and thickness (d) is then introduced between the plates of capacitor. The new energy of the system is given by **[CG PMT 2015]**

(a) KV_0 (b) K^2V_0 (c) $\frac{V_0}{K}$ (d) $\frac{V_0}{K^2}$

11. A capacitor of capacitance 100 μF is charged by connecting it to a battery of voltage 12 V with internal resistance 2 Ω. The time after which 99% of the maximum charge is stored on the capacitor is **[UP CPMT 2015]**

(a) 0.92 ms (b) 0.72 ms (c) 0.34 ms (d) 0.54 ms

12. An infinite number of identical capacitors each of capacitance 1 μF are connected as in adjoint figure. Then, the equivalent capacitance between A and B is **[UP CPMT 2015]**



(a) 1 μF (b) 2 μF (c) $\frac{1}{2}$ μF (d) ∞

13. A parallel plate capacitor is charged and then isolated. The effect of increasing the plate separation on charge, potential and capacitance respectively are **[KCET 2015]**

(a) constant, decreases, increases
(b) constant, decreases, decreases
(c) constant, increases, decreases
(d) increases, decreases, decreases

14. A spherical shell of radius 10 cm is carrying a charge q. If the electric potential at distances 5 cm, 10 cm and 15 cm from the centre of the spherical shell is V_1, V_2 and V_3 respectively, then **[KCET 2015]**

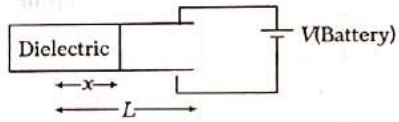
(a) $V_1 = V_2 > V_3$ (b) $V_1 > V_2 > V_3$
(c) $V_1 = V_2 < V_3$ (d) $V_1 < V_2 < V_3$

15. An electron of mass M_e, initially at rest, moves through a certain distance in a uniform electric field in time t_1. A proton of mass M_p also intially at rest, takes time t_2 to move through an equal distance in this uniform electric field. Neglecting the effect of gravity, the ratio t_2 / t_1 is nearly equal to **[AIIMS 2015]**

(a) 1 (b) $\sqrt{\frac{M_p}{M_e}}$

(c) $\sqrt{\frac{M_e}{M_p}}$ (d) $\frac{M_e}{M_p}$

16. Consider the diagram,



A parallel plate capacitor has the plate width t and length L while the separation between the plates is d. The capacitor is connected to a battery of voltage rating V. A dielectric which carefully occupy, the space between the plates of the capacitor is slowly inserted between the plates. When length x of the dielectric slab is introduced into the capacitor, then energy stored in the system is **[UP CPMT 2015]**

(a) $\frac{\varepsilon_0 t V^2}{2d} L$ (b) $\frac{\varepsilon_0 t V^2}{2d} L[x+1]$

(c) $\frac{\varepsilon_0 t V^2}{2d}[L + x(K-1)]$ (d) $\frac{\varepsilon_0^2 t^2 V^2}{2d^2}[L^2 + x + 1]$

17. A conducting sphere of radius R is given a charge Q. The electric potential and the electric field at the centre of the sphere respectively are **[CBSE AIPMT 2014]**

(a) zero and $\frac{Q}{4\pi\varepsilon_0 R^2}$ (b) $\frac{Q}{4\pi\varepsilon_0 R}$ and zero

(c) $\frac{Q}{4\pi\varepsilon_0 R}$ and $\frac{Q}{4\pi\varepsilon_0 R^2}$ (d) Both are zero

18. Two concentric spheres kept in air have radii R and r. They have similar charge and equal surface charge density σ. The electrical potential at their common centre is (where, ε_0 = permittivity of free space) **[MHT CET 2014]**

(a) $\frac{\sigma(R+r)}{\varepsilon_0}$ (b) $\frac{\sigma(R-r)}{\varepsilon_0}$

(c) $\frac{\sigma(R+r)}{2\varepsilon_0}$ (d) $\frac{\sigma(R+r)}{4\varepsilon_0}$

19. Two charges of equal magnitude q are placed in air at a distance $2a$ apart and third charge $-2q$ is placed at mid-point. The potential energy of the system is (where, ε_0 = permittivity of free space) **[MHT CET 2014]**

(a) $-\frac{q^2}{8\pi\varepsilon_0 a}$ (b) $-\frac{3q^2}{8\pi\varepsilon_0 a}$ (c) $-\frac{5q^2}{8\pi\varepsilon_0 a}$ (d) $-\frac{7q^2}{8\pi\varepsilon_0 a}$

20. Consider two concentric spherical metal shells of radii r_1 and r_2 ($r_2 > r_1$). If the outer shell has a charge q and the inner one is grounded, then the charge on the inner shell is **[WB JEE 2014]**

(a) $\frac{-r_2}{r_1}q$ (b) zero (c) $\frac{-r_1}{r_2}q$ (d) $-q$

21. What is the electric potential at a distance of 9 cm from 3 nC? **[KCET 2014]**

(a) 270 V (b) 3 V
(c) 300 V (d) 30 V

22. A hollow sphere of radius 0.1 m has a charge of 5×10^{-8} C. The potential at a distance of 5 cm from the centre of the sphere is

$$\left(\frac{1}{4\pi\varepsilon_0} = 9\times10^9\ \text{Nm}^2\ \text{C}^{-2}\right)$$ **[J & K CET 2013]**

(a) 4000 V (b) 4500 V (c) 5000 V (d) 6000 V

23. Three capacitors 3 μF, 6 μF and 6 μF are connected in series to a source of 120 V. The potential difference in volt, across the 3 μF capacitor will be **[WB JEE 2014]**

(a) 24 V (b) 30 V (c) 40 V (d) 60 V

24. The capacitance of two concentric spherical shells of radii R_1 and $R_2 (R_2 > R_1)$ is **[EAMCET 2014]**

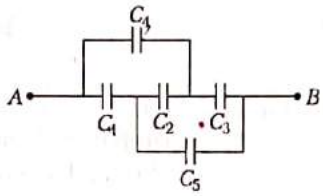
(a) $4\pi\varepsilon_0 R_2$ (b) $4\pi\varepsilon_0 \frac{(R_2-R_1)}{R_1R_2}$

(c) $4\pi\varepsilon_0 \frac{R_1R_2}{(R_2-R_1)}$ (d) $4\pi\varepsilon_0 R_1$

25. Two capacitors of 10 pF and 20 pF are connected to 200 V and 100 V sources, respectively. If they are connected by the wire, then what is the common potential of the capacitors? **[KCET 2014]**

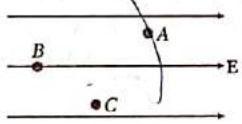
(a) 133.3 V (b) 150 V (c) 300 V (d) 400 V

26. In the given figure, the capacitors C_1, C_3, C_4 and C_5 have a capacitance 4 μF each. If the capacitor C_2 has a capacitance 10 μF, then effective capacitance between A and B will be **[UK PMT 2014]**



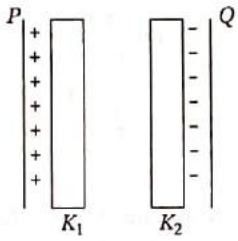
(a) 2 μF (b) 4 μF
(c) 6 μF (d) 8 μF

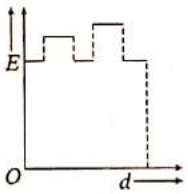
27. A, B and C are three points in a uniform electric field. The electric potential is **[NEET 2013]**

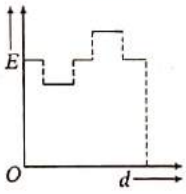


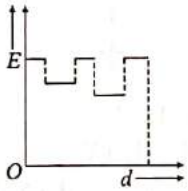
(a) maximum at A
(b) maximum at B
(c) maximum at C
(d) same at all the three points A, B and C

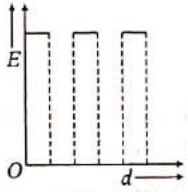
28. Two thin dielectric slabs of dielectric constants K_1 and K_2, $(K_1 < K_2)$ are inserted between the plates of a parallel plate capacitor, as shown in the figure alongside. The variation of electric field E between the plates with distance d as measured from plate P is correctly shown by **[CBSE AIPMT 2014]**



(a)


(b)


(c)


(d)


29. Two identical capacitors are first connected in series and then in parallel. The ratio of equivalent capacitance is **[J & K CET 2013]**

(a) 1 : 1 (b) 1 : 2
(c) 1 : 3 (d) 1 : 4

30. Two capacitors having capacitances C_1 and C_2 are charged with 120 V and 200 V batteries respectively. When they are connected in parallel now, it is found that the potential on each one of them is zero. Then, **[EAMCET 2013]**

(a) $5\ C_1 = 3\ C_2$ (b) $8\ C_1 = 5\ C_2$
(c) $9\ C_1 = 5\ C_2$ (d) $3\ C_1 = 5\ C_2$

31. A small oil drop of mass 10^{-6} kg is hanging in at rest between two plates separated by 1 mm having a potential difference of 500 V. The charge on the drop is ($g = 10\ \text{ms}^{-2}$) **[Karnataka CET 2013]**

(a) 2×10^{-9} C (b) 2×10^{-11} C
(c) 2×10^{-6} C (d) 2×10^{-9} C

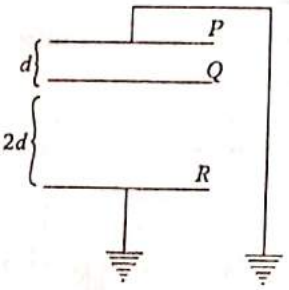
32. Two metal spheres of radii 0.01 m and 0.02 m are given a charge of 15 mC and 45 mC, respectively. They are then connected by a wire. The final charge on the first sphere is ... $\times 10^{-3}$ C. **[Karnataka CET 2013]**

(a) 40 (b) 30
(c) 20 (d) 10

33. The concentric spheres of radii R and r have positive charges q_1 and q_2 with equal surface charge densities. What is the electric potential at their common centre? **[Karnataka CET 2013]**

(a) $\frac{\sigma}{\varepsilon_0}(R + r)$ (b) $\frac{\sigma}{\varepsilon_0}(R - r)$
(c) $\frac{\sigma}{\varepsilon_0}\left(\frac{1}{R} + \frac{1}{r}\right)$ (d) $\frac{\sigma}{\varepsilon_0}\left(\frac{1}{R}\right)$

34. See the diagram, area of each plate is 2.0 m^2 and $d = 2 \times 10^{-3}$ m. A charge of 8.85×10^{-8} C is given to Q. Then, the potential of Q becomes **[Karnataka CET 2013]**



(a) 13 V (b) 10 V
(c) 6.67 V (d) 8.825 V

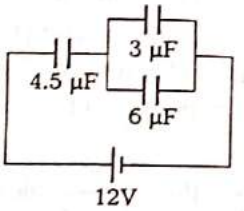
35. A soap bubble is charged to a potential 12 V. If its radius is doubled, then the potential of the bubble becomes **[Kerala CET 2013]**

(a) 12 V (b) 24 V
(c) 3 V (d) 6 V
(e) 9 V

36. A sphere of 4 cm radius is suspended with in a hollow sphere of 6 cm radius. If the inner sphere i charged to a potential 3 esu while the outer sphere earthed, then the charge on the inner sphere will b **[UP CPMT 20**

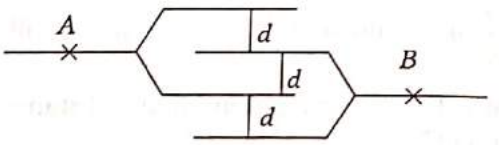
(a) $\frac{1}{4}$ esu (b) 30 esu
(c) 36 esu (d) 54 esu

37. In the adjoining figure, the potential difference across the 4.5 μF capacitor is **[UP CPMT 20**



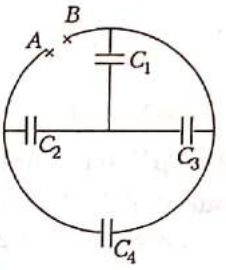
(a) 4 V (b) 6 V (c) 8 V (d) 4.5 V

38. The equivalent capacity between points A and B in figure will be, while capacitance of each capacitor i 3 μF. **[UP CPMT 201**



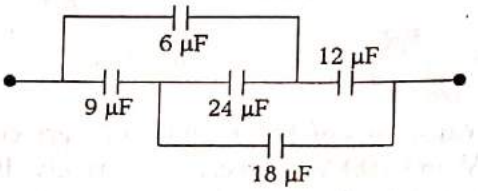
(a) 2 μF (b) 4 μF
(c) 7 μF (d) 9 μF

39. In the arrangement of capacitors shown in figure, each capacitor is of 9 μF, then the equivalent capacitance between the points A and B is **[UP CPMT 2012**



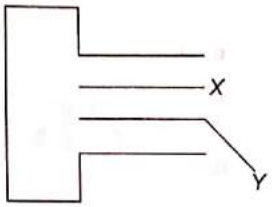
(a) 9 μF (b) 18 μF
(c) 4.5 μF (d) 15 μF

40. The equivalent capacitance between points A and B will be **[BCECE (Mains) 2012]**



(a) 10 μF (b) 15 μF
(c) 10.8 μF (d) 69 μF

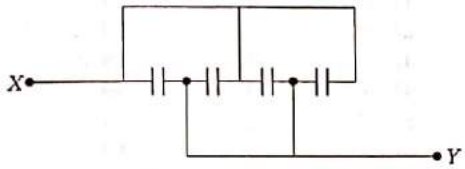
41. Four metallic plates each with a surface area of one side A are placed at a distance d from each other as shown in figure. Then, the capacitance of the system between X and Y is



[BCECE (Mains) 2012]

(a) $\frac{2\varepsilon_0 A}{d}$ (b) $\frac{2\varepsilon_0 A}{3d}$ (c) $\frac{3\varepsilon_0 A}{d}$ (d) $\frac{3\varepsilon_0 A}{2d}$

42. Four capacitors each of capacity 8 μF are connected with each other as shown in figure. The equivalent capacitance between points X and Y will be **[BHU 2012]**



(a) 2 μF (b) 8 μF (c) 16 μF (d) 32 μF

43. Four point charges $-Q$, $-q$, $2q$ and $2Q$ are placed, one at each corner of the square. The relation between Q and q for which the potential at the centre of the square is zero, is **[CBSE AIPMT 2012]**

(a) $Q=-q$ (b) $Q=-\frac{1}{q}$ (c) $Q=q$ (d) $Q=\frac{1}{q}$

44. A spherical drop of capacitance 1 μF is broken into eight drops of equal radius. Then, the capacitance of each small drop is **[AIIMS 2012]**

(a) $\frac{1}{2}$ μF (b) $\frac{1}{4}$ μF (c) $\frac{1}{8}$ μF (d) 8 μF

45. A capacitor having capacity of 2 μF is charged to 200 V and then the plates of the capacitor are connected to a resistance wire. The heat produced in joule will be **[Manipal 2012]**

(a) 2×10^{-2} (b) 4×10^{-2} (c) 4×10^{4} (d) 4×10^{10}

46. The potential of a large liquid drop when eight liquid drops are combined is 20 V. Then, the potential of each single drop was **[Manipal 2012]**

(a) 10 V (b) 7.5 V
(c) 5 V (d) 2.5 V

47. The electric field in a certain region is given by $\mathbf{E} = 5\hat{\mathbf{i}} - 3\hat{\mathbf{j}}$ kV/m. The potential difference $V_B - V_A$ between points A and B, having coordinates (4, 0, 3) m and (10, 3, 0) m respectively, is equal to **[AMU 2012]**

(a) 21 kV (b) – 21 kV
(c) 39 kV (d) – 39 kV

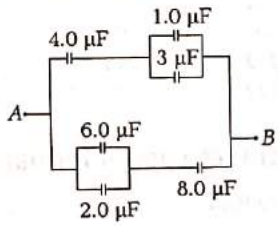
48. In a parallel plate capacitor with plate area A and charge Q, the force on one plate because of the charge on the other is equal to **[AMU 2012]**

(a) $\frac{Q^2}{\varepsilon_0 A^2}$ (b) $\frac{Q^2}{2\varepsilon_0 A^2}$ (c) $\frac{Q^2}{\varepsilon_0 A}$ (d) $\frac{Q^2}{2\varepsilon_0 A}$

49. Two capacitors 3 μF and 4 μF, are individually charged across a 6 V battery. After being disconnected from the battery, they are connected together with the negative plate of one attached to the positive plate of the other. What is the final total energy stored **[AMU 2012]**

(a) 1.26×10^{-4} J (b) 2.57×10^{-4} J
(c) 1.26×10^{-6} J (d) 2.57×10^{-6} J

50. The equivalent capacitance between A and B for the combination of capacitors shown in figure, where all capacitances are in microfarad is **[AFMC 2012]**



(a) 6.0 μF (b) 4.0 μF (c) 2.0 μF (d) 3.0 μF

Answers

Check point 2.1

1. (a)	2. (a)	3. (c)	4. (c)	5. (b)	6. (b)	7. (c)	8. (b)	9. (a)	10. (c)
11. (b)	12. (a)	13. (c)	14. (a)	15. (a)					

Check point 2.2

1. (c)	2. (b)	3. (b)	4. (b)	5. (a)	6. (c)	7. (b)	8. (c)	9. (d)	10. (a)
11. (a)	12. (c)	13. (a)	14. (a)	15. (c)					

Check point 2.3

1. (b)	2. (b)	3. (c)	4. (a)	5. (c)	6. (c)	7. (b)	8. (d)	9. (c)	10. (a)
11. (d)	12. (a)	13. (c)	14. (d)	15. (a)					

Check point 2.4

1. (d)	2. (c)	3. (d)	4. (c)	5. (a)	6. (c)	7. (a)	8. (c)	9. (d)	10. (d)
11. (d)	12. (d)	13. (b)	14. (b)	15. (a)					

Check point 2.5

1. (c)	2. (b)	3. (c)	4. (d)	5. (c)	6. (c)	7. (b)	8. (a)	9. (b)	10. (a)
11. (c)	12. (c)	13. (d)	14. (a)	15. (c)	16. (a)	17. (a)	18. (b)	19. (a)	20. (d)

A. Taking it together

1. (c)	2. (a)	3. (a)	4. (a,d)	5. (d)	6. (a)	7. (a)	8. (c)	9. (c)	10. (c)
11. (c)	12. (b)	13. (a)	14. (b)	15. (a)	16. (b)	17. (c)	18. (d)	19. (a)	20. (a)
21. (a)	22. (b)	23. (a)	24. (a)	25. (b)	26. (a)	27. (d)	28. (c)	29. (a)	30. (c)
31. (a)	32. (c)	33. (d)	34. (c)	35. (c)	36. (b)	37. (d)	38. (b)	39. (b)	40. (c)
41. (b)	42. (b)	43. (a)	44. (c)	45. (d)	46. (a)	47. (a)	48. (d)	49. (a)	50. (b)
51. (b)	52. (a)	53. (c)	54. (d)	55. (c)	56. (d)	57. (a)	58. (d)	59. (c)	60. (d)
61. (c)	62. (b)	63. (c)	64. (b)	65. (a)	66. (d)	67. (a)	68. (b)	69. (a)	70. (c)
71. (b)	72. (a)	73. (a)	74. (b)	75. (c)	76. (b)	77. (a)	78. (c)	79. (b)	80. (d)
81. (d)	82. (c)	83. (d)	84. (d)	85. (b)	86. (d)	87. (a)	88. (b)	89. (a)	90. (d)
91. (d)	92. (b)	93. (c)	94. (b)	95. (d)	96. (a)	97. (c)	98. (d)	99. (b)	100. (c)
101. (b)	102. (d)	103. (a)	104. (a)	105. (d)	106. (b)	107. (b)	108. (b)	109. (d)	110. (d)
111. (c)	112. (a)	113. (c)	114. (b)	115. (d)	116. (b,c)	117. (c)	118. (d)	119. (b)	120. (b)
121. (b)	122. (d)	123. (a)	124. (a)	125. (d)	126. (a)	127. (c)	128. (b)	129. (c)	130. (c)
131. (b)	132. (a)	133. (c)	134. (d)	135. (c)	136. (d)	137. (c)	138. (d)	139. (c)	140. (a)
141. (d)	142. (c)	143. (c)	144. (a)	145. (c)	146. (c)	147. (b)	148. (c)	149. (c)	150. (a)
151. (a)	152. (b)	153. (b)	154. (b)	155. (b)	156. (a)	157. (c)	158. (a)	159. (c)	160. (a)
161. (d)	162. (d)	163. (c)	164. (d)	165. (c)	166. (b)	167. (c)	168. (a)	169. (b)	170. (c)
171. (b)	172. (a)	173. (b)	174. (a)	175. (b)	176. (b)	177. (b)	178. (c)	179. (d)	180. (a)
181. (b)	182. (d)	183. (d)	184. (d)	185. (b)	186. (b)	187. (c)	188. (d)	189. (b)	190. (d)
191. (a)	192. (a)	193. (c)	194. (b)	195. (c)	196. (d)	197. (a)	198. (c)	199. (b)	200. (c)
201. (b)	202. (c)	203. (c)	204. (d)	205. (c)	206. (c)	207. (d)	208. (b)	209. (c)	210. (d)
211. (a)	212. (d)	213. (b)	214. (b)	215. (a)	216. (d)	217. (c)	218. (a)	219. (c)	220. (b)
221. (a)	222. (c)	223. (a)	224. (a)	225. (c)	226. (c)	227. (b)	228. (c)	229. (c)	

B. Medical entrance special format questions

Assertion and reason

1. (b)	2. (e)	3. (c)	4. (b)	5. (d)	6. (d)	7. (c)	8. (d)	9. (c)	10. (c)
11. (b)	12. (a)	13. (b)	14. (c)	15. (a)	16. (a)				

Match the columns

1. (A → p,q; B → s; C → s; D → r)
2. (A → p,q,r; B → s; C → p,q,r)
3. (A → r; B → p; C → p; D → q)
4. (A → s; B → p; C → r; D → q)

C. Medical entrances' gallery

1. (c)	2. (c)	3. (d)	4. (a)	5. (b)	6. (d)	7. (b)	8. (b)	9. (b)	10. (a)
11. (a)	12. (b)	13. (c)	14. (a)	15. (b)	16. (c)	17. (b)	18. (a)	19. (d)	20. (c)
21. (c)	22. (b)	23. (d)	24. (c)	25. (a)	26. (b)	27. (b)	28. (b)	29. (d)	30. (d)
31. (b)	32. (c)	33. (a)	34. (c)	35. (d)	36. (c)	37. (c)	38. (d)	39. (d)	40. (a)
41. (d)	42. (d)	43. (a)	44. (a)	45. (b)	46. (a)	47. (c)	48. (d)	49. (a)	50. (a)

Hints and explanations

Check point 2.1

1. (*a*) Using the relation $W = qV_a$

where, V_a is the electric potential at point A.

We have, $W = (4 \times 10^{-6})(10^4) = 4 \times 10^{-2}$ J

2. (*a*) $V = \frac{Kq}{R}$ i.e., $V \propto \frac{1}{R}$

$\therefore$ Potential on smaller sphere will be more.

3. (*c*) Electric potential at a distance r due to point charge Q is $V = \frac{KQ}{r}$ and electric field at same point $E = \frac{KQ}{r^2}$

$$\Rightarrow \quad E = \frac{KQ}{(KQ/V)^2} = \frac{V^2}{KQ} = \frac{(Q \times 10^{11})^2}{KQ} = 4\pi\varepsilon_0 Q \times 10^{22} \text{ V/m}$$

$$\left(\because K = \frac{1}{4\pi\varepsilon_0}\right)$$

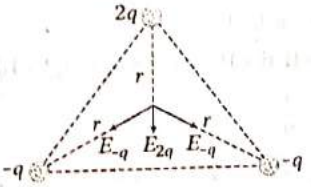
4. (*c*) The electrical potential produced by the nucleus at the positions of the electron.

$$V = 9 \times 10^9 \times \frac{Q}{r} = 9 \times 10^9 \times \frac{(+1.6 \times 10^{-19})}{0.53 \times 10^{-10}} = 27.2 \text{ V}$$

5. (*b*) Obviously, from charge configuration, at the centre electric field is non-zero. Potential at the centre due to $2q$ charge $V_{2q} = \frac{2q}{r}$ and potential due to $-q$ charge $V_{-q} = -\frac{q}{r}$

(where, r = distance of centre point)

$\therefore$ Total potential, $V = V_{2q} + V_{-q} + V_{-q} = 0$



6. (*b*) The electric field intensity E and electric potential V are related as $E = 0$ and for V = constant, $\frac{dV}{dr} = 0$

This imply that electric field intensity, $E = 0$.

7. (*c*) Potential difference, $\Delta V = \frac{N}{q} = \frac{2}{20} = 0.1$ V

8. (*b*) Potential at A = Potential due to $(+q)$ charge + Potential to $(-q)$ charge

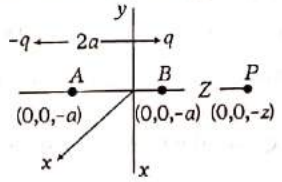
$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{\sqrt{a^2 + b^2}} + \frac{1}{4\pi\varepsilon_0} \frac{(-q)}{\sqrt{a^2 + b^2}} = 0$$

9. (*a*) $V_A = 10^{10}\left[\frac{-5 \times 10^{-6}}{15 \times 10^{-2}} + \frac{2 \times 10^{-6}}{5 \times 10^{-2}}\right] = \frac{1}{15} \times 10^6$ V

$$V_B = 10^{10}\left[\frac{2 \times 10^6}{15 \times 10^6} - \frac{5 \times 10^{-6}}{5 \times 10^{-2}}\right] = -\frac{13}{15} \times 10^6 \text{ V}$$

$$\therefore \quad W = V_A - V_B = 3 \times 10^{-6}\left[\frac{1}{15} \times 10^{-6} - \left(-\frac{13}{15} \times 10^6\right)\right] = 2.8 \text{J}$$

10. (*c*) Potential at P due to $(+q)$ charge



$$V_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{(z - a)}$$

Potential at P due to $(-q)$ charge,

$$V_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{-q}{((z + a)}$$

Total potential at P due (AB) electric dipole,

$$V = V_1 + V_2$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{(z - a)} - \frac{1}{4\pi\varepsilon_0} \frac{q}{(z + a)}$$

$$= \frac{2qa}{4\pi\varepsilon_0 (z^2 - a^2)}$$

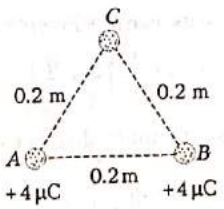
11. (*b*) Electric field, $E = \frac{V}{d} = \frac{10}{2 \times 10^{-2}} = 500$ N/C

12. (*a*) We have, $V = E \times d$

$\Rightarrow$ Distance, $d = \frac{V}{E} = \frac{3000}{500} = 6$ m

13. (*c*) Potential at C, $V = 2\left(\frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}\right)$

$$= \left(9 \times 10^9 \times \frac{4 \times 10^{-6}}{0.2}\right) \times 2 = 36 \times 10^4 \text{ V}$$



14. (*a*) The electric potential $V(x, y, z) = 4x^2$ V

Now, $\mathbf{E} = -\left(\hat{\mathbf{i}}\frac{\partial V}{\partial x} + \hat{\mathbf{j}}\frac{\partial V}{\partial y} + \hat{\mathbf{k}}\frac{\partial V}{\partial z}\right)$

Now, $\frac{\partial V}{\partial x} = 8x, \frac{\partial V}{\partial y} = 0$ and $\frac{\partial V}{\partial z} = 0$

Hence, $\mathbf{E} = -8x\hat{\mathbf{i}}$, so at point (1m, 0, 2m)

$\mathbf{E} = -8\hat{\mathbf{i}}$ V/m or 8 along negative X-axis.

15. (*a*) We have, electric field,

$$E = -\frac{dV}{dx} = -\frac{d}{dx}(5x^2 + 10x - 9) = -10x - 10$$

$\therefore (E)_{x=1} = -10 \times 1 - 10 = -20$ V/m

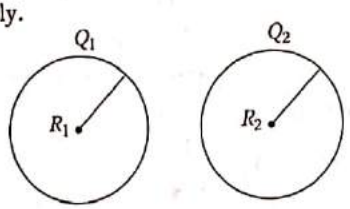
Check point 2.2

1. (*c*) Work done in displacing a charge particle is given by $W_{12} = q(V_2 - V_1)$ and the line integral of electrical field from point 1 to 2 gives potential difference $V_2 - V_1 = -\int_1^2 E \cdot dl$

For equipotential surface, $V_2 - V_1 = 0$ and $W = 0$.

4. (*b*) Since, potential inside the hollow sphere is same as that on the surface.

5. (*a*) Let Q_1 and Q_2 are the charges on sphere of radii R_1 and R_2, respectively.



Surface charge denstiy, $\sigma = \frac{\text{Charge}}{\text{Area}}$

According to given problem, $\sigma_1 = \sigma_2$

$$\frac{Q_1}{4\pi R_1^2} = \frac{Q_2}{4\pi R_2^2} \quad \therefore \frac{Q_1}{Q_2} = \frac{R_1^2}{R_2^2} \qquad \text{...(i)}$$

In case of a charged sphere, $V_s = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$

$$\therefore \quad V_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{R_1}, V_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q_2}{R_2}$$

$$\Rightarrow \quad \frac{V_1}{V_2} = \frac{Q_1}{R_1} \times \frac{R_2}{Q_2} = \frac{Q_1}{Q_2} \times \frac{R_2}{R_1}$$

$$= \left(\frac{R_1}{R_2}\right)^2 \times \left(\frac{R_2}{R_1}\right) = \frac{R_1}{R_2} \qquad \text{(using Eq. (i))}$$

6. (*c*) Let q' be the charge on the inner shell. Then, the potential of the inner is

V = Potential due to its own q' + Potential due to the outer charge

$$= \frac{1}{4\pi\varepsilon}\left(\frac{q}{r_1} + \frac{q}{r_2}\right)$$

But $V = 0$, because the inner shell is earthed

$$\therefore \frac{q_1}{r_1} + \frac{q_2}{r_2} = 0 \quad q' = -q\left(\frac{r_1}{r_2}\right)$$

7. (*b*) Potential inside the sphere will be same as that on its surface *i.e.*, $V = V_{surface} = \frac{q}{10}$ stat volt, $V_{out} = \frac{q}{15}$ stat volt

$$\therefore \quad \frac{V_{out}}{V} = \frac{2}{3} \Rightarrow V_{out} = \frac{2}{3}V$$

8. (*c*) Inside a conducting body, potential is same everywhere and equals to the potential of its surface.

9. (*d*) If charge acquired by the smaller sphere is Q, then its potential $120 = \frac{KQ}{2}$. ...(i)

Whole charge comes to outer sphere.

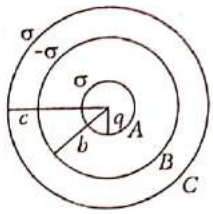
Also, potential of the outer sphere,

$$V = \frac{KQ}{6} \qquad \text{...(ii)}$$

From Eq. (i) and (ii), we get

$$V = 40 \text{ V}$$

10. (*a*) $V_A = \frac{\sigma}{\varepsilon_0}(a - b + c)$



$$V_B = \frac{\sigma}{\varepsilon_0}\left(\frac{a^2}{c} - b + c\right)$$

$$\Rightarrow \quad V_C = \frac{\sigma}{\varepsilon_0}\left(\frac{a^2}{c} - \frac{b^2}{c} + c\right)$$

On putting $c = a + b \Rightarrow V_A = V_C \neq V_B$

12. (*c*) Potential, $V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r} = 9 \times 10^9 \times \frac{500 \times 10^{-6}}{10} = 4.5 \times 10^6$ V

13. (*a*) The kinetic energy gained by a charge of q coulomb through a potential difference of V volt is

$K = qV$ Joule

$K = (1.6 \times 10^{-19})(2000) = 3.2 \times 10^{-16}$ J

We have $K = \frac{1}{2}mv^2$

$$v = \sqrt{\frac{2K}{m}} = \frac{2 \times 3.2 \times 10^{-6}}{9 \times 10^{-31}} = \frac{8}{3} \times 10^7 \text{ m/s}$$

14. (*a*) Speed of the particle, $v = \sqrt{\frac{2QV}{m}}$

$$\Rightarrow \quad u \propto \sqrt{Q} \Rightarrow \frac{v_A}{v_B} = \sqrt{\frac{Q_A}{Q_B}} = \sqrt{\frac{16q}{4q}} = \frac{2}{1}$$

15. (*c*) The kinetic energy of the electron

$$k = eV = \frac{1}{2}mv^2$$

$$\Rightarrow \quad V = \frac{1}{2}\frac{mv^2}{e}$$

Here, $v = \frac{c}{10} = \frac{3\times10^8}{10} = 3\times10^7$ m/s

$$V = \frac{1}{2}\frac{(9.0\times10^{-31})(3.0\times10^{-7})^2}{(1.6\times10^{-19})}$$

Potential, $V = 2531\text{V}$

Check point 2.3

1. (*b*) Δ PE = Work done by external agent

$= (V_f q - V_i q), V_f > V_i \Rightarrow \Delta\text{PE} < 0$ *i.e.*, PE will increase.

2. (*b*) Potential energy of the system will be given by

$$= \frac{(-e)(-e)}{4\pi\varepsilon_0 r} = \frac{e^2}{4\pi\varepsilon_0 r}$$

As r decreases, potential energy increases.

3. (*c*) Work done, $W = U_f - U_i = 9\times10^9 \times Q_1Q_2\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$

$$\Rightarrow W = 9\times10^9 \times 12\times10^{-6} \times 8\times10^{-6}\left[\frac{1}{4\times10^{-2}} - \frac{1}{10\times10^{-2}}\right]$$

$= 12.96\text{ J} \approx 13\text{J}$

4. (*a*) Since, the proton is moving against the direction of electric field, so work is one by the proton against electric field. It implies that electric field does negative work on the proton. Again, proton is moving in electric field from low potential region to high potential region, hence its potential energy increases.

5. (*c*) Electric potential energy, $U = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{d}$

$$\therefore \quad U = \frac{(9\times10^9)\times(1.6\times10^{-19})\times(-1.6\times10^{-19})\text{ J}}{10^{-10}}$$

$= -9\times10^9 \times 1.6\times10^{-9}\text{ eV} = -14.4\text{ eV}$

Note In the solution given all the values are positive. It is important to mention the sign.

6. (*c*) Potential energy as well as force are positive, if there is repulsion between the particles and negative, if there is attraction.

We take of only the magnitude of values when discussing decrease or increase of energy.

As $U = \frac{Q_1Q_2}{4\pi\varepsilon_0 r}$,

Plus or minus *i.e.*, whether both are of the same sign or different, if r decreases, the value increase. Therefore, (c) is wrong.

7. (*b*) Potential energy of charges Q_1 and Q_2 at 10 cm apart,

$$U_i = \frac{1}{4\pi\varepsilon_0}\frac{12\times10^{-6}\times5\times10^{-6}}{0.1}$$

$$= \frac{9\times10^9\times60\times10^{-12}}{0.1} = 54\times10^{-1} = 5.4\text{ J}$$

Potential energy of charge Q_1 and Q_2 At 6 cm apart,

$$U_2 = \frac{9\times10^9\times60\times10^{-12}}{0.06} = 9\text{ J}$$

$\therefore$ Work done = (9 – 5.4) J = 3.6 J

8. (*d*) We have, $AB + AC = 12$ cm ...(i)

$AB\cdot AC = 32\text{ cm}^2$

$\therefore AB - AC = \sqrt{(AB+AC)^2 - 4AB\cdot AC}$

$AB - AC = 4$

From Eqs. (i) and (ii), we get

$AB = 8$ cm, $AC = 4$ cm

Potential energy at point A,

$$U_A = \frac{1}{4\pi\varepsilon_0}q_1q_2\left[\frac{1}{AB} + \frac{1}{AC}\right]$$

$$= \frac{9\times10^9\times4\times10^{-12}}{10^{-2}}\left(\frac{1}{8} + \frac{1}{4}\right) = 1.35\text{ J}$$

9. (*c*) Potential energy, $U = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1Q_2}{r_1} + \frac{Q_2Q_3}{r_2} + \frac{Q_1Q_2}{r_3}\right]$

Net potential energy, $U_{\text{net}} = \frac{3}{4\pi\varepsilon_0}\cdot\frac{q^2}{l}$

10. (*a*) Potential energy of the system,

$$U = K\frac{Qq}{l} + \frac{Kq^2}{l} + \frac{KqQ}{l} = 0$$

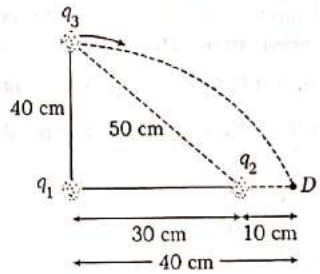
$$\Rightarrow \quad \frac{Kq}{l}(Q + q + Q) = 0 \Rightarrow Q = -\frac{q}{2}$$

11. (*d*) Length of the diagonal of a cube having each side b is $\sqrt{3}\,b$. So, distance of centre of cube from each vertex is $\frac{\sqrt{3}b}{3}$

Hence, potential energy of the given system of charge is

$$U = 8\times\left[\frac{1}{4\pi\varepsilon_0}\cdot\frac{(-q)(q)}{\sqrt{3}b/2}\right] = \frac{-4q^2}{\sqrt{3}\pi\varepsilon_0 b}$$

12. (*a*) Change in potential energy $(\Delta U) = U_f - U_i$



$\Rightarrow$ Potential energy,

$$\Delta U = \frac{1}{4\pi\varepsilon_0}\left[\left(\frac{q_1q_3}{0.4} + \frac{q_2q_3}{0.1}\right) - \left(\frac{q_1q_3}{0.4} + \frac{q_2q_3}{0.5}\right)\right]$$

$$\Rightarrow \quad \Delta U = \frac{1}{4\pi\varepsilon_0}[8q_2q_3] = \frac{q_3}{4\pi\varepsilon_0}(8q_2)$$

$\therefore \quad K = 8q_2$

13. (*c*) Torque, $\tau_{\max} = \text{PE}$

$= 2(2l)E = 2\times10^{-6}\times0.01\times5\times10^5 = 10\times10^{-3}$N-m

14. (*d*) Work done, $W = pE(1-\cos\theta)$

$= pE(1-\cos180°)$

$W = pE[1-(-1)] = 2pE$

15. (*a*) Given, $q_1 = 1C$, $q_2 = 2C$, $q_3 = 3C$ and $r_1 = 100$ cm $= 1$m

Initial PE of system, $U_1 = \frac{1}{4\pi\varepsilon_0 r_1}[q_1q_2 + q_2q_3 + q_3q_1]$

$$= \frac{9\times10^9}{1}[1\times2 + 2\times3 + 3\times1] = 99\times10^9 \text{ J}$$

When $r_2 = 50$ cm $= 0.5$ m

Final PE of system, $U_2 = \frac{1}{4\pi\varepsilon_0 r_2}[q_1q_2 + q_2q_3 + q_3q_1]$

$$= \frac{9\times10^9}{0.5}[1\times2 + 2\times3 + 3\times1] = 2\times99\times10^9 \text{ J}$$

Work done, $U_2 - U_1 = 2\times99\times10^9 - 99\times10^9$

$$W = 99\times10^9 \text{ J} = 9.9\times10^{10} \text{ J}$$

Check point 2.4

1. (*d*) Electrostatic field at the surface of a conductor is normal to the surface.

2. (*c*) Volume of 8 small drops = Volume of big drop

$$8\times\frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3 \Rightarrow R = 2r$$

As capacity is proportional to r, hence capacity become 2 times.

3. (*d*) We have, $C = \frac{\varepsilon_0 A}{d} \Rightarrow C \propto \frac{1}{d}$.

Therefore, the capacity of parallel plate condenser depends in the separation between the plates.

4. (*c*) Since, aluminium is a metal, therefore field inside this will be zero. Hence, it would not affect the field in between the two plates, so capacity $= \frac{q}{V} = \frac{q}{Ed}$ remains unchanged.

5. (*a*) When a lamp is connected to DC line with a capacitor. If will form an open circuit. Hence the lamp will not glow.

6. (*c*) Capacity of a spherical conductor $C = 4\pi\varepsilon_0 R$.

7. (*a*) The potential difference across the parallel plate capacitor is 10 V–(–10 V)= 20 V.

Capacitance $= \frac{Q}{V} = \frac{40}{20} = 2$F

8. (*c*) For spherical conductor,

$$C = 4\pi\varepsilon_0 R$$

$\because$ $V = \frac{4}{3}\pi R^3$ and $A = 4\pi R^2$

$\therefore$ $R = \frac{3V}{A}$

$\therefore$ Capacitance, $C = 12\pi\varepsilon_0 \frac{V}{A}$

9. (*d*) Given, $2\pi R = 2 \Rightarrow R = \frac{1}{\pi}$

For sphere $C = 4\pi\varepsilon_0 KR$

$\Rightarrow$ $C = \frac{1}{9\times10^9}\times\frac{1}{\pi}\times80$ (for water $K = 80$)

Capacitance of sphere, $C = 2828.28$ pF

10. (*d*) $C = \frac{\varepsilon_0 A}{d}$ As $A \to \frac{1}{2}$ times and $d \to 2$ times

So, $C \to \frac{1}{4}$ times *i.e.*, $C' = \frac{1}{4}C = \frac{12}{3} = 3\,\mu$F

11. (*d*) Capacitance, $C_{medium} = KC_{air} \Rightarrow K = \frac{C_{medium}}{C_{air}} = \frac{110}{50} = 2.20$

12. (*d*) Charge on capacitor for when its potential becomes 10 V.

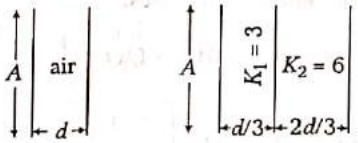
$$Q = CV = 500\times10^{-6}\times10 = 5000\times10^{-6}\mu\text{C}$$

$\therefore$ Required time $= \frac{5000\times10^{-6}}{100\times10^{-6}} = 50$ s

13. (*b*) $C = \frac{\varepsilon_0 A}{d} = 1$pF and $C = \frac{K\varepsilon_0 A}{2d} = 2$ pF

$\therefore$ $K = 4$.

14. (*b*)



Capacitance in air,

$$C_{air} = \frac{\varepsilon_0 A}{d} = 9, \quad \frac{1}{C_{med}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d_1}{K_1\varepsilon_0 A} + \frac{d}{K_2\varepsilon_0 A}$$

$\Rightarrow$ Capacitance in medium, $C_{med} = \frac{K_1K_2\varepsilon_0 A}{K_1d_2 + K_2d_1}$

$$= \frac{3\times6\times\varepsilon_0 A}{3\times2d/3 + 6\times d/3} = \frac{18}{4}\times9 = 40.5 \text{ pF}$$

15. (*a*) Parallel plate capacitor,

$$C = K\varepsilon_0 A/d$$

As given in figure, for series combination,

$$\frac{1}{C'} = \frac{1}{\frac{\varepsilon_0 A}{\frac{d}{2}}} + \frac{1}{\frac{2\varepsilon_0 A}{\frac{d}{2}}} \Rightarrow C = \frac{4}{3}\frac{\varepsilon_0 A}{d}.$$

Check point 2.5

1. (*c*) Capacitor are in series, $\frac{1}{C'} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} = \frac{3}{C}$

Capacitance, $C' = \frac{C}{3}$

Voltage of the combination,

$$V' = V_1 + V_2 + V_3 = V + V + V = 3V$$

2. (*b*) Charge on capacitor A, $Q_1 = \frac{C_1}{C_1 + C_2}Q_n$

$\Rightarrow$ $Q_1 = \frac{6C}{6C + 3C}\times9q = 6q$

Charge on capacitor B, $Q_2 = \frac{C_2}{C_1 + C_2}Q_n$

$\Rightarrow$ $Q_2 = \frac{3C}{6C + 3C}\times9q = 3q$

3. (*c*) In series, $\frac{1}{C} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$

$\Rightarrow$ Capacitance, $C = \frac{2}{3}$F

4. (*d*) Equivalent capacitance $= \frac{2\times3}{2+3} = \frac{6}{5}\mu F$

Total charge by $Q = CV = \frac{6}{5}\times1000 = 1200\mu C$

Potential (V) across $2\mu F$ is $V = \frac{Q}{C} = \frac{1200}{2} = 600$ V

$\therefore$ Potential on internal plates $= 1000 - 600 = 400$ V

5. (*c*) Capacitor are in series,

$$\frac{1}{C_{eq}} = \frac{1}{1} + \frac{1}{2} + \frac{1}{8}$$

$$\Rightarrow \quad C_{eq} = \frac{8}{13}\mu F$$

Total charge, $Q = C_{eq}V = \frac{8}{13}\times13 = 8\mu C$

Potential difference across $2\mu F$ capacitor $= \frac{8}{2} = 4V$

6. (*c*) The given arrangement becomes an arrangement of $(n-1)$ capacitors connected in parallel. So, $C_R = (n-1)C$

7. (*b*) Capacitance, $C_1 = \frac{C}{4}$ (series)

Capacitance, $C_2 = 4C$ (parallel)

The ratio of capacitance,

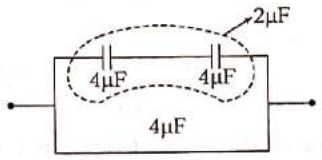
$$\therefore \quad \frac{C_1}{C_2} = \frac{C/4}{4C} = \frac{1}{16}$$

8. (*a*) Capacitance $C_{max} = nC = 3\times3 = 9\mu F$

Minimum capacitance,

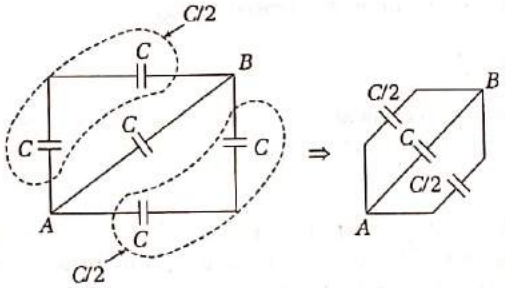
$$C_{min} = \frac{C}{n} = \frac{3}{3} = 1\mu F$$

9. (*b*) The given circuit can be drawn as follows



Effective capacitance, $C_{AB} = 2 + 4 = 6\mu F$

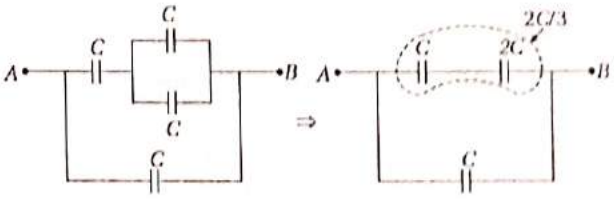
10. (*a*) The given circuit can be simplified as follows



Equivalent capacitance between A and B is

$$C_{AB} = \frac{C}{2} + \frac{C}{2} + C = 2C$$

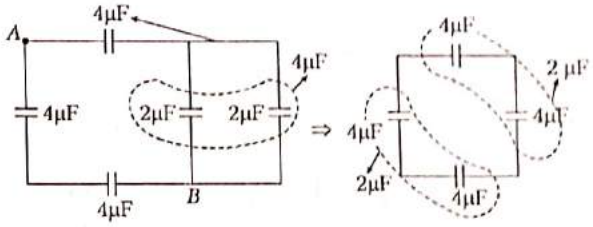
11. (*c*) The given circuit can be simplified as follows



Equivalent capacitance between A and B is

$$C_{AB} = \frac{2C}{3} + C = \frac{5}{3}C$$

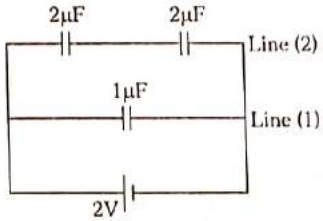
12. (*c*) The given circuit can be simplified as follows



Equivalent capacitance between A and B is $C_{AB} = 2 + 2 = 4\mu F$

13. (*d*) Potential difference across both the lines is same *i.e.*, $2V$. Hence charge flowing in line 2.

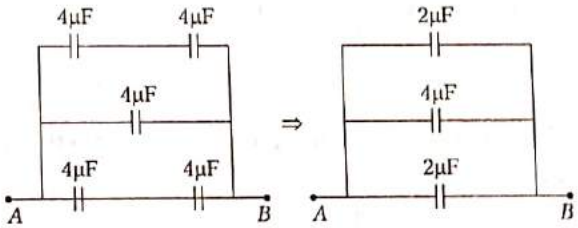
$$Q = \left(\frac{2\times2}{2+2}\right) = 2\mu C$$



So charge on each capaction in line (2) is $2\mu C$

Charge line 1 $Q = 2\times1 = 2\mu$ C

14. (*a*)



Therefore, capacitor $2\mu F$, $4\mu F$ and $2\mu F$ are in parallel.

So, equivalent capacitance between A and B

$$C_{AB} = 2 + 4 + 2 = 8\mu F$$

15. (*c*) The energy strored in a capacitor is

$$U = \frac{1}{2}CV^2$$

or Potential difference, $V = \sqrt{\frac{2U}{C}} = \sqrt{\frac{2\times50}{100\times10^{-6}}} = 1000$ V

16. (*a*) Potential energy of charged parallel plate capacitor

$U_0 = \frac{Q^2}{2C}$

when a slab of dielectric K is inserted then $C' = CK$

The new potential energy,

$$U = \frac{Q^2}{2C'} = \frac{Q^2}{2CK} = \frac{U_0}{K}$$

17. (*a*) In series capacitance, $C' = \left(\frac{C_1}{n_1}\right)$

and $V = uV$

Energy, $U' = \frac{1}{2}C'V'^2 = \frac{1}{2}\left(\frac{C_1}{n_1}\right)(uV)^2$

In parallel capacitance $C'' = n_2 C_2$

and $V'' = V$

$$U'' = \frac{1}{2}C''V''^2 = \frac{1}{2}(n_2C_2)V^2$$

$$\frac{1}{2}\left(\frac{C_1}{n_1}\right)(uV)^2 = \frac{1}{2}(n_2C_2)V^2 \quad \Rightarrow C_2 = \frac{16C_1}{n_1n_2}$$

18. (*b*) Energy of capacitor $U = \frac{1}{2}\frac{Q^2}{C}$

$$\therefore \quad 1.21\,U = \frac{1}{2}\frac{(Q+2)^2}{C}$$

$$\therefore \quad \frac{121}{1} = \frac{(Q+2)^2}{Q^2}$$

$$\Rightarrow \quad \sqrt{\frac{121}{1}} = \frac{Q+2}{Q}$$

$$\Rightarrow \quad 1.1Q = Q + 2$$

Charge on the capacitor,

$$Q = 20\text{ C}$$

19. (*a*) Initial energy $= \frac{1}{2}\times 1\times 10^{-6}\times(30)^2 = 450\times 10^{-6}$ J

Final energy $= \frac{1}{2}(C_1 + C_2)V^2_{\text{common}}$ $\quad \left(\because V = \frac{V_1C_1 + V_2C_2}{C_1 + C_2}\right)$

$$= \frac{1}{2}\times 3\times 10^{-6}\times(10)^2 = 150\times 10^{-6}\text{ J}$$

Loss of energy $= (450 - 150)\times 10^{-6}$ J $= 300\times 10^{-6}$ J $= 300\,\mu$ J

20. (*d*) Initial energy stored in the capacitor,

$$U_i = \frac{1}{2}CV^2 = \frac{1}{2}C\times(50)^2 = \frac{1}{2}C(50)^2 \quad \ldots\text{(i)}$$

After 2s, when the potential drops by 10 V, the final potential is 40 V.

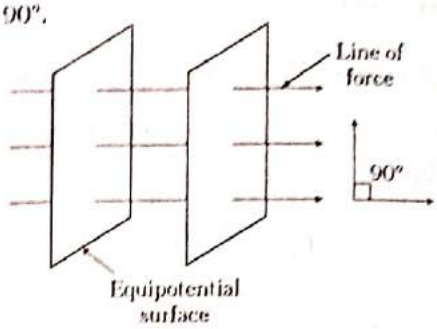
Final energy stored in the capacitor

$$U_f = \frac{1}{2}C(40)^2 \quad \ldots\text{(ii)}$$

Fraction of energy stored $= \frac{U_f}{U_i} = \frac{\frac{1}{2}C(40)^2}{\frac{1}{2}C(50)^2} = \left(\frac{40}{50}\right)^2 = 0.64$

A. Taking it together

1. (*c*) Angle between equipotential surface and line of force is 90°.



2. (*a*) Potential, $V = E\cdot d$

$\therefore$ Distance from the point charge, $d = \frac{V}{E} = \frac{3000}{500} = 6$ m

3. (*a*) $\because$ Potential difference $= \frac{\text{Work done}}{\text{Charge}}$

$\therefore \quad W = qV$

$10 = 5\times V$

$\therefore \quad V = 2$ V

4. (*a, d*) A negative charge when moves from higher potential to lower potential, its velocity increases.

7. (*a*) Work done, $W_{A\to B} = U_B - U_A = q(V_B - V_A)$

Potential difference, $V_B - V_A = \frac{W_{A-B}}{q}$

$$= \frac{10\times 10^{-3}}{5\times 10^{-6}} = 2000\text{V} = 2\text{kV}$$

8. (*c*) Capacity of parallel plate capacitor is given by, $C = \frac{\varepsilon_0 A}{d}$

$$\therefore \quad \varepsilon_0 = \frac{Cd}{A}$$

$$\text{Unit} = \frac{\text{farad}\times\text{metre}}{\text{metre}^2} = \frac{\text{farad}}{\text{metre}}$$

11. (*c*) The electric dipole potential varies as $1/r$ at large distance is not true.

12. (*b*) Energy, $U = \frac{1}{2}\frac{q^2}{V}$

$\therefore$ It is in the form of potential energy.

13. (*a*) $U = \frac{Q^2}{2C}$

When a slab of dielectric constant K is inserted then $C' = Ck$

$$U' = \frac{Q^2}{2C'} = \frac{Q^2}{2Ck} \Rightarrow U' = \frac{U_0}{k}$$

14. (*b*) Potential at a point in a field is defined as the amount of work done in bringing a unit positive test charge (q) from infinity to that point along any arbitrary path.

Potential, $V = \frac{W}{q_0}$

or Work done, $W = qV$

15. (*a*) Force between the plates of a parallel plate capacitor is given by

$$|F| = \frac{\sigma^2 A}{2\varepsilon_0} = \frac{Q^2}{2\varepsilon_0 A} = \frac{CV^2}{2d}$$

16. (*b*) The torque acting on a dipole is given by $\tau = \mathbf{p} \times \mathbf{E}$, the potential energy of the dipole is given by $U = -\mathbf{p} \cdot \mathbf{E}$ and the resultant force on the dipole is zero.

17. (*c*) The direction of electric field is always perpendicular to one equipotential surface maintained at high electrostatic potential to other equipotential surface maintained at low electrostatic potential.

The positively charged particle experiences electrostatic force along the direction of electric field *i.e.*, from high electrostatic potential to low electrostatic potential. Thus, the work is done by the electric field on the positive charge, hence electrostatic potential energy of the positive charge decreases.

18. (*d*) Electrostatic field at the surface of a conductor is normal to the surface.

19. (*a*) In this problem, the collection of charges, whose total sum is not zero, with regard to great distance can be considered as a point charge. The equipotentials due to point charge are spherical in shape as electric potential due to point charge q is given by

$$V = k_e \frac{q}{r}$$

This suggest that electric potentials due to point charge is same for all equidistant points. The locus of these equidistant points, which are at same potential, form spherical surface.

20. (*a*) Electron is moving in opposite direction of field so field will produce an accelerating effect on electron.

21. (*a*) Capacitance of a metallic sphere,

$$c = 4\pi\varepsilon_0 r = 1 \times 10^{-6} \Rightarrow r = 10^{-6} \times 9 \times 10^9 = 9\,\text{km}$$

22. (*b*) Unit of E in SI system is $E = \frac{F}{q_0}$ = newton/coulomb

As, $$E = -\frac{dV}{dr},$$

So, unit of E is also volt/metre

Also, $q = CV$

$\therefore$ $q = CEd$ $(\because V = Ed)$

$$E = \frac{q}{Cd} = \frac{qV}{qd} = \frac{K}{qd} \quad \left(\because C = \frac{q}{V}\right)$$

$$= \frac{\text{joule}}{\text{coulomb-metre}}$$

while J/C is unit of electric potential.

23. (*a*) Electric field is negative of potential gradient *i.e.*,

$$E = -\frac{dV}{dx} = -\frac{d}{dx}(5x^2 + 10x - 9)$$

$$= -10x - 10$$

$$\therefore \quad (E)_{x=1} = -10 \times 1 - 10 = -20 \text{ V/m}$$

24. (*a*) Inside a sphere, potential remains uniform.

25. (*b*) For charged spherical conductor, potential of the sphere at its surface

$$= V_{\text{surface}} = \frac{q}{5} \text{ stat volt}$$

$$V_{\text{out}} = \frac{q}{30} \text{ stat volt} \quad \therefore \quad \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{1}{6}$$

Potential at a point distant 30 from the centre

$$V_{\text{out}} = \frac{1}{6} V_{\text{in}} \Rightarrow V_{\text{out}} = \frac{1}{6} \text{V}$$

26. (*a*) Electric field is given by

$$E = -\frac{\Delta V}{\Delta r} = \frac{30 - (-10)}{2 \times 10^{-2}} = 2000 \text{ V/m}$$

27. (*d*) Potential at a point due to electric dipole

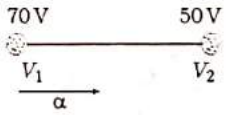
$$V = \frac{p \cos\theta}{r^2}$$

If $\theta = 0°$, then V = max

If $\theta = 180°$, then V = min

28. (*c*) $U = -pE\cos\theta$, U is minimum at $\theta = 0°$.

29. (*a*) Kinetic energy gained by α-particle



$$\text{KE} = q \cdot \Delta V = q(V_1 - V_2) = 2e(V_1 - V_2)$$

$$= 2 \times 1.6 \times 10^{-19}\,(70 - 50) = 40 \text{ eV}$$

30. (*c*) $\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}m\,(at)^2 = \frac{1}{2}m\left(\frac{qE}{m}\right)^2 t^2 \quad \left(\because a = \frac{qE}{m}\right)$

$$= \frac{E^2 q^2 t^2}{2m}$$

31. (*a*) Work done, $W = U_f - U_i = \frac{q_1 q_2}{4\pi\varepsilon_0}\left[\frac{1}{r_f} - \frac{1}{r_i}\right]$

$$= (9 \times 10^9)(10^{-12})(12)(8)\left[\frac{1}{0.06} - \frac{1}{0.1}\right] = 5.8 \text{ J}$$

32. (*c*) Capacitance, $C = (4\pi\varepsilon_0 R)$

$$= \frac{1}{9 \times 10^9} \times 6408 \times 10^3 = 712\,\mu\text{F}$$

33. (*d*) If metallic slab fills the complete space between the plates or both plates are joined through a matellic wire, then capacitance became infinite.

34. (*c*) Energy stored, $W = \frac{Q^2}{2C}$

$\therefore$ $W \propto Q^2$ (if C is constant)

or $$\frac{W}{W'} = \frac{Q^2}{(2Q)^2} = \frac{Q^2}{4Q^2}$$

$\therefore$ $W' = 4W$

35. (*c*) Heat produced in capacitor = Energy of charged capacitor

$$= \frac{1}{2}CV^2 = \frac{1}{2} \times (2 \times 10^{-6}) \times (100)^2 = 0.01 \text{ J}$$

36. (*b*) The energy stored, $U = \frac{1}{2}(nC)V^2$

37. (*d*) Change in energy of condenser

$$= \Delta U = U_2 - U_1$$

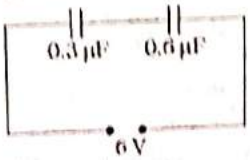
$$= \frac{1}{2}C_2V^2 - \frac{1}{2}C_1V^2$$

$$= \frac{V^2}{2}(C_2 - C_1)$$

$$= \frac{(100)^2}{2}(10 \times 10^{-6} - 2 \times 10^{-6})$$

$$= 4 \times 10^{-3}\ \text{J}$$

38. (*b*) In series, charge remains same on both capacitors



$\therefore$ $U = \frac{Q^2}{2C}$ or $U \propto \frac{1}{C}$

$\therefore$ $\frac{U_1}{U_2} = \frac{C_2}{C_1} = \frac{0.6}{0.3}$

$\therefore$ $U_1 : U_2 = 2 : 1$

39. (*b*) In parallel combination, $C_{net} = C_1 + C_2$

$$V = \frac{q_{net}}{C_{net}} = \frac{C_1V_0}{C_1 + C_2}$$

40. (*c*) q remains same, while V decreases.

41. (*b*) Potential on parallel plate capacitor, $V = \frac{Q}{C}$.

Also, capacity of parallel plate capacitor is given by

$$C = \frac{\varepsilon_0 KA}{d}$$

$\therefore$ $V = \frac{Qd}{\varepsilon_0 KA}$

$\Rightarrow$ $V \propto d$

So, on increasing the distance between plates of capacitor, the potential difference between plates also increases.

42. (*b*) Potential difference $= V_i - V_0 = \frac{q_i}{4\pi\varepsilon_0}\left(\frac{1}{r_i} - \frac{1}{r_0}\right)$

If q_i is positive, $V_i - V_0$ = positive or $V_i > V_0$.

43. (*a*) Dielectric constant,

$$K = \frac{\text{Permittivity of medium}}{\text{Permittivity of free space}}$$

$$K = \frac{\varepsilon}{\varepsilon_0}$$

$\therefore$ Permittivity $\varepsilon = K\varepsilon_0$

$$= 81 \times 8.86 \times 10^{-12}$$

$$= 7.16 \times 10^{-10}\ \text{MKS units}$$

44. (*c*) Both the conductors carry equal and opposite charges. So, after connecting by a wire, there will be no charge in any conductor. Hence, all the stored energy will be destroyed.

Loss of energy $= 2\left(\frac{1}{2}CV^2\right) = CV^2$

45. (*d*) Net charge distributes in direct ratio of capacity (or radius in case of spherical conductor)

$\therefore$ $q'_A = q_{net}\left[\frac{4}{4+6}\right] = 120\left[\frac{4}{4+6}\right]$

$= 48\ \mu\text{C}$

So, $\Delta q = 80 - 48 = 32\ \mu\text{C}$

46. (*a*) Force, $F_e = qE = q\left(\frac{V}{d}\right) = (4 \times 10^{-6})\left(\frac{2000}{2 \times 10^{-3}}\right) = 4\text{N}$

47. (*a*) $\left(\frac{q}{R}\right)_1 = \left(\frac{q}{R}\right)_2$

$\therefore$ $V_1 = V_2$

So, the charge will not fall at all.

48. (*d*) In redistribution of charge there is always loss of energy, unless their potentials are same or $Q_1R_2 = Q_2R_1$.

49. (*a*) As per the given condition in the question, electricity will flow from the smaller sphere to the bigger one when the smaller one is placed inside the bigger one and connected by wire.

50. (*b*) Work done, $W = \Delta U = U_f - U_i = (3U) - U = 2U$

51. (*b*) Potential, $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

Here, $V = 2V_{+ve} + 2V_{-ve}$

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{2q}{L} - \frac{2q}{L\sqrt{5}}\right]$$

The electric potential at mid-point of A,

$$V = \frac{2q}{4\pi\varepsilon_0 L}\left(1 - \frac{1}{\sqrt{5}}\right)$$

52. (*a*) The potential at the centre of the sphere is 80 V because it remains same at each point under the metallic hollow sphere.

53. (*c*) The energy stored in the capacitor,

$$U = \frac{1}{2}CV^2$$

$$U = \frac{1}{2}\left(\frac{A\varepsilon_0}{d}\right)(Ed)^2 \qquad \left(\because C = \frac{A\varepsilon_0}{d} \text{ and } V = Ed\right)$$

$$U = \frac{1}{2}\varepsilon_0 E^2 Ad$$

54. (*d*) Given, $q_1 = 5$ mC, $q_2 = 10$ mC, and $r = 0.5$ m

Work done, $W = \frac{1}{4\pi\varepsilon_0 k}\frac{q_1q_2}{r}$

$$= 9 \times 10^9 \times \frac{5 \times 10^{-6} \times 10 \times 10^{-6}}{0.5}$$

$$= 9 \times 10^{-1}\ \text{J}$$

55. (*c*) Here, $q = 4 \times 10^{-3}$ C, $E = 5$ V/m, $t = 10$ s, $m = 2 \times 10^{-3}$

The KE of charged particle $= \dfrac{q^2E^2t^2}{2m}$

$$= \frac{(4 \times 10^{-3})^2 \times (5)^2 \times (10)^2}{2 \times 2 \times 10^{-3}}$$

$\therefore$ KE = 10 J

56. (*d*) Ionisation potential (V) of mercury is the energy required to strip it of an electron. The electric field strength is given by $E = \dfrac{V}{d}$.

where, d is distance between plates creating electric field.

Given, $V = 10.39$ V, $E = 1.5 \times 10^6$ V/m

Distance travelled by electron to gain ionisation energy

$$\therefore \quad d = \frac{V}{E} = \frac{10.39}{1.5 \times 10^6} \text{ m}$$

57. (*a*) By using charge conservation,

$$0.2 \times 600 = (0.2 + 1)\,V$$

$$V = \frac{0.2 \times 600}{1.2} = 100\text{V}$$

58. (*d*) Here, $q = 8 \times 10^{-18}$ C, $C = 100\,\mu\text{F} = 10^{-4}$ F

Potential, $V = \dfrac{q}{C} = \dfrac{8 \times 10^{-18}}{10^{-4}} = 8 \times 10^{-14}$ V

Work done $= \dfrac{1}{2}qV = \dfrac{1}{2} \times 8 \times 10^{-18} \times 8 \times 10^{-14} = 32 \times 10^{-32}$ J

59. (*c*) We have, $K = \dfrac{t}{t - d'}$

$$2 = \frac{1}{1 - d'} \Rightarrow d' = \frac{1}{2} \text{ mm}$$

So now distance $= 3 + \dfrac{1}{2} = 3.5$ mm

60. (*d*) Electric field, $E = -\dfrac{dV}{dr} = -\dfrac{d}{dx}(3x^2) = -6x$

At point $x = 2$

Electric field, $E = -12\ \text{Vm}^{-1}$

61. (*c*) Kinetic energy, $K = \dfrac{1}{2}mv^2 = eV$

$\Rightarrow$ Speed of the electron $v = \sqrt{\dfrac{2eV}{m}}$.

62. (*b*) Kinetic energy,

$$= q_0(V_A - V_B) = 1.6 \times 10^{-19}(70 - 50) = 3.2 \times 10^{-18}\text{J}$$

63. (*c*) By using, $W = Q \cdot \Delta V$

Potential difference,

$$\Delta V = \frac{W}{Q} = \frac{2}{20} = 0.1\text{ V}$$

64. (*b*) By using, KE $= QV \Rightarrow 4 \times 10^{20} \times 1.6 \times 10^{-19} = 0.25 \times V$

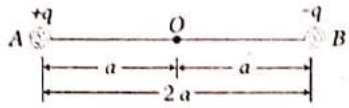
Potential difference, $V = \dfrac{4 \times 10^{20} \times 1.6 \times 10^{-19}}{0.25} = 256\text{V}$

65. (*a*) By using KE $= QV \Rightarrow$ KE $= 1.6 \times 10^{-19} \times 100$

$= 1.6 \times 10^{-17}$ J

66. (*d*) Conducting surface behaves as equipotential surface.

67. (*a*) Potential at O due to charge at A



$$\therefore \quad V_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{a}$$

Potential at O due to charge at B,

$$V_2 = \frac{1}{4\pi\varepsilon_0} \frac{(-q)}{a}$$

Potential at mid-point O,

$$V = \frac{1}{4\pi\varepsilon_0} \frac{q}{a} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{(-q)}{a} = 0$$

68. (*b*) As electric potential of spheres are same, is

$$V_A = V_B$$

$$\therefore \quad \frac{\sigma_A \cdot a}{\varepsilon_0} = \frac{\sigma_B b}{\varepsilon_0} \quad \text{or} \quad \frac{\sigma_A}{\sigma_B} = \frac{b}{a}$$

69. (*a*) Given, $C_1 = 2\mu\text{F}$, $V_1 = 50\text{V}$

$C_2 = 1\mu\text{F}$, $V_2 = 20$ V

Common potential,

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{2 \times 50 + 1 \times 20}{2 + 1} = 40\text{ V}$$

Loss of energy,

$$\Delta v = \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2$$

$$= \frac{1}{2}\frac{2 \times 1}{(2 + 1)}(50 - 20)^2 \times 10^{-6}$$

$$= 300 \times 10^{-6} = 300\mu\text{ J}$$

70. (*c*) A, B, C, D, E lies on equipotential surface. On sphere, the surface potential is same. So,

$$W_{AB} = W_{AC} = W_{AD} = W_{AE} = q(V_f - V_i) = \text{zero}$$

71. (*b*) We have, $E_0 = -\dfrac{dV}{dx}$ or $dV = -E_0\,dx$

On integrating both sides, we get

$$\int dV = -\int E_0\,dx \Rightarrow V_x = -xE_0$$

72. (*a*) Potential energy of charges Q_1 and Q_2 at 10 cm apart

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{12 \times 10^{-6} \times 5 \times 10^{-6}}{0.1}$$

$$= \frac{9 \times 10^9 \times 60 \times 10^{-12}}{0.1} = 54 \times 10^{-1} = 5.4\text{ J}$$

At 6 cm apart

$$\text{PE} = \frac{9 \times 10^9 \times 60 \times 10^{-12}}{0.06} = 9\text{ J}$$

$\therefore$ Work done $= (9 - 5.4) = 3.6$ J

73. (a) Force, $F = qE = q\left(\frac{V}{d}\right)$

$\therefore$ Potential difference between two points

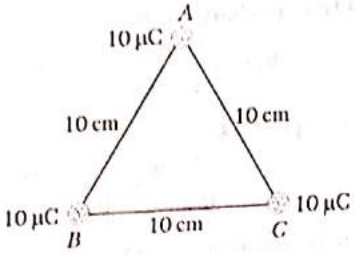
$$V = \frac{F \cdot d}{q} = \frac{(3000)(10^{-2})}{3} = 10\text{ V}$$

74. (b) When charge particle enters in a potential field, then

$$\frac{1}{2}mv^2 = qV$$

$$\therefore \quad v = \sqrt{\left(\frac{2qV}{m}\right)} \quad \therefore \frac{v_A}{v_B} = \sqrt{\frac{q}{4q}} = \frac{1}{2}$$

75. (c)



$$U_{AB} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(10)(10) \times 10^{-12}}{10 \times 10^{-2}}$$

$$U_{BC} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(10)(10) \times 10^{-12}}{10 \times 10^{-2}}$$

$$U_{AC} = \frac{1}{4\pi\varepsilon_0} \frac{(10)(10) \times 10^{-12}}{10 \times 10^{-2}}$$

$$\therefore \quad U_{\text{total}} = U_{AB} + U_{BC} + U_{CA}$$

$$= \frac{3}{4\pi\varepsilon_0}\left[\frac{100 \times 10^{-12}}{10 \times 10^{-2}}\right] = 27\text{ J}$$

76. (b) When charge particle enters in uniform electric field, then force on charged particle, $F = qE$

Also, $\quad F = ma$

$\therefore \quad ma = qE$

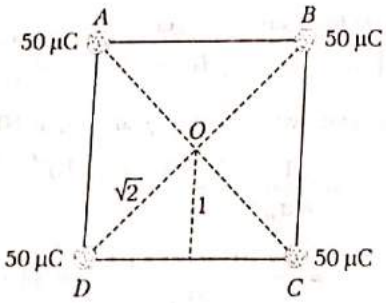
or Acceleration of the particle,

$$a = \frac{qE}{m} = \frac{3 \times 10^{-3} \times 80}{20 \times 10^{-3}} = 12\text{ m/s}^2$$

So, from equations of motion,

$$v = u + at = 20 + 12 \times 3 = 56\text{ m/s}$$

77. (a)



Potential, $V_A = V_B = V_C = V_D = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(50 \times 10^{-6})}{(\sqrt{2})}$$

$\therefore$ Potential at the centre of square,

$$V_0 = 4V$$

$$V_0 = 4 \times \left[\frac{9 \times 10^9 \times 50 \times 10^{-6}}{\sqrt{2}}\right] = 90\sqrt{2} \times 10^4\text{ V}$$

Work done in bringing a charge ($q = 50\ \mu$C) from ∞ to centre of the square is

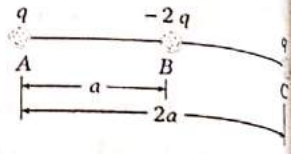
$$\therefore \quad W = qV_0 = 50 \times 10^{-6} \times 90\sqrt{2} \times 10^4 = 64\text{ J}$$

78. (c)

$$U_{AB} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(q)(-2q)}{a}$$

$$U_{BC} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(-2q)(q)}{a}$$

$$U_{CA} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(q)(q)}{2a}$$



Potantial energy of the system

$$\therefore \quad U_{\text{system}} = \frac{1}{4\pi\varepsilon_0} \cdot \left[\frac{-2q^2}{a} - \frac{2q^2}{a} + \frac{q^2}{2a}\right]$$

$$U_{\text{system}} = \frac{-7q^2}{8\pi\varepsilon_0 a}$$

79. (b) When α-particle is accelerated through a potential difference V, then kinetic energy of α-particle

$$K = qV = (2e)V\text{ J} = 2 \times 16 \times 10^{-19} \times 10^6\text{ J}$$

$$= \frac{2 \times 1.6 \times 10^{-19} \times 10^6}{1.6 \times 10^{-19}}\text{ eV} = 2\text{ MeV}$$

80. (d) Since we know that, $K = \frac{p^2}{2m}$

Also, $\quad K = qV \ \therefore \ p = \sqrt{2mqV}$

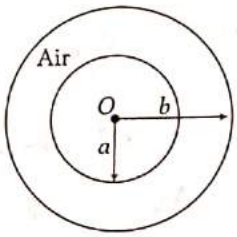
Ratio of momenta,

$$\therefore \quad \frac{p_e}{p_\alpha} = \sqrt{\frac{m_e q_e}{m_\alpha q_\alpha}} = \sqrt{\frac{m_e}{2m_\alpha}} \qquad (\because q_\alpha = 2q_e)$$

81. (d) Velocity, $v = at = \left(\frac{qE}{m}\right)t \Rightarrow v \propto \frac{q}{m}$ or $\text{KE} \propto \frac{q^2}{m^2}$

$\therefore$ The ratio of kinetic energy, $\frac{K_1}{K_2} = \left(\frac{2}{1/2}\right)^2 = 16$

82. (c) Capacity of spherical condenser when outer sphere is earthed,



$$C_1 = 4\pi\varepsilon_0 \cdot \frac{ab}{b-a}$$

Capacity of spherical condenser when inner sphere is earthed

$$C_2 = 4\pi\varepsilon_0 b + \frac{4\pi\varepsilon_0 ab}{b-a}$$

$\therefore$ Difference in their capacity $= C_2 - C_1 = 4\pi\varepsilon_0 b$

83. (*d*) Work done, $W = U_f - U_i$

Here $U_f = 0$

$$\therefore \quad W = -U_i = -\frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_2}{r} + \frac{q_2q_3}{r} + \frac{q_3q_1}{r}\right]$$

$$= -\frac{(9\times10^9)}{0.1}[(1)(-2) + (1)(4) + (-2)(4)]\times10^{-12}$$

$$= 0.54\ \text{J}$$

84. (*d*) $$\frac{1}{2}mv^2 = \frac{q_1q_2}{4\pi\varepsilon_0}\left(\frac{1}{r_i} - \frac{1}{r_f}\right)$$

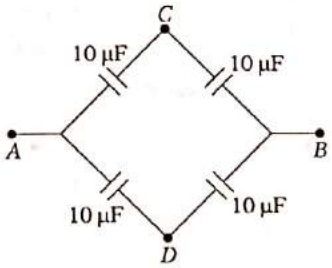
$$\frac{1}{2}\times2\times10^{-3}\times v^2 = (10^{-9})(9\times10^9)(0.9)$$

Speed of particle $v = \sqrt{8.1\times10^3}$ m/s = 90 m/s

85. (*b*) Work done, $W = U_i - U_f = 6\left[\frac{1}{4\pi\varepsilon_0}\cdot\frac{q\cdot q}{r}\right] - 0 = \frac{6q^2}{4\pi\varepsilon_0 r}$

86. (*d*) Given circuit is balanced Wheatstone bridge circuit.

$$\frac{10\times10}{10+10} = 5\,\mu\text{F}$$



$$\frac{10\times10}{10+10} = 5\,\mu\text{F}$$

Their is no current in branch *CD*.

So, equivalent capacitance between *AB*,

$$C_{AB} = 5 + 5 = 10\,\mu\text{F}$$

87. (*a*) When capacitors are connected in series, then

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{3} + \frac{1}{9} + \frac{1}{18}$$

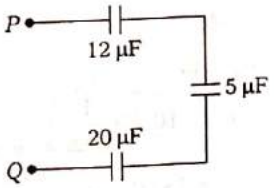
$$\frac{1}{C_S} = \frac{1}{2} \Rightarrow C_S = 2\,\mu\text{F}$$

When capacitors are joined is parallel, then

$$C_P = 3 + 9 + 18 = 30\,\mu\text{F}$$

$$\therefore \quad \frac{C_S}{C_P} = \frac{2}{30} = \frac{1}{15}$$

88. (*b*) In circuit, condenser of capacity 2 μF and 3 μF are in parallel. Their resultant capacity is 5 μF.

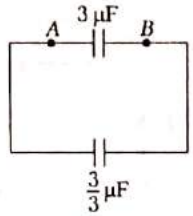


Now, capacitor 12 μF, 5 μF and 20 μF are in series. So, their resultant capacity,

$$\frac{1}{C} = \frac{1}{5} + \frac{1}{20} + \frac{1}{12} = \frac{1}{3}$$

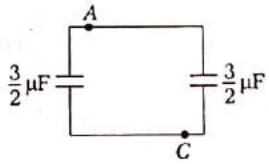
$$\therefore \quad C = 3\,\mu\text{F}$$

89. (*a*) Capacitance between *A* and *B*



$$C_{AB} = 3 + 1 = 4\,\mu\text{F}$$

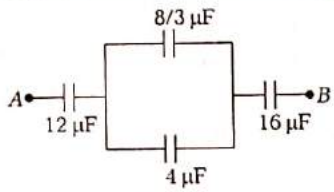
Capacitance between *A* and *C*



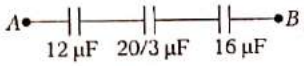
$$C_{AC} = \frac{3}{2} + \frac{3}{2} = 3\,\mu\text{F}$$

$$\therefore \quad \frac{C_{AB}}{C_{AC}} = \frac{4}{3}$$

90. (*d*) Given circuit can be reduced as following.



Step (1)



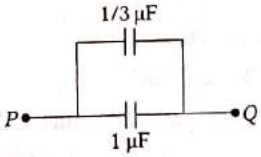
Step (2)

Hence, equivalent capacitance between *A* and *B*

$$\frac{1}{C_{AB}} = \frac{1}{12} + \frac{1}{20/3} + \frac{1}{16}$$

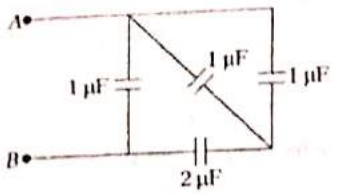
$$\therefore \quad C_{AB} = \frac{240}{71}\,\text{F}$$

91. (*d*) Given circuit can be simplified as shown

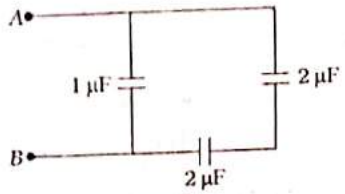


$$\therefore \quad C_{PQ} = \frac{1}{3} + 1 = \frac{4}{3}\,\mu\text{F}$$

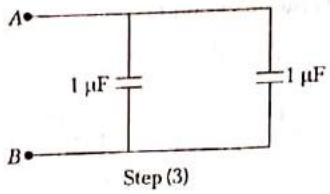
92. (*b*) Given circuit can be simplified as follows



Step (1)



Step (2)

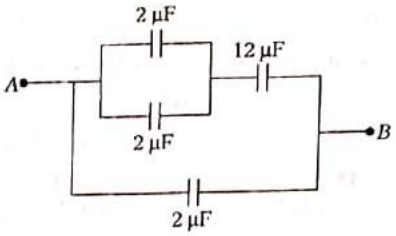


Step (3)

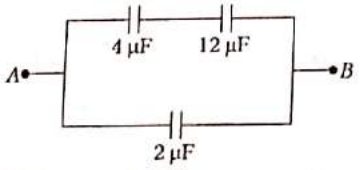
So, equivalent capacitance between A and B,

$$C_{AB} = 1 + 1 = 2\,\mu F$$

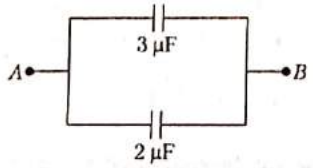
93. (*c*) Given circuit can be expressed as



Step (1)



Step (2)



Step (3)

So, net capacitance between AB,

$$C_{AB} = 3 + 2 = 5\,\mu F$$

94. (*b*) Capacitors C_1 and C_2 are in parallel and, they are in series with C_3, then equivalent capacity between A and B,

$$C = \frac{C_P \times C_3}{C_P + C_3} = \frac{15 \times 4}{15 + 4} = \frac{60}{19} = 3.2\,\mu F$$

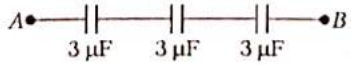
95. (*d*) The situation can be simplified as follows

$\therefore$ Equivalent capacity between A and B,

$$C_{AB} = \frac{2}{1 + 2 + 1} = \frac{2}{4}$$

$$\therefore \quad C_{AB} = 0.5\,\mu F$$

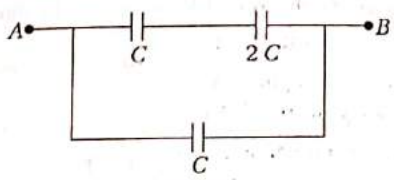
96. (*a*) The circuit can be redrawn as



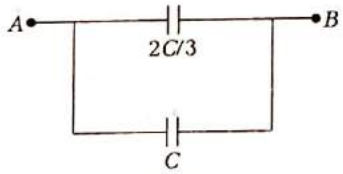
So, the capacitance between the point A and B,

$$C_{AB} = \frac{3}{3} = 1\,\mu F$$

97. (*c*) Given circuit can be redrawn as follows,



Step (1)



Step (2)

Equivalent capacitance between A and B,

$$C_{AB} = \frac{2C}{3} + C = \frac{5}{3}C$$

98. (*d*) $\quad C_S = \dfrac{C_0}{7}$

and $\quad C_P = 7C_0$

$\therefore \quad C_P = 49C_S$

Here, C_0 = capacity of one capacitor.

99. (*b*) Potential, $V = \dfrac{q}{C}$ or $V \propto \dfrac{1}{C}$

V has reduced to $\dfrac{1}{8}$th its original value. Therefore, C has increased 8 times.

100. (*c*) Capacitance, $C = \dfrac{K\varepsilon_0 A}{d}$ or $C \propto \dfrac{K}{d}$

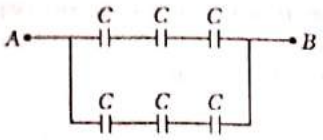
$$\frac{C'}{C} = \frac{Kd_1}{d_2}$$

Here, $K = 4.0$; $\quad d_2 = 2d_1$

$$\therefore \quad C' = (10^{-12})\left(\frac{4.0}{2}\right)$$

$$= 2 \times 10^{-12}\ F$$

101. (*b*)



The effective capacity of the combination,

$$\frac{2C}{3} = 4\,\mu F$$

$\therefore$ Capacitance, $C = 6\,\mu F$

102. (*d*) By such an arrangement, two capacitors in parallel are formed.

$$\therefore \quad C_{net} = 2C = 2\left(\frac{\varepsilon_0 A}{d}\right)$$

103. (*a*) According to the question $8\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi R^3$

$\therefore$ Radius, $R = 2r$

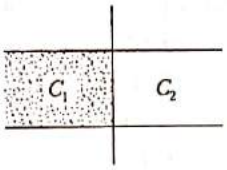
Capacitance, $C = 4\pi\varepsilon_0 Rr$

$$C' = 4\pi\varepsilon_0 2r;\ C' = 2C$$

$\therefore$ Therefore, the capacitance of the bigger drop as campared to each smaller drop is 2 times.

104. (*a*) Capacity of the capactior

$$C = C_1 + C_2 = KC_1 + C_2 \qquad [\because \text{parallel}]$$
$$= 4 \times 5 + 5 = 25\,\mu F$$



105. (*d*) Capacity of capacitor $C = \frac{\varepsilon_0 A}{d}$

$$\Rightarrow \quad C \propto \frac{1}{d}$$

$\therefore$ Capacitance, $C' = 2C$

$\therefore$ Extra charge flow, $q = (2CV - CV) = CV$

$\therefore$ Work done, $W = qV = (CV)V = CV^2$

106. (*b*) $\frac{C'}{C} = \frac{\varepsilon_0 A/(d-t)}{\varepsilon_0 A/d} = \frac{d}{d-t}$

$$\Rightarrow \quad = \frac{d}{(5/7)d} = \frac{7}{5}$$

$\therefore$ Capacitance, $C' = \frac{7}{5} \times 25\,\mu F = 35\,\mu F$

107. (*b*) On introducing dielectric K in a parallel plate capacitor, its capacity becomes,

$$C' = KC_0$$

$$\therefore \quad C' = 5C_0$$

Also, energy stored, $W_0 = \frac{q^2}{2C_0}$

$$\therefore \quad W' = \frac{q^2}{2C'} = \frac{q^2}{2 \times 5C_0}$$

$$\therefore \quad \frac{W_0}{W'} = \frac{5}{1}$$

$$\therefore \quad W' = \frac{W_0}{5}$$

108. (*b*) $\frac{C'}{C} = \frac{\varepsilon_0 A/d-t}{\varepsilon_0 A/d} = \frac{d}{d-t} = \frac{d}{d/2} = 2:1$

$\left(\because \text{in question, thickness is } b = \frac{d}{2}\right)$

109. (*d*) When charging battery is disconnected then charge remains constant, V becomes $\frac{1}{K}$ times and E becomes $\frac{1}{K}$ times.

So, new value of charge is Q_0.

New value of $V = \frac{V_0 \times 3}{9} = \frac{V_0}{3}$

New value of $E = \frac{E_0 \times 3}{9} = \frac{E_0}{3}$

110. (*d*) Here capacitors of capacity $4\,\mu F$, each are in parallel, their equivalent capacity is $8\,\mu F$. Now, there is a combination of three capacitors in series of capacity $20\,\mu F$, $8\,\mu F$ and $12\,\mu F$ so, their resultant capacity,

$$\frac{1}{C} = \frac{1}{20} + \frac{1}{8} + \frac{1}{12}$$

$$\therefore \quad C = \frac{120}{31}\,\mu F$$

Total charge, $Q = CV = \frac{120}{31} \times 300 = 1161\,\mu C$

$\because$ Charge through $4\,\mu F$ capacitor $= \frac{1161}{2} = 580\,\mu C$

and potential difference across $4\,\mu F$ condenser,

$$V = \frac{q}{C} = \frac{580}{4} = 145\text{ V}$$

111. (*c*) As capacitors C_1 and C_2 are in series, then there should be equal charge on them *i.e.*,

$$\text{Charge on } C_1 = \text{Charge on } C_2$$

$$\therefore \quad C_1(V_A - V_D) = C_2(V_D - V_B)$$

$$\text{or} \quad C_1(V_1 - V_D) = C_2(V_D - V_2)$$

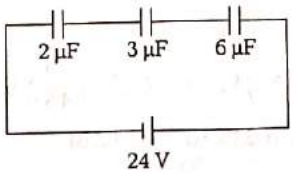
$$\text{or} \quad C_1V_1 - C_1V_D = C_2V_D - C_2V_2$$

$$\text{or} \quad V_D(C_1 + C_2) = C_1V_1 + C_2V_2$$

$\therefore$ The potential difference of point D,

$$V_D = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

112. (*a*) Equivalent capacitance of circuit,



$$C_{eq} = \frac{6}{3+2+1} = 1\,\mu F$$

Total charge, $Q = 1 \times 24 = 24\,\mu C$

Now, potential difference across $6\,\mu F$ capacitor

$$= \frac{24}{6} = 4\,V$$

113. (*c*) Given circuit can be reduced as

Let potential at P be V_P and potential at B be V_B.

As capacitors $3\,\mu F$ and $6\,\mu F$ are in series, they have same charge.

$\therefore$ Charge on $3\,\mu F$ = Charge on $6\,\mu F$

$\therefore$ $C_1V_1 = C_2V_2$

or $3(1200 - V_P) = 6(V_P - V_B)$

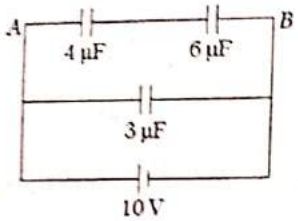
As B point is attached to earth.

So, $V_B = 0$

So, $1200 - V_P = 2V_P$

$\therefore$ $V_P = 400\,V$

114. (*b*) The circuit can be redrawn as



Here, $4\,\mu F$ and $6\,\mu F$ are in series. So, charge is same on both.

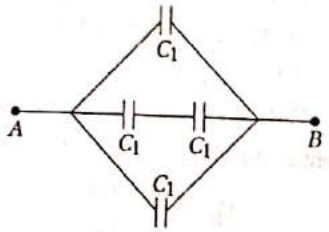
Now, equivalent capacity between A and B,

$$C_{AB} = \frac{6 \times 4}{6+4} = 2.4\,\mu F$$

So, charge on $4\,\mu F$ capacitor,

$$Q = C_{AB} \times 10$$
$$= 2.4 \times 10 = 24\,\mu C$$

115. (*d*) Equivalent capacitance between A and B,



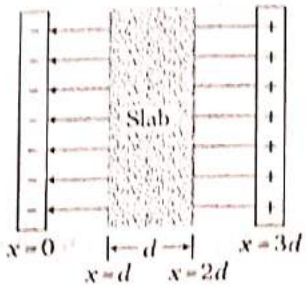
$$C_{AB} = C_1 + \frac{C_1}{2} + C_1 = \frac{5}{2}C_1$$

As charge $Q = CV$

So, $1.5\,\mu C = \frac{5}{2}C_1 \times 6 \Rightarrow C_1 = \frac{15}{15} \times 10^{-6}$

$$= 0.1 \times 10^{-6}\,F = 0.1\,\mu F$$

116. (*b,c*) After the introduction of dielectric slab, direction of electric field remains perpendicular to plate and is directed from positive to negative plate.



Electric field in air $= \frac{\sigma}{\varepsilon_0}$

and electric field in dielectric $= \frac{\sigma}{K\varepsilon_0}$

Positive plate is at higher potential and negative plate is at lower potential. So, electric potential increases continuously as we more from $x = 0$ to $x = 3d$.

117. (*c*) Potential, $V = \frac{q_{net}}{C_{net}} \Rightarrow 40 = \frac{(2)(200)}{2+C} \Rightarrow 80 + 40\,C = 40$

$\therefore$ Capacity of second condenser $C = 8\,\mu F$

118. (*d*) We have, $q_i = q_f$

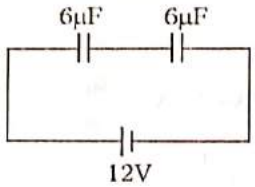
$$C_1V_1 + C_2V_2 = (C_1 + C_2)V$$

$\therefore$ $(2)(4) + (C)5 = (2 + C)4.6$

$$0.4\,C = 1.2$$

$$C = 3 \text{ units}$$

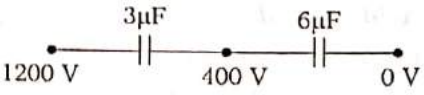
119. (*b*) The simple circuit will be as under.



The potential drop 12 V will be equally distribute.

Potential difference on capacitor of $6\,\mu F$ is 6 V. So, the voltage across a $2\,\mu F$ capacitor is 6 V.

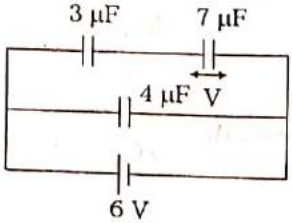
120. (*b*) PD across charge $4\,\mu F$.



$$V_2 = \left(\frac{3}{3+6}\right) \times 1200 = \frac{3}{9} \times 1200 = 400 \text{ volt}$$

$\therefore$ Charge, $q = 4 \times 400 = 1600\,\mu C$

121. (*b*) In parallel, potential difference is same and in series, it distributes in inverse ratio of capacity.



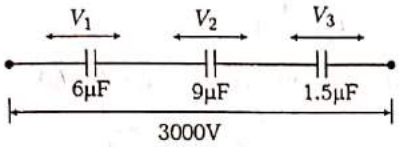
Potential difference

$$V = 6\left(\frac{3}{3+7}\right) = 1.8 \text{ V}$$

$\therefore$ Charge stored in $5\,\mu F$ capacitor will be

$$q = CV = (5\,\mu F)(1.8\,V) = 9\,\mu C$$

122. (*d*) In series, potential difference distributes in inverse ratio of capacities. Hence,



$$V_1 : V_2 : V_3 = \frac{1}{6} : \frac{1}{9} : \frac{1}{1.5} = 1.5 : 1 : 6$$

$$V_2 = (3000)\left(\frac{1}{1+1.5+6}\right) = 352.9 \text{ V}$$

123. (*a*) In the given figure capacitors $3\,\mu F$, $3\,\mu F$ and $3\,\mu F$ (between R and S) are in series. If Q is the charge on each of these capacitors, then

$$30 = \frac{Q}{3\mu} + \frac{Q}{3\mu} + \frac{Q}{3\mu}$$

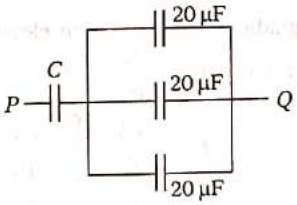
$$Q = 30\,\mu C$$

Potential difference between R and S

$$= \frac{Q}{C} = \frac{30\,\mu}{3\mu} \text{ volt}$$

$$V = 10 \text{ volt}$$

124. (*a*)



$$C' = C_1 + C_2 + C_3$$
$$= 20 + 20 + 20 = 60\,\mu F$$

$$\frac{1}{C''} = \frac{1}{C} + \frac{1}{C'}$$

$$\frac{1}{30} = \frac{1}{C} + \frac{1}{60}$$

$$\frac{1}{C} = \frac{1}{30} - \frac{1}{60} \Rightarrow \frac{1}{C} = \frac{2-1}{60}$$

$$\frac{1}{C} = \frac{1}{60} \Rightarrow C = 60\,\mu F$$

125. (*d*) Potential difference across C is 10 V.

$\therefore$ $q = CV = 60\,\mu C$

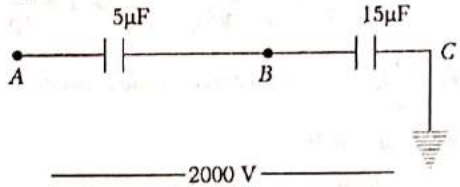
126. (*a*) Potential difference across $4\,\mu F$ capacitor is 9 V and potential difference across $6\,\mu F$ capacitor will be

$(12 - 9)$ V = 3 V.

$\therefore$ Charge stored in capacitor of $6\,\mu F$

$$q = CV = 6 \times 3 = 18\,\mu C$$

127. (*c*) The given circuit can be redrawn as follow



$$(V_A - V_B) = \left(\frac{15}{5+15}\right) \times 2000$$

$$(V_A - V_B) = 1500 \text{ V}$$

$$2000 - V_B = 1500 \text{ V}$$

$$V_B = 500 \text{ V}$$

128. (*b*) Resultant capacitance between A and B

$$C = \frac{(C_1 + C_2) \times C_3}{(C_1 + C_2) + C_3} = \frac{(5+10) \times 4}{(5+10) + 4} = \frac{60}{19} = 3.2\,\mu F$$

129. (*c*) Work done $= \frac{1}{2}\left(\frac{3C}{2}\right)V^2 = \frac{3CV^2}{4}$

130. (*c*) Common potential, $V = \frac{6 \times 20 + 3 \times 0}{(6+3)} = \frac{120}{9}$ V

So, charge on $3\,\mu F$ capacitor

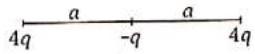
$$Q_2 = 3 \times 10^{-6} \times \frac{120}{9} = 40\,\mu C$$

131. (*b*) The energy stored when the plates are fully charged

$$U = \frac{1}{2}(2C)V^2 = CV^2 = \left(\frac{\varepsilon_0 A}{d}\right)(V^2)$$

132. (*a*) When key is closed, then capacitor becomes short circuited and bulb will light up. But when capacitor becomes fully charged, it act as open circuit and this time bulb will not glow.

133. (*c*) At the present position all the charges are in equilibrium. But when they displaced slightly from their present position, they do not return back. So, they are in unstable equilibrium position.



134. (*d*) We have, $V \propto \frac{q}{R}$ $\left[\because V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{1}{R}\right]$

$\because$ $\sigma = \frac{q}{4\pi R^2}$

$\therefore$ $q \propto \sigma R^2$

or $V \propto \sigma R$

Potential is same $V_1 = V_2$

$\therefore$ $\sigma_1 R_1 = \sigma_2 R_2$

$\Rightarrow$ Ratio potential is same $\frac{\sigma_1}{\sigma_2} = \frac{R_2}{R_1} = \frac{5}{4}$

135. (*c*) $V = \frac{q}{4\pi\varepsilon_0 r} - \frac{q}{4\pi\varepsilon_0 3r} = \frac{q}{6\pi\varepsilon_0 r}$

Electric field intensity at a distance $3r$ is given by

$$E = \frac{q}{4\pi\varepsilon_0 (3r)^2} = \frac{q}{4\pi\varepsilon_0 \cdot 9r^2}$$

Thus, $\frac{E}{V} = \frac{q(6\pi\varepsilon_0 r)}{4\pi\varepsilon_0 9r^2(q)} = \frac{1}{6r} \Rightarrow E = \frac{\varsigma}{6\rho}$

136. (*d*) Area $= \frac{1}{2}QV$ = energy stored in the capacitor.

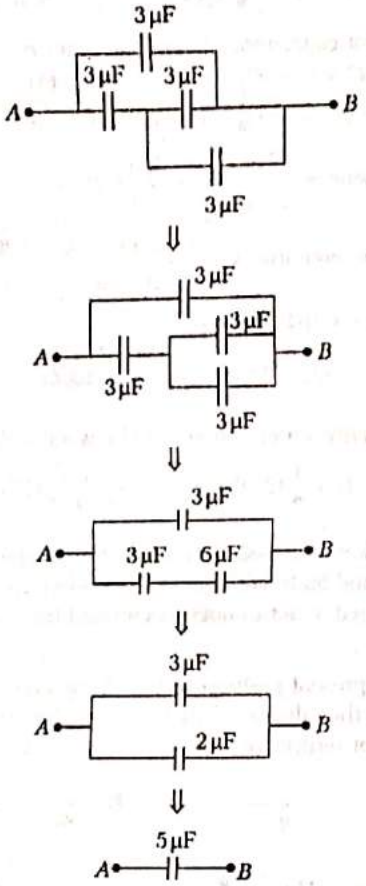
137. (*c*) Charge, $q = nCV$

$$1 = n \times 1 \times 10^{-6} \times 110$$

Number of capacitors, $n = \frac{1}{110 \times 10^{-6}}$

$$n = \frac{100000}{11} \approx 9090$$

138. (*d*)



The effective capacitance between point A and B.

139. (*c*) Charge, $Q = CV = 500\,\mu F \times 10\,V = 5 \times 10^{-3}\,C$

Now, $Q = qt$ or $t = \frac{Q}{q}$

or $t = \frac{5 \times 10^{-3}}{100 \times 10^{-6}} = \frac{1}{20} \times 1000\,s = 50\,s$

140. (*a*) By using, $\frac{1}{2}m(v_1^2 - v_2^2) = QV$

$$\frac{1}{2} \times 10^{-3}[v_1^2 - (0.2)^2] = 10^{-8}(600 - 0)$$

Velocity of the ball at the point A, $v_1 = 22.8\,cms^{-1}$

141. (*d*) The total capacity of capacitor,

$$C = \frac{9 \times 4.5}{13.5}\mu F$$

or $C = 3\mu F$

Charge, $Q = CV = 3 \times 12\,\mu C = 36\,\mu C$

Potential difference across $4.5\,\mu F = \frac{36}{4.5}V = 8V$

142. (*c*) In series combination,

$$\frac{1}{C_1} = \frac{1}{1} + \frac{1}{2} + \frac{1}{4} = \frac{4+2+1}{4} = \frac{7}{4}$$

$$\Rightarrow \quad C_1 = \frac{4}{7}\mu F$$

In parallel combination, $C_1 = 1 + 2 + 4 = 7\mu F$

$$\therefore \quad \frac{C_1}{C_2} = \frac{4/7}{7} = \frac{4}{49}$$

143. (*c*) Electric force, $qE = ma$

$$\Rightarrow \quad a = \frac{qE}{m}$$

$$a = \frac{1.6 \times 10^{-19} \times 1 \times 10^3}{9 \times 10^{-31}}$$

$$= \frac{1.6 \times 10^{15}}{9}\,ms^{-2}$$

Initlally speed, $u = 5 \times 10^6$ and Final speed, $v = 0$

$\therefore$ From $v^2 = u^2 - 2as$

$\Rightarrow$ Distance travelled by electrón, $s = \frac{u^2}{2a}$

$$s = \frac{(5 \times 10^6)^2 \times 9}{2 \times 1.6 \times 10^{15}} = 7\,cm$$

144. (*a*) Potential gradient is related with electric field according to the following relation $E = \frac{-dV}{dr}$.

$$\mathbf{E} = -\frac{\partial V}{\partial r} = \left[-\frac{\partial V}{\partial x}\hat{\mathbf{i}} - \frac{\partial V}{\partial y}\hat{\mathbf{j}} - \frac{\partial V}{\partial x}\hat{\mathbf{k}}\right]$$

$$= [\hat{\mathbf{i}}(2xy + z^3) + \hat{\mathbf{j}}x^2 + \hat{\mathbf{k}}3xz^2]$$

145. (*c*) Potential energy of the system,

$$\frac{-kqQ}{x} - \frac{kQq}{x} + \frac{kq^2}{2x} = 0$$

$$\Rightarrow \quad = \frac{-4kqQ + kq^2}{2x}$$

$$\Rightarrow \quad kq^2 = 4\,kQq$$

Ratio $\frac{q}{Q} = 4$

146. (*c*) Electrostatic potential energy, $U = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r}$

Here, $q_1 = q_2 = 1.6 \times 10^{-19}\,C$

and $r = 9 \times 10^{-15}\,m$

$$\therefore \quad U = \frac{9 \times 10^9 \times 1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}{9 \times 10^{-15}}$$

$$= 2.56 \times 10^{-14}\,J$$

147. (*b*) When capacitors are connected in parallel, then

$$C_P + C + C + C = 3C$$

Acoording to question,

$\therefore$ In series, $C_s = \dfrac{3C \times C}{3C + C} \Rightarrow 3.75 = \dfrac{3C \times C}{4C}$

$\Rightarrow \quad C = 5\,\mu F$

148. (*c*) The work done by a electrostatic force is given by $W_{12} = q(V_2 - V_1)$. Here initial and final potentials are same in all three cases and same charge is moved, so work done is same in all three cases.

149. (*c*) In this problem, the electric field intensity E and electric potential V are related as $E = -\dfrac{dV}{dr}$.

Electric field intensity $E = 0$ suggest that $\dfrac{dV}{dr} = 0$

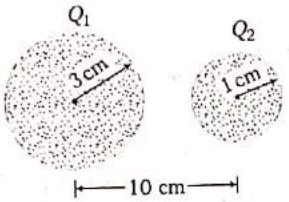
This imply that V = constant.

Thus, $E = 0$ inside the charged conducting sphere causes , the same electrostatic potential 100 V at any point inside the sphere.

Note V equals zero does not necessary imply that $E = 0$ *e.g.*, the electric potential at any point on the perpendicular bisector due to electric dipole is zero but E not.

$E = 0$ does not necessary imply that $V = 0$ *e.g.*, the electric field intensity at any point inside the charged spherical shell is zero but there may exist non-zero electric potential.

150. (*a*) Let Q_1 and Q_2 be the charge on the two conducting spheres of radii $R_1 (= 3\text{ cm})$ and $R_2 (= 1\text{ cm})$, respectively.



$\therefore$ Potential, $V_1 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q_1}{R_1} \Rightarrow 10 = \dfrac{9\times10^9 \times Q_1}{3\times10^{-2}}$

$\Rightarrow$ Charge, $Q_1 = \dfrac{10\times3\times10^{-2}}{9\times10^9}$ ...(i)

and $\quad V_2 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q_2}{R_2} \Rightarrow 10 = \dfrac{9\times10^9 \times Q_2}{1\times10^{-2}}$

$\Rightarrow$ Charge, $Q_2 = \dfrac{10\times1\times10^{-2}}{9\times10^9}$ ...(ii)

According to Coulomb's law,

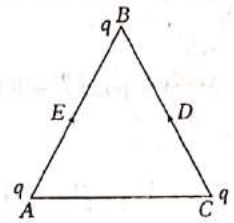
Force of repulsion between them is

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r^2} = \frac{9\times10^9 \times 10\times3\times10^{-2}\times10\times1\times10^{-2}}{9\times10^9\times9\times10^9\times(10\times10^{-2})^2}$$

(using Eqs. (i) and (ii))

$$= \left(\frac{1}{3}\right)\times10^{-9}\text{ N}$$

151. (*a*) We have $AC = BC$



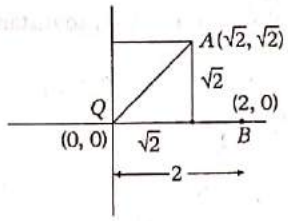
$\therefore \quad V_D = V_E$

Workdone in taking a charge Q from D to E

$$W = Q(V_E - V_D)$$

$$W = 0$$

152. (*b*)



$$AO = \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2}$$

$$AO = \sqrt{4} = 2 \text{ and } BO = 2$$

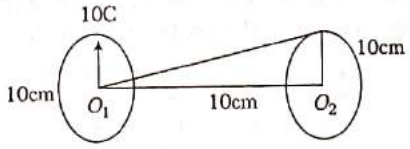
Potential difference the points A and B

$$= V_A - V_B$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{AO} - \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{BO}$$

$$= 0 \qquad [\because AO = BO]$$

153. (*b*)



Potential at the centre of the ring 1 is

$$V_1 = \frac{1}{4\pi\varepsilon_0}\left[\frac{10}{10} + \frac{5}{\sqrt{(10)^2 + (10)^2}}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{10}{10} + \frac{5}{10\sqrt{2}}\right]$$

Potential at the centre of the ring 2 is

$$V_2 = \frac{1}{4\pi\varepsilon_0}\left[\frac{5}{10} + \frac{10}{\sqrt{(10)^2 + (10)^2}}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{5}{10} + \frac{10}{10\sqrt{2}}\right]$$

Work done, $W = q(V_1 - V_2)$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{10}{10} + \frac{5}{10\sqrt{2}} - \frac{5}{10} + \frac{10}{10\sqrt{2}}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{5}{10} - \frac{5}{10\sqrt{2}}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{2} - \frac{1}{2\sqrt{2}}\right] = \frac{q}{8\pi\varepsilon_0}\left[\frac{\sqrt{2}-1}{\sqrt{2}}\right]$$

154. (*b*) As shown in figure component of dipole moment along the line OP will be

$$p' = p\cos\theta$$

Hence, electric potential at point P will be

$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p\cos\theta}{r^2}$$

155. (*b*) With in $r = 2R_0$ total charge is $q/2$.

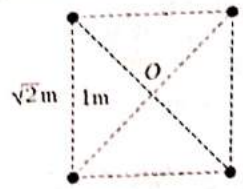
Hence, option (b) is incorrect.

156. (*a*) $U = -pE\cos\theta$

It has minimum value when $\theta = 0°$

$$U_{min} = -pE\cos 0° = -pE$$

157. (*c*) Length of each side of square is $\sqrt{2}$ m, so distance of its centre from each corner is



Potential at the centre,

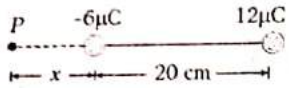
$$V = 9\times10^9\left[\frac{10\times10^{-6}}{1} + \frac{5\times10^{-6}}{1} - \frac{3\times10^{-6}}{1} + \frac{8\times10^{-6}}{1}\right]$$

$$= 1.8\times10^5 \text{ V}$$

158. (*a*) By using $W = Q(\mathbf{E}\cdot\Delta\mathbf{r})$

$$\Rightarrow \quad W = Q[(e_1\hat{\mathbf{i}} + e_2\hat{\mathbf{j}} + e_3\hat{\mathbf{k}})\cdot(a\hat{\mathbf{i}} + b\hat{\mathbf{j}})] = Q(e_1a + e_2b)$$

159. (*c*) Point P will lie near the charge which is smaller in magnitude *i.e.*, -6μC. Hence, potential at P,



$$V = \frac{1}{4\pi\varepsilon_0}\frac{(-6\times10^{-6})}{x} + \frac{1}{4\pi\varepsilon_0}\frac{(12\times10^{-6})}{(0.2\times x)} = 0 \Rightarrow x = 0.2 \text{ m}$$

160. (*a*) Work done, $W = 3\times10^{-6}(V_A - V_B)$, where

$$V_A = 10^{10}\left[\frac{(-5\times10^6)}{15\times10^{-2}} + \frac{2\times10^6}{5\times10^{-2}}\right] = \frac{1}{15}\times10^6 \text{ V}$$

and $$V_B = 10^{10}\left[\frac{(2\times10^{-6})}{15\times10^{-2}} - \frac{5\times10^6}{5\times10^{-2}}\right] = -\frac{13}{15}\times10^6 \text{ V}$$

$$\therefore \quad W = 3\times10^{-6}\left[\frac{1}{15}\times10^6 - \left(-\frac{13}{15}\times10^6\right)\right] = 2.8 \text{ J}$$

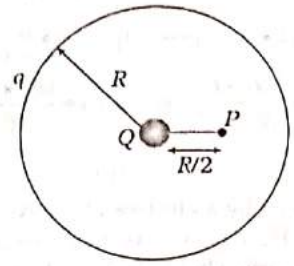
161. (*d*) $E_x = -\dfrac{dV}{dx} = -(-5) = 5$, $E_y = \dfrac{dV}{dy} = -3$

and $E_z = \dfrac{dV}{dz} = -\sqrt{15}$

$$E_{net} = \sqrt{E_x^2 + E_y^2 + E_z^2} = \sqrt{(5)^2 + (3)^2 + (-\sqrt{15})^2} = 7$$

162. (*d*) Electric potential at P;

$$V = \frac{kQ}{R/2} + \frac{kq}{R} = \frac{2Q}{4\pi\varepsilon_0 R} + \frac{q}{4\pi\varepsilon_0 R}$$



163. (*c*) Charge on the 3 μF

$$q_3 = \left(\frac{C_3}{C_2 + C_3}\right)\cdot Q;$$

$$q_3 = \left(\frac{3}{3+2}\right)\times 80 = \frac{3}{5}\times 80 = 48\mu\text{C}$$

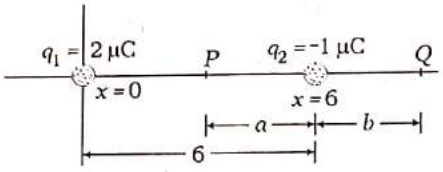
164. (*d*) $C_{air} = \dfrac{\varepsilon_0 A}{d}$, with dielectric slab $C' = \dfrac{\varepsilon_0 A}{\left(d - t + \dfrac{t}{K}\right)}$

Given, $C' = \dfrac{4}{3}C \Rightarrow \dfrac{\varepsilon_0 A}{\left(d - t + \dfrac{t}{K}\right)} = \dfrac{4}{3}\times\dfrac{\varepsilon_0 A}{d}$

$\Rightarrow$ The dielectric constant of the slab

$$K = \frac{4t}{4t - d} = \frac{4(d/2)}{4[(d/2) - d]} = 2$$

165. (*c*) Let potential will be zero at two points P and Q, then



$$\frac{1}{4\pi\varepsilon_0}\times\left[\frac{2\times10^{-6}}{(6-a)} + \frac{(-1\times10^{-6})}{a}\right] = 0$$

$\therefore \quad a = 2$

So, distance of P from origin, $x = 6 - 2 = 4$

At external point Q,

$$\frac{1}{4\pi\varepsilon_0}\times\left[\frac{2\times10^{-6}}{(6+b)} + \frac{(-1\times10^{-6})}{b}\right] = 0$$

$\therefore \quad b = 6$

So, distance of Q from origin, $x = 6 + 6 = 12$

166. (*b*) Let radius of big drop be R and radius of small drops be r,

$\therefore$ Volume of big drop = 8 × volume of small drops

or $\quad \dfrac{4}{3}\pi R^3 = 8\times\dfrac{4}{3}\pi r^3$ or $R = 2r$

Potential of big drop,

$$V = \frac{Q}{C} = \frac{8q}{(8)^{1/3}C} \quad (q \text{ is charge on small drop})$$

$\therefore \quad V = (8)^{2/3} v_{small}$

$\therefore \quad \dfrac{V_{big}}{v_{small}} = (8)^{2/3} = \dfrac{4}{1}$

167. (*c*) We have, $8\left(\dfrac{4}{3}\pi r^3\right) = \dfrac{4}{3}\pi R^3, \Rightarrow R = 2r, \; Q = 8q$

Further, $V = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{q}{r} = 50$ V

and Potential of the large drop $V' = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{Q}{R} = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{8q}{2r}$

$= 4 \times 50 = 200$ V

168. (*a*) $U = \dfrac{1}{4\pi\varepsilon_0}\left[\dfrac{Qq}{R} + \dfrac{qq}{R} + \dfrac{Qq}{R}\right]$

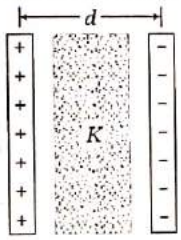
$\because \quad U = 0$

$2Qq + q^2 = 0$ or $Q = -\dfrac{q}{2}$

169. (*b*) Potential of the system

$$V = \frac{\text{Net charge}}{\text{Net capacity}} = \frac{q_1 + q_2}{4\pi\varepsilon_0 (r_1 + r_2)}$$

170. (*c*) Potential difference between plates A and B,



V = Potential difference in air + Potential difference in medium

$V = \dfrac{\sigma}{\varepsilon_0}(d - t) + \dfrac{\sigma}{K\varepsilon_0}t$

$\therefore \; V = \dfrac{\sigma}{\varepsilon_0}\left[d - t + \dfrac{t}{K}\right] = \dfrac{Q}{A\varepsilon_0}\left(d - t + \dfrac{t}{K}\right) \qquad \left(\because \sigma = \dfrac{Q}{A}\right)$

$\therefore$ Capacitance, $C = \dfrac{Q}{V} = \dfrac{Q}{\dfrac{Q}{A\varepsilon_0}\left(d - t + \dfrac{t}{K}\right)}$

$= \dfrac{\varepsilon_0 A}{d - t + \dfrac{t}{K}} = \dfrac{\varepsilon_0 A}{d - t\left(1 - \dfrac{1}{K}\right)}$

171. (*b*) According to question,

Capacity of spherical condenser = Capacity of parallel plate capacitor

$\therefore \quad 4\pi\varepsilon_0 r = \dfrac{\varepsilon_0 A}{d}$

$\therefore \quad d = \dfrac{A}{4\pi r} = \dfrac{\pi R^2}{4\pi r} = \dfrac{\pi(20 \times 10^{-3})^2}{4\pi \times 1} = 0.1$ mm

172. (*a*) Three capacitors are in series therefore, their resultant capacity is given by

$$\frac{1}{C_S} = \frac{1}{\left(\dfrac{\varepsilon_0 K_1 A}{d_1}\right)} + \frac{1}{\left(\dfrac{\varepsilon_0 K_2 A}{d_2}\right)} + \frac{1}{\left(\dfrac{\varepsilon_0 K_3 A}{d_3}\right)}$$

or $\dfrac{1}{C_S} = \dfrac{d_1}{\varepsilon_0 K_1 A} + \dfrac{d_2}{\varepsilon_0 K_2 A} + \dfrac{d_3}{\varepsilon_0 K_3 A}$

$\dfrac{1}{C_S} = \dfrac{1}{\varepsilon_0 A}\left(\dfrac{d_1}{K_1} + \dfrac{d_2}{K_2} + \dfrac{d_3}{K_3}\right)$

$\therefore \quad C_S = \dfrac{\varepsilon_0 A}{\left(\dfrac{d_1}{K_1} + \dfrac{d_2}{K_2} + \dfrac{d_3}{K_3}\right)}$

173. (*b*) $\dfrac{1}{C_{eq}} = \dfrac{1}{C} + \dfrac{1}{2C} + \dfrac{1}{4C} + \ldots\infty$

$A \dashv C \vdash 2C \vdash 4C \vdash \infty B$

$= \dfrac{1}{C}\left[1 + \dfrac{1}{2} + \dfrac{1}{2^2} + \ldots\infty\right] = \dfrac{1}{C}\left[\dfrac{1}{1 - \dfrac{1}{2}}\right] = \dfrac{2}{C}$

The equivalent capacitance between A and B,

$$C_{eq} = \frac{C}{2}$$

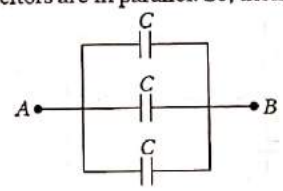
174. (*a*) The given figure is equivalent to two identical capacitors in parallel combination

$\therefore \quad C = \dfrac{\varepsilon_0 A}{d} + \dfrac{\varepsilon_0 A}{d} = \dfrac{2\varepsilon_0 A}{d}$

175. (*b*) Given plates are equivalent to 3 identical capacitors in parallel combination. Hence, equivalent capacitance

$$C_P = C + C + C = 3C = 3\frac{\varepsilon_0 A}{d}$$

176. (*b*) Three capacitors are in parallel. So, their equivalent capacity



$C_P = C + C + C = 3C$

177. (*b*) We have,

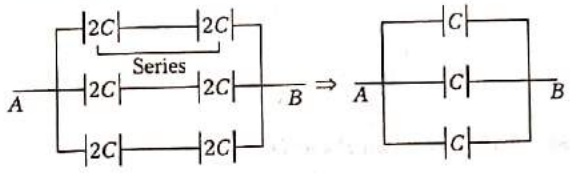
$\dfrac{\varepsilon_0 A}{d} = \dfrac{\varepsilon_0 A}{(d + \Delta d) - t + \dfrac{t}{K}}$ or $K = \dfrac{t}{t - \Delta d}$

Dielectric constant, $K = \dfrac{4 \times 10^{-3}}{4 \times 10^{-3} - 3.5 \times 10^{-3}}$

$K = \dfrac{4 \times 10^{-3}}{0.5 \times 10^{-3}} = \dfrac{40}{5} = 8$

$\Rightarrow \quad K = 8$

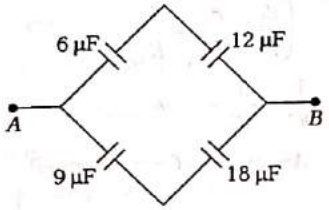
178. (*c*) This diagram is symmetrical about line AB, hence point on perpendicular of AB *i.e.*, CD and D are at same potential, remove capacitor between C and D, D and E.



The equivalent capacitance, $C_{eq} = C + C + C = 3C$

179. (*d*) Given circuit is balanced Wheatstone bridge circuit. Now, capacitor of capacity 6 μF, 12 μF are in series and 9 μF, 18 μF are also in series.

$$\Rightarrow \quad \frac{6\times 12}{6+12} = 4\,\mu F$$

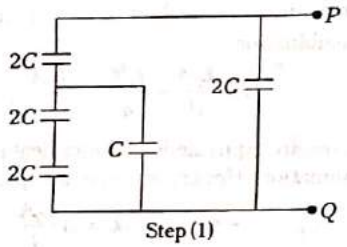


$$\Rightarrow \text{ and } \frac{9\times 18}{9+18} = 6\,\mu F$$

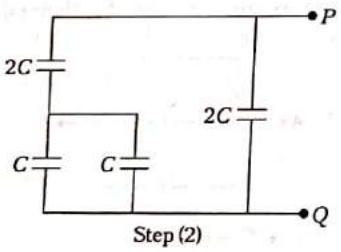
∴ Equivalent capacitance between A and B is

$$C_{AB} = 4 + 6 = 10\,\mu F$$

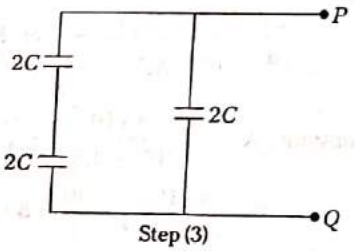
180. (*a*) The given circuit can be simplified in following way.



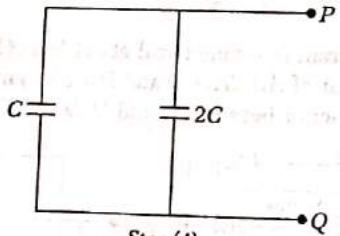
Step (1)



Step (2)



Step (3)

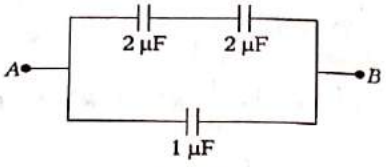


Step (4)

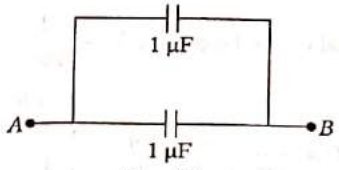
So, capacity between P and Q,

$$C_{PQ} = 2C + C = 3C$$

181. (*b*) Given circuit can be redrawn as follows.



Step (1)



Step (2)

So, equivalent capacitance between A and B,

$$C_{AB} = 1 + 1 = 2\,\mu F$$

182. (*d*) Capacitors C_1 and C_2 are in series with C_3 in parallel with them.

Now,
$$C_1 = \frac{K_1\varepsilon_0(A/2)}{(d/2)} = \frac{K_1\varepsilon_0 A}{d}$$

$$C_2 = \frac{K_2\varepsilon_0(A/2)}{(d/2)} = \frac{K_2\varepsilon_0 A}{d} \quad \text{and} \quad C_3 = \frac{K_3\varepsilon_0 A}{2d}$$

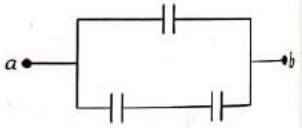
$$C_{\text{equivalent}} = C_3 + \frac{C_1C_2}{C_1 + C_2}$$

$$= \frac{K_3\varepsilon_0 A}{2d} + \frac{\left(\frac{K_1\varepsilon_0 A}{d}\right)\left(\frac{K_2\varepsilon_0 A}{d}\right)}{\frac{K_1\varepsilon_0 A}{d} + \frac{K_2\varepsilon_0 A}{d}}$$

$$= \frac{\varepsilon_0 A}{d}\left(\frac{K_3}{2} + \frac{K_1K_2}{K_1 + K_2}\right)$$

So, none option is correct.

183. (*d*) In the given figure, the metallic plates forms a combination of two capacitors in series with one capacitor in parallel,



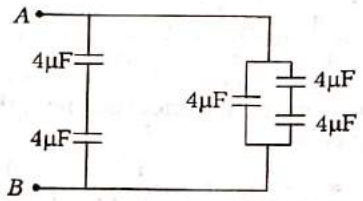
Let capacity of each capacitor be C, then equivalent capacitance between a and b,

$$C_{ab} = \frac{C}{2} + C = \frac{3C}{2}$$

$$\because \quad C = \frac{\varepsilon_0 A}{d}$$

$$\therefore \quad C_{ab} = \frac{3}{2}\frac{\varepsilon_0 A}{d}$$

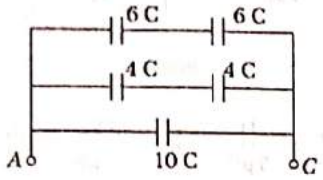
184. (*d*) The simple circuit is as shown below,



$$C_{AB} = (2 + 6)\,\mu F = 8\,\mu F$$

185. (*b*) The given combination is a balanced Wheatstone bridge in parallel with 10 C

Capacitance, $C_1 = \frac{6 \times 6}{6+6} = 3\,C$



Capacitance $C_2 = \frac{4 \times 4}{4+4} = 2\,C$

The equivalent capacitance between the points A and C.

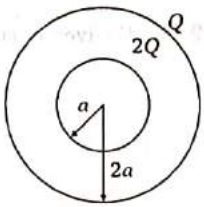
$$C_{AC} = (3+2+10)C \Rightarrow C_{AC} = 15C$$

186. (*b*) Potential difference between two points,

$$V_A - V_B = \int_A^B (2\hat{i} - 4\hat{j}) \cdot (dx\hat{i} + dy\hat{j})$$

$$= [2x - 4y]_{(0,0)}^{(3m,\,4m)} = -10\text{ V}$$

187. (*c*) $\frac{K(2Q)}{2a} + \frac{KQ}{2a} = 5\text{ V}$



or $\frac{K(3Q)}{2a} = 5$

$\therefore$ $\frac{KQ}{a} = \frac{10}{3}$

Now potential of inner sphere,

$$V_m = \frac{K2Q}{a} + \frac{KQ}{2a} = \frac{5}{2}\frac{KQ}{a} = \frac{5}{2}\left(\frac{10}{3}\right) = \frac{25}{3}\text{ V}$$

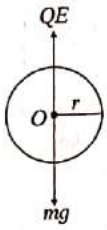
188. (*d*) We have, $V = \frac{kq}{r} \Rightarrow 120 = \frac{Kq}{2}$

$\therefore$ $Kq = 240$ units

When it is made to touch the bigger sphere, whole charge will transfer to bigger sphere.

$\therefore$ $V = \frac{Kq}{6} = \frac{240}{6} = 40\text{ V}$

189. (*b*) Let radius of drop is r. According to Millikan, for balance of drop $QE = mg$ or $Q\frac{V}{d} = \left[\left(\frac{4}{3}\pi r^3\right)\rho\right]g$



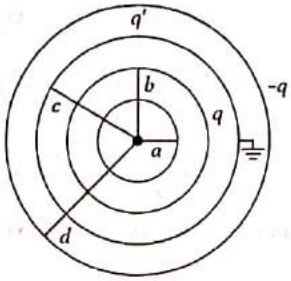
where, V is potential difference and ρ is density of drop.

$\therefore$ $\frac{Q_1}{Q_2} = \left(\frac{r_1}{r_2}\right)^3 \times \frac{V_2}{V_1}$

$\therefore$ $\frac{Q}{Q_2} = \left(\frac{r}{r/2}\right)^3 \times \frac{600}{2400} = 2 \Rightarrow Q_2 = Q/2$

190. (*d*) Shell c is now earthed.

$\because$ $V_c = 0$



$\therefore$ $\frac{K(q+q')}{3a} - \frac{Kq}{4a} = 0$

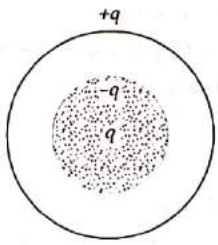
$\therefore$ $q' = -\frac{q}{4}$

Now, potential difference,

$$V_a - V_c = \frac{Kq}{2a} - \frac{Kq/4}{3a} - \frac{Kq}{4a} - 0 = \frac{Kq}{6a}$$

191. (*a*) Potential difference only depends upon the inner charge. So, if the shell is given a charge of $-3Q$, the new potential difference between the same two surface is V.

192. (*a*) The induced charges will be as under.



Potential at point O (the centre),

$$V_0 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r} - \frac{q}{R} + \frac{q}{2R}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{1}{2R}\right]$$

193. (*c*) Potential at point P,

$$V = \frac{KQ}{R} - \frac{Kq}{x} = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R} - \frac{q}{x}\right)$$

194. (*b*) Here circuit is equivalent to two capacitors in parallel

$\therefore$ $C_{eq} = C_1 + C_2$

$$= \frac{\varepsilon_0 A}{d} + \frac{\varepsilon_0 A}{d} = \frac{2\varepsilon_0 A}{d}$$

$\therefore$ Energy stored $= \frac{1}{2}C_{eq}V^2$

$$= \frac{1}{2}\left(\frac{2\varepsilon_0 A}{d}\right)V^2$$

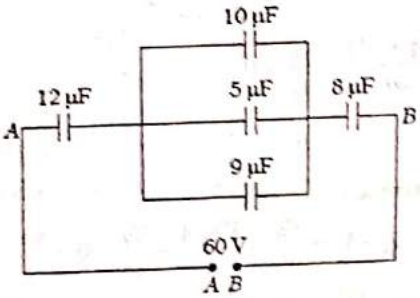
$$= \frac{8.86 \times 10^{-12} \times 50 \times 10^{-4} \times 12 \times 12}{3 \times 10^{-3}}$$

$$= 2.1 \times 10^{-9}\text{ J}$$

195. (*c*) In figure, there is combination of two capacitors in parallel

$$\therefore \quad C_p = C_1 + C_2$$
$$= \frac{K_1\varepsilon_0(A/2)}{d} + \frac{K_2\varepsilon_0(A/2)}{d}$$
$$= \frac{2\varepsilon_0(A/2)}{d} + \frac{4\varepsilon_0(A/2)}{d}$$
$$= 2\times\frac{10}{2} + 4\times\frac{10}{2} = 30\ \mu F$$

196. (*d*) Given circuit can be redrawn as follows.



Equivalent capacitance of the circuit,

$$C_{AB} = \frac{24}{2+1+3} = 4\ \mu F$$

Total charge given by battery,

$$q = C_{AB}\cdot V = 4\times 60 = 240\ \mu C$$

Charge on 5 μF capacitor,

$$q_2 = \left(\frac{5}{10+5+9}\right)\times 240 = 50\ \mu C$$

197. (*a*) We have, $K_i + U_i = K_f + U_f$

$$0 + 0 = \frac{1}{2}mv^2 + (-e)\frac{KQ}{R}$$

$\therefore$ Velocity, $v = \sqrt{\dfrac{2KQe}{mR}}$

198. (*c*) Charge on first capacitor $= C(-V) = -CV$

Charge on second capacitor $= (2C)(2V) = 4CV$

$\therefore$ Total charge on both capacitors

$$= 4CV - CV = 3CV$$

$\therefore$ Common potential on them $= \dfrac{3CV}{3C} = V$

$\therefore$ Energy $= \dfrac{1}{2}(3C)V^2 = \dfrac{3}{2}CV^2$

199. (*b*) Charge, $q_1 = C_1V = (15)\times 100 = 1500\ \mu C$

Charge, $q_2 = C_2V = (1)\times 100 = 100\ \mu C$

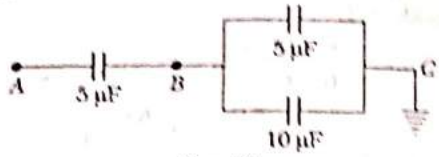
$\therefore$ Net charge, $q_{net} = q_1 + q_2 = 1600\ \mu C$

When dielectric is removed,

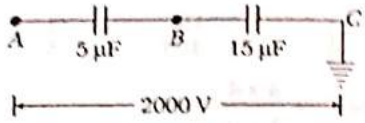
$$C'_1 = \frac{C_1}{K} = \frac{15}{15} = 1\mu F$$

Now, common potential, $V = \dfrac{q}{C'_1 + C_2} = \dfrac{1600}{1+1} = 800$ V

200. (*c*) Given circuit can be redrawn as



Step (1)



Step (2)

Potential defference between A and B

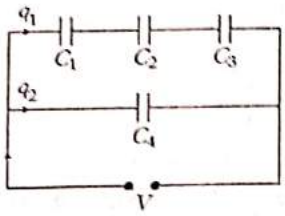
i.e., $\quad V_A - V_B = \left(\dfrac{15}{5+15}\right)\times 2000$

$\therefore \quad V_A - V_B = 1500$ V

$\therefore \quad 2000 - V_B = 1500$ V

$\therefore \quad V_B = 500$ V

201. (*b*) Given circuit can be reduced to



For series combination of C_1, C_2, C_3 resultant capacity,

$$C_S = \frac{6C}{6C + 3C + 2C} \quad \text{or} \quad C_S = \frac{6C}{11}$$

Now the ratio of the charges on C_2 and C_4,

$$\frac{q_1}{q_2} = \frac{C_S}{C_4} = \frac{6C/11}{4C} = \frac{3}{22}$$

202. (*c*) Potential difference between two equipotential surfaces A and B.

$$V_A - V_B = kq\left(\frac{1}{r_A} - \frac{1}{r_B}\right)$$
$$= kq\left(\frac{r_B - r_A}{r_A r_B}\right) = \frac{kqt_1}{r_A r_B}$$

or $\quad t_1 = \dfrac{(V_A - V_B)r_A r_B}{kq}$

or $\quad t_1 \propto r_A r_B$

Similarly, $\quad t_2 \propto r_B r_C$

Since, $r_A < r_B < r_C$, therefore, $r_A r_B < r_B r_C$,

$\therefore \quad t_1 < t_2$

203. (*c*) We know that,

$$qE = mg$$
$$\frac{qQ}{\varepsilon_0 A} = mg$$

or the charge of the oil drop,

$$q = \frac{\varepsilon_0 A m g}{Q} = \frac{8.85\times10^{-12}\times2\times10^{-2}\times2.5\times10^{-7}\times10}{5\times10^{-7}}$$

$$= 8.85\times10^{-13}\text{C}$$

204. (*d*) Current flows through $2\,\Omega$ resistance from left to right, is given by

$$I = \frac{V}{R+r} = \frac{2.5\text{V}}{2+0.5} = 1\text{ A}$$

The potential difference across $2\,\Omega$ resistance,

$$V = IR = 1\times2 = 2\text{V}$$

Since, capacitor is in series with $2\,\Omega$ resistance, so it also has 2 V potential difference across it.

The charge on capacitor,

$$q = CV = (4\mu\text{F})\times2\text{V} = 8\mu\text{C}$$

Note The potential difference across 2Ω resistance solely occurs across capacitor as no potential drop occurs across 10Ω resistance.

205. (*c*) The capacitance of parallel plate capacitor filled with dielectric block has thickness d_1 and dielectric constant K_2 is given by

$$C_1 = \frac{K_1\varepsilon_0 A}{d_1}$$

Similarly, capacitance of parallel plate capacitor filled with dielectric block has thickness d_2 and dielectric constant K_2 is given by

$$C_2 = \frac{K_2\varepsilon_0 A}{d_2}$$

Since, the two capacitors are in series combination, the equivalent capacitance is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

or
$$C = \frac{C_1C_2}{C_1+C_2} = \frac{\frac{K_1\varepsilon_0 A}{d_1}\frac{K_2\varepsilon_0 A}{d_2}}{\frac{K_1\varepsilon_0 A}{d_1}+\frac{K_2\varepsilon_0 A}{d_2}} = \frac{K_1K_2\varepsilon_0 A}{K_1d_2+K_2d_1} \quad \text{...(i)}$$

But the equivalent capacitances is given by

$$C = \frac{K\varepsilon_0 A}{d_1+d_2}$$

On comparing, we have

$$K = \frac{K_1K_2(d_1+d_2)}{K_1d_2+K_2d_1}$$

206. (*c*) Minimum number of condensers in each row $= \frac{3000}{500} = 6$

If C_S is capacity of 6 condensers in a row,

$$\frac{1}{C_S} = \frac{1}{1}+\frac{1}{1}+\frac{1}{1}+\frac{1}{1}+\frac{1}{1}+\frac{1}{1} = 6 \Rightarrow C_S = \frac{1}{6}\mu\text{F}$$

Let there be m such rows in parallel.

Total capacity $= m\times C_s$

$$2 = m\times\frac{1}{6}$$

$\therefore$ $m = 12$

Total number of capacitors $= 6\times12 = 72$

207. (*d*) ***Case* I** When the capacitors are joined in series

$$U_{\text{series}} = \frac{1}{2}\frac{C_1}{n_1}(4V)^2$$

***Case* II** When the capacitors are joined in parallel

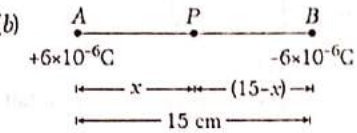
$$U_{\text{parallel}} = \frac{1}{2}(n_2C_2)V^2$$

Given, $U_{\text{series}} = U_{\text{parallel}}$

or
$$\frac{1}{2}\frac{C_1}{n_1}(4V)^2 = \frac{1}{2}(n_2C_2)V^2 \Rightarrow C_2 = \frac{16C_1}{n_2n_1}$$

Note For the equivalent capacitance of the combination is equal to the separation between two plates *i.e.*, $d_1 + d_2$ and dielectric constant K.

208. (*b*)



Let the potential be zero at point P at a distance x, from the charge $+6\times10^{-6}$ C at A as shown in the figure. Potential at P

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{6\times10^{-6}}{x} + \frac{(-4\times10^{-6})}{(15-x)}\right]$$

$$0 = \frac{1}{4\pi\varepsilon_0}\left[\frac{6\times10^{-6}}{x} - \frac{4\times10^{-6}}{15-x}\right]$$

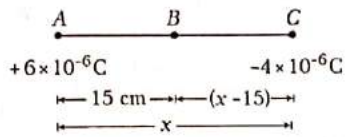
$$0 = \frac{6\times10^{-6}}{x} - \frac{4\times10^{-6}}{15-x}$$

$$\frac{6\times10^{-6}}{x} = \frac{4\times10^{-6}}{15-x} \Rightarrow 6(15-x) = 4x$$

$$90 - 6x = 4x \text{ or } 10x = 90$$

$$\Rightarrow \quad x = \frac{90}{10} = 9\text{ cm}$$

The other possibility is that point of zero potential P may lie on AB produced at a distance x from the charge $+6\times10^{-6}$ C at A as shown in the figure.



Potential at P,

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{6\times10^{-6}}{x} + \frac{(-4\times10^{-6})}{(x-15)}\right]$$

$$0 = \frac{1}{4\pi\varepsilon_0}\left[\frac{6\times10^{-6}}{x} - \frac{4\times10^{-6}}{x-15}\right]$$

$$0 = \frac{6\times10^{-6}}{x} = \frac{4\times10^{-6}}{x-15}$$

$$\Rightarrow \quad \frac{6}{x} = \frac{4}{x-15}$$

$$6x - 90 = 4x \Rightarrow 2x = 90 \text{ or } x = \frac{90}{2} = 45\text{ cm}$$

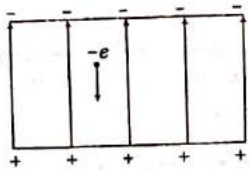
Thus, electrostatic potential is zero at 9 cm and 45 cm from the charge $+6\times10^{-6}$ C at A. Also, potential to be zero at infinity.

209. (*c*) We have, potential difference, $V_A - V_0 = -\int_0^A E_x dx$

Potential difference, $V_A - V_0 = -\int_0^2 30x^2 dx = -30\frac{2^3}{3} = -80$ V

210. (*d*) As the field is upward, so the negatively charged electron experiences a downward force of the magnitude of eE, where E is the magnitude of the electric field.

The acceleration of the electron is $a_e = \frac{eE}{m_e}$.



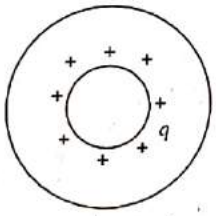
where, m_e is the mass of the electron,

Starting from rest, the time taken by the electron to fall through a distance h is given by

$$t_e = \sqrt{\frac{2h}{a_e}} = \sqrt{\frac{2hm_e}{eE}} = \sqrt{\frac{2\times1.5\times10^{-2}\times9.1\times10^{-31}}{1.6\times10^{-19}\times2\times10^4}}$$

$$= 2.9\times10^{-9}\text{ s}$$

211. (*a*) When a charge density is given to inner cylinder an electric field will be produced between the inner and outer cylinder. Hence a potential difference will appear between the two cylinders.



212. (*d*) We have, $V_x = \frac{2}{x^2 - 4}$

Electric field at $x = 4$,

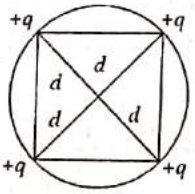
Electric field, $E = \frac{-dV}{dx} = \frac{20}{(x^2-4)^2}(2x - 0) = \frac{160}{144} = \frac{10}{9}$

213. (*b*) Net electrostatic energy, $U = \frac{kQq}{a} + \frac{kq^2}{a} + \frac{kQq}{a\sqrt{2}} = 0$

$$\Rightarrow \quad = \frac{kq}{a}\left(Q + q + \frac{Q}{\sqrt{2}}\right) = 0$$

$\Rightarrow$ The value of $Q = -\frac{2q}{2+\sqrt{2}}$

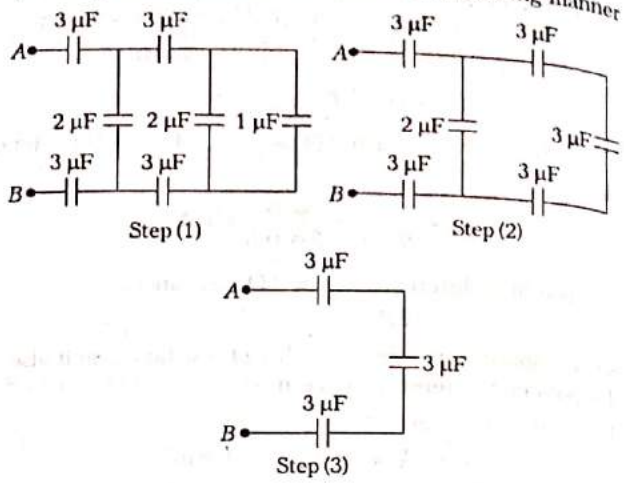
214. (*b*) Potential at centre due to all charges,



$$V = \frac{1}{4\pi\varepsilon_o}\left[\frac{q}{d} + \frac{q}{d} + \frac{q}{d} + \frac{q}{d}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{4q}{d} \text{ in SI unit} = \frac{4q}{d} \text{ in CGS unit}$$

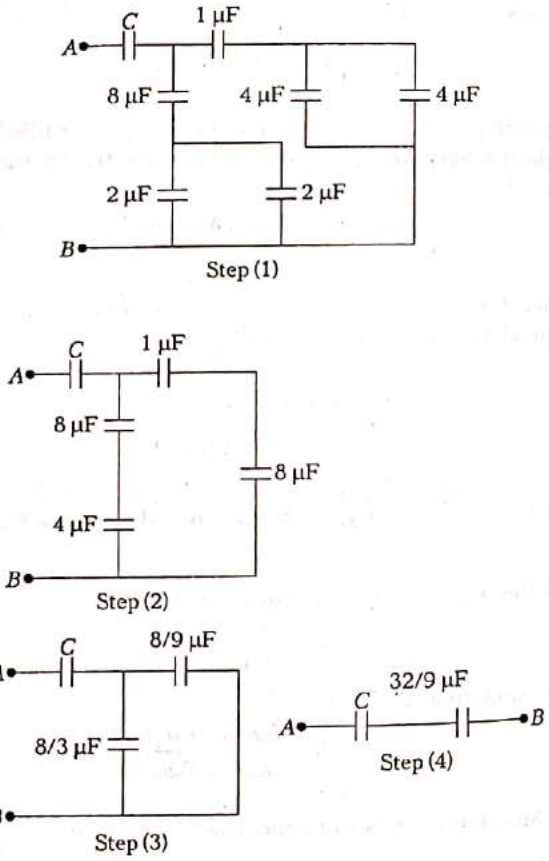
215. (*a*) The given circuit can be reduced in following manner



$\therefore$ Resultant capacity between A and B

i.e., $C_{AB} = 1\mu F$

216. (*d*) Given circuit can be reduced in following manner

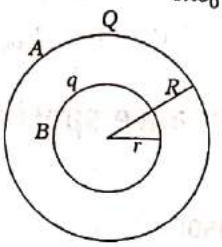


So, equivalent capacitance between A and B,

$$C_{eq} = 1 = \frac{\frac{32}{9}\times C}{\frac{32}{9} + C}$$

$\therefore$ The value of $C = \frac{32}{23}\mu F$

217. (*c*) The potential of Q at the surface $A = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R}$, The potential of q at the surface $A = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R}$



The potential at B is due to Q inside $= \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R}$

The potential at B due to $q = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r}$

$\therefore$ Potential at A, $V_A = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R}+\frac{q}{R}\right)$

Potential at B, $V_B = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R}+\frac{q}{r}\right)$

$\therefore \quad V_B - V_A = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r}-\frac{q}{R}\right)$

218. (*d*) Capacitance, $C = \frac{\varepsilon_0 A}{d}$

When capacitor is half filled with a dielectric capacitance

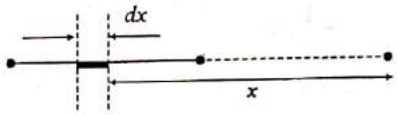
$$C = \frac{\varepsilon_0 A}{2d} + \frac{\varepsilon_0(5A)}{2d} = \frac{\varepsilon_0 A}{2d}(1+5) = \frac{6\varepsilon_0 A}{2d} = \frac{3\varepsilon_0 A}{d}$$

$$\Rightarrow \quad \Delta C = C' - C = \frac{3\varepsilon_0 A}{d} - \frac{\varepsilon_0 A}{d} = \frac{2\varepsilon_0 A}{d}$$

Percentage change in capacitance,

$$\frac{\Delta C}{C} = \frac{\frac{2\varepsilon_0 A}{d}}{\frac{\varepsilon_0 A}{d}} \times 100\% = 200\%$$

219. (*c*) We have, $dQ = \left(\frac{Q}{2l}\right)\cdot dx$



Potential of small part, $dV = \frac{1}{4\pi\varepsilon_0}\cdot\frac{dQ}{x} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{2l}\cdot\frac{dx}{x}$

$\therefore$ Potential at P, $V = \int_{x=l}^{x=3l} dV = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{2l}\ln(3)$

220. (*b*) Work done, $W_{AB} = U_B - U_A = q_0(V_B - V_A)$

Potential, $V_B - V_A = -\int_A^B E\,dr$

$$= -\int_{3a}^{2a}\frac{\lambda}{2\pi\varepsilon_0 r}dr = \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{3}{2}\right)$$

$\therefore$ Work done, $W_{A\to B} = \frac{q_0\lambda}{2\pi\varepsilon_0}\ln\left(\frac{3}{2}\right)$

221. (*a*) $\Delta U = U_f - U_i$

$$= \left[\frac{Kq_1q_2}{0.3}+\frac{Kq_1q_3}{0.4}+\frac{Kq_2q_3}{0.1}\right] - \left[\frac{Kq_1q_2}{0.3}+\frac{Kq_1q_3}{0.4}+\frac{Kq_2q_3}{0.5}\right]$$

$$= 8Kq_2q_3 = 8q_2\frac{q_3}{4\pi\varepsilon_0}$$

According to the question the change in the potential energy is $\frac{q_3}{4\pi\varepsilon_0}K$

$\therefore \quad K = 8q_2$

222. (*c*) $q_A = \frac{Q}{2}$

When *B* is earthed

$$V_B = 0$$

$$\therefore \quad \frac{Kq_B}{a} + \frac{K(Q/2)}{d} = 0$$

$$\therefore \quad q_B = \frac{-Qa}{2d}$$

When *C* is earthed

$$V_C = 0$$

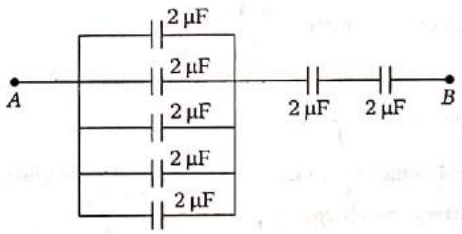
$$\therefore \quad \frac{Kq_C}{a} + \frac{K(Q/2)}{d} - \frac{K(Qa/2d)}{d} = 0$$

$$\therefore \quad q_C = \frac{Qa^2}{2d^2} - \frac{Qa}{2d} = \frac{Qa}{2d}\left(\frac{a}{d}-1\right)$$

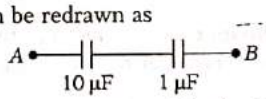
223. (*a*) At unstable equilibrium position potential energy is maximum. But $U = qV$

Therefore C is stable or unstable, it will depend on the sign of q, placed at C.

224. (*a*) From concept of series and parallel combination, we can easily find that in option (a) the resultant capacity is $\frac{10}{11}\mu$F.

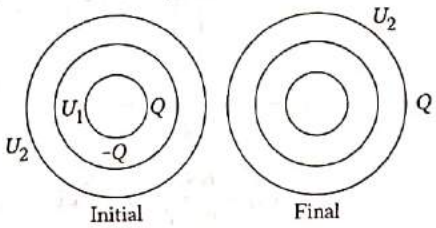


The circuit can be redrawn as



$\therefore$ Equivalent capacity, $C_{eq} = \frac{10\times 1}{10+1} = \frac{10}{11}\mu$F

225. (*c*) Heat produced $= U_i - U_f = (U_1 + U_2) - U_2 = U_1$



Heat produced $= \frac{q^2}{2C} = \frac{q^2}{2\left[4\pi\varepsilon_0\left\{\frac{a\times 2a}{2a-a}\right\}\right]} = \frac{Kq^2}{4a}$

226. (c) $F = \frac{K \cdot q^2}{l^2}$ or $\frac{Kq^2}{l} = F \cdot l = W$

Now, $W = U_f - U_i = \frac{3K \cdot q^2}{2l} - \frac{3Kq^2}{l}$

$$= -\frac{3Kq^2}{2l} = -\frac{3}{2}Fl$$

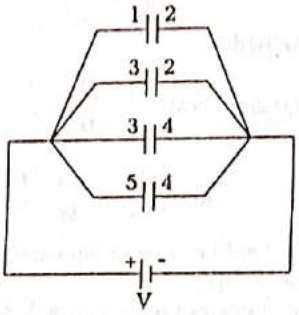
227. (b) Applying charge distribution law on C_1 and C_2 at steady state. Charge on C_1,

$$Q_1 = \left(\frac{C_1}{C_1 + 2C_1}\right) \times Q = \frac{Q}{3}$$

Charge on C_2,

$$Q_2 = \left(\frac{2C_1}{C_1 + 2C_1}\right) \times Q = \frac{2}{3}Q$$

228. (c) Given circuit can be redrawn as shown.



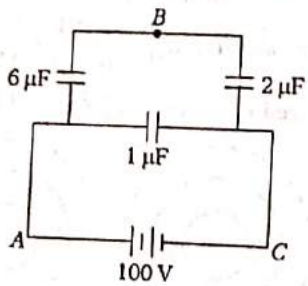
Capacity of each capacitor is $C = \frac{\varepsilon_0 A}{d}$.

So, magnitude of charge on each capacitor = Magnitude of charge on each plate $= \frac{\varepsilon_0 A}{d}V$.

As plate 1 is connected with +ve terminal of battery, so charge on $= +\frac{\varepsilon_0 A}{d} \cdot V$

Plate 4 comes twice and it is connected with positive terminal of battery. So charge on plate 4 $= -\frac{2\varepsilon_0 AV}{d}$

229. (c) The given circuit can be reduced as follows. (Resistance does not matter in considering equivalent capacitance).



$$C_{eq} = \frac{6 \times 2}{6 + 2} + 1 = \frac{5}{2}\mu F$$

Total charge, $Q = CV = \frac{5}{2} \times 100 = 250\ \mu C$

So, charge in 6 μF branch $= CV = \left(\frac{6 \times 2}{6 + 2}\right) \times 100 = 150\ \mu C$

$\therefore$ $V_{AB} = \frac{150}{6} = 25$ V

and $V_{BC} = 100 - V_{AB} = 75$ V

B. Medical entrance special format question

Assertion and reason

2. (e) Electric potential of a charge conductor depents not only on the amount of charge and volume but also on the shape of the conductor. Hence, if their shapes are different, they may have different electric potential.

4. (b) Potential energy of a system of two charges, $U = K\frac{q_1 q_2}{r}$.

$\therefore$ When two positive point charges move away from other then their potential energy decreases and work done by F_{ca} always be expressed in terms of a potential energy, when the particle moves from a point.

5. (d) Positive charge flows from higher potential to lower potential.

6. (d) For non-uniformly charged ring with $Q_{net} = 0$

$$V_x = 0 \text{ but } E_x \neq 0$$

7. (c) Five forces of equal magnitude are acting on $-Q$. When they are added as per polygon law, their vector sum is zero.

8. (d) Battery is disconnected from the capacitor.

So, Q = constant

$$\text{Energy} = \frac{Q^2}{2C} = \frac{Q^2 d}{2\varepsilon_0 A}$$

$\Rightarrow$ Energy $\propto d$

9. (c) Energy supplied by battery $= qV = (CV)V = CV^2$

Energy stored $= \frac{1}{2}CV^2$

$\therefore$ Energy lost $= CV^2 - \frac{1}{2}CV^2$

$$= \frac{1}{2}CV^2$$

Therefore, half energy is lost.

10. (c) Increase in capacity of a capacitor does not depend on the conductivity of conducting plate.

11. (b) $C = \frac{\varepsilon_0 kA}{d} \Rightarrow C \propto \frac{k}{d}$

$$\therefore \quad \frac{C_1}{C_2} = \frac{k_1}{d_1} \times \frac{d_1}{k_2} = \frac{k}{d} \times \frac{d/2}{3k} = \frac{1}{6}$$

$$C_2 = 6C_1$$

Again for capacity of a capacitor $C = \frac{k\varepsilon_0 A}{d}$

Therefore, capacity of a capacitor depends upon the medium between two plates of capacitor.

12. (a) Charge on both will become zero.

$\therefore\ U_f = 0$

Hence, U_i is completely lost.

14. (c) Work done, $W = q = (V_1 - V_2)$

Resultant electric potential at the equatorial potential

$$(V_1 - V_2) = 0$$

$$\therefore \quad W = 0$$

Match the Columns

1. $U \propto \frac{1}{r}$ and $F \propto \frac{1}{r^2}$

2. $q = CV, U = \frac{1}{2}CV^2, C = \frac{K\varepsilon_0 A}{d}$ or $\frac{\varepsilon_0 A}{d-t}$

3. A dielectric slab is then inserted to fill the space between the plates and battery is removed then

Quantity	Battery is removed
Capacity	$C' = Kc$
Charge	$Q' = Q$
Potential	$V' = V/K$
Intensity	$E' = E/K$
Energy	$U' = U/K$

4. (A) Capacitance $C = \frac{\varepsilon_0 A}{d}$

$$= \frac{8.85 \times 10^{-12} \times 0.20}{0.01} = 177 \times 10^{-10}$$

(B) Charge on each plate $q = CV = 3000 \times 177 \times 10^{-10}$

$$= 5.31 \times 10^{-7}$$

(C) $C = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}}$

$$5.31 \times 10^{-10} = \frac{8.85 \times 10^{-12} \times 0.20}{0.01 - 0.01 + \frac{0.01}{K}}$$

$$K = \frac{5.31 \times 10^{-10} \times 0.01}{8.85 \times 10^{-12} \times 0.20}$$

Dielectric constant $= K = 3$

(D) $C = \frac{q}{V} \Rightarrow C = \frac{5.31 \times 10^{-7}}{1000} \Rightarrow C = 5.31 \times 10^{-10}$

C. Medical entrance's gallery

1. (c) Consider the given figure,

When the switch S is connected to point 1, then initial energy stored in the capacitor can be given as

$$= \frac{1}{2}(2\mu F) \times V^2.$$

When the switch S is connected to point 2, energy dissipated on connection across $8\,\mu F$ will be

$$= \frac{1}{2}\left(\frac{C_1 C_2}{C_1 + C_2}\right).V^2 = \frac{1}{2} \times \frac{2\mu F \times 8\mu F}{10\,\mu F} \times V^2$$

$$= \frac{1}{2} \times (1.6\,\mu F) \times V^2$$

Therefore, % loss of energy $= \frac{1.6}{2} \times 100 = 80\%$.

2. (c) Given capacitor is equivalent to capacitors K_1, K_2 and K_3 in parallel and part of K_4 in series with them.

$$\frac{1}{C_{eq}} = \frac{1}{(C_1 + C_2 + C_3)} + \frac{1}{C_4}$$

$$\frac{1}{\frac{K\varepsilon_0 A}{d}} = \frac{1}{\frac{2}{3}[K_1 + K_2 + K_3]\frac{\varepsilon_0 A}{d}} + \frac{1}{2(K_4)\frac{\varepsilon_0 A}{d}}$$

$$\frac{2}{K} = \frac{3}{[K_1 + K_2 + K_3]} + \frac{1}{K_4}$$

3. (d) When a parallel plate air capacitor connected to a cell of emf V, then charge stored will be

$$q = CV$$

$$\Rightarrow \quad V = \frac{q}{C}$$

Also energy stored is $U = \frac{1}{2}CV^2 = \frac{q^2}{2C}$

As the battery is disconnected from the capacitor, the charge will not be destroyed *i.e.*, $q' = q$ with the introduction of dielectric in the gap of capacitor, so the new capacitance will be

$$C' = CK$$

$$\Rightarrow \quad V' = \frac{q}{C'} = \frac{q}{CK}$$

The new energy stored will be

$$U' = \frac{q^2}{2CK}$$

$$\Delta U = U' - U = \frac{q^2}{2C}\left(\frac{1}{K} - 1\right) = \frac{1}{2}CV^2\left(\frac{1}{K} - 1\right)$$

Hence, option (d) is incorrect.

4. (a) The equivalent capacitance between the points P and N is given by

$$\frac{1}{C_{eq}} = \frac{1}{(10 + 20)} + \frac{1}{30} = \frac{2}{30} = \frac{1}{15}$$

$$\therefore \quad C_{eq} = 15\,\mu F$$

Charge on equivalent capacitor having capacitance $15\,\mu F$ is

$$Q = C_{eq}V = 15 \times 30 = 450\,\mu C$$

5. (b) As, potential of spherical charge (Q) having radius r is given by $V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r}$

Also, capacitance, $C = \frac{Q}{V} = \frac{Q}{\frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r}}$

$$\therefore \quad C = 4\pi\varepsilon_0 r$$

If $C = 50 \times 10^{-12}$ F, then

Radius of the sphere, $r = \dfrac{50 \times 10^{-12}}{4\pi\varepsilon_0}$

$$= 50 \times 10^{-12} \times 9 \times 10^{9} = 50 \times 9 \times 10^{-3}$$

$$= 450 \times 10^{-3}\ \text{m} = 0.450\ \text{m} = 45\ \text{cm}$$

6. (*d*) There are 91 plates forming 90 capacitors.
All the capacitors are connected in parallel.
The equivalent capacitance is

$$C_{eq} = (90)\,C = (90)\,(3\,\text{pF}) = 270\ \text{pF}$$

7. (*b*) We have, $mg = qE$

$$q = \frac{mg}{E} = \frac{mg}{V/d} = \frac{1.96 \times 10^{-15} \times 10}{(400/0.02)}$$

$$= \frac{1.96 \times 10^{-14}}{2 \times 10^{4}} = 0.98 \times 10^{-18}$$

$$= 9.8 \times 10^{-19} = 6e$$

8. (*b*) At distance of closest approach, total energy of particle is converted into potential energy.
Let charge on α-particle is q_1 and charge on nucleus is q_2, then in first case

$$K = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_0} \quad \text{...(i)}$$

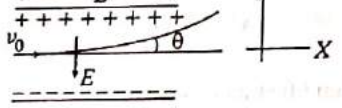
In second case, (let distance of closest approach is r_0')

$$2K = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_0'} \quad \text{...(ii)}$$

On dividing Eq. (ii) by Eq. (i), we get

$$\frac{2K}{K} = \frac{\dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1 q_2}{r_0'}}{\dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1 q_2}{r_0}} = \frac{1}{r_0'} \times \frac{r_0}{1} = \frac{r_0}{r_0'} \Rightarrow r_0' = \frac{r_0}{2}$$

9. (*b*) Initial velocity of the electron along *x*-direction is given by



$$u_x = v_0$$

As applied electric field is vertical

$$v_x = u_x = v_0 \Rightarrow u_y = 0 \text{ and } v_y = u_y + a_y t$$

or $$v_y = 0 + \frac{eE}{m} \times \frac{L}{v_0} \quad (\because L = v_0 t)$$

or $$v_y = \frac{eEL}{mv_0}$$

We can write $\tan\theta = \dfrac{v_y}{v_x}$

$$\Rightarrow \quad \tan\theta = \frac{eEL}{mv_0} \times \frac{1}{v_0} = \frac{eEL}{mv_0^2}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{eEL}{mv_0^2}\right)$$

10. (*a*) Energy stored in a parallel plate capacitor of capacitance and separated by a distance *d* is

$$V_0 = \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)V^2$$

If a slab of dielectric constant *K* and thickness *d* is introduc between plates of capacitor, then

$$C' = \frac{\varepsilon_0 A}{d - d\left(1 - \dfrac{1}{K}\right)} = \frac{K\varepsilon_0 A}{d}$$

∴ The new energy of the system is given by

$$U' = \frac{1}{2}C'V^2 = \frac{1}{2}\left(\frac{K\varepsilon_0 A}{d}\right)V^2 = K\left(\frac{1}{2}\frac{\varepsilon_0 A}{d}V^2\right) = KV_0$$

⇒ The new energy of the system, $U' = KV_0$

11. (*a*) The time constant,

$$\tau = CR = 100 \times 2 = 200\ \mu\text{s}$$

The charge on capacitor after time *t*,

$$q = \varepsilon C\,(1 - e^{-t/RC})$$

According to question, $q = 0.99\,\varepsilon C \Rightarrow 0.99 = 1 - e^{-t/200\,\mu s}$

$$\Rightarrow \quad \frac{t}{200\ \mu\text{s}} = \ln(0.01)$$

This gives $t = 920\ \mu\text{s} = 0.92$ ms

12. (*b*) The equivalent capacitances of each row of the capacitor are

$$1, \frac{1}{2}, \frac{1}{4} \ldots, \infty$$

These capacitors are in parallel combination.
So, the equivalent capacitance,

$$C_{eq} = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots \infty$$

$$= \frac{1}{1 - \dfrac{1}{2}} \quad \left(\because \text{sum of infinite GP series, } S = \frac{a}{1-}\right.$$

$$= 2\,\mu\text{F}$$

13. (*c*) After separation charge is constant.

Capacity, $C = \dfrac{\varepsilon_0 A}{d}$

Capacitance decreases with increase in distance and $V = \dfrac{Q}{C}$

Potential increases with decrease in capacitance (*C*).

14. (*a*) Inside the spherical shell potential is same

$$\Rightarrow \quad V_1 = V_2 \Rightarrow V \propto \frac{1}{d}$$

i.e., $$V_1 = V_2 > V_3$$

15. (*b*) $s = ut + \dfrac{1}{2}at^2 = \dfrac{1}{2}at^2$ or $t = \sqrt{\dfrac{2S}{a}}$

As *s* is same.

$$\therefore \quad t \propto \frac{1}{\sqrt{a}} \Rightarrow \frac{t_2}{t_1} = \sqrt{\frac{a_1}{a_2}} = \sqrt{\frac{F_e/M_e}{F_p/M_p}} = \sqrt{\frac{M_p}{M_e}}$$

16. (*c*) When length x of the dielectric slab is placed between the plates of the capacitor, then

$$C_1 = \frac{K\varepsilon_0 t x}{d} \qquad \left(\because C = K\frac{\varepsilon_0 A}{d}\right)$$

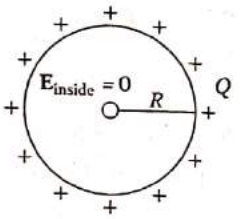
and similarly $C_2 = \frac{\varepsilon_0 t(L-x)}{d}$

$\therefore$ Net capacitance, $C_{net} = C_1 + C_2 = \frac{\varepsilon_0 t}{d}[L + x(K-1)]$

Thus, energy stored in the capacitor,

$$U = \frac{1}{2}CV^2 = \frac{1}{2}C_{net}V^2 = \frac{\varepsilon_0 t V^2}{2d}[L + x(K-1)]$$

17. (*b*) $\mathbf{E}_{inside} = 0$ and $V_{inside} = V_{surface} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$



18. (*a*) Potential due to a uniformly charged sphere is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} \qquad ...(i)$$

Also, $$\sigma = \frac{Q}{4\pi R^2} = \frac{Q\varepsilon_0}{4\pi\varepsilon_0 R^2}$$

$$\frac{\sigma R^2}{\varepsilon_0} = \frac{1\times Q}{4\pi\varepsilon_0} \qquad ...(ii)$$

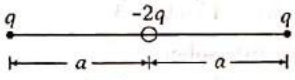
From Eqs. (i) and (ii), we get $V = \frac{\sigma R}{\varepsilon_0}$

Potential due to sphere (1), $V_1 = \frac{\sigma R}{\varepsilon_0}$

Potential due to sphere (2), $V_2 = \frac{\sigma r}{\varepsilon_0}$

Net potential at centre, $V = V_1 + V_2 = \frac{\sigma}{\varepsilon_0}(R + r)$

19. (*d*) Potential energy of the system,



$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q(-2q)}{a} + \frac{q(-2q)}{a} + \frac{qq}{2a}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{-2q^2}{a} - \frac{2q^2}{a} + \frac{q^2}{2a}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{-4q^2}{a} + \frac{q^2}{2a}\right] = \frac{-7q^2}{8\pi\varepsilon_0 a}$$

20. (*c*) Let the charge on the inner shell be q'.

The total charge will be zero.

So, $$\frac{Kq'}{r_1} + \frac{Kq}{r_2} = 0 \qquad (\because r_2 > r_1)$$

$$\Rightarrow \quad q' = \frac{-Kq/r_2}{K/r_1} \Rightarrow q' = -\left(\frac{r_1}{r_2}\right)q$$

21. (*c*) Given, $q = 3\text{ nC} = 3\times10^{-9}$ C

and $r = 9\text{ cm} = 9\times10^{-2}$ m

We know that, potential due to point charge is given by

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} = \frac{9\times10^9\times3\times10^{-9}}{9\times10^{-2}} = 3\times10^2$$

$\Rightarrow$ Electric potential, $V = 300$ V

22. (*b*) Potential, $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

$$\therefore \quad V = \frac{(9\times10^{9})\times(5\times10^{-8})}{0.1} = 4500\text{ V}$$

23. (*d*) The combination of three charges in series,

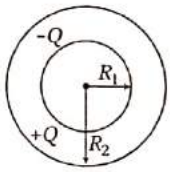
$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \Rightarrow \frac{1}{C} = \frac{1}{3} + \frac{1}{6} + \frac{1}{6} \Rightarrow C = \frac{6}{4} = 1.5\,\mu\text{F}$$

The charge of this circuit, $q = CV = 1.5\times120 = 180\,\mu$C

The potential difference across the $3\,\mu$F

$$q = CV, \quad V = \frac{q}{C} = \frac{180}{3} = 60\text{ V}$$

24. (*c*) We know that,



Concentric spherical shells ($R_2 > R_1$),

$$V_{R_1} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R_1} \qquad ...(i)$$

$$V_{R_2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R_2} \qquad ...(ii)$$

The common potential of the capacitors,

$$V = V_{R_1} - V_{R_2} = \frac{1}{4\pi\varepsilon_0}\cdot Q\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

If the capacitance of two concentric spherical shell be C, then

$$C = \frac{Q}{V} = \frac{Q}{\frac{Q}{4\pi\varepsilon_0}} = \frac{Q}{\frac{Q}{4\pi\varepsilon_0}\left(\frac{R_2 - R_1}{R_2R_1}\right)} = 4\pi\varepsilon_0\frac{R_1R_2}{(R_2 - R_1)}$$

25. (*a*) Given, $C_1 = 10\text{ pF} = 10\times10^{-12}$F

$C_2 = 20\text{ pF} = 20\times10^{-12}$F

$V_1 = 200$ V and $V_2 = 100$ V

where, C_1 = capacitance of 1st capacitor

C_2 = capacitance of 2nd capacitor

V_1 = voltage across 1st capacitor

V_2 = voltage across 2nd capacitor

We know that, $V_1 = \frac{q_1}{C_1}$ and $V_2 = \frac{q_2}{C_2} \Rightarrow q_1 = V_1C_1$ ...(i)

$q_2 = V_2C_2$ ...(ii)

So, common potential of capacitors,

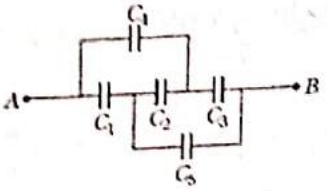
$$V = \frac{q_1+q_2}{C_1+C_2} = \frac{V_1C_1+V_2C_2}{C_1+C_2}$$

$$= \frac{200\times10\times10^{-12}+100\times20\times10^{-12}}{10\times10^{-12}+20\times10^{-12}}$$

$$= \frac{200\times10+100\times20}{10+20}$$

$$= \frac{2000+2000}{30} = \frac{4000}{30} = 133.3\text{ V}$$

26. (*b*)



Clearly, the capacitors form Wheatstone bridge arrangement.

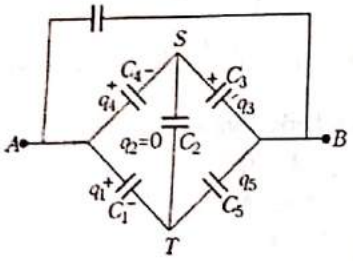
$$C_1 = C_2 = C_3 = C_4$$

$$\Rightarrow \quad \frac{C_4}{C_1} = \frac{C_3}{C_5}$$

By symmetry, $q_1 = q_4$, $q_3 = q_5$

Also, $q_1 + q_4 = q_3 + q_5$

$\Rightarrow \quad q_1 = q_4 = q_5 = q_3 \Rightarrow q_2 = 0$



Hence, the bridge is balanced. The point S and T are in the same potential. No charge can accumulate on C_2 which thus become ineffective.

Between A and B two series combination (C_4, C_3) and (C_1, C_5) are connected in parallel.

Then, effective capacitance,

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{4} + \frac{1}{4} \Rightarrow C' = \frac{4}{2} = 2\mu\text{F}$$

Similarly, $C'' = 2\mu\text{F}$

Hence, equivalent capacitance between A and B is

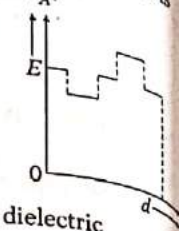
$$C = C' + C'' = 2 + 2 = 4\mu\text{F}$$

Due to balanced Wheatstone bridge arrangement, charge q on C_2 is zero.

Hence, net capacitance between A and B will be 4μF.

27. (*b*) The electric field maximum at B, because electric field is directed along decreasing potential $V_B > V_C > V_A$.

28. (*b*) Graph 2 will be the right graph, of the electric field inside the dielectrics will be less than the electric field outside the dielectrics. The electric field inside the dielectric could not be zero.



As, $K_2 > K_1$ the drop in electric field for K_2 dielectric must be more than K_1.

29. (*d*) As the capacitors are identical,

$$\therefore \quad C_1 = C_2 = C$$

$$C_3 = \frac{C_1C_2}{C_1+C_2} = \frac{C\cdot C}{2C} = \frac{C}{2}$$

$$C_P = C + C = 2C$$

$$\therefore \quad \frac{C_S}{C_P} = 1:4$$

30. (*d*) Given, $V_1 = 120$ V and $V_2 = 200$ V

Here given potential is zero for each capacitor, then

$$\frac{C_1V_1 - C_2V_2}{C_1+C_2} = 0;\ C_1V_1 = C_2V_2;$$

$$120\,C_1 = 200\,C_2$$

$$12\,C_1 = 20\,C_2;\ 3\,C_1 = 5\,C_2$$

31. (*b*) As $F = mg = qE$

$$\therefore \quad E = \frac{mg}{q} \text{ or } q = \frac{mg}{E}$$

Also, $E = \frac{V}{d}$

$$\therefore \quad q = \frac{mgd}{V} = \frac{(10^{-6}\times10\times1\times10^{-3})}{500} = 2\times10^{-11}\text{ C}$$

32. (*c*) When two metal spheres are joined by a wire, charge flows from one at higher potential to the other at lower potential, till they acquire the same potential.

$$\therefore \quad V_1 = V_2$$

$$\frac{q_1}{4\pi\varepsilon_0 r_1} = \frac{q_2}{4\pi\varepsilon_0 r_2}$$

$$\Rightarrow \quad \frac{r_1}{r_2} = \frac{q_1}{q_2} = \frac{15\text{ mC}}{45\text{ mC}} = \frac{1}{3}$$

Final charge on first sphere,

$$q_1 = \frac{1}{3}(q_1+q_2) = \frac{1}{3}\times60 = 20\text{ mC}$$

33. (*a*) Electric potential at their common centre is given by

$$V = \frac{\sigma}{\varepsilon_0}(R+r)$$

where, σ is surface charge density.

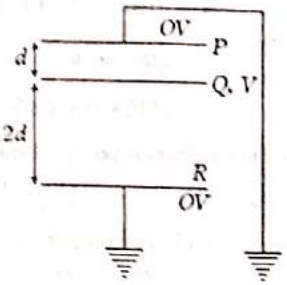
34. (*c*) In the given arrangement, plate Q is common for two capacitors which are connected in parallel.

$$\therefore \quad C_{\text{eff}} = C_P = C_1 + C_2$$

$$\Rightarrow \quad C_P = \frac{\varepsilon_0 A}{d} + \frac{\varepsilon_0 A}{2d} = \frac{3\varepsilon_0 A}{2d}$$

The given arrangement can be shown as

Let V be the potential difference across the capacitor, which is equal to the potential of the plate Q.



$$\text{Potential } V = \frac{Q}{C_p} = \frac{8.85\times10^{-8}}{\frac{3}{2}\frac{\varepsilon_0 A}{d}} = \frac{2d\times8.85\times10^{-8}}{3\times8.85\times2}$$

$$= \frac{2\times2\times10^{-3}\times8.85\times10^{-8}}{3\times8.85\times10^{-12}\times2} = 6.67$$

35. (*d*) We know that the potential,

$$V = \frac{q}{4\pi\varepsilon_0 r}$$

Here, $r_1 = r$ and $r_2 = 2r$

So, $V_1 = \frac{q}{4\pi\varepsilon_0 r}$ and $V_2 = \frac{q}{4\pi\varepsilon_0(2r)}$

Hence $\frac{V_1}{V_2} = \frac{q}{4\pi\varepsilon_0 r}\times\frac{4\pi\varepsilon_0(2r)}{q} \Rightarrow \frac{V_1}{V_2} = \frac{2}{1}; V_2 = \frac{V_1}{2}$

In question $V_1 = 12\,V$ (given)

So, $V_2 = \frac{12}{2} = 6V$

36. (*c*) Let the charge on the inner sphere be Q, then the charge induced on the inner surface of the outer sphere is $-Q$.

$\therefore$ Electric potential V of the inner sphere is given by

$$V = \frac{Q}{4} - \frac{Q}{6}$$

But $V = 3$

$\therefore$ $3 = \frac{Q}{4} - \frac{Q}{6}$

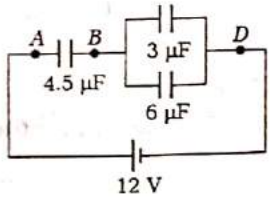
$\Rightarrow$ $Q\left[\frac{1}{4} - \frac{1}{6}\right] = 3$

$\Rightarrow$ $\frac{Q}{12} = 3$

$\Rightarrow$ Charge on the inner sphere $Q = 36$ esu

37. (*c*) The given circuit capacitance between B and D is

$$C_1 = (3+6)\,\mu F = 9\mu F$$



Capacitance between A and D,

$$\frac{1}{C_2} = \frac{1}{4.5} + \frac{1}{9} = \frac{2+1}{9} = \frac{3}{9} = \frac{1}{3}$$

$\Rightarrow$ $C_2 = 3\mu F$

Charge on $C_2 = (3\mu F)(12\,V) = 36\mu C$

So, the potential difference between A and B

$$= \frac{\text{Charge}}{\text{Capacitance}} = \frac{36\,\mu C}{4.5\,\mu F} = 8\,V$$

38. (*d*) It will act like three capacitors which are connected in parallel order, so equivalent capacitance,

$$C = C_1 + C_2 + C_3 = 3 + 3 + 3 = 9\mu F$$

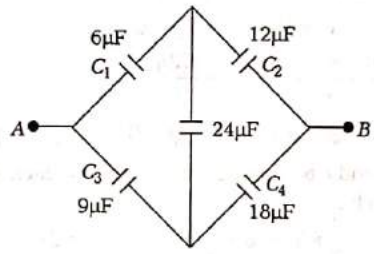
39. (*d*) $C_{13} = C_1 + C_3 = 9 + 9 = 18\mu F$

$$C_{2\text{-}13} = \frac{C_2\cdot C_{13}}{C_2 + C_{13}} = \frac{9\times18}{9+18} = 6\,\mu F$$

Equivalent capacitance between the point A and B.

$$C_{eq} = C_{2\text{-}13} + C_4 = 6 + 9 = 15\,\mu F$$

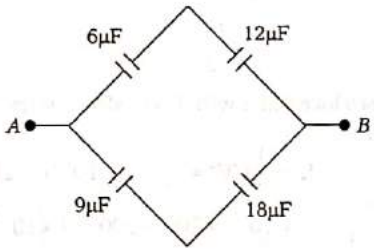
40. (*a*) The given circuit can be redrawn as



It is a balance Wheatstone bridge type network

i.e., $\frac{C_1}{C_2} = \frac{C_3}{C_4} = \frac{1}{2}$

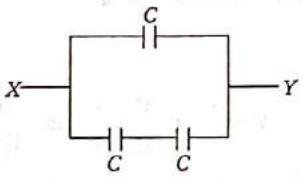
$\therefore$ 24 μF capacitor can be neglected.



Hence, equivalent capacitance between A and B

$$= 4 + 6 = 10\,\mu F$$

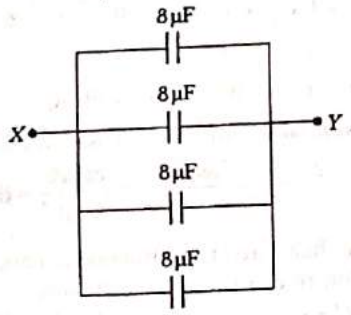
41. (*d*) The given circuit can be redrawn as



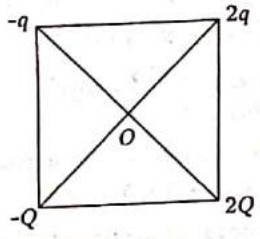
$$C_{eq} = \frac{3C}{2} = \frac{3\varepsilon_0 A}{2d}$$

42. (*d*) The given circuit can be redrawn as

$\therefore \quad C_{xy} = 4 \times 8 = 32\mu F$



43. (*a*)



If potential at centre is zero, then

$$V_1 + V_2 + V_3 + V_4 = 0$$

$$-\frac{KQ}{r} - \frac{Kq}{r} + \frac{K2Q}{r} + \frac{K2q}{r} = 0$$

$$-Q - q + 2q + 2Q = 0 \Rightarrow Q = -q$$

44. (*a*) Let R and r be the radii of bigger and each smaller drop, respectively.

$\therefore \quad \frac{4}{3}\pi R^3 = 8 \times \frac{4}{3}\pi r^3 \Rightarrow R = 2r$ ...(i)

The capacitance of a smaller spherical drop is

$C = 4\pi\varepsilon_0 r$ ...(ii)

The capacitance of bigger drop is

$C' = 4\pi\varepsilon_0 R = 2 \times 4\pi\varepsilon_0 r$ $(\because R = 2r)$

$= 2C$ (from Eq. (ii))

$\therefore \quad C = \frac{C'}{2} = \frac{1}{2}\mu F$ $(\because C' = 1\mu F)$

45. (*b*) Heat produced in a wire is equal to energy stored in capacitor.

$$H = \frac{1}{2}CV^2 = \frac{1}{2} \times (2 \times 10^{-6}) \times (200)^2$$

$$= 10^{-6} \times 200 \times 200 = 4 \times 10^{-2} J$$

46. (*a*) Volume of 8 drops = Volume of big drop

$\therefore \quad \left(\frac{4}{3}\pi r^3\right) \times 8 = \frac{4}{3}\pi R^3 \Rightarrow 2r = R$ ...(i)

According to charge conservation,

$8q = Q$ ...(ii)

We have,

$$V \propto \frac{1}{r} \text{ or } \frac{V_1}{V_2} = \frac{r_2}{r_1} \Rightarrow \frac{20}{V_2} = \frac{2r}{r}$$

$\Rightarrow$ Potential on each drop, $V_2 = 10$ V

47. (*c*) The electric field $\mathbf{E} = 5\hat{\mathbf{i}} - 3\hat{\mathbf{j}}$ kV/m

$$E = \sqrt{25 + 9} = \sqrt{34} \text{ kV/m}$$

$$d = \sqrt{(10-4)^2 + (3-0)^2}$$

$$= \sqrt{(6)^2 + (3)^2}$$

$$= \sqrt{(36 + 9} = \sqrt{45}$$

Potential difference between points A and B,

$$V = E \cdot d = \sqrt{34} \times \sqrt{45} = 39 \text{ kV}$$

48. (*d*) Force between the plates of a parallel plate capacitor

$$|F| = \frac{V^2 A}{2\varepsilon_0} = \frac{Q^2}{2\varepsilon_0 A}$$

49. (*a*) Store energy in capacitor of 3 μF,

$$U_1 = \frac{1}{2} \times C_1 V^2 = \frac{1}{2} \times 3 \times (6)^2 \times 10^{-6}$$

$$= 54 \times 10^{-6} \text{ J}$$

Store energy in capacitor of 4 μF,

$$U_2 = \frac{1}{2}C_2 V^2 = \frac{1}{2} \times 4 \times (6)^2 \times 10^{-6} = 72 \times 10^{-6} \text{ J}$$

When both capacitors are connected in series,

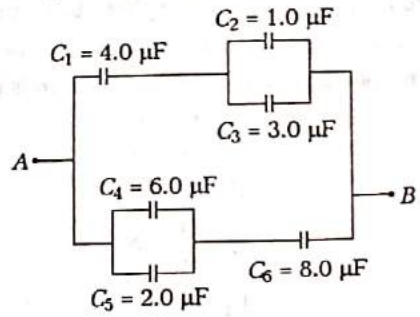
$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{3 \times 4}{3 + 4} = \frac{12}{7}\mu F$$

Energy lost

$$U = \frac{1}{2}C_{eq}(V_1 - V_2)^2 = \frac{1}{2} \times \frac{12}{7} \times (0)^2 \times 10^{-6} = 0$$

Total energy $= U_1 + U_2 = 1.26 \times 10^{-4}$ J

50. (*a*) In given figure, C_2 and C_3 are in parallel.



$\therefore \quad C' = C_2 + C_3 = 4\mu F$

As C' and C_1 are in series, $\frac{1}{C''} = \frac{1}{C'} + \frac{1}{C_1} = \frac{1}{4} + \frac{1}{4}$

$\Rightarrow \quad C'' = 2\mu F$

Similarly, C_4 and C_5 are in parallel $C''' = 6 + 2 = 8\mu F$

C''' and C_6 are in series $\frac{1}{C''''} = \frac{1}{C'''} + \frac{1}{C_6} = \frac{1}{8} + \frac{1}{8}$

$\Rightarrow \quad C'''' = 4\mu F$

Now, C'''' and C'' are in parallel,

$\therefore \quad C = 4\mu F + 2\mu F = 6\mu F$

CHAPTER 03

Current electricity

We have read that the directional flow of charge in a conductor under a potential difference maintained between the ends of the conductor constitutes an **'electric current'** in the conductor. In this chapter, we will study about the basic properties of electric current, property of batteries and how they cause current and energy transfer in a circuit. In this analysis, we will use the concepts of current, potential difference, resistance and electromotive force.

1 Electric current

Electric current is defined as the rate of flow of charge through any cross-sectional area of the conductor. It is denoted by I.

If electric current is steady, then it can be expressed by

$$I = \frac{\text{Total charge flowing } (q)}{\text{Total time taken } (t)} \Rightarrow I = \frac{q}{t} = \frac{ne}{t}$$

where, n = number of carriers of electricity and e = electronic charge.

But current is not always steady. *So, it can be defined in two ways*

(i) **Average current** If a charge Δq flows through a conductor in the time interval t to $(t + \Delta t)$, then average current is defined as $I_{av} = \frac{\Delta q}{\Delta t}$.

(ii) **Instantaneous current** Current at any instant of time is called instantaneous current. If a charge dq flows through a conductor in small time dt, *i.e.*, limit of Δt tending to zero, then $I_{inst} = \lim_{\Delta t \to 0} \frac{\Delta q}{\Delta t} = \frac{dq}{dt}$

Important points about electric current

(i) Current is a scalar quantity.

(ii) Its SI unit is ampere (A) and CGS unit is emu and is called biot (Bi) or ab ampere. $|A| = \left(\frac{1}{10}\right)$ Bi (ab amp).

(iii) One ampere is the current through a wire, if a charge of one coulomb flows through any cross-section of the wire in one second.

$$1 \text{ ampere (A)} = \frac{1 \text{ coulomb (C)}}{1 \text{ second (s)}} = 1 \text{ coulomb per second} = 1 \text{ Cs}^{-1}$$

Inside

1) Electric current
- Current density

2) Ohm's law
- Combination of resistances

3) Kirchhoff's laws
- Electrical energy and power
- Heating effect of current

4) Measuring instruments for current and voltage
- Wheatstone's bridge
- Meter bridge
- Potentiometer

(iv) As a matter of convention, the direction of flow of positive charge gives the direction of current. This is called **conventional current**. The direction of flow of electrons gives the direction of **electronic current**. Therefore, the direction of electric current is opposite to that of conventional current.

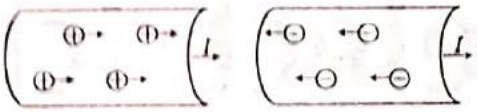


Fig. 3.1 Flow of electronic current is equivalent to the flow of conventional current in opposite direction

(v) As charge is conserved and current is the rate of flow of charge *i.e.*, charge entering per second at one end of conductor is equal to the charge leaving per second the other end.

Note

1. In case a charge q is revolving in a circle of radius r with uniform velocity v, current is given by
$$I = \frac{q}{t} = qf = \frac{q}{2\pi/\omega} = \frac{qv}{2\pi r}.$$
2. Total charge in time interval t_1 and t_2 can be given as
$$Q = \int_{t_1}^{t_2} I\,dt$$
= Area under the graph I *versus* t in the interval t_1 to t_2 as shown in the figure.

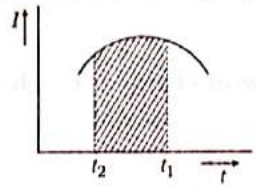


Fig. 3.2 Area under I versus t graph

So, average current in this interval,
$$I_{av} = \frac{Q}{t_2 - t_1} = \frac{\int_{t_1}^{t_2} Q \cdot dt}{t_2 - t_1} = \frac{\text{Area under the } I \text{ versus } t \text{ graph}}{\text{Time interval}}.$$

Example 3.1 *How many electrons pass through a bulb in 1 min, if the current is 400 mA. ($e = 1.6 \times 10^{-19}$ C)?*

Sol. Current, $I = 400$ mA $= 400 \times 10^{-3}$ A

Time, $t = 1$ min $= 60$ s

Charge, $e = 1.6 \times 10^{-19}$ C

Thus, charge passing through the bulb in 1 min is
$$q = I \times t = 400 \times 10^{-3} \times 60 \text{ C}$$
Also, $q = ne$

$\therefore$ Number of electrons,
$$n = \frac{q}{e} = \frac{400 \times 10^{-3} \times 60}{1.6 \times 10^{-19}} = 1.5 \times 10^{20} \text{ electrons}$$

Example 3.2 *10^{20} electrons, each having a charge of 1.6×10^{-19} C, pass from a point A towards another point B in 0.1 s. What is the current in ampere? What is its direction?*

Sol. Given, $n = 10^{20}$, $e = 1.6 \times 10^{-19}$ C and $t = 0.1$ s

As we know,
$$\text{Current, } I = \frac{q}{t} = \frac{ne}{t} = \frac{10^{20} \times 1.6 \times 10^{-19} \text{ C}}{0.1 \text{ s}} = 160 \text{ A}$$
Direction of current is from point A to B (in the direction of electrons)

Example 3.3 *A wire carries a current of 2 A. What is the charge that has flowed through its cross-section in 1.0 s? How many electrons does this correspond to?*

Sol. Given, $i = 2$ A and $t = 1.0$ s

As, current, $i = \frac{q}{t}$

$\therefore$ $q = it = (2 \text{ A})(1 \text{ s}) = 2$ C

Also, $q = ne$

$\therefore$ Number of electrons, $n = \frac{q}{e} = \frac{2}{1.6 \times 10^{-19}} = 1.25 \times 10^{19}$

Example 3.4 *If an electron revolves in a circle of radius $\pi/2$ with uniform speed 6×10^5 m/s. Find the electric current. (Take, $\pi^2 = 10$)*

Sol. Time period is given by $t = \frac{2\pi r}{v}$.

Here, $r = \frac{\pi}{2}$ cm and $v = 6 \times 10^5$ m/s.

So, electric current, $i = \frac{ev}{2\pi r} = \frac{1.6 \times 10^{-19} \times 6 \times 10^5}{2\pi \times \frac{\pi}{2} \times 10^{-2}}$
$$= 9.6 \times 10^{-13} \text{ A}$$

Example 3.5 *If the amount of charge flowed in time t through cross-section of wire is $q = \beta t - \gamma t^2$, where β and γ are constant*

(i) Find the current in terms of t.

(ii) Sketch i versus t graph.

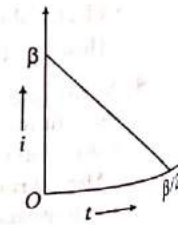
Sol. Given $q = \beta t - \gamma t^2$

(i) As we know, current in a wire is given by
$$i = \frac{dq}{dt} = \frac{d}{dt}(\beta t - \gamma t^2) = \beta - 2\gamma t$$
So, at $t = 0$, $i = \beta$

and when $i = 0$, then $t = \frac{\beta}{2\gamma}$.

(ii) To plot a graph i *versus* t, we will compare the given current with the equation of a straight line, *i.e.*,
$$y = mx + c$$
So, according to this, we have
$$y = \beta - 2\gamma t$$
Which means a straight line with negative slope *i.e.*, -2γ and positive intercept, *i.e.*, β.

So, the plot will be



Example 3.6 *The current in a wire varies with time according to the relation,*

$$i = (3A) + (2\ A/s)\ t$$

(i) *How many coulombs of charge pass through a cross-section of the wire in the time interval between $t = 0$ and $t = 4$ s?*

(ii) *What constant current would transport the same charge in the same time interval?*

Sol. Given, $i = 3(A) + 2(A/s)\,t$

(i) Current through a cross-sectional area is given by $i = \frac{dq}{dt}$

$\therefore \quad dq = idt$

On integrating both sides between the time interval given *i.e.*, 0 to 4 s, we get

$$\int_0^q dq = \int_0^4 idt$$

$$\therefore \text{Charge, } q = \int_0^4 (3 + 2t)\,dt = \left(3t + \frac{2t^2}{2}\right)_0^4$$

$$= [3t + t^2]_0^4 = [12 + 16] = 28 \text{ C}$$

(ii) Therefore, the current flowing through wire would be

$$i = \frac{q}{t} = \frac{28}{4} = 7 \text{ A}$$

Current density

Current density at any point inside a conductor is defined as "*the amount of charge flowing per second through a unit area held normal to the direction of the flow of charge at that point.*"

Current density is a vector quantity and its direction is along the motion of the positive charge as shown in Fig. 3.3 (a).

$$\boxed{\text{Current density, } \mathbf{J} = \frac{q/t}{A} = \frac{I}{\mathbf{A}}}$$

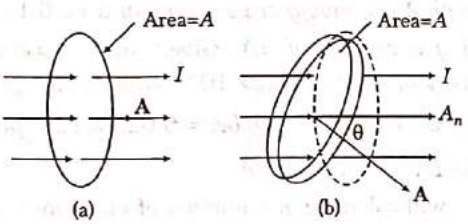


Fig. 3.3 Direction of current density

If the cross-sectional area A is not perpendicular to the current and normal to the area makes an angle θ with the direction of the current as shown in Fig. 3.3 (b), then

$$A_n = A\cos\theta$$

$$\therefore \text{Current density, } \mathbf{J} = \frac{I}{A_n} = \frac{I}{A\cos\theta} \text{ or } I = \mathbf{J}\cdot\mathbf{A}$$

If area is finite,

$$\boxed{I = \int \mathbf{J}\cdot d\mathbf{A}}$$

where, $d\mathbf{A}$ is a small element of the given surface.

The SI unit of current density is ampere per square metre *i.e.*, Am^{-2} and its dimensional formula are $[\text{AL}^{-2}]$.

Example 3.7 *An electron beam has an aperture of 2 mm^2. A total of 7×10^{16} electrons flow through any perpendicular cross-section per second. Calculate the current density in the electron beam.*

Sol. As we know, current density in the electron beam,

$$J = \frac{I}{A} = \frac{ne}{t} / A = \frac{ne}{tA}$$

Substituting the values, we get

$$= \frac{7 \times 10^{16} \times 1.6 \times 10^{-19}}{2 \times 10^{-6}} = 5.6 \times 10^3 \text{ Am}^{-2}$$

Electric Currents in Conductors

The electric current in a conductor can be explained by an electron theory. In an atom of a substance, the electrons in the orbits close to the nucleus are bound to it under the strong attraction of the nuclear positive charge, but the electrons far from the nucleus experience a very feeble force. Hence, the outer electrons can be removed easily from the atom (by rubbing or by heating the substance). In fact, a few outer electrons, leave their atoms and move freely within the substance (in the vacant spaces between the atoms). These electrons called **free electrons** or **conduction electrons**, carry the charge in the substance from one place to the other.

Therefore, the electrical conductivity of a solid substance depends upon the number of free electrons in it. In metals, this number is quite large ($\approx 10^{29}$ /m^3). Hence, metals are good conductors of electricity. **Silver** is the best **conductor** of electricity than are copper, gold and aluminium, respectively.

There are some other materials in which the electrons will be bound and they will not be accelerated, even if the electric field is applied, *i.e.*, **no current on applying electric field**. Such materials are called **insulators**, *e.g.* wood, plastic, rubber, etc.

Drift velocity

It is defined as "*the average velocity with which the free electrons in a conductor get drifted towards the positive end of the conductor under the influence of an electric field applied across the conductor.*"

It is given by $\boxed{v_d = \frac{eE}{m}\tau}$

where, e = charge on electron, E = electric field,
m = mass of the electron and τ = relaxation time.

The electric current relates with drift velocity as

$$\boxed{i = neAv_d}$$

$$\Rightarrow \quad v_d = \frac{i}{neA} = \frac{\tau}{ne}$$

Hence, current density is also given by

$$\boxed{J = \frac{i}{A} = nev_d}$$

The direction of drift velocity for electrons in a metal is opposite to that of applied electric field. The drift velocity of electron is very small of the order of 10^{-4} ms^{-1} as compared to thermal speed ($\approx 10^5$ m/s) of electron at room temperature.

Relaxation time (τ)

As free electrons move in a conductor they continuously collide with positive ions. The time interval between two successive collisions of electrons with the positive ions in the metallic lattice is defined as relaxation time.

$$\tau = \frac{\text{mean free path}}{\text{rms velocity of electrons}} = \frac{\lambda}{v_{rms}}$$

With rise in temperature v_{rms} increases, consequently τ decreases.

Mobility (μ)

Drift velocity per unit electric field is called mobility of electron, *i.e.*, $\mu = \frac{v_d}{E} = \frac{e\tau}{m}$.

Its unit is m^2/V-s.

Mobility of free electrons is independent of electric field.

Example 3.8 *Find the current flow through a copper wire of length 0.2 m, area of cross-section 1 mm^2, when connected to a battery of 4 V. Given that electron mobility is $4.5 \times 10^{-6} m^2V^{-1}s^{-1}$ and charge on electron is 1.6×10^{-19} C. The number of density of electron in copper is 8.5×10^{28} m^{-3}.*

Sol. Given, length of copper wire, $l = 0.2$ m

Cross-sectional area, $A = 1$ mm^2 $= 10^{-6}$m^2

Potential difference, $V = 4$ V

Electron mobility, $\mu = 4.5 \times 10^{-6}$ m^{2}s^{-1}V^{-1}

Charge of an electron, $e = 1.6 \times 10^{-19}$C

Number of density of electron, $n = 8.5 \times 10^{28}$m^{-3}

We know that electric field set up across the conductor,

$$E = \frac{V}{l} = \frac{4}{0.2} = 20 \text{ Vm}^{-1}$$

Mobility of electrons is $\mu = \frac{v_d}{E}$

$$\Rightarrow \quad v_d = 4.5 \times 10^{-6} \times 20 = 9 \times 10^{-5} \text{ m/s}$$

So, the current flow through a copper wire is

$$I = nev_d A$$
$$= 8.5 \times 10^{+28} \times 1.6 \times 10^{-19} \times 10^{-6} \times 9 \times 10^{-5}$$
$$= 1.224 \text{ A}$$

Example 3.9 *An aluminium wire of diameter 0.24 cm is connected in series to a copper wire of diameter 0.16 m. The wire carry an electric current 10 A. Find (i) current density in the aluminium wire, (ii) drift velocity of electrons in the copper wire.*

(Given, number of electrons per cubic metre volume of copper = 8.4×10^{28}.)

Sol. (i) Radius of Al wire, $r = \frac{0.24}{2} = 0.12$ cm $= 0.12 \times 10^{-2}$ m

Area of cross-section of Al wire,

$$A = \pi r^2 = 3.14 \times (0.12 \times 10^{-2})^2 = 4.5 \times 10^{-6} \text{ m}^2$$

$\therefore$ Current density, $J = \frac{I}{A} = \frac{10}{4.5 \times 10^{-6}}$

$$= 2.2 \times 10^6 \text{ Am}^{-2}$$

(ii) Area of cross-section of copper wire is

$$A = \pi \times (0.08 \times 10^{-2})^2 = 2 \times 10^{-6} \text{ m}^2$$

Also, $n = 8.4 \times 10^{28}$ m^{-3}, $e = 1.6 \times 19^{-19}$ C, $I = 10$ A

$\therefore$ Drift velocity of electrons in the copper wire,

$$v_d = \frac{I}{enA} = \frac{10}{1.6 \times 10^{-19} \times 8.4 \times 10^{28} \times 2 \times 10^{-6}}$$
$$= 3.7 \times 10^{-4} \text{ ms}^{-1}$$

Example 3.10 *What is the drift velocity of electrons in a si wire of length 2 m, having cross-sectional area 6.14×10^{-6} and carrying a current of 5A. Given, atomic weight of si = 108, density of silver = 9.5×10^3 kg/m^3, charge of elec = 1.6×10^{-9} C, Avogadro's number = 6.023×10^{26} per kg ato*

Sol. Use the formula, $I = nev_d A$

First, we will calculate, n = number of electrons per unit volume.

Imagine the volume of silver to be 1 m^3.

Now, its mass = density × volume = $7.5 \times 10^3 \frac{\text{kg}}{\text{m}^3} \times 1 \text{ m}^3$

$$= 7.5 \times 10^3 \text{ kg}$$

Now, number of moles of silver

$$= \frac{7.5 \times 10^3}{108 \times 10^3} = 0.069 \cong 6.9 \times 10^4 \text{ mol}$$

So, number of silver atoms in this get by multiplying by the number of moles by Avogadro's number.

i.e., number of silver atoms $= \frac{7.5 \times 10^3 \times 6.023 \times 10^{23}}{108 \times 10^{-3}}$

$$= 4.18 \times 10^{28} \text{ atoms}$$

Now, since the valency of silver is one, we can assume each atom of silver contributes one electron. So, finally

$$\Rightarrow \quad n = \frac{7.5 \times 10^3 \times 6.023 \times 10^{23}}{108 \times 10^{-3}}$$

$$= 4.18 \times 10^{28} \text{ per m}^3$$

Given, $I = 5$ A and $A = 6.14 \times 10^{-6}$ m^2

So, we have drift velocity, $v_d = \frac{I}{neA}$

$$= \frac{5}{4.18 \times 10^{28} \times 1.602 \times 10^{-19} \times 6.14 \times 10^{-6}}$$

$$= 1.216 \times 10^{-4} \text{ m/s}$$

Check point 3.1

1. The current through a wire depends on time as $I = 3t^2 + 2t + 5$. The charge flowing through the cross-section of the wire in time from $t = 0$ to $t = 2$ s is
(a) 22 C (b) 20 C
(c) 18 C (d) 5 C

2. The charge of an electron is 1.6×10^{-19} C. How many electrons strike the screen of a cathode ray tube each second when the beam current is 16 mA?
(a) 10^{17} (b) 10^{19}
(c) 10^{-19} (d) 10^{-17}

3. A conductor carries a current of 0.2 A. In 30 s how many electrons will flow through the cross-section of the conductor ($q = 1.6 \times 10^{-19}$ C)?
(a) 0.375×10^{19} (b) 375×10^{19}
(c) 3.75×10^{19} (d) 37.5×10^{19}

4. In a closed circuit, the current I (in ampere) at an instant of time t (in second) is given by $I = 4 - 0.08t$. The number of electrons flowing in 50 s through the cross-section of the conductor is
(a) 1.25×10^{19} (b) 6.25×10^{20}
(c) 5.25×10^{19} (d) 2.25×10^{20}

5. Drift velocity v_d varies with the intensity of electric field as per the relation.
(a) $v_d \propto E$ (b) $v_d \propto \frac{1}{E}$
(c) v_d = constant (d) $v_d \propto E^2$

6. When the current i is flowing through a conductor, then the drift velocity is v. If $2i$ current is flowed through the same metal but having double the area, then the drift velocity will be
(a) $v/4$ (b) $v/2$
(c) v (d) $4v$

7. A metallic resistor is connected across a battery. If the number of collisions of the free electrons with the lattice is somehow decreased in the resistor (for example, by cooling it), then the current will
(a) increase (b) decrease
(c) remain constant (d) become zero

8. When current flows through a conductor, then the order of drift velocity of electrons will be
(a) 10^{10} cms^{-1} (b) 10^{-2} cms^{-1}
(c) 10^4 cms^{-1} (d) 10^{-1} cms^{-1}

9. The drift velocity does not depend upon
(a) cross-section of the wire
(b) length of the wire
(c) number of the free electrons
(d) magnitude of the current

10. The number density of free electron in a copper conductor is 8.5×10^{28} m^{-3}. How long does an electron take to drift from one end of a wire, 3.0 m long to its other end? The area of cross-section of the wire is 2.0×10^{-6} m^2 and it is carrying a current of 3.0 A.
(a) 2.73×10^4 s (b) 4.73×10^4 s
(c) 5×10^4 s (d) 6×10^8 s

2 Ohm's law

It states that, "the current I flowing through a conductor is always directly proportional to the potential difference V across the ends of the conductor", provided that the physical conditions (temperature, mechanical strain, etc) are kept constant. Mathematically,

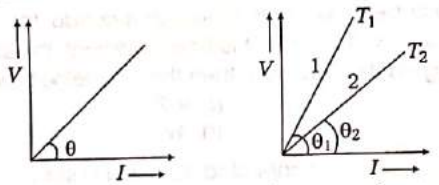
$$I \propto V \quad \text{or} \quad V \propto I$$

or

$$\boxed{V = IR} \qquad \text{...(i)}$$

where, R is resistance of the conductor.

Graph between V and I for a metallic conductor is a straight line as shown. At different temperature V-I curves are different.



(a) Slope of the line $= \tan\theta = \frac{V}{I} = R$

(b) Here, $\tan\theta_1 > \tan\theta_2$
So, $R_1 > R_2$
i.e., $T_1 > T_2$

Fig. 3.4

Note The substances which obey Ohm's law are known as ohmic substances. The devices or substances which don't obey Ohm's law *e.g.*, gases, crystal rectifiers, thermionic valve, transistors, etc, are known as non-ohmic or non-linear conductors.

Resistance and resistivity

The property of a substance by virtue of which it opposes the flow of current through it, is known as the resistance.

Resistance is defined as the ratio of the potential difference applied across the ends of the conductor to the current flowing through it.

Mathematically, $R = \frac{V}{I}$

The resistance of the conductor depends upon the following factors

(a) It is directly proportional to the length of the conductor.

$$R \propto l \qquad \text{...(i)}$$

(b) It is inversely proportional to the area of the cross-section of the conductor.

$$R \propto \frac{1}{A} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$R \propto \frac{l}{A} \quad \Rightarrow \quad R = \rho\frac{l}{A} \qquad \text{...(iii)}$$

where, ρ is the constant of proportionality known as **resistivity or specific resistance** of the conductor.

Resistivity is the intrinsic property of the substance. It is independent of shape and size of the body and it depends upon the nature of the material.

Example 3.11 *The resistance of a wire is R ohm. What will be its new resistance if it is stretched to n times its original length?*

Sol. Suppose the initial length of the wire is l and becomes l' after stretching. The initial cross-section of the wire is A, and becomes A' after stretching. Then, initial volume of the wire Al and after stretching it will be $A'l'$. But the volume of the wire does not changing on stretching.

i.e.,

$$Al = A'l'$$

$$\frac{A}{A'} = \frac{l'}{l} = n$$

If the specific resistance of the material of the wire be ρ, then the resistance of the wire before stretching is

$$R = \rho\,(l/A) \qquad \text{...(i)}$$

If the resistance becomes R' after stretching, then

$$R' = \rho\,(l'/A) \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{R'}{R} = \left(\frac{l'}{l}\right)\left(\frac{A}{A'}\right) = n^2 \qquad \text{...(iii)}$$

$$\boxed{R' = n^2 R}$$

Example 3.12 *Resistance of a conductor of length l and area of cross-section A is R. If its length is doubled and area of cross-section is halved, then find its new resistance.*

Sol. Initial length $= l$, area $= A$

So, initial resistance, $R = \rho\frac{l}{A}$

Final length, $l' = 2l$, area, $A' = \frac{A}{2}$

New resistance, $R' = \rho\frac{l'}{A'} = \rho\frac{2l}{\left(\frac{A}{2}\right)} = 4\rho\frac{l}{A} = 4R$

Unit and dimensional formula of resistance

The SI unit of resistance is ohm and is denoted by symbol Ω.

$$1 \text{ ohm } (\Omega) = \frac{1 \text{ volt (V)}}{1 \text{ ampere (A)}} = 1 \text{ volt/ampere (or V/A)}.$$

The resistance of a conductor is said to be one ohm, if one ampere of current flows, when a potential difference of one volt is applied across the ends of the conductor.

Its dimensional formula of electrical resistance is $[ML^2T^{-3}A^{-2}]$

Note Any conducting material that offers some resistance is known as resistor.

Conductance and conductivity

Conductance

Reciprocal of resistance is known as conductance, $C = 1/R$. Its unit is $1/\Omega$ or Ω^{-1} or 'Siemen'.

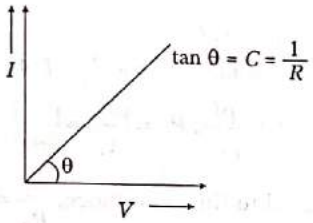


Fig. 3.5

Conductivity

Reciprocal of resistivity is called conductivity (σ), *i.e.*,

$\boxed{\sigma = \frac{1}{\rho}}$ Its unit is mho/m.

The dimensional formula of conductivity is $[M^{-1}L^{-3}T^{3}A^{2}]$.

Vector form of Ohm's law

Electric field in a conductor of length l and having potential difference V at its ends is given by

$$V = El$$

Using relation (i) and (iii), we get

$$V = IR = \frac{I\rho l}{A} = El \Rightarrow E = \frac{I}{A}\rho$$

$$\therefore \quad \mathbf{E} = \rho \mathbf{J}$$

or $$\boxed{\mathbf{J} = \sigma \mathbf{E}} \quad \text{...(iv)}$$

The above equation is the vector form of Ohm's law.

Also, current density,

$$J = nev_d = ne\left(\frac{eE\tau}{m}\right)$$

$$J = \left(\frac{ne^2\tau}{m}\right)E$$

Comparing with Eq. (iv), we get

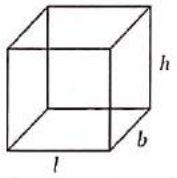
$$\text{Conductivity, } \sigma = \frac{ne^2\tau}{m}$$

and resistivity, $\rho = \frac{1}{\sigma} = \frac{m}{ne^2\tau}$.

Example 3.13 *All the edges of a block with parallel faces are unequal. Its longest edge is four times its shortest edge. Find the ratio of the maximum to the minimum resistance between parallel faces.*

Sol. Let l = longest edge, b = shortest edge,

$$l/b = 4$$



Resistance between different faces of area

(i) Area, $A_1 = (l \times b)$

$\therefore$ Resistance, $R_1 = \rho\frac{l}{A} = \frac{\rho h}{lb}$

(ii) Area, $A_2 = (b \times h)$

$\therefore$ Resistance, $R_2 = \frac{\rho l}{bh} = R_{max}$

(iii) Area, $A_3 = (l \times h)$

$\therefore$ Resistance, $R_3 = \frac{\rho b}{l \times h} = R_{min}$

$$\therefore \frac{R_{max}}{R_{min}} = \frac{l^2}{b^2} = \frac{(4b)^2}{b^2} = 16$$

Example 3.14 *Calculate the electric field in a copper wire of cross-sectional area 2.0 mm^2 carrying a current of 1A. The conductivity of copper is 6.25×10^7 Sm^{-1}.*

Sol. Given, area $A = 2\,mm^2 = 2 \times 10^{-6}\,m^2$

Current, $I = 1$ A, $\sigma = 6.25 \times 10^7\ Sm^{-1}$

As current density, $J = \frac{I}{A} = \sigma E$

$\therefore$ Electric field, $E = \frac{I}{A\sigma} = \frac{1}{2 \times 10^{-6} \times 6.25 \times 10^7}$

$$= 8 \times 10^{-3}\ Vm^{-1}$$

Example 3.15 *A copper wire has a resistance of 10Ω and an area of cross-section 1 mm^2. A potential difference of 10V exists across the wire. Calculate the drift speed of electrons, if the number of electrons per cubic metre in copper is 8×10^{28} electrons.*

Sol. Given, $R = 10\Omega$, $A = 1\,mm^2 = 10^{-6}\,m^2$

$V = 10$ V, $n = 8 \times 10^{28}$ electrons/m^3

Now, electric current, $I = enAv_d$

Using Ohm's law,

$$\therefore \quad \frac{V}{R} = enAv_d$$

or $$v_d = \frac{V}{enAR}$$

$$= \frac{10}{1.6 \times 10^{-19} \times 8 \times 10^{28} \times 10^{-6} \times 10}$$

$$= 0.078 \times 10^{-3}\ ms^{-1} = 0.078\ mm/s$$

Example 3.16 *A current of 1 ampere is flowing through a copper wire of length 0.1 m and cross-section $1 \times 10^{-6} m^2$.*

(i) *If the specific resistance of copper be $1.7 \times 10^{-8}\ \Omega$-m, calculate the potential difference across the ends of the wire.*

(ii) *Determine current density in the wire.*

(Given, density of copper $= 8.9 \times 10^3$ kg m^{-3}, atomic weight $= 63.5$, $N = 6.02 \times 10^{26}$ per kg-atom.

Sol. Given, $I = 1$ A, $l = 0.1$ m, $A = 1 \times 10^{-6}$ m^2,

$$\rho = 1.7 \times 10^{-8}\ \Omega\text{-m}, d = 8.9 \times 10^3 \text{ kg m}^{-3}.$$

(i) Resistance of wire is, $R = \frac{\rho l}{A} = \frac{1.7 \times 10^{-8} \times 0.1}{1 \times 10^{-6}}$

$$= 1.7 \times 10^{-3}\ \Omega$$

$\therefore$ Potential difference, $V = IR = 1 \times 1.7 \times 10^{-3} = 1.7 \times 10^{-3}$ V

(ii) Current density, $J = \frac{I}{A} = \frac{1}{1 \times 10^{-6}} = 1 \times 10^6$ Am^{-2}

Example 3.17 *Find the time of relaxation between collision and free path of electrons in copper at room temperature. (Given, resistivity of copper $= 1.7 \times 10^{-8}\ \Omega$-m, density of electrons in copper $= 8.5 \times 10^{28}$ m^{-3}, charge on an electron $= 1.6 \times 10^{-19}$ C, mass of electron $= 9.1 \times 10^{-31}$ kg and drift velocity of free electrons $= 1.6 \times 10^{-4}$ ms^{-1}.)*

Sol. Given, $\rho = 1.7 \times 10^{-8}\ \Omega$-m, $n = 8.5 \times 10^{28}$ m^{-3},

$$e = 1.6 \times 10^{-19} \text{ C}, m_e = 9.1 \times 10^{-31} \text{ kg},$$

$$v_d = 1.6 \times 10^{-4} \text{ ms}^{-1}$$

We know that, $\rho = \frac{m_e}{ne^2\tau}$

$\therefore$ Relaxation time,

$$\tau = \frac{m_e}{e^2 n\rho} = \frac{9.1 \times 10^{-31}}{(1.6 \times 10^{-19})^2 \times 8.5 \times 10^{28} \times 1.7 \times 10^{-8}}$$

$$= 2.5 \times 10^{-14} \text{ s}$$

Mean free path of electron (distance covered between two collisions)

$$= v_d\tau = 1.6 \times 10^{-4} \times 2.5 \times 10^{-14}$$

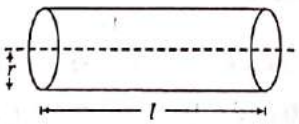
$$= 4 \times 10^{-18} \text{ m}$$

Example 3.18 *A wire has a resistance R. Find new resistance,*

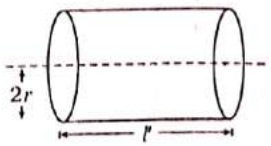
(i) *if radius of cross-section of a cylindrical wire is doubled, then find ratio of initial to final resistance.*

(ii) *if length of wire is increased by 10%, then find the percentage increase in its resistance.*

(iii) *if length of wire is increased by 20%, then find the percentage increase in its resistance.*



Sol. (i) According to the question, we draw a figure given below



We have, $A_1 l_1 = A_2 l_2$

$$\pi r^2 l = \pi (2r)^2 l_2 \Rightarrow l_2 = l/4$$

Resistance, $R_1 = \frac{\rho l}{\pi r^2}$, $R_2 = \frac{\rho l_2}{A_2} = \frac{\rho l/4}{4\pi r^2} = \frac{1}{16}\frac{\rho l}{\pi x^2}$

Ratio of initial to final resistance, $\frac{R_1}{R_2} = 16$.

(ii) Given, $l_1 = l$, $A_1 = A$, $l_2 = l + 10\%$ of l

$$= l + 0.1l = 1.1l, A_2 = A'$$

We have $A_1 l_1 = A_2 l_2 \Rightarrow Al = A \times 1.1 \times l$

Area, $A' = \frac{A}{1.1}$

Resistance, $R = \frac{\rho l}{A}$, $R' = \frac{\rho l_2}{A_2} = \frac{\rho \times l \times 1.1}{A/1.1} = \frac{\rho l}{A}(1.1)^2$

$$\frac{R'}{R} = (1.1)^2$$

$$\% \text{ change} = \left(\frac{R'}{R} - 1\right) \times 100 = (1.21 - 1) \times 100 = 21\%$$

(iii) Given, $l_1 = l$, $A_1 = A$

$$l_2 = l\left(1 + \frac{20}{100}\right) = \frac{6l}{5}, A_2 = A'$$

We have, $A_1 l_1 = A_2 l_2$

$$Al = A'\frac{6l}{5} \Rightarrow A' = \frac{5A}{6}$$

$$\therefore \quad \frac{R_2}{R_1} = \frac{\frac{\rho l_2}{A_2}}{\frac{\rho l_1}{A_1}} = \frac{l_2}{l_1} \cdot \frac{A_1}{A_2} = \frac{6l/5}{l} \cdot \frac{A}{5A/6} = \frac{36}{25}$$

Increase in the resistance,

$$\left(\frac{R_2}{R_1} - 1\right) \times 100 = \left(\frac{36}{25} - 1\right) \times 100 = 44\%$$

Temperature dependence of resistivity

Resistivity of a material depends on the temperature. As,

$$\rho \propto \frac{1}{\tau}$$

When temperature increases in a conductor, average speed of free electrons increases, hence relaxation time decreases. Thus, resistivity increases.

Resistivity of a metal conductor is given by

$$\boxed{\rho = \rho_0 [1 + \alpha (T - T_0)]}$$

where, ρ = resistivity at temperature T, ρ_0 = resistivity at temperature T_0, and α is temperature coefficient of resistivity

$$\Rightarrow \quad \alpha = \frac{\rho - \rho_0}{\rho_0 (T - T_0)}$$

In terms of resistance,

$$R \propto \rho$$

$$\therefore \quad R = R_0 [1 + \alpha (T - T_0)]$$

$$\Rightarrow \quad \alpha = \frac{R - R_0}{R_0 (T - T_0)}$$

where, R_0 = resistance of conductor at 0°C
R_t = resistance of conductor at t°C.

Important points related to resistivity

For metal, α is positive as their resistivity increases with rise in temperature. The graph of ρ_T plotted against ρ would be a straight line as in Fig. 3.6 (a).

For semiconductor, α is negative as their resistivity decreases with rise in temperature.

Variation in resistivity with the temperature for semiconductor is shown in Fig.3.6 (b). The resistivity of **alloys** also increases with rise in temperature. The graph between resistivity and temperature is shown in Fig. 3.6 (c).

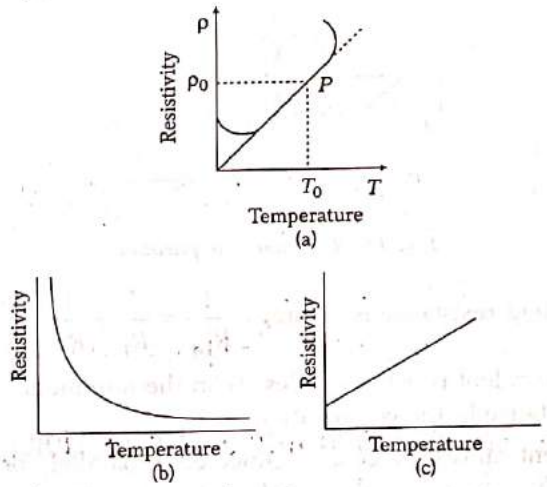


Fig. 3.6 Resistivity as the function of temperature for (a) metals, (b) semiconductors and (c) alloys

Example 3.19 *The resistance of a thin silver wire is 1.0 Ω at 20°C. The wire is placed in a liquid bath and its resistance rises to 1.2 Ω. What is the temperature of the bath? α for silver is 3.8×10^{-3} per °C.*

Sol. We know, $R(T) = R_0[1 + \alpha(T - T_0)]$

Here, $R(T) = 1.2\,\Omega$, $R_0 = 1.0\,\Omega$

$\alpha = 3.8 \times 10^{-3}$ per °C and $T_0 = 20°\text{C}$

Substituting the values, we have

$$1.2 = 1.0[1 + 3.8 \times 10^{-3}(T - 20)]$$

or $$3.8 \times 10^{-3}(T - 20) = 0.2$$

Solving this, we get $T = 72.6°\text{C}$

Example 3.20 *Resistance of platinum wire in a platinum resistance thermometer at melting ice, boiling water and at a hot bath are 6 Ω, 6.5Ω and 6.2 Ω, respectively. Find temperature of hot bath.*

Sol. Given, $R_0 = 6\,\Omega$, $R_{100} = 6.5\,\Omega$ and $R_t = 6.2\,\Omega$

The working principle of platinum resistance thermometer (t) is given by

$$t = \frac{R_t - R_0}{R_{100} - R_0} \times 100°\text{C} = \frac{6.2 - 6}{6.5 - 6} \times 100°\text{C}$$

$$= \frac{0.2}{0.5} \times 100°\text{C} = 40°\text{C}$$

Example 3.21 *The temperature coefficient of resistance of a wire is 0.00145 °C^{-1}. At 100 °C its resistance is 2 Ω. At what temperature the resistance of the wire will be 3 Ω?*

Sol. Using the relation, $R = R_0(1 + \alpha T)$

$$R_1 = R_0(1 + \alpha t_1)$$

$$\therefore \quad 2 = R_0[1 + 0.00145 \times 100] \quad \text{...(i)}$$

and $$R_2 = R_0[1 + \alpha t_2]$$

$$\therefore \quad 3 = R_0[1 + 0.00145 \times t_2] \quad \text{...(ii)}$$

On dividing Eq. (ii) by Eq. (i), we get

$$3/2 = \frac{1 + 0.00145 t_2}{1 + 0.00145 \times 100}$$

or $$t_2 = 394.8°\text{C} = 667.8\text{K}$$

Colour code for carbon resistor

In electrical and electronic circuits, there are two major types of resistors, which are being used, *i.e.*, wire bound resistors and carbon resistor, offering resistances over a wide range. Wire bound resistors have resistances upto few hundred ohm, whereas for higher resistances, resistors are mostly made up of carbon.

To know the value of resistance of carbon resistors, colour code is used. These codes are printed in the form of set of rings or strips. By reading the values of colour bands, we can estimate the value of resistance.

The carbon resistor has normally four coloured rings or bands say A, B, C and D as shown in the following Fig. 3.7.

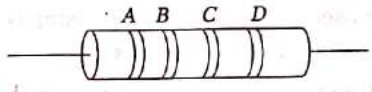


Fig. 3.7 Carbon resistor

Colour bands *A* and *B* indicates the first two significant figures of resistance in ohm.

B* and *C indicates the decimal multiplier, *i.e.*, the number of zeros that follows the two significant figures *A* and *B*.

B* and *D indicates the tolerance in percent as per the indicated value, in other words, it represents the percentage accuracy of the indicated value.

The tolerance in the case of gold is ±5 % and in silver is ±10 %. If only three bands are marked on carbon resistance, then it indicates a tolerance of 20%. (*i.e.*, no colour)

$R = AB \times C \pm D\%$, where D is tolerance.

Colour code for carbon resistors

Letters as an aid to memory	Colour	Figure (*A*, *B*)	Multiplier (*C*)
B	Black	0	10^0
B	Brown	1	10^1
R	Red	2	10^2
O	Orange	3	10^3
Y	Yellow	4	10^4
G	Green	5	10^5
B	Blue	6	10^6
V	violet	7	10^7
G	Grey	8	10^8
W	White	9	10^9

Note To remember the sequence of colour code following sentences should be kept in memory. B B ROY Great Britain Very Good Wife.

Example 3.22 *How will you represent a resistance of 3700 Ω ± 10% by colour code?*

Sol. The value of carbon resistance = 3700 Ω ± 10%

$\Rightarrow \quad R = 37 \times 10^2 \pm 10\%$

The colour assigned to numbers 3, 7 and 2 are orange, violet and red. For ± 10% accuracy, the colour is silver.

Hence, the bands of colour on carbon resistance in sequence are orange, violet, red and silver.

Combination of resistances

There are two types of combination of resistances, *i.e.*, series and parallel combination.

1. Series combination

In series combination, resistances are connected end-to-end, *i.e.*, second end of first resistor is connected to first end of the second resistor and so on.

Same current flows through each resistance but potential difference distributes in the ratio of their resistance, *i.e.*, $V \propto R$. So, the total potential drop is equal to the sum of potential applied across the combination.

Equivalent resistance in series combination is given by

$$\boxed{R_{eq} = R_1 + R_2 + R_3}$$

i.e., equivalent resistance is greater than the maximum value of resistance in the combination.

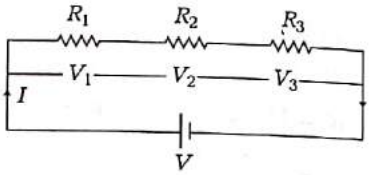


Fig. 3.8 *Resistances in series*

If n identical resistances are connected in series, then $R_{eq} = nR$ and potential difference across each resistance, $V' = V/n$.

2. Parallel combination

In this combination, first end of all resistors are connected to one point and second ends of all resistors are connected to other point.

Same potential difference appears across each resistance but current distributes in the reverse ratio of their resistance *i.e.*, $I \propto \frac{V}{R}$. So, the total current is equal to the sum of currents through each resistance.

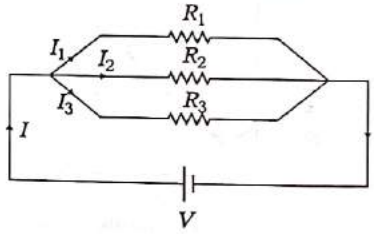


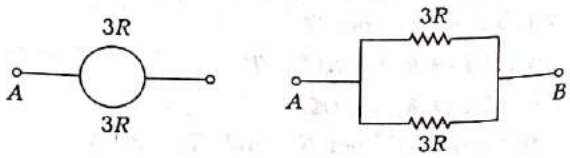
Fig. 3.9 *Resistances in parallel*

Equivalent resistance is given by $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

i.e., equivalent resistance is less than the minimum value of resistance in the combination.

In n identical resistance are connected in parallel, then $R_{eq} = \frac{R}{n}$ and current through each resistance is, $I' = \frac{I}{n}$.

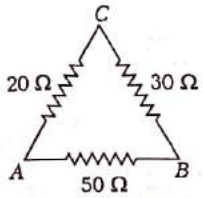
Example 3.23 *A wire of resistance 6R is bent in the form of a circle. What is the effective resistance between the ends of the diameter?*

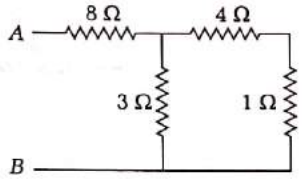


Sol. As shown in figure, the two resistances of value $3R$ each are in parallel with each other. So, the resistance between the ends A and B of a diameter is

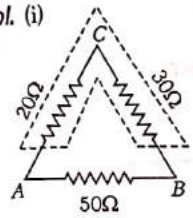
$$R' = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{3R \times 3R}{3R + 3R} = \frac{9R^2}{6R} = \frac{3}{2}R.$$

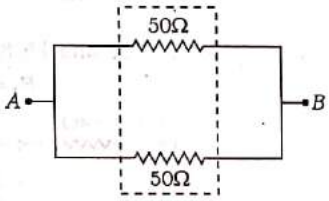
Example 3.24 *Find the equivalent resistance between A and B in the following cases*

(i)


(ii)


Sol. (i)





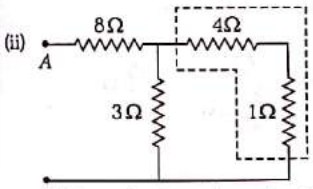
From the above dotted region, we have 20 Ω and 30 Ω are in series,

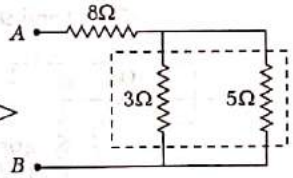
$\Rightarrow R = 20 + 30 = 50\ \Omega$

As per dotted region 50 Ω and 50 Ω are in parallel,

$$\Rightarrow R_{eq} = \frac{50 \times 50}{50 + 50} = \frac{2500}{100} = 25\ \Omega$$

(ii)



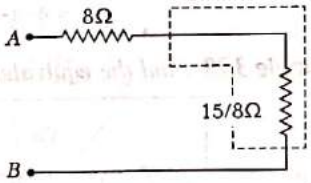


From the above dotted region, 4 Ω and 1Ω are in series,

$R = 4 + 1 = 5\ \Omega$

From the dotted region, 3 Ω and 5Ω are in parallel,

$$\Rightarrow \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{3} + \frac{1}{5} \Rightarrow R = \frac{15}{8}\Omega$$



Hence, from the dotted region, we have 8 Ω and 15/8Ω are in series,

$R_{eq} = 15/8 + 8 = 79/8\Omega$

Example 3.25 *A letter 'A' consists of a uniform wire of resistance 1 ohm per cm. The sides of letter are each 20 cm long and the cross-piece in the middle is 2 cm long while apex angle is 60°. Find the resistance of the letter between the two ends of the legs.*

Sol. Clearly it is given that, $AB = BC = CD = DE = BD = 10$ cm

and $R_1 = R_2 = R_3 = R_4 = R_5 = 2\,\Omega$

As R_2 and R_3 are in series, their combined resistance

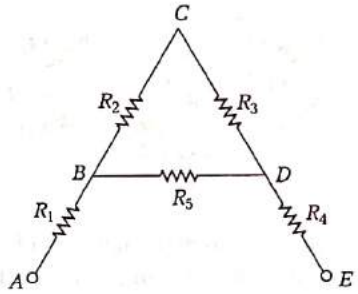
$$= 2 + 2 = 4\,\Omega$$

This combination is in parallel with R_5 $(= 2\,\Omega)$.

Hence, resistance between points of B and D is given by

$$\frac{1}{R} = \frac{1}{4} + \frac{1}{2}$$

$$\frac{1}{R} = \frac{6}{8} \Rightarrow R = \frac{4}{3}$$

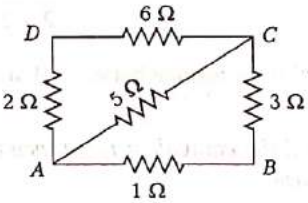


Now, resistance R_1, R, R_4 form a series combination. So, resistance between the ends A and E is

$$R' = 4 + \frac{4}{3} + 4$$

$$R' = 9.34\,\Omega$$

Example 3.26 *Find in the given network of resistors, the equivalent resistance between the points A and B, between A and D.*



Sol. *Resistance between A and D*

As we can see from the circuit, 1 Ω and 3Ω are in series,

$\Rightarrow$ $R_1 = 1 + 3 = 4\Omega$

Now, R_1 and 5 Ω are in parallel, $R_2 = \dfrac{4 \times 5}{4 + 5} = \dfrac{20}{9}\Omega$

Now, R_2 and 6 Ω are in series, $R_3 = 20/9 + 6 = \dfrac{74}{9}\Omega.$

Now, R_3 and 2 Ω are in parallel, $R_4 = \dfrac{74 \times 2}{92} = \dfrac{37}{23}\Omega.$

Resistance between A and B

The resistors AD $(= 2\,\Omega)$ and DC $(= 6\,\Omega)$ are in series to give a total resistance $R' = 8\,\Omega$. The resistance R' $(= 8\,\Omega)$ and the resistor AC $(= 5\,\Omega)$ are in parallel. Their equivalent resistance is

$$R'' = \frac{5 \times 8}{5 + 8} = \frac{40}{13}\Omega$$

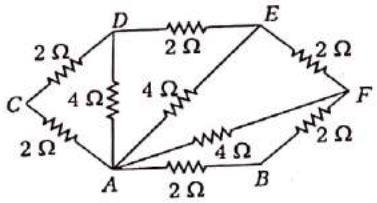
Now, R'' and 3Ω are in series,

$$R''' = R'' + 3 = \frac{40}{13} + 3 = \frac{79}{13}$$

Now, R''' and $1\,\Omega$ are in parallel resistance between A and B,

$$(R'''') = \frac{1 \times 79/13}{1 + 79/13} = \frac{79}{92}\Omega$$

Example 3.27 *Find the effective resistance between points A and B for the network shown in the figure below.*



Sol. At points A and D a series combination of $2\,\Omega$, $2\,\Omega$ resistance (along AC and CD) is in parallel with $4\,\Omega$ resistance (along AD), therefore resistance between A and D

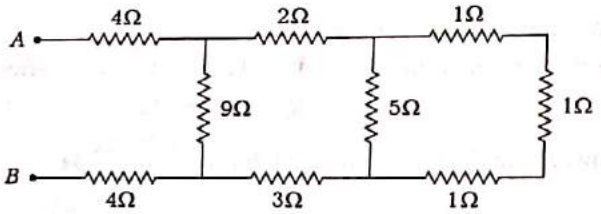
$$= \frac{1}{\frac{1}{2+2} + \frac{1}{4}} = 2\,\Omega$$

Similarly, resistance between A and $F = \dfrac{1}{\frac{1}{2+2} + \frac{1}{4}} = 2\,\Omega$

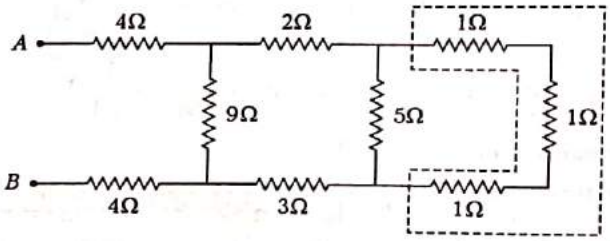
Finally, resistance between A and $B = \dfrac{1}{\frac{1}{2+2} + \frac{1}{2}} = \dfrac{4}{3}\Omega$

Thus, the effective resistance between A and B is $\dfrac{4}{3}\Omega$.

Example 3.28 *Find the equivalent resistance between A and B in the following case*

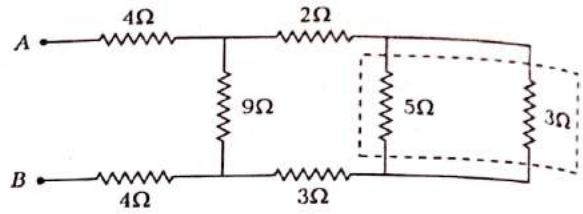


Sol.



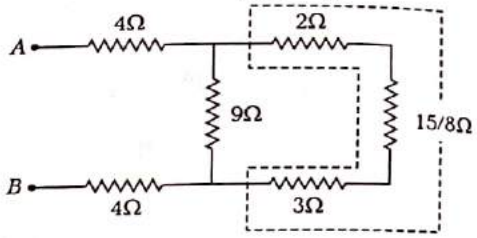
As per the dotted region, 1Ω, 1Ω and 1Ω are in series,

$$R = 1 + 1 + 1 = 3\,\Omega$$



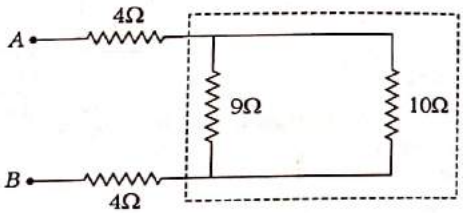
$\Rightarrow$ 3Ω and 5Ω are in parallel,

$$R_1 = \frac{3 \times 5}{3+5} = \frac{15}{8}\Omega$$

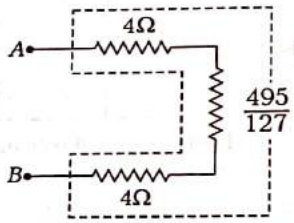


$\Rightarrow$ 2Ω, 3Ω and $15/8\Omega$ are in series,

$$R_2 = 2 + 3 + \frac{15}{8} = \frac{55}{8}\,\Omega$$



$\Rightarrow$ $55/8\ \Omega$ and $9\ \Omega$ are in parallel,



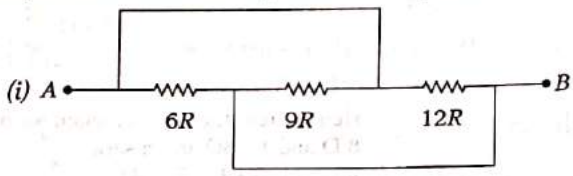
$$\Rightarrow R = \frac{\frac{55}{8} \times 9}{\frac{55}{8} + 9} = \frac{495}{127}\,\Omega$$

4Ω, 4Ω and $495/127\Omega$ are in series,

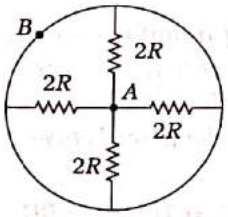
$$R_{eq} = 4 + 4 + 495/127 \quad \Rightarrow \quad R = \frac{1511}{127}\,\Omega$$

Example 3.29 *Find the equivalent resistance between A and B*

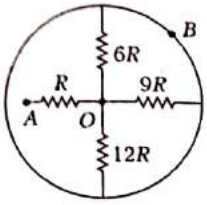
(i)



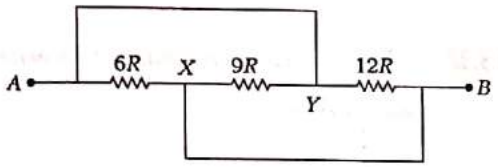
(ii)

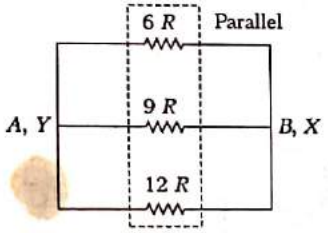


(iii)



Sol. (i) The points connected by a wire are at same potential or same points. Then, redraw the diagram, by placing the points of same potential at one place and then solve for equivalent resistance.



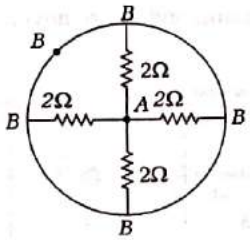


Hence, from the new figure A and Y are at same potential; B and X are at same potential.

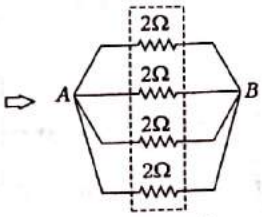
$$\Rightarrow \quad \frac{1}{R_{eq}} = \frac{1}{6R} + \frac{1}{9R} + \frac{1}{12R} = \frac{13}{36R}$$

$$R_{eq} = \frac{36R}{13} = 2.77R$$

(ii)

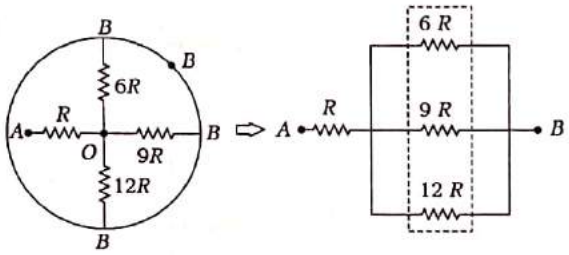


Similarly, placing the points of same potential at one place, then the equivalent resistance is

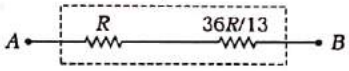


Parallel, $R_{eq} = \frac{2}{2} = 1\ \Omega$

(iii)

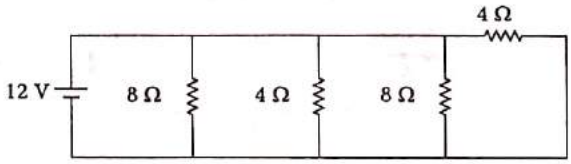


Parallel, $\frac{1}{R'} = \frac{1}{6R} + \frac{1}{9R} + \frac{1}{12R} = \frac{13}{36R} \Rightarrow R' = \frac{36}{13}R$

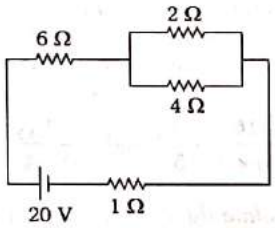


Series, $R_{eq} = R + \frac{36R}{13} = \frac{49}{13}R = 3.76R.$

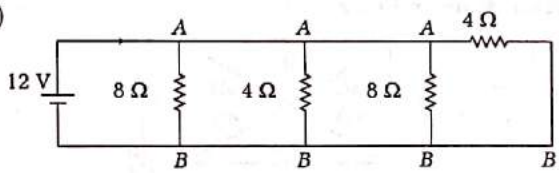
Example 3.30 *(i) Determine the current supplied by the battery in the circuit as shown*

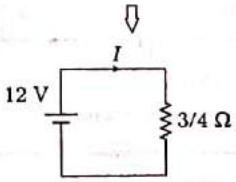


(ii) Find currents in resistances 2Ω and 4Ω.



Sol. (i)





All resistances in the circuit are in parallel,

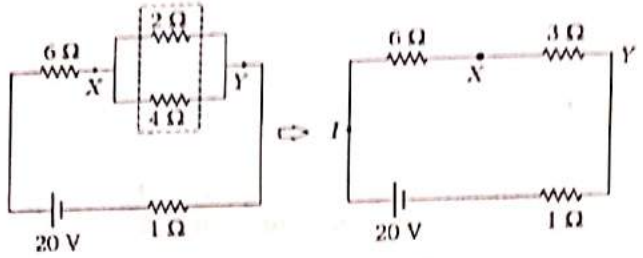
$$\Rightarrow \quad \frac{1}{R_{eq}} = \frac{1}{4} + \frac{1}{8} + \frac{1}{4} + \frac{1}{8}$$

$$= \frac{1+2+1+2}{8} = \frac{6}{8} = \frac{3}{4}$$

$$R_{eq} = 4/3\ \Omega$$

So, current supplied by battery, $I = \frac{V}{R} = \frac{12 \times 3}{4} = 9\ \text{A}$

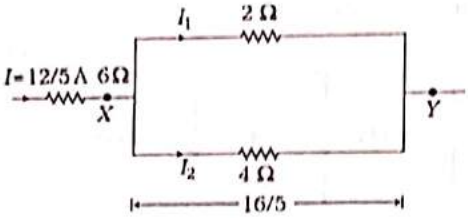
(ii) According to the circuit, parallel, $\frac{4\times 2}{4+2}=\frac{8}{6}=\frac{4}{3}\Omega$



$\Rightarrow$ $$R_{eq}=6+1+\frac{4}{3}=\frac{25}{3}\Omega$$

$\therefore$ Current supplied by battery $=\frac{20\times 3}{25}=\frac{12}{5}$ A.

In order to calculate the current in resistance 2 Ω and 4 Ω, we can redraw the circuit as

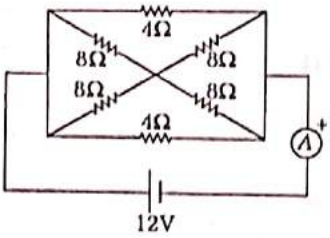


Potential difference across XY, $V_{XY}=\left(\frac{12}{5}\text{A}\right)\left(\frac{4}{3}\Omega\right)=\frac{16}{5}$ V

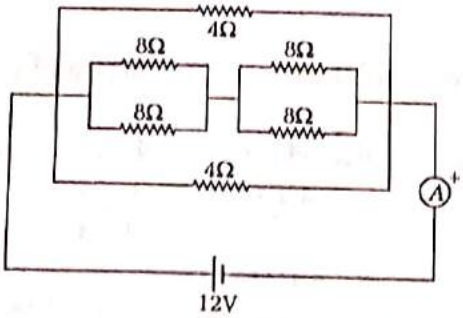
2 Ω and 4 Ω are parallel across XY. Hence, have same potential.

$\Rightarrow I_1=\frac{V_{XY}}{2}=\frac{16}{5\times 2}=\frac{8}{5}$ A and $I_2=\frac{V_{XY}}{4}=\frac{16}{5\times 4}=\frac{4}{5}$ A

Example 3.31. *Calculate the current shown by the ammeter A in the circuit shown in figure.*



Sol. The circuit given can be redrawn as



From the figure, the two 8 Ω resistances connected in parallel, equivalent resistance $=\frac{8\times 8}{8+8}=4\,\Omega$

These two combinations are connected in series, equivalent resistance $=4+4=8\,\Omega$.

Now, we have resistances of 4 Ω, 8 Ω and 4 Ω connected in parallel, so

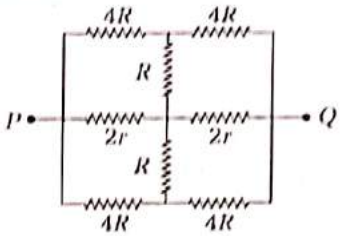
$\Rightarrow$ $$\frac{1}{R}=\frac{1}{4}+\frac{1}{8}+\frac{1}{4}=\frac{5}{4}\quad\text{or}\quad R=\frac{4}{5}\Omega$$

Also, $V=12$V (given)

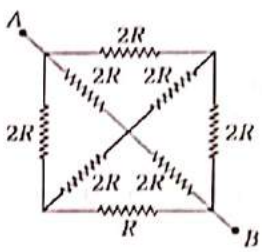
$\therefore$ Current, $I=\frac{V}{R}=\frac{12}{4/5}=15$ A

Example 3.32 *Find the equivalent resistance between P and Q*

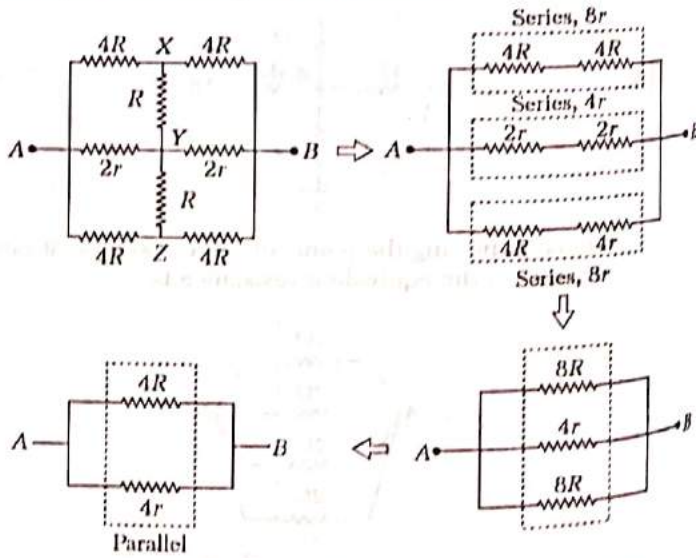
(i)



(ii)

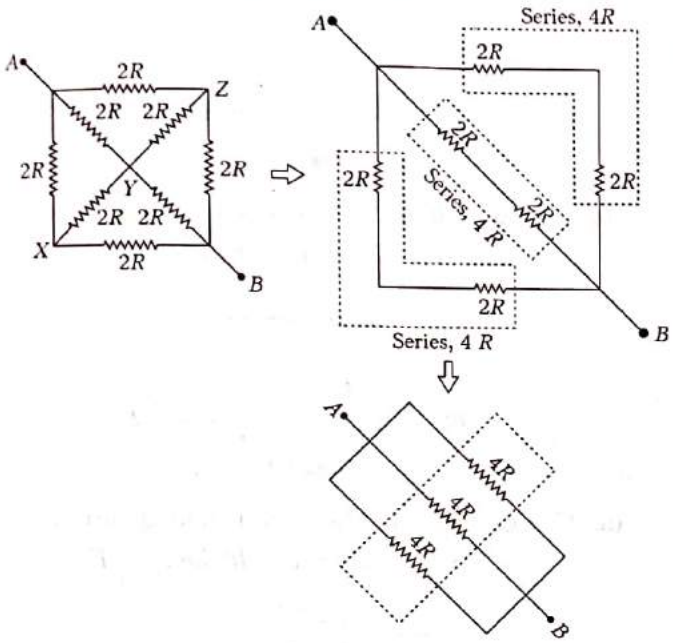


Sol. (i) It can be seen that, in this diagram is symmetrical about PQ, so points on the perpendicular bisector of PQ, *i.e.*, X, Y and Z are at same potential, so in type of diagrams, to same the equivalent resistance, we can remove the resistances at the same potential, *i.e.*, the resistances between X and Y, Y and Z, are redundant, and are removed.



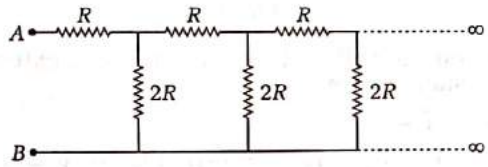
$\Rightarrow R_{eq}=\frac{4R\cdot 4r}{4R+4r}=\frac{4Rr}{R+r}$ $\because$ Parallel, 8R, 8R in parallel, $R'_{eq}=2R$

(ii) Similarly as option (a), we see that there is symmetry about AB and X, Y and Z are at same potential: remove resistance between X and Y; Y and Z.

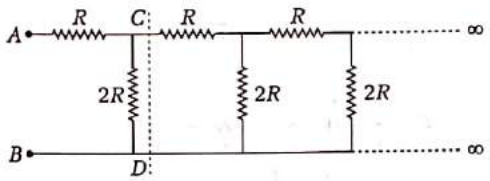


$\Rightarrow$ Parallel, $R_{eq} = \frac{2R}{3}$

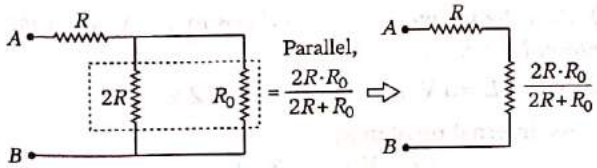
Example 3.33 *Find the equivalent resistance between A and B.*



Sol.



Here, there are infinite pairs of R and $2R$. Suppose, the equivalent resistance is R_0 between C and D, *i.e.*, excluding one pair near AB (since, pairs are infinite, equivalent resistance will remain same, if we include pair near AB.)



$$\Rightarrow \quad R_{eq} = R_0 = R + \frac{2RR_0}{2R + R_0}$$

$$(R_0 - R)(2R + R_0) = 2RR_0$$

$$2RR_0 - 2R^2 + R_0^2 - RR_0 = 2RR_0$$

$$R_0^2 - RR_0 - 2R^2 = 0$$

$$R_0 = \frac{R \pm \sqrt{R^2 + 8R^2}}{2} = \frac{R \pm 3R}{2} = 2R \text{ or } -R$$

Equivalent resistance between A and $B = 2R$ (equivalent resistance cannot be negative).

Cells, emf and internal resistance

An electric cell is a source of energy that maintains a continuous flow of charge in a circuit. Electric cell changes chemical energy into electrical energy.

Electromotive force (emf) of a cell (ε)

Electric cell has to do some work in maintaining the current through a circuit. The work done by the cell in moving unit positive charge through the whole circuit (including the cell) is called the **electromotive force (emf)** of the cell.

If during the flow of q coulomb of charge in an electric circuit, the work done by the cell is W, then

$$\text{emf of the cell, } \boxed{\varepsilon = \frac{W}{q}}$$

Its unit is joule/coulomb or volt.

If $W = 1$ joule and $q = 1$ coulomb, then $\varepsilon = 1$ volt, *i.e.*, if in the flow of 1 coulomb of charge, the work done by the cell is 1 joule, then the emf of the cell is 1 volt.

Internal resistance (r)

Internal resistance of a cell is defined as the resistance offered by the electrolyte of the cell to the flow of current through it. It is denoted by r. Its unit is ohm.

Internal resistance of a cell depends on the following factors:

(*i*) It is directly proportional to the separation between the two plates of the cell.

(*ii*) It is inversely proportional to the plates area dipped into the electrolyte.

(*iii*) It depends on the nature, concentration and temperature of the electrolyte and increases with increase in concentration.

Terminal potential difference (V)

Terminal potential difference of a cell is defined as the potential difference between the two terminals of the cell in a closed circuit (*i.e.*, when current is drawn from the cell). It is represented by V and its unit is volt.

Terminal potential difference of a cell is always less than the emf of the cell. In closed circuit, the current flows through the circuit including the cell, due to internal

resistance of the cell, there is some fall of potential. This is the amount of potential by which the terminal potential difference is less than the emf of the cell.

Relation between terminal potential difference, emf of the cell and internal resistance of a cell

(i) If no current is drawn from the cell, *i.e.*, the cell is in open circuit, so emf of the cell will be equal to the terminal potential difference of the cell.

$$I = 0 \quad \text{or} \quad V = \varepsilon$$

(ii) Consider a cell of emf ε and internal resistance r is connected across an external resistance R.

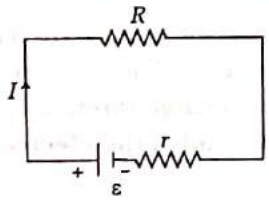


Fig. 3.10

Current drawn from the cell, $I = \dfrac{\varepsilon}{R + r}$ ...(i)

where, ε = emf of the cell, R = external resistance and r = internal resistance of a cell

Now from Ohm's law,

$$V = IR \Rightarrow I = \frac{V}{R} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we have

$$\frac{V}{R} = \frac{\varepsilon}{R + r}$$

$$\Rightarrow \quad r = \left(\frac{\varepsilon}{V} - 1\right)R$$

Charging of cell

During charging of a cell, the positive terminal (electrode) of the cell is connected to positive terminal of battery charger and negative terminal (electrode) of the cell is

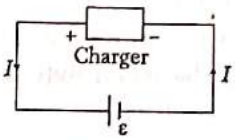


Fig. 3.11

connected to negative terminal of battery charger. In this process, current flows from positive electrode to negative electrode of the cell. From figure above,

$$V = \varepsilon + Ir$$

Thus, the terminal potential difference of a cell becomes greater than the emf of the cell.

The potential drop across internal resistance of the cell is called lost voltage, as it is not indicated by a voltmeter. Its value is equal to Ir.

Potential difference (V) across the terminals of a cell

(i) When the cell is in open circuit,

$$V = V_P - V_Q = E, \text{ i.e., } V = E$$

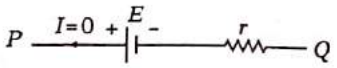


Fig. 3.12

(ii) Discharging of cell when cell is supplying current,

$$\therefore \quad V = V_P - V_Q = E - Ir, \text{ i.e., } V < E$$

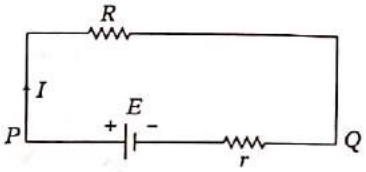


Fig. 3.13

(iii) Charging of cell when cell is taking current.

$$V = V_P - V_Q = E + Ir, \text{ i.e., } V > E$$

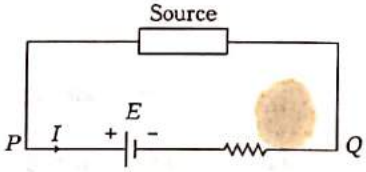


Fig. 3.14

(iv) When the cell is short circuited *i.e.*, external resistance is zero,

$$\Rightarrow \quad I = E/r$$

$$V = V_P - V_Q = E - Ir = 0 \text{ i.e., } V = 0$$

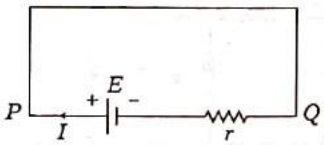


Fig. 3.15

Example 3.34 *The reading on a high resistance voltmeter when a cell is connected across it is 3 V. When the terminals of the cell are also connected to a resistance of 4 Ω, then the voltmeter reading drops to 1.2 V. Find the internal resistance of the cell.*

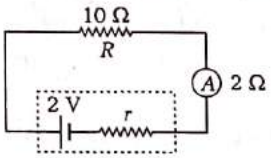
Sol. Here, $E = 3$ V, $R = 4\,\Omega$ and $V = 1.2$ V

As, internal resistance,

$$r = R\left(\frac{E - V}{V}\right) = 4\left(\frac{3 - 1.2}{1.2}\right)\Omega = 6\,\Omega$$

Example 3.35 *A battery of emf 2 V and internal resistance r is connected in series with a resistor of 10 Ω through an ammeter of resistance 2 Ω. The ammeter reads 50 mA. Draw the circuit diagram and calculate the value of r.*

Sol. Total resistance $= 10 + 2 + r = (12 + r)\,\Omega$



Now, current $= 50\text{ mA} = 50 \times 10^{-3}\text{A}$

and emf $= 2$ V.

So, $$\text{resistance} = \frac{\text{emf}}{\text{current}}$$

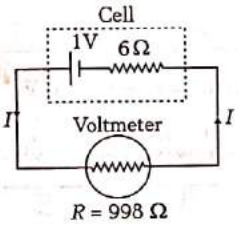
$$12 + r = \frac{2}{50 \times 10^{-3}} = 40$$

$$\Rightarrow \quad r = 40 - 12 = 28\,\Omega$$

Example 3.36 *A voltmeter of resistance 994 Ω is connected across a cell of emf 1 V and internal resistance 6 Ω. Find the potential difference across the voltmeter, that across the terminals of the cell and percentage error in the reading of the voltmeter.*

Sol. Here, $E = 1$ V, $r = 6\,\Omega$

Resistance of voltmeter, $R = 994\,\Omega$



Current in the circuit is

$$I = \frac{E}{R + r} = \frac{1\text{V}}{(994 + 6)\,\Omega} = 1 \times 10^{-3}\text{A}$$

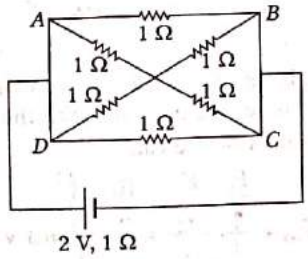
The potential difference across the voltmeter is

$$V = IR = 1 \times 10^{-3} \times 994 = 9.98 \times 10^{-1}\text{V}$$

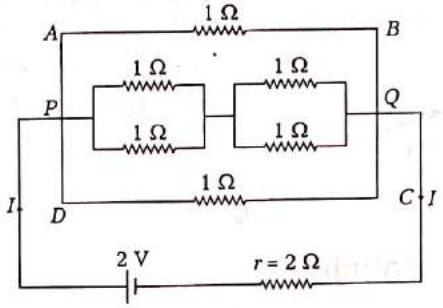
The same will be the potential difference across the terminals of the cell. The voltmeter used to measure the emf of the cell will read 0.998 V. Hence, the percentage error is

$$\frac{E - V}{E} \times 100 = \frac{1 - 0.998}{1} \times 100 = 0.2\%$$

Example 3.37 *Find the current drawn from a cell of emf 2 V and internal resistance 2 Ω connected to the network given below.*



Sol. The equivalent circuit is shown below.



Resistance in arm $AB = 1\,\Omega$

Resistance in arm $PQ = \frac{1 \times 1}{1 + 1} + \frac{1 \times 1}{1 + 1} = \frac{1}{2} + \frac{1}{2} = 1\,\Omega$

Resistance in arm $DC = 1\,\Omega$

These three resistances are connected in parallel.

Their equivalent reistance R is given by

$$\frac{1}{R} = \frac{1}{1} + \frac{1}{1} + \frac{1}{1} = \frac{3}{1} \quad \text{or} \quad R = \frac{1}{3}\,\Omega$$

Current drawn fron the cell, $$I = \frac{E}{R + r} = \frac{2V}{\left(\frac{1}{3} + 2\right)\Omega}$$

$$= \frac{2 \times 3}{7} = \frac{6}{7}\text{A}$$

Grouping of cells

Series grouping

In series grouping anode of one cell is connected to cathode of other cell and so on. If n identical cells are connected in series, then

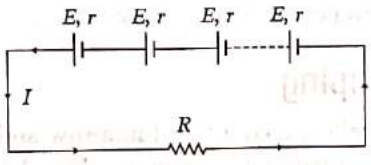


Fig. 3.16

(i) Equivalent emf of the combination, $E_{eq} = nE$.

(ii) Equivalent internal resistance, $r_{eq} = nr$.

(iii) Main current = Current from each cell $= I = \frac{nE}{R + nr}$.

(iv) Potential difference across external resistance, $V = IR$.

(v) Potential difference across each cell, $V' = \frac{V}{n}$.

If external resistance is much higher than the total internal resistance, then cells should be connected in series to get the maximum current.

Note If dissimilar plates of cells are connected together their emf's are added to each other while, if their similar plates are connected together their emf's are subtractive. While their internal resistance is always additive.

Fig. 3.17

Parallel grouping

In parallel grouping all anodes are connected at one point and all cathodes are connected together at other point.

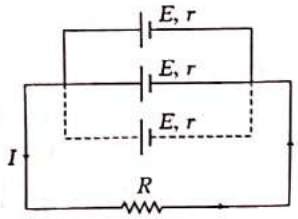


Fig. 3.18

If n identical cells are connected in parallel, then

(i) Equivalent emf, $E_{eq} = E$.

(ii) Equivalent internal resistance, $R_{eq} = r/n$.

(iii) Main current, $I = \dfrac{E}{R + r/n}$.

(iv) Current from each cell, $I' = \dfrac{I}{n}$.

(v) Potential difference across external resistance = Potential difference across each cell = $V = IR$.

If external resistance is much smaller than the net internal resistance, then cells should be connected in parallel to get the maximum current.

Mixed grouping

If n identical cells are connected in a row and such m rows are connected in parallel as shown in Fig. 3.17, then

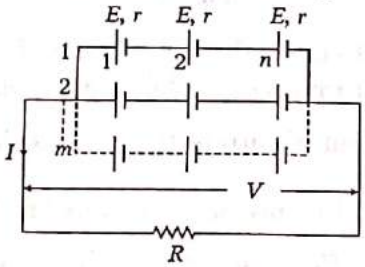


Fig. 3.19

(i) Equivalent emf of the combination, $E_{eq} = nE$.

(ii) Equivalent internal resistance of the combination,

$$r_{eq} = \frac{nr}{m}$$

(iii) Main current flowing through the load,

$$I = \frac{nE}{R + \dfrac{nr}{m}} = \frac{mnE}{mR + nr}$$

(iv) Potential difference across load, $V = IR$.

(v) Current from each cell, $I' = \dfrac{I}{n}$

(vi) Potential difference across each cell, $' = \dfrac{V}{n}$.

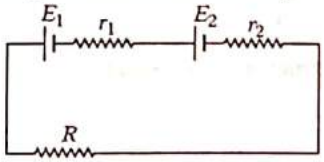
(vii) Total number of cells = mn

In mixed grouping of a cell, the current through the external resistance would be maximum, if the external resistance is equal to the total internal resistance of the cells.

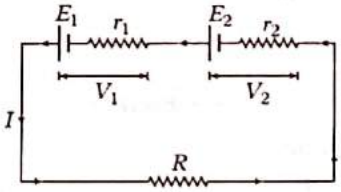
Example 3.38. *In the circuit shown in figure,*

$$E_1 = 10\,V, E_2 = 4\,V, r_1 = r_2 = 1\,\Omega \text{ and } R = 2\,\Omega.$$

Find the potential difference across battery 1 and battery 2.



Sol. Net emf of the circuit $= E_1 - E_2 = 6$ V



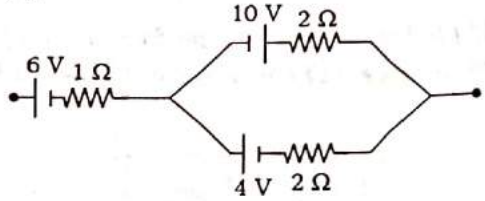
Total resistance of the circuit $= R + r_1 + r_2 = 4\,\Omega$

$\therefore$ Current in the circuit, $I = \dfrac{\text{Net emf}}{\text{Total resistance}} = \dfrac{6}{3} = 1.5$ A

Now, $V_1 = E_1 - Ir_1 = 10 - (1.5)(1) = 8.5$ V

and $V_2 = E_2 + Ir_2 = 4 + (1.5)(1) = 5.5$ V

Example 3.39 *Find the emf and internal resistance of a single battery which is equivalent to a combination of three batteries as shown in figure.*



Sol. The given combination consists of two batteries in parallel and resultant of these two in series with the third one.

For parallel combination we can apply,

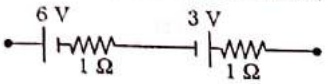
$$E_{eq} = \frac{\dfrac{E_1}{r_1} - \dfrac{E_2}{r_2}}{\dfrac{1}{r_1} + \dfrac{1}{r_2}} = \frac{\dfrac{10}{2} - \dfrac{4}{2}}{\dfrac{1}{2} + \dfrac{1}{2}} = 3 \text{ V}$$

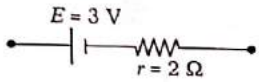
Further,

$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} = \frac{1}{2} + \frac{1}{2} = 1$$

$\therefore$ $r_{eq} = 1\,\Omega$

Now, this is in series with the third one, *i.e.*,





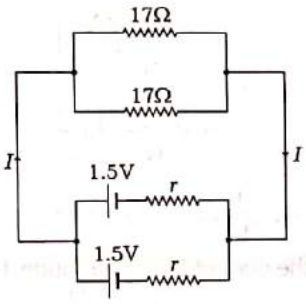
The equivalent emf of these two is (6 – 3) V or 3 V and the internal resistance will be (1 + 1) or 2 Ω.

Example 3.40 *Two identical cells of emf 1.5 V each joined in parallel provide supply to an external circuit consisting of two resistances of 17 Ω each joined in parallel. A very high resistance voltmeter reads the terminal voltage of cells to be 1.4 V. Calculate the internal resistance of each cell.*

Sol. Here, $E = 1.5$ V, $V = 1.4$ V

Resistance of external circuit = Total resistance of two resistances of 17 Ω connected in parallel.

or $$R = \frac{R_1R_2}{R_1 + R_2} = \frac{17 \times 17}{17 + 17}\,\Omega = 8.5\,\Omega$$



Let r' be the total internal resistance of the two cells. Then,

$$r' = R\left(\frac{E - V}{V}\right) = 8.5\left(\frac{1.5 - 1.4}{1.4}\right)\Omega = 0.6\,\Omega.$$

As the two cells of internal resistance $r\,\Omega$ each have been connected in parallel, therefore.

$$\frac{1}{r'} = \frac{1}{r} + \frac{1}{r} \text{ or } \frac{1}{0.6} = \frac{2}{r} \text{ or } r = 0.6 \times 2 = 1.2\,\Omega$$

Example 3.41 *Find the minimum number of cells required to produce an electric current of 1.5 A through a resistance of 30 Ω. Given that the emf of each cell is 1.5 V and internal resistance 1.0 Ω.*

Sol. As, $\frac{nr}{m} = R$

$\therefore$ $$\frac{n \times 1}{m} = 30 \text{ or } n = 30 \text{ m}$$

$$\text{Current, } I = \frac{nE}{2R} \text{ or } 1.5 = \frac{n \times 1.5}{2 \times 30} \text{ or } n = 60$$

$\therefore$ $m = 60/30 = 2$ and $mn = 120$

Example 3.42 *36 cells each of internal resistance 0.5 Ω and emf 1.5 V each are used to send current through an external circuit of 2 Ω resistance. Find the best mode of grouping them and the current through the external circuit.*

Sol. Here, $E = 1.5$ V, $r = 0.5\,\Omega$, $R = 2\,\Omega$

Total number of cells, $mn = 36$ ...(i)

For maximum current in the mixed grouping,

$$\frac{nr}{m} = R \text{ or } \frac{n \times 0.5}{m} = 2 \quad \text{... (ii)}$$

Multiplying Eqs. (i) and (ii), we get

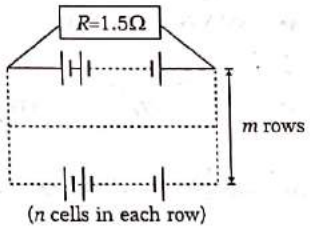
$$0.5n^2 = 72 \text{ or } n^2 = 144$$

$\therefore$ $$n = 12 \text{ and } m = \frac{36}{12} = 3$$

Thus, for maximum current there should be three rows in parallel, each containing 12 cells in series.

$$\therefore \text{ Maximum current} = \frac{mnE}{mR + nr} = \frac{36 \times 1.5}{3 \times 2 + 12 \times 0.5} = 4.5 \text{ A}$$

Example 3.43 *12 cells, each of emf 1.5 V and internal resistance of 0.5 Ω, are arranged in m rows each containing n cells connected in series, as shown. Calculate the values of n and m for which this combination would send maximum current through an external resistance of 1.5 Ω.*



Sol. For maximum current through the external resistance,

external resistance = total internal resistance of cells

or $$R = \frac{nr}{m}$$

$\therefore$ $$1.5 = \frac{n \times 0.5}{\frac{12}{n}} \quad [\because mn = 12]$$

or $$36 = n^2 \text{ or } n = 6 \text{ and } m = 2$$

Check point 3.2

1. Calculate the amount of charge flowing in 2 min in a wire of resistance 10 Ω when a potential difference of 20 V is applied between its ends

(a) 120 C (b) 240 C
(c) 20 C (d) 4 C

2. If e, τ and *m* respectively, represent the electron density, relaxation time and mass of the electron, then the resistance of a wire of length *l* and area of cross-section *A* will be

(a) $\frac{ml}{ne^2\tau A}$ (b) $\frac{m\tau^2 A}{ne^2 l}$

(c) $\frac{ne^2\tau A}{2ml}$ (d) $\frac{ne^2 A}{2m\tau l}$

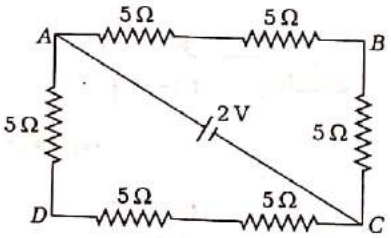
3. The following four wires are made of the same material and are at the same temperature. Which one of them has highest electrical resistance?

(a) Length = 50 cm, diameter = 0.5 mm
(b) Length = 100 cm, diameter = 1 mm
(c) Length = 200 cm, diameter = 2 mm
(d) Length = 300 cm, diameter = 3 mm

4. Carbon resistors, used in electronic circuits are marked for their value of resistance and tolerance by a colour code. A given resistor has colour scheme brown, red, green and gold. Its value in ohm is

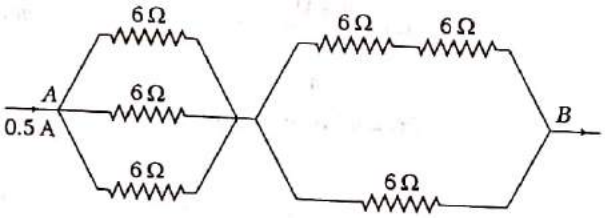
(a) $52 \times 10^6 \pm 10\%$ (b) $24 \times 10^5 \pm 5\%$
(c) $12 \times 10^4 \pm 10\%$ (d) $12 \times 10^5 \pm 5\%$

5. The potential difference between points *A* and *B* of the following figure is



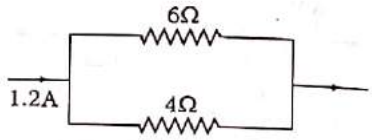
(a) $\frac{2}{3}$ V (b) $\frac{8}{9}$ V (c) $\frac{4}{3}$ V (d) 2 V

6. Resistance of 6 Ω each are connected in the manner shown in figure. With the current 0.5 A, the potential difference $V_A - V_B$ is



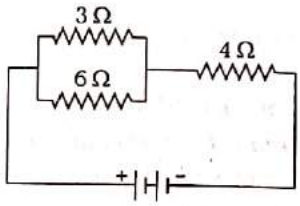
(a) 3.6 V (b) 6.0 V (c) 3.0 V (d) 7.2 V

7. In the figure given below, the current passing through 6 Ω resistor is



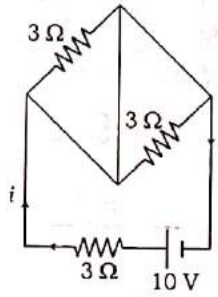
(a) 0.40 A (b) 0.48 A (c) 0.72 A (d) 0.80 A

8. In the figure, current through the 3 Ω resistor is 0.8 A, then potential drop through 4 Ω resistor is



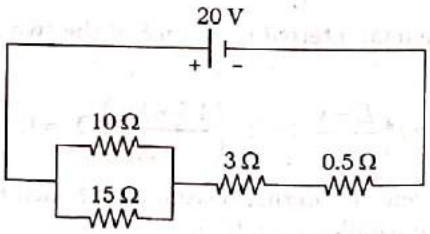
(a) 9.6 V (b) 2.6 V (c) 4.8 V (d) 1.2 V

9. Current *i* as shown in the circuit will be



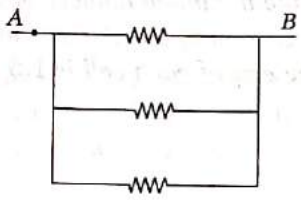
(a) 10 A (b) $\frac{10}{3}$ A (c) zero (d) infinite

10. In the circuit, the current flowing through 10 Ω resistance is



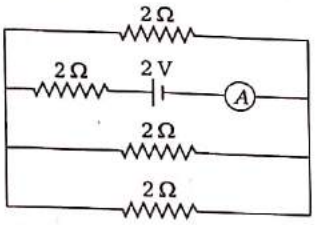
(a) 12 A (b) 1.2 A (c) 0.8 A (d) 0.4 A

11. If all the resistors shown have the value 2 ohm each the equivalent resistance over *AB* is



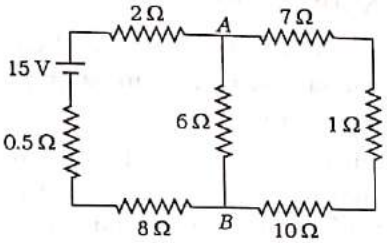
(a) 2 ohm (b) 4 ohm
(c) $1\frac{2}{3}$ ohm (d) $2\frac{2}{3}$ ohm

12. The reading of the ammeter as per figure shown is



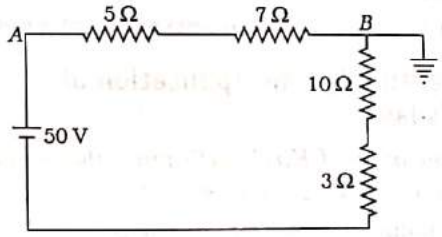
(a) $\frac{1}{8}$ A (b) $\frac{3}{4}$ A (c) $\frac{1}{2}$ A (d) 2 A

13. The current drawn from the battery in circuit diagram shown is



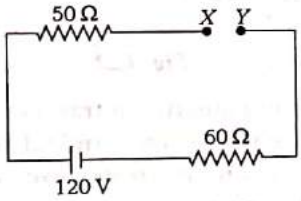
(a) 1 A (b) 2 A (c) 1.5 A (d) 3 A

14. In the circuit shown, the point *B* is earthed. The potential at the point *A* is



(a) 14 V (b) 24 V (c) 26 V (d) 50 V

15. In the circuit shown, potential difference between *X* and *Y* will be

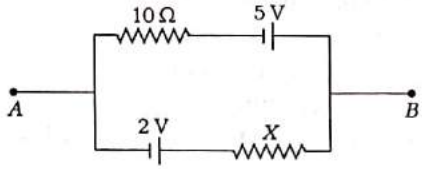


(a) zero (b) 20 V (c) 60 V (d) 120 V

16. A current of 0.2 A passes through a cell of emf 1.5 volt having internal resistance of 0.15 ohm. The potential difference measured in volt, across both the ends of the cell will be

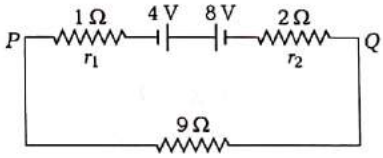
(a) 1.35 (b) 1.50 (c) 1.00 (d) 1.20

17. If V_{AB} is 4 V in the given figure, then resistance *X* will be



(a) 5 Ω (b) 10 Ω (c) 15 Ω (d) 20 Ω

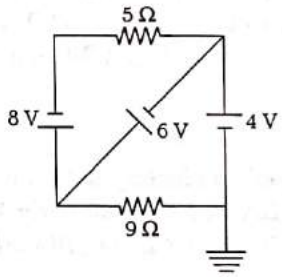
18. Two batteries of emf 4 V and 8 V with internal resistances 1 Ω and 2 Ω are connected in a circuit with a resistance of 9 Ω as shown in figure. The current and potential difference between the points *P* and *Q* are



(a) $\frac{1}{3}$ A and 3 V (b) $\frac{1}{6}$ A and 4 V

(c) $\frac{1}{9}$ A and 9 V (d) $\frac{1}{2}$ A and 12 V

19. The current through the 5 Ω resistor is



(a) 10 A (b) 1 A (c) 2.5 A (d) 0.4 A

20. Two cells, each of emf *E* and internal resistance *r* are connected in parallel between the resistance *R*. The maximum energy given to the resistor will be, only when

(a) $R = r/2$ (b) $R = r$ (c) $R = 2r$ (d) $R = 0$

3 Kirchhoff's laws

Many electric circuits cannot be reduced to simple series-parallel combinations. Kirchhoff's laws are used to solve complicated electric circuits. These rules are basically the expressions of conversion of electric charge and of energy. For example, two circuits that cannot be so broken down are shown in Fig. 3.20.

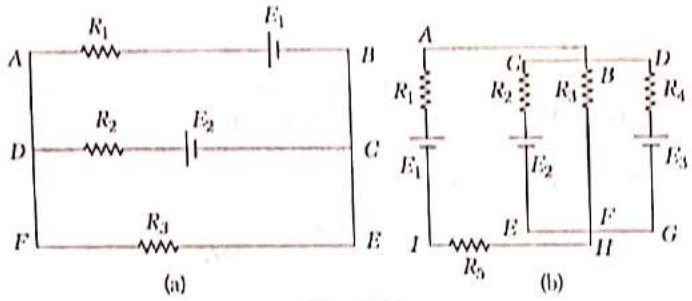


Fig. 3.20

However, it is always possible to analyse such circuits by applying two rules, devised by Kirchhoff in 1845 and 1846. The two terms related to Kirchhoff's laws are given below.

Junction

A junction in a circuit is a point, where three or more conductors meet. Junctions are also called nodes or branch points. For example, in Fig. 3.20 (a) points D and C are junctions. Similarly, in Fig. 3.20 (b) points B and F are junctions.

Loop

A loop is any closed conducting path. For example, in Fig. 3.20 (a) *ABCDA*, *DCEFD* and *ABEFA* are loops. Similarly, in Fig. 3.20 (b), *CBFEC*, *BDGFB* are loops.

Kirchhoff's junction rule

The algebraic sum of the currents meeting at a point or at a junction in an electric circuit is always zero.

i.e.,

$$\sum_{\text{junction}} i = 0$$

This law can also be written as, "the sum of all the currents directed towards a point in a circuit is equal to the sum of all the currents directed away from that point."

This law is also known as Kirchhoff's current law (KCL).

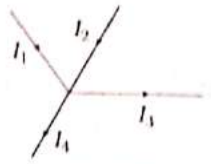


Fig. 3.21

Thus, in Fig. 3.21 according to KCL, $I_1 + I_2 = I_3 + I_4$

The junction rule is based on **conservation of electric charge.**

Kirchhoff's loop rule

This law states that the algebraic sum of charges in potential around any closed loop involving resistors and cells in the loop is zero.

It means that, in any closed part of an electrical circuit, the algebraic sum of the emfs is equal to the algebraic sum of the products of the resistances and currents flowing through them. It is also known as **loop rule.**

i.e.,

$$\sum_{\text{closed loop}} \Delta V = 0$$

This law is also known as **Kirchhoff's voltage law (KVL).**

The loop rule is based on **conservation of energy.**

Sign convention for the application of Kirchhoff's law

For the application of Kirchhoff's laws, the following sign conventions are to be considered.

(i) The change in potential in traversing a resistance in the direction of current is $-IR$ while in the opposite direction is $+IR$.

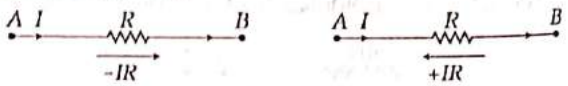


Fig. 3.22

(ii) The change in potential in traversing an emf source from negative to positive terminal is $+E$ while in the opposite direction $-E$ irrespective of the direction of current in the circuit.

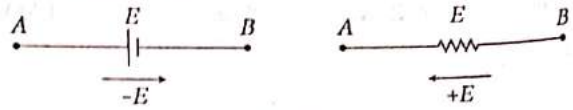


Fig. 3.23

To calculate potential difference between two points by using Kirchhoff's law

While using these rules to calculate the potential difference, the following points should be considered

(i) Start from a point on the loop and go along the loop, either anti-clockwise or clockwise, to reach the same point again, but balance currents at junction as per KCL.

(ii) If moving along the direction of the current, there will be potential drop across a resistance and if moving in the opposite direction, there will be potential gain.

(iii) The net sum of all these potential differences should be zero, using the KVL rule.

Now, let us consider a circuit as shown in Fig. 3.24.

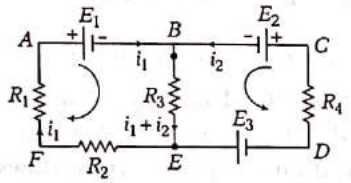


Fig. 3.24

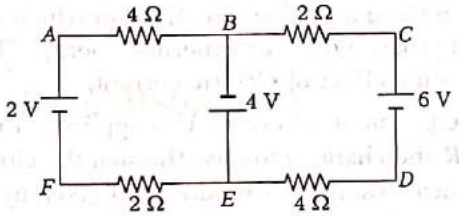
From point A to point D

(i) Path $ABCD, V_A - E_1 + E_2 + i_2R_4 = V_D$

(ii) Path $ABED, V_A - E_1 - (i_1 + i_2)R_3 - E_3 = V_D$

(iii) Path $AFED, V_A + i_1R_1 + i_1R_2 - E_3 = V_D$

Example 3.44 *Find currents in different branches of the electric circuit shown in figure.*



Sol. Applying Kirchhoff's first law (junction law) at junction B,

$$i_1 = i_2 + i_3 \quad ...(i)$$

Applying Kirchhoff's second law in loop 1 ($ABEFA$),

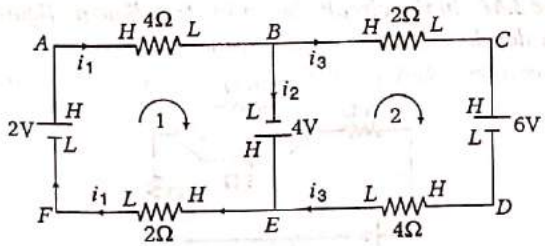
$$-4i_1 + 4 - 2i_1 + 2 = 0 \quad ...(ii)$$

Applying Kirchhoff's second law in loop 2 ($BCDEB$),

$$-2i_3 - 6 - 4i_3 - 4 = 0 \quad ...(iii)$$

Solving Eqs. (i), (ii) and (iii), we get

$$i_1 = 1 \text{ A}$$



$$i_2 = \frac{8}{3} \text{ A} \Rightarrow i_3 = -\frac{5}{3} \text{ A}$$

Here, negative sign of i_3 implies that current i_3 is in opposite direction of what we have assumed.

Example 3.45 *In example above, find the potential difference between points F and C.*

Sol. Let us reach from F to C *via* A and B,

$$V_F + 2 - 4i_1 - 2i_3 = V_C$$

$$\therefore \quad V_F - V_C = 4i_1 + 2i_3 - 2$$

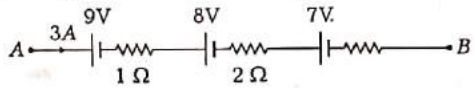
Substituting, $i_1 = 1$ A

and $i_3 = -(5/3)$ A, we get

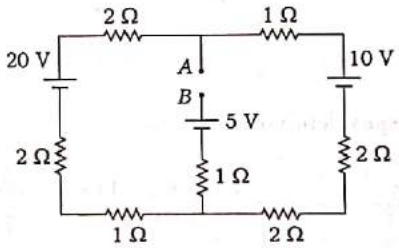
$$V_F - V_C = -(4/3) \text{ V}$$

Here, negative sign implies that $V_F < V_C$.

Example 3.46 *(i) Find the potential difference between A and B.*



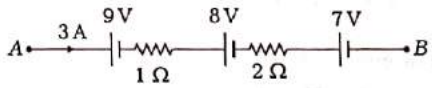
(ii)



(a) Find the potential difference between the points A and B.

(b) Find current through 20 V cell, if points A and B are connected.

Sol. (i)

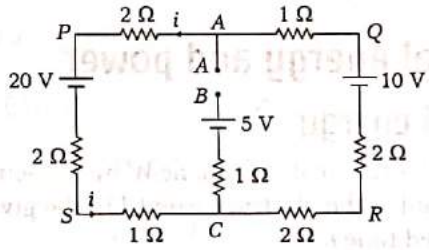


When we move from A to B, using KVL,

$$V_A - 9 - 3 \times 1 - 8 - 3 \times 2 - 7 = V_B$$

$$\Rightarrow \quad V_A - V_B = 33 \text{ V}$$

(ii) (a)



No current flows in the branch CB as AB is not connected.

Let current in the circuit $APSCRQA$ be i.

Using KVL,

$$V_A - i \times 2 - 20 - i \times 2 - i \times 1 - i \times 2 - i \times 2 + 10 - i \times 1 = V_A$$

$$\Rightarrow 10i = -10 \Rightarrow \quad i = -1 \text{A}$$

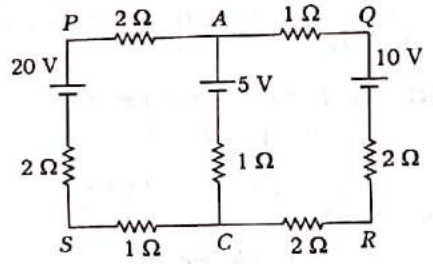
i.e., direction of i is opposite.

Now, A to B path will be $APSCB$,

$$V_A - i \times 2 - 20 - i \times 2 - i \times 1 + 5 = V_B$$

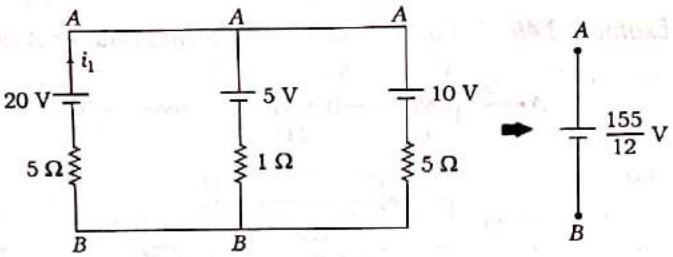
$$V_A - V_B = 15 + 5i = 15 + 5 \times (-1) = 10\text{ V}$$

(b) Now if the points A and B are connected.



Resistance 2 Ω, 2 Ω and 1 Ω are in series.

and resistance 1 Ω, 2 Ω and 2 Ω are in series.



So, the equivalent voltage will be

$$E = \frac{\frac{20}{5} + \frac{5}{1} + \frac{10}{5}}{\frac{1}{5} + \frac{1}{1} + \frac{1}{5}} = \frac{4+5+2}{\frac{1+5+1}{5}} = \frac{11 \times 5}{7} = \frac{55}{7}\text{ V}$$

$\therefore$ Potential difference between points A and B,

$$V_A - V_B = \frac{55}{7}\text{ V}$$

Cell of emf 20 V,

$$V_A - V_B = 20 - 5\,i_1$$

$$\frac{55}{7} = 20 - 5\,i_1 \Rightarrow 5\,i_1 = 20 - \frac{55}{7} = 17$$

Current through cell of 20 V, $i_1 = 17\text{ A}$

Electrical energy and power

Electrical energy

It is defined as the total work done W by the source of emf V in maintaining the electric current I in the given circuit for a specified time t.

According to Ohm's law, we have

$$V = IR$$

Total charge that crosses the resistor is given by

$$q = It$$

Energy gained is given by

$$E = W = Vq = V\,(It) = VIt$$

$$= [IR]\,It = I^2 R\,t \qquad [\because V = IR]$$

$$= \left[\frac{V}{R}\right]^2 Rt = \frac{V^2 t}{R} \qquad \left(\because I = \frac{V}{R}\right)$$

The SI unit of electrical energy is joule (J), where 1 joule

$$= 1\text{ volt} \times 1\text{ ampere} \times 1\text{ sec} = 1\text{ watt} \times 1\text{ sec}$$

Electrical Power

It is defined as the rate of electrical energy supplied per unit time to maintain flow of electric current through a conductor.

Mathematically, $\quad P = VI = I^2 R = \dfrac{V^2}{R}$

The SI unit of power is watt (W), where

$$1\text{ watt} = 1\text{ volt} \times 1\text{ ampere} = 1\text{ ampere-volt.}$$

It can be defined as the power of an electric circuit is called **one watt**, if one ampere current flows in it against a potential difference of one volt. The bigger units of electrical power are kilowatt (kW) and megawatt (MW) where, $1\text{kW} = 1000\text{ W}$ and $1\text{ MW} = 10^6\text{ W}$

Commercial unit of electrical power is horse power (hp) where, 1hp = 746 watt.

Heating effects of current

An electric current through a resistor increases its thermal energy. Also these are other situations in which an electric current can produce or absorb thermal energy. This effect is called heating effect of electric current.

When some potential difference V is applied across a resistance R and charge q to flow through the circuit in time t, the heat absorbed or produced in given by

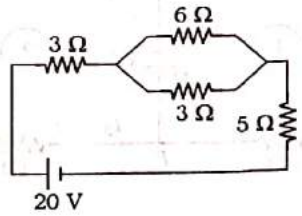
$$W = qV = \text{Vit} = i^2 Rt = \frac{V^2 t}{R}\text{ joule}$$

or $$= \frac{Vit}{4.2} = \frac{i^2 Rt}{4.2} = \frac{V^2 t}{4.2R}\text{ cal}$$

where, J is the joule's mechanical equivalent of heat (4.21 J/cal).

These relations are also called **Joule's law of heating**.

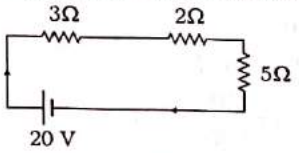
Example 3.47 *In the circuit shown in the following figure, find the heat developed across each resistance in 2 s.*



Sol. The 6 Ω and 3 Ω resistances are in parallel. So, their combined resistance is

$$\frac{1}{R}=\frac{1}{6}+\frac{1}{3}=\frac{1}{2} \quad \text{or} \quad R=2\,\Omega$$

The equivalent simple circuit can be drawn as shown.

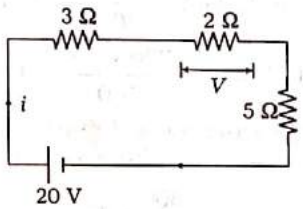


Current in the circuit,

$$i=\frac{\text{Net emf}}{\text{Total resistance}}=\frac{20}{3+2+5}=2\text{ A}$$

$$V=iR=(2)(2)=4\text{ V}$$

i.e., potential difference across 6 Ω and 3 Ω resistances are 4 V. Now,



$$H_{3\Omega}=i^2Rt=(2)^2(3)(2)=24\text{ J}$$

$$H_{6\Omega}=\frac{V^2}{R}t=\frac{(4)^2}{6}(2)=\frac{16}{3}\text{ J}$$

$$H_{3\Omega}=\frac{V^2}{R}t=\frac{(4)^2(2)}{3}=\frac{32}{3}\text{ J}$$

and $$H_{5\Omega}=i^2Rt=(2)^2(5)(2)=40\text{ J}$$

Electricity consumption

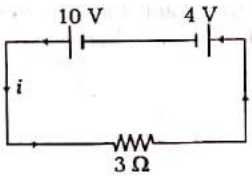
To measure the electrical energy consumed commercially, joule is not sufficient. So, to express electrical energy consumed commercially a special unit **kilo-watt-hour** is used in place of joule. It is also called 1 unit of electrical energy.

1 kilowatt hour or 1 unit of electrical energy is the amount of energy dissipated in 1 hour in a circuit, when the electric power in the circuit is 1 kilowatt.

$$1\text{ kilowatt hour (kWh)}=3.6\times10^6\text{ joule (J)}$$

Note **Resistance of electrical appliance** On electrical appliances (bulbs, geysers, heaters, etc.) wattage, voltage printed are called rated values. The resistance of any electrical appliance can be calculated by rated power and rated voltage by using $R=V_R^2/P_R$.

Example 3.48 *In the circuit shown in the following figure, find*



(i) the power supplied by 10 V battery.

(ii) the power consumed by 4 V battery and

(iii) the power dissipated in 3 Ω resistance.

Sol. Net emf of the circuit = (10 − 4) V = 6 V

Total resistance of the circuit = 3 Ω

$$\therefore \text{ Current in the circuit, } i=\frac{\text{Net emf}}{\text{Total resistance}}=\frac{6}{3}=2\text{ A}$$

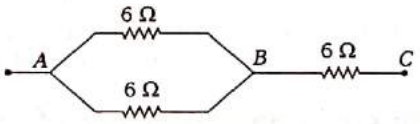
(i) Power supplied by 10 V battery $=Ei=(10)(2)$
$=20\text{ W}$

(ii) Power consumed by 4 V battery $=Ei=(4)(2)=8\text{ W}$

(iii) Power consumed by 3 Ω resistance $=i^2R=(2)^2(3)$
$=12\text{ W}$

Note Here we can see that total power supplied by 10 V battery (*i.e.*,20 W) = power consumed by 4 V battery and 3 Ω resistance. Which proves that conservation of energy holds good in electric circuits also.

Example 3.49 *In the following figures, each of the three resistances, the rating of 24 W and resistance of 6 Ω. Find the maximum power rating of the circuit.*



Sol. To find maximum current i,

we use, $$P=i^2R \Rightarrow i^2=\frac{P}{R}=\frac{24}{6}=4 \Rightarrow i=2\text{ A}$$

Resistance between A and B, $\Rightarrow \frac{1}{R}=\frac{1}{6}+\frac{1}{6}=\frac{2}{6} \Rightarrow R=3\,\Omega$

and resistance between A and $C=3+6=9\,\Omega$

If we make 2 A current flow through the given circuit 1 A will flow through each of 6 Ω in parallel and 6 Ω in series, so now will fuse. This is maximum current circuit can hold.

So, power of circuit $=I^2R=2\times2\times9=36\text{ W}$

Power consumption in a combination of bulbs

Series combination of bulbs

(i) Total power consumed is given by

$$\frac{1}{P_{eq}}=\frac{1}{P_1}+\frac{1}{P_2}+\ldots$$

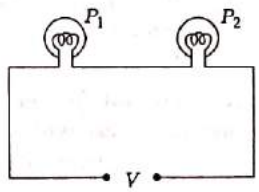


Fig. 3.25 *Series combination of two bulbs*

(ii) P_{consumed} (brightness) $\propto V \propto R \propto \frac{1}{P_{\text{rated}}}$, *i.e.*, in series combination bulb of lesser wattage will give more bright light and potential difference appearing across it will be more.

Parallel combination of bulbs

(i) Total power consumed is given by

$$P_{\text{total}} = P_1 + P_2 + \ldots.$$

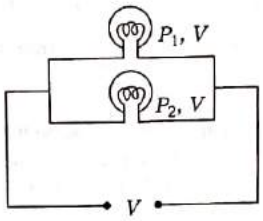


Fig. 3.26 Parallel combination of two bulbs

(ii) P_{consumed} (brightness) $\propto P_{\text{rated}} \propto I \propto \frac{1}{R}$, *i.e.*, in parallel combination, bulb of greater wattage will give more bright light and more current will pass through it.

Applications of heating effect of current

(i) Filament of electric bulb is made up of tungsten which has high resistivity and high melting point.

(ii) Electric devices having heating elements like heater, geyser or press are made up of nichrome which has high resistivity and high melting point.

(iii) Fuse wire is made up of tin-lead alloy, since it should have low melting point and high resistivity. It is used in series as a safety device in an electric circuit and is designed, so as to melt and thereby open the circuit, if the current exceeds a predetermined value due to some fault.

Example 3.50 *Two bulbs having rating of 60 W, 220 V and 100 W, 220 V are joined (i) in series, (ii) in parallel. Which of the two will glow brighter in each case?*

Sol. Suppose both are used in 220 V condition in both the cases. When they are in parallel, both will receive 220 V, so they will flow with full wattage. In case of **parallel combination,** same voltage will be there, hence heat produced will be proportional to $1/R$. As, bulb of greater wattage will glow more. Hence, 100 W bulb will glow brighter. In case of **series combination,** same current flows through, hence heat produced will be proportional to R ($P = i^2R$). As we know higher the wattage, lower the resistance, then 60 W bulb will have higher resistance. That means more heat will be produced in 60 W bulb in this case, so this bulb will glow brighter.

Example 3.51 *Two bulbs having rating of 40 W, 220 V and 100 W, 220 V are joined in series and alternately, (i) 300 V and (ii) 440 V is applied. Find out which will fuse in each case.*

Sol. We first have to find out the maximum current each bulb can bear. This can be calculated from rating of the bulb,

$$P = VI$$

$$\Rightarrow \quad 40 = 200\,I$$

$$\Rightarrow \quad I = \frac{40}{220} = 0.18\text{ A}$$

This is maximum current 40 W bulb can bear.

Similarly, 60 W bulb can bear $\frac{60}{220} = 0.27$ A.

Now, find out resistance of each bulb, $R = \frac{V^2}{P}$.

$\Rightarrow$ Resistance of 40 W bulb $= \frac{220}{40} \times 220 = 1210\,\Omega$

Resistance of 100 W bulb $= \frac{220 \times 220}{100} = 484\,\Omega$

These are joined in series, so total resistance $= 1694\,\Omega$

Current in each when joined with 300 V

i.e., $= \frac{300}{1694} = 17.7$

This current will flow in each, so now will fuse as the less than their maximum permissible current they are joined under 440 V in series then current will be $= \frac{440}{1694} = 26$A.

This current is less than maximum permissible current of 100 W bulb but more than 40W bulb. Hence, 40 W bulb will be fused and 100 W bulb will remain safe.

Example 3.52 *In the above example, if we join the bulbs in parallel and 300 V is applied, which of the two bulbs will fuse?*

Sol. When they are joined in parallel and 300 V is applied on them, then both will get 300 V. Since, their rating is 220 V, naturally current flowing through them will be more than maximum possible. Hence, both will fuse out.

Example 3.53 *Two coils of power 60 W and 100 W and both operating at 220 V takes time 2 min and 1.5 min separately to boil certain amount of water. If they are joined (i) in series and (ii) in parallel, then find the ratio of time taken by them to boil the same water in the two cases.*

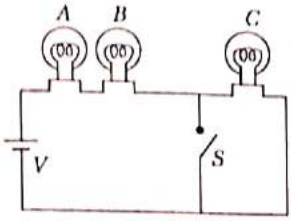
Sol. When joined in series, then total power, P.

$$= \frac{P_1 P_2}{P_1 + P_2} = \frac{60 \times 100}{160} = 37.5\text{ W}$$

When joined in parallel, then total power $= 60 + 100 = 160$ W

Time taken to boil the water will be inversely proportional to power, so ratio of time taken in the two cases will be 160 : 37.5.

Example 3.54 *Figure shows three identical bulbs A, B and C which are connected to a battery of supply voltage V. When the switch S is closed, then discuss the change in*



(i) the illumination of the three bulbs.

(ii) the power dissipated in the circuit.

Sol. When the switch is open,

$$V_A = V_B = V_C = V/3$$

and $$P_A = P_B = P_C = \frac{(V/3)^2}{R} = \frac{V^2}{9R} = P \text{ (say)}$$

(i) When the switch is closed, then the bulb C is short circuited and hence these will be no current through C. So, $P_C = 0$

$$V_A = V_B = \frac{V}{2}$$

So, $$P_A = P_B = \frac{(V/2)^2}{R} = \frac{V^2}{4R} = \frac{9}{4}P$$

Therefore, the intensity of illumination of each of the bulb A and B become 9/4 times. The initial value but the intensity of the bulb C becomes zero.

(ii) The power dissipated in the circuit before closing the switch is

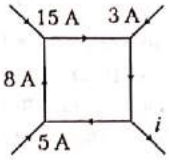
$$P_i = P_A + P_B + P_C = 3P$$

The power dissipated after closing the switch is

$$P_f = P_A + P_B + P_C$$
$$= \frac{9}{4}P + \frac{9}{4}P + 0 = \frac{9}{2}P = \frac{3}{2}P_i$$

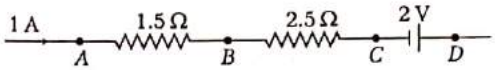
Check point 3.3

1. The figure shows a network of currents. The current i will be



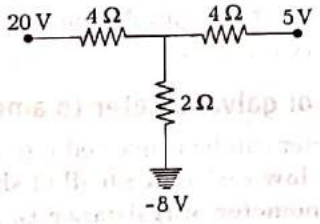
(a) 3 A (b) 13 A
(c) 23 A (d) – 3 A

2. In the circuit element given here, if the potential at point $B, V_B = 0$, then the potentials of A and D are given as



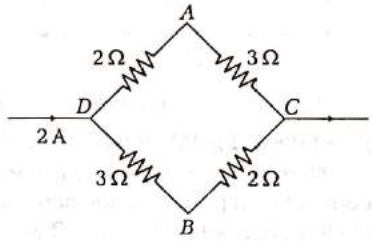
(a) $V_A = -1.5$ V, $V_D = +2$ V
(b) $V_A = -1.5$ V, $V_D = +0.5$ V
(c) $V_A = +1.5$ V, $V_D = +0.5$ V
(d) $V_A = +1.5$ V, $V_D = -0.5$ V

3. Three resistances are connected to form a T-shape as shown in the figure. Then, the current in the 4 Ω resistor is



(a) 0.93 A
(b) 4.5 A
(c) 2.5 A
(d) 1.57 A

4. A current of 2 A flows in resistances as shown. The potential difference $V_A - V_B$ will be



(a) + 4 V (b) + 2 V (c) + 1 V (d) – 1 V

5. How much work is required to carry a 6 μC charge from the negative terminal to the positive terminal of a 9 V battery?

(a) 54×10^{-3} J (b) 54×10^{-6} J
(c) 54×10^{-9} J (d) 54×10^{-12} J

6. Two resistors R and $2R$ are connected in series in an electric circuit. The thermal energy developed in R and $2R$ are in the ratio

(a) 1 : 2 (b) 2 : 1 (c) 1 : 4 (d) 4 : 1

7. The resistor of resistance R is connected to 25 V supply and heat produced in it is 25 Js^{-1}. The value of R is

(a) 225 Ω (b) 1 Ω
(c) 25 Ω (d) 50 Ω

8. Just as electricity is supplied at 220 V for domestic use in India, it is supplied at 110 V in USA. If a resistance of 60 W bulb for use in India is R, then that of 60 W bulb for use in USA will be

(a) $R/4$ (b) $R/2$
(c) R (d) $2R$

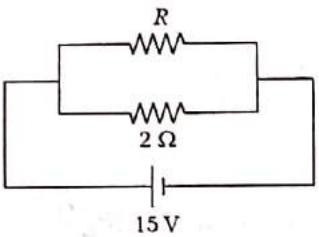
9. If R_1 and R_2 are respectively, the filament resistances of a 200 W bulb and 100 W bulb designed to operate on the same voltage, then
(a) R_1 is two times R_2 (b) R_2 is two times R_1
(c) R_2 is four times R_1 (d) R_1 is four times R_2

10. The electric bulb have tungsten filaments of same length. If one of them gives 60 W and other 100 W, then
(a) 100 W bulb has thicker filament
(b) 60 W bulb has thicker filament
(c) Both filaments are of same thickness
(d) it is not possible to get different wattage unless the lengths are different

11. A resistor of resistance R is connected to an ideal battery. If the value of R is decreased, then the power dissipated in the resistor will
(a) increase (b) decrease
(c) remain unchanged (d) None of these

12. How many calories of heat will be produced approximately in 210 watt electric bulb in 5 min.
(a) 800000 cal (b) 63000 cal
(c) 1050 cal (d) 15000 cal

13. If in the circuit, power dissipation is 150 W, then R is



(a) 2 Ω (b) 6 Ω (c) 5 Ω (d) 4 Ω

14. A wire when connected to 220 V mains supply has power dissipation P_1. Now, the wire is cut into two equal pieces which are connected in parallel to the same supply. Power dissipation in this case is P_2. Then, $P_2 : P_1$ is
(a) 1 (b) 4 (c) 2 (d) 3

15. Two electric bulbs one, if 200 V-40 W and other 200 V-100 W are connected in series to a 200 V line, then the potential drop across
(a) the two bulbs is the same
(b) Both the bulbs is 200 V
(c) 40 W bulb is greater than the potential drop across 100 W bulb
(d) 100 W bulb is greater than the potential drop across 40 W bulb

16. One kilowatt hour is equal to
(a) 36×10^5 J (b) 36×10^3 J
(c) 10^3 J (d) 10^5 J

17. Three identical bulbs are connected in series and these together dissipate a power P. If now the bulbs are connected in parallel, then the power dissipated will be
(a) $\frac{P}{3}$ (b) $3P$
(c) $9P$ (d) $\frac{P}{9}$

18. A and B are two bulbs connected in parallel. If A is glowing brighter than B, then the relation between R_A and R_B is
(a) $R_A = R_B$ (b) $R_B > R_A$
(c) $R_A > R_B$ (d) None of these

19. Electric bulb 50 W 100 V glowing at full power are to be used in parallel with battery 120 V, 10 Ω. Maximum number of bulbs that can be connected so that glow in full power is
(a) 2 (b) 8
(c) 4 (d) 6

20. Some electric bulbs are connected in series across a 220 V supply in a room. If one bulb is fused, then remaining bulbs are connected again in series (after removing the fused bulb) across the same supply. The illumination in the room will
(a) increase
(b) decrease
(c) remain the same
(d) not continuous

4 Measuring instruments for current and voltage

There are various instruments like galvanometer, ammeter and voltmeter which can be used to detect current and voltage, depending on the range.

Galvanometer

It is an instrument used to detect small current passing through it by showing deflection. It does not measure current. It can be converted into voltmeter (for measuring voltage) and ammeter (for measuring current).

Ammeter

It is an instrument used to measure current and is always connected in series with the element through which current is to be measured. Smaller the resistance of an ammeter more accurate will be its reading as it will not to change the circuit current. An ammeter is said to be ideal, if its resistance r is zero.

Conversion of galvanometer to ammeter

A galvanometer can be converted into an ammeter by connecting a **low resistance (called shunt S) in parallel to the galvanometer of resistance G.** Hence, only a small amount of current pass through galvanometer and remaining pass through the shunt.

G and S are parallel and hence have equal potential difference, *i.e.*, $i_g G = (i - i_g)S$.

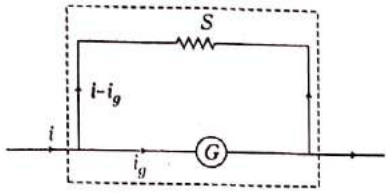


Fig. 3.27 Ammeter

$\therefore$ **Required resistance,** $S = \dfrac{i_g}{(i - i_g)} G$

Example 3.55 *What shunt resistance is required to make the 1 mA, 20 Ω galvanometer into an ammeter with a range of 0 to 50 mA?*

Sol. Here, $i_g = 1\text{mA} = 10^{-3}$ A, $G = 20\,\Omega$

$i = 50 \times 10^{-3}$ A

Substituting in $S = \left(\dfrac{i_g}{i - i_g}\right) G$

$$= \frac{(10^{-3})(20)}{(50 \times 10^{-3}) - (10^{-3})}$$

$$= 0.408\,\Omega$$

Voltmeter

It is a device used to measure potential difference and is always connected in parallel with the circuit element across which potential difference is to be measured. Greater the resistance of voltmeter, more accurate will be the reading, as only small amount of current pass through it, by not changing the circuit current. A voltmeter is said to be ideal, if its resistance is infinite.

Conversion of galvanometer into voltmeter

A galvanometer can be converted into voltmeter by **connecting a large resistance *R* in series with the galvanometer.**

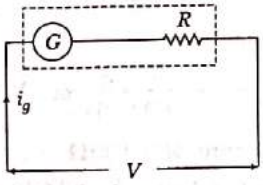


Fig. 3.28 Voltmeter

According to Ohm's law, $V = i_g (G + R)$

or required resistance, $\boxed{R = \dfrac{V}{i_g} - G.}$

Example 3.56 *How can we make a galvanometer with $G = 20\,\Omega$ and $i_g = 1$ mA into a voltmeter with a maximum range of 10 V?*

Sol. Using $R = \dfrac{V}{i_g} - G,$

We have, $R = \dfrac{10}{10^{-3}} - 20 = 9980\,\Omega$

Thus, a resistance of 9980 Ω is to be connected in series with the galvanometer to convert it into the voltmeter of desired range.

Wheatstone bridge

It is an arrangement of four resistances used to measure one of them in terms of the other three as shown in Fig. 3.29.

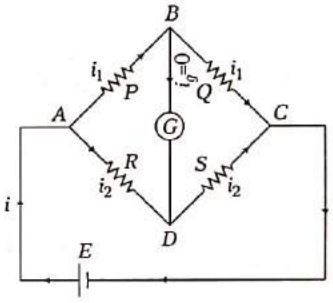


Fig. 3.29 A Wheatstone bridge

The bridge is said to be balanced when deflection in galvanometer is zero ($I_g = 0$), *i.e.*, no current flows through the galvanometer (branch *BD*). In the balanced condition,

$$\boxed{\frac{P}{Q} = \frac{R}{S}}$$

On mutually changing the position of cell and galvanometer, this condition will not change.

Note Different forms of Wheatstone bridge.

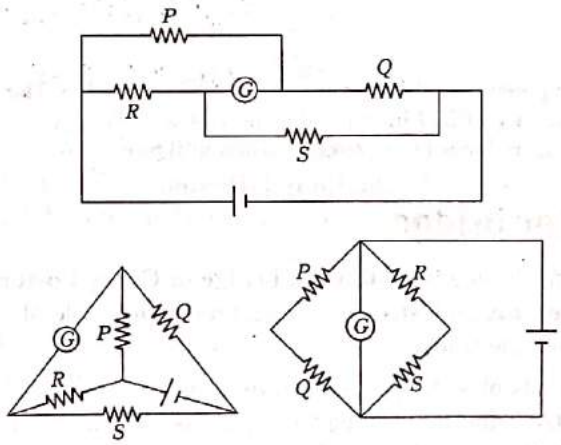
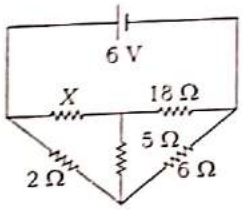


Fig. 3.30

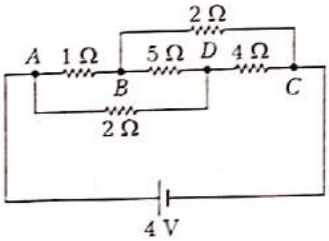
Example 3.57 *Find out the magnitude of resistance X in the circuit shown in figure, when no current flows through the 5 Ω resistor.*



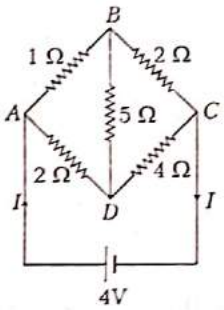
Sol. As no current flows through the middle 5 Ω resistor, the circuit represents a balanced Wheatstone bridge.

$$\therefore \quad \frac{X}{18} = \frac{2}{6} \text{ or } X = \frac{2}{6} \times 18 = 6\,\Omega$$

Example 3.58 *Calculate the current drawn from the battery by the network of resistors shown in figure.*



Sol. The given network is equivalent to the circuit shown in figure.



Now, $$\frac{1\,\Omega}{2\,\Omega} = \frac{2\,\Omega}{4\,\Omega}$$

i.e., $$\frac{P}{Q} = \frac{R}{S}$$

The given circuit is a balanced Wheatstone bridge. The resistance of 5 Ω in arm *BD* is ineffective. The equivalent circuit reduces to the circuit shown in figure.

Meter bridge

A meter bridge is **slide wire bridge** or **Carey Foster bridge**. It is an instrument work on the principle of Wheatstone bridge.

It consists of a straight and uniform wire along a meter scale (AC) and by varying the taping point *B* a shown in Fig. 3.31, the bridge is balanced.

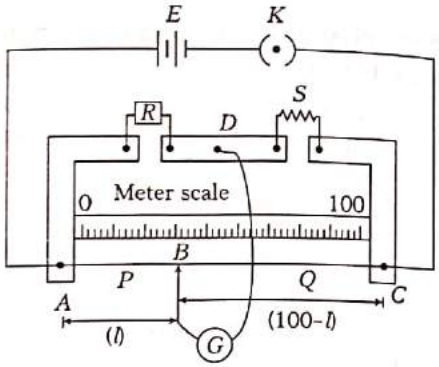


Fig. 3.31 *Measuring unknown resistance using meter bridge*

∴ At balancing situation of bridge,

$$\Rightarrow \quad \frac{P}{Q} = \frac{R}{S} \Rightarrow \frac{l}{100-l} = \frac{R}{S} \Rightarrow S = \frac{100-l}{l} \times R$$

Applications of meter bridge

(i) It is used to measure an unknown resistance from $S = \frac{R(100-l)}{l}$.

(ii) To compare the two unknown resistances $\frac{R}{S} = \frac{l}{100-l}$.

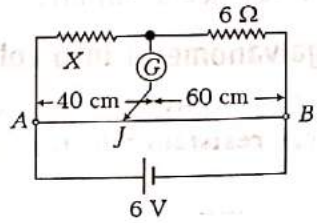
Example 3.59 *In the following circuit, a meter bridge is shown in its balanced state. The meter bridge wire has a resistance of 1 Ω-cm. Calculate the value of the unknown resistance X and the current drawn from the battery of negligible internal resistance.*

Sol. In balanced condition, no current flows through the galvanometer.

Here, P = resistance of wire $AJ = 40\,\Omega$

Q = resistance of wire $BJ = 60\,\Omega$

$R = X, S = 6\,\Omega$



In the balanced condition,

$$\frac{P}{Q} = \frac{R}{S} \text{ or } \frac{40}{60} = \frac{X}{6} \text{ or } X = 4\,\Omega$$

Total resistance of wire $AB = 100\,\Omega$

Total resistance of resistances X and 6 Ω connected in series

$$= 4 + 6 = 10\,\Omega$$

This series combination is in parallel with wire *AB*.

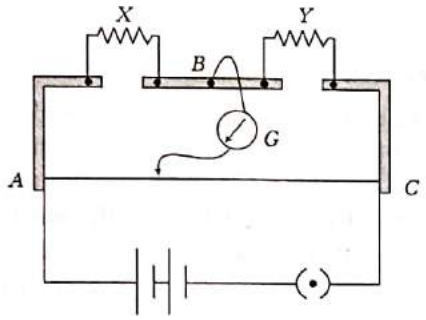
$$\therefore \text{Equivalent resistance} = \frac{10 \times 100}{10 + 100} = \frac{100}{11}\,\Omega$$

emf of the battery = 6 V

∴ Current drawn from the battery,

$$I = \frac{\text{emf}}{\text{resistance}} = \frac{6}{100/11} = 0.66 \text{ A}$$

Example 3.60 *The given figure shows the experimental set up of a meter bridge. The null point is found to be 60 cm away from the end A with X and Y in position as shown. When a resistance of 15 Ω is connected in series with Y, then the null point is found to shift by 10 cm towards the end A of the wire. Find the position of null point, if a resistance of 30 Ω were connected in parallel with Y.*



Sol. In first case,

$$\frac{X}{Y} = \frac{60}{40} \text{ or } \frac{X}{Y} = \frac{3}{2}$$

In second case,

$$\frac{X}{Y+15} = \frac{50}{50} = 1$$

∴ $$\frac{X}{Y} \times \frac{Y+15}{X} = \frac{3}{2} \times 1$$

or $$1 + \frac{15}{Y} = \frac{3}{2}$$

or $$Y = 30\,\Omega$$

$$X = \frac{3}{2}Y = \frac{3}{2} \times 30 = 45\,\Omega$$

When a resistance of 30 Ω is connected in parallel with Y, then the resistance in the right gap becomes

$$Y' = \frac{30Y}{30+Y} = \frac{30 \times 30}{30+30} = 15\,\Omega$$

Suppose the null point occurs at l cm from end A.

Then,

$$\frac{X}{15} = \frac{l}{100-l}$$

or $$\frac{45}{15} = \frac{l}{100-l}$$

or $$300 - 3l = l$$

or $$4l = 300$$

or $$l = 75 \text{ cm}$$

Potentiometer

The potentiometer is an instrument that can be used to measure the emf or the internal resistance of an unknown source. It is a device which does not draw any current through the circuit to measure the potential difference. Hence, it is equivalent to an ideal voltmeter. It also has a number of other useful applications.

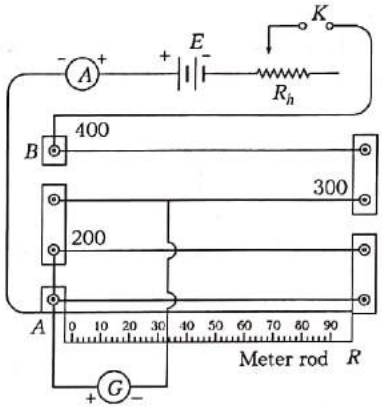


Fig. 3.32 Potentiometer

Principle of potentiometer

The principle of potentiometer is that, when through a wire of uniform cross-section and composition, a constant amount of current flows, then the potential drop across the wire is directly proportional to its length, *i.e.*,

$$V \propto l \Rightarrow V = kl \quad \text{...(i)}$$

where, k is the constant of proportionality.

Also, by Ohm's law,

$$V = IR = \rho \frac{l}{A} I \quad \text{...(ii)}$$

If the current I is constant, then $\frac{\rho I}{A}$ would be constant.

So, comparing Eqs. (i) and (ii), we will have

$$\Rightarrow \quad k = \rho \frac{I}{A}$$

By putting this in Eq. (ii)

$V = kl$, the principle of potentiometer.

'k' is also known as **potential gradient**, which means potential drop per unit length of the potentiometer wire

i.e., $$k = \frac{V}{l}$$

The SI unit of potential gradient is Vm^{-1} and CGS unit is Vcm^{-1}.

Applications of potentiometer

(i) To compare the emfs of two cells

Consider two cells of emfs E_1 and E_2 is to be compared. The positive poles of both the cells are connected to terminal A of potentiometer and the negative pole of both cells are connected to terminals 1 and 2 of a 2-way key, while its common terminal is connected to a jockey J through a galvanometer G. A battery E, ammeter A, rheostat R_h and 1-way key K are connected between the terminals A and B of the potentiometer.

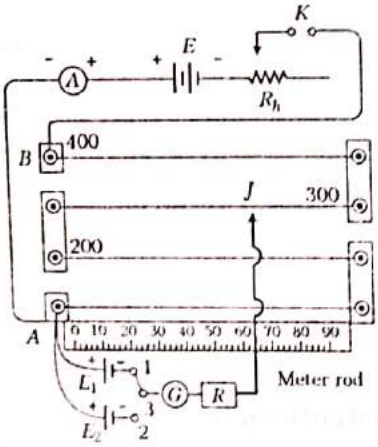


Fig. 3.33 Circuit for comparing emf's of two cells

Now to compare emfs of two cells having a constant current passing through the wire between terminals A and B. The current is kept constant by using rheostat. If the plug is put in the gap between terminals 1 and 3 of 2-way key, then the emf E_1 of the cell is given by

$$E_1 = (xl_1)\,I \qquad \text{...(i)}$$

where, x = resistance per unit length of potentiometer wire

[$\because$ l_1 = balancing length]

Now, when the key is put in the gap between terminals 2 and 3 after removing it from the gap between 1 and 3, then the emf E_2 is given by

$$E_2 = (xl_2)I \qquad [\because l_2 = \text{balancing length}] \text{ ...(ii)}$$

From Eqs. (i) and (ii), we have

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

(ii) To measure internal resistance of a cell

Now, to find the internal resistance r of a cell of emf E. Let E' be emf of the battery. A constant current I is maintained through the potentiometer wire with the help of rheostat.

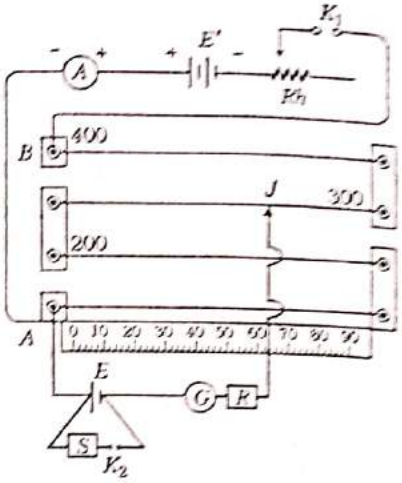


Fig. 3.34 Circuit for determining internal resistance of cell

If the plug in key K_2 is kept out and the jockey J is moved on the potentiometer wire to balance the emf E of the cell whose internal resistance r is to be determined.

Suppose l_1 be the balancing length of the potentiometer wire between point A and jockey J. If x is resistance per unit length of wire, then emf of cell is given by

$$E = xl_1I \qquad \text{...(i)}$$

Introduce some resistance, says S from the resistance box and now put in the plug key K_2. The potential difference V between the two poles of the cell is given by

$$V = xl_2I \qquad \text{...(ii)}$$

[$\because$ l_2 = balancing length]

On dividing Eq. (i) by Eq. (ii), we have

$$\frac{E}{V} = \frac{l_1}{l_2}$$

The internal resistance of the cell is given by

$$r = \left(\frac{E}{V} - 1\right)S$$

Now, substituting values of E/V in above equation, we get

$$\Rightarrow \qquad r = \left(\frac{l_1}{l_2} - 1\right)S$$

where, S is the resistance of the resistance box.

Note

1. The sensitiveness of potentiometer means the smallest potential difference that can be measured with its help.
2. A potentiometer can also be used to compare unknown resistances and to calibrate a voltmeter or an ammeter.
3. A balance point is obtained on the potentiometer wire, if the fall of potential along the potentiometer wire due to driving cell is greater than the emf of the cell to be balanced.

Example 3.61 *A potentiometer wire is 10 m long and has a resistance of 18 Ω. It is connected to a battery of emf 5 V and internal resistance 2 Ω. Calculate the potential gradient along the wire.*

Sol. Here, $l = 10\text{ m}$, $R = 18\,\Omega$, $E = 5\text{ V}$, $r = 2\,\Omega$

Current through the potentiometer wire,

$$I = \frac{E}{R+r} = \frac{5}{18+2} = \frac{5}{20} = \frac{1}{4}\text{A}$$

$$\therefore \text{ Potential gradient} = \frac{IR}{l} = \frac{1}{4} \times \frac{18}{10} = 0.45\,\text{Vm}^{-1}$$

Example 3.62 *A cell can be balanced against 110 cm and 100 cm of potentiometer wire respectively, when in open circuit and when short circuited through a resistance of 10 Ω. Find the internal resistance of the cell.*

Sol. Given, $l_1 = 110\text{ cm}$, $l_2 = 100\text{ cm}$, $R = 10\,\Omega$ and $r = ?$

$$\therefore \quad r = \left(\frac{l_1 - l_2}{l_2}\right)R \Rightarrow r = \frac{110-100}{100} \times 10 = 1\,\Omega$$

Example 3.63 *In a potentiometer arrangement, a cell of emf 2.25 V gives a balance point at 30.0 cm length of the wire. If the cell is replaced by another cell and the balance point shifts to 60.0 cm, then what is the emf of the second cell?*

Sol. Here, $E_1 = 2.25\text{ V}$, $l_1 = 30.0\text{ cm}$, then

$l_1 = 60.0\text{ cm}$, $E_2 = ?$

As we know that in case of potentiometer, the potential gradient remains constant.

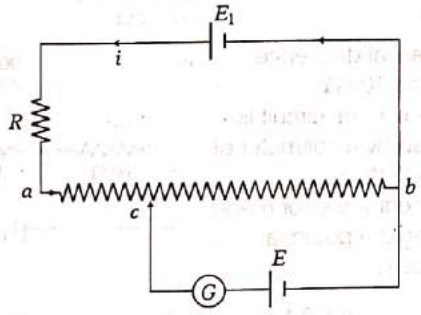
So, $\quad E \propto l$

$$\therefore \quad \frac{E_1}{E_2} = \frac{l_1}{l_2} \Rightarrow \frac{2.25}{E_2} = \frac{30.0}{60.0}$$

$$\therefore \quad E_2 = \frac{2.25 \times 60}{30} = 4.5\text{ V}$$

Example 3.64 *A potentiometer wire of length 100 cm has a resistance of 10 Ω. It is connected in series with a resistance and a cell of emf 2 V and of negligible internal resistance. A source of emf 10 mV is balanced against a length of 40 cm of the potentiometer wire. What is the value of external resistance?*

Sol. From the theory of potentiometer,



$V_{cb} = E$, if no current is drawn from the battery

or $$\left(\frac{E_1}{R + R_{ab}}\right)R_{cb} = E$$

Here, $E_1 = 2\text{ V}$, $R_{ab} = 10\,\Omega$

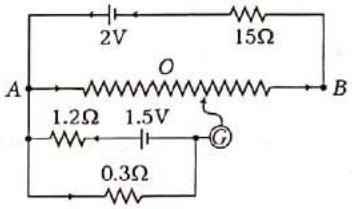
$$R_{cb} = \left(\frac{40}{100}\right) \times 10 = 4\,\Omega$$

and $E = 10 \times 10^{-3}\text{ V}$

Substituting in above, we get

$$R = 790\,\Omega$$

Example 3.65 *AB is 1 m long uniform wire of 10 Ω resistance. Other data are as shown in figure. Calculate (i) potential gradient along AB and (ii) length AO, when galvanometer shows no deflection.*



Sol. (i) Total resistance of the primary circuit

$= 15 + 10 = 25\,\Omega$, emf $= 2\text{ V}$

$\therefore$ Current in the wire AB,

$$I = \frac{2}{25} = 0.08\text{ A}$$

Potential difference across the wire AB

= Current × Resistance of wire AB

$= 0.08 \times 10 = 0.8\text{ V}$

Potential gradient

$$= \frac{\text{Potential difference}}{\text{Length}} = \frac{0.8}{100} = 0.008\text{ V cm}^{-1}$$

(ii) Resistance of secondary circuit $= 1.2 + 0.3 = 1.5\,\Omega$

emf $= 1.5\text{ V}$

Current in the secondary circuit $= \dfrac{1.5}{1.5} = 1.0\text{ A}$

The same is the current in 0.3 Ω resistor.

Potential difference between points A and O

= Potential difference across 0.3 Ω resistor in the zero deflection condition

= Current × Resistance

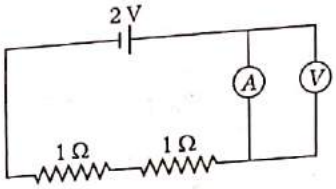
$= 1.0 \times 0.3 = 0.3\text{ V}$

$$\text{Length } AO = \frac{\text{Potential difference}}{\text{Potential gradient}}$$

$$= \frac{0.3\text{ V}}{0.008\text{ V cm}^{-1}} = 37.5\text{ cm}$$

Check point 3.4

1. In the circuit shown, A and V are ideal ammeter and voltmeter, respectively. Reading of the voltmeter will be

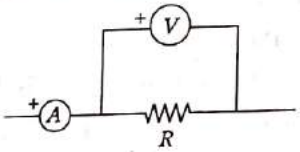


(a) 2 V (b) 1 V
(c) 0.5 V (d) zero

2. The net resistance of a voltmeter should be large to ensure that
(a) it does not get overheated
(b) it does not draw emmissive current
(c) it can measure large potential difference
(d) it does not appreciably change the potential difference to be measured.

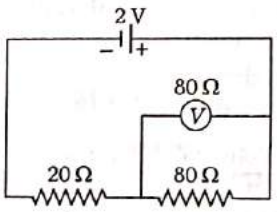
3. Two galvanometers A and B require 3 mA and 6 mA respectively, to produce the same deflection of 10 divisions. Then,
(a) A is more sensitive than B
(b) B is more sensitive than A
(c) A and B are equally sensitive
(d) sensitiveness of B is twice that of A

4. An ammeter A, a voltmeter V and a resistance R are connected as shown in the figure. If the voltmeter reading is 1.6 V and the ammeter reading is 0.4 A, then R is



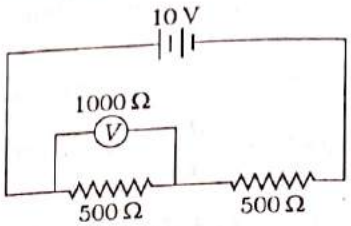
(a) equal to 4 Ω (b) greater than 4 Ω
(c) less than 4 Ω (d) between 3 Ω and 4 Ω

5. In the following circuit, the emf of the cell is 2 V and the internal resistance is negligible. The resistance of the voltmeter is 80 Ω. The reading of the voltmeter will be



(a) 0.80 V (b) 1.60 V
(c) 1.33 V (d) 2.00 V

6. What is the reading of voltmeter in the figure?

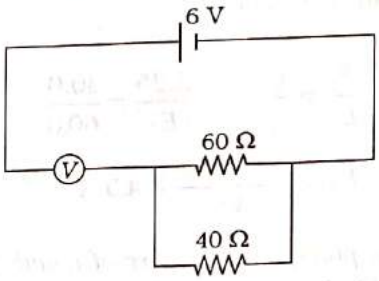


(a) 3 V (b) 2 V (c) 5 V (d) 4 V

7. A galvanometer of 25 Ω and having full scale deflection for a current of 10 mA is changed into voltmeter of range 100 V by connecting a resistance R in series with the galvanometer. The resistance R in ohm is
(a) 10000 (b) 975
(c) 10025 (d) 9975

8. An ammeter and a voltmeter are joined in series to a cell. Their readings are A and V, respectively. If a resistance is now joined in parallel with the voltmeter
(a) Both A and V will decrease
(b) Both A and V will increase
(c) A will decrease, V will increase
(d) A will increase, V will decrease

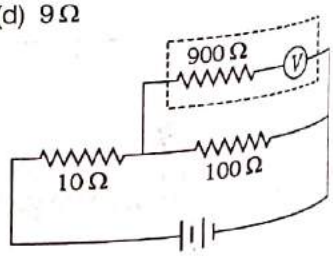
9. In the circuit shown in the figure, the voltmeter reading is



(a) 2.4 V (b) 3.4 V
(c) 4.0 V (d) 6.0 V

10. To send 10% of the main current through a moving coil galvanometer of resistance 99 Ω, the shunt required is
(a) 9.9 Ω (b) 10 Ω
(c) 11 Ω (d) 9 Ω

11. The potential difference across the 100 Ω resistance in the circuit is measured by a voltmeter of 900 Ω resistance. The percentage error made in reading the potential difference is



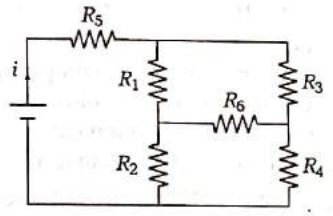
(a) $\frac{10}{9}$ (b) 0.1 (c) 1.0 (d) 10.0

12. A microammeter has a resistance of 100 Ω and a full scale range of 50 μA. It can be used as a voltmeter or as a higher range ammeter provided a resistance is added to it. Pick the correct range and resistance combinations.
(a) 50 V range with 10 kΩ resistance in series
(b) 10 V range with 200 kΩ resistance in series
(c) 10 mA range with 1Ω resistance in parallel
(d) None of the above

13. The percentage error in measuring resistance with a meter bridge can be minimized by adjusting the balancing point close to
(a) 20 cm (b) 50 cm (c) 80 cm (d) 100 cm

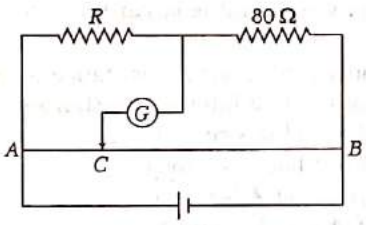
14. When an additional resistance of 1980 Ω is connected in series with a voltmeter, then the scale division reads 100 times larger value. Resistance of the voltmeter is
(a) 10 Ω (b) 20 Ω (c) 30 Ω (d) 40 Ω

15. In the given circuit, it is observed that the current I is independent of the value of the resistance R_6. Then, the resistance values must satisfy



(a) $R_1R_2R_3 = R_3R_4R_5$ (b) $\frac{1}{R_5} + \frac{1}{R_6} = \frac{1}{R_1 + R_2} + \frac{1}{R_3 + R_4}$
(c) $R_1R_4 = R_2R_3$ (d) $R_1R_3 = R_2R_4 = R_5R_6$

16. AB is a wire of uniform resistance. The galvanometer G shows no current when the length $AC = 20$ cm and $CB = 80$ cm. The resistance R is equal to

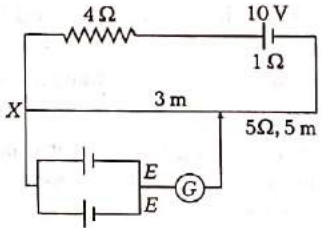


(a) 320 Ω (b) 8 Ω (c) 20 Ω (d) 40 Ω

17. A potentiometer is used for the comparison of emf of two cells E_1 and E_2. For cell E_1, the no deflection point is obtained at 20 cm and for E_2, the no deflection point is obtained at 30 cm. The ratio of their emfs will be
(a) 2/3
(b) 3/2
(c) 1
(d) 2

18. In a potentiometer experiment, the galvanometer shows no deflection when a cell is connected across 60 cm of the potentiometer wire. If the cell is shunted by a resistance of 6 Ω, the balance is obtained across 50 cm of the wire. The internal resistance of the cell is
(a) 0.5 Ω
(b) 0.6 Ω
(c) 1.2 Ω
(d) 1.5 Ω

19. A resistance of 4 Ω and a wire of length 5 m and resistance 5Ω are joined in series and connected to a cell of emf 10 V and internal resistance 1 Ω. A parallel combination of two identical cells is balanced across 3 m of the wire. The emf E of each cell is



(a) 1.5 V
(b) 3.0 V
(c) 0.67 V
(d) 1.33 V

20. Potentiometer wire of length 1 m is connected in series with 490 Ω resistance and 2 V battery. If 0.2 mV/ cm is the potential gradient, then the resistance of the potentiometer wire is
(a) 4.9 Ω
(b) 7.9 Ω
(c) 5.9 Ω
(d) 6.9 Ω

Chapter exercises

A. Taking it together

(Assorted questions of the chapter for advanced level practice)

1. The terminal potential difference of a cell is greater than its emf when it is
(a) being discharged
(b) open circuit
(c) being charged
(d) being either charged or discharged

2. Identify the material commonly used for making coil of a resistance box.
(a) Molybdenum (b) Manganese
(c) Manganin (d) Magnesium

3. Which of the following is a vector quantity?
(a) Current density (b) Current
(c) Wattless current (d) Power

4. Electric field and current density have relation
(a) $E \propto J$ (b) $E \propto J^2$ (c) $E \propto \frac{1}{J^2}$ (d) $E^2 \propto \frac{1}{J}$

5. Consider the following two statements.
A. Kirchhoff's junction law follows from the conservation of charge.
B. Kirchhoff's loop law follows from the conservation of energy.
Which of the following is correct?
(a) Both A and B are wrong
(b) A is correct but B is wrong
(c) A is wrong but B is correct
(d) Both A and B are correct

6. For measurement of potential difference, potentiometer is preferred in comparison to voltmeter because
(a) potentiometer is more sensitive than voltmeter
(b) the resistance of potentiometer is less than voltmeter
(c) potentiometer is cheaper than voltmeter
(d) potentiometer does not take current from the circuit

7. What is immaterial for an electric fuse wire?
(a) Its specific resistance (b) Its radius
(c) Its length (d) Current flowing through it

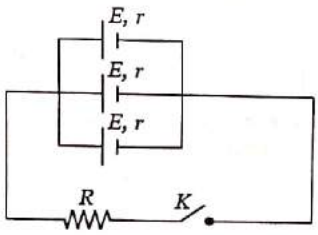
8. Conductivity increases in the order of
(a) Al, Ag, Cu (b) Al, Cu, Ag
(c) Cu, Al, Ag (d) Ag, Cu, Al

9. By mistake, a voltmeter is connected in series and an ammeter in parallel. When the circuit is switched on
(a) Both ammeter and voltmeter will be damaged
(b) Neither the ammeter nor the voltmeter will be damaged
(c) Only the ammeter will be damaged
(d) Only the voltmeter will be damaged

10. If E is the emf of a cell of internal resistance r and external resistance R, then potential difference across R is given as
(a) $V = E/(R + r)$ (b) $V = E$
(c) $V = E/(1 + r/R)$ (d) $V = E/(1 + R/r)$

11. Two non-ideal batteries are connected in series. Consider the following statements.
A. The equivalent emf is larger than either of the two emfs.
B. The equivalent internal resistance is smaller than either of the two internal resistances.
(a) Each of A and B is correct
(b) A is correct but B is wrong
(c) B is correct but A is wrong
(d) Each of A and B is wrong

12. When n cells are joined in parallel combination as shown, the strength of the current i is given by



(a) $\frac{nE}{R + nr}$ (b) $\frac{E}{R + (r/n)}$
(c) $\frac{E}{r + Rn}$ (d) None of these

13. A student has 10 resistors of resistance r each. The minimum resistance made by him from given resistors is

(a) $10r$ (b) $\frac{r}{10}$ (c) $\frac{r}{100}$ (d) $\frac{r}{5}$

14. A wire has resistance $12\,\Omega$. It is bent in the form of a circle. The effective resistance between the two points on any diameter is equal to

(a) $12\,\Omega$ (b) $6\,\Omega$ (c) $3\,\Omega$ (d) $24\,\Omega$

15. A steady current (i) is flowing through a conductor of uniform cross-section. Any segment of the canductor has

(a) zero charge
(b) only positive charge
(c) only negative charge
(d) charge proportional to current i

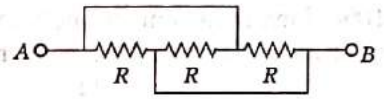
16. There are n cells, each of emf E and internal resistance r, connected in series with an external resistance R. One of the cells is wrongly connected, so that it sends current in the opposite direction. The current flowing in the circuit is

(a) $\frac{(n-1)E}{(n+1)r+2}$ (b) $\frac{(n-1)E}{nr+R}$
(c) $\frac{(n-2)E}{nr+R}$ (d) $\frac{(n-2)E}{(n-2)r+R}$

17. The maximum power dissipated in an external resistance R, when connected to a cell of emf E and internal resistance r, will be

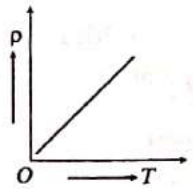
(a) $\frac{E^2}{r}$ (b) $\frac{E^2}{2r}$ (c) $\frac{E^2}{3r}$ (d) $\frac{E^2}{4r}$

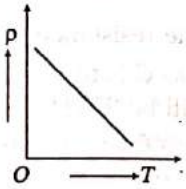
18. The resistance across A and B in the figure below will be

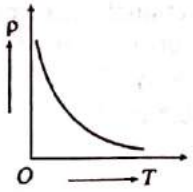


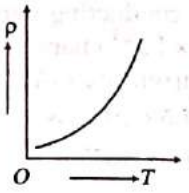
(a) $3R$ (b) R (c) $\frac{R}{3}$ (d) None of these

19. The temperature (T) dependence of resistivity (ρ) of a semiconductor is represented by

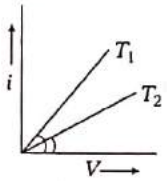
(a)


(b)


(c)


(d)


20. The current-voltage graph for a given metallic wire at two different temperatures T_1 and T_2 is shown in the figure. The temperatures T_1 and T_2 are related as



(a) $T_1 > T_2$ (b) $T_1 < T_2$
(c) $T_1 = T_2$ (d) $T_1 > 2T_2$

21. Which of the following characteristics of electrons determines the current in a conductor? **[NCERT Exemplar]**

(a) Drift velocity alone
(b) Thermal velocity alone
(c) Both drift velocity and thermal velocity
(d) Neither drift nor thermal velocity

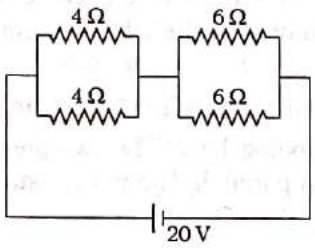
22. An ammeter and a voltmeter of resistance R are connected in series to an electric cell of negligible internal resistance. Their readings are A and V, respectively. If another resistance R is connected in parallel with the voltmeter, then

(a) Both A and V will increase
(b) Both A and V will decrease
(c) A will decrease and V will increase
(d) A will increase and V will decrease

23. The resistance of a wire is $10\,\Omega$. Its length is increased by 10% by stretching. The new resistance will now be

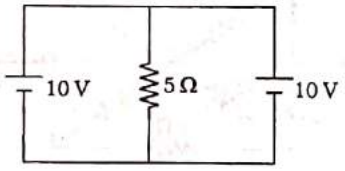
(a) $12\,\Omega$ (b) $1.2\,\Omega$ (c) $13\,\Omega$ (d) $11\,\Omega$

24. Four resistances are connected in circuit in the given figure. The electric current flowing through $4\,\Omega$ and $6\,\Omega$ resistance is respectively



(a) 2 A and 4 A (b) 1 A and 2 A
(c) 1 A and 1 A (d) 2 A and 2 A

25. Current through the $5\,\Omega$ resistor is



(a) 2A (b) 4A
(c) zero (d) 1A

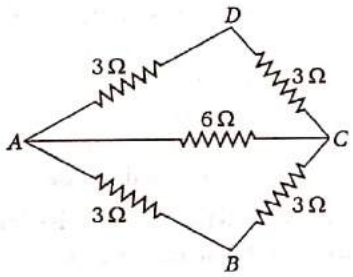
26. A cell which has an emf 1.5 V is connected in series with an external resistance of 10 Ω. If the potential difference across the cell is 1.25 V, then the internal resistance of the cell is (in Ω)
(a) 2 (b) 0.25 (c) 1.5 (d) 0.3

27. A piece of wire of resistance 4 Ω is bent through 180° at its mid-point and the two halves are twisted together, then the resistance is
(a) 8 Ω (b) 1 Ω (c) 2 Ω (d) 5 Ω

28. A wire of resistance R is divided into 10 equal parts. These parts are connected in parallel, the equivalent resistance of such connection will be
(a) 0.01 R (b) 0.1 R
(c) 10 R (d) 100 R

29. Three resistors each of 2 Ω are connected together in a triangular shape. The resistance between any two vertices will be
(a) 4/3 Ω (b) 3/4 Ω (c) 3 Ω (d) 6 Ω

30. The effective resistance between the points A and B in the figure is

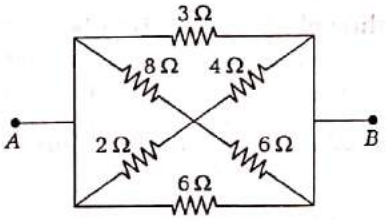


(a) 5 Ω (b) 2 Ω (c) 3 Ω (d) 4 Ω

31. Two resistances are joined in parallel whose resultant is $\frac{6}{8}$ Ω. One of the resistance wire is broken and the effective resistance becomes 2 Ω. Then, the resistance in ohm of the wire that got broken was
(a) 3/5 (b) 2 (c) 6/5 (d) 3

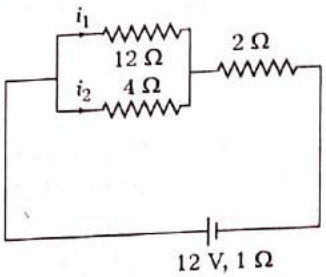
32. A wire of resistance 9Ω is broken in two parts. The length ratio being 1 : 2. The two pieces are connected in parallel. The net resistance will be
(a) 2 Ω (b) 3 Ω (c) 4 Ω (d) 6 Ω

33. In the network shown, the equivalent resistance between A and B is



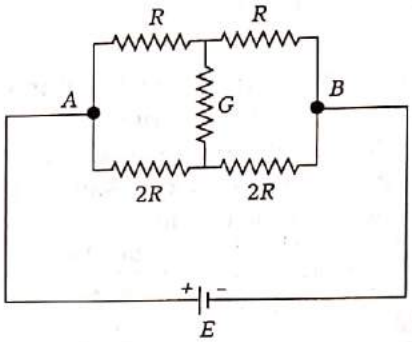
(a) $\frac{4}{3}$ Ω (b) $\frac{3}{4}$ Ω (c) $\frac{24}{17}$ Ω (d) $\frac{17}{24}$ Ω

34. In the circuit shown, the currents i_1 and i_2 are



(a) $i_1 = 1.5$ A, $i_2 = 0.5$ A (b) $i_1 = 0.5$ A, $i_2 = 1.5$ A
(c) $i_1 = 1$ A, $i_2 = 3$ A (d) $i_1 = 3$ A, $i_2 = 1$ A

35. Consider the following statements regarding the network shown in the figure.



(1) The equivalent resistance of the network between points A and B is independent of value of G.
(2) The equivalent resistance of the network between points A and B is $4/3\,R$.
(3) The current through G is zero.

Which of the above statement(s) is/are true?
(a) (1) alone (b) (2) alone
(c) Both (2) and (3) (d) (1), (2) and (3)

36. To send 10% of main current through a moving coil galvanometer of resistance 99Ω, shunt required is
(a) 9 Ω (b) 11 Ω
(c) 10 Ω (d) 9.9 Ω

37. A 1250 W heater operates at 115 V. What is the resistance of the heating coil?
(a) 1.6 Ω (b) 13.5 Ω
(c) 1250 Ω (d) 10.6 Ω

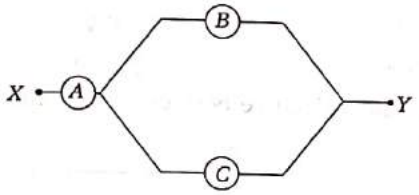
38. The resistance of a wire at 20°C is 20Ω and at 500°C is 60Ω. At which temperature its resistance will be 25Ω?
(a) 50°C (b) 60°C
(c) 70°C (d) 80°C

39. A conducting wire of cross-sectional area 1 cm^2 has 3×10^{23} charge carriers per $metre^3$. If wire carries a current 24 mA, then drift velocity of carriers is
(a) 5×10^{-2} ms^{-1} (b) 0.5 ms^{-1}
(c) $5\times 10^{-3} ms^{-1}$ (d) $5\times 10^{-6} ms^{-1}$

40. At room temperature, copper has free electron density of $8.4\times 10^{28}\,m^{-3}$. The electron drift velocity in a copper conductor of cross-sectional area of $10^{-6}\,m^2$ and carrying a current of 5.4 A, will be

(a) $4\,ms^{-1}$ (b) $0.4\,ms^{-1}$
(c) $4\,cms^{-1}$ (d) $0.4\,mms^{-1}$

41. Three voltmeters A, B and C having resistances R, $1.5\,R$ and $3R$ respectively, are used in a circuit as shown. When a potential difference is applied between X and Y, the readings of the voltmeters are V_1, V_2 and V_3 respectively. Then,



(a) $V_1 = V_2 = V_3$ (b) $V_1 < V_2 = V_3$
(c) $V_1 > V_2 > V_3$ (d) $V_1 > V_2 = V_3$

42. A metal rod of length 10 cm and a rectangular cross-section of 1cm × (1/2) cm is connected to a battery across opposite faces. The resistance will be **[NCERT Exemplar]**

(a) maximum when the battery is connected across 1 cm × (1/2) cm faces
(b) maximum when the battery is connected across 10 cm × (1) cm faces
(c) maximum when the battery is connected across 10 cm × (1/2) cm faces
(d) same irrespective of the three faces

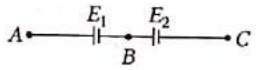
43. Two cells of emfs approximately 5V and 10V are to be accurately compared using a potentiometer of length 400 cm. **[NCERT Exemplar]**

(a) The battery that runs the potentiometer should have voltage of 8V
(b) The battery of potentiometer can have a voltage of 15 V and R adjusted so that the potential drop across the wire slightly exceeds 10 V
(c) The first portion of 50 cm of wire itself should have a potential drop of 10 V
(d) Potentiometer is usually used for comparing resistances and not voltages

44. The resistivity of a potentiometer wire is $40\times 10^{-8}\,\Omega$-m and its area of cross-section is $8\times 10^{-6}\,m^2$. If 0.2 A current is flowing through the wire, the potential gradient will be

(a) 10^{-2} V/m
(b) 10^{-1} V/m
(c) 3.2×10^{-2} V/m
(d) 1 V/m

45. Two cells of emfs E_1 and $E_2 (E_1 > E_2)$ are connected as shown in figure.



When a potentiometer is connected between A and B, the balancing length of the potentiometer wire is 300 cm. On connecting the same potentiometer between A and C, the balancing length is 100 cm. The ratio $\frac{E_1}{E_2}$ is

(a) 3 : 1 (b) 1 : 3
(c) 2 : 3 (d) 3 : 2

46. A voltmeter of resistance 998 Ω is connected across a cell of emf 2 V and internal resistance 2 Ω. The potential difference across the voltmeter is

(a) 1.99 V (b) 3.5 V
(c) 5 V (d) 6 V

47. A wire 50 cm long and 1 mm^2 in cross-section carries a current of 4 A when connected to a 2 V battery. The resistivity of the wire is

(a) $2\times 10^{-7}\,\Omega$-m (b) $5\times 10^{-7}\,\Omega$-m
(c) $4\times 10^{-6}\,\Omega$-m (d) $1\times 10^{-6}\,\Omega$-m

48. Three resistances P, Q, R each of 2Ω and an unknown resistance S form the four arms of a Wheatstone's bridge circuit. When a resistance of 6Ω is connected in parallel to S, the bridge gets balanced. What is the value of S?

(a) 2Ω (b) 3Ω
(c) 6Ω (d) 1Ω

49. A 2 V battery, a 990 Ω resistor and a potentiometer of 2 m length, all are connected in series of the resistance of potentiometer wire is 10 Ω, then the potential gradient of the potentiometer wire is

(a) $0.05\,Vm^{-1}$ (b) $0.5\,Vm^{-1}$
(c) $0.01\,Vm^{-1}$ (d) $0.1\,Vm^{-1}$

50. The electron with charge ($q = 1.6\times 10^{-19}$ C) moves in an orbit of radius 5×10^{-11} m with a speed of $2.2\times 10^{6}\,ms^{-1}$, around an atom. The equivalent current is

(a) 1.12×10^{-6} A (b) 1.12×10^{-3} A
(c) 1.12×10^{-9} A (d) 1.12 A

51. Two resistance wires on joining in parallel, the resultant resistance is $\frac{6}{5}\,\Omega$. One of the wire breaks, the effective resistance is 2 Ω. The resistance of the broken wire is

(a) $\frac{3}{5}\Omega$ (b) 2 Ω (c) $\frac{6}{5}\Omega$ (d) 3 Ω

52. A potentiometer having the potential gradient of 2 mV/cm is used to measure the difference of potential across a resistance of 10 Ω in same circuit. If a length of 50 cm of the potentiometer wire is required to get the null point, then the current passing through the 10 Ω resistor is (in mA)

(a) 1 (b) 2 (c) 5 (d) 10

53. The n rows each containing m cells in series are joined in parallel. Maximum current is taken from this combination across an external resistance of 3 Ω resistance. If the total number of cells used are 24 and internal resistance of each cell is 0.5 Ω, then

(a) $m = 8, n = 3$ (b) $m = 6, n = 4$
(c) $m = 12, n = 2$ (d) $m = 2, n = 12$

54. A 100 V voltmeter of internal resistance 20 kΩ in series with a high resistance R is connected to a 110 V line. The voltmeter reads 5 V, the value of R is

(a) 210 kΩ (b) 315 kΩ (c) 420 kΩ (d) 440 kΩ

55. A cell supplies a current i_1 through a resistance R_1 and a current i_2 through a resistance R_2. The internal resistance of this cell is

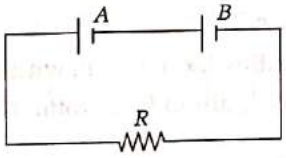
(a) $R_2 - R_1$ (b) $\dfrac{i_1R_2 - i_2R_1}{i_1 - i_2}$

(c) $\dfrac{i_2R_2 - i_1R_1}{i_1 - i_2}$ (d) $\left(\dfrac{i_1 + i_2}{i_1 - i_2}\right)\sqrt{R_1R_2}$

56. Out of five resistances of $R\,\Omega$ each 3 are connected in parallel and are joined to the rest 2 in series. Find the resultant resistance.

(a) $(3/7)R\,\Omega$ (b) $(7/3)R\,\Omega$
(c) $(7/8)R\,\Omega$ (d) $(8/7)R\,\Omega$

57. Two batteries A and B each of emf 2V are connected in series to an external resistance $R = 1\,\Omega$. If the internal resistance of battery A is 1.9 Ω and that B is 0.9 Ω. What is the potential difference between the terminals of battery A?

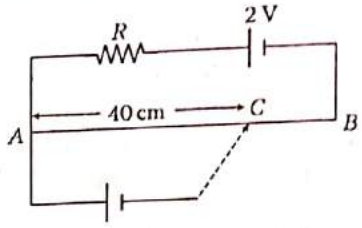


(a) 2 V (b) 3.8 V
(c) zero (d) 4.8 V

58. For a cell of emf 2 V, a balance is obtained for 50 cm of the potentiometer wire. If the cell is shunted by a 2 Ω resistor, the balance is obtained across 40 cm of the wire. Find the internal resistance of the cell

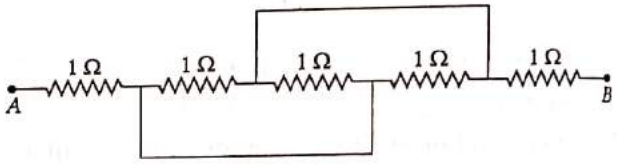
(a) 0.25 Ω (b) 0.50 Ω
(c) 0.80 Ω (d) 1.00 Ω

59. AB is a potentiometer wire of length 100 cm and its resistance is 10 Ω. It is connected in series with a resistance $R = 40\ \Omega$ and a battery of emf 2 V and negligible internal resistance. If a source of unknown emf E is balanced by 40 cm length of the potentiometer wire, the value of E is



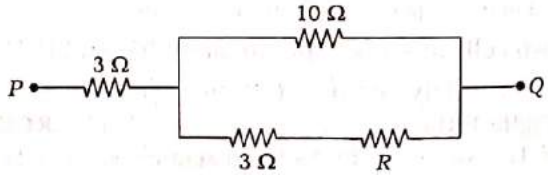
(a) 0.8 V (b) 1.6 V
(c) 0.08 V (d) 0.16 V

60. Equivalent resistance between the points A and B is (in Ω)



(a) $\dfrac{1}{5}$ (b) $\dfrac{5}{4}$
(c) $\dfrac{7}{3}$ (d) $\dfrac{7}{2}$

61. In the circuit shown here, what is the value of the unknown resistor R, so that the total resistance of the circuit between points P and Q is also equal to R?



(a) 3 Ω (b) $\sqrt{39}\,\Omega$
(c) $\sqrt{69}\,\Omega$ (d) 10 Ω

62. Two wires of same metal have the same length but their cross-sections are in the ratio 3 : 1. They are joined in series. The resistance of the thicker wire is 10 Ω. The total resistance of the combination will be

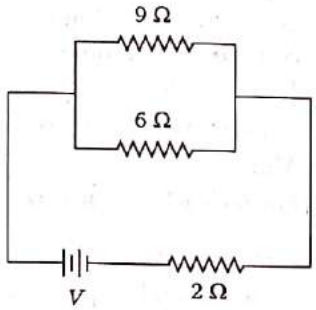
(a) 40 Ω (b) $\dfrac{40}{3}\,\Omega$

(c) $\dfrac{5}{2}\,\Omega$ (d) 100 Ω

63. The masses of the three wires of copper are in the ratio 5 : 3 : 1 and their lengths are in the ratio 1 : 3 : 5. The ratio of their electrical resistances is

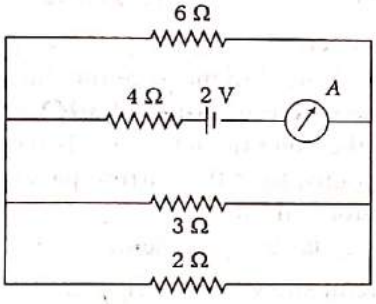
(a) 5 : 3 : 1 (b) $\sqrt{125}$: 15 : 1
(c) 1 : 15 : 125 (d) 1 : 3 : 5

64. If power dissipated in the 9Ω resistor in the circuit shown is 36 W, the potential difference across the 2Ω resistor is



(a) 8 V (b) 10 V (c) 2 V (d) 4 V

65. The reading of the ammeter in the following figure will be



(a) 0.8 A (b) 0.6 A (c) 0.4 A (d) 0.2 A

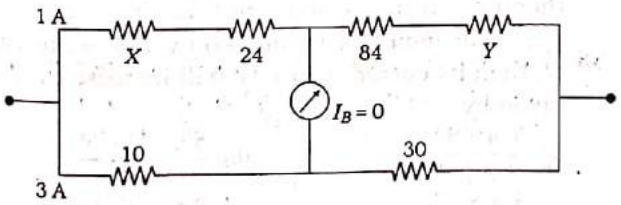
66. A wire of length 100 cm is connected to a cell of emf 2 V and negligible internal resistance. The resistance of the wire is 3Ω. The additional resistance required to produce a potential drop of 1 milli volt per cm is

(a) 60 Ω (b) 47 Ω (c) 57 Ω (d) 35 Ω

67. Two uniform wires A and B are of same metal and have equal masses. The radius of wire A is twice that of wire B. The total resistance of A and B when connected in parallel is

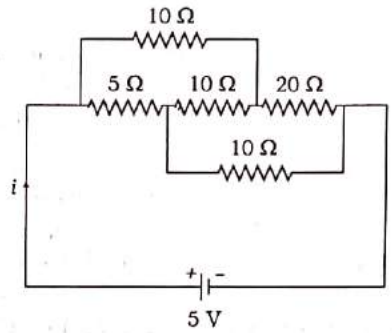
(a) 4 Ω when the resistance of wire A is 4.25 Ω
(b) 5 Ω when the resistance of wire A is 4 Ω
(c) 4 Ω when the resistance of wire B is 4.25 Ω
(d) 5 Ω when the resistance of wire B is 4 Ω

68. In the given circuit, the resistances are given in ohm. The current through the 10 Ω resistance is 3 A while that through the resistance X is 1 A. No current passes through the galvanometer. The values of the unknown resistances X and Y are respectively (in ohm)



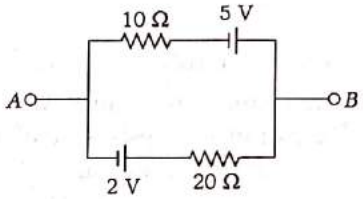
(a) 14 and 54 (b) 12 and 6 (c) 6 and 12 (d) 6 and 6

69. The current i drawn from the 5 V source will be



(a) 0.33 A (b) 0.5 A (c) 0.67 A (d) 0.17 A

70. The current in the given circuit is



(a) 0.3 A (b) 0.4 A (c) 0.1 A (d) 0.2 A

71. Two resistors 400Ω and 800Ω are connected in series with a 6 V battery. The potential difference measured by voltmeter of 10 kΩ across 400Ω resistor is

(a) 2 V (b) 1.95 V (c) 3.8 V (d) 4 V

72. A battery of emf E has an internal resistance r. A variable resistance R is connected to the terminals of the battery. A current i is drawn from the battery. V is the terminal potential difference. If R alone is gradually reduced to zero, which of the following best describes i and V?

(a) i approaches zero, V approaches E
(b) i approaches $\frac{E}{r}$, V approaches zero
(c) i approaches $\frac{E}{r}$, V approaches E
(d) i approaches infinity, V approaches E

73. Two resistances are connected in two gaps of a meter bridge. The balance point is 20 cm from the zero end. A resistance of 15 Ω is connected in series with the smaller of the two. The null point shifts to 40 cm. The value of the smaller resistance in ohm is

(a) 3 (b) 6
(c) 9 (d) 12
(e) None of these

74. A battery of four cells in series, each having an emf of 1.4 V and an internal resistance of 2 Ω is to be used to charge a small 2 V accumulator of negligible internal resistance. What is the charging current?

(a) 0.1 A (b) 0.2 A (c) 0.3 A (d) 0.45 A

75. The length of a wire of a potentiometer is 100 cm, and the emf of its cell is E volt. It is employed to measure the emf of a battery whose internal resistance is $0.5\,\Omega$. If the balance point is obtained at $l = 30$ cm from the positive end, the emf of the battery is

(a) $\frac{30E}{100.5}$

(b) $\frac{30E}{100-0.5}$

(c) $\frac{30(E-0.5i)}{100}$, where i is the current in the potentiometer wire

(d) $\frac{30E}{100}$

76. When a resistance of $100\,\Omega$ is connected in series with a galvanometer of resistance R, then its range is V. To double its range, a resistance of $1000\,\Omega$ is connected in series. Find the value of R.

(a) $700\,\Omega$ (b) $800\,\Omega$
(c) $900\,\Omega$ (d) $100\,\Omega$

77. Two cells with the same emf E and different internal resistances r_1 and r_2 are connected in series to an external resistance R. The value of R, so that the potential difference across the first cell be zero, is

(a) $\sqrt{r_1 r_2}$ (b) $r_1 + r_2$
(c) $r_1 - r_2$ (d) $\frac{r_1 + r_2}{2}$

78. Two wires of the same material but of different diameters carry the same current i. If the ratio of their diameters is 2 : 1, then the corresponding ratio of their mean drift velocities will be

(a) 4 : 1 (b) 1 : 1 (c) 1 : 2 (d) 1 : 4

79. Two bulbs consume same power when operated at 200 V and 300 V, respectively. When these bulbs are connected in series across a DC source of 500 V, then

(a) ratio of potential differences across them is 3/2
(b) ratio of potential differences across them is 9/4
(c) ratio of powers consumed across them is 4/9
(d) ratio of powers consumed across them is 2/3

80. A factory is served by a 220 V supply line. In a circuit protected by a fuse marked 10 A, the maximum number of 100 W lamps in parallel that can be turned on, is

(a) 11 (b) 22 (c) 33 (d) 66

81. Two bulbs marked 60 W, 220 V and 100 W, 220 V are connected in series and the series combination is now connected across a 220 V main supply. The power dissipated in the circuit is

(a) 37.5 W (b) 75 W (c) 80 W (d) 40 W

82. A tap supplies water at 22°C, a man takes 1 L of water per min at 37°C from the geyser. The power of geyser is

(a) 525 W (b) 1050 W (c) 1775 W (d) 2100 W

83. The mean free path of electrons in a metal is 4×10^{-8} m. The electric field which can give on an average 2 eV energy to an electron in the metal will be in unit of Vm^{-1}?

(a) 8×10^{7} (b) 5×10^{-11} (c) 8×10^{-11} (d) 5×10^{7}

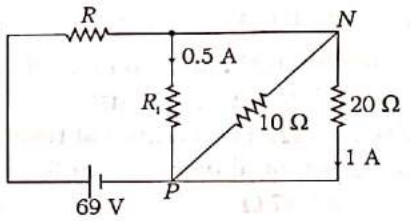
84. You are given two resistances R_1 and R_2. By using them singly, in series and in parallel, you can obtain four resistances of 1. 5Ω, 2Ω, 6Ω and 8Ω. The values of R_1 and R_2 are

(a) $1\Omega, 7\Omega$ (b) $1.5\Omega, 6.5\Omega$
(c) $3\Omega, 5\Omega$ (d) $2\Omega, 6\Omega$

85. A potentiometer having the potential gradient of 2 mVcm^{-1} is used to measure the difference of potential across a resistance of $10\,\Omega$. If a length of 50 cm of the potentiometer wire is required to get the null points, then the current passing through $10\,\Omega$ resistor is (in mA)

(a) 1 (b) 2 (c) 5 (d) 10

86. In the circuit shown in the figure,



(a) The current through NP is 0.5 A
(b) The value of $R_1 = 40\,\Omega$
(c) The value of $R = 14\,\Omega$
(d) The potential difference across $R = 49$ V

87. A galvanometer of resistance $50\,\Omega$ is connected to a battery of 3 V along with a resistance of $2950\,\Omega$ in series. A full scale deflection of 30 divisions is obtained in the galvanometer. In order to reduce, this deflection to 20 divisions the resistance in series should be

(a) $4450\,\Omega$ (b) $5050\,\Omega$
(c) $5550\,\Omega$ (d) $6050\,\Omega$

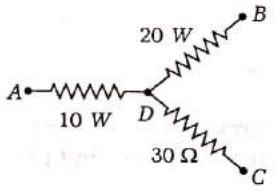
88. When a galvanometer is shunted by resistance S, then its current capacity increases n times. If the same galvanometer is shunted by another resistance S', then its current capacity will increase by n' is given by

(a) $\frac{(n+1)S}{S'}$ (b) $\frac{S(n-1)+S'}{S'}$
(c) $\frac{n+S}{S'}$ (d) $\frac{S(n-1)-S'}{S'}$

89. The tungsten filaments of two electric bulbs are of the same length. If one of them gives 25 W power and the other 60 W power, then
(a) Both the filaments are of same thickness
(b) 25 W bulb has thicker filament
(c) 60 W bulb has thicker filament
(d) Both the filaments have same cross-sectional area

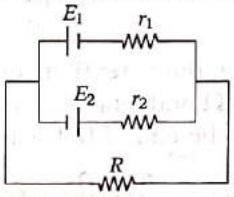
90. Three unequal resistors in parallel are equivalent to a resistance 1 Ω. If two of them are in the ratio 1 : 2 and if no resistance value is fractional, then the largest of the three resistances in ohms is
(a) 4 (b) 6
(c) 8 (d) 12

91. In the circuit given here, the points A, B and C are 70 V, zero, 10 V, respectively. Then,



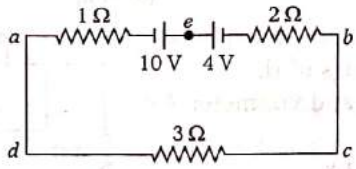
(a) the point D will be at a potential of 60 V
(b) the point D will be at a potential of 20 V
(c) currents in the paths AD, DB and DC are in the ratio of 1:2:3
(d) currents in the paths AB, DB and DC are in the ratio of 3:2:1

92. The current in the resistance R will be zero if



(a) $E_1 r_1 = E_2 r_2$
(b) $\frac{E_1}{r_1} = \frac{E_2}{r_2}$
(c) $(E_1 + E_2) r_1 = E_1 r_2$
(d) $(E_1 - E_2) r_1 = E_2 r_1$

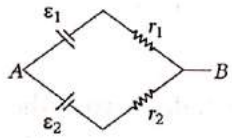
93. The magnitude and direction of the current in the circuit shown will be



(a) $\frac{7}{3}$ A from a to e (b) $\frac{7}{3}$ A from b to e
(c) 1A from b to e (d) 1A from a to e

94. Consider a current carrying wire (current I) in the shape of a circle. Note that as the currrent progresses along the wire, the direction of J (current density) changes in an exact manner, while the current I remains unaffected. The agent that is essentially responsible for **[NCERT Exemplar]**
(a) source of emf
(b) electric field produced by charges accumulated on the surface of wire
(c) the charges just behind a given segment of wire which push them just the right way by repulsion
(d) the charges ahead

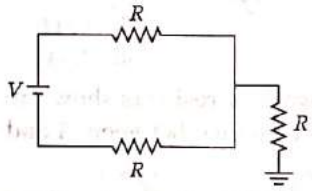
95. Two batteries of emf ε_1 and $\varepsilon_2 (\varepsilon_2 > \varepsilon_1)$ and internal resistances r_1 and r_2 respectively are connected in parallel as shown in figure. **[NCERT Exemplar]**



(a) Two equivalent emf ε_{eq} of the two cells is between ε_1 and ε_2, *i.e.*, $\varepsilon_1 < \varepsilon_{eq} < \varepsilon_2$
(b) The equivalent emf ε_{eq} is smaller than ε_1
(c) The ε_{eq} is given by $\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2$ always
(d) ε_{eq} is independent of internal resistances r_1 and r_2

96. A resistance R is to be measured using a meter bridge, student chooses the standard resistance S to be 100 Ω. He finds the null point at $l_1 = 2.9$ cm. He is told to attempt to improve the accuracy. Which of the following is a useful way? **[NCERT Exemplar]**
(a) He should measure l_1 more accurately
(b) He should change S to 1000 Ω and repeat the experiment
(c) He should change S to 3 Ω and repeat the experiment
(d) He should given up hope of a more accurate measurement with a meter bridge

97. The current drawn from the battery shown in the figure is

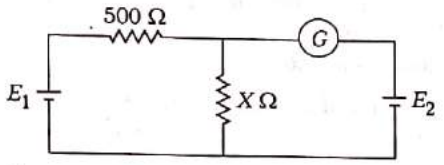


(a) $\frac{V}{R}$ (b) $\frac{V}{2R}$ (c) $\frac{2V}{R}$ (d) $\frac{3V}{2R}$

98. Two non-ideal unidentical batteries are connected in parallel with positive terminals. Consider the following statements
A. The equivalent emf is smaller than either of the two emfs.

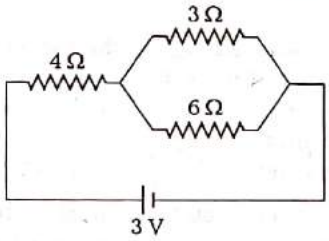
B. The equivalent internal resistance is smaller than either of the two internal resistances.
(a) Both A and B are correct
(b) A is correct but B is wrong
(c) B is correct but A is wrong
(d) Both A and B are wrong

99. In the below circuit, the battery E_1 has an emf of 12 V and zero internal resistance. While the battery E_2 has an emf of 2 V.



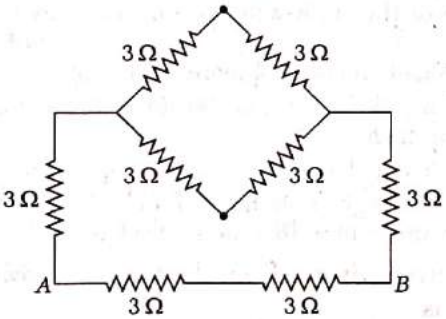
If the galvanometer G reads zero, then the value of the resistance X in ohms is
(a) 250 (b) 100
(c) 50 (d) 200

100. The potential drop across the 3 Ω resistor is



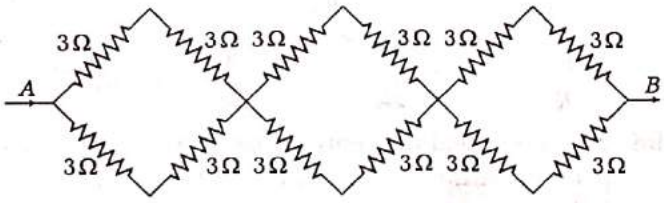
(a) 1 V (b) 1.5 V (c) 2 V (d) 3 V

101. Equivalent resistance between A and B will be



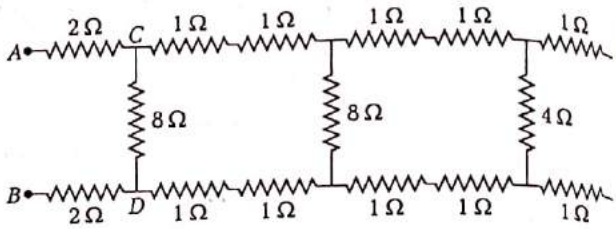
(a) 2 Ω (b) 18 Ω
(c) 6 Ω (d) 3.6 Ω

102. In the network of resistors shown in the figure, the equivalent resistance between A and B is



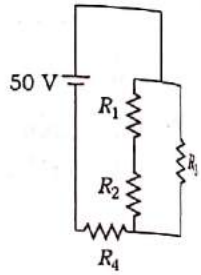
(a) 54 Ω (b) 18 Ω
(c) 36 Ω (d) 9 Ω

103. In the figure shown, the total resistance between A and B is



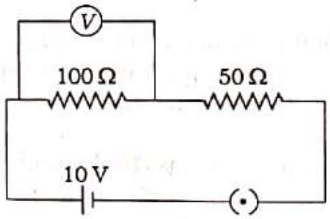
(a) 12 Ω (b) 4 Ω (c) 6 Ω (d) 8 Ω

104. The potential difference in volt across the resistance R_3 in the circuit shown in figure, is ($R_1 = 15\Omega, R_2 = 15\Omega, R_3 = 30\Omega, R_4 = 35\Omega$)



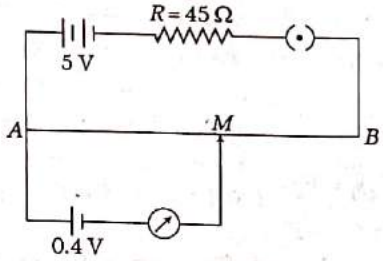
(a) 5
(b) 7.5
(c) 15
(d) 12.5

105. In the given circuit, the voltmeter records 5 V. The resistance of the voltmeter in Ω is



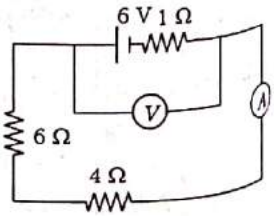
(a) 200 (b) 100
(c) 10 (d) 50

106. In given figure, the potentiometer wire AB has a resistance of 5 Ω and length 10 m. The balancing length AM for the emf of 0.4 V is



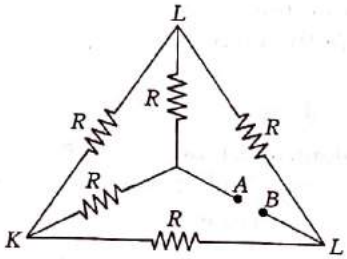
(a) 0.4 m (b) 4 m
(c) 0.8 m (d) 8 m

107. In the circuit shown here, the readings of the ammeter and voltmeter are



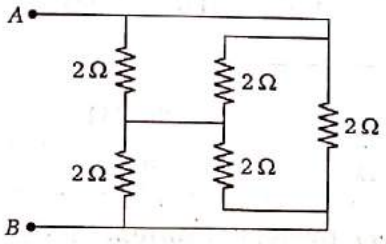
(a) 6 A, 60 V
(b) 0.6 A, 6 V
(c) 6 A, 6 V
(d) 6/11 A, 6/11 V

108. Each of the resistance in the network shown in the figure is equal to R. The resistance between the terminals A and B is



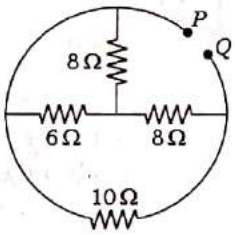
(a) R (b) $5R$ (c) $3R$ (d) $6R$

109. Find the equivalent resistance across AB.



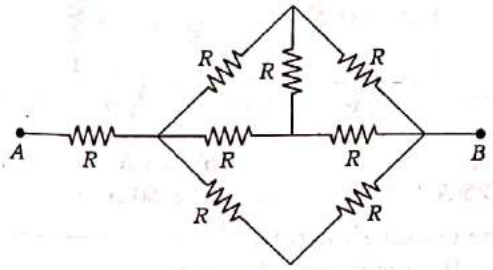
(a) 1 Ω (b) 2 Ω (c) 3 Ω (d) 4 Ω

110. The equivalent resistance between P and Q in the figure is approximately



(a) 6 Ω (b) 5 Ω (c) 7.5 Ω (d) 20 Ω

111. In the given network of resistances, the effective resistance between A and B is

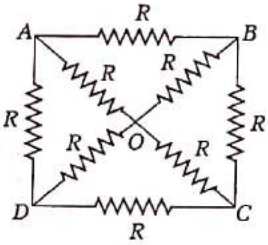


(a) $\frac{5}{3}R$ (b) $\frac{8}{3}R$ (c) $5R$ (d) $8R$

112. A source of emf $E = 15$ V and having negligible internal resistance is connected to a variable resistance, so that the current in the circuit increases with time as $i = 1.2t + 3$. Then, the total charge that will flow in first five second will be

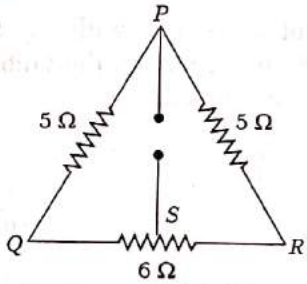
(a) 10 C (b) 20 C
(c) 30 C (d) 40 C

113. The effective resistance between points A and C for the network shown in figure is



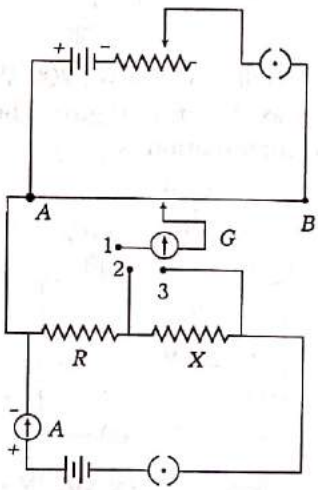
(a) $\frac{2}{3}R$ (b) $\frac{3}{2}R$ (c) $2R$ (d) $\frac{1}{2R}$

114. Three resistances 5Ω, 5Ω and 6Ω are connected as shown in figure. If the point S divides the resistance 6Ω into two equal halves, the resistance between points P and S is



(a) 11Ω (b) 8Ω (c) 6Ω (d) 10Ω
(e) 4 Ω

115. A potentiometer circuit is set up as shown. The potential gradient across the potentiometer wire, is k volt/cm and the ammeter, present in the circuit, reads 1.0 A when two way key is switched off. The balance points, when the key between the terminals (i) 1 and 2 (ii) 1 and 3, is plugged in, are found to be at lengths l_1 cm and l_2 cm, respectively. The magnitudes of the resistors R and X in ohm, are then, respectively equal to

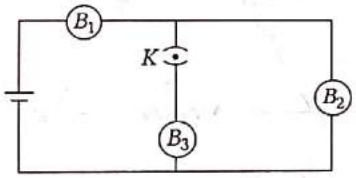


(a) $k(l_2 - l_1)$ and kl_2 (b) kl_1 and $k(l_2 - l_1)$
(c) $k(l_2 - l_1)$ and kl_2 (d) kl_1 and kl_2

116. An electric immersion heater of 1.08 kW is immersed in water. After the water has reached a temperature of 100°C, how much time will be required to produce 100 g of steam?
(a) 50 s (b) 420 s
(c) 105 s (d) 210 s

117. A moving coil galvanometer is converted into an ammeter reading upto 0.03 A by connecting a shunt of resistance $4r$ across it and into an ammeter reading upto 0.06 A when a shunt of resistance r is connected across it. What is the maximum current which can be sent through this galvanometer, if no shunt is used?
(a) 0.01 A (b) 0.02 A
(c) 0.03 A (d) 0.04 A

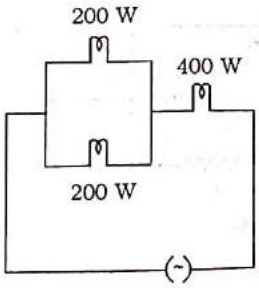
118. B_1, B_2 and B_3 are the three identical bulbs connected to a battery of steady emf with key K closed. What happens to the brightness of the bulbs B_1 and B_2 when the key is opened?



(a) Brightness of the bulb B_1 increases and that of B_2 decreases
(b) Brightness of the bulbs B_1 and B_2 increases
(c) Brightness of the bulb B_1 decreases and B_2 increases
(d) Brightness of the bulbs B_1 and B_2 decreases

119. The scale of a galvanometer of resistance 100 Ω contains 25 divisions. It gives a deflection of one division on passing a current of 4×10^{-4} A. The resistance (in ohm) to be added to it, so that it may become a voltmeter of range 2.5 V is
(a) 150 (b) 170
(c) 110 (d) 220

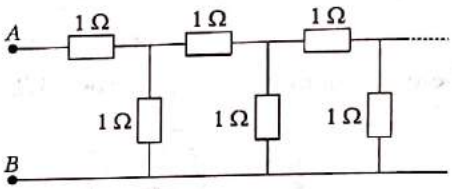
120. Three electric bulbs of 200 W, 200 W and 400 W are connected as shown in figure. The resultant power of the combination is



(a) 800 W (b) 400 W
(c) 200 W (d) 600 W

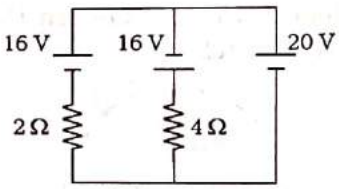
121. Two electric bulbs rated 50 W and 100 W are glowing at full power, when used in parallel with a battery of emf 120 V and internal resistance 10 Ω. The maximum number of bulbs that can be connected in the circuit when glowing at full power, is
(a) 6 (b) 4 (c) 2 (d) 8

122. The equivalent resistance between points A and B of an infinite network of resistances, each of 1 Ω, connected as shown, is



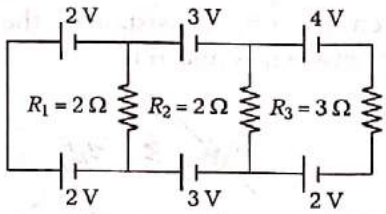
(a) infinite (b) 2 Ω
(c) $\frac{1+\sqrt{5}}{2}\Omega$ (d) zero

123. In the given figure, the current through the 20 V battery is



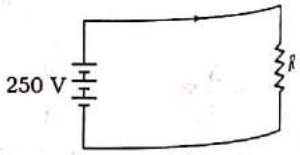
(a) 11 A (b) 12 A
(c) 7 A (d) 14 A

124. The current in resistance R_3 in the given circuit is



(a) 1 A (b) 2/3 A
(c) 0.25 A (d) 0.50 A

125. In the circuit shown in figure, the resistance R has a value that depends on the current. Specifically R is 20 Ω when i is zero and the amount of increase in resistance is numerically equal to one-half of the current. What is the value of current I in circuit?



(a) 15 A (b) 10 A
(c) 20 A (d) 5 A

126. The charge flowing in a conductor varies with time as $Q = at - bt^2$, then the current

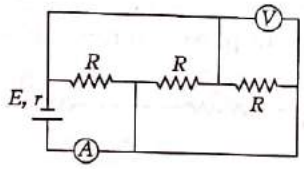
(a) reaches a maximum and then decreases

(b) falls to zero after $t = \frac{a}{2b}$

(c) changes at a rate of $(-2b)$

(d) Both (b) and (c)

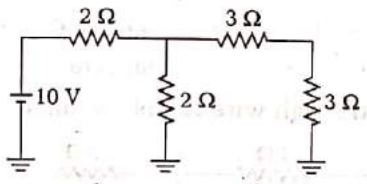
127. In the circuit shown in figure, ammeter and voltmeter are ideal. If $E = 4$ V, $R = 9\,\Omega$ and $r = 1\,\Omega$, then readings of ammeter and voltmeter are



(a) 1 A, 3 V (b) 2 A, 3 V

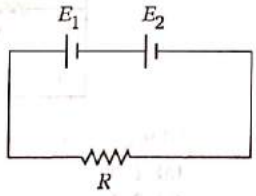
(c) 3 A, 4 V (d) 4 A, 4 V

128. In the circuit shown, the current in 3 Ω resistance is



(a) 1 A (b) $\frac{1}{7}$ A (c) $\frac{5}{7}$ A (d) $\frac{15}{7}$ A

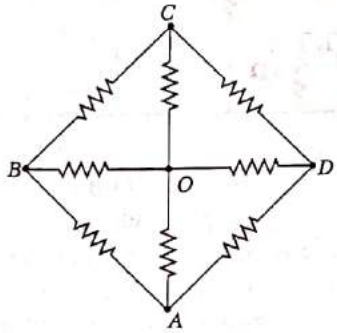
129. Under what conditions current passing through the resistance R can be increased by short circuiting the battery of emf E_2. The internal resistances of the two batteries are r_1 and r_2, respectively.



(a) $E_2 r_1 > E_1(R + r_2)$ (b) $E_1 r_2 < E_2(R + r_1)$

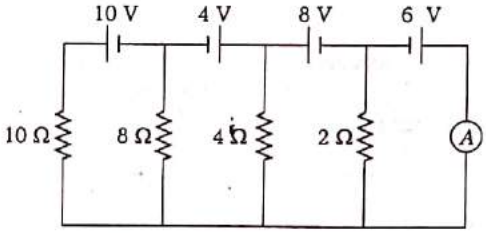
(c) $E_2 r_2 < E_1(R + r_2)$ (d) $E_1 r_2 > E_2(R + r_1)$

130. In the arrangement shown, the magnitude of each resistance is 1 Ω. The equivalent resistance between O and A is given by



(a) $\frac{14}{13}\,\Omega$ (b) $\frac{3}{4}\,\Omega$ (c) $\frac{2}{3}\,\Omega$ (d) $\frac{5}{6}\,\Omega$

131. Find the reading of the ideal ammeter connected in the given circuit. Assume that the cells have negligible internal resistance



(a) 0.8 A (b) 0.25 A (c) 1.95 A (d) 1.0 A

132. A moving coil galvanometer has 150 equal divisions. Its current sensitivity is 10 divisions per milliampere and voltage sensitivity is 2 divisions per millivolt. In order that each division reads 1 V the resistance in ohm needed to be connected in series with the coil will be

(a) 99995 (b) 9995

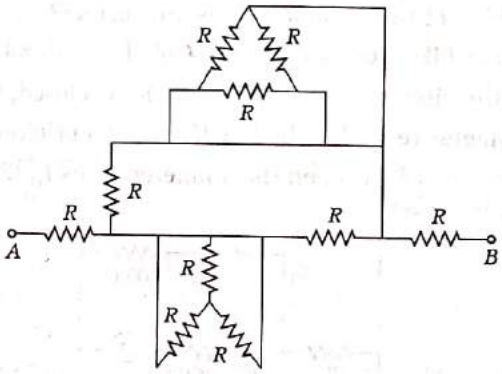
(c) 10^3 (d) 10^5

(e) None of these

133. It takes 16 min to boil some water in an electric kettle. Due to some defect it becomes necessary to remove 10% turns of heating coil of the kettle. After repairs, how much time will it take to boil the same mass of water?

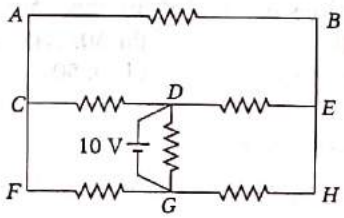
(a) 17.7 min (b) 14.4 min (c) 20.9 min (d) 13.9 min

134. Equivalent resistance between points A and B is



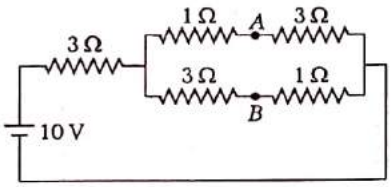
(a) $3R$ (b) $4R$ (c) $\frac{5R}{2}$ (d) $\frac{7R}{2}$

135. All resistances shown in circuit are 2 Ω each. The current in the resistance between D and E is



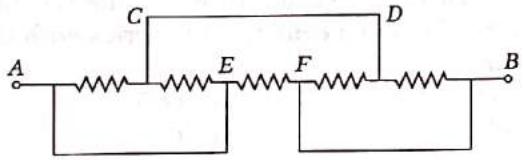
(a) 5 A (b) 2.5 A (c) 1 A (d) 7.5 A

136. A battery of emf 10 V is connected to a group of resistances as shown in figure. The potential difference $V_A - V_B$ between the points A and B is



(a) −2 V (b) 2 V
(c) 5 V (d) $\frac{20}{11}$ V

137. If $V_A - V_B = V_0$ and the value of each resistance is R, then



I. net resistance between AB is $\frac{R}{2}$

II. net resistance between AB is $\frac{3R}{5}$

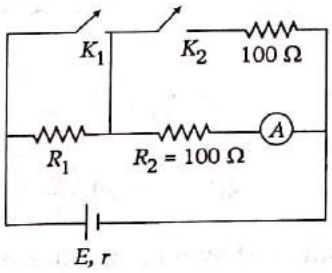
III. current through CD is $\frac{V_0}{R}$

IV. current through EF is $\frac{2V_0}{3R}$

Which of the option/options are correct?

(a) I and II (b) I and III (c) Only I (d) All of these

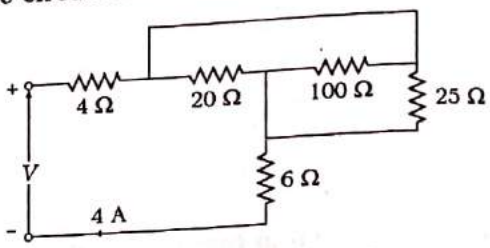
138. In the circuit shown, when key K_1 is closed, then the ammeter reads I_0 whether K_2 is open or closed. But when K_1 is open the ammeter reads $I_0/2$, when K_2 is closed.



Assuming that ammeter resistance is much less than R_2, the values of r and R_1 in ohms are

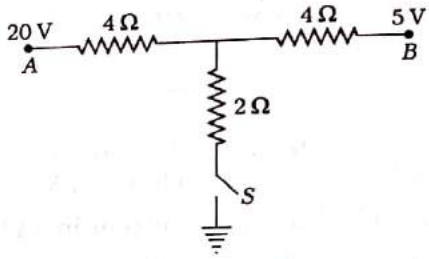
(a) 100, 50 (b) 50, 100
(c) 0, 100 (d) 0, 50

139. In the circuit shown in figure, V must be



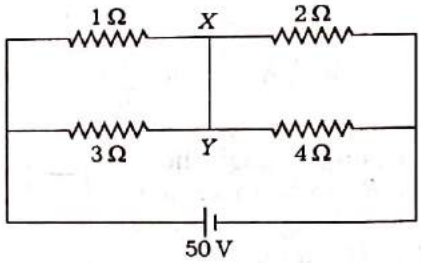
(a) 50 V (b) 80 V
(c) 100 V (d) 1290 V

140. As the switch S is closed in the circuit shown in figure, current passing through it is



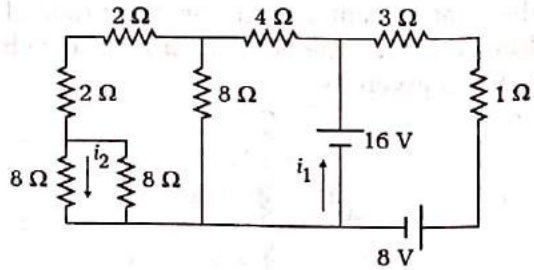
(a) 4.5 A (b) 6.0 A
(c) 3.0 A (d) zero

141. Current through wire XY of circuit shown is



(a) 1 A (b) 4 A
(c) 2 A (d) 3 A

142. In the circuit shown in figure, the ratio of currents i_1/i_2 is



(a) 2 (b) 8
(c) 0.5 (d) 4

B. Medical entrance special format questions

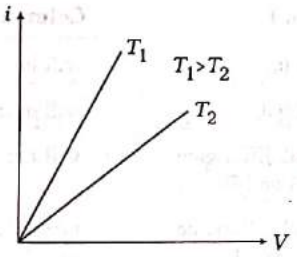
Assertion and reason

Directions (Q. Nos. 1-19) *These questions consist of two statements each linked as Assertion and Reason. While answering these questions you are required to choose any one of the following five responses.*

(a) If both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are true but Reason is not correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If Assertion is false but Reason is true.
(e) If both Assertion and Reason are false.

1. **Assertion** $V = iR$ is Ohm's law.
Reason $V - i$ graph is always a straight line passing through origin.

2. **Assertion** Current *versus* potential difference ($i - V$) graph for a conductor at two different temperatures T_1 and T_2 is shown in figure. Hence, $T_1 > T_2$.



Reason Resistance of a conductor increases with rise in temperature.

3. **Assertion** If a current flows through a wire of non-uniform cross-section, potential difference per unit length of wire is same throughout the length of wire.
Reason Current through the wire is same at all cross-sections.

4. **Assertion** In our houses when we start switching on different light buttons, main current goes on increasing.
Reason Different connections in houses are in parallel. When we start switching on different light buttons, then net resistance of the circuit decreases. Therefore, main current increases.

5. **Assertion** If a current is flowing through a conducting wire of non-uniform cross-section, drift speed and resistance both will increase at a section where cross-sectional area is less.
Reason Current density at such sections is more where cross-sectional area is less.

6. **Assertion** Kilovolt-ampere (kVA) and kilowatt-hour have the same dimensions.
Reason Both are the units of energy.

7. **Assertion** If by mistake, a voltmeter is connected in series, then circuit will burn.
Reason Current will drastically decrease in the circuit.

8. **Assertion** Resistance of an ammeter is less than the resistance of a milliammeter.
Reason Value of shunt required in case of ammeter is more than a milliammeter.

9. **Assertion** A resistance wire is broken into four pieces and all are connected in parallel. Then, net resistance becomes 1/16 times.
Reason In parallel net resistance is less than the smallest value of individual resistance.

10. **Assertion** Potential difference across the terminals of a battery is always less than the emf of the battery.
Reason During discharging of a battery potential difference across to terminals of a battery is
$$V = E - ir$$
$$\therefore \quad V < E$$

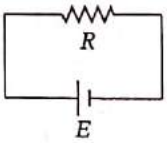
11. **Assertion** Inside a conductor, electrons have no motion in the absence of some potential difference across it.
Reason In the absence of potential difference no electrostatic force will act on the electrons inside the conductor.

12. **Assertion** Current between two points in an electrical circuit always flows from higher potential to lower potential.
Reason During charging of a battery current inside the battery flows from negative terminal to positive terminal.

13. **Assertion** In case of potentiometer experiment, if emfs of known and unknown both batteries are increased two times, then null point length will remain unchanged.
Reason Null point length does not depend on emf of either of the two batteries.

14. **Assertion** In potentiometer experiment, null point cannot be obtained if emf of unknown battery is more than the emf of known battery.
Reason By increasing the emf of known battery, null point length increases.

15. Assertion In the circuit shown in figure, battery is ideal. If a resistance R_0 is connected in parallel with R, then power across R will increase.

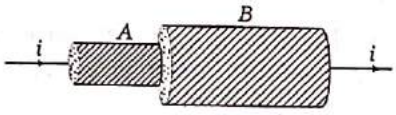


Reason Current drawn from the battery will increase.

16. Assertion Two 60 W bulbs are first connected in series and then in parallel with same battery. Total power produced in second case will become four times. Rated voltage is same for two bulbs.

Reason In series total power produced will be 30 W and in parallel 120 W.

17. Assertion Two resistance wires shown in figure are of same material. They have equal length. More heat is generated in wire A.



Reason In series $H \propto R$ and resistance of wire A is more.

18. Assertion Two identical bulbs when connected across a battery, produce a total power P. When they are connected across the same battery in series total power consumed will be $\frac{P}{4}$.

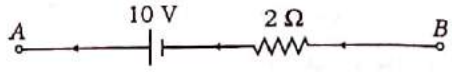
Reason In parallel, $P = P_1 + P_2$ and in series $P = \frac{P_1 P_2}{P_1 + P_2}$.

Match the columns

1. Rated power of a bulb at V voltage is P. Now, same voltage V is applied in all conditions mentioned in Column I. Match this Column I with Column II in which actual total power consumed is given.

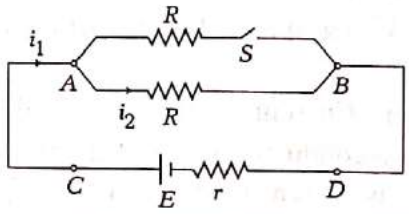
	Column I		Column II
A.	Two bulbs are connected in parallel.	(p)	P
B.	Two bulbs are connected in series.	(q)	$2P$
C.	Two bulbs are connected in parallel and one bulb in series with this combination.	(r)	$\frac{P}{2}$
D.	A group of two-two bulbs in parallel are mutually connected in series.	(s)	None

2. In the part of circuit shown in figure, match the following two columns for V_{AB}



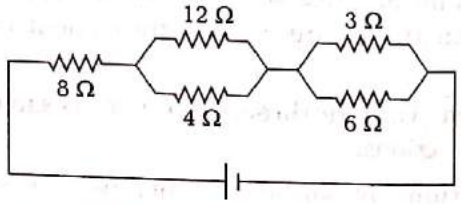
	Column I		Column II
A.	If $i = 5$ A	(p)	$V_{AB} = 0$ V
B.	If $i = -5$ A	(q)	$V_{AB} = 10$ V
C.	If $i = 0$	(r)	$V_{AB} = 20$ V
D.	If points A and B are connected by a conducting wire	(s)	$V_{AB} = -10$ V

3. In the circuit diagram shown in figure match the following two columns when switch S is closed.



	Column I		Column II
A.	Current (i_1)	(p)	will increase
B.	Current (i_2)	(q)	will decrease
C.	Potential difference across A and B	(r)	will remain same
D.	Potential difference across C and D	(s)	may increase or decrease. It will depend on value of r

4. In the circuit diagram shown in figure, potential difference across 3 Ω resistance is 20 V. Then, match the following two columns.



	Column I		Column II
A.	Potential difference across 6 Ω resistance	(p)	30 V
B.	Potential difference across 4 Ω resistance	(q)	40 V
C.	Potential difference across 12 Ω resistance	(r)	20 V
D.	Potential difference across 8 Ω resistance	(s)	80 V

C. Medical entrances' gallery

(Collection of questions asked in NEET & various medical entrance exams)

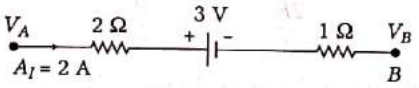
1. A potentiometer wire is 100 cm long and a constant potential difference is maintained across it. Two cells are connected in series first to support one another and then in opposite direction. The balance points are obtained at 50 cm and 10 cm from the positive end of the wire in the two cases. The ratio of emfs is **[NEET 2016]**

(a) 5 : 4 (b) 3 : 4 (c) 3 : 2 (d) 5 : 1

2. The charge flowing through a resistance R varies with time t as $Q = at - bt^2$, where a and b are positive constants. The total heat produced in R is **[NEET 2016]**

(a) $\frac{a^3R}{3b}$ (b) $\frac{a^3R}{2b}$ (c) $\frac{a^3R}{b}$ (d) $\frac{a^3R}{6b}$

3. The potential difference $(V_A - V_B)$ between the points A and B in the given figure is **[NEET 2016]**



(a) -3 V (b) +3 V (c) +6 V (d) +9 V

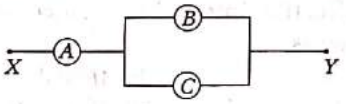
4. A filament bulb (500 W, 100 V) is to be used in a 230 V main supply. When a resistance R is connected in series, it works perfectly and the bulb consumes 500 W. The value of R is **[NEET 2016]**

(a) 230 Ω (b) 46 Ω (c) 26 Ω (d) 13 Ω

5. A potentiometer wire has length 4 m resistance 8 Ω. The resistance that must be connected in series with the wire and an accumulator of emf 2 V, so as to get a potential gradient of 1 mV per cm on the wire is **[CBSE AIPMT 2015]**

(a) 32 Ω (b) 40 Ω (c) 44 Ω (d) 48 Ω

6. A, B and C are voltmeters of resistance R, 1.5 R and 3R respectively, as shown in the figure. When some potential difference is applied between X and Y, then the voltmeter readings are V_A, V_B and V_C, respectively. Then, **[CBSE AIPMT 2015]**

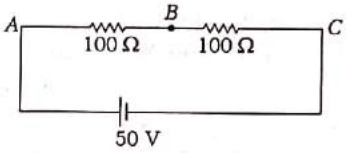


(a) $V_A = V_B = V_C$ (b) $V_A \neq V_B = V_C$
(c) $V_A = V_B \neq V_C$ (d) $V_A \neq V_B \neq V_C$

7. Across a metallic conductor of non-uniform cross-section, a constant potential difference is applied. The quantity which remain (s) constant along the conductor is **[CBSE AIPMT 2015]**

(a) current density (b) current
(c) drift velocity (d) electric field

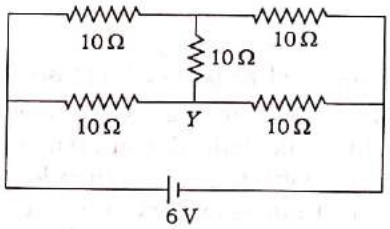
8. Consider the diagram shown below. **[AIIMS 2015]**



A voltmeter of resistance 150 Ω is connected across A and B. The potential drop across B and C measured by voltmeter is

(a) 29 (b) 27 V (c) 31 V (d) 30 V

9. Each resistor shown in the figure has a resistance of 10 Ω and the battery has the emf 6 V. What will be the current supplied by the battery? **[UK PMT 2015]**



(a) 0.6 A (b) 1.2 A (c) 1.8 A (d) 0.3 A

10. A 1Ω resistance in series with an ammeter is balanced by 75 cm of potentiometer wire. A standard cell of emf 1.02 V is balanced by 50 cm. The ammeter shows a reading of 1.5 A. Then, the error in ammeter reading is **[EAMCET 2015]**

(a) 0.03 A (b) 3A (c) 1.3 A (d) 0.3 A

11. The range of voltmeter is 10 V and its internal resistance is 50 Ω. To convert it to a voltmeter of range 15 V, how much resistance is to be added? **[EAMCET 2015]**

(a) Add 25 Ω resistor in parallel
(b) Add 25 Ω resistor in series
(c) Add 125 Ω resistor in parallel
(d) Add 125 Ω resistor in series

12. Identify the wrong statement. **[Kerala CEE 2015]**

(a) Charge is a vector quantity
(b) Current is a scalar quantity
(c) Charge can be quantised
(d) Charge is additive in nature
(e) Charge is conserved

13. When the rate of flow of charge through a metallic conductor of non-uniform cross-section is uniform, then the quantity that remains constant along the conductor is **[Kerala CEE 2015]**

(a) current density (b) electric field
(c) electric potential (d) drift velocity
(e) current

14. The resistance of a carbon resistor of colour code Red-Red Green Silver is (in kΩ) **[Kerala CEE 2015]**

(a) $2200 \pm 5\%$ (b) $2200 \pm 10\%$
(c) $220 \pm 10\%$ (d) $220 \pm 5\%$
(e) $2200 \pm 1\%$

15. The slope of the graph showing the variation of potential difference V on X-axis and current Y-axis gives conductor **[Kerala CEE 2015]**

(a) resistance (b) resistivity
(c) reciprocal of resistance (d) conductivity
(e) impedance

16. Two wires of equal length and equal diameter and having resistivities ρ_1 and ρ_2 are connected in series. The equivalent resistivity of the combination is **[Guj. CET 2015]**

(a) $\frac{\rho_1 + \rho_2}{2}$ (b) $\rho_1 + \rho_2$
(c) $\frac{\rho_1 \rho_2}{\rho_1 + \rho_2}$ (d) $\sqrt{\rho_1 \rho_2}$

17. A galvanometer of resistance 50 Ω is connected to a battery of 8 V along with a resistance of 3950 Ω in series. A full scale deflection of 30 divisions is obtained in the galvanometer . In order to reduce this deflection to 15 divisions, the resistance in series should be ... Ω. **[Guj. CET 2015]**

(a) 1950 (b) 7900
(c) 2000 (d) 7950

18. Choose the correct statement. **[CG PMT 2015]**

(a) Kirchhoff's first law of electricity is based on conservation of charge while the second law is based on conservation of energy
(b) Kirchhoff's first law of electricity is based on conservation of energy while the second law is based on conservation of charge
(c) Kirchhoff's both laws are based on conservation of charge
(d) Kirchhoff's both laws are based on conservation of energy

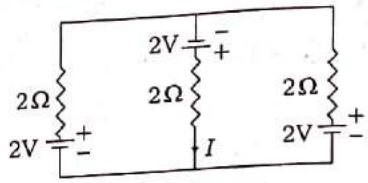
19. A metal plate weighing 750 g is to be electroplated with 0.05% of its weight of silver. If a current of 0.8 A is used, find the time (approx) needed for depositing the required weight of silver (ECE of silver is 11.8×10^{-7} kgC^{-1}) **[CG PMT 2015]**

(a) 5 min 32 s (b) 6 min 37 s
(c) 4 min 16 s (d) 6 min 10 s

20. A DC ammeter has resistance 0.1 Ω and its current ranges 0-100 A. If the range is to be extended to 0-500, then the following shunt resistance will be required **[CG PMT 2015]**

(a) 0.010 Ω (b) 0.011 Ω
(c) 0.025 Ω (d) 0.25 Ω

21. The current I shown in the circuit is **[WB JEE 2015]**

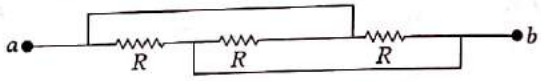


(a) 1.33 A (b) zero (c) 2 A (d) 1 A

22. A metal wire of circular cross-section has a resistance R_1. The wire is now stretched without breaking, so that its length is doubled and the density is assumed to remain the same. If the resistance of the wire now becomes R_2, then $R_2 : R_1$ is **[WB JEE 2015]**

(a) 1 : 1 (b) 1 : 2 (c) 4 : 1 (d) 1 : 4

23. Consider the combination of resistor,



The equivalent resistance between a and b is **[UP CPMT 2015]**

(a) $\frac{R}{6}$ (b) $\frac{2R}{3}$ (c) $\frac{R}{3}$ (d) $3R$

24. A potentiometer wire of length 100 cm has a resistance of 10 Ω. It is connected in series with a resistance and a cell of emf 2V having negligible internal resistance. A source of emf 10 mV is balanced against a length of 40 cm of the potentiometer wire. The value of external resistance is **[UP CPMT 2015]**

(a) 760 Ω (b) 640 Ω
(c) 790 Ω (d) 840 Ω

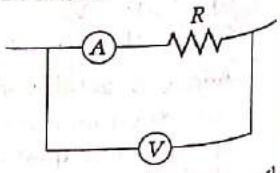
25. Three resistances 2 Ω, 3 Ω and 4 Ω are connected in parallel. The ratio of currents passing through them when a potential difference is applied across its ends will be **[KCET 2015]**

(a) 5 : 4 : 3 (b) 6 : 3 : 2
(c) 4 : 3 : 2 (d) 6 : 4 : 3

26. Four identical cells of emf ε and internal resistance r are to be connected in series. Suppose, if one of the cell is connected wrongly, then the equivalent emf and effective internal resistance of the combination is **[KCET 2015]**

(a) $2E$ and $4r$ (b) $4E$ and $4r$
(c) $2E$ and $2r$ (d) $4E$ and $2r$

27. In the circuit shown alongside, the ammeter and the voltmeter readings are 3A and 6V, respectively. Then, the value of the resistance R is **[KCET 2015]**

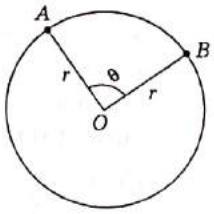


(a) $< 2\,\Omega$ (b) $2\,\Omega$ (c) $\geq 2\,\Omega$ (d) $> 2\,\Omega$

28. The resistance of a bulb filament is 100 Ω at a temperature of 100°C. If its temperature coefficient of resistance be 0.005 per °C, then its resistance will become 200 Ω at a temperature **[KCET 2015]**

(a) 500°C (b) 300°C
(c) 200°C (d) 400°C

29. A and B are the two points on a uniform ring of radius r. The resistance of the ring is R and $\angle AOB = \theta$ as shown in the figure. The equivalent resistance between points A and B is **[Guj. CET 2015]**



(a) $\frac{R(2\pi-\theta)}{4\pi}$ (b) $\frac{R\theta}{2\pi}$
(c) $R\left(1-\frac{\theta}{2\pi}\right)$ (d) $\frac{R}{4\pi^2}(2\pi-\theta)\theta$

30. The dimensions of mobility of charge carriers are **[Kerala CEE 2014]**

(a) $[M^{-2}T^2A]$ (b) $[M^{-1}T^2A]$
(c) $[M^{-2}T^3A]$ (d) $[M^{-1}T^3A]$
(e) $[M^{-1}T^3A^{-1}]$

31. The temperature coefficient of resistance of an alloy used for making resistor is **[Kerala CEE 2014]**

(a) small and positive (b) small and negative
(c) large and positive (d) large and negative
(e) zero

32. A wire of resistance 4 Ω is stretched to twice its original length. In the process of stretching, its area of cross-section gets halved. Now, the resistance of the wire is **[EAMCET 2014]**

(a) 8 Ω (b) 16 Ω (c) 1 Ω (d) 4 Ω

33. A carbon film resistor has colour code green, black, violet, gold. The value of the resistor is **[KCET 2014]**

(a) 50 MΩ (b) 500 MΩ
(c) 500 ± 5% MΩ (d) 500 ± 10% MΩ

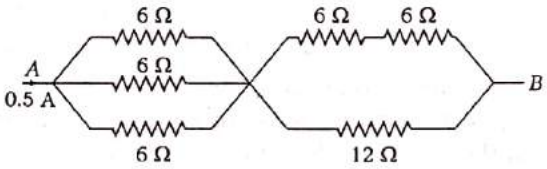
34. A uniform wire of resistance 9 Ω is joined end-to-end to form a circle. Then, the resistance of the circular wire between any two diametrically points is **[Kerala CEE 2014]**

(a) 6 Ω (b) 3 Ω (c) $\frac{9}{4}$ Ω (d) $\frac{3}{2}$ Ω
(e) 1 Ω

35. The equivalent resistance of two resistors connected in series is 6 Ω and their parallel equivalent resistance is $\frac{4}{3}$ Ω. What are the values of resistances? **[KCET 2014]**

(a) 4 Ω, 6 Ω (b) 8 Ω, 1 Ω
(c) 4 Ω, 2 Ω (d) 6 Ω, 2 Ω

36. Six resistances are connected as shown in figure. If total current flowing is 0.5 A, then the potential difference $V_A - V_B$ is **[EAMCET 2014]**

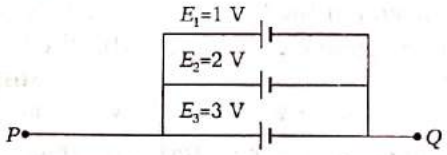


(a) 8 V (b) 6 V (c) 2 V (d) 4 V

37. Four cells, each of emf E and internal resistance r, are connected in series across an external resistance R. By mistake one of the cells is connected in reverse. Then, the current in the external circuit is **[WB JEE 2014]**

(a) $\frac{2E}{4r+R}$ (b) $\frac{3E}{4r+R}$ (c) $\frac{3E}{3r+R}$ (d) $\frac{2E}{3r+R}$

38. A circuit consists of three batteries of emf $E_1 = 1$ V, $E_2 = 2$ V and $E_3 = 3$ V and internal resistance 1 Ω, 2 Ω and 1 Ω respectively which are connected in parallel as shown in figure. The potential difference between points P and Q is **[WB JEE 2014]**



(a) 1.0 V (b) 2.0 V
(c) 2.2 V (d) 3.0 V

39. Two resistors of resistances 2 Ω and 6 Ω are connected in parallel. This combination is then connected to a battery of emf 2 V and internal resistance 0.5 Ω. What is the current flowing through the battery? **[KCET 2014]**

(a) 4 A (b) $\frac{4}{3}$ A (c) $\frac{4}{17}$ A (d) 1 A

40. The resistance in the two arms of the meter bridge are 5Ω and R Ω, respectively. When the resistance R is shunted with an equal resistance, then the new balance point is at 1.6 l_1. The resistance R is **[CBSE AIPMT 2014]**

(a) 10 Ω (b) 15 Ω (c) 20 Ω (d) 25 Ω

41. A potentiometer circuit has been set up for finding the internal resistance of a given cell. The main battery, used across the potentiometer wire, has an emf of 2.0 V and a negligible internal resistance. The potentiometer wire itself is 4m long. When the resistance R connected across the given cell, then has values of

(i) infinity (ii) 9.5 Ω

The balancing lengths of the potentiometer wire are found to be 3 m and 2.85 m, respectively.

The value of internal resistance of the cell is [CBSE AIPMT 2014]

(a) $0.25\,\Omega$ (b) $0.95\,\Omega$ (c) $0.5\,\Omega$ (d) $0.75\,\Omega$

42. In an ammeter, 0.2% of main current passes through the galvanometer. If resistance of galvanometer is G, then the resistance of ammeter will be [CBSE AIPMT 2014]

(a) $\frac{1}{499}G$ (b) $\frac{499}{500}G$ (c) $\frac{1}{500}G$ (d) $\frac{500}{499}G$

43. In Wheatstone bridge, three resistors P, Q and R are connected in three arms in order and 4th arm of resistance s, is formed by two resistors s_1 and s_2 connected in parallel. The condition for bridge to be balanced is, $\frac{P}{Q}$ [MHT CET 2014]

(a) $\frac{R(s_1+s_2)}{s_1s_2}$ (b) $\frac{s_1s_2}{R(s_1+s_2)}$ (c) $\frac{R\,s_1s_2}{(s_1+s_2)}$ (d) $\frac{(s_1+s_2)}{R\,s_1s_2}$

44. An electron in potentiometer experiences a force 2.4×10^{-19} N. The length of potentiometer wire is 6m. The emf of the battery connected across the wire is (electronic charge $=16\times10^{-19}$ C) [MHT CET 2014]

(a) 6 V (b) 9 V (c) 12 V (d) 15 V

45. A galvanometer having internal resistance $10\,\Omega$ requires 0.01 A for a full scale deflection. To convert this galvanometer to a voltmeter of full scale deflection at 120 V, we need to connect a resistance of [UK PMT 2014]

(a) $1190\,\Omega$ in series (b) $11990\,\Omega$ in parallel
(c) $12010\,\Omega$ in series (d) $12010\,\Omega$ in parallel

46. In potentiometer experiment, a cell of emf 1.25 V gives balancing length of 30 cm. If the cell is replaced by another cell, then balancing length is found to be 40 cm. What is the emf of second cell? [KCET 2014]

(a) $\simeq 1.5$ V (b) $\simeq 1.67$ V (c) $\simeq 1.47$ V (d) $\simeq 1.37$ V

47. Potentiometer measures the potential difference more accurately than a voltmeter because [UK PMT 2014]

(a) it does not draw current from external circuit
(b) it draws a heavy current from external circuit
(c) it has a wire of high resistance
(d) it has a wire of low resistance

48. In a potentiometer experiment, the balancing with a cell is at length 240 cm. On shunting the cell with a resistance of $2\,\Omega$, the balancing becomes 120 cm. The internal resistance of the cell is [UK PMT 2014]

(a) $1\,\Omega$ (b) $0.5\,\Omega$ (c) $4\,\Omega$ (d) $2\,\Omega$

49. A galvanometer has a coil of resistance $100\,\Omega$ and gives full scale deflection for 30 mA current. If it is to work as a voltmeter of 30 V, the resistance required to be added is [UK PMT 2014]

(a) $500\,\Omega$ (b) $900\,\Omega$
(c) $1000\,\Omega$ (d) $1800\,\Omega$

50. When 4 A current flows for 2 min in an electroplating experiment, then m gram of silver is deposited. Then the amount (in gram) of silver deposited by 6 A current flowing for 40 s is [NEET 2013]

(a) $4m$ (b) $\frac{m}{2}$ (c) $2m$ (d) $\frac{m}{4}$

51. A wire of resistance $4\,\Omega$ is stretched to twice its original length. The resistance of stretched wire would be [NEET 2013]

(a) $2\,\Omega$ (b) $4\,\Omega$ (c) $8\,\Omega$ (d) $16\,\Omega$

52. The internal resistance of a 2.1 V cell which gives a current of 0.2 A through a resistance of $10\,\Omega$ is [NEET 2013]

(a) $0.2\,\Omega$ (b) $0.5\,\Omega$ (c) $0.8\,\Omega$ (d) $1.0\,\Omega$

53. The resistances of the four arms P, Q, R and S in a Wheatstone's bridge are $10\,\Omega$, $30\,\Omega$, $30\,\Omega$ and $90\,\Omega$, respectively. The emf and internal resistance of the cell are 7V and $5\,\Omega$, respectively. If the galvanometer resistance is $50\,\Omega$, then the current drawn from the cell will be [NEET 2013]

(a) 1.0 A (b) 0.2 A (c) 0.1 A (d) 2.0 A

54. An electron revolves in a circle at the rate of 10^{19} rounds per second. The equivalent current is ($e=1.6\times10^{-19}$ C) [J & K CET 2013]

(a) 1.0 A (b) 1.6 A (c) 2.0 A (d) 2.6 A

55. A silver wire of radius 0.1 cm carries a current of 2A. If the charge density in silver is $5.86\times10^{28}\ m^{-3}$, then the drift velocity is [J & K CET 2013]

(a) $0.2\times10^{-3}\ ms^{-1}$ (b) $0.4\times10^{-4}\ ms^{-1}$
(c) $0.68\times10^{-4}\ ms^{-1}$ (d) $7\times10^{-4}\ ms^{-1}$

56. A 1 m long wire of diameter 0.31 mm has a resistance of $4.2\,\Omega$. If it is replaced by another wire of same material of length 1.5 m and diameter 0.155 mm, then the resistance of wire is [J & K CET 2013]

(a) $25.2\,\Omega$ (b) $0.6\,\Omega$ (c) $26.7\,\Omega$ (d) $0.8\,\Omega$

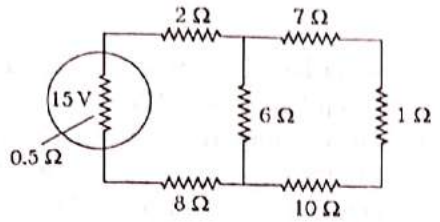
57. 24 cells of emf 1.5 V each having internal resistance of 1 ohm are connected to an external resistance of 1.5 ohms. To get maximum current, [J & K CET 2013]

(a) all cells are connected in series combination
(b) all cells are connected in parallel combination
(c) 4 cells in each row are connected in series and 6 such rows are connected in parallel
(d) 6 cells in each row are connected in series and 4 such rows are connected in parallel

58. The temperature coefficient of the resistance of a wire is 0.00125 per °C. At 300 K its resistance is 1 Ω. The resistance of wire will be 2 Ω at **[J&K CET 2013]**

(a) 1154 K (b) 1100 K
(c) 1400 K (d) 1127 K

59. The emf of a cell E is 15 V as shown in the figure with an internal resistance of 0.5 Ω. Then, the value of the current drawn from the cell is **[EAMCET 2013]**



(a) 3 A (b) 2 A
(c) 5 A (d) 1 A

60. Copper and carbon wires are connected in series and the combined resistor is kept at 0°C. Assuming the combined resistance does not vary with temperature, the ratio of the resistances of carbon and copper wires at 0°C is (Temperature coefficients of resistivity of copper and carbon respectively are 4×10^{-3}/°C and -0.5×10^{-3} /°C) **[EAMCET 2013]**

(a) 4 (b) 8
(c) 6 (d) 2

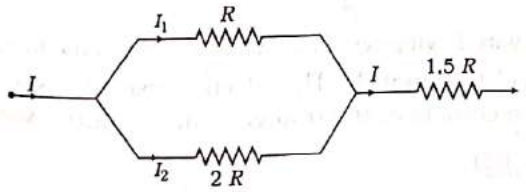
61. Three conductors draw currents of 1 A, 2 A snd 3 A respectively, when connected in turn across a battery. If they are connected in series and the combination is connected across the same battery, then the current drawn will be **[Karnataka CET 2013]**

(a) $\frac{2}{7}$ A (b) $\frac{3}{7}$ A
(c) $\frac{4}{7}$ A (d) $\frac{5}{7}$ A

62. Masses of three wires of copper are in the ratio of 1 : 3 : 5 and their lengths are in the ratio of 5 : 3 : 1 The ratio of their electrical resistances is **[Karnataka CET 2013]**

(a) 1 : 3 : 5 (b) 5 : 3 : 1
(c) 1 : 15 : 125 (d) 125 : 15 : 1

63. In the circuit diagram, heat produces in R, $2R$ and 1.5 R are in the ratio of **[Karnataka CET 2013]**



(a) 4 : 2 : 3 (b) 8 : 4 : 27
(c) 2 : 4 : 3 (d) 27 : 8 : 4

64. Which one of the following electrical meter has the smallest resistance?

(a) Ammeter (b) Milliammeter
(c) Galvanometer (d) Voltmeter
(d) Millivoltmeter

65. Two wires of the same material having equal area of cross-section have length L and $2L$. Their respective resistances are in the ratio **[Kerala CET 2013]**

(a) 2 : 1 (b) 1 : 1
(c) 1 : 2 (d) 1 : 3
(e) 4 : 1

66. Two bulbs 60 W and 100 W designed for voltage 220 V are connected in series across 220 V source. The net power dissipated is **[Kerala CET 2013]**

(a) 80 W (b) 160 W (c) 37.5 W (d) 60 W
(e) 120 W

67. The drift speed of electrons in copper wire of diameter and length l is v. If the potential difference across the wire is doubled, then the new drift speed becomes **[Kerala CET 2013]**

(a) v (b) $2v$ (c) $3v$ (d) $v/2$
(e) $v/4$

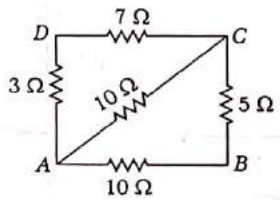
68. A potentiometer wire of length 10 m and resistance 10 Ω per metre is connected in series with a resistance box and a 2 volt battery. If a potential difference of 100 mV is balanced across the whole length of potentiometer wire, then the resistance introduced in the resistance box will be **[MP PMT (2013)]**

(a) 1900 Ω (b) 900 Ω
(c) 190 Ω (d) 90 Ω

69. If a wire is stretched to four times its length, then the specific resistance of the wire will **[MP PMT 2013]**

(a) become 4 times (b) become 1/4 times
(c) become 16 times (d) remain the same

70. For the circuit shown in figure given below, the equivalent resistance between points A and B is **[MPPMT 2013]**

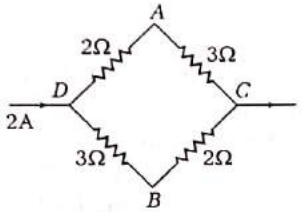


(a) 10 Ω (b) 5 Ω
(c) $\frac{10}{3}$ Ω (d) 2 Ω

71. Two resistors of 6Ω and 9Ω are connected in series to a 120 V source. The power consumed by 6Ω resistor is **[MP PMT 2013]**

(a) 384 W (b) 616 W
(c) 1500 W (d) 1800 W

72. A current of 2A flows in the arrangement of conductors as shown in below figure. The potential difference between points A and $B(V_A - V_B)$ will be **[UP CPMT 2013]**



(a) +1 V (b) −1 V
(c) +2 V (d) −2 V

73. A cell of emf E and internal resistance r supplies current for the same time t through external resistance R_1 and R_2 separately. If the heat developed in both the cases is the same, then the internal resistance r will be **[UP CPMT 2013]**

(a) $r = \sqrt{R_1 + R_2}$ (b) $r = \sqrt{R_1 \times R_2}$
(c) $r = \frac{R_1 + R_2}{2}$ (d) $r = \frac{1}{R_1} + \frac{1}{R_2}$

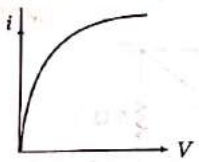
74. When a current of (2.5 ± 0.5) A flows through a wire, it develops a potential difference of (20 ± 1) V, then the resistance of wire is **[UP CPMT 2012]**

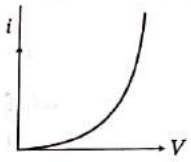
(a) $(8 \pm 2)\Omega$ (b) $(8 \pm 1.6)\Omega$
(c) $(8 \pm 1.5)\Omega$ (d) $(8 \pm 3)\Omega$

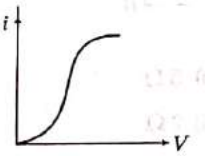
75. To draw the maximum current from a combination of cells, how should be the cells be grouped? **[UP CPMT 2012]**

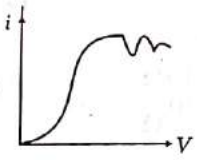
(a) Parallel
(b) Series
(c) Depends upon the relative values of internal and external resistance
(d) Mixed grouping

76. The variation between V-i has shown by graph for heating filament **[UP CPMT 2012]**

(a)


(b)


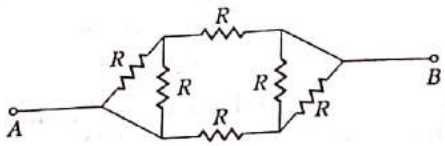
(c)


(d)


77. Two bulbs when connected in parallel to a source take 60 W each, the power consumed, when they are connected in series with the same source is **[UP CPMT 2012]**

(a) 15 W (b) 30 W (c) 60 W (d) 120 W

78. The equivalent resistance between A and B of network shown in figure is **[UP CPMT 2013]**



(a) $\frac{3R}{4}$ (b) $\frac{4R}{3}$ (c) $6R$ (d) $2R$

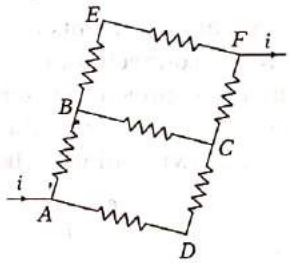
79. A galvanometer having resistance of 50 Ω requires a current of 100 μA to give full scale deflection. How much resistance is required to convert it into an ammeter of range of 10 A? **[BCECE (Mains) 2012]**

(a) $5 \times 10^{-3}\Omega$ in series (b) $5 \times 10^{-4}\ \Omega$ in parallel
(c) $10^5\Omega$ in series (d) $10^5\Omega$ in parallel

80. Two cells when connected in series are balanced on 8 m on a potentiometer. If the cells are connected with polarities of one of the cells is reversed, then they balance on 2 m. The ratio of emfs of two cells is **[BCECE (Mains) 2012]**

(a) 3 : 4 (b) 4 : 3
(c) 3 : 5 (d) 5 : 3

81. In the given circuit diagram if each resistance is of 10 Ω, then the current in arm AD will be **[BCECE (Mains) 2012]**



(a) $\frac{i}{5}$ (b) $\frac{2i}{5}$
(c) $\frac{3i}{5}$ (d) $\frac{4i}{5}$

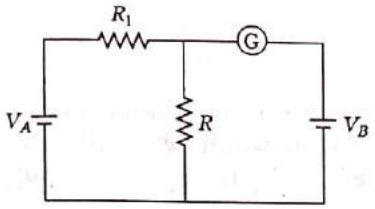
82. When current i is flowing through a conductor, the drift velocity is v. If the value of current through the conductor and its area of cross-section is doubled, then new drift velocity will be **[BCECE Mains 2012]**

(a) $4v$ (b) $\frac{v}{2}$ (c) $\frac{v}{4}$ (d) v

83. A wire having resistance 12 Ω is bent in the form of an equilateral triangle. The effective resistance between any two corners of the triangle will be **[BCECE Mains 2012]**

(a) 6 Ω (b) $\frac{8}{3}\Omega$
(c) 9 Ω (d) 12 Ω

84. In the circuit shown, the cells A and B have negligible resistances. For $V_A = 12$ V, $R_1 = 500\ \Omega$ and $R = 100\ \Omega$ the galvanometer (G) shows no deflection. The value of V_B is **[CBSE AIPMT 2012]**



(a) 4 V (b) 2 V (c) 12 V (d) 6 V

85. A millivoltmeter of 25 mV range is to be converted into an ammeter of 25 A range. The value (in ohm) of necessary shunt will be **[CBSE AIPMT 2012]**

(a) 0.001 (b) 0.01 (c) 1 (d) 0.05

86. If voltage across a bulb rated 220 V-100 W drops by 2.5% of its rated value, then the percentage of the rated value by which the power would decrease is **[CBSE AIPMT 2012]**

(a) 20% (b) 2.5% (c) 5% (d) 10%

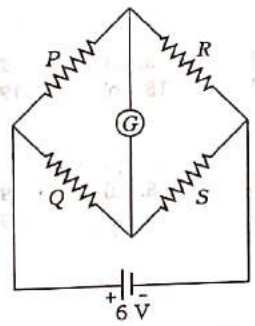
87. $6\ \Omega$ and $12\ \Omega$ resistors are connected in parallel. This combination is connected in series with a 10 V battery and $6\ \Omega$ resistor. What is the potential difference between the terminals of the $12\ \Omega$ resistor? **[AIIMS 2012]**

(a) 4 V (b) 16 V (c) 2 (d) 8 V

88. A voltmeter of range 2 V and resistance $300\ \Omega$ cannot be converted into ammeter of range **[Manipal 2012]**

(a) 1 A (b) 1 mA (c) 100 mA (d) 10 mA

89. In the Wheatstone network given, $P = 10\ \Omega$, $Q = 20\ \Omega$, $R = 15\ \Omega$, $S = 30\ \Omega$, the current passing through the battery (of negligible internal resistance) is **[Manipal 2012]**

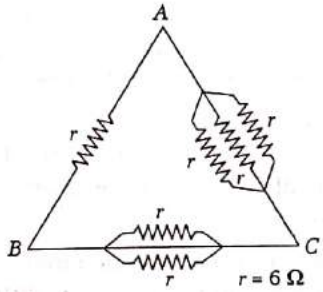


(a) 0.36 A (b) zero (c) 0.18 A (d) 0.72 A

90. A current of 5 A is passing through a metallic wire of cross-sectional area 4×10^{-6} m^2. If the density of charge carriers of the wire is 5×10^{26} m^{-3}, the drift velocity of the electrons will be **[Manipal 2012]**

(a) 1×10^2 ms^{-1} (b) 1.56×10^{-2} ms^{-1}
(c) 1.56×10^{-3} ms^{-1} (d) 1×10^{-2} ms^{-1}

91. Six resistances each of value $r = 5\ \Omega$ are connected between points A, B and C as shown in the figure. If R_1, R_2 and R_3 are the net resistance between A and B, between B and C and between A and C respectively, then $R_1 : R_2 : R_3$ will be equal to **[AMU 2012]**

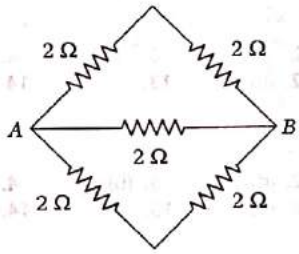


(a) 6 : 3 : 2 (b) 1 : 2 : 3
(c) 5 : 4 : 3 (d) 4 : 3 : 2

92. Charge passing through a conductor of cross-section area $A = 0.3$ m^2 is given by $q = 3t^2 + 5t + 2$ in coulomb, where t is in second. What is the value of drift velocity at $t = 2$s ? (Given, $n = 2 \times 10^{25}$/m^3) **[AIIMS 2012]**

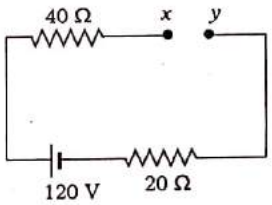
(a) 0.77×10^{-5}m/s
(b) 1.77×10^{-5}m/s
(c) 2.08×10^{5}m/s
(d) 0.57×10^{5}m/s

93. Each resistance shown in figure is $2\ \Omega$. The equivalent resistance between A and B is **[AFMC 2012]**



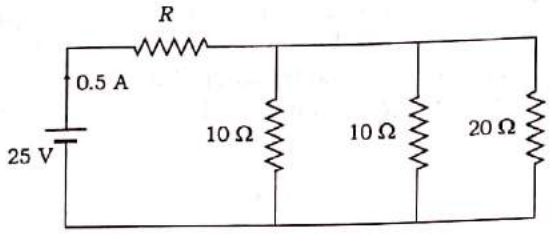
(a) $2\ \Omega$ (b) $1\ \Omega$
(c) $4\ \Omega$ (d) $5\ \Omega$

94. In the circuit shown, the potential difference between x and y will be **[JCECE 2012]**



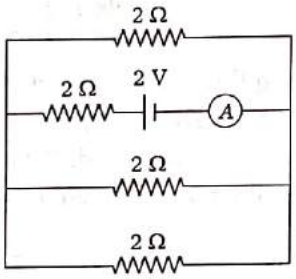
(a) zero (b) 120 V
(c) 60 V (d) 20 V

95. For the circuit shown in figure, **[JCECE 2012]**



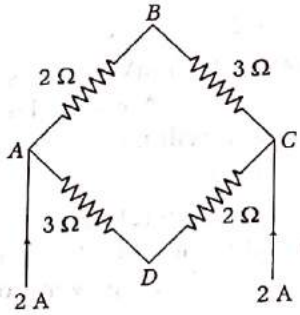
(a) resistance $R = 46\Omega$
(b) current through 20Ω resistance is 0.1A
(c) potential difference across the middle resistance is 2 V
(d) All of the above are true

96. An ammeter connected in the circuit as shown in figure shows a reading of **[BHU Screening 2012]**



(a) $\frac{1}{2}$ A
(b) $\frac{3}{4}$ A
(c) $\frac{1}{8}$ A
(d) 2 A

97. If a current of 2 A flows through resistances connected as shown in figure, the potential difference $V_A - V_B$ is **[BHU Screening 2012]**



(a) −1 V
(b) +1 V
(c) −2 V
(d) +2 V

Answers

Check point 3.1

1. (a)	2. (a)	3. (c)	4. (b)	5. (a)	6. (c)	7. (b)	8. (b)	9. (b)	10. (a)

Check point 3.2

1. (b)	2. (a)	3. (a)	4. (d)	5. (c)	6. (c)	7. (b)	8. (c)	9. (b)	10. (b)
11. (d)	12. (b)	13. (a)	14. (b)	15. (d)	16. (d)	17. (d)	18. (a)	19. (d)	20. (a)

Check point 3.3

1. (c)	2. (d)	3. (b)	4. (c)	5. (b)	6. (a)	7. (c)	8. (a)	9. (b)	10. (a)
11. (b)	12. (d)	13. (b)	14. (b)	15. (c)	16. (a)	17. (c)	18. (b)	19. (c)	20. (a)

Check point 3.4

1. (d)	2. (d)	3. (a)	4. (b)	5. (c)	6. (d)	7. (d)	8. (d)	9. (d)	10. (c)
11. (b)	12. (b)	13. (c)	14. (b)	15. (c)	16. (c)	17. (a)	18. (c)	19. (b)	20. (a)

A. Taking it together

1. (c)	2. (c)	3. (a)	4. (a)	5. (d)	6. (d)	7. (c)	8. (b)	9. (c)	10. (c)
11. (b)	12. (b)	13. (b)	14. (b)	15. (a)	16. (c)	17. (d)	18. (c)	19. (c)	20. (b)
21. (a)	22. (c)	23. (a)	24. (d)	25. (a)	26. (a)	27. (b)	28. (a)	29. (a)	30. (b)
31. (c)	32. (a)	33. (a)	34. (b)	35. (d)	36. (b)	37. (d)	38. (d)	39. (c)	40. (d)
41. (a)	42. (a)	43. (b)	44. (a)	45. (d)	46. (a)	47. (d)	48. (b)	49. (c)	50. (b)
51. (d)	52. (d)	53. (c)	54. (c)	55. (c)	56. (b)	57. (c)	58. (b)	59. (d)	60. (c)
61. (c)	62. (a)	63. (c)	64. (b)	65. (c)	66. (c)	67. (a)	68. (d)	69. (b)	70. (c)
71. (b)	72. (b)	73. (c)	74. (d)	75. (d)	76. (c)	77. (c)	78. (d)	79. (c)	80. (b)
81. (a)	82. (b)	83. (d)	84. (d)	85. (d)	86. (b,c,d)	87. (a)	88. (b)	89. (c)	90. (b)
91. (c)	92. (b)	93. (d)	94. (b)	95. (a)	96. (c)	97. (b)	98. (c)	99. (b)	100. (a)
101. (d)	102. (d)	103. (d)	104. (c)	105. (b)	106. (d)	107. (d)	108. (a)	109. (a)	110. (b)
111. (a)	112. (c)	113. (a)	114. (e)	115. (b)	116. (d)	117. (b)	118. (c)	119. (a)	120. (a)
121. (b)	122. (c)	123. (a)	124. (b)	125. (b)	126. (b)	127. (a)	128. (c)	129. (d)	130. (b)
131. (c)	132. (b)	133. (b)	134. (c)	135. (b)	136. (b)	137. (b)	138. (d)	139. (b)	140. (a)
141. (c)	142. (b)								

B. Medical entrance special format questions

Assertion and reason

1. (c)	2. (d)	3. (d)	4. (a)	5. (b)	6. (e)	7. (d)	8. (b)	9. (b)	10. (d)
11. (e)	12. (e)	13. (c)	14. (c)	15. (d)	16. (a)	17. (a)	18. (a)		

Match the columns

1. (A → q; B → r; C → s; D → p)
2. (A → p; B → r; C → q; D → p)
3. (A → p; B → q; C → q; D → q)
4. (A → r; B → p; C → p; D → s)

C. Medical entrances' gallery

1. (c)	2. (d)	3. (d)	4. (c)	5. (a)	6. (a)	7. (b)	8. (c)	9. (a)	10. (a)
11. (b)	12. (a)	13. (e)	14. (b)	15. (c)	16. (a)	17. (a)	18. (a)	19. (b)	20. (c)
21. (a)	22. (c)	23. (c)	24. (c)	25. (d)	26. (a)	27. (b)	28. (b)	29. (d)	30. (b)
31. (a)	32. (b)	33. (c)	34. (c)	35. (c)	36. (d)	37. (a)	38. (b)	39. (d)	40. (b)
41. (c)	42. (a)	43. (a)	44. (b)	45. (b)	46. (b)	47. (a)	48. (d)	49. (b)	50. (b)
51. (d)	52. (b)	53. (b)	54. (b)	55. (c)	56. (a)	57. (d)	58. (d)	59. (d)	60. (b)
61. (*)	62. (d)	63. (b)	64. (a)	65. (c)	66. (c)	67. (b)	68. (c)	69. (d)	70. (c)
71. (a)	72. (a)	73. (b)	74. (a)	75. (c)	76. (a)	77. (b)	78. (c)	79. (b)	80. (d)
81. (b)	82. (d)	83. (b)	84. (b)	85. (a)	86. (c)	87. (a)	88. (b)	89. (a)	90. (b)
91. (c)	92. (b)	93. (a)	94. (b)	95. (d)	96. (b)	97. (d)			

Hints and explanations

Check point 3.1

1. (*a*) Current, $I = \frac{dq}{dt} = 3t^2 + 2t + 5$

$$q = \int_{t=0}^{t=2} (3t^2 + 2t + 5)\, dt$$

Charge, $q = [t^3 + t^2 + 5t]_0^2 = 22\text{ C}$

2. (*a*) Current, $i = \frac{q}{t}$

or $\quad i = \frac{ne}{t} \Rightarrow 16 \times 10^{-3} = \frac{n \times 1.6 \times 10^{-19}}{1}$

$\therefore$ Number of electrons, $n = 10^{17}$

3. (*c*) We have current, $i = \frac{ne}{t}$

$\therefore$ Number of electrons, $n = \frac{it}{e} = \frac{0.2 \times 30}{1.6 \times 10^{-19}} = 3.75 \times 10^{19}$

4. (*b*) Let the number of electrons $= N$

$$I = 4 - 0.08\,t$$

$$\Rightarrow \quad \frac{dq}{dt} = 4 - 0.08\,t$$

$$\Rightarrow \quad q = \int_0^{50} (4 - 0.08t)\, dt$$

$$\Rightarrow \quad Ne = \left[4t - \frac{0.08t^2}{2}\right]_0^{50} = 100\text{ C}$$

$$\Rightarrow \quad N = \frac{100}{e} = \frac{100}{1.6 \times 10^{-19}}$$

$$= 6.25 \times 10^{20}$$

5. (*a*) Drift velocity, $v_d = \frac{e}{m} \times \frac{V}{l}\tau$ or $v_d = \frac{e}{m} \cdot \frac{El}{l}\tau \quad [\because V = El]$

$\therefore \quad v_d \propto E$

6. (*c*) Drift velocity, $v_d = \frac{J}{ne} \Rightarrow v_d \propto J$

$\Rightarrow \quad J_1 = \frac{i}{A}$ and $J_2 = \frac{2i}{2A} = \frac{i}{A} = J_1$

$\therefore \quad (v_{d_1}) = (v_{d_2}) = v$

8. (*b*) The order of drift velocity is $10^{-4}\text{ ms}^{-1} = 10^{-2}\text{ cms}^{-1}$

10. (*a*) Drift velocity,

$$v_d = \frac{i}{neA} = \frac{3}{8.5 \times 10^{28} \times 1.6 \times 10^{-19} \times 2 \times 10^{-6}}$$

$$= 0.11 \times 10^{-3}\text{ ms}^{-1}$$

Time $(t) = \frac{\text{length of the wire}}{v_d} = \frac{3}{0.11 \times 10^{-3}} = 2.73 \times 10^4\text{ s}$

Check point 3.2

1. (*b*) We have, $i = \frac{V}{R} = \frac{q}{t}$

$\therefore$ Charge, $q = \frac{Vt}{R} = \frac{20 \times 2 \times 60}{10} = 240\text{ C}$

2. (*a*) Resistance, $R = \rho\frac{l}{A}$ and resistivity, $\rho = \frac{m}{ne^2\tau}$

$\therefore \quad R = \frac{ml}{ne^2\tau A}$

3. (*a*) $R = \rho\frac{l}{A}$

Since, wires are of same material, so resistivity ρ of all wire will be same.

Therefore, $R \propto \frac{l}{A} \Rightarrow R \propto \frac{l}{\pi r^2}$

$$R \propto \frac{l}{\left(\frac{d}{2}\right)^2}$$

For length = 50 cm and diameter = 0.5 mm

Resistance will be maximum.

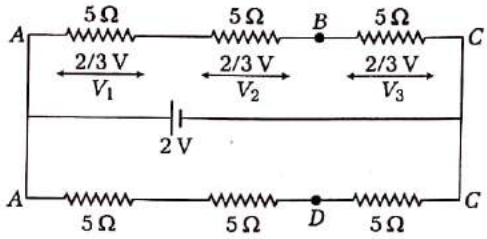
4. (*A*)

A	B	C	D
Brown	Red	Green	Gold
↓	↓	↓	↓
1	2	10^5	5%

$R = AB \times C \pm D$

$R = 12 \times 10^5 \pm 5\%$

5. (*c*) Given circuit can be redrawn as follows



$\therefore$ Potential difference across AB is $\frac{2}{3} + \frac{2}{3} = \frac{4}{3}$ V

6. (*c*) The given circuit can be reduced to

Equivalent resistance between A and B,

$$R_{AB} = 2 + 4 = 6\,\Omega$$

$\therefore \quad V_A - V_B = (6)(0.5) = 3\text{ V}$

7. (*b*) Potential difference across the circuit

$$= i\left(\frac{R_1 R_2}{R_1 + R_2}\right) = 1.2 \times \left(\frac{6 \times 4}{6 + 4}\right) = 2.88\,\text{V}$$

So, current through 6Ω resistance $= \frac{2.88}{6} = 0.48\text{A}$

Alternative solution

From current division law,

$$i_1 = \left(\frac{R_2}{R_1 + R_2}\right) i$$

$$= 1.2\left(\frac{4}{6+4}\right) = 1.2\left(\frac{4}{10}\right) = 0.48\,\text{A}$$

8. (*c*) Current divides according to resistance, so current in 6Ω resistance is $\frac{0.8}{2} = 0.4\,\text{A}$

So, total current in circuit is $0.8 + 0.4 = 1.2\,\text{A}$

$\therefore$ Potential drop across $4\Omega = 1.2 \times 4 = 4.8\,\text{V}$

9. (*b*) Two resistances are short circuited.

So, only third circuit will be considered and hence $i = V/R = 10/3\,\text{A}$

10. (*b*) Net resistance, R_N is

$$R_N = \frac{10 \times 15}{10 + 15} + 3 + 0.5 = 9.5$$

$$\therefore \quad I = \frac{V}{R_N} = \frac{20}{9.5} = 2.1\text{A}$$

Current through $10\,\Omega$ resistance is

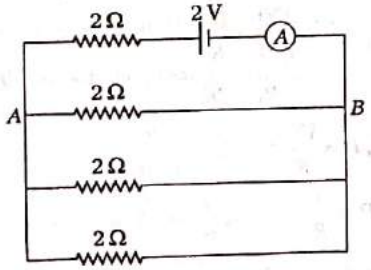
$$I_1 = \left(\frac{15}{10 + 15}\right) \times 2.1 = 1.2\,\text{A}$$

11. (*d*) Resistance, $R_{AB} = \frac{R}{3} + R$

$$= \frac{2}{3} + 2$$

$$R_{AB} = \frac{8}{3} = 2\frac{2}{3}\Omega$$

12. (*b*) The circuit can be redrawn as follows



Resistance across AB,

$$R_{AB} = \frac{2}{1 + 1 + 1} = \frac{2}{3}\Omega$$

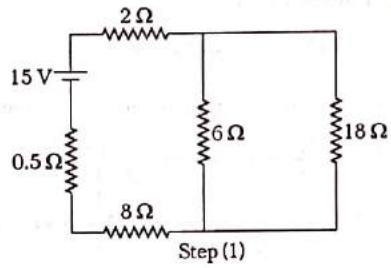
Total resistance of circuit,

$$R_T = 2 + \frac{2}{3} = \frac{8}{3}\Omega$$

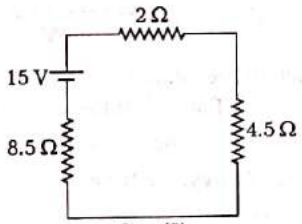
Current through ammeter,

$$i = \frac{2}{(8/3)} = \frac{6}{8} = \frac{3}{4}\,\text{A}$$

13. (*a*) The circuit can be reduced as follows



Step (1)



Step (2)

So, equivalent resistance across battery,

$$R_{eq} = 8.5 + 2 + 4.5 = 15\Omega$$

Hence, current from the battery

$$i = \frac{15}{15} = 1\text{A}$$

14. (*b*) As B is connected to the earth, so potential of B is $V_B = 0$.

Now, current in the given circuit,

$$i = \frac{50}{5 + 7 + 10 + 3} = 2\,\text{A}$$

Potential difference between A and B is

$$V_A - V_B = 2 \times 12 \quad \text{or} \quad V_A - 0 = 24$$

$$V_A = 24\,\text{V}$$

15. (*d*) It is an open circuit and for open circuit, terminal potential difference = emf of cell, *i.e.*, $V = E$

16. (*d*) We have

$$V = E - ir = 1.5 - 2 \times 0.15$$

$$V = 1.20\,\text{V}$$

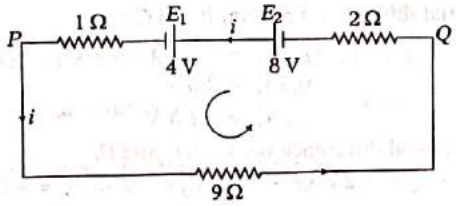
17. (*d*) Potential difference between A and B is given by

$$V_A - V_B = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}$$

$$\therefore \quad 4 = \frac{5X + 2 \times 10}{X + 10}$$

$$\therefore \quad X = 20\Omega$$

18. (*a*)



$$E_1 = 4\text{ V} \quad \text{and} \quad E_2 = 8\text{ V}$$

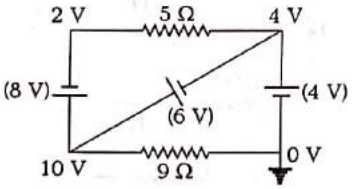
So, $E_2 > E_1$ current flows from Q to P.

$\therefore \quad i = \frac{1}{3}\text{ A}$

$\therefore$ Potential difference across

$$PQ = \frac{1}{3} \times 9 = 3\text{ V}$$

19. (*d*) The potentials of different points are as shown in figure,



Current through 5Ω resistance

$$= \frac{\text{Potential difference}}{\text{Resistance}} = \frac{(4-2)}{5} = 0.4\text{ A}$$

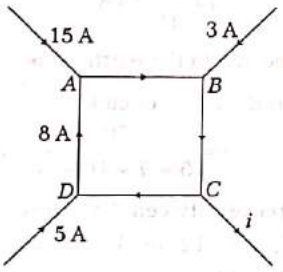
20. (*a*) For maximum energy, we have

External resistance of the circuit

= Equivalent internal resistance of the circuit, *i.e.*, $R = \frac{r}{2}$

Check point 3.3

1. (*c*)



Applying Kirchhoff's first law at junction A, B, C, D

At A, $\quad i_{AB} = 15 + 8 = 23\text{A}$

At B, $\quad i_{BC} = 23 + 3 = 26\text{A}$

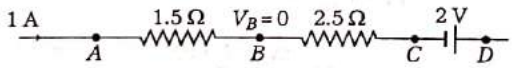
At D, $\quad i_{CD} = 8\text{A} - 5\text{A} = 3\text{A}$

At C, $\quad i_{CD} + i = i_{BC}$

or $\quad 3 + i = 26$

$\therefore \quad i = 23\text{ A}$

2. (*d*)



Potential difference between A and B,

$$V_A - V_B = 1 \times 1.5 \text{ or } V_A - 0 = 1.5$$

$\therefore \quad V_A = 1.5\text{ V}$

Potential difference between B and C,

$$V_B - V_C = 1 \times 2.5 = 2.5\text{ V}$$

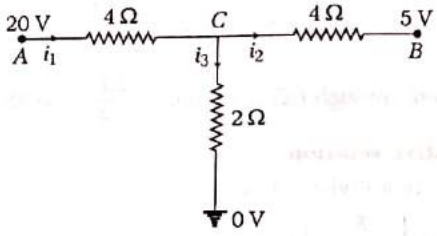
$\therefore \quad 0 - V_C = 2.5\text{ V}$

$\therefore \quad V_C = -2.5\text{ V}$

So, potential difference between C and D,

$$V_C - V_D = -2\text{ V or } -2.5 - V_D = -2 \text{ or } V_D = -0.5\text{ V}$$

3. (*b*) Let at junction C, potential is V



$$\frac{V_A - V_C}{2} + \frac{V_B - V_C}{4} = \frac{V_C - V}{2}$$

$$\frac{20 - V}{2} + \frac{5 - V}{4} = \frac{V - 0}{2}$$

$$V = 9\text{V}$$

Current, $\quad i = \frac{9}{2} = 4.5\text{A}$

4. (*c*) Equal current (of 1 A) will be distributed in the two branches.

$$V_D - V_A = 2 \times 1 = 2\text{ V}$$

and $\quad V_D - V_B = 3 \times 1 = 3\text{ V}$

$\therefore \quad V_A - V_B = +1\text{V}$

5. (*b*) Work done, $W = qV = 6 \times 10^{-6} \times 9 = 54 \times 10^{-6}$ J

6. (*a*) In series i is same, $H = i^2Rt$. Therefore, $H \propto R$.

7. (*c*) Resistance, $R = \frac{V^2}{P} \Rightarrow \frac{25 \times 25}{25} = 25\,\Omega$

8. (*a*) Resistance, $R = \frac{V^2}{P}$ or $R \propto V^2$

$$\Rightarrow \frac{R_1}{R_2} = \frac{V_1^2}{V_2^2} \Rightarrow \frac{R}{R_2} = 4 \Rightarrow R_2 = \frac{R}{4}$$

9. (*b*) Power, $P = \frac{V^2}{R}$

$\therefore \quad P \propto \frac{1}{R}$

So, $\quad \frac{P_1}{P_2} = \frac{R_2}{R_1}$

$$\frac{200}{100} = \frac{R_2}{R_1}$$

$$R_2 = 2R_1$$

10. (*a*) Power, $P = \frac{V^2}{R}$

$\therefore \quad P \propto \frac{1}{R}$

Also, resistance of wire $\propto \frac{1}{(\text{radius of filament})^2}$

$\therefore \quad P \propto (\text{radius of filament})^2$

So, 100 W bulb has thicker filament.

12. (*d*) $H = \frac{V^2t}{R \times J}\text{ cal} = \frac{Pt}{J} = \frac{210 \times 5 \times 60}{4.2} = 15000\text{ cal}$

13. (*b*) $P = \frac{V^2}{R} \Rightarrow 150 = \frac{(15)^2}{\left(\frac{2R}{2+R}\right)}$

$\therefore \quad \frac{2+R}{2R} = \frac{2}{3}$

$\therefore$ Resistance, $R = 6\Omega$

14. (*b*) $R' = \frac{R}{4}$ and $P = \frac{V^2}{R}$

$\therefore \quad P' = 4P$

15. (*c*) $R_{40} > R_{100}$. In series, potential difference distributes in direct ratio of resistance.

16. (*a*) $1 \text{ kWh} = 1000 \times 60 \times 60 = 36 \times 10^5 \text{ W-s} = 36 \times 10^5 \text{ J}$

17. (*c*) When bulbs are in series, $P = \frac{V^2}{3R}$ ...(i)

When bulbs are connected in parallel,

$$P' = \frac{V^2}{(R/3)} = \frac{3V^2}{R} = 3 \times 3P \quad \text{[from Eq. (i)]}$$

$$= 9P$$

18. (*b*) In parallel, V is same, $P = \frac{V^2}{R}$ or $P \propto \frac{1}{R}$

19. (*c*) When each bulb is glowing at full power.

Current from each bulb, $i = \frac{50}{100} = \frac{1}{2}$ A

So, main current, $i = \frac{n}{2}$ A

Also, $E = V + ir$

$$120 = 100 + \left(\frac{n}{2}\right) \times 10$$

$$n = 4$$

20. (*a*) $P = \frac{V^2}{R}$, R_{net} will decrease. Therefore, P will increase.

Check point 3.4

1. (*d*) Resistance is parallel with voltmeter, therefore it is zero.

4. (*b*) If voltmeter is ideal, then R should be $\frac{1.6}{0.4} = 4\Omega$. If it is non-ideal R should be greater than 4Ω.

5. (*c*) Total resistance of given circuit

$$= \frac{80}{1+1} + 20 = 40 + 20 = 60\Omega$$

$\therefore$ Main current, $i = \frac{2}{60} = \frac{1}{30}$ A

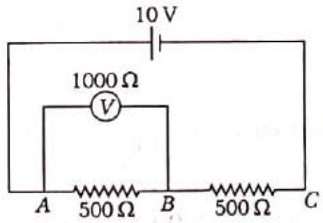
Now, in parallel, there are two resistances of 80Ω each. (one of voltmeter and other 80Ω resistance) So, current is equally distributed in 80Ω resistance and voltmeter, So, $\frac{1}{60}$ A.

Current flows through each.

$\therefore$ Potential difference across 80Ω resistance

$$= \frac{1}{60} \times 80 = 1.33 \text{ V}$$

6. (*d*)



Equivalent resistance of circuit,

$$R_{eq} = 500 + \frac{1000}{3} = \frac{2500}{3}$$

$\therefore$ Current drawn from the cell,

$$i = \frac{10}{2500/3} = \frac{3}{250} \text{ A}$$

Reading of voltmeter $= \frac{3}{250} \times \frac{1000}{3} = 4$ V

7. (*d*) $V = IR$

$$100 = (10 \times 10^{-3})(25 + R)$$

$\therefore$ Resistance, $R = 9975\Omega$

8. (*d*) When same resistance is connected in parallel with voltmeter the effective resistance get decreased.

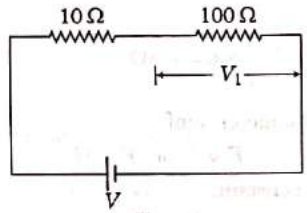
10. (*c*) We have, $i_g G = (i - i_g)S$

$$10 \times 99 = (90)S$$

$$S = \frac{10 \times 99}{90}$$

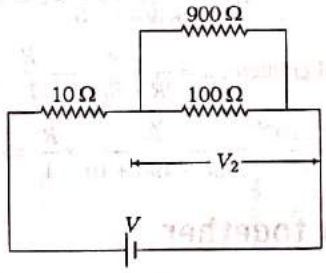
Shunt, $S = 11\Omega$

11. (*c*)



Before connecting the voltmeter, let the potential difference across 100Ω is V_1.

$$\therefore \quad V_1 = \left(\frac{100}{100+10}\right) \times V = \frac{10}{11} V$$



After connecting the voltmeter across 100Ω.

Equivalent resistance $= \frac{100 \times 900}{100 + 900} = 90\Omega$

Let this time potential difference is V_2.

$$\therefore \quad V_2 = \left(\frac{90}{90+10}\right)V = \frac{9}{10}V$$

$$\text{Magnitude of \% error} = \frac{\frac{10}{11}V - \frac{9}{10}V}{\frac{10}{11}V} \times 100 = 1.0$$

12. (*b*) (a) $50 \neq (50 \times 10^{-6})(100 + 10^4)$. Therefore, (a) is wrong.

(b) $10 \approx (50 \times 10^{-6})(100 + 200 \times 10^3)$. Therefore, (b) is correct.

(c) $\frac{50 \times 10^{-6}}{10 \times 10^{-3} - 50 \times 10^{-6}} \neq \frac{1}{100}$. Therefore, (c) is wrong.

14. (*b*) $(i_g RV)(100) = i_g (R_V + 1980)$

$\therefore$ Resistance, $R = 20\,\Omega$

15. (*c*) $\frac{R_3}{R_4} = \frac{R_1}{R_2}$ (balanced Wheatstone bridge)

or $R_1 R_4 = R_2 R_3$

16. (*c*) The galvanometer shows no current, it means this is a balanced Wheatstone bridge, So,

$$\frac{R}{80} = \frac{AC}{BC} = \frac{20}{80}$$

$\therefore$ Resistance, $R = 20\,\Omega$

17. (*a*) In potentiometer, the ratio of emfs is equal to the ratio of no deflection lengths.

$$\frac{E_1}{E_2} = \frac{l_1}{l_2} = \frac{2}{3}$$

18. (*c*) $r = \frac{(l_1 - l_2)}{l_2} \times R$

$$= \left(\frac{60-50}{50}\right) \times 6 = 1.2\,\Omega$$

19. (*b*) For potentiometer, emf

$$E \propto l \text{ or } E = kl$$

where, k is a constant.

Also, $E = \frac{V}{l} = \frac{iR}{L} \times l$

$$\therefore \quad E = \frac{E'}{(R_1 + R_2 + r)} \times \frac{R}{L} \times l$$

$$\therefore \quad E = \frac{10}{5+4+1} \times \frac{5}{5} \times 3 = 3\text{ V}$$

20. (*a*) Potential gradient, $x = \frac{e}{(R + R_h + r)} \cdot \frac{R}{L}$

$$\Rightarrow \quad \frac{0.2 \times 10^{-3}}{10^{-2}} = \frac{2}{(R + 490 + 0)} \times \frac{R}{1} \Rightarrow R = 4.9\,\Omega$$

A) Taking it together

1. (*c*) During the charging of battery terminal, potential difference is always greater than emf of circuit.

$$V = E + ir$$

3. (*a*) Although current is denoted by arrow but it is a scalar quantity, and power is also scalar quantity, while current density (J) is a vector quantity.

4. (*a*) Current density relates with electric field as

$$J = \sigma E = \frac{E}{\rho} i \Rightarrow J \propto E$$

where, σ = conductivity and ρ = resistivity or specific resistance of substance.

6. (*d*) Potentiometer works on null deflection method. In balance condition, no current flow in secondary circuit.

7. (*c*) We cannot increase the rating of fuse wire of lower value just by increasing its length. For it, we shall have to make fuse wire thick.

8. (*b*) Specific resistance of silver, copper and aluminium are $1.6 \times 10^{-8}\,\Omega$-m, $1.7 \times 10^{-8}\,\Omega$-m and $2.7 \times 10^{8}\,\Omega$-m, respectively.

Thus, correct sequence is Al, Cu, Ag.

9. (*c*) Voltmeter has high resistance. So, most of the main current will flow through ammeter which is in parallel. So, it will burn out. No damage will occur to voltmeter.

10. (*c*) $V = E - ir = E - \left(\frac{E}{R+r}\right) r = E\left(\frac{R}{R+r}\right) = \frac{E}{1 + r/R}$

13. (*b*) The minimum resistance can be achieved when we connect all resistances in parallel.

So, equivalent resistance of combination $= \frac{r}{10}$.

14. (*b*) When wire is bent in the form of a circle, then it is equivalent to two resistances in parallel,

$$R_{eq} = \frac{12}{2} = 6\,\Omega$$

15. (*a*) As steady current is flowing through the conductor, hence the number of electrons entering from one end and outgoing from the other end of any segment is equal. Hence, charge will be zero.

16. (*c*) Out of n cells, two cells will cancel out each others emf. So, net emf $= (n-2)E$.

Total resistance $= R + nr$

Current, $i = \frac{(n-2)E}{nr + R}$

17. (*d*) Current, $i = \frac{E}{r + R}$

$$\text{Power, } P = i^2 R \Rightarrow P = \frac{E^2 R}{(r+R)^2}$$

Power will be maximum when $r = R$

$$P_{max} = \frac{E^2}{4r}$$

18. (*c*) Three resistances are in parallel, so effectively the resistance would be equal to

$$\frac{1}{R_{eff}} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} = \frac{3}{R}$$

So, $R_{eff} = \frac{R}{3}$

19. (*c*) The resistivity of a semiconductor decreases with increase in temperature exponentially.

Hence, option (c) is correct.

20. (*b*) Slope of the V-i curve at any point is equal to resistance at that point.

From the curve, slope for T_1 > slope for T_2

$$R_{T_1} < R_{T_2}$$

⇒ Also at higher temperature resistance will be higher, so

$$T_2 > T_1$$

21. (*a*) The relationship between current and drift speed is given by

$$i = neAv_d$$

Here, I is the current and v_d is the drift velocity.

So, $I \propto v_d$

Thus, only drift velocity determines the current in a conductor.

22. (*c*) Current in the circuit will increase because another resistance is connected in parallel to the circuit and hence potential drop across other ammeter will therefore decrease. So, the potential difference over voltmeter will increase because total of potential difference over ammeter and voltmeter is equal to emf (constant).

23. (*a*) If there is a increase in length, then

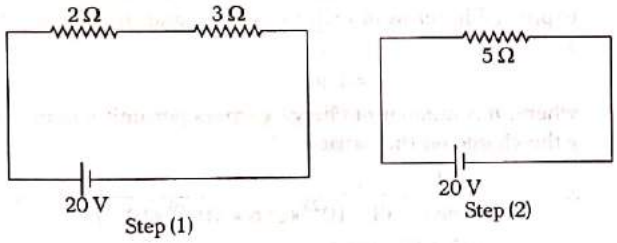
$R = \rho L/A$, let assume the wire is a cylindrical on stretching the wire, the volume of the wire remains same.

So, new resistance $\rho L^2/A'$

$$\Rightarrow \quad \frac{R'}{R} = \left(\frac{l'}{l}\right) = \left(\frac{1.1l}{l}\right)^2 = 1.21$$

$$\therefore \quad R' = 1.21R = 1.21 \times 10 = 12.1\Omega$$

24. (*d*) Circuit can be reduced as follows.



So, equivalent resistance of circuit,

$$R_{eq} = 5\Omega$$

$\therefore$ Current in the circuit $= \dfrac{20}{5} = 4\,\text{A}$

As in parallel, current is divided according to resistance, so, current flowing through each resistance = 2 A.

25. (*a*) Potential difference = 10 V

So, $i = \dfrac{10}{5} = 2\,\text{A}$

26. (*a*) Internal resistance, $r = R\left(\dfrac{E}{V} - 1\right)$

$$= (10)\left(\frac{1.5}{1.25} - 1\right) = 2\Omega$$

27. (*b*) For twisted wire, there are two halves each of resistance 2Ω in parallel.

So, $R_{eq} = \dfrac{2 \times 2}{2 + 2} = 1\Omega$

28. (*a*) When wire is divided in 10 equal parts, then each part will have a resistance $= R/10 = 0.01R = r$.

Let equivalent resistance be r_R, then

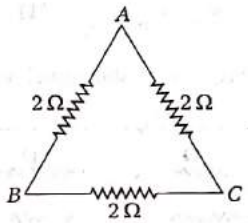
$$\frac{1}{r_R} = \frac{1}{r} + \frac{1}{r} + \frac{1}{r} + \ldots 10 \text{ times}$$

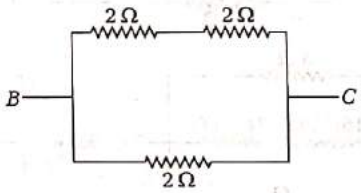
$$\therefore \quad \frac{1}{r_R} = \frac{10}{r} = \frac{10}{(R/10)} = \frac{100}{R}$$

$$\therefore \quad r_R = \frac{R}{100} = 0.01R$$

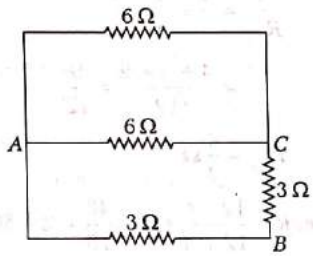
29. (*a*) Equivalent resistance between B and C.

$$R_{BC} = \frac{(2+2) \times 2}{2+2+2} = \frac{8}{6} = \frac{4}{3}\Omega$$

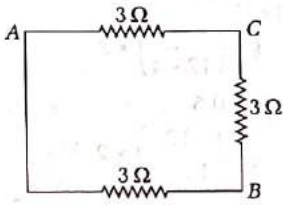




30. (*b*) Given circuit can be reduced to



So, equivalent resistance between points A and B is equal to



$$R_{eq} = \frac{6 \times 3}{6 + 3} = 2\Omega$$

31. (*c*) Given, $\frac{R_1 R_2}{R_1 + R_2} = \frac{6}{8}$ ...(i)

When one resistance is broken say R_2 is broken, then

$$R_1 = 2\Omega \quad ...(ii)$$

From Eqs. (i) and (ii), we get

$$R_2 = \frac{6}{5}\Omega$$

32. (*a*) We have $R = \rho\frac{l}{A}$

$$R \propto l$$

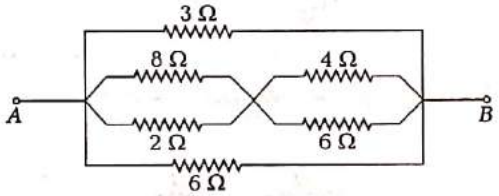
$$\frac{R_1}{R_2} = \frac{l_1}{l_2} = \frac{1}{2}$$

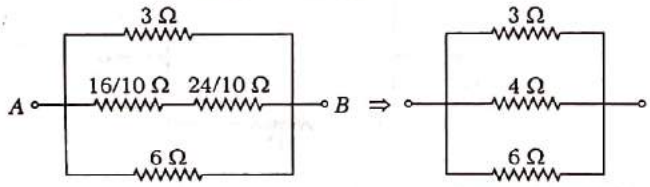
$$\therefore \quad R_1 = \frac{1}{3} \times 9 = 3\Omega$$

$$R_2 = \frac{2}{3} \times 9 = 6\Omega$$

Net resistance, $R_{net} = \frac{R_1 R_2}{R_1 + R_2} = 2\Omega$

33. (*a*) The simple circuit is as shown below





$$\Rightarrow \quad \frac{1}{R_{eq}} = \frac{1}{3} + \frac{1}{4} + \frac{1}{6}$$

$$= \frac{4+3+2}{12} = \frac{9}{12}$$

$$R_{eq} = \frac{4}{3}\Omega$$

34. (*b*) Resistance, $R = \frac{12 \times 4}{12+4} + 2 = \frac{48}{16} + 2 = 5\Omega$

Electric current, $i = \frac{E}{R+r} = \frac{12}{6} = 2$ A

$$i_1 = \left(\frac{4}{12+4}\right) \times 2$$

$$i_1 = 0.5$$

and $i_2 = \left(\frac{12}{12+4}\right) \times 2$

$$i_2 = 1.5$$

$$\therefore \quad i_2 = 1.5 \text{ A}, i_1 = 0.5 \text{ A}$$

35. (*d*) $R_{eff} = \frac{(P+Q)(R+S)}{(P+Q+R+S)} = \frac{(R+R)(2R+2R)}{(R+R+2R+2R)}$

The equivalent resistance of the network between points A and B.

$$R_{eff} = \frac{4}{3}R$$

In this case, equivalent resistance of the network between points A and B is independent of the value of G.

36. (*b*) Let resistance, $R = 100\Omega$

$$\therefore \quad R' = 100 + 100 \times \frac{10}{100} = 110\Omega$$

$\therefore$ Required shunt, $\Delta R = R' - R = 110 - 99 = 11\Omega$

37. (*d*) Given, $V = 115$ V

and $P = 1250$ W

We know, $R = \frac{V^2}{P}$

So, $R = \frac{(115)^2}{1250} = 10.58\ \Omega$

Resistance, $R = 10.6\ \Omega$

38. (*d*) Resistance, $R_t = R_0(1 + \alpha t)$

$$20 = R_0(1 + 20\alpha) \text{ and } 60 = R_0(1 + 500\alpha)$$

From this equation, we can find

$$R_0 = 18.33\Omega$$

$$\alpha = 4.54 \times 10^{-3}$$

$$R_t = 25\Omega$$

Again $R_t = R_0(1 + \alpha t)$

$$25 = 18.33(1 + 4.54 \times 10^{-3} t)$$

We find $t = 80°$ C

39. (*c*) The current i crossing area of cross-section A, can be expressed in terms of drift velocity v_d and the moving charges as

$$i = nev_d A$$

where, n is number of charge carriers per unit volume and e the charge on the carrier.

$$\therefore \quad v_d = \frac{i}{neA} = \frac{24 \times 10^{-3}}{(3 \times 10^{23})(1.6 \times 10^{-19})(10^{-4})}$$

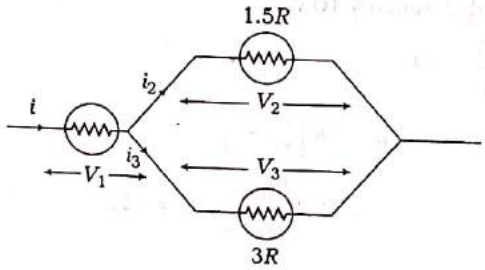
$$= 5 \times 10^{-3} \text{ms}^{-1}$$

40. (*d*) Drift velocity in a copper conductor,

$$v_d = \frac{i}{neA} = \frac{5.4}{8.4 \times 10^{28} \times 1.6 \times 10^{-19} \times 10^{-6}}$$

$$= 0.4 \times 10^{-3} \text{ms}^{-1} = 0.4 \text{ mms}^{-1}$$

41. (*a*) Here, $V_2 = V_3$



i.e., $i_2 \times 15R = 3R \times i_3$

and $i_2 + i_3 = i$

$\Rightarrow$ $i_2 = \frac{2i}{3}$ and $i_3 = \frac{i}{3}$

Now, $V_1 = iR$

$V_2 = \frac{2i}{3} \times 15R = iR$

$V_3 = \frac{i}{3} \times 3R = iR$

i.e., $V_1 = V_2 = V_3$

42. (*a*) The resistance of wire is given by

$$R = \rho \frac{l}{A}$$

For greater value of R, l must be higher and A should be lower and it is possible only when the battery is connected across $1\text{cm} \times \left(\frac{1}{2}\right)$ cm (area of cross-section A).

43. (*b*) In potentiometer experiment, the emf of a cell can be measured, if the potential drop along the potentiometer wire is more than the emf of the cell to be determined. Here value of emfs of two cells are given as 5V and 10V, therefore the potential drop along the potentiometer wire must be more than 10 V.

44. (*a*) Potential gradient $= \frac{V}{L} = \frac{iR}{L} = \frac{i\rho L}{L} = \frac{i\alpha}{A}$

$$= \frac{0.2 \times 40 \times 10^{-8}}{8 \times 10^{-6}} = 10^{-2}\text{V/m}$$

45. (*d*) When potentiometer is connected between A and B, then it measures only E_1 and when connected between A and C, then it measures $E_1 - E_2$.

$\therefore$ $\frac{E_1}{E_1 - E_2} = \frac{l_1}{l_2}$

$\frac{E_1 - E_2}{E_1} = \frac{l_2}{l_1}$

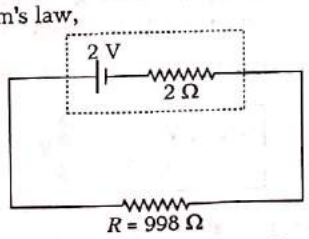
$\Rightarrow$ $1 - \frac{E_2}{E_1} = \frac{100}{300}$

$\Rightarrow$ $\frac{E_2}{E_1} = 1 - \frac{1}{3}$

$\Rightarrow$ $\frac{E_2}{E_1} = \frac{2}{3}$

$\Rightarrow$ $\frac{E_1}{E_2} = \frac{3}{2}$

46. (*a*) From Ohm's law,



$$i = \frac{E}{R + r} = \frac{2}{998 + 2} = 2 \times 10^{-3}\text{A}$$

Potential difference across the voltmeter is

$$V = iR = (2 \times 10^{-3}) \times 998 = 1.996\text{ V}$$

47. (*d*) Resistance, $R = \frac{V}{i} = \frac{2\text{ V}}{4\text{ A}} = 0.5\,\Omega$

Resistance, $R = \frac{\rho l}{A} \Rightarrow 0.5 = \frac{\rho\, 0.5}{1 \times 10^{-6}}$

Resistivity of wire

$$\rho = \frac{0.5 \times 10^{-6}}{0.5} = 10^{-6}\ \Omega\text{-m}$$

48. (*b*) In order to balance the bridge, the effective value of S must be equal to $2\,\Omega$.

So, $2 = \frac{S \times 6}{S + 6}$

$$2S + 12 = 6S \Rightarrow 4S = 12 \Rightarrow S = 3\,\Omega$$

49. (*c*) Current in the potential wire $= \frac{2}{990 + 10} = 2 \times 10^{-3}$ A

Potential drop over wire $= 2 \times 10^{-3} \times 10 = 2 \times 10^{-2}$ $[\because V = IR]$

Potential gradient $= \frac{2 \times 10^{-2}}{2} = 10^{-2}$ V/m $\left[\because k = \frac{V}{l}\right]$

50. (*b*) Current, $i = qf = q\left(\frac{v}{2\pi r}\right) = \frac{1.6 \times 10^{-19}\ (2.2 \times 10^{6})}{(2\pi)\ (5 \times 10^{-11})}$

$$= 1.12 \times 10^{-3}\text{A}$$

51. (*d*) Suppose resistances of wires are R_1 and R_2, then

$$\frac{6}{5} = \frac{R_1 R_2}{R_1 + R_2}$$

If R_2 breaks, then $R_1 = 2\Omega$

Hence, $\frac{6}{5} = \frac{2 \times R_2}{2 + R_2} \Rightarrow R_2 = 3\Omega$

52. (*d*) We have $k \times l = i \times R$

$(2 \times 10^{-3})(50) = 10 \times i$

The current passing through the resistor,

$$i = 10 \times 10^{-3}\text{ A} = 10\text{ mA}$$

53. (*c*) Total cells $= m \times n = 24$ ...(i)

For maximum current in the circuit,

$R = \frac{mr}{n} \Rightarrow 3 = \frac{m}{n} \times (0.5) \Rightarrow m = 6n$ ...(ii)

On solving Eqs. (i) and (ii), we get $m = 12$ and $n = 2$

54. (*c*) Current, $I_V = \frac{V_V}{R_V} = \frac{5}{20 \times 10^3} = 0.25 \times 10^{-3}\text{A}$

$\therefore$ $V = I_V(R + r)$

$110 = 0.25 \times 10^{-3}(R + 20)$

$R = 420\text{ k}\Omega$

55. (*c*) We have $i_1 = \frac{E}{r + R_1} \Rightarrow i_2 = \frac{E}{r + R_2}$

From these two equations, we get

$$r = \frac{i_2 R_2 - i_1 R_1}{i_1 - i_2}$$

56. (*b*) $\frac{1}{R_P} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} = \frac{3}{R}$

$R_P = \frac{R}{3}\Omega \Rightarrow R_S = R + R = 2R\Omega$

$R_{net} = R_P + R_S$

$R_{net} = 2R + \frac{R}{3} = \frac{7R}{3}\Omega$

57. (*c*) $i = \frac{2+2}{1+1.9+0.9} = \frac{4}{3.8}$ A

For cell *A*,

$E = V + ir$

$V = E - ir = 2 - \frac{4}{3.8} \times 1.9$

$V = 0$ (zero)

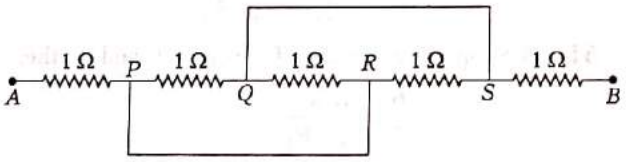
58. (*b*) Internal resistance

$$r = \left(\frac{l_2}{l_1} - 1\right)R = \left(\frac{50}{40} - 1\right) \times 2 = 0.50\Omega$$

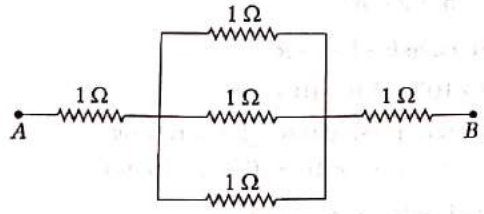
59. (*d*) $E = \frac{e}{(R + R_h + r)} \cdot \frac{R}{L} \times l$

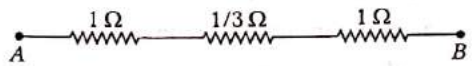
$= \frac{2}{(10 + 40 + 0)} \times \frac{10}{1} \times 0.4 = 0.16\text{V}$

60. (*c*)



In circuit, resistance between *PQ*, *QR* and *RS* are in parallel. Now circuit reduces to





$\therefore \quad R_{AB} = 1 + 1 + \frac{1}{3} = \frac{7}{3}\Omega$

61. (*c*) Resistance, $R = 3 + \frac{10\,(3 + R)}{13 + R}$

Solving this equation, we get

$R = \sqrt{69}\,\Omega$

62. (*a*) Resistance, $R = \rho \cdot \frac{l}{A}$

For same meterial and same length,

$\frac{R_2}{R_1} = \frac{A_1}{A_2} = \frac{3}{1}$

$\therefore \quad R_2 = 3R_1$

Resistance of thick wire, $R_1 = 10\,\Omega$ (given)

$\therefore$ Resistance of thin wire, $R_2 = 3 \times 10 = 30\,\Omega$

Total resistance of series combination $= 10 + 30 = 40\,\Omega$

63. (*c*) Given, $l_1 = 1k$, $l_2 = 3k$, $l_3 = 5k$

and $m_1 = 5m$, $m_2 = 3m$, $m_3 = 1m$

Resistance, $R = \frac{\rho l}{A}$

where, ρ is resistivity of the material of conductor.

So, $R_1 : R_2 : R_3 = \frac{l_1}{A_1} : \frac{l_2}{A_2} : \frac{l_3}{A_3} = \frac{l_1^2}{V_1} : \frac{l_2^2}{V_2} : \frac{l_3^2}{V_3}$

$= \frac{l_1^2}{m_1} : \frac{l_2^2}{m_2} : \frac{l_3^2}{m_3} = \frac{1}{5} : \frac{9}{3} : \frac{25}{1} = 1 : 15 : 125$

64. (*b*) Electric power, $P = i^2R$

$\therefore \quad$ Current, $i = \sqrt{\frac{P}{R}}$

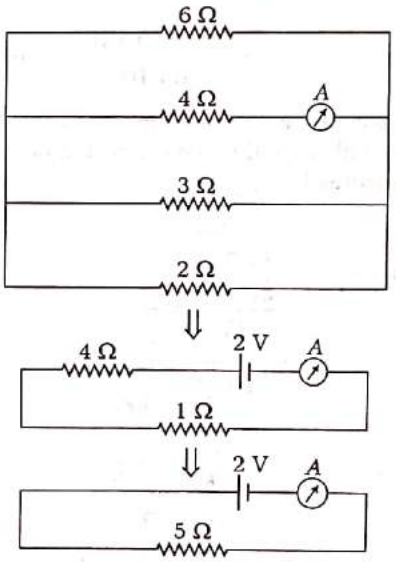
For resistance of 9Ω,

$i_1 = \sqrt{\frac{36}{9}} = \sqrt{4} = 2$ A

$i_2 = \frac{i_1 \times R}{6} = \frac{2 \times 9}{6} = 3\text{A}$

$i = i_1 + i_2 = 2 + 3 = 5\text{A} \Rightarrow V_2 = iR_2 = 5 \times 2 = 10\text{V}$

65. (*c*) The equivalent circuit can be drawn as

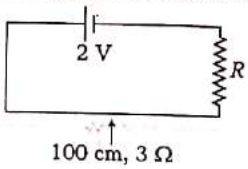


Current $= \frac{2}{5} = 0.4$ A

66. (*c*) Total potential drop across the given wire

$= (1 \times 10^{-3})(10^2) = 0.1$ V

Therefore, potential difference across *R* should be 1.9 V



Now, $\frac{1.9}{0.1} = \frac{R}{3}$ or $R = 57\,\Omega$

67. (*a*) We have $r_A = 2r_B$

$\therefore$ $A_A = 4A_B$ or $l_A = \frac{l_B}{4}$

$\therefore$ $R_B = 16R_A$

$$R_{net} = \left(\frac{16}{17}\right)R_A = \frac{R_B}{17}$$

If $R_{net} = 4$, then $R_A = 4.25\Omega$ or $R_B = 68\Omega$

Balanced condition for Wheatstone's bridge.

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

68. (*d*) We have, $\frac{24+x}{84+y} = \frac{10}{30} = \frac{1}{3}$ ...(i)

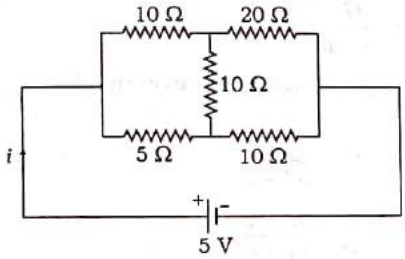
$$\frac{3}{1} = \frac{24+84+x+y}{10+30}$$

$$= \frac{108+x+y}{40} \quad ...(ii)$$

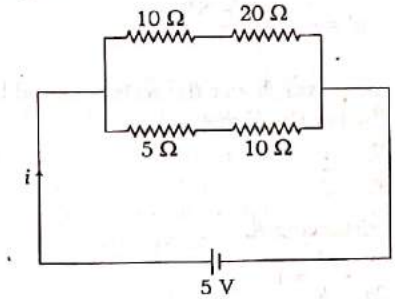
Solving this equation, we get

$x = y = 6\Omega$

69. (*b*) The given circuit can be redrawn as



It is a balanced Wheatstone bridge and hence, no current flows in the middle resistor, so equivalent circuit would be as shown if figure.



10 Ω and 20 Ω resistances are in series,

$\therefore$ $R' = 10\Omega + 20\Omega = 30\Omega$

Similarly, 5Ω and 10Ω are in series, $R'' = 15\Omega$

Now, R' and R'' are in parallel,

$\therefore$ $R = \frac{15 \times 30}{15+30} = 10\Omega$

So, $i = \frac{V}{R} = \frac{5}{10} = 0.5$ A

70. (*c*) We know that when current flow is same then resistors are connected in series, hence resultant resistance is

$R' = R_1 + R_2 = 10\Omega + 20\Omega = 30\Omega$

Also since, cells are connected in opposite directions, the resultant emf is

$E = E_1 - E_2 = 5\text{ V} - 2\text{ V} = 3\text{ V}$

From Ohm's law, $E = iR$

$\Rightarrow$ $i = \frac{E}{R} = \frac{3}{30} = 0.1$ A

71. (*b*) Here, the resistances of 400 Ω and 10000 Ω are in parallel, their effective resistance R_P will be

$$R_P = \frac{400 \times 10000}{400 + 10000} = \frac{5000}{13}\Omega$$

Total resistance of the circuit

$$= \frac{5000}{13} + 800 = \frac{15400}{13}\Omega$$

Current in the circuit, $i = \frac{6}{15400/13} = \frac{39}{7700}$ A

Potential difference across voltmeter

$$= iR_P = \frac{39}{7700} \times \frac{5000}{13} = 1.95\text{ V}$$

72. (*b*) Current, $i = \frac{E}{R+r}$ when R decreases to 0, $i = \frac{E}{r}$.

73. (*c*) $\frac{P}{Q} = \frac{20}{80}$

$$\frac{P+15}{Q} = \frac{40}{60} \Rightarrow \frac{P}{Q} + \frac{15}{Q} = \frac{2}{3}$$

$$\frac{1}{4} + \frac{15}{Q} = \frac{2}{3} \qquad \frac{15}{Q} = \frac{5}{12} \Rightarrow Q = 36\Omega$$

Resistance, $P = \frac{1}{4} \times 36 = 9\Omega$

74. (*d*) Effective voltage on charging $= 4 \times 1.4 - 2 = 3.6$ V

Total resistance $= 8\Omega \Rightarrow i = V/R = 3.6/8 = 0.45$ A

75. (*d*) Since, on balancing no current flows through the battery so its internal resistance will not affected.

Potential gradient $= \frac{E}{100}$

So, balancing emf $= \frac{E}{100} \times 30 = \frac{30E}{100}$

76. (*c*) When a resistance of 100 Ω is connected in series, then the current flowing will be

$$i = \frac{V}{100+R} \quad ...(i)$$

When a resistance of 1000 Ω is connected in series, then its range of galvanometer double.

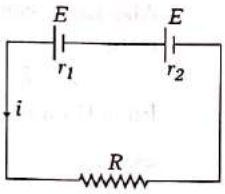
Current, $i = \frac{2V}{1100+R}$ ...(ii)

From Eqs. (i) and (ii), we get

$$\frac{V}{100+R} = \frac{2V}{1100+R}$$

$\Rightarrow$ $R = 900\Omega$

77. (*c*) There are two batteries with emf E each and the internal resistances r_1 and r_2, respectively.



Hence, we have $i(R + r_1 + r_2) = 2E$

Thus, $$i = \frac{2E}{R + r_1 + r_2}$$

Now, the potential difference across the first cell would be equal to $V = E - ir_1$. From the question, $V = 0$, hence,

$E = ir_1 = \frac{2Er_1}{R + r_1 + r_2}$, thus $R + r_1 + r_2 = 2r_1$, hence $R = r_1 - r_2$.

78. (*d*) Drift velocity, $v_d = \frac{i}{nAe} = \frac{i \times 4}{n\pi D^2 e}$

i.e., $$v_d \propto \frac{1}{D^2}$$

$$\therefore \quad \frac{v_{d_1}}{v_{d_2}} = \frac{D_2^2}{D_1^2} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

79. (*c*) Power, $$P = \frac{V^2}{R}$$

$$\therefore \quad R = \frac{V^2}{P} \text{ or } R \propto V^2$$

i.e., $$\frac{R_1}{R_2} = \left[\frac{200}{300}\right]^2 = \frac{4}{9}$$

When connected in series, then potential drop and power consumed are in the ratio of their resistances. So,

$$\frac{P_1}{P_2} = \frac{V_1}{V_2} = \frac{R_1}{R_2} = \frac{4}{9}$$

80. (*b*) Current required by each bulb,

$$i = \frac{P}{V} = \frac{100}{220} \text{ A}$$

If n bulbs are joined in parallel, then

$$ni = i_{fuse}$$

or $$n \times \frac{100}{220} = 10$$

or $$n = 22$$

81. (*a*) When connected in series power dissipated,

$$P_S = \frac{P_1P_2}{P_1 + P_2} = \frac{100 \times 60}{100 + 60} = \frac{6000}{160} = 37.5 \text{ W}$$

82. (*b*) Rise in temperature = 15°C

Amount of water = 1 l = 1000 cc = 1000 g

Heat supplied in cal = $1000 \times 15 \times 1$

In joule = $1000 \times 15 \times 4.2$

Power = $\frac{1000 \times 15 \times 4.2}{60}$ = 1050 W (Joule/sec)

83. (*d*) Work done by electric force on electron on an average on mean per path

$$\therefore \quad Eq \times 4 \times 10^{-8} = 2q \times 1 \text{ V}$$

$$E = \frac{2}{4 \times 10^{-8}} = 0.5 \times 10^8$$

$$= 5 \times 10^7 \text{ V/m}$$

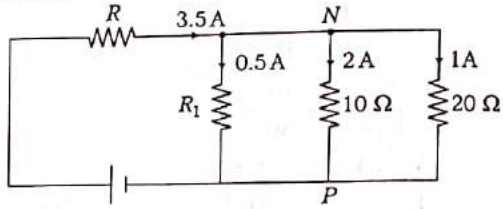
84. (*d*) By joining 2 Ω and 6 Ω in parallel, we get 1.5 Ω and joining them in series we get 8 Ω.

So, values of R_1 and R_2 will be 2Ω and 6Ω.

85. (*d*) $10 \times i = 2 \times 50$

$$i = \frac{100}{10} = 10 \text{ mA}$$

86. (*b*) In parallel current distribution in inverse ratio of resistance.



$$R_1 = 40\Omega$$

Further $$V_{NP} = 20\text{V} \Rightarrow V_R = 69 - 20 = 49\text{V}$$

$$R = \frac{49}{3.5} = 19\Omega$$

88. (*b*) We have

shunt, $S = \frac{G}{(n-1)}$ ...(i)

where, G is resistance of galvanometer.

Again,

$$S' = \frac{G}{(n'-1)} \quad \text{...(ii)}$$

$$\therefore \quad \frac{S}{S'} = \frac{(n'-1)}{(n-1)}$$

$$\frac{S(n-1)}{S'} = (n'-1)$$

$$n' = \frac{S(n-1) + S'}{S'}$$

89. (*c*) Higher the power, lower the resistance and lower the resistance, thicker the element.

90. (*b*) Given, $\frac{R_1}{R_2} = \frac{1}{2}$

Let, third resistance is R.

So, $$\frac{1}{a} + \frac{1}{2a} + \frac{1}{R} = 1$$

$$\therefore \quad a = \frac{3R}{2(R-1)}$$

As resistance is not fractional. (given)

$$\therefore \quad \frac{R}{R-1} = 2$$

So, $$R = 2$$

$$\therefore \quad a = 3 \text{ and } 2a = 6$$

91. (*c*) Applying Kirchhoff's law at point D, we get

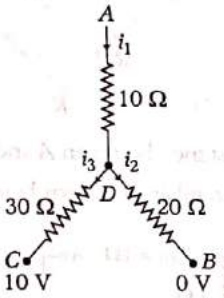
$$I_1 = I_2 + I_3$$

$$\frac{V_A - V_D}{10} = \frac{V_D - 0}{20} + \frac{V_D - V_C}{30}$$

or $\quad 70 - V_D = \frac{V_D}{2} + \frac{V_D - 10}{3}$

$\Rightarrow \quad V_D = 40\,\text{V} \Rightarrow i_1 = \frac{70-40}{10} = 3\text{A}$

$i_2 = \frac{40-0}{20} = 2\text{A}$



and $\quad i_3 = \frac{40-10}{30} = 1\text{A}$

92. (*b*) $E_{net} = \frac{E_1/r_1 - E_2/r_2}{1/r_1 + 1/r_2}$

Current through R will be zero if $E_{net} = 0$

$\therefore \quad \frac{E_1}{r_1} = \frac{E_2}{r_2}$

93. (*d*) $i = \frac{\text{Net emf}}{\text{Net resistance}} = \frac{10-4}{1+2+3} = 1\text{A}$ from a to e.

94. (*b*) Electric field provides force to the electrons to change direction due to current density. It is all into the centripetal force in circular motion which changes the direction of velocity in this type of motion.

95. (*a*) E_{eq} will be less than E_2 and more than E_1.

We know that in the given circuit,

$$E_{eq} = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}$$

Here, E_{eq} is weighted average of E_1 and E_2.

Naturally, its value will lie between the value of E_1 and E_2.

96. (*c*) We know that percentage of error can be minimised by adjusting the balance point near the middle of the meter scale *i.e.*, around 50 cm. This can be achieved by adjusting the value of S. For balance point to be at 2.9, we have the equation.

$$\frac{R}{S} = \frac{2.9}{97.1} \Rightarrow R = \frac{2.9}{97.1} \times 100 = 3$$

The value of R is 3 Ω.

For balance point to be in the middle,

$$\frac{R}{S} = \frac{1}{1}$$

$$R = S$$

Here, $\quad R = 3\Omega$

$\Rightarrow \quad S = 3\Omega$

So, the value of S should be changed from 100 Ω to 3 Ω to obtain reading in the mean position.

97. (*b*) No current will flow through the grounded wire.

98. (*c*) $E_{eq} = \frac{E_1/r_1 + E_2/r_2}{1/r_1 + 1/r_2} = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2} = E_1\left[\frac{r_2 + r_1(E_2/E_1)}{r_1 + r_2}\right]$

Therefore, E_{eq} can be greater than, less than or equal to E_1 depending upon the condition whether $E_2 > E_1$, $E_2 = E_1$ or $E_2 < E_1$.

99. (*b*) We have, $I_1 = 0$

$\therefore \quad V_X = E_2 = 2\,\text{V}$

$$\frac{V_{500\,\Omega}}{V_X} = \frac{(12-2)}{2} = \frac{500}{X}$$

$\therefore \quad X = 100\,\Omega$

100. (*a*) Equivalent resistance of circuit,

$$R = 4 + \frac{3 \times 6}{3+6} = 6\Omega$$

and current through battery,

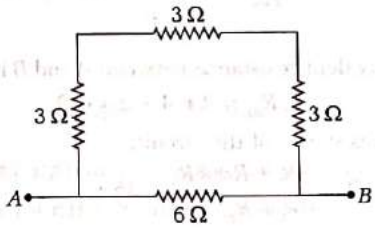
$$i = \frac{3}{6} = 0.5\,\text{A}$$

$\therefore \quad V = iR = 0.5 \times 6 = 3\,\text{V}$

Potential difference over $4\Omega = 3\,\text{V}$

Potential difference across the resistor of 3Ω over $6\Omega = 4 - 3 = 1$ V

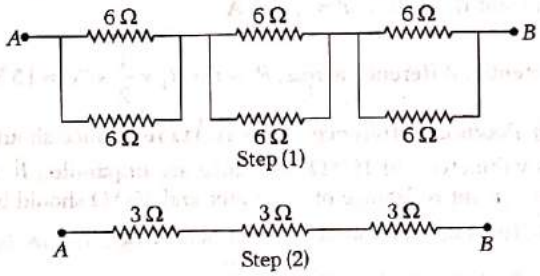
101. (*d*) The given circuit can be reduced to



So, equivalent resistance between A and B,

$$R_{AB} = \frac{9 \times 6}{9+6} = \frac{9 \times 6}{15} = \frac{18}{5} \Rightarrow R_{AB} = 3.6\Omega$$

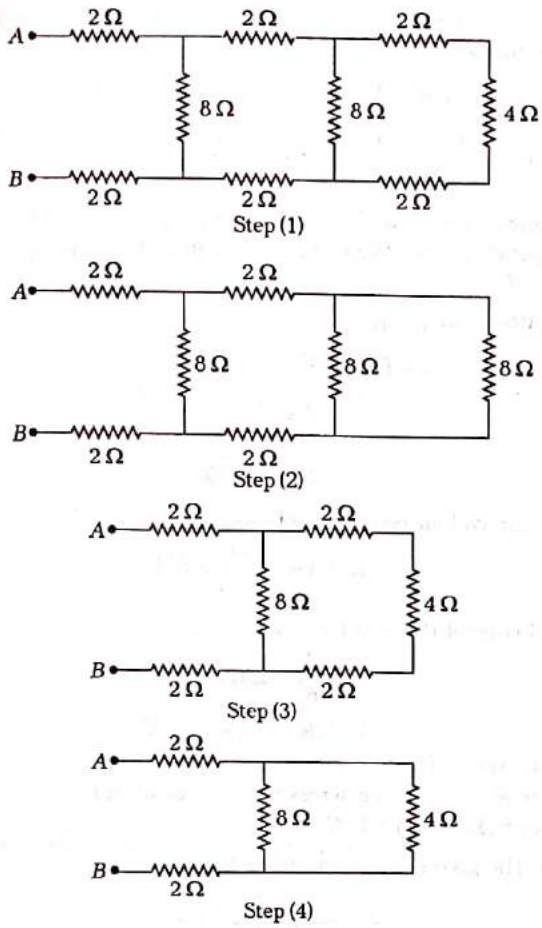
102. (*d*) The network can be redrawn as follows



Now, there are three resistances in series.

So, equivalent resistance, $R_{eq} = 3 + 3 + 3 = 9\Omega$

103. (*d*)



Now, equivalent resistance between A and B is

$$R_{eq} = 2 + 4 + 2 = 8\,\Omega$$

104. (*c*) Total resistance of the circuit,

$$R = R_4 + \frac{(R_1 + R_2) \times R_3}{(R_1 + R_2) + R_3} = 35 + \frac{(15 + 15) \times 30}{(15 + 15) + 30}$$

$$= 35 + \frac{30 \times 30}{30 + 30} = 50\,\Omega$$

Current in circuit, $i = \frac{50}{50} = 1\,\text{A}$

Current through R_3, $i' = i_2 = \frac{1}{2}\,\text{A}$

Potential difference across, $R_3 = i' \times R_3 = \frac{1}{2} \times 30 = 15\,\text{V}$

105. (*b*) Potential difference across $100\,\Omega$ resistance should be 5V as voltmeter and $100\,\Omega$ resistance are in parallel. It means equivalent resistance of voltmeter and $100\,\Omega$ should be $50\,\Omega$. So, resistance of voltmeter must be $100\,\Omega$.

106. (*d*) Emf of cell, $E = Xl = \frac{V}{l} = \frac{iR}{L} \times l$

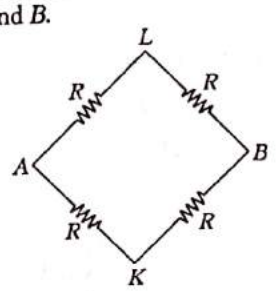
$$\therefore \quad E = \frac{e}{(R_1 + R_2 + r)} \times \frac{R}{L} \times l$$

$$0.4 = \frac{5}{(5 + 45 + 0)} \times \frac{5}{10} \times l$$

$$\therefore \quad l = 8\,\text{m}$$

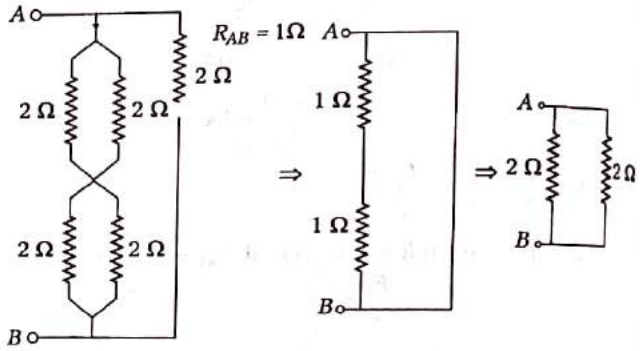
108. (*a*) The given circuit forms a balanced Wheatstone bridge between points A and B.

i.e.,



So, effective resistance between A and B is R.

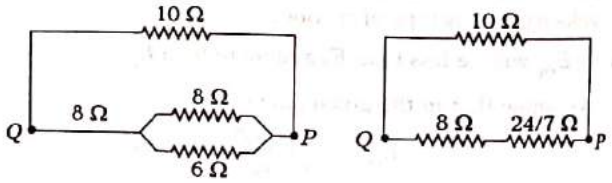
109. (*a*) The simple circuit is as shown below,



$\therefore$ The equivalent resistance across AB,

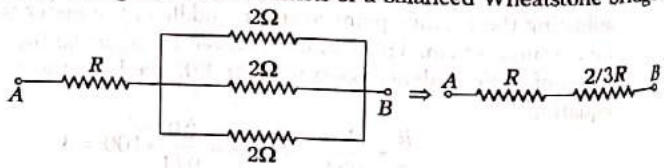
$$R_{eq} = \frac{2 \times 2}{2 + 2} = 1\Omega$$

110. (*b*) The simple circuit is as shown below.



$$R_{net} = \frac{10 \times (8 + 24/7)}{10 + (8 + 24/7)} = 5.3\Omega$$

111. (*a*) The given circuit consists of a balanced Wheatstone bridge.



$$\therefore \quad R_{net} = R + \frac{2}{3}R = \frac{5}{3}R$$

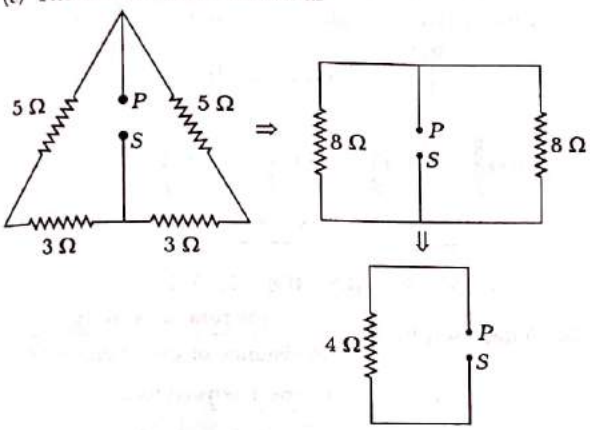
112. (*c*) $q = \int_0^5 i\,dt = \int_0^5 (1.2t + 3)\,dt = 30\,\text{C}$

113. (*a*) As $R_{AB} = R_{AO}$ and $R_{BC} = R_{OC}$, so points O and B will be at same potential and hence, resistance R_{OB} becomes ineffective. Similarly, as $R_{AO} = R_{AD}$ and $R_{OC} = R_{DC}$, resistance R_{OD} becomes ineffective.

So, excluding R_{BO} and R_{OD}, equivalent resistance R_{eq} of the given network between points A and C will be given by

$$\frac{1}{R_{eq}} = \frac{1}{2R} + \frac{1}{2R} + \frac{1}{2R}, \quad i.e., \; R_{eq} = \frac{2}{3}R$$

114. (*e*) The circuit can be drawn as



Resistance between points P and $S = 4\,\Omega$

115. (*b*) In the first case potential difference balance against resistance $R = k_1 l_1$

Potential difference against, $R + X = kl_1$

Potential difference over $X\Omega = k(l_2 - l_1)$

116. (*d*) Power = 1.08 kW = 1080 W

$P \times t = E \qquad 1080 \times t = m \times L$

$1080 \times t = 100 \times 540 \times 4.2$

$t = 210$ s

117. (*b*) For an ammeter, $\frac{i_g}{i} = \frac{S}{G+S}$

$\Rightarrow \qquad i_g G = (i - i_g)S$

$\therefore \qquad i_g G = (0.03 - i_g)\,4r \qquad$ ...(i)

and $\qquad i_g G = (0.06 - i_g)r \qquad$ ...(ii)

From Eqs. (i) and (ii), we get $\qquad 0.12 - 4i_g = 0.06 - i_g$

Maximum current through the galvanometer, $i_g = 0.02$ A

118. (*c*) When key K is opened, bulb B_2 will not draw any current from the source. So, that terminal voltage of source increases. Hence, power consumed by bulb increases. So, light of the bulb becomes more. The brightness of bulb B_1 decreases.

119. (*a*) Maximum current possible in galvanometer

$= 25 \times 4 \times 10^{-4}$ A $= 10^{-2}$ A

So, $\qquad 10^{-2} = \frac{2.5}{100 + R}$

$100 + R = 250$

Resistance, $R = 150\,\Omega$

120. (*a*) Let, the bulb 400 W is having resistance value of R. For 200 W, necessary value of resistance will be $2R$.

Total value of resistance in the circuit will be $R + R = 2R$

If I is the maximum current in the circuit, then $I^2R = 400$

through 200 W each, current will be $I/2$ and power will be

$$\frac{I_2}{4} \times 2R = \frac{I^2R}{2} = \frac{400}{2} = 200 \text{ W}$$

Power circuit as a whole $= I^2 \times 2R = 2 \times I^2R = 2 \times 400$

$= 800$ W

121. (*b*) Maximum current possible in bulb $= \frac{50}{100} = 0.5$ A

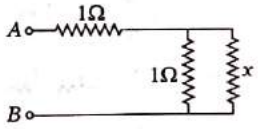
Resistance of each bulb $= \frac{V^2}{P} = \frac{100 \times 100}{50} = 200\,\Omega$

If n be the number of bulb possible, then total resistance of circuit $= \frac{200}{n} + 10$

Maximum current in the circuit $= 0.5 \times n$

So $\quad \frac{120}{\frac{200}{n} + 10} = 0.5n \Rightarrow n = 4$

122. (*c*) Let $R_{AB} = x$. Then,



$$R_{AB} = 1 + \frac{x}{1+x}$$

or $\qquad x = 1 + \frac{x}{1+x}$

$\therefore \qquad x + x^2 = 1 + x + x \quad$ or $\quad x^2 - x - 1 = 0$

$$x = \frac{1 + \sqrt{1+4}}{2} = \frac{1+\sqrt{5}}{2}\,\Omega$$

123. (*a*) Potential difference across 4Ω resistance $(20 + 16)V$

current through 4Ω resistance

$= 36V = \frac{36}{4} = 9$ A $\qquad$ [from top to bottom]

Similarly, current through 2Ω resistance $= \frac{20 - 46}{2} = 2$ A

Therefore, total current through 20 volt battery will be 11 A.

124. (*b*) Current through R_1 and R_2 comes out to be zero (potential difference = 0)

$\therefore$ Current through, $R_3 = \frac{\text{Net emf}}{\text{Total resistance}}$

$$= \frac{(4+3+2) - (2+3+2)}{3} = \frac{2}{3}\text{ A}$$

125. (*b*) According to the question, $R = \left(20 + \frac{i}{2}\right)$

Now, current $i = \frac{250}{R} = \frac{250}{20 + (i/2)}$

Solving, we get

$i = 10$ A

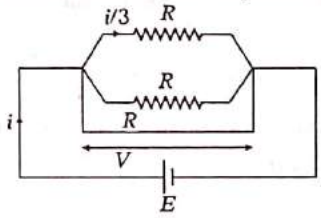
126. (*b*) According to question,

$$Q = at - bt^2 \Rightarrow i = \frac{dQ}{dt} = a - 2bt$$

$$i = 0 \text{ at } t = \frac{a}{2b}$$

$$\frac{di}{dt} = -2b$$

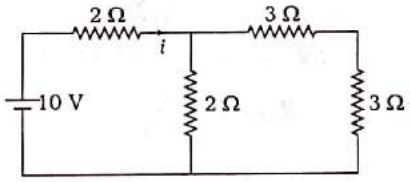
127. (*a*) The given three resistors are in parallel.



$\therefore$ Current, $i = \dfrac{E}{r + R/3} = \dfrac{4}{1+3} = 1\text{A}$

$$V = \frac{i}{3}R \text{ or } E - ir = \left(\frac{1}{3}\right)(9) = 3\text{ V}$$

128. (*c*) Resistance, $R_{net} = 2 + \dfrac{6\times 2}{6+2} = \dfrac{7}{2}\Omega$



$\therefore$ $= \dfrac{10}{(7/2)} = \dfrac{20}{7}\text{ A}$

$$i_{3\Omega} = \left(\frac{2}{2+6}\right) i = \frac{1}{4}\cdot i = \frac{5}{7}\text{ A}$$

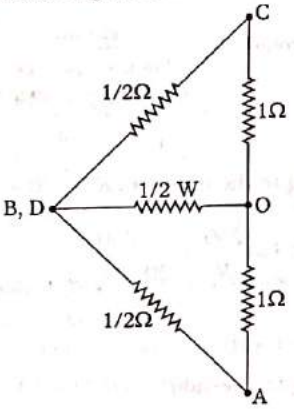
129. (*d*) Current, $i_1 = \dfrac{E_1 + E_2}{r_1 + r_2 + R}$

and $i_2 = \dfrac{E_1}{R + r_1}$

$i_2 > i_1$

$\therefore$ $\dfrac{E_1}{R + r_1} > \dfrac{E_1 + E_2}{r_1 + r_2 + R}$ or $E_1 r_2 > E_2(R + r_1)$

130. (*b*) Both B and D are symmetrically located with respect to points O. Hence, the figure can be folded as shown in figure.

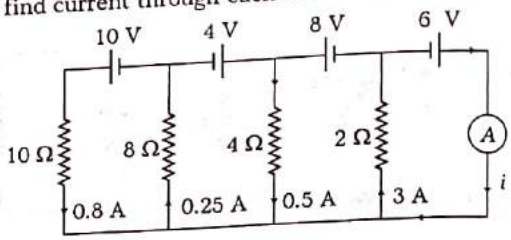


The given circuit forms Wheatstone bridge.

$\therefore$ $R_S = \dfrac{1+1}{2} = \dfrac{3}{2}\Omega$

Resistance, $R_{net} = \dfrac{\frac{3}{2}\times\frac{3}{2}}{\frac{3}{2}+\frac{3}{2}} = \dfrac{3}{4}\Omega$

131. (*c*) By finding potential difference across any resistance, we can find current through each resistance directly.



$\therefore$ $i = 3 + 0.25 - 0.5 - 0.8 = 1.95$ A

132. (*b*) Voltage sensitivity $= \dfrac{\text{Current sensitivity}}{\text{Resistance of galvanometer } G}$

$$G = \frac{\text{Current sensitivity}}{\text{Voltage sensitivity}}$$

$$G = \frac{10}{2} = 5\Omega$$

Full scale deflection current, $i_g = \dfrac{150}{10} = 15$ mA

Voltage to be measured, $V = iR = 150 \times 1 = 150$ V

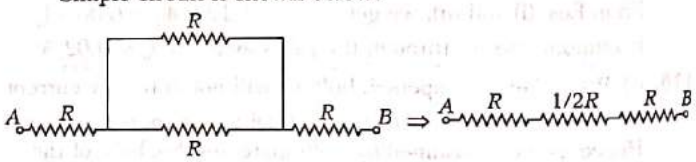
Hence, $R = \dfrac{V}{i_g} - G = \dfrac{150}{15\times 10^{-3}} - 5 = 9995\Omega$

133. (*b*) We have $\dfrac{V^2}{R}\times 16 = H$ $[\because H = P\times t]$

$$\frac{V^2}{0.9R}\times t = H \Rightarrow \frac{V^2}{R}\times 16 = \frac{V^2}{0.9R}\times t$$

Time, $t = 16\times 0.9 = 14.4$ min

134. (*c*) Topmost and bottommost figures are short-circuited. Simple circuit is shown below.



$$R_{eq} = R + R + \frac{R}{2} = \frac{5R}{2}$$

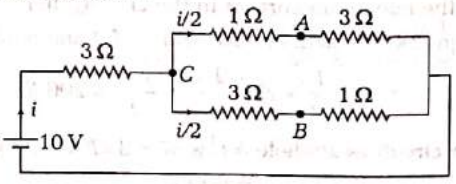
135. (*b*) Due to balanced Wheatstone bridge, resistance between A and B can be removed.

The current between D and E,

$$I_{DE} = \frac{10\text{ V}}{R_{DE} + R_{HG}} = \frac{10}{2+2} = 2.5\text{ A}$$

136. (*b*) Resistance between upper branch and lower branch in parallel part is same, so equal amount of current flows through them.

Let main current is i.



$\therefore$ $i = \dfrac{V}{R_{eq}}$

Equivalent resistance of circuit,

$$R_{eq} = 3 + 2 = 5\,\Omega \Rightarrow i = \frac{10}{5} = 2\,A$$

So, current in each branch = 1A

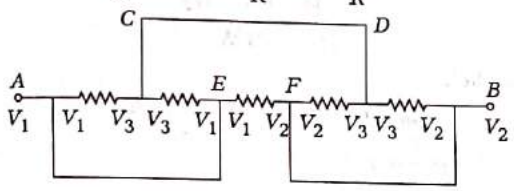
Now, $V_C - V_A = 1 \times 1 = 1\,V$ ...(i)

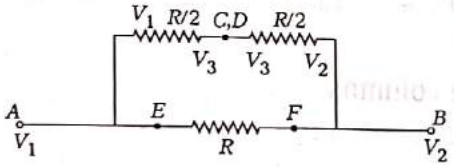
also $V_C - V_B = 1 \times 3 = 3\,V$ ...(ii)

Solving Eqs. (i) and (ii), we have

$$V_A - V_B = 3 - 1 = 2\,V$$

136. (*a, c*) $R_{AB} = \frac{R}{2} \Rightarrow I_{CD} = \frac{V_0}{R}, I_{EF} = \frac{V_0}{R}$





138. (*d*) When K_1 is closed, R_1 is short-circuited.

When K_2 is open, $i_0 = \frac{E}{r + R_2} = \frac{E}{r + 100}$ ...(i)

When K_2 is closed,

$$i_0 = \frac{1}{2}\left[\frac{E}{r + 50}\right] \quad ...(ii)$$

From these two equations, we get $r = 0$

When K_1 is open and K_2 is closed.

$$\frac{i_0}{2} = \frac{E}{2(R_1 + 50)} \quad ...(iii)$$

From Eqs. (i) and (iii), we have

$$R_1 = 50\,\Omega$$

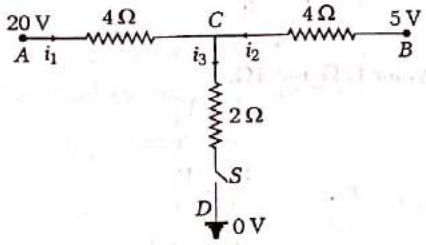
139. (*b*) In the circuit, we can see that 20 Ω, 100 Ω and 25 Ω are in parallel.

Net resistance of circuit $= 4 + 6 + \frac{1}{\frac{1}{20} + \frac{1}{100} + \frac{1}{25}} = 20\,\Omega$

$\therefore$ $V = iR = 4 \times 20 = 80\,V$

140. (*a*) Let the potential at junction C, is V.

Applying Kirchhoff's junction law at C



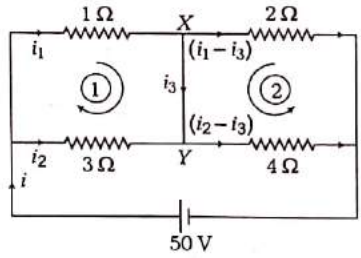
or $\frac{V_A - V_C}{2} + \frac{V_B - V_C}{4} = \frac{V_C - V_D}{2}$

or $\frac{20 - V}{2} + \frac{5 - V}{4} = \frac{V - 0}{2}$ or $5V = 45$

$\therefore$ $V = 9\,V$

$\therefore$ Current through switch S, $i_3 = \frac{9}{2} = 4.5\,A$

141. (*c*)



Let current through XY is i_3. Applying Kirchhoff's law to loops (1) and (2),

$$i_1 + 0 \times i_3 - 3i_2 = 0$$

$\therefore$ $i_1 = 3i_2$ ...(i)

and $-2(i_1 - i_3) + 4(i_2 + i_3) = 0$

So, $2i_1 - 4i_2 = 6i_3$ ...(ii)

Also, $50 = 1i_1 - 2(i_1 - i_3)$

$50 = i_1 - 2(i_1 - i_3)$

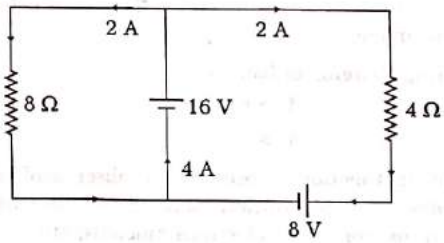
$\therefore$ $3i_1 - 2i_3 = 50$ ...(iii)

From Eqs. (i), (ii) and (iii), we get

$$i_3 = 2\,A$$

142. (*b*) $i_1 = 4\,A$

Further dividing 2 A across different resistors we can find that



$$i_2 = \frac{2}{4} = \frac{1}{2}\,A$$

Hence, $\frac{i_1}{i_2} = \frac{8}{1}$

B. Medical entrance special format questions

Assertion and reason

1. (*c*) $\frac{V}{i}$ = constant = R for ohmic circuits is Ohm's law. Further, only for ohmic circuits V - i graph is a straight line passing through origin.

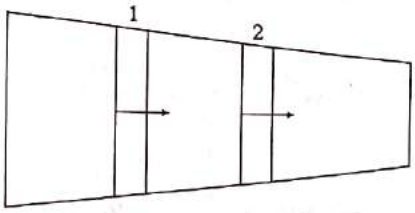
2. (*d*) $R = \frac{V}{I}$

$\therefore$ $R_2 > R_1$

Resistance of a conductor increases with increase in temperature. Hence,

$$T_2 > T_1$$

3. (*d*) R_1 and R_2 are resistances per unit length.



$$i_1 = i_2$$
$$V_1 = i_1R_1$$
and $$V_2 = i_2R_2$$
$$R_2 > R_1 \qquad \left(\text{as } R \propto \frac{1}{A}\right)$$
Hence, $$V_2 > V_1$$

5. (*b*) We have, v_d, R and $J \propto \frac{1}{A}$

6. (*e*) kVA is the unit of power. While kWh is the unit of energy.

7. (*d*) Resistance of voltmeter is high. If it is connected in series, then current will decrease but circuit will not burn.

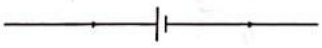
8. (*b*) The resistance of milliammeter becomes high because of increased no of turns of coil, so the torque produced in the coil is not decreased due to low value of current. Milliammeter generally do not have shunt because the main current is already very low. Even if there is shunt, it value is kept high so that current diversion through it is least in case of milliammeter.

10. (*d*) During charging of battery,

$$V = E + iR$$
$$\therefore \quad V > E$$

11. (*e*) Only drift motion is absent in the absence of potential difference across it. Further, there will be electrostatic force on electrons from other electrons, nucleus, etc.

12. (*e*) Current through a resistance wire flows from higher potential to lower potential. But inside cell it flows from lower to higher potential.



Further, during charging of a battery current flows in the direction shown in above figure.

13. (*c*) $$i\rho l = E_U$$
$$\therefore \quad l = \frac{E_U}{i\rho}$$

E_U and i both will become two times. Therefore, l will remain unchanged.

14. (*c*) Reason is false. If we increase emf of known battery, potential gradient increases, so less length will be required to equalise emf of unknown battery. So, null point length will decrease.

15. (*d*) In both cases potential difference across R is E.

$$\therefore \quad P = \frac{E^2}{R}$$

In second case net resistance will decrease. Therefore, main current will increase.

16. (*b*) In series,

$$\frac{1}{P} = \frac{1}{P_1} + \frac{1}{P_2}$$
$$\frac{1}{P} = \frac{1}{60} + \frac{1}{60}$$
$$P_{\text{eff}} = 30 \text{ W}$$

In parallel,

$$P_{\text{eff}} = P_1 + P_2 = 60 + 60 = 120 \text{ W}$$

17. (*a*) $R = \rho\frac{l}{A}$ or $R \propto \frac{1}{A}$

Area of cross-section of first wire is less. Hence, its resistance is more.

Match the columns

1. In parallel, $P = P_1 + P_2$

and in series $\frac{1}{P} = \frac{1}{P_1} = \frac{1}{P_2}$

2. During discharging, $V = E - ir$

During charging, $V = E + ir$

When $i = 0$, $V = E$ and $V = 0$, if short-circuited.

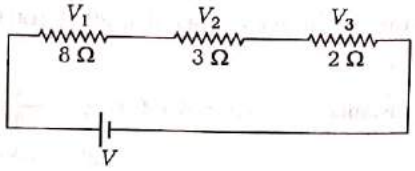
3. In parallel connections net resistance will decrease. Therefore, main current i_1 will increase.

$$V_{AB} = V_{CD} = E - i_1r$$

With increase in i_1, V_{AB} or V_{CD} will decrease. Further, $i_2 = \frac{V_{AB}}{R}$

V_{AB} is decreasing. Therefore, i_2 will also decrease.

4. The simple circuit is shown in figure.



Also, 3Ω and 6Ω are in parallel connection.

Hence, potential difference across 6Ω is also 20 V.

and, for 8Ω, $\frac{V}{8} = \frac{20}{3} + \frac{20}{6} = \frac{60}{6} = 10$

$$V = 80 \text{ V}$$

Similarly, for 12Ω and 4Ω,

$$\frac{V}{8} = \frac{V_2}{3}$$
$$\Rightarrow \quad \frac{80}{8} = \frac{V_2}{3}$$
$$\Rightarrow \quad V_2 = 30 \text{ V}$$

C. Medical entrances' gallery

Entrance corner

1. (*c*) According to question, emf of the cell is directly proportional to the balancing length, *i.e.*, $E \propto l$ ...(i)

Now, in the first case, cells are connected in series to support one another, *i.e.*,

Net emf $= E_1 + E_2$

From Eq (i), $E_1 + E_2 = 50$ cm (given) ...(ii)

Again cells are connected in series but in opposite direction *i.e.*,

Net emf $= E_1 - E_2$

From Eq. (i), $E_1 - E_2 = 10$ cm (given) ...(iii)

From Eqs. (ii) and (iii),

$$\frac{E_1 + E_2}{E_1 - E_2} = \frac{50}{10} \Rightarrow \frac{E_1}{E_2} = \frac{5+1}{5-1} = \frac{6}{4} = \frac{3}{2}$$

2. (*d*) Given, charge $Q = at - bt^2$

$\because$ We know that current, $I = \frac{dQ}{dt}$ ...(i)

So, Eq. (i) can be written as

$$I = \frac{d}{dt}(at - bt^2) \Rightarrow I = a - 2bt \quad ...(ii)$$

For maximum value of t, $\frac{dI}{dt} = 0$ or $\frac{d}{dt}(at - bt^2) = 0$

$\Rightarrow \quad a - 2bt = 0$

$\therefore \quad t = \frac{a}{2b}$ ...(ii)

$\because$ The total heat produced (H) can be given as

$$H = \int_0^t I^2 R\, dt = \int_0^{a/2b} (a - 2bt)^2 R\, dt \quad \left(\because t = \frac{a}{2b}\right)$$

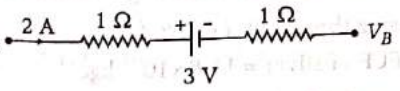
$$= \int_0^{a/2b} (a^2 + 4b^2t^2 - 4abt) R dt$$

$$H = \left[a^2 t + 4b^2 \frac{t^3}{3} - \frac{4abt^2}{2}\right]_0^{a/2b} R$$

Solving above equation, we get

$\Rightarrow \quad H = \frac{a^3 R}{6b}$

3. (*d*)



Applying KVL,

$$V_A + \Sigma V = V_B + 2\times2 + 2\times1$$

$$V_A - V_B - 3 = 4 + 2; V_A - V_B = 9\text{ V}$$

4. (*c*) If a rated voltage and power are given,

Then, $P_{rated} = \frac{V_{rated}^2}{R}$

$\therefore$ Current in the bulb, $i = \frac{P}{V} \Rightarrow i = \frac{500}{100} = 5$ A

$\therefore$ Resistance of bulb, $R_b = \frac{100 \times 100}{500} = 20\,\Omega$

$\because$ Resistance R is connected in series,

$\therefore$ Curent, $i = \frac{E}{R_{net}} = \frac{230}{R + R_b} \Rightarrow R + 20 = \frac{230}{5} = 46$

$\therefore \quad R = 26\,\Omega$

5. (*a*) Given, $l = 4$ m,

R = potentiometer wire resistance $= 8\,\Omega$

Potential gradient $= \frac{dV}{dl} = 1\text{ mVcm}^{-1}$

So, for 400 cm, $\Delta V = 400 \times 1 \times 10^{-3} = 0.4$ V

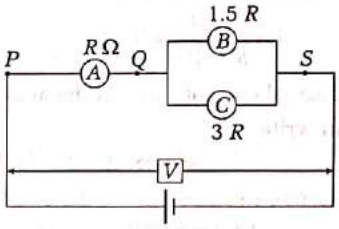
Let a resistor R_S is connected in series, so as

$$\Delta V = \frac{V}{R + R_S} \times R$$

$$\Rightarrow \quad 0.4 = \frac{2}{8 + R} \times 8$$

$$\Rightarrow \quad 8 + R = \frac{16}{0.4} = 40 \Rightarrow R = 32\,\Omega$$

6. (*a*) The equivalent resistance between Q and S is given by



$$\frac{1}{R'} = \frac{1}{1.5R} + \frac{1}{3R} = \frac{2+1}{3R} \Rightarrow R' = R$$

Now, $V_{PQ} = V_A = IR$

Also, $V_{QS} = V_B = V_C = IR$

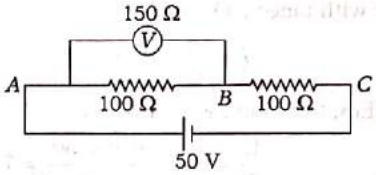
Hence, $V_A = V_B = V_C$

7. (*b*) As the cross-sectional area of the conductor is non-uniform, so current density will be different.

As, $I = JA$ ...(i)

It is clear from Eq. (i), when area increases the current density decreases so the number of flow of electrons will be same and thus the current will be constant.

8. (*c*) When voltmeter is connected across A and B, the equivalent resistance of the circuit is



$$R_{eq} = 100 + \frac{150 \times 100}{100 + 150}$$

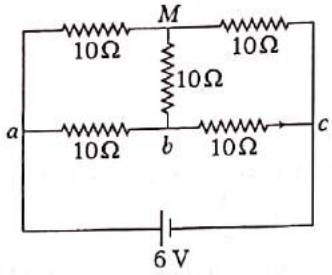
$$= 100 + \frac{15000}{250} = 100 + 60 = 160\,\Omega$$

$\therefore$ Current, $i = \frac{50}{160} = \frac{5}{16}$ A

Therefore, potential drop across B and C is

$$V_{BC} = i R_{BC} = \frac{5}{16} \times 100 = \frac{500}{16} = 31.25\text{ V}$$

9. (*a*) Figure can be redrawn as



As, $\frac{10}{10} = \frac{10}{10}$ (from figure)

So, circuit is said to be Wheatstone bridge in balanced condition and hence current through arm *Mb* is zero, *i.e.*, no current flows.

Now, the equivalent resistance of the circuit is given by

$$\frac{1}{R_{eq}} = \frac{1}{20} + \frac{1}{20} = \frac{2}{20} \Rightarrow R_{eq} = 10\,\Omega$$

∴ Current supplied by the battery,

$$i = \frac{V}{R} = \frac{6}{10} = 0.6\,\text{A}$$

10. (*a*) Suppose, actual current through the ammeter is *I*.

Now, we can write

$$Ir = kl_1 \Rightarrow I \times (1) = k(75) \quad \text{...(i)}$$

where, *k* is a constant.

Similarly, $1.02 = k\,(50)$ ...(ii)

From Eqs. (i) and (ii), we get

$$\frac{I}{1.02} = \frac{75}{50} = \frac{3}{2}$$

$$\Rightarrow \quad I = \frac{3}{2}(1.02) = 3\,(0.51) = 1.53\,\text{A}$$

Thus, error in ammeter reading is

$$\Delta I = (1.53 - 1.5)\,\text{A} = 0.03\,\text{A}$$

11. (*b*) The current in case of voltmeter range 10 V and resistance 50 Ω is

$$I = \frac{10}{50} \quad \text{...(i)}$$

Let *X* is the resistance connected in order to make a voltmeter with range 15 V is

$$I = \frac{15}{X} \quad \text{...(ii)}$$

Equating Eqs. (i) and (ii), we get

$$\frac{15}{X} = \frac{10}{50} = \frac{(15) \times (50)}{(10)} = 75\,\Omega$$

Then, $R = 75 - 50 = 25\,\Omega$ must be connected in series, because $V \propto R$ when current is constant.

13. (*e*) Current remains constant throughout the metallic conductor. Current density $J = \frac{I}{A}$ is not constant because cross-sectional area is variable parameter. Drift velocity $v_d = \frac{I}{neA}$ is not constant. Since, $v_d \propto \frac{1}{A}$.

14. (*b*) Given,

Colour coding of resistance = Red, Red, Green, Silver

First band value = 2

Second band value = 2

Third band value = 10^5

Fourth band value = ± 10%

Now, $R = 22 \times 10^5 \pm 10\%\,\Omega$

$$R = 2200 \pm 10\%\,\text{k}\Omega$$

15. (*c*) Slope $= \frac{I}{V}$

$\because \quad V = IR$

$\therefore \quad \frac{I}{V} = \frac{1}{R}$

Hence, slope is reciprocal of resistance.

16. (*a*) The resistance of two wires are

$$R_1 = \rho_1 \frac{l}{A} \text{ and } R_2 = \rho_2 \frac{l}{A}$$

Now, equivalent resistance of series connection of wire

$$R_{eq} = R_1 + R_2 = \rho_1 \frac{l}{A} + \rho_2 \frac{l}{A} = \frac{l}{A}(\rho_1 + \rho_2)$$

The equivalent resistance R_{eq} can be given by $R_{eq} = \rho_{eq} \frac{l}{A}$

$$\Rightarrow \quad \rho_{eq} \frac{l}{A} = \frac{l}{A}(\rho_1 + \rho_2)$$

Hence, $\rho_{eq} = \frac{\rho_1 + \rho_2}{2}$

17. (*a*) Total resistance = 50 + 3950 = 4000 Ω

For this circuit, deflection of resistance, *R* be

$$\Rightarrow \quad \frac{4000}{R} = \frac{30}{15} \Rightarrow R = \frac{4000}{2} = 2000\,\Omega$$

Then, resistance should be in series = 2000 − 50 = 1950 Ω

18. (*a*) Kirchhoff's first law states that algebraic sum of currents meeting at a point in a circuit is zero. It is based on conservation of charge. Kirchhoff's second law states that the algebraic sum of potential differences around a closed loop is zero. It is based on the law of conservation of energy.

19. (*b*) Using the formula, $m = ZI$, where m = mass of silver deposited = 0.05% of 750 g = 0.375 g = 3.75×10^{-4} kg

Current passing through it, $I = 0.8$ A

$$Z = \text{ECE of silver} = 11.8 \times 10^{-7}\ \text{kgc}^{-1}$$

∴ The time needed for depositing silver is given by

$$t = \frac{m}{ZI} = \frac{3.75 \times 10^{-4}}{11.8 \times 10^{-7} \times 0.8} = 397\,\text{s} = 6\,\text{min}\ 37\,\text{s}.$$

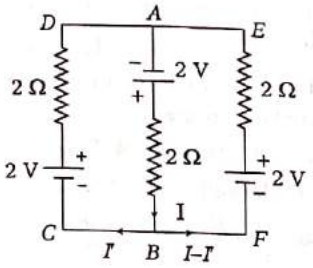
20. (*c*) The shunt resistance required is given by

$$S = \left(\frac{I_{dc}}{I - I_{dc}}\right) R_{dc} = \left(\frac{100}{500 - 100}\right) \times 0.1 = \frac{100}{400} \times 0.1$$

$\therefore \quad S = 0.025\,\Omega$

If this value of shunt resistance is connected in parallel with the DC ammeter, then the range will be extended to 0-500 A.

21. (*a*) The circuit diagram can be redrawn as the potential between A and B is



For the loop $ABCDA$, $+2-2I'+2-2I=0$ ...(i)

For the loop $ABFEA$,

$$2-2I+2-2(I-I')=0$$
$$4-2I-2I+2I'=0$$
$$4-4I+2I'=0$$
$$2=2I-I' \quad ...(ii)$$

From Eqs. (i) and (ii), we get

$$2=2I-I',\ 2=I+I'$$
$$4=3I,\ I=4/3=1.3\text{ A}$$

22. (*c*) As we know that, $R_1=\rho\dfrac{l}{a}=\rho\dfrac{l^2}{V}$

where, l = length of wire

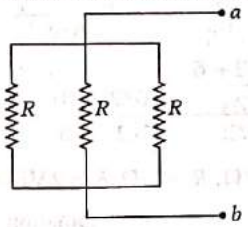
a = area of cross-section of the wire

and V = volume of the wire.

As, $R_1 \propto l^2$

$$\Rightarrow \quad \frac{R_1}{R_2}=\left(\frac{l_1}{l_2}\right)^2=\left(\frac{1}{2}\right)^2 \Rightarrow R_2:R_1=4:1$$

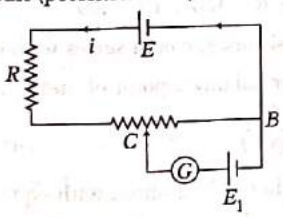
23. (*c*) The combination of resistors can be radrawn as



Therefore, equivalent resistance is given by

$$\frac{1}{R_{eq}}=\frac{1}{R}+\frac{1}{R}+\frac{1}{R} \Rightarrow R_{eq}=\frac{R}{3}$$

24. (*c*) See the circuit (potentiometer)



Hence, $V_{CB}=E$ (if no current is drawn from the battery)

$$\Rightarrow \quad \left[\frac{E_1}{R+R_{AB}}\right]R_{CB}=E$$

Given, $E_1=2\text{ V}$, $R_{AB}=10\,\Omega$

$$\therefore \quad R_{CB}=\frac{40}{100}\times 10=4\,\Omega \text{ and } E=10\times 10^{-3}\text{ V}$$

On solving Eq. (i), we get $R=790\,\Omega$

25. (*d*) We know that potential drop across a resistance,

$$V=IR \Rightarrow V=\text{constant}$$

i.e., $$I\propto\frac{1}{R}$$

$$I_1:I_2:I_3=\frac{1}{R_1}:\frac{1}{R_2}:\frac{1}{R_3}=\frac{1}{2}:\frac{1}{3}:\frac{1}{4}=6:4:3$$

As, $V\propto I$

$\therefore$ $V_1:V_2:V_3=6:4:3$

26. (*a*) Total internal resistance does not change

$\therefore$ $R'=4r$

Net emf $E'=E(n-2m)$

Here, n = total number of cells = 4

m = wrong connection = 1

$E'=E[4-2]$, *i.e.*, $E'=2E$

27. (*b*) Given, $I=3\text{ A}$, $V=6\text{ V}$

We know that, $V=IR$

$$R=\frac{V}{I} \Rightarrow R=\frac{6}{3},\ R=2\,\Omega$$

If the ammeter and voltmeter have resistance, *i.e.*, $R<2\,\Omega$

28. (*b*) Given, $R_1=100\,\Omega$

$$R_2=200\,\Omega \Rightarrow T_1=100°\text{ C}$$
$$\alpha=0.005\text{ per °C}$$

We know that new resistance of the bulb filament,

$$R_2=R_1[1+\alpha(T_2-T_1)]$$
$$200=100\,(1+0.005\,(T_2-100)]$$
$$2=1+[0.005\,(T_2-100)]$$
$$0.005\,(T_2-100)=1$$
$$T_2-100=\frac{1}{0.005} \Rightarrow T_2-100=\frac{1000}{5}$$
$$T_2-100=200 \Rightarrow T_2=200+100=300°\text{ C}$$

29. (*d*) Consider the ring as two parts. As two resistances joined in parallel between two points A and B, then two resistances would be

$$R_1=\frac{R}{2\pi r}\cdot r\theta=\frac{R}{2\pi}\theta$$

and $$R_2=\frac{R}{2\pi r}r(2\pi-\theta)=\frac{R}{2\pi}(2\pi-\theta)$$

Now, equivalent or effective resistance between A and B,

$$R_{eq}=\frac{R_1\times R_2}{R_1+R_2}$$

$$\Rightarrow \quad R_{eq}=\frac{\frac{R}{2\pi}\theta\times\frac{R}{2\pi}(2\pi-\theta)}{\frac{R}{2\pi}[\theta+2\pi-\theta]}=\left[\frac{\frac{R^2\theta(2\pi-\theta)}{4\pi^2}}{\frac{2\pi R}{2\pi}}\right]$$

$$=\frac{R^2\theta(2\pi-\theta)}{4\pi^2}\times\frac{2\pi}{2\pi R}=\frac{R\theta(2\pi-\theta)}{4\pi^2}$$

30. (*b*) As, mobility $= \dfrac{\text{drift velocity}}{\text{electric field}}$

$$\Rightarrow \mu = \frac{v_d}{E} = \frac{\text{metre}^2}{\text{volt-second}} = \frac{[L]^2}{\text{(joule/coulomb)-second}}$$

$$= \frac{[L]^2}{\dfrac{[\text{kg-metre}^2\text{-sec}^{-2}]\text{-second}}{\text{ampere-second}}}$$

$$= \frac{[L^2]}{[ML^2T^{-2}A^{-1}]} = [M^{-1}T^2A]$$

32. (*b*) Resistance of wire, $R = \rho\dfrac{l}{A}$

Given, $R_1 = 4\Omega,\ l_1 = l,\ A_1 = A,\ R_2 = ?$

$l_2 = 2l,\ A_2 = A/2,\ \rho = \text{constant}$

$$\therefore \quad \frac{R_1}{R_2} = \frac{l_1/A_1}{l_2/A_2} = \frac{l_1 \times A_2}{l_2 \times A_1}$$

$$\Rightarrow \quad \frac{4}{R_2} = \frac{l \times A/2}{2l \times A} \Rightarrow \frac{4}{R_2} = \frac{1}{4} \Rightarrow R_2 = 16\Omega$$

33. (*c*) Corresponding to the colours of the first and second bands *i.e.*, green and black, the figures are 5 and 0.

Corresponding to the colour of third band, *i.e.*, violet, the multiplier is 10^7. Therefore, the value of the resistance is $50 \times 10^7\ \Omega$. The gold colour of the fourth band indicates the tolerance of ±5%.

So, the value of the resistor is written as

$$50 \times 10^7\ \Omega \pm 5\% = 500 \times 10^6\Omega + 5\% = 500 \pm 5\%\ \text{M}\Omega$$

34. (*c*) The required resistance $= \dfrac{\dfrac{R}{2} \times \dfrac{R}{2}}{\dfrac{R}{2} + \dfrac{R}{2}} = \dfrac{R}{4} = \dfrac{9}{4}\Omega$

35. (*c*) Let the values of resistances be R_1 and R_2, respectively.

When R_1 and R_2 resistances are in series, then

$$R_1 + R_2 = 6 \text{ (according to question)} \quad \text{...(i)}$$

When R_1 and R_2 resistances are parallel, then

$$\frac{R_1 R_2}{R_1 + R_2} = \frac{4}{3} \quad \text{...(ii)}$$

From the Eq. (i), we get

$$\frac{R_1 R_2}{6} = \frac{4}{3} \Rightarrow R_1 R_2 = 4 \times 2 \Rightarrow R_1 R_2 = 8 \quad \text{...(iii)}$$

We know that,

$$R_1 - R_2 = \sqrt{(R_1 + R_2)^2 - 4R_1R_2} = \sqrt{36 - 4 \times 8}$$

$$\Rightarrow \quad R_1 - R_2 = \sqrt{4}$$

$$\Rightarrow \quad R_1 - R_2 = 2 \quad \text{...(iv)}$$

From Eqs. (i) and (iv), we get

$$R_1 = 4\Omega,\ R_2 = 2\Omega$$

36. (*d*) Given, 6Ω, 6Ω and 6Ω are in parallel.

So, $\quad \dfrac{1}{R} = \dfrac{1}{6} + \dfrac{1}{6} + \dfrac{1}{6} = \dfrac{3}{6}$

$\therefore \quad R = \dfrac{6}{3} = 2\Omega$

Now, 6Ω and 6Ω are in series.

So, $\quad R' = 6 + 6 = 12\Omega$

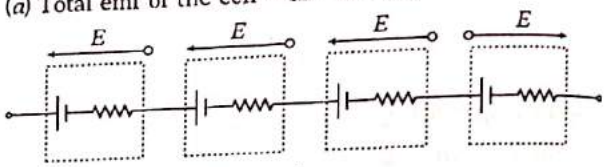
$$\therefore \quad \frac{1}{R''} = \frac{1}{R'} + \frac{1}{12} = \frac{1}{12} + \frac{1}{12} = \frac{2}{12} \Rightarrow R'' = 6\Omega$$

The required resistance of the circuit $= 2 + 6 = 8\Omega$.

So, the potential difference,

$$V_A - V_B = iR = 0.5 \times 8 = 4\text{ V}$$

37. (*a*) Total emf of the cell $= 3E - E = 2E$



Total internal resistance $= 4r$

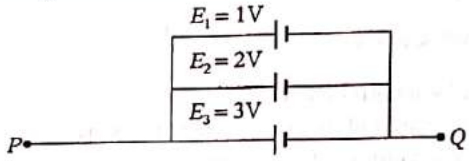
$\therefore$ Total resistance of the circuit $= 4r + R$

So, the current in the external circuit

$$i = \frac{2E}{4r + R} \qquad \left(\because i = \frac{V}{R}\right)$$

38. (*b*) As, resistances 1Ω, 2Ω and 1Ω are in parallel.

So, the required internal resistance,



$$\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{3} = \frac{1}{1} + \frac{1}{2} + \frac{1}{1} = \frac{2 + 1 + 2}{2} \Rightarrow r = \frac{2}{5}\Omega$$

The potential difference between points P and Q,

$$E = \frac{\dfrac{E_1}{r_1} + \dfrac{E_2}{r_2} + \dfrac{E_3}{r_3}}{1/r} = \frac{\dfrac{1}{1} + \dfrac{2}{2} + \dfrac{3}{1}}{5/2}$$

$$= \frac{\dfrac{2 + 2 + 6}{2}}{5/2} = \frac{10/2}{5/2} = \frac{10}{5} \times \frac{2}{2} = 2\text{ V}$$

39. (*d*) Given, $R_1 = 2\Omega$, $R_2 = 6\Omega$, $E = 2\text{ V}$, $r = 0.5\Omega$, $I = ?$

$\because \quad R_1, R_2$ are in parallel combination.

So, $\quad \dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2} = \dfrac{1}{2} + \dfrac{1}{6} = \dfrac{3+1}{6} \Rightarrow R = \dfrac{6}{4} = 15\Omega$

Then, the current in the circuit,

$$I = \frac{E}{r + R} = \frac{2}{0.5 + 15} = \frac{2}{20} = 1\text{A}$$

(as internal resistance, r is in series with other resistance)

40. (*b*) **Case I** For balanced point of meter bridge,

$$\frac{5}{R} = \frac{l_1}{100 - l_1} \quad \text{...(i)}$$

Case II When R is shunted with equal resistance, *i.e.*, R

$$\frac{1}{R'} = \frac{1}{R} + \frac{1}{R} \Rightarrow R' = R/2$$

$$\therefore \quad \frac{5}{R/2} = \frac{16\,l_1}{100 - 16\,l_1} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$2\left(\frac{l_1}{100-l_1}\right)=\frac{16\,l_1}{100-16\,l_1}\Rightarrow l_1=25\text{ cm}$$

From Eq. (i), $11\frac{5}{R}=\frac{25}{75}\Rightarrow R=15\,\Omega$

41. (*c*) As, r = internal resistance,

and $r=R_1\left(\frac{l-l_1}{l_1}\right)\Rightarrow\frac{rl_1}{R_1}=l-l_1$ ...(i)

Also, $r=R_2\left(\frac{l-l_2}{l_2}\right)\Rightarrow\frac{rl_2}{R_2}=l-l_2$ ...(ii)

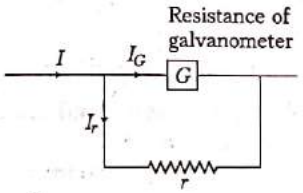
Subtracting Eq. (ii) from Eq. (i), we get

$$l-l_2-l+l_1=r\left[\frac{l_2}{R_2}-\frac{l_1}{R_1}\right]$$

As, $R_1=\infty\Rightarrow l_1=3\text{ m}$ and $R_2=5\Rightarrow l_2=2.85\text{ cm}$

$\Rightarrow\quad(3-2.85)=r\left[\frac{2.85}{9.5}-0\right]\Rightarrow r=0.5\,\Omega$

42. (*a*) As, current $I_G=\frac{2}{1000}I$



Also, $I_r=\left(\frac{G}{G+r}\right)I$

We know that potential across G and shunt r are same.

$\therefore\quad V_G=V_r\Rightarrow I_G(G)=I_r\,r$

$\Rightarrow\quad\frac{2IG}{1000}=\frac{GI}{G+r}(r)\Rightarrow 2(G+r)=1000\,r$

$\Rightarrow\quad G+r=500\,r\Rightarrow\frac{G}{r}+1=500$

$\Rightarrow\quad G/r=499\Rightarrow r=\frac{1}{499}G$

43. (*a*) Balanced condition for Wheatstone bridge,

$$\frac{P}{Q}=\frac{R}{S}\quad\text{...(i)}$$

where, $\frac{1}{s}=\frac{1}{s_1}+\frac{1}{s_2}\Rightarrow s=\frac{s_1s_2}{s_1+s_2}$ ...(ii)

From Eqs. (i) and (ii), we get

$$\frac{P}{Q}=\frac{R(s_1+s_2)}{s_1s_2}$$

44. (*b*) Force experienced by electron, $F=qE$

where, E = electric field intensity.

$\therefore\quad 24\times10^{-19}=16\times10^{-19}E\Rightarrow E=15\text{ NC}^{-1}$

Moreover, E = potential gradient

$$E=\frac{dV}{dl}\Rightarrow E\times l=V\Rightarrow 15\times6=9\text{ V}$$

45. (*b*) Resistance, $R=\frac{V}{i_g}-G=\frac{120}{0.01}-10=12000-10$

$$R=11990\,\Omega$$

To convert a galvanometer into a voltmeter, high resistance of value $11990\,\Omega$ should be connected in parallel.

46. (*b*) Given, first balancing length $l_1=30$ cm

Second balancing length, $l_2=40\text{ cm}\Rightarrow E_1=125\text{ V}$

$E_2=?$

So, according to the principle of potentiometer,

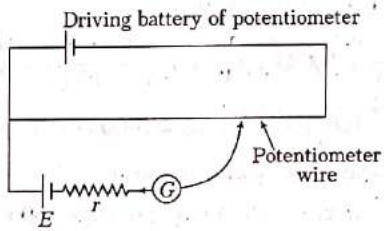
$E_1=Kl_1$ ...(i)

and $E_2=Kl_2$ ...(ii)

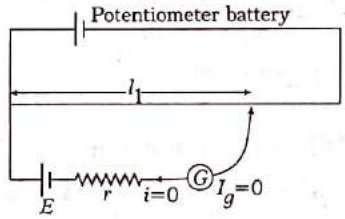
$\because\quad\frac{E_1}{E_2}=\frac{Kl_1}{Kl_2}\Rightarrow\frac{125}{E_2}=\frac{30}{40}$

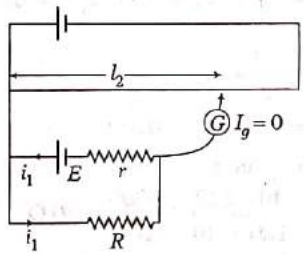
$\Rightarrow\quad E_2=\frac{125\times40}{30}\Rightarrow E_2=\frac{5}{3}=1666\text{ V}\approx1.67\text{ V}$

47. (*a*) Potentiometer measures the potential difference more accurately than a voltmeter because while measuring emf (electromotive force), it does not draw any current from the source of known emf. Potentiometer has its own battery which maintains potential drop across the potentiometer wire.



48. (*d*) As, $kl_1=E-ri=E-r(0)=E$





$\Rightarrow\quad kl_1=E$ ...(i)

and $kl_2=E-ri_1=Ri_1$

$\Rightarrow\quad i_1=\frac{E}{R+r}\Rightarrow kl_2=\frac{RE}{R+r}$ ...(ii)

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{l_1}{l_2}=\frac{R+r}{R}=1+\frac{r}{R}\Rightarrow r=R\left(\frac{l_1}{l_2}-1\right)$$

We know that for internal resistance,

$$r = \left(\frac{l_1}{l_2} - 1\right)$$

where, l_1 = balancing length of potentiometer wire = 240 cm

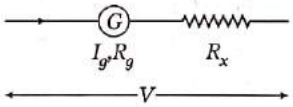
l_2 = balancing length after shunting = 120 cm

R = shunting resistance = $2\,\Omega$.

Putting all the values, we get

Internal resistance, $r = \left(\frac{240-1}{120}\right) \times 2 = (2-1) \times 2 = 2\,\Omega$

49. (*b*) Let, R_x resistance be connected in series to convert galvanometer to voltmeter.



According to the equation, $R_g = 100\,\Omega$, $I_g = 30$ mA

Current corresponding to full scale deflection.

Now, we can write, $V = I_g \times R_g + I_g R_x = I_g(R_g + R_x)$ ...(i)

Given, $V = 30$

From Eq. (i), we get

$$30 = (30 \text{ mA})\,(100\,\Omega + R_x)$$

$$\Rightarrow \frac{30}{(30 \text{ mA})} = 100\,\Omega + R_x \Rightarrow \frac{30\text{V}}{30 \times 10^{-3}\text{A}} = 100\,\Omega + R_x$$

$$\Rightarrow 10^3\,\Omega = 100\,\Omega + R_x \Rightarrow R_x = 1000\,\Omega - 100\,\Omega = 900\,\Omega$$

50. (*b*) By Faraday's law of electrolysis, $m = Zit$

Given, $m_1 = m$ gram, $i_1 = 4$ A, $i_2 = 6$ A, $t_1 = 120$ s

$t_2 = 40$ s, $m_2 = ?$

$$\because \quad \frac{m_1}{m_2} = \frac{i_1 t_1}{i_2 t_2} \Rightarrow \frac{m}{m_2} = \frac{4 \times 120}{6 \times 40}$$

$$\Rightarrow \quad m_2 = \frac{m \times 6 \times 40}{4 \times 120} = \frac{m}{2}$$

51. (*d*) As $R \propto l^2$ (when wire is stretched),

$$\therefore \quad R' = n^2 R = 2^2 \times 4 = 4 \times 4 = 16\,\Omega$$

52. (*b*) As $I = \frac{E}{R+r}$ or $E = I(R+r) \Rightarrow 21 = 0.2\,(10 + r)$

$$10 + r = \frac{21}{2} \times 10$$

$$\therefore \quad r = 10.5 - 10 = 0.5\,\Omega$$

53. (*b*) Effective resistance,

$$R_{\text{eff}} = \frac{40 \times 120}{120 + 40} = \frac{4800}{160} = 30\,\Omega$$

$\therefore$ Current $\quad I = \frac{7}{(30+5)} = \frac{7}{35} = 0.2$ A $\quad \left(\because I = \frac{E}{R+r}\right)$

54. (*b*) Current, $I = qf$

Given, $q = 16 \times 10^{-19}$ C and $f = 10^{19}$

$\therefore \quad I = 1.6$ A

55. (*c*) $\quad v_d = \frac{I}{nAe}$

Given, $\quad I = 2$ A, $n = 5.86 \times 10^{28}\,\text{m}^{-3}$

$$A = \pi r^2 = \pi(0.1 \times 10^{-2})^2,\ e = 16 \times 10^{-19}\text{C}$$

$\therefore$ Drift velocity, $v_d = 0.68 \times 10^{-4}$ ms^{-1}

56. (*a*) As $R \propto l/d^2 \Rightarrow \frac{R_2}{R_1} = \frac{l_2 d_1^2}{l_2 d_2^2} = 25.2\,\Omega$

57. (*d*) Suppose m rows are connected in parallel and each row contains n identical cells (each cell having $E = 1.5$ V and $r = 1\,\Omega$)

For maximum current in the external resistance R, the necessary condition

$$R = \frac{nr}{m}$$

$$1.5 = \frac{n \times 1}{m}$$

$$1.5\,m = n \quad \text{...(i)}$$

Total cells $= 24 = n \times m$ ...(ii)

On solving Eqs. (i) and (ii), we get

$$m = 4$$

$$n = 4$$

Therefore, 6 cells in each row are connected in series and 4 such rows are connected in parallel.

58. (*d*) $\quad R_1 = R_0\,(1 + \alpha t)$

$$1 = R_0\,(1 + 0.00125 \times 27) \quad \text{...(i)}$$

$$2 = R_0\,(1 + 0.00125 \times t) \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$t = 848°\,\text{C} = 1121\text{K}$$

Note None of the four options are correct, but option (*d*) is close to correct answer.

59. (*d*) We know that,

$$i = \frac{E}{r + R_{\text{eff}}} = \frac{15}{0.15 + 14.5} = \frac{15}{14.65} \cong 1\text{A}$$

60. (*b*) Given, temperature coefficient of carbon,

$$\alpha_1 = 4 \times 10^{-3}/°\,\text{C}$$

Temperature coefficient of copper,

$$\alpha_2 = -0.5 \times 10^{-3}/°\,\text{C}$$

Hence, $R_1\alpha_1 = -R_2\alpha_2$

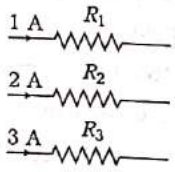
$$\frac{R_2}{R_1} = \frac{-4 \times 10^{-3}}{0.5 \times 10^{-3}} \quad \Rightarrow \quad \frac{R_2}{R_1} = \frac{8}{1}$$

61. (*) Let the three conductors are R_1, R_2 and R_3 and the current drawn by them are 1A, 2 A and 3 A respectively, when connected in turn across a battery.

$\therefore \quad V = R_1 \quad V = 2R_2$ and $V = 3R_3$

So, $\quad R_1 = V,\ R_2 = \frac{V}{2}$ and $R_3 = \frac{V}{3}$

When the conductors are connected in series, across the same battery, then



$$V = I[R_1 + R_2 + R_3]$$

$$\Rightarrow \quad V = I\left[V + \frac{V}{2} + \frac{V}{3}\right] \Rightarrow 1 = I\left[\frac{6+3+2}{6}\right] \text{ or } I = \frac{6}{11}\text{A}$$

62. (*d*) $R \propto l^2/m \Rightarrow R_1 : R_2 : R_3 = \frac{l_1^2}{m_1} : \frac{l_2^2}{m_2} : \frac{l_3^2}{m_3}$

$$= \frac{25}{1} : \frac{9}{3} : \frac{1}{5} = 125 : 15 : 1$$

63. (*b*) We have, $I_1 = \frac{I \times 2R}{3R} = \frac{2I}{3}$

$\therefore \quad H_1 = I_1^2 R = \frac{4I^2}{9} \times R$ ...(i)

Also, $I_2 = \frac{I \times R}{3R} = \frac{I}{3}$

$\therefore \quad H_2 = I_2^2 (2R) = \frac{I^2}{9} \times 2R$ ...(ii)

and $H_3 = I^2 (1.5R)$ ...(iii)

From Eqs. (i), (ii) and (iii), we get

$$H_1 : H_2 : H_3 = \frac{4I^2}{9} \times R : \frac{I^2}{9} \times 2R : I^2 \times 1.5R$$

$$= \frac{4}{9} : \frac{2}{9} : 1.5 = 4 : 2 : 13.5 = 8 : 4 : 27$$

64. (*a*) A milliammeter will have greater resistance than ammeter because (1) number of turns in the coil in milliammeter is increased to compensate low value of current to create large deflection. (2) milliammeter does not have usually a shunt in parallel because main current is very small. Ammeter usually have a shunt in parallel which makes overall resistance low.

65. (*c*) We know that the resistance of any wire,

$$R = \frac{\rho L}{A}$$

In given case $L_1 = L$, $L_2 = 2L$, ρ and A are constants.

Hence, $\frac{R_1}{R_2} = \frac{L_1}{L_2} \Rightarrow \frac{R_1}{R_2} = \frac{L}{2L} \Rightarrow \frac{R_1}{R_2} = \frac{1}{2}$

66. (*c*) Power when connected in series,

$$\frac{1}{P} = \frac{1}{P_1} + \frac{1}{P_2}$$

Given, $P_1 = 60$ W and $P_2 = 100$ W

Hence, $P = \frac{60 \times 100}{60 + 100} \Rightarrow P = \frac{6000}{160} \Rightarrow P = 37.5$ W

67. (*b*) We know drift speed, $v_\alpha = \frac{eE\tau}{m} = \frac{e\tau}{m}\frac{v}{l}$

Here, $v_\alpha \propto v$

Given, condition $v_1 = v'$ and $V_2 = 2v$

Hence, $\frac{v_\alpha}{vV_\alpha} = \frac{v}{2v} \Rightarrow v'_\alpha = 2v_\alpha$

68. (*c*) Potential across potentiometer wire

$$= V \times I = \frac{100 \times 10^{-3} \times 10}{10^{-2}} = 0.1\text{V} \Rightarrow 0.1 = \frac{R}{100 + R} \times 2$$

Resistance, $R = 190\,\Omega$

69. (*d*) Specific resistance does not depend upon length.

70. (*c*) $7\,\Omega$ and $3\,\Omega$ are in series, so total $10\,\Omega$ and $10\,\Omega$ are in parallel, so total resistance = 5Ω. Now, $5\,\Omega$ and $5\,\Omega$ are in series, so they make $10\,\Omega$, $10\,\Omega$ and $10\,\Omega$ are in parallel. So, $R_{net} = \frac{10}{3}\Omega$.

71. (*a*) Equivalent resistance in series $= 6 + 9 = 15\Omega$

Current flow in circuit, $i = \frac{V}{R} = \frac{120}{15} = 8$ A

Voltage in $6\,\Omega$ resistor, $V = iR = 8 \times 6 = 48$ V.

Power consumed by $6\,\Omega$ resistor,

$$P = \frac{V^2}{R} = \frac{48 \times 48}{6} = 8 \times 48 = 384 \text{ W}$$

72. (*a*) Current through each arm DAC and $DBC = 1$A

$$V_D - V_A = 2 \text{ and } V_D - V_B = 3$$

$$\therefore \quad V_A - V_B = 1\text{V}$$

73. (*b*) Heat $= i^2Rt$

$$\therefore \quad \left(\frac{E}{R_1 + r}\right)^2 R_1 = \left(\frac{E}{R_2 + r}\right)^2 R_2 \quad [\because t \text{ is the same}]$$

On solving, we get

$$(R_1 - R_2)r^2 = (R_1 - R_2)R_1R_2 \Rightarrow r = \sqrt{R_1 R_2}$$

74. (*a*) We have, $R = \frac{V}{I} = \frac{20}{2.5} = 8\,\Omega$, $\frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I}$

$$= \frac{1}{20} + \frac{0.50}{2.5} = \frac{1}{4}$$

$$\Delta R = \frac{R}{4} = \frac{8}{4} = 2\,\Omega \Rightarrow R = (8 \pm 2)\,\Omega$$

75. (*c*) We have $R = \frac{nr}{m}$, so grouping of cells depends upon the relative values of internal and external resistance.

76. (*a*) As the current in heater filament increases, it gets more heated. Hence, its temperature increases and resistance also increases, due to which the current decreases.

77. (*b*) In series, $P = \frac{P_1P_2}{P_1 + P_2}$

Given, $P_1 = P_2 = 60$

$\therefore$ Power, $P = \frac{60 \times 60}{60 \times 60} = 30$ W

79. (*b*) To convert a galvanometer into an ammeter, a resistance (shunt) is required, connected in parallel to the galvanometer.

By using, $S = \frac{Gi_g}{i - i_g} = \frac{50 \times 100 \times 10^{-6}}{(10 - 100 \times 10^{-6})} \approx 5 \times 10^{-4}\Omega$

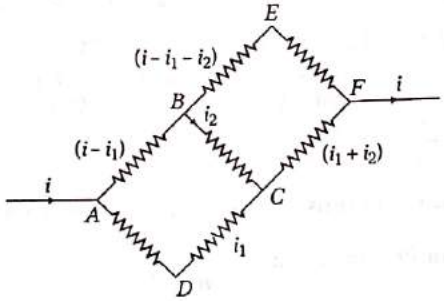
80. (*d*) By using the relation,

$$\frac{E_1}{E_2} = \frac{l_1 + l_2}{l_1 - l_2} = \frac{(8+2)}{(8-2)} = \frac{5}{3} \qquad \begin{pmatrix} E' + E'' = E_1 \\ E' - E'' = E_2 \end{pmatrix}$$

81. (*b*) The current distribution in the circuit can be shown as Applying Kirchhoff's law in mesh $ABCDA$,

$$-10(i - i_1) - 10\,i_2 + 20\,i_1 = 0$$

$$3i_1 - i_2 = i \quad \text{...(i)}$$



and in Mesh *BEFCB*

$$-20\,(i-i_1-i_2)+10(i_1+i_2)+10i_2=0$$

$\Rightarrow$ $$3i_1+4i_2=2i \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$i_1=\frac{2i}{5},\ i_2=\frac{i}{5},\ i_{AD}=\frac{2i}{5}$$

82. (*d*) The drift velocity, $v_d=\dfrac{J}{ne}$

(where, J is current density $=\dfrac{i}{A}$)

So, $v_d \propto j$

$\therefore$ $J_1=\dfrac{i}{A}$ and $J_2=\dfrac{2i}{2A}=\dfrac{i}{A}=J_1$

So, $(v_d)_i=(v_d)_2=v$

83. (*b*) As resistance, $R \propto$ length

$\therefore$ Resistance of each arm $=\dfrac{12}{3}=4\,\Omega$

So, effective resistance between any two corners,

$$R_{\text{eff}}=\frac{4\times 8}{4+8}=\frac{32}{12}=\frac{8}{3}\,\Omega$$

84. (*b*) Applying Kirchhoff's law, $500I+100I=12$

So, $I=\dfrac{12\times 10^{-2}}{6}=2\times 10^{-2}\text{A}$

Hence, $V_B=100\,(2\times 10^{-2})=2\,\text{V}$

85. (*a*) The full scale deflection current,

$$i_g=\frac{25\text{ mV}}{G}\text{ ampere}$$

where, G is the resistance of the meter.

The value of shunt required for converting it into ammeter of range 25 A is $S=\dfrac{i_gG}{i-i_g}$.

So that $S=\dfrac{25\text{ mV}}{25}=0.001\,\Omega$

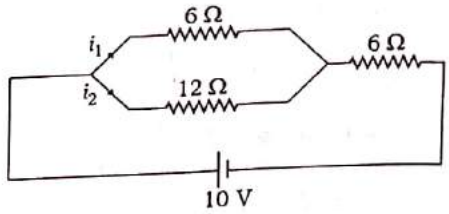
86. (*c*) Power, $P=\dfrac{V^2}{R}$

For small variation, $\dfrac{\Delta P}{P}\times 100\%=\dfrac{2\times\Delta V}{V}\times 100\%$

$$=2\times 2.5=5\%$$

Therefore, power would decrease by 5%.

87. (*a*)



Resistance, $R=\dfrac{6\times 12}{6+12}=\dfrac{6\times 12}{18}=4\,\Omega$

Total resistance,

$$R_{\text{eq}}=6+4=10\,\Omega$$

Current, $i=\dfrac{V}{R}=\dfrac{10}{10}=1\text{A}$

The current in $12\,\Omega$ resistor is

$$i_2=i\left(\frac{R_1}{R_1+R_2}\right)=1\times\left(\frac{6}{6+12}\right)\Rightarrow i_2=\frac{1}{3}$$

The potential difference in $12\,\Omega$ resistor,

$$V=i\times 2R=\frac{1}{3}\times 12=4\,\text{V}$$

88. (*b*) $I_g=\dfrac{2}{300}\text{A}=\dfrac{2}{300}\times 1000\text{ mA}=\dfrac{20}{3}\text{ mA}$

$$I_g=6.67\text{ mA}$$

As range of ammeter cannot be decreased but can be increased only, therefore, the instrument cannot be converted to measure the range 1 mA.

89. (*a*) The balanced condition for Wheatstone bridge is $\dfrac{P}{Q}=\dfrac{R}{S}$.

As is obvious from the given values, no, current flows through galvanometer is zero.

Now, P and R are in series, so

Resistance, $R_1=P+R=10+15=25\,\Omega$

Similarly, Q and S are in series, so

Resistance, $R_2=R+S=20+30=50\,\Omega$

Net resistance of the network as R_1 and R_2 are in parallel,

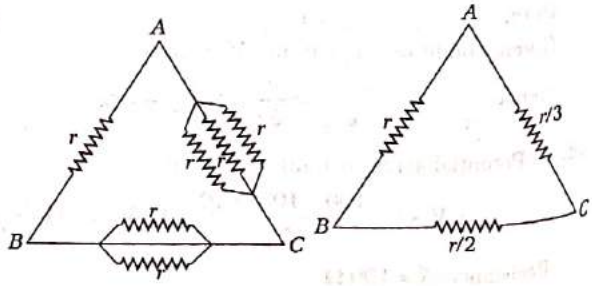
$$i=\frac{V}{R}=\frac{0.6}{50}=0.36\text{ A}$$

90. (*b*) Drift velocity, $v_d=\dfrac{I}{neA}$

$\Rightarrow$ $$v_d=\frac{5}{(5\times 10^{26})\times(16\times 10^{-19})\times(4\times 10^{-6})}$$

$$=1.56\times 10^{-2}\text{ ms}^{-1}$$

91. (*c*)



Resistance between points A and B,

$$R_{AB} = r \| \left(\frac{r}{3} + \frac{r}{2}\right) = \frac{\left(r \times \frac{5}{6} r\right)}{\left(r + \frac{5}{6} r\right)} = \frac{5}{11} r$$

Resistance between points B and C,

$$R_{BC} = \frac{r}{2} \| \left(r + \frac{r}{3}\right) = \frac{\frac{r}{2} \times \frac{4r}{3}}{\frac{r}{2} + \frac{4r}{3}} = \frac{4}{11} r$$

Resistance between points C and D,

$$R_{CA} = \frac{r}{3} \| \left(\frac{r}{2} + r\right) = \frac{\left(\frac{r}{3} \times \frac{3r}{2}\right)}{\frac{r}{3} + \frac{3r}{2}} = \frac{3}{11} r$$

$$R_{AB} : R_{BC} : R_{CA} = 5 : 4 : 3$$

92. (*b*) Given, $A = 0.3\ \text{m}^2$, $n = 2 \times 10^{25}\ /\text{m}^3$

$$q = 3t^2 + 5t + 2$$

$$i = \frac{dq}{dt} = 6t + 5 = 17$$

We have current, $i = ne A v_d$

Drift velocity,

$$v_d = \frac{i}{ne A} = \frac{17}{2 \times 10^{25} \times 1.6 \times 10^{-19} \times 0.3}$$

$$= \frac{17}{0.96 \times 10^{-6}} = 1.77 \times 10^{-5}\ \text{ms}^{-1}$$

93. (*a*) Given circuit is a balanced Wheatstone bridge. So, diagonal resistance of $2\,\Omega$ will be ineffective.

Equivalent resistance of upper arm $= 2 + 2 = 4\Omega$

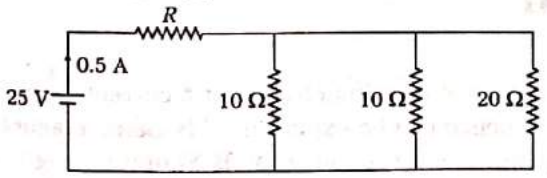
Equivalent resistance of lower arm $= 2 + 2 = 4\Omega$

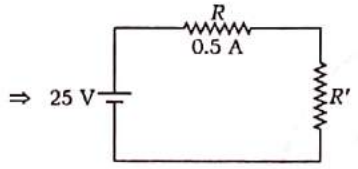
$$\therefore \quad R_{AB} = \frac{4 \times 4}{4 + 4} = 2\Omega$$

94. (*b*) In open circuit, current through the circuit $i = 0$

Potential difference between x and y will be $V = E = 120$ V

95. (*d*) $\frac{1}{R'} = \frac{1}{10} + \frac{1}{10} + \frac{1}{20}$





$$\Rightarrow \quad R' = \frac{20}{5} = 4\Omega$$

Now, using Ohm's law, $i = \frac{25}{R + R'}$

$$\Rightarrow \quad 0.5 = \frac{25}{R + 4}$$

$$\Rightarrow \quad R + 4 = \frac{25}{0.5}$$

$$\Rightarrow \quad R = 50 - 4 = 46\Omega$$

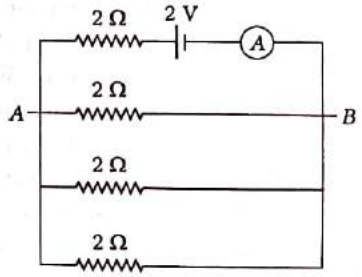
Current through $20\,\Omega$ resistor $= \frac{0.5 \times 5}{20 + 5} = \frac{2.5}{25} = 0.1\text{A}$

Potential difference across middle resistor

= Potential difference across $20\,\Omega = 20 \times 0.1 = 2$ V

96. (*b*) The given circuit can be shown as the resistance between A and $B = \frac{2}{3}\Omega$.

Total resistance of the circuit



$$= 2 + \frac{2}{3} = \frac{8}{3}\Omega$$

So, the reading of ammeter,

$$i = \frac{V}{R} = \frac{2}{8/3} = \frac{6}{8} = \frac{3}{4}\text{A}$$

97. (*d*) The resistances of branches ABC and ADC are in parallel. The current entering at node A will equally divided in each parallel branch.

$$\therefore \quad V_A - V_B = IR = 1 \times 2 = 2\text{ V}$$

Point A is at greater potential with respect to point B, so

$$V_A - V_B = +2\text{ V}$$

CHAPTER

4

Magnetic effect of current and moving charges

In earlier unit, we have studied about the electric current, its measurement and the thermal effects of current. In this unit, we shall study about magnetic effects of current. Earlier, it was thought that there is no connection between electricity and magnetism. However in the year 1820, Oersted realised that electricity and magnetism were related to each other. He showed experimentally that the electric current through a straight wire causes noticeable deflection of the magnetic compass needle held near the wire. He also found that the iron fillings sprinkled around the wire, arrange themselves in concentric circles with the wire as centre in the plane perpendicular to the wire. This shows that the magnetic field is associated with a moving charge or a current carrying conductor. The branch of physics which deals with the magnetism due to electric current is called **electromagnetism.**

Inside

1) Magnet field
- Biot-Savart's law (magnetic field due to a current carrying conductor)
- Application of Biot-Savart's law

2) Ampere's circuital law
- Applications of ampere's circuital law

3) Force on a moving charge in a uniform magnetic field

4) Force on a current carrying conductor in a magnetic field
- Force between two parallel current carrying conductors
- Magnetic force between two moving charges
- The moving coil galvanometer (MCG) current sensitivity

1 Magnetic field

The space in the surroundings of a magnet, moving charges or a current carrying conductor in which its magnetic influence can be experienced is called **magnetic field.** Magnetic field is a vector quantity and is denoted by **B.** SI unit of magnetic field is weber/m^2 or tesla (T) and its CGS unit is gauss or maxwell/cm^2, where 1 tesla $= 10^4$ gauss.

Magnetic field due to moving charges (Oersted's experiment)

Oersted, found experimentally that a magnetic field is established around a current carrying conductor just as it occur around a magnet. In the experiment, he placed a magnetic compass near a current carrying conductor and observed that the needle of the compass shows deflection. Also, when the current is reversed, the needle shows deflection in opposite direction.

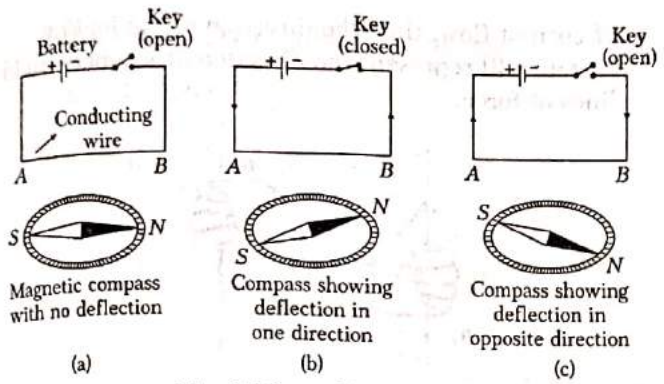


Fig. 4.1 Oersted's experiment

Thus, from the above figure, he concluded that a magnetic field is established around a current carrying wire because of which such deflections are visible.

Oersted experiment shows the connectivity between the magnetic field and electric current and as electric current means moving charges, so it can also be concluded that moving charges also produce magnetic field in their surroundings.

Biot-Savart's law (magnetic field due to a current carrying conductor)

Biot-Savart's law is an experimental law predicted by Biot and Savart. This law deals with the magnetic field produced at a point due to a small current element (a part of any conductor carrying current).

Consider a wire XY carrying current I. Let dl be infinitesimal element of the conductor, dB be magnetic field at point P at a distance r from the element.

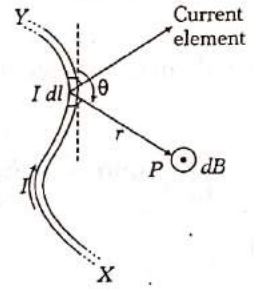


Fig. 4.2 A long current carrying conductor

Biot-Savart's law states that, the magnitude of the magnetic field induction ($d\mathbf{B}$) at the point due to the current element is

- directly proportional to the current flowing through the conductor *i.e.*, $dB \propto I$ ...(i)
- directly proportional to the length of the element *i.e.*, $dB \propto dl$ ...(ii)
- directly proportional to the sine of angle between the length of element and line joining the element to the point *i.e.*, $dB \propto \sin\theta$ ...(iii)
- inversely proportional to the square of distance between the element and the point *i.e.*, $dB \propto 1/r^2$ ...(iv)

Combining all the above four relations, we get

$$\boxed{d\mathbf{B} \propto \frac{Idl\sin\theta}{r^2}}$$

This relation is called **Biot-Savart's law.**

If conductor is placed in air or vacuum, then magnitude of magnetic field is given by

$$\boxed{|d\mathbf{B}| = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}}$$

where, $\frac{\mu_0}{4\pi}$ is a proportionality constant.

We call μ_0 the permeability of free space (or vacuum).

SI unit of magnetic field is N/A-m or tesla (T) or weber/m^2 or gauss,

where, $1 \text{ gauss} = 10^{-4}$ T.

In SI unit, $\mu_0 = 4\pi \times 10^{-7}$ Tm/A or Wb/A-m and its dimensions are $[MLT^{-2}A^{-2}]$. The value of $\frac{\mu_0}{4\pi}$ in CGS system is unity.

In vector form, Biot-Savart's law can be written as

$$d\mathbf{B} \propto \frac{I(d\mathbf{l}\times\mathbf{r})}{|\mathbf{r}|^3} = \frac{\mu_0}{4\pi}\cdot\frac{Id\mathbf{l}\times\mathbf{r}}{r^3} \quad ...(i)$$

The direction of $d\mathbf{B}$ is represented by the right hand screw rule or right hand thumb rule.

Similarly, magnetic field induction at point P due to current through entire wire is

$$\mathbf{B} = \int \frac{\mu_0}{4\pi}\cdot\frac{I(d\mathbf{l}\times\mathbf{r})}{r^3}$$

or

$$\boxed{\mathbf{B} = \int \frac{\mu_0}{4\pi}\cdot\frac{Id\mathbf{l}\sin\theta}{r^2}}$$

Biot-Savart's law in a medium

If the conductor is placed in a medium, then vector form of magnetic field is given as

$$d\mathbf{B} = \frac{\mu}{4\pi}\frac{I(d\mathbf{l}\times\mathbf{r})}{|\mathbf{r}|^3} = \frac{\mu_0\mu_r}{4\pi}\frac{I(d\mathbf{l}\times\mathbf{r})}{r^3}$$

where, μ_r = relative permeability $= \frac{\mu}{\mu_0}$

and μ = absolute permeability of the medium.

For air or vacuum $\mu_r = 1$.

Biot-Savart's law in terms of current density J

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\cdot\frac{(\mathbf{J}\times\mathbf{r})}{r^3}dV \quad \left(\because J = \frac{I}{A} = \frac{IdA}{Adl} = \frac{Idl}{dV}\right)$$

Biot-Savart's law in terms of charge (q) and its velocity (v)

A moving charge constitutes current and hence, associated magnetic field is given as,

$$d\mathbf{B} = \frac{\mu_0}{4\pi} \cdot \frac{q(\mathbf{v} \times \mathbf{r})}{r^3} \qquad \left(\because Idl = \frac{q}{dt} dl = q\frac{dl}{dt} = qv\right)$$

Special conditions of Biot-Savart's law

Special conditions of Biot-Savart's law are given below

(i) The magnetic field due to the current element can also be written as,

$$d\mathbf{B} = \frac{1}{4\pi\varepsilon_0 c^2} \frac{i\, d\mathbf{l} \times \mathbf{r}}{r^3}$$

where, $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$ = speed of light in vacuum.

(ii) When $\theta = 0°$ or $180°$, then magnetic field produced due to the current will be zero.

(iii) When $\theta = 90°$ *i.e.*, r is perpendicular to dl, then magnetic field will be $d\mathbf{B} = \frac{\mu_0 i dl}{\pi r^2}$ which is maximum.

Rules to find direction of magnetic field

(i) **Right hand palm rule** If we spread our right hand in such a way that thumb is towards the direction of current and fingers are towards that point where we have to find the direction of field, then the direction of field will be perpendicular to the palm.

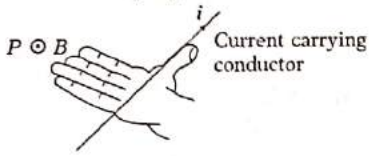


Fig. 4.3 Right hand palm rule

(ii) **Maxwell's right handed screw rule** If a right handed cork screw is rotated such that its tip moves in the direction of flow of current through the conductor, then the rotation of the head of the screw gives the direction of magnetic field.

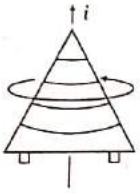


Fig. 4.4 Right handed screw rule

(iii) **Right hand thumb rule** If a straight current carrying conductor is held in the right hand such that the thumb of the hand represents the direction of current flow, then the direction of the folding fingers will represent the direction of magnetic field lines of force.

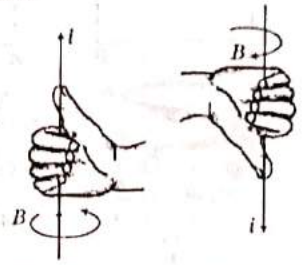
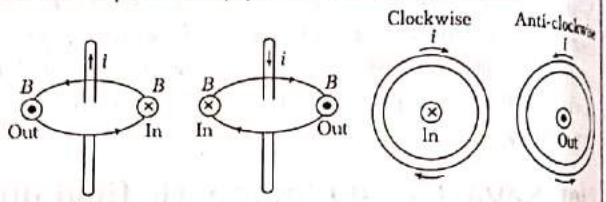


Fig. 4.5 Right hand thumb rule

Note

1. If magnetic field is directed perpendicular and into the plane of the paper, it is represented by ⊗ (cross) whereas if magnetic field is directed perpendicular and out of the plane of the paper, it is represented by ⊙ (dot).
2.



Direction of magnetic fields

In Magnetic field is away from the observer or perpendicular inwards.
Out Magnetic field is towards the observer or perpendicular outwards.

Applications of Biot-Savart's law

Applications of Biot-Savart's law are given below

1. Magnetic field surrounding a thin, straight current carrying conductor

Magnetic field due to a current carrying wire at a point P which lies at a perpendicular distance d from the wire as shown is given as

$$\boxed{B = \frac{\mu_0}{4\pi} \frac{i}{d} (\sin\alpha + \sin\beta)}$$

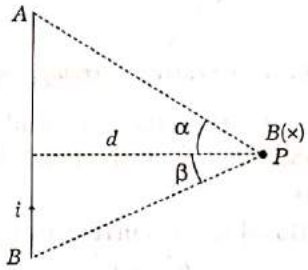


Fig. 4.6 A current carrying wire subtends angle α and β at a point P

Note down the following points regarding the above equation.

(i) For an infinitely long straight wire, $\alpha = \beta = 90°$

$\therefore \quad \sin\alpha + \sin\beta = 2 \quad$ or $\quad B = \frac{\mu_0}{2\pi} \frac{i}{d}$

(ii) When wire is semi-infinite (at the foot of long wire), $\alpha_1 = 0°$ and $\beta = \frac{\pi}{2}$

$$\therefore \quad B = \frac{\mu_0 i}{4\pi d}\left(\sin 0° + \sin\frac{\pi}{2}\right) \Rightarrow B = \frac{\mu_0 i}{4\pi d}$$

(iii) For axial position of wire *i.e.*, when point P lies on axial position of current carrying conductor, then magnetic field at P

$$B = 0$$

(iv) $B \propto 1/d$ *i.e.*, B-d graph for an infinitely long straight wire is a rectangular hyperbola as shown in figure.

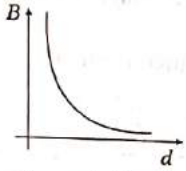
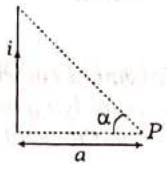


Fig. 4.7 B-d graph for an infinitely long straight wire

Magnetic field for different configurations of the straight conductors

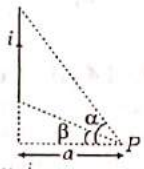
Magnitude and direction of magnetic fields associated with different configurations of straight conductors are given below.

(i)



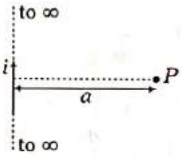
$$B_P = \frac{\mu_0 i}{4\pi a}[\sin\alpha]\otimes$$

(ii)



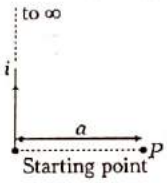
$$B_P = \frac{\mu_0 i}{4\pi a}[\sin\alpha - \sin\beta]\otimes$$

(iii)



$$B_P = \frac{\mu_0 i}{2\pi a}\otimes$$

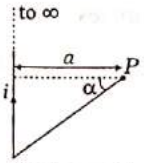
(iv)



Starting point

$$B_P = \frac{\mu_0 i}{4\pi a}\otimes$$

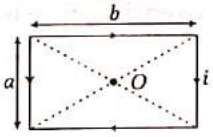
(v)



Starting point

$$B_P = \frac{\mu_0 i}{4\pi a}[\sin\alpha + 1]\otimes$$

(vi)



$$B_0 = \frac{8\mu_0 i}{4\pi}\frac{\sqrt{a^2+b^2}}{ab}\otimes$$

When $a = b$, $B_0 = \frac{8\sqrt{2}\,\mu_0 i}{4\pi a}$

Example 4.1 *A current of 10 A is flowing east to west in a long wire kept in the east-west direction. Find magnetic field in a horizontal plane at a distance of (i) 10 cm north, (ii) 20 cm south from the wire, and in a vertical plane at a distance of (iii) 40 cm downwards, (iv) 50 cm upwards.*

Sol. The magnitude of the magnetic field at a distance of r metre from a long wire carrying a current of i ampere is given by

$$B = \frac{\mu_0}{2\pi}\frac{i}{r}$$

where, $\frac{\mu_0}{2\pi} = 2\times10^{-7}\ \text{T m A}^{-1}$.

(i) The magnetic field in a horizontal plane at a distance of 10 cm (= 0.10 m) north from the wire is

$$B_N = (2\times10^{-7}\,\text{T m A}^{-1})\frac{10\ \text{A}}{0.10\ \text{m}}$$
$$= 2\times10^{-5}\ \text{T}$$

The current in the wire is from east to west. So, according to the right hand palm rule no. 1, the direction of the field at the point towards north will be downwards in a vertical plane.

(ii) The magnetic field at a distance of 20 cm (= 0.20 m) south of the wire is

$$B_S = (2\times10^{-7})\frac{10}{0.2} = 1\times10^{-5}\ \text{T}$$

The direction of the field will be upward in the vertical plane.

(iii) The field at 40 cm (= 0.40 m) from the wire downwards in the vertical plane is

$$B_D = (2\times10^{-7})\frac{10}{0.40} = 5\times10^{-6}\ \text{T}$$

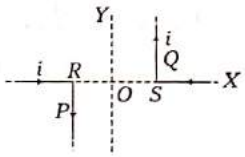
The field will be in a horizontal plane pointing south.

(iv) The field at 50 cm (= 0.50 m) above the wire in the vertical plane is

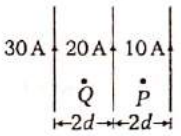
$$B_U = (2\times10^{-7})\frac{10}{0.50} = 4\times10^{-6}\ \text{T}$$

The field will be in a horizontal plane pointing north.

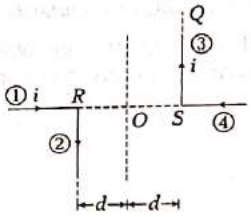
Example 4.2 *(i) A pair of stationary and infinitely long bent wires are placed in the XY-plane as shown. The wires carry currents 10 A each. The segments P and Q are parallel to the Y-axis such as OS = OR = 0.02 m. Find the magnitude and direction of the magnetic field at the origin O.*



(ii) Three long wires, carrying currents 10 A, 20 A and 30 A are placed parallel to each other as shown. Points P and Q are in the midway of wires. Find ratio of magnetic fields at P and Q.



Sol. (i) Consider the wires as shown below



Magnetic field $B_1 = B_4 = 0$ $[\because \theta = 0°]$

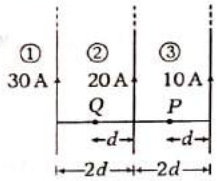
Magnetic field $B_2 = B_3 = \dfrac{\mu_0 i}{4\pi d}, \odot$

Magnetic field at the centre O,

$$B_O = B_2 + B_3 = \frac{\mu_0 i}{2\pi d} = \frac{\mu_0}{2\pi} \times \frac{10}{0.02} = \frac{\mu_0}{4\pi} \times 1000$$

$$= 10^{-7} \times 10^3 = 10^{-4}\ \text{T}$$

(ii)



Magnetic field at the point Q,

$$B_Q = \frac{\mu_0}{2\pi}\left[\frac{30}{d} - \frac{20}{d} - \frac{10}{3d}\right] = \frac{\mu_0}{2\pi} \cdot \frac{20}{3d} \otimes$$

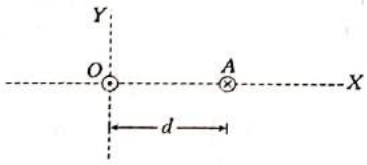
Magnetic field at the point P,

$$B_P = \frac{\mu_0}{2\pi}\left[\frac{30}{3d} + \frac{20}{d} - \frac{10}{d}\right] = \frac{\mu_0}{2\pi} \cdot \frac{20}{d} \otimes$$

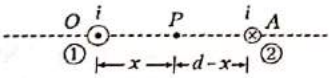
The ratio of magnetic fields at P and Q is

$$\therefore \quad \frac{B_P}{B_Q} = 3$$

Example 4.3 *Two long wires carrying same currents in opposite directions are placed at separation d as shown. Predict variation of magnetic field as one moves from one wire to another along X-axis excluding the points O and A.*



Sol.



Between the wires *Magnetic field at P.*

Due to (1), $B_1 = \dfrac{\mu_0 i}{2\pi x}$, upward

Due to (2), $B_2 = \dfrac{\mu_0 i}{2\pi(d - x)}$, upward

$$\therefore \quad B_P = B_1 + B_2 = \frac{\mu_0 i}{2\pi}\left(\frac{1}{x} + \frac{1}{d - x}\right) = \frac{\mu_0 i}{2\pi} \cdot \frac{d}{x(d - x)}, \text{ upward}$$

B_p is minimum, if $x(d - x)$ is maximum.

and $x(d - x)$ is maximum, if $x = d - x \Rightarrow x = d/2$

(the product of two parts is maximum, if parts are equal)

As, $x = \dfrac{d}{2}$, B is minimum, $(B_P)_{\min} = \dfrac{2\mu_0 i}{\pi d}$, upward

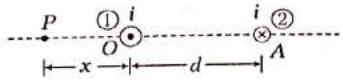
As, $x \to 0,\ B \to \infty$

$x \to d,\ B \to \infty$

From $x = 0$ to $x = d/2$, magnetic field decreases reaching to minimum value at $x = d/2$.

From $x = d/2$ to $x = d$, magnetic field increases and tends to infinite at $x = d$.

Left of O: At x distance from O.



At P: $B_1 = \dfrac{\mu_0 i}{2\pi x}$, downward,

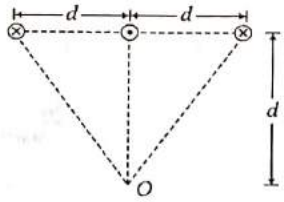
$$B_2 = \frac{\mu_0 i}{2\pi(x + d)}, \text{ upward}$$

Magnetic field at the point P,

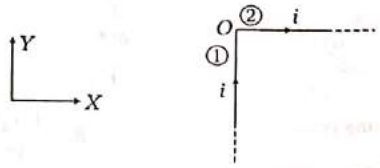
$$B_P = \frac{\mu_0 i}{2\pi}\left(\frac{1}{x} - \frac{1}{x + d}\right), \text{ downward}$$

Example 4.4 *(i) A very long wire carrying a current i is bent at right angles. Find magnetic field at a point lying on a perpendicular to the wire, drawn through the point of bending, at a distance d from it.*

(ii) Three long wires carrying same current are placed as shown. Find magnetic field at O.



Sol. (i) A very long wire is bent at right angles

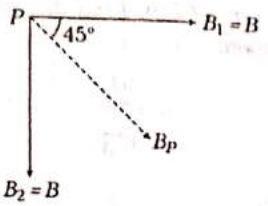


Magnetic field at point P at height d above point O,

Due to wire (1), $B_1 = \dfrac{\mu_0 i}{4\pi d}$, towards $+X$-axis

Due to wire (2), $B_2 = \dfrac{\mu_0 i}{4\pi d}$, towards $-Y$-axis

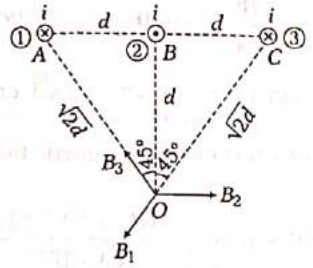
B_1 is perpendicular to B_2.



Magnetic field at the point P,

$$B_P = \sqrt{2}B = \frac{\sqrt{2}\mu_0 i}{4\pi d} = \frac{\mu_0 i}{2\sqrt{2}\pi d}$$

(ii)

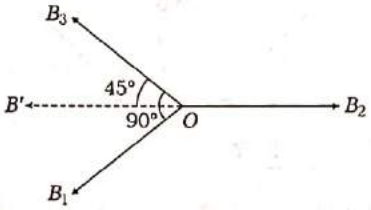


Magnetic field at the point O,

Due to wire (1), $B_1 = \dfrac{\mu_0 i}{2\pi\sqrt{2}d}$

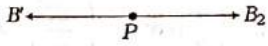
Due to wire (2), $B_2 = \dfrac{\mu_0 i}{2\pi d}$

Due to wire (3), $B_3 = \dfrac{\mu_0 i}{2\pi\sqrt{2}d} = B_1$



Resultant of B_1 and B_3 is

$$B' = 2B_1 \cos 45° = \frac{\mu_0 i}{2\pi d}$$



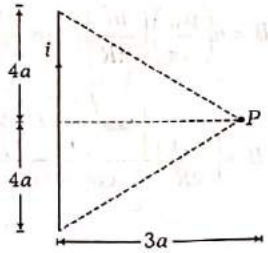
$$B' = B_2$$

∴ Magnetic field at the centre O,

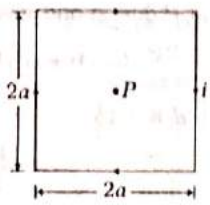
$$B_O = 0$$

Example 4.5 *Evaluate magnitude and direction of magnetic field at a point P in the following cases.*

(i)

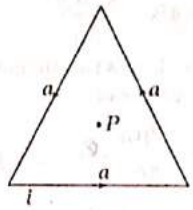


(ii)



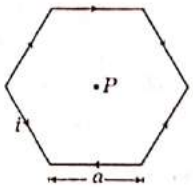
P is the centre of square.

(iii)



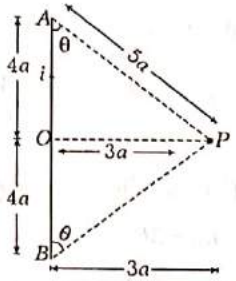
P is the centre of equilateral triangle.

(iv)



P is the centre of regular hexagon.

Sol. (i)



Magnetic field at point P,

$$B_P = \frac{\mu_0 i}{4\pi a'}[\cos\theta_1 + \cos\theta_2]$$

$$= \frac{\mu_0 i}{4\pi(3a)}[\cos\theta + \cos\theta] = \frac{\mu_0 i}{6\pi a}\cos\theta$$

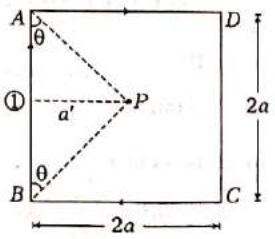
From the figure, $PA = \sqrt{(3a)^2 + (4a)^2}$

$$PA = \sqrt{25a^2} \Rightarrow PA = 5a$$

From the figure, $\cos\theta = \dfrac{4}{5}$

∴ Magnetic field, $B_P = \dfrac{\mu_0 i}{6\pi a} \cdot \dfrac{4}{5} = \dfrac{2\mu_0 i}{15\pi a}, \otimes$

(ii)



Magnetic field at P due to one side of square, say AB,

$$B_1 = \frac{\mu_0 i}{4\pi a'}(\cos\theta + \cos\theta)$$

$$a' = d, \theta = 45°$$

$$B_1 = \frac{\mu_0 i}{4\pi a}\cdot 2\cdot\frac{1}{\sqrt{2}} = \frac{\mu_0 i}{2\sqrt{2}\pi a}$$

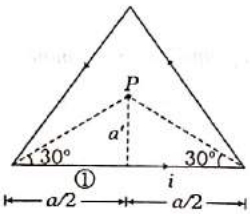
Magnetic field due to square loop,

$$B_P = 4B_1 = \frac{\sqrt{2}\mu_0 i}{\pi a}$$

Since, current is clockwise, magnetic field at P will be inside the plane of paper.

$$B_P = \frac{\sqrt{2}\mu_0 i}{\pi a}, \otimes$$

(iii)



From the figure,

$$\tan 30° = \frac{a'}{a/2} \Rightarrow a' = \frac{a}{2\sqrt{3}} \Rightarrow \theta_1 = \theta_2 = 30°$$

Magnetic field, $B_1 = \frac{\mu_0 i}{4\pi a'}[\cos 30° + \cos 30°]$

$$B = \frac{\mu_0 i}{4\pi\cdot\frac{a}{2\sqrt{3}}}\cdot 2\cdot\frac{\sqrt{3}}{2} = \frac{3\mu_0 i}{2\pi a}$$

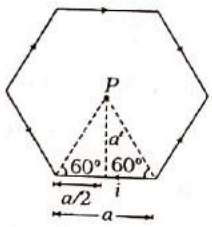
So magnetic field at P due to triangular loop,

$$B_P = 3B_1 = \frac{9\mu_0 i}{2\pi a}$$

Since, current is anti-clockwise, magnetic field at P is outside the plane of paper.

$$B_P = \frac{9\mu_0 i}{2\pi a}, \otimes$$

(iv)



Magnetic field,

$$B_1 = \frac{\mu_0 i}{4\pi a'}(\cos 60° + \cos 60°)$$

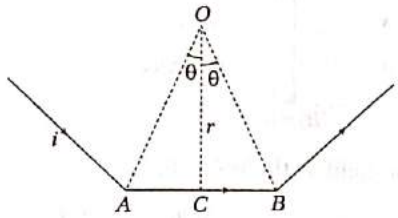
$$= \frac{\mu_0 i}{4\pi\cdot\frac{a}{2}\cdot\tan 60°}\cdot 2\cos 60° = \frac{\mu_0 i}{2\pi a\sqrt{3}}$$

Magnetic field at the centre P,

$$B_P = 6B_1 = \frac{\sqrt{3}\mu_0 i}{\pi a}, \otimes$$

Example 4.6 *A wire shaped to a regular hexagon of side 2 cm carries a current of 2 A. Find the magnetic field at the centre of the hexagon.*

Sol.



From geometry of the figure, we have

$$\Rightarrow \quad \frac{BC}{OC} = \tan\theta \qquad [BC = 1 \text{ cm}]$$

$$\therefore \quad \frac{1}{r} = \tan 30° = \frac{1}{\sqrt{3}} \Rightarrow r = \sqrt{3} \text{ cm}$$

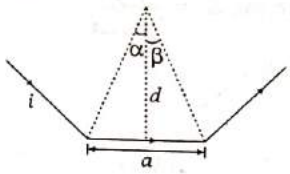
Net magnetic field at O = 6 times the magnetic field due to one side.

$$\therefore \quad B = 6\left[\frac{\mu_0}{2\pi}\frac{i}{r}(\sin\theta + \sin\theta)\right] = \frac{6(2\times 10^{-7})(2)}{\sqrt{3}\times 10^{-2}}\left(\frac{1}{2}+\frac{1}{2}\right)$$

$$= 1.38\times 10^{-4} \text{ T}$$

Example 4.7 *A rectangular polygon of n sides is formed by bending a wire of total length $2\pi R$ which carries a current i. Find the magnetic field at the centre of the polygon.*

Sol. One side of the polygon is



$$a = \frac{2\pi R}{n}$$

Angle, $$\alpha = \beta = \frac{\left(\frac{2\pi}{n}\right)}{2} = \frac{\pi}{n} \Rightarrow \frac{d}{(a/2)} = \cot\alpha$$

$$\therefore \quad d = \left(\frac{a}{2}\right)\cot\alpha = \left(\frac{\pi R}{n}\right)\cot\left(\frac{\pi}{n}\right)$$

All sides of the polygon produce the magnetic field at the centre in same direction (here). Hence, net magnetic field,

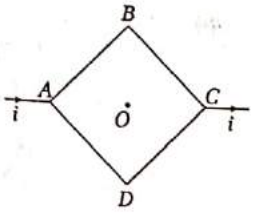
$$B = (n)\,(\text{magnetic field due to one side})$$

$$= n\left\{\frac{\mu_0}{4\pi}\frac{i}{d}(\sin\alpha + \sin\beta)\right\}$$

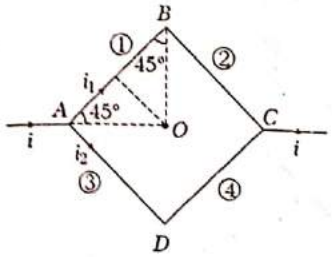
or $$B = n\left(\frac{\mu_0}{4\pi}\right)\left(\frac{in}{\pi R}\right)\left(\tan\frac{\pi}{n}\right)\left(2\sin\frac{\pi}{n}\right)$$

or $$B = \left(\frac{\mu_0 i}{2R}\right)\frac{\left\{\sin\left(\frac{\pi}{n}\right)/\frac{\pi}{n}\right\}^2}{\cot(\pi/n)}$$

Example 4.8 *Consider the following figure in which ABCD is a square of edge a. Resistance of the wire ABC is R_0 and that of ADC is $2R_0$. Find magnitude and direction of magnetic field at the centre O of the square.*



Sol.



Given, $R_{ABC} = R_1 = R_0, R_{ADC} = R_2 = 2R_0$

$$i_1 = \frac{2R_0}{R_0 + 2R_0}, i = \frac{2i}{3}, i_2 = i - i_1 = \frac{i}{3}$$

Magnetic field $B_1 = \frac{\mu_0 i_1}{4\pi d}(\cos 45° + \cos 45°) = \frac{\mu_0 i_1}{2\sqrt{2}\,\pi d}$

$$= \frac{\mu_0 (2i/3)}{2\sqrt{2}\pi\left(\frac{a}{2}\right)} = \frac{2\mu_0 i}{3\sqrt{2}\pi a} = B_2, \otimes$$

Magnetic field $B_3 = \frac{\mu_0 i_2}{4\pi d}(\cos 45° + \cos 45°)$

$$= \frac{\mu_0 i_2}{2\sqrt{2}\,\pi d} = \frac{\mu_0 (i/3)}{2\sqrt{2}\pi\left(\frac{a}{2}\right)} = \frac{\mu_0 i}{3\sqrt{2}\pi a} = B_4, \odot$$

Magnetic field at the centre O,

$$B_O = (B_1 + B_2) - (B_3 + B_4) = \frac{4\mu_0 i}{3\sqrt{2}\pi a} - \frac{2\mu_0 i}{3\sqrt{2}\pi a} = \frac{\sqrt{2}\,\mu_0 i}{3\pi a}, \otimes$$

2. Magnetic field at the centre of a circular current carrying coil

Consider a circular current carrying coil of radius a and carrying current i. Magnetic field at the centre O, due to the current element dl is

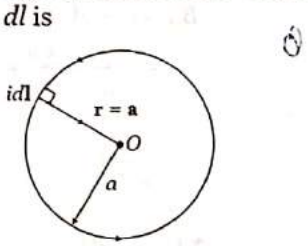


Fig. 4.8 Circular current carrying coil

$$d\mathbf{B} = \frac{\mu_0 I}{4\pi} \cdot \frac{d\mathbf{l} \times \mathbf{r}}{r^3}$$

where, $d\mathbf{l} \perp \mathbf{r}$, so $\theta = 90°$

$$dB = \frac{\mu_0}{4\pi}\frac{i\,dl \sin 90°}{a^2} \qquad [\because i d\mathbf{l} \perp \mathbf{r}]$$

Therefore, magnetic field at the centre due to the whole circular loop is

$$B = \int dB = \frac{\mu_0 i}{4\pi a^2}\int dl = \frac{\mu_0 i}{4\pi a^2}(2\pi a)$$

$$\boxed{B = \frac{\mu_0 i}{2\pi a}}$$

Direction of this field is outward perpendicular to the plane of the paper.

Note If the loop have N turns, then magnetic field, $B = \frac{\mu_0 N I}{2\pi a}$

Magnetic field due to an arc

Magnetic field due to an arc of a circular current carrying coil at the centre is given by,

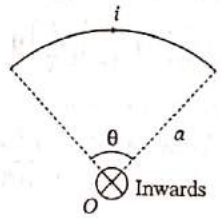
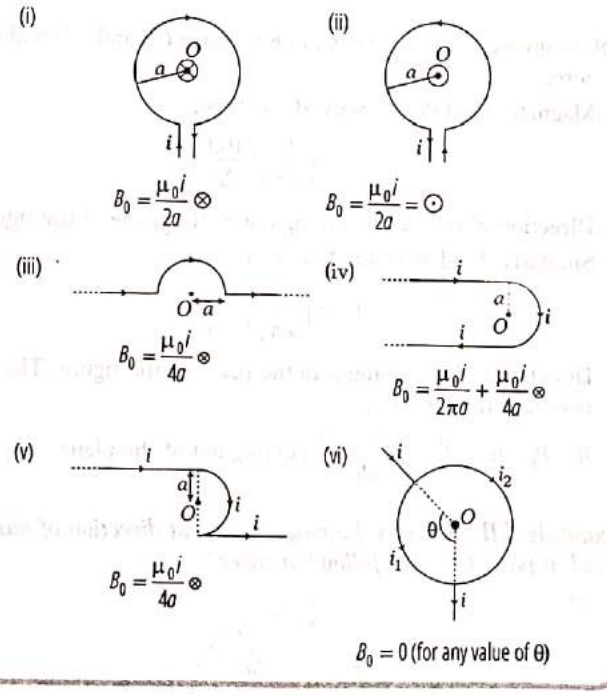


Fig. 4.9 Magnetic field due to an arc

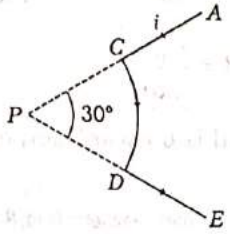
$$B = \left(\frac{\theta}{2\pi}\right)\frac{\mu_0 i}{2a} = \frac{\mu_0}{4\pi}\left(\frac{i}{a}\right)\theta \text{ or } \boxed{B = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{i}{a}\right)\theta}$$

Here, θ is to be substituted in radians.

Magnetic field for different configurations of the circular coil



Example 4.9 *A current path shaped as shown in figure produces a magnetic field at P, the centre of the arc. If the arc subtends an angle of 30° and the radius of the arc is 0.6 m. What is the magnitude of the field at P, if the current is 3.0 A?*



Sol. Magnetic field at centre of the circular loop,

$$B = \frac{\mu_0 N i}{2R}$$

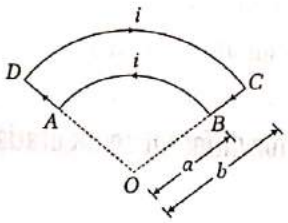
Magnetic field due to an arc of a circle at the centre is

$$B = \left(\frac{\theta}{2\pi}\right)\frac{\mu_0 i}{2R} = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{i}{R}\right)\theta$$

Here, $\theta = 30°$ and $i = 3$ A, $R = 0.6$ m

$$B = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{3}{0.6}\right)\left(\frac{\pi}{6}\right) = \frac{10^{-7} \times 3 \times \pi}{0.6 \times 6} = 2.6 \times 10^{-7}\ \text{T}$$

Example 4.10 *Figure shows a current loop having two circular arcs joined by two radial lines. Find the magnetic field B at the centre O.*



Sol. Magnetic field at point O, due to wires CB and AD will be zero.

Magnetic field due to wire BA will be,

$$B_1 = \left(\frac{\theta}{2\pi}\right)\left(\frac{\mu_0 i}{2a}\right)$$

Direction of field $\mathbf{B}_1$ is coming out of the plane of the figure. Similarly, field at O due to arc DC will be

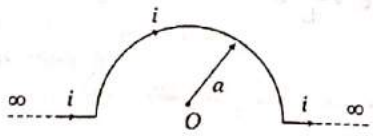
$$B_2 = \left(\frac{\theta}{2\pi}\right)\left(\frac{\mu_0 i}{2b}\right)$$

Direction of $\mathbf{B}_2$ is going into the plane of the figure. The resultant field at O is,

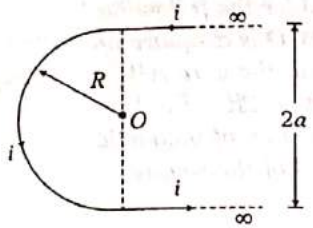
$$B = B_1 - B_2 = \frac{\mu_0\, i\, \theta\, (b-a)}{4\pi\, ab}, \text{ coming out of the plane}$$

Example 4.11 *What is the magnitude and direction of magnetic field at point O in the following cases?*

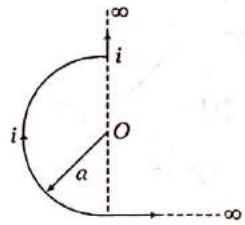
(i)



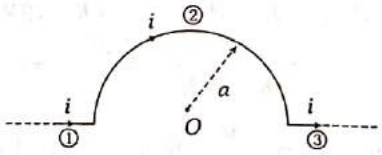
(ii)



(iii)



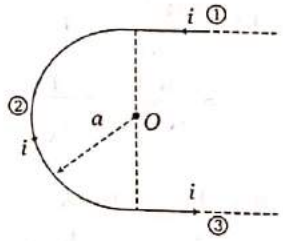
Sol. (i)



Since point O is along the length of straight wires 1 and 3, hence $B_1 = B_3 = 0$

Magnetic field $B_O = B_2 = \frac{\mu_0 i}{4a}, \otimes$

(ii)



Since, point O is the foot of long wires 1 and 3,

$$\therefore \quad B_1 = B_3 = \frac{\mu_0 i}{4\pi a}, \odot$$

Due to semi-circle (2),

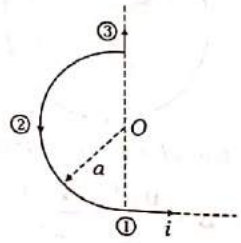
$$B_2 = \frac{\mu_0 i}{4a}, \odot$$

Magnetic field at the point O,

$$B_O = B_1 + B_3 + B_2$$

$$= \frac{\mu_0 i}{2\pi a} + \frac{\mu_0 i}{4a} = \frac{\mu_0 i}{4a}\left(\frac{2}{\pi} + 1\right), \odot$$

(iii)



Magnetic field, $B_1 = \frac{\mu_0 i}{4\pi a}, \odot$

Magnetic field, $B_2 = \frac{\mu_0 i}{4a}, \odot$

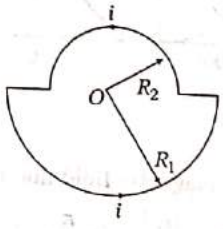
Magnetic field, $B_3 = 0$

Magnetic field at the centre point O,

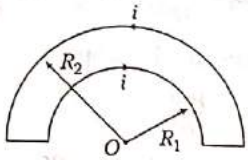
$$B_O = B_1 + B_2 = \frac{\mu_0 i}{4\pi R} + \frac{\mu_0 i}{4R} = \frac{\mu_0 i}{4R}\left(\frac{1}{\pi} + 1\right), \odot$$

Example 4.12 *Find the magnitude and direction of magnetic field at point O in the following cases.*

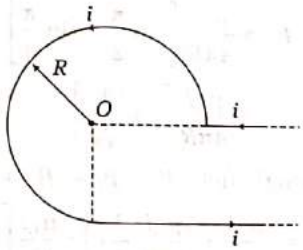
(i)



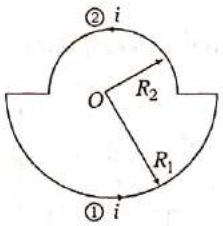
(ii)



(iii)



Sol. (i)



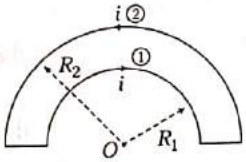
Magnetic field at O due to straight wires is zero.

$$B_1 = \frac{\mu_0 i}{4R_1}, \odot \Rightarrow B_2 = \frac{\mu_0 i}{4R_2}, \odot$$

Magnetic field at the centre point O,

$$B_O = B_1 + B_2 = \frac{\mu_0 i}{4}\left(\frac{1}{R_1} + \frac{1}{R_2}\right), \odot$$

(ii)



Magnetic field at O due to straight wires is zero.

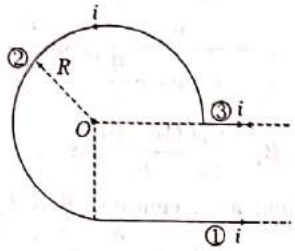
$$B_1 = \frac{\mu_0 i}{4R_1}, \otimes \text{ and } B_2 = \frac{\mu_0 i}{4R_2}, \odot$$

$$B_1 > B_2$$

Magnetic field at the centre O,

$$B_O = B_1 - B_2 = \frac{\mu_0 i}{4}\left(\frac{1}{R_1} - \frac{1}{R_2}\right), \otimes$$

(iii)



Magnetic field, $B_1 = \frac{\mu_0 i}{4\pi R}, \odot$

Magnetic field, $B_2 = \frac{\mu_0 i}{2R} \cdot \frac{3}{4}, \odot$

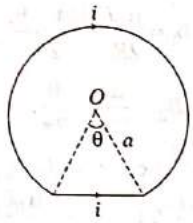
Magnetic field, $B_3 = 0$

Magnetic field at the centre, $B_O = B_1 + B_2$

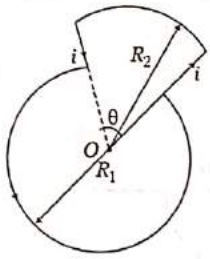
$$= \frac{\mu_0 i}{4R}\left(\frac{1}{\pi} + \frac{3}{2}\right), \odot$$

Example 4.13 *Evaluate the magnitude and direction of magnetic field at point O in the following cases.*

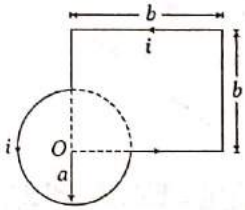
(i)



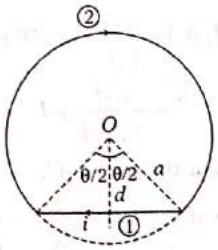
(ii)



(iii)



Sol. (i)



From the figure, $d = a\cos\frac{\theta}{2}$

$$B_1 = \frac{\mu_0 i}{4\pi d}\left(\sin\frac{\theta}{2} + \sin\frac{\theta}{2}\right)$$

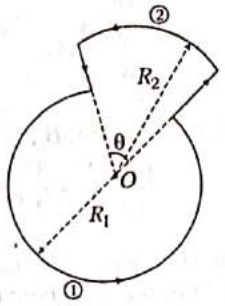
$$= \frac{\mu_0 i}{4\pi \cdot a\cos\frac{\theta}{2}} \cdot 2\sin\frac{\theta}{2} = \frac{\mu_0 i}{2\pi a}\tan\frac{\theta}{2}, \otimes$$

$$B_2 = \frac{\mu_0 i}{2a}\frac{(2\pi - \theta)}{2\pi}, \otimes$$

Magnetic field at the centre O, $B_O = B_1 + B_2$

$$= \frac{\mu_0 i}{2\pi a}\left(\tan\frac{\theta}{2} + \pi - \frac{\theta}{2}\right), \otimes$$

(ii)



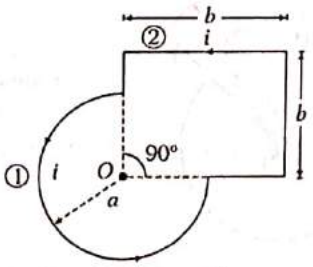
Magnetic field, $B_1 = \frac{\mu_0 i}{2R_1} \cdot \frac{(2\pi - \theta)}{2\pi}, \odot$

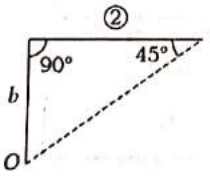
Magnetic field, $B_2 = \frac{\mu_0 i}{2R_2} \cdot \frac{\theta}{2\pi}, \odot$

Magnetic field at the centre O, $B_O = B_1 + B_2$

$$= \frac{\mu_0 i}{4\pi R_1}(2\pi - \theta) + \frac{\mu_0 i}{4\pi R_2}\theta, \odot$$

(iii)





Magnetic field, $B_1 = \frac{\mu_0 i}{2a} \cdot \frac{3}{4} = \frac{3\mu_0 i}{8a}, \odot$

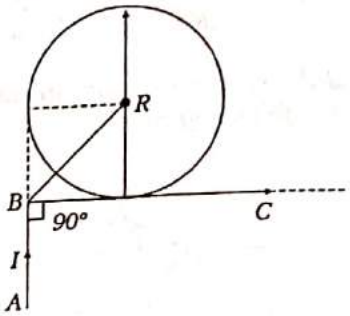
Magnetic field, $B_2 = \frac{\mu_0 i}{4\pi b}(\cos 90° + \cos 45°)$

$$= \frac{\mu_0 i}{4\sqrt{2}\pi b} = B_3, \odot$$

Magnetic field at the centre O,

$$B_O = B_1 + (B_2 + B_3) = \frac{3\mu_0 i}{8a} + \frac{\mu_0 i}{2\sqrt{2}\pi b}, \odot$$

Example 4.14 *What is the magnetic field at the centre of the circular loop (as shown in figure), when a single wire is bent to form a circular loop and also extends to form straight section?*



Sol. From the figure, magnetic field due to AB,

$$B_1 = \frac{\mu_0 I}{4\pi R}\left[\sin\frac{\pi}{2} - \sin\frac{\pi}{4}\right] = \frac{\mu_0 I}{4\pi R}\left[1 - \frac{1}{\sqrt{2}}\right]$$

Magnetic field due to circular loop,

$$B_2 = \frac{\mu_0 I}{2R}$$

Magnetic field due to straight wire BC,

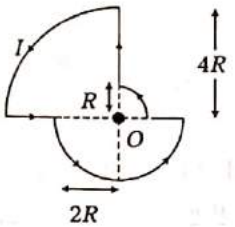
$$B_3 = \frac{\mu_0 I}{4\pi R}\left[\sin\frac{\pi}{2} + \sin\frac{\pi}{4}\right]$$

$$= \frac{\mu_0 I}{4\pi R}\left[1 + \frac{1}{\sqrt{2}}\right]$$

$\therefore$ Resultant magnetic field, $B = B_1 + B_2 + B_3$

$$= \left(\frac{\mu_0 I}{2R} + \frac{2\mu_0 I}{4\pi R}\frac{1}{\sqrt{2}}\right) = \frac{\mu_0 I}{2R}\left[1 + \frac{1}{\sqrt{2}\,\pi}\right]$$

Example 4.15 *Find the magnetic field at the centre O of the loop shown in the figure.*



Sol. Due to straight parts, there is no magnetic field at O. There are three fields at O. $\because B = \frac{\mu_0 i}{4\pi R}\theta$

$B_1 = \frac{\mu_0 i}{4\pi(4R)} \times \frac{\pi}{2}$, due to arc of radius $4R$

$B_2 = \frac{\mu_0 i}{4\pi(2R)}(\pi)$, due to arc of radius $2R$

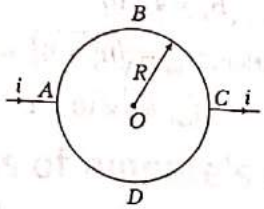
$B_3 = \frac{\mu_0 i}{4\pi R}\left(\frac{\pi}{2}\right)$, due to arc of radius R

As the current is anti-clockwise in all three arcs, therefore, all the fields are perpendicular to plane and outwards.

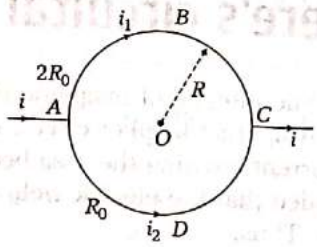
$\Rightarrow$ $B_{net} = B_1 + B_2 + B_3$

$\therefore$ $B_{net} = \frac{\mu_0 i}{R}\left[\frac{1}{32} + \frac{1}{8} + \frac{1}{8}\right] = \frac{9\,\mu_0 i}{32R}$

Example 4.16 *In the given figure, resistance of wire ABC is double of wire ADC. Find the magnitude and direction of magnetic field at the centre O.*



Sol.



Given, $R_{ABC} = R_1 = 2R_0, R_{ADC} = R_2 = R_0$

$$i_1 = \frac{R_0}{R_0 + 2R_0} \cdot i = \frac{i}{3} \text{ and } i_2 = i - i_1 = \frac{2i}{3}$$

Magnetic field, $B_1 = \frac{\mu_0 i_1}{2R} \cdot \frac{1}{2} = \frac{\mu_0 i}{12R}, \otimes$

Magnetic field, $B_2 = \frac{\mu_0 i_2}{2R} \cdot \frac{1}{2} = \frac{\mu_0 i}{6R}, \odot$

Magnetic field at the centre $O, B_O = B_2 - B_1 = \frac{\mu_0 i}{12R}, \odot$

3. Magnetic field at the axis of a circular current carrying coil

If a coil of radius a is carrying current i, then magnetic field on its axis at a distance r from its centre is given by

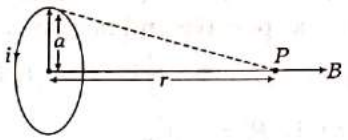


Fig. 4.10 Magnetic field on the axis of a current carrying coil

$$\boxed{B_{\text{axis}} = \frac{\mu_0}{4\pi} \frac{2\pi Nia^2}{(a^2 + r^2)^{3/2}}}$$

where, N = number of turns in coil.

(i) At centre, $r = 0 \Rightarrow B_{\text{centre}} = \frac{\mu_0}{4\pi} \cdot \frac{2\pi Ni}{a} = \frac{\mu_0 Ni}{2a} = B_{\max}$

(ii) At far points,

$$r >> a, B \approx \frac{\mu_0}{4\pi}\left(\frac{2\pi Nia^2}{r^3}\right) = \frac{\mu_0}{2}\frac{Nia^2}{r^3} \Rightarrow B \propto \frac{1}{r^3}$$

(iii) B-r graph shows the variation of magnetic field at various position on the axis of circular current carrying coil. Magnetic field is maximum at the centre and decreases as we move away from the centre on the axis of the loop.

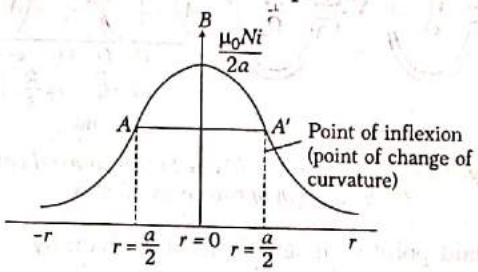


Fig. 4.11 B-r graph at the axis of coil (ring)

Direction of magnetic field on the axis of the circular coil

Direction of magnetic field on the axis of a circular loop can be obtained using the right hand thumb rule. If the fingers are curled along the current, then the stretched thumb will point towards the magnetic field as shown in figures given below.

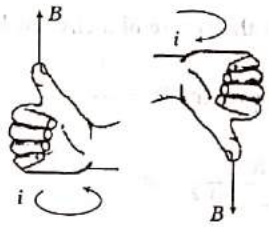


Fig. 4.12 Right hand thumb rule

Magnetic field due to circular current carrying coil apart from axis

The magnetic field at a point not on the axis is mathematically difficult to calculate. We show qualitatively in figure the magnetic field lines due to a circular current which will give some idea of the field.

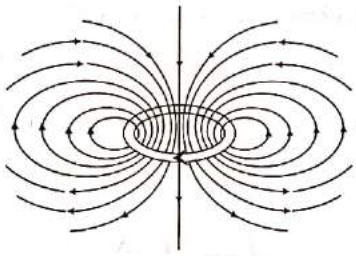


Fig. 4.13 Magnetic field lines of a current carrying loop

4. Magnetic field due to a set of two circular current carrying coils (case of Helmholtz)

Consider the setup of two co-axial coils of same radius such that distance between their centres is equal to their radius. This arrangement is called Helmholtz.

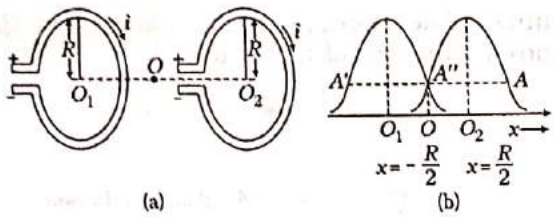


Fig. 4.14 *(a) Magnetic field due to two co-axial coils (b) B-R graph of two co-axial coils*

At axial mid-point O, magnetic field is given by

$$B = \frac{8\mu_0 Ni}{5\sqrt{5}\,R} = 0.716\,\frac{\mu_0 Ni}{R} = 1.432\,B_{max}$$

where, $\boxed{B_{max} = \frac{\mu_0 Ni}{2R}}$

Example 4.17 *The magnetic field B due to a current carrying circular loop of radius 12 cm at its centre is 0.5×10^{-4} T. Find the magnetic field due to this loop at a point on the axis at a distance of 5.0 cm from the centre.*

Sol. Magnetic field at the centre of a circular loop is, $B_1 = \frac{\mu_0 i}{2R}$

and that at an axial point, $B_2 = \frac{\mu_0 iR^2}{2(R^2 + x^2)^{3/2}}$

Thus, $\frac{B_2}{B_1} = \frac{R^3}{(R^2 + x^2)^{3/2}}$ or $B_2 = B_1\left[\frac{R^3}{(R^2 + x^2)^{3/2}}\right]$

Substituting the values, we have

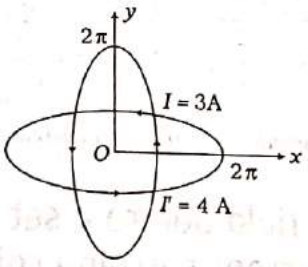
$$B_2 = (0.5\times 10^{-4})\left[\frac{(12)^3}{(144 + 25)^{3/2}}\right] = 3.9\times 10^{-5}\ \text{T}$$

Example 4.18 *Two concentric coils, each of radius equal to 2π cm, are placed at right angles to each other. Current of 3 and 4 A respectively, are flowing through the two coils. What will be the magnetic induction in Wbm^{-2} at the centre of the coils [$\mu = 4\pi\times 10^{-7}$ Wb $(Am)^{-1}$]?*

Sol. Field at the centre of the loop is given by

$$B = \frac{\mu_0 I}{2R}$$

where, R is radius given, $R = 2\pi$ cm $= 2\pi\times 10^{-2}$ m



$$B_x = \frac{\mu_0}{2}\cdot\frac{I_1}{2\pi\times 10^{-2}}$$

$\because$ $I_1 = 3$ A

$\therefore$ $$B_x = \frac{\mu_0}{2}\cdot\frac{3\times 10^2}{2\pi} = 3\times 10^{-5}\text{T}$$

$$B_y = \frac{\mu_0}{2}\cdot\frac{I_2}{2\pi\times 10^{-2}},\quad \because I_2 = 4\text{ A}$$

$$B_y = 4\times 10^{-5}$$

$\therefore$ Magnetic induction, $B_{net} = \sqrt{B_x^2 + B_y^2} = \sqrt{(3^2 + 4^2)\times 10^{-10}}$

$$B_{net} = 5\times 10^{-5}\ \text{T}$$

2 Ampere's circuital law

It states that the line integral of magnetic field, **B** around any closed path (Amperian loop) or circuit is equal to μ_0 times the total current crossing the area bounded by the closed path provided that the electric field inside the loop remains constant. Thus,

$$\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0 (i_{net})$$

In the above relation, we will use a sign convention given by right hand rule. By this rule, fingers of the right hand are curled in the sense, the boundary is traversed in the loop integral $\oint \mathbf{B}\cdot d\mathbf{l}$. Now, the direction of the thumb gives the sense in which the current I is treated as positive.

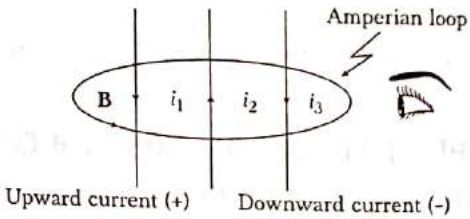


Fig. 4.15 *Sign convention for the current*

In this way, currents i_1, i_2 and i_3 shown in the diagram are respectively taken as positive and negative.

$\therefore$ $i_{net} = i_1 + i_2 - i_3 \Rightarrow \oint \mathbf{B}\cdot d\mathbf{l} = \mu_0 (i_1 + i_2 - i_3)$

Its simplified form is, $Bl = \mu_0 i_{net}$

This simplified equation can be used under the following conditions

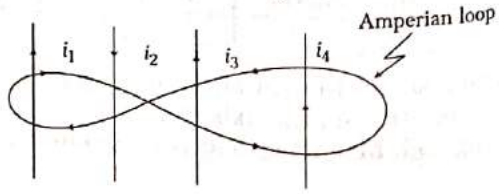
(i) At every point of the closed path $\mathbf{B} \parallel d\mathbf{l}$.

(ii) Magnetic field has the same magnitude B at all places on the closed path.

(iii) Ampere circuital law holds for steady currents which do not change with time.

Example 4.19 *Write equation for Ampere's circuital law for the Amperial loop as shown below.*

(Traverse in the direction shown by arrow mark but on it.)



Sol. If observed from the top left side portion of the loop is traversed clockwise, while the other portion is traversed anti clockwise. Therefore, by sign convention

$$i_1 \longrightarrow (+),\ i_2 \longrightarrow (-)$$
$$i_3 \longrightarrow (+),\ i_4 \longrightarrow (+)$$

Thus, $\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0 (i_1 + i_3 + i_4 - i_2)$

Applications of ampere's circuital law

Ampere's circuital law can be applied to calculate magnetic field associated with symmetric distributed current carrying configurations, *which are discussed as follows*

1. Magnetic field due to a cylindrical wire

(i) **Outside the cylinder** ($r > R$) To find magnetic field outside at point P. Now, we will assume Amperian loops as shown below.

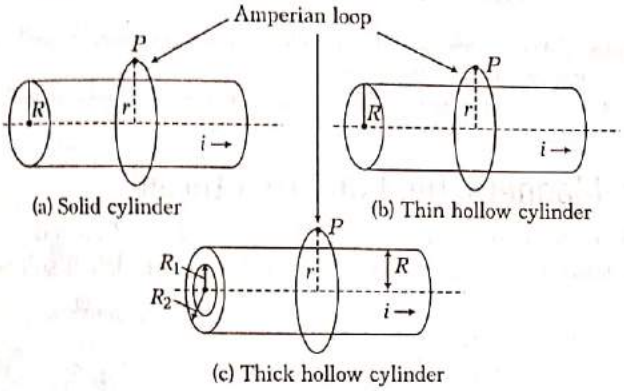


Fig. 4.16 Applications of Ampere's circuital law for a point P, outside the cylinder

In all the above cases, magnetic field outside the wire at P,

$$\oint \mathbf{B}\cdot d\mathbf{l} = B(2\pi r)$$

$$\Rightarrow \quad \boxed{B_{\text{out}} = \frac{\mu_0 i}{2\pi r}}$$

In all the above cases, $\boxed{B_{\text{surface}} = \frac{\mu_0 i}{2\pi R}}$

(ii) **Inside the hollow cylinder** ($r < R$) Magnetic field inside the hollow cylinder is zero because no current is enclosed by Amperian loops as shown below

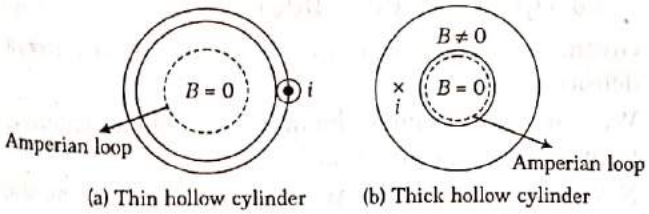


Fig. 4.17 Magnetic field inside the hollow cylinder

(iii) **Inside the solid cylinder** ($r < R$) Current i' enclosed by Amperian loop is lesser than the total current (i).

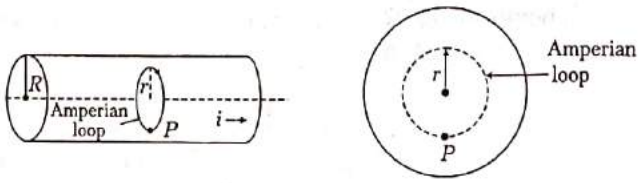


Fig. 4.18 Amperian loop inside a solid cylinder

Current density is uniform *i.e.*, $J = J'$

$$\Rightarrow \quad i' = i \times \frac{A'}{A} = i\left(\frac{r^2}{R^2}\right)$$

Hence, at inside point $\oint \mathbf{B}_{\text{in}}\cdot d\mathbf{l} = \mu_0 i'$

$$\Rightarrow \quad \boxed{B_{\text{in}} = \frac{\mu_0}{2\pi}\cdot\frac{ir}{R^2}}$$

(iv) **Inside the thick portion of hollow cylinder** Current enclosed by the Amperian loop of radius r is given by

$$i' = i \times \frac{A'}{A} = i \times \frac{(r^2 - R_1^2)}{(R_2^2 - R_1^2)}$$

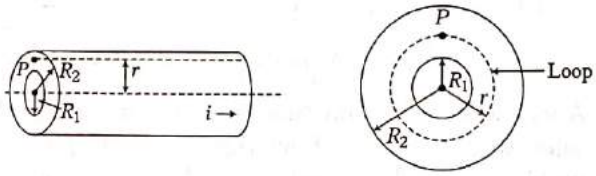


Fig. 4.19 Amperian loop inside the thick portion of the hollow cylinder

Hence, at point P, $\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0 i'$

$$\Rightarrow \quad \boxed{B_{\text{in}} = \frac{\mu_0 i}{2\pi r}\cdot\frac{(r^2 - R_1^2)}{(R_2^2 - R_1^2)}}$$

Variation in magnetic field with radius

(i) The variation in magnetic field due to infinite long solid cylindrical conductor along its radius is as shown in figure.

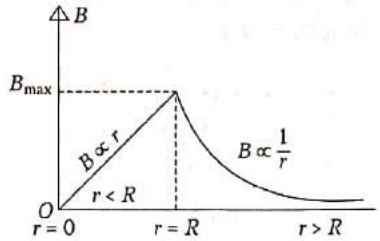


Fig. 4.20 B versus r graph for solid cylinder

Note that inside the wire $B \to 0$ as $r \to 0$.

Also, the magnetic field is continuous at the surface of the wire.

(ii) The variation in magnetic field due to infinite long hollow cylindrical conductor along its radius is as shown in figure.

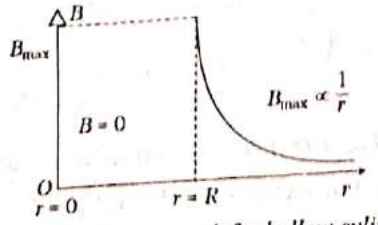


Fig. 4.21 B versus r graph for hollow cylinder

Magnetic field outside the cylindrical conductor does not depend upon nature (thick/thin or solid/hollow) of the conductor as well as its radius of cross-section.

2. Magnetic field due to a solenoid

A cylindrical coil of many wounded turns of insulated wire with generally diameter of the coil smaller than its length is called a solenoid.

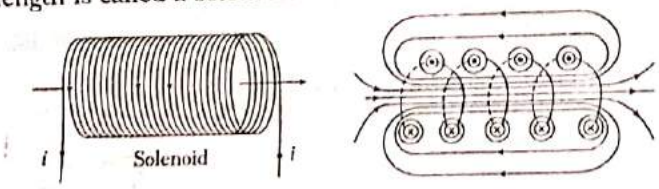


Fig. 4.22 Solenoid and its magnetic field

A magnetic field is produced around and within the solenoid as shown in above figure. The magnetic field within the tightly wound solenoid (ideal solenoid) is uniform and parallel to the axis of solenoid.

Outside the ideal solenoid magnetic field is zero as shown in the figure below.

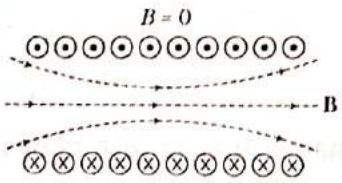


Fig. 4.23 Magnetic field (outside) due to ideal solenoid

(i) **Finite length solenoid** If N = total number of turns, l = length of the solenoid, n = number of turns per unit length = N/l

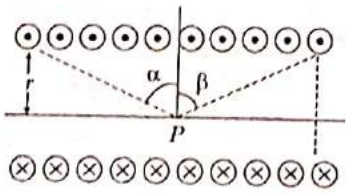


Fig. 4.24 Magnetic field due to finite length solenoid

Magnetic field inside the solenoid at point P is given by

$$\boxed{B = \frac{\mu_0 ni}{2}[\sin\alpha + \sin\beta]}$$

(ii) **Infinite length solenoid** If the solenoid is of infinite length and the point is well inside it *i.e.*, $\alpha = \beta = (\pi/2)$.

So, $\boxed{B_{in} = \mu_0 ni}$

If the solenoid is of infinite length and the point is near one end *i.e.*, $\alpha = 0$ and $\beta = (\pi/2)$, so

$$\boxed{B_{end} = \frac{1}{2}(\mu_0 ni)} \qquad \left(B_{end} = \frac{1}{2}B_{in}\right)$$

(iii) Variation in magnetic field with distance d from its centre

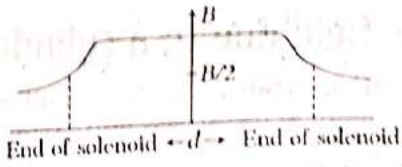


Fig. 4.25 B versus distance d graph for solenoid

Note 1. Magnetic field produced by solenoid is directed along its axis.
2. Magnetic field inside the solenoid is uniform.
3. Magnetic field outside the volume of the ideal solenoid (tightly wound) approaches to zero.

3. Magnetic field due to a toroid

A toroid can be considered as a ring shaped closed solenoid. Hence, it is like an endless cylindrical solenoid.

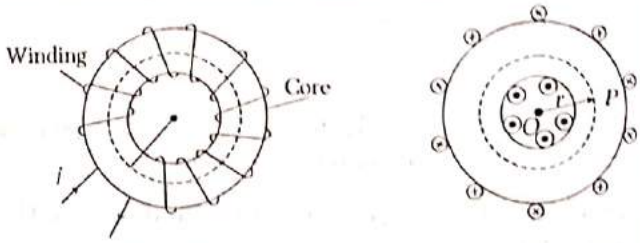


Fig. 4.26 Toroid and its magnetic field

Consider a toroid having n turns per unit length. Magnetic field at a point P in the figure is given as

$$\boxed{B = \frac{\mu_0 Ni}{2\pi r} = \mu_0 ni} \quad \text{where } n = \frac{N}{2\pi r}$$

Note

1. Magnetic field outside the volume of toroid is always zero.
2. Magnetic field at the centre of toroid is always zero.

4. Magnetic field due to an infinitely large carrying current sheet

Given, an infinite sheet of current with linear current density J (A/m).

We want to determine the magnetic field at distance d from the sheet as shown in Fig. (a).

Now, consider a square Amperian loop of side l as shown in the Fig. (b).

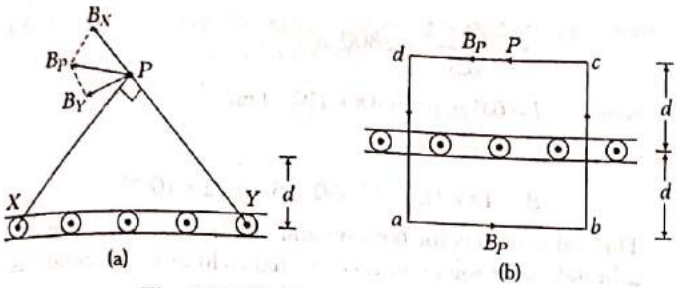


Fig. 4.27 (a) Magnetic field at point P
(b) Amperian loop around the sheet

Consider two strips X and Y of the sheet situated symmetrically on two sides of the point P. At this point magnetic field due to strip X is $B_X \perp X_P$ similarly $B_Y \perp Y_P$. Resultant of B_X and B_Y is parallel to XY and resultant of field due to whole sheet bc in this direction. Let the resultant field be B_P.

From Ampere's law,

$$\int_a^b \mathbf{B}_P \cdot d\mathbf{l} + \int_b^c \mathbf{B}_P \cdot d\mathbf{l} + \int_c^d \mathbf{B}_P \cdot d\mathbf{l} + \int_d^a \mathbf{B}_P \cdot d\mathbf{l} = \mu_0 i$$

Since, $\mathbf{B}_P \perp d\mathbf{l}$ along the path $b \to c$ and $d \to a$, therefore

$$\int_b^c \mathbf{B}_P \cdot d\mathbf{l} = 0; \int_d^a \mathbf{B}_P \cdot d\mathbf{l} = 0$$

Also, $B \parallel dl$ along the path $a \to b$ and $c \to d$, thus

$$\int_a^b \mathbf{B}_P \cdot d\mathbf{l} + \int_d^a \mathbf{B}_P \cdot d\mathbf{l} = 2B_P l$$

The current enclosed by the loop is $i = Jl$.

Therefore,

$$2\mathbf{B}_P l = \mu_0 (Jl) \text{ or } \mathbf{B}_P = \frac{\mu_0 J}{2} \quad \text{(from Ampere's law)}$$

or

$$\boxed{B = \frac{\mu_0 J}{2}}$$

Example 4.20 *A 0.8 m long solenoid has 800 turns and has a field density of 2.52×10^{-3} T at its centre. Find the current in the solenoid.*

Sol. Number of turns per unit length,

$$n = \frac{N}{l} = \frac{800}{0.8\text{ m}} = 1000 \text{ turns/m}$$

We know that, $B = \mu_0 ni$

$\therefore$ Current in the solenoid, $i = \dfrac{B}{\mu_0 n} = \dfrac{2.52 \times 10^{-3}}{4\pi \times 10^{-7} \times 1000} = 2.0\text{ A}$

Example 4.21 *A copper wire having a resistance of 0.01 Ω per metre is used to wind 400 turns solenoid of radius 1.0 cm and length 20 cm. Find the emf of a battery which when connected across the solenoid would produce a magnetic field of 10^{-2} T near the centre of the solenoid.*

Sol. Length of wire used $= 2\pi r \times$ No. of turns

$$= 2\pi \times 1.0 \times 10^{-2} \times 400\text{ m}$$

Resistance per unit length $= 0.01\ \Omega/\text{m}$

$\therefore$ Total resistance of wire,

$$R = 2\pi \times 1.0 \times 10^{-2} \times 400 \times 0.01 = 8\pi \times 10^{-2}\ \Omega$$

Number of turns per unit length, $n = \dfrac{400}{20 \times 10^{-2}} = 2000\text{ m}^{-1}$

As, magnetic field $B = \mu_0 ni = \mu_0 n \dfrac{V}{R}$

$\therefore$ emf of the battery,

$$\therefore \quad V = \frac{BR}{\mu_0 n} = \frac{10^{-2} \times 8\pi \times 10^{-2}}{4\pi \times 10^{-7} \times 2000} = 1\text{ V}$$

Example 4.22 *A coil wrapped around a toroid has inner radius of 20.0 cm and an outer radius of 25.0 cm. If the wire wrapping makes 800 turns and carries a current of 12.0 A. Find the maximum and minimum values of the magnetic field within the toroid?*

Sol. Let a and b denote the inner and outer radii of the toroid. Then, maximum value of magnetic field,

$$B_{max} = \mu_0 nI = \frac{\mu N}{2\pi a} I = \frac{4\pi \times 10^{-7} \times 800 \times 12.0}{2\pi \times 20.0 \times 10^{-2}}$$

$$= 9.6 \times 10^{-3}\text{ T} = 9.6\text{ mT}$$

Minimum value of magnetic field, $B_{min} = \mu_0 nI = \mu_0 \dfrac{NI}{2\pi b}$

$$B_{min} = \frac{4\pi \times 10^{-7} \times 800 \times 12.0}{2\pi \times 25.0 \times 10^{-2}} = 7.68 \times 10^{-3}\text{T} = 7.68\text{ mT}$$

Example 4.23 *A long straight wire of radius a carries a steady current I. The current is uniformly distributed across its cross-section. Find the ratio of the magnetic field at $\frac{a}{2}$ and 2a.*

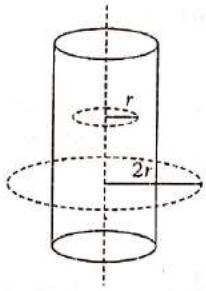
Sol. Current density, $J = \dfrac{I}{\pi a^2}$

From Ampere's circuital law, $\oint \mathbf{B} \cdot d\mathbf{l} = \mu \cdot I_{enclosed}$

For $r < a$, $B \times 2\pi r = \mu_0 \times J \times \pi r^2 \Rightarrow B = \dfrac{\mu_0 J}{\pi a^2} \times \dfrac{r}{2}$

At $r = a/2$, $\quad B_1 = \dfrac{\mu_0 I}{4\pi a}$

For $r > a$, $B \times 2\pi r = \mu_0 I \Rightarrow B = \dfrac{\mu_0 I}{2\pi r}$



At $r = 2a$, $B_2 = \dfrac{\mu_0 i}{4\pi a}$

So, $\dfrac{B_1}{B_2} = 1$

Example 4.24 *A solenoid 50 cm long has 4 layers of windings of 350 turns each. The radius of the lowest layer is 1.4 cm. If the current carried is 6.0 A. Find the magnitude of* **B** *(i) near the centre of the solenoid on its axis and off its axis, (ii) near its ends on its axis, (iii) outside the solenoid near its centre.*

Sol. (*i*) The magnitude of the magnetic field at or near the centre of the solenoid is given by

$$B = \mu_0 n I$$

where, n is the number of turns per unit length.

This expression for B can also be used, if the solenoid has more than one layer of windings because the radius of wire does not enter this equation. Therefore,

$$n = \frac{\text{No. of turns per layer} \times \text{No. of layers}}{\text{length of the solenoids}}$$

$$= \frac{350 \times 4}{0.5} = 2800\ \text{m}^{-1}$$

Now, $I = 6.0\ \text{A}, \mu_0 = 4\pi \times 10^{-7}\ \text{TmA}^{-1}$

$n = 2800\ \text{m}^{-1}$

$\therefore \quad B = 4\pi \times 10^{-7} \times 2800 \times 6\text{T} = 2.1 \times 10^{-2}\text{T}$

This value of B is for both on and off its axis. Since, for an infinitely long solenoid, the internal field near the centre is uniform over the entire cross-section.

(ii) Magnetic field at the ends of the solenoid is B_{end}

$$= \frac{\mu_0 n I}{2} = 1.05 \times 10^{-2}\text{T}$$

(iii) The outside field near the centre of a long solenoid is negligible compared to the internal field.

Check point 4.1

1. Which of the following gives the value of magnetic field due to a small current element according to Biot-Savart's law?

(a) $\frac{i\Delta l \sin\theta}{r^2}$ (b) $\frac{\mu_0}{4\pi}\frac{i\Delta l \sin\theta}{r}$

(c) $\frac{\mu_0}{4\pi}\frac{i\Delta l \sin\theta}{r^2}$ (d) $\frac{\mu_0}{4\pi}\frac{i\Delta l \sin\theta}{r^3}$

2. Magnetic field at a distance r from an infinitely long straight conductor carrying steady current varies as

(a) $1/r^2$ (b) $1/r$

(c) $1/r^3$ (d) $1/\sqrt{r}$

3. The strength of the magnetic field at a distance r near a long straight current carrying wire is B. The field at a distance $r/2$ will be

(a) $\frac{B}{2}$ (b) $\frac{B}{4}$ (c) $2B$ (d) $4B$

4. Two parallel long wires carry currents i_1 and i_2 with $i_1 > i_2$. When the currents are in the same direction, then the magnetic field at a point midway between the wires is 10 μT. If the direction of i_2 is reversed, then the field becomes 30 μT. The ratio i_1/i_2 is

(a) 4 (b) 3

(c) 2 (d) 1

5. The current is flowing in south direction along a power line. The direction of magnetic field above the power line (neglecting earth's field) is

(a) south (b) east

(c) north (d) west

6. Two infinitely long, thin, insulated, straight wires lie in the XY-plane along the X and Y-axes, respectively. Each wire carries a current I respectively, in the positive X-direction and positive Y-direction. The magnetic field will be zero at all points on the straight line with equation.

(a) $y = x$ (b) $y = -x$

(c) $y = x - 1$ (d) $y = -x + 1$

7. The magnetic field produced at the centre of a current carrying circular coil of radius r, is

(a) directly proportional to r

(b) inversely proportional to r

(c) directly proportional to r^2

(d) inversely proportional to r^2

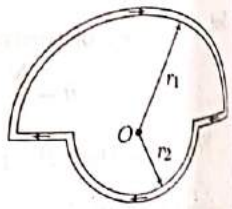
8. An arc of a circle of radius R subtends at an angle $\pi/2$ at the centre. It carries a current i. The magnetic field at the centre will be

(a) $\frac{\mu_0 i}{2R}$ (b) $\frac{\mu_0 i}{8R}$ (c) $\frac{\mu_0 i}{4R}$ (d) $\frac{2\mu_0 i}{5R}$

9. A particle carrying a charge equal to 100 times. The charge on an electron is rotating one rotation per second in a circular path of radius 0.8 m. The value of the magnetic field produced at the centre will be
(μ_0 = permeability for vacuum)

(a) $\frac{10^{-7}}{\mu_0}$ (b) $10^{-17}\mu_0$ (c) $10^{-6}\mu_0$ (d) $10^{-7}\mu_0$

10. In the figure shown, there are two semicircles of radii r_1 and r_2 in which a current i is flowing. The magnetic induction at the centre O will be



(a) $\frac{\mu_0 i}{4}(r_1 + r_2)$ (b) $\frac{\mu_0 i}{4}(r_1 - r_2)$

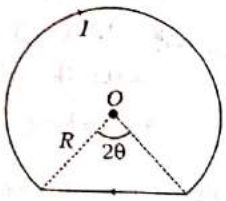
(c) $\frac{\mu_0 i}{4}\left[\frac{r_1 + r_2}{r_1 r_2}\right]$ (d) $\frac{\mu_0 i}{4}\left[\frac{r_2 - r_1}{r_1 r_2}\right]$

11. A current of 0.1 A circulates around a coil of 100 turns and having a radius equal to 5 cm. The magnetic field set up at the centre of the coil is ($\mu_0 = 4\pi \times 10^{-7}$ Wb/A-m)

(a) $5\pi \times 10^{-5}$ T (b) $8\pi \times 10^{-5}$ T

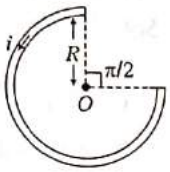
(c) $4\pi \times 10^{-5}$T (d) $2\pi \times 10^{-5}$ T

12. A current I flows through a closed loop as shown in figure. The magnetic field at the centre O is



(a) $\frac{\mu_0 I}{2\pi R}(\pi - \theta + \tan\theta)$ (b) $\frac{\mu_0 I}{2\pi R}(\pi - \theta + \sin\theta)$
(c) $\frac{\mu_0 I}{2\pi R}(\theta + \sin\theta)$ (d) None of these

13. A current i ampere flows in a circular arc of wire whose radius is R, which subtends an angle $3\pi/2$ radian at its centre. The magnetic induction B at the centre is



(a) $\frac{\mu_0 i}{R}$ (b) $\frac{\mu_0 i}{2R}$
(c) $\frac{2\mu_0 i}{R}$ (d) $\frac{3\mu_0 i}{8R}$

14. Magnetic field due to a ring having n turns at a distance x on its axis is proportional to (if r = radius of ring)
(a) $\frac{r}{(x^2 + r^2)}$ (b) $\frac{r}{(x^2 + r^2)^{3/2}}$
(c) $\frac{nr^2}{(x^2 + r^2)^{3/2}}$ (d) $\frac{n^2 r^2}{(x^2 + r^2)^{3/2}}$

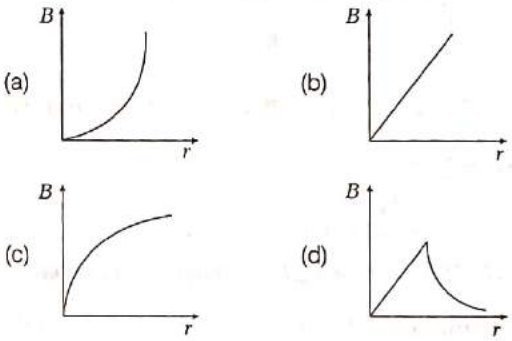
15. The ratio of the magnetic field at centre of a current carrying coil of the radius a and at a distance a from centre of the coil and perpendicular to the axis of coil is
(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$ (c) $\frac{1}{2\sqrt{2}}$ (d) $2\sqrt{2}$

16. A circular current carrying coil has a radius R. The distance from the centre of the coil on the axis of the coil, where the magnetic induction is 1/8th of its value at the centre of coil is
(a) $\sqrt{3}R$ (b) $R/\sqrt{3}$ (c) $(2/\sqrt{3})R$ (d) $R/2\sqrt{3}$

17. Two concentric coils of 10 turns each are situated in the same plane. Their radii are 2 cm and 4 cm and they carry respectively, 0.2 and 0.3 A current in opposite directions. The magnetic field (in Wb/m^2) at the centre is
(a) $(35/4)\mu_0$ (b) $(\mu_0/80)$
(c) $(7/80)\mu_0$ (d) $(25/2)\mu_0$

18. A given length of a wire carries a steady current. It is bent first to form a circular plane coil of one turn. If a loop of same length is now bent more sharply to give a double loop of smaller radius, then the magnetic field at the centre caused by the same current is
(a) a quarter of its first value (b) unaltered
(c) four times of its first value (d) two times of its first value

19. Which of the following figure shows the magnetic flux density B at a distance r from a long straight rod carrying a steady current i ?



20. In using Ampere's law, to find the magnetic field inside a straight long solenoid, the loop that is taken is
(a) a circular loop, co-axial with the solenoid
(b) a rectangular loop in a plane is perpendicular to the axis of the solenoid
(c) a rectangular loop in a plane containing the axis of the solenoid, the loop being totally within the solenoid
(d) a rectangular loop in a plane containing the axis of the solenoid, the loop being partly inside the solenoid and partly outside it

3 Force on a moving charge in a uniform magnetic field

If a particle carrying a positive charge q and moving with velocity v enters a magnetic field B, then it experiences a force F_m which is given by the expression

$$\boxed{\mathbf{F}_m = q(\mathbf{v} \times \mathbf{B})}$$

(i) The magnitude of $\mathbf{F}_m$ is

$$F_m = Bqv \sin\theta$$

where, θ is the angle between $\mathbf{v}$ and $\mathbf{B}$.

(ii) $\mathbf{F}_m$ is zero, when
(a) $B = 0$ *i.e.*, no magnetic field is present.
(b) $q = 0$ *i.e.*, particle is neutral.
(c) $v = 0$ *i.e.*, charged particle is at rest or
(d) $\theta = 0°$ or $180°$ *i.e.*, Force = 0

(iii) $\mathbf{F}_m$ is maximum at $\theta = 90°$ and this maximum value is Bqv.

Rules to find the direction of magnetic force (F_m)

From the property of cross product we can infer that $\mathbf{F}_m$ is perpendicular to both $\mathbf{v}$ and $\mathbf{B}$ or it is perpendicular to the

plane formed by **v** and **B**. *The exact direction of* $\mathbf{F}_m$ *can be given by any of the following methods.*

(i) Direction of $\mathbf{F}_m$ = (sign of q) (direction of $v \times B$) ...(i)

We can say that, if q is positive in Eq. (i), magnetic force is along $\mathbf{v} \times \mathbf{B}$ and if q is negative magnetic force is in a direction opposite to $\mathbf{v} \times \mathbf{B}$.

(ii) **Fleming's left hand rule** According to this rule, the forefinger, the central finger and the thumb of the left hand are stretched in such a way that they are mutually perpendicular to each other. If the central finger shows the direction of velocity of positive charge (**v**) and forefinger shows the direction of magnetic field (**B**), then the thumb will give the direction of magnetic force ($\mathbf{F}_m$).

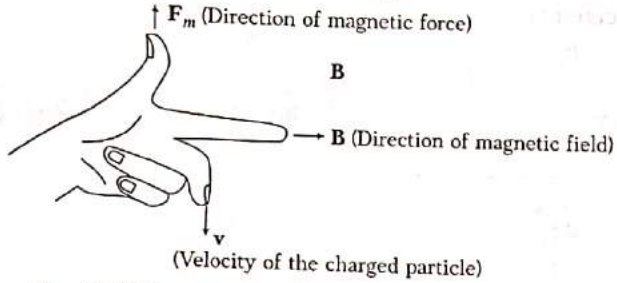


Fig. 4.28 Directions of $\mathbf{F}_m$ *by Flemings left hand rule*

(c) **Right hand rule** Wrap the fingers of your right hand around the line perpendicular to the plane of **v** and **B** as shown in figure, so that they curl around with the sense of rotation from **v** to **B** through the smaller angle between them. Your thumb, then points in the direction of the force $\mathbf{F}_m$ on a positive charge.

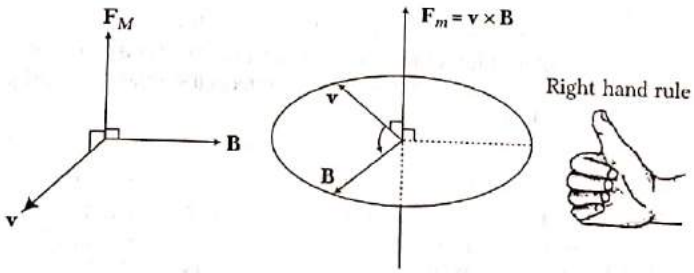


Fig. 4.29 Direction of F_m *by right hand rule*

Note $\mathbf{F}_m \perp \mathbf{v}$ or $\mathbf{F} \perp \frac{d\mathbf{s}}{dt}$. Therefore, $\mathbf{F}_m \perp d\mathbf{s}$ or the work done by the magnetic force in a static magnetic field is zero.

$$W_{F_m} = 0$$

Example 4.25 *A charged particle of specific charge (i.e., charge per unit mass) 0.2 C/kg has velocity* $2\hat{\mathbf{i}} - 3\hat{\mathbf{j}}$ ms^{-1} *at some instant in a uniform magnetic field* $5\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$ *(tesla). Find the acceleration of the particle at this instant.*

Sol. The acceleration of the particle, $\mathbf{a} = \frac{\mathbf{F}}{m} = \frac{q}{m}(\mathbf{v} \times \mathbf{B})$

Given, $\frac{q}{m} = 0.2$ C/kg

$$\mathbf{v} = (2\hat{\mathbf{i}} - 3\hat{\mathbf{j}})\,\text{ms}^{-1}$$

and $\mathbf{B} = (5\hat{\mathbf{i}} + 2\hat{\mathbf{j}})$ T

Therefore, $\mathbf{a} = 0.2(2\hat{\mathbf{i}} - 3\hat{\mathbf{j}}) \times (5\hat{\mathbf{i}} \times 2\hat{\mathbf{j}})\,\text{ms}^{-2}$

$$= 0.2\,[4\hat{\mathbf{k}} + 15\hat{\mathbf{k}}]\,\text{ms}^{-2}$$

$\Rightarrow$ $\mathbf{a} = 3.8\,\hat{\mathbf{k}}\,\text{ms}^{-2}$

Example 4.26 *When a proton has a velocity* $\mathbf{v} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) \times 10^6\ ms^{-1}$, *it experiences a force* $\mathbf{F} = -(1.28 \times 10^{-13}\ \hat{\mathbf{k}})\,N$. *When its velocity is along the Z-axis, then it experiences a force along the X-axis. What is the magnetic field?*

Sol. Magnetic force is,

$$\mathbf{F}_m = q\,(\mathbf{v} \times \mathbf{B})$$

For $\mathbf{F}_m$ to be along $(-\hat{\mathbf{k}})$ magnetic field must be along $(-\hat{\mathbf{j}})$.

Now, substituting proper values, we get

$$-(1.28 \times 10^{-13}\ \hat{\mathbf{k}}) = (1.6 \times 10^{-19})\,[(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) \times (-B_0\hat{\mathbf{j}})] \times 10^6$$

$\therefore$ $1.28 = 1.6 \times 2 \times B_0$

or $B_0 = \frac{1.28}{3.2} = 0.4$

Therefore, the magnetic field is, $B = -B_0\hat{\mathbf{j}} = (-0.4\,\hat{\mathbf{j}})$ T

Example 4.27 *A charged particle is projected in a magnetic field*

$$\mathbf{B} = (3\,\hat{\mathbf{i}} + 4\hat{\mathbf{j}}) \times 10^{-2}\ \text{T}$$

The acceleration of the particle is found to be

$$\mathbf{a} = (x\hat{\mathbf{i}} + 2\hat{\mathbf{j}})\,ms^{-2}$$

Find the value of x.

Sol. As we have read, $\mathbf{F}_m \perp \mathbf{B}$

i.e., the acceleration $\mathbf{a} \perp \mathbf{B}$ or $\mathbf{a} \cdot \mathbf{B} = 0$

or $(x\hat{\mathbf{i}} + 2\hat{\mathbf{j}}) \cdot (3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}) \times 10^{-2} = 0$

or $(3x + 8) \times 10^{-2} = 0$

$\therefore$ Value of $x = -\frac{8}{3}$

Example 20.28 *A magnetic field of* $(4.0 \times 10^{-3}\ \hat{\mathbf{k}})T$ *exerts a force* $(4.0\,\hat{\mathbf{i}} + 3.0\,\hat{\mathbf{j}}) \times 10^{-10}$ *N on a particle having a charge* 10^{-9} *C and moving in the XY-plane. Find the velocity of the particle.*

Sol. Given, $\mathbf{B} = (4 \times 10^{-3}\hat{\mathbf{k}})$ T, $q = 10^{-9}$ C

and magnetic force, $\mathbf{F}_m = (4.0\,\hat{\mathbf{i}} + 3.0\,\hat{\mathbf{j}}) \times 10^{-10}$ N

Let velocity of the particle in XY-plane be

$$\mathbf{v} = v_x\hat{\mathbf{i}} + \mathbf{v}_y\hat{\mathbf{j}}$$

Then, from the relation,

$$\mathbf{F}_m = q\,(\mathbf{v} \times \mathbf{B})$$

We have, $(4.0\,\hat{\mathbf{i}} + 3.0\,\hat{\mathbf{j}}) \times 10^{-10} = 10^{-9}\,[(v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}}) \times (4 \times 10^{-3}\hat{\mathbf{k}})]$

$$= (4v_y \times 10^{-12}\,\hat{\mathbf{i}} - 4v_x \times 10^{-12}\hat{\mathbf{j}})$$

Comparing the coefficients of $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$, we have

$$4 \times 10^{-10} = 4v_y \times 10^{-12}$$

$$\therefore \quad v_y = 10^2 \text{ ms}^{-1} = 100 \text{ ms}^{-1}$$

and $$3.0 \times 10^{-10} = -4v_x \times 10^{-12}$$

$$\therefore \quad v_x = -75 \text{ ms}^{-1}$$

$$\therefore \quad \mathbf{v} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} = (-75\hat{\mathbf{i}} + 100\hat{\mathbf{j}}) \text{ m/s}$$

Example 4.29 *A charged particle carrying charge, $q = 1\mu C$ moves in uniform magnetic field with velocity, $v_1 = 10^6\ ms^{-1}$ at an angle 45° with X-axis in the xy-plane and experiences a force $F_1 = 5\sqrt{2}$ mN along the negative Z-axis. When the same particle move with velocity, $v_2 = 10^6\ ms^{-1}$ along the Z-axis then it experiences a force F_2 in y-direction. Find*

(i) magnitude and direction of the magnetic field.

(ii) the magnitude of the force F_2.

Sol. F_2 is in y-direction when velocity is along Z-axis. Therefore, magnetic field should be along X-axis. So, let

$$\mathbf{B} = B_0\hat{\mathbf{i}}$$

(i) Given, $$\mathbf{v}_1 = \frac{10^6}{\sqrt{2}}\hat{\mathbf{i}} + \frac{10^6}{\sqrt{2}}\hat{\mathbf{j}}$$

and $$\mathbf{F}_1 = -5\sqrt{2} \times 10^3\ \hat{\mathbf{k}}$$

From the equation,

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$$

We have,

$$(-5\sqrt{2} \times 10^{-3})\hat{\mathbf{k}} = (10^{-6})\left[\left(\frac{10^6}{\sqrt{2}}\hat{\mathbf{i}} + \frac{10^6}{\sqrt{2}}\hat{\mathbf{j}}\right) \times (B_0\hat{\mathbf{i}})\right]$$

$$= -\frac{B_0}{\sqrt{2}}\hat{\mathbf{k}}$$

$$\therefore \quad \frac{B_0}{\sqrt{2}} = 5\sqrt{2} \times 10^{-3}$$

or $$B_0 = 10^{-2}\text{T}$$

Therefore, the magnetic field is

$$\mathbf{B} = (10^{-2}\ \hat{\mathbf{i}})\text{T}$$

(ii) $$F_2 = B_0 q v_2 \sin 90°$$

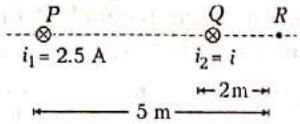
As the angle between $\mathbf{B}$ and $\mathbf{v}$ in this case is 90°.

$$\therefore \quad F_2 = (10^{-2})(10^6)(10^{-6}) = 10^{-2}\text{ N}$$

Example 4.30 *Two long parallel wires carrying current 2.5 A and i in the same direction (directed into plane of the paper) are held at P and Q respectively, such that they are perpendicular to the plane of paper. The points P and Q are located at distance of 5 m and 2 m respectively, from a collinear point R.*

(i) An electron moving with a velocity of 4×10^5 m/s along positive x-direction experience a force of magnitude 3.2×10^{-20} N at the point R. Find the value of i.

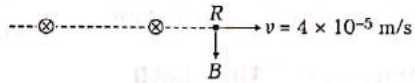
(ii) Find all positions at which a third long parallel wire carrying a current of magnitude 2.5 A may be placed, so that magnetic field at R is zero.



Sol. (i) Magnetic field at R due to wires,

$$B = \frac{\mu_0}{2\pi}\left[\frac{i_1}{5} + \frac{i_2}{2}\right] = 2 \times 10^{-7}\left[\frac{2.5}{5} + \frac{i}{2}\right]$$

$$= 10^{-7}(1 + i), \text{ along } -Y\text{-axis}$$



Angle between $\mathbf{v}$ and $\mathbf{B}$ is 90°,

Force, $F_m = Bqv$

$$\Rightarrow 3.2 \times 10^{-20} = 10^{-7}(1 + i)(1.6 \times 10^{-19})(4 \times 10^5)$$

$$\Rightarrow \quad 1 + i = 5$$

$\therefore$ Current, $i = 4$ A

(b) Third wire can be placed left of R or right of R at distance d. If it is placed left of R, current in it should be upward or placed right of R, current in it should be in downward direction.

$$B_R = 0$$

$$\Rightarrow \quad \frac{\mu_0}{2\pi}\left[\frac{2.5}{5} + \frac{4}{2}\right] = \frac{\mu_0}{2\pi}\frac{2.5}{d}$$

$\therefore$ Position, $d = 1$ m

Motion of a charged particle in uniform magnetic field

As we know that, the magnetic force on a charged particle is perpendicular to its velocity. Thus, this force does not do any work on the particle. Hence, the kinetic energy or the speed of the particle does not change due to the magnetic force.

Let a charged particle q is thrown in magnetic field $\mathbf{B}$ with a velocity $\mathbf{v}$ as shown in the figure. The magnetic force acting on the particle is given by $F = Bqv \sin\theta$, where θ is the angle between the velocity and the magnetic field.

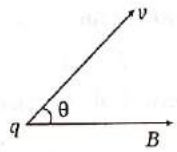


Fig. 4.30 *Charged particle with velocity v at angle θ with the magnetic field*

Depending on the initial conditions, the charged particle can follow different trajectories in a region of uniform magnetic field. Let us take them up one by one.

Case 1. When θ is 0° or 180°,

As discussed earlier $\mathbf{F}_m = 0$, when θ is either 0° or 180°. Hence, path of the charged particle is a straight line

(undeviated) when it enters parallel or anti-parallel to magnetic field, as shown below.

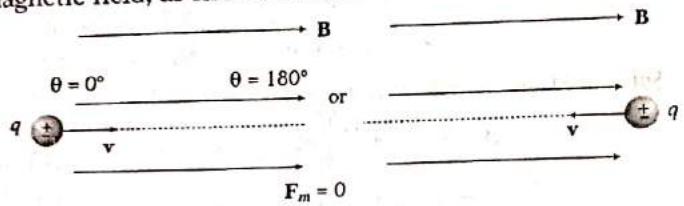


Fig. 4.31 A charged particle moving parallel or anti-parallel to magnetic field

Case 2. When θ is 90°, then path

When θ = 90°, the magnetic force is,

$$F_m = Bqv \sin 90° = Bqv.$$

This magnetic force is perpendicular to the velocity at every instant. Hence, path is a **circle**. The necessary centripetal force is provided by the magnetic force. Hence, if r be the radius of the circle, then

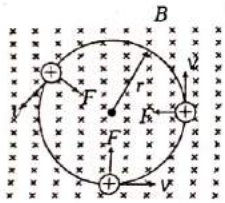


Fig. 4.32 A charged particle is moving in a circular path

$$\frac{mv^2}{r} = Bqv \text{ or } r = \frac{mv}{Bq}$$

This expression of r can be written in following different ways

$$r = \frac{mv}{Bq} = \frac{p}{Bq} = \frac{\sqrt{2Km}}{Bq} \Rightarrow \boxed{\frac{\sqrt{2qVm}}{Bq}}$$

Here, p = momentum of particle and

$$K = \text{KE of particle} = \frac{p^2}{2m} \text{ or } p = \sqrt{2Km}$$

We also know that, if the charged particle is accelerated by a potential difference of V volts, it acquires a KE given by

$$K = qV$$

(i) Further, time period of the circular path will be

$$T = \frac{2\pi r}{v} = \frac{2\pi\left(\frac{mv}{Bq}\right)}{v} \Rightarrow \boldsymbol{T = \frac{2\pi m}{Bq}}$$

(ii) Angular speed (ω) of the particle is,

$$\omega = \frac{2\pi}{T} = \frac{Bq}{m}$$

$$\therefore \quad \boldsymbol{\omega = \frac{Bq}{m}}$$

(iii) Frequency of rotation is,

$$f = \frac{1}{T} \text{ or } \boldsymbol{f = \frac{Bq}{2\pi m}}$$

Note When a charged particle is projected perpendicular to a magnetic field,
(i) Its path is circular in a plane perpendicular to the plane of magnetic field.
(ii) The speed and kinetic energy of the particle remains constant.
(iii) The velocity and momentum of the particle changes only in direction.
(iv) The time period of revolution, angular velocity and frequency of revolution is independent of velocity of the particle and radius of circular path.

Important points related to motion of a charge particle in uniform magnetic field

The speed of the particle in magnetic field does not change but the particle gets deviated. The devitation θ can be found in two ways.

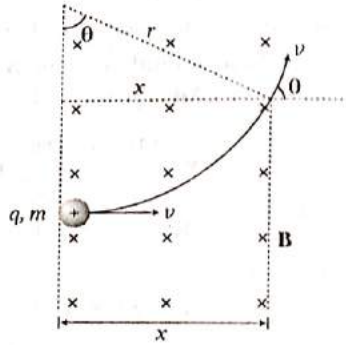


Fig. 4.33 Deviation (θ) of the charged particle in magnetic field

(i) After time t, deviation will be

$$\theta = \omega t = \left(\frac{Bq}{m}\right)t$$

as $$\omega = \frac{Bq}{m}$$

(ii) In terms of the length of the magnetic field (*i.e.*, when the particle leaves the magnetic field) the deviation will be,

$$\theta = \sin^{-1}\left(\frac{x}{r}\right)$$

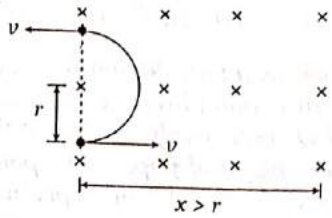


Fig. 4.34 Charged particle having 180° deviation

Since, $\sin\theta \geq 1$, this relation can be used only when $x \leq r$.

For $x > r$, then deviation will be 180° as shown in Fig. 4.36.

Example 4.31 *What is the smallest value of B that can be set up at the equator to permit a proton of speed 10^7 ms^{-1} to circulate around the earth? ($R = 6.4 \times 10^6$ m, $m_p = 1.67 \times 10^{-27}$ kg).*

Sol. From the relation, $R = \frac{mv}{Bq}$

We have, $$B = \frac{mv}{qR}$$

Substituting the values, we have

$$B = \frac{(1.67 \times 10^{-27})(10^7)}{(1.6 \times 10^{-19})(6.4 \times 10^6)} = 1.6 \times 10^{-8}\ \text{T}$$

Example 4.32 *A proton and an α-particle, accelerated through same potential difference, enter in a region of uniform magnetic field with their velocities perpendicular to the field. Compare the radii of circular paths followed by them.*

Sol. Let mass of proton $= m$, charge of proton $= e$

Now, mass of α-particle $= 4\,m$, charge of α-particle $= 2e$

When a charge q is accelerated by V volts, it acquires a kinetic energy $E_k = qV$

∴ Momentum is given by $mv = \sqrt{2mE_k} = \sqrt{2mqV}$

Radius, $r = \frac{mv}{qB}$ or $r = \frac{\sqrt{2mqV}}{qB} = \sqrt{\frac{2mV}{qB^2}}$

Thus, $\frac{r_p}{r_\alpha} = \sqrt{\frac{2mV}{eB^2}} \times \sqrt{\frac{2eB^2}{2(4m)V}} = \frac{1}{\sqrt{2}}$

Example 4.33 *(i) A charged particle having mass m and charge q is accelerated by a potential difference V, it flies through a uniform transverse magnetic field B. The field occupies a region of space d. Find the time interval for which it remains inside the magnetic field.*

(ii) An α-particle is accelerated by a potential difference of 10^4 V. Find the change in its direction of motion if it enters normally in a region of thickness 0.1 m having transverse magnetic induction of 0.1 T. ($m_\alpha = 6.4 \times 10^{-27}$ kg).

(iii) A 10 g bullet having a charge of 4 μC is fired at a speed of 270 ms^{-1} in a horizontal direction. A vertical magnetic field of 500 μT exists in the space. Find the deflection of the bullet due to the magnetic field as it travels through 100 m. Make appropriate approximations.

Sol. (i) Kinetic energy of the charged particle,

$$K = \frac{1}{2}mv^2 = qV$$

$$v = \sqrt{\frac{2qV}{m}}$$

$$\sin\theta = \frac{d}{R} = \frac{d}{mv/Bq} = \frac{Bqd}{mv}$$

$$\Rightarrow \quad \theta = \sin^{-1}\left(\frac{qBd}{mv}\right)$$

$$\text{Time interval, } t = \frac{\theta}{\omega} = \frac{m\theta}{Bq} = \frac{m}{qB}\sin^{-1}\left(\frac{qBd}{m}\right)$$

(ii) We have, $V = 10^4$ V, $q = 2e = 2 \times 1.6 \times 10^{-19}$ C

$d = 0.1$ m, $B = 0.1$T and $m_\alpha = 6.4 \times 10^{-27}$ kg

Kinetic energy, $K = \frac{1}{2}m_\alpha v^2 = qV = 2eV$

$$\Rightarrow \quad v = \sqrt{\frac{4eV}{m_\alpha}}$$

$$\therefore \quad R = \frac{m_\alpha v}{Bq} = \frac{m_\alpha v}{2eB}$$

Also, $$\sin\theta = \frac{d}{R} = \frac{d}{m_\alpha v/B \cdot 2e} = \frac{2eBd}{m_\alpha v}$$

$$= \frac{2eBd}{m_\alpha\sqrt{\frac{4eV}{m_\alpha}}} = \sqrt{\frac{e}{m_\alpha V}} \cdot Bd$$

$$= \sqrt{\frac{1.6 \times 10^{-19}}{6.4 \times 10^{-27} \times 10^4}} \times 0.1 \times 0.1 = \frac{1}{2}$$

∴ Change in direction $\theta = 30°$

(iii) We have, $m = 10$ g $= 10 \times 10^{-3} = 10^{-2}$ kg

$q = 4\ \mu\text{C} = 4 \times 10^{-6}$ C, $v = 270$ m/s

Magnetic field, $B = 500\ \mu\text{T} = 500 \times 10^{-6} = 5 \times 10^{-4}$ T

$d = 100$ m

$$\text{Radius, } R = \frac{mv}{Bq} = \frac{10^{-2} \times 270}{5 \times 10^{-4} \times 4 \times 10^{-6}} = 13.5 \times 10^8\ \text{m}$$

$$\sin\theta = \frac{d}{R} = \frac{100}{13.5 \times 10^8} = \frac{10^{-6}}{13.5}$$

θ is very small.

$$\text{Deflection} = \frac{d^2}{2R} = \frac{(100)^2}{2 \times 13.5 \times 10^8} = 3.7 \times 10^{-6}\ \text{m}$$

Case 3. When the charged particle is moving at an angle to the field other than 0°, 90° or 180°

In this case velocity can be resolved in two components, one along **B** and another perpendicular to **B**. Let the two components be $v_\parallel$ and $v_\perp$.

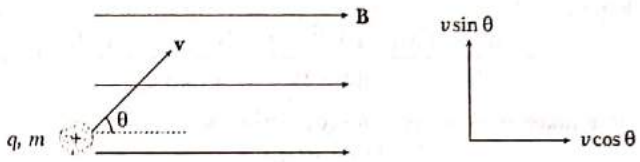


Fig. 4.35 Components of velocity of the charged particle

Then, $v_\parallel = v\cos\theta$

and $v_\perp = v\sin\theta$

The components perpendicular to field ($v_\perp$) gives a circular path and the component parallel to field ($v_\parallel$) gives a straight line path. The resultant path is a helix as shown in figure.

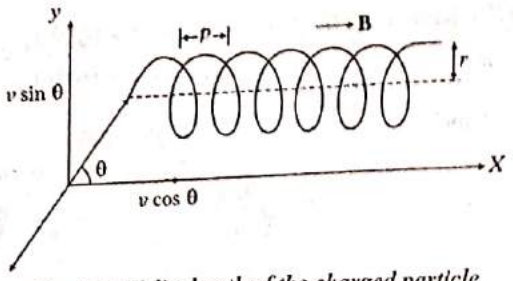


Fig. 4.36 Helical path of the charged particle

(i) The radius of this helical path is,

$$r = \frac{mv_{\perp}}{Bq} = \frac{mv \sin\theta}{Bq}$$

(ii) Time period and frequency do not depend on velocity and so they are given by,

$$T = \frac{2\pi m}{Bq} \quad \text{and} \quad f = \frac{Bq}{2\pi m}$$

(iii) There is one more term associated with a helical path, that is pitch (p) of the helical path. Pitch is defined as the distance travelled along magnetic field in one complete cycle. *i.e.*,

$$p = v_{\parallel}T \quad \text{or} \quad p = (v\cos\theta)\frac{2\pi m}{Bq}$$

$$\therefore \quad p = \frac{2\pi mv\cos\theta}{Bq}$$

Note After every pitch, the particle touches the X-axis (as shown in Fig. 4.36), *i.e.*, $X = np$, where, $n = 0, 1, 2, \ldots$

Example 4.34 *A beam of protons with a velocity of 4×10^5 ms^{-1} enters in a region of uniform magnetic field of 0.3 T. The velocity makes an angle of 60° with the magnetic field. Find the radius of the helical path taken by the proton beam and the pitch of the helix.*

Sol. Velocity component along the field $v_{\parallel} = 4 \times 10^5 \times \cos 60°$

$= 2 \times 10^5$ ms^{-1}

and $v_{\perp} = (4 \times 10^5)\sin 60° = 2\sqrt{3} \times 10^5$ ms^{-1}

Proton will describe a circle in plane perpendicular to magnetic field with radius,

$$r = \frac{mv_{\perp}}{qB} = \frac{(1.67 \times 10^{-27}\text{ kg}) \times (2\sqrt{3} \times 10^5\text{ ms}^{-1})}{(1.6 \times 10^{-19}\text{C}) \times (0.3\text{ T})} = 1.2\text{ cm}$$

Time taken to complete one revolution is

$$T = \frac{2\pi r}{v_{\perp}} = \frac{2 \times 3.14 \times 0.012}{2\sqrt{3} \times 10^5}\text{ s}$$

Because of $v_{\parallel}$ protons will also move in the direction of magnetic field.

Pitch of helix $= v_{\parallel} \times T$

$$= \frac{2 \times 10^5 \times 2 \times 3.14 \times 0.012}{2\sqrt{3} \times 10^5}\text{ m} = 0.044\text{ m} = 4.4\text{ cm}$$

Example 4.35 *A beam of protons with a velocity 6×10^5 ms^{-1} enters a uniform magnetic field of 0.4 T at an angle of 37° to the magnetic field. Find the radius of the helical path taken by proton beam. Also, find the pitch of helix.*

Use, $\sin 37° = 3/5$ *and* $\cos 37° = 4/5$, $m_p \cong 1.6 \times 10^{-27}$ kg

Sol. Radius of helical path, $R = \dfrac{mv\sin\theta}{Bq}$

$$= \frac{(1.6 \times 10^{-27})(6 \times 10^5)(\sin 37°)}{(0.4)(1.6 \times 10^{-19})}$$

$$= 0.045\text{ cm}$$

Time period, $T = \dfrac{2\pi m}{Bq} = \dfrac{2\pi(1.6 \times 10^{-27})}{0.4(1.6 \times 10^{-19})} = 5\pi \times 10^{-8}$ s

Pitch of helical path

$$p = v\cos\theta T = (6 \times 10^5)(\cos 37°)(5\pi \times 10^{-8})$$

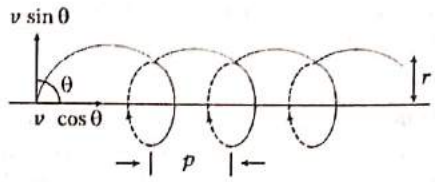
$$= 30\pi \times 0.8 \times 10^{-3} = 2.4\pi \times 10^{-2}\text{m} = 7.5\text{ cm}$$

Example 4.36 *A beam of protons with a velocity 4×10^5 ms^{-1} enters a uniform magnetic field of 0.3 T at an angle of 60° to the magnetic field. Find the pitch of the helix (which is the distance travelled by a proton in the beam parallel to the magnetic field during one period of the rotation).*
Mass of the proton $= 1.67 \times 10^{-27}$ *kg*

Sol. When a charged particle is projected at an angle θ to a magnetic field, the component of velocity parallel to the field is $v\cos\theta$ while perpendicular to the field is $v\sin\theta$, so the particle will move in a circle of radius

$$r = \frac{m(v\sin\theta)}{qB} = \frac{1.67 \times 10^{-27} \times 4 \times 10^5 \times (\sqrt{3}/2)}{1.6 \times 10^{-19} \times 0.3}$$

$$= \frac{2}{\sqrt{3}} \times 10^{-2}\text{m} = 1.2\text{ cm}$$



and time period, $T = \dfrac{2\pi r}{v\sin\theta} = \dfrac{2\pi \times 1.2 \times 10^{-2}}{4 \times 10^5 \times (\sqrt{3}/2)}$

$$= 2.175 \times 10^{-7}\text{s}$$

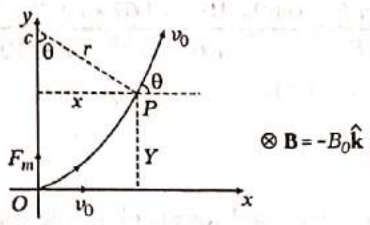
So, pitch of the helix, $p = v\cos\theta \times T$

$$= 4 \times 10^5 \times \frac{1}{2} \times 2.175 \times 10^{-7}$$

i.e., $\quad p = 4.35 \times 10^{-2}\text{m} = 4.35\text{ cm}$

Example 4.37 *A particle of specific charge α enters a uniform magnetic field* $\mathbf{B} = -B_0\hat{\mathbf{k}}$ *with velocity* $\mathbf{v} = v_0\hat{\mathbf{i}}$ *from the origin. Find the time dependence of velocity and position of the particle.*

How to Proceed *In such type of problems first of all see the angle between* **v** *and* **B**. *Because it only decides the path of the particle. Here, the angle is 90°. Therefore, the path is a circle. If it is a circle, see the plane of the circle (perpendicular to the magnetic field). Here the plane is x-y. Then see the sense of the rotation.*



$OC = CP$ = radius of circle

Here, it will be anticlockwise as shown in figure, because at origin the magnetic force is along positive y-direction (which can be seen from Fleming's left hand rule). Find the deviation and radius of the particle.

$$\theta = \omega t = B_0\ \alpha\ t \quad \text{and} \quad r = \frac{V_0}{B_0\alpha}$$

Now according to the figure, find $\mathbf{v}(t)$ *and* $\mathbf{r}(t)$.

Sol. Velocity of the particle at any time t is,

$$\mathbf{v}(t) = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} = v_0\cos\theta\hat{\mathbf{i}} + v_0\sin\theta\hat{\mathbf{j}}$$

or $$\mathbf{v}(t) = v_0\cos(B_0\alpha t)\hat{\mathbf{i}} + v_0\sin(B_0\alpha t)\hat{\mathbf{j}}$$

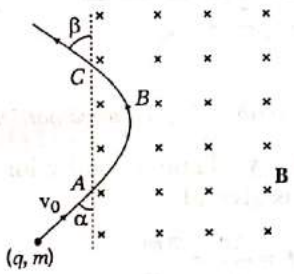
Position of particle at time t is,

$$\mathbf{r}(t) = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} = r\sin\theta\,\hat{\mathbf{i}} + (r - r\cos\theta)\hat{\mathbf{j}}$$

Substituting the values of r and θ, we have

$$\mathbf{r}(t) = \frac{v_0}{B_0\alpha}[\sin(B_0\alpha t)\hat{\mathbf{i}} + \{1 - \cos(B_0\alpha t)\}\hat{\mathbf{j}}]$$

Example 4.38 *A charged particle* (q, m) *enters a uniform magnetic field* **B** *at angle* α *as shown in figure with speed* v_0. *Find,*

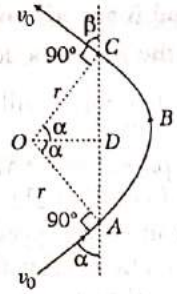


(a) the angle β *at which it leaves the magnetic field,*

(b) time spent by the particle in magnetic field and

(c) the distance AC.

Sol. (a) Here, velocity of the particle is in the plane of paper while the magnetic field is perpendicular to the paper inwards, *i.e.*, angle between **v** and **B** is 90°. So, the path is a circle. The radius of the circle is, $r = \dfrac{mv_0}{Bq}$.



O is the centre of the circle. In ΔAOC,

$$\angle OCD = \angle OAD$$

or $$90° - \beta = 90° - \alpha$$

$\therefore$ $$\beta = \alpha$$

(b) $$\angle COD = \angle DOA = \alpha$$

$$(\text{as } \angle OCD = \angle OAD = 90° - \alpha)$$

$\therefore$ $$\angle AOC = 2\alpha$$

or $$\text{length } ABC = r(2\alpha) = \frac{2mv_0}{Bq}.\alpha$$

$\therefore$ $$t_{ABC} = \frac{ABC}{v_0} = \frac{2m\alpha}{Bq}$$

Alternate method

$$t_{ABC} = \left(\frac{T}{2\pi}\right)(2\alpha) = \left(\frac{\alpha}{\pi}\right)\cdot T$$

$$= \left(\frac{\alpha}{\pi}\right)\left(\frac{2\pi m}{Bq}\right) = \frac{2\alpha m}{Bq}$$

(c) Distance $$AC = 2(AD) = 2(r\sin\alpha)$$

$$= \frac{2mv_0}{Bq}\sin\alpha$$

Motion of a charged particle is combined electric and magnetic field : Lorent'z force

Consider a point charge q moving with velocity **v** is located at position vector **r** at a given time t. If electric field **E** and magnetic field **B** are existing at that point, then force on the electric charge q is given by

$$\boxed{\mathbf{F} = q[\mathbf{E} + \mathbf{v}\times\mathbf{B}] = \mathbf{F}_{\text{electric}} + \mathbf{F}_{\text{magnetic}}}$$

This force was first given by H. A Lorentz, hence it is called **Lorentz force.**

Here, magnetic force $\mathbf{F}_m = q(\mathbf{v}\times\mathbf{B}) = Bqv\sin\theta$ and electric force $\mathbf{F}_e = q\mathbf{E}$.

The direction of magnetic force is same as $\mathbf{v}\times\mathbf{B}$, if charge is positive and opposite to $\mathbf{v}\times\mathbf{B}$, if charge q is negative.

***Case* I When v , E and B are all collinear**

In this situation, the magnetic force on it will be zero and only electric force will act. So, $a = \frac{\mathbf{F}}{m} = \frac{q\mathbf{E}}{m}$.

The particle will pass through the field following a straight line path (parallel field) with change in its speed. So, in this situation speed, velocity, momentum and kinetic energy all will change without change in direction of motion as shown below

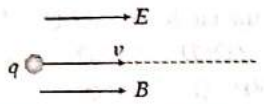


Fig. 4.37 Motion of a charged particle with $\mathbf{v} \parallel \mathbf{E} \parallel \mathbf{B}$

***Case* II When v, E and B are mutually perpendicular**

In this situation, if **E** and **B** are such that

$\mathbf{F} = \mathbf{F}_e + \mathbf{F}_m = 0$ *i.e.*, $\mathbf{a} = (\mathbf{F}/m) = \mathbf{0}$

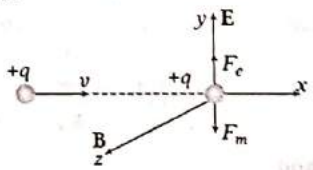


Fig. 4.38 Motion of a charged particle with $\mathbf{v} \perp \mathbf{E} \perp \mathbf{B}$

As shown in figure, the particle will pass through the field with same velocity, without any deviation in its path.

In this situation, as $F_e = F_m$ *i.e.*, $qE = qvB$,

$$\boxed{v = E/B}$$

This principle is used in 'velocity-selector' to get a charged beam having a specific velocity.

Note If only magnetic field is present, we have to put $\mathbf{E} = 0 \Rightarrow \mathbf{F} = q(\mathbf{v} \times \mathbf{B})$.

Similarly, if $\mathbf{B} = 0 \Rightarrow \mathbf{F} = q\mathbf{E}$

Example 4.39 *A charge particle having charge 2 C is thrown with velocity* $2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$ *inside a region having* $\mathbf{E} = 2\hat{\mathbf{j}}$ *and magnetic field* $5\hat{\mathbf{k}}$. *Find the initial Lorentz force acting on the particle.*

Sol. Lorentz Force is given by,

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

$$\Rightarrow \quad \mathbf{F} = 2\,[2\hat{\mathbf{j}} + (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) \times 5\hat{\mathbf{k}}]$$

$$= 2\,[2\hat{\mathbf{j}} + 10\,(-\hat{\mathbf{j}}) + 15\hat{\mathbf{i}}] = 30\hat{\mathbf{i}} - 16\hat{\mathbf{j}}$$

Example 4.40 *A proton beam passes without deviation through a region of space, where there are uniform transverse mutually perpendicular electric and magnetic fields with* $E = 220\ kV/m$ *and* $B = 50\ mT$. *Then, the beam strikes a grounded target. Find the force imparted by the beam on the target, if the beam current is equal to* $i = 0.80\ mA$.

Sol. Since, proton is moving in a straight line, hence net force is zero.

$$\therefore \quad qE = Bqv \Rightarrow v = \frac{E}{B}$$

Also, current associated with the beam,

$$i = ne \Rightarrow n = i/e$$

where, n is number of protons/time.

Momentum of a proton $= mv$

Force, $F = n\,m\,v$

$$= \frac{i\,m\,E}{eB} = \frac{0.80 \times 10^{-3} \times 1.67 \times 10^{-27} \times 220 \times 10^{3}}{1.6 \times 10^{-19} \times 50 \times 10^{-3}}$$

$$= 3.6 \times 10^{-5}\ \text{N}$$

Cyclotron

Cyclotron is a device used for accelerating positively charged particle (like α-particles, deutrons, etc.) by the help of uniform magnetic field upto energy of the order of MeV. It consists of two hollow metallic dees D_1 and D_2. These are placed in a uniform magnetic field perpendicular to the plane of dees. An alternating voltage is applied between the dees.

As, it uses the combination of both the fields (electric and magnetic), which are perpendicular to each other, hence, are called crossed fields. The charged particle to be accelerated is produced at centre point between the dees. The particle accelerate along circular path to acquire enough energy to carry out nuclear disintegration, etc.

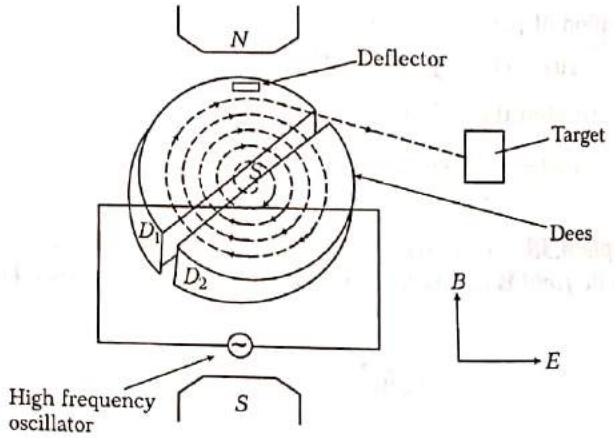


Fig. 4.39 Path described by a charged particle in a cyclotron

Cyclotron frequency Time taken by ion to describe a semicircular path is given by

$$t = \frac{\pi r}{v} = \frac{\pi m}{qB} \qquad \left(\because r = \frac{mv}{qB}\right)$$

If T = time period of oscillating electric field, then

$$T = 2t = \frac{2\pi m}{qB}$$

$\therefore$ Cyclotron frequency, $\nu = \frac{1}{T} = \frac{qB}{2\pi m}$

Note Cyclotron is based on the fact that the time period of one revolution is independent of its speed or radius of the orbit.

Maximum energy of particle Maximum energy gained by the charged particle,

$$E_{max} = \left(\frac{q^2B^2}{2m}\right) r_0^2$$

where, r_0 is maximum radius of the circular path followed by positive ion.

Note

1. Cyclotron is suitable only for accelerating heavy particles like proton, deuteron, α-particle, etc. Electrons cannot be accelerated by the cyclotron because the mass of the electron is small and a small increase in energy of the electron makes the electron move with a very high speed. As a result of it, the electrons go quickly out of step with oscillating electric field.
2. When a positive ion is accelerated by the cyclotron, then it moves with greater and greater speed. As the speed of ion becomes comparable with that of light, the mass of the ion increase according to the relation,

$$m = \frac{m_0}{\sqrt{(1-v^2/c^2)}}$$

where, m_0 = the rest mass of the ion
m = the mass of the ion while moving with velocity v
and c = velocity of light.

Now, the time taken by the ion to describe semicircular path is

$$t = \frac{\pi m}{qB} = \frac{\pi}{qB} \cdot \frac{m_0}{\sqrt{(1-v^2/c^2)}}$$

It shows, that as v increases, t increases. It means, the positive ion will take longer time to describe semicircular path than the time for half-cycle of oscillating electric field. As a result of it, the ion will not arrive in the gap between the two dees exactly at the instant, the polarity of the two dees is reversed and hence, will not be accelerated further. Therefore, the ion cannot move with a speed beyond a certain limit in a cyclotron.

Example 4.41 *The magnetic field applied in a cyclotron is 3.5 T. What will be the frequency of electric field that must be applied between the dees in order to accelerate protons?*

Sol. The frequency of the field,

$$f = \frac{Bq}{2\pi m} = \frac{3.5 \times 1.6 \times 10^{-19}}{2 \times 3.14 \times 1.6 \times 10^{-27}} = 5.57 \times 10^7 \text{ Hz}$$

Example 4.42 *Magnetic field applied on a cyclotron is 0.7 T and radius of its dees is 1.8 m. What will be energy of the emergent protons in MeV?*

Sol. Energy of the emergent protons,

$$E = \frac{q^2B^2r^2}{2m} = \frac{(1.6 \times 10^{-19})^2 \times (0.7)^2 \times (1.8)^2}{2 \times 1.67 \times 10^{-27}} \text{ J}$$

$$= \frac{(1.6 \times 10^{-19})^2 \times (0.7)^2 \times (1.8)^2}{2 \times 1.67 \times 10^{-27} \times 1.6 \times 10^{-13}} \text{ MeV} = 76 \text{ MeV}$$

Example 4.43 *Deuterons in a a circle of radius 32 cm. Just before emerging from the . The frequency of the applied alternating voltage is 10 MHz. Find*

(i) the magnetic flux density (i.e., the magnetic field).

(ii) the energy and speed of the deuterons upon emergence.

Sol. (i) Frequency of the applied emf = Cyclotron frequency

or $f = \frac{Bq}{2\pi m}$

$$\therefore B = \frac{2\pi mf}{q}$$

$$= \frac{(2)(3.14)(2\times 1.67\times 10^{-27})(10\times 10^6)}{1.6\times 10^{-19}} = 1.30 \text{ T}$$

(ii) The speed of deuterons on the emergence from the cyclotron, $v = \frac{BqR}{m} = 2\pi fR$

$$= (2)(3.14)(10\times 10^6)(32\times 10^{-2})$$

$$= 2.01\times 10^7 \text{ ms}^{-1}$$

$\therefore$ Energy of deuterons $= \frac{1}{2}mv^2$

$$= \frac{1}{2}\times (2\times 1.67\times 10^{-27})(2.01\times 10^7)^2 \text{ J}$$

$$= 4.22 \text{ MeV}$$

Check point 4.2

1. A strong magnetic field is applied on a stationary electron, then the electron
(a) moves in the direction of the field
(b) moves in an opposite direction of the field
(c) remains stationary
(d) starts spinning

2. A particle of mass m and charge Q moving with a velocity $\mathbf{v}$ enters a region of uniform magnetic field of induction $\mathbf{B}$. Then, its path in the region is
(a) always circular
(b) circular, if $\mathbf{v} \times \mathbf{B} = 0$
(c) circular, if $\mathbf{v} \cdot \mathbf{B} = 0$
(d) None of these

3. An electron is moving on a circular path of radius r with speed v in a transverse magnetic field B. e/m for it will be
(a) $\frac{v}{Br}$
(b) $\frac{B}{rv}$
(c) Bvr
(d) $\frac{vr}{B}$

4. An electron and a proton with equal momentum enter perpendicularly into a uniform magnetic field, then
(a) the path of proton shall be more curved than that of electron
(b) the path of proton shall be less curved than that of electron
(c) Both are equally curved
(d) path of both will be straight line

5. A charged particle travels along a straight line with a speed v in a region, where both electric field **E** and magnetic field **B** are present. It follows that
(a) $|\mathbf{E}| = |\mathbf{B}|$ and the two fields are perpendicular
(b) $|\mathbf{E}| = v|\mathbf{B}|$ and the two fields are perpendicular
(c) $|\mathbf{B}| = v|\mathbf{E}|$ and the two fields are parallel
(d) $|\mathbf{B}| = v|\mathbf{E}|$ and the two fields are perpendicular

6. A proton, a deuteron and an α-particle, having the same kinetic energy, are moving in circular trajectories in a constant magnetic field. If r_p, r_d and r_α denote respectively the radii of the trajectories of these particles, then
(a) $r_\alpha = r_p < r_d$ (b) $r_\alpha > r_d > r_p$
(c) $r_\alpha = r_d > r_p$ (d) $r_p = r_d = r_\alpha$

7. When a charged particle enters in a uniform magnetic field, then its kinetic energy
(a) remains constant (b) increases
(c) decreases (d) becomes zero

8. A proton of energy 8 eV is moving in a circular path in uniform magnetic field. The energy of an alpha particle moving in the same magnetic field and along the same path will be
(a) 4 eV (b) 2 eV
(c) 8 eV (d) 6 eV

9. A charged particle of mass m and charge q describes circular motion of radius r in a uniform magnetic field of strength B. The frequency of revolution is
(a) $\frac{Bq}{2\pi m}$ (b) $\frac{Bq}{2\pi rm}$
(c) $\frac{2\pi m}{Bq}$ (d) $\frac{Bm}{2\pi q}$

10. A proton and an electron both moving with the same velocity v enter into a region of magnetic field directed perpendicular to the velocity of the particles. They will now move in circular orbits such that
(a) their time periods will be same
(b) the time period for proton will be higher
(c) the time period for electron will be higher
(d) their orbital radii will be same

11. If a charged particle is describing a circle of radius r in a magnetic field with a time period T. Then,
(a) $T^2 \propto r^3$ (b) $T^2 \propto r$ (c) $T \propto r^2$ (d) $T \propto r^0$

12. A particle is projected in a plane perpendicular to a uniform magnetic field. The area bounded by the path described by the particle is proportional to
(a) the velocity (b) the momentum
(c) the kinetic energy (d) None of these

13. Lorentz force can be calculated by using the formula
(a) $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$ (b) $\mathbf{F} = q(\mathbf{E} - \mathbf{v} \times \mathbf{B})$
(c) $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \cdot \mathbf{B})$ (d) $\mathbf{F} = q(\mathbf{E} \times \mathbf{v} + \mathbf{B})$

14. If electron velocity is $(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$ and it is subjected to a magnetic field $4\hat{\mathbf{k}}$, then
(a) speed of electron will change
(b) path of electron will change
(c) Both (a) and (b)
(d) None of the above

15. A beam of protons with a velocity of 4×10^5 ms^{-1} enters a uniform magnetic field of 0.3 T at an angle of 60° to the magnetic field. The radius of helical path taken by proton beam is
(a) 0.036 m (b) 0.012 m
(c) 0.024 m (d) 0.048 m

16. A proton and a deuteron both having the same kinetic energy, enter perpendicularly into a uniform magnetic field B. For motion of proton and deuteron on circular path of radius R_p and R_d respectively, the correct statement is
(a) $R_d = \sqrt{2}R_p$ (b) $R_d = R_p/\sqrt{2}$
(c) $R_d = R_p$ (d) $R_d = 2R_p$

17. If a charged particle at rest experiences no electromagnetic force, then the
(a) electric field must be zero
(b) magnetic field must be zero
(c) electric field may or may not be zero
(d) None of the above

18. If a charged particle projected in a gravity free room deflects, then
(a) there must be an electric field
(b) there must be a magnetic field
(c) Both fields cannot be zero
(d) None of the above

19. Two ions having masses in the ratio 1 : 1 and charges 1 : 2 are projected into uniform magnetic field and perpendicular to the field of a cyclotron with speeds in the ratio 2 : 3. The ratio of the radii of circular paths along which the two particles move is
(a) 4 : 3 (b) 2 : 3
(c) 3 : 1 (d) 1 : 4

20. Which particles will have minimum frequency of revolution when projected in a cyclotron with the same velocity perpendicular to a magnetic field?
(a) Li^+ (b) Electron
(c) Proton (d) He^+

4 Force on a current carrying conductor in a magnetic field

When a current carrying conductor is placed in a magnetic field, then the conductor experiences a force in a direction perpendicular to both the direction of magnetic field and the direction of current flowing in the conductor.

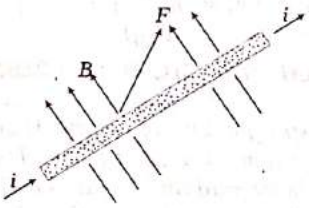


Fig. 4.40 Current carrying conductor in a uniform magnetic field

The magnetic force is, $F = ilB \sin\theta$

In vector form, $\boxed{\mathbf{F} = i\,(\mathbf{l} \times \mathbf{B})}$ ...(i)

where, B = intensity of magnetic field

i = current in the conductor

l = length of the conductor

and θ = angle between the length of conductor and direction of magnetic field.

Cases

I. If $\theta = 90°$ or $\sin\theta = 1$, then $F = ilB$ (maximum). Therefore, force will be maximum when the conductor carrying current is perpendicular to magnetic field.

II. If $\theta = 0°$ or $\sin\theta = 0$, then $F = ilB \times 0 = 0$ Thus, the force will be zero, when the current carrying conductor is parallel to the field.

Direction of force on a current carrying conductor in a magnetic field

The direction of this force can be found out either by Fleming's left hand rule or by right hand palm rule as discussed below.

Fleming's left hand rule

The direction of $\mathbf{F}_m$ or F can be given by Fleming's left hand rule. According to this rule, the forefinger, the central finger and the thumb of the left hand are stretched in such a way that they are mutually perpendicular to each other. If the central finger shows the direction of current (or $\mathbf{l}$) and forefinger shows the direction of magnetic field ($\mathbf{B}$), then the thumb will give the direction of magnetic force ($\mathbf{F}_m$).

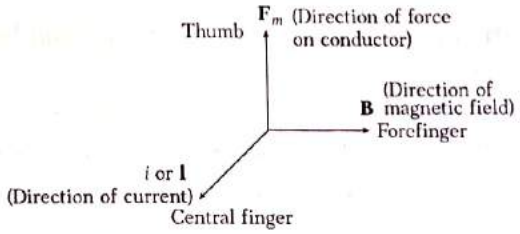


Fig. 4.41 Fleming's left hand rule

Right hand palm rule

Stretch the fingers and thumb of right hand at right angles to each other. Then, if the fingers point in the direction of field $\mathbf{B}$ and thumb in the direction of current i, then normal to the palm will point in the direction of force.

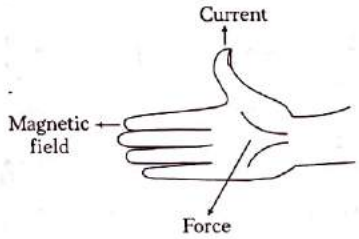


Fig. 4.42 Right hand palm rule for force on a current carrying conductor

Magnetic force on an arbitrarily shaped wire

For the magnetic force on an arbitrarily shaped wire segment, let us consider the magnetic force exerted on a small segment of vector length $d\mathbf{l}$.

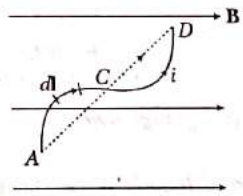


Fig. 4.43 Arbitary shaped wire in a uniform magnetic field

$$d\mathbf{F}_m = i\,(d\mathbf{l} \times \mathbf{B}) \quad \text{...(ii)}$$

To calculate the total force $\mathbf{F}_m$ acting on the wire shown in figure, we integrate Eq. (ii) over the length of the wire.

$$\mathbf{F}_m = i\left(\int_A^D d\mathbf{l}\right) \times \mathbf{B} \quad \text{...(iii)}$$

But the quantity $\int_A^D d\mathbf{l}$ represents the vector sum of all length elements from A to D. From the polygon law of vector addition, the sum equals the vector $\mathbf{l}$ directed from A to D. Thus,

$$\mathbf{F}_m = i\,(\mathbf{l} \times \mathbf{B})$$

or we can write, $\mathbf{F}_{ACD} = \mathbf{F}_{AD} = i\,(A\mathbf{D} \times \mathbf{B})$ in uniform field. Here, AD = effective length of the wire (left).

Magnetic force on a closed current carrying loop

When a closed current carrying loop is placed in uniform magnetic field, then the magnetic force on it is always zero. As the vector sum of $d\mathbf{l}$ is always zero, *i.e.*, $\oint d\mathbf{l} = 0$

$$\therefore \qquad \mathbf{F} = i\oint d\mathbf{l} \times \mathbf{B} = 0$$

Magnetic force on different configurations of the curve

Now, consider magnetic forces on current carrying conductors of different configurations as shown below. Here, magnetic force only depends upon effect length of the wires.

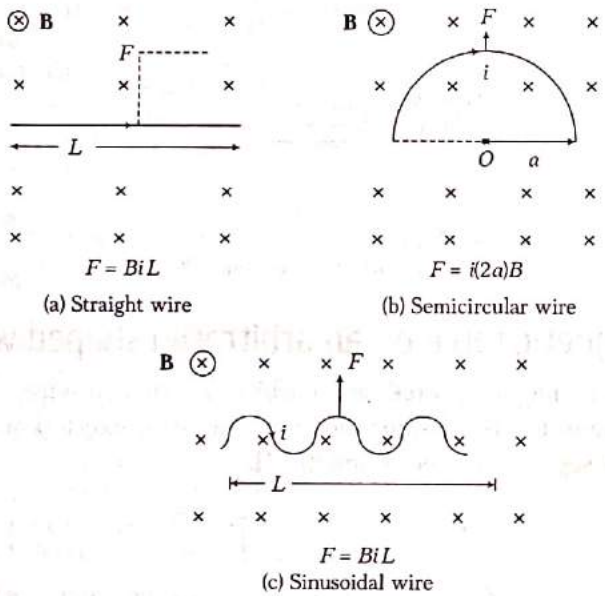


Fig. 4.44

Example 4.44 *A wire of length 'a' carries a current i along the Y-axis. A magnetic field exists given by*

$$\mathbf{B} = B_0(3\,\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}})\ T$$

Calculate magnetic force in vector form and its magnitude.

Sol. Length of the wire, $\mathbf{l} = a\hat{\mathbf{j}}$

Magnetic force on the wire,

$$\mathbf{F} = i(\mathbf{l} \times \mathbf{B}) = ia\,B_0(\hat{\mathbf{j}} \times (3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}})\hat{\mathbf{j}}$$

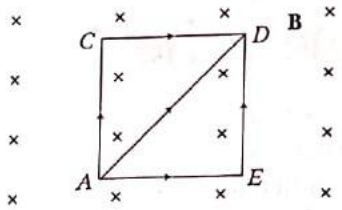
$$= ia\,B_0[-3\hat{\mathbf{k}} + 0 + \hat{\mathbf{i}}]$$

$$= B_0 ia\,(\hat{\mathbf{i}} - 3\hat{\mathbf{k}})$$

$$|\mathbf{F}| = F + B_0 ia\sqrt{(1)^2 + (-3)^2}$$

$$= \sqrt{10}B_0 ia$$

Example 4.45 *A square of side 2.0 m is placed in a uniform magnetic field* $\mathbf{B} = 2.0\,T$ *in a direction perpendicular to the plane of the square inwards. Equal current, i = 3.0 A is flowing in the directions shown in figure. Find the magnitude of magnetic force on the loop.*



Sol. Force on wire ACD = Force on AD = Force on AED

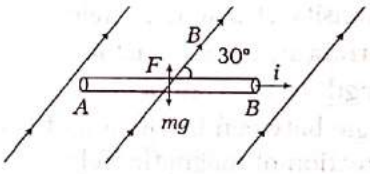
All these forces are acting along EC.

$\therefore$ Net force on the loop = 3 ($\mathbf{F}_{AD}$)

or $\quad F_{net} = 3(i)\,(AD)\,(B) = (3)\,(3.0)\,(2\sqrt{2})\,(2.0)\,\text{N} = 36\sqrt{2}\ \text{N}$

Example 4.46 *A straight wire of length 30 cm and mass 60 mg lies in a direction 30° east of north. The earth's magnetic field at this site is horizontal and has a magnitude of 0.8 G. What current must be passed through the wire, so that it may float in air?*

Sol. As shown in figure, if a current i is passed through the wire from end A towards B it will experience a force $Bi\,L\sin\theta$ vertically up and hence will float, if

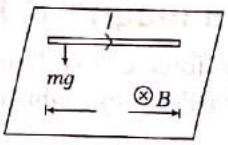


$$Bil\sin\theta = mg$$

i.e., $$i = \frac{mg}{Bl\sin\theta}$$

$$= \frac{60 \times 10^{-6} \times 10}{0.8 \times 10^{-4} \times 30 \times 10^{-2} \times 1/2)} = 50\ \text{A}$$

Example 4.47 *A straight wire of mass 200 g and length 1.5 m carries a current of 2 A. It is suspended in mid air by a uniform horizontal magnetic field B. What is the magnitude of the magnetic field?*



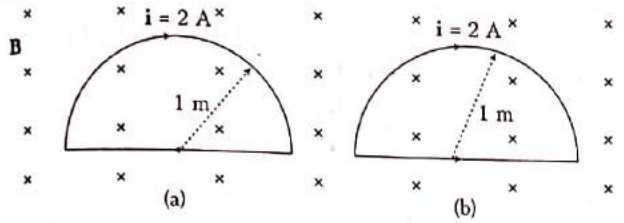
Sol. Applying Fleming's rule, we find that upward force F of magnitude IlB acts. For mid air suspension this must be balanced by the force due to gravity.

$$\therefore \qquad mg = I\,lB \ \Rightarrow\ B = \frac{mg}{Il}$$

Given, $m = 200\ \text{g} = 0.2\ \text{kg}$, $g = 9.8\ \text{ms}^{-2}$, $I = 2\text{A}$, $l = 1.5\ \text{m}$

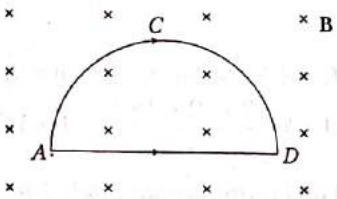
We have, $$B = \frac{0.2 \times 9.8}{2 \times 1.5} = 0.65\ \text{T}$$

Example 4.48 *In the figure shown a semicircular wire loop is placed in a uniform magnetic field B = 1.0 T. The plane of the loop is perpendicular to the magnetic field. Current i = 2A flows in the loop in the directions shown. Find the magnitude of the magnetic force in both the cases (a) and (b). The radius of the loop is 1.0 m.*



Sol. Refer Fig. (a) It forms a closed loop and the current completes the loop. Therefore, net force on the loop in uniform field should be zero.

Refer Fig. (b) In this case although it forms a closed loop but current does not complete the loop. Hence, net force is not zero.



$$\mathbf{F}_{ACD} = \mathbf{F}_{AD}$$

$$\therefore \quad \mathbf{F}_{loop} = \mathbf{F}_{ACD} + \mathbf{F}_{AD} = 2\mathbf{F}_{AD}$$

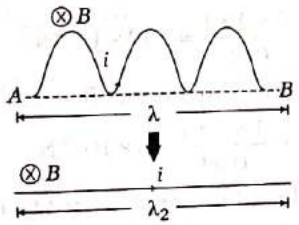
$$\therefore \quad |\mathbf{F}_{loop}| = 2\,|\mathbf{F}_{AD}|$$

$$= 2ilB \sin\theta \qquad (\because l = 2r = 2.0 \text{ m})$$

$$= (2)(2)(2)(1)\sin 90° = 8 \text{ N}$$

Example 4.49 *A wire, carrying a current i is kept in the xy-plane along the curve* $y = (2\text{ cm})\,A\sin\left(\frac{2\pi}{\lambda}x\right)$. *A magnetic field B exists in the z-direction. Find the magnitude of the magnetic force on the portion of the wire between x = 0 and x = λ/2.*

Sol. Consider the sinusoidal shaped wire as shown below



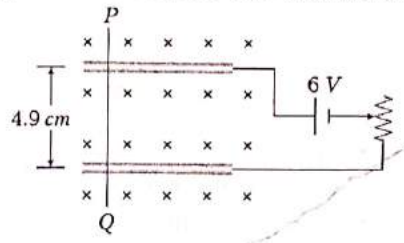
Effective length of the wire for $x = \lambda/2$ is

$$\text{Left} = \frac{\lambda}{2}$$

∴ Magnetic force on the wire,

$$F = i(\text{left})B = i\left(\frac{\lambda}{2}\right)B = \frac{iB\lambda}{2}, \text{ upward.}$$

Example 4.50 *A wire PQ of mass 10 g is at rest on two parallel metal rails. The separation between the rails is 4.9 cm. A magnetic field of 0.80 T is applied perpendicular to the plane of the rails, directed downwards. The resistance of the circuit is slowly decreased. When the resistance decreases to below 20 Ω, the wire PQ begins to slide on the rails. Calculate the coefficient of friction between the wire and the rails.*



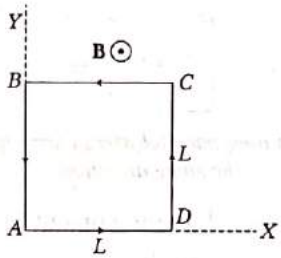
Sol. Wire *PQ* begins to slide when magnetic force is just equal to the force of friction *i.e.*, $\mu\, mg = ilB \sin\theta$ $\quad (\because \theta = 90°)$

Here, $\quad i = \frac{E}{R} = \frac{6}{20} = 0.3$ A

$$\therefore \quad \mu = \frac{ilB}{mg} = \frac{(0.3)(4.9\times 10^{-2})(0.8)}{(10\times 10^{-3})(9.8)} = 0.12$$

Example 4.51 *The magnetic field existing in a region is given by,* $\mathbf{B} = B_0\left(\frac{x}{L} + \frac{x^2}{L^2}\right)\hat{\mathbf{k}}$. *A square loop of edge L carrying a current i is placed with its edges parallel to the X and Y-axes. Find the net magnetic force experienced by the loop.*

Sol. Consider the situation shown below



For wire *AB*,

$$x = 0,\ \mathbf{B} = 0,\ F_{AB} = 0$$

For wire *CD*,

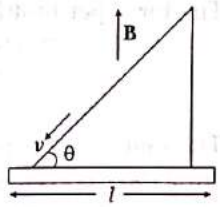
$$x = L,\ \mathbf{B} = B_0\left(\frac{x}{L} + \frac{x^2}{L^2}\right)\hat{\mathbf{k}} = 2B_0\hat{\mathbf{k}}$$

$F_{CD} = 2B_0 iL$, towards right

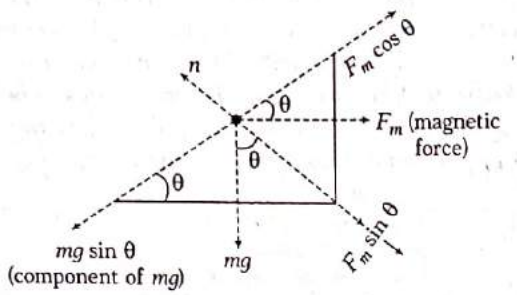
Force on *BC* is equal and opposite to force on *DA*.

$\therefore (F_{net})_{loop} = F_{CD} = 2B_0\, iL$, towards right

Example 4.52 *A conducting rod of length and mass m is moving down a smooth inclined plane of inclination θ with constant velocity v. A current i is flowing in the conductor in a direction perpendicular to paper inwards. A vertically upwards magnetic field* **B** *exists in space. Then, find magnitude of magnetic field* **B**.



Sol. Consider the forces acting on the rod as given below



Hence, from figure,

$$F_m \cos\theta = mg \sin\theta \quad ...(i)$$

Also,

$$F_m = Bil \quad ...(ii)$$

From Eqs. (i) and (ii), we get

$$B = \frac{mg}{il} \tan\theta$$

Force between two parallel current carrying conductors

Consider two long wires 1 and 2 kept parallel to each other at a distance r and carrying currents i_1 and i_2 respectively in the same direction.

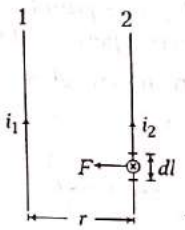


Fig. 4.45 Two long parallel wires carrying currents in the same direction

Magnetic field on wire 2 due to current in 1 is,

$$B = \frac{\mu_0}{2\pi} \cdot \frac{i_1}{r} \quad \text{(in } \otimes \text{ direction)}$$

Magnetic force on a small element dl of wire 2 due to this magnetic field is, $d\mathbf{F} = i_2(d\mathbf{l} \times \mathbf{B})$

Magnitude of this force is,

$$dF = i_2[(dl)(B)\sin 90°] = i_2(dl)\left(\frac{\mu_0}{2\pi}\frac{i_1}{r}\right) = \frac{\mu_0}{2\pi} \cdot \frac{i_1 i_2}{r} \cdot dl$$

Direction of this force is along $d\mathbf{l} \times \mathbf{B}$ or towards the wire 1.

The force per unit length of wire 2 due to wire 1 is,

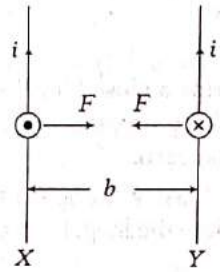
$$\boxed{\frac{dF}{dl} = \frac{\mu_0}{2\pi}\frac{i_1 i_2}{r}}$$

The same force acts on wire 1 due to wire 2. The wires attract each other, if currents in the wires are flowing in the same direction and they repel each other, if the currents are in opposite directions.

Example 4.53 *Currents of 10 A and 2 A are passed through two parallel wires A and B respectively, in opposite directions. If the wire A is infinitely long and length of the wire B is 2m, then the find the force acting on the conductor B which is situated at 10 cm distance from A.*

Sol. Force on a conductor of length l carrying current I_2 and placed at a distance r parallel to another infinitely long conductor carrying current I_1 is

$$F = \frac{\mu_0}{4\pi} \cdot \frac{2I_1 I_2}{r} l$$

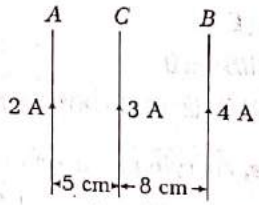


Here,

$$I_1 = 10\text{ A}, I_2 = 2\text{ A}, l = 2\text{ m}, r = 10\text{ cm} = 0.1\text{ m}$$

$$\therefore \quad F = 10^{-7} \times \frac{2 \times 10 \times 2 \times 2}{0.1} = 8 \times 10^{-5}\text{ N}$$

Example 4.54 *A and B are two infinitely long straight parallel conductors. C is another straight conductor of length 1m kept parallel to A and B as shown in the figure. Then, find the magnitude and direction of the force experienced by C*



Sol. The force between the conductors is attractive, if the currents in them are in the same direction and repulsive, if the currents are in opposite directions.

Mutual force between conductors A and C,

$$F_1 = \frac{\mu_0}{2\pi}\frac{I_1 I_2 l}{r}$$

$$= \frac{\mu_0}{2\pi}\frac{2 \times 3 \times 1}{0.05} = 2.40 \times 10^{-5}\text{N} \quad \text{(towards } A\text{)}$$

Mutual force between conductors B and C

$$= \frac{\mu_0}{2\pi} \times \frac{4 \times 3 \times 1}{0.08} = 3 \times 10^{-5}\text{ N} \quad \text{(towards } B\text{)}$$

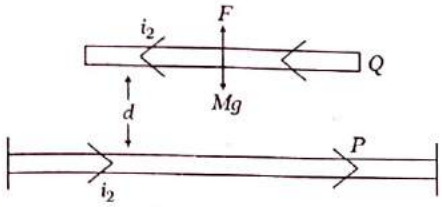
Hence, the resultant force experienced by C

$$= (3 - 2.4) \times 10^{-5}\text{ N} = 0.6 \times 10^{-5}\text{ N} \quad \text{(towards } B\text{)}$$

Example 4.55 *A long horizontal wire P carries a current of 50 A. It is rigidly fixed. Another fine wire Q is placed directly above and parallel to P. The weight of wire Q is 0.075 Nm^{-1} and carries a current of 25 A. Find the perpendicular distance of wire Q from P so that wire Q remains suspended due to the magnetic repulsion. Also indicate the direction of current in Q with respect to P.*

Sol. As force per unit length between two parallel current carrying wires separated by a distance d is given by

$$\frac{dF}{dL} = \frac{\mu_0}{4\pi}\frac{2i_1 i_2}{d}$$



and is repulsive if the current in the wires is in opposite direction (otherwise attractive).

So, in order that wire Q may remain suspended, the force F on it must be repulsive and equal to its weight *i.e.*, the current in the two wires must be opposite directions and

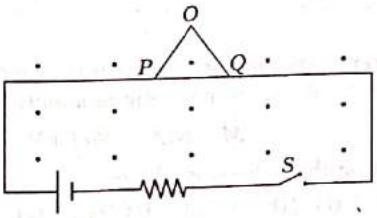
$$\Rightarrow \quad \boxed{F = Mg \text{ i.e., } \frac{F}{L} = \frac{Mg}{L}}$$

$$\Rightarrow \quad \frac{\mu_0}{4\pi}\frac{2i_1 i_2}{d} = \frac{Mg}{L} \qquad \left[\text{as } \frac{dF}{dL} = \frac{\mu_0}{4\pi}\frac{2i_1 i_2}{d}\right]$$

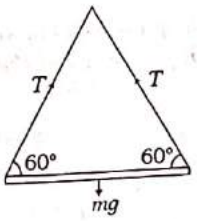
$$\Rightarrow \quad d = 10^{-7} \times \frac{2 \times 50 \times 25}{0.075} = \frac{1}{3} \times 10^{-2}\text{m}$$

$$\left(\text{as, } \frac{Mg}{L} = 0.075\,\text{Nm}^{-1}\right)$$

Example 4.56 *The figure shows a rod PQ of length 20cm and mass 100g suspended through a fixed point O by two threads of lengths 20 cm each. A magnetic field of strength 0.5T exists in the vicinity of the wire PQ as shown in figure. The wire connecting PQ with the battery are lose and exert no force on PQ. (a) Find the tension in the threads when the switch S is open. (b) A current of 2A is established when the switch S is closed. Find the tension in the threads now.*



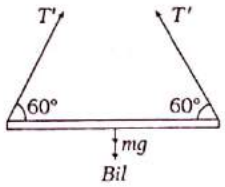
Sol. (a) When switch S is open



$$2T \sin 60° = mg$$

$$T = \frac{mg}{\sqrt{3}} = \frac{100 \times 10^{-3} \times 10}{\sqrt{3}} = \frac{1}{\sqrt{3}}\text{N}$$

(b) When switch S is closed, current i is in wire PQ. Magnetic force on PQ, $F_m = Bil$, downward



$$2T' \sin 60° = mg + Bil$$

$$T' = \frac{mg + Bil}{\sqrt{3}}$$

$$= \frac{100 \times 10^{-3} \times 10 + 0.5 \times 2 \times 0.2}{\sqrt{3}}$$

$$= \frac{1 + 0.2}{\sqrt{3}} = \frac{1.1}{\sqrt{3}}\text{N}$$

Magnetic force between two moving charges

Consider two charges q_1 and q_2 are moving with velocities v_1 and v_2 respectively and at any instant the distance between them is r.

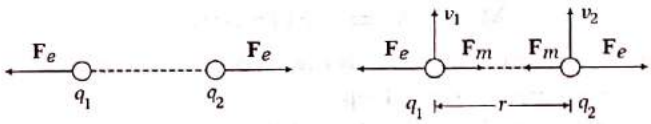


Fig. 4.46 Two moving charges

A magnetic force F_m will appear between them alongwith the electric force.

i.e., $$\boxed{F_m = \frac{\mu_0}{4\pi}\frac{q_1 q_2 v_1 v_2}{r^2}} \quad \text{...(i)}$$

Electric force between them, is

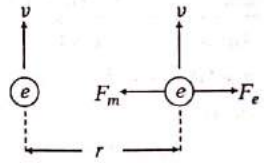
$$F_e = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we have

$$\frac{F_m}{F_e} = \left(\frac{v}{c}\right)^2 \qquad \left(\because \mu_0\varepsilon_0 = \frac{1}{c^2}\right)$$

So, $\frac{F_m}{F_e} = \left(\frac{v}{c}\right)^2$ for charges moving with same velocity

Example 4.57 *Two electrons move parallel to each other with an equal velocity v = electrons. Calculate the ratio of forces of magnetic and electrical of the electrons.*



Sol. Electrical force between electrons,

$$F_e = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e \cdot e}{r^2}, \text{ repulsion}$$

Magnetic force between electrons,

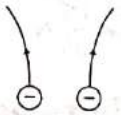
$$F_m = \frac{\mu_0}{4\pi} \cdot \frac{e \cdot e \cdot v \cdot v}{r^2}, \text{ attraction}$$

$$\therefore \quad \frac{F_m}{F_e} = \mu_0 \varepsilon_0 v^2 = \frac{v^2}{c^2} \qquad \left(\because c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}\right)$$

$$= \left(\frac{200 \times 10^3}{3 \times 10^8}\right)^2 = 4.4 \times 10^{-7}$$

$$F_m << F_e$$

The path of electrons will be as shown below,



Magnetic dipole moment

Magnetic moment associated with a current carrying loop is given as,

$$\mathbf{M} = Ni\mathbf{A} \Rightarrow |\mathbf{M}| = NiA$$

Here, N = number of turns in the loop,

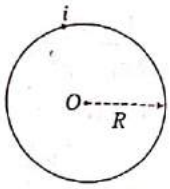
i = current in the loop

and A = area of cross-section of the loop.

It is a vector quantity directed along area vector (**A**) of the loop with anti-clockwise sense as positive. Its SI unit is A-m^2.

Example 4.58 *Find the magnetic moment of a thin round loop with current if the radius of the loop is equal to $R = 100$ mm and the magnetic induction at its centre is equal to $B = 8.0\mu T$.*

Sol.



Magnetic field at centre O

$$B = \frac{\mu_0 i}{2R} \Rightarrow i = \frac{2RB}{\mu_0}$$

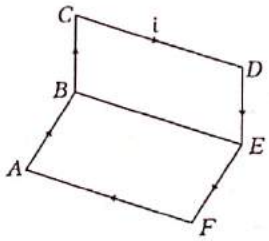
Magnetic moment of loop

$$M = iA = i \cdot \pi R^2 = \frac{2RB}{\mu_0}\pi R^2$$

$$= \frac{2BR^3}{\mu_0/\pi} = \frac{2 \times 8 \times 10^{-6} \times (0.1)^3}{4 \times 10^{-7}}$$

$$= 4 \times 10^{-2} \text{ Am}^{-2}$$

Example 4.59 *Find the magnitude of magnetic moment of the current carrying loop ABCDEFA. Each side of the loop is 10 cm long and current in the loop is $i = 2.0$ A*



Sol. By assuming two equal and opposite currents in BE, two current carrying loops ($ABEFA$ and $BCDEB$) are formed. Their magnetic moments are equal in magnitude but perpendicular to each other. Hence,

$$M_{net} = \sqrt{M^2 + M^2} = \sqrt{2}M$$

$$\because \quad M = iA = 2 \times (l \times b)$$

$$= 2 \times (0.1)(0.1) = 0.02 \text{ A-m}^2$$

$$N_{net} = (\sqrt{2})(0.02) = 0.028 \text{ A-m}^2$$

Example 4.60 *In the Bohr model of the hydrogen atom, the electron circulates around the nucleus in a path of radius 5×10^{-11} m at a frequency of 6.8×10^{15} Hz.*

(i) What value of magnetic field is set-up at the centre of the orbit?

(ii) What is the equivalent magnetic moment ?

Sol. (i) An electron moving around the nucleus is equivalent to a current,

$$i = qf$$

Magnetic field at the centre,

$$B = \frac{\mu_0 i}{2R} = \frac{\mu_0 q f}{2R}$$

Substituting the values, we have

$$B = \frac{(4\pi \times 10^{-7})(1.6 \times 10^{-19})(6.8 \times 10^{15})}{2 \times 5.1 \times 10^{-11}} = 13.4 \text{ T}$$

(ii) The current carrying circular loop is equivalent to a magnetic dipole, with magnetic moment,

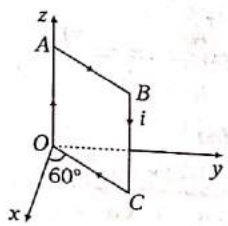
$$M = NiA = (Nqf\pi R^2)$$

Substituting the values, we have

$$M = (1)(1.6 \times 10^{-19})(6.8 \times 10^{15})(3.14)(5.1 \times 10^{-11})^2$$

$$= 8.9 \times 10^{-24} \text{ A-m}^2$$

Example 4.61 *A square loop OABCO of side l carries a current i. It is placed as shown in figure. Find the magnetic moment of the loop.*



Sol. Magnetic moment of the loop can be written as,

$$\mathbf{M} = i(\mathbf{BC} \times \mathbf{CO})$$

Here, $\mathbf{BC} = -l\,\hat{\mathbf{k}}$

$$\mathbf{CO} = -l\cos 60°\,\hat{\mathbf{i}} - l\sin 60°\,\hat{\mathbf{j}}$$

$$= -\frac{l}{2}\hat{\mathbf{i}} - \frac{\sqrt{3}l}{2}\hat{\mathbf{j}}$$

$\therefore$ $$\mathbf{M} = i\left[(-l\,\hat{\mathbf{k}}) \times \left(-\frac{l}{2}\hat{\mathbf{i}} - \frac{\sqrt{3}l}{2}\hat{\mathbf{j}}\right)\right]$$

or $$\mathbf{M} = \frac{il^2}{2}(\hat{\mathbf{j}} - \sqrt{3}\,\hat{\mathbf{i}})$$

Torque on current carrying loop in a magnetic field

When a current carrying coil is placed in a magnetic field, the coil experiences a torque.

Consider a rectangular current carrying coil *PQRS* having *N* turns and area *A*, when this coil is placed in a uniform field **B**, in such a way that the normal ($\hat{\mathbf{n}}$) to the coil makes an angle θ with the direction of **B**. The coil experiences a torque given by

$$\boxed{\tau = NBiA\sin\theta}$$

In vector form $\tau = \mathbf{M} \times \mathbf{B}$

Special cases

(i) τ is zero when θ = 0 *i.e.*, when the plane of the coil is perpendicular to the field.

(ii) τ is maximum when θ = 90° *i.e.*, the plane of the coil is parallel to the field $\tau_{max} = NBiA$

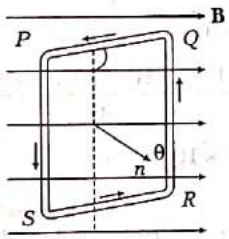
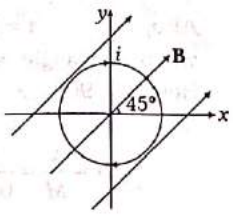


Fig. 4.47 Current carrying loop at an angle θ with magnetic field

Note

1. Expression of torque is applicable for all shapes.
2. Axis of rotation is parallel to τ, for loops relating freely in magnetic field.

Example 4.62 *A circular loop of radius R = 20 cm is placed in a uniform magnetic field* **B** *= 2 T in xy-plane as shown in figure. The loop carries a current i = 1.0 A in the direction shown in figure. Find the magnitude of torque acting on the loop.*



Sol. Magnitude of torque is given by

$$|\tau| = (0.04\,\pi)(2)\sin 90°$$

Here, $M = NiA = (1)(1.0)(\pi)(0.2)^2 = (0.04\pi)A\text{-m}^2$

$B = 2T$ and θ = angle between **M** and **B** = 90°

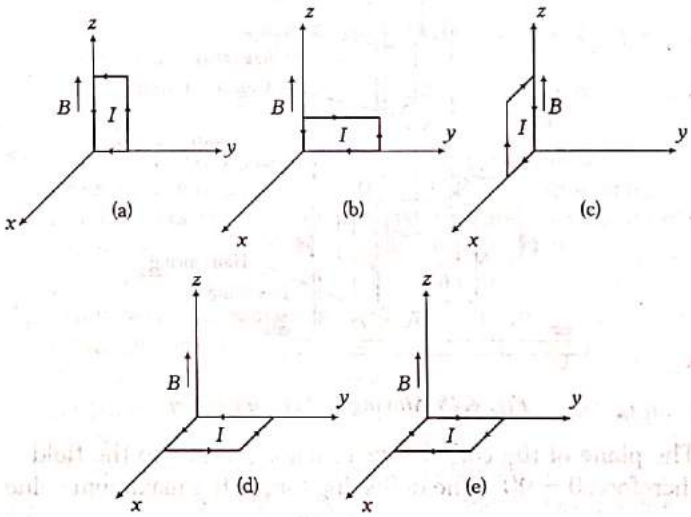
$\therefore$ $|\tau| = (0.04\pi)(2)\sin 90° = 0.25\,\text{N-m}$

Note Here the angle 45° is given just to confuse the students. **M** is along negative *z*-direction (perpendicular to paper inwards) while **B** is in *x*-*y* plane. So, the angle between **M** and **B** is 90° not 45°. So, far as only magnitude of τ is concerned there is no use of giving 45°. But when the direction of torque is desired then this angle is used. Because in that case you will write, $\mathbf{B} = 2\cos 45°\hat{\mathbf{i}} + 2\sin 45°\hat{\mathbf{j}} = \sqrt{2}(\hat{\mathbf{i}} + \hat{\mathbf{j}})$ T

and $\mathbf{M} = -(0.04\,\pi)\,\hat{\mathbf{k}}$ A-m²

$\therefore$ $\tau = \mathbf{M} \times \mathbf{B} = (0.04\sqrt{2}\pi)\,(-\hat{\mathbf{j}} + \hat{\mathbf{i}})$ or $\tau = 0.18\,(\hat{\mathbf{i}} - \hat{\mathbf{j}})$

Example 4.63 *A uniform magnetic field of 3000 G is established along the positive z-direction. A rectangular loop of sides 10 cm and 5 cm carries a current of 12 A. What is the torque on the loop in the different cases shown in figure. What is the force in each case? Which case corresponds to stable equilibrium?*



Sol. Here, *B* = uniform magnetic field = 3000 G along *Z*-axis

$= 3000 \times 10^{-4}$ T = 0.3 T

$\therefore$ *A* = area of rectangular loop

$= l \times b = 10 \times 5 = 50\,\text{cm}^2 = 50 \times 10^{-4}\,\text{m}^2$

Torque on the loop is given by, $\tau = IAB\sin\theta$

where, θ is the angle between the normal to the plane of the loop and the direction of magnetic field.

(a) Here, θ = 90°

$\therefore$ $\tau = 0.3 \times 12 \times 50 \times 10^{-4} \times \sin 90° = 1.8 \times 10^{-2}$ N-m

and acts along negative *y*-direction.

$\tau = \mathbf{M} \times \mathbf{B}$

(b) θ = 90°

$\therefore$ $\tau = 0.3 \times 12 \times 50 \times 10^{-4} \times \sin 90° = 1.8 \times 10^{-2}$ N-m

and acts along *Y'*-axis or *Y*-axis.

(c) θ = 90°

$\therefore$ $\tau = 0.3 \times 12 \times 50 \times 10^{-4} \times \sin 90° = 1.8 \times 10^{-2}$ N-m

and acts along −*x*-direction.

(d) Here, $\theta = 0°$

$\therefore \quad \tau = BIA \sin 0° = 0$

(e) $\theta = 180°$

$\therefore \tau = 0$

Net force on a planar loop in a uniform magnetic field is always zero, so force is zero in each case.

Case (d) corresponds to stable equilibrium M is aligned with B.

The moving coil galvanometer (MCG) current sensitivity

It is a device which is based on the principle when a current carrying coil is placed in a magnetic field it experiences a torque.

The MCG consists of a multi-turn coil free to rotate about a vertical axis, in a uniform radial magnetic field. There is a cylindrical soft iron core to increase the sensitivity of the MCG.

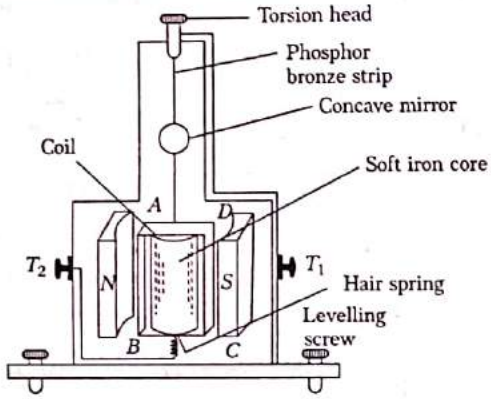


Fig. 4.48 Moving coil galvanometer

The plane of the coil always remains parallel to the field therefore, $\theta = 90°$. The deflecting torque has maximum value

$$\tau = NiAB \quad ...(i)$$

where, the symbols have their usual meaning, since the field is radial by design, we have taken $\sin\theta \approx 1$ in the above expression for the torque.

As coil deflects, a restoring torque is set-up in the suspension fibre. If α is the angle of twist the restoring torque is

$$\tau_{rest} = k\alpha \quad ...(ii)$$

where, k is the torsional constant of the fibre. When the coil is in equilibrium $NBiA = k\alpha \Rightarrow i = G\alpha$

$$\Rightarrow \quad \boxed{\alpha \propto I}$$

where, $G = \dfrac{k}{NBA}$ is the galvanometer constant.

This linear relationship between i and α makes the moving coil galvanometer useful for current measurement and detection.

Current sensitivity of the galvanometer is the deflection per unit current flowing through it. It is given by,

$$\therefore \quad I_s = \frac{\theta}{I} = \left(\frac{NAB}{k}\right)$$

Its unit is rad/A or div/A.

Voltage sensitivity is the deflection per unit voltage.

It is given by, $V_S = \dfrac{\theta}{V} = \left(\dfrac{NAB}{k}\right)\dfrac{I}{V}$

$$\Rightarrow \quad V_S = \frac{NAB}{k} \times \frac{I}{IR} = \frac{NAB}{kR} \quad (\because V = IR)$$

$$\text{Voltage sensitivity} = \frac{\text{Current sensitivity}}{R}$$

Its unit is rad/V or div/V.

Example 4.64 *The coil of a moving coil galvanometer has an effective area of $5 \times 10^{-2} m^2$. It is suspended in a magnetic field of 2×10^{-2} Wbm^{-2}. If the torsional constant of the suspension fibre is 4×10^{-9} $Nm\ deg^{-1}$, then find its current sensitivity in degree per microampere.*

Sol. Here, $N = 1, A = 5 \times 10^{-2} m^2$

$B = 2 \times 10^{-2} Wbm^{-2}, k = 4 \times 10^{-9}$ Nm deg^{-1}

$$\text{Current sensitivity} = \frac{NBA}{k} = \frac{1 \times 2 \times 10^{-2} \times 5 \times 10^{-2}}{4 \times 10^{-9}}$$

$$= 0.25 \times 10^6 \text{ deg A}^{-1} = 0.25 \text{ deg } \mu \text{ A}^{-1}$$

Example 4.65 *A moving coil galvanometer has 100 turns and each turns has as area of 2 cm^2. The magnetic field produced by the magnet is 0.01 T. The deflection in the galvanometer coil is 0.5 rad when a current of 10 mA is passed through it. Find the torsional constant of the spiral spring.*

Sol. We have, $i = \dfrac{k}{NAB}\theta \Rightarrow k = \dfrac{NABi}{\theta}$

or $$k = \frac{100 \times 2 \times 10^{-4} \times 0.01 \times 10 \times 10^{-3}}{0.05}$$

$$\therefore \quad k = 4.0 \times 10^{-5} \text{ Nm rad}^{-1}$$

Example 4.66 *A current of 5.0 A is passed through the coil of a galvanometer having 500 turns and each turn has an average area of 3×10^{-4} m^2. If a torque of 1.5 N-m is required for this coil carrying same current to set it parallel to a magnetic field, then calculate the strength of the magnetic field.*

Sol. The magnetic moment of a current loop,

$$M = NiA = 500 \times 0.5 \times 3 \times 10^{-4} = 0.075 \text{ A-m}^2$$

Also, $\tau = \mathbf{M} \times \mathbf{B}$ of $|\tau| = MB \sin\theta$

where, θ = angle between B and A.

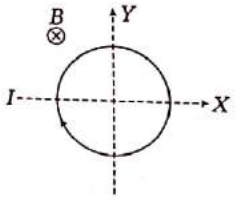
Here, $\theta = 90°$

$$\therefore \quad \tau = MB \sin 90°;$$

$$B = \frac{\tau}{M} = \frac{1.5}{0.075} = 20 \text{ T}$$

Check point 4.3

1. A conducting loop carrying a current I is placed in a uniform magnetic field pointing into the plane of the paper as shown. The loop will have a tendency to



(a) contract
(b) expand
(c) move towards +ve X-axis
(d) move towards –ve X-axis

2. Two parallel wires carrying currents in the same direction attract each other because of
(a) potential difference between them
(b) mutual inductance between them
(c) electric force between them
(d) magnetic force between them

3. Two parallel conductor A and B of equal length carry currents I and $10I$, respectively, is the same direction, then
(a) A and B will repel each other with same force
(b) A and B with attract each other with same force
(c) A will attract B but B will repel A
(d) A and B will attract each other with different forces

4. Two thin long parallel wires, separated by a distance d carry a current of (i) ampere in the same direction. They will
(a) attract each other with a force of $\mu_0 i / 2\pi d^2$
(b) repel each other with a force of $\mu_0 i^2 / 2\pi d^2$
(c) attract each other with a force of $\mu_0 i^2 / 2\pi d$
(d) repel each other with a force of $\mu_0 i / 2\pi d$

5. Two long conductors, separated by a distance d carry currents I_1 and I_2 in the same direction. They exert a force F on each other. Now the current in one of them is increased to two times and its direction is reversed. The distance is also increased to $3d$. The new value of the force between them is
(a) $-\frac{F}{3}$ (b) $\frac{F}{3}$ (c) $\frac{2F}{3}$ (d) $\frac{-2F}{3}$

6. In two parallel wires A and B, 10 and 2 A current respectively, is flowing in opposite directions. The distance between the wires is 10 cm. If the wire A is of infinite length and wire B is of 2 m length, then the force acting on the wire B will be
(a) 8×10^{-5} N (b) 4×10^{-5} N
(c) 4×10^{-7} N (d) 8×10^{-7} N

7. The force between two long parallel wires A and B carrying current is 0.004 Nm^{-1}. The conductors are 0.01 m apart. If the current in conductor A is twice that of conductor B, then the current in the conductor B would be
(a) 5 A (b) 50 A (c) 10 A (d) 100 A

8. A metallic loop is placed in a magnetic field. If a current is passed through it, then
(a) the ring will feel a force of attraction
(b) the ring will feel a force of repulsion
(c) it will move to and fro about its centre of gravity
(d) None of the above

9. A current carrying loop is placed in a uniform magnetic field. The torque acting on it does not depend on
(a) shape of the loop (b) area of the loop
(c) number of turns in the loop (d) strength of the current

10. Current i is carried in a wire of length. If the wire is turned into a circular coil, the maximum magnitude of torque in a given magnetic field B will be
(a) $\frac{L^2B^2}{2}$ (b) $\frac{L^2B}{2}$ (c) $\frac{L^2iB}{4\pi}$ (d) $\frac{L^2B}{4\pi}$

11. A circular coil of 20 turns and radius 10 cm is placed in uniform magnetic field of 0.10 T normal to the plane of the coil. If the current in coil is 5 A, then the torque acting on the coil will be
(a) 31.4 Nm (b) 3.14 Nm (c) 0.314 Nm (d) zero

12. The pole pieces of the magnet used in a pivoted coil galvanometer are
(a) plane surfaces of a bar magnet
(b) plane surfaces of a horse-shoe magnet
(c) cylindrical surfaces of a bar magnet
(d) cylindrical surfaces of a horse-shoe magnet.

13. In a moving coil galvanometer, the deflection of the coil θ is related to the electrical current i by the relation
(a) $i \propto \tan\theta$ (b) $i \propto \theta$
(c) $i \propto \theta^2$ (d) $i \propto \sqrt{\theta}$

14. In order to increase the sensitivity of a moving coil galvanometer one should decrease
(a) the strength of its magnet
(b) the torsional constant of its suspension
(c) the number of turns in its coil
(d) the area of its coil

15. Two galvanometers A and B require 3 mA and 5 mA respectively to produce the same deflection of I_0 division. Then
(a) A is more sensitive than B
(b) B is more sensitive than A
(c) A and B are equally sensitive
(d) sensitiveness of B is 5/3 times of that of A

Chapter exercises

A. Taking it together

(Assorted questions of the chapter for advanced level practice)

1. Biot-Savart law indicates that the moving electrons (velocity **v**) produce a magnetic field **B** such that **[NCERT Exemplar]**

(a) **B** is perpendicular to **v**
(b) **B** is parallel to **v**
(c) it obeys inverse cube law
(d) it is along the line joining the electron and point of observation

2. A current flows in a conductor from east to west. The direction of the magnetic field at a point above the conductor is

(a) towards north (b) towards south
(c) towards east (d) towards west

3. In a cyclotron, a charged particle **[NCERT Exemplar]**

(a) undergoes acceleration all the time
(b) speeds up between the dees because of the magnetic field
(c) speeds up in a dee
(d) slows down within a dee and speeds up between dees

4. An electron is projected with uniform velocity along the axis of a current carrying long solenoid. Which of the following is true? **[NCERT Exemplar]**

(a) The electron will be accelerated along the axis
(b) The electron path will be circular about the axis
(c) The electron will experience a force at 45° to the axis and hence execute a helical path
(d) The electron will continue to move with uniform velocity along the axis of the solenoid

5. A proton is moving along the negative direction of X-axis in a magnetic field directed along the positive direction of Y-axis. The proton will be deflected along the negative direction of

(a) X-axis (b) Y-axis
(c) Z-axis (d) None of these

6. Let ε_0 denote the permittivity of the vacuum and μ_0 that of the permeability of the vacuum. If M = mass, L = length, T = time and I = electric current, then the dimensional formula of the

(a) $\varepsilon_0 = [M^{-1}L^{-3}T^2I]$ (b) $\varepsilon_0 = [M^{-1}L^{-3}T^4\ I^2]$
(c) $\mu_0 = [MLT^{-2}I^{-2}]$ (d) $\mu_0 = [ML^2T^{-1}I]$

7. In a co-axial, straight cable, the central conductor and the outer conductor carry equal currents in opposite directions. The magnetic field is zero

(a) outside the cable
(b) inside the inner conductor
(c) inside the outer conductor
(d) in between the two conductors

8. A current carrying circular loop of radius R is placed in the xy-plane with centre at the origin. Half of the loop with $x > 0$ is now bent, so that it now lies in the yz-plane. Then, **[NCERT Exemplar]**

(a) The magnitude of magnetic moment now diminishes
(b) The magnetic moment does not change
(c) The magnitude of **B** at (0,0, z), $z > R$ increases
(d) The magnitude of **B** at (0,0, z), $z >> R$ is unchanged

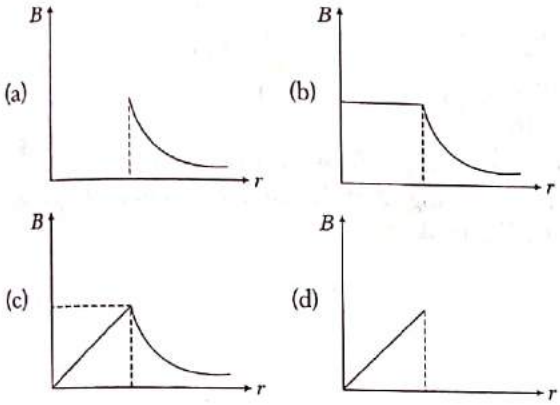
9. A particle of charge q and mass m moves in a circular orbit of radius r with angular speed ω. The ratio of the magnitude of its magnetic moment to that of its angular momentum depends on

(a) ω and q (b) ω, q and m
(c) q and m (d) ω and m

10. The maximum energy of a deuteron coming out a cyclotron is 20 MeV. The maximum energy of proton that can be obtained from this accelerator is

(a) 10 MeV (b) 20 MeV
(c) 30 MeV (d) 40 MeV

11. A long thin hollow metallic cylinder of radius R has a current i ampere. The magnetic induction B away from the axis at a distance r from the axis varies as shown in



12. In hydrogen atom, an electron is revolving in the orbit of radius 0.53 Å with 6.6×10^{15} rot/s. Magnetic field produced at the centre of the orbit is

(a) 0.125 Wb/m^2 (b) 1.25 Wb/m^2
(c) 12.5 Wb/m^2 (d) 125 Wb/m^2

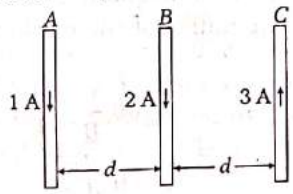
13. A particle of charge q and velocity v passes undeflected through a space with non-zero electric field E and magnetic field B. The undeflecting conditions will hold, if

(a) signs of both q and E are reversed
(b) signs of both q and B are reversed
(c) Both B and E are changed in magnitude but keeping the product of B and E fixed
(d) Both B and E are doubled in magnitude

14. The magnetic field at the centre of a circular coil of radius r carrying current I is B_1. The field at the centre of another coil of radius $2r$ carrying same current I is B_2. The ratio B_1/B_2 is

(a) 1/2 (b) 1
(c) 2 (d) 4

15. Three long straight wires A, B and C are carrying current as shown in figure. Then, the resultant force on B is directed



(a) towards A
(b) towards C
(c) perpendicular to the plane of paper and outward
(d) perpendicular to the plane of paper and inward

16. A wire of length 2 m carrying a current of 1 A is bend to form a circle. The magnetic moment of the coil is (in A-m^2)

(a) 2π (b) $\pi/2$ (c) $\pi/4$ (d) $1/\pi$

17. A current carrying conductor of length l is bent into two loops one by one. First loop has one turn of wire and the second loop has two turns of wire. Compare the magnetic fields at the centre of the loops.

(a) $B' = 4B$ (b) $4B' = B$ (c) $2B' = B$ (d) $B' = 2B$

18. Two charged particles traverse identical helical paths in a completely opposite sense in a uniform magnetic field $\mathbf{B} = B_0\hat{\mathbf{k}}$. **[NCERT Exemplar]**

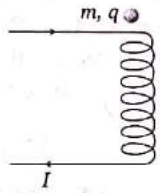
(a) They have equal z-components of moment
(b) They must have equal charges
(c) They necessarily represent a particle, anti-particle pair
(d) The charge to mass ratio satisfy

$$\left(\frac{e}{m}\right)_1 + \left(\frac{e}{m}\right)_2 = 0$$

19. When a certain length of wire is turned into one circular loop, then the magnetic field produced at the centre of coil due to some current flowing, is B_1. If the same wire is turned into three loops to make a circular coil, then the magnetic induction at the centre of this coil for the same current will be

(a) B_1 (b) $9B_1$ (c) $3B_1$ (d) $27B_1$

20. A long solenoid carrying a current I is placed with its axis vertical as shown in the figure. A particle of mass m and charge q is released from the top of the solenoid. Its acceleration is (g being acceleration due to gravity)



(a) greater than g (b) less than g
(c) equal to g (d) None of these

21. A proton moves at a speed $v = 2 \times 10^6$ m/s in a region of constant magnetic field of magnitude $B = 0.05$ T. The direction of the proton when it enters this field is $\theta = 30°$ to the field. When you look along the direction of the magnetic field, then the path is a circle projected on a plane perpendicular to the magnetic field. How far will the proton move along the direction of B when two projected circles have been completed?

(a) 4.35 m (b) 0.209 m (c) 2.82 m (d) 2.41 m

22. An electric current i enters and leaves a uniform circular wire of radius a through diametrically opposite points. A charged particle q moving along the axis of the circular wire passes through its centre at speed v. The magnetic force acting on the particle when it passes through the centre has a magnitude

(a) $qv\frac{\mu_0 i}{2a}$ (b) $qv\frac{\mu_0 i}{2\pi a}$

(c) $qv\frac{\mu_0 i}{a}$ (d) zero

23. A particle of mass m and charge q moves with a constant velocity v along the positive x-direction. It enters a region containing a uniform magnetic field B directed along the negative z-direction, extending from $x = a$ to $x = b$. The minimum value of v required, so that the particle can just enter the region $x > b$ is

(a) qbB/m (b) $q(b-a)B/m$

(c) qaB/m (d) $q(b+a)B/2m$

24. A proton of mass 1.67×10^{-27} kg and charge 1.6×10^{-19} C is projected in xy-plane with a speed of 2×10^{6} m/s at an angle of 60° to the X-axis. If a uniform magnetic field of 0.14 T is applied along the Y-axis, then the path of the proton is

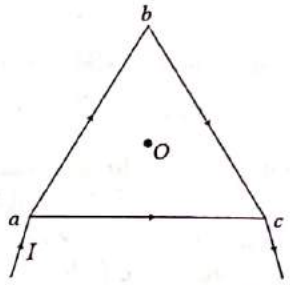
(a) a circle of radius 0.2 m and time period $\pi\times 10^{-7}$ s

(b) a circle of radius 0.1 m and time period $2\pi\times 10^{-7}$ s

(c) a helix of radius 0.07 m and time period 0.5×10^{-6}s

(d) a helix of radius 0.14 m and time period 1.0×10^{-7}s

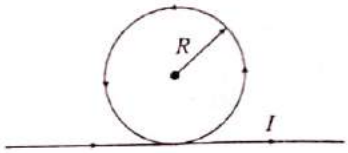
25. An equilateral triangle of side length l is formed from a piece of wire of uniform resistance. The current I is fed as shown in the figure. Then, the magnitude of the magnetic field at its centre O is



(a) $\frac{\sqrt{3}\mu_0 I}{2\pi l}$ (b) $\frac{3\sqrt{3}\mu_0 I}{2\pi l}$

(c) $\frac{\mu_0 I}{2\pi l}$ (d) zero

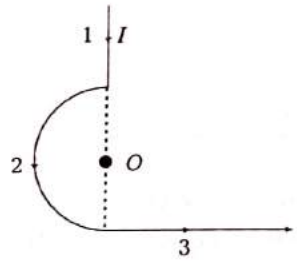
26. An infinitely long conductor is bent into a circle as shown in figure. It carries a current I ampere and the radius of loop is R metre. The magnetic induction at the centre of loop is



(a) $\frac{\mu_0 2I}{4\pi R}(\pi+1)$ (b) $\frac{\mu_0 2I}{4\pi R}(\pi-1)$

(c) $\frac{\mu_0 I}{8\pi R}(\pi+1)$ (d) zero

27. Magnetic field produced at the point O due to current flowing in an infinite wire shaped as shown in the figure is



(a) $\frac{\mu_0 I}{4\pi R}$ (b) $\frac{\mu_0 I}{4R}-\frac{\mu_0 I}{4\pi R}$

(c) $\frac{\mu_0 I}{4R}+\frac{\mu_0 I}{2\pi R}$ (d) $\frac{\mu_0 I}{4R}+\frac{\mu_0 I}{4\pi R}$

28. Two identical coils carrying equal currents, have a common centre and their planes are at right angles to each other. Find the ratio of the magnitudes of the resultant magnetic field at the centre and the field due to one coil alone.

(a) 2 : 1 (b) 1 : 1

(c) $1:\sqrt{2}$ (d) $\sqrt{2}:1$

29. A circular flexible loop of wire of radius r carrying a current I is placed in a uniform magnetic field B perpendicular to the plane of the circle, so that wire comes under tension. If B is doubled, then tension in the loop

(a) remains unchanged (b) is doubled

(c) is halved (d) becomes 4 times

30. An electron moves in a circular orbit with a uniform speed v. It produces a magnetic field B at the centre of the circle. The radius of the circle is proportional to

(a) $\frac{B}{v}$ (b) $\frac{v}{B}$

(c) $\sqrt{\frac{v}{B}}$ (d) $\sqrt{\frac{B}{v}}$

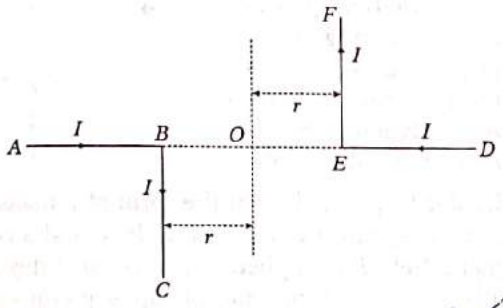
31. Two wires of same length are shaped into a square and a circle. If they carry same current, then ratio of the magnetic moment is

(a) $2:\pi$ (b) $\pi:2$ (c) $\pi:4$ (d) $4:\pi$

32. Two particles X and Y having equal charges, after being accelerated through the same potential difference, enter a region of uniform magnetic field and describe circular path of radius R_1 and R_2, respectively. The ratio of mass of X to that of Y is

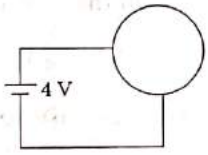
(a) $\left(\frac{R_1}{R_2}\right)^{1/2}$ (b) $\frac{R_2}{R_1}$ (c) $\left(\frac{R_1}{R_2}\right)^{2}$ (d) $\frac{R_1}{R_2}$

33. Two long thin wires ABC and DEF are arranged as shown in the figure. The magnitude of the magnetic field at O is



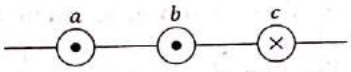
(a) $\frac{\mu_0 I}{4\pi r}$ (b) $\frac{\mu_0 I}{2\pi r}$ (c) $\frac{\mu_0 I}{2\sqrt{2}\pi r}$ (d) zero

34. A circular conductor of uniform resistance per unit length, is connected to a battery of 4 V. The total resistance of the conductor is 4 Ω. The net magnetic field at the centre of the conductor is



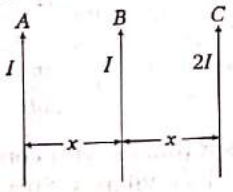
(a) $\frac{\mu_0}{2}$ (b) $\frac{8\mu_0}{3}$ (c) $2\mu_0$ (d) zero

35. Figure shows, three long straight wires parallel and equally spaced with identical currents. Then, the force acting on each wire due to the other is



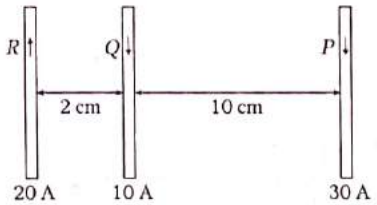
(a) $F_a > F_b > F_c$ (b) $F_b > F_c > F_a$
(c) $F_c > F_a > F_b$ (d) $F_b > F_a > F_c$

36. A, B and C are parallel conductors of equal length carrying currents I, I and $2I$, respectively. Distance between A and B is x. Distance between B and C is also x. F_1 is the force exerted by B on A. F_2 is the force exerted by C on A. Choose the correct answer.



(a) $F_1 = 2F_2$ (b) $F_2 = 2F_1$ (c) $F_1 = F_2$ (d) $F_1 = -F_2$

37. Three long, straight and parallel wires carrying currents are arranged as shown in figure. The force experienced by 10 cm length of wire Q is



(a) 1.4×10^{-4} N towards the right
(b) 1.4×10^{-4} N towards the left
(c) 2.6×10^{-4} N towards the right
(d) 2.6×10^{-4} N towards the left

38. A current of 10 A is flowing in a wire of length 1.5 m. A force of 15 N acts on it when it is placed in a uniform magnetic field of 2 T. The angle between the magnetic field and the direction of the current is

(a) 30° (b) 45°
(c) 60° (d) 90°

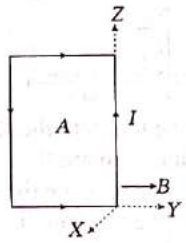
39. An ionised gas contains both positive and negative ions initially at rest. If it is subjected simultaneously to an electric field along the $+x$-direction and a magnetic field along the $+z$-direction, then

(a) positive ions deflect towards $+y$-direction and negative ions $-y$-direction
(b) all ions deflect towards $+y$-direction
(c) all ions deflect towards $-y$-direction
(d) positive ions deflect towards $-y$-direction and negative ions towards $+y$-direction

40. A charged particle P leaves the origin with speed $v = v_0$, at some inclination with the X-axis. There is a uniform magnetic field B along the X-axis. P strikes a fixed target T on the X-axis for a minimum value of $B = B_0$. P will also strike T, if

(a) $B = 2B_0$, $v = 2v_0$ (b) $B = 2B_0$, $v = v_0$
(c) Both are correct (d) Both are wrong

41. The rectangular coil of area A is in a field B. Find the torque about the Z-axis when the coil lies in the position shown and carries a current I.

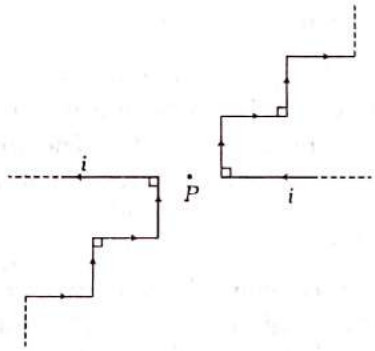


(a) IAB in negative Z-axis
(b) IAB in positive Z-axis
(c) $2IAB$ in positive Z-axis
(d) $2IAB$ in negative Z-axis

42. In hydrogen atom, the electron is making 6.6×10^{15} rev/s around the nucleus in an orbit of radius 0.528 Å. The magnetic moment (A-m^2) will be

(a) 1×10^{-15} (b) 1×10^{-10}
(c) 1×10^{-23} (d) 1×10^{-27}

43. Two infinitely long conductors carrying equal currents are shaped as shown in figure. The short sections are all of equal lengths. The point P is located symmetrically with respect to the two conductors. The magnetic field at P due to any one conductor is B. The total field at P is

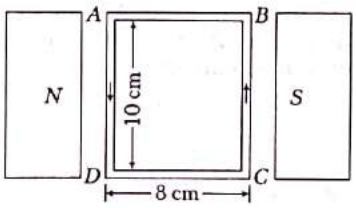


(a) zero (b) B
(c) $\sqrt{2}\,B$ (d) $2B$

44. A particle of mass m and having a positive charge q is projected from origin with speed v_0 along the positive X-axis in a magnetic field $\mathbf{B} = -B_0\hat{\mathbf{k}}$, where B_0 is a positive constant. If the particle passes through (0, y, 0), then y is equal to

(a) $-\dfrac{2mv_0}{qB_0}$ (b) $\dfrac{mv_0}{qB}$ (c) $-\dfrac{mv_0}{qB}$ (d) $\dfrac{2mv_0}{qB_0}$

45. A 100 turns coil shown in figure carries a current of 2 A in a magnetic field $B = 0.2$ Wb/m^2. The torque acting on the coil is



(a) 0.32 N-m tending to rotate the side AD out of the page
(b) 0.32 N-m tending to rotate the side AD into the page
(c) 0.0032 N-m tending to rotate the side AD out of the page
(d) 0.0032 N-m tending to rotate the side AD into the page

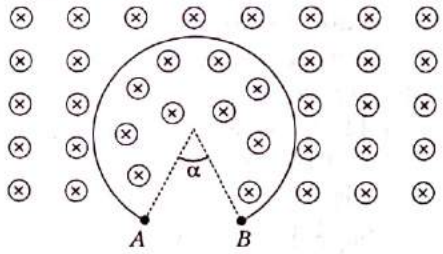
46. H^+, He^+ and O^{2+} ions having same kinetic energy pass through a region of space filled with uniform magnetic field B directed perpendicular to the velocity of ions. The masses of the ions H^+, He^+ and O^{2+} are respectively, in the ratio 1 : 4 : 16. As a result

(a) H^+ ions will be deflected most
(b) O^{2+} ions will be deflected least
(c) He^+ and O^{2+} ions will suffer same deflection
(d) All ions will suffer the same deflection

47. A battery is connected between two points A and B on the circumference of a uniform conducting ring of radius r and resistance R. One of the arc AB of the ring subtends an angle θ at the centre. The value of the magnetic field produced at the centre due to the current in the ring is

(a) proportional to $2(180° - \theta)$
(b) inversely proportional to r
(c) zero, only if $\theta = 180°$
(d) zero for all values of θ

48. A circular loop which is in the form of a major arc of a circle is kept in the horizontal plane and a constant magnetic field B is applied in the vertical direction such that the magnetic lines of forces go into the plane. If R is the radius of circle and it carries a current I in the clockwise direction, then the force on the loop will be



(a) $BIR \tan \alpha$ (b) $2\,BIR \cos(\alpha/2)$
(c) $2\,BIR \sin(\alpha/2)$ (d) None of these

49. Two protons are projected simultaneously from a fixed point with the same velocity v into a region, where there exists a uniform magnetic field. The magnetic field strength at B and it is perpendicular to the initial direction of v. One proton starts at time $t = 0$ and another proton at $t = \dfrac{\pi m}{2qB}$. The separation between them at time $t = \dfrac{\pi m}{qB}$ (where, m and q are the mass and charge of proton), will be approximately

(a) $2\dfrac{mv}{qB}$ (b) $\dfrac{\sqrt{2}\,mv}{qB}$
(c) $\dfrac{mv}{qB}$ (d) $\dfrac{mv}{2qB}$

50. A disc of radius R rotates with constant angular velocity ω about its own axis. Surface charge density of this disc varies as $\sigma = \alpha r^2$, where r is the distance

from the centre of disc. Determine the magnetic field intensity at the centre of disc.

(a) $\mu_0 \alpha\omega R^3$ (b) $\frac{\mu_0 \alpha\omega R^3}{6}$

(c) $\frac{\mu_0 \alpha\omega R^3}{8}$ (d) $\frac{\mu_0 \alpha\omega R^3}{3}$

51. A rigid circular loop of radius r and mass m lies in the xy-plane on a flat table and has a current i flowing in it. At this particular place the earth's magnetic field is $\mathbf{B} = B_x\hat{\mathbf{i}} + B_z\hat{\mathbf{k}}$. The value of i, so that the loop starts tilting is

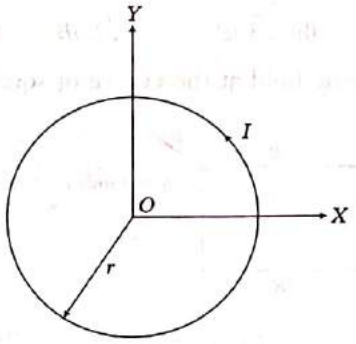
(a) $\frac{mg}{\pi r\sqrt{B_x^2 + B_z^2}}$ (b) $\frac{mg}{\pi r B_x}$

(c) $\frac{mg}{\pi r B_z}$ (d) $\frac{mg}{\pi r\sqrt{B_x B_z}}$

52. Two circular coils 1 and 2 are made from the same wire but the radius of the 1st coil is twice that of the 2nd coil. What is the ratio of potential difference applied across them, so that the magnetic field at their centres is the same?

(a) 3 (b) 4 (c) 6 (d) 2

53. A circular loop of mass m and radius r is lying in a horizontal (xy-plane) table as shown in figure. A uniform magnetic field B is applied parallel to X-axis. The current I in the loop, so that its one edge just lifts from the table, is



(a) $\frac{mg}{\pi r^2 B}$ (b) $\frac{mg}{\pi r B}$ (c) $\frac{mg}{2\pi r B}$ (d) $\frac{\pi r B}{mgl}$

54. A charged particle with specific charge (charge per unit mass $= \frac{q}{m} = S$) moves undeflected through a region of space containing mutually perpendicular and uniform electric and magnetic fields E and B. When the E field is switched off, then the particle will move in a particular path of radius

(a) $\frac{E}{BS}$ (b) $\frac{ES}{B}$

(c) $\frac{ES}{B^2}$ (d) $\frac{E}{B^2 S}$

55. The coil of a moving coil galvanometer has an effective area of 4×10^{-2} m^2. It is suspended in a magnetic field of 5×10^{-2} Wbm^{-2}. If deflection in the galvanometer coil is 0.2 rad when a current of 5 mA is passed through it, then

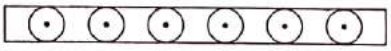
(a) torsional constant is 5×10^{-5} N-m rad^{-1}

(b) current sensitivity is 40 rad A^{-1}

(c) torsional constant is 3×10^{-3} N-m rad^{-1}

(d) current sensitivity is 40 deg A^{-1}

56. A large metal sheet carries an electric current along its surface. Current per unit length is λ.

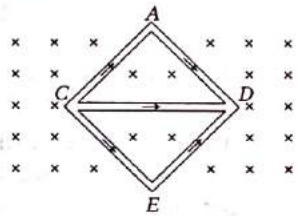


Magnetic field induction near the metal sheet is

(a) $\lambda\mu_0$ (b) $\frac{\lambda\mu_0}{2}$

(c) $\frac{\lambda\mu_0}{2\pi}$ (d) $\frac{\mu_0}{2\pi\lambda}$

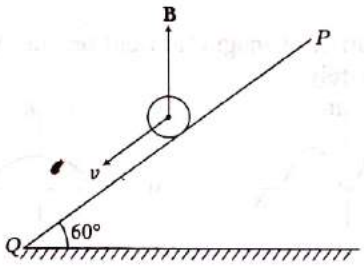
57. Some current $i = 2$A is flowing in a wire frame as shown in figure. The frame is a combination of two equilateral triangles ACD and CDE of side 1 m. It is placed in uniform magnetic field $B = 4$ T acting perpendicular to the plane of frame. The magnitude of magnetic force acting on the frame is



(a) 24 N (b) zero

(c) 16 N (d) 8 N

58. A conducting stick of length $2L$ and mass m is moving down a smooth inclined plane of inclination 60° with constant speed 5m/s. A current 2A is flowing in the conductor perpendicular to the paper inwards. A vertically upward magnetic field B exists in space there. The magnitude of magnetic field $\mathbf{B}$ is



(a) $\frac{mg}{4L}$ (b) $\frac{mg}{L}$

(c) $\frac{\sqrt{3}\,mg}{4L}$ (d) $\frac{\sqrt{3}mg}{2L}$

59. A charge q is moving with a velocity $\mathbf{v}_1 = 1\hat{\mathbf{i}}$ m/s at a point in a magnetic field and experiences a force $\mathbf{F} = q[-\hat{\mathbf{j}} + 1\hat{\mathbf{k}}]$ N. If the charge is moving with a velocity $\mathbf{v}_2 = 2\hat{\mathbf{j}}$ m/s at the same point, then it experiences a force $\mathbf{F}_2 = q(1\hat{\mathbf{i}} - 1\hat{\mathbf{k}})$ N. The magnetic induction $\mathbf{B}$ at that point is

(a) $(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$ Wb/m^2 (b) $(\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}})$ Wb/m^2
(c) $(-\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}})$ Wb/m^2 (d) $(\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}})$ Wb/m^2

60. A square frame of side 1 m carries a current I, produces a magnetic field B at its centre. The same current is passed through a circular coil having the same perimeter as the square. The magnetic field at the centre of the circular coil is B'. The ratio B/B' is

(a) $\frac{8}{\pi^2}$ (b) $\frac{8\sqrt{2}}{\pi^2}$ (c) $\frac{16}{\pi^2}$ (d) $\frac{16}{\sqrt{2}\pi^2}$

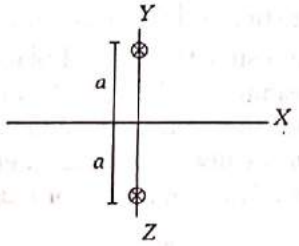
61. The magnetic field existing in a region is given by

$$\mathbf{B} = \mathbf{B}_0\left[1 + \frac{x}{l}\right]\hat{\mathbf{k}}$$

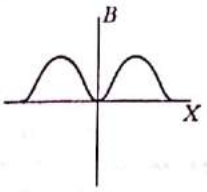
A square loop of edge l and carrying current I is placed with its edges parallel to the X and Y-axes. The magnitude of the net magnetic force experienced by the loop is

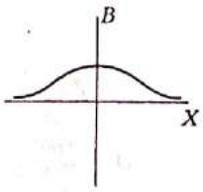
(a) $2B_0Il$ (b) zero (c) B_0Il (d) $4B_0Il$

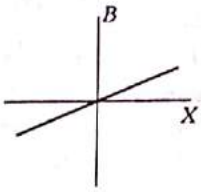
62. Two straight infinitely long current carrying wires are kept along Z-axis at the coordinates $(0, a, 0)$ and $(0, -a, 0)$ respectively, as shown in the figure. The current in each of the wire is equal and along negative Z-axis (into the plane of the paper).

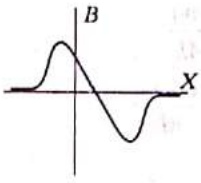


The variation of magnetic field on the X-axis will be approximately

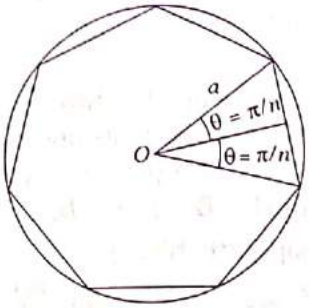
(a)


(b)


(c)


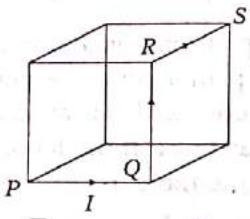
(d)


63. A wire bent in the form of a regular polygon of n sides, is inscribed in a circle of radius a. If i ampere is the current flowing in the wire, then the magnetic field at the centre of the circle is



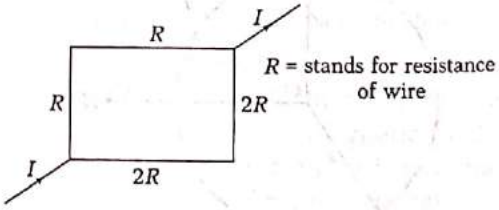
(a) $\frac{\mu_0 i}{2\pi a}\tan\frac{\pi}{n}$ (b) $\frac{\mu_0 ni}{2\pi a}\tan\frac{\pi}{n}$
(c) $\frac{2}{\pi}\frac{ni}{a}\mu_0\tan\frac{\pi}{n}$ (d) $\frac{ni}{2a}\mu_0\tan\frac{\pi}{n}$

64. A wires $PQRS$ carrying a current I runs along three edges of a cube of side l as shown in figure. There exists a uniform magnetic field of magnitude B along one of the sides of the cube. The magnitude of the force acting on the wire is



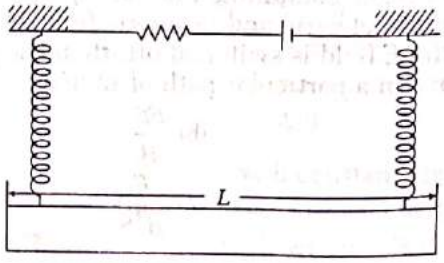
(a) zero (b) $\sqrt{3}\,IB$ (c) $\sqrt{2}I\,lB$ (d) $2IlB$

65. The magnetic field at the centre of square of side a is



(a) $\frac{\sqrt{2}\,\mu_0}{\pi a}$ (b) $\frac{\sqrt{2}\,\mu_0 I}{3\pi a}$ (c) $\frac{2}{3}\frac{\mu_0 I}{a}$ (d) zero

66. A straight rod of mass m and length L is suspended from the identical springs as shown in figure. The spring is stretched a distance x_0 due to the weight of the wire.



The circuit has total resistance R. When the magnetic field perpendicular to the plane of paper is switched on, then springs are observed to extend further by the same distance. The magnetic field strength is

(a) $\frac{2mgR}{LE}$ (b) $\frac{mgR}{LE}$ (c) $\frac{mgR}{2LE}$ (d) $\frac{mgR}{E}$

67. A particle of specific charge $q/m = \pi$ C/kg is projected from the origin towards positive X-axis with a velocity of 10 m/s in a uniform magnetic field $\mathbf{B} = -2\hat{\mathbf{k}}$T. The velocity $\mathbf{v}$ of the particle after time $t = \frac{1}{6}$ s will be

(a) $(5\hat{\mathbf{i}} + 5\sqrt{3}\,\hat{\mathbf{j}})$ m/s (b) $10\hat{\mathbf{j}}$ m/s

(c) $(5\sqrt{3}\,\hat{\mathbf{i}} + 5\hat{\mathbf{j}})$ m/s (d) $-10\hat{\mathbf{j}}$ m/s

68. A charged particle enters into a uniform magnetic field with velocity v_0 perpendicular to it, the length of magnetic field is $x = (\sqrt{3}/2)R$, where R is the radius of the circular path of the particle in the field. The magnitude of change in velocity of the particle when it comes out of the field is

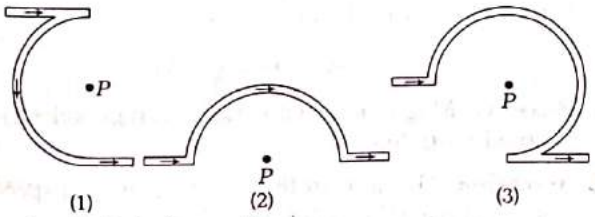
(a) $2v_0$ (b) $\frac{v_0}{2}$ (c) $\frac{\sqrt{3}v_0}{2}$ (d) v_0

69. A proton moving with a constant velocity passes through a region of space without any change in its velocity. If $\mathbf{E}$ and $\mathbf{B}$ represent the electric and magnetic fields respectively, then this region of space may have

(a) $E = 0, B = 0$ (b) $E = 0, B \neq 0$

(c) $E \neq 0, B = 0$ (d) $E \neq 0, B \neq 0$

70. Figure here shows three cases, in all cases the circular path has radius r and straight ones are infinitely long. For same current, the magnetic field at the centre P in cases 1, 2 and 3 have the ratio



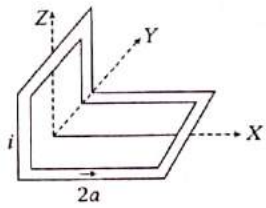
(a) $\left(-\frac{\pi}{2}\right):\left(\frac{\pi}{2}\right):\left(\frac{3\pi}{4}-\frac{1}{2}\right)$

(b) $\left(-\frac{\pi}{2}+1\right):\left(\frac{\pi}{2}+1\right):\left(\frac{3\pi}{4}+\frac{1}{2}\right)$

(c) $-\frac{\pi}{2}:\frac{\pi}{2}:3\frac{\pi}{4}$

(d) $\left(-\frac{\pi}{2}-1\right):\left(\frac{\pi}{2}-\frac{1}{4}\right):\left(\frac{3\pi}{4}+\frac{1}{2}\right)$

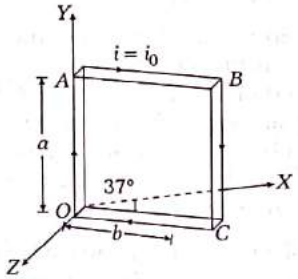
71. A non-planar loop of conducting wire carrying a current I is placed as shown in the figure. Each of the straight sections of the loop is of length $2a$. The magnetic field due to this loop at the point $P(a, 0, a)$ in the direction



(a) $\frac{1}{\sqrt{2}}(-\hat{\mathbf{j}} + \hat{\mathbf{k}})$ (b) $\frac{1}{\sqrt{3}}(-\hat{\mathbf{j}} + \hat{\mathbf{k}} + \hat{\mathbf{i}})$

(c) $\frac{1}{\sqrt{3}}(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$ (d) $\frac{1}{\sqrt{2}}(\hat{\mathbf{i}} + \hat{\mathbf{k}})$

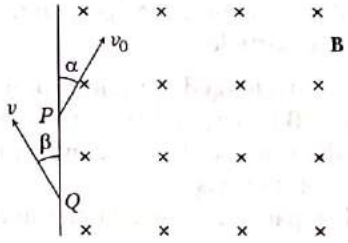
72. A rectangular loop consists of N closed wrapped turns and has dimensions $a \times b$. The loop is hinged along the Y-axis. What is the magnitude of the torque exerted on the loop by a uniform magnetic field $B = B_0$ directed along the X-axis when current $i = i_0$ in the direction shown. The torque acting on the loop is



(a) $-\frac{4Ni_0abB_0\hat{\mathbf{j}}}{5}$ (b) $\frac{4Ni_0abB_0\hat{\mathbf{i}}}{5}$

(c) $-\frac{4Ni_0abB_0J}{3}\hat{\mathbf{j}}$ (d) $\frac{-2Ni_0abB_0\hat{\mathbf{j}}}{5}$

73. A particle of charge $-q$ and mass m enters a uniform magnetic field $\mathbf{B}$ (perpendicular to paper inwards) at P with a speed v_0 at an angle α and leaves the field at Q with speed v at angle β as shown in the figure. Then,



(a) $\alpha = \beta$

(b) $v = v_0$

(c) $PQ = \frac{2mv_0 \sin\alpha}{Bq}$

(d) particle remains in the field for time $t = \frac{2m(\pi - \alpha)}{Bq}$

74. A square coil of edge L having n turns carries a current i. It is kept on a smooth horizontal plate. A uniform magnetic field B exists in a direction parallel to an edge. The total mass of the coil is M. What should be the minimum value of B for which the coil will start tipping over?

(a) $\frac{Mg}{niL}$ (b) $\frac{Mg}{2niL}$

(c) $\frac{Mg}{4niL}$ (d) $\frac{2Mg}{niL}$

75. A long straight wire along the Z-axis carries a current I in the negative z-direction. The magnetic vector field $\mathbf{B}$ at a point having coordinates (x, y) in the $z = 0$ plane is

(a) $\frac{\mu_0 I(y\hat{\mathbf{i}} - x\hat{\mathbf{j}})}{2\pi(x^2 + y^2)}$ (b) $\frac{\mu_0 I(x\hat{\mathbf{i}} + y\hat{\mathbf{j}})}{2\pi(x^2 + y^2)}$

(c) $\frac{\mu_0 I(x\hat{\mathbf{j}} - y\hat{\mathbf{i}})}{2\pi(x^2 + y^2)}$ (d) $\frac{\mu_0 I(x\hat{\mathbf{i}} - y\hat{\mathbf{j}})}{2\pi(x^2 + y^2)}$

B. Medical entrance special format questions

Assertion and reason

Directions (Q. Nos. 1-20) *These questions consist of two statements each linked as Assertion and Reason. While answering these questions you are required to choose any one of the following five responses*

(a) If both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are true but Reason is not correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If Assertion is false but Reason is true.
(e) If both Assertion and Reason are false.

1. **Assertion** Two infinitely long wires A and B carry unequal currents both in inward direction.

Then, there is only one point (excluding the points at infinity), where net magnetic field is zero.

Reason That point lies between points A and B.

2. **Assertion** When a charged particle moves perpendicular to a uniform magnetic field then its momentum remains constant.

Reason Magnetic force acts perpendicular to the velocity of the particle.

3. **Assertion** If a charged particle is projected in a region, where $\mathbf{B}$ is perpendicular to velocity of projection, then the net force acting on the particle is independent of its mass.

Reason The particle is performing uniform circular motion and force acting on it is $\frac{mv^2}{r}$.

4. **Assertion** If a charged particle enters from outside in uniform magnetic field, then it will keep on rotating in a circular path.

Reason Magnetic force is always perpendicular to velocity vector.

5. **Assertion** An α-particle and a deuteron having same kinetic energy enter in a uniform magnetic field perpendicular to the field. Then, radius of circular path of α-particle will be more.

Reason $\frac{q}{m}$ ratio of an α-particle is more than the $\frac{q}{m}$ ratio of a deuteron.

6. **Assertion** A charged particle is rotating in a circular path in uniform magnetic field. Then, plane of circle is perpendicular to the magnetic field.

Reason Circular motion is a two-dimensional motion.

7. **Assertion** In a uniform magnetic field $\mathbf{B} = B_0\hat{\mathbf{k}}$, if velocity of a charged particle is $v_0\hat{\mathbf{i}}$ at $t = 0$, then it can have the velocity $v_0\hat{\mathbf{j}}$ at some other instant.

Reason In uniform magnetic field, acceleration of a charged particle is always zero.

8. **Assertion** If velocity of charged particle in a uniform magnetic field at some instant is $(a_1\hat{\mathbf{i}} - a_2\hat{\mathbf{j}})$ and at some other instant is $(b_1\hat{\mathbf{i}} + b_2\hat{\mathbf{j}})$, then

$$a_1^2 + a_2^2 = b_1^2 + b_2^2$$

Reason Magnetic force cannot change velocity of a charged particle.

9. **Assertion** In non-uniform magnetic field speed of a charged particle varies.

Reason Work done by magnetic force on a charged particle is always zero.

10. **Assertion** Path of a charged particle in uniform magnetic field cannot be a parabola, if no other forces (other than magnetic force) are acting on the particle.

Reason For parabolic path, a constant acceleration is required.

11. Assertion A beam of electron can pass undeflected through a region of **E** and **B**.

Reason Force on moving charged particle due to magnetic field may be zero in some cases.

12. Assertion If the path of a charged particle in a region of uniform electric and magnetic field is not a circle, then its kinetic energy may remain constant.

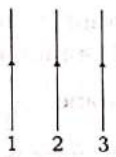
Reason In a combined electric and magnetic field region, a moving charge experiences a net force $\mathbf{F} = q\mathbf{E} + q(\mathbf{v} \times \mathbf{B})$, where symbols have their usual meanings.

13. Assertion Magnetic field (**B**) and electric field (**E**) are present in a region. Net force on a charged particle in this region is zero, if

$$\mathbf{E} = \mathbf{B} \times \mathbf{v}$$

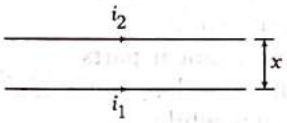
Reason $\frac{E}{B}$ has the dimensions of velocity.

14. Assertion Three infinitely long current carrying wires have equal currents and they are equally spaced. The magnitude of magnetic force on all three is same.



Reason Net force on wire-2 is zero.

15. Assertion Upper wire shown in figure is fixed. At a certain distance x, lower wire can remain in equilibrium.



Reason The above equilibrium of lower wire is stable equilibrium.

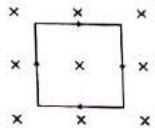
16. Assertion A flexible wire loop of irregular shape carrying current when placed in a uniform external magnetic field acquires a circular shape.

Reason For a given perimeter circular shape is having the greatest area.

17. Assertion At the centre of a circular current carrying loop (I_1), there is an infinitely long straight current carrying wire (I_2) perpendicular to the plane of the circle. Then, magnetic force of attraction between two is zero.

Reason Magnetic field of I_1 at centre is inwards, parallel to I_2.

18. Assertion Net torque in the current carrying loop placed in a uniform magnetic field (pointing inwards) is zero.



Reason Magnetic moment (**M**) is inwards.

19. Assertion A charged particle is rotating in a circle. Then, magnetic field (**B**) at centre of circle and magnetic moment (**M**) produced by motion of charged particle are parallel to each other.

Reason **M** and **B** are always parallel to each other.

20. Assertion Current sensitivity of a galvanometer is directly proportional to the current through the coil.

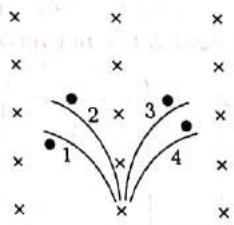
Reason Voltage sensitivity is inversely proportional to voltage.

Match the columns

1. A charged particle enters in a uniform magnetic field perpendicular to it. Now, match the following two columns.

	Column I		Column II
(A)	Speed of the particle	(p)	is constant
(B)	Velocity of the particle	(q)	is not constant
(C)	Acceleration of the particle	(r)	is maximum
(D)	Force on the particle	(s)	is minimum

2. Four particles α-particle, deuteron and electron and a Cl^- ion enter in a transverse magnetic field perpendicular to it with same kinetic energy. Their paths are as shown in figure. Now, match the following two columns.

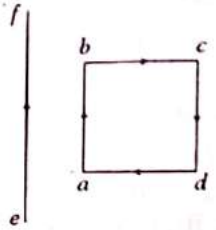


	Column I		Column II
(A)	Deuteron	(p)	path-1
(B)	α-particle	(q)	path-2
(C)	Electron	(r)	path-3
(D)	Cl^-	(s)	path-4

3. A charged particle is rotating in uniform circular motion in a uniform magnetic field. Let r = radius of circle, v = speed of particle, K = kinetic energy, a = magnitude of acceleration, p = magnitude of linear momentum, $\frac{q}{m} = \alpha$ = specific charge and ω = angular speed. Then, match the following two columns.

	Column I		Column II
(A)	If v is doubled	(p)	r will become two times
(B)	If K is doubled	(q)	ω will become two times
(C)	If p is doubled	(r)	a will become two times
(D)	If α is doubled	(s)	None

4. A square current carrying loop *abcd* is placed near an infinitely long another current carrying wire *ef*. Now, match the following two columns.



	Column I		Column II
(A)	Net force on *bc* and *da*	(p)	zero
(B)	Net force on *ab* and *cd*	(q)	non-zero
(C)	Net force on complete loop *abcd*	(r)	rightwards
(D)	Net force on *ab*	(s)	leftwards

C. Medical entrances' gallery

(Collection of questions asked in NEET & various medical entrance exams)

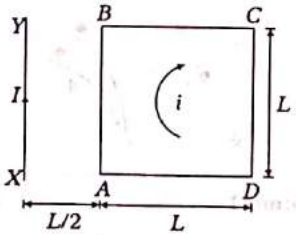
1. A long wire carrying a steady current is bent into a circular loop of one turn. The magnetic field at the centre of the loop is B. It is then bent into a circular coil of n turns. The magnetic field at the centre of this coil of n turns will be **[NEET PHASE II 2016]**

(a) nB (b) n^2B
(c) $2nB$ (d) $2n^2B$

2. An electron is moving in a circular path under the influence of a transverse magnetic field of 3.57×10^{-2} T. If the value of e/m is 1.76×10^{11} C/kg, the frequency of revolution of the electron is **[NEET PHASE II 2016]**

(a) 1 GHz (b) 100 MHz
(c) 62.8 MHz (d) 6.28 MHz

3. A square loop *ABCD* carrying a current i, is placed near and coplanar with a long straight conductor *XY* carrying a current I, the net force on the loop will be **[NEET 2016]**

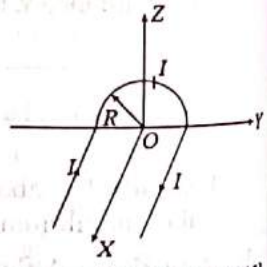


(a) $\frac{\mu_0 I i}{2\pi}$ (b) $\frac{2\mu_0 I i L}{3\pi}$
(c) $\frac{\mu_0 I i L}{2\pi}$ (d) $\frac{2\mu_0 I i}{3\pi}$

4. A long straight wire of radius a carries a steady current I. The current is uniformly distributed over its cross-section. The ratio of the magnetic fields **B** and **B'** at radial distances $\frac{a}{2}$ and $2a$ respectively, from the axis of the wire is **[NEET 2016]**

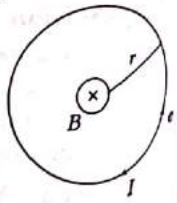
(a) $\frac{1}{2}$ (b) 1 (c) 4 (d) $\frac{1}{4}$

5. A wire carrying current I has the shape as shown in adjoining figure. Linear parts of the wire are very long and parallel to X-axis while semicircular portion of radius R is lying in yz-plane. Magnetic field at point O is **[CBSE AIPMT 2015]**



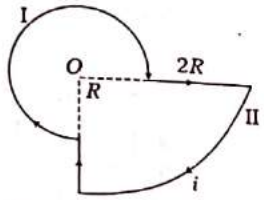
(a) $B = \frac{\mu_0}{4\pi}\frac{I}{R}(\pi\hat{\mathbf{i}} + 2\hat{\mathbf{k}})$ (b) $B = -\frac{\mu_0}{4\pi}\frac{I}{R}(\pi\hat{\mathbf{i}} - 2\hat{\mathbf{k}})$
(c) $B = -\frac{\mu_0}{4\pi}\frac{I}{R}(\pi\hat{\mathbf{i}} + 2\hat{\mathbf{k}})$ (d) $B = \frac{\mu_0}{4\pi}\frac{I}{R}(\pi\hat{\mathbf{i}} - 2\hat{\mathbf{k}})$

6. An electron moving in a circular orbit of radius r makes n rotations per second. The magnetic field produced at the centre has magnitude **[CBSE AIPMT 2015]**



(a) $\frac{\mu_0 n e}{2\pi r}$ (b) zero
(c) $\frac{\mu_0 n^2 e}{r}$ (d) $\frac{\mu_0 n e}{2r}$

7. Consider the circular loop having current i and with central point O. The magnetic field at the central point O is [AIIMS 2015]



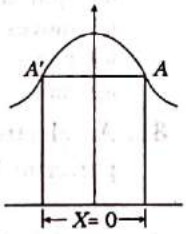
(a) $\frac{2\mu_0 i}{3\pi R}$ acting downward (b) $\frac{5\mu_0 i}{12R}$ acting downward
(c) $\frac{6\mu_0 i}{11R}$ acting downward (d) $\frac{3\mu_0 i}{7R}$ acting upward

8. A proton is projected with a speed of 3×10^6 m/s horizontally from east to west. An uniform magnetic field of strength 2×10^{-3} T exist in the vertically upward direction. What would be the acceleration of proton? [UK PMT 2015]
(a) 11.6×10^{11} m/s^2 (b) 17.4×10^{11} m/s
(c) 5.8×10^{11} m/s^2 (d) 2.9×10^{11} m/s^2

9. The magnetic field at the centre of a circular coil carrying current I ampere is B. If the coil is bent into smaller circular coil of n turns, then its magnetic field at the centre is B'. The ratio between B' and B is [Kerala CEE 2015]
(a) 1 : 1 (b) n : 1
(c) n^2 : 1 (d) $2n$: 1
(e) $(n+1)$: 1

10. Equal currents are passing through two very long and straight parallel wires in the same direction. They will [Guj. CET 2015]
(a) attract each other
(b) repel each other
(c) lean towards each other
(d) Neither attract nor repel each other

11. The variation of magnetic field (B) due to circular coil as the distance X varies is shown in the graph. Which of the following is false? [CG PMT 2015]



(a) Points A and A' are known as points of zero curvature
(b) B varies linearly with X at points A and A'
(c) $\frac{dB}{dt} = 0$ at points A and A'
(d) $\frac{d^2B}{dt^2} = 0$ at points A and A'

12. Two particles A and B having equal charges, after being accelerated through the same potential difference enter into a region of uniform magnetic field and the particles describe circular paths of radii R_1 and R_2, respectively. The ratio of the masses of A and B is [WB JEE 2015]
(a) $\sqrt{R_1/R_2}$ (b) R_1/R_2 (c) $(R_1/R_2)^2$ (d) $(R_2/R_1)^2$

13. There is a ring of radius r having linear charge density λ and rotating with a uniform angular velocity ω. The magnetic field produced by this ring at its own centre would be [UP CPMT 2015]
(a) $\frac{\lambda\omega^2}{2-\mu_0}$ (b) $\frac{\mu_0\lambda^2\omega}{\sqrt{2}}$ (c) $\frac{\mu_0\lambda\omega}{2}$ (d) $\frac{\mu_0\lambda}{2\omega^2}$

14. Two particles A and B having equal charges + 6 C after being accelerated through the same potential difference, enter in a region of uniform magnetic field and describe circular paths of radii 2 cm and 3 cm, respectively. The ratio of mass of A to that of B is [Manipal 2015]
(a) 1/3 (b) 1/2 (c) 4/9 (d) 9/5

15. A proton beam enters a magnetic field of 10^{-4} Wb/m^2 normally. If the specific charge of the proton is 10^{11} C/kg and its velocity is 10^9 m/s, then the radius of the circle described will be [KCET 2015]
(a) 100 m (b) 0.1 m (c) 1 m (d) 10 m

16. A cyclotron is used to accelerate [KCET 2015]
(a) Only negatively charged particles
(b) neutron
(c) Both positively and negatively charged particles
(d) Only positively charged particles

17. Two parallel beams of positron moving in the same direction will [Manipal 2015]
(a) not interact with each other
(b) repel each other
(c) attract each other
(d) be deflected normal to the plane containing two beams

18. Two concentric coils each of radius equal to 2π cm are placed at right angles to each other. If 3A and 4A are the currents flowing through the two coils, respectively. The magnetic induction (in Wb/m^2) at the centre of the coils will be [KCET 2015]
(a) 5×10^{-5} (b) 12×10^{-5}
(c) 7×10^{-5} (d) 10^{-5}

19. The conducting loop in the form of a circle is placed in a uniform magnetic field with its plane perpendicular to the direction of the field. An emf will be induced in the loop, if [WB JEE 2015]
(a) it is translated parallel to itself
(b) it is rotated about one of its diameters
(c) it is rotated about its own axis which is parallel to the field
(d) the loop is deformed from the original shape

20. A solenoid has length 0.4 cm, radius 1 cm and 400 turns of wire. If a current of 5 A is passed through this solenoid, then what is the magnetic field inside the solenoid? **[KCET 2014]**

(a) 6.28×10^{-4} T (b) 6.28×10^{-3} T
(c) 6.28×10^{-7} T (d) 6.28×10^{-6} T

21. A toroid having 200 turns carries a current of 1A. The average radius of the toroid is 10 cm. The magnetic field at any point in the open space inside the toroid is **[Kerala CEE 2014]**

(a) 4×10^{-3} T (b) zero
(b) 0.5×10^{-3} T (d) 3×10^{-3} T
(e) 2×10^{-3} T

22. When a proton is released from rest in a room, it starts with an initial acceleration a_0 towards west. When it is projected towards north with a speed v_0 it moves with an initial acceleration $3a_0$ towards west. The electric and magnetic fields in the room are **[NEET 2013]**

(a) $\frac{ma_0}{e}$ west, $\frac{2ma_0}{ev_0}$ up (b) $\frac{ma_0}{e}$ west, $\frac{2ma_0}{ev_0}$ down
(c) $\frac{ma_0}{e}$ east, $\frac{3ma_0}{ev_0}$ up (d) $\frac{ma_0}{e}$ east, $\frac{3ma_0}{ev_0}$ down

23. The magnetic field due to a current carrying circular loop of radius 3 cm at a point on the axis at a distance of 4 cm from the centre is 54 μT. What will be its value at the centre of the loop? **[UK PMT 2014]**

(a) 200 μT (b) 250 μT (c) 125 μT (d) 75 μT

24. Two identical long conducting wires *AOB* and *COD* are placed at right angle to each other, with one above other such that *O* is their common point for the two. The wires carry I_1 and I_2 currents, respectively. Point *P* is lying at distance *d* from *O* along a direction perpendicular to the plane containing the wires. The magnetic field at the point *P* will be **[CBSE AIPMT 2014]**

(a) $\frac{\mu_0}{2\pi d}\left(\frac{I_1}{I_2}\right)$ (b) $\frac{\mu_0}{2\pi d}(I_1 + I_2)$
(c) $\frac{\mu_0}{2\pi d}(I_1^2 - I_2^2)$ (d) $\frac{\mu_0}{2\pi d}(I_1^2 + I_2^2)^{1/2}$

25. A proton of mass *m* and charge *q* is moving in a plane with kinetic energy *E*. If there exists a uniform magnetic field *B*, perpendicular to the plane motion, the proton will move in a circular path of radius **[WB JEE 2014]**

(a) $\frac{2Em}{qB}$ (b) $\frac{\sqrt{2Em}}{qB}$
(c) $\frac{\sqrt{Em}}{2qB}$ (d) $\frac{\sqrt{2Eq}}{qB}$

26. If the velocity of charged particle has both perpendicular and parallel components while moving through a magnetic field, then what is the path followed by a charged particle? **[KCET 2014]**

(a) Circular (b) Elliptical (c) Linear (d) Helical

27. A particle with charge *q* is moving along a circle of radius *R* with uniform speed *v*. The associated magnetic moment μ is given by **[EAMCET 2014]**

(a) $\frac{1}{2}v^2R$ (b) $\frac{1}{4}qvR$
(c) $\frac{1}{2}qvR$ (d) $\frac{1}{2}q^2vR$

28. A charged particle of mass *m* and charge *q* moves along a circular path of radius *r* that is perpendicular to a magnetic field *B*. The time taken by the particle to complete one revolution is **[UK PMT 2014]**

(a) $\frac{2\mu mq}{B}$ (b) $\frac{2\pi q^2B}{m}$
(c) $\frac{2\pi qB}{m}$ (d) $\frac{2\pi m}{qB}$

29. Magnetic induction produced at the centre of a circular loop carrying current is *B*. The magnetic moment of the loop of radius *R* is (where, μ_0 = permeability of free space) **[MHT CET 2014]**

(a) $\frac{BR^2}{2\pi\mu_0}$ (b) $\frac{2\pi BR^3}{\mu_0}$ (c) $\frac{BR^2}{2\pi\mu_0}$ (d) $\frac{2\pi BR^2}{\mu_0}$

30. In cyclotron, for a given magnet, radius of the semicircle traced by positive ion is directly proportional to (where, v = velocity of positive ion) **[MHT CET 2014]**

(a) v^{-2} (b) v^{-1} (c) v (d) v^2

31. When a magnetic field is applied on a stationary electron, then it **[Kerala CEE 2014]**

(a) remains stationary
(b) spins about its own axis
(c) moves in the direction of the field
(d) moves perpendicular to the direction of the field
(e) moves opposite to the direction of the field

32. An electron in a circular orbit of radius 0.05 nm performs 10^{16} rev/s. The magnetic moment due to this rotation of electron is (in A-m^2). **[WB JEE 2014]**

(a) 2.16×10^{-23} (b) 3.21×10^{-22}
(c) 3.21×10^{-24} (d) 1.26×10^{-23}

33. A circular coil of radius 10 cm and 100 turns carries a current 1 A. What is the magnetic moment of the coil? **[KCET 2014]**

(a) 3.142×10^4 A-m^2 (b) 10^4 A-m^2
(c) 3.142 A-m^2 (d) 3 A-m^2

34. Two thin long conductors separated by a distance d carry currents I_1 and I_2 in the same direction. They exert a force F on each other. Now, the current in one of them is increased to two times and its direction is reversed. The distance is also increased to $3d$. The new value of force between them is **[UK PMT 2014]**

(a) $-2F$ (b) $\frac{F}{3}$ (c) $-\frac{2F}{3}$ (d) $-\frac{F}{3}$

35. A charged particle experiences magnetic force in the presence of magnetic field. Which of the following statement is correct? **[KCET 2014]**

(a) The particle is moving and magnetic field is perpendicular to the velocity
(b) The particle is moving and magnetic field is parallel to the velocity
(c) The particle is stationary and magnetic field is perpendicular to the velocity
(d) The particle is stationary and magnetic field is parallel to the velocity

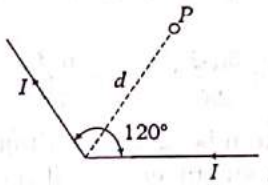
36. The ratio of magnetic dipole moment of an electron of charge e and mass m in Bohr's orbit in hydrogen atom to its angular momentum is **[MHT CET 2014]**

(a) $\frac{e}{m}$ (b) $\frac{m}{e}$ (c) $\frac{2m}{e}$ (d) $\frac{e}{2m}$

37. A wire of length L metre carrying a current I ampere is bent in the form of a circle. The magnitude of the magnetic moment is **[EAMCET 2014]**

(a) $\frac{L^2I^2}{4\pi}$ (b) $\frac{LI}{4\pi}$ (c) $\frac{L^2I}{4\pi}$ (d) $\frac{LI}{4\pi}$

38. A long conducting wire carrying a current I is bent at 120° (see figure). The magnetic field B at a point P on the right bisector of bending angle at a distance d from the bend is
(μ_0 is the permeability of free space) **[MP PMT 2014]**



(a) $\frac{2\mu_0 I}{2\pi d}$ (b) $\frac{\mu_0 I}{2\pi d}$ (c) $\frac{\mu_0 I}{\sqrt{3}\pi d}$ (d) $\frac{\sqrt{3}\mu_0 I}{2\pi d}$

39. A current loop in a magnetic field **[NEET 2013]**

(a) experiences a torque whether the field is uniform or non-uniform in all orientations
(b) can be in equilibrium in one orientations
(c) can be equilibrium in two orientations, both the equilibrium states are unstable
(d) can be in equilibrium in two orientations, one stable while the other is unstable

40. A proton and helium nucleus are shot into a magnetic field at right angles to the field with same kinetic energy. Then, the ratio of their radii is **[KCET 2013]**

(a) 1 : 1 (b) 1 : 2 (c) 2 : 1 (d) 1 : 4

41. Two charged particles have charges and masses in the ratio 2 : 3 and 1 : 4 respectively. If they enter a uniform magnetic field and move with the same velocity, then the ratio of their respective time periods of revolution is **[Kerala CEE 2013]**

(a) 3 : 8 (b) 1 : 4 (c) 3 : 5 (d) 1 : 6
(e) 2 : 5

42. A metal rod of length l cuts across a uniform magnetic field B with a velocity v. If the resistance of the circuit of which the rod forms a part is r, then the force required to move the rod is **[Kerala CEE 2013]**

(a) $\frac{B^2l^2v}{r}$ (b) $\frac{Blv}{r}$ (c) $\frac{B^2lv}{r}$ (d) $\frac{B^2l^2v^2}{r}$
(e) $\frac{Blv^2}{r}$

43. A current of 2 A is made to flow through a coil which has only one turn. The magnetic field produced at the centre is $4\pi \times 10^{-6}$ Wb/m^2. The radius of the coil is **[MP PMT 2013]**

(a) 0.0001m (b) 0.01m
(c) 0.1m (d) 0.001 m

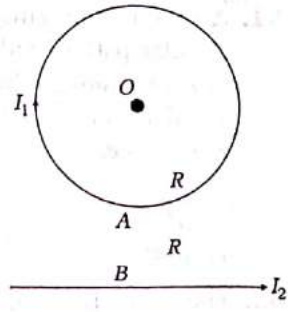
44. A long straight wire is carrying a current of 12 A. The magnetic field at a distance of 8 cm is
($\mu_0 = 4\pi \times 10^{-7}$ NA^2) **[J & K CET 2013]**

(a) 2×10^{-4} Wb/m^2 (b) 3×10^{-5} Wb/m^2
(c) 4×10^{-4} Wb/m^2 (d) 4×10^{-5} Wb/m^2

45. The magnetic field at a point on the axis of a long solenoid having 5 turns per cm length when a current of 0.8 A flows through it is **[J & K CET 2013]**

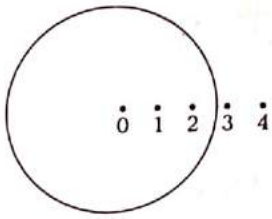
(a) 5.024×10^{-8} Wb/m^2 (b) 6.024×10^{-8} Wb/m^2
(c) 7.024×10^{-8} Wb/m^2 (d) 8.024×10^{-8} Wb/m^2

46. In the diagram, I_1, I_2 are the strength of the currents in the loop and straight conductors respectively, $OA = AB = R$. The net magnetic field at the centre O is zero. Then, the ratio of the currents in the loop and the straight conductor is **[KCET 2013]**



(a) π (b) 2π (c) $\frac{1}{\pi}$ (d) $\frac{1}{2\pi}$

47. The adjacent figure shows, the cross-section of a long rod with its length perpendicular to the plane of the paper. It carries constant current flowing along its length. B_1, B_2, B_3 and B_4 respectively, represent the magnetic fields due to the current in the rod at points 1, 2, 3 and 4 lying at different separations from the centre O, as shown in the figure. Which of the following shall hold true? **[AMU 2012]**



(a) $B_1 > B_2 \neq 0$ (b) $B_2 > B_3 \neq 0$
(c) $B_1 = B_2 = B_3 \neq 0$ (d) $B_3 > B_4 \neq 0$

48. Two straight wires each 10 cm long are parallel to one another and separated by 2 cm. When the current flowing in them is 30 A and 40 A respectively, then the force experienced by either of the wires is **[J & K CET 2013]**

(a) 1.2×10^{-3} N (b) 12×10^{-3} N
(c) 11.2×10^{-3} N (d) 10.2×10^{-3} N

49. A conductor of length 5 cm is moved parallel to itself with a speed of 2 m/s, perpendicular to a uniform magnetic field of 10^{-3} Wb/m^2. The induced emf generated is **[J & K CET 2013]**

(a) 2×10^{-3} V (b) 1×10^{-3} V
(c) 1×10^{-4} V (d) 2×10^{-4} V

50. A wire of one metre length carries a constant current. The wire is bent to form a circular loop. The magnetic field at the centre of this loop is B. The same is now bent to form a circular loop of smaller radius having four turns. The magnetic field at the centre of this new loop will be **[BHU 2012]**

(a) $\frac{B}{2}$ (b) $4B$ (c) $\frac{B}{4}$ (d) $16B$

51. A proton is moving in a uniform magnetic field B in a circular path of radius a in a direction perpendicular to Z-axis along which field B exists. Calculate the angular momentum. If the radius is a and charge on proton is e. **[Manipal 2012]**

(a) $\frac{Be}{a^2}$ (b) eB^2a
(c) a^2eB (d) aeB

52. The magnetic field in a certain region of space is given by $\mathbf{B} = 8.35 \times 10^{-2}\,\hat{\mathbf{i}}$ T. A proton is shot into the field with velocity $\mathbf{v} = (2 \times 10^5\,\hat{\mathbf{i}} + 4 \times 10^5\,\hat{\mathbf{j}})$ m/s. The proton follows a helical path in the field. The distance moved by proton in the X-direction during the period of one revolution in the yz-plane will be (mass of proton $= 1.67 \times 10^{-27}$ kg) **[AMU 2012]**

(a) 0.053 m (b) 0.136 m (c) 0.157 m (d) 0.236 m

53. A planar coil having 12 turns carries 15 A current. The coil is oriented with respect to the uniform magnetic field $\mathbf{B} = 0.2\hat{\mathbf{i}}$ T such that its directed area is $\mathbf{A} = 0.04\hat{\mathbf{i}}$ m^2. The potential energy of the coil in the given orientation is **[AMU 2012]**

(a) 0 (b) + 0.72 J (c) +1.44 J (d) −1.44 J

54. A current i ampere flows along the inner conductor of a co-axial cable and returns along the outer conductor of the cable, then the magnetic induction at any point outside the conductor at a distance r metre from the axis is **[JCECE 2012]**

(a) ∞ (b) zero (c) $\frac{\mu_0}{4\pi}\frac{2i}{r}$ (d) $\frac{\mu_0}{4\pi}\frac{2\pi i}{r}$

55. Two parallel long wires carry currents i_1 and i_2 with $i_1 > i_2$. When the currents are in the same direction, then the magnetic field midway between the wires is 10 μT. When the direction of i_2 is reversed, then it becomes 40 μT. Then, ratio of i_1/i_2 is **[JCECE 2012]**

(a) 3 : 4 (b) 5 : 3 (c) 7 : 11 (d) 11 : 7

56. A charged particle with a velocity 2×10^3 ms^{-1} passes undeflected through electric field and magnetic fields in mutually perpendicular directions. The magnetic field is 1.5 T. The magnitude of electric field will be **[Karnataka CET 2013]**

(a) 1.5×10^3 NC^{-1} (b) 2×10^3 NC^{-1}
(c) 3×10^3 NC^{-1} (d) 1.33×10^3 NC^{-1}

57. Two similar coils of radius R are lying concentrically with their planes at right angles to each other. The current flowing in them are I and $2I$, respectively, The resultant magnetic field induction at the centre will be **[CBSE PMT 2012]**

(a) $\frac{\sqrt{5}\mu_0 I}{2R}$ (b) $\frac{3\mu_0 I}{2R}$ (c) $\frac{\mu_0 I}{2R}$ (d) $\frac{\mu_0 I}{R}$

58. When an electron beam passes through an electric field they gain kinetic energy. If the same electron beam passes through a magnetic field, then their **[BHU 2012]**

(a) energy and momentum both ramain unchanged
(b) potential energy increases
(c) momentum increases
(d) kinetic energy increases

59. If a steel wire of length l and magnetic moment M is bent into a semicircular are, the the new magnetic moment is **[JCECE 2012]**

(a) $M \times l$ (b) $\frac{M}{l}$ (c) $\frac{2M}{\pi}$ (d) M

Answers

Check point 4.1

1. (c)	2. (b)	3. (c)	4. (c)	5. (d)	6. (a)	7. (b)	8. (b)	9. (b)	10. (c)
11. (c)	12. (a)	13. (d)	14. (c)	15. (d)	16. (a)	17. (b)	18. (d)	19. (d)	20. (d)

Check point 4.2

1. (c)	2. (c)	3. (a)	4. (c)	5. (b)	6. (a)	7. (d)	8. (a)	9. (a)	10. (b)
11. (d)	12. (c)	13. (b)	14. (b)	15. (b)	16. (a)	17. (a)	18. (c)	19. (a)	20. (a)

Check point 4.3

1. (b)	2. (d)	3. (b)	4. (c)	5. (d)	6. (a)	7. (c)	8. (d)	9. (a)	10. (c)
11. (d)	12. (d)	13. (b)	14. (b)	15. (a)					

A. Taking it together

1. (a)	2. (a)	3. (a)	4. (d)	5. (c)	6. (c)	7. (a)	8. (a)	9. (c)	10. (d)
11. (a)	12. (c)	13. (c)	14. (c)	15. (a)	16. (d)	17. (a)	18. (d)	19. (b)	20. (c)
21. (a)	22. (d)	23. (b)	24. (c)	25. (d)	26. (a)	27. (d)	28. (c)	29. (b)	30. (c)
31. (c)	32. (c)	33. (d)	34. (d)	35. (b)	36. (c)	37. (c)	38. (a)	39. (c)	40. (a)
41. (b)	42. (c)	43. (a)	44. (d)	45. (a)	46. (a,c)	47. (d)	48. (c)	49. (b)	50. (b)
51. (b)	52. (b)	53. (b)	54. (d)	55. (,ab)	56. (b)	57. (a)	58. (c)	59. (a)	60. (b)
61. (c)	62. (d)	63. (b)	64. (c)	65. (b)	66. (b)	67. (a)	68. (d)	69. (a,b,d)	70. (a)
71. (d)	72. (a)	73. (a,b,c,d)	74. (b)	75. (a)					

B. Medical entrance special format questions

Assertion and reason

1. (b)	2. (d)	3. (c)	4. (d)	5. (e)	6. (b)	7. (c)	8. (c)	9. (d)	10. (b)
11. (b)	12. (b)	13. (b)	14. (d)	15. (c)	16. (d)	17. (a)	18. (a)	19. (c)	20. (b)

Match the columns

1. (A → p; B → q; C → q; D → q,r)
2. (A → q; B → p; C → s; D → r)
3. (A → p; B → q; C → p; D → q)
4. (A → p; B → q, s; C → q, s; D → q,s)

C. Medical entrances' gallery

1. (b)	2. (a)	3. (d)	4. (b)	5. (c)	6. (d)	7. (b)	8. (c)	9. (c)	10. (a)
11. (d)	12. (b)	13. (c)	14. (c)	15. (a)	16. (d)	17. (c)	18. (a)	19. (b)	20. (b)
21. (b)	22. (b)	23. (b)	24. (d)	25. (b)	26. (d)	27. (c)	28. (d)	29. (b)	30. (c)
31. (a)	32. (d)	33. (c)	34. (c)	35. (a)	36. (d)	37. (c)	38. (d)	39. (d)	40. (a)
41. (a)	42. (a)	43. (c)	44. (b)	45. (a)	46. (d)	47. (d)	48. (a)	49. (c)	50. (d)
51. (c)	52. (c)	53. (d)	54. (b)	55. (b)	56. (c)	57. (a)	58. (a)	59. (a)	

Hints and explanations

Check point 4.1

3. (*c*) Magnetic field at distance r near a straight current carrying wire is given by

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2i}{r} \Rightarrow B \propto \frac{1}{r}$$

$$\therefore \quad \frac{B_1}{B_2} = \frac{r_2}{r_1} \text{ or } \frac{B}{B_2} = \frac{r/2}{r}$$

$$\therefore \quad B_2 = 2B$$

4. (*c*) According to the question,

$$\frac{\mu_0}{2\pi r}\left(\frac{1}{i_2} - \frac{1}{i_1}\right) = 10 \times 10^{-6} \quad \text{...(i)}$$

and

$$\frac{\mu_0}{2\pi r}\left(\frac{1}{i_2} + \frac{1}{i_1}\right) = 30 \times 10^{-6} \quad \text{...(ii)}$$

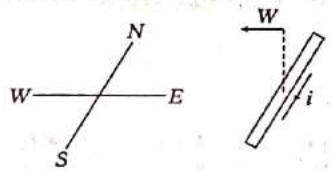
On dividing Eq. (ii) by Eq. (i), we get

$$\frac{i_1 + i_2}{i_1 - i_2} = \frac{3}{1}$$

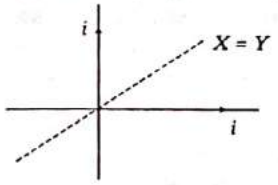
Solving this equation, we get

$$\frac{i_1}{i_2} = 2$$

5. (*d*) From Fleming's right-hand rule, the direction of current is towards West.



6. (*a*) Along the line $X = Y$, magnetic fields due to two wires will be equal and opposite.



7. (*b*) Magnetic field at the centre of coil is, $B = \frac{\mu_0 N i}{2r}$

8. (*c*) Magnetic field at the centre of circular coil of N turns is given by

$$B = \frac{\mu_0 N i}{2r} = \frac{4\pi \times 10^{-7} \times 100 \times 0.1}{2 \times 5 \times 10^{-2}} = 4\pi \times 10^{-5}\ \text{T}$$

11. (*b*) Magnetic field at the centre of arc is given by

$$B = \frac{\mu_0}{4\pi}\frac{\theta i}{r} = \frac{\mu_0}{4\pi} \times \frac{\pi}{2} \times \frac{i}{R} = \frac{\mu_0 i}{8R}$$

9. (*b*) Current produced in wire,

$$i = \frac{q}{t} = \frac{100 \times e}{1}$$

$$\therefore \quad i = 100\,e$$

Magnetic field at the centre of circular path

$$= \frac{\mu_0}{4\pi} \cdot \frac{2\pi i}{r} = \frac{\mu_0}{4\pi} \cdot \frac{2\pi \times 100\,e}{r}$$

$$= \frac{\mu_0 \times 200 \times 1.6 \times 10^{-19}}{4 \times 0.8} = 10^{-17}\mu_0$$

10. (*c*) The direction of magnetic field produced due to both semicircular parts will be perpendicular to the paper and inwards.

Also, $B = B_1 + B_2 = \frac{\mu_0 i}{4r_1} + \frac{\mu_0 i}{4r_2} = \frac{\mu_0 i}{4}\left(\frac{r_1 + r_2}{r_1 r_2}\right)$

12. (*a*) The magnetic field at the centre O

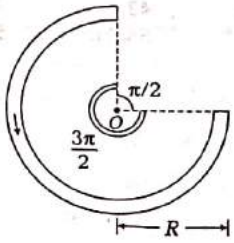
$$B_0 = B_1 + B_2$$

$$= \frac{\mu_0 I}{2R}\left[\frac{2\pi - 2\theta}{2\pi}\right] + \frac{\mu_0 I}{4\pi r}[\sin\theta + \sin\theta]$$

$$= \frac{\mu_0 I}{2R}\left[\frac{\pi - \theta}{\pi}\right] + \frac{\mu_0 I}{4\pi(R\cos\theta)}(2\sin\theta)$$

$$= \frac{\mu_0 I}{2\pi R}[\pi - \theta + \tan\theta]$$

13. (*d*) Magnetic field induction at O is is given by



$$B = \frac{3}{4}\text{ (Magnetic field due to whole circle)}$$

$$= \frac{3}{4}\left(\frac{\mu_0 i}{2R}\right) = \frac{3\mu_0 i}{8R}$$

14. (*c*) Magnetic field due to a ring having n turns at a distance x on its axis is given by

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi nir^2}{(x^2 + r^2)^{3/2}}$$

$$\therefore \quad B \propto \frac{nr^2}{(x^2 + r^2)^{3/2}}$$

15. (*d*) Ratio of magnetic field at the centre of a current carrying coil and at a distance a from the centre is

$$\frac{B_c}{B_a} = \left(1 + \frac{x^2}{a^2}\right)^{3/2}$$

Here, $x = a$

$$\therefore \quad \frac{B_c}{B_a} = \left(1 + \frac{a^2}{a^2}\right)^{3/2} = 2^{3/2} = 2\sqrt{2}$$

16. (*a*) According to the question,

$$\frac{\mu_0 IR^2}{2(x^2 + R^2)^{3/2}} = \frac{1}{8}\left(\frac{\mu_0 I}{2R}\right)$$

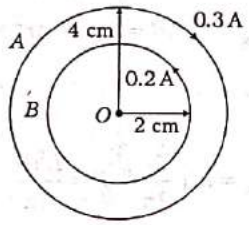
$$\therefore \quad \frac{R^2}{(x^2 + R^2)^{3/2}} = \frac{1}{8R}$$

or $(x^2 + R^2)^{3/2} = 8R^3$

or $x^2 + R^2 = 4R^2$

$\therefore \quad x = \sqrt{3}R$

17. (*b*) Suppose the coils are like,



At O,

$$B_1 = \frac{\mu_2(0.2)}{2(2)} = \frac{\mu_0}{20}, \odot$$

$$B_2 = \frac{\mu_0(0.3)}{2(4)} = \frac{3\mu_0}{80}, \otimes$$

Net magnetic field, $B_0 = B_1 - B_2 = \frac{\mu_0}{20} - \frac{3\mu_0}{80}$

$$= \frac{4\mu_0 - 3\mu_0}{80} = \frac{\mu_0}{80}, \odot$$

18. (*d*) The magnetic field, $B = \frac{\mu_0 N_i}{2R}$ or $B \propto \frac{N}{2R}$

Number of turns will become two times while radius remains half.

19. (*d*) Inside the rod, $B \propto r$ and outside it $B \propto \frac{1}{r}$.

Thus, graph (d) is correct option.

Check point 4.2

1. (*c*) On static charge, there is no effect of magnetic field, so electron remains stationary.

2. (*c*) Path is circular when $\mathbf{v} \perp \mathbf{B}$ or $\mathbf{v} \cdot \mathbf{B} = 0$.

3. (*a*) Radius, $r = \frac{mv}{qB}$

For electron, $r = \frac{mv}{eB}$

$$\therefore \quad \frac{e}{m} = \frac{v}{Br}$$

4. (*c*) Radius of circular path of charged particle is given by

$$r = \frac{mv}{qB} = \frac{p}{qB}$$

Since, electron and proton both have same momentum, therefore the circular path of both will have the same radius.

5. (*b*) As, $F_e = F_m$, $qE = Bqv$

$$\therefore \quad E = Bv$$

6. (*a*) Radius, $r = \frac{\sqrt{2Km}}{Bq}$ or $r \propto \frac{\sqrt{m}}{q}$

$$\left(\frac{\sqrt{m}}{q}\right)_\alpha : \left(\frac{\sqrt{m}}{q}\right)_p : \left(\frac{\sqrt{m}}{q}\right)_d = \frac{\sqrt{4}}{2} : \frac{\sqrt{1}}{1} : \frac{\sqrt{2}}{1} = 1:1:\sqrt{2}$$

7. (*d*) Work done by magnetic force is zero.

8. (*a*) Radius of circular path of an alpha particle,

$$r = \frac{\sqrt{2Km}}{Bq} \text{ or } K \propto \frac{q^2}{m} \quad \text{(for same radius)}$$

9. (*a*) For charged particle in uniform magnetic field, time period is given by

$$T = \frac{2\pi m}{qB}$$

$$\therefore \quad \text{Frequency, } \nu = \frac{1}{T} = \frac{qB}{2\pi m}$$

10. (*b*) When charge particle move in circular path into a region of magnetic field, then time period of charged particle is given by, $T = \frac{2\pi m}{qB}$

$\therefore \quad T \propto m$ (q and B are same)

$\because \quad m_p > m_e$

$\therefore$ Time period of proton > Time period of electron.

11. (*d*) Time period of charged particle is, $T = \frac{2\pi m}{Bq}$ or T is independent of r.

12. (*c*) Radius of particle, $r = \frac{\sqrt{2Km}}{Bq}$

$$\therefore \quad A = \pi r^2 = \frac{2\pi Km}{B^2 q^2} \text{ or } A \propto K$$

13. (*a*) Lorentz force, $\mathbf{F}_{net} = \mathbf{F}_e + \mathbf{F}_m = q\,\mathbf{E} + q\,(\mathbf{v} \times \mathbf{B})$

$$= q\,(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

14. (*b*) Velocity is in *xy*-plane and magnetic field along *Z*-axis. Therefore, path of the electron will be a circle. Magnetic force cannot change the speed of a particle.

15. (*b*) Radius of helical path taken by proton beam

$$r = \frac{Mv \sin 60°}{Bq} = \frac{(1.67 \times 10^{-27}) \times (4 \times 10^5) \sin 60°}{0.3 \times 1.6 \times 10^{-19}}$$

$$= 0.012\ \text{m}$$

16. (*a*) For charged particle on circular path

$$\frac{mv^2}{r} = qvB$$

$$\therefore \quad r = \frac{mv}{qB}$$

For proton,

$$R_p = \frac{mv}{qB} = \frac{\sqrt{2m_pE}}{qB}$$

For deuteron,

$$R_d = \frac{\sqrt{2\,m_dE}}{qB}$$

$$\therefore \quad \frac{R_d}{R_p} = \sqrt{\frac{m_d}{m_p}} = \sqrt{2}$$

$$\therefore \quad R_d = \sqrt{2}R_p$$

19. (*a*) Radius of the circular path, $r = \frac{mv}{qB}$

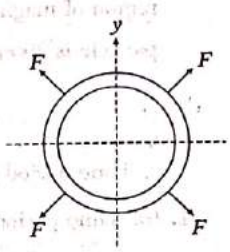
$$\Rightarrow \quad \frac{r_1}{r_2} = \frac{m_1v_1}{m_2v_2} \times \frac{q_2}{q_1} = \frac{1 \times 2}{1 \times 3} \times \frac{2}{1} = \frac{4}{3}$$

20. (*a*) Frequency of revolution, $f = \frac{Bq}{2\pi m}$ *i.e.*, $f \propto \frac{q}{m}$

As q is same for all given particles, $f \propto \frac{1}{m}$. Mass of Li^+ is maximum, therefore frequency will be minimum.

Check point 4.3

1. (*b*) Applying Fleming's left-hand rule, if magnetic field is perpendicular to paper inwards and current in the loop is clockwise, the magnetic force F on each element of loop is radially outwards and loop has a tendency to expand outwards.



Also, when a current carrying loop placed in uniform magnetic field, then net force on it is zero and loop cannot have translation motion.

2. (*d*) The parallel wires carrying currents in the same direction attract each other because magnetic forces on the two wires act towards each other.

3. (*b*) Two parallel conductor A and B of equal length carry currents I and $10I$, respectively is the same direction, then A and B attract each other with same force.

4. (*c*) Force per unit length,

$$\frac{F}{l} = \frac{\mu_0}{4\pi} \cdot \frac{2\,i_1i_2}{d}$$

$$\frac{F}{l} = \frac{\mu_0}{4\pi} \cdot \frac{2i^2}{d} \qquad (i = i_1 = i_2)$$

$$\frac{F}{l} = \frac{\mu_0}{4\pi} \cdot \frac{2\,i^2}{d} = \frac{\mu_0}{2\pi} \cdot \frac{i^2}{d} \qquad \text{(attractive)}$$

5. (*d*) Force between two conductors,

$$F \propto \frac{I_1I_2}{d}$$

When I_1 is changed to $2\,I_1$ and d is changed to $3d$.

$$\therefore \quad F' \propto \frac{(2\,I_1)\,(I_2)}{3d} \propto \frac{2}{3}F$$

As direction of current is reversed so, $F' = -2F/3$.

6. (*a*) $F = ilB = \frac{(2)\,(2)\,(2 \times 10^{-7})\,(10)}{0.1} = 8 \times 10^{-5}\ \text{N}$

7. (*c*) Force per unit length,

$$\frac{F}{l} = \frac{\mu_0}{2\pi}\frac{i_Ai_B}{r}$$

Given, $\quad i_A = 2i_B$

$$\therefore \quad (0.004) = \frac{2 \times 10^{-7}}{0.01}(2i_B^2)$$

$$\therefore \quad i_B = 10\ \text{A}$$

10. (*c*) Torque, $\tau_{max} = NiAB = 1 \times i \times \pi r^2 \times B$

$$\left(\because 2\pi r = L \Rightarrow r = \frac{L}{2\pi}\right)$$

$$\tau_{max} = \pi i\left(\frac{L}{2\pi}\right)^2 B = \frac{L^2iB}{4\pi}$$

11. (*d*) Torque acting on the coil is given by

$$\tau = NiBA \sin\theta$$

As, magnetic field is normal to plane of coil.

So, $\quad \theta = 0°$

$$\therefore \quad \tau = 0$$

12. (*d*) The pole pieces of the magnet used in a pivoted coil galvanometer are cylindrical surfaces of a horse-shoe magnet.

13. (*b*) The electrical current in moving coil galvanometer

$$i = \frac{C\theta}{NAB}$$

$$i \propto \theta$$

14. (*b*) We have sensitivity of a moving coil galvanometer

$$S = \frac{NAB}{C}$$

where, C is the torsional constant of its suspension.

In order to increase the sensitivity of a moving coil galvanometer one should decrease the torsional constant of its suspension.

15. (*a*) Sensitivity, $S = \frac{\theta}{i}$

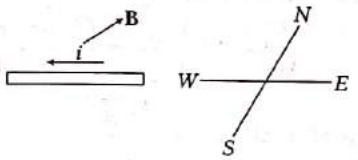
$$\frac{S_A}{S_B} = \frac{i_B}{i_A} \Rightarrow \frac{S_A}{S_B} = \frac{5}{3} \Rightarrow S_A > S_B$$

A. Taking it together

1. (*a*) In Biot-Savart's law, magnetic field $\mathbf{B} \| \mathbf{i}\, dl \times \mathbf{r}$ and $\mathbf{i}d\mathbf{l}$ due to flow of electron is in opposite direction of $\mathbf{v}$ and by direction of cross product of two vectors,

$$\mathbf{B} \perp \mathbf{v}$$

2. (*a*) The direction of current is shown in the figure.



According to Fleming's left-hand rule, direction of magnetic field produced will be towards north.

3. (*a*) *The charged particle undergoes acceleration as*

(i) speeds up between the dees because of the oscillating electric field and

(ii) speed remain the same inside the dees because of the magnetic field but direction undergoes change continuously.

4. (*d*) Due the magnetic force, electron is projected with uniform velocity along the axis of a current carrying long solenoid, $F = -evB \sin 180° = 0$ ($\theta = 0°$) as magnetic field and velocity are parallel. The electron will continue to move with uniform velocity along the axis of the solenoid.

5. (*c*) $\because$ Force, $\mathbf{F}_m = q(\mathbf{v} \times \mathbf{B})$

$\because$ A proton is moving along the negative direction of X-axis in a magnetic field directed along the positive direction of Y-axis.

So, proton will be deflected along the negative direction of Z-axis.

8. (*a*) The direction of magnetic moment of circular loop of radius R is placed in the xy-plane is along z-direction and given by $M = I(\pi r^2)$, when half of the loop with $x > 0$ is now bent, so that it now lies in the yz-plane, the magnitudes of magnetic moment of each semicircular loop of radius R lie in the xy-plane and the yz-plane is $M' = I(\pi r^2)/4$ and the direction of magnetic moments are along z-direction and x-direction, respectively.

Their resultant

$$M_{net} = \sqrt{M'^2 + M'^2} = \sqrt{2}\, M' = \sqrt{2}\, I(\pi r^2)/4$$

So, $M_{net} < M$ or M diminishes.

11. (*a*) For hollow metallic cylinder, magnetic field inside is zero while, outside and magnetic field is inversely proportional to distance from centre of cylinder.

$$B_{in} = 0 \quad \text{...(i)}$$

$$B_{out} = \frac{\mu_0 i}{2\pi r} \quad \text{...(ii)}$$

Eqs. (i) and (ii) both are shown by graph (a).

12. (*c*) Magnetic field produced at the centre of the orbit,

$$B = \frac{\mu_0}{4\pi}\frac{2\pi i}{r} \Rightarrow i = \frac{q}{t} = qv$$

$$\therefore \quad B = \frac{\mu_0}{4\pi}\frac{2\pi(qv)}{r}$$

$$= \frac{4\pi \times 10^{-7} \times 2 \times 3.14 \times 1.6 \times 10^{-19} \times 6.6 \times 10^{15}}{4\pi \times 0.53 \times 10^{-10}}$$

$$= 12.5 \text{ Wb m}^{-2}$$

14. (*c*) We have, $B_1 = \frac{\mu_0 I}{2r}$, $B_2 = \frac{\mu_0 I}{4r}$

$$\therefore \quad \frac{B_1}{B_2} = 2$$

15. (*a*) Magnetic force on B from both the wires is towards A.

16. (*d*) Given, $2\pi R = 2.0$

$$\therefore \quad R = \frac{1.0}{\pi} \text{ m}$$

$$M = iA = (i)(\pi R^2)$$

$$= \frac{1}{\pi} \text{ A-m}^2 = (1)(\pi)\left(\frac{1.0}{\pi}\right)^2 = \frac{1}{\pi} \text{ A-m}^2$$

17. (*a*) **In first case**

$$l = 2\pi r_1$$

At centre, $B = \frac{\mu_0 i}{2r_1}$,

In second case

$$l = 2 \times 2 \times \pi r_2 \Rightarrow r_2 = r_1/2$$

$$\therefore \quad B' = \frac{2 \times \mu_0 i}{2r_2} \quad (\because \text{there are 2 turns})$$

$$\Rightarrow \quad \frac{B'}{B} = \frac{2r_1}{r_2} = 4 \Rightarrow B' = 4B$$

18. (*d*) For given pitch d correspond to charge particle, we have,

$$\frac{q}{m} = \frac{2\pi v \cos\theta}{qB} = \text{constant}$$

Since, charged particles traverse identical helical paths in a completely opposite sense in a uniform magnetic field $\mathbf{B}$, LHS for two particles should be same and of opposite sign. Therefore,

$$\left(\frac{e}{m}\right)_1 + \left(\frac{e}{m}\right)_2 = 0$$

19. (*b*) When wire is turned into n circular loops, then magnetic field produced is

$$B' = n^2 B = (3)^2 B_1 = 9B_1$$

21. (*a*) Distance = 2 (pitch)

$$= 2\left(\frac{2\pi m}{Bq}\right) v\cos\theta$$

$$= \frac{4 \times \pi \times 1.6 \times 10^{-27} \times 2 \times 10^6 \times \cos 30°}{0.05 \times 1.6 \times 10^{-19}}$$

$$= 4.35 \text{ m}$$

22. (*d*) Velocity vector is parallel to magnetic field vector.

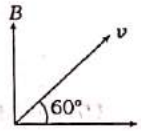
23. (*b*) We have, $(b-a) = r = \frac{mv}{Bq}$

$$\therefore \quad v_{min} = \frac{(b-a)Bq}{m}$$

24. (*c*) Radius of circular pair,

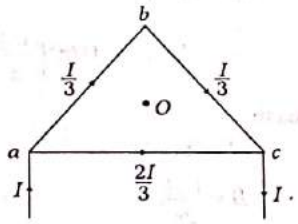
$$r = \frac{mv\cos 60°}{Bq}$$

$$= \frac{(1.67 \times 10^{-27})(2 \times 10^6)\cos 60°}{0.14 \times 1.6 \times 10^{-19}} = 0.07 \text{ m}$$



and $T = \frac{2\pi m}{Bq} = \frac{(2\pi)(1.67 \times 10^{-27})}{0.14 \times 1.6 \times 10^{-19}} = 0.5 \times 10^{-6}$ s

25. (*d*) Distribution of current will be as shown in below.



At point O, $\mathbf{B}_{abc} = -\mathbf{B}_{ac}$

$\therefore$ Net magnetic field at O is zero.

26. (*a*) B = (due to circular wire) + (due to straight wire)

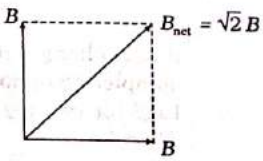
$$= \frac{\mu_0 I}{2R} + \frac{\mu_0}{2\pi}\frac{I}{R} = \frac{\mu_0 I}{2R}\left(1 + \frac{1}{\pi}\right) = \frac{2\mu_0 I}{4R\pi}(1+\pi)$$

27. (*d*) Magnetic field at O due to wire 1 will be zero.

$$B_{net} = B_2 + B_3$$

$$= \frac{\mu_0 I}{4R} + \frac{\mu_0 I}{4\pi R}$$

28. (*c*) The magnitude of the resultant is $B_{net} = \sqrt{2}B$



The ratio of the magnitude of the resultant magnetic field at the centre and the field.

$$\Rightarrow \quad \frac{B_{at\ centre}}{B_{net}} = \frac{1}{\sqrt{2}}$$

30. (*c*) Magnetic field, $B = \frac{\mu_0 i}{2R} = \frac{\mu_0 (ef)}{2R} = \frac{(\mu_0)(e)\left(\frac{v}{2\pi R}\right)}{R}$

or Radius, $R = \sqrt{\frac{v}{B}}$

31. (*c*) Let length of each wire is L.

For square length of each side is $L/4$

Area of square $= \left(\frac{L}{4}\right)^2 = \frac{L^2}{16}$

For circle, $L = 2\pi r \Rightarrow r = \frac{L}{2\pi}$

Area of circle $= \pi r^2 = \pi\left(\frac{L}{2\pi}\right)^2 = \frac{L^2}{4\pi}$

As, magnetic moment, $M = iA$

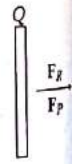
$$\therefore \quad \frac{M_{square}}{M_{circle}} = \frac{A_{square}}{A_{circle}} = \frac{L^2/16}{L^2/4\pi} = \frac{\pi}{4}$$

32. (*c*) Radius of particle, $r = \frac{\sqrt{2mk}}{qB} \Rightarrow r \propto \sqrt{m}$

$$\therefore \quad \frac{m_1}{m_2} = \left(\frac{R_1}{R_2}\right)^2$$

33. (*d*) At O, due to wire AB and DE, magnetic field will be zero. The combined effect of BC and DF is equivalent to that of an infinitely long wire.

37. (*c*) Force on wire Q due to wire R,



$$F_R = \frac{4\pi \times 10^{-7}}{4\pi} \times \frac{2 \times 20 \times 10}{0.02} \times 0.1$$

$$= 20 \times 10^{-5} \text{ N} \quad \text{(towards right)}$$

Force on wire Q due to wire P,

$$F_P = \frac{4\pi \times 10^{-7}}{4\pi} \times \frac{2 \times 30 \times 10}{0.1} \times 0.1$$

$$= 6 \times 10^{-5} \text{ N} \quad \text{(towards right)}$$

Net force on Q,

$$F = F_R + F_P$$

$$= 20 \times 10^{-5} + 6 \times 10^{-5} = 26 \times 10^{-5} \text{ N}$$

$$= 2.6 \times 10^{-4} \text{ N} \quad \text{(towards right)}$$

38. (*a*) Force on the wire, $F = ilB\sin\theta$

$$\sin\theta = \frac{F}{ilB} = \frac{15}{10 \times 1.5 \times 2} = \frac{1}{2}$$

$\therefore$ The angle between the magnetic field and direction of the current, $\theta = 30°$.

39. (*c*) If q is positive, due to electric field, their velocity will become towards positive x-axis.

Now, $\mathbf{F}_m = q(\mathbf{v} \times \mathbf{B})$

where, $\mathbf{v} \rightarrow$ towards $\hat{\mathbf{i}}$

and $\mathbf{B} \rightarrow$ towards $\hat{\mathbf{k}}$

$\therefore$ $\mathbf{F}_m \rightarrow$ towards $\hat{\mathbf{j}}$

Similarly, we can see with $-q$ charge.

40. (*a*) For minimum value of B, it is colliding after one pitch. It will collide the target again, if pitch is halved or remains same.

$$p = (v \cos \theta)\left(\frac{2\pi m}{Bq}\right) \text{ or } p \propto \frac{v}{B}$$

42. (*c*) We know that, current, $i = qv$

$$= 1.6 \times 10^{-19} \times 6.6 \times 10^{15} = 10.5 \times 10^{-4} \text{ A}$$

$\therefore$ Area, $A = \pi R^2 = 3.14 \times (0.528)^2 \times 10^{-20}$ m^2

$\therefore$ Magnetic moment, $M = iA$

$$= 10.5 \times 10^{-4} \times 3.14 \times (0.528)^2 \times 10^{-20}$$

$$= 10 \times 10^{-24} \text{ units } = 1 \times 10^{-23} \text{ units}$$

43. (*a*) All magnetic fields are cancelling each other.

44. (*d*) $2r = 2\dfrac{mv_0}{qB_0}$

45. (*a*) Torque acting on the coil $\tau = NiBA$

$$= 100 \times 2 \times 0.2 \times (0.08 \times 0.1) = 0.32 \text{N-m}$$

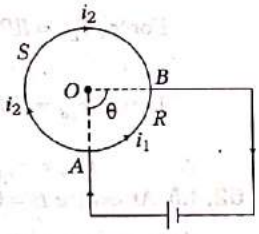
Direction of torque τ is given by the vector $\mathbf{M} \times \mathbf{B}$.

46. (*a,c*) Radius, $r = \dfrac{\sqrt{2\,mk}}{qB}$

$\therefore$ $r \propto \dfrac{\sqrt{m}}{q}$

$\therefore$ $r_H : r_{He} : r_O = \dfrac{\sqrt{1}}{1} : \dfrac{\sqrt{4}}{1} : \dfrac{\sqrt{16}}{2} = 1:2:2$

47. (*d*) Let l_1, l_2 be the lengths of two parts ARB and ASB of the ring, and ρ be resistance per unit length of the conductor,



The resistance of the portion ARB will be $R_1 = l_1\rho$

The resistance of the portion ASB will be $R_2 = l_2\rho$

Let i_1, i_2 be the currents in ARB and ASB, respectively.

Since, these parts are in parallel, the potential difference across their ends will be same.

$\therefore$ $i_1R_1 = i_2R_2$

or $i_1l_1\rho = i_2l_2\rho$

or $i_1l_1 = i_2l_2$ ...(i)

According to Biot-Savart's law, the magnetic fields at the centre O due to current through circular ring ARB and ASB will be

$$B_1 = \frac{\mu_0}{4\pi}\frac{i_1l_1 \sin 90°}{r^2}$$

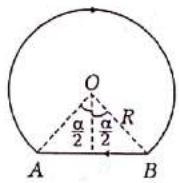
and $$B_2 = \frac{\mu_0}{4\pi}\frac{i_2l_2 \sin 90°}{r^2}$$

Since, $i_1l_1 = i_2l_2$

$\therefore$ $B_1 = B_2$

According to Fleming's right hand rule, the directions of B_1 and B_2 are opposite to each other, hence the resultant magnetic field at O will be zero.

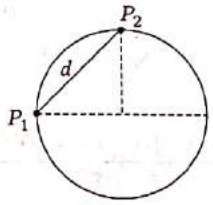
48. (*c*) From the figure,



$$AB = 2R \sin\frac{\alpha}{2}$$

$$\therefore \text{ Force} = IB\left(2R \sin\frac{\alpha}{2}\right) = 2BIR \sin\frac{\alpha}{2}$$

49. (*b*) Given, time, $t = \dfrac{\pi m}{Bq} = \dfrac{T}{2}$ of proton



Sepearation, $d = \sqrt{2}r = \dfrac{\sqrt{2}mv}{Bq}$

50. (*b*) As, $dA = (2\pi r)\,dr$

$$dq = \sigma \cdot dA = (2\pi\alpha r^3) \cdot dr$$

$$i = (dq)\,f = (dq)\frac{\omega}{2\pi} = (\alpha\omega r^3)\,dr$$

$$dB = \frac{\mu_0 i}{2r} = \frac{\alpha\mu_0\omega r^2 dr}{2}$$

$\therefore$ $$B = \int_0^R dB = \frac{\mu_0\alpha\omega R^3}{6}$$

51. (*b*) As $(mg)(r) = MB_x = I(\pi r^2)B_x$

$\therefore$ Current, $i = \dfrac{mg}{\pi r B_x}$

52. (*b*) At the centre of coil-1,

$$B_1 = \frac{\mu_0}{4\pi} \times \frac{2\pi i_1}{r_1} \quad \text{...(i)}$$

At the centre of coil-2,

$$B_2 = \frac{\mu_0}{4\pi} \times \frac{2\pi i_2}{r_2} \quad \text{...(ii)}$$

But $B_1 = B_2$

$\therefore$ $\dfrac{\mu_0}{4\pi}\dfrac{2\pi i_1}{r_1} = \dfrac{\mu_0}{4\pi}\dfrac{2\pi i_2}{r_2}$ or $\dfrac{i_1}{r_1} = \dfrac{i_2}{r_2}$

As $r_1 = 2r_2$

$\therefore$ $\dfrac{i_1}{2r_2} = \dfrac{i_2}{r_2}$ or $i_1 = 2i_2$

Now, ratio of potential differences,

$$\frac{V_2}{V_1} = \frac{i_2 \times r_2}{i_1 \times r_1} = \frac{i_2 \times r_2}{2i_2 \times 2r_2} = \frac{1}{4} \Rightarrow \frac{V_1}{V_2} = \frac{4}{1}$$

53. (*b*) As, $(mgr) = MB = (i)(\pi r^2)B$

$\therefore \quad i = \dfrac{mg}{\pi r B}$

54. (*d*) Velocity, $v = \dfrac{E}{B}$ and radius of path, $r = \dfrac{mv}{Bq} = \dfrac{E/B}{B \cdot S} = \dfrac{E}{B^2 S}$

55. (*a,b*) Here, $N = 1, A = 4 \times 10^{-2}\ \text{m}^2$

$B = 5 \times 10^{-2}\ \text{Wb/m}^2$

$i = 5 \times 10^{-3}\text{A}, \theta = 0.2\ \text{rad}, K = ?, I_s = ?$

We have, $i = \dfrac{K}{NAB}\theta$

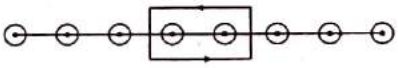
$\Rightarrow \quad K = \dfrac{NABi}{Q} = \dfrac{1 \times 4 \times 10^{-2} \times 5 \times 10^{-2} \times 5 \times 10^{-3}}{0.2}$

$= 5 \times 10^{-5}\ \text{N-m rad}^{-1}$

Current sensitivity,

$I_s = \dfrac{NBA}{K} = \dfrac{1 \times 5 \times 10^{-2} \times 4 \times 10^{-2}}{5 \times 10^{-5}} = 40\ \text{rad A}^{-1}$

56. (*b*) Applying Ampere's circuital law in the closed loop as shown in figure,



$2Bl = \mu_0(\lambda \cdot l)$

$\therefore$ Magnetic field induction, $B = \dfrac{\mu_0 \lambda}{2}$

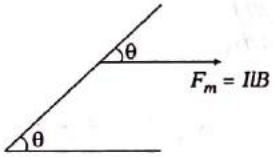
57. (*a*) We have, $\mathbf{F}_{CAD} = \mathbf{F}_{CD} = \mathbf{F}_{CED}$

$\therefore$ Net force on frame $= 3\,\mathbf{F}_{CD}$

$= 3\,(Bil) = 3 \times 2 \times 1 \times 4 = 24\ \text{N}$

58. (*c*) According to the question,

$mg \sin\theta = iLB \cos\theta$



$\therefore$ Magnitude of magnetic field, $B = \dfrac{mg}{IL}\tan\theta = \dfrac{\sqrt{3}mg}{4L}$

59. (*a*) Let the magnetic field is,

$\mathbf{B} = B_1\hat{\mathbf{i}} + B_2\hat{\mathbf{j}} + B_3\hat{\mathbf{k}}$

Applying $\mathbf{F}_m = q(\mathbf{v} \times \mathbf{B})$ two times, we have

$q[-\hat{\mathbf{j}} + \hat{\mathbf{k}}] = q[(\hat{\mathbf{i}}) \times (B_1\hat{\mathbf{i}} + B_2\hat{\mathbf{j}} + B_3\hat{\mathbf{k}})] = q[B_2\hat{\mathbf{k}} - B_3\hat{\mathbf{j}}]$

On comparing two sides, we get

$B_2 = 1$ and $B_3 = 1$

Further,

$q[\hat{\mathbf{i}} - \hat{\mathbf{k}}] = q[(\hat{\mathbf{j}}) \times (B_1\hat{\mathbf{i}} + B_2\hat{\mathbf{j}} + B_3\hat{\mathbf{k}})]$

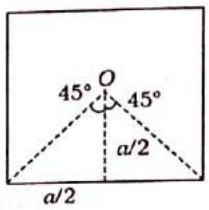
$= q[-B_1\hat{\mathbf{k}} + B_3\hat{\mathbf{i}}]$

Again comparing, we get

$B_1 = 1$ and $B_3 = 1$

$\therefore \quad \mathbf{B} = (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})\ \text{Wb/m}^2$

60. (*b*) According to question, $l = 2\pi R \Rightarrow R = \dfrac{l}{2\pi}$



and $\quad l = 4a \Rightarrow a = \dfrac{l}{4}$

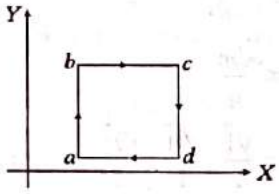
$B' = \dfrac{\mu_0 i}{2R} = \dfrac{\mu_0 i}{2 \cdot \dfrac{l}{2\pi}} = \dfrac{\mu_0 \pi i}{l}$

$B = 4\left[\dfrac{\mu_0}{4\pi} \cdot \dfrac{i}{a/2}(\sin 45° + \sin 45°)\right]$

$= \dfrac{2\sqrt{2}\,\mu_0 i}{\pi a} = \dfrac{8\sqrt{2}\,\mu_0 i}{\pi l}$

$\therefore \quad \dfrac{B}{B'} = \dfrac{8\sqrt{2}}{\pi^2}$

61. (*c*) $\mathbf{F}_{bc} + \mathbf{F}_{da} = 0$



Force, $\mathbf{F}_{ab} = IlB_0\left(1 + \dfrac{x}{l}\right) = IB_0 l + IB_0 x$ (towards right)

Force, $\mathbf{F}_{cd} = IlB_0\left[1 + \dfrac{x + l}{l}\right] = 2IB_0 l + IB_0 x$ (towards left)

$\therefore \quad \mathbf{F}_{\text{net}} = F_{cd} - F_{ab} = IB_0 l$ (towards left)

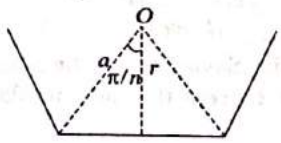
62. (*d*) At centre $B = 0$.

At $x \to \alpha, B \to 0$ between zero and zero there will be a maximum value.

Along X-axis for $x > 0$, B_{net} is downwards.

Along X-axis for $x < 0$, B_{net} is upwards.

63. (*b*) Radius, $r = a \cos\dfrac{\pi}{n}$



$\therefore$ Magnetic field,

$B = n\left[\dfrac{\mu_0}{4\pi} \cdot \dfrac{i}{a \cos \pi/n}\left(\sin\dfrac{\pi}{n} + \sin\dfrac{\pi}{n}\right)\right] = \dfrac{\mu_0 n i}{2\pi a}\tan\left(\dfrac{\pi}{n}\right)$

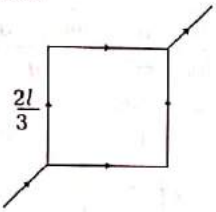
64. (*c*) The magnitude of the force acting on the wire,

$$F_{net} = \sqrt{F^2 + F^2} = \sqrt{2}\,F = \sqrt{2}\,IlB$$

65. (*b*) Magnetic field,

$$B = 2\left[\frac{\mu_0}{4\pi}\frac{2I/3}{a/2}(\sin 45° + \sin 45°) - \frac{\mu_0}{4\pi}\cdot\frac{I/3}{a/2}(\sin 45° + \sin 45°)\right]$$

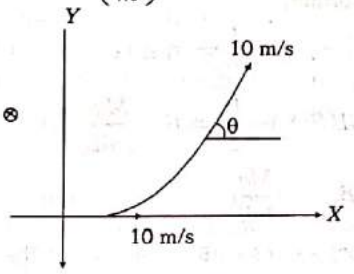
$$= \frac{\sqrt{2}\,\mu_0 I}{3\pi a}$$



66. (*b*) As, $2kx_0 = mg = ILB = \left(\frac{E}{R}\right)\cdot LB$

$\therefore$ The magnetic field, $B = \frac{mgR}{EL}$

67. (*a*) Angle, $\theta = \omega t = \left(\frac{Bq}{m}\right)t$

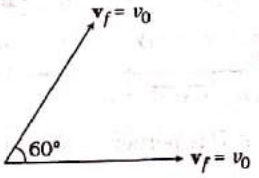


$$= 2 \times \pi \times \frac{1}{6} = \frac{\pi}{3} \text{ or } 60°$$

Velocity, $\mathbf{v} = 10\cos 60°\,\hat{\mathbf{i}} + 10\sin 60°\,\hat{\mathbf{j}}$

$$= (5\hat{\mathbf{i}} + 5\sqrt{3}\,\hat{\mathbf{j}})\text{ m/s}$$

68. (*d*) Deviation, $\theta = \sin^{-1}\left(\frac{L}{R}\right) = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60°$



Velocity of the particle when it comes out of the field,

$$|\Delta\mathbf{v}| = 2v_0 \sin\frac{\theta}{2}$$

$$= 2v_0 \sin 30° = v_0$$

69. (*a, b, d*) As proton is moving with constant velocity, so acceleration is zero.

So, $E = 0$ and $B = 0$

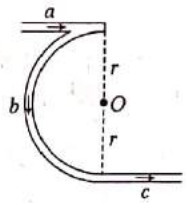
When $E = 0$ and $B \neq 0$, but parallel to the motion of proton, there will be no net force acting on proton.

When, $E \neq 0$ and $B \neq 0$ but electric force and magnetic force cancel each other, then also velocity will remain unchanged.

70. (*a*) For first figure,

magnetic field at O due to straight part (a),

$$B_a = \frac{\mu_0}{4\pi}\cdot\frac{i}{r}$$



Magnetic field at O due to circular part (b),

$$B_b = \frac{\mu_0}{4\pi}\frac{\pi i}{r}$$

Magnetic field at O due to straight part (c),

$$B_c = \frac{\mu_0}{4\pi}\frac{i}{r}$$

So, net magnetic field at the centre of first figure,

$$B_1 = B_b + B_c - B_a = \frac{\mu_0}{4\pi}\frac{\pi i}{r} \quad \text{....(i)}$$

For second figure, magnetic field at O due to two straight and one circular part is

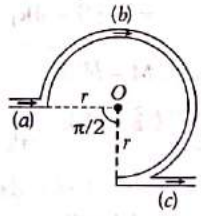
$$B_2 = \frac{\mu_0}{4\pi}\frac{\pi i}{r} \quad \text{...(ii)}$$

For third figure, magnetic field at O due to straight part (a),

$$B_a = 0$$

Magnetic field at O due to circular part (b),

$$B_b = \frac{\mu_0}{4\pi}\frac{(2\pi - \pi/2)i}{r}$$



Magnetic field at C due to straight part (c),

$$B_c = \frac{\mu_0}{4\pi}\frac{i}{r}$$

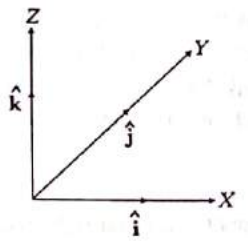
So, net magnetic field at centre of third figure,

$$B_3 = \frac{\mu_0}{4\pi}\cdot\frac{i}{r}\left(\frac{3\pi}{2} - 1\right) \quad \text{...(iii)}$$

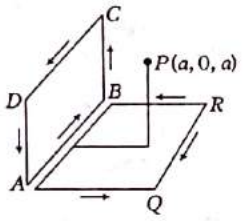
From Eqs. (i), (ii) and (iii), we get

$$B_1 : B_2 : B_3 = \pi : \pi : \left(\frac{3\pi}{2} - 1\right) = -\frac{\pi}{2} : \frac{\pi}{2} : \left(\frac{3\pi}{4} - \frac{1}{2}\right)$$

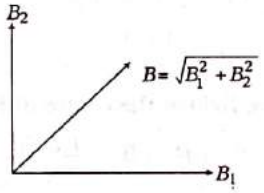
71. (*d*)



We can consider the non-planar loop as made by two loops *ABCDA* and *AQRBA* as shown in figure.



Magnetic field due to loop *ABCDA* will be along *X*-axis and due to loop *AQRBA* along *Z*-axis, magnitude of magnetic field due to both loops will be equal.



Therefore, direction of resultant magnetic field at *P* will be along $\frac{1}{\sqrt{2}}(\hat{i} + \hat{k})$.

72. (*a*) Area vector, $\mathbf{A} = \mathbf{CO} \times \mathbf{OA}$

$$= (-b\cos 37°\,\hat{i} - b\sin 37°\,\hat{k}) \times (a\hat{j})$$

$$= -\frac{ab}{5}[4(\hat{i} \times \hat{j}) + 3(\hat{k} \times \hat{j})]$$

$$= -\frac{ab}{5}[4\hat{k} - 3\hat{i}]$$

$$= \frac{ab}{5}[3\hat{i} - 4\hat{k}]$$

Magnetic moment, $\mathbf{M} = Ni_0\mathbf{A}$

$$= \frac{Ni_0ab}{5}(3\hat{i} - 4\hat{k})$$

$$\mathbf{B} = B_0\hat{i}$$

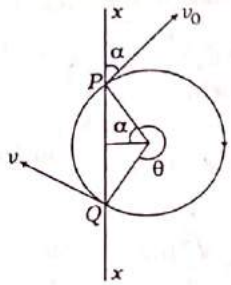
$$\tau = \mathbf{M} \times \mathbf{B} = \frac{Ni_0abB_0}{5}[3(\hat{i} \times \hat{i}) - 4(\hat{k} \times \hat{i})]$$

$$= -\frac{4Ni_0abB_0}{5}\hat{j}$$

The loop will rotate in clockwise direction as seen from above.

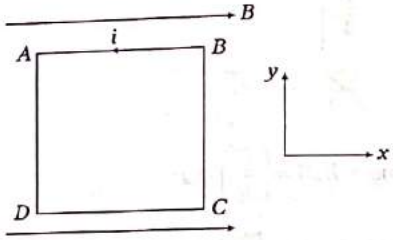
73. (*a, b, c, d*) From the question, $PQ = 2r\sin\alpha = \frac{2mv_0}{Bq}\sin\alpha$

$\because \quad \theta = (2\pi - 2\alpha)$



$$\therefore \quad t = \frac{\theta}{\omega} = \frac{2(\pi - \alpha)}{(Bq/m)} = \frac{2m(\pi - \alpha)}{Bq}$$

74. (*b*)



Deflecting torque due to current,

$$\tau_1 = (niA)B = \sin 90° = niL^2B$$

Restoring torque,

$$\tau_2 = Mg\frac{L}{2} \Rightarrow \tau_1 \geq \tau_2$$

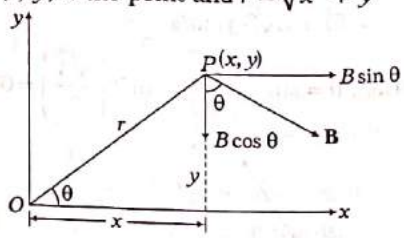
$$niL^2B \geq Mg\frac{L}{2} \Rightarrow B \geq \frac{Mg}{2niL}$$

$$B_{\min} = \frac{Mg}{2niL}$$

Note **M** is along + *Z*-axis and **B** is along + *X*-axis. The magnetic moment vector **M** tries to align along **B** through smaller angle to have minimum potential energy so coil rotates clockwise about line *BC*.

75. (*a*) In the $z = 0$ plane, the situation is as follows

Here, *P* (*x*, *y*) is the point and $r = \sqrt{x^2 + y^2}$



Magnetic field at *P* is perpendicular to *OP*, it is shown in figure.

So, $\quad \mathbf{B} = B\sin\theta\,\hat{i} - B\cos\theta\,\hat{j}$

Here, $\quad B = \frac{\mu_0}{2\pi}\frac{I}{r}$

$$\sin\theta = \frac{y}{r}, \cos\theta = \frac{x}{r}$$

$$\therefore \quad \mathbf{B} = \frac{\mu_0 I}{2\pi r}\left(\frac{y}{r}\hat{i} - \frac{x}{y}\hat{j}\right) = \frac{\mu_0}{2\pi}\frac{I(y\hat{i} - x\hat{j})}{(x^2 + y^2)}$$

B. Medical entrance special format questions

Assertion and reason

1. (*b*) To the right of *B*, both fields are downwards and to the left of *A* both fields are upwards.

2. (*d*) Velocity will change but speed will remain constant. Hence, momentum will change.

3. (*c*) Path will be uniform circular, only if field is uniform.

5. (*e*) As, $r = \frac{\sqrt{2Km}}{Bq}$ or $r \propto \frac{\sqrt{m}}{q}$

$$\left(\frac{\sqrt{m}}{q}\right) = \frac{\sqrt{4}}{2} = 1 \Rightarrow \left(\frac{\sqrt{m}}{q}\right)_d = \frac{\sqrt{2}}{1} = \sqrt{2}$$

$\therefore$ $r_d > r_\alpha$

Further, $\left(\frac{q}{m}\right)_\alpha = \frac{2}{4} = \frac{1}{2}$ and $\left(\frac{q}{m}\right)_d = \frac{1}{2}$

$\therefore$ $\left(\frac{q}{m}\right)_\alpha = \left(\frac{q}{m}\right)_d$

7. (*c*) Acceleration of charged particle in uniform field is non-zero but speed remains constant. In given condition, particle will rotate in a circular path in *xy*-plane. Therefore, if velocity is $v_0\hat{\mathbf{i}}$ at some instant, then it can have the value $v_0\hat{\mathbf{j}}$ at other instant also.

8. (*c*) Magnetic force can change the velocity but not the speed. Speeds at two instants must be same.

11. (*b*) $\mathbf{F}_m = 0$, if $\theta = 0°$ or $180°$

Further, if $\mathbf{F}_e + \mathbf{F}_m = 0$ or $q\mathbf{E} + q(\mathbf{v} \times \mathbf{B}) = 0$

Path is undeflected.

12. (*b*) Kinetic energy will remain constant, if

$$\mathbf{F}_e + \mathbf{F}_m = 0$$

13. (*b*) As, $q\mathbf{E} + q(\mathbf{v} \times \mathbf{B}) = 0$

$\therefore$ $\mathbf{E} = -(\mathbf{v} \times \mathbf{B}) = (\mathbf{B} \times \mathbf{v})$

14. (*d*) Wire-2 is equally attracted by wires-1 and 3.

15. (*c*) Magnetic force of attraction on lower wire is upwards while its weight is downwards. At a certain distance *x* these two forces are equal. If the lower wire is displaced upwards from this position, then magnetic force will increase but weight will remain same. Therefore, net force is upwards or equilibrium is unstable.

16. (*d*) It may or may not take a circular shape depending upon the direction of **B**.

18. (*a*) **M** and **B** are parallel (both inwards). Therefore, $\tau = 0$.

19. (*c*) If a positive charged particle is rotating in a circle in clockwise direction, then equivalent current is also clockwise. Therefore, **B** at centre and **M** are both inwards. Hence, they are parallel. But at some other point they may not be parallel.

Match the columns

2. As, $r = \frac{\sqrt{2Km}}{Bq} \Rightarrow r \propto \frac{\sqrt{m}}{q}$

3. As, $r = \frac{mv}{Bq} = \frac{p}{Bq} = \frac{\sqrt{2Km}}{Bq} \Rightarrow \omega = \frac{Bq}{m}$

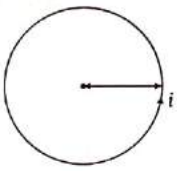
4. Magnetic field over the loop from the straight wire is perpendicular to paper inwards.

C. Medical entrances' gallery

1. (*b*) **Key idea** $B_{\text{centre}} = \frac{n \cdot \mu_0 i}{2R}$

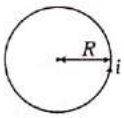
(for a circular coil)

where, *n* is number of turns is circular coil



$$B = \frac{\mu_0 i}{2R} = \frac{\mu_0 i(2\pi)}{2l} = \frac{\mu_0 \pi i}{l}$$

$$= \frac{\mu_0 n i}{2\left(\frac{l}{2n\pi}\right)} = \frac{n^2 \mu_0 \pi i}{l} = n^2 B$$



2. (*a*) As we know that, radius of a charged particle in a magnetic field *B* is given by

$$r = \frac{mv}{qB} \quad \text{...(i)}$$

where, r = charge on the particle

v = speed of the particle

$\therefore$ The time taken to complete the circle,

$$T = \frac{2\pi r}{v} \Rightarrow \frac{T}{2\pi} = \frac{m}{qB} \quad \text{(from Eq. (i))}$$

$$\therefore \quad \omega = \frac{2\pi}{T} = \frac{qB}{m}$$

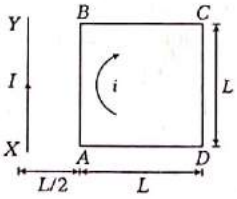
$\because$ $q = e$ and $\frac{e}{m} = 1.76 \times 10^{11}$ C/kg

$$B = 3.57 \times 10^{-2}\text{T} \Rightarrow \frac{2\pi}{T} = \frac{eB}{m}$$

$$f = \frac{1}{2\pi}\frac{e}{m}B = \frac{1}{2\pi} \times 1.76 \times 10^{11} \times 3.57 \times 10^{-2} \left(\because \frac{1}{T} = f\right)$$

$$= 1.0 \times 10^9 \text{ Hz} = 1 \text{ GHz}$$

3. (*d*) Consider the given figure.



From the above figure, it can be seen that the direction of current in a long straight conductor *XY* and arm *AB* of a square loop *ABCD* are in the same direction. So, there exist

a force of attraction between the two, which will be experienced by F_{BA} as

$$F_{BA} = \frac{\mu_0 IiL}{2\pi\left(\frac{L}{2}\right)}$$

In the case of XY and arm CD, the direction of currents are in the opposite direction. So, there exist a force of repulsion which will be experienced by CD as

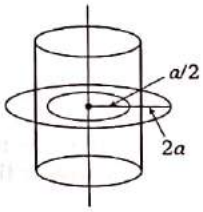
$$F_{CD} = \frac{\mu_0 IiL}{2\pi\left(\frac{3L}{2}\right)}$$

Therefore, net force on the loop $ABCD$ will be

$$F_{\text{loop}} = F_{BA} - F_{CD} = \frac{\mu_0 IiL}{2\pi}\left[\frac{1}{(L/2)} - \frac{1}{(3L/2)}\right]$$

$$F_{\text{loop}} = \frac{2\mu_0 iI}{3\pi}$$

4. (*b*) Consider two Amperian loops of radius $\frac{a}{2}$ and $2a$ as shown in the figure. Applying Ampere's circuital law for these loops, we get



$$\int \mathbf{B}\, d\mathbf{L} = \mu_0 I_{\text{enclosed}}$$

For the smaller loop,

$$\Rightarrow \quad \mathbf{B} \times 2\pi\frac{a}{2} = \mu_0 \times \frac{I}{\pi a^2} \times \pi\left(\frac{a}{2}\right)^2 = \mu_0 I \times \frac{1}{4} = \frac{\mu_0 I}{4}$$

$\Rightarrow$ $\mathbf{B} = \frac{\mu_0 I}{4\pi a}$, at distance $\frac{a}{2}$ from the axis of the wire.

Similarly, for bigger Amperian loop.

$$\mathbf{B}' \times 2\pi(2a) = \mu_0 I$$

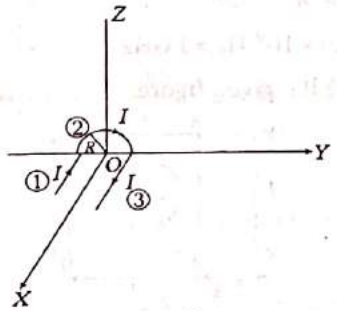
(total current enclosed by Amperian loop is 2)

$$\Rightarrow \quad \mathbf{B}' = \frac{\mu_0 I}{4\pi a}$$

At distance $2a$ from the axis of the wire.

So, ratio of $\frac{\mathbf{B}}{\mathbf{B}'} = \frac{\mu_0 I}{4\pi a} \times \frac{4\pi a}{\mu_0 I} = 1$

5. (*c*) The magnetic field in the different regions is given by



$$B_1 = \frac{\mu_0}{4\pi} \times \frac{I}{R}(-\hat{\mathbf{k}}),\ B_3 = \frac{\mu_0 I}{4\pi R}(-\hat{\mathbf{k}})$$

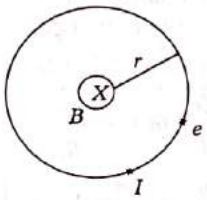
$$B_2 = \frac{\mu_0 I}{4R}(-\hat{\mathbf{i}})$$

The net magnetic field at the centre O is

$$B = B_1 + B_2 + B_3 = \frac{\mu_0 I}{4\pi R}(-2\hat{\mathbf{k}}) + \frac{\mu_0 I}{4R}(-\hat{\mathbf{i}}) = -\frac{\mu_0 I}{4\pi R}(2\hat{\mathbf{k}} + \pi\hat{\mathbf{i}})$$

6. (*d*) Current, $I = \frac{q}{t}$

So, for an electron revolving in a circular orbit of radius r.



$$q = e \text{ and } t = T$$

$$\Rightarrow \quad I = \frac{e}{T} = \frac{e}{2\pi/\omega} = \frac{\omega e}{2\pi} = \frac{2\pi ne}{2\pi} = ne$$

The magnetic field produced at the centre is

$$B = \frac{\mu_0 I}{2R} = \frac{\mu_0 ne}{2r}$$

7. (*b*) The angle subtended by circular part of the conductor is $3\pi/2$ or 270°.

The net magnetic field at point O is

$$B_{\text{net}} = B_1 + B_2$$

where, B_1 = magnetic field due to arc II and B_2 = magnetic field due to arc I

$$B_{\text{net}} = \frac{\mu_0 i}{4\pi \times 3R} \times \frac{\pi}{2} + \frac{\mu_0 i}{4\pi R} \times 3\frac{\pi}{2} = \frac{5\mu_0 i}{12R} \quad \text{(downward)}$$

8. (*c*) Given, speed of proton, $v = 3 \times 10^6$ m/s

Magnetic field, $B = 2 \times 10^{-3}$ T

Now, force (magnetic) on proton,

$$F = qvB\sin\theta$$
$$= 1.6 \times 10^{-19} \times 3 \times 10^6 \times 2 \times 10^{-3} \times \sin 90°$$
$$= 3.2 \times 3 \times 10^{-16} = 9.6 \times 10^{-6}\ \text{N}$$

$\therefore$ Acceleration of proton,

$$a = \frac{F}{m} = \frac{9.6 \times 10^{-16}\ \text{N/kg}}{1.67 \times 10^{-27}} = \frac{9.6}{1.67} \times 10^{11}\ \text{m/s}^2$$
$$= 5.74 \times 10^{11}\ \text{m/s}^2 = 5.8 \times 10^{11}\ \text{m/s}^2$$

9. (*c*) In the first condition, magnetic field,

$$B = \frac{\mu_0 i}{2r} \quad \text{...(i)}$$

In the second condition, magnetic field,

$$B' = \frac{\mu_0 ni}{2r/n} \Rightarrow B' = n^2\frac{\mu_0 i}{2r}$$

From Eq. (i), we get

$$B' = n^2 B \Rightarrow \frac{B'}{B} = \frac{n^2}{1}$$

10. (*a*) Two infinitely long equal wire carrying equal currents in same direction will attract each other.

11. (*d*) Points A and A' are the inflection points, where the sign of curvature changes.

At these points (*i.e.*, A and A')

$$\frac{dB}{dt} \text{ as well as } \frac{d^2B}{dt^2} \text{ are zero.}$$

12. (*b*) Let the masses of two particles are m_1 and m_2. As the changes are of the same magnitude and being accelerated through same potential, so these charges enter into the magnetic field with the same speeds (let v). Now, radii of the circular paths followed by two changes is given by

$$R_1 = \frac{m_1 v}{qB} \text{ and } R_2 = \frac{m_2 v}{qB} \Rightarrow \frac{R_1}{R_2} = \frac{m_1}{m_2}$$

13. (*c*) The magnetic field produced by the ring at its centre is

$$B = \frac{\mu_0 I}{2r} \quad \text{...(i)}$$

where, I = current through the ring

$$= \frac{Q}{T} = \frac{\frac{Q}{2\pi}}{\omega} = \frac{Q\omega}{2\pi} \quad \text{...(ii)}$$

Here, Q = total change on the ring

and T = time period of the ring

According to the question, $\lambda = \frac{Q}{2\pi r} \Rightarrow Q = \lambda \times 2\pi r$ ...(iii)

From Eqs. (i), (ii) and (iii), we get

$$B = \frac{\mu_0 \lambda \omega}{2}$$

14. (*c*) Since, $r = \frac{\sqrt{2mqV}}{qB} = \sqrt{\frac{2mV}{qB^2}} \Rightarrow \frac{r_1}{r_2} = \sqrt{\frac{m_1}{m_2}} \Rightarrow \frac{m_1}{m_2} = \frac{r_1^2}{r_2^2}$

Hence, $$\frac{m_1}{m_2} = \frac{(2)^2}{(3)^2} = \frac{4}{9}$$

15. (*a*) Given, $B = 10^{-4}$ Wb/m^2,

$$\frac{q}{m} = 10^{11} \text{C/kg} \quad \text{and} \quad v = 10^9 \text{ m/s}$$

We know that, radius of the circle,

$$r = \frac{mv}{qB} \Rightarrow r = \frac{v}{\left(\frac{q}{m}\right)B}$$

$$r = \frac{10^9}{(10^{11})(10^{-4})} \Rightarrow r = 100 \text{ m}$$

16. (*d*) As, electrons cannot be accelerated in a cyclotron. A large increase in their energy increase their velocity to a very large extent. This throws the electrons out of step with the oscillating field, while, neutron being electrically neutral, cannot be accelerated in a cyclotron.

So, cyclotron is used to accelerate only positively charged particle.

17. (*c*) Two parallel beams of position moving in the same direction set up two parallel currents flowing in the same direction. Hence, they attract each other.

18. (*a*) Given, $I_1 = 3$ A, $I_2 = 4$ A; $R = 2\pi$ cm $= 2\pi \times 10^{-2}$m

We know that, magnetic field of a coil, $B = \frac{\mu_0 I}{2R}$

Now, $B_1 = \frac{\mu_0 I_1}{2R} = \frac{\mu_0}{2} \times \frac{3}{2\pi \times 10^{-2}} = \frac{\mu_0}{4\pi} \times \frac{3}{10^{-2}}$

$$= 10^{-7} \times \frac{3}{10^{-2}} = 3 \times 10^{-5} \text{T}$$

Similarly, $B_2 = \frac{\mu_0 I_2}{2R} = \frac{\mu_0}{2} \times \frac{4}{2\pi \times 10^{-2}} = \frac{\mu_0}{4\pi} \times \frac{4}{10^{-2}}$

$$= 10^{-7} \times \frac{4}{10^2} = 4 \times 10^{-5} \text{T}$$

Now, net magnetic field at centre of a coil,

$$B = \sqrt{B_1^2 + B_2^2}$$

$$\Rightarrow \quad B = \sqrt{(3 \times 10^{-5})^2 + (4 \times 10^{-5})^2}$$

$$\Rightarrow \quad B = \sqrt{10^{-10}(9 + 16)}$$

$$\Rightarrow \quad B = 5 \times 10^{-5} \text{T} = 5 \times 10^{-5} \text{ Wb/m}^2$$

19. (*b*) Whenever, there is a change in the magnetic flux crossing through the loop, an emf induced in it and hence there is an amount of induced current through the loop. If loop is rotated about one of its diameter, the flux through it varies and causing the emf induced in it.

20. (*b*) The magnetic field inside the solenoid is given by

$$B = \mu_0 n I$$

where, n = number of turns per unit length

and I = current in coil

Now, we have

$$n = \frac{N}{L} \cdot r = \frac{400}{0.4 \times 10^{-2}} \times 1 \times 10^{-2} = 1000$$

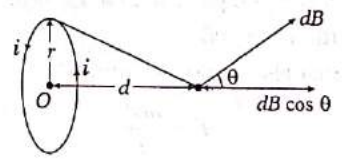
and $$\mu_0 = 4\pi \times 10^{-7} \text{ T-m/A}$$

$$\therefore \quad B = 4 \times 3.14 \times 1000 \times 5 \times 10^{-7}$$

$$= 62800 \times 10^{-7} = 6.28 \times 10^{-3} \text{ T}$$

21. (*b*) Magnetic field **B** at any point in the open space inside the toroid is zero, because the amperean loop enclose net current equals zero.

23. (*b*)



As magnetic field at axis of current carrying loop is given by

$$B = \frac{\mu_0 i r^2}{2(r^2 + d^2)^{3/2}} \quad \text{...(i)}$$

where, i = current in loop, r = radius of loop

d = distance on axis

and B = magnetic field of current carrying loop

Now, B_1 = magnetic field at centre, $d = 0$

Now, $$B_1 = \frac{\mu_0 i r^2}{2(r^2 + 0^2)^{3/2}} = \frac{\mu_0 i r^2}{2r^3} \quad ...(ii)$$

On dividing Eqs. (i) and (ii), we get

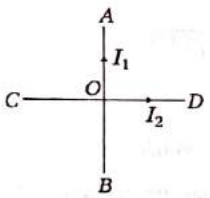
$$\frac{B}{B_1} = \frac{\mu_0 i r^2 \times 2 \times (r^2)^{3/2}}{2(r^2 + d^2)^{3/2} \times \mu_0 i r^2}$$

$$\Rightarrow \quad \frac{B}{B_1} = \frac{27}{(25)^{3/2}}$$

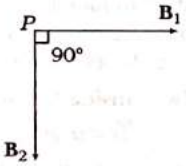
$$\Rightarrow \quad \frac{54\mu T}{B_1} = \frac{27}{5^3}$$

$$B_1 = \frac{54 \times 125}{27} \Rightarrow B_1 = 250\,\mu T$$

24. (*d*)



The point P is lying at a distance d along the Z-axis.



$$|\mathbf{B}_1| = \frac{\mu_0}{2\pi}\frac{I_1}{d} \text{ and } |\mathbf{B}_2| = \frac{\mu_0}{2\pi}\frac{I_2}{d}$$

$$B_{net} = \sqrt{B_1^2 + B_2^2}$$

$$B_{net} = \frac{\mu_0}{2\pi}\frac{1}{d}(I_1^2 + I_2^2)^{1/2}$$

25. (*b*) Given, kinetic energy = E

Mass = m

Magnetic field = B

and charge = q

We know that, $F = qvB \sin\theta$

(motion of a charged particle in a uniform magnetic field)

If $\theta = 90°$, then $F = qvB$ ...(i)

We know that also (centripetal force),

$$F = \frac{mv^2}{r} \quad ...(ii)$$

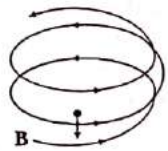
From Eqs. (i) and (ii), we get

$$qvB = \frac{mv^2}{r}, r = \frac{mv}{qB} \quad \left(\because E = \frac{1}{2}mv^2, v = \sqrt{\frac{2E}{m}}\right)$$

$$\therefore \quad r = \frac{m\sqrt{\frac{2E}{m}}}{qB}$$

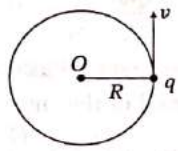
$$\because \quad r = \frac{\sqrt{2Em}}{qB}$$

26. (*d*) Suppose, the velocity v of the particle entering the field instead of being perpendicular to **B** makes an angle with it.



Then, v may be resolved into two components, $v_{\parallel} = v\cos\theta$ parallel to **B** and $v_{\perp} = v\sin\theta$ perpendicular to **B**. The component $v_{\parallel}$ gives a linear path and the component $v_{\perp}$ gives a circular path to the particle. The resultant of these two is a helical path whose axis is parallel to the magnetic field.

27. (*c*) As, number of revolutions per second by the charge q is given as

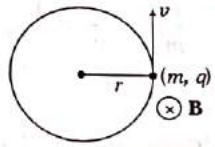


$$N_{\text{per sec}} = \frac{1}{T} = \frac{v}{2\pi R}$$

Current through circular path, $i = q(1/T) = vq/2\pi R$

Magnetic moment, $\mu = iA = \frac{vq}{2\pi R}(\pi R^2) \Rightarrow \mu = \frac{1}{2}vqR$

28. (*d*) Consider the diagram, where a charged particle of mass m and charge q moves along a circular path



anti-clockwise as shown, magnetic field B is assumed to be into plane of paper and of constant magnitude.

Clearly, centripetal force for the circular movement of the charge will be provided by magnetic Lorentz force $q(v \times B)$.

Hence, $$\frac{mv^2}{r} = qvB\sin 90°$$

$$\Rightarrow \quad \frac{mv^2}{r} = qvB$$

$$\Rightarrow \quad \text{speed} = v = \frac{qBr}{m} \quad ...(i)$$

$$\Rightarrow \quad r = \frac{mv}{qB}$$

Time period of the periodic motion = T = time taken to complete one revolution

$$\Rightarrow \quad T = \frac{2\pi}{\omega} = \frac{2\pi}{v/r} = \frac{2\pi r}{v} = \frac{2\pi}{v}\left(\frac{mv}{qB}\right) \quad \text{[from Eq. (i)]}$$

$$= \frac{2\pi m}{qB}$$

29. (*b*) $\because$ Magnetic moment,

$$M = \text{Current} \times \text{Area of enclosed by loop}$$
$$= I \times A \quad \text{...(i)}$$

and magnetic induction at the centre of circular loop,

$$B = \frac{\mu_0 I}{2R} \Rightarrow I = \frac{2BR}{\mu_0} \quad \text{...(ii)}$$

Here, $A = \pi R^2$ ...(iii)

Substituting Eqs. (ii) and (iii) in Eq. (i), we get

$$M = \frac{2BR}{\mu_0} \times \pi R^2 = \frac{2\pi BR^3}{\mu_0}$$

30. (*c*) In cyclotron, force on charge = centripetal force

$$\therefore \quad Bqv = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{Bq} \Rightarrow r \propto v$$

31. (*a*) When a magnetic field is applied on a stationary electron, then it remains stationary.

Because $F = q(\mathbf{v} \times \mathbf{B})$

If $v = 0 \Rightarrow F = 0$

32. (*d*) Given, $r = 0.05 \text{ nm} = 0.05 \times 10^{-9}\text{m}$, $(\because 1 \text{ nm} = 10^{-9}\text{m})$

$k = 10^{16}$ rev/s $\Rightarrow e = 1.6 \times 10^{-19}$ C

We know that, the magnetic moment, $M = Ai$

$$M = \pi r^2 \times ne \quad (\because A = \pi r^2)$$
$$= 3.14 \times (0.05 \times 10^{-9})^2 \times 10^{16} \times 1.6 \times 10^{-19}$$

or $= 1.26 \times 10^{-23}$ A-m^2

33. (*c*) Given, radius of coil, $r = 10 \text{ cm} = 10 \times 10^{-2}$ m

Number of turns, $N = 100$ turns, current in coil, $i = 1$ A

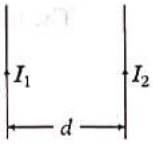
Magnetic moment, $M = ?$

By magnetic dipole moment of current loop,

$M = NiA$, where $A = \pi r^2$

$$M = Ni\pi r^2 = 100 \times 1 \times 3.142 \times (10 \times 10^{-2})^2$$
$$= 100 \times 3 \times 3.142 \times 100 \times 10^{-4} = 3.142 \text{ A-m}^2$$

34. (*c*) The situation is shown in the figure



Given wires are long.

Now, B_1 = magnetic field at I_2 (second wire) due to current in the first wire

$$= \left[\frac{\mu_0}{2\pi d} I_1\right]$$

The field is perpendicularly inward.

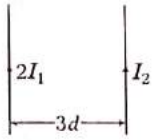
Now, force on the second wire due to first wire,

$$\mathbf{F}_{21} = I_2(\mathbf{l} \times \mathbf{B}_2) \quad \text{(towards left)}$$
$$= I_2 l B_1 \sin 90°$$
$$F_{21} = I_2 l B_1$$

where, l is the length of the wire and d is separation between the two wires $F_{21} = I_2 l\left(\frac{\mu_0 I_1}{2\pi d}\right) - \frac{\mu_0 I_1 I_2 l}{2\pi d}$

According to question, $|\mathbf{F}_{21}| = F \Rightarrow F = \frac{\mu_0 I_1 I_2 l}{2\pi d}$

Now, consider the situation, where the current in first wire is doubled and reversed.



Proceeding as previous case, we can write

$$F' = \frac{\mu_0 (2I_1)(I_2) l}{2\pi d \times 3}$$

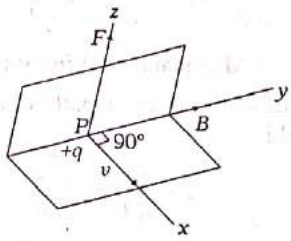
Here, force on second wire will be towards right.

So, $$\frac{F'}{F} = \left[\frac{\frac{\mu_0 (2I_1)(I_2) l}{2\pi d \times 3}}{\frac{\mu_0 I_1 I_2 l}{2\pi d}}\right]$$

Here, negative sign is due to opposite directions of F' and F.

$$\frac{F'}{F} = \left[\frac{2I_1 I_2}{3I_1 I_2}\right] = -\frac{2}{3} \Rightarrow F' = -\frac{2F}{3}$$

35. (*a*) By Fleming's left-hand rule, the particle is moving and magnetic field is perpendicular to the velocity.



36. (*d*) Angular momentum, $L = m_e vr$ ...(i)

Therefore, orbital motion of electron is equivalent to a current,

$$I = e(1/T)$$

Period of revolution of electron is given by, $T = \frac{2\pi r}{v}$

$$\therefore \quad I = e\left(\frac{1}{2\pi r / v}\right) = \frac{ev}{2\pi r}$$

Area of electron orbit, $A = \pi r^2$

Magnetic dipole moment of the atom

$$M = \frac{ev}{2\pi r} \times \pi r^2 = evr$$

Using Eq. (i), we have

$$M = \left(\frac{e}{2m_e}\right) L \quad \left[\frac{M}{L} = \frac{e}{2m}\right]$$

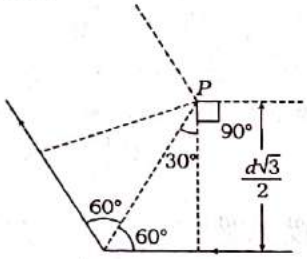
37. (*c*) Circumference of circle $= 2\pi R$

$\therefore \quad 2\pi R = L \Rightarrow R = \left(\frac{L}{2\pi}\right)$

Area, $A = \pi R^2 = \frac{\pi L^2}{4\pi^2} = \frac{L^2}{4\pi}$

Magnetic moment, $(\mu) = IA = \frac{IL^2}{4\pi}$

38. (*d*) We know that,



The magnetic field at a point P,

$$B_{net} = 2\left[\frac{\mu_0 I}{4\pi r}(\sin\theta_1 + \sin\theta_2)\right]$$

$$= 2\left[\frac{\mu_0}{4\pi} \times \frac{I}{\frac{d\sqrt{3}}{2}} \times (\sin 90° + \sin 30°)\right]$$

$$= 2\left[\frac{\mu_0}{4\pi} \times \frac{2I}{d\sqrt{3}} \times \left(1 + \frac{1}{2}\right)\right] = 2\left[\frac{\mu_0}{4\pi} \times \frac{2I}{d\sqrt{3}} \times \frac{3}{2}\right] = \frac{\sqrt{3}\mu_0 I}{2\pi d}$$

39. (*d*) For parallel **M** is stable and for antiparallel is unstable.

40. (*a*) The radius of the circular path, of a charges particle in magnetic field

$$r = \frac{mv}{qB} = \sqrt{\frac{2mE}{qB}}$$

Here, kinetic energy for proton and helium is same and both are moving in the same magnetic field

$\therefore \quad r \propto \frac{\sqrt{m}}{q}$

So, $\frac{r_P}{r_{He}} = \frac{\frac{\sqrt{m_P}}{q_P}}{\frac{\sqrt{m_{He}}}{q_{He}}} = \sqrt{\frac{m_P}{m_{He}}} \times \frac{q_{He}}{q_P} = \sqrt{\frac{m}{4m}} \cdot \frac{2q}{q} = \frac{1}{1}$

43. (c) We know that, magnetic field at centre of a coil

$$B = \frac{\mu_0 Ni}{2R}$$

Given, $i = 2$ A; $N = 1$

So, $R = \frac{\mu_0 Ni}{2B}$

$$R = \frac{4\pi \times 10^{-7} \times 1 \times 2}{2 \times 4\pi \times 10^{-6}} \Rightarrow R = \frac{1}{10} = 0.1 \text{ m}$$

44. (*b*) $\therefore$ Magnetic field, $B = \frac{\mu_0 I}{2\pi r}$

Given, $\mu = 4\pi \times 10^{-7}$ N/A^2, $r = 8$ cm $= 8 \times 10^{-2}$m, $I = 12$A

$$B = \frac{4\pi \times 10^{-7}}{2\pi} \times \frac{12}{8 \times 10^{-2}} \Rightarrow B = 3 \times 10^{-5} \text{ Wb/m}^2$$

45. (*a*) Given, $\mu_0 = 4\pi \times 10^{-7}$ N/A^2, $n = 5$ turns/cm, $I = 0.8$A

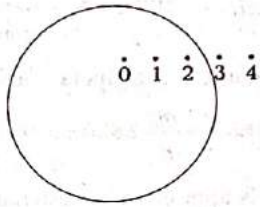
We know that, $B = \mu_0 nI$

$$B = 5.024 \times 10^{-8} \text{ Wb/m}^2$$

46. (*d*) Given that the net magnetic field at the centre O is zero. Therefore, magnetic field at O due to circular coil and straight conductor must be equal and opposite in direction.

$\therefore \quad \frac{\mu_0 I_1}{2R} = \frac{\mu_0 I_2}{2\pi(2R)} \Rightarrow \frac{I_1}{I_2} = \frac{1}{2\pi}$

47. (*d*)



Magnetic field outside the long rod,

$\because \quad B = \frac{\mu_0 i}{2\pi r}$

$\therefore \quad B_3 > B_4 \neq 0$

50. (*d*) The magnetic field at the centre of new loop

$$B' = n^2 B$$

where, n is the number of turns in the loop

$\therefore \quad B' = (4)^2 B$ or $B' = 16B$

51. (*c*) Under uniform magnetic field, force evB acts on proton and provides the necessary centripetal force $\frac{mv^2}{a}$.

$$\frac{mv^2}{a} = evB \Rightarrow v = \frac{aeB}{m}$$

Now, angular momentum,

$$J = r \times p = a \times aeB = a^2 eB$$

52. (*c*) Given, $B = 8.35 \times 10^{-2}\hat{\mathbf{i}}$T, $v = (2 \times 10^5\hat{\mathbf{i}} + 4 \times 10^5\hat{\mathbf{j}})$

The distance covered by proton, $p = T(v) = 2\pi \frac{m}{qB}(v)$

$$= 2 \times 3.14 \times \frac{1.67 \times 10^{-27}}{1.6 \times 10^{-19} \times 8.35 \times 10^{-2}\hat{\mathbf{i}}} \times (2 \times 10^5\hat{\mathbf{i}} + 4 \times 10^5\hat{\mathbf{j}})$$

$\Rightarrow \quad p = 0.157$m

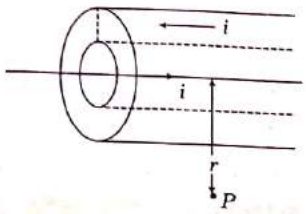
53. (*d*) Given, $n = 12$ turns, $I = 15$ A, $B = 0.2\,\hat{\mathbf{i}}$,

$A = -0.04\,\hat{\mathbf{i}}$ m^2

$\therefore$ Potential energy, $U = nIAB$

$= 12 \times 15 \times (-0.04) \times 0.2 = -1.44$ J

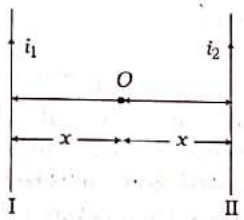
54. (*b*) The described condition can be shown as



The magnetic field at P due to inner and outer conductors are equal and opposite. Hence, the net magnetic field at P will be zero.

55. (*b*) When the current in the wires is in same direction. Magnetic field at mid-point O due to I and II wires are respectively

$$B_{\mathrm{I}} = \frac{\mu_0}{4\pi}\frac{2i_1}{x} \otimes \quad \text{and} \quad B_{\mathrm{II}} = \frac{\mu_0}{4\pi}\cdot\frac{2i_2}{x} \odot$$



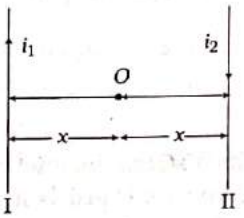
So, the net magnetic field at O

$$B_{\text{net}} = \frac{\mu_0}{4\pi}\cdot\frac{2}{x}(i_1 - i_2)$$

$$\Rightarrow \quad 10\times10^{-6} = \frac{\mu_0}{4\pi}\cdot\frac{2}{x}(i_1 - i_2) \quad \text{...(i)}$$

When the direction of i_2 is reversed,

$$B_{\mathrm{I}} = \frac{\mu_0}{4\pi}\cdot\frac{2i_1}{x} \otimes \quad \text{and} \quad B_{\mathrm{II}} = \frac{\mu_0}{4\pi}\cdot\frac{2i_2}{x} \otimes$$



So, net magnetic field at O

$$B_{\text{net}} = \frac{\mu_0}{4\pi}\cdot\frac{2}{x}(i_1 + i_2)$$

$$\Rightarrow \quad 40\times10^{-6} = \frac{\mu_0}{4\pi}\cdot\frac{2}{x}(i_1 + i_2) \quad \text{...(ii)}$$

On dividing Eq. (ii) by Eq. (i), we get

$$\frac{i_1 + i_2}{i_1 - i_2} = \frac{4}{1} \Rightarrow \frac{i_1}{i_2} = \frac{5}{3}$$

56. (*c*) The charged particle goes undeflected through both the field, therefore, force experience by charged particle due magnetic field must be equal to the force experienced by the charge particle due to electric field, *i.e.*, $F_m = F_e$

or $\quad evB\sin\theta = eE$

Given, $\quad v = 2\times10^2\ \text{ms}^{-1}$

$B = 1.5\ \text{T and } \theta = 90°$

Hence, $E = vB\sin\theta = 2\times10^3\times1.5\times\sin 90°$

$= 3\times10^3\ \text{V/m}$

57. (*a*) The magnetic field (B) at the centre of circular current carrying coil of radius R and current I, $B = \frac{\mu_0 I}{2R}$

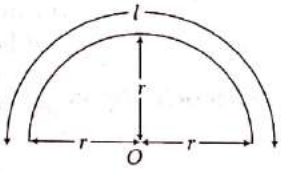
Similarly, if current $= 2I$, then

Magnetic field $= \frac{\mu_0 2I}{2R} = 2B$

So, resultant magnetic field

$$= \sqrt{B^2 + (2B)^2} = \sqrt{5B^2} = \sqrt{5}B = \frac{\mu_0 I\sqrt{5}}{2R}$$

59. (*a*) When wire is bent in the form of semicircular arc then, $l = \pi r$



$\therefore$ The radius of semicircular arc, $r = l/\pi$

Distance between two end points of semicircular wire,

$$= 2r = \frac{2l}{\pi}$$

$\therefore$ Magnetic moment of semicircular wire

$$= m\times 2r = m\times\frac{2l}{\pi} = \frac{2}{\pi}ml$$

But ml is the magnetic moment of straight wire *i.e.*,

$$ml = M$$

$\therefore$ New magnetic moment $= \frac{2}{\pi}M$

CHAPTER

05 Magnetism and matter

A Greek philosopher, Thales of Miletus had observed in 600 BC that a naturally occuring ore of iron attracted small pieces of iron towards it. This ore was found in the district of Magnesia in Asia Minor in Greece. Hence, the ore was named magnetite. Later, William Gilbert first suggested that earth behaves as a huge magnet which causes the alignment of compass needles, then oersted discovered that moving charges are the sources of magnetic fields. So, the science of magnetism was known long before the 19th century.

The phenomenon of attraction of small bits of iron, steel, cobalt, nickel, etc., towards the ore was called magnetism. Matters like (solid, liquid or gases) show magnetism and hence are classified into different categories according to their magnetic properties. In this chapter, we shall learn about magnetism and magnetisation of matter followed by knowledge of magnet, magnetic dipole and their magnetic fields. We will also discuss about the earth's magnetism.

Inside

1) Magnet
- Magnetic field lines
- Magnetic dipole
- Coulomb's law for magnetism
- Magnetic field strength at a point due to magnetic dipole or bar magnet
- Current carrying loop as a magnetic dipole
- Bar magnet in a uniform magnetic field

2) Earth's magnetism
- Elements of earth's magnetism
- Neutral points
- Tangent galvanometer
- Vibration magnetometer

3) Magnetic induction and magnetic materials
- Some important terms used in magnetism
- Classification of substances on the basis of magnetic behaviour
- Curie's law
- Atomic model of magnetism
- Hysteresis

1 Magnet

A magnet is a material or object that exhibits a strong magnetic field and has a property to attract some specific materials like iron towards it. The magnetic field is invisible but is responsible for properties of a magnet.

Magnets are of two types

(i) Natural magnets

(ii) Artificial magnets

Natural magnets are generally irregular in shape and weaker in strength. On the other hand, artificial magnets may have desired shape and strength. A bar magnet, a horse shoe magnet, compass needle, etc., all are examples of artificial magnet.

Bar magnet

A bar magnet consists of two equal and non-separable magnetic poles. One pole is designated as north pole (N) and the other as south pole (S). These poles are separated at a small distance. But they are not exactly at the ends. The distance between two

poles of a bar magnet is known as **magnetic length of a magnet**. It's direction is from S-pole of the magnet to N-pole and is represented by $2l$. It is sometimes also known as effective length (L_e) of the magnet and is less than its geometric length (L_g).

For a bar magnet, $L_e = \left(\frac{5}{6}\right) L_g$.

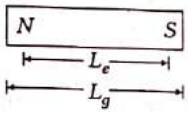


Fig. 5.1 Bar magnet

General properties of magnet

Directional property

When a magnet is suspended freely, then it points in the earth's N-S direction (in magnetic meridian).

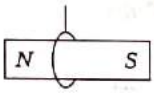


Fig. 5.2 Freely suspended bar magnet

Monopole Non-existance

If a magnet is broken into number of pieces then each piece behaves as an individual magnet rather than isolated poles. This means that monopoles do not exist.

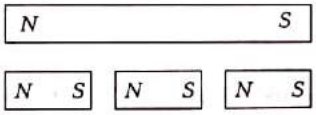


Fig. 5.3 If a bar magnet is broken, each piece behaves as a small magnet.

Attractive/repulsive properties

Like magnetic poles repel each other and unlike magnetic poles attract each other.

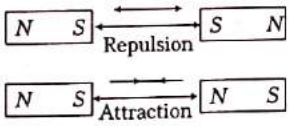


Fig. 5.4 Like poles repel and unlike poles attract

Note Repulsion is a sure test of magnetism.

Magnetic field lines

The magnetic field lines of a magnetic field are the imaginary lines which continuously represent the direction of that magnetic field.

Properties of magnetic field lines

(i) The magnetic field lines of a magnet form closed continuous loops. This property is unlike electric field lines, which begin from a positive charge and terminate at the negative charge or escape to infinity.

(ii) The tangent, at any point of a magnetic field line, represents, the directionof net magnetic field 'B' at the point.

(iii) At any point tangent to the magnetic field line represents the direction of net magnetic field (**B**) at that point.

(iv) The tangent at any point of a magnetic field line respect the directions of net magnetic field **B** at that point.

(v) The larger the number of field lines crossing per unit area, the stronger is the magnitude of the magnetic field B.

(vi) The magnetic field lines do not intersect, for if they did, the direction of the magnetic field would not be unique at the point of intersection.

(vii) The direction of field lines in from N to S if they are outside the magnet and from S to N, if they are inside the magnet.

(viii) Fig. 5.5 (a) shows the uniform magnetic field lines and Fig. 5.5 (b) shows non-uniform magnetic field lines.

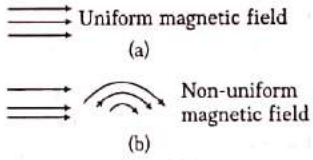


Fig. 5.5

Different pattern of magnetic field lines

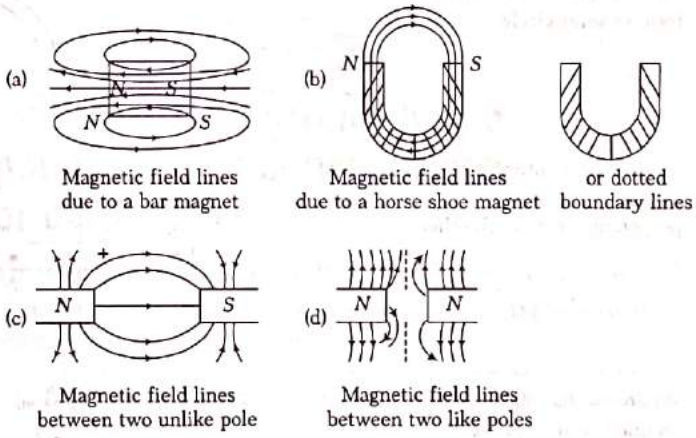


Fig. 5.6 Different patterns of magnetic field lines

Magnetic Dipole

A magnetic dipole is an arrangement which consists two magnetic poles of equal and opposite strengths separated at a small distance. A bar magnet, a compass needle, etc., are the examples of magnetic dipoles.

Pole strength

Like electric charge, we have equivalent analogous in magnetism which is known as pole strength. Thus, **pole strength can be defined as the strength of a magnetic pole to attract magnetic materials towards itself.** It is a scalar quantity and its SI unit is ampere-metre (A-m). The strength of N and S-pole of a magnet is conventionally represented by $+m$ and $-m$, respectively. It depends on the nature of material and area of cross-section of the magnet.

Strength of N and S-pole of a magnet is always equal and opposite ($+m$ and $-m$).

Magnetic dipole moment

It is the product of the strength of either of the pole strength and the magnetic length of the magnet. It is represented by **M**. It is a vector quantity.

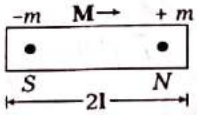
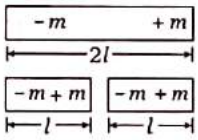
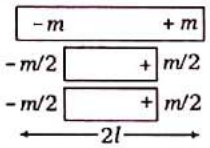
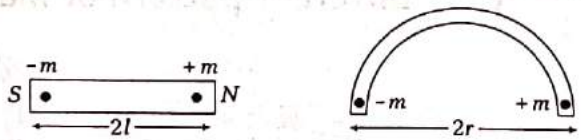
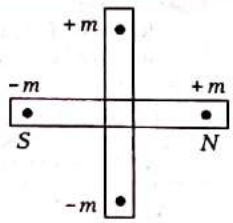
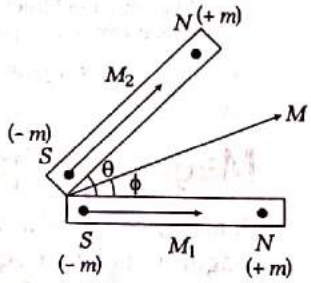


Fig. 5.7

The direction of magnetic dipole moment is same as that of 2l. Therefore, $\mathbf{M} = m(2\mathbf{l})$

Its SI unit is ampere-metre^2 (A-m^2).

Pole strength and magnetic dipole moment in special cases

Special Cases	Figure	Effect on Pole strength	Formula for new magnetic dipole moment
If bar magnet is cut into two equal pieces such that the length of each piece becomes half		Remains unchanged	$M' = m \cdot \frac{2l}{2} = \frac{M}{2}$ (becomes half)
If bar magnet is cut into two equal pieces such that the width of each piece becomes half		Pole strength of each piece becomes half	$M' = \left(\frac{m}{2}\right)(2l) = \frac{M}{2}$ (becomes half)
If bar magnet is bent in the form of semi-circle		Remains unchanged	$M' = m(2r)[\because 2l = \pi r]$ $M' = m \times 2\left(\frac{2l}{\pi}\right) = \frac{2M}{\pi}$ $\left(\text{becomes } \frac{2}{\pi} \text{ times}\right)$
When two identical bar magnets are joined perpendicular to each other		Remains unchanged	$M = \sqrt{M_1^2 + M_2^2} = \sqrt{2}M$
When two bar magnets are inclined at an angle θ.		Remains unchanged	Resultant magnetic moment, $M' = \sqrt{M_1^2 + M_2^2 + 2M_1M_2\cos\theta}$ Angle made by resultant magnetic moment (M) with M_1 is given by, $\tan\phi = \frac{M_2\sin\theta}{M_1 + M_2\cos\theta}$

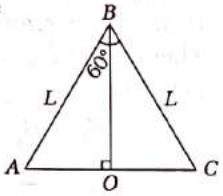
Example 5.1 *Consider a short magnetic dipole of magnetic length 10 cm. Find its geometric length.*

Sol. Geometric length of a magnet is $\frac{6}{5}$ times its magnetic length.

$\therefore$ Geometric length $= \frac{6}{5} \times 10 = 12$ cm

Example 5.2 *A thin bar magnet of length 2L is bent at the mid-point, so that the angle between them is 60°. Find the new length of the magnet.*

Sol. On bending the magnet, the length of the magnet,



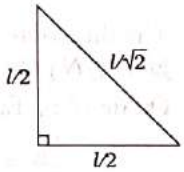
$$AC = AO + OC = L\sin\left(\frac{\theta}{2}\right) + L\sin\left(\frac{\theta}{2}\right)$$

$$= 2L\sin 30° = 2L \times \frac{1}{2} = L$$

Example 5.3 *The length of a magnetised steel wire is l and its magnetic moment is M. It is bent into the shape of L with two sides equal. What will be the new magnetic moment?*

Sol. If m is strength of each pole, then magnetic moment $M = m \times l$

When the wire is bent into L shape, effective distance between the poles



$$= \sqrt{\left(\frac{l}{2}\right)^2 + \left(\frac{l}{2}\right)^2} = \frac{l}{\sqrt{2}}$$

$\therefore$ New magnetic moment

$$M' = m \times \frac{l}{\sqrt{2}} = \frac{M}{\sqrt{2}} \quad (m \text{ will remain unchanged})$$

Coulomb's law for magnetism

This law states that the force of attraction or repulsion between two magnetic poles is directly proportional to the product of their pole strength (m) and inversely proportional to the square of the distance between them.

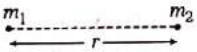


Fig. 5.8 Two magnetic poles separated by a distance r

Let pole strength of either pole be m, then magnetic force between these two isolated poles kept at separation r will be

$$F \propto \frac{m_1 \times m_2}{r^2} \Rightarrow \boxed{F = \frac{\mu_0}{4\pi}\frac{m_1 \times m_2}{r^2}}$$

where, $\frac{\mu_0}{4\pi} = 10^{-7}$ N/A^2

Important points about the Coulomb's law for magnetism are as follows

(i) Force will be attractive, if one pole is north and other south, *i.e.* opposite poles.

(ii) Force will be repulsive, if both poles are of same type (*i.e.*, north-north or south-south).

Example 5.4 *Two magnetic poles, one of which is four times stronger than the other, exert a force of 10 gf on each other when placed at a distance of 20 cm. Find the strength of each pole.*

Sol. Let the pole strength of the two dipoles be m and $4m$

Here, $F = 10\text{gf} = 10 \times 10^{-3}\text{ kg f} = 10 \times 10^{-3} \times 9.8\text{N}$,

and $r = 20\text{ cm} = 0.2\text{m}$

Using Coulomb's law of magnetism, $F = \frac{\mu_0}{4\pi} \cdot \frac{m_1 m_2}{r^2}$

Substituting the values, $10 \times 10^{-3} \times 9.8 = -\frac{10^{-7} \times m \times 4m}{(0.2)^2}$

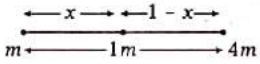
or $m^2 = \frac{10 \times 9.8 \times (0.2)^2 \times 10^4}{4} = 9800$

or $m_1 = m = 98.9$ Am

and $m_2 = 4m = 4 \times 98.9 = 396$ Am

Example 5.5 *Two similar magnetic poles, having pole strengths in the ratio 1 : 3 and placed 1 m apart. Find the point where a unit pole experiences no net force due to these two poles.*

Sol. Let the pole strengths of the two magnetic poles be m and $3m$. Suppose the required point is located at distance x from the first pole. Then at this point,



Force on unit pole due to first pole

= Force on unit pole due to second pole

or $\frac{\mu_0}{4\pi} \cdot \frac{m \times 1}{x^2} = \frac{\mu_0}{4\pi} \cdot \frac{3m \times 1}{(1-x)^2}$

or $3x^2 = (1-x)^2$ or $\sqrt{3}x = 1 - x$

or $x = \frac{1}{1+\sqrt{3}} = 0.366$ m

Magnetic field strength at a point due to magnetic dipole or bar magnet

The strength of a magnetic field at any point is defined by of unit strength the force experienced by a hypothetical north pole placed at that point

i.e., $\mathbf{B} = \frac{\mathbf{F}}{m}$, where m is the pole strength of hypothetical north pole. Magnetic field strength is a vector quantity.

The direction of magnetic field **B** is the direction along which hypothetical north pole would tend to move, if free to do so. We have used the word hypothetical north pole in the above discussion because an isolated magnetic pole does not exist.

Let us now calculate the magnetic field strength at different points of magnetic dipole (or bar magnet).

1. When point lies on axial line of a bar magnet

Let $2l$ be the magnetic length of a bar magnet with its centre at O. The magnetic dipole moment of the magnet is M, where $M = m \times 2l$. The distance of the observation point P on the axial line from the centre of the magnet be $OP = r$.

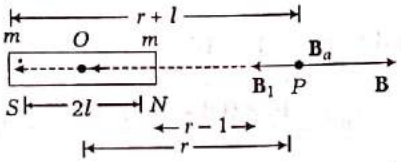


Fig. 5.9 Magnetic field of bar magnet on axial line

If m is the strength of each pole, then magnetic field strength at P due to N-pole of magnet is given by

$$\mathbf{B}_1 = \frac{\mu_0}{4\pi} \times \frac{m \times 1}{(NP)^2} = \frac{\mu_0}{4\pi} \frac{m}{(r-l)^2}, \text{ along } OP.$$

Magnetic field strength at P due to S-pole of magnet,

$$\mathbf{B}_2 = \frac{\mu_0}{4\pi} \times \frac{m \times 1}{(SP)^2} = \frac{\mu_0}{4\pi} \frac{m}{(r+l)^2}, \text{ along } PO$$

$\therefore$ Magnetic field strength at P due to the bar magnet,

$$B_a = B_1 - B_2 = \frac{\mu_0}{4\pi} \frac{2Mr}{(r^2 - l^2)^2}$$

(superposition – sign due to opposite direction)

When the magnet is short, $l^2 << r^2$, such that l^2 is neglected, then,

$$B_a = \frac{\mu_0}{4\pi} \frac{2Mr}{r^4} \Rightarrow \boxed{B_a = \frac{\mu_0}{4\pi} \frac{2M}{r^3}} \quad \text{...(i)}$$

The direction of $\mathbf{B}_a$ is along OP.

2. When point lies on equatorial line of a bar magnet

In figure given below, the point P is shown on equatorial line of the same bar magnet, where $OP = r$.

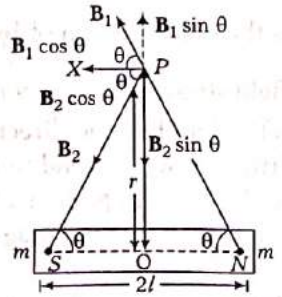


Fig. 5.10 Magnetic field of bar magnet on equitorial line

Magnetic field strength at P due to N-pole of magnet is,

$$\mathbf{B}_1 = \frac{\mu_0}{4\pi} \frac{m \times 1}{(NP)^2} = \frac{\mu_0}{4\pi} \frac{m}{(r^2 + l^2)}, \text{ from } N \text{ pole to point } P.$$

Magnetic field strength at P due to S-pole of magnet,

$$\mathbf{B}_2 = \frac{\mu_0}{4\pi} \frac{m \times 1}{(SP)^2} = \frac{\mu_0}{4\pi} \frac{m}{(r^2 + l^2)}, \text{ from point } P \text{ to pole } S$$

As, $B_1 = B_2$ in magnitude, their vertical components $B_1 \sin\theta$ along OP produced and $B_2 \sin\theta$ along PO will cancel out. However, horizontal components along PX will add. Therefore, magnetic field strength at P due to the bar magnet,

$$B_e = B_1 \cos\theta + B_2 \cos\theta = 2 B_2 \cos\theta, \text{ along } PX.$$

$$\therefore \quad B_e = \frac{\mu_0}{4\pi} \frac{m \times 2l}{(r^2 + l^2)^{3/2}} \Rightarrow B_e = \frac{\mu_0}{4\pi} \frac{M}{(r^2 + l^2)^{3/2}}$$

If the magnet is short, $l^2 << r^2$, such that l^2 is neglected.

$$\therefore \quad B_e = \frac{\mu_0}{4\pi} \frac{M}{(r^2)^{3/2}} \Rightarrow \boxed{B_e = \frac{\mu_0}{4\pi} \frac{M}{r^3}} \quad \text{...(ii)}$$

The direction of B_2 is along PX, a line parallel to line joining N pole to S pole.

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{B_a}{B_e} = 2 \Rightarrow B_a = 2B_e$$

Hence, **magnitude of magnetic field due to a short bar magnet at any point on the axial line of magnet is twice the magnetic field at a point at the same distance on the equatorial line of the magnet.**

3. When point makes angle θ with axis of a bar magnet

At an angle θ with the axis of magnet, the magnetic field at point P at a distance r from centre of magnet is

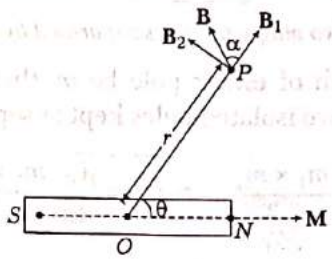


Fig. 5.11

$$\boxed{B = \frac{\mu_0}{4\pi} \cdot \frac{M}{r^3} \sqrt{1 + 3\cos^2\theta}}$$

and $\quad \tan\alpha = \frac{1}{2} \tan\theta$

For axial position of point P, $\theta = 0°$ and for equatorial position, $\theta = 90°$.

Note The magnetic potential due to a magnetic diople at distance r is

$$V = \frac{\mu_0}{4\pi} \cdot \frac{M\cos\theta}{r^2}$$

Now, (a) on the axis of magnet $\theta = 0°$

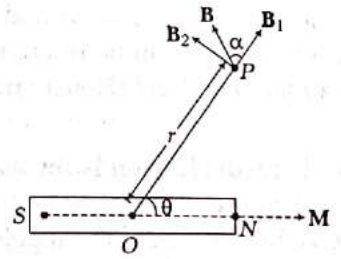
$$\therefore \quad V = \frac{\mu_0}{4\pi}\frac{M}{r^2}$$

(b) On the neutral axis, $\theta = 90°$,

$$\therefore \quad V = 0$$

Example 5.6 *Find the magnetic field due to a dipole of magnetic moment 3 Am^2 at a point 5 m away from it in the direction making angle of 45° with the dipole axis.*

Sol. The condition given in the figure can be drawn as



So, the magnetic field at point P is,

$$B = \frac{\mu_0}{4\pi}\frac{M}{r^3}\sqrt{1 + 3\cos^2\theta}$$

Here, $M = 3\ Am^2$, $r = 5$ m, $\theta = 45°$

$$\therefore \quad B = 10^{-7} \times \frac{3}{(5)^3}\sqrt{1 + 3\cos^2(45°)}$$

$$= 10^{-7} \times \frac{3}{125} \times \sqrt{1 + 1.5}$$

$$= 10^{-7} \times \frac{3}{125} \times 1.58 = 3.79 \times 10^{-9}\ \text{T}$$

Also, $\tan\alpha = \frac{1}{2}\tan\theta = \frac{1}{2}\tan 45° = \frac{1}{2} \Rightarrow \alpha = \tan^{-1}(0.5)$

Example 5.7 *A bar magnet of length 0.1 m has pole strength of 50 A-m. Calculate the magnetic field at distance of 0.2 m from its centre on*

(i) its axial line and (ii) its equatorial line.

Sol. Here, $m = 50$ A-m, $r = 0.2$ m, $2l = 0.1$ m or $l = 0.05$ m

$\therefore$ Magnetic dipole moment,

$$M = m(2l) = 50 \times 0.1 = 5\ \text{A-m}^2$$

$$\text{(i) } B_{axial} = \frac{\mu_0}{4\pi}\frac{2Mr}{(r^2 - l^2)^2} = \frac{10^{-7} \times 2 \times 5 \times 0.2}{(0.2^2 - 0.05^2)^2} = 1.42 \times 10^{-4}\text{T}$$

$$\text{(ii) } B_{equi} = \frac{\mu_0}{4\pi}\frac{M}{(r^2 + l^2)^{3/2}} = \frac{10^{-7} \times 5}{(0.2^2 + 0.05^2)^{3/2}}$$

$$= 5.71 \times 10^{-5}\text{T}$$

Formula with $(r^2 - l^2)$ and $(\sigma^2\rho l^2)$ are used because here $r >> l$ does not apply.

Example 5.8 *Calculate the magnetic induction at a point 1 Å away from a proton, measured along its axis of spin. The magnetic moment of the proton is 1.4×10^{-26} A-m^2.*

Sol. On the axis of a magnetic dipole, magnetic induction is given by

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2M}{r^3}$$

Substituting the values, we get

$$B = \frac{(10^{-7})(2)(1.4 \times 10^{-26})}{(10^{-10})^3} = 2.8 \times 10^{-3}\ \text{T} = 2.8\ \text{mT}$$

Example 5.9 *A short bar magnet has a magnetic moment of $0.48 JT^{-1}$. Give the direction and magnitude of the magnetic field produced by the magnet at a distance of 10 cm from the centre of magnet on (i) the axis (ii) the equatorial lines (normal bisector) of the magnet.*

Sol. (i) When the point lies on the axis, then let B_1 be the magnetic field at P.

$$r = 10\ \text{cm} = 0.1\ \text{m}$$

$$\therefore \quad B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2M}{r^3} = 10^{-7} \times \frac{2 \times 0.48}{(0.1)^3}$$

$$= 0.96 \times 10^{-4}\text{T from } S \text{ pole to } N \text{ pole}$$

(ii) Let B_2 be the magnetic field at point P on the equatorial line.

$$\therefore \quad B_2 = \frac{\mu_0}{4\pi} \cdot \frac{M}{r^3} = 10^{-7} \times \frac{0.48}{(0.1)^3}$$

$$= 0.48 \times 10^{-4}\text{T} = 0.48\ \text{G along from } N \text{ pole to } S \text{ pole.}$$

Magnetic field due to a hypothetical monopole

At any point, magnetic field due to hypothetical monopole is given by

$$B = \frac{F_m}{m_0}$$

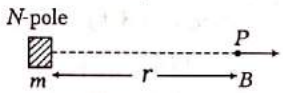


Fig. 5.12 *Magnetic field due to a monopole*

So, at a point P situated at distance r from a monopole, magnetic field is given by

$$\boxed{B = \frac{\mu_0}{4\pi}\frac{m}{r^2}}$$

Important points about magnetic field due to monopole

(i) It is away from pole, if it is N-pole as shown in Fig. 5.13

(ii) It is towards pole, if it is S-pole, as shown in Fig. 5.13

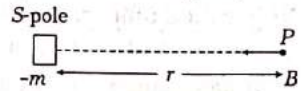


Fig. 5.13 *Magnetic field due to S-pole*

Current carrying loop as a magnetic dipole

Consider a planar loop of wire carrying current as shown in Fig. 5.14. Looking from the upper face, current is anti-clockwise. Therefore, it has a north polarity (N).

Looking from the lower face of the loop current is clockwise (S). Therefore, it has a south polarity. Thus, the current carrying loop behaves as a system of two equal and opposite magnetic poles and can be considered as a magnetic dipole.

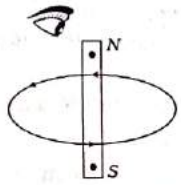


Fig. 5.14 Current loop with north and south polarity

Magnetic dipole moment of current carrying loop

The magnetic dipole moment of the current carrying loop (M) is directly proportional to

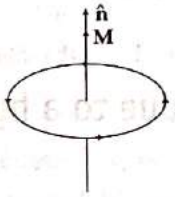


Fig. 5.15 Magnetic dipole moment of current loop

(i) strength of current (I) through the loop and

(ii) area (A) enclosed by the loop

i.e., $M \propto I$ and $M \propto A$

$\therefore$ $M = kIA$ ... (i)

where, k is proportionality constant

If we define unit magnetic dipole moment as that of a small single turn loop of unit area carrying unit current, then from Eq. (i), we get

$$1 = k \times 1 \times 1 \quad \text{or} \quad k = 1$$

$\therefore$ From Eq. (i), $M = IA$

For N turns $\boxed{M = NIA}$

In vector form, $\boxed{\mathbf{M} = NIA\hat{\mathbf{n}}}$

where, $\hat{\mathbf{n}}$ is unit vector perpendicular to the plane of the loop.

Here, the factor NI is called ampere turns of current loop. So, magnetic dipole moment of current loop

= Ampere turns × loop area

Note Direction of magnetic moment is given by right hand rule.

Example 5.10 *A current of 6A is flowing through a 20 turns circular coil of radius 5 cm. The coil lies in the xy-plane. What is the magnitude and direction of the magnetic dipole moment associated with it?*

Sol. Magnetic dipole moment is given by

$$M = NIA$$

Here, $N = 20, I = 6\text{A}, r = 5 \text{ cm} = 0.05 \text{ cm}$

$\therefore$ $M = (20) \times (6) \times \frac{22}{7} \times (0.05)^2 = 0.94 \text{ Am}^2$

The direction of magnetic dipole moment is perpendicular to the plane of the coil. Hence, it is along Z-axis.

Magnetic dipole moment of a revolving electron in an atom

The circular motion of an electron around the positively charged nucleus of an atom can be treated as a current loop producing a magnetic field. Hence, it behaves like a magnetic dipole.

The (negatively charged) electron is revolving anti-clockwise and so the current is clockwise. Hence, according to the right-hand rule, the dipole moment **M** is perpendicular to the plane of the current loop and is directed downwards.

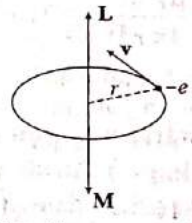


Fig. 5.16 Orbital magnetic moment of a revolving electron.

Let m_e be the mass and $-e$ the charge of an electron revolving with speed v in a circular orbit of radius r.

The magnitude of the magnetic dipole moment **M** associated with the revolving electron is

$$\mathbf{M} = IA = \frac{ev}{2\pi r} \times \pi r^2 = \frac{evr}{2} \quad \left(\because I = \frac{e}{T} = \frac{ev}{2\pi r}\right) \quad \text{... (i)}$$

The magnitude of the orbital angular momentum **L** of electron,

$$L = m_e vr \quad \text{... (ii)}$$

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{M}{L} = \frac{e}{2m_e} \quad \text{... (iii)}$$

Thus, **the ratio of the magnitude of the magnetic dipole moment to the magnitude of the angular momentum of the revolving electron is a constant. This ratio is called the gyromagnetic ratio.** Its value for an electron is 8.8×10^{10} C/kg.

The vector form of Eq. (iii) can be written as

$$\boxed{\mathbf{M} = -\left(\frac{e}{2m_e}\right)\mathbf{L}}$$

The minus sign means that **M** is directed opposite to **L**.

Now, according to Bohr's quantisation principle, the angular momentum of a revolving electron assumes discrete values only. That means, it is an integral multiple of $\frac{h}{2\pi}$, where h is a fundamental constant of quantum mechanics named after Max Planck, known as Planck's constant having value 6.626×10^{-34} J-s. Therefore, angular momentum can be written as

$$L = m_e vr = \frac{nh}{2\pi} \quad \text{...(iv)}$$

where, n is an integer of values $n > 0$ and is also known as principle quantum number.

Now, from Eqs. (iii) and (iv), we get,

$$\frac{M}{nh/2\pi} = \frac{e}{2m_e} \Rightarrow M = \frac{neh}{4\pi m_e} \quad \text{...(v)}$$

When $n = 1, M = \mu$ (the elementary magnetic dipole moment), thus

$$\therefore \quad \mu = \frac{eh}{4\pi m_e} \quad \text{...(vi)}$$

The elementary magnetic moment of a revolving electron is also known as Bohr magneton (μ).

Now, substituting $e = 1.6 \times 10^{-19}$ C, $h = 6.626 \times 10^{-34}$ J-s, $\pi = 3.14$ and $m_e = 9.01 \times 10^{-31}$ kg in Eq. (vi), we get,

$$\mu = \frac{1.6 \times 10^{-19} \times 6.626 \times 10^{-34}}{4 \times 3.14 \times 9.01 \times 10^{-31}} \text{A-m}^2$$

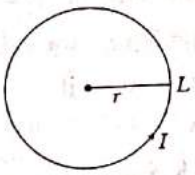
$$= 9.27 \times 10^{-27} \text{ A-m}^2$$

$\therefore$ 1 Bohr magneton $= 9.27 \times 10^{-27}$ A-m^2

Example 5.11 *A current I flows in a conducting wire of length L. If we bent it in a circular form, then calculate its magnetic dipole moment.*

Sol. Let a wire of length L is bent in a circular form of radius r.

Then, $\quad 2\pi r = L \Rightarrow r = \frac{L}{2\pi} \quad$ ...(i)



The magnetic dipole moment of a circular ring,

$M = IA$ (where, A is area of the ring)

or $\quad M = I\pi r^2 \quad$ ...(ii)

On putting the value of r from Eq. (i) in Eq. (ii), we get

$$M = I\pi\left(\frac{L}{2\pi}\right)^2$$

$$\Rightarrow \quad M = I\pi \times \frac{L^2}{4\pi^2}$$

$$\Rightarrow \quad M = \frac{IL^2}{4\pi} \text{ A-m}^2$$

Example 5.12 *The electron in hydrogen atom moves with a speed of 2.2×10^6 m/s in an orbit of radius 5.3×10^{-11} cm. Find the magnetic moment of the orbiting electron.*

Sol. Frequency of revolution, $f = \frac{v}{2\pi r}$

The moving charge is equivalent to a current loop, given by

$$I = f \times e \quad \text{or} \quad I = \frac{ev}{2\pi r}$$

If A be the area of the orbit, then the magnetic moment of the orbiting electron is,

$$M = IA = \left(\frac{ev}{2\pi r}\right)(\pi r^2) = \frac{evr}{2}$$

Putting the values, we get

$$M = \frac{(1.6 \times 10^{-19})(2.2 \times 10^6)(5.3 \times 10^{-11} \times 10^{-2})}{2}$$

$$= 9.3 \times 10^{-26} \text{A-m}^2$$

Bar magnet as an equivalent solenoid

The magnetic field lines for a bar magnet and a current carrying solenoid resemble very closely. Therefore, a bar magnet can be thought as a large number of circulating currents in analogy with a solenoid. The expression for the magnetic field due to a solenoid is,

$$B = \frac{\mu_0}{4\pi}\frac{2m}{r^3}$$

where r is the distance between the centre of solenoid to the required point.

This expression is same as the expression for magnetic field of a bar magnet at its axial point. Thus, a bar magnet and a solenoid produce similar magnetic fields.

Therefore magnetic field lines associated with a solenoid are also similar to a bar magnet as shown in the figure.

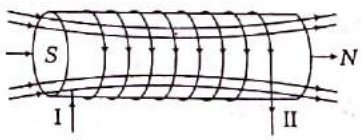


Fig. 5.17 *Magnetic field lines due to a solenoid carrying current*

Example 5.13 *A closely wound solenoid of 800 turns and area of cross-section $2.5 \times 10^4 m^2$ carries a current of 3.0 A. Explain the sense in which the solenoid acts like a bar magnet. What is its associated magnetic moment?*

Sol. It is given that number of turns, $N = 800$ and

Area of cross-section, $A = 2.5 \times 10^4 \text{ m}^2$

Now, magnetic moment of the solenoid is

$$M = NIA = 800 \times 3.0 \times 2.5 \times 10^{-4}$$
$$= 0.60 \text{JT}^{-1}$$

Which acts along the axis of the solenoid in the direction related to the sense of flow of current according to right handed screw rule.

Bar magnet in uniform magnetic field

When a bar magnet is placed in a uniform magnetic field, torque acts on the magnet. Also, magnetic potential energy is associated with the magnet due to its orientation as discussed in following sections.

Torque on bar magnet in a uniform magnetic field

Let us consider a bar magnet of length $2l$ placed in a uniform magnetic field B. Let the magnetic axis of the bar magnet makes angle θ with the field **B**, as shown in the figure.

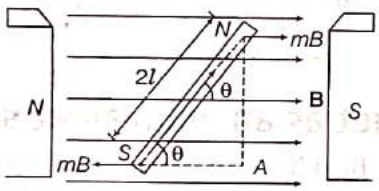


Fig. 5.18 *Torque on a bar magnet*

Then, Force on N-pole $= mB$, along **B**

Force on S-pole $= mB$, opposite to **B**

where, m = strength of each pole and

B = strength of magnetic field

These forces being equal and opposite form a couple, which tends to rotate the magnet clockwise, so as to align it along **B**. So, the moment of couple or torque is given by,

$$\tau = \text{Force} \times \text{Perpendicular distance}$$

$$\Rightarrow \quad \tau = mB \times NA \quad \text{...(i)}$$

In ΔNAS, $\sin\theta = \dfrac{NA}{NS} = \dfrac{NA}{2l}$

$$\therefore \quad NA = 2l \sin\theta$$

Eq. (i) becomes, $\tau = mB \times 2l \sin\theta$

Also, $M = m \times 2l$

$$\Rightarrow \quad \boxed{\tau = MB\sin\theta} \quad \text{...(ii)}$$

In vector form, we can rewrite this equation as $\boxed{\tau = \mathbf{M} \times \mathbf{B}}$

The direction of τ is perpendicular to the plane containing **M** and **B** and is given by right handed screw rule.

Work done or potential energy of a magnetic dipole (bar magnet) in a magnetic field

The torque acting on the dipole tends to align it in the direction of the field. Work has to be done in rotating the dipole against the action of the magnetic torque. This work done is stored in the form of potential energy of the dipole.

Now, a small amount of work done in rotating the dipole through a small angle $d\theta$ is given by,

$$dW = \tau\, d\theta = MB\sin\theta \cdot d\theta \quad \text{[from Eq. (ii)]}$$

Total work done in rotating the dipole from $\theta = \theta_0$ to $\theta = \theta$ can be obtained by integrating the above expression.

$\therefore$ Then, potential energy of the dipole is given by,

$$U = W = -MB(\cos\theta - \cos\theta_0)$$

Let us assume that, $\theta_0 = 90°$

So, $\quad U = W = -MB(\cos\theta - \cos 90°)$

Therefore, $\boxed{U = -MB\cos\theta}$

In vector notation, we may rewrite this equation as

$$\boxed{U = -\mathbf{M} \cdot \mathbf{B}}$$

Special Cases

(i) When dipole is perpendicular to magnetic field its potential energy is zero *i.e.*, when $\theta = 90°$

$$\Rightarrow \quad U = -MB\cos\theta = -MB\cos 90° = 0$$

(ii) When the magnetic dipole is aligned along the magnetic field, then it is in stable equilibrium having minimum potential energy *i.e.*, when $\theta = 0°$

$$\Rightarrow \quad U = -MB\cos\theta = -MB\cos 0° = -MB$$

(iii) When $\theta = 180°$.

$$\Rightarrow \quad U = -MB\cos\theta = -MB\cos 180° = MB$$

In this condition, potential energy is maximum.

Example 5.14 *A magnet of magnetic moment $50\hat{\mathbf{i}}$ A-m^2 is placed along the X-axis in a magnet field $\mathbf{B} = (0.5\hat{\mathbf{i}} + 3\hat{\mathbf{j}})T$. Find the torque acting on the magnetic field.*

Sol. $\because$ Torque, $\tau = \mathbf{M} \times \mathbf{B}$

where, M is magnetic moment and B is magnetic field

Given, $\quad M = 50\hat{\mathbf{i}}$ A-m^2, $B = 0.5\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$ T

$$\therefore \quad \tau = 50\hat{\mathbf{i}} \times (0.5\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$$
$$\tau = 150(\hat{\mathbf{i}} \times \hat{\mathbf{j}})$$

Using $\hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}}$, we have

$$\tau = 150\,\hat{\mathbf{k}} \text{ N-m}$$

Example 5.15 *A bar magnet when placed at an angle of 30° to the direction of magnetic field of 5×10^{-2} T, experiences a moment of couple 2.5×10^{-6} N-m. If the length of the magnet is 5 cm, then what will be its pole strength?*

Sol. Here, $\theta = 30°, B = 5 \times 10^{-2}$ T

and $\tau = 2.5 \times 10^{-6}$ N-m,

and $2l = 5\text{cm} = 0.05$ m; $m = ?$

$\therefore$ Torque $\tau = MB \sin\theta = m(2l)B \sin\theta$

$$\Rightarrow \quad m = \frac{\tau}{B(2l)\sin\theta} = \frac{2.5 \times 10^{-6}}{5 \times 10^{-2}(0.05)\sin 30°}$$

$$\therefore \quad m = 2 \times 10^{-3} \text{ A-m}$$

Example 5.16 *The work done in turning a magnet of magnetic moment 'M' by an angle 90° from the meridian is 'n' times the corresponding work done to turn it through an angle of 60°. What is the value of n?*

Sol. Here, $W_1 = MB(\cos 0° - \cos 90°) = MB(1 - 0) = MB$

Similarly, $W_2 = MB(\cos 0° - \cos 60°)$

$$= MB\left(1 - \frac{1}{2}\right) = \frac{MB}{2}$$

$$\therefore \quad W_1 = 2W_2 \Rightarrow n = 2$$

Example 5.17 *A bar magnet of magnetic moment 2.0 $A\text{-}m^2$ is free to rotate about a vertical axis through its centre. The magnet is released from rest from the east-west position. Find the kinetic energy of the magnet as it takes the north-south position. The horizontal component of the earth's magnetic field is $B = 25\ \mu T$. Earth's magnetic field is from south to north.*

Sol. Gain in kinetic energy = loss in potential energy

Thus, $\text{KE} = U_i - U_f$

As $U = -MB \cos\theta$

$$\therefore \quad \text{KE} = -MB \cos\left(\frac{\pi}{2}\right) - (-MB \cos 0°) = MB$$

Substituting the values, we have,

$$\text{KE} = (2.0)(25 \times 10^{-6})\text{ J} = 50\ \mu\text{J}$$

Example 5.18 *A short bar magnet of magnetic moment $m = 0.32 JT^{-1}$ is placed in a uniform magnetic field 0.15 T. If the bar magnet is free to rotate in the plane of the field, which orientation would correspond to its (i) stable and (ii) unstable equilibrium? What is the potential energy of the magnet in each case?*

Sol. (i) When **m** is parallel to **B**, then the magnet is in stable equilibrium,

$\therefore \quad \theta = 0°$

$\therefore$ PE in this case is given by,

$$U = -\mathbf{m} \cdot \mathbf{B} = -mB \cos\theta$$
$$= -0.32 \times 0.15 \times 1 = -0.048\text{J}$$

(ii) When **m** is anti-parallel to **B**, then the magnet will be in unstable equilibrium,

$\therefore \quad \theta = 180°$

Thus potential energy in this case is given by

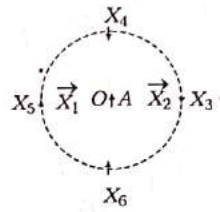
$$U = -\mathbf{m} \cdot \mathbf{B} = -mB \cos 180°$$
$$= -0.32 \times 0.15 \times (-1)$$
$$= +0.048\text{J}$$

Example 5.19 *Consider the situation shown in the diagram, where A small magnetised needle A is placed at a centre marked as O. The direction of its magnetic moment is indicated by arrow. The other arrows show different positions (and orientations of the magnetic moment) of another identical magnetised needle X.*

(i) In which configuration, the system is not in equilibrium?

(ii) In which configuration is the system in (a) stable and (b) unstable equilibrium?

(iii) Which configuration corresponds to the lowest potential energy among all the configurations shown.



Sol. Potential energy arises due to the PE of dipole X in magnetic field of A.

Magnetic field due to A,

On end position, $\mathbf{B}_A = \frac{\mu_0}{4\pi} \cdot \frac{2M}{r^3}$, along **M**

On broadside position, $B_P' = \frac{\mu_0}{4\pi} \cdot \frac{M}{r^3}$, opposite to **M**

The PE, $U = -\mathbf{M} \cdot \mathbf{B} = -MB \cos\theta$

For stable equilibrium, U is minimum *i.e.*, $\theta = 0$

For unstable equilibrium, U is maximum *i.e.*, $\theta = 180°$

(i) In X_1 and X_2 when not in equilibrium

(ii) (a) X_5, X_6 in stable equilibrium and (b) X_4, X_3 in unstable equilibrium.

(iii) X_6 corresponds to lowest PE,

$$U = -MB \cos\theta$$
$$= -M \cdot \frac{\mu_0}{4\pi}\frac{2M}{r^3} \cos 0° = -\frac{\mu_0 \cdot 2M^2}{4\pi r^3} = U_{\min}$$

Oscillations of a freely suspended magnet

When a small bar magnet of magnetic moment **M** is placed in a uniform magnetic field **B** such as it is free to vibrate in a horizontal plane of magnetic field **B** about a vertical axis passing through its centre of mass. This bar magnet oscillates. The restoring torque in this case will be,

$$\tau = -MB\theta \quad (\because \text{For small oscillation, } \sin\theta \simeq \theta)$$

The deflecting torque on the magnet is

$$\tau = I\alpha = I\frac{d^2\theta}{dt^2}$$

where I is the moment of inertia of the magnet about the axis of rotation and $\frac{d^2\theta}{dt^2}$ is the angular acceleration.

In equilibrium, deflecting torque = Restoring torque

or $$\frac{d^2\theta}{dt^2} = \frac{-MB\theta}{I} = -\omega^2\theta, \text{ where } \omega = \sqrt{\frac{MB}{I}}$$

i.e., angular acceleration $\frac{d^2\theta}{dt^2} \propto$ angular displacement θ

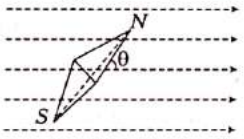
So, the oscillations of a freely suspended magnet in a uniform magnetic field are simple harmonic. The period of vibration is given by

$$\boxed{T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{MB}}}$$

Magnetic field B can be calculated from above equation and is given as

$$\boxed{B = \frac{4\pi^2 I}{MT^2}}$$

Example 5.20 *A magnetic needle is free to oscillate in a uniform magnetic field as shown in figure. The magnetic moment of magnetic needle is 7.2 Am^2 and moment of inertia $I = 6.5 \times 10^{-6}$ kg m^2. The number of oscillations performed in 5 s is 10. Calculate the magnitude of magnetic field?*



Sol. Here, $T = \frac{\text{Number of revolutions}}{\text{Time taken}} = \frac{5}{10} = 0.5$ s

$$M = 7.2\text{Am}^2, I = 6.5 \times 10^{-6} \text{ kgm}^2$$

As, $$T = 2\pi\sqrt{\frac{I}{MB}} \quad \text{or} \quad T^2 = 4\pi^2\frac{I}{MB}$$

The magnitude of the magnetic field is

$$B = \frac{4\pi^2 I}{MT^2} = \frac{4 \times (3.14)^2 \times 6.5 \times 10^{-6}}{7.2 \times (0.5)^2} = 1.42 \times 10^{-4} \text{ T}$$

Comparison between an electric dipole and a magnetic dipole: The electrostatic analogue

The behaviour of a magnetic dipole (may be a bar magnet also) is similar to the behaviour of an electric dipole. The only difference is that the electric dipole moment **P** is replaced by magnetic dipole moment **M** and the constant $\frac{1}{4\pi\varepsilon_0}$ is replaced by $\frac{\mu_0}{4\pi}$.

The table given below gives a comparison between an electric dipole and a magnetic dipole.

S. N.	Physical quantity to be compared	Electric dipole	Magnetic dipole
1.	Dipole moment	$p = q(2l)$	$M = m(2l)$
2.	Direction of dipole moment	From negative charge to the positive charge	From south to north pole
3.	Net force in uniform field	0	0
4.	Net torque in uniform field	$\tau = \mathbf{p} \times \mathbf{E}$	$\tau = \mathbf{M} \times \mathbf{B}$
5.	Field at far away point on the axis	$\frac{1}{4\pi\varepsilon_0} \cdot \frac{2p}{r^3}$ (along **p**)	$\frac{\mu_0}{4\pi} \cdot \frac{2M}{r^3}$ (along **M**)
6.	Field at far away point on perpendicular bisector	$\frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{r^3}$ (opposite to **p**)	$\frac{\mu_0}{4\pi} \cdot \frac{M}{r^3}$ (opposite to **M**)
7.	Potential energy	$U_\theta = -\mathbf{p} \cdot \mathbf{E} = -pE\cos\theta$	$U_\theta = -\mathbf{M} \cdot \mathbf{B} = -MB\cos\theta$
8.	Work done in rotating the dipole	$W_{\theta_1-\theta_2} = pE(\cos\theta_1 - \cos\theta_2)$	$W_{\theta_1-\theta_2} = MB(\cos\theta_1 - \cos\theta_2)$

Note In the table, θ is the angle between field (**E** or **B**) and dipole moment (**p** or **M**).

Magnetism and Gauss's law

The Gauss's law of magnetism states that, "*the surface integral of a magnetic field over a closed surface is zero i.e., the net magnetic flux through any closed surface is always zero.*"

$$\oint \mathbf{B} \cdot d\mathbf{S} = 0$$

Consequences of Gauss's law

(i) Consider a Gaussian surface (any closed surface) enclosing one of the poles (say south) of the magnet is shown in figure. Here, the number of field lines entering the Gaussian surface is same as the number of lines leaving it. *i.e.*, net magnetic flux for the whole Gaussian surface is zero.

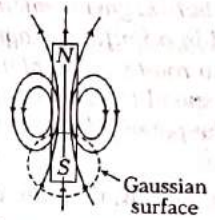


Fig. 5.19 Magnetic field lines are continues

(ii) The Gauss's theorem in magnetism implies that isolated magnetic poles do not exist.

(iii) In case of open surface $\int_S \mathbf{B} \cdot d\mathbf{S} = \phi_B$ (magnetic flux associated with the surface)

(iv) The magnetic poles always exist as unlike pairs of equal strengths.

Check point 5.1

1. Magnetic length is
   (a) less than geometric length
   (b) equal to geometric length
   (c) greater than geometric length
   (d) None of the above

2. Magnetic lines of force due to a bar magnet do not intersect because
   (a) a point always has a single net magnetic field
   (b) the lines have similar charges and so repel each other
   (c) the lines always diverge from a single point
   (d) None of the above

3. The unit of pole strength is
   (a) A-m (b) Am^{-1} (c) Am^{-2} (d) $A\text{-}m^2$

4. A bar magnet of magnetic moment M_1 is axially cut into two equal parts. If these two pieces are arranged perpendicular to each other, the resultant magnetic moment is M_2. Then, the value of M_1/M_2 is
   (a) $\frac{1}{2\sqrt{2}}$ (b) 1 (c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{2}$

5. At a point on the right bisector of a magnetic dipole, the magnetic
   (a) potential varies as $1/r^2$
   (b) potential is zero at all points on the right bisector
   (c) field varies as r^2
   (d) field is perpendicular to the axis of dipole

6. The ratio of the magnetic fields due to small bar magnet in end position an broad side on position is (at equal distance from the magnet)
   (a) 1/4 (b) 1/2 (c) 1 (d) 2

7. Two solenoids acting as short bar magnets P and Q are arranged such that their centres are on the X-axis and are separated by a large distance. The magnetic axes of P and Q are along X and Y-axes, respectively. At a point R, midway between their centres, if B is the magnitude of induction due to Q, then the magnitude of total induction at R due to the both magnets is
   (a) $3B$ (b) $\sqrt{5}B$ (c) $\frac{\sqrt{5}}{2}B$ (d) B

8. The intensity of magnetic field due to an isolated pole of strength m at a point distant r from it will be proportional to
   (a) $\frac{m}{r^2}$ (b) mr^2 (c) $\frac{r^2}{m}$ (d) $\frac{m}{r}$

9. A particle of charge q and mass m moves in a circular orbit of radius r with angular speed ω. The ratio of the magnitude of its magnetic moment to that of its angular momentum is
   (a) $-\frac{q}{2m}$ (b) $\frac{q\omega r^2}{2}$ (c) $\frac{q\omega}{2mr^2}$ (d) $\frac{q\omega r^2}{2m}$

10. A bar magnet of magnetic moment **M**, is placed in a magnetic field of induction **B**. The torque exerted on it is
    (a) $\mathbf{M}\times\mathbf{B}$ (b) $-\mathbf{B}\cdot\mathbf{M}$
    (c) $\mathbf{M}\cdot\mathbf{B}$ (d) $\mathbf{M}+\mathbf{B}$

11. The couple acting on a magnet of length 10 cm and pole strength 15 A-m, kept in a field of $B = 2\times10^{-5}$ T, at an angle of 30° is
    (a) 1.5×10^{-5} N-m (b) 1.5×10^{-3} N-m
    (c) 1.5×10^{-2} N-m (d) 1.5×10^{-6} N-m

12. A bar magnet is held at right angle to a uniform magnetic field. The couple acting on the magnet is to be halved by rotating it from this position. The angle of rotation is
    (a) 60° (b) 45°
    (c) 30° (d) 75°

13. If a bar magnet of magnetic moment M is freely suspended in a uniform magnetic field of strength B, then the work done in rotating the magnet through an angle θ is
    (a) $MB(1-\sin\theta)$ (b) $MB\sin\theta$
    (c) $MB\cos\theta$ (d) $MB(1-\cos\theta)$

14. The effect due to uniform magnetic field on a freely suspended magnetic needle is
    (a) Both torque and net force are present
    (b) torque is present but no net force
    (c) Both torque and net force are absent
    (d) net force is present but not torque

15. The net magnetic flux through any closed surface kept in a magnetic field is
    (a) zero (b) $\frac{\mu_0}{4\pi}$ (c) $4\pi\mu_0$ (d) $\frac{4\mu_0}{\pi}$

2 Earth's magnetism

Our earth behaves as a huge powerful magnet. The value of magnetic field on the surface of earth is a few tenths of a gauss ($1\ G = 10^{-4}$ T) and its strength varies from place to place on the earth's surface. The earth's magnetic south pole is located near the geographic north pole and the earth's magnetic north pole is located near the geographic south pole.

In fact, the configuration of the earth's magnetic field is very much similar to the one that would be achieved by burying a gigantic bar magnet deep in the interior of the earth.

The axis of earth's magnet makes an angle of 11.5° with the earth's rotational axis. The magnetic lines of force around the earth is shown in figures.

Some definitions related to earth's magnetism are

(i) **Geographic axis** The straight line passing through the geographical north and south poles of the earth is called its geographic axis. It is the axis of rotation of the earth.

(ii) **Magnetic axis** The straight line passing through the magnetic north and south poles of the earth is called the magnetic axis.

(iii) **Magnetic equator** It is the great circle on the earth perpendicular to the magnetic axis.

(iv) **Magnetic meridian** The vertical plane in the direction of B is called magnetic meridian.

(v) **Geographic meridian** The vertical plane passing through the line joining the geographical north and south poles is called the geographic meridian.

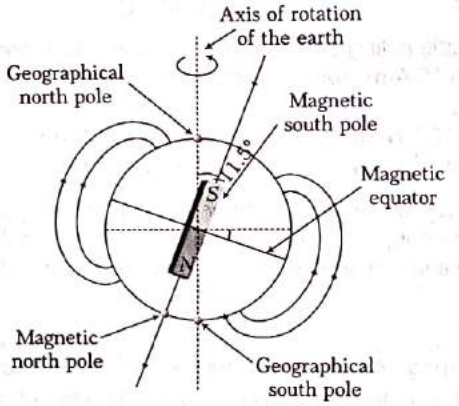


Fig. 5.20 Magnetic field of the earth

Note Earth's magnetic field varies irregularly from place to place and at a place it varies with time also.

Elements of earth's magnetism

The earth's magnetic field at a place can be completely described by three parameters which are called elements of earth's magnetic field. *These three elements are*

(i) Magnetic declination or Angle of declination (α)

At any place the acute angle between the magnetic meridian and the geographical meridian is called **angle of declination** α. The value of α is small in India. It is $0°41'$ E for Delhi and $0°58'$ W for Mumbai. This means that at this place compass needle tells true north very accurately.

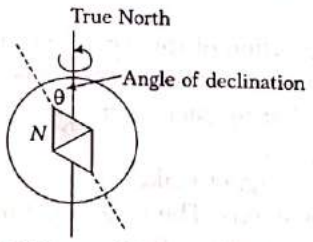


Fig. 5.21 Determining angle of declination

(ii) Magnetic Inclination or Angle of dip (θ)

The angle of dip (θ) at a place is the angle between the direction of earth's magnetic field (B) and horizontal line in the magnetic meridian. At earth's magnetic poles, the magnetic field of earth is vertical *i.e.*, angle of dip is 90°, the freely suspended magnetic needle is vertical there. At magnetic equator, field is horizontal or angle of dip is 0°.

(iii) Horizontal and vertical component of earth's magnetic field

Let B_e be the net magnetic field at some point. H and V be the horizontal and vertical components of B_e. Let θ is the angle of dip at the some place, then we can see that,

$$H = B_e \cos\theta \quad \text{...(i)}$$

and

$$V = B_e \sin\theta \quad \text{...(ii)}$$

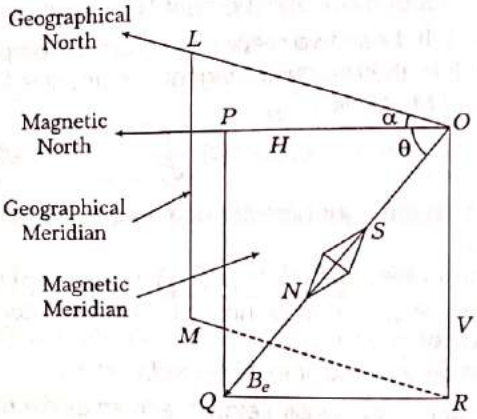


Fig. 5.22 Elements of earth's magnetic field

Squaring and adding Eqs. (i) and (ii), we get

$$B_e = \sqrt{H^2 + V^2}$$

Further, dividing Eqs. (ii) by Eq. (i), we get

$$\theta = \tan^{-1}\left(\frac{V}{H}\right)$$

By knowing H and θ at some place we can find B_e and V at that place.

Note

1. At magnetic equator, $H = B_e \cos 0° = B_e$ and at poles $H = B_e \cos 90° = 0$. Similarly, at magnetic equator $V = B_e \sin 0° = 0$ and at poles $V = B_e \sin 90° = B_e$.
2. $\tan\theta = 2\tan\phi$, where ϕ is magnetic lattitude of place.
3. Magnetic maps show variation of magnetic elements from place to place. Some important lines drawn on magnetic maps are
   (i) **Isoclinic lines** These are the lines joining points of equal dip or inclination. A line joining places of zero dip is called aclinic line or magnetic equator.
   (ii) **Isogonic lines** These are the lines joining places of equal declination. The line joining place of zero declination is called agonic line.
   (iii) **Isodynamic lines** These are the lines joining places having the same value of horizontal component of earth's magnetic field.

Example 5.21 *A compass needle of magnetic moment 60 A-m² is pointing geographical north at a certain place. It experiences a torque of 1.2×10^{-3} N-m. The horizontal component of earth's magnetic field at that place is 40 μWb/m². What is the angle of declination at that place?*

Sol. A compass needle in stable equilibrium position points towards magnetic north *i.e.*, along the horizontal component H of earth's magnetic field. When it is turned through the angle of declination α, so as to point geographical north, then it experiences a torque of magnitude $MH \sin\alpha$.

$\therefore MH \sin\alpha = 1.2\times 10^{-3}$ N-m (given)

Here, $M = 60$ A-m²,

$H = 40\times 10^{-6}$ Wb/m²

$$\therefore \quad \sin\alpha = \frac{1.2\times 10^{-3}}{60\times 40\times 10^{-6}} = 0.5 \therefore \alpha = 30°$$

Example 5.22 *In the magnetic meridian of a certain place, the horizontal component of earth's magnetic field is 0.26 G and the dip angle is 60°. Find*

(i) vertical component of earth's magnetic field.

(ii) the net magnetic field at this place.

Sol. Given, $H = 0.26$ G and $\theta = 60°$

(i) $\tan\theta = \dfrac{V}{H}$

$\therefore \quad V = H\tan\theta = (0.26)\tan 60° = 0.45$ G

(ii) $H = B_e \cos\theta$

$$\therefore \quad B_e = \frac{H}{\cos\theta} = \frac{0.26}{\cos 60°} = 0.52 \text{ G}$$

Example 5.23 *The horizontal and vertical component of earth's field at a place are 0.22 gauss and 0.38 gauss, respectively. Calculate the angle of dip and resultant intensity of earth's field.*

Sol. Here, $H = 0.22$G, $V = 0.38$G

Now, $\tan\theta = \dfrac{V}{H} = \dfrac{0.38}{0.22} = 1.7272$

$\therefore$ Angle of dip, $\delta = 59° 56'$

Resultant magnetic field of the earth is

$$B = \sqrt{H^2 + V^2} = \sqrt{0.22^2 + 0.38^2} = 0.427 \text{ G}$$

Example 5.24 *At a certain location in Africa, a compass points 12° west of the geographic north. The north tip of the magnetic needle of a dip circle placed in the plane of magnetic meridian points 60° above the horizontal. The horizontal component of the earth's field is measured to be 0.16 G. Specify the direction and magnitude of the earth's field at the location.*

Sol. Using the relation, $H = B_e \cos\theta$, we get the magnitude of **B** given by

$$B_e = \frac{H}{\cos\theta} = \frac{0.16}{\cos 60°} = \frac{0.16}{\left(\frac{1}{2}\right)}$$

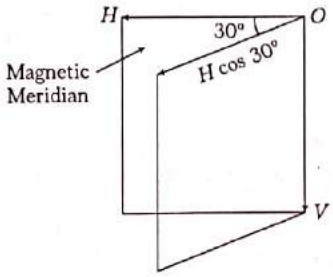
$$= 0.16\times 2 = 0.32 \text{ G}$$

or $\quad B = 0.32\times 10^{-4}$ T $\quad [\because 1\text{G} = 10^{-4}\text{ T}]$

Direction of B The earth's field lies in a vertical plane 12° west of geographic meridian at an angle of 60° above the horizontal line.

Example 5.25 *A magnetic needle suspended in a vertical plane at 30° from the magnetic meridian makes an angle of 45° with the horizontal. Find the true angle of dip.*

Sol. In a vertical plane at 30° from the magnetic meridian, the horizontal component is,



$$H' = H\cos 30°$$

While vertical component is still V. Therefore, apparent dip will be given by $\tan\theta' = \dfrac{V}{H'} = \dfrac{V}{H\cos 30°}$

but $\quad \dfrac{V}{H} = \tan\theta'$ $\quad$ (where θ = true angle of dip)

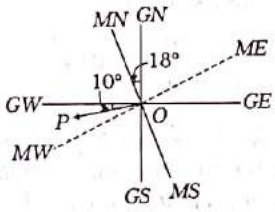
$$\therefore \quad \tan\theta' = \frac{\tan\theta}{\cos 30°}$$

$$\therefore \quad \theta = \tan^{-1}[\tan\theta' \cos 30°]$$

$$= \tan^{-1}[(\tan 45°)(\cos 30°)] \approx 41°$$

Example 5.26 *A ship is to reach a place 10° south of west. In which direction should it be steered if the declination at the place is 18° west of north?*

Sol. As the ship has to reach a place 10° south of west *i.e.*, along OP, so it should be steered west of magnetic north at angle of $90° - 18° + 10° = 82°$



Example 5.27 *A dip circle shows an apparent dip of 45° at a place where the true dip is 30°. If the dip circle is rotated through 90°, what apparent dip will it show?*

Sol. θ_1 and θ_2 are the angles of dip in two arbitary planes which are perpendicular to each other.

Here, $\theta_1 = 45°$ and $\theta = 30°$

As, $\cot^2\theta = \cot^2\theta_1 + \cot^2\theta_2$

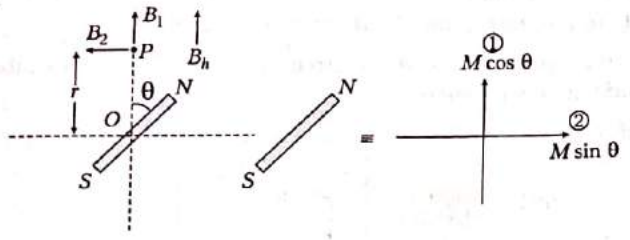
where θ is ture dip.

$\therefore \quad \cot^2 30° = \cot^2 45° + \cot^2\theta_2$

$$\cot^2\theta_2 = 3 - 1 = 2 \Rightarrow \cot\theta_2 = 1.414 \therefore \theta_2 = 35.2$$

Example 5.28 *A short magnet ($M = 4 \times 10^{-2}$ A-m^2) lying in a horizontal plane with its north-pole points 37° east of north. Find the net horizontal field at a point of the magnet 0.1 m away from it ($B_h = 11\,\mu$ T) ($\sin 37° = 3/5$, $\cos 37° = 4/5$)*

Sol.



Due to magnet at P

$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2M\cos\theta}{r^3}$$

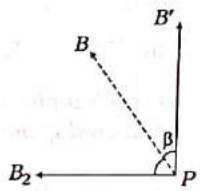
$$= \frac{10^{-7} \times 2 \times 4 \times 10^{-2} \times \frac{4}{5}}{(0.1)^3} = 6.4 \times 10^{-6}\text{T}$$

$$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{M\sin\theta}{r^3}$$

$$= \frac{10^{-7} \times 4 \times 10^{-2} \times \frac{3}{5}}{(0.1)^3} = 2.4 \times 10^{-6}\text{T}$$

Since, B_1 and B_h are in same direction

$$B' = B_1 + B_h = 6.4 \times 10^{-6} + 11 \times 10^{-6} = 17.4 \times 10^{-6}\text{T}$$

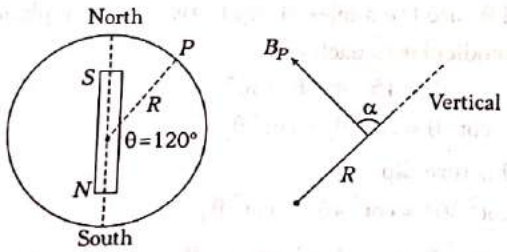


$$B_P = \sqrt{B'^2 + B_2^2} = \sqrt{(17.4)^2 + (2.4)^2} \times 10^{-6}$$

$$= \sqrt{308.52} \times 10^{-6}\text{T}$$

$$\tan\beta = \frac{B_2}{B'} = \frac{2.4 \times 10^{-6}}{17.4 \times 10^{-6}} = 0.14$$

Example 5.29 *The earth's magnetic field at geomagnetic poles has a magnitude 8×10^{-5} T. Find the magnitude and the direction of the field at a point on the earth's surface where the radius makes an angle of 120° with the axis of the earth's assumed magnetic dipole. What is the inclination (dip) at this point?*



Sol. The geomagnetic poles are in end-on-position.

The magnetic field at geomagnetic poles

$$B' = \frac{\mu_0}{4\pi} \cdot \frac{2M}{R^3}, \; B' = 8 \times 10^{-5}\text{ T}$$

The magnetic field at point P

$$B_P = \frac{\mu_0 M}{4\pi R^3}(1 + 3\cos^2\theta)^{1/2}$$

$$= \frac{B'}{2}(1 + 3\cos^2\theta)^{1/2}$$

$$= \frac{8 \times 10^{-5}}{2}[1 + 3\cos^2 120°]^{1/2}$$

$$= 4 \times 10^{-5}\left[1 + \frac{3}{4}\right]^{\frac{1}{2}}$$

$$\tan\alpha = \frac{1}{2}\tan\theta = \frac{1}{2}\tan 120°$$

$$= \frac{1}{2} \times (-\sqrt{3}) = -\frac{\sqrt{3}}{2}$$

$$\alpha = \tan^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$

Dip ϕ is an angle made by the earth's magnetic field with the horizontal plane

$$\phi = \alpha - 90° = \tan^{-1}\left(-\frac{\sqrt{3}}{2}\right) - 90°$$

Neutral points

When we trace magnetic lines of force around a magnet using a compass needle, we obtain the resultant of the magnetic field of magnet and that of the earth. In the plot of the resultant magnetic field, we come across points at which field (B) due to the magnet becomes equal and opposite to the horizontal component (H) of the earth's field *i.e.*, $B = H$. Therefore, the net magnetic field at these points will be zero.

So, **the points where net magnetic field due to the magnet and due to the earth is zero are called neutral points.**

A small compass needle placed at a neutral point shall experience no force/torque. Therefore, it can set itself in any direction, which may be different from usual N-S direction.

(i) When a magnet is placed with its north pole towards geographic south, then neutral points lie on axial line of the magnet.

At each neutral point,

$$B_1 = \frac{\mu_0}{4\pi}\frac{2Mr}{(r^2 - l^2)^2} = H$$

For a short magnet ($l << r$), $H = \frac{\mu_0}{4\pi}\frac{2M}{r^3}$

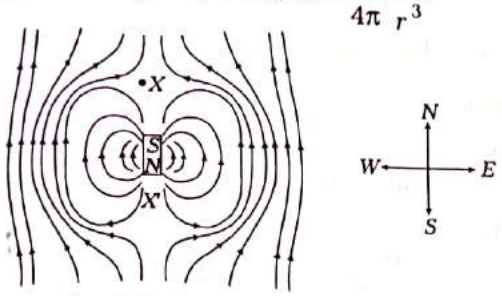


Fig. 5.23 Neutral points on axial line

(ii) When a magnet is placed with its north pole towards geographic north, neutral points lie on equitorial line of the magnet.

At each neutral point, $B_2 = \frac{\mu_0}{4\pi}\frac{M}{(r^2 + l^2)^{3/2}} = H$

For a short magnet ($l << r$), $H = \frac{\mu_0}{4\pi}\frac{M}{r^3}$

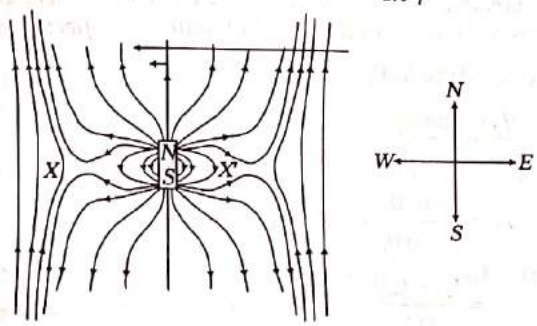


Fig. 5.24 Neutral points on equiatorial line

Note When the magnet is held vertically on the board there will be only one neutral point on a horizontal board. In the other orientations, there will be two neutral points.

Example 5.30 *A bar magnet 30 cm long is placed in the magnetic meridian with its north pole pointing south. The neutral point is observed at a distance of 30 cm from its one end. Calculate the pole strength of the magnet. Given horizontal component of earth's field = 0.34 G.*

Sol. Here, $2l = 30$ cm or $l = 15$ cm $= 0.15$ m,

$r = 30$ cm $= 0.30$ m

and $H = 0.34\text{G} = 0.34 \times 10^{-4}$ T

When magnet is placed with its north pole pointing south, then neutral point is obtained on its axial line.

$$\therefore \quad B_{\text{axial}} = H \quad \text{or} \quad \frac{\mu_0}{4\pi} \times \frac{2Mr}{(r^2 - l^2)^2} = H$$

or
$$M = \frac{4\pi}{\mu_0} \times \frac{H(r^2 - l^2)^2}{2r}$$

$$= \frac{1}{10^{-7}} \times \frac{0.34 \times 10^{-4} \times (0.30^2 - 0.15^2)^2}{2 \times 0.30}$$

$$= \frac{0.34 \times 10^{-4} \times (0.0675)^2}{10^{-7} \times 2 \times 0.30} = 2.582 \text{ A-m}^2$$

The pole strength of the magnet,

$$m = \frac{M}{2l} = \frac{2.582}{0.30} = 8.606 \text{ A-m}$$

Example 5.31 *A short bar magnet is placed with its north pole pointing north. The neutral point is 10 cm away from the centre of the magnet. If H = 0.4 G. Calculate the magnetic moment of the magnet.*

Sol. When north pole of the magnet points towards magnetic north, null point is obtained on perpendicular bisector of the magnet. Simultaneously, magnetic field due to the bar magnet should be equal to the horizontal component of earth's magnetic field H.

Thus, $$H = \frac{\mu_0}{4\pi} \cdot \frac{M}{r^3} \text{ or } M = \frac{Hr^3}{(\mu_0/4\pi)}$$

Substituting the values, we have

$$M = \frac{(0.4 \times 10^{-4})(10 \times 10^{-2})^3}{10^{-7}} = 0.4 \text{ A-m}^2$$

Tangent law

Tangent law states that, if a magnet is placed in two magnetic fields right angle to each other, then it will be acted upon by two couples tending to rotate it in opposite directions. It will be deflected through an angle θ, such that the two couples balance each other.

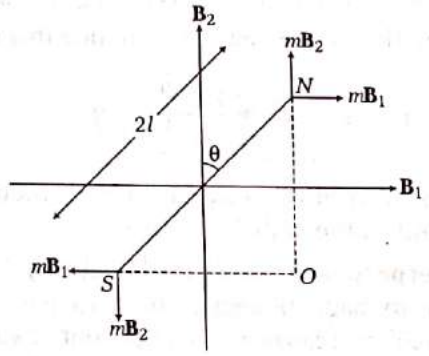


Fig. 5.25 Tangent law

And the tangent of the angle of deflection gives the ratio of the two fields as,

where θ is angle between magnet and magnetic field B_2.

$$\boxed{\tan\theta = \frac{B_1}{B_2}}$$

Applications of tangent law

Tangent galvanometer

It is a device used to measure very small current. It is a moving magnet type galvanometer and works on the principle of tangent law.

In case of tangent galvanometer, a magnetic compass needle is placed horizontally at the centre of a vertical fixed current-carrying coil whose plane is in the magnetic meridian.

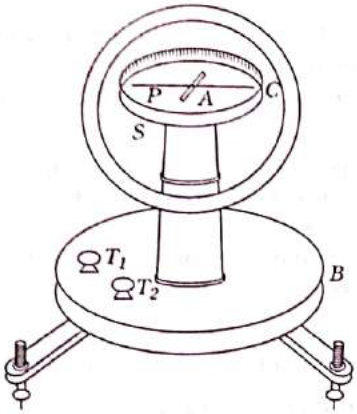


Fig. 5.26 Tangent galvanometer

As shown in figure, pointer P is rigidly attached to compass needle and perpendicular to it. The compass needle together with pointer can rotate freely about the vertical axis.

When no current flows through the coil, its plane is parallel to compass needle and coincides with magnetic meridian. When current passes through the coil, it produces magnetic field at the centre which deflects the compass needle and pointer deflects through the same angle.

The magnetic field produced at centre due to a coil is,

$$\boxed{B_c = \frac{\mu_0 NI}{2r}}$$

where N is number of turns in the coil, r is radius of coil and I is current in the coil.

Field B_c is perpendicular to plane of coil, *i.e.*, perpendicular to magnetic meridian,
i.e., perpendicular to horizontal component of earth's magnetic field.

The resultant horizontal magnetic field

$$B = \sqrt{B_c^2 + H^2}$$

$$\Rightarrow \quad \tan\theta = \frac{B_c}{H} \quad \text{(tangent law of perpendicular field)}$$

The needle will stay in equilibrium when its length is parallel to B.

The deflection of the needle from its original position θ is given by,

$$H\tan\theta = B_c = \frac{\mu_0 NI}{2r}$$

$$\Rightarrow \quad I = \frac{2rH}{\mu_0 N}\tan\theta = k\tan\theta$$

where $k = \dfrac{2rH}{\mu_0 N}$ is constant for a given galvanometer at given place and is called reduction factor of galvanometer.

Sensitivity

Sensitivity means, the measure of change in the deflection produced by a unit current.

It is given by,

$$\frac{d\theta}{dI} = \frac{1}{k\sec^2\theta} = \frac{1}{k\left(1+\dfrac{I^2}{k^2}\right)}$$

For good sensitivity, the change in deflection should be large for a given fractional change of current.

Note

1. The tangent galvanometer is most sensitive when deflection is around 45°.

Example 5.32 *In a tangent galvanometer, when a current of 10 mA is passed, the deflection is 31°. By what percentage, the current has to be increased, so as to produce a deflection of 42°?*

Sol. Here, $I_1 = 10\text{mA}$, $\theta_1 = 31°$, $I_2 = ?$, $\theta_2 = 42°$

As, $$\frac{I_2}{I_1} = \frac{\tan\theta_2}{\tan\theta_1}$$

$$\therefore \quad I_2 = I_1\frac{\tan\theta_2}{\tan\theta_1} = 10\times\frac{\tan 42°}{\tan 31°}$$

$$= \frac{10\times 0.9}{0.6} = 15\text{ mA}$$

Percentage increase in current,

$$\frac{(I_2 - I_1)}{I_1}\times 100 = \frac{(15-10)}{10}\times 100 = 50\%$$

Example 5.33 *The coil of a tangent galvanometer of radius 12 cm is having 200 turns. If the horizontal component of earth's magnetic field is 25 μT. Find the current which gives a deflection of 60°.*

Sol. Given, r = radius of coil = 12 cm = 0.12 m,

N = Number of turns of coil = 200,

$H = 25\,\mu\text{T} = 25\times 10^{-6}\text{T}$

and $\phi = 60°$

As we know, $I = k\tan\theta$

where, $$k = \frac{2rH}{\mu_0 N}$$

$$\therefore \quad I = \frac{2rH}{\mu_0 N}\tan\theta$$

$$= \frac{2\times 0.12\times 25\times 10^{-6}\times\tan 60°}{4\pi\times 10^{-7}\times 200}$$

$$= 0.042\text{ A}$$

Vibration magnetometer

Vibration magnetometer is an instrument which is used to find magnetic moment of bar magnet and to compare magnetic fields of two magnets. This device works on the principle that whenever a freely suspended magnet in a uniform magnetic field is disturbed from its equilibrium position, it starts vibrating about the mean position.

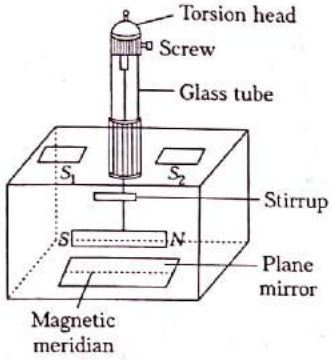


Fig. 5.27 Vibration magnetometer

Time period of oscillation of experimental bar magnet (with magnetic moment M) in earth's magnetic field (H) is given by the formula, $T = 2\pi\sqrt{\frac{I}{MH}}$, where I = moment of inertia of short bar magnet.

(i) **Comparison of horizontal components of earth's magnetic field at two places**

$T = 2\pi\sqrt{\frac{I}{MH}}$, since I and M of the magnet are constant

So, $$T^2 \propto \frac{1}{H} \Rightarrow \frac{H_1}{H_2} = \frac{T_2^2}{T_1^2}$$

(ii) **Determination of magnetic moment of a magnet**

The experimental (given) magnet is put into vibration magnetometer and its time period T is determined.

Now, $$T = 2\pi\sqrt{\frac{I}{MH}} \Rightarrow M = \frac{4\pi^2 I}{H \cdot T^2}$$

(iii) **Comparison of magnetic moment of two magnets of same size and mass**

As, $T = 2\pi\sqrt{\frac{I}{M.H}}$, here I and H are constants

So, $$M \propto \frac{1}{T^2}$$

$$\Rightarrow \frac{M_1}{M_2} = \frac{T_2^2}{T_1^2}$$

(iv) **Comparison of magnetic moments by sum and difference method**

Sum position

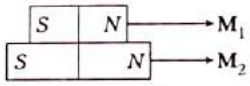


Fig. 5.28

Net magnetic moment, $M_s = M_1 + M_2$

Net moment of inertia, $I_s = I_1 + I_2$

Time period of oscillation of this pair in earth's magnetic field H,

$$T_s = 2\pi\sqrt{\frac{I_s}{M_s H}} = 2\pi\sqrt{\frac{I_1 + I_2}{(M_1 + M_2)H}} \quad ...(i)$$

$$\Rightarrow \text{Frequency, } \nu_s = \frac{1}{2\pi}\sqrt{\frac{(M_1 + M_2)H}{I_s}}$$

Difference position This method can be used when magnets are placed with their poles in opposite direction.

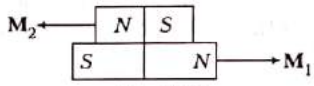


Fig. 5.29

Net magnetic moment,

$$M_d = M_1 - M_2$$

Net moment of inertia, $I_d = I_1 + I_2$

and $$T_d = 2\pi\sqrt{\frac{I_d}{M_d H}} = 2\pi\sqrt{\frac{I_1 + I_2}{(M_1 - M_2)H}} \quad ...(ii)$$

$$\text{Frequency, } \nu_d = \frac{1}{2\pi}\sqrt{\frac{(M_1 - M_2)H}{(I_1 + I_2)}}$$

(ii) **Comparison of two magnetic fields** Suppose we wish to compare the magnetic fields B_1 and B_2 at some point P due to two magnets. For this vibration magnetometer is so placed that the centre of its magnet lies on P. Now one of the given magnets is placed at some known distance from P in the magnetic meridian, such that point P lies on its axial line and its north pole points north. In this position the fields B_1 at P produced by the magnet will be in the direction of H. Hence, the magnet suspended in the magnetometer will vibrate in the resultant magnetic field $(H + B_1)$. Its period of vibration is noted, say it is T_1, then

$$T_1 = 2\pi\sqrt{\frac{I}{M(H + B_1)}}$$

Now the first magnet is replaced by the second magnet and the second magnet is placed in the same position and again the time period is noted.

If the field produced at P due to this magnet be B_2 and the new time period be T_2, then

$$T_2 = 2\pi\sqrt{\frac{I}{M(H+B_2)}}$$

Finally the time period of the magnetometer under the influence of the earth's magnetic field alone is determined. Let it be T, then

$$T = 2\pi\sqrt{\frac{I}{MH}}$$

Solving above three equations for T, T_1 and T_2 we can show that

$$\boxed{\frac{B_1}{B_2} = \frac{(T^2 - T_1^2)\,T_2^2}{(T^2 - T_2^2)\,T_1^2}}$$

Example 5.34 *A bar magnet of length 5 cm, width 3 cm and height 2 cm takes 5 s to complete an oscillation in vibration magnetometer placed in a horizontal magnetic field of 20 μT. The mass of this bar magnet is 250 g. (a) Find the magnetic moment of the magnet. (b) If the magnet is put in the magnetometer with its 0.5 cm edge horizontal, what would be the new time period?*

Sol. (a) Moment of inertia of magnet is given by ,

$$I = \frac{m(l^2 + b^2)}{12}$$

where m = mass of magnet

$$\therefore \quad I = \frac{250(5^2 + 3^2)\times 10^{-4}\times 10^{-3}}{12} = 7.08\times 10^{-5}\ \text{kgm}^2$$

Also, $T = 2\pi\sqrt{\frac{I}{MH}}$ $\therefore$ $M = \frac{4\pi^2 I}{HT^2}$

On putting values, we get

$$M = \frac{4\times(3.14)^2\times(7.08\times 10^{-5})}{20\times 10^{-6}\times 5\times 5} = 5.58\ \text{Am}^2$$

(b) New moment of inertia is given by $I' = \frac{m_0(l^2 + h^2)}{12}$

$$\therefore \quad T' = 2\pi\sqrt{\frac{I'}{MH}}$$

$$\Rightarrow \frac{T'}{T} = \sqrt{\frac{I'}{I}} = \sqrt{\frac{l^2 + h^2}{l^2 + b^2}} = \sqrt{\frac{5^2 + (0.5)^2}{5^2 + 3^2}} \Rightarrow \frac{T'}{T} = 0.86$$

$$\therefore \quad T' = T\times 0.86 = 5\times 0.86 = 4.30\ \text{s}$$

Example 5.35 *A magnetic needle performs 20 oscillations per minute in a horizontal plane. If the angle of dip be 30°, then how many oscillations per minute will this needle perform in vertical north-south plane and in vertical east-west plane?*

Sol. In horizontal plane, the magnetic needle oscillates in horizontal component H.

$$\therefore \quad T = 2\pi\sqrt{\frac{I}{MH}}$$

In the vertical north-south plane (magnetic meridian), the needle oscillates in the total earth's magnetic field B_e and in vertical east-west plane (plane perpendicular to the magnetic meridian) it oscillates only in earth's vertical component V. If its time period be T_1 and T_2, then

$$T_1 = 2\pi\sqrt{\frac{I}{MB_e}} \quad \text{and} \quad T_2 = 2\pi\sqrt{\frac{I}{MV}}$$

From above equations, we can find

$$\frac{T_1^2}{T^2} = \frac{H}{B_e} \quad \text{or} \quad \frac{n_1^2}{n^2} = \frac{B_e}{H}$$

Similarly, $\frac{n_2^2}{n^2} = \frac{V}{H}$

Further, $\frac{B_e}{H} = \sec\theta = \sec 30° = \frac{2}{\sqrt{3}}$

and $\frac{V}{H} = \tan\theta = \tan 30° = \frac{1}{\sqrt{3}}$

$$\therefore \quad n_1^2 = (n)^2\left(\frac{B_e}{H}\right) = (20)^2\left(\frac{2}{\sqrt{3}}\right)$$

or $n_1 = 21.5$ oscillations/min

and $n_2^2 = (n)^2\left(\frac{V}{H}\right) = (20)^2\left(\frac{1}{\sqrt{3}}\right)$

$\therefore$ $n_2 = 15.2$ oscillations/min

Example 5.36 *A magnet performs 15 oscillations per minute in a horizontal plane, where angle of dip is 60° and earth's total field is 0.5 G. At another place, where total field is 0.6 G the magnet performs 20 oscillations per minute. What is the angle of dip at this place?*

Sol. As, $H = B\cos\theta$

where, H = horizontal component of earth's magnetic field,

B = total earth's magnetic field

and θ = angle of dip

Then, $H_1 = B_1\cos\theta_1$ and $H_2 = B_2\cos\theta_2$

Further, $T_1 = 2\pi\sqrt{\frac{I}{MH_1}} = 2\pi\sqrt{\frac{T}{MB_1\cos\theta_1}}$

and $T_2 = 2\pi\sqrt{\frac{I}{MH_2}} = 2\pi\sqrt{\frac{I}{MB_2\cos\theta_2}}$

$$\because \quad \frac{T_1^2}{T_2^2} = \frac{B_2\cos\theta_2}{B_1\cos\theta_1}$$

$$\therefore \quad \cos\theta_2 = \frac{B_1}{B_2}\times\frac{T_1^2}{T_2^2}\cos\theta_1$$

$$= \frac{B_1}{B_2}\times\left(\frac{\nu_2}{\nu_1}\right)^2\cos\theta_1$$

$$= \left(\frac{0.5}{0.6}\right)\left(\frac{20}{15}\right)^2\cos 60° = 0.74$$

$$\therefore \quad \theta_2 = \cos^{-1}(0.74) = 42.2°$$

Example 5.37 *The time period of vibration of two magnets in sum position (magnets placed with similar poles on one side one above the other) is 3s. When polarity of weaker magnet is reversed the combination makes 12 oscillations per minute. What is the ratio of magnetic moments of two magnets?*

Sol. Here, $T_1 = 3\text{s}, T_2 = \frac{1}{12}\text{min} = \frac{60\text{s}}{12} = 5\text{s}$

$$\frac{M_1}{M_2} = \frac{T_2^2 + T_1^2}{T_2^2 - T_1^2} \quad \therefore \frac{M_1}{M_2} = \frac{5^2 + 3^2}{5^2 - 3^2} = \frac{34}{16} = \frac{17}{8}$$

Example 5.38 *A small bar magnet having a magnetic moment of 9×10^{-3} A-m^2 is suspended at its centre of gravity by a light torsionless string at a distance of 10^{-2} m vertically above a long, straight horizontal wire carrying a current of 1.0 A from east to west. Find the frequency of oscillation of the magnet about its equilibrium position. The moment of inertia of the magnet is 6×10^{-9} kg-m^2. ($H = 3 \times 10^{-5}$ T).*

Sol. The magnetic moment of the bar magnet is,

$$M = 9 \times 10^{-9} \text{ A-m}^2$$

The magnitude of the magnetic field at the location of the magnet due to current carrying wire is,

$$B = \frac{\mu_0}{2\pi}\frac{I}{r} = \frac{(2 \times 10^{-7})(1.0)}{10^{-2}}$$

$$= 2 \times 10^{-5} \text{ Wb/m}^2 \qquad (S \text{ to } N)$$

The earth's horizontal magnetic field is,

$$H = 3 \times 10^{-5} \text{ T} = 3 \times 10^{-5} \text{ Wb/m}^2$$

$$\therefore \qquad B + H = 5 \times 10^{-5} \text{ Wb/m}^2$$

The frequency of oscillation will be,

$$\nu = \frac{1}{2\pi}\sqrt{\frac{M(B+H)}{I}} \quad [\text{here, } I = \text{moment of inertia}]$$

$$= \frac{1}{2\pi}\sqrt{\frac{(9 \times 10^{-9}) \times (5 \times 10^{-5})}{6 \times 10^{-9}}} \qquad [S \text{ to } N]$$

$$= 1.38 \times 10^{-3} \text{ Hz}$$

Example 5.39 *A thin rectangular magnet suspended freely has a period of oscillation equal to T. Now, it is broken into two equal halves (each having half of the original length) and one piece is made to oscillate freely in the same field. If its period of oscillation is T. Then, find the ratio T/T.*

Sol. When magnet is divided into two equal parts, the magnetic dipole moment,

$$M' = \text{pole strength} \times \frac{l}{2} = \frac{M}{2} \quad [\text{pole strength remain same}]$$

Also, the mass of magnet becomes half *i.e.*, $m' = \frac{m}{2}$.

Moment of inertia of magnet, $I = \frac{ml^2}{12}$.

New moment of inertia, $I' = \frac{1}{12}\left(\frac{m}{2}\right)\left(\frac{l}{2}\right)^2 = \frac{ml^2}{12 \times 8}$

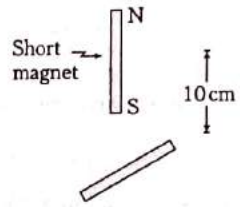
$$\Rightarrow \qquad I' = \frac{I}{8}$$

Now, $$T = 2\pi\sqrt{\left(\frac{I}{MB}\right)}$$

and $$T' = 2\pi\sqrt{\left(\frac{I'}{M'B}\right)} = 2\pi\sqrt{\left(\frac{I/8}{MB/2}\right)}$$

$$T' = \frac{T}{2} \Rightarrow \frac{T'}{T} = \frac{1}{2}$$

Example 5.40 *The time period of the magnetic in an oscillation magnetometer in the earth magnetic field is 2 s. A short bar magnet is placed to the north of the magnetometer, at a separation 10 cm from the oscillating magnet, with its north pole pointing towards north. The time period becomes half. Calculate the magnetic moment of this short magnet. $B_h = 12\mu T$.*



Sol. Time period $T = 2\pi\sqrt{\frac{I}{MB}} \Rightarrow T \propto \frac{1}{\sqrt{H}} \Rightarrow B \propto \frac{1}{T^2}$

Let M is magnet moment due to short magnet and B' is a magnetic field due to short magnet, along south to north

Given, $T_1 = 2$ s, $B_h = 12\,\mu T$

$$T_2 = 1 \text{ s}, B_h = B + B' = 12 + B'$$

$$\frac{(B + B')}{B} = \frac{T_1^2}{T_2^2}$$

$$\frac{12 + B'}{12} = \left(\frac{2}{1}\right)^2 = 4$$

$$B' = 36\,\mu T$$

$$B' = \frac{\mu_0}{4\pi}\frac{2M}{r^3}$$

$$\Rightarrow \qquad 36 \times 10^{-6} = 10^{-7} \times \frac{2M}{(0.10)^3}$$

$$M = 0.18 \text{ Am}^2$$

Check point 5.2

1. The earth's magnetic field is
   (a) 10^{-4} T (b) 10^{-5} T
   (c) 10^{-6} T (d) None of these
2. Magnetic meridian is a
   (a) point (b) horizontal plane
   (c) vertical plane (d) line along *N-S*
3. The angle between the magnetic meridian and geographical meridian is called
   (a) angle of dip (b) angle of declination
   (c) magnetic moment (d) power of magnetic field
4. Angle of dip at the equator is
   (a) 0° (b) 30° (c) 60° (d) 90°
5. Earth's magnetic field always has a horizontal component except at
   (a) magnetic equator (b) magnetic pole
   (c) geographical north pole (d) everywhere
6. If $H = \frac{1}{\sqrt{3}}V$, then find angle of dip. (where, symbols have their usual meanings)
   (a) 60° (b) 30° (c) 45° (d) 90°
7. Let V and H be the vertical and horizontal components of earth's magnetic field at any point on earth. Near the north pole
   (a) $V >> H$ (b) $V << H$ (c) $V = H$ (d) $V = H = 0$
8. If a magnet is suspended at angle 30° to the magnetic meridian, then the dip needle makes angle of 45° with the horizontal. The real dip is
   (a) $\tan^{-1}(\sqrt{3}/2)$ (b) $\tan^{-1}(\sqrt{3})$
   (c) $\tan^{-1}\left(\frac{3}{\sqrt{2}}\right)$ (d) $\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$
9. The dip at a place is δ. For measuring it, the axis of the dip needle is perpendicular to the magnetic meridian. If the axis of the dip needle makes an angle θ with the magnetic meridian, then the apparent dip will be given by
   (a) $\tan\delta \operatorname{cosec}\theta$ (b) $\tan\delta \sin\theta$
   (c) $\tan\delta \cos\theta$ (d) $\tan\delta \sec\theta$
10. At a neutral point
    (a) field of magnet is zero
    (b) field of earth is zero
    (c) field of magnet is perpendicular to field to earth
    (d) None of the above
11. Tangent galvanometer measures
    (a) capacitance (b) current
    (c) resistance (d) potential difference
12. Two tangent galvanometers *A* and *B* are identical except in their number of turns. They are connected in series. On passing a current through them, deflections of 60° and 30° are produced. The ratio of the number of turns in *A* and *B* is
    (a) 1 : 3 (b) 3 : 1 (c) 1 : 2 (d) 2 : 1
13. Vibration magnetometer is used for comparing
    (a) magnetic fields (b) earth's field
    (c) magnetic moments (d) All of these
14. The time period of a freely suspended bar magnet in a field is 2 s. It is cut into two equal parts along its axis, then the time period is
    (a) 4 s (b) 0.5 s (c) 2 s (d) 0.25 s
15. A bar magnet suspended freely in a uniform magnetic field is vibrating with a time period of 3 s. If the field strength is increased to 4 times of the earlier field strength, then the time period (in second) will be
    (a) 12 (b) 6 (c) 1.5 (d) 0.75

3 Magnetic induction and magnetic materials

Several experiments have been done which showed that magnetic lines get modified due to the presence of certain materials in the magnetic field. Few substances such as O_2, air, platinum, aluminium, etc., show a very small increase in the magnetic flux passing through them, when placed in a magnetic field. Such substances are called **paramagnetic substances.**

Few other substances such as H_2, H_2O, Cu, Zn, Sb, etc., show a very small decrease in flux and are said to be **diamagnetic.** There are other substances like Fe, Co etc., through which the flux increases to a larger value and are known as **ferromagnetic substances.**

Some important terms used to describe the properties of magnetic materials

(i) Magnetic induction (*B*)

The number of magnetic lines of force inside a magnetic substance crossing per unit area normal to their direction is called the **magnetic induction** or **magnetic flux density** inside the substance. It is denoted by **B**. The SI unit of **B** is tesla (T) or weber/metre^2 (Wb/m^2). The CGS unit is gauss (G).

$$1 \text{ Wb/m}^2 = 1 \text{ T} = 10^4 \text{ G}$$

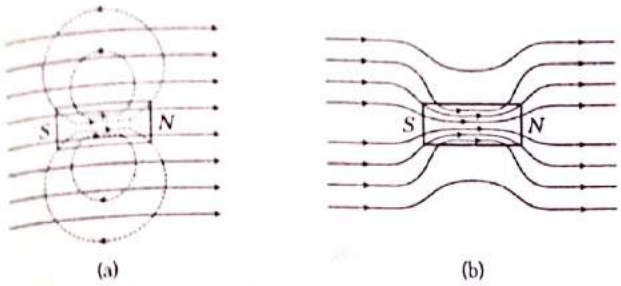


Fig. 5.30 Magnetic field lines of a magnetic material

(ii) Intensity of magnetisation (*I*)

"Intensity of magnetisation (I) is defined as the magnetic moment per unit volume of the magnetised substance." This basically represents the extent to which the substance is magnetised. Thus, $\mathbf{I} = \frac{M}{V}$

The SI unit of **I** is ampere/metre (A/m). Its dimensional formula is $[M^0L^{-1}I^0A]$

Note For bar magnet, $I = m / A$.

(ii) Magnetic intensity or magnetic field strength (*H*)

The capability of the magnetising field to magnetise the substance is expressed by means of a vector **H**, called the **magnetic intensity** of the field. It is defined through the vector relation,

$$\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{I}$$

The SI unit of **H** is same as that of **I** *i.e.*, ampere/metre (A/m). The CGS unit is oersted.

Note

1. In vacuum, $\mathbf{I} = 0 \therefore \mathbf{H} = \frac{\mathbf{B}}{\mu_0}$.
2. The magnetic field intensity of solenoid is $H = nI$. It is independent of the material of the core of a solenoid.
3. When magnetic material is kept inside the solenoid, then $\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_M$
   where, $\mathbf{B}_0$ = magnetic field due to magnetising field.
   and $\mathbf{B}_m$ = magnetic field due to magnetisation of the material.

(iv) Magnetic permeability (μ)

"It is defined as the ratio of the magnetic induction **B** inside the magnetised substance to the magnetic intensity **H** of the magnetising field *i.e.*,

$$\mu = \frac{\mathbf{B}}{\mathbf{H}}$$

It is basically a measure of conduction of magnetic lines of force through a substance. The SI unit of magnetic permeability is weber/ampere-metre (Wb/A-m) or tesla metre-ampere^{-1} (TmA^{-1}).

(v) Relative magnetic permeability (μ_r)

It is the ratio of the magnetic permeability μ of the substance to the permeability of free space.

Thus, $$\mu_r = \frac{\mu}{\mu_0}$$

μ_r is a pure ratio, hence dimensionless. For vacuum its value is 1.

μ_r can also be defined as the ratio of the magnetic field B in the substance when placed in magnetic field B_0. Thus,

$$\mu_r = \frac{B}{B_0}$$

For paramagnetic substance, $\mu_r > 1$.
For diamagnetic substance, $\mu_r < 1$ and
For ferromagnetic substance, $\mu_r >> 1$.

(vi) Magnetic susceptibility (χ_m)

It is the property of the substance which shows how easily a substance can be magnetised when kept in a magnetising field. It can also be defined as the ratio of intensity of magnetisation (I) in a substance to the magnetic intensity (H) applied to the substance *i.e.*,

$$\chi_m = \frac{I}{H}.$$

It is a scalar quantity with no units and dimensions.
For paramagnetic substance, χ is small and positive.
For diamagnetic substance, χ is small and negative.

(vii) Relation between μ_r and χ_m

We have, $\mathbf{B} = \mu_0(\mathbf{I} + \mathbf{H})$

or $B = \mu_0 H\left(\frac{I}{H} + 1\right)$ or $B = B_0(\chi_m + 1) \Rightarrow \frac{B}{B_0} = \chi_m + 1$

But $\frac{B}{B_0} = \frac{\mu}{\mu_0} = \mu_r$ = relative permeability

$\therefore$ $\mu_r = \chi_m + 1$

Example 5.41 *The magnetic moment of a magnet (15 cm × 2 cm × 1 cm) is 1.2 A-m^2. Calculate its intensity of magnetisation.*

Sol. Given, magnetic moment, $M = 1.2$ A-m^2

Volume, $V = (15 \times 2 \times 1) \times 10^{-6}\text{m}^3 = 30 \times 10^{-6}\text{ m}^3$

$\therefore$ Intensity of magnetisation,

$$I = \frac{M}{V} = \frac{1.2}{(15 \times 2 \times 1) \times 10^{-6}} = 4 \times 10^4 \text{Am}^{-1}$$

Example 5.42 *Relative permeability of iron is 5500, what will be its magnetic susceptibility.*

Sol. As we know that, $\mu_r = 1 + \chi_m \Rightarrow 5500 = 1 + \chi_m$

$\Rightarrow$ $\chi_m = 5500 - 1 = 5499$

Example 5.43 *The magnetising field of 20 CGS units produces a flux of 2400 CGS units in a bar of iron of cross-section 0.2 cm^2. Calculate the (i) permeability and (ii) susceptibility of the bar.*

Sol. ∴ Magnetic field,

$$B = \frac{\phi}{A} = \frac{2400 \times 10^{-8}}{0.2 \times 10^{-4}} = 1.20 \text{ Wb/m}^2$$

(i) The permeability of the bar material,

$$\mu = \frac{B}{H} = \frac{1.2}{20/4\pi \times 10^{-3}} = 7.54 \times 10^{-4} \text{ H/m}$$

(ii) The magnetic susceptibility and permeability of a material are related with each other as,

$$\mu = \mu_0(1 + \chi_m) \text{ or } \chi_m = \frac{\mu}{\mu_0} - 1 = \frac{7.54 \times 10^{-4}}{4\pi \times 10^{-7}} - 1 = 599$$

Example 5.44 *A solenoid having 2000 turns/m has a core of a material with relative permeability 220. The area of core is 4 cm^2 and carries a current of 5A. Calculate (a) Magnetic intensity (b) Magnetic field (c) Magnetisation (I) of the core. Also calculate the pole strength developed.*

Sol. Given $n = 2000$ turns, $I = 5$A, $\mu_r = 220$, $A = 4$ cm^2

(a) Magnetic intensity, $H = nI = 2000 \times 5 = 10000$ A/m

(b) Magnetic field, $B = \mu H = \mu_0 \mu_r H$

$$= 4\pi \times 10^{-7} \times 220 \times 10{,}000 = 88\pi \times 10^{-2} \text{ T}$$

(c) As we know, $B = B_0(H + I)$

$$88\pi \times 10^{-2} = 4\pi \times 10^{-7}(10000 + I)$$

$$\Rightarrow \quad 2.20 \times 10^6 = 10^4 + I$$

$$\therefore \quad I = (2.20 \times 10^6 - 10^4) = 2.19 \times 10^6 \text{ A/m}$$

(d) Pole strength, $m = IA = (2.19 \times 10^6)(4 \times 10^{-4}) = 876$ A-m

Example 5.45 *The space within a current carrying solenoid is filled with magnesium having magnetic susceptibility. $\chi_{Mg} = 1.2 \times 10^{-5}$. What will be the percentage increase in magnetic field?*

Sol. Magnetic field without magnesium is $B_0 = \mu_0 H$

With magnesium, $B = \mu H = \mu_0(1 + \chi)H$

$$\therefore \% \text{ increase} = \left(\frac{B}{B_0} - 1\right) \times 100$$

$$= \chi_{mg} \times 100 = 1.2 \times 10^{-5} \times 100 = 1.2 \times 10^{-3}$$

Example 5.46 *Consider a bar magnet having pole strength 2 A-m, magnetic length 4 cm and area of cross-section 1 cm^2. Find*

(i) the magnetisation I.

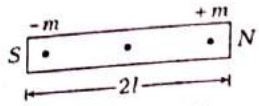
(ii) the magnetic intensity H and

(iii) the magnetic field at the centre of magnet.

Sol. Given, $m = 2$ A-m, $2l = 4$ cm, $A = 1$ cm^2

(i) Magnetisation $I = \frac{M}{V} = \frac{m \cdot 2l}{A \cdot 2l} = \frac{m}{A} = \frac{2}{1 \times 10^{-4}} = 2 \times 10^4$ A/m

The direction will be from S to N-pole

(ii) 

At centre, $H_N = \frac{\frac{\mu_0}{4\pi}\frac{m}{l^2}}{\mu_0} = \frac{m}{4\pi l^2}$, along north pole

$$H_S = \frac{m}{4\pi l^2}, \text{ along south pole}$$

$$H = H_N + H_S = \frac{m}{2\pi l^2} = \frac{2}{2\pi(2 \times 10^{-2})^2}$$

$$= \frac{1}{4\pi} \times 10^4 \frac{A}{m}, \text{ along south pole}$$

(iii) Magnetic field at the centre of magnet, $\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{I})$

$$= 4\pi \times 10^{-7}\left[-\frac{1}{4\pi} \times 10^4 + 2 \times 10^4\right]$$

$$= -10^{-3} + 8\pi \times 10^{-3} = (8\pi - 1) \times 10^{-3}$$

$$= 2.4 \times 10^{-2} \text{ T, along north pole}$$

Classification of substances on the basis of magnetic behaviour

All substances (whether solid, liquid or gases) may be classified into three categories in terms of their magnetic behaviour (i) paramagnetic, (ii) diamagnetic and (iii) ferromagnetic. *In the following section we will discussed them in details.*

1. Diamagnetic substances

Diamagnetic substances are those which are repelled by a magnetic field.

Examples of such substances are bismuth, antimony, gold, quartz, water, alcohol, etc.

They have the following properties.

(i) These substances when placed in a magnetic field, acquire feeble magnetisation in a direction opposite to that of the applied field. Thus, the lines of induction inside the substance is smaller than that outside to it.

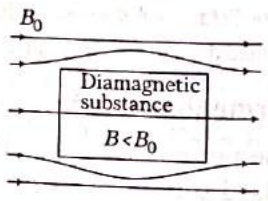


Fig. 5.31 *Magnetic field inside a diamagnetic substance*

(ii) In a non-uniform field these substances move from stronger to weaker parts of the external field. As they acquire magnetisation in opposite direction, hence, these are weekly repelled by the field.

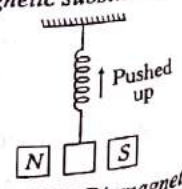


Fig. 5.32 *Diamagnetic substance in external field*

(iii) If a diamagnetic liquid is filled in a narrow U-tube and one limb is placed in between the pole of an electromagnet, then the level of liquid depresses when the field is switched on.

(iv) The relative permeability μ_r is slightly less than 1 *i.e.*, $\mu_r < 1$

(v) The susceptibility χ_m of such substances is always small and negative. It is constant and does not vary with field or the temperature as shown in figure.

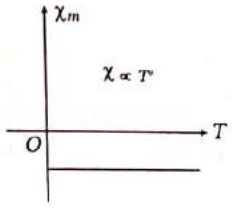


Fig. 5.33 χ_m *versus T graph for diamagnetic substances*

(vi) Here, intensity of magnetisation varies inversely with magnetic field strength i.e., $-I \propto H$ *as shown in figure.*

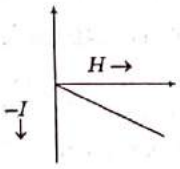


Fig. 5.34 I versus H graph for diamagnetic substances

Note Diamagnetism is a universal property, but in most of the substances, the effect is so weak that they behave like paramagnetic or ferromagnetic, etc.

2. Paramagnetic substances

Paramagnetic substances are those which are attracted by a magnetic field.

Examples of such substances are platinum, aluminium, chromium, manganese, $CuSO_4$ solution, etc.

They have the following properties

(i) The substances when placed in a magnetic field, acquire a feeble magnetisation in the same sense as the applied field.

Thus, the magnetic inductance inside the substance is slightly greater than outside to it.

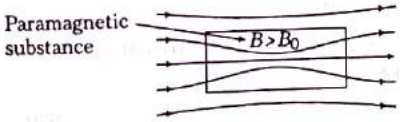


Fig. 5.35 Magnetic field inside a paramagnetic substance

(ii) These substances are attracted towards regions of stronger magnetic field when placed in a non-uniform magnetic field.

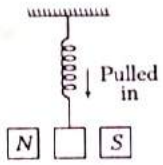


Fig. 5.36 Paramagnetic substance in external field

Thus, they move from weaker to stronger parts of the field.

(iii) If a paramagnetic liquid is filled in a narrow U-tube and one limb is placed in between the pole pieces of an electromagnet such that the level of the liquid is in line with the field, then the liquid will rise in the limb as the field is switched on.

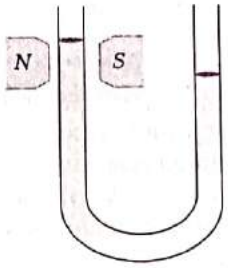


Fig. 5.37

(iv) For paramagnetic substances, the relative permeability μ_r is slightly greater than one.

(v) Magnetic susceptibility is positive.

(vi) *I-H curve can be given as*

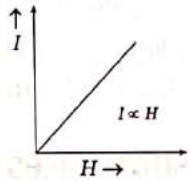


Fig. 5.38 I versus H graph for paramagnetic substances

Curie's law

According to this law, the magnetic susceptibility of paramagnetic substances is inversely proportional to its absolute temperature *i.e.*,

$$\chi_m \propto \frac{1}{T} \Rightarrow \chi_m = \frac{C}{T};$$

where C is Curie constant, T = absolute temperature in kelvin. On increasing temperature, the magnetic susceptibility of paramagnetic material decreases and *vice-versa* as shown is figure. *This variation can be shown as*

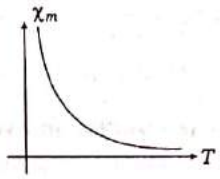


Fig. 5.39 χ_m *versus T graph for paramagnetic substances*

Example 5.47 *The magnetic susceptibility of a paramagnetic material at −73°C is 0.0075. Find its value at −173°C.*

Sol. Magnetic susceptibility,

$$\chi_{m_1} = 0.0075, T_1 = -73°\text{C} = (-73 + 273)\text{ K} = 200\text{ K}$$
$$\chi_{m_2} = ?, T_2 = -173°\text{C} = (-173 + 273)\text{ K} = 100\text{ K}$$

According to Curie's law, $\chi_m \propto \frac{1}{T}$

$\therefore$ Ratio of magnetic susceptibilities, $\frac{\chi_{m_2}}{\chi_{m_1}} = \frac{T_1}{T_2} = \frac{200}{100} = 2$

$$\Rightarrow \quad \chi_{m_2} = 2\chi_{m_1} = 2 \times 0.0075 = 0.015$$

Example 5.48 *A solenoid having 5000 turns/m carries a current of 2A. An aluminium ring at temperature 300K inside the solenoid provides the core. (a) If the magnetisation I is 2×10^{-2} A/m, find the susceptibility of aluminium at 300K. (b) If temperature of the aluminium ring is 320K, what will be the magnetisation?*

Sol. (a) $H = I = 5000 \times 2 = 10^4$ A/m

$$I = \chi H$$
$$\chi = \frac{I}{H} = \frac{2 \times 10^{-2}}{10^4} = 2 \times 10^{-6}$$

(b) According to Curie law

$$\chi = \frac{c}{T} \Rightarrow \frac{\chi_2}{\chi_1} = \frac{T_2}{T_1}$$
$$\chi_2 = \frac{T_2}{T_1}\chi_1 = \frac{320}{300} \times 2 \times 10^{-6} = 2.13 \times 10^{-6}$$

Magnetisation at 320 K

$$I = \chi_2 H = 2.13 \times 10^{-6} \times 10^4 = 2.13 \times 10^{-2}\text{ A/m}$$

3. Ferromagnetic substances

Ferromagnetic substances are those which are strongly attracted by a magnetic field. Examples of such substances are iron, nickel, steel, cobalt and their alloys. These substances resemble to a higher degree with paramagnetic substances as regard with their behaviour.

They have following properties

(i) These substances are strongly magnetised by even a weak magnetic field.

(ii) The relative permeability is very large and is of the order of hundreds and thousands. Here $B >> B_0$.

Fig. 5.40 Magnetic field inside ferromagnetic substance

(iii) The susceptibility is positive and very large.

(iv) Magnetic susceptibility of ferromagnetic substance does not change according to Curie law.

(v) Susceptibility decreases steady with the rise in temperature. Above a certain temperature known as **Curie temperature,** the ferromagnetic substances become paramagnetic. It is 1000°C for iron, 770°C for steel, 360°C for nickel and 1150°C for cobalt.

(vi) *Its I-H curve can be drawn as*

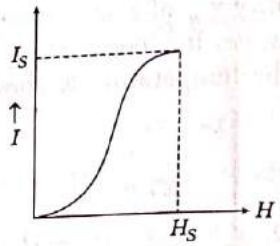


Fig. 5.41 I versus H curve for ferromagnetic substance

Curie-Weiss law

When temperature increases above the Curie temperature, a ferromagnetic substance becomes an ordinary paramagnetic substance whose magnetic susceptibility obeys the Curie-Weiss law as given below

$$\chi_m = \frac{C'}{T - T_C}$$

where, T_C = Curie temperature, C' = constant

Variation of χ_m *versus* T is shown in the figure.

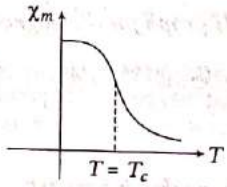


Fig. 5.42 χ_m versus T graph for a magnetic substance

Atomic model of magnetism

We are familiar with the model of an atom which has a nucleus that contains the protons and neutrons and electron in the orbit around the nucleus. Within the atom, the electrons behave as if they are magnet.

The three properties of atoms that give rise to magnetic dipole moment are discussed below.

(i) The electrons moving around the nucleus in the orbits act as small current loops and contribute magnetic moments.

(ii) The spinning electron has an intrinsic magnetic dipole moment.

(iii) The nucleus contribute to magnetic moment due to the motion of charge within the nucleus. The magnitude of nuclear moments is about 10^{-3} times that of electronic moments or the spin magnetic moments as the later two are of the same order. Major contribution in magnetic moment of an atom is produced by electron spin as the net

contribution of the orbital revolution is very small. This is because most of the electrons pair off in such a way that they produce equal and opposite orbital magnetic moment and they cancel out. Although these electrons also try to pair up with their opposite spins but in case of spin motion of an electron it is not always possible to form equal and opposite pairs.

Explanation of magnetism in magnetic substances

On the basis of the above discussed properties, we can *explain the origin of magnetic materials as follow.*

(i) Diamagnetism

The property of diamagnetism is generally found in those substances whose atoms (or molecules) have even number of electrons which form pairs. "The net magnetic moment of an atom of a diamagnetic substance is thus zero." When a diamagnetic substance is placed in an external magnetic field, the spin motion of electrons is so modified that the electrons which produce the magnetic moments in the direction of external field slow down while the electrons which produce magnetic moments in opposite direction get accelerated.

Thus, a net magnetic moment is induced in the opposite direction of applied magnetic field. Hence, the substance is magnetised opposite to the external field.

Note Diamagnetism is temperature independent.

(ii) Paramagnetism

The property of paramagnetism is found in those substances, whose atoms or molecules have an excess of electrons, spinning in the same direction. Hence, atoms of paramagnetic substances have a permanent magnetic moment and behave like tiny bar magnets. In the absence of external magnetic field, the atomic magnets are randomly oriented and net magnetic moment is, thus zero.

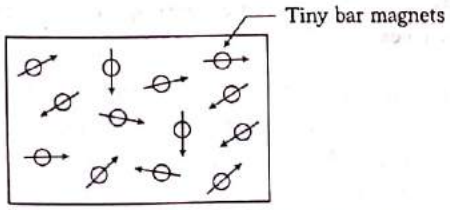


Fig. 5.43 In the absence of external magnetic field

When paramagnetic substance is placed in an external magnetic field, then each atomic magnet experiences a torque which tends to turn the magnet in the direction of the field. The atomic magnets are thus, aligned in the direction of the field. Thus, the whole substance is magnetised in the direction of the external magnetic field.

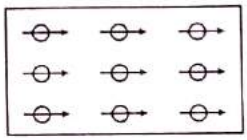


Fig. 5.44 In presence of external magnetic field

As the temperature of substance is increased, the thermal agitation disturbs the magnetic alignment of the atoms. Thus, we can say that paramagnetism is temperature dependent.

(iii) Ferromagnetism

Iron like elements and their alloys are known as ferromagnetic substances. The susceptibility of these substances is in several thousands.

Like paramagnetic substances, atoms of ferromagnetic substances have a permanent magnetic moment and behave like tiny magnets. But in ferromagnetic substances, the atoms form innumerable small effective regions called **domains**.

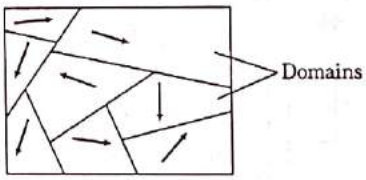


Fig. 5.45 Unmagnetised material

The size of the domain vary from about 10^{-6} cm^3 to 10^{-2} cm^3. Each domain has 10^{17} to 10^{21} atoms whose magnetic moments are aligned in the same direction. In an unmagnetised ferromagnetic specimen, the domains are oriented randomly, so that their resultant magnetic moment is zero.

When the specimen is placed in a magnetic field, then the resultant magnetisation may increase in two different ways.

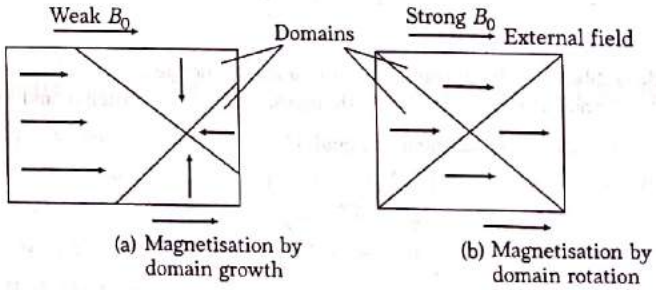
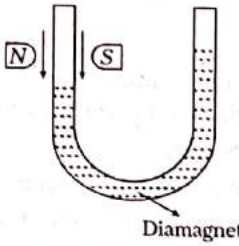
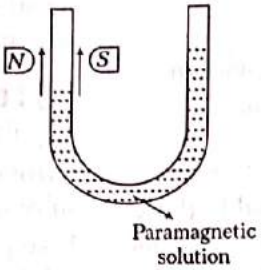
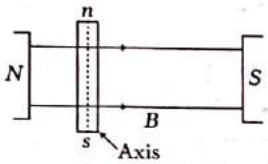
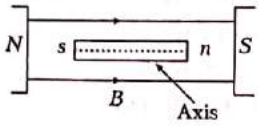
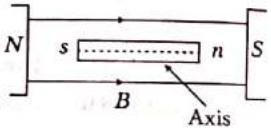
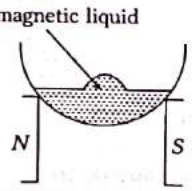
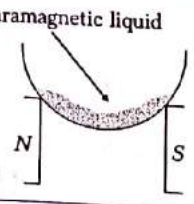


Fig. 5.46 Ferromagnetic substance in (a) weak B_0 and (b) strong B_0

(a) The domains which are oriented favourably with respect to the field will increase in size. Whereas those oriented opposite to the external field will reduce.

(b) The domains rotate towards the field direction.

Comparative study of magnetic materials

S.No.	Diamagnetic substances	Paramagnetic substances	Ferromagnetic substances
1.	These substances when placed in a magnetic field, acquire feeble magnetism opposite to the direction of the magnetic field.	These substances when placed in a magnetic field, acquire feeble magnetism in the direction of the magnetic field.	These substances when placed in a magnetic field are strongly magnetised in the direction of the field.
2.	These substances are repelled by a magnet.	These substances are feebly attracted by a magnet.	These substances are strongly attracted by a magnet.
3.	When a diamagnetic solution is poured into a U-tube and one arm is placed between the poles of strong magnet, then the level of solution in that arm is lowered. 	The level of the paramagnetic solution in that arm rises. 	No liquid is ferromagnetic.
4.	If a rod of diamagnetic material is suspended freely between two magnetic poles, then its axis becomes perpendicular to the magnetic field. 	Paramagnetic rod becomes parallel to the magnetic field. 	Ferromagnetic rod also becomes parallel to the magnetic field. 
5.	In non-uniform magnetic field, the diamagnetic substances are attracted towards the weaker fields *i.e.*, they move from stronger to weaker magnetic field.	In non-uniform magnetic field, they move from weaker to stronger part of the magnetic field slowly.	In non-uniform magnetic field, they move from weaker to stronger magnetic field rapidly.
6.	Their permeability is less than one ($\mu < 1$). $0 < \mu_r < 1, \mu < \mu_0$	Their permeability is slightly greater than one ($\mu > 1$). $1 < \mu_r < 1 + \varepsilon, \mu > \mu_0$	Their permeability is much greater than one ($\mu >> 1$). $\mu_r >> 1, \mu >> \mu_0$
7.	Their susceptibility is small and negative. $[-1 < \chi_m < 0]$. It is independent of temperature.	Their susceptibility is small and positive. $(0 < \chi_m < \varepsilon)$. It is inversely proportional to absolute temperature which is Curie's law *i.e.*, $\chi \propto \frac{1}{T}$.	Their susceptibility is large and positive. $(\chi_m >> 1)$. They also follow Curie's law. *i.e.*, $\chi \propto \frac{1}{T - T_C}$. At Curie temperature, ferromagnetic substances change into paramagnetic substances.
8.	Shape of diamagnetic liquid in a glass crucible and kept over two magnetic poles. 	Shape of paramagnetic liquid in a glass crucible and kept over two magnetic poles. 	No liquid is ferromagnetic.
9.	In these substances, the magnetic lines of force are farther than in air.	In these substances, the magnetic lines of force are closer than in air.	In these substances, magnetic lines of force are much closer than in air.
10.	The resultant magnetic moment of these substances is zero.	These substances have a permanent magnetic moment.	These substances also have a permanent magnetic moment.

Hysteresis

The word hysteresis means delayed. The phenomenon of the lagging of magnetic induction behind the magnetising field is called **hysteresis.**

(i) The plot of intensity of magnetisation (I) of ferromagnetic substances *versus* magnetic intensity (H) for a complete cycle of magnetisation and demagnetisation is called **hysteresis loop.**

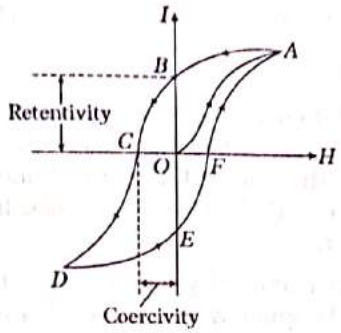


Fig. 5.47 Hysteresis loop for a ferromagnetic substance

(ii) It is clear from the above figure that intensity of magnetising field (H) is increased, the intensity of magnetisation also increases. This is because more and more domains are aligned in the direction of applied field.

(iii) When all domains are aligned material is magnetically saturated. Beyond this, if intensity of magnetising field (H) is increased, then intensity of magnetisation (I) does not increase.

(iv) When intensity of magnetising field (H) is decreased intensity of magnetisation also decreases but it lags behind H. Therefore, when H becomes zero but I does not reduce to zero *i.e.*, the curve does not retrace itself.

(v) Now this value of intensity of magnetisation (I) which is left in the material at $H = 0$, is called **retentivity** or **remanence** (BO). This occurs because all domains do not dealign even, if $H = 0$.

(vi) When the magnetising field is applied in reverse direction and its intensity H, is increased. The material starts demagnetising. The value of magnetising field needed to reduce magnetisation to zero is called **coercivity** (OC).

(vii) If reverse magnetising field is increased further, the material becomes saturated. Now, if the magnetising field is reduced after attaining the reverse saturation, the cycle repeats itself.

(viii) The area enclosed by the loop represents loss of energy during a cycle of magnetisation and demagnetisation.

(ix) Unit of area of I-H loop is A^2m^{-2}.

(x) Unit of area of BH loop
$= \mu_0 \times$ unit of area of I-H loop.

(xi) Area under B-H loop is equal to the energy loss per cycle per unit volume.

Example 5.49 *The hysteresis loss for a specimen of iron weighing 15 kg is equivalent to 300 Jm^{-3} $cycle^{-1}$. Find the loss of energy per hour at 25 cycle s^{-1}. Density of iron is 7500 kg m^{-3}.*

Sol. Let Q be the energy dissipated per unit volume per hysteresis cycle in the given sample. Then the total energy lost by the volume V of the sample in time t will be

$$W = Q \times V \times \nu \times t$$

where ν is the number of hysteresis cycles per second.

Here, $Q = 300\ \text{Jm}^{-3}\ \text{cycle}^{-1}$, $\nu = 25\ \text{cycle s}^{-1}$, $t = 1\text{h} = 3600\ \text{s}$

Volume, $V = \dfrac{\text{Mass}}{\text{Density}} = \dfrac{15}{7500}\ \text{m}^3$

$\therefore$ Hysteresis loss, $W = 300 \times \dfrac{15}{7500} \times 25 \times 3600\ \text{J} = 54000\ \text{J}$

Example 5.50 *The coercivity of a certain permanent magnet is 4.0×10^4 Am^{-1}. The magnet is placed inside a solenoid 20 cm long and having 700 turns and a current is passed in the solenoid to demagnetise it completely. Find the current.*

Sol. The coercivity of $4 \times 10^4\ \text{Am}^{-1}$ of the permanent magnet implies that a magnetic intensity $H = 4 \times 10^4\ \text{Am}^{-1}$ is required to be applied in opposite direction to demagnetise the magnet.

Here $n = \dfrac{700}{20\ \text{cm}} = \dfrac{700}{20 \times 10^{-2}\ \text{m}} = 3500\ \text{turns/m}$

As $H = nI$

$\therefore$ Current, $I = \dfrac{H}{n} = \dfrac{4 \times 10^4}{3500} = 11.5\ \text{A}$

Magnetic properties of soft Iron and steel

A comparison of the magnetic properties of ferromagnetic substances can be made by the comparison of the shapes and sizes of their hysteresis loops.

These are two types of ferromagnetic materials.

(i) **Soft magnetic materials** These are those materials which have low retentivity, coercivity and small hysteresis loss *e.g.*, Soft iron.

(ii) **Hard magnetic materials** These are those materials which have high retentivity, coercivity and large hysteresis loss. *e.g.*, steel.

Hysteresis loop for soft iron and steel can be drawn as below.

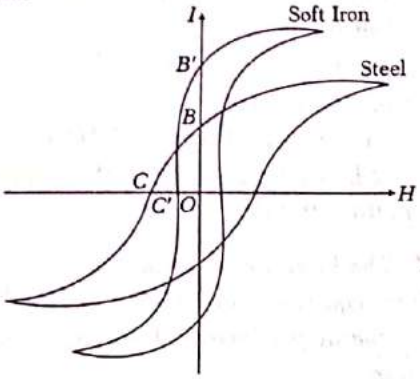


Fig. 5.48 Hysteresis loop for soft iron and steel

Following three conclusions can be drawn from their hysteresis loops

(i) Retentivity of soft iron is more than the retentivity of steel.

(ii) Coercivity of soft iron is less than the coercivity of steel.

(iii) Area of hysteresis loop (*i.e.*, hysteresis loss) in soft iron is smaller than that in steel.

Choice of magnetic materials

The choice of a magnetic material for different uses is decided from the hysteresis curve of a specimen of the material.

(i) **Permanent magnets** The materials for a permanent magnet should have

(a) high retentivity (so that the magnet is strong) and

(b) high coercivity (so that the magnetising is not wiped out by stray magnetic fields). As the material in this case is never put to cyclic changes of magnetisation, hence hysteresis is immaterial. From the point of view of these facts **steel** is more suitable for the construction of permanent magnets than soft iron.

Modern permanent magnets are made of 'cobalt-steel', alloys 'ticonal'.

(ii) **Electromagnets** The materials for the construction of electromagnets should have,

(a) high initial permeability,

(b) low hysteresis loss.

From the view point of these facts, **soft iron** is an ideal material for this purpose.

(iii) **Transformer cores and telephone diaphragms** As the magnetic material used in these cases is subjected to cyclic changes. Thus, the essential requirements for the selection of the material are

(a) high initial permeability,

(b) low hysteresis loss to prevent the breakdown.

Electromagnet

As we know that a current carrying solenoid behaves like a bar magnet. If we place a soft iron rod in the solenoid, then the magnetism of the solenoid increases hundreds of times and the solenoid is called an electromagnet. It is a temporary magnet.

An electromagnet is made by winding closely a number of turns of insulated copper wire over a soft iron straight rod or a horse shoe rod. On passing current through this solenoid, a magnetic field is produced in the space within the solenoid.

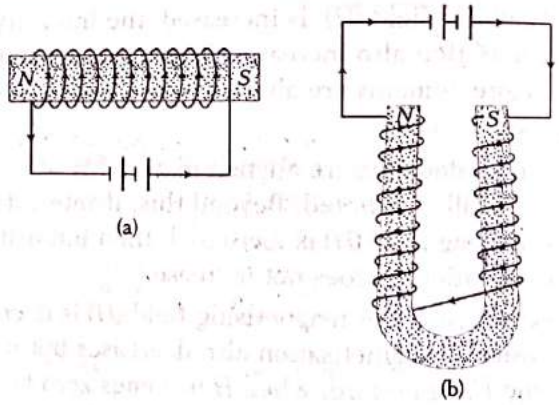


Fig. 5.49 An electromagnet

Applications of Electromagnets

(i) Electromagnets are used in electric bell, transformer, telephone diaphragms, etc.

(ii) In medical field they are used in extracting bullets from the human body.

(iii) Large electromagnets are used in cranes for lifting and transferring big machines and parts.

Check point 5.3

1. Which one of the following is a non-magnetic substance?
   (a) Iron (b) Nickel
   (c) Cobalt (d) Brass

2. The SI unit of magnetic permeability is
   (a) Am^{-1}
   (b) A-m
   (c) Henry m^{-1}
   (d) No unit, it is a dimensionless number

3. The unit of magnetic susceptibility is
   (a) H (b) Wb/m
   (c) A/m (d) None of these

4. The relation connecting B, H and I in SI system is
   (a) $B = H + I$ (b) $B = H - I$
   (c) $B = \mu_0 (H + I)$ (d) $B = \mu_0 (H - I)$

5. An example of a diamagnetic substance is
   (a) aluminium (b) copper
   (c) iron (d) nickel

6. The universal property of all substances is
   (a) diamagnetism
   (b) ferromagnetism
   (c) paramagnetism
   (d) All of these

7. Which magnetic materials have negative susceptibility?
   (a) Diamagnetic materials (b) Paramagnetic materials
   (c) Ferromagnetic materials (d) All of these

8. Identify the paramagnetic substance.
   (a) Iron (b) Aluminium
   (c) Nickel (d) Hydrogen

9. Which of the following is true?
   (a) Diamagnetism is temperature dependent
   (b) Paramagnetism is temperature dependent
   (c) Paramagnetism is temperature independent
   (d) None of the above

10. Magnetic permeability is maximum for
    (a) diamagnetic substance (b) paramagnetic substance
    (c) ferromagnetic substance (d) All of these

11. The temperature at which a ferromagnetic material becomes paramagnetic is called a
    (a) neutral temperature (b) Curie temperature
    (c) inversion temperature (d) critical temperature

12. Substances in which the magnetic moment of a single atom is not zero, is known as
    (a) diamagnetism (b) ferromagnetism
    (c) paramagnetism (d) ferrimagnetism

13. Liquid oxygen remains suspended between two pole faces of a magnet because it is
    (a) diamagnetic (b) paramagnetic
    (c) ferromagnetic (d) antiferromagnetic

14. The only property possessed by ferromagnetic substance is
    (a) hysteresis
    (b) susceptibility
    (c) directional property
    (d) attracting magnetic substances

15. The permanent magnet is made from which one of the following substances?
    (a) Diamagnetic (b) Paramagnetic
    (c) Ferromagnetic (d) Electromagnetic

Chapter exercises

A. Taking it together

(Assorted Questions of the Chapter for Advanced Level Practice)

1. A magnet is placed in iron powder and then taken out, then maximum iron powder is at
(a) some distance away from north pole
(b) some distance away from south pole
(c) the middle of the magnet
(d) the end of the magnet

2. A permanent magnet
(a) attracts all substances
(b) attracts only magnetic substances
(c) attracts magnetic substances and repels all non-magnetic substances
(d) attracts non-magnetic substances and repels magnetic substances

3. Magnetic field is measured by
(a) pyrometer (b) hydrometer
(c) thermometer (d) fluxmeter

4. Lines which represent places of constant angle of dip are called
(a) isobaric lines (b) isogonic lines
(c) isoclinic lines (d) isodynamic lines

5. A line passing through places having zero value of magnetic dip is called
(a) isoclinic line (b) agonic line
(c) isogonic line (d) aclinic line

6. Aclinic lines are the lines joining places of
(a) zero dip (b) equal dip
(c) zero declination (d) equal declination

7. The arms of a deflection magnetometer in the tan B position are placed
(a) east-west (b) north-south
(c) north-east (d) south-west

8. If the current is doubled, the deflection is also doubled in
(a) a tangent galvanometer
(b) a moving coil galvanometer
(c) Both (a) and (b)
(d) None of the above

9. Which of the following is diamagnetic?
(a) Aluminium (b) Quartz
(c) Nickel (d) Bismuth

10. The permeability of paramagnetic substance is
(a) slightly more than vacuum
(b) slightly less than vacuum
(c) much more than vacuum
(d) None of the above

11. Unit of magnetic flux density (or magnetic induction) is
(a) tesla (b) weber/metre^2
(c) newton/ampere-metre (d) All of these

12. Magnetic field intensity is defined as
(a) magnetic moment per unit volume
(b) magnetic induction force acting on a unit magnetic pole
(c) number of lines of force crossing per unit area
(d) number of lines of force crossing per unit volume

13. Permeability is defined as the ratio between
(a) magnetic induction and susceptibility
(b) magnetic induction and magnetising field
(c) magnetising field and magnetic induction
(d) magnetising field and susceptibility

14. Hysteresis loss for steel is ..... that for iron.
(a) lesser than (b) equal to
(c) greater than (d) Either (b) or (c)

15. Hysteresis is exhibited by a ..... substance.
(a) paramagnetic (b) ferromagnetic
(c) diamagnetic (d) All of these

16. Which of the following materials has got the maximum retentivity?
(a) Copper (b) Zinc
(c) Soft iron (d) Hard iron

17. The area enclosed by a hysteresis loop is a measure of
(a) retentivity (b) susceptibility
(c) permeability (d) energy loss per cycle

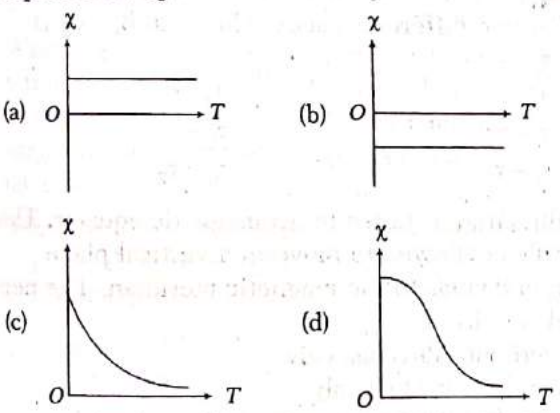
18. Which of the following is the most suitable material for making permanent magnet?
(a) Steel (b) Soft iron
(c) Copper (d) Nickel

19. The materials suitable for making electromagnets should have
(a) high retentivity and high coercivity
(b) low retentivity and low coercivity
(c) high retentivity and low coercivity
(d) low retentivity and high coercivity

20. Which of the following is most suitable for the core of electromagnets?
(a) Iron (b) Steel
(c) Soft iron (d) Cu-Ni alloy

21. A magnetic needle kept in a non-uniform magnetic field experiences
(a) a force and torque
(b) a force but not a torque
(c) a torque but not a force
(d) Neither a torque nor a force

22. The variation of magnetic susceptibility (χ) with temperature for a diamagnetic substance is best represented by



23. The angle between the earth's magnetic and the earth's geographic axis is
(a) zero (b) 11.5°
(c) 23° (d) None of these

24. If a magnet is hanged with its magnetic axis, then it stops in
(a) magnetic meridian
(b) geometric meridian
(c) angle of dip
(d) None of the above

25. A dip needle in a plane perpendicular to magnetic meridian will remain
(a) vertical
(b) horizontal
(c) in any direction
(d) at an angle of dip to the horizontal

26. A dip circle is at right angles to the magnetic meridian. What will be the apparent dip?
(a) 0° (b) 30°
(c) 60° (d) 90°

27. A compass needle which is allowed to move in a horizontal plane is taken to a geomagnetic pole. It will
(a) stay in north-south direction only
(b) stay in east-west direction only
(c) become rigid showing no movement
(d) stay in any position

28. When the N-pole of a bar magnet points towards the south and S-pole towards the north, the null points are at the
(a) magnetic axis
(b) magnetic centre
(c) perpendicular divider of magnetic axis
(d) N and S poles

29. Due to earth's magnetic field charged cosmic ray particles
(a) require greater kinetic energy to reach the equator than pole
(b) require less kinetic energy to reach the equator than pole
(c) can never reach the pole
(d) can never reach the equator

30. A magnetic needle, suspended horizontally by an unspun silk fibre, oscillates in the horizontal plane, because of a restoring force originating mainly from
(a) the torsion of the silk fibre
(b) the force of gravity
(c) the horizontal component of earth's magnetic field
(d) All the above factors

31. An electron moving around the nucleus with an angular momentum l has a magnetic moment
(a) $\frac{e}{m}l$ (b) $\frac{e}{2m}l$
(c) $\frac{2e}{m}l$ (d) $\frac{e}{2\pi m}l$

32. A vibration magnetometer is placed at the south pole, then the time period will be
(a) zero
(b) infinity
(c) same as at magnetic equator
(d) same as at any other place on earth

33. Which of the following statements are true about the magnetic susceptibility χ_m of paramagnetic substance?
(a) Value of χ_m is inversely proportional to the absolute temperature of the sample
(b) χ_m is positive at all temperature
(c) χ_m is negative at all temperature
(d) χ_m does not depend on the temperature of the sample

34. The angle which the total magnetic field of earth makes with the surface of the earth is called
(a) declination (b) magnetic meridian
(c) geographic meridian (d) inclination

35. Resultant force acting on a diamagnetic material in a magnetic field is in direction
(a) from stronger to the weaker part of the magnetic field
(b) from weaker to the stronger part of the magnetic field
(c) perpendicular to the magnetic field
(d) in the direction making 60° to the magnetic field

36. The mathematical equation for magnetic field lines of force is
(a) $\nabla \cdot \mathbf{B} = 0$ (b) $\nabla \cdot \mathbf{B} \neq 0.1$
(c) $\nabla \cdot \mathbf{B} > 0$ (d) $\nabla \cdot \mathbf{B} < 0$

37. Two lines of force due to a bar magnet
(a) intersect at the neutral point
(b) intersect near the poles of the magnet
(c) intersect on the equatorial axis of the magnet
(d) do not intersect at all

38. What happens to the force between magnetic poles when their pole strength and the distance between them both gets doubled?
(a) Force increases to two times the previous value
(b) No change
(c) Force decreases to half the previous value
(d) Force increases to four times the previous value

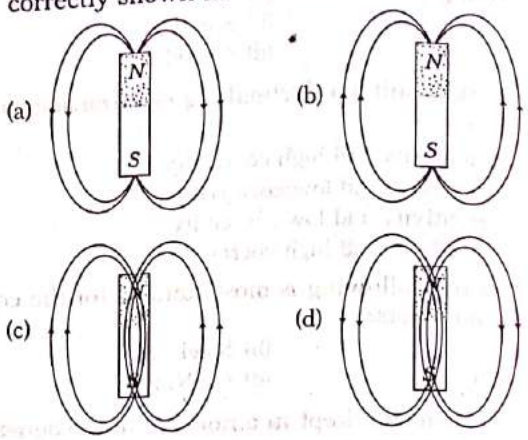
39. If a magnet of pole strength m is divided into four parts such that the length and width of each part is half that of initial one, then the pole strength of each part will be
(a) $m/4$ (b) $m/2$
(c) $m/8$ (d) $4m$

40. Two magnets have the same length and the same pole strength. But one of the magnets has a small hole at its centre. Then
(a) both have equal magnetic moment
(b) one with hole has smaller magnetic moment
(c) one with hole has larger magnetic moment
(d) one with hole loses magnetism through the hole

41. The magnetic field at a distance d from a short bar magnet in longitudinal and transverse positions are in the ratio
(a) 1:1 (b) 2:3
(c) 2:1 (d) 3:2

42. When a diamagnetic substance is brought near north or south pole of a bar magnet, it is
(a) attracted by the poles
(b) repelled by the poles
(c) attracted by the north pole and repelled by the south pole
(d) repelled by the north pole and attracted by the south pole

43. The magnetic field lines due to a bar magnet are correctly shown in



44. Susceptibility is positive and large for a
(a) paramagnetic substance
(b) ferromagnetic substance
(c) non-magnetic substance
(d) diamagnetic substance

45. Torques τ_1 and τ_2 are required for a magnetic needle to remain perpendicular to the magnetic fields B_1 and B_2 at two different places. The ratio B_1/B_2 is
(a) $\frac{\tau_2}{\tau_1}$ (b) $\frac{\tau_1}{\tau_2}$
(c) $\frac{\tau_1+\tau_2}{\tau_1-\tau_2}$ (d) $\frac{\tau_1-\tau_2}{\tau_1+\tau_2}$

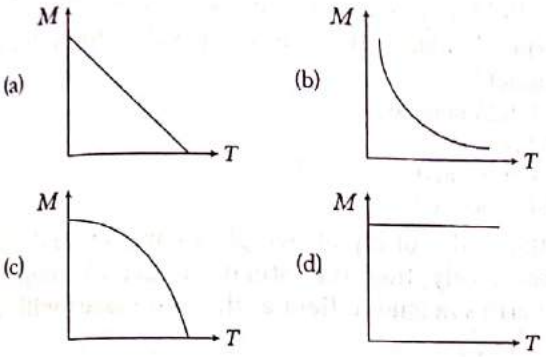
46. A dip circle is taken to geomagnetic equator. The needle is allowed to move in a vertical plane perpendicular to the magnetic meridian. The needle will stay in
(a) horizontal direction only
(b) vertical direction only
(c) any direction except vertical and horizontal
(d) None of the above

47. At the magnetic north pole of the earth, the values of the horizontal component H and the angle of dip θ are
(a) $H = 0, \theta = 45°$ (b) $H = B_e, \theta = 0°$
(c) $H = 0, \theta = 90°$ (d) $H = B_e, \theta = 90°$

48. When the magnetic inclination (dip) was measured at various places on earth, in one of the following countries it was found to be zero. Which one was it?
(a) Pakistan (b) Brazil
(c) Scotland (d) Canada

49. In a deflection magnetometer, the needle is short and the pointer is long because, the
(a) needle cannot be made long
(b) circular scale cannot be made short
(c) needle must be in a uniform field
(d) pointer must be in a non-uniform field

50. A curve between magnetic moment and temperature of magnet is



51. The tangents deflection produced in tan A and tan B positions by a short magnet at equal distances are in the ratio

(a) 1 : 1 (b) 2 : 1
(c) 1 : 2 (d) None of these

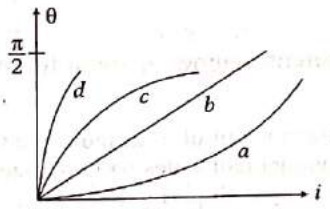
52. The relative permeability is represented by μ_r and the susceptibility is denoted by χ for a magnetic substance. Then for a paramagnetic substance

(a) $\mu_r < 1, \chi < 0$ (b) $\mu_r < 1, \chi > 0$
(c) $\mu_r > 1, \chi < 0$ (d) $\mu_r > 1, \chi > 0$

53. When a piece of a ferromagnetic substance is put in a uniform magnetic field,the flux density inside it is four times the flux density away from the piece. The magnetic permeability of the material is

(a) 1 (b) 2 (c) 3 (d) 4

54. Which of the four graphs may best represent the current- deflection relation in a tangent galvanometer?



(a) d (b) a (c) c (d) b

55. If a diamagnetic solution is poured into a U-tube and one arm of this U-tube placed between the poles of a strong magnet with the meniscus in a line with the field, then the level of the solution will

(a) rise (b) fall
(c) oscillate slowly (d) remain as such

56. A paramagnetic liquid is filled in a glass U-tube of which one limb is placed between the pole pieces of an electromagnet. When the field is switched on the liquid in the limb which is in the field will

(a) rise (b) fall
(c) remain stationary (d) first rise and then fall

57. The magnetic moment of a magnet of length 10 cm and pole strength 4.0 Am will be

(a) 0.4 Am^2 (b) 1.6 Am^2
(c) 20 Am^2 (d) 8.0 Am^2

58. All the magnetic materials loss their magnetic properties when

(a) dipped in water
(b) dipped in oil
(c) brought near a piece of iron
(d) strongly heated

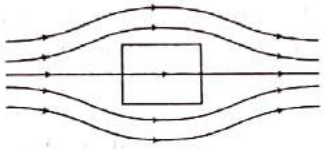
59. A ferromagnetic material is heated above its Curie temperature. Which one is a correct statement?

(a) Ferromagnetic domains are perfectly arranged
(b) Ferromagnetic domains become random
(c) Ferromagnetic domains are not influenced
(d) Ferromagnetic material changes into diamagnetic material

60. Above the Curie temperature, the susceptibility of a ferromagnetic substance varies

(a) directly as the absolute temperature
(b) inversely as the absolute temperature
(c) inversely as the square of absolute temperature
(d) directly as the square of absolute temperature

61. The given figure represents a material which is



(a) paramagnetic (b) diamagnetic
(c) ferromagnetic (d) None of these

62. A magnetic wire of dipole moment $4\pi Am^2$ bent in the form of semi-circle. The new magnetic moment is

(a) $4\pi Am^2$ (b) $8\pi Am^2$
(c) $4 Am^2$ (d) None of these

63. A magnet of magnetic moment M and pole strength m is cut in two equal parts along the axis of magnet, then magnetic moment of each part will be

(a) M (b) $M/2$ (c) $M/4$ (d) $2M$

64. Two identical thin bar magnets each of length l and pole strength m are placed at right angles to each other with north pole of one touching south pole of the other. Magnetic moment of the system is

(a) ml (b) $2ml$ (c) $\sqrt{2}ml$ (d) $\frac{1}{2}ml$

65. A short bar magnet placed with its axis at 30° with a uniform external magnetic field of 0.16 T experiences a torque of magnitude 0.032 J. The magnetic moment of the bar magnet will be

(a) 0.23 J/T (b) 0.40 J/T
(c) 0.80 J/T (d) zero

66. A bar magnet when placed at an angle of 30° to the direction of magnetic field induction of 5×10^{-2} T, experiences a moment of couple 25×10^{-6} N-m. If the length of the magnet is 5 cm, its pole strength is
(a) 2×10^{-2} A-m (b) 5×10^{-2} A-m
(c) 2 A-m (d) 5 A-m

67. A bar magnet of magnetic moment 3 A-m^2 is placed in a uniform magnetic induction field of 2×10^{-5} T. If each pole of the magnet experiences a force of 6×10^{-4} N, the length of the magnet is
(a) 0.5 m (b) 0.3 m
(c) 0.2 m (d) 0.1 m

68. A toroid of n turns, mean radius R and cross-sectional radius a carries current I. It is placed on a horizontal table taken as XY-plane. Its magnetic moment m **[NCERT Exemplar]**
(a) is non-zero and points in the Z-direction by symmetry
(b) points along the axis of the toroid ($\mathbf{m} = m\,\hat{\phi}$)
(c) is zero, otherwise there would be a field falling as $\frac{1}{r^3}$ at large distances outside the toroid
(d) is pointing radially outwards

69. A magnet of magnetic moment M is situated with its axis along the direction of a magnetic field of strength B. The work done is rotating it by an angle of 180° will be
(a) $-MB$ (b) $+MB$
(c) zero (d) $+2MB$

70. A magnet of magnetic moment 2 J/T is aligned in the direction of magnetic field of 0.1 T. What is the net work done to bring the magnet normal to the magnetic field?
(a) 0.1 J (b) 0.2 J (c) 1 J (d) 2 J

71. A planar coil having 15 turns carries 20 A current. The coil is oriented with respect to the uniform magnetic field $\mathbf{B} = 0.5\,\text{T}$ such that its directed area is $\mathbf{A} = -0.04\,\hat{\mathbf{i}}\,\text{m}^2$. The potential energy of the coil in the given orientation is
(a) 0 (b) + 0.72 J (c) 6 J (d) −1.44 J

72. The magnetic field of the earth can be modelled by that of a point dipole placed at the centre of the earth. The dipole axis makes an angle of 11.3° with the axis of the earth. At Mumbai, declination is nearly zero. Then, **[NCERT Exemplar]**
(a) the declination varies between 11.3° W to 11.3° E
(b) the least declination is 0°
(c) the plane defined by dipole axis and the earth axis passes through Greenwich
(d) declination averaged over the earth must be always negative

73. The magnetic field on the axis of a short bar magnet at a distance of 10 cm is 0.2 oersted. What will be the field at a point, distant 5 cm on the line perpendicular to the axis and passing through the magnet?
(a) 0.025 oersted
(b) 0.2 oersted
(c) 0.4 oersted
(d) 0.8 oersted

74. If the angles of dip at two places are 30° and 45° respectively, then the ratio of horizontal components of earth's magnetic field at the two places will be
(a) $\sqrt{3} : \sqrt{2}$
(b) $1 : \sqrt{2}$
(c) $1 : \sqrt{3}$
(d) 1 : 2

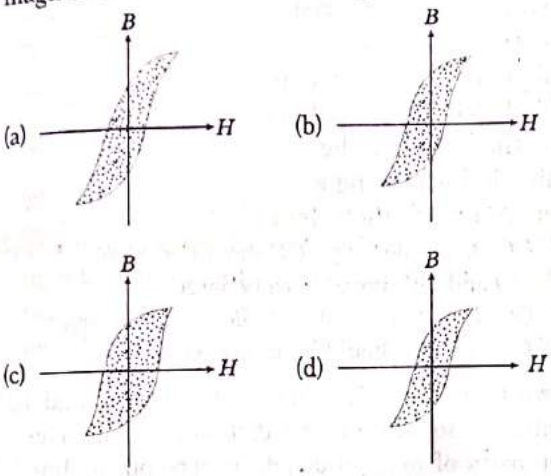
75. The earth's magnetic field at a certain place has a horizontal component 0.3 G and the total strength 0.5 G. The angle of dip is
(a) $\tan^{-1}\left(\frac{3}{4}\right)$ (b) $\sin^{-1}\left(\frac{3}{4}\right)$
(c) $\tan^{-1}\left(\frac{4}{3}\right)$ (d) $\sin^{-1}\left(\frac{3}{5}\right)$

76. At a certain place, the angle of dip is 30° and the horizontal component of earth's magnetic field is 0.50 oersted. The earth's total magnetic field (in oersted) is
(a) $\sqrt{3}$
(b) 1
(c) $1/\sqrt{3}$
(d) $\frac{1}{2}$

77. In a permanent magnet at room temperature, **[NCERT Exemplar]**
(a) magnetic moment of each molecule is zero
(b) the individual molecules have non-zero magnetic moment which are all perfectly aligned
(c) domains are partially aligned
(d) domains are all perfectly aligned

78. A dip needle lies initially in the magnetic meridian when it shows an angle of dip θ at a place. The dip circle is rotated through an angle x in the horizontal plane and then it shows an angle of dip θ′. Then $\frac{\tan\theta'}{\tan\theta}$ is
(a) $1/\cos x$
(b) $1/\sin x$
(c) $1/\tan x$
(d) $\cos x$

79. For different substances hysteresis (*B-H*) curves are given as shown in figure. For making temporary magnet which of the following is best?

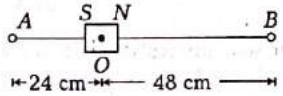


80. A bar magnet is oscillating in the earth's magnetic field with time period T. If its mass is increased four times, then its time period will be
(a) $4T$ (b) $2T$ (c) T (d) $T/2$

81. When 2 A current is passed through a tangent galvanometer, it gives a deflection of 30°. For 60° deflection the current must be
(a) 1 A (b) $2\sqrt{3}$ A (c) 4 A (d) 6 A

82. Two tangent galvanometers having coils of the same radius are connected in series. A current flowing in them produces deflections of 60° and 45° respectively. The ratio of the number of turns in the coils is
(a) $4\sqrt{3}$ (b) $(\sqrt{3}+1)/1$ (c) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$ (d) $\frac{\sqrt{3}}{1}$

83. A bar magnet of length 3 cm has points A and B along its axis at distances of 24 cm and 48 cm on the opposite sides. Ratio of magnetic fields at these points will be



(a) 8 (b) $\frac{1}{2\sqrt{2}}$ (c) 3 (d) 4

84. The magnetic moment produced in a substance of 1g is 6×10^{-7} Am^2. If its density is 5 g/cm^3, then the intensity of magnetisation in A/m will be
(a) 8.3×10^{6} (b) 3.0
(c) 1.2×10^{-7} (d) 3×10^{-6}

85. A short bar magnet is arranged with its north pole pointing geographical north. It is found that the horizontal component of earth's magnetic induction (B_H) is balanced by the magnetic induction of the magnet at a point which is at a distance of 20 cm from its centre. The magnetic moment of the magnet is (if $H = 4\times10^{-5}$ Wbm^{-2})
(a) 3.2 Am^2 (b) 1.6 Am^2
(c) 6.4 Am^2 (d) 0.8 Am^2

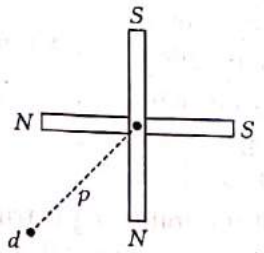
86. A long magnetic needle of length $2L$, magnetic moment M and pole strength m units is broken into two at the middle. The magnetic moment and pole strength of each piece will be
(a) $(M/2), m/2$ (b) $M, m/2$
(c) $M/2, m$ (d) M, m

87. Consider the two idealised systems (i) a parallel plate capacitor with large plates with small separation and (ii) a long solenoid of length $L >> R$, radius of cross-section. In (i) **E** is ideally treated as a constant between plates and zero outside. In (ii) magnetic field is constant inside the solenoid and zero outside. These idealised assumptions, however, contradict fundamental laws as below **[NCERT Exemplar]**
(a) Case (i) contradicts Gauss's law for electrostatic fields
(b) Case (ii) contradicts Gauss's law for magnetic fields
(c) Case (i) agrees with $\oint \mathbf{E}\cdot d\mathbf{l} = 0$.
(d) Case (ii) contradicts $\oint \mathbf{H}\cdot d\mathbf{l} = I_{em}$

88. Due to a small magnet, intensity at a distance x in the end on position is 9 gauss. What will be the intensity at a distance $\frac{x}{2}$ on broad side on position?
(a) 9 gauss (b) 4 gauss (c) 36 gauss (d) 4.5 gauss

89. A magnet oscillating in a horizontal plane has a time period of 2 s at a place where the angle of dip is 30° and 3 s at another place where the angle of dip is 60°. The ratio of resultant magnetic fields at the two places is
(a) $\frac{4\sqrt{3}}{7}$ (b) $\frac{4}{9\sqrt{3}}$ (c) $\frac{9}{4\sqrt{3}}$ (d) $\frac{9}{\sqrt{3}}$

90. Two short magnets of equal dipole moments M are fastened perpendicularly at their centres (figure). The magnitude of the magnetic field at a distance d from the centre on the bisector of the right angle is

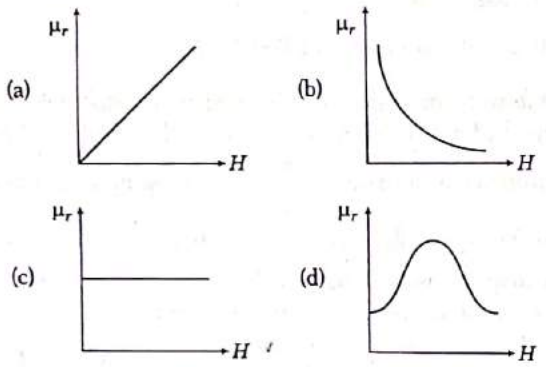


(a) $\frac{\mu_0}{4\pi}\frac{M}{d^3}$ (b) $\frac{\mu_0}{4\pi}\frac{\sqrt{2}M}{d^3}$
(c) $\frac{\mu_0}{4\pi}\frac{2\sqrt{2}M}{d^3}$ (d) $\frac{\mu_0}{4\pi}\frac{2M}{d^3}$

91. Two bar magnets of the same mass, length and breadth having magnetic moments M and $2M$ are joined together pole to pole and suspended in a vibration magnetometer. The time period of oscillation is 3 s. If the polarity of one of the magnets is reversed, the time period of oscillation will be
(a) $\sqrt{3}$ s
(b) $3\sqrt{3}$ s
(c) 3 s
(d) 6 s

92. A thin rectangular magnet suspended freely has a period of oscillation 4 s. If it is broken into two halves each having half their initial length, then when suspended similarly, the time period of oscillation of each part will be
(a) 4 s (b) 2 s
(c) 1 s (d) $2\sqrt{2}$ s

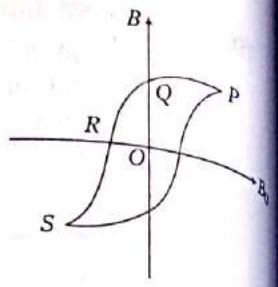
93. For ferromagnetic material, the relative permeability (μ_r), *versus* magnetic intensity (H) has the following shape



94. Two magnets of same size and mass make respectively 10 and 15 oscillations per minute at certain place. The ratio of their magnetic moments is
(a) 4 : 9 (b) 9 : 4
(c) 2 : 3 (d) 3 : 2

95. There are four light-weight-rod samples A, B, C and D separately suspended by threads. A bar magnet is slowly brought near each sample and the following observations are noted.
(i) A is feebly repelled
(ii) B is feebly attracted
(iii) C is strongly attracted
(iv) D remains unaffected
Which one of the following is true?
(a) C is of a diamagnetic material
(b) D is of a ferromagnetic material
(c) A is of a non-magnetic material
(d) B is of a paramagnetic material

96. The figure illustrates how B, the flux density inside a sample of unmagnetised ferromagnetic material varies with B_0, the magnetic flux density in which the sample is kept. For the sample to be suitable for making a permanent magnet, then



(a) OQ should be large, OR should be small
(b) OQ and OR should both be large
(c) OQ should be small and OR should be large
(d) OQ and OR should both be small

97. Two like magnetic poles of strength 10 and 40 SI units are separated by a distance 30 cm. The intensity of magnetic field is zero on the line joining them
(a) At a point 10 cm from the stronger pole
(b) At a point 20 cm from the stronger pole
(c) At the mid-point
(d) At infinity

98. A magnet makes 40 oscillations per minute at a place having magnetic field intensity of 0.1×10^{-5} T. At another place, it takes 2.5 s to complete one vibration. The value of earth's horizontal field at that place is
(a) 0.25×10^{-6} T (b) 0.36×10^{-6} T
(c) 0.66×10^{-8} T (d) 1.2×10^{-6} T

99. A circular coil of radius 20 cm and 20 turns of wire is mounted vertically with its plane in magnetic meridian. A small magnetic needle is placed at the centre of the coil and it is deflected through 45° when a current is passed through the coil. Horizontal component of earth's field is 0.37×10^{-4} T. The current in coil is
(a) 0.6 A (b) 6 A
(c) 6×10^{-3} A (d) 0.06 A

100. A dip circle is so that its needle moves freely in the magnetic meridian. In this position, the angle of dip is 40°. Now, the dip circle is rotated so that the plane in which the needle moves makes an angle of 30° with the magnetic meridian. In this position the needle will dip by an angle
(a) 40° (b) 30°
(c) more than 40° (d) less than 40°

101. An iron rod of 0.2 cm^2 cross-sectional area is subjected to a magnetising field of 1200 Am^{-1}. The susceptibility of iron is 599. The magnetic flux produced is
(a) 0.904 Wb (b) 1.81×10^{-5} Wb
(c) 0.904×10^{-5} Wb (d) 5.43×10^{-5} Wb

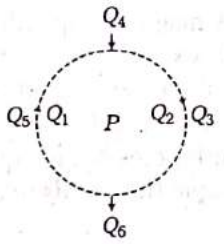
102. A paramagnetic sample shows a net magnetisation of $8\ Am^{-1}$ when placed in an external magnetic field of 0.6 T at a temperature of 4 K. When the same sample is placed in an external magnetic field of 0.2 T at a temperature of 16 K, the magnetisation will be **[NCERT Exemplar]**

(a) $\frac{32}{3} Am^{-1}$ (b) $\frac{2}{3} Am^{-1}$
(c) $6 Am^{-1}$ (d) $2.4\ Am^{-1}$

103. The plane of a dip circle is set in the geographic meridian and the apparent dip is δ_1. It is then set in vertical plane perpandicular to the geographic meridian. The appartent dip angle is δ_2. The declination θ at the plane is

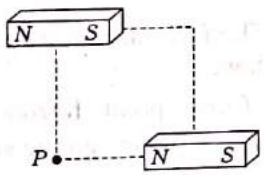
(a) $\theta = \tan^{-1}(\tan\delta_1 \tan\delta_2)$
(b) $\theta = \tan^{-1}(\tan\delta_1 + \tan\delta_2)$
(c) $\theta = \tan^{-1}\left(\frac{\tan\delta_1}{\tan\delta_2}\right)$
(d) $\theta = \tan^{-1}(\tan\delta_1 - \tan\delta_2)$

104. The figure shows the various positions (labelled by subscripts) of small magnetsed needles P and Q. The arrows show the direction of their magnetic moment. Which configuration corresponds to the lowest potential energy of all the configurations shown?



(a) PQ_3 (b) PQ_4
(c) PQ_5 (d) PQ_6

105. Two short magnets of magnetic moment $1000\ Am^2$ are placed as shown at the corners of a square of side 10 cm. The net magnetic induction at P is



(a) 0.1 T (b) 0.2 T
(c) 0.3 T (d) 0.4 T

106. Two magnets are held together in a vibration magnetometer and are allowed to oscillate in the earth's magnetic field with like poles together. 12 oscillations per minute are made but for unlike poles together only 4 oscillations per minute are executed. The ratio of their magnetic moments is

(a) 3 : 1 (b) 1 : 3
(c) 3 : 5 (d) 5 : 4

107. A magnet is suspended in such a way that it oscillates in the horizontal plane. It makes 20 oscillations per minute at a place where dip angle is 30° and 15 oscillations per minute at a place where dip angle is 60°. The ratio of earth's magnetic fields at two places is

(a) $3\sqrt{3} : 8$ (b) $16 : 9\sqrt{3}$
(c) 4 : 9 (d) $2\sqrt{2} : 3$

108. Two identical short bar magnets, each having magnetic moment M, are placed a distance of $2d$ apart with axes perpendicular to each other in a horizontal plane. The magnetic induction at a point midway between them is

(a) $\frac{\mu_0}{4\pi}(\sqrt{2})\frac{M}{d^3}$ (b) $\frac{\mu_0}{4\pi}(\sqrt{3})\frac{M}{d^3}$
(c) $\left(\frac{2\mu_0}{\pi}\right)\frac{M}{d^3}$ (d) $\frac{\mu_0}{4\pi}(\sqrt{5})\frac{M}{d^3}$

109. A short magnet oscillating in vibration magnetometer with a frequency 10 Hz. A downward current of 15 A is established in a long vertical wire placed 20 cm to the West of the magnet. The new frequency of the short magnet is (the horizontal component of earth's magnetic field is 12μ)

(a) 4 Hz (b) 2.5 Hz
(c) 9 Hz (d) 15 Hz

110. Two bar magnets having same geometry with magnetic moments M and $2M$ are placed in such a way that their similar poles are on the same side, then its time period of oscillation is T_1. Now, if the polarity of one of the magnets is reversed, then time period of oscillation is T_2, then

(a) $T_1 < T_2$ (b) $T_1 > T_2$
(c) $T_1 = T_2$ (d) $T_1 = \infty,\ T_1 = 0$

111. The length of a magnet is large compared to its width and breadth. The time period of its oscillation in a vibration magnetometer is 2 s. The magnet is cut along its length into three equal parts and three parts are then placed on each other with their like poles together. The time period of this combination will be

(a) 2 s (b) $\frac{2}{3}$ s
(c) $2\sqrt{3}$ s (d) $\frac{2}{\sqrt{3}}$ s

B. Medical entrance special format questions

Assertion and reason

Directions (Q. Nos. 1-19) *These questions consist of two statements each linked as Assertion and Reason. While answering these questions you are required to choose any one of the following five responses.*

(a) If both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are true but Reason is not correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If Assertion is false but Reason is true.
(e) If both Assertion and Reason are false.

1. **Assertion** The poles of a magnet cannot be separated by breaking it into two pieces.
Reason The magnetic moment will be reduced to half when a magnet is broken into two equal pieces.

2. **Assertion** It is not necessary that every magnet has one north pole and one-south pole.
Reason It is a basic fact that magnetic poles occur in pairs.

3. **Assertion** Basic difference between an electric line and magnetic line of force is that former is discontinuous and the latter is continuous or endless.
Reason No electric lines of forces exist inside a charged conductor but magnetic lines do exist inside a magnet.

4. **Assertion** The net magnetic flux coming out of a closed surface is always zero.
Reason Unlike poles of equal strength exist together.

5. **Assertion** Horizontal component of earth's magnetic field (H) has been chosen as a magnetic element instead of the vertical component (V).
Reason Most of our experiments are performed in horizontal configuration. So, H is more relevant.

6. **Assertion** At neutral point, a compass needle may point out in any arbitrary direction.
Reason Magnetic field of earth is balanced by field due to magnet at the neutral point.

7. **Assertion** Steel is not attracted by a magnet.
Reason Steel is not a magnetic substance.

8. **Assertion** Relative magnetic permeability has no units and no dimensions.
Reason $\mu_r = \mu/\mu_0$, where the symbols have their standard meaning.

9. **Assertion** If one arm of a U-tube containing a diamagnetic solution is placed in between the poles of a strong magnet with the level in line with the field, the level of the solution falls.
Reason Diamagnetic substances are repelled by the magnetic field.

10. **Assertion** The earth's magnetic field is due to iron present in its core.
Reason At a high temperature magnet loses its magnetic property or magnetism.

11. **Assertion** Earth's magnetic field inside a closed iron box is less as compared to the outside.
Reason The magnetic permeability of iron is low.

12. **Assertion** To protect any instrument from external magnetic field, it is put inside an iron box.
Reason Iron is a magnetic substance.

13. **Assertion** χ_m - T graph for a diamagnetic material is a straight line parallel to T-axis.
Reason This is because susceptibility of a diamagnetic material is not affected by temperature.

14. **Assertion** When radius of a circular wire carrying current is doubled, its magnetic moment becomes four times.
Reason Magnetic moment is directly proportional to area of the loop.

15. **Assertion** A magnet suspended freely in a uniform magnetic field experiences no net force, but a torque that tends to align the magnet along the field.
Reason Net force, $mB - mB = 0$. But the forces on north and south poles being equal, unlike and parallel make up a couple that tends to align the magnet, along the field.

16. **Assertion** Time period of vibration of a pair of magnets in sum position is always smaller than in difference position.
Reason $T = 2\pi\sqrt{I/MH}$, where symbols have their standard meaning.

17. **Assertion** The ferromagnetic substances do not obey Curie's law.
Reason At Curie point ferromagnetic substances start behaving as a paramagnetic substances.

18. **Assertion** A soft iron core is used in a moving coil galvanometer to increase the strength of magnetic field.
Reason From soft iron more number of the magnetic lines of force passes.

19. **Assertion** The properties of paramagnetic and ferromagnetic substances are not affected by heating.
Reason As temperature rises, the alignment of molecular magnets gradually decreases.

Match the columns

1. With reference to magnetic dipole, match the terms of Column I with the terms of Column II and choose the correct option from the codes given below.

Column I	Column II
(A) Dipole moment	(p) $-M \cdot B$
(B) Equatiorial field for a short dipole	(q) $M \times B$
(C) Axial field for a short dipole	(r) $-\mu_0 m/4\pi r^3$
(D) External field : Torque	(s) m
(E) External field : Energy	(t) $\mu_0 2m/4\pi r^3$

2. Consider the expression for magnetic potential energy U_m obtained in previous question, match the terms of Column I with the terms of Column II and choose the correct option from the codes given below.

Column I	Column II
(A) Potential energy at $\theta = 90°$	(p) Minimum
(B) Potential energy at $\theta = 0°$	(q) Maximum
(C) Potential energy at $\theta = 180°$	(r) Zero

3. Match the terms of Column I with the items of Column II and choose the correct option from the codes given below.

Column I	Column II
(A) Negative susceptibility	(p) Ferromagnetic
(B) Positive and small susceptibility	(q) Diamagnetic
(C) Positive and large susceptibility	(r) Paramagnetic

4. Match the terms of Column I with the items of Column II and choose the correct option from the codes given below.

Column I	Column II
(A) Diamagnetic	(p) $\mu >> \mu_0, \mu_r >> 1$ and $\chi >> 1$
(B) Paramagnetic	(q) $-1 \le \chi < 0, \le \mu_r < 1$ and $\mu < \mu_0$
(C) Ferromagnetic	(r) $0 < \chi < \varepsilon, 1 < \mu_r, < 1 + \varepsilon$ and $\mu > \mu_0$

C. Medical entrances' gallery

(Collection of questions asked in NEET & various medical entrance exams)

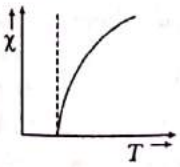
1. A bar magnet is hung by a thin cotton thread in a uniform horizontal magnetic field and is in equilibrium state. The energy required to rotate it by 60° is W. Now the torque required to keep the magnet in this new position is **[NEET 2016]**

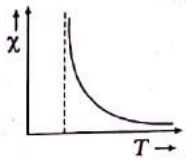
(a) $\frac{W}{\sqrt{3}}$ (b) $\sqrt{3}W$ (c) $\frac{\sqrt{3}W}{2}$ (d) $\frac{2W}{\sqrt{3}}$

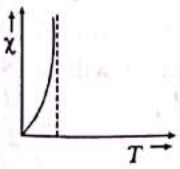
2. The magnetic susceptibility is negative for **[AIMPT 2016]**

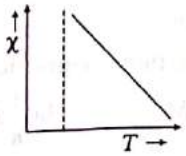
(a) paramagnetic material only
(b) ferromagnetic material only
(c) paramagnetic and ferromagnetic materials
(d) diamagnetic material only

3. The variation of magnetic susceptibility with the temperature of a ferromagnetic material can be ploted as **[AIIMS 2015]**

(a) 

(b) 

(c) 

(d) 

4. If r be the distance of a point on the axis of a magnetic dipole from its centre then the magnetic field at such a point is proportional to **[UK PMT 2015]**

(a) $\frac{1}{r}$ (b) $\frac{1}{r^2}$ (c) $\frac{1}{r^3}$ (d) None of these

5. A bar magnet has a coercivity of 4×10^3 Am^{-1}. It is placed inside a solenoid of length 12 cm having 60 turns in order to demagnetise it. The amount of current that should be passed through the solenoid is **[EAMCET 2015]**

(a) 16A (b) 8A (c) 4A (d) 2A

6. The effective length of magnet is 31.4 cm and its pole strength is 0.8 Am. The magnetic moment, if it is bent in the form of a semicircle is ..... A-m^2. **[Guj. CET 2015]**

(a) 1.2 (b) 1.6 (c) 0.16 (d) 0.12

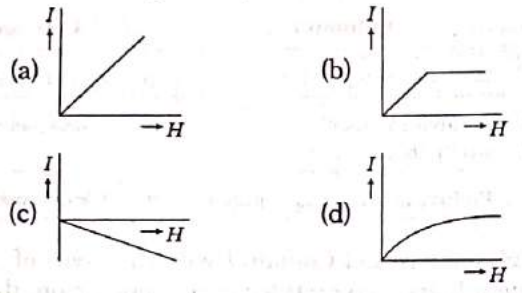
7. At a place on earth, the vertical component of earth's magnetic field is $\sqrt{3}$ times its horizontal component. The angle of dip at this place is ... **[J&K CET 2011]**

(a) 60° (b) 30° (c) 45° (d) 0°

8. A tangent galvanometer has a coil of 50 turns and a radius of 20 cm. The horizontal component of earth's magnetic field is $B_H = 3 \times 10^{-5}$T. What will be the current which gives a deflection of 45°? **[Guj. CET 2015]**

(a) 0.39 A (b) 0.29 A (c) 0.19 A (d) 0.09 A

9. The correct curve between intensity of magnetisation (I) and magnetic field (H) for a ferromagnetic substance is given by **[UP CPMT 2015]**



10. A bar magnet with magnetic moment 2.5×10^3 JT^{-1} is rotating in horizontal plane in the space containing magnetic induction $B = 4\times10^{-5}$T. The work done in rotating the magnet slowly from a direction parallel to the field to a direction 45° from the field, is (in joule) **[UP CPMT 2015]**

(a) 0.0 (b) 0.2 (c) 0.03 (d) 0.02

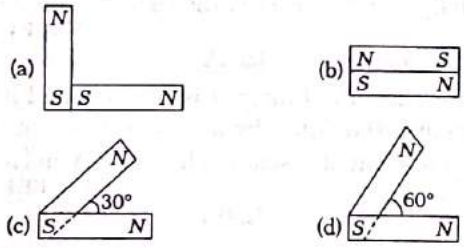
11. Core of electromagnets are made of ferromagnetic material which has **[KCET 2015]**

(a) low permeability and high retentivity
(b) high permeability and low retentivity
(c) low permeability and low retentivity
(d) high permeability and high retentivity

12. If the magnetising field on a ferromagnetic material is increased, its permeability **[AIMS 2015]**

(a) decrease (b) increase
(c) is unaffected (d) may be increase or decrease

13. Following figures show the arrangement of bar magnets in different configurations. Each magnet has magnetic dipole moment m. Which configuration has highest net magnetic dipole moment? **[CBSE AIPMT 2014]**



14. A bar magnet of moment M and pole strength m is cut into parts of equal lengths. The magnetic moment and pole strength of either part is **[UK PMT 2014]**

(a) $\frac{M}{2}, \frac{m}{2}$ (b) $M, \frac{m}{2}$ (c) $\frac{M}{2}, m$ (d) M, m

15. A susceptibility of a certain magnetic material is 400. What is the class of the magnetic material? **[KCET 2014]**

(a) Diamagnetic (b) Paramagnetic
(c) Ferromagnetic (d) Ferroelectic

16. A paramagnetic sample shows a net magnetisation of 0.8 Am^{-1}, when placed in an external magnetic field of strength 0.8 T at a temperature 5K. When the same sample is placed in an external magnetic field of 0.4 T at a temperature of 20K. The magnetisation is **[EAM CET 2014]**

(a) 0.8 Am^{-1} (b) 0.8 Am^{2}
(c) 0.1 Am^{-2} (d) 0.1 Am^{-1}

17. Nickel shows ferromagnetic property at room temperature. If the temperature is increased beyond curie temperature, then it will show **[UK PMT 2014]**

(a) paramagnetic (b) anti-ferromagnetism
(c) diamagnetism (d) no magnetic property

18. The intensity of magnetisation of a bar magnet is 5×10^4 Am^{-1}. The magnetic length and the area of cross-section of the magnet are 12 cm and 1 cm^{-2} respectively. The magnitude of magnetic moment of this bar magnet (in SI unit) is **[WB JEE 2014]**

(a) 0.6 (b) 1.3
(c) 1.2 (d) 2.4

19. The magnetic susceptibility of a material of a rod is 299. Permeability of vacuum μ_0 is $4\pi\times10^{-7}$ Hm^{-1}. Absolute permeability of the material of the rod is **[EAMCET 2014]**

(a) 3771×10^{-7} Hm^{-1} (b) 3771×10^{-5} Hm^{-1}
(c) 3770×10^{-6} Hm^{-1} (d) 3771×10^{-8} Hm^{-1}

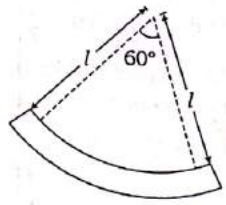
20. A wire of length L metre carrying a current I ampere is bent in the form of a circle. The magnitude of the magnetic moment is **[EAMCET 2014]**

(a) $\frac{L^2I^2}{4\pi}$ (b) $\frac{LI}{4\pi}$ (c) $\frac{L^2I}{4\pi}$ (d) $\frac{LI}{4\pi}$

21. An electron in a circular orbit of radius 0.05 nm performs 10^{16} revolutions per second. The magnetic moment due to this rotation of electron is (in Am^2). **[WB JEE 2014]**

(a) 2.16×10^{-23} (b) 3.21×10^{-22}
(c) 3.21×10^{-24} (d) 1.26×10^{-23}

22. A bar magnet of length l and magnetic dipole moment M is bent in the form of an arc as shown in figure.



The new magnetic dipole moment will be **[NEET 2013]**

(a) M (b) $\frac{3}{\pi}M$ (c) $\frac{2}{\pi}M$ (d) $\frac{M}{2}$

23. The horizontal and vertical components of earth's magnetic field at a place are 0.3 G and 0.52G. The earth's magnetic field and the angle of dip are **[J&K CET 2013]**

(a) 0.3 G and $\delta = 30°$ (b) 0.4 G and $\delta = 40°$
(c) 0.5 G and $\delta = 50°$ (d) 0.6 G and $\delta = 60°$

24. A bar magnet of pole strength 10 A-m is cut into two equal parts breadthwise. The pole strength of each magnet is **[J&K CET 2013]**

(a) 5 A-m (b) 10 A-m (c) 15 A-m (d) 20 A-m

25. A short magnet of magnetic moment M, is placed on a straight line. The ratio of magnetic induction fields B_1, B_2, B_3 values on this line at points which are at distance 30 cm, 60 cm and 90 cm respectively from the centre of the magnet is **[EAMCET 2013]**

(a) 27 : 3.37 : 1 (b) 37.3 : 1 : 27
(c) 27 : 8 : 3.37 (d) 1 : 2 : 3

26. A bar magnet of moment of inertia I is vibrated in a magnetic field of induction 0.4×10^{-4} T. The time period of vibration is 12 s. The magnetic moment of the magnet is 120 Am^2. The moment of inertia of the magnet is (in kgm^2) approximately **[EAMCET 2013]**

(a) 172.8×10^{-4} (b) 2.1×10^{-2}
(c) 1.57×10^{-2} (d) 1728×10^{-2}

27. On heating a ferromagnetic substance above curie temperature **[MP PMT 2013]**

(a) becomes paramagnetic
(b) becomes diamagnetic
(c) remains ferromagnetic with constant magnetic susceptibility
(d) becomes electromagnetic

28. The work done in turning a magnet of magnetic moment M by an angle of 90° from the magnetic meridian is n times the corresponding work done to turn it through an angle of 60°. The value of n is **[UP CPMT 2013]**

(a) 1 (b) 2 (c) 1/2 (d) 1/4

29. A dip needle vibrates in a vertical plane perpendicular to magnetic meridian. The time of vibration is found to be 25, the same needle is then allowed to vibrate in the horizontal plane and the time period is again found to be 25. Then, the angle of dip is **[UPCPMT 2012]**

(a) 0° (b) 30° (c) 45° (d) 90°

30. If the dipole moment of a short bar magnet is 1.25 A-m^2, the magnetic field on its axis at a distance of 0.5 m from the centre of the magnet is **[BCECE 2012]**

(a) 1×10^{-4} $NA^{-1}m^{-1}$ (b) 2×10^{-6} $NA^{-1}m^{-1}$
(c) 4×10^{-2} $NA^{-1}m^{-1}$ (d) 6.64×10^{-8} $NA^{-1}m^{-1}$

31. The horizonatal component of earth's magnetic field at a place is 3×10^{-4} T and the dip is $\tan^{-1}\left(\frac{4}{3}\right)$.

A metal rod of length 0.25 m is placed in N-S direction and is moved at a constant speed of 10 cms^{-1} towards the east. The emf induced in the rod will be **[BCECE Mains 2012]**

(a) 1 μV (b) 5 μV (c) 7 μV (d) 10 μV

32. **Assertion** Susceptibility is defined as the ratio of intensity of magnetisation I to magnetic intensity H.

Reason Greater the value of susceptibility smaller the value of intensity of magnetisation I. **[AIIMS 2012]**

(a) If both Assertion and Reason and true and Reason is the correct explanation of Assertion
(b) If both Assertion and Reason are true but Reason is not the correct explanation of Assertion
(c) If Assertion is true but Reason is false
(d) If both Assertion and Reason are false

33. If a magnet is suspended at an angle 30° to the magnetic meridian, the dip needle makes angle of 45° with the horizontal. The real dip is **[Manipal 2012]**

(a) $\tan^{-1}(\sqrt{3}/2)$ (b) $\tan^{-1}(\sqrt{3})$
(c) $\tan^{-1}(3/\sqrt{2})$ (d) $\tan^{-1}(2/\sqrt{3})$

34. If a steel wire of length l and magnetic moment M is bent into a semicircular arc, then the new magnetic moment is **[JCECE 2012]**

(a) $M \times l$ (b) $\frac{M}{l}$ (c) $\frac{2M}{\pi}$ (d) M

35. An iron rod of volume 10^{-4} m^3 and relative permeability 1000 is placed inside a long solenoid having 5 turns/cm. If a current of 0.5 A is passed through the solenoid, then the magnetic moment of the rod is **[BCECE 2012]**

(a) 20 Am^2 (b) 25 Am^2
(c) 30 Am^2 (d) 35 Am^2

36. Two tangent galvanometers A and B have coils of radii 8 cm and 16 cm respectively and resistance 8 Ω each. They are connected in parallel with a cell of emf 4V and negligible internal resistance. The deflections produced in the tangent galvanometers A and B are 30° and 60° respectively. If A has 2 turns, then B must have **[Manipal 2012]**

(a) 18 turns (b) 12 turns
(c) 6 turns (d) 2 turns

Answers

Check point 5.1

1. (a)	2. (d)	3. (a)	4. (d)	5. (b)	6. (d)	7. (b)	8. (a)	9. (a)	10. (a)
11. (a)	12. (a)	13. (d)	14. (b)	15. (a)					

Check point 5.2

1. (a)	2. (c)	3. (b)	4. (a)	5. (b)	6. (a)	7. (a)	8. (d)	9. (a)	10. (b)
11. (b)	12. (b)	13. (d)	14. (c)	15. (c)					

Check point 5.3

1. (d)	2. (c)	3. (d)	4. (c)	5. (b)	6. (a)	7. (a)	8. (b)	9. (b)	10. (c)
11. (b)	12. (c)	13. (b)	14. (a)	15. (c)					

A. Taking it together

1. (d)	2. (b)	3. (d)	4. (c)	5. (d)	6. (a)	7. (b)	8. (b)	9. (d)	10. (a)
11. (d)	12. (c)	13. (b)	14. (c)	15. (b)	16. (c)	17. (d)	18. (a)	19. (c)	20. (c)
21. (a)	22. (b)	23. (b)	24. (a)	25. (a)	26. (d)	27. (d)	28. (a)	29. (c)	30. (c)
31. (b)	32. (b)	33. (a,b)	34. (d)	35. (a)	36. (a)	37. (d)	38. (b)	39. (b)	40. (b)
41. (c)	42. (b)	43. (d)	44. (b)	45. (b)	46. (d)	47. (c)	48. (b)	49. (c)	50. (c)
51. (b)	52. (d)	53. (d)	54. (c)	55. (b)	56. (a)	57. (a)	58. (d)	59. (b)	60. (b)
61. (b)	62. (d)	63. (b)	64. (c)	65. (b)	66. (a)	67. (d)	68. (c)	69. (d)	70. (b)
71. (c)	72. (a)	73. (d)	74. (a)	75. (c)	76. (c)	77. (d)	78. (a)	79. (d)	80. (b)
81. (d)	82. (d)	83. (a)	84. (b)	85. (a)	86. (c)	87. (b)	88. (c)	89. (c)	90 (b)
91. (b)	92. (b)	93. (d)	94. (a)	95. (d)	96. (b)	97. (b)	98. (b)	99. (a)	100. (a)
101. (b)	102. (b)	103. (c)	104. (d)	105. (a)	106. (d)	107. (b)	108. (d)	109. (d)	110. (a)
111. (b)									

B. Medical entrance special format questions

Assertion and reason

1. (b)	2. (d)	3. (b)	4. (a)	5. (a)	6. (a)	7. (c)	8. (a)	9. (a)	10. (d)
11. (c)	12. (b)	13. (a)	14. (a)	15. (a)	16. (b)	17. (b)	18. (a)	19. (e)	

Match the columns

1. (A → s, B → r, C → t, D → q, E → p)
2. (A → r, B → p, C → q)
3. (A → q, B → r, C → p)
4. (A → q, B → r, C → p)

C. Medical entrances' gallery

1. (b)	2. (d)	3. (b)	4. (c)	5. (b)	6. (c)	7. (a)	8. (c)	9. (b)	10. (c)
11. (b)	12. (a)	13. (c)	14. (c)	15. (c)	16. (d)	17. (a)	18. (a)	19. (a)	20. (c)
21. (d)	22. (b)	23. (d)	24. (a)	25. (a)	26. (a)	27. (a)	28. (b)	29. (c)	30. (b)
31. (d)	32. (c)	33. (d)	34. (c)	35. (b)	36. (b)				

Hints and explanations

Check point 5.1

2. (*d*) Magnetic lines of force due to a bar magnet do not intersect because if they intersect then it means, there are two directions of magnetic field intensity which is impossible.

4. (*d*) Initial magnetic moment $= M_1$

When cut in two parts, magnetic moment of each part

$$M' = \frac{M_1}{2}$$

When place perpendicular, effective magnetic moment

$$M_2 = \sqrt{2}M' = \sqrt{2 \times \frac{M_1}{2}}$$

$$\therefore \quad \frac{M_1}{M_2} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

7. (*b*) Since for magnet P, axis lies along X-axis and for magnet Q, axis is along Y-axis. The point R is along axial line w.r.t. magnet P, and is along equatorial line w.r.t. magnet Q.

Hence, magnetic field due to magnet Q

$$B_Q = \frac{\mu_0}{4\pi}\frac{M}{x^3} = B \quad [R \Rightarrow \text{equatorial point}] \quad \text{...(i)}$$

Magnetic field due to magnet P

$$B_P = \frac{\mu_0}{4\pi}\frac{2M}{x^3} = 2B \quad [R = \text{axial point}] \quad \text{...(ii)}$$

As at point R magnetic field due to P and Q magnet are perpendicular to each other,

$B_R \Rightarrow$ Net magnetic field at R due to magnet P and is given as Q,

$$B_R \Rightarrow \sqrt{B_P^2 + B_Q^2} = \sqrt{B^2 + (2B)^2} = \sqrt{5}B$$

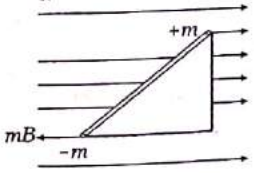
9. (*a*) The relation between magnetic moment (μ_l) and angular momentum (l) is

$$\mu_l = -\frac{q}{2m}l \Rightarrow \frac{\mu_l}{l} = -\frac{q}{2m}$$

The negative sign indicates that the angular momentum of the electron is opposite in direction to the magnetic moment.

10. (*a*) When a bar magnet of magnetic moment **M** is placed in a magnetic field of induction **B**, then net force on it is

$$F_R = mB + (-m)B = 0$$



While torque, $\tau = mB \times 2l \sin\theta = MB\sin\theta$ (as $M = 2ml$)

i.e., $\tau = \mathbf{M} \times \mathbf{B}$

11. (*a*) As, $\tau = MB\sin\theta = (m \times 2l) \times 2 \times 10^{-5} \sin 30°$

$$= 15 \times 10 \times 10^{-2} \times 2 \times 10^{-5} \times \frac{1}{2} = 1.5 \times 10^{-5}\,\text{N-m}$$

12. (*a*) As, $\tau = MB\sin\theta$

where, $\theta = 90° \Rightarrow \tau = MB$

Given, $\tau_2 = \frac{1}{2}\tau_1 \Rightarrow MB\sin\theta = \frac{1}{2}MB \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = 30°$

$\therefore$ Angle of rotation $= 90° - 30° = 60°$

13. (*d*) Work done is rotating the magnet through an angle θ from initial position (*i.e.*, $\theta_1 = 0°$) is given by

$$W = MB(\cos\theta_1 - \cos\theta) = MB(\cos 0° - \cos\theta) = MB(1 - \cos\theta)$$

15. (*a*) According to Gauss' theorem in magnetism, surface integral of magnetic field intensity over a surface (closed or open) is always zero *i.e.*, $\oint \mathbf{B} \cdot d\mathbf{A} = 0$

Check point 5.2

1. (*a*) It has been established that earth magnetic field is fairly uniform. The strength of this field at the surface of the earth is approximately 10^{-4} tesla or 1 gauss.

3. (*b*) The angle between the magnetic meridian and the geographic meridian is called magnetic declination.

6. (*a*) Magnetic dip or magnetic inclination is given by

$$\tan\theta = \frac{V}{H} \quad \text{...(i)}$$

where, V and H are vertical and horizontal components of earth's magnetic field, respectively.

Given, $H = \frac{1}{\sqrt{3}}V \Rightarrow \frac{V}{H} = \sqrt{3}$ ...(ii)

From Eqs. (i) and (ii), we get

$$\tan\theta = \sqrt{3} \Rightarrow \theta = 60°$$

8. (*d*) As, $\tan\delta' = \frac{\tan\delta}{\cos\theta} = \frac{\tan 45°}{\cos 30°}$

$$\tan\delta' = \frac{1}{\sqrt{3}/2} = \frac{2}{\sqrt{3}} \Rightarrow \delta' = \tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$$

9. (*a*) As, $\tan\delta = \frac{V}{H}$ and $\tan\delta_1 = \frac{V}{H\cos(90° - \theta)}$

$$= \frac{\tan\delta}{\cos(90° - \theta)} = \tan\delta \operatorname{cosec}\theta$$

12. (*b*) As the current and the other factors are same for both the galvanometers, so

$$N \propto \tan\theta \Rightarrow \frac{N_1}{N_2} = \frac{\tan 60°}{\tan 30°} \text{ or } \frac{N_1}{N_2} = \frac{\sqrt{3}}{1/\sqrt{3}} = 3:1$$

14. (*c*) As, time period $T = 2\pi\sqrt{\frac{I}{MB}}$

$$T = 2\text{s}, I' = \frac{I}{2}, M' = \frac{M}{2} \Rightarrow T' = T \Rightarrow T' = 2\text{s}$$

15. (*c*) As, the time period, $T = 2\pi\sqrt{\frac{I}{MB}}$

or $$T \propto \frac{1}{\sqrt{B}} \Rightarrow \frac{T_2}{T_1} = \sqrt{\frac{B_1}{B_2}}$$

or $$\frac{T_2}{3} = \sqrt{\frac{1}{4}} \text{ or } T_2 = 1.5\text{s}$$

Check point 5.3

2. (*c*) Magnetic permeability,

$$\mu = \frac{B}{H}$$

Its SI unit is $\frac{\text{Wb/m}^2}{\text{A/m}} = \frac{\text{V}\times\text{s}}{\text{A}\times\text{m}} = \frac{\Omega\times\text{s}}{\text{m}} = \frac{\text{H}}{\text{m}}$

14. (*a*) Some peculiar properties of ferromagnetic materials are commenly displayed by curve of **B** against **H** which is called *B-H* curve or hysteresis is *B* loop. Dia and para do not show these properties.

15. (*c*) If a magnet retains its attracting power for a long time it is said to be permanent, otherwise temporary. Permanent magnets are made of ferromagnetic substances.

A) Taking it together

1. (*d*) Magnetic pole strength is stronger at end part of magnet, so maximum iron powder is collected at the end point of magnet.

2. (*b*) A permanent magnet has large retentivity and coercivity so it attracts only magnetic substances.

4. (*c*) Lines which represent places of constant angle of dip are called isoclinic lines. The line joining places of zero dip is called aclinic line (or magnetic equator). At all places upon this line, a freely suspended magnet will remain horizontal.

5. (*d*) A line passing through places having zero value of magnetic dip is called aclinic line.

11. (*d*) SI unit of magnetic induction is,

$$B = \frac{\text{Unit of } \phi}{\text{Unit of } \Delta A} = \frac{\text{Wb}}{\text{m}^2} = \text{Tesla}$$

Also, SI unit of B is weber/metre^2 and Newton/amp-metre.

12. (*c*) Magnetic moment per unit volume is intensity of magnetisation, and number of lines of force crossing per unit area is intensity of magnetic field. So right option is (c).

18. (*a*) Steel has more retentivity and coercivity, so it is used for making permanent magnet.

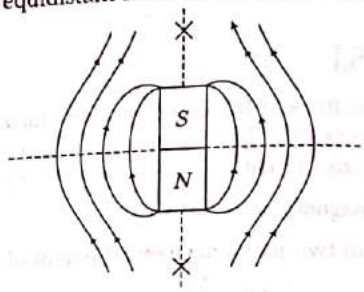
21. (*a*) In non-uniform magnetic field, a magnetic needle experiences a force and a torque.

22. (*b*) For a diamagnetic substance, χ is small, negative and independent of temperature.

24. (*a*) A freely hanged magnet stays with its magnetic axis parallel to magnetic meridian.

25. (*a*) A dip needle in a plane perpendicular to magnetic meridian will remain always vertical.

28. (*a*) If the north pole of a magnet points south, the fields of the magnet and the earth will point in opposite directions along the axis of the magnet, so two neutral points are obtained which are equidistant from the magnet on its axis.



31. (*b*) An electron moving around the nucleus has a magnetic moment μ given by

$$\mu = \frac{e}{2m} l$$

where, l is the magnitude of the angular momentum of the circulating electron around the nucleus. The smallest value of μ is called the bohr magneton μ_B and its value is $\mu_B = 9.27 \times 10^{-24}\ \text{JT}^{-1}$.

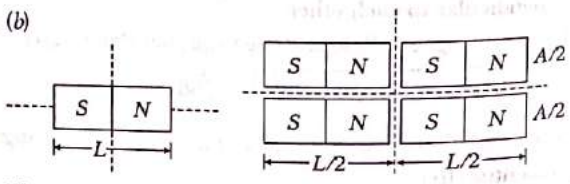
35. (*a*) Resultant force acting on a diamagnetic material in a magnetic field is in direction from stronger to the weaker part of the magnetic field.

38. (*b*) Let pole strength be m_1 and m_2 and distance between them is r, then force between magnetic poles, $F \propto \frac{m_1 m_2}{r^2}$

In second case, $F' \propto \frac{(2m_1)(2m_2)}{(2r)^2}, \propto \frac{m_1 m_2}{r^2}$

$\therefore$ $$F' = F$$

39. (*b*)



The situation is summerised in figure, we see that magnetic pole strength of each part becomes half *i.e.*, $m/2$.

41. (*c*) For longitudinal positions magnetic field will be $B_1 \propto \frac{2M}{d^3}$

For transverse positions magnetic field will be $B_2 \propto \frac{M}{d^3}$

$\therefore$ $$\frac{B_1}{B_2} = 2:1$$

45. (*b*) Torque, $\tau = MB\sin\theta$

$$\tau_1 = MB_1 \sin 90° = MB_1$$

$$\tau_2 = MB_2 \sin 90° = MB_2 \text{ or } \frac{MB_1}{MB_2} = \frac{\tau_1}{\tau_2}$$

$\therefore$ $$\frac{B_1}{B_2} = \frac{\tau_1}{\tau_2}$$

50. (*c*) Magnetism of a magnet falls with rise of temperature and becomes practically zero above curie temperature.

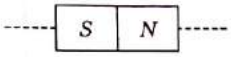
53. (*d*) Given, $B = 4B_0$

$$\therefore \quad \frac{B}{B_0} = 4 \Rightarrow \mu_r = 4 \qquad \left[\because \mu_r = \frac{B}{B_0}\right]$$

60. (*b*) Above the Curie temperature, ferromagnetic substance become paramagnetic in nature.

62. (*d*) The magnetic moment will be zero because it becomes an open circuit.

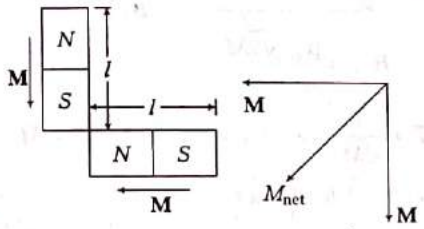
63. (*b*) If magnet is cut along the axis of magnet of length l, then new pole strength becomes half *i.e.*, $m' = \frac{m}{2}$ and new length remains same *i.e.*, $l' = l$.



$\therefore$New magnetic moment

$$M' = \frac{m}{2} \times l = \frac{ml}{2} = \frac{M}{2}$$

64. (*c*) Net magnetic moment of the system is given by



$$M = \sqrt{M^2 + M^2} = \sqrt{2}M = \sqrt{2}\, ml$$

65. (*b*) Magnetic moment of the bar magnet is given by

$$\tau = MB \sin\theta$$

$$\therefore \quad M = \frac{\tau}{B \sin\theta}$$

$$= \frac{0.032}{0.16 \times \sin 30°} = \frac{0.032 \times 2}{0.16} = 0.4\ \text{J T}^{-1}$$

66. (*a*) As, $\tau = MB \sin\theta$

$$\tau = m(2L)\, B \sin\theta$$

$$m = \frac{\tau}{(2l)\, B \sin\theta} = \frac{25 \times 10^{-6}}{(0.05)(5 \times 10^{-2}) \sin 30°} = 2 \times 10^{-2}\ \text{A-m}$$

67. (*d*) Force experienced by each pole of bar magnet is given by

$$F = mB$$

where, m is pole strength of bar magnet

but $\quad M = mL$

$$\therefore \quad F = \frac{M}{L} \times B \Rightarrow 6 \times 10^{-4} = \frac{3}{L} \times 2 \times 10^{-5}$$

$$\therefore \quad L = 0.1\ \text{m}$$

68. (*c*) In case of toroid, the magnetic field is only confined inside the body of toroid in the form of concentric magnetic lines of force and there is no magnetic field outside the body of toroid. This is because the loop encloses no current. Thus, the magnetic moment of toroid is zero.

In general, if we take r as a large distance outside the toroid, then $m \propto \frac{1}{r^3}$. But this case is not possible here.

69. (*d*) Work done in rotating the magnet from angle θ_1 to angle θ_2 is given by,

$$W = MB(\cos\theta_1 - \cos\theta_2)$$

Here, $\theta_1 = 0°$ and $\theta_2 = 180°$

$$W = MB(\cos 0° - \cos 180°) = 2MB$$

70. (*b*) Work done in rotating the magnet in uniform magnetic field is given by

$$W = MB(\cos\theta_1 - \cos\theta_2)$$

Here, $\theta_1 = 0°, \theta_2 = 90°$

$$\therefore \quad W = 2 \times 0.1(\cos 0° - \cos 90°)$$

$$= 2 \times 0.1(1 - 0) = 0.2\ \text{J}$$

71. (*c*) Here, $\mathbf{B} = 0.5\ \text{T}; I = 20\ \text{A}; N = 12; \mathbf{A} = -0.04\ \hat{\mathbf{i}}\,\text{m}^2$

Magnetic moment of the coil,

$$\mathbf{M} = NI\mathbf{A}$$

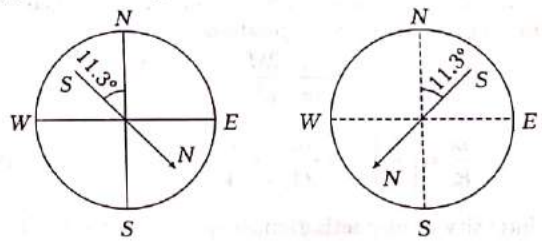
$$= (15)(20\ \text{A})(-0.04\ \hat{\mathbf{i}}\,\text{m}^2) = -12\hat{\mathbf{i}}\,\text{Am}^2$$

The potential energy of the coil in the given orientation is

$$U = -\mathbf{M}\cdot\mathbf{B} = -(-12\hat{\mathbf{i}}\,\text{Am}^2)\cdot(0.5\hat{\mathbf{i}}\,\text{T}) = +6\,\text{J}$$

72. (*a*) For the earth's magnetism, the magnetic field lines of the earth resemble that of a hypothetical magnetic dipole located at the centre of the earth.

The axis of the dipole does not coincide with the axis of rotation of the earth but is presently tilted by approximately 11.3° with respect to the later. This results into two situations as given in the figure ahead.



Hence, the declination varies between 11.3° W to 11.3° E.

73. (*d*) $\therefore \quad \frac{B}{0.2} = \frac{M/(5)^3}{2M/(10)^3} = \frac{(10/5)^3}{2} = 0.8$ oersted

74. (*a*) As, $\frac{H_1}{H_2} = \frac{B\cos\theta_1}{B\cos\theta_2} = \frac{\cos\theta_1}{\cos\theta_2} = \frac{\cos 30°}{\cos 45°} = \frac{\sqrt{3}}{2} \times \sqrt{2} = \frac{\sqrt{3}}{\sqrt{2}}$

75. (*c*) Total strength of magnetic field

$$B^2 = H^2 + V^2$$

$$\therefore \quad V = \sqrt{B^2 - H} = \sqrt{(0.5)^2 - (0.3)^2} = 0.4$$

$$\tan\phi = \frac{V}{H} = \frac{0.4}{0.3} = \frac{4}{3}$$

$$\therefore \quad \phi = \tan^{-1}\left(\frac{4}{3}\right)$$

76. (*c*) As, $H = B_e \cos\theta$

$$\frac{1}{2} = B_e \frac{\sqrt{3}}{2}$$

$$\therefore \quad B_e = \frac{1}{\sqrt{3}}\ \text{oersted}$$

77. (*d*) As we know a permanent magnet is a substance which at room temperature retain ferromagnetic property for a long period of time.

The individual atoms in a ferromagnetic material possess a dipole moment as in a paramagnetic material.

However, they interact with one another in such a way that they spontaneously align themselves in a common direction over a macroscopic volume called domain. Thus, we can say that in a permanent magnet at room temperature, domains are all perfectly aligned.

79. (*d*) For a temporary magnet, the hysteresis loop should be long and narrow.

80. (*b*) $T \propto \sqrt{I}$

By increasing mass to four times, the moment of inertia will also increase four times. Then $T' = \sqrt{4I} = 2\sqrt{I}$

81. (*d*) For a tangent galvanometer, if I amp current flows through coil, then this current is proportional to angle of deflection (of the needle).

$$I \propto \tan\theta$$

$$\therefore \quad \frac{I_1}{I_2} = \frac{\tan\theta_1}{\tan\theta_2} \Rightarrow \frac{2}{I_2} = \frac{\tan 30°}{\tan 60°}$$

$$\therefore \quad I_2 = 6\text{A}$$

82. (*d*) In series current is same.

$$\Rightarrow \quad \frac{M_1}{M_2} = \frac{\tan\theta_1}{\tan\theta_2} \text{ or } \frac{N_1}{N_2} = \frac{\tan 60°}{\tan 45°} = \sqrt{3}$$

83. (*a*) Magnetic field due to a bar magnet at a distance r from the centre of magnet on axial position

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2M}{r^3}$$

$$\therefore \quad \frac{B_1}{B_2} = \left(\frac{r_2}{r_1}\right)^3 = \left(\frac{48}{24}\right)^3 = \frac{8}{1} = 8$$

84. (*b*) Intensity of magnetisation,

$$I = \frac{M}{V} = -\frac{M}{\text{mass/density}}$$

Given, $\quad$ mass $= 1\text{ g} = 10^{-3}\text{kg}$

and $\quad$ density $= 5\text{g/cm}^3 = \dfrac{5\times10^{-3}\text{kg}}{(10^{-2})^3} = 5\times10^3\text{kg/m}^3$

Hence, $\quad I = \dfrac{6\times10^{-7}\times5\times10^3}{10^{-3}} = 3\text{ A/m}$

85. (*a*) $H = \dfrac{\mu_0}{4\pi}\cdot\dfrac{M}{r^3} \Rightarrow 4\times10^{-5} = 10^{-7}\times\dfrac{M}{(20)^3\times10^{-6}}$

$$M = \frac{(20)^3\times4\times10^{-5}\times10^{-6}}{10^{-7}}$$

$$M = 3.2\text{ A-m}^2$$

87. (*b*) As Gauss' law states, $\oint_S \mathbf{E}\cdot d\mathbf{S} = \dfrac{q}{\varepsilon_0}$ for electrostatic field. It does not contradict for electrostatic fields as the electric field lines do not form continuous closed path.

According to Gauss' law in magnetic field, $\oint_S \mathbf{B}\cdot d\mathbf{S} = 0$

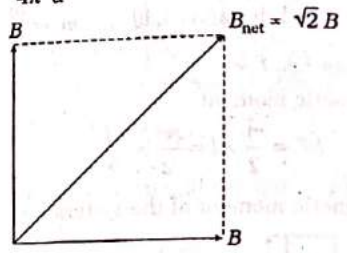
It contradicts for magnetic field, because there is a magnetic field inside the solenoid and no field outside the solenoid carrying current but the magnetic field lines form the closed path.

88. (*c*) In CGS, $B_{\text{axial}} = 9 = \dfrac{2M}{x^3}$ ...(i)

$$B_{\text{equatorial}} = \frac{M}{\left(\frac{x}{2}\right)^2} = \frac{8M}{x^3} \quad \text{...(ii)}$$

From Eq. (i) and (ii), $B_{\text{equatorial}} = 36$ gauss

90. (*b*) $\quad B = \dfrac{\mu_0}{4\pi}\dfrac{M}{d^3}$



$$\therefore \quad B_{\text{net}} = \frac{\mu_0}{4\pi}\frac{\sqrt{2}M}{d^3}$$

91. (*b*) $\quad T \propto \dfrac{1}{\sqrt{M}} \Rightarrow M_1 = 3M$ and $M_2 = M$

$$\therefore \quad T_2 = \sqrt{3}\,T_1 = 3\sqrt{3}\text{ s}$$

94. (*a*) Time period of vibrating magnet is given by

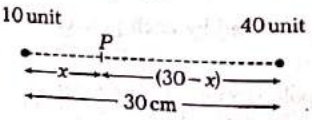
$$T = 2\pi\sqrt{\left(\frac{I}{MH}\right)}$$

$$\therefore \quad \frac{T_1}{T_2} = \sqrt{\left(\frac{M_2}{M_1}\right)}$$

$$\therefore \quad \frac{M_1}{M_2} = \frac{T_2^2}{T_1^2} = \frac{(60/15)^2}{(60/10)^2} = \frac{4}{9}$$

95. (*d*) Paramagnetic materials will be feebly attract, diamagnetic material will be feebly repelled and ferromagnetic material will be strongly attracted.

97. (*b*) Suppose magnetic field is zero at point P which lies at a distance x from 10 unit pole.



Hence, at P

$$\frac{\mu_0}{4\pi}\cdot\frac{10}{x^2} = \frac{\mu_0}{4\pi}\cdot\frac{40}{(30-x)^2} \Rightarrow x = 10\text{ cm}$$

So, from stronger pole distance is 20 cm.

98. (*b*) Time period, $T = 2\pi\sqrt{\dfrac{I}{MB}}$

$$\therefore \quad T \propto \frac{1}{\sqrt{B}}$$

$$\therefore \quad \frac{T_1}{T_2} = \sqrt{\frac{B_2}{B_1}}$$

$$\Rightarrow \quad \frac{60/40}{2.5} = \sqrt{\frac{B_2}{0.1\times10^{-5}}}$$

$$\therefore \quad B_2 = 0.36\times10^{-6}\ \text{T}$$

99. (*a*) The needle is deflected through 45° means magnetic field at the centre of circular coil is equal (in magnitude) to the horizontal component of earth's magnetic field.

$$\frac{\mu_0 NI}{2r} = H$$

$$\therefore \quad I = \frac{2rH}{\mu_0 N} = \frac{2\times0.2\times0.37\times10^{-4}}{(4\pi\times10^{-7})(20)} \approx 0.6\ \text{A}$$

100. (*a*) Apparent dip, $\theta' = \tan^{-1}\left(\frac{V'}{H'}\right)$

$$= \tan^{-1}\left(\frac{V}{H\cos 30°}\right) = \tan^{-1}\left(\frac{2}{\sqrt{3}}\cdot\theta\right)$$

Here, θ = true dip = 40°

101. (*b*) As, $\mu = \mu_0(1+\chi)$ or $\mu = 4\pi\times10^{-7}(1+599)$

or $\quad \mu = 7.536\times10^{-4}\ \text{TmA}^{-1}$

$$B = \mu H = 7.536\times10^{-4}\times1200\ \text{T}$$

$\therefore$ Hence, $\quad f = BA = 7.536\times10^{-4}\times1200\times0.2\times10^{-4}\ \text{Wb}$

$$= 1.81\times10^{-5}\ \text{Wb}$$

102. (*b*) As Curie law explains, we can deduce a formula for the relation between magnetic field induction, temperature and magnetisation.

i.e., $\quad I\,(\text{magnetisation}) \propto \frac{B\,(\text{magnetic field induction})}{t\,(\text{temperature in kelvin})}$

$$\Rightarrow \quad \frac{I_2}{I_1} = \frac{B_2}{B_1}\times\frac{t_1}{t_2}$$

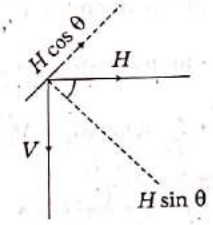
Here $\quad I_1 = 8\,\text{Am}^{-1}$

$B_1 = 0.6\text{T},\ T_1 = 4\ \text{K}$

$B_2 = 0.2\,\text{T},\ T_2 = 16\,\text{K},\ I_2 = ?$

$$\Rightarrow \quad \frac{0.2}{0.6}\times\frac{4}{16} = \frac{I_2}{8} \Rightarrow I_2 = 8\times\frac{1}{12} = \frac{2}{3}\,\text{Am}^{-1}$$

103. (*c*) Here, $\tan\delta_1 = \frac{V}{H\cos\theta}$



$$\tan\delta_2 = \frac{V}{H\cos(90°-\theta)} = \frac{V}{H\sin\theta}$$

$$\frac{\tan\delta_1}{\tan\delta_2} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

or $\quad \theta = \tan^{-1}\left(\frac{\tan\delta_1}{\tan\delta_2}\right)$

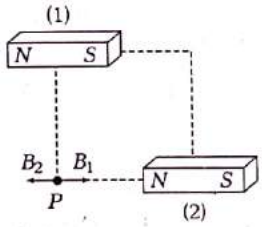
104. (*d*) As potential energy is given as

$U = -MB(1-\cos\theta) \Rightarrow U = -MB$ [when $\theta = 0°$, U = minimum]

Hence, **M** and **B** parallel to each other for minimum potential energy.

PQ_6 configuration provides $\theta = 0°$, $U_{min} = -MB$

105. (*a*) Point P lies on equatorial line of magnet (1) and axial line of magnet (2) as shown



$$B_1 = \frac{\mu_0}{4\pi}\cdot\frac{M}{d^3} = 10^{-7}\times\frac{1000}{(0.1)^3} = 0.1\text{T}$$

$$B_2 = \frac{\mu_0}{4\pi}\cdot\frac{2M}{d^3} = 10^{-7}\times\frac{2\times1000}{(0.1)^3} = 0.2\text{T}$$

$$\therefore \quad B_{net} = B_2 - B_1 = 0.1\text{T}$$

106. (*d*) $\quad \frac{\nu_1}{\nu_2} = \frac{12}{4} = \frac{3}{1}$

$$\therefore \quad \frac{T_1}{T_2} = \frac{1}{3}$$

Further, $\quad T \propto \frac{1}{\sqrt{M}}$

$$\therefore \quad \sqrt{\frac{M_1 - M_2}{M_1 + M_2}} = \frac{1}{3}$$

This equation gives, $\frac{M_1}{M_2} = \frac{5}{4}$

107. (*b*) Time period of suspended magnet

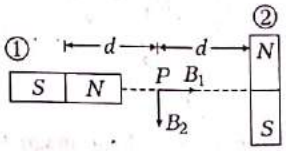
$$T = 2\pi\sqrt{\frac{I}{MB\cos\delta}} \Rightarrow \nu = \frac{1}{2\pi}\sqrt{\frac{MB\cos\delta}{I}}$$

$$\nu \propto \sqrt{B\cos\delta}$$

or $\quad B \propto \frac{\nu^2}{\cos\delta}$

$$\therefore \quad \frac{B_1}{B_2} = \frac{400}{\cos 30°}\times\frac{\cos 60°}{225} = \frac{16\times2}{9\times\sqrt{3}}\times\frac{1}{2} = 16:9\sqrt{3}$$

108. (*d*) At point P net magnetic field $B_{net} = \sqrt{B_1^2 + B_2^2}$



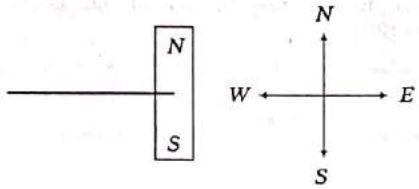
where $\quad B_1 = \frac{\mu_0}{4\pi}\cdot\frac{2M}{d^3}$

and $\quad B_2 = \frac{\mu_0}{4\pi}\cdot\frac{M}{d^3} \Rightarrow B_{net} = \frac{\mu_0}{4\pi}\cdot\frac{\sqrt{5}M}{d^3}$

109. (*d*) Frequency, $\nu = \frac{1}{2\pi}\sqrt{\frac{MH}{I}}$

$\therefore \quad \nu' = \frac{1}{2\pi}\sqrt{\frac{M(B+H)}{I}}$

where, B = Magnetic field due to downward conductor



Then, $B = \frac{\mu_0}{4\pi}\cdot\frac{2I}{a}$

$B = 10^{-7} \times \frac{2\times 15}{20\times 10^{-2}}$

$B = 15\ \mu T$

$\therefore \quad \frac{\nu'}{\nu} = \sqrt{\frac{B+B_H}{B}}$

$\frac{\nu'}{10} = \sqrt{\frac{15+12}{12}} = 1.5$

$\nu' = 15\ Hz$

110. (*a*) Using the formula for time period for magnetic system

$$T = 2\pi\sqrt{\left(\frac{I}{MH}\right)}$$

$\Rightarrow \quad T \propto \frac{1}{\sqrt{M}}$ ...(i)

When similar poles placed at same side, then

$$M_1 = M + 2M = 3M$$

So, from Eq. (i)

$T_1 \propto \frac{1}{\sqrt{3M}}$ ...(ii)

When the polarity of a magnet is reversed, then

$$M_2 = 2M - M = M$$

So, from Eq . (i)

$T_2 \propto \frac{1}{\sqrt{M}}$ ...(iii)

Now, dividing Eq. (ii) by Eq. (iii), we get

$$\frac{T_1}{T_2} = \frac{\sqrt{M}}{\sqrt{3M}} = \frac{1}{\sqrt{3}}$$

$\Rightarrow \quad T_2 = \sqrt{3}\,T_1$

Hence, $T_1 < T_2$

111. (*b*) Initially, the time period of the magnet

$$T = 2\pi\sqrt{\left(\frac{I}{MB}\right)}$$

For each part of magnet, its moment of inertia is $\frac{I}{27}$ and its magnetic moment $= M/3$.

When magnet is cut along its length, into three equal parts and placed on each other, then new moment of inertia of system

$$I' = \frac{I}{27}\times 3 = \frac{I}{9}$$

and magnetic moment of system

$$M' = \frac{M}{3}\times 3 = M$$

$\therefore$ Time period of system

$$T' = 2\pi\sqrt{\frac{I'}{M'B}} = \frac{1}{3}\times 2\pi\sqrt{\frac{1}{B}} = \frac{T}{3} = \frac{2}{3}\ s$$

B) Medical entrances' Special format Questions.

1. (*b*) When we break magnet into two pieces its poles does not get separated as it again becomes a magnet. Also, when magnet is broken into two equal pieces, its magnetic moments gets reduced to half.

4. (*a*) According to Gauss's law in magnetism, $\oint B\cdot dS = 0$

8. (*a*) Relative magnetic permeability is given by

$$\mu_r = \frac{\mu}{\mu_0}$$

It is a dimensionless quantity.

9. (*a*) Diamagnetic substances are repelled by a magnet. The level of a solution falls when one arm of a U-tube containing a diamagnetic solution is placed in between the poles of a strong magnet.

12. (*b*) Infact, iron is a ferromagnetic material.

13. (*a*) The susceptibility of diamagnetic material is small and negative. It is independent of temperature.

14. (*a*) Magnetic moment of circular loop, $M = NIA$

where, $A = \pi r^2$ when $r' = 2r$

$$M' = NI(2\pi r)^2 = 4M$$

15. (*a*) A magnet does not experience any force since net force, $mB - mB = 0$. The forces on NS poles are equal, unlike and parallel makes a couple to align the magnet.

16. (*b*) Time period in sum position, $T_s = 2\pi\sqrt{\frac{I_s}{M_s H}}$

where, $I_s = I_1 + I_2$ and $M_s = M_1 + M_2$

$\therefore \quad T_s = 2\pi\sqrt{\frac{I_1+I_2}{(M_1+M_2)H}}$

Time period in difference position,

$$T_d = 2\pi\sqrt{\frac{I_1+I_2}{(M_1-M_2)H}} \Rightarrow T_S < T_D$$

17. (*b*) The susceptibility of ferromagnetic substances varies inversely with temperature. After curie point the susceptibility of ferromagnetic substance varies inversely with its absolute temperature. These substances obey's Curie's law only above its curie point.

18. (a) The core of a transformer undergoes cycles of magnetisation again and again. The energy loss is lesser and hence it is used in moving coil galvanometer.

19. (e) The properties of a substance is due to alignment of molecules in it. When these substances are heated, molecules acquire kinetic energy. Some of the molecules get back to the closed chain arrangement (leading to zero resultant). So they lose their magnetic property. Hence the properties of paramagnetic and ferromagnetic substances are effected by heating.

Match the columns

2. Since, the magnetic potential energy U_m is given by,

$$U_m = \mathbf{m}\cdot\mathbf{B} = -mB\cos\theta$$

A. For $\theta = 90° \Rightarrow U_m = -mB\cos 90° = 0$

B. For $\theta = 0°, U_m = -mB\cos 0° = -mB$

The potential energy is minimum and hence the needle is in most stable position.

C. For $\theta = 180°, U_m = -mB\cos 180° = +mB$

3. In terms of the susceptibility χ, a material is diamagnetic, if χ is negative, para-if χ is positive and small and ferro-if χ is large and positive.

4.

Diamagnetic	Paramagnetic	Ferromagnetic
$-1 \le \chi < 0$	$0 < \chi < \varepsilon$	$\chi >> 1$
$0 \le \mu_r < 1$	$1 < \mu_r < 1+\varepsilon$	$\mu_r >> 1$
$\mu < \mu_0$	$\mu > \mu_0$	$\mu >> \mu_0$

Medical entrance's gallery

1. (b) ∵ Work done in rotating the magnet

$$W = MB(\cos\theta_0 - \cos\theta)$$

where, M = magnetic moment of the magnet and

B = magnetic field

$$\Rightarrow \quad W = MB(\cos 0° - \cos 60°) = MB\left(1-\frac{1}{2}\right) \Rightarrow \frac{MB}{2} \quad \text{...(i)}$$

$$\therefore \quad MB = 2W$$

Torque on a magnet in this position is given by,

$$\tau = \mathbf{M}\times\mathbf{B} = MB\cdot\sin\theta = 2W\cdot\sin 60° \quad \text{[from Eq. (i)]}$$

$$= 2W\frac{\sqrt{3}}{2} = W\sqrt{3}$$

2. (d) As we know the relation between the magnetic permeability and susceptibility of material is, $\mu_r = 1+\chi_m$ ...(i)

∵ For diamagnetic substances, $\mu_r < 1$

So, according to Eq. (i), the magnetic susceptibility (χ_m) of diamagnetic substance will be negative.

While in case of para and ferromagnetic substances subsceptibility is positive.

3. (b) The magnetic susceptibility of a ferromagnetic substance decreases with increase in temperature and above curie's temperature, the substance behaves like paramagnetic one. Hence, option (b) is correct.

4. (c) As we know that, the magnetic field at a point due to the dipole is given by $B = \frac{\mu_0}{4\pi}\cdot\frac{M}{r^3}\sqrt{1+3\cos^2\theta}$

where, M is magnetic dipole moment.

$$\Rightarrow \quad B \propto \frac{1}{r^3}$$

5. (b) The coercivity of bar magnet $= 4\times10^3\ \text{Am}^{-1}$

Length of solenoid = 12 cm; Number of turns = 60

The coercivity of 12 cm length of solenoid

$$= 4\times10^3\times12\times10^{-2} = 480$$

Now, current through each turn to demagnetise $= \frac{480}{60} = 8\ \text{A}$

6. (c) The effective length of magnet = 31.4 cm

Pole strength, $m = 0.8$ Am

Length of semi-circle $= \pi\frac{D}{2} = l$

where, D = diameter of circle

$$\Rightarrow \quad D = \frac{2l}{\pi} = \frac{2\times31.4\times10^2}{3.14} \Rightarrow D = 20\times10^{-2}\ \text{m}$$

Now, the magnetic moment

$$= ml = mD = 0.8\times20\times10^{-2}$$

$$= 16\times10^{-2} = 0.16\ \text{A-m}^2$$

7. (a) As, vertical component of Earth's magnetic field is

$$B_v = \sqrt{3}B_H \quad \text{[Given]}$$

Angle of dip at any place is given by,

$$\tan\delta = \frac{B_v}{B_H} = \frac{\sqrt{3}B_H}{B_H} = \sqrt{3}$$

$$\Rightarrow \quad \delta = \tan^{-1}(\sqrt{3}) = 60°$$

8. (c) Given, $B_H = 3\times10^{-6}$ T, $\theta = 45°$, $n = 50$

and $r = 20\times10^{-2}$ m

As, the current, $I = \frac{2rB_H}{\mu_0 n}\tan\theta$

$$= \frac{2\times20\times10^{-2}\times3\times10^{-5}\times\tan 45°}{4\pi\times10^{-7}\times50}$$

$$= \frac{120\times10^{-7}}{4\pi\times10^{-7}\times50} = \frac{3}{5\pi} = \frac{3}{5\times3.14} = \frac{3}{15.7} = 0.19\ \text{A}$$

9. (b) The intensity of magnetisation increases for a ferromagnetic substance with the applied magnetic field and become constant, when the domains get perfectly aligned with respect to the external magnetic fields.

10. (c) The work done (in rotating) by external agent

$$= \text{change in potential energy}$$

$$= -MB\cos\theta_2 - (-MB\cos\theta_1)$$

$$= -MB(\cos 45° - \cos 0°)$$

$$= \left(1-\frac{1}{\sqrt{2}}\right)MB = \frac{0.41}{\sqrt{2}}\times\frac{1}{10} \approx 0.03\ \text{J}$$

11. (b) Core of electromagnets are made up of ferromagnetic material which has high permeability and low retentivity.

12. (*a*) Magnetic permeability of substance, $\mu = \frac{B}{H} \Rightarrow \mu \propto \frac{1}{H}$

where, H is the magnetising or magnetic field strength. So, the magnetising field of a ferromagnetic material is increasing thus its, permeability decreased.

13. (*c*) a. $M_1 = m\sqrt{2}$

b. $\overset{m}{\underset{m}{\leftrightarrows}} \Rightarrow M_2 = 0$

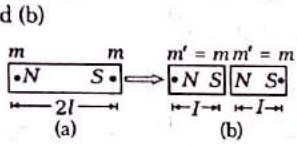
c. $M_3 = m\sqrt{(1+\cos 30°)2}$

$$= m\sqrt{\left(1+\frac{\sqrt{3}}{2}\right)2} = m\sqrt{2+\sqrt{3}}$$

d. $M_4 = 2m\cos 30° = m\sqrt{3}$

14. (*c*) The bar magnet is shown in the diagram

For fig. (a) and (b)



Magnetic moment for (a) is $M = m\,(2l)$ = magnetic moment

Magnetic moment for (b) is of the first

$$M' = m'(l)$$

Now, $$\frac{M'}{M} = \frac{m'(l)}{m(2l)} = \frac{1}{2}\left(\frac{m'}{m}\right) \quad ...(i)$$

When magnet is cut as shown in the diagram, pole strength will not be affected. Hence, $m' = m$

$\Rightarrow \quad \frac{m'}{m} = 1 \Rightarrow M' = \frac{M}{2}$ [from Eq. (i)]

15. (*c*) The class of the magnetic material is ferromagnetic.

16. (*d*) For paramagnetic sample (Curie's law) $I \propto B/T$

where, $I_1 = 0.8$ A/m and $B_1 = 0.8$ T

and $T_1 = 5$ K $\Rightarrow B_2 = 0.4$ T

$T_2 = 20$ K $\Rightarrow I_2 = ?$

$\Rightarrow \frac{I_1}{I_2} = \frac{B_2/T_1}{B_2/T_2} \Rightarrow \frac{I_1}{I_2} = \frac{B_1 \times T_2}{B_2 \times T_1}$

$\Rightarrow \frac{0.8}{I_2} = \frac{0.8\times 20}{0.4\times 5} \Rightarrow I_2 = \frac{0.4\times 5}{2} = 0.1\,\text{Am}^{-1}$

17. (*a*) When temperature increases beyond Curie temperature, the disorder of the magnetic moments of the material increases, hence alignment of magnetic moments are disturbed which caused material to behave as paramagnetic.

18. (*a*) We know that intensity of magnetisation, $I = \frac{M}{V}$

where, M = magnetic moment, V = volume

So, $M = IV = 5\times 10^4 \times \frac{12}{100} \times \frac{1}{(100)^2}$

$$= 60\times 10^4 \times 10^{-6} = 0.6\,\text{A-m}^2$$

19. (*a*) The given, $\chi_m = 299$

and $\mu_0 = 4\pi\times 10^{-7}$ Hm, $\mu = ?$

We know that, $\mu = \mu_0(1+\chi_m)$

$\therefore \quad \mu = 4\pi\times 10^{-7}(1+299) \Rightarrow = 4\times\frac{22}{7}\times 10^{-7}\times 300$

$$= \frac{2.6400}{7}\times 10^{-7} = 3771.4\times 10^{-7}\,\text{Hm}^{-1} \text{ or}$$

$$\cong 3771\times 10^{-7}\,\text{Hm}^{-1}$$

20. (*c*) Circumference of circle $= 2\pi R$

$\therefore \quad 2\pi R = L \Rightarrow R = \left(\frac{L}{2\pi}\right)$

Area, $A = \pi R^2 = \frac{\pi L^2}{4\pi^2} = \frac{L^2}{4\pi}$

Magnetic moment, $(\mu) = IA = \frac{IL^2}{4\pi}$

21. (*d*) Given, $r = 0.05$ nm $= 0.05\times 10^{-9}$m [$\because$ 1 nm $= 10^{-9}$ m]

$n = 10^{16}$ revolutions

and $e = 16\times 10^{-19}$ C

We know that the magnetic moment, $M = AI$

$$M = \pi r^2 \times ne = 3.14\times(0.05\times 10^{-9})^2\times 10^{16}\times 1.6\times 10^{-19}$$

or $= 1.26\times 10^{-23}$ A-m^2

22. (*b*) Let magnetic pole strength be m then $M = ml$

$$M' = m\left(2r\frac{\sin 60}{2}\right)^{\circ} \text{ where } r\left(\frac{\pi}{3}\right) = l$$

$$\Rightarrow \quad M' = 2m\left(\frac{3l}{\pi}\right)\left(\frac{1}{2}\right) = \frac{3ml}{\pi} = \frac{3M}{\pi}$$

23. (*d*) As, $B_E = \sqrt{B_H^2 + B_V^2}$ and $\tan\delta = \frac{B_v}{B_H}$

Given, $B_H = 0.3$ G, $B_V = 0.52$G

$$B_E = \sqrt{(0.30)^2 + (0.52)^2} = \sqrt{0.36} = 0.6\,\text{G}$$

and $\delta = \tan^{-1}\frac{B_V}{B_H} = \tan^{-1}\frac{0.52}{0.3} = 60°$

24. (*a*) When magnet is cut into equal parts, area of cross-section becomes half. Hence, pole strength becomes half.

25. (*a*) We know that the magnetic field induction at a point,

$$B = \frac{\mu_0}{4\pi}\frac{M}{r^3}$$

According to question $B_1 : B_2 : B_3 = \frac{1}{(0.3)^3} : \frac{1}{(0.6)^3} : \frac{1}{(0.9)^3}$

$$= 27 : 3.37 : 1$$

26. (*a*) Given, $M = 120$ Am^2, T=12 s and $B = 0.4\times 10^{-4}$ T

We know that, $T = 2\pi\sqrt{\frac{I}{MB}}$ (squaring both sides)

So, $I = \frac{T^2 MB}{4\pi^2}$ or $I = \frac{(12)^2\times(120)\times(0.4\times 10^{-4})}{4\times 3.14\times 3.14}$

$$I = 175.2\times 10^{-4}\,\text{kgm}^2$$

In this question, the option (a) is approximately equivalent to 175.2×10^{-4}, so the answer is $I = 172.8\times 10^{-4}$ kgm^2.

27. (*a*) On heating a ferromagnetic substance above curie temperature, it becomes paramagnetic.

29. (*c*) As $T_1 = 2\pi\sqrt{\frac{I}{MV}}$, $T_2 = 2\pi\sqrt{\frac{I}{MH}}$

$$V = H\tan\delta \Rightarrow \frac{V}{H} = \tan\delta$$

But, $\frac{V}{H} = 1$

$\therefore$ $\tan\delta = 1 \Rightarrow \delta = 45°$

30. (*b*) The magnetic field at a point on the axis of a bar magnet is,

$$B = \frac{\mu_0}{4\pi}\frac{2M}{r^3} = 10^{-7} \times \frac{2\times 1.25}{(0.5)^3} = 2\times 10^{-6}\ \text{NA}^{-1}\,\text{m}^{-1}$$

31. (*d*) Rod is moving towards east, so induced emf across its end will be

$$B = B_V vl = (B_H \tan\theta)vl \qquad [\because \tan\theta = B_V/B_H]$$

$$= 3\times 10^{-4} \times \frac{4}{3} \times 10 \times 10^{-2} \times 0.25$$

$$= 10^{-5}\text{V} = 10\mu\text{V}$$

32. (*c*) From the relation, susceptibility of the material is

$$\chi_m = \frac{I}{H} \Rightarrow \chi_m \propto I$$

Thus, it is obvious that greater the value of susceptibility of a material greater will be the value of intensity of magnetisation *i.e.*, more easily it can be magnetised.

33. (*d*) $\tan\delta' = \frac{\tan\delta}{\cos\theta} = \frac{\tan 45°}{\cos 30°}$; $\tan\delta' = \frac{1}{\sqrt{3}/2} = \frac{2}{\sqrt{3}}$

$\therefore$ $\delta' = \tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$

34. (*c*) When wire is bent in the form of semicircular arc then, $l = \pi r$

$\therefore$ The radius of semicircular arc, $r = l/\pi$

Distance between two end points of semicircular wire

$$= 2r = \frac{2l}{\pi}$$

$\therefore$ Magnetic moment of semicircular wire

$$= m \times 2r = m \times \frac{2l}{\pi} = \frac{2}{\pi}ml$$

But ml is the magnetic moment straight wire *i.e.*, $ml = M$

$\therefore$ New magnetic moment $= \frac{2}{\pi}M$

35. (*b*) We know that, $B = \mu_0 H + \mu_0 I$

$$\therefore \quad I = \frac{B - \mu_0 H}{\mu_0} = \frac{\mu H - \mu_0 H}{\mu_0} = \left(\frac{\mu}{\mu_0} - 1\right)H$$

$$I = (\mu_r - 1)H$$

For a solenoid having n turns/length and current I.

$$H = I$$

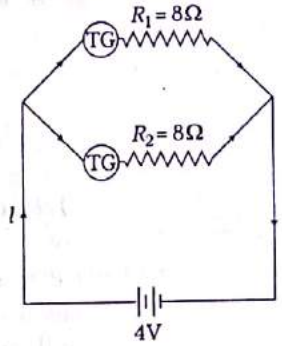
$$I = (\mu_r - 1)nI' = (1000 - 1)500 \times 0.5$$

$$= 2.5 \times 10^5\text{Am}^{-1}$$

$\therefore$ Magnetic moment, $M = lV = 2.5 \times 10^5 \times 10^{-4} = 25\,\text{A-m}^2$

36. (*b*) Current in tangent galvanometer

$$I = \frac{2rH}{\mu_0 N}\tan\theta$$



Here, R_1 and R_2 are in parallel

$$\therefore \quad \frac{1}{R_{net}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_2 + R_1}{R_1 R_2} = \frac{8+8}{8\times 8}$$

$$R_{net} = 4\Omega$$

Hence, $I = \frac{V}{R} = \frac{4}{4} = 1\text{A}$

From Eq. (i), we get

$$\frac{r\tan\theta}{N} = \frac{\mu_0 I}{2H}$$

$$\therefore \quad \frac{r_A \tan\theta_A}{N_A} = \frac{r_B \tan\theta_B}{N_B}$$

$$\Rightarrow \quad \frac{8\times 1}{\sqrt{3}\times 2} = \frac{16\times\sqrt{3}}{N_B}$$

$$\therefore \quad N_B = 12\,\text{turns}$$

CHAPTER

06 Electromagnetic induction

The phenomenon of generating current/emf in a conducting circuit by change in strength, position or orientation of an associated external magnetic field is called **electromagnetic induction**. The emf, so developed is called **induced emf**. If the conductor is in the form of a closed circuit, a current flows in the circuit. This is called **induced current**. It is the reverse process of magnetic field produced by electric current. EMI was discovered by Michael Faraday in 1831. In this chapter, we will study about the Lenz's law, motional emf and inductances.

Inside

1) Magnetic flux
- Faraday's laws of electromagnetic induction
- Lenz's law and conservation of energy

2) Motional electromotive force
- General form of motional emf
- Fleming's right hand thumb rule to find the direction of induced current
- Induction of field

3) Self-induction
- Kirchhoff's second law with an inductor

4) Mutual induction
- Mutual inductance for two concentric coils
- Mutual inductance of two long co-axial solenoids
- Mutual inductance of a solenoid surrounded by a coil
- Coefficient of coupling
- Combination of mutual inductances
- Growth and decay of current in *L-R* circuit

1) Magnetic flux

The total number of magnetic field lines crossing through any surface, normally when it is placed in a magnetic field is known as the magnetic flux of that surface.

Consider an element of area $d\mathbf{S}$ on an arbitrary shaped surface as shown in Fig. 6.1. If the magnetic field at this element is $\mathbf{B}$, then the magnetic flux through the element is

$$d\phi_B = \mathbf{B}\cdot d\mathbf{S} \Rightarrow d\phi_B = BdS\cos\theta$$

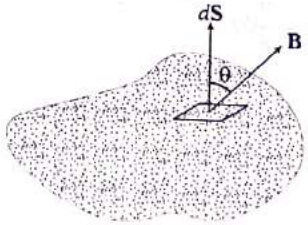


Fig. 6.1

Here, $d\mathbf{S}$ is a vector that is perpendicular to the surface and has a magnitude equal to the area dS and θ is the angle between $\mathbf{B}$ and $d\mathbf{S}$ at that element. In general, $d\phi_B$ varies from element to element.

The total magnetic flux through the surface is the sum of the contributions from the individual area elements.

$$\therefore \quad \phi_B = \int BdS\cos\theta = \int \mathbf{B}\cdot d\mathbf{S}$$

Note

(i) As magnetic lines of force are closed curves (*i.e.*, monopoles do not exist), total magnetic flux linked with a closed surface is always zero.

(ii) A special case, in which **B** is uniform over a plane surface with total area *S*, magnetic flux is given by $\phi_B = BS\cos\theta$

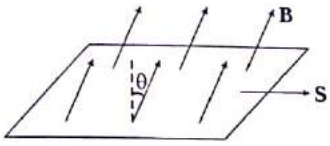
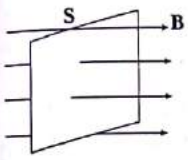
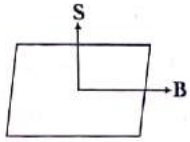
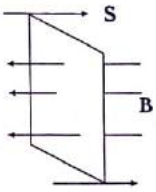


Fig. 6.2 A plane of surface area S placed on uniform field B

Specific conditions

For $\theta = 0°$	For $\theta = 90°$	For $\theta = 180°$
		
B is parallel to surface vector *i.e.*, **S**.	**B** is perpendicular to surface vector *i.e.*, **S**	**B** is antiparallel to surface vector, *i.e.*, **S**.
$\Rightarrow \phi_B = BS\cos 0° = BS$	$\Rightarrow \phi_B = BS\cos 90° = 0$	$\Rightarrow \phi_B = BS\cos 180° = BS(-1) = -BS$
$\Rightarrow \phi_B$ = maximum		

Units and dimension of magnetic flux

(i) Magnetic flux is a scalar quantity.

(ii) The SI unit of magnetic flux is **tesla-metre**^{2} (1 T-m^2). This unit is called **weber (Wb)**.

$$1 \text{ Wb} = 1 \text{ T-m}^2 = 1 \text{ N-m/A}$$

The CGS unit of flux is maxwell (Mx).

$$1 \text{Mx} = 1 \text{ gauss-cm}^2$$

$$1 \text{ Wb} = 1 \text{T} \times 1 \text{m}^2 = 10^4 \text{G} \times 10^4 \text{cm}^2$$

or $\quad 1 \text{ Wb} = 10^8 \text{ Mx}$

Thus, unit of magnetic field is also weber/m^2 (1Wb/m^2)

or $\quad 1 \text{ T} = 1 \text{ Wb/m}^2$

(iii) The dimensional formula of magnetic flux is $[ML^2T^{-2}A^{-1}]$.

Example 6.1 *A uniform magnetic field exists in the space* $\mathbf{B} = B_1\hat{\mathbf{i}} + B_2\hat{\mathbf{j}} - B_3\hat{\mathbf{k}}$. *Find the magnetic flux through an area* **S**, *if the area* **S** *is in yz-plane.*

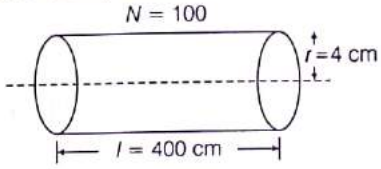
Sol. Since, the field is uniform, we can use formula, $\phi_B = \mathbf{B}\cdot\mathbf{S}$

Area **S** is in *yz*-plane, it means $\mathbf{S} = S\hat{\mathbf{i}}$

Hence, $\phi_B = (B_1\hat{\mathbf{i}} + B_2\hat{\mathbf{j}} - B_3\hat{\mathbf{k}})\cdot(S\hat{\mathbf{i}}) = B_2S$.

Example 6.2 *A long solenoid of radius 4 cm, length 400 cm carries a current of 3 A. The total number of turns is 100. Assuming ideal solenoid, find the flux passing through a circular surface having centre on axis of solenoid of radius 3 cm and is perpendicular to the axis of solenoid (i) inside and (ii) at the end of solenoid.*

Sol.



Number of turns per unit length is given by

$$n = \frac{N}{l} = \frac{100}{4} = 25 \text{ turns/m}$$

(i) Magnetic field of a solenoid at a point inside is

$$B = \mu_0 ni, \; S = \pi r_1^2 \text{ and } \theta = 0°$$

Magnetic flux

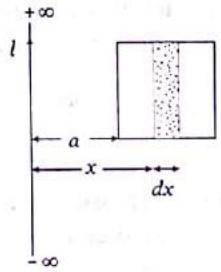
$$\phi_B = BS\cos\theta = \mu_0 ni\, \pi r_1^2 \cos 0°$$
$$= 4\pi \times 10^{-7} \times 25 \times 3 \times \pi \times (3 \times 10^{-2})^2$$
$$= 0.27 \times 10^{-6} \text{ Wb} = 0.27\mu \text{ Wb}$$

(ii) At the end, magnetic field of solenoid is given by

$$= \frac{1}{2}\mu_0 ni.$$

$$\because \quad \phi = \frac{0.27}{2} = 0.135\,\mu\text{Wb}$$

Example 6.3 *A long straight wire carrying current I and a square conducting wire loop of side l , at a distance 'a' from current wire as shown in the figure. Both the current wire and loop are in the plane of paper. Find the magnetic flux of current wire, passing through the loop.*



Sol. As the field of current wire passing through the loop is same in direction (normally inward) but not uniform in magnitude. So, we will use integration method for finding the flux.

The small flux through a thin rectangular strip of length *l* and width *dx*, is given by

$$d\phi_B = \mathbf{B}_x \cdot d\mathbf{S} = B_{(x)} dS \cos 180°.$$

Magnetic field due to a long straight wire carrying current *I* is given by $B = \frac{\mu_0 I}{2\pi x}$, and area, $dS = l \times dx$.

$$\therefore \quad \phi_B = \int d\phi_B = -\int \frac{\mu_0}{2\pi}\frac{Il}{x}\cdot dx = -\frac{\mu_0}{2\pi} Il[\log_e x]_{x=a}^{x=a+l}$$

$$= -\frac{\mu_0}{2\pi}\cdot Il \log_e \frac{a+I}{a}$$

Faraday's laws of electromagnetic induction

There are two laws of electromagnetic induction given by Faraday's law as below:

Faraday's first law

Whenever the amount of magnetic flux linked with a circuit changes, an emf is induced in the circuit.

The actual number of magnetic lines passing through the circuit does not matter of the values of the induced emf. Induced current is determined by the rate at which the magnetic flux changes.

Faraday's second law

The magnitude of the induced emf in a circuit is equal to the rate of change of magnetic flux through the circuit. Mathematically, Faraday's second law can be expressed as,

Induced emf $\propto$ Rate of change of magnetic flux

$$e = \frac{-d\phi_B}{dt}$$

$$\left[\because \text{rate of change of magnetic flux} = \frac{\phi_2 - \phi_1}{t_2 - t_1}\right]$$

The negative sign in above relation indicates that the induced emf in the loop due to changing flux always opposes the change in the magnetic flux.

In the case of a closely wound coil of N turns, the change of flux associated with each turn, is same. Therefore, the expression for the total induced emf is given by

$$\boxed{e = -N\frac{d\phi_B}{dt}}$$

The induced emf can be increased by increasing the number of turns N of a closed coil.

The magnetic flux can be changed in the following ways

(i) The magnitude of **B** can change with time.
Thus, time-varying magnetic field is
$$B = B(t)$$

(ii) The current producing the magnetic field can change with time. Thus, time-varying current is
$i = i(t)$

(iii) The area enclosed by the loop can change with time. This can be done by pulling a loop inside (or outside) a magnetic field. By doing, so the area enclosed by loop (hatched area) can be changed.

Fig. 6.3

(iv) The angle θ between **B** and the normal to the loop can change with time.
This can be done by rotating a loop in a magnetic field.

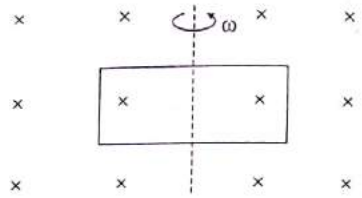


Fig. 6.4

(v) Any combination of the above can occur.

Example 6.4 *A coil with an average diameter of 0.02 m is placed perpendicular to a magnetic field of 6000 T. If the induced emf is 11 V. When the magnetic field is changed to 1000 T in 4 s. What is the number of turns in the coil?*

Sol. Given, radius of coil, $r = \frac{0.02}{2} = 0.01\text{m}$

$$B_1 = 6000\text{ T}, B_2 = 1000\text{ T}, t = 4\text{ s}, e = 11\text{V}$$

$$\text{emf, } e = N\frac{(\phi_2 - \phi_1)}{t} = NA\frac{B_2 - B_1}{t}$$

$$= N\pi r^2 \frac{B_2 - B_1}{t}$$

$$11 = N \cdot \frac{22}{7} \times (0.01)^2 \times \frac{(6000 - 1000)}{4}$$

Number of turns,

$$N = \frac{11 \times 7 \times 4}{22 \times (0.01)^2 \times 5000} = 28$$

Induced current and induced charge

Induced current

When the magnetic flux passing through a loop is changed, an induced emf and hence an induced current is produced in the circuit.

If R is the resistance of the circuit, then induced current is given by

$$i = \frac{e}{R} = \frac{1}{R}\left(\frac{-d\phi_B}{dt}\right) \quad \text{...(i)}$$

If induced current is produced in a coil rotated in a uniform magnetic field, then

$$I = \frac{NBA\,\omega \sin\omega t}{R} = I_0 \sin\omega t$$

where, $I_0 = \frac{NBA\,\omega}{R}$ = peak value of induced current,

N = number of turns in the coil,

B = magnetic field, ω = angular velocity of rotation,

A = area of cross-section of the coil.

Induced charge

When the current starts flowing in the circuit, flow of charges also takes place. Charge flown in the circuit in time dt will be given by

$$dq = idt = \frac{1}{R}(-d\phi_B) \quad \text{...(ii)}$$

Thus, for a time interval Δt we can write,

$$e = -\frac{\Delta\phi_B}{\Delta t}, \quad i = \frac{1}{R}\left(\frac{-\Delta\phi_B}{\Delta t}\right)$$

and $$\Delta q = \frac{1}{R}(-\Delta\phi_B) \quad \text{...(iii)}$$

Direction of induced current

The direction of induced current in a loop may be obtained by using

$$e = -\frac{d\phi}{dt} \text{ and } i = \frac{1}{R}\left(-\frac{d\phi}{dt}\right)$$

So, the steps to decide the direction of induced current are as follows

(i) Define a positive direction for the area vector **S**.

(ii) From the directions of **S** and the magnetic field **B**, determine the sign of ϕ_B and its time rate of change *i.e.*, $\frac{d\phi_B}{dt}$.

(iii) If $\frac{d\phi_B}{dt}$ is positive, *i.e.*, flux is increasing, sign of induced current is negative and *vice-versa*. In this way, we will determine the sign of the induced current.

(iv) Finally, determine the direction of induced current using your right hand. Curl the fingers of your right hand around **S**, with your right thumb in the direction of **S**. If the induced current is positive, it is in the same direction as your curled fingers and if it is negative, it is in the opposite direction.

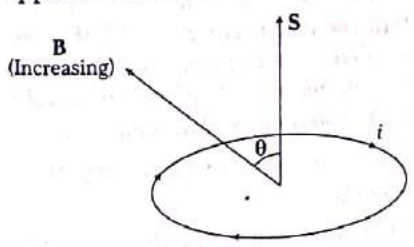


Fig. 6.5 [$\theta < 90°, \phi_B > 0, \frac{d\phi_B}{dt} > 0$, *so induced current is negative*]

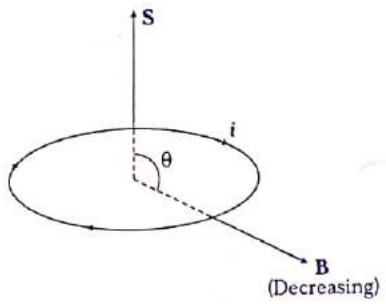


Fig. 6.6

[$\theta > 90°, \phi_B < 0, \frac{d\phi_B}{dt} > 0$, so induced current is negative]

Example 6.5 *A coil consists of 200 turns of wire having a total resistance of 2.0 Ω. Each turn is a square of side 18 cm, and a uniform magnetic field directed perpendicular to the plane of the coil is turned on. If the field changes linearly from 0 to 0.5 T in 0.80 s, then what is the magnitude of induced emf and current in the coil while the field is changing?*

Sol. Given, $N = 200$ turns, $R = 2.0\,\Omega$, change in magnetic field, $\Delta B = 0.5$ T, side of square, $l = 18$ cm $= 18 \times 10^{-2}$m, $\Delta t = 0.80$ s

From the Faraday's law,

Induced emf, $|e| = -\frac{Nd\phi_B}{dt} = (N \times S)\frac{\Delta B}{\Delta t}$ [$\because \phi_B = \mathbf{B}\cdot\mathbf{S}$]

Area, $S = l^2 = (18 \times 10^{-2})^2$

$$= \frac{(200)(18 \times 10^{-2})^2 (0.5 - 0)}{0.8} = 4.05 \text{ V}$$

$\therefore$ Further, induced current, $i = \frac{|e|}{R} = \frac{4.05}{2} \approx 2.0$ A

Example 6.6 *The magnetic flux threading a metal ring varies with time t according to* $\phi_B = 3(at^3 - bt^2)$ *T-m*^{2} *with* $a = 2.00\,s^{-3}$ *and* $b = 6.00\,s^{-2}$. *The resistance of the ring is* 3.0 Ω. *Determine the maximum current induced in the ring during the interval from t = 0 to t = 2.0 s.*

Sol. Given, magnetic flux $\phi_B = 3(at^3 - bt^2)$ T-m^2

where, $a = 2\text{s}^{-3}, b = 6\text{s}^{-2}$

$\therefore$ Induced emf, $|e| = \left|\frac{d\phi_B}{dt}\right| = 9at^2 - 6bt$

$\therefore$ Induced current, $i = \frac{|e|}{R} = \frac{9at^2 - 6bt}{3} = 3at^2 - 2bt$

For current to be maximum, $\frac{di}{dt} = 0$

$\therefore$ $6at - 2b = 0$ or $t = \frac{b}{3a}$

i.e., at $t = \frac{b}{3a}$, current is maximum.

Substituting the values in Eq. (b), the maximum current is

$$i_{max} = 3a\left(\frac{b}{3a}\right)^2 - 2b\left(\frac{b}{3a}\right)$$

$$= \frac{b^2}{3a} - \frac{2b^2}{3a} = \frac{-b^2}{3a}$$

Magnitude of this maximum current will be $i_{max} = \frac{b^2}{3a}$.

Substituting the given values of a and b, we have

$$i_{max} = \frac{(6)^2}{3(2)} = 6.0 \text{ A}$$

Example 6.7 *A square loop of side 10 cm and resistance 0.5 Ω is placed vertically in the east-west plane. A uniform magnetic field of 0.10T is set up across the plane in the north-east direction. The magnetic field is decreased to zero in 0.70 s at a steady state. What is the magnitude of current in this time-interval?*

Sol. Given, $B = 0.10$ T, area of square loop $= 10 \times 10 = 100 \text{ cm}^2$

$= 10^{-2}\text{m}^2$

As the magnetic field is set up across the plane in the north-east direction, then $\theta = 45°$

The initial magnetic flux is given by

$$\phi_B = BS\cos\theta$$

$$\therefore \quad \phi = \frac{0.1 \times 10^{-2}}{\sqrt{2}} \text{ Wb}$$

Final flux, $\phi_{min} = 0$ (given)

The change in flux is brought about in 0.70 s, *i.e.*, $\Delta t = 0.70$s.

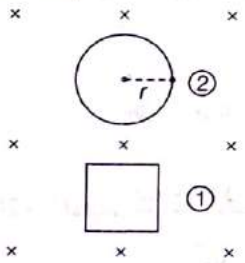
The magnitude of the induced emf is

$$e = \frac{\Delta\phi_B}{\Delta t} = \frac{|\phi - 0|}{\Delta t} = \frac{10^{-3}}{\sqrt{2} \times 0.7} \cong 1 \text{ mV}$$

The magnitude of induced current is $I = \frac{e}{R} = \frac{10^{-3}}{0.5} = 2$ mA

Example 6.8 *A wire of length l in the form of a square loop lies in a plane normal to a magnetic field B_0. If this wire is converted into a circular loop in time, then find the average induced emf.*

Sol.



Total length of circle will be $l = 2\pi r \Rightarrow r = \frac{l}{2\pi}$

Side of square, $L = \frac{l}{4}$.

Using the relation, $\phi_B = \mathbf{B} \cdot \mathbf{S} = BS\cos\theta$

Here, $\theta = 0$

$\Rightarrow \quad \phi_B = BS$

Magnetic flux of square loop will be

$$\phi_1 = B_0 L^2 = \frac{B_0 l^2}{16}$$

Magnetic flux of circular loop will be

$$\phi_2 = B_0 \pi r^2 = \frac{B_0 \pi l^2}{4\pi^2} = \frac{B_0 l^2}{4\pi}$$

Average induced emf is given by

$$\bar{e} = -\frac{\phi_2 - \phi_1}{\Delta t} = -\frac{B_0 l^2}{t_0}\left(\frac{1}{4\pi} - \frac{1}{16}\right) = \frac{B_0 l^2}{16\pi t_0}(4 - \pi)$$

Example 6.9 *Suppose a coil of area $5\,m^2$, resistance 10 Ω and number of turns 200 held perpendicular to a uniform magnetic field of strength 0.4T. The coil is now turned through 180° in time 1s. What is*

(i) average induced emf

(ii) average induced current

(iii) total charge that flows through a given cross-section of the coil?

Sol. Given, area of coil, $S = 5\text{m}^2$, resistance, $R = 10\Omega$, number of turns, $N = 200$, magnetic field, $B = 0.4$T, $\Delta t = 1$s.

When the plane of coil is perpendicular to the magnetic field *i.e.*, $\theta_i = 0°$ and after it is rotated through 180°, then $\theta_f = 180°$.

$\Rightarrow$ Initial flux, $\phi_i = NBS\cos 0° = NBS$

$= 200 \times 0.4 \times 5 = 400$ Wb

and final flux $= NBS\cos 180° = -NBS = -400$ Wb

$\Rightarrow$ Change in flux, $|\Delta\phi_B|$

$= NBS - (-NBS) = 2NBS = 800$ Wb

(i) Average induced emf $(\varepsilon) = \frac{|\Delta\phi|}{\Delta t} = \frac{2NBA}{\Delta t} = 800$ V

(ii) Average current $= \frac{\varepsilon}{R} = \frac{2NBA}{R\Delta t} = 80$ A

(iii) Average charge $= \frac{\Delta Q}{\Delta t} = \frac{2NBA}{R\Delta t}$

$\Rightarrow$ Total charge $\Delta Q = \frac{2NBA}{R} = 80$ C

Example 6.10 *Through a long solenoid of diameter 4.1 cm, having 100 turns per cm, a current I = 1A is flowing. At its centre, a 60 turns closely packed coil of diameter 3.1 cm is placed such that the coil is coaxial with the long solenoid. The current in the solenoid is reduced to zero at a steady rate in 10 ms. What is the magnitude of emf induced in the coil while the current in the solenoid is changing?*

Sol. Initially, magnetic flux passing through the coil (one turn), $\phi_1 = \mathbf{B} \cdot \mathbf{A} = BA\cos 0°$.

Magnetic field at a point inside the solenoid is given by $B = \mu_0 nI$, where n is number of turns per metre,

$$\therefore \qquad \phi_1 = \mu_0 nI \times \frac{\pi d^2}{4}$$

Here, $n = 10000$ turns per metre,

$I = 1\text{A}, d = 3.1$ cm

$$\therefore \qquad \phi_1 = \mu_0 nI \frac{\pi d^2}{4} = 4\pi \times 10^{-7} \times 1 \times 10000 \times \pi \times \frac{(3.1 \times 10^{-2})^2}{4}$$

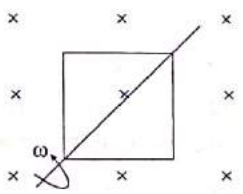
$$= 0.947 \times 10^{-5} \text{ Wb}$$

Finally, the flux becomes zero because the current reduces to zero.

Thus, induced emf, $e = \frac{|\Delta\phi|}{\Delta t} = \frac{0.947 \times 10^{-5}}{10 \times 10^{-3}} = 9.47 \times 10^{-4}$V

The total emf $= N \times e = 60 \times 9.47 \times 10^{-4} = 568.2 \times 10^{-4}$V

Example 6.11 *A square loop of edge b having M turns is rotated with a uniform angular velocity ω about one of its diagonals which is kept fixed in a horizontal position. A uniform magnetic field B_0 exists in the vertical direction.*



Find (i) the emf induced in the coil as a function of time t.

(ii) the maximum emf induced.

(iii) the average emf induced in the loop over a long period.

(iv) if resistance of loop is R, amount of charge flown in time $t = 0$ to $t = 2T$.

(v) heat produced in time $t = 0$ to $t = 2T$.

Sol. (i) Initially, plane of loop is $\perp^{ar}$ to the field. Let at any time t, normal to loop makes an angle θ with magnetic field $\theta = \omega t$.

The flux passing through loop,

$$\phi_B = MB_0 S \cos\theta = MB_0 b^2 \cos\omega t$$

$$\Rightarrow \qquad e = -\frac{d\phi_B}{dt} = MB_0 b^2 \omega \sin\omega t = e_0 \sin\omega t,$$

where, $e_0 = MB_0 b^2 \omega$

(ii) For emf to be maximum, then $\sin\omega t$ should be equal to 1.

$$\Rightarrow \qquad e_{max} = e_0$$

(iii) Long period means one time period, *i.e.*, $0 \to T$

We know that average emf is given as

$$\bar{e} = \frac{\int_0^T e\,dt}{\int_0^T dt} = \frac{1}{T} \cdot e_0 \int_0^T \sin\omega t\,dt = 0 \qquad \left(\because \int_0^T \sin\omega t\,dt = 0\right)$$

(iv) Using the relation, induced current,

$$i = \frac{e}{R} = \frac{e_0}{R} \sin\omega t = i_0 \sin\omega t$$

where, $i_0 = \frac{e_0}{R}$.

We know, $q = i\,dt$

So, for time $0 \to 2T$,

$$q = \int_0^{2T} i\,dt = i_0 \int_0^{2T} \sin\omega t\,dt = 0$$

(v) Heat (H) generated in a loop during time $0 \to 2T$, is given as

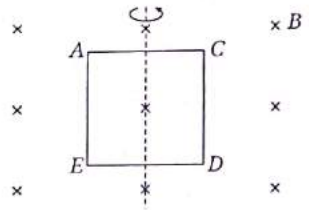
$$H = \int_0^{2T} \frac{e^2}{R} dt = \frac{e_0^2}{0} \int_0^{2T} \sin^2\omega t\,dt$$

$$= \frac{e_0^2}{R} \cdot 2 \cdot \frac{T}{2} = \frac{e_0^2}{R} \cdot \frac{2\pi}{\omega} = \frac{2\pi e_0^2}{\omega R} \qquad \left(\because T = \frac{2\pi}{\omega}\right)$$

Example 6.12 *A square loop ACDE of area 20 cm^2 and resistance 5 Ω is rotated in a magnetic field* **B** *= 2 T through 180°,*

(i) in 0.01 s and *(ii) in 0.02 s.*

Find the magnitude of e, i and Δq in both the cases.



Sol. Let us take the area vector **S** perpendicular to plane of loop inwards. So initially, $d\mathbf{S} \uparrow\uparrow \mathbf{B}$ and when it is rotated by 180°, $\mathbf{S} \uparrow\downarrow \mathbf{B}$.

Hence, initial flux passing through the loop,

$$\phi_i = BS\cos 0° = (2)(20 \times 10^{-4})(1)$$

$$= 4 \times 10^{-3} \text{ Wb}$$

Flux passing through the loop when it is rotated by 180°,

$$\phi_f = BS\cos 180°$$

$$= (2)(20 \times 10^{-4})(-1) = -4 \times 10^{-3} \text{ Wb}$$

Therefore, change in flux,

$$\Delta\phi_B = \phi_f - \phi_i = -8 \times 10^{-3} \text{ Wb}$$

(i) Given at $\Delta t = 0.01$ s, $R = 5\,\Omega$

Using the relation,

$$|e| = \left|-\frac{\Delta\phi_B}{\Delta t}\right| = \frac{8 \times 10^{-3}}{0.01} = 0.8 \text{ V}$$

or induced current $i = \frac{|e|}{R} = \frac{0.8}{5} = 0.16$ A

and charge $\Delta q = i\Delta t = 0.16 \times 0.01 = 1.6 \times 10^{-3}$ C

(ii) Similarly, given at $\Delta t = 0.02$ s, $R = 5\,\Omega$

$$\therefore \qquad |e| = \left|-\frac{\Delta\phi_B}{\Delta t}\right| = \frac{8 \times 10^{-3}}{0.02} = 0.4 \text{ V}$$

$$\therefore \qquad i = \frac{|e|}{R} = \frac{0.4}{5} = 0.08 \text{ A}$$

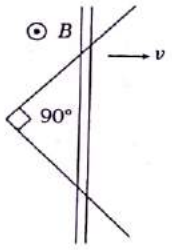
and charge $\Delta q = i\Delta t = (0.08)(0.02) = 1.6 \times 10^{-3}$ C

Example 6.13 *The two conducting rails are placed perpendicular to each other, such that their ends are joined as shown in figure. A conducting bar is now placed over the rails and start moving with constant velocity v starting from the vertex at time t = 0.*

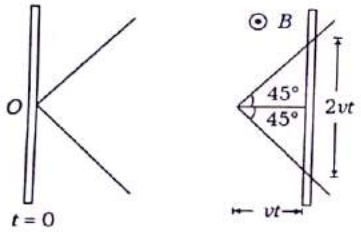
(i) The flux through the triangle (isosceles) by the rails and bar at $t = t_0$.

(ii) The emf around the triangle at that time.

(iii) In what manner does the emf around the triangle vary with time.



Sol.



(i) At time $= t$, area of loop, $S = \frac{1}{2} \cdot vt \cdot 2vt = v^2t^2$

($\because$ area of isosceles triangle $= \frac{1}{2} \times$ base $\times$ height)

Using the relation,

Magnetic flux $\phi_B = BS = Bv^2t^2$ and at, $t = t_0 \Rightarrow \phi = Bv^2t_0^2$

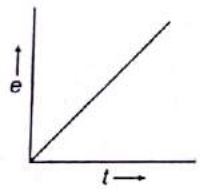
(ii) The emf developed around the triangle at to,

further, $e = \left|\frac{d\phi_B}{dt}\right| = Bv^2 \cdot 2t = 2Bv^2t$

At $t = t_0$, $e = 2Bv^2t_0$

(iii) According to the relation obtained for the emf

i.e., $e = 2Bv^2t \Rightarrow e \propto t$



Hence, emf varies linearly with time.

Lenz's law and conservation of energy

The negative sign in Faraday's equation of electromagnetic induction describes the direction in which induced emf drives the current. This direction is easily determined with the help of Lenz's law, an associated principle with Faraday's law.

Lenz's law give the same result as the sign rule we introduced in Faraday's law but is often easier to use. Lanz's law states that, **the direction of any magnetic induction effect is such as to oppose the cause of effect.**

This law is based upon law of conservation of energy. As the induced emf opposes the change in flux, work has to be done against the opposition offered by induced emf/current in changing the flux. The work done appears as electrical energy in the circuit.

Direction of induced current with the help of Lenz's law

Direction of induced current can be determined by checking whether the flux through a conducting loop or circuit is increasing or decreasing.

- If flux is decreasing, the magnetic field due to induced current will be along the existing magnetic field.
- If flux is increasing, the magnetic field due to induced current will be opposite to existing magnetic field.

Important points based on Lenz's law

The following are some important points that will explain the direction of induced current according to Lenz's law.

(i) Consider a conducting loop is being pulled inside a magnetic field, hence area of loop inside the magnetic field increases.

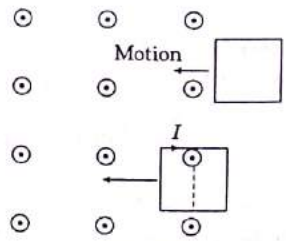


Fig. 6.7

From the above figure, it is clear that the magnetic flux through loop is increasing and hence magnetic field due to induced current will be opposite to the existing magnetic field *i.e.*, inside the plane of paper, thus direction of induced current will be clockwise.

(ii) Consider a loop is being pulled out of magnetic field, area of loop inside field decreases and hence flux ($\phi = BS$) passing through loop also decreases.

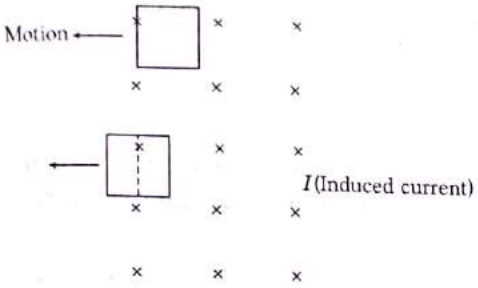


Fig. 6.8

So, magnetic field due to induced current will be along the existing magnetic field, *i.e.*, inside the plane of paper, thus direction of induced current will be clockwise.

(iii) Consider a loop is placed near a current carrying loop wire. The magnetic field due to long wire at distance x from it is given by

$$B = \frac{\mu_0 i}{2\pi x}$$

$$\Rightarrow \quad B \propto i$$

$$\Rightarrow \quad \phi \propto B \propto i$$

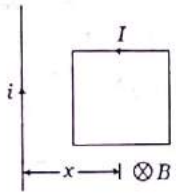


Fig. 6.9

If the current in long wire increases, then

ϕ also increases. The direction of magnetic field due to long wire will be ⊗. In this case, ϕ is increasing, magnetic field due to induced current will be opposite to existing magnetic field, *i.e.*⊙, hence direction of induced current is anti-clockwise.

(iv) If a conducting loop is brought away from a current carrying straight wire.

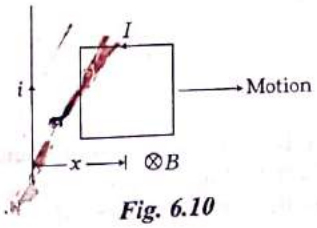


Fig. 6.10

As the loop is moving away so, flux through the loop will decrease, magnetic field due to induced current will be along the existing magnetic field, *i.e.*, ⊗, hence direction of induced current is clockwise.

(v) Consider two conducting loops are placed in a magnetic field.

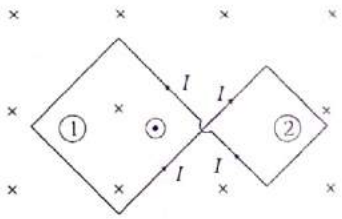


Fig. 6.11

In this case, if magnetic field is increasing, then flux through each loop will increase, hence magnetic field due to induced current will be opposite to the existing magnetic field, *i.e.* ⊙, hence induced current will be anti-clockwise but single current will flow in the wire. The induced emf in loop 1 will be greater due to its bigger area, so single current will flow according to current in loop 1. Induced current in 1 will be anti-clockwise and in loop 2 will be clockwise.

(vi) (a) When north pole moves towards ring, then flux will increase, induced current will oppose this, so north pole will be formed in loop as seen by observer.

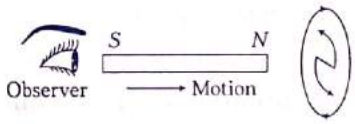


Fig. 6.12

Induced current will be anti-clockwise.

(b) When north pole moves away from ring, then flux will decrease, induced current will oppose this, so south pole will be formed in loop as seen by observer.

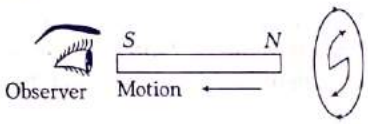


Fig. 6.13

Induced current will be clockwise.

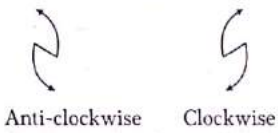


Fig. 6.14

(c) Similar observations (as in case (a) can be observed when south pole moves towards ring. So, induced current will be clockwise here.

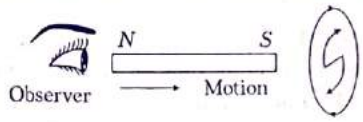


Fig. 6.15

(d) Similar observations (as in case (b) can be observed when south pole moves away from the ring. So, induced current in this case will be anti-clockwise.

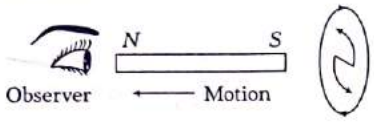


Fig. 6.16

(vii) Two co-axial circular loops, identical to each other, carry equal amount of currents and circulating in the same direction.

(a) When loops approaches each other, then the flux linked with each coil will increase.

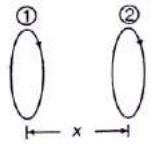


Fig. 6.17

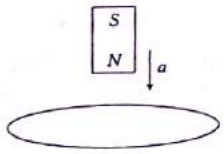
Magnetic field due to 1 ① at ② will be given by

$$B = \frac{\mu_0 N i R^2}{2(R^2 + x^2)^{3/2}}$$

So, x decreases, B will increase ϕ also increases. The induced current in each coil will try to reduce flux and will be opposite to original current. So, current in each coil will decrease.

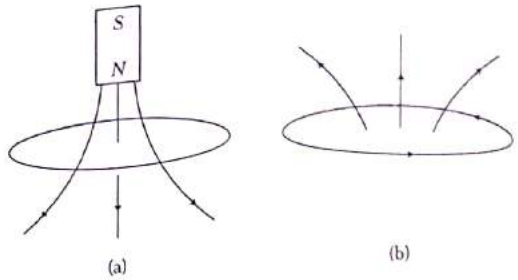
(b) Now, loops move away from each other, then the flux linked with coil will decrease and so the induced current will try to increase the flux and hence current in each coil will increase.

Example 6.14 *A bar magnet is freely falling along the axis of a circular loop as shown in figure. State whether its acceleration 'a' is equal to, greater than or less than the acceleration due to gravity g.*



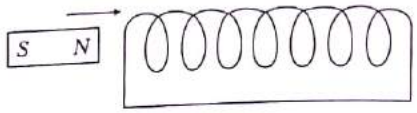
Sol. Acceleration of the bar magnet will be less than acceleration due to gravity ($a < g$) because according to Lenz's law, whatever may be the direction of induced current, it will oppose the cause. Here, the cause is, the free fall of magnet and so the induced current will oppose it and the acceleration of magnet will be less than the acceleration due to gravity g.

Alternate method This can be understood in a different manner. When the magnet falls downwards with its north pole downwards. The magnetic field lines passing through the coil in the downward direction increase. Since, the induced current opposes this, the upper side of the coil will become north pole, so that field lines of coil's magnetic field are upwards. Now, like poles repel each other. Hence, $a < g$.

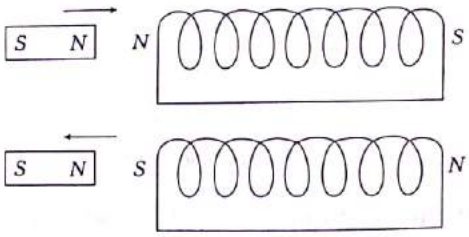


If this circular loop had a slot, an emf in that case will be induced but no current will flow *i.e.*, $a = g$.

Example 6.15 *A bar magnet is brought near a solenoid as shown in figure. Will the solenoid attract or repel the magnet?*

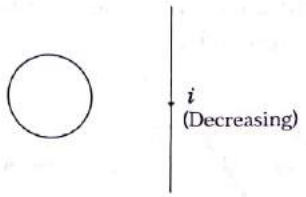


Sol. When the magnet is brought near the solenoid, then according to Lenz's law, both repel each other. On the other hand, if the magnet is moved away from the solenoid, then it attracts the magnet. When the magnet is brought near the solenoid, then the nearer side becomes the same pole and when it is moved away it becomes the opposite pole as shown in figure.

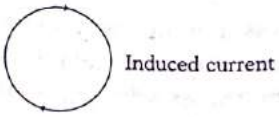


It can also be understood by increasing or decreasing field lines as in Example 6.14.

Example 6.16 *A circular loop is placed near a current carrying conductor as shown in figure. Find the direction of induced current, if the current, in the wire is decreasing.*

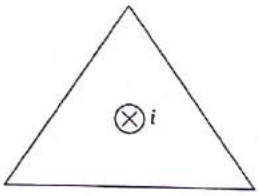


Sol. In this case loop is placed to the right of current carrying wire (not to the left as it appears, because if you move in the direction of current, loop lies to the right).

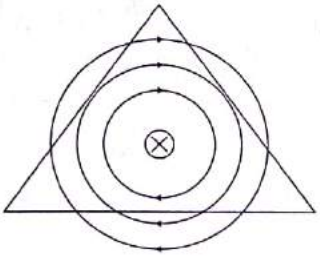


Now, the current is decreasing, therefore induced current in the loop is clockwise (S) as shown in figure.

Example 6.17 *A current carrying straight wire passes inside a triangular coil as shown in figure. The current in the wire is perpendicular to paper inwards. Find the direction of the induced current in the loop, if current in the wire is increased.*

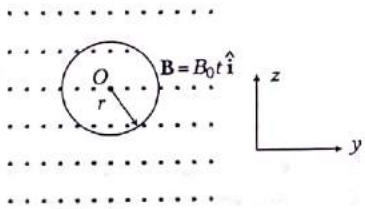


Sol. Magnetic field lines around the current carrying wire are as shown in figure below. Since, the lines are tangential to the loop ($\theta = 90°$), the flux passing through the loop is zero, whether the current is increased or decreased. Hence, change in flux is zero. Therefore, induced current in the loop will be zero.



Example 6.18 *A uniform magnetic field* $\mathbf{B} = B_0 t\hat{\mathbf{i}}$ *in a region exists. A circular conducting loop of radius r and resistance R is placed with its plane in yz-plane. Determine the current through the loop and sense of the current.*

Sol. The flux linked with the loop is



We know that magnetic flux is given as

$$\phi_B = \mathbf{B} \cdot \mathbf{S} = (B_0 t)(\pi r^2)\cos 0°$$

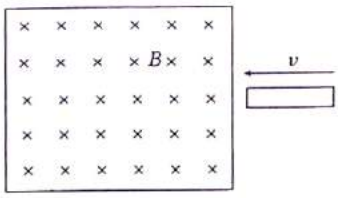
$$\Rightarrow \qquad \phi_B = B_0 \pi r^2 t \quad \text{and} \quad \frac{d\phi}{dt} = B_0 \pi r^2$$

Further, induced emf, $\varepsilon = \dfrac{-d\phi}{dt} = -B_0 \pi r^2$

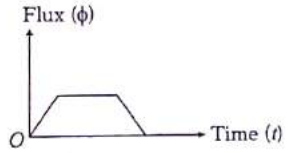
Therefore, current, $i = \dfrac{|\varepsilon|}{R} = \dfrac{B_0 \pi r^2}{R}$

By Lenz's law, i should be clockwise, so that it can oppose the increase in magnetic field.

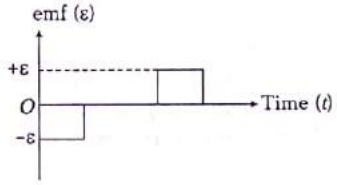
Example 6.19 *Through a uniform B field, A small rectangular loop is moving towards left with constant velocity as shown in the figure. Counting of time t begins at the moment, the loop starts entering the field. Plot the variation of flux through the loop with respect to time. Also, plot the variation of induced emf w.r.t. time t.*



Sol. As the loop is moving with constant velocity, the flux will be increasing linearly, as long as the loop will be entering the B field. Once the loop has completely entered the B field the flux will become constant. Now, when the loop will start coming out the flux will linearly decrease to zero value.

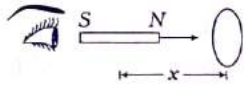


Since, emf induced, $\varepsilon = -\dfrac{d\phi}{dt}$, it means emf is negative of the slope of the ϕ-t graph. So, emf is negative constant in the beginning, then it becomes zero and then it becomes positive constant and again it becomes zero.



Example 6.20 *Through a conducting coil along its axis, a short bar magnet is rapidly pulled with uniform velocity with, with its north pole entering the coil first. Plot the variation of (i) flux, (ii) induced current and (iii) power dissipated in coil with time.*

Sol. (i)

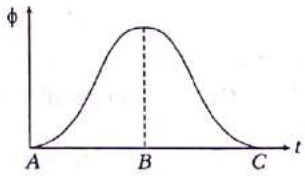


Magnetic field due to magnet is given as

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2M}{x^3}$$

$$\Rightarrow \qquad B \propto \frac{1}{x^3} \Rightarrow \phi \propto \frac{1}{x^3}$$

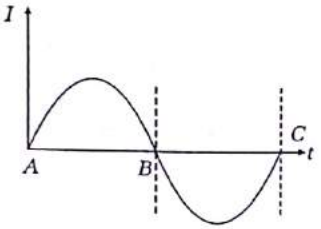
When a bar magnet is pulled rapidly through a conducting coil, then flux first increases, reaching to a maximum value and then decreases.



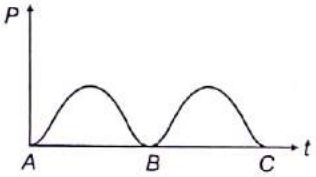
(ii) *Induced current versus time t*

When flux increases, then north pole is formed, current is anti-clockwise.

When flux decreases, then south pole is formed, current is clockwise. (Taking anti-clockwise current as positive.)

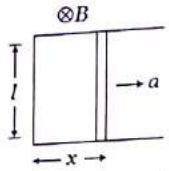


(iii) As power ∝ induced current. The corresponding power *versus t* can be drawn as



Example 6.21 *In a uniform magnetic field, a π shaped metal frame is located perpendicular to the plane of the conductor and varying with time at the rate $(dB/dt) = 0.20\,T/s$. A conducting connector starts moving with an acceleration $a = 30\ cm/s^2$ along the parallel bars of the frame. The length of the connector is equal to $l = 44$ cm. Find the emf induced in the loop $t = 1$ s after the beginning of the motion, if at the moment $t = 0$, the loop area and the magnetic induction are equal to zero.*

Sol.



We know, according to kinematic equation, distance travelled $x = ut + \frac{1}{2}at^2$. Here, initial velocity, $u = 0$, acceleration, $a = b = 30\ cm/s^2$

So, in time t, distance travelled by conductor, $x = \frac{1}{2}bt^2$

Flux passing through loop, $\phi = BS = Blx = \frac{Blbt^2}{2}$

Here, B and t both are variable.

So, induced emf, $e = \left|\frac{d\phi}{dt}\right| = \frac{lb}{2}\left[\frac{dB}{dt}t^2 + B \cdot 2t\right]$

$$= \frac{0.4 \times 0.3}{2}(0.2 \times 2^2 + 2B)$$

$$= 0.06\,(0.8 + 2B)$$

Given, $\frac{dB}{dt} = 0.2\ T/s \Rightarrow dB = 0.2dt$

On integrating both sides, we get

$$\int_0^B dB = 0.2\int_0^t dt \Rightarrow B = 0.2\,t = 0.2 \times 1 = 0.2$$

So, induced emf, $e = 0.06(0.8 + 0.2) = 0.06\ V = 6 \times 10^{-2}\ V$

Check point 6.1

1. The magnetic flux linked with a vector area **A** in a uniform magnetic field **B** is

(a) $\mathbf{B} \times \mathbf{A}$ (b) AB (c) $\mathbf{B} \cdot \mathbf{A}$ (d) $\frac{B}{A}$

2. The unit of magnetic flux is

(a) Wbm^{-2} (b) Wb (c) H (d) Am^{-1}

3. The dimensions of magnetic flux are

(a) $[MLT^{-2}A^{-2}]$ (b) $[ML^2T^{-2}A^{-2}]$
(c) $[ML^2T^{-1}A^{-2}]$ (d) $[ML^2T^{-2}A^{-1}]$

4. The magnetic flux linked with a coil, in webers is given by the equation $\phi = 3t^2 + 4t + 9$. Then, the magnitude of induced emf at $t = 2$ s will be

(a) 2 V (b) 4 V (c) 8 V (d) 16 V

5. A coil having an area A_0 is placed in a magnetic field which changes from B_0 to $4B_0$ in time interval t. The emf induced in the coil will be

(a) $3A_0B_0/t$ (b) $4A_0B_0/t$ (c) $3B_0/A_0t$ (d) $4B_0/A_0t$

6. The magnetic flux ϕ (in weber) in a closed circuit of resistance $10\,\Omega$ varies with time t (in second) according to equation $\phi = 6t^2 - 5t + 1$. The magnitude of induced current at $t = 0.25$ s is

(a) 1.2 A (b) 0.8 A (c) 0.6 A (d) 0.2 A

7. When the current through a solenoid increases at a constant rate, then the induced current

(a) is a constant and is in the direction of the inducing current
(b) is a constant and is opposite to the direction of the inducing current
(c) increase with time and is in the direction of inducing current
(d) increase with time and is opposite to the direction of inducing current

8. The magnetic flux across a loop of resistance $10\,\Omega$ is given by $\phi = 5t^2 - 4t + 1$ Wb. How much current is induced in the loop after 0.2 s?

(a) 0.4 A (b) 0.2 A
(c) 0.04 A (d) 0.02 A

9. A circular ring of diameter 20 cm has a resistance 0.01 Ω. How much charge will flow through the ring, if it is rotated from a position perpendicular to a uniform magnetic field of $B = 2$ T to a position parallel to field?
(a) 4 C (b) 6.28 C (c) 3.14 C (d) 2.5 C

10. In a circuit with a coil of resistance 2Ω, the magnetic flux changes from 2.0 Wb to 10.0 Wb in 0.2 s. The charge that flows in the coil during this time is
(a) 5.0 C (b) 4.0 C (c) 1.0 C (d) 0.8 C

11. The direction of induced emf during electromagnetic induction is given by
(a) Faraday's law (b) Lenz's law
(c) Maxwell's (d) Ampere's law

12. Two different loops are concentric and lie in the same plane. The current in the outer loop is clockwise and increasing with time. The induced current in the inner loop, is
(a) clockwise
(b) zero
(c) counter-clockwise
(d) in a direction that depends on the ratio of the loop radii

13. Lenz's law is due to conservation of
(a) charge
(b) momentum
(c) energy
(d) current

14. The north pole of a long horizontal bar magnet is being brought closer to a vertical conducting plane along the perpendicular direction. The direction of the induced current in the conducting plane will be
(a) horizontal
(b) vertical
(c) clockwise
(d) anti-clockwise

15. There is a uniform magnetic field directed perpendicular and into the plane of the paper. An irregular shaped conducting loop is slowly changing into a circular loop in the plane of the paper. Then,
(a) current is induced in the loop in the anti-clockwise direction
(b) current is induced in the loop in the clockwise direction
(c) AC is induced in the loop
(d) No current is induced in the loop

2 Motional electromotive force

So far we have considered the cases in which an emf is induced in a stationary circuit placed in a time-varying magnetic field. In this section, we will study about **motional emf,** *i.e.*, the emf induced in a conductor when it is moving through a constant magnetic field. Now, let us consider, a straight conductor of length l as shown in Fig. 6.17 which is moving through a uniform magnetic field directed into the page. For simplicity, we assume that the conductor is moving in a direction perpendicular to the field with constant velocity under the influence of some external agent.

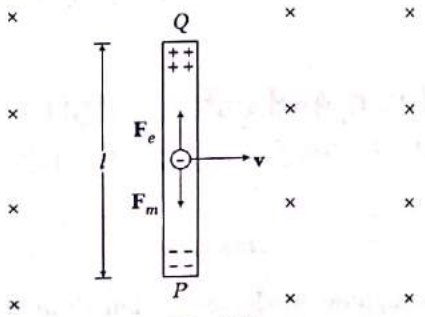


Fig. 6.17

Due to motion of conductor in magnetic field a potential difference is maintained between the ends of the conductor as long as the conductor continues to move through the uniform magnetic field.

This potential difference is called **motional electromagnetic force,** which is denoted by e, and is given by

$$\boxed{e = Bvl} \quad \text{...(i)}$$

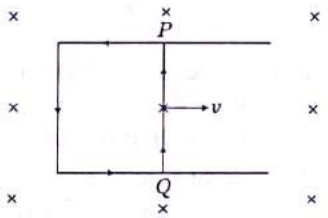


Fig. 6.18

If R is the resistance of the circuit, then current will be written as

$$\Rightarrow \quad i = \frac{e}{R} = \frac{Bvl}{R} \quad \text{...(ii)}$$

General form of motional emf

To find motional emf for a conductor of any shape, moving in a uniform or non-uniform magnetic field, we will consider any small element $d\mathbf{l}$ of a conductor, then the contribution de to the emf is the magnitude of $d\mathbf{l}$ multiplied by the component of $(\mathbf{v} \times \mathbf{B})$ which is parallel to $d\mathbf{l}$, *i.e.*,

$$de = (\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{l}$$

$$\Rightarrow \quad e = \oint (\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{l} \quad \text{[motional emf, closed conducting loop]}$$

Important points related to motional emf

Some important points related to motional emf are given below.

(i) For a semicircular conducting loop of radius R with the centre at O and moving with velocity v, then the emf is given by

$$e = Bv(2R)$$

$$\Rightarrow \quad V_P - V_Q = 2BvR$$

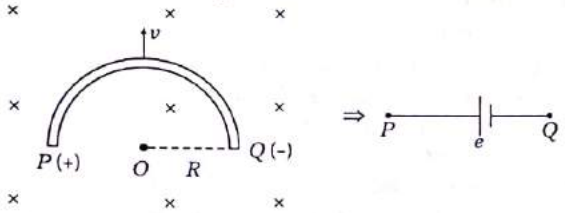


Fig. 6.19

(ii) If a conducting rod AB moves with a velocity v in xy-plane, then we can resolve the velocity vector in two components as shown in Fig. 6.20.

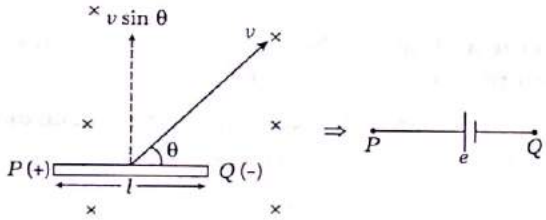


Fig. 6.20

No emf will be induced, due to $v \cos\theta$. So, the net effect of emf will be due to $v \sin\theta$,
which is given by $\boldsymbol{e = B(v \sin\theta)l}$.

(iii) For a circular loop of radius R moving with velocity v, we can replace its two ends lying on opposite sides of diameter with a battery of emf e. Then, the emf is given by

$$\boldsymbol{e = Bv(2R) = 2BvR}$$

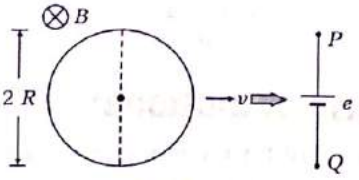


Fig. 6.21

(iv) For a irregular shape, a conducting body of length l, we can assume it, as a straight conductor and replace it with a battery of emf e as shown in Fig. 6.22.

Then, $\boldsymbol{e = B(v \cos\theta)l}$

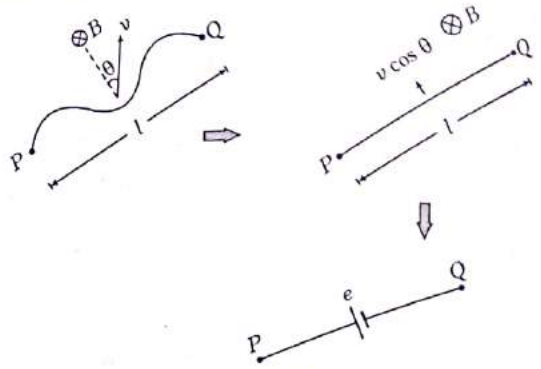


Fig. 6.22

(v) When magnetic field varies at every point of conductor which is moving with speed v, then the induced emf will be given by

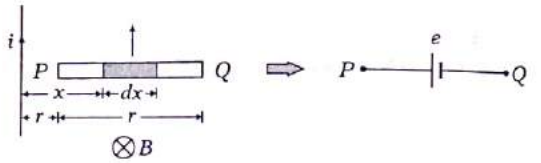


Fig. 6.23

$$de = Bv\,dx,$$

where, B is the magnetic field due to a long wire at distance x and is equal to $\frac{\mu_0 i}{2\pi x}\otimes$.

$$\Rightarrow \quad e = \frac{\mu_0 iv}{2\pi}\int_r^{r+l}\frac{dx}{x} = \frac{\mu_0 iv}{2\pi}\ln\left(1+\frac{l}{r}\right)$$

$$\Rightarrow \quad V_P - V_Q = e$$

Motional emf induced in a rotating bar

(i) A conducting rod of length l rotates with a constant angular speed ω about a pivot at one end. A uniform magnetic field B is directed perpendicular to the plane of rotation as shown in Fig. 6.24.

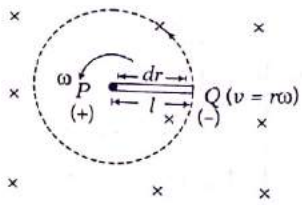


Fig. 6.24

Let us suppose an element of length dr at a distance r from P. The induced emf in this segment is

$$de = Bvr\,dr = B(r\omega)dr$$

Because every segment of the rod is moving perpendicular to B, an emf de of same form is generated across each.

So, summing the emfs induced across the ends of the rod is

$$e = \int_0^l de = \int_0^l Br\omega dr$$

$$\Rightarrow \quad e = \frac{1}{2}B\omega l^2$$

$$= Bl^2\pi v = \frac{Bl^2\pi}{T} \qquad [\because 2\pi\nu = \omega]$$

where, ν = frequency (revolution per second),
T = time period

(ii) If a conducting disc of radius r rotates with constant angular velocity ω about its axis in a uniform magnetic field parallel to its axis of rotation as shown in Fig. 6.25.

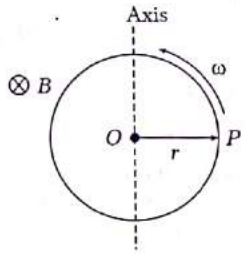


Fig. 6.25

Take a rod between centre and point P.

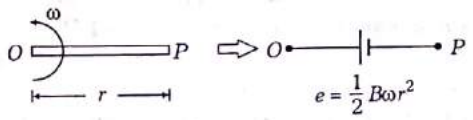


Fig. 6.26

As, we have already deduced the emf for a rod rotating in a constant magnetic field.

So, the induced emf is given by, $e = \frac{1}{2}B\omega r^2$.

That means, disc is equivalent to rod.

Fleming's right hand thumb rule to find the direction of induced current

The direction of motional emf or current can be given by Fleming's right hand thumb rule. If we stretch the thumb, the forefinger and the central finger of right hand in such a way that all are mutually perpendicular to each other and if thumb represents the direction of motion of the conductor, the forefinger represents the direction magnetic field, then central finger will represent the direction of induced current as shown in Fig. 6.27.

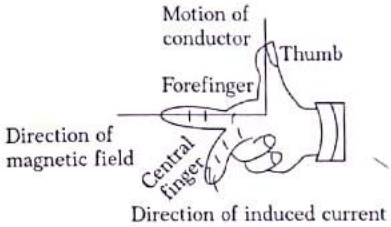
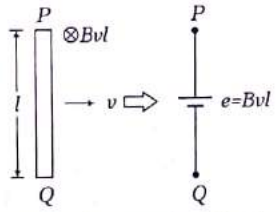


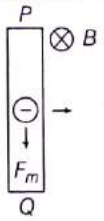
Fig. 6.27

Example 6.22 *A conducting rod of unit length moves with a velocity of 5m/s in a direction perpendicular to its length and perpendicular to a uniform magnetic field of magnitude 0.4T. Find the emf induced between the ends of the stick.*

Sol.



The negative terminal is that end towards which electrons are moving due to magnetic force and, the magnetic force on electron is acting along Q, hence Q is negative terminal of battery.

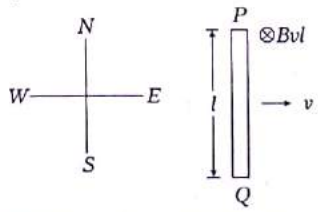


As, we know that induced emf for a moving rod is given by

$$\Rightarrow \quad e = Bvl = 0.4 \times 5 \times 1 = 2\text{ V}$$

Example 6.23 *The horizontal component of the earth's magnetic field at a place is 4.0×10^{-4} T and the dip is 45°. A metal rod of length 20 cm is placed in the north-south direction and is moved at a constant speed of 5 cm/s towards east. Calculate the emf induced in the rod.*

Sol.



According to the question, the velocity v, length l and B_v (vertical component of earth magnetic field) are mutually perpendicular using the relation,

$$e = B_v vl \qquad \left[\because \tan\phi = \frac{B_v}{B_h}\right]$$

$$= B_h \tan\phi vl$$

$$= 4 \times 10^{-4} \tan 45° \times 0.05 \times 0.20 = 4 \times 10^{-6}\text{ V}$$

There will be no induced emf due to B_h (horizontal component of earth magnetic field) because it is parallel to length of rod.

Example 6.24 *A ring of radius 2m translates in its plane with a constant velocity 5m/s. A uniform magnetic field 0.1 T exists in the space in a direction perpendicular to the plane of the ring. Consider different pairs of diametrically opposite points on the ring.*

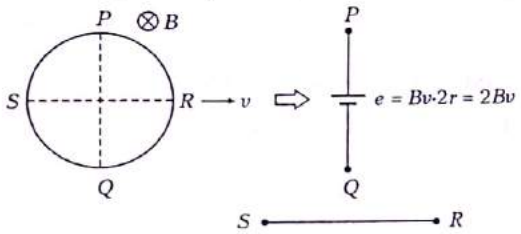
(i) Between which pair of points is the emf maximum? What is the value of this maximum emf?

(ii) Between which pair of points is the emf maximum? What is the value of this minimum emf?

Sol. Given, radius, $r = 2\,\text{m}$, velocity, $v = 5\,\text{ms}^{-1}$

Constant magnetic field, $B = 0.1\,\text{T}$

According to the question,

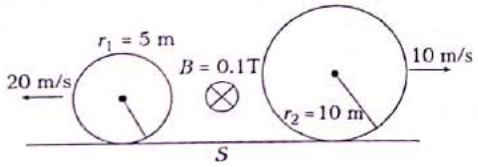


From figure, we can see that emf is maximum at PQ and minimum at RS as PQ is perpendicular to field and RS along the field.

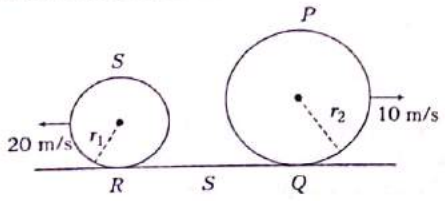
Using the relation for induced emf for a rod,

$$V_A - V_B = 2Bvr$$
$$e_{max} = 2Bvr$$
$$= 2 \times 0.1 \times 5 \times 2 = 2\,\text{V}$$
$$V_C - V_D = 0$$
$$\Rightarrow \quad e_{min} = 0$$

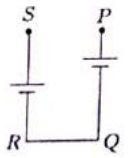
Example 6.25 *Two rings of radii 5m and 10m move in opposite directions with velocity 20 m/s and 10 m/s respectively, on a conducting surface S. There is a uniform magnetic field of magnitude 0.1 T perpendicular to the plane of the rings. Find the potential difference between the highest points of the two rings.*



Sol. The above situation can be drawn as



or



As we know, induced emf in a rod is given by $e = Bvl$.

So, for rod PQ, $e = Bvl = 0.1 \times 10 \times 10 = 10\,\text{V}$

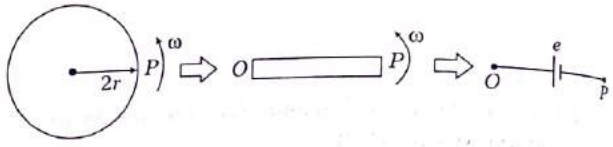
For rod RS, $e = Bvl = 0.1 \times 20 \times 5 = 10\,\text{V}$

So, the net emf produced in above figure is given by

$$V_P - 10\text{V} - 10\text{V} = V_S \;\Rightarrow\; V_P - V_S = 20\,\text{V}$$

Example 6.26 *A vertical disc of diameter 10 cm makes 20 revolutions per second about a horizontal axis passing through its centre. A uniform magnetic field 10^{-1} T acts perpendicular to the plane of the disc. Calculate the potential difference between its centre and rim in volts.*

Sol. As we know, disc is equivalent to a rod of length r. Divide disc into rods, each rod can be replaced by a battery and these batteries are would be parallel in arrangement.



According to the question, radius of disc,

$$r = \frac{10}{2} = 5\,\text{cm} = 0.05\,\text{m}$$

Angular velocity, $\omega = 20 \times 2\pi = 40\,\pi\,\text{rad/s}$

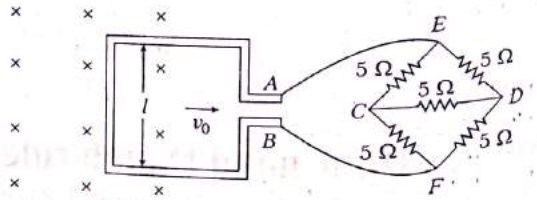
Magnetic field, $B = 10^{-1}\,\text{T}$

The emf of a disc is given as

$$e = \frac{1}{2}B\omega r^2 = \frac{1}{2}(10^{-1})(40\pi)(0.05)^2$$

$$\Rightarrow \quad V_O - V_A = 1.57 \times 10^{-2}\,\text{V}$$

Example 6.27 *A square metal wire loop of side 20 cm and resistance 2Ω is moved with a constant velocity v_0 in a uniform magnetic field of induction $B = 1\,\text{Wb/m}^2$ as shown in the figure. The magnetic field lines are perpendicular to the plane of the loop. The loop is connected to a network of resistance each of value 5Ω. The resistances of the lead wires BF and AE are negligible. What should be the speed of the loop, so as to have a steady current of 2 mA in the loop? Give the direction of current in the loop.*



Sol. From the figure, we see that, network $CEOF$ is balanced Wheatstone bridge, so no current will flow in branch CD.

So, the equivalent resistance of $CEOF$ network is

$$R_{eq} = \frac{10 \times 10}{10 + 10} = 5\,\Omega$$

Resistance of loop $= 2\,\Omega$

$$R_{total} = 2 + R_{eq} = 2 + 5 = 7\,\Omega$$

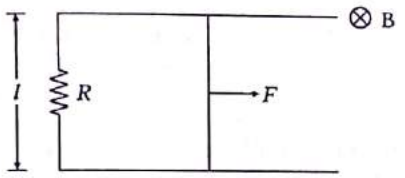
We know that induced emf, $e = Bv_0 l$

and induced current, $I = \frac{e}{R_{total}} = \frac{Bv_0 l}{7} \Rightarrow 2 \times 10^{-3} = \frac{1 \times v_0 \times 0.2}{7}$

$\Rightarrow \quad v_0 = 7 \times 10^{-2}$ m/s = 7 cm/s

As, flux is decreasing, so induced current I will be clockwise.

Example 6.28 *Figure shows the top view of a rod that can slide without friction. The resistor is 6.0 Ω and a 2.5 T magnetic field is directed perpendicularly downward into the paper. Let l =1.20 m.*



(i) *Calculate the force F required to move the rod to the right at a constant speed of 2.0 ms^{-1}.*

(ii) *At what rate is energy delivered to the resistor?*

(iii) *Show that this rate is equal to the rate of work done by the applied force.*

Sol. Given, resistance, $R = 6.0\Omega$

Magnetic field, $B = 2.57$ T

Length of the rod, $l = 1.20$ m

The motional emf in the rod is given as, $e = Bvl$

or $\quad e = (2.5)(2.0)(1.2)$ V = 6.0 V

The current in the circuit, $i = \frac{e}{R} = \frac{6.0}{6.0} = 1.0$ A

(i) The magnitude of force F required will be equal to the magnetic force acting on the rod, which opposes the motion.

$\therefore \quad F = F_m = lvB = ilB$

or $\quad F = (1.0)(1.2)(2.5)$ N = 3 N

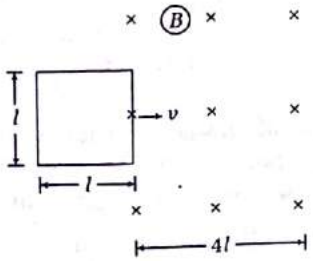
(ii) Rate by which energy is delivered to the resistor is

$$P_1 = i^2R = (1)^2 (6.0) = 6 \text{ W}$$

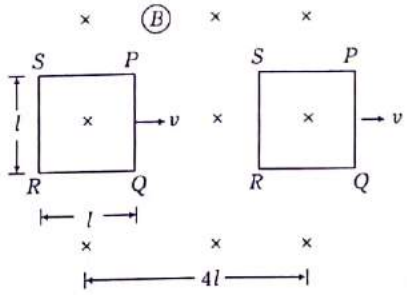
(iii) The rate by which work is done by the applied force is,

$$P_2 = Fv = (3)(2.0) = 6 \text{ W} \Rightarrow P_1 = P_2$$

Example 6.29 *A square loop of side l being moved towards right at a constant speed v as shown in figure. The front edge enters the magnetic field B at t = 0. The width of field is 4l. Sketch induced emf versus time graph.*



Sol.



From, $t = 0$ to $t = \frac{l}{v}$

Induced emf,

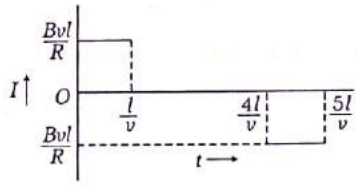
$e = Bvl$ and $I = \frac{e}{R} = \frac{Bvl}{R}$, which is anti-clockwise in direction.

Again from $t = \frac{l}{v}$ to $t = \frac{4l}{v}$, emf $e = 0$

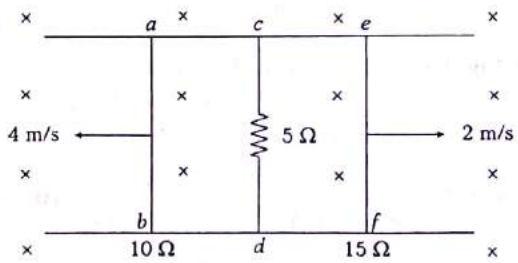
$t = \frac{4l}{v}$ to $t = \frac{5l}{v}$ emf $e = Bvl$

So, induced current $I = \frac{e}{R} = \frac{Bvl}{R}$, which is clockwise in direction now.

So, the plot of induced emf *versus* time is given as



Example 6.30 *Two parallel rails with negligible resistance are 10 cm apart. They are connected by a 5Ω resistor. The circuit also contains two metal rods having resistances of 10 Ω and 15 Ω along the rails. The rods are pulled away from the resistor at constant speeds 4.00 m/s and 2 m/s, respectively. A uniform magnetic field of magnitude 0.01 T is applied perpendicular to the plane of the rails. Determine the current in the 5 Ω resistor.*



Sol. Here, two conductors are moving in uniform magnetic field. So, we will use the motional approach. The rod *ab* will act as a source of emf,

$$e_1 = Bvl = (0.01)(4)(0.1) = 4 \times 10^{-3} \text{ V}$$

and internal resistance, $r_1 = 10\Omega$

Similarly, rod *ef* will also act as a source of emf,

$$e_2 = (0.01)(2)(0.1) = 2 \times 10^{-3} \text{ V}$$

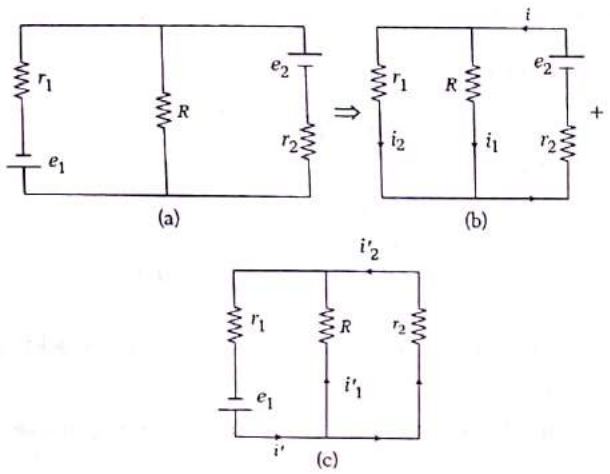
and internal resistance, $r_2 = 15\Omega$

From right hand rule we can see that

$$V_b > V_a \text{ and } V_e > V_f$$

Now, either by applying Kirchhoff 's laws or applying principle of superposition (discussed in the chapter of current electricity) we can find current through 5 Ω resistor. We will here use the superposition principle. You solve it by using Kirchhoff 's laws.

Now, in the figures, $R = 5\,\Omega$, $r_1 = 10\,\Omega$, $r_2 = 15\,\Omega$, $e_1 = 4 \times 10^{-3}$ V and $e_2 = 2 \times 10^{-3}$ V.



Refer Fig. (b) Net resistance of the circuit

$$= r_2 + \frac{Rr_1}{R + r_1} = 15 + \frac{10 \times 5}{10 + 5} = \frac{55}{3}\,\Omega$$

$\therefore$ Current, $i = \dfrac{e_2}{\text{Net resistance}}$

$$= \frac{2 \times 10^{-3}}{55/3} = \frac{6}{55} \times 10^{-3} \text{ A}$$

$\therefore$ Current through R,

$$i_1 = \left(\frac{r_1}{R + r_1}\right) i = \left(\frac{10}{10 + 5}\right)\left(\frac{6}{55} \times 10^{-3}\right) \text{A}$$

$$= \frac{4}{55} \times 10^{-3} \text{ A} = \frac{4}{55} \text{ mA}$$

Refer Fig. (c) Net resistance of the circuit

$$= r_1 + \frac{Rr_2}{R + r_2} = 10 + \frac{5 \times 15}{5 + 15} = \frac{55}{4}\,\Omega$$

$\therefore$ Current, $i' = \dfrac{e_1}{\text{Net resistance}} = \dfrac{4 \times 10^{-3}}{55/4} = \dfrac{16}{55} \times 10^{-3}$ A

$\therefore$ Current through R, $i'_1 = \left(\dfrac{r_2}{R + r_2}\right) i'$

$$= \left(\frac{15}{15 + 5}\right)\left(\frac{16}{55}\right) \times 10^{-3} \text{ A} = \frac{12}{55} \text{ mA}$$

From superposition principle net current through 5 Ω resistor is

$$i'_1 - i_1 = \frac{8}{55} \text{ mA from } d \text{ to } c$$

Induction of field

The modern view of electromagnetic induction states that induction occurs whether or not a conducting wire or material medium is present.

In general case, Faraday's law can be stated as, "An electric field is induced in any region of space in which a magnetic field is changing with time."

This law is also true if magnetic and electric fields are interchanged, *i.e.*,

"A magnetic field is induced in any region of space in which an electric field is changing with time".

In each case the strength of the induced field is proportional to the rate of change of the inducing field. Induced electric and magnetic fields are at right angles to each other.

Induced electric field

As per Faraday's law, due to change in magnetic field, an electric field is induced.

Let us understand this by considering a ring of radius r. A uniform magnetic field B is perpendicular to its plane.

So, time rate of change of magnetic field is $\dfrac{dB}{dt}$, then the flux associated will be ϕ. And the induced emf will be $e = \dfrac{-d\phi_B}{dt}$.

Hence, an induced current will flow in anti-clockwise direction, which means, if there is a current present in the ring, then there must be an electric field. And this electric field is produced by the changing magnetic field.

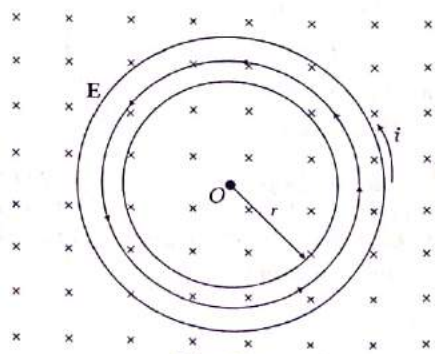


Fig. 6.28

Suppose a particle of charge q_0 is moving around the ring in a circular path. So, the work done by the induced electric field in one revolution is given by

$W = q_0 e$, where e is the induced emf ...(i)

We also know,

$$W = \oint \mathbf{F} \cdot d\mathbf{l} = q_0 \cdot \oint \mathbf{E} \cdot d\mathbf{l} \quad \text{... (ii)}$$

Equating Eqs. (i) and (ii), we get

$$q_0 e = q_0 \oint \mathbf{E} \cdot d\mathbf{l} \Rightarrow \quad e = \oint \mathbf{E} \cdot d\mathbf{l} \quad \text{... (iii)}$$

Eq. (iii) is the relationship between induced emf and induced electric field.

From the Faraday's law, $e = \dfrac{-d\phi_B}{dt}$ ... (iv)

From Eqs. (iii) and (iv), we get

$$\oint \mathbf{E} \cdot d\mathbf{l} = \frac{-d\phi_B}{dt} \quad \text{... (v)}$$

This is known as integral form of Faraday's law of electromagnetic induction.

Note In case of electromagnetic induction line intergral of induced emf E around a close path is not zero, *i.e.*, induced electric field is not conservative. In such a field, work done in moving a charge round or close path is not zero.

Example 6.31 *A uniform magnetic field exists in a circular region of radius R centred at O. The field is perpendicular to the plane of paper and its strength varies with time as $B = B_0 t$. Find the induced electric field at a distance r from the centre for (i) $r < R$, (ii) $r > R$. Also, plot a graph between $|E|$ and r for both the cases.*

Sol. As we know, induced electric field is given by

$$\int \mathbf{E} \cdot d\mathbf{l} = \frac{-d\phi_B}{dt}$$

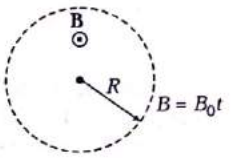
(i) For, $r < R$. Consider a ring of radius $r(< R)$ centered at O. The induced field at the periphery of the ring is circular such that,

$$\oint \mathbf{E} \cdot d\mathbf{l} = E \times 2\pi r \quad \text{...(i)}$$

($\because dl$ = circumference of the ring $= 2\pi r$)

As, magnetic flux, $\phi_B = \mathbf{B} \cdot \mathbf{S} = BS\cos\theta$

Here, $\theta = 0°$; so $\phi_B = (B_0 t)\pi r^2$ [$\because B = B_0 t$]



$$\therefore \quad \frac{d\phi_B}{dt} = B_0 \pi r^2 \quad \text{... (ii)}$$

Equating Eqs. (i) and (ii), we get

$$\therefore \quad E \times 2\pi r = -B_0 \pi r^2$$

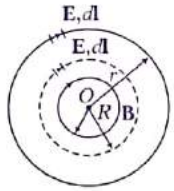
$$E = \frac{-B_2 r}{2} \Rightarrow E \propto r$$

(the sign comes, when clockwise electric field lines are developed)

(ii) For, $r > R$. Consider a ring of radius $r(> R)$ centred at O. Again following the same procedure,

$$\oint \mathbf{E} \cdot d\mathbf{l} = E \times 2\pi r \quad \text{... (iii)}$$

Similarly, flux, $\phi_B = (B_0 t)\pi R^2$ (as field is present only in region $0 < r < R$). ... (iv)



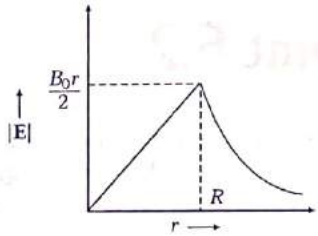
$$\Rightarrow \quad \frac{d\phi_B}{dt} = B_0 \pi R^2$$

Equating Eqs. (iii) and (iv), we get

$$\Rightarrow \quad E 2\pi r = -B_0 \pi R^2$$

$$\Rightarrow \quad E = \frac{-B_0 R^2}{2r} \Rightarrow E \propto \frac{1}{r}$$

Hence, plot between $|\mathbf{E}|$ and r will be



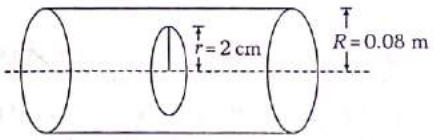
Example 6.32 *The current in an ideal, long solenoid is varied at a uniform rate of 0.02 A/s. The solenoid has 1000 turns/s and its radius is 8 cm.*

(i) Consider a circle of radius 2 cm inside the solenoid with its axis coinciding with the axis of the solenoid. Write the change in the magnetic flux through this circle in 4 s.

(ii) Find the electric field induced at a point on the circumference of the circle.

(iii) Find the electric field induced at a point outside the solenoid at a distance 9 cm from its axis.

Sol.



Given, for solenoid, $n = 1000$ m/s

and $\dfrac{di}{dt} = 0.02$ A/s

$$\Rightarrow \quad di = 0.02\, dt \quad \text{...(i)}$$

(i) Magnetic field due to solenoid is given as

$$B = \mu_0 n i$$

Flux through circle of radius r, $\phi_B = B\pi r^2$

($\because \phi_B = \mathbf{B} \cdot \mathbf{S}$)

$$= \mu_0 n i \pi r^2$$

$$\Rightarrow \quad d\phi_B = \mu_0 n \pi r^2 i$$

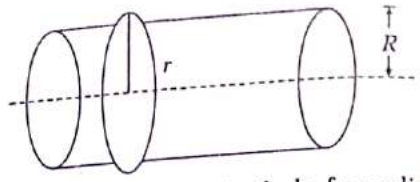
$$= \mu_0 n \pi r^2 (0.02) dt \quad [\because \text{using Eq. (i)}]$$

$$\Rightarrow \quad \phi = \mu_0 n\pi r^2 (0.02)\int_0^4 dt$$
$$= 4\pi \times 10^{-7} \times 1000 \times \pi(0.02)^2(0.02 \times 4)$$
$$= 1.26 \times 10^{-7} \text{ Wb}$$

(ii) Electric field induced is related to emf as, $\oint \mathbf{E} \cdot d\mathbf{l} = \frac{d\phi}{dt}$

$$\Rightarrow \quad E \cdot 2\pi r = \mu_0 n\pi r^2 \frac{di}{dt}$$
$$E = \frac{\mu_0 nr}{2}\frac{di}{dt}$$
$$= \frac{4\pi \times 10^{-7} \times 1000 \times 0.02}{2} \times 0.02$$
$$= 8\pi \times 10^{-8} \text{ V/m}$$

(iii)



As discussed in (i), similarly for radius outside the solenoid,

$$\oint \mathbf{E} \cdot d\mathbf{l} = \frac{d}{dt}(\mu_0 ni\,\pi R^2)$$
$$\Rightarrow \quad E \cdot 2\pi r = \mu_0 n \frac{di}{dt}\pi R^2 \quad \Rightarrow \quad E = \frac{\mu_0 nR^2}{2r}\frac{di}{dt}$$
$$= \frac{4\pi \times 10^{-7} \times 1000 \times (0.08)^2 \times 0.02}{2 \times 0.09}$$
$$= 28\pi \times 10^{-8} \text{ V/m}$$

Check point 6.2

1. A conducting rod of length l is falling with a constant velocity v perpendicular to a uniform horizontal magnetic field B. A potential difference between its two ends will be
(a) $2\,Blv$ (b) Blv
(c) $\frac{1}{2}Blv$ (d) $B^2 l^2 v^2$

2. A wire of length 50 cm moves with a velocity of 300 m/min, perpendicular to a magnetic field. If the emf induced in the wire is 2 V, then the magnitude of the field in tesla is
(a) 2 (b) 5
(c) 0.8 (d) 2.5

3. A 10 m wire kept in east-west direction is falling with velocity 5 m/s perpendicular to the field 0.3×10^4 Wb/m^2. The induced emf across the terminal will be
(a) 0.15 V (b) 1.5 mV
(c) 1.5 V (d) 15.0 V

4. A boat is moving due east in a region, where the earth's magnetic field is 5.0×10^{-5} NA^{-1}m^{-1} due north and horizontal. The boat carried a vertical areal 2 m long. If the speed of the boat is 1.50 ms^{-1}, then the magnitude of the induced emf in the wire of areal is
(a) 1 mV (b) 0.75 mV
(c) 0.50 mV (d) 0.15 mV

5. The magnitude of the earth's magnetic field at a place is B_0 and the angle of dip is δ. A horizontal conductor of length l lying along the magnetic north-south moves eastwards with a velocity v. The emf induced across the conductor is
(a) zero (b) $B_0 lv \sin\delta$
(c) $B_0 lv$ (d) $B_0 lv \cos\delta$

6. A conducting circular loop is placed in a uniform magnetic field of induction B tesla with its plane normal to the field. Now, the radius of the loop starts shrinking at the rate (dr/dt). Then, the induced emf at the instant when the radius is r, is
(a) $\pi rB(dr/dt)$ (b) $2\pi rB(dr/dt)$
(c) $\pi r^2(dB/dt)$ (d) $(\pi r^2/2)B(dr/dt)$

7. Two similar circular loops carry equal currents in the same direction. On moving the coils further apart, the electric current will
(a) remain unaltered
(b) increases in one and decreases in the second
(c) increase in both
(d) decrease in both

8. A circular coil of mean radius of 7 cm and having 4000 turns is rotated at the rate of 1800 rev/min in the earth's magnetic field (B = 0.5 gauss), the maximum emf induced in coil will be
(a) 1.158 V (b) 0.57 V
(c) 0.29 V (d) 5.8 V

9. A coil of N turns and mean cross-sectional area A is rotating with uniform angular velocity ω about an axis at right angle to uniform magnetic field B. The induced emf E in the coil will be
(a) $NBA\sin\omega t$
(b) $NB\omega\sin\omega t$
(c) $NB/A\sin\omega t$
(d) $NBA\omega\sin\omega t$

10. A 0.1 m long conductor carrying a current of 50 A is held perpendicular to a magnetic field of 1.25 mT. The mechanical power required to move the conductor with a speed of 1 ms^{-1} is
(a) 62.5 mW (b) 625 mW
(c) 6.25 mW (d) 12.5 mW
(e) 125 mW

11. A metal rod of length 2 m is rotating with an angular velocity of 100 rads^{-1} in a plane perpendicular to a uniform magnetic field of 0.3 T. The potential difference between the ends of the rod is

(a) 30 V (b) 40 V
(c) 60 V (d) 600 V

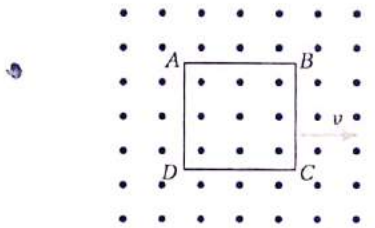
12. A conducting rod of length l is moving in a transverse magnetic field of strength B with velocity v. The resistance of the rod is R. The current in the rod is

(a) $\frac{Blv}{R}$ (b) Blv
(c) zero (d) $\frac{B^2v^2l^2}{R}$

13. A rectangular coil rotates about an axis normal to the magnetic field. If E_m is the maximum value of the induced emf, then the instantaneous emf when the plane of the coil makes an angle of 45° with the magnetic field is

(a) $\frac{1}{2}E_m$ (b) $\frac{1}{4}E_m$
(c) $\frac{1}{\sqrt{2}}E_m$ (d) E_m

14. A metallic square loop $ABCD$ is moving in its own plane with velocity v in a uniform magnetic field perpendicular to its plane as shown in the figure. An electric field is induced



(a) in AD but not in BC (b) in BC but not in AD
(c) Neither in AD nor in BC (d) in both AD and BC

15. Consider the following statements.

A. An emf can be induced by moving a conductor in a magnetic field.

B. An emf can be induced by changing the magnetic field.

(a) Both A and B are true (b) A is true but B is false
(c) B is true but A is false (d) Both A and B are false

3 Self-induction

Self-induction is the property of a coil by virtue of which the coil opposes any change in the strength of current flowing through it by inducing an emf in itself.

This induced emf is also called **back emf**. When the current in a coil is switched on, then the self-induction opposes the growth of the current and when it is switched off, then the self-induction opposes the decay of the current. Hence, self-induction is also known as **inertia of electricity.**

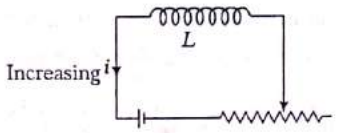


Fig. 6.29

A circuit or part of a circuit, that is designed to have a particular induction is called an inductor. The usual symbol for an inductor is shown in Fig. 6.30.

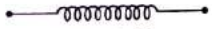


Fig. 6.30

Thus, an inductor is a circuit element which opposes the change in current through it. It may be a circular coil, solenoid, etc.

Coefficient of self-induction

Definition using magnetic flux

If i is the strength of current flowing through a coil at any time and ϕ is the amount of magnetic flux linked with all the turns of the coil at that time, then it is found that

$$\phi \propto i \text{ or } \phi = Li \Rightarrow \boldsymbol{L = \frac{\phi}{i}}$$

where, L is a constant of proportionality and is called **coefficient of self-induction** or **self-inductance of the coil**. The value of L depends on number of turns N, area of cross-section A and nature of material of the core on which coil is wound.

For N number of turns in coil, $L = \frac{N\phi}{i}$.

Definition using induced emf

If a current i is passed in a circuit and it is changed with a rate di/dt, then the induced emf e produced in the circuit is directly proportional to the time rate of change of current. Thus,

$$e \propto \frac{di}{dt}$$

When the proportionality sign is removed, the same constant L again comes here.

Hence, $$e = -L\frac{di}{dt}$$

The minus sign here is a reflection of Lenz's law. It says that the self-induced emf in a circuit opposes any change in the current in that circuit.

Average induced emf e is given by

$$e = -L\frac{di}{dt} = -\frac{L(i_2 - i_1)}{\Delta t}.$$

From the above equation, we can write

$$L = \left|\frac{-e}{di/dt}\right| \quad \text{...(i)}$$

If $\frac{di}{dt} = 1$, then $L = e$

So, **the coefficient of self-induction** or **self-inductance of a coil is equal to the emf induced in the coil when time rate of change of current through the coil is unity.** Inductance is a scalar quantity.

Some important points regarding coefficient of self-induction

(i) The SI unit of L is henry.

(ii) According to Eq. (i), we can write

$$1 \text{ henry} = \frac{1 \text{ volt}}{1 \text{ ampere/second}} = \frac{1 \text{ volt-second}}{\text{ampere}}$$

Self-inductance of a coil is said to be one henry (H) when a current change at the rate of 1 ampere/second through the coil induces an emf of 1 volt in the coil.

(iii) Dimensions of self-inductance L is $[ML^2T^{-2}A^{-2}]$.

Kirchhoff's second law with an inductor

Let us consider a circuit as shown in Fig. 6.31.

According to Kirchhoff's second law (loop rule), when we go through an inductor in the same direction as the assumed current, we encounter a voltage drop equal to $L\,di/dt$, where di/dt is to be substituted with sign.

e.g., in the loop shown in figure, Kirchhoff's second law gives the equation,

$$\Rightarrow \quad E - iR - L\frac{di}{dt} = 0$$

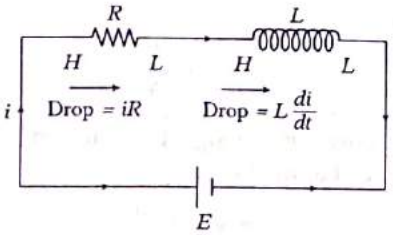
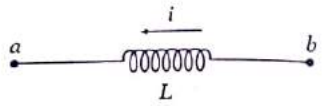


Fig. 6.31

Example 6.33 *The inductor shown in figure has inductance 0.54 H and carries a current in the direction shown that is decreasing at a uniform rate* $\frac{di}{dt} = -0.03$ *A/s.*



(i) Find the self-induced emf.

(ii) Which end of the inductor a or b is at a higher potential?

Sol. (i) Given, inductance, $L = 0.54$ H

$$\frac{di}{dt} = -0.03$$

Self-induced emf is given as

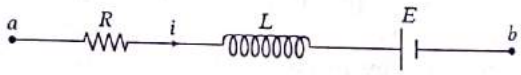
$$e = -L\frac{di}{dt} = (-0.54)(-0.03) \text{ V}$$

$$= 1.62 \times 10^{-2} \text{ V}$$

(ii) We know that, $V_{ba} = L\frac{di}{dt} = -1.62 \times 10^{-2}$ V

Since, V_{ba} $(= V_b - V_a)$ is negative. It implies that $V_a > V_b$ or a is at higher potential.

Example 6.34 *In the circuit diagram shown in figure,* $R = 10\,\Omega$, $L = 5$ *H,* $E = 20$ *V,* $i = 2$ *A. This current is decreasing at a rate of* -1.0 *A/s. Find* V_{ab} *at this instant.*



Sol. Potential difference across inductor is given as

$$V_L = L\frac{di}{dt} = (5)(-1.0) = -5 \text{ V}$$

Now, using Kirchhoff's second law, $V_a - iR - V_L - E = V_b$

$$\therefore \quad V_{ab} = V_a - V_b = E + iR + V_L$$

$$= 20 + (2)(10) - 5 = 35 \text{ V}$$

Note In the above example, as the current is decreasing, the inductor can be replaced by a source of emf $e = \left|L \cdot \frac{di}{dt}\right| = 5$ V in such a manner that this emf supports the decreasing current, or it sends the current in the circuit in the same direction as the existing current. So, positive terminal of this source is towards b. Thus, the given circuit can be drawn as

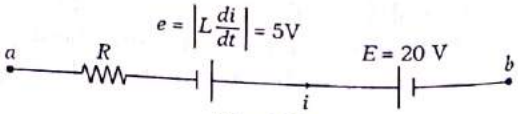


Fig. 6.32

Now, we can find V_{ab}.

Example 6.35 *In the figure shown* $i = 10\,e^{-4t}$ *A. Find* V_L *and* V_{ab}.

Sol. Potential difference across the inductor is given as

$$V_L = +L\frac{di}{dt} = (2)\frac{d}{dt}(10e^{-4t}) = -80\,e^{-4t}$$

Further, by using Kirchhoff's second law,

$$V_a - iR - L\frac{di}{dt} = V_b$$

$$\therefore\ V_a - V_b = iR + L\frac{di}{dt} \quad \text{or} \quad V_{ab} = (10e^{-4t})(4) - 80e^{-4t} = -40e^{-4}t$$

Self-inductance of a coil

Consider a coil of radius r and current i is flowing through the coil. If number of turns in the coil is N, then magnetic field at the centre is given as

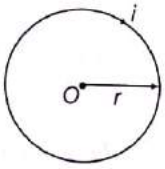


Fig. 6.33

$$\Rightarrow \quad B_{\text{centre}} = \frac{\mu_0 Ni}{2r} \quad \text{and} \quad \phi_B = BS$$

where, S = area of cross-sectional of the coil = πr^2

So, $$\phi_B = \frac{\mu_0 Ni}{2r} \times \pi r^2 \Rightarrow \phi_B = \frac{\mu_0 \pi Nir}{2}$$

Now, net flux for the coil of N number of turns,

$$\Rightarrow \quad \phi_B = N\phi_0 = \frac{N \times \mu_0 \pi Nir}{2} = \frac{\mu_0 \pi N^2 ir}{2}$$

We know that, $L = \dfrac{\phi_B}{i} \Rightarrow \boxed{L = \dfrac{\mu_0 N^2 \pi r}{2} \text{ henry}}$

Self-inductance of a solenoid

Let us find the inductance of a uniformly wound solenoid having N turns and length l. Assume that l is much longer than the radius of the windings and that the core of the solenoid is air. We can assume that the interior magnetic field due to a current 'i' is uniform and given by equation,

$$\Rightarrow \quad B = \mu_0 ni = \mu_0\left(\frac{N}{l}\right)i$$

Fig. 6.34

where, $n = \dfrac{N}{l}$ is the number of turns per unit length.

The magnetic flux through each turn is

$$\phi_B = BS = \mu_0 \frac{NS}{l} i$$

Here, S is the cross-sectional area of the solenoid.

Now, as we know, $L = \dfrac{N\phi_B}{i} = \dfrac{N}{i}\left(\dfrac{\mu_0 NSi}{l}\right) = \dfrac{\mu_0 N^2 S}{l}$

$$\Rightarrow \quad \boxed{L = \frac{\mu_0 N^2 S}{l}}$$

This result shows that L depends on dimensions (S, l) and is proportional to the square of the number of turns.

$$\therefore \quad L \propto N^2$$

Because $N = nl$, we can also express the result in the form,

$$L = \mu_0 \frac{(nl)^2}{l} S = \mu_0 n^2 Sl = \mu_0 n^2 V \quad \text{or} \quad L = \mu_0 n^2 V$$

Here, $V = Sl$ is the volume of the solenoid.

Note If the space inside the solenoid is filled with a material of relative permeability μ_r, then $L = \mu_0 \mu_r n^2 Sl$.

Example 6.36 *(i) Calculate the inductance of an air core solenoid containing 300 turns, if the length of the solenoid is 25 cm and its cross-sectional area is 4 cm^2.*

(ii) Calculate the self-induced emf in the solenoid, if the current through it is decreasing at the rate of 50 A/s.

Sol. (i) The inductance of a solenoid is given by

$$L = \frac{\mu_0 N^2 S}{l}$$

where, S is the cross-sectional area of solenoid.

Substituting the values, we have

$$L = \frac{(4\pi \times 10^{-7})(300)^2(4 \times 10^{-4})}{(25 \times 10^{-2})} \text{H}$$

$$= 1.81 \times 10^{-4} \text{ H}$$

(ii) The self-induced emf is given by

$$e = -L\frac{di}{dt}$$

Here, $\dfrac{di}{dt} = -50\,\text{As}^{-1}$

$\therefore \quad e = -(1.81 \times 10^{-4})(-50) = 9.05 \times 10^{-3}$ V

or $\quad e = 9.05$ mV

Energy stored in an inductor

The energy of a capacitor is stored in the electric field between its plates. Similarly, an inductor has the capability of storing energy in its magnetic field.

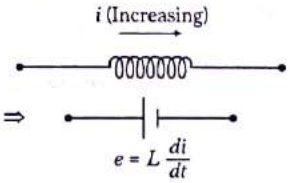


Fig. 6.35

An increasing current in an inductor causes an emf between its terminals.

The work done per unit time is power,

$$P = \frac{dW}{dt} = -ei = -Li\frac{di}{dt}$$

From, $dW = -dU$ or $\frac{dW}{dt} = -\frac{dU}{dt}$, we have

$$\frac{dU}{dt} = Li\frac{di}{dt}$$

or $$dU = Li\,di$$

The total energy U supplied while the current increases from zero to a final value i is

$$U = L\int_0^i i\,di = \frac{1}{2}Li^2$$

$$\therefore \quad \boxed{W = U = \frac{1}{2}Li^2}$$

Energy stored per unit volume in magnetic field is known as energy density.

$$\therefore \quad \boxed{\text{Energy density} = \frac{U}{V} = \frac{1}{2}\frac{B^2}{\mu_0}}$$

Thus, if $i = 1$ A, then $2W = L$

Hence, **the coefficient of self-inductance is equal to twice the work done in establishing a flow of one ampere current in the circuit.**

Note Time constant in R-L circuit is $\tau = \frac{L}{R}$.

Example 6.37 *What inductance would be needed to store 1 kWh of energy in a coil carrying a 200 A current?* ($1\ kWh = 3.6\times 10^6\ J$)

Sol. We have, $i = 200$ A

and $U = 1$ kWh $= 3.6\times 10^6$ J

$$\therefore \text{ Self-inductance, } L = \frac{2U}{i^2} = \frac{2\,(3.6\times 10^6)}{(200)^2} = 180 \text{ H}$$

$$\left(\because U = \frac{1}{2}Li^2\right)$$

Example 6.38 *Two coils having self-inductances, $L_1 = 5\,mH$ and $L_2 = 1\,mH$. The current in the coil is increasing at same constant rate at a certain instant and the power supplied to the coils is also same. Find the ratio of*

(i) induced voltage

(ii) current

(iii) energy stored in two coils at that instant

Sol. Given, $L_1 = 5$ mH and $L_2 = 1$ mH

(i) As we know, induced voltage is given by, $e = \frac{Ldi}{dt}$

$$\Rightarrow \quad \frac{e_1}{e_2} = \frac{L_1(di/dt)}{L_2(di/dt)} = \frac{L_1}{L_2} = \frac{5}{1} = 5:1 \quad \text{...(i)}$$

(ii) Power in the coil is given by

$$P = ei$$

Here, $$P_1 = P_2 \Rightarrow e_1 i_1 = e_2 i_2 \Rightarrow \frac{i_1}{i_2} = \frac{e_2}{e_1}$$

Using Eq. (i), we can write, $\frac{i_1}{i_2} = \frac{1}{5} = 1:5$

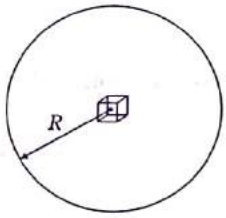
(iii) Energy stored in a coil is given by

$$U = \frac{1}{2}Li^2$$

$$\Rightarrow \quad \frac{U_1}{U_2} = \frac{(1/2)L_1 i_1^2}{(1/2)L_2 i_2^2} = \frac{L_1}{L_2}\left(\frac{e_2}{e_1}\right)^2 = \frac{5}{1}\left(\frac{1}{5}\right)^2 = 1:5$$

Example 6.39 *Suppose a cube of volume $2mm^3$ is placed at the centre of a circular loop of radius 5cm carrying current 2A. Find the magnetic energy stored inside the cube.*

Sol.



Magnetic field at the centre of the circular loop is given by $B = \frac{\mu_0 i}{2R}$

We know, energy density, $\mu = \frac{B^2}{2\mu_0}$

and energy stored in the cube will be given by

$$U = \mu V_0 = \frac{B^2}{2\mu_0}V_0$$

where, V_0 is the volume of the cube.

Substituting the values in the above equation,

$$= \frac{1}{2\mu_0}\times\left(\frac{\mu_0 i}{2R}\right)^2 V_0 = \frac{\mu_0 i^2 V_0}{8R^2}$$

$$= \frac{4\pi\times 10^{-7}\times 2^2\times 2\times 10^{-9}}{8\times(0.05)^2} = 16\,\pi\times 10^{-14}\text{ J}$$

Combination of self-inductances

(i) In series

If several inductances are connected in series and there is interactions through self-inductance only as shown in Fig. 6.36,

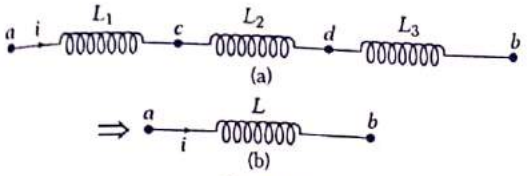


Fig. 6.36

Then their equivalent inductance is given by

Equivalent inductance, $L = L_1 + L_2 + L_3$

(ii) In parallel

When several inductances are connected in parallel as shown in the following figure 6.37, then their equivalent inductance can be calculated as

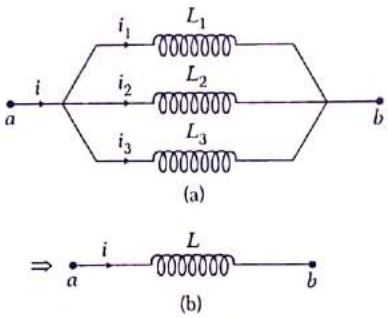


Fig. 6.37

The equivalent inductance L is given by

$$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

Check point 6.3

1. The SI unit of inductance, the henry, can be written as
(a) weber/ampere (b) volt-second/ampere
(c) joule/(ampere)^2 (d) ohm-second

2. A long solenoid has 500 turns. When a current of 2 A is passed through it, then the resulting magnetic flux linked with each turn of the solenoid is 4×10^{-3} Wb. The self-inductance of the solenoid is
(a) 1.0 H (b) 4.0 H (c) 2.5 H (d) 2.0 H

3. If a current of 10 A changes in one second through a coil, and the induced emf is 10 V,then the self-inductance of the coil is
(a) $\frac{2}{5}$ H (b) $\frac{4}{5}$ H
(c) $\frac{5}{4}$ H (d) 1 H

4. During a current change from 2 A to 4 A in 0.5 s, 8 V of emf is developed in a coil. The coefficient of self-induction is
(a) 1 H (b) 2 H (c) 4 H (d) 8 H

5. The current passing through a choke coil of 5 H is decreasing at the rate of 2 A/s. The emf developing across the coil is
(a) 10 V (b) –10 V
(c) 2.5 V (d) –2.5 mV

6. In a coil of self-inductance 0.5 H, the current varies at a constant rate from zero to 10 A in 2 s. The emf generated in the coil is
(a) 10 V (b) 5 V
(c) 2.5 V (d) 1.25 V

7. The self-inductance of a long solenoid cannot be increased by
(a) increasing its area of cross-section
(b) increasing its length
(c) changing the medium with greater permeability
(d) increasing the current through it

8. Self-inductance of a coil is 50 mH. A current of 1 A passing through the coil reduces to zero at steady rate in 0.1 s, the self-induced emf is
(a) 5 V (b) 0.05 V (c) 50 V (d) 0.5 V

9. The self-inductance of a coil is L. Keeping the length and area same, the number of turns in the coil is increased to four times. The self-inductance of the coil will now be
(a) $\frac{1}{4}L$ (b) L (c) $4L$ (d) $16L$

10. In circular coil, when number of turns is doubled and resistance becomes (1/4)th of initial, then inductance becomes
(a) 4 times (b) 2 times (c) 8 times (d) No change

11. The self-inductance of solenoid of length L, area of cross-section S and having N turns is
(a) $\frac{\mu_0 N^2 S}{L}$ (b) $\frac{\mu_0 NS}{L}$ (c) $\mu_0 N^2 LS$ (d) $\mu_0 NLS$

12. A solenoid has 2000 turns would over a length of 0.30 m. The area of its cross-section is $1.2 \times 10^{-3} m^2$. If an initial current of 2 A in the solenoid is reversed in 0.25 s, then the emf induced in the coil is
(a) 6×10^{-4} V (b) 4.8×10^{-3} V
(c) 6×10^{-2} V (d) 32.1×10^{-2} V

13. A 50 mH coil carries a current of 2 A. The energy stored in joules is
(a) 1 (b) 0.1 (c) 0.05 (d) 0.5

14. In an inductor of inductance $L = 100$ mH, a current of $I = 10$ A is flowing. The energy stored in the inductor is
(a) 5 J (b) 10 J (c) 100 J (d) 1000 J

15. Two pure inductors each of self-inductance L are connected in parallel but are well separated from each other. The total inductance is
(a) $2L$ (b) L (c) $\frac{L}{2}$ (d) $\frac{L}{4}$

4 Mutual induction

Consider two neighbouring coils of wire as shown in Fig. 6.38. A current flowing in coil 1 produces magnetic field and hence a magnetic flux through coil 2. If the current in coil 1 changes, then the flux through coil 2 changes as well. According to Faraday's law, this induces an emf in coil 2. In this way, a change in the current in one circuit can induce a current in a second circuit. This phenomenon is known as **mutual induction.**

In other words, the phenomena according to which an opposing emf is produced in a coil as a result of change in current a magnetic flux linked with a neighbouring coil is called mutual induction.

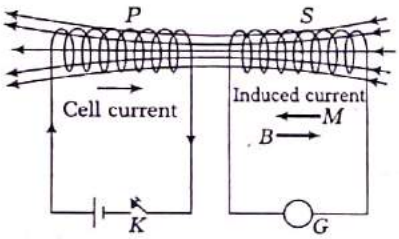


Fig. 6.38

Like the self-inductance (L), two circuits has mutual inductance (M).

Coefficient of mutual inductance

Definition using magnetic flux

Suppose the circuit 1 has a current i_1 flowing in it. Then, total flux $N_2\phi_{B_2}$ linked with circuit 2 is proportional to the current in circuit 1.

Thus, $N_2\,\phi_{B_2} \propto i_1$ or $N_2\phi_{B_2} = Mi_1$

Here, the proportionality constant M is known as coefficient of mutual inductance 'M' of the two circuits.

Thus, $$M = \frac{N_2\phi_{B_2}}{i_1}$$

From this expression M can be defined as **the total flux $N_2\phi_{B_2}$ linked with circuit 2 per unit current in circuit 1.**

Definition using induced emf

If we change the current in circuit 1 at a rate di_1/dt, an induced emf e_2 is developed in circuit 1, which is proportional to the rate di_1/dt.

Thus, $e_2 \propto di_1/dt$ or $e_2 = -Mdi_1/dt$ ...(i)

Here, the proportionality constant is again M. Minus sign indicates that e_2 is in such a direction that it opposes any change in the current in circuit 1. From the above equation, we can write

$$M = \left|\frac{-e_2}{di_1/dt}\right|$$

If $\frac{di}{dt} = 1$, then $M = e$. Thus, coefficient of mutual induction or mutual inductance of two coils is equal to the emf induced in one coil when rate time of change of current through the other coil is unity.

Some important points regarding the coefficient of mutual inductance

(i) The SI unit of coefficient of mutual inductance is **henry** (1H).

Coefficient of mutual induction or mutual inductance of two coils is said to be **one henry**, when a current change at the rate of one ampere/second in one coil induces an emf of one volt in the other coil.

(ii) M depends upon closeness of the two circuits, their orientations and sizes and the number of turns, etc.

(iii) Its dimensions are $[ML^2T^{-2}A^{-2}]$.

(iv) **Reciprocity theorem** This theorem states that mutual inductance due to secondary coil on primary coil is equal to the mutual induction due to primary on secondary coil, *i.e.*, $M_{21} = M_{12} = M$

Using Eq. (i),

$$e_2 = -M(di_1/dt)$$

and $$e_1 = -M(di_2/dt)$$

$\Rightarrow$ $$M_{12} = \frac{N_2\phi_{B_2}}{i_1}$$

and $$M_{21} = \frac{N_1\phi_{B_1}}{i_2}$$

(v) Average induced emf, $\bar{e} = -M\frac{\Delta i}{\Delta t} = -\frac{-M(i_2 - i_1)}{\Delta t}$.

(vi) *A good approach for calculating the mutual inductance of two circuits consists of the following steps*

(a) Assume anyone of the circuits as primary (first) and the other as secondary (second).

(b) Suppose a current i_1 flows through the primary circuit.

(c) Determine the magnetic field **B** produced by the current i_1.

(d) Obtain the magnetic flux ϕ_{B_2}.

(e) With the flux known, the mutual inductance can be found from

$$M = \frac{N_2\phi_{B_2}}{i_1}$$

Mutual inductance for two concentric coils

Let us consider a coil having N_1 turns and of radius r is surrounded by another coil having N_2 turns and of radius R as shown in the Fig. 6.39 ($R >> r$).

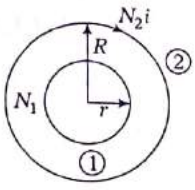


Fig. 6.39

Magnetic field due to coil 2 at centre, $B_2 = \frac{\mu_0 N_2 i}{2R}$

Flux through coil 1, $\phi_{B_1} = N_1 B_2 S = \frac{N_1 \mu_0 N_2 i}{2R} \pi r^2 = Mi$

$$\Rightarrow \quad \boxed{M = \frac{\mu_0 N_1 N_2 \pi r^2}{2R}}$$

Mutual inductance of two long co-axial solenoids

Fig. 6.40 shows two long co-axial solenoids, each of length l. Let n be the number of turns per unit length of inner solenoid S_1 of radius r_1 and n_1 is number of turns per unit length of outer solenoid S_2 of radius r_2.

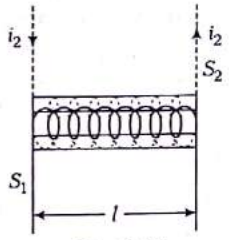


Fig. 6.40

Magnetic field due to S_1, $B_1 = \mu_0 n_1 i$

Flux through S_2,

$$\phi_B = (n_2 l) B_1 \pi r_1^2$$
$$= (n_2 l) \mu_0 n_1 i \pi r_1^2 = Mi$$

$$\Rightarrow \quad M = \mu_0 n_1 n_2 \pi r_1^2 l$$

$$\boxed{M = \frac{\mu_0 N_1 N_2 \times S}{l}}$$

where, S is the area of cross-section of inner solenoid.

Note We do not take the area of cross-section of outer solenoid because there is no magnetic field between the two solenoids.

Mutual inductance of a solenoid surrounded by a coil

Let N_1 be the number of turns of solenoid of radius r_1 and N_2 be the number of turns of coil of radius r_2.

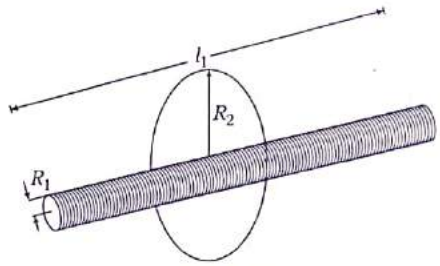


Fig. 6.41

Magnetic field due to S,

$$B = \mu_0 (N_1 / l_1) i$$

Then, flux through coil is

$$\phi_B = N_2 BS = N_2 \mu_0 \frac{N_1}{l} i \pi R_1^2 = Mi$$

$$\Rightarrow \quad \boxed{M = \frac{\mu_0 N_1 N_2 \pi R_1^2}{l}} \quad \left(\because M = \frac{N_2 \phi_B}{i}\right)$$

From the above relation, we see that, M is independent of radius R_2. This is because, solenoid's magnetic field is confined to its interior.

Note Mutual inductance of a small circular loop and a rectangular loop ($r << a, b$)

$$\boxed{M = \frac{\mu_0}{4\pi} \frac{8\sqrt{a^2 + b^2}}{ab} \pi r^2}$$

Mutual inductance between two coils depends upon the geometry of two coils and their geometrical orientation with respect to each other. To find mutual inductance for a given arrangement, we assume a current flowing through one of the coils and find the flux through the other coil. Then, using the formula $\phi = MI$, mutual inductance M can be calculated.

Coefficient of coupling

The coefficient of coupling of two coils gives a measure of the manner in which the two coils are coupled together. If L_1 and L_2 are the self-inductances of two coils and M is their mutual inductance, then their coefficient of coupling is given by

$$K = \frac{M}{\sqrt{L_1 L_2}}$$

It is also defined as

$$K = \frac{\text{magnetic flux linked in secondary coil}}{\text{magnetic flux linked in primary coil}}$$

where, $0 \le K \le 1$

So, when $K = 0$, there will be no coupling,
$0 < K < 1$, for loose coupling
and $K = 1$, for maximum coupling. It occurs when the two coils are wound over each other over a ferromagnetic core. The value of K is a number, depending on the geometry of the coils and their relative closeness.

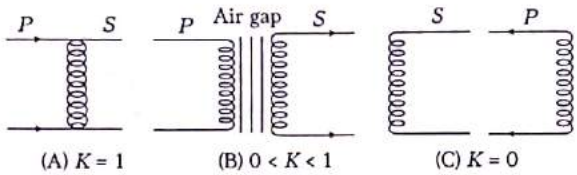


Fig. 6.42

Combination of mutual inductances

(i) **In series** If two coils of self-inductances L_1 and L_2 having mutual inductance M are in series, situated close to each other, then net inductance will be

$$L_S = L_1 + L_2 \pm 2M$$

(ii) **In parallel** If two coils of self-inductances L_1 and L_2 having mutual inductance M, are connected in parallel, situated close to each other, then

$$L_P = \frac{L_1 L_2 - M^2}{L_1 + L_2 \pm 2M}.$$

Example 6.40 *Calculate the mutual inductance between two coils when a current 2A changes to 6A in 0.2 s and induces an emf of 20mV in the secondary coil.*

Sol. We know that emf is related to the mutual inductance by

$$e = M\frac{\Delta i}{\Delta t}$$

Substituting the given values,

$$20 \times 10^{-3} = \frac{M(6-2)}{0.2} = \frac{M \times 4}{0.2} = 20M$$

$$\Rightarrow \quad M = 10^{-3}\text{H} = 1\text{mH}$$

Example 6.41 *A straight solenoid has 50 turns per cm in primary and 200 turns in the secondary. The area of cross-section of the solenoid is 4 cm^2. Calculate the mutual inductance.*

Sol. The magnetic field at any point inside the straight solenoid of primary with n_1 turns per unit length carrying a current i_1 is given by the relation,

$$B = \mu_0 n_1 i_1$$

The magnetic flux through the secondary of N_2 turns each of area S is given as

$$N_2\phi_2 = N_2(BS) = \mu_0 n_1 N_2 i_1 S$$

$$\therefore \quad M = \frac{N_2\phi_2}{i_1} = \mu_0 n_1 N_2 S$$

Substituting the values,

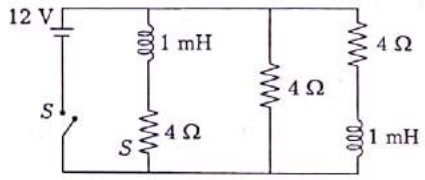
$$M = (4\pi \times 10^{-7})\left(\frac{50}{10^{-2}}\right)(200)(4 \times 10^{-4})$$

$$= 5 \times 10^{-4}\text{ H}$$

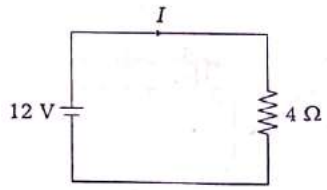
Growth and decay of current in *L-R* circuit

If a circuit containing a pure inductor L and a resistor R in series with a battery and a key is closed, then the current through the circuit rises exponentially and reaches upto a certain maximum value (steady state). If circuit is opened from its steady state condition, then current through the circuit decreases exponentially. In case of growth of current at time $t = 0$, inductor offers infinite resistance and at time $t = \infty$ (steady state) inductor offers zero resistance. This type of circuit is called L-R circuit.

Example 6.42 *Find current through the battery*
(i) just after the switch is closed.
(ii) long after the switch has been closed.

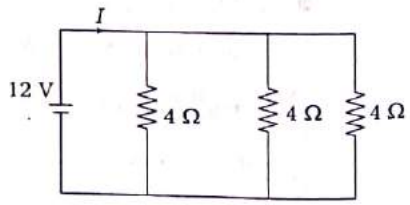


Sol. (i) Just after closing the switch, inductor offers infinite resistance. So, the circuit will become as



So, the current will be $I = \frac{V}{r} = \frac{12}{4} = 3\text{ A}$

(ii) After long time, inductor offers zero resistance. So, the equivalent circuit will be



Net resistance, $R_{eq} = \frac{4 \times 4 \times 4}{4 + 4 + 4} = \frac{64}{12} = 5.33$

$\therefore$ Current, $I = \frac{V}{R} = \frac{12}{5.33}$ or $\frac{12 \times 12}{64} = \frac{144}{64} = 2.25\text{ A}$

Growth of current

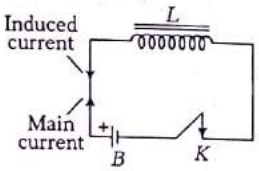
Let us consider a circuit as shown in Fig. 6.43, when the switch K is closed, then the value of current at any instant of time t after closing the circuit (when current is rising) is given by

$$i = i_0\left[1 - e^{-\frac{R}{L}t}\right] \quad ...(i)$$

where, $i_0 = i_{max} = \frac{E}{R}$ = steady state current at $t = \infty$

and time constant $\tau_L = L/R$ [time constant]

So, the Eq. (i) can be re-written as $i = i_0(l - e^{-t/\tau_L})$



Growth of current

Fig. 6.43

Here, τ_L refers at time equal to one time constant the current has risen to $(1 - 1/e)$ or about 63% of its final value i_0. The i-t graph will be shown as Fig. 6.44.

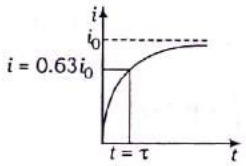


Fig. 6.44

Let us have an insight into the behaviour of an L-R circuit from energy considerations. The instantaneous rate at which the source delivers energy to the circuit ($P = Ei$) is equal to the instantaneous rate at which energy is dissipated in the resistor ($= i^2R$) plus the rate at which energy is stored in the inductor ($= iV_{bc} = Li\, di/dt$)

or $$\frac{d}{dt}\left(\frac{1}{2}Li^2\right) = Li\frac{di}{dt}$$

Thus, $$Ei = i^2R + Li\frac{di}{dt}$$

Note At any time from Fig. 6.53, $V_R + V_L = E$

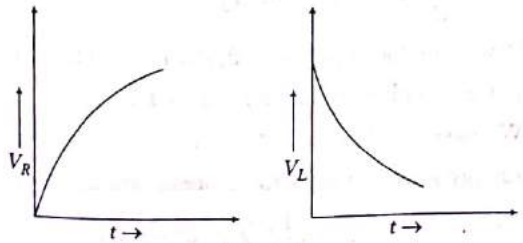


Fig. 6.45

Decay of current

Let us consider the Fig. 6.46, in which now the key K is open. Now, the current will decrease with time.

During the decaying of current, the value of current at any instant of time t after opening from the steady state condition is given by $i = i_0 e^{-\frac{R}{L}t} = i_0 e^{-t/\tau_L}$

Here, τ_L is the time for current to decrease to $1/e$ or about 37% of its original value.

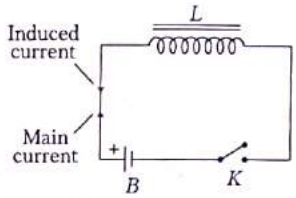


Fig. 6.46 Decay of current

The i-t graph is shown as Fig 6.47.

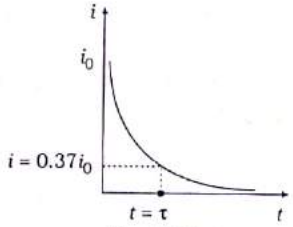


Fig. 6.47

The energy that is needed to maintain the current during this decay is provided by energy stored in the magnetic field. Thus, the rate at which energy is dissipated in the resistor = rate at which the stored energy decreases in magnetic field of inductor. This is given by

$$i^2R = -\frac{dU}{dt} = -\frac{d}{dt}\left(\frac{1}{2}Li^2\right) = Li\left(-\frac{di}{dt}\right) \text{ or } i^2R = Li\left(\frac{-di}{dt}\right)$$

Note At any time, t, $V_R = V_L$

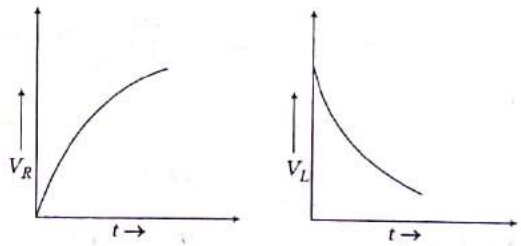


Fig. 6.48

Example 6.43 *A coil of resistance 20 Ω and inductance 0.5 H is switched to DC 200 V supply. Calculate the rate of increase of current*

(i) at the instant of closing the switch and

(ii) after one time constant.

(iii) find the steady state current in the circuit.

Sol. (i) This is the case of growth of current in an L-R circuit. Hence, current at time t is given by

$$i = i_0\,(1 - e^{-t/\tau_L})$$

Rate of increase of current, $\dfrac{di}{dt} = \dfrac{i_0}{\tau_L} e^{-t/\tau_L}$

At $t = 0$, $\quad \dfrac{di}{dt} = \dfrac{i_0}{\tau_L} = \dfrac{E/R}{L/R} = \dfrac{E}{L}$

Substituting the value, we have

$$\frac{di}{dt} = \frac{200}{0.5} = 400 \text{ A/s}$$

(ii) At, $t = \tau_L$, $\dfrac{di}{dt} = (400)e^{-1} = (0.37)\,(400) = 148 \text{ As}^{-1}$

(iii) The steady state current in the circuit,

$$i_0 = \frac{E}{R} = \frac{200}{20} = 10 \text{ A}$$

Example 6.44 *Self-inductance 0.8×10^{-4} H of a uniformly wound solenoid, having resistance 3 Ω is broken up into two identical coils. Those coils are connected in series across a 6 V battery of negligible resistance. Find time constant and steady state current.*

Sol. For a solenoid, self-inductance is given by

$$L = \frac{\mu_0 N^2 \pi r^2}{l}$$

where, r = radius of solenoid, l = length of solenoid,

N = number of turns

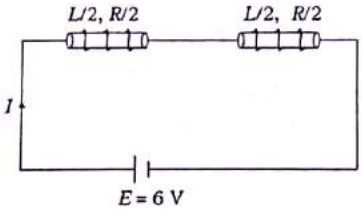
and we know that, resistance is given by

$$R = \rho \frac{l}{A} = \rho \frac{N 2\pi r}{A}$$

So, on breaking it into two parts, now inductance and resistance is given as

$$N' = \frac{N}{2},\ l' = \frac{l}{2} \Rightarrow L' = \frac{\mu_0 (N/2)^2 \pi r^2}{l/2} = \frac{L}{2}$$

$$R' = \frac{\rho (N/2) \times 2\pi r}{A} = R$$



So, the equivalent inductance, $L_{eq} = L_1 + L_2$

$$= L/2 + L/2 = L$$

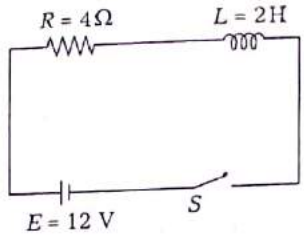
and equivalent resistance, $R_{eq} = R_1 + R_2 = R/2 + R/2 = R$

So, the time constant,

$$\tau = \frac{L_{eq}}{R_{eq}} = \frac{0.8 \times 10^{-4}}{3} = 0.267 \times 10^{-4} \text{ s}$$

and current, $I = \dfrac{E}{R_{eq}} = \dfrac{E}{R} = \dfrac{6}{3} = 2 \text{ A}$

Example 6.45



At $t = 0$, switch S is closed, calculate

(i) initial rate of increase of current, i.e., $\dfrac{di}{dt}$ at $t = 0$.

(ii) $\dfrac{di}{dt}$ at time when current in the circuit is 0.5A.

(iii) Current at $t = 0.6$ s.

(iv) rate at which energy of magnetic field is increasing, rate of heat produced in resistance and rate at which energy is supplied by battery when $i = 0.5$A.

(v) energy stored in inductor in steady state.

Sol. We know that

$$E = iR + L\,di/dt$$

$$\frac{di}{dt} = \frac{E - iR}{L}$$

Current in the circuit at any time t,

$$i = i_0(1 - e^{-t/\tau_L})$$

where, $i_0 = E/R$, $\tau_L = L/R$

(i) Rate of increase of current $\dfrac{di}{dt} = \dfrac{E - iR}{L}$, when $t = 0$, $i = 0$

$$\left.\frac{di}{dt}\right|_{t=0} = \frac{E}{L} = \frac{12}{2} = 6 \text{ A/s}$$

(ii) At $t = 0.5$ s, $\dfrac{di}{dt} = \dfrac{E - iR}{L} = \dfrac{12 - 0.5 \times 4}{2} = \dfrac{12 - 2}{2} = 5 \text{ A/s}$

(iii) At $t = 0.6$ s, current $i = \dfrac{E}{R}(1 - e^{-\frac{Rt}{L}})$

$$= \frac{12}{4}(1 - e^{-\frac{4 \times 0.6}{0.5}}) = 3(1 - e^{2.4})\text{A}$$

(iv) Energy stored in inductor in steady state is given by

$$U = \frac{1}{2}Li^2 \Rightarrow \frac{dU}{dt} = Li\frac{di}{dt}$$

$$\frac{di}{dt} = \frac{E - iR}{L} = \frac{12 - 0.5 \times 4}{2} = 5 \text{ A/s}$$

$$\frac{dU}{dt} = 2 \times 0.5 \times 5 = 5 \text{ J/s}$$

Power produced per second, $P = i^2R = (0.5)^2 \times 4 = 1 \text{ J/s}$

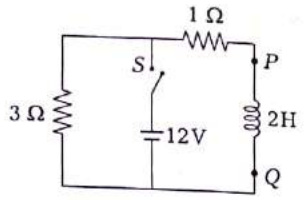
Power supplied by battery $= Ei = 0.5 \times 12 = 6 \text{ J/s}$

(v) We have $i_0 = E/R = 12/4 = 3$ A

Energy stored in inductor in steady state,

$$U = \frac{1}{2}Li_0^2 = \frac{1}{2} \times 2 \times (3)^2 = 9 \text{ J/s}$$

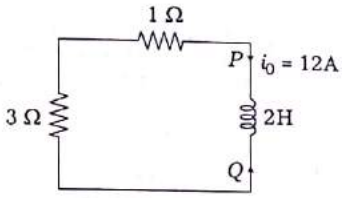
Example 6.46



The switch is closed for a long time and then opened at time $t = 0$. Find the initial voltage across L after $t = 0$, which end is at higher potential P or Q?

Sol. Initially, current will be, $i_0 = \frac{12}{1} = 12\text{A}$

So, the above circuit would be now as



Current at any time is given by $i = i_0\, e^{-t/\tau_L}$ and $i_0 = 12\text{A}$

$\because \quad \tau_L = L/R = \frac{2}{(1+3)} = \frac{2}{4} = 0.5$

Current, $\quad i = 12e^{-t/0.5}$

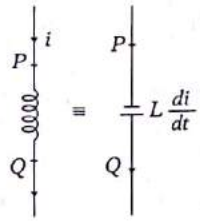
$\Rightarrow \quad \frac{di}{dt} = 12e^{-t/0.5}\left(-\frac{1}{0.5}\right) \Rightarrow \left|\frac{di}{dt}\right| = 24e^{-2t}$

So, $\quad V_L = L\frac{di}{dt} = 48e^{-2t}$

At $\quad t = 0, V_L = 48\text{V}$

i.e., current is decreasing,

$\Rightarrow$



$\Rightarrow \quad V_Q > V_P$

Applications of EMI: eddy current

When a changing magnetic flux is applied to a bulk piece of conducting material, then circulating current induced in the body of the conductor, are called eddy currents. Because the resistance of the bulk conductor is usually low, eddy currents often have large magnitudes and heat up the conductor.

Eddy currents are always produced in a plane perpendicular to the direction of magnetic field. This current shows both heating and magnetic effects.

The magnitude of eddy current is given by

$$i = \frac{\text{induced emf}}{\text{resistance}} = \frac{e}{R}$$

According to Faraday's law,

$$e = -\frac{d\phi}{dt}, \quad i = -\frac{d\phi/dt}{R}$$

The direction of eddy currents can be given by Lenz's law or by Fleming's right hand rule.

However, their flow patterns resemble swirling eddies in water. That is why, they are called **eddy currents**. These were discovered by Foucault in the year 1895 and hence, they are also named as **Foucault current**, *e.g.*, when we move a metal plate out of a magnetic field, then the relative motion of the field and the conductor again induces a current in the conductor. The conducting electrons building up the induced current whirl around within the plate as, if they were caught in an eddy of water.

Effects of eddy current

Eddy currents are produced inside the iron cores of the rotating armatures of electric motors and dynamos and also in the cores of transformers, which experience flux changes, when they are in use. The currents causes unnecessary heating and wastage of power and the heat produced may even damage the insulation of coils.

Methods to Reduce eddy current

Eddy currents are minimised by using laminations of metal to make a metal core. The laminations are separated by an insulating material. The plane of the laminations must be arranged parallel to the magnetic field, so that they cut across the eddy current paths. This arrrangement reduces the strength of eddy currents.

Applications of eddy current

Eddy currents are useful in many ways. *Some of the important applications of eddy currents are given below*

(i) Electromagnetic damping

In order to bring the moving coil of a galvanometer immediately to rest, we make the use of electromagnetic damping which uses eddy currents to bring the coil to rest. When the coil oscillates, then the eddy currents generated in the core oppose the motion and bring the coil to rest.

(ii) Induction furnace

In an induction furnace, high temperature can be produced by using eddy currents. We generally use induction furnace in preparation of alloys by melting the constituents of metal.

A coil is wound over the metal which needs to be melted and through the coil, we pass high frequency alternating current. The eddy current generated in the metal produces high temperature to melt the metal.

(iii) Electric power meter

Old electric power meters (analog type) had a metallic disc. The disc rotates due to generation of eddy currents which are produced due to sinusoidally varying current in the coil.

(iv) Magnetic breaking in electronic trains

Some electric powered trains make use of strong electromagnets which are situated above the rails. These electromagnets are used to produce eddy current in the rails which oppose the motion of the train and thus stop it. In this case, as there is no mechanical linkage, the breaking effect is smooth.

Check point 6.4

1. Two coils X and Y are placed in a circuit such that when the current changes by 2 A in coil X. The magnetic flux changes by 0.4 Wb in Y. The value of mutual inductance of the coils is
(a) 0.2 H (b) 5 H (c) 0.8 H (d) 4 H

2. Two coils have a mutual inductance 0.005 H. The current changes in the first coil according to equation $I = I_0 \sin \omega t$, where $I_0 = 10$ A and $\omega = 100\pi$ rads^{-1}. The maximum value of emf in the second coil is (in volt)
(a) 2π (b) 5π (c) π (d) 4π

3. The mutual inductance between a primary and secondary circuits is 0.5 H. The resistance of the primary and the secondary circuits are 20 Ω and 5 Ω, respectively. To generate a current of 0.4 A in the secondary, current in the primary must be changed at the rate of
(a) 4.0 As^{-1} (b) 1.6 As^{-1} (c) 16.0 As^{-1} (d) 8.0 As^{-1}

4. The coefficient of mutual induction between two circuits is equal to the, emf produced in one circuit when the current in the second circuit is
(a) kept steady at 1 A
(b) cut-off at 1 A level
(c) changed at the rate of 1 As^{-1}
(d) changed from 1 As^{-1} to 2 As^{-1}

5. Two circuits have coefficient of mutual induction of 0.09 H. Average emf induced in the secondary by a change of current from 0 to 20 A in 0.006 s in the primary will be
(a) 120 V (b) 80 V (c) 200 V (d) 300 V

6. A current is varying at the rate of 3 A/s in a coil generates an emf of 8 mV in a nearby coil. The mutual inductance of the two coils is
(a) 2.66 ¬H (b) 2.66×10^{-3} mH
(c) 2.66 H (d) 0.266 H

7. A solenoid is placed inside another solenoid, the length of both being equal carrying same magnitude of current. The parameters like radius and number of turns are in the ratio 1: 2 for the two solenoids. The mutual inductance on each other would be
(a) $M_{12} = M_{21}$ (b) $M_{12} = 2M_{21}$
(c) $2M_{12} = M_{21}$ (d) $M_{12} = 4M_{21}$

8. Two coils of self-inductances 2 mH and 8 mH are placed, so close together that the effective flux in one coil is completely linked with the other. The mutual inductance between these coils is
(a) 4 mH (b) 16 mH
(c) 10 mH (d) 6 mH

9. Two coils of self-inductances L_1 and L_2 are placed closer to each other, so that total flux in one coil is completely linked with other. If M is mutual inductance between them, then
(a) $M = L_1L_2$ (b) $M = L_1 / L_2$
(c) $M = \sqrt{L_1L_2}$ (d) $M = (L_1L_2)^2$

10. An ideal coil of 10 H is joined in series with a resistance of 5Ω and a battery of 5 V. 2 s after joining, the current flowing in ampere in the circuit will be
(a) e^{-1} (b) $(1-e^{-1})$ (c) $(1-e)$ (d) e

11. An L-R circuit has a cell of emf E, which is switched on at time $t = 0$. The current in the circuit after a long time will be
(a) zero (b) $\frac{E}{R}$ (c) $\frac{E}{L}$ (d) $\frac{E}{\sqrt{L^2 + R^2}}$

12. During current growth in an L-R circuit the time constant is the time in which the magnitude of current becomes
(a) I_0 (b) $I_0/2$ (c) $0.63\, I_0$ (d) $0.37\, I_0$

13. An L-R circuit with a battery is connected at $t = 0$. Which of the following quantities is not zero just after the connection?
(a) Current in the circuit
(b) Magnetic field energy
(c) Power delivered by the battery
(d) Emf induced in the inductor

14. Eddy currents are produced when
(a) a metal is kept in varying magnetic field
(b) a metal is kept in a steady magnetic field
(c) a circular coil is placed in a magnetic field
(d) through a circular coil current is passed

15. Which of the following is not an application of eddy currents?
(a) Induction furnace (b) Galvanometer damping
(c) Speedometer of automobiles (d) X-ray crystallography

Chapter exercises

A Taking it together

(Assorted questions of the chapter for advanced level practice)

1. If L, C and R represent the physical quantities inductance, capacitance and resistance respectively, only the following combination does not have the dimensions of frequency

(a) $\frac{1}{RC}$ (b) $\frac{R}{L}$ (c) $\frac{1}{\sqrt{LC}}$ (d) $\frac{C}{L}$

2. The self-inductance L of a solenoid of length l and area of cross-section A, with a fixed number of turns N increases as **[NCERT Exemplar]**

(a) Both l and A increase
(b) l decreases and A increases
(c) l increases and A decreases
(d) Both l and A decrease

3. In electromagnetic induction, the induced charge in a coil is independent of

(a) change in the flux
(b) time taken to change the flux
(c) resistance in the circuit
(d) None of the above

4. A coil and a bulb are connected in series with a DC source, a soft iron core is then inserted in the coil, then

(a) intensity of the bulb remains the same
(b) intensity of the bulb decreases
(c) intensity of the bulb increases
(d) the bulb ceases to glow

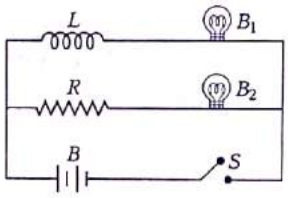
5. A magnet is brought towards a coil (i) speedly (ii) slowly, then the induced emf/induced charge will be respectively

(a) more in first case/more in first case
(b) more in first case/equal in both case
(c) less in first case/more in second case
(d) less in first case/equal in both case

6. In a coil of area 10 cm^2 and 10 turns, a magnetic field is directed perpendicular to the plane and is changing at the rate of 10^8 gauss/s. The resistance of the coil is 20 Ω. The current in the coil will be

(a) 5 A (b) 0.5 A (c) 0.05 A (d) 5×10^8 A

7. An inductor L, a resistance R and two identical bulbs, B_1 and B_2 are connected to a battery through a switch S as shown in the figure. Which of the following statements gives the correct description of the happenings when the switch S is closed?



(a) The bulb B_2 lights up earlier than B_1 and finally both the bulbs shine equally bright
(b) B_1 lights up earlier and finally both the bulbs acquire equal brightness
(c) B_2 lights up earlier and finally B_1 shines brighter than B_2
(d) B_1 and B_2 light up together with equal brightness all the time

8. Two identical circular loops of metal wires are lying on a table without touching each other. Loop A carries a current which increases with time. In response, the loop B

(a) remains stationary
(b) is attracted by loop A
(c) is repelled by loop A
(d) rotates about its centre of mass with CM fixed

9. A coil having 500 square loops each of side 10 cm is placed normal to a magnetic flux which increases at the rate of 1.0 Ts^{-1}. The induced emf in volts is

(a) 0.1 (b) 0.5
(c) 1 (d) 5

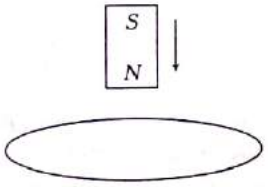
10. A coil having an area 2 m^2 is placed in a magnetic field which changes from 1 Wbm^{-2} to 4 Wbm^{-2} in an interval of 2 s. The emf induced in the coil will be

(a) 4 V (b) 3 V
(c) 1.5 V (d) 2 V

11. The circular loops of equal radii are placed coaxially at some separation. The first is cut and a battery is inserted in between to drive a current in it. The current changes slightly because of the variation in resistance with temperature.
During this period, the two loops
(a) attract each other
(b) repel each other
(c) do not exert any force on each other
(d) attract or repel each other depending on the sense of the current

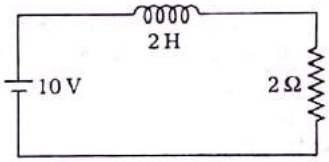
12. A small, conducting circular loop is placed inside a long solenoid carrying a current. The plane of the loop contains the axis of the solenoid.
If the current in the solenoid is varied, then the current induced in the loop is
(a) clockwise
(b) anti-clockwise
(c) zero
(d) clockwise or anti-clockwise depending on whether the resistance is increased or decreased

13. The north pole of a magnet is brought near a metallic ring as shown in the figure. The direction of induced current in the ring will be



(a) anti-clockwise
(b) clockwise
(c) first anti-clockwise and then clockwise
(d) first clockwise and then anti-clockwise

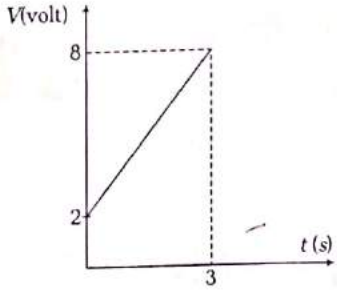
14. In the figure, magnetic energy stored in the coil is



(a) zero
(b) infinite
(c) 25 J
(d) None of the above

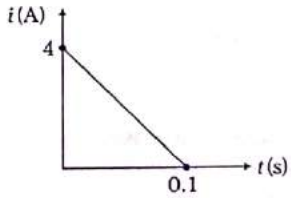
15. An emf of 15 V is applied in a circuit coil containing 5 H inductance and 10 Ω resistance. The ratio of the currents at time $t = \infty$ and $t = 1$ s is
(a) $\frac{e^{1/2}}{e^{1/2}-1}$
(b) $\frac{e^2}{e^2-1}$
(c) $1-e^{-1}$
(d) e^{-1}

16. A circuit element is placed in a closed box. At time $t = 0$, a constant current generator supplying a current of 1 A is connected across the box. Potential difference across the box varies according to graph shown in the figure. The element in the box is



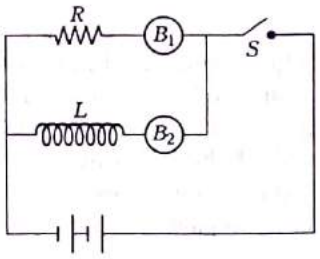
(a) a resistance of 2 Ω
(b) a battery of emf 6 V
(c) an inductance of 2 H
(d) a capacitance of 0.5 F

17. Some magnetic flux is changed from a coil of resistance 10 Ω. As a result an induced current is developed in it, which varies with time as shown in figure. The magnitude of change in flux through the coil in Wb is



(a) 2
(b) 4
(c) 6
(d) None of these

18. Figure shows two bulbs B_1 and B_2 resistor R and inductor L, when the switch S is turned off



(a) Both B_1 and B_2 die out promptly
(b) Both B_1 and B_2 die out with some delay
(c) B_2 dies out promptly but B_1 with some delay
(d) B_1 dies out promptly but B_2 with some delay

19. A coil of 40 Ω resistance, 100 turns and radius 6 mm is connected to ammeter of resistance 160 Ω. Coil is placed perpendicular to the magnetic field. When coil is taken out of the field, 32 μC charge flows through it. The intensity of magnetic field will be
(a) 6.55 T
(b) 5.66 T
(c) 2.55 T
(d) 0.566 T

20. A coil has an area of $0.05\ m^2$ and it has 800 turns. It is placed perpendicularly in a magnetic field of strength $4 \times 10^{-5}\ Wb\ m^{-2}$, and is rotated through 90° in 0.1 s. The average emf induced in the coil is

(a) 0.056 V (b) 0.046 V
(c) 0.026 V (d) 0.016 V

21. In a magnetic field of 0.05 T, area of a coil changes from $101\ cm^2$ to $100\ cm^2$ without changing the resistance which is $2\ \Omega$. The amount of charge that flows during this period is

(a) 2.5×10^{-6} C
(b) 2×10^{-6} C
(c) 10^{-6} C
(d) 8×10^{-6} C

22. The resistance and inductance of series circuit are 5Ω and 20 H, respectively. At the instant of closing the switch, the current is increasing at the rate $4\ As^{-1}$. The supply voltage is

(a) 20 V (b) 80 V
(c) 120 V (d) 100 V

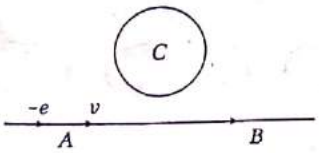
23. The number of turns in the coil of an AC generator is 5000 and the area of the coil is $0.25\ m^2$. The coil is rotated at the rate of 100 cycles/s in a magnetic field of $0.2\ Wm^{-2}$. The peak value of the emf generated is nearly

(a) 786 kV (b) 440 kV
(c) 220 kV (d) 157 kV

24. A wheel with ten metallic spokes each 0.50 m long is rotated with a speed of 120 rev/min in a plane normal to the earth's magnetic field at the place. If the magnitude of the field is 4.0 gauss, the induced emf between the axle and the rim of the wheel is equal to

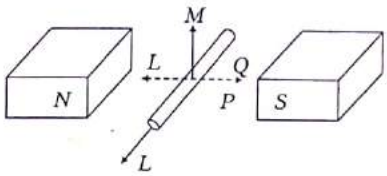
(a) 1.256×10^{-3} V
(b) 6.28×10^{-4} V
(c) 1.256×10^{-4} V
(d) 6.28×10^{-5} V

25. An electron moves along the line AB which lies in the same plane as a circular loop of conducting wire as shown in figure. What will be the direction of the current induced (if any) in the loop?



(a) No current will be induced
(b) The current will be clockwise
(c) The current will be anti-clockwise
(d) The current will change direction as the electron passes through

26. An electric potential difference will be induced between ends of the conductor shown in the diagram, when the conductor moves in the direction

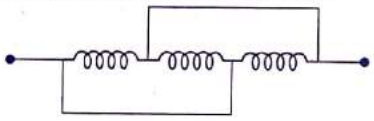


(a) P (b) Q (c) L (d) M

27. A long horizontal metallic rod with length along the east-west direction is falling under gravity. The potential difference between its two ends will

(a) be zero (b) be constant
(c) increase with time (d) decrease with time

28. All the three inductors have inductance 3.0 H. The equivalent inductance of the circuit is



(a) 1 H (b) 2 H (c) 3 H (d) 9 H

29. Two inductances connected in parallel are equivalent to a single inductance of 1.5 H and when connected in series are equivalent to a single inductance of 8 H. The difference in their inductance is

(a) 3 H (b) 7.5 H (c) 2 H (d) 4 H

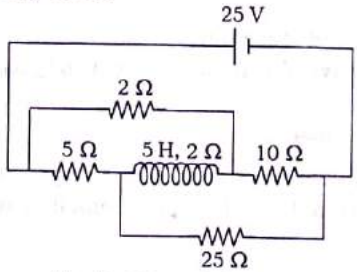
30. An inductance L and a resistance R are first connected to a battery. After some time the battery is disconnected but L and R remain connected in a closed circuit. Then the current reduces to 37% of its initial value in

(a) RL second (b) $\frac{R}{L}$ second
(c) $\frac{L}{R}$ second (d) $\frac{1}{LR}$ second

31. The time constant of an inductance coil is 2.0×10^{-3} s. When a $90\ \Omega$ resistance is joined in series, then the time constant becomes 0.5×10^{-3} s. The inductance and resistance of the coil are

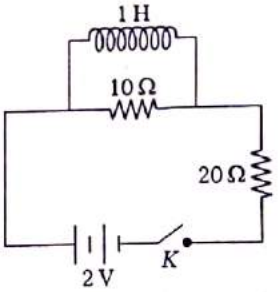
(a) 30 mH, $30\ \Omega$ (b) 30 mH, $60\ \Omega$
(c) 60 mH, $30\ \Omega$ (d) 60 mH, $60\ \Omega$

32. In the circuit shown, what is the energy stored in the coil at steady state?



(a) 21.3 J (b) 42.6 J (c) Zero (d) 213 J

33. In the following figure, what is the final value of current in the 10 ohm resistor when the plug of key K is inserted?



(a) (3/10) A
(b) (3/20) A
(c) (3/11) A
(d) Zero

34. A square loop of side l, resistance R is placed in a uniform magnetic field B acting normally to the plane of the loop. If we attempt to pull it out of the field with a constant velocity v, then the power needed is

(a) $BRlv$
(b) $\frac{B^2l^2v^2}{R}$
(c) $\frac{Bl^2v^2}{R}$
(d) $\frac{Bvl}{R}$

35. A square of side L metres lies in the xy-plane in a region, where the magnetic field is given by $\mathbf{B} = B_0(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$ T, where B_0 is constant. The magnitude of flux passing through the square is **[NCERT Exemplar]**

(a) $2B_0L^2$ Wb
(b) $3B_0L^2$ Wb
(c) $4B_0L^2$ Wb
(d) $\sqrt{29}B_0L^2$ Wb

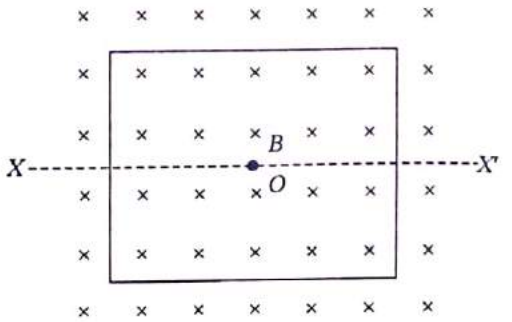
36. A conducting loop of area 5.0 cm^2 is placed in a magnetic field which varies sinusoidally with time as $B = 0.2 \sin 300\, t$. The normal to the coil makes an angle of 60° with the field. The emf induced at $t = (\pi / 900)$s

(a) 7.5×10^{-3} V
(b) zero
(c) 15×10^{-3} V
(d) 20×10^{-3} V

37. An infinitely long cylinder is kept parallel to a uniform magnetic field **B** directed along positive Z-axis. The direction of induced current as seen from the Z-axis will be

(a) clockwise of the positive z-axis
(b) anti positive Z-axis clockwise of the positive z-axis
(c) zero
(d) along the magnetic field

38. A rectangular coil is placed in a region having a uniform magnetic field B perpendicular to the plane of the coil. An emf will not be induced in the coil, if the



(a) magnetic field is increased uniformly
(b) magnetic field is switched off
(c) coil is rotated about an axis XX'
(d) coil is rotated about an axis perpendicular to the plane of the coil and passing through its centre O

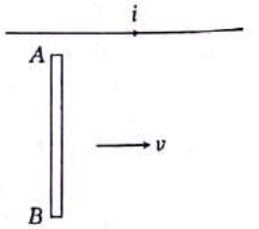
39. If a coil of 40 turns and area 4.0 cm^2 is suddenly removed from a magnetic field, then it is observed that a charge of 2.0×10^{-4} C flows into the coil. If the resistance of the coil is 80Ω, then the magnetic flux density in Wb/m^2 is

(a) 0.5
(b) 1.0
(c) 1.5
(d) 2.0

40. A coil of wire of a certain radius has 600 turns and a self-inductance of 108 mH. The self-inductance of a 2nd similar coil of 500 turns will be

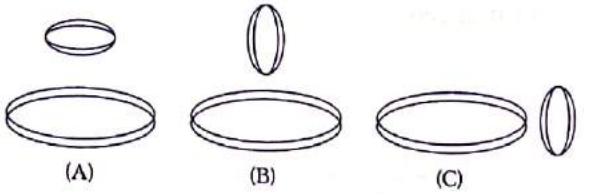
(a) 74 mH
(b) 75 mH
(c) 76 mH
(d) 77 mH

41. The current carrying wire and the rod AB are in the same plane. The rod moves parallel to the wire with a velocity v. Which one of the following statements is true about induced emf in the rod?



(a) End A will be at lower potential with respect to B
(b) A and B will be at the same potential
(c) There will be no induced emf in the rod
(d) Potential at A will be higher than that at B

42. Two circular coils can be arranged in any of the three situations shown in the figure. Their mutual inductance will be

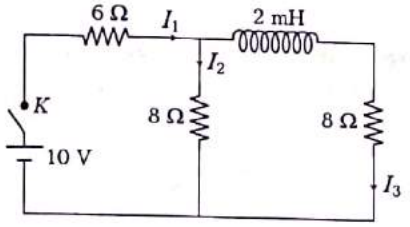


(a) maximum in situation (A)
(b) maximum in situation (B)
(c) maximum in situation (C)
(d) the same in all situations

43. A coil of inductance 300 mH and resistance 2 Ω is connected to a source of voltage 2 V. The current reaches half of its steady state value in
(a) 0.15 s (b) 0.3 s (c) 0.05 s (d) 0.1 s

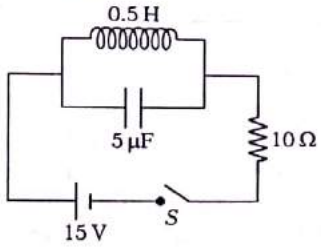
44. An inductor of 2 H and a resistance of 10 Ω are connected in series with a battery of 5 V. The initial rate of change in current is
(a) 0.5 As^{-1} (b) 2.0 As^{-1}
(c) 2.5 As^{-1} (d) 0.25 As^{-1}

45. In the circuit shown in the figure, what is the value of I_1 just after pressing the key K?



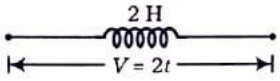
(a) 5/7 A (b) 5/11 A
(c) 1 A (d) None of these

46. For in the circuit shown in figure, current through the battery at $t = 0$ and $t = \infty$ is



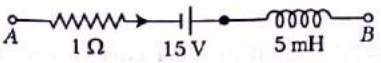
(a) 1.5 A, 1.5 A (b) 0, 0
(c) 1, 1.5 A (d) 1.5 A, 0

47. A variable voltage $V = 2t$ is applied across an inductor of inductance $L = 2$ H as shown in figure. Then,



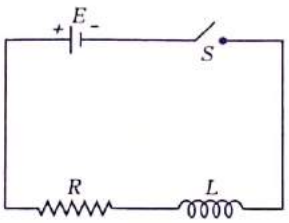
(a) current *versus* time graph is a parabola
(b) energy stored in magnetic field at $t = 2$ s is 4 J
(c) potential energy at time $t = 1$ s in magnetic field is increasing at a rate of 1 J s^{-1}
(d) energy stored in magnetic field is zero all the time.

48. The network shown in the figure is a part of a complete circuit. If at a certain instant the current i is 5 A and decreasing at the rate of 10^3 As^{-1}, then $V_B - V_A$ is

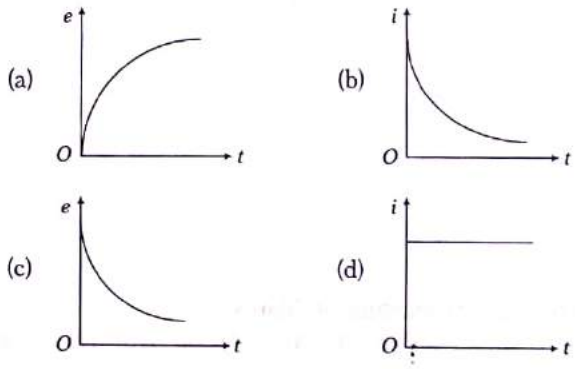


(a) 5 V (b) 10 V (c) 15 V (d) 20 V

49. Switch S of the circuit shown in figure is closed at $t = 0$.

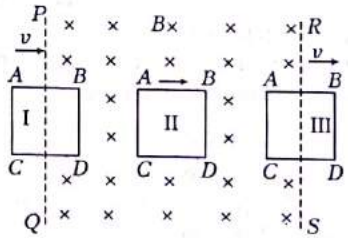


If emf in L is e and i is the current flowing through the circuit at time t, which of the following graphs is correct?



50. Coefficient of coupling between two coils of self-inductances L_1 and L_2 is unity. It means
(a) 50% flux of L_1 is linked with L_2
(b) 100% flux of L_1 is linked with L_2
(c) $\sqrt{L_1}$ time of flux of L_1 is linked with L_2
(d) None of the above

51. A square loop $ABCD$ of edge 'a' moves to the right with a velocity v, parallel to AB. There is a uniform magnetic field of magnitude B, directed into the paper, in the region between PQ and RS only. I, II and III are three positions of the loop.

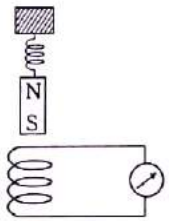


(i) The emf induced in the loop has magnitude Bav in all three positions.
(ii) Induced emf is zero in position II.
(iii) The induced emf is anti-clockwise in position I
(iv) The induced emf is clockwise in position III.
(a) (i), (iii)
(b) (ii), (iii), (iv)
(c) (i), (ii)
(d) (iii), (iv)

52. A rectangular, a square, a circular and an elliptical loop, all in the XY-plane, are moving out of a uniform magnetic field with a constant velocity $\mathbf{v} = u\hat{\mathbf{i}}$. The magnetic field is directed along the negative Z-axis direction. The induced emf during the passage of these loops, out of the field region, will not remain constant for

(a) the rectangular, circular and elliptical loops
(b) the circular and the elliptical loops
(c) only the elliptical loop
(d) any of the four loops

53. A magnet N-S is suspended from a spring and while it oscillates, the magnet moves in and out of the coil C. The coil is connected to a galvanometer G.



Then, as the magnet oscillates,

(a) G shows deflection to the left and right with constant amplitude
(b) G shows deflection on one side
(c) G shows no deflection
(d) G shows deflection to the left and right but the amplitude steadily decreases

54. A coil is suspended in a uniform magnetic field, with the plane of the coil parallel to the magnetic lines of force. When a current is passed through the coil it starts oscillating, then it is very difficult to stop. But, if an aluminium plate is placed near to the coil, then it stops. This is due to

(a) development of air current when the plate is placed
(b) induction of electrical charge on the plate
(c) shielding of magnetic lines of force as aluminium is a paramagnetic material
(d) electromagnetic induction in the aluminium plate give rise to electromagnetic damping

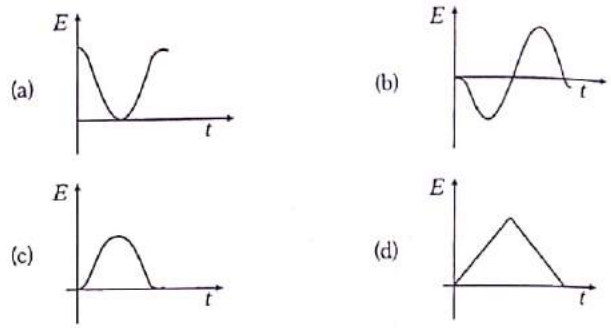
55. Two conducting circular loops of radii R_1 and R_2 are placed in the same plane with their centres coinciding. If, then $R_1 >> R_2$, then the mutual inductance M between them will be directly proportional to

(a) R_1 / R_2 (b) R_2 / R_1
(c) R_1^2 / R_2 (d) R_2^2 / R_1

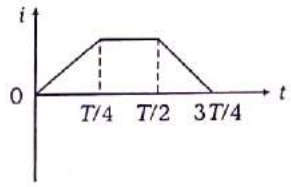
56. A short-circuited coil is placed in a time-varying magnetic field. Electrical power is dissipated due to the current induced in the coil. If the number of turns were to be quadrupled and the wire radius halved, then the electrical power dissipated would be

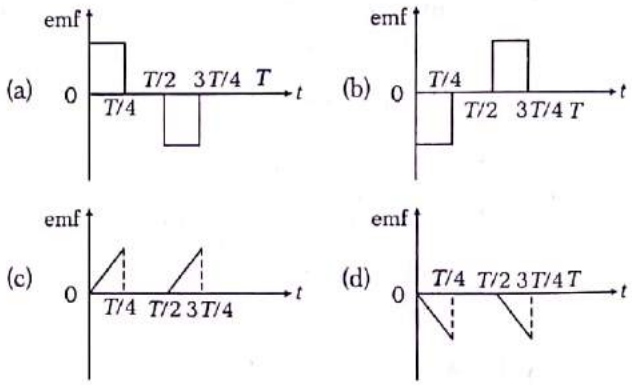
(a) halved (b) the same
(c) doubled (d) quadrupled

57. The variation of induced emf (E) with time (t) in a coil, if a short bar magnet is moved along its axis with a constant velocity is best represented as

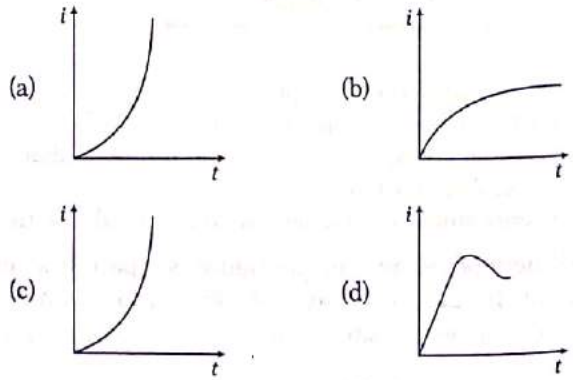


58. The current i in a coil varies with time as shown in the figure. The variation of induced emf with time would be



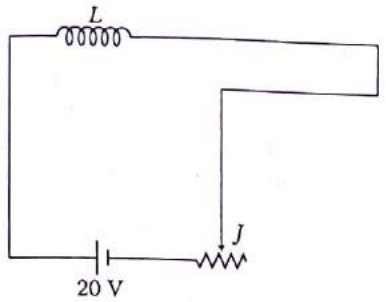


59. When a battery is connected across a series combination of self-inductance L and resistance R, then the variation in the current i with time t is best represented by



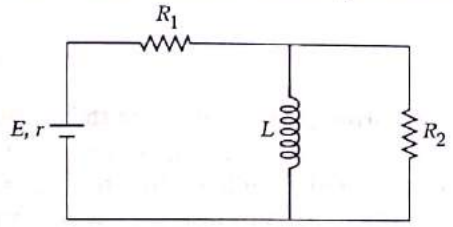
60. In the circuit shown in the figure, the jockey J is being pulled towards right, so that the resistance in the circuit is increasing. Its a value at some instant is $5\,\Omega$.

The current in the circuit at this instant will be



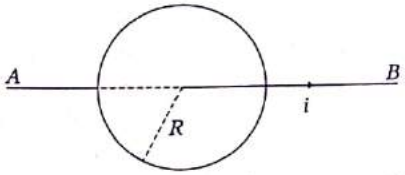
(a) 4 A
(b) less than 4 A
(c) more than 4 A
(d) may be less than or more than 4 A depending on the value of L

61. The value of time constant for the given circuit is



(a) $\frac{L}{R_1 + r + R_2}$ (b) $\frac{L}{(R_1 + r)}$
(c) $\frac{L(R_1 + R_2 + r)}{(R_1 + r)R_2}$ (d) None of these

62. An infinitely long conductor AB lies along the axis of a circular loop of radius R. If the current in the conductor AB varies at the rate of x ampere/second, then the induced emf in the loop is



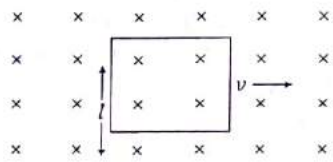
(a) $\frac{\mu_0 xR}{2}$ (b) $\frac{\mu_0 xR}{4}$ (c) $\frac{\mu_0 \pi xR}{2}$ (d) zero

63. A physicist works in a laboratory where the magnetic field is 2 T. She wears a necklace of enclosing area 0.01 m^2 in such a way that the plane of the necklace is normal to the field and is having a resistance $R = 0.01\ \Omega$. Because of power failure, the field decays to 1 T in time 10^{-3} s. Then, what is the total heat produced in her necklace?
(a) 10 J (b) 20 J (c) 30 J (d) 40 J

64. A wheel with ten metallic spokes each 0.50 m long is rotated with a speed of 120 rev min^{-1} in a plane normal to the earth's magnetic field at the plane. If the magnitude of the field is 0.4 G, the induced emf between the axle and the rim of the wheel is equal to
(a) 1.256×10^{-3} V (b) 6.28×10^{-4} V
(c) 1.256×10^{-4} V (d) 6.28×10^{-5} V

65. An aeroplane in which the distance between the tips of the wings is 50 m is flying horizontally from east to west with the speed of 360 kmh^{-1} over a place where the vertical component of the earth's magnetic field is 2.0×10^{-4} Wbm^{-2}. The potential difference between the tips of the wings would be
(a) 0.1 V (b) 0.01 V (c) 0.2 V (d) 1.0 V

66. A conducting square loop of side l and resistance R is moving out of the plane with a uniform velocity v perpendicular to one of its sides. A uniform and constant magnetic field B exists along the perpendicular to the plane of the loop as shown in figure.



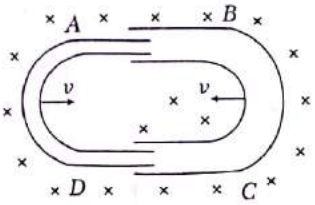
The current induced in the loop is
(a) $\frac{Blv}{R}$ clockwise (b) $\frac{Blv}{R}$ anti-clockwise
(c) $\frac{2Blv}{R}$ anti-clockwise (d) zero

67. Two rails of a railway track, insulated from each other and the ground, are connected to a millivoltmetre. What is the reading of the millivoltmetre when a train travels at a speed of 20 ms^{-1} along the track? Given that the vertical component of earth's magnetic field is 0.2×10^{-4} Wbm^{-2} and the rails are separated by 1 m
(a) 4 mV (b) 0.4 mV
(c) 80 mV (d) 10 mV

68. A pair of coils of turns n_1 and n_2 are kept close together. Current passing through the first is reduced at the rate r, and emf 3 mV is developed across the other coil. If the second coil carries current which is then reduced at the rate $2r$, then the emf produced across the first coil will be
(a) $\frac{6n_1}{n_2}$ mV (b) $\frac{6n_2}{n_1}$ mV (c) 6 mV (d) 3/2 mV

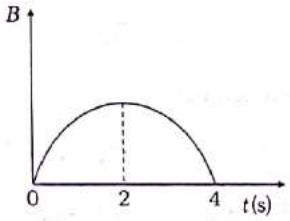
69. A loop made of straight edges has six corners at A (0, 0, 0), B (L, 0, 0), C (L, L, 0), D (0, L, 0), E (0, L, L) and F (0, 0, L). A magnetic field $B = B_0(\hat{\mathbf{i}} + \hat{\mathbf{k}})$ T is present in the region. The flux passing through the loop $ABCDEFA$ (in that order) is **[NCERT Exemplar]**
(a) B_0L^2 Wb (b) $2B_0L^2$ Wb (c) $\sqrt{2}B_0L^2$ Wb (d) $4B_0L^2$ Wb

70. One conducting U-tube can slide inside another as shown in figure, maintaining electrical contacts between the tubes. The magnetic field B is perpendicular to the plane of the figure. If each tube moves towards the other at a constant speed u, then the emf induced in the circuit in terms of B, l and v, where l is the width of each tube, will be



(a) zero (b) $2Blv$
(c) Blv (d) $-Blv$

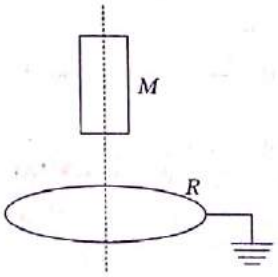
71. In a closed loop, which has some inductance but negligible resistance, uniform but time varying magnetic field is applied directed into the plane of the loop. Variation of field with time is shown. Initially, current in the loop was zero. Then,



(a) emf induced in the loop is zero at $t = 2$ s
(b) current in the loop will be maximum at $t = 2$ s
(c) direction of emf in the loop will change at $t = 2$ s
(d) None of the above

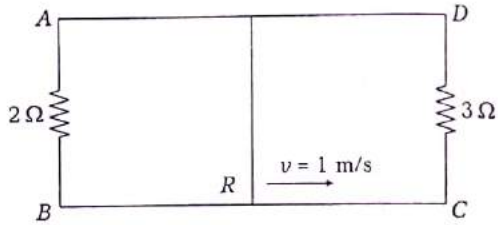
72. A magnetic field given by $B(t) = 0.2t - 0.05t^2$ tesla is directed perpendicular to the plane of a circular coil containing 25 turns of radius 1.8 cm and whose total resistance is 1.5 Ω. The power dissipation at 3 s is approximately
(a) 1.37 μW (b) 7 μW (c) zero (d) 4 μW

73. A small magnet M is allowed to fall through a fixed horizontal conducting ring R. Let g be the acceleration due to gravity. The acceleration of M, a will be



(a) $< g$ when it is above R and moving towards R
(b) $> g$ when it is above R and moving towards R
(c) $< g$ when it is below R and moving away from R
(d) $> g$ when it is below R and moving away from R

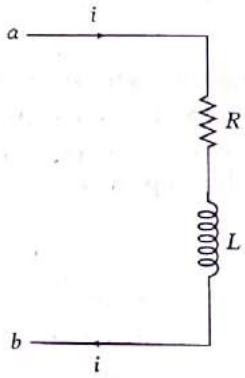
74. A rectangular loop with a sliding connector of length 10 cm is situated in uniform magnetic field perpendicular to plane of loop. The magnetic induction is 0.1 T and resistance of connector (R) is 1Ω. The sides AB and CD have resistances 2Ω and 3 Ω, respectively. Find the current in the connector during its motion with constant velocity 1 ms^{-1}.



(a) $\frac{1}{242}$ A (b) $\frac{1}{220}$ A
(c) $\frac{1}{55}$ A (d) $\frac{1}{440}$ A

75. When the current in the portion of the circuit shown in the figure is 2 A and increasing at the rate of 1 As^{-1}, then the measured potential difference $V_{ab} = 8$ V. However, when the current is 2 A and decreasing at the rate of 1 As^{-1}, then the measured potential difference $V_{ab} = 4$ V.

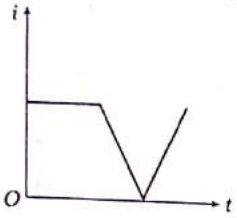
The values of R and L are



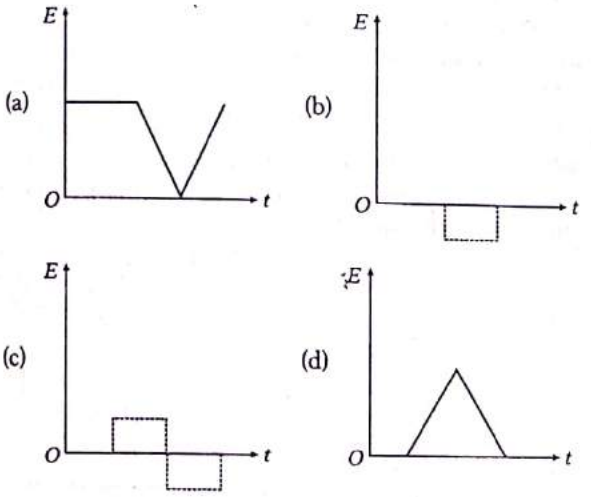
(a) 3 Ω and 2 H respectively
(b) 3 Ω and 3 H respectively
(c) 2 Ω and 1 H respectively
(d) 3 Ω and 1 H respectively

76. Two different coils have self-inductances $L_1 = 8$ mH and $L_2 = 2$ mH. The current in one coil is increased at a constant rate. The current in second coil is also increased at the same constant rate. At certain instant of time, the power given to the coils is the same. At that time, the current, the induced voltage and the energy stored in the first coil are i_1, V_1 and W_1 respectively. Corresponding values for the second coil at the same instant are i_2, V_2 and W_2, respectively. Then, choose the wrong option.
(a) $\frac{i_1}{i_2} = \frac{1}{4}$ (b) $\frac{i_1}{i_2} = 4$ (c) $\frac{W_1}{W_2} = \frac{1}{4}$ (d) $\frac{V_2}{V_1} = \frac{1}{4}$

77. The current i in an induction coil varies with time t according to the graph shown in figure.

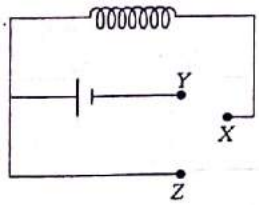


Which of the following graphs shows the induced emf (E) in the coil with time?



78. A coil of inductance $L = 50\,\mu H$ and resistance $= 0.5\,\Omega$ is connected to a battery of emf $= 5$ V.
A resistance of $10\,\Omega$ is connected parallel to the coil. Now, at same instant the connection of the battery is switched off. Then, the amount of heat generated in the coil after switching off the battery is
(a) 1.25 mJ (b) 2.5 mJ (c) 0.65 mJ (d) 0.12 mJ

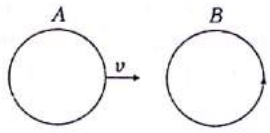
79. In the circuit shown, the coil has inductance and resistance. When X is joined to Y, the time constant is τ during growth of current. When the steady state is reached, then heat is produced in the coil at a rate P. If X is now joined to Z, then choose the correct statement.



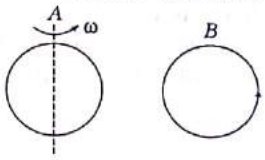
(a) the total heat produced in the coil is $P\tau$
(b) the total heat produced in the coil is $\frac{1}{2}P\tau$
(c) the total heat produced in the coil is $2P\tau$
(d) the data given is not sufficient to reach a conclusion

80. A cylindrical bar magnet is rotated about its axis. A wire is connected from the axis and is made to touch the cylindrical surface through a contact. Then, **[NCERT Exemplar]**
(a) a direct current flows in the ammeter A
(b) no current flows through the ammeter A
(c) an alternating sinusoidal current flows through the ammeter A with a time period $T = \frac{2\pi}{\omega}$
(d) a time varying non-sinusoidal current flows through the ammeter A

81. There are two coils A and B as shown in figure. A current starts flowing in B as shown, when A is moved towards B and stops when A stops moving. The current in A is counter-clockwise. B is kept stationary when A moves. We can infer that **[NCERT Exemplar]**
(a) there is a constant current in the clockwise direction in A
(b) there is a varying current in A
(c) there is no current in A
(d) there is a constant current in the counter-clockwise direction in A



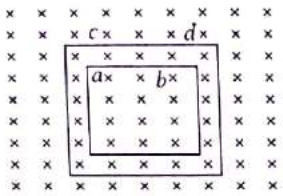
82. Same as above problem except, the coil A is made to rotate about a vertical axis (figure). No current flows in B, if A is at rest. The current in coil A, when the current in B (at $t = 0$) is counter-clockwise and the coil A is as shown at this instant, $t = 0$, is **[NCERT Exemplar]**
(a) constant current clockwise
(b) varying current clockwise
(c) varying current counter-clockwise
(d) constant current counter-clockwise



83. A rectangular loop of sides 10 cm and 5 cm with a cut is stationary between the pole pieces of an electromagnet.

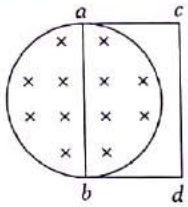
The magnetic field of the magnet is normal to the loop. The current feeding the electromagnet is reduced, so that the field decreased from its initial value of 0.2 T at the rate of $0.02\,\Omega$. If the cut is joined and the loop has a resistance of 2.0Ω, then the power dissipated by the loop as heat is
(a) 5 nW
(b) 4 nW
(c) 3 nW
(d) 2 nW

84. The figure shows certain wire segments joined together to form a coplanar loop. The loop is placed in a perpendicular magnetic field in the direction going into the plane of the figure. The magnitude of the field increases with time. I_1 and I_2 are the currents in the segments *ab* and *cd*. Then,



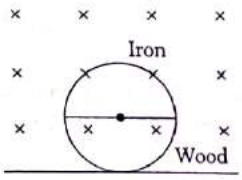
(a) $I_1 > I_2$
(b) $I_1 < I_2$
(c) I_1 is in the direction *ba* and I_2 is in the direction *cd*
(d) I_1 is in the direction *ab* and I_2 is in the direction *dc*

85. A uniform magnetic field B exists in a cylindrical region of radius 10 cm as shown in figure. A uniform wire of length 80 cm and resistance 4.0 Ω is bent into a square frame and is placed with one side along a diameter of the cylindrical region. If the magnetic field increases at a constant rate of 0.010 T/s, then the current induced in the frame



(a) 3.9×10^{-5} A (b) 4.0×10^{5} A
(c) 4.1×10^{-5} A (d) 3.9×10^{-4} A

86. Figure shows a circular wheel of radius 10.0 cm whose upper half, is made of iron and the lower half of wood as shown in figure. The two junctions are joined by an iron rod. A uniform magnetic field B of magnitude 2.0×10^{-4} T exists in the space above the central line as suggested by the figure. The wheel is set into pure rolling on the horizontal surface. If it takes 2.0 s for the iron part to come down and the wooden part to go up, then the average emf induced during this period is

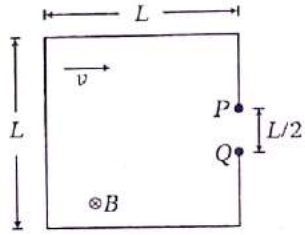


(a) 1.57×10^{-6} V (b) 1.5×10^{5} V
(c) 15.7×10^{-6} V (d) 1.55×10^{-6} V

87. A wire of length 10 cm translates in a direction making an angle of 60° with its length. The plane of motion is perpendicular to a uniform magnetic field of 1.0 T that exists in the space. The emf induced between the ends of the rod, if the speed of translation is 20 cm/s is

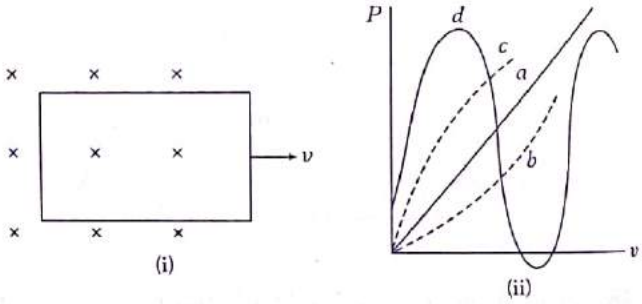
(a) 1.7×10^{-3} V (b) 17×10^{-3} V
(c) 0.17×10^{-3} V (d) 1.7×10^{-4} V

88. The loop shown moves with a velocity v in a uniform magnetic field of magnitude B, directed into the paper. The potential difference between P and Q is e



(i) $e = BLv / 2$
(ii) $e = BLv$
(iii) P is positive with respect to Q
(iv) Q is positive with respect to P
(a) (ii), (iii) (b) (i), (iv) (c) (i), (iii) (d) (ii), (iv)

89. A conducting loop being pulled out of magnetic field with a speed v. Which of the following plots may represent the power delivered by the pulling agent as a function of the speed v?

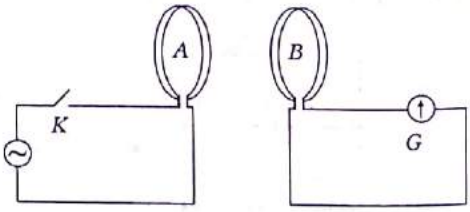


(a) *a* (b) *b* (c) *c* (d) *d*

90. A square loop of wire with side length 10 cm is placed at angle of 45° with a magnetic field that changes uniformly from 0.1 T to zero in 0.7s. The induced current in the loop (its resistance is 1Ω) is

(a) 1.0 mA (b) 2.5 mA (c) 3.5 mA (d) 4.0 mA

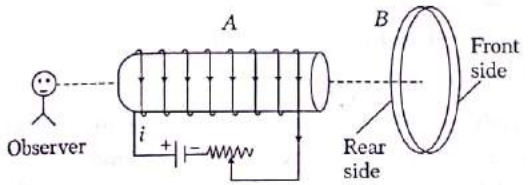
91. The diagram below shows two coils A and B placed parallel to each other at a very small distance. Coil A is connected to an AC supply. G is a very sensitive galvanometer. When the key is closed



(a) constant deflection will be observed in the galvanometer for 50 Hz supply
(b) visible small variations will be observed in the galvanometer for 50 Hz input
(c) oscillations in the galvanometer may be observed when the input AC voltage has a frequency of 1 to 2 Hz
(d) No variation will be observed in the galvanometer even when the input AC voltage is 1 or 2 Hz

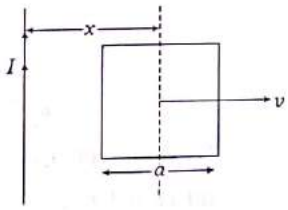
92. A magnet is dropped down an infinitely long vertical copper tube,
(a) the magnet moves with continuously increasing velocity and ultimately acquires a constant terminal velocity
(b) the magnet moves with continuously decreasing velocity and ultimately comes to rest
(c) the magnet moves with continuously increasing velocity but constant acceleration
(d) the magnet moves with continuously increasing velocity and acceleration

93. An aluminium ring B faces an electromagnet A. The current I through A can be altered



(a) whether I increases or decreases, B will not experience any force
(b) if I decreases, A will repel B
(c) if I increases, A will attract B
(d) if I increases, A will repel B

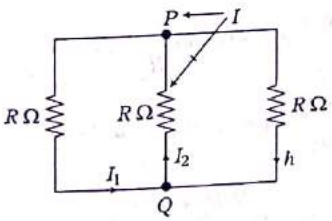
94. A conducting square frame of side a and a long straight wire carrying current I are located in the same plane as shown in the figure. The frame moves to the right with a constant velocity v.



The emf induced in the frame will be proportional to

(a) $\frac{1}{(2x-a)^2}$ (b) $\frac{1}{(2x+a)^2}$
(c) $\frac{1}{(2x-a)(2x+a)}$ (d) $1/x^2$

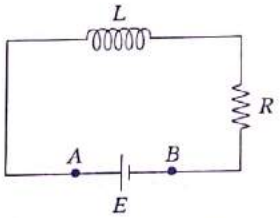
95. A rectangular loop has a sliding connector PQ of length l and resistance $R\,\Omega$ and it is moving with a speed v as shown. The set up is placed in a uniform magnetic field going into the plane of the paper. The three currents I_1, I_2 and I are



(a) $I_1 = I_2 = \frac{Blv}{6R}, I = \frac{Blv}{3R}$ (b) $I_1 = -I_2 = \frac{Blv}{R}, I = \frac{2Blv}{R}$
(c) $I_1 = I_2 = \frac{Blv}{3R}, I = \frac{2Blv}{3R}$ (d) $I_1 = I_2 = I = \frac{Blv}{R}$

96. Two concentric coils each of radius equal to 2π cm are placed at right angles to each other. 3 A and 4 A are the currents flowing in each coil, respectively. The magnetic induction in Wb / m^2 at the centre of the coils will be ($\mu_0 = 4\pi \times 10^{-7}$ Wb / A-m)
(a) 12×10^{-5} (b) 10^{-5}
(c) 5×10^{-5} (d) 7×10^{-5}

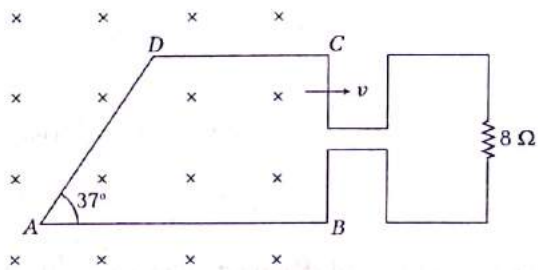
97. An inductor ($L = 100$ mH), a resistor ($R = 100\Omega$) and a battery ($E = 100$V) are initially connected in series as shown in the figure. After a long time, the battery is disconnected after short-circuiting the points A and B. The current in the circuit 1 ms after the short-circuit is



(a) eA (b) 0.1 A
(c) 1 A (d) $1/e$ A

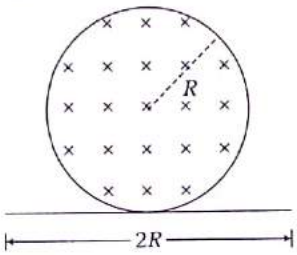
98. Three solenoid coils of same dimension, same number of turns and same number of layers of windings are taken. Coil 1 with inductance L_1 was wound using a wire of resistance 11 Ω / m, coil 2 with inductance L_2 was wound using the similar wire but the direction of winding was reversed in each layer, coil 3 with inductance L_3 was wound using a superconducting wire. The self-inductance of the coils L_1, L_2 and L_3 are
(a) $L_1 = L_2 = L_3$
(b) $L_1 = L_2, L_3 = 0$
(c) $L_1 = L_3, L_2 = 0$
(d) $L_1 > L_2 > L_3$

99. The loop $ABCD$ is moving with velocity v towards right. The magnetic field is 4 T. The loop is connected to a resistance of 8 Ω. If steady current of 2 A flows in the loop, then value of v, if loop has resistance of 4 Ω, is
(Given, $AB = 30$ cm, $AD = 30$ cm, $\sin 37° = \frac{3}{5}$)



(a) $\frac{50}{3}$ ms^{-1} (b) 20 ms^{-1}
(c) 10 ms^{-1} (d) $\frac{100}{3}$ ms^{-1}

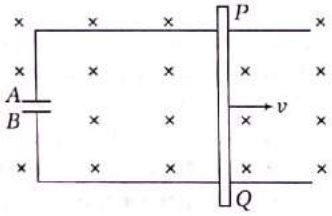
100. A uniform but time varying magnetic field is present in a circular region of radius R. The magnetic field is perpendicular and into the plane of the loop and the magnitude of field is increasing at a constant rate α. There is a straight conducting rod of length $2R$ placed as shown in figure.



The magnitude of induced emf across the rod is

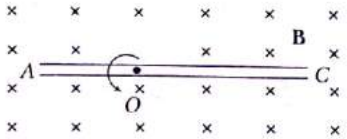
(a) $\pi R^2\alpha$ (b) $\frac{\pi R^2\alpha}{2}$ (c) $\frac{R^2\alpha}{\sqrt{2}}$ (d) $\frac{\pi R^2\alpha}{4}$

101. A conducting rod PQ of length $L = 1.0$ m is moving with a uniform speed $v = 2\ \text{ms}^{-1}$ in a uniform magnetic field $B = 4.0$ T directed into the paper. A capacitor of capacity $C = 10\ \mu\text{F}$ is connected as shown in figure. Then,



(a) $q_A = +80\ \mu\text{C}$ and $q_B = -80\ \mu\text{C}$
(b) $q_A = -80\ \mu\text{C}$ and $q_B = +80\ \mu\text{C}$
(c) $q_A = 0 = q_B$
(d) charge stored in the capacitor increases exponentially with time

102. A conducting rod AC of length $4l$ is rotated about a point O in a uniform mangnetic field **B** directed into the paper. $AO = l$ and $OC = 3l$. Then,



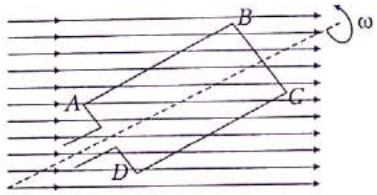
(a) $V_A - V_O = \frac{B\omega l^2}{2}$ (b) $V_O - V_C = \frac{7}{2}B\omega l^2$

(c) $V_A - V_C = 4B\omega l^2$ (d) $V_C - V_O = \frac{9}{2}B\omega l^2$

103. The current in an L-R circuit builds upto $\frac{3}{4}$th of its steady state value in 4 s. The time constant of this circuit is

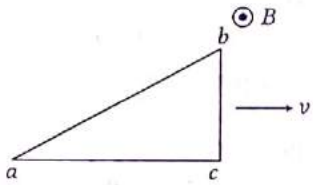
(a) $\frac{1}{\ln 2}$ s (b) $\frac{2}{\ln 2}$ s (c) $\frac{3}{\ln 2}$ s (d) $\frac{4}{\ln 2}$ s

104. A rectangular coil $ABCD$ which is rotated at a constant angular velocity about an horizontal as shown in the figure. The axis of rotation of the coil as well as the magnetic field B are horizontal. Maximum current will flow in the circuit when the plane of the coil is



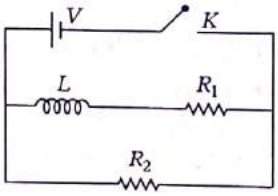
(a) inclined at 30° to the magnetic field
(b) perpendicular to the magnetic field
(c) inclined at 45° to the magnetic field
(d) parallel to the magnetic field

105. A right-angled triangle abc, made from a metallic wire, moves at a uniform speed v in its plane as shown in figure. A uniform magnetic field B exists in the perpendicular direction. The emf induced in the loop abc, in the segment bc, in the segment ac and in the segment ab are



(a) zero; $vB(bc)$,+ve at c, zero, $vB(bc)$,+ve at a
(b) $vB(bc)$,+ve at c, zero; zero; $vB(bc)$,+ve at a
(c) zero; zero; $vB(bc)$,+ve at c, $vB(bc)$,+ve at a
(d) $vB(bc)$,+ve at c, $vB(bc)$,+ve at a; zero; zero

106. In the circuit shown below, the key K is closed at $t = 0$. The current through the battery is



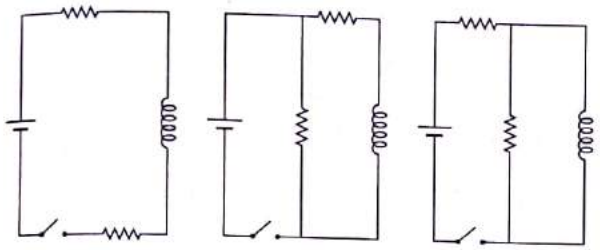
(a) $\frac{V(R_1+R_2)}{R_1R_2}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$

(b) $\frac{V(R_1+R_2)}{\sqrt{R_1^2+R_2^2}}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$

(c) $\frac{V}{R_2}$ at $t = 0$ and $\frac{V(R_1+R_2)}{R_1R_2}$ at $t = \infty$

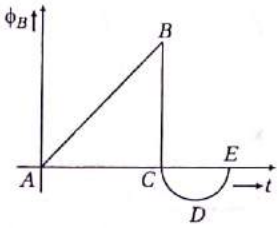
(d) $\frac{V}{R_2}$ at $t = 0$ and $\frac{V(R_1+R_2)}{\sqrt{R_1^2+R_2^2}}$ at $t = \infty$

107. The figure shows three circuits with identical batteries, inductors and resistors. Rank the circuits, in the decreasing order, according to the current through the battery (i) just after the switch is closed and (ii) a long time later



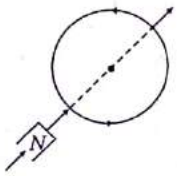
(a) (i) $i_2 > i_3 > i_1$, $(i_1 = 0)$ (ii) $i_2 > i_3 > i_1$
(b) (i) $i_2 < i_3 < i_1$, $(i_1 \neq 0)$ (ii) $i_2 > i_3 > i_1$
(c) (i) $i_2 = i_3 = i_1$, $(i_1 = 0)$ (ii) $i_2 < i_3 < i_1$
(d) (i) $i_2 = i_3 > i_1$, $(i_1 \neq 0)$ (ii) $i_2 > i_3 > i_1$

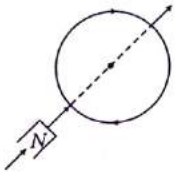
108. The graph shows the variation in magnetic flux $\phi(t)$ with time through a coil. Which of the statements given below is not correct?

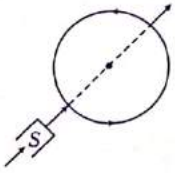


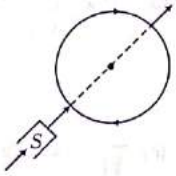
(a) There is a change in the direction as well as magnitude of the induced emf between B and D
(b) The magnitude of the induced emf is maximum between B and C
(c) There is a change in the direction as well as magnitude of induced emf between A and C
(d) The induced emf is not zero at B

109. Which of the following figure correctly depicts the Lenz's law. The arrows show the movement of the labelled pole of a bar magnet into a closed circular loop and the arrows on the circle show the direction of the induced current

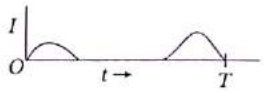
(a)


(b)


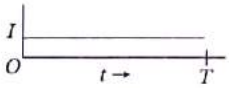
(c)


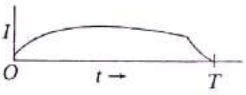
(d)


110. A metallic ring is dropped down, keeping its plane perpendicular to a constant and horizontal magnetic field. The ring enters the region of magnetic field at $t = 0$ and completely emerges out at $t = T$ sec. The current in the ring varies as

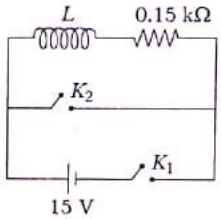
(a)


(b)

(c)


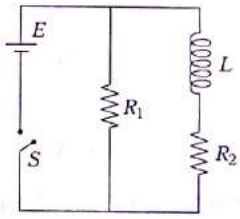
(d)


111. An inductor ($L = 0.03$ H) and a resistor ($R = 0.15\Omega$) are connected in series to a battery of 15 V emf in a circuit shown below. The key K_1 has been kept closed for a long time. Then, at $t = 0$, K_1 is opened and key K_2 is closed simultaneously. At $t = 1$ m/s the current in the circuit will be ($e^5 \cong 150$)



(a) 100 mA (b) 67 mA (c) 6.7 mA (d) 0.67 mA

112. An inductor of inductance $L = 400$ mH and resistors of resistances $R_1 = 2\Omega$ and $R_2 = 2\Omega$ are connected to a battery of emf 12 V as shown in the figure. The internal resistance of the battery is negligible. The switch S is closed at $t = 0$. The potential drop across L as a function of time is

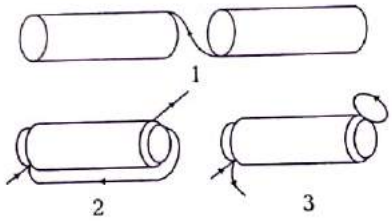


(a) $6e^{-5t}$ V (b) $\frac{12}{t}e^{-3t}$ V (c) $6\left(1 - e^{\frac{-1}{0.2}}\right)$ V (d) $12e^{-5t}$ V

113. There are two solenoids of same length and inductance L but their diameters differ to the extent that one can just fit into the other. They are connected in three different ways in series.

(1) They are connected in series but separated by large distance.
(2) They are connected in series with one inside the other and senses of the turns coinciding.
(3) Both are connected in series with one inside the other with senses of the turns opposite,

as depicted in figures 1, 2 and 3, respectively. The total inductance of the solenoids in each of the case 1, 2 and 3 are respectively.



(a) 0, $4L_0, 2L_0$
(b) $4L_0, 2L_0, 0$
(c) $2L_0, 0, 4L_0$
(d) $2L_0, 4L_0, 0$

114. A rectangular loop of length l and breadth b is placed at distance of x from infinitely long wire carrying current i such that the direction of current is parallel to breadth. If the loop moves away from the current wire in a direction perpendicular to it with a velocity v, then magnitude of the emf in the loop is (μ_0 = permeability of free space)

(a) $\frac{\mu_0 iv}{2\pi x}\left(\frac{l+b}{b}\right)$ (b) $\frac{\mu_0 i^2 v}{4\pi^2 x}\log\left(\frac{b}{l}\right)$

(c) $\frac{\mu_0 ilbv}{2\pi x(l+x)}$ (d) $\frac{\mu_0 ilbv}{2\pi}\log\left(\frac{x+l}{x}\right)$

115. A circular coil of one turn of radius 5.0 cm is rotated about a diameter with a constant angular speed of 80 revolutions per minute. A uniform magnetic field $B = 0.01$ T exists in a direction perpendicular to the axis of rotation, the maximum emf induced, the average emf induced in the coil over a long period and the average of the squares of emf induced over a long period is

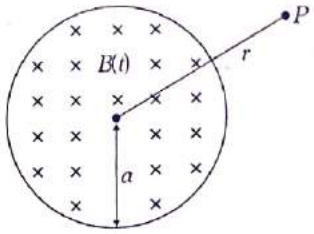
(a) 6.4×10^{-4} V, zero, 2.2×10^{-7} V^2
(b) 6.6×10^{-4} V, zero, 2.0×10^{-7} V^2
(c) 6.8×10^{-4} V, zero 2.5×10^{-7} V^2
(d) 6.4×10^{-4} V, zero 2.0×10^{-6} V^2

116. A non-conducting ring having charge q uniformly distributed over its circumference is placed on a rough horizontal surface. A vertical time varying magnetic field $B = 4t^2$ is switched on at time $t = 0$. Mass of the ring is m and radius is R.

The ring starts rotating after 2 s, the coefficient of friction between the ring and the table is

(a) $\frac{4qmR}{g}$ (b) $\frac{2qmR}{g}$ (c) $\frac{8qR}{mg}$ (d) $\frac{qR}{2mg}$

117. A uniform but time-varying magnetic field $B(t)$ exists in a cylindrical region of radius a and is directed into the plane of the paper as shown. The magnitude of the induced electric field at point P at a distance r from the centre of the circular region

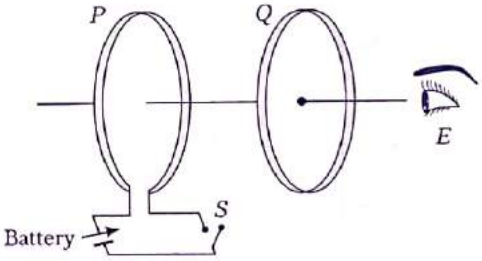


(a) is zero (b) decreases as $\frac{1}{r}$

(c) increases as r (d) decreases as $\frac{1}{r^2}$

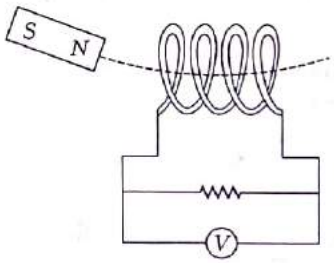
118. As shown in the figure, P and Q are two co-axial conducting loops separated by some distance. When the switch S is closed, a clockwise current I_P flows in P (as seen by E) and an induced current I_{Q_1} flows in Q. The switch remains closed for a long time. When S is opened, a current I_{Q_2} flows in Q.

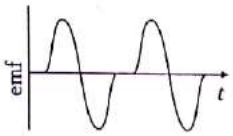
Then, the directions of I_{Q_1} and I_{Q_2} (as seen by E) are

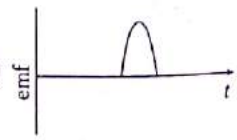


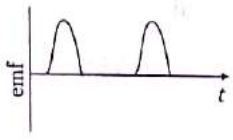
(a) respectively clockwise and anti-clockwise
(b) both clockwise
(c) both anti-clockwise
(d) respectively anti-clockwise and clockwise

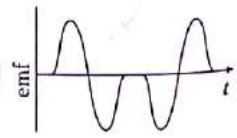
119. A magnet is made to oscillate with a particular frequency, passing through a coil as shown in figure. The time variation of the magnitude of emf generated across the coil during one cycle.



(a) 

(b) 

(c) 

(d) 

B Medical entrance special format questions

Assertion and reason

Directions (Q. Nos. 1-17) *These questions consists of two statements each printed as Assertion and Reason. While answering these questions you are required to choose anyone of the following five responses.*

(a) If both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If Assertion is false but Reason is true.
(e) If Both Assertion and Reason are false.

1. **Assertion** Magnetic flux and the electric flux have the same units.

   Reason Flux passing through a surface gives an idea about the field lines crossing that surface.

2. **Assertion** In the phenomenon of mutual induction, self-induction of each of the coils persists.

   Reason Self-induction arises when strength of current in same coil changes. In mutual induction, current is changing in both the individual coils.

3. **Assertion** When two magnets are brought closer to each other, then they will always repel each other.

   Reason According to Lenz's law induced effects always opposes the cause.

4. **Assertion** Mutual inductance of two coils depends on the distance between the coils and their orientation.

   Reason It does not depend on the magnetic material filled between the coils.

5. **Assertion** Coefficient of self-induction of an inductor depends upon the rate of change of current passing through it.

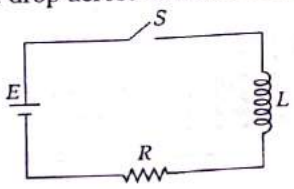
   Reason From, $e = -L\dfrac{di}{dt}$

   We can see that, $L = \dfrac{-e}{(di/dt)}$ or $L \propto \dfrac{1}{(di/dt)}$

6. **Assertion** If a magnet is brought closer to a current carrying loop along its axis, then current always decreases in the loop.

   Reason Magnet is repelled by the loop.

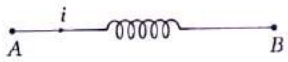
7. **Assertion** In the figure, just after closing the switch the potential drop across inductor is maximum.



   Reason The rate of change of current just after closing the switch is maximum.

8. **Assertion** If current shown in the figure is increasing, then

$$V_A > V_B$$



   Reason If current passing through an inductor is constant, then both ends of the inductor are at same potential.

9. **Assertion** If two inductors are in parallel, then current in them distributes in inverse ratio of their inductance.

   Reason In parallel, potential difference remains constant.

10. **Assertion** An induced emf of 2 V is developed in a circular loop, if current in the loop is changed at a rate of 4 As^{-1}. If 4 A of current is passed through this loop, then flux linked with this coil will be 2 Wb.

    Reason Flux linked with the coil is

$$|\phi| = \left|\frac{e}{di/dt}\right| i$$

11. **Assertion** If a straight wire is moved in a magnetic field, no emf will be induced across its two ends as the circuit is not closed.

    Reason Since, the circuit is not closed, induced current will be zero.

12. **Assertion** If a loop is placed in a non-uniform (with respect to position) magnetic field, then induced emf is produced in the loop.

    Reason In a non-uniform magnetic field, magnetic flux passing through the loop will change. Therefore, induced emf is produced.

13. **Assertion** If current passing through a circular loop is doubled, then magnetic flux linked with the circular loop will also become two times.

    Reason No flux will link through the coil by its own current.

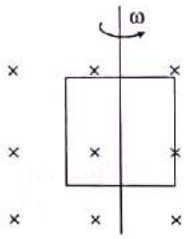
14. **Assertion** Two concentric conducting rings of different radii are placed in space. The mutual inductance of both the rings is maximum, if the rings are coplanar.

    Reason For two co-axial conducting rings of different radii, the magnitude of magnetic flux in one ring due to current in other ring is maximum when both rings are coplanar.

15. Assertion The induced emf and current will be same in two identical loops of copper and aluminium when rotated with same speed in the same magnetic field.

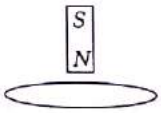
Reason Induced emf is proportional to rate of change of magnetic field while induced current depends on resistance of wire.

16. Assertion A conducting loop is rotated in a uniform magnetic field with constant angular velocity ω as shown in figure. At time $t = 0$, plane of the loop is perpendicular to the magnetic field. Induced emf produced in the loop is maximum when plane of loop is parallel to magnetic field.



Reason When plane of loop is parallel to magnetic field, then magnetic flux passing through the loop is zero.

17. Assertion A magnet is dropped along the axis of a circular conducting loop as shown in figure. Then, acceleration of magnet is always less than g.



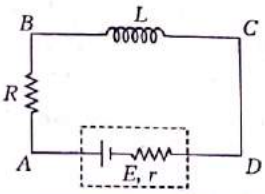
Reason When magnet is above the loop, then it will repel the magnet and when it is below the loop, then it will attract the magnet.

Match the columns

1. Match the following columns.

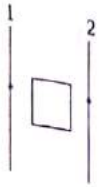
	Column I		Column II
A.	Tesla	p.	$[ML^2A^{-2}T^{-2}]$
B.	Weber	q.	$[MLA^{-2}T^{-1}]$
C.	Weber m^{-2}	r.	$[MA^{-1}T^{-2}]$
D.	Henry	s.	None

2. In the circuit shown in figure, $E = 10$ V, $r = 1\ \Omega$, $R = 4\ \Omega$ and $L = 5$ H. The circuit is closed at time $t = 0$. Then, match the following two columns.



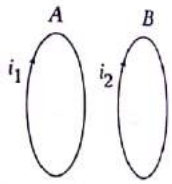
	Column I		Column II
A.	V_{AD} at $t = 0$	p.	4 V
B.	V_{BC} at $t = 0$	q.	8 V
C.	V_{AB} at $t = \infty$	r.	0 V
D.	V_{AD} at $t = \infty$	s.	10 V

3. A square loop is symmetrically placed between two infinitely long current carrying wires in the same direction. Magnitude of currents in both the wires are same. Now, match the following two columns.



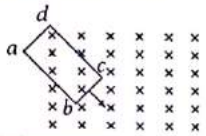
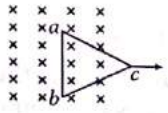
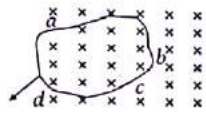
	Column I		Column II
A.	Loop is moved towards right	p.	Induced current in the loop is clockwise.
B.	Loop is moved towards left	q.	Induced current in the loop is anti-clockwise.
C.	Wire-1 is moved towards left	r.	Induced current in the loop is zero.
D.	Wire-2 is moved towards right	s.	Induced current in the loop is non-zero.

4. Two co-axial identical circular current carrying loops are shown in figure, currents in them are in the same directions. Now, match the following two columns.



	Column I		Column II
A.	Current i_1 is increased	p.	Loops will attract each other
B.	Current i_2 is decreased	q.	Loops will repel each other
C.	Loop-1 is moved towards loop-2	r.	Current i_1 will increase
D.	Loop-2 is moved away from loop-1	s.	Current i_2 will increase

5. Match the items of Column I with those of Column II.

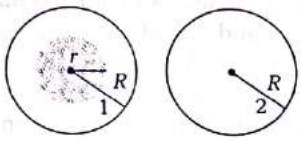
	Column I (Planar loops of different shapes)		Column II (Direction of induced current)
A.		p.	*b a c b*
B.		q.	*c d a b c*
C.		r.	*b c d a b*

C. Medical entrances' gallery

(Collection of questions asked in NEET & various medical entrance exams)

1. A long solenoid has 1000 turns. When a current of 4 A flows through it, then the magnetic flux linked with each turn of the solenoid is 4×10^{-3} Wb. The self-inductance of the solenoid is **[NEET 2016]**
   (a) 3 H (b) 2 H
   (c) 1 H (d) 4 H

2. A uniform magnetic field is restricted within a region of radius r. The magnetic field changes with time at a rate $\frac{dB}{dt}$. Loop 1 of radius $R > r$ encloses the region r and loop 2 of radius R is outside the region of magnetic field as shown in the figure. Then, the emf generated is **[NEET 2016]**



   (a) Zero in loop 1 and zero in loop 2
   (b) $-\frac{dB}{dt}\pi r^2$ in loop 1 and $-\frac{dB}{dt}\pi r^2$ in loop 2
   (c) $-\frac{dB}{dt}\pi R^2$ in loop 1 and zero in loop 2
   (d) $-\frac{dB}{dt}\pi r^2$ in loop 1 and zero in loop 2

3. The self-inductance of a coil having 500 turns is 50 mH. The magnetic flux through the cross-sectional area of the coil while current through it is 8 mA is found to be **[AIIMS 2015]**
   (a) 4×10^{-4} Wb (b) 0.04 Wb
   (c) 4 μWb (d) 40 mWb

4. The phase difference between the flux linked with a coil rotating in a uniform magnetic field and induced emf produced in it is **[AIIMS 2015]**
   (a) $\pi/2$ (b) $\pi/3$
   (c) $-\pi/6$ (d) π

5. A rectangular copper coil is placed in a uniform magnetic field of induction 40 mT with its plane perpendicular to the field. The area of the coil is shrinking at a constant rate of $0.5\ m^2s^{-1}$. The emf induced in the coil is **[EAMCET 2015]**
   (a) 10 mV (b) 20 mV (c) 80 mV (d) 40 mV

6. Changing magnetic fields can set up current loops in nearby metal bodies and the currents are called as **[Kerala CEE 2015]**
   (a) eddy currents (b) flux currents
   (c) alternating currents (d) leaking currents
   (e) wattless currents

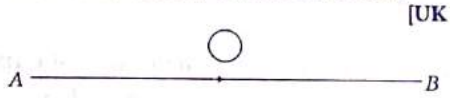
7. A rod of 10 cm length is moving perpendicular to uniform magnetic field of intensity $5\times 10^{-4}\ Wbm^{-2}$. If the acceleration of the rod is $5\ ms^{-1}$, then the rate of increase of induced emf is **[Guj. CET 2015]**
   (a) $25\times 10^{-4}Vs^{-1}$ (b) $2.5\times 10^{-4}Vs^{-1}$
   (c) $20\times 10^{-4}Vs^{-1}$ (d) $20\times 10^{-4}Vs^{-1}$

8. The identical loops of copper and aluminium are moving with the same speed in a magnetic field. Which of the following statements is true? **[CG PMT 2015]**
   (a) Induced emf and induced current are same in both loops
   (b) Induced emf remains same, induced current changes in both loops
   (c) Induced emf changes but induced current remains same in both loops
   (d) Induced emf will be more in aluminium loop

9. A straight conductor 0.1 m long moves in a uniform magnetic field 0. 1 T. The velocity of the conductor is $15\ ms^{-1}$ and is directed perpendicular to the field. The emf induced between the two ends of the conductor is **[WB JEE 2015]**
   (a) 0.10 V (b) 0.15 V (c) 1.50 V (d) 15 V

10. The initial rate of increase of current, when a battery of emf 6 V is connected in series with an inductance of 2 H and resistance 12 Ω, is **[CG PMT 2015]**
    (a) $0.5\ As^{-1}$ (b) $1\ As^{-1}$ (c) $3\ As^{-1}$ (d) $3.5\ As^{-1}$

11. The current flows from A to B as shown in the figure. What is the direction of current in circle? **[UK PMT 2014]**



    (a) Clockwise (b) Anti-clockwise
    (c) Straight line (d) None of these

12. A very small circular loop of radius a is initially (at $t=0$) coplanar and concentric with a much larger fixed circular loop of radius b. A constant current I flows in the larger loop. The smaller loop is rotated with a constant angular speed ω about the common diameter. The emf induced in the smaller loop as a function of time t is **[WB JEE 2014]**
    (a) $\frac{\pi a^2\mu_0 I}{2b}\omega\cos\omega t$ (b) $\frac{\pi a^2\mu_0 I}{2b}\omega\sin\omega^2 t^2$
    (c) $\frac{\pi a^2\mu_0 I}{2b}\omega\sin\omega t$ (d) $\frac{\pi a^2\mu_0 I}{2b}\omega\sin^2\omega t$

13. A straight conductor of length 0.4 m is moved with a speed of $7\ ms^{-1}$ perpendicular to a magnetic field of intensity $0.9\ Wbm^{-2}$. The induced emf across the conductor is **[UK PMT 2014]**
    (a) 5.04 V (b) 1.26 V (c) 2.52 V (d) 25.2 V

14. The current in a self-inductance $L = 40$ mH is to be increased uniformly from 1A to 11 A in 4 ms. The emf induced in the inductor during the process is **[UK PMT 2014]**

(a) 100 V (b) 0.4 V (c) 40 V (d) 440 V

15. The induced emf in a coil of 10 H inductance in which current varies from 9 A to 4 A in 0.2 s is **[J & K CET 2013]**

(a) 200 V (b) 250 V
(c) 300 V (d) 350 V

16. A conductor of length 5 cm is moved parallel to itself with a speed of 2 m/s, perpendicular to a uniform magnetic field of 10^{-3} Wb/m^3. The induced emf generated is **[J & K CET 2013]**

(a) 2×10^{-3}V (b) 1×10^{-3}V
(c) 1×10^{-4}V (d) 2×10^{-4}V

17. Two identical circular coils A and B are kept on a horizontal tube side by side without touching each other. If the current in the coil A increases with time, in response, the coil B **[Karnataka CET 2013]**

(a) is attracted by A (b) remains stationary
(c) is repelled (d) rotates

18. A rectangular coil of 100 turns and size 0.1 m× 0.05 m is placed perpendicular to a magnetic field of 0.1 T. If the field drops to 0.05 T in 0.05 s, the magnitude of the emf induced in the coil is **[Karnataka CET 2013]**

(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) $\sqrt{0.6}$ (d) $\sqrt{6}$

19. Electromagnetic induction is not used in **[Kerala CET 2013]**

(a) speedometer (b) transformer
(c) AC generator (d) induction furnace
(e) room heater

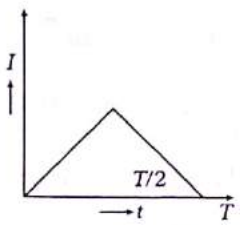
20. Two coils have the mutual inductance of 0.05 H. The current changes in the first coil as $I = I_0 \sin \omega t$, where $I_0 = 1$ A and $\omega = 100\pi$ rad/s. The maximum emf induced in secondary coil is **[UP CPMT 2013]**

(a) 2.5 V (b) 10 V (c) 6πV (d) 5πV

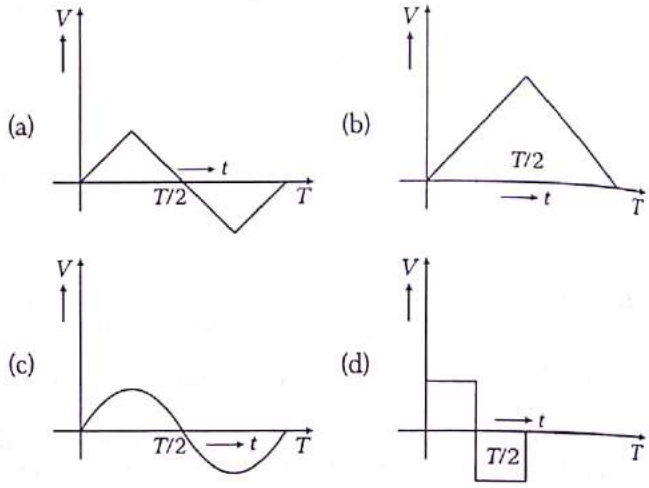
21. The magnetic flux linked with a circuit of resistance 100 Ω increases from 10 to 60 Wb. The amount of induced charge that flows in the circuit is (in coulomb) **[UP CPMT 2012]**

(a) 0.5 (b) 5 (c) 50 (d) 100

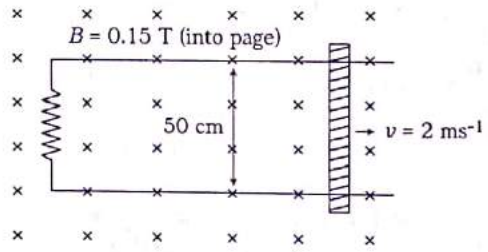
22. The current (I) in the inductance is varying with time according to the plot shown in figure.



Which one of the following is the correct variation of voltage with time in the coil? **[CBSE AIPMT 2012]**



23. As shown in the figure, a metal rod makes contact with a partial circuit and completes the circuit. The circuit area is perpendicular to a magnetic field with $B = 0.15$ T. If the resistance of the total circuit is 3 Ω, the force needed to move the rod as indicated with a constant speed of 2 ms^{-1} will be equal to **[AMU 2012]**



(a) 3.75×10^{-3} N
(b) 2.75×10^{-3} N
(c) 6.57×10^{-4} N
(d) 4.36×10^{-4} N

24. If emf induced in a coil is 2 V by changing the current in it from 8 A to 6 A in 2×10^{-3} s, then the coefficient of self-induction is **[AFMC 2012]**

(a) 2×10^{-3} H (b) 10^{-3} H
(c) 0.5×10^{-3} H (d) 4×10^{-3} H

25. A wire of length 1m is moving at a speed of 2 ms^{-1} perpendicular to its length in a homogeneous magnetic field of 0.5T. If the ends of the wire are joined to a circuit of resistance 6 Ω, then the rate at which work is being done to keep the wire moving at constant speed is **[JCECE 2012]**

(a) 1 W (b) $\frac{1}{3}$ W
(c) $\frac{1}{6}$ W (d) $\frac{1}{12}$ W

Answers

Check point 6.1

1. (c)	2. (b)	3. (d)	4. (d)	5. (a)	6. (d)	7. (b)	8. (b)	9. (b)	10. (b)
11. (b)	12. (c)	13. (c)	14. (d)	15. (a)					

Check point 6.2

1. (b)	2. (c)	3. (b)	4. (d)	5. (b)	6. (b)	7. (c)	8. (b)	9. (d)	10. (c)
11. (c)	12. (c)	13. (c)	14. (d)	15. (a)					

Check point 6.3

1. (b)	2. (a)	3. (d)	4. (b)	5. (a)	6. (c)	7. (d)	8. (d)	9. (d)	10. (a)
11. (a)	12. (d)	13. (b)	14. (a)	15. (c)					

Check point 6.4

1. (a)	2. (b)	3. (a)	4. (c)	5. (d)	6. (a)	7. (a)	8. (a)	9. (c)	10. (b)
11. (b)	12. (c)	13. (a)	14. (a)	15. (d)					

A. Taking it together

1. (d)	2. (b)	3. (b)	4. (b)	5. (b)	6. (a)	7. (c)	8. (c)	9. (d)	10. (b)
11. (d)	12. (c)	13. (a)	14. (c)	15. (b)	16. (d)	17. (a)	18. (c)	19. (d)	20. (d)
21. (a)	22. (b)	23. (d)	24. (d)	25. (d)	26. (d)	27. (c)	28. (a)	29. (d)	30. (c)
31. (c)	32. (c)	33. (d)	34. (b)	35. (c)	36. (a)	37. (c)	38. (d)	39. (b)	40. (b)
41. (d)	42. (a)	43. (d)	44. (c)	45. (a)	46. (a)	47. (a,b,c)	48. (c)	49. (c)	50. (b)
51. (b)	52. (b)	53. (d)	54. (d)	55. (d)	56. (b)	57. (b)	58. (b)	59. (b)	60. (c)
61. (c)	62. (d)	63. (a)	64. (d)	65. (d)	66. (a)	67. (b)	68. (c)	69. (b)	70. (b)
71. (a,c)	72. (d)	73. (a,c)	74. (b)	75. (a)	76. (b)	77. (c)	78. (d)	79. (b)	80. (b)
81. (d)	82. (a)	83. (a)	84. (d)	85. (a)	86. (a)	87. (b)	88. (c)	89. (b)	90. (a)
91. (c)	92. (a)	93. (d)	94. (c)	95. (c)	96. (c)	97. (d)	98. (b)	99. (d)	100. (d)
101. (a)	102. (c)	103. (b)	104. (d)	105. (a)	106. (c)	107. (a)	108. (d)	109. (a)	110. (b)
111. (d)	112. (d)	113. (d)	114. (c)	115. (b)	116. (c)	117. (b)	118. (d)	119. (a)	

B. Medical entrance special format questions

Assertion and reason

1. (d)	2. (b)	3. (d)	4. (c)	5. (d)	6. (b)	7. (a)	8. (b)	9. (a)	10. (a)
11. (d)	12. (e)	13. (c)	14. (a)	15. (d)	16. (b)	17. (a)			

Match the columns

1. (A $\rightarrow$ r; B $\rightarrow$ s; C $\rightarrow$ r; D $\rightarrow$ p)
2. (A $\rightarrow$ s; B $\rightarrow$ s; C $\rightarrow$ q; D $\rightarrow$ q)
3. (A $\rightarrow$ p, s; B $\rightarrow$ q, s; C $\rightarrow$ p, s; D $\rightarrow$ q, s)
4. (A $\rightarrow$ q; B $\rightarrow$ p; C $\rightarrow$ q, r; D $\rightarrow$ p,s)
5. (A $\rightarrow$ r; B $\rightarrow$ p; C $\rightarrow$ q)

C. Medical entrances' gallery

1. (c)	2. (c)	3. (a)	4. (a)	5. (b)	6. (a)	7. (b)	8. (b)	9. (b)	10. (c)
11. (d)	12. (c)	13. (c)	14. (a)	15. (b)	16. (c)	17. (c)	18. (*)	19. (e)	20. (d)
21. (a)	22. (d)	23. (a)	24. (a)	25. (c)					

*None of the option is correct.

Hints and explanations

Check point 6.1

1. (*c*) Magnetic flux linked with a vector area **A** in a uniform magnetic field **B** is given by $\phi = \mathbf{B} \cdot \mathbf{A}$

2. (*b*) Magnetic flux is a scalar quantity. Its SI unit is weber.

3. (*d*) Dimensional formula of magnetic flux $= [ML^2 T^{-2} A^{-1}]$

4. (*d*) Magnitude of induced emf, $e = -\frac{d\phi}{dt}$

$$= -\frac{d}{dt}(3t^2 + 4t + 9) = -(6t + 4)$$

$$= -[6 \times (2) + 4] = -16$$

$\therefore \quad |e| = 16$ V

5. (*a*) As, $|e| = \frac{\Delta\phi}{\Delta t} = \frac{4B_0 A_0 - B_0 A_0}{t} = \frac{3B_0 A_0}{t}$

6. (*d*) Induced emf, $e = \left|\frac{d\phi}{dt}\right| = |12t - 5|$

At $\quad t = 0.25$ s $\Rightarrow e = 2$ V

$\therefore$ Induced current, $i = \frac{e}{R} = 0.2$ A

8. (*b*) Magnetic flux, $\phi = (5t^2 - 4t + 1)$ Wb

$\therefore \quad \frac{d\phi}{dt} = 10t - 4 \text{ Wbs}^{-1}$

The induced emf, $e = \frac{-d\phi}{dt} = -(10t - 4)$

At, $t = 0.2$ s, $e = -(10 \times 0.2 - 4) = 2$ V

The induced current, $I = \frac{e}{R} = \frac{2\text{ V}}{10\,\Omega} = 0.2$ A

9. (*b*) As, charge, $|\Delta q| = \frac{\Delta\phi}{R} = \frac{(2)\left(\frac{\pi}{4}\right)(0.2)^2}{0.01} = 6.28$ C

10. (*b*) As, charge, $\Delta Q = \frac{\Delta\phi}{R} = \frac{(10-2)}{2} = 4$ C

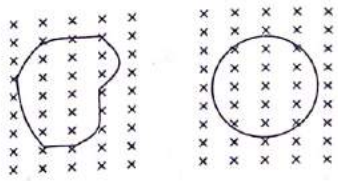
12. (*c*) According to Lenz's law, the induced current will be in such a direction, so that it opposes the change due to which it is produced.

13. (*c*) Lenz's law is based upon law of conservation of energy.

14. (*d*) According to Lenz's law, it will be anti-clockwise.

15. (*a*) Due to change in the shape of the loop, the magnetic flux linked with the loop increases. Hence, current is induced in the loop in such a direction that it opposes the increases in flux.

Therefore, induced current flows in the anti-clockwise direction.



Check point 6.2

1. (*b*) The potential difference between two ends of conducting rods will be Bvl.

2. (*c*) When a wire of length l moves with velocity v, perpendicular to a magnetic field B, then the induced emf is produced. The magnitude of induced emf is given by $|\varepsilon| = Blv$.
Given $l = 50$ cm $= 0.5$ m, $v = 300$ m/min $= 5$ m/s

$$|\varepsilon| = 2\text{V or } B = \frac{|\varepsilon|}{lv} = \frac{2}{0.5 \times 5} = 0.8\text{ T}$$

3. (*b*) Induced emf $= Blv = 0.3 \times 10^{-4} \times 10 \times 5$
$= 1.5 \times 10^{-3}$ V $= 1.5$ mV

4. (*d*) Induced emf $= B \times v \times l$
$= 5.0 \times 10^{-5} \times 1.50 \times 2 = 10.0 \times 10^{-5} \times 1.5$
$= 1.5 \times 10^{-4} = 0.15$ mV

5. (*b*) When a conductor lying along the magnetic north-south, then moves eastwards, it will cut vertical component of B_0. So, induced emf,

$$e = vB_v\, l = v(B_0 \sin\delta l) = B_0 lv \sin\delta$$

6. (*b*) Induced emf, $e = B \cdot \frac{dS}{dt} = 2\pi r B \cdot \frac{dr}{dt}$

where, $\frac{dr}{dt}$ is the rate at which radius of loop is shrinking,

7. (*c*) Two circular loops carrying current in the same direction will attract each other. If they are now separated, then current will increase in both in accordance with Lenz's law.

8. (*b*) Induced emf, $e_0 = \omega NBS = (2\pi\nu)NB(\pi r^2) = 2 \times \pi^2 \nu NBr^2$
$= 2 \times (3.14)^2 \times \frac{1800}{60} \times 4000 \times 0.5 \times 10^{-4} \times (7 \times 10^{-2})^2 = 0.57$ V

10. (*c*) Here, $l = 0.1$ m, $v = 1$ ms^{-1}

$$I = 50 \text{ A and } B = 1.25 \text{ mT} = 1.25 \times 10^{-3} \text{ T}$$

The induced emf is, $\varepsilon = Blv$.
The mechanical power is,
$P = \varepsilon I = BlvI = 1.25 \times 10^{-3} \times 0.1 \times 1 \times 50$
$= 6.25 \times 10^{-3}$ W $= 6.25$ mW

11. (c) Induced emf, when rod rotates in a vertical plane perpendicular to magnetic field is given as

$$e = \frac{1}{2}Bl^2\omega = \frac{1}{2} \times 0.3 \times 2^2 \times 100 = 60 \text{ V}$$

13. (c) $E = E_m \cos 45°$,

where, E_m is the maximum value of induced emf.

$$\therefore \quad E = \frac{E_m}{\sqrt{2}}$$

Check point 6.3

1. (b) $U = \frac{1}{2}Li^2$, $[L] = \left[\frac{U}{i^2}\right]$

and $$L = \frac{e}{di/dt}$$

$$L = \phi_B / i$$

$$\Rightarrow \quad [L] = [\phi/i][L] = [edt/di] = \frac{\text{volt-second}}{\text{ampere}}$$

2. (a) We have, $N\phi_B = Li$

Self-inductance, $L = \frac{N\phi_B}{i} = \frac{500 \times 4 \times 10^{-3}}{2} = 1\text{H}$

3. (d) Self-inductance of coil,

$$|e| = L\frac{di}{dt}$$

$$\therefore \quad 10 = L \times \frac{10}{1}$$

$\therefore$ Self-inductance, $L = 1$ H

4. (b) Self-inductance, $L = \left|\frac{e}{di/dt}\right| = \frac{8}{(2/0.5)} = 2$ H

5. (a) Given, $\frac{di}{dt} = 2$ A/s and $L = 5$ H

$\therefore$ The emf developing across the coil,

$$e = L\frac{di}{dt} = 5 \times 2 = 10 \text{ V}$$

6. (c) We have, $\frac{\Delta i}{\Delta t} = \frac{10}{2} = 5$ V

emf, $e = L\frac{\Delta i}{\Delta t}$

$$= 0.5 \times 5 = 2.5\text{V}$$

7. (d) The self-inductance of a long solenoid is given by

$$L = \frac{\mu_r \mu_0 n^2 A}{l}$$

It is clear that the self-inductance of a long solenoid does not depend upon the current flowing through it.

8. (d) Here, $L = 50 \times 10^{-3}$ H

$$\frac{dI}{dt} = \frac{(1-0)}{0.1} = 10$$

emf, $e = \frac{L \cdot dI}{dt} = 50 \times 10^{-3} \times 10$

$$= 50 \times 10^{-2} = 0.5\text{V}$$

9. (d) Self-inductance of coil is directly proportional to square of number of turns in the coil, *i.e.*,

$$L \propto N^2$$

$$\therefore \quad \frac{L_1}{L_2} = \frac{N_1^2}{N_2^2}$$

$$\therefore \quad \frac{L}{L_2} = \frac{N^2}{(4N)^2}$$

$$\therefore \quad L_2 = 16L$$

10. (a) $L \propto N^2$

So, when number of turns is doubled L' will become 4 times of L, *i.e.*, $L' = 4L$ or 4 times.

11. (a) Self-inductance of solenoid is given by

$$\frac{\mu_0 N^2 S}{L}$$

12. (d) Induced emf, $e = \frac{Li}{dt} = \frac{\mu_0 N^2 S}{l}\frac{di}{dt}$

$$= \frac{4\pi \times 10^{-7} \times (2000)^2 \times 1.2 \times 10^{-3}}{0.30} \times \left(\frac{4}{0.25}\right) = 321 \times 10^{-2} \text{ V}$$

13. (b) Energy stored in the coil,

$$E = \frac{1}{2}Li^2 = \frac{1}{2} \times 50 \times 10^{-3} \times 4 = 0.1 \text{ J}$$

14. (a) $E = \frac{1}{2}Li^2 = \frac{1}{2}(100 \times 10^{-3})(10)^2 = 5$ J

15. (c) Equivalent inductance, $\frac{1}{L_{eq}} = \frac{1}{L} + \frac{1}{L}$

$\therefore$ The total inductance, $L_{eq} = \frac{L}{2}$

Check point 6.4

1. (a) We have, $M = \frac{\Delta\phi_2}{\Delta i_1}$

$$M = \frac{0.4}{2}$$

$\therefore$ Mutual inductance, $M = 0.2$ H

2. (b) Value of induced emf in coil,

$$e = M\frac{di}{dt}$$

$$= 0.005 \times \frac{d}{dt}(i_0 \sin \omega t)$$

$$= 0.005 \times i_0 \omega \cos \omega t \qquad [\text{for } e_{max}, \cos \omega t = 1]$$

$$\therefore \quad e_{max} = 0.005 \times 10 \times 100\pi$$

$$= 5\pi$$

3. (a) emf, $e_2 = M\left(\frac{di_1}{dt}\right) = i_2 R_2$

$\therefore$ Current $\left(\frac{di_1}{dt}\right) = \frac{i_2 R_2}{M} = \frac{(0.4)(5)}{0.5} = 4 \text{ As}^{-1}$

5. (d) Average emf, $e = M\frac{di}{dt} = 0.09 \times \frac{20}{0.006} = 300$ V

6. (*a*) As, $|e| = M\frac{di}{dt} \Rightarrow 8\times10^{-3} = M\times3$

Mutual inductance, $M = 2.66$ mH

7. (*a*) We have, $M = K\sqrt{L_1L_2}$

For perfect coupling, $K = 1 \Rightarrow M_{12} = M_{21}$

8. (*a*) Mutual inductance between coils is

$$M = K\sqrt{L_1L_2}$$

$$\Rightarrow \quad M = 1\sqrt{2\times10^{-3}\times8\times10^{-3}} \quad [\because K = 1]$$

$$= 4\times10^{-3} = 4 \text{ mH}$$

9. (*c*) Mutual inductance between two coils,

$$M = -\frac{e_2}{(di_1/dt)} = -\frac{e_1}{(di_2/dt)}$$

Also, $e_1 = -L_1\frac{di_1}{dt}$ and $e_2 = -L_2\frac{di_2}{dt}$

$$\therefore \quad M^2 = \frac{e_1e_2}{\left(\frac{di_1}{dt}\right)\left(\frac{di_2}{dt}\right)} = L_1L_2 \Rightarrow M = \sqrt{L_1L_2}$$

10. (*b*) Growth of current in the circuit is given by

$$i = i_0(1 - e^{-Rt/L})$$

where, i_0 is peak value of current and

$$i_0 = \frac{5}{5} = 1\text{A}$$

$$\therefore \quad i = 1(1 - e^{-5\times2/10}) = (1 - e^{-1})\text{ A}$$

11. (*b*) In case of growth of current in a L-R circuit, the current in the circuit grows exponentially with time 0 to the maximum value at, $i_0 = E/R$.

12. (*c*) Time interval, during which the current in an inductive circuit rises to 63% of its maximum value is defined as time constant.

A) Taking it together

1. (*d*) We have, $\tau_c = CR$, $\tau_L = \frac{L}{R}$ and $\omega = \frac{1}{\sqrt{LC}}$

2. (*b*) The self-inductance of a long solenoid of cross-sectional area A and length l, having n turns per unit length, filled the inside of the solenoid with a material of relative permeability (*e. g.*, soft iron, which has a high value of relative permeability) is given by

$$L = \mu_r\mu_0n^2Al = \frac{\mu_r\mu_0N^2A}{l}$$

3. (*b*) Induced emf, $e = -\frac{d\phi}{dt}$

But, $e = iR$ and $i = \frac{dq}{dt}$

$$\therefore \quad \frac{dq}{dt}R = -\frac{d\phi}{dt} \text{ or } dq = -\frac{d\phi}{R}$$

So, induced charge in a coil is independent of time taken to change the flux.

4. (*b*) There will be self-induction effect when soft iron core is inserted in the coil. So, intensity of bulb increases.

5. (*b*) The magnitude of induced emf is directly proportional to the rate of change of magnetic flux. Also, induced charge is $dq = -\frac{d\phi}{R}$, which is independent of time.

6. (*a*) Induced current is given by

$$i = \frac{e}{R} = -\frac{N(d\phi/dt)}{R} = -\frac{N}{R}\left(\frac{d\phi}{dt}\right)$$

$$= \frac{10\times10^8\times10^{-4}\times10^{-4}\times10}{20} = 5\text{ A}$$

7. (*c*) In L-R circuit current grows exponentially. So, B_2 lights up earlier and finally B_1 shines brighter than B_2.

8. (*c*) Apply Lenz's law, then loop B is repelled by loop A.

9. (*d*) Induced emf, $e = -N\frac{d\phi}{dt}$

$$|e| = N\frac{d\phi}{dt} = N\frac{d}{dt}(BA\cos\theta) = N\frac{dB}{dt}\cdot A\cos\theta$$

$$= 500\times1\times(10\times10^{-2})^2\cos0°$$

$$= 5\text{ V}$$

10. (*b*) Induced emf, $e = -\frac{d\phi}{dt}$

$$|e| = \frac{d}{dt}(BA) = A\frac{dB}{dt} = 2\times\frac{(4-1)}{2} = 3\text{ V}$$

12. (*c*) $\phi = 0$. Therefore, $\Delta\phi = 0$.

13. (*a*) The magnet gets repelled. Therefore, north pole is formed on the top side and hence current will flow in anti-clockwise direction.

14. (*c*) Current in the circuit, $i = V/R = 10/2 = 5\text{A}$

$\therefore$ Magnetic energy stored in the coil,

$$U = \frac{1}{2}Li^2 = \frac{1}{2}\times2\times(5)^2 = 25\text{ J}$$

15. (*b*) Time constant, $\tau_L = \frac{L}{R} = \frac{1}{2}$

We have, $i = i_0(1 - e^{-t/\tau_L})$

$$\therefore \quad \frac{i_0}{i} = \frac{1}{1 - e^{-t/\tau_L}}$$

Substituting, $t = 1$s and $\tau_L = \frac{1}{2}$ s, we get

$$\frac{i_0}{i} = \frac{e^2}{e^2 - 1}$$

16. (*d*) For a capacitor, $i = \left(\frac{dq}{dt}\right) = C\cdot\left(\frac{dV}{dt}\right)$

i.e., i = constant, if $\frac{dV}{dt}$ = constant

$$\therefore \text{ Capacitance, } C = \frac{i}{\left(\frac{dV}{dt}\right)} = \frac{1}{2} = 0.5\text{ F}$$

17. (*a*) Charge induced in coil is given as

$$dq = \frac{d\phi_B}{R} = i\,dt = \text{Area under } i\text{-}t \text{ graph}$$

$$\therefore \quad d\phi_B = (\text{Area under } i\text{-}t \text{ graph})R$$

$$= \frac{1}{2} \times 4 \times 0.1 \times 10 = 2 \text{ Wb}$$

19. (*d*) We have, $\Delta q = -\frac{N}{R}\Delta\phi$ and $\Delta\phi = \Delta B \cdot A\cos\theta$

$$\therefore \quad 32 \times 10^{-6} = -\frac{100}{(160+40)}(0 - B) \times \pi \times (6 \times 10^{-3})^2 \times \cos 0°$$

$\therefore$ Intensity of magnetic field, $B = 0.566$ T

20. (*d*) Induced emf,

$$e = \left|\frac{-NBA(\cos\theta_2 - \cos\theta_1)}{\Delta t}\right|$$

$$= \left|-\frac{800 \times 4 \times 10^{-5} \times 0.05(\cos 90° - \cos 0°)}{0.1}\right| = 0.016 \text{ V}$$

21. (*a*) $\therefore$ Magnetic flux, $\phi = B \cdot A$

$\therefore$ Change in flux, $d\phi = B \cdot dA$

$$= 0.05\,(101 - 100) \times 10^{-4} = 5 \times 10^{-6} \text{ Wb}$$

$$\therefore \quad \text{Charge, } dq = \frac{d\phi}{R} = \frac{5 \times 10^{-6}}{2} = 2.5 \times 10^{-6} \text{ C}$$

22. (*b*) For growth of current in L-R circuit, current is given by

$$i = i_0(1 - e^{-Rt/L})$$

$$\therefore \quad \frac{di}{dt} = -i_0\left(-\frac{R}{L}\right)e^{-Rt/L} = \frac{i_0 R}{L}e^{-Rt/L}$$

At $\quad \frac{di}{dt} = \frac{i_0 R}{L} = \frac{E}{L} \Rightarrow 4 = \frac{E}{20}$

$\therefore \quad E = 20 \times 4 = 80$ V

23. (*d*) Peak value of emf generated in generator is given by

$$e_0 = \omega NBA$$
$$= (2\pi\nu)NBA$$
$$= 2 \times 3.14 \times 100 \times 5000 \times 0.2 \times 0.25$$
$$= 157 \text{ kV}$$

24. (*d*) emf, $e = Bl^2\pi\nu$

$$= 0.4 \times 10^{-4} \times (0.5)^2 \times (3.14) \times \frac{120}{60} = 6.28 \times 10^{-5} \text{V}$$

25. (*d*) If electron is moving from left to right, the flux linked with the loop (which is into the page) will first increase and then decrease as the electron passes through it. So the induced current in the loop will be first anti-clockwise and will change direction as the electron passes by.

26. (*d*) **B**, **v** and **L** should be mutually perpendicular.

27. (*c*) Induced emf, $e = Bvl$

$\therefore \quad e \propto v$

Also, $\quad v = 0 + gt$

$\therefore \quad e \propto gt$

So, potential difference between its two ends will increase with time.

28. (*a*) In the given circuit, three inductances are in parallel, their equivalent inductance is given by

$$\frac{1}{L_{eq}} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} \text{ or } L_{eq} = \frac{3}{3} = 1\text{H}$$

29. (*d*) $\therefore$ In parallel, $L_P = \frac{L_1 L_2}{L_1 + L_2}$

$$(L_1 + L_2)1.5 = L_1 L_2$$

and in series, $L_S = L_1 + L_2 = 8$

$$\therefore \quad L_1 L_2 = 12$$

$$(L_1 - L_2)^2 = (L_1 + L_2)^2 - 4L_1L_2$$

$$L_D = \sqrt{(8)^2 - 4 \times 12}$$

$$L_D = 4\text{H}$$

30. (*c*) Time constant, $\tau = \frac{L}{R}$

It is the time interval during which the current after opening an inductive circuit falls to 37% of its maximum value.

31. (*c*) Time constant, $\tau = \frac{L}{R} = 2.0 \times 10^{-3}$ s

and $\quad \frac{L}{R + 90} = 0.5 \times 10^{-3}$ s

Solving these two equations, we get

$$L = 60 \text{ mH and } R = 30\,\Omega$$

32. (*c*) Four resistances form a balanced Wheatstone bridge. Therefore, energy stored in the coil is zero.

33. (*d*) The whole current will pass through the inductor. So, current in 10 Ω resistor is zero.

34. (*b*) Power, $\quad P = Fv = ilBv$

$$= \left(\frac{Bvl}{R}\right)lBv = \frac{B^2l^2v^2}{R}$$

35. (*c*) The magnetic flux linked with uniform surface of area A in uniform magnetic field is given by

$$\phi = \mathbf{B} \cdot \mathbf{A}$$

Here, $\quad A = L^2\hat{\mathbf{k}}$ and $B = B_0(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$ T

$$\therefore \quad \phi = \mathbf{B} \cdot \mathbf{A} = B_0(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}) \cdot L^2\hat{\mathbf{k}} = 4B_0L^2 \text{ Wb}$$

36. (*a*) $\therefore$ Magnetic flux,

$$\phi = BA\cos\theta = 0.2\sin(300t) \times 5 \times 10^{-4} \times \cos 60°$$
$$= 5 \times 10^{-5}\sin(300t)$$

$$\therefore \text{ emf, } e = \frac{d\phi}{dt} = 5 \times 10^{-5} \times 300\cos(300t)$$

$$= 15 \times 10^{-2}\cos\left(300 \times \frac{\pi}{900}\right)$$

$$= 1.5 \times 10^{-2}\cos(\pi/3) = 0.75 \times 10^{-2} = 7.5 \times 10^{-3} \text{ V}$$

37. (*c*) Since, the magnetic field is uniform, therefore there will be no change in flux, hence no current will be induced.

39. (*b*) Charge, $\Delta Q = \frac{\Delta\phi}{R} = \frac{n \times BA}{R}$

$\Rightarrow$ Magnetic flux density,

$$B = \frac{\Delta Q \cdot R}{nA} = \frac{2 \times 10^{-4} \times 80}{40 \times 4 \times 10^{-4}} = 1 \text{ Wb/m}^2$$

40. (*b*) We have, $L \propto N^2$

$$\frac{L_B}{L_A} = \left(\frac{n_B}{n_A}\right)^2$$

Self-inductance, $L_B = \left(\frac{500}{600}\right)^2 \times 108 = 75$ mH

42. (*a*) Mutual inductance between two coils depend on their degree of flux linkage *i.e.*, the fraction of flux linked with one coil which is, when some current passes through the other coil. In figure (A) two coils with their planes are parallel. In this situation, maximum flux passes.

43. (*d*) During growth of current in the coil,

$$i = i_0(1 - e^{-Rt/L})$$

For $\quad i = \frac{i_0}{2}$

$\Rightarrow \quad \frac{i_0}{2} = i_0\,(1 - e^{-Rt/L})$

$\therefore \quad t = 0.693\,\frac{L}{R} = 0.693 \times \frac{300 \times 10^{-3}}{2} = 0.1$ s

44. (*c*) Growth of current in the circuit is

$$i = i_0(1 - e^{-Rt/L})$$

$\Rightarrow \quad \frac{di}{dt} = \frac{d}{dt}\,i_0 - \frac{d}{dt}\,(i_0 e^{-Rt/L})$

$\therefore \quad \frac{di}{dt} = 0 - i_0\left(-\frac{R}{L}\right)e^{-Rt/L} = \frac{i_0 R}{L}\,e^{-Rt/L}$

Initially, $t = 0$,

$\therefore \quad \frac{di}{dt} = \frac{i_0 R}{L} = \frac{E}{L} = \frac{5}{2} = 2.5\,\text{As}^{-1}$

45. (*a*) Just after pressing the key K, the inductor offers infinite resistance. So, net resistance will be 14Ω.

$\therefore \quad I = V/R = 5/7\text{A}$

46. (*a*) At $t = 0$, current will flow from the capacitor and at $t = \infty$ current flows from the inductor. So, current in both cases will be 1.5 A.

47. (*a, b, c*) We have, $V = 2t$

$\therefore \quad V = L \cdot \frac{di}{dt} = 2t$ or $2 \cdot \frac{di}{dt} = 2t$ or $\frac{di}{dt} = t$

or $\quad (di) = t\,(dt)$

On integrating, we get $i = \frac{t^2}{2}$

i.e., i-t graph is a parabola.

At $t = 2$ s, $i = 2$ A

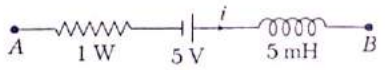
$\therefore$ Potential energy, $U = \frac{1}{2}Li^2 = \frac{1}{2} \times 2 \times 4 = 4$ J

$$\frac{dU}{dt} = Li\left(\frac{di}{dt}\right) = (2)\left(\frac{t^2}{2}\right)(t) = t^3$$

At $\quad t = 1$

Potential energy, $\frac{dU}{dt} = 1\ \text{Js}^{-1}$

48. (*c*) Applying Kirchhoff's second law to above circuit,



$$V_A - iR + 15 - L\frac{di}{dt} = V_B$$

or $\quad V_A - 5 \times 1 + 15 - (5 \times 10^{-3})(-10^3) = V_B$

$\therefore \quad V_B - V_A = 15$ V

49. (*c*) At $t = 0$, induced emf e is maximum and is equal to applied emf, but current i is zero.

After lapse of time, current through the circuit increases but induced emf decreases.

50. (*b*) Two coils are said to be magnetically coupled, if full or a part of the flux produced by one link with the other. Let L_1 and L_2 be the self-inductances of the coils and M be their mutual inductances, then

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

When 100% flux produced by one coil links with the other, then mutual inductance between the two is maximum and is given by

$$M = \sqrt{L_1 L_2}$$

51. (*b*) In position II, change in flux = 0, emf = 0.

In position I and III, $e = Bav$.

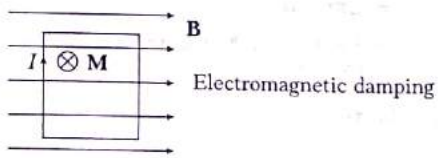
In position I, flux is increasing, hence anti-clockwise emf.

If position III, flux is decreasing, hence clockwise emf.

52. (*b*) Induced emf, $|e| = \frac{d\phi}{dt} = \frac{BdS}{dt}$

Now, as the square loop and rectangular loop move out of magnetic field, $\frac{BdS}{dt}$ is constant, therefore $|e|$ is constant. But in case of circular and elliptical loops, $\frac{dS}{dt}$ changes. Therefore, $|e|$ does not remain constant.

54. (*d*)



55. (*d*) Mutual inductance between two coils in the same plane with their centres coinciding is given by

$$M = \frac{\mu_0}{4\pi}\left(\frac{2\pi^2 R_2^2 N_1 N_2}{R_1}\right) \text{henry}$$

56. (*b*) Power, $P = \frac{e^2}{R}$, hence $e = -\left(\frac{d\phi}{dt}\right)$, where $\phi = NBS$

$\therefore \quad e = -NS\left(\frac{dB}{dt}\right)$. Also, $R \propto \frac{l}{r^2}$

where, R = resistance, r = radius, l = length

$\therefore \quad P \propto \frac{N^2 r^2}{l} \Rightarrow \frac{P_1}{P_2} = 1$

57. (*b*) As the magnet moves towards the coil, the magnetic flux increases (non-linearly). Also, there is a change in polarity of induced emf when the magnet passes on to the other side of the coil.

58. (*b*) emf, $e \propto -\frac{di}{dt}$.

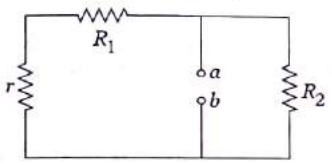
59. (*b*) $i = i_0\left(1 - e^{-\frac{R}{L}t}\right)$. At $t = 0, i = 0$.

At $t >> \tau (= L/R)$, $i = i_0$.

60. (*c*) Since, you are decreasing current in the circuit by increasing resistance of the circuit.

Induced emf across inductor will support 20 V battery. Hence, net emf of the circuit is greater than 20 V or current in the circuit is more than 4 A.

61. (*c*) After short-circuiting the battery.



Resistance, $R_{ab} = \frac{R_2(R_1 + r)}{R_2 + R_1 + r}$

$\therefore$ Time constant, $\tau_L = \frac{L}{R_{net}} = \frac{L(R_1 + R_2 + r)}{(R_1 + r)R_2}$

62. (*d*) Magnetic lines on circular loop, due to current in wire AB are tangential to its plane.

$\therefore$ Magnetic flux, $\phi_B = 0$ or induced emf, $\frac{d\phi_B}{dt} = 0$

63. (*a*) Induced emf, $|e| = \frac{\Delta\phi}{\Delta t} = \frac{(2-1)(0.01)}{10^{-3}} = 10\,\text{V}$

$\therefore$ Current, $i = \frac{e}{R} = 10^3\,\text{A}$

Heat produced, $H = eit = (10)(10^3)(10^{-3}) = 10\,\text{J}$

64. (*d*) Induced emf between the axle and the rim is given by

$e = \frac{1}{2}Bl^2\omega = \pi l^2 fB$ [as $\omega = 2\pi f$]

$= 0.4 \times 10^{-4} \times (0.5)^2 \times (3.14) \times \frac{120}{60} = 6.28 \times 10^{-5}$ V

65. (*d*) As, potential difference,

$V = Bvl = (2.0 \times 10^{-4})\left(360 \times \frac{5}{18}\right)(50) = 1.0$ V

66. (*a*) If the loop remains inside the region of magnetic field, no current will be induced. But, if it moves out of the plane, then the induced current is given by $i = \frac{Blv}{R}$ and its direction will be clockwise (as per Lenz's law).

67. (*b*) As, Potential difference, $e = Bvl$

$= (0.2 \times 10^{-4})(20)(1)$ V

$= 0.4$ mV

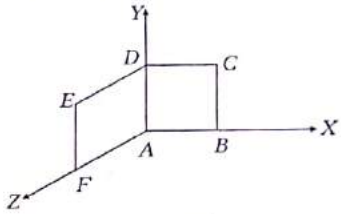
68. (*c*) $M_{12} = M_{21}$, when rate of change of current is doubled, induced emf also becomes two times.

Thus, emf produced across the first coil is 6 mV.

69. (*b*) Here, loop $ABCDA$ lies in XY-plane whose area vector $\mathbf{A}_1 = L^2\hat{\mathbf{k}}$, whereas loop $ADEFA$ lies in YZ-plane whose area vector $\mathbf{A}_2 = L^2\hat{\mathbf{i}}$.

Also, the magnetic flux linked with uniform surface of area A in uniform magnetic field is given by

$\phi = \mathbf{B} \cdot \mathbf{A}$



$\mathbf{A} = \mathbf{A}_1 + \mathbf{A}_2 = (L^2\hat{\mathbf{k}} + L^2\hat{\mathbf{i}})$

and $\mathbf{B} = B_0(\hat{\mathbf{i}} + \hat{\mathbf{k}})$ T

Now, $\phi = \mathbf{B} \cdot \mathbf{A} = B_0(\hat{\mathbf{i}} + \hat{\mathbf{k}}) \cdot (L^2\hat{\mathbf{k}} + L^2\hat{\mathbf{i}}) = 2B_0L^2$ Wb

70. (*b*) Two emfs are induced in the closed circuit each of value Blv. Their two emfs are additive. So, $E_{net} = 2Blv$.

71. (*a, c*) $e = -\frac{d\phi}{dt} = -A \cdot \left(\frac{dB}{dt}\right) = -A$ (slope of B-t graph)

At $t = 2$, slope is zero and it changes its sign.

72. (*d*) Magnetic flux,

$\phi = NAB = (25)(\pi)(1.8 \times 10^{-2})^2[0.2t - 0.05t^2]$

$= 0.025(0.2t - 0.05t^2)$

Induced emf, $e = \left|\frac{d\phi}{dt}\right| = |0.025(0.2 - 2 \times 0.05t)|$

At $t = 3$ s, $e = 0.0025$ V

$i = \frac{e}{R} = 0.0016$ A

$\therefore$ Power dissipation, $P = ei = 4 \times 10^{-6}$ W $= 4\,\mu$W

73. (*a, c*) In both the cases (a) and (c) the induced current in the ring will be in such a direction that it attracts the magnet.

74. (*b*) $e = Bvl = (0.1)(1)(0.1) = \frac{1}{100}$ V

Net resistance $= 1 + \frac{2 \times 3}{2 + 3} = \frac{11}{5}\Omega$

$\therefore$ Current, $i = \left(\frac{1/100}{11/5}\right) = \frac{1}{220}$ A

75. (*a*) From the figure, $V_a - iR - L\frac{di}{dt} = V_b$

$\therefore$ $V_a - V_b = iR + L\frac{di}{dt}$

According to given conditions,

$8 = 2R + L$ ...(i)

$4 = 2R - L$ ...(ii)

Solving these two equations, we get

$R = 3\,\Omega$ and $L = 2$ H

76. (*b*) $V = L\dfrac{di}{dt}$ or $V \propto L\left(\text{as } \dfrac{di}{dt} \rightarrow \text{same}\right)$

$\therefore \dfrac{V_1}{V_2} = \dfrac{L_1}{L_2} = 4, \qquad P = Vi = \text{constant}$

$\therefore \quad i \propto \dfrac{1}{V}$ or $\dfrac{i_1}{i_2} = \dfrac{V_2}{V_1} = \dfrac{1}{4} \Rightarrow W = \dfrac{1}{2}Li^2$

$\therefore \quad \dfrac{W_1}{W_2} = \dfrac{L_1}{L_2}\left(\dfrac{i_1}{i_2}\right)^2 = (4)\left(\dfrac{1}{16}\right) = \dfrac{1}{4}$

$\because \quad \dfrac{i_1}{i_2} \neq 4$

Thus, wrong option is (b).

77. (*c*) Induced emf, $E = -L\dfrac{di}{dt}$

or induced emf $\propto -$ slope of i-t graph. So, graph (c) is correct.

78. (*d*) Energy, $U = \dfrac{1}{2}Li_0^2 = \dfrac{1}{2}L\left(\dfrac{V}{R}\right)^2$

$$= \frac{1}{2} \times 50 \times 10^{-6}\left(\frac{5}{0.5}\right)^2 \text{ J} = 2.5 \text{ mJ}$$

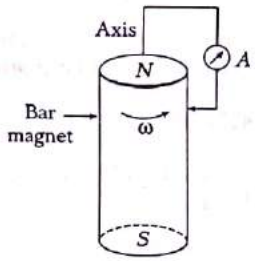
Now, after switching off the battery this energy is dissipated in coil (resistance = 0.5 Ω) and resistance (10 Ω) in the ratio of their resistances ($H = i^2Rt$ or $H \propto R$). Therefore, heat generated in the coil.

$$\left(\frac{5}{0.5}\right)^2\left(\frac{0.5 \times 10}{0.5 + 10}\right) 2.5 \text{ mJ} = 0.12 \text{ mJ}$$

79. (*b*) Power, $P = (i_0)^2R$, *i.e.*, $(i_0)^2 = \dfrac{P}{R}$ and $I = \dfrac{L}{R}$

Total heat produced, $U = \dfrac{1}{2}Li_0^2 = \dfrac{1}{2}(\tau R)\left(\dfrac{P}{R}\right) = \dfrac{1}{2}P\tau$

80. (*b*) When cylindrical bar magnet is rotated about its axis, no change in flux linked with the circuit takes place, consequently no emf induces and hence no current flows through the ammeter A.



81. (*d*) When A stops moving the current in B become zero, possible only if the current in A is constant. If the current in A would be variable, then there must be an induced emf (current) in B even, if A stops moving.

82. (*a*) When the current in B (at $t = 0$) is counter-clockwise and the coil A is considered above to it. The counter-clockwise flow of the current in B is equivalent to north pole of magnet and magnetic field lines are emanating upward to coil A. When coil A start rotating at $t = 0$, then the current in A is constant along clockwise direction by Lenz's rule.

83. (*a*) Here, area, $A = 10 \times 5 = 50 \text{ cm}^2 = 50 \times 10^{-4} \text{ m}^2$

$$\frac{dB}{dt} = 0.2 \text{ Ts}^{-1} \Rightarrow R = 2\Omega$$

$\therefore \quad$ emf, $e = \dfrac{d\phi}{dt} = A \cdot \dfrac{dB}{dt} = 50 \times 10^{-4} \times 0.02 = 10^{-4}$ V

Power dissipated in the form of heat

$$= \frac{e^2}{R} = \frac{10^{-4} \times 10^{-4}}{2} = 0.5 \times 10^{-8} \text{ W} = 5 \times 10^{-9} \text{ W} = 5 \text{ nW}$$

84. (*d*) Cross ⊗ magnetic field passing from the closed loop is increasing. Therefore, from Lenz's law, induced current will produce dot ⊙ magnetic field. Hence, induced current is anti-clockwise.

In that case, $k = 1$(unity)

85. (*a*) Only half area of ring will be considered.

$\therefore \phi_B = BS = B \cdot \dfrac{\pi r^2}{2}$

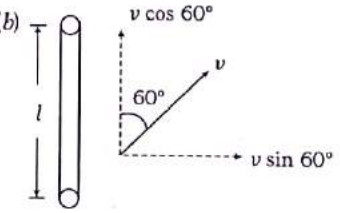
$\therefore \quad e = \dfrac{d\phi_B}{dt} = \dfrac{\pi r^2}{2} \cdot \dfrac{dB}{dt} = \dfrac{\pi(0.1)^2}{2} \times 0.01 = \dfrac{\pi}{2} \times 10^{-4}$ V

$\therefore \quad i = \dfrac{e}{R} = \dfrac{\frac{\pi}{2} \times 10^{-4}}{4} = \dfrac{\pi}{8} \times 10^{-4} = 3.9 \times 10^{-5}$ A

86. (*a*) $\therefore$ emf, $|\bar{e}| = \dfrac{\Delta\phi}{\Delta t} = \dfrac{BS - 0}{\Delta t} = \dfrac{2 \times 10^{-4} \times \pi \times (0.1)^2}{2 \times 2}$

$\Rightarrow \quad |\bar{e}| = 1.57 \times 10^{-5}$ V

87. (*b*)

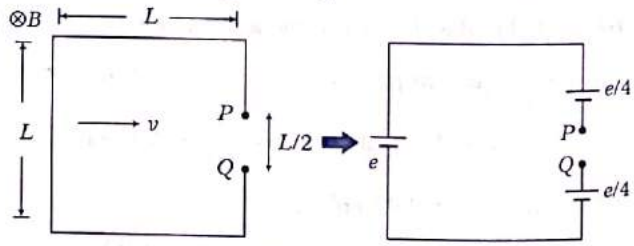


Due to $v \cos 60°$, $e = 0$

Due to $v \sin 60°$, $e = B(v \sin 60°)l$

$$= \frac{\sqrt{3}}{2}Bvl = \frac{\sqrt{3}}{2} \times 1 \times 0.2 \times 0.1 = 17 \times 10^{-3} \text{ V}$$

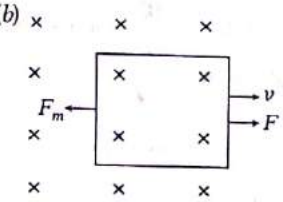
88. (*c*)



$e = BLv$

$$V_P + \frac{e}{4} - e + \frac{e}{4} = V_Q \Rightarrow V_P - V_Q = \frac{e}{2} = \frac{BLv}{2}$$

89. (*b*)



Induced emf $= Bvl$

Induced current, $i = \frac{Bvl}{R}$

Magnetic force on loop, $F_m = Bil = \frac{B^2v^2l^2}{R}$

Since, loop is moving with constant speed,

$\Rightarrow \quad F = F_m$

Input power, $P = \mathbf{F}\cdot\mathbf{v} = Fv = \frac{B^2v^2l^2}{R}$

P *versus* v graph will be like,

$y \propto x^2$ (parabola, open upward)

90. (*a*) Area of square loop, $S = 10\text{ cm} \times 10\text{ cm}$

$\Rightarrow S = 100\text{ cm}^2 = 100\times10^{-4}\text{ m}^2 = 10^{-2}\text{m}^2$

Initial magnetic flux linked with loop,

$\phi_{B_1} = B_1S\cos\phi = 0.1\times10^{-2}\times\cos 45°$

$$= \frac{0.1\times10^{-2}\times1}{\sqrt{2}} = \frac{10^{-3}}{\sqrt{2}}\text{ Wb}$$

Final magnetic flux linked with loop,

$\phi_{B_2} = 0\text{ Wb}$ $[\because B_2 = 0]$

The induced emf in the loop, $\varepsilon = -\frac{d\phi}{dt} = -\frac{(\phi_2-\phi_1)}{t}$

$$= -\frac{\left(0-\frac{10^{-3}}{\sqrt{2}}\right)}{0.7} = \frac{10^{-3}}{0.7\times\sqrt{2}} \cong 10^{-3}\text{ V}$$

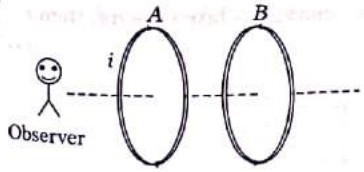
The induced current in the loop,

$$I = \frac{\varepsilon}{R} = \frac{10^{-3}\text{V}}{1\Omega} = 10^{-3}\text{A} = 1.0\text{ mA}$$

91. (*c*) At low frequency of 1 to 2 Hz also, oscillations may be observed.

92. (*a*) If bar magnet is falling vertically through the hollow region of long vertical copper tube, then the magnetic flux linked with the copper tube (due to 'non-uniform' magnetic field of magnet) changes and eddy currents are generated in the body of the tube by Lenz's law. The eddy currents oppose the falling of the magnet which, therefore experiences a retarding force. The retarding force increases with increasing velocity of the magnet and finally equals the weight of the magnet. The magnet, then attains a constant final terminal velocity, *i.e.*, magnet ultimately falls with zero acceleration in the tube.

93. (*d*)

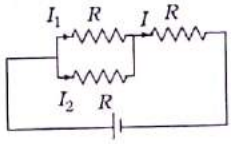


If current through A increases, magnetic field (×) linked with coil B increases. Hence, anti-clockwise current induces in coil B. As shown in figure both the currents produce repulsive effect.

94. (*c*) As the emf induced $= B_1vl - B_2vl$

$$= \frac{\mu_0 I}{2\pi(x-a/2)}lv - \frac{\mu_0 I}{2\pi(x+a/2)}lv \propto \frac{1}{(2x-a)(2x+a)}$$

95. (*c*) Circuit can be reduced as



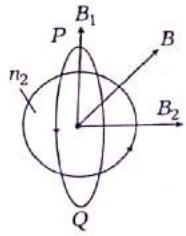
$e = VlB$

$$I = \frac{e}{3R/2} = \frac{2vlB}{3R}$$

$$\Rightarrow \quad I_1 = I_2 = \frac{I}{2} = \frac{vlB}{3R}$$

96. (*c*) Magnetic field at P, $B_1 = \frac{\mu_0 I_2}{2R}$

$$= \frac{4\pi\times10^{-7}\times4}{2\times0.02\pi} = 4\times10^{-5}\text{ Wb/m}^2$$



Magnetic field, at Q,

$$B_2 = \frac{\mu_0 I_1}{2R} = \frac{4\pi\times10^{-7}\times3}{2\times0.02\pi} = 3\times10^{-5}\text{ Wb/m}^2$$

$$\therefore \quad B = \sqrt{B_1^2 + B^2{}_2}$$

$$= \sqrt{(4\times10^{-5})^2 + (3\times10^{-5})^2} = 5\times10^{-5}\text{ Wb/m}^2$$

97. (*d*) During decay of current,

$$i = i_0 e^{-\frac{Rt}{L}} = \frac{E}{R}e^{-\frac{Rt}{L}} = \frac{100}{100}e^{-\frac{100\times10^{-3}}{100\times10^{-3}}} = \frac{1}{e}\text{A}$$

98. (*b*) The self-inductance of a coil, $L = \mu_0 n^2 Sl$, where μ_0 = permeability of air,

n = number of turns per unit length,

S = area of cross-section and

l = length of the solenoid

This depends on the geometry of the inductor such as cross-sectional area, length and number of turns and not on the material, even if it is made of a superconducting material. If the superconductor is below the critical temperature, then the current will continuously flow and the inductance may not have the property of inductance any more.

$\therefore \quad L_1 = L_2, L_3 = 0$

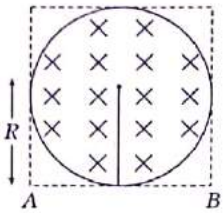
99. (*d*) Current, $I = \frac{\text{Potential difference}}{\text{Resistance}}$

$\therefore$ Potential difference $= 2\times12 = 24\text{ V} = Bvl$

Here, $l = AD\sin 37° = 0.3\times\frac{3}{5} = 0.18\text{ m}$

$\therefore$ Velocity, $v = \frac{24}{Bl} = \frac{24}{4\times0.18} = \frac{100}{3}\text{ ms}^{-1}$

100. (*d*) Induced emf, $V_{BA} = \int \mathbf{E}\cdot d\mathbf{l} = \frac{1}{4}\left(\frac{d\phi}{dt}\right)$



$$= \frac{1}{4}(\pi R^2)\cdot\frac{dB}{dt} = \frac{\pi R^2 \alpha}{4} \qquad [\because \phi = B\cdot A \text{ and } \frac{dB}{dt} = \alpha]$$

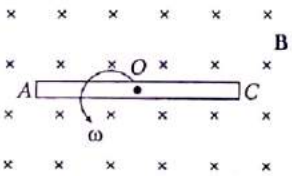
101. (*a*) According to Fleming's right hand rule, P is at higher potential and Q is at lower potential. Therefore, A is positively charged and B is negatively charged.

Also, charge, $Q = CV = C(Bvl)$

$$= 10 \times 10^{-6} \times 4 \times 2 \times 1 = 80\ \mu\text{C}$$

$\therefore \qquad q_A = 80\ \mu\text{C}$ and $q_B = -80\ \mu\text{C}$

102. (*c*) For rotating rod, induced emf, $v = \frac{1}{2}Bl^2\omega$



For part AO, $\quad V_{OA} = V_O - V_A = \frac{1}{2}Bl^2\omega$

For part OC, $\quad V_{OC} = V_O - V_C = \frac{1}{2}B(3l)^2\omega$

$\therefore \qquad V_A - V_C = 4Bl^2\omega$

103. (*b*) In L-R circuit, for growth of current,

$$i = i_0(1 - e^{-Rt/L}) \text{ or } \frac{3}{4}i_0 = i_0(1 - e^{-t/\tau_L}) \text{ (given)}$$

$$\text{(where, } \tau_L = \frac{L}{R} = \text{time constant)}$$

$$\therefore \qquad e^{-t/\tau_L} = 1 - \frac{3}{4} = \frac{1}{4}$$

$$e^{t/\tau_L} = 4 \text{ or } \frac{t}{\tau_L} = \log_e 4$$

$$\tau_L = \frac{t}{\log_e 4} = \frac{4}{2\log_e 2} \Rightarrow \tau_L = \frac{2}{\log_e 2}\text{ s}$$

104. (*d*) As the coil is rotated, angle θ (angle which is normal to the coil makes with $\mathbf{B}$ at any instant t) changes, therefore magnetic flux ϕ linked with the coil changes and hence an emf is induced in the coil. At this instant t, if e is the emf induced in the coil, then

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(NSB\cos\omega t)$$

where, N is number of turns in the coil.

or $\qquad e = -NSB\frac{d}{dt}(\cos\omega t) = -NSB(-\sin\omega t)\,\omega$

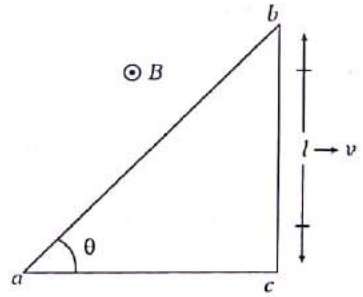
or $\qquad e = NSB\omega\sin\omega t \qquad$ ...(i)

The induced emf will be maximum when, $\sin\omega t$ *i.e.*, maximum

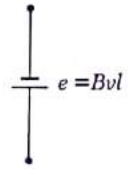
$\therefore \qquad e_{max} = e_0 = NSB\omega \times 1$ or $\quad e = e_0\sin\omega t$

Therefore, e would be maximum, hence current is maximum (as $i_0 = e_0/R$), when $\theta = 90°$ *i.e.*, normal to plane of coil is perpendicular to the field or plane of coil is parallel to magnetic field.

105. (*a*)

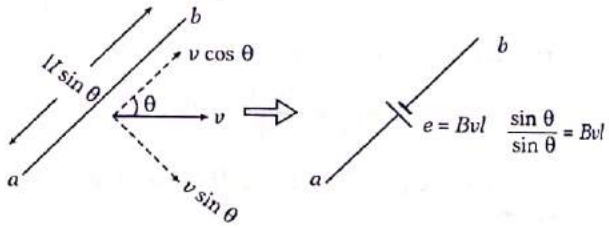


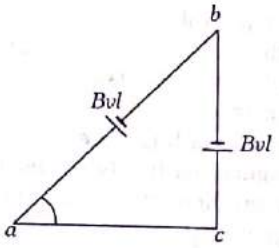
For segment bc, emf



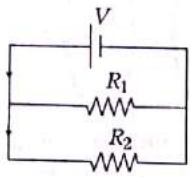
For segment ac, emf $ac: e = 0$

For segment ab, emf ab:





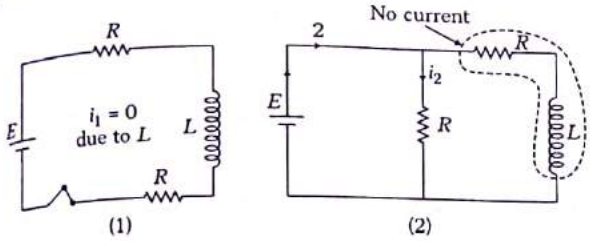
106. (*c*) At $t = 0$ inductor behaves as broken wire, then $i = \frac{V}{R_2}$.

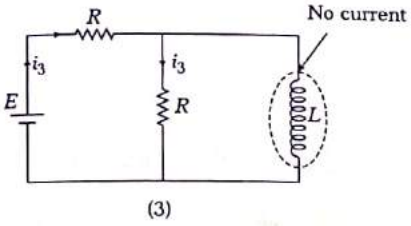


At $i = \infty$, inductor behaves as a conducting wire,

$$i = \frac{V}{R_1R_2/(R_1+R_2)} = \frac{V(R_1+R_2)}{R_1R_2}$$

107. (a) (i) Just before closing the switch,

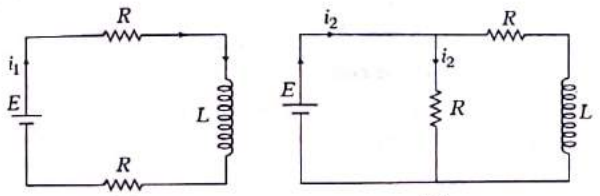




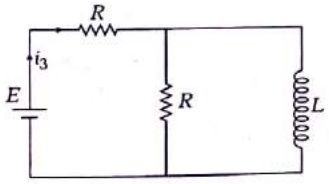
$$i_1 = 0,\ i_2 = \frac{E}{R},\ i_3 = \frac{E}{2R},$$

So, $\quad i_2 > i_3 > i_1\,[i_1 = 0]$.

(ii) After a long time closing the switch,



$R_{eq} = 2R$ $\qquad$ $R_{eq} = \frac{R}{2}$



$R_{eq} = 2R$

As, $\quad i = \frac{E}{R}$

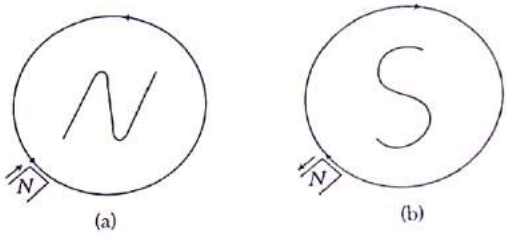
$\Rightarrow \quad i_1 = E/2R,\ i_2 = 2E/R,\ i_3 = E/R$

Therefore, $i_2 > i_3 > i_1$.

108. (d) At B, flux is maximum, so from $|e| = \frac{d\phi}{dt}$ at B, $|e| = 0$

109. (a) When the north pole of a bar magnet moves towards the coil, the induced current in the coil flows in a direction such that the coil presents its north pole to the bar magnet as shown in Fig. (a).

Therefore, the induced current flows in the coil in the anti-clockwise direction. When the north pole of a bar magnet moves away from the coil, then the induced current in the coil flows in a direction such that the coil presents its south pole to the bar magnet as shown in Fig. (b).



Therefore, induced current flows in the coil in the clockwise direction.

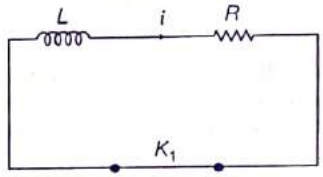
110. (b) When ring enters and leaves, then the field polarity of induced emf is opposite. Also, during the stay of ring completely in the field there is no induction.

111. (d) When switch K_1 closed for long time current through inductor, $I = \frac{E}{R} = \frac{15}{0.15 \times 10^3} = 0.1\text{A}$

When K_1 opened and K_2 is closed, then

$$i = Ie^{-t/r}$$

$$t = 1\,\text{ms} = 1 \times 10^{-3}\text{s}$$



$$\therefore \quad \tau_L = \frac{L}{R} = \frac{0.03}{0.15 \times 10^3} = \frac{10^{-3}}{5}$$

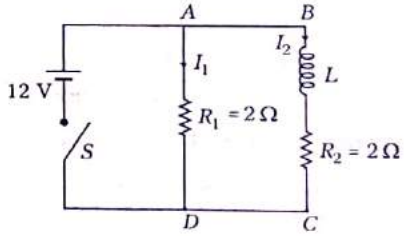
$$\therefore \quad i = 0.1e^{\left(\frac{-1\times10^{-3}}{\frac{10^{-3}}{5}}\right)} = 0.1e^{-5}$$

$$i = \frac{0.1}{e^5} = \frac{0.1}{150} = \frac{1}{15} \times 10^{-2} \times 10^3\text{mA}$$

$$i = 0.67\ \text{mA}$$

112. (d) $E(\text{across } BC) = L\frac{dI_2}{dt} + R_2 I_2$ [using Kirchhoff's law] ...(i)

$$I_2 = I_0(I - e^{-t/t_0})$$



At $t = 0$, $\quad I_0 = \frac{E}{R_2} = \frac{12}{2} = 6\text{A}$

$$\tau_L = t_0 = \frac{L}{R} = \frac{400 \times 10^{-3}}{2\Omega} = 0.2\,\text{s}$$

$\therefore \quad I_2 = 6(1 - e^{-t/0.2})$ $\qquad$ [using Eq. (i)]

Potential drop area $L = E - R_2 I_2$

$$= 12 - 2 \times 6(1 - e^{-t/0.2}) = 12e^{-t/0.2} = 12e^{-5t}\ \text{V}$$

113. (*d*) When two solenoids of inductance L_0 are connected in series at large distance and current i is passed through them, then the total flux linkage $\phi_{total} = L_0 i + L_0 i$.

If L be the equivalent inductance of the system, then

$$\phi_{total} = Li$$

$$\therefore \quad Li = L_0 i + L_0 i \quad \text{or} \quad L = 2L_0$$

When solenoids are connected in series with one inside the other and senses of the turns coinciding, then there will be a mutual inductance L between them. In this case, the resultant induced emf in the coils is the sum of the emf's e_1 and e_2 in the respective coils, *i.e.*,

$$e = e_1 + e_2$$

$$= \left(-L_0 \frac{di}{dt} \pm L_0 \frac{di}{dt}\right) + \left(-L_0 \frac{di}{dt} \pm L_0 \frac{di}{dt}\right)$$

where, (+) sign is for positive coupling and (–) sign for negative coupling.

But, $\quad e = -L \dfrac{di}{dt}$

$$\therefore \quad -L \frac{di}{dt} = -L_0 \frac{di}{dt} - L_0 \frac{di}{dt} \pm 2L_0 \frac{di}{dt}$$

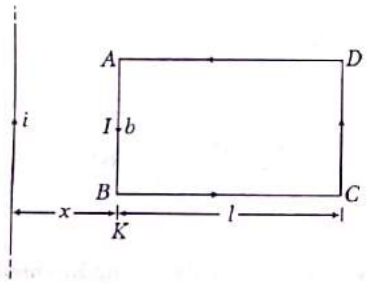
i.e., $\quad L = L_0 + L_0 + 2L_0$

$= 4L_0$ [for positive coupling]

When solenoids are connected in series with one inside the other with senses of the turn opposite, then there is a negative coupling.

So, $\quad L = L_0 + L_0 - 2L_0 = 0$

114. (*c*) We can show the situation as



Since, loop is moving away from the wire, so the direction of current in the loop will be as shown in the figure.

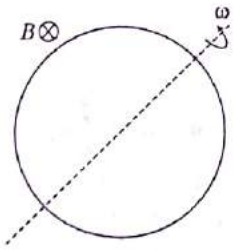
Net magnetic field on the loop due to wire,

$$B = \frac{\mu_0 i}{2\pi}\left(\frac{1}{x} - \frac{1}{l+x}\right) = \frac{\mu_0 i l}{2\pi x(l+x)}$$

So, the magnitude of the emf in the loop,

$$e = vBb = \frac{\mu_0 i l v b}{2\pi x(l+x)}$$

115. (*b*)



Let at any time, normal to loop makes an angle θ with magnetic field

$$\phi_B = NBS\cos\theta = NBS\cos\omega t$$

$$e = -\frac{d\phi_B}{dt} = NBS\,\omega\sin\omega t = e_0 \sin\omega t$$

$$\Rightarrow e_{max} = e_0 = NSB\omega = 1 \times 0.01 \times \pi(5)^2 \times 10^{-4} \times \frac{80 \times 2\pi}{60}$$

$$= 6.6 \times 10^{-4}\,\text{V}$$

Over a long period, *i.e.* one time period, average of induced emf is

$$\bar{e} = \frac{\int_0^T e\,dt}{T} = 0$$

$$\overline{e^2} = \frac{\int_0^T e^2 dt}{T} = \frac{e_0^2}{2} = 2.0 \times 10^{-7}\,\text{V}^2$$

116. (*c*) As, $\quad E\,l = \dfrac{d\phi_B}{dt}$

or $\quad E\,(2\pi R) = \pi R^2 \cdot \dfrac{dB}{dt} = \pi R^2\,(8t)$

$$\therefore \quad E = 4Rt$$

$$F = qE = 4qRt \quad \text{[tangential]}$$

$$\tau_F = 4qR^2 t$$

$$\tau_f = (\mu mgR)$$

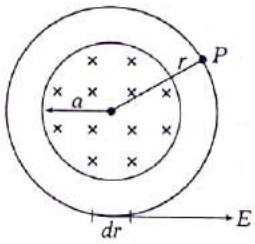
When $\tau_F > \tau_f$, ring will start rotating.

At $\quad t = 2$ s,

$$8qR^3 = \mu mgR$$

$\therefore$ Coefficient of friction, $\mu = \dfrac{8\,qR}{mg}$

117. (*b*)



Draw a concentric circle of radius r. The induced electric field (E) at any point on the circle is equal to that at P.

For this circle, induced emf

$$e = \oint \mathbf{E} \cdot d\mathbf{l} = \left|\frac{d\phi}{dt}\right| = S\left|\frac{dB}{dt}\right|$$

$$\therefore \quad E = \oint dl = \pi a^2 \left|\frac{dB}{dt}\right| \quad \left[\text{but } \oint dl = 2\pi r\right]$$

$$\therefore \quad E \times (2\pi r) = \pi a^2 \left|\frac{dB}{dt}\right|$$

$$\therefore \quad E = \frac{a^2}{2r}\left|\frac{dB}{dt}\right|$$

$$\Rightarrow \quad E \propto \frac{1}{r}$$

118. (*d*) When switch S is closed magnetic field lines passing through Q increases in the direction from right to left.

So, according to Lenz's law, induced current in Q, *i.e.*, I_{Q_1} will flow in such a direction, so that the magnetic field lines due to I_{Q_1} passes from left to right through Q. This is possible when I_{Q_1} flows in anti-clockwise direction as seen by E. Opposite is the case when switch S is opened, *i.e.*, I_{Q_2} will be clockwise as seen by E.

119. (*a*) As the north pole approaches, a north pole is developed at the face, *i.e.*, the current flows anti-clockwise. Finally, when it completes the oscillation, no emf is present. Now, south pole approaches the other side, *i.e.*, RHS, the current flows clockwise to repel the south pole.

This means the current is anti-clockwise at the LHS as before. The break occurs when the pendulum is at the extreme and momentarily stationary.

B) Medical entrance special format questions

Assertion and reason

1. (*d*) Electric flux = ES and magnetic flux = BS. Both have different units.

2. (*b*) The total number of magnetic lines of force passing through an area placed in magnetic field is equal to magnetic flux linked with the area.

Mutual inductance is the phenomenon according to which on opposing emf produces. Flux in a coil as a result of change in the current or magnetic flux linked with neighbouring coil. But when two coils are coupled, in emf produced inductively addition to induced due to mutual inductance, induced emf is also produced in each of the two coils due to self-inductance.

3. (*d*) According to Lenz's law, the direction of induced emf or current in a circuit in such as to oppose the cause that produces it.

4. (*c*) (i) Mutual inductance of two coils depends on the distance between the coils and their orientation.

(ii) Mutual inductance depends on the magnetic permeability of medium between the coils or nature of material on which two coils are wound.

5. (*d*) Self-inductance does not depend upon current flowing or change in current flowing but it depends upon of turns N. Area of cross-section (A) and permeability of medium (μ).

We have, $e = -L\dfrac{di}{dt}$.

6. (*b*) According to Lenz's law, induction effects always oppose the cause.

7. (*a*) If a circuit containing a pure inductor L and resistor R in series with a battery and a key is closed, then the circuit current through the circuit rises exponentially and reaches upto a certain maximum value.

8. (*b*) If moving from left to right, current is increasing, then battery $\left(\text{induced emf} = L\cdot\dfrac{di}{dt}\right)$ will produce right to left current *i.e.*, its positive terminal is on left hand side or $V_A > V_B$.

Further i = constant $\Rightarrow \dfrac{di}{dt} = 0$ or $V_L = 0$.

9. (*a*) $V = L\dfrac{di}{dt}$ = constant

$\therefore \quad L\,(di)$ = constant $\Rightarrow Li$ = constant

or $\quad i \propto \dfrac{1}{L}$

10. (*a*) $L = \dfrac{-e}{di/dt} \Rightarrow L = 2/4 = 1/2$

On putting values, we get

$$\phi = Li \Rightarrow \phi = \frac{1}{2}\times 4 = 2\,\text{Wb}$$

11. (*d*) If a conducting rod of length l moving with a uniform velocity v perpendicular to a uniform magnetic field B, then induced emf will be $e = Bvl$.

12. (*e*) In non-uniform magnetic field, magnetic flux will be obtained by integration, but it will not vary with time.

13. (*c*) $\phi = Li \Rightarrow \phi \propto i$

15. (*d*) Since, both the loops are identical (same area and number of turns) and moving with a same speed in same magnetic field. Therefore, same emf is induced in both the coils. But the induced current will be more in the copper loop as its resistance wire be lesser as compared to that of the aluminium loop.

16. (*b*) $\phi = BA\cos\omega t$

At $\quad t = 0,\ \phi$ = maximum

$$|e| = \left|\frac{d\phi}{dt}\right| = B\,\omega A\sin\omega t$$

$|e|_{max}$ when $\omega t = 90°$, *i.e.*, loop is rotated 90°, or it is parallel to magnetic field. At this instant $\phi = 0$.

17. (*a*) This is in accordance with Lenz's law. When magnet is above the loop, then north pole is formed and hence will repel and when it is below the loop, then it will attract the magnet.

Match the columns

1. A. Tesla $\rightarrow [MT^{-2}A^{-1}]$
   B. Weber $\rightarrow [ML^2T^{-2}A^{-1}]$
   C. Weber/m^2 $\rightarrow [MA^{-1}T^{-2}]$
   D. Henry $\rightarrow [ML^2A^{-2}T^{-2}]$

2. V_{AD} at $t = 0 = E = 10$ V
   V_{BC} at $t = 0 = E = 10$ V
   V_{AB} at $t = \infty$

$$I = \frac{E}{R+r} = \frac{10}{5} = 2\,\text{A}$$

$$\therefore \quad V_{AB} = V_A - V_B = I\times R = 2\times 4 = 8\,\text{V}$$

V_{AD} at $t = \infty$

$$V_{AD} = V_A - V_D = E - Ir = 10 - (2\times 1) = 8\,\text{V}$$

3. In all of the cases, the induced current in the loop is non-zero. The direction of induced current can be determined by Lenz's law. When loop is moved towards right, then induced current in the loop is clockwise. When loop is moved towards left, then induced current in the loop is anti-clockwise.

4. When current i_2 is decreased, then flux through A decreases. According to Lenz's law, attraction occurs between A and B. Similarly, when current i_1 is increased, then loops will repel each other. As distance between the loop decreases, M decreases. Hence, loops will repel each other and current i_1 will increase, when loop 1 is moved towards loop 2. When loop 2 is moved away, loops will attract and i_2 will increase.

5. A. The magnetic flux through the rectangular loop *abcd* increases, due to the motion of the loop into the region of magnetic field. The induced current must flow along the path *bcdab*, so that it opposes the increasing flux.
   B. Due to the outward motion, magnetic flux through the triangular loop *abc* decreases due to which the induced current flows along *bacd*, so as to oppose the change in flux.
   C. As the magnetic flux decreases due to motion of the irregular shaped loop *abcd* out of the region of magnetic field, the induced current flows along *cdabc*, so as to oppose change in flux.

C Medical entrance's gallery

1. (*c*) Given, number of turns of solenoid, $N = 1000$

Current, $I = 4$ A

Magnetic flux, $\phi_B = 4 \times 10^{-3}$ Wb

$\because$ Self-inductance of solenoid is given by

$$L = \frac{\phi_B N}{I} \quad \text{...(i)}$$

Substituting the given values in Eq. (i), we get

$$L = \frac{4 \times 10^{-3} \times 1000}{4} = 1 \text{ H}$$

2. (*c*) Induced emf in the region is given by

$$|e| = \frac{d\phi}{dt}$$

where, $\phi = BA = \pi r^2 B$

$$\Rightarrow \quad \frac{d\phi}{dt} = -\pi r^2 \frac{dB}{dt}$$

Rate of change of magnetic flux associated with loop 1,

$$e_1 = -\frac{d\phi_1}{dt} = -\pi r^2 \frac{dB}{dt}$$

Similarly, e_2 = emf associated with loop 2

$$= -\frac{d\phi_2}{dt} = 0 \quad (\because \phi_2 = 0)$$

3. (*a*) Given, $n = 500$

$$L = 50 \text{ mH} = 50 \times 10^{-3} \text{ H}$$

and $i = 8 \text{ mA} = 8 \times 10^{-3}$ A

The magnetic flux linked with the coil is

$$\phi = Li = 50 \times 10^{-3} \times 8 \times 10^{-3}$$
$$= 400 \times 10^{-6} = 4 \times 10^{-4} \text{ Wb}$$

4. (*a*) The flux is given by

$$\phi = \mathbf{B} \cdot \mathbf{A}$$
$$\phi = |\mathbf{B}||\mathbf{A}| \cos\theta$$

Also, induced emf, $\varepsilon = -\frac{d\phi}{dt} = \frac{d[|\mathbf{B}||\mathbf{A}|\cos\theta]}{dt}$

$$= -|\mathbf{B}||\mathbf{A}| \frac{d(\cos\theta)}{dt} = |\mathbf{B}||\mathbf{A}| \sin\theta \cdot \omega$$

$$\Rightarrow \quad \varepsilon = BA\,\omega \cos(\theta - \pi/2)$$

5. (*b*) Given, $B = 40$ mT $= 40 \times 10^{-3}$T

Rate of shrinking of area $= 0.5 \text{ m}^2\text{s}^{-1}$

$$\Rightarrow \quad \frac{dA}{dt} = 0.5 \text{ m}^2\text{s}^{-1}$$

As, $\phi = BA\cos\theta = BA$ $\quad [\because \cos\theta = \cos 0° = 1]$

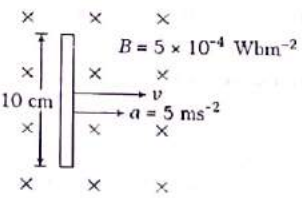
On differentiating with w.r.t. to t,

$$\frac{d\phi}{dt} = B \cdot \frac{dA}{dt} = e$$

$$\Rightarrow \quad e = \text{induced emf} = 40 \times 10^{-3} \times 0.5 = 20 \times 10^{-3}\text{V}$$
$$= 20 \text{ mV}$$

6. (*a*) Changing magnetic fields can set up current loop in nearby metal bodies and the current is called as eddy current.

7. (*b*) Consider a rod of 10 cm length is moving perpendicular to uniform magnetic field as shown below.



The emf induced in the rod, $\varepsilon = vBl$

Differentiating with respect to time t, we get

Rate of increase of induced emf $= \frac{d\varepsilon}{dt}$

$$= \frac{dv}{dt} Bl = aBl$$
$$= 5 \times 5 \times 10^{-4} \times 10 \times 10^{-2}$$
$$= 250 \times 10^{-6} = 2.5 \times 10^{-4} \text{ Vs}^{-1}$$

8. (*b*) Induced emf in both the loops will be same as

$$e = \frac{-d\phi}{dt} = B\,\omega \sin\omega t$$

Since, B and ω are same for both the cases, induced emf will be same.

However, induced current is given by

$$I = \frac{e}{R}$$

So, it will depend upon the value of resistance, lower the resistance higher will be the current. Therefore, induced current is different in both the loops.

9. (*b*) Given, length of conductor, $l = 0.1\,\text{m}$

Mangetic field, $B = 0.1\,\text{T}$

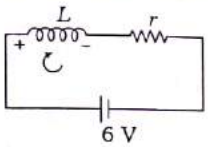
Velocity of conductor, $v = 15\,\text{ms}^{-1}$

The angle between v and B is 90°.

When v and B are mutually perpendicular, then emf (induced) is given by

$$\varepsilon = vBl = 15 \times 0.1 \times 0.1 = \frac{15}{100} = 0.15\,\text{V}$$

10. (*c*) Applying KVL in the circuit, we can write



$$6 - L\frac{dI}{dt} - Ir = 0$$

At initial stage, $I = 0$

$$\Rightarrow \quad 6 = L\frac{dI}{dt}$$

or $$\frac{dI}{dt} = \frac{6}{L} = \frac{6}{2} = 3\ \text{As}^{-1}$$

11. (*d*) As magnetic field through circle is constant because current through straight wire is constant. Magnetic flux through circle is not changing with time.

$$\frac{d\phi_B}{dt} = 0$$

$$\Rightarrow \quad |E| = \left|\frac{d\phi_B}{dt}\right| = 0 \Rightarrow i = \frac{E}{R} = \frac{0}{R} = 0$$

12. (*c*) We know that, $e = NBA\,\omega \sin \omega t$

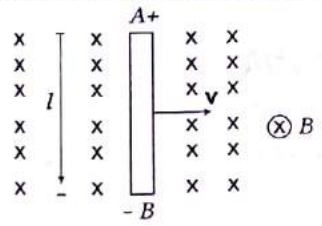
where, N = number of loops = 1

$$B = \frac{\mu_0 I}{2b}\ \text{N/A-m} \Rightarrow A = \pi a^2\ \text{m}^2$$

$$\therefore \quad e = \frac{\mu_0 I}{2b}(\pi a^2)\,\omega \sin \omega t$$

$$\Rightarrow \quad = \frac{\pi a^2 \mu_0 I}{2b} \cdot \omega \sin \omega t$$

13. (*c*) Consider the diagram, where a conductor AB of length l moving perpendicular to an inward magnetic field B.



Value of induced emf,

$$|E_{\text{ind}}| = |l(\mathbf{v} \times \mathbf{B})|$$

$$\Rightarrow \quad = lvB \sin 90° \qquad [\because v \perp B]$$

$$E_{\text{ind}} = vBl \qquad \text{...(i)}$$

Given, $v = 7\,\text{ms}^{-1}$, $l = 0.4$ m and $B = 0.9\,\text{Wbm}^{-2}$

Now, from Eq. (i), we get

$$E_{\text{ind}} = (7\ \text{ms}^{-1})(0.9\ \text{Wbm}^{-2})(0.4\ \text{m})$$

$$= 7(0.9)(0.4)\ \text{V} = 2.52\ \text{V}$$

14. (*a*) Consider the inductor of inductance L.

The current flowing through the inductor is i.

Now, we can write $\phi = Li$

where, ϕ is magnetic flux linked with the inductor

$$\Rightarrow \quad \frac{d\phi}{dt} = L\frac{di}{dt}$$

Given, $L = 40$ mH

dt = change in time $= t_2 - t_1 = 4$ ms $= \Delta t$

$di = i_2 - i_1 = 11 - 1 = 10\ \text{A} = \Delta i$

So, $$\frac{d\phi}{dt} = \frac{\Delta\phi}{\Delta t} = L\frac{\Delta i}{\Delta t} = (40\ \text{mH})\frac{(10)}{4\ \text{ms}}$$

$$= 10 \times 10 = 100 \qquad \text{...(i)}$$

According to Faraday's law of electromagnetic induction,

Emf induced, $$e = -\frac{d\phi}{dt}$$

$$\Rightarrow \quad |e| = \frac{\Delta\phi}{\Delta t} = 100\ \text{V} \qquad [\text{from Eq. (i)}]$$

15. (*b*) $\because$ Induced emf, $e = -L\dfrac{di}{dt}$

Given, $L = 10$ H, $\Delta i = 9 - 4 = 5$ A, $dt = 0.2$ s

emf, $$e = 10 \times \frac{5}{0.2} = 250\,\text{V}$$

16. (*c*) As, $e = Bvl$

Here, $B = 10^{-3}\ \text{Wb/m}^3$

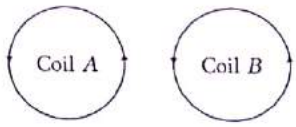
$v = 2\ \text{m/s}$

$l = 5\,\text{cm} = 0.05\,\text{m}$

$\therefore$ emf, $e = (10^{-3})(2)(0.05) = 1 \times 10^{-4}$ V

17. (*c*) As the current in coil A changes, the magnetic flux linked with coil B changes. Direction of flux in B will be downwards, hence current induces in coil B is in anti-clockwise direction according to Lenz's law.

The direction of induced current in coil B is opposite to that of the direction of current in coil A as shown in figure



Due to currents in opposite directions in the near by sides of the coil, the coil B is repelled.

18. (*) Given, $n = 100$ turns, $A = 0.1 \times 0.05\ \text{m}^2$

$B_1 = 0.1$ T, $B_2 = 0.05$ T, $dt = 0.05$ s

We know that,

$$e = \left|\frac{-d\phi}{dt}\right| = \left|\frac{d}{dt}(n\,B\,A\cos\theta)\right| = \left|\frac{nA\,dB\cos\theta}{dt}\right|$$

Here, $\theta = 0°$

$\therefore \quad e = \left|\frac{nAdB}{dt}\right| = \left|\frac{100 \times 0.1 \times 0.05 \times (0.1 - 0.05)}{0.05}\right|$

$\Rightarrow \quad e = 0.5$ V

None of the option is correct.

19. (*e*) Electromagnetic induction is not used in room heater because it works on the principle of heating effect of electric current.

20. (*d*) Given, $M = 0.05$ H and $I = I_0 \sin \omega t$

$\therefore \quad \frac{dI}{dt} = I_0 \cos \omega t(\omega)$

$\Rightarrow \left(\frac{dI}{dt}\right)_{max} = I_0(\omega) \times 1 = 1 \times 100\ \pi$ A/s

So, maximum emf, $e_{max} = M\left(\frac{dI}{dt}\right)_{max} = 0.05 \times 100\pi = 5\pi$ V

21. (*a*) The amount of induced charge is given by

$$q = \frac{1}{R}\Delta\phi = \frac{1}{100}(60 - 10)$$

[where, $\phi = 10$ Wb, $\phi_2 = 60$Wb, $R = 100\,\Omega$]

$\therefore$ Charge, $\quad q = \frac{50}{100} = 0.5$ C

22. (*d*) For inductor as we know induced voltage

For $\quad t = 0$ to $t = T/2$

$$V = L\frac{dI}{dt} = L\frac{d}{dt}\left(\frac{2I_0 t}{T}\right) = \text{constant}$$

For $\quad t = T/2$ to $t = T$

$$V = \frac{LdI}{dt}\left(\frac{-2I_0 t}{T}\right) = -\text{constant}$$

Therefore, answer will represent by graph (d).

23. (*a*) Given, $B = 0.15$ T, $l = 50 \times 10^{-2}$ cm and $v = 2\ \text{ms}^{-1}$

$\therefore \quad$ emf, $e = Bvl$

$e = 0.15 \times 2 \times 50 \times 10^{-2} = 0.15$

Current, $i = \frac{e}{R} = \frac{0.15}{3} = 5 \times 10^{-2}$

Force, $F = Bil = 0.15 \times 5 \times 10^{-2} \times 50 \times 10^{-2}$

$= 3.75 \times 10^{-3}$ N

24. (*a*) Induced emf, $e = 2$ V

$i_1 = 8$ A, $i_2 = 6$ A

$\Delta t = 2 \times 10^{-3}$ s

Coefficient of self-induction,

$$L = \frac{e}{\Delta i / \Delta t} = \frac{-2}{(6-8)/2 \times 10^{-3}}$$

$$= \frac{-2 \times 2 \times 10^{-3}}{-2}$$

$$= 2 \times 10^{-3}\ \text{H}$$

25. (*c*) Rate of doing work $= \frac{W}{t} = P$

But, $\quad P = Fv$

Also, $\quad F = Bil = B\left(\frac{Bvl}{R}\right)l$

$\left[\because \text{induced current, } i = \frac{Bvl}{R}\right]$

$\therefore$ Power, $P = B\left(\frac{Bvl}{R}\right)lv = \frac{B^2v^2l^2}{R}$

$$= \frac{(0.5)^2 \times (2)^2 \times (1)^2}{6} = \frac{1}{6}\ \text{W}$$

CHAPTER 7

Alternating current

We have already studied in our previous chapters about electric and magnetic fields and how energy can be stored in capacitors and inductors. In this chapter, we will study how energy stored in one location can be transferred to another location, so that it can be put to use. Most of the electric power generated and used in the world is in the form of alternating current. Here, we will study about some alternating current system that transfers energy efficiently and we will also discuss some of the devices that make use of that energy.

1 Types of current

There are two types of current which flows in any of the electrical appliances. *These are as follow*

Direct current (DC)

Currents whose direction does not change with time through a load, are known as direct current (DC). The graph given below shows that the direction of direct current (I) or voltage (V) does not change with time (t).

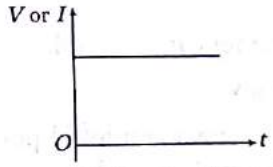


Fig. 7.1 voltage or current vs time graph for a DC

Alternating current (AC)

Currents whose direction changes periodically through a load are known as alternating currents and the voltage is known as alternating voltage (AC voltage)

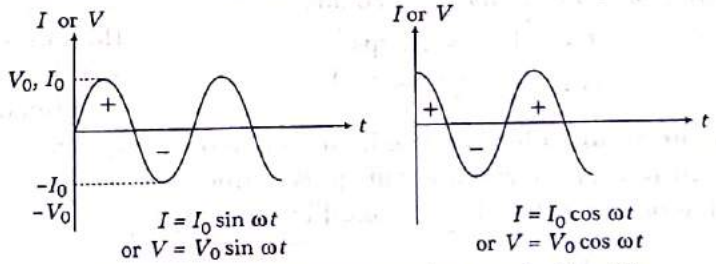


Fig. 7.2 Voltage or current vs time graph of an AC

Inside

1) Type of current
   - Mean or average value of an alternating current
   - Root mean square value of an alternating current
   - Form factor
   - Peak factor
2) Representation of current and emf as rotating vectors — Phasors
   - Different types of alternating current circuits
   - Inductor as low pass filter
   - AC voltage applied to a series L-C-R cicruit
   - Parallel circuit (Rejector circuit)
3) Power in an AC circuit
   - Wattless current
   - L-C oscillators
4) Choke coil
   - Transformer
   - Electric generator or dynamo

Here, the magnitude of current or voltage increases from zero to a maximum value, then decreases to zero and reverses in direction, increases to a maximum in this direction and then decrease to zero. The complete set of variations is known as a **cycle**. Thus, during one-half of the cycle the current or voltage flows in one direction, whereas in the next half-cycle, it flows in the opposite direction.

The instantaneous value of AC is given by

$$I = I_0 \sin\omega t$$

where, I = current at any instant, t

I_0 = maximum or peak value of AC

ω = angular frequency

Alternating emf or voltage

The instantaneous value of alternating emf or voltage is given by

$$V = V_0 \sin\omega t$$

where, V = voltage at any time, t

V_0 = maximum or peak value of alternating voltage

ω = angular frequency

Note Alternating current, alternating voltage or emf, flux etc. all are sinusoidal waves.

Some definitions related to alternating current or voltage

(i) **Time period** The time taken to complete one cycle of variations is called time period or periodic time. It is given by the formula,

$$T = \frac{2\pi}{\omega}$$

where, 2π = angular displacement in a complete cycle

ω = angular frequency

(ii) **Frequency** The number of cycles completed per second by an alternating current is called its frequency and is denoted by f.

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

The frequency of AC in India is 50 Hz *i.e.*,

$$f = 50 \text{ Hz} = 50 \text{ cps}$$

or $$\omega = 2\pi f \approx 314 \text{ rads}^{-1}$$

(iii) **Peak value or amplitude** The maximum value of alternating current or alternating voltage is defined as peak value or amplitude. It is denoted by I_0 or V_0.

(iv) **Phase** Phase is a physical quantity representing both the instantaneous value and direction of alternating quantity at any instant. It is a dimensionless quantity and its unit is radian. If $I = I_0 \sin(\omega t + \phi)$, then the argument of $\sin(\omega t + \phi)$ is called its phase.

(v) **Phase difference** The difference between the phases of current and voltage is called phase difference. If alternating voltage and current is given by the relation,

$$V = V_0 \sin(\omega t + \phi_1) \text{ and } I = I_0 \sin(\omega t + \phi_2)$$

Then, phase difference, $\phi = \phi_2 - \phi_1$ (relative to voltage)

and $\phi = \phi_1 - \phi_2$ (relative to current)

Some important points related to alternating current

(i) An AC is produced by AC generator or AC dynamo and is represented by the symbol ⊖.

(ii) The AC is converted into DC with the help of rectifier while DC is converted into AC with the help of invertor.

(iii) It cannot produce electroplating or electrolysis.

(iv) It is measured with the help of 'hot wire ammeter'.

(v) An AC can be transmitted over long distances without much power loss.

(vi) In an AC (sinusoidal) current or voltage can have *following four values*

(a) Instantaneous value (b) Peak value (I_0 or V_0)

(c) rms value (I_{rms} or V_{rms}) (d) Average value

Mean or average value of an alternating current

Average or mean value of AC is defined as that value of direct current which sends the same amount of charge in a circuit in the same time as is sent by the given alternating current in its half-cycle (*i.e.*, $T/2$). It is denoted by I_{av} or I_m.

Let an alternating current be represented by $I = I_0 \sin\omega t$, where I_0 is the peak value. Clearly, the **mean value of the current over a complete cycle is zero.** It has no significance. Hence, the mean value of alternating current is defined as its average over half a cycle. For positive half cycle,

$$I_{mean} = \frac{1}{T/2}\int_0^{T/2} I\,dt$$

Putting $I = I_0 \sin \omega t$ and $T = (2\pi/\omega)$, we get

$$I_{mean} = \frac{2}{T}\int_0^{\pi/\omega} I_0 \sin \omega t \, dt = \frac{\omega}{\pi}\frac{I_0}{\omega}[-\cos \omega t]_0^{\pi/\omega}$$

$$= -\frac{I_0}{\pi}[\cos \pi - \cos 0] = -\frac{I_0}{\pi}[-1-1]$$

Thus, $\boxed{I_{mean} = \frac{2}{\pi} I_0 = 0.637 I_0 = 63.7\% \text{ of } I_0}$

...[Mean current over positive half cycle]

Similarly, the mean value of alternating emf is $2E_0/\pi$ for positive half cycle. For negative half cycle the mean value of alternating current is $-2I_0/\pi$ and the average value of emf for negative half cycle is $-2E_0/\pi$. So, the average value of emf of current for complete cycle is zero.

In the same way, mean value of alternating emf (E_{mean})

$$\boxed{E_{mean} = \frac{2E_0}{\pi} = 0.637 E_0 = 63.7\% \text{ of } E_0}$$

Average values of some important expressions of current and voltage

(i) When $I = I_1 \sin\omega t + I_2 \cos \omega t$
and $V = V_1 \sin\omega t + V_2 \cos\omega t$
where I_1, I_2, V_1 and V_2 are constants,
then $I_{av} = 0$ and $V_{av} = 0$

(ii) Similarly when $I = I_0 + I_1 \sin\omega t + I_2 \cos\omega t$
and $V = V_0 + V_1 \sin\omega t + V_2 \cos\omega t$
then, $I_{av} = I_0$ and $V_{av} = V_0$

(iii) When $I = I_0 + I_1 \sin\omega t + I_2 \cos 2\omega t$
and $V = V_0 + V_1 \sin\omega t + V_2 \cos 2\omega t$
then $I_{av} = I_0$ and $V_{av} = V_0$

(iv) When $I = I_1 \sin\omega t + I_2 \sin 2\omega t + I_3 \sin 3\omega t + I_0$
and $V = V_1 \sin\omega t + V_2 \sin 2\omega t + V_3 \sin 3\omega t + V_0$
then $I_{av} = 0 + I_0 = I_0$ and $V_{av} = V_0$

Example 7.1 *If the peak value of a current in 50Hz AC. circuit is 7.07 A. What is the mean value of current over half a cycle and the value of current 1/300 s after it was zero?*

Sol. As $I_m = \frac{2I_0}{\pi}$, where, I_0 = peak value of current = 7.07A

$\therefore \quad I_m = 0.637 \times 7.07 = 4.5\text{A}$

Also, at $t = \frac{1}{300}$ s

$$I = I_0 \sin \omega t = I_0 \sin 2\pi f t \qquad (\because \omega = 2\pi f)$$

$$\therefore \quad I = 7.07 \sin\left(2\pi \times 50 \times \frac{1}{300}\right) = 7.07 \sin\frac{\pi}{3}$$

$$= 7.07 \times \frac{\sqrt{3}}{2} = 6.12\text{A}$$

Example 7.2 *Find the average value in the following cases*

(i) $i = 4 + 3\cos \omega t$
(ii) $V = 5\sin \omega t + 3\cos \omega t$
(iii) $i = \sin \omega t + 2\sin 2\omega t + 3\sin 3\omega t$
(iv) $V = \cos \omega t + 3\cos 2\omega t + 3\cos 3\omega t + 2$

Sol. (i) Average value of current $I_{av} = I_0 = 4$ unit
(ii) Average value of the voltage $V_{av} = 0$
(iii) Average value of current $I_{av} = 0$
(iv) Average value of voltage $V_{av} = 2$ unit

Root mean square value of an alternating current

It is defined as that value of a direct current which produces the same amount of heating effect in a given resistor as is produced by the given alternating current when passed for the same time during a complete cycle. It is denoted by I_{rms}, I_V or I_{eff}.

It is also called virtual value or effective value of AC.

Instantaneous value of alternating current

$$I = I_0 \sin \omega t$$

If dH is small amount of heat produced in time dt in resistor R, then $dH = I^2 R \, dt$ in complete cycle, the total heat produced is

$$\int dH = \int_0^T I^2 R \, dt$$

$$H = \int_0^T (I_0 \sin \omega t) R^2 \, dt$$

On solving we get

$$\boxed{H = \frac{I_0^2 RT}{2}} \qquad ...(i)$$

If I_{rms} is rms value of alternating current and H is the heat produced by rms current, then

$$H = I_{rms}^2 RT \qquad ...(ii)$$

From Eq. (i) and (ii)

$$\boxed{I_{rms} = \frac{I_0}{\sqrt{2}}}$$

or $\boxed{I_{rms} = 0.707 I_0}$

or $\boxed{I_{rms} = 70.7\% \text{ of } I_0}$

In the same way, rms value of alternating emf

$$E_{rms} = \frac{E_0}{\sqrt{2}} = 0.707 E_0 = 70.7\% \text{ of } E_0$$

AC ammeter and AC voltmeter always measure the virtual value of AC or alternating voltage. Also, the power supply which we are getting in our homes as 220 V AC is the rms voltage and its voltage amplitude is, $V_0 = \sqrt{2} V_{rms} = 311$ V

This is the reason why AC is more dangerous than DC as 311 V will cause more harm to the human body than 220 V DC. The different values I_0, I_{av} and I_{rms} are shown in figure,

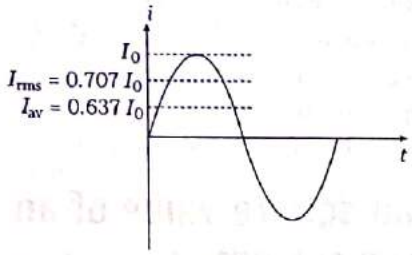


Fig. 7.3 rms and average value on the same graph

rms values for some important expressions of current and voltage

(i) When $I = I_1 \sin\omega t + I_2 \cos\omega t$
where I_1 and I_2 are constant currents, then

$$I_{rms} = \sqrt{\frac{I_1^2}{2} + \frac{I_2^2}{2}}$$

Similarly when $V = V_1 \sin\omega t + V_2 \cos\omega t$
where V_1 and V_2 are constant voltages, then

$$V_{rms} = \sqrt{\frac{V_1^2}{2} + \frac{V_2^2}{2}}$$

(ii) When $I = I_0 + I_1 \sin\omega t + I_2 \cos\omega t$
and $V = V_0 + \sin\omega t + V_2 \cos 2\omega t$

$$I_{rms} = \sqrt{I_0^2 + \frac{I_1^2}{2} + \frac{I_2^2}{2}} \text{ and } V_{rms} = \sqrt{V_0^2 + \frac{V_1^2}{2} + \frac{V_2^2}{2}}$$

(iii) When $V = V_1 \sin\omega t + V_2 \sin 2\omega t + V_3 \sin 3\omega t + V_0$
and $I = I_1 \sin\omega t + I_2 \sin 2\omega t + I_3 \sin 3\omega t + I_0$

them, $V_{rms} = \sqrt{\frac{V_1^2}{2} + \frac{V_2^2}{2} + \frac{V_3^2}{2} + V_0^2}$

and $I_{rms} = \sqrt{\frac{I_1^2}{2} + \frac{I_2^2}{2} + \frac{I_3^2}{2} + I_0^2}$

Example 7.3

(a) The peak voltage of an AC supply is 300 V. What is the rms voltage?

(b) The rms value of current in an AC circuit is 10 A. What is the peak current?

Sol. (a) Here, peak value of AC supply, $V_0 = 300\,\text{V}$

$\therefore$ Using the relation $V_{rms} = \frac{V_0}{\sqrt{2}}$

We get, $V_{rms} = \frac{300}{\sqrt{2}} = 212.1\,\text{V}$

(b) Let I_0 be the peak value of the current.

Here, I_{rms} = rms value of current = 10 A

Then, $I_{rms} = \frac{I_0}{\sqrt{2}}$

$\therefore$ $I_0 = \sqrt{2}\, I_{rms} = 1.414 \times 10 = 14.14\,\text{A}$

Example 7.4 *If $V = 220\sqrt{2}\sin(314t - \phi)$, calculate (a) peak and rms value of the voltage (b) average voltage for half time-period and (c) frequency of AC.*

Sol. Given, $V = 220\sqrt{2}\sin(314t - \phi)$ ...(i)

Comparing with general equation, we get

(a) Peak voltage, $V_0 = 220\sqrt{2}\,\text{V}$

rms voltage, $V_{rms} = \frac{V_0}{\sqrt{2}} = \frac{220\sqrt{2}}{\sqrt{2}} = 220\,\text{V}$

(b) Average voltage $\overline{V} = \frac{2}{\pi}V_0 = \frac{2}{\pi} \times 220\sqrt{2} = \frac{440\sqrt{2}}{\pi}\,\text{V}$

(c) As, $\omega = 314\,\text{rad s}^{-1}$

$\Rightarrow$ $2\pi f = 314 \Rightarrow f = 50\,\text{Hz}$

Example 7.5 *If the current in an AC circuit is represented by the equation, $I = 5\sin(300t - \pi/4)$*

Here, t is in second and I is in ampere. Calculate,

(i) peak and rms value of current.

(ii) frequency of AC.

(iii) average current.

Sol. (i) Comparing the given equation with general equation, we have

$$I = I_0 \sin(\omega t \pm \phi)$$

$\therefore$ The peak value, $I_0 = 5\,\text{A}$

and $I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{5}{\sqrt{2}} = 3.535\,\text{A}$

(ii) Angular frequency,

$$\omega = 300\,\text{rads}^{-1}$$

$\therefore$ $\nu = \frac{\omega}{2\pi} = \frac{300}{2\pi} \approx 47.75\,\text{Hz}$

(iii) Average current, $I_{av} = \left(\frac{2}{\pi}\right)I_0 = \left(\frac{2}{\pi}\right)(5) = 3.18\,\text{A}$

Example 7.6 *The voltage supplied to a circuit is given by $V = V_0 t^{3/2}$, where t is time in second. Find the rms value of voltage for the period, t = 0 to t = 1 s.*

Sol. The mean square voltage,

$$\overline{V}^2 = \frac{1}{t}\int_0^t V^2 dt = \frac{1}{t}\int_0^1 V_0^2 t^3 dt$$

$$= \frac{V_0^2}{t}\left[\frac{t^4}{4}\right]_0^1 = \left[\frac{V_0^2 t^3}{4}\right]_0^1 = \frac{V_0^2}{4}$$

$\therefore$ $V_{rms} = \sqrt{\overline{V}^2} = \frac{V_0}{2}$

Example 7.7 *Calculate rms value of curent and voltages for the following cases*

(i) $i = 4 + 3\sin\omega t$

(ii) $V = 5 + 2\cos\omega t$

(iii) $i = 2 + 3\sin\omega t + 2\cos 2\omega t$

(iv) $V = \cos\omega t + 2\cos 2\omega t + 3\cos 3\omega t$

Sol. (i) rms value of current

$$I_{rms} = \sqrt{I_0^2 + \frac{I_1^2}{2}} = \sqrt{4^2 + \frac{(3)^2}{2}} = 4.52 \text{ unit}$$

(ii) rms value of voltage

$$V_{rms} = \sqrt{V_0^2 + \frac{V_1^2}{2}} = \sqrt{5^2 + \frac{(2)^2}{2}} = 5.19 \text{ unit}$$

(iii) rms value of current

$$I_{rms} = \sqrt{I_0^2 + \frac{I_1^2}{2} + \frac{I_2^2}{2}} = \sqrt{2^2 + \frac{3^2}{2} + \frac{2^2}{2}} = 3.24 \text{ unit}$$

(iv) rms value of voltage

$$V_{rms} = \sqrt{\frac{V_1^2}{2} + \frac{V_2^2}{2} + \frac{V_3^2}{2}} = \sqrt{\frac{1^2}{2} + \frac{2^2}{2} + \frac{3^2}{2}} = 2.64 \text{ unit}$$

Form factor

Form factor is defined as the ratio of rms value of AC to its average value during half-cycle *i.e.*,

$$\text{Form factor, } (R_f) = \frac{\text{rms value}}{\text{average value}} = \frac{V_0/\sqrt{2}}{2V_0/\pi} = \frac{\pi}{2\sqrt{2}} = 1.11$$

Peak factor

The ratio of peak value and rms value is called peak factor. It is given by R_p, $R_p = \dfrac{V_0}{V_0/\sqrt{2}} = \sqrt{2} = 1.41$

Calculation of mean and rms value for some specific cases of alternating current

Sinusoidal waveform

(i) $I_{rms} = \dfrac{I_0}{\sqrt{2}}$

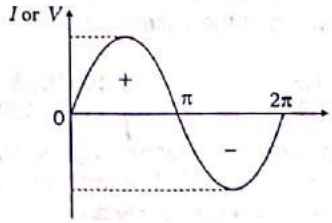


Fig. 7.4 Sinusoidal waveform

(ii) $I_{av} = \dfrac{2I_0}{\pi}$ for half-cycle

$I_{av} = 0$ for one complete cycle

(iii) $R_f = \dfrac{\pi}{2\sqrt{2}} = 1.11$

(iv) $R_p = \sqrt{2} = 1.41$

where, R_f = form factor; R_p = peak factor

Square or rectangular waveform

(i) $I_{rms} = I_0$

for one complete cycle

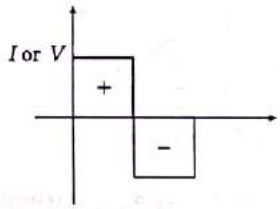


Fig. 7.5 Square or rectangular waveform

(ii) $I_{av} = I_0$ for half-cycle

$I_{av} = 0$

for one complete cycle

(iii) $R_f = \dfrac{I_0}{I_0} = 1$

(iv) $R_p = \dfrac{I_0}{I_0} = 1$

Half-wave rectified

(i) $I_{rms} = \dfrac{I_0}{2}$

for one complete cycle

(ii) $I_{av} = \dfrac{I_0}{\pi}$

for one complete cycle

$I_{av} = \dfrac{2I_0}{\pi}$ for half-cycle

(iii) $R_f = \dfrac{I_0/2}{I_0/\pi} = \dfrac{\pi}{2} = 1.57$

(iv) $R_p = \dfrac{I_0}{I_0/2} = 2$

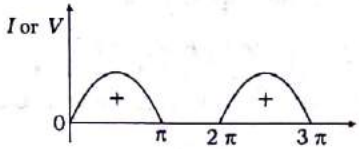


Fig. 7.6 Half-wave rectified waveform

Full wave rectified

(i) $I_{rms} = \frac{I_0}{\sqrt{2}}$ for one complete cycle

(ii) $I_{av} = \frac{2I_0}{\pi}$

(iii) $R_f = \frac{I_0/\sqrt{2}}{2I_0/\pi} = \frac{\pi}{2\sqrt{2}} = 1.11$

(iv) $R_p = \frac{I_0}{I_0/\sqrt{2}} = \sqrt{2}$

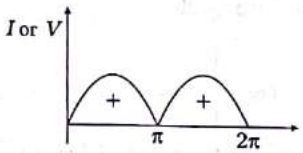


Fig. 7.7 Full wave rectified waveform

For a current varying wave as shown in the graph

(i) $I_{rms} = \left[\frac{\int_0^T i^2 dt}{\int_0^T dt}\right]^{1/2} = \frac{I_0}{\sqrt{3}}$

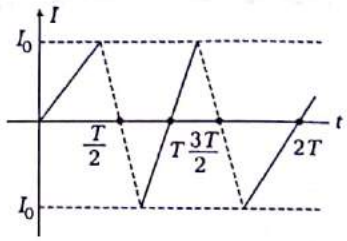


Fig. 7.8

(ii) $I_{av} = \left[\frac{\int_0^T i dt}{\int_0^T dt}\right]$

$= \text{Area of } i\text{-}t \text{ graph for } \frac{t = 0 \text{ to } t = T}{T} = \frac{I_0}{2}$

(iii) $R_f = \frac{I_0/\sqrt{3}}{I_0/2}$

$= \frac{2}{\sqrt{3}} = 1.15$

(iv) $R_p = \frac{I_0}{I_0/\sqrt{3}} = \sqrt{3}$

Check point 7.1

1. The frequency of a sinusoidal wave
$E = 0.40 \cos [2000t + 0.80]$ would be
(a) 1000π Hz (b) 2000 Hz
(c) 20 Hz (d) $1000/\pi$ Hz

2. Frequency of AC mains in India is
(a) 30 cps (b) 50 cps (c) 60 cps (d) 120 cps

3. A 220 V AC is more dangerous than 220 V DC because
(a) the AC attracts
(b) the DC repels
(c) the body offers less resistance to AC
(d) peak voltage for AC is much larger than 220 V

4. Alternating current is transmitted to take places
(a) at high voltage and low current
(b) at high voltage and high current
(c) at low voltage and low current
(d) at low voltage and high current

5. An AC voltage is given by $E = E_0 \sin \frac{2\pi t}{T}$
Then, the mean value of voltage calculated over any time interval of $T/2$ second
(a) is always zero (b) is never zero
(c) is always $(2E_0/\pi)$ (d) may be zero

6. 220 V, 50 Hz, AC is applied to a resistor. The instantaneous value of voltage is
(a) $220\sqrt{2}\sin 100\pi t$ (b) $220\sin 100\pi t$
(c) $220\sqrt{2}\sin 50\pi t$ (d) $220\sin 50\pi t$

7. The instantaneous current in an AC circuit is $I = \sqrt{2}\sin(50t + \pi/4)$. The rms value of current is
(a) $\sqrt{2}$ A (b) 50 A (c) 90 A (d) 1 A

8. The peak value of an alternating current is 5 A and its frequency is 60 Hz. Find its rms value and time taken to reach the peak value of current starting from zero.
(a) 3.536 A, 4.167 ms (b) 3.536 A, 15 ms
(c) 6.07 A, 10 ms (d) 2.536 A, 4.167 ms

9. If an alternating voltage is represented as $E = 141\sin(628t)$, then the rms value of the voltage and the frequency are respectively.
(a) 141V, 628 Hz (b) 100 V, 50 Hz
(c) 100 V, 100 Hz (d) 141 V, 100 Hz

10. An alternating current in a circuit is given by $I = 20\sin(100\pi t + 0.05\pi)$ A. The rms value and the frequency of current respectively are
(a) 10A and 100 Hz (b) 10A and 50 Hz
(c) $10\sqrt{2}$ A and 50 Hz (d) $10\sqrt{2}$ A and 100 Hz

2 Representation of current and emf as rotating vectors—Phasors

To simplify the study of AC circuit, alternating quantities (*i.e.*, I or V) are represented as rotating vectors, with the angle between the vectors equal to the phase difference between the current and emf or voltage. These rotating vectors representing current and alternating voltage are called phasors.

- The length of the vector represents the maximum or peak value *i.e.*, I_0 and V_0.
- The projection of the vector on fixed axis gives the instantaneous value of alternating current and alternating emf. If ($I = I_0 \sin \omega t$ and $V = V_0 \sin \omega t$), projection is taken on Y-axis. If ($I = I_0 \cos \omega t$ and $V = V_0 \cos \omega t$), projection is taken on X-axis.
- A phasor is a geometric entity and not a real physical quantity that helps us to describe and analyse physical quantities that vary sinusoidally with time.
- A diagram representing alternating current and alternating emf (of same frequency) as rotating vectors (phasors) with the phase angle between them is called **phasor diagram.**

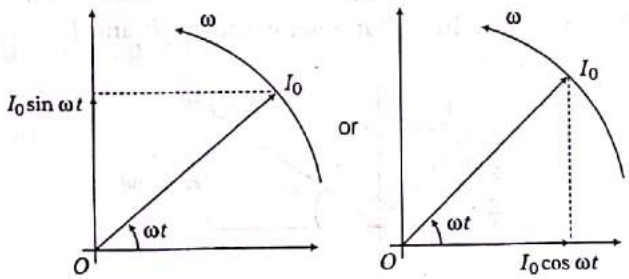


Fig. 7.9 Phasor diagram

Different types of alternating current circuits

In this section, we will derive voltage current relations for individual as well as combined circuit elements carrying a sinusoidal current. We will consider resistors, inductors and capacitors.

1. Pure resistor in an AC circuit (Resistive circuit or *R*-circuit)

Suppose a resistor of resistance R is connected to an AC source of emf with instantaneous value (E) is given by

$$E = E_0 \sin \omega t \quad \text{...(i)}$$

Suppose E is the potential drop across resistance (R), then

$$E = IR \quad \text{...(ii)}$$

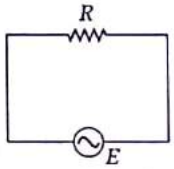


Fig. 7.10 AC voltage applied to a resistor

Instantaneous emf = Instantaneous value of potential drop

∴ From Eqs. (i) and (ii), we have

$$IR = E = E_0 \sin \omega t \quad \text{or} \quad I = \frac{E}{R} = \frac{E_0 \sin \omega t}{R}$$

$$I = I_0 \sin \omega t \qquad \left[\because I_0 = \frac{E_0}{R}\right] \text{...(iii)}$$

Comparing $I_0 = E_0 / R$ with ohm's law, we find that resistors work equally well for both AC and DC voltages.

From Eqs. (i) and (iii), we get that there is zero phase difference between instantaneous alternating current and instantaneous alternating emf (i.e. they are in same phase)

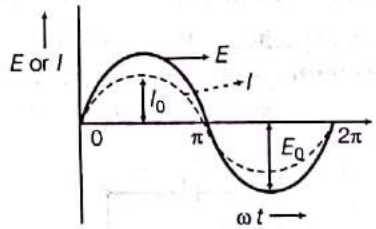


Fig. 7.11 Graph of E and I of a resistive circuit

Phasor Diagram

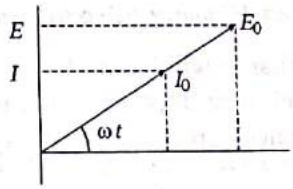


Fig. 7.12 Phasor diagram for a purely resistive circuit

Here, peak values E_0 and I_0 are represented by vectors rotating with angular velocity ω with respect to horizontal reference. Their projections on vertical axis give their instantaneous value.

Note Since, there is no phase difference between E and I, so R behaves same in AC and DC (Direct Current). Steady current is also called DC.

Variation of resistance with frequency of source

Figure given below shows the graph of R as a function of angular frequency ω. The resistance R, does not depend upon angular frequency of the source.

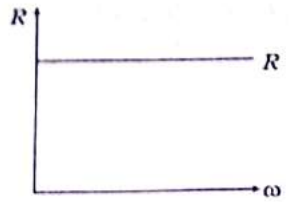


Fig. 7.13 R remains constant with ω or f

Example 7.8 *A 200 Ω resistor is connected to a 220 V, 50 Hz AC supply. Calculate rms value of current in the circuit. Also find phase difference between voltage and the current.*

Sol. Given $V_{rms} = 220$ V, $f = 50$ Hz, $R = 200\,\Omega$

Current in the purely resistive AC circuit $I = \frac{V}{R}$

∴ rms value of current

$$I_{rms} = \frac{V_{rms}}{R} = \frac{220}{200} = 1.1 \text{ A}$$

In purely resistive AC circuit, current and voltage are in phase. Therefore phase difference is zero.

2. Pure capacitor in an AC circuit (Capacitive circuit or *C*-circuit)

Let us consider a capacitor with capacitance C be connected to an AC source with emf having instantaneous value.

$$E = E_0 \sin\omega t \quad \text{...(i)}$$

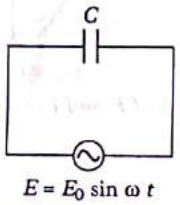


Fig. 7.14 An AC source connected to a capacitor

Due to this emf, charge will be produced and it will charge the plates of capacitor with positive and negative charge. If potential difference across the plates of capacitor is V, then

$$V = \frac{q}{C} \text{ or } q = CV$$

The instantaneous value of current in the circuit,

$$I = \frac{dq}{dt} = \frac{d}{dt}(CE) \qquad [\because V = E]$$

$$= \frac{d}{dt}(CE_0 \sin\omega t) \qquad [\because E = E_0 \sin\omega t]$$

$$= CE_0 \cos\omega t \times \omega$$

$$= \frac{E_0}{1/\omega C}\cos\omega t$$

$$[\because \cos\omega t = \sin(\pi/2 + \omega t)]$$

$$\Rightarrow \quad I = \frac{E_0}{1/\omega C}\sin(\omega t + \pi/2) \quad \text{...(ii)}$$

I will be maximum when $\sin(\omega t + \pi/2) = 1$, so that $I = I_0$

where peak value of current, $I_0 = \frac{E_0}{1/\omega C}$

$$\therefore \quad \boxed{I = I_0 \sin(\omega t + \pi/2)} \quad \text{...(iii)}$$

From Eqs. (i) and (iii), it is clear that in a perfect capacitor, the current leads the voltage by a phase angle of $\pi/2$ (90°) or the voltage lags behind the current by a phase angle of $\pi/2$ (90°).

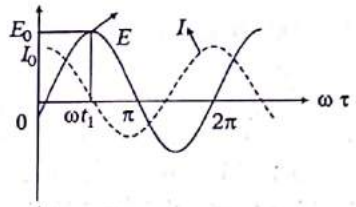


Fig. 7.15 Graph of E and I of a capacitive circuit

Phasor Diagram

The phase representing peak emf E_0 makes an angle ωt in anti-clockwise direction with respect to horizontal axis. As current leads the voltage by 90°, the phasor representing I_0 is turned 90° anti-clockwise with the phasor representing E_0. The projections of these phasors on the vertical axis give instantaneous values of E and I.

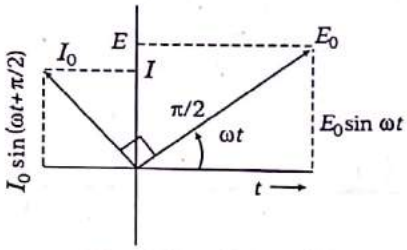


Fig. 7.16 Phasor diagram for purely capacitive circuit

Capacitive Reactance (X_C)

Instantaneous value of alternating current through a capacitor is given by

$$I = \frac{E_0}{1/\omega C}\sin(\omega t + \pi/2) = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

Comparing with Ohm's law we get, $I_0 = \frac{E_0}{1/\omega C}$

$$\boxed{X_C = \frac{1}{\omega C}}$$

where, X_C is called capacitive reactance.

The opposing nature of capacitor to the flow of alternating current is called **capacitive reactance.**

If f is the frequency of the alternating current, then

$$\boxed{X_C = \frac{1}{2\pi f C}}$$

The dimension of capacitive reactance is same as that of resistance and its SI unit is ohm (Ω). The capacitive reactance limits the amplitude of the current in a purely capacitive circuit in the same way as the resistance limits the current in a purely resistive circuit. It is inversely proportional to the capacitance and frequency of the current. Thus, if frequency of AC increases, then its capacitive reactance decreases.

When capacitor is connected to DC source,

$$X_C = \frac{1}{\omega C} = \frac{1}{0} = \infty$$

[$\because$ For DC, $\omega = 2\pi f = 0$, as $f = 0$]

Thus, capacitor blocks DC and acts as open circuit while passing AC of high frequency.

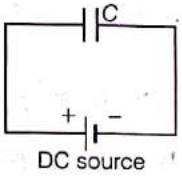


Fig. 7.17 Capacitor connected to a DC source

Variation of capacitive reactance with frequency

The graph of X_C as function of angular frequency ω can be drawn as

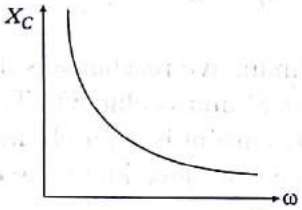


Fig. 7.18 Capacitive reactance X_C, decreases with frequency

Capacitor as high pass filter

The capacitive reactance of a capacitor is inversely proportional both to the capacitance C and to angular frequency ω. The greater the capacitance and the higher the frequency, the smaller is the capacitive reactance X_C. Capacitors tend to pass high frequency current and to block low frequency current, just the opposite of inductors. The device that passes signals of high frequency is called a **high pass filter**.

Example 7.9 *A 60 μF capacitor is connected to a 110 V, 60 Hz AC supply determine the rms value of the current in the circuit.*

Sol. As $\quad I_{rms} = \dfrac{V_{rms}}{X_C}$

where, $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi f C}$

Here, $\quad C = 60\,\mu\text{F} = 60 \times 10^{-6}\text{F}, V_{rms} = 110\text{V}, f = 60\,\text{Hz}$

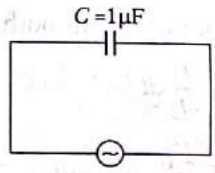
So, $\quad X_C = \dfrac{1}{2 \times \pi \times 60 \times 60 \times 10^{-6}}\,\Omega$

$\therefore \quad I_{rms} = 110 \times 2 \times 3.14 \times 3600 \times 10^{-6}$

$= 2.49\text{ A}$

Example 7.10 *An alternating voltage $V = 200\sqrt{2}\sin(100t)V$ is connected to a 1 μF capacitor through an AC ammeter. What will be the reading of the ammeter?*

Sol. Consider the purely capacitive circuit as shown below.



Given, $\quad V = 200\sqrt{2}\sin(100t)$ ...(i)

Comparing it with general equation, we get

$V = V_0 \sin(\omega t)$ ...(ii)

$V_0 = 200\sqrt{2}\,V, \omega = 100\text{ rad s}^{-1}$

$\therefore$ Reactance $X_C = \dfrac{1}{\omega C} = \dfrac{1}{100 \times 10^{-6}} = 10^4\,\Omega$

Peak value of current, $I_0 = \dfrac{V_0}{X_C} = \dfrac{200\sqrt{2}}{10^4} = 2\sqrt{2} \times 10^{-2}\text{ A}$

rms value of current

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{2\sqrt{2} \times 10^{-2}}{\sqrt{2}} = 20 \times 10^{-3}\text{A} = 20\text{mA}$$

Reading of the ammeter will be same as $I_{rms} = 20$ mA.

3. Pure inductor in an AC circuit (Inductive circuit or *L*-circuit)

Suppose an inductor with self-inductance (L) is connected to AC source with instantaneous emf (E) is given by

$$E = E_0 \sin\omega t \quad \text{...(i)}$$

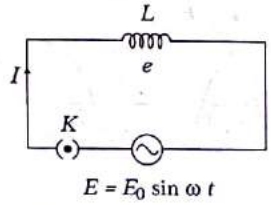


Fig. 7.19 An AC source connected to an inductor

When key is closed, then current begins to grow because magnetic flux linked with it changes and an emf is induced which opposes the applied emf.

According to Lenz's law,

$$e = -L\frac{dI}{dt}$$

where, e is induced emf and $\frac{dI}{dt}$ is the rate of change of current.

To maintain the flow of current in the circuit, applied voltage must be equal and opposite to the induced emf.

$$E = -e$$

$$\therefore \quad E = -\left(-\frac{LdI}{dt}\right) = \frac{LdI}{dt} \quad \text{or} \quad dI = \frac{E}{L}dt$$

Integrating the above equation on both sides

$$\int dI = \int \frac{E}{L}dt$$

$$I = \int \frac{E_0 \sin\omega t}{L}dt \qquad [\because E = E_0 \sin\omega t]$$

$$\Rightarrow \quad I = \frac{E_0}{L}\left[\frac{-\cos\omega t}{\omega}\right] = -\frac{E_0}{\omega L}\cos\omega t$$

$$\left[\because \sin\left(\frac{\pi}{2} - \omega t\right) = \cos\omega t\right]$$

$$\Rightarrow \quad I = -\frac{E_0}{\omega L}\sin(\pi/2 - \omega t)$$

$$\Rightarrow \quad I = \frac{E_0}{\omega L}\sin(\omega t - \pi/2) \qquad \text{...(ii)}$$

If $\sin(\omega t - \pi/2) = \text{maximum} = 1$, then $I = I_0$

where, peak value of current, $I_0 = \frac{E_0}{\omega L}$

$$\Rightarrow \quad \boxed{I = I_0 \sin(\omega t - \pi/2)} \qquad \text{...(iii)}$$

From Eqs. (i) and (iii), it is clear that in a pure inductor, the current lags behind the voltage by a phase angle of $\pi/2$ (90°) or the voltage leads the current by a phase angle of 90° ($\pi/2$).

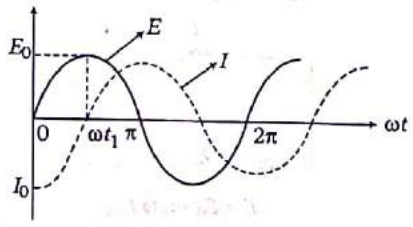


Fig. 7.20 Graph of E and I with time

Phasor Diagram

The phasor representing peak emf E_0 makes an angle ωt in anti-clockwise direction from horizontal axis. As current lags behind the voltage by 90°, so the phasor representing I_0 is turned 90° clockwise with the direction of E_0.

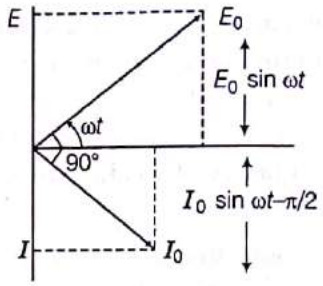


Fig. 7.21 Phasor diagram for purely inductive circuit

Inductive Reactance (X_L)

The opposing nature of inductor to the flow of current is called inductive reactance.

$$\text{As, } I = \frac{E_0}{\omega L}\sin(\omega t - \pi/2) \quad \text{or} \quad I_0 = E_0/\omega L$$

Comparing with Ohm's law, $I_0 = \frac{E_0}{R}$. The quantity ωL is analogous to the resistance and is denoted by X_L.

So, $\quad X_L = \omega L$

X_L is called inductive reactance.

If f is the frequency of AC source, then

$$\boxed{X_L = \omega L = 2\pi f L} \qquad \left[\because \omega = \frac{2\pi}{T} = 2\pi f\right]$$

The dimension of inductive reactance is the same as that of resistance and its SI unit is ohm (Ω). The inductive reactance limits the current in a purely inductive circuit in the same way as the resistance limits the current in a purely resistive circuit.

The inductive reactance is directly proportional to the inductance and to the frequency of the AC current.

Thus if the frequency of AC increases, its inductive reactance also increases.

If inductor is connected to DC source, then for DC

$$f = 0 \qquad \left(\because f = \frac{1}{T}\right)$$

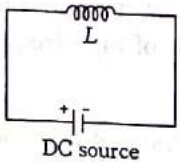


Fig. 7.22 Inductor connected to a DC source

Here, f is frequency.

$$\therefore \quad X_L = 0$$

Therefore, inductor passes DC and blocks AC of very high frequency.

Variation of inductive reactance with frequency

The graph of X_L as function of angular frequency ω can be drawn as shown below clearly X_2 linearly increases with ω.

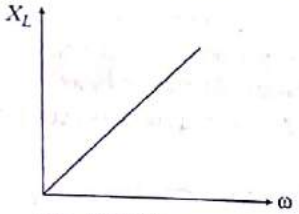


Fig. 7.23 X_L versus ω

Inductor as low pass filter

If an oscillating voltage of a given amplitude V_0 is applied across an inductor, the resulting current will have a smaller amplitude I_0 for larger value of ω. Since, X_L is proportional to frequency, a high frequency voltage applied to the inductor gives only a small current while a lower frequency voltage of the same amplitude gives rise to a larger current. Inductors are used in some circuit applications, such as power supplies and radio interference filters to block high frequencies while permitting lower frequencies to pass through. A circuit device that uses an inductor for this purpose is called a **low pass filter.**

Example 7.11 *An ideal inductor of inductance 50 μH is connected to an AC source of 220 V, 50 Hz. Find the inductive reactance.*

Sol. As inductive reactance, $X_L = \omega L = 2\pi f L$

where, L is inductance $= 50\,\mu\text{H} = 50 \times 10^{-6}$ H

f = source frequency = 50 Hz

$$\therefore \quad X_L = 2 \times \pi \times 50 \times 50 \times 10^{-6}$$

$$= 2\pi \times 25 \times 10^{-4}\,\Omega$$

$$= 5\pi \times 10^{-3}\,\Omega = 5\pi\,\text{m}\Omega$$

Example 7.12 *A 44 mH inductor is connected to 220 V, 50 Hz AC supply. Determine the rms value of the current in the circuit.*

Sol. As $I_{rms} = \dfrac{V_{rms}}{X_L}$

where, $X_L = \omega L = 2\pi f L$ is the reactance of the inductor

Here, $f = 50$ Hz, $L = 44\text{mH} = 44 \times 10^{-3}$ H, $V_{rms} = 220$V

$$\therefore \quad X_L = 2\pi \times 50 \times 44 \times 10^{-3} = 13.82\,\Omega$$

$\therefore$ rms value of current in the circuit $I_{rms} = \dfrac{220}{13.82} = 15.9$A

AC voltage applied to a series *L-C-R* circuit

Suppose that an inductor (L), capacitor (C) and resistor (R) are connected in series to an AC source. I is the current passed through this circuit. As R, L, C are in series, therefore at any instant through the three elements, AC has the same amplitude and phase. Let it be represented by

$$I = I_0 \sin\omega t$$

However, voltage across each element bears a different phase relationship with the current.

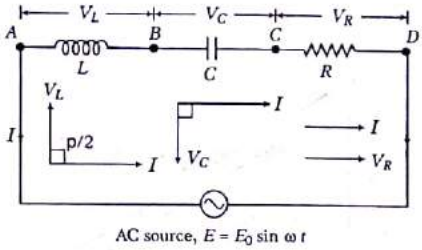


Fig. 7.24 An AC source connected to series L-C-R circuit

$V_L = I_0 X_L$ [V_L is maximum voltage across L]

$V_C = I_0 X_C$ [V_C is maximum voltage across C]

$V_R = I_0 R$ [V_R is maximum voltage across R]

Inside phasor diagrams of each L, C and R are given. To form phasor diagram for series L-C-R circuit, combine all these phasor diagrams.

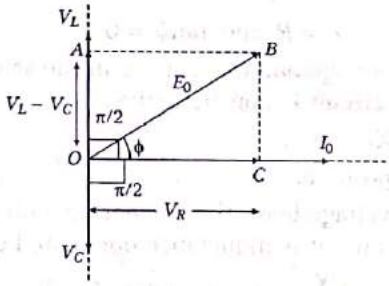


Fig. 7.25 Phasor diagram of a series L-C-R circuit

Since, voltage (V_L) is in upward direction and voltage (V_C) in downward direction, so net voltage upto point A is $V_L - V_C$ (assuming $V_L > V_C$) and net maximum voltage is E_0.

From phasor diagram,

$$OB = \sqrt{(OC)^2 + (CB)^2} = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$E_0 = \sqrt{(I_0 R)^2 + (I_0 X_L - I_0 X_C)^2} \quad [\because OB = E_0]$$

$$E_0 = I_0 \sqrt{R^2 + (X_L - X_C)^2}$$

$$\therefore \quad \boxed{Z = \frac{E_0}{I_0} = \sqrt{R^2 + (X_L - X_C)^2}}$$

Here, Z is called **impedance.**

Impedance

It is the total resistance of a circuit applied in the path of alternating current. It is given by

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

From phasor diagram, it is clear that voltage leads the current by angle ϕ.

$\therefore$ From ΔOCB,

$$\tan\phi = \frac{CB}{OC} = \frac{V_L - V_C}{V_R} = \frac{I_0 X_L - I_0 X_C}{I_0 R}$$

$$\tan\phi = \frac{X_L - X_C}{R}$$

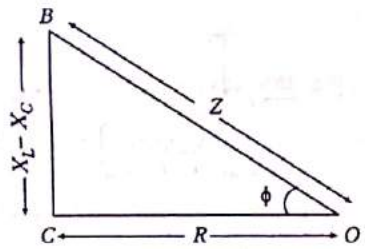


Fig. 7.26 Impedance triangle of an L-C-R circuit

$\therefore$ The alternating emf in the series L-C-R circuit would be represented by $E = E_0 \sin(\omega t + \phi)$.

Special Cases

(i) When $X_L = X_C$,

$\therefore$ $Z = R$ and $\tan\phi = 0$ $[\because \phi = 0°]$

Hence, voltage and current are in the same phase. The AC circuit is non-inductive.

(ii) When $X_L > X_C$,

$\tan\phi$ is positive.

Hence, voltage leads the current by a phase angle ϕ. The AC circuit is inductance dominated circuit.

(iii) When $X_C > X_L$,

$\tan\phi$ is negative.

Hence, voltage lags behind the current by a phase angle ϕ. The AC circuit is capacitance dominated circuit.

The variation of Z with frequency, ω is shown in figure below. At $\omega = \omega_0$, $X_L = X_C$, $Z = R$ = minimum.

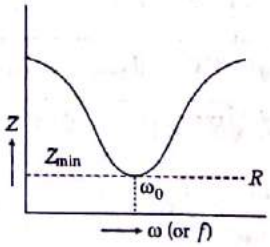


Fig. 7.27 Z vs ω for a series L-C-R circuit

Reduction of *L-C-R* Circuit to *L-C, R-C, L-R* Circuit

The general formula of L-C-R series circuit can be reduced to apply it on L-C, R-C, L-R, circuit

L-C circuit For L-C-R circuit, impedance $Z = \sqrt{R^2 + (X_L - X_C)^2}$

For C-C circuit, there is no R, so, we can ignore terms related to R.

For L-C circuit, $Z = \sqrt{(X_L - X_C)^2}$ or $(X_L - X_C)$

- If $X_L > X_C$ then circuit will behave like pure inductor circuit and voltage will lead by current by $\pi/2$.
- If $X_L < X_C$, then circuit will behave like pure capacitor circuit and voltage will lag behind the current by $\pi/2$.
- If $X_L = X_C$, then $Z = 0$ in this condition, current in the circuit will be maximum.

R-C circuit For L-C-R circuit, impedance $Z = \sqrt{R^2 + (X_L - X_C)^2}$

For R-C circuit there is no L, so we can ignore the term related to L.

$\therefore$ For R-C circuit $Z = \sqrt{R^2 + X_C^2}$

The phase difference between current and emf

$$\tan\phi = \frac{X_C}{R}$$

Here, the current leads the voltage by ϕ

$\therefore$ $I = I_0 \sin(\omega t + \phi)$

L-R circuit In L-C-R circuit, impedance $Z = \sqrt{R^2 + (X_L - X_C^2)}$

In L-R circuit there is no C, so we can ignore the term related to C.

For L-R circuit $Z = \sqrt{R^2 + X_L^2}$

The phase difference between current and emf $\tan\phi = X_L/R$

Here the emf leads the current by ϕ.

$\therefore$ $I = I_0 \sin(\omega t - \phi)$

(i) Resonance

In series L-C-R circuit, when phase (ϕ) between current and voltage is zero, the circuit is said to be **is resonance.**

As applied frequency increases, then

$$X_L = \omega L, \ X_L \text{ increases}$$

and

$$X_C = \frac{1}{\omega C}, \ X_C \text{ decreases.}$$

At some frequency (ω_r), $X_L = X_C$

$$\Rightarrow \quad X_L = \omega_0 L \text{ and } X_C = \frac{1}{\omega_0 C}$$

The frequency at which X_C and X_L becomes equal is called **resonant frequency.**

$$\therefore \quad \omega_0 L = \frac{1}{\omega_0 C} \text{ or } \omega_0^2 = \frac{1}{LC} \text{ or } (2\pi f_0)^2 = \frac{1}{LC}$$

[$\because \omega_0 = 2\pi\nu_r$, where f_0 is resonating frequency]

$$2\pi f_0 = \frac{1}{\sqrt{LC}} \text{ or } f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Resonant frequency is independent of the resistance of the circuit. At resonating frequency,

$$Z = R = \text{Minimum}$$

$$\therefore \quad I = \frac{V}{Z} = \text{Maximum}$$

Since, Z is minimum, therefore I will be maximum.

These circuits are used for voltage amplification and as selector circuits in wireless telegraphy.

Response curves of series *L-C-R* circuit

The impedance of an *L-C-R* circuit depends on the frequency. The dependance is shown in figure. The frequency is taken on logarithmic scale because of its wide range.

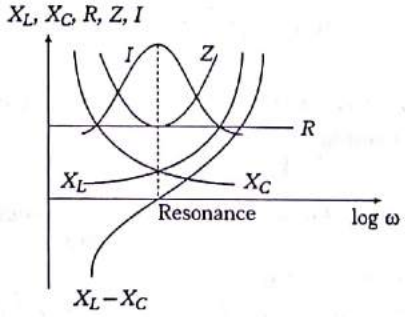


Fig. 7.28 Impedance (Z) vs log ω graph for a series L-C-R circuit

From the figure, we can see that at resonance,

(i) $X_L = X_C$ or $\omega_0 = \dfrac{1}{\sqrt{LC}}$

(ii) $Z = Z_{min} = R$ and

(iii) I is maximum.

Quality factor (*Q*-factor)

It is the measure of sharpness of the resonance of an *L-C-R* circuit. It is defined as, the ratio of voltage developed across the inductance or capacitance at resonance to the impressed voltage, which is the **voltage applied across *R***.

$$Q\text{-factor} = \frac{\text{Voltage across } L \text{ (or } C)}{\text{Voltage across } R}$$

$$\boxed{Q\text{-factor} = \frac{V_L \text{ or } V_C}{V_R} = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

$$= \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC}$$

It is also defined as,

$$Q\text{-factor} = 2\pi \times \frac{\text{Maximum energy stored}}{\text{Energy dissipation}}$$

Q is just a number having no dimensions, it can also be called **voltage multiplication factor** of the circuit.

From the above relation, it is clear that as R increases, (or current decreases) Q-factor of the circuit decreases which can be shown in graph below.

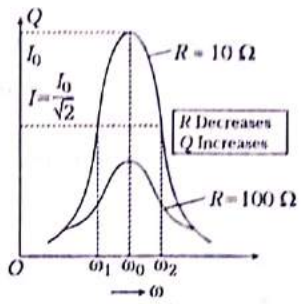


Fig. 7.29 Q vs ω graph of an L-C-R circuit

The electronic circuit with high Q values would respond to a very narrow range of frequencies and *vice-versa*. The higher value of Q, the narrower and sharper is the resonance and greater is the current as shown below.

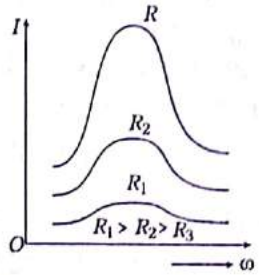


Fig. 7.30 I vs ω graph of an L-C-R circuit

Also, Q-factor or $Q = \dfrac{\omega_0}{\omega_1 - \omega_2}$

where, ω_1 and ω_2 are frequencies when current decreases to 0.707 times of the peak value of current (Fig. 7.30).

We may write, $\omega_1 = \omega_0 + \Delta\omega \Rightarrow \omega_2 = \omega_r - \Delta\omega$

The difference $\omega_1 - \omega_2 = 2\Delta\omega$ is often called the bandwidth of the circuit.

The quantity $\left(Q = \dfrac{\omega_0}{2\Delta\omega}\right)$, is regarded as a measure of **sharpness of resonance**, *i.e.*, Q-factor is defined as the ratio of resonant angular frequency to bandwidth of the circuit.

The smaller the bandwidth ($\Delta\omega$), the sharper or narrower is the resonance.

Note

(i) *Q*-factor denotes the sharpness of tuning.

(ii) High *Q*-factor indicates lower rate of energy loss.

(iii) Higher value of *Q*-factor indicates sharper peak in the current.

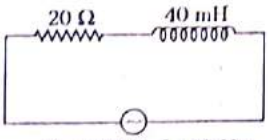
(iv) $R = 0$, *Q*-factor = infinity

Example 7.13 *A sinusoidal voltage of frequency 60 Hz and peak value 150 V is applied to a series L-R circuit, where $R = 20\,\Omega$ and $L = 40$ mH.*

(a) Compute T, ω, X_L, Z and ϕ

(b) Compute the amplitudes of current, V_R, V_L and V_0.

Sol. (a) Consider a series L-R circuit as shown below



$V_0 = 150$ V, $f = 60$ Hz

As $\quad T = \frac{1}{f} = \frac{1}{60}$ s

$\therefore \quad \omega = 2\pi f = 377 \text{ rads}^{-1}$

$\because \quad X_L = \omega L = (377)(0.040) = 15.08\,\Omega$

$\Rightarrow \quad Z = \sqrt{X_L^2 + R^2} = 25.05\,\Omega$

Phase angle, $\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}(0.754) = 37°$

(b) Amplitudes (maximum value) are,

$$I_0 = \frac{V_0}{Z} = \frac{150}{25.05} \approx 6\text{A}$$

$\Rightarrow \quad (V_0)_R = I_0 R = 120$ V and $(V_0)_L = I_0 X_L = 90.5$ V

$\therefore \quad V_0 = \sqrt{(V_0)_R^2 + (V_0)_L^2} = \sqrt{(120)^2 + (90.5)^2} = 150.3$ V

Example 7.14 *A 100 Ω resistance is connected in series with a 4 H inductor. The voltage across the resistor is, $V_R = (2.0V)\sin(10^3 \text{ rads}^{-1})$.*

(i) Find the expression of circuit current.

(ii) Find the inductive reactance.

(iii) Derive an expression for the voltage across the inductor.

Sol. (i) Current in the circuit $I = \frac{V_R}{R} = \frac{(2.0\text{ V})\sin(10^3 \text{ rad s}^{-1})t}{100}$

$= (2.0\times 10^{-2}\text{ A})\sin(10^3 \text{ rad s}^{-1})t$

(ii) Inductive reactance $X_L = \omega L = (10^3 \text{ rad s}^{-1})(4\text{ H})$

$= 4.0\times 10^3\,\Omega$

(iii) The amplitude of voltage across inductor,

$$V_0 = I_0 X_L = (2.0\times 10^{-2}\text{ A})(4.0\times 10^3\,\Omega) = 80\text{ V}$$

In an AC, voltage across the inductor leads the current by 90° or $\frac{\pi}{2}$ rad. Hence,

$$V_L = V_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= (80\text{ V})\sin\left\{(10^3 \text{ rad s}^{-1})t + \frac{\pi}{2}\text{ rad}\right\}$$

Note The amplitude of voltage across the resistor (= 2.0 V) is not same as the amplitude of the voltage across the inductor (= 80 V), even though the amplitude of the current through both devices is the same.

Example 7.15 *An alternating emf 200 virtual volts at 50 Hz is connected to a circuit of resistance 1 Ω and inductance 0.01 H. What is the phase difference between the current and the emf in the circuit? Also find the virtual current in the circuit.*

Sol. In case of an AC, the voltage leads the current in phase by an angle,

$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$$

Here, $\quad X_L = \omega L = (2\pi f L) = (2\pi)(50)(0.01) = \pi\,\Omega$

and $\quad R = 1\,\Omega$

$\therefore \quad \phi = \tan^{-1}(\pi) \approx 72.3°$

Further, $\quad I_{rms} = \frac{V_{rms}}{|Z|} = \frac{V_{rms}}{\sqrt{R^2 + X_L^2}}$

Substituting the values we have,

$$I_{rms} = \frac{200}{\sqrt{(1)^2 + (\pi)^2}} = 60.67\text{ A}$$

Example 7.16 *A resistance and inductance are connected in series across a voltage,*

$$V = 283 \sin 314t$$

The current is found to be $4\sin\left(314t - \frac{\pi}{4}\right)$. Find the values of the inductance and resistance.

Sol. In L-R series circuit current lags the voltage by an angle,

$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$$

Here, $\quad \phi = \frac{\pi}{4}$

$\therefore \quad X_L = R$ or $\omega L = R \qquad [\omega = 314 \text{ rad s}^{-1}]$

$\therefore \quad 314\,L = R \qquad$ ...(i)

Further, $\quad V_0 = I_0|Z|$

$\therefore \quad 283 = 4\sqrt{R^2 + X_L^2}$

or $\quad R^2 + (\omega L)^2 = \left(\frac{283}{4}\right)^2 = 5005.56$

or $\quad 2R^2 = 5005.56 \qquad [\text{as } \omega L = R]$

$\therefore \quad R = 50\,\Omega$

and from Eq. (i), $\quad L = 0.16$ H

Example 7.17 *A long solenoid connected to a 12 V DC source passes a steady current of 2 A. When the solenoid is connected to an AC source of 12 V at 50 Hz, the current flowing is 1A. Calculate inductance of the solenoid.*

Sol. Remember that the solenoid mentioned here is not just a solenoid but has the resistance also due to its wire material. Now, in a DC, the solenoid doesn't offer any reactance after a few milliseconds, however in the beginning it offers infinite resistance. So, the current by a DC into an inductor is, $I = \frac{V}{R}$

or $\quad R = \frac{12}{2} = 6\,\Omega$

But when an AC is connected to the same inductor, the reactance will also come into picture.

So, $$I = \frac{V}{\sqrt{R^2 + \omega^2 L^2}}$$

Here, $I = 1\text{A}, V = 12\text{ V}, R = 6\Omega$

$\omega = 2\pi 50 = 100\,\pi\,[\text{rads}^{-1}], L = ?$

or $$10 = \frac{12}{\sqrt{6^2 + (100\,\pi L)^2}} \Rightarrow L = 33\text{ mH}$$

Example 7.18 *A 100 μF capacitor in series with a 40 Ω resistance is connected to 110 V, 60 Hz supply.*

(a) What is the maximum current in the circuit?

(b) What is the time lag between the current maximum and the voltage maximum?

Sol. (a) Peak value of voltage $E_0 = \sqrt{2}\,E_{rms} = \sqrt{2} \times 110\text{ V}$

Let I_0 = maximum current in the circuit

$$\therefore \quad I_0 = \frac{E_0}{Z} = \frac{E_0}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}$$

or $$\frac{155.1}{\sqrt{1600 + 703.84}} = 3.23\text{ A}$$

(b) In an *R-C* circuit, the voltage lags behind the current by the phase angle ϕ given by

$$\tan\phi = \frac{X_C}{R} = \frac{1}{\omega CR} = \frac{1}{120\pi \times 10^{-4} \times 40}$$

$$= 0.6628 = \tan 33.5°$$

$$\therefore \quad \phi = 33.5° = 33.5 \times \frac{\pi}{180}\text{ rad}$$

∴ Time between the current maximum and the voltage maximum is given by

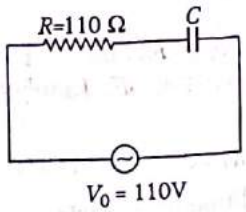
$$t = \frac{\phi}{\omega} = \frac{\frac{33.5\,\pi}{180}}{120\pi}$$

$$= 1.55 \times 10^{-3}\text{ s}$$

$$= 1.55\text{ ms}$$

Example 7.19 *A circuit containing of a capacitor and an active resistance $R = 110\ \Omega$ connected in series is fed an alternating voltage with amplitude $V_0 = 110V$. In this case, the amplitude of current is equal to $I_0 = 0.50A$. Find the phase difference between the current and the voltage fed.*

Sol. Consider an *RC* circuit as shown below



Given, $I_0 = 0.50\text{A}$

$$\therefore \quad I_0 = \frac{V_0}{Z} \Rightarrow 0.5 = \frac{110}{Z} \Rightarrow Z = 220\Omega$$

$$\Rightarrow \quad Z^2 = R^2 + X_C^2 \Rightarrow (220)^2 = (110)^2 + X_C^2$$

$$\Rightarrow \quad X_C^2 = (220)^2 - (110)^2 = 330 \times 110$$

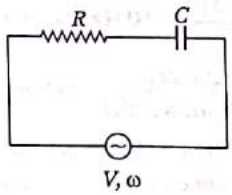
$$\Rightarrow \quad X_C = 110\sqrt{3}\,\Omega$$

Now we can write

$$\tan\phi = \frac{X_C}{R} = \frac{110\sqrt{3}}{110} = \sqrt{3} \Rightarrow \phi = 60°$$

Example 7.20 *An AC voltage source is applied across an R-C circuit. Angular frequency of the source is ω, resistance is R and capacitance is C. The current registered is I. If now the frequency of source is changed to ω/2 (but maintaining the same voltage), the current in the circuit is found to be two third. Calculate the ratio of reactance to resistance at the original frequency ω.*

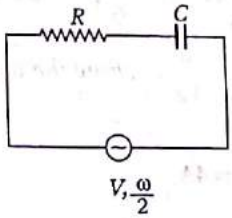
Sol. Consider the *R-C* circuit as shown below.



$$\because X_C = \frac{1}{\omega C} \Rightarrow Z = \sqrt{R^2 + X_C^2}$$

$$\therefore \quad I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + X_C^2}}$$

When frequency is changed to $\frac{\omega}{2}$.



The value of new capacitive reactance is

$$X_C' = \frac{1}{(\omega/2)C} = \frac{2}{\omega C} = 2X_C$$

New impedance, $Z' = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + (2X_C)^2}$

$$\Rightarrow \quad I' = \frac{V}{\sqrt{R^2 + 4X_C^2}}$$

It is given that $I' = \frac{2}{3}I$

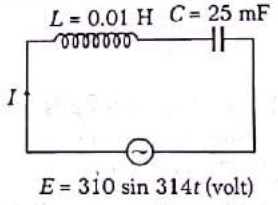
$$\Rightarrow \frac{2}{3} \times \frac{V}{\sqrt{R^2 + X_C^2}} = \frac{V}{\sqrt{R^2 + 4X_C^2}}$$

$$\Rightarrow \quad 4R^2 + 16X_C^2 = 9R^2 + 9X_C^2$$

$$\Rightarrow \quad 7X_C^2 = 5R^2 \Rightarrow \frac{X_C}{R} = \sqrt{\frac{5}{7}}$$

Example 7.21 *A coil of inductance 0.01H is connected in series with a capacitor of capacitance 25 μF with an AC source whose emf is given by $E = 310 \sin 314t$ (volt). What is the reactance of the circuit?*

Sol. Consider a series *L-C circuit as shown below*



Given, $E = 310 \sin 314t$ (volt)

Comparing it with general equation, $E = E_0 \sin \omega t$

We get,

$$\omega = 2\pi f = 314$$

or $$f = \frac{314}{2\pi} = 50\,\text{Hz}$$

∴ Inductive reactance,

$$X_L = \omega L = 2\pi f.L$$
$$= 314 \times 0.01 = 3.14\,\Omega$$

Capacitive reactance

$$X_C = \frac{1}{\omega C} = \frac{1}{314 \times 25 \times 10^{-6}} = 127.3\,\Omega$$

∴Net Reactance,

$$X = X_C - X_L = (127.3 - 3.14)\Omega = 124.16\,\Omega$$

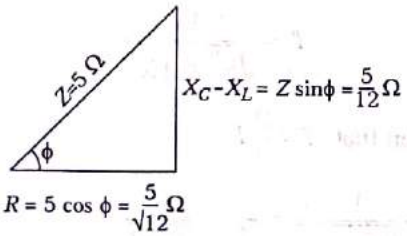
Example 7.22 *A series LCR circuit is connected across a source of emf $E = 20 \sin\left(100\pi t - \frac{\pi}{6}\right)$. The current from the supply is $I = 4 \sin\left(100\pi t + \frac{\pi}{12}\right)$. Draw the impedance triangle for the circuit.*

Sol. Given, $E_0 = 20\text{V}, I_0 = 4\text{A}$

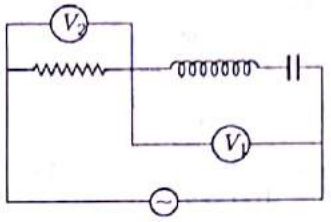
$$\therefore \quad Z = \frac{E_0}{I_0} = 5\,\Omega$$

The current leads emf by $\phi = \frac{\pi}{12} - \left(-\frac{\pi}{6}\right) = \frac{\pi}{4}$

The impedance triangle can be drawn as



Example 7.23 *If the reading of voltmeter V_1 is 30 V, what is the reading of voltmeter V_2?*



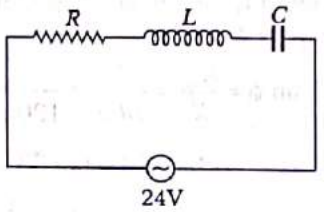
Sol. AC voltmeter reads rms value of voltage. RMS voltage applied $= \frac{V_0}{\sqrt{2}} = 40\text{V}$

Reading of voltmeter V_2 is X, then $V_{rms} = \sqrt{V_1^2 + V_2}$

or $$\sqrt{X^2 + 30^2} = 40\text{V} \Rightarrow X = 26.45\text{V}$$

Example 7.24 *An inductor coil, a capacitor and an AC source of rms voltage 24 V are connected in series. When the frequency of the sources is varied, a maximum rms current of 6 A is observed. If this inductor coil is connected to a battery of emf 12 V and internal resistance 4.0 Ω. What will be the current?*

Sol. Consider an *L-C-R* series circuit as shown below

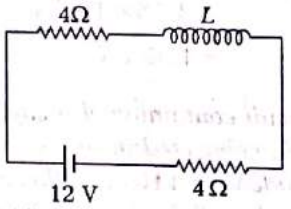


Maximum current will occur at series resonance

At series resonance, $Z_{min} = R_{max} = 6\text{A}$

$$\Rightarrow \quad I_{max} = \frac{V}{Z_{min}} \Rightarrow 6 = \frac{24}{R} \Rightarrow R = 4\,\Omega$$

Now, this inductor coil is connected with a battery of emf 12 V and internal resistance 4 Ω



In DC circuit, inductor offers zero resistance,

∴ Current, $I = \frac{12}{4+4} = 1.5\text{A}$

Example 7.25 *A coil of inductance 0.4 mH is connected to a capacitor of capacitance 400 pF. To what wavelength, is this circuit tuned?*

Sol. Given, $L = 0.4\,\text{mH} = 0.4 \times 10^{-3}\,\text{H}$, $C = 400\,\text{pF} = 4 \times 10^{-10}\text{F}$

This is the condition of resonance.

∴ At resonsnace, frequency,

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\times 3.14} \times \frac{1}{\sqrt{0.4\times 10^{-3}\times 4\times 10^{-10}}}$$

$$= \frac{10^7}{6.28\times 4}\text{ Hz}$$

If the speed of electromagnetic wave is v then for tuning the circuit wavelength required would be

$$\lambda = \frac{v}{f} = \frac{3\times 10^8 \text{ms}^{-1}}{\frac{10^7}{6.28\times 4\text{ Hz}}} = 753.6 \text{ m}$$

Example 7.26 *A 200 km telephone wire has capacity of 0.014 μF km^{-1}. If it carries an alternating current of frequency 50 kHz, what should be the value of an inductance required to be connected in series so that impedance is minimum?*

Sol. Capacitance, $C = 0.014\times 200\,\mu\text{F} = 2.8\times 10^{-16}\text{F}$

Frequency, $f = 50\text{k Hz} = 50\times 10^3$ cycles s^{-1}

The impedance will be minimum at resonant frequency, when applied frequency will be same as resonant frequency ($f_0 = f$).

The resonant frequency, $f_0 = \frac{1}{2\pi\sqrt{LC}} \Rightarrow L = \frac{1}{4\pi^2 f_0^2 C}$

$$L = \frac{1}{4\times\frac{22}{7}\times\frac{22}{7}\times(50\times 10^3)^2\times(2.8\times 10^{-6})} = 0.36\times 10^{-5}\text{ H}$$

Example 7.27 *Find the voltage across the various elements, i.e., resistance, capacitance and inductance which are in series and having values 1000 Ω, 1 μF and 2.0 H respectively. Given emf as,*

$$V = 100\sqrt{2}\sin 1000\,t \text{ V}$$

Sol. The rms value of voltage across the source, $V_{rms} = \frac{V_0}{\sqrt{2}}$

Comparing the given equation with general equation,

$$V_0 = 100\sqrt{2},\ \omega = 1000 \text{ rad s}^{-1}$$

Also, $R = 1000\,\Omega,\ C = 1\,\mu\text{F} = 1\times 10^{-6}\text{F}$

$L = 2\text{H}$

$$V_{rms} = \frac{100\sqrt{2}}{\sqrt{2}} = 100 \text{ V}$$

$$\therefore\ I_{rms} = \frac{V_{rms}}{|Z|} = \frac{V_{rms}}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V_{rms}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$= \frac{100}{\sqrt{(1000)^2 + \left(1000\times 2 - \frac{1}{1000\times 1\times 10^{-6}}\right)^2}}$$

$$= 0.0707 \text{ A}$$

The current will be same everywhere in the circuit, therefore,

PD across resistor

$$V_R = I_{rms} R = 0.0707\times 1000 = 70.7 \text{ V}$$

PD across inductor

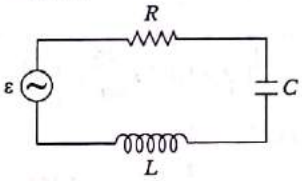
$$V_L = I_{rms} X_L = 0.0707\times 1000\times 2 = 141.4 \text{ V}$$

and PD across capacitor

$$V_C = I_{rms} X_C = 0.0707\times\frac{1}{1\times 1000\times 10^{-6}} = 70.7 \text{ V}$$

Note The rms voltages do not add directly as, $V_R + V_L + V_C = 282.8$ V which is not the source voltage 100 V. The reason is that these voltages are not in phase and can be added by vector or by phasor algebra.

Example 7.28 *Figure here, shows a series L-C-R circuit connected to a variable frequency 230 V source. L = 5.0 H, C = 80 μF and R = 40 Ω.*



(a) Determine the source frequency which drives the circuit in resonance.

(b) Obtain the impedance of the circuit and the amplitude of current at the resonating frequency.

(c) Determine the rms potential drops across the three elements of the circuit. Show that the potential drop across the L-C combination is zero at the resonating frequency.

Sol. (a) If ω_0 be the resonant angular frequency = source frequency at resonance, then

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5\times 80\times 10^{-6}}} = 50 \text{ rad s}^{-1}$$

$$f_0 = \frac{\omega_0}{2\pi} = \frac{50}{2\times 3.142} = 7.96 \text{ Hz}$$

(b) At resonance, $Z = R = 40\,\Omega$

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{230}{40} = 5.75 \text{ A and } I_0 = \frac{V_0}{Z} = \frac{\sqrt{2}V_{rms}}{Z}$$

$$= \frac{\sqrt{2}\times 230}{40} = 8.13 \text{ A}$$

(c) The rms potential drop across R is given by

$$V_{rms} = I_{rms} R = 5.75\times 40 = 230 \text{ V}$$

The rms potential drop across L is given by

$$V_{L\,rms} = I_{rms} X_L = I_{rms}(\omega_0 L)$$

$$= 5.75\times 50\times 5 = 1437.5 \text{ V}$$

The rms potential drop across C is given by

$$V_{rms} = I_{rms} X_C = I_{rms}\times\frac{1}{\omega_0 C} = 5.75\times\frac{1}{50\times 80\times 10^{-6}}$$

$$= 1437.5 \text{ V}$$

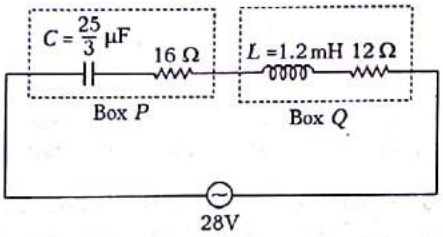
The rms potential drop across L-C is given by

$$V_{LC} = V_L - V_C = 1437.5 - 1437.5 = 0$$

Example 7.29 *A box P and a coil Q are connected is series with an AC source of variable frequency. The emf of the source is constant at 28 V. The frequency is so adjusted that the maximum current flows in P and Q. Find*

(a) impedance of P and Q at this frequency

(b) voltage across P and Q.



Sol. Current is amximum *i.e.*, case of series resonance

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1.2\times10^{-3}\times\frac{25}{3}\times10^{-6}}} = 10^4 \text{ rad s}^{-1}$$

At resonance, $X_C = X_L$

$$X_C = \frac{1}{\omega_0 C} = \frac{1}{10^4\times\frac{25}{3}\times10^{-6}} = 12\,\Omega = X_L$$

Resistance R_1 and R_2 are in series, so $R_{net} = R_1 + R_2$

$$\therefore\ I = \frac{V}{R_1+R_2} = \frac{28}{16+12} = \frac{28}{28} = 1\text{A}$$

(a) $Z_P = \sqrt{(16)^2 + (X_C)^2} = \sqrt{(16)^2 + (12)^2} = 20\,\Omega$

$Z_Q = \sqrt{(12)^2 + (X_L)^2} = \sqrt{(12)^2 + (12)^2} = 12\sqrt{2}\,\Omega$

(b) $V_P = I Z_P = 1\times 20 = 20\text{V}$

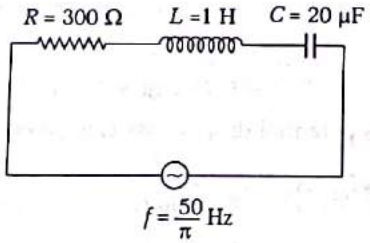
$V_Q = I Z_Q = 1\times 12\sqrt{2} = 12\sqrt{2}\text{ V}$

Example 7.30

(a) In a series L-C-R circuit with an AC source, $R = 300\,\Omega$, $C = 20\,\mu F$, $L = 1.0\,H$, $V_0 = 50\sqrt{2}\,V$ and $f = 50/\pi$ Hz. Find (i) the rms current in the circuit and (ii) the rms voltage across each element.

(b) Consider the situation of the previous part. Find the average electric field energy stored in the capacitor and the average magnetic field energy stored in the coil.

Sol. Consider the series *L-C-R* circuit as shown below



It is given that $V_0 = 50\sqrt{2}$ V

and $\omega = 2\pi f = 2\pi\times\frac{50}{\pi} = 100 \text{ rad s}^{-1}$

$\therefore\quad X_L = \omega L = 100\times1 = 100\,\Omega$

$\therefore\quad X_C = \frac{1}{\omega C} = \frac{1}{100\times20\times10^{-6}} = 500\,\Omega$

$\Rightarrow\quad Z = \sqrt{R^2 + (X_C - X_L)^2}$

$= \sqrt{(300)^2 + (500-100)^2} = 500\,\Omega$

$\therefore$ Peak value of current $I_0 = \frac{V_0}{Z} = \frac{50\sqrt{2}}{500} = 0.1\sqrt{2}\text{ A}$

(i) $\therefore\ I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{0.1\sqrt{2}}{\sqrt{2}} = 0.1\text{A}$

(ii) rms voltage across each element is

$V_R = I_{rms}R = 0.1\times300 = 30\text{V}$

$V_L = I_{rms}X_L = 0.1\times100 = 10\text{V}$

and $V_C = I_{rms}X_C = 0.1\times500 = 50\text{V}$

(b) The average electric field energy stored in capacitor is given by

$$U_C = \frac{1}{2}CV_C^2 = \frac{1}{2}\times20\times10^{-6}\times(50)^2 = 25\text{mJ}$$

also the average magnetic field energy stored in the coil is given by

$$U_L = \frac{1}{2}LI_{rms}^2 = \frac{1}{2}\times1\times(0.1)^2 = 5\text{mJ}$$

Example 7.31 *A series L-C-R circuit containing a resistance of $120\,\Omega$ has angular frequency 4×10^5 rads^{-1}. At resonance the voltages across resistance and inductance are 60 V and 40 V respectively. Find the values of L and C. At what angular frequency the current in the circuit lags the voltage by $\pi/4$?*

Sol. At resonance, $X_L - X_C = 0$ and $Z = R = 120\,\Omega$

$\therefore\quad I_{rms} = \frac{(V_R)_{rms}}{R} = \frac{60}{120} = \frac{1}{2}\text{A}$

Also, $I_{rms} = \frac{(V_L)_{rms}}{\omega L}$

$\therefore\quad L = \frac{(V_L)_{rms}}{\omega I_{rms}} = \frac{40}{(4\times10^5)\left(\frac{1}{2}\right)}$

$= 2.0\times10^{-4}\text{ H} = 0.2\text{ mH}$

The resonance frequency is given by, $\omega = \frac{1}{\sqrt{LC}}$ or

$C = \frac{1}{\omega^2 L}$ Substituting the values, we have

$$C = \frac{1}{(4\times10^5)^2\,(2.0\times10^{-4})} = 3.125\times10^{-8}\text{ F}$$

Current lags the voltage by 45°, when $\tan 45° = \frac{\omega L - \frac{1}{\omega C}}{R}$

Substituting the values of *L, C, R* and tan 45°

We get, $\omega = 8\times10^5 \text{ rad s}^{-1}$

Parallel circuit (Rejector circuit)

Let us consider an alternating source connected across an inductance L in parallel with a capacitor C.

The resistance in series with the inductance is R and with the capacitor is zero.

(Every inductor has some non-zero internal resistance while in case of a capacitor, it can be neglected).

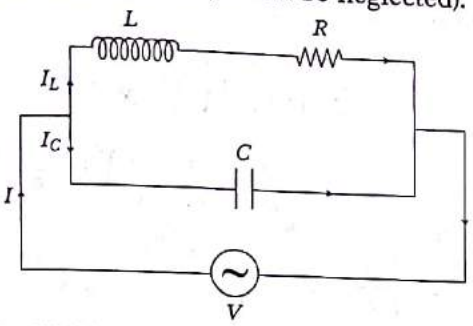


Fig. 7.31 A capacitor (C) is in parallel with series combination of R and L

Let the instantaneous value of emf applied be V and the corresponding currents are I, I_L and I_C. Then,

$$I = I_L + I_C \Rightarrow \frac{1}{Z} = \frac{1}{R + j\omega L} + j\omega C$$

$\frac{1}{Z}$ is known as **admittance** (Y). Admittance is defined as the reciprocal of the impedance. Its unit is mho.

Therefore,

$$Y = \frac{1}{Z} = \frac{R - j\omega L}{R^2 + \omega^2 L^2} + j\omega C$$

$$= \frac{R + j(\omega CR^2 + \omega^3 L^2 C - \omega L)}{R^2 + \omega^2 L^2}$$

∴ The magnitude of the admittance,

$$|Y| = \left|\frac{1}{Z}\right| = \frac{\sqrt{R^2 + (\omega CR^2 + \omega^3 L^2 C - \omega L)^2}}{R^2 + \omega^2 L^2}$$

The admittance will be minimum, when

$$\omega CR^2 + \omega^3 L^2 C - \omega L = 0$$

or

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

It gives the condition of resonance and the corresponding frequency,

$$\boxed{f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}}$$

is known as resonance frequency. At resonance frequency, admittance is minimum or the impedance is maximum. Thus, the parallel circuit does not allow this frequency from the source to pass in the circuit. Due to this reason the circuit with such a frequency is known as **rejector circuit.**

Note If $R = 0$, resonance frequency is $\frac{1}{2\pi\sqrt{LC}}$ same as resonance frequency in series L-C-R circuit or acceptor circuit.

At resonance, the reactive component of **Y** vanishes or **Y** is real. The reciprocal of the admittance is called the **parallel resistor or** the **dynamic resistance.** The dynamic resistance is thus, reciprocal of the real part of the admittance.

∴ Dynamic resistance $= \frac{R^2 + \omega^2 L^2}{R}$

Substituting, $\omega^2 = \frac{1}{LC} - \frac{R^2}{L^2}$ (resonance frequency)

We have, dynamic resistance $= \frac{L}{CR}$

∴ Peak current through the supply $= \frac{V_0}{L/CR} = \frac{V_0 CR}{L}$

The peak current through capacitor $= \frac{V_0}{1/\omega C} = \omega C V_0$. The ratio of the peak current through capacitor and through the supply is known as **Q-factor.**

Thus, $\boxed{Q\text{-factor} = \frac{V_0 \omega C}{V_0 CR/L} = \frac{\omega L}{R}}$

This is basically the measure of current magnification. The rejector circuit at resonance exhibits current magnification of $\omega L/R$, similar to the voltage magnification of the same ratio exhibited by the series acceptor circuit at resonance.

Fig. (a) shows the variation of current I with angular frequency, ω in parallel LC circuit (*i.e.*, $R = 0$) and Fig. (b) shows the variation of impedance Z with ω.

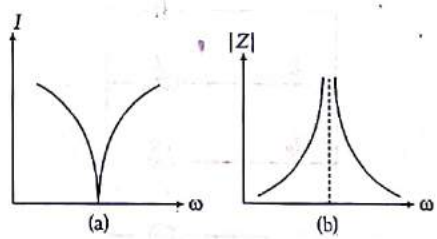


Fig. 7.32 For a parallel LC circuit (a) I vs ω (b) (Z) vs ω

Note At resonance the current through the supply and voltage are in phase, while the current through the capacitor leads the voltage by 90°.

Example 7.32 *A capacitor of capacitance 250 pF is connected in parallel with a choke coil having inductance of 1.6×10^{-2} H and resistance 20 Ω. Calculate*

(a) the resonance frequency and

(b) the circuit impedance at resonance.

Sol. (a) The resonance frequency of a rejector L-C-R circuit is given by,

$$f=\frac{1}{2\pi}\sqrt{\frac{1}{LC}-\frac{R^2}{L^2}}$$

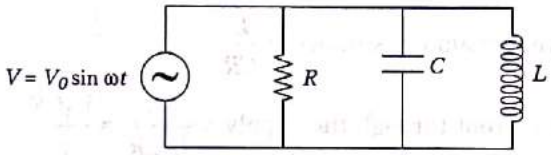
$$=\frac{1}{2\pi}\sqrt{\frac{1}{(1.6\times10^{-2})(250\times10^{-12})}-\frac{(20)^2}{(1.6\times10^{-2})^2}}$$

$$=7.96\times10^4\ \text{Hz}$$

(b) The circuit impedance at resonance is given by,

$$Z=\frac{L}{CR}=\frac{1.6\times10^{-2}}{(250\times10^{-12})(20)}=3.2\times10^6\,\Omega$$

Example 7.33 *For the circuit shown in figure, find the instantaneous current through each element.*



Sol. The three current equations are,

$$V=I_R R,\ V=L\frac{dI_L}{dt}\ \text{and}\ \frac{dV}{dt}=\frac{1}{C}I_C \qquad ...(i)$$

The steady state solutions of Eq. (i) are,

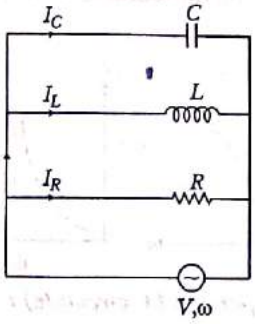
$$I_R=\frac{V_0}{R}\sin\omega t\equiv(I_0)_R\sin\omega t$$

$$I_L=-\frac{V_0}{\omega L}\cos\omega t\equiv-\frac{V_0}{X_L}\cos\omega t\equiv-(I_0)_L\cos\omega t$$

and $$I_C=V_0\omega C\cos\omega t\equiv\frac{V_0}{X_C}\cos\omega t\equiv(I_0)_C\cos\omega t$$

where, the reactance X_L and X_C are as defined.

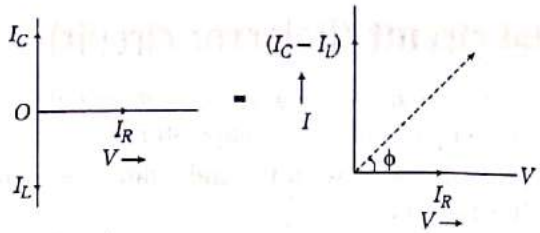
Example 7.34 *AC voltage source (V, ω) is applied across a parallel LC circuit as shown in the figure. Find the impedance of the circuit and phase of current.*



Sol. Current through the resistor, $I_R=V/R$, in phase with voltage

Current through the inductor, $I_L=V/X_L$, lags voltage by $\pi/2$

Current through the capacitor, $I_C=V/X_C$, leads voltage by $\pi/2$



Assuming

Resultant current, $I=[I_R^2+(I_C-I_L)^2]^{1/2}$

$$\Rightarrow\quad \frac{V}{Z}=\left[\left(\frac{V}{R}\right)^2+\left(\frac{V}{X_C}-\frac{V}{X_L}\right)^2\right]^{1/2}$$

$$\Rightarrow\quad \frac{1}{Z}=\left[\frac{1}{R^2}+\left(\frac{1}{X_C}-\frac{1}{X_L}\right)^2\right]^{1/2}$$

Impedance, $$Z=\left[\frac{1}{R^2}+\left(\omega C-\frac{1}{\omega L}\right)^2\right]^{-1/2}$$

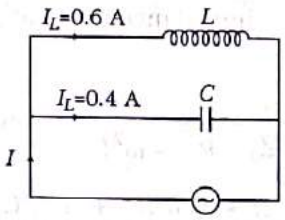
Phase difference, $$\tan\phi=\frac{I_C-I_L}{I_R}=\frac{\frac{1}{X_C}-\frac{1}{X_L}}{1/R}$$

$$\phi=\tan^{-1}\left[R\left(\omega C-\frac{1}{\omega L}\right)\right]$$

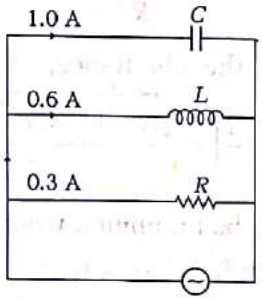
This means current leads the voltage.

Example 7.35 *Find the current drawn from the source in each of the circuits as given below*

(i)



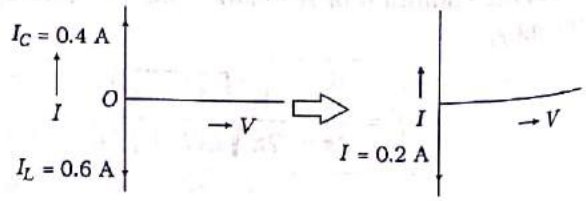
(ii)



Sol. (i) Current in inductor lags the voltage by $\pi/2$.

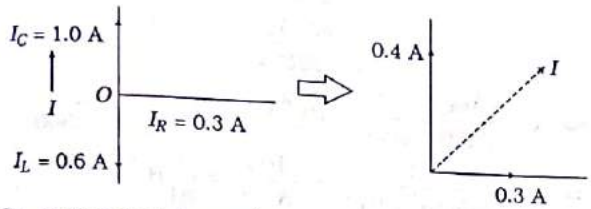
Current in capacitor leads the voltage by $\pi/2$.

The phasor diagram can be drawn as



By vector algebra, current from source = 0.2 A

(ii) Current in resistor is in phase with voltage. The phasor diagram can be drawn as

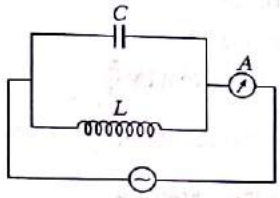


So, net current drawn from the source is

$$i = \sqrt{(0.3)^2 + (0.4)^2} = 0.5\,\text{A}$$

Current from source = 0.5 A

Example 7.36 *An LC circuit ($L = 0.01$H, $C = 1\mu F$) is connected to an AC source of variable frequency. If the frequency is varied from 1 kHz to 2 kHz, then show the consequent variation of impedance by a rough sketch.*



Sol. Impedance (Z) of the crcuit is

$$\frac{1}{Z} = \frac{1}{X_C} - \frac{1}{X_L}$$

At resonance, $X_L = X_C$

$$\Rightarrow \quad \omega L = \frac{1}{\omega C}$$

$$\Rightarrow \quad \omega = \omega_0 = \frac{1}{\sqrt{LC}}$$

$$\Rightarrow \quad f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{(0.01)\times 10^{-6}}}$$

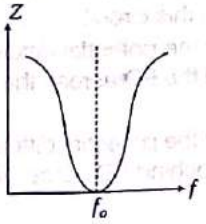
$$= 1592 \text{ Hz} = 1.592 \text{ kHz}$$

At $f = f_0$, impedance is minimum and equal to zero.

At $f < f_0, X_C > X_L$, as frequency increases, impedance increases

$f = f_0, X_C > X_L$ impedance is maximum, $Z = \infty$

when $f > f_0, X_C < X_L$, as frequency increases, impedance decreases as shown below



Example 7.37 *Inductance (L), capacitance (C) and resistance (R) are constanied in a box. When 250 V DC is applied to the terminals of the box, a current of 1.0A flows in the circuit. When an AC source of 250 V_{rms} at 2250 rad sec^{-1} is connected, a current of 1.25 A_{rms} flows. It is observed that the current rises with frequency and becomes maximum at 4500 rad sec^{-1}. Find the values of L, C and R. Draw the circuit diagram.*

Sol. When DC source is applied, current = 1A

In DC circuit, inductor offers zero resistance and capacitor offers infinite resistance, R and C cannot be in series.

$$I = \frac{V_{DC}}{R} \Rightarrow 1 = \frac{250}{R} \Rightarrow R = 250\,\Omega$$

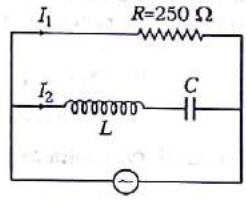
For AC source,

impedance, $Z = \dfrac{V_{rms}}{I_{rms}} = \dfrac{250}{1.25} = 200\,\Omega$

As $Z < R$, R L and C cannot be in series.

R cannot be in series with C as explained earlier.

As current is maximum at resonance and current increases with frequency, R cannot be in series with L. In fact, L and C must be in series.



$$I_1 = \frac{250}{250} = 1\text{ A, in phase with voltage}$$

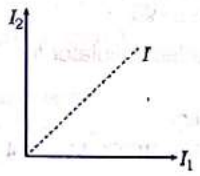
impedance of lower branch,

$$Z' = \left|\left(\omega L - \frac{1}{\omega C}\right)\right|$$

$$\Rightarrow \quad I_2 = \frac{250}{Z'}$$

The phase difference between I_2 and voltage is $\pi/2$.

The angle between I_1 and I_2 is 90°.



$$I_1^2 + I_2^2 = I^2 \Rightarrow (1)^2 + I_2^2 = (1.25)^2$$

$$\Rightarrow \quad I^2 = (1.25)^2 - (1)^2 = (2.25)(0.25)$$

$$\Rightarrow \quad I_2 = 1.5 \times 0.5 = 0.75\text{A}$$

$$\Rightarrow \quad I_2 = \frac{V}{Z'}$$

$$\therefore \quad Z' = \frac{V}{I_2}$$

$$\Rightarrow \quad \left|\omega L - \frac{1}{\omega C}\right| = \frac{250}{0.75} \Rightarrow \left|\frac{\omega^2 LC - 1}{\omega C}\right| = \frac{1000}{3}$$

$$\Rightarrow \quad \left|\frac{(2250)^2 LC - 1}{2250\,C}\right| = \frac{1000}{3} \quad \text{...(i)}$$

At resonance, I_2 is maximum, Z' is minimum

$$\Rightarrow \quad Z' = \omega L - \frac{1}{\omega C} = 0$$

$$\Rightarrow \quad \frac{1}{LC} = \omega^2 = \omega_r^2 = (4500)^2 \quad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get

$$\left|\frac{\left(\frac{2250}{4500}\right)^2 - 1}{2250\,C}\right| = \frac{1000}{3}$$

$$\Rightarrow \quad C = \frac{3/4}{2250} \times \frac{3}{1000} = 10^{-6}\,\text{F} = 1\mu\text{F} \Rightarrow LC = \frac{1}{(4500)^2}$$

$$\Rightarrow \quad L = \frac{1}{(4500)^2 \times 10^{-6}} = \frac{100}{45 \times 45} = \frac{4}{81}\,\text{H}$$

$$\therefore \quad R = 250\,\Omega,\ L = 4/81\,\text{H},\ C = 1\ \mu\text{F}$$

Check point 7.2

1. Ohm's law expressed as $E = IR$
(a) can never be applied to AC
(b) applies to AC in the same manner as to DC
(c) always applies to AC circuits when Z is substituted for R
(d) tells us that $E_{\text{eff}} = 0.707\,(E_{\text{max}})$ for AC

2. An alternating current of rms value 10 A is passed through a $12\,\Omega$ resistor. The maximum potential difference across the resistor is
(a) 20 V (b) 90 V
(c) 169.68 V (d) None of these

3. The reactance of a $25\,\mu$F capacitor at the AC frequency of 4000 Hz is
(a) $\frac{5}{\pi}\Omega$ (b) $\sqrt{\frac{5}{\pi}}\Omega$ (c) $10\,\Omega$ (d) $\sqrt{10}\,\Omega$

4. The capacity of a pure capacitor is 1 F. In DC circuits, its effective resistance will be
(a) zero (b) infinite
(c) $1\,\Omega$ (d) $1/2\,\Omega$

5. In an AC circuit containing only capacitance, the current
(a) leads the voltage by 180°
(b) remains in phase with the voltage
(c) leads the voltage by 90°
(d) lags the voltage by 90°

6. A capacitor is a perfect insulator for
(a) direct current
(b) alternating current
(c) direct as well as alternating current
(d) None of the above

7. In an AC circuit, an alternating voltage $e = 200\sqrt{2}\sin 100t$ V is connected to a capacitor of capacity $1\,\mu$F. The rms value of the current in the circuit is
(a) 100 mA (b) 200 mA (c) 20 mA (d) 10 mA

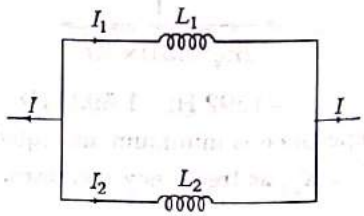
8. The reactance of a coil when used in the domestic AC power supply (220 V, 50 cycles per second) is $50\,\Omega$. The inductance of the coil is nearly
(a) 2.2 H (b) 1.6 H (c) 0.22 H (d) 0.16 H

9. The unit of inductance is
(a) A (V-s)^{-1} (b) JA^{-1} (c) V-sA^{-1} (d) V-As^{-1}

10. In the case of an inductor
(a) voltage lags the current by $\frac{\pi}{2}$
(b) voltage leads the current by $\frac{\pi}{2}$
(c) voltage leads the current by $\frac{\pi}{3}$
(d) voltage leads the current by $\frac{\pi}{4}$

11. An inductive coil has a resistance of $100\,\Omega$. When an AC signal of frequency 1000 Hz is applied to the coil, the voltage leads the current by 45°. The inductance of the coil is
(a) $\frac{1}{10\pi}$ (b) $\frac{1}{20\pi}$ (c) $\frac{1}{40\pi}$ (d) $\frac{1}{60\pi}$

12. Two inductors L_1 and L_2 are connected in parallel and a time varying current flows as shown in figure. Then the ratio of currents I_1/I_2 at any time t is



(a) L_1/L_2 (b) L_2/L_1 (c) $\frac{L_1}{(I_1 + I_2)^2}$ (d) $\frac{L_2}{(I_1 + I_2)^2}$

13. An inductance and a resistance are connected in series with an AC potential. In this circuit
(a) the current and the potential difference across the resistance lead the PD across the inductance by phase angle $\pi/2$
(b) the current and the potential difference across the resistance lag behind PD across the inductance by an angle $\pi/2$

(c) the current and the potential difference across the resistance lag behind the PD across the inductance by an angle π

(d) the PD across the resistance lags behind the PD across the inductance by an angle $\pi/2$ but the current in the resistance leads the PD across inductance by $\pi/2$

14. In *L-R* circuit, resistance is $8\,\Omega$ and inductive reactance is 6Ω, then impedance is

(a) 2Ω (b) $14\,\Omega$
(c) $4\,\Omega$ (d) $10\,\Omega$

15. In an AC circuit, the current lags behind the voltage by $\frac{\pi}{3}$. The components in the circuit may be

(a) *R* and *L* (b) *L* and *C* (c) *R* and *C* (d) only *R*

16. In an AC circuit, a resistance of *R* ohm is connected in series with an inductance *L*. If phase angle between voltage and current be 45°, the value of inductive reactance will be

(a) *R* /4
(b) *R* /2
(c) *R*
(d) cannot be found with the given data

17. In a circuit containing *R* and *L*, as the frequency of the impressed AC increases, the impedance of the circuit

(a) decreases
(b) increases
(c) remains unchanged
(d) first increases and then decreases

18. An AC voltage is applied to a resistance *R* and an inductor *L* in series. If *R* and the inductive reactance are both equal to 3Ω, the phase difference between the applied voltage and the current in the circuit is

(a) $\pi/4$ (b) $\pi/2$
(c) zero (d) $\pi/6$

19. In an *L-R* circuit, the value of *L* is $\left(\frac{0.4}{\pi}\right)$ H and the value of *R* is $30\,\Omega$. If in the circuit, an alternating emf of 200 V at 50 cycle s^{-1} is connected, the impedances of the circuit and current will be

(a) $11.4\,\Omega$, 17.5 A (b) $30.7\,\Omega$, 6.5 A
(c) $40.4\,\Omega$, 5 A (d) $50\,\Omega$, 4 A

20. The instantaneous values of current and voltage in an AC circuit are given by

$$I = 6\sin\left(100\,\pi t + \frac{\pi}{4}\right)$$

$$V = 5\sin\left(100\,\pi t - \frac{\pi}{4}\right), \text{ then}$$

(a) current leads the voltage by 45°
(b) voltage leads the current by 90°
(c) current leads the voltage by 90°
(d) voltage leads the current by 45°

21. In an *L-C-R* series circuit the AC voltage across *R*, *L* and *C* come out as 10 V, 10 V and 20 V respectively. The voltage across the entire combination will be

(a) 30 V (b) $10\sqrt{3}$ V (c) 20 V (d) $10\sqrt{2}$ V

22. With increase in frequency of an AC supply, the impedance of an *L-C-R* series circuit

(a) remains constant
(b) increases
(c) decreases
(d) decreases at first, becomes minimum and then increases

23. A sinusoidal voltage of peak value 300 V and an angular frequency $\omega = 400$ rads^{-1} is applied to series *L-C-R* circuit, in which $R = 3\Omega$, $L = 20$ mH and $C = 625\mu$F. The peak current in the circuit is

(a) $30\sqrt{2}$ A (b) 60A (c) 100A (d) $60\sqrt{2}$ A

24. The value of current at resonance in a series *L-C-R* circuit is affected by the value of

(a) *R* only (b) *C* only (c) *L* only (d) *L*, *C* and *R*

25. An *L-C-R* series circuit is connected to a source of alternating current. At resonance the applied voltage and current flowing through the circuit will have a phase difference of

(a) zero (b) $\pi/4$ (c) $\pi/2$ (d) π

26. A series *L-C-R* circuit is operated at resonance. Then

(a) voltage across *R* is minimum
(b) impedance is minimum
(c) impedance is maximum
(d) current amplitude is minimum

27. An *L-C-R* series circuit is under resonance. If I_m is current amplitude V_m is voltage amplitude, *R* is the resistance, *Z* is the impedance, X_L is the inductive reactance and X_C is the capacitive reactance, then

(a) $I_m = \frac{V_m}{Z}$ (b) $I_m = \frac{V_m}{X_L}$ (c) $I_m = \frac{V_m}{X_C}$ (d) $I_m = \frac{V_m}{R}$

28. In an *L-C-R* series, AC circuit at resonance

(a) the capacitive reactance is more than the inductive
(b) the capacitive reactance equals the inductive reactance
(c) the capacitive reactance is less than the inductive reactance
(d) the power dissipated is minimum

29. An *L-C-R* series circuit, connected to a source *E*, is at resonance. Then,

(a) the voltage across *R* is zero
(b) the voltage across *R* equals applied voltage
(c) the voltage across *C* is zero
(d) the voltage across *C* equals applied voltage

30. The reciprocal of impedance is called

(a) reactance (b) admittance
(c) inductance (d) conductance

3 Power in an AC circuit

For electric circuits power is defined as the product of voltage and current.

In AC circuit, both emf and current changes continuously with respect to time, so in AC circuit, we have to calculate average power in complete cycle ($0 \to T$).

Hence, average power is defined as the average of instantaneous power in an AC circuit over a full cycle. Its SI unit is watt.

Instantaneous power, $P = VI$ ...(ii)

$[\because V = V_0 \sin\omega t, I = I_0 \sin(\omega t + \phi)]$

Here, V and I are instantaneous voltage and current, respectively.

Total work done or energy spent in maintaining current over one full cycle is given by

$$W = \frac{V_0 I_0 T}{2}\cos\phi$$

Thus, average power associated in AC circuit is given by

$$P_{av} = \frac{W}{T} = \frac{V_0 I_0}{2}\cos\phi$$

$$P_{av} = \frac{V_0}{\sqrt{2}} \cdot \frac{I_0}{\sqrt{2}}\cos\phi$$

$$\boxed{P_{av} = V_{rms} I_{rms}\cos\phi = V_V I_V \cos\phi}$$

Here, $\cos\phi$ is power factor.

Power factor is defined as cosine of the angle of lag or lead (*i.e.* $\cos\phi$). It is also defined as the ratio of resistance and impedance.

$$\boxed{\cos\phi = \frac{P_{av}}{V_{rms} I_{rms}} = \frac{\text{True power}}{\text{Apparent power}}}$$

It is said to be leading, if current leads voltage, lagging, if current lags voltage. Thus, a power factor of 0.5 lagging means current lags the voltage by 60° (as $\cos^{-1} 0.5 = 60°$).

The product of V_{rms} and I_{rms} gives the **apparent power** (or **virtual power**). While the **true power** is obtained by multiplying the apparent power by the power factor $\cos\phi$. Thus,

Apparent power $= V_{rms} \times I_{rms}$

and **True power = apparent power × power factor**

Special cases

(*i*) **AC circuit containing R only**

Here, $\phi = 0°$, $P_{av} = V_{rms} I_{rms} \cos 0°$

$\Rightarrow \quad P_{av} = V_{rms} I_{rms}$ [maximum]

(*ii*) **AC circuit containing L only**

$$\phi = \frac{\pi}{2} \text{ or } P_{av} = V_{rms} I_{rms} \cos\frac{\pi}{2} \Rightarrow P_{av} = 0$$

So, average power in L is zero.

(*iii*) **AC circuit containing C only**

$$\phi = \frac{\pi}{2} \text{ or } P_{av} = V_{rms} I_{rms} \cos\frac{\pi}{2} = 0$$

So, average power in C is zero.

(*iv*) **AC circuit containing L and R**

$$\tan\phi = \frac{\omega L}{R}, \quad \cos\phi = \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$$

$$\Rightarrow \quad P_{av} = V_{rms} I_{rms} \cdot \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$$

(*v*) **AC circuit containing C and R**

$$\tan\phi = \frac{1/\omega C}{R}, \quad \cos\phi = \frac{R}{\sqrt{R^2 + 1/\omega^2 C^2}}$$

$$\Rightarrow \quad P_{av} = V_{rms} I_{rms} \frac{R}{\sqrt{R^2 + 1/\omega^2 C^2}}$$

(*vi*) **AC circuit containing L, C and R**

$$\tan\phi = \frac{\omega L - 1/\omega C}{R}$$

$$\Rightarrow \quad \cos\phi = \frac{R}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

$$\Rightarrow \quad P_{av} = V_{rms} I_{rms} \cdot \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

Half power points in series *L-C-R* circuit

The values of ω at which the power input is half of its maximum value are called half power points.

At these points,

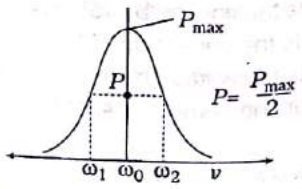


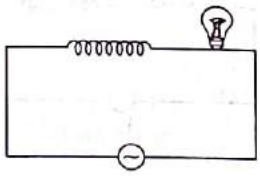
Fig. 7.33 Half power points in series L-C-R circuit

$$\omega = \omega_0 \pm \Delta\omega = \omega_0 \pm \frac{1}{2}\left(\frac{2\Delta\omega}{\omega_0}\right)\omega_0 = \omega_0\left(1 \pm \frac{1}{2Q}\right)$$

The amplitude of the current falls by a factor of $\frac{1}{\sqrt{2}}$ and hence, the power ($P \propto I^2$) goes down by half.

The separation between these two half power points is $2\Delta\omega = \frac{\omega_0}{Q}$. This separation is called the full-width at half maximum and is $\frac{1}{Q}$ th fraction of the resonant frequency.

Example 7.38 *An iron cored coil is connected in series with an electric bulb, with an AC source, as shown in figure. As the iron piece is taken out of the coil, how will the brightness of bulb change?*



Sol. As the iron rod is taken out of the coil, the self-inductance of the coil decreases, the impedance of circuit decreases, hence current increases. So, power consumed (and hence brightness of bulb) by bulb increases.

Example 7.39 *A light bulb has the rating 200 W 220 V. Find (i) resistance of the bulb filament (ii) rms value of current flowing through the filament.*

Sol. (i) Resistance of the bulb,

$$R = \frac{V^2}{P} = \frac{220 \times 220}{200} = \frac{22 \times 22}{2} = 242\,\Omega$$

(ii) The rms value of current $= \frac{P}{V} = \frac{200}{220} = \frac{10}{11}\text{A} = 0.9\text{A}$

Example 7.40 *A series L-C-R circuit with $R = 20\,\Omega$, $L = 1.5$ H and $C = 35\,\mu$F is connected to a variable frequency 200 V, AC supply. When the frequency of the supply equals the natural frequency of the circuit, what is the average power transferred to the circuit in one complete cycle?*

Sol. When the frequency of the supply equals the natural frequency of the circuit, the circuit is said to be in resonance. At resonance,

$$Z = R = 20\,\Omega$$

Since, the *L-C-R* circuit is resistive, so the phase angle between the current and voltage is zero.

i.e., $\phi = 0°$ and $I_{rms} = \frac{E_{rms}}{Z} = \frac{200}{20} = 10$ A

P = average power transferred per cycle.

$\therefore$ $P = I_{rms} E_{rms} \cos 0°$

$= 10 \times 200 = 2000$ W $= 2.0$ kW

Example 7.41 *A 100 Ω resistor is connected to a 220 V, 50 Hz AC supply.*

(i) What is the rms value of current in the circuit?

(ii) What is the net power consumed over a full cycle?

Sol. (i) R_{rms} value of current in the circuit,

$$I_{rms} = \frac{V_{rms}}{R} = \frac{220}{100} = 2.2 \text{ A}$$

(ii) We know that the power dissipated in an AC circuit is given by $P = V_{rms} I_{rms} \cos\phi$

where, ϕ is the phase difference between current and voltage

In a circuit containing resistor only, $\phi = 0°$, thus $\cos\phi = \cos 0° = 1$

$\therefore$ $P = V_{rms} I_{rms}$

$= 220 \times 2.2 = 484$ W

Example 7.42 *A series L-C-R circuit is connected across an AC source $E = 10\sin\left[100\pi t - \frac{\pi}{6}\right]$. Current from the supply is $I = 2\sin\left[100\pi t + \frac{\pi}{12}\right]$, what is the average power dissipated?*

Sol. Phase difference between voltage and current,

$$\phi = \frac{\pi}{12} - \left(-\frac{\pi}{6}\right) = \frac{\pi}{4}$$

$\Rightarrow$ Power factor $= \cos\phi = \frac{1}{\sqrt{2}}$

Average power dissipated,

$$= \frac{V_m I_m}{2}\cos\phi = \frac{10 \times 2}{2} \times \frac{1}{\sqrt{2}} = 5\sqrt{2}\text{ W}$$

Example 7.43 *An AC circuit containing 800 mH inductor and a 60 μF capacitor is in series with 15 Ω resistance. They are connected to 230 V, 50 Hz AC supply. Obtain average power transferred to each element and total power absorbed.*

Sol. Reactance of the inductor,

$$X_L = \omega L = 2\pi\nu L = 2 \times 3.14 \times 50 \times 800 \times 10^{-3} = 251.3\,\Omega$$

Capacitive reactance,

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C} = \frac{1}{2 \times 3.14 \times 50 \times 60 \times 10^{-6}} = 53.05\,\Omega$$

$\therefore$ Impedance,

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{15^2 + (251.3 - 53.05)^2}$$

$\Rightarrow$ $Z = 198.8\,\Omega$

$\therefore$ $I_{rms} = \frac{V_{rms}}{Z} = \frac{230}{198.8} = 1.157$ A

Note that the inductor and capacitor do not consume power over a cycle.

Now, average power dissipated in the resistor.

$= I_{rms}^2 \cdot R = (1.157)^2 \times 15$

$= 20.07$ W

Total power consumed $= V_{rms} \cdot I_{rms} \cdot \cos\phi$

$$= V_{rms} \cdot I_{rms} \cdot \frac{R}{Z}$$

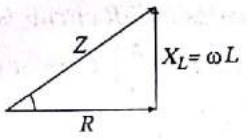
$$= \frac{230 \times 1.157 \times 15}{198.8} = 20.07 \text{ W}$$

Note that the total power consumed always is same as that consumed in resistor.

Example 7.44 *A 60 cycle AC circuit has a resistance of 200 Ω and inductor of 100 mH. What is the power factor? What capacitance placed in the circuit will make the power factor unity?*

Sol. $Z = \sqrt{R^2 + (\omega L)^2}$

$$= \sqrt{(200)^2 + (2\pi 60 \times 100 \times 10^{-3})^2} = 203.52\ \Omega$$



Power factor, $\cos\phi = \frac{R}{Z} = \frac{200}{203.52} = 0.983$

Now, for the power factor to be unit, $\phi = 0$ that means

$X_L = X_C$ or, $\omega L = \frac{1}{\omega C}$

$\therefore \quad C = \frac{1}{\omega^2 L} = \frac{1}{(2\pi 60)^2 \times 100 \times 10^{-3}}$

$\therefore \quad C = 70.36\ \mu\text{F}$

Example 7.45 *An L-C-R series circuit with 100 Ω resistance is connected to an AC source of 200 V and angular frequency 300 rads^{-1}. When only the capacitance is removed, the current lags behind the voltage by 60°. When only the inductance is removed, the current leads the voltage by 60°. Calculate the current and the power dissipated in the L-C-R circuit.*

Sol. When capacitance is removed,

$\tan\phi = \frac{X_L}{R}$ or $\tan 60° = \frac{X_L}{R}$

$\therefore \quad X_L = \sqrt{3}\,R$ ...(i)

When inductance is removed,

$\tan\phi = \frac{X_C}{R}$ or $\tan 60° = \frac{X_C}{R}$

$\therefore \quad X_C = \sqrt{3}\,R$ ...(ii)

From Eqs. (i) and (ii) we see that, $X_C = X_L$

So, the *L-C-R* circuit is in resonance.

Hence, $Z = R$

$\therefore \quad I_{rms} = \frac{V_{rms}}{Z} = \frac{200}{100} = 2\text{A}$

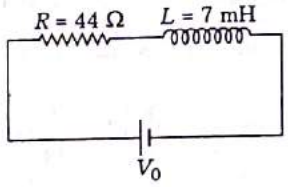
Power dissipated $\langle P \rangle = V_{rms} I_{rms} \cos\phi$

At resonance, current and voltage are in phase or $\phi = 0°$

$\therefore \quad \langle P \rangle = (200)(2)(1) = 400$ W

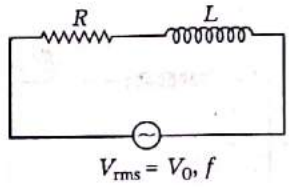
Example 7.46 *A solenoid with inductance L = 7 mH and active resistance R = 44 Ω is first connected to a source of direct voltage V_0 and then to source of sinusoidal voltage with effective value $V = V_0$. At what frequency of the oscillator will the power consumed by the solenoid be η = 5.0 times less than in the former case?*

Sol. When a solenoid is connected to a source of direct voltage, V_0



Power consumed, $P_1 = \frac{V_0^2}{R}$

When it is connected to a sinusoidal voltage, effective impedance comes into role.



$\therefore \quad Z = \sqrt{R^2 + X_L^2}$

Now, power consumed will be

$$P_2 = V_{rms} I_{rms} \cos\phi = V_0 \cdot \frac{V_0}{Z} \cdot \frac{R}{Z} = \frac{V_0^2 R}{Z^2}$$

Given, $P_2 = \frac{P_1}{\eta}$

$\Rightarrow \quad \frac{V_0^2 R}{Z^2} = \frac{V_0^2}{\eta R} \quad \Rightarrow \eta R^2 = Z^2 = R^2 + X_L^2$

$\Rightarrow \quad X_L^2 = (\eta - 1)R^2 \Rightarrow (2\pi f L)^2 = (\eta - 1)R^2$

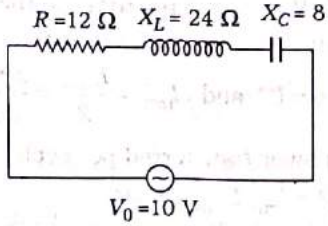
$\therefore$ Frequency of the oscillator $f = \frac{R}{2\pi L}\sqrt{\eta - 1}$

$$= \frac{44}{2 \times \frac{22}{7} \times 7 \times 10^{-3}} \sqrt{5-1} = 2 \times 10^3 \text{ Hz} = 2\text{kHz}$$

Example 7.47 *Consider the following R-L-C circuit in which R = 12 Ω, X_L = 24 Ω, X_C = 8 Ω. The emf of source is given by $V = 10\sin(100\pi t)$V.*

(i) Find the energy dissipated in 10 min.

(ii) If resistance is removed from the circuit and value of inductance is doubled, express variation of current with time t in the new circuit.



Sol. (i) Impedance,

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(12)^2 + (24-8)^2} = 20\ \Omega$$

$$\cos\phi = \frac{R}{Z} = \frac{12}{20} = \frac{3}{5}$$

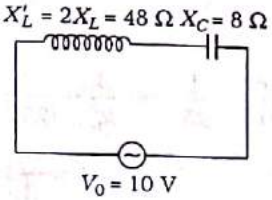
Power consumed,

$$P = \frac{1}{2}V_0 I_0 \cos\phi = \frac{1}{2}V_0 \cdot \frac{V_0}{Z}\cos\phi$$

$$= \frac{1}{2}\frac{(10)^2}{(20)} \times \frac{3}{5} = 1.5\,\text{W}$$

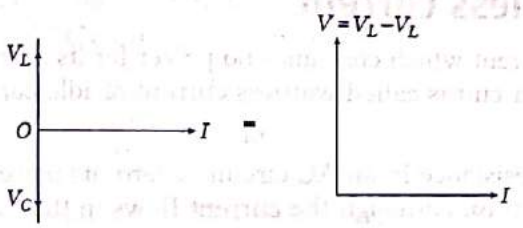
Energy dissipated, $E = Pt = (1.5)(10 \times 60) = 900\ \text{J}$

(ii)



Impedance, $Z = X'_L - X_C = 48 - 8 = 40\Omega$

$\therefore$ Current, $I_0 = \frac{V_0}{Z} = \frac{10}{40} = \frac{1}{4}\text{A}$

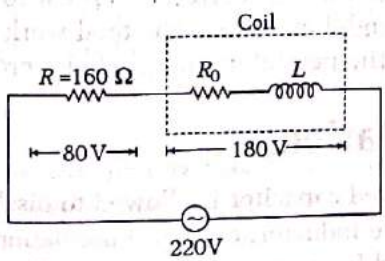


Current lags the voltage by $\pi/2$.

$$I = I_0 \sin(\omega t - \pi/2) = \frac{1}{4}\sin(100\pi t - \pi/2)$$

Example 7.48 *A series circuit consisting of an inductance free resistance $R = 0.16\ k\Omega$ and a coil with active resistance is connected to the mains with effective voltage $V = 220V$. Find the heat power generated in the coil, if the effective voltage values across the resistance R and the coil are equal to $V_1 = 80V$ and $V_2 = 180V$ respectively.*

Sol.



Current in the circuit,

$$V_r = I_{rms}R$$

$$\Rightarrow \quad 80 = I_{rms} \times 160 \Rightarrow I_{rms} = \frac{1}{2}\text{A}$$

Now, $\quad V_{R_0} = I_{rms}R_0 = \frac{R_0}{2}$

and $\quad V_L = I_{rms}X_L = \frac{X_L}{2}$

$$\Rightarrow \quad (V_R + V_{R_0})^2 + V_L^2 = (220)^2$$

$$\Rightarrow \quad \left(80 + \frac{R_0}{2}\right)^2 + \left(\frac{X_L}{2}\right)^2 = (220)^2$$

Also, $\quad V_{R_0}^2 + V_L^2 = (180)^2$

$$\Rightarrow \left(\frac{R_0}{2}\right)^2 + \left(\frac{X_L}{2}\right)^2 = (180)^2$$

Subtracting Eqs. (i) and (ii), we get

$$\left(80 + \frac{R_0}{2}\right)^2 - \left(\frac{R_0}{2}\right)^2 = (220)^2 - (180)^2$$

$$\Rightarrow (80 + R_0)(80) = (400)(40)$$

Power consumed in coil,

$$P = I_{rms}^2 R_0 = \left(\frac{1}{2}\right)^2 \times 120 = 30\,\text{W}$$

Example 7.49 *A current of 4 A flows in a coil when connected to a 12 V DC source. If the same coil is connected to a 12 V, 50 rads^{-1} AC source, a current of 2.4 A flows in the circuit. Determine the inductance of the coil. Also find the power developed in the circuit, if a 2500 μF capacitor is connected in series with the coil.*

Sol. (i) A coil consists of an inductance (*L*) and a resistance (*R*). In DC, only resistance is effective. Hence,

$$R = \frac{V}{I} = \frac{12}{4} = 3\Omega$$

In AC, $\quad I_{rms} = \frac{V_{rms}}{Z} = \frac{V_{rms}}{\sqrt{R^2 + \omega^2 L^2}}$

$$\therefore \quad L^2 = \frac{1}{\omega^2}\left[\left(\frac{V_{rms}}{I_{rms}}\right)^2 - R^2\right]$$

$$\therefore \quad L = \frac{1}{\omega}\sqrt{\left(\frac{V_{rms}}{I_{rms}}\right)^2 - R^2}$$

Substituting the values, we have

$$L = \frac{1}{50}\sqrt{\left(\frac{12}{2.4}\right)^2 - (3)^2} = 0.08\ \text{H}$$

(ii) When capacitor is connected to the circuit, the impedance is,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Here, $\quad R = 3\Omega$

$$X_L = \omega L = (50)(0.08) = 4\Omega$$

and $\quad X_C = \frac{1}{\omega C} = \frac{1}{(50)(2500 \times 10^{-6})} = 8\Omega$

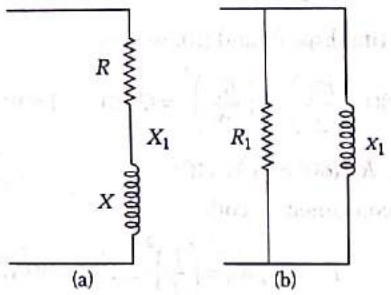
$\therefore \quad Z = \sqrt{(3)^2 + (4-8)^2} = 5\,\Omega$

Now, $\quad P = V_{rms} I_{rms} \cos\phi$

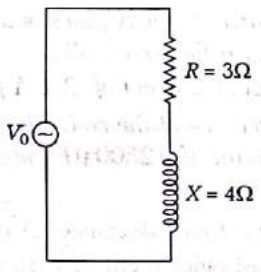
$$= V_{rms} \times \frac{V_{rms}}{Z} \times \frac{R}{Z} = \left(\frac{V_{rms}}{Z}\right)^2 \times R$$

Substituting the values, we have $P = \left(\frac{12}{5}\right)^2 \times 3 = 17.28$ W

Example 7.50 *The series and parallel circuits shown in figure have the same impedance and the same power factor. If $R = 3\Omega$ and $X = 4\Omega$, find the values of R_1 and X_1. Also, find the impedance and power factor.*



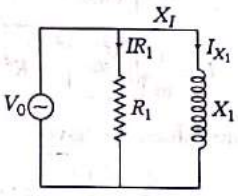
Sol.



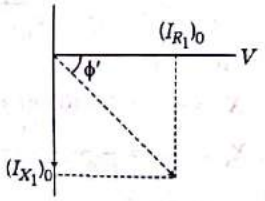
Impedance, $Z = \sqrt{R^2 + X^2} = 5\Omega$

Power factor, $\cos\phi = \frac{R}{Z} = \frac{3}{5} \Rightarrow \phi = \cos^{-1}(0.6)$

$\Rightarrow \quad \tan\phi = \frac{X}{R} = \frac{4}{3}$



Now, we have, $(I_{R_1})_1 = \frac{V_0}{R_1}$, $(I_{X_1})_0 = \frac{V_0}{X_1}$



$$\therefore \quad I_0 = \sqrt{(I_{R_1})_0^2 + (I_{X_1})_0^2} = \sqrt{\left(\frac{V_0}{R_1}\right)^2 + \left(\frac{V_0}{X_1}\right)^2} \quad \text{...(i)}$$

$$\Rightarrow \quad \frac{V_0}{Z} = V_0\sqrt{\frac{1}{R_1^2} + \frac{1}{X_1^2}} \Rightarrow Z = \frac{1}{\sqrt{\frac{1}{R_1^2} + \frac{1}{X_1^2}}} = 5$$

$$\tan\phi' = \frac{(I_{X_1})_0}{(I_{R_1})_0} = \frac{V_0/X_1}{V_0/R_1} = \frac{R_1}{X_1} = \tan\phi = \frac{4}{3} \quad \text{...(ii)}$$

Putting $X_1 = \frac{3}{4}R_1$ in Eq. (i)

$$\Rightarrow \quad \frac{1}{R_1^2} + \frac{1}{\left(\frac{3}{4}R_1\right)^2} = \frac{1}{5^2}$$

$$\Rightarrow \quad \frac{1}{R_1^2}\left(1 + \frac{16}{9}\right) = \frac{1}{25} \Rightarrow \frac{1}{R_1^2} = \frac{9}{(25)^2}$$

$$\Rightarrow \quad R_1 = \frac{25}{3}\Omega$$

$$\Rightarrow \quad X_1 = \frac{3}{4}R_1 = \frac{25}{4}\Omega$$

Wattless current

The current which consumes no power for its maintainance in the circuit is called wattless current or idle current.

or

If the resistance in an AC circuit is zero, its power factor will be zero. Although the current flows in the circuit, yet the average power remains zero. *i.e.*, there is no energy dissipation in the circuit. Such a circuit is called the wattless circuit and the current flowing is called the wattless current.

If the circuit contains either inductance or capacitance only, then phase difference between current and voltage is 90°, *i.e.*, $\phi = 90°$. The average power in such a circuit is

$$P_{av} = V_{rms} \times I_{rms} \times \cos\phi$$
$$= V_{rms} \times I_{rms} \times \cos 90° = 0$$

The concept of wattless current is similar to that of a frictionless pendulum, where the total work done by gravity upon the pendulum in a cycle is zero.

L-C oscillations

When a charged capacitor is allowed to discharge through a non-resistive inductor, electrical oscillations of constant amplitude and frequency are produced. These oscillations are called *L-C* oscillations.

Working

When a capacitor is supplied with an AC current, it gets charged.

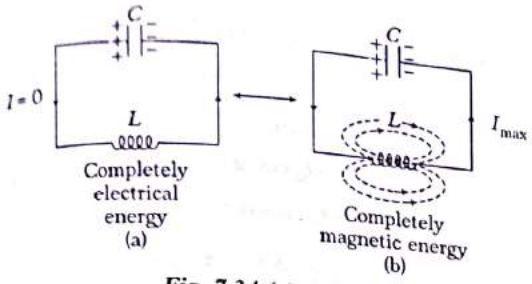


Fig. 7.34 (a) and (b)

When this charged capacitor is connected with an inductor, current flows through inductor, giving rise to magnetic flux, hence induced emf is produced in the circuit. Due to this, the charge (or energy) on the capacitor decreases and an equivalent amount of energy is stored in the inductor in the form of magnetic field when the discharging of the capacitor completes, current and magnetic flux linked with L starts decreasing. Therefore, an induced emf is produced which recharges the capacitor in opposite direction. This process of charging and discharging of capacitor is repeated and energy taken once from source keeps on oscillating between C and L.

The equation of LC oscillation is given by,

$$\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0$$

where, $q = q_0 \cos(\omega t + \phi)$

And the charge oscillates with a frequency,

$$\nu = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

The L-C oscillations discussed above are not realistic for the two reasons.

(i) Every inductor has some resistance. The effect of this resistance will introduce a damping effect on the charge and current in the circuit and the oscillations finally die away.

(ii) Even, if the resistance is zero, the total energy of the system would not remain constant. It is radiated away from the system in the form of electromagnetic waves. In fact, radio and TV transmitters depend on this radiation.

Example 7.51 *A charged 30 μF capacitor is connected to a 27 mH inductor. What is the angular frequency of free oscillations of the circuit?*

Sol. Angular frequency of free oscillations of L-C oscillations is

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{27\times10^{-3}\times30\times10^{-6}}}$$

$$= 1.11\times10^3 \text{ rad s}^{-1}$$

Example 7.52 *A radio can tune over the frequency range of a portion of MW broadcast band (800 kHz to 1200 kHz). If its L-C circuit has an effective inductance of 200 μH, what must be the range of its variable capacitor?*

Sol. For tuning, the natural frequency, *i.e.*, the frequency of free oscillations of the LC circuit should be equal to the frequency of the radiowave. Now for

$$f = \frac{1}{2\pi\sqrt{LC}}, \text{ we get}$$

$$f_1 = \frac{1}{2\pi\sqrt{LC_1}} \quad \text{or} \quad C_1 = \frac{1}{4\pi^2 L f_1^2}$$

or $$C_1 = \frac{1}{4\times9.87\times2\times10^{-4}\times(8\times10^5)^2}$$

$$= 197.8\times10^{-12}\text{ F} = 197.8\text{ pF}$$

Similarly, $$C_2 = \frac{1}{4\pi^2 L f_2^2} = \frac{1}{4\times9.87\times2\times10^{-4}\times(12\times10^5)^2}$$

or $$C_2 = 87.95\times10^{-12}\text{ F} = 87.95\text{ pF}$$

Example 7.53 *In an L-C circuit $L = 3.3\,H$ and $C = 840\,pF$. At $t = 0$ charge on the capacitor is $105\,\mu C$ and maximum. Compute the following quantities at $t = 2$ ms.*

(i) The energy stored in the capacitor.

(ii) The total energy in the circuit.

(iii) The energy stored in the inductor.

Also, write the equation for the variation of current in the inductor with time.

Sol. Given, $L = 3.3\text{ H}$, $C = 840\times10^{-12}\text{ F}$

and $q_0 = 105\times10^{-6}\text{ C}$

The angular frequency of L-C oscillations is,

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{3.3\times840\times10^{-12}}}$$

$$= 1.9\times10^4 \text{ rads}^{-1}$$

Charge stored in the capacitor at time t, would be,

$$q = q_0 \cos\omega t$$

(i) At $t = 2\times10^{-3}$ s

$$q = (105\times10^{-6})\cos(1.9\times10^4)(2\times10^{-3})$$

$$= 100.3\times10^{-6}\text{ C}$$

∴ Energy stored in the capacitor

$$U_C = \frac{1}{2}\frac{q^2}{C} = \frac{(100.3\times10^{-6})^2}{2\times840\times10^{-12}} = 6.0\text{ J}$$

(ii) Total energy in the circuit

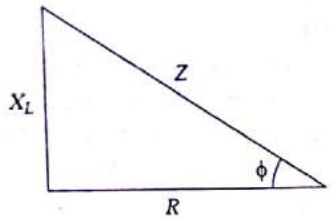
$$U = \frac{1}{2}\frac{q_0^2}{C} = \frac{(105\times10^{-6})^2}{2\times840\times10^{-12}} = 6.56\text{ J}$$

(iii) Energy stored in inductor in the given time

= total energy in circuit − energy stored in capacitor

$= (6.56 - 6.0)\text{ J} = 0.56\text{ J}$

$$\therefore \quad I_{rms} = \frac{V_{rms}}{Z} = \frac{220}{311} = 0.707 \text{ A}$$



and from the impedance diagram drawn above,

$$\cos\phi = \frac{R}{Z} = \frac{220}{311} = 0.707$$

$\therefore$ The power absorbed in the circuit, $P = V_{rms} I_{rms} \cos\phi$

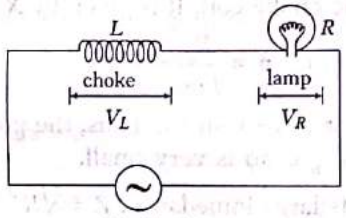
$$= (220)(0.707)(0.707) \text{ W}$$
$$= 110.08 \text{ W}$$

(b) When the resistance and choke are in parallel, the entire power is absorbed in resistance, as the choke (having zero resistance) absorbs no power.

$$\therefore \quad P = \frac{V_{rms}^2}{R} = \frac{(220)^2}{220} = 220 \text{ W}$$

Example 7.56 *A choke coil is needed to operate an arc lamp at 160 V (rms) and 50 Hz. The lamp has an effective resistance of 5 Ω when running at 10 A (rms). Calculate the inductance of the choke coil. If the same arc lamp is to be operated on 160 V (DC), what additional resistance is required? Compare the power loses in both cases.*

Sol. For lamp, $(V_{rms})_R = (i_{rms})(R) = 10 \times 5 = 50$ V



In series,

$$(V_{rms})^2 = (V_{rms})_R^2 + (V_{rms})_L^2$$

$$\therefore \quad (V_{rms})_L = \sqrt{(V_{rms})^2 - (V_{rms})_R^2}$$

$$= \sqrt{(160)^2 - (50)^2} = 152 \text{ V}$$

As $\quad (V_{rms})_L = (I_{rms}) X_L = (I_{rms})(2\pi f L)$

$$\therefore \quad L = \frac{(V_{rms})_L}{(2\pi f)(I_{rms})}$$

Substituting the values, $L = \dfrac{152}{(2\pi)(50)(10)}$

$$= 4.84 \times 10^{-2} \text{ H}$$

Now, when the lamp is operated at 160 V DC and instead of choke let an additional resistance R' is put in series with it then,

$$V = I(R + R')$$

or $\quad 160 = 10(5 + R')$

$\therefore \quad R' = 11\Omega$

In case of AC, as the choke has no resistance, power loss in choke is zero.

In case of DC, the loss in additional resistance R' is,

$$P = I^2 R' = (10)^2 (11) = 1100 \text{ W}$$

Transformer

It is a device which is used to change the alternating voltage. *The transformers are of the following types*

Step-up transformer

It is used to increase the AC voltages.

Step-down transformer

It is used to decrease the AC voltages.

Principle

Transformer is based on the principle of mutual induction, *i.e.*, whenever a amount of magnetic flux linked with a coil changes, an emf is induced in the neighbouring coil.

Construction

It consists of two coils, primary coil (P) and secondary coil (S), insulated from each other wounded on soft iron core, often the primary coil is the input coil and secondary coil is the output coil. These soft iron cores are laminated to minimise eddy current loss.

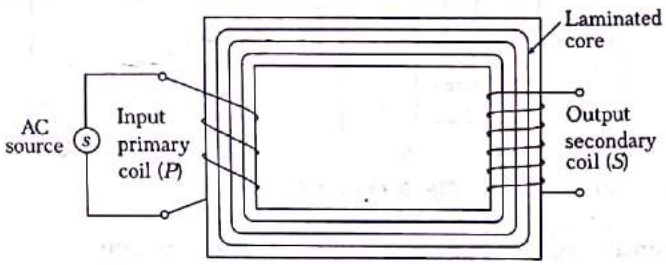


Fig. 7.37 *Construction of a transformer*

Theory and working

The value of the emf induced in secondary coil due to alternating voltage applied to primary depends on the number of turns in the secondary. We consider an **ideal transformer** in which the primary has **negligible resistance** and all the flux in the core links both primary and secondary windings. Let ϕ be the flux in each turn in the core at time t due to current in the primary when a voltage V_P is applied to it. Then, the induced emf or voltage ε_S in the secondary with N_S turns is

$$\varepsilon_S = -N_S \frac{d\phi}{dt} \quad \text{...(i)}$$

The alternating flux ϕ also induces an emf, called back emf in the primary. This is

$$\varepsilon_P = -N_P \frac{d\phi}{dt} \quad \text{...(ii)}$$

But $\varepsilon_P = V_P$. If this were not so, the primary current would be infinite, since the primary has zero resistance (as considered). If the secondary is an open circuit or the current taken from it is small, then to a good approximation.

$$\varepsilon_S = V_S$$

where, V_S is the voltage across the secondary.

Therefore, Eqs. (i) and (ii) can be written as,

$$V_S = -N_S \frac{d\phi}{dt} \quad \text{...(iii)}$$

$$V_P = -N_P \frac{d\phi}{dt} \quad \text{...(iv)}$$

From Eqs. (iii) and (iv), we have

$$\frac{V_S}{V_P} = \frac{N_S}{N_P} \quad \text{...(v)}$$

The above relation is based on three assumptions:

(i) The primary resistance and current are small.

(ii) The same flux links both the primary and the secondary as there is very little leakage of magnetic flux.

(iii) The secondary current is small.

Efficiency of a transformer is defined as the ratio of output power to the input power.

i.e.,

$$\eta = \frac{\textbf{Output power}}{\textbf{Input power}} = \frac{V_S I_S}{V_P I_P}$$

If the transformer is assumed to be 100% efficient (no energy losses), the power input is equal to the power output, and since $P = IV$,

$$I_P V_P = I_S V_S \quad \text{...(vi)}$$

Although, some energy is always lost, still this is a good approximation, since a well designed transformer may have an efficiency of more than 95%.

Combining Eqs. (v) and (vi), we have

$$\frac{I_P}{I_S} = \frac{V_S}{V_P} = \frac{N_S}{N_P} \quad \text{...(vii)}$$

Since, I and V both oscillate with the same frequency as the AC source, Eq. (vii) also gives the ratio of the amplitudes or rms values of corresponding quantities.

Now, we can observe how a transformer affects the voltage and current. We have

$$V_S = \left(\frac{N_S}{N_P}\right) V_P \text{ and } I_S = \left(\frac{N_P}{N_S}\right) I_P \quad \text{...(viii)}$$

i.e., if the secondary coil has a greater number of turns, then the primary (*i.e.,* $N_S > N_P$), the voltage is stepped up ($V_S > V_P$). This type of arrangement is called a **step-up transformer**. However, in this arrangement, there is less current in the secondary than in the primary (*i.e.,* $N_P/N_S < 1$ and $I_S < I_P$). If the secondary coil has less turns than the primary (*i.e.,* $N_S < N_P$), we have a **step-down transformer**. In this case, $V_S < V_P$ and $I_S > I_P$, *i.e.,* the voltage is stepped down, (or reduced), and the current is increased. The equations obtained above apply to ideal transformers (without any energy losses).

Note The ratio $k = \frac{N_s}{N_p}$, is also known as transformation ratio.

Energy losses in transformers

In actual transformers, small energy losses do occur due to the following reasons.

(*i*) **Flux leakage** There is always some leakage of flux *i.e.,* not all of the flux due to primary passes through the secondary due to poor design of the core or the air gaps in the core. It can be reduced by winding the primary and secondary coils one over the other.

(*ii*) **Resistance of the windings** The wire used for the windings has some resistance and so, energy is lost due to heat produced in the wire (I^2R). In high current, low voltage windings, these are minimised by using thick wire.

(*iii*) **Eddy currents** The alternating magnetic flux induces eddy currents in the iron core and causes heating. The effect is reduced by having a laminated core.

(*iv*) **Hysteresis** The magnetisation of the core is repeatedly reversed by an alternating magnetic field. The resulting expenditure of energy in the core appears as heat and is kept to a minimum by using a magnetic material which has a low hysteresis loss.

(*v*) **Magnetostriction** It is the humming noise of a transformer.

Uses of transformers

Transformers are used in almost all AC operations. *Some of them are given below*

(*i*) In the induction furnaces.

(*ii*) In voltage regulators for TV, computer, refrigerator etc.

(*iii*) A step-down transformer is used for the purpose of weldings.

(*iv*) In the transmission of AC over long distances.

Example 7.57 *In a step-down transformer having primary to secondary turn ratio 10 : 1, the input voltage applied is 250 V and output current is 10 A. Assuming 100% efficiency, calculate the*

(i) voltage across secondary coil

(ii) current in primary coil

(iii) power output

Sol. Given, $\frac{N_S}{N_P} = \frac{1}{10}, V_P = 250\text{V}, I = 8\text{A}$

(i) $\frac{V_S}{V_P} = \frac{N_S}{N_P} \Rightarrow V_S = \frac{N_S}{N_P} \times V_P = \frac{1}{10} \times 250 = 25\text{V}$

(ii) $\frac{I_P}{I_S} = \frac{N_S}{N_P} \Rightarrow I_P = \frac{N_S}{N_P} I_S = \frac{1}{10} \times 10 = 1\text{A}$

(iii) Power output $= V_S I_S = 25 \times 10 = 250\text{W}$

Example 7.58 *A 10 kW transformer has 20 turns in the primary and 100 turns in the secondary circuit. An AC voltage $V_1 = 600 \sin 314\, t$ is applied to the primary. Find (i) the maximum value of flux and (ii) the maximum value of secondary voltage.*

Sol. (i) Flux linked with each turn of primary,

$$\phi = BA\cos\omega t = \phi_0 \cos\omega t$$

Here, $\phi_0 = BA =$ maximum value of flux linked with each turn

$$\therefore\ V_1 = -N_1 \frac{d\phi}{dt} = -N_1 \frac{d}{dt}(\phi_0 \cos\omega t) = \omega N_1 \phi_0 \sin\omega t$$

Peak value of $V_1 = V_0 = \omega N_1 \phi_0$

or $\phi_0 = \frac{V_0}{\omega N_1}$

Given, $V_1 = 600 \sin 314\, t = V_0 \sin\omega t$

$\therefore\ V_0 = 600\text{V},\ \omega = 314\ \text{rad s}^{-1}$

Hence, $\phi_0 = \frac{600}{314 \times 20} = 0.0955\text{Wb}$

(ii) $\frac{V_2^0}{V_1^0} = \frac{N_2}{N_1}$

$\therefore$ Maximum value of secondary voltage is,

$$V_2^0 = \frac{N_2}{N_1} V_1^0 = \frac{100}{20} \times 600 = 3000\text{V}$$

Example 7.59 *(i) The primary of a transformer has 400 turns while the secondary has 2000 turns. If the power output from the secondary at 1100 V is 12.1 kW, calculate the primary voltage. (ii) If the resistance of the primary is 0.2 Ω and that of the secondary is 2.0 Ω and the efficiency of the transformer is 90 % calculate the heat losses in the primary and the secondary coils.*

Sol. (i) Given, $N_1 = 400, N_2 = 2000, V_2 = 1100\text{V}$

$$\text{As } V_1 = V_2 \cdot \frac{N_1}{N_2} = 1100 \times \frac{400}{2000} = 220\text{V}$$

(ii) Resistance of primary, $R_1 = 0.2\ \Omega$

Resistance of secondary, $R_2 = 2.0\Omega$

Output power $= V_2 I_2 = 12.1\text{kW} = 12100\text{W}$

$\therefore$ Current in the secondary, $I_2 = \frac{P_2}{V_2} = \frac{12100}{1100} = 11\text{A}$

As, efficiency $= \frac{\text{output power}}{\text{input power}}$

$$\Rightarrow \quad \frac{90}{100} = \frac{12100\text{W}}{\text{Input power}}$$

Input power, $P_1 = \frac{12100 \times 100}{90} = 13.44 \times 10^3\text{W}$

Also, input power, $P_1 = V_1 I_1$

$\therefore$ Current in the primary,

$$I_1 = \frac{P_1}{V_1} = \frac{13.44 \times 10^3}{220} = 61.1\text{A}$$

Power loss in the primary,

$$= I_1^2 R_1 = (61.1)^2 \times 0.2 = 746.61\text{W}$$

Power loss in the secondary,

$$= I_2^2 R_2 = (11)^2 \times 2.0 = 242\text{W}$$

Electric generator or dynamo

Electric generator is a device which is used to convert mechanical energy to electrical energy. Generator provide nearly all of the power for electric power grids.

The reverse conversion of electrical energy to mechanical energy is done by electric motor.

Electric generator are of two types

1. AC generator
2. DC generator

1. AC generator or AC dynamo

It is used to produce alternating current energy from mechanical energy.

(i) Principle

It is based on the phenomenon of **electromagnetic induction**, *i.e.*, whenever amount of magnetic flux linked with a coil changes, an emf is induced in the coil. The direction of induced current is given by Fleming's right-hand rule.

(ii) Construction

AC generator has following parts

(a) **Armature** *ABCD* is a rectangular armature coil consisting of a large number of turns of insulated copper wire wound over a laminated soft iron core.

(b) **Field magnets** N and S are the pole pieces of a strong electromagnet in which the armature coil is rotated. Axis of rotation is perpendicular to the magnetic field lines.

(c) **Slip rings** R_1 and R_2 are two hollow metallic rings which is connected by the two ends of armature coil. These rings rotate with the rotation of the coil.

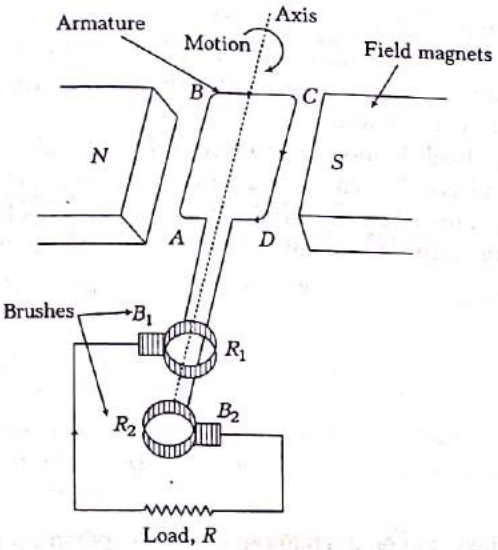


Fig. 7.38 Construction of an AC generators

(d) **Brushes** B_1 and B_2 are two flexible metal plates. They are fixed and are kept in light contact with R_1 and R_2 respectively. It is used to pass on current from the armature coil to the external load resistance R.

(e) **Source of energy** The armature coil is rotated with the help of turbine or any other device connected to it. The rotational kinetic energy of the turbine is rotational kinetic energy of the turbine is converted into electrical energy by the AC generator.

(iii) Theory and working

As the armature coil is rotated in the magnetic field, angle θ between the field and normal to the coil changes continuously. Hence, magnetic flux linked with the coil also changes and therefore emf is induced in the coil. Induced current in the external circuit changes direction after every half rotation of the coil. Hence, the induced current is of alternating in nature. The emf, so induced is given by the formula, $E = E_0 \sin\omega t$.

where, $E_0 = E_{max} = NAB\omega$

ω = angular frequency of the coil

N = number of turns in the coil

A = area enclosed by each turn of the coil

B = strength of magnetic field

The variation of E with time in AC generator is shown in figure below

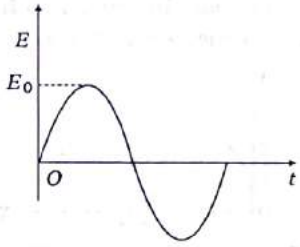


Fig. 7.39 Variation of emf in AC generator

Note If AC is produced by a generator having a large number of poles, then its frequency $\nu = \frac{\text{Number of poles} \times \text{rotation per second}}{2} = \frac{P \times n}{2}$

where, P is the number of poles, n is the rotational frequency of the coil.

Example 7.60 *An AC generator consists of a coil of 1000 turns each of area 100 cm^2 and rotating at an angular speed of 100 rpm in a uniform magnetic field of 3.6×10^{-2} T. Find the peak and rms value of emf induced in the coil.*

Sol. Given, number of turns $N = 1000$,

$A = 100 \text{ cm}^2 = 100 \times 10^{-4} \text{ m}^2 = 10^{-2} \text{ m}^2$

Frequency, $\nu = 100 \text{ rpm} = \frac{100}{60} \text{ rps} = \frac{5}{3} \text{ rps}$

Magnetic field, $B = 3.6 \times 10^{-2}$ T

$\Rightarrow$ Peak value of emf, $E_0 = NAB\omega = NAB(2\pi\nu)$

$= 1000 \times 10^{-2} \times 3.6 \times 10^{-2} \times 2 \times \pi \times \frac{5}{3} = 3.77$ V

E_{rms} or $E_V = \frac{E_0}{\sqrt{2}} = 2.67$ V

2. DC generator or dc dynamo

It is used for producing direct current energy from mechanical energy.

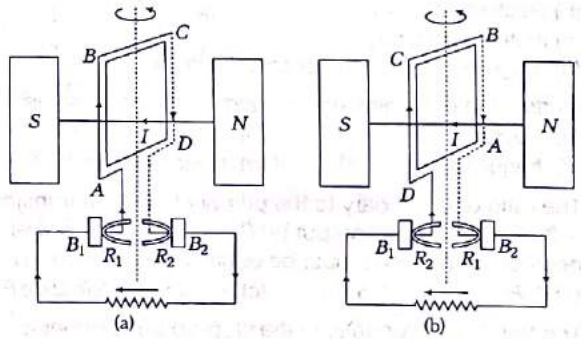


Fig. 7.40 (a) and (b) Constructions of DC generator

The principle and working of DC generator is same as that of AC generator. The only difference is that slip ring arrangement of AC generator is replaced by split ring arrangement or commutator arrangement in DC generator.

The magnitude of emf induced in DC generator is also

$$E = E_0 \sin\omega t$$

However, the direction of E is not reversed in the second half cycle because after half the rotation of the coil, R_1 goes in contact with B_2 and R_2 goes in contact with B_1. The variation of E with time in DC generator is shown in figure below.

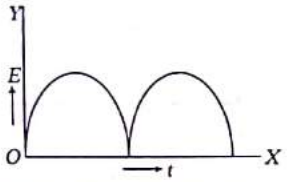


Fig. 7.41 Variation of emf in DC generator

Advantages of AC over DC

Advantages of AC over DC are given below

(i) The generation of AC is easy and economical.
(ii) AC can easily be converted into DC with the help of rectifiers.
(iii) In AC, energy loss is minimum, so it can be transmitted over large distances.
(iv) Wide range of voltages are obtained by the use of transformer.
(v) AC machines have low maintainance cost.

Disadvantages of AC over DC

Disadvantages of AC over DC are given below.

(i) An AC is transmitted from surface of the conductor and hence need several strands of thin wire insulated from each other.
(ii) AC shock is more attractive, while DC shock is repulsive. So, AC is more harmful.
(iii) AC cannot be used in electroplating process because here constant current is needed which is given by DC.

Check point 7.4

1. Choke coil is a device having
(a) low inductance and low resistance
(b) high inductance high resistance
(c) low inductance and high resistance
(d) high inductance and low resistance

2. What is increased in step-down transformer?
(a) Voltage (b) Current
(c) Power (d) Current density

3. A transformer works on the principle of
(a) self-induction
(b) electrical inertia
(c) mutual induction
(d) magnetic effect of the electrical current

4. Quantity that remains unchanged in a transformer is
(a) voltage (b) current
(c) frequency (d) None of these

5. The ratio of secondary to the primary turns in a transformer is 3 : 2. If the power output be P, then the input power neglecting all losses must be equal to
(a) $5P$ (b) $1.5P$ (c) P (d) $(2/5)P$

6. The transformation ratio in the step-up transformer is
(a) 1
(b) greater than one
(c) less than one
(d) the ratio greater or less than one depends on the other factors

7. In a transformer, the number of turns in primary and secondary are 500 and 2000 respectively. If current in primary is 48 A, the current in the secondary is
(a) 12 A (b) 24 A
(c) 48 A (d) 144 A

8. The core used in a transformer and other electromagnetic devices is laminated so that
(a) ratio of voltage in the primary and secondary may be increased
(b) energy loss due to eddy currents may be minimised
(c) the weight of the transformer may be reduced
(d) residual magnetism in the core may be reduced

9. Which of the following is constructed on the principle of electromagnetic induction?
(a) Galvanometer
(b) Electric motor
(c) Generator
(d) Voltmeter

10. When speed of a DC generator decreases, the armature current
(a) increases
(b) decreases
(c) does not change
(d) increases and decreases continuously

Chapter exercises

A. Taking it together

(Assorted questions of the chapter for advanced level practice)

1. The resistance of a coil for DC is 5 Ω. In case of AC, the resistance will
   (a) remain 5 Ω (b) decrease
   (c) increase (d) be zero

2. In non-resonant circuit, what will be the nature of the circuit for frequencies higher than the resonant frequency?
   (a) Resistive (b) Capacitive
   (c) Inductive (d) None of these

3. A choke coil has
   (a) high inductance and low resistance
   (b) low inductance and high resistance
   (c) high inductance and high resistance
   (d) low inductance and low resistance

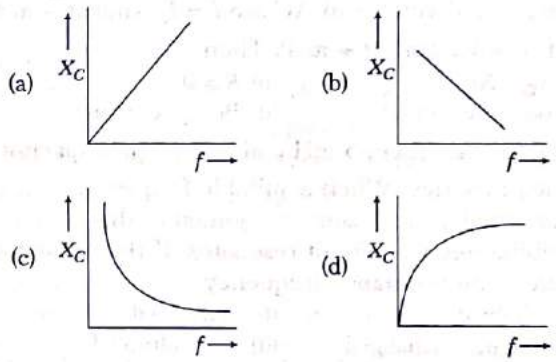
4. A circuit contains a capacitor and inductance each with negligible resistance. The capacitor is initially charged and the charging battery is disconnected. At subsequent time, the charge on the capacitor will
   (a) increase exponentially (b) decrease exponentially
   (c) decrease linearly (d) remain constant

5. A choke is preferred to a resistance for limiting current in AC circuit because
   (a) choke is cheap
   (b) there is no wastage of power
   (c) choke is compact in size
   (d) choke is a good absorber of heat

6. The frequency for which a 5 μF capacitor has a reactance of $\frac{1}{1000}$ Ω is given by
   (a) $\frac{100}{\pi}$ MHz (b) $\frac{1000}{\pi}$ Hz (c) $\frac{1}{1000}$ Hz (d) 1000 Hz

7. What will be the approximate resistance offered by a capacitor of 10 μF and frequency 100 Hz?
   (a) 160 Ω (b) 1600 Ω
   (c) 16 Ω (d) None of these

8. Which of the following curves correctly represent the variation of capacitive reactance (X_C) with frequency (f)?



9. L, C and R denote inductance, capacitance and resistance respectively. Pick out the combination which does not have the dimensions of frequency
   (a) $\frac{1}{RC}$ (b) $\frac{R}{L}$ (c) $\frac{1}{\sqrt{LC}}$ (d) $\frac{C}{L}$

10. An AC source is connected to a capacitor. The current in the circuit is I. Now a dielectric slab is inserted into the capacitor, then the new current is
    (a) equal to I (b) more than I
    (c) less than I (d) may be more than or less than I

11. An alternating voltage is connected in series with a resistance R and an inductance L. If the potential drop across the resistance is 200 V and across the inductance is 150 V, then the applied voltage is
    (a) 350 V (b) 250 V
    (c) 500 V (d) 300 V

12. An L-R circuit has $R = 10\ \Omega$ and $L = 2$ H. If 120 V, 60 Hz AC voltage is applied, then current in the circuit will be
    (a) 0.32 A (b) 0.16 A
    (c) 0.48 A (d) 0.80 A

13. A complex current wave is given by $i = (5 + 5 \sin 100\ \pi t)$ A. Its average value over one time period is given as
(a) 10 A (b) 5 A
(c) $\sqrt{50}$ A (d) 0

14. If the rms current in a 50 Hz AC circuit is 5 A, the value of the current 1/300 s after its value becomes zero is **[NCERT Exemplar]**
(a) $5\sqrt{2}$ A (b) $5\sqrt{3/2}$ A
(c) 5 / 6 A (d) $5/\sqrt{2}$ A

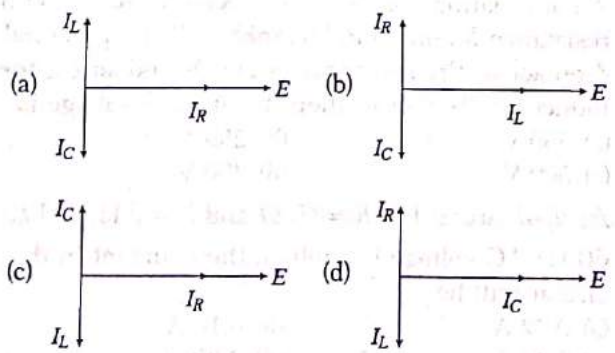
15. The peak value of an alternating emf E given by
$$E = E_0 \cos \omega t$$
is 10 V and frequency is 50 Hz. At time $t = (1/600)$ s, the instantaneous value of emf is
(a) 10 V (b) $5\sqrt{3}$ V
(c) 5 V (d) 1 V

16. Current and voltage in AC are $I = I_0 \sin(\omega t - \pi/4)$ and $V = V_0 \sin(\omega t + \pi/4)$, Then
(a) $X_L > X_C$ (b) $R = 0$
(c) Both are correct (d) Both are wrong

17. A 10 Ω resistance, 5 mH coil and 10 μF capacitor are joined in series. When a suitable frequency alternating current source is joined to this combination, the circuit resonates. If the resistance is halved, the resonance frequency
(a) is halved (b) is doubled
(c) remains unchanged (d) in quadrupled

18. The resonant frequency of a circuit is f. If the capacitance is made 4 times the initial value, then the resonant frequency will become
(a) $f/2$ (b) $2f$
(c) f (d) $f/4$

19. An alternating emf is applied across a parallel combination of a resistance R, capacitance C and an inductance L. If I_R, I_L and I_C are the currents through R, L and C respectively, then the diagram which correctly represents, the phase relationship among I_R, I_L, I_C and source emf E, is given by



20. An AC supply gives 30 V_{rms} which passes through a 10 Ω resistance. The power dissipated in it is
(a) $90\sqrt{2}$ W (b) 90 W
(c) $45\sqrt{2}$ W (d) 45 W

21. An alternating potential $V = V_0 \sin \omega t$ is applied across a circuit. As a result the current $I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$ flows in it. The power consumed in the circuit per cycle is
(a) zero (b) $0.5\, V_0 I_0$
(c) $0.707\, V_0 I_0$ (d) $1.414\, V_0 I_0$

22. A direct current of 2 A and an alternating current having a maximum value of 2A flow through two identical resistances. The ratio of heat produced in the two resistances will be
(a) 1 : 1 (b) 1 : 2
(c) 2 : 1 (d) 4 : 1

23. In a heating arrangement, an alternating current having a peak value of 28 A is used. To produce the same heat energy, if the constant current is used, its magnitude must be
(a) about 14 A (b) about 28 A
(c) about 20 A (d) Cannot say

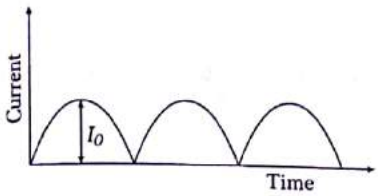
24. A lamp consumes only 50% of peak power in an AC circuit. What is the phase difference between the applied voltage and circuit current
(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$
(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

25. 110 V_{rms} is applied across a series circuit having resistance 11 Ω and impedance 22 Ω. The power consumed is
(a) 275 W (b) 366 W
(c) 550 W (d) 1100 W

26. A 20 V AC is applied to a circuit consisting of a resistance and a coil with a negligible resistance. If the voltage across the resistance is 12 V, the voltage across the coil is
(a) 16 V (b) 10 V
(c) 8 V (d) 6 V

27. To reduce the resonant frequency in an L-C-R series circuit with a generator **[NCERT Exemplar]**
(a) the generator frequency should be reduced
(b) another capacitor should be added in parallel to the first
(c) the iron core of the inductor should be removed
(d) dielectric in the capacitor should be removed

28. The output sinusoidal current *versus* time curve of a rectifier is shown in the figure. The average value of output current in this case is



(a) 0 (b) $\frac{I_0}{2}$ (c) $\frac{2I_0}{\pi}$ (d) I_0

29. An alternating voltage is given by

$$e = e_1 \sin \omega t + e_2 \cos \omega t$$

Then, the root mean square value of voltage is given by

(a) $\sqrt{e_1^2 + e_2^2}$ (b) $\sqrt{e_2 e_2}$

(c) $\sqrt{\frac{e_1 e_2}{2}}$ (d) $\sqrt{\frac{e_1^2 + e_2^2}{2}}$

30. Voltage and current in an AC circuit are given by $V = 5 \sin\left(100\pi t - \frac{\pi}{6}\right)$ and $I = 4 \sin\left(100\pi t + \frac{\pi}{6}\right)$

(a) voltage leads the current by 30°
(b) current leads the voltage by 30°
(c) current leads the voltage by 60°
(d) voltage leads the current by 60°

31. An alternating voltage $E = 200\sqrt{2} \sin(100t)$ is connected to a 1 μF capacitor through an AC ammeter. The reading of the ammeter shall be

(a) 10 mA (b) 20 mA (c) 40 mA (d) 80 mA

32. A coil having an inductance of $\frac{1}{\pi}$ henry is connected in series with a resistance of 300 Ω. If 20 V and a 200 Hz source are impressed across the combination, the value of the tangent of the phase angle between the voltage and the current is

(a) $\frac{5}{4}$ (b) $\frac{4}{5}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

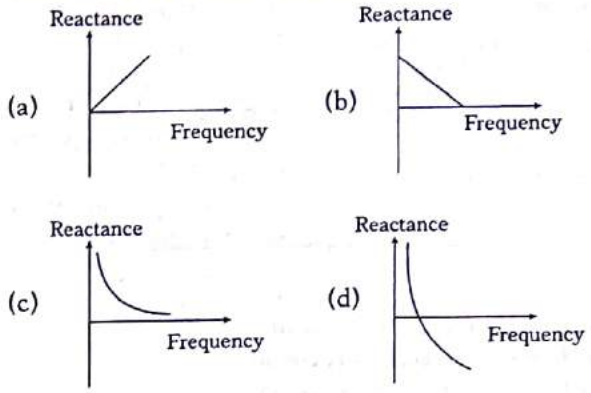
33. A condenser of capacity 20 μF is first charged and then discharged through a 10 mH inductance. Neglecting the resistance of the coil, the frequency of the resulting vibrations will be

(a) 356 cycles/s (b) 35.6 cycles/s
(c) 356×10^3 cycles/s (d) 3.56 cycles/s

34. An electric current has both DC and AC components. DC component of 8A and AC component is given as $I = 6 \sin \omega t$. So I_{rms} value of resultant current is

(a) 8.05 A (b) 9.05 A
(c) 11.58 A (d) 13.58 A

35. Which of the shown graphs may represent the reactance of a series L-C combination?



36. Two coils have a mutual inductance 0.005 H. The alternating current changes in the first coil according to equation $I = I_0 \sin \omega t$, where $I_0 = 10$ A and $\omega = 100\pi$ rads^{-1}. The maximum value of emf in the second coil is (in volt)

(a) 2π (b) 5π (c) π (d) 4π

37. Two identical electric heaters each marked 1000 W, 220 V are connected in series. This combination is connected to an AC supply of 220 V. What will be their combined rate of heating? (Assume that resistance of each heater remains constant)

(a) 2000 W (b) 1000 W
(c) 500 W (d) 250 W

38. In an L-R circuit, the inductive reactance is equal to the resistance R of the circuit. An emf $E = E_0 \cos(\omega t)$ is applied to the circuit. The power consumed in the circuit is

(a) $\frac{E_0^2}{\sqrt{2}R}$ (b) $\frac{E_0^2}{4R}$ (c) $\frac{E_0^2}{2R}$ (d) $\frac{E_0^2}{8R}$

39. A choke coil and capacitor are connected in series and the current through the combination is maximum for AC of frequency n. If they are connected in parallel, at what frequency is the current through the combination minimum?

(a) n (b) $n/2$
(c) $2n$ (d) None of these

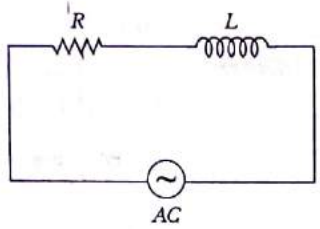
40. In a transformer, the coefficient of mutual inductance between the primary and the secondary coil is 0.2 H. When the current changes by 5 A/s in the primary, the induced emf in the secondary will be

(a) 5 V (b) 1 V
(c) 25 V (d) 10 V

41. In a transformer, number of turns in the primary are 140 and that in the secondary are 280. If current in primary is 4 A, then that in the secondary is

(a) 4 A (b) 2 A (c) 6 A (d) 10 A

42. A circuit contains resistance R and an inductance L in series. An alternating voltage $V = V_0 \sin \omega t$ is applied across it. The currents in R and L respectively will be

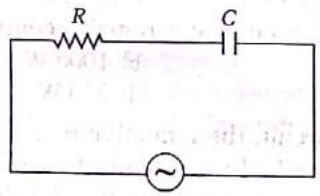


(a) $I_R = I_0 \cos \omega t, I_L = I_0 \cos \omega t$
(b) $I_R = -I_0 \sin \omega t, I_L = I_0 \cos \omega t$
(c) $I_R = I_0 \sin \omega t, I_L = -I_0 \cos \omega t$
(d) None of the above

43. A transformer has 500 primary turns and 10 secondary turns. If the secondary has a resistive load of 15 Ω, the currents in the primary and secondary respectively, are

(a) 0.16 A, 3.2×10^{-3} A (b) 3.2×10^{-3} A, 0.16 A
(c) 0.16 A, 0.16 A (d) 3.2×10^{-3} A, 3.2×10^{-3} A

44. A 50 Hz, AC source of 20 V is connected across R and C as shown in figure. The voltage across R is 12 V. The voltage across C is



(a) 8 V
(b) 16 V
(c) 10 V
(d) not possible to determine unless value of R and C are given

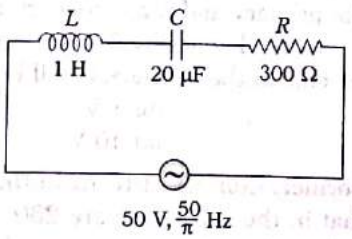
45. In an AC circuit, V and I are given by

$V = 150 \sin(150t)$ V and $I = 150 \sin\left(150t + \frac{\pi}{3}\right)$ A.

The power dissipated in the circuit is

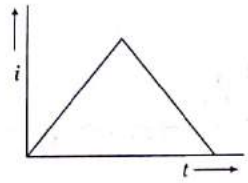
(a) 106 W (b) 150 W (c) 5625 W (d) zero

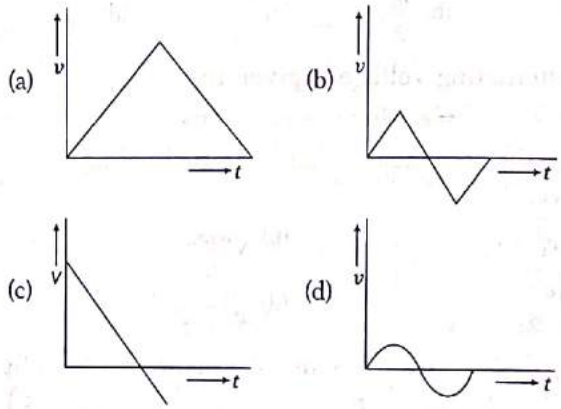
46. In the series L-C-R circuit shown, the impedance is



(a) 200 Ω (b) 100 Ω (c) 300 Ω (d) 500 Ω

47. An alternating current I in an inductance coil varies with time according to the graph given below. Which one of the following graphs gives the variation of voltage with time?





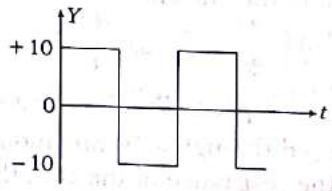
48. A resistor and a capacitor are connected in series with an AC source. If the potential drop across the capacitor is 5 V and that across resistor is 12 V, then applied voltage is

(a) 13 V (b) 17 V (c) 5 V (d) 12 V

49. When a voltage measuring device is connected to AC mains, the meter shows the steady input voltage of 220 V. This means **[NCERT Exemplar]**

(a) input voltage cannot be AC voltage, but a DC voltage
(b) maximum input voltage is 220 V
(c) the meter reads not v but $< v^2 >$ and is calibrated to read $\sqrt{< v^2 >}$
(d) the pointer of the meter is stuck by some mechanical defect

50. The rms voltage of the wave form shown is



(a) 10 V (b) 7 V
(c) 6.37 V (d) None of these

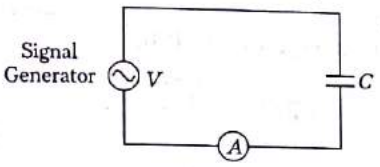
51. Using an AC voltmeter, the potential difference in the electrical line in a house is found to be 234 V. If the line frequency is known to be 50 cycles per second, the equation for the line voltage is

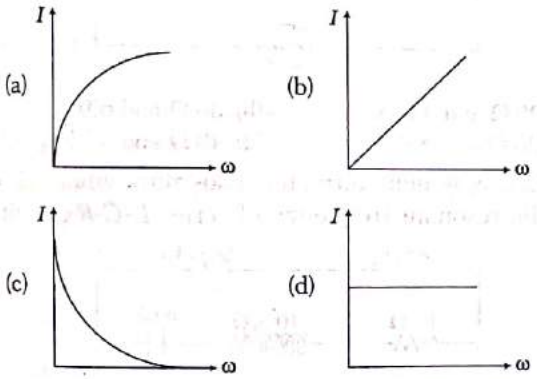
(a) 165 sin (200πt) (b) 234 sin (100πt)
(c) 331 sin (100πt) (d) 440 sin (200πt)

52. The output of a step-down transformer is measured to be 24 V when connected to a 12 W light bulb. The value of the peak current is **[NCERT Exemplar]**
(a) $1/\sqrt{2}$ A (b) $\sqrt{2}$ A (c) 2 A (d) $2\sqrt{2}$ A

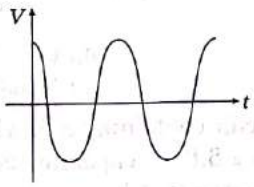
53. The rms value of an AC of 50 Hz is 10 A. The time taken by an alternating current in reaching from zero to maximum value and the peak value of current will be
(a) 2×10^{-2} s and 14.14 A (b) 1×10^{-2} s and 7.07 A
(c) 5×10^{-3} s and 7.07 A (d) 5×10^{-3} s and 14.14 A

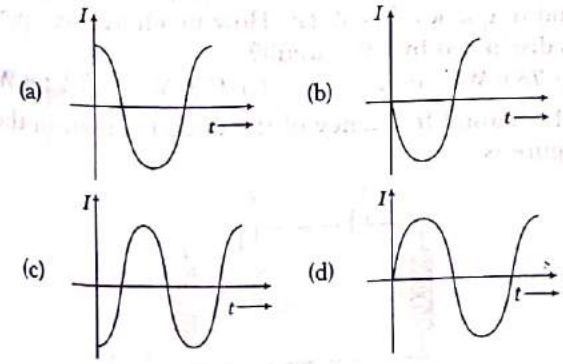
54. A constant voltage at different frequencies is applied across a capacitance C as shown in the figure. Which of the following graphs correctly depicts the variation of current with frequency?





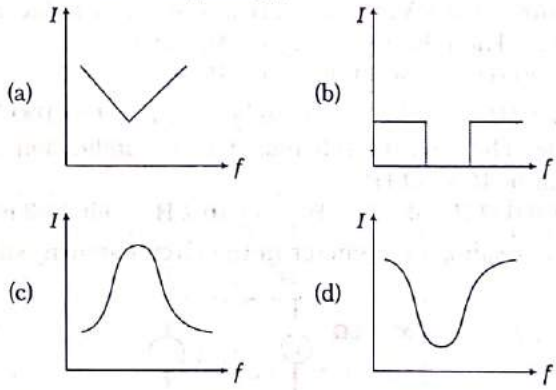
55. The voltage across a pure inductor is represented in figure. Which one of the following curves in the figure will represent the current?





56. An inductance of 1 mH, a condenser of 10 μF and a resistance of 50 Ω are connected in series. The reactances of inductor and condensers are same. The reactance of either of them will be
(a) 100 Ω (b) 30 Ω
(c) 3.2 Ω (d) 10 Ω

57. An AC source of variable frequency f is connected to an L-C-R series circuit. Which one of the graphs in figure represents the variation of current I in the circuit with frequency f?



58. The armature of DC motor has 20 Ω resistance. It draws current of 1.5 A when run by 220 V DC supply. The value of back emf induced in it will be
(a) 150 V (b) 170 V (c) 180 V (d) 190 V

59. A group of electric lamps having a total power rating of 1000 W is supplied by an AC voltage $E = 200 \sin(310\,t + 60°)$, then the rms value of the circuit current is
(a) 10 A (b) $5\sqrt{2}$ A
(c) 20 A (d) $10\sqrt{2}$ A

60. An alternating voltage $V = 30 \sin 50t + 40 \cos 50t$ is applied to a resistor of resistance 10 Ω. The rms value of current through resistor is
(a) $\frac{5}{\sqrt{2}}$ A (b) $\frac{10}{\sqrt{2}}$ A
(c) 5 A (d) None of these

61. An alternating voltage $V = 140 \sin 50t$ is applied to a resistor of 10 Ω. This voltage produces ΔH heat in the resistor in time Δt. To produce the same heat in the same time, required DC current is
(a) 14 A (b) about 20 A
(c) about 10 A (d) None of these

62. In a certain circuit current changes with time according to $i = 2\sqrt{t}$. Root mean square value of current between $t = 2$ to $t = 4$ s will be
(a) 3 A (b) $3\sqrt{3}$ A
(c) $2\sqrt{3}$ A (d) $\sqrt{3}$ A

63. The power factor of an R-L circuit is $\frac{1}{\sqrt{2}}$. If the frequency of AC is doubled, what will be the power factor?

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{1}{\sqrt{5}}$ (c) $\frac{1}{\sqrt{7}}$ (d) $\frac{1}{\sqrt{11}}$

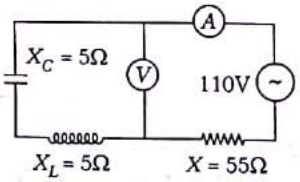
64. When a DC voltage of 200V is applied to a coil of self inductance $(2\sqrt{3}/\pi)$H, a current of 1A flows through it. But by replacing DC source with AC source of 200V, the current in the coil is reduced to 0.5A. Then the frequency of AC supply is

(a) 30 Hz (b) 60 Hz (c) 75 Hz (d) 50 Hz

65. One 10 V, 60 W bulb is to be connected to 100 V line. The required self-inductance of indication coil will be (f = 50 Hz)

(a) 0.052 H (b) 2.42 H (c) 16.2 H (d) 16.2 mH

66. The reading of ammeter in the circuit shown will be



(a) 2A (b) 2.4 A (c) Zero (d) 1.7 A

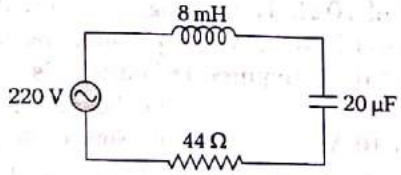
67. In series L-C-R circuit voltage drop across resistance is 8 V and across inductor is 6 V and across capacitor is 12 V. Then

(a) voltage of the source will be leading in the circuit
(b) voltage drop across each element will be less than the applied voltage
(c) power factor of the circuit will be 3/4
(d) None of the above

68. In a series L-C-R circuit, resistance $R = 10\ \Omega$ and the impedance $Z = 10\Omega$. The phase difference between the current and the voltage is

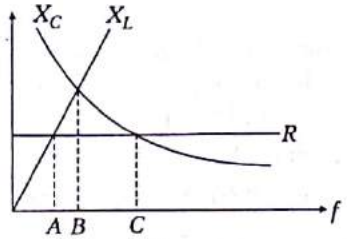
(a) 0° (b) 30° (c) 45° (d) 60°

69. For the series L-C-R circuit shown in the figure, what is the resonance frequency and the current at the resonating frequency?



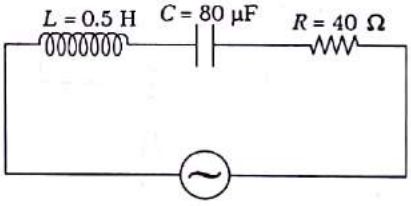
(a) 2500 rad s^{-1} and $5\sqrt{2}$ A
(b) 2500 rad s^{-1} and 5 A
(c) 2500 rad s^{-1} and $\frac{5}{\sqrt{2}}$ A
(d) 25 rad s^{-1} and $5\sqrt{2}$ A

70. The figure shows variation of R, X_L and X_C with frequency f in a series L, C, R circuit. Then, for what frequency point, the circuit is inductive?



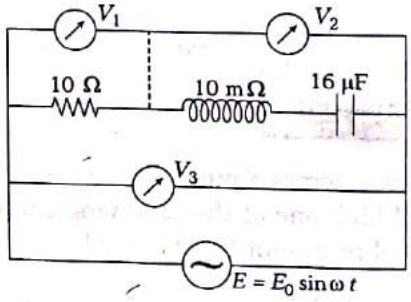
(a) A (b) B (c) C (d) All points

71. In the given figure, a series L-C-R circuit is connected to a variable frequency source of 230 V. The impedance and amplitude of the current at the resonating frequency will be



(a) 20 Ω and 4.2 A (b) 30 Ω and 6.9 A
(c) 25 Ω and 5.8 A (d) 40 Ω and 5.75 A

72. In figure, which voltmeter reads zero, when ω is equal to the resonant frequency of series L-C-R circuit?

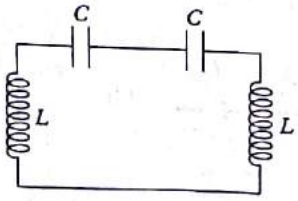


(a) V_1 (b) V_2
(c) V_3 (d) None of these

73. An R-L-C circuit containing a 52 Ω resistor, a 230 mH inductor, and a 8.8 μF capacitor is driven by an AC voltage source that has an amplitude of 150 V and frequency f = 80 Hz. How much average power is dissipated by this circuit?

(a) 78.6 W (b) Zero (c) 19.6 W (d) 24.8 W

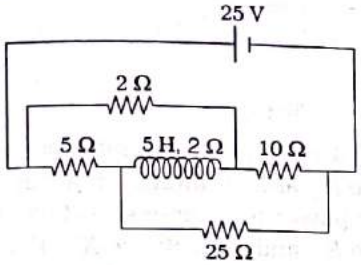
74. The natural frequency of the circuit shown in the figure is



(a) $\frac{1}{2\pi\sqrt{LC}}$ (b) $\frac{1}{2\pi\sqrt{2LC}}$

(c) $\frac{2}{2\pi\sqrt{LC}}$ (d) None of these

75. In the circuit shown what is the energy stored in the coil at steady state?



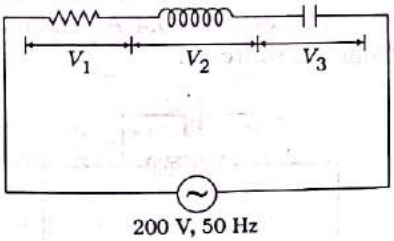
(a) 21.3 J (b) 42.6 J (c) Zero (d) 213 J

76. A loss-free transformer having 100 turns in primary is used to transmit 10 kW of power. The input voltage is 200 V and power is transmitted at 5 kV. The currents in the primary and secondary of the transformer are

(a) 2 A and 50 A (b) 50 A and 2 A

(c) 25 A and 4 A (d) 12.5 A and 8 A

77. For series L-C-R AC circuit shown in figure, the readings of V_1 and V_3 are same and each equal to 100 V. Then



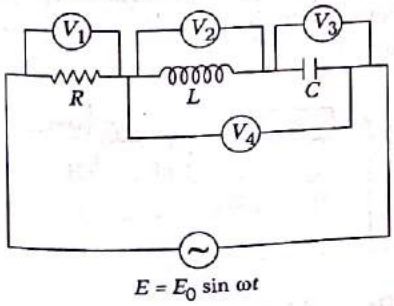
(a) the reading V_2 is 200 V

(b) the reading of V_2 is 0

(c) the circuit is in resonant mode and resonant frequency is 50 Hz

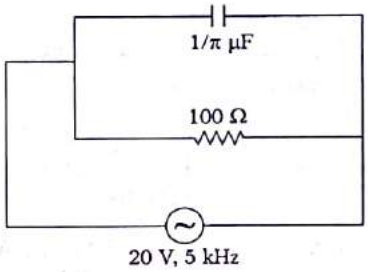
(d) the inductive and capacitive reactance are equal

78. In the given figure, which voltmeter will read zero voltage at resonant frequency?



(a) V_1 (b) V_2 (c) V_3 (d) V_4

79. A signal generator supplies a sine wave of 20 V, 5 kHz to the circuit shown in the figure. Then choose the wrong statement.



(a) The current in the resistive branch is 0.2 A

(b) The current in the capacitive branch is 0.126 A

(c) Total line current is ≈ 0.283 A

(d) Current in both the branches is same

80. An AC source is 120 V-60 Hz. The value of voltage after $\frac{1}{720}$ s from start will be

(a) 20.2 V (b) 42.4 V (c) 84.8 V (d) 106.8 V

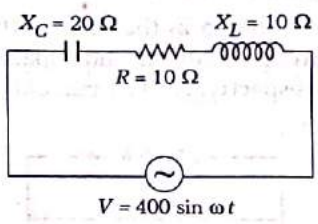
81. A pure resistive circuit element X when connected to an AC supply of peak voltage 200 V, gives a peak current of 5 A. A second circuit element Y, when connected to the same AC supply also gives the same value of peak current but the current lags behind by 90°. If the series combination of X and Y is connected to the same supply, what will be the rms value of current?

(a) $\frac{10}{\sqrt{2}}$ A (b) $\frac{5}{\sqrt{2}}$ A (c) $\frac{5}{2}$ A (d) 5 A

82. A coil, a capacitor and an AC source of rms voltage 24 V are connected in series. By varying the frequency of the source, a maximum rms current of 6 A is observed. If this coil is connected to a battery of emf 12 V and internal resistance 4 Ω, the maximum current through it will be

(a) 2.4 A (b) 1.8 A (c) 1.5 A (d) 1.2 A

83. In the L-C-R circuit as shown in figure.



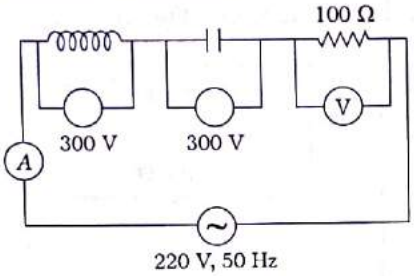
(a) Current will lead the voltage

(b) Rms value of current is 20 A

(c) Power factor of the circuit is $\frac{1}{\sqrt{2}}$

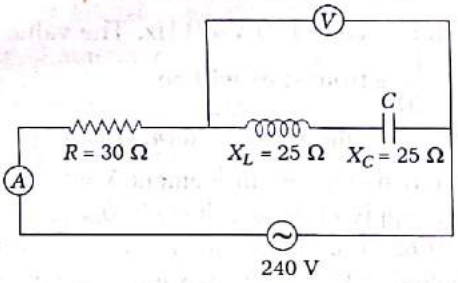
(d) Voltage drop across resistance is 200 V

84. In the circuit shown below what will be the reading of the voltmeter and ammeter?



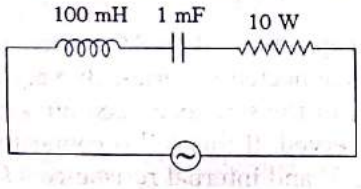
(a) 200 V, 1 A (b) 800 V, 2 A
(c) 220 V, 2 A (d) 220 V, 2.2 A

85. In the circuit shown in figure neglecting source resistance, the voltmeter and ammeter readings will be respectively



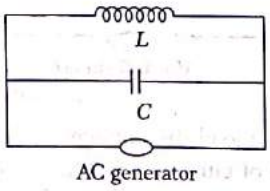
(a) 0 V, 3 A (b) 150 V, 3 A
(c) 150 V, 6 A (d) 0 V, 8 A

86. The following series L-C-R circuit, when driven by an emf source of angular frequency 70 kilo-radians per second, the circuit effectively behaves like



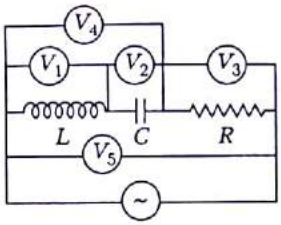
(a) purely resistive circuit
(b) series R-L circuit
(c) series R-C circuit
(d) series L-C circuit with $R = 0$

87. In the circuit shown in the figure , the alternating currents through inductor and capacitor are 1.2 A and 1.0 A respectively. The current drawn from the generator is



(a) 0.4 A (b) 0.2 A (c) 1.0 A (d) 1.2 A

88. In the adjoining AC circuit the voltmeter whose reading will be zero at resonance is



(a) V_1 (b) V_2 (c) V_3 (d) V_4

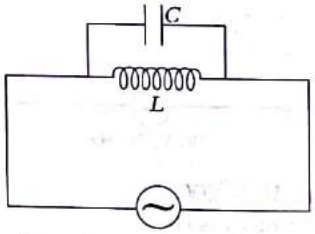
89. An alternating current generator has an internal resistance R_g and an internal reactance X_g. It is used to supply power to a passive load consisting of a resistance R_g and a reactance X_L. For maximum power to be delivered from the generator to the load, the value of X_L is equal to **[NCERT Exemplar]**

(a) zero (b) X_g (c) $-X_g$ (d) R_g

90. Which of the following combinations should be selected for better tuning of an L-C-R circuit used for communication? **[NCERT Exemplar]**

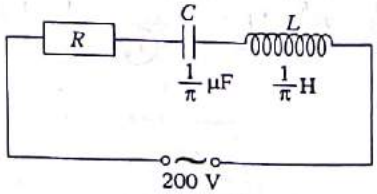
(a) $R = 20\,\Omega$, $L = 1.5$ H, $C = 35\,\mu$F
(b) $R = 25\,\Omega$, $L = 2.5$ H, $C = 45\,\mu$F
(c) $R = 15\,\Omega$, $L = 3.5$ H, $C = 30\,\mu$F
(d) $R = 25\,\Omega$, $L = 1.5$ H, $C = 45\,\mu$F

91. For the circuit as shown in the figure the current through the inductor is 1.6 A, while the current through the condenser is 0.4 A. Then, the current drawn from the source is



(a) $I = 2\sqrt{2}$A (b) $I = 1.65$ A
(c) $I = 1.2$ A (d) $I = 2.0$ A

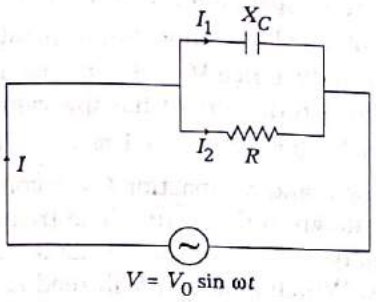
92. In the circuit shown in figure, the supply has a constant rms value V but variable frequency f. The frequency at which the voltage drop across R is maximum is



(a) 100 Hz (b) 500 Hz
(c) 300 Hz (d) None of these

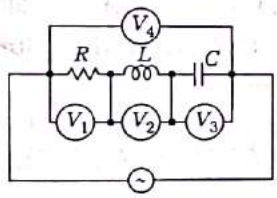
93. When an AC voltage, of variable frequency is applied to series L-C-R circuit, the current in the circuit is the same at 4 kHz and 9 kHz. The current in the circuit is maximum at
(a) 5 kHz (b) 6.5 kHz (c) 4.2 kHz (d) 6 kHz

94. In the given AC circuit



(a) current I_2 and V are is same phase
(b) current I_2 leads I_1 by 90°
(c) current I leads I_2 by $\theta < 90°$
(d) current I leads I_1 by $\theta < 90°$

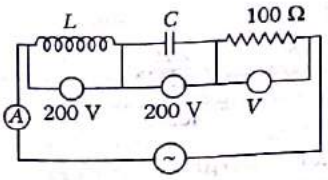
95. An ideal resistance R, ideal inductance, L, ideal capacitance C and AC voltmeters V_1, V_2, V_3 and V_4 are connected to an AC source as shown. At resonance



(a) Reading in V_3 = reading in V_1
(b) Reading in V_1 = reading in V_2
(c) Reading in V_2 = reading in V_4
(d) Reading in V_2 = reading in V_3

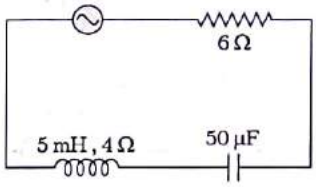
96. An AC voltage source of variable angular frequency ω and fixed amplitude V connected in series with a capacitance C and an electric bulb of resistance R (inductance zero). When ω is increased
(a) The bulb glows dimmer
(b) The bulb glows brighter
(c) Total impedance of the circuit is unchanged
(d) Total impedance of the circuit increases

97. The readings of ammeter and voltmeter in the following circuit are respectively



(a) 2 A, 200 V (b) 1.5 A, 100 V
(c) 2.7 A, 220 V (d) 2.2 A, 220 V

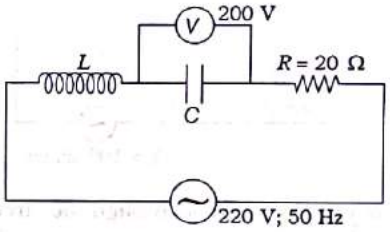
98. In the circuit shown, the AC source has voltage $V = 20 \cos(\omega t)$ volt with $\omega = 2000$ rads^{-1} the amplitude of the current will be nearest to



(a) 2 A (b) 3.3 A (c) $2/\sqrt{5}$A (d) $\sqrt{5}$A

99. In a series L-C-R circuit the voltage across resistance, capacitance and inductance is 10V each. If the capacitance is short circuited, the voltage across the inductance will be
(a) $\frac{10}{\sqrt{2}}$ V (b) 10 V (c) $10\sqrt{2}$ V (d) 20 V

100. In the circuit shown, rms current is 11 A. The potential difference across the inductor is

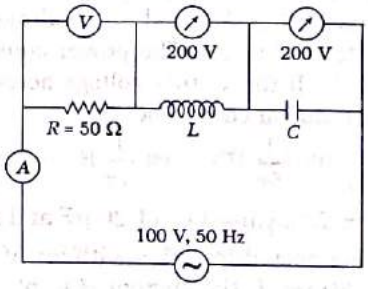


(a) 220 V (b) 0 V (c) 300 V (d) 200 V

101. An inductor of reactance 1 Ω and a resistor of 2 Ω are connected in series to the terminals of a 6V (rms) AC source. The power dissipated in the circuit is **[NCERT Exemplar]**
(a) 8 W (b) 12 W (c) 14.4 W (d) 18 W

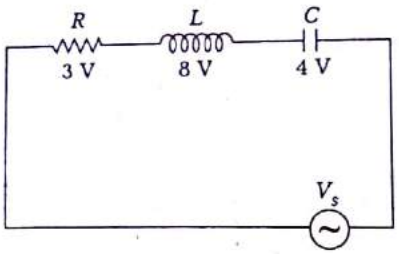
102. An AC circuit consists of a resistance and a choke coil in series. The resistance is of 220 Ω and choke coils is of 0.7 H. The power absorbed from 220 V and 50 Hz, source connected with the circuit, is
(a) 55 W (b) 110 W (c) 220 W (d) 440 W

103. In the series L-C-R circuit, the voltmeter and ammeter readings are respectively



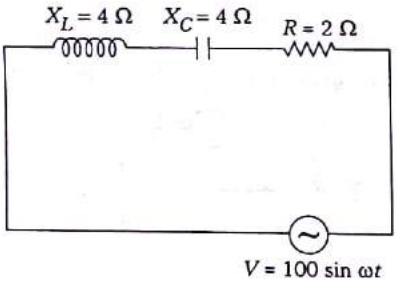
(a) V = 200 V, I = 4 A (b) V = 150 V, I = 2 A
(c) V = 100 V, I = 5 A (d) V = 100 V, I = 2 A

104. Current in resistance is 1 A, then



(a) $V_s = 5$ V
(b) impedance of network is $5\,\Omega$
(c) power factor of given circuit is (0.6) lagging (current is lagging)
(d) All of the above

105. Which of the following statement is correct regarding the AC circuit shown in the adjacent figure?



(a) The rms value of current through the circuit is $i_{rms} = 5\sqrt{2}$ A
(b) The phase difference between source emf and current is $\phi = \cos^{-1}\left(\frac{1}{3}\right)$
(c) Average power dissipated in the circuit is 500 W
(d) None of the above

106. An *L-C-R* series circuit with a resistance of $100\,\Omega$ is connected to an AC source of 200 V (rms) and angular frequency 300 rad s^{-1}. When only the capacitor is removed, the current lags behind the voltage by 60°. When only the inductor is removed the current leads the voltage by 60°. The average power dissipated in original *L-C-R* circuit is
(a) 50 W (b) 100 W (c) 200 W (d) 400 W

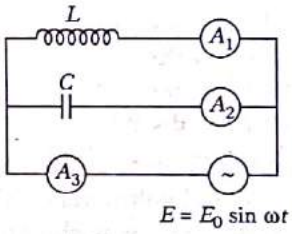
107. A virtual current of 4 A and 50 Hz flows in an AC circuit containing a coil. The power consumed in the coil is 240 W. If the virtual voltage across the coil is 100 V. Its inductance will be
(a) $\frac{1}{3\pi}$ H (b) $\frac{1}{5\pi}$ H (c) $\frac{1}{7\pi}$ H (d) $\frac{1}{9\pi}$ H

108. An inductor L, a capacitor of 20 μF and a resistor of $10\,\Omega$ are connected in series with an AC source of frequency 50 Hz. If the current is in phase with the voltage, then the inductance of the inductor is
(a) 2.00 H (b) 0.51 H (c) 1.5 H (d) 0.99 H

109. An *L-C-R* series circuit consists of a resistance of $10\,\Omega$ a capacitor of reactance $60\,\Omega$ and an inductor coil. The circuit is found to resonate when put across a 300 V, 100 Hz supply. The inductance of coil is (take, $\pi = 3$)
(a) 0.1 H (b) 0.01 H (c) 0.2 H (d) 0.02 H

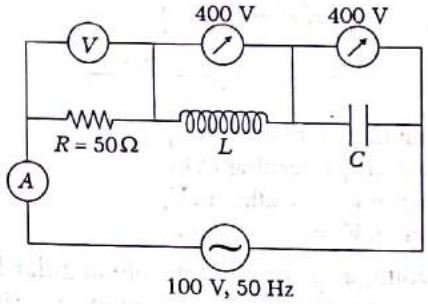
110. A capacitor of capacitance 1 μF is charged to a potential of 1V. It is connected in parallel to an inductor of inductance 10^{-3}H. The maximum current that will flow in the circuit has the value
(a) $\sqrt{1000}$ mA (b) 1 A (c) 1 mA (d) 1000 mA

111. An inductor L and a capactior C are connected in the circuit as shown in the figure. The frequency of the power supply is equal to the resonant frequency of the circuit. Which ammeter will read zero ampere



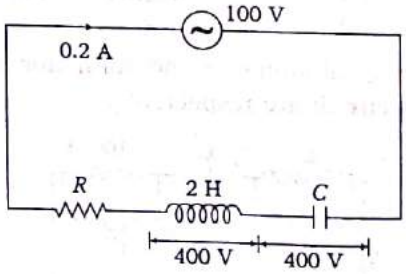
(a) A_1 (b) A_2 (c) A_3 (d) None of these

112. In the series *L-C-R* circuit, the voltmeter and ammeter readings are



(a) $V = 100$ V, $I = 2$ A (b) $V = 100$ V, $I = 5$A
(c) $V = 1000$ V, $I = 2$ A (d) $V = 300$ V, $I = 1$ A

113. Which of the following options is correct with respect to the circuit diagram given below?



(a) $R = 400\,\Omega$, $C = 0.5\,\mu$F
(b) $R = 500\,\Omega$, $C = 0.5\,\mu$F
(c) $R = 500\,\Omega$, $C = 1\,\mu$F
(d) $R = 400\,\Omega$, $C = 0.1\,\mu$F

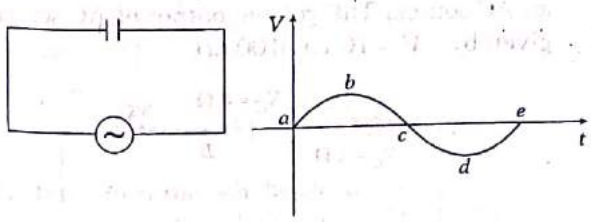
114. When 100 V DC is applied across a solenoid, a current of 1 A flows in it. When 100 V AC is applied across the same solenoid the current drops to 0.5 A. If the frequency of the AC source is 50 Hz, the impedance and inductance of the solenoid are
(a) 200 Ω and 0.55 H (b) 100 Ω and 0.86 H
(c) 200 Ω and 1.0 H (d) 1100 Ω and 0.93 H

115. An ideal choke takes a current of 8 A when connected to an AC source of 100 V and 50 Hz. A pure resistor under the same conditions takes a current of 10 A. If two are connected in series to an AC supply of 100 V and 40 Hz, then the current in the series combination of above resistor and inductor is
(a) 10 A (b) 5 A
(c) $5\sqrt{2}$ A (d) $10\sqrt{2}$ A

116. An AC source is connected with a resistance *(R)* and an uncharged capacitance *C*, in series. The potential difference across the resistor is in phase with the initial potential difference across the capacitor for the first time at the instant (assume that at $t = 0$, emf is zero).
(a) $\frac{\pi}{4\omega}$ (b) $\frac{2\pi}{\omega}$ (c) $\frac{\pi}{2\omega}$ (d) $\frac{3\pi}{2\omega}$
where, ω is the angular frequency.

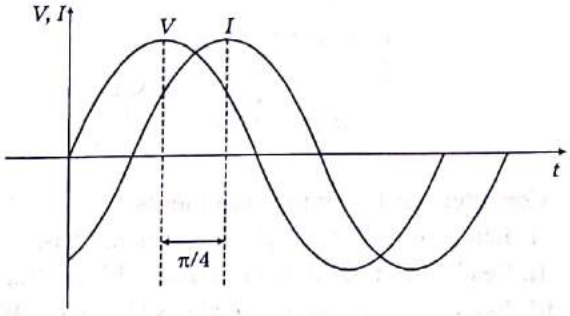
117. Current through an AC series *L-C-R* circuit is 2 A if operated at resonant frequency, and 1 A if operated at 50% less than resonant frequency. The current (in A) if the frequency is 100% more than the resonant frequency, is
(a) $\sqrt{2}$ (b) 1
(c) $\sqrt{3}$ (d) Data insufficient

118. For an AC circuit containing capacitor only, the applied AC voltage waveform is shown in figure.



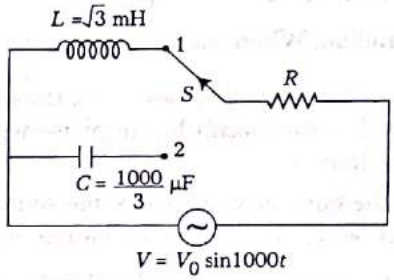
For this situation, mark the correct statement(s).
(a) As *V* increases from *a* to *b*, the charging of capacitor takes place
(b) As *V* increases from *a* to *b*, the current in circuit decreases from maximum to zero value
(c) As *V* decreases from *b* to *c*, the capacitor discharges
(d) As *V* decreases from *b* to *c* charging of capacitor takes place

119. An AC voltage $V = V_0 \sin 100\,t$ is applied to the circuit, the phase difference between current and voltage is found to be $\frac{\pi}{4}$, then



(a) $R = 100\,\Omega$, $C = 1\,\mu F$ (b) $R = 1\,k\Omega$, $C = 10\,\mu F$
(c) $R = 10\,k\Omega$, $L = 1$ H (d) $R = 1\,k\Omega$, $L = 10$ H

120. In the given AC circuit, when switch *S* is at position 1, the source emf leads current by π/6. Now, if the switch is at position 2, then

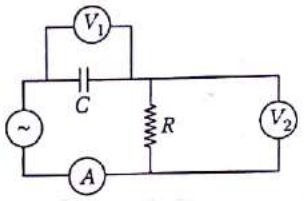


(a) current leads source emf by $\frac{\pi}{4}$
(b) current leads source emf by $\frac{\pi}{3}$
(c) source emf leads current by $\frac{\pi}{4}$
(d) source emf leads current by $\frac{\pi}{3}$

121. An AC ammeter is used to measure current in a circuit. When a given direct current passes through the circuit, the AC ammeter reads 3A. When another alternating current passes through the circuit, the AC ammeter reads 4A. Then the reading of this ammeter, if DC and AC flow through the circuit simultaneously, is
(a) 3A (b) 4A (c) 7A (d) 5A

122. An *L-C* circuit contains a 20 mH inductor and a 50μF capacitor with an initial charge of 10 mC. The resistance of the circuit is negligible. At the instant the circuit is closed be $t = 0$. At what time is the energy stored completely magnetic
(a) $t = 0$ (b) $t = 1.54$ ms (c) $t = 3.14$ ms (d) $t = 6.28$ ms

123. The diagram shows a capacitor C and a resistor R connected in series to an AC source V_1 and V_2 are voltmeters and A is an ammeter



Consider the following statements

I. Readings in A and V_2 are always in phase

II. Reading in V_1 is ahead in phase with reading in V_2

III. Readings in A and V_1 are always in phase. Which of these statements are/is correct

(a) I only (b) II only
(c) I and II only (d) II and III only

124. When an alternating voltage of 220 V is applied across a device P, a current of 0.25 A flows through the circuit and it leads the applied voltage by an angle $\frac{\pi}{2}$ radian. When the same voltage source is connected across another device Q, the same current is observed in the circuit but in phase with the applied voltage.

What is the current when the same source is connected across a series combination of P and Q?

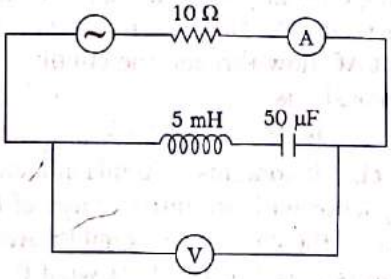
(a) $\frac{1}{4\sqrt{2}}$ A lagging in phase by $\frac{\pi}{4}$ with voltage

(b) $\frac{1}{4\sqrt{2}}$ A leading in phase by $\frac{\pi}{4}$ with voltage

(c) $\frac{1}{\sqrt{2}}$ A leading in phase by $\frac{\pi}{4}$ with voltage

(d) $\frac{1}{\sqrt{2}}$ A leading in phase by $\frac{\pi}{6}$ with voltage

125. In the circuit shown in figure, the AC source gives a voltage $V = 20 \cos(2000t)$. Neglecting source resistance, the voltmeter and ammeter readings will be (approximately)



(a) 4 V, 2.0 A (b) 0 V, 2A
(c) 0 V, 1.4 A (d) 8 V, 2.0 A

126. A fully charged capacitor C with initial charge q_0 is connected to a coil of self inductance L at $t = 0$. The time at which the energy is stored equally between the electric and the magnetic fields is

(a) $\pi\sqrt{LC}$ (b) $\frac{\pi}{4}\sqrt{LC}$
(c) $2\pi\sqrt{LC}$ (d) $\sqrt{LC}$

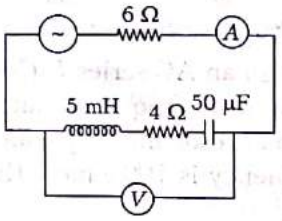
127. A circuit draws 330 W from a 110 V, 60 Hz AC line. The power factor is 0.6 and the current lags the voltage. The capacitance of a series capacitor that will result in a power factor of unity is equal to

(a) 31 μF (b) 54 μF
(c) 151 μF (d) 201 μF

128. An arc lamp requires a direct current of 10 A at 80 V to function. If it is connected to a 220 V(rms), 50 Hz AC supply, the series inductor needed for it to work is close to

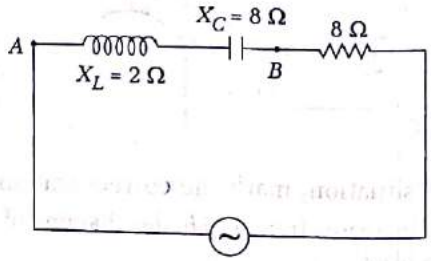
(a) 0.08 H (b) 0.044 H
(c) 0.065 H (d) 80 H

129. In the circuit shown in the figure, the ac source gives a voltage $V = 20 \cos(200t)$. Neglecting source resistance, the voltmeter and ammeter reading will be



(a) 0 V, 0.47 A (b) 1.68 V, 0.47 A
(c) 0 V, 1.4 A (d) 5.6 V, 1.4 A

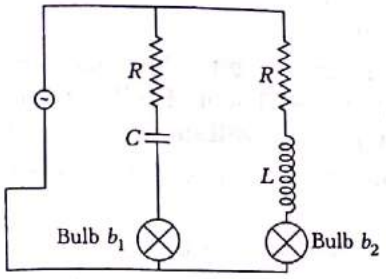
130. An inductor ($X_L = 2\ \Omega$), a capacitor ($X_C = 8\ \Omega$) and a resistance ($R = 8\ \Omega$) are connected in series with an AC source. The voltage output of AC source is given by $V = 10 \cos(100\pi t)$.



The instantaneous potential difference between points A and B, when the applied voltage is 3/5th of the maximum value of applied voltage is

(a) 0 V (b) 6 V
(c) 8 V (d) None of these

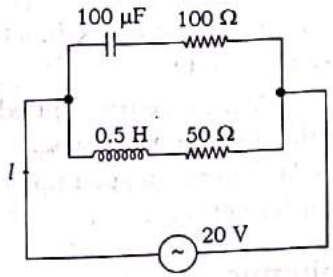
131. Two identical incandescent light bulbs are connected as shown in figure. When the circuit is an AC voltage source of frequency f, which of the following observations will be correct



(a) Both bulbs will glow alternatively
(b) Both bulbs will glow with same brightness provided $f = \frac{1}{2\pi}\sqrt{(1/LC)}$
(c) Bulb b_1 will light up initially and goes OFF, bulb b_2 will be ON constantly
(d) Bulb b_1 will blink and bulb b_2 will be ON constantly

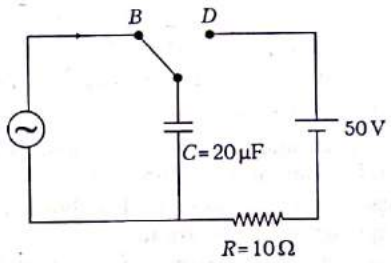
132. A series R-C circuit is connected to AC voltage source. Consider two cases; (A) when C is without a dielectric medium and (B) when C is filled with dielectric of constant 4. The current I_R through the resistor and voltage V_C across the capacitor are compared in the two cases. Which of the following is/are true
(a) $I_R^A > I_R^B$ (b) $I_R^A < I_R^B$ (c) $V_C^A > V_C^B$ (d) $V_C^A < V_C^B$

133. In the given circuit, the AC source has $\theta = 100 \text{ rad s}^{-1}$. considering the inductor and capacitor to be ideal, the correct choice (s) is (are)



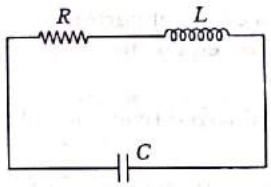
(a) The current through the circuit, I is 0.3 A
(b) The current through the circuit, I is $0.3\sqrt{2}$ A
(c) The voltage across 100 Ω resistor $= 10\sqrt{2}$ V
(d) The voltage across 50 Ω resistor $= 10$ V

134. At time $t = 0$, terminal A in the circuit shown in the figure is connected to B by a key and alternating current $I(t) = I_0 \cos(\omega t)$, with $I_0 = 1$A and $\omega = 500 \text{ rads}^{-1}$ starts flowing in it with the initial direction shown in the figure. At $t = \frac{7\pi}{6\omega}$, the key is switched from B to D. Now onwards only A and D are connected. A total charge Q flows from the battery to charge the capacitor fully. If $C = 20\ \mu, R = 10\ \Omega$ and the battery is deal with emf of 50 V, identify the correct statement(s).

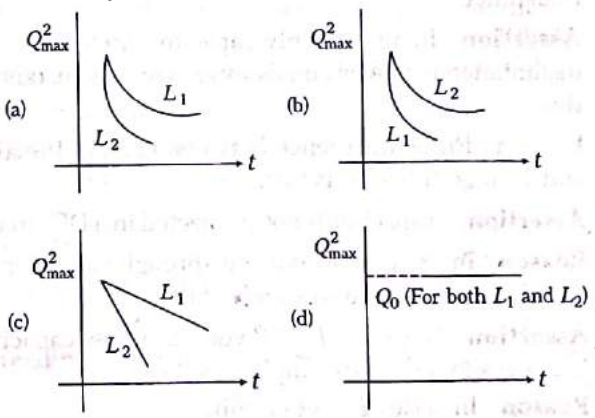


(a) Magnitude of the maximum charge on the capacitor before $t = \frac{7\pi}{6\omega}$ is 1×10^{-3}C
(b) The current in the left part of the circuit just before $t = \frac{7\pi}{6\omega}$ is clockwise
(c) Immediately after A is connected to D. The current in R is 10A
(d) $Q = 2 \times 10^{-3}$C

135. A L-C-R circuit is equivalent to a damped pendulum. In an L-C-R circuit the capacitor is charged to Q_0 and then connected to the L and R as shown below.



If a student plots graphs of the square of maximum charge (Q_{max}^2) on the capacitor with time (t) for two different values L_1 and L_2 ($L_1 > L_2$) of L, then which of the following represents this graph correctly (plots are schematic and not drawn to scale)



B. Medical entrance special format questions

Assertion and reason

Directions (Q. Nos. 1-15) *These questions consists of two statements each linked as Assertion and Reason. While answering these question you are required to choose any one of the following five responses.*

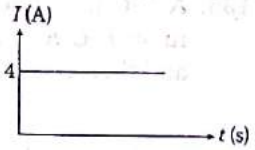
(a) If both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If Assertion is false but Reason is true.
(e) If both Assertion and Reason are false.

1. **Assertion** Average value of current in half the cycle of an AC circuit can't be zero.

   Reason For positive half cycle average value of current is $\frac{2}{\pi} i_0$, where i_0 is the peak value current.

   In the time interval from t_1 to t_2 average value of current will be zero.

2. **Assertion** Current *versus* time graph is as shown in figure, rms value of current is 4 A.

   Reason For a constant current, rms current is equal to that constant value.

3. **Assertion** Inductive reactance of an inductor in DC circuit is zero.

   Reason Angular frequency of DC circuit is zero.

4. **Assertion** If an inductor coil is connected to DC source, the current supplied by it is I_1. If the same coil is connected with an AC source of same voltage. Then current is I_2. Then $I_2 < I_1$.

   Reason In AC circuit, inductor coil offers more resistance.

5. **Assertion** In an AC, only capacitor circuit has instantaneous power equals to zero at any instant of time.

   Reason Phase difference between current function and voltage function is 90°.

6. **Assertion** A capacitor is not connected in a DC circuit.

   Reason In DC circuit, current through capacitor circuit becomes zero in steady state.

7. **Assertion** In series, *L-C-R* voltage across capacitor is always less than the applied voltage.

   Reason In series *L-C-R* circuit,

   $$V = \sqrt{V^2 + (V_L - V_C)^2}$$

8. **Assertion** When a ferromagnetic rod is inserted inside an inductor, then current in *L-C-R*, alternating circuit will decrease.

   Reason By inserting the ferromagnetic rod inside the inductor, coefficient of self induction and hence the net impedance will increases.

9. **Assertion** At resonance, power factor of *L-C-R* series circuit is 1.

   Reason At resonance, $X_C = X_L$.

10. **Assertion** At frequency greater than resonance frequency circuit is inductive in nature.

    Reason $X_L \propto \omega$.

11. **Assertion** In *L-C-R* series AC circuit, $X_L = X_C = R$ at a given frequency. When frequency is doubled, the impedance of the circuit is $\frac{\sqrt{13}}{2} R$.

    Reason The given frequency is resonance frequency.

12. **Assertion** Average power in an AC circuit is given by $P = I_{rms}^2 R$

    Reason In one full cycle, net power is dissipated only along a resistor.

13. **Assertion** In one complete cycle, power is consumed only across a resistance in series *L-C-R* circuit.

    Reason Average power consumed across an inductor or a capacitor is zero.

14. **Assertion** At resonance, power factor of series *L-C-R* circuit is zero.

    Reason At resonance, current function and voltage functions are in same phase.

15. **Assertion** An AC can be transmitted over long distances without much power loss.

    Reason An AC can be stepped up or down with the help of a transformer.

Match the Columns

1. Angular frequency ω in an AC, *L-C-R* series circuit is gradually increased. Then, match the following two columns.

	Column I		Column II
A.	Capacitive reactance	(p)	will continuously increase
B.	Inductive reactance	(q)	will continuously decrease
C.	Resistance	(r)	will remain constant
D.	Total impedance	(s)	will first decrease then increase

2. Match the following two columns for L-C-R series AC circuit.

	Column I		Column II
A.	At resonance frequency	(p)	Power factor = 0
B.	No resistance in the circuit	(q)	Power factor = 1
C.	Only resistance in the circuit	(r)	Circuit is capacitor
D.	Frequency greater than the resonance frequency	(s)	Circuit is inductive

3. In a series L-C-R, AC circuit assuming that symbols have their usual meanings match the following two columns.

	Column I		Column II
A.	If R is decreased	(p)	I will decrease
B.	If ω is decreased	(q)	I will increase
C.	If X_L is increased	(r)	I will first decrease, then increase
D.	If Z is increased	(s)	Can't say

4. In an AC, series L-C-R circuit, $R = X_L = X_C$ and applied AC voltage is V. Then match the following two columns.

	Column I		Column II
A.	V_R	(p)	zero
B.	V_C	(q)	V
C.	V_{RL}	(r)	$\sqrt{2}\,V$
D.	V_{CL}	(s)	$2\,V$

5. In an AC series L-C-R circuit, applied voltage is

$$V = \{100\sqrt{2}\sin(\omega t + 45°)\}\ \text{V}$$

Given that, $R = 30\,\Omega$, $X_L = 50\,\Omega$ and $X_C = 10\,\Omega$

Now match the following two columns.

	Column I		Column II
A.	Current in the circuit	(p)	120 SI units
B.	Power dissipated in the circuit	(q)	60 SI units
C.	Potential difference across resistance	(r)	2 SI units
D.	Potential difference across inductance	(s)	None

C. Medical entrances' gallery

(Collection of questions asked in NEET & various medical entrance exams)

1. A filament bulb (500W, 100V) is to be used in a 230 V main supply. When a resistance R is connected in series, it works perfectly and the bulb consumes 500W. The value of R is **[NEET (II) 2016]**
(a) $230\,\Omega$ (b) $46\,\Omega$ (c) $26\,\Omega$ (d) $13\,\Omega$

2. Which of the following combinations should be selected for better tuning of an L-C-R circuit used for communication? **[NEET (II) 2016]**
(a) $R = 20\,\Omega$, $L = 1.5$ H, $C = 35\,\mu$F
(b) $R = 25\,\Omega$, $L = 2.5$ H, $C = 45\,\mu$F
(c) $R = 15\,\Omega$, $L = 3.5$ H, $C = 30\,\mu$F
(d) $R = 25\,\Omega$, $L = 1.5$ H, $C = 45\,\mu$F

3. The potential differences across the resistance, capacitance and inductance are 80 V, 40 V and 100 V respectively in an L-C-R circuit. The power factor of this circuit is **[NEET (II) 2016]**
(a) 0.4 (b) 0.5 (c) 0.8 (d) 1.0

4. A $100\,\Omega$ resistance and a capacitor of $100\,\Omega$ reactance are connected in series across a 220 V source. When the capacitor is 50% charged, the peak value of the displacement current is **[NEET (II) 2016]**
(a) 2.2 A (b) 11 A (c) 4.4 A (d) $11\sqrt{2}$ A

5. A small signal voltage $V(t) = V_0 \sin \omega t$ is applied across an ideal capacitor C **[NEET 2016]**
(a) over a full cycle the capacitor C does not consume any energy from the voltage source
(b) current $I(t)$ is in phase with voltage $V(t)$
(c) current $I(t)$ leads voltage $V(t)$ by 180°
(d) current $I(t)$, lags voltage $V(t)$ by 90°

6. An inductor 20 mH, a capacitor 50 μF and a resistor $40\,\Omega$ are connected in series across a source of emf $V = 10 \sin 340\,t$. The power loss in AC circuit is **[NEET 2016]**
(a) 0.67 W (b) 0.76 W (c) 0.89 W (d) 0.51 W

7. A transformer is used to light 100 W-110 V lamp from 220 V mains. If the main current is 0.5 A, the efficiency of the transformer is **[KCET 2015]**
(a) 96% (b) 90%
(c) 99% (d) 95%

8. An alternating voltage given as, $V = 100\sqrt{2}\sin 100\,t$ V is applied to a capacitor of 1 μF. The current reading of the ammeter will be equal to ..... mA. **[Guj. CET 2015]**
(a) 20 (b) 10 (c) 40 (d) 80

9. An inductor coil is connected to a 12 V battery and drawing a current 24 A. This coil is connected to capacitor and an AC source of rms voltage rating 24 V in the series connection. The phase difference between current and emf is zero. The rms current through the circuit would found to be **[UP CPMT 2015]**
(a) 48 A (b) 36 A (c) 12 A (d) 24 A

10. A resistance R draws power P when connected to an AC source. If an inductance is now placed in series with the resistance, such that the impedance of the circuit becomes Z the power drawn will be **[CBSE AIPMT 2015]**
(a) $P\left(\frac{R}{Z}\right)^2$ (b) $P\sqrt{\frac{R}{Z}}$ (c) $P\left(\frac{R}{Z}\right)$ (d) P

11. A condenser of 250 μF is connected in parallel to a coil of inductance of 0.16 mH, while its effective resistance is 20Ω. Determine the resonant frequency. **[AIIMS 2015]**
(a) 9×10^4 Hz (b) 16×10^7 Hz
(c) 8×10^5 Hz (d) 9×10^3 Hz

12. The electric current in AC circuit is given by the relation $i = 3\sin\omega t + 4\cos\omega t$. The rms value of the current in the circuit in ampere is **[EAMCET 2015]**
(a) $\frac{5}{\sqrt{2}}$ (b) $5\sqrt{2}$ (c) $\frac{\sqrt{2}}{5}$ (d) $\frac{1}{\sqrt{2}}$

13. In an L-C-R series resonant circuit, the capacitance is changed from C to $4C$. For the same resonant frequency, the inductance should be changed from L to **[Kerala CEE 2015]**
(a) $2L$ (b) $\frac{L}{2}$ (c) $4L$ (d) $\frac{L}{4}$ (e) $\frac{L}{8}$

14. In a circuit, L, C and R are connected in series with an alternating voltage source of frequency f. The current leads the voltage by 45°. The value of C is **[Manipal 2015]**
(a) $\frac{1}{\pi f(2\pi fL+R)}$ (b) $\frac{1}{\pi f(2\pi fL-R)}$
(c) $\frac{1}{2\pi f(2\pi fL-R)}$ (d) $\frac{1}{2\pi f(2\pi fL+R)}$

15. The average power dissipated in a pure inductor is **[KCET 2015]**
(a) $\frac{VI^2}{4}$ (b) $\frac{1}{2}VI$ (c) zero (d) VI^2

16. A transformer having efficiency of 90% is working on 200 V and 3 kW power supply. If the current in the secondary coil is 6 A, then the voltage across the secondary coil and the current in the primary coil respectively are **[CBSE AIPMT 2014]**
(a) 300 V, 15 A (b) 450 V, 15 A
(c) 450 V, 13.5 A (d) 600 V, 15 A

17. A dynamo converts **[Kerala CEE 2014]**
(a) mechanical energy into thermal energy
(b) electrical energy into thermal energy
(c) thermal energy into electrical energy
(d) mechanical energy into electrical energy
(e) electrical energy into mechanical energy

18. Transformer is used to **[Kerala CEE 2014]**
(a) convert AC to DC voltage
(b) convert DC to AC voltage
(c) obtain desired DC power
(d) obtain desired AC voltage and current
(e) obtain desired DC voltage and current

19. A step-up transformer operates on a 230 V line and supplies a current of 2 A. The ratio of primary and secondary winding is 1 : 25. The primary current is **[UK PMT 2014]**
(a) 12.5 A (b) 50 A (c) 8.8 A (d) 25 A

20. A step-down transformer has 50 turns on secondary and 1000 turns on primary winding. If a transformer is connected to 220 V, 1 A AC source, then what is output current of the transformer? **[KCET 2014]**
(a) $\frac{1}{20}$ A (b) 20 A
(c) 100 A (d) 2 A

21. In L-C-R series circuit, an alternating emf e and current i are given by the equations $e = 100\sin(100t)$ V.

$$i = 100\sin\left(100t + \frac{\pi}{3}\right)\text{mA}$$

The average power dissipated in the circuit will be **[MHT CET 2014]**
(a) 100 W (b) 10 W
(c) 5 W (d) 2.5 W

22. The average power dissipated in AC circuit is 2 W. If a current flowing through a circuit is 2 A, impedance is 1Ω, then what is the power factor of the circuit? **[KCET 2014]**
(a) 0.5 (b) 1 (c) Zero (d) $\frac{1}{\sqrt{2}}$

23. In an L-C-R series circuit, the potential difference between the terminals of the inductance is 60 V, between the terminals of the capacitor is 30 V and that across the resistance is 40 V. Then, the supply voltage will be equal to **[UK PMT 2014]**
(a) 10 V (b) 50 V (c) 70 V (d) 130 V

24. An alternating emf given by equation

$$e = 300\sin[(100\pi)t]\text{ V}$$

is applied to a resistance 100 Ω. The rms current through the circuit is (in amperes) **[EAMCET 2014]**
(a) $\frac{3}{\sqrt{2}}$ (b) $\frac{9}{\sqrt{2}}$ (c) 3 (d) $\frac{6}{\sqrt{2}}$

25. A series L-C-R circuit contains inductance 5 mH, capacitance 2 μF and resistance 10 Ω. If a frequency AC source is varied, then what is the frequency at which maximum power is dissipated? **[KCET 2014]**

(a) $\frac{10^5}{\pi}$ Hz (b) $\frac{10^{-5}}{\pi}$ Hz

(c) $\frac{2}{\pi} \times 10^5$ Hz (d) $\frac{5}{\pi} \times 10^3$ Hz

26. The alternating current in a circuit is given by $I = 50 \sin 314t$. The peak value and frequency of the current are **[J & K CET 2013]**

(a) $I_0 = 25$ A and $f = 100$ Hz (b) $I_0 = 50$ A and $f = 50$ Hz
(c) $I_0 = 50$ A and $f = 100$ Hz (d) $I_0 = 25$ A and $f = 50$ Hz

27. A 50 Hz AC signal is applied in a circuit of inductance of $(1/\pi)$ H and resistance 2100 Ω. The impedance offered by the circuit is **[J & K CET 2013]**

(a) 1500 Ω (b) 1700 Ω (c) 2102 Ω (d) 2500 Ω

28. If the alternating current $I = I_1 \cos\omega t + I_2 \sin\omega t$, then the rms current is given by **[J & K CET 2013]**

(a) $\frac{I_1 + I_2}{\sqrt{2}}$ (b) $\frac{|I_1 + I_2|}{\sqrt{2}}$ (c) $\sqrt{\left(\frac{I_1^2 + I_2^2}{2}\right)}$ (d) $\sqrt{\frac{I_1^2 + I_2^2}{\sqrt{2}}}$

29. A 0.01 H inductor and $\sqrt{3}\,\pi\Omega$ resistance are connected in series with a 220 V, 50 Hz AC source. The phase difference between the current and emf is **[EAMCET 2013]**

(a) $\frac{\pi}{2}$ rad (b) $\frac{\pi}{6}$ rad (c) $\frac{\pi}{3}$ rad (d) $\frac{\pi}{4}$ rad

30. A coil of self-inductance L is connected in series with a bulb B and an AC source. Brightness of the bulb decreases when **[NEET 2013]**

(a) frequency of the AC source is decreased
(b) number of turns in the coil is reduced
(c) a capacitance of reactance $X_C = X_L$ is included in the same circuit
(d) an iron rod is inserted in the coil

31. For a transformer, the turns ratio is 3 and its efficiency is 0.75. The current flowing in the primary coil is 2 A and the voltage applied to it is 100 V. Then the voltage and the current flowing in the secondary coil are ..... respectively. **[Karnataka CET 2013]**

(a) 150 V, 1.5 A (b) 300 V, 0.5 A
(c) 300 V, 1.5 A (d) 150 V, 0.5A

32. In R-L-C series circuit, the potential differences across each element is 20 V. Now the value of the resistance alone is doubled, then PD across R, L and C respectively. **[Karnataka CET 2013]**

(a) 20 V, 10 V, 10 V (b) 20 V, 20 V, 20 V
(c) 20 V, 40 V, 40 V (d) 10 V, 20 V, 20 V

33. A series combination of resistor (R), capacitor (C) is connected to an AC source of angular frequency ω. Keeping the voltage same, if the frequency is changed to $\omega/3$, the current becomes half of the original current. Then, the ratio of the capacitive reactance and resistance at the former frequency is **[Karnataka CET 2013]**

(a) $\sqrt{0.6}$ (b) $\sqrt{3}$ (c) $\sqrt{2}$ (d) $\sqrt{6}$

34. If both the resistance and the inductance in an LR AC series circuit are doubled the new impedance will be **[Kerala CET 2013]**

(a) halved (b) fourfold
(c) doubled (d) quadrupled
(e) unchanged

35. A L-C-R circuit with $L = 1.00$ mH, $C = 10$ μF and $R = 50\Omega$, is driven with 5 V AC voltage. At resonance, the current through the circuit is **[Kerala CET 2013]**

(a) 0.2 A (b) 0.25 A (c) 0.15 A (d) 0.1 A
(e) 0.3 A

36. For a series L-C-R circuit, the rms values of voltage across various components are $V_L = 90$ V, $V_C = 60$ V and $V_R = 40$ V. The rms value of the voltage applied to the circuit is **[MP PMT 2013]**

(a) 190 V (b) 110 V (c) 70 V (d) 50 V

37. An air core coil and an electric bulb are connected in series with an AC source. If an iron rod is put in the coil, then the intensity of light of the bulb will **[MP PMT 2013]**

(a) remain same (b) increase
(c) decrease (d) first decrease then increase

38. The power factor of an AC circuit having resistance R and inductance L connected in series to an AC source of angular frequency ω is **[UP CPMT 2013]**

(a) zero (b) $\omega L/R$

(c) $\frac{R}{\sqrt{R^2 + \omega^2 L^2}}$ (d) $R/\omega L$

39. The self inductance of a choke coil is 10 mH. When it is connected with a 10 V DC source, then the loss of power is 20 W. When its is connected with 10 V AC source loss of power is 10 W. The frequency of AC source will be **[BCECE (Mains) 2012]**

(a) 80 Hz (b) 100 Hz (c) 120 Hz (d) 220 Hz

40. If in a L-R series circuit the power factor is $\frac{1}{2}$ and $R = 100\Omega$, then the value of L is, when AC mains is used **[BCECE (Mains) 2012]**

(a) π H (b) $\frac{\sqrt{3}}{\pi}$ H (c) $\frac{\pi}{\sqrt{3}}$ H (d) $\frac{\sqrt{2}}{\pi}$ H

Hints and explanations

Check point 7.1

1. (*d*) $2\pi f = 2000$

$\therefore\ f = \frac{1000}{\pi}$ Hz

6. (*a*) Here, $V_{rms} = 220\text{V}, v = 50$ Hz

Peak value of voltage, $V_0 = \sqrt{2}V_{rms} = 220\sqrt{2}$ V

The instantaneous value of voltage is

$$V = V_0 \sin 2\pi vt = 220\sqrt{2}\sin 2\pi \times 50t$$

$$= 220\sqrt{2}\sin 100\pi t$$

7. (*d*) $I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}} = 1\text{A}$

8. (*a*) $I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{5}{\sqrt{2}} = 3.536\text{A}$

Time taken, $T = \frac{1}{4 \times 60} = 4.167\text{ms}$

9. (*c*) $E = 141\sin(628t)$

$E_{rms} = \frac{E_0}{\sqrt{2}} = \frac{141}{141} = 100\text{V}$ and $2\pi f = 628$

$\Rightarrow\ f = 100$ Hz

10. (*c*) $I = 20\sin(100\pi t + 0.05\pi)$

$\therefore\ I_{rms} = \frac{20}{\sqrt{2}} = 10\sqrt{2}$ A

$\omega = 100\pi \Rightarrow f = 50\text{Hz}$

Check point 7.2

2. (*c*) Maximum potential difference, $V_0 = I_0 R$

$= (I_{rms}\sqrt{2})R$

$= (10\sqrt{2})(12) = 169.68$ V

3. (*a*) Capacitive reactance,

$$X_C = \frac{1}{2\pi v C} = \frac{1}{2\pi \times 4000 \times 25 \times 10^{-6}} = \frac{5}{\pi}\Omega$$

4. (*b*) In DC circuits, $f = 0$

$$\therefore\quad X_C = \frac{1}{2\pi(0)C} = \frac{1}{0} = \infty$$

7. (*c*) Given, $e = 200\sqrt{2}\sin 100t$

and $C = 1\mu\text{F}$

$E_{rms} = 200$ V

$$X_C = \frac{1}{\omega C} = \frac{1}{1 \times 10^{-6} \times 100} = 10^4\ \Omega$$

$$i_{rms} = \frac{E_{rms}}{X_C} \Rightarrow i_{rms} = \frac{200}{10^4} = 2 \times 10^{-2}\text{ A} = 20\text{ mA}$$

8. (*d*) $X_L = (2\pi fL)$

$$\therefore\quad L = \frac{X_L}{2\pi f} = \frac{50}{(2\pi)(50)} = 0.16\text{ H}$$

9. (*c*) Induced emf, $e = L\frac{di}{dt}$

$$\therefore\quad L = \frac{e}{(di/dt)} \Rightarrow L = \frac{\text{Volt-second}}{\text{Ampere}}$$

10. (*b*) In case of an inductor, voltage leads the current by $\frac{\pi}{2}$.

11. (*b*) $\tan\phi = \frac{X_L}{R}$

$$\therefore\quad \tan 45° = \frac{L\omega}{R}$$

$$\Rightarrow\quad L = \frac{R}{\omega} = \frac{100}{2\pi \times 1000} \qquad [\because \tan 45° = 1]$$

$$\Rightarrow\quad L = \frac{1}{20\pi}$$

12. (*b*) As, $I \propto \frac{1}{\omega L} \therefore \frac{I_1}{I_2} = \frac{\omega L_1}{\omega L_2} = \frac{L_1}{L_2}$

$$\therefore\quad \frac{I_1}{I_2} = \frac{X_{L_2}}{X_{L_1}} = \frac{L_2}{L_1}$$

14. (*d*) $Z = \sqrt{R^2 + X_L^2} = 10\,\Omega$

17. (*b*) impedance, $Z = \sqrt{R^2 + (2\pi fL)^2}$

As f increases, Z will increase.

18. (*a*) $\tan\phi = \frac{X_L}{R} = \frac{L\omega}{R}$

$$\tan\phi = \frac{3\Omega}{3\Omega} \Rightarrow \tan\phi = 1$$

$$\phi = \tan^{-1}(1) \Rightarrow \phi = 45°$$

$$\phi = \frac{\pi}{4}\text{ rad}$$

19. (*d*) Given, $X_L = \omega L = 2\pi fL = 2\pi \times 50 \times \frac{0.4}{\pi} = 40\,\Omega$

$R = 30\,\Omega$

$Z = \sqrt{R^2 + X_L^2} = \sqrt{(30)^2 + (40)^2} = 50\,\Omega$

$I_{rms} = \frac{V_{rms}}{Z} = \frac{200}{50} = 4$ A

20. (c) The phase difference between instantaneous value of I and V is

$$\frac{\pi}{4} - \left(-\frac{\pi}{4}\right) = \frac{\pi}{2}$$

Hence current leads the voltage by 90°.

21. (d) $V = \sqrt{V_R^2 + (V_L - V_C)^2} = 10\sqrt{2}\,\text{V}$

23. (b) The impedance of the circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$X_L = \omega L = 400 \times 20 \times 10^{-3} = 8\,\text{H}$$

$$X_C = \frac{1}{\omega C} = \frac{1}{400 \times 625 \times 10^{-6}} = 4\text{F}$$

$$Z = \sqrt{(3)^2 + (8-4)^2} = 5$$

$$I = \frac{E}{Z} = \frac{300}{5} = 60\text{ A}$$

28. (b) At resonance (series resonant circuit).

$$X_L = X_C$$

$\Rightarrow Z_{min} = R$, *i.e*, circuit behaves as resistive circuit.

29. (b) Equation of voltage, $E = \sqrt{V_R^2 + (V_L - V_C)^2}$

At resonance (series circuit)

$$V_L = V_C \Rightarrow E = V_R$$

i.e,whole applied voltage appeared across the resistance.

Check point 7.3

1. (b) $P = VI$

$$I = \frac{P}{V} = \frac{550}{220} = 2.5\text{ A}$$

4. (c) Voltage amplitude, $V_0 = 100$ V, current amplitude $i_0 = 100\text{ mA} = 100 \times 10^{-3}$ A and phase difference between i and V is $\phi = \frac{\pi}{3} = 60°$.

Now power dissipated is given by

$$P = \frac{V_0 i_0}{2} \cos\phi$$

$$= \frac{100 \times 100 \times 10^{-3}}{2} \times \cos 60° = 2.5\text{ W}$$

6. (d) In case of a pure inductance, $R = 0$

$\therefore$ Power factor $= \cos\phi = \frac{R}{Z} = \frac{0}{X} = 0$

So, $P_{av} = E_{rms} \cdot i_{rms} \times 0 = 0$

8. (b,c) Power factor of an AC circuit is given by

$$\cos\phi = \frac{R}{Z}$$

For circuit containing an ideal resistance only, $Z = R$

$\therefore$ $\cos\phi = 1$

For circuit containing an ideal inductance, $R = 0$

$\therefore$ $\cos\phi = \frac{0}{X_L} = 0$

9. (b) $Z = \sqrt{(3)^2 + (4)^2} = 5\,\Omega$

$$\cos\phi = \frac{R}{Z} = 0.6$$

11. (b) From $L = \frac{e}{di/dt}$

$\Rightarrow$ $[L] = [edt/di]$

12. (b) The energy stored in an inductor $U = \frac{1}{2}LI^2$

13. (a) $E = \frac{1}{2}LI^2 = \frac{1}{2}(100 \times 10^{-3})(10)^2 = 5\text{ J}$

Check point 7.4

2. (b) In step down transformer,

$$V_P > V_S$$

but, $\frac{V_P}{V_S} = \frac{I_s}{I_P}$

$\therefore$ $I_S > I_P$

So, current in the secondary coil is greater than the primary.

3. (c) A transformer works on the principle of mutual induction.

5. (c) If we neglect all losses, then input power = output power.

6. (b) Transformation ratio

$$k = \frac{N_S}{N_P} = \frac{V_S}{V_P}$$

For step up transformer,

$$N_S > N_P \quad i.e., \quad V_S > V_P$$

Hence $k > 1$

7. (a) As $\frac{N_1}{N_2} = \frac{I_2}{I_1} \Rightarrow I_2 = \left(\frac{N_1}{N_2}\right) I_1 = \left(\frac{500}{2000}\right) \times 48 = 12\text{ A}$

A. Taking it together

1. (c) In case of AC, $Z = \sqrt{R^2 + (\omega L)^2}$, while in case of DC it is only R.

2. (c) In a pure inductive circuit current lags the voltage by $\frac{\pi}{2}$.

3. (a) A choke coil has high inductance and low resistance, due to large inductance L of the coil, the current in the circuit is decreased appreciably. Due to small resistance of the coil, the power loss in the choke, coil is less.

5. (b) In an AC circuit, the coil of high inductance and negligible resistance used to control current, is called the choke coil. The power factor of such a coil is given by

$$\cos\phi = \frac{R}{\sqrt{R^2 + \omega^2 L^2}} \approx \frac{R}{\omega L} \qquad (\text{as } R << \omega L)$$

As $R << \omega L$, $\cos\phi$ is very small. Thus, the power absorbed by the coil is very small. The only loss of energy is due to hysteresis in the iron core, which is much less than the loss of energy in the resistance that can also reduce the current if placed instead of the choke coil.

6. (*a*) Capacitive reactance, $X_C = \frac{1}{2\pi \nu C}$

$$\Rightarrow \quad \frac{1}{1000} = \frac{1}{2\pi \times \nu \times 5 \times 10^{-6}}$$

$$\therefore \quad \nu = \frac{100}{\pi} \text{ MHz}$$

7. (*a*) $X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 100 \times 10^{-5}} = 160\ \Omega$

10. (*b*) By introducing the slab, C will increase. Therefore, X_C will decrease or I will increase.

11. (*b*) Applied voltage, $V = \sqrt{V_R^2 + V_L^2}$

$$\therefore \quad V = \sqrt{(200)^2 + (150)^2} = 250 \text{ V}$$

12. (*b*) $I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (2\pi fL)^2}}$

$$= \frac{120}{\sqrt{(10)^2 + (2\pi \times 60 \times 2)^2}} = 0.16 \text{ A}$$

13. (*b*) $< i >_T = < 5 >_R + < 5 \sin 100\ \pi t >_I$

$$= 5 + 0 = 5 \text{ A}$$

14. (*b*) Given, $\nu = 50$ Hz, $I_{rms} = 5$A

$$t = \frac{1}{300} \text{ s}$$

We have to find $I(t)$

$$I_0 = \text{Peak value} = \sqrt{2}\ I_{rms} = \sqrt{2} \times 5$$

$$= 5\sqrt{2} \text{ A}$$

$$I = I_0 \sin \omega t = 5\sqrt{2} \sin 2\pi \nu t = 5\sqrt{2} \sin 2\pi \times 50 \times \frac{1}{300}$$

$$= 5\sqrt{2} \sin \frac{\pi}{3} = 5\sqrt{2} \times \frac{\sqrt{3}}{2} = 5\sqrt{3/2} \text{ A}$$

15. (*b*) $E = 10 \cos (2\pi ft) = 10 \cos \left[2\pi \times 50 \times \frac{1}{600}\right]$

$$= 10 \cos \frac{\pi}{6} = 5\sqrt{3} \text{ V}$$

16. (*c*) Phase difference between current and voltage is $\pi/4$, with voltage leading.

17. (*c*) Resonant frequency, $f = \frac{1}{2\pi \sqrt{LC}}$

does not depend on resistance.

18. (*a*) Resonant frequency, $f = \frac{1}{2\pi\sqrt{LC}}$

$$\therefore \quad f \propto \frac{1}{\sqrt{C}} \quad \text{or} \quad \frac{f_1}{f_2} = \sqrt{\frac{C_2}{C_1}}$$

$$\Rightarrow \quad \frac{f}{f_2} = \sqrt{\frac{4C}{C}} \Rightarrow f_2 = \frac{f}{2}$$

19. (*c*) Current in inductance I_L lags behind, current in resistance I_R is in phase with voltage, while current in capacitance I_C leads by a phase of $\pi/2$.

20. (*b*) Power dissipated in circuit

$$P = \frac{V_{rms}^2}{R} = \frac{(30)^2}{10} = 90 \text{ W}$$

22. (*c*) $\frac{H_{DC}}{H_{DC}} = \left(\frac{i_{DC}}{i_{rms}}\right)^2 (\because H \propto i^2) = \left(\frac{2}{\sqrt{2}}\right)^2 = 2$

23. (*c*) $I_0 = 28$ A

$$\therefore \quad I_{rms} = \frac{I_0}{\sqrt{2}} \simeq 20 \text{ A}$$

24. (*b*) $P = \frac{1}{2} V_0 I_0 \cos \phi$

$$\Rightarrow \quad P = P_{peak} \cos \phi$$

$$\Rightarrow \quad \frac{1}{2}(p_{peak}) = P_{peak} \cos \phi$$

$$\Rightarrow \quad \cos \phi = \frac{1}{2} \Rightarrow \phi = \frac{\pi}{3}$$

25. (*a*) $P = V_{rms} I_{rms} \cos \phi$

$$= V_{rms}\left(\frac{V_{rms}}{Z}\right)\left(\frac{R}{Z}\right) = \frac{(110)^2\,(11)}{(22)^2} = 275 \text{ W}$$

26. (*a*) $V^2 = V_R^2 + V_L^2$

$$\therefore \quad V_L = \sqrt{V^2 - V_R^2}$$

$$= \sqrt{(20^2 - (12)^2} = 16 \text{ V}$$

27. (*b*) We know that resonant frequency in an L-C-R circuit is given by

$$\nu_0 = \frac{1}{2\pi\sqrt{LC}}$$

Now to reduce ν_0 either we can increase L or we can increase C. To increase capacitance, we must connect another capacitor parallel to the first.

28. (*c*) Average value of output current is given by

$$I_{av} = \frac{\int_0^{T/2} I\,dt}{\int_0^{T/2} dt} = \frac{\int_0^{T/2} I_0 \sin \omega t\,dt}{T/2}$$

$$= \frac{2I_0}{T}\left[-\frac{\cos \omega t}{\omega}\right]_0^{T/2}$$

$$= \frac{2I_0}{T}\left[-\frac{\cos(\omega T/2)}{\omega} + \frac{\cos 0°}{\omega}\right]$$

$$= \frac{2I_0}{\omega T}[-\cos \pi + \cos 0°] = \frac{2I_0}{2\pi}[1 + 1] = \frac{2I_0}{\pi}$$

29. (*d*) $e = \sqrt{e_1^2 + e_2^2} \sin(\omega t + \phi)$

where $\cos \phi = \frac{e_1}{\sqrt{e_1^2 + e_2^2}}$ and $\sin \phi = \frac{e_2}{\sqrt{e_1^2 + e_2^2}}$

i.e., $e_0 = \sqrt{e_1^2 + e_2^2}$

or $e_{rms} = \frac{\sqrt{e_1^2 + e_2^2}}{\sqrt{2}}$

30. (c) Phase different between current and voltage,

$$\Delta\phi = \phi_2 - \phi_1 = \frac{\pi}{6} - \left(-\frac{\pi}{6}\right) = \frac{\pi}{3}$$

31. (b) Reading of ammeter $= i_{rms}$

$$i_{rms} = \frac{V_{rms}}{X_C} = \frac{V_0}{\sqrt{2}} \times \omega C$$

$$= \frac{200\sqrt{2} \times 100 \times (1 \times 10^{-6})}{\sqrt{2}}$$

$$= 2 \times 10^{-2} \text{ A} = 20 \text{ mA}$$

32. (d) $\tan\phi = \frac{X_L}{R}$ (In case of L-R circuit)

$$= \frac{L\omega}{R} = \frac{L2\pi f}{R} = \frac{2 \times 200}{300} = \frac{4}{3}$$

33. (a) $\omega = \frac{1}{\sqrt{LC}}$

$$\therefore \quad f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{20 \times 10^{-6} \times 10 \times 10^{-3}}} = 356 \text{ Hz}$$

34. (b) $I_{rms} = \sqrt{(I^2)} = \sqrt{(8 + 6\sin\omega t)^2}$

$\Rightarrow I_{rms} = \sqrt{((64 + 96\sin\omega t + 36\sin^2\omega t))}$

$\Rightarrow I_{rms} = \sqrt{(64) + 96(\sin\omega t) + 36(\sin^2\omega t)}$

$\Rightarrow I_{rms} = \sqrt{64 + 0 + 36 \times 0.5} = 9.05$

Since, $(\sin^2\omega t) = 0.5$

36. (b) Value of induced emf in coil

$$e = M\frac{dI}{dt}$$

$$= 0.005 \times \frac{d}{dt}(I_0 \sin\omega t)$$

$$= 0.005 \times I_0\, \omega \cos\omega t$$

$$\therefore \quad e_{max} = 0.005 \times 10 \times 100\pi = 5\pi$$

37. (c) For only a resistance, it hardly matters, whether the source is DC or AC. In series,

$$P = \frac{P_1P_2}{P_1 + P_2}$$

38. (b) $X_L = R \;\therefore Z = \sqrt{2}R$

$$P = \left(\frac{V_{rms}}{Z}\right)^2 \cdot R = \left(\frac{E_0/\sqrt{2}}{\sqrt{2}R}\right)^2 \cdot R = \frac{E_0^2}{4R}$$

40. (b) Induced emf in the circuit, $e = M\frac{di}{dt} = (0.2)(5) = 1$ V

41. (b) $\frac{N_P}{N_S} = \frac{V_P}{V_S} = \frac{i_S}{i_P}$

$$\therefore \quad i_S = \frac{N_P i_P}{N_S} = \frac{4 \times 140}{280} = 2\text{A}$$

42. (d) $I_R = I_L = I$

But they have a phase difference $0 < \phi < 90°$, with applied voltage.

43. (b) We have, $\frac{N_S}{N_P} = \frac{i_P}{i_S}$

$$\Rightarrow \quad \frac{10}{500} = \frac{i_P}{i_S} \Rightarrow \frac{i_P}{i_S} = \frac{1}{50} \Rightarrow i_S = 50\, i_P$$

This condition is satisfied only when current in primary 3.2×10^{-3} A and in secondary 0.16 A.

44. (b) $V_C = \sqrt{V^2 - V_R^2} = 16\text{V}$

45. (c) Given, $V = 150 \sin(150t)$ V

and $\quad I = 150 \sin(150t + \pi/3)$ A

$\therefore \quad I_0 = 150$ A and $V_0 = 150$ V

Power, $\quad P = \frac{1}{2}V_0 I_0 \cos\phi$

$\Rightarrow \quad P = 0.5 \times 150 \times 150 \times \cos 60°$

$\Rightarrow \quad P = 5625$ W

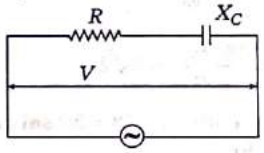
46. (d) $X_L = 2\pi fL = 2\pi\left(\frac{50}{\pi}\right) \times 1 = 100\,\Omega$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi\left(\frac{50}{\pi}\right) 20 \times 10^{-6}} = 500\,\Omega$$

impedance, $\quad Z = \sqrt{R^2 + (X_C - X_L)^2}$

$\Rightarrow \quad Z = \sqrt{(300)^2 + (400)^2} = 500\,\Omega$

48. (a) Let the applied voltage be V volt



Here, $\quad V_R = 12\text{V}, V_C = 5\text{V}$

$$V = \sqrt{V_R^2 + V_C^2} = \sqrt{(12)^2 + (5)^2}$$

$$= \sqrt{144 + 25} = \sqrt{169} = 13\text{V}$$

49. (c) The voltmeter connected to AC mains reads mean value ($<v^2>$) and is calibrated in such a way that it gives value of $<v^2>$, which is multiplied by form factor to give rms value.

50. (a) $V_{rms} = \sqrt{V_{av}} = \sqrt{\frac{1}{T}\int_0^T V^2\,dt} = \sqrt{\frac{1}{T}\int_0^T 10^2\,dt} = 10$ V

51. (c) Peak voltage, $V_0 = \sqrt{2}\,V_{rms} = 331\text{V}$

$\omega = 2\pi f = 100\,\pi$

$\Rightarrow V_L = 331\sin(100\,\pi\,t)$

where, V_L is line voltage.

52. (a) Secondary voltage $V_S = 24$ V

Power associated with secondary, $P_S = 12$ W

$$I_S = \frac{P_S}{V_S} = \frac{12}{24} = \frac{1}{2}\text{A} = 0.5\text{ A}$$

Peak value of the current in the secondary

$$I_0 = I_S\sqrt{2}$$

$$= (0.5)(1.414) = 0.707 = \frac{1}{\sqrt{2}}\text{A}$$

53. (*d*) Time taken by the current to reach the maximum value.

$$t = \frac{T}{4} = \frac{1}{4f} = \frac{1}{4 \times 50} = 5 \times 10^{-3}\ \text{s}$$

and $\quad I_0 = \sqrt{2} I_{rms} = 10\sqrt{2} = 14.14\ \text{A}$

54. (*b*) For capacitive circuits, $X_C = \dfrac{1}{\omega C}$

$$\therefore \quad I = \frac{V}{X_C} = V\omega C \Rightarrow I \propto \omega$$

56. (*d*) According to problem, $X_L = X_C$

$$\omega L = \frac{1}{\omega C}$$

$$\therefore \quad \omega^2 = \frac{1}{LC} = \frac{1}{\sqrt{10^{-3} \times 10 \times 10^{-6}}}$$

$$\omega = \frac{1}{\sqrt{10^{-8}}} = 10^4$$

$$\therefore \quad X_L = \omega L = 10^4 \times 10^{-3} = 10\ \Omega$$

57. (*c*) At resonance frequency, current is maximum.

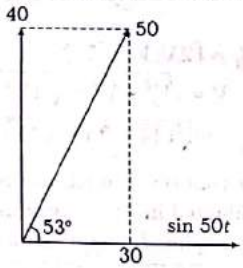
58. (*d*) For DC motor, $I = \dfrac{V_S - V_B}{R}$ and $V_B \propto \omega$

$$1.5 = \frac{220 - V_B}{20} \Rightarrow V_B = 190\ \text{V}$$

59. (*b*) $P = V_{rms} I_{rms} = \dfrac{V_0}{\sqrt{2}} I_{rms}$

$$\therefore \quad I_{rms} = \frac{1000\sqrt{2}}{200} = 5\sqrt{2}$$

60. (*a*) $V = 30 \sin 50t + 40 \cos 50t = 50 \sin (50t + 53°)$



$$I_{rms} = \frac{V_{rms}}{Z} = \frac{50/\sqrt{2}}{10} = \frac{5}{\sqrt{2}}\ \text{A}$$

61. (*c*) $I_0 = \dfrac{140}{10} = 14\ \text{A}$

$$\therefore \quad I_{rms} = \frac{I_0}{\sqrt{2}} \approx 10\ \text{A}$$

$\therefore I_{DC}$ required will be approximately 10 A.

62. (*c*) $i_{rms}^2 = \dfrac{\int i^2\, dt}{\int dt}$

$$= \frac{\int_2^4 (4t)\, dt}{\int_2^4 dt} = \frac{4\int_2^4 t\, dt}{2} = 2\left[\frac{t^2}{2}\right]_2^4 = 12\ \text{A}^2$$

$$\Rightarrow i_{rms} = 2\sqrt{3}\ \text{A}$$

63. (*b*) Power factor, $\cos\phi = \dfrac{1}{\sqrt{2}} = \dfrac{R}{\sqrt{R^2 + \omega^2 L^2}}$

This gives, $\quad \omega L = R$

when ω is doubled, we find

$$\cos\phi' = \frac{R}{\sqrt{R^2 + 4\omega^2 L^2}} = \frac{R}{\sqrt{R^2 + 4R^2}} = \frac{1}{\sqrt{5}}$$

64. (*d*) Resistance of coil $(R) = \dfrac{200\text{V}}{1\text{A}} = 200\,\Omega$

With AC source, $I = \dfrac{200}{\sqrt{R^2 + X_L^2}}$ or $0.5 = \dfrac{200}{\sqrt{R^2 + X_L^2}}$

$$\Rightarrow \quad R^2 + (2\pi f L)^2 = (400)^2$$

$$\Rightarrow \quad \left(2\pi f \times \frac{2\sqrt{3}}{\pi}\right)^2 = (400)^2 - (200)^2 = 200 \times 600$$

$$\Rightarrow \quad 4f\sqrt{3} = 2\sqrt{3} \times 100 \Rightarrow f = 50\ \text{Hz}$$

65. (*a*) From $P = VI = \dfrac{V^2}{R}$

$$\Rightarrow \quad I = \frac{P}{V} = 6\text{A}$$

$$\Rightarrow \quad R = \frac{V^2}{P} = \frac{100}{60} = \frac{5}{3}\,\Omega$$

Now, $\quad I = \dfrac{V}{Z}$

$$\Rightarrow \quad 6 = \frac{100}{\sqrt{\left(\frac{5}{3}\right)^2 + (2\pi f L)^2}}$$

Solving this equation we get, $L = 0.052$ H

66. (*c*) Given $X_L = X_C = 5\Omega$, this is the condition of resonance. So $V_L = V_C$, so net voltage across L and C combination will be zero.

67. (*d*) Since, $V_C > V_L$

$$\therefore \quad X_C > X_L$$

Hence, current will lead the voltage.

$$V = \sqrt{V^2 + (V_C - V_L)^2} = 10\ \text{V} < V_C$$

$$\cos\phi = \frac{R}{Z} = \frac{8}{10} = \frac{4}{5}$$

68. (*a*) impedance,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\therefore \quad 10 = \sqrt{(10)^2 + (X_L - X_C)^2}$$

$$\Rightarrow \quad 100 = 100 + (X_L - X_C)^2$$

$$\Rightarrow \quad X_L - X_C = 0 \qquad \text{...(i)}$$

Let ϕ is the phase difference between current and voltage

$$\therefore \quad \tan\phi = \frac{X_L - X_C}{R}$$

$$\therefore \quad \tan\phi = \frac{0}{R}$$

$$\phi = 0 \qquad \text{[From Eq. (i)]}$$

69. (b) Resonance frequency, $\omega = \frac{1}{\sqrt{LC}}$

$$= \frac{1}{\sqrt{8 \times 10^{-3} \times 20 \times 10^{-6}}}$$

$$= 2500 \text{ rads}^{-1}$$

Resonance current, $I = \frac{V}{R} = \frac{220}{44} = 5\text{A}$

70. (c) At point A, $X_C > X_L$

at point B, $X_C = X_L$

at point C, $X_C < X_L$

So at C, circuit is inductive.

71. (d) At resonance,

$$X_L = X_C$$

$$\therefore \quad Z = R = 40\,\Omega$$

$$I_{rms} = \frac{V_{rms}}{Z} = 5.75\text{A}$$

72. (b) At resonance,

$$X_L = X_C$$

$$\therefore \quad V = V_R$$

Hence, V_2 will be zero

73. (a) $Z = \sqrt{(52)^2 + \left[230 \times 10^{-3} \times 2\pi \times 80 - \frac{1}{2\pi \times 80 \times 8.8 \times 10^{-6}}\right]^2}$

$= 122\,\Omega$

$$P = \left(\frac{V_{rms}}{Z}\right)^2 \cdot R = \left(\frac{150}{122}\right)^2 (52) = 78.6 \text{ W}$$

74. (a) $C_{net} = C/2$ (in series) and $L_{net} = 2L$ (in series)

$$\therefore \quad \omega = \frac{1}{\sqrt{L_{net}C_{net}}} \text{ or } f = \frac{1}{2\pi\sqrt{(2L)\cdot\left(\frac{C}{2}\right)}} \text{ or } f = \frac{1}{2\pi\sqrt{LC}}$$

75. (c) Four resistances form a balanced Wheatstone bridge. Current flowing in the coil will be zero and hence energy stored in the coil is zero.

76. (b) $N_P = 100$, $P_i = 10$ kW, $V_i = 200$ V

$$\therefore \quad I_P = \frac{P_i}{V_i} = \frac{10000}{200} = 50\text{A}$$

Input power = Output power (Neglecting all losses)

$$\therefore \quad I_S = \frac{P_i}{V_0} = \frac{10000}{5000} = 2\text{A}$$

77. (a, c, d) In resonance $V_L = V_C$ and $V = V_R$

79. (a,c,d) $X_C = \frac{1}{\omega C} = \frac{1}{2\pi \times 5 \times 10^3 \times \frac{1}{\pi} \times 10^{-6}} = 100\,\Omega$

Since $X_C = R$

$$\therefore \quad I_C = I_R = \frac{20}{100} = 0.2 \text{ A}$$

$$\Rightarrow \quad I = \sqrt{(0.2)^2 + (0.2)^2} = 0.283\text{A}$$

81. (c) $R = X_L = \frac{200}{5} = 40\,\Omega$

$$\therefore \quad Z = \sqrt{R_f^2 + X_L^2} = 40\sqrt{2}\,\Omega$$

$$\Rightarrow \quad I_{rms} = \frac{V_{rms}}{Z} = \frac{200/\sqrt{2}}{40\sqrt{2}} = \frac{5}{2}\text{A}$$

82. (c) Let R be the resistance of coil + capacitor. Then,

$$R = \frac{24}{6} = 4\,\Omega$$

In the second case,

$$I_{max} = \frac{12}{4+4} = \frac{12}{8} = 1.5 \text{ A}$$

83. (a, b, c, d) $X_C > X_L$. Hence, current will lead the voltage.

$$Z = \sqrt{R^2 + (X_C - X_L)^2} = 10\sqrt{2}\,\Omega$$

$$\therefore \quad I_{rms} = \frac{V_{rms}}{Z} = \frac{400/\sqrt{2}}{10\sqrt{2}} = 20 \text{ A}$$

$$\cos\phi = \frac{R}{Z} = \frac{1}{\sqrt{2}}$$

$$V_R = I_{rms}R = (20)(10) = 200 \text{ V}$$

84. (d) Taking, $V^2 = V_R^2 + (V_L - V_C)^2$

According to the figure, we conclude that

$$V_L = V_C$$

$$\therefore \quad V_R = V = 220 \text{ V}$$

Reading of voltmeter = 220 V

Reading of ammeter, $I = \frac{220}{100} = 2.2$ A

85. (d) The voltages V_L and V_C are equal and opposite so, voltmeter reading will be zero.

Also, $R = 30\,\Omega$, $X_L = X_C = 25\,\Omega$

So, $I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$

$$= \frac{V}{R} = \frac{240}{30} = 8 \text{ A}$$

86. (b) Resonance frequency,

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{100 \times 10^{-6} \times 1 \times 10^{-6}}} = 10^5 \text{ rads}^{-1}$$

Now, given $\omega = 70$ kilo-rads^{-1} = 70000 rads^{-1}

$\Rightarrow \quad \omega_0 > \omega$

As $X_L \propto \omega$ and $X_C \propto \frac{1}{\omega}$

$\therefore \quad X_L > X_C$

The circuit will be R-L circuit.

88. (d) At resonance net voltage across L and C is zero.

89. (c) For delivering maximum power from the generator to the load, total internal reactance must be equal to conjugate of total external reactance.

Hence, $X_{int} = {}^*X_{ext}$

$\Rightarrow \quad X_g = (X_L)^* = -X_L$

$\Rightarrow \quad X_L = -X_g$

90. (*c*) Quality factor (Q) of an L-C-R circuit is given by,

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

where R is resistance, L is inductance and C is capacitance of the circuit. To make Q high,

R should be low, L should be high and C should be low.

These conditions are best satisfied by the values given in option (c).

Note We should be careful while writing formula for quality factor, because we are considering series L-C-R circuit.

91. (*c*) Current through inductor and through condenser are out of phase.

So, $I = (I_L - I_C) = 1.2\text{A}$

92. (*b*) Voltage drop across R is maximum at resonance, *i.e.*, at

$$\omega = \frac{1}{\sqrt{LC}} \quad \text{or} \quad f = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{\left(\frac{1}{\pi}\right)\left(\frac{1}{\pi}\times 10^{-6}\right)}} = 500 \text{ Hz}$$

93. (*d*) $I_1 = I_2$

$$\therefore \quad \omega_1 L - \frac{1}{\omega_1 C} = \frac{1}{\omega_2 C} - \omega_2 L$$

Solving this, we get

$$\frac{1}{\sqrt{LC}} = \text{resonance frequency} = \sqrt{\omega_1\omega_2}$$

At resonance frequency, current will be maximum.

94. (*a*) The given AC circuit is the combination of two pure parallel circuits with the applied voltage. In which I_2 is in phase with V and I_1 leads V by 90°.

95. (*d*) At resonance,

Voltage across L = Voltage across C

$\therefore$ Reading in V_2 = Reading in V_3.

96. (*b*) $I_{rms} = \dfrac{V_{rms}}{\sqrt{R^2 + \dfrac{1}{\omega^2 c^2}}}$

when ω increases, I_{rms} increases so the bulb glows brighter.

97. (*d*) $V = \sqrt{V_R^2 + (V_L - V_C)^2}$

$\because \quad V_R = V$

$\because \quad V_L = V_C$

$\therefore$ Reading of voltmeter = 220V

Reading of ammeter, $I_{rms} = \dfrac{E_{rms}}{Z} = \dfrac{220}{100} = 2.2\text{ A}$

98. (*a*) Total resistance of the circuit,

$$R = 6 + 4 = 10\,\Omega$$

Capacitive reactance,

$$X_C = \frac{1}{\omega C} = \frac{1}{2000 \times 50 \times 10^{-6}} = 10\,\Omega$$

Inductive reactance,

$$X_L = \omega L = 2000 \times 5 \times 10^{-3} = 10\,\Omega$$

$$\therefore \quad Z = \sqrt{R^2 + (X_L - X_C)^2} = 10\,\Omega$$

Amplitude of current, $I_0 = \dfrac{V_0}{Z} = \dfrac{20}{10} = 2\text{A}$

99. (*a*) $X_C = X_L = R$

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

but $V_L = V_C$

$\therefore \quad V = V_R = 10$

When capacitor is short circuited, $V_C = 0$

and $V_R = V_L$ (as $R = X_L$)

$\therefore \quad V = \sqrt{V_R^2 + V_L^2}$ or $10 = \sqrt{2}\,V_L$

$\therefore \quad V_L = \dfrac{10}{\sqrt{2}}\text{ V}$

100. (*d*) $V_C = 200$ V [given]

$V_R = (11)(20) = 220$ V

$V = 220$ V [applied]

$\therefore \quad V_L = 200$ V

because $V = \sqrt{V_R^2 + (V_L - V_C)^2}$

101. (*c*) Given, $X_L = 1\Omega$, $R = 2\Omega$

$E_{rms} = 6$ V, $P_{av} = ?$

Average power dissipated in the circuit

$$P_{av} = V_{rms}\, I_{rms} \cos\phi \quad \text{...(i)}$$

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{V_{rms}}{Z}$$

$$Z = \sqrt{R^2 + X_L^2}$$

$$= \sqrt{4+1} = \sqrt{5}$$

$$I_{rms} = \frac{6}{\sqrt{5}}\text{ A}$$

$$\cos\phi = \frac{R}{Z} = \frac{2}{\sqrt{5}}$$

$$P_{av} = 6 \times \frac{6}{\sqrt{5}} \times \frac{2}{\sqrt{5}} \quad \text{[from Eq. (i)]}$$

$$= \frac{72}{\sqrt{5}\sqrt{5}} = \frac{72}{5} = 14.4\text{ W}$$

102. (*b*) $X_L = (2\pi f L) = 2\pi \times 50 \times 0.7 = 220\,\Omega$

$$\therefore \quad Z = \sqrt{R^2 + X_L^2} = 220\sqrt{2}\,\Omega$$

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{220}{220\sqrt{2}} = \frac{1}{\sqrt{2}}\text{ A}$$

$$P = I_{rms}^2 \cdot R = \frac{1}{2} \times 220 = 110\text{ W}$$

103. (*d*) $V_L = V_C \Rightarrow X_L = X_C$ or circuit shown is in resonance.

$\therefore \quad V_{applied} = V_R = 100$ V

$$I = \frac{V_R}{R} = 2\text{A}$$

104. (*d*) $V_S^2 = (3)^2 + (8-4)^2$; $V_S = 5$V

Now, $Z = \dfrac{V_S}{I} = \dfrac{5}{1} = 5\,\Omega$

Also, $V_R = IR$ or $R = \frac{3}{1} = 3\,\Omega$

So $\text{PF} = \frac{R}{Z} = \frac{3}{5} = 0.6$ as $V_L > V_C$

$\Rightarrow$ I lags V, so this is a lagging nature.

105. (d) $X_C = X_L$

$\therefore\ Z = R = 2\Omega \Rightarrow I_{rms} = \frac{V_{rms}}{Z} = \frac{100/\sqrt{2}}{2} = 25\sqrt{2}\,\text{A}$

Phase difference will be zero.

Average power $= V_{rms}\, I_{rms} \cos\phi = \frac{100}{\sqrt{2}} \times 25\sqrt{2} \times \cos 0°$

$= 2500$ W

106. (d) Phase angle, $\tan\phi = \frac{X_L}{R} = \frac{X_C}{R}$

$\Rightarrow \quad \tan 60° = \frac{X_L}{R} = \frac{X_C}{R} \Rightarrow X_L = X_C = \sqrt{3}\,R$

i.e., $Z = \sqrt{R^2 + (\sqrt{3}\,R - \sqrt{3}\,R)^2} \Rightarrow Z = R$

So, average power, $P = \frac{V^2}{R} = \frac{200 \times 200}{100} = 400$ W

107. (b) Power consumed in the coil

$P = I_{rms}^2\, R$

$\therefore \quad R = \frac{P}{I_{rms}^2} = \frac{240}{16} = 15\,\Omega$

Impedance, $Z = \frac{V}{I} = \frac{100}{4} = 25\,\Omega$

Now, $X_L = \sqrt{Z^2 - R^2}$

$= \sqrt{(25)^2 - (15)^2} = 20\,\Omega$

$\therefore \quad 2\pi\nu L = 20$

$L = \frac{20}{2\pi \times 50} = \frac{1}{5\pi}$ H

108. (b) In an L-C-R circuit, the current and the voltage are in phase ($\phi = 0$), when

$\tan\phi = \frac{\omega L - \frac{1}{\omega C}}{R} = 0$ or $\omega L = \frac{1}{\omega C}$

or $L = \frac{1}{\omega^2 C}$

Here, $\omega = 2\pi f = 2 \times 3.14 \times 50\ \text{s}^{-1} = 314\ \text{s}^{-1}$

and $C = 20\ \mu\text{F} = 20 \times 10^{-6}$ F

$\therefore \quad L = \frac{1}{(314\ \text{s}^{-1})^2 \times (20 \times 10^{-6}\ \text{F})} = 0.51$ H

109. (a) Angular velocity,

$\omega_0 = 2\pi\nu = 2\pi \times 100$

$\omega_0 = 2 \times 3 \times 100$ $\quad [\because \pi = 3]$

$= 600\ \text{rads}^{-1}$ $\quad$ ...(i)

Further, $\omega_0 = \frac{1}{\sqrt{LC}}$

Also, $X_C = \frac{1}{C\omega_0} = 60\ \Omega$

$\Rightarrow \quad C = \frac{1}{\omega_0 \times 60} = \frac{1}{600 \times 60} \Rightarrow C = \frac{1}{36 \times 10^3}$ F

So, put values in Eq. (i), we get

$600 = \frac{1}{\sqrt{L\left(\frac{1}{36 \times 10^3}\right)}}$

$\Rightarrow \quad 36 \times 10^4 = \frac{36 \times 10^3}{L} \Rightarrow L = \frac{36 \times 10^3}{36 \times 10^4} = \frac{1}{10} = 0.1$H

110. (a) Change on the capacitor,

$q_0 = CV = 1 \times 10^{-6} \times 1 = 10^{-6}$ C

Here, $q = q_0 \sin\omega t$

or $I_0 = \omega q_0 =$ maximum current

Now, $\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-9}}} = (10^9)^{1/2}$

$\therefore \quad I_0 = (10^9)^{1/2} \times (1 \times 10^{-6}) = \sqrt{1000}$ mA

112. (a) $V = \sqrt{V_R^2 + (V_L - V_C)^2}$

As $V_L = V_C = 400$ V

$\therefore \quad V = V_R$

Reading of voltmeter = 100 V

Reading of ammeter, $I_{rms} = \frac{V_{rms}}{Z} = \frac{100}{50} = 2$A

113. (b) Circuit is in resonance,

$\therefore \quad V = V_R = IR$

or $R = \frac{V}{I} = \frac{100}{0.2} = 500\,\Omega$

$X_L = \frac{V_L}{I} = \frac{400}{0.2} = 2000\,\Omega = \omega L$

$\therefore \quad \omega = 1000\ \text{rads}^{-1}$

Further, $X_L = X_C = 2000\,\Omega = \frac{1}{\omega C}$

$\therefore \quad C = \frac{1}{2000\,\omega} = 0.5\,\mu$F

114. (a) impedance $= \frac{V_{AC}}{I_{AC}} = \frac{100}{0.5} = 200\,\Omega = Z$

$R = \frac{V_{DC}}{I_{DC}} = \frac{100}{1} = 100\,\Omega$

$X_L = \sqrt{Z^2 - R^2} = 100\sqrt{3}\,\Omega = (2\pi f L)$

$\therefore \quad L = \frac{100\sqrt{3}}{2\pi f} = \frac{100 \times 1.732}{2 \times 3.14 \times 50} = 0.55$ H

115. (c) $X_L = \frac{100}{8}\,\Omega = 12.5\,\Omega$ with 50 Hz frequency and with 40 Hz frequency

$X_L' = 12.5 \times \frac{40}{50} = 10\,\Omega$ $\quad$ [as $X_L \propto \omega$]

$R = \frac{100}{10} = 10\,\Omega$

$\therefore \quad Z = \sqrt{R^2 + (X_L')^2} = 10\sqrt{2}\,\Omega$

$I = \frac{100}{10\sqrt{2}} = 5\sqrt{2}$ A

116. (*d*) Let $V = V_0 \sin \omega t$ [as $V = 0$ at $t = 0$]

Then $V_R = V_0 \sin \omega t$

and $V_C = V_0 \sin(\omega t - \pi/2)$

V and V_R are in same phase. While V_C lags V (or V_R) by 90°.

Now V_R is in same phase with initial potential difference across the capacitor for the first time when,

$$\omega t = -\frac{\pi}{2} + 2\pi = \frac{3\pi}{2}$$

$$\therefore \quad t = \frac{3\pi}{2\omega}$$

117. (*b*) At resonance, $X_C = X_L$

If ω is reduced to half,

$$X_C' = 2X_C \qquad \left[\text{as } X_C \propto \frac{1}{\omega}\right]$$

and $$X_L' = \frac{X_L}{2} \qquad [\text{as } X_L \propto \omega]$$

If ω is made two times,

$$X_C' = \frac{X_C}{2} \text{ and } X_L' = 2X_L$$

i.e, value of Z will remain unchanged current should be 1 A.

118. (*a, b, c*) $V = V_0 \sin \omega t$

$$q = CV = CV_0 \sin \omega t = q_0 \sin \omega t$$

$$i = \frac{dq}{dt} = q_0 \omega \cos \omega t = i_0 \cos \omega t$$

119. (*b*) In the given graph current is leading the voltage by 45°. Therefore circuit is *C-R*.

$$\tan \phi = \frac{X_C}{R} \text{ and } \phi = \frac{\pi}{4}$$

$$\therefore \quad X_C = R$$

$$\Rightarrow \quad \omega CR = 1$$

$$\Rightarrow \quad CR = \frac{1}{\omega} = \frac{1}{100}$$

When, $R = 1\text{k}\Omega = 10^3\,\Omega$, then

$$C = \frac{1}{10} = 10^{-5}\text{F} = 10\,\mu\text{F}$$

120. (*a*) In position-1

$$\tan\frac{\pi}{6} = \frac{\omega L}{R} = \frac{(1000)(\sqrt{3} \times 10^{-3})}{R} \text{ or } \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{R}$$

$$\therefore \quad R = 3\,\Omega$$

In position-2

$$\tan \phi = \frac{X_C}{R} = \frac{(1/\omega C)}{R}$$

$$= \frac{1\Big/\left(1000 \times \frac{1000}{3} \times 10^{-6}\right)}{3} = 1$$

$$\therefore \quad \phi = \frac{\pi}{4}$$

121. (*d*) Quantity of heat liberated in the ammeter of resistance *R*.

(i) Due to direct current of 3 ampere $= [(3)^2 R/J]$

(ii) Due to alternating current of 4 ampere

$$= [(4)^2 R/J]$$

Total heat produced per second $= \frac{(3)^2 R}{J} + \frac{(4)^2 R}{J} = \frac{25R}{J}$

Let the equivalent alternating current be I virtual ampere; then

$$\frac{I^2 R}{J} = \frac{25R}{J} \quad \text{or} \quad I = 5\text{A}$$

122. (*b*) For *L-C*, oscillator, $T = 2\pi\sqrt{LC}$

From one extreme to another- when electric field is maximum magnetic field is zero. When electric field is zero, magnetic field is maximum.

At time $t = \frac{T}{4}$, energy stored is completely magnetic

$$\therefore \text{ Time, } t = \frac{T}{4} = \frac{2\pi\sqrt{LC}}{4}$$

$$\Rightarrow \quad t = \frac{\pi\sqrt{20 \times 10^{-3} \times 50 \times 10^{-6}}}{2} = \frac{\pi\sqrt{1 \times 10^{-6}}}{2}$$

$$\Rightarrow \quad t = 1.57\,\text{ms}$$

123. (*b*) In *RC* series circuit voltage across the capacitor leads the voltage across the resistance by $\frac{\pi}{2}$.

124. (*b*) *P* is a capacitor, $X_C = \frac{220}{0.25} = 880\,\Omega$

Q is resistance, $R = \frac{220}{0.25} = 880\,\Omega$

When *P* and *Q* are in series.

$$Z = \sqrt{R^2 + X_C^2}$$

$$= 880\sqrt{2}\,\Omega$$

$$\phi = \tan^{-1}\left(\frac{X_C}{R}\right) = 45° \quad \text{(current leading)}$$

$$\Rightarrow \quad I = \frac{V}{Z} = \frac{1}{4\sqrt{2}}\text{A}$$

125. (*c*) $X_L = \omega L = (2000)(5 \times 10^{-3}) = 10\,\Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{(2000)(50 \times 10^{-6})} = 10\,\Omega$$

Since $X_C = X_L$

$$\therefore \quad I = \frac{V}{Z} = \frac{20/\sqrt{2}}{10} \quad \text{or } I = \sqrt{2}\text{ A and } V = 0$$

126. (*b*) In *LC* oscillation energy is transferred *C* to *L* or *L* to *C*, maximum energy in *L* is $= \frac{1}{2}LI_{max}^2$. Maximum energy in *C* is

$$= \frac{q_{max}^2}{2C}$$

Equal energy will be when

$$\frac{1}{2}LI^2 = \frac{1}{2}\frac{1}{2}LI_{max}^2$$

$$I = \frac{1}{\sqrt{2}}I_{max}$$

$$I = I_{max} \sin \omega t = \frac{1}{\sqrt{2}} I_{max}$$

$$\omega t = \frac{\pi}{4}$$

or $2\frac{\pi}{T}t = \frac{\pi}{4}$ or $t = \frac{T}{8} \Rightarrow t = \frac{1}{8} 2\pi\sqrt{LC} = \frac{\pi}{4}\sqrt{LC}$

127. (*b*) Resistance of circuit,

$$R = \frac{V^2}{P} = \frac{110 \times 110}{330} = \frac{110}{3}\Omega$$

1st case Power factor, $\cos\phi = 0.6$

Since, current lags the voltage thus, the circuit contains resistance and inductance.

$$\therefore \quad \cos\phi = \frac{R}{\sqrt{R^2 + X_L^2}} = 0.6$$

$$\Rightarrow \quad R^2 + X_L^2 = \left(\frac{R}{0.6}\right)^2$$

$$\Rightarrow \quad X_L^2 = \frac{R^2}{(0.6)^2} - R^2$$

$$\Rightarrow \quad X_L^2 = \frac{R^2 \times 0.64}{0.36}$$

$$\therefore \quad X_L = \frac{0.8R}{0.6} = \frac{4R}{3} \quad \text{...(i)}$$

IInd case Now $\cos\phi = 1$ [Given]

Therefore, circuit is purely resistive, *i.e.*, it contains only resistance. This is the condition of resonance in which

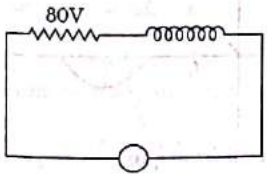
$$X_L = X_C$$

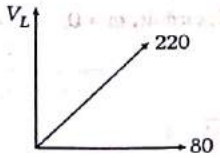
$$\therefore \quad X_C = \frac{4R}{3} = \frac{4}{3} \times \frac{110}{3} = \frac{440}{9}\Omega \quad \text{[from Eq. (i)]}$$

$$\Rightarrow \quad \frac{1}{2\pi fC} = \frac{440}{9}\Omega$$

$$\therefore \quad C = \frac{9}{2 \times 3.14 \times 60 \times 440} = 0.000054\text{F} = 54\mu\text{F}$$

128. (*c*) $R = \frac{80}{10} = 8\Omega$





Also, $V_L^2 + 80^2 = 220^2$

$$V_L^2 = (220 + 80)(220 - 80)$$

$$= 300 \times 140$$

$$\Rightarrow \quad V_L = 204.9$$

$$\Rightarrow \quad I_{rms} X_L = 204.9$$

$$\Rightarrow \quad \frac{220}{\sqrt{64 + X_L^2}} X_L = 204.9$$

$$\Rightarrow \quad X_L = 19.9 \Rightarrow L = 0.065\text{ H}$$

129. (*d*) $Z = \sqrt{(R)^2 + (X_L - X_C)^2}$

$$R = 10\Omega, X_L = \omega L = 2000 \times 5 \times 10^{-3} = 10\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2000 \times 50 \times 10^{-6}} = 10\ \Omega \text{ i.e., } Z = 10\Omega$$

Maximum current, $i_0 = \frac{V_0}{Z} = \frac{20}{10} = 2\text{A}$

Hence, $i_{rms} = \frac{2}{\sqrt{2}} = 1.4\text{A}$

and $V_{rms} = 4 \times 141 = 5.64\text{V}$

130. (*b*) $Z = \sqrt{R^2 + (X_C - X_L)^2} = 10\,\Omega$

$$\cos\phi = \frac{R}{Z} = \frac{4}{5}$$

$$\therefore \quad \phi = 37°$$

$$I_0 = \frac{V_0}{Z} = \frac{10}{10} = 1\text{A}$$

$$\therefore \quad I = 1\cos(100\pi t + 37°)$$

$$V = 10\cos(100\pi t)$$

The applied voltage is $\frac{3}{5}$ th of the maximum applied voltage when, or $100\,\pi t = 53°$

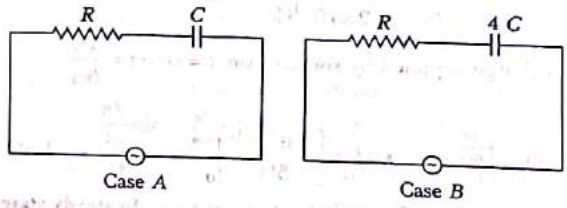
$$\therefore \quad I = 1\cos(53° + 37°) = 0$$

$$\therefore \quad V_R = 0$$

$$\therefore \quad V_{AB} = V_{applied} = \frac{3}{5} \times 10 = 6\text{V}$$

131. (*a*) This is a parallel circuit, for oscillation, the energy in L and C will be alternately maximum.

132. (*b, c*) $Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$



$I_R^A = \frac{V}{Z}$ $\quad Z' < Z$

$I_R^B = \frac{V}{Z}$ $\quad I_R^A < I_R^B$

$v_R^A < V_R^B$

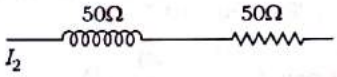
So $V_C^A > V_C^B$ $\quad \because V_R^2 + V_C^2 = V_0^2$

133. (*a, c*) $X_L = \omega L = 10 \times 0.5 = 50\,\Omega$

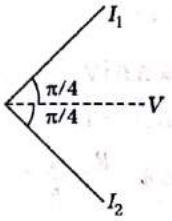
$$X_C = \frac{1}{\omega C} = \frac{1}{100 \times 100 \times 10^{-6}} = 100\,\Omega$$

$$Z_1 = 100\sqrt{2};\ I_1 = \frac{20}{100\sqrt{2}} = \frac{1}{5\sqrt{2}}$$

$$V \text{ across } 100\Omega = \frac{1}{5\sqrt{2}} \times 100 = \frac{20}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = 10\sqrt{2}\text{ V}$$



Phase difference between I_1 and V,



$$\cos\phi_1 = \frac{R_1}{Z_1} = \frac{100}{100\sqrt{2}}$$

$\Rightarrow \quad \phi = \pi/4$

Thereofre, I_1 lead V

$$Z_2 = 50\sqrt{2};\ I_2 = \frac{20}{50\sqrt{2}} = \frac{2}{5\sqrt{2}}\text{ A}$$

$$V \text{ across } 50\Omega = \frac{2}{50\sqrt{2}} \times 50 = \frac{20}{\sqrt{2}} = 10\sqrt{2}\text{ V} \Rightarrow \phi_2 = \pi/4$$

I_2 lag V by $\pi/4$

$\Rightarrow \quad I_{Net} = \sqrt{I_1^2 + I_2^2}$

$$I = \sqrt{\frac{1}{25 \times 2} + \frac{2}{25 \times 2}} = \sqrt{\frac{5}{50}} = \frac{1}{\sqrt{10}} \Rightarrow I = 0.316\text{ A}$$

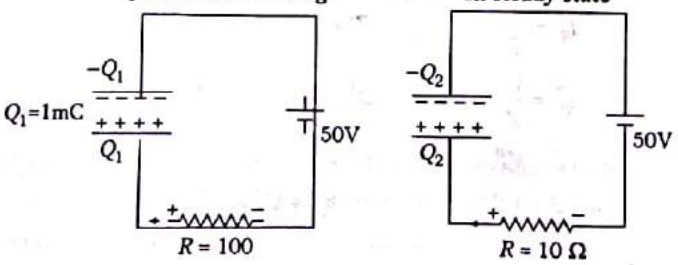
134. (*c, d*) Charge on capacitor will be maximum at $t = \frac{\pi}{2\omega}$.

$$Q_{max} = 2 \times 10^{-3}\text{C}$$

(A) charge supplied by source from $t = 0$ to $t = \frac{7\pi}{6\omega}$

$$Q = \int_0^{\frac{7\pi}{6\omega}} \cos(500)dt = \left[\frac{\sin 500t}{500}\right]_0^{\frac{7\pi}{6\omega}} = \frac{\sin\frac{7\pi}{6}}{500} = -1\text{mC}$$

Just after switching **In steady state**



Apply KVL just after switching,

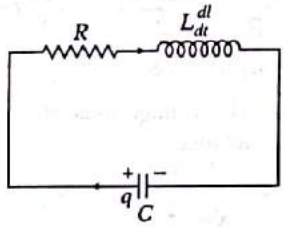
$$50 + \frac{Q_1}{C} = IR = 0$$

$\Rightarrow \quad I = 10\text{A}$

In steady state $Q_2 = 1\text{mC}$

Net charge flown from battery $Q = 2\text{mC}$

135. (*a*) At any time t apply KVL, $\frac{q}{C} - iR - L\frac{di}{dt} = 0$



$$i = -\frac{dq}{dt}$$

$$\Rightarrow \quad \frac{q}{C} + \frac{dq}{dt}R + \frac{Ld^2q}{dt^2} = 0$$

$$\Rightarrow \quad \frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{q}{LC} = 0$$

from damped harmonic oscillator, the amplitude is given by $A = A_0 e^{-\frac{dt}{2m}}$, for general equation of double differential equation $\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$

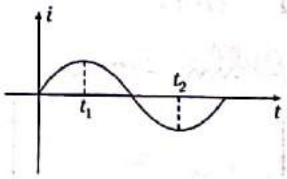
$$\Rightarrow \quad Q_{max}^{(t)} = Q_0 e^{-\frac{Rt}{2L}} \Rightarrow Q_{max}^2 = Q_0^2 e^{-\frac{Rt}{L}}$$

Lesser the self-inductance, faster will be damping hence.

B. Medical entrance special format questions

Assertion and reason

1. (*d*)



In the time interval from t_1 to t_2 average value of current will be zero.

3. (*a*) $X_L = \omega L$. In DC circuit, $\omega = 0$

$\therefore X_L = 0$

4. (*a*) $Z_{DC} = r$

$$Z_{AC} = \sqrt{r^2 + X_L^2}$$

Here, r = internal resistance of the coil.

Also, $\quad I \propto \frac{1}{Z}$

$\therefore \quad I_1 > I_2$

5. (*d*) If $V = V_0 \sin \omega t$, then current function will lead the voltage function by 90°.

$$I = I_0 \cos \omega t$$

∴ Instantaneous power,

$$P = VI = V_0 I_0 \sin \omega t \cos \omega t$$

or $P \neq 0$ at all times.

6. (*d*) Capacitor is connected in DC circuit for charging of capacitor.

7. (*d*) $V_R \leq V$, but V_C or V_L can be greater than V also.

8. (*e*) $Z = \sqrt{R^2 + (X_L - X_C)^2}$

By inserting the ferromagnetic rod, L and therefore X_L will increase. But value of Z and therefore I may increase, decrease or remain same.

9. (*a*) At resonance, $X_C = X_L$ and $Z = R$.

∴ Power factor, $\cos \phi = \frac{R}{Z} = 1$

10. (*a*) At $\omega = \omega_r$, $X_L = X_C$

At $\omega > \omega_r$, $X_L > X_C$ as,

$X_L \propto \omega$ and $X_L \propto \frac{1}{\omega}$

11. (*b*) When frequency is doubled,

$$X_L = 2R \quad [\text{as } X_L \propto \omega]$$

and $$X_C = \frac{R}{2} \quad \left[\text{as } X_C \propto \frac{1}{\omega}\right]$$

$$\therefore \quad Z = \sqrt{R^2 + \left(2R - \frac{R}{2}\right)^2} = \frac{\sqrt{13}}{2} R$$

12. (*b*) $P = V_{rms} I_{rms} \cos \phi$

$$= (I_{rms} Z)(I_{rms})\left(\frac{R}{Z}\right) = I_{rms}^2 R$$

14. (*d*) At resonance, $\phi = 0$

∴ $\cos \phi = 1$ or power factor = 1

Match the columns

1. $X_C = \frac{1}{\omega C} \propto \frac{1}{\omega}$, $X_L = \omega L \propto L$

R is not a function of ω.

Z is minimum at resonance frequency.

2. Power factor, $\cos \phi = \frac{R}{Z}$

At resonance frequency, $X_L = X_C$

At frequency > resonance frequency, $X_L > X_C$ as $X_L \propto \omega$

3. $I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$

4. $$Z = R \quad [\text{as } X_L = X_C]$$

$$\therefore \quad I = \frac{V}{Z} = \frac{V}{R}$$

Now, $$V_R = IR = V$$

$$V_C = IX_C = V = V_L$$

$$V_{RL} = \sqrt{V^2 + V^2} = \sqrt{2}\ \text{V}$$

$$V_{LC} = V - V = 0$$

5. As, $Z = \sqrt{R^2 + (X_L - X_C)^2} = 50\ \Omega$

$$I = \frac{V}{Z} = \frac{100\ \text{V}}{50\ \Omega} = 2\ \text{A}$$

$$P = I^2 R = 120\ \text{W}$$

$$V_R = IR = 60\ \text{V}$$

$$V_L = IX_L = 100\ \text{V}$$

C. Medical entrances' gallery

1. (*c*) If a rated voltage and power are given,

then $$P_{rated} = \frac{V_{rated}^2}{R}$$

∴ Current in the bulb, $I = \frac{P}{V}$ [∵ $P = VI$]

$$I = \frac{500}{100} = 5\text{A}$$

∴ Resistance of bulb, $R_b = \frac{100 \times 100}{500} = 20\Omega$

∵ Resistance R is connected in series.

∴ Current, $I = \frac{E}{R_{net}} = \frac{230}{R + R_b}$

⇒ $$R + 20 = \frac{230}{5} = 46$$

∴ $$R = 26\Omega$$

2. (*b*) For better tuning, peak of current growth must be sharp. This is ensured by a high value of quality factor Q.

Now, quality factor is given by $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

From the given options highest value of Q is associated with $R = 15\Omega$, $L = 3.5$ H and $C = 30\ \mu$F

3. (*c*) Power factor of the L-C-R circuit

$$= \cos\phi = \frac{R}{Z} = \frac{IR}{IZ} = \frac{80}{\sqrt{(X_L - X_C)^2 + R^2}} = \frac{80}{\sqrt{(IX_L - IX_C)^2 + (IR)^2}}$$

$$= \frac{80}{\sqrt{(100 - 40)^2 + (80)^2}} = \frac{80}{\sqrt{(6)^2 + (80)^2}} = \frac{80}{100} = 0.8$$

4. (*a*) impedance of the R-C circuit, $Z = \sqrt{R^2 + X_C^2}$

where, $R = 100\Omega$ and $X_C = 100\Omega$

⇒ $$Z = \sqrt{(100)^2 + (100)^2} = 100\sqrt{2}\Omega$$

Peak value of the current,

$$I_{max} = \frac{V_{max}}{Z} = \frac{220\sqrt{2}}{100\sqrt{2}} = 2.2\ \text{A}$$

5. (*a*) For an AC circuit containing capacitor only, the phase difference between current and voltage will be $\frac{\pi}{2}$ (*i.e.*, 90°).

In this case current is ahead of voltage by $\frac{\pi}{2}$.

Hence, power in this case is given by

$$P = VI\cos\phi$$

(ϕ = phase difference between voltage and current)

$$P = VI\cos 90° = 0$$

6. (*d*) Given, inductance, $L = 20$ mH

Capacitance, $C = 50\ \mu F$

Resistance, $R = 40\ \Omega$

emf, $V = 10\sin 340\,t$

$\therefore$ Power loss in AC circuit will be given as

$$P_{av} = I_V^2 R = \left[\frac{E_V}{Z}\right]^2 . R$$

$$= \left(\frac{10}{\sqrt{2}}\right)^2 \cdot 40\left[\frac{1}{40^2 + \left(340\times 20\times 10^{-3} - \frac{1}{340\times 50\times 10^{-6}}\right)^2}\right]$$

$$= \frac{100}{2}\times 40\times \frac{1}{1600 + (6.8 - 58.8)^2}$$

$$= \frac{2000}{1600 + 2704} \approx 0.46\text{ W} \approx 0.51\text{ W}$$

7. (*b*) Given, output power = 100 W

$V = 220$ V, $I = 0.5$ A

We know that,

$$\text{Efficiency} = \frac{\text{Output power}}{\text{Input power}}\times 100\% = \frac{100}{220\times 0.5}\times 100 = 90\%$$

8. (*b*) Given, $V = 100\sqrt{2}\sin 100\,t \Rightarrow \omega = 100$

$C = 1\mu F$

The peak voltage, $V_0 = 100\sqrt{2}$

$\therefore$ rms voltage, $V_{rms} = \frac{V_0}{\sqrt{2}} = \frac{100\sqrt{2}}{\sqrt{2}} = 100$ V

$\therefore$ current reading of the ammeter,

$$I = \frac{V_{rms}}{X_C} = \frac{V_{rms}}{\left(\frac{1}{\omega C}\right)} = \frac{100\text{ V}}{\frac{1}{100\times 1\times 10^{-6}}}$$

$$= 100\times 100\times 10^{-6} = 10^{-2}\text{ A}$$

$$= 10^{-2}\times\frac{10}{10} = 10\times 10^{-3}\text{ A} = 10\text{ mA}$$

9. (*a*) The resistance of inductor (coil), $R = \frac{12}{24} = 0.5\ \Omega$

If phase difference between current and emf is zero, then reactance of circuit will must be zero.

The impedance of the circuit equals to resistance *i.e.*, $Z = 0.5\ \Omega$.

Now, rms current, $I_{rms} = \frac{V_{rms}}{Z} = \frac{24}{0.5} = 48$ A

10. (*a*) When a resistor is connected to an AC source. The power drawn will be

$$P = V_{rms}\cdot I_{rms} = V_{rms}\cdot\frac{V_{rms}}{R} \Rightarrow V_{rms}^2 = PR$$

When an inductor is connected in series with the resistor, then the power drawn will be

$$P' = V_{rms}\cdot I_{rms}\cos\phi$$

where, ϕ = phase difference

$$\therefore \quad P' = \frac{V_{rms}^2}{R}\cdot\frac{R^2}{Z^2} = P\cdot R\cdot\frac{R}{Z^2}$$

$$\Rightarrow \quad P' = \frac{P\cdot R^2}{Z^2} = P\left(\frac{R}{Z}\right)^2$$

11. (*c*) The resonant frequency is given by, $f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$

where, R = resistance and L = inductance

$$\Rightarrow f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$= \frac{1}{2\times 3.14}\sqrt{\frac{1}{250\times 10^{-6}\times 0.16\times 10^{-3}} - \frac{20\times 20}{(0.16\times 10^{-3})^2}}$$

$$= 8\times 10^5\text{ Hz}$$

12. (*a*) The current through the AC circuit is given by

$i = 3\sin\omega t + 4\cos\omega t$

The maximum value of current is

$$I_{max} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5\text{ unit}$$

The rms value of current, $I_{rms} = \frac{I_{max}}{\sqrt{2}} = \frac{5}{\sqrt{2}}$ unit

13. (*d*) We know that,

In L-C-R circuit, $f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$

For the same resonant frequency, the inductance should be changed from L to $\frac{L}{4}$.

14. (*d*) The phase difference ϕ between current and voltage is given by

$$\tan\phi = \frac{X_L - X_C}{R}$$

$$\Rightarrow \quad \frac{X_L - X_C}{R} = \tan 45° = 1 \Rightarrow X_C = X_L + R$$

$$\Rightarrow \quad \frac{1}{2\pi fC} = 2\pi fL + R$$

$$\Rightarrow \quad C = \frac{1}{2\pi f(2\pi fL + R)}$$

15. (*c*) In the inductor circuit, phase difference (ϕ) = 90° or $\frac{\pi}{2}$

We know that, $P_{av} = V_{rms}I_{rms}\cos\phi = V_{rms}I_{rms}\cos\frac{\pi}{2}$

$$\therefore \quad P_{av} = 0$$

16. (*b*) Initial power = 3 kW = 3000 W

As efficiency is 90%, then final power

$$= 3000\times\frac{90}{100} = 2700\text{ W}$$

$$\Rightarrow \quad P_1 = V_1I_1 = 3000\text{ W} \quad \text{...(i)}$$

$$P_2 = V_2I_2 = 2700\text{ W} \quad \text{...(ii)}$$

So, $V_2 = \frac{2700}{6} = \frac{900}{2} = 450$ V

and $I_1 = \frac{3000}{200} = 15$ A

17. (*d*) A dynamo convert mechanical energy into electrical energy.

18. (*d*) Transformer is used to obtain desired AC voltage and current because a transformer is a device based on the principle of mutual induction, which is used for converting large AC at low voltage into small current at high voltage.

19. (*b*) As we know that for transformer, $\frac{V_1}{V_2} = \frac{I_2}{I_1} = \frac{N_1}{N_2}$

where all the symbols have their usual meanings. (Suffix 1 is for primary and 2 is for the secondary)

$\Rightarrow \quad \frac{N_1}{N_2} = \frac{1}{25} = \frac{2}{I}$

where, I = primary side current.

$\therefore \quad I = 50$ A

20. (*b*) We know that,

Power in the secondary coil = Power in the primary coil

$$V_S \times N_P = V_P \times N_S \Rightarrow \frac{N_s}{N_P} = \frac{V_S}{V_P}$$

where, V_S = voltage across secondary coil

V_P = voltage across primary coil

N_P = number of turns is primary coil

N_S = number of turns in secondary coil

Given, $N_S = 50$ turns, $N_P = 1000$ turns

$V_P = 220$ V, $I_S = ?$

$$\frac{50}{1000} = \frac{V_S}{220}$$

$\Rightarrow \quad V_S = \frac{50 \times 220}{1000} = 11$ V

But $\quad V_S I_S = V_P I_P$

$\Rightarrow \quad 11 \times I_S = 220 \times 1$

$\Rightarrow \quad I_S = \frac{220}{11} = 20$ A

21. (*d*) Average power, $P_{av} = E_{rms} I_{rms} \cos\phi$

$$= \frac{E_0}{\sqrt{2}} \cdot \frac{I_0}{\sqrt{2}} \cos\phi = \frac{100 \text{ V}}{\sqrt{2}} \times \frac{100 \times 10^{-3} \text{ A}}{\sqrt{2}} \cos\frac{\pi}{3}$$

$$= \frac{100 \times 100}{2} \times 10^{-3} \times \frac{1}{2} = 2.5 \text{ W}$$

22. (*a*) As average power, $\bar{P} = 2$ W, $I_{rms} = 2$ A,

impedance, $Z = 1\Omega$

Power factor $(\cos\phi) = ?$

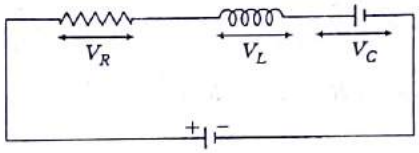
We know that, $\bar{P} = V_{rms} \times I_{rms} \cdot \cos\phi$

$\Rightarrow \quad \bar{P} = I_{rms}^2 Z \cdot \cos\phi \qquad [\because V_{rms} = I_{rms} \cdot Z]$

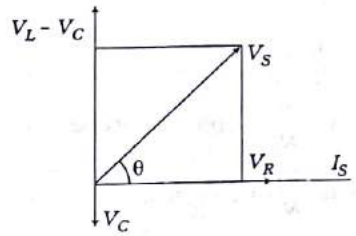
$$\cos\phi = \frac{\bar{P}}{I_{rms}^2 \cdot Z} = \frac{2}{(2)^2 \times 1} = \frac{2}{4} = \frac{1}{2} = 0.5$$

23. (*b*) Given, $V_L = 60$ V, $V_C = 30$ V, $V_R = 40$ V

According to L- C-R series circuit, we know that



In phasor diagram, V_C and V_L are in anti-phase to each other due to their 90° leading and lagging relationship with the circuit current I_S.



So, $V_S = \sqrt{(V_L - V_C)^2 + V_R^2} = \sqrt{(60-30)^2 + (40)^2}$

$= \sqrt{(30)^2 + (40)^2} = \sqrt{900 + 1600} = \sqrt{2500} = 50$ V

24. (*a*) The given equation, $e = 300 \sin[(100\pi)t]$ ...(i)

We know that, $e = e_0 \sin\omega t$ ...(ii)

On comparing Eqs. (i) and (ii), we get

$e_0 = 300$ V and $R = 100\Omega$

The rms current through the circuit $I_{rms} = ?$

$$I_{rms} = \frac{e_0}{\sqrt{2}R} = \frac{300}{\sqrt{2} \times 100} = \frac{3}{\sqrt{2}} \text{ A}$$

25. (*d*) We know that,

Resonant frequency is given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{5 \times 10^{-3} \times 2 \times 10^{-6}}}$$

where, $L = 5 \times 10^{-3}$ H and $C = 2 \times 10^{-6}$ F

$\Rightarrow \quad f_0 = \frac{1}{2\pi\sqrt{10^{-8}}} = \frac{1}{2\pi \times 10^{-4}}$

$= \frac{10^4}{2\pi}$ Hz $= \frac{5}{\pi} \times 10^3$

26. (*b*) From standard equation, we have

$I = I_0 \sin\omega t$ ...(i)

Given, $I = 50 \sin 314t$ ...(ii)

Comparing Eqs. (i) and (ii), we get

$I_0 = 50$ A, $\omega = 2\pi f = 314 \Rightarrow f = \frac{314}{2 \times 3.14} = 50$ Hz

27. (*c*) impedance, $Z = \sqrt{R^2 + X_L^2}$, $X_L = \omega L = 2\pi f L$

Given, $R = 2100\,\Omega$, $f = 50$ Hz, $L = \frac{1}{\pi}$ H

$\Rightarrow \quad Z = \sqrt{(2100)^2 + (2 \times 50)^2} = \sqrt{(2100)^2 + (100)^2}$

$\Rightarrow \quad Z = 2102\,\Omega$

29. (*b*) $\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{2\pi\nu L}{R}\right)$

$$= \tan^{-1}\left(\frac{2\pi \times 50 \times 0.01}{\sqrt{3}\,\pi}\right) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}\text{ rad}$$

30. (*d*) As, $Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (2\pi f\nu L)^2}$

As $I = \frac{V}{Z}, P = I^2R$

i.e., $V\uparrow, L\uparrow \Rightarrow Z\uparrow, I\downarrow$ and $P\downarrow$

31. (*b*) $\frac{N_S}{N_P} = 3$

$$\frac{V_S}{V_P} = \frac{I_P}{I_S} = \frac{N_S}{N_P}$$

$$V_S = V_P \cdot \frac{N_S}{N_P} = 100 \times 3 = 300\text{ V}$$

$$\Rightarrow \quad I_S = I_P \cdot \frac{N_P}{N_S} = 2 \times \frac{1}{3} \approx 0.5\text{ A}$$

32. (*a*) Given, the potential difference across each element is same *i.e.*, 20 V, so the circuit is at resonance and we have

$$V = V_R = 20\text{ V}$$

If the value of R is doubled, then value of I reduced to half

$$\therefore \quad V_L = V_C = \frac{IX_L}{2} = \frac{IX_C}{2} = \frac{20}{2} = 10\text{ V}$$

[$\because$ circuit is at resonance]

and $V_R = V = 20$ V

33. (*a*) $I_{rms} = \frac{V_{rms}}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}$ ...(i)

$$\frac{I_{rms}}{2} = \frac{V_{rms}}{\sqrt{R^2 + \left[\left(\frac{1}{\omega C/3}\right)\right]^2}} = \frac{V_{rms}}{\sqrt{R^2 + \frac{9}{\omega^2C^2}}} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$3R^2 = \frac{5}{\omega^2C^2}$$

$$\frac{1}{\omega C/R} = \sqrt{\frac{3}{5}}$$

or $\frac{X_C}{R} = \sqrt{\frac{3}{5}}$ or $\frac{X_C}{R} = \sqrt{0.6}$

34. (*c*) For L-R circuit, $Z = \sqrt{R^2 + X_L^2}$

Here, $Z_1 = \sqrt{(R_1)^2 + (X_{L_2})^2}$

and $Z_2 = \sqrt{(R_2)^2 + (X_{L_2})^2}$

Given, $R_2 = 2R$ and $X_{L_2} = 2X_L$

$$Z_2 = \sqrt{(2R)^2 + (2X_L)^2} = 2Z_1$$

35. (*d*) Given, $L = 1.00$ mH, $C = 1.0\ \mu$F and $R = 50\ \Omega$ and $V = 5$V

At resonance, $I = \frac{V}{R} = \frac{5}{50}$

$$\Rightarrow \quad I = 0.1\text{ A}$$

36. (*d*) We know that for series L-C-R in circuit.

$$V_{rms} = \sqrt{V_R^2 + (V_L - V_C)^2}$$

Given, $V_L = 90$ V, $V_C = 60$ V

and $V_R = 40$ V

Now, $V_{rms} = \sqrt{40^2 + (90 - 60)^2} = 50$ V

37. (*c*) If an iron rod is puts in the coil, then the intensity of light of bulb will decrease because the iron rod increase the resistance.

38. (*c*) Power factor, $\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + \omega^2L^2}}$

39. (*a*) With DC, power, $P = \frac{V^2}{R}$

$$R = \frac{(10)^2}{20} = 5\,\Omega$$

With AC power, $P = \frac{V_{rms}^2 R}{Z^2} \Rightarrow Z^2 = \frac{(10)^2 \times 5}{10} = 50\,\Omega$

The impedance, $Z^2 = R^2 + 4\pi^2\nu^2L^2$

$$50 = (5)^2 + 4(3.14)^2 \times \nu^2 \times (10 \times 10^{-3})^2$$

$$\nu = 80\text{ Hz}$$

40. (*b*) Power factor, $\cos\phi = \frac{1}{2} \Rightarrow \phi = 60°$

$$\therefore \quad \tan 60° = \frac{\omega L}{R} \Rightarrow \sqrt{3} = \frac{2\pi\nu \times L}{100}$$

$$L = \frac{\sqrt{3} \times 100}{2\pi \times 50} = \frac{\sqrt{3}}{\pi}\text{ H}$$

41. (*c*) Here, phase difference,

$$\tan\phi = \frac{X_L - X_C}{R}$$

$$\tan\frac{\pi}{3} = \frac{X_L - X_C}{R}$$

When L is removed, $\sqrt{3} = \frac{X_C}{R} \Rightarrow X_C = \sqrt{3}R$

When C is removed, $\tan\frac{\pi}{3} = \sqrt{3} = \frac{X_L}{R}$

$$X_L = R\sqrt{3}$$

Hence, in resonant circuit, $\tan\phi = \frac{\sqrt{3}R - \sqrt{3}R}{R} = 0$

$$\phi = 0$$

$\therefore$ Power factor, $\cos\phi = 1$

It is the condition of resonance therefore phase difference between voltage and current is zero and power factor is $\cos\phi = 1$

42. (*d*) For inductor as we know induced voltage leads the current

For $t = 0$ to $t = T/2$

$$V = L\frac{dI}{dt} = L\frac{d}{dt}\left(\frac{2I_0t}{T}\right) = \text{constant}$$

For $t = T/2$ to $t = T$

$$V = \frac{LdI}{dt}\left(\frac{-2I_0t}{T}\right) = -\text{constant}$$

Therefore, answer will represents by graph (d).

43. (*a*) From transformer ratio,

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}$$

$$\Rightarrow \quad V_S = \frac{V_P \times N_S}{N_P}$$

$$= \frac{220 \times 40000}{200} = 44000 \text{ V}$$

Potential difference per turn is

$$\frac{V_S}{N_S} = \frac{44000}{40000} = 1.1 \text{ V}$$

44. (*a*) $\eta = \dfrac{\text{Output power}}{\text{Input power}}$

$$\Rightarrow \frac{80}{100} = \frac{20 \times 120}{1000 \times I}$$

$$\Rightarrow \quad I = \frac{20 \times 120 \times 100}{1000 \times 80} = 3 \text{ A}$$

45. (*c*) Input power, $VI = 200 \times 10 = 2000$ W

Output power = Efficiency × Input power

$$= \frac{40}{100} \times 2000 = 800 \text{ W}$$

Power loss in heating the armature

$$= 2000 - 800 = 1200 \text{ W}$$

$\therefore \quad I^2R = 1200$

or $\quad R = \dfrac{1200}{I^2} = \dfrac{1200}{10 \times 10}$

or $\quad R = 12\,\Omega$

46. (*c*) Emf of AC source $E = E_0 \sin(100\,t)$

We have, $\omega = 100$ and $\phi = \dfrac{\pi}{4} = 45°$

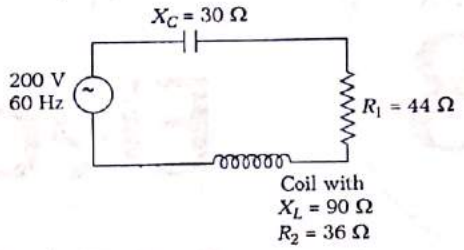
$$\tan\phi = \frac{\omega L}{R}$$

$$\Rightarrow \quad 1 = \frac{100 \times L}{R}$$

$$R = 100 \text{ L}$$

If $L = 10$ H, then $R = 1\text{k}\Omega$

47. (*a*)



We have, $X_L = 90\,\Omega$, $X_C = 30\,\Omega$

and $\quad R = R_1 + R_2 = 44 + 36 = 80\,\Omega$

impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$= \sqrt{(80)^2 + (60)^2}$$

$$Z = \sqrt{6400 + 3600} = 100\,\Omega$$

$$I = \frac{V}{Z} = \frac{200}{100} = 2 \text{ A}$$

$$P_{av} = I^2R = (2)^2 \times 80 = 320 \text{ W}$$

48. (*c*) For step-up transformer, $V_S > V_P$ and $I_S < I_P$

For an ideal transformer,

$$V_S I_S = V_P I_P$$

$\therefore \quad 240000\, I_S = 100 \times 4000$

or $\quad I_S = 1.67$ A

49. (*a*) We know that, $I_{WL} = I_{rms} \sin\phi$

$$\Rightarrow \sqrt{3} = 2\sin\phi \text{ or } \sin\phi = \frac{\sqrt{3}}{2} \Rightarrow \phi = 60°$$

So, power factor $= \cos\phi = \cos 60° = \dfrac{1}{2}$