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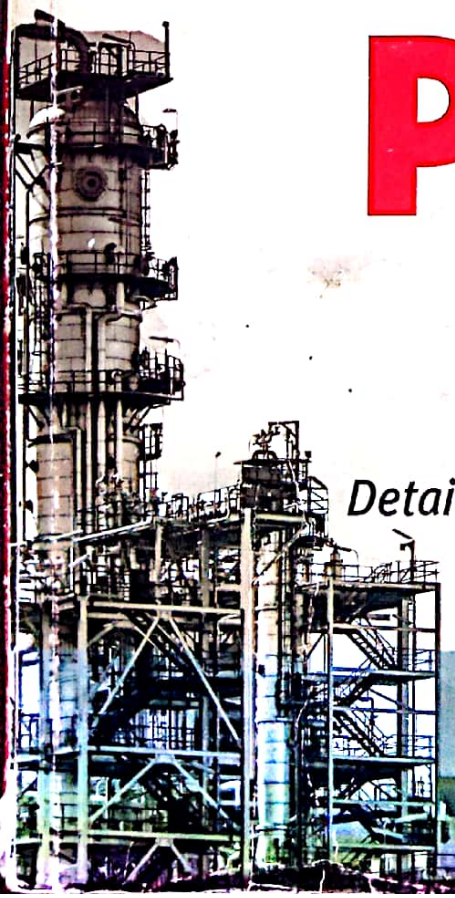
NEET PREPARATORY SERIES

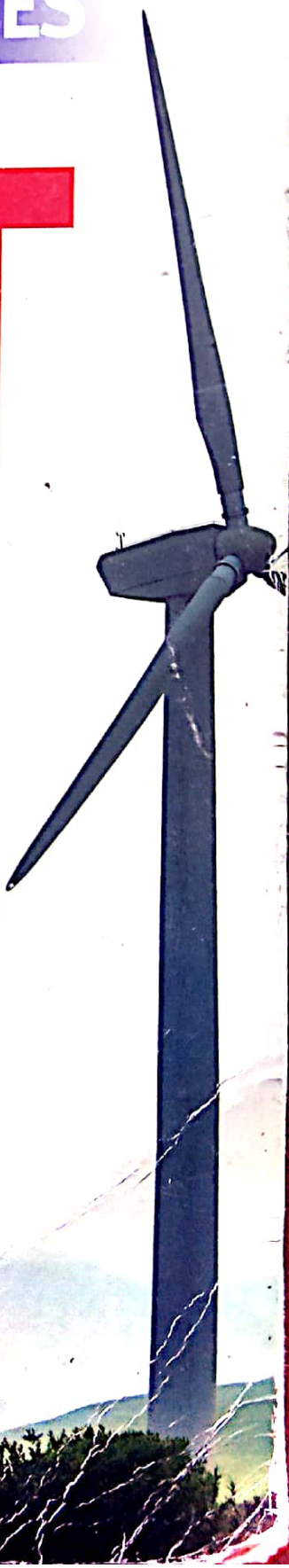
NEET

National Eligibility cum Entrance Test

Objective PHYSICS

VOL 1





Detailed study Material & Practice Exercises for NEET, AIIMS, JIPMER etc.

DC PANDEY

CHAPTER

0

Basics Mathematics

1 Trigonometry

Trigonometry is one of the important branch of mathematics which deal with relations of sides and angles of triangle and also with the relevant functions of any angle.

Consider a ray OA. If this ray revolves about its end point O in anti-clockwise direction and takes position OB, then we say that the angle $\angle AOB$ has been generated *as shown in the following figure.*

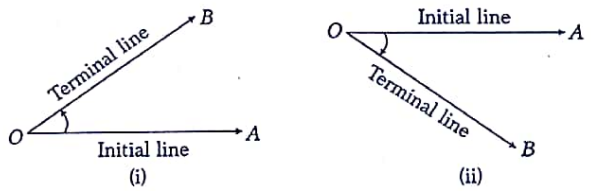


Fig. 0.1 (i) Positive angle *(ii) Negative angle*

Or simply say that angle is a measure or an amount of revolution of a given ray about its initial point.

The angle is positive (or negative), if the initial line revolves in anti-clockwise (or clockwise) direction to get the terminal line.

System of measurement of angles

(i) Sexagesimal system

In this system, a right angle is divided into 90 equal parts, called degree.

Thus, 1 right angle = 90° (degrees)

1° = 60′ (minutes)

1′ = 60″ (seconds)

Inside

1) Trigonometry
2) Calculus (Differentiation)
3) Intergration
4) Graph

(ii) Circular system

In this system, the unit of measurement is radian.

$$\theta = \frac{l}{r} \text{ radian}$$

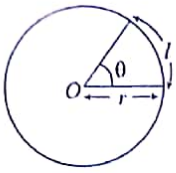


Fig. 0.2 Circular system

$$1 \text{ right angle} = \frac{\pi}{2} \text{ rad (radian)}$$

1 straight angle = π rad

and 1 complete angle = 2π rad

One radian (denoted by 1^c) is the measure of an angle subtended at the centre of a circle by an arc of length equal to the radius of the circle.

$$1 \text{ rad } (1^c) = \frac{180°}{\pi} \approx 57°17'45'' \approx 57.3°$$

Example 0.1 *Find the radian measures corresponding to the following degree measures.*

(*i*) 75° (*ii*) −37°30′ (*iii*) 5°37′30″

Sol. We have, $1^c = \frac{180°}{\pi} \Rightarrow 1° = \left(\frac{\pi}{180}\right)^c$

(i) $75° = \left(75 \times \frac{\pi}{180}\right)^c = \left(\frac{5\pi}{12}\right)^c$

(ii) $-37°30' = -\left(37\frac{1}{2}\right)^\circ = -\left(\frac{75}{2}\right)^\circ$

$$= -\left(\frac{75}{2} \times \frac{\pi}{180}\right)^c = -\left(\frac{5\pi}{24}\right)^c$$

(iii) $5°37'30'' = \left(5\frac{5}{8}\right)^\circ = \left(\frac{45}{8}\right)^\circ = \left(\frac{45}{8} \times \frac{\pi}{180}\right)^c = \left(\frac{\pi}{32}\right)^c$

Example 0.2 *Find the degree measures corresponding to the following radian measures*

(*i*) $\left(\frac{2\pi}{15}\right)^c$ (*ii*) $\left(\frac{\pi}{8}\right)^c$ (*iii*) $(-2)^c$

Sol. We have, $1^c = \left(\frac{180}{\pi}\right)^\circ$

(i) $\left(\frac{2\pi}{15}\right)^c = \left(\frac{2\pi}{15} \times \frac{180}{\pi}\right)^\circ = 24°$

(ii) $\left(\frac{\pi}{8}\right)^c = \left(\frac{\pi}{8} \times \frac{180}{\pi}\right)^\circ = \left(\frac{45}{2}\right)^\circ = \left(22\frac{1}{2}\right)^\circ = 22°\left(\frac{1}{2} \times 60\right)'$

$= 22°30'$

(iii) $(-2)^c = \left(-2 \times \frac{180}{\pi}\right)^\circ = -\left(2 \times \frac{180}{22} \times 7\right)^\circ = -\left(114\frac{6}{11}\right)^\circ$

$$= -114°\left(\frac{6}{11} \times 60\right)' = -114°\left(32\frac{8}{11}\right)'$$

$$= -114°32'\left(\frac{8}{11} \times 60\right)'' = -[114°32'44'']$$

Trigonometrical ratios (or *T*-ratios)

Consider the two fixed lines $X'OX$ and YOY' intersecting each other at right angle at point O *as shown in the following figure*

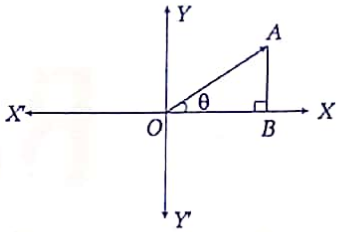


Fig. 0.3 Trigonometrical ratios

Then,

(i) Intersection point O is called origin.

(ii) $X'OX$ and YOY' are called X-axis and Y-axis, respectively.

(iii) The portion XOY, YOX', $X'OY'$ and $Y'OX$ are known as I, II, III and IV quadrants, respectively.

Now, consider that the revolving line OA has traced out an angle θ in anti-clockwise direction (in I quadrant).

From, point A draw $AB \perp OX$ which results a right angled ΔABO, where AB = perpendicular, OA = hypotenuse and OB = base.

The three sides of right angled triangle are related to each other through side different ratios, called trigonometrical ratio or T-ratios, *which are given as*

(i) $\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{AB}{OA}$ (from Fig. 0.3)

(ii) $\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{OB}{OA}$

(iii) $\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{AB}{OB}$

(iv) $\cot\theta = \frac{\text{Base}}{\text{Perpendicular}} = \frac{OB}{AB}$

(v) $\sec\theta = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{OA}{OB}$

(vi) $\text{cosec}\,\theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{OA}{AB}$

Fundamental of *T*-ratios or trigonometric functions

For any acute angle say θ (< 90°), *the functions are given as*

(i) $\text{cosec}\,\theta = \frac{1}{\sin\theta}$

(ii) $\sec\theta = \frac{1}{\cos\theta}$

(iii) $\cot\theta = \dfrac{1}{\tan\theta}$

(iv) $\sin^2\theta + \cos^2\theta = 1$

(v) $1 + \tan^2\theta = \sec^2\theta$

(vi) $1 + \cot^2\theta = \text{cosec}^2\theta$

Signs of trigonometric ratios in various quadrants

II only sin and cosec are + ve	I All + ve
III only tan and cot are + ve	IV only cos and sec are + ve

Fig. 0.4

(i) In I quadrant, all T-ratios are positive.

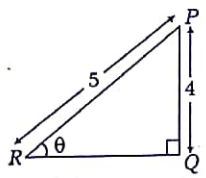
(ii) In II quadrant, $\sin\theta$ is + ve, $\cos\theta$ and $\tan\theta$ are – ve .

(iii) In III quadrant, $\tan\theta$ is + ve, $\sin\theta$ and $\cos\theta$ are – ve .

(iv) In IV quadrant, $\cos\theta$ is + ve, $\sin\theta$ and $\tan\theta$ are – ve.

Example 0.3 *If* $\sin\theta = \dfrac{4}{5}$, *where* θ *lies in the first quadrant, then find all the other T-ratios.*

Sol. Let ΔPQR be right angled triangle, right angled at Q



$$\sin\theta = \frac{4}{5} \qquad \text{(Given)}$$

$\therefore$ $PR = 5$ and $PQ = 4$

On applying Pythagoras theorem in ΔPQR, we have

$$(PR)^2 = (PQ)^2 + (QR)^2$$

$$\Rightarrow \quad (5)^2 = (4)^2 + (QR)^2$$

$$\Rightarrow \quad QR = \sqrt{25-16} = 3$$

[Taking positive square root]

Now, $\cos\theta = \dfrac{QR}{PR} = \dfrac{3}{5}$,

$\tan\theta = \dfrac{PQ}{QR} = \dfrac{4}{3}$,

$\cot\theta = \dfrac{QR}{PR} = \dfrac{3}{4}$,

$\sec\theta = \dfrac{PR}{QR} = \dfrac{5}{3}$ and $\text{cosec}\,\theta = \dfrac{PR}{PQ} = \dfrac{5}{4}$

Values of T-ratios of some standard angles

Angle (θ)	0°	30° $\left(=\frac{\pi}{6}\right)$	45° $\left(=\frac{\pi}{4}\right)$	60° $\left(=\frac{\pi}{3}\right)$	90° $\left(=\frac{\pi}{2}\right)$	120° $\left(=\frac{2\pi}{3}\right)$	135° $\left(=\frac{3\pi}{4}\right)$	150° $\left(=\frac{5\pi}{6}\right)$	180° (= π)
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	–1
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	–1	$-\frac{1}{\sqrt{3}}$	0

T-ratios of allied angles

In trigonometry two angles are said to be allied angles when their sum or difference is a multiple of 90°. *The T-ratios of the following allied angles are as*

(i) When angle (say) θ is negative, then

(a) $\sin(-\theta) = -\sin\theta$

(b) $\cos(-\theta) = \cos\theta$

(c) $\tan(-\theta) = -\tan\theta$

(ii) When angle θ is less than 90° (*i.e.*, lies in I quadrant), then

(a) $\sin(90° - \theta) = \cos\theta$

(b) $\cos(90° - \theta) = \sin\theta$

(c) $\tan(90° - \theta) = \cot\theta$

(iii) When angle θ lies between 90° and 180° (II quadrant), then

(a) • $\sin(90° + \theta) = \cos\theta$
• $\cos(90° + \theta) = -\sin\theta$
• $\tan(90° + \theta) = -\cot\theta$

(b) • $\sin(180° - \theta) = \sin\theta$
• $\cos(180° - \theta) = -\cos\theta$
• $\tan(180° - \theta) = -\tan\theta$

(iv) When angle θ lies between 180° and 270° (III quadrant), then

(a) • $\sin(180° + \theta) = -\sin\theta$
• $\cos(180° + \theta) = -\cos\theta$
• $\tan(180° + \theta) = \tan\theta$

(b) • $\sin(270° - \theta) = -\cos\theta$
• $\cos(270° - \theta) = -\sin\theta$
• $\tan(270° - \theta) = \cot\theta$

(v) When angle θ lies between 270° and 360° (IV quadrant), then

(a) • $\sin(270° + \theta) = -\sin\theta$
• $\cos(270° + \theta) = \sin\theta$
• $\tan(270° + \theta) = -\cot\theta$

(b) • $\sin(360° - \theta) = -\sin\theta$
• $\cos(360° - \theta) = \cos\theta$
• $\tan(360° - \theta) = -\tan\theta$

Example 0.4 *Find the value of*

(*i*) $\sin(-45°)$ (*ii*) $\tan 225°$

(*iii*) $\cos 300°$ (*iv*) $\sec 120°$

Sol.

(i) $\sin(-45°) = -\sin 45°$ $[\because \sin(-\theta) = -\sin\theta]$

$= -\frac{1}{\sqrt{2}}$ $[\because \sin 45° = \frac{1}{\sqrt{2}}]$

(ii) $\tan 225° = \tan(270° - 45°) = \cot 45°$

$[\because \tan(270° - \theta) = \cot\theta]$

$= 1$ $[\because \cot 45° = 1]$

(iii) $\cos 300° = \cos(270° + 30°) = \sin 30°$

$[\because \cos(270° + \theta) = \sin\theta]$

$= \frac{1}{2}$ $[\because \sin 30° = \frac{1}{2}]$

(iv) $\sec 120° = \sec(180° - 60°) = -\sec 60°$

$[\because \sec(180° - \theta) = -\sec\theta]$

$= -2$

Some important formulae of trigonometry

- $\sin(A + B) = \sin A \cos B + \cos A \sin B$
- $\sin(A - B) = \sin A \cos B - \cos A \sin B$
- $\cos(A + B) = \cos A \cos B - \sin A \sin B$
- $\cos(A - B) = \cos A \cos B + \sin A \sin B$
- $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$
- $\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$
- $\sin 2A = 2 \sin A \cos A = \dfrac{2 \tan A}{1 + \tan^2 A}$
- $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1$

  $= 1 - 2\sin^2 A = \dfrac{1 - \tan^2 A}{1 + \tan^2 A}$
- $\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$
- $\sin 3A = 3 \sin A - 4 \sin^3 A$
- $\cos 3A = 4 \cos^3 A - 3 \cos A$
- $\tan 3A = \dfrac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$
- $\sin(A + B) + \sin(A - B) = 2 \sin A \cos B$
- $\sin(A + B) - \sin(A - B) = 2 \cos A \sin B$
- $\cos(A + B) + \cos(A - B) = 2 \cos A \cos B$
- $\cos(A + B) - \cos(A - B) = -2 \sin A \sin B$
- $\sin C + \sin D = 2 \sin\left(\dfrac{C + D}{2}\right) \cdot \cos\left(\dfrac{C - D}{2}\right)$
- $\sin C - \sin D = 2 \cos\left(\dfrac{C + D}{2}\right) \cdot \sin\left(\dfrac{C - D}{2}\right)$
- $\cos C + \cos D = 2 \cos\left(\dfrac{C + D}{2}\right) \cdot \cos\left(\dfrac{C - D}{2}\right)$
- $\cos C - \cos D = -2 \sin\left(\dfrac{C + D}{2}\right) \cdot \sin\left(\dfrac{C - D}{2}\right)$

Example 0.5 *Find the value of* (*i*) $\sin 15°$ (*ii*) $\tan 75°$

Sol. (i) We have,

$\sin 15° = \sin(45° - 30°)$

$= \sin 45° \cos 30° - \cos 45° \sin 30°$

$[\because \sin(A - B) = \sin A \cos B - \cos A \sin B]$

$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

$(\because \sin 45° = \cos 45° = \frac{1}{\sqrt{2}}, \cos 30° = \frac{\sqrt{3}}{2}$ and $\sin 30° = \frac{1}{2})$

$= \frac{\sqrt{3} - 1}{2\sqrt{2}}$

(ii) We have, $\tan 75° = \tan(45° + 30°)$

$= \dfrac{\tan 45° + \tan 30°}{1 - \tan 45° \cdot \tan 30°}$ $\left(\because \tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \cdot \tan B}\right)$

$= \dfrac{1 + \frac{1}{\sqrt{3}}}{1 - 1 \cdot \frac{1}{\sqrt{3}}}$ $\left(\because \tan 45° = 1 \text{ and } \tan 30° = \frac{1}{\sqrt{3}}\right)$

$= \dfrac{\sqrt{3} + 1}{\sqrt{3} - 1}$

Check point 0.1

1. Find the radian measures corresponding to the following degree measures.

(i) 25° (ii) – 47°30′

(iii) 39°22′30″

Ans. (i) $\frac{5\pi}{36}$ (ii) $-\frac{19\pi}{72}$ (iii) $\frac{11}{16}$

2. Find the degree measures corresponding to the following radian measures.

(i) $\left(\frac{18\pi}{5}\right)^C$ (ii) $(-3)^C$

(iii) $\left(-\frac{5\pi}{6}\right)^C$ (iv) $\left(\frac{9\pi}{5}\right)^C$

Ans. (i) 648° (ii) – 171°49′5″ (iii) – 150°

(iv) 329°

3. Find sin θ and tan θ if $\cos\theta = -\frac{12}{13}$ and θ lies in the third quadrant.

Ans. $\sin\theta = -\frac{5}{13}$ and $\tan\theta = \frac{5}{12}$

4. Find the values of other five *T*-ratios if $\tan\theta = -\frac{3}{4}$, θ in quadrant II.

Ans. $\sin\theta = \frac{3}{5}$, $\cos\theta = -\frac{4}{5}$, $\operatorname{cosec}\theta = \frac{5}{3}$, $\sec\theta = -\frac{5}{4}$ and $\cot\theta = -\frac{4}{3}$

5. Find the values of the following *T*-ratios

(i) cosec 315° (ii) cos 210°

(iii) sin(– 330°)

Ans. (i) $-\sqrt{2}$ (ii) $-\frac{\sqrt{3}}{2}$ (iii) $\frac{1}{2}$

6. Find the value of

(i) sec 165° (ii) cot 105°

Ans. (i) $(\sqrt{2}-\sqrt{6})$ (ii) $\frac{1+\sqrt{3}}{1-\sqrt{3}}$

2 Calculus (Differentiation)

Differentiation in calculus, is the process of finding the derivative. The derivative is the instantaneous rate of change of a function with respect to one of its variable. This is equivalent to finding the slope of the tangent line to the function at a point.

Variable

A quantity, which can take different values, is called a variable quantity. A variable is usually represented by x, y, z etc.

Constant

A quantity, whose value remains unchanged during mathematical operations, is called a constant quantity. The integers, fractions such as π, e, etc are all constant.

Function

A quantity y is called a function of a variable x, if corresponding to any given value of x, there exists a single definite value of y. The phrase 'y is function of x' is represented as $y = f(x)$.

e.g., Consider that y is a function of the variable x which is given by $y = 4x^2 + 3x + 7$ and $y = \sin x + e^x$

Here, we will treat x as independent variable and y as dependent variable, *i.e.*, the value y depends on x. If we change the value of x, then y will change.

Physical meaning of $\frac{dy}{dx}$

(i) The ratio of small change in the function y and the variable x is called the average rate of change of y w.r.t. x.

e.g. If a body covers a small distance Δs in small time Δt, then average velocity of the body, $v_{av} = \frac{\Delta s}{\Delta t}$

(ii) The limiting value of $\frac{\Delta y}{\Delta x}$, when $\Delta x \to 0$, *i.e.*,

$\underset{\Delta x \to 0}{Lt} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$, is called the instantaneous rate of change of y w.r.t. x.

Thus, the differentiation of a function w.r.t. a variable implies the instantaneous rate of change of the function w.r.t. that variable.

e.g., Instantaneous velocity of the body,

$$\underset{\Delta t \to 0}{Lt} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

Theorems of differentiation

(i) If c is constant, then $\frac{d}{dx}(c) = 0$

(ii) If $y = cu$, where C is a constant and v is a function of x, then $\frac{dy}{dx} = \frac{d}{dx}(c \cdot v) = c\frac{dv}{dx}$

(iii) If $y = x^n$, where n is a real number, then $\frac{dy}{dx} = \frac{d}{dx}(x^n) = nx^{n-1}$

(iv) If $y = u \pm v \pm w$, where u, v and w are functions of x, then $\frac{dy}{dx} = \frac{d}{dx}(u \pm v \pm w) = \frac{du}{dx} \pm \frac{dv}{dx} \pm \frac{dw}{dx}$

(v) **Product rule** If $y = u \cdot v$, where u and v are functions of x, then $\frac{dy}{dx} = \frac{d}{dx}(u \cdot v) = u\frac{dv}{dx} + v\frac{du}{dx}$

(vi) **Division rule** If $y = \frac{u}{v}$, where u and v are functions of x, then $\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Example 0.6 *Differentiate the following functions*

(i) $y = x^{-3}$ (ii) $6x^5 + 4x^3 - 3x^2 + 2x - 7$

(iii) $y = (x+2)(x^2+1)$ (iv) $y = \frac{(x^3+4)}{(x+1)}$

Sol. (i) We have, $y = x^{-3}$

On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = -3x^{-3-1} \quad \left(\because \frac{d}{dx}x^n = nx^{n-1}\right)$$

$$= -3x^{-4} = \frac{-3}{x^4}$$

(ii) Let $y = 6x^5 + 4x^3 - 3x^2 + 2x - 7$

On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = 6\frac{d}{dx}x^5 + 4\frac{d}{dx}x^3 - 3\frac{d}{dx}x^2 + 2\frac{d}{dx}x - \frac{d}{dx}7$$

$$\left(\because \frac{d}{dx}(c \cdot v) = c \cdot \frac{dv}{dx}\right)$$

$$\Rightarrow \frac{dy}{dx} = 6 \cdot 5x^{5-1} + 4 \cdot 3x^{3-1} - 3 \cdot 2x^{2-1} + 2 \cdot x^{1-1} - 0$$

$$\left(\because \frac{d}{dx}x^n = nx^{n-1} \text{ and } \frac{d}{dx}c = 0\right)$$

$$\Rightarrow \frac{dy}{dx} = 30x^4 + 12x^2 - 6x + 2 \times 1 \quad (\because x^{1-1} = x^0 = 1)$$

$$\Rightarrow \frac{dy}{dx} = 30x^4 + 12x^2 - 6x + 2$$

(iii) We have, $y = (x+2)(x^2+1)$

On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = (x+2)\frac{d}{dx}(x^2+1) + (x^2+1)\frac{d}{dx}(x+2)$$

(By product rule)

$$= (x+2)(2x+0) + (x^2+1)(1+0)$$

$$= 2x(x+2) + x^2 + 1 = 2x^2 + 4x + x^2 + 1 = 3x^2 + 4x + 1$$

(iv) We have, $y = \frac{x^3+4}{x+1}$

On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = \frac{(x+1)\frac{d}{dx}(x^3+4) - (x^3+4)\frac{d}{dx}(x+1)}{(x+1)^2}$$

(By division rule)

$$\frac{dy}{dx} = \frac{(x+1)(3x^2+0) - (x^3+4)(1+0)}{(x+1)^2}$$

$$= \frac{3x^2(x+1) - x^3 - 4}{(x+1)^2} = \frac{3x^3 + 3x^2 - x^3 - 4}{(x+1)^2}$$

$$= \frac{2x^3 + 3x^2 - 4}{(x+1)^2}$$

Formulae for differential coefficient of trigonometric, logarithmic and exponential function

- $\frac{d}{dx}(\sin x) = \cos x$
- $\frac{d}{dx}(\cos x) = -\sin x$
- $\frac{d}{dx}(\tan x) = \sec^2 x$
- $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$
- $\frac{d}{dx}(\sec x) = \sec x \cdot \tan x$
- $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cdot \cot x$
- $\frac{d}{dx}(\log x) = \frac{1}{x}$
- $\frac{d}{dx}(e^x) = e^x$

Example 0.7 *Differentiate the following*

(i) $y = \sin x + e^x$

(ii) $y = 3x^2 + \log x + 4e^x + 5$

(iii) $y = e^x \cdot \tan x$

Sol. (i) We have, $y = \sin x + e^x$

On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(\sin x + e^x) = \cos x + e^x$$

(ii) We have, $y = 3x^2 + \log x + 4e^x + 5$

On differentiating both sides w.r.t. x^2, we get

$$\frac{dy}{dx} = \frac{d}{dx}(3x^2 + \log x + 4e^x + 5)$$

$$= 3\frac{d}{dx}x^2 + \frac{d}{dx}(\log x) + 4\frac{d}{dx}e^x + \frac{d}{dx}5$$

$$= 3 \cdot 2x^{2-1} + \frac{1}{x} + 4e^x + 0 = 6x + \frac{1}{x} + 4e^x$$

(iii) We have, $y = e^x \cdot \tan x$

On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = e^x \cdot \frac{d}{dx}\tan x + \tan x \frac{d}{dx}e^x \quad \text{(By product rule)}$$

$$= e^x \cdot \sec^2 x + \tan x \cdot e^x = e^x(\sec^2 x + \tan x)$$

Chain rule

Chain rule is applied, when the given function is the function of function, *i.e.*, a function is in the form of $f[g(x)]$

$$\therefore \quad \frac{d}{dx}f[g(x)] = f'[g(x)] \cdot g'(x)$$

Example 0.8 *Differentiate* $\sin(x^2 + 5)$ *w.r.t. x.*

Sol. Let $y = \sin(x^2 + 5)$. On differentiating both sides w.r.t. 'x', we get

$$\frac{dy}{dx} = \frac{d}{dx}\sin(x^2 + 5) = \cos(x^2 + 5) \cdot \frac{d}{dx}(x^2 + 5)$$

$$= \cos(x^2 + 5) \cdot (2x + 0) = 2x\cos(x^2 + 5)$$

Maxima and minima

Let $y = f(x)$, where y is a function of x.

For y to be minimum or maximum, put $\frac{dy}{dx} = 0$, then find x.

If $\frac{d^2y}{dx^2} < 0$, y is maximum and if $\frac{d^2y}{dx^2} > 0$, y is minimum.

Note Most of time, it is known from physical situation whether the quantity is a maximum or minimum; therefore, there is no need to check maximum or minimum with the help of $\frac{d^2y}{dx^2}$.

Example 0.9 *Divide a number 1000 in two parts such that there product is maximum.*

Sol. Let the two parts be x and $(1000 - x)$.

$\therefore$ Their product, $P = x(1000 - x) \Rightarrow P = 1000x - x^2$

On differentiating both sides w.r.t. x, we get

$$\frac{dP}{dx} = 1000 - 2x \quad \text{...(i)}$$

For P to be maximum or minimum,

$$\frac{dP}{dx} = 0 \Rightarrow 1000 - 2x = 0 \Rightarrow x = 500$$

On differentiating both sides of Eq. (i), w.r.t. x, we get

$$\frac{d^2P}{dx^2} = -2 < 0$$

$\therefore$ P is maximum at $x = 500$

On dividing equally, the two parts are (500, 500).

Remembering point

- All the problems of maxima/minima cannot be solved by the above methods *e.g.*, $y = x^2$, y is maximum when x is maximum.
- If $y = \sin x$, by simple observation, y is maximum if $\sin x$ is maximum, *i.e.*, $\sin x = 1$. The value of sine or cosine functions lies between -1 and $+1$.
- The product of the two parts is the maximum when the parts are equal.

Applications of differentiation in physics

(i) If the displacement is a function of time t, to find the velocity, differentiate s w.r.t. t.

$$\boxed{s = f(t),\ v = \frac{ds}{dt}}$$

(ii) If the velocity is a function of time t, to find acceleration, differentiate v w.r.t. t

$$v = f(t),\ a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{ds}{dt}\right) = \frac{d^2s}{dt^2}$$

(iii) If the velocity is a function of displacement, s to find the acceleration, differentiate v w.r.t. t and use the expression $a = v\frac{dv}{ds}$

(iv) *Consider the motion along the X-axis*

- If $v > 0$, s is increasing, then the particle is moving along the positive X-axis.
- If $v < 0$, s is decreasing, then the particle is moving along the negative X-axis.
- If $a > 0$, $v > 0$, then speed is increasing along the positive X-axis.
- If $a > 0$, $v < 0$, then speed is decreasing along the negative X-axis.
- If $a < 0$, $v > 0$, then the speed is decreasing along the positive X-axis.
- If $a < 0$, $v < 0$, then the speed is increasing along the negative X-axis.

Note

1. If v and a have same sign, then the speed is increasing.
2. If v and a have opposite sign, then the speed is decreasing.

Example 0.10 *The displacement of a particle as a function of time t is given by* $s = \alpha + \beta t + \gamma t^2 + \delta t^4$, *where* α, β, γ *and* δ *are constants. Find the ratio of the initial velocity to the initial acceleration.*

Sol. First find the velocity and acceleration in terms of time t, then use $t = 0$ to find the initial values.

$$s = \alpha + \beta t + \gamma t^2 + \delta t^4 \quad \text{(Given)}$$

On differentiating both sides w.r.t. t, we get

$$\frac{ds}{dt} = \beta + 2\gamma t + 4\delta t^3$$

$$\Rightarrow \quad v = \beta + 2\gamma t + 4\delta t^3 \quad \left(\because \frac{ds}{dt} = v\right) \quad \text{...(i)}$$

On differentiating both sides of Eq. (i), w.r.t. t, we get

$$\frac{dv}{dt} = 2\gamma + 4.3\,\delta t^2 \Rightarrow \frac{dv}{dt} = 2\gamma + 12\,\delta t^2 \qquad \left(\because \frac{dv}{dt} = a\right)$$

At $t = 0$, $v = \beta$ and $a = 2\gamma$

$$\therefore \quad \frac{\text{Initial velocity}}{\text{Initial acceleration}} = \frac{\beta}{2\gamma}$$

Example 0.11 *The position of a particle moving along the x-axis varies with time t as $x = 6t - t^2 + 4$. Find the time interval during which the particle is moving along the positive x-direction.*

Sol. Given, $x = 6t - t^2 + 4$

$$v = \frac{dx}{dt} = 6 - 2t + 0 = 2(3 - t)$$

At $t < 3$ sec, $v > 0$, then the particle is moving along the positive x-direction.

At $t > 3$, $v < 0$, then particle is moving along the negative x-direction.

At $t = 3$, $v = 0$

For time-interval $t = 0$ to $t = 3$, then the particle is moving along the positive x-direction.

Check Point 0.2

1. Differentiate the following function

(i) $y = 3x^4 + 2\frac{1}{x^2} + \log x$ (ii) $y = (x^2 + 1)(x + 2)$

(iii) $y = \frac{3x^2}{x + 1}$ (iv) $y = \sin x$

(v) $y = \tan(x^2 + 3x + 1)$ (vi) $y = 3\cos x + 7e^x + \log(x^2 + 1)$

Ans. (i) $12x^3 - 4\frac{1}{x^3} + \frac{1}{x}$ (ii) $3x^2 + 4x + 1$

(iii) $\frac{3x^2 + 6x}{(x + 1)^2}$ (iv) $\cos x$

(v) $(2x + 3)\sec^2(x^2 + 3x + 1)$ (vi) $-3\sin x + 7e^x + \frac{2x}{x^2 + 1}$

2. A particle is moving with velocity $v = t^3 - 6t^2 + 4$, where v is in m/s and t is in seconds. At what time will the velocity be maximum/minimum and what is it equal to?

Ans. $v_{max} = 4$ m/sec at $t = 0$ s and $v_{min} = -28$ m/s at $t = 4$ s

3. If the time and displacement of particle along the positive X-axis are related as $t = (x^2 - 1)^{\frac{1}{2}}$, then find the acceleration in terms of x.

Ans. $\frac{1}{x^3}$

3 Integration

Integration means summation. Integration is the process of finding the function, whose derivative is given. In other word, integration is the reverse process of differentiation. Its symbol is $\int$. Consider a function $f(x)$, whose derivative w.r.t. x is equal to $f'(x)$, then $f(x) + c$ is called integration of $f'(x)$, where c is called constant of integration. Symbolically, it is written as $\int f'(x)dx = f(x) + c$

Here, $f'(x)\,dx$ is called element of integration and $\int$ is called indefinite integral.

Some basic formulae of integration

- $\int x^n dx = \frac{x^{n+1}}{n+1} + c;\ n \neq -1 \qquad \left(\because \frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = x^n\right)$
- $\int 1\,dx = x + c \qquad \left(\because \frac{d}{dx}x = 1\right)$
- $\int \sin x\,dx = -\cos x + c \qquad \left(\because \frac{d}{dx}-(\cos x) = 2\sin x\right)$
- $\int \cos x\,dx = \sin x + c \qquad \left(\because \frac{d}{dx}(\sin x) = \cos x\right)$
- $\int \frac{1}{x}dx = \log_e x + c \qquad \left(\because \frac{d}{dx}\log_e x = \frac{1}{x}\right)$
- $\int e^x dx = e^x + c \qquad \left(\because \frac{d}{dx}e^x = e^x\right)$

Example 0.12 *Evaluate*

(i) $\int (e^x + \frac{1}{x} + 2x^2 + 3)\,dx$ (ii) $\int\left(\cos x + 3x^{1/2} + \frac{3}{x} + \frac{4}{x^2}\right)dx$

Sol. (i) Let $I = \int\left(e^x + \frac{1}{x} + 2x^2 + 3\right)dx$

$$= \int e^x dx + \int \frac{1}{x}dx + 2\int x^2 dx + 3\int 1\,dx$$

$$= e^x + \log_e x + 2\cdot\frac{x^{2+1}}{2+1} + 3x + C$$

$$= e^x + \log_e x + \frac{2}{3}x^3 + 3x + C$$

(ii) Let $I=\int\left(\cos x+3x^{1/2}+\frac{3}{x}+\frac{4}{x^2}\right)dx$

$$=\int\cos x\,dx+3\int x^{1/2}dx+3\int\frac{1}{x}dx+4\int\frac{1}{x^2}dx$$

$$=\sin x+3\cdot\frac{x^{(1/2)+1}}{(1/2)+1}+3\log_e x+4\int x^{-2}dx+C$$

$$=\sin x+\frac{3\times 2}{3}x^{3/2}+3\log_e x+4\frac{x^{-2+1}}{-2+1}+C$$

$$=\sin x+2x^{3/2}+3\log_e x-\frac{4}{x}+C$$

Some important step in integration

$$I=\int f'(ax+b)\,dx=\frac{f(ax+b)}{\left[\frac{d}{dx}(ax+b)\right]}$$

Example 0.13 *Evaluate the following*

(i) $\int(2x+1)^3dx$ (ii) $\int\left(\frac{1}{a-x}\right)dx$

(iii) $\int(x^2+3x+4)^4dx$ (iv) $\int\sin(2x^2)\,dx$

Sol. (i) Let $I=\int(2x+1)^3dx=\frac{(2x+1)^{3+1}}{(3+1)\left[\frac{d}{dx}(2x+1)\right]}+C$

$$=\frac{(2x+1)^4}{4\cdot 2}+C=\frac{(2x+1)^4}{8}+C$$

(ii) Let $I=\int\left(\frac{1}{a-x}\right)dx=\frac{\log(a-x)}{\frac{d}{dx}(a-x)}+C$

$$=\frac{\log(a-x)}{-1}+C=-\log(a-x)+C$$

(iii) Let $I=\int(x^2+3x+4)^4dx$

$$=\frac{(x^2+3x+4)^{4+1}}{(4+1)\frac{d}{dx}(x^2+3x+4)}+C$$

$$=\frac{(x^2+3x+4)^5}{5(2x+3)}+C$$

(iv) Let $I=\int\sin(2x^2)\,dx$

$$=\frac{-\cos(2x^2)}{\frac{d}{dx}(2x^2)}+C=\frac{-\cos(2x^2)}{4x}+C$$

Definite integral

A definite integral has definite value.

or

When a function is integrated between a lower limit and an upper limit, is called a definite integral.

If $\frac{d}{dx}f(x)=f'(x)$, then $\int_a^b f'(x)\,dx$ is called definite integral, where a and b are called lower and upper limit, respectively of variable x. After carrying out integration, then the result is evaluated between upper and lower limits as shown below

$$\int_a^b f'(x)\,dx=|f(x)|_a^b=f(b)-f(a)$$

Note 1. $e=2.71, \pi=\frac{22}{7}=3.14$ 2. $360°=2\pi$ radian, $\frac{\pi}{2}$ radian $=90°$

Example 0.14 *Evaluate*

(i) $\int_0^2(4x^3+2x^2+2x+1)\,dx$

(ii) $\int_0^{\pi/4}(\sin x+\cos x)\,dx$

(iii) $\int_2^4\frac{dx}{x}$

Sol. (i) Let $I=\int_0^2(4x^3+2x^2+2x+1)\,dx$

$$=4\int_0^2x^3dx+2\int_0^2x^2dx+2\int_0^2x\,dx+\int_0^2 1\,dx$$

$$=4\left|\frac{x^4}{4}\right|_0^2+2\left|\frac{x^3}{3}\right|_0^2+2\left|\frac{x^2}{2}\right|_0^2+|x|_0^2$$

$$=4\left(\frac{2^4-0^4}{4}\right)+2\left(\frac{2^3-0^3}{3}\right)+2\left(\frac{2^2-0^2}{2}\right)+(2-0)$$

$$=16+\frac{2}{3}\times 8+4+2$$

$$\Rightarrow\quad I=22+\frac{16}{3}=\frac{66+16}{3}=\frac{82}{3}$$

(ii) Let $I=\int_0^{\pi/4}(\sin x+\cos x)\,dx$

$$=\int_0^{\pi/4}\sin x\,dx+\int_0^{\pi/4}\cos x\,dx$$

$$=|-\cos x|_0^{\pi/4}+|\sin x|_0^{\pi/4}$$

$$=-\cos\frac{\pi}{4}+\cos 0+\sin\frac{\pi}{4}-\sin 0$$

$$=-\frac{1}{\sqrt{2}}+1+\frac{1}{\sqrt{2}}-0=1$$

(iii) Let $I=\int_2^4\frac{dx}{x}=|\log_e x|_2^4$

$$=\log_e 4-\log_e 2$$

$$=\log_e\left(\frac{4}{2}\right)=\log_e 2\quad\left[\because\log_e a-\log_e b=\log_e\left(\frac{a}{b}\right)\right]$$

Example 0.15 *Evaluate the following*

(i) $\int_1^2 e^{(x+4)}dx$ (ii) $\int_0^{\pi/4} \cos(2x^2 + x)\,dx$

(iii) $\int_1^2 \frac{dx}{(3x+4)}$ (iv) $\int_0^4 (3x^2 + 4x + 5)\,dx$

Sol. (i) Let $I = \int_1^2 e^{(x+4)}dx = \left|\frac{e^{x+4}}{\frac{d}{dx}(x+4)}\right|_1^2$

$$= \left|\frac{e^{x+4}}{1}\right|_1^2 = e^{2+4} - e^{1+4} = e^6 - e^5 = e^5(e-1)$$

(ii) Let $I = \int_0^{\pi/4} \cos(2x^2 + x)\,dx$

$$= \left|\frac{\sin(2x^2+x)}{\frac{d}{dx}(2x^2+x)}\right|_0^{\pi/4} = \left|\frac{\sin(2x^2+x)}{4x+1}\right|_0^{\pi/4}$$

$$= \frac{\sin\left(2.\frac{\pi^2}{16} + \frac{\pi}{4}\right)}{4\cdot\frac{\pi}{4}+1} - \frac{\sin(2\times 0^2 + 0)}{4\times 0 + 1}$$

$$= \frac{\sin\left(\frac{\pi^2+2\pi}{8}\right)}{(\pi+1)} - \frac{\sin 0}{1}$$

$$= \frac{\sin\left(\frac{\pi^2+2\pi}{8}\right)}{\pi+1} - 0 = \left(\frac{1}{\pi+1}\right)\sin\left(\frac{\pi^2+2\pi}{8}\right)$$

(iii) Let $I = \int_1^2 \frac{dx}{(3x+4)} = \left|\frac{\log(3x+4)}{\frac{d}{dx}(3x+4)}\right|_1^2$

$$= \left|\frac{\log(3x+4)}{3}\right|_1^2$$

$$= \frac{\log(3\times 2+4) - \log(3\times 1+4)}{3}$$

$$= \frac{1}{3}[\log 10 - \log 7] = \frac{1}{3}\log\left(\frac{10}{7}\right)$$

(iv) Let $I = \int_0^4 (3x^2 + 4x + 5)\,dx$

$$= \left|3\frac{x^{2+1}}{2+1} + \frac{4x^{1+1}}{1+1} + 5x\right|_0^4$$

$$= \left|3\frac{x^3}{3} + 4\frac{x^2}{2} + 5x\right|_0^4$$

$$= |x^3 + 2x^2 + 5x|_0^4$$

$$= 4^3 + 2\times 4^2 + 5\times 4 - 0$$

$$= 64 + 32 + 20 = 64 + 52 = 116$$

Application of integration in physics

We Know that, $V = \frac{dS}{dt}$, $a = \frac{dV}{dt}$ or $V\frac{dV}{ds}$

If the displacement is given and we have to find the velocity and acceleration, then we use differentiation.

If the acceleration is given and we have to find the velocity and displacement, then we use integration.

Example 0.16 *A particle is moving under constant acceleration $a = 3t + 4t^2$. If the position and velocity of the particle at start, i.e., $t = 0$ are x_0 and v_0, respectively, then find the displacement and velocity on a function of time t.*

Sol.

We have, $a = 3t + 4t^2$

$$\frac{dv}{dt} = 3t + 4t^2 \qquad \left(\because a = \frac{dV}{dt}\right)$$

$$\Rightarrow \int_{v_0}^{v} dv = \int_0^t (3t + 4t^2)\,dt \qquad \text{(Integrating both sides)}$$

$$\Rightarrow |v|_{v_0}^{v} = \left|3\frac{t^2}{2} + 4\frac{t^3}{3}\right|_0^t \Rightarrow (v - v_0) = \frac{3}{2}(t^2 - 0) + \frac{4}{3}(t^3 - 0)$$

$$\Rightarrow v = v_0 + \frac{3}{2}t^2 + \frac{4}{3}t^3 \qquad \text{...(i)}$$

$$\Rightarrow \frac{dx}{dt} = v_0 + \frac{3}{2}t^2 + \frac{4}{3}t^3 \qquad \left(\because v = \frac{dx}{dt}\right)$$

$$\Rightarrow \int_{x_0}^{x} dx = \int_0^t \left(v_0 + \frac{3}{2}t^2 + \frac{4}{3}t^3\right)dt$$

$$\Rightarrow x - x_0 = \left|v_0 t + \frac{3}{2}\cdot\frac{t^3}{3} + \frac{4}{3}\cdot\frac{t^4}{4}\right|_0^t = v_0 t + \frac{1}{2}\cdot t^3 + \frac{1}{3}t^4 - 0$$

$$\Rightarrow x - x_0 = v_0 t + \frac{1}{2}\cdot t^3 + \frac{1}{3}t^4 \Rightarrow x = x_0 + v_0 t + \frac{1}{2}t^3 + \frac{1}{3}t^4$$

Average value

If the velocity is variable and depends on time t, to find the average value of velocity for time interval $t = t_1$ to $t = t_2$.

Let $v = f(t)$; $\bar{v} = \frac{\int_{t_1}^{t_2} v\,dt}{\int_{t_1}^{t_2} dt}$

To find the average value of square of velocity

$$\overline{v^2} = \frac{\int_{t_1}^{t_2} v^2 dt}{\int_{t_1}^{t_2} dt} \Rightarrow \overline{v^4} = \frac{\int_{t_1}^{t_2} v^4 dt}{\int_{t_1}^{t_2} dt}$$

If velocity is a function of displacement, $v = f(x)$, for average of v from $x = x_1$ to $x = x_2$

$$\Rightarrow \qquad \bar{v} = \frac{\int_{x_1}^{x_2} v\, dt}{\int_{x_1}^{x_2} dt}$$

The above procedure can be applied to find the average value of any quantity like velocity, acceleration, force, etc.

Example 0.17 *The velocity of a particle is given by $v = v_0 \sin \omega t$, where v_0 is constant and $\omega = 2\pi / T$. Find the average velocity in time interval $t = 0$ to $t = T/2$*

Sol. We have, $v = v_0 \sin \omega t$

$$\text{Average velocity, } \bar{v} = \frac{\int_0^{T/2} v_0 \sin(\omega t)\, dt}{\int_0^{T/2} dt}$$

$$= \frac{v_0 \left| -\cos(\omega t) \right|_0^{T/2}}{\omega \left| t_0 \right|_0^{T/2}}$$

$$= \frac{\frac{v_0}{\omega}\left[-\cos\left(\omega \frac{T}{2}\right) + \cos 0\right]}{\left(\frac{T}{2} - 0\right)}$$

$$= \frac{\frac{V_0}{\frac{2\pi}{T}}\left[-\cos\left(\frac{2\pi}{T}\cdot\frac{T}{2}\right) + 1\right]}{\frac{T}{2}} \left(\because \omega = \frac{2\pi}{T}, \text{given}\right)$$

$$= \frac{2v_0 T}{2\pi}\frac{(1 - \cos\pi)}{T} = \frac{v_0}{\pi}[1 - (1)] \; (\because \cos\pi = \cos 180° = -1)$$

$$= \frac{v_0}{\pi}(1 + 1) = \frac{2v_0}{\pi}$$

Check Point 0.3

1. Evaluate

(i) $\int \left(\sin x + \frac{1}{x} + 2\frac{1}{x^2} + 3x^3\right) dx$

(ii) $\int (3\cos x + e^x + 4x^2 + x + 5)\, dx$

Ans. (i) $-\cos x + \log x - \frac{2}{x} + \frac{3}{4}x^4 + C$

(ii) $3\sin x + e^x + \frac{4}{3}x^3 + \frac{x^2}{2} + 5x + C$

2. Evaluate the following

(i) $\int (2x^2 + 4x + 1)^2 dx$ (ii) $\int \frac{1}{x^2 + 2} dx$

(iii) $\int \cos(x + 2)\, dx$

Ans. (i) $\frac{1}{12}\frac{(2x^2 + 4x + 1)^3}{(x + 1)} + C$ (ii) $\frac{1}{2x}\log(x^2 + 2) + C$

(iii) $\sin(x + 2) + C$

3. Evaluate

(i) $\int_1^3 \left(4x + \frac{1}{x} + 1\right) dx$ (ii) $\int_0^{\pi/4} (\sin x - \cos x)\, dx$

Ans. (i) $18 + \log 3$ (ii) 1

4. Evaluate the following

(i) $\int_0^2 \frac{dx}{(x^2 + 4x + 1)}$

(ii) $\int_1^3 (4x^3 + 3x^2 + 2x + 1)\, dx$

Ans. (i) $\frac{1}{8}\log 13$ (ii) 116

5. A particle is moving in a straight line under acceleration $a = kt$, where k is a constant. Find the velocity in term of t. The motion starts from rest.

Ans. $\frac{kt^2}{2}$

6. A particle is moving in a straight line such that its velocity varies as $v = v_0 e^{-\lambda t}$, where α and t are constant. Find the average velocity during the time interval in which the velocity decrease from v_0 to $\frac{v_0}{2}$.

Ans. $\frac{v_0}{2}\log_e 2$

4 Graphs

(i) If a graph is concave up (curved upward), the slope is increasing.

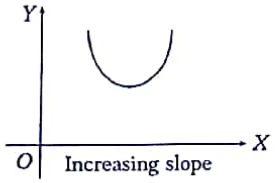


Fig. 0.5

(ii) If a graph is concave downward (curved downward), the slope is decreasing.

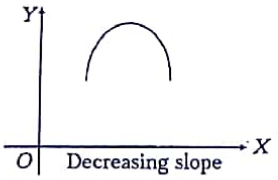


Fig. 0.6

(iii) If the graph is a straight line, the slope is constant.

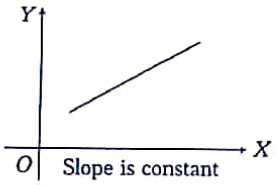


Fig. 0.7

(iv) The general equation of a straight line is of the form $y = mx + c$ where, m is the slope of line, $m = \tan\theta$ and c is the intercept on the Y-axis.

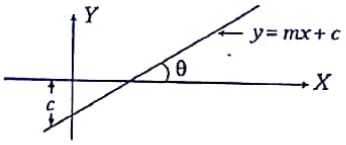


Fig. 0.8

(a) If m is + ve, *i.e.*, $0° < \theta < 90°$, then lines will be of the type.

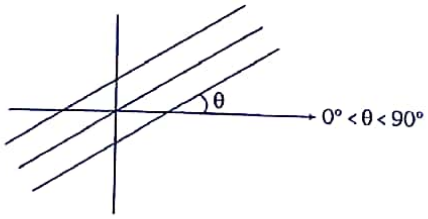


Fig. 0.9

(b) If m is -ve, *i.e.*, $90° < \theta < 180°$, then the lines will be of the type.

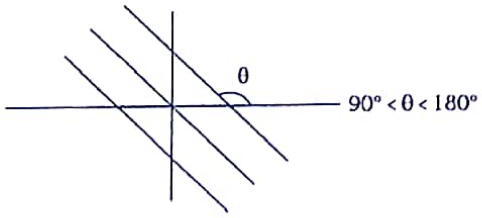


Fig. 0.10

(c) If c is + ve, then the lines will cut the Y-axis above the origin.

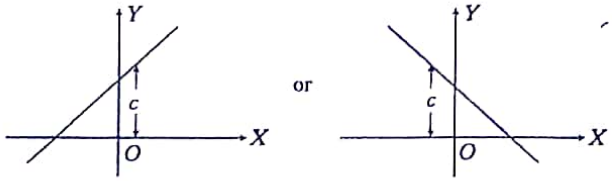


Fig. 0.11

(d) If c is -ve, then the lines will cut the Y-axis below the origin.

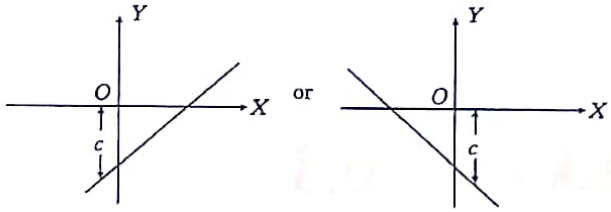


Fig. 0.12

(e) If $c = 0$, then the lines will pass through the origin.

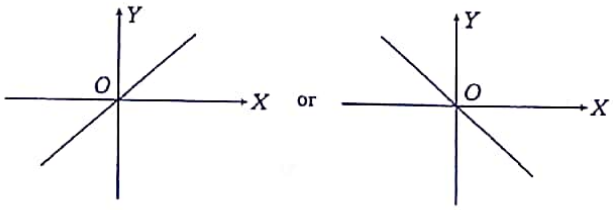


Fig. 0.13

(v) **Parabola** *Some standard forms of parabola are as follows*

(a) $y^2 = kx$, a parabola passing through the origin and open rightward.

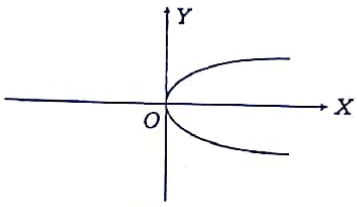


Fig. 0.14

(b) $y^2 = -kx$, a parabola passing through the origin and open leftward.

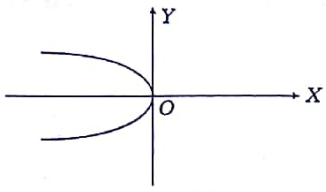


Fig. 0.15

(c) $x^2 = ky$, a parabola passing through the origin and open upward.

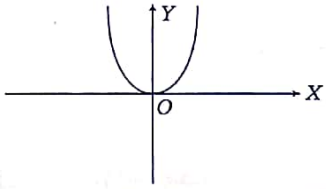


Fig. 0.16

(d) $x^2 = -ky$, a parabola passing through the origin and open downward.

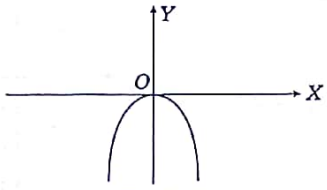


Fig. 0.17

(vi) **Exponential graph** *The most popular graph based on exponential (e) are*

(a) $y = e^{-x}$

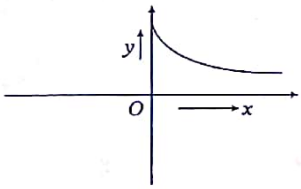


Fig. 0.18

(b) $y = 1 - e^{-x}$

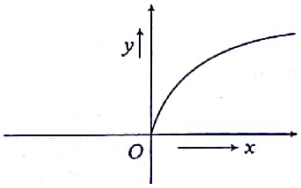


Fig. 0.19

(vii) **Circle** If equation of circle is $x^2 + y^2 = a^2$, where centre of circle $\equiv (0, 0)$ and radius of circle $= a$

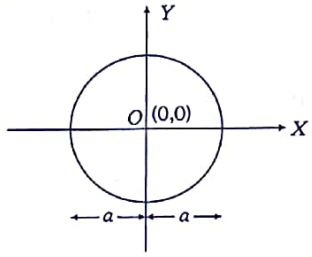


Fig. 0.20

(viii) **Ellipse** Equation of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\ (a > b)$, where, coefficient of $x^2 \neq$ coefficient of y^2.

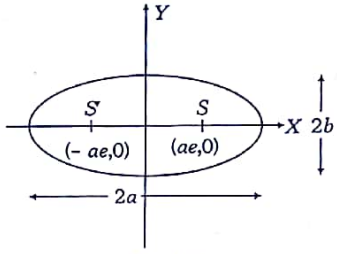


Fig. 0.21

Here, $2a$ is major axis and $2b$ is minor axis.

Eccentricity, $e = \sqrt{1 - \frac{b^2}{a^2}}$

For ellipse, $e < 1$

Focus, $S \equiv (ae, 0)$, $S' \equiv (-ae, 0)$

Area of ellipse $= \pi ab$

(ix) **Hyperbola** Equation of hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

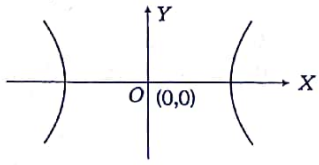


Fig. 0.22

(x) **Rectangular Hyperbola** If $x \propto \frac{1}{y}$ or $xy =$ constant, then

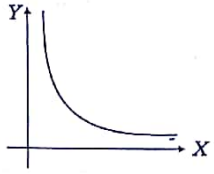
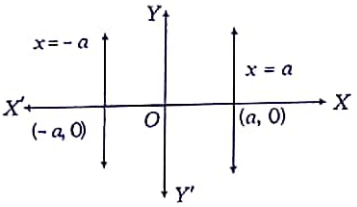
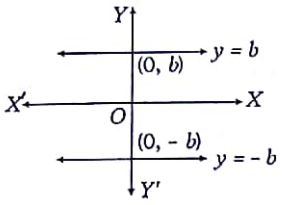
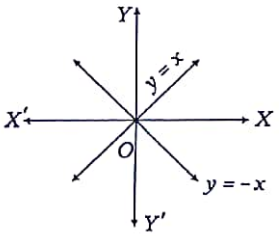
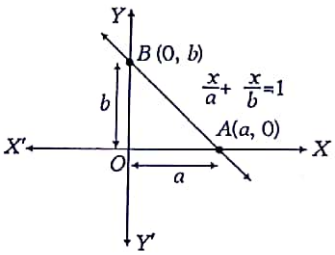
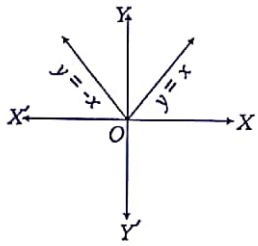
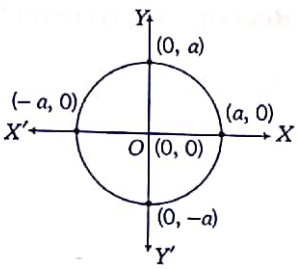
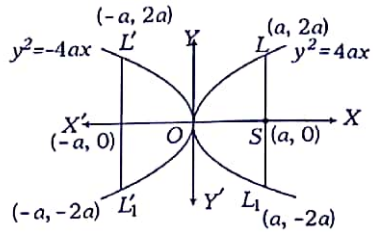
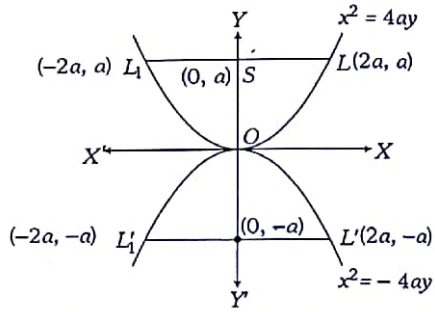
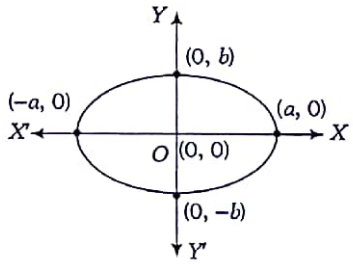
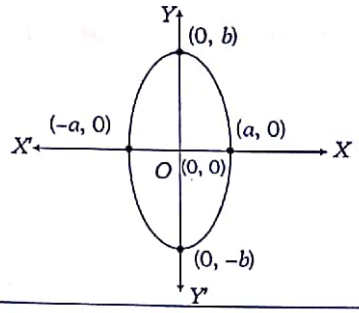
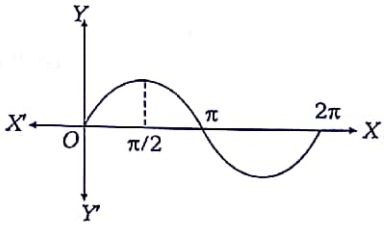
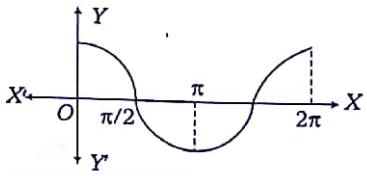


Fig. 0.23

Sketches of some standard curves

Equation of curve	Diagram/Sketch
1. (a) **Straight lines** $x = a$ and $x = -a$, where $a > 0$	
(b) **Straight lines** $y = b$ and $y = -b$, where $b > 0$	
2. **Straight lines** $y = x$ and $y = -x$	
3. **Straight lines** $\frac{x}{a} + \frac{y}{b} = 1$, $a \neq b$ $x + y = a$, $a = b$	
4. **Modulus function** $y = \lvert x \rvert$ $y = \begin{cases} x, \text{ for } x \geq 0 \\ -x, \text{ for } x < 0 \end{cases}$	
5. **Circle** $x^2 + y^2 = a^2$ **Centre** $= (0, 0)$ **Radius** $= a$	

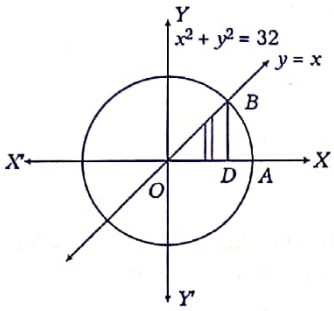
Equation of curve	Diagram/Sketch
6. (a) **Parabola** $y^2 = 4ax$ and $y^2 = -4ax$ **Vertex,** $O = (0, 0)$, **Focus,** $S = (a, 0)$ and $(-a, 0)$ **Length of latusrectum** $= 4a$	
(b) **Parabola** $x^2 = 4ay$ and $x^2 = -4ay$ **Vertex,** $O = (0, 0)$, **Focus,** $S = (0, a)$ and $(0, -a)$ **Length of latusrectum** $= 4a$	
7. (a) **Ellipse** $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, when $a > b$ **Vertices** $= (\pm a, 0)$, **Centre,** $O = (0, 0)$	
(b) **Ellipse** $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, when $a < b$ **Vertices** $= (0, \pm b)$, **Centre,** $O = (0, 0)$	
8. (a) **Sine function** $y = \sin x$	
(b) **Cosine function** $y = \cos x$	

Example 0.18 *Find the area of the region in the first quadrant enclosed by the X-axis, then the line $y = x$ and the circle $x^2 + y^2 = 32$.*

Sol. We have, circle $x^2 + y^2 = 32$ ...(i)

and line $y = x$ ...(ii)

It is clear from the figure that region for finding area is *OABO*.



On putting the value of y from Eq. (ii) in Eq. (i), we get

$$x^2 + x^2 = 32$$

$$\Rightarrow \quad 2x^2 = 32 \Rightarrow x^2 = 16 \Rightarrow x = \pm 4$$

From Eq. (ii), we get $y = \pm 4$

Thus, line and circle intersect each other at two points (4, 4) and (– 4, – 4). So, coordinates of $A(4\sqrt{2}, 0)$, and $B(4, 4)$ in I quadrant.

Now, area of *OABO* = Area of *ODBO* + Area of *DABD*

$$= \int_0^4 y\,dx_{\text{line}} + \int_4^{4\sqrt{2}} y\,dx_{\text{circle}}$$

$$= \int_0^4 x dx + \int_4^{4\sqrt{2}} \sqrt{32 - x^2}\,dx \text{ [From Eq. (i) and Eq. (ii)]}$$

$$= \left[\frac{x^2}{2}\right]_0^4 + \int_4^{4\sqrt{2}} \sqrt{(4\sqrt{2})^2 - x^2}\,dx$$

$$= \frac{4^2 - 0^2}{2} + \left[\frac{x}{2}\sqrt{32 - x^2} + \frac{32}{2}\sin^{-1}\left(\frac{x}{4\sqrt{2}}\right)\right]_4^{4\sqrt{2}}$$

$$= \frac{16}{2} + \left[\frac{4\sqrt{2}}{2}\sqrt{32-32} - \frac{4}{2}\sqrt{32-16} + \frac{32}{2}\sin^{-1}\frac{4\sqrt{2}}{4\sqrt{2}} - \frac{32}{2}\sin^{-1}\frac{4}{4\sqrt{2}}\right]$$

$$= 8 + 0 - 2 \times 4 + 16\sin^{-1}(1) - 16\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$= 8 - 8 + 16\left(\frac{\pi}{2}\right) - 16\left(\frac{\pi}{4}\right)$$

$$= 8\pi - 4\pi = 4\pi \text{ sq unit}$$

Hence, the required area of region is 4π sq units.

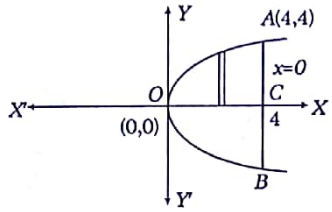
Example 0.19 *Find the area of region bounded by the curve $y^2 = 4x$ and the line $x = 4$.*

Sol. Given curve is a parabola, $y^2 = 4x$ ...(i)

Which is of the form $y^2 = 4aX$ having vertex (0, 0)

and line, $x = 4$ ...(ii)

Then, the region for which we have to find area is *OBCAO*, also region *OCAO* is symmetrical about *X*-axis.



On putting the value of x from Eq. (ii) in Eq. (i), we get

$$y^2 = 4(4) = 16 \Rightarrow y = \pm 4$$

$\therefore$ Area of bounded region *OACBO* = 2(Area of region *OACO*)

($\because$ Parabola is symmetrical about *X*-axis)

$$= 2\int_0^4 y(\text{parabola})\,dx = 2\int_0^4 2 \cdot x^{1/2}\,dx \quad \text{[From Eq. (i)]}$$

$$= 4\int_0^4 x^{1/2}dx = 4\left[\frac{x^{3/2}}{3/2}\right]_0^4 = 4 \cdot \frac{2}{3}[x^{3/2}]_0^4$$

$$= \frac{8}{3}[4^{3/2} - 0] = \frac{8}{3} \times 4\sqrt{4} = \frac{8}{3} \times 4 \times 2 = \frac{64}{3} \text{ sq units}$$

Hence, the required area is $\frac{64}{3}$ sq units

Example 0.20 *Find the area of the region bounded by the line $y = 3x + 2$, then the X-axis and the ordinates $x = -1$ and $x = 1$.*

Sol. Given lines are $y = 3x + 2$ ...(i)

$y = 0$ [on *X*-axis] ...(ii)

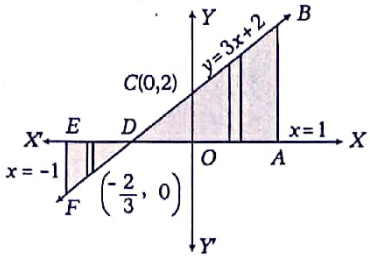
$x = -1$ ...(iii)

and $x = 1$ ...(iv)

Now, table for $y = 3x + 2$

x	0	$-\frac{2}{3}$
y	2	0

The given region bounded by $y = 3x + 2$, *X*-axis and the ordinates $x = -1$ and $x = 1$ is represented by shaded region.



$\therefore$ Required Area

$= \text{Area of region } EFDE + \text{Area of region } ABDA$

$$= \left| \int_{-1}^{-2/3} y_1\, dx \right| + \int_{-2/3}^{-1} y_2\, dx$$

$$= \left| \int_{-1}^{-2/3} (3x+2)\, dx \right| + \int_{-2/3}^{1} (3x+2)\, dx$$

$$= \left| \left[\frac{3x^2}{2} + 2x \right]_{-1}^{-2/3} \right| + \left[\frac{3x^2}{2} + 2x \right]_{-2/3}^{1}$$

$$= \left| \left\{ \frac{3}{2}\left(\frac{4}{9}\right) - \frac{4}{3} \right\} - \left(\frac{3}{2} - 2 \right) \right| + \left[\left(\frac{3}{2} + 2 \right) - \left\{ \frac{3}{2}\left(\frac{4}{9}\right) - \frac{4}{3} \right\} \right]$$

$$= \left| \frac{2}{3} - \frac{4}{3} + \frac{1}{2} \right| + \left[\frac{7}{2} - \frac{2}{3} + \frac{4}{3} \right]$$

$$= \left| \frac{4 - 8 + 3}{6} \right| + \left[\frac{21 - 4 + 8}{6} \right]$$

$$= \left| -\frac{1}{6} \right| + \frac{25}{6} = \frac{1}{6} + \frac{25}{6} = \frac{26}{6}$$

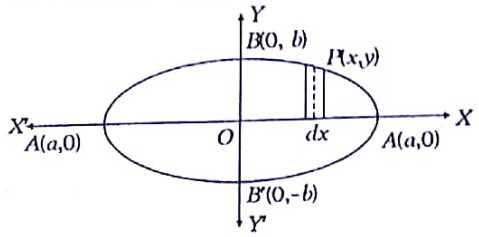
$$= \frac{13}{3} \text{ sq units}$$

Hence, the required area is $\frac{13}{3}$ sq units.

Example 0.21 *Find the area of the region bounded by the ellipse* $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Sol. We have, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ...(i)

Since all power of x and y both are even in the equation of the curve. So, it is symmetrical about the axes as shown in figure.



Area enclosed by the ellipse = 4 [Area enclosed by the ellipse and coordinate axes in 1st quadrant]

$$\Rightarrow \quad A = 4\int_0^a y\, dx = 4\int_0^a \frac{b}{a}\sqrt{a^2 - x^2}\, dx \qquad \text{[From Eq. (i)]}$$

$$= \frac{4b}{a}\left[\frac{1}{2} x\sqrt{a^2 - x^2} + \frac{1}{2} a^2 \sin^{-1}\frac{x}{a} \right]_0^a$$

$$= \frac{4b}{a}\left\{ 0 + \frac{1}{2} a^2 \sin^{-1}(1) \right\} = \frac{4b}{a} \times \frac{1}{2} a^2 \left(\frac{\pi}{2} \right) = \pi ab \text{ sq units}$$

Check Point 0.4

1. Find the area of region bounded by the curve $y = x^3$, $y = x + 6$ and $x = 0$.

Ans. 10 sq units

2. Find the area of the region included between the parabola $y = \frac{3x^2}{4}$ and the line $3x - 2y + 12 = 0$.

Ans. 27 sq units

3. Draw the rough sketch of the graph of the curve $\frac{x^2}{4} + \frac{y^2}{9} = 1$ and evaluate the area of the region under the curve and above the X-axis.

Ans. 3π sq units

4. Find the area of the region bounded by $y = \sqrt{x}$ and $y = x$.

Ans. $\frac{1}{6}$ sq units

CHAPTER 01

Units, dimensions & error analysis

In this chapter, we will discuss about units and dimensions of different physical quantities and error that occur in measurement.

Physical quantities

All the quantities which are used to describe the laws of physics are called physical quantities, *e.g.*, length, mass, volume, etc.

To express the measurement of a physical quantity, we need to know two things as given below.

(i) The unit in which the quantity is measured.

(ii) The numerical value or the magnitude of the quantity.

i.e., the number of times that unit is contained in the given physical quantity = nu

$$\therefore \qquad n \propto \frac{1}{u} \Rightarrow nu = \text{constant}$$

n = numerical value of the physical quantity and u = size of unit

We may write as, $\boxed{n_1u_1 = n_2u_2}$

where, n_1 and n_2 are values of the physical quantity in two different units u_1 and u_2.

Inside

1) Units
- System of unit

2) Dimensions of physical quantities
- Applications of dimensional analysis
- Defects or limitations of dimensional analysis

3) Significant figure
- Rules to determine significant figure
- Mathematical operations of significant figures
- Rounding off
- Order of magnitude

4) Error in measurement
- Expression of errors
- Combination of errors

1 Units

The standard amount of a physical quantity chosen to measure the physical quantity of same kind is called a physical unit.

The essential requirements of physical unit are given below.

(i) It should be of suitable size.

(ii) It should be easily accessible.

(iii) It should not vary with time.

(iv) It should be easily reproducible.

(v) It should not depend on physical conditions like pressure, volume, etc.

Systems of unit

A complete set of units which is used to measure all kinds of fundamental and derived quantities is called a system of units.

Some of the commonly used systems of units are as follows

1. CGS system

In this system, the units of length, mass and time are centimetre (cm), gram (g) and second (s), respectively. The unit of force is dyne and that of work or energy is erg.

2. FPS system

In this system, the units of length, mass and time are foot, pound and second respectively. The unit of force in this system is poundal.

3. MKS system

In this system, the units of length, mass and time are metre (m), kilogram (kg) and second (s), respectively. The unit of force is newton (N) and that of work or energy is joule (J).

4. International system (SI) of units

This system of units helps in revolutionary changes over the MKS system and is known as **rationalised MKS system.** It is helpful to obtain all the physical quantities in physics.

Note

1. The FPS system is not a metric system. This system is not in much use these days.
2. The drawback of CGS system is that many of the derived units on this system are inconveniently small.
3. The advantages of MKS system is that some of the derived units are of convenient size.

Example 1.1 *The acceleration due to gravity is 9.8 ms^{-2}. Give its value in ft s^{-2}.*

Sol. As 1 m = 3.28 ft

$\therefore$ $9.8\ m/s^2 = 9.8 \times 3.28\ ft\ s^{-2} = 32.14\ ft\ s^{-2} \approx 32\ ft\ s^{-2}$

Example 1.2 *The value of gravitational constant G in MKS system is 6.67×10^{-11} N-m^2 kg^{-2}. What will be its value in CGS.*

Sol. $G = 6.67 \times 10^{-11}\ Nm^2\ kg^{-2}$

$= 6.67 \times 10^{-11}\ (kg\ ms^{-2})\ m^2\ kg^{-2}$

$= 6.67 \times 10^{-11}\ (m^3)\ (s^{-2})\ (kg^{-1})$

$= 6.67 \times 10^{-11}\ (10^2\ cm)^3\ (s)^{-2}\ (10^3\ g)^{-1}$

$= 6.67 \times 10^{-8}\ cm^3\ g^{-1}\ s^{-2}$

$= 6.67 \times 10^{-8}$ dyne-cm^2 g^{-2}

Fundamental quantities and fundamental units

Those physical quantities which are independent of other physical quantities and not defined in terms of other physical quantities, are called fundamental quantities or base quantities. The units of these quantities are called fundamental or base units.

Table below gives the seven fundamental quantities and their SI units.

Base quantity	SI units		
	Name	Symbol	Definition
Length	Metre	m	The metre is the length of the path travelled by light in vacuum during a time interval of 1/299792458 of a second.
Mass	Kilogram	kg	The kilogram is equal to the mass of the international prototype of the kilogram (a platinum-iridium alloy cylinder) kept at international Bureau of Weights and Measures at Sevres, near Paris, France.
Time	Second	s	The second is the duration of 9192631770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom.
Electric current	Ampere	A	The ampere is that constant current which if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 m apart in vacuum, would produce a force between these conductors equal to 2×10^{-7} N/m of length.
Thermo-dynamic temperature	Kelvin	K	The kelvin is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water.
Amount of substance	Mole	mol	The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kg of carbon-12.
Luminous intensity	Candela	cd	The candela is the luminous intensity in a given direction of a source that emits monochromatic radiation of frequency 540×10^{12} Hz and that has a radiant intensity in that direction of 1/683 W/Sr.

Derived quantities and derived units

The quantities which can be expressed in terms of the fundamental quantities are called derived quantities. The units of these quantities are called derived units.

e.g., Unit of speed = ms^{-1} can be derived from fundamental unit *i.e.*, unit of length and time as

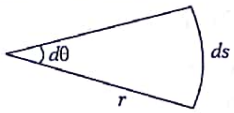
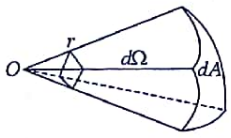
$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$= \frac{m}{s} = ms^{-1}$$

Supplementary quantities and supplementary units

Other than fundamental and derived quantities, there are two more quantities called as supplementary quantities. The units of these quantities are known as supplementary units.

Table below gives the supplementary quantities and their SI units.

Supplementary quantity	SI units		
	Name	Symbol	Definition
Plane angle	Radian	rad	One radian is the angle subtended at the centre by an arc equal in length to the radius of the circle.  *i.e.*, $d\theta = \frac{ds}{r}$
Solid angle	Steradian	sr	One steradian is the solid angle subtended at the centre of a sphere, by that surface of the sphere, which is equal in area, to the square of radius of the sphere.  *i.e.*, $d\Omega = \frac{dA}{r^2}$

Some other units (Not contained in SI units)

Length

(i) 1 micron = 1 μm = 10^{-6} m
(ii) 1 nanometre = 1 nm = 10^{-9} m
(iii) 1 Angstrom = 1 Å = 10^{-10} m = 10^{-8} cm = 10^{-4} μ
(iv) 1 fermi = 1 fm = 10^{-15} m
(v) 1 astronomical unit = 1 AU = 1.496×10^{11} m
(vi) 1 light year = 1 ly = 9.467×10^{15} m
(vii) 1 Parsec = 3.08×10^{16} m = 3.26 ly = 206267 AU

Mass

(i) 1 quintal = 100 kg
(ii) 1 tonne or 1 metric ton = 1000 kg = 10 quintal
(iii) 1 megagram = 10^3 kg
(iv) 1 gigagram = 10^6 kg
(v) 1 teragram = 10^9 kg
(vi) 1 slug = 14.57 kg
(vii) 1 pound = 1 lb = 0.4536 kg

Time

(i) 1 millisecond = 10^{-3} s
(ii) 1 microsecond = 10^{-6} s
(iii) 1 shake = 10^{-8} s
(iv) 1 nanosecond = 10^{-9} s
(v) 1 picosecond = 10^{-12} s
(vii) 1 hour = 60 min = 3600 s
(viii) 1 day = 24 hours = 86400 s
(ix) 1 year = 365 days = 3.15×10^7 s
(x) 1 century = 100 years

Common SI Prefixes and Symbols for Multiples and Sub-multiples

Multiple			Sub-multiple		
Factor	Prefix	Symbol	Factor	Prefix	Symbol
10^{18}	Exa	E	10^{-18}	atto	a
10^{15}	Peta	P	10^{-15}	femto	f
10^{12}	Tera	T	10^{-12}	pico	p
10^{9}	Giga	G	10^{-9}	nano	n
10^{6}	Mega	M	10^{-6}	micro	μ
10^{3}	Kilo	k	10^{-3}	milli	m
10^{2}	Hecto	h	10^{-2}	centi	c
10^{1}	Deca	da	10^{-1}	deci	d

Example 1.3 *The wavelength of a light is of the order of 6400 Å. Express this in micron and metre.*

Sol. As $1\,\text{Å} = 10^{-10}$ m

$\therefore$ Wavelength of light $= 6400\,\text{Å} = 6400 \times 10^{-10}$ m

$= 6.4 \times 10^{-7}$ m

Also, 1 micron $= 10^{-6}$ metre

$\therefore$ Wavelength of light (in micron) $= \dfrac{6.4 \times 10^{-7}}{10^{-6}}$ micron

$= 0.64$ micron

Example 1.4 *How many microns are there in 1 light year?*

Sol. 1 ly $= 9.46 \times 10^{15}$ m

As, $1\text{m} = 10^6$ micron

$\therefore$ 1 ly $= 9.46 \times 10^{15} \times 10^6$ micron

$= 9.46 \times 10^{21}$ micron

$\approx 10^{22}$ micron (approx)

Example 1.5 *How many microseconds are there in 10 minutes?*

Sol. 1 second $= 10^6$ microseconds

10 minutes $= 10 \times 60$ seconds

$= 10 \times 60 \times 10^6 = 6 \times 10^8$ microseconds

Example 1.6 *Calculate the angle of*

(i) $1°$ *(degree)*

(ii) $1'$ *(minute of arc or arc minute) and*

(iii) $1''$ *(second of arc or arc sec)*

in radian. (Use $360° = 2\pi$ *rad,* $1° = 60'$ *and* $1' = 60''$*)*

Sol. (i) $1° = \dfrac{2\pi}{360}$ rad $= \dfrac{\pi}{180}$ rad $= \dfrac{22}{7 \times 180}$ rad $= 1.746 \times 10^{-2}$ rad

(ii) 1 arc min $= 1' = \dfrac{1°}{60} = \dfrac{1}{60} \times \dfrac{\pi}{180}$ rad $= 2.91 \times 10^{-4}$ rad

(iii) 1 arc sec $= 1'' = \dfrac{1'}{60} = \dfrac{1°}{60 \times 60} = \dfrac{1}{60 \times 60} \times \dfrac{\pi}{180}$ rad

$= 4.85 \times 10^{-6}$ rad

2 Dimensions of physical quantities

The dimensions of a physical quantity are the powers (or exponents) to which the fundamental quantities must be raised to represent that quantity completely.

e.g., Density $= \dfrac{\text{Mass}}{\text{Volume}} = \dfrac{\text{Mass}}{(\text{Length})^3}$

or $\quad$ Density = (Mass) (Length)$^{-3}$ $\quad$ ...(i)

Thus, the dimensions of density are 1 in mass and -3 in length. The dimensions of all other fundamental quantities are zero.

Dimensional representation of physical quantities

For convenience, the fundamental quantities are represented by one letter symbols. The dependence of all other physical quantities on these base quantities can be expressed in terms of their dimensions. Thus, the seven dimensions of physical world are represented as follows.

[M]	for mass
[L]	for length
[T]	for time
[A]	for electric current
[K] or [θ]	for thermodynamic temperature
[cd]	for luminous intensity
[mol]	for amount of substance

The physical quantity that is expressed in terms of the base quantities is enclosed in square brackets.

Thus, from Eq. (i) dimensions of density can be represented as $[\text{ML}^{-3}]$.

Dimensional formula and dimensional equation

The expression of a physical quantity in terms of its dimensions is called its dimensional formula. *e.g.*, Dimensional formula for density is $[\text{ML}^{-3}\text{T}^0]$, the dimensional formula of force is $[\text{MLT}^{-2}]$ and that for acceleration is $[\text{M}^0\text{LT}^{-2}]$.

An equation which contains a physical quantity on one side and its dimensional formula on the other side, is called the dimensional equation of that quantity.

Dimensional equations for a few physical quantities are given below.

Speed $\quad [v] = [\text{M}^0\text{LT}^{-1}]$

Area $\quad [A] = [\text{M}^0\text{L}^2\text{T}^0]$

Force $\quad [F] = [\text{MLT}^{-2}]$ etc.

The physical quantities having same derived units have same dimensions.

Dimensional formulae of some physical quantities

The table given below gives the dimensional formulae and SI units of some physical quantities frequently used in physics.

S. No.	Physical Quantity	SI Units	Dimensional Formula
1.	Velocity = displacement/time	m/s	$[M^0LT^{-1}]$
2.	Acceleration = velocity/time	m/s^2	$[M^0LT^{-2}]$
3.	Force = mass × acceleration	$kg\text{-}m/s^2$ = newton or N	$[MLT^{-2}]$
4.	Work = force × displacement	$kg\text{-}m^2/s^2$ = N-m = joule or J	$[ML^2T^{-2}]$
5.	Energy	J	$[ML^2T^{-2}]$
6.	Torque = force ×perpendicular distance	N-m	$[ML^2T^{-2}]$
7.	Power = work/time	J/s or watt	$[ML^2T^{-3}]$
8.	Momentum = mass × velocity	kg-m/s	$[MLT^{-1}]$
9.	Impulse = force × time	N-s	$[MLT^{-1}]$
10.	Angle = arc/radius	radian or rad	$[M^0L^0T^0]$
11.	Strain = $\frac{\Delta L}{L}$ or $\frac{\Delta V}{V}$	No units	$[M^0L^0T^0]$
12.	Stress = force/area	N/m^2	$[ML^{-1}T^{-2}]$
13.	Pressure = force/area	N/m^2	$[ML^{-1}T^{-2}]$
14.	Modulus of elasticity = stress/strain	N/m^2	$[ML^{-1}T^{-2}]$
15.	Frequency = 1/time period	per second or hertz (Hz)	$[M^0L^0T^{-1}]$
16.	Angular velocity = angle/time	rad/s	$[M^0L^0T^{-1}]$
17.	Moment of inertia = (mass) × $(\text{distance})^2$	$kg\text{-}m^2$	$[ML^2T^0]$
18.	Surface tension = force/length	N/m	$[ML^0T^{-2}]$
19.	Gravitational constant = $\frac{\text{force} \times (\text{distance})^2}{(\text{mass})^2}$	$N\text{-}m^2/kg^2$	$[M^{-1}L^3T^{-2}]$
20.	Angular momentum	$kg\text{-}m^2/s$	$[ML^2T^{-1}]$
21.	Coefficient of viscosity	$N\text{-}s/m^2$	$[ML^{-1}T^{-1}]$
22.	Planck's constant	J-s	$[ML^2T^{-1}]$
23.	Specific heat (s)	J/kg-K	$[M^0\ L^2T^{-2}\ \theta^{-1}]$
24.	Coefficient of thermal conductivity (K)	watt/m-K	$[M\ LT^{-3}\ \theta^{-1}]$
25.	Gas constant (R)	J/mol-K	$[M\ L^2T^{-2}\ \theta^{-1}mol^{-1}]$
26.	Boltzmann constant (k)	J/K	$[M\ L^2T^{-2}\ \theta^{-1}]$
27.	Wien's constant (b)	m-K	$[\ L\theta]$
28.	Stefan's constant (σ)	$watt/m^2\text{-}K^4$	$[M\ L^0T^{-3}\ \theta^{-4}]$
29.	Electric charge	C	[AT]
30.	Electric intensity	N/C	$[M\ LT^{-3}A^{-1}]$
31.	Electric potential	volt (V)	$[M\ L^2T^{-3}A^{-1}]$

S. No.	Physical Quantity	SI Units	Dimensional Formula
32.	Capacitance	farad (F)	$[M^{-1}L^{-2}T^4A^2]$
33.	Permittivity of free space	$C^2N^{-1}m^{-2}$	$[M^{-1}L^{-3}T^4A^2]$
34.	Electric dipole moment	C-m	[LTA]
35.	Resistance	Ohm	$[ML^2T^{-3}A^{-2}]$
36.	Magnetic field	tesla (T) or weber/m^2 (Wb/m^2)	$[MT^{-2}A^{-1}]$
37.	Coefficient of self-induction	henry (H)	$[ML^2T^{-2}A^{-2}]$
38.	Magnetic flux	Wb (weber)	$[ML^2T^{-2}A^{-1}]$
39.	Permeability of free space	Hm^{-1}	$[MLT^{-2}A^{-2}]$
40.	Magnetic moment	Am^2	$[M^0L^2T^0A]$

Quantities having same dimensions

S. No.	Quantities	Dimensions
1.	Strain, refractive index, relative density, angle, solid angle, phase, distance gradient, relative permeability, relative permittivity, angle of contact, Reynolds number, coefficient of friction, mechanical equivalent of heat, electric susceptibility etc.	$[M^0L^0T^0]$
2.	Mass and inertia	$[M^1L^0T^0]$
3.	Momentum and impulse	$[M^1L^1T^{-1}]$
4.	Thrust, force, weight, tension, energy gradient.	$[M^1L^1T^{-2}]$
5.	Pressure, stress, Young's modulus, bulk modulus, shear modulus, modulus of rigidity, energy density.	$[M^1L^{-1}T^{-2}]$
6.	Angular momentum and Planck's constant (h).	$[M^1L^2T^{-1}]$
7.	Acceleration, g and gravitational field intensity.	$[M^0L^1T^{-2}]$
8.	Surface tension, free surface energy (energy per unit area), force gradient, spring constant.	$[M^1L^0T^{-2}]$
9.	Latent heat and gravitational potential.	$[M^0L^2T^{-2}]$
10.	Thermal capacity, Boltzmann constant, entropy.	$[ML^2T^{-2}K^{-1}]$
11.	Work, torque, internal energy, potential energy, kinetic energy, moment of force, (q^2/C), (LI^2), (qV), (V^2C), (I^2Rt), $\left(\frac{V^2}{R}t\right)$, (VIt), (pV), (RT), (mL), $(mc\Delta T)$.	$[M^1L^2T^{-2}]$
12.	Frequency, angular frequency, angular velocity, velocity gradient, radioactivity. $\left(\frac{R}{L}\right)$, $\left(\frac{1}{RC}\right)$, $\left(\frac{1}{\sqrt{LC}}\right)$.	$[M^0L^0T^{-1}]$
13.	$\left(\frac{1}{g}\right)^{1/2}$, $\left(\frac{m}{k}\right)^{1/2}$, $\left(\frac{R}{g}\right)^{1/2}$, $\left(\frac{L}{R}\right)$, (RC), $(\sqrt{LC})$ time	$[M^0L^0T^1]$
14.	Power (VI), (I^2R), (V^2R).	$[ML^2T^{-3}]$

Example 1.7 *Find the dimensional formulae of*

(i) coefficient of viscosity, η *(ii) charge, q*

(iii) potential, V *(iv) capacitance, C and*

(v) resistance, R

Some of the equations containing these quantities are

$$F = -\eta A\left(\frac{\Delta v}{\Delta l}\right), \quad q = It, \quad U = VIt,$$

$$q = CV$$

and $V = IR$

where, A is the area, v is the velocity, l is the length, I is the electric current, t is the time and U is the energy.

Sol. (i) $\eta = -\frac{F}{A}\frac{\Delta l}{\Delta v}$

$\therefore$ $[\eta] = \frac{[F][l]}{[A][v]} = \frac{[MLT^{-2}][L]}{[L^2][LT^{-1}]} = [ML^{-1}T^{-1}]$

(ii) $q = It$

$\therefore$ $[q] = [I][t] = [AT]$

(iii) $U = VIt$

$\therefore$ $V = \frac{U}{It}$

or $[V] = \frac{[U]}{[I][t]} = \frac{[ML^2T^{-2}]}{[A][T]} = [ML^2T^{-3}A^{-1}]$

(iv) $q = CV$

$\therefore$ $C = \frac{q}{V}$

or $[C] = \frac{[q]}{[V]} = \frac{[AT]}{[ML^2T^{-3}A^{-1}]} = [M^{-1}L^{-2}T^4A^2]$

(v) $V = IR$

$\therefore$ $R = \frac{V}{I}$

or $[R] = \frac{[V]}{[I]} = \frac{[ML^2T^{-3}A^{-1}]}{[A]} = [ML^2T^{-3}A^{-2}]$

Example 1.8 *If force (F), velocity (v) and time (T) are taken as fundamental units, then find the dimension of mass.*

Sol. We know that,

$$F = ma \Rightarrow F = \frac{mv}{t} \Rightarrow m = \frac{Ft}{v}$$

$\therefore$ Dimensions of $m = \frac{[F][T]}{[v]} = [Fv^{-1}T]$

Example 1.9 *If C and R denote capacitance and resistance, then find the dimension of CR.*

Sol. The capacitance of a conductor is defined as the ratio of the charge given to the rise in the potential of the conductor

$$C = \frac{q}{V} = \frac{q^2}{W} \qquad \left(V = \frac{W}{q}\right)$$

$$C = \frac{\text{ampere}^2\text{-sec}^2}{\text{kg-metre}^2/\text{sec}^2}$$

Hence, dimensions of C are $[M^{-1}L^{-2}T^4A^2]$.

From Ohm's law, $V = iR$, therefore dimensions of resistance

$$R = \frac{V}{i} = \frac{\text{Volt}}{\text{Ampere}}$$

$$= \text{kg-metre}^2\text{sec}^{-3}\text{ampere}^{-2}$$

$\therefore$ Dimensions of $R = [ML^2T^{-3}A^{-2}]$

$\therefore$ Dimensions of $RC = [M^{-1}L^{-2}T^4A^2][ML^2T^{-3}A^{-2}]$

$= [M^0L^0T]$

Example 1.10 *For which of the following quantities does ratios are dimensionless?*

(i) $\frac{Work}{Energy}$ *(ii)* $\sin\theta$ *(iii)* $\frac{Momentum}{Time}$

Sol. (i) Since work and energy both have the same dimensions $[ML^2T^{-2}]$, their ratio is a dimensionless quantity.

(ii) $\sin\theta$, here θ represents an angle. An angle is the ratio of two lengths *i.e.*, arc length and radius. Therefore θ is dimensionless hence $\sin\theta$ is dimensionless.

(iii) $\left[\frac{\text{Momentum}}{\text{Time}}\right] = \left[\frac{MLT^{-1}}{T}\right] = [MLT^{-2}]$

Hence, the given ratio is not dimensionless.

Example 1.11 *In the formula $x = 3yz^2$, x and y have dimensions of capacitance and magnetic induction respectively, then find the dimensions of y.*

Sol. $x = 3yz^2$

$$y = \frac{x}{3z^2} = \frac{\text{Capacitance}}{(\text{Magnetic induction})^2}$$

$$y = \frac{[M^{-1}L^{-2}T^2\theta^2]}{[M^1T^{-1}\theta^{-1}]^2} = [M^{-3}L^{-2}T^4\theta^4]$$

Applications of dimensional analysis

The method of studying a physical phenomenon on the basis of dimensions is called dimensional analysis.

The three main uses of a dimensional analysis are described in detail in the following sections.

1. Checking the dimensional consistency of equation

Every physical equation should be dimensionally balanced. This is called the principle of homogeneity. This principle states that the dimensions of each term on both sides of an equation must be the same. On this basis we can judge whether a given equation is correct or not. But a dimensionally correct equation may or may not be physically correct.

e.g., in the physical expression $s = ut + \frac{1}{2}at^2$, the dimensions of s, ut and $\frac{1}{2}at^2$ all are same.

Note The physical quantities separated by the symbols +, −, =, >, < etc., have the same dimension

Example 1.12 *Show that the expression of the time period T of a simple pendulum of length l given by $T = 2\pi\sqrt{l/g}$ is dimensionally correct.*

Sol. Given, $$T = 2\pi\sqrt{\frac{l}{g}}$$

Dimensionally, $$[T] = \sqrt{\frac{[L]}{[LT^{-2}]}} = [T]$$

As in the above equation, the dimensions of both sides are same. Therefore, the given formula is dimensionally correct.

Example 1.13 *Check the correctness of the relation $s = ut + \frac{1}{2}at^2$ where u is initial velocity, a is the acceleration, t is the time and s is the displacement.*

Sol. Writing the dimensions of either side of the given equation.

LHS = s = displacement = $[M^0LT^0]$

RHS = ut = velocity × time = $[M^0LT^{-1}][T] = [M^0LT^0]$

and $$\frac{1}{2}at^2 = (\text{acceleration}) \times (\text{time})^2$$
$$= [M^0LT^{-2}][T]^2 = [M^0LT^0]$$

As LHS = RHS, formula is dimensionally correct.

Example 1.14 *Write the dimensions of a and b in the relation,*

$$P = \frac{b - x^2}{at}$$

where P is power, x the distance and t the time.

Sol. The given equation can be written as,

$$Pat = b - x^2$$

Now, $$[Pat] = [b] = [x^2]$$

or $$[b] = [x^2] = [M^0L^2T^0]$$

and $$[a] = \frac{[x^2]}{[Pt]} = \frac{[L^2]}{[ML^2T^{-3}][T]} = [M^{-1}L^0T^2]$$

Example 1.15 *The velocity v of a particle depends upon the time t according to the equation $v = a + bt + \frac{c}{d+t}$. Write the dimensions of a, b, c and d.*

Sol. *From principle of homogeneity*

$$[a] = [v] \text{ or } [a] = [LT^{-1}]$$
$$[bt] = [v]$$

or $$[b] = \frac{[v]}{[t]} = \frac{[LT^{-1}]}{[T]} \text{ or } [b] = [LT^{-2}]$$

Similarly, $$[d] = [t] = [T]$$

Further, $$\frac{[c]}{[d+t]} = [v] \text{ or } [c] = [v][d+t]$$

or $$[c] = [LT^{-1}][T]$$

or $$[c] = [L]$$

∴ Dimensions of $a = [LT^{-1}]$

Dimensions of $b = [LT^{-2}]$

Dimension of $c = [L]$

Dimension of $d = [T]$

Example 1.16 *The following equation gives a relation between the mass m_1, kept on a surface of area A and the pressure p exerted on this area.*

$$p = \frac{(m_1 + m_2)x}{A}$$

What must be the dimensions of the quantities x and m_2?

Sol. Since all the terms of a mathematical equation should have the same dimensions.

Therefore, $$[p] = \left[\frac{(m_1 + m_2)x}{A}\right] \quad \text{...(i)}$$

Only the quantities having same dimensions and nature can be added to each other.

Here, m_2 is added to mass m_1.

Hence $[m_2] = [m_1] = [M]$

Also the quantity obtained by the addition of m_1 and m_2 would have the same dimensions as that of mass.

∴ $$[m_1 + m_2] = [M]$$

Now going back to Eq. (i)

$$[p] = \frac{[m_1 + m_2][x]}{[A]}$$

$$\Rightarrow \quad [ML^{-1}T^{-2}] = \frac{[M][x]}{[L^2]} \Rightarrow [ML^{-1}T^{-2}] = [ML^{-2}][x]$$

$$\Rightarrow \quad \frac{[ML^{-1}T^{-2}]}{[ML^{-2}]} = [x] \Rightarrow [x] = [LT^{-2}]$$

Hence, the quantity x represents acceleration. In this example, it is the acceleration due to gravity, g. $(m_1 + m_2)g$ represents the weight exerted by two masses m_1, m_2 on the area A.

2. To convert a physical quantity from one system of units to other system of units

This is based on the fact that the product of the numerical value (n) and its corresponding unit (u) is a constant, *i.e.*,

$$n(u) = \text{constant} \quad \text{or} \quad n_1[u_1] = n_2[u_2]$$

Suppose the dimensions of a physical quantity are a in mass, b in length and c in time. If the fundamental units in

one system are M_1, L_1 and T_1 and in the other system are M_2, L_2 and T_2 respectively. Then, we can write

$$n_1[M_1^a L_1^b T_1^c] = n_2[M_2^a L_2^b T_2^c] \quad ...(i)$$

$$n_2 = n_1 \frac{u_1}{u_2} = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

Here, n_1 and n_2 are the numerical values in two system of units respectively. Using Eq. (i), we can convert the numerical value of a physical quantity from one system of units into the other system.

Example 1.17 *Find the value of 100 J on a system which has 20 cm, 250 g and half minute as fundamental units of length, mass and time.*

Sol. The dimensional formula of work is $=[ML^2T^{-2}]$

The convert a physical quantity from one system of units to other system of units, we use the following formula

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

$$\therefore \quad n_2 = 100\left[\frac{1\text{ kg}}{250\text{ g}}\right]^1\left[\frac{1\text{ m}}{20\text{ cm}}\right]^2\left[\frac{1\text{ s}}{0.5\text{ min}}\right]^{-2}$$

$$= 100\left[\frac{1000\text{ g}}{250\text{ g}}\right]^1\left[\frac{100\text{ cm}}{20\text{ cm}}\right]^2\left[\frac{1\text{ s}}{30\text{ s}}\right]^{-2}$$

$$= 100 \times 4 \times 25 \times 30 \times 30$$

$$= 9 \times 10^6 \text{ new units}$$

Example 1.18 *The value of gravitational constant is $G = 6.67 \times 10^{-11}$ N-m^2/kg^2 in SI units. Convert it into CGS system of units.*

Sol. The dimensional formula of G is $[M^{-1}L^3T^{-2}]$.

The convert a physical quantity from one system of units to other system of units, we use the following formula

$$n_1[M_1^{-1}L_1^3T_1^{-2}] = n_2[M_2^{-1}L_2^3T_2^{-2}]$$

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^{-1}\left[\frac{L_1}{L_2}\right]^3\left[\frac{T_1}{T_2}\right]^{-2}$$

$$n_2 = 6.67 \times 10^{-11}\left[\frac{1\text{ kg}}{10^{-3}\text{ kg}}\right]^{-1}\left[\frac{1\text{ m}}{10^{-2}\text{ m}}\right]^3\left[\frac{1\text{ s}}{1\text{ s}}\right]^{-2}$$

or $\quad n_2 = 6.67 \times 10^{-8}$

Thus, value of G in CGS system of units is 6.67×10^{-8} dyne cm^2/g^2.

3. Deducing relation between the physical quantities

If we know the factors on which a given physical quantity depends, we can find a formula relating to those factors.

Example 1.19 *The frequency (f) of a stretched string depends upon the tension F (dimensions of force), length l of the string and the mass per unit length, μ of string. Derive the formula for frequency.*

Sol. Suppose, the frequency f depends on the tension raised to the power a, length raised to the power b and mass per unit length raised to the power c.

Then, $\quad f \propto (F)^a (l)^b (\mu)^c$

or $\quad f = k (F)^a (l)^b (\mu)^c \quad ...(i)$

Here, k is a dimensionless constant.

Thus, $\quad [f] = [F]^a [l]^b [\mu]^c$

or $\quad [M^0L^0T^{-1}] = [MLT^{-2}]^a [L]^b [ML^{-1}]^c$

or $\quad [M^0L^0T^{-1}] = [M^{a+c}L^{a+b-c}T^{-2a}]$

For dimensional balance, the dimensions on both sides should be same.

Thus, $\quad a + c = 0 \quad ...(ii)$

$a + b - c = 0 \quad ...(iii)$

and $\quad -2a = -1 \quad ...(iv)$

Solving these three equations, we get

$$a = \frac{1}{2}, \quad c = -\frac{1}{2} \quad \text{and} \quad b = -1$$

Substituting these values in Eq. (i), we get

$$f = k(F)^{1/2}(l)^{-1}(\mu)^{-1/2} \quad \text{or} \quad f = \frac{k}{l}\sqrt{\frac{F}{\mu}}$$

Experimentally, the value of k is found to be $\frac{1}{2}$.

Hence, $\quad f = \frac{1}{2l}\sqrt{\frac{F}{\mu}}$

Example 1.20 *The centripetal force F acting on a particle moving uniformly in a circle may depend upon mass (m), velocity (v) and radius (r) of the circle. Derive the formula for F using the method of dimensions.*

Sol. Let $\quad F = k (m)^x (v)^y (r)^z \quad ...(i)$

Here, k is a dimensionless constant of proportinality.

Writing the dimensions of RHS and LHS in Eq. (i), we have

$$[MLT^{-2}] = [M]^x [LT^{-1}]^y [L]^z$$

$$= [M^xL^{y+z}T^{-y}]$$

Equating the powers of M, L and T of both sides, we have

$$x = 1, \; y = 2 \text{ and } y + z = 1$$

or $\quad z = 1 - y = -1$

Putting the values in Eq. (i), we get

$$F = kmv^2r^{-1} = k\frac{mv^2}{r}$$

$$F = \frac{mv^2}{r} \quad \text{(where } k = 1\text{)}$$

Defects or limitations of dimensional analysis

The method of dimensions has the following limitations.

(i) The value of dimensionless constant involved in a formula cannot be deduced from this method.

(ii) By this method the equation containing trigonometrical, expanential and logarithmic terms cannot be analysed.

(iii) This method does not work when physical quantity depends on more than three variables because we only have three equations by equalising the power of *M*, *L* and *T*.

(iv) If dimensions are given, physical quantity may not be unique. *e.g.*, work, energy and torque all have the same dimensional formula $[ML^2T^{-2}]$.

(v) It gives no information whether a physical quantity is a scalar or a vector.

Check point 1.1

1. In the SI system, the unit of temperature is
(a) degree centigrade (b) kelvin
(c) degree celsius (d) degree Fahrenheit

2. Dimensions of surface tension are
(a) $[M^2L^2T^{-2}]$ (b) $[M^2LT^{-2}]$ (c) $[MT^{-2}]$ (d) $[MLT^{-2}]$

3. The dimensions of impulse are equal to that of
(a) force (b) linear momentum
(c) pressure (d) angular momentum

4. Which of the following is not equal to watt?
(a) joule/second (b) ampere × volt
(c) $(\text{ampere})^2 \times \text{ohm}$ (d) ampere/volt

5. Which of the following does not possess the same dimensions as that of pressure?
(a) Stress (b) Bulk modulus
(c) Thrust (d) Energy density

6. What is the dimensional formula of gravitational constant?
(a) $[ML^2T^{-2}]$ (b) $[ML^{-1}T^{-1}]$
(c) $[M^{-1}L^3T^{-2}]$ (d) None of these

7. If *C* and *R* denote capacitor and resistance, the dimensions of *CR* are
(a) $[M^0L^0T]$
(b) $[ML^0T]$
(c) $[M^0L^0T^2]$
(d) not expressible in terms of M, L and T

8. Which one of the following have same dimensions?
(a) Torque and force
(b) Potential energy and force
(c) Torque and potential energy
(d) Planck's constant and linear momentum

9. Which of the following is a dimensional constant?
(a) Poission's ratio (b) Refractive index
(c) Relative density (d) Gravitational constant

10. The force *F* on a sphere of radius *a* moving in a medium with velocity *v* is given by $F = 6\pi\,\eta a\,v$. The dimensions of η are
(a) $[ML^{-3}]$ (b) $[MLT^{-2}]$ (c) $[MT^{-1}]$ (d) $[ML^{-1}T^{-1}]$

11. Density of liquid in CGS system is 0.625 g cm^{-3}. What is its magnitude in SI system?
(a) 0.625 (b) 0.0625
(c) 0.00625 (d) 625

12. Joule × second is the unit of
(a) energy (b) momentum
(c) angular momentum (d) power

13. $[ML^2T^{-3}A^{-1}]$ is the dimensional formula for
(a) capacitance (b) resistance
(c) resistivity (d) potential difference

14. The dimensional representation of specific resistance in terms of charge *Q* is
(a) $[ML^3T^{-1}Q^{-2}]$ (b) $[ML^2T^{-2}Q^2]$
(c) $[MLT^{-2}Q^{-1}]$ (d) $[ML^2T^{-2}Q^{-1}]$

15. The dimensional formula for Planck's constant and angular momentum is
(a) $[ML^2T^{-2}]$ and $[MLT^{-1}]$ (b) $[ML^2T^{-1}]$ and $[ML^2T^{-1}]$
(c) $[ML^3T^{-1}]$ and $[ML^2T^{-2}]$ (d) $[MLT^{-1}]$ and $[MLT^{-2}]$

16. Which of the following pairs has the same units?
(a) Wavelength and Rydberg constant
(b) Relative velocity and relative density
(c) Thermal capacity and Boltzmann constant
(d) Time period and acceleration gradient

17. Which one of the following is not the dimensionless quantity?
(a) Planck's constant (b) Dielectric constant
(c) Solid angle (d) Strain

18. Given that $y = a\cos\left(\frac{t}{p} - qx\right)$, where *t* represents time in following statements is true?
(a) The unit of x is same as that of q
(b) The unit of x is same as that of p
(c) The unit of t is same as that of q
(d) The unit of t is same as that of p

19. The dimensions of $\frac{a}{b}$ in the equation $p = \frac{a - t^2}{bx}$, where p is pressure, x is distance and t is time, are
(a) $[M^2LT^{-3}]$ (b) $[MT^{-2}]$ (c) $[LT^{-3}]$ (d) $[ML^3T^{-1}]$

20. The equation of a wave is given by $y = a\sin\omega\left(\frac{x}{v} - k\right)$ where ω is angular velocity and v is the linear velocity. The dimension of k will be
(a) $[T^{-2}]$ (b) $[T^{-1}]$ (c) $[T]$ (d) $[LT]$

21. If 'muscle times speed equals power', then what is the ratio of the SI unit and the CGS unit of muscle?
(a) 10^5 (b) 10^3 (c) 10^7 (d) 10^{-5}

22. The dimensions of $\frac{1}{2}\varepsilon_0 E^2$ (ε_0 is the permittivity of the space and E is electric field), is
(a) $[ML^2T^{-1}]$ (b) $[ML^{-1}T^{-2}]$ (c) $[ML^2T^{-2}]$ (d) $[MLT^{-1}]$

23. If p represents radiation pressure, c represent speed of light and Q represents radiation energy striking a unit area per second, then non-zero integers x, y and z such that $p^xQ^yc^z$ is dimensionless are
(a) $x = 1, y = 1, z = -1$ (b) $x = 1, y = -1, z = 1$
(c) $x = -1, y = 1, z = 1$ (d) $x = 1, y = 1, z = 1$

24. The units of length, velocity and force are doubled. Which of the following is the correct change in the other units?
(a) Unit of time is doubled
(b) Unit of mass is doubled
(c) Unit of momentum is doubled
(d) Unit of energy is doubled

25. Assuming that the mass m of the largest stone that can be moved by a flowing river depends upon the velocity v of the water, its density ρ and the acceleration due to gravity g. Then, m is directly proportional to
(a) v^3 (b) v^4 (c) v^5 (d) v^6

3 Significant figure

The significant figures are normally those digits in a measured quantity which are known reliably or about which we have confidence in our measurement plus one additional digit that is uncertain.

e.g., If length of some object is 185.2 cm, then it has four significant figures. The digits 1,8 and 5 are reliable and digit 2 is uncertain.

Note Significant figures indicate the precision of the measurement which depends on the least count of the measuring instrument.

Rules to determine significant figure

For determining number of significant figures, we use the following rules :

Rule 1 All non-zero digits are significant. *e.g.*, $x = 2567$ has four significant figures.

Rule 2 The zeros appearing between two non-zero digits are significant, no matter where the decimal point is, if any. *e.g.*, 6.028 has 4 significant figures.

Rule 3 If the number is less than 1, the zero(s) on the right of decimal point but to the left of first non-zero digit are not significant.

e.g., 0.0042 has two significant digits.

Rule 4 The terminal or trailing zero(s) in a number without a decimal point are not significant. Thus, 426 m = 42600 cm = 426000 mm has three significant figures.

Rule 5 In a number with decimal, zeros to the right of last non-zero digit are significant.

e.g., 4.600 or 0.002300 have four significant figures each.

Point of confusion and its remedy

Suppose we change the units,then we will write

$$2.30 \text{ m} = 23.0 \text{ cm} = 2300 \text{ mm} = 0.00230 \text{ km}$$

When we are writing 2300 mm, then from Rule-4, we would conclude erroneously that the number has two significant figures, while in fact it has three significant figures and a mere change of units cannot change the number of significant figures.

To remove such ambiguities in determining the number of significant figures, apply following rule.

Rule 6 The power of 10 is irrelevant to the determination of significant figures. *e.g.*, in the measurements

$$2.30 \text{ m} = 2.30 \times 10^2 \text{ cm}$$
$$= 2.30 \times 10^3 \text{ mm}$$
$$= 2.30 \times 10^{-3} \text{ km}$$

The significant figures are three in each measurement, because all zeros appearing in the base number in the scientific notation (in the power of 10) are not significant.

Rule 7 A choice of change of different units does not change the number of significant digits or figures in a measurement.

e.g., the length 7.03 cm has three significant figures. But in different units, the same value can be written as, 0.0703 m or 70.3 mm. All these measurements have the same number of significant figures (digits 7, 0 and 3) namely three.

Rule 8 The exact numbers appearing in the mathematical formulae of various physical quantities have infinite number of significant figures. *e.g.*, perimeter of a square is given by $4 \times$ side. Here, 4 is an exact number and has infinite number of significant figures.

$\therefore$ It can be written as 4.0, 4.00, 4.0000 as per the requirement.

Some significant figures of measured values given in the table below.

Measured value	Number of significant figures	Rule
12376	5	1
6024.7	5	2
0.071	2	3
410 m	2	4
2.40	3	5
1.6×10^{10}	2	6
In 2 $(l + b)$, digit 2	Infinite	8

Example 1.21 *How many significant figures are there in the measured values.*

(*i*) 227.2 *g*, (*ii*) 3600 *g*

(*iii*) 0.00602 *g* (*iv*) 2.50×10^{10} *g*

Sol. (i) 227.2 g has all the non-zero digits. Hence, it has four significant figures.

(ii) According to rule number 4, trailing zeroes are not significant. Hence, 3600 g has 2 significant figures.

(iii) According to the rule number 3, the zeroes at the beginning are not significant. Hence, 0.00602 g has 3 significant figures.

(iv) According to the rule number 6, it has 3 significant figures.

Mathematical operations of significant figures

The result of a mathematical operation involving measured values of quantities cannot be more accurate than the measured value themselves. So, certain rules have to be followed while doing mathematical operations with significant figures so that precision in final result is consistent with the precision of the original measured values.

Addition and subtraction

Suppose in the measured values to be added or subtracted, the least number of significant digits after the decimal is n. Then, in the sum or difference also, the number of significant digits after the decimal should be n.

e.g., $1.2 + 3.45 + 6.789 = 11.439 \approx 11.4$

Here, the least number of significant digits after the decimal is one. Hence, the result will be 11.4 (when rounded off to smallest number of decimal places).

e.g., $12.63 - 10.2 = 2.43 \approx 2.4$

Example 1.22 *Add* 6.75×10^3 *cm to* 4.52×10^2 *cm with regard to significant figures.*

Sol. Let $a = 6.75 \times 10^3$ cm

$b = 4.52 \times 10^2$ cm $= 0.452 \times 10^3$ cm

$= 0.45 \times 10^3$ cm (upto 2 places of decimal)

$\therefore$ Addition of significant figures

$a + b = (6.75 \times 10^3 + 0.45 \times 10^3)$ cm $= 7.20 \times 10^3$ cm

Example 1.23 *Two sticks of lengths* 12.132 *cm and* 10.2 *cm are placed end to end. Find their total length with due regard to decimal places.*

Sol. Length of first stick = 12.132 cm (5 significant figures)

Length of second stick = 10.2 cm (3 significant figures)

$\therefore$ Total length of two sticks $= 12.132 + 10.2 = 22.332$

The answer should be rounded off with least number of significant digits after the decimal.

$\therefore$ Total length of two sticks will be 22.3 cm

Multiplication or division

Suppose in the measured values to be multiplied or divided, the least number of significant digits be n, then in the product or quotient, the number of significant digits should also be n.

e.g., $1.2 \times 36.72 = 44.064 \approx 44$

The least number of significant digits in the measured values are two. Hence, the result when rounded off to two significant digits become 44. Therefore, the answer is 44.

e.g., $\dfrac{1100}{10.2} = 107.8431373 \approx 110$

As 1100 has minimum number of significant figures (*i.e.*, 2), therefore the result should also contain only two significant digits. Hence, the result when rounded off to two significant digits becomes 110.

e.g., $\dfrac{1100 \text{ m/s}}{10.2 \text{ m/s}} = 107.8431373 \approx 108$

Note In this case answer becomes 108. Think why?

Example 1.24 *A thin wire has a length of* 21.7 *cm and radius* 0.46 *cm. Calculate the volume of the wire to correct significant figures.*

Sol. Given, $l = 21.7$ cm, $r = 0.46$ mm $= 0.046$ cm

Volume of wire, $V = \pi r^2 l = \dfrac{22}{7}(0.046)^2 (21.7)$

$= 0.1443 \text{ cm}^3 = 0.14 \text{ cm}^3$

Example 1.25 *The time taken by a pendulum to complete 25 vibrations is 88.0 s. Find the time period of the pendulum in seconds upto appropriate significant figures.*

Sol. Time period of oscillation $= \dfrac{\text{Total time taken}}{\text{Number of oscillations}}$

$$= \frac{88.0}{25}\text{ s} = 3.52\text{ s}$$

Out of the two quantities given in the data, 25 is exact hence has infinite significant figures. Therefore, the answer should be reported to three significant figures *i.e.*, 3.52 s.

Example 1.26 *The voltage across a lamp is 6.32 V when the current passing through it is 3.4 A. Find the power consumed to appropriate significant figures.*

Sol. Voltage across a lamp = 6.32 V (3 significant figure)

Current flowing through lamp = 3.4A (2 significant figure)

$\therefore$ Power consumed, $P = VI = (6.32)(3.4) = 21.488$ W

Answer should have minimum number of significant figure. Here, the minimum number of significant figure is 2.

$\therefore$ Power consumed = 21 W

Example 1.27 *5.74 g of substance occupies 1.2 cm^3. Express its density by keeping the significant figures in view.*

Sol. Here, mass $(m) = 5.74$ g; volume $(V) = 1.2$ cm^3

As density, $\rho = \dfrac{\text{mass}}{\text{volume}} = \dfrac{5.74\text{ g}}{1.2\text{ cm}^3} = 4.783\text{ g cm}^3$

As mass has 3 significant digits and volume has 2 significant digits, therefore, as per rule, density will have only two significant digits, rounding off, we get $\rho = 4.8$ cm^{-3}.

Rounding off

The process of omitting the non significant digits and retaining only the desired number of significant digits, incorporating the required modifications to the last significant digit is called rounding off the number. In physics, calculation is a vital part and during that we shall reduce the number to the required extent and that is why there is a need to round off numbers.

Like mathematical operations of significant figures, rounding off numbers also follow certain rules.

Rules for rounding off a measurement

Following are the rules for rounding off a measurement.

Rule 1 If the number lying to the right of cut-off digit is less than 5, then the cut-off digit is retained as such. However, if it is more than 5, then the cut-off digit is increased by 1.

e.g., $x = 6.24$ is rounded off to 6.2 to two significant digits and $x = 5.328$ is rounded off to 5.33 to three significant digits.

Rule 2 *If the insignificant digit to be dropped is 5 then the rule is*

(i) if the preceding digit is even, the insignificant digit is simply dropped.

(ii) if the preceding digit is odd, the preceding digit is raised by 1.

e.g., $x = 6.265$ is rounded off to $x = 6.26$ to three significant digits and $x = 6.275$ is rounded off to $x = 6.28$ to three significant digits.

Rule 3 The exact numbers like π, 2, 3 and 4 etc., that appear in formulae and are known to have infinite significant figures, can be rounded off to a limited number of significant figures as per the requirement.

Example 1.28 Round off the following numbers upto three significant figures.

(*i*) 2.520 (*ii*) 4.645 (*iii*) 22.78 (*iv*) 36.35

Sol. (i) 2.520 : Since 0 is less than 5, preceding digit is left unchanged. Hence, 2.52.

(ii) 4.645 : Since the digit to be dropped is 5 and the preceding digit 4 is even. Hence, 4.64.

(iii) 22.78 : Since the digit to be dropped is 8 and is greater than 5, therefore the preceding digit 7, is raised by 1. Hence, 22.8.

(iv) 36.35 : Since the digit to be dropped is 5 and the preceding digit 3 is odd, we can write the answer as 36.4.

Example 1.29 *The length and the radius of a cylinder measured with slide callipers are found to be 4.54 cm and 1.75 cm, respectively. Calculate the volume of the cylinder.*

Sol. Length of cylinder, $h = 4.54$ cm (3 significant figures)

Radius of cylinder, $r = 1.75$ cm (3 significant figures)

$\therefore$ Volume of cylinder $= \pi r^2 h = 3.14 \times (1.75)^2 \times 4.54$ cm^3

$= 43.657775$ cm$^3 = 43.6$ cm^3

(Rounded off upto 3 significant figures)

Order of magnitude

Any physical quantity can be expressed in the form of $a \times 10^b$ (in terms of magnitude), where a is a number lying between 1 and 10 and b is any negative or positive exponent of 10, then the exponent b is called the **order of magnitude** of the physical quantity. And the expression of a quantity as $a \times 10^b$ is called **scientific notation.**

e.g., the speed of light is given as 3.00×10^8 m/s. So, the order of magnitude of the speed of light is 8.

The order of magnitude, gives an estimate of the magnitude of the quantity. The charge on an electron is 16×10^{-19} C. Therefore, we can say that the charge possessed by an electron is of the order 10^{-19} or its order of magnitude is -19.

Example 1.30 *What is the order of magnitude of the distance of the sun from the earth in SI unit?*

Sol. Distance of sun from the earth = 1 AU $= 1.496 \times 10^{11}$ m

Order of magnitude = 11

Check point 1.2

1. What is the number of significant figures in 0.0310×10^3?
   (a) 2 (b) 3
   (c) 4 (d) 6
2. The number of significant figures in 11.118×10^{-6} V is
   (a) 3 (b) 4
   (c) 5 (d) 6
3. In which of the following numerical values, all zeros are significant?
   (a) 0.2020 (b) 20.2
   (c) 2020 (d) None of these
4. What is the number of significant figure in $(3.20 + 4.80) \times 10^5$?
   (a) 5 (b) 4
   (c) 3 (d) 2
5. Subtract 0.2 J from 7.26 J and express the result with correct number of significant figures
   (a) 7.1 (b) 7.06
   (c) 7 (d) None of these
6. The length, breadth and thickness of rectangular sheet of metal are 4.234 m, 1.005 m and 2.01 cm, respectively. The volume of the sheet to correct significant figures is
   (a) $0.0855\ m^3$ (b) $0.086\ m^3$
   (c) $0.08556\ m^3$ (d) $0.08\ m^3$
7. Multiply 107.88 by 0.610 and express the result with correct number of significant figures.
   (a) 65.8068 (b) 64.807 (c) 65.81 (d) 65.8
8. The radius of a thin wire is 0.16 mm. The area of cross-section of the wire in mm^2 with correct number of significant figures is
   (a) 0.08 (b) 0.080 (c) 0.0804 (d) 0.080384
9. What is the value of $[(5.0 \times 10^{-6})\ (5.0 \times 10^{-8})]$ with due regards to significant digits?
   (a) 25×10^{-14} (b) 25.0×10^{-14}
   (c) 2.50×10^{-13} (d) 250×10^{-15}
10. When 97.52 is divided by 2.54, the correct result is
    (a) 38.3937 (b) 38.394 (c) 65.81 (d) 38.4

4 Error in measurement

We use different kinds of instruments for measuring various quantities. However, these measurements always has a degree of uncertainty related to it. This uncertainty is called as error in the measurement. Thus, the difference between the measured value and the true value of a quantity is known as the error of measurement.

$\therefore$ $\boxed{\text{Error} = \text{True value} - \text{Measured value}}$

Errors may arise from different sources and are usually classified as follows

1. Systematic errors

These are the errors whose causes are known to us. They can be either positive or negative.

Some of the sources of systematic errors are as follows

(i) Instrumental errors

These errors are due to imperfect design or erroneous manufacture or misuse of the measuring instrument.

These are of following types

(a) **Zero error** If the zero mark of vernier scale does not coincide with the zero mark of the main scale, the instrument is said to have zero error. A metre scale having worn off zero mark also has zero error.

(b) **Least count or permissible error** This error is due to the limitation imposed by the least count of the measuring instrument. It is an uncertainty associated with the resolution of the measuring instrument.

(c) **Constant error** The errors which affect each observation by the same amount are called constant errors. Such errors are due to faulty calibration of the scale of the measuring instrument.

(d) **Backlash error** Backlash error occurs in screw gauge, when we try to rotate the screw very fast to measure a reading. Due to this, there is some slipping between the different screws instead of the rotation, which gives an incorrect reading to avoid this we should rotate the screw slowly in only one direction.

(ii) Causes of systematic errors

Following are few causes of these errors

(a) Instrumental errors may be due to erroneous instruments. These errors can be reduced by using more accurate instruments and applying zero correction, when required.

(b) Sometimes errors arise on account of ignoring certain facts. *e.g.*, in measuring time period of simple pendulum error may creap because no consideration is taken of air resistance. These errors can be reduced by applying proper corrections to the formula used.

(c) Change in temperature, pressure, humidity etc., may also sometimes cause errors in the result. Relevant corrections can be made to minimise their effects.

2. Random error

The errors which occur irregularly and at random, in magnitude and direction are called random errors. The causes of random errors are not known. Hence, it is not possible to remove them completely. These errors may arise due to a variety of reasons.

e.g., the reading of a sensitive beam balance may change by the vibrations caused in the building, due to persons moving in the laboratory or vehicles running nearby. The random error can be minimised by repeating the observation a large number of times and taking the arithmetic mean of all the observations. The mean value would be very close to the most accurate reading. Thus,

$$a_{\text{mean}} = \frac{a_1 + a_2 + \ldots + a_n}{n} = \frac{1}{n}\sum_{i=1}^{n} a_i$$

Expression of errors

Errors can be expressed in following way

1. Absolute error

The difference between the true value and the measured value of a quantity is called an absolute error. Usually the mean value a_m is taken as the true value. So, if

$$\boxed{a_m = \frac{a_1 + a_2 + \ldots + a_n}{n}}$$

Then by definition, absolute errors in the measured values of the quantity are,

$$\Delta a_1 = a_m - a_1$$
$$\Delta a_2 = a_m - a_2$$
$$\ldots \quad \ldots \quad \ldots$$
$$\Delta a_n = a_m - a_n$$

Absolute error may be positive or negative.

Mean absolute error

It is the arithmetic mean of the magnitudes of absolute errors. Thus,

$$\Delta a_{\text{mean}} = \frac{|\Delta a_1| + |\Delta a_2| + \ldots + |\Delta a_n|}{n} = \frac{1}{n}\sum_{i=1}^{n} |\Delta a_i|$$

This final result of measurement can be written as

$$\boxed{a = a_m \pm \Delta a_{\text{mean}}}$$

This implies that value of a is likely to lie between $a_m + \Delta a_{\text{mean}}$ and $a_m - \Delta a_{\text{mean}}$.

2. Relative or fractional error

The ratio of mean absolute error to the mean value of the quantity measured is called relative or fractional error.

Thus, $$\boxed{\text{Relative error} = \frac{\Delta a_{\text{mean}}}{a_m}}$$

3. Percentage error

When the relative error is expressed in per cent, it is called percentage error. It is denoted by δa.

Thus, $$\boxed{\delta a = \frac{\Delta a_{\text{mean}}}{a_m} \times 100\%}$$

Example 1.31 *The length of a rod as measured in an experiment is found to be 2.48 m, 2.46 m, 2.49 m, 2.49 m and 2.46 m. Find the average length, the absolute error in each observation and the percentage error.*

Sol. Average length = Arithmetic mean of the measured values

$$x_{\text{mean}} = \frac{2.48 + 2.46 + 2.49 + 2.49 + 2.46}{5} = \frac{12.38}{5} = 2.476 \text{ m}$$

$\therefore$ True value, $x_{\text{mean}} = 2.48$ m

Absolute errors in various measurements,

$$|\Delta x_1| = |x_1 - x_{\text{mean}}| = 2.48 - 2.48 = 0.00 \text{ m}$$
$$|\Delta x_2| = |2.46 - 2.48| = 0.02 \text{ m}$$
$$|\Delta x_3| = |2.49 - 2.48| = 0.01 \text{ m}$$
$$|\Delta x_4| = |2.49 - 2.48| = 0.01 \text{ m}$$
$$|\Delta x_5| = |2.46 - 2.48| = +0.02 \text{ m}$$

$$\text{Mean absolute error} = \frac{|\Delta x_1| + |\Delta x_2| + |\Delta x_3| + \ldots + |\Delta x_5|}{5}$$
$$= \frac{(0.00 + 0.02 + 0.01 + 0.01 + 0.02)}{5} = \frac{0.06}{5}$$

$$\Delta x_{\text{mean}} = 0.01 \text{ m}$$

Thus, $x = 2.48 \pm 0.01$ m

$$\text{Percentage error, } \delta x = \frac{\Delta x_{\text{mean}}}{x} \times 100$$
$$= \frac{0.01}{2.48} \times 100 = 0.40\%$$

Example 1.32 *The diameter of a wire as measured by a screw gauge was found to be 2.620, 2.625, 2.630, 2.628 and 2.626 cm. Calculate*

(i) mean value of diameter,
(ii) absolute error in each measurement,
(iii) mean absolute error,
(iv) fractional error,
(v) percentage error and
(vi) express the result in terms of percentage error.

Sol. (i) Mean value of diameter,

$$a_m = \frac{2.620 + 2.625 + 2.630 + 2.628 + 2.626}{5}$$

$$= 2.6258 \text{ cm} = 2.626 \text{ cm}$$

(rounding off to three decimal places)

(ii) Taking a_m as the true value, the absolute errors in different observations are

$$\Delta a_1 = 2.626 - 2.620 = +0.006 \text{ cm}$$
$$\Delta a_2 = 2.626 - 2.625 = +0.001 \text{ cm}$$
$$\Delta a_3 = 2.626 - 2.630 = -0.004 \text{ cm}.$$
$$\Delta a_4 = 2.626 - 2.628 = -0.002 \text{ cm}$$
$$\Delta a_5 = 2.626 - 2.626 = 0.000 \text{ cm}$$

(iii) Mean absolute error,

$$\Delta a_{\text{mean}} = \frac{|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + |\Delta a_4| + |\Delta a_5|}{5}$$

$$= \frac{0.006 + 0.001 + 0.004 + 0.002 + 0.000}{5}$$

$$= 0.0026 = 0.003$$

(rounding off to three decimal places)

(iv) Fractional error $= \pm \frac{\Delta a_{\text{mean}}}{a_m} = \pm \frac{0.003}{2.626} = \pm 0.001$

(v) Percentage error $= \pm 0.001 \times 100 = \pm 0.1\%$

(vi) Diameter of wire can be written as,

$$d = 2.626 \text{ cm} \pm 0.1\%$$

Example 1.33 *The refractive index (n) of glass is found to have the values 1.49, 1.50, 1.52, 1.54 and 1.48. Calculate*

(i) the mean value of refractive index,
(ii) absolute error in each measurement,
(iii) mean absolute error,
(iv) fractional error, and
(v) percentage error.

Sol. (i) Mean value of refractive index,

$$n_m = \frac{1.49 + 1.50 + 1.52 + 1.54 + 1.48}{5}$$

$$= 1.505 = 1.51$$

(rounded off to two decimal places)

(ii) Taking n_m as the true value, the asbolute errors in different observations are,

$$\Delta n_1 = 1.51 - 1.49 = +0.02$$
$$\Delta n_2 = 1.51 - 1.50 = +0.01$$
$$\Delta n_3 = 1.51 - 1.52 = -0.01$$
$$\Delta n_4 = 1.51 - 1.54 = -0.03$$
$$\Delta n_5 = 1.51 - 1.48 = +0.03$$

(iii) Mean absolute error,

$$\Delta n_{\text{mean}} = \frac{|\Delta n_1| + |\Delta n_2| + |\Delta n_3| + |\Delta n_4| + |\Delta n_5|}{5}$$

$$= \frac{0.02 + 0.01 + 0.01 + 0.03 + 0.03}{5} = 0.02$$

(iv) Fractional error $= \frac{\pm \Delta n_{\text{mean}}}{n_m} = \frac{\pm 0.02}{1.51} = \pm 0.0132$

(v) Percentage error $= (\pm 0.0132 \times 100) = \pm 1.32\%$

Combination of errors

Most of our experiments require us to use the measured values, of a number of different physical quantities. We then put them in appropriate formula, to calculate the required quantity. We must therefore know how the errors in all the measurements combine and appear in the final quantity.

1. Errors in sum or difference

Let $x = a \pm b$.

Further, let Δa is the absolute error in the measurement of a, Δb is the absolute error in the measurement of b and Δx is the absolute error in the measurement of x.

Then, $$x + \Delta x = (a \pm \Delta a) \pm (b \pm \Delta b)$$
$$= (a \pm b) \pm (\pm \Delta a \pm \Delta b)$$
$$= x \pm (\pm \Delta a \pm \Delta b)$$

or $$\Delta x = \pm \Delta a \pm \Delta b$$

The four possible values of Δx are $(\Delta a - \Delta b)$, $(\Delta a + \Delta b)$, $(-\Delta a - \Delta b)$ and $(-\Delta a + \Delta b)$. Therefore, the maximum absolute error in x is

$$\boxed{\Delta x = \pm(\Delta a + \Delta b)}$$

i.e., the maximum absolute error in sum and difference of two quantities is equal to sum of the absolute errors in the individual quantities.

Example 1.34 *The volumes of two bodies are measured to be $V_1 = (10.2 \pm 0.02)\ cm^3$ and $V_2 = (6.4 \pm 0.01)\ cm^3$. Calculate sum and difference in volumes with error limits.*

Sol. Given, $V_1 = (10.2 \pm 0.02)\ \text{cm}^3$ and $V_2 = (6.4 \pm 0.01)\ \text{cm}^3$

$$\Delta V = \pm(\Delta V_1 + \Delta V_2)$$
$$= \pm\ (0.02 + 0.01)\ \text{cm}^3 = \pm 0.03\ \text{cm}^3$$
$$V_1 + V_2 = (10.2 + 6.4)\ \text{cm}^3 = 16.6\ \text{cm}^3$$

and $$V_1 - V_2 = (10.2 - 6.4)\ \text{cm}^3 = 3.8\ \text{cm}^3$$

Hence, sum of volumes $= (16.6 \pm 0.03)\ \text{cm}^3$

and difference of volumes $= (3.8 \pm 0.03)\ \text{cm}^3$

2. Errors in a product

Let $x = ab$

Then, $(x \pm \Delta x) = (a \pm \Delta a)(b \pm \Delta b)$

or $$x\left(1 \pm \frac{\Delta x}{x}\right) = ab\left(1 \pm \frac{\Delta a}{a}\right)\left(1 \pm \frac{\Delta b}{b}\right)$$

or $$1 \pm \frac{\Delta x}{x} = 1 \pm \frac{\Delta b}{b} \pm \frac{\Delta a}{a} \pm \frac{\Delta a}{a} \cdot \frac{\Delta b}{b} \quad (\text{as } x = ab)$$

or $$\pm \frac{\Delta x}{x} = \pm \frac{\Delta a}{a} \pm \frac{\Delta b}{b} \pm \frac{\Delta a}{a} \cdot \frac{\Delta b}{b}$$

Here, $\frac{\Delta a}{a} \cdot \frac{\Delta b}{b}$ is a small quantity, so can be neglected.

Hence, $$\pm \frac{\Delta x}{x} = \pm \frac{\Delta a}{a} \pm \frac{\Delta b}{b}$$

Possible values of $\frac{\Delta x}{x}$ are $\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right), \left(\frac{\Delta a}{a} - \frac{\Delta b}{b}\right), \left(-\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$ and $\left(-\frac{\Delta a}{a} - \frac{\Delta b}{b}\right)$.

Hence, maximum possible value of

$$\boxed{\frac{\Delta x}{x} = \pm\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)}$$

Therefore, maximum fractional error in product of two (or more) quantities is equal to sum of fractional errors in the individual quantities.

3. Error in division

Let $x = \frac{a}{b}$

Then, $$x \pm \Delta x = \frac{a \pm \Delta a}{b \pm \Delta b}$$

or $$x\left(1 \pm \frac{\Delta x}{x}\right) = \frac{a\left(1 \pm \frac{\Delta a}{a}\right)}{b\left(1 \pm \frac{\Delta b}{b}\right)}$$

or $$\left(1 \pm \frac{\Delta x}{x}\right) = \left(1 \pm \frac{\Delta a}{a}\right)\left(1 \pm \frac{\Delta b}{b}\right)^{-1} \quad \left(\text{as } x = \frac{a}{b}\right)$$

As $\frac{\Delta b}{b} << 1$, so expanding binomially, we get

$$\left(1 \pm \frac{\Delta x}{x}\right) = \left(1 \pm \frac{\Delta a}{a}\right)\left(1 \mp \frac{\Delta b}{b}\right)$$

or $$1 \pm \frac{\Delta x}{x} = 1 \pm \frac{\Delta a}{a} \mp \frac{\Delta b}{b} \pm \frac{\Delta a}{a} \cdot \frac{\Delta b}{b}$$

Here, $\frac{\Delta a}{a} \cdot \frac{\Delta b}{b}$ is a small quantity, so can be neglected.

Hence, $$\pm \frac{\Delta x}{x} = \pm \frac{\Delta a}{a} \mp \frac{\Delta b}{b}$$

Possible values of $\frac{\Delta x}{x}$ are $\left(\frac{\Delta a}{a} - \frac{\Delta b}{b}\right), \left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right), \left(-\frac{\Delta a}{a} - \frac{\Delta b}{b}\right)$ and $\left(-\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$. Therefore, the maximum value of

$$\boxed{\frac{\Delta x}{x} = \pm\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)}$$

Or, the maximum value of fractional error in division of two quantities is equal to the sum of fractional errors in the individual quantities.

Example 1.35 *Calculate focal length of a spherical mirror from the following observations. Object distance $u = (50.1 \pm 0.5)$ cm and image distance $v = (20.1 \pm 0.2)$ cm.*

Sol. Formula for focal length of a spherical mirror

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

or $$f = \frac{uv}{u+v}$$

$$= \frac{(50.1)(20.1)}{(50.1 + 20.1)} = 14.3 \text{ cm}$$

Also, $$\frac{\Delta f}{f} = \pm\left[\frac{\Delta u}{u} + \frac{\Delta v}{v} + \frac{\Delta u + \Delta v}{u+v}\right]$$

$$= \pm\left[\frac{0.5}{50.1} + \frac{0.2}{20.1} + \frac{0.5 + 0.2}{50.1 + 20.1}\right]$$

$$= [0.00998 + 0.00995 + 0.00997]$$

$$= \pm (0.0299)$$

$\therefore$ $\Delta f = 0.0299 \times 14.3 = 0.428 = 0.4$ cm

$\therefore$ $f = (14.3 \pm 0.4)$ cm

4. Error in quantity raised to some power

Let $$x = \frac{a^n}{b^m}$$

Then, $$\ln(x) = n \ln(a) - m \ln(b)$$

Differentiating both sides, we get

$$\frac{dx}{x} = n\frac{da}{a} - m\frac{db}{b}$$

In terms of fractional error we may write,

$$\pm \frac{\Delta x}{x} = \pm n\frac{\Delta a}{a} \mp m\frac{\Delta b}{b}$$

Therefore, maximum value of

$$\boxed{\frac{\Delta x}{x} = \pm\left(n\frac{\Delta a}{a} + m\frac{\Delta b}{b}\right)}$$

Example 1.36 *The radius of sphere is measured to be (2.1 ± 0.5) cm. Calculate its surface area with error limits.*

Sol. Surface area, $S = 4\pi r^2 = (4)\left(\frac{22}{7}\right)(2.1)^2$

$$= 55.44 = 55.4 \text{ cm}^2$$

Further, $\frac{\Delta S}{S} = 2\frac{\Delta r}{r}$ or $\Delta S = 2\left(\frac{\Delta r}{r}\right)(S)$

$$= \frac{2\times 0.5\times 55.4}{2.1} = 26.38 = 26.4 \text{ cm}^2$$

$\therefore$ $S = (55.4 \pm 26.4) \text{ cm}^2$

Example 1.37 *The mass and density of a solid sphere are measured to be (12.4 ± 0.1) kg and (4.6 ± 0.2) kg/m³. Calculate the volume of the sphere with error limits.*

Sol. Here, $m \pm \Delta m = (12.4 \pm 0.1)$ kg

and $\rho \pm \Delta\rho = (4.6 \pm 0.2) \text{ kgm}^{-3}$

Volume, $V = \frac{m}{\rho} = \frac{12.4}{4.6} = 2.69 \text{ m}^3 = 2.7 \text{ m}^3$

(rounding off to one decimal place)

Now, $\frac{\Delta V}{V} = \pm\left(\frac{\Delta m}{m} + \frac{\Delta\rho}{\rho}\right)$

or $\Delta V = \pm\left(\frac{\Delta m}{m} + \frac{\Delta\rho}{\rho}\right)\times V = \pm\left(\frac{0.1}{12.4} + \frac{0.2}{4.6}\right)\times 2.7$

$$= \pm 0.14$$

$\therefore$ $V \pm \Delta V = (2.7 \pm 0.14) \text{ m}^3$

Example 1.38 *A thin copper wire of length L increase in length by 2% when heated from T_1 to T_2. If a copper cube having side 10 L is heated from T_1 to T_2 what will be the percentage change in*

(i) area of one face of the cube and

(ii) volume of the cube

Sol. (i) Area, $A = 10\text{L} \times 10\text{L} = 100\text{L}^2$

Percentage change in area

$$= \frac{\Delta A}{A}\times 100 = 2\times\frac{\Delta L}{L}\times 100 = 2\times 2\% = 4\%$$

$$\left(\because \frac{\Delta A}{A} = \frac{\Delta 100}{100} + 2\frac{\Delta L}{L} = 0 + 2\frac{\Delta L}{L} = 2\frac{\Delta L}{L}\right)$$

Note Constants do not have any error in them.

(ii) Volume, $V = 10\text{L} \times 10\text{L} \times 10\text{L} = 1000\text{L}^3$

Percentage change in volume

$$= \frac{\Delta V}{V}\times 100$$

$$= 3\frac{\Delta L}{L} = 3\times 2\% = 6\%$$

Conclusion The maximum percentage change will be observed in volume, lesser in area and the least (minimum) change will be observed in length or radius.

Example 1.39 *Calculate percentage error in determination of time period of a pendulum*

$$T = 2\pi\sqrt{\frac{l}{g}}$$

where, l and g are measured with ± 1% and ± 2% errors.

Sol. $\frac{\Delta T}{T}\times 100 = \pm\left(\frac{1}{2}\times\frac{\Delta l}{l}\times 100 + \frac{1}{2}\times\frac{\Delta g}{g}\times 100\right)$

$$= \pm\left(\frac{1}{2}\times 1 + \frac{1}{2}\times 2\right) = \pm 1.5\ \%$$

Example 1.40 *Find the relative error in Z, if $Z = \frac{A^4B^{1/3}}{CD^{3/2}}$ and the percentage error in the measurements of A, B, C and D are 4%, 2%, 3% and 1%, respectively.*

Sol. $\frac{\Delta Z}{Z} = 4\left(\frac{\Delta A}{A}\right) + \frac{1}{3}\left(\frac{\Delta B}{B}\right) + \frac{\Delta C}{C} + \frac{3}{2}\left(\frac{\Delta D}{D}\right)$

Given that, $\frac{\Delta A}{A}\times 100 = 4$,

$$\frac{\Delta B}{B}\times 100 = 2,$$

$$\frac{\Delta C}{C}\times 100 = 3$$

and $\frac{\Delta D}{D}\times 100 = 1$

$\therefore$ $\frac{\Delta Z}{Z}\times 100 = (4\times 4) + \left(\frac{1}{3}\times 2\right) + 3 + \left(\frac{3}{2}\times 1\right)$

$$= 16 + \frac{2}{3} + 3 + \frac{3}{2}$$

$$= 21.16\%$$

The percentage error in the measurement of Z is 21.16%. Therefore, the relative error in Z is 0.2116.

Check point 1.3

1. If error in measuring diameter of a circle is 4%, the error in measuring radius of the circle would be
(a) 2% (b) 8%
(c) 4% (d) 1%

2. The heat generated in a wire depends on the resistance, current and time. If the error in measuring the above are 1%, 2% and 1%, respectively. The maximum error in measuring the heat is
(a) 8% (b) 6% (c) 18% (d) 12%

3. A force F is applied on a square plate of side L. If the percentage error in the determination of L is 2% and that in F is 4%. What is the permissible error in pressure?
(a) 8% (b) 6% (c) 4% (d) 2%

4. A cuboid has volume $V = l \times 2l \times 3l$, where l is the length of one side. If the relative percentage error in the measurement of l is 1%, then the relative percentage error in measurement of V is
(a) 18% (b) 6% (c) 3% (d) 1%

5. The length of a rod is (11.05 ± 0.2) cm. What is the length of the two rods?
(a) (22.1 ± 0.05) cm (b) (22.1 ± 0.1) cm
(c) (22.10 ± 0.05) cm (d) (22.10 ± 0.2) cm

6. Three measurements are made as 18.425 cm, 7.21 cm and 5.0 cm. The addition should be written as
(a) 30.635 cm (b) 30.64 cm
(c) 30.63 cm (d) 30.6 cm

7. If the error in the measurement of momentum of a particle is (+ 100%), then the error in the measurement of kinetic energy is
(a) 100% (b) 200%
(c) 300% (d) 400%

8. A body travels uniformly a distance of (13.8 ± 0.2) m in a time (4.0 ± 0.3) s. The velocity of the body within error limit is
(a) (3.45 ± 0.2) ms^{-1} (b) (3.45 ± 0.3) ms^{-1}
(c) (3.45 ± 0.4) ms^{-1} (d) (3.45 ± 0.5) ms^{-1}

9. The radius of a ball is (5.2 ± 0.2) cm. The percentage error in the volume of the ball is (approximately).
(a) 11% (b) 4%
(c) 7% (d) 9%

10. The values of two resistors are (5.0 ± 0.2) kΩ and (10.0 ± 0.1) kΩ. What is the percentage error in the equivalent resistance when they are connected in parallel?
(a) 2% (b) 5% (c) 7% (d) 10%

Chapter exercises

A Taking it together

(Assorted questions of the chapter for advanced level practice)

1. If dimensions of A and B are different, then which of the following operation is valid?

(a) $\frac{A}{B}$ (b) $e^{-A/B}$ (c) $A-B$ (d) $A+B$

2. The diameter of a wire is measured to be 0.0250×10^{-4} m. The number of significant figures in the measurement is

(a) five (b) four (c) three (d) nine

3. Dimensional formula for electromotive force is same as that for

(a) potential (b) current (c) force (d) energy

4. The number of significant figures in 0.06900 is **[NCERT Exemplar]**

(a) 5 (b) 4 (c) 2 (d) 3

5. The sum of the numbers 436.32, 227.2 and 0.301 in appropriate significant figures is **[NCERT Exemplar]**

(a) 663.821 (b) 664 (c) 663.8 (d) 663.82

6. The dimensional formula for magnetic flux is

(a) $[ML^2T^{-2}A^{-1}]$ (b) $[ML^3T^{-2}A^{-2}]$

(c) $[M^0L^{-2}T^{-2}A^{-2}]$ (d) $[ML^2T^{-1}A^2]$

7. A force F is given by $F = at + bt^2$, where t is time. The dimensions of a and b are

(a) $[MLT^{-3}]$ and $[MLT^{-4}]$ (b) $[MLT^{-4}]$ and $[MLT^{-3}]$

(c) $[MLT^{-1}]$ and $[MLT^{-2}]$ (d) $[MLT^{-2}]$ and $[MLT^0]$

8. If the dimensions of a physical quantity are given by $[M^aL^bT^c]$, then the physical quantity will be

(a) force, if $a = 0, b = -1, c = -2$

(b) pressure if $a = 1, b = -1, c = -2$

(c) velocity if $a = 1, b = 0, c = -1$

(d) acceleration if $a = 1, b = 1, c = -2$

9. Three measurements are made as 18.425 cm, 7.21 cm and 5.0 cm. The addition should be written as

(a) 30.635 cm (b) 30.64 cm (c) 30.63 cm (d) 30.6 cm

10. What is the units of $k = \frac{1}{4\pi\varepsilon_0}$?

(a) $C^2N^{-1}m^{-2}$ (b) Nm^2C^{-2} (c) Nm^2C^2 (d) Unitless

11. The radius of a circle is 2.12 m. Its area according to the rule of significant figures is

(a) $14.1124\ m^2$ (b) $14.112\ m^2$

(c) $14.11\ m^2$ (d) $14.1\ m^2$

12. If the value of resistance is 10.845 ohm and the value of current is 3.23 amp, the value of potential with significant numbers would be

(a) 35.0 V (b) 3.50 V

(c) 35.029 V (d) 35.030 V

13. The position of the particle moving along Y-axis is given as $y = At^2 - Bt^3$, where y is measured in metre and t in second. Then, the dimensions of B are

(a) $[LT^{-2}]$ (b) $[LT^{-1}]$ (c) $[LT^{-3}]$ (d) $[MLT^{-2}]$

14. The length, breadth and thickness of a block are given by $l = 12$ cm, $b = 6$ cm and $t = 2.45$ cm. The volume of the block according to the idea of significant figures should be

(a) $1 \times 10^2\ cm^3$ (b) $2 \times 10^2 cm^3$

(c) $1.764 \times 10^2 cm^3$ (d) None of these

15. Out of the following four dimensional quantities, which one qualifies to be called a dimensional constant?

(a) Acceleration due to gravity

(b) Surface tension of water

(c) Weight of a standard kilogram mass

(d) The velocity of light in vacuum

16. The random error in the arithmetic means of 100 observations is x, then random error in the arithmetic mean of 400 observation would be

(a) $4x$ (b) $\frac{1}{4}x$ (c) $2x$ (d) $\frac{1}{2}x$

17. A physical quantity Q is calculated according to the expression

$$Q = \frac{A^3 B^3}{C\sqrt{D}}$$

If percentage errors in A, B, C, D are 2%, 1%, 3% and 4% respectively. What is the percentage error in Q?

(a) ± 8% (b) ± 10% (c) ± 14% (d) ± 12%

18. With usual notation, the following equation, said to give the distance covered in the nth second. *i.e.*,

$S_n = u + a\frac{(2n-1)}{2}$ is

(a) only numerically correct
(b) only dimensionally correct
(c) Both dimensionally and numerically
(d) Neither numerically nor dimensionally correct

19. The velocity v of a particle at time t is given by $v = at + \frac{b}{t+c}$, where a, b and c are constants. The dimensions of a, b and c are, respectively.

(a) $[LT^{-2}]$, [L] and [T] (b) $[L^2]$, [T] and $[LT^2]$
(c) LT^2, LT and L (d) [L], [LT] and $[T^2]$

20. If the random error in the arithmetic mean of 50 observations is α, then the random error in the arithmetic mean of 150 observations would be

(a) α (b) 3α (c) $\frac{\alpha}{3}$ (d) 2α

21. Velocity v is given by $v = at^2 + bt + c$, where t is time. What are the dimensions of a, b and c respectively?

(a) $[LT^{-3}]$, $[LT^{-2}]$ and $[LT^{-1}]$
(b) $[LT^{-1}]$, $[LT^{-2}]$ and $[LT^{-3}]$
(c) $[LT^{-2}]$, $[LT^{-3}]$ and $[LT^{-1}]$
(d) $[LT^{-1}]$, $[LT^{-3}]$ and $[LT^{-2}]$

22. The square root of the product of inductance and capacitance has the dimensions of

(a) length (b) time
(c) mass (d) no dimension

23. The frequency of vibration of string is given by $f = \frac{p}{2l}\left[\frac{F}{m}\right]^{1/2}$. Here, p is number of segments in the string and l is the length. The dimensional formula for m will be

(a) $[M^0LT^{-1}]$ (b) $[ML^0T^{-1}]$
(c) $[ML^{-1}T^0]$ (d) $[M^0L^0T^0]$

24. The numbers 2.745 and 2.735 on rounding off to 3 significant figures will give **[NCERT Exemplar]**

(a) 2.75 and 2.74 (b) 2.74 and 2.73
(c) 2.75 and 2.73 (d) 2.74 and 2.74

25. The mass and volume of a body are 4.237 g and 2.5 cm^3, respectively. The density of the material of the body in correct significant figures is **[NCERT Exemplar]**

(a) 1.6048 g cm^{-3} (b) 1.69 g cm^{-3}
(c) 1.7 g cm^{-3} (d) 1.695 g cm^{-3}

26. The length and breadth of a rectangular sheet are 16.2 cm and 10.1 cm, respectively. The area of the sheet in appropriate significant figures and error is **[NCERT Exemplar]**

(a) 164 ± 3 cm^2 (b) 163.62 ± 2.6 cm^2
(c) 163.6 ± 2.6 cm^2 (d) 163.62 ± 3 cm^2

27. Which of the following pairs of physical quantities does not have same dimensional formula? **[NCERT Exemplar]**

(a) Work and torque
(b) Angular momentum and Planck's constant
(c) Tension and surface tension
(d) Impulse and linear momentum

28. Measure of two quantities along with the precision of respective measuring instrument is $A = 2.5 ms^{-1} \pm 0.5 ms^{-1}$, $B = 0.10 s \pm 0.01 s$. The value of AB will be **[NCERT Exemplar]**

(a) (0.25 ± 0.08) m (b) (0.25 ± 0.5) m
(c) (0.25 ± 0.05) m (d) (0.25 ± 0.135) m

29. Which of the following measurement is most precise? **[NCERT Exemplar]**

(a) 5.00 mm (b) 5.00 cm (c) 5.00 m (d) 5.00 km

30. The mean length of an object is 5 cm. Which of the following measurements is most accurate? **[NCERT Exemplar]**

(a) 4.9 cm (b) 4.805 cm (c) 5.25 cm (d) 5.4 cm

31. Young's modulus of steel is 1.9×10^{11} Nm^{-2}. When expressed in CGS units of dyne/cm^2, it will be equal to ($1N = 10^5$ dyne, $1m^2 = 10^4 cm^2$) **[NCERT Exemplar]**

(a) 1.9×10^{10} (b) 1.9×10^{11} (c) 1.9×10^{12} (d) 1.9×10^{13}

32. If the energy (E), velocity (v) and force (F) be taken as fundamental quantities, then the dimensions of mass will be

(a) $[Fv^{-2}]$ (b) $[Fv^{-1}]$ (c) $[Ev^{-2}]$ (d) $[Ev^2]$

33. If force F, length L and time T are taken as fundamental units, the dimensional formula for mass will be

(a) $[FL^{-1}T^2]$ (b) $[FLT^{-2}]$ (c) $[FL^{-1}T^{-1}]$ (d) $[FL^5T^2]$

34. From the dimensional consideration, which of the following equations is correct?

(a) $T = 2\pi\sqrt{\frac{R^3}{GM}}$ (b) $T = 2\pi\sqrt{\frac{GM}{R^3}}$
(c) $T = 2\pi\sqrt{\frac{GM}{R^2}}$ (d) $T = 2\pi\sqrt{\frac{R^2}{GM}}$

35. If voltage $V = (100 \pm 5)$ V and current $I = (10 \pm 0.2)$ A, the percentage error in resistance R is
(a) 5.2% (b) 25% (c) 7% (d) 10%

36. A wire has a mass (0.3 ± 0.003) g, radius (0.5 ± 0.005) mm and length (0.6 ± 0.006) cm. The maximum percentage error in the measurement of its density
(a) 1 (b) 2 (c) 3 (d) 4

37. If $x = 10.0 \pm 0.1$ and $y = 10.0 \pm 0.1$, then $2x - 2y$ is equal to
(a) (0.0 ± 0.1) (b) Zero (c) (0.0 ± 0.4) (d) (20 ± 0.2)

38. The dimensional formula for molar thermal capacity is same as that of
(a) gas constant (b) specific heat
(c) Boltzmann's constant (d) Stefan's constant

39. Dimensions of Ohm are same as
(a) $\frac{h}{e}$ (b) $\frac{h^2}{e}$ (c) $\frac{h}{e^2}$ (d) $\frac{h^2}{e^2}$
(where, h is Planck's constant and e is charge)

40. The equation of state of some gases can be expressed as

$$\left(p + \frac{a}{V^2}\right)(V - b) = RT$$

where, p is the pressure, V is the volume, T is the absolute temperature and a, b and R are constants. The dimensions of a are
(a) $[ML^5T^{-2}]$ (b) $[ML^{-1}T^{-2}]$ (c) $[L^3]$ (d) $[L^6]$

41. Using mass (M), length (L), time (T) and current (A) as fundamental quantities, the dimensions of permeability is
(a) $[M^{-1}LT^{-2}A]$ (b) $[ML^2T^{-2}A^{-1}]$
(c) $[MLT^{-2}A^{-2}]$ (d) $[MLT^{-1}A^{-1}]$

42. Let g be the acceleration due to gravity at earth's surface and K the rotational kinetic energy of the earth. Suppose the earth's radius decreases by 2%. Keeping mass to be constant, then
(a) g increases by 2% and K increases by 2%
(b) g increases by 4% and K increases by 4%
(c) g increases by 4% and K increases by 2%
(d) g increases by 2% and K increases by 4%

43. In a system of units, the units of mass, length and time are 1 quintal, 1 km and 1 h, respectively. In this system 1 N force will be equal to
(a) 1 new unit (b) 129.6 new unit
(c) 427.6 new unit (d) 60 new unit

44. Given that $\int \frac{dx}{\sqrt{2ax - x^2}} = a^n \sin^{-1}\left[\frac{x-a}{a}\right]$
where, a = constant. Using dimensional analysis, the value of n is
(a) 1 (b) zero
(c) -1 (d) None of these

45. If momentum of an object is increased by 10%, then its kinetic energy will increase by
(a) 20% (b) 21% (c) 40% (d) 19%

46. The magnetic force on a point charge is
$\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$
Here, q = electric charge
$\mathbf{v}$ = velocity of point charge
$\mathbf{B}$ = magnetic field
The dimensions of $\mathbf{B}$ are
(a) $[MLT^{-1}A]$ (b) $[M^2LT^{-2}A^{-1}]$
(c) $[MT^{-2}A^{-1}]$ (d) None of these

47. A capillary tube is attached horizontally to a constant heat arrangement. If the radius of the capillary tube is increased by 10%, then the rate of flow of liquid will change nearly by
(a) +10% (b) + 46% (c) −10% (d) − 40%

48. By what percentage should the pressure of a given mass of a gas be increased so as to decrease its volume by 10% at a constant temperature?
(a) 5% (b) 7.2% (c) 12.5% (d) 11.1%

49. In measuring electric energy, 1kWh is equal to
(a) 3.6×10^4 J (b) 3.6×10^6 J
(c) 7.3×10^6 J (d) None of these

50. A quantity is given by $X = \frac{\varepsilon_0 lV}{t}$, where V is the potential difference and l is the length. Then, X has dimensional formula same as that of
(a) resistance (b) charge (c) voltage (d) current

51. The length of a strip measured with a metre rod is 10.0 cm. Its width measured with a vernier calipers is 1.00 cm. The least count of the metre rod is 0.1 cm and that of vernier calipers 0.01 cm. What will be error in its area?
(a) ±13% (b) ± 7% (c) ± 4% (d) ± 2%

52. The length of cylinder is measured with a metre rod having least count 0.1 cm. Its diameter is measured with vernier calipers having least count 0.01 cm. Given that length is 5.0 cm and radius is 2.0 cm. The percentage error in the calculated value of the volume will be
(a) 1.5% (b) 2.5% (c) 3.5% (d) 4%

53. The length of a uniform rod is 100.0 cm. If length is measured with a meter rod having least count 1 mm and radius is measured with vernier callipers having least count 0.1 mm, the percentage error in calculated volume of cylinder is
(a) 2.1% (b) 3% (c) 2.01% (d) 3.2%

54. You measure two quantities as $A = 1.0$ m ± 0.2 m, $B = 2.0$m ± 0.2 m. We should report correct value for $\sqrt{AB}$ as **[NCERT Exemplar]**
(a) 1.4m ± 0.4m (b) 1.41m ± 0.15 m
(c) 1.4 m ± 0.3 m (d) 1.4m ± 0.2m

55. If momentum (p), area (A) and time (T) are taken to be fundamental quantities, then energy has the dimensional formula **[NCERT Exemplar]**

(a) $[pA^{-1}T^{1}]$ (b) $[p^2AT]$
(c) $[pA^{-1/2}T]$ (d) $[pA^{1/2}T^{-1}]$

56. If E = energy, G = gravitational constant, I = impulse and M = mass, then dimensions of $\frac{GIM^2}{E^2}$ are same as that of

(a) time (b) mass (c) length (d) force

57. The relation $p = \frac{\alpha}{\beta} e^{\frac{-\alpha Z}{k\theta}}$, where p is pressure, Z is distance, k is Boltzmann constant and θ is temperature. The dimensional formula of β will be

(a) $[M^0L^2T^0]$ (b) $[ML^2T]$ (c) $[ML^0T^{-1}]$ (d) $[M^0L^2T^{-1}]$

58. If E, M, L and G denote energy, mass, angular momentum and gravitational constant respectively, then the quantity (E^2L^2/M^5G^2) has the dimensions of

(a) angle (b) length
(c) mass (d) None of these

59. A uniform wire of length L, diameter D and density ρ is stretched under a tension T. The correct relation between its fundamental frequency f, the length L and the diameter D is

(a) $f \propto \frac{1}{LD}$ (b) $f \propto \frac{1}{L\sqrt{D}}$
(c) $f \propto \frac{1}{D^2}$ (d) $f \propto \frac{1}{LD^2}$

60. If the energy $E = G^p h^q c^r$, where G is the universal gravitational constant, h is the Planck's constant and c is the velocity of light, then the values of p, q and r are, respectively

(a) – 1/2, 1/2 and 5/2 (b) 1/2, – 1/2 and – 5/2
(c) – 1/2, 1/2 and 3/2 (d) 1/2, 1/2 and – 3/2

61. A gas bubble formed from an explosion under water oscillates with a period T proportional to $p^a d^b E^c$, where p is pressure, d is the density of water and E is the total energy of explosion. The values of a, b and c are

(a) $a = 1,\ b = 1, c = 2$ (b) $a = 1, b = 2, c = 1$
(c) $a = \frac{5}{6}, b = \frac{1}{2}, c = \frac{1}{3}$ (d) $a = -\frac{5}{6}, b = \frac{1}{2}, c = \frac{1}{3}$

B) Medical entrance special format questions

Assertion and reason

Directions (Q. Nos. 1-17) *These questions consists of two statements each printed as Assertion and Reason. While answering these question you are required to choose any one of the following five responses.*

(a) If both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If Assertion is false but Reason is true.

1. Assertion Method of dimension cannot be used for deriving formulae containing trigonometrical ratios.

Reason This is because trigonometrical ratios have no dimensions.

2. Assertion Number of significant figures in 0.005 is one and that in 0.500 are three.

Reason This is because zeros before decimal are non significant.

3. Assertion When we change the unit of measurement of a quantity, its numerical value changes.

Reason Smaller the unit of measurement smaller is its numerical value.

4. Assertion Pressure has the dimensions of energy density.

Reason Energy density $= \frac{\text{energy}}{\text{volume}} = \frac{[ML^2T^{-2}]}{[L^3]}$

$= [ML^{-1}T^{-2}] =$ pressure

5. Assertion When percentage errors in the measurement of mass and velocity are 1% and 2% respectively, the percentage error in KE is 5%.

Reason KE or $E = \frac{1}{2}mv^2$, $\frac{\Delta E}{E} = \frac{\Delta m}{m} + \frac{2\Delta v}{v}$

6. Assertion The error in the measurement of radius of the sphere is 0.3%. The permissible error in its surface area is 0.6%.

Reason The permissible error is calculated by the formula

$$\frac{\Delta A}{A} = 4\frac{\Delta r}{r}.$$

7. Assertion The light year and wavelength consist of dimensions of length.

Reason Both light year and wavelength represent time.

8. Assertion Out of two measurements $l = 0.7$ m and $l = 0.70$ m, the second one is more accurate.

Reason In every measurement, the last digit is not accurately known.

9. Assertion A screw gauge having a smaller value of pitch has greater accuracy.

Reason The least count of screw gauge is directly proportional to the number of divisions on circular scale.

10. Assertion L/R and CR both have same dimensions.

Reason L/R and CR both have dimensions of time.

11. Assertion If $x = \frac{a^n}{b^m}$ the $\frac{\Delta x}{x} = n\left(\frac{\pm \Delta a}{a}\right) - m\left(\frac{\pm \Delta b}{b}\right)$

The change in a or b *i.e.*, Δa or Δb may be comparable to a and b.

Reason The above relation is valid when $\Delta a << a$ and $\Delta b << b$.

12. Assertion Systematic errors and random errors fall in the same group of errors.

Reason Both systematic and random errors are based on the cause of error.

13. Assertion Absolute error may be negative or positive.

Reason Absolute error is the difference between the real value and the measured value of a physical quantity.

14. Assertion

$$\sqrt{\frac{\text{Magnetic dipole moment} \times \text{moment induction}}{\text{Moment of inertia}}}$$

Dimensional formula $[M^0L^0T]$

Reason The given dimension is that of frequency.

15. Assertion $\sqrt{\frac{\text{Modulus of elasticity}}{\text{Density}}}$ has the unit ms^{-1}.

Reason Acceleration has the dimensions of $\frac{1}{(\sqrt{\varepsilon_0 \mu_0})\, t}$.

Match the columns

1. Match the following columns.

Column I	Column II
(A) R/L	(p) Time
(B) C/R	(q) Frequency
(C) E/B	(r) Speed
(D) $\sqrt{\varepsilon_0 \mu_0}$	(s) None

2. Match the following columns.

Column I	Column II
(A) Stress	(p) Pressure
(B) Strain	(q) Energy density
(C) Modulus of elasticity	(r) Angle
(D) Torque	(s) Energy

3. Suppose force (F), area (A) and time (T) are the fundamental units, then the match the following columns.

Column I	Column II
(A) Work	(p) $[A^{1/2}T^{-1}]$
(B) Moment of inertia	(q) $[FA^{1/2}]$
(C) Velocity	(r) $[FA^{1/2}T^2]$

4. Match the following columns.

Column I	Column II
(A) Electrical resistance	(p) $[M^{-1}L^{-2}T^4A^2]$
(B) Capacitance	(q) $[ML^2T^{-2}A^{-2}]$
(C) Magnetic field	(r) $[ML^2T^{-3}A^{-2}]$
(D) Inductance	(s) $[MT^{-2}A^{-1}]$

5. Match the following two columns.

Column I	Column II
(A) GM_eM_s	(p) $[M^2L^2T^{-3}]$
(B) $\frac{3RT}{M}$	(q) $[ML^3T^{-2}]$
(C) $\frac{F^2}{q^2B^2}$	(r) $[L^2T^{-2}]$
(D) $\frac{GM_e}{R_e}$	(s) None

6. Match list I with list II and select the correct answer.

A.	Spring constant	1.	$[M^1L^2T^{-2}]$
B.	Pascal	2.	$[M^0L^0T^{-1}]$
C.	Hertz	3.	$[M^1L^0T^{-2}]$
D.	Joule	4.	$[M^1L^{-1}T^{-2}]$

C Medical entrances' gallery

(Collection of questions asked in NEET & various medical entrance exams)

1. In terms of time t and distance x, the force F is given by $F = A\sin Ct + B\cos Dx$, then dimensions of $\frac{A}{B}$ and $\frac{C}{D}$ are given by **[UK PMT 2015]**

(a) $[M^0L^0T^0]$, $[M^0LT^{-1}]$ (b) $[MLT^{-2}]$, $[M^0L^0T^{-1}]$
(c) $[MLT^{-2}]$, $[M^0L^{-1}T^0]$ (d) $[M^0LT^{-1}]$, $[M^0L^0T^0]$

2. The scalar quantity among the following is **[Kerala CEE 2015]**

(a) weight of body (b) temperature gradient
(c) elementary area (d) magnetic field strength
(e) electric potential

3. The wrong unit conversion among the following is **[Kerala CEE 2015]**

(a) 1 angstrom $= 10^{-10}$m
(b) 1 fermi $= 10^{-15}$ m
(c) 1 light year $= 9.46 \times 10^{15}$ m
(d) 1 parsec $= 3.08 \times 10^{16}$ m
(e) 1 astronomical unit $= 1.496 \times 10^{-11}$ m

4. The mass of the liquid flowing per second per unit area of cross-section of the tube is proportional to (pressure difference across the ends)^n and (average velocity)^m of the liquid. Which one of the following relation is correct? **[CG PMT 2015]**

(a) $m = n$ (b) $m = -n$ (c) $m^2 = n$ (d) $m = -n^2$

5. The ratio of the dimensions of Planck's constant and that of moment of inertia has the dimensions of **[KCET 2015]**

(a) angular momentum (b) time
(c) velocity (d) frequency

6. In terms of basic units of mass (M), length (L), time (T) and charge (Q), the dimensions of magnetic permeability of vacuum (μ_0) would be **[AIIMS 2015]**

(a) $[MLQ^{-2}]$ (b) $[LT^{-1}Q^{-1}]$
(c) $[ML^2T^{-1}Q^{-2}]$ (d) $[LTQ^{-1}]$

7. The dimensional formula for electric flux is **[AIIMS 2015]**

(a) $[ML^3I^{-1}T^{-3}]$ (b) $[M^2L^2I^{-1}T^{-2}]$
(c) $[ML^3I^{1}T^{-3}]$ (d) $[ML^{-3}I^{-1}T^{-3}]$

8. If energy (E), velocity (v) and time (T) are chosen as the fundamental quantities, the dimensional formula of surface tension will be **[CBSE AIPMT 2015]**

(a) $[Ev^{-2}T^{-1}]$ (b) $[Ev^{-1}T^{-2}]$
(c) $[Ev^{-2}T^{-2}]$ (d) $[E^{-2}v^{-1}T^{-3}]$

9. Match the following two columns. **[Guj. CET 2015]**

	Column I		Column II
A.	Electrical resistance	1.	$[ML^3T^{-3}A^{-2}]$
B.	Electrical potential	2.	$[ML^2T^{-3}A^{-2}]$
C.	Specific resistance	3.	$[ML^2T^{-3}A^{-1}]$
D.	Specific conductance	4.	None

(a) A-2, B-3,C-1, D-4 (b) A-2, B-4, C-3, D-1
(c) A-1, B-2, C-4, D-3 (d) A-1, B-3, C-2, D-4

10. The three physical quantities x, y and z have units g cm^{2}s^{-5}, g s^{-1} and cms^{-2}, respectively. The relation between x, y and z is **[AFMC 2015]**

(a) $x = yz^2$ (b) $x = y^2z$
(c) $y^2 = xz$ (d) $z = x^2y$

11. The unit of universal gas constant is **[WB JEE 2014]**

(a) watt/K (b) dyne/°C
(c) erg/K (d) newton/°R

12. Unit of emf is **[WB JEE 2014]**

(a) joule/ampere (b) volt/ampere
(c) $\frac{\text{henry -ampere}}{\text{second}}$ (d) joule/coulomb

13. The dimensional formula for Reynold's number is **[MHT CET 2014]**

(a) $[L^0M^0T^0]$ (b) $[LMT]$
(c) $[L^{-1}MT]$ (d) $[LMT^{-1}]$

14. The relation between force F and density d is $F = \frac{x}{\sqrt{d}}$.

The dimension of x is **[MHT CET 2014]**

(a) $[L^{-1/2}M^{3/2}T^{-2}]$ (b) $[L^{-1/2}M^{1/2}T^{-2}]$
(c) $[L^{-1}M^{3/2}T^{-2}]$ (d) $[L^{-1}M^{1/2}T^{-2}]$

15. In which of the following pairs, the two physical quantities have different dimensions? **[WB JEE 2014]**

(a) Planck's constant and angular momentum
(b) Impulse and linear momentum
(c) Moment of inertia and moment of a force
(d) Energy and torque

16. If the absolute errors in two physical quantities A and B are a and b respectively, then the absolute error in the value of $A - B$ is **[EAMCET 2014]**

(a) $b - a$ (b) $a \neq b$
(c) $a + b$ (d) $a - b$

17. If force (F), velocity (v) and time (T) are taken as fundamental units, then the dimensions of mass is **[CBSE AIPMT 2014]**

(a) $[FvT^{-1}]$ (b) $[FvT^{-2}]$ (c) $[Fv^{-1}T^{-1}]$ (d) $[Fv^{-1}T]$

18. If the unit of force is kN, the length is 1 km and time 100 s, then what will be the unit of mass? **[KCET 2014]**

(a) 1000 kg (b) 1 kg (c) 10000 kg (d) 100 kg

19. The dimensional formula of magnetic flux is **[Guj CET 2014]**

(a) $[ML^2T^{-2}A^{-1}]$ (b) $[ML^2T^{-3}A^{-1}]$
(c) $[M^{-1}L^{-2}T^2A]$ (d) $[ML^3T^{-2}A^{-1}]$

20. The dimensional formula for electric field is **[Guj CET 2014]**

(a) $[ML^2T^{-3}A^{-1}]$ (b) $[ML^2T^{-3}A^{-2}]$
(c) $[MLT^{-3}A^{-1}]$ (d) $[M^0L^0T^0A^0]$

21. If n denotes a positive integer, h the Planck's constant, q the charge and B the magnetic field, then the quantity $\left[\frac{nh}{2\pi qB}\right]$ has the dimension of **[WB JEE 2014]**

(a) area (b) length
(c) speed (d) acceleration

22. In an experiment four quantities a, b, c and d are measured with percentage error 1%, 2%, 3% and 4%, respectively. Quantity P is calculated as follows

$$P = \frac{a^3b^2}{cd}\ \%, \text{ Error in } P \text{ is}$$

[NEET 2013]

(a) 14% (b) 10% (c) 7% (d) 4%

23. Which of the following physical quantity unit is not a fundamental unit? **[J&K CET 2013]**

(a) Length (b) Mass
(c) Magnetic field (d) Current

24. The dimensional formula for rate of doing work is **[Kerala CEE 2013]**

(a) $[ML^2T^{-3}]$ (b) $[ML^{-3}T^2]$ (c) $[M^2L^2T^2]$ (d) $[MLT^{-2}]$
(e) $[M^3L^3T^3]$

25. The density of glass is 2.8 gram/cc in CGS system. The value of density in SI unit is **[Kerala CEE 2013]**

(a) 2.8×10^{-3} (b) 2.8×10^{-2} (c) 2.8×10^2 (d) 2.8×10^6
(e) 2.8×10^3

26. The dimensional formula of electric potential is **[J&K CET 2013]**

(a) $[ML^2T^{-3}A^{-1}]$ (b) $[M^{-1}L^2T^{-2}A]$
(c) $[M^{-1}L^2T^{-2}A^{-1}]$ (d) $[ML^2T^{-2}A]$

27. In the equation $\left(\frac{1}{p\beta}\right) = \frac{y}{k_BT}$, where p is the pressure, y is the distance, k_B is Boltzmann constant and T is the temperature. Dimensions of β are **[EAMCET 2013]**

(a) $[M^{-1}L^1T^2]$ (b) $[M^0L^2T^0]$
(c) $[M^1L^{-1}T^{-2}]$ (d) $[M^0L^0T^0]$

28. Which one of the following is not correct? **[Karnataka CET 2013]**

(a) Dimensional formula of thermal conductivity (K) is $[M^1L^1T^{-3}K^{-1}]$
(b) Dimensional formula of potential (V) is $[M^1L^2T^3A^{-1}]$
(c) Dimensional formula of permeability of free space (μ_0) is $[M^1L^1T^{-2}A^{-2}]$
(d) Dimensional formula of RC is $[M^0L^0T^{-1}]$

29. A physical quantity X is defined by the formula

$$X = \frac{IFv^2}{WL^3}$$

where I is moment of inertia, F is force, v is velocity, W is work and L is length, the dimensions of X are **[MP PMT 2013]**

(a) $[MLT^{-2}]$ (b) $[MT^{-2}]$
(c) $[ML^2T^{-3}]$ (d) $[LT^{-1}]$

30. A physical quantity X is given by

$$X = \frac{2k^3l^2}{m\sqrt{n}}$$

The percentage error in the measurements of k, l, m and n are 1%, 2%, 3% and 4%, respectively. The value of X is uncertain by **[AMU 2012]**

(a) 8% (b) 10%
(c) 12% (d) None of these

31. The quantities A and B are related by the relation, $m = A/B$, where m is the linear density and A is the force. The dimensions of B are of **[BCECE 2012]**

(a) pressure (b) latent heat
(c) work (d) None of these

32. A physical quantity is given by $X = [M^aL^bT^c]$. The percentage error in measurement of M, L and T are α, β and γ, respectively. Then, the maximum % error in the quantity X is **[AFMC 2012]**

(a) $a\alpha + b\beta + c\gamma$ (b) $a\alpha + b\beta - c\gamma$
(c) $\frac{a}{\alpha} + \frac{b}{\beta} + \frac{c}{\gamma}$ (d) None of these

Answers

Check point 1.1

1. (b)	2. (c)	3. (b)	4. (d)	5. (c)	6. (c)	7. (a)	8. (c)	9. (d)	10. (d)
11. (d)	12. (c)	13. (d)	14. (a)	15. (b)	16. (b)	17. (a)	18. (d)	19. (b)	20. (c)
21. (a)	22. (b)	23. (b)	24. (c)	25. (d)					

Check point 1.2

1. (b)	2. (c)	3. (b)	4. (c)	5. (c)	6. (a)	7. (d)	8. (b)	9. (a)	10. (d)

Check point 1.3

1. (c)	2. (b)	3. (a)	4. (c)	5. (d)	6. (d)	7. (c)	8. (b)	9. (a)	10. (b)

A. Taking it together

1. (a)	2. (c)	3. (a)	4. (b)	5. (c)	6. (a)	7. (a)	8. (b)	9. (d)	10. (b)
11. (d)	12. (a)	13. (c)	14. (b)	15. (d)	16. (b)	17. (c)	18. (c)	19. (a)	20. (c)
21. (a)	22. (b)	23. (c)	24. (d)	25. (c)	26. (a)	27. (c)	28. (a)	29. (a)	30. (a)
31. (c)	32. (c)	33. (a)	34. (a)	35. (c)	36. (d)	37. (c)	38. (a)	39. (c)	40. (a)
41. (c)	42. (b)	43. (b)	44. (b)	45. (b)	46. (c)	47. (b)	48. (d)	49. (b)	50. (d)
51. (d)	52. (b)	53. (a)	54. (d)	55. (d)	56. (a)	57. (a)	58. (d)	59. (a)	60. (a)
61. (d)									

B. Medical entrance special format questions

Assertion and reason

1. (a)	2. (c)	3. (c)	4. (a)	5. (a)	6. (c)	7. (c)	8. (b)	9. (c)	10. (a)
11. (d)	12. (a)	13. (a)	14. (d)	15. (b)					

Match the columns

1. $(A \to q, B \to p, C \to r, D \to s)$
2. $(A \to p, B \to r, C \to q, D \to s)$
3. $(A \to q, B \to r, C \to p)$
4. $(A \to r, B \to p, C \to s, D \to q)$

C. Medical entrances' gallery

1. (a)	2. (e)	3. (e)	4. (b)	5. (d)	6. (a)	7. (a)	8. (c)	9. (a)	10. (a)
11. (c)	12. (c)	13. (a)	14. (a)	15. (c)	16. (c)	17. (d)	18. (c)	19. (a)	20. (c)
21. (a)	22. (a)	23. (c)	24. (a)	25. (a)	26. (a)	27. (b)	28. (b)	29. (b)	30. (d)
31. (b)	32. (a)								

Hints and explanations

Check point 1.1

3. (*b*) Impulse = change in linear momentum.

7. (*a*) *CR* is time constant.

11. (*d*) We know that density $= \frac{\text{Mass}}{\text{Volume}}$

In CGS units, $d = 0.625 \text{ g cm}^{-3}$

In SI units, $d = \frac{0.625 \times 10^{-3} \text{ kg}}{10^{-6} \text{m}^3}$

$= 625 \text{ kg m}^{-3}$

12. (*c*) Since $(mvr) = n \cdot \frac{h}{2\pi}$

and $E = h\nu$

So, unit of h = joule second = angular momentum

14. (*a*) Since, $R = \frac{\rho l}{A}$, where ρ is specific resistance.

$\therefore$ $[\rho] = \left[\frac{RA}{l}\right], R = \frac{V}{i}, V = \frac{W}{Q}$

$[\rho] = [ML^3T^{-1}\theta^{-2}]$

17. (*a*) Solid angle, strain and dielectric constant are dimensionless constant.

19. (*b*) $p = \frac{a - t^2}{bx}$, where p-pressure, t-time

$[pbx] = [a] = [t^2]$

Hence, $[b] = \frac{[t^2]}{[px]}$

Dimensions of $\frac{a}{b} = [px] = [MT^{-2}]$

20. (*c*) ωk is dimensionless.

21. (*a*) Muscle × Speed = Power

or Muscle $= \frac{\text{Power}}{\text{Speed}} = \frac{\text{Work}}{\text{Time} \times \text{Speed}}$

$= \frac{[ML^2T^{-2}]}{[T][LT^{-1}]} = [MLT^{-2}]$

= Mass × Acceleration = Force

Now, $\frac{\text{SI unit of force}}{\text{CGS unit of force}} = \frac{\text{kg} \times \text{m} \times \text{s}^{-2}}{\text{g} \times \text{cm} \times \text{s}^{-2}}$

$= 10^3 \times 10^2 = 10^5$

23. (*b*) Since $p^x Q^y c^z$ is dimensionless. Therefore,

$[ML^{-1}T^{-2}]^x [MT^{-3}]^y [LT^{-1}]^z = [M^0L^0T^0]$

Only option (b) satisfies this expression.

So, $x = 1, y = -1, z = 1$

24. (*c*) Since units of length, velocity and force are doubled.

Hence, $[\text{m}] = \frac{[\text{force}][\text{time}]}{[\text{velocity}]}$,

$[\text{time}] = \frac{[\text{length}]}{[\text{velocity}]}$

Hence, unit of mass, and time remains same.

Momentum is doubled.

25. (*d*) $m \propto v^a \rho^b g^c$. Writing the dimensions on both sides

$[M] = [LT^{-1}]^a [ML^{-3}]^b [LT^{-2}]^c$

$[M] = [M^b L^{a-3b+c} T^{-a-2c}]$

$\therefore$ $b = 1$

$a - 3b + c = 0 \Rightarrow -a - 2c = 0$

Solving these, we get

$a = 6$

Hence, $m \propto v^6$

Check point 1.2

1. (*b*) We shall count number of significant digit only in 0.0310 which is 3.

3. (*b*) Only in 20.2 all zero are significant because in this digit zero lies between two digits.

7. (*d*) Out of 107.88 and 0.610, least number of significant digit is 3, so the product must contain 3 significant digit. So the right answer is 65.8.

8. (*b*) $R = 0.16$ mm

Hence, $A = \pi R^2 = \frac{22}{7} \times (0.16)^2 = 0.080384$

Since radius has two significant figure so answer also will have two significant figures.

9. (*a*) The product must have two significant digit as both the figures being multiplies have two significant digits.

10. (*d*) As 2.54 has least number of significant digit which is 3 so the result must have 3 digits as significant.

Check Point 1.3

1. (*c*) As $d = 2r$

$\therefore \quad \Delta d = \Delta r$

2. (*b*) $H = i^2 R t$

$\therefore$ % error in H = 2 (% error in i) + (% error in R) + (% error in t)

3. (*a*) $p = \frac{F}{A} = \frac{F}{L^2} = FL^{-2}$

% error in pressure = (% error in F) + 2 (error in L)

$= (4\%) + 2(2\%) = 8\%$

4. (*c*) Volume of cuboid, $V = l \times 2l \times 3l = 6l^3$

$$\therefore \quad \frac{\Delta V}{V} \times 100 = 3\left(\frac{\Delta l}{l}\right) = 3\% \qquad \left[\because \frac{\Delta l}{l} \times 100 = 1\%\right]$$

7. (*c*) Since error in measurement of momentum is + 100%.

$$\therefore \quad p_1 = p,\ p_2 = 2p$$

$$K_1 = \frac{p^2}{2m},\ K_2 = \frac{(2p)^2}{2m}$$

$$\% \text{ in } K = \left(\frac{K_2 - K_1}{K_1}\right) \times 100$$

$$= \left(\frac{4-1}{1}\right) \times 100 = 300\%$$

9. (*a*) Radius of ball = 5.2 cm

$$V = \frac{4}{3}\pi R^3$$

$$\frac{\Delta V}{V} = 3\left(\frac{\Delta R}{R}\right)$$

$$\left(\frac{\Delta V}{V}\right) \times 100 = 3\left(\frac{0.2}{5.2}\right) \times 100 = 11\%$$

A) Taking it together

6. (*a*) $[\phi] = [BS] = [MT^{-2}A^{-1}]\,[L^2] = [ML^2T^{-2}A^{-1}]$

13. (*c*) As, $y = B[T^3] \Rightarrow [L] = B[T^3] \Rightarrow B = [LT^{-3}]$

14. (*b*) Using the relation for volume,

$$V = \text{Length} \times \text{Breadth} \times \text{Thickness}$$

$$= 12 \times 6 \times 2.45 = 176.4 \text{ cm}^3$$

$$V = 1.764 \times 10^2 \text{cm}^3$$

The minimum number of significant figure is 1 in breadth. Hence, the volume will contain only one significant figure. Therefore, $V = 2 \times 10^2$ cm^3.

16. (*b*) Since error is measured for 400 observations instead of 100 observations. So error will reduce by 1/4 factor.

17. (*c*) $Q = \frac{A^3B^3}{C\sqrt{D}}$

$$\frac{\Delta Q}{Q} = 3\left(\frac{\Delta A}{A}\right) + 3\left(\frac{\Delta B}{B}\right) + \left(\frac{\Delta C}{C}\right) + \frac{1}{2}\left(\frac{\Delta D}{D}\right)$$

Given that, $\frac{\Delta A}{A} \times 100 = 2,\ \frac{\Delta B}{B} \times 100 = 1$

$$\frac{\Delta C}{C} \times 100 = 3,\ \frac{\Delta D}{D} \times 100 = 4$$

$$\therefore \quad \frac{\Delta Q}{Q} \times 100 = (3 \times 2) + (3 \times 1) + (3) + \frac{1}{2} \times (4)$$

$$= \pm 14\%$$

21. (*a*) Dimensions of velocity are $[V] = [L][T^{-1}]$

So, dimensions of $[at^2] = [LT^{-1}]$

$\Rightarrow \quad [a]\,[T^2] = [LT^{-1}]$

$\Rightarrow \quad [a] = [LT^{-3}]$

Dimensions of $[bt] = [LT^{-1}] \Rightarrow [b]\,[T] = [LT^{-1}]$

$\Rightarrow \quad [b] = [LT^{-2}]$

Dimensions of $[c] = [LT^{-1}]$

22. (*b*) $f = \frac{1}{2\pi\sqrt{LC}}$

or $\sqrt{LC} = \frac{1}{2\pi f} = \frac{T}{2\pi}$

Thus, $\sqrt{LC}$ has the dimensions of time.

23. (*c*) m is mass per unit length.

24. (*d*) Rounding off 2.745 to 3 significant figures it would be 2.74. Rounding off 2.735 to 3 significant figures it would be 2.74.

25. (*c*) In this question, density should be reported to two significant figures.

$$\text{Density} = \frac{4.237\text{g}}{2.5 \text{ cm}^3} = 1.6948$$

On rounding off the number, we get density = 17

26. (*a*) Given, length $l = (16.2 \pm 0.1)$ cm

Breadth $b = (10.1 \pm 0.1)$ cm

Area $A = l \times b$

$= (16.2 \text{ cm}) \times (10.1 \text{ cm}) = 163.62 \text{ cm}^2$

Rounding off to three significant digits, area $A = 164$ cm^2

$$\frac{\Delta A}{A} = \frac{\Delta l}{l} + \frac{\Delta b}{b} = \frac{0.1}{16.2} + \frac{0.1}{10.1}$$

$$= \frac{1.01 + 1.62}{16.2 \times 10.1} = \frac{2.63}{163.62}$$

$$\Rightarrow \quad \Delta A = A \times \frac{2.63}{163.62} = 163.62 \times \frac{2.63}{163.62}$$

$$= 2.63 \text{ cm}^2$$

$\Delta A = 3$ cm^2 (By rounding off to one significant figure)

$\therefore$ Area, $A = A \pm \Delta A = (164 \pm 3)$ cm^2.

27. (*c*) (a) Work = force × distance $= [MLT^{-2}][L] = [ML^2T^{-2}]$

Torque = force × distance $= [ML^2T^{-2}]$

(b) Angular momentum $= mvr = [M][LT^{-1}][L] = [ML^2T^{-1}]$

$$\text{Planck's constant} = \frac{E}{\nu} = \frac{[ML^2T^{-2}]}{[T^{-1}]} = [ML^2T^{-1}]$$

(c) Tension = force = $[MLT^{-2}]$

$$\text{Surface tension} = \frac{\text{force}}{\text{length}} = \frac{[MLT^{-2}]}{[L]} = [ML^0T^{-2}]$$

(d) Impulse = force × time = $[MLT^{-2}][T] = [MLT^{-1}]$

Momentum = mass × velocity = $[M][LT^{-1}] = [MLT^{-1}]$

Note One should not be confused with the similar form tension in both the physical quantities-surface tension and tension. **Dimensional formula** for both of them is not same.

28. (*a*) Given, $A = 2.5\ ms^{-1} \pm 0.5\ ms^{-1}, B = 0.10\ s \pm 0.01s$

$$x = AB = (2.5)(0.10) = 0.25\ m$$

$$\frac{\Delta x}{x} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

$$= \frac{0.5}{2.5} + \frac{0.01}{0.10} = \frac{0.05 + 0.025}{0.25} = \frac{0.075}{0.25}$$

$\Delta x = 0.075 = 0.08$ m, rounding off to two significant figures.

$$AB = (0.25 \pm 0.08)\ m$$

29. (*a*) All given measurements are correct upto two decimal places. As here 5.00 mm has the smallest unit and the error in 5.00 mm is least (commonly taken as 0.01 mm if not specified), hence, 5.00 mm is most precise.

Note In solving these type of questions, we should be careful about units although their magnitude is same.

30. (*a*) Given length, $l = 5$ cm

Now, checking the errors with each options one by one, we get

$$\Delta l_1 = 5 - 4.9 = 0.1\text{cm}$$
$$\Delta l_2 = 5 - 4.805 = 0.195\ \text{cm}$$
$$\Delta l_3 = 5.25 - 5 = 0.25\ \text{cm}$$
$$\Delta l_4 = 5.4 - 5 = 0.4\ \text{cm}$$

Error Δl_1 is least.

Hence, 4.9 cm is most accurate.

31. (*c*) Given, Young's modulus, $Y = 1.9 \times 10^{11}\ N/m^2$

$1\ N = 10^5$ dyne

Hence, $Y = 19 \times 10^{11} \times 10^5\ dyne/m^2$

We know that 1 m = 100 cm

$\therefore$ $Y = 19 \times 10^{11} \times 10^5\ dyne/(100)^2\ cm^2$

$= 1.9 \times 10^{16-4}\ dyne/cm^2$

$Y = 19 \times 10^{12}\ dyne/cm^2$

Note While we are going through units conversion, we should keep in mind that proper relation between units are mentioned.

32. (*c*) Energy $= \frac{1}{2}mv^2$

$$[m] = \frac{[E]}{[v^2]} = [Ev^{-2}]$$

33. (*a*) $[FL^{-1}T^2] = [MLT^{-2}][L^{-1}][T^2] = [M]$

34. (*a*) Taking, $T = 2\pi\sqrt{\frac{R^3}{GM}}$

Substituting the dimensions, LHS = [T]

$$\text{RHS } 2\pi\left|\sqrt{\frac{R^3}{GM}}\right| = \sqrt{\frac{[L]^3}{[M^{-1}L^3T^{-2}][M]}} = \sqrt{[T]^2} = [T]$$

Thus, LHS = RHS for $T = 2\pi\sqrt{\frac{R^3}{GM}}$

35. (*c*) Given, voltage, $V = (100 \pm 5)$ V

Current, $I = (10 \pm 0.2)$ A

From Ohm's law, $V = IR$

$\therefore$ Resistance, $R = \frac{V}{I}$

Maximum percentage error in resistance

$$\left(\frac{\Delta R}{R} \times 100\right) = \left(\frac{\Delta V}{V} \times 100\right) + \left(\frac{\Delta I}{I} \times 100\right)$$

$$= \left(\frac{5}{100} \times 100\right) + \left(\frac{0.2}{10} \times 100\right)$$

$$= 5 + 2 = 7\%$$

36. (*d*) Density, $\rho = \frac{m}{\pi r^2 L}$

$$\therefore \quad \frac{\Delta\rho}{\rho} \times 100 = \left[\frac{\Delta m}{m} + \frac{2\Delta r}{r} + \frac{\Delta L}{L}\right] \times 100$$

After substituting the values, we get the maximum percentage error in density = 4%.

39. (*c*) Dimension of (ohm) R

$$= \frac{h}{e^2};\ (e = \text{charge} = \text{current} \times \text{time})$$

$$= \frac{[Et]}{[it]^2} = \frac{P}{i^2} = (R) \text{ as } P = \left(\frac{E}{t}\right)$$

41. (*c*) $B = \mu NI \Rightarrow \mu = \frac{B}{NI}$

$$Bqv = F$$

$$\Rightarrow \quad B = \frac{F}{qv} \text{ so } \mu = \frac{F}{qvNI}$$

$$\mu = [MLT^{-2}A^{-2}]$$

42. (*b*) $g = \frac{GM}{R^2}, K = \frac{1}{2}I\omega^2 = \frac{L^2}{2I}$

Further, L will remain constant.

$$\therefore \quad K \propto \frac{1}{I} \text{ or } K \propto \frac{1}{\frac{2}{5}MR^2}$$

or $K \propto R^{-2}$ and $g \propto R^{-2}$

43. (*b*) $[\text{Force}] = [MLT^{-2}]$

$$\therefore \quad 1N = \left(\frac{1}{100}\right)\left(\frac{1}{1000}\right)(3600)^2 = 129.6 \text{ units.}$$

46. (*c*) Magnetic force, $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$

or $F = qvB\sin\theta$

$$\therefore \quad [B] = \left[\frac{F}{qv}\right] = \frac{[MLT^{-2}]}{[AT][LT^{-1}]} = [MT^{-2}A^{-1}]$$

47. (*b*) Volume of liquid coming out of the tube per second,

$$V = \frac{p\pi r^4}{8\eta l} \Rightarrow \frac{V_2}{V_1} = \left(\frac{r_2}{r_1}\right)^4$$

$$\Rightarrow \quad V_2 = V_1\left(\frac{110}{100}\right)^4 = V_1(1.1)^4 = 1.4641V_1 \text{ unit}$$

$$\Rightarrow \quad \frac{\Delta V}{V_1} = \frac{V_2 - V_1}{V_1}$$

$$= \frac{1.4641V_1 - V_1}{V_1} \simeq 46\%$$

48. (*d*) When T is constant, pV = constant. When volume is decreased by 10% that is volume becomes $\frac{90}{100}$, the pressure must become 100/90. Thus, percentage increase in pressure

$$= \frac{(100-90)\times 100}{90} = 11.1\%$$

51. (*d*) Area of strip = lb

$$\therefore \quad \left(\frac{\Delta A}{A}\right)\times 100 = \left(\frac{\Delta l}{l}\right)\times 100 + \left(\frac{\Delta b}{b}\right)\times 100$$

$$= \frac{0.1}{10}\times 100 + \frac{0.01}{1}\times 100 = \pm 2\%$$

52. (*b*) Volume of cylinder

$$V = \pi r^2 L, \; r = \left(\frac{D}{2}\right)$$

$$\therefore \quad \left(\frac{\Delta V}{V}\right)\times 100 = 2\left(\frac{\Delta D}{D}\right)\times 100 + \left(\frac{\Delta L}{L}\right)\times 100$$

$$= 2\left(\frac{0.01}{4.0}\right)\times 100 + \left(\frac{0.1}{5}\right)\times 100 = 2.5\%$$

54. (*d*) Given, $A = 1.0\text{ m} \pm 0.2\text{ m}$, $B = 2.0\text{ m} \pm 0.2\text{ m}$

Let, $Y = \sqrt{AB} = \sqrt{(1.0)(2.0)} = 1.414$ m

Rounding off to two significant digit $Y = 1.4$ m

$$\frac{\Delta Y}{Y} = \frac{1}{2}\left[\frac{\Delta A}{A} + \frac{\Delta B}{B}\right]$$

$$= \frac{1}{2}\left[\frac{0.2}{1.0} + \frac{0.2}{2.0}\right] = \frac{0.6}{2\times 2.0}$$

$$\Rightarrow \quad \Delta Y = \frac{0.6Y}{2\times 2.0} = \frac{0.6\times 1.4}{2\times 2.0} = 0.212$$

Rounding off to one significant digit $\Delta Y = 0.2$ m

Thus, correct value for $\sqrt{AB} = r + \Delta r = 1.4 \pm 0.2$ m

55. (*d*) Given, fundamental quantities are momentum (p), area (A) and time (T).

We can write energy E as

$$E \propto p^a A^b T^c \Rightarrow E = kp^a A^b T^c$$

where, k is dimensionless constant of proportionality.

Dimensions of $E = [E] = [ML^2T^{-2}]$ and $[p] = [MLT^{-1}]$

$$[A] = [L^2] \Rightarrow [T] = [T]$$

$$[E] = [K][p]^a[A]^b[T]^c$$

Putting all the dimensions, we get

$$[ML^2T^{-2}] = [MLT^{-1}]^a[L^2]^b[T]^c$$

$$= [M^aL^{2b+a}T^{-a+c}]$$

By principle of homogeneity of dimensions,

$$a = 1, \; 2b + a = 2$$

$$\Rightarrow \quad 2b + 1 = 2 \Rightarrow b = 1/2$$

$$-a + c = -2$$

$$\Rightarrow \quad c = -2 + a = -2 + 1 = -1$$

Hence, $E = pA^{1/2}T^{-1}$

57. (*a*) In the given equation, $\frac{\alpha Z}{k\theta}$ should be dimensionless.

$$\therefore \quad \alpha = \frac{k\theta}{Z} \Rightarrow [\alpha] = \frac{[ML^2T^{-2}K^{-1}][K]}{[L]} = [MLT^{-2}]$$

and $p = \frac{\alpha}{\beta}$

$$\Rightarrow \quad [\beta] = \left[\frac{\alpha}{p}\right] = \frac{[MLT^{-2}]}{[ML^{-1}T^{-2}]} = [M^0L^2T^0]$$

58. (*d*) The dimensions of $E = [ML^2T^{-2}]$

Dimensions of M = [M]

Dimensions of L = $[ML^2T^{-1}]$

Dimensions of G = $[M^{-1}L^3T^{-2}]$

$$\therefore \text{ Dimensions of } \left[\frac{E^2L^2}{M^5G^2}\right] = \frac{[ML^2T^{-2}]^2[ML^2T^{-1}]^2}{[M]^5[M^{-1}L^3T^{-2}]^2}$$

$$= [ML^2T^{-2}]$$

59. (*a*) The fundamental frequency is $f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$

$$\Rightarrow \quad f = \frac{1}{2L}\sqrt{\frac{T}{\rho\pi\frac{D^2}{4}}} = \frac{1}{LD}\sqrt{\frac{T}{\pi\rho}}$$

$$\therefore \quad f \propto \frac{1}{LD} \quad \text{(as } T, \pi \text{ and } \rho \text{ are constants)}$$

61. (*d*) Given, $T \propto p^a d^b E^c$

We have $[M^0L^0T] = k[ML^{-1}T^{-2}]^a[ML^{-3}]^b[ML^2T^{-2}]^c$

where, k is a constant.

On comparing dimensions of similar terms, we have

$$[M^0L^0T] = k[M^{a+b+c}L^{-a-3b+2c}T^{-2a-2c}]$$

On comparing powers of M, we have

$$0 = a + b + c \quad \text{...(i)}$$

On comparing powers of L, we have

$$0 = -a - 3b + 2c \quad \text{...(ii)}$$

On comparing powers of T, we have

$$1 = -2a - 2c \quad \text{...(iii)}$$

On solving Eqs. (i), (ii) and (iii), we have

$$a = -\frac{5}{6}, \; b = \frac{1}{2}, \; c = \frac{1}{3}$$

B) Medical entrance special format questions

Assertion and reason

4. (*a*) Pressure and energy density have same dimensions, μ, $[M^1L^{-1}T^{-2}]$.

5. (*a*) $KE = \frac{1}{2}mv^2 \Rightarrow \frac{\Delta E}{E} = \frac{\Delta m}{m} + \frac{2\Delta v}{v}$

$$\Rightarrow \frac{\Delta E}{E} \times 100 = \left(\frac{\Delta m}{m}\right) \times 100 + 2\left(\frac{\Delta v}{v} \times 100\right) = 1 + 2(2) = 5\%$$

6. (*c*) $A = 4\pi r^2$

$$\frac{\Delta A}{A} \times 100 = 2\frac{\Delta r}{r} \times 100 \Rightarrow 2(0.3)\% = 0.6\%$$

10. (*a*) L/R and CR both have dimensions of time.

15. (*b*) $\sqrt{\frac{\text{Modulus of Elasticity}}{\text{Density}}} = \sqrt{\frac{ML^{-1}T^{-2}}{ML^{-3}T^0}} = m/s$

Match the column

3. $[A] = [L^2]$

$\therefore \quad [L] = [A^{1/2}]$

$[T] = [T] \Rightarrow [F] = [MLT^{-2}]$

$\therefore \quad [M] = [FL^{-1}T^2] = [FA^{1/2}T^2]$

Now, $[W] = [FL] = [FA^{1/2}]$

$[I] = [ML^2] = [FA^{-1/2}T^2A] = [FA^{1/2}T^2]$

$[V] = [LT^{-1}] = [A^{1/2}T^{-1}]$

C) Medical entrances' gallery

1. (*a*) Given, $F = A\sin Ct + B\cos Dx$ ...(i)

where, t = time and x = distance

As, we know that trigonometric ratios are dimensionless. This implies

$\sin Ct$ = dimensionless and $\cos Dx$ = dimensionless

Also, $[C] = \left[\frac{1}{t}\right] = [T^{-1}]$ and $[D] = \left[\frac{1}{x}\right] = [L^{-1}]$

As, Eq. (i) represents the force. So, A and B both have the dimensions as that of force. So, A/B is dimensionless, i.e. $[M^0L^0T^0]$

While, $\left[\frac{C}{D}\right] = \left[\frac{T^{-1}}{L^{-1}}\right] = [M^0LT^{-1}]$

2. (*e*) Electric potential is a scalar quantity.

3. (*e*) Option (e) is wrong because 1 astronomical unit $= 1.5 \times 10^{11}$m

4. (*b*) According to the question,

we have, $\frac{m}{t \cdot A} \propto p^n \cdot v^m$ or $m/t \cdot A = kp^n v^m$

where, k is proportionality constant.

Using principle of homogeneity, we get

$$[ML^{-2}T^{-1}] = k[ML^{-1}T^{-2}]^n \cdot [LT^{-1}]^m$$

or $$[ML^{-2}T^{-1}] = k[M]^n[L]^{-n+m}[T]^{-2n}$$

Equating both sides, we find $n = -m$ or $m = -n$

5. (*d*) We know that, energy of an emitted particle,

$$E = h\nu \Rightarrow h = \frac{E}{\nu}$$

Planck's constant

$$h = \frac{[ML^2T^{-2}]}{T^{-1}} = [ML^2T^{-1}] \quad ...(i)$$

and moment of inertia,

$$I = mr^2 \Rightarrow I = [ML^2] \quad ...(ii)$$

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{h}{I} = \left[\frac{ML^2T^{-1}}{ML^2}\right] = [T^{-1}] = \frac{1}{T}$$

i.e., $\frac{h}{I} = [T^{-1}]$ = frequency of a particle

6. (*a*) The force per unit length experienced due to two wires in which current is flowing in the same direction is given by

$$\frac{dF}{dl} = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{d} \Rightarrow \frac{[MLT^{-2}]}{[L]} = [\mu_0]\frac{[A^2]}{[L]}$$

$$\Rightarrow \frac{[MLT^{-2}]}{[L]} = \mu_0\left[\frac{Q^2}{T^2L}\right] \Rightarrow \mu_0 = [MLQ^{-2}]$$

7. (*a*) As electric flux is given by

$$\phi_E = Eds = \frac{F}{q}ds$$

$\therefore$ Dimensions of $\phi = \left[\frac{MLT^{-2}}{IT}\right][L^2] = [ML^3T^{-1}T^{-3}]$

8. (*c*) We know that surface tension $(S) = \frac{\text{Force [F]}}{\text{Length [L]}}$

So, $[S] = \frac{[MLT^{-2}]}{[L]} = [ML^0T^{-2}]$

Energy (E) = Force × displacement $\Rightarrow [E] = [ML^2T^{-2}]$

Velocity $(v) = \frac{\text{displacement}}{\text{time}} \Rightarrow [v] = [LT^{-1}]$

As, $S \propto E^a v^b T^c$

where, a, b, c are constants.

From the principle of homogeneity,

$$[LHS] = [RHS]$$

$$\Rightarrow [ML^0T^{-2}] = [ML^2T^{-2}]^a[LT^{-1}]^b[T]^c$$

$$\Rightarrow [ML^0T^{-2}] = [M^aL^{2a+b}T^{-2a-b+c}]$$

Equating the power on both sides, we get

$$a = 1, 2a + b = 0, b = -2$$

$$\Rightarrow -2a - b + c = -2$$

$$\Rightarrow c = (2a + b) - 2 = 0 - 2 = -2$$

So, $[S] = [Ev^{-2}T^{-2}] = [Ev^{-2}T^{-2}]$

9. (*a*) The dimensions of electrical resistance

$$R=\frac{V}{I}=\frac{\left(\frac{W}{q}\right)}{I}=\frac{\frac{W}{It}}{(I)}$$

$$=\frac{W}{I^2t}=[ML^2T^{-2}\,T^{-1}\,A^{-2}]=[ML^2\,T^{-3}\,A^{-2}]$$

Then, (A) → (2)

The dimensions of electrical potential

$$V=\frac{W}{q}=\frac{W}{It}=[ML^2T^{-2}A^{-1}T^{-1}]=[ML^2\,T^{-3}\,A^{-1}]$$

Then, (B) → (3)

The dimensions of specific resistance

$$\rho=R\frac{l}{A}=[ML^2T^{-3}A^{-2}]\,[L]=[ML^3T^{-3}A^{-2}]$$

Thus, (C) → (1) and the dimensions of specific conductance

$$\sigma=\frac{1}{\rho}=\frac{1}{[ML^3T^{-3}A^{-2}]}=[M^{-1}L^{-3}T^3A^2]$$

= not given in column

Thus, (D) → (4)

10. (*a*) Given, $x = g\ cm^2\ s^{-5}=[ML^2T^{-5}]$

$y = g\ s^{-1}=[ML^0T^{-1}]$

and $z = cms^{-2}=[M^0LT^{-2}]$

Now, $z^2=[M^0L^2T^{-4}]$

and $yz^2=[ML^0T^{-1}][M^0\ L^2T^{-4}]$

$=[ML^2T^{-5}]=x$

i.e., $x = yz^2$

11. (*c*) Ideal gas law is given by

$$pV=RT$$

$$\therefore\quad R=\frac{pV}{T}=\frac{\text{Unit of work}}{\text{Unit of temperature}}$$

$$\Rightarrow\quad R=\frac{\text{erg}}{\text{K}}$$

12. (*c*) From the formula, $\varepsilon=\frac{LdI}{dt}\Rightarrow\varepsilon=\frac{\text{henry-ampere}}{\text{second}}$

13. (*a*) Reynold's number describes the ratio of inertial force per unit area to viscous force per unit area for a flowing fluid.

Thus, Reynold's number is the ratio of two physical quantity of same dimension which cancel out each other. Hence, Reynold's number is dimensionless $[M^0L^0T^0]$ quantity.

14. (*a*) Substituting dimensions, $[MLT^{-2}]=\frac{x}{\sqrt{[ML^{-3}]}}$

$$\Rightarrow\quad x=[M^{3/2}L^{-1/2}T^{-2}]$$

15. (*c*) (a) Planck's constant $=\frac{\text{Energy}}{\text{Frequency}}=\frac{[ML^2T^{-2}]}{[T^{-1}]}=[ML^2T^{-1}]$

Angular momentum = Moment of inertia × Angular velocity

$=[ML^2]\times[T^{-1}]=[ML^2T^{-1}]$

(b) Impulse = Force × Time $=[MLT^{-2}][T]=[MLT^{-1}]$

and linear momentum = Mass × Velocity

$=[M][LT^{-1}]=[MLT^{-1}]$

(c) Moment of inertia = Mass × (Distance)^2

$=[ML^2]=[M]\,[L^2]$

and moment of force = Force × Distance

$=[MLT^{-2}][L]=[ML^2T^{-2}]$

(d) Energy $=[ML^2T^{-2}]$ and torque $=[ML^2T^{-2}]$

16. (*c*) The absolute error in the value $A - B$ will be $a + b$.

17. (*d*) We know that

$$F=ma\ \Rightarrow\ F=\frac{mv}{t}\ \Rightarrow\ m=\frac{Ft}{v}$$

$$\therefore\ \text{Dimensions of [M]}=\frac{[F][T]}{[v]}=[Fv^{-1}T]$$

18. (*c*) Let x kg be unit of mass, then

$$f=(x\,\text{kg})(1000\ \text{m})(100\ \text{s})^{-2}$$

$$\Rightarrow\quad 1000=x\times1000\times\frac{1}{10000}$$

$$\therefore\quad x=10000\ \text{kg}=10^4\ \text{kg}$$

19. (*a*) Dimensions of ϕ_B = Unit of B × Unit of A

$$=\frac{\text{newton}}{\text{ampere-metre}}\times\text{metre}^2$$

$$=\frac{\text{newton}\times\text{metre}}{\text{ampere}}$$

$$=\frac{\text{kg ms}^{-2}\times\text{m}}{\text{A}}=\text{kgm}^2\text{s}^{-2}\text{A}^{-1}=[ML^2T^{-2}A^{-1}]$$

20. (*c*) The unit of electric field E is

$$=\frac{\text{Newton}}{\text{Coulomb}}=\frac{\text{kg-metre sec}^{-2}}{\text{ampere-sec}}=\text{kg-metre sec}^{-3}\ \text{amp}^{-1}$$

$\therefore$ Dimensions of electric field are $[MLT^{-3}A^{-1}]$.

21. (*a*) $\left[\frac{nh}{2\pi qB}\right]$, where n and 2π are dimensionless quantities.

So, $[h]=[mvr]\ [qB]=\left[\frac{qvB}{v}\right]=\left[\frac{F}{v}\right]$

$$\therefore\quad\left[\frac{mvr}{F/v}\right]=\frac{[mvr][v]}{[F]}=\frac{[mv^2r]}{\left[\frac{mv^2}{r}\right]}=[r^2]=\text{Area}$$

22. (*a*) Here, $P=\frac{a^3b^2}{cd}$

$$\therefore\ \frac{\Delta P}{P}\times100=\left(\frac{3\Delta a}{a}+\frac{2\Delta b}{b}+\frac{\Delta c}{c}+\frac{\Delta d}{d}\right)\times100$$

$$=3\frac{\Delta a}{a}\times100+2\frac{\Delta b}{b}\times100+\frac{\Delta c}{c}\times100+\frac{\Delta d}{d}\times100$$

$$=3\times1+2\times2+3+4$$

$$=3+4+3+4=14\%$$

23. (*c*) Standards which are used to measure the fundamental quantities are called as fundamental units which are length, mass, time, temperature, current, luminous intensity, quantity of matter. Hence, magnetic field is not a fundamental quantity.

24. (*a*) From the definition, we know that the rate of doing work is called power.

So that,

Dimension of rate of doing work = dimension of power

$$=[ML^2T^{-3}]$$

25. (*a*) Given density of glass in CGS system= 2.8 g/cc

Value of density in SI system = 28 g/cm^3

$$=\frac{2.8\times10^{-3}\,\text{kg}}{10^{-6}\,\text{m}^3}$$

$$=2.8\times10^{-3}\,\text{kgm}^{-3}$$

26. (*a*) From Ohm's law $V=IR=[A\cdot ML^2T^{-3}A^{-2}]=[ML^2T^{-3}A^{-1}]$

27. (*b*) Given equation, $\frac{1}{p\beta}=\frac{y}{k_BT}$

where, p = pressure, y = distance,

k_B = Boltzmann constant and T = temperature

$$\text{Dimension of }[\beta]=\frac{[\text{Dimensions of }k_B]\,[\text{Dimensions of }T]}{[\text{Dimensions of }p]\,[\text{Dimensions of }y]}$$

$$=\frac{[ML^2T^{-3}][T]}{[ML^{-1}T^{-2}][L]}=[M^0L^2T^0]$$

28. (*b*) Dimensional formula of potential $[V]=[M\,L^2T^{-3}A^{-1}]$

Dimensional formula of $[RC]=[M^0\,L^0T]$.

29. (*b*) We knows,

Dimensions of moment of inertia $=[ML^2]$

Dimensions of moment of force $F=[MLT^{-2}]$

Dimensions of moment of velocity $(v)=[LT^{-2}]$

Dimensions of moment of work $(W)=[ML^2T^{-2}]$

Dimensions of moment of length $(l)=[L]$

$$\text{Here, Dimensions of }[X]=\frac{\text{Dimensions of }[IFv^2]}{\text{Dimensions of }[WL^3]}$$

$$=\frac{[ML^2]\,[MLT^{-2}]\,[LT^{-2}]^2}{[ML^2T^{-2}]\,[L^3]}$$

$$=\frac{[M^2L^5T^{-4}]}{[ML^{-5}T^{-2}]}=[MT^{-2}]$$

30. (*d*) A physical quantity

$$x=\frac{2K^3l^2}{m\sqrt{n}}$$

$$\text{Percentage error in }x=\left(3\times1+2\times2+3\times1+2\times\frac{1}{2}\right)=11\%$$

31. (*b*) Given, $m=\frac{A}{B}$

$\therefore$ Dimensions of $B=[B]=\frac{[A]}{[m]}$

Here, A = force $=[MLT^{-2}]$

and m = linear density = mass per unit length $=\frac{[M]}{[L]}$

$$\therefore\quad [B]=\frac{[MLT^{-2}]}{[ML^{-1}]}=[L^2T^{-2}]$$

This is same dimension as that of latent heat.

32. (*a*) $X=[M^aL^bT^c]$

Maximum % error in $X=a\,\alpha+b\,\beta+c\,\gamma$

CHAPTER 02

Vectors

In physics, we study a large number of physical quantities. These physical quantities can either have magnitude or magnitude and direction both. On this basis, we have broadly categorised physical quantities into two categories : scalars and vectors. In this chapter, we will study about the vector quantities and their operations in detail.

Inside

1) Scalar and vector quantities
- General points regarding vectors
- Types of vectors
- Multiplication and division of vectors by scalars

2) Addition of two vectors

3) Subtraction of two vectors
- Resolution of vectors
- Position vector and displacement vector

4) Product of two vectors
- Scalar product of two vectors
- Vector product of two vectors

1) Scalar and vector quantities

Scalar quantities

A physical quantity which can be described completely by its magnitude only and does not require a direction is called scalar quantity. Addition, subtraction, multiplication or division of scalar quantities can be done according to the general rules of algebra. Mass, volume, density, etc., are few examples of scalar quantities.

Vector quantities

A physical quantity which has magnitude and direction and obeys the triangle law of vector addition or equivalently the parallelogram law of vector addition is called a vector quantity. Displacement, velocity, acceleration, etc., are few examples of vectors.

Note The physical quantity current has both magnitude and direction but it is still a scalar as it disobeys the laws of vector algebra.

General points regarding vectors

General points regarding vectors are as follows

Vector notation

Usually a vector is represented by a bold capital letter with an arrow (or without arrow) over it, as $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, $\vec{\mathbf{C}}$ or simply **A, B, C.**

The magnitude of a vector **A** is represented by A or $|\mathbf{A}|$.

Graphical representation of a vector

Graphically a vector is represented by an arrow drawn to a chosen scale, parallel to the direction of the vector. The length of the arrow represents the magnitude and the tip of the arrow (arrow-head) represents the direction.

Suppose that a car A is running with a speed of 10 m/s towards east; and another car B is running with a speed of 20 m/s towards north-east. These velocities can be represented by vectors drawn in Fig. 2.1.

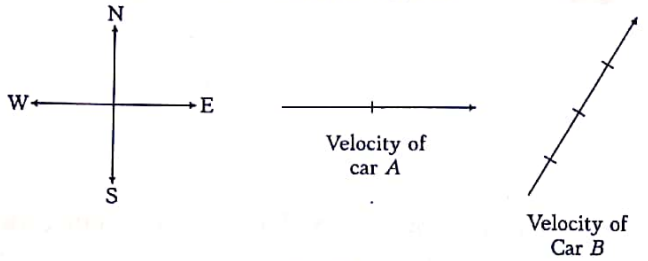


Fig. 2.1

To draw these vectors it has been assumed that 1 cm length represents a velocity of 5 m/s. The velocity vector of car A is an arrow of length 2 cm and its tip is directed towards east, while that of B is an arrow of length 4 cm with its tip directed towards north-east.

Angle between two vectors (θ)

Angle between two vectors, *i.e.*, smaller of two angles between the vectors when they are placed tail to tail.

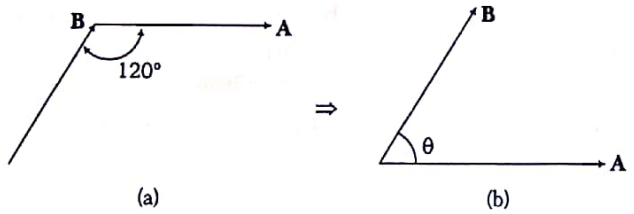


Fig. 2.2

For example, in Fig. 2.2 angle between **A** and **B** is 60° not 120°. Because in Fig. (a) their tails are not together while in Fig. (b) they are drawn as we desire.

If a vector is displaced parallel to itself, it does not change.

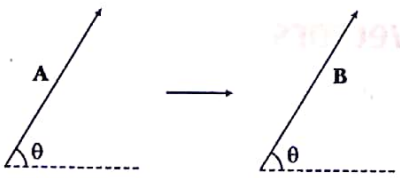


Fig. 2.3

∴ **B = A**

If a vector is rotated by an angle ($\theta \neq 2n\pi$, $n = 1, 2, 3, \ldots$), the vector is changed.

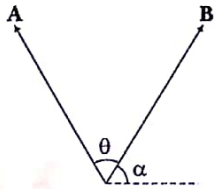


Fig. 2.4

∴ **B ≠ A**

Types of vectors

Different types of vectors are given below.

(i) **Polar vectors** These are those vectors which have a starting point or a point of application.
e.g., Displacement, force, etc.

(ii) **Axial vectors** These are those vectors which represent rotational effect and act along the axis of rotation in accordance with right hand screw rule.
e.g., Angular velocity, angular acceleration, torque, etc.

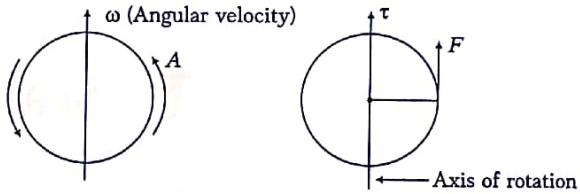


Fig. 2.5 Axial vectors

(iii) **Equal vectors** These are those vectors which have equal magnitude and same direction. In Fig. 2.5, **A** and **B** are equal vectors. *i.e.*, **A = B**

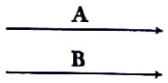


Fig. 2.6 Equal vectors

(iv) **Parallel vectors** These are those vectors which have same direction and magnitude may equal or different.

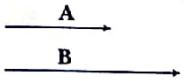


Fig. 2.7 Parallel vectors

(v) **Anti-parallel vectors** These are those vectors which have opposite direction and magnitude may equal or different.

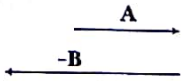


Fig. 2.8 Anti-parallel vectors

(vi) **Collinear vectors** These are those vectors which act along same line, *i.e.*, vectors lying in the same line. Angle between them can be zero or 180°. *e.g.*,

(i) ⟵⟶ ($\theta = 0°$) (ii) ⟶⟶ ($\theta = 0°$)

(iii) ⟵⟵ ($\theta = 180°$) (iv) ⟶⟵ ($\theta = 180°$)

(vii) **Zero vector** A vector having zero magnitude is known as zero vector. Its direction is not specified and hence is arbitrary. It is represented by **0**.

e.g., Displacement, velocity and acceleration of particle at rest or acceleration of a particle moving with uniform velocity.

(viii) **Unit vector** A vector whose magnitude is unit and points in a particular directions, is called unit vector. It is represented by $\hat{\mathbf{A}}$ (A cap or A hat). The unit vector along the direction of **A** is

$$\hat{\mathbf{A}} = \frac{\text{Vector}}{\text{Magnitude of the vector}}$$

$$= \frac{\mathbf{A}}{|\mathbf{A}|} \quad (\text{Here, } \mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}})$$

$\therefore \quad \mathbf{A} = \hat{\mathbf{A}}\,|\mathbf{A}|,$

where, $|\mathbf{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$

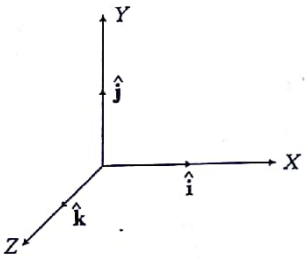


Fig. 2.9

In cartesian coordinates, $\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}$ are the unit vectors along X-axis, Y-axis and Z-axis, respectively.

where, $|\hat{\mathbf{i}}| = |\hat{\mathbf{j}}| = |\hat{\mathbf{k}}| = 1$

Note A unit vector is used to specify the direction of a vector.

(ix) **Coplanar vector** These are those vector which always lie in the same plane.

(x) **Negative vector** If the direction of a vector is reversed, the sign of the vector is reversed. It is called 'negative vector' of the original vector.

In Fig. 2.10, **B** is negative vector of **A**. *i.e.*, $\mathbf{A} = -\mathbf{B}$.

Fig. 2.10

Example 2.1 *Check whether the vector* $\left(\frac{\hat{\mathbf{i}}}{\sqrt{2}} + \frac{\hat{\mathbf{j}}}{\sqrt{2}}\right)$ *is a unit vector or not.*

Sol. A unit vector is a vector with magnitude 1.

The magnitude of given vector is

$$\left|\frac{\hat{\mathbf{i}}}{\sqrt{2}} + \frac{\hat{\mathbf{j}}}{\sqrt{2}}\right| = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = \sqrt{\frac{1}{2} + \frac{1}{2}} = \sqrt{1} = 1$$

As magnitude of given vector is 1.

$\therefore$ It is a unit vector.

Example 2.2 *Find the unit vector of* $4\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$.

Sol. We know that, for unit vector $\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$

$$= 4\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$$

$\therefore A_x = 4,\ A_y = -3,\ A_z = 1$

$$|\mathbf{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2} = \sqrt{(4)^2 + (-3)^2 + (1)^2} = \sqrt{26}$$

$\therefore$ Unit vector, $\hat{\mathbf{A}} = \frac{\mathbf{A}}{|\mathbf{A}|} = \frac{4\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}}}{\sqrt{26}}$

Example 2.3 *If vectors* $\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ *and* $\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - a\hat{\mathbf{k}}$ *are equal vectors, then find the value of a.*

Sol. Two vectors are said to be equal if their magnitude is equal

i.e., $\mathbf{A} = \mathbf{B}$

Here, $\mathbf{A} = \hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ and $\mathbf{B} = \hat{\mathbf{i}} - 3\hat{\mathbf{j}} - a\hat{\mathbf{k}}$

$\Rightarrow \quad \hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}} = \hat{\mathbf{i}} - 3\hat{\mathbf{j}} - a\hat{\mathbf{k}}$

Comparing on both sides, we get

$$a = -5$$

Example 2.4 *If* $A = 4\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$ *and* $B = 24\hat{\mathbf{i}} + 7\hat{\mathbf{j}}$, *find the vector having the same magnitude as B and parallel to A.*

Sol. $|\mathbf{B}| = \sqrt{24^2 + 7^2} = 25$

$$\hat{\mathbf{A}} = \frac{4\hat{\mathbf{i}} + 3\hat{\mathbf{j}}}{5}$$

Required vector, $\mathbf{r} = 25\frac{(4\hat{\mathbf{i}} + 3\hat{\mathbf{j}})}{5} = 20\hat{\mathbf{j}} + 15\hat{\mathbf{j}}$

Multiplication and division of vectors by scalars

The product of a vector **A** and a scalar m is a vector $m\mathbf{A}$ whose magnitude is m times the magnitude of **A** and which is in the direction or opposite to **A** according as the scalar m is positive or negative. Thus, $m(\mathbf{A}) = m\mathbf{A}$

Further, if m and n are two scalars, then

$$(m+n)\mathbf{A} = m\mathbf{A} + n\mathbf{A} \text{ and } m(n\mathbf{A}) = n(m\mathbf{A}) = (mn)\mathbf{A}$$

The division of vector **A** by a non-zero scalar m is defined as the multiplication of **A** by $\frac{1}{m}$.

Check point 2.1

1. Which is not a vector quantity?
(a) Current (b) Displacement
(c) Velocity (d) Acceleration

2. Pressure is
(a) scalar (b) vector
(c) Both (a) and (b) (d) None of these

3. Which of the following is not the vector quantity?
(a) Torque (b) Displacement
(c) Dipole moment (d) Electric flux

4. Which of the following is a vector?
(a) Pressure (b) Surface tension
(c) Moment of inertia (d) None of these

5. Surface area is
(a) scalar (b) vector
(c) Neither scalar nor vector (d) Both (a) and (b)

6. Which is a vector quantity?
(a) Angular momentum (b) Work
(c) Potential energy (d) Electric current

7. Which one is a vector quantity?
(a) Temperature (b) Momentum
(c) Work (d) Speed

8. Which is a vector quantity?
(a) Work (b) Power
(c) Torque (d) Gravitational constant

9. Find the vector that must be added to the vector $\hat{i} - 3\hat{j} + 2\hat{k}$ and $3\hat{i} + 6\hat{j} - 7\hat{k}$ so that the resultant vector is a unit vector along the y-axis.
(a) $-4\hat{i} - 2\hat{j} + 5\hat{k}$ (b) $-4\hat{i} + 2\hat{j} + 5\hat{k}$
(c) $4\hat{i} - 2\hat{j} + 5\hat{k}$ (d) $-4\hat{i} - 2\hat{j} - 5\hat{k}$

10. A vector multiplied by the number 0, results into
(a) 0 (b) **A** (c) **0** (d) $\hat{\mathbf{A}}$

11. Which of the following represents a unit vector?
(a) $\frac{|\mathbf{A}|}{\mathbf{A}}$ (b) $\frac{\mathbf{A}}{|\mathbf{A}|}$ (c) $\frac{\mathbf{A}}{\mathbf{A}}$ (d) $\frac{|\mathbf{A}|}{|\mathbf{A}|}$

12. Unit vector does not have any specified
(a) direction (b) magnitude
(c) unit (d) All of these

13. The magnitude of $\hat{i} + \hat{j}$ is
(a) 2 (b) 0 (c) $\sqrt{2}$ (d) 4

14. The unit vector along $\hat{i} + \hat{j}$ is
(a) $\hat{k}$ (b) $\hat{i} + \hat{j}$ (c) $\frac{\hat{i} + \hat{j}}{\sqrt{2}}$ (d) $\frac{\hat{i} + \hat{j}}{2}$

15. What happens, when we multiply a vector by (–2)?
(a) direction reverses and unit charges
(b) direction reverses and magnitude is doubled
(c) direction remains unchanged and unit changes
(d) None of the above

2 Addition of two vectors

Vectors cannot be added by simple laws of algebra, which are applicable to scalars. To add two vectors, we must follow certain laws. These laws are described below.

1. Triangle law of vector addition of two vectors

It states that if two vectors acting on a particle at the same time are represented in magnitude and direction by the two sides of a triangle taken in same order, then their resultant is represented in magnitude and direction by the third side of the triangle taken in opposite order.

As is evident from the figure that the resultant **R** is the same irrespective of the order in which the vectors **A** and **B** are taken.

Thus,

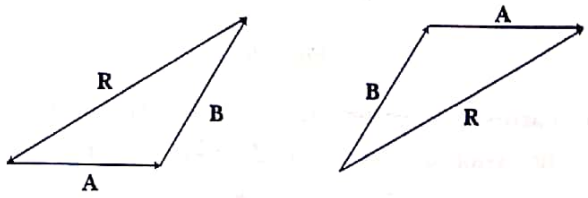


Fig. 2.11

$$\mathbf{R} = \mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$$

This is the geometrical method of vector addition.

2. Parallelogram law of vector addition

It states that if two vectors acting on a particle at the same time can be represented in magnitude and direction by the two adjacent sides of a parallelogram drawn from a point, then their resultant vector is represented in magnitude and direction by the diagonal of the parallelogram draw from the same point.

(i) Magnitude of resultant vector

Let **R** be the resultant of two vectors **A** and **B**. According to parallelogram law of vector addition, the resultant **R** is the diagonal of the parallelogram of which **A** and **B** are the adjacent sides as shown in figure.

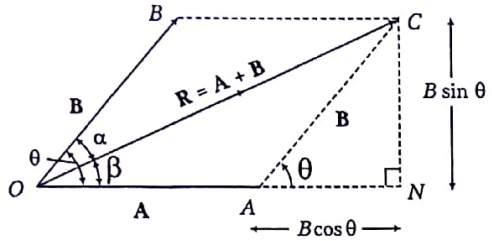


Fig. 2.12

Magnitude of **R** is given by

$$\boxed{R = \sqrt{A^2 + B^2 + 2AB\cos\theta}}$$

Here, θ = angle between **A** and **B**.

Special cases

(a) Resultant of two vectors will be maximum when they are parallel, *i.e.*, angle between then is zero.

or $\quad R_{max} = A + B$

(b) Resultant of two vectors will be minimum when they are anti-parallel, *i.e.*, angle between them is 180°.

or $\quad R_{min} = A - B$

(c) Resultant of two vectors of unequal magnitude can never be zero.

(ii) Direction of resultant vector

Let θ be the angle between **A** and **B**, then

$$|\mathbf{A} + \mathbf{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

If R makes on angle α or β with **A** or **B**, then

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

$$\tan\beta = \frac{A\sin\theta}{B + A\cos\theta}$$

Example 2.5 *Two forces whose magnitude are in the ratio 3 : 5 give a resultant of 28 N. If the angle of their inclination is 60°, find the magnitude of each force.*

Sol. Let A and B be the two forces, then $A = 3x$; $B = 5x$

$$R = 28\,\text{N} \quad \text{and} \quad \theta = 60°$$

Thus, $\quad \dfrac{A}{B} = \dfrac{3}{5}$

Now, $\quad R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$$\therefore \quad 28 = \sqrt{(3x)^2 + (5x)^2 + 2(3x)(5x)\cos 60°}$$

$$28 = \sqrt{9x^2 + 25x^2 + 15x^2} = 7x \quad \text{or} \quad x = 4$$

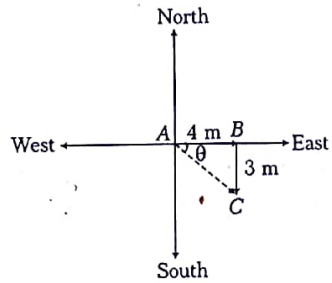
$$\therefore \quad F_A = 3 \times 4 = 12\,\text{N} \quad \text{and} \quad B = 5 \times 4 = 20\,\text{N}$$

Example 2.6 *A boy walks 4 m east and then 3 m south. Find the displacement of the boy.*

Sol. This can be easily done using triangle method of vector addition. In the figure shown

AB = displacement along east

BC = displacement along south



Then result displacement is given by the third side of ΔABC

$$\mathbf{AC} = \mathbf{AC} + \mathbf{BC}$$

$$|\mathbf{AC}| = \sqrt{AB^2 + BC^2} = \sqrt{16 + 9} = 5\,\text{m}$$

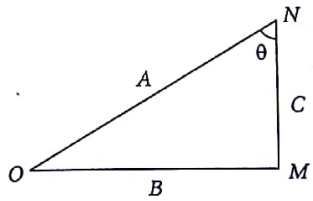
Let **AC** make angle θ with east direction.

Then $\quad \tan\theta = \dfrac{3}{4} \Rightarrow \theta = \tan^{-1}\left(\dfrac{3}{4}\right)$

Thus, the boy is actually displaced by 5 m at an angle $\tan^{-1}\left(\dfrac{3}{4}\right)$ south of east.

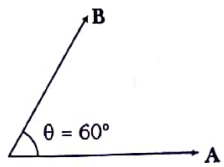
Example 2.7 *If* **A** = **B** + **C** *have scalar magnitudes of 5, 4, 3 units respectively, then find the angle between* **A** *and* **C**.

Sol. Here, triangle *OMN* is given with vectors **A**, **B** and **C** are its adjacent sides.

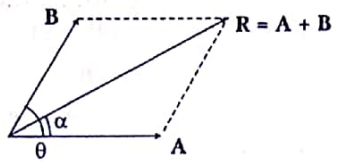


As, $\quad \cos\theta = \dfrac{MN}{ON} \Rightarrow \theta = \cos^{-1}\left(\dfrac{|\mathbf{C}|}{|\mathbf{A}|}\right) = \cos^{-1}\left(\dfrac{3}{5}\right)$

Example 2.8 *Find the sum of vectors* **A** *and* **B** *as shown in the figure, also find the direction of sum vector. Given A = 4 unit and B = 3 unit.*



Sol. According to the question, we draw the following figure.



Resultant of the vectors **A** and **B**

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$= \sqrt{16 + 9 + 2 \times 4 \times 3\cos 60°} = \sqrt{37} \text{ unit}$$

∴ Direction of the sum vector,

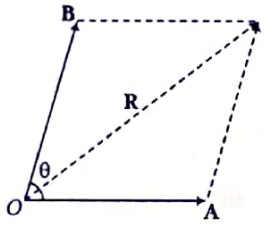
$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta} = \frac{3\sin 60°}{4 + 3\cos 60°} = 0.472$$

∴ $\alpha = \tan^{-1}(0.472) = 25.3°$

Thus, resultant of $|\mathbf{A}|$ and $|\mathbf{B}|$ is $\sqrt{37}$ unit at angle 25.3° from **A** in the direction shown in figure.

Example 2.9 *Two equal vectors have a resultant equal to either of the two. Find the angle between them.*

Sol. Let two vectors are **A** and **B**, inclined at an angle θ.



Resultant of the two vectors **A** and **B**

$$|\mathbf{R}| = \sqrt{|\mathbf{A}|^2 + |\mathbf{B}|^2 + 2|\mathbf{A}||\mathbf{B}|\cos\theta} \quad \text{...(i)}$$

Observing θ = 0°, gives

$$|\mathbf{R}| = |\mathbf{A}| + |\mathbf{B}|, \text{ which is true}$$

Let, $|\mathbf{A}| = |\mathbf{B}| = a$

According to the question, $|\mathbf{R}| = a$

From Eq. (i),

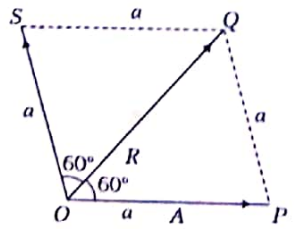
$$a = \sqrt{a^2 + a^2 + 2aa\cos\theta}$$

$\Rightarrow$ $a^2 = a^2 + a^2 + 2a^2\cos\theta$

$\Rightarrow$ $2a^2\cos\theta = -a^2 \Rightarrow \cos\theta = -1/2$

$\Rightarrow$ $-\cos\theta = 1/2 \Rightarrow \cos(180° - \theta) = \cos 60°$

$\Rightarrow$ $\theta = 120°$



Geometrically,

$$\angle SOP = 120°$$

Polygon law of vector addition for more than two vectors

It states that if any number of vectors, acting on a particle at the same time are represented in magnitude and direction by various sides of an open polygon taken in the same order, then their resultant is represented in magnitude and direction by the closing side of the polygon taken in opposite order.

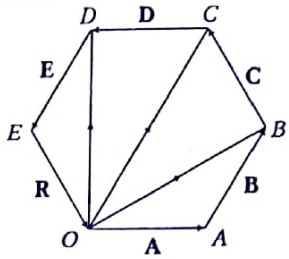


Fig. 2.13

Thus, in the figure

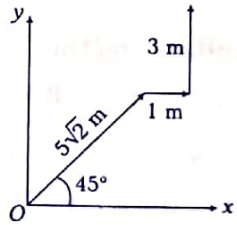
$$\mathbf{OE} = \mathbf{OA} + \mathbf{AB} + \mathbf{BC} + \mathbf{CD} + \mathbf{DE}$$

∴ $\boxed{\mathbf{R} = \mathbf{A} + \mathbf{B} + \mathbf{C} + \mathbf{D} + \mathbf{E}}$

Note

1. Resultant of two vectors is always located in their common plane.
2. Vector addition is commutative, *i.e.*, $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$
3. Vector addition is associative
   i.e., $\mathbf{A} + (\mathbf{B} + \mathbf{C}) = (\mathbf{A} + \mathbf{B}) + \mathbf{C}$
4. If vectors are of unequal magnitude then minimum three coplanar vectors are required for zero resultant.

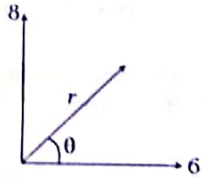
Example 2.10 *Find the resultant of three vectors direction shown in the figure.*



Sol. From the figure

On x-axis, $x = 5\sqrt{2}\cos 45° + 1 = 5 + 1 = 6\text{m}$

On y-axis, $y = 5\sqrt{2}\sin 45° + 3 = 5 + 3 = 8\text{m}$



Resultant of force vectors

$$r = \sqrt{x^2 + y^2} = \sqrt{(6)^2 + (8)^2} = 10\text{m}$$

$$\tan\theta = \frac{y}{x} = \frac{8}{6} = \frac{4}{3} \Rightarrow \theta = \tan^{-1}\left(\frac{4}{3}\right)$$

3 Subtraction of two vectors

Negative of a vector say $-\mathbf{A}$ is a vector of the same magnitude as vector $\mathbf{A}$ but pointing in a direction opposite to that of $\mathbf{A}$.

Thus, $\mathbf{A} - \mathbf{B}$ can be written as $\mathbf{A} + (-\mathbf{B})$ or $\mathbf{A} - \mathbf{B}$ is really the vector addition of $\mathbf{A}$ and $-\mathbf{B}$.

Fig. 2.14

Suppose angle between two vectors $\mathbf{A}$ and $\mathbf{B}$ is θ. Then angle between $\mathbf{A}$ and $-\mathbf{B}$ will be $180° - \theta$ as shown in Fig. 2.15 (b).

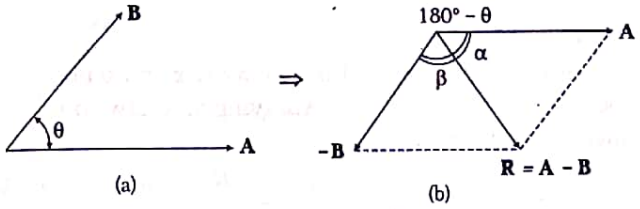


Fig. 2.15

Magnitude of resultant vector $\mathbf{R} = \mathbf{A} - \mathbf{B}$ will be thus given by

$$\mathbf{R} = |\mathbf{A} - \mathbf{B}| = \sqrt{A^2 + B^2 + 2AB\cos(180° - \theta)}$$

or $$R = \sqrt{A^2 + B^2 - 2AB\cos\theta} \quad \text{...(i)}$$

For direction of resultant vector $\mathbf{R}$ we will either calculate angle α or β, where,

$$\tan\alpha = \frac{B\sin(180° - \theta)}{A + B\cos(180° - \theta)} = \frac{B\sin\theta}{A - B\cos\theta} \quad \text{...(ii)}$$

or $$\tan\beta = \frac{A\sin(180° - \theta)}{B + A\cos(180° - \theta)} = \frac{A\sin\theta}{B - A\cos\theta} \quad \text{...(iii)}$$

Special cases

(i) If two vectors have equal magnitudes, *i.e.*, $|\mathbf{A}| = |\mathbf{B}| = a$, and θ is the angle between them, then

$$|\mathbf{A} - \mathbf{B}| = \sqrt{a^2 + a^2 - 2a^2\cos\theta} = 2a\sin\left(\frac{\theta}{2}\right)$$

Special case If $\theta = 60°$, then $2a\sin\left(\frac{\theta}{2}\right) = a$

i.e., $|\mathbf{A} - \mathbf{B}| = |\mathbf{A}| = |\mathbf{B}| = a$ at $\theta = 60°$

(ii) If two vectors are such that their sum and their difference have equal magnitude, then angle between the given vectors $(\theta) = 90°$

i.e., $|\mathbf{A} + \mathbf{B}| = |\mathbf{A} - \mathbf{B}|$

then, $\cos\theta = 0$ or $\theta = 90°$

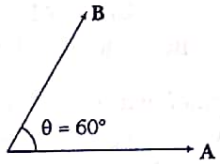
(iii) If $\mathbf{A} + \mathbf{B} = \mathbf{A} - \mathbf{B}$

then, $\mathbf{B} = \mathbf{0}$ (a null vector)

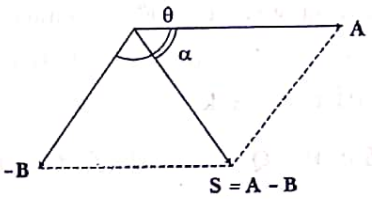
Note

1. The vector subtraction does't follow commutative law, *i.e.*, $\mathbf{A} - \mathbf{B} \neq \mathbf{B} - \mathbf{A}$.
2. The vector subtraction doesn't follow associative law, *i.e.*, $\mathbf{A} - (\mathbf{B} - \mathbf{C}) \neq (\mathbf{A} - \mathbf{B}) - \mathbf{C}$

Example 2.11 *Find the subtraction of vector* $\mathbf{A}$ *and* $\mathbf{B}$ *as shown in the figure, also find the direction of subtraction vector. Given* $A = 4$ *unit and* $B = 3$ *unit.*



Sol. According to the question, we draw the following figure.



Resultant of the vectors $\mathbf{A}$ and $\mathbf{B}$

$$R = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$
$$= \sqrt{16 + 9 - 2 \times 4 \times 3\cos 60°}$$
$$= \sqrt{13} \text{ unit}$$

and direction of the subtraction vector

$$\tan\alpha = \frac{B\sin\theta}{A - B\cos\theta} = \frac{3\sin 60°}{4 - 3\cos 60°} = 1.04$$

$$\therefore \quad \alpha = \tan^{-1}(1.04) = 46.1°$$

Thus, $\mathbf{A} - \mathbf{B}$ is $\sqrt{13}$ unit at 46.1° from $\mathbf{A}$ in the direction shown in figure.

Example 2.12 *Obtain the magnitude of* $2\mathbf{A} - 3\mathbf{B}$, *if* $\mathbf{A} = \hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ *and* $\mathbf{B} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$.

Sol. $2\mathbf{A} - 3\mathbf{B} = 2(\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}) - 3(2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}) = -4\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 7\hat{\mathbf{k}}$

$\therefore$ Magnitude of

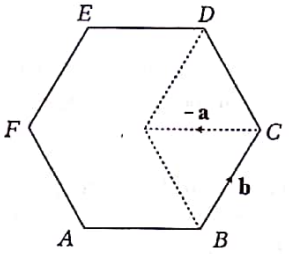
$$2\mathbf{A} - 3\mathbf{B} = \sqrt{(-4)^2 + (5)^2 + (-7)^2}$$
$$= \sqrt{16 + 25 + 49} = \sqrt{90}$$

Example 2.13 *If* **a** *and* **b** *are the vectors* **AB** *and* **BC**, *determined by the adjacent sides of a regular hexagon. What are the vectors determined by the other sides taken in order.*

Sol. Given, $\mathbf{AB} = \mathbf{a}$ and $\mathbf{BC} = \mathbf{b}$

From the method of vector addition (or subtraction) we can show that,

$$\mathbf{CD} = \mathbf{b} - \mathbf{a}$$



Then, $\mathbf{DE} = -\mathbf{AB} = -\mathbf{a}$

$\mathbf{EF} = -\mathbf{BC} = -\mathbf{b}$ and $\mathbf{FA} = -\mathbf{CD} = \mathbf{a} - \mathbf{b}$

Example 2.14 *The initial and final position vectors for a particle are* $(-3\ m)\,\hat{\mathbf{i}} + (2\ m)\,\hat{\mathbf{j}} + (8\ m)\,\hat{\mathbf{k}}$ *and* $(9\ m)\,\hat{\mathbf{i}} + (2\ m)\,\hat{\mathbf{j}} + (-8\ m)\,\hat{\mathbf{k}}$ *respectively, find the displacement of the particle.*

Sol. Displacement of particle = Final position − Initial position

$$= [(9m)\,\hat{\mathbf{i}} + (2m)\,\hat{\mathbf{j}} + (-8m)\,\hat{\mathbf{k}}] - [(-3m)\,\hat{\mathbf{i}} + (2m)\,\hat{\mathbf{j}} + (8m)\,\hat{\mathbf{k}}]$$

$$= (12\ m)\,\hat{\mathbf{i}} + (-16\ m)\,\hat{\mathbf{k}}$$

Example 2.15 *If* $|\mathbf{P} + \mathbf{Q}| = |\mathbf{P} - \mathbf{Q}|$, *find the angle between* **P** *and* **Q**.

Sol. Let the angle be θ.

Given, $|\mathbf{P} + \mathbf{Q}| = |\mathbf{P} - \mathbf{Q}|$

$\because$ Magnitude of both sides are also equal.

$$\therefore \quad \sqrt{P^2 + Q^2 + 2PQ\cos\theta} = \sqrt{P^2 + Q^2 + 2PQ\cos(180° - \theta)}$$

$$4PQ\cos\theta = 0 \Rightarrow \cos\theta = 0$$

$$\Rightarrow \quad \theta = 90°$$

Resolution of vectors

The resolution of a vector is opposite to vector addition. If a vector is resolved into two vectors whose combined effect is the same as that of the given vector, then the resolved vectors are called the components of the given vector.

Resolution of vectors into rectangular components

When a vector is splitted into components which are at right angle to each other then the components are called rectangular or orthogonal components of that vectors.

Refer Fig. (a)

We have resolved a two dimensional vector (in XY-plane) **R** in mutually perpendicular directions x and y.

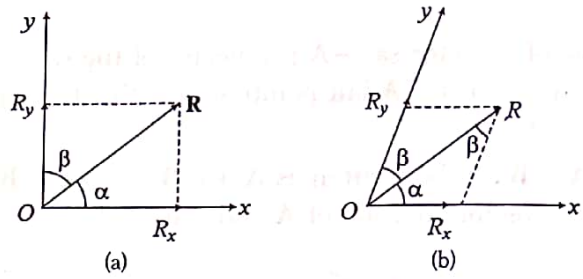


Fig. 2.16

Component along X-axis $= R_x = R\cos\alpha$ or $R\sin\beta$ and

Component along Y-axis $= R_y = R\cos\beta$ or $R\sin\alpha$.

If $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ be the unit vectors along X and Y-axes respectively, we can write

$$\mathbf{R} = R_x\hat{\mathbf{i}} + R_y\hat{\mathbf{j}}$$

Refer Fig. (b)

Vector **R** has been resolved in two axes x and y not perpendicular to each other. Applying sine law in the triangle shown , we have

$$\frac{R}{\sin[180° - (\alpha + \beta)]} = \frac{R_x}{\sin\beta} = \frac{R_y}{\sin\alpha}$$

or $$R_x = \frac{R\sin\beta}{\sin(\alpha + \beta)} \text{ and } R_y = \frac{R\sin\alpha}{\sin(\alpha + \beta)}$$

If $\alpha + \beta = 90°$, $R_x = R\sin\beta$ and $R_y = R\sin\alpha$

Also, $$\tan\alpha = \frac{R_y}{R_x} \text{ or } \alpha = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

Rectangular components of a vector in three dimensions

Let R_x, R_y and R_z are the components of resultant vector R in X, Y and Z axes respectively, and $\hat{\mathbf{i}}, \hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ are unit vectors along these directions. Then a vector R and its magnitude can be written as

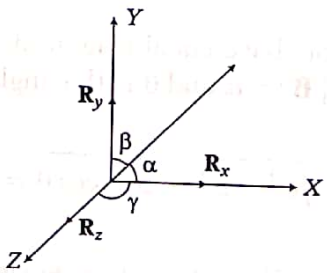


Fig. 2.17

$$R = \mathbf{R}_x + \mathbf{R}_y + \mathbf{R}_z \text{ or } R = R_x\hat{\mathbf{i}} + R_y\hat{\mathbf{j}} + R_z\hat{\mathbf{k}}$$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

This vector **R** makes an angle of

$$\alpha = \cos^{-1}\left(\frac{R_x}{R}\right) \text{ with } X\text{-axis.}$$

$$\beta = \cos^{-1}\left(\frac{R_y}{R}\right) \text{ with } Y\text{-axis.}$$

and $$\gamma = \cos^{-1}\left(\frac{R_z}{R}\right) \text{ with } Z\text{-axis.}$$

Note

1. A vector can be resolved into maximum infinite number of components.
   For example, $10\hat{\mathbf{i}} = \hat{\mathbf{i}} + \hat{\mathbf{i}} + \hat{\mathbf{i}} \ldots$ 10 times
   or $= \frac{\hat{\mathbf{i}}}{2} + \frac{\hat{\mathbf{i}}}{2} + \frac{\hat{\mathbf{i}}}{2} \ldots$ 20 times
2. Maximum number of rectangular components of a vector in a plane (2-dimensions) is two. But maximum number of rectangular components in space (3-dimensions) is three which are along X, Y and Z-axes.
3. A vector is independent of the orientation of axes but the components of that vector depends upon the orientation of axes.
4. The component of a vector along its perpendicular direction is always zero.

Example 2.16 *A vector is given by* $\mathbf{A} = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$. *Find the magnitude of* **A**, *unit vector along* **A** *and angles made by* **A** *with coordinate axes.*

Sol. We have, magnitude $|\mathbf{A}| = A = \sqrt{A_x^2 + A_y^2 + A_z^2}$

$$\sqrt{(3)^2 + (4)^2 + (t)^2} = 5\sqrt{2}$$

Unit vector, $$\hat{\mathbf{A}} = \frac{\mathbf{A}}{|\mathbf{A}|} = \frac{3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 5\hat{\mathbf{k}}}{5\sqrt{2}}$$

Angles made by **A** with coordinate axis

$$\cos\alpha = \frac{A_x}{|\mathbf{A}|} = \frac{3}{5\sqrt{2}} \Rightarrow \alpha = \cos^{-1}\left(\frac{3}{5\sqrt{2}}\right)$$

$$\cos\beta = \frac{A_y}{|\mathbf{A}|} = \frac{4}{5\sqrt{2}} \Rightarrow \beta = \cos^{-1}\left(\frac{4}{5\sqrt{2}}\right)$$

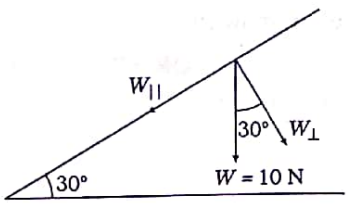
$$\cos\gamma = \frac{A_z}{|\mathbf{A}|} = \frac{5}{5\sqrt{2}}$$

$$\Rightarrow \quad \gamma = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$$

α, β and γ angles made by **A** with x, y, z axes, respectively.

Example 2.17 *Resolve a weight of 10 N in two directions which are parallel and perpendicular to a slope inclined at 30° to the horizontal.*

Sol. Component perpendicular to the plane



$$W_\perp = W\cos 30°$$
$$= (10)\frac{\sqrt{3}}{2} = 5\sqrt{3}\text{ N}$$

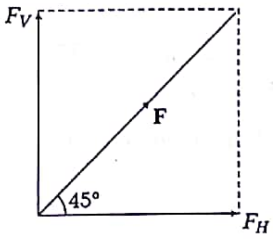
and component parallel to the plane

$$W_{||} = W\sin 30° = (10)\left(\frac{1}{2}\right) = 5\text{ N}$$

Example 2.18 *Resolve horizontally and vertically a force* $F = 8$ *N which makes an angle of* 45° *with the horizontal.*

Sol. Horizontal component of **F** is

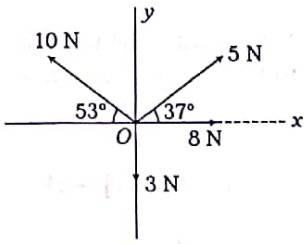
$$F_H = F\cos 45°$$
$$= (8)\left(\frac{1}{\sqrt{2}}\right) = 4\sqrt{2}\text{ N}$$



and vertical component of **F** is

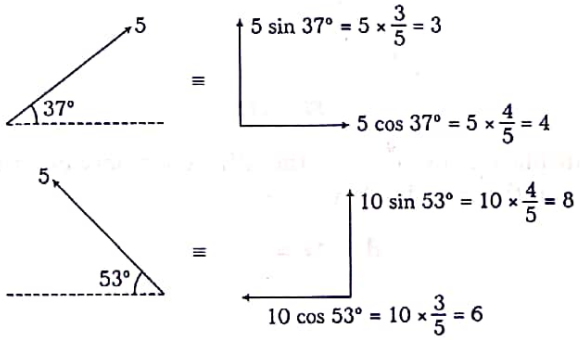
$$F_V = F\sin 45°$$
$$= (8)\left(\frac{1}{\sqrt{2}}\right)$$
$$= 4\sqrt{2}\text{ N}$$

Example 2.19 *Find the resultant of the following forces.*



$$\left[\sin 37° = \frac{3}{5}, \cos 37° = \frac{4}{5}, \sin 53° = \frac{4}{5}, \cos 53° = \frac{3}{5}\right]$$

Sol. Resolve the inclined vectors.



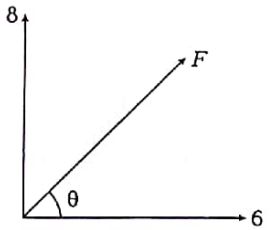
Now combine the components in x and y directions, by taking the proper sign.

$$F_x = 4 - 6 + 8 = 6$$

and $$F_y = 3 + 8 - 3 = 8$$

Resultant force, $F = \sqrt{(6)^2 + (8)^2} = 10\,\text{N}$

Direction of force, $\tan\theta = \dfrac{F_y}{F_x} = \dfrac{8}{6} = \dfrac{4}{3} \Rightarrow \theta = 53°$



The resultant of these forces is 10 N at 53° with the positive X-axis.

Example 2.20 *Find the angle that the vector* $\mathbf{A} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}$ *makes with* Y*-axis.*

Sol. According to the resolution of the vector

$$\cos\theta = \frac{A_y}{A} = \frac{3}{\sqrt{(2)^2 + (3)^2 + (-1)^2}} = \frac{3}{\sqrt{14}}$$

$$\therefore \quad \theta = \cos^{-1}\left(\frac{3}{\sqrt{14}}\right)$$

Position vector and displacement vector

If the coordinates of point A be (x, y, z), then the vector joining point A to the origin is called a position vector.

Position vector is given by

$$\boxed{\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}}$$

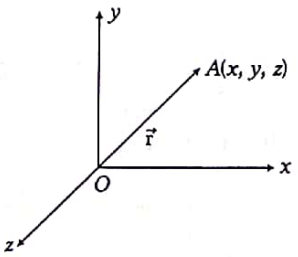


Fig. 2.18

The displacement vector is the difference between the final and the initial position vector.

$$\therefore \quad \boxed{\mathbf{d} = \Delta\mathbf{r} = \mathbf{r}_f - \mathbf{r}_i}$$

If the particle is displaced from (x_1, y_1, z_1) to (x_2, y_2, z_2), the displacement vector is

$$\boxed{\mathbf{d} = (x_2 - x_1)\,\hat{\mathbf{i}} + (y_2 - y_1)\,\hat{\mathbf{j}} + (z_2 - z_1)\,\hat{\mathbf{k}}}$$

Example 2.21 *A particle is moving from point* $A(1, 2, 3)$ *to the point* $B(4, 6, 9)$. *Find its displacement vector.*

Sol. The particle is moving from point $A(1, 2, 3)$ to the point $B(4, 6, 9)$

$\therefore$ According to the formula of displacement vector

$$\mathbf{d} = (x_2 - x_1)\hat{\mathbf{i}} + (y_2 - y_1)\hat{\mathbf{j}} + (z_2 - z_1)\hat{\mathbf{k}} \quad \text{...(i)}$$

where, $x_1, y_1, z_1 = 1, 2, 3$

$x_2, y_2, z_3 = 4, 6, 9$

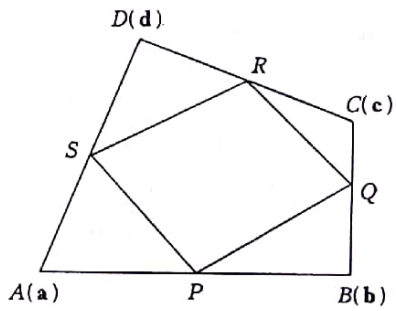
Putting these values, in Eq. (i), we get

$$\mathbf{d} = (4 - 1)\hat{\mathbf{i}} + (6 - 2)\hat{\mathbf{j}} + (9 - 3)\hat{\mathbf{k}}$$

$$= 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$$

Example 2.22 *If the mid-points of the consecutive sides of any quadrilateral are connected by straight line segments, prove that the resulting quadrilateral is a parallelogram.*

Sol. Let **a**, **b**, **c** and **d** be the position vectors of A, B, C and D.



Hence, position vectors of mid-points P, Q, R and S will be

$$\frac{\mathbf{a}+\mathbf{b}}{2}, \quad \frac{\mathbf{b}+\mathbf{c}}{2}, \quad \frac{\mathbf{c}+\mathbf{d}}{2} \quad \text{and} \quad \frac{\mathbf{d}+\mathbf{a}}{2}$$

We have,

$\mathbf{PQ}$ = position vector of Q-position vector of P

$$= \frac{\mathbf{b}+\mathbf{c}}{2} - \frac{\mathbf{a}+\mathbf{b}}{2} = \frac{\mathbf{c}-\mathbf{a}}{2} \quad \text{...(i)}$$

and $\mathbf{SR}$ = Position vector of R-position vector of S

$$= \frac{\mathbf{c}+\mathbf{d}}{2} - \frac{\mathbf{d}+\mathbf{a}}{2} = \frac{\mathbf{c}-\mathbf{a}}{2} \quad \text{...(ii)}$$

From Eq. (i) and (ii), we get

$$\mathbf{PQ} = \mathbf{SR}$$

Similarly, we can show that

$$\mathbf{QR} = \mathbf{PS}$$

$\therefore$ $PQRS$ is a parallelogram.

Check point 2.2

1. $\mathbf{A}+\mathbf{B}$ can also be written as
(a) $\mathbf{A}-\mathbf{B}$ (b) $\mathbf{B}-\mathbf{A}$ (c) $\mathbf{B}+\mathbf{A}$ (d) $\mathbf{B}\cdot\mathbf{A}$

2. If $\mathbf{P}+\mathbf{Q}=\mathbf{0}$, then which of the following is necessarily true?
(a) $\mathbf{P}=\mathbf{0}$ (b) $\mathbf{P}=-\mathbf{Q}$ (c) $\mathbf{Q}=\mathbf{0}$ (d) $\mathbf{P}=\mathbf{Q}$

3. Two vectors having magnitudes 8 and 10 can have maximum and minimum value of magnitude of their resultant as
(a) 12, 6 (b) 10, 3
(c) 18, 2 (d) None of these

4. $(\mathbf{P}+\mathbf{Q})$ is a unit vector along X-axis. If $\mathbf{P}=\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}}$, then what value is $\mathbf{Q}$?
(a) $\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}}$ (b) $\hat{\mathbf{j}}-\hat{\mathbf{k}}$
(c) $\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$ (d) $\hat{\mathbf{j}}+\hat{\mathbf{k}}$

5. For the resultant of two vectors to be maximum what must be the angle between them
(a) 0° (b) 60°
(c) 90° (d) 180°

6. Minimum number of forces of unequal magnitude whose vector sum can equal to zero is
(a) Two (b) Three
(c) Four (d) Any

7. Given that $\mathbf{P}+\mathbf{Q}+\mathbf{R}=0$. Which of the following statement is true?
(a) $|\mathbf{P}|+|\mathbf{Q}|=|\mathbf{R}|$ (b) $|\mathbf{P}+\mathbf{Q}|=|\mathbf{R}|$
(c) $|\mathbf{P}|-|\mathbf{Q}|=|\mathbf{R}|$ (d) $|\mathbf{P}-\mathbf{Q}|=|\mathbf{R}|$

8. $\mathbf{A}=2\hat{\mathbf{i}}+\hat{\mathbf{j}}$, $\mathbf{B}=3\hat{\mathbf{j}}-\hat{\mathbf{k}}$ and $\mathbf{C}=6\hat{\mathbf{i}}-2\hat{\mathbf{k}}$.
Value of $\mathbf{A}-2\mathbf{B}+3\mathbf{C}$ would be
(a) $20\hat{\mathbf{i}}+5\hat{\mathbf{j}}+4\hat{\mathbf{k}}$ (b) $20\hat{\mathbf{i}}-5\hat{\mathbf{j}}-4\hat{\mathbf{k}}$
(c) $4\hat{\mathbf{i}}+5\hat{\mathbf{j}}+20\hat{\mathbf{k}}$ (d) $5\hat{\mathbf{i}}+4\hat{\mathbf{j}}+10\hat{\mathbf{k}}$

9. If $\mathbf{A}=4\hat{\mathbf{i}}-3\hat{\mathbf{j}}$ and $\mathbf{B}=6\hat{\mathbf{i}}+8\hat{\mathbf{j}}$, then magnitude and direction of $\mathbf{A}+\mathbf{B}$ with X-axis will be
(a) 5, $\tan^{-1}(3/4)$
(b) $5\sqrt{5}$, $\tan^{-1}(1/2)$
(c) 10, $\tan^{-1}(5)$
(d) 25, $\tan^{-1}(3/4)$

10. A truck travelling due north at 20 ms^{-1} turns west and travels at the same speed. The change in its velocity will be
(a) 40 ms^{-1} N-W
(b) $20\sqrt{2}$ ms^{-1} N-W
(c) 40 ms^{-1} S-W
(d) $20\sqrt{2}$ ms^{-1} S-W

11. Resultant of two vectors $\mathbf{A}$ and $\mathbf{B}$ is given by $|\mathbf{R}|=\{|\mathbf{A}|-|\mathbf{B}|\}$, angle between $\mathbf{A}$ and $\mathbf{B}$ will be
(a) 90° (b) 180°
(c) 0° (d) None of these

12. If $|\mathbf{A}|=2$ and $|\mathbf{B}|=4$ and angle between them is 60°, then $|\mathbf{A}-\mathbf{B}|$
(a) $\sqrt{13}$ (b) $3\sqrt{3}$
(c) $\sqrt{3}$ (d) $2\sqrt{3}$

13. If $\mathbf{A}$ and $\mathbf{B}$ are two vectors such that $|\mathbf{A}+\mathbf{B}|=2|\mathbf{A}-\mathbf{B}|$, the angle between vectors $\mathbf{A}$ and $\mathbf{B}$ is
(a) 45° (b) 60°
(c) 30° (d) data insufficient

14. If $\mathbf{P}+\mathbf{Q}=\mathbf{P}-\mathbf{Q}$, then
(a) $\mathbf{P}=\mathbf{0}$ (b) $\mathbf{Q}=\mathbf{0}$ (c) $\mathbf{P}=1$ (d) $|\mathbf{Q}|=1$

15. Position of a particle in a rectangular coordinate system is (3, 2, 5). Then its position vector will be
(a) $3\hat{\mathbf{i}}+5\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ (b) $3\hat{\mathbf{i}}+2\hat{\mathbf{j}}+5\hat{\mathbf{k}}$
(c) $5\hat{\mathbf{i}}+3\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ (d) None of these

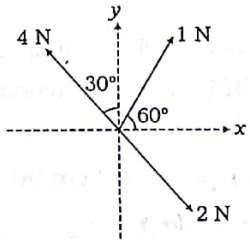
16. If a particle moves from point P (2, 3, 5) to point Q (3, 4, 5). Its displacement vector is given by
(a) $\hat{\mathbf{i}}+\hat{\mathbf{j}}+10\hat{\mathbf{k}}$ (b) $\hat{\mathbf{i}}+\hat{\mathbf{j}}+5\hat{\mathbf{k}}$
(c) $\hat{\mathbf{i}}+\hat{\mathbf{j}}$ (d) $2\hat{\mathbf{i}}+4\hat{\mathbf{j}}+6\hat{\mathbf{k}}$

17. At what angle should the two forces $2P$ and $\sqrt{2}P$ act so that the resultant force is $P\sqrt{10}$
(a) 45° (b) 60° (c) 90° (d) 120°

18. What vector must be added to the sum of two vectors $2\hat{\mathbf{i}}-\hat{\mathbf{j}}+3\hat{\mathbf{k}}$ and $3\hat{\mathbf{i}}-2\hat{\mathbf{j}}-2\hat{\mathbf{k}}$. So that the resultant is a unit vector along z-axis?
(a) $5\hat{\mathbf{i}}+\hat{\mathbf{k}}$ (b) $-5\hat{\mathbf{i}}+3\hat{\mathbf{j}}$
(c) $3\hat{\mathbf{j}}+5\hat{\mathbf{k}}$ (d) $-3\hat{\mathbf{j}}+2\hat{\mathbf{k}}$

19. Three vectors each of magnitude A are acting at a point such that angle between any two consecutive vectors in same plane is 60°. The magnitude of their resultant is
(a) $2A$ (b) $\sqrt{2}A$
(c) $\sqrt{3}A$ (d) $\sqrt{6}A$

20. Three forces acting on a body are shown in the figure. To have the resultant force only along the y-direction, the magnitude of the minimum additional force needed is



(a) $\frac{\sqrt{3}}{4}$ N (b) $\sqrt{3}$ N (c) 0.5 N (d) 1.5 N

4 Product of two vectors

The multiplication of two vector quantities cannot be done by simple algebraic method. The product of two vectors may be a scalar as well as a vector.

If the product of two vectors is a scalar quantity then it is called 'scalar product' (or dot product); if the product is a vector quantity then it is called 'vector product' (or cross product).

Scalar product of two vectors

The scalar product (or dot product) of two vectors is defined as the product of their magnitude with cosine of the angle between them.

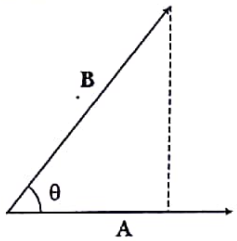


Fig. 2.19

Thus, if there are two vectors **A** and **B** having angle θ between them, then their scalar product is written as

$$\mathbf{A}\cdot\mathbf{B} = AB\cos\theta \quad \text{(a scalar quantity)}$$

e.g., Work done $(W) = \mathbf{F}\cdot\mathbf{s}$ and power $(P) = \mathbf{F}\cdot\mathbf{V}$

Note The scalar or dot product of two vectors **A** and **B** is denoted by **A · B** and is read as **A** dot **B**.

Important points regarding dot product

The following points should be remembered regarding the dot product.

(i) $\mathbf{A}\cdot\mathbf{B} = \mathbf{B}\cdot\mathbf{A}$

(ii) $\mathbf{A}\cdot(\mathbf{B}+\mathbf{C}) = \mathbf{A}\cdot\mathbf{B} + \mathbf{A}\cdot\mathbf{C}$

(iii) $\mathbf{A}\cdot\mathbf{A} = A^2$

(iv) $\mathbf{A}\cdot\mathbf{B} = A(B\cos\theta) = A$ (component of **B** along **A**)

or $\mathbf{A}\cdot\mathbf{B} = B(A\cos\theta) = B$ (component of **A** along **B**)

(v) $\hat{\mathbf{i}}\cdot\hat{\mathbf{i}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{k}} = (1)(1)\cos 0° = 1$

(vi) $\hat{\mathbf{i}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{k}} = \hat{\mathbf{i}}\cdot\hat{\mathbf{k}} = (1)(1)\cos 90° = 0$

(vii) $(a_1\hat{\mathbf{i}} + b_1\hat{\mathbf{j}} + c_1\hat{\mathbf{k}})\cdot(a_2\hat{\mathbf{i}} + b_2\hat{\mathbf{j}} + c_2\hat{\mathbf{k}})$

$$= a_1a_2 + b_1b_2 + c_1c_2$$

(viii) $\cos\theta = \dfrac{\mathbf{A}\cdot\mathbf{B}}{AB}$ (cosine of angle between **A** and **B**)

(ix) Two vectors are perpendicular if their dot product is zero. $(\theta = 90°)$

(x) Dot product of two vectors will be maximum when vectors are parallel, $\theta = 0$ $(\mathbf{A}\cdot\mathbf{B})_{max} = AB$

Projection of *A* along *B* (Components of dot product)

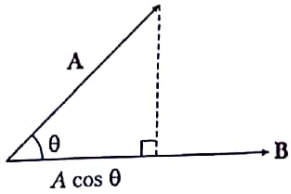


Fig. 2.20

(i) **In scalar form** Projection or scalar component of **A** along **B**

$$= A\cos\theta = A\times\frac{\mathbf{A}\cdot\mathbf{B}}{AB} = \frac{\mathbf{A}\cdot\mathbf{B}}{B} = \mathbf{A}\cdot\hat{\mathbf{B}}$$

(ii) **In vector form** Projection or vector component of **A** along **B**

$$= (A\cos\theta)\,\hat{\mathbf{B}} = \left(A\times\frac{\mathbf{A}\cdot\mathbf{B}}{AB}\right)\hat{\mathbf{B}} = \frac{\mathbf{A}\cdot\mathbf{B}}{B}\cdot\hat{\mathbf{B}} = (\mathbf{A}\cdot\hat{\mathbf{B}})\,\hat{\mathbf{B}}$$

Example 2.23 *Find the component of vector* $\mathbf{A}+\mathbf{B}$ *along*
(i) X-axis, *(ii)* **C**.
Given $\mathbf{A} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}}$, $\mathbf{B} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{k}}$ *and* $\mathbf{C} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$.

Sol. $\mathbf{A}+\mathbf{B} = (\hat{\mathbf{i}} - 2\hat{\mathbf{j}}) + (2\hat{\mathbf{i}} + 3\hat{\mathbf{k}}) = 3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

(i) Component of $\mathbf{A}+\mathbf{B}$ along X-axis is 3.

(ii) Component of $\mathbf{A}+\mathbf{B} = \mathbf{R}$ (say) along **C** is

$$\therefore \quad \mathbf{R}\cdot\mathbf{C} = RC\cos\theta$$

$$R\cos\theta = \frac{\mathbf{R}\cdot\mathbf{C}}{C} = \frac{(3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})\cdot(\hat{\mathbf{i}} + \hat{\mathbf{j}})}{\sqrt{(1)^2 + (1)^2}}$$

$$= \frac{3-2}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

Example 2.24 *Work done by a force* **F** *on a body is* $W = \mathbf{F}\cdot\mathbf{d}$, *where* **d** *is the displacement of body. Given that under a force* $\mathbf{F} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$ *N a body is displaced from position vector* $\mathbf{r}_1 = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}})$ *m to the position vector* $\mathbf{r}_2 = (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$ *m. Find the work done by this force.*

Sol. The body is displaced from $\mathbf{r}_1$ to $\mathbf{r}_2$. Therefore, displacement of the body is

$$\mathbf{d} = \mathbf{r}_2 - \mathbf{r}_1 = (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) - (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}})$$

$$= (-\hat{\mathbf{i}} - 2\hat{\mathbf{j}})\,\text{m}$$

Now, work done by the force is

$$W = \mathbf{F}\cdot\mathbf{d}$$

$$= (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})\cdot(-\hat{\mathbf{i}} - 2\hat{\mathbf{j}})$$

$$= (2)(-1) + (3)(-2) = -8\ \text{J}$$

Example 2.25 *Find the angle between two vectors*

$$\mathbf{A} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}} \text{ and } \mathbf{B} = \hat{\mathbf{i}} - \hat{\mathbf{k}}.$$

Sol.
$$A = |\mathbf{A}| = \sqrt{(2)^2 + (1)^2 + (-1)^2} = \sqrt{6}$$
$$B = |\mathbf{B}| = \sqrt{(1)^2 + (-1)^2} = \sqrt{2}$$
$$\mathbf{A} \cdot \mathbf{B} = (2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}) \cdot (\hat{\mathbf{i}} - \hat{\mathbf{k}})$$
$$= (2)(1) + (-1)(-1) = 3$$

Now, $$\cos\theta = \frac{\mathbf{A} \cdot \mathbf{B}}{AB} = \frac{3}{\sqrt{6} \cdot \sqrt{2}} = \frac{3}{\sqrt{12}} = \frac{\sqrt{3}}{2}$$

$\therefore$ $$\theta = 30°$$

Example 2.26 *Prove that the vectors* $\mathbf{A} = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$ *and* $\mathbf{B} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ *are mutually perpendicular.*

Sol.
$$\mathbf{A} \cdot \mathbf{B} = (2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}}) \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$$
$$= (2)(1) + (-3)(1) + (1)(1)$$
$$= 0 = AB\cos\theta$$

$\therefore$ $\cos\theta = 0$ (as $A \neq 0, B \neq 0$)

or $\theta = 90°$ ($\because \cos 90° = 0$)

or the vectors **A** and **B** are mutually perpendicular.

Example 2.27 *Find the (i) scalar component and (ii) vector component of* $\mathbf{A} = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ *on* $\mathbf{B} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$.

Sol. (i) Scalar component of **A** along **B** is

$$A\cos\theta = \mathbf{A} \cdot \hat{\mathbf{B}} = (3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 5\hat{\mathbf{k}}) \cdot \frac{(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})}{\sqrt{3}}$$
$$= \frac{3 + 4 + 5}{\sqrt{3}} = \frac{12}{\sqrt{3}} = 4\sqrt{3}$$

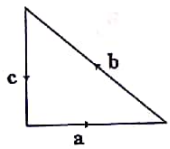
(ii) Vector component of **A** along **B** is

$$(A\cos\theta)\hat{\mathbf{B}} = (\mathbf{A} \cdot \hat{\mathbf{B}})\hat{\mathbf{B}}$$
$$= (4\sqrt{3})\frac{(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})}{\sqrt{3}} = 4\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$$

Example 2.28 *Show that the vectors* $\mathbf{a} = 3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\mathbf{b} = \hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ *and* $\mathbf{c} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 4\hat{\mathbf{k}}$ *form a right angled triangle. Also find the magnitude of* **a**, **b** *and* **c**.

Sol. Given, $\mathbf{a} = 3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ and $\mathbf{b} = \hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$

$$\mathbf{c} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 4\hat{\mathbf{k}}$$



We have $\mathbf{b} + \mathbf{c} = (\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}) + (2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 4\hat{\mathbf{k}})$

$$= 3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}} = \mathbf{a}$$

Hence, **a**, **b** and **c** are coplanar.

Also, we observe that no two of these vectors are parallel, therefore, the given vectors form a triangle. Further,

$$\mathbf{a} \cdot \mathbf{c} = (3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}) \cdot (2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 4\hat{\mathbf{k}}) = 0$$

Dot product of two non-zero vectors is zero. Hence, they are perpendicular so they form a right angled triangle.

$\therefore$ Magnitude of a, $|\mathbf{a}| = \sqrt{9 + 4 + 1} = \sqrt{14}$,

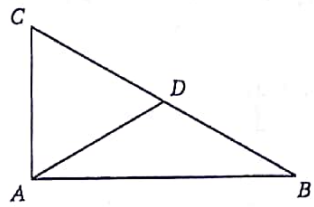
Magnitude of b, $|\mathbf{b}| = \sqrt{1 + 9 + 25} = \sqrt{35}$

and magnitude of c, $|\mathbf{c}| = \sqrt{4 + 1 + 16} = \sqrt{21}$

Example 2.29. *Prove that the mid-point of the hypotenuse of right angled triangle is equidistant from its vertices.*

Sol. Here, $\angle CAB = 90°$, let D be the mid-point of hypotenuse, we have

$$\mathbf{BD} = \mathbf{DC}$$
$$\mathbf{AB} = \mathbf{AD} + \mathbf{DB}$$
$$\mathbf{AC} = \mathbf{AD} + \mathbf{DC} = \mathbf{AD} + \mathbf{BD} \quad \ldots \text{(i)}$$



Since, $\angle BAC = 90°$

$\therefore$ $$\mathbf{AB} \perp \mathbf{AC}$$
$$(\mathbf{AD} + \mathbf{DB}) \cdot (\mathbf{AD} + \mathbf{BD}) = 0$$
$$(\mathbf{AD} - \mathbf{BD}) \cdot (\mathbf{AD} + \mathbf{BD}) = 0$$
$$AD^2 - BD^2 = 0$$

$\therefore$ $AD = BD$ also, $BD = DC$

$\because$ D is mid-point of BC.

Thus, $|AD| = |BD| = |DC|$.

Hence, its proved that the mid-point of the hypotenuse of right angled triangle is equidistant from its vertices.

Example 2.30. *Find the projection of* $\mathbf{A} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$ *on* $\mathbf{B} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$.

Sol. Projection of **A** on **B** $= A\cos\theta$ (θ = angle between **A** and **B**)

$$= \frac{\mathbf{A} \cdot \mathbf{B}}{B} = (2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}) \cdot \frac{(\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}})}{\sqrt{(1)^2 + (-2)^2 + (1)^2}} = \frac{2 + 2 + 1}{\sqrt{6}} = \frac{5}{\sqrt{6}}$$

Vector product of two vectors

The vector product or cross product of two vectors is defined as a vector having magnitude equal to the product of their magnitudes with the sine of angle between them, and its direction is perpendicular to the plane containing both the vectors.

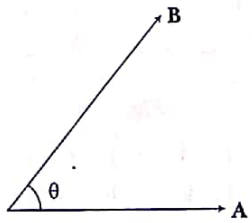


Fig. 2.21

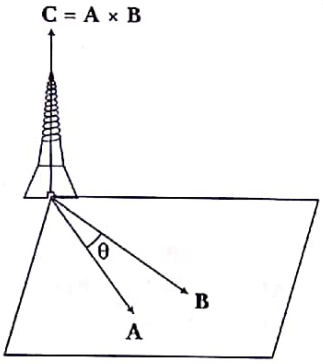
Thus, if **A** and **B** are two vectors, then their vector product i.e., $\mathbf{A} \times \mathbf{B}$ is a vector **C** defined by $\mathbf{C} = \mathbf{A} \times \mathbf{B} = AB \sin\theta\, \hat{\mathbf{n}}$.

where $\hat{\mathbf{n}}$ is a unit vector perpendicular to the plane of **A** and **B**.

Right-hand thumb rule

The direction of **C** (or of $\hat{\mathbf{n}}$) is perpendicular to the plane containing **A** and **B** and its sense is decided by right hand screw rule.

If the fingers of the right hand be curled in the direction in which vector **A** must be turned through the smaller included angle θ to coincide with the direction of vector **B**, the thumb points in the direction of **C** as shown in Fig. 2.19.



Plane of A and B

Fig. 2.22

e.g., Torque $(\tau) = \mathbf{r} \times \mathbf{F}$ and velocity $(\mathbf{v}) = \omega \times \mathbf{r}$

Important points about vector product

(i) $\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A}$

(ii) The magnitude of cross product of two parallel vectors is zero, as $|\mathbf{A} \times \mathbf{B}| = AB \sin\theta$ and $\theta = 0°$ for two parallel vectors. Thus,

$$\hat{\mathbf{i}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}} \times \hat{\mathbf{k}} = 0$$

(iii) If two vectors are perpendicular to each other, we have $\theta = 90°$ and therefore, $\sin\theta = 1$ So that $\mathbf{A} \times \mathbf{B} = AB\, \hat{\mathbf{n}}$. The vectors **A**, **B** and $\mathbf{A} \times \mathbf{B}$ thus form a right handed system of mutually perpendicular vectors. It follows at once from the above that in case of the orthogonal triad of unit vectors $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ (each perpendicular to each other)

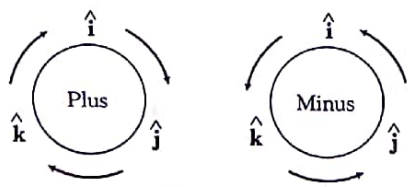


Fig. 2.23

$$\hat{\mathbf{i}} \times \hat{\mathbf{j}} = -\hat{\mathbf{j}} \times \hat{\mathbf{i}} = \hat{\mathbf{k}}$$

$$\hat{\mathbf{j}} \times \hat{\mathbf{k}} = -\hat{\mathbf{k}} \times \hat{\mathbf{j}} = \hat{\mathbf{i}}$$

and $$\hat{\mathbf{k}} \times \hat{\mathbf{i}} = -\hat{\mathbf{i}} \times \hat{\mathbf{k}} = \hat{\mathbf{j}}$$

(iv) $\mathbf{A} \times (\mathbf{B} + \mathbf{C}) = \mathbf{A} \times \mathbf{B} + \mathbf{A} \times \mathbf{C}$

(v) A vector product can be expressed in terms of rectangular components of the two vectors and put in the determinant form as may be seen from the following

Let $\mathbf{A} = a_1\hat{\mathbf{i}} + b_1\hat{\mathbf{j}} + c_1\hat{\mathbf{k}}$

and $\mathbf{B} = a_2\mathbf{i} + b_2\mathbf{j} + c_2\mathbf{k}$

Putting it in determinant form, we have

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

It may be noted that the scalar components of the first vector **A** occupy the middle row of the determinant.

(vi) A unit vector $(\hat{\mathbf{n}})$ perpendicular to **A** as well as **B** is given by $\hat{\mathbf{n}} = \dfrac{\mathbf{A} \times \mathbf{B}}{|\mathbf{A} \times \mathbf{B}|}$

(vii) If **A**, **B** and **C** are coplanar, then $[\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})] = 0$

(viii) Angle between $(\mathbf{A} + \mathbf{B})$ and $(\mathbf{A} \times \mathbf{B})$ is 90°.

(ix) Two vectors can be shown parallel to one another if

(a) The coefficient of $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ of both the vectors bear a constant ratio. For example, a vector $\mathbf{A} = a_1\hat{\mathbf{i}} + b_1\hat{\mathbf{j}} + c_1\hat{\mathbf{k}}$ is parallel to another vector $\mathbf{B} = a_2\hat{\mathbf{i}} + b_2\hat{\mathbf{j}} + c_2\hat{\mathbf{k}}$ if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

(b) The cross product of both the vectors is zero. For instance **A** and **B** are parallel to each other, if

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{k}} & \hat{\mathbf{j}} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

(x) The area of triangle bounded by vectors **A** and **B** is $\frac{1}{2}|\mathbf{A} \times \mathbf{B}|$.

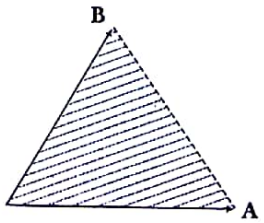


Fig. 2.24

Area of triangle *ABC* if position vector of *A* is **a** position vector of *B* is **b** and position vector of *C* is **c**.

Area of triangle $ABC = \frac{1}{2}|\mathbf{a} \times \mathbf{b} + \mathbf{b} \times \mathbf{c} + \mathbf{c} \times \mathbf{a}|$

(xi) Area of parallelogram shown in figure is,

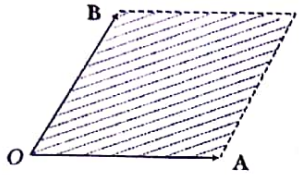


Fig. 2.25

$\therefore$ Area of parallelogram $= |\mathbf{A} \times \mathbf{B}| = \frac{1}{2}|d_1 \times d_2|$

where d_1 and d_2 are diagonals.

Example 2.31 *Let a force* **F** *be acting on a body free to rotate about a origin and let* **r** *the position vector of any point on the line of action of the force. Then torque* (τ) *of this force about origin is defined as*

$$\tau = \mathbf{r} \times \mathbf{F}$$

Given, $\mathbf{F} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}})\ N$

and $\mathbf{r} = (\hat{\mathbf{i}} - \hat{\mathbf{j}} + 6\hat{\mathbf{k}})\ m$

Find the torque of this force.

Sol. Given, $\mathbf{F} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}})$N and $\mathbf{r} = (\hat{\mathbf{i}} - \hat{\mathbf{j}} + 6\hat{\mathbf{k}})$m

$\therefore$ Torque $\tau = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & -1 & 6 \\ 2 & 3 & -1 \end{vmatrix}$

$$= \hat{\mathbf{i}}(1-18) + \hat{\mathbf{j}}(12+1) + \hat{\mathbf{k}}(3+2)$$

or $\tau = (-17\hat{\mathbf{i}} + 13\hat{\mathbf{j}} + 5\hat{\mathbf{k}})$ N-m

Example 2.32 *Find a unit vector perpendicular to* $\mathbf{A} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$ *and* $\mathbf{B} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$ *both.*

Sol. Given, $\mathbf{A} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\mathbf{B} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$

As we have read, $\mathbf{C} = \mathbf{A} \times \mathbf{B}$ is a vector perpendicular to both **A** and **B**. Hence, a unit vector $\hat{\mathbf{n}}$ perpendicular to **A** and **B** can be written as

$$\hat{\mathbf{n}} = \frac{\mathbf{C}}{C} = \frac{\mathbf{A} \times \mathbf{B}}{|\mathbf{A} \times \mathbf{B}|}$$

Here, $\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 3 & 1 \\ 1 & -1 & 1 \end{vmatrix}$

$$= \hat{\mathbf{i}}(3+1) + \hat{\mathbf{j}}(1-2) + \hat{\mathbf{k}}(-2-3)$$

$$= 4\hat{\mathbf{i}} - \hat{\mathbf{j}} - 5\hat{\mathbf{k}}$$

Further, $|\mathbf{A} \times \mathbf{B}| = \sqrt{(4)^2 + (-1)^2 + (-5)^2} = \sqrt{42}$

$\therefore$ The desired unit vector is $\hat{\mathbf{n}} = \frac{\mathbf{A} \times \mathbf{B}}{|\mathbf{A} \times \mathbf{B}|}$

or $\hat{\mathbf{n}} = \frac{1}{\sqrt{42}}(4\hat{\mathbf{i}} - \hat{\mathbf{j}} - 5\hat{\mathbf{k}})$

Example 2.33 *Show that the vector* $\mathbf{A} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ *is parallel to a vector* $\mathbf{B} = 3\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$.

Sol. A vector **A** is parallel to an another vector **B** if it can be written as

$$\mathbf{A} = m\mathbf{B}$$

Here, $\mathbf{A} = (\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}) = \frac{1}{3}(3\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 6\hat{\mathbf{k}})$

$(\because \mathbf{B} = 3\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 6\hat{\mathbf{k}})$

$\therefore$ $\mathbf{A} = \frac{1}{3}\mathbf{B}$

This implies that **A** is parallel to **B** and magnitude of **A** is 1/3 times the magnitude of **B**.

Example 2.34 *Prove that*

$$|\mathbf{a} \times \mathbf{b}|^2 = a^2b^2 - (\mathbf{a} \cdot \mathbf{b})^2$$

Sol. Let $|\mathbf{a}| = a$, $|\mathbf{b}| = b$

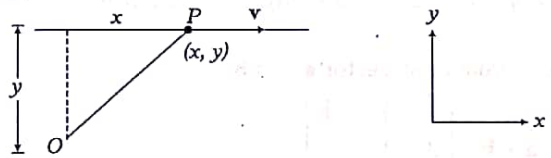
and θ be the angle between them.

$\therefore$ $|\mathbf{a} \times \mathbf{b}|^2 = (ab \sin\theta)^2 = a^2b^2 \sin^2\theta$

$$= a^2b^2(1 - \cos^2\theta) = a^2b^2 - (a \cdot b \cos\theta)^2$$

$$= a^2b^2 - (\mathbf{a} \cdot \mathbf{b})^2 = \text{RHS}$$

Example 2.35 *A particle moves on a given line with a constant speed v. At a certain time it is at a point P on its straight line path. O is fixed point. Show that* ($\mathbf{OP} \times \mathbf{v}$) *is independent of the position P.*



Sol. Let $\mathbf{v} = v\hat{\mathbf{i}} \Rightarrow \mathbf{OP} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$

Take $\mathbf{OP} \times \mathbf{v} = (x\hat{\mathbf{i}} + y\hat{\mathbf{j}}) \times v\hat{\mathbf{i}}$

$= -yv\hat{\mathbf{k}}$ ($\because y$ is constant)

which is independent of position.

Example 2.36 *If* $\mathbf{a} \times \mathbf{b} = \mathbf{b} \times \mathbf{c} \neq \mathbf{0}$ *with* $\mathbf{a} \neq -\mathbf{c}$ *then show that* $\mathbf{a} + \mathbf{c} = k\mathbf{b}$, *where k is scalar.*

Sol. Given, $\mathbf{a} \times \mathbf{b} = \mathbf{b} \times \mathbf{c}$

$$\mathbf{a} \times \mathbf{b} = -\mathbf{c} \times \mathbf{b}$$

$\therefore$ $\mathbf{a} \times \mathbf{b} + \mathbf{c} \times \mathbf{b} = 0$

$$(\mathbf{a} + \mathbf{c}) \times \mathbf{b} = 0$$

$\therefore$ $\mathbf{a} \times \mathbf{b} \neq 0$, $\mathbf{b} \times \mathbf{c} \neq 0$, **a**, **b**, **c**, **d** are non-zero vectors.

$$(\mathbf{a} + \mathbf{c}) \neq \mathbf{0}$$

Hence, $\mathbf{a} + \mathbf{c}$ is parallel to **b**.

$\therefore$ $\mathbf{a} + \mathbf{c} = k\mathbf{b}$ (k-scalar)

Hence proved.

Example 2.37 *Let* **A**, **B** *and* **C** *be unit vectors. Suppose that* $\mathbf{A} \cdot \mathbf{B} = \mathbf{A} \cdot \mathbf{C} = 0$ *and that the angle between* **B** *and* **C** *is* $\pi/6$ *then prove that*

$$\mathbf{A} = \pm 2(\mathbf{B} \times \mathbf{C})$$

Sol. Since, $\mathbf{A}\cdot\mathbf{B}=0$, $\mathbf{A}\cdot\mathbf{C}=0$

Hence, $(\mathbf{B}+\mathbf{C})\cdot\mathbf{A}=0$

So, **A** is perpendicular to $(\mathbf{B}+\mathbf{C})$ and **A** is a unit vector perpendicular to the plane of vectors **B** and **C**.

$$\mathbf{A}=\frac{\mathbf{B}\times\mathbf{C}}{|\mathbf{B}\times\mathbf{C}|}$$

where, $|\mathbf{B}\times\mathbf{C}|=|\mathbf{B}||\mathbf{C}|\sin\theta$

$$=|\mathbf{B}||\mathbf{C}|\sin\frac{\pi}{6} \qquad \left(\because \sin\theta=\frac{\pi}{6}\right)$$

$$=1\times1\times\frac{1}{2}=\frac{1}{2}$$

$$\therefore \quad \mathbf{A}=\frac{\mathbf{B}\times\mathbf{C}}{|\mathbf{B}\times\mathbf{C}|}=\pm2\,(\mathbf{B}\times\mathbf{C})$$

Example 2.38 *If the diagonals of a parallelogram are $2\hat{\mathbf{i}}$ and $2\hat{\mathbf{j}}$, then find its area.*

Sol. Let $\mathbf{A}=2\hat{\mathbf{i}}$ and $\mathbf{B}=2\hat{\mathbf{j}}$

$$\text{Area}=\frac{1}{2}|\mathbf{A}\times\mathbf{B}|=\frac{1}{2}[2\hat{\mathbf{i}}\times2\hat{\mathbf{j}}]$$

$$=\frac{1}{2}[4(\hat{\mathbf{i}}\times\hat{\mathbf{j}})]=\frac{1}{2}|4\hat{\mathbf{k}}| \qquad (\because \hat{\mathbf{i}}\times\hat{\mathbf{j}}=\hat{\mathbf{k}})$$

$$=2 \text{ sq units.}$$

Example 2.39 *If $\mathbf{a}=3\hat{\mathbf{i}}+\hat{\mathbf{j}}-4\hat{\mathbf{k}}$, $\mathbf{b}=6\hat{\mathbf{i}}+5\hat{\mathbf{j}}-2\hat{\mathbf{k}}$, then find the area of a triangle whose adjacent sides are determined by* **a** *and* **b**.

Sol. Cross product of vector **a** and **b**

$$\mathbf{a}\times\mathbf{b}=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 3 & 1 & -4\\ 6 & 5 & -2\end{vmatrix}$$

$$=\hat{\mathbf{i}}(-2+20)-\hat{\mathbf{j}}(-6+24)+\hat{\mathbf{k}}(15+6)=18\hat{\mathbf{i}}-18\hat{\mathbf{j}}+9\hat{\mathbf{k}}$$

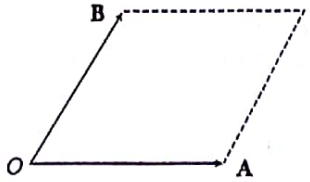
Magnitude of **a** and **b**

$$|\mathbf{a}\times\mathbf{b}|=\sqrt{(18)^2+(-18)^2+(9)^2}=\sqrt{729}=27$$

$$\therefore \text{ Area of } \Delta=\frac{1}{2}|\mathbf{a}\times\mathbf{b}|=\frac{27}{2}=13.5 \text{ sq unit}$$

Example 2.40 *The adjacent sides of a paralellelogram is given by two vectors* **A** *and* **B** *where $\mathbf{A}=5\hat{\mathbf{i}}-4\hat{\mathbf{j}}+3\hat{\mathbf{k}}$ and $\mathbf{B}=3\hat{\mathbf{i}}-2\hat{\mathbf{j}}-\hat{\mathbf{k}}$. Calculate the area of parallelogram.*

Sol. Here, **A** and **B** represents the adjacent side of a parallelogram.



$$\mathbf{A}=5\hat{\mathbf{i}}-4\hat{\mathbf{j}}+3\hat{\mathbf{k}}$$

$$\mathbf{B}=3\hat{\mathbf{i}}-2\hat{\mathbf{j}}-\hat{\mathbf{k}}$$

Area of parallelogram $=|\mathbf{A}\times\mathbf{B}|$

$$\therefore \quad \mathbf{A}\times\mathbf{B}=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 5 & -4 & 3\\ 3 & -2 & -1\end{vmatrix}$$

$$=\hat{\mathbf{i}}(4+6)-\hat{\mathbf{j}}(-5-9)+\hat{\mathbf{k}}(-10+12)$$

$$=10\hat{\mathbf{i}}+14\hat{\mathbf{j}}+2\hat{\mathbf{k}}$$

$$\Rightarrow \quad |\mathbf{A}\times\mathbf{B}|=\sqrt{(10)^2+(14)^2+(2)^2}$$

$$=\sqrt{300}=10\sqrt{3} \text{ sq units}$$

Check point 2.3

1. The work done by a force $\mathbf{F}=(\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}})$ N, to displace a body from position *A* to position *B* is [The position vector of *A* is $\mathbf{r}_1=(\hat{\mathbf{i}}+3\hat{\mathbf{j}}+\hat{\mathbf{k}})$ m and the position vector of *B* is $\mathbf{r}_2=(2\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}})$ m]

(a) 5 J (b) 3 J
(c) 2 J (d) 10 J

2. The condition $(\mathbf{a}\cdot\mathbf{b})^2=a^2b^2$ is satisfied when

(a) **a** is parallel to **b** (b) $\mathbf{a}\neq\mathbf{b}$
(c) $\mathbf{a}\cdot\mathbf{b}=1$ (d) $\mathbf{a}\perp\mathbf{b}$

3. The modulus of the vector product of two vectors is $\frac{1}{\sqrt{3}}$ times their scalar product. The angle between vectors is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$

4. Three vector **A**, **B** and **C** satisfy the relation $\mathbf{A}\cdot\mathbf{B}=0$ and $\mathbf{A}\cdot\mathbf{C}=0$. Then the vector **A** is parallel to

(a) **B** (b) **C**
(c) $\mathbf{B}\cdot\mathbf{C}$ (d) $\mathbf{B}\times\mathbf{C}$

5. What is the dot product of two vectors of magnitude 3 and 5, if angle between them is 60°?

(a) 5.2 (b) 7.5
(c) 8.4 (d) 8.6

6. When $\mathbf{A}\cdot\mathbf{B}=-|A||B|$, then

(a) **A** and **B** are perpendicular to each other
(b) **A** and **B** act in the same direction
(c) **A** and **B** act in the opposite direction
(d) **A** and **B** can act in any direction

7. The vector projection of a vector $3\hat{i} + 4\hat{k}$ on Y-axis is
(a) 5 (b) 4 (c) 3 (d) zero

8. In a clockwise system
(a) $\hat{j} \times \hat{k} = \hat{i}$ (b) $\hat{k} \cdot \hat{i} = 1$
(c) $\hat{i} \cdot \hat{i} = 0$ (d) $\hat{j} \times \hat{j} = 1$

9. $|\mathbf{A} \times \mathbf{B}| = \sqrt{3}\,\mathbf{A} \cdot \mathbf{B}$, then the value of $|\mathbf{A} + \mathbf{B}|$ is
(a) $\left(A^2 + B^2 + \frac{AB}{\sqrt{3}}\right)^{1/2}$ (b) $A + B$
(c) $(A^2 + B^2 + \sqrt{3}\,AB)^{1/2}$ (d) $(A^2 + B^2 + AB)^{1/2}$

10. If $|\mathbf{A}| = 2, |\mathbf{B}| = 5$ and $|\mathbf{A} \times \mathbf{B}| = 8$. Angle between **A** and **B** is acute, then $\mathbf{A} \cdot \mathbf{B}$ is
(a) 6 (b) 3 (c) 4 (d) 7

11. Find the torque of a force $\mathbf{F} = -3\hat{i} + 2\hat{j} + \hat{k}$ acting at the point $\mathbf{r} = 8\hat{i} + 2\hat{j} + 3\hat{k}$
(a) $14\hat{i} - 38\hat{j} + 16\hat{k}$
(b) $4\hat{i} + 4\hat{j} + 6\hat{k}$
(c) $-14\hat{i} + 38\hat{j} - 16\hat{k}$
(d) $-4\hat{i} - 17\hat{j} + 22\hat{k}$

12. What is the unit vector perpendicular to the following vectors $2\hat{i} + 2\hat{j} - \hat{k}$ and $6\hat{i} - 3\hat{j} + 2\hat{k}$?
(a) $\frac{\hat{i} + 10\hat{j} - 18\hat{k}}{5\sqrt{17}}$ (b) $\frac{\hat{i} - 10\hat{j} + 18\hat{k}}{5\sqrt{17}}$
(c) $\frac{\hat{i} - 10\hat{j} - 18\hat{k}}{5\sqrt{17}}$ (d) $\frac{\hat{i} + 10\hat{j} + 18\hat{k}}{5\sqrt{17}}$

13. The angle between vectors $(\mathbf{A} \times \mathbf{B})$ and $(\mathbf{B} \times \mathbf{A})$ is
(a) zero (b) π
(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

14. What is the value of linear velocity, if $\omega = 3\hat{i} - 4\hat{j} + \hat{k}$ and $\mathbf{r} = 5\hat{i} - 6\hat{j} + 6\hat{k}$?
(a) $6\hat{i} - 2\hat{j} + 3\hat{k}$ (b) $6\hat{i} - 2\hat{j} + 8\hat{k}$
(c) $4\hat{i} - 13\hat{j} + 6\hat{k}$ (d) $-18\hat{i} - 13\hat{j} + 2\hat{k}$

15. The area of the parallelogram determined by two adjacent sides as $\mathbf{A} = 2\hat{i} + \hat{j} - 3\hat{k}$ and $\mathbf{B} = 12\hat{j} - 2\hat{k}$ is approximately
(a) 43 (b) 56
(c) 38 (d) 74

Chapter exercises

A) Taking it together

(Assorted questions of the chapter for advanced level practice)

1. The vector quantity among the following is
(a) mass (b) time
(c) distance (d) displacement

2. A vector is added to an equal and opposite vector of similar nature, forms a
(a) unit vector (b) position vector
(c) null vector (d) displacement vector

3. Out of the following quantities, which is scalar?
(a) Displacement (b) Momentum
(c) Potential energy (d) Torque

4. Which of the following is a unit vector?
(a) $\hat{\mathbf{i}} + \hat{\mathbf{j}}$ (b) $\cos\theta\,\hat{\mathbf{i}} - \sin\theta\,\hat{\mathbf{j}}$
(c) $\sin\theta\,\hat{\mathbf{i}} + 2\cos\theta\,\hat{\mathbf{j}}$ (d) $\frac{1}{\sqrt{3}}(\hat{\mathbf{i}} + \hat{\mathbf{j}})$

5. The component of a vector along any other direction is
(a) always less than its magnitude
(b) always greater than its magnitude
(c) always equal to its magnitude
(d) None of the above

6. The expression $\left(\frac{1}{\sqrt{2}}\hat{\mathbf{i}} + \frac{1}{\sqrt{2}}\hat{\mathbf{j}}\right)$ is a
(a) unit vector
(b) null vector
(c) vector of magnitude $\sqrt{2}$
(d) scalar

7. The position vector of a moving particle at time t is $\mathbf{r} = 3\hat{\mathbf{i}} + 4t\hat{\mathbf{j}} - t\hat{\mathbf{k}}$. Its displacement during the time interval $t = 1$ s to $t = 3$ s is
(a) $\hat{\mathbf{j}} - \hat{\mathbf{k}}$
(b) $3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - \hat{\mathbf{k}}$
(c) $9\hat{\mathbf{i}} + 36\hat{\mathbf{j}} - 27\hat{\mathbf{k}}$
(d) None of the above

8. Vector $\mathbf{P} = 6\hat{\mathbf{i}} + 4\sqrt{2}\hat{\mathbf{j}} + 4\sqrt{2}\hat{\mathbf{k}}$ makes angle from Z-axis equal to
(a) $\cos^{-1}\left(\frac{\sqrt{2}}{5}\right)$ (b) $\cos^{-1}(2\sqrt{2})$
(c) $\cos^{-1}\left(\frac{2\sqrt{2}}{5}\right)$ (d) None of these

9. Given $\mathbf{A} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\mathbf{B} = -\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}}$, then $(\mathbf{A} - \mathbf{B})$ will make angle with A
(a) 0° (b) 180° (c) 90° (d) 60°

10. Which of the following statement is true?
(a) When the coordiante axes are translated the component of a vector in a plane changes.
(b) When the coordinate axes are rotated thorugh some angle components of the vector change but the vector's magnitude remains constant.
(c) Sum of **a** and **b** is **R**. If the magnitude of **a** alone is increased angle between **b** and **R** decreases.
(d) The cross product of $3\hat{\mathbf{i}}$ and $4\hat{\mathbf{j}}$ is 12.

11. Component of the vector $\mathbf{A} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$ along the vector $\mathbf{B} = (\hat{\mathbf{i}} + \hat{\mathbf{j}})$ is
(a) $\frac{5}{\sqrt{2}}$ (b) $4\sqrt{2}$
(c) $\frac{\sqrt{2}}{3}$ (d) None of these

12. A vector $\mathbf{P} = 3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + a\hat{\mathbf{k}}$ is perpendicular to the vector $\mathbf{Q} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$. The value of a is
(a) 2 (b) 1 (c) 4 (d) 3

13. If three vectors along coordinate axis represent the adjacent sides of a cube of length b, then the unit vector along its diagonal passing through the origin will be
(a) $\frac{\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}}{\sqrt{2}}$ (b) $\frac{\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}}{\sqrt{36}}$ (c) $\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ (d) $\frac{\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}}{\sqrt{3}}$

14. The angle between the two vectors $-2\hat{i}+3\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}-4\hat{k}$ is
(a) 45° (b) 90°
(c) 30° (d) 60°

15. The angle between the vector $2\hat{i}+4\hat{k}$ and the Y-axis is
(a) 0° (b) 90° (c) 45° (d) 180°

16. What is the angle between $\hat{i}+\hat{j}+\hat{k}$ and $\hat{j}$?
(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{3}$ (d) None of these

17. Which of the following is the unit vector perpendicular to **A** and **B**?
(a) $\frac{\hat{A}\times\hat{B}}{AB\sin\theta}$ (b) $\frac{\hat{A}\times\hat{B}}{AB\cos\theta}$
(c) $\frac{\mathbf{A}\times\mathbf{B}}{AB\sin\theta}$ (d) $\frac{\mathbf{A}\times\mathbf{B}}{AB\cos\theta}$

18. If $\frac{|\mathbf{a}+\mathbf{b}|}{|\mathbf{a}-\mathbf{b}|}=1$, then angle between **a** and **b** is
(a) 0° (b) 45° (c) 90° (d) 60°

19. A man walks 20 m at an angle 60° north of east. How far towards east has he travelled?
(a) 10 m (b) 20 m (c) $20\sqrt{3}$ m (d) $10\sqrt{3}$ m

20. If **A** = **B**, then which of the following is not correct?
(a) $\hat{A}=\hat{B}$ (b) $|\mathbf{A}|=|\mathbf{B}|$
(c) $A\hat{B}=B\hat{A}$ (d) $\mathbf{A}+\mathbf{B}=\hat{A}+\hat{B}$

21. The area of the parallelogram represented by the vectors $\mathbf{A}=2\hat{i}+3\hat{j}$ and $\mathbf{B}=\hat{i}+4\hat{j}$ is
(a) 14 units (b) 7.5 units (c) 10 units (d) 5 units

22. The forces, which meet at one point but their lines of action do not lie on one plane, are called
(a) non-coplanar non-concurrent forces
(b) non-coplanar concurrent forces
(c) coplanar concurrent forces
(d) coplanar non-concurrent forces

23. If the angle between two vectors **A** and **B** is 120°, its resultant C will be
(a) $C=|A-B|$ (b) $C<|A-B|$
(c) $C>|A-B|$ (d) $C=|A+B|$

24. The condition under which the vectors $(\mathbf{a}+\mathbf{b})$ and $(\mathbf{a}-\mathbf{b})$ are parallel is
(a) $\mathbf{a}\perp\mathbf{b}$ (b) $|\mathbf{a}|=|\mathbf{b}|$
(c) $\mathbf{a}\neq\mathbf{b}$ (d) **a** is parallel to **b**

25. A force $\mathbf{F}=(6\hat{i}-8\hat{j}+10\hat{k})$ N produces an acceleration of 1 ms^{-2} in a body. The mass of body would be
(a) 200 kg (b) 20 kg (c) $10\sqrt{2}$ kg (d) $6\sqrt{2}$ kg

26. The vector sum of two forces is perpendicular to their vector differences. In that case, the forces
(a) are not equal to each other in magnitude
(b) are parallel
(c) are perpendicular
(d) are equal to each other in magnitude

27. The resultant of **A** and **B** makes an angle α with **A** and β with **B**, then
(a) $\alpha<\beta$ (b) $\alpha>\beta$ if $A<B$
(c) $\alpha<\beta$ if $A=B$ (d) $\alpha<\beta$ if $A<B$

28. If two vectors are equal and their resultant is also equal to one of them, then the angle between the two vectors is
(a) 60° (b) 120° (c) 90° (d) 0°

29. If **A** and **B** are two non-zero vectors having equal magnitude, the angle between the vectors **A** and **A** − **B** is
(a) 0°
(b) 90°
(c) 180°
(d) dependent on the orientation of **A** and **B**

30. The angle between $\mathbf{A}=\hat{i}+\hat{j}$ and $\mathbf{B}=\hat{i}-\hat{j}$ is **[NCERT Exemplar]**
(a) 45° (b) 90° (c) −45° (d) 180°

31. Five equal forces of 10 N each are applied at one point and all are lying in one plane. If the angles between them are equal, the resultant force will be
(a) zero (b) 10 N (c) 20 N (d) $10\sqrt{2}$ N

32. Two vectors **A** and **B** inclined at angle θ have a resultant **R** which makes an angle ϕ with **A**. If the directions of **A** and **B** are interchanged and resultant will have the same
(a) magnitude
(b) direction
(c) magnitude as well as direction
(d) Neither

33. What is correct?
(a) $|\mathbf{a}-\mathbf{b}|=|\mathbf{a}|-|\mathbf{b}|$ (b) $|\mathbf{a}-\mathbf{b}|\leq|\mathbf{a}|-|\mathbf{b}|$
(c) $|\mathbf{a}-\mathbf{b}|\geq|\mathbf{a}|-|\mathbf{b}|$ (d) $|\mathbf{a}-\mathbf{b}|>|\mathbf{a}|-|\mathbf{b}|$

34. Resultant of which of a following may be equal to zero?
(a) 10 N, 10 N, 10 N (b) 10 N, 10 N, 25 N
(c) 10 N, 10 N, 35 N (d) None of these

35. A man first moves 3 m due east, then 6 m due north and finally 7 m due west, then the magnitude of the resultant displacement is (in metre)
(a) $\sqrt{16}$ (b) $\sqrt{24}$ (c) $\sqrt{52}$ (d) $\sqrt{94}$

36. The resultant of two forces $3P$ and $2P$ is R. If the first force is doubled then the resultant is also doubled. The angle between the two forces is
(a) 60° (b) 120° (c) 90° (d) 180°

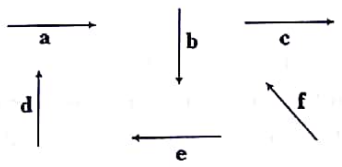
37. Two forces 8 N and 12 N act at 120°. The third force required to keep the body in equilibrium is
(a) 4 N (b) $4\sqrt{7}$ N
(c) 20 N (d) None of these

38. If $A \cdot B = 0$ and $A \times B = 1$, then A and B are
(a) perpendicular unit vectors
(b) parallel unit vector
(c) parallel
(d) anti-parallel

39. If $\mathbf{P} + \mathbf{Q} = \mathbf{R}$ and $|\mathbf{P}| = |\mathbf{Q}| = \sqrt{3}$ and $|\mathbf{R}| = 3$, then the angle between $\mathbf{P}$ and $\mathbf{Q}$ is
(a) $\pi/4$ (b) $\pi/6$
(c) $\pi/3$ (d) $\pi/2$

40. The component of a vector $\mathbf{r}$ along X-axis will have maximum value if **[NCERT Exemplar]**
(a) $\mathbf{r}$ is along positive Y-axis
(b) $\mathbf{r}$ is along positive X-axis
(c) $\mathbf{r}$ makes an angle of 45° with the X-axis
(d) $\mathbf{r}$ is along negative Y-axis

41. Six vectors have magnitude and direction as indicated in the figure. Which of the following statements is true?



(a) $\mathbf{b} + \mathbf{e} = \mathbf{f}$ (b) $\mathbf{b} + \mathbf{c} = \mathbf{f}$
(c) $\mathbf{d} + \mathbf{c} = \mathbf{f}$ (d) $\mathbf{d} + \mathbf{e} = \mathbf{f}$

42. A particle starting from the origin (0, 0) moves in a straight line in the (x, y) plane. Its coordinates at a later time are $(\sqrt{3}, 3)$. The path of the particle makes with the X-axis an angle of
(a) 30° (b) 45° (c) 60° (d) 0°

43. Consider a $\mathbf{F} = 4\hat{\mathbf{i}} - 3\hat{\mathbf{j}}$. Another vector perpendicular to $\mathbf{F}$ is
(a) $4\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$ (b) $6\hat{\mathbf{i}}$ (c) $7\hat{\mathbf{k}}$ (d) $3\hat{\mathbf{i}} - 4\hat{\mathbf{j}}$

44. The angle between vectors $(\mathbf{A} \times \mathbf{B})$ and $(\mathbf{B} \times \mathbf{A})$ is
(a) zero (b) π (c) $\pi/4$ (d) $\pi/2$

45. A vector perpendicular to both the vectors $2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ and X-axis is
(a) $\hat{\mathbf{j}} + 5\hat{\mathbf{j}}$ (b) $\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$ (c) $5\hat{\mathbf{j}} + \hat{\mathbf{k}}$ (d) $\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$

46. Unit vector perpendicular to vector $\mathbf{A} = -3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$ and $\mathbf{B} = 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$ both is
(a) $\dfrac{3\hat{\mathbf{j}} - 2\hat{\mathbf{k}}}{\sqrt{13}}$ (b) $\dfrac{3\hat{\mathbf{k}} - 2\hat{\mathbf{j}}}{\sqrt{13}}$
(c) $\dfrac{-\hat{\mathbf{j}} + 2\hat{\mathbf{k}}}{\sqrt{13}}$ (d) $\dfrac{\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}}{\sqrt{13}}$

47. Unit vector parallel to the resultant of vector $8\hat{\mathbf{i}}$ and $8\hat{\mathbf{j}}$ will be
(a) $(24\hat{\mathbf{i}} + 5\hat{\mathbf{j}})/13$ (b) $(12\hat{\mathbf{i}} + 5\hat{\mathbf{j}})/13$
(c) $(6\hat{\mathbf{i}} + 5\hat{\mathbf{j}})/13$ (d) None of these

48. A force of 5 N acts on a particle along a direction making an angle of 60° with vertical. Its vertical component be
(a) 10 N (b) 3 N
(c) 4 N (d) 2.5 N

49. If $\mathbf{A} = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$ and $\mathbf{B} = 7\hat{\mathbf{i}} + 24\hat{\mathbf{j}}$, the vector having the same magnitude as $\mathbf{B}$ and parallel to $\mathbf{A}$ is
(a) $5\hat{\mathbf{i}} + 20\hat{\mathbf{j}}$ (b) $15\hat{\mathbf{i}} + 10\hat{\mathbf{j}}$
(c) $20\hat{\mathbf{i}} + 15\hat{\mathbf{j}}$ (d) $15\hat{\mathbf{i}} + 20\hat{\mathbf{j}}$

50. The vector that must be added to the vector $\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ and $3\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 7\hat{\mathbf{k}}$ so that the resultant vector is a unit vector along the Y-axis is
(a) $4\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ (b) $-4\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$
(c) $3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ (d) Null vector

51. A vector is represented by $3\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$. Its length in XY-plane is
(a) 2 (b) $\sqrt{14}$
(c) $\sqrt{10}$ (d) $\sqrt{5}$

52. Let $\mathbf{A} = \hat{\mathbf{i}}A\cos\theta + \hat{\mathbf{j}}A\sin\theta$ be any vector. Another vector $\mathbf{B}$ which is perpendicular to $\mathbf{A}$ can be expressed as
(a) $\hat{\mathbf{i}}B\cos\theta - \hat{\mathbf{j}}B\sin\theta$ (b) $\hat{\mathbf{i}}B\sin\theta - \hat{\mathbf{j}}B\cos\theta$
(c) $\hat{\mathbf{i}}B\cos\theta + \hat{\mathbf{j}}B\sin\theta$ (d) $\hat{\mathbf{i}}B\sin\theta + \hat{\mathbf{j}}B\cos\theta$

53. If two vectors $2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $-4\hat{\mathbf{i}} - 6\hat{\mathbf{j}} - \lambda\hat{\mathbf{k}}$ are parallel to each other, then value of λ is
(a) 0 (b) 2
(c) 3 (d) 4

54. What is the angle between $(\mathbf{P} + \mathbf{Q})$ and $(\mathbf{P} \times \mathbf{Q})$?
(a) Zero (b) $\dfrac{\pi}{2}$
(c) $\dfrac{\pi}{4}$ (d) π

55. If a unit vector is represent by $0.5\hat{\mathbf{i}} + 0.8\hat{\mathbf{j}} + c\hat{\mathbf{k}}$, then the value of c is
(a) 1 (b) $\sqrt{0.11}$
(c) $\sqrt{0.01}$ (d) $\sqrt{0.39}$

56. If $\mathbf{A} = a_1\hat{\mathbf{i}} + b_1\hat{\mathbf{j}}$ and $\mathbf{B} = a_2\hat{\mathbf{i}} + b_2\hat{\mathbf{j}}$, the condition that they are perpendicular to each other is
(a) $\dfrac{a_1}{b_1} = -\dfrac{b_2}{a_2}$ (b) $a_1b_1 = a_2b_2$
(c) $\dfrac{a_1}{a_2} = -\dfrac{b_1}{b_2}$ (d) None of these

57. For what value of x, will the two vectors $\mathbf{A} = 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - x\hat{\mathbf{k}}$ and $\mathbf{B} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} - 3\hat{\mathbf{k}}$ are perpendicular to each other?

(a) $x = -2/3$ (b) $x = 3/2$ (c) $x = -4/3$ (d) $x = 2/3$

58. **A** and **B** are two vectors given by $\mathbf{A} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$ and $\mathbf{B} = 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$. The magnitude of the component of **A** along **B** is

(a) $\frac{5}{\sqrt{2}}$ (b) $\frac{3}{\sqrt{2}}$ (c) $\frac{8}{\sqrt{5}}$ (d) $\frac{5}{\sqrt{13}}$

59. The values of x and y for which vectors $\mathbf{A} = (6\hat{\mathbf{i}} + x\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$ and $\mathbf{B} = (5\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - y\hat{\mathbf{k}})$ may be parallel are

(a) $x = 0, y = \frac{2}{3}$ (b) $x = \frac{36}{5}, y = \frac{5}{3}$

(c) $x = -\frac{15}{3}, y = \frac{23}{5}$ (d) $x = -\frac{36}{5}, y = \frac{15}{4}$

60. The angles which the vector $\mathbf{A} = 3\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ makes with the coordinate axes are

(a) $\cos^{-1}\frac{3}{7}, \cos^{-1}\frac{6}{7}$ and $\cos^{-1}\frac{2}{7}$

(b) $\cos^{-1}\frac{4}{7}, \cos^{-1}\frac{5}{7}$ and $\cos^{-1}\frac{3}{7}$

(c) $\cos^{-1}\frac{3}{7}, \cos^{-1}\frac{4}{7}$ and $\cos^{-1}\frac{1}{7}$

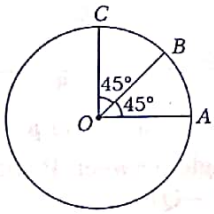
(d) None of the above

61. A cyclist is moving on a circular path with constant speed v. What is the change in its velocity after it has described an angle of 60°

(a) $v\sqrt{2}$ (b) $\frac{v}{2}$

(c) $v\sqrt{3}$ (d) v

62. Find the resultant of three vectors **OA**, **OB** and **OC** shown in the following figure. Radius of the circle is R.



(a) $2R$ (b) $R(1+\sqrt{2})$ (c) $R\sqrt{2}$ (d) $R(\sqrt{2}-1)$

63. Resultant of two vectors of equal magnitude A is

(a) $\sqrt{3}A$ at 60° (b) $\sqrt{2}A$ at 90°

(c) $2A$ at 120° (d) A at 180°

64. If $a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$ is a unit vector and it is perpendicular to $\hat{\mathbf{i}} + \hat{\mathbf{j}}$, then value of a and b is

(a) 1, 0 (b) −2, 0

(c) 0.5, −0.5 (d) None of these

65. If $\mathbf{a} + \mathbf{b} + \mathbf{c} = 0$, then $\mathbf{a} \times \mathbf{b}$ is equal to

(a) $\mathbf{b} \times \mathbf{c}$ (b) $\mathbf{c} \times \mathbf{b}$

(c) $\mathbf{a} \times \mathbf{c}$ (d) None of these

66. Consider a force vector $\mathbf{F} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$. Another vector perpendicular of **F** is

(a) $4\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$ (b) $6\hat{\mathbf{i}}$

(c) $2\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}}$ (d) $3\hat{\mathbf{i}} - 4\hat{\mathbf{j}}$

67. A vector perpendicular to both the vector $2\hat{\mathbf{i}} - 3\hat{\mathbf{j}}$ and $3\hat{\mathbf{i}} - 2\hat{\mathbf{j}}$ is

(a) $\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ (b) $\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$

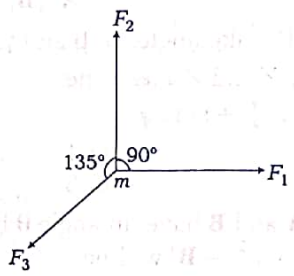
(c) $6\hat{\mathbf{k}}$ (d) $\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$

68. If $\mathbf{A} = 4\hat{\mathbf{i}} - 3\hat{\mathbf{j}}$ and $\mathbf{B} = 6\hat{\mathbf{i}} + 8\hat{\mathbf{j}}$, then magnitude and direction of **A** + **B** with X-axis will be

(a) 5, $\tan^{-1}(3/4)$ (b) $5\sqrt{5}$, $\tan^{-1}(1/2)$

(c) 10, $\tan^{-1}(5)$ (d) 25, $\tan^{-1}(3/4)$

69. When a force F acts on a body of mass m, the acceleration produced in the body is a. If three equal forces $F_1 = F_2 = F_3 = F$ act on the same body as shown in figure the acceleration produced is



(a) $(\sqrt{2}-1)a$ (b) $(\sqrt{2}+1)a$

(c) $\sqrt{2}a$ (d) a

70. The direction cosines of vector (**A** − **B**), if $\mathbf{A} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\mathbf{B} = 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ are

(a) $0, \frac{1}{\sqrt{5}}, \frac{-2}{\sqrt{5}}$ (b) $0, \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}$

(c) $0, 0, \frac{1}{\sqrt{5}}$ (d) None of these

71. Two vectors having equal magnitudes A make an angle θ with each other. The magnitude and direction of the resultant are respectively

(a) $2A\cos\frac{\theta}{2}$, along bisector

(b) $A\cos\frac{\theta}{2}$, at 45° from one vector

(c) $2A\sin\frac{\theta}{2}$, along bisector

(d) $A\cos\frac{\theta}{2}$, along bisector

72. 12 different coplanar forces (all of equal magnitude) maintain a body in equilibrium, then the angle between any two adjacent forces is
(a) 15° (b) 30° (c) 45° (d) 60°

73. Given that $\mathbf{A} + \mathbf{B} + \mathbf{C} = 0$, out of three vectors two are equal in magnitude and the magnitude of third vector is $\sqrt{2}$ times that of either of two having equal magnitude. Then angle between vectors are given by
(a) 30°, 60°, 90° (b) 45°, 45°, 90°
(c) 90°, 135°, 45° (d) 90°, 135°, 135°

74. Two vector **A** and **B** are such that $\mathbf{A} + \mathbf{B} = \mathbf{C}$ and $\mathbf{A}^2 + \mathbf{B}^2 = \mathbf{C}^2$.
If θ is the angle between **A** and **B**, then the correct statement is
(a) $\theta = \pi$ (b) $\theta = \frac{2\pi}{3}$ (c) $\theta = 0$ (d) $\theta = \frac{\pi}{2}$

75. Given $\mathbf{A} = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$ and $\mathbf{B} = 6\hat{\mathbf{i}} + 8\hat{\mathbf{j}}$, which of the following statement is correct?
(a) $\mathbf{A} \times \mathbf{B} = 0$ (b) $\frac{|\mathbf{A}|}{|\mathbf{B}|} = \frac{1}{2}$
(c) $|\mathbf{A}| = 15$ (d) $\mathbf{A} \cdot |\mathbf{B}| = 48$

76. If a vector **P** make angles α, β and γ respectively with the *X*, *Y* and *Z* axes. Then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma =$
(a) 0 (b) 1
(c) 2 (d) 3

77. If vectors **A** and **B** have an angle θ between them, then value of $|\hat{\mathbf{A}} - \hat{\mathbf{B}}|$ will be
(a) $2\cos\frac{\theta}{2}$ (b) $2\tan\frac{\theta}{2}$
(c) $2\sin\frac{\theta}{2}$ (d) None of these

78. A particle undergoes three successive displacements given by $s_1 = \sqrt{2}$ m north-east, $s_2 = 2$ m due south and $s_3 = 4$ m, 30° north of west, then magnitude of net displacement is
(a) $\sqrt{14 + 4\sqrt{3}}$ (b) $\sqrt{14 - 4\sqrt{3}}$
(c) $\sqrt{4}$ (d) None of these

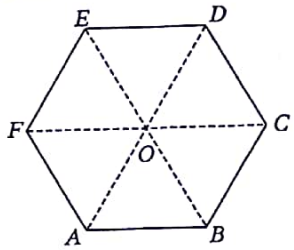
79. The resultant of **A** and **B** is $\mathbf{R}_1$. On reversing the vector **B**, the resultant becomes $\mathbf{R}_2$. What is the value of $R_1^2 + R_2^2$?
(a) $A^2 + B^2$ (b) $A^2 - B^2$
(c) $2(A^2 + B^2)$ (d) $2(A^2 - B^2)$

80. If the sum of two unit vectors is a unit vector, then magnitude of difference in two unit vectors is
(a) $\sqrt{2}$ (b) $\sqrt{3}$
(c) $1/\sqrt{2}$ (d) $\sqrt{5}$

81. The sum of two vectors **A** and **B** is at right angles to their difference. Then
(a) $A = B$ (b) $A = 2B$
(c) $B = 2A$ (d) None of these

82. A vector having magnitude 30 unit makes equal angles with each of *X*, *Y* and *Z*-axes. The components of vector along each of *X*, *Y* and *Z*-axes are
(a) $10\sqrt{3}$ unit (b) $20\sqrt{3}$ unit (c) $15\sqrt{3}$ unit (d) 10 unit

83. In the figure shown, *ABCDEF* is a regular hexagon. What is the value of **AB** + **AC** + **AD** + **AE** + **AF**?

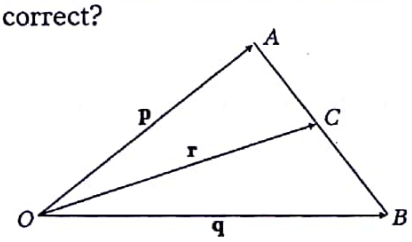


(a) **AO** (b) **2AO** (c) **4AO** (d) **6AO**

84. If **A** is a unit vector in a given direction then, the value of $\hat{\mathbf{A}} \cdot \frac{d\hat{\mathbf{A}}}{dt}$ is
(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) 2

85. If $\mathbf{F}_1$ and $\mathbf{F}_2$ are two vectors of equal magnitudes *F* such that $|\mathbf{F}_1 \cdot \mathbf{F}_2| = |\mathbf{F}_1 \times \mathbf{F}_2|$, then $|\mathbf{F}_1 + \mathbf{F}_2|$ equals to
(a) $\sqrt{(2+\sqrt{2})}\,F$ (b) $2F$
(c) $F\sqrt{2}$ (d) None of these

86. Figure shows three vectors **p**, **q** and **r** where *C* is the mid-point of *AB*. Then which of the following relation is correct?



(a) $\mathbf{p} + \mathbf{q} = 2\mathbf{r}$ (b) $\mathbf{p} + \mathbf{q} = \mathbf{r}$ (c) $\mathbf{p} - \mathbf{q} = 2\mathbf{r}$ (d) $\mathbf{p} - \mathbf{q} = \mathbf{r}$

87. What is the angle between **P** and the cross product of (**P** + **Q**) and (**P** − **Q**)?
(a) 90° (b) $\tan^{-1}(P/Q)$
(c) $\tan^{-1}(Q/P)$ (d) 0°

88. The resultant of two vectors **P** and **Q** is **R**. If *Q* is doubled, the new resultant is perpendicular to **P**. Then *R* equals
(a) *P* (b) $(P + Q)$ (c) *Q* (d) $(P - Q)$

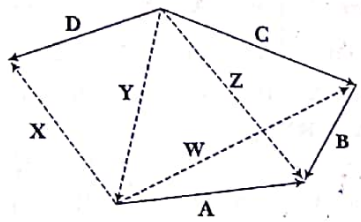
89. The angle between the vector **A** and **B** is θ. The value of the triple product $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{A})$ is
(a) A^2B (b) zero (c) $A^2B\sin\theta$ (d) $A^2B\cos\theta$

90. The sum of the magnitudes of two forces acting at a point is 18 and the magnitude of their resultant is 12. If the resultant is at 90° with the force of smaller magnitude, what are the magnitudes of forces?
(a) 12, 6 (b) 14, 4 (c) 5, 13 (d) 10, 8

91. A vector **a** is turned without a change in its length through a small angles $d\theta$. The value of $|\Delta \mathbf{a}|$ and Δa are, respectively.
(a) 0, $a\, d\theta$ (b) $a \cdot d\theta$, 0
(c) 0, 0 (d) None of these

92. A force $\mathbf{F} = -K(y\hat{\mathbf{i}} - x\hat{\mathbf{j}})$ (where K is a positive constant) acts on a particle moving in the XY-plane. Starting from the origin, the particle is taken along the positive X-axis to the point $(a, 0)$ and then parallel to the Y-axis to the point (a, a). The total work done by the forces **F** on the particle is
(a) $-2Ka^2$ (b) $2Ka^2$ (c) $-Ka^2$ (d) Ka^2

93. In the diagram shown in figure



(a) $\mathbf{X} = \mathbf{A} + \mathbf{B} - \mathbf{C} + \mathbf{D}$ (b) $\mathbf{Y} = \mathbf{B} + \mathbf{C} - \mathbf{A}$
(c) $\mathbf{Z} = \mathbf{B} + \mathbf{C}$ (d) $\mathbf{W} = \mathbf{A} + \mathbf{B}$

94. If **a** and **b** are two vectors, then the value of $(\mathbf{a} + \mathbf{b}) \times (\mathbf{a} - \mathbf{b})$ is
(a) $2(\mathbf{b} \times \mathbf{a})$ (b) $-2(\mathbf{b} \times \mathbf{a})$
(c) $\mathbf{b} \times \mathbf{a}$ (d) $\mathbf{b} \times \mathbf{a}$

95. Given that $\mathbf{A} + \mathbf{B} = \mathbf{C}$ and that **C** is perpendicular to **A** further if $|\mathbf{A}| = |\mathbf{C}|$ then what is the angle between **A** and **B**?
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{3\pi}{4}$ (d) π

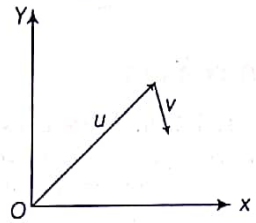
96. The resultant of two forces, one double the other in magnitude, is perpendicular to the smaller of the two forces. The angle between the two forces is
(a) 120° (b) 135° (c) 90° (d) 150°

97. If $\mathbf{A} + \mathbf{B} = \mathbf{C}$, $|\mathbf{A}| = 2|\mathbf{B}|$ and $\mathbf{B} \cdot \mathbf{C} = 0$, then
(a) $|\mathbf{A} + \mathbf{C}| = |\mathbf{A} + \mathbf{B}|$ (b) $|\mathbf{A} + \mathbf{C}| = \mathbf{B}$
(c) $\mathbf{A} \cdot \mathbf{B} < 0$ (d) $\mathbf{A} \cdot \mathbf{C}$ may be zero

98. Two unit vector when added give a unit vector. Then choose the correct statement.
(a) Magnitude of their difference is $\sqrt{3}$
(b) Magnitude of their difference is 1
(c) Angle between the vectors is 90°
(d) Angle between the sum and the difference of the two vectors is 90°

99. Which one of the following statements is true? **[NCERT Exemplar]**
(a) A scalar quantity is the one that is conserved in a process
(b) A scalar quantity is the one that can never take negative values
(c) A scalar quantity is the one that does not vary from one point to another in space
(d) A scalar quantity has the same value for observers with different orientation of the axes.

100. Figure shows the orientation of two vectors **u** and **v** in the XY-plane.
If $\mathbf{u} = a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$ and $\mathbf{v} = p\hat{\mathbf{i}} + q\hat{\mathbf{j}}$
Which of the following is correct? **[NCERT Exemplar]**



(a) a and p are positive while b and q are negative
(b) a, p and b are positive while q is negative
(c) a, q and b are positive while p is negative
(d) a, b, p and q are all positive

101. The velocity of a particle is $\mathbf{v} = 6\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$. The component of the velocity parallel to vector $\mathbf{a} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ in vector form is
(a) $6\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ (b) $2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$
(c) $\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ (d) $6\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$

102. The resultant **R** of vectors **P** and **Q** is perpendicular to **P** and $\mathbf{R} = \mathbf{P}$ both, then angle between $|\mathbf{P}|$ and $|\mathbf{Q}|$ is
(a) 45° (b) 135°
(c) 120° (d) All of these

103. The value of $\hat{\mathbf{i}} \times (\hat{\mathbf{i}} \times \mathbf{a}) + \hat{\mathbf{j}} \times (\hat{\mathbf{j}} \times \mathbf{a}) + \hat{\mathbf{k}} \times (\hat{\mathbf{k}} \times \mathbf{a})$ is
(a) $\mathbf{a}$ (b) $\mathbf{a} \times \hat{\mathbf{k}}$
(c) $-2\mathbf{a}$ (d) $-\mathbf{a}$

104. What is the angle between P and the resultant of $(\mathbf{P} + \mathbf{Q})$ and $(\mathbf{P} - \mathbf{Q})$?
(a) Zero (b) $\tan^{-1}(P/Q)$
(c) $\tan^{-1}(Q/P)$ (d) $\tan^{-1}(P - Q)/(P + Q)$

105. If $\mathbf{a}_1$ and $\mathbf{a}_2$ are two non-collinear unit vectors and if $|\mathbf{a}_1 + \mathbf{a}_2| = \sqrt{3}$, then the value of $(\mathbf{a}_1 - \mathbf{a}_2) \cdot (2\mathbf{a}_1 - \mathbf{a}_2)$ is
(a) 2 (b) $\frac{3}{2}$
(c) $\frac{1}{2}$ (d) 1

106. There are N coplanar vectors each of magnitude V. Each vector is inclined to the preceding vector at angle $\frac{2\pi}{N}$. What is the magnitude of their resultant?

(a) $\frac{V}{N}$ (b) V (c) Zero (d) $\frac{N}{V}$

107. Three equal masses of 1 kg each are placed at the vertices of an equilateral triangle PQR and a mass of 2 kg is placed at the centroid O of the triangle which is at a distance of $\sqrt{2}$ m from each of the vertices of the triangle. The force, in newton, acting on the mass of 2 kg is

(a) 2 (b) 1 (c) 1 (d) zero

108. At what angle must the two forces $(x + y)$ and $(x - y)$ act so that the resultant may be $\sqrt{x^2 + y^2}$?

(a) $\cos^{-1}\left[-\frac{x^2+y^2}{2(x^2-y^2)}\right]$ (b) $\cos^{-1}\left[-\frac{-2(x^2-y^2)}{x^2+y^2}\right]$

(c) $\cos^{-1}\left[-\frac{(x^2+y^2)}{(x^2-y^2)}\right]$ (d) $\cos^{-1}\left[-\frac{(x^2-y^2)}{(x^2+y^2)}\right]$

B) Medical entrance special format questions

Assertion and reason

Directions (Q. Nos. 1-20) *These questions consists of two statements each printed as Assertion and Reason. While answering these question you are required to choose any one of the following five responses*

(a) If both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If Assertion is false but Reason is true.
(e) If both Assertion and Reason are false.

1. **Assertion** Finite angular displacement is not a vector quantity.
Reason It does not obey the vector laws.

2. **Assertion** Small displacement is a vector quantity.
Reason Pressure and surface tension are also vector quantities.

3. **Assertion** Vector addition of two vectors is always greater than their vector subtraction.
Reason At $\theta = 90°$, addition and subtraction of two vector are equal.

4. **Assertion** Vector product of two vectors may be greater than, equal to or less than the scalar product.
Reason At $\theta = 45°$, two are equal.

5. **Assertion** Angle between $\hat{\mathbf{i}} + \hat{\mathbf{j}}$ and $\hat{\mathbf{i}}$ is 45°.
Reason $\hat{\mathbf{i}} + \hat{\mathbf{j}}$ is equally inclined to both $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ and the angle between $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ is 90°.

6. **Assertion** Component of **A** along **B** is equal to component of **B** along **A**.
Reason Component of **A** along **B** is $A\cos\theta$, where θ is the angle between two vectors.

7. **Assertion** Component of **A** along **B** is equal to component of **B** along **A**.
Reason Value of component is always less than the magnitude of vector.

8. **Assertion** $(\mathbf{A} + \mathbf{B}) \cdot (\mathbf{A} - \mathbf{B})$ is always positive.
Reason This is positive if $|\mathbf{A}| > |\mathbf{B}|$.

9. **Assertion** $\mathbf{A} \times \mathbf{B}$ is perpendicular to both $\mathbf{A} + \mathbf{B}$ as well as $\mathbf{A} - \mathbf{B}$.
Reason $\mathbf{A} + \mathbf{B}$ as well as $\mathbf{A} - \mathbf{B}$ lie in the plane containing **A** and **B** while $\mathbf{A} \times \mathbf{B}$ lies perpendicular to the plane containing **A** and **B**.

10. **Assertion** If angle between **a** and **b** is 30°. Then angle between $2\mathbf{a}$ and $-\frac{\mathbf{b}}{2}$ will be 150°.
Reason Sign of dot product of two vectors tells you whether angle between two vectors is acute or obtuse.

11. **Assertion** If $|\mathbf{A}| = |\mathbf{B}|$, then $(\mathbf{A} + \mathbf{B})$, $(\mathbf{A} - \mathbf{B})$ and $(\mathbf{A} \times \mathbf{B})$ are three mutually perpendicular vectors.
Reason Dot product of a null vector with any other vector is always zero.

12. **Assertion** $(\mathbf{A} \times \mathbf{B}) \cdot (\mathbf{B} \times \mathbf{A})$ is $-A^2B^2\sin^2\theta$. Here θ is the angle between **A** and **B**.
Reason $(\mathbf{A} \times \mathbf{B})$ and $(\mathbf{B} \times \mathbf{A})$ are two anti-parallel vectors provided **A** and **B** are neither parallel nor anti-parallel.

13. **Assertion** We can find angle between two vectors by using the relation,
$$\theta = \sin^{-1}\left(\frac{|\mathbf{A} \times \mathbf{B}|}{AB}\right)$$
Reason $\frac{|\mathbf{A} \times \mathbf{B}|}{AB}$ is always positive.

14. Assertion If θ be the angle between **A** and **B**, then

$$\tan\theta = \frac{\mathbf{A}\times\mathbf{B}}{\mathbf{AB}}.$$

Reason $\mathbf{A}\times\mathbf{B}$ is perpendicular to **AB**.

Match the columns

1. For component of a vector $\mathbf{A} = (3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}})$, match the following columns.

	Column I		Column II
(A)	X-axis	(p)	5 unit
(B)	Along another vector $(2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}})$	(q)	3 unit
(C)	Along $(6\hat{\mathbf{i}} + 8\hat{\mathbf{j}} - 10\hat{\mathbf{k}})$	(r)	Zero
(D)	Along another vector $(-3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 5\hat{\mathbf{k}})$	(s)	None

2. Two vectors **A** and **B** have equal magnitude x. Angle between them is 60°. Then match the following two columns.

	Column I		Column II
(A)	$\lvert\mathbf{A}+\mathbf{B}\rvert$	(p)	$\frac{\sqrt{3}}{2}x^2$
(B)	$\lvert\mathbf{A}-\mathbf{B}\rvert$	(q)	x
(C)	$\mathbf{A}\cdot\mathbf{B}$	(r)	$\sqrt{3}x$
(D)	$\lvert\mathbf{A}\times\mathbf{B}\rvert$	(s)	None

3. Vector **A** is pointing eastwards and vector **B** northwards. If $|\mathbf{A}| = |\mathbf{B}|$ then match the following two columns.

	Column I		Column II
(A)	$(\mathbf{A}+\mathbf{B})$	(p)	North-east
(B)	$(\mathbf{A}-\mathbf{B})$	(q)	Vertically upwards
(C)	$(\mathbf{A}\times\mathbf{B})$	(r)	Vertically downwards
(D)	$(\mathbf{A}\times\mathbf{B})\times(\mathbf{A}\times\mathbf{B})$	(s)	None

4. A vector has a magnitude x. If it is rotated by an angle θ, then magnitude of change in vector is nx. Match the following two columns.

	Column I		Column II
(A)	$\theta = 60°$	(p)	$n = \sqrt{3}$
(B)	$\theta = 90°$	(q)	$n = 1$
(C)	$\theta = 120°$	(r)	$n = \sqrt{2}$
(D)	$\theta = 180°$	(s)	$n = 2$

5. If θ is the angle between two vectors **A** and **B**, then match the following two columns.

	Column I		Column II
(A)	$\mathbf{A}\cdot\mathbf{B} = \lvert\mathbf{A}\times\mathbf{B}\rvert$	(p)	$\theta = 90°$
(B)	$\mathbf{A}\cdot\mathbf{B} = B^2$	(q)	$\theta = 0°$ or $180°$
(C)	$\lvert\mathbf{A}+\mathbf{B}\rvert = \lvert\mathbf{A}-\mathbf{B}\rvert$	(r)	$\mathbf{A} = \mathbf{B}$
(D)	$\lvert\mathbf{A}\times\mathbf{B}\rvert = AB$	(s)	None

C Medical entrances' gallery

(Collection of questions asked in NEET & various medical entrance exams)

1. If the magnitude of sum of two vectors is equal to the magnitude of difference of the two vectors, the angle between these vectors is **[NEET 2016]**

(a) 90° (b) 45° (c) 180° (d) 0°

2. If a vector $2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 8\hat{\mathbf{k}}$ is perpendicular to the vector $4\hat{\mathbf{j}} - 4\hat{\mathbf{i}} + \alpha\hat{\mathbf{k}}$. Then, the value of α is **[Manipal 2015]**

(a) 1/2 (b) −1 (c) −1/2 (d) 1

3. The angle θ between the vector $\mathbf{p} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and unit vector along X-axis is **[MHT CET 2014]**

(a) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (b) $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$

(c) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (d) $\cos^{-1}\left(\frac{1}{2}\right)$

4. Consider three vectors $\mathbf{A} = \hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$, $\mathbf{B} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ and $\mathbf{C} = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$. A vector X of the form $\alpha\mathbf{A} + \beta\mathbf{B}$ (α and β are numbers) is perpendicular to **C**. The ratio of α and β is **[EAMCET 2014]**

(a) 1 : 1 (b) 2 : 1
(c) −1 : 1 (d) 3 : 1

5. Two equal vectors have a resultant equal to either of the two. The angle between them is **[UK PMT 2014]**

(a) 90° (b) 60°
(c) 120° (d) 0°

6. Which of the following not a vector quantity? **[KCET 2014]**

(a) Weight (b) Nuclear spin
(c) Momentum (d) Potential energy

7. The scalar product of two vectors $\mathbf{A} = 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $\mathbf{B} = -\hat{\mathbf{j}} + \hat{\mathbf{k}}$, is given by [J&K CET 2013]

(a) $\mathbf{A} \cdot \mathbf{B} = 3$ (b) $\mathbf{A} \cdot \mathbf{B} = 4$
(c) $\mathbf{A} \cdot \mathbf{B} = -4$ (d) $\mathbf{A} \cdot \mathbf{B} = -3$

8. If $\mathbf{A} \cdot \mathbf{B} = \mathbf{A} \times \mathbf{B}$, then angle between $\mathbf{A}$ and $\mathbf{B}$ is [AMFC 2012]

(a) 45° (b) 30°
(c) 60° (d) 90°

9. If a vector $\mathbf{A}$ having a magnitude of 8 is added to a vector $\mathbf{B}$ which lies along X-axis, then the resultant of two vectors lies along Y-axis and has magnitude twice that of $\mathbf{B}$. The magnitude of $\mathbf{B}$ is [JCECE 2012]

(a) $\frac{6}{\sqrt{5}}$ (b) $\frac{12}{\sqrt{5}}$
(c) $\frac{16}{\sqrt{5}}$ (d) $\frac{8}{\sqrt{5}}$

Answers

Check point 2.1

1. (a)	2. (a)	3. (d)	4. (d)	5. (a)	6. (a)	7. (b)	8. (c)	9. (a)	10. (c)
11. (b)	12. (c)	13. (c)	14. (c)	15. (b)					

Check point 2.2

1. (c)	2. (b)	3. (c)	4. (b)	5. (a)	6. (b)	7. (b)	8. (b)	9. (b)	10. (d)
11. (b)	12. (d)	13. (d)	14. (b)	15. (b)	16. (c)	17. (a)	18. (b)	19. (a)	20. (c)

Check point 2.3

1. (a)	2. (a)	3. (a)	4. (d)	5. (b)	6. (c)	7. (d)	8. (a)	9. (d)	10. (a)
11. (d)	12. (c)	13. (b)	14. (d)	15. (a)					

A. Taking it together

1. (d)	2. (c)	3. (c)	4. (b)	5. (a)	6. (a)	7. (d)	8. (c)	9. (a)	10. (b)
11. (a)	12. (c)	13. (d)	14. (b)	15. (b)	16. (d)	17. (c)	18. (c)	19. (a)	20. (d)
21. (d)	22. (b)	23. (c)	24. (d)	25. (c)	26. (d)	27. (b)	28. (b)	29. (d)	30. (b)
31. (a)	32. (a)	33. (c)	34. (a)	35. (c)	36. (b)	37. (b)	38. (a)	39. (c)	40. (b)
41. (d)	42. (c)	43. (c)	44. (b)	45. (c)	46. (a)	47. (d)	48. (d)	49. (d)	50. (b)
51. (c)	52. (b)	53. (b)	54. (b)	55. (b)	56. (a)	57. (a)	58. (c)	59. (b)	60. (a)
61. (d)	62. (b)	63. (a,b)	64. (d)	65. (a)	66. (c)	67. (c)	68. (b)	69. (a)	70. (a)
71. (a)	72. (b)	73. (d)	74. (d)	75. (a,b)	76. (c)	77. (c)	78. (b)	79. (c)	80. (b)
81. (a)	82. (a)	83. (d)	84. (a)	85. (a)	86. (a)	87. (a)	88. (c)	89. (b)	90. (c)
91. (b)	92. (c)	93. (b,c)	94. (a)	95. (c)	96. (a)	97. (c)	98. (d,a)	99. (d)	100. (b)
101. (b)	102. (b)	103. (c)	104. (a)	105. (b)	106. (c)	107. (d)	108. (a)		

B. Medical entrance special format questions

Assertion and reason

1. (a)	2. (c)	3. (d)	4. (b)	5. (b)	6. (d)	7. (d)	8. (d)	9. (a)	10. (b)
11. (b)	12. (b)	13. (c)	14. (e)						

Match the columns

1. (A → q, B → r, C → s, D → s)
2. (A → r, B → q, C → s, D → p)
3. (A → p, B → s, C → q, D → s)
4. (A → q, B → r, C → p, D → s)
5. (A → s, B → q, r, C → p, D → p)

C. Medical entrances' gallery

1. (a)	2. (c)	3. (a)	4. (a)	5. (c)	6. (d)	7. (d)	8. (a)	9. (d)

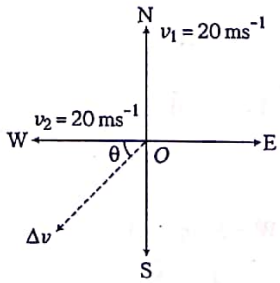
Hints and explanations

Check point 2.1

1. (*a*) Current does not obey law of vector algebra. Hence, it is not a vector quantity.
2. (*a*) Pressure is a scalar quantity.
3. (*d*) Electric flux is a scalar quantity.
4. (*d*) Pressure and surface tension are scalar quantities. Whereas moment of inertia is neither a scalar nor a vector quantity. It is a tensor quantity.
5. (*a*) Surface area only has magnitude. Therefore, it is a scalar quantity.
6. (*a*) Angular momentum is a vector quantity.
7. (*b*) Momentum is the product of mass and velocity. Mass is a scalar quantity and velocity is a vector quantity.
   So, momentum is a vector quantity.
8. (*c*) Torque is defined as $\tau = \mathbf{r} \times \mathbf{F}$. So, it is a vector quantity.
9. (*a*) $(\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) + (3\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 7\hat{\mathbf{k}}) + \mathbf{r} = \hat{\mathbf{j}}$
   $\Rightarrow \quad \mathbf{r} = -4\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$
10. (*c*) When a vector is multiplied by zero, it results into zero vector, *i.e.*, a vector having zero magnitude. It is written as $\vec{0}$.
11. (*b*) A unit vector is represented by $\hat{\mathbf{A}}$ where
    $$\hat{\mathbf{A}} = \frac{\mathbf{A}}{|\mathbf{A}|}$$
12. (*c*) Unit vector does not have any unit.
13. (*c*) Let the given vector be **A** where
    $$\mathbf{A} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$$
    $$\therefore \quad |\mathbf{A}| = \sqrt{(1)^2 + (1)^2} = \sqrt{2}$$
14. (*c*) Let $\mathbf{A} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$
    $$|\mathbf{A}| = \sqrt{(1)^2 + (1)^2} = \sqrt{2}$$
    $\therefore$ Unit vector along **A** will be $\hat{\mathbf{A}}$ given by
    $$\hat{\mathbf{A}} = \frac{\mathbf{A}}{|\mathbf{A}|} = \frac{\hat{\mathbf{i}} + \hat{\mathbf{j}}}{\sqrt{2}}$$
15. (*b*) When a vector is multiplied by a negative scalar number, then its magnitude gets change and direction gets reversed.

Check point 2.2

1. (*c*) As vector addition is commutative. So, $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$
2. (*b*) The sum of two vectors is zero only when one vector is equal in magnitude but opposite in direction to that of other vector.
3. (*c*) Maximum resultant $R_{max} = |A + B| = 18$
   Minimum resultant $R_{min} = |A - B| = 2$
4. (*b*) Given, $P = \hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$
   and $P + Q = \hat{\mathbf{i}} \Rightarrow Q = \hat{\mathbf{i}} - \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}} = \hat{\mathbf{j}} - \hat{\mathbf{k}}$
5. (*a*) Resultant of two vectors will be maximum when they are parallel, *i.e.*, angle between them is zero.
6. (*b*) Minimum three forces of unequal magnitude are required to make vector sum equal to zero.
7. (*b*) If $P + Q + R = 0$
   then $|P + Q| = |R|$
8. (*b*) $A = 2\hat{\mathbf{i}} + \hat{\mathbf{j}}$, $B = 3\hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $C = 6\hat{\mathbf{i}} - 2\hat{\mathbf{k}}$,
   $$A - 2B + 3C = (2\hat{\mathbf{i}} + \hat{\mathbf{j}}) - 2(3\hat{\mathbf{j}} - \hat{\mathbf{k}}) + 3(6\hat{\mathbf{i}} - 2\hat{\mathbf{k}})$$
   $$= 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 6\hat{\mathbf{j}} + 2\hat{\mathbf{k}} + 18\hat{\mathbf{i}} - 6\hat{\mathbf{k}}$$
   $$= 20\hat{\mathbf{i}} - 5\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$$
9. (*b*) $A + B = 10\hat{\mathbf{i}} + 5\hat{\mathbf{j}}$
   $\therefore |A + B| = \sqrt{(10)^2 + (5)^2} = 5\sqrt{5}$
   Angle of $(A + B)$ with X-axis, $\theta = \tan^{-1}\left(\frac{5}{10}\right) = \tan^{-1}\left(\frac{1}{2}\right)$
10. (*d*) From the figure
    Velocity of truck in north direction $\mathbf{v}_1 = 20\hat{\mathbf{j}}$
    Velocity of truck in west direction
    $$\mathbf{v}_2 = -20\hat{\mathbf{i}} \Rightarrow \Delta\mathbf{v} = \mathbf{v}_2 - \mathbf{v}_1 = -20(\hat{\mathbf{i}} + \hat{\mathbf{j}})$$



Change in velocity

$|\Delta \mathbf{v}| = 20\sqrt{2}$ and direction will be given by

$$\theta = \tan^{-1}(1) = 45, \text{ i.e., S-W}$$

11. (*b*) Resultant of two vector A and B

$$R = (|A| - |B|)$$

then $\theta = 180°$

12. (*d*) $|A - B| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$

$$= \sqrt{4 + 16 - 2 \times 2 \times 4 \times \frac{1}{2}}$$

$$= \sqrt{12} = 2\sqrt{3}$$

13. (*d*) We have, $|A + B| = 2|A - B|$

$$\therefore \quad \sqrt{A^2 + B^2 + 2AB\cos\theta} = 2\sqrt{A^2 + B^2 - 2AB\cos\theta}$$

From this equation we cannot find θ.

14. (*b*) If $\mathbf{P} + \mathbf{Q} = \mathbf{P} - \mathbf{Q}$

then, = 0 (a null vector)

15. (*b*) The position vector of a particle in a rectangular coordinate system can be written as $3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$.

16. (*c*) Displacement vector $S = r_Q - r_P$

$$= (3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 5\hat{\mathbf{k}}) - (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}) = \hat{\mathbf{i}} + \hat{\mathbf{j}}$$

17. (*a*) $P\sqrt{10} = \sqrt{4P^2 + 2P^2 + 4\sqrt{2}P^2\cos\theta}$

$$\therefore \quad \theta = 45$$

18. (*b*) $A = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ and $B = 3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$

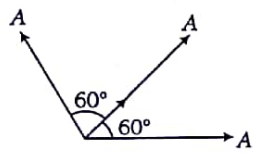
Resultant, $R = \hat{\mathbf{k}} = A + B + C$

$$\hat{\mathbf{k}} = (2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 3\hat{\mathbf{k}}) + (3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 2\hat{\mathbf{k}}) + C$$

$$\hat{\mathbf{k}} = 5\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}} + C$$

$$C = -5\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$$

19. (*a*)



20. (*c*) According to given problem there should be $\Sigma F_x = 0$

In the given figure net force along X-axis.

$$\mathbf{F} = 1\cos 60°\,\hat{\mathbf{i}} + 2\sin 30°\,\hat{\mathbf{i}} - 4\sin 30°\,\hat{\mathbf{i}}$$

$$= \frac{1}{2}\hat{\mathbf{i}} + 1\hat{\mathbf{i}} - 2\hat{\mathbf{i}}$$

$$\Rightarrow \quad \mathbf{F} = -\frac{1}{2}\hat{\mathbf{i}} = -0.5\hat{\mathbf{i}}$$

Check point 2.3

1. (*a*) Work done, $W = F \cdot (r_2 - r_1)$

Position, $r_2 - r_1 = \hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

$$W = (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) \cdot (\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}) = 1 - 2 + 6 = 5\,\text{J}$$

2. (*a*) $(a \cdot b)^2 = a^2b^2\cos^2\theta = a^2b^2$ (Given)

$$\therefore \quad \theta = 0°$$

$\Rightarrow$ a is parallel to b.

3. (*a*) $AB\sin\theta = \frac{1}{\sqrt{3}}AB\cos\theta$

$$\therefore \quad \tan\theta = \frac{1}{\sqrt{3}} \text{ or } \theta = 30°$$

4. (*d*) Given, $A \cdot B = 0$

$$\therefore \quad A \perp B$$

$$A \cdot C = 0$$

$$\therefore \quad A \perp C$$

i.e., A is $\perp$ to both B and C.

5. (*b*) We have $\mathbf{A} \cdot \mathbf{B} = AB\cos\theta$

$$= (5)(3)\cos 60° = 7.5$$

6. (*c*) We have $\mathbf{A} \cdot \mathbf{B} = AB\cos\theta$

Here, $A \cdot B = -AB$, i.e., $\cos\theta = -1$

$$\therefore \quad \theta = 180°$$

So, $\mathbf{A}$ and $\mathbf{B}$ acts in the opposite direction.

7. (*d*) As the multiple of $\hat{\mathbf{j}}$ in the given vector is zero therefore this vector lies in XZ-plane and projection of this vector on Y-axis is zero.

8. (*a*) In a clockwise system,

$$\hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}}, \quad \hat{\mathbf{j}} \times \hat{\mathbf{k}} = \hat{\mathbf{i}}, \quad \hat{\mathbf{k}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}}$$

9. (*d*) $AB\sin\theta = \sqrt{3}AB\cos\theta$ or $\tan\theta = \sqrt{3}$

$$\therefore \quad \theta = 60°$$

Now, $|\mathbf{A} + \mathbf{B}| = \sqrt{A^2 + B^2 + 2AB\cos 60°}$

$$= \sqrt{A^2 + B^2 + AB}$$

10. (*a*) We have $A \times B = AB\sin\theta$

$$8 = (5)(2)\sin\theta$$

$$\therefore \quad \sin\theta = 4/5 \text{ or } \cos\theta = 3/5$$

$$\therefore \quad \mathbf{A} \cdot \mathbf{B} = AB\cos\theta = (5)(2)(3/5) = 6$$

11. (*d*) Torque of the force, $\tau = \mathbf{r} \times \mathbf{F}$

So,
$$\tau = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 8 & 2 & 3 \\ -3 & 2 & 1 \end{vmatrix}$$

$$= \hat{\mathbf{i}}(2 - 6) - \hat{\mathbf{j}}(8 + 9) + \hat{\mathbf{k}}(16 + 6)$$

$$= -4\hat{\mathbf{i}} - 17\hat{\mathbf{j}} + 22\hat{\mathbf{k}}$$

12. (*c*) $\mathbf{A} = 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $\mathbf{B} = 6\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

$$\mathbf{C} = \mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 2 & -1 \\ 6 & -3 & 2 \end{vmatrix}$$

$$= \hat{\mathbf{i}}(4 - 3) - \hat{\mathbf{j}}(4 + 6) + \hat{\mathbf{k}}(-6 - 12)$$

$$= \hat{\mathbf{i}} - 10\hat{\mathbf{j}} - 18\hat{\mathbf{k}}$$

Unit vector perpendicular to both **A** and **B**

$$= \frac{\hat{i} - 10\hat{j} - 18\hat{k}}{\sqrt{(1)^2 + (10)^2 + (18)^2}} = \frac{\hat{i} - 10\hat{j} - 18\hat{k}}{5\sqrt{17}}$$

13. (*b*) $(\mathbf{A} \times \mathbf{B})$ and $(\mathbf{B} \times \mathbf{A})$ are parallel and opposite to each other. So the angle will be π.

14. (*d*) Velocity $\mathbf{v} = \omega \times \mathbf{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -4 & 1 \\ 5 & -6 & 6 \end{vmatrix} = -18\hat{i} - 13\hat{j} + 2\hat{k}$

15. (*a*) Area of parallelogram $= |\mathbf{A} \times \mathbf{B}|$

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -3 \\ 0 & 12 & -2 \end{vmatrix}$$

$$= \hat{i}(-2 + 36) - \hat{j}(-4) + \hat{k}(24)$$

$$= 34\hat{i} + 4\hat{j} + 24\hat{k}$$

$$|\mathbf{A} \times \mathbf{B}| = \sqrt{(34)^2 + (4)^2 + (24)^2} = 41.8 \approx 43$$

A) Taking it together

1. (*d*) Displacement has both direction as well as magnitude. So, it is a vector quantity.

2. (*c*) When a vector is added to another vector of similar nature having equal magnitude and opposite sign, it forms null vector.

3. (*c*) Potential energy is a scalar quantity.

4. (*b*) Unit vector has a magnitude equal to 1.

Here, $\sqrt{\cos^2\theta + \sin^2\theta} = 1$

$\therefore$ Option (b) is the correct answer.

5. (*a*) The component of a vector is always less than its magnitude along any other direction.

6. (*a*) Let $\mathbf{A} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$

$$|\mathbf{A}| = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = \sqrt{\frac{2}{2}} = 1$$

As the magnitude of given vector is 1.

$\therefore$ It is a unit vector.

7. (*d*) Displacement, $S = r_f - r_i = r_3 - r_1$

where r_f and r_i is the final and initial position vector of a particle during the time interval $t = 1$s to $t = 3$s

$\therefore$ $r_3 - r_1 = (3\hat{i} + 12\hat{j} - 3\hat{k}) - (3\hat{i} + 4\hat{j} - \hat{k}) = 8\hat{j} - 2\hat{k}$

8. (*c*) $\mathbf{P} = 6\hat{i} + 4\sqrt{2}\hat{j} + 4\sqrt{2}\hat{k}$

Let **P** makes angle γ from Z-axis

$\therefore$ $\gamma = \cos^{-1}\left(\frac{P_z}{P}\right)$

Here $P_Z = 4\sqrt{2}$ and $P = \sqrt{(6)^2 + (4\sqrt{2})^2 + (4\sqrt{2})^2} = 10$

$\therefore$ $\gamma = \cos^{-1}\left(\frac{4\sqrt{2}}{10}\right) = \cos^{-1}\left(\frac{2\sqrt{2}}{5}\right)$

9. (*a*) $\mathbf{A} - \mathbf{B} = (\hat{i} + \hat{j} + \hat{k}) - (-\hat{i} - \hat{j} - \hat{k})$

$= 2\hat{i} + 2\hat{j} + 2\hat{k} - 2\mathbf{A}$

i.e., $\mathbf{A} - \mathbf{B}$ and **A** are parallel.

10. (*b*) When the coordinate axes are rotated only the components of the vector changes whereas magnitude remains constant.

11. (*a*) Component of **A** along **B** $= a\cos\theta = \frac{\mathbf{A}\cdot\mathbf{B}}{B} = \frac{2+3}{\sqrt{1+1}} = \frac{5}{\sqrt{2}}$

12. (*c*) Since vector P is perpendicular to the vector Q

$\therefore$ $\mathbf{P}\cdot\mathbf{Q} = 0 \Rightarrow (3\hat{i} - 2\hat{j} + a\hat{k})\cdot(2\hat{i} + \hat{j} - \hat{k}) = 0$

$\Rightarrow$ $6 - 2 - a = 0 \Rightarrow a = 4$

13. (*d*) Diagonal vector, $A = b\hat{i} + b\hat{j} + b\hat{k}$

or $A = \sqrt{b^2 + b^2 + b^2} = \sqrt{3}b$

$\therefore$ $\mathbf{A} = \frac{\mathbf{A}}{|\mathbf{A}|} = \frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$

14. (*b*) Let $\mathbf{A} = -2\hat{i} + 3\hat{j} + \hat{k}$

$\mathbf{B} = \hat{i} + 2\hat{j} - 4\hat{k}$

$\therefore$ $\mathbf{A}\cdot\mathbf{B} = (-2\hat{i} + 3\hat{j} + \hat{k})\cdot(\hat{i} + 2\hat{j} - 4\hat{k}) = -2 + 6 - 4 = 0$

Since dot product of two vectors is zero so vectors will be perpendicular each other

15. (*b*) The given vector is in XZ-plane. Hence, this is perpendicular to Y-axis.

16. (*d*) Angle $\theta = \cos^{-1}\left(\frac{\mathbf{A}\cdot\mathbf{B}}{AB}\right) = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

18. (*c*) Given, $\frac{|a + b|}{|a - b|} = 1$

i.e., $|a + b| = |a - b|$

or $a^2 + b^2 + 2ab\cos\theta = a^2 + b^2 - 2ab\cos\theta$

$\therefore$ $\cos\theta = 0°$ or $\theta = 90°$

19. (*a*) $20\cos 60° = 10$ m

21. (*d*) Area $= |\mathbf{A} \times \mathbf{B}|$

Here $\mathbf{A} \times \mathbf{B} = (2\hat{i} + 3\hat{j}) \times (\hat{i} + 4\hat{j})$

$= 8\hat{k} - 3\hat{k} = 5\hat{k}$

$\therefore$ Area $= |\mathbf{A} \times \mathbf{B}| = 5$ units

23. (*c*) Resultant of two vectors lies between $|A + B|$ and $|A - B|$.

24. (*d*) Their cross product should be zero.

$(\mathbf{a} + \mathbf{b}) \times (\mathbf{a} - \mathbf{b}) = 0$

$\therefore$ $2(\mathbf{a} \times \mathbf{b}) = 0$

So a is parallel to b.

25. (*c*) Force, $F = ma$

$\therefore$ Mass $m = \frac{F}{a} = \frac{6\hat{i} - 8\hat{j} + 10\hat{k}}{1}$

$|m| = \sqrt{(6)^2 + (8)^2 + (10)^2}$

$= \sqrt{36 + 64 + 100}$

$= \sqrt{200} = 10\sqrt{2}$ kg

26. (*d*) We have $(\mathbf{A}+\mathbf{B})\cdot(\mathbf{A}-\mathbf{B}) = 0$

$\therefore\ \mathbf{A}\cdot\mathbf{A}+\mathbf{B}\cdot\mathbf{A}-\mathbf{A}\cdot\mathbf{B}-\mathbf{B}\cdot\mathbf{B} = 0$

$\therefore\ A^2 + AB\cos\theta - AB\cos\theta - B^2 = 0$

$\therefore\ A = B$

27. (*b*) Resultant is inclined towards a vector having greater magnitude.

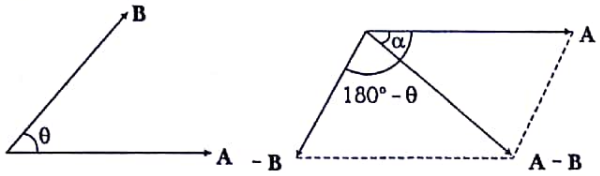
28. (*b*) We have $R = A = B$

$\therefore\ R^2 = R^2 + R^2 + 2RR\cos\theta$

or $\cos\theta = -\frac{1}{2}$

$\therefore\ \theta = 120°$

29. (*d*) Suppose angle between two vectors **A** and **B** of equal magnitude is θ. Then angle between **A** and **A** − **B** will be $\frac{180°-\theta}{2}$ or $90° - \frac{\theta}{2}$. Hence this angle will depend on the angle between **A** and **B** or θ.



30. (*b*) Given, $\mathbf{A} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$

and $\mathbf{B} = \hat{\mathbf{i}} - \hat{\mathbf{j}}$

We know that

$$\mathbf{A}\cdot\mathbf{B} = |\mathbf{A}||\mathbf{B}|\cos\theta$$

$$\Rightarrow\quad (\hat{\mathbf{i}}+\hat{\mathbf{j}})\cdot(\hat{\mathbf{i}}-\hat{\mathbf{j}}) = (\sqrt{1+1})(\sqrt{1+1})\cos\theta$$

where θ is the angle between **A** and **B**

$$\Rightarrow\quad \cos\theta = \frac{1-0+0-1}{\sqrt{2}\sqrt{2}} = \frac{0}{2} = 0$$

$$\Rightarrow\quad \theta = 90°$$

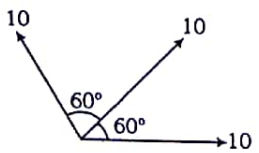
31. (*a*) When drawn as per polygon law of vector addition, then will form a closed regular polygon. Hence resultant will be zero.

32. (*a*) Only the magnitude will remain same.

33. (*c*) (**a** − **b**) is nothing but addition of **a** and −**b**.

So, the magnitude of **a** − **b** will lie between |**a**| + |**b**| and |**a**| − |**b**|.

34. (*a*)



35. (*c*) Resultant $S = 3\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 7\hat{\mathbf{i}} = -4\hat{\mathbf{i}} + 6\hat{\mathbf{j}}$

$\therefore\ |S| = \sqrt{16+36} = \sqrt{52}$ m

36. (*b*) Resultant $R = \sqrt{9P^2 + 4P^2 + 12P^2\cos\theta}$ ...(i)

$2R = \sqrt{36P^2 + 4P^2 + 24P^2\cos\theta}$ ...(ii)

Solve them $\cos\theta = -\frac{1}{2}$

$\therefore\ \theta = 120°$

37. (*b*) Third force should be equal and opposite to the resultant of the given vectors.

38. (*a*) Given, $A\cdot B = 0$

$$A \perp B = 1 \Rightarrow A \times B = 1$$

$$AB\sin\theta = 1 \Rightarrow AB\sin 90° = 1$$

or $$AB = 1 \Rightarrow A = 1 \text{ and } B = 1$$

So, A and B are perpendicular unit vectors.

39. (*c*) Resultant $R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$$3 = \sqrt{(\sqrt{3})^2 + (\sqrt{3})^2 + 2(\sqrt{3})(\sqrt{3})\cos\theta}$$

Squaring both sides

$$9 = 3 + 3 + 6\cos\theta$$

$$\cos\theta = \frac{1}{2} \Rightarrow \theta = 60° \text{ or } \frac{\pi}{3}$$

40. (*b*) Let **r** makes an angle θ with positive x-axis component of **r** along X-axis

$$r_x = |\mathbf{r}|\cos\theta$$

$$(r_x)_{\text{maximum}} = |\mathbf{r}|(\cos\theta)_{\text{maximum}}$$

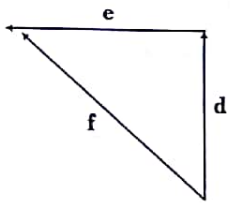
$$= |\mathbf{r}|\cos 0° = |\mathbf{r}|$$

($\because$ cos θ is maximum of θ = 0°)

As $\theta = 0°$,

r is along positive x-axis.

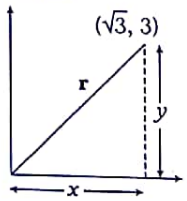
41. (*d*)



$$\mathbf{f} = \mathbf{d} + \mathbf{e}$$

42. (*c*) Final position vector of a particle, $\mathbf{r} = \sqrt{3}\mathbf{i} + 3\hat{\mathbf{j}}$

Angle of path from X-axis,



$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{3}{\sqrt{3}}\right) = \tan^{-1}(\sqrt{3}) = 60°$$

43. (*c*) **F** lies in XY-plane. Hence, $7\hat{\mathbf{k}}$ which is perpendicular to XY-plane is also perpendicular to **F**.

44. (*b*) The angle between vectors (**A** × **B**) and (**B** × **A**) is 180°.

45. (*c*) Dot product of two perpendicular vector should be zero. The only vector which will be perpendicular to both the given vector will be $5\hat{\mathbf{j}} + \hat{\mathbf{k}}$.

46. (*a*) Unit vector is given by $\hat{\mathbf{n}} = \dfrac{\mathbf{A}\times\mathbf{B}}{|\mathbf{A}\times\mathbf{B}|}$

$$\mathbf{A}\times\mathbf{B} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ -3 & -2 & -3 \\ 2 & 4 & 6 \end{vmatrix}$$

$$= \hat{\mathbf{i}}(-12+12) - \hat{\mathbf{j}}(-18+6) + \hat{\mathbf{k}}(-12+4) = 12\hat{\mathbf{j}} - 8\hat{\mathbf{k}}$$

$$|\mathbf{A}\times\mathbf{B}| = \sqrt{(12)^2+(8)^2} = \sqrt{208} = 4\sqrt{13}$$

$$\therefore \quad \hat{\mathbf{n}} = \frac{12\hat{\mathbf{j}} - 8\hat{\mathbf{k}}}{4\sqrt{13}} = \frac{3\hat{\mathbf{j}} - 2\hat{\mathbf{k}}}{\sqrt{13}}$$

47. (*d*) Resultant of two given vectors will be

$$\sqrt{(8)^2+(8)^2} = 8\sqrt{2}$$

$\therefore$ Unit vector parallel to the given vectors will be

$$\frac{\hat{\mathbf{i}}+\hat{\mathbf{j}}}{\sqrt{2}}$$

48. (*d*) Force $F_v = F\cos\theta = 5\cos 60 = 2.5\,\text{N}$

49. (*d*) $\mathbf{r} = |B|\hat{\mathbf{A}}$

50. (*b*) $\hat{\mathbf{j}} = (\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) + (3\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 7\hat{\mathbf{k}}) + \mathbf{C}$

Hence, $\mathbf{C} = -4\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$

51. (*c*) In XY-plane, vector is $3\hat{\mathbf{i}} + \hat{\mathbf{j}}$

$\therefore$ Length in XY-plane $= \sqrt{9+1} = \sqrt{10}$

52. (*b*) Dot product of two perpendicular vectors should be zero.

$\therefore$ $\mathbf{B} = \hat{\mathbf{i}}B\sin\theta - \hat{\mathbf{j}}B\cos\theta$

$[\because (\hat{\mathbf{i}}A\cos\theta + \hat{\mathbf{j}}A\sin\theta)(\hat{\mathbf{i}}B\sin\theta - \hat{\mathbf{j}}B\cos\theta) = 0]$

53. (*b*) The coefficients of $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ should be a constant ratio.

or $\quad \dfrac{2}{-4} = \dfrac{3}{-6} = \dfrac{1}{-\lambda}$ or $\lambda = 2$

54. (*b*) $\mathbf{P}\times\mathbf{Q}$ is perpendicular to the plane formed by $\mathbf{P}$ and $\mathbf{Q}$.

$\mathbf{P}+\mathbf{Q}$ lies in this plane. Hence $\mathbf{P}+\mathbf{Q}$ is perpendicular to $\mathbf{P}\times\mathbf{Q}$.

55. (*b*) For a unit vector, $\sqrt{(0.5)^2+(0.8)^2+c^2} = 1$

On solving, we get

$$c = \sqrt{0.11}$$

56. (*a*) For vectors to be perpendicular, dot product should be zero.

$\therefore \quad (a_1\hat{\mathbf{j}} + b_1\hat{\mathbf{j}})\cdot(a_2\hat{\mathbf{i}} + b_2\hat{\mathbf{j}}) = 0$

or $\quad \dfrac{a_1}{b_1} = -\dfrac{b_2}{a_2}$

57. (*a*) For vectors to be perpendicular,

$$\mathbf{A}\cdot\mathbf{B} = 0$$

$\Rightarrow \quad (2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - x\hat{\mathbf{k}})\cdot(2\hat{\mathbf{i}} - \hat{\mathbf{j}} - 3\hat{\mathbf{k}}) = 0$

$\Rightarrow \quad 4 - 2 + 3x = 0$

$\Rightarrow \quad 3x = -2$ or $x = \dfrac{-2}{3}$

58. (*c*) Component of $\mathbf{A}$ along $\mathbf{B} = \dfrac{\mathbf{A}\cdot\mathbf{B}}{B}$

$$= \frac{4+12}{\sqrt{(2)^2+(4)^2}} = \frac{16}{2\sqrt{5}} = \frac{8}{\sqrt{5}}$$

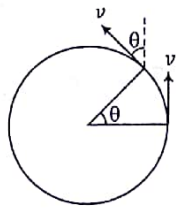
59. (*b*) For vectors to be parallel, ratio of coefficient should be same.

$\Rightarrow \quad \dfrac{6}{5} = \dfrac{x}{6} = \dfrac{-2}{-y} \Rightarrow x = \dfrac{36}{5}, y = \dfrac{10}{6} = \dfrac{5}{3}$

60. (*a*) Vector $\mathbf{A} = \sqrt{(3)^2+(6)^2+(2)^2} = 7$

$$\alpha = \cos^{-1}\left(\frac{3}{7}\right), \ \beta = \cos^{-1}\left(\frac{6}{7}\right), \gamma = \cos^{-1}\left(\frac{2}{7}\right)$$

61. (*d*) $v_1 = v\hat{\mathbf{j}}$



$v_2 = -v\sin\theta\hat{\mathbf{j}} + v\cos\theta\hat{\mathbf{j}}$

$\Delta v = v_2 - v_1$

$= -v\sin\theta\,\hat{\mathbf{i}} + v(\cos\theta - 1)\hat{\mathbf{j}}$

$|\Delta v| = 2v\sin\dfrac{\theta}{2}$

$= 2v\sin\dfrac{60°}{2} = 2v\sin 30°$

$= 2v \times \dfrac{1}{2} = v$

62. (*b*) $R_{\text{net}} = R + \sqrt{R^2+R^2} = R + \sqrt{2}R = R(\sqrt{2}+1)$

63. (*a, b*) Resultant of two vector $A_R = 2A\cos\dfrac{\theta}{2}$

64. (*d*) $(a\hat{\mathbf{i}} + b\hat{\mathbf{j}})\cdot(\hat{\mathbf{i}} + \hat{\mathbf{j}}) = 0$ or $a + b = 0$ ...(i)

Further, $\sqrt{a^2+b^2} = 1$

or $\quad a^2 + b^2 = 1$ ...(ii)

Solving, Eqs. (i) and (ii), we get

$$a = \pm\frac{1}{\sqrt{2}} \text{ and } b = \pm\frac{1}{\sqrt{2}}$$

65. (*a*) We have, $\mathbf{a}+\mathbf{b}+\mathbf{c} = 0$

$\therefore \quad \mathbf{a}+\mathbf{c} = -b$

or $\quad (\mathbf{a}+\mathbf{c})\times\mathbf{b} = -\mathbf{b}\times\mathbf{b} = 0$

or $\quad (\mathbf{a}\times\mathbf{b}) + (\mathbf{c}\times\mathbf{b}) = 0$

or $\quad \mathbf{a}\times\mathbf{b} = \mathbf{b}\times\mathbf{c}$

66. (*c*) $\mathbf{F}\cdot\mathbf{F}' = 0$. Here $\mathbf{F}'$ = unknown force.

$\therefore \quad \mathbf{F}' = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}}$

67. (*c*) For vectors to be perpendicular, $\mathbf{A}\cdot\mathbf{B} = 0$

$\therefore$ The appropriate vector will be $6\hat{\mathbf{k}}$.

68. (*b*) $\mathbf{A}+\mathbf{B} = 10\hat{\mathbf{i}} + 5\hat{\mathbf{j}}$

$\therefore \quad |\mathbf{A}+\mathbf{B}| = \sqrt{100+25} = 5\sqrt{5}$

Angle of $\mathbf{A}+\mathbf{B}$ with X-axis,

$$\theta = \tan^{-1}\left(\frac{5}{10}\right) = \tan^{-1}\left(\frac{1}{2}\right)$$

69. (*a*) Acceleration $a = \frac{F}{m}$

Resultant of F_1 and F_2 will be

$F_{12} = \sqrt{2}F$ (in opposite direction of F_3)

Now, resultant of F_{12} and F_3 will be,

$$F_{net} = (\sqrt{2} - 1)F$$

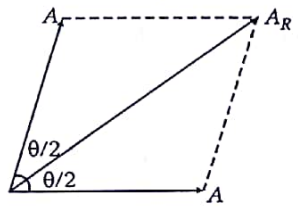
$$\therefore \quad a' = \frac{F_{net}}{m} = (\sqrt{2} - 1)\frac{F}{m} = (\sqrt{2} - 1)a$$

70. (*a*) We have, $\mathbf{A} - \mathbf{B} = \hat{\mathbf{j}} - 2\hat{\mathbf{k}} = C$

$$C = \sqrt{1+4} = \sqrt{5}$$

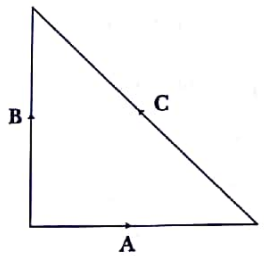
$$\cos\alpha = \frac{0}{\sqrt{5}} = 0, \cos\beta = \frac{1}{\sqrt{5}} \text{ and } \cos\gamma = \frac{-2}{\sqrt{5}}$$

71. (*a*) Resultant will be $2A\cos\frac{\theta}{2}$ along bisector.



72. (*b*) For a given condition $\theta = \frac{360°}{n} = \frac{360°}{12} = 30$

73. (*d*) Angle between **A** and **B** is 90°.



Between **B** and **C** is 135° and that between **A** and **C** is 135°.

74. (*d*) $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$ ...(i)

From Eq. (i), we get $R^2 = C^2 = A^2 + B^2 + 2AB\cos\theta = A^2 + B^2$

$$\Rightarrow \quad \cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2}$$

75. (*a,b*) $|\mathbf{A}| = \sqrt{9+16} = 5$ and $|\mathbf{B}| = \sqrt{36+64} = 10$. **B** = 2**A** or **B** is parallel to **A**. Ratio of their coefficients are equal, so they are parallel. Or their cross product is zero.

76. (*c*) $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$

$$= 1 - \cos^2\alpha + 1 - \cos^2\beta + 1 - \cos^2\gamma$$

$$= 3 - (\cos^2\alpha + \cos^2\beta + \cos^2\gamma) = 3 - 1 = 2$$

77. (*c*) $|\hat{\mathbf{A}} - \hat{\mathbf{B}}| = \sqrt{1 + 1 - 2\times1\times1\times\cos\theta} = 2\sin\frac{\theta}{2}$

Angle between $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$ is also θ, but their magnitudes are 1.

78. (*b*) We have, $S_1 = (\sqrt{2}\cos 45°)\hat{\mathbf{i}} + (\sqrt{2}\sin 45°)\hat{\mathbf{j}} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$

$S_2 = -2\hat{\mathbf{j}}$ and $S_3 = (-4\cos 30°)\hat{\mathbf{i}} + (4\sin 30°)\hat{\mathbf{j}}$

$$= -2\sqrt{3}\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$$

Now, $S = S_1 + S_2 + S_3 = (1 - 2\sqrt{3})\hat{\mathbf{i}} + \hat{\mathbf{j}}$

$\therefore$ The magnitude is $\sqrt{(1-2\sqrt{3})^2 + (1)^2} = \sqrt{14 - 4\sqrt{3}}$

79. (*c*) We have, $R_1^2 = A^2 + B^2 + 2AB\cos\theta$

and $R_2^2 = A^2 + B^2 - 2AB\cos\theta$

$$\therefore \quad R_1^2 + R_2^2 = 2(A^2 + B^2)$$

80. (*b*) Sum of two unit vectors is a unit vector, means angle between those two unit vectors is 120°.

$$\therefore \quad |S| = \sqrt{1 + 1 - 2\times1\times1\times\cos 120°} = \sqrt{3}$$

81. (*a*) $\mathbf{R}\cdot\mathbf{S} = (\mathbf{A} + \mathbf{B})(\mathbf{A} - \mathbf{B})$

$$= \mathbf{A}\cdot\mathbf{A} - \mathbf{A}\cdot\mathbf{B} + \mathbf{B}\cdot\mathbf{A} - \mathbf{B}\cdot\mathbf{B}$$

or $0 = A^2 - B^2$ (as $\mathbf{A}\cdot\mathbf{B} = \mathbf{B}\cdot\mathbf{A}$)

$\therefore \quad A^2 = B^2$ or $A = B$

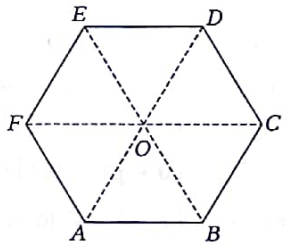
82. (*a*) We have, $A_x = A_y = A_z$

Now, Resultant $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$

$$= \sqrt{3}A_x$$

$$\therefore \quad A_x = \frac{A}{\sqrt{3}} = \frac{30}{\sqrt{3}} = 10\sqrt{3}$$

83. (*d*) **AB + AC + AD + AE + AF**



$$= \mathbf{AB} + (\mathbf{AB} + \mathbf{BC}) + (\mathbf{AB} + \mathbf{BC} + \mathbf{CD}) + (\mathbf{AB} + \mathbf{BC} + \mathbf{CD} + \mathbf{DE}) + \mathbf{AF}$$

$$= \mathbf{AB} + (\mathbf{AB} + \mathbf{BC} + \mathbf{CD} + \mathbf{DE} + \mathbf{EF})$$

$$+ \mathbf{AB} + \mathbf{BC} + \mathbf{CD} + \mathbf{DE} + \mathbf{EF}$$

$$\mathbf{AB} = -\mathbf{DE}, \mathbf{BC} = -\mathbf{EF} = 3(\mathbf{AB} + \mathbf{BC} + \mathbf{CD}) = 3\mathbf{AD} = 6\mathbf{AO}$$

84. (*a*) Since **A** is a unit vector in a given direction. It should be a constant unit vector.

or $\frac{d\mathbf{A}}{dt} = 0$

85. (*a*) $F\cdot F\cdot\cos\theta = F\cdot F\cdot\sin\theta$ or $\tan\theta = 1$ or $\theta = 45°$

$$|\mathbf{F}_1\cdot\mathbf{F}_2| = \sqrt{F^2 + F^2 + 2\cdot F\cdot F\cdot\cos 45°}$$

$$= (\sqrt{2+\sqrt{2}})F$$

86. (*a*) From the figure $\mathbf{OB} + \mathbf{BC} = \mathbf{r}$ or $\mathbf{q} + \mathbf{BC} = \mathbf{r}$ ...(i)

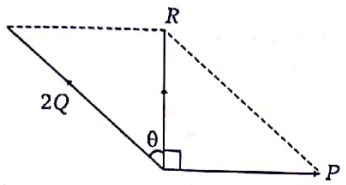
$$\mathbf{OA} + \mathbf{AC} = \mathbf{r}$$

or $\mathbf{p} + \mathbf{AC} = \mathbf{r}$ ...(ii)

Adding these two equations, we get, $\mathbf{p} + \mathbf{q} = 2\mathbf{r}$, as **AC** and **BC** are equal and opposite vectors.

87. (*a*) Cross product of $(\mathbf{P} + \mathbf{Q})$ and $(\mathbf{P} - \mathbf{Q})$ is perpendicular to the plane formed by $(\mathbf{P} + \mathbf{Q})$ and $(\mathbf{P} - \mathbf{Q})$.

88. (*c*) We have $2Q\sin\theta = P$, as $R \perp P$



Now, $R = \sqrt{P^2 + Q^2 + 2PQ\cos(90° + \theta)}$

$= \sqrt{P^2 + Q^2 - 2PQ\sin\theta}$

$= \sqrt{(2Q\sin\theta)^2 + Q^2 - 2(2Q\sin\theta)Q\sin\theta}$

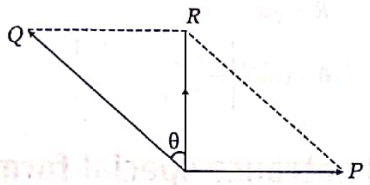
$= \sqrt{4Q^2\sin^2\theta + Q^2 - 4Q^2\sin^2\theta}$

$= Q$

89. (*b*) $\mathbf{B} \times \mathbf{A}$ is perpendicular to $\mathbf{A}$. Hence $\mathbf{A}\cdot(\mathbf{B}\times\mathbf{A})$ will be zero.

90. (*c*) We have, $P + Q = 18$ ...(i)

and $R = 12$



$Q\sin\theta = P$

and $Q\cos\theta = R$

Squaring and adding, we get

$P^2 + R^2 = Q^2$

or $Q^2 - P^2 = R^2 = 144$ ...(iii)

Solving the above equation, we get $P = 5$ and $Q = 13$.

91. (*b*) $|\Delta\mathbf{a}| = \sqrt{a^2 + a^2 - 2\cdot a\cdot a\cdot\cos(d\theta)} = 2a\sin\left(\frac{d\theta}{2}\right)$

For small angles $\sin\frac{d\theta}{2} \approx \frac{d\theta}{2}$

$\therefore \quad |\Delta\mathbf{a}| = 2a \times \frac{d\theta}{2} = a\cdot d\theta$

92. (*c*) Total work done $W = \int \mathbf{F}\cdot d\mathbf{r} = \int -K(y\hat{\mathbf{i}} + x\hat{\mathbf{j}})\cdot(dx\hat{\mathbf{i}} + dy\hat{\mathbf{i}})$

$= \int K(ydx + xdy) = -K\int_{(0,0)}^{(a,a)} d(xy) = -K[(xy)]_{(0,0)}^{(a,a)} = -Ka^2$

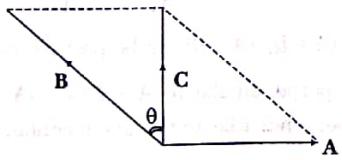
93. (*b, c*) Apply polygon law,

We get $Y = B + C - A$ and $Z = B + C$.

94. (*a*) $(\mathbf{a}+\mathbf{b})\times(\mathbf{a}-\mathbf{b}) = \mathbf{a}\times\mathbf{a} - \mathbf{a}\times\mathbf{b} + \mathbf{a}\times\mathbf{a} - \mathbf{b}\times\mathbf{b}$

$= 2(b \times a)$

95. (*c*) From the figure, $B\cos\theta = C$ and $B\sin\theta = A$

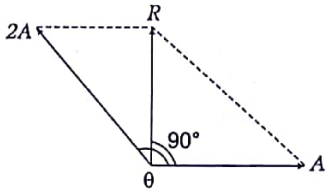


But $A = C$

$\therefore B\cos\theta = B\sin\theta$ or $\theta = 45°$

$\therefore$ Angle between **A** and **B** is 135°

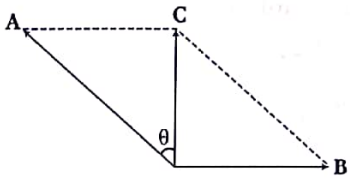
96. (*a*) We have, $\tan 90° = \frac{2A\sin\theta}{A + 2A\cos\theta} = \infty$



$\therefore \quad A + 2A\cos\theta = 0$

or $\cos\theta = -\frac{1}{2}$ or $\theta = 120°$

97. (*c*) Given, $\mathbf{B}\cdot\mathbf{C} = 0$



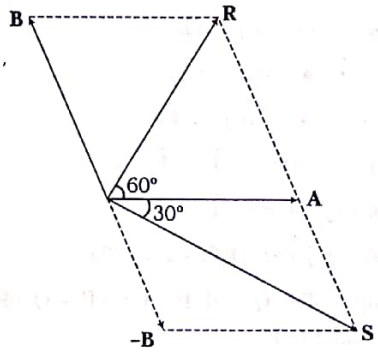
$\therefore \quad \mathbf{B} \perp \mathbf{C}$

$A\sin\theta = B$

or $\sin\theta = \frac{B}{A} = \frac{1}{2}$

$\therefore \quad \theta = 30°$ or $\mathbf{A}\cdot\mathbf{B} < 0$

98. (*d, a*) Resultant vector $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$



we have,

$1 = \sqrt{1 + 1 + 2\cos\theta}$

or $\cos\theta = -\frac{1}{2}$

$\therefore \quad \theta = 120°$

$S = \sqrt{A^2 + B^2 - 2AB\cos\theta}$

$= \sqrt{1 + 1 - 2\times1\times1\times\left(-\frac{1}{2}\right)} = \sqrt{3}$

99. (*d*) A scalar quantity is independent of direction hence has the same value for observers with different orientations of the axes.

100. (*b*) Clearly from the diagram, $\mathbf{u} = a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$

As **u** is in the first quadrant, hence both components a and b will be positive.

For $\mathbf{v} = p\,\hat{\mathbf{i}} + q\,\hat{\mathbf{j}}$, as it is in positive x-direction and located downward hence x-component p will be positive and y-component q will be negative.

101. (*b*) Magnitude of component of **v** along a

$$= \frac{\mathbf{v}\cdot\mathbf{a}}{a} = \frac{6+2-2}{\sqrt{3}} = 2\sqrt{3}$$

Now, $\hat{\mathbf{a}} = \dfrac{\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}}{\sqrt{3}}$

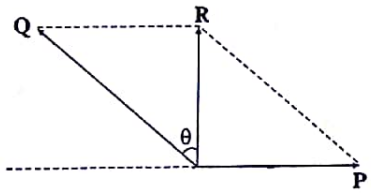
$\therefore$ Component in vector form

$$= 2\sqrt{3}\hat{\mathbf{a}}$$

$$= (2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$$

102. (*b*) We have, $Q\sin\theta = P$

and $Q\cos\theta = R = P$



Dividing these two equations we get,

$$\tan\theta = 1$$

$\therefore$ $\theta = 45°$

$\therefore$ Angle between **P** and **Q** is 135°

103. (*c*) Suppose $\mathbf{a} = a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{j}} + a_3\hat{\mathbf{k}}$

Now, $(\hat{\mathbf{i}} \times \mathbf{a}) = a_2\hat{\mathbf{k}} - a_3\hat{\mathbf{j}}$

Now, $\hat{\mathbf{i}} \times (\hat{\mathbf{i}} \times \mathbf{a}) = -a_2\hat{\mathbf{j}} - a_3\hat{\mathbf{k}}$

Similarly, $\hat{\mathbf{j}} \times (\hat{\mathbf{j}} \times \hat{\mathbf{a}}) = -a_1\hat{\mathbf{i}} - a_3\hat{\mathbf{k}}$

and $\hat{\mathbf{k}} \times (\hat{\mathbf{j}} \times \mathbf{a}) = -a_1\hat{\mathbf{i}} - a_2\hat{\mathbf{j}}$

$\therefore$ $\hat{\mathbf{i}}(\hat{\mathbf{i}} \times \mathbf{a}) + \hat{\mathbf{j}}(\hat{\mathbf{j}} \times \mathbf{a}) + \hat{\mathbf{k}}(\hat{\mathbf{k}} \times \mathbf{a}) = -2\mathbf{a}$

104. (*a*) Resultant of $(\mathbf{P} + \mathbf{Q})$ and $(\mathbf{P} - \mathbf{Q})$ is $\mathbf{P} + \mathbf{Q} + \mathbf{P} - \mathbf{Q}$ or $2\mathbf{P}$ which is parallel to **P**.

So angle between P and $2P$ will be zero.

105. (*b*) Since, $\mathbf{a}_1$ and $\mathbf{a}_2$ are non-collinear

$\therefore$ $a_1 = a_2 = 1$

and $|\mathbf{a}_1 + \mathbf{a}_2| = \sqrt{3}$

$\Rightarrow$ $a_1^2 + a_2^2 + 2a_1a_2\cos\theta = (\sqrt{3})^2$

$\Rightarrow$ $1 + 1 + 2\cos\theta = 3 \Rightarrow \cos\theta = \dfrac{1}{2}$

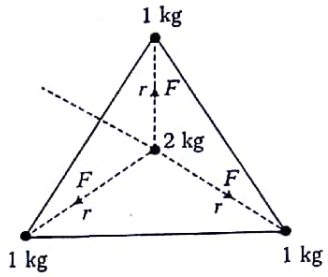
Now $(\mathbf{a}_1 - \mathbf{a}_2)\cdot(2\mathbf{a}_1 - \mathbf{a}_2) = 2a_1^2 - a_1.a_2 - 2a_1.a_2 + a_2^2$

$$= 2a_1^2 + a_2^2 - 3a_1a_2\cos\theta$$

$$= 2 + 1 - \frac{3}{2} = 3/2$$

106. (*c*) Since each of N-coplanar vector is inclined at $\dfrac{2\pi}{N}$ to the preceding. Hence, they will form a closed polygon. Therefore their resultant must be zero.

107. (*d*)



All forces are equal. Angle between two forces = 120°

Resultant of any two forces is equal and opposite to third force.

108. (*a*) Resultant $R^2 = A^2 + B^2 + 2AB\cos\theta$

Substituting, $A = (x + y), B = (x - y)$

and $R = \sqrt{x^2 + y^2}$

we get, $\theta = \cos^{-1}\left[-\dfrac{(x^2 + y^2)}{2(x^2 - y^2)}\right]$

B) Medical entrance special format question

Assertion and reason

1. (*a*) If a particle is rotating about on axis on itself, the orientation of the particle is not same under a different order of addition of angular displacement, so it cannot be regarded as vector quantity.

$$\mathbf{d}\theta_1 + \mathbf{d}\theta_2 = \mathbf{d}\theta_2 + \mathbf{d}\theta_1$$

but $\theta_1 + \theta_2 \neq \theta_2 + \theta_1$

2. (*c*) Pressure and surface tension are scalar quantities.

3. (*d*) $|\mathbf{A} + \mathbf{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

and $|\mathbf{A} - \mathbf{B}| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$

$$\theta = 90°$$

$$|A + B| = |A - B| = \sqrt{A^2 + B^2}$$

4. (*b*) $|\mathbf{A} \times \mathbf{B}| = AB\sin\theta$ and $\mathbf{A}\cdot\mathbf{B} = AB\cos\theta$

5. (*b*) $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ are mutually perpendicular to each other and angle between $(\hat{\mathbf{i}} + \hat{\mathbf{j}})$ and $\hat{\mathbf{i}}$ is 45°.

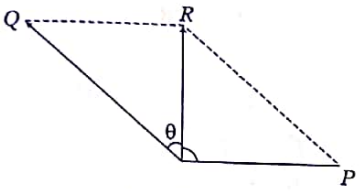
6. (*d*) Component of A along $B = \dfrac{\mathbf{A}\cdot\mathbf{B}}{|\mathbf{B}|} = \dfrac{AB\cos\theta}{B} = A\cos\theta$

Component of **B** along $A = \dfrac{\mathbf{A}\cdot\mathbf{B}}{|\mathbf{A}|} = \dfrac{AB\cos\theta}{A} = B\cos\theta$

8. (*d*) Value of $(A + B)\cdot(A - B)$ can be positive or negative.

9. (*a*) $(\mathbf{A} \times \mathbf{B})$ is perpendicular to $(\mathbf{A} + \mathbf{B})$ and $(\mathbf{A} - \mathbf{B})$ because $(\mathbf{A} \times \mathbf{B})$ lies perpendicular to the plane containing **A** and **B**.

10. (*b*) $\mathbf{A}\cdot\mathbf{B} = AB\cos\theta$. Here AB is always positive value of $\cos\theta$ determines the sign of $\mathbf{A}\cdot\mathbf{B}$.



11. (*b*) $(\mathbf{A}+\mathbf{B})\cdot(\mathbf{A}-\mathbf{B}) = A^2 - B^2$

This dot product is zero only if $A = B$.
Therefore they are perpendicular. Further, $(\mathbf{A}\times\mathbf{B})$ is perpendicular to both $(\mathbf{A}+\mathbf{B})$ and $(\mathbf{A}-\mathbf{B})$.

12. (*b*) Angle between $\mathbf{A}\times\mathbf{B}$ and $\mathbf{B}\times\mathbf{A}$ is 180°.

$\therefore\ (\mathbf{A}\times\mathbf{B})\cdot(\mathbf{B}\times\mathbf{A}) = (AB\sin\theta)(AB\sin\theta)(\cos 180°)$

$= -A^2B^2\sin^2\theta$

13. (*c*) $\dfrac{|\mathbf{A}\times\mathbf{B}|}{AB}$ is $\dfrac{AB\sin\theta}{AB} = \sin\theta$

14. (*e*) $\dfrac{\mathbf{A}\times\mathbf{B}}{\mathbf{A}\cdot\mathbf{B}} = \dfrac{AB\sin\theta\,\hat{\mathbf{n}}}{AB\cos\theta} = \tan\theta\,\hat{\mathbf{n}}$

where $\hat{\mathbf{n}}$ is unit vector perpendicular to both **A** and **B**.

However $\dfrac{|\mathbf{A}\times\mathbf{B}|}{\mathbf{A}\cdot\mathbf{B}} = \tan\theta$

Match the column

1. $(2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}})$ is perpendicular to **A**, because the dot product of these two vectors is zero.

Further $(6\hat{\mathbf{i}} + 8\hat{\mathbf{j}} - 10\hat{\mathbf{k}})$ vector is parallel to **A**. So component of **A** along this vector is magnitude of **A** which is $5\sqrt{2}$ unit.

The last vector *i.e.*, $(-3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 5\hat{\mathbf{k}})$ is anti-parallel to **A** along this vector is negative of magnitude of **A** or $-5\sqrt{2}$ unit.

2. $|\mathbf{A}+\mathbf{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

Here $|A| = |B| = x \Rightarrow \theta = 60°$

$\therefore\ |A+B| = \sqrt{3}x$

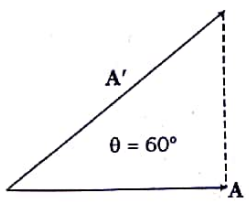
$|A-B| = \sqrt{A^2 + B^2 - 2AB\cos\theta} = x$

$\mathbf{A}\cdot\mathbf{B} = AB\cos\theta = \dfrac{x^2}{2}$

$|A\times B| = |A||B|\sin\theta\,\hat{\mathbf{n}} = \dfrac{\sqrt{3}}{2}x^2$

3. Using right hand rule, direction of $(A\times B)$ will be vertically upwards $(A+B)$ will point towards north east.

4. (a) When $\theta = 60°$

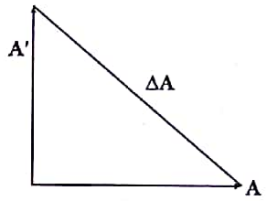


$\Rightarrow\ \mathbf{A} = \mathbf{A}'$

$\therefore\ \Delta A = \sqrt{x^2 + x^2 - 2\cdot x\cdot x\cdot\cos 60°}$

$\therefore\ n = 1$

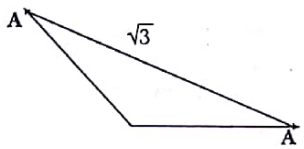
(b) When $\theta = 90°$



$\Rightarrow$ Change in A, $\Delta A = \sqrt{x^2 + x^2} = \sqrt{2}x \Rightarrow n = \sqrt{2}$

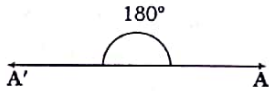
(c) When $\theta = 120°$

$\therefore\ n = \sqrt{3}$



(d) When $\theta = 180°$

$\therefore\ n = 2$



5. (a) $A\cdot B = AB\cos\theta$

$|A\times B| = AB\sin\theta$

$A\cdot B = |A\times B|$

Only when $\theta = 45°$

(b) $A\cdot B = B^2$

When $A = B$ or $A\cdot B = AB\cos\theta$

$\theta = 0$ or $\theta = 180°$

(c) $|A+B| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$|A-B| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$

when $\theta = 90°$

$|A+B| = |A-B|$

(d) when $\theta = 90°$

then, $|A\times B| = AB$

C) Medical entrances' gallary

1. (*a*) Suppose two vectors are **P** and **Q**.

It is given that

$$|\mathbf{P}+\mathbf{Q}| = |\mathbf{P}-\mathbf{Q}|$$

Let angle between **P** and **Q** is ϕ.

$\therefore\ P^2 + Q^2 + 2PQ\cos\phi = P^2 + Q^2 - 2PQ\cos\phi$

$\Rightarrow \quad 4PQ \cos\phi = 0$

$\Rightarrow \quad \cos\phi = 0 \quad [\because P, Q \neq 0]$

$\Rightarrow \quad \phi = \frac{\pi}{2} = 90°$

2. (*c*) Dot product of these two vectors should be equal to zero as they are perpendicular to each other.

$\therefore \quad \mathbf{A}\cdot\mathbf{B} = -8 + 12 + 8\alpha = 0$

$8\alpha = -4 \Rightarrow \alpha = \frac{-1}{2}$

3. (*a*) The angle between

$\mathbf{p} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$

and X-axis, $\mathbf{x} = \hat{\mathbf{i}}$ is given by 3

$$\cos\theta = \frac{\mathbf{p}\cdot\mathbf{x}}{|\mathbf{p}||\mathbf{x}|} = \frac{(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})\cdot(\hat{\mathbf{i}})}{\sqrt{1^2 + 1^2 + 1^2}\cdot\sqrt{1^2}} = \frac{1}{\sqrt{3}}$$

$\therefore \quad \theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

4. (*a*) Vector $\mathbf{X}$ of the form $\alpha\mathbf{A} + \beta\mathbf{B}$.

$\mathbf{X} = \alpha\mathbf{A} + \beta\mathbf{B}$

$= \alpha(\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}) + \beta(\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}})$

$\mathbf{X} = \hat{\mathbf{i}}(\alpha + \beta) + \hat{\mathbf{j}}(\alpha - \beta) + \hat{\mathbf{k}}(-2\alpha + \beta)$

A vector $\mathbf{X}$ is perpendicular to $\mathbf{C}$, *i.e.*, $\mathbf{X}\cdot\mathbf{C} = 0$

$[\hat{\mathbf{i}}(\alpha + \beta) + \hat{\mathbf{j}}(\alpha - \beta) + \hat{\mathbf{k}}(-2\alpha + \beta)]\cdot[2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}] = 0$

$\Rightarrow \quad 2(\alpha + \beta) - 3(\alpha - \beta) + 4(-2\alpha + \beta) = 0$

$\Rightarrow \quad 2\alpha + 2\beta - 3\alpha + 3\beta - 8\alpha + 4\beta = 0$

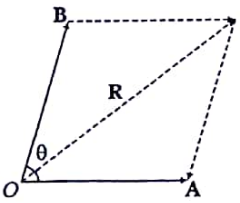
$\Rightarrow \quad -9\alpha + 9\beta = 0$

or $\quad \alpha = \beta$

$\Rightarrow \quad \frac{\alpha}{\beta} = \frac{1}{1}$

or $\quad \alpha : \beta = 1 : 1$

5. (*c*) Let two vectors are $\mathbf{A}$ and $\mathbf{B}$, inclined at an angle θ.



Resultant of the two vectors $\mathbf{A}$ and $\mathbf{B}$

$$|\mathbf{R}| = \sqrt{|\mathbf{A}|^2 + |\mathbf{B}|^2 + 2|\mathbf{A}||\mathbf{B}|\cos\theta} \quad \text{...(i)}$$

Observing $\theta = 0°$ gives

$|\mathbf{R}| = |\mathbf{A}| + |\mathbf{B}|$, which is true

Let, $\quad |\mathbf{A}| = |\mathbf{B}| = a$

According to the question, $|\mathbf{R}| = a$

From Eq. (i),

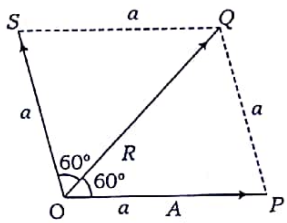
$a = \sqrt{a^2 + a^2 + 2aa\cos\theta}$

$\Rightarrow \quad a^2 = a^2 + a^2 + 2a^2\cos\theta$

$\Rightarrow \quad 2a^2\cos\theta = -a^2$

$\Rightarrow \quad \cos\theta = -1/2$

$\Rightarrow \quad \theta = 120°$



Geometrically, $\quad \angle SOP = 120°$

6. (*d*) Potential energy is not a vector quantity.

7. (*d*) Given, $\mathbf{A} = 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$

and $\quad \mathbf{B} = -\hat{\mathbf{j}} + \hat{\mathbf{k}}$

Scalar product $\mathbf{A}\cdot\mathbf{B} = (2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}})\cdot(-\hat{\mathbf{j}} + \hat{\mathbf{k}})$

Using $\quad \hat{\mathbf{i}}\cdot\hat{\mathbf{i}} = 1, \hat{\mathbf{j}}\cdot\hat{\mathbf{j}} = 1, \hat{\mathbf{k}}\cdot\hat{\mathbf{k}} = 1$

we have $\quad \mathbf{A}\cdot\mathbf{B} = -2 - 1 = -3$

8. (*a*) Given, $\quad \mathbf{A}\cdot\mathbf{B} = \mathbf{A}\times\mathbf{A}$

$AB\cos\theta = AB\sin\theta$

$\tan\theta = 1$

$\tan\theta = \tan 45°$

$\theta = 45°$

9. (*d*) According to given condition,

$\mathbf{A} + \mathbf{B}\hat{\mathbf{i}} = \mathbf{R}\hat{\mathbf{j}}$ (where, $\mathbf{R}$ is the resultant vector)

Also, $\quad \mathbf{R} = 2\mathbf{B}$

$\therefore \quad \mathbf{A} + \mathbf{B}\hat{\mathbf{i}} = 2\mathbf{B}\hat{\mathbf{j}}$ or $\mathbf{A} = 2\mathbf{B}\hat{\mathbf{j}} - \mathbf{B}\hat{\mathbf{i}}$

$\therefore \quad A^2 = 4B^2 + B^2 = 5B^2$

Here, $\quad A = 8$

$\therefore \quad 64 = 5B^2$

$\Rightarrow \quad B = \sqrt{\frac{64}{5}} = \frac{8}{\sqrt{5}}$

HAPTER
03

Motion in one dimension

Motion is defined as the change in position of an object with time. When the object moves along a single axis the motion is known as **one dimensional motion** and such a motion is along a straight line only, which may be horizontal, vertical or slanted. In this chapter, we will learn about motion using the concepts of velocity and acceleration along with the basic physics of one dimensional motion.

Frame of reference

A system of coordinate axes which defines the position of a particle or an event in two or three dimensional space is called a frame of reference. The simplest frame of reference is of course the familiar cartesian system of coordinates, in which the position of the particle is specified by its three coordinates x, y and z.

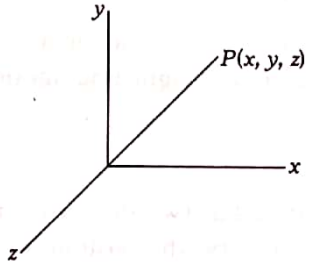


Fig. 3.1 Frame of reference for position of a particle P

In a simple word, a frame of reference is the frame in which an observer sits and make observations.

Types of frame of reference

Frame of references are of two types.

(i) **Inertial frame of reference** A non-accelerating frame of reference is called an inertial frame of reference. A frame of reference which is either at rest or moving with a constant velocity is an inertial frame of reference.

(ii) **Non-inertial frame of reference** An accelerating frame of reference is called a non-inertial frame of reference.

If observer is sit in accelerated moving frame, then frame is called a non-inertial frame of reference.

Inside

1) Rest and motion
   - Some basic terms related to motion
2) Kinematic equations for uniformly accelerated motion
3) Motion under gravity
   - Equations for motion under gravity
4) Non-uniformly accelerated motion
5) Graphical representation of motion
6) Relative velocity
   - Different cases of relative velocity
   - Examples of relative motion

1 Rest and motion

If the position of an object does not change w.r.t. its surrounding with the passage of time, it is said to be at rest. *e.g.*, Book lying on the table, a person sitting on a chair etc. And if the position of an object is continuously changing w.r.t. its surrounding, then it is said to be in the state of motion. *e.g.*, The walking man, crawling insects, water flowing down a dam' etc.

Rest and motion are relative terms

Rest and motion are always relative but never absolute. It means an object, can be at rest for an observer but the same object can be in motion when observed by other observer. *e.g.*, a person sitting in his house is at rest w.r.t. earth but is in motion w.r.t. moon.

Classification of motion

On the basis of the number of coordinates required to specify of the object, motion of object can be classified as

One dimensional motion

The motion of an object is considered as one dimensional, if only one coordinate is needed to specify the position of the object.

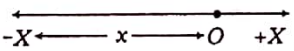


Fig. 3.2 *One dimensional motion*

In one dimensional motion, the object moves along a straight line. *e.g.*, A boy running on a straight line, motion of freely falling body etc.

Two dimensional motion

The motion of an object is considered as two dimensional, if two coordinates are needed to specify the position of the object.

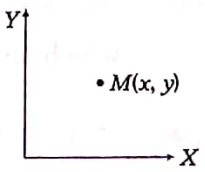


Fig. 3.3 *Two dimensional motion*

In two dimensional motion, the object moves in a plane. *e.g.*, A satellite revolving around the earth, motion of billiard ball, etc.

Three dimensional motion

The motion of an object is considered as three dimensional if all the three coordinates are needed to specify the position of the object.

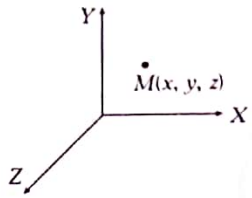


Fig. 3.4 *Three dimensional motion*

In three dimensional motion, the object moves in a space. *e.g.*, Butterfly flying in garden, the motion of water molecules, etc.

Some basic terms related to motion

1. Point object

An object is considered as point object if the size of the object is much smaller than the distance, it moves in a reasonable duration of time. *e.g.*, Earth can be considered as a point object during its revolution around sun because it covers the much larger distance than its own size.

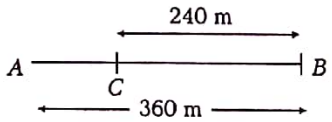
2. Distance and displacement

Distance

The length of the path covered by the object in a given time-interval, is known as its distance. Distance is a scalar quantity. The unit of distance is metre in SI or MKS and centimetre in CGS. Its dimensional formula is $[M^0LT^0]$.

Example 3.1 *A scooter is moving along a straight line AB covers a distance of 360 m in 24 s and returns back from B to C and coveres 240 m in 18 s. Find the total distance travelled by the scooter.*

Sol. From the above question, we draw the following figure.



Hence, to find out the total path distance, it does not matter what the time taken by a scooter to reach at *B* and the time taken to return at *C*.

$\therefore$ Total distance $= AB + BC = 360 + 240 = 600\,\text{m}$

Example 3.2 *A wheel completes 2000 revolutions to cover the 9.5 km distance. Find the diameter of the wheel.*

Sol. Given, number of revolutions $n = 2000$

Distance $x = 9.5\,\text{km} = 9.5 \times 10^3\,\text{m} = 9500\,\text{m}$

$\because$ Distance covered in n revolutions is equal to the circumference of the wheel.

$\therefore \quad x = n \cdot 2\pi r \Rightarrow x = n \cdot \pi D$ ($\because$ Diameter $= 2 \times$ radius)

$9500 = 2000 \times \pi \times D$

$$\Rightarrow \quad \text{Diameter, } D = \frac{9500}{2000 \times 3.14} = 1.5\,\text{m}$$

Displacement

The change in position vector of an object in a particular direction is termed as displacement. In other words, displacement is the shortest distance between the initial and final position of the moving object.

If $\mathbf{x}_1$ and $\mathbf{x}_2$ are the initial and final position vectors of particle respectively. Then, displacement of the particle is given by

$$\boxed{\Delta\mathbf{x} = \mathbf{x}_2 - \mathbf{x}_1}$$

- if $\mathbf{x}_2 > \mathbf{x}_1$, then $\Delta\mathbf{x}$ is positive;
- if $\mathbf{x}_1 > \mathbf{x}_2$, then $\Delta\mathbf{x}$ is negative;
- if $\mathbf{x}_1 = \mathbf{x}_2$, then $\Delta\mathbf{x}$ is zero.

i.e., the displacement of an object in motion can be positive, negative or zero while distance can never be negative or zero.

Displacement is a vector quantity as it possesses both, the magnitude and direction. The unit of displacement is metre in SI or MKS and centimetre in CGS. Its dimensional formula is $[M^0LT^0]$.

To understand distance and displacement clearly, let us consider the following example.

Suppose a person (moving body) moves from A to B (4 km) towards East and from B to C (3 km) due North as shown in figure, then the distance travelled by the person is

$$AB + BC = 4 \text{ km} + 3 \text{ km} = 7 \text{ km}$$

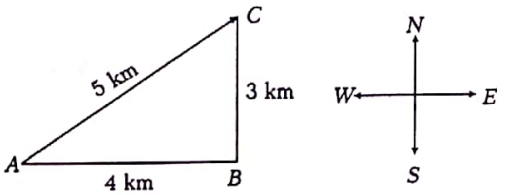


Fig. 3.5

But the displacement of the person is AC which can be calculated by Pythagoras' theorem

i.e., $$AC^2 = AB^2 + BC^2$$

$$= (4)^2 + (3)^2 = 16 + 9 = 25$$

$$\Rightarrow \quad AC = 5 \text{ km}$$

Distance vs Displacement

1. The magnitude of displacement may or may not be equal to the distance traversed by an object.
2. The magnitude of the displacement for a course of motion may be zero but the corresponding distance can never be zero.
3. If a particle moves in a straight line without change in direction, the magnitude of displacement is equal to the distance travelled otherwise displacement is always less than distance. Thus,

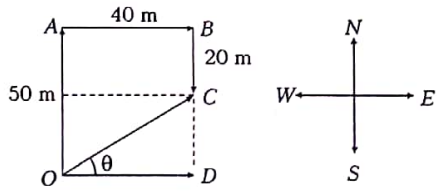
$$|\text{Displacement}| \le \text{Distance}$$

Example 3.3 *A man starts from his home and walks 50 m towards north, then he turns towards east and walks 40 m and then reaches to his office after moving 20 m towards south.*

(*i*) *What is the total distance covered by the man from his home to office?*

(*ii*) *What is his displacement from his home to office?*

Sol. Let O is a point of home then according to the question, the man moves from O to A (50 m) towards north, then from A to B (40 m) towards east and from B to C (20 m) towards south as shown in figure.



(*i*) Total distance travelled by the man is

$$OA + AB + BC = 50 + 40 + 20 = 110 \text{ m}$$

(ii) Displacement of the person is OC, which can be calculated by Pythagoras theorem, *i.e.*,

$$OC^2 = OD^2 + CD^2 = (40)^2 + (30)^2$$

$$= 1600 + 900 = 2500$$

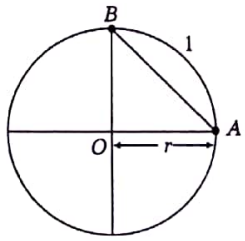
$$\Rightarrow \quad OC = 50 \text{ m}$$

Example 3.4 *An object covers 1/4 of the circular path, what will be the ratio of the distance and displacement of the object?*

Sol. Distance covered by object = 1/4 of the circular path

$= AB$ from path (1)

$= 1/4$ part of circumference of circular path $= \dfrac{2\pi r}{4} = \dfrac{\pi r}{2}$



Displacement = Short distance between initial position (A) and final position (B)

$$AB = \sqrt{OA^2 + OB^2} = \sqrt{r^2 + r^2} = r\sqrt{2}$$

$$\therefore \quad \frac{\text{Distance}}{\text{Displacement}} = \frac{\pi r/2}{r\sqrt{2}} = \frac{\pi}{2\sqrt{2}}$$

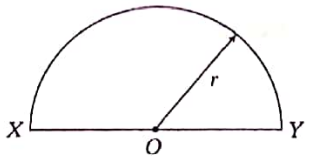
Example 3.5 *Displacement of a person moving from X to Y along a semicircular path of radius r is 200 m. What is the distance travelled by him?*

Sol. Given, displacement = 200 m

Distance travelled by the person from X to Y is equal to the circumference of the semicircular path

$$\therefore \quad x = 2\pi D \Rightarrow x = \pi r \quad \left(\because \text{Diameter} = \frac{\text{Radius}}{2}\right) \text{...(i)}$$

According to the question, the shortest distance between the final position Y and initial position X is $XY = 2r$.



$\therefore$ The displacement traversed by the person is $2r$.

$\therefore \quad 200 = 2r \Rightarrow r = 100$ m

Putting $r = 100$ m in Eq. (i), we get

Distance $x = \pi \times 100 = 3.14 \times 100 \Rightarrow x = 314$ m

Example 3.6 *An athlete completes one round of a circular track of diameter 200 m in 40 s. What will be the distance covered and the displacement at the end of 2 min 20 s?*

Sol. Diameter of circular track, $d = 200$ m

Circumference of circular track

$$= 2\pi r = \pi \times (d)$$

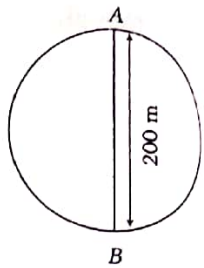
$$= \frac{22}{7} \times 200 = \frac{4400}{7} \text{ m}$$

Time for completing one round = 40 s.

Time in which athlete moved = 2 min and 20 s = 140 s.

In 40 s distance covered by athlete

$$= \frac{4400}{7} \text{ m}$$



$\therefore$ Distance covered by athlete in 140 s

$$= \frac{4400}{7} \times \frac{140}{40} = 2200 \text{ m}$$

$\therefore$ Distance covered = 2200 m.

As the athlete returns to the initial point A in 40 s, this displacement = 0

In 40 s, the no. of round around the track = 1

$\therefore$ In 140 s, the no. of rounds around the track $= \frac{140}{40} = 3\frac{1}{2}$

For each complete round the displacement is 0.

$\therefore$ For 3 complete rounds, the displacement will be 0.

$\therefore$ The final displacement will be due to 1/2 round.

Thus, his displacement = diameter of circular track = 200 m

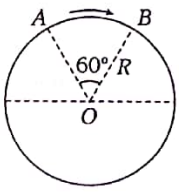
$\therefore$ Displacement after 2 min 20 s = 200 m

Check point 3.1

1. Which of the following is a one-dimensional motion?
(a) Landing of an aircraft
(b) Earth revolving around the sun
(c) Motion of wheels of moving train
(d) Train running on a straight track

2. A person moves towards East for 3 m, then towards North for 4 m and then moves vertically up by 5 m. What is his distance now from the starting point?
(a) $5\sqrt{2}$ m (b) 5 m
(c) 10 m (d) 20 m

3. A particle moves in a circle of radius R from A to B, as shown in figure. The distance covered by the object is



(a) $\frac{\pi R}{3}$ (b) $\frac{\pi R}{2}$
(c) $\frac{\pi R}{4}$ (d) πR

4. A person moves 30 m North and then 20 m towards East and finally $30\sqrt{2}$ m in South-West direction. The displacement of the person from the origin will be
(a) 10 m along North (b) 10 m long South
(c) 10 m along West (d) Zero

5. An aeroplane flies 400 m North and 300 m South and then flies 1200 m upwards then net displacement is
(a) 1200 m (b) 1300 m
(c) 1400 m (d) 1500 m

6. A wheel of radius 1 m rolls forward half a revolution on a horizontal ground. The magnitude of the displacement of the point of the wheel initially in contact with the ground is
(a) 2π (b) $\sqrt{2}\pi$ (c) $\sqrt{\pi^2 + 4}$ (d) π

7. The three initial and final position of a man on the X-axis are given as
(i) (−8 m, 7 m) (ii) (7 m, −3 m)
(iii) (−7 m, 3 m)
Which pair gives the negative displacement?
(a) (i) (b) (ii) (c) (iii) (d) (i) and (iii)

8. A particle moves along a circular path of radius R. The distance and displacement of a particle after one complete revolution is
(a) 0, $2\pi r$ (b) $2\pi r$, 0
(b) 0, πr (d) πr, 0

9. A particle starts from the origin, goes along X-axis to the point (20 m, 0) and then returns along the same line to the point (−20 m, 0). The distance and displacement of the particle during the trip are
(a) 40 m, 0 (b) 40 m, 20 m
(d) 40 m, − 20 m (d) 60 m, − 20 m

10. The numerical ratio of displacement to the distance covered is always
(a) less than one (b) equal to one
(c) equal to or less than one (d) equal to or greater than one

3. Speed

The time rate of distance travelled by an object in any direction is called speed of the object.

$$\boxed{\text{Speed } (v) = \frac{\text{Distance travelled}}{\text{Time taken}}}$$

It is a scalar quantity.

The unit of speed in SI or MKS system is ms^{-1} and in CGS system it cms^{-1}. Its dimensional formula is $[M^0LT^{-1}]$.

For a moving body, speed is always positive and can never be negative or zero.

Average speed

The ratio of the total distance travelled by the object to the total time taken is called average speed of the object.

i.e., $$\boxed{\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}}$$

Average speed of particles in different cases

Case I. If a particle travels distance $s_1, s_2, s_3, \ldots$, etc., with speeds $v_1, v_2, v_3, \ldots$, etc., in same direction, then the distance travelled $= s_1 + s_2 + s_3 + \ldots$

$$\text{Total time taken} = \frac{s_1}{v_1} + \frac{s_2}{v_2} + \frac{s_3}{v_3} + \ldots$$

$$\text{Average speed, } v_{av} = \frac{s_1 + s_2 + \ldots}{\left(\frac{s_1}{v_1} + \frac{s_2}{v_2} + \ldots\right)}$$

If $s_1 = s_2 = s$, *i.e.*, the body covers equal distance with different speeds, then

$$v_{av} = \frac{2s}{s\left[\frac{1}{v_1} + \frac{1}{v_2}\right]}$$

$$\boxed{v_{av} = \frac{2v_1v_2}{v_1 + v_2}}$$

Case II. If a particle travels with speeds $v_1, v_2, v_3, \ldots$, etc., during time intervals $t_1, t_2, t_3, \ldots$, etc., then total distance travelled, $s = v_1t_1 + v_2t_2 + v_3t_3 + \ldots$

Total time taken $= t_1 + t_2 + t_3 + \ldots$

So, average speed, $$\boxed{v_{av} = \frac{v_1t_1 + v_2t_2 + v_3t_3 + \ldots}{t_1 + t_2 + t_3 + \ldots}}$$

Case III. If $t_1 = t_2 = t_3 = \ldots = t_n$, then we have

$$v_{av} = \frac{(v_1 + v_2 + \ldots + v_n)t}{nt}$$

$$\boxed{v_{av} = \frac{v_1 + v_2 + \ldots + v_n}{n}}$$

Instantaneous speed

The speed of a particle at any instant of time is known as its instantaneous speed.

$$\boxed{\text{Instantaneous speed} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}}$$

where, s represents distance.

Example 3.7 *The distance covered by an object (in metre) is given by*

$$s = 8t^3 - 7t^2 + 5t$$

Find its speed at $t = 2$ s.

Sol. As speed $= \frac{\text{Distance}}{\text{Time}}$

$$\therefore \quad \text{Speed} = \frac{ds}{dt}$$

$$= \frac{d}{dt}(8t^3 - 7t^2 + 5t) = 24t^2 - 14t + 5$$

$\therefore$ Speed of object at any instant of time is $(24t^2 - 14t + 5)ms^{-1}$

At $t = 2$

Speed $= 24(2)^2 - 14(2) + 5 = 73\ ms^{-1}$

Example 3.8 *Abdul while driving to school, computes the average speed for his trip to be 20 kmh^{-1}. On his return trip along the same route, there is less traffic and the average speed is 40 kmh^{-1}. What is the average speed for Abdul's trip?*

Sol. Let t_1 and t_2 be the time taken by Abdul to go to school and come back from the school respectively. Let s be its distance covered one way, then

$$t_1 = \frac{s}{20}\text{ h and } t_2 = \frac{s}{40}\text{ h}$$

$$\text{Total time taken} = t_1 + t_2 = \frac{s}{20} + \frac{s}{40} = \frac{3s}{40}\text{ h}$$

Total distance covered $= 2s$

$$\therefore \text{Average speed} = \frac{2s}{3s} \times 40 = \frac{80}{3} = 26.67\ kmh^{-1}$$

Example 3.9 *A car moves from X to Y with a uniform speed v_u and returns to Y with a uniform speed v_d. Find average speed for this round trip.*

Sol. We know that, average speed $= \frac{\text{Distance travelled}}{\text{Time taken}}$

Let t_1 and t_2 be times taken by the car to go from X to Y and then from Y to X, respectively.

Then, $$t_1 + t_2 = \frac{XY}{v_u} + \frac{XY}{v_d} = XY\left(\frac{v_u + v_d}{v_uv_d}\right)$$

Total distance travelled $= XY + XY = 2XY$

Therefore, average speed of the car for this round trip is

$$v_{av} = \frac{2XY}{XY\left(\frac{v_u + v_d}{v_uv_d}\right)} \quad \text{or} \quad v_{av} = \frac{2v_uv_d}{v_u + v_d}$$

Example 3.10 *A p 'e travelled half the distance with a speed v_0. The rem 'g part of the distance was covered with speed v_1 for half the · and with speed v_2 for the other half of the time. Find the 'age speed of the particle.*

Sol. If s is the total dis · travelled by the particle, then

$$\frac{s}{2} = v_0 t_1 \qquad t_1 = \frac{s}{2v_0}$$

If t is the time taken by particle to travel remaining distance $s/2$, then

$$\frac{s}{2} = \frac{v_1 t}{2} + \frac{v_2 t}{2} = (v_1 + v_2)\frac{t}{2} \quad \text{or} \quad t = \frac{s}{(v_1 + v_2)}$$

$$\text{Average speed} = \frac{s}{t_1 + t} = \frac{s}{\frac{s}{2v_0} + \frac{s}{(v_1 + v_2)}} = \frac{2v_0(v_1 + v_2)}{v_1 + v_2 + 2v_0}$$

Example 3.11 *A car covers the first half of the distance between two places at a speed of 40 kmh^{-1} and second half at 60 kmh^{-1} Calculate the average speed of the car.*

Sol. Given, speed in first half, $v_1 = 40\ \text{kmh}^{-1}$.

Speed in second half, $v_2 = 60\ \text{kmh}^{-1}$.

$\because$ Car covers equal distance with different speeds.

$\therefore$ Average speed of car

$$v_{av} = \frac{2v_1 v_2}{v_1 + v_2}$$

$$v_{av} = \frac{2(40)(60)}{40 + 60} = 48\ \text{kmh}^{-1}$$

4. Velocity

The rate of change of position or displacement of an object with time is called the velocity of that object.

i.e.,

$$\boxed{\text{Velocity} = \frac{\text{Displacement}}{\text{Time}}}$$

Velocity is a vector quantity. The unit of velocity in SI or MKS system is ms^{-1} and in CGS system it is cms^{-1}. Its dimensional formula is $[M^0LT^{-1}]$. The velocity of an object is taken to be positive if the object is moving towards the right of the origin and is taken to be negative if the object is moving towards the left of the origin.

Velocity vs Speed

1. Velocity of an object can be changed by changing the object's speed or direction of motion or both.
2. For an object in a time interval (t); $|\text{Velocity}| \le \text{Speed}$
   i.e., the magnitude of velocity of an object is always equal to or less than its speed.
3. If a body is moving in a straight line, then the magnitude of its speed and velocity will be equal.
4. Average velocity could be zero or positive or negative but average speed is always positive for a moving body.

Average velocity

The ratio of the total displacement to the total time taken is called average velocity.

$$\boxed{\text{Average velocity} = \frac{\text{Total displacement } (\Delta x)}{\text{Total time } (\Delta t)}}$$

If velocity of the object changes at a uniform rate, then

$$\text{Average velocity} = \frac{\text{Initial velocity} + \text{Final velocity}}{2}$$

Note For a given time interval average velocity has single value while average speed can have many values depending on path followed.

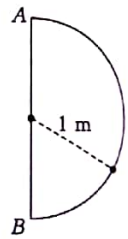
Instantaneous velocity

The velocity of a particle at any instant of time is known as its instantaneous velocity.

$$\boxed{v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}}$$

Note The magnitude of average velocity may be less than or equal to the average speed for a particular motion. But the magnitude of instantaneous velocity is always equal to the instantaneous speed for a particular instant.

Example 3.12 *In one second a particle goes from point A to point B moving in a semicircle as shown in figure. Find the magnitude of average velocity.*



Sol. Given, $t = 1$ s and radius $R = 1.0$ m.

The shortest distance between the final position (B) and initial position A is AB, which is the displacement of the particle.

$\therefore$ Total displacement $(AB) = 2R = 2 \times 1.0 = 2$ m.

$$\therefore \text{Average velocity } |v_{av}| = \frac{\text{Total displacement}}{\text{Total time taken}}$$

$$= \frac{AB}{\Delta t} = \frac{2.0}{1.0}\ \text{ms}^{-1} = 2\ \text{ms}^{-1}$$

Example 3.13 *A farmer has to go 500 m due north, 400 m due east and 200 m due south to reach his field. If he takes 20 min to reach the field,*

(i) what distance he has to walk to reach the field ?

(ii) what is the displacement from his house to the field ?

(iii) what is the average speed of farmer during the walk ?

(iv) what is the average velocity of farmer during the walk ?

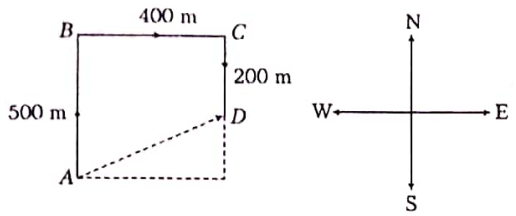
Sol. (i) Distance $= AB + BC + CD$

$$= (500 + 400 + 200) = 1100\ \text{m}$$

(ii) Displacement $= AD = \sqrt{(AB - CD)^2 + BC^2}$

$$= \sqrt{(500 - 200)^2 + (400)^2}$$

$$= 500\ \text{m}$$



(iii) Average speed $= \dfrac{\text{Total distance}}{\text{Total time}}$

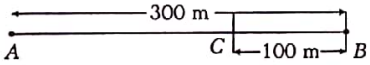
$$= \frac{1100}{20} = 55 \text{ m/min}$$

(iv) Average velocity $= \dfrac{AD}{t} = \dfrac{500}{20} = 25$ m/min (along AD)

Example 3.14 *Joseph jogs from one end A to the other end B of a straight 300 m road in 2 min 50 s and then turns around and jogs 100 m back to point C in another 1 min. What are Joseph's average speeds and velocities in jogging (i) from A to B and (ii) from A to C?*

Sol. Given, Distance covered = 300 m,

Time = 2 min 50 s = (2 × 60) + 50 s = 170 s



(i) Average speed $= \dfrac{\text{Distance covered}}{\text{Time}} = \dfrac{300}{170} = 1.76 \text{ ms}^{-1}$

Average velocity $= \dfrac{\text{Displacement along } AB}{\text{Time}}$

$$= \frac{300}{170} = 1.76 \text{ ms}^{-1} \text{ along } AB$$

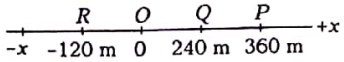
(ii) When Joseph turns around from B to C towards A.

Average speed $= \dfrac{\text{Distance covered}}{\text{Time}} = \dfrac{400}{230} = 1.7 \text{ ms}^{-1}$

Average velocity $= \dfrac{\text{Displacement } AC}{\text{Time}} = \dfrac{200}{230}$ along AC

$$= 0.87 \text{ ms}^{-1} \text{ along } AC$$

Example 3.15 *A car is moving along a straight line, say OP in the given figure. It moves from O to P in 18 s and return from P to Q in 6.0s. What are the average velocity and average speed of the car in going (i) from O to P and (ii) from O to P and back to Q?*



Sol. (i) Average velocity $= \dfrac{\text{Total displacement}}{\text{Total time interval}}$

$$v_{av} = \frac{+360 \text{ m}}{18 \text{ s}} = +20 \text{ ms}^{-1}$$

Average speed $= \dfrac{\text{Path length}}{\text{Time interval}} = \dfrac{360 \text{ m}}{18 \text{ s}} = 20 \text{ ms}^{-1}$

Thus, in this case, the average speed is equal to the magnitude of the average velocity.

(ii) In this case,

Average velocity $= \dfrac{\text{Total displacement}}{\text{Total time}} = \dfrac{+240 \text{ m}}{(18 + 6.0) \text{ s}}$

$$= +10 \text{ ms}^{-1}$$

Average speed $= \dfrac{\text{Path length}}{\text{Time interval}} = \dfrac{OP + PQ}{\Delta t}$

$$= \frac{(360 + 120)\text{m}}{24 \text{ s}} = 20 \text{ ms}^{-1}$$

Example 3.16 *The position of an object moving along an x-axis is given by $x = 3t - 4t^2 + t^3$, where x is in meters and t in seconds. Find the position of the object at the following values of t : (i) 2s, (ii) 4s; (iii) What is the object's displacement between t = 0 s and t = 4 s?; and (iv) What is its average velocity for the time interval from t = 2 s to t = 4 s?*

Sol. Using $x = 3t - 4t^2 + t^3$ with SI units,

(i) Substituting $t = 2$ s in Eq. (i), we get

$$x_2 = 3(2) - 4(2)^2 + (2)^3 = -2 \text{ m}.$$

Thus, the position of the object at $t = 2$ s is $x = -2$ m.

(ii) Substituting $t = 4$ s in Eq. (i), we get

$$x_4 = 3(4) - 4(4)^2 + (4)^3 = 12 - 64 + 64 = 12 \text{ m}.$$

Thus, the position of the object at $t = 4$ s is $x_4 = 12$ m.

(iii) The displacement of the object between $t = 0$ s and $t = 4$ s can be calculated as follows :

The position of the object at $t = 0$ s is $x = 0$.

Δx = Final position - Initial position $= x_4 - x_0$

$= 12 \text{ m} - 0 \text{ m} = 12 \text{ m}$

Hence, the displacement between $t = 0$ s and $t = 4$ s is 12 m

(iv) The displacement of the object from $t = 2$ s to $t = 4$ s is

$$\Delta x = x_4 - x_2 = 12 \text{ m} - (-2 \text{ m}) = 14 \text{ m}.$$

The time interval from $t = 2$ s to $t = 4$ s is

$$\Delta t = 4 \text{ s} - 2 \text{ s} = 2 \text{ s}.$$

The average velocity of the object from $t = 2$ s to $t = 4$ s given is by the relation

$$\Delta_{av} = \frac{\Delta x}{\Delta t} = \frac{14 \text{ m}}{2 \text{ s}} = 7 \text{ ms}^{-1}$$

Example 3.17 *The velocity of particle moving in the positive direction of X-axis varies as $v = \alpha\sqrt{x}$, where α is a positive constant. Assuming that at moment t = 0, the particle was located at the point x = 0.*

Find

(i) the time dependence of the velocity of the particle.

(ii) the mean velocity of the particle averaged over the time that the particle takes to cover first s meters of the path.

Sol. (i) Given, the velocity of the particle moving in the positive direction of X-axis.

$$v = \alpha\sqrt{x} \Rightarrow \frac{dx}{dt} = \alpha\sqrt{x}$$

By integrating $\int_0^x \frac{dx}{\sqrt{x}} = \alpha \int_0^t dt$

$$\Rightarrow \quad \frac{x^{-1/2+1}}{-(1/2)+1} = \alpha t \Rightarrow \quad x = \frac{\alpha^2 t^2}{4}$$

Time dependence of the velocity of the particle

$$v = \frac{dx}{dt} = \frac{2\alpha^2 t}{4} = \frac{1}{2}\alpha^2 t$$

(ii) Distance, $x = \frac{\alpha^2 t^2}{4}$; for s distance $s = \frac{\alpha^2 t^2}{4}$

Time taken to cover first s distance, $t = \sqrt{\frac{4s}{\alpha^2}}$

The mean velocity of the particle,

$$v_{av} = \frac{s}{t} = \frac{s}{\sqrt{4s/\alpha^2}} = \frac{\alpha\sqrt{s}}{2}$$

Check point 3.2

1. A car has to cover the distance 60 km. If half of the total time it travels with speed 80 kmh^{-1} and in rest half time its speed becomes 40 kmh^{-1}, the average speed of car will be
(a) 60 kmh^{-1} (b) 80 kmh^{-1}
(c) 120 kmh^{-1} (d) 180 kmh^{-1}

2. During the first 18 min of a 60 min trip, a car has an average speed of 11 ms^{-1}. What should be the average speed for remaining 42 min so that car is having an average speed of 21 ms^{-1} for the entire trip?
(a) 25.3 ms^{-1} (b) 29.2 ms^{-1}
(c) 31 ms^{-1} (d) 35.6 ms^{-1}

3. A man walks on a straight road from his home to a market 2.5 km away with a speed of 5 kmh^{-1}. Finding the market closed, he instantly turns and walks back home with a speed of 7.5 kmh^{-1}. The average speed of the man over the interval of time 0 to 40 min is equal to
(a) 5 kmh^{-1} (b) $\frac{25}{4}$ kmh^{-1}
(c) $\frac{30}{4}$ kmh^{-1} (d) $\frac{45}{8}$ kmh^{-1}

4. A particle is constrained to move on a straight line path. It returns to the starting point after 10 s. The total distance covered by the particle during this time is 30 m. Which of the following statements about the motion of the particle is true?
(a) Displacement of the particle is zero
(b) Average speed of the particle is 3 ms^{-1}
(c) Displacement of the particle is 30 m
(d) Both (a) and (b)

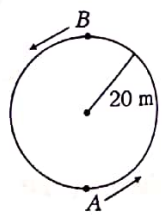
5. A 150 m long train is moving with a uniform velocity of 45 kmh^{-1}. The time taken by the train to cross a bridge of length 850 m is
(a) 56 s (b) 68 s (c) 80 s (d) 92 s

6. The displacement of a particle starting from rest (at $t = 0$) is given by $s = 6t^2 - t^3$. The time in seconds at which the particle will attain zero velocity again is
(a) 2 (b) 4
(c) 6 (d) 8

7. An insect crawls a distance of 4 m along North in 10 s and then a distance of 3 m along East in 5 s. The average velocity of the insect is
(a) $\frac{7}{15}$ ms^{-1} (b) $\frac{1}{5}$ ms^{-1}
(c) $\frac{1}{3}$ ms^{-1} (d) $\frac{4}{5}$ ms^{-1}

8. A point traversed 3/4 th of the circle of radius R in time t. The magnitude of the average velocity of the particle in this time interval is
(a) $\frac{\pi R}{t}$ (b) $\frac{3\pi R}{2t}$
(c) $\frac{R\sqrt{2}}{t}$ (d) $\frac{R}{\sqrt{2}t}$

9. A boy is running over a circular track with unifrom speed of 10 ms^{-1}. What is the average velocity for movement of boy from A to B (in ms^{-1})?



(a) $\frac{10}{\pi}$ (b) $\frac{40}{\pi}$
(c) 10 (d) None of these

10. The displacement x of an object is given as a function of time, $x = 2t + 3t^2$. The instantaneous velocity of the object at $t = 2$ s is
(a) 16 ms^{-1} (b) 14 ms^{-1}
(c) 10 ms^{-1} (d) 12 ms^{-1}

5. Acceleration

The time rate of change of velocity of a body is called acceleration.

$$\therefore \quad \text{Acceleration} = \frac{\text{Change in velocity } (\Delta v)}{\text{Time interval } (\Delta t)}$$

If in a given time interval the velocity of a body changes from u to v, then acceleration a is expressed as

$$a = \frac{\text{Final velocity} - \text{Initial velocity}}{\text{Time interval}} = \frac{v-u}{t}$$

It is a vector quantity, denoted by a and its SI unit is ms^{-2} and CGS is cms^{-2}.

Its dimensional formula is $[M^0LT^{-2}]$. Its direction is same as that of change in velocity (not of the velocity).

There are three possible ways by which change in velocity may occur

(i) **When only direction of velocity changes**, then acceleration is perpendicular to velocity. *e.g.*, Uniform circular motion.

(ii) **When only magnitude of velocity changes**, then acceleration is parallel or anti-parallel to velocity. *e.g.*, Motion under gravity.

(iii) **When both magnitude and direction of velocity changes**, then acceleration has two components one is perpendicular to velocity and another parallel or anti-parallel to velocity. *e.g.*, Projectile motion.

Retardation

When the velocity of a body increases with time, acceleration is positive and when the velocity of a body decreases with time (*i.e.*, $u > v$), then acceleration becomes negative. Negative acceleration is also called **deceleration** or **retardation.**

Average acceleration

When an object is moving with a variable acceleration, then the average acceleration of the object for the given motion is defined as the ratio of the total change in velocity of the object during motion to the total time taken.

$$\text{Average acceleration} = \frac{\text{Total change in velocity}}{\text{Total time taken}}$$

Note The average acceleration can be positive or negative depending upon the sign of change of velocity. It is zero if the change in velocity of the object in the given interval of time is zero.

Instantaneous acceleration

The acceleration of an object at a given instant of time or at a given point during the motion, is called its instantaneous acceleration.

$$\mathbf{a} = \lim_{\Delta t \to 0} = \frac{\Delta \mathbf{v}}{\Delta t} = \frac{d\mathbf{v}}{dt}$$

Key points regarding acceleration motion

(i) A body falling down from a height or a body rolling down on a smooth inclined plane, has uniform acceleration.

(ii) If a car travelling along a straight road increases its speed by unequal amounts in equal intervals of time, then the car is said to be moving with non- uniform acceleration.

(iii) The acceleration is created by accelerator of the vehicles and the applications of breaks give the uniform deceleration to the vehicles. The acceleration produced in spring-block system is non-uniform acceleration.

(iv) *Instantaneous acceleration of an object*

(a) is equal to the first time derivative of velocity at the given instant.

(b) is equal to the second time derivative of position of the object at the given instant.

(v) If a particle is accelerated for time t_1 with acceleration a_1 and for time t_2 with acceleration a_2, then average acceleration is

$$a_{av} = \frac{a_1t_1 + a_2t_2}{t_1 + t_2}$$

(vi) Acceleration can also be written as

$$a = \frac{dv}{dt}$$

$$= \frac{dv}{dx} \cdot \frac{dx}{dt} = v\left(\frac{dv}{dx}\right)$$

Example 3.18 The distance travelled by a body is proportional to the square of time. What type of motion this body has?

Sol. Let x be the distance travelled in time t. Then,

$$x \propto t^2 \quad \text{(given)}$$

$$x = kt^2 \quad \text{(Here, } k = \text{constant of proportionality)}$$

We know that velocity is given

$$v = \frac{dx}{dt} = 2\,kt$$

and acceleration is given by

$$a = \frac{dv}{dt} = 2\,k \qquad \text{(constant)}$$

Thus, the body has uniform accelerated motion.

Example 3.19 *Give examples where*

(i) the velocity is in opposite direction to the acceleration,

(ii) the velocity of the particle is zero but its acceleration is not zero,

(iii) the velocity is perpendicular to the acceleration.

Sol. (i) A particle thrown upwards has its velocity in opposite direction to its acceleration (g, downwards).

(ii) When the particle is released from rest from a certain height, its velocity is zero, while acceleration is g acting downwards. Similarly, at the extreme position of a pendulum velocity is zero, while acceleration is not zero.

(iii) In uniform circular motion velocity is perpendicular to its radial or centripetal acceleration.

Example 3.20 *The velocity of a particle is given by* $\mathbf{v} = (2t^2 - 3t + 10)\,ms^{-1}$. *Find the instantaneous acceleration at* $t = 5$ *s.*

Sol. Given, $\mathbf{v} = (2t^2 - 3t + 10)\,\text{ms}^{-1}$

$$\Rightarrow \quad \mathbf{a}_{in} = \frac{dv}{dt} = (4t - 3)\,\text{ms}^{-2}$$

If $\quad t = 5\text{s}, \quad \mathbf{a}_{in} = 5 \times 4 - 3 = 17\,\text{ms}^{-2}$

Example 3.21 *A particle is moving with a velocity of* $v = (3 + 6t + 9t^2)\,cms^{-1}$. *Find out*

(a) the acceleration of the particle at $t = 3$ *s.*

(b) the displacement of the particle in the interval $t = 5$ *s to* $t = 8$ *s.*

Sol. (a) Velocity of the moving particle $v = (3 + 6t + 9t^2)$

Acceleration of particle, $a = \dfrac{dv}{dt} = (6 + 18t)\,\text{cms}^{-2}$

At $t = 3$ s, $\quad a = (6 + 18 \times 3)\,\text{cm/s}^2 = 60\,\text{cms}^{-2}$

(b) Given, velocity $v = (3 + 6t + 9t^2)$ cm/s

or $\quad \dfrac{ds}{dt} = (3 + 6t + 9t^2)$ or $ds = (3 + 6t + 9t^2)\,dt$

$\therefore$ By integration $\displaystyle\int_0^s ds = \int_5^8 (3 + 6t + 9t^2)\,dt$

The displacement of the particle in the interval $t = 5$ s to $t = 8$ s

$$s = [3t + 3t^2 + 3t^3]_5^8 \quad \text{or} \quad s = 1287\,\text{cm}$$

Example 3.22 *The motion of a particle along a straight line is described by the function* $x = (2t - 3)^2$ *where* x *is in metre and* t *is in second.*

(i) Find the position, velocity and acceleration at $t = 2$ *s.*

(ii) Find velocity of the particle at origin.

Sol. (i) Position of the particle, $x = (2t - 3)^2$

At $t = 2$ s, $\quad x = (2 \times 2 - 3)^2 = 1.0$ m

Velocity, $\quad v = \dfrac{dx}{dt} = 4\,(2t - 3)\,\text{ms}^{-1}$

At $t = 2$ s, $\quad v = 4\,(2 \times 2 - 3) = 4\,\text{ms}^{-1}$

and acceleration of the particle, $a = \dfrac{dv}{dt} = 8\,\text{ms}^{-2}$

(ii) At origin, $x = 0$ or $(2t - 3)^2 = 0$

Velocity of the particle at origin, $v = 4 \times 0 = 0$

Example 3.23 *The radius vector of a point depends on time* t, *as*

$$\mathbf{r} = \mathbf{c}t + \frac{\mathbf{b}t^2}{2}$$

where $\mathbf{c}$ *and* $\mathbf{b}$ *are constant vectors. Find the modulus of velocity and acceleration at any time* t.

Sol. (i) Velocity $\mathbf{v} = \dfrac{d\mathbf{r}}{dt} = \mathbf{c} + t\,\mathbf{b}$

Modulus of velocity vector will be

$$|\mathbf{v}| = \sqrt{c^2 + b^2t^2 + 2cbt\cos\theta}$$

Here c and b are modulus of $\mathbf{c}$ and $\mathbf{b}$ and θ is the angle between $\mathbf{c}$ and $\mathbf{b}$ which can be written as

$$c^2 = \mathbf{c}\cdot\mathbf{c},\ b^2 = \mathbf{b}\cdot\mathbf{b}$$

and $\quad cb\cos\theta = \mathbf{c}\cdot\mathbf{b}$

Hence $\quad |\mathbf{v}| = \sqrt{\mathbf{c}\cdot\mathbf{c} + \mathbf{b}\cdot\mathbf{b}t^2 + 2\mathbf{c}\cdot\mathbf{b}t}$

(ii) Acceleration $\quad \mathbf{a} = \dfrac{d\mathbf{v}}{dt} = \mathbf{b}$

Hence, $\quad |\mathbf{a}| = |\mathbf{b}|$

Example 3.24 *A particle is moving in a straight line. Its displacement at any instant* t *is given by* $x = 10t + 15t^3$, *where* x *is in metres and* t *is in seconds. Find*

(i) the average acceleration in the interval $t = 0$ *to* $t = 2$*s and*

(ii) instantaneous acceleration at $t = 2$ *s.*

Sol. The given equation, $x = 10t + 15t^3$

and the variables are (i) $t = 0$ to $t = 2$ s (ii) $t = 2$ s

We need to clarify all the variables.

Velocity of particle, $v = \dfrac{dx}{dt}$

While calculating instantaneous velocity, we need to differentiate. Differentiating x w.r.t. time.

$$\mathbf{v} = \frac{d}{dt}(10t + 15t^3) = 10 + 45t^2$$

At $t = 0$, $\mathbf{v}_0 = 10 + 45(0) = 10\,\text{ms}^{-1}$

At $t = 2$ s,

$$\mathbf{v}_2 = 10 + 45 \times (2)^2 = 10 + 180 = 190\,\text{ms}^{-1}$$

$$\Delta\mathbf{v} = \mathbf{v}_2 - \mathbf{v}_1 = 190 - 10 = 180\,\text{ms}^{-1}$$

$$\Delta t = 2 - 0 = 2\,\text{s}$$

$$\therefore \quad \mathbf{a}_{av} = \frac{\Delta\mathbf{v}}{\Delta t} = \frac{180}{2} = 90\,\text{ms}^{-2}$$

Now, differentiate, v with respect to time to find the acceleration of the particle.

$$\mathbf{a} = \frac{d}{dt}(10 + 45t^2) = 90t$$

At $t = 2$ s, $\mathbf{a} = 90 \times 2 = 180 \text{ ms}^{-2}$

The instantaneous acceleration of a particle at 2s is 180 ms^{-2}.

Example 3.25 *(i) What does* $\left|\frac{d\mathbf{v}}{dt}\right|$ *and* $\frac{d|\mathbf{v}|}{dt}$ *represent?*
(ii) Can these be equal?
(iii) Can $\frac{d|\mathbf{v}|}{dt} = 0$ *while* $\left|\frac{d\mathbf{v}}{dt}\right| \neq 0$*?*
(iv) Can $\frac{d|\mathbf{v}|}{dt} \neq 0$ *while* $\left|\frac{dv}{dt}\right| = 0$*?*

Sol. (i) $\left|\frac{d\mathbf{v}}{dt}\right|$ is the magnitude of total acceleration. While $\frac{d|\mathbf{v}|}{dt}$ represents the time rate of change of speed (called the tangential acceleration, a component of total acceleration) as $|\mathbf{v}| = v$.

(ii) These two are equal in case of one dimensional motion (without change in direction).

(iii) In case of uniform circular motion speed remains constant while velocity changes.

Hence, $\frac{d|\mathbf{v}|}{dt} = 0$ while $\left|\frac{d\mathbf{v}}{dt}\right| \neq 0$.

(iv) $\frac{d|\mathbf{v}|}{dt} \neq 0$ implies that speed of particle is not constant.

Velocity cannot remain constant if speed is changing.

Hence, $\left|\frac{d\mathbf{v}}{dt}\right|$ cannot be zero in this case. So, it is not possible to have $\left|\frac{d\mathbf{v}}{dt}\right| = 0$ while $\frac{d|\mathbf{v}|}{dt} \neq 0$

Check point 3.3

1. Acceleration of a particle changes when
(a) direction of velocity changes
(b) magnitude of velocity changes
(c) Both (a) and (b)
(d) speed changes

2. If a particle moves with an acceleration, then which of the following can remain constant?
(a) Both speed and velocity (b) Neither speed nor velocity
(c) Only the velocity (d) Only the speed

3. The average velocity of a body moving with uniform acceleration travelling a distance of 3.06 m is 0.34 ms^{-1}. If the change in velocity of the body is 0.18 ms^{-1} during this time, its uniform acceleration is
(a) 0.01 ms^{-2} (b) 0.02 ms^{-2}
(c) 0.03 ms^{-2} (d) 0.04 ms^{-2}

4. The displacement x of a particle at time t along a straight line is given by $x = \alpha - \beta t + \gamma t^2$. The acceleration of the particle is
(a) $-\beta$ (b) $-\beta + 2\gamma$ (c) 2γ (d) -2γ

5. A car travelling with a velocity of 80 km/h slowed down to 44 km/h in 15 s. The retardation is
(a) 0.67 ms^{-2} (b) 1 ms^{-2}
(c) 1.25 ms^{-2} (d) 1.5 ms^{-2}

6. A body is moving with velocity 30 ms^{-1} towards cast. After 10 s its velocity becomes 40 ms^{-1} towards north. The average acceleration of the body is
(a) 7 ms^{-2} (b) $\sqrt{7}$ ms^{-2} (c) 5 ms^{-2} (d) 1 ms^{-2}

7. The displacement (in metre) of a particle moving along X-axis is given by $x = 18t + 5t^2$. The average acceleration during the interval $t_1 = 2$ s and $t_2 = 4$ s is
(a) 13 ms^{-2} (b) 10 ms^{-2} (c) 27 ms^{-2} (d) 37 ms^{-2}

8. The distance traversed by a particle moving along a straight line is given by $x = 180t + 50t^2$ metre. The acceleration of the particle is
(a) 180 ms^{-2} (b) 580 ms^{-2}
(c) 100 ms^{-2} (d) 50 ms^{-2}

9. The displacement (in metre) of a particle moving along X-axis is given by $x = 18t + 5t^2$. The instantaneous acceleration is
(a) 18 ms^{-2} (b) 10 ms^{-2} (c) 5 ms^{-2} (d) 1 ms^{-2}

10. A particle's velocity changes from $(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}})\text{ms}^{-1}$ to $(2\hat{\mathbf{i}} - 3\hat{\mathbf{j}})\text{ms}^{-1}$ in 2 s. The acceleration in ms^{-2} is
(a) $-(\hat{\mathbf{i}} + 5\hat{\mathbf{j}})$ (b) $(\hat{\mathbf{i}} + 5\hat{\mathbf{j}})/2$
(c) zero (d) $(-3\hat{\mathbf{j}})$

2 Kinematic equations for uniformly accelerated motion

When a body is moving along a straight line with uniform acceleration, then its motion is called **uniformly accelerated motion.** We can establish the relation between velocity of the body, acceleration of the body and the distance travelled by the body in a particular time interval by a set of equations. These equations are known as **kinematic equations** or **equations of motion.**

The three equations of motion on a straight line are

(i) $v = u + at$ (ii) $s = ut + \frac{1}{2}at^2$

(iii) $v^2 - u^2 = 2as$

where, u is the initial velocity of the body, a is the uniform acceleration of the body, v is the final velocity of the body after t second and s is the distance travelled in this time.

- Distance travelled in nth second

$$\boxed{s_n = u + \frac{1}{2}a(2n-1)}$$

where, s_n = distance covered by a body in nth second.

Note

1. If initial position vector of a particle is $\mathbf{r}_0$, then position vector at time t can be written as
$$\mathbf{r} = \mathbf{r}_0 + \mathbf{s} = \mathbf{r}_0 + \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$
2. **Stopping distance** When brakes are applied to a moving vehicle, the distance it travels before stopping is called stopping distance. It is an important factor for road safety. It is given by
$$\boxed{s = \frac{u^2}{2a}}$$
where u is initial velocity and a is the retardation produced by brakes. Stopping distance is proportional to the square of the initial velocity. Stopping distance is an important facter considered in setting speed limits.

Key points regarding kinematic equations

Following points are worthnoting in case of one dimensional motion with constant acceleration.

(i) If the motion starts from rest then initial velocity is taken as zero *i.e.*, $u = 0$.

(ii) If the object comes to rest after the motion then final velocity is taken as zero *i.e.*, $v = 0$.

(iii) If velocity of moving object increases with time, then acceleration is taken as positive and if velocity decreases with time, acceleration is taken as negative.

(iv) If velocity and acceleration both have same sign like $v > 0$; $a > 0$ or $v < 0$; $a < 0$ then particle have speeding up. Similarly velocity and acceleration both have opposite sign like $v < 0$; $a > 0$ or $v > 0$; $a < 0$. Then particle have speeding down.

(v) If the motion of a particle is taking place in a straight line, there is no need of using vector addition in equation of motion. We can directly use the equations. Just by taking one direction as the positive (and opposite to it as negative) and then substituting u, a etc with sign.

Normally we take vertically upward direction positive (and downward negative) and horizontally rightwards positive (or leftwards negative). Sign convention for (a) motion in vertical direction (b) motion in horizontal direction is shown in figure.

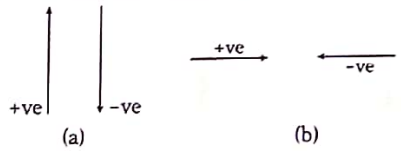


Fig. 3.6 Sign convention for vertical and horizontal direction.

Example 3.26 *A car was moving at a rate of 18 kmh^{-1}. When the brakes were applied, it comes to rest in a distance of 100 m. Calculate the retardation produced by the brakes.*

Sol. Given, $v = 0$, $u = 18\ \text{kmh}^{-1} = 5\ \text{ms}^{-1}$, $s = 100\ \text{m}$

Using the equation of motion

$$v^2 - u^2 = 2as \quad \text{...(i)}$$

$$\therefore \quad -u = 2as \quad (\because v = 0)$$

$$\Rightarrow \quad s = -\frac{u^2}{2a} \Rightarrow a = \frac{-5 \times 5}{2 \times 100} = -\frac{1}{8} = -0.125\ \text{ms}^{-2}$$

So, the retardation produced by the brakes is 0.125 ms^{-2}.

Example 3.27 *Two cars start off a race with velocities 2 ms^{-1} and 4 ms^{-1} travel in straight line with uniform accelerations 2 ms^{-2} and 1 ms^{-2} respectively. What is the length of the path if they reach the final point at the same time?*

Sol. Let both particles reach at same position in same time t then from $s = ut + \frac{1}{2}at^2$

For 1st particle, $s = 4(t) + \frac{1}{2}(1)t^2 = 4t + \frac{t^2}{2}$,

For 2nd particle, $s = 2(t) + \frac{1}{2}(2)t^2 = 2t + t^2$

Equating above equations, we get $4t + \frac{t^2}{2} = 2t + t^2 \Rightarrow t = 4\ \text{s}$

Substituting value of t in above equations, we get

$$s = 4(4) + \frac{1}{2}(1)(4)^2 = 16 + 8 = 24\ \text{m}$$

Example 3.28 *A body starting from rest has an acceleration of 4 ms^{-2}. Calculate distance travelled by it in 5th second.*

Sol. Given, $u = 0$, $a = 4\ ms^{-2}$

Distance travelled in 5th second

$$s_n = u + \frac{1}{2}a(2n-1)$$

$$s_5 = 0 + \frac{1}{2} \times 4(2 \times 5 - 1) = \frac{1}{2} \times 4(9) = \frac{36}{2} = 18\ m$$

Example 3.29 *A train, travelling at 20 kmh^{-1} is approaching a platform. A bird is sitting on a pole on the platform. When the train is at a distance of 2 km from pole, brakes are applied which produce a uniform deceleration in it. At that instant the bird flies towards the train at 60 kmh^{-1} and after touching the nearest point on the train flies back to the pole and then flies towards the train and continues repeating itself. Calculate how much distance the bird covers before the train stops?*

Sol. For retardation of train $v^2 = u^2 + 2as$

$$\Rightarrow \quad 0 = (20)^2 + 2(a)(2) \Rightarrow a = -100\ kmh^{-2}$$

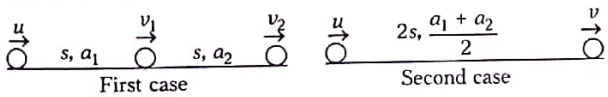
Time required to stop the train

$$v = u + at \Rightarrow 0 = 20 - 100t \Rightarrow t = \frac{1}{5}\ h$$

For bird, speed $= \dfrac{\text{Distance}}{\text{Time}} \Rightarrow s_B = v_B \times T = 60 \times \dfrac{1}{5} = 12\ km$

Example 3.30 *A particle starts with an initial velocity and passes successively over the two halves of a given distance with accelerations a_1 and a_2 respectively. Show that the final velocity is the same as if the whole distance is covered with a uniform acceleration $\dfrac{(a_1 + a_2)}{2}$.*

Sol. In the first case,



$$v_1^2 = u^2 + 2a_1 s \quad \text{...(i)}$$

$$v_2^2 = v_1^2 + 2a_2 s \quad \text{...(ii)}$$

Adding Eqs. (i) and (ii), we have

$$v_2^2 = u^2 + 2\left(\frac{a_1 + a_2}{2}\right)(2s) \quad \text{...(iii)}$$

In the second case

$$v^2 = u^2 + 2\left(\frac{a_1 + a_2}{2}\right)(2s) \quad \text{...(iv)}$$

From Eqs. (iii) and (iv) we can see that

$$v_2 = v$$

Example 3.31 *In a car race, car A takes a time t less than car B at the finish point and passes the finishing point with speed v more than that of the car B. Assuming that both the cars starts from rest and travel with constant acceleration a_1 and a_2 respectively. Show that $v = \sqrt{a_1 a_2}\, t$.*

Sol. Let A takes t_1 second, then according to the given problem B will take $(t_1 + t)$ seconds. Further, let v_1 be the velocity of B at finishing point, then velocity of A will be $(v_1 + v)$. Writing equations of motion for A and B.

$$v_1 + v = a_1 t_1 \quad \text{...(i)}$$

$$v_1 = a_2(t_1 + t) \quad \text{...(ii)}$$

From these two equations, we get

$$v = (a_1 - a_2)t_1 - a_2 t \quad \text{...(iii)}$$

Total distance travelled by both the cars is equal.

or $\quad s_A = s_B$

or $\quad \dfrac{1}{2}a_1 t_1^2 = \dfrac{1}{2}a_2(t_1 + t)^2$ or $t_1 = \dfrac{\sqrt{a_2}\, t}{\sqrt{a_1} - \sqrt{a_2}}$

Substituting this value of t_1 in Eq. (iii), we get the desired result.

or $\quad v = (\sqrt{a_1 a_2})\, t$

Example 3.32 *A particle starts from rest and moves under constant acceleration in a straight line. Find the ratio of displacement (i) in successive second and (ii) in successive time interval t_0.*

Sol. (i) Displacement in 1 s or 1st second

$$s_1 = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}a(1)^2 = \frac{1}{2}a$$

or $\quad s_1 = u + \dfrac{1}{2}a(2t - 1) = 0 + \dfrac{1}{2}a(2 \times 1 - 1) = \dfrac{1}{2}a$

Displacement in the 2nd second

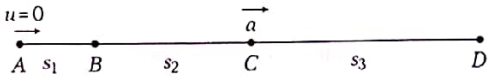
$$s_2 = u + \frac{1}{2}a(2t - 1) = 0 + \frac{1}{2}a(2 \times 2 - 1) = \frac{3}{2}a$$

Displacement in the 3rd second

$$s_3 = 0 + \frac{1}{2}a(2 \times 3 - 1) = \frac{5}{2}a$$

$$s_1 : s_2 : s_3 : \ldots = \frac{1}{2}a : \frac{3}{2}a : \frac{5}{2}a : \ldots = 1 : 3 : 5 : \ldots$$

(ii)



A to B : Displacement in the first t_0 second

$$s = ut + \frac{1}{2}at^2 \Rightarrow s_1 = 0 + \frac{1}{2}at_0^2 = \frac{1}{2}at_0^2$$

A to C $\quad t = t_0 + t_0 = 2t_0$

$$s_1 + s_2 = 0 + \frac{1}{2}a(t_0 + t_0)^2 = 4\,\frac{1}{2}at_0^2$$

Displacement in the next t_0 second

$$s_2 = \frac{3}{2}at_0^2$$

A to D $\quad t = t_0 + t_0 + t_0 = 3t_0$

$$s_1 + s_2 + s_3 = 0 + \frac{1}{2}a(t_0 + t_0 + t_0)^2 = \frac{9}{2}at_0^2$$

Displacement in the next t_0 second

$$s_3 = \frac{5}{2}at_0^2 \Rightarrow s_1 : s_2 : s_3 = 1 : 3 : 5$$

Check point 3.4

1. An object is moving with velocity $10\ ms^{-1}$. A constant force acts for 4 s on the object and gives it a speed of $2\ ms^{-1}$ in opposite direction. The acceleration produced is
(a) $3\ ms^{-2}$ (b) $-3\ ms^{-2}$
(c) $6\ ms^{-2}$ (d) $-6\ ms^{-2}$

2. Velocity of a body moving along a straight line with uniform acceleration (a) reduces by $\frac{3}{4}$ of its initial velocity in time t_0. The total time of motion of the body till its velocity becomes zero is
(a) $\frac{4}{3}t_0$ (b) $\frac{3}{2}t_0$ (c) $\frac{5}{3}t_0$ (d) $\frac{8}{3}t_0$

3. The distance travelled by a particle is proportional to the square of time, then the particle travels with
(a) uniform acceleration (b) uniform velocity
(c) Both of these (d) speed changes

4. The displacement of a body in 8 s starting from rest with an acceleration of $20\ cms^{-2}$ is
(a) 64 m (b) 64 cm (c) 640 cm (d) 0.064 m

5. A particle starts with a velocity of $2\ ms^{-1}$ and moves in a straight line with a retardation of $0.1\ ms^{-2}$. The first time at which the particle is 15 m from the starting point is
(a) 10 s (b) 20 s (c) 30 s (d) 40 s

6. A particle starts from rest, accelerates at $2\ ms^{-2}$ for 10 s and then goes for constant speed for 30 s and then decelerates at $4\ ms^{-2}$ till it stops after next. What is the distance travelled by it?
(a) 750 m (b) 800 m (c) 700 m (d) 850 m

7. The motion of a particle is described by the equation $v = at$. The distance travelled by the particle in the first 4 s is
(a) $4a$ (b) $12a$ (c) $6a$ (d) $8a$

8. A body is moving with uniform velocity of $8\ ms^{-1}$. When the body just crossed another body, the second one starts and moves with uniform acceleration of $4\ ms^{-2}$. The time after which two bodies meet, will be
(a) 2 s (b) 4 s (c) 6 s (d) 8 s

9. Two bodies A and B start from rest from the same point with a uniform acceleration of $2\ ms^{-2}$. If B starts one second later, then the two bodies are separated, at the end of the next second, by
(a) 1 m (b) 2 m (c) 3 m (d) 4 m

10. The displacement of a particle moving in a straight line is described by the relation, $s = 6 + 12t - 2t^2$. Here, s is in metre and t is in second. The distance covered by particle in first 5 s is
(a) 20 m (b) 32 m (c) 24 m (d) 26 m

11. A train accelerating uniformly from rest attains a maximum speed of $40\ ms^{-1}$ in 20 s. It travels at this speed for 20 s and is brought to rest uniform retardation in further 40 s. What is the average velocity during this period?
(a) $80/3\ ms^{-1}$ (b) $40\ ms^{-1}$ (c) $25\ ms^{-1}$ (d) $30\ ms^{-1}$

12. A particle starts from rest and traverses a distance l with uniform acceleration, then moves uniformly over a further distance $2l$ and finally comes to rest after moving a further distance $3l$ under uniform retardation. Assuming entire motion to be rectilinear motion the ratio of average speed over the journey to the maximum speed on its ways is
(a) 1/5 (b) 2/5 (c) 3/5 (d) 4/5

13. A body travelling with uniform acceleration crosses two points A and B with velocities $20\ ms^{-1}$ and $30\ ms^{-1}$ respectively. The speed of the body at mid-point of A and B is
(a) $25\ ms^{-1}$ (b) $25.5\ ms^{-1}$
(c) $24\ ms^{-1}$ (d) $10\sqrt{6}\ ms^{-1}$

14. The velocity of a particle moving in the positive direction of X-axis varies as $v = 5\sqrt{x}$. Assuming that at $t = 0$, particle was at $x = 0$. What is the acceleration of the particle?
(a) $12.5\ ms^{-2}$ (b) $7.5\ ms^{-2}$
(c) $5\ ms^{-2}$ (d) $2.5\ ms^{-2}$

15. If a body starts from rest and travels 120 cm in the 6th second, then what is the acceleration?
(a) $0.20\ ms^{-2}$ (b) $0.027\ ms^{-2}$
(c) $0.218\ ms^{-2}$ (d) $0.03\ ms^{-2}$

3 Motion under gravity

The objects falling towards the earth under the gravitational force alone, are called freely falling objects and such fall is called **free fall.**

Whenever an object falls towards the earth, an acceleration is involved, this acceleration is due to the earth's gravitational pull and is called **acceleration due to gravity.** The value of acceleration due to gravity near the earth surface is $9.8\ ms^{-2}$. It is independent of the mass of freely falling objects and is denoted by g.

Though the value of g is independent of freely falling mass, a feather falls much slowly than a coin when released from a height. This is due to the resistance offered by air to the falling mass. If both the bodies were released at the same time in vacuum (no air resistance), they would reach the earth surface within the same duration of time.

Equations for motion under gravity

When the objects fall under the influence of gravity, its motion is uniformly accelerated motion. Hence, equations of motion are applicable in this case also.

Equation for motion under gravity are given below.

(i) **If particle is projected vertically upwards**

In this case, applicable kinematics relations are

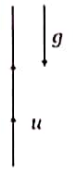


Fig. 3.7

$$v = u - gt \quad ...(i)$$

$$h = ut - \frac{1}{2}gt^2 \quad ...(ii)$$

$$v^2 = u^2 - 2gh \quad ...(iii)$$

Here, h is the vertical height of the particle in upward direction.

In this case, acceleration due to gravity is taken as negative.

At maximum height (say h), $v = 0$

($\because$ at maximum height the particle stops moving upwards that means its velocity becomes zero)

$\therefore$ From the Eq. (i), $u = gt$

or $\quad t = \frac{u}{g}$, is called **time of ascent.**

For motion under gravity, for the same distance the time taken to go up is same as time taken to fall down.

$\therefore$ Time of ascent = Time of descent

Total flight time $T = 2 \times$ time of ascent or decent

$$\boxed{\text{Total flight time } (T) = \frac{2u}{g}}$$

From the Eqs. (ii) and (iii)

$$h = \frac{1}{2}gt^2 \quad \text{and} \quad u^2 = 2gh \quad \text{or} \quad h = \frac{u^2}{2g}$$

(ii) **If particle is projected vertically downward with some velocity from some height.**

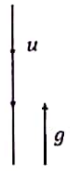


Fig. 3.8

In this case, $\quad v = u + gt \quad ...(i)$

$$h = ut + \frac{1}{2}gt^2 \quad ...(ii)$$

$$v^2 = u^2 + 2gh \quad ...(iii)$$

Here, h is the vertical height of particle in downward direction.

In this case, acceleration due to gravity is taken as positive.

(iii) **If a body is dropped from some height.**

In this case initial velocity is taken zero ($u = 0$), so equations of motion are

$$v = gt \quad ...(i)$$

$$h = \frac{1}{2}gt^2 \quad ...(ii)$$

$$v^2 = 2gh \quad ...(iii)$$

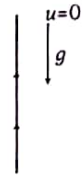


Fig. 3.9

(iv) *For fast calculation in objective problems, remember the following results*

(a) Maximum height attained by a particle, thrown upwards from ground

$$h = \frac{u^2}{2g}$$

(b) Velocity of particle at the time of striking the ground when released ($u = 0$) from a height h is

$$v = \sqrt{2gh}$$

(c) In above (b) time of collision with ground

$$t = \sqrt{\frac{2h}{g}}$$

Example 3.33 *Velocity and acceleration of a particle at time $t = 0$ are $\mathbf{u} = (2\hat{i} + 3\hat{j})$ m/s and $\mathbf{a} = (4\hat{i} + 2\hat{j})$ m/s^2 respectively. Find the velocity and displacement of particle at $t = 2$ s.*

Sol. Here, acceleration $\mathbf{a} = (4\hat{i} + 2\hat{j})$ m/s^2 is constant. So, we can apply

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t \quad \text{and} \quad \mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

Substituting the proper values, we get

$$\mathbf{v} = (2\hat{i} + 3\hat{j}) + (2)(4\hat{i} + 2\hat{j})$$

$$= (10\hat{i} + 7\hat{j}) \text{ m/s}$$

and

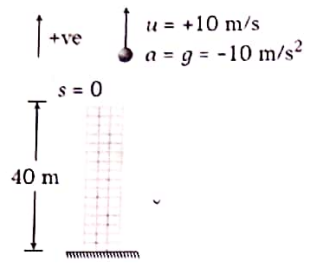
$$\mathbf{s} = (2)(2\hat{i} + 3\hat{j}) + \frac{1}{2}(2)^2(4\hat{i} + 2\hat{j})$$

$$= (12\hat{i} + 10\hat{j}) \text{ m}$$

Therefore, velocity and displacement of particle at $t = 2$ s are $(10\hat{i} + 7\hat{j})$ m/s and $(12\hat{i} + 10\hat{j})$ m respectively.

Example 3.34 *A ball is thrown upwards from the top of a tower 40 m high with a velocity of 10 m/s. Find the time when it strikes the ground. (Take $g = 10$ m/s^2)*

Sol. According to the question, we draw the following figure



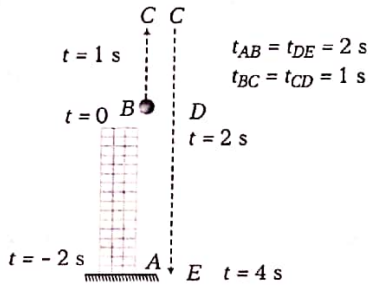
Given, $u = +10\text{ m/s}$, $a = -10\text{ m/s}^2$ and $s = -40\text{ m}$ (at the point where stone strikes the ground)

Substituting in $s = ut + \frac{1}{2}at^2$, we have

$-40 = 10t - 5t^2$ or $5t^2 - 10t - 40 = 0$ or $t^2 - 2t - 8 = 0$

Solving this we have $t = 4\text{ s}$ and -2 s. Taking the positive value $t = 4\text{ s}$.

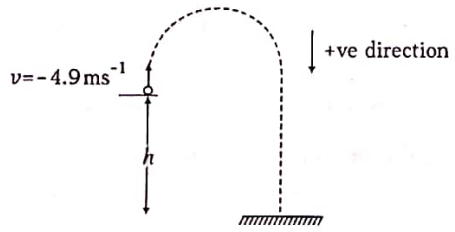
Note The significance of $t = -2\text{ s}$ can be understood by following figure



Example 3.35 *A pebble is thrown vertically upward from a bridge with an initial velocity of 4.9 ms^{-1}. It strikes the water after 2 s. If acceleration due to gravity is 9.8 ms^{-2} (i) what is the height of the bridge? (ii) with what velocity does the pebble strike the water?*

Sol. Let height of the bridge be h, then

$$h = -4.9 \times 2 + \frac{1}{2} \times 9.8 \times (2)^2 \Rightarrow h = 9.8\text{ m}$$



velocity with which ball will hit the ground

$$v = u + at \Rightarrow v = -4.9 + 9.8 \times 2 = 14.7\text{ ms}^{-1}$$

Example 3.36 *A rocket is fired vertically up from the ground with a resultant vertical acceleration of 10 ms^{-2}. The fuel is finished in 1 minute and it contiues to move up.*

(i) What is the maximum height reached?

(ii) After finishing fuel, calculate the time for which it continues its upwards motion. (Take $g = 10\text{ ms}^{-2}$)

Sol. (i) The distance travelled by the rocket during burning of fuel (1 minute = 60 s) in which resultant acceleration is vertically upwards and is 10 ms^{-2} will be

$h_1 = 0 \times 60 + (1/2) \times 10 \times 60^2 = 18000\text{ m} = 18\text{ km}$ and

velocity acquired by it will be

$v = 0 + 10 \times 60 = 600\text{ ms}^{-1}$.

Now after 1 min the rocket moves vertically up with initial velocity of 600 ms^{-1} and acceleration due to gravity opposes its motion. So, it will go to a height h_2 from this point, till its velocity becomes zero such that $0 = (600)^2 - 2gh_2$ or $h_2 = 18000\text{ m} = 18\text{ km}$ ($g = 10\text{ ms}^{-2}$)

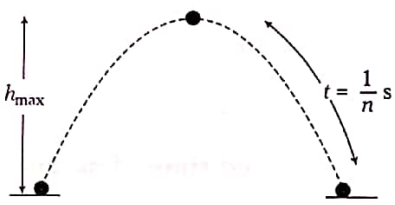
So the maximum height reached by the rocket from the ground, $H = h_1 + h_2 = 18 + 18 = 36\text{ km}$

(ii) As after burning of fuel the initial velocity attained will be 600 ms^{-1} and gravity opposes the motion of rocket, so from 1st equation of motion time taken by it till it velocity $v = 0$

$$0 = 600 - gt \Rightarrow t = 60\text{ s}$$

Example 3.37 *A Juggler throws balls into air. He throws one ball whenever the previous one is at its highest point. How high does the balls rise if he throws n balls each second? Acceleration due to gravity is g.*

Sol. Juggler throws n balls in one second so time interval between two consecutive throws is $t = \frac{1}{n}$ s



Each ball takes $\frac{1}{n}$ s to reach maximum height.

So, $$h_{max} = \frac{1}{2} \times gt^2 = \frac{1}{2} \times g\left(\frac{1}{n}\right)^2 \Rightarrow h_{max} = \frac{g}{2n^2}$$

Example 3.38 *From an elevated point A, a stone is projected vertically upwards. When the stone reaches a distance h below A, its velocity is double of what it was at a height h above A. Show that the greatest height attained by the stone is $\frac{5}{3}h$.*

Sol. Let u be the velocity with which the stone is projected vertically upwards.

Given that, $v_{-h} = 2v_h$ or $(v_{-h})^2 = 4v_h^2$

$$\therefore \quad u^2 - 2g(-h) = 4(u^2 - 2gh)$$

$$\therefore \quad u^2 = \frac{10gh}{3}$$

Now, Maximum height $h_{max} = \frac{u^2}{2g} = \frac{5h}{3}$

Example 3.39 *A ball is thrown vertically upwards with a velocity of 20 ms^{-1} from the top of a multistorey building. The height of the point from where the ball is thrown is 25 m from the ground. How long will it be before the ball hits the ground (Take, $g = 10$ ms^{-2})?*

Sol. Let us take the Y-axis in the vertically upward direction with zero at the ground.

Now, $v_0 = +20\,\text{ms}^{-1}$, $a = -g = -10\,\text{ms}^{-2}$, $v = 0\,\text{ms}^{-1}$

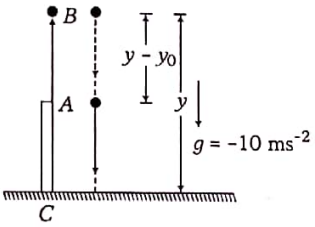
If the ball rises to height y from the point of launch, then using the equation $v^2 = v_0^2 + 2a(y - y_0)$

We get $\quad 0 = (20)^2 + 2(-10)(y - y_0)$

Solving, we get, $(y - y_0) = 20$ m

There are two ways of solving the problem

Method I We split two parts the upward motion (A to B) and the downward motion (B to C) and calculate the corresponding time taken t_1 and t_2.



Since, the velocity at B is zero, we have

$$v = v_0 + gt$$
$$0 = 20 - 10t_1$$
$$t_1 = 2\text{ s}$$

This is the time in going from A to B. From B or the point of the maximum height, the ball falls freely under the acceleration due to gravity. The ball is moving in negative y-direction. We use equation

$$y = y_0 + v_0 t + \frac{1}{2}at^2$$

We have, $\quad y_0 = 45$ m, $y = 0$, $v_0 = 0$, $a = -g = -10$ ms^{-2}

$$0 = 45 + \frac{1}{2}(-10)t_2^2 \Rightarrow t_2 = 3\text{ s}$$

Total time taken by the ball before it hits the ground

$$= t_1 + t_2 = 2\text{ s} + 3\text{ s} = 5\text{ s}$$

Method II The total time taken can also be calculated by noting the coordinates of initial and final positions of the ball with respect to the origin chosen and using equation

$$y = y_0 + v_0 t + \frac{1}{2}at^2$$

Now, $y_0 = 25$ m, $y = 0$ m, $v_0 = 20$ ms^{-1}, $a = -10$ ms^{-2}

$$\therefore \quad 0 = 25 + 20t + \left(\frac{1}{2}\right)(-10)t^2 \Rightarrow 5t^2 - 20t - 25 = 0$$

Solving this quadratic equation for t, we get

$$t = 5\text{ s}$$

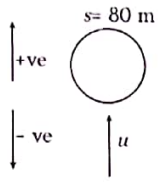
Note The second method is better since we do not have to worry about the path of the motion as the motion is under constant acceleration.

Example 3.40 *A ball is thrown upwards from the ground with an initial speed u. The ball is at a height of 80 m at two times, the time interval being 6s. Find the value of u.*

Sol. Here, $a = g = -10$ ms^{-2} and $s = 80$ m

Substituting the values, in $s = ut + \frac{1}{2}at^2$,

We have
$$80 = ut - 5t^2$$



or $\quad 5t^2 - ut + 80 = 0$

or $\quad t = \dfrac{u + \sqrt{u^2 - 1600}}{10}$ and $t = \dfrac{u - \sqrt{u^2 - 1600}}{10}$

Now, it is given that

$$\frac{u + \sqrt{u^2 - 1600}}{10} - \frac{u - \sqrt{u^2 - 1600}}{10} = 6 \Rightarrow \frac{\sqrt{u^2 - 1600}}{5} = 6$$

$$\Rightarrow \sqrt{u^2 - 1600} = 30 \Rightarrow u^2 - 1600 = 900$$

$$\therefore \quad u^2 = 2500 \Rightarrow u = \pm 50\text{ ms}^{-1}$$

Ignoring the negative sign, we have

$$u = 50\text{ ms}^{-1}$$

Example 3.41 *A particle of mass 1 kg has a velocity of 2 m/s. A constant force of 2 N acts on the particle for 1 s in a direction perpendicular to its initial velocity. Find the velocity and displacement of the particle at the end of 1 s.*

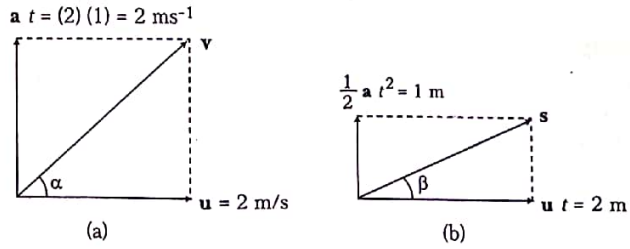
Sol. Force acting on the particle is constant. Hence, acceleration of the particle will also remain constant.

$$a = \frac{F}{m} = \frac{2}{1} = 2\text{ms}^{-2}$$

Since, acceleration is constant. We can apply

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t \quad \text{and} \quad \mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

Refer Fig. (a) $\qquad \mathbf{v} = \mathbf{u} + \mathbf{a}t$



Here, $\mathbf{u}$ and $\mathbf{a}t$ are two mutually perpendicular vectors. So

$$|\mathbf{v}| = \sqrt{(|\mathbf{u}|)^2 + (|\mathbf{a}t|)^2} = \sqrt{(2)^2 + (2)^2} = 2\sqrt{2}\text{ ms}^{-1}$$

$$\alpha = \tan^{-1}\frac{|\mathbf{a}t|}{|\mathbf{u}|} = \tan^{-1}\left(\frac{2}{2}\right) = \tan^{-1}(1) = 45°$$

Thus, velocity of the particle at the end of 1s is $2\sqrt{2}$ ms^{-1} at an angle of 45° with its initial velocity.

Refer Fig. (b) $\quad \mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

Here, $\mathbf{u}t$ and $\frac{1}{2}\mathbf{a}t^2$ are also two mutually perpendicular vectors. So,

$$|\mathbf{s}| = \sqrt{(|\mathbf{u}t|)^2 + (|\tfrac{1}{2}\mathbf{a}t^2|)^2} = \sqrt{(2)^2 + (1)^2} = \sqrt{5}\text{ m}$$

and $$\beta = \tan^{-1}\frac{|(1/2)\mathbf{a}t^2|}{|\mathbf{u}t|} = \tan^{-1}\left(\frac{1}{2}\right)$$

Thus, displacement of the particle at the end of 1s is $\sqrt{5}$ m at an angle of $\tan^{-1}\left(\frac{1}{2}\right)$ from its initial velocity.

Example 3.42 *An open elevator is ascending with zero acceleration. The speed $v = 10$ m/s. A ball is thrown vertically up by a boy when he is at a height $h = 10$ m from the ground. The velocity of projection is $v = 30$ m/s with respect to elevator. Find*

(i) the maximum height attained by the ball.

(ii) the time taken by the ball to meet the elevator again.

(iii) time taken by the ball to reach the ground after crossing the elevator.

Sol. (i) Absolute velocity of ball = 40 m/s (upwards)

$\therefore \quad h_{max} = h_i + h_f$

Here, h_i = initial height = 10 m

and h_f = further height attained by ball

$$= \frac{u^2}{2g} = \frac{(40)^2}{2\times 10} = 80\text{ m}$$

$\therefore \quad h_{max} = (10 + 80)\text{ m} = 90\text{ m}$

(ii) The ball will meet the elevator again when displacement of lift = displacement of ball

or $\quad 10\times t = 40\times t - \frac{1}{2}\times 10\times t^2$ or $t = 6$ s

(iii) Let t_0 be the total time taken by the ball to reach the ground. Then,

$$-10 = 40\times t_0 - \frac{1}{2}\times 10\times t_0^2$$

Therefore, time taken by the ball to reach the ground after crossing the elevator $= (t_0 - t) = 2.24$ s

Example 3.43 *A particle is thrown vertically upwards from the surface of the earth. Let T_P be the time taken by the particle to travel from a point P above the earth to its highest point and back to the point P.*
Similarly, let T_Q be the time taken by the particle to travel from another point Q above the earth to its highest point and back to the same point Q. If the distance between the points P and Q is H, find the expression for acceleration due to gravity in terms of T_P, T_Q and H.

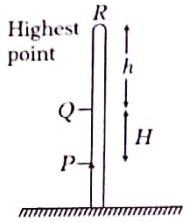
Sol. Time taken from point P to point P

$$T_P = T_{PR} + T_{RP}$$

Here, $T_{PR} = T_{RP}$ then $T_{PP} = 2T_{PR}$

Using second equation of motion,

$$(H + h) = \frac{1}{2}gT_{PR}^2 \Rightarrow T_{PR} = \sqrt{2\frac{(H+h)}{g}}$$



Then similarly, time taken from point Q to point Q

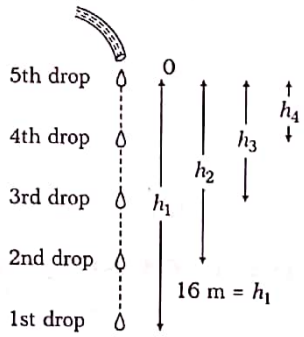
$$T_Q = 2\sqrt{\frac{2h}{g}} \Rightarrow T_P^2 = \frac{8(h+H)}{g} \text{ and } T_Q^2 = \frac{8h}{g}$$

$$\Rightarrow \quad T_P^2 = T_Q^2 + \frac{8H}{g} \Rightarrow g = \frac{8H}{T_P^2 - T_Q^2}$$

Example 3.44 *From the top of a building, 16 m high water drops are falling at equal intervals of time such that when the first drop reaches the ground, the fifth drop just starts. Find the distance between the successive drops at that instant.*

Sol. Let the interval of time be t_0.

First drop is released at $t = 0$, second drop at $t = t_0$, third drop at $t = 2t_0$, fourth drop at $t = 3t_0$, fifth drop at $t = 4t_0$. Therefore, first drop has fallen for time $4t_0$, second drop for $3t_0$, third drop for $2t_0$, fourth drop for t_0 and fifth drop is about to fall. The location of drops are as shown in the figure.



For 1st drop, $h_1 = 16 = \frac{1}{2}g(4t_0)^2 \Rightarrow \frac{1}{2}gt_0^2 = 1$

For 2nd drop, $h_2 = \frac{1}{2}g(3t_0)^2 = 9$ m

For 3rd drop, $h_3 = \frac{1}{2}g(2t_0)^2 = 4$ m

For 4th drop, $h_4 = \frac{1}{2}gt_0^2 = 1$m

For 5th drop, $h_5 = 0$

Separation between drops

1st and 2nd : $h_1 - h_2 = 7$ m

2nd and 3rd : $h_2 - h_3 = 5$m

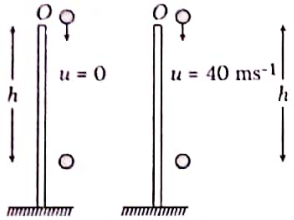
3rd and 4th : $h_3 - h_4 = 3$m

4th and 5th : $h_4 - h_5 = 1$ m

Note If the 1st drop is at the ground and the 5th drop is about to fall, the time for which the first drop has fallen $(5-1)t_0 = 4t_0$, where t_0 is the regular interval of time.

Example 3.45 *A ball is dropped from the top of a tower. After 2 s another ball is thrown vertically downwards with a speed of 40 ms^{-1}. After how much time and at what distance below the top of tower the balls meet?*

Sol. Let the balls meet at distance h below the top of water t second after dropping of first ball. The second ball takes time $(t-2)$ seconds.



For first ball, $h = \frac{1}{2} g t^2$ ...(i)

For second ball, $h = 40(t-2) + \frac{1}{2} g (t-2)^2$ ...(ii)

Eqs. (i) and (ii), we get

$$40(t-2) + \frac{1}{2} g (t-2)^2 = \frac{1}{2} g t^2$$

$$40(t-2) = \frac{1}{2} g [t^2 - (t-2)^2]$$

$$40(t-2) = \frac{1}{2} \times 10 (2t-2) \times 2$$

$$4t - 8 = 2t - 2 \Rightarrow t = 3 \text{ s}$$

Distance below the top of tower the balls meet

$$h = \frac{1}{2} g t^2 = \frac{1}{2} \times 10 \times 3^2 = 45 \text{ m}$$

4 Non-uniformly accelerated motion

When acceleration of particle is not constant, motion is known as non-uniformly accelerated motion. Then basic equations of velocity and acceleration can be written as

(i) $\mathbf{v} = \frac{d\mathbf{s}}{dt}$ or sometimes $\mathbf{v} = \frac{d\mathbf{r}}{dt}$ (ii) $\mathbf{a} = \frac{d\mathbf{v}}{dt}$

(iii) $d\mathbf{s} = \mathbf{v}\, dt$ (iv) $d\mathbf{v} = \mathbf{a}\, dt$

For one dimensional motion, above relations can be written as under.

(i) $v = \frac{ds}{dt}$ (ii) $a = \frac{dv}{dt} = v \frac{dv}{ds}$

(iii) $ds = v\, dt$ and (iv) $dv = a dt$ or $v\, dv = a\, ds$

Such problems can be solved either by differentiation or integration (with some boundary conditions).

$s\text{-}t \longrightarrow v\text{-}t \longrightarrow a\text{-}t$ (Differentiation)

$a\text{-}t \longrightarrow v\text{-}t \longrightarrow s\text{-}t$ (Integration with boundary conditions)

Note

1. By boundary condition, we mean that velocity or displacement at some time (usually at $t = 0$) should be known to us. Otherwise we cannot find constant of integration.
2. Equation $a = v\, dv/ds$ or $v\, dv = a\, ds$ is useful when acceleration displacement equation is known and velocity displacement equation is required.

Example 3.46 *Velocity-time equation of a particle moving in a straight line is,*

$$v = (10 + 2t + 3t^2) \quad \text{(SI Units)}$$

Find,

(i) displacement of particle from the mean position at time $t = 1$ s, if it is given that displacement is 20 m at time $t = 0$.

(ii) acceleration-time equation.

Sol. (i) The given equation can be written as,

$$v = \frac{ds}{dt} = (10 + 2t + 3t^2)$$

or $ds = (10 + 2t + 3t^2)\, dt$

or $\int_{20}^{s} ds = \int_0^1 (10 + 2t + 3t^2)\, dt$

or $s - 20 = [10t + t^2 + t^3]_0^1$

or $s = 20 + 12 = 32$ m

(ii) Acceleration-time equation can be obtained by differentiating the given equation w.r.t. time. Thus,

$$a = \frac{dv}{dt} = \frac{d}{dt}(10 + 2t + 3t^2) \quad \text{or} \quad a = 2 + 6t$$

Example 3.47 *Displacement-time equation of a particle moving along X-axis is*

$$x = 20 + t^3 - 12t \quad \text{(SI Units)}$$

(i) Find, position and velocity of particle at time $t = 0$.

(ii) State whether the motion is uniformly accelerated or not.

(iii) Find position of particle when velocity of particle is zero.

Sol. (i) Given, displacement-time equation of a particle P

$$x = 20 + t^3 - 12t \quad ...(i)$$

At $t = 0$, $x = 20 + 0 - 0 = 20$ m

Velocity of particle at time t can be obtained by differentiating Eq. (i) w.r.t. time *i.e.*,

$$v = \frac{dx}{dt} = 3t^2 - 12 \quad ...(ii)$$

At $t = 0$, $v = 0 - 12 = -12$ m/s

(ii) Differentiating Eq. (ii) w.r.t. time t, we get the acceleration,

$$a = \frac{dv}{dt} = 6t$$

As acceleration is a function of time, the motion is non-uniformly accelerated.

(iii) Substituting $v = 0$ in Eq. (ii), we have

$$0 = 3t^2 - 12$$

Positive value of t comes out to be 2 s from this equation. Substituting $t = 2$ s in Eq. (i), we have,

$$x = 20 + (2)^3 - 12(2) \text{ or } x = 4 \text{ m}$$

Check point 3.5

1. Free fall of an object (in vacuum) is a case of motion with
(a) uniform velocity (b) uniform acceleration
(b) variable acceleration (d) constant momentum

2. With what speed should a body be thrown upwards so that the distances traversed in 5th second and 6th second are equal?
(a) $5.84\,ms^{-1}$ (b) $49\,ms^{-1}$ (c) $\sqrt{98}\,ms^{-1}$ (d) $98\,ms^{-1}$

3. If a ball is thrown vertically upwards with speed u, the distance covered during the last t second of its ascent is
(a) $ut - (gt^2/2)$ (b) $(u + gt)t$
(c) ut (d) $gt^2/2$

4. A person throws balls into air after every second. The next ball is thrown when the velocity of the first ball is zero. How high do the ball rise above his hand?
(a) 2 m (b) 5 m (c) 8 m (d) 10 m

5. A body thrown vertically up from the ground passes the height of 10.2 m twice in an interval of 10 s. What was its initial velocity?
(a) $52\,ms^{-1}$ (b) $61\,ms^{-1}$ (c) $45\,ms^{-1}$ (d) $26\,ms^{-1}$

6. If a stone is thrown up with a velocity of $9.8\,ms^{-1}$, then how much time will it take to come back?
(a) 1 s (b) 2 s (c) 3 s (d) 4 s

7. A stone falls freely under gravity. The total distance covered by it in the last second of its journey equals the distance covered by it in first 3 s of its motion. The time for which the stone is in air is
(a) 5 s (b) 12 s (c) 15 s (d) 8 s

8. A stone thrown upward with a speed u from the top of the tower reaches the ground with a speed $3u$. The height of the tower is
(a) $3u^2/g$ (b) $4u^2/g$ (c) $6u^2/g$ (d) $9u^2/g$

9. A body is thrown vertically upwards from the top A of tower. It reaches the ground in t_1s. If it is thrown vertically downwards from A with the same speed it reaches the ground in t_2 s. If it is allowed to fall freely from A, then the time it takes to reach the ground is given by
(a) $t = \frac{t_1 + t_2}{2}$ (b) $t = \frac{t_1 - t_2}{2}$
(c) $t = \sqrt{t_1 t_2}$ (d) $t = \sqrt{\frac{t_1}{t_2}}$

10. A body is projected with a velocity u. It passes through a certain point above the ground after t_1 second. The time interval after which the body passes through the same point during the return journey is
(a) $\left(\frac{u}{g} - t_1^2\right)$ (b) $2\left(\frac{u}{g} - t_1\right)$
(c) $\left(\frac{u}{g} - t_1\right)$ (d) $\left(\frac{u^2}{g^2} - t_1\right)$

11. A helicopter, moving vertically upwards, releases a packet when it is at a certain height above the ground. The packet initially moves upwards for a time t_1 and then falls downwards for a time t_2 until it reaches the ground. Then
(a) $t_1 < t_2$ (b) $t_1 = t_2$
(c) $t_1 > t_2$ (d) Data insufficient

12. A ball P is dropped vertically and another ball Q is thrown horizontally with the same velocities from the same height and at the same time. If air resistance is neglected, then
(a) ball P reaches the ground first
(b) ball Q reaches the ground first
(c) Both reach the ground at the same time
(d) the respective masses of the two balls will decide the time

13. A particle is dropped under gravity from rest from a height h ($g = 9.8\,ms^{-2}$) and it travels a distance $9h/25$ in the last second, the height h is
(a) 100 m (b) 122.5 m (c) 145 m (d) 167.5 m

14. A body dropped from the top of a tower covers a distance $7x$ in the last second of its journey, where x is the distance covered in first second. How much time does it take to reach the ground?
(a) 3 s (b) 4 s (c) 5 s (d) 6 s

15. A body falls from a height $h = 200$ m. The ratio of distance travelled in each 2 s, during $t = 0$ to $t = 6$ s of the journey is
(a) 1 : 4 : 9 (b) 1 : 2 : 4 (c) 1 : 3 : 5 (d) 1 : 2 : 3

16. A particle is thrown vertically upwards. Its velocity at half of the height is $10\,ms^{-1}$. Then, the maximum height attained by it is (take, $g = 10\,ms^{-2}$)
(a) 16 m (b) 10 m (c) 20 m (d) 40 m

17. When a ball is thrown up vertically with velocity v_0, it reaches a maximum height of h. If one wishes to triple the maximum height, then the ball should be thrown with velocity.
(a) $\sqrt{3}\,v_0$ (b) $3\,v_0$ (c) $9\,v_0$ (d) $3/2\,v_0$

18. A man in a balloon rising vertically with an acceleration of $4.9\,ms^{-2}$ releases a ball 2 s after the balloon is let go from the ground. The greatest height above the ground reached by the ball is ($g = 9.8\,ms^{-2}$)
(a) 14.7 m (b) 19.6 m (c) 9.8 m (d) 24.5 m

19. A body falling from the rest has a velocity v after it falls through a height h. The distance it has to fall down further for its velocity to become double, will be
(a) $8h$ (b) $6h$ (c) $4h$ (d) $5h$

20. A ball is released from height h and another from $2h$. The ratio of time taken by the two balls to reach the ground is
(a) $1 : \sqrt{2}$ (b) $\sqrt{2} : 1$ (c) 2 : 1 (d) 1 : 2

21. An aeroplane is moving with a velocity u. It drops a packet from a height h. The time t taken by the packet in reaching the ground will be
(a) $\sqrt{\left(\frac{2g}{h}\right)}$ (b) $\sqrt{\left(\frac{2u}{g}\right)}$ (c) $\sqrt{\left(\frac{h}{2g}\right)}$ (d) $\sqrt{\left(\frac{2h}{g}\right)}$

22. The displacement of a particle moving in a straight line depends on time as $x = \alpha t^3 + \beta t^2 + \gamma t + \delta$.
The ratio of initial acceleration to its initial velocity depends
(a) only on α and γ (b) only on β and γ
(c) only on α and β (d) only on α

23. The acceleration of a particle is increasing linearly with time t as bt. The particle starts from the origin with an initial velocity v_0. The distance travelled by the particle in time t will be
(a) $v_0 t + \frac{1}{6} bt^3$ (b) $v_0 t + \frac{1}{3} bt^3$
(c) $v_0 t + \frac{1}{3} bt^2$ (d) $v_0 t + \frac{1}{2} bt^2$

24. The acceleration a in ms^{-2}, of a particle is given by $a = 3t^2 + 2t + 2$, where t is the time. If the particle starts out with a velocity $v = 2\ ms^{-1}$ at $t = 0$, then the velocity at the end of 2 s is
(a) $12\ ms^{-1}$ (b) $14\ ms^{-1}$
(c) $16\ ms^{-1}$ (d) $18\ ms^{-1}$

25. A particle is moving such that $s = t^3 - 6t^2 + 18t + 9$, where s is in metres and t is in seconds. The minimum velocity attained by the particle is
(a) $29\ ms^{-1}$ (b) $5\ ms^{-1}$ (c) $6\ ms^{-1}$ (d) $12\ ms^{-1}$

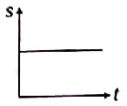
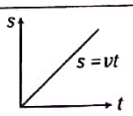
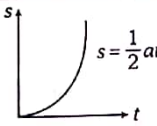
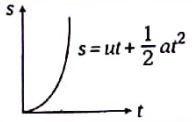
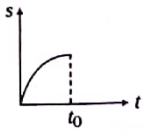
5 Graphical representation of motion

Motion of a point or body or a particle in all aspects can be shown with the help of the graph, such as displacement-time graph and velocity-time graph, etc. The tabular forms of displacement-time and velocity-time graphs are given for one dimensional motion.

1. Displacement-time graph

(i) Displacement-time graph gives instantaneous value of displacement at any instant.
(ii) The slope of tangent drawn to the graph at any instant of time gives the instantaneous velocity at that instant.
(iii) The s-t graph cannot make sharp turns.

Different Cases of Displacement-Time Graph

Different Cases	s-t Graph	Main Features of Graph
At rest		Slope $= v = 0$
Uniform motion		Slope = constant, v = constant or $a = 0$
Uniformly accelerated motion with $u = 0$, $s = 0$ at $t = 0$		$u = 0$, *i.e.*, Slope of s-t graph at $t = 0$ should be zero.
Uniformly accelerated motion with $u \neq 0$ but $s = 0$ at $t = 0$		Slope of s-t graph gradually goes on increasing.
Uniformly retarded motion		θ is decreasing so, v is decreasing, a is negative.

2. Velocity-time graph

(i) Velocity-time graph gives the instantaneous value of velocity at any instant.
(ii) The slope of tangent drawn on graph gives instantaneous acceleration.
(iii) Area under v-t graph with time axis gives the value of displacement covered in given time.
(iv) The v-t curve cannot take sharp turns.

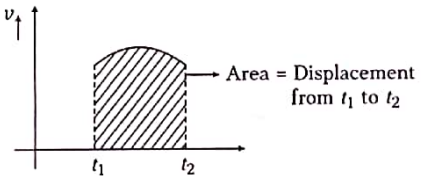
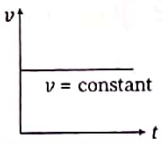
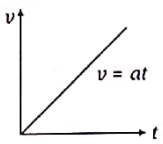
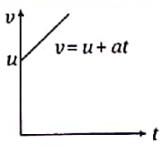
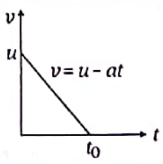
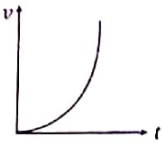


Fig. 3.10

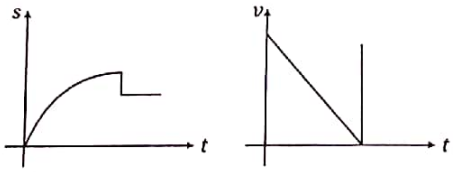
Different Cases in Velocity-Time Graph

Different Cases	v-t Graph	Main Features of Graph
Uniform motion		(i) $\theta = 0°$ (ii) v = constant (iii) Slope of v-t graph $= a = 0$
Uniformly accelerated motion with $u = 0$ and $s = 0$ at $t = 0$		Slope of v-t graph is constant so, a = constant, $u = 0$ *i.e.*, $v = 0$ at $t = 0$
Uniformly accelerated motion with $u \neq 0$ but $s = 0$ at $t = 0$		Positive constant acceleration because θ is constant and $< 90°$ but the initial velocity of the particle is positive.
Uniformly decelerated motion		Slope of v-t graph $= -a$ (retardation)

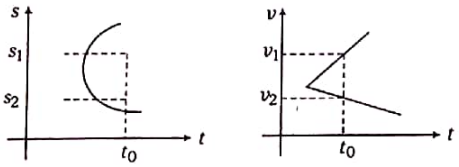
Different Cases	v-t Graph	Main Features of Graph
Non-uniformly accelerated motion		Slope of v-t graph increases with time. θ is increasing, so, acceleration is increasing.
Non-uniformly decelerating motion		θ is decreasing, so acceleration is decreasing.

Note

1. Slope of s-t or v-t graphs can never be infinite at any point, because infinite slope of s-t graphs means infinite velocity. Similarly, infinity slope of v-t graph means infinite acceleration. Hence, the following graphs are not possible.



2. At one time, two values of velocity or displacement are not possible. Hence, the following graphs are not acceptable.



3. Slope of displacement-time graph gives velocity $\left(\text{as } v = \frac{ds}{dt}\right)$.

4. Slope of velocity-time graph gives acceleration $\left(\text{as } a = \frac{dv}{dt}\right)$.

3. Acceleration-time graph

The area of the a-t graph between time t_1 to t_2 gives the change in velocity.

$$\therefore \quad a = \frac{dv}{dt}$$

$$\Rightarrow \quad dv = a\,dt \Rightarrow \int_{v_1}^{v_2} dv = \int_{t_1}^{t_2} a\,dt$$

$$v_2 - v_1 = \int_{t_1}^{t_2} a\,dt$$

Change in velocity = Area of the a-t graph

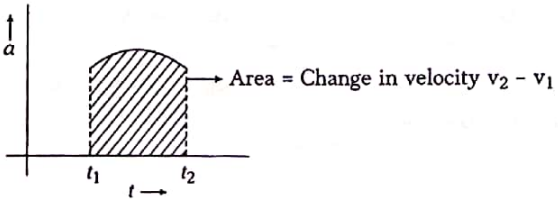
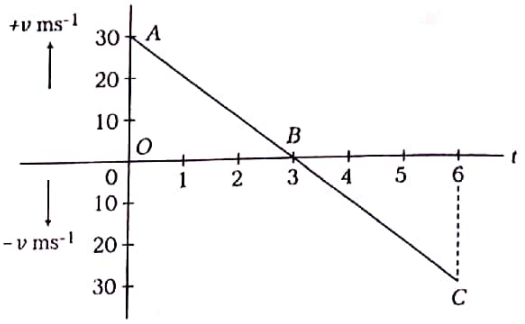


Fig. 3.11 Acceleration-time graph

Example 3.48 With the help of the given velocity-time graph, find the

(i) displacement in first three seconds and

(ii) acceleration for the graph.



Sol. (i) Displacement in first three seconds = Area of triangle OAB

$$= \frac{1}{2}(OB) \times (OA)$$

$$= \frac{1}{2}(3) \times (30) = +45 \text{ m}$$

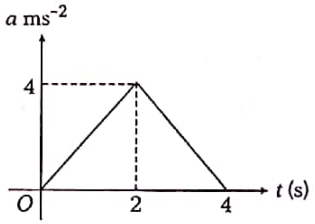
(ii) Acceleration = Slope of v-t graph

As, v-t graph is a straight line. So, consider the slope of line AB.

$$\therefore \text{ Slope of line } AB = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 30}{3} = -10 \text{ ms}^{-2}$$

So, the acceleration is negative.

Example 3.49 *Acceleration-time graph of a particle moving in a straight line is shown in figure. Velocity of particle at time $t = 0$ is 2 ms^{-1}. Find velocity at the end of fourth second.*



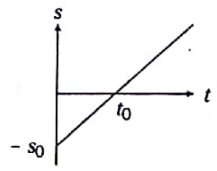
Sol. According to acceleration time-graph, $dv = a\,dt$

or change in velocity = area under a-t graph

Hence, $$v_f - v_i = \frac{1}{2}(4)(4) = 8 \text{ ms}^{-1}$$

$$\therefore \quad v_f = v_i + 8 = (2 + 8) \text{ ms}^{-1} = 10 \text{ ms}^{-1}$$

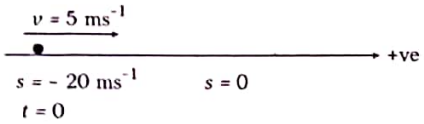
Example 3.50 *Displacement-time graph of a particle moving in a straight line is as shown in figure. State whether the motion is accelerated or not. Describe the motion in detail. Given, $s_0 = 20$ m and $t_0 = 4$ s.*



Sol. Slope of s-t graph is constant. Hence, velocity of particle is constant. Further at time $t = 0$, displacement of the particle from the mean position is $-s_0$ or -20 m. Velocity of particle,

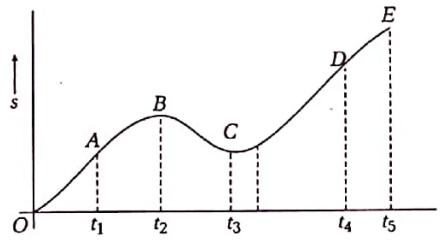
$$v = \text{slope}$$

$$= \frac{s_0}{t_0} = \frac{20}{4} = 5 \text{ ms}^{-1}$$

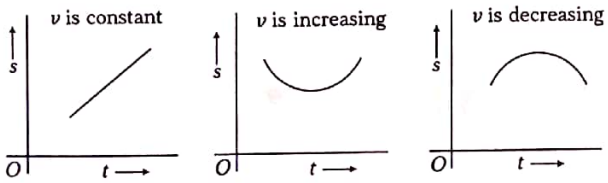


Motion of the particle is as shown in figure. At $t = 0$, particle is at -20 m and has a constant velocity of 5 ms^{-1}. At $t_0 = 4$ s, particle will pass through its mean position.

Example 3.51 *A particle is moving along the X-axis and its position-time graph is shown. Determine the sign of acceleration.*



Sol. By observing the s-t graph, we can determine the sign of acceleration. Recall if the graph is concave upwards, the slope is increasing; if it is concave downward, the slope is decreasing; and if the graph is straight line, the slope is constant.



OA : Slope is increasing, v is increasing and a is positive.

AB : Slope is constant, v is constant and $a = 0$.

BC : Slope is decreasing, v is decreasing and a is negative.

CD : Slope is increasing, v is increasing and a is positive.

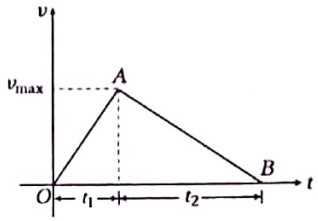
DE : Slope is constant, v is constant and $a = 0$.

Example 3.52 *A car accelerates from rest at a constant rate α for some time, after which it decelerates at a constant rate β to come to rest. If the total time elapsed is t second evaluate (i) the maximum velocity reached and (ii) the total distance travelled.*

Sol. (i) Let the car accelerates for time t_1 and decelerates for time t_2. Then,

$$t = t_1 + t_2 \quad \text{...(i)}$$

and corresponding velocity-time graph will be as shown in figure.



From the graph,

$$\alpha = \text{slope of line } OA = \frac{v_{max}}{t_1} \quad \text{or} \quad t_1 = \frac{v_{max}}{\alpha} \quad \text{...(ii)}$$

$$\text{and} \quad \beta = -\text{slope of line } AB = \frac{v_{max}}{t_2} \quad \text{or} \quad t_2 = \frac{v_{max}}{\beta} \quad \text{...(iii)}$$

From Eqs. (i), (ii) and (iii), we get

$$\frac{v_{max}}{\alpha} + \frac{v_{max}}{\beta} = t$$

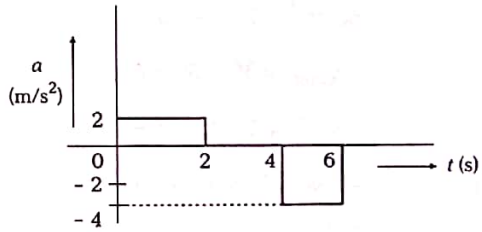
$$\text{or} \quad v_{max}\left(\frac{\alpha+\beta}{\alpha\beta}\right) = t \quad \text{or} \quad v_{max} = \frac{\alpha\beta t}{\alpha+\beta}$$

(ii) Total distance = displacement = area under v-t graph

$$= \frac{1}{2} \times t \times v_{max} = \frac{1}{2} \times t \times \frac{\alpha\beta t}{\alpha+\beta}$$

$$\text{or} \quad \text{Distance} = \frac{1}{2}\left(\frac{\alpha\beta t^2}{\alpha+\beta}\right)$$

Example 3.53 *The acceleration versus time graph of a particle moving along a straight line is shown in the figure. Draw the respective velocity-time graph.*



Assuming at $t = 0$, $v = 0$.

Sol. From $t = 0$ to $t = 2$ s, $a = +2$ m/s^2 $\Rightarrow$ $v = at = 2t$

or v-t graph is a straight line passing through origin with slope 2 m/s^2.

At the end of 2 s,

$$v = 2 \times 2 = 4 \text{ m/s}$$

From $t = 2$ to 4 s, $a = 0$.

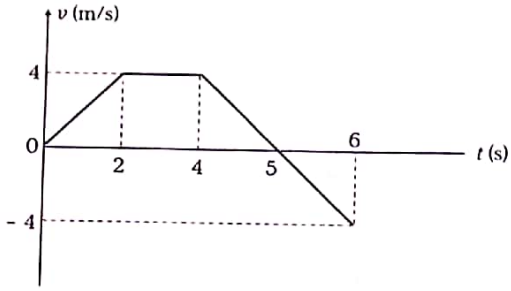
Hence, $v = 4$ m/s will remain constant.

From $t = 4$ to 6 s, $a = -4$ m/s^2. Hence,

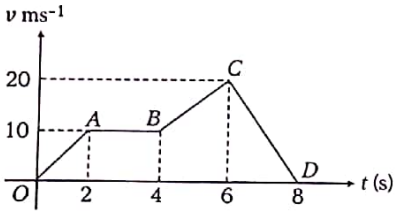
$$v = u - at = 4 - 4t \quad \text{(with } t = 0 \text{ at 4 s)}$$

$v = 0$ at $t = 1$ s or at 5 s from origin.

At the end of 6 s (or $t = 2$ s) $v = -4$ m/s. Corresponding v-t graph is as shown below



Example 3.54 *Velocity-time graph of a particle moving in a straight line is shown in figure.*



Plot the corresponding displacement-time graph of the particle if at time $t = 0$, displacement $s = 0$.

Sol. Displacement = Area under velocity-time graph

Hence, $s_{OA} = \frac{1}{2} \times 2 \times 10 = 10$ m

$s_{AB} = 2 \times 10 = 20$ m

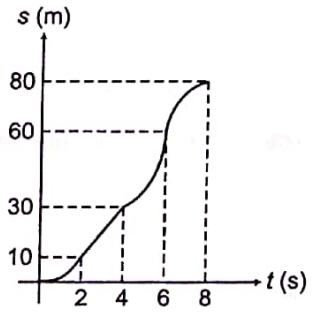
or $s_{OAB} = 10 + 20 = 30$ m

$s_{BC} = \frac{1}{2} \times 2(10 + 20) = 30$ m

or $s_{OABC} = 30 + 30 = 60$ m

and $s_{CD} = \frac{1}{2} \times 2 \times 20 = 20$ m

or $s_{OABCD} = 60 + 20 = 80$ m



Between 0 to 2 s and 4 to 6 s motion is accelerated, hence displacement-time graph is a parabola. Between 2 to 4 s motion is uniform, so displacement-time graph will be a straight line. Between 6 to 8 s motion is decelerated hence displacement-time graph is again a parabola but inverted in shape.

At the end of 8 s velocity is zero, therefore, slope of displacement- time graph should be zero. The corresponding graph is shown in figure.

Example 3.55 *A rocket is fired vertically upwards with a net acceleration of 4 ms^{-2} and initial velocity zero. After 5 s its fuel is finished and it decelerates with g. At the highest point its velocity becomes zero. Then, it accelerates downwards with acceleration g and return back to ground. Plot velocity-time and displacement-time graphs for the complete journey. (Take $g = 10\ ms^{-2}$)*

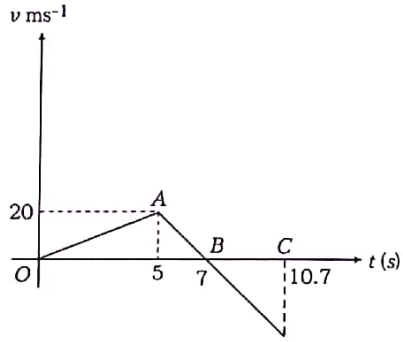
Sol. In the graphs,

$$v_A = at_{OA} = (4)(5) = 20\ ms^{-1}$$

$$v_B = 0 = v_A - gt_{AB}$$

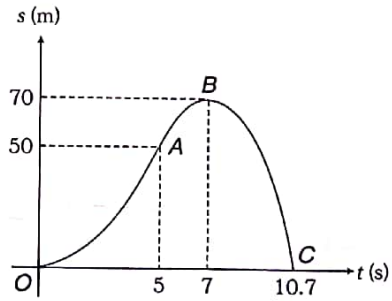
$$\therefore \quad t_{AB} = \frac{v_A}{g} = \frac{20}{10} = 2\text{ s}$$

$$\therefore \quad t_{OAB} = (5 + 2)\text{ s} = 7\text{ s}$$



Now, s_{OAB} = area under v-t graph between 0 to 7 s

$$= \frac{1}{2}(7)(20) = 70\text{ m}$$



Now, $|s_{OAB}| = |s_{BC}| = \frac{1}{2} g t_{BC}^2$

$$\therefore \quad 70 = \frac{1}{2}(10)\, t_{BC}^2$$

$$\therefore \quad t_{BC} = \sqrt{14} = 3.7\text{ s}$$

$$\therefore \quad t_{OABC} = 7 + 3.7 = 10.7\text{ s}$$

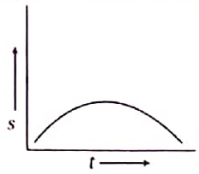
Also s_{OA} = area under v-t graph between OA

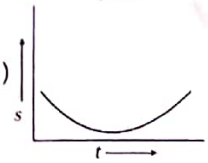
$$= \frac{1}{2}(5)(20) = 50\text{ m}$$

Check point 3.6

1. Which of the following graph represents the uniform motion?

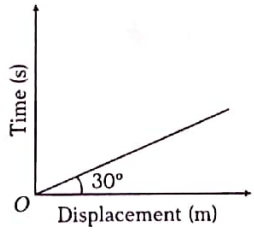
(a)

(b) 

(c) 

(d) None of these

2. From the displacement-time graph find out the velocity of a moving body.



(a) $\frac{1}{\sqrt{3}}$ ms^{-1} (b) 3 ms^{-1} (c) $\sqrt{3}$ ms^{-1} (d) $\frac{1}{3}$ ms^{-1}

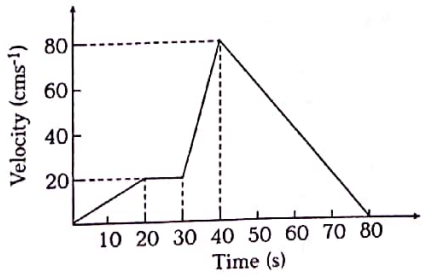
3. The distance-time graph of a particle at time t makes angle 45° with the time axis. After one second, it makes angle 60° with the time axis. What is the average acceleration of the particle?

(a) $\sqrt{3} - 1$ (b) $\sqrt{3} + 1$
(c) $\sqrt{3}$ (d) 1

4. The graph between displacement and time for a particle moving with uniform acceleration is a

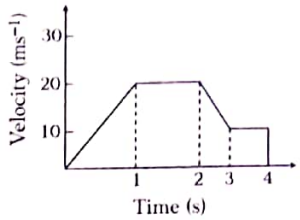
(a) straight line with a positive slope
(b) parabola
(c) ellipse
(d) straight line parallel to time axis

5. The v-t graph of a moving object is shown in the figure. The maximum acceleration is



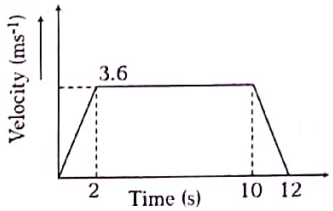
(a) 1 cms^{-2} (b) 2 cms^{-2} (c) 3 cms^{-2} (d) 6 cms^{-2}

6. The variation of velocity of a particle with time moving along a straight line is illustrated in the adjoining figure. The distance travelled by the particle in 4 s is



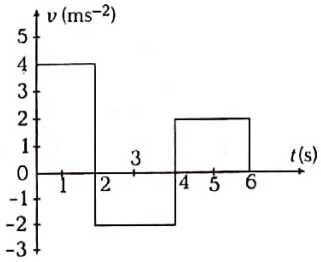
(a) 60 m (b) 55 m
(c) 25 m (d) 30 m

7. A lift is going up. The variation in the speed of the lift is as given in the graph. What is the height to which the lift takes the passengers?



(a) 3.6 m
(b) 28.8 m
(c) 36.0 m
(d) Cannot be calculated from the above graph

8. The velocity-time graph of a body moving in a straight line is shown in the figure. The displacement and distance travelled by the body in 6 s are respectively

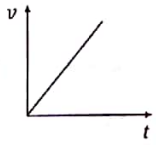


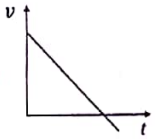
(a) 8 m, 16 m (b) 16 m, 32 m
(c) 16 m, 16 m (d) 8 m, 18 m

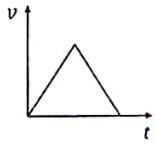
9. The x-t equation is given as $x = 2t + 1$. The corresponding v-t graph is

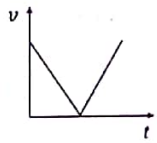
(a) a straight line passing through origin
(b) a straight line not passing through origin
(c) a parabola
(d) None of the above

10. Which of the following graphs correctly represents velocity-time relationship for a particle released from rest to fall freely under gravity?

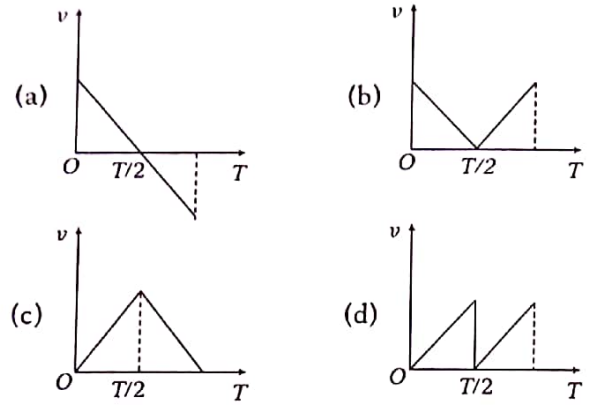
(a) 

(b) 

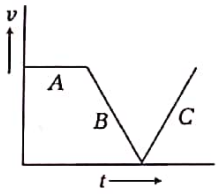
(c) 

(d) 

11. A particle projected vertically upwards returns to the ground in time T. Which graph represents the correct variation of velocity (v) against time (t)?

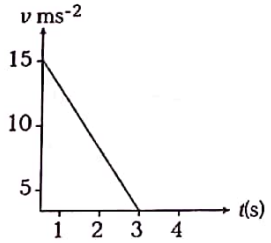


12. The velocity-time graph of a body is shown in the figure. It implies that at point B



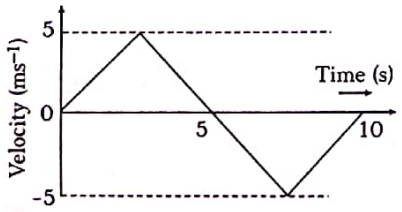
(a) the is zero
(b) there is a force towards motion
(c) there is a force which opposes motion
(d) there is only gravitational force

13. The velocity-time graph is shown in the figure, for a particle. The acceleration of particle is



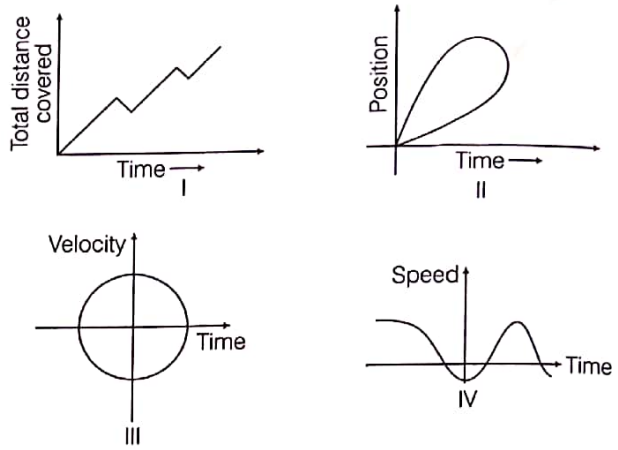
(a) $22.5\,ms^{-2}$ (b) $5\,ms^{-2}$
(c) $-5\,ms^{-2}$ (d) $-3\,ms^{-2}$

14. The v-t plot of a moving object is shown in the figure. The average velocity of the object during the first 10 s is



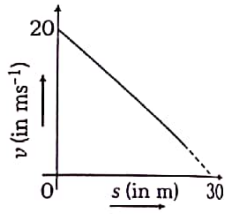
(a) zero (b) $2.5\,ms^{-1}$
(c) $5\,ms^{-1}$ (d) $2\,ms^{-1}$

15. Which of the following graphs cannot possibly represent one dimensional motion of a particle



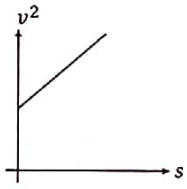
(a) I and II (b) II and III
(c) II and IV (d) All four

16. If the velocity v of a particle moving along a straight line decreases linearly with its displacement s from $20\,ms^{-1}$ to a value approaching zero at $s = 30$ m, then acceleration of the particle at $v = 10$ m is



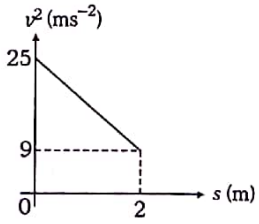
(a) $\frac{2}{3}\,ms^{-2}$ (b) $-\frac{2}{3}\,ms^{-2}$ (c) $\frac{20}{3}\,ms^{-2}$ (d) $-\frac{20}{3}\,ms^{-2}$

17. v^2 *versus* s-graph of a particle moving in a straight line is shown in the figure. From the graph some conclusions are drawn. State which statement is wrong?



(a) The given graph shows a uniformly accelerated motion
(b) Initial velocity of particle is zero
(c) Corresponding s-t graph will be a parabola
(d) None of the above

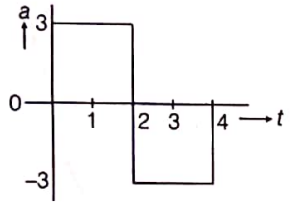
18. A graph between the square of the velocity of a particle and the distance s moved by the particle is shown in the figure. The acceleration of the particle is



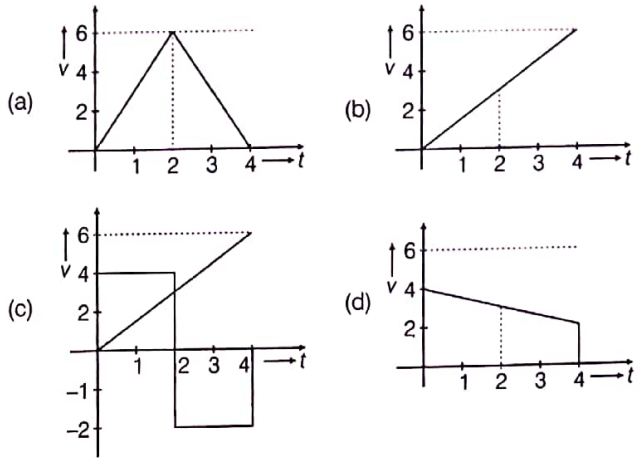
(a) $-8\,ms^{-2}$ (b) $-4\,ms^{-2}$
(c) $-16\,ms^{-2}$ (d) None of these

19. The area under acceleration time graph gives
(a) distance travelled (b) change in acceleration
(c) force acting (d) change in velocity

20. A particle starts from rest at $t = 0$ and undergoes an acceleration a in ms^{-2} with time t in seconds which is as shown.



Which one of the following plot represents velocity v in ms^{-1} *versus* time t in seconds



6 Relative velocity

The term relative is frequently used for comparison of displacement, velocity and acceleration of the two objects. The time rate of change of relative position of one object with respect to another is called **relative velocity.**

Let two objects A and B are moving along the + ve direction on X-axis. At time t, their displacement from the origin be x_A and x_B.

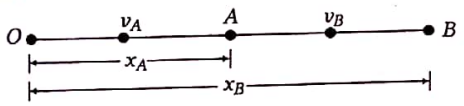


Fig. 3.12

$\therefore$ Their velocities are $v_A = \dfrac{dx_A}{dt}$ and $v_B = \dfrac{dx_B}{dt}$

The displacement of B relative to A.

$$x_{BA} = x_B - x_A$$

Rate of change of relative displacement of w.r.t. time is

$$\frac{d(\mathbf{x}_{BA})}{dt} = \frac{d}{dt}(\mathbf{x}_B - \mathbf{x}_A)$$

$$\Rightarrow \quad \frac{d\mathbf{x}_{BA}}{dt} = \frac{d\mathbf{x}_B}{dt} - \frac{d\mathbf{x}_A}{dt}$$

$$\therefore \quad \boxed{\mathbf{v}_{BA} = \mathbf{v}_B - \mathbf{v}_A}$$

Similarly, relative acceleration of A with respect to B is

$$\boxed{\mathbf{a}_{AB} = \mathbf{a}_A - \mathbf{b}_B}$$

If it is a one dimensional motion we can treat the vectors as scalars just by assigning the positive sign to one direction and negative to the other. So, in case of a one dimensional motion the above equations can be written as

$$v_{AB} = v_A - v_B$$

and $$a_{AB} = a_A - a_B$$

Further, we can see that

$$\mathbf{v}_{AB} = -\mathbf{v}_{BA}$$

or $$\mathbf{a}_{BA} = -\mathbf{a}_{AB}$$

Different cases of relative velocity

Case I When the two objects move with equal velocities, *i.e.* $v_A = v_B$ or $v_B - v_A = 0$. It means, the two objects stay at constant distance apart during the whole journey.

In this case, the position-time graph of two objects are parallel straight lines.

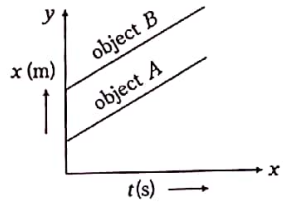


Fig. 3.13

Case II If both objects A and B move along parallel straight lines in the opposite direction, then relative velocity of B w.r.t. A is given as

$$\mathbf{v}_{BA} = \mathbf{v}_B - (-\mathbf{v}_A) = \mathbf{v}_B + \mathbf{v}_A$$

and the relative velocity of A w.r.t. B is given by

$$\mathbf{v}_{AB} = \mathbf{v}_A - (-\mathbf{v}_B) = \mathbf{v}_A + \mathbf{v}_B$$

Case III If $v_A < v_B$, $v_A - v_B$ is negative.

Then, $x - x_0 < 0 \Rightarrow x < x_0$

where, x_0 = initial displacement of object A w.r.t. B

and x = displacement of object A w.r.t. B at time t

i.e., $(x - x_0)$ is negative. It means the separation between the two objects will go on decreasing and two objects will meet and object B will overtake object A at this time. In this case, relative velocity of A w.r.t. B, *i.e.*,

$$v_{AB} = v_A - v_B$$
$$= -v_{BA}$$

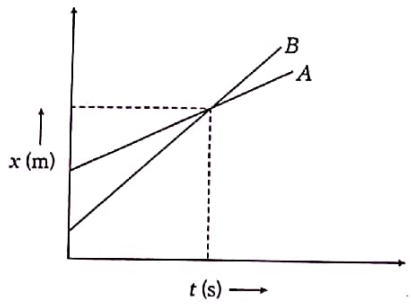


Fig. 3.14

Case IV If $v_A > v_B$, $v_A - v_B$ is positive.

Then, $x - x_0 > 0$ *i.e.*, $(x - x_0)$ is positive.

It means the separation between the two objects will go on increasing with time *i.e.*, the separation $(x - x_0)$ between them will increase by an amount $(v_A - v_B)$ after each unit of time. *Therefore, their position-time graphs will open out gradually as shown below.*

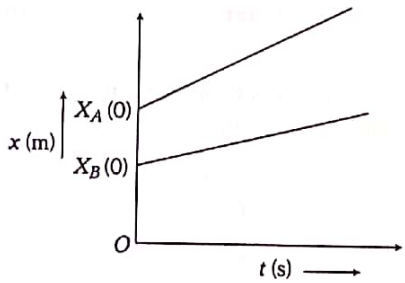


Fig. 3.15

Example 3.56 *Seeta is moving due east with a velocity of 1 ms^{-1} and Geeta is moving due west with a velocity of 2 ms^{-1}. What is the velocity of Seeta with respect to Geeta?*

Sol. It is a one dimensional motion. So, let us choose the east direction as positive and the west as negative.
Now given that

$$v_S = \text{velocity of Seeta} = 1 \text{ ms}^{-1}$$

and $$v_G = \text{velocity of Geeta} = -2 \text{ ms}^{-1}$$

Thus, $$v_{SG} = \text{velocity of Seeta with respect to Geeta}$$
$$= v_S - v_G = 1 - (-2) = 3 \text{ ms}^{-1}$$

Hence, velocity of Seeta with respect to Geeta is 3 m/s due east.

Example 3.57 *Two parallel rail tracks run North-South. Train A moves North with a speed of 54 kmh^{-1} and train B moves South with a speed of 90 kmh^{-1}. Find the relative velocity of B w.r.t. A.*

Sol. Let positive direction of motion be from South to North.

Given, $$\mathbf{v}_A = +54 \text{ kmh}^{-1} = 15 \text{ ms}^{-1},$$
$$\mathbf{v}_B = -90 \text{ kmh}^{-1} = -25 \text{ ms}^{-1}$$

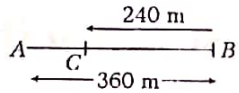
$\therefore$ The relative velocity of B w.r.t. A

$$\mathbf{v}_{BA} = \mathbf{v}_B - \mathbf{v}_A = -25 - 15 = -40 \text{ ms}^{-1}$$

i.e., the train B appears to A to move with a speed of 40 ms^{-1} from North to South.

Example 3.58 *A man A moves due to East with velocity 6 ms^{-1} and another man B moves in N-30°E with 6 ms^{-1}. Find the velocity of B w.r.t. A.*

Sol. Given, $$\mathbf{v}_A = 6\hat{\mathbf{i}}$$



$$\mathbf{v}_B = v_B \cos 60° \hat{\mathbf{i}} + v_B \sin 60° \hat{\mathbf{j}}$$
$$= 6\left(\frac{1}{2}\right)\hat{\mathbf{i}} + 6\left(\frac{\sqrt{3}}{2}\right)\hat{\mathbf{j}} = 3\hat{\mathbf{i}} + 3\sqrt{3}\hat{\mathbf{j}}$$

To find the velocity,

$$\mathbf{v}_{BA} = \mathbf{v}_B - \mathbf{v}_A = (3\hat{\mathbf{i}} + 3\sqrt{3}\hat{\mathbf{j}}) - 6\hat{\mathbf{i}} = -3\hat{\mathbf{i}} + 3\sqrt{3}\hat{\mathbf{j}}$$
$$|\mathbf{v}_{BA}| = \sqrt{(-3)^2 + (3\sqrt{3})^2}$$
$$= \sqrt{9 + 27} = \sqrt{36} = 6 \text{ ms}^{-1}$$

Here, $\hat{\mathbf{i}}$ is –ve and $\hat{\mathbf{j}}$ is +ve. So, second quadrant is possible.

Direction,

$$\tan \alpha = \frac{\text{coefficient of } \hat{\mathbf{j}}}{\text{coefficient of } \hat{\mathbf{i}}} = \frac{3\sqrt{3}}{-3} = -\sqrt{3} \Rightarrow \alpha = 60°$$

Example 3.59 *Buses A and B are moving in the same direction with speed 20 ms^{-1} and 15 ms^{-1} respectively. Find the relative velocity of A w.r.t. B and relative velocity of B w.r.t. A.*

Sol. Let their direction of motion of bus A is $\hat{\mathbf{i}}$, then

$$\mathbf{v}_A = (20 \text{ ms}^{-1})\hat{\mathbf{i}} \quad \text{and} \quad \mathbf{v}_B = (15 \text{ ms}^{-1})\hat{\mathbf{i}}$$

(i) Relative velocity of A w.r.t. B

$$\mathbf{v}_{AB} = \mathbf{v}_A - \mathbf{v}_B = (\text{actual velocity of } A) - (\text{velocity of } B)$$
$$= (20 \text{ ms}^{-1})\hat{\mathbf{i}} - (15 \text{ ms}^{-1})\hat{\mathbf{i}} = (5 \text{ ms}^{-1})\hat{\mathbf{i}}$$

(ii) Relative velocity of B w.r.t. A

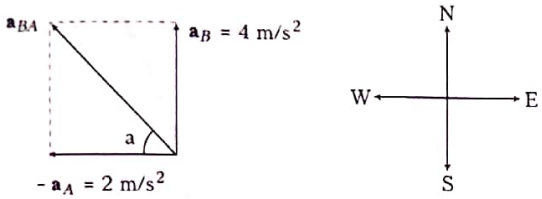
$$\mathbf{v}_{BA} = \mathbf{v}_B - \mathbf{v}_A = (\text{Actual velocity of } B) - (\text{velocity of } A)$$
$$= (15 \text{ ms}^{-1})\hat{\mathbf{i}} - (20 \text{ ms}^{-1})\hat{\mathbf{i}} = (-5 \text{ ms}^{-1})\hat{\mathbf{i}} = (5 \text{ ms}^{-1})(-\hat{\mathbf{i}})$$

Example 3.60 *Car A has an acceleration of 2 m/s^2 due east and car B, 4 m/s^2 due north. What is the acceleration of car B with respect to car A?*

Sol. It is a two dimensional motion. Therefore,

$\mathbf{a}_{BA}$ = acceleration of car B with respect to car A

$= \mathbf{a}_B - \mathbf{a}_A$



Here, $\mathbf{a}_B$ = acceleration of car, $B = 4$ m/s² (due north)

and $\mathbf{a}_A$ = acceleration of car $A = 2$ m/s² (due east)

$$|\mathbf{a}_{BA}| = \sqrt{(4)^2 + (2)^2}$$

$$= 2\sqrt{5} \text{ m/s}^2$$

and $$\alpha = \tan^{-1}\left(\frac{4}{2}\right) = \tan^{-1}(2)$$

Thus, $\mathbf{a}_{BA}$ is $2\sqrt{5}$ m/s² at an angle of $\alpha = \tan^{-1}(2)$ from west towards north.

Example 3.61 *A police van moving on a highway with a speed of 30 kmh^{-1} fires a bullet at a thief car which is speeding away in the same direction with a speed of 190 kmh^{-1}. If the muzzle speed of the bullet is 150 ms^{-1}, find speed of the bullet with respect to the thief's car.*

Sol. Let, v_b is velocity of bullet v_p is velocity of police van and v_t is velocity of thief's car.

Then, speed of the bullet with respect to the thief's car

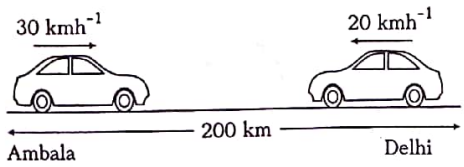
$$v_{bp} = v_b - v_p$$

$$v_b = v_{bp} + v_p = 150 \times \frac{18}{5} \text{kmh}^{-1} + 30 \text{kmh}^{-1} = 570 \text{kmh}^{-1}$$

$$v_{bt} = v_b - v_t = 570 \text{kmh}^{-1} - 190 \text{kmh}^{-1} = 380 \text{kmh}^{-1}$$

Example 3.62 *Delhi is at a distance of 200 km from Ambala. Car A set out from Ambala at a speed of 30 kmh^{-1} and car B set out at the same time from Delhi at a speed of 20 kmh^{-1}. When they will meet each other? What is the distance of that meeting point from Ambala?*

Sol. Relative velocity, $\mathbf{v}_{AB} = \mathbf{v}_A - \mathbf{v}_B = 30 - (-20) = 50 \text{kmh}^{-1}$



They will meet after time t

$$t = \frac{s}{v_{AB}} = \frac{200}{50} = 4 \text{ h}$$

Distance from Ambala where they will meet

$$x = 30 \times 4 = 120 \text{ km}$$

Example 3.63 *Two car travelling towards each other on a straight road at velocity 10 ms^{-1} and 12 ms^{-1} respectively. When they are 150 m apart, both the drivers apply their brakes and each car decelerates at 2 ms^{-2} until it stops. How far apart will they be when both of them come to a halt?*

Sol. Let x_1 and x_2 be the distance travelled by the car before they stop under deceleration. From IIIrd equation of motion

$$v^2 = u^2 + 2as$$

$\Rightarrow$ $$0 = (10)^2 - 2 \times 2x_1 \Rightarrow x_1 = 25 \text{ m}$$

and $$0 = (12)^2 - 2 \times 2x_2 \Rightarrow x_2 = 36 \text{ m}$$

Total distance covered by the two cars

$$= x_1 + x_2 = 25 + 36 = 61 \text{ m}$$

Distance between the two cars when they stop

$$= 150 - 61 = 89 \text{ m}$$

Examples of relative motion

1. Relative velocity of rain

Consider a man walking East with velocity $\mathbf{v}_m$ represented by OA. Let the rain be falling vertically downwards with velocity $\mathbf{v}_r$, represented by OB. To find the relative velocity of rain w.r.t. man (*i.e.*, $\mathbf{v}_{rm}$) being the man at rest by imposing a velocity $-\mathbf{v}_m$ on man and apply this velocity on rain also.

Now the relative velocity of rain w.r.t. man will be the resultant velocity of $\mathbf{v}_r$ (= OB) and $-\mathbf{v}_m$ (= OA), which will be represented by diagonal OC of rectangle $OACB$

$\therefore$ $$\mathbf{v}_{rm} = \sqrt{v_r^2 + v_m^2 + 2v_r v_m \cos 90°}$$

$$= \sqrt{v_r^2 + v_m^2}$$

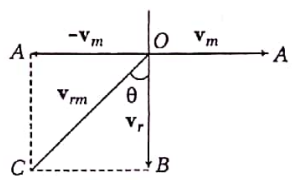


Fig. 3.16 Relative velocity of rain

If θ is the angle which $\mathbf{v}_{rm}$ makes with the vertical direction, then

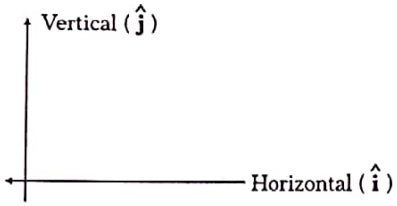
$$\tan\theta = \frac{BC}{OB} = \frac{\mathbf{v}_m}{\mathbf{v}_r} \quad \text{or} \quad \theta = \tan^{-1}\left(\frac{\mathbf{v}_m}{\mathbf{v}_r}\right)$$

Here, angle θ is from vertical towards West and is written as θ, West of vertical.

Note In the above problem if the man wants to protect himself from the rain, he should hold his umbrella in the direction of relative velocity of rain w.r.t. man i.e., the umbrella should be held making an angle θ from West or vertical.

Example 3.64 *To a man walking at the rate of 3 km/h the rain appears to fall vertically. When he increases his speed to 6 km/h it appears to meet him at an angle of 45° with vertical. Find the speed of rain.*

Sol. Let $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ be the unit vectors in horizontal and vertical directions respectively.



Let velocity of rain

$$\mathbf{v}_r = a\hat{\mathbf{i}} + b\hat{\mathbf{j}} \quad \text{...(i)}$$

Then speed of rain will be

$$|\mathbf{v}_r| = \sqrt{a^2 + b^2} \quad \text{...(ii)}$$

In the first case $\mathbf{v}_m$ = velocity of man $= 3\hat{\mathbf{i}}$

$\therefore \quad \mathbf{v}_{rm} = \mathbf{v}_r - \mathbf{v} = (a-3)\hat{\mathbf{i}} + b\hat{\mathbf{j}}$

It seems to be in vertical direction. Hence,

$$a - 3 = 0 \quad \text{or} \quad a = 3$$

In the second case $\quad \mathbf{v}_{rm} = 6\hat{\mathbf{i}}$

$\therefore \quad \mathbf{v}_{rm} = (a-6)\hat{\mathbf{i}} + b\hat{\mathbf{j}} = -3\hat{\mathbf{i}} + b\hat{\mathbf{j}}$

This seems to be at 45° with vertical. Hence, $|b| = 3$

Therefore, from Eq. (ii) speed of rain is

$$|\mathbf{v}_r| = \sqrt{(3)^2 + (3)^2} = 3\sqrt{2} \text{ km/h}$$

2. Crossing the river

To cross the river over shortest distance, *i.e.*, to cross the river straight, the man should swim upstream making an angle θ with OB such that, OB gives the direction of resultant velocity ($\mathbf{v}_{mR}$) of velocity of swimmer and velocity of river water as shown in figure. Let us consider

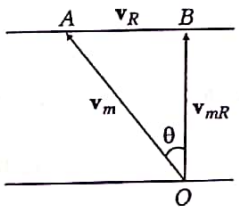


Fig. 3.17 Crossing the river

$AB = \mathbf{v}_R$ (velocity of river water)
$OA = \mathbf{v}_m$ (velocity of man in still river water)
$OB = \mathbf{v}_{mR}$ (relative velocity of man w.r.t. river)

$$\mathbf{v}_{mR} = \sqrt{\mathbf{v}_m^2 - \mathbf{v}_R^2}$$

In ΔOAB, $\quad \sin\theta = \dfrac{\mathbf{v}_R}{\mathbf{v}_m}$

where, θ is the angle made by man with shortest distance OB,

$$\tan\theta = \frac{\mathbf{v}_R}{\mathbf{v}_{mR}} = \frac{\mathbf{v}_R}{\sqrt{\mathbf{v}_m^2 - \mathbf{v}_R^2}}$$

(a) **Time taken to cross the river** If d be the width of the river, then time taken cross to the river given by

$$\boxed{t_1 = \frac{d}{\mathbf{v}_{mR}} = \frac{d}{\sqrt{\mathbf{v}_m^2 - \mathbf{v}_R^2}}}$$

(b) **To cross the river in possible shortest time** The man should go along OA. Now the swimmer will be going along OB, which is the direction of resultant velocity of $\mathbf{v}_m$ and $\mathbf{v}_R$.

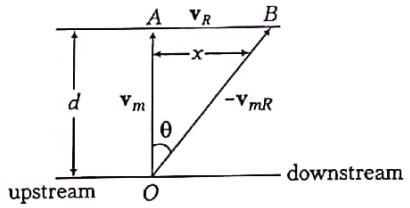


Fig. 3.18 Cross the river in possible shortest time

In ΔOAB $\quad \tan\theta = \dfrac{AB}{OA} = \dfrac{\mathbf{v}_R}{\mathbf{v}_m}$ and $\mathbf{v}_{mR} = \sqrt{\mathbf{v}_m^2 + \mathbf{v}_R^2}$

Time of crossing the river,

$$t = \frac{d}{\mathbf{v}_m} = \frac{OB}{\mathbf{v}_{mR}} \Rightarrow t = \frac{\sqrt{x^2 + d^2}}{\sqrt{\mathbf{v}_m^2 + \mathbf{v}_R^2}}$$

The boat will be reading the point B instead of point A. If $AB = x$, then $\tan\theta = \dfrac{\mathbf{v}_R}{\mathbf{v}_m} = \dfrac{x}{d} \Rightarrow x = \dfrac{d\mathbf{v}_R}{\mathbf{v}_m}$

In this case, the man will reach the opposite bank at a distance AB downstream.

Drift

It is defined as the displacement of man in the direction of river flow as shown

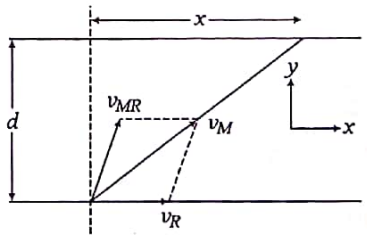


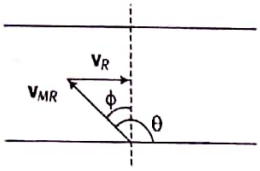
Fig. 3.19

It is simply the displacement along x-axis. During the period the man crosses the river. ($v_{MR}\cos\theta + v_R$) is the components of velocity of man in the direction of river flow and this component of velocity is responsible for drift along the river flow. If drift is x then

$$x = (v_{MR}\cos\theta + v_R) \times \frac{d}{v_{MR}\sin\theta}$$

Note If $v_R > v_{MR}$ then it is not possible to have zero drift. In this case, the minimum drift corresponding to shortest possible path is non-zero and the condition for minimum drift can be proved to be

$\cos\theta = -\frac{v_{MR}}{v_R}$ or $\sin\phi = \frac{v_{MR}}{v_R}$ for minimum but non-zero drift.



Example 3.65 *A man crosses a river in a boat. If he cross the river in minimum time he takes 10 min with a drift 120 m. If he crosses the river taking shortest path, he takes 12.5 min, find*

(a) width of the river.

(b) velocity of the boat with respect to water.

(c) speed of the current.

Sol. Let v_r = velocity of river

v_{br} = velocity of river in still water and

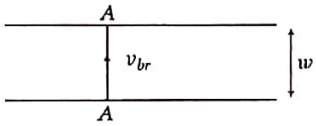
w = width of river

Given, $t_{min} = 10$ min or $\frac{w}{v_{br}} = 10$ ...(i)

Drift in this case will be,

$x = v_r t$

$\therefore \quad 120 = 10 v_r$ ...(ii)



For minimum time

Shortest path is taken when v_b is along AB. In this case,

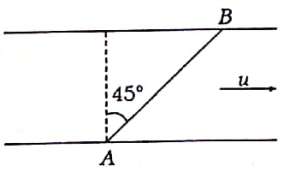
$$v_b = \sqrt{v_{br}^2 - v_r^2}$$

Now, $$12.5 = \frac{w}{v_b} = \frac{w}{\sqrt{v_{br}^2 - v_r^2}} \quad \text{...(iii)}$$

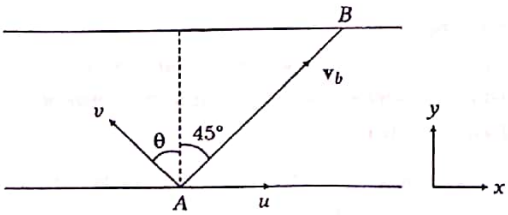
Solving these three equations, we get

$v_{br} = 20$ m/min, $v_r = 12$ m/min and $w = 200$ m.

Example 3.66 *A man wants to reach point B on the opposite bank of a river flowing at a speed as shown in figure. What minimum speed relative to water should the man have so that he can reach point B ? In which direction should he swim ?*



Sol. Let v be the speed of boatman in still water.



Resultant of v and u should be along AB. Components of $\mathbf{v}_b$ (absolute velocity of boatman) along x and y directions are,

$$v_x = u - v\sin\theta \quad \text{and} \quad v_y = v\cos\theta$$

Further, $$\tan 45° = \frac{v_y}{v_x}$$

or $$1 = \frac{v\cos\theta}{u - v\sin\theta}$$

$\therefore$ $$v = \frac{u}{\sin\theta + \cos\theta} = \frac{u}{\sqrt{2}\sin(\theta + 45°)}$$

v is minimum at,

$$\theta + 45° = 90° \quad \text{or} \quad \theta = 45° \quad \text{and} \quad v_{min} = \frac{u}{\sqrt{2}}$$

Example 3.67 *A man can row a boat with 4 km/h in still water. If he is crossing a river where the current is 2 km/h.*

(a) In what direction will his boat be headed if he wants to reach a point on the other bank, directly opposite to starting point?

(b) If width of the river is 4 km, how long will the man take to cross the river, with the condition in part (a)?

(c) In what direction should he head the boat if he wants to cross the river in shortest time and what is this minimum time?

(d) How long will it take him to row 2 km up the stream and then back to his starting point?

Sol. (a) Given, that $v_{br} = 4$ km/h and $v_r = 2$ km/h

$$\therefore \quad \theta = \sin^{-1}\left(\frac{v_r}{v_{br}}\right) = \sin^{-1}\left(\frac{2}{4}\right) = \sin^{-1}\left(\frac{1}{2}\right) = 30°$$

Hence, to reach the point directly opposite to starting point he should head the boat at an angle of 30° with AB or $90° + 30° = 120°$ with the river flow.

(b) Time taken by the boatman to cross the river

w = width of river = 4 km

$v_{br} = 4$ km/h

and $\theta = 30°$

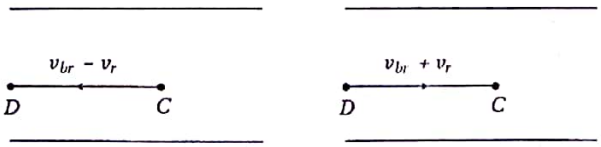
$$\therefore \quad t = \frac{4}{4\cos 30°} = \frac{2}{\sqrt{3}} \text{ h.}$$

(c) For shortest time $\theta = 0°$

and $$t_{min} = \frac{w}{v_{br}\cos 0°} = \frac{4}{4} = 1 \text{ h}$$

Hence, he should head his boat perpendicular to the river current for crossing the river in shortest time and this shortest time is 1 h.

(d) $t = t_{CD} + t_{DC}$

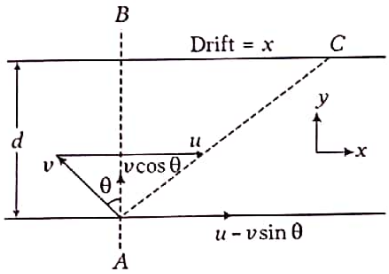


or $$t = \frac{CD}{v_{br} - v_r} + \frac{DC}{v_{br} + v_r} = \frac{2}{4-2} + \frac{2}{4+2} = 1 + \frac{1}{3} = \frac{4}{3} \text{ h}$$

Example 3.68 *A boat moves relative to water with a velocity v is n times less than the river flow u. At what angle to the stream direction must the boat move to minimize drifting?*

Sol. In this problem, one thing should be carefully noted that the velocity of boat is less than the river flow velocity. Hence boat cannot reach the point directly opposite to its starting point *i.e.*, drift can never be zero.

Suppose boat starts at an angle θ from the normal direction up stream as shown.



Component of velocity of boat along the river,

$$v_x = u - v\sin\theta$$

and velocity perpendicular to the river,

$$v_y = v\cos\theta,$$

time taken to cross the river is

$$t = \frac{d}{v_y} = \frac{d}{v\cos\theta}$$

$$\text{Drift } x = (v_x)\,t = (u - v\sin\theta)\frac{d}{v\cos\theta}$$

$$= \frac{ud}{v}\sec\theta - d\tan\theta$$

The drift x is minimum, when $\frac{dx}{d\theta} = 0$

or $$\left(\frac{ud}{v}\right)(\sec\theta.\tan\theta) - d\sec^2\theta = 0$$

$$\frac{u}{v}\sin\theta = 1 \Rightarrow \sin\theta = \frac{v}{u}$$

So, for minimum drift, the boat must move at an angle

$$\theta = \sin^{-1}\left(\frac{v}{u}\right) = \sin^{-1}\left(\frac{1}{n}\right)$$

from normal direction.

3. Minimum distance between two bodies in motion

When two bodies are in motion, the questions like, the minimum distance between them or the time when one body overtakes the other can be solved easily by the principle of relative motion. In these type of problems one body is assumed to be at rest and the relative motion of the other body is considered.

By assuming so two body problem is converted into one body problem and the solution becomes easy.

Following example will illustrate the statement.

Example 3.69 *Car A and car B start moving simultaneously in the same direction along the line joining them. Car A with a constant acceleration $a = 4\ ms^{-2}$, while car B moves with a constant velocity $v = 1\ ms^{-1}$. At time $t = 0$, car A is 10 m behind car B. Find the time when car A overtakes car B.*

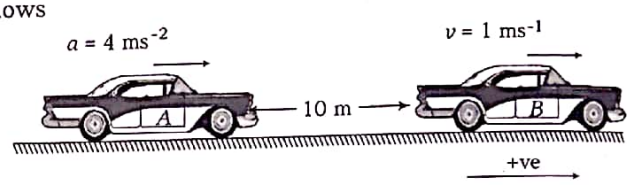
Sol. Given, $u_A = 0$, $u_B = 1\ ms^{-1}$, $a_A = 4\ ms^{-2}$ and $a_B = 0$

Assuming car B to be at rest, we have

$$u_{AB} = u_A - u_B = 0 - 1 = -1\ ms^{-1}$$

$$a_{AB} = a_A - a_B = 4 - 0 = 4\ ms^{-2}$$

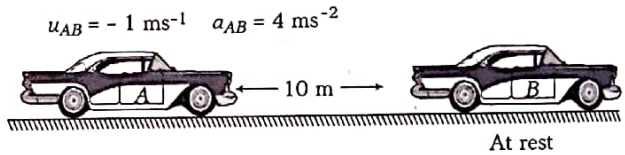
Now, the problem can be assumed in simplified form as follows



Substituting the proper values in equation

$$s = ut + \frac{1}{2}at^2$$

we get, $$10 = -t + \frac{1}{2}(4)(t^2) \text{ or } 2t^2 - t - 10 = 0$$



or $$t = \frac{1 \pm \sqrt{1+80}}{4} = \frac{1 \pm \sqrt{81}}{4} = \frac{1 \pm 9}{4}$$

or $$t = 2.5\text{ s and } -2\text{ s}$$

Ignoring the negative value, the desired time is 2.5 s.

Note The above problem can also be solved without using the concept of relative motion as under.

At the time when A overtakes B, $s_A = s_B + 10$

$$\therefore \quad \frac{1}{2} \times 4 \times t^2 = 1 \times t + 10 \quad \text{or} \quad 2t^2 - t - 10 = 0$$

Which on solving gives $t = 2.5$ s and -2 s, the same as we found above. As per my opinion, this approach (by taking absolute values) is more suitable in case of two body problem in one dimensional motion. Let us see one more example in support of it.

Example 3.70 *An open lift is moving upwards with velocity $10\,ms^{-1}$ It has an upward acceleration of $2\ ms^{-2}$. A ball is projected upwards with velocity $20\ ms^{-1}$ relative to ground. Find*

(i) time when ball again meets the lift.

(ii) displacement of lift and ball at that instant.

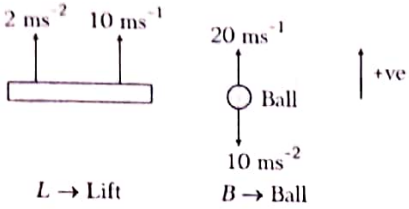
(iii) distance travelled by the ball upto that instant.

(Take, $g = 10\ ms^{-2}$)

Sol. (i) At the time when ball again meets the lift, $s_L = s_B$

$$\therefore \quad 10t + \frac{1}{2} \times 2 \times t^2 = 20t - \frac{1}{2} \times 10t^2$$

Solving this equation, we get



$$t = 0 \quad \text{and} \quad t = \frac{5}{3}\text{ s}$$

$\therefore$ Ball will again meet the lift after $\frac{5}{3}$ s.

(ii) At this instant $s_L = s_B = 10 \times \frac{5}{3} + \frac{1}{2} \times 2 \times \left(\frac{5}{3}\right)^2$

$$= \frac{175}{9}\text{ m} = 19.4\text{ m}$$

(iii) For the ball $u \uparrow \downarrow a$. Therefore we will first find t_0, the time when its velocity becomes zero.

$$t_0 = \left|\frac{u}{a}\right| = \frac{20}{10} = 2\text{ s}$$

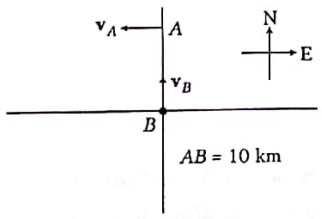
As $t\left(= \frac{5}{3}\text{ s}\right) < t_0$, distance and displacement are equal

or $\qquad d = 19.4$ m

Concept of relative motion is more useful in two body problem in two (or three) dimensional motion. This can be understood by the following example.

Example 3.71 *Two ships A and B are 10 km apart on a line running South to North. Ship A farther North is streaming West at 20 kmh^{-1} and ship B is streaming North at 20 kmh^{-1}. What is their distance of closest approach and how long do they take to reach it?*

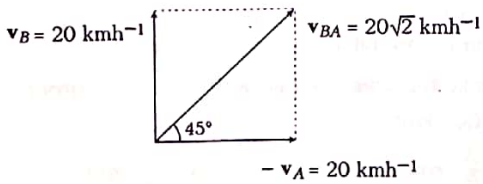
Sol. Ships A and B are moving with same speed 20 kmh^{-1} in the directions shown in figure. It is a two dimensional, two body problem with zero acceleration. Let us find $\mathbf{v}_{BA}$.



$$\mathbf{v}_{BA} = \mathbf{v}_B - \mathbf{v}_A$$

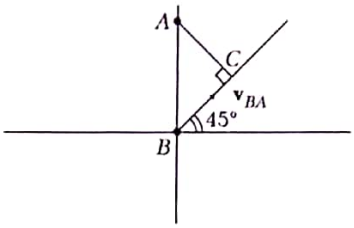
Here, $\qquad |\mathbf{v}_{BA}| = \sqrt{(20)^2 + (20)^2} = 20\sqrt{2}$ kmh^{-1}

i.e., v_{BA} is $20\sqrt{2}$ kmh^{-1} at an angle of 45° from East towards North. Thus, the given problem can be simplified as



A is at rest and B is moving with $\mathbf{v}_{BA}$ in the direction shown in figure. Therefore, the minimum distance between the two is

$$s_{\min} = AC = AB \sin 45° = 10\left(\frac{1}{\sqrt{2}}\right)\text{km} = 5\sqrt{2}\text{ km}$$



and the desired time is

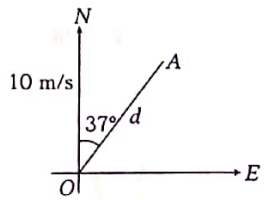
$$t = \frac{BC}{|\mathbf{v}_{BA}|} = \frac{5\sqrt{2}}{20\sqrt{2}} \quad (BC = AC = 5\sqrt{2}\text{ km})$$

$$= \frac{1}{4}\text{ h} = 15\text{ min}$$

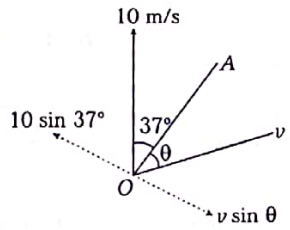
4. Aircraft wind problems

This is similar to river boat problem. The only difference is that $\mathbf{v}_{mR}$ is replaced by $\mathbf{v}_{aw}$ (velocity of aircraft with respect to wind or velocity of aircraft in still air), $\mathbf{v}_R$ is replaced by $\mathbf{v}_w$ (velocity of wind) and $\mathbf{v}_m$ is replaced by $\mathbf{v}_a$ (absolute velocity of aircraft). Further, $\mathbf{v}_a = \mathbf{v}_{aw} + \mathbf{v}_w$. The following example will illustrate the theory.

Example 3.72 *An aeroplane has to go from a point O to another point A, at distance d due 37° East of North. A wind is blowing due North at a speed of 10 ms^{-1}. The air speed of the plane is v. (i) Find the direction in which the pilot should head the plane to reach the point A. (ii) Find the time taken by the plane to go from O to A.*



Sol. (i) If a particle moves in a straight line, the velocity of particle perpendicular to straight line should be zero. If an aeroplane moves along OA, the pilot should head the plane towards right of line OA.



Perpendicular to line OA

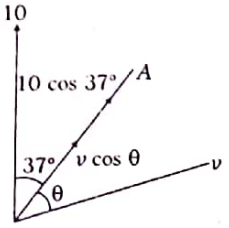
$v \sin\theta = 10 \sin 37° = 6$

$$\sin\theta = \frac{6}{v} \Rightarrow \theta = \sin^{-1}\left(\frac{6}{v}\right)$$

The pilot should head the plane at angle

$$\sin^{-1}\left(\frac{6}{v}\right) \text{ East of line } OA$$

(ii) Along line OA



Time taken by the plane to move from O to A

$$t = \frac{d}{10\cos 37° + v\cos\theta} = \frac{d}{8 + v\cos\theta}$$

Example 3.73 *An aircraft flies at 400 km/h in still air. A wind of $200\sqrt{2}$ km/h is blowing from the south. The pilot wishes to travel from A to a point B north east of A. Find the direction he must steer and time of his journey if $AB = 1000$ km.*

Sol. Given that $v_w = 200\sqrt{2}$ km/h.

$v_{aw} = 400$ km/h and $\mathbf{v}_a$ should be along AB or in north-east direction. Thus, the direction of $\mathbf{v}_{aw}$ should be such as the resultant of $\mathbf{v}_w$ and $\mathbf{v}_{aw}$ is along AB or in north-east direction.

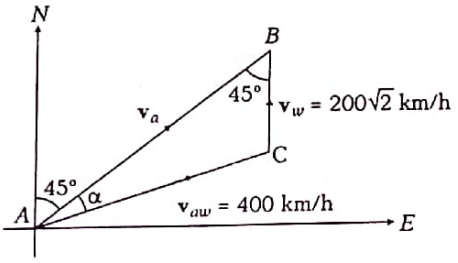


Fig. 3.18

Let $\mathbf{v}_{aw}$ makes an angle α with AB as shown in Fig. 3.32. Applying sine law in triangle ABC, we get

$$\frac{AC}{\sin 45°} = \frac{BC}{\sin\alpha}$$

or
$$\sin\alpha = \left(\frac{BC}{AC}\right)\sin 45°$$
$$= \left(\frac{200\sqrt{2}}{400}\right)\frac{1}{\sqrt{2}}$$
$$= \frac{1}{2}$$

$\therefore$
$$\alpha = 30°$$

Therefore, the pilot should steer in a direction at an angle of $(45° + \alpha)$ or 75° from north towards east.

Further,
$$\frac{|\mathbf{v}_a|}{\sin(180° - 45° - 30°)}$$
$$= \frac{400}{\sin 45°}$$

or
$$|\mathbf{v}_a| = \frac{\sin 105°}{\sin 45°} \times (400) \text{ km/h}$$
$$= \left(\frac{\cos 15°}{\sin 45°}\right)(400) \text{ km/h}$$
$$= \left(\frac{0.9659}{0.707}\right)(400) \text{ km/h}$$
$$= 546.47 \text{ km/h}$$

$\therefore$ The time of journey from A to B is

$$t = \frac{AB}{|\mathbf{v}_a|} = \frac{1000}{546.47} \text{ h}$$
$$t = 1.83 \text{ h}$$

Check point 3.7

1. A train is moving towards East and a car is along North, both with same speed. The observed direction of car to the passenger in the train is
(a) East-North direction (b) West-North direction
(c) South-East direction (d) None of these

2. A 100 m long train crosses a man travelling at 5 kmh^{-1}, in opposite direction , in 7.2 s then the velocity of train is
(a) 40 ms^{-1} (b) 25 ms^{-1} (c) 20 ms^{-1} (d) 45 ms^{-1}

3. Two bodies are held separated by 9.8 m vertically one above the other. They are released simultaneously to fall freely under gravity. After 2 s the relative distance between them is .
(a) 4.9 m (b) 19.6 m (c) 9.8 m (d) 39.2 m

4. A particle (A) moves due North at 3 kmh^{-1} and another particle (B) moves due West at 4 kmh^{-1}. The relative velocity of A with respect to B is (tan 37° = 3/ 4)
(a) 5 kmh^{-1}, 37° North of East (b) 5 kmh^{-1}, 37° East of North
(c) $5\sqrt{2}$ kmh^{-1}, 53° East of North (d) $5\sqrt{2}$ kmh^{-1}, 53° North of East

5. A man standing on a road has to hold his umbrella at 30° with the vertical to keep the rain away. He throws the umbrella and starts running at 10 kmh^{-1}. He finds that raindrops are hitting his head vertically.
What is the speed of rain with respect to ground?
(a) $10\sqrt{3}$ kmh^{-1} (b) 20 kmh^{-1}
(c) $\frac{20}{\sqrt{3}}$ kmh^{-1} (d) $\frac{10}{\sqrt{3}}$ kmh^{-1}

6. A stationary man observes that the rain is falling vertically downward. When he starts running with a velocity of $12\,kmh^{-1}$, he observes that the rain is falling at an angle 60° with the vertical. The actual velocity of rain is

(a) $12\sqrt{3}\,kmh^{-1}$ (b) $6\sqrt{3}\,kmh^{-1}$

(c) $4\sqrt{3}\,kmh^{-1}$ (d) $2\sqrt{3}\,kmh^{-1}$

7. A boy is running on the plane road with velocity v with a long hollow tube in his hand. The water is falling vertically downwards with velocity u. At what angle to the vertical, he must inclined the tube so that the water drops enter it without touching its sides?

(a) $\tan^{-1}\left(\frac{v}{u}\right)$ (b) $\sin^{-1}\left(\frac{v}{u}\right)$

(c) $\tan^{-1}\left(\frac{u}{v}\right)$ (d) $\cos^{-1}\left(\frac{v}{u}\right)$

8. The speed of boat is $5\,kmh^{-1}$ in still water. It crosses a river of width 1 km along the shortest possible path in 15 min. Then, velocity of river will be

(a) $4.5\,kmh^{-1}$ (b) $4\,kmh^{-1}$

(c) $1.5\,kmh^{-1}$ (d) $3\,kmh^{-1}$

9. A ship X moving due North with speed v observes that another ship Y is moving due West with same speed v. The actual velocity of Y is

(a) $\sqrt{2}v$ towards South-West

(b) $\sqrt{2}v$ towards North-West

(c) $\sqrt{2}v$ towards South-East

(d) v towards North-East

10. A river is flowing from West to East at a speed of 8 m per min. A man on the South bank of the river, capable of swimming at 20 m/min in still water, wants to swim across the river in the shortest time. He should swim in a direction.

(a) due North (b) 30° East of North

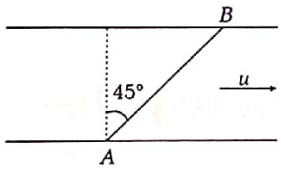
(c) 30° West of North (d) 60° East of North

11. The rowing speed of a man relative to water is $5\,kmh^{-1}$ and the speed of water flow is $3\,kmh^{-1}$. At what angle to the river flow should he head if he wants to reach a point on the other bank, directly opposite to starting point?

(a) 127° (b) 143°

(c) 120° (d) 150°

12. A man wants to reach point B on the opposite bank of a river flowing at a speed as shown in figure. With what minimum speed and in which direction should the man swim relative to water so that he can reach point B?



(a) u, 45° North-West (b) u, 45° North-East

(c) $\frac{u}{\sqrt{2}}$, 45° North-West (d) $\frac{u}{\sqrt{2}}$, 45° North-East

13. Two trains are each 50 m long moving parallel towards each other at speeds $10\,ms^{-1}$ and $15\,ms^{-1}$ respectively, at what time will they pass each other?

(a) 8 s (b) 4 s

(c) 2 s (d) 6 s

14. A man is 25 m behind a bus, when bus starts accelerating at $2\,ms^{-2}$ and man starts moving with constant velocity of $10\,ms^{-1}$. Time taken by him to board the bus is

(a) 2 s (b) 3 s (c) 4 s (d) 5 s

15. A ball is dropped from the top of a building 100 m high. At the same instant another ball is thrown upwards with a velocity of $40\,ms^{-1}$ from the bottom of the building. The two balls will meet after

(a) 5 s (b) 2.5 s

(c) 2 s (d) 3 s

Chapter exercises

A Taking it together

(Assorted questions of the chapter for advanced level practice)

1. A boy walks to his school at a distance of 6 km with constant speed of 2.5 kmh^{-1} and walks back with a constant speed of 4 kmh^{-1}. His average speed for round trip expressed in kmh^{-1}, is
(a) 24/13 (b) 40/13 (c) 3 (d) 1/2

2. A man walks on a straight road from his home to a market 2.5 km away with a speed of 5 kmh^{-1}. Finding the market closed, he instantly turns and walks back home with a speed of 7.5 kmh^{-1}. The average speed of the man over the interval of time 0 to 50 min is equal to
(a) 5 kmh^{-1} (b) $\frac{25}{4}$ kmh^{-1}
(c) $\frac{30}{4}$ kmh^{-1} (d) $\frac{45}{8}$ kmh^{-1}

3. The distance travelled by a particle starting from rest and moving with an acceleration $\frac{4}{3}$ ms^{-2}, in the third second is
(a) $\frac{10}{3}$ m (b) $\frac{19}{3}$ m
(c) 6 m (d) 4 m

4. A particle moves in a straight line with a constant acceleration. It changes its velocity from 10 ms^{-1} to 20 ms^{-1} while passing through a distance 135 m in t second. The value of t (in second) is
(a) 12 (b) 9
(c) 10 (d) 1.8

5. A particle moves along X-axis as
$$x = 4(t-2) + a(t-2)^2$$
Which of the following is true ?
(a) The initial velocity of particle is 4
(b) The acceleration of particle is $2a$
(c) The particle is at origin at $t = 0$
(d) None of the above

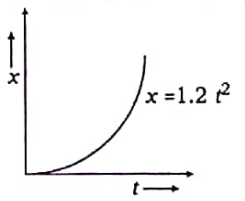
6. A car moving with a velocity of 10 ms^{-1} can be stopped by the application of a constant force F in a distance of 20 m. If the velocity of the car is 30 ms^{-1}, it can be stopped by this force in
(a) $\frac{20}{3}$ m (b) 20 m (c) 60 m (d) 180 m

7. A vehicle travels half the distance l with speed v_1 and the other half with speed v_2, then its average speed is **[NCERT Exemplar]**
(a) $\frac{v_1 + v_2}{2}$ (b) $\frac{2v_1 + v_2}{v_1 + v_2}$ (c) $\frac{2v_1v_2}{v_1 + v_2}$ (d) $\frac{L(v_1 + v_2)}{v_1v_2}$

8. The x and y-coordinates of a particle at any time t are given by $x = 7t + 4t^2$ and $y = 5t$, where x and y are in metre and t in seconds. The acceleration of particle at $t = 5$ s is
(a) zero (b) 8 ms^{-2} (c) 20 ms^{-2} (d) 40 ms^{-2}

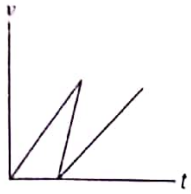
9. A body A starts from rest with an acceleration a_1. After 2 seconds, another body B starts from rest with an acceleration a_2. If they travel equal distance in the 5th second, after the start of A, then the ratio $a_1 : a_2$ is equal to
(a) 5 : 9 (b) 5 : 7
(c) 9 : 5 (d) 9 : 7

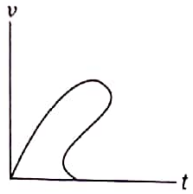
10. Figure given shows the distance-time graph of the motion of a car. It follows from the graph that the car is

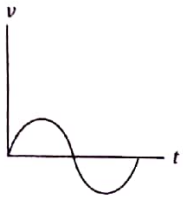


(a) at rest
(b) in uniform motion
(c) in non-uniform acceleration
(d) uniformly accelerated

11. Which of the following speed-time (v-t) graphs is physically not possible?

(a) 

(b) 

(c) 

(d) All of these

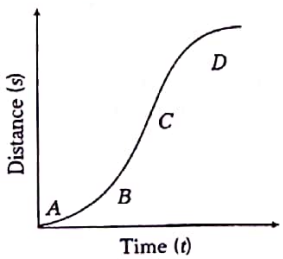
12. The displacement (x)-time (t) graph of a particle is shown in figure. Which of the following is correct?

(a) Particle starts with zero velocity and variable acceleration
(b) Particle starts with non-zero velocity and variable acceleration
(c) Particle starts with zero velocity and uniform acceleration
(d) Particle starts with non-zero velocity and uniform acceleration

13. The velocity of a body depends on time according to the equation $v = \frac{t^2}{10} + 20$. The body is undergoing

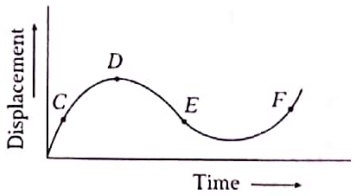
(a) Uniform acceleration
(b) Uniform retardation
(c) Non-uniform acceleration
(d) Zero acceleration

14. A particle shows distance-time curve as given in this figure. The maximum instantaneous velocity of the particle is around the point



(a) A (b) B (c) C (d) D

15. The displacement-time graph of a moving particle is shown below. The instantaneous velocity of the particle is negative at the point.



(a) E (b) F
(c) C (d) D

16. The velocity v of a particle as a function of its position (x) is expressed as $v = \sqrt{c_1 - c_2 x}$, where c_1 and c_2 are positive constants. The acceleration of the particle is

(a) c_2 (b) $-\frac{c_2}{2}$
(c) $c_1 - c_2$ (d) $\frac{c_1 + c_2}{2}$

17. A person walks up a stalled escalator in 90 s. When just standing on the same moving escalator, he is carried in 60 s. The time it would take him to walk up the moving escalator will be

(a) 27 s (b) 50 s
(c) 18 s (d) 36 s

18. A car starts moving along a line, first with acceleration $a = 5\ \text{ms}^{-2}$ starting from rest, then uniformly and finally decelerating at the same rate a and comes to rest. The total time of motion is 25 s. The average speed during the time is $20\ \text{ms}^{-1}$. How long does particle move uniformly?

(a) 10 s (b) 12 s (c) 20 s (d) 15 s

19. The displacement (x) of a particle depends on time t as $x = \alpha t^2 - \beta t^3$. Choose the incorrect statements from the following.

(a) The particle never returns to its starting point
(b) The particle comes to rest after time $\frac{2\alpha}{3\beta}$
(c) The initial velocity of the particle is zero
(d) The initial acceleration of the particle is zero

20. A body starts from rest, with uniform acceleration a. The acceleration of the body as function of time t is given by the equation $a = pt$, where p is a constant, then the displacement of the particle in the time interval $t = 0$ to $t = t_1$ will be

(a) $\frac{1}{2} pt_1^3$ (b) $\frac{1}{3} pt_1^2$
(c) $\frac{1}{2} pt_1^2$ (d) $\frac{1}{6} pt_1^3$

21. A ball is dropped onto the floor from a height of 10 m. It rebounds to a height of 5 m. If the ball was in contact with the floor for 0.01 s, what was its average acceleration during contact?

(Take $g = 10\ ms^{-2}$)

(a) $2414\ ms^{-2}$ (b) $1735\ ms^{-2}$
(c) $3120\ ms^{-2}$ (d) $4105\ ms^{-2}$

22. Particle A is moving along X-axis. At time $t = 0$, it has velocity of $10\ ms^{-1}$ and acceleration $-4\ ms^{-2}$. Particle B has velocity of $20\ ms^{-1}$ and acceleration $-2\ ms^{-2}$. Initially both the particles are at origin. At time $t = 2$ s distance between the two particles is

(a) 24 m (b) 36 m (c) 20 m (d) 42 m

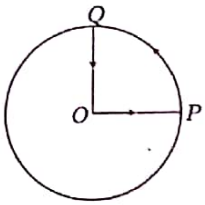
23. At a metro station, a girl walks up a stationary escalator in time t_1. If she remains stationary on the escalator, then the escalator take her up in time t_2. The time taken by her to walk up on the moving escalator will be **[NCERT Exemplar]**

(a) $\frac{(t_1 + t_2)}{2}$ (b) $\frac{t_1 t_2}{(t_2 - t_1)}$ (c) $\frac{t_1 t_2}{(t_2 + t_1)}$ (d) $t_1 - t_2$

24. The displacement of a body along X-axis depends on time as $\sqrt{x} = t + 1$. Then the velocity of body

(a) increases with time (b) decreases with time
(c) independent of time (d) None of these

25. A cyclist starts from the centre O of a circular park of radius one kilometre, reaches the edge P of the park, then cycles along the circumference and returns to the centre along QO as shown in the figure. If the round trip takes ten minutes, the net displacement and average speed of the cyclist (in metre and kilometre per hour) is



(a) 0, 1 (b) $\frac{\pi + 4}{2}$, 0
(c) 21.4, $\frac{\pi + 4}{2}$ (d) 0, 21.4

26. A particle moves along a straight line OX. At a time t (in seconds) the distance $x = 40 + 12t - t^3$. How long would the particle travel before coming to rest?

(a) 24 m (b) 40 m
(c) 56 m (d) 16 m

27. Two boys are standing at the ends A and B of a ground where $AB = a$. The boy at B starts running in a direction perpendicular to AB with velocity v_1. The boy at A starts running simultaneously with velocity v and catches the other boy in a time t, where t is

(a) $a / \sqrt{v^2 + v_1^2}$ (b) $\sqrt{a^2 / (v^2 - v_1^2)}$
(c) $a / (v - v_1)$ (d) $a / (v + v_1)$

28. A bullet emerges from a barrel of length 1.2 m with a speed of $640\ ms^{-1}$. Assuming constant acceleration, the approximate time that it spends in the barrel after the gun is fired is

(a) 4 ms (b) 40 ms (c) 400 μs (d) 1 s

29. From the top of a tower, 80 m high from the ground a stone is thrown in the horizontal direction with a velocity of $8\ ms^{-1}$. The stone reaches the ground after a time t and falls at a distance of d from the foot of the tower. Assuming $g = 10\ ms^{-2}$, the time t and distance d are given respectively by

(a) 6s, 64 m (b) 6s, 48 m (c) 4s, 32 m (d) 4s, 16 m

30. A boggy of uniformly moving train is suddenly detached from train and stops after covering some distance. The distance covered by the boggy and distance covered by the train in the same time has relation.

(a) Both will be equal
(b) First will be half of second
(c) First will be 1/4 of second
(d) No definite ratio

31. A man is 45 m behind the bus when the bus start accelerating from rest with acceleration $2.5\ ms^{-2}$. With what minimum velocity should the man start running to catch the bus

(a) $12\ ms^{-1}$ (b) $14\ ms^{-1}$ (c) $15\ ms^{-1}$ (d) $16\ ms^{-1}$

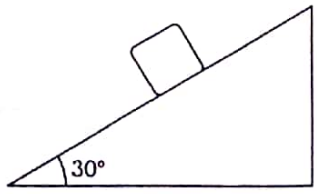
32. A body moves for a total of nine second starting from rest with uniform acceleration and then with uniform retardation, which is twice the value of acceleration and then stops. The duration of uniform acceleration is

(a) 3 s (b) 4.5 s (c) 5 s (d) 6 s

33. A point initially at rest moves along X-axis. Its acceleration varies with time as $a = (6t + 5)\ ms^{-2}$. If it starts from origin, then the distance covered in 2 s is

(a) 20 m (b) 18 m (c) 16 m (d) 25 m

34. The time taken by a block of wood (initially at rest) to slide down a smooth inclined plane 9.8 m long (angle of inclination is 30°) is



(a) $\frac{1}{2}$ s (b) 2 s (c) 4 s (d) 1 s

35. A particle moves a distance x in time t according to equation $x = (t + 5)^{-1}$. The acceleration of particle is proportional to

(a) $(\text{velocity})^{3/2}$ (b) $(\text{distance})^2$

(c) $(\text{distance})^{-2}$ (d) $(\text{velocity})^{2/3}$

36. A ball is thrown vertically upward with a speed v from a height h metre above the ground. The time taken for the ball to hit ground is

(a) $\frac{v}{g}\sqrt{1-\frac{2hg}{v^2}}$ (b) $\frac{v}{g}\sqrt{1+\frac{2hg}{v^2}}$

(c) $\sqrt{1+\frac{2hg}{v^2}}$ (d) $\frac{v}{g}\left[1+\sqrt{1+\frac{2hg}{v^2}}\right]$

37. The position of a particle along X-axis at time t is given by $x = 2 + t - 3t^2$. The displacement and the distance travelled in the interval, $t = 0$ to $t = 1$ are respectively

(a) 2, 2 (b) −2, 2.5 (c) 0, 2 (d) −2, 2.1

38. A stone is allowed to fall freely from rest. The ratio of the time taken to fall through the first metre and the second metre distance is

(a) $\sqrt{2}-1$ (b) $\sqrt{2}+1$

(c) $\sqrt{2}$ (d) None of these

39. Which of the following represents uniformly accelerated motion?

(a) $x=\sqrt{\frac{t+a}{b}}$ (b) $x=\frac{t+a}{b}$

(c) $t=\sqrt{\frac{x+a}{b}}$ (d) $x=\sqrt{t+a}$

40. A particle moves along a straight line. Its position at any instant is given by $x = 32t - \frac{8t^3}{4}$, where x is in metre and t is in second. Find the acceleration of the particle at the instant when particle is at rest.

(a) $-16\,\text{ms}^{-2}$ (b) $-27.6\,\text{ms}^{-2}$

(c) $32\,\text{ms}^{-2}$ (d) $16\,\text{ms}^{-2}$

41. A point moves in a straight line so that its displacement x at time t is given by $x^2 = t^2 + 1$. Its acceleration is

(a) $1/x$ (b) $1/x^3$

(c) $-1/x^2$ (d) $-1/x^3$

42. The displacement x of a particle varies with time t as $x = ae^{-\alpha t} + be^{\beta t}$, where a, b, α and β are positive constants. The velocity of the particle will

(a) go on decreasing with time

(b) be independent of α and β

(c) drop to zero when $\alpha = \beta$

(d) go on increasing with time

43. The ratios of the distance traversed, in successive intervals of time by a body, falling from rest, are

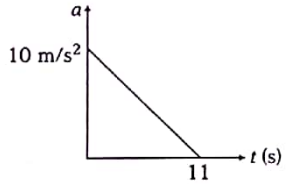
(a) 1 : 3 : 5 : 7 : 9 :...

(b) 2 : 4 : 6 : 8 : 10 :...

(c) 1 : 4 : 7 : 10 : 13 :...

(d) None of the above

44. A particle starts from rest. Its acceleration (a) *versus* time (t) graph is as shown in the figure. The maximum speed of the particle will be



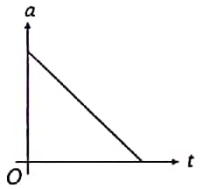
(a) $110\,\text{ms}^{-1}$

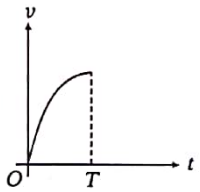
(b) $55\,\text{ms}^{-1}$

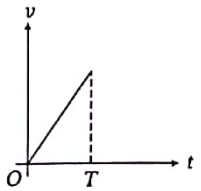
(c) $550\,\text{ms}^{-1}$

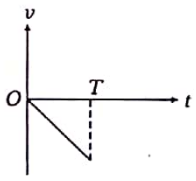
(d) $660\,\text{ms}^{-1}$

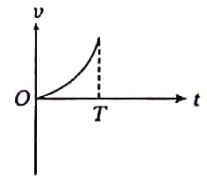
45. The acceleration (a)-time (t) graph for a particle moving along a straight starting from rest is shown in figure. Which of the following graph is the best representation of variation of its velocity (v) with time (t)?



(a)


(b)


(c)


(d)


46. A point moves with uniform acceleration and v_1, v_2 and v_3 denote the average velocities in the three successive intervals of time t_1, t_2 and t_3. Which of the following relations is correct?

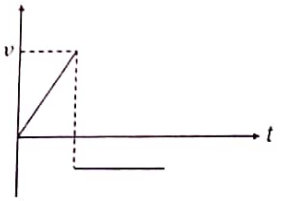
(a) $(v_1 - v_2) : (v_2 - v_3) = (t_1 - t_2) : (t_2 + t_3)$

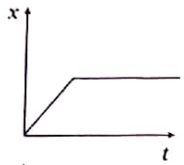
(b) $(v_1 - v_2) : (v_2 - v_3) = (t_1 + t_2) : (t_2 + t_3)$

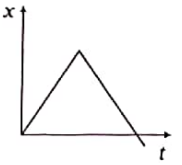
(c) $(v_1 - v_2) : (v_2 - v_3) = (t_1 - t_2) : (t_1 - t_3)$

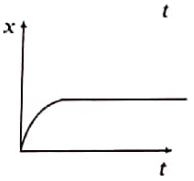
(d) $(v_1 - v_2) : (v_2 - v_3) = (t_1 - t_2) : (t_2 - t_3)$

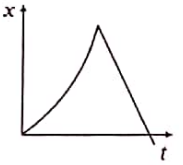
47. The velocity-time graph for a particle moving along X-axis is shown in the figure. The corresponding displacement-time graph is correctly shown by



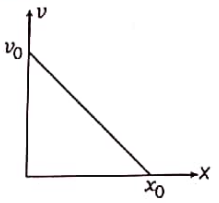
(a) 

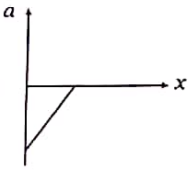
(b) 

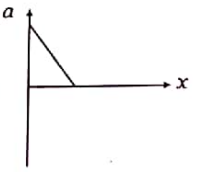
(c) 

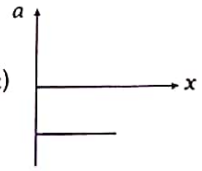
(d) 

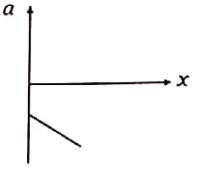
48. The given graph shows the variation of velocity with displacement. Which one of the graphs given below correctly represents the variation of acceleration with displacement?



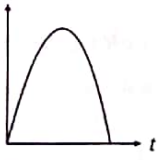
(a) 

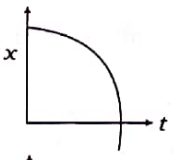
(b) 

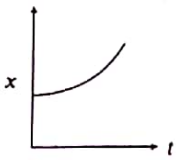
(c) 

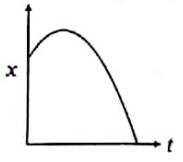
(d) 

49. The displacement x of a particle in a straight line motion is given by $x = 1 - t - t^2$. The correct representation of the motion is

(a) 

(b) 

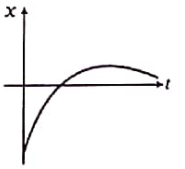
(c) 

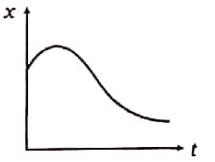
(d) 

50. The vertical height of point P above the ground is twice that of Q. A particle is projected downward with a speed of 5 ms^{-1} from P and at the same time another particle is projected upward with the same speed from Q. Both particles reach the ground simultaneously, then

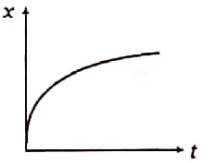
(a) $PQ = 30$ m
(b) time of flight of stones = 3 s
(c) Both (a) and (b) are correct
(d) Both (a) and (b) are wrong

51. Among the four graph shown in the figure there is only one graph for which average velocity over the time interval (O, T) can vanish for a suitably chosen T. Which one is it? **[NCERT Exemplar]**

(a) 

(b) 

(c)

(d) 

52. A lift is coming from 8th floor and is just about to reach 4th floor. Taking ground floor as origin and positive direction upwards for all quantities, which one of the following is correct? **[NCERT Exemplar]**

(a) $x < 0, v < 0, a > 0$
(b) $x > 0, v < 0, a < 0$
(c) $x > 0, v < 0, a > 0$
(d) $x > 0, v > 0, a < 0$

53. In one dimensional motion, instantaneous speed v satisfies $0 \le v < v_0$. **[NCERT Exemplar]**

(a) The displacement in time T must always take non-negative values
(b) The displacement x in time T satisfies $-v_0 T < x < v_0 T$
(c) The acceleration is always a non-negative number
(d) The motion has no turning points

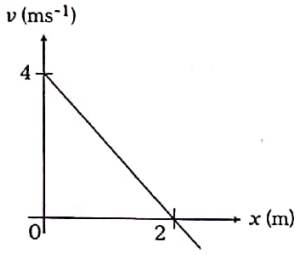
54. The displacement of a particle is given by $x = (t - 2)^2$ where x is in metre and t in second. The distance covered by the particle in first 4 seconds is **[NCERT Exemplar]**

(a) 4 m (b) 8 m
(c) 12 m (d) 16 m

55. A parachutist after bailing out falls 50 m without friction. When parachute opens, it decelerates at 2 ms^{-2}. At what height did he bail out

(a) 293 m (b) 111m
(c) 91 m (d) 182 m

56. The velocity (v) of a particle moving along X-axis varies with its position x as shown in figure. The acceleration (a) of particle varies with position (x) as



(a) $a^2 = x + 3$ (b) $a = 2x^2 + 4$
(c) $2a = 3x + 5$ (d) $a = 4x - 8$

57. A car A moves along north with velocity 30 km/h and another car B moves along east with velocity 40 km/h. The relative velocity of A with respect to B is
(a) 50 km/h North-East
(b) 50 km/h North-West
(c) 50 km/h at angle $\tan^{-1}(3/4)$ North of West
(d) 50 km/h at angle $\tan^{-1}(4/3)$ West of North

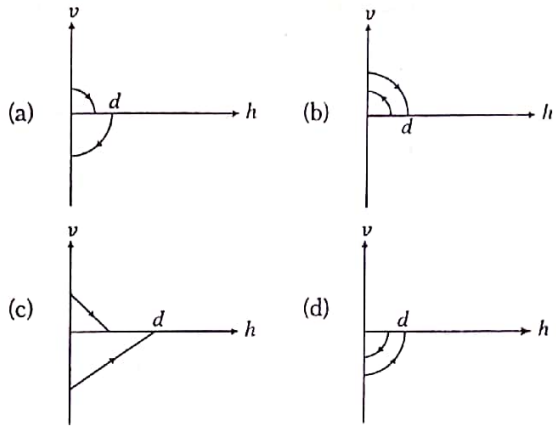
58. Rain is falling vertically downward with velocity 4 m/s. A man is moving horizontally with velocity 3 m/s, the velocity of rain with respect to man is
(a) 5 m/s at an angle $\tan^{-1}(4/3)$ with horizontal
(b) 5 m/s at an angle $\tan^{-1}(3/4)$ with vertical
(c) 5 m/s at an angle $\tan^{-1}(4/3)$ with vertical
(d) Both (a) and (b)

59. A ship is travelling due east at a speed of 15 km/h. Find the speed of a boat heading 30° east of north if it appears always due north from the ship.
(a) 30 km/h (b) $\frac{15\sqrt{3}}{2}$ km/h
(c) $10\sqrt{3}$ km/h (d) 20 km/h

60. A man takes 3 h to cover a certain distance along the flow and takes 6 h to cover the same distance opposite to flow. In how much time, he will cross this distance in still water.
(a) 3.5 h (b) 4 h (c) 4.5 h (d) 5 h

61. A river 500 m wide is flowing at a rate of 4 m/s. A boat is sailing at a velocity of 10 m/s with respect to the water in a direction perpendicular to the river. The time taken by the boat to reach the opposite bank
(a) 30 s (b) 40 s
(c) 50 s (d) 60 s

62. A ball is dropped vertically from a height d above the ground. It hits the ground and bounces up vertically to a height $d/2$. Neglecting subsequent motion and air resistance, its velocity v varies with the height h above the ground can be plotted as



63. Driver of a train moving at a speed v_1 sights a freight train at a distance d ahead of him on the same track moving in the same direction with a slow speed v_2. He puts on the brakes and gives his train a constant deceleration α. Then, there will be no collision if
(a) $d > \left(\frac{v_1 - v_2}{2\alpha}\right)$ (b) $d < \frac{(v_1 - v_2)^2}{2\alpha}$
(c) $d > \frac{(v_1 - v_2)^2}{2\alpha}$ (d) None of these

64. The speed of a boat is 5 km/h in still water. It crosses a river of width 1 km along the shortest possible path in 15 min. The velocity of the river water is
(a) 1 km/h (b) 3 km/h (c) 4 km/h (d) 5 km/h

65. Two cars A and B are travelling in the same direction with velocities v_1 and $v_2 (v_1 > v_2)$. When the car A is at a distance d ahead of the car B, the driver of the car A applied the brake producing a uniform retardation a. There will be no collision when
(a) $d < \frac{(v_1 - v_2)^2}{2a}$ (b) $d < \frac{v_1^2 - v_2^2}{2a}$
(c) $d > \frac{(v_1 - v_2)^2}{2a}$ (d) $d > \frac{v_1^2 - v_2^2}{2a}$

66. Water drops fall at regular intervals from a tap which is 5 m above the ground. The third drop is leaving the tap at the instant the first drop touches the ground. How far above the ground is the second drop at that instant?
(a) 2.50 m (b) 3.75 m (c) 4.00 m (d) 1.25 m

67. A body is thrown vertically up with a velocity u. It passes three points A, B and C in its upward journey with velocities $\frac{u}{2}, \frac{u}{3}$ and $\frac{u}{4}$, respectively. The ratio of the separations between points A and B and between B and C *i.e.*, $\frac{AB}{BC}$ is
(a) 1 (b) 2 (c) $\frac{10}{7}$ (d) $\frac{20}{7}$

68. A particle moving along x-axis has acceleration f, at time t, given $f = f_0\left(1 - \frac{t}{T}\right)$, where f_0 and T are constants. The particle at $t = 0$ has zero velocity. When $f = 0$, the particle's velocity (v_x) is

(a) $\frac{1}{2} f_0 T$ (b) $f_0 T$ (c) $\frac{1}{2} f_0 T^2$ (d) $f_0 T^{-2}$

69. The position x of a particle with respect to time t along X-axis is given by $x = 9t^2 - t^3$, where x is in metres and t in second. What will be the position of this particle when it achieves maximum speed along the positive x-direction?

(a) 24 m (b) 32 m (c) 54 m (d) 81 m

70. Two particles P and Q simultaneously start moving from point A with velocities 15 ms^{-1} and 20 ms^{-1} respectively. The two particles move with accelerations equal in magnitude but opposite in direction. When P overtakes Q at B then its velocity is 30 ms^{-1}. The velocity of Q at point B will be

(a) 30 ms^{-1} (b) 5 ms^{-1} (c) 20 ms^{-1} (d) 15 ms^{-1}

71. A body falling from a high Minaret travels 40 m in the last 2 seconds of its fall to ground. Height of Minaret in meter is (take, $g = 10$ ms^{-2})

(a) 60 (b) 45 (c) 80 (d) 50

72. A small block slides without friction down an inclined plane starting from rest. Let S_n be the distance travelled from time $t = n - 1$ to $t = n$. Then $\frac{S_n}{S_{n+1}}$ is

(a) $\frac{2n-1}{2n}$ (b) $\frac{2n+1}{2n-1}$ (c) $\frac{2n-1}{2n+1}$ (d) $\frac{2n}{2n+1}$

73. A particle located at $x = 0$ at time $t = 0$, starts moving along the positive x-direction with a velocity 'v' that varies as $v = \alpha\sqrt{x}$. The displacement of the particle varies with time as

(a) t (b) $t^{1/2}$ (c) t^3 (d) t^2

74. A body falls freely from the top of a tower. It covers 36% of the total height in the last second before striking the ground level. The height of the tower is

(a) 50 m (b) 75 m (c) 100 m (d) 125 m

75. An elevator car whose floor to ceiling distance is 2.7m starts ascending with a constant acceleration of 1.2 ms^{-2}. 2 s after the start, a bolt falls from the ceiling of the car. The free fall time of the bolt is ($g = 9.8$ ms^{-2})

(a) $\sqrt{\frac{2.7}{9.8}}$ s (b) $\sqrt{\frac{5.4}{9.8}}$ s (c) $\sqrt{\frac{5.4}{8.6}}$ s (d) $\sqrt{\frac{5.4}{11}}$ s

B Medical entrance special format questions

Assertion and reason

Directions (Q. Nos. 1-20) *These questions consists of two statements each printed as Assertion and Reason. While answering these question you are required to choose any one of the following five responses.*

(a) If both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If Assertion is false but Reason is true.

1. Assertion Acceleration of a moving particle can change its direction without any change in direction of velocity.

Reason If the direction of change in velocity vector changes, direction of acceleration vector also changes.

2. Assertion An object may have varying speed without having varying velocity.

Reason If the velocity is zero at an instant, the acceleration may not be zero at that instant.

3. Assertion Magnitude of average velocity is equal to average speed, if velocity is constant.

Reason If velocity is constant, then there is no change in the direction of motion.

4. Assertion In the equation, $s = u + at - \frac{1}{2}a$ where, s is the distance travelled by uniformly accelerated body in tth second.

Reason The above equation is dimensionally incorrect.

5. Assertion A body is momentarily at rest at the instant it reverses the direction.

Reason A body cannot have acceleration if its velocity is zero at a given instant of time.

6. Assertion The average velocity of a particle having initial and final velocity $\mathbf{v}_1$ and $\mathbf{v}_2$ is $\mathbf{v}_1 + \mathbf{v}_2 / 2$

Reason If $\mathbf{r}_1$ and $\mathbf{r}_2$ be the initial and final displacement in time t, then $\mathbf{v}_{av} = \frac{\mathbf{r}_1 - \mathbf{r}_2}{t}$.

7. Assertion The v-t graph perpendicular to time axis is not possible in practice.

Reason Infinite acceleration cannot be realised in practice.

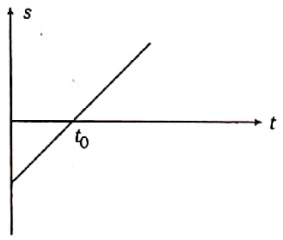
8. **Assertion** If velocity-time equation of a particle moving in a straight line is quadratic in time, then displacement-time equation cannot be linear.

   Reason If displacement-time is quadratic in time, then velocity-time is linear.

9. **Assertion** Distance between two particles moving with constant velocities always remains constant.

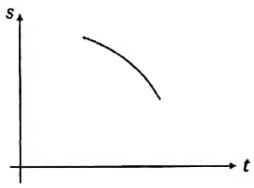
   Reason In the above case, relative motion between them is uniform.

10. **Assertion** In the s-t diagram as shown in figure, the body starts moving in positive direction but not form $s = 0$.



   Reason At $t = t_0$, velocity of body changes its direction of motion.

11. **Assertion** In the s-t graph as shown in figure, velocity of particle is negative and acceleration is positive.



   Reason Slope of s-t graph is negative and increasing in magnitude.

12. **Assertion** A body of mass 4 kg has an initial velocity $5\hat{i}$ ms^{-1}. It is subjected to a force of $4\hat{j}$ N. The displacement of body from origin after 4 s will be 215 m.

   Reason The equation $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ can be applied to obtain $\mathbf{v}$ if $\mathbf{a}$ is constant.

13. **Assertion** Particle A is moving Eastwards and particle B Northwards with same speed. Then, velocity of A with respect to B is in South-East direction.

   Reason Relative velocity between them is zero as their speeds are same.

14. **Assertion** On a curved path, average speed of a particle can never be equal to average velocity.

   Reason Average speed is total distance travelled divided by total time. Whereas average velocity is, final velocity plus initial velocity divided by two.

15. **Assertion** If a particle is thrown upwards, then distance travelled in last second of upward journey is independent of the velocity of projection.

   Reason In last second, distance travelled is 4.9 m. (Take, $g = 9.8$ ms^{-2})

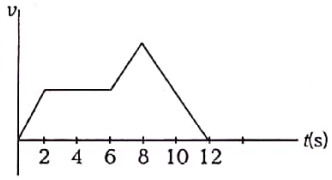
16. **Assertion** If acceleration of a particle moving in a straight line varies as $a \propto t^n$, then $S \propto t^{n+2}$

   Reason If a-t graph is a straight line,then s-t graph may be a parabola.

17. **Assertion** A lift is ascending with decreasing speed means acceleration of lift is downwards.

   Reason A body always moves in the direction of its acceleration.

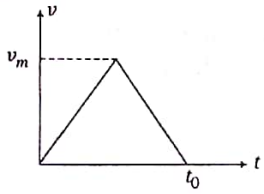
18. **Assertion** A body is moving along a straight line such that its velocity varies with a time as shown in figure. Magnitude of displacement of the body from $t = 0$ to $t = 12$ s is the same as the distance travelled by it in the given time duration.



   Reason For unidirectional motion of a body,

$$|\text{displacement}| = \text{distance}$$

19. **Assertion** In the v-t diagram as shown in figure, average velocity between the interval $t = 0$ and $t = t_0$ is independent of t_0.



   Reason Average velocity in the given interval is $\frac{1}{2} v_m$.

Match the columns

1. Match the following columns.

	Column I		Column II
(A)	$d\mathbf{v}/dt$	(p)	Acceleration
(B)	$d\lvert\mathbf{v}\rvert/dt$	(q)	Magnitude of acceleration
(C)	$\frac{d\mathbf{r}}{dt}$	(r)	Velocity
(D)	$\left\lvert\frac{d\mathbf{r}}{dt}\right\rvert$	(s)	Magnitude of velocity
		(t)	Rate of change of speed

2. In the s-t equation ($s = 10 + 20\,t - 5t^2$) match the following columns.

	Column I		Column II
(A)	Distance travelled in 3 s	(p)	−20 units
(B)	Displacement in 1 s	(q)	15 units
(C)	Initial acceleration	(r)	25 units
(D)	Velocity at 4 s	(s)	−10 units

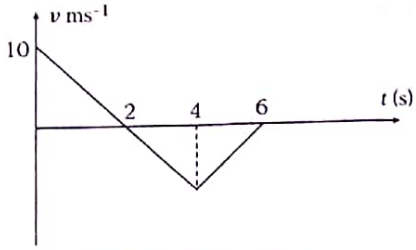
3. Velocity of a particle is in negative direction with constant acceleration in positive direction. Then, match the following columns.

	Column I		Column II
(A)	Velocity-time graph	(p)	Slope → negative
(B)	Acceleration-time graph	(q)	Slope → positive
(C)	Displacement-time graph	(r)	Slope → zero
		(s)	|Slope| → increasing
		(t)	|Slope| → decreasing
		(u)	|Slope| → constant

4. Match the following columns.

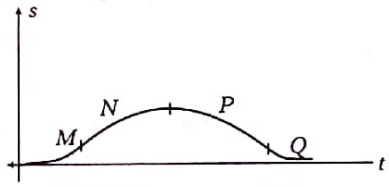
	Column I		Column II
(A)	Constant positive acceleration	(p)	Speed may increase
(B)	Constant negative acceleration	(q)	Speed may decrease
(C)	Constant displacement	(r)	Speed is zero
(D)	Constant slope of a-t graph	(s)	Speed must increase
		(t)	Speed must decrease

5. For the velocity-time graph as shown in the figure, in a time interval from $t = 0$ to $t = 6$ s, match the following columns.



	Column I		Column II
(A)	Change in velocity	(p)	− 5/3 SI unit
(B)	Average acceleration	(q)	− 20 SI unit
(C)	Total displacement	(r)	− 10 SI unit
(D)	Acceleration at $t = 3$ s	(s)	− 5 SI unit

6. Let us call a motion, A when velocity is positive and increasing, A^{-1} when velocity is negative and increasing. R when velocity is positive and decreasing and R^{-1} when velocity is negative and decreasing. Now match the following two columns for the given s-t graph.



	Column I		Column II
(A)	M	(p)	A^{-1}
(B)	N	(q)	R^{-1}
(C)	P	(r)	A
(D)	Q	(s)	R

C Medical entrances' gallery

(Collection of questions asked in NEET & various medical entrance exams)

1. If the velocity of a particle is $v = At + Bt^2$, where A and B are constants, then the distance travelled by it between 1s and 2s is **[NEET 2016]**

(a) $3A + 7B$ (b) $\frac{3}{2}A + \frac{7}{3}B$ (c) $\frac{A}{2} + \frac{B}{3}$ (d) $\frac{3}{2}A + 4B$

2. A particle of unit mass undergoes one-dimensional motion such that its velocity varies according to $v(x) = \beta\, x^{-2n}$, where β and n are constants and x is the position of the particle. The acceleration of the particle as a function of x, is given by **[CBSE AIPMT 2015]**

(a) $-2n\beta^2\, x^{-2n-1}$ (b) $-2n\beta^2\, x^{-4n-1}$

(c) $-2\beta^2\, x^{-2n+1}$ (d) $-2n\beta^2\, x^{-4n+1}$

3. A ball is dropped from a bridge 122.5 m above a river. After the ball has been falling for 2s, a second ball is thrown straight down after it. What must the initial velocity of the second ball be so that both hit the water at the same time? **[AIIMS 2015]**

(a) 40 ms^{-1} (b) 55.5 ms^{-1} (c) 26.1 ms^{-1} (d) 9.6 ms^{-1}

4. A ball is thrown vertically upwards from the ground with a speed of 25.2 ms^{-1}. How long does it take to reach its highest point and how high does it rise? (Take, $g = 9.8\ ms^{-2}$) **[UK PMT 2015]**

(a) 2.75 s, 3.24 m (b) 25.7 s, 34.2 m

(c) 2.57 s, 32.4 m (d) 27.5 s, 3.42 m

5. A particle moves in an XY-plane in such a way that its x and y-coordinates vary with time according to

$$x(t) = t^3 - 32t,\ y(t) = 5t^2 + 12$$

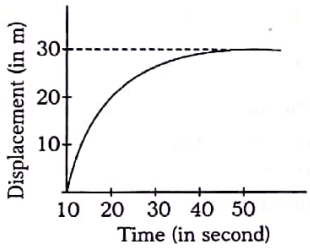
Find the acceleration of the particle, if $t = 3$ s. **[UK PMT 2015]**

(a) $9\hat{i} + 5\hat{j}$ (b) $18\hat{i} + 10\hat{j}$ (c) $18\hat{i} - 5\hat{j}$ (d) $-18\hat{i} + 10\hat{j}$

6. A vehicle moving with a constant acceleration from A to B in a straight line AB, has velocities u and v at A and B, respectively. C is the mid-point of AB. If time taken to travel from A to C is twice the time taken to travel from C to B, then the velocity of the vehicle v at B is **[EAMCET 2015]**

(a) $5u$ (b) $6u$ (c) $7u$ (d) $8u$

7. The displacement of a particle as a function of time is shown in figure. It indicates that **[Kerala CEE 2015]**

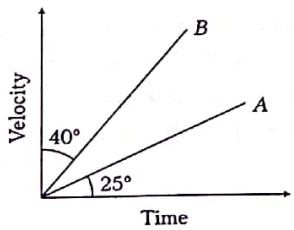


(a) the velocity of the particle is constant throughout
(b) the acceleration of the particle is constant throughout
(c) the particle starts with a constant velocity and is accelerated
(d) the particle starts from rest and is accelerated throughout
(e) the motion is retarded and finally the particle stops

8. A car starts from rest and accelerates uniformly to a speed of 180 kmh^{-1} in 10 s. The distance covered by the car in the time interval is **[Manipal 2015]**

(a) 200 m (b) 300 m (c) 500 m (d) 250 m

9. The velocity-time graph for two bodies A and B are shown in figure. Then, the acceleration of A and B are in the ratio **[KCET 2015]**



(a) $\sin 25°$ to $\sin 50°$ (b) $\tan 25°$ to $\tan 40°$
(c) $\cos 25°$ to $\cos 50°$ (d) $\tan 25°$ to $\tan 50°$

10. A particle is moving such that its position coordinates (x, y) are (2 m, 3 m) at time $t = 0$, (6 m, 7 m) at time $t = 2s$ and (13 m, 14 m) at time $t = 5$ s. Average velocity vector (v_{av}) from $t = 0$ to $t = 5$ s is **[CBSE AIPMT 2014]**

(a) $\frac{1}{5}(13\hat{i} + 14\hat{j})$ (b) $\frac{7}{3}(\hat{i} + \hat{j})$ (c) $2(\hat{i} + \hat{j})$ (d) $\frac{11}{5}(\hat{i} + \hat{j})$

11. A ball thrown vertically upwards after reaching a maximum height h returns to the starting point after a time of 10 s. Its displacement after 5 s is **[Kerala CEE 2014]**

(a) h (b) $2h$ (c) $10h$ (d) $20h$

12. A police jeep is chasing with velocity of 45 kmh^{-1}, a thief in another jeep moving with velocity 153 kmh^{-1}. Police fires a bullet with muzzle velocity of 180 ms^{-1}. The velocity with which it will strike the car of the thief is **[EAMCET 2014]**

(a) 150 ms^{-1} (b) 27 ms^{-1} (c) 450 ms^{-1} (d) 250 ms^{-1}

13. A particle moves with constant acceleration along a straight line starting from rest. The percentage increase in its displacement during the 4th second compared to that in the 3rd second is **[WB JEE 2014]**

(a) 33% (b) 40% (c) 66% (d) 77%

14. A car covers the first half of the distance between the two places at 40 kmh^{-1} and another half at 60 kmh^{-1}. The average speed of the car is **[UK PMT 2014]**

(a) 40 kmh^{-1} (b) 48 kmh^{-1} (c) 50 kmh^{-1} (d) 60 kmh^{-1}

15. A particle starts moving from rest with uniform acceleration. It travels a distance x in first 2 s and distance y in the next 2 s. Then, **[EAMCET 2014]**

(a) $y = 3x$ (b) $y = 4x$ (c) $y = x$ (d) $y = 2x$

16. At time $t = 0$, two bodies A and B are at the same point. A moves with constant velocity v and B starts from rest and moves with constant acceleration. Relative velocity of B w.r.t. A when the bodies meet each other is **[EAMCET 2014]**

(a) $\frac{v}{2}$ (b) $\frac{v}{3}$ (c) v (d) $2v$

17. A car moves from A to B with a speed of 30 kmh^{-1} and from B to A with a speed of 20 kmh^{-1}. What is the average speed of the car? **[KCET 2014]**

(a) 25 kmh^{-1} (b) 24 kmh^{-1} (c) 50 kmh^{-1} (d) 10 kmh^{-1}

18. A body starts from rest and moves with constant acceleration for t second. It travels a distance x_1 in first half of time and x_2 in next half of time, then **[KCET 2014]**

(a) $x_2 = x_1$ (b) $x_2 = 2x_1$
(c) $x_2 = 3x_1$ (d) $x_2 = 4x_1$

19. The acceleration of a moving body is found from the **[Kerala CEE 2014]**

(a) area under velocity-time graph
(b) area under displacement-time graph
(c) slope of distance-time graph
(d) slope of velocity-time graph
(e) area under acceleration-time graph

20. A stone falls freely under gravity. It covers distances h_1, h_2 and h_3 in the first 5 s, the next 5 s and the next 5 s respectively.

The relation between h_1, h_2 and h_3 is [NEET 2013]

(a) $h_1 = 2h_2 = 3h_3$ (b) $h_1 = \frac{h_2}{3} = \frac{h_3}{5}$

(c) $h_2 = 3h_1$ and $h_3 = 3h_2$ (d) $h_1 = h_2 = h_3$

21. The motion of a particle in straight line is an example of [J&K CET 2013]

(a) constant velocity motion
(b) uniformly accelerated motion
(c) non-uniformly accelerated motion
(d) zero velocity motion

22. The velocity-time graph of particle comes out to be a non-linear curve. The motion is [J&K CET 2013]

(a) uniform velocity motion
(b) uniformly accelerated motion
(c) non-uniform accelerated motion
(d) Nothing can be said about the motion

23. A person reaches a point directly opposite on the other bank of a river. The velocity of the water in the river is 4 ms^{-1} and the velocity of the person in still water is 5 ms^{-1}. If the width of the river is 84.6 m, time taken to cross the river in seconds is [EAMCET 2013]

(a) 28.2 (b) 9.4 (c) 2 (d) 84.6

24. A body is thrown vertically upward from a point A 125 m above the ground. It goes up to a maximum height of 250 m above the ground and passes through A on its downward journey. The velocity of the body when it is at a height of 70 m above the ground is (Take, $g = 10\ ms^{-2}$) [EAMCET 2013]

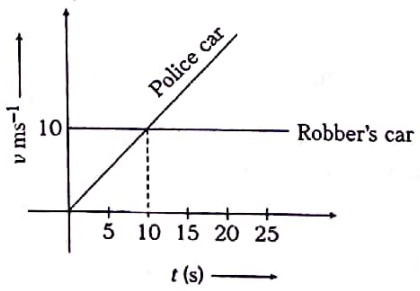
(a) 50 ms^{-1} (b) 60 ms^{-1} (c) 80 ms^{-1} (d) 20 ms^{-1}

25. A particle is moving eastwards with a velocity of 5 m/s. If in 10 s the velocity changes by 5 m/s northwards, what is the average acceleration in this time? [MP PMT 2013]

(a) $\frac{1}{\sqrt{2}}$ m/s^2 (North-West) (b) $\frac{1}{\sqrt{2}}$ m/s^2 (North-East)

(c) $\sqrt{2}$ m/s^2 (North-West) (d) $\sqrt{2}$ m/s^2 (North-East)

26. The velocity-time graph of robber's car and a chasing police car are shown in the following graph. Police car crosses the robber's car in time [UP CPMT 2013]



(a) 10 s after it starts (b) 15 s after it starts

(c) 20 s after it starts (d) Never crosses

27. Initial speed of an α particle inside a tube of length 4m is 1 kms^{-1}, if it is accelerated in the tube and comes out with a speed of 9 kms^{-1}, then the time for which the particle remains inside the tube is [BCECE 2013]

(a) 8×10^{-3} s
(b) 8×10^{-4} s
(c) 80×10^{-3} s
(d) 800×10^{-3} s

28. A body X is projected upwards with a velocity of 98 ms^{-1}, after 4 s, a second body Y is also projected upwards with the same initial velocity. Two bodies will meet after [BCECE 2012]

(a) 8 s (b) 10 s

(c) 12 s (d) 14 s

29. Let $\mathbf{r}_1(t) = 3t\hat{\mathbf{i}} + 4t^2\hat{\mathbf{j}}$

and $\mathbf{r}_2(t) = 4t^2\hat{\mathbf{i}} + 3t^2\hat{\mathbf{j}}$

represent the positions of particles 1 and 2, respectively, as function of time t, $\mathbf{r}_1(t)$ and $\mathbf{r}_2(t)$ are in metre and t in second. The relative speed of the two particles at the instant $t = 1$s, will be [AMU 2012]

(a) 1 m/s
(b) $3\sqrt{2}$ m/s
(c) $5\sqrt{2}$ m/s
(d) $7\sqrt{2}$ m/s

30. The motion of a particle along a straight line is described by equation [CBSE AIPMT 2012]

$$x = 8 + 12t - t^3$$

where, x is in metre and t in second. The retardation of the particle when its velocity becomes zero, is

(a) 24 ms^{-2}
(b) zero
(c) 6 ms^{-2}
(d) 12 ms^{-2}

31. A scooter starts from rest have an acceleration of 1 ms^{-2} while a car 150 m behind it starts from rest with an acceleration of 2 ms^{-2}. After how much time the car catches up with the scooter? [BHU 2012]

(a) $\sqrt{700}$ s
(b) $\sqrt{300}$ s
(c) $\sqrt{150}$ s
(d) None of the above

32. A particle moves along with X-axis. The position x of particle with respect to time t from origin given by $x = b_0 + b_1t + b_2t^2$. The acceleration of particle is [AIIMS 2012]

(a) b_0 (b) b_1

(c) b_2 (d) $2b_2$

Answers

Check point 3.1

1. (d)	2. (a)	3. (a)	4. (c)	5. (a)	6. (c)	7. (b)	8. (b)	9. (d)	10. (c)

Check point 3.2

1. (a)	2. (a)	3. (d)	4. (d)	5. (c)	6. (b)	7. (c)	8. (c)	9. (d)	10. (b)

Check point 3.3

1. (c)	2. (d)	3. (b)	4. (c)	5. (a)	6. (c)	7. (b)	8. (c)	9. (b)	10. (d)

Check point 3.4

1. (b)	2. (a)	3. (a)	4. (c)	5. (a)	6. (a)	7. (d)	8. (b)	9. (c)	10. (d)
11. (c)	12. (c)	13. (b)	14. (a)	15. (c)					

Check point 3.5

1. (b)	2. (b)	3. (d)	4. (b)	5. (a)	6. (b)	7. (a)	8. (b)	9. (c)	10. (b)
11. (a)	12. (c)	13. (b)	14. (b)	15. (c)	16. (b)	17. (a)	18. (a)	19. (c)	20. (a)
21. (d)	22. (b)	23. (a)	24. (c)	25. (c)					

Check point 3.6

1. (a)	2. (c)	3. (a)	4. (b)	5. (d)	6. (b)	7. (c)	8. (a)	9. (b)	10. (a)
11. (a)	12. (c)	13. (c)	14. (a)	15. (d)	16. (d)	17. (b)	18. (b)	19. (d)	20. (a)

Check point 3.7

1. (b)	2. (d)	3. (c)	4. (b)	5. (b)	6. (c)	7. (a)	8. (d)	9. (b)	10. (a)
11. (a)	12. (c)	13. (b)	14. (d)	15. (b)					

A. Taking it together

1. (b)	2. (c)	3. (a)	4. (b)	5. (b)	6. (d)	7. (c)	8. (b)	9. (a)	10. (d)
11. (d)	12. (a)	13. (c)	14. (c)	15. (a)	16. (b)	17. (d)	18. (d)	19. (d)	20. (d)
21. (a)	22. (a)	23. (c)	24. (a)	25. (d)	26. (c)	27. (b)	28. (b)	29. (c)	30. (b)
31. (c)	32. (d)	33. (b)	34. (b)	35. (a)	36. (d)	37. (d)	38. (b)	39. (c)	40. (b)
41. (b)	42. (d)	43. (a)	44. (b)	45. (a)	46. (b)	47. (d)	48. (a)	49. (b)	50. (c)
51. (b)	52. (a)	53. (b)	54. (b)	55. (a)	56. (d)	57. (c)	58. (d)	59. (a)	60. (b)
61. (c)	62. (a)	63. (c)	64. (b)	65. (c)	66. (b)	67. (d)	68. (a)	69. (c)	70. (b)
71. (b)	72. (c)	73. (d)	74. (d)	75. (d)					

B. Medical entrance special format questions

Assertion and reason

1. (d)	2. (d)	3. (b)	4. (b)	5. (c)	6. (d)	7. (a)	8. (b)	9. (d)	10. (c)
11. (d)	12. (b)	13. (c)	14. (c)	15. (a)	16. (b)	17. (c)	18. (a)	19. (a)	

Match the columns

1. (A $\rightarrow$ p, B $\rightarrow$ t, C $\rightarrow$ r, D $\rightarrow$ s)
2. (A $\rightarrow$ r, B $\rightarrow$ p, C $\rightarrow$ s, D $\rightarrow$ p)
3. (A $\rightarrow$ q, u, B $\rightarrow$ r, u, C $\rightarrow$ p, t)
4. (A $\rightarrow$ p, q, B $\rightarrow$ p, q, C $\rightarrow$ r, D $\rightarrow$ p, q)
5. (A $\rightarrow$ r, B $\rightarrow$ p, C $\rightarrow$ r, D $\rightarrow$ s)
6. (A $\rightarrow$ r, B $\rightarrow$ s, C $\rightarrow$ p, D $\rightarrow$ q)

C. Medical entrances' gallery

1. (b)	2. (b)	3. (c)	4. (c)	5. (b)	6. (c)	7. (e)	8. (d)	9. (d)	10. (d)
11. (a)	12. (a)	13. (b)	14. (b)	15. (a)	16. (c)	17. (b)	18. (d)	19. (d)	20. (b)
21. (b)	22. (c)	23. (a)	24. (b)	25. (c)	26. (c)	27. (b)	28. (c)	29. (c)	30. (d)
31. (b)	32. (d)								

Hints and explanations

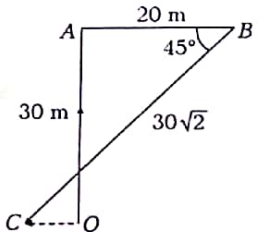
Check point 3.1

2. (*a*) Distance from starting point $= \sqrt{(3)^2 + (4)^2 + (5)^2} = 5\sqrt{2}$ m

3. (*a*) Distance = Length $AB = 2\pi R \times \frac{60}{360} = \frac{\pi R}{3}$

4. (*c*) From figure, $\mathbf{OA} = 0\hat{\mathbf{i}} + 30\hat{\mathbf{j}}$, $\mathbf{AB} = 20\hat{\mathbf{i}} + 0\hat{\mathbf{j}}$,

$$\mathbf{BC} = -30\sqrt{2}\cos 45°\,\hat{\mathbf{i}} - 30\sqrt{2}\sin 45°\,\hat{\mathbf{j}}$$
$$= -30\,\hat{\mathbf{i}} - 30\hat{\mathbf{j}}$$



∴ Net displacement,

$$\mathbf{OC} = \mathbf{OA} + \mathbf{AB} + \mathbf{BC} = -10\hat{\mathbf{i}} + 0\hat{\mathbf{j}}$$
$$|\mathbf{OC}| = 10 \text{ m}$$

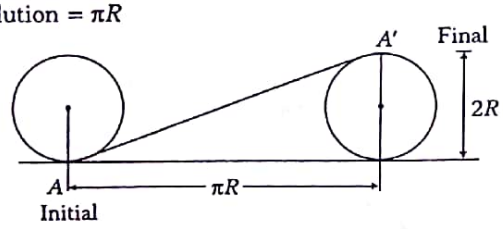
5. (*a*) An aeroplane flies 400 m north and 300 m south so the net displacement is 100 m towards north.

Then it flies 1200 m upward so, $r = \sqrt{(100)^2 + (1200)^2}$

$$= 1204 \text{ m} \simeq 1200 \text{ m}$$

The option should be 1204 m, because this value mislead one into thinking that net displacement is in upward direction only.

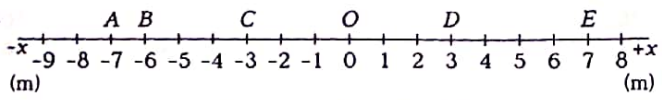
6. (*c*) Horizontal distance covered by the wheel in half revolution $= \pi R$



So, the displacement of the point which was initially in contact with ground $= AA' = \sqrt{(\pi R)^2 + (2R)^2}$

$$= R\sqrt{\pi^2 + 4} = \sqrt{\pi^2 + 4} \qquad (\text{as } R = 1\text{ m})$$

7. (*b*)



(i) The displacement of the man from A to E is $\Delta x = x_2 - x_1 = 7\text{m} - (-8\text{ m}) = +15$ m directed in the positive x-direction.

(ii) The displacement of the man from E to C is $\Delta x = -3\text{ m} - (7\text{ m}) = -10$ m directed in the negative x-direction.

(iii) The displacement of the man from B to D is $\Delta x = 3\text{ m} - (-7\text{ m}) = +10$ m directed in the positive x-axis.

10. (*c*) Since, displacement is always less than or equal to distance but never greater than distance. Hence, numerical ratio of displacement to the distance covered is always equal to or less than one.

Check point 3.2

1. (*a*) $v_{av} = \frac{\text{Total displacement}}{\text{Total time}} = \frac{80t + 40t}{2t} = 60 \text{ kmh}^{-1}$

2. (*a*) $21 = \frac{11 \times 18 + 42 \times v}{60}$

∴ $v = 25.3 \text{ ms}^{-1}$

3. (*d*) Man walks from his home to market with a speed of 5 kmh^{-1}. Distance = 2.5 km and time $= \frac{2.5}{5} = \frac{1}{2}$ h = 30 min

and he returns back with speed of 7.5 km/h in rest of time of 10 min.

$$\text{Distance} = 7.5 \times \frac{10}{60} = 1.25 \text{ km}$$

So, $\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}}$

$$= \frac{(2.5 + 1.25)\text{ km}}{(40/60)\text{ h}} = \frac{45}{8} \text{ kmh}^{-1}$$

4. (*d*) Displacement of the particle will be zero because it comes back to its starting point.

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}} = \frac{30 \text{ m}}{10 \text{ s}} = 3 \text{ ms}^{-1}$$

5. (*c*) $t = \frac{d}{V} = \frac{150 + 850}{45 \times \frac{5}{18}} = 80$ s

6. (*b*) Displacement of the particle

$$s = 6t^2 - t^3$$

Velocity of the particle

$$v = \frac{ds}{dt} = \frac{d}{dt}(6t^2 - t^3)$$
$$v = 12t - 3t^2$$

For $\quad v = 0 \Rightarrow 12t = 3t^2 \Rightarrow t = 4$ s

7. (c) $v_{av} = \dfrac{\text{Total displacement}}{\text{Total time}} = \dfrac{\sqrt{(4)^2 + (3)^2}}{10 + 5} = \dfrac{1}{3}\ \text{ms}^{-1}$

8. (c) $v_{av} = \dfrac{\text{Displacement}}{\text{Time}} = \dfrac{\sqrt{2}R}{t}$

9. (d) $v_{av} = \dfrac{\text{Displacement}}{\text{Time}} = \dfrac{2R}{(\pi R/v)} = \dfrac{2v}{\pi} = \dfrac{20}{\pi}\ \text{ms}^{-1}$

10. (b) $x = 2t + 3t^2$

$$v = \frac{dx}{dt} = 2 + 6t$$

For $t = 2$ s, $v = 2 + 6(2) = 14$ m/s

Check point 3.3

1. (c) Acceleration is a vector quantity. So, it changes when both magnitude and direction of velocity changes.

3. (b) Time $= \dfrac{\text{Distance}}{\text{Average velocity}} = \dfrac{3.06}{0.34} = 9$ s

Acceleration $= \dfrac{\text{Change in velocity}}{\text{Time}} = \dfrac{0.18}{9} = 0.02\ \text{m/s}^2$

4. (c) $x = \alpha - \beta t + \gamma t^2$

$$v = \frac{dx}{dt} = \frac{d}{dt}(\alpha - \beta t + \gamma t^2) = -\beta + 2\gamma t$$

$$a = \frac{dv}{dt} = \frac{d}{dt}(-\beta + 2\gamma t) = 2\gamma$$

5. (a) $a = \dfrac{\text{Change in velocity}}{\text{Time taken}} = \dfrac{v_f - v_i}{t}$

$$= \frac{\left(44 \times \frac{5}{18}\right) - \left(80 \times \frac{5}{18}\right)}{15} = -0.67\ \text{m/s}^2$$

Negative sign represents the retardation.

6. (c) Average acceleration $= \dfrac{\text{Change in velocity}}{\text{Time taken}}$ or $a = \dfrac{|v_f - v_i|}{\Delta t}$

$$\Rightarrow \quad a = \frac{\sqrt{30^2 + 40^2}}{10} = 5\ \text{m/s}^2$$

7. (b) $x = 18t + 5t^2$

$$v = \frac{dx}{dt} = \frac{d}{dt}(18t + 5t^2) = 18 + 10t$$

$\therefore \quad v = 10t + 18$

At $t_1 = 2$ s, $v_1 = 10(2) + 18 = 38$ m/s

At $t_2 = 4$ s, $v_2 = 10(4) + 18 = 58$ m/s

$\therefore \quad a = \dfrac{v_2 - v_1}{t} = \dfrac{20}{2} = 10\ \text{m/s}^2$

8. (c) $x = 180t + 50t^2$

$$v = \frac{dx}{dt} = 180 + 100t$$

$$a = \frac{dv}{dt} = 100\ \text{m/s}^2$$

9. (b) $a = \dfrac{dv}{dt}$

Here, $x = 18t + 5t^2$

$$\therefore \quad v = \frac{dx}{dt} = \frac{d}{dt}(18t + 5t^2) = 18 + 10t$$

$$\Rightarrow \quad a = \frac{dv}{dt} = \frac{d}{dt}(18 + 10t) = 10\ \text{m/s}^2$$

10. (d) $a = \dfrac{\mathbf{v}_f - \mathbf{v}_i}{\Delta t} = \dfrac{(2\hat{\mathbf{i}} - 3\hat{\mathbf{j}}) - (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}})}{2} = \dfrac{-6\hat{\mathbf{j}}}{2} = -3\hat{\mathbf{j}}\ \text{ms}^{-2}$

Check point 3.4

1. (b) $v = u + at$ or $-2 = 10 + 4a$ or $a = -3\ \text{ms}^{-2}$

2. (a) $\dfrac{u}{4} = u - at_0 \qquad \left(\therefore a = \dfrac{3u}{4t_0}\right)$

or $\dfrac{u}{a} = \dfrac{4}{3}t_0$

Now, $0 = u - at$ or $t = \dfrac{u}{a} = \dfrac{4}{3}t_0$

3. (a) $S \propto t^2$ [Given]

$\therefore \quad s = kt^2$

Acceleration, $a = \dfrac{d^2s}{dt^2} = 2k$ [constant]

It means the particle travels with uniform acceleration.

4. (c) $s = \dfrac{1}{2}at^2 = \dfrac{1}{2} \times (0.2)(64) = 6.4$ m $= 640$ cm

5. (a) $15 = 2t - \dfrac{1}{2} \times (0.1)t^2$ or $t = 10$ s

6. (a) $s = s_1 + s_2 + s_3$

$$= \frac{1}{2} \times 2 \times (10)^2 + (2)(10)(30) + \frac{1}{2} \times 4 \times (5)^2$$

$$= 100 + 600 + 50 = 750\ \text{m}$$

7. (d) $s = \dfrac{1}{2}at^2 \Rightarrow s = \dfrac{1}{2} \times a \times (4)^2 \Rightarrow s = 8a$

8. (b) Displacements of both should be equal.

or, $8t = \dfrac{1}{2} \times 4 \times t^2$ or $t = 4$ s

9. (c) $s_B = \dfrac{1}{2} \times 2 \times (1)^2 = 1$ m

$$s_A = \frac{1}{2} \times 2 \times (2)^2 = 4\ \text{m}$$

$\therefore \quad s_A - s_B = 3$ m

10. (d) $v = \dfrac{ds}{dt} = 12 - 4t$

Comparing with $v = u + at$, $u = 12\ \text{ms}^{-1}$ and $a = -4\ \text{ms}^{-2}$

Velocity will become zero at time, $0 = 12 - 4t_0$ or $t_0 = 3$ s.

Since, the given time $t = 5$ s is greater than $t_0 = 3$ s

distance > | displacement |

Distance $d = |s_{0-t_0}| + |s_{t-t_0}| = \dfrac{u^2}{2|a|} + \dfrac{1}{2}|a|(t - t_0)^2$

$$= \frac{(12)^2}{8} + \frac{1}{2} \times 4 \times (2)^2 = 26\ \text{m}$$

11. (*c*) $40 = (20)\,a_1$,

$a_1 = 2\ \text{ms}^{-2}$

Further $40 = (40)\,a_2$

$\therefore$ $a_2 = 1\,\text{ms}^{-2}$

Therefore acceleration is $2\ \text{ms}^{-2}$ and retardation is $1\,\text{ms}^{-2}$.

Now, $s_1 = \frac{1}{2} a_1 t_1^2 = \frac{1}{2} \times 2 \times (20)^2 = 400$ m

$s_2 = v_{max}\, t_2 = 40 \times 20 = 800$ m

$s_3 = \frac{v_{max}^2}{2a_2} = \frac{(40)^2}{2 \times 1} = 800$ m

Now average velocity $= \frac{\text{Total displacement}}{\text{Total time}}$

$= \frac{400 + 800 + 800}{20 + 20 + 40} = 25\ \text{ms}^{-1}$

12. (*c*) Let v_m be the maximum speed,

$v_m = a_1 t_1$ and $v_m = \sqrt{2a_1 l}$ $(\because u = 0)$

$t_2 = \frac{2l}{v_m}$ and $v_m = a_2 t_3 = \sqrt{2a_2(3l)}$

Now, average speed, $v_{av} = \frac{l + 2l + 3l}{t_1 + t_2 + t_3}$

$$v_{av} = \frac{6l}{(v_m/a_1) + (2l/v_m) + (v_m/a_2)}$$

$$= \frac{6l}{\left(\frac{v_m}{v_m^2/2l}\right) + \left(\frac{2l}{v_m}\right) + \left(\frac{v_m}{v_m^2/6l}\right)}$$

$$= \frac{6l}{(10l/v_m)} = \frac{3v_m}{5}$$

$\Rightarrow$ $\frac{v_{av}}{v_m} = \frac{3}{5}$

13. (*b*) $(30)^2 = (20)^2 + 2a\,(2s)$ or $2as = 250$

Now, $v^2 = (u)^2 + 2as = v^2 = (20)^2 + 250$

$\Rightarrow$ $v^2 = 650$

$\therefore$ $v = 25.5\ \text{ms}^{-1}$

14. (*a*) $v^2 = 25x$. Comparing with $v^2 = 2as$, we have, $a = 12.5\,\text{ms}^{-2}$

15. (*c*) $S_n = u + \frac{a}{2}(2n - 1) \Rightarrow 1.2 = 0 + \frac{a}{2}(2 \times 6 - 1)$

$\Rightarrow$ $a = \frac{1.2 \times 2}{11} = 0.218\ \text{ms}^{-2}$

Check point 3.5

1. (*b*) Free fall of an object (in vacuum) is a case of motion with uniform acceleration.

2. (*b*) Total time taken in upward journey should be 5 s

$\therefore$ $u = gt = 9.8 \times 5 = 49\ \text{ms}^{-1}$

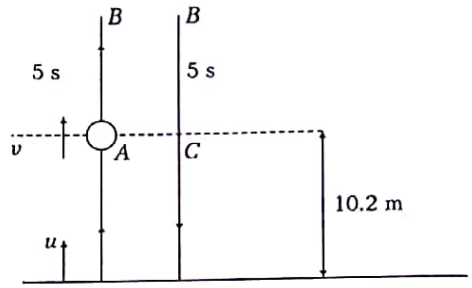
3. (*d*) Let t second of upward journey = first t second of downward journey (with zero initial velocity)

$\therefore$ Desired distance $= \frac{1}{2} gt^2$

4. (*b*) Time taken to reach maximum height is 1 s.

Height = free fall distance in 1 s $= \frac{1}{2} gt^2 = 5$ m

5. (*a*) $t_{ABC} = 10$ s $\Rightarrow$ $t_{AB} = 5$ s



At B, velocity becomes zero. Hence at A velocity should be $50\ \text{ms}^{-1}$.

Now, $(50)^2 = (u^2) - 2 \times 10 \times 10.2$

$\therefore$ $u = 52\ \text{ms}^{-1}$

6. (*b*) $t = \frac{2u}{g} \Rightarrow t = \frac{2 \times 9.8}{10} = 1.96 \approx 2$ s

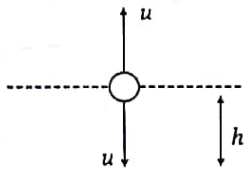
7. (*a*) $S_t\text{th} = u + at - \frac{1}{2} a$

$\Rightarrow$ $0 + 10\,t - 5 = \frac{1}{2} \times 10 \times (3)^2 \Rightarrow t = 5$ s

8. (*b*) $v^2 = u^2 + 2gh$

$\Rightarrow$ $(3u)^2 = (-u)^2 + 2gh \Rightarrow h = \frac{4u^2}{g} \downarrow +\text{ve}$

9. (*c*) Taking downward direction as the positive direction.



$+h = -ut_1 + \frac{1}{2} gt_1^2$ ...(i)

$+h = ut_2 + \frac{1}{2} gt_2^2$ ...(ii)

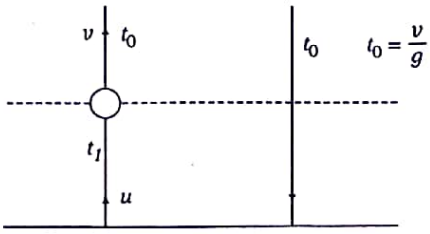
Multiplying Eq. (i) by t_2 and Eq. (ii) by t_1 and adding, we get

$h\,(t_1 + t_2) = \frac{1}{2} gt_1t_2\,(t_1 + t_2)$ or $h = \frac{1}{2} gt_1t_2$

For free fall from rest, $h = \frac{1}{2} gt^2$

$\therefore$ $t^2 = t_1t_2 \Rightarrow t = \sqrt{t_1t_2}$

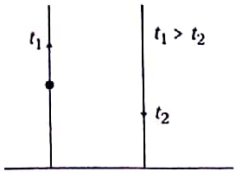
10. (*b*) Velocity of particle of this instant will be $v = (u - gt_1)$.



Now the desired time interval will be $\frac{2v}{g}$.

or $$\frac{2(u - gt_1)}{g} = 2\left(\frac{u}{g} - t_1\right)$$

11. (*a*)



12. (*c*) Vertical component of velocities of both the balls are same and equal to zero.

So, $$t = \sqrt{\frac{2h}{g}}.$$

13. (*b*) Let h be distance the covered in t second $\Rightarrow h = \frac{1}{2}gt^2$

Distance covered in tth second $= \frac{1}{2}g(2t - 1)$

$\Rightarrow$ $$\frac{9h}{25} = \frac{g}{2}(2t - 1)$$

From above two equations, $h = 122.5$ m

14. (*b*) In first second distance travelled,

$$x = \frac{1}{2} \times g \times t^2 = 5 \text{ m}$$

(for $g = 10$ ms^{-2}) in last second $7x = 35$ m

Now, $$S_t\text{th} = \left(u + at - \frac{1}{2}a\right)$$

$\Rightarrow$ $$35 = 0 + 10 \times t - \frac{1}{2} \times 10$$

$\therefore$ $t = 4$ s

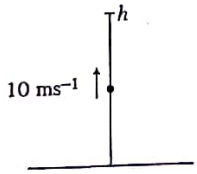
15. (*c*) $$h_1 = \frac{1}{2} \times g \times (2)^2 = 2g$$

$\Rightarrow$ $$h_2 = \frac{1}{2} \times g \times (4)^2 - 2g = 6g$$

$$h_3 = \frac{1}{2} \times g \times (6)^2 - 8g = 10g$$

$\therefore$ $h_1 : h_2 : h_3 = 1 : 3 : 5$

16. (*b*) $h = \frac{(10)^2}{2g} = 5$ m



$\therefore$ Total height $= 2h = 10$ m

17. (*a*) $H_{max} \propto v^2 \Rightarrow v \propto \sqrt{H_{max}}$

i.e., to triple the maximum height, ball should be thrown with velocity $\sqrt{3}\, v_0$.

18. (*a*) After 2 s $v = 4.9 \times 2 = 9.8$

and $$h = \frac{1}{2} \times 4.9 \times (2)^2 = 9.8 \text{ m}$$

Greatest height, $h_{max} = h + \frac{v^2}{2g} = 14.7$ m

19. (*c*) $v = \sqrt{2gh}$ or $v \propto \sqrt{h}$

$\Rightarrow$ $$\frac{v_1}{v_2} = \sqrt{\frac{h_1}{h_2}} \Rightarrow \left(\frac{v_1}{v_2}\right)^2 = \frac{h}{h_2} \qquad (\because h_1 = h)$$

$\Rightarrow$ $$\left(\frac{v}{2v}\right)^2 = \frac{h}{h_2} \qquad \left\{\begin{array}{l}\because v_1 = v \\ \therefore v_2 = 2v\end{array}\right\}$$

$\Rightarrow$ $h_2 = 4h$

20. (*a*) $t = \sqrt{\frac{2h}{g}}$ or $t \propto \sqrt{h} \Rightarrow \frac{t_1}{t_2} = \sqrt{\frac{h}{2h}} = 1 : \sqrt{2}$

21. (*d*) The initial velocity of airplane is horizontal. Hence, the vertical component of velocity of packet will be zero.

$\therefore$ $$h = \frac{1}{2}gt^2$$

$\Rightarrow$ $$t = \sqrt{\frac{2h}{g}}$$

22. (*b*) $v = 3\alpha t^2 + 2\beta t + \gamma : v_{t=0} = v_i = \gamma$

$a = 6\alpha t + 2\beta : a_{t=0} = a_i = 2\beta$

$\therefore$ $$\frac{v_i}{a_i} = \frac{\gamma}{2\beta}$$

23. (*a*) $a = bt$

$\therefore$ $$\int_{v_0}^{v} dv = \int a\,dt = \int_0^t bt \cdot dt$$

$\therefore$ $$v = v_0 + \frac{bt^2}{2}$$

Further integrating we get,

$$s = v_0 t + \frac{bt^3}{6}$$

24. (*c*) $dv = a\,dt$

$\therefore$ $$\int_2^v dv = \int_0^2 (3t^2 + 2t + 2)\,dt = [t^3 + t^2 + 2t]_0^2$$

or $v = 18$ ms^{-1}

25. (*c*) $s = t^3 - 6t^2 + 18t + 9$

$$v = \frac{ds}{dt} = 3t^2 - 12t + 18$$

For v to be minimum or maximum

$$a = \frac{dv}{dt} = 6t - 12 = 0 \Rightarrow t = 2 \text{ s}$$

At $t = 2$ s

$$\frac{d^2v}{dt^2} = 6 > 0, \text{ i.e., } v \text{ is minimum at } t = 2 \text{ s}$$

$$v_{min} = 3(2)^2 - 12(2) + 18$$
$$= 6 \text{ m/s}$$

Check point 3.6

1. (*d*) Uniform motion means uniform velocity or constant slope of *s-t* graph.

2. (*c*) In first instant, you will apply $v = \tan\theta$ and say

$$v = \tan 30° = \frac{1}{\sqrt{3}}\ \text{ms}^{-1}$$

But it is wrong because formula $v = \tan\theta$ is valid when angle is measured with time axis.

Here angle is taken from displacement axis. So, angle from time axis = 90° – 30° = 60°

Now $$v = \tan 60° = \sqrt{3}\ \text{ms}^{-1}$$

3. (*a*) v_i = slope of *s-t* graph = $\tan 45° = 1\ \text{ms}^{-1}$

v_f = slope of *s-t* graph = $\tan 60° = \sqrt{3}\ \text{ms}^{-1}$

Now $$a_{av} = \frac{v_f - v_i}{\Delta t} = \frac{\sqrt{3}-1}{1} = (\sqrt{3}-1)\ \text{units}$$

4. (*b*) The graph between displacement and time for a particle moving with uniform acceleration is a parabola.

5. (*d*) Maximum acceleration means maximum change in velocity in minimum time interval.

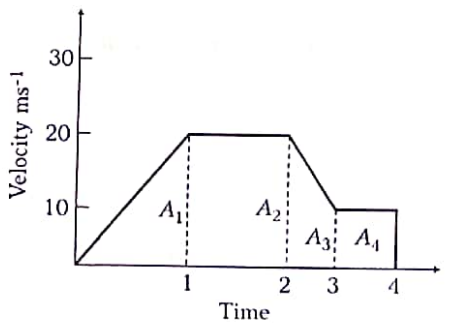
In time interval $t = 30$ to $t = 40$ s

$$a = \frac{\Delta v}{\Delta t} = \frac{80-20}{40-30} = \frac{60}{10} = 6\ \text{cms}^{-2}$$

6. (*b*) Distance = Area under *v-t* graph = $A_1 + A_2 + A_3 + A_4$

Displacement = positive area – negative area

and distance = Σ (Areas)

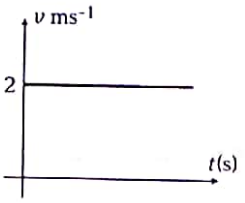


$$= \frac{1}{2}\times 1\times 20 + (20\times 1) + \frac{1}{2}(20+10)\times 1 + (10\times 1)$$

$$= 10 + 20 + 15 + 10 = 55\ \text{m}$$

7. (*c*) Area of trapezium $= \frac{1}{2}\times 3.6\times(12+8) = 36\ \text{m}$

9. (*b*) $v = \frac{dx}{dt} = 2\ \text{ms}^{-1}$ = constant



10. (*a*) Velocity will continuously increase (starting from rest).

11. (*a*) Velocity first decreases in upwards direction, then increases in downward direction.

12. (*c*) In region *B*, velocity is decreasing. So, a force will act which opposes the motion of a body.

13. (*c*) a = slope of *v-t* graph $= -\frac{15}{3} = -5\ \text{ms}^{-2}$

14. (*a*) Since, total displacement is zero, hence average velocity is also zero.

15. (*d*) I is not possible because total distance covered by a particle cannot decrease with time. II is not possible because at a particular time, position cannot have two values. III is not possible because at a particular time, velocity cannot have two values. IV is not possible because speed can never be negative.

16. (*d*) $a = v\left(\frac{dv}{ds}\right) = (10)\left(-\frac{20}{30}\right) = -\frac{20}{3}\ \text{ms}^{-2}$

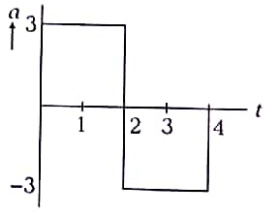
17. (*b*) For uniformly accelerated motion, $v^2 = u^2 + 2as$, *i.e.*, v^2 *versus* s graph is a straight line with intercept u^2 and slope $2a$. Since intercept is non-zero, initial velocity is non-zero.

18. (*b*) $v^2 = u^2 + 2as$, slope $= 2a = -\frac{16}{2} = -8\ \text{ms}^{-2}$

or $a = -4\ \text{ms}^{-2}$

19. (*d*) The area under acceleration-time graph gives change in velocity.

20. (*a*)



Taking the motion from 0 to 2 s

$u = 0,\ a = 3\ \text{ms}^{-2},\ t = 2s,\ v = ?$

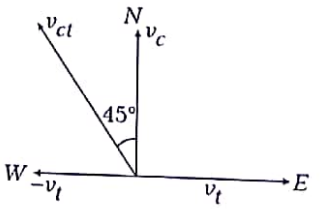
$v = u + at = 0 + 3\times 2 = 6\ \text{ms}^{-1}$

Taking the motion from 2s to 4s

$v = 6 + (-3)(2) = 0\ \text{ms}^{-1}$

Check point 3.7

1. (*b*) $v_{ct} = v_c - v_t \Rightarrow v_{ct} = v_c + (-v_t)$



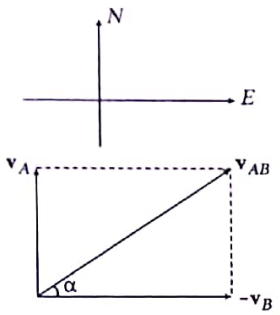
Velocity of car w.r.t. train (v_{ct}) is towards West-North

2. (*d*) $7.2 = \frac{100}{(v+5)(5/18)}$ or $v = 45\ \text{ms}^{-1}$

3. (*c*) Both the particles will fall same distance in same time interval.

So, the relative separation will remain unchanged.

4. (*b*) $\mathbf{v}_{AB} = \mathbf{v}_A - \mathbf{v}_B$



$$v_{AB} = \sqrt{v_A^2 + v_B^2} = 5\,\text{kmh}^{-1}$$

$$\alpha = \tan^{-1}\left(\frac{v_A}{v_B}\right) = \tan^{-1}\left(\frac{3}{4}\right) = 37° \text{ East of North}$$

5. (*b*) Velocity of rain is at 30° in vertical direction. So, its horizontal component is $v_R \sin 30° = \frac{v_R}{2}$. When man starts walking with 10 kmh^{-1} rain appears vertical. So horizontal component $\frac{v_R}{2}$ is balanced by his speed of 10 kmh^{-1}. Thus,

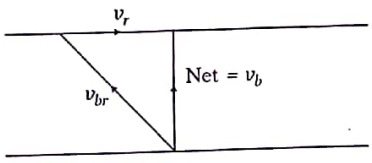
$\frac{v_R}{2} = 10$ or $v_R = 20$ kmh^{-1}

6. (*c*) $\tan 60° = \frac{v_H}{v_V}$ or $\sqrt{3} = \frac{12}{v_V}$

$\therefore$ $v_V = 4\sqrt{3}$ kmh^{-1}

7. (*a*) $\tan\theta = \frac{v_H}{v_V} = \frac{v}{u}$ or $\theta = \tan^{-1}\left(\frac{v}{u}\right)$

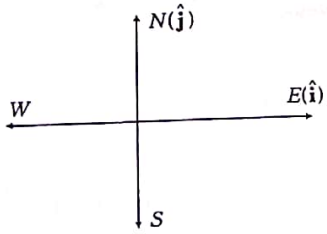
8. (*d*) $v_b = \frac{1}{1/4} = 4$ kmh^{-1}



$v_{br} = 5$ kmh^{-1}

$\therefore$ $v_r = v_{br} - v_b = 3$ kmh^{-1}

9. (*b*) $\mathbf{v}_x = -v\,\hat{\mathbf{j}}$



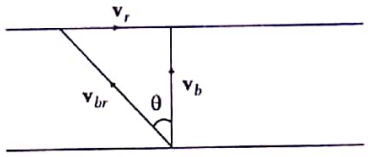
$\therefore$ $\mathbf{v}_Y = \mathbf{v}_{YX} + \mathbf{v}_X = -v\hat{\mathbf{i}} + v\hat{\mathbf{j}}$

$\therefore$ $|\mathbf{v}_Y| = \sqrt{2}\,v$

Direction of $\mathbf{v}_Y$ is North-West.

10. (*a*) For shortest time, one should swim at right angles to river current.

11. (*a*) $\sin\theta = \frac{v_r}{v_{br}} = \frac{3}{5}$

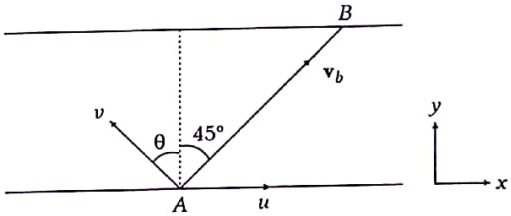


$\therefore$ $\theta = 37°$

The required angle is therefore

$$90° + \theta = 90° + 37° = 127°$$

12. (*c*) Let v be the speed of boatman in still water. Resultant of v and u should be along AB. Components of $\mathbf{v}_b$ (absolute velocity of boatman) along x and y directions are,



$$v_x = u - v\sin\theta \text{ and } v_y = v\cos\theta$$

Further, $\tan 45° = \frac{v_y}{v_x}$ or $1 = \frac{v\cos\theta}{u - v\sin\theta}$

$\therefore$ $v = \frac{u}{\sin\theta + \cos\theta} = \frac{u}{\sqrt{2}\sin(\theta + 45°)}$

v is minimum at,

$$\theta + 45° = 90° \text{ or } \theta = 45° \text{ and } v_{\min} = \frac{u}{\sqrt{2}}$$

13. (*b*) $v_r = 10 + 15 = 25$ ms^{-1}

where, v_r is relative velocity

$\therefore$ $t = \frac{50 + 50}{25} = 4$ s

14. (*d*) Displacement of man = displacement of bus + 25

or $10t = \frac{1}{2} \times 2 \times t^2 + 25$ or $10t = t^2 + 25$

Solving this equation, we get $t = 5$ s.

15. (*b*) Relative acceleration = 0, relative velocity is 40 ms^{-1} and relative separation is 100 m.

$\therefore$ $t = \frac{100}{40} = 2.5$ s

A) Taking it together

1. (*b*) Average speed of a boy $= \frac{2v_1v_2}{v_1 + v_2} = \frac{2 \times 2.5 \times 4}{2.5 + 4}$

$$= \frac{20}{6.5} = \frac{40}{13} \text{ km/h}$$

2. (*c*) Man walks from his home to market with a speed of 5 kmh^{-1}. Distance = 2.5 km and time $= \frac{d}{v} = \frac{2.5}{5} = \frac{1}{2}$

and he returns back with speed of 7.5 kmh^{-1} in rest a time of 20 min.

$$\text{Distance} = 7.5 \times \frac{20}{60} = 2.5 \text{ km}$$

$$\text{So, Average speed} = \frac{\text{Total distance}}{\text{Total time}} = \frac{(2.5 + 2.5)}{(50/60)\text{ h}} = \frac{30}{4} \text{ km/h}$$

3. (*a*) $S_n = u + \frac{a}{2}(2n - 1)$

$$\Rightarrow S_3 = 0 + \frac{4/3}{2}(2 \times 3 - 1)$$

$$\Rightarrow S_3 = \frac{10}{3} \text{ m}$$

4. (*b*) $v^2 = u^2 + 2as$

$$a = \frac{v^2 - u^2}{2s} = \frac{(20)^2 - (10)^2}{2 \times 135} = \frac{300}{270} = \frac{10}{9} \text{ ms}^{-2}$$

From first equation of motion

$$v = u + at \Rightarrow t = \frac{v - u}{a} = \frac{20 - 10}{10/9} = \frac{10}{10/9} = 9 \text{ s}$$

5. (*b*) $x = 4(t - 2) + a(t - 2)^2$

At $t = 0, x = -8 + 4a = 4a - 8$

$$v = \frac{dx}{dt} = 4 + 2a(t - 2)$$

At $t = 0, v = 4 - 4a = 4(1 - a)$

But acceleration, $a = \frac{d^2x}{dt^2} = 2a$

6. (*d*) $S \propto u^2$. If u becomes 3 times, then S will become 9 times *i.e.*, $9 \times 20 = 180$ m

7. (*c*) Time taken to travel first half distance $t_1 = \frac{l/2}{v_1} = \frac{l}{2v_1}$

Time taken to travel second half distance $t_2 = \frac{l}{2v_2}$

$$\text{Total time} = t_1 + t_2 = \frac{l}{2v_1} + \frac{l}{2v_2} = \frac{l}{2}\left[\frac{1}{v_1} + \frac{1}{v_2}\right]$$

We know that, v_{av} = Average speed

$$= \frac{\text{total distance}}{\text{total time}} = \frac{l}{\frac{l}{2}\left[\frac{1}{v_1} + \frac{1}{v_2}\right]} = \frac{2v_1v_2}{v_1 + v_2}$$

8. (*b*) $a = \sqrt{a_x^2 + a_y^2} = \left[\left(\frac{d^2x}{dt^2}\right)^2 + \left(\frac{d^2y}{dt^2}\right)^2\right]^{1/2}$

Here $\frac{d^2y}{dt^2} = 0$

Hence, $a = \frac{d^2x}{dt^2} = 8 \text{ ms}^{-2}$

9. (*a*) Distance travelled by body A in 5th second and distance travelled by body B in 3rd second of its motion are equal.

$$0 + \frac{a_1}{2}(2 \times 5 - 1) = 0 + \frac{a_2}{2}(2 \times 3 - 1)$$

$$9a_1 = 5a_2 \Rightarrow \frac{a_1}{a_2} = \frac{5}{9}$$

10. (*d*) Since $x = 1.2t^2$, comparing it with equation of motion $x = \frac{1}{2}at^2$.

Thus the motion is uniformly accelerated.

11. (*d*) In all the given graphs, we have more than one values of speed corresponding to a single time. The graph must have a unique value of speed corresponding to a single time. Thus all the three graphs are not possible.

12. (*a*) From the given x-t graph, it is clear that

At $t = 0, \frac{dx}{dt} = 0$

$\therefore$ Velocity $v = \frac{dx}{dt} = 0$, at $t = 0$

As time passes $\frac{dx}{dt}$ increases

$\therefore$ Velocity and hence acceleration changes.

13. (*c*) It is given that

$$v = \frac{t^2}{10} + 20$$

Differentiating both sides w.r.t. time, we get

$$\frac{dv}{dt} = \frac{2t}{10} + 0 = \frac{t}{5}$$

$\therefore$ Acceleration, $a = \frac{dv}{dt} = \frac{t}{5} \neq$ constant

Hence, it is a case of non-uniform acceleration.

14. (*c*) In the given distance-time graph at the point C, slope of the graph is maximum thus at this point C instantaneous velocity of the particle is maximum.

15. (*a*) $\because$ Slope of displacement-time graph at the point E is negative. Thus, at the point E the instantaneous velocity of the particle is negative.

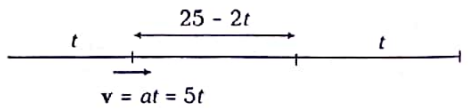
16. (*b*) $v^2 = c_1 - c_2 x$, comparing with $v^2 = u^2 + 2as$ we have

$$a = -\frac{c_2}{2}$$

17. (*d*) $v_1 = \frac{s}{90}, v_2 = \frac{s}{60}$

Now, $t = \frac{s}{v_1 + v_2} = \frac{s}{\frac{s}{90} + \frac{s}{60}} = \frac{90 \times 60}{90 + 60} = 36 \text{ s}$

18. (*c*) Average velocity $= \frac{\text{Total displacement}}{\text{Total time}}$



or $$20 = \frac{\frac{1}{2} \times a \times t^2 + (at)(25 - 2t) + \frac{1}{2} \times a \times t^2}{25}$$

Solving this equation with $a = 5 \text{ ms}^{-2}$ we get, $t = 5$ s

Thus, the particle moved uniformly for $(25 - 2t)$ or 15 s.

19. (*d*) $x = 0$ at $t = 0$ and $t = \alpha / \beta$. So, the particle returns to starting point at $t = \alpha / \beta$.

$$v = \frac{dx}{dt} = 2\alpha t - 3\beta t^2$$

At $t = 0$, $v = 0$ *i.e.*, initial velocity of particle is zero.

$$v = 0 \text{ at } t = 0 \text{ and } t = \frac{2\alpha}{3\beta}$$

Thus, the particle comes to rest after time

$$t = 2\alpha / 3\beta \Rightarrow a = \frac{dv}{dt} = 2\alpha - 6\beta t$$

At $t = 0$, $a = 2$, $a \neq 0$

20. (*d*) Integrate twice to convert *a-t* equation into *s-t* equation.

$$\because \quad a = \frac{v}{t} \Rightarrow v = at$$

$$\Rightarrow \quad \int dv = \int a \cdot dt \Rightarrow v = \int pt \cdot dt = \frac{pt^2}{2}$$

$$\because \quad s = v \times t \Rightarrow ds = v\, dt$$

$$\Rightarrow \quad \int ds = \int v\, dt \Rightarrow s = \frac{1}{6} pt^3$$

21. (*a*) $a_{av} = \frac{v_f - v_i}{\Delta t} = \frac{v_f - v_i}{\Delta t}$ (as they are in opposite direction)

$$= \frac{\sqrt{2gh_f} + \sqrt{2gh_i}}{\Delta t} = \frac{\sqrt{2 \times 10 \times 5} + \sqrt{2 \times 10 \times 10}}{0.01}$$

$$\approx 2414 \text{ ms}^{-2}$$

22. (*a*) At $t = 2$ s : $x_A = 10 \times 2 - \frac{1}{2} \times 4 \times (2)^2 = 12$ m

and $\quad x_B = 20 \times 2 - \frac{1}{2} \times 2 \times (2)^2 = 36$ m

$\therefore$ Distance between A and B at that instant is 24 m.

23. (*c*) In this question, we have to find net velocity with respect to the Earth that will be equal to velocity of the girl plus velocity of escalator.

Let displacement is L, then

velocity of girl $v_g = \frac{L}{t_1}$

velocity of escalator $v_e = \frac{L}{t_2}$

Net velocity of the girl $= v_g + v_e = \frac{L}{t_1} + \frac{L}{t_2}$

If t is total time taken in covering distance L, then

$$\frac{L}{t} = \frac{L}{t_1} + \frac{L}{t_2}$$

$$\Rightarrow \quad t = \frac{t_1 t_2}{t_1 + t_2}$$

24. (*a*) $\sqrt{x} = t + 1$

Squaring both sides, we get $x = (t + 1)^2 = t^2 + 2t + 1$

Differentiating it w.r.t. time t, we get, $\frac{dx}{dt} = 2t + 2$

Velocity, $v = \frac{dx}{dt} = 2t + 2$

25. (*d*) Net displacement = 0 and total distance = $OP + PQ + QO$

$$= 1 + \frac{2\pi \times 1}{4} + 1 = \frac{14.28}{4} \text{ km}$$

Average speed $= \frac{14.28}{4 \times 10 / 60} = \frac{6 \times 14.28}{4} = 21.42$ km/h

26. (*c*) Distance travelled by the particle is $x = 40 + 12t - t^3$

We know that, speed is defined as the rate of change of distance *i.e.*,

$$v = \frac{dx}{dt}$$

$$\therefore \quad v = \frac{d}{dt}(40 + 12t - t^3) = 0 + 12 - 3t^2$$

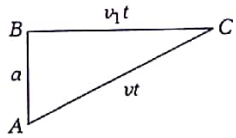
But final velocity, $v = 0$

$$\therefore \quad 12 - 3t^2 = 0 \Rightarrow t^2 = \frac{12}{3} = 4 \Rightarrow t = 2\text{s}$$

Hence, distance travelled by the particle before coming to rest is given by

$$x = 40 + 12(2) - (2)^3 = 40 + 24 - 8 = 64 - 8 = 56 \text{ m}$$

27. (*b*) Let two boys meet at point C after time 't' from the starting. Then $AC = ut$, $BC = v_1 t$ (see figure)



$$(AC)^2 = (AB)^2 + (BC)^2 \Rightarrow v^2 t^2 = a^2 + v_1^2 t^2$$

By solving, we get $t = \sqrt{\frac{a^2}{v^2 - v_1^2}}$

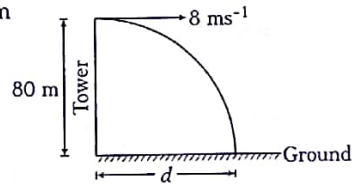
28. (*a*) $s = 12$ m, $v = 640$ ms^{-1}

$a = ?$; $u = 0$; $t = ?$ $\quad 2as = v^2 - u^2$

$$\Rightarrow 2a \times 12 = 640 \times 640 \Rightarrow a = \frac{8 \times 64 \times 100}{3}$$

$$v = u + at \Rightarrow t = \frac{v}{a} = \frac{640 \times 3}{8 \times 64 \times 100} = 37.5 \times 10^{-3} \text{ s} \approx 40 \text{ ms}$$

29. (*c*) Here, $u = 8$ ms^{-1}, $h = 80$ m



The time taken by the stone to reach the ground is

$$t = \sqrt{\frac{2h}{g}}$$

$$= \sqrt{\frac{2 \times 80 \text{ m}}{10 \text{ ms}^{-2}}} = 4 \text{ s}$$

Distance, $d = u \times t = 8 \text{ ms}^{-1} \times 4 \text{ s} = 32$ m

30. (*b*) Let a be the retardation of boggy, then distance covered by it be S. If u is the initial velocity of boggy after detaching from train (*i.e.*, uniform speed of train)

$$v^2 = u^2 + 2as \Rightarrow 0 = u^2 - 2as \Rightarrow s_b = \frac{u^2}{2a}$$

Time taken by boggy to stop

$$v = u + at \Rightarrow 0 = u - at \Rightarrow t = \frac{u}{a}$$

In this time t distance travelled by train $= s_t = ut = \frac{u^2}{a}$

Hence, ratio $\frac{s_b}{s_t} = \frac{1}{2}$.

31. (*c*) Let man will catch the bus after t second. So, he will cover distance ut.

Similarly distance travelled by the bus will be $\frac{1}{2}at^2$

For the given condition

$$ut = 45 + \frac{1}{2}at^2 = 45 + 1.25t^2 \qquad [\text{As } a = 2.5\ \text{ms}^{-2}]$$

$$\Rightarrow \qquad u = \frac{45}{t} + 1.25t$$

To find the minimum value of u

$$\frac{du}{dt} = 0 \Rightarrow \frac{du}{dt} = \frac{-45}{t^2} + 1.25 = 0$$

so we get $t = 6$ s, then

$$u = \frac{45}{6} + (1.25 \times 6) = 15\ \text{ms}^{-1}$$

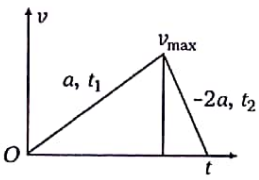
32. (*d*) Let acceleration is a and retardation is $-2a$

Then for acceleration motion

$$t_1 = \frac{v}{a} \qquad \text{...(i)}$$

For retarding motion

$$t_2 = \frac{v}{2a} \qquad \text{...(ii)}$$



Given $t_1 + t_2 = 9 \Rightarrow \frac{v}{a} + \frac{v}{2a} = 9$

$$\Rightarrow \qquad \frac{3v}{2a} = 9 \Rightarrow \frac{v}{a} = 6$$

Hence, duration of acceleration, $t_1 = \frac{v}{a} = 6$ s.

33. (*b*) Given, acceleration $a = 6t + 5$

$$\therefore \qquad a = \frac{dv}{dt} = 6t + 5,\ dv = (6t + 5)dt$$

Integrating it, we have $\int_0^v dv = \int_0^t (6t + 5)dt$

$v = 3t^2 + 5t + C$, where C is a constant of integration.

When $t = 0$, $v = 0$ so $C = 0$

$$\therefore \qquad v = \frac{ds}{dt} = 3t^2 + 5t \quad \text{or} \quad ds = (3t^2 + 5)\,dt$$

Integrating it within the conditions of motion, *i.e.*, as t changes from 0 to 2s, s changes from 0 to s, we have

$$\int_0^s ds = \int_0^2 (3t^2 + 5t)dt$$

$$\therefore \qquad s = \left[t^3 + \frac{5}{2}t^2\right]_0^2 = 8 + 10 = 18\,\text{m}$$

34. (*b*) For one dimensional motion along a plane

$$S = ut + \frac{1}{2}at^2 \Rightarrow 9.8 = 0 + \frac{1}{2}g\sin 30°\, t^2 \Rightarrow t = 2\,\text{s}$$

35. (*a*) Given, $x = \frac{1}{t+5}$

Differentiating both sides w.r.t. t, we get

$$\frac{dx}{dt} = -\frac{1}{(t+5)^2} \times 1$$

$$\therefore \text{ Velocity, } v = \frac{dx}{dt} = -\frac{1}{(t+5)^2} \qquad \text{...(i)}$$

Again differentiating both sides w.r.t. t, we get

$$\frac{dv}{dt} = \frac{1 \times 2}{(t+5)^3}(1) = \frac{2}{(t+5)^3} \qquad \text{...(ii)}$$

Now, from Eqs. (i) and (ii), we get

$$\frac{dv}{dt} = -2v \times \frac{1}{(t+5)}$$

$$= (-2v)(\sqrt{v}) = -2(v)^{3/2}$$

$$\therefore \text{ Acceleration, } a = \frac{dv}{dt} = -2v^{3/2} \Rightarrow a \propto (\text{velocity})^{3/2}$$

36. (*d*) $-h = vt - \frac{1}{2}gt^2$ or $gt^2 - 2vt - 2h = 0$

$$\therefore \qquad t = \frac{2v + \sqrt{4v^2 + 8gh}}{2g} = \frac{v}{g}\left[1 + \sqrt{1 + \frac{2hg}{v^2}}\right]$$

37. (*d*) $x = 2 + t - 3t^2$, $v = \frac{dx}{dt} = 1 - 6t$, velocity will become zero at time to $0 = 1 - 6t_0$ or $t_0 = \frac{1}{6}$ s,

Since, the given time $t = 1$ s is greater than $t_0 = \frac{1}{6}$ s,

distance > | displacement |

Displacement, $S = x_f - x_i$

$$= (2 + 1 - 3) - (2 + 0 - 0) = -2\,\text{m}$$

Distance, $d = |S_{0-t_0}| + |S_{t-t_0}|$

$$= \frac{u^2}{2|a|} + \frac{1}{2}|a|(t - t_0)^2$$

On comparing $v = 1 - 6t$ with $v = u + at$, we have $u = 1\,\text{ms}^{-1}$ and $a = -6\,\text{ms}^{-2}$

$$\therefore \text{ Distance} = \frac{(1)^2}{2 \times 6} + \frac{1}{2} \times 6 \times \left(1 - \frac{1}{6}\right)^2 = 2.1\ \text{m}$$

38. (*b*) $1 = \frac{1}{2}g\,t_1^2$ or $t_1 = \sqrt{\frac{2}{g}}$

$$2 = \frac{1}{2}g\,t^2 \quad \text{or} \quad t = \sqrt{\frac{4}{g}}$$

but $\qquad t_2 = t - t_1 = \sqrt{\frac{4}{g}} - \sqrt{\frac{2}{g}}$

$$\therefore \qquad \frac{t_1}{t_2} = \frac{\sqrt{2/g}}{\sqrt{4/g} - \sqrt{2/g}} = \frac{\sqrt{2}}{2 - \sqrt{2}}$$

$$= \frac{\sqrt{2}\,(2 + \sqrt{2})}{2} = (\sqrt{2} + 1)$$

39. (*c*) $t = \sqrt{\frac{x+a}{b}}$ or $(x + a) = bt^2$ or $x = -a + bt^2$

Comparing this equation with general equation of uniformly accelerated motion,

$$s = s_i + ut + \frac{1}{2}at^2$$

we see that $s_i = -a$, $u = 0$ and acceleration $= 2b$.

40. (*b*) $v = \frac{dx}{dt} = 32 - 6t^2$, $v = 0$ at $t = 2.30$ s

$$a = \frac{dv}{dt} = -12t$$

At 2.30 s $\quad a = -27.6\,\text{ms}^{-2}$

41. (*b*) $x^2 = t^2 + 1$ or $2x \cdot \frac{dx}{dt} = 2t$ or $x \cdot \frac{dx}{dt} = t$

$$\therefore \quad v = \frac{dx}{dt} = \frac{t}{x} = \frac{t}{\sqrt{t^2+1}}$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = \frac{\sqrt{t^2+1} - \frac{t^2}{\sqrt{t^2+1}}}{(t^2+1)} = \frac{1}{(t^2+1)^{3/2}} = \frac{1}{x^3}$$

42. (*d*) Given, $x = ae^{-\alpha t} + be^{\beta t}$

So, velocity, $v = \frac{dx}{dt} = -a\alpha e^{-\alpha t} + b\beta e^{\beta t}$

$$a = \frac{dv}{dt} = a\alpha^2 e^{-\alpha t} + b\beta^2 e^{\beta t} = +\text{ve all time.}$$

$\therefore$ v will go on increasing.

43. (*a*) Here, $u = 0$, $a = g$

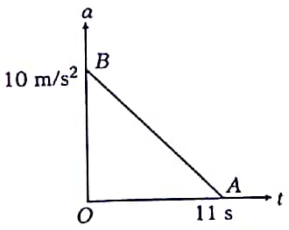
Distance travelled in nth second is given by

$$D_n = u + \frac{a}{2}(2n - 1)$$

$\therefore \quad D_n \propto (2n - 1)$

$\therefore \quad D_1 : D_2 : D_3 : D_4 : D_5 \ldots = 1 : 3 : 5 : 7 : 9 : \ldots$

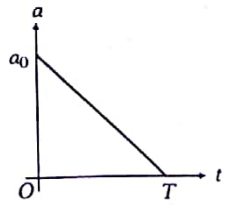
44. (*b*) The area under acceleration time graph gives change in velocity.



As acceleration is zero at the end of 11 s.

i.e., $\quad v_{max} =$ Area of $\Delta OAB = \frac{1}{2} \times 11 \times 10 = 55\,\text{ms}^{-1}$

45. (*a*) Given acceleration-time graph is shown in figure.



Let $a(t = 0) = a_0 =$ constant

From the graph, we have

$$\frac{t}{T} + \frac{a}{a_0} = 1$$

$\Rightarrow \quad a_0 t + aT = a_0 T \quad$ ...(i)

Also, we know that

$$a = \frac{dv}{dt} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we have

$$\frac{a_0 T - a_0 t}{T} = \frac{dv}{dt}$$

Integrating both sides, we get

$$\int_0^t (a_0 T - a_0 t)\,dt = T\int_0^v dv$$

$$\Rightarrow \quad a_0 Tt - \frac{a_0 t^2}{2} = Tv \Rightarrow v = a_0 t - \frac{a_0}{2T}t^2$$

Therefore, v-t graph must be parabolic.

Now, at $t = 0$, $v = 0$

$$t = T, v = \frac{a_0 T}{2} = \text{constant}$$

These conditions are satisfied by graph (a).

46. (*b*) Average velocity in uniformly accelerated motion is given by,

$$v_{av} = \frac{s}{t} = \frac{ut + \frac{1}{2}at^2}{t} = u + \frac{1}{2}at$$

Now, $\quad v_1 = u + \frac{1}{2}at_1$, $\quad v_2 = (u + at_1) + \frac{1}{2}at_2$

and $\quad v_3 = u + a(t_1 + t_2) + \frac{1}{2}at_3$

$\therefore \; v_1 - v_2 : v_2 - v_3 = (t_1 + t_2) : (t_2 + t_3)$

47. (*d*) Motion is first accelerated in positive direction, then uniform in negative direction.

48. (*a*) Given line has positive intercept but negative slope. So its equation can be written as

$$v = -mx + v_0 \left[\text{where, } m = \tan\theta = \frac{v_0}{x_0}\right] \quad \text{...(i)}$$

By differentiating with respect to time, we get

$$\frac{dv}{dt} = -m\frac{dx}{dt} = -mv$$

Now substituting the value of v from Eq. (i), we get

$$\frac{dv}{dt} = -m[-mx + v_0] = m^2x - mv_0$$

$$\therefore \quad a = m^2x - mv_0$$

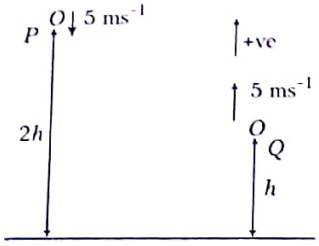
i.e., the graph between a and x should have positive slope but negative intercept on a-axis. So, graph (a) is correct.

49. (*b*) $\frac{dx}{dt} = v = -1 - 2t$

Comparing with $v = u + at$, we have, $u = -1\,\text{ms}^{-1}$ and $a = -2\,\text{ms}^{-2}$

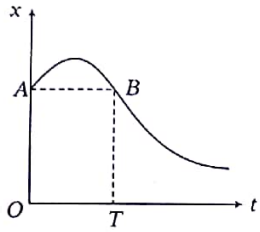
At $t = 0$, $x = 1$ m. Then, u and a both are negative. Hence, x-coordinate of particle will go on decreasing.

50. (c) $-h = 5t - 5t^2$ and $-2h = -5t - 5t^2$



From these two equations we get, $t = 3$ s and h = distance between P and $Q = 30$ m.

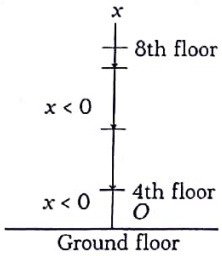
51. (b) In graph (b) for one value of displacement there are two different points of time. Hence, for one time, the average velocity is positive and for other time is equivalent negative.



As there are opposite velocities in the interval 0 to T hence average velocity can vanish in (b). This can be seen in the figure alongside.

Here, $OA = BT$ (same displacement) for two different points of time.

52. (a) As the lift is coming in downward directions displacement will be negative. We have to see whether the motion is accelerating or retarding.



We know that due to downward motion, displacement will be negative. When the lift reaches 4th floor and is about to stop hence, motion is retarding in nature hence, $x < 0$; $a > 0$.

As displacement is in negative direction, velocity will also be negative *i.e.*, $v < 0$.

This can be shown on the adjacent graph.

53. (b) For maximum and minimum displacement we have to keep in mind the magnitude and direction of maximum velocity.

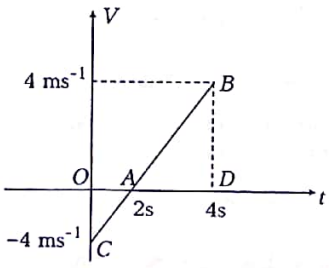
As maximum velocity in positive direction is v_0 maximum velocity in opposite direction is also v_0.

Maximum displacement in one direction $= v_0T$

Maximum displacement in opposite directions $= -v_0T$

Hence, $-v_0T < x < v_0T$.

54. (b) Given, $x = (t-2)^2$



Velocity, $v = \frac{dx}{dt} = \frac{d}{dt}(t-2)^2 = 2(t-2)\ \text{ms}^{-1}$

Acceleration, $a = \frac{dv}{dt} = \frac{d}{dt}[2(t-2)] = 2(1-0) = 2\ \text{ms}^{-2}$

When, $t = 0$; $v = -4\ \text{ms}^{-1}$, $t = 2$ s; $v = 0\ \text{ms}^{-1}$

$t = 4$ s; $v = 4\ \text{ms}^{-1}$

v-t graph is shown in adjacent diagram.

Distance travelled = area of the graph

$= |\text{area } OAC| + \text{area } ABD$

$= \left|\frac{4 \times 2}{2}\right| + \frac{1}{2} \times 2 \times 4 = 8$ m

55. (a) After bailing out from point A parachutist falls freely under gravity. The velocity acquired by it will be v.

From $v^2 = u^2 + 2as = 0 + 2 \times 9.8 \times 50 = 980$

[As $u = 0$, $a = 9.8\ \text{ms}^{-2}$, $s = 50$ m]

At point B, parachute opens and it moves with retardation of $2\ \text{ms}^{-2}$ and reach at ground (Point C with velocity of $3\ \text{ms}^{-1}$).

For the part BC by applying the equation $v^2 = u^2 + 2as$

$v = 3\ \text{ms}^{-1}$, $u = \sqrt{980}\ \text{ms}^{-1}$, $a = -2\ \text{ms}^{-2}$, $s = h$

$\Rightarrow \quad (3)^2 = (\sqrt{980})^2 + 2 \times (-2) \times h$

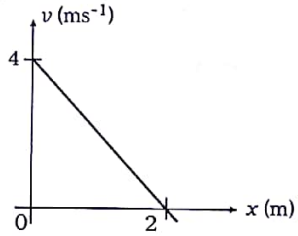
$\Rightarrow \quad 9 = 980 - 4h$

$\Rightarrow \quad h = \frac{980 - 9}{4} = \frac{971}{4} = 242.7 \cong 243$ m

So, the total height by which parachutist bails out

$= 50 + 243 = 293$ m

56. (d) Given velocity (v)-position (x) graph is shown in figure.



We know that acceleration $a = \frac{v\,dv}{dx}$ ...(i)

From the graph, we have

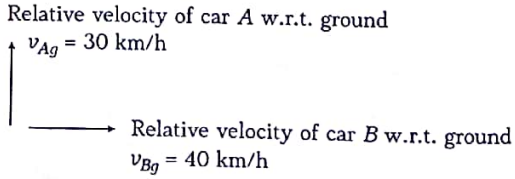
$\frac{x}{2} + \frac{v}{4} = 1 \Rightarrow 2x + v = 4$ ...(ii)

$\Rightarrow \quad \frac{dv}{dx} = -2$ ...(iii)

From Eqs. (i), (ii) and (iii)

$a = (4 - 2x)(-2) = -8 + 4x \Rightarrow a = 4x - 8$

57. (c) Relative velocity of car A w.r.t. ground



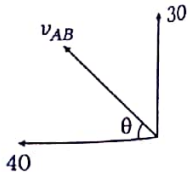
∴ Relative velocity of A w.r.t. B

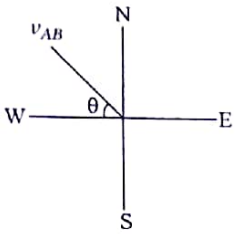
$\mathbf{v}_{AB} = \mathbf{v}_{Ag} - \mathbf{v}_{Bg} = \mathbf{v}_{Ag} + (-\mathbf{v}_{Bg})$

$|\mathbf{v}_{AB}| = \sqrt{(30)^2 + (40)^2} = 50$ km/h

$\tan\theta = \frac{30}{40} = \frac{3}{4}$

$\theta = \tan^{-1}(3/4)$





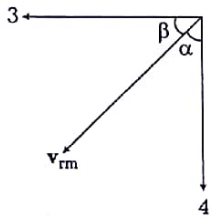
50 km/h, at $\tan^{-1}(3/4)$ N of W

58. (*d*) Relative velocity of man w.r.t. ground $v_{mg} = 3$ m/s

Relative velocity of rain w.r.t. ground, $v_{rg} = 4$ m/s

Relative velocity of rain w.r.t. main $\mathbf{v}_{rm} = ?$

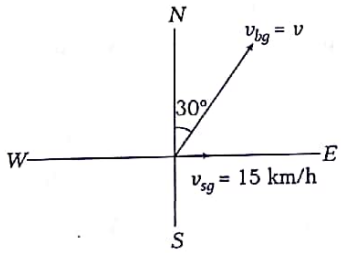
$$\mathbf{v}_{rm} = \mathbf{v}_{rg} - \mathbf{v}_{mg} = \mathbf{v}_{rg} + (-\mathbf{v}_{mg})$$



$$\mathbf{v}_{rm} = 5\,\text{m/s},\ \tan\alpha = \frac{3}{4},\ \tan\beta = \frac{4}{3}$$

$v_{rm} = 5$ m/s at $\tan^{-1}\left(\frac{3}{4}\right)$ with vertical or at $\tan^{-1}\left(\frac{4}{3}\right)$ with horizontal.

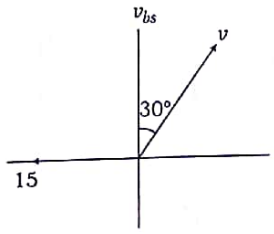
59. (*a*)



where $\mathbf{v}_{bg}$ is the velocity of boat w.r.t. ground and $\mathbf{v}_{sg}$ is the velocity of ship w.r.t. ground.

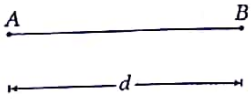
Relative velocity of boat w.r.t. ship v_{bs} is along north.

v_{bs} is resultant of v_{bg} and v_{bg} and v_{sg} in opposite direction.



$$v\cos 30° = v_{bs}$$
$$v\sin 30° = 15 \Rightarrow v = 30\,\text{km/h}$$

60. (*b*)



$u : v_{rg}$ (velocity of river w.r.t. ground)

$v : v_{mr}$ (velocity of man w.r.t. river)

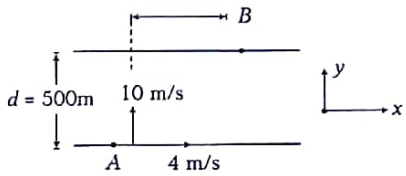
Along the flow, $v_{mg} = v + u,\ 3 = \dfrac{d}{v+u}$ ...(i)

where v_{mg} is velocity of man w.r.t. ground

Opposite to the flow, $v_{mg} = v - u,\ 6 = \dfrac{d}{v-u}$ ...(ii)

$$t_0 = \frac{d}{v} = 4\text{h}$$

61. (*c*)

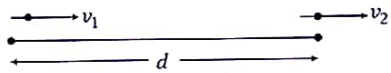


y : displacement = velocity × time

$$500 = 10t \Rightarrow t = 50\text{s}$$

62. (*a*) For the given condition, initial height $h = d$ and velocity of the ball is zero. When the ball moves downward, its velocity increases and it will be maximum, when the ball hits the ground and just after the collision, it becomes half and in opposite direction. As the ball moves upwards, its velocity again decreases and becomes zero at height $d/2$. This explanation match with graph (a).

63. (*c*) Relative velocity of first train with respect to other $= (v_1 - v_2)$



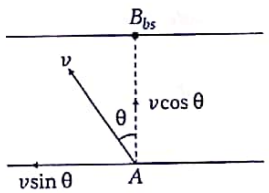
$$(0) = (v_1 - v_2)^2 - 2\alpha\, s$$

where, α = relative retardation

$$s = \frac{(v_1 - v_2)^2}{2\alpha}$$

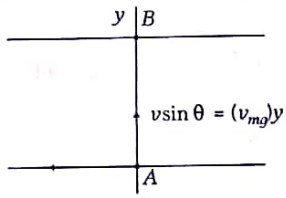
For avoiding collision $d > \dfrac{(v_1 - v_2)^2}{2\alpha}$

64. (*b*)



$u : v_{rg}$ (Velocity of river w.r.t. ground)

$v : v_{mr}$ (velocity of boat w.r.t. river)



$$(v_{mr})_x = (v\sin\theta - u)$$

To solve problems, covert all velocities with respect to the ground.

$$x : (v\sin\theta - u)t = 0 \Rightarrow \sin\theta = \frac{u}{v}$$

$$y : d = v\cos\theta t \Rightarrow t = \frac{d}{v\cos\theta} = \frac{d}{v\sqrt{1-\sin^2\theta}}$$

$$= \frac{d}{v\sqrt{1-\frac{u^2}{v^2}}} = \frac{d}{\sqrt{v^2-u^2}}$$

$$t = 15\,\text{min} = 1\frac{5}{60} = \frac{1}{4}\text{h}$$

$$t = \frac{d}{\sqrt{v^2-u^2}} \Rightarrow \frac{1}{2} = \frac{1}{\sqrt{5^2-u^2}}$$

$$u = 3\text{km/h}$$

65. (*c*) Initial relative velocity $= v_1 - v_2$, Final relative velocity $= 0$

From $\quad v^2 = u^2 - 2as \Rightarrow 0 = (v_1 - v_2)^2 - 2 \times a \times s$

$$\Rightarrow \quad s = \frac{(v_1 - v_2)^2}{2a}$$

If the distance between two cars is s, then collision will take place. To avoid collision $d > s$

$$\therefore \quad d > \frac{(v_1 - v_2)^2}{2a}$$

where, d = actual initial distance between two cars.

66. (*b*) Time taken by first drop to reach the ground $t = \sqrt{\frac{2h}{g}}$

$$\Rightarrow \quad t = \sqrt{\frac{2\times 5}{10}} = 1\text{s}$$

As the water drops fall at regular intervals from a tap therefore time difference between any two drops $= \frac{1}{2}$ s.

In this given time, distance of second drop from the tap

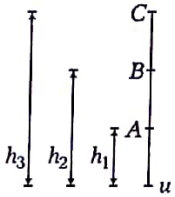
$$= \frac{1}{2}g\left(\frac{1}{2}\right)^2 = \frac{10}{8} = 1.25\text{ m}$$

Its distance from the ground $= 5 - 1.25 = 3.75$ m

67. (*d*) $A \Rightarrow \frac{u^2}{4} - u^2 = -2gh_1$

$B \Rightarrow \frac{u^2}{9} - u^2 = -2gh_2$

$C \Rightarrow \frac{u^2}{16} - u^2 = -2gh_3$



$$\therefore \quad AB = \frac{u^2}{2g}\left(\frac{8}{9} - \frac{3}{4}\right) = \frac{u^2}{2g}\cdot\frac{5}{36}$$

$$BC = \frac{u^2}{2g}\left(\frac{15}{16} - \frac{8}{9}\right) = \frac{u^2}{2g}\cdot\frac{7}{144}$$

$$\therefore \quad \frac{AB}{BC} = \frac{5}{36}\times\frac{144}{7} = \frac{20}{7}$$

68. (*a*) Acceleration $f = f_0\left(1 - \frac{t}{T}\right)$

$$\frac{dv}{dt} = f_0\cdot\left(1 - \frac{t}{T}\right) \qquad \left(\because f = \frac{dv}{dt}\right)$$

$$\Rightarrow \quad dv = f_0\cdot\left(1 - \frac{t}{T}\right)dt$$

$$\Rightarrow \quad \int dv = \int f_0\left(1 - \frac{t}{T}\right)dt$$

$$v = f_0 t - f_0\frac{t^2}{2T} + c \qquad \text{...(i)}$$

where, c is constant

when $t = 0, v = 0$ thus from Eq. (i) $c = 0$

$$v = f_0 t - \frac{f_0}{T}\cdot\frac{t^2}{2} \qquad \text{...(ii)}$$

As $\quad f = f_0\left(1 - \frac{t}{T}\right)$

when, $f = 0$

$$\therefore \quad f_0\left(1 - \frac{t}{T}\right) = 0; \quad f_0 \neq 0$$

$$\therefore \quad t = T$$

Putting $t = T$ in Eq. (ii)

$$v = f_0 T - \frac{f_0}{T}\cdot\frac{T^2}{2} = \frac{f_0 T}{2}$$

69. (*c*) The position x of a particle w.r.t. to time t along X-axis

$$x = 9t^2 - t^3 \qquad \text{...(i)}$$

Differentiating Eq. (i) w.r.t. time, we get speed, *i.e.*,

$$v = \frac{dx}{dt} = \frac{d}{dt}(9t^2 - t^3) \Rightarrow v = 18t - 3t^2 \qquad \text{...(ii)}$$

Again differentiating Eq. (ii) w.r.t. time, we get acceleration, *i.e.*,

$$a = \frac{dv}{dt} = \frac{d}{dt}(18t - 3t^2) \Rightarrow a = 18 - 6t \qquad \text{...(iii)}$$

Now, when speed of particle is maximum, its acceleration is zero.

$$a = 0 \Rightarrow 18 - 6t = 0 \Rightarrow t = 3\text{ s}$$

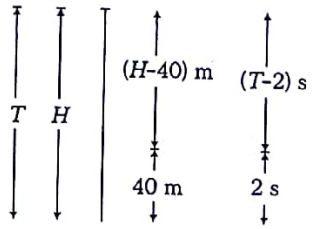
Putting in Eq. (i), we obtain position of particle at that time

$$x = 9(3)^2 - (3)^3 = 9(9) - 27 = 81 - 27 = 54\text{ m}$$

70. (*b*) For P, $30 = 15 + at$ or $at = 15\text{ ms}^{-1}$

For Q, $v = 20 - at$ or $v = 20 - 15 = 5\text{ ms}^{-1}$

71. (*b*) Let height of minaret be H and body take time T to fall from top to bottom.



$$H = \frac{1}{2}gT^2 \qquad \text{...(i)}$$

In last 2 s body travels distance of 40 m so in $(T - 2)$ second distance travelled $= (H - 40)$ m

$$(H - 40) = \frac{1}{2}g(T - 2)^2$$

By solving Eqs. (i) and (ii), $T = 3$ s and $H = 45$ m

72. (c) $S_n = \frac{1}{2} g \cos\theta (2n-1)$,

$S_{n+1} = \frac{1}{2} g \cos\theta \{2(n+1)-1\}$

$\frac{S_n}{S_{n+1}} = \frac{2n-1}{2n+1}$

73. (d) $v = \alpha\sqrt{x} \Rightarrow \frac{dx}{dt} = \alpha\sqrt{x}$

$\Rightarrow \quad \frac{dx}{\sqrt{x}} = \alpha dt$

By integrating both sides

$\int x^{-1/2} dx = \int \alpha dt$

$\frac{\sqrt{x}}{1/2} = \alpha t \Rightarrow \sqrt{x} = \frac{1}{2}\alpha t$

$\Rightarrow \quad x = \frac{1}{4}\alpha^2 t^2$

$\therefore \quad x \propto t^2$

74. (d) Let height of tower is h and body takes t time to reach to ground when it fall freely.

$\therefore \quad h = \frac{1}{2} g t^2$ ...(i)

In last second *i.e.*, tth second body travels $= 0.36h$

It means in rest of the time *i.e.*, in $(t-1)$ second it travels

$= h - 0.36h = 0.64h$

Now applying equation of motion for $(t-1)$ second

$0.64h = \frac{1}{2} g (t-1)^2$ ...(ii)

From Eqs. (i) and (ii) we get, $t = 5$ s and $h = 125$ m

75. (d) Relative to lift,

$u_r = 0, a_r = (9.8 + 1.2) = 11 \text{ ms}^{-2}$ $s_r = \frac{1}{2} a_r t^2$

$\therefore \quad 2.7 = \frac{1}{2} \times 11 \times t^2 \Rightarrow t = \sqrt{\frac{5.4}{11}}$ s

B Medical entrance special format questions

Assertion and reason

1. (d) $a = \frac{v_f - v_i}{\Delta t} = \frac{dv}{dt}$ *i.e.*, direction of acceleration is same as that of change in velocity vector, or in the direction of Δv.

2. (d) If speed varies, then velocity will definitely vary. When a particle is thrown upwards, at highest point $a \neq 0$ but $v = 0$.

5. (c) When a particle is released from rest, $v = 0$ but $a \neq 0$

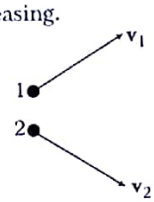
7. (a) $a = \frac{dv}{dt}$ = slope of v-t graph.

Perpendicular to t-axis, slope $= \infty$ $\therefore a = \infty$

8. (b) v-$t \xrightarrow{\text{Intergration}} s$-$t$; s-$t \xrightarrow{\text{Differentiation}} v$-$t$

9. (d) v_1 and v_2 = constant

But distance is increasing.



10. (c) Slope of s-t graph = velocity = positive

At $t = 0$, $s \neq 0$. Further at $t = t_0 : s = 0, v \neq 0$.

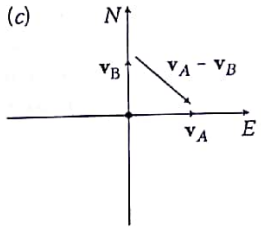
11. (d) Slope is negative. Therefore velocity is negative. Slope (therefore velocity) is increasing in magnitude. Therefore acceleration is also negative.

12. (b) $a = \frac{F}{m} = 1\hat{j}$ N

$s = ut + \frac{1}{2} a t^2 = 20\hat{i} + 8\hat{j}$

$\therefore \quad |s| = 21.5$ m

13. (c)



14. (c) On a curved path, distance > displacement

$\therefore$ average speed > average velocity

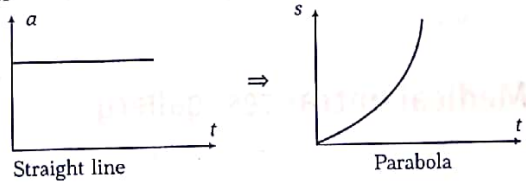
Further, average velocity $= \frac{\text{Total displacement}}{\text{Total time}}$

15. (a) Last second of upward journey is the first second of downward journey.

$\therefore$ Distance travelled $= \frac{1}{2} g t^2 = \frac{1}{2} \times 9.8 \times (1)^2 = 4.9$ m

16. (b) By differentiating a-t equation two times, we will get s-t equation.

Further



17. (c) Ascending means velocity is in upward direction. Speed is decreasing. It means acceleration is downwards. Further, the body moves in the direction of velocity.

18. (a) Distance = displacement for motion in straight line.

19. (a) Average velocity $= \frac{\text{Displacement}}{\text{Time}} = \frac{\text{Area of } v\text{-}t \text{ graph}}{\text{Time}}$

$= \frac{(1/2) v_m t_0}{t_0} = \frac{1}{2} v_m$

Match the columns

1. $\frac{d|\mathbf{v}|}{dt}$ is rate of change of speed of the particle.

   $\frac{d|\mathbf{r}|}{dt}$ in the rate by which distance of particle from the origin is changing.

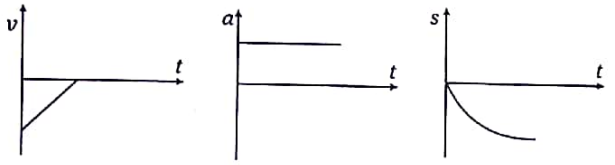
2. At $t = 3$ s, $s = 10 + 20(3) - 5(3)^2 = 25$ unit

$$v = \frac{ds}{dt} = 20 - 10t$$

At $t = 4$ s, $v = 20 - 10(4) = -20$ unit

$$a = \frac{dv}{dt} = -10 \text{ units}$$

3. Corresponding v-t, a-t and s-t graphs are as shown below.



4. With constant positive acceleration, speed will increase when velocity is positive, speed will decrease, if velocity is negative. Similarly, with constant negative acceleration speed will increase if velocity is negative and speed will decrease if velocity is positive.

5. $v_i = +10 \text{ ms}^{-1}$ and $v_f = 0$

$$\therefore \quad \Delta v = v_f - v_i = -10 \text{ ms}^{-1}$$

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{-10}{6} = \frac{-5}{3} \text{ ms}^{-2}$$

Total displacement = area under v-t graph (with sign)

$$= \frac{1}{2} \times 10 \times 2 - \frac{1}{2} \times 2 \times 10 - \frac{1}{2} \times 2 \times 10$$

$$= -10 \text{ m}$$

And acceleration = slope of v-t graph

$$= -\frac{10}{2} = -5 \text{ ms}^{-2}$$

6. For M, slope of s-t graph is positive and increasing. Therefore, velocity of the particle is positive and increasing. Hence, it is A type motion. Similarly, N, P and Q can be observed from the slope.

C) Medical entrances' gallery

1. (*b*) Velocity of the particle is given as

$$v = At + Bt^2$$

where, A and B are constants.

$$\Rightarrow \quad \frac{dx}{dt} = At + Bt^2 \qquad \left[\because v = \frac{dx}{dt}\right]$$

$$\Rightarrow \quad dx = (At + Bt^2)\,dt$$

Integrating both sides, we get

$$\int_{x_1}^{x_2} dx = \int_1^2 (At + Bt^2)dt$$

$$\Rightarrow \quad \Delta x = x_2 - x_1 = A\int_1^2 t\,dt + B\int_1^2 t^2 dt$$

$$= A\left[\frac{t^2}{2}\right]_1^2 + B\left[\frac{t^3}{3}\right]_1^2$$

$$= \frac{A}{2}(2^2 - 1^2) + \frac{B}{3}(2^3 - 1^3)$$

$\therefore$ Distance travelled between 1s and 2s is

$$\Delta x = \frac{A}{2} \times (3) + \frac{B}{3}(7) = \frac{3A}{2} + \frac{7B}{3}$$

2. (*b*) Given, $\quad v = \beta x^{-2n}$

$$a = \frac{dv}{dt} = \frac{dx}{dt} \cdot \frac{dv}{dx}$$

$$\Rightarrow \quad a = v\frac{dv}{dx} = (\beta x^{-2n})(-2n\beta x^{-2n-1})$$

$$\Rightarrow \quad a = -2n\beta^2 x^{-4n-1}$$

3. (*c*) Let the ball hit water in t second.

For first ball, $\quad s = ut + \frac{1}{2}at^2$

$$122.5 = 0 + \frac{1}{2} \times 9.8 \times t^2 = 4.9\,at^2$$

$$t + \sqrt{\frac{122.5}{4.9}} = \sqrt{25} = 5 \text{ s}$$

For second ball,

$$122.5 = u(5 - 2) + \frac{1}{2} \times 9.8 \times (5 - 2)^2$$

$$= 3u + 44.1$$

$$\Rightarrow \quad 3u = 122.5 - 44.1$$

$$3u = 78.4$$

$$\Rightarrow \quad u = 26.1 \text{ ms}^{-1}$$

4. (*c*) Given, upward velocity of ball = 25.2 ms^{-1}

Height attained by the ball,

$$H = \frac{u^2}{2g} = \frac{25.2 \times 25.2}{2 \times 9.8} = 32.4 \text{ m}$$

Now, time taken by the ball to attain 32.4 m is

$$t = \frac{u}{g} = \frac{25.2}{9.8} = 2.57\text{s}$$

5. (*b*) The coordinates of the particle is given as

$$x(t) = t^3 - 32t \text{ and } y(t) = 5t^2 + 12$$

where, t = time

The velocities at time t,

$$v_x(t) = \frac{dx(t)}{dt} = 3t^2 - 32$$

$$\Rightarrow \quad a_x(t) = \frac{dv_x(t)}{dt} = 6t - 0 = 6t$$

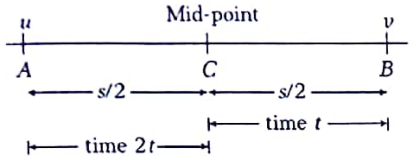
At $t = 3$, $a_x = 6 \times 3 = 18$

and $\quad v_y(t) = \frac{dy(t)}{dt} = 10t + 0 = 10t$

At $t = 3$, $a_y = 10$

Thus, $\quad a = a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}} = 18\hat{\mathbf{i}} + 10\hat{\mathbf{j}}$

6. (*c*) According to question, the diagram can be drawn as



For motion from A to C

$$\frac{s}{2} = 2ut + \frac{1}{2}a \times (2t)^2 = 2ut + 2at^2$$

Also, for motion from C to B. (Let initial velocity be v')

$$\frac{s}{2} = (u + 2at)t + \frac{1}{2}at^2 \qquad (\because v' = u + 2at)$$

$$= ut + 2at^2 + \frac{1}{2}at^2$$

Now, $\quad 2ut + 2at^2 = ut + 2at^2 + \frac{1}{2}at^2 \Rightarrow u = \frac{at}{2}$

For overall motion,

$$v = u + 3at = \frac{at}{2} + 3at = \frac{7at}{2} = 7u \qquad \left(\because u = \frac{at}{2}\right)$$

7. (*e*) At first, the slope is decreasing therefore, the motion is retarded. Finally, the displacement becomes constant and the body stops.

8. (*d*) $u = 0, v = 180\ \text{kmh}^{-1} = 50\ \text{ms}^{-1}$

Time taken, $t = 10\text{s}, a = \frac{v-u}{t} = \frac{50}{10} = 5\ \text{ms}^{-2}$

Distance covered by the car $\quad s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 5 \times (10)^2$

$$= \frac{500}{2} = 250\ \text{m}$$

9. (*d*) The slope of velocity-time graph gives acceleration,

i.e., $\quad \frac{a_A}{a_B} = \frac{\tan\theta_1}{\tan\theta_2}$

$\Rightarrow \quad \frac{a_A}{a_B} = \frac{\tan 25°}{\tan 50°}$

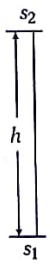
10. (*d*) Average velocity, $v_{av} = \frac{\text{Displacement } (\Delta r)}{\text{Time taken } (\Delta t)}$

Displacement in $t = 0$ to 5s is

$$\Delta r = (13 - 2)\hat{i} + (14 - 3)\hat{j} = 11\hat{i} + 11\hat{j}$$

$$v_{av} = \frac{\Delta r}{\Delta t} = \frac{11}{5}(\hat{i} + \hat{j})$$

11. (*a*) The position of an object is always expressed w.r.t. some reference point. If the initial position of an object w.r.t. a reference points is s_1 and after sometime, it changes to s_2, then the magnitude of the displacement of the object is $s_2 - s_1$. If the value of s_2 is greater than s_1, then the displacement is positive, but if the value of s_2 is less than s_1, then the displacement is negative.



So, its displacements is zero ($\because s_1 = s_2 = h$)

12. (*a*) Let v_{PG} = velocity of police w.r.t. ground

v_{TG} = velocity of thief w.r.t. ground

v_{TP} = velocity of thief w.r.t. police $= v_{TG} - v_{PG}$

$$= \left(\frac{153 - 45}{18}\right) \times 5 = 30\ \text{ms}^{-1}$$

v_{BC} = velocity of bullet w.r.t. car

$= 180 - 30 = 150\ \text{ms}^{-1}$

13. (*b*) We know that $s_{nth} = u + \frac{1}{2}a(2n - 1)$

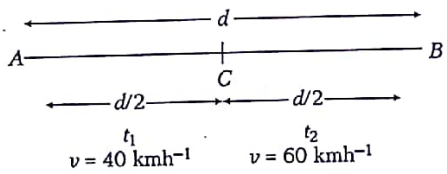
$$s_{3rd} = 0 + \frac{1}{2}a(2 \times 3 - 1) = \frac{5}{2}a \qquad (\text{for } n = 3\text{s})$$

$$s_{4rd} = 0 + \frac{1}{2}a(2 \times 4 - 1) = \frac{7}{2}a \qquad (\text{for } n = 4\text{s})$$

So, the percentage increase $= \frac{s_{4th} - s_{3rd}}{s_{3rd}} \times 100$

$$= \frac{\frac{7}{2}a - \frac{5}{2}a}{\frac{5}{2}a} \times 100 = \frac{\frac{2a}{2}}{\frac{5}{2}a} \times 100 = 2 \times 20 = 40\%$$

14. (*b*) As from the question,



Average speed of the car v_{av} is given by

$$\frac{\text{Total distance travelled by the body } (d)}{\text{Total time taken } (t_1 + t_2)} \qquad \text{...(i)}$$

So, now time taken in 1st half of the distance,

$$t_1 = \text{Time} = \frac{\text{Distance}}{\text{Velocity}} = \frac{d}{2 \times 40}$$

$$\left(\because \text{distance} = \frac{d}{2}, \text{velocity} = 40\ \text{kmh}^{-1}\right)$$

$$\Rightarrow \quad t_1 = \frac{d}{80}$$

Time taken for 2nd half, $t_2 = \frac{d}{2 \times \text{velocity}}$

$$\left(\because \text{distance} = \frac{d}{2}, \text{velocity} = 60\ \text{kmh}^{-1}\right)$$

So, $\quad t_2 = \frac{d}{2 \times 60} = \frac{d}{120}$

Now, average velocity, $v_{av} = \frac{d}{\frac{d}{80} + \frac{d}{120}}$ [From Eq. (i)]

$$= \frac{d \times 80 \times 120}{200d} = 48\ \text{kmh}^{-1}$$

15. (*a*) The distance covered in 2 s, $x = \frac{1}{2}a(2)^2 = 2a$

The distance covered in next 2 s,

$$y = \frac{1}{2}a(4)^2 - \frac{1}{2}a(2)^2 = 6a$$

Now, $\quad x/y = \frac{2a}{6a} = \frac{1}{3} \Rightarrow y = 3x$

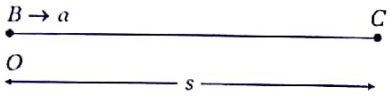
16. (*c*) $A \rightarrow v_A$ (constant velocity)

$B \rightarrow a$ (constant acceleration)

C be the fixed point at which both A and B meet in time t

Now, $s = v_A t$ and $v_B = at$

$A \rightarrow v_A = v$



$$s = \frac{1}{2}at^2 = v_A t$$

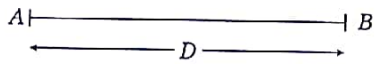
$$a = \frac{2v_A}{t} \Rightarrow v_B = at = 2v_A$$

v_{BA} = velocity of B w.r.t. A

$\Rightarrow \quad v_{BA} = v_B - v_A = 2v_A - v_A = v_A$

$\Rightarrow \quad v_{BA} = v \qquad (\because v_A = v)$

17. (*b*) v_{AB} = speed of car from A to B



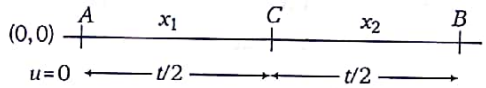
$v_{AB} = 30 \text{ kmh}^{-1}$

v_{BA} = speed of car from B to A

$v_{BA} = 20 \text{ kmh}^{-1}$

$$v_{av} = \frac{2v_{AB}\, v_{BA}}{v_{AB} + v_{BA}} = 24 \text{ kmh}^{-1}$$

18. (*d*)



a = constant

Let A be the original point

From equation of motion,

$$x = ut + \frac{1}{2}at^2 = \frac{1}{2}at^2 \qquad (\because u = 0)$$

Here, x is the position of the particle.

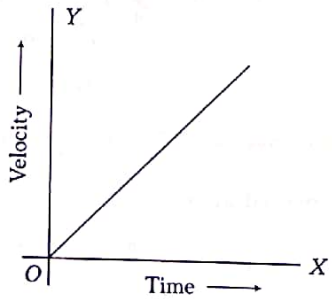
For point C, $\quad x_1 = \frac{1}{2}a(t/2)^2 \Rightarrow x_1 = \frac{1}{8}at^2 \qquad$ ...(i)

For point B, $\quad x_2 = \frac{1}{2}a(t)^2 \qquad$ ...(ii)

From Eqs. (i) and (ii), we get

$$2x_2 = 8x_1 \Rightarrow x_2 = 4x_1$$

19. (*d*) The acceleration of a moving body is found from the slope of velocity-time graph.



$$a = \frac{dv}{dt} = \text{slope of } v\text{-}t \text{ curve}$$

20. (*b*) The relation between h_1, h_2 and h_3 is

$$h_1 = \frac{h_2}{3} = \frac{h_3}{5} \Rightarrow h_1 : h_2 : h_3 = 1 : 3 : 5$$

21. (*b*) When velocity of a body changes by equal amount in equal intervals of time are same, then the body is said to have uniform acceleration, this holds true for straight line motion.

22. (*c*) Velocity-time graph gives the instantaneous value of velocity at any instant. For non-uniformly accelerated motion, v-t graph is non-linear.

23. (*a*) Given, velocity of water $v_w = 4 \text{ ms}^{-1}$

Velocity of person $v_p = 5 \text{ ms}^{-1}$, width of the river = 84.6 m

We know that time taken, $t = \dfrac{s}{\sqrt{v_w^2 - v_p^2}}$

$$\Rightarrow \quad t = \frac{84.6}{\sqrt{25 - 16}}$$

$$\Rightarrow \quad t = \frac{84.6}{3} \cong 28.25$$

24. (*b*) The initial velocity of the body

$$v^2 = 2gh$$

$$u = \sqrt{2gh} = \sqrt{2 \times 10 \times 125} = \sqrt{2500} = 50 \text{ ms}^{-1}$$

The final velocity from equation of motion

$$v^2 = u^2 + 2gh$$

$$v = \sqrt{(50)^2 + 2 \times 10 \times 55}$$

$$v = \sqrt{2500 + 1100} = \sqrt{3600} = 60 \text{ ms}^{-1}$$

25. (*c*) From first equation of motion, $v = u + at$ consider $u = 0$ and v = m/s towards east wards and then consider $u = 5$ and $v = 5$ m/s towards northwards, we get a and a_2 and taking average, we get

$$a = \sqrt{2} \text{ ms}^{-2} \text{ (North-west)}$$

26. (*c*) From graph, velocity of robber's car = 10 ms^{-1}

Let police car crosses it after t second.

Distance travelled by robber's car = 10 t m

Police car is moving with a constant acceleration of 1 ms^{-2} as it attains a velocity of 10 ms^{-1} in 10 s after starting from rest.

Distance travelled in t second $= \frac{1}{2} \cdot a \cdot t^2 = \frac{1}{2}t^2$

When the police car crosses the robber's car, distance travelled by the both cars should be same from the starting point of chase.

$$\therefore \quad \frac{1}{2}t^2 = 10\,t \Rightarrow t = 20 \text{ s}$$

27. (*b*) Initial speed, $u = 1 \text{ kms}^{-1} = 1000 \text{ ms}^{-1}$

Final speed, $v = 9 \text{ kms}^{-1} = 9000 \text{ ms}^{-1}$

By using the relation,

$$v^2 = u^2 + 2as$$

$$(9000)^2 = (1000)^2 + 2 \times a \times 4 \Rightarrow a = 10^7 \text{ ms}^{-2}$$

$\therefore$ The time for which the particle remains in the tube

$$v = u + at$$

$$\Rightarrow \quad t = \frac{v-u}{a} = \frac{9000-1000}{10^7} = 8 \times 10^{-4}\ \text{s}$$

28. (*c*) Let t second be the time of flight of the first body after meeting, then $(t-4)$ second will be the time of flight of the second body.

Since, $\quad h_1 = h_2$

$$\therefore \quad 98t - \frac{1}{2}gt^2 = 98\,(t-4)\,g\,(t-4)^2$$

On solving, $t = 12$ s

29. (*c*) Given,

$$r_1(t) = 3t\,\hat{\mathbf{i}} + 4t^2\,\hat{\mathbf{j}}$$

$$\therefore \quad \frac{dr_1}{dt} = 3\hat{\mathbf{i}} + 8t\hat{\mathbf{j}}$$

At $\quad t = 1$ s

$$v_1 = \frac{dr_1}{dt} = 3\hat{\mathbf{i}} + 8\hat{\mathbf{j}}$$

Again, $\quad \mathbf{r}_2(t) = 4t^2\hat{\mathbf{i}} + 3t\hat{\mathbf{j}}$

$$\frac{d\mathbf{r}_2}{dt} = 8t\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$$

At $\quad t = 1$ s

$$v_2 = \frac{d\mathbf{r}_2}{dt} = 8\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$$

Relative velocity $= v_1 - v_2 = -5\hat{\mathbf{i}} + 5\hat{\mathbf{j}}$

$$= \sqrt{(5)^2 + (5)^2} = 5\sqrt{2}\ \text{ms}^{-1}$$

30. (*d*) Given, $x = 8 + 12t - t^3$

We know, $v = \dfrac{dx}{dt}$ and $a = \dfrac{dv}{dt}$

So, $\quad v = 12 - 3t^2$

and $\quad a = -6t$

At $\quad t = 2$ s

$$v = 0 \text{ and } a = -6t$$

$$\Rightarrow \quad a = -12\ \text{ms}^{-2}$$

So, retardation of the particle $= 12\ \text{ms}^{-2}$.

31. (*b*) Let after time t, car catches the scooter and the distance travelled by scooter in time t

$$x = \frac{1}{2} \times (1) \times t^2 = \frac{t^2}{2} \quad \text{...(i)}$$

The distance travelled by car in time t

$$x + 150 = \frac{1}{2} \times 2 \times t^2 = t^2 \quad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get

$$t = \sqrt{300}\ \text{s}$$

32. (*d*) Distance, $x = b_0 + b_1 t + b_2 t^2$

$$\text{Velocity, } v = \left(\frac{dx}{dt}\right) = b_1 + 2b_2 t$$

$$\text{Acceleration, } \alpha = \frac{d^2x}{dt^2} = 2\,b_2$$

CHAPTER

04

Motion in a plane and projectile motion

In this chapter, we will study motion of those particles which do not move in a straight line rather, they move in a plane. In our daily life examples of motion in a plane are motion of a football, circular motion, motion of a stone fired from a gulel, etc.

Inside

1) Motion in a plane (Two dimensional motion)
- Position vector
- Velocity vector
- Displacement vector
- Acceleration vector

2) Projectile
- Projectile motion
- Projectile projected obliquely on the surface of the earth
- Projectile fire at an angle with the vertical

3) Projectile from a point above the ground
- Shooting a freely falling target

1 Motion in a plane (Two dimensional motion)

Motion of an object is called two dimensional motion when two of the coordinates (x-y, y-z or z-x) from the three coordinates (x, y, z) are required to specify the position of the object in space, changes with time. In two dimensional motion the object moves in xy-plane, yz-plane or zx-plane therefore it is called motion in a plane. *e.g.,* Projectile motion, circular motion, etc.

When an object moves in a plane or in two dimensional motion, different physical quantities related with it changes with time, *e.g.,* Position vector, displacement vector, velocity vector and acceleration vector. In xy-plane, these physical quantities are studied in terms of their x, y-components.

Position vector

A vector that extends from a reference point to the point at which particle is located is called position vector.

Let **r** be the position vector of a particle P located in a plane with reference to the origin O in xy-plane as shown in figure alongside.

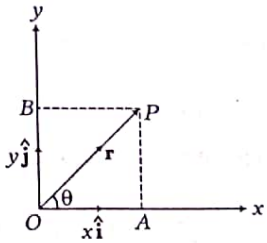


Fig. 4.1 Representation of position vector

$$\text{OP} = \text{OA} + \text{OB}$$

Position vector, $\boxed{\mathbf{r} = x\hat{i} + y\hat{j}}$

Direction of this position vector **r** is given by the angle θ with x-axis, where $\tan\theta = \left(\frac{y}{x}\right)$

$$\Rightarrow \quad \boxed{\theta = \tan^{-1}\left(\frac{y}{x}\right)}$$

In three dimensions, the position vector is represented as

$$\boxed{\mathbf{r} = x\hat{i} + y\hat{j} + z\hat{k}}$$

Example 4.1 *A particle moves in a plane such that its coordinates changes with time as $x = at$ and $y = bt$, where a and b are constants. Find the position vector of the particle and its direction at any time t.*

Sol. Coordinates of the particle are $x = at$ and $y = bt$

∴ Position vector of the particle at any time t is

$$\mathbf{r} = x\hat{i} + y\hat{j} = (at)\hat{i} + (bt)\hat{j}$$

Direction of **r**, $\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{bt}{at}\right) = \tan^{-1}\left(\frac{b}{a}\right)$

Displacement vector

Consider a particle moving in xy-plane with a uniform velocity **v** and point O as an origin for measuring time and position of the particle. Let, the particle be at positions A and B at timings t_1 and t_2 respectively. The position vectors are $\text{OA} = \mathbf{r}_1$ and $\text{OB} = \mathbf{r}_2$.

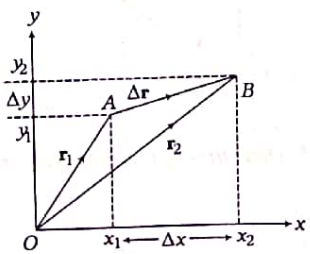


Fig. 4.2 Representation of displacement vector

Then, the displacement of the object in time interval $(t_2 - t_1)$ is **AB**. From triangle law of vector addition, we have

$$\text{OA} + \text{AB} = \text{OB}$$

$$\Rightarrow \quad \text{AB} = \text{OB} - \text{OA}$$

$$\text{AB} = \mathbf{r}_2 - \mathbf{r}_1 \quad \text{...(i)}$$

If the coordinates of the particle at points A and B are (x_1, y_1) and (x_2, y_2), then

$$\therefore \quad \mathbf{r}_1 = x_1\hat{i} + y_1\hat{j} \quad \text{and} \quad \mathbf{r}_2 = x_2\hat{i} + y_2\hat{j}$$

Substituting the values of $\mathbf{r}_1$ and $\mathbf{r}_2$ in Eq. (i), we have

$$\text{AB} = (x_2\hat{i} + y_2\hat{j}) - (x_1\hat{i} + y_1\hat{j})$$

Displacement, $\text{AB} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$

Displacement, $\boxed{\Delta\mathbf{r} = \Delta x\hat{i} + \Delta y\hat{j}}$

$\Rightarrow$ Magnitude of displacement, $|\Delta\mathbf{r}| = \sqrt{(\Delta x)^2 + (\Delta y)^2}$

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

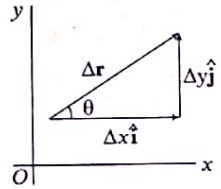


Fig. 4.3 Component of displacement

Direction of the displacement vector **Δr** is given by

$$\tan\theta = \frac{\Delta y}{\Delta x} \Rightarrow \boxed{\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right)}$$

where θ = angle made by **Δr** with x-axis.

Similarly, in three dimensions the displacement can be represented as

$$\boxed{\Delta\mathbf{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}}$$

Note Magnitude of displacement (Δr) between two points is always less than or equal to distance (s) between corresponding points.

i.e., $\Delta r \le s$

Example 4.2 *An object moves from position (3,4) to (6,5) in the xy-plane. Find the magnitude and direction of displacement vector of the particle.*

Sol. Positions of the particle are

$$\mathbf{r}_1 = x_1\hat{i} + y_1\hat{j} = 3\hat{i} + 4\hat{j} \quad \text{and} \quad \mathbf{r}_2 = x_2\hat{i} + y_2\hat{j} = 6\hat{i} + 5\hat{j}$$

∴ Displacement vector, $\Delta\mathbf{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$

$$= (6 - 3)\hat{i} + (5 - 4)\hat{j} = 3\hat{i} + \hat{j}$$

∴ Magnitude of displacement vector,

$$|\Delta\mathbf{r}| = \sqrt{(3)^2 + (1)^2} = \sqrt{10}$$

Direction of **Δr** with x-axis,

$$\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right) = \tan^{-1}\left(\frac{1}{3}\right) \approx 18.43°$$

Instantaneous acceleration, $\mathbf{a} = a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}}$
where,

a_x = magnitude of instantaneous acceleration in x-direction $= \dfrac{dv_x}{dt}$

a_y = magnitude of instantaneous acceleration in y-direction $= \dfrac{dv_y}{dt}$

The magnitude of instantaneous acceleration is given by

$$\boxed{a = \sqrt{a_x^2 + a_y^2}}$$

If acceleration **a** makes an angle θ with X-axis, then

$$\tan\theta = \frac{a_y}{a_x} \Rightarrow \boxed{\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right)}$$

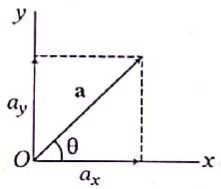


Fig. 4.7 Direction of instantaneous acceleration

In three dimensions, we can write

$$\mathbf{a} = a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}} + a_z\hat{\mathbf{k}}$$

Example 4.6 *The position of a particle is given by*

$$\mathbf{r} = 3t\hat{\mathbf{i}} + 2t^2\hat{\mathbf{j}} + 8\hat{\mathbf{k}}$$

where, t is in seconds and the coefficients have the proper units for r to be in metres.

(i) Find v (t) and a (t) of the particles.

(ii) Find the magnitude and direction of v(t) and a(t) at t = 1 s.

Sol. Position of particle, $\mathbf{r} = 3t\hat{\mathbf{i}} + 2t^2\hat{\mathbf{j}} + 8\hat{\mathbf{k}}$

The formula used

(i) $v(t) = \dfrac{dr}{dt}$ and $a(t) = \dfrac{dv}{dt}$

(ii) $v(t) = \sqrt{v_x^2 + v_y^2}$ and $\theta = \tan^{-1}\left(\dfrac{v_y}{v_x}\right)$

$$\therefore \quad \mathbf{v}(t) = \frac{d}{dt}(3t\hat{\mathbf{i}} + 2t^2\hat{\mathbf{j}} + 8\hat{\mathbf{k}}) = 3\hat{\mathbf{i}} + 4t\hat{\mathbf{j}}$$

$$\mathbf{a}(t) = \frac{dv}{dt} = 4\hat{\mathbf{j}}$$

At $t = 1$ s

Velocity, $v(t) = 3\hat{\mathbf{i}} + 4t\hat{\mathbf{j}}$

$$\therefore \quad v = \sqrt{(3)^2 + (4)^2} = 5 \text{ ms}^{-1}$$

$$\Rightarrow \text{Direction of } v(t) = \theta = \tan^{-1}\left(\frac{4}{3}\right) = 53° \text{ with } X\text{-axis}$$

Direction of $a(t)$, at $t = 1$ s

$$\theta' = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{4}{0}\right) = \tan^{-1}(\infty) = 90° \text{ with } X\text{-axis}$$

Motion in plane with uniform acceleration

A body is said to be moving with uniform acceleration if its velocity vector suffers the same change in the same interval of time however small.

Let an object is moving in xy-plane and its acceleration **a** is constant. At time $t = 0$, the velocity of an object be $\mathbf{v}_0$ (say) and **v** be the velocity at time t.

According to definition of average acceleration, we have

$$\mathbf{a} = \frac{\mathbf{v} - \mathbf{v}_0}{t - 0} = \frac{\mathbf{v} - \mathbf{v}_0}{t} \Rightarrow \boxed{\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t}$$

In terms of rectangular components, we can express it as

$$v_x = v_{0x} + a_x t \quad \text{and} \quad v_y = v_{0y} + a_y t$$

It can be concluded that each rectangular component of velocity of an object moving with uniform acceleration in a plane depends upon time as if it were the velocity vector of one-dimensional uniformly accelerated motion.

Path of particle under constant acceleration

Now, we can also find the position vector (**r**). Let $\mathbf{r}_0$ and **r** be the position vectors of the particle at time $t = 0$ and $t = t$ and their velocities at these instants be $\mathbf{v}_0$ and **v** respectively. Then, the average velocity is given by

$$\mathbf{v}_{av} = \frac{\mathbf{v}_0 + \mathbf{v}}{2}$$

Displacement is the product of average velocity and time interval.

It is expressed as

$$\mathbf{r} - \mathbf{r}_0 = \left(\frac{\mathbf{v} + \mathbf{v}_0}{2}\right)t = \left[\frac{(\mathbf{v}_0 + \mathbf{a}t) + \mathbf{v}_0}{2}\right]t$$

$$\Rightarrow \quad \mathbf{r} - \mathbf{r}_0 = \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$$

$$\Rightarrow \quad \boxed{\mathbf{r} = \mathbf{r}_0 + \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2}$$

In terms of rectangular components, we have

$$x\hat{\mathbf{i}} + y\hat{\mathbf{j}} = x_0\hat{\mathbf{i}} + y_0\hat{\mathbf{j}} + (v_{0x}\hat{\mathbf{i}} + v_{0y}\hat{\mathbf{j}})t + \frac{1}{2}(a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}})t^2$$

Now, equating the coefficients of $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$,

$$\boxed{x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 \text{ ..... along } x\text{-axis}}$$

and

$$\boxed{y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 \text{..... along } y\text{-axis}}$$

Note Motion in a plane (two-dimensional motion) can be treated as two separate simultaneous one-dimensional motions with constant acceleration along two perpendicular directions.

Example 4.7 *A particle starts from origin at $t = 0$ with a velocity of $15\hat{\mathbf{i}}\ ms^{-1}$ and moves in xy-plane under the action of a force which produces a constant acceleration of $15\hat{\mathbf{i}} + 20\hat{\mathbf{j}}\ ms^{-2}$. Find the y-coordinate of the particle at the instant its x-coordinate is 180 m.*

Sol. The position of the particle is given by

$$r(t) = v_0 t + \frac{1}{2}at^2 = 15\hat{\mathbf{i}}\,t + \frac{1}{2}(15\hat{\mathbf{i}} + 20\hat{\mathbf{j}})t^2$$

$$= (15t + 7.5t^2)\hat{\mathbf{i}} + 10\hat{\mathbf{j}}\,t^2$$

$$\therefore \quad x(t) = 15t + 7.5\,t^2 \quad \text{and} \quad y(t) = 10t^2$$

If $x(t) = 180\text{ m}$, $t = ?$

$$180 = 15t + 7.5t^2 \Rightarrow t = 4\text{ s}$$

$\therefore$ y-coordinate, $y(t) = 10 \times 16 = 160\text{ m}$

Example 4.8 *An object has a velocity, $\mathbf{v} = (2\hat{\mathbf{i}} + 4\hat{\mathbf{j}})\ ms^{-1}$ at time $t = 0$ s. It undergoes a constant acceleration $a = (\hat{\mathbf{i}} - 3\hat{\mathbf{j}})\text{ ms}^{-2}$ for 4s. Then*

(i) Find the coordinates of the object if it is at origin at $t = 0$

(ii) Find the magnitude of its velocity at the end of 4s.

Sol. (i) Here initial position of the object

$$\mathbf{r}_0 = x_0\hat{\mathbf{i}} + y_0\hat{\mathbf{j}} = 0\hat{\mathbf{i}} + 0\hat{\mathbf{j}}$$

Initial velocity, $\mathbf{v}_0 = v_{0_x}\hat{\mathbf{i}} + v_{0_y}\hat{\mathbf{j}} = 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$

Acceleration $\quad \mathbf{a} = a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}} = \hat{\mathbf{i}} - 3\hat{\mathbf{j}}$

And $t = 4$ s

Let the final coordinates of the object be (x, y). Then according to the equation for the path of particle under constant acceleration.

$$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$$

$$= 0 + 2 \times 4 + \frac{1}{2}(1) \times 4^2$$

$$\Rightarrow \quad x = 16\text{ m}$$

and $\quad y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$

$$= 0 + 4 \times 4 + \frac{1}{2}(-3) \times 4^2$$

$$\Rightarrow \quad y = -8\text{ m}$$

Therefore the object lies at $(16\hat{\mathbf{i}} - 8\hat{\mathbf{j}})$ at $t = 4$ s.

(ii) Using equation

$$\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$$

$$\Rightarrow \quad \mathbf{v} = (2\hat{\mathbf{i}} + 4\hat{\mathbf{j}}) + (\hat{\mathbf{i}} - 3\hat{\mathbf{j}}) \times 4$$

$$= (2\hat{\mathbf{i}} + 4\hat{\mathbf{j}}) + (4\hat{\mathbf{i}} - 12\hat{\mathbf{j}}) = (2 + 4)\hat{\mathbf{i}} + (4 - 12)\hat{\mathbf{j}}$$

$$\Rightarrow \quad \mathbf{v} = 6\hat{\mathbf{i}} - 8\hat{\mathbf{j}}$$

$\therefore$ Magnitude of velocity, $|\mathbf{v}| = \sqrt{6^2 + 8^2} = 10\text{ ms}^{-1}$

Its direction with X-axis, $\theta = \tan^{-1}\left(\frac{-8}{6}\right) \approx -53°$

Check Point 4.1

1. The x and y components of a position vector **P** have numerical values 5 and 6 respectively. Direction and magnitude of vector **P** are

(a) $\tan^{-1}\left(\frac{6}{5}\right)$ and $\sqrt{61}$ (b) $\tan^{-1}\left(\frac{5}{6}\right)$ and $\sqrt{61}$

(c) 60° and 8 (d) 30° and 9

2. An object moves from positions (6, 8) to (12, 10) in the x-y plane. Magnitude and direction of displacement is

(a) $\sqrt{40}$ and 18.43° (b) $\sqrt{40}$ and 71.56°

(c) 10 and 53° (d) $\sqrt{244}$ and 53°

3. A particle moves in xy-plane from positions (2 m, 4m) to (6m, 8m) is 2 s. Magnitude and direction of average velocity is

(a) $\sqrt{2}\text{ms}^{-1}$ and 45° (b) $2\sqrt{2}\text{ms}^{-1}$ and 45°

(c) $4\sqrt{2}\text{ms}^{-1}$ and 30° (d) $3\sqrt{2}\text{ms}^{-1}$ and 60°

4. The displacement of an object along the three axes are given by, $x = 2t^2$, $y = t^2 - 4t$ and $z = 3t - 5$. The initial velocity of the particle is

(a) 10 units (b) 12 units

(c) 5 units (d) 2 units

5. A particle moves along the positive branch of the curve $y = \frac{x^2}{2}$ where $x = \frac{t^2}{2}$, x and y are measured in metres and t in second. At $t = 2$ s, the velocity of the particle is

(a) $2\hat{\mathbf{i}} - 4\hat{\mathbf{j}}\text{ ms}^{-1}$ (b) $4\hat{\mathbf{i}} + 2\hat{\mathbf{j}}\text{ ms}^{-1}$

(c) $2\hat{\mathbf{i}} + 4\hat{\mathbf{j}}\text{ ms}^{-1}$ (d) $4\hat{\mathbf{i}} - 2\hat{\mathbf{j}}\text{ ms}^{-1}$

6. The position vector of a particle is

$$\mathbf{r} = a\sin\omega t\ \hat{\mathbf{i}} + a\cos\omega t\ \hat{\mathbf{j}}$$

The velocity of the particle is

(a) parallel to position vector

(b) perpendicular to position vector

(c) directed towards origin

(d) directed away from the origin

7. The position vector of an object at any time t is given by $3t^2\hat{\mathbf{i}} + 6t\hat{\mathbf{j}} + \hat{\mathbf{k}}$. Its velocity along y-axis has the magnitude

(a) $6t$ (b) 6 (c) 0 (d) 9

8. The height y and distance x along the horizontal for a body projected in the xy-plane are given by $y = 8t - 5t^2$ and $x = 6t$. The initial speed of projection is

(a) 8 ms^{-1} (b) 9 ms^{-1}

(c) 10 ms^{-1} (d) $(10/3)\text{ ms}^{-1}$

9. The coordinates of a moving particle at any time t are given by $x = ct^2$ and $y = bt^2$. The speed of the particle is given by

(a) $2t\sqrt{c^2+b^2}$ (b) $\frac{2t}{\sqrt{c^2+b^2}}$

(c) $t\sqrt{c^2+b^2}$ (d) $\frac{t}{\sqrt{c^2+b^2}}$

10. The coordinates of a moving particle at any time t are given by, $x = 2t^3$ and $y = 3t^3$. Acceleration of the particle is given by

(a) $468t$ (b) $t\sqrt{468}$ (c) $234t^2$ (d) $t\sqrt{234}$

11. The position of a particle moving in the xy-plane at any time t is given by $x = (3t^2 - 6t)$ m, $y = (t^2 - 2t)$ m. Select the correct statement about the moving particle from the following.

(a) The acceleration of the particle is zero at $t = 0$ s

(b) The velocity of the particle is zero at $t = 0$ s

(c) The velocity of the particle is zero at $t = 1$ s

(d) The velocity and acceleration of the particle are zero

12. A particle's velocity changes from $(2\hat{i} + 3\hat{j})$ ms^{-1} in to $(3\hat{i} - 2\hat{j})$ ms^{-1} in 2 s. Its average acceleration in ms^{-2} is

(a) $-(\hat{i} + 5\hat{j})$ (b) $(\hat{i} + 5\hat{j})/2$ (c) zero (d) $(\hat{i} - 5\hat{j})/2$

13. A particle has an initial velocity of $4\hat{i} + 3\hat{j}$ and an acceleration of $0.4\hat{i} + 0.3\hat{j}$. Its speed after 10 s is

(a) 10 units (b) 7 units

(c) $7\sqrt{2}$ units (d) 8.5 units

14. A body lying initially at point (3, 7) starts moving with a constant acceleration of $4\hat{i}$. Its position after 3 s is given by the coordinates

(a) (7, 3) (b) (7, 18) (c) (21, 7) (d) (3, 7)

15. The initial position of an object at rest is given by $3\hat{i} - 8\hat{j}$. It moves with constant acceleration and reaches to the position $2\hat{i} + 4\hat{j}$ after 4 s. What is its acceleration?

(a) $-\frac{1}{8}\hat{i} + \frac{3}{2}\hat{j}$ (b) $2\hat{i} - \frac{1}{8}\hat{j}$

(c) $-\frac{1}{2}\hat{i} + 8\hat{j}$ (d) $8\hat{i} - \frac{3}{2}\hat{j}$

2 Projectile

A projectile is the name given to any object or body which once released into the space with some initial velocity, moves under the effect of gravity alone.

Projectile motion

If a constant force (and hence constant acceleration) acts on a particle at an angle θ ($\neq 0°$ or $180°$) with the direction of its initial velocity ($\neq$ zero), the path followed by the particle is a parabola and the motion of the particle is called projectile motion. Projectile motion is a two dimensional motion, *i.e.*, motion of the particle is constrained in a plane.

If a particle move in horizontal as well as vertical motion simultaneously the motion of particle is known as projectile motion.

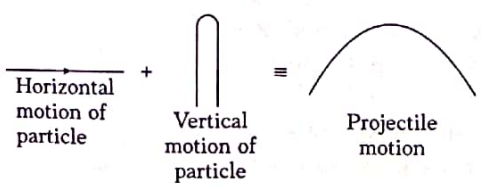


Fig. 4.8

Note

1. If an angle between acceleration and velocity is θ and where $0° < \theta < 180°$, then particle is executing projectile motion.
2. In projectile motion change in velocity of particle in magnitude and direction both are acting simultaneously.

When a particle is thrown obliquely near the earth's surface it moves in a parabolic path, provided the particle remains close to the surface of earth and the air resistance is negligible. This is an example of projectile motion.

Projectile projected obliquely on the surface of the earth

Let OX be a horizontal line on the ground, OY be a vertical line perpendicular to ground and O be the origin for XY-axes on a plane.

Suppose an object be projected from point O with velocity (u), making an angle (θ) with the horizontal direction OX, such that $x_0 = 0$ and $y_0 = 0$ when $t = 0$.

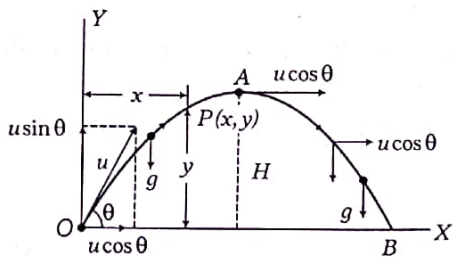


Fig. 4.9 Oblique projectile motion

While resolving velocity (u) into two components, we get

(i) $u\cos\theta$ along OX and (ii) $u\sin\theta$ along OY.

As there is no force acting in horizontal direction so the horizontal component of velocity ($u\cos\theta$) is constant throughout the motion, so there is no acceleration in the horizontal direction (if air resistance is assumed to be zero).

The vertical components of velocity ($u \sin\theta$) decreases continuously with height, from O to A, due to downward force of gravity and becomes zero. At this point, the object attains maximum height, now it has only horizontal component of velocity. From point A, the object starts to fall down and reaches at point B on the ground.

Let us make ourselves familiar with certain terms used in projectile motion.

1. Equation of path of projectile

Suppose at any time t, the object reaches at point P (x, y).
So, x = horizontal distance travelled by object in time t.
y = vertical distance travelled by object in time t.

Motion along horizontal direction

The velocity of the object in horizontal direction *i.e.*, OX is constant, so the acceleration a_x in horizontal direction is zero.

$\therefore$ Position of the object at time t along horizontal direction is given by, $x = x_0 + u_x t + \frac{1}{2} a_x t^2$

But $x_0 = 0$, $u_x = u\cos\theta$, $a_x = 0$ and $t = t$

$\therefore$ $$x = u\cos\theta\, t$$

$\Rightarrow$ $$\boxed{\text{Time, } t = \frac{x}{u\cos\theta}} \quad \text{...(i)}$$

Motion along vertical direction

The vertical velocity of the object is decreasing from O to P due to gravity, so acceleration a_y is $-g$.

$\therefore$ Position of the object at any time t along the vertical direction *i.e.*, OY is given by $y = y_0 + u_y t + \frac{1}{2} a_y t^2$

But $y_0 = 0$, $u_y = u\sin\theta$, $a_y = -g$ and $t = t$

So, $$y = u\sin\theta\, t + \frac{1}{2}(-g)\, t^2 = u\sin\theta\, t - \frac{1}{2} g t^2 \quad \text{...(ii)}$$

Substituting the value of t from Eq. (i) in Eq. (ii), we get

$$y = u\sin\theta\left(\frac{x}{u\cos\theta}\right) - \frac{1}{2} g\left(\frac{x}{u\cos\theta}\right)^2$$

$$= x\tan\theta - \frac{1}{2} g\left(\frac{x}{u\cos\theta}\right)^2$$

Vertical displacement,

$$\boxed{y = x\tan\theta - \left(\frac{1}{2}\frac{g}{u^2\cos^2\theta}\right)x^2}$$

This equation represents a parabola and is known as **equation of trajectory of a projectile.**

Example 4.9 *A body is projected with a velocity of $20\,ms^{-1}$ in a direction making an angle of 60° with the horizontal. Determine its (i) position after 0.5 s and (ii) the velocity after 0.5 s.*

Sol. Given, $u = 20\ ms^{-1}$, $\theta = 60°$; $t = 0.5$ s

(i) Since, horizontal distance,

$$x = (u\cos\theta)t = (20\cos 60)\times 0.5 = 5\text{ m}$$

Similarly, vertical distance,

$$y = (u\sin\theta)t - \frac{1}{2} g t^2$$

$$= (20\sin 60°)\times 0.5 - \frac{1}{2}\times 9.8\times (0.5)^2 \Rightarrow y = 7.43\text{ m}$$

(ii) Velocity along horizontal direction,

$$v_x = u\cos\theta = 20\cos 60° = 10\ ms^{-1}$$

Velocity along vertical direction,

$$v_y = u\sin\theta - gt = 20\sin 60° - 9.8\times 0.5 = 12.42\ ms^{-1}$$

Example 4.10 *A stone is thrown with a speed of $10\ ms^{-1}$ at an angle of projection 60°. Find its height above the point of projection when it is at a horizontal distance of 3 m from the thrower? (Take $g = 10\ ms^{-2}$)*

Sol. Considering the equation of trajectory,

$$y = (\tan\theta_0)\, x - \frac{g}{2(v_0^2\cos^2\theta_0)} x^2$$

Here, $\theta_0 = 60°$, $v_0 = 10\ ms^{-1}$, $x = 3$ m

$\therefore$ $$y = (\tan 60°)\times 3 - \frac{10}{2\,(100\cos^2 60°)}(3)^2$$

$$= 3\sqrt{3} - \frac{9}{5} = \frac{15\sqrt{3} - 9}{5}\text{ m} = 3.396\text{ m}$$

2. Time of flight of projectile

It is defined as the total time for which projectile is in flight *i.e.*, time during the motion of projectile is from O to B. It is denoted by T.

Time of flight consists of two parts such as

(i) Time taken by an object to go from point O to A. It is also known as **time of ascent** (t_a).

(ii) Time taken by an object to go from point A to B. It is also known as **time of descent** (t_d).

Total time can be expressed as

$$T = t_a + t_d = 2t \Rightarrow t = T/2 \qquad [\because t_a = t_d = t]$$

The vertical component of velocity of the projectile becomes zero at the highest point H.

Let us consider vertical upward motion of the object from O to H, we have

$$u_y = u\sin\theta,\ a_y = -g,\ t = T/2 \text{ and } v_y = 0$$

Since, $$v_y = u_y + a_y t \Rightarrow 0 = u\sin\theta - g\frac{T}{2}$$

$\therefore$ $$\boxed{\text{Time of flight, } T = \frac{2u\sin\theta}{g}}$$

Example 4.11 *A cricket ball is thrown at a speed of 28 ms^{-1} in a direction 30° above the horizontal. Calculate the time taken by the ball to return to the same level.*

Sol. Given, speed $u = 28\ ms^{-1}$ and $\theta = 30°$

∴ The time taken by the ball to return the same level is

$$T = \frac{2u \sin\theta}{g} = \frac{2 \times 28 \times \sin 30°}{9.8} = \frac{28}{9.8} = 2.9\ s.$$

3. Maximum height of a projectile

It is defined as the maximum vertical height attained by the projectile above the point of projection during its flight. It is denoted by H.

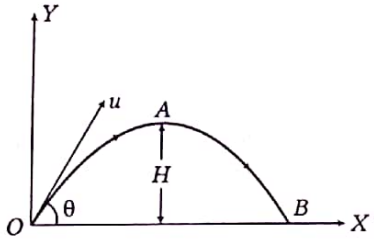


Fig. 4.10 *Maximum height of projectile*

Let us consider the vertical upward motion of the projectile from O to A.

We have,

$$u_y = u \sin\theta,\ a_y = -g,\ y_0 = 0,\ y = H,$$

$$t = \frac{T}{2} = \frac{u \sin\theta}{g}$$

Using this relation, $y = y_0 + u_y t + \frac{1}{2} a_y t^2$

We have $H = 0 + u \sin\theta \frac{u \sin\theta}{g} + \frac{1}{2}(-g)\left(\frac{u \sin\theta}{g}\right)^2$

$$= \frac{u^2}{g}\sin^2\theta - \frac{1}{2}\frac{u^2 \sin^2\theta}{g}$$

Maximum height, $\boxed{H = \frac{u^2 \sin^2\theta}{2g}}$

Example 4.12 *Assume that a ball is kicked at an angle of 60° with the horizontal, so if the horizontal component of its velocity is 19.6 ms^{-1}, determine its maximum height.*

Sol. Given, $\theta = 60°$

Horizontal component of velocity $= u \cos 60° = 19.6\ ms^{-1}$

$$\therefore \quad u = \frac{19.6}{\cos 60°} = \frac{19.6}{0.5} = 39.2\ ms^{-1}$$

Therefore, maximum height,

$$H = \frac{u^2 \sin^2 60°}{2g} = \frac{(39.2)^2}{2 \times 9.8} \times \left(\frac{\sqrt{3}}{2}\right)^2 = 58.8\ m$$

4. Horizontal range of a projectile

The horizontal range of a projectile is defined as the horizontal distance covered by the projectile during its time of flight. It is denoted by R.

If the object having uniform velocity $u \cos\theta$ (*i.e.*, horizontal component) and the time of flight is T, then the horizontal range covered by the projectile.

$$\therefore \quad OB = R = u \cos\theta \times T = u \cos\theta \times 2u \frac{\sin\theta}{g}$$

Horizontal range, $\boxed{R = \frac{u^2 \sin 2\theta}{g}}$ $(\because \sin 2\theta = 2 \sin\theta \cos\theta)$

The horizontal range will be maximum, if angle of projection is 45°.

∴ Maximum horizontal range, $R_m = \frac{u^2}{g}$

For same value of initial velocity, horizontal range of projectile is same for complementary angles.

So, $R_{30°} = R_{60°}$ or $R_{20°} = R_{70°}$

Some important points related to projectile motion

(i) Velocity of the projectile at any instant t

$$\mathbf{v} = v_x \hat{\mathbf{i}} + v_y \hat{\mathbf{j}} = 4\cos\theta \hat{\mathbf{i}} + (4\sin\theta - gt)\hat{\mathbf{j}}$$

$$|\mathbf{v}| = \sqrt{u^2 + g^2t^2 - 2ugt\sin\theta}$$

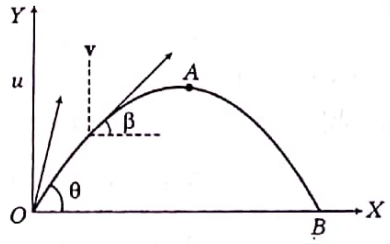


Fig. 4.11

- This velocity makes an angle β with the horizontal given by

$$\tan\beta = \frac{v_y}{v_x} = \frac{u\sin\theta - gt}{u\cos\theta}$$

- When the projectile reaches at point B put $t = T$.

(ii) When a projectile is thrown upward, its kinetic energy decreases, potential energy increase and the total energy always remains constant.

(iii) Total energy of projectile = Kinetic energy + Potential energy

$$= \frac{1}{2}mv^2\cos^2\theta + \frac{1}{2}mv^2\sin^2\theta = \frac{1}{2}mv^2$$

(iv) In projectile motion speed (and hence kinetic energy) is minimum at highest point.

Speed = (cos θ) times the speed of projection

and kinetic energy = ($\cos^2\theta$) times the initial kinetic energy

Here, θ = angle of projection

(v) In projectile motion it is sometimes better to write the equations of H, R and T in terms of u_x and u_y as under.

$$T = \frac{2u_y}{g}, \quad H = \frac{u_y^2}{2g} \quad \text{and} \quad R = \frac{2u_x u_y}{g}$$

(vi) In projectile motion $H = R$, when $u_y = 4u_x$ or $\tan\theta = 4$.

(vii) Equation of trajectory can also be written as

$$y = x\left(1 - \frac{x}{R}\right)\tan\theta$$

where R is horizontal range.

(viii) All the above expressions for T, H and R are derived by neglecting air resistance. If air resistance is considered then values may differ slightly.

(ix) Due to air resistance when a net speed of projectile decreases, then R decreases, T increases and H decreases. Reverse is the case when net speed increases.

Example 4.13 *An object is projected with a velocity of 30 ms^{-1} at an angle of 60° with the horizontal. Determine the horizontal range covered by the object.*

Sol. Given, initial velocity, $u = 30\ ms^{-1}$

Angle of projection, $\theta = 60°$

Therefore, the horizontal range (or distance) covered by the object will be given as

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{(30)^2 \sin(60°)}{g}$$

$$= \frac{(30)^2 \sin 60°}{9.8} = 79.53\ m$$

$$\Rightarrow \quad R = 79.53\ m$$

Example 4.14 *A projectile has a range of 40 m and reaches a maximum height of 10 m. Find the angle at which the projectile is fired.*

Sol. Range of a projectile $R = \frac{u_0^2 \sin 2\theta_0}{g} = 40\ m$

$$H = \frac{u_0^2 \sin^2\theta_0}{2g} = 10\ m$$

Dividing the two, we get, $\frac{2(\sin 2\theta_0)}{\sin^2\theta_0} = 4$

$$\frac{4\sin\theta_0\cos\theta_0}{\sin^2\theta_0} = 4$$

$$\Rightarrow \quad \tan\theta_0 = 1 \Rightarrow \theta_0 = 45°$$

Example 4.15 *Find the angle of projection of a projectile for which the horizontal range and maximum height are equal.*

Sol. Given, $R = H$

$$\therefore \quad \frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin^2\theta}{2g}$$

$$\text{or} \quad 2\sin\theta\cos\theta = \frac{\sin^2\theta}{2}$$

$$\text{or} \quad \frac{\sin\theta}{\cos\theta} = 4 \quad \text{or} \quad \tan\theta = 4$$

$$\therefore \quad \theta = \tan^{-1}(4)$$

Example 4.16 *There are two angles of projection for which the horizontal range is the same. Show that the sum of the maximum heights for these two angles is independent of the angle of projection.*

Sol. There are two angles of projection α and 90°– α for which the horizontal range R is same.

Now, $$H_1 = \frac{u^2 \sin^2\theta}{2/g}$$

and $$H_2 = \frac{u^2 \sin^2(90° - \theta)}{2/g} = \frac{u^2 \cos^2\theta}{2/g}$$

Therefore, $$H_1 + H_2 = \frac{u^2}{2g}(\sin^2\theta + \cos^2\theta) = \frac{u^2}{2g}$$

Clearly the sum of the heights for the two angles of projection is independent of the angles of projection.

Example 4.17 *Prove that the maximum horizontal range is four times the maximum height attained by the projectile; when fired at an inclination so as to have maximum horizontal range.*

Sol. For θ = 45°, the horizontal range is maximum and is given by

$$R_{max} = \frac{u^2}{g}$$

Maximum height attained,

$$H_{max} = \frac{u^2 \sin^2 45°}{2g} = \frac{u^2}{4g} = \frac{R_{max}}{4}$$

$$\text{or} \quad R_{max} = 4H_{max}$$

Projectile fired at an angle with the vertical

Let a particle be projected vertically with an angle θ with vertical and it's muzzle speed (*i.e.*, speed of projection) is u.

Clearly, the angle made by the velocity of projectile at point of projection with horizontal is (90°–θ).

In this case

(*i*) Time of flight $= \frac{2u\sin(90° - \theta)}{g} = \frac{2u}{g}\cos\theta$

(*ii*) Maximum height $= \dfrac{u^2 \sin^2(90° - \theta)}{2g} = \dfrac{u^2 \cos^2 \theta}{2g}$

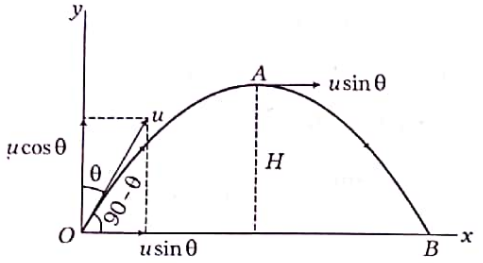


Fig. 4.12 Projectile fired at an angle with the vertical

(*iii*) Horizontal range

$$= \frac{u^2}{g} \sin 2(90° - \theta) = \frac{u^2}{g} \sin(180° - 2\theta) = \frac{u^2}{g} \sin 2\theta$$

(*iv*) Equation of path of projectile

$$y = x \tan(90° - \theta) - \frac{1}{2} \frac{gx^2}{u^2 \cos^2(90° - \theta)}$$

$$= x \cot \theta - \frac{gx^2}{2u^2 \sin^2 \theta}$$

(*v*) Velocity at any time t

$$v = \sqrt{u^2 + g^2t^2 - 2ugt \sin(90° - \theta)}$$

$$= \sqrt{u^2 + g^2 t^2 - 2u\, gt \cos \theta}$$

This velocity makes an angle β with the horizontal direction, then

$$\tan \beta = \frac{u \sin(90° - \theta) - gt}{u \cos(90° - \theta)} = \frac{u \cos \theta - gt}{u \sin \theta}$$

Example 4.18 *A football is kicked at an angle of 30° with the vertical, so if the horizontal component of its velocity is 20 ms^{-1}, determine its maximum height.*

Sol. Given, $\theta = 30°$

Horizontal component of velocity $= u \sin 30° = 20\ \text{ms}^{-1}$

$$u = \frac{20}{\sin 30°} = \frac{20}{1/2} = 40\ \text{ms}^{-1}$$

Therefore, maximum height,

$$H = \frac{u^2 \cos^2 30°}{2g} = \frac{(40)^2}{2 \times 9.8} \times \left(\frac{\sqrt{3}}{2}\right)^2 = 61.22\ \text{m}$$

Check Point 4.2

1. At the top of the trajectory of a projectile, the directions of its velocity and acceleration are
(a) parallel to each other
(b) antiparallel to each other
(c) inclined to each other at an angle of 45°
(d) perpendicular to each other

2. At the top of the trajectory of a particle, the acceleration is
(a) maximum (b) minimum
(c) zero (d) g

3. In the motion of a projectile falling freely under gravity, its (neglect air friction)
(a) total mechanical energy is conserved
(b) momentum is conserved
(c) mechanical energy and momentum both are conserved
(d) None is conserved

4. When a stone is projected which remains constant?
(a) Angular momentum
(b) Linear momentum
(c) Vertical component of velocity
(d) Horizontal component of velocity

5. A stone is projected with speed of 50 ms^{-1} at an angle of 60° with the horizontal. The speed of the stone at highest point of trajectory is
(a) 75 ms^{-1} (b) 25 ms^{-1}
(c) 50 ms^{-1} (d) cannot find

6. A football player throws a ball with a velocity of 50 ms^{-1} at an angle 30° from the horizontal. The ball remains in the air for (Take, $g = 10$ ms^{-2}).
(a) 2.5 s (b) 1.25 s (c) 5 s (d) 0.625 s

7. A particle is projected with a velocity of 20 ms^{-1} at an angle of 60° to the horizontal. The particle hits the horizontal plane again during its journey. What will be the time of impact?
(a) 3.53 s (b) 2.4 s
(c) 1.7 s (d) 1s

8. If 2 balls are projected at angles 45° and 60° and the maximum heights reached are same, what is the ratio of their initial velocities?
(a) $\sqrt{2} : \sqrt{3}$ (b) $\sqrt{3} : \sqrt{2}$
(c) 3 : 2 (d) 2 : 3

9. If the initial velocity of a projection be doubled, keeping the angle of projection same, the maximum height reached by it will
(a) remain the same (b) be doubled
(c) become four times (d) be halved

10. For a projectile, the ratio of maximum height reached to the square of flight time is (Take, $g = 10$ ms^{-2})
(a) 5 : 4 (b) 5 : 2
(c) 5 : 1 (d) 10 : 1

11. A particle is projected from ground with speed u and at an angle θ with horizontal. If at maximum height from ground, the speed of particle is 1/2 times of its initial velocity of projection, then find its maximum height attained.

(a) $\frac{u^2}{g}$ (b) $\frac{2u^2}{g}$ (c) $\frac{u^2}{2g}$ (d) $\frac{3u^2}{8g}$

12. A projectile, thrown with velocity v_0 at an angle α to the horizontal, has a range R. It will strike a vertical wall at a distance $R/2$ from the point of projection with a speed of

(a) v_0 (b) $v_0 \sin\alpha$ (c) $v_0 \cos\alpha$ (d) $\sqrt{\frac{gR}{2}}$

13. A particle is projected at an angle of 45° with a velocity of $9.8\ ms^{-1}$. The horizontal range will be (Take, $g = 9.8\ ms^{-2}$)

(a) 9.8 m (b) 4.9 m (c) $\frac{9.8}{\sqrt{2}}$ (d) $9.8\sqrt{2}$

14. Two projectiles A and B are projected with same speed at angles 30° and 60° to be horizontal then, which one is wrong?

(a) $R_A = R_B$ (b) $H_B = 3H_A$
(c) $T_B = \sqrt{3}T_A$ (d) None of these

15. A projectile fired with initial velocity u at some angle θ has a range R. If the initial velocity be doubled at the same angle of projection, then the range will be

(a) $2R$ (b) $R/2$ (c) R (d) $4R$

16. An object is thrown along a direction inclined at an angle of 45° with the horizontal direction. The horizontal range of the particle is equal to

(a) vertical height (b) twice the vertical height
(c) thrice the vertical height (d) four times the vertical height

17. An object is projected at an angle of 45° with the horizontal. The horizontal range and the maximum height reached will be in the ratio

(a) 1:2 (b) 2:1
(c) 1:4 (d) 4:1

18. The horizontal range of a projectile is $4\sqrt{3}$ times of its maximum height. The angle of projection will be

(a) 60° (b) 37° (c) 30° (d) 45°

19. A ball is projected with a velocity $20\sqrt{3}\ ms^{-1}$ at angle 60° to the horizontal. The time interval after which the velocity vector will make an angle 30° to the horizontal is (Take, $g = 10\ ms^{-2}$)

(a) 5 s (b) 2 s (c) 1 s (d) 3 s

20. A projectile is thrown with a velocity of $10\ ms^{-1}$ at an angle of 60° with horizontal. The interval between the moments when speed is $\sqrt{5g}$ m/s is (Take, $g = 10\ ms^{-2}$).

(a) 1 s (b) 3 s
(c) 2 s (d) 4 s

3 Projectile from a point above the ground

Projectile projected horizontally from a tower

Assume that an object is thrown horizontally with some velocity u from a point O, on a tower of height h above the ground level and after time t, it reaches ground at point E.

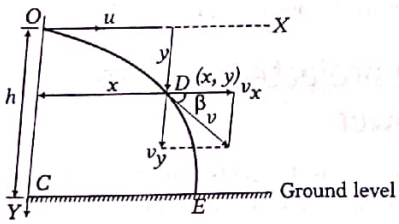


Fig. 4.13 Horizontal projectile

Horizontal components	Vertical components
$u_x = u$	$u_y = 0$
$a_x = 0\ (\because F_x = 0)$	$a_y = -g$

Different terms related to this projectile motion are

(i) Equation of trajectory $y = \frac{1}{2}\left(\frac{g}{u^2}\right)x^2$

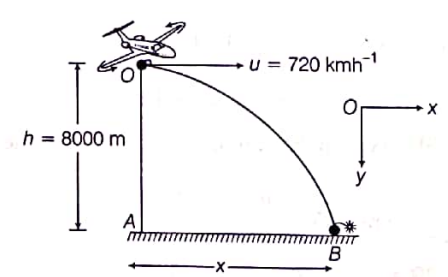
(ii) Time of flight $T = \sqrt{\frac{2h}{g}}$

(iii) Horizontal range $R = u\sqrt{\frac{2h}{g}}$

(iv) Velocity of projectile at any instant $v = \sqrt{u^2 + g^2t^2}$

Example 4.19 *A bomb is released from an aeroplane flying at a speed of 720 kmh^{-1} in the horizontal direction 8000 m above the ground. At what horizontal distance from the initial position of aeroplane it strikes the ground?*

Sol.



According to the figure, during motion of the bomb from O to B

$$u = 720 \times \frac{5}{18} = 200\ ms^{-1} \Rightarrow y = h = \frac{1}{2}gt^2$$

$\Rightarrow \quad 8000 = \frac{1}{2} \times 10t^2 \Rightarrow t = 40\text{ s}$

$\Rightarrow \quad x = ut = 200 \times 40 = 8000\text{ m}$

Example 4.20 *A body is thrown horizontally from the top of a tower and strikes the ground after three seconds at an angle of 45° with the horizontal. Then find*

(i) the height of the tower

(ii) the speed of projection of the body.

Sol. (i) Let H be the height of the tower

The time of flight, $T_f = \sqrt{\frac{2H}{g}} = 3\text{ s}$

$\Rightarrow \quad H = \frac{g \times (3)^2}{2} = \frac{9.8 \times 9}{2} = 44.1\text{ m}$

(ii) Let the speed of projection be v_0.

Then for horizontal projection

$v_x = v_0 \Rightarrow v_y = -gt$

At $\quad t = T_f = 3\text{ s}, \; v_y = -9.8 \times 3 = -29.4\text{ ms}^{-1}$

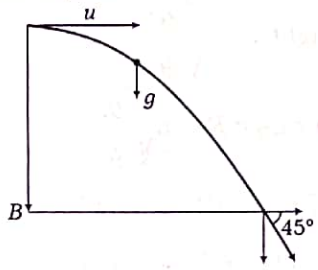
The angle which the final velocity makes with the horizontal $= \theta = 45°$ (Given)

$\Rightarrow \quad -\tan 45° = \frac{-v_y}{v_x} \Rightarrow v_y = v_x$

So, $\quad v_x = 29.4\text{ ms}^{-1}$

Example 4.21 *A body is thrown horizontally from the top of a tower and strikes the ground after three seconds at an angle of 45° with the horizontal. Find the height of the tower and the speed with which the body was projected. (Take, $g = 9.8\text{ ms}^{-2}$)*

Sol. As shown in the figure.



$u_y = 0$ and $a_y = g = 9.8\text{ ms}^{-2}$

$\Rightarrow \quad s_y = u_y t + \frac{1}{2} a_y t^2$

$= 0 \times 3 + \frac{1}{2} \times 9.8 \times (3)^2 = 44.1\text{ m}$

Further, $v_y = u_y + a_y t = 0 + (9.8)(3) = 29.4\text{ ms}^{-1}$

As the resultant velocity v makes an angle of 45° with the horizontal so,

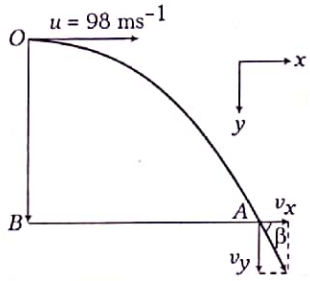
$$\tan 45° = \frac{v_y}{v_x}$$

or $\quad 1 = \frac{29.4}{v_x}$ or $v_x = 29.4\text{ ms}^{-1}$

Therefore, the speed with which the body was projected (horizontally) is $u = v_x = 29.4\text{ ms}^{-1}$.

Example 4.22 *A projectile is fired horizontally with a velocity of 98 ms^{-1} from the top of a hill 490 m high. Find :*

(i) the time taken by the projectile to reach the ground,



(ii) the distance of the point where the particle hits the ground from foot of the hill, and

(iii) the velocity with which the projectile hits the ground.

($g = 9.8\text{ ms}^{-2}$)

Sol. In this problem we cannot apply the formulae of R, H and T directly. Here, it will be more convenient to choose x and y directions as shown in figure.

Here, $u_x = 98\text{ ms}^{-1}$, $a_x = 0$, $u_y = 0$ and $a_y = g$.

(i) At A, $s_y = 490\text{ m}$. So, applying

$$s_y = u_y t + \frac{1}{2} a_y t^2$$

$\therefore \quad 490 = 0 + \frac{1}{2}(9.8)t^2 \quad \therefore \; t = 10\text{ s}$

(ii) $BA = s_x = u_x t + \frac{1}{2} a_x t^2$ or $BA = (98)(10) + 0 \quad (\because a_x = 0)$

or $\quad BA = 980\text{ m}$

(iii) Horizontal velocity $v_x = u_x = 98\text{ ms}^{-1}$

Vertical velocity $v_y = u_y + a_y t$

$= 0 + (9.8)(10) = 98\text{ ms}^{-1}$

$\therefore$ Resultant velocity $v = \sqrt{v_x^2 + v_y^2} = \sqrt{(98)^2 + (98)^2}$

$= 98\sqrt{2}\text{ ms}^{-1}$

and $\quad \tan\beta = \frac{v_y}{v_x} = \frac{98}{98} = 1 \quad \therefore \; \beta = 45°$

Thus, the projectile hits the ground with a velocity $98\sqrt{2}\text{ ms}^{-1}$ at an angle of $\beta = 45°$ with horizontal as shown in the above figure.

Projectile projected upward from a tower

Consider a projectile projected upward at an angle (θ) from point O which is situated on a tower at height h above the ground. Now, from the diagram, we have

$$u_x = u\cos\theta, \; a_x = 0$$

$$u_y = u\sin\theta, \; a_y = -g$$

(i) Equation of horizontal motion,

$$x = u\cos\theta t \quad \text{...(i)}$$

(ii) Equation of verticle motion

$$-h = u\sin\theta t - \frac{1}{2}gt^2 \quad ...(ii)$$

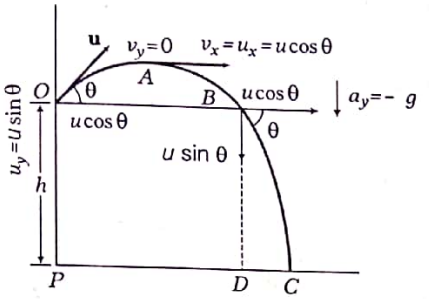


Fig. 4.14 Projectile projected upward from a tower

(iii) Time of flight $T = \frac{u\sin\theta}{g} \pm \sqrt{\frac{u^2\sin^2\theta}{g^2} + \frac{2h}{g}}$

(iv) Horizontal distance covered (in time of flight T)

$$PC = (u\cos\theta)T$$

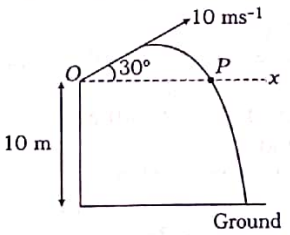
(v) Horizontal distance covered from the top of tower

$$OB = \frac{u^2\sin 2\theta}{g}$$

In such case for range PC to become maximum, θ should be 45°.

Example 4.23 *A boy playing on the roof of a 10 m high building throws a ball with a speed of 10 ms^{-1} at an angle of 30° with the horizontal. How far from the throwing point will the ball be at the height of 10 m from the ground?*

Sol. The ball will be at point P when it is at a height of 10 m from the ground. So, we have to find distance OP, which can be calculated direct considering it is a projectile on a level OX at a height h from the horizontal level.



$$OP = R = \frac{u^2\sin 2\theta}{g} \Rightarrow R = \frac{10^2 \times \sin(2\times 30°)}{10} = 8.66 \text{ m}$$

Projectile projected downward from a tower

Consider a projectile projected downward at an angle θ from point O which is on a tower of height h above the ground.

Now from the diagram we have

$$u_x = u\cos\theta,\ a_x = 0 \Rightarrow u_y = -u\sin\theta,\ a_y = -g$$

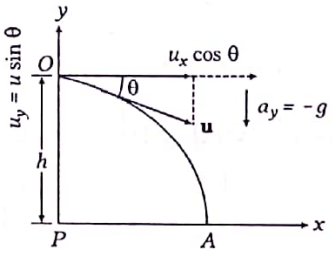


Fig. 4.15 Projectile projected downward from a tower

(i) Equation of motion

$$-h = (-u\sin\theta)\,t + \frac{1}{2}(-g)t^2$$

or $$gt^2 + (2u\sin\theta)t - 2h = 0$$

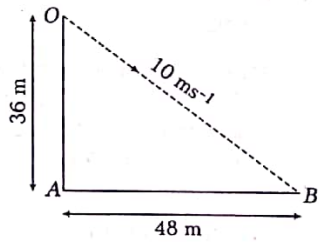
(ii) Time of flight $T = \frac{-2u\sin\theta}{2g} \pm \frac{\sqrt{4u^2\sin^2\theta + 8gh}}{2g}$

(iii) Horizontal distance covered from the base of tower

$$PA = (u\cos\theta)T$$

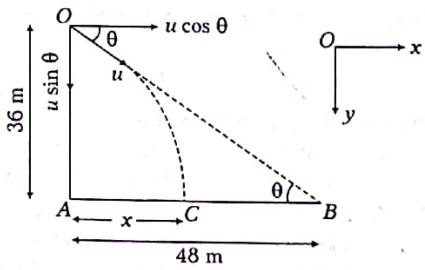
Example 4.24 *A boy standing on the top of a tower 36 m high has to throw a packet to his friend standing on the ground 48 m horizontally away. If he throws a packet directly aiming the friend with a speed of 10 ms^{-1}, how short will the packet fall?*

Sol. The packet will strike at point C instead of reaching to friend B because the packet will move on a parabolic path instead of straight line.



From geometry, $\tan\theta = \frac{36}{48} = \frac{3}{4}$

$\Rightarrow$ $\sin\theta = \frac{3}{5}, \cos\theta = \frac{4}{5}$



For O to C, $S_y = u\sin\theta t + \frac{1}{2}gt^2 \Rightarrow 36 = 10\times\frac{3}{5}t + \frac{1}{2}\times 10t^2 = 6t + 5t^2 \Rightarrow 5t^2 + 6t - 36 = 0$

At $t = 2.15$ s, $S_x = u\cos\theta t = 10\times\frac{4}{5}\times 2.15 = 17.2$ m $= AC$ $\therefore$ $BC = AB - AC = 48 - 17.2 = 30.8$ m

The packet will fall at distance 30.8 m infront of his friend.

Shooting a freely falling target

Aiming a target situated at some height is also an example of projectile motion. For example consider a stone of a gulel which aimed at a mango A as shown in figure.

Suddenly the mango starts falling and at the same time stone is fired from the gulel. The stone hits the mango at points A'. Reason behind this is stone and mango falls through same height in same time and stone always hits the mango.

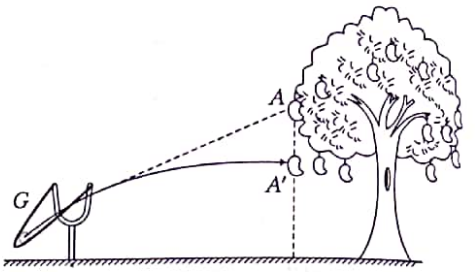


Fig. 4.16 Shooting a freely falling target

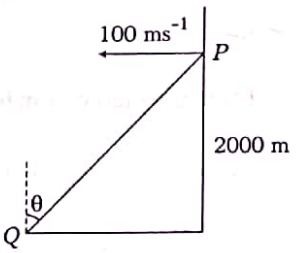
Check Point 4.3

1. A bomb is dropped from an aeroplane moving horizontally at constant speed. If air resistance is neglected then the bomb
(a) falls on the earth exactly below the aeroplane
(b) falls on the earth behind the aeroplane
(c) falls on the earth ahead of the aeroplane
(d) flies with the aeroplane

2. A body is projected horizontally with a velocity of $4\,ms^{-1}$ from the top of a high tower. The velocity of the body after 0.7 s is nearly (Take, $g = 10\,ms^{-2}$)
(a) $10\,ms^{-1}$ (b) $8\,ms^{-1}$ (c) $19.2\,ms^{-1}$ (d) $11\,ms^{-1}$

3. A particle is projected horizontally with speed $20\,ms^{-1}$ from the top of a tower. After what time velocity of particle will be at 45° angle from initial direction of projection.
(a) 1 s (b) 2 s (c) 3 s (d) 4 s

4. An aeroplane is travelling horizontally at a height of 2000 m from the ground. The aeroplane, when at a point P, drops a bomb to hit a stationary target Q on the ground. In order that the bomb hits the target, what angle θ must the line PQ make with the vertical? ($g = 10\,ms^{-2}$)



(a) 45° (b) 30° (c) 60° (d) 90°

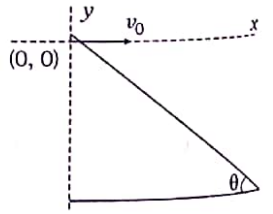
5. An aeroplane is flying at a constant height of 1960 m with speed $600\,kmh^{-1}$ above the ground towards point directly over a person struggling in flood water. At what angle of sight with the vertical should the pilot release a survival kit if it is to reach the person in water? ($g = 9.8\,ms^{-2}$)
(a) 45° (b) 30° (c) 60° (d) 90°

6. A bomber plane moves horizontally with a speed of $500\,ms^{-1}$ and a bomb released from it strikes the ground in 10 s. Angle at which it strikes the ground will be (Take, $g = 10\,ms^{-2}$).
(a) $\tan^{-1}\left(\frac{1}{5}\right)$ (b) 60° (c) 45° (d) $\tan^{-1}(5)$

7. A ball is projected horizontally from the top of a tower with a velocity v_0. It will be moving at an angle of 60° with the horizontal after time.
(a) $\frac{v_0}{\sqrt{3}g}$ (b) $\frac{\sqrt{3}\,v_0}{g}$ (c) $\frac{v_0}{g}$ (d) $\frac{v_0}{2g}$

8. An aeroplane is flying in horizontal direction with a velocity of $600\,kmh^{-1}$ and at a height of 1960 m. When it is vertically above a point A on the ground a body is dropped from it, the body strikes the ground at point B. Then the distance AB will be
(a) 3.33 km (b) 4.33 km (c) 5.33 km (d) 6.33 km

9. A man standing on a hill top projects a stone horizontally with speed v_0 as shown in figure. Taking the coordinate system as given in the figure. The coordinates of the point where the stone will hit the hill surface are



(a) $\left(\frac{2v_0^2\tan\theta}{g}, -\frac{2v_0^2\tan^2\theta}{g}\right)$ (b) $\left(\frac{2v_0^2}{g}, -\frac{2v_0^2\tan^2\theta}{g}\right)$
(c) $\left(\frac{2v_0^2\tan\theta}{g}, -\frac{2v_0^2}{g}\right)$ (d) $\left(\frac{2v_0^2\tan^2\theta}{g}, -\frac{2v_0^2\tan\theta}{g}\right)$

10. A ball is dropped from a height of 49 m. The wind is blowing horizontally. Due to wind a constant horizontal acceleration is provided to the ball. Choose the correct statement (s). (Take, $g = 9.8\,ms^{-2}$)
(a) Path of the ball is circular one
(b) Path of the ball is a curved one
(c) The time taken by the ball to reach the ground is 3.16 s
(d) Actual distance travelled by the ball is less than 49 m

Chapter exercises

A. Taking it together

(Assorted questions of the chapter for advanced level practice)

1. In a two dimensional motion, instantaneous speed v_0 is a positive constant. Then which of the following are necessarily true? **[NCERT Exemplar]**

(a) The average velocity is not zero at any time
(b) Average acceleration must always vanish
(c) Displacements in equal time intervals are equal
(d) Equal path lengths are traversed in equal intervals

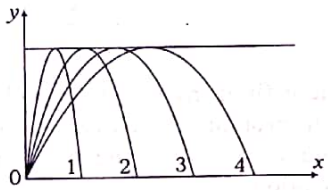
2. In a two dimensional motion, instantaneous speed v_0 is a positive constant. Then, which of the following are necessarily ture? **[NCERT Exemplar]**

(a) The acceleration of the particle is zero
(b) The acceleration of the particle is bounded
(c) The acceleration of the particle is necessarily in the plane of motion
(d) The particle must be undergoing a uniform circular motion.

3. A particle velocity changes from $(2\hat{i} - 3\hat{j})\ \text{ms}^{-1}$ to $(3\hat{i} - 2\hat{j})\ \text{ms}^{-1}$ in 2 s. If its mass is 1 kg, the acceleration (ms^{-2}) is

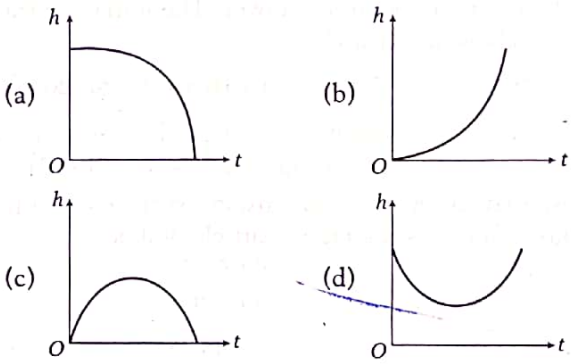
(a) $-(\hat{i} + \hat{j})$
(b) $(\hat{i} + \hat{j})/2$
(c) zero
(d) $(\hat{i} - \hat{j})/2$

4. Figure shows four paths for a kicked football. Ignoring the effects of air on the flight, rank the paths according to initial horizontal velocity component highest first.

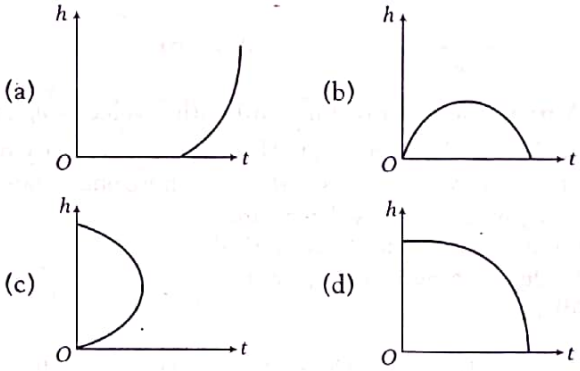


(a) 1, 2, 3, 4
(b) 2, 3, 4, 1
(c) 3, 4, 1, 2
(d) 4, 3, 2, 1

5. Which of the following is the graph between the height (h) of a projectile and time (t), when it is projected from the ground?



6. Which of the following is the altitude-time graph for a projectile thrown horizontally from the top of the tower?



7. Two projectiles A and B are thrown from the same point with velocities v and $v/2$, respectively. If B is thrown at an angle 45° with horizontal, what is the inclination of A when their ranges are the same?

(a) $\sin^{-1}\left(\frac{1}{4}\right)$
(b) $\frac{1}{2}\sin^{-1}\left(\frac{1}{4}\right)$
(c) $2\sin^{-1}\left(\frac{1}{4}\right)$
(d) $\frac{1}{2}\sin^{-1}\left(\frac{1}{8}\right)$

8. Two stones having different masses m_1 and m_2 are projected at an angle α and $(90° - \alpha)$ with same speed from same point. The ratio of their maximum heights is

(a) 1 : 1 (b) 1 : tan α (c) tan α : 1 (d) $\tan^2 \alpha$: 1

9. A projectile is thrown at an angle θ with the horizontal and its range is R_1. It is then thrown at an angle θ with vertical and the range is R_2, then

(a) $R_1 = 4R_2$ (b) $R_1 = 2R_2$
(c) $R_1 = R_2$ (d) None of these

10. The range of a particle when launched at an angle of 15° with the horizontal is 1.5 km. What is the range of the projectile when launched at an angle of 45° to the horizontal?

(a) 1.5 km (b) 3 km (c) 6 km (d) 0.75 km

11. A body is thrown horizontally from the top of a tower of height 5 m. It touches the ground at a distance of 10m from the foot of the tower. The initial velocity of the body is ($g = 10 \text{ms}^{-2}$)

(a) 2.5 ms^{-1} (b) 5 ms^{-1} (c) 10 ms^{-1} (d) 20 ms^{-1}

12. Velocity and acceleration of a particle at some instant of time are $\mathbf{v} = (3\hat{i} + 4\hat{j})\,\text{ms}^{-1}$ and $\mathbf{a} = -(6\hat{i} + 8\hat{j})\,\text{ms}^{-2}$ respectively. At the same instant particle is at origin. Maximum x-coordinate of particle will be

(a) 1.5 m (b) 0.75 m
(c) 2.25 m (d) 4 m

13. A particle moves in the XY-plane according to the law $x = kt$, $y = kt(1 - \alpha t)$, where k and α are positive constants and t is time. The trajectory of the particle is

(a) $y = kx$ (b) $y = x - \dfrac{\alpha x^2}{k}$
(c) $y = -\dfrac{\alpha x^2}{k}$ (d) $y = \alpha x$

14. A projectile is thrown upward with a velocity v_0 at an angle α to the horizontal. The change in velocity of the projectile when it strikes the same horizontal plane is

(a) $v_0 \sin\alpha$ vertically downward
(b) $2v_0 \sin\alpha$ vertically downward
(c) $2v_0 \sin\alpha$ vertically upward
(d) zero

15. The equation of projectile is $y = \sqrt{3}x - \dfrac{g}{2}x^2$, the angle of its projection is

(a) 90° (b) zero
(c) 60° (d) 30°

16. The maximum range of a gun on horizontal terrain is 1 km. If $g = 10 \text{ m s}^{-2}$, what must be the muzzle velocity of the shell?

(a) 400 ms^{-1} (b) 200 ms^{-1}
(c) 100 ms^{-1} (d) 50 ms^{-1}

17. Two paper screens A and B are separated by a distance of 100 m. A bullet pierces A and then B. The hole in B is 10 cm below the hole in A. If the bullet is travelling horizontally at the time of hitting A, then the velocity of the bullet at A is

(a) 100 ms^{-1} (b) 200 ms^{-1}
(c) 600 ms^{-1} (d) 700 ms^{-1}

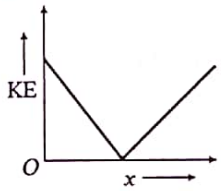
18. A body is projected at an angle of 30° with the horizontal with momentum p. At its highest point the magnitude of the momentum is

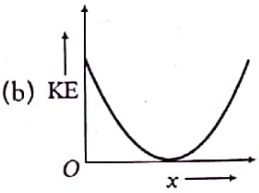
(a) $\dfrac{\sqrt{3}}{2}p$ (b) $\dfrac{2}{\sqrt{3}}p$
(c) p (d) $\dfrac{p}{2}$

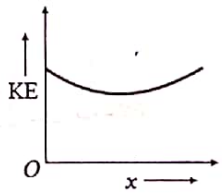
19. The maximum height attained by a projectile is increased by 10% by increasing its speed of projection, without changing the angle of projection. The percentage increases in the horizontal range will be

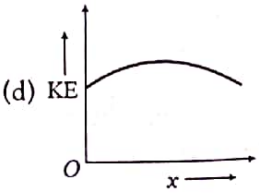
(a) 20% (b) 15% (c) 10% (d) 5%

20. A ball is thrown up with a certain velocity at an angle θ to the horizontal. The kinetic energy KE of the ball varies with horizontal displacement x as

(a)


(b)


(c)


(d)


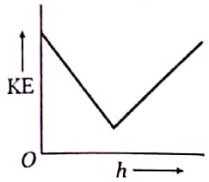
21. A ball is thrown at different angles with the same speed u and from the same point. It has the same range in both cases. If y_1 and y_2 be the heights attained in the two cases, then $y_1 + y_2$ equals to

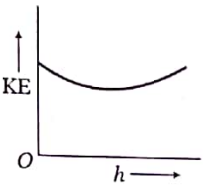
(a) $\dfrac{u^2}{g}$ (b) $\dfrac{2u^2}{g}$
(c) $\dfrac{u^2}{2g}$ (d) $\dfrac{u^2}{4g}$

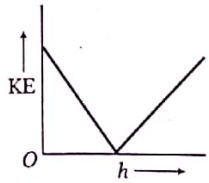
22. A projectile is fired from level ground at an angle θ above the horizontal. The elevation angle ϕ of the highest point as seen from the launch point is related to θ by the relation

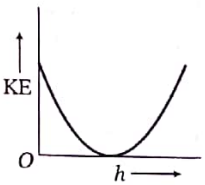
(a) $\tan\phi = \dfrac{1}{4}\tan\theta$ (b) $\tan\phi = \tan\theta$
(c) $\tan\phi = \dfrac{1}{2}\tan\theta$ (d) $\tan\phi = 2\tan\theta$

23. A ball is thrown up with a certain velocity at an angle θ to the horizontal. The kinetic energy KE of the ball varies with height h as

(a) 

(b) 

(c) 

(d) 

24. A body of mass m is thrown upwards at an angle θ with the horizontal with velocity v. While rising up the velocity of the mass after t seconds will be

(a) $\sqrt{(v\cos\theta)^2 + (v\sin\theta)^2}$ (b) $\sqrt{(v\cos\theta - v\sin\theta)^2 - gt}$

(c) $\sqrt{v^2 + g^2t^2 - (2v\sin\theta)\,gt}$ (d) $\sqrt{v^2 + g^2t^2 - (2v\cos\theta)\,gt}$

25. A projectile is thrown with an initial velocity of $(a\hat{i} + b\hat{j})\ \text{ms}^{-1}$. If the range of the projectile is twice the maximum height reached by it, then

(a) $a = 2b$ (b) $b = a$ (c) $b = 2a$ (d) $b = 4a$

26. A body projected with velocity u at projection angle θ has horizontal range R. For the same velocity and projection angle, its range on the moon surface will be $(g_{moon} = g_{earth}/6)$

(a) $36R$ (b) $\frac{R}{36}$ (c) $\frac{R}{16}$ (d) $6R$

27. Three balls of same masses are projected with equal speeds at angle 15°, 45°, 75°, and their ranges are respectively R_1, R_2 and R_3, then

(a) $R_1 > R_2 > R_3$ (b) $R_1 < R_2 < R_3$

(c) $R_1 = R_2 = R_3$ (d) $R_1 = R_3 < R_2$

28. A man can throw a stone such that it acquires maximum horizontal range 80 m. The maximum height to which it will rise for the same projectile in metre is

(a) 10 (b) 20 (c) 40 (d) 50

29. The ratio of the speed of a projectile at the point of projection to the speed at the top of its trajectory is x. The angle of projection with the horizontal is

(a) $\sin^{-1} x$ (b) $\cos^{-1} x$

(c) $\sin^{-1}(1/x)$ (d) $\cos^{-1}(1/x)$

30. The velocity at the maximum height of a projectile is half of its initial velocity of projection (u). Its range on horizontal plane is

(a) $\frac{3u^2}{g}$ (b) $\frac{3}{2}\cdot\frac{u^2}{g}$ (c) $\frac{u^2}{3g}$ (d) $\frac{\sqrt{3}}{2}\cdot\frac{u^2}{g}$

31. A projectile is thrown from a point in a horizontal plane such that the horizontal and vertical velocities are $9.8\ \text{ms}^{-1}$ and $19.6\ \text{ms}^{-1}$. It will strike the plane after covering distance of

(a) 39.2 m (b) 19.6 m (c) 9.8 m (d) 4.9 m

32. A stone is projected in air. Its time of flight is 3 s and range is 150 m. Maximum height reached by the stone is (Take, $g = 10\ \text{ms}^{-2}$)

(a) 37.5 m (b) 22.5 m (c) 90 m (d) 11.25 m

33. The greatest height to which a man can throw a stone is h. The greatest distance to which he can throw it will be

(a) $\frac{h}{2}$ (b) h (c) $2h$ (d) $3h$

34. The range of a projectile when launched at angle θ is same as when launched at angle 2θ. What is the value of θ?

(a) 15° (b) 30° (c) 45° (d) 60°

35. A boy throws a ball with a velocity u at an angle θ with the horizontal. At the same instant he starts running with uniform velocity to catch the ball before it hits the ground. To achieve this he should run with a velocity of

(a) $u\cos\theta$ (b) $u\sin\theta$

(c) $u\tan\theta$ (d) $u\sec\theta$

36. Galileo writes that for angle of projection of a projectile at angle (45° − θ) and (45° + θ), the horizontal ranges described by the projectile are in the ratio of (if θ ≤ 45°)

(a) 2 : 1 (b) 1 : 2 (c) 1 : 1 (d) 2 : 3

37. If time of flight of a projectile is 10 s. Range is 500 m. The maximum height attained by it will be

(a) 125 m (b) 50 m (c) 100 m (d) 150 m

38. Four bodies P, Q, R and S are projected with equal velocities having angles of projection 15°, 30°, 45° and 60° with the horizontal respectively. The body having shortest range is

(a) P (b) Q (c) R (d) S

39. A stone is thrown at an angle θ with the horizontal reaches a maximum height H. Then the time of flight of stone will be

(a) $\sqrt{\frac{2H}{g}}$ (b) $2\sqrt{\frac{2H}{g}}$

(c) $\frac{2\sqrt{2H\sin\theta}}{g}$ (d) $\frac{\sqrt{2H\sin\theta}}{g}$

40. For a given velocity, a projectile has the same range R for two angles of projection. If t_1 and t_2 are the time of flight in the two cases, then $t_1 = t_2$ is equal to

(a) $\frac{2R}{g}$ (b) $\frac{R}{g}$ (c) $\frac{4R}{g}$ (d) $\frac{R}{2g}$

41. A projectile can have same range from two angles of projection with same initial speed. If h_1 and h_2 be the maximum heights, then

(a) $R = \sqrt{h_1 h_2}$ (b) $R = \sqrt{2 h_1 h_2}$
(c) $R = 2\sqrt{h_1 h_2}$ (d) $R = 4\sqrt{h_1 h_2}$

42. A cricket ball is hit for a six the bat at an angle of 45° to the horizontal with kinetic energy K. At the highest point, the kinetic energy of the ball is

(a) zero (b) K (c) $K/2$ (d) $K/\sqrt{2}$

43. The equation of motion of a projectile is

$$y = 12x - \frac{3}{4}x^2$$

What is the range of the projectile?

(a) 12 m (b) 16 m (c) 20 m (d) 24 m

44. A particle is thrown with a speed u at an angle θ with the horizontal. When the particle makes an angle ϕ with the horizontal, its speed changes to v where

(a) $v = u\cos\theta$ (b) $v = u\cos\theta\cos\phi$
(c) $v = u\cos\theta\sec\phi$ (d) $v = u\sec\theta\cos\phi$

45. The equation of projectile is $Y = \sqrt{3}X - \frac{1}{2}gX^2$. The velocity of projection is

(a) $1\ \text{ms}^{-1}$ (b) $2\ \text{ms}^{-1}$ (c) $3\ \text{ms}^{-1}$ (d) $1.2\ \text{ms}^{-1}$

46. A ball of mass m is projected from the ground with an initial velocity u making an angle of θ with the vertical. What is the change in velocity between the point of projection and the highest point?

(a) $u\cos\theta$ downward (b) $u\cos\theta$ upward
(c) $u\sin\theta$ upward (d) $u\sin\theta$ downward

47. A ball is projected from ground with a speed of $20\ \text{ms}^{-1}$ at an angle of 45° with horizontal. There is a wall of 25 m height at a distance of 10 m from the projection point. The ball will hit the wall at a height of

(a) 5 m (b) 7.5 m (c) 10 m (d) 12.5 m

48. A particle (A) is dropped from a height and another particle (B) is thrown in horizontal direction with speed of 5 m/s from the same height. The correct statement is

(a) both particles will reach at ground simultaneously
(b) both particles will reach at ground with same speed
(c) particle (A) will reach at ground first with respect to particle (B)
(d) particle (B) will reach at ground first with respect to particle (A)

49. A bullet is to be fired with a speed of $2000\ \text{ms}^{-1}$ to hit a target 200 m away on a level ground. If $g = 10\ \text{ms}^{-2}$, the gun should be aimed

(a) directly at the target (b) 5 cm below the target
(c) 5 cm above the target (d) 2 cm above the target

50. An aeroplane moving horizontally with a speed of $720\ \text{kmh}^{-1}$ drops a food pocket, while flying at a height of 396.9 m. The time taken by a food pocket to reach the ground and its horizontal range is (Take $g = 9.8\ \text{ms}^{-2}$)

(a) 3 s and 2000 m (b) 5 s and 500 m
(c) 8 s and 1500 m (d) 9 s and 1800 m

51. A boy can throw a stone up to a maximum height of 10 m. The maximum horizontal distance that the boy can throw the same stone up to will be

(a) $20\sqrt{2}$ m (b) 10 m (c) $10\sqrt{2}$ m (d) 20 m

52. At the height 80 m, an aeroplane is moving with $150\ \text{ms}^{-1}$. A bomb is dropped from it so as to hit a target. At what distance from the target should the bomb be dropped (Given $g = 10\ \text{ms}^{-2}$)

(a) 605.3 m (b) 600 m (c) 80 m (d) 230 m

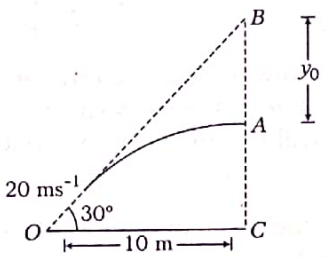
53. A particle moves along a parabolic path $y = -9x^2$ in such a way that the x component of velocity remains constant and has a value $1/3\ \text{ms}^{-1}$. The acceleration of the particle is

(a) $\frac{1}{3}\ \text{ms}^{-2}$ (b) $3\ \text{ms}^{-2}$ (c) $\frac{2}{3}\ \text{ms}^{-2}$ (d) $2\ \text{ms}^{-2}$

54. A projectile has the maximum range 500 m. If the projectile is thrown up a smooth inclined plane of 30° with the same (magnitude) velocity, the distance covered by it along the inclined plane till it stops will be

(a) 250 m (b) 500 m (c) 750 m (d) 1000 m

55. A ball is thrown from a point O aiming a target at angle 30° with the horizontal so that the ball hits the target at B but the ball hits at point A, a vertical distance h below B. If the initial velocity of the ball is $20\ \text{ms}^{-1}$ and the horizontal distance between O and C is 10 m. Find the value of h.



(a) $\frac{g}{6}$ m (b) $\frac{g}{10}$ m
(c) $\frac{g}{3}$ m (d) $\frac{g}{12}$ m

56. The horizontal range of a projectile fired at an angle of 15° is 50 m. If it is fired with the same speed at an angle of 45°, its range will be **[NCERT Exemplar]**

(a) 60 m (b) 71 m (c) 100 m (d) 141 m

57. A cricket fielder can throw the cricket ball with a speed v_0. If he throws the ball while running with speed u at an angle θ to the horizontal, what is the effective angle to the horizontal at which the ball is projected in air as seen by a spectator? **[NCERT Exemplar]**

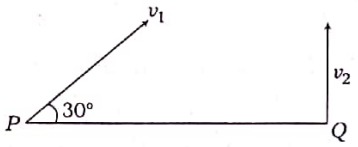
(a) $\tan^{-1}\left[\frac{v_0 \cos\theta}{u + v_0 \sin\theta}\right]$ (b) $\tan^{-1}\left[\frac{v_0 \sin\theta}{u + v_0 \cos\theta}\right]$

(c) $\tan^{-1}\left[\frac{u}{v_0 \cos\theta + v_0 \sin\theta}\right]$ (d) $\tan^{-1}\left[\frac{v_0 \sin\theta + v_0 \cos\theta}{u}\right]$

58. Two stones are projected so as to reach the same distance from the point of projection on a horizontal surface. The maximum height reached by one exceeds the other by an amount equal to half the sum of the height attained by them. Then angle of projection of the stone which attains smaller height is

(a) 45° (b) 60° (c) 30° (d) $\tan^{-1}(3/4)$

59. A projectile A is thrown at an angle 30° to the horizontal from point P. At the same time, another projectile B is thrown with velocity v_2 upwards from the point Q vertically below the highest point A would reach. For B to collide with A the ratio $\frac{v_2}{v_1}$ should be



(a) $\frac{\sqrt{3}}{2}$ (b) 2 (c) $\frac{1}{2}$ (d) $\frac{2}{\sqrt{3}}$

60. The equation of motion of a projectile are given by $x = 36\,t$ m and $2y = 96\,t - 9.8\,t^2$ m. The angle of projection is

(a) $\sin^{-1}\left(\frac{4}{5}\right)$ (b) $\sin^{-1}\left(\frac{3}{5}\right)$

(c) $\sin^{-1}\left(\frac{4}{3}\right)$ (d) $\sin^{-1}\left(\frac{3}{4}\right)$

61. A ball is thrown from a point with a speed v_0 at an angle of projection θ. From the same point and at the same instant, a person starts running with a constant speed $v_0/2$ to catch the ball. Will the person be able to catch the ball? If yes, what should be the angle of projection?

(a) Yes, 60° (b) Yes, 30° (c) No (d) Yes, 45°

62. An arrow is shot into air. Its range is 200 m and its time of flight is 5 s. If $g = 10\ \text{ms}^{-2}$, then horizontal component of velocity and the maximum height will be respectively

(a) 20 ms^{-1}, 62.50 m (b) 40 ms^{-1}, 31.25 m

(c) 80 ms^{-1}, 62.5 m (d) None of these

63. The initial velocity of a particle of mass 2 kg is $(4\hat{\mathbf{i}} + 4\hat{\mathbf{j}})\ \text{ms}^{-1}$. A constant force of $-20\,\hat{\mathbf{j}}$ N is applied on the particle. Initially the particle was at (0, 0). Find the x-coordinate of the point where its y-coordinate is again zero.

(a) 3.2 m (b) 6 m (c) 4.8 m (d) 1.2 m

64. Two balls are thrown simultaneously from ground with same velocity of 10 ms^{-1} but different angles of projection with horizontal. Both balls fall at same distance $5\sqrt{3}$ m from point of projection. What is the time interval between balls striking the ground?

(a) $(\sqrt{3} - 1)$ s (b) $(\sqrt{3} + 1)$ s (c) $\sqrt{3}$ s (d) 1 s

65. A piece of marble is projected from earth's surface with velocity of $19.6\sqrt{2}\ \text{ms}^{-1}$ at 45°. 2 s later its velocity makes an angle α with horizontal, where α is

(a) 45° (b) 30° (c) 60° (d) 0°

66. An object of mass m is projected with a momentum p at such an angle that its maximum height is 1/4th of its horizontal range. Its minimum kinetic energy in its path will be

(a) $\frac{p^2}{8m}$ (b) $\frac{p^2}{4m}$ (c) $\frac{3p^2}{4m}$ (d) $\frac{p^2}{m}$

67. A particle is projected with a velocity of 30 ms^{-1}, at an angle of $\theta_0 = \tan^{-1}\left(\frac{3}{4}\right)$. After 1 s, the particle is moving at an angle θ to the horizontal, where $\tan\theta$ will be equal to (Take, $g = 10\ \text{ms}^{-2}$)

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

68. A body is projected from the ground with a velocity $\mathbf{v} = (3\hat{\mathbf{i}} + 10\hat{\mathbf{j}})\text{ms}^{-1}$. The maximum height attained and the range of the body respectively are (given $g = 10\ \text{ms}^{-2}$)

(a) 5 m and 6 m (b) 3 m and 10 m

(c) 6 m and 5 m (d) 3 m and 5 m

69. A bomber plane moves horizontally with a speed of 500 ms^{-1} and a bomb released from it, strikes the ground in 10 s. Angle at which it strikes the ground will be ($g = 10\ \text{ms}^{-2}$)

(a) $\tan^{-1}\left(\frac{1}{5}\right)$ (b) $\tan^{-1}\left(\frac{1}{2}\right)$

(c) $\tan^{-1}(1)$ (d) $\tan^{-1}(5)$

70. A ball rolls off the edge of a horizontal table top 4 m high. If it strikes the floor at a point 5 m horizontally away from the edge of the table, what was its speed at the instant it left the table?

(a) 2.5 ms^{-1} (b) 3.5 ms^{-1} (c) 5.5 ms^{-1} (d) 6.5 ms^{-1}

71. A ball is projected upwards from the top of a tower with velocity 25 ms^{-1} making an angle of 30° with the horizontal. If the height of the tower is 70 m, after what time from the instant of throwing, will the ball reach the ground? (Take, $g = 10\ ms^{-2}$).

(a) 2 s (b) 5 s (c) 7 s (d) 9 s

72. From the top of a tower of height 40 m, a ball is projected upwards with a speed of 20 ms^{-1} at an angle of elevation of 30°. The ratio of the total time taken by the ball to hit the ground to its time of flight (time taken to come back to the same elevation) is (Take, $g = 10\ ms^{-2}$)

(a) 2 : 1 (b) 3 : 1 (c) 3 : 2 (d) 1.5 : 1

73. The coordinates of a moving particle at any time t are given by $x = ct$ and $y = bt^2$. The speed of the particle is given by

(a) $2t\sqrt{b^2 - c^2}$ (b) $\sqrt{4b^2t^2 + c^2}$

(c) $2t(b + c)$ (d) $2t(b - c)$

74. Two particles A and B are projected simultaneously from a fixed point of the ground. Particle A is projected on a smooth horizontal surface with speed v, while particle B is projected in air with speed $\frac{2v}{\sqrt{3}}$. If particle B hits the particle A, the angle of projection of B with the vertical is

(a) 30° (b) 60°

(c) Both (a) and (b) (d) 45°

75. An object is projected with a velocity of 20 ms^{-1} making an angle of 45° with horizontal. The equation for trajectory is $h = Ax - Bx^2$, where h is height, x is horizontal distance. A and B are constants. The ratio $A : B$ is (Take, $g = 10\ ms^{-2}$)

(a) 1 : 5 (b) 5 : 1

(c) 1 : 40 (d) 40 : 1

76. A particle is projected from horizontal making an angle of 53° with initial velocity 100 ms^{-1}. The time taken by the particle to make angle 45° from horizontal is

(a) 14 s (b) 2 s

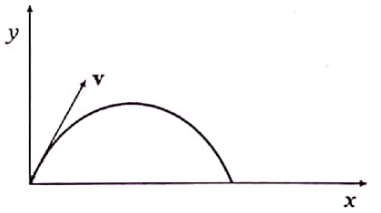
(c) Both (*a*) and (*b*) (d) None of these

77. A ground to ground projectile is at point A at $t = \frac{T}{3}$, is at point B at $t = \frac{5T}{6}$ and reaches the ground at $t = T$. The difference in heights between points A and B is

(a) $\frac{gT^2}{6}$ (b) $\frac{gT^2}{12}$

(c) $\frac{gT^2}{18}$ (d) $\frac{gT^2}{24}$

78. If the instantaneous velocity of a particle projected as shown in figure is given by $\mathbf{v} = a\hat{i} + (b - ct)\hat{j}$ where a, b, and c are positive constants, the range on the horizontal plane will be



(a) $2ab/c$ (b) ab/c (c) ac/b (d) $a/2bc$

79. A ball is thrown from the ground to a wall 3 m high at a distance of 6 m and falls 18 m away from the wall, the angle of projection of ball is

(a) $\tan^{-1}\left(\frac{3}{2}\right)$ (b) $\tan^{-1}\left(\frac{2}{3}\right)$ (c) $\tan^{-1}\left(\frac{1}{2}\right)$ (d) $\tan^{-1}\left(\frac{3}{4}\right)$

80. The horizontal range and maximum height attained by a projectile are R and H respectively. If a constant horizontal acceleration $a = g/4$ is imparted to the projectile due to wind, then its horizontal range and maximum height will be

(a) $(R + H), \frac{H}{2}$ (b) $\left(R + \frac{H}{2}\right), 2H$

(c) $(R + 2H), H$ (d) $(R + H), H$

81. A large number of bullets are fired in all directions with same speed v. What is the maximum area on the ground on which these bullets will spread?

(a) $\pi\frac{v^2}{g}$ (b) $\pi\frac{v^4}{g^2}$

(c) $\pi^2\frac{v^4}{g^2}$ (d) $\pi^2\frac{v^2}{g^2}$

82. A cart is moving horizontally along a straight line with a constant speed of 30 ms^{-1}. A projectile is to be fired from the moving cart in such a way that it will return to the cart (at the same point on cart) after the cart has moved 80 m. At what velocity (relative to the cart) must be projectile be fired? (Take, $g = 10\ ms^{-2}$)

(a) 10 ms^{-1} (b) $\frac{20}{3}\ ms^{-1}$

(c) $\frac{40}{3}\ ms^{-1}$ (d) $\frac{80}{3}\ ms^{-1}$

83. A particle is projected from the ground at an angle of θ with the horizontal with an initial speed of u. Time after which velocity vector of the projectile is perpendicular to the initial velocity.

(a) $u/g \sin\theta$ (b) $u/g \cos\theta$

(c) $2u/g \sin\theta$ (d) $2u \tan\theta$

84. Two particles are simultaneously projected in opposite directions horizontally from a given point in space where gravity g is uniform. If u_1 and u_2 be their initial speeds, then the time t after which their velocities are mutually perpendicular is given by

(a) $\frac{\sqrt{u_1 u_2}}{g}$ (b) $\frac{\sqrt{u_1^2 + u_2^2}}{g}$

(c) $\frac{\sqrt{u_1 (u_1 + u_2)}}{g}$ (d) $\frac{\sqrt{u_2 (u_1 + u_2)}}{g}$

85. A hill is 500 m high. Supplies are to be sent across the hill using a canon that can hurl packets at a speed of 125 ms^{-1} over the hill. The canon is located at a distance of 800 m from the foot of hill and can be moved on the ground at a speed of 2 ms^{-1}; so that its distance from the hill can be adjusted. What is the shortest time in which a packet can reach on the ground across the hill? (Take, $g = 10\ ms^{-2}$)

[NCERT Exemplar]

(a) 10 s (b) 25 s (c) 35 s (d) 45 s

86. A ball is rolled off the edge of a horizontal table at a speed of 4 ms^{-1}. It hits the ground after 0.4 s. Which statement given below is true

(a) It hits the ground at a horizontal distance 1.6 m from the edge of the table
(b) The speed with which it hits the ground is 4.0 ms^{-1}
(c) Height of the table 1m
(d) It hits the ground at an angle of 60° to the horizontal

87. A jet aeroplane is flying at a constant height of 2 km with a speed 360 kmh^{-1} above the ground towards a target and releases a bomb. After how much time it will hit the target and what will be the horizontal distance of the aeroplane from the target so that the bomb should hit the target? (Take $g = 10\ ms^{-2}$)

(a) 10 s, 1 km (b) 20 s, 2 km
(c) 30 s, 3 km (d) 40 s, 4 km

88. Two second after projection, a projectile is travelling in a direction inclined at 30° to be horizontal. After 1 more second it is travelling horizontally. Then (Take, $g = 10\ ms^{-2}$)

(a) the velocity of projection is $20\sqrt{3}\ ms^{-1}$
(b) the angle of projection is 30° with horizontal
(c) Both (a) and (b) are correct
(d) Both (a) and (b) are wrong

89. A projectile is fired at an angle of 30° to the horizontal such that the vertical component of its initial velocity is 80 ms^{-1}. Its time of flight is T. Its velocity at $t = T/4$ has a magnitude of nearly

(a) 200 ms^{-1} (b) 300 ms^{-1}
(c) 100 ms^{-1} (d) None of these

90. A very broad elevator is going up vertically with a constant acceleration of 2 ms^{-2}. At the instant when its velocity is 4 ms^{-1}, a ball is projected from the floor of the lift with a speed of 4 ms^{-1} relative to the floor at an elevation of 30°. The time taken by the ball to return the floor is (Take, $g = 10\ ms^{-2}$)

(a) $\frac{1}{2}$ s (b) $\frac{1}{3}$ s
(c) $\frac{1}{4}$ s (d) 1 s

91. A projectile is thrown with some initial velocity at an angle α to the horizontal. Its velocity when it is at the highest point is $(2/5)^{1/2}$ times the velocity when it is at height half of the maximum height. Find the angle of projection α with the horizontal.

(a) 30° (b) 45° (c) 60° (d) 37°

92. A body of mass 1 kg is projected with velocity 50 ms^{-1} at an angle of 30° with the horizontal. At the highest point of its path a force 10 N starts acting on body for 5 s vertically upward besides gravitational force, what is horizontal range of the body? (Take, $g = 10\ ms^{-2}$)

(a) $125\sqrt{3}$ m (b) $200\sqrt{3}$ m (c) 500 m (d) $250\sqrt{3}$ m

93. A grasshopper can jump maximum distance 1.6 m. How far can it go in 10 s?

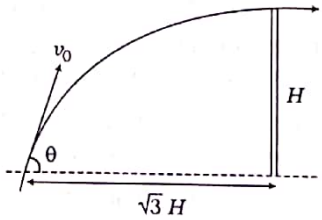
(a) $5\sqrt{2}$ m (b) $10\sqrt{2}$ m (c) $20\sqrt{2}$ m (d) $40\sqrt{2}$ m

94. A ball rolls off top of a stair way with a horizontal velocity $u\ ms^{-1}$. If the steps are h metres high and b metres wide, the ball will just hit the edge of nth step if n equals to

(a) $\frac{hu^2}{gb^2}$ (b) $\frac{u^2 g}{gb^2}$ (c) $\frac{2hu^2}{gb^2}$ (d) $\frac{2u^2 g}{hb^2}$

95. A projectile is thrown at an angle θ such that it is just able to cross a vertical wall at its highest point as shown in the figure.

The angle θ at which the projectile is thrown is given by



(a) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (b) $\tan^{-1}\sqrt{3}$

(c) $\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (d) $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$

96. Two particles are projected from the same point with same speed u at angles of projection α and β from horizontal. The maximum heights attained by them are h_1 and h_2 respectively, R is the range for both. If t_1 and t_2 their times of flight respectively, then

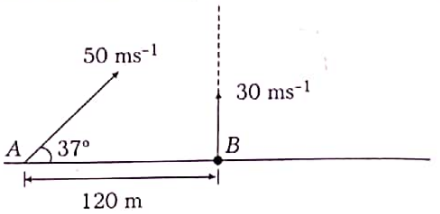
(a) $\alpha + \beta = \frac{\pi}{2}$

(b) $R = 4\sqrt{h_1 h_2}$

(c) $\tan\alpha = \frac{t_1}{t_2} = \sqrt{h_1 h_2}$

(d) None of the above

97. Balls A and B are thrown from two points lying on the same horizontal plane separated by a distance 120 m. Which of the following statement (s) is/are correct?



(a) The two balls can never meet
(b) The balls can meet if the ball B is thrown 1 s later
(c) The two balls meet at a height of 45 m
(d) None of the above

B. Medical entrance special format questions

Assertion and reason

Directions (Q. Nos. 1-13) *These questions consists of two statements each printed as Assertion and Reason. While answering these question you are required to choose any one of the following four responses*

(a) If both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If Assertion is false but Reason is true.

1. Assertion In projectile motion, if time of flight is made two times, then maximum height will become four times.

Reason $T \propto \sin\theta$ and $H \propto \sin^2\theta$, where θ is angle of projection.

2. Assertion If in a projectile motion, we take air friction into consideration, then $t_{ascent} < t_{descent}$.

Reason During ascent, magnitude of retardation is greater than magnitude of acceleration during descent.

3. Assertion In case of projectile motion, the magnitude of rate of change of velocity is variable.

Reason In projectile motion, magnitude of velocity first decreases and then increases during the motion.

4. Assertion At highest point of a projectile, dot product of velocity and acceleration is zero.

Reason At highest point, velocity and acceleration are mutually perpendicular.

5. Assertion A particle is projected with speed u at an angle θ with the horizontal. At any time during motion, speed of particle is v at angle α with the vertical, then $v\sin\alpha$ is always constant throughout the motion.

Reason In case of projectile motion, magnitude of radial acceleration at topmost point is maximum.

6. Assertion In projectile motion, if time of flight is 4 s, then maximum height will be 20 m. (Take, $g = 10\ \text{ms}^{-2}$)

Reason Maximum height $= \frac{gT}{2}$.

7. Assertion For projection angle $\tan^{-1}(4)$, the horizontal and maximum height of a projectile are equal.

Reason The maximum range of projectile is directly proportional to square of velocity and inversely proportional to acceleration due to gravity.

8. Assertion A particle in xy-plane is governed by $x = a\sin\omega t$ and $y = a\cos\omega t$ where a as well as ω are constants, then the particle will have parabolic motion.

Reason A particle under the influence of mutually perpendicular velocities has curvilinear motion.

9. Assertion If a particle is projected vertically upwards with velocity u, the maximum height attained by the particle is h_1. The same particle is projected at angle 30° from horizontal with the same speed u. Now the maximum height is h_2. Thus $h_1 = 4h_2$

Reason In first case, $v = 0$ at highest point and in second case, $v \neq 0$ at highest point.

10. Assertion On the surface of moon, value of g is $\frac{1}{6}$th the value on the surface of earth. A particle is projected as projectile under similar condition on the surface of moon and on the surface of earth. Then values of T, H and R on the surface of moon will become six times.

Reason T, H and $R \propto \frac{1}{g}$

11. **Assertion** In projectile motion, the angle between instantaneous velocity vector and acceleration vector can be anything between 0 to π (excluding the limiting case.)

Reason In projectile motion, acceleration vector is always pointing vertically downwards. (Neglect air friction).

12. **Assertion** Particel-1 is dropped from a tower and particle-2 is projected horizontal from the same tower. Then both the particles reach the ground simultaneously.

Reason Both the particles strike the ground with different speeds.

13. **Assertion** At height 20 m from ground, velocity of a projectile is $\mathbf{v} = (20\hat{\mathbf{i}} + 10\hat{\mathbf{j}})\ \text{ms}^{-1}$. Here, $\hat{\mathbf{i}}$ is horizontal and $\hat{\mathbf{j}}$ is vertical. Then, the particle is at the same height after 4 s.

Reason Maximum height of particle from ground is 40 m. (Take, $g = 10\ \text{ms}^{-2}$)

Match the columns

1. A particle is projected from ground with velocity u at angle θ from horizontal. Match the following two columns.

Column I	Column II
(A) Average velocity between initial and final points	(p) $u\sin\theta$
(B) Change in velocity between initial and final points	(q) $u\cos\theta$
(C) Change in velocity between initial and peak points	(r) zero
(D) Average velocity between initial and highest points	(s) None

2. Given that u_x = horizontal component of initial velocity of a projectile, u_y = vertical component of initial velocity, R = horizontal range, T = time of flight and H = maximum height of projectile. Now match the following two columns.

Column I	Column II
(A) u_x is doubled, u_y is halved	(p) H will remain unchanged
(B) u_y is doubled u_x is halved	(q) R will remain unchanged
(C) u_x and u_y both are doubled	(r) R will become four times
(D) Only u_y is doubled	(s) H will become four times

3. A particle is projected horizontally from a tower with velocity $10\ \text{ms}^{-1}$. Taking, $g = 10\ \text{ms}^{-2}$. Match the following two columns at time $t = 1$ s.

Column I	Column II
(A) Horizontal component of velocity	(p) 5 SI units
(B) Vertical component of velocity	(q) 10 SI unit
(C) Horizontal displacement	(r) 15 SI unit
(D) Vertical displacement	(s) 20 SI unit

4. Trajectory of particle in a projectile motion is given as $y = x - \frac{x^2}{80}$. Here, x and y are in metre. For this projectile motion match the following with $g = 10\ \text{ms}^{-2}$.

Column I	Column II
(A) Angle of projection	(p) 20 m
(B) Angle of velocity with horizontal after 4 s	(q) 80 m
(C) Maximum height	(r) 45°
(D) Horizontal range	(s) $\tan^{-1}\left(\frac{1}{2}\right)$

C. Medical entrances' gallery

(Collection of questions asked in NEET & various medical entrance exams)

1. A particle moves so that its position vector is given by $\mathbf{r} = \cos\omega t\hat{\mathbf{x}} + \sin\omega t\ \hat{\mathbf{y}}$, where, ω is a constant.

Which of the following is true? **[NEET 2016]**

(a) Velocity and acceleration both are parallel to **r**
(b) Velocity is perpendicular to **r** and acceleration is directed towards to origin
(c) Velocity is perpendicular to **r** and acceleration is directed away from the origin
(d) Velocity and acceleration both are perpendicular to **r**

2. A particle is projected with an angle of projection θ to the horizontal line passing through the points (P, Q) and (Q, P) referred to horizontal and vertical axes (can be treated as X-axis and Y-axis respectively).

The angle of projection can be given by **[AIIMS 2015]**

(a) $\tan^{-1}\left[\frac{P^2 + PQ + Q^2}{PQ}\right]$ (b) $\tan^{-1}\left[\frac{P^2 + Q^2 - PQ}{PQ}\right]$

(c) $\tan^{-1}\left[\frac{P^2 + Q^2}{2PQ}\right]$ (d) $\sin^{-1}\left[\frac{P^2 + Q^2 + PQ}{2PQ}\right]$

3. An object is thrown towards the tower which is at a horizontal distance of 50 m with an initial velocity of $10\ \text{ms}^{-1}$ and making an angle 30° with the horizontal.

The object hits the tower at certain height. The height from the bottom of the tower, where the object hits the tower is (Take, $g = 10\ \text{ms}^{-2}$) **[EAMCET 2015]**

(a) $\frac{50}{\sqrt{3}}\left[1-\frac{10}{\sqrt{3}}\right]$ m (b) $\frac{50}{3}\left[1-\frac{10}{\sqrt{3}}\right]$ m

(c) $\frac{100}{\sqrt{3}}\left[1-\frac{10}{\sqrt{3}}\right]$ m (d) $\frac{100}{3}\left[1-\frac{10}{\sqrt{3}}\right]$ m

4. The range of a projectile is R when the angle of projection is 40°. For the same velocity of projection and range, the other possible angle of projection is **[Kerala CEE 2015]**

(a) 45° (b) 50° (c) 60° (d) 40°

5. A particle is projected with a velocity v so that its horizontal range twice the greatest height attained. The horizontal range is **[KCET 2015]**

(a) $\frac{4v^2}{5g}$ (b) $\frac{v^2}{g}$ (c) $\frac{v^2}{2g}$ (d) $\frac{2v^2}{3g}$

6. If the angle of projection of a projector with same initial velocity exceed or fall short of 45° by equal amount α, then the ratio of horizontal ranges is **[Kerala CEE 2014]**

(a) 1 : 2 (b) 1 : 3 (c) 1 : 4 (d) 1 : 1
(e) $1:\sqrt{2}$

7. A particle is moving such that its position coordinates (x, y) are $(2m, 3m)$ at time $t = 0$, $(6m, 7m)$ at time $t = 2$ s and $(13m, 14m)$ at time $t = 5$ s. Average velocity vector (v_{av}) from $t = 0$ to 5 s is **[CBSE AIPMT 2014]**

(a) $\frac{1}{5}(13\hat{i}+14\hat{j})$ (b) $\frac{7}{3}(\hat{i}+\hat{j})$

(c) $2(\hat{i}-\hat{j})$ (d) $\frac{11}{5}(\hat{i}+\hat{j})$

8. A cricket ball thrown across a field is at heights h_1 and h_2 from the point of projection at times t_1 and t_2 respectively after the throw. The ball is caught by a fielder at the same height as that of projection. The time of flight of the ball in this journey is **[WB JEE 2014]**

(a) $\left(\frac{h_1t_2^2-h_2t_1^2}{h_1t_2-h_2t_1}\right)$ (b) $\left(\frac{h_1t_2^2-h_2t_1^2}{h_1t_2-h_2t_1}\right)$

(c) $\left(\frac{h_1t_2^2-h_2t_1^2}{h_1t_2-h_2t_1}\right)$ (d) None of these

9. For an object thrown at 45° to the horizontal, the maximum height H and horizontal range R are related as **[UK PMT 2014]**

(a) $R = 16H$ (b) $R = 8H$ (c) $R = 4H$ (d) $R = 2H$

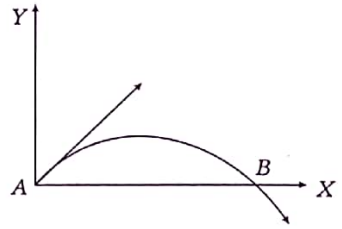
10. A body is projected horizontally from the top of a tower with a velocity of $10\ \text{ms}^{-1}$. If it hits the ground at an angle of 45°, then the vertical component of velocity when it hits ground (in ms^{-1}) is **[EAMCET 2014]**

(a) $10\sqrt{2}$ (b) $5\sqrt{2}$ (c) 5 (d) 10

11. A body is projected with an angle θ. The maximum height reached is h. If the time of flight is 4 s and $g = 10\ \text{ms}^{-2}$, then value of h is **[EAMCET 2014]**

(a) 40 m (b) 20 m
(c) 5 m (d) 10 m

12. The velocity of a projectile at the initial point A is $(2\hat{i}+3\hat{j})\ \text{ms}^{-1}$. Its velocity (in ms^{-1}) at point B is **[NEET 2013]**



(a) $-2\hat{i}-3\hat{j}$ (b) $-2\hat{i}+3\hat{j}$
(c) $2\hat{i}-3\hat{j}$ (d) $2\hat{i}+3\hat{j}$

13. A projectile is thrown with initial velocity u_0 and angle 30° with the horizontal. If it remains in the air for 1s, what was its initial velocity? **[J & K CET 2013]**

(a) $19.6\ \text{ms}^{-1}$ (b) $9.8\ \text{ms}^{-1}$ (c) $4.9\ \text{ms}^{-1}$ (d) $1\ \text{ms}^{-1}$

14. A projectile is projected at $10\ \text{ms}^{-1}$ by making at an angle 60° to the horizontal. After some time, its velocity makes and angle of 30° to the horizontal. Its speed at this instant is **[KCET 2013]**

(a) $\frac{10}{\sqrt{3}}$ (b) $10\sqrt{3}$ (c) $\frac{5}{\sqrt{3}}$ (d) $5\sqrt{3}$

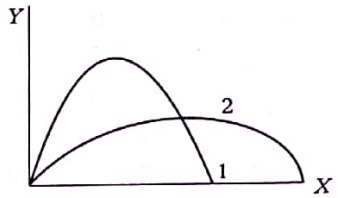
15. Two particles are projected upwards with the same initial velocity v_0 in two different angles of projection such that their horizontal ranges are the same. The ratio of the heights of their highest point will be **[UP CPMT 2013]**

(a) $\tan^2\theta_1$ (b) $v_0^2\sin^2\theta_1$
(c) $v_0\sin\theta_1$ (d) $v_0/\cos\theta_1$

16. The velocity vector of the motion described by the position vector of a particle, $r = 2t\hat{i} + t^2\hat{j}$ is given by **[J & K CET 2013]**

(a) $v = 2\hat{i} + 2t\hat{j}$ (b) $v = 2t\hat{i} + 2t\hat{j}$
(c) $v = t\hat{i} + t^2\hat{j}$ (d) $v = 2\hat{i} + t^2\hat{j}$

17. Trajectories of two projectiles are shown in figure, let T_1 and T_2 be the periods and u_1 and u_2 are their speeds of projections, then **[UP CPMT 2012]**



(a) $T_2 > T_1$ (b) $T_1 = T_2$
(c) $u_1 > u_2$ (d) $u_1 < u_2$

18. The horizontal range and the maximum height of a projectile are equal. The angle of projection of the projectile is **[CBSE AIPMT 2012]**

(a) $\theta = \tan^{-1}\left(\frac{1}{4}\right)$ (b) $\theta = \tan^{-1}(4)$

(c) $\theta = \tan^{-1}(2)$ (d) $\theta = 45°$

19. If for the same range, the two heights attained are 20 m and 80 m, then the range will be **[BHU 2012]**

(a) 20 m (b) 40 m (c) 120 m (d) 160 m

20. A ball thrown by one player reaches the other in 2 s. The maximum height attained by the ball above the point of projection will be (take, $g = 10\ \text{ms}^{-2}$) **[BHU 2012]**

(a) 2.5 m (b) 5 m (c) 7.5 m (d) 10 m

Answers

Check point 4.1

1. (a)	2. (a)	3. (b)	4. (c)	5. (c)	6. (b)	7. (b)	8. (c)	9. (a)	10. (b)
11. (c)	12. (d)	13. (a)	14. (c)	15. (a)					

Check point 4.2

1. (d)	2. (d)	3. (a)	4. (d)	5. (b)	6. (c)	7. (a)	8. (b)	9. (c)	10. (a)
11. (d)	12. (c)	13. (a)	14. (d)	15. (d)	16. (d)	17. (d)	18. (c)	19. (b)	20. (c)

Check point 4.3

1. (a)	2. (b)	3. (b)	4. (a)	5. (c)	6. (a)	7. (b)	8. (a)	9. (a)	10. (c)

A. Taking it together

1. (d)	2. (c)	3. (b)	4. (d)	5. (c)	6. (d)	7. (d)	8. (d)	9. (c)	10. (b)
11. (c)	12. (b)	13. (b)	14. (b)	15. (c)	16. (c)	17. (d)	18. (a)	19. (c)	20. (c)
21. (c)	22. (c)	23. (a)	24. (c)	25. (c)	26. (d)	27. (d)	28. (b)	29. (d)	30. (d)
31. (a)	32. (d)	33. (c)	34. (b)	35. (a)	36. (c)	37. (a)	38. (a)	39. (b)	40. (a)
41. (d)	42. (c)	43. (b)	44. (c)	45. (b)	46. (a)	47. (b)	48. (a)	49. (c)	50. (d)
51. (d)	52. (a)	53. (d)	54. (b)	55. (a)	56. (c)	57. (b)	58. (c)	59. (c)	60. (a)
61. (a)	62. (b)	63. (a)	64. (a)	65. (d)	66. (b)	67. (d)	68. (a)	69. (a)	70. (c)
71. (c)	72. (a)	73. (b)	74. (a)	75. (d)	76. (c)	77. (d)	78. (a)	79. (b)	80. (d)
81. (b)	82. (c)	83. (a)	84. (a)	85. (d)	86. (a)	87. (b)	88. (c)	89. (d)	90. (b)
91. (c)	92. (d)	93. (c)	94. (c)	95. (c)	96. (d)	97. (c)			

B. Medical entrance special format Questions

Assertion and reason

1. (a)	2. (a)	3. (d)	4. (a)	5. (b)	6. (c)	7. (b)	8. (d)	9. (b)	10. (a)
11. (a)	12. (b)	13. (b)							

Match the columns

1. (A → q, B → s, C → p, D → s)
2. (A → q, B → q, s, C → r, s, D → s)
3. (A → q, B → q, C → q, D → p)
4. (A → r, B → r, C → p, D → q)

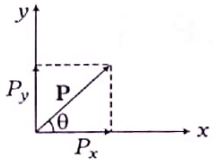
C. Medical entrances' gallery

1. (b)	2. (a)	3. (a)	4. (b)	5. (a)	6. (d)	7. (d)	8. (c)	9. (c)	10. (d)
11. (b)	12. (c)	13. (b)	14. (a)	15. (a)	16. (a)	17. (d)	18. (b)	19. (d)	20. (b)

Hints and explanations

Check point 4.1

1. (*a*) Let **P** be as shown in the figure, then according to the given information



$$P_x = 5,\ P_y = 6$$

$\therefore$ $|\mathbf{P}| = \sqrt{P_x^2 + P_y^2} = \sqrt{25 + 36}$

$\Rightarrow$ $|\mathbf{P}| = \sqrt{61}$

And $\tan\theta = \dfrac{P_y}{P_x} = \dfrac{6}{5} \Rightarrow \theta = \tan^{-1}\left(\dfrac{6}{5}\right)$

Thus, **P** having magnitude $\sqrt{61}$ lies in XY-plane at an angle $\tan^{-1}\left(\dfrac{6}{5}\right)$ to the x-axis.

2. (*a*) Positions of the particle are

$$\mathbf{r}_1 = 6\hat{\mathbf{i}} + 8\hat{\mathbf{j}} \text{ and } \mathbf{r}_2 = 12\hat{\mathbf{i}} + 10\hat{\mathbf{j}}$$

$\therefore$ $\Delta\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1 = 6\hat{\mathbf{i}} + 2\hat{\mathbf{j}} \Rightarrow |\Delta\mathbf{r}| = \sqrt{40}$

$$\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right) = \tan^{-1}\left(\frac{1}{3}\right) = 18.43°$$

3. (*b*) Displacement, $\Delta\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1 = 4\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$

$\therefore$ $\mathbf{v}_{av} = \dfrac{\Delta\mathbf{r}}{\Delta t} = \dfrac{4\hat{\mathbf{i}} + 4\hat{\mathbf{j}}}{2} = 2(\hat{\mathbf{i}} + \hat{\mathbf{j}})\ \text{ms}^{-1}$

$\Rightarrow$ Magnitude of velocity,

$$|\mathbf{v}_{av}| = 2\sqrt{1^2 + 1^2} = 2\sqrt{2}\ \text{ms}^{-1}$$

Direction, $\theta = \tan^{-1}\left(\dfrac{\Delta v_y}{\Delta v_x}\right) = \tan^{-1}\left(\dfrac{2}{2}\right) = \tan^{-1} 1 = 45°$

4. (*c*) $v_x = 4t,\ v_y = 2t - 4,\ v_z = 3$

At $t = 0,\ v_x = 0,\ v_y = -4$ and $v_z = 3$

$\therefore$ $v = \sqrt{v_y^2 + v_z^2} = 5$ units

5. (*c*) As, $v_x = \dfrac{dx}{dt} = \dfrac{2t}{2} = t$

At 2 s, $v_x = 2\ \text{ms}^{-1}$

Further, $y = \dfrac{x^2}{2} = \dfrac{(t^2/2)^2}{2} = \dfrac{t^4}{8} \Rightarrow v_y = \dfrac{dy}{dt} = \dfrac{t^3}{2}$

At $t = 2\text{ s},\ v_y = 4\ \text{ms}^{-1}$

6. (*b*) Given, $\mathbf{r} = (a\sin\omega t)\,\hat{\mathbf{i}} + (a\cos\omega t)\,\hat{\mathbf{j}}$

$\therefore$ $\mathbf{v} = \dfrac{dr}{dt} = (-a\omega\sin\omega t)\,\hat{\mathbf{j}} + (a\omega\cos\omega t)\,\hat{\mathbf{i}}$

$\mathbf{r}\cdot\mathbf{v} = 0 \Rightarrow \mathbf{r} \perp \mathbf{v}$

7. (*b*) Position vector, $\mathbf{r} = 3t^2\hat{\mathbf{i}} + 6t\hat{\mathbf{j}} + \hat{\mathbf{k}}$

$\therefore$ Velocity vector, $\mathbf{v} = \dfrac{d\mathbf{r}}{dt} = 6t\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + 0 = 6t\hat{\mathbf{i}} + 6\hat{\mathbf{j}}$

$\therefore$ Magnitude of the velocity of the object along Y-axis = 6

8. (*c*) $v_y = \dfrac{dy}{dt} = 8 - 10t,\ v_x = \dfrac{dx}{dt} = 6$

At $t = 0,\ v_y = 8\ \text{ms}^{-1}$ and $v_x = 6\ \text{ms}^{-1}$

$\therefore$ $v = \sqrt{v_x^2 + v_y^2} = 10\ \text{ms}^{-1}$

9. (*a*) Given, $x = ct^2$

$\therefore$ $v_x = \dfrac{dx}{dt} = 2ct$

Also, $y = bt^2$

$\therefore$ $v_y = \dfrac{dy}{dt} = 2bt$

$\therefore$ $v = \sqrt{v_x^2 + v_y^2} = 2t\sqrt{c^2 + b^2}$

10. (*b*) Given, $x = 2t^3$

$\therefore$ $v_x = \dfrac{dx}{dt} = 6t^2 \Rightarrow a_x = \dfrac{dv_x}{dt} = 12t$

Also, $y = 3t^3$

$\Rightarrow$ $v_y = \dfrac{dy}{dt} = 9t^2 \Rightarrow a_y = \dfrac{dv_y}{dt} = 18t$

$\therefore$ Acceleration, $a = \sqrt{a_x^2 + a_y^2} = t\sqrt{468}$

11. (*c*) $v_x = \dfrac{dx}{dt} = 6t - 6,\ a_x = \dfrac{dv_x}{dt} = 6\ \text{ms}^{-2}$

$v_y = 2t - 2,\ a_y = \dfrac{dv_y}{dt} = 2\ \text{ms}^{-2}$

At $t = 1\text{ s}$, v_x and v_y both are zero. Hence, net velocity is zero

12. (*d*) $\mathbf{a}_{av} = \dfrac{\Delta\mathbf{v}}{\Delta t} = \dfrac{3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}}}{2} = \dfrac{\hat{\mathbf{i}} - 5\hat{\mathbf{j}}}{2}$

13. (*a*) $\mathbf{v} = \mathbf{u} + \mathbf{a}t = (4\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) + (0.4\hat{\mathbf{i}} + 0.3\hat{\mathbf{j}})(10) = (8\hat{\mathbf{i}} + 6\hat{\mathbf{j}})$

$\therefore$ $v = \sqrt{(8)^2 + (6)^2} = 10$ units

14. (*c*) Position vector of the body, $\mathbf{r} = 3\hat{\mathbf{i}} + 7\hat{\mathbf{j}}$

$\because$ acceleration of the body

$\mathbf{a} = 4\hat{\mathbf{i}}$ and $t = 3\text{ s}$

using $\mathbf{s} = \mathbf{u}_0 t + \dfrac{1}{2}\mathbf{a}t^2 = 0 \times 3 + \dfrac{4\hat{\mathbf{i}}}{2} \times 3 \times 3$

$\Rightarrow$ $\mathbf{s} = 18\hat{\mathbf{i}}$

$\therefore$ New position vector of the body

$$\mathbf{r}_1 = 3\hat{\mathbf{i}} + 7\hat{\mathbf{j}} + 18\hat{\mathbf{i}} = 21\hat{\mathbf{i}} + 7\hat{\mathbf{j}}$$

Co-ordinates (21, 7)

15. (*a*) Initial position vector, $\mathbf{r}_1 = 3\hat{\mathbf{i}} - 8\hat{\mathbf{j}}$

Final position vector, $\mathbf{r}_2 = 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$

Change in position

$$\Delta\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1 = 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 3\hat{\mathbf{i}} + 8\hat{\mathbf{j}} = -\hat{\mathbf{i}} + 12\hat{\mathbf{j}}$$

Using $\quad \mathbf{s} = \mathbf{u}_0 t + \frac{1}{2}\mathbf{a}t^2$

$$\Rightarrow \quad -\hat{\mathbf{i}} + 12\hat{\mathbf{j}} = 0 + \frac{1}{2}\mathbf{a}(4)^2 \Rightarrow \mathbf{a} = \frac{-\hat{\mathbf{i}} + 12\hat{\mathbf{j}}}{8} = \frac{-\hat{\mathbf{i}}}{8} + \frac{3}{2}\hat{\mathbf{j}}$$

Check point 4.2

1. (*d*) Velocity is horizontal and acceleration is vertical download. Therefore, the direction of its velocity and acceleration are perpendicular to each other.

2. (*d*) Acceleration throughout the projectile motion remains constant and equal to g.

3. (*a*) An external force by gravity is present throughout the motion. So, momentum will not be conserved.

4. (*d*) $a_x = 0 \quad \therefore u_x = \text{constant}$

5. (*b*) At highest point, vertical component of velocity is zero. Only horizontal component of velocity is present.

$v_x = u\cos\theta = 50\cos(60) = 25\ \text{ms}^{-1}$

6. (*c*) Time of flight, $T = \frac{2u\sin\theta}{g} = \frac{2 \times 50 \times \sin 30°}{10} = 5\ \text{s}$

7. (*a*) The particle hits the horizontal plane again at time.

$$T = \frac{2u\sin\theta}{g} = \frac{40 \times \sin 60°}{9.8} = 3.53\ \text{s}$$

8. (*b*) $\frac{u_1^2 \sin^2 45°}{2g} = \frac{u_2^2 \sin 60°}{2g}$

$$\therefore \quad \frac{u_1}{u_2} = \frac{\sin 60°}{\sin 45°} = \frac{\sqrt{3}/2}{(1/\sqrt{2})} = \sqrt{3} : \sqrt{2}$$

9. (*c*) $\quad H = \frac{u^2 \sin^2\theta}{2g} \Rightarrow H \propto u^2$

If initial velocity be doubled, then maximum height reached by the projectile will become four times.

10. (*a*) Maximum height, $H = \frac{u^2 \sin^2\theta}{2g}$

and Time of flight, $T = \frac{2u\sin\theta}{g}$

So, $\quad \frac{H}{T^2} = \frac{u^2 \sin^2\theta/2g}{4u^2 \sin^2\theta/g^2} = \frac{g}{8} = \frac{5}{4}$

11. (*d*) Given at maximum height

$u\cos\theta = \frac{1}{2}u \Rightarrow \cos\theta = \frac{1}{2} \quad \therefore \theta = 60°$

$$\therefore \quad H = \frac{u^2 \sin^2\theta}{2g} = \frac{u^2 \sin^2 60°}{2g} = \frac{3u^2}{8g}$$

12. (*c*) Projectile will strike at highest point of its path with velocity $v_0 \cos\alpha$.

14. (*d*) $T \propto \sin\theta, \frac{T_A}{T_B} = \frac{\sin 30°}{\sin 60°} = \frac{1}{\sqrt{3}}$ or $T_B = \sqrt{3}\,T_A$

$$H \propto \sin^2\theta, \frac{H_A}{H_B} = \frac{\sin^2 30°}{\sin^2 60°} = \frac{1}{3} \quad \text{or} \quad H_B = 3H_A$$

As, $\quad R_\theta = R_{90°-\theta}$

$\therefore \quad R_A = R_B$

15. (*d*) $\quad R = \frac{u^2 \sin 2\theta}{g}$

$\therefore \quad R \propto u^2$

If initial velocity be doubled at some angle of projection, then range will become four times.

16. (*d*) $\frac{R}{H} = \frac{\frac{u^2 \sin 2\theta}{g}}{\frac{u^2 \sin^2\theta}{2g}} = \frac{2\sin\theta\cos\theta}{g} \times \frac{2g}{\sin^2\theta} = 4\cot\theta$

$\Rightarrow \quad R = 4H\cot\theta$ if $\theta = 45°$ then $R = 4H\cot(45°) = 4H$

17. (*d*) $R = 4H\cot\theta$

If $\theta = 45°$, then $4H \Rightarrow \frac{R}{H} = \frac{4}{1}$

18. (*c*) Given range, $R = 4\sqrt{3}H$

$$\Rightarrow \frac{2u^2 \sin\theta\cos\theta}{g} = 4\sqrt{3}\left(\frac{u^2 \sin^2\theta}{2g}\right) \Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$$

$\therefore \quad \theta = 30°$.

19. (*b*) $\tan 30° = \frac{v_y}{v_x} = \frac{u_y - gt}{u_x}$

$$= \frac{(20\sqrt{3}\sin 60°) - 10t}{(20\sqrt{3}\cos 60°)}$$

or $\quad 10 = 30 - 10t$

$\therefore \quad t = 2\ \text{s}$

20. (*c*) $\quad v^2 = v_y^2 + v_x^2$ or $5g = (u_y - gt)^2 + u_x^2$

Since, $\quad v_y = u\sin 30° = 10\sin 30° = 5\sqrt{3}\ \text{ms}^{-1}$

$v_x = u\cos 30° = 10\cos 30° = 5\ \text{ms}^{-1}$

or $\quad 50 = (5\sqrt{3} - 10t)^2 + (5)^2$

$\therefore \quad (5\sqrt{3} - 10t) = \pm 5$

$$t_1 = \frac{5\sqrt{3} + 15}{10} \quad \text{and} \quad t_2 = \frac{5\sqrt{3} - 5}{10}$$

$\therefore \quad t_1 - t_2 = 2\text{s}$

Check point 4.3

1. (*a*) Velocity of the bomb is same as velocity of the plane. Therefore, the bomb falls exactly below the plane.

2. (*b*) $v_y = gt = 7\ \text{ms}^{-1}, v_x = 4\ \text{ms}^{-1}$

$$\therefore \quad v = \sqrt{v_x^2 + v_y^2} \approx 8\ \text{ms}^{-1}$$

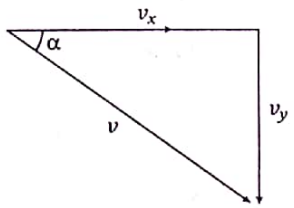
3. (*b*) Let horizontal is x and downwards is y-direction.

After time t, $v_x = u_x = 20\ \text{ms}^{-1}$

Velocity in y-direction

$$v_y = u_y + a_y t = 0 + gt = gt$$

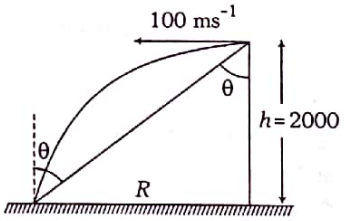
$$\tan\alpha = \frac{v_y}{v_x} = \frac{gt}{20} \quad \text{if } \alpha = 45°$$



Then, $1 = \dfrac{10 \times t}{20} \Rightarrow t = 2\ \text{s}$

4. (*a*) Let t be the time taken by bomb to hit the target.

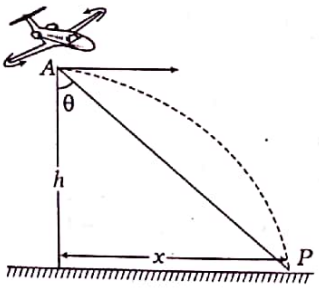
$$h = 2000 = \frac{1}{2}gt^2 \Rightarrow t = 20\ \text{s}$$



$$R = ut = (100)(20) = 2000\ \text{m}$$

$$\because \quad \tan\theta = \frac{R}{h} = \frac{2000}{2000} = 1 \Rightarrow \theta = 45°$$

5. (*c*) Plane is flying at a speed $= 600 \times \dfrac{5}{18} = \dfrac{500}{3}\ \text{ms}^{-1}$ horizontally (at a height 1960 m)



Time taken by the kit to reach the ground

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 1960}{9.8}} = 20\ \text{s}$$

in this time the kit will move horizontally by

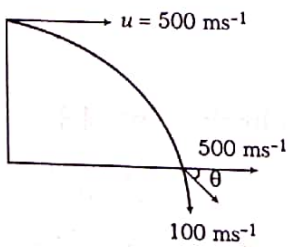
$$x = ut = \frac{500}{3} \times 20 = \frac{10000}{3}\ \text{m}$$

So the angle of sight

$$\tan\theta = \frac{x}{h} = \frac{10000}{3 \times 1960} = \frac{10}{5.88} = 1.7 = \sqrt{3} \quad \text{or} \quad \theta = 60°$$

6. (*a*) Horizontal component of velocity, $v_x = 500\ \text{ms}^{-1}$ and vertical component of velocity while striking the ground can be given as

$$v_y = 0 + 10 \times 10 = 100\ \text{ms}^{-1}$$



∴ Angle with which it strikes the ground

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{100}{500}\right) = \tan^{-1}\left(\frac{1}{5}\right)$$

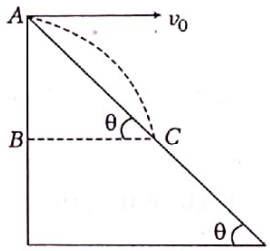
7. (*b*) $\tan 60° = \dfrac{v_V}{v_H} = \dfrac{gt}{v_0}$

$$\therefore \quad t = \frac{\sqrt{3}\,v_0}{g}$$

8. (*a*) $t = \sqrt{\dfrac{2h}{g}} = \sqrt{\dfrac{2 \times 1960}{9.8}} = 20\ \text{s}$

$$\therefore \text{Horizontal distance} = 600 \times \frac{20}{60 \times 60}\ \text{km} = 3.33\ \text{km}$$

9. (*a*) $\dfrac{AB}{BC} = \dfrac{\frac{1}{2}gt^2}{v_0 t} = \tan\theta$

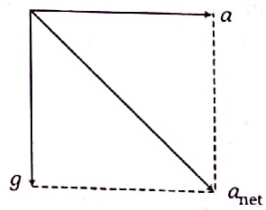


$$\therefore \quad t = \frac{2v_0 \tan\theta}{g}$$

Now, x-coordinate $= v_0 t = \dfrac{2v_0^2 \tan\theta}{g}$

and y-coordinate $= -\dfrac{1}{2}gt^2 = -\dfrac{2v_0^2 \tan^2\theta}{g}$

10. (*c*) As initial velocity is zero. Particle will move in a straight line along a_{net}.



Further, $t = \sqrt{\dfrac{2h}{g}} = \sqrt{\dfrac{2 \times 49}{9.8}} = \sqrt{10} = 3.16\ \text{s}$

Clearly distance travelled by the ball is more than 49m.

A) Taking it together

1. (*d*) We know that

$$\text{speed, } v_0 = \frac{\text{total distance travelled}}{\text{time taken}}$$

Hence, total distance travelled = Path length

= (speed) × time taken

2. (*c*) As given motion is two dimensional motion and given that instantaneous speed v_0 is positive constant. Acceleration is rate of change of velocity (instantaneous speed) hence it will also be in the plane of motion.

3. (*b*) $\mathbf{a} = \dfrac{v_f - v_i}{t} = \dfrac{(3\hat{\mathbf{i}} - 2\hat{\mathbf{i}}) - (+2\hat{\mathbf{j}} - 3\hat{\mathbf{j}})}{2} = \dfrac{\hat{\mathbf{i}} + \hat{\mathbf{j}}}{2}$

4. (*d*) $R = \frac{u^2 \sin 2\theta}{g} = \frac{2u_x u_y}{g}$

$\therefore$ Range $\propto$ horizontal initial velocity component (u_x)

In path 4 range is maximum as football has maximum horizontal velocity component in this path.

7. (*d*) Given, $R_A = R_B$

$$\Rightarrow \quad \frac{v^2 \sin 2\theta}{g} = \frac{(v/2)^2 \sin 90°}{g}$$

$$\Rightarrow \quad \sin 2\theta = \frac{1}{4} \text{ or } \theta = \frac{1}{2}\sin^{-1}\left(\frac{1}{8}\right)$$

$$\therefore \quad \theta = \frac{1}{2}\sin^{-1}\left(\frac{1}{8}\right)$$

8. (*d*) Maximum height, $H = \frac{u^2 \sin^2 \alpha}{2g} \Rightarrow H \propto \sin^2 \alpha$

$$\therefore \quad \frac{H_1}{H_2} = \frac{\sin^2 \alpha}{\sin^2(90° - \alpha)} = \tan^2 \alpha$$

9. (*c*) We know that, $R_\theta = R_{90° - \theta}$

10. (*b*) $R_{15°} = \frac{u^2 \sin(2 \times 15°)}{g} = \frac{u^2}{2g} = 1.5$ km

$$\Rightarrow R_{45°} = \frac{u^2 \sin(2 \times 45°)}{g} = \frac{u^2}{g} = 1.5 \times 2 = 3 \text{ km}$$

11. (*c*) $S = u \times \sqrt{\frac{2h}{g}} \Rightarrow 10 = u\sqrt{2 \times \frac{5}{10}} \Rightarrow u = 10 \text{ ms}^{-1}$

12. (*b*) $u_x = 3 \text{ ms}^{-1}$, $a_x = -6 \text{ ms}^{-1}$

$$\therefore \quad X_{max} = \frac{u_x^2}{2|a_x|} = \frac{9}{12} = 0.75 \text{ m}$$

13. (*b*) $x = kt, \quad t = \frac{x}{k}$

Now, $y = k\left(\frac{x}{k}\right)\left(1 - \alpha \cdot \frac{x}{k}\right)$ or $y = x - \frac{\alpha x^2}{k}$

14. (*b*) $\Delta v = a\,\Delta t$ (as a = constant)

$$= (-g\hat{j})\left(\frac{2v_0 \sin\alpha}{g}\right) = (-2v_0 \sin\alpha)\hat{j}$$

i.e., change in velocity is $2v_0 \sin\alpha$, vertically downwards.

15. (*c*) Compare the given equation with

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta} \Rightarrow \tan\theta = \sqrt{3} \Rightarrow \theta = 60°$$

16. (*c*) $R_{max} = \frac{u^2}{g}$ at $\theta = 45°$

$$\therefore \quad u = \sqrt{gR_{max}} = 100 \text{ ms}^{-1}$$

17. (*d*) $h = \frac{1}{2}gt^2$ (in vertical direction)

$$\therefore \quad t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 0.1}{10}} = 0.141 \text{ s}$$

Now in horizontal direction, $v_x = \frac{S_x}{t} = \frac{100}{0.141} \approx 700 \text{ ms}^{-1}$

18. (*a*) At highest point velocity will remain $v\cos 30°$ or $\frac{\sqrt{3}\,v}{2}$.

Therefore, momentum will also remain $\frac{\sqrt{3}\,p}{2}$.

19. (*c*) H and R both are proportional to u^2. Hence, percentage increases in horizontal range will also be 10%.

20. (*c*) At highest point, kinetic energy will be minimum but not zero.

21. (*c*) $y_1 + y_2 = \frac{u^2 \sin^2\theta}{2g} + \frac{u^2 \sin^2(90° - \theta)}{2g} = \frac{u^2}{2g}$

22. (*c*) $\tan\phi = \frac{H}{R/2} = \frac{2H}{R} = \frac{(u^2 \sin^2\theta/g)}{u^2 \sin 2\theta/g}$

$$\tan\phi = \frac{\sin^2\theta}{2\sin\theta\cos\theta} = \frac{\tan\theta}{2}$$

23. (*a*) $K = K_0 - mgh$, Here K = kinetic energy at height h, K_0 = initial kinetic energy. Variation of K with h is linear. At highest point kinetic energy is not zero.

24. (*c*) Instantaneous velocity of rising mass after t s will be

$$v_t = \sqrt{v_x^2 + v_y^2}$$

where, $v_x = v\cos\theta$ = Horizontal component of velocity

$v_y = v\sin\theta - gt$ = Vertical component of velocity

$$\Rightarrow \quad v_t = \sqrt{(v\cos\theta)^2 + (v\sin\theta - gt)^2}$$

$$\Rightarrow \quad v_t = \sqrt{v^2 + g^2t^2 - (2v\sin\theta)\,gt}$$

25. (*c*) $R = 2H$ or $\frac{2u_x u_y}{g} = \frac{2u_y^2}{2g}$

or $2u_x = u_y$ or $2a = b$

26. (*d*) $R \propto \frac{1}{g} \quad \therefore R_{moon} = 6R_{earth}$

27. (*d*) At 45° range is maximum. At 15° and 75° ranges are equal as $R_\theta = R_{90° - \theta}$

28. (*b*) $\theta = 45°$, $R_{max} = \frac{u^2}{g} = 80$

$$\therefore \quad u^2 = 800 \text{ ms}^{-2}$$

Now, $H = \frac{u^2 \sin^2 45°}{2g} = \frac{(800)(1/2)}{20} = 20$ m

29. (*d*) Speed of projectile at the point of projection = u

Speed of projectile at the top of its trajectory = $u\cos\theta$.

$$\frac{u}{u\cos\theta} = x \text{ or } \cos\theta = \frac{1}{x} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{x}\right)$$

30. (*d*) Given, $u\cos\theta = \frac{u}{2} \Rightarrow \theta = 60°$

Now, $R = \frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin 120°}{g} = \frac{\sqrt{3}\,u^2}{2g}$

31. (*a*) $R = \frac{2u_x u_y}{g} = \frac{2 \times 9.8 \times 19.6}{9.8} = 39.2$ m

32. (*d*) Time of flight, $T = \frac{2u_y}{g} = 3$

$\therefore \quad u_y = 15 \text{ ms}^{-1}$

Now, $H = \frac{u_y^2}{2g} = \frac{(15)^2}{20} = 11.25 \text{ m}$

33. (*c*) Maximum height $\frac{u^2}{2g} = h$ (given)

Now, $R_{max} = \frac{u^2 \sin 90°}{g} = \frac{u^2}{g} = 2h$

34. (*b*) Range of projectile launched at an angle θ is same as the range of projectile launched at angle 2θ.

$\Rightarrow \quad \frac{u^2 \sin 2(2\theta)}{g} = \frac{u^2 \sin 2\theta}{g}$

$\Rightarrow \quad \sin 2(2\theta) = \sin 2\theta$

$\Rightarrow \quad 2 \sin 2\theta \cos 2\theta = 2 \sin\theta \cos\theta$

$4 \sin\theta \cos\theta \cos 2\theta = 2 \sin\theta \cos\theta$

$\Rightarrow \quad 4 \cos 2\theta = 2 \Rightarrow \cos 2\theta = \frac{1}{2}$

$\Rightarrow \quad \cos 2\theta = \cos 60°$

$\Rightarrow \quad 2\theta = 60°$

$\Rightarrow \quad \theta = 30°$

35. (*a*) Velocity of boy should be equal to the horizontal component of velocity of ball *i.e.*, $u \cos\theta$.

36. (*c*) For angle $(45° - \theta)$, $R = \frac{u^2 \sin(90° - 2\theta)}{g} = \frac{u^2 \cos 2\theta}{g}$

For angle $(45° + \theta)$, $R = \frac{u^2 \sin(90° + 2\theta)}{g} = \frac{u^2 \cos 2\theta}{g}$

37. (*a*) Time of flight, $T = \frac{2u \sin\theta}{g} = 10 \text{ s}$

$\Rightarrow \quad u \sin\theta = 50 \text{ ms}^{-1}$

$\therefore \quad H = \frac{u^2 \sin^2\theta}{2g} = \frac{(u \sin\theta)^2}{2g} = \frac{50 \times 50}{2 \times 10} = 125 \text{ m}$

38. (*a*) When the angle of projection is very far from 45° then range will be minimum. Therefore, the body *P* with angle of projection 15° will have a shortest range.

39. (*b*) $H = \frac{u^2 \sin^2\theta}{2g}$ and $T = \frac{2u \sin\theta}{g} \Rightarrow T^2 = \frac{4u^2 \sin^2\theta}{g^2}$

$\therefore \quad \frac{T^2}{H} = \frac{8}{g} \Rightarrow T = \sqrt{\frac{8H}{g}} = 2\sqrt{\frac{2H}{g}}$

40. (*a*) $R = \frac{u^2 \sin 2\theta}{g}$ at angles θ and 90° − θ

Now, $t_1 = \frac{2u \sin\theta}{g}$

and $t_2 = \frac{2u \sin(90° - \theta)}{g} = \frac{2u \cos\theta}{g}$

$\therefore \quad t_1 t_2 = \frac{2}{g}\left[\frac{u^2 \sin 2\theta}{g}\right] = \frac{2R}{g}$

41. (*d*) $R = \frac{u^2 \sin\theta}{g}$ at angles θ and 90° − θ

Now, $h_1 = \frac{u^2 \sin^2\theta}{2g}$ and $h_2 = \frac{u^2 \sin^2(90° - \theta)}{2g} = \frac{u^2 \cos^2\theta}{2g}$

$h_1 h_2 = \left(\frac{u^2 \sin 2\theta}{g}\right)^2 \cdot \frac{1}{16} = \frac{R^2}{16}$

$\therefore \quad R = 4\sqrt{h_1 h_2}$

42. (*c*) At highest point speed will remain $\frac{1}{\sqrt{2}}$ times $(= u \cos 45°)$.

Therefore kinetic energy will become 1/2 times.

43. (*b*) Put $y = 0$, $0 = 12x - \frac{3}{4}x^2 \Rightarrow x = 16 \text{ m}$

44. (*c*) Horizontal component of velocity remains unchanged.

Hence, $v \cos\phi = u \cos\theta$

$\therefore \quad v = u \cos\theta \sec\phi$

45. (*b*) Comparing with $y = x \tan\theta - \frac{gx^2}{2u^2 \cos^2\theta}$ we have,

$\tan\theta = \sqrt{3}$ or $\theta = 60°$ and $u^2 \cos^2\theta = 1$

or $u = \sec\theta = \sec 60° = 2 \text{ ms}^{-1}$

46. (*a*) $\Delta v = a\Delta t = a \cdot \left(\frac{T}{2}\right) = (-g\hat{j})\left(\frac{u \cos\theta}{g}\right) = (-u \cos\theta)\hat{j}$

Therefore, change in velocity is $u \cos\theta$ in downward direction.

47. (*b*) Since, $u \sin\theta \times t = 10 \text{ m}$

$\Rightarrow \quad 20 \sin 45° t = 10$

$\Rightarrow \quad t = \frac{10}{20 \sin 45°} = \frac{1}{\sqrt{2}} \text{ s}$

Now, $y = (20 \sin 45°)t - \frac{1}{2}gt^2$

$= 20 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times 10 \times \frac{1}{2} = 7.5 \text{ m}$

48. (*a*) For both cases $t = \sqrt{\frac{2h}{g}}$ = constant.

Because vertical downward component of velocity will be zero for both the particles.

49. (*c*) Let *t* be time taken by the bullet to hit the target.

$\therefore \quad 200 \text{ m} = 2000 \text{ ms}^{-1} t \Rightarrow t = \frac{200 \text{ m}}{2000 \text{ ms}^{-1}} = \frac{1}{10} \text{ s}$

For vertical motion, here, $u = 0$

$\therefore \quad h = \frac{1}{2}gt^2$

$h = \frac{1}{2} \times 10 \times \left(\frac{1}{10}\right)^2 = \frac{1}{20} \text{ m} = 5 \text{ cm}$

∴ Gun should be aimed 5 cm above the target.

50. (*d*) $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 396.9}{9.8}} \approx 9 \text{ s}$

and $u = 720 \text{ km/h} = 200 \text{ m/s}$

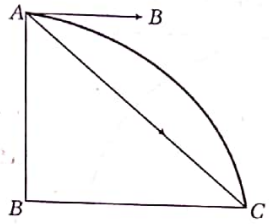
$\therefore \quad R = u \times t = 200 \times 9 = 1800 \text{ m}$

51. (*d*) $h_{max} = \frac{u^2}{2g} = 10$ $(\because \theta = 90°)$

$\Rightarrow \quad u^2 = 200 \Rightarrow R_{max} = \frac{u^2}{g} = 20$ m

52. (*a*) The horizontal distance covered by bomb.

$BC = v_H \times \sqrt{\frac{2h}{g}} = 150\sqrt{\frac{2 \times 80}{10}} = 600$ m



$\therefore$ The distance of target from dropping point of bomb,

$AC = \sqrt{(AB)^2 + (BC)^2} = \sqrt{(80)^2 + (600)^2} = 605.3$ m

53. (*d*) Comparing with the trajectory of projectile in which particle is projected from certain height horizontally ($\theta = 0°$).

$$y = x \tan\theta - \frac{gx^2}{2u^2 \cos^2\theta}$$

Putting $\theta = 0°$ and $g = a = 18u^2 = 2$ ms^{-2}

$$\left(\because y = -9x^2, u = \frac{1}{3} \text{ ms}^{-1}\right)$$

54. (*b*) $\frac{u^2}{g} = R_{max} = 500$

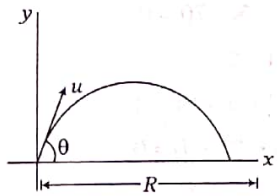
Now, $s = \frac{u^2}{2a} = \frac{u^2}{2g \sin 30°} = \frac{u^2}{g} = 500$ m

55. (*a*) In moving horizontal distance 10 m, the ball will fall by distance $\frac{1}{2}gt^2$.

$\Rightarrow \quad 10 = 20 \times \cos 30° t = 20 \times \frac{\sqrt{3}}{2} t \Rightarrow t = \frac{1}{\sqrt{3}}$ s

$AB = h = \frac{1}{2}gt^2 = \frac{1}{2} \times g \times \left(\frac{1}{\sqrt{3}}\right)^2 = \frac{g}{6}$ m

56. (*c*) We know that, where θ is angle of projection



Given, $\theta = 15°$ and $R = 50$ m

Range, $R = \frac{u^2 \sin 2\theta}{g}$

Putting all the given values in the formula, we get

$\Rightarrow \quad R = 50\text{m} = \frac{u^2 \sin(2 \times 15°)}{g}$

$\Rightarrow \quad 50 \times g = u^2 \sin 30° = u^2 \times \frac{1}{2} \Rightarrow 50 \times g \times 2 = u^2$

$\Rightarrow \quad u^2 = 50 \times 9.8 \times 2 = 100 \times 9.8 = 980$

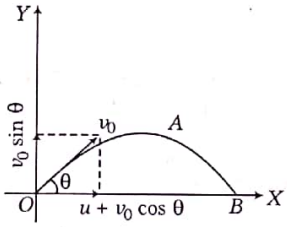
$\Rightarrow \quad u = \sqrt{980} = \sqrt{49 \times 20} = 7 \times 2 \times \sqrt{5}$ms^{-1}

$= 14 \times 2.23$ ms^{-1} $= 31.304$ ms^{-1}

For $\theta = 45°$; $R = \frac{u^2 \sin 2 \times 45°}{g} = \frac{u^2}{g}$ $(\because \sin 90° = 1)$

$\Rightarrow \quad R = \frac{(14\sqrt{5})^2}{g} = \frac{14 \times 14 \times 5}{9.8} = 100$ m

57. (*b*) Consider the adjacent diagram.



Initial velocity in, x-direction, $u_x = u + v_0 \cos\theta$

u_y = Initial velocity in y-direction $= v_0 \sin\theta$

where angle of projection is θ.

Now, we can write

$$\tan\theta = \frac{u_y}{u_x} = \frac{u_0 \sin\theta}{u + u_0 \cos\theta} \Rightarrow \theta = \tan^{-1}\left(\frac{v_0 \sin\theta}{u + v_0 \cos\theta}\right)$$

58. (*c*) It in given that, $H_1 - H_2 = \frac{H_1 + H_2}{2}$ or $H_1 = 3H_2$

$\therefore \frac{u^2 \sin^2\theta}{2g} = 3\left\{\frac{u^2 \sin^2(90° - \theta)}{2g}\right\}$

$\tan^2\theta = 3$

$\therefore \quad \tan\theta = \sqrt{3}$ or $\theta = 60°$

Therefore, the other angle is $90° - \theta$ or $30°$.

59. (*c*) Vertical component of velocity of A should be equal to vertical velocity of B.

or $\quad v_1 \sin 30° = v_2$ or $\frac{v_1}{2} = v_2$

$\therefore \quad \frac{v_2}{v_1} = \frac{1}{2}$

60. (*a*) It in given that, $x = 36t$

$\therefore \quad v_x = \frac{dx}{dt} = 36$ ms^{-1} $\Rightarrow y = 48t - 4.9t^2$

$\therefore \quad v_y = 48 - 9.8t$

at $\quad t = 0 \; v_x = 36$ ms^{-1}

and $\quad v_y = 48$ ms^{-1}

So, angle of projection, $\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{4}{3}\right)$

or $\quad \theta = \sin^{-1}\left(\frac{4}{5}\right)$

61. (*a*) Person will catch the ball if his velocity will be equal to horizontal component of velocity of the ball.

$$\frac{v_0}{2} = v_0 \cos\theta \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = 60°$$

62. (*b*) Time of flight, $T = \frac{2u_y}{g}$

$$\therefore \quad u_y = \frac{gT}{2} = 25 \text{ ms}^{-1}$$

Now, $H = \frac{u_y^2}{2g} = \frac{(25)^2}{20} = 31.25$ m

Further, $R = u_x T$

$$\therefore \quad u_x = \frac{R}{T} = 40 \text{ ms}^{-1}$$

63. (*a*) Comparing with projectile motion we can see that it is like a projectile motion, with $u_x = 4$ ms^{-1}, $u_y = 4$ ms^{-1} and $a_y = 10$ ms^{-2}.

x-coordinate = Range $= \frac{2u_x u_y}{g} = \frac{2 \times 4 \times 4}{10} = 3.2$ m

64. (*a*) Given, $5\sqrt{3} = \frac{(10)^2 \sin 2\theta}{g}$ or $\sin 2\theta = \frac{\sqrt{3}}{2}$

$$\therefore \quad 2\theta = 60° \text{ or } \theta = 30°$$

Two different angles of projection are therefore θ and $90° - \theta$ or 30° and 60°.

$$\therefore \quad T_1 = \frac{2u \sin 30°}{g} = 1 \text{ s}$$

$$\therefore \quad T_2 = \frac{2u \sin 60°}{g} = \sqrt{3} \text{ s}$$

$$\therefore \quad \Delta t = T_2 - T_1 = (\sqrt{3} - 1) \text{ s}$$

65. (*d*) $u_x = 19.6$ ms^{-1} and $u_y = 19.6$ ms^{-1}. After 2 s, vertical component of velocity v_y will become zero. So particle is at its highest point.

66. (*b*) It is given that

$$H = \frac{R}{4}$$

$$\therefore \quad \frac{u^2 \sin^2\theta}{2g} = \frac{2u^2 \sin\theta \cos\theta}{4g} \text{ or } \tan\theta = 1$$

$$\therefore \quad \theta = 45°$$

At highest point, momentum will remain $\frac{p}{\sqrt{2}}$.

$$\therefore \quad K = \frac{(p/\sqrt{2})^2}{2m} = \frac{p^2}{4m}$$

67. (*d*) $u_x = u \cos\theta_0 = 30 \times \frac{4}{5} = 24$ ms^{-1}

and $u_y = u \sin\theta_0 = 30 \times \frac{3}{5} = 18$ ms^{-1}

After 1 s, u_x will remain as it is and u_y will decrease by 10 ms^{-1} or it will remain 8 ms^{-1}

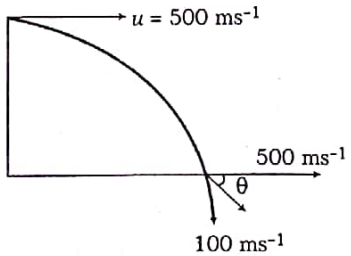
$$\therefore \quad \tan\theta = \frac{v_y}{v_x} = \frac{8}{24} = \frac{1}{3}$$

68. (*a*) $v = 3\hat{i} + 10\hat{j}$, $v_x = 3$, $v_y = 10$

$$H = \frac{v_y^2}{2g} = \frac{100}{2 \times 10} = 5 \text{ m}$$

$$R = v_x \times T = v_x \times \frac{2v_y}{g} = 6 \text{ m}$$

69. (*a*) Horizontal component of velocity, $v_x = 500$ ms^{-1} and vertical component of velocity while striking the ground.

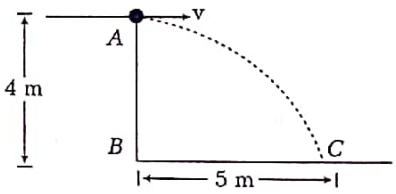


$$v_y = 0 + 10 \times 10 = 100 \text{ ms}^{-1}$$

$\therefore$ Angle with which it strikes the ground

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{100}{500}\right) = \tan^{-1}\left(\frac{1}{5}\right)$$

70. (*c*) Using, $h = \frac{1}{2}gt^2$, we have



$$h_{AB} = \frac{1}{2} g t_{AC}^2$$

or $t_{AC} = \sqrt{\frac{2h_{AB}}{g}} = \sqrt{\frac{2 \times 4}{9.8}} = 0.9$ s

Further, $BC = vt_{AC}$

or $v = \frac{BC}{t_{AC}} = \frac{5}{0.9} = 5.55$ ms^{-1}

71. (*c*) $s_y = u_y t + \frac{1}{2}a_y t^2$ or $-70 = 25t - 5t^2$

$$\Rightarrow \quad 5t^2 - 25t - 70 = 0$$

$$\Rightarrow \quad t^2 - 5t - 14 = 0$$

$$\Rightarrow \quad t^2 - 7t + 2t - 14 = 0$$

$$\Rightarrow \quad t(t - 7) + 2(t - 7) = 0$$

$$\Rightarrow \quad t = 7, t = -2$$

$$\therefore \quad t = 7 \text{ s}$$

72. (*a*) From $s_y = u_y t + \frac{1}{2}a_y t^2$ we have,

$$-40 = (20 \sin 30°)t - 5t^2 \text{ or } t^2 - 2t - 8 = 0$$

Solving this, we get $t = 4$ s

$$T = \frac{2u \sin\theta}{g} = \frac{2 \times 20 \times \sin 30°}{10} = 2 \text{ s} \Rightarrow \frac{t}{T} = 2$$

73. (*b*) Given, $x = ct \Rightarrow v_x = \frac{dx}{dt} = c$

Also, $y = bt^2 \Rightarrow v_y = \frac{dy}{dt} = 2bt$

$\therefore$ Speed $= |\mathbf{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{(c)^2 + (2bt)^2} = \sqrt{c^2 + 4b^2t^2}$

74. (*a*) Their horizontal components should be same.

$\therefore \quad \frac{2v}{\sqrt{3}} \cdot \cos\theta = v$ or $\theta = 30°$

75. (*d*) Standard equation of projectile motion

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

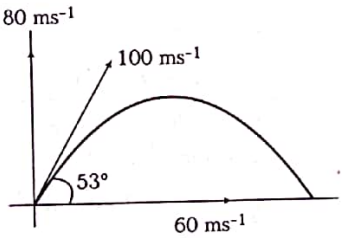
Comparing with given equation

$$A = \tan\theta \text{ and } B = \frac{g}{2u^2\cos^2\theta}$$

So, $\frac{A}{B} = \frac{\tan\theta \times 2u^2\cos^2\theta}{g} = 40$

(As, $\theta = 45°$, $u = 20\ \text{ms}^{-1}$, $g = 10\ \text{ms}^{-2}$)

76. (*c*) Component 60 ms^{-1} will remain unchanged. Velocity will make 45° with horizontal when vertical component also becomes $\pm$ 60 ms^{-1}.



Using, $v = u + at$ (in vertical direction)

$+60 = 80 + (-10)\,t_1$

$\therefore \quad t_1 = 2\text{ s} \Rightarrow -60 = 80 + (-10)\,t_2$

$\therefore \quad t_2 = 14\text{ s}$

77. (*d*) As, $T = \frac{2u_y}{g} \quad \therefore u_y = \frac{gT}{2}$

$$\Rightarrow \quad h_A = u_y t_A - \frac{1}{2}gt_A^2 = u_y\left(\frac{2u_y}{3g}\right) - \frac{1}{2}g\left(\frac{2u_y}{3g}\right)^2$$

$$= \frac{4}{9}\frac{u_y^2}{g} = \left(\frac{4}{9g}\right)\left(\frac{gT}{2}\right)^2 = \frac{gT^2}{9} \qquad \left[\because u_y = \frac{gT}{2}\right]$$

$$\Rightarrow \quad h_B = u_y\left(\frac{5}{6} \times \frac{2u_y}{g}\right) - \frac{1}{2} \times g \times \left(\frac{5}{6} \times \frac{2u_y}{g}\right)^2$$

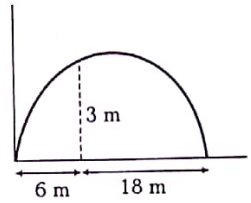
$$= \frac{5}{18}\frac{u_y^2}{g} = \frac{5}{18g}\left(\frac{gT}{2}\right)^2 = \frac{5}{72}gT^2$$

$$\therefore \quad h_A - h_B = \frac{gT^2}{24}$$

78. (*a*) $u_x = a, u_y = b, g = c$

$\because$ Horizontal range, $R = \frac{2u_x u_y}{g} = \frac{2ab}{c}$

79. (*b*) Given, $R = \frac{u^2\sin 2\theta}{g} = 24$ ...(i)



$\because \quad y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$

$3 = 6\tan\theta - \frac{36g}{2u^2\cos^2\theta}$ ...(ii)

From Eq. (i), $\frac{g}{u^2} = \frac{\sin 2\theta}{24} = \frac{\sin\theta\cos\theta}{12}$

Substituting in Eq. (ii) we have

$$3 = 6\tan\theta - \frac{3}{2}\tan\theta = \frac{9}{2}\tan\theta \Rightarrow \theta = \tan^{-1}\left(\frac{2}{3}\right)$$

80. (*d*) $H' = H$ (as vertical component of acceleration has not changed)

$$R' = u_x T + \frac{1}{2}a_x T^2 = R + \frac{1}{2} \times \frac{g}{4} \times \frac{4u^2\sin^2\theta}{g^2}$$

$$= R + \frac{u^2\sin^2\theta}{2g} = (R + H)$$

81. (*b*) Area in which bullet will spread= πr^2

For maximum area, $r = R_{max} = \frac{v^2}{g}$ (when $\theta = 45°$)

Maximum area $= \pi R_{max}^2 = \pi\left(\frac{v^2}{g}\right)^2 = \frac{\pi v^4}{g^2}$

82. (*c*) The time taken by cart to cover 80 m

$$\frac{s}{v} = \frac{80}{30} = \frac{8}{3}\text{ s}$$

The projectile must be fired (relative to cart) vertically upwards.

$a = -g = -10\ \text{ms}^{-2}, v' = 0$ and $t = \frac{8/3}{2} = \frac{4}{3}$ s

$\therefore \quad v' = u + at$ or $0 = u - 10 \times \frac{4}{3}$ or $u = \frac{40}{3}\ \text{ms}^{-1}$

83. (*a*) Since, $\mathbf{v} \perp \mathbf{u}$ or $\mathbf{v} \cdot \mathbf{u} = 0$

or $(\mathbf{u} + a\,t) \cdot \mathbf{u} = 0$

or $\mathbf{u} \cdot \mathbf{u} + (a \cdot \mathbf{u})\,t = 0$

or $u^2 + gut\cos(90° + \theta) = 0$

(angle between **u** and **u** is $90° + \theta$)

or $u - gt\sin\theta = 0$ or $t = \frac{u}{g\sin\theta}$

84. (*a*) Given, $\mathbf{v}_1 \perp \mathbf{v}_2$

$\therefore \quad \mathbf{v}_1 \cdot \mathbf{v}_2 = 0$

or $(u_1\hat{\mathbf{i}} - gt\,\hat{\mathbf{j}}) \cdot (-u_2\hat{\mathbf{i}} - gt\,\hat{\mathbf{j}}) = 0$

$\therefore \quad g^2t^2 = u_1u_2$ or $t = \frac{\sqrt{u_1u_2}}{g}$

85. (*d*) Given, speed of packets = 125 ms^{-1}

Height of the hill = 500 m.

To cross the hill, the vertical component of the velocity should be sufficient to cross such height.

$$u_y \geq \sqrt{2gh}$$

$$\geq \sqrt{2 \times 10 \times 500} \geq 100 \text{ ms}^{-1}$$

But $\quad u^2 = u_x^2 + u_y^2$

∴ Horizontal component of initial velocity,

$$u_x = \sqrt{u^2 - u_y^2} = \sqrt{(125)^2 - (100)^2}$$

$$= 75 \text{ ms}^{-1}$$

Time taken to reach the top of the hill,

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 500}{10}} = 10 \text{ s}$$

Time taken to reach the ground from the top of the hill

$$t' = t = 10 \text{ s}$$

Horizontal distance travelled in 10 s

$$x = u_x \times t = 75 \times 10$$

$$= 750 \text{ m}$$

∴ Distance through which canon has to be moved

$$= 800 - 750 = 50 \text{ m}$$

Speed with which canon can move = 2 ms^{-1}

∴ Time taken by canon $= \frac{50}{2}$

$\Rightarrow \quad t'' = 25$ s

∴ Total time taken by a packet to reach on the ground

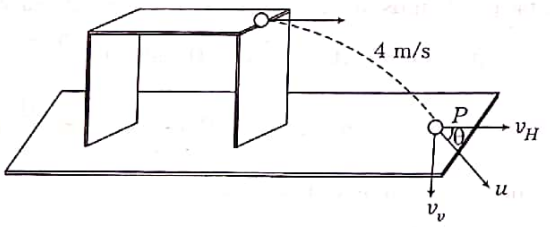
$= t'' + t + t' = 25 + 10 + 10 = 45$ s

86. (*a*) Vertical component of velocity of ball at point P

$$u_V = 0 + gt = 10 \times 0.4 = 4 \text{ ms}^{-1}$$

Horizontal component of velocity = initial velocity

$\Rightarrow \quad v_H = 4$ m/s



So, the speed with which it hits the ground

$$v = \sqrt{v_H^2 + v_V^2} = 4\sqrt{2} \text{ m/s}$$

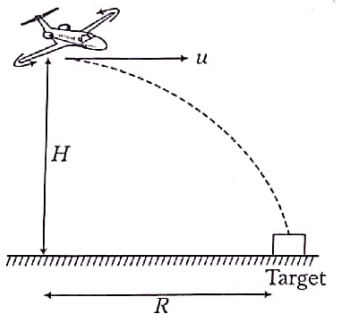
and $\quad \tan\theta = \frac{v_V}{v_H} = \frac{4}{4} = 1 \Rightarrow \theta = 45°$

It means the ball hits the ground at an angle of 45° to the horizontal.

Height of the table $h = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times (0.4)^2 = 0.8$ m

Horizontal distance travelled by the ball from the edge of table $h = ut = 4 \times 0.4 = 1.6$ m

87. (*b*) Given, $H = 2000$ m, $u = 360$ kmh^{-1}



$$u = 360 \times \frac{5}{18} = 100 \text{ ms}^{-1}$$

The time taken by the bomb to hit the target is

$$T = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2 \times 2000}{10}} = 20 \text{ s}$$

The horizontal distance of the aeroplane from the target

$$R = u\sqrt{\frac{2H}{g}} = 100 \times 20 = 2000 \text{ m} = 2 \text{ km}$$

88. (*c*) $T/2 = 2 + 1 = 3$ s or $T = 6$ s

$$\therefore \quad \frac{2u_y}{g} = 6$$

$$\therefore \quad u_y = 30 \text{ ms}^{-1}$$

Further, $\tan 30° = \frac{v_y}{v_x} = \frac{u_y - gt}{u_x} = \frac{30 - 20}{u_x}$

or $\quad u_x = 10\sqrt{3} \text{ ms}^{-1}$

or $\quad u = \sqrt{u_x^2 + u_y^2} = 20\sqrt{3} \text{ ms}^{-1}$

$$\tan\theta = \frac{u_y}{u_x} = \frac{30}{10\sqrt{3}} = 1/\sqrt{3} \text{ or } \theta = 30°$$

89. (*d*) $\tan 30° = \frac{u_y}{u_x} \quad \therefore \quad \frac{1}{\sqrt{3}} = \frac{80}{u_x}$

or $\quad u_x = 80\sqrt{3} \text{ ms}^{-1}$

$$T = \frac{2u_y}{g} = \frac{2 \times 80}{10} = 16 \text{ s}$$

At $t = \frac{T}{4} = 4$ s $\Rightarrow v_x = u_x = 80\sqrt{3}$ ms^{-1} and $v_y = u_y + a_y t$

or $\quad v_y = 80 + (-10)(4) = 40 \text{ ms}^{-1}$

$\therefore \quad$ Speed $= \sqrt{(80\sqrt{3})^2 + (40)^2} = 144.2 \text{ ms}^{-1}$

90. (*b*) Let us see the motion relative to elevator,

$$a_r = a_b - a_e = (-10) - (+2) = -12 \text{ ms}^{-2}$$

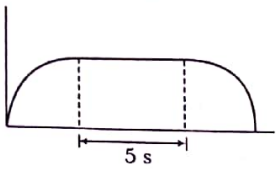
Now, $\quad T = \frac{2u_y}{a_r} = \frac{2 \times u\sin\theta}{a_r} = \frac{2 \times 4 \times \sin 30°}{12} = \frac{1}{3}$ s

91. (*c*) $(u\cos\alpha) = \sqrt{\frac{2}{5}}\sqrt{(u\cos\alpha^2) + \{(u\sin\alpha)^2 - 2gh\}}$

Here, $\quad h = \frac{H}{2} = \frac{u^2\sin^2\alpha}{4g}$

Solving this equation we get, $\alpha = 60°$

92. (*d*) For 5 s, weight of the body is balanced by the given force. Hence it will move in a straight line as shown.



$$R = \frac{u^2 \sin 2\theta}{g} + (u\cos\theta)(5)$$

$$= \frac{(50)^2 \cdot \sin 60°}{10} + (50 \times \cos 30°)(5) = 250\sqrt{3}\text{ m}$$

93. (*c*) $R_{max} = \frac{u^2}{g} = 1.6$ m (at $\theta = 45°$)

or $u = \sqrt{16} = 4\text{ ms}^{-1}$

Now, $T = \frac{2u_y}{g} = \frac{2(4/\sqrt{2})}{10} = \frac{4}{5\sqrt{2}}$ s

$\therefore$ Total distance travelled in 10 s $= 1.6 \times \frac{10}{4/5\sqrt{2}} = 20\sqrt{2}$ m

94. (*c*) If the ball hits the *n*th step, the horizontal and vertical distances traversed are *nb* and *nh* respectively. Let *t* be the time taken by the ball for these horizontal and vertical displacement. Then velocity along horizontal direction remains constant = *u*, initial vertical velocity is zero

$\therefore$ $nb = ut$ ...(i)

$nh = 0 + (1/2)\,gt^2$ ...(ii)

From Eqs. (i) and (ii), we get by eliminating *t*,

$$nh = (1/2)\,g\,(nb/u)^2 \Rightarrow n = \frac{2hu^2}{gb^2}$$

95. (*c*) From the given diagram,

$$\frac{R/2}{H} = \frac{\sqrt{3}H}{H} = \sqrt{3}$$

or $\frac{(v_0^2 \sin\theta\cos\theta)/g}{(v_0^2 \sin^2\theta)/2g} = \sqrt{3} \Rightarrow 2\cot\theta = \sqrt{3}$

or $\tan\theta = \frac{2}{\sqrt{3}}$ or $\theta = \tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$

96. (*d*) (a) Range becomes equal at complimentary angle. Hence

$\beta = 90° - \alpha$

(b) $h_1 = \frac{u^2 \sin^2\alpha}{2g}$

$\Rightarrow h_2 = \frac{u^2 \cos^2\alpha}{2g}$ (as $\beta = 90 - \alpha$)

$\therefore 4\sqrt{h_1 h_2} = \frac{2u^2 \sin\alpha\cos\alpha}{g} = \frac{u^2 \sin 2\alpha}{g} = R$

(c) $\frac{t_1}{t_2} = \frac{(2u\sin\alpha/g)}{(2u\cos\alpha/g)} = \tan\alpha$

(d) $\sqrt{\frac{h_1}{h_2}} = \tan\alpha$

97. (*c*) Two balls will meet if,

$(50\cos 37°)\,t_A = 120$ or $t_A = 3$ s

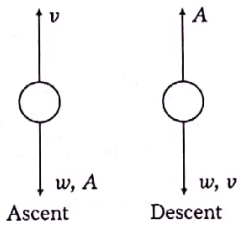
Vertical component of *A* is also 50 sin 37° or 30 ms^{-1}, so they will meet if thrown simultaneously.

$$h_A = h_B = 30 \times 3 - \frac{1}{2} \times 10 \times (3)^2 = 45\text{ m}$$

B) Medical entrance special format questions

Assertion and reason

2. (*a*) v = velocity, w = weight and A = air resistance



3. (*d*) $\left|\frac{dv}{dt}\right| = |a| = 9.8\text{ ms}^{-2}$ = constant and it is true that in case of projectile motion, the magnitude of velocity first decreases and then increases during the motion.

4. (*a*) Velocity is horizontal and acceleration is vertical. *i.e.*, both are perpendicular to each other and hence, their dot product is zero.

5. (*b*) **Assertion** $v \sin\alpha$ = horizontal component of velocity = constant

Reason $a_r = \sqrt{g^2 - a_t^2}$

At highest point $a_t = 0$. Therefore, a_r is maximum.

6. (*c*) $H = \frac{u^2 \sin^2\theta}{2g}$ and $T = \frac{2u\sin\theta}{g}$

$\therefore$ $u\sin\theta = \frac{gT}{2}$

$\therefore$ Maximum height, $H = \frac{(g^2T^2/4)}{2g} = \frac{gT^2}{8} = \frac{g(4)^2}{8} = 20$ m

7. (*b*) $H = R$ or $\frac{u^2 \sin^2\theta}{2g} = \frac{2u^2 \sin\theta\cos\theta}{g}$ or $\tan\theta = 4$

9. (*b*) $h_1 = \frac{u^2}{2g}$, $h_2 = \frac{u^2 \sin^2 30°}{2g} = \frac{u^2}{8g}$

12. (*b*) $t_1 = t_2 = \sqrt{\frac{2h}{g}}$ (h = height of tower)

$v_1 = \sqrt{2gh}$

While $v_2 = \sqrt{v_1^2 + v_0^2}$ (v_0 = initial horizontal velocity)

Therefore, both assertion and reason are correct but reason is not the correct explanation of assertion.

13. (*b*) $t = \frac{2u_y}{g} = \frac{2 \times 20}{10} = 4$ s

$$H = 20 + \frac{u_y^2}{2g} = 20 + \frac{(20)^2}{2 \times 10} = 40\text{ m}$$

Match the columns

2. $R = \frac{2u_x u_y}{g}, H = \frac{u_y^2}{2g}$

3. In horizontal projectile motion,

Horizontal component of velocity $u_x = u = 10\ \text{ms}^{-1}$

Vertical component of velocity $u_y = gt = 10 \times 1 = 10\ \text{ms}^{-1}$

Horizontal displacement $= u \times t = 10 \times (1) = 10$ m

Vertical displacement $= \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times (1)^2 = 5$ m

4. Comparing with the standard equation of projectile,

$$y = x \tan\theta - \frac{gx^2}{2u^2 \cos\theta}$$

We get, $\theta = 45°$ and $u = 20\sqrt{2}\ \text{ms}^{-1}$

Time period of this projectile is 4 s. Hence, after 4 s, velocity vector will again make an angle of 45° with horizontal.

So, maximum height, $H = \frac{u^2 \sin^2\theta}{2g} = \frac{(20\sqrt{2})^2 \sin^2(45°)}{2 \times 10}$

$$H = \frac{400 \times 2 \times \frac{1}{2}}{20} = \frac{400}{20} = 20 \text{ m}$$

and horizontal range, $R = \frac{u^2 \sin 2\theta}{g} = \frac{(20\sqrt{2})^2 \sin 2(45°)}{10}$

$$= \frac{400 \times 2 \times 1}{10} \qquad (\because \sin 90° = 1)$$

$\Rightarrow$ $R = 80$ m

C Medical entrances' gallery

1. (*b*) Position vector of the particle is given by

$$\mathbf{r} = \cos\omega t\,\mathbf{x} + \sin\omega t\,\mathbf{y}$$

where, ω is a constant.

Velocity of the particle is

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \frac{d}{dt}(\cos\omega t\,\mathbf{x} + \sin\omega t\,\mathbf{y})$$

$$= (-\sin\omega t)\,\omega\mathbf{x} + (\cos\omega t)\,\omega\mathbf{y}$$

$$= -\omega(\sin\omega t\,\mathbf{x} - \cos\omega t\,\mathbf{y})$$

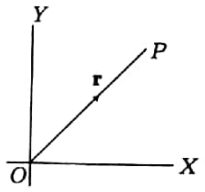
Acceleration of the particles

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d}{dt}[-\omega\sin\omega t\,\mathbf{x} + \omega\cos\omega t\,\mathbf{y}]$$

$$= -\omega^2\cos\omega t\,\mathbf{x} - \omega^2\sin\omega t\,\mathbf{y}$$

$\Rightarrow$ $\mathbf{a} = -\omega^2\mathbf{r} = \omega^2(-\mathbf{r})$

Assuming the particle as P, then its position vector is directed as shown in the diagram.



Therefore, acceleration is directed towards $-\mathbf{r}$ that is towards O (origin)

$$\mathbf{v}.\mathbf{r} = -\omega(\sin\omega t\,\mathbf{x} - \cos\omega t\,\mathbf{y}).(\cos\omega t\,\mathbf{x} + \sin\omega t\,\mathbf{y})$$

$$= -\omega[\sin\omega t.\cos\omega t + 0 + 0 - \sin\omega t.\cos\omega t]$$

$$= -\omega(0) = 0$$

$\Rightarrow \mathbf{v} \perp \mathbf{r}$ $\qquad [\because \mathbf{x} \perp \mathbf{y}]$

Thus, velocity is perpendicular to $\mathbf{r}$.

2. (*a*) The equation of trajectory.

$$y = x\tan\alpha\left[1 - \frac{x}{R}\right] \text{ gives}$$

$$Q = P\tan\theta\left[1 - \frac{P}{R}\right] \qquad \text{...(i)}$$

and $$P = Q\tan\theta\left[1 - \frac{Q}{R}\right] \qquad \text{... (ii)}$$

On dividing we get, $\frac{Q^2}{P^2} = \frac{[1 - P/R]}{[1 - Q/R]}$

$$\frac{1}{R}[P^3 - Q^3] = P^2 - Q^2$$

$$R = \frac{P^3 - Q^3}{P^2 - Q^2} = \frac{P + PQ + Q^2}{P + Q}$$

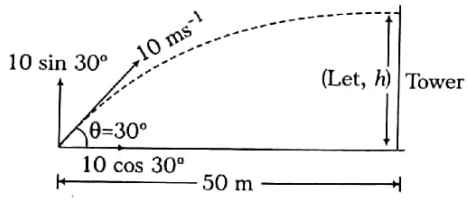
Now, $$\frac{Q}{P} = \tan\theta\left[1 - \frac{P(P+Q)}{P^2 + PQ + Q^2}\right]$$

$$= \tan\theta\left[\frac{P^2 + PQ + Q^2 - P^2 - PQ}{P^2 + PQ + Q^2}\right]$$

$\Rightarrow$ $$\tan\theta = \frac{P^2 + Q^2 + PQ}{PQ}$$

$\Rightarrow$ $$\theta = \tan^{-1}\left[\frac{P^2 + PQ + Q^2}{PQ}\right]$$

3. (*a*) Consider the diagram.



For horizontal motion,

$$50 = 10\cos 30° \times t$$

$\Rightarrow$ $$t = \frac{5}{\cos 30°} = \frac{5}{\frac{\sqrt{3}}{2}} = \frac{10}{\sqrt{3}}$$

For vertical motion,

$$h = 10\sin 30° \times \frac{10}{\sqrt{3}} - \frac{1}{2} \times 10 \times \left(\frac{10}{\sqrt{3}}\right)^2$$

$$= 10 \times \frac{1}{2} \times \frac{10}{\sqrt{3}} - \frac{100}{\sqrt{3}}\frac{5}{\sqrt{3}} = \frac{100}{\sqrt{3}}\left(\frac{1}{2} - \frac{5}{\sqrt{3}}\right)$$

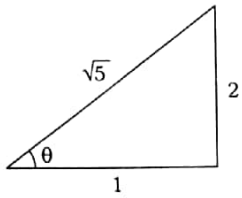
$$= \frac{50}{\sqrt{3}}\left[1 - \frac{10}{\sqrt{3}}\right] \text{ m}$$

4. (*b*) Range of projectile is same for θ and $90° - \theta$.

Given, $\theta = 40°$

Another angle $= 90° - \theta = 90° - 40° = 50°$

5. (*a*) Given, a particle having horizontal range is twice the greatest height attained by it, *i.e.*,



$$R = 2H$$

$$\frac{v^2 \sin 2\theta}{g} = \frac{2v^2 \sin^2\theta}{2g}$$

$$\Rightarrow \quad 2\sin\theta\cos\theta = \sin^2\theta \Rightarrow \tan\theta = 2$$

$$\Rightarrow \quad \sin\theta = \frac{2}{\sqrt{5}} \Rightarrow \cos\theta = \frac{1}{\sqrt{5}}$$

$$\therefore \quad R = \frac{v^2 \sin 2\theta}{g} = \frac{v^2 \times 2\sin\theta\cos\theta}{g}$$

$$\Rightarrow \quad R = v^2 \times \frac{2}{g} \times \frac{2}{\sqrt{5}} \times \frac{1}{\sqrt{5}}$$

i.e., $$R = \frac{4v^2}{5g}$$

6. (*d*) Given, $\theta_1 = \theta_2 = 45°$

$$\theta_1 + \theta_2 = (45° + \alpha) + (45° - \alpha) = 90°$$

$$u_1 = u_2 = u \Rightarrow R_1 = R_2 = ?$$

$$R_1 = \frac{u_1^2 \sin(\theta_1 + \theta_2)}{g}$$

$$\Rightarrow \quad R_1 = \frac{u^2 \sin 90°}{g} = \frac{u^2}{g} \quad \text{...(i)}$$

Similarly, $$R_2 = \frac{u_2^2 \sin(\theta_1 + \theta_2)}{g}$$

$$\Rightarrow \quad R_2 = \frac{u^2 \sin 90°}{g} = \frac{u^2}{g} \quad \text{...(ii)}$$

The ratio of horizontal ranges

$$\frac{R_1}{R_2} = \frac{u^2/g}{u^2/g} = 1:1 \Rightarrow \quad R_1 : R_2 = 1:1$$

7. (*d*) Average velocity, $$\mathbf{v}_{av} = \frac{\text{Net displacement}}{\text{Time taken}}$$

$$= \frac{(13-2)\hat{\mathbf{i}} + (14-3)\hat{\mathbf{j}}}{5}$$

$$= \frac{11\hat{\mathbf{i}} + 11\hat{\mathbf{j}}}{5} = \frac{11}{5}(\hat{\mathbf{i}} + \hat{\mathbf{j}})$$

8. (*c*) For vertically moment,

$$h_1 = u\sin\theta t_1 - \frac{1}{2}gt_1^2 \quad \text{(for } h_1\text{)}$$

$$\Rightarrow \quad t_1 = \frac{h_1 + \frac{1}{2}gt_1^2}{u\sin\theta} \quad \text{...(i)}$$

$$\Rightarrow \quad h_2 = u\sin\theta t_2 - \frac{1}{2}gt_2^2 \quad \text{(for } h_2\text{)}$$

$$\Rightarrow \quad t_2 = \frac{h_2 + \frac{1}{2}gt_2^2}{u\sin\theta} \quad \text{...(ii)}$$

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{t_1}{t_2} = \frac{h_1 + \frac{1}{2}gt_1^2/u\sin\theta}{h_2 + \frac{1}{2}gt_2^2/u\sin\theta} \Rightarrow h_1t_2 - h_2t_1 = \frac{g}{2}(t_1t_2^2 - t_1^2t_2)$$

The time of flight of the ball,

$$T = \frac{2u\sin\theta}{g} = \frac{2}{g}(u\sin\theta) \quad \text{[from Eq. (i)]}$$

$$= \frac{2}{g}\left(\frac{h_1 + 1/2\, gt_1^2}{t_1}\right) = \frac{2}{t_1}\left(\frac{h_1}{g} + \frac{t_1^2}{2}\right)$$

$$= \frac{h_1}{t_1} \times \frac{2}{g} + t_1 = \frac{h_1}{t_1} \times \left(\frac{t_1t_2^2 - t_1^2t_2}{h_1t_2 - h_2t_1}\right) + t_1$$

$$= \frac{h_1t_1t_2^2 - h_1t_1^2t_2 + h_1t_1^2t_2 - h_2t_1^3}{t_1(h_1t_2 \;\; h_2t_1)} = \left(\frac{h_1t_2^2 - h_2t_1^2}{h_1t_2 - h_2t_1}\right)$$

9. (*c*) As we know that maximum height of a projectile is given by

$$H_{max} = \frac{u^2 \sin^2\theta}{2g}$$

where, u = initial velocity of projectile

g = acceleration due to gravity

and θ = angle of projection.

As from question,

$$H_{max} = \frac{v^2 \sin^2 45°}{2g} \quad \text{...(i) (as } u = v,\ \theta = 45°\text{)}$$

Now, range of a projectile is given by

$$R = \frac{u^2 \sin 2\theta}{g} \Rightarrow R = \frac{v^2 \sin(2 \times 45°)}{g}$$

$$\Rightarrow \quad R = \frac{v^2 \sin 90°}{g} \quad \text{...(ii)}$$

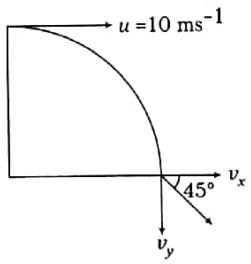
On dividing Eq. (i) by Eq. (ii), we get

$$\frac{H_{max}}{R} = \frac{v^2 \sin^2 45° \times g}{2g \times v^2 \sin 90°} = \frac{1}{4 \times 1}$$

$$\Rightarrow \quad R = 4H_{max} = 4H$$

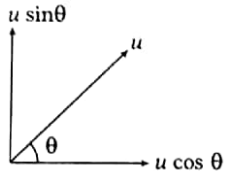
10. (*d*) The horizontal component of velocities

$$v_x = u = 10 \text{ ms}^{-1}$$



Now, $\tan\theta = \tan 45° = \frac{v_y}{v_x} = \frac{v_y}{10} \Rightarrow v_y = 10 \text{ ms}^{-1}$

11. (*b*) As, $t = \frac{2u\sin\theta}{g} \Rightarrow 4 = \frac{2u\sin\theta}{g}$ ...(i)



$\Rightarrow \quad u\sin\theta = 2g$ and $h = \frac{u^2\sin^2\theta}{2g} = \frac{4g^2}{2g}$ [from Eq.(i)]

$\Rightarrow \quad h = 20$ m

12. (*c*) From the question's figure, the X-component remain unchanged, while the Y-component is reverse. Then, the velocity at point B is $(2\hat{\mathbf{i}} - 3\hat{\mathbf{j}})\,\text{ms}^{-1}$.

13. (*b*) Time of flight, $T = \frac{2u\sin\theta}{g}$

Given, $u = u_0$, $T = 1$ s, $\theta = 30°$, $g = 9.8\,\text{ms}^{-2}$

$u_0 = u = g = 9.8\,\text{ms}^{-1}$

14. (*a*) As the velocity makes an angle of 30° with horizontal, so the horizontal component of velocity at the instant will be $v\cos 30°$.

$\Rightarrow \quad v\cos 30° = 5 \Rightarrow v = \frac{5}{\cos 30°} = \frac{5}{\sqrt{3}/2} = \frac{10}{\sqrt{3}}\,\text{ms}^{-1}$

15. (*a*) As the horizontal ranges are the same.

$\therefore \quad \frac{v_0^2\sin 2\theta}{g} = \frac{v_0^2\sin 2\theta_2}{g}$

So, $\sin 2\theta_1 = \sin 2\theta_2$

or $2\theta_1 = \pi - 2\theta_2$

$\Rightarrow \quad \theta_1 + \theta_2 = \pi/2$

$\therefore \quad (h_1)_{max} = \frac{v_0^2\sin^2\theta_1}{2g}$

and $(h_2)_{max} = \frac{v_0^2\sin^2\theta_2}{2g}$

$\therefore \quad \frac{(h_1)_{max}}{(h_2)_{max}} = \frac{\sin^2\theta_1}{\sin^2\theta_2} = \frac{\sin^2\theta_1}{\cos^2\theta_1} = \tan^2\theta_1$

16. (*a*) Given $\mathbf{r} = 2t\,\hat{\mathbf{i}} + t^2\hat{\mathbf{j}}$

Velocity vector $\mathbf{v} = \frac{d\mathbf{r}}{dt}$ and using $\frac{d}{dx}x^n = nx^{n-1}$

We have, $\frac{d\mathbf{r}}{dt} = 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$

17. (*d*) Maximum height and time of height depends upon the vertical component of initial velocity.

$H_1 > H_2 \Rightarrow u_{y_1} > u_{y_2}$

Range $R_2 > R_1$

So, $u_2 > u_1$

18. (*b*) Given, $R = H$

Range, $R = \frac{u^2(2\sin\theta\cos\theta)}{g}$

Height, $H = \frac{u^2\sin^2\theta}{2g}$

Hence, $\frac{u^2(2\sin\theta\cos\theta)}{g} = \frac{u^2\sin^2 a}{2g}$

$2\cos\theta = \frac{\sin\theta}{2} \Rightarrow \tan\theta = 4 \Rightarrow \theta = \tan^{-1}(4)$

19. (*d*) According to given condition,

$\frac{h_1}{h_2} = \frac{\sin^2\theta}{\sin^2(90° - \theta)} = \frac{20}{80}$

$\Rightarrow \quad \tan^2\theta = \frac{1}{4} \Rightarrow \tan\theta = \frac{1}{2}$

$\sin\theta = \frac{1}{\sqrt{5}}$ and $\cos\theta = \frac{2}{\sqrt{5}}$

So, $h = \frac{u^2\sin^2\theta}{2g}$

$\Rightarrow \quad 20 = \frac{u^2}{10g}$ or $\frac{u^2}{g} = 200$

$\therefore$ The range, $R = \frac{u^2 \times 2\sin\theta\cos\theta}{g} = \frac{200 \times 2 \times 2}{\sqrt{5} \times \sqrt{5}}$

$= \frac{200 \times 4}{5} = 160$ m

20. (*b*) Since, the ball reaches from one player to another in 2 s, so the time period of the flight, $T = 2$ s

$\Rightarrow \quad \frac{2u\sin\theta}{g} = 2$ s

Here, u is the initial velocity and θ is the angle of projection.

$\Rightarrow \quad u\sin\theta = g$ ...(i)

Now, we know that the maximum height of the projection

$H = \frac{u^2\sin^2\theta}{2g}$ or $H = \frac{(u\sin\theta)^2}{2g}$

On putting the value of $u\sin\theta$ from Eq. (i), we have

$H = \frac{g^2}{2g} = \frac{g}{2}$ or $H = \frac{g}{2} = \frac{10}{2}$ m or $H = 5$ m

CHAPTER 05

Laws of motion

We normally observe around us, a number of objects or bodies at rest or in motion. On observing we can find that, the objects at rest do not move by themselves or the objects in motion do not come to rest by themselves but they require some external force to do so. *e.g.*, to move a book kept on a table, we need to push or pull it, or to stop a vehicle in motion, breaks are required.

The factors which are necessary for causing motion or change in motion are termed as force. This cause of motion (force) and effects of motion are governed by Newton's laws of motion.

In this chapter, we will discuss the motion of a body by taking into consideration the cause of motion *i.e.*, the external force which produces the motion or change the motion.

Inside

1) Force
- Inertia
- Momentum

2) Newton's laws of motion
- Newton's Ist law of motion
- Newton's IInd law of motion
- Impulse
- Newton's IIIrd law of motion

3) Law of conservation of linear momentum

4) Forces in equilibrium
- Newton's Ist law for forces in equilibrium

5) Common forces in mechanics
- Free body diagram

6) Apparent weight of a man in a lift

7) Applications of Newton's laws of motion
- Motion of bodies connected through strings
- Bodies attached through pulley

8) Force of friction

9) Equation of motion of a body on a rough inclined plane

1) Force

Force is an effort in the form of push or pull causing or tending to cause motion, change in motion or deformations in a body.

There are basically two types of forces which are commonly observed.

1. Distant forces (Field forces)

The forces acting between two or more objects, which do not require the physical contact between the objects are called distant forces or field forces. Gravitational force between two bodies, electrostatic force between two charges, weak forces and nuclear forces are examples of distant forces. Weight ($w = mg$) of a body comes in this category.

2. Contact forces

The forces acting between two or more objects, which require the physical contact between the objects are called contact forces. Friction force, normal reaction, tension, spring forces etc are some examples of contact forces.

Inertia

The term inertia means resistance or opposition to the change of state. It is defined as the inherent property of a body by virtue of which it remains in its state of rest or of uniform motion in a straight line. This term was first used by Galileo.

There are three types of inertia

(i) Inertia of rest

It is defined as the tendency of a body to remain in its position of rest. *i.e.,* A body at rest remains at rest and can not start moving on its own.

(ii) Inertia of motion

It is defined as the tendency of a body to remain in its state of uniform motion along a straight line. *i.e.,* A body in uniform motion can neither gets accelerated nor get retarded on its own, also it cannot stop on its own.

(iii) Inertia of direction

It is defined as inability of a body to change by itself its direction of motion.

Relation between mass and inertia

Mass of a body is the measurement of its inertia. A body with greater mass shows greater inertia, *i.e.,* it is more difficult to change its state of rest or uniform motion as compared to that of a body having smaller mass.

Quantitatively, the inertia of an object is measured by its mass. Thus, the SI unit for mass as well as inertia is kilogram (kg), whereas the units in the CGS system and in the BE system for mass or inertia are gram (g) and slug (sl), respectively.

Momentum

Momentum of a body is the quantity of motion contained in the body. It is measured as the product of mass and velocity of a body. It is represented by p.

i.e., $\boxed{\text{Momentum } (p) = \text{Mass } (m) \times \text{Velocity } (v)}$

SI unit of momentum is $kg\text{-}ms^{-1}$ and CGS unit of momentum is $g\text{-}cms^{-1}$.

The dimensional formula of momentum is $[MLT^{-1}]$.

It is a vector quantity.

2 Newton's laws of motion

Newton's laws of motion are the three physical laws that, together laid the foundation for classical mechanics. These three law of motion were first proposed by Sir Isaac Newton.

Newton's Ist law of motion

This law states that, "every body continues in its state of rest or of uniform motion in a straight line unless it is compelled by some external force to change its state". Thus, it can be concluded that if the net external force on a body is zero, its acceleration is zero. Acceleration can be non-zero only if there is a net external force on the body. Newton's first law defines force qualitatively.

Newton's IInd law of motion

This law states that, "the rate of change of momentum of a body is directly proportional to the external force applied on the body and the change takes place in the direction of the applied force."

We may also state Newton's second law of motion as

"If the unbalanced external force (net force) acts on a body, the body accelerates. The direction of acceleration is the same as the direction of the net force."

Calculating force with the help of Newton's IInd law

Let **F** be external force applied on the body in the direction of motion of the body for time interval Δt, then the velocity of a body of mass m changes from $\mathbf{v}$ to $\mathbf{v} + \Delta\mathbf{v}$ *i.e.* change in momentum, $\Delta\mathbf{p} = m\Delta\mathbf{v}$.

According to Newton's second law.

$$\mathbf{F} \propto \frac{\Delta\mathbf{p}}{\Delta t} \quad \text{or} \quad \mathbf{F} = k\frac{\Delta\mathbf{p}}{\Delta t}$$

where k is a constant of proportionality.

If limit $\Delta t \to 0$, then the term $\frac{\Delta\mathbf{p}}{\Delta t}$ becomes the derivative $\frac{d\mathbf{p}}{dt}$.

Thus, $$\mathbf{F} = k\frac{d\mathbf{p}}{dt}$$

For a body of fixed mass m, we have

$$\mathbf{F} = k\frac{d(m\mathbf{v})}{dt} = km\frac{d\mathbf{v}}{dt} \Rightarrow \mathbf{F} = km\mathbf{a}$$

If v is fixed and m is variable then $F = \frac{kd(mv)}{dt} = \frac{kvdm}{dt}$

$\because$ $k = 1$, then $F = vdm/dt$

Now, **a unit force may be defined as the force which produces unit acceleration in a body of unit mass.**

So, $F = 1, m = 1, a = 1$

So, $k = 1$

So, $\boxed{\text{Force, } \mathbf{F} = m\mathbf{a}}$

In scalar form, this equation can be written as, $F = ma$.

The force is a vector quantity.

Note The slope of momentum-time graph is equal to force on the particle *e.g.*,

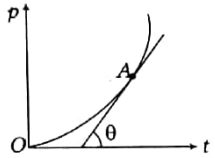


***Fig. 5.1** Momentum-time graph*

At point *A*, $F = \frac{dp}{dt} = \text{slope} = \tan\theta$

This graph depicts the motion of a body on which increasing force is acting.

Unit of force

- In SI system, **absolute unit** of force is **newton.**

One newton is defined as that much force which produces an acceleration of 1 ms^{-2} in a body of mass 1 kg.

$$1\text{ N} = 1\text{ kg} \times 1\text{ ms}^{-2}$$
$$1\text{ N} = 1\text{ kg ms}^{-2}$$

In CGS system, absolute unit of force is **dyne**. One dyne is that much of force which produces an acceleration of 1 cms^{-2} in a body of mass 1 g.

$$1\text{ dyne} = 1\text{ g} \times 1\text{ cms}^{-2}$$
$$1\text{ dyne} = 1\text{ g cms}^{-2}$$

- In MKS unit, **gravitational unit** of force is kilogram weight (kg-wt). One kg-wt is that much of force which produces an acceleration of 9.80 ms^{-2} in a body of mass 1 kg. It is also known as kilogram-force (kgf).

$$1\text{ kg-wt} = 1\text{ kgf} = 9.8\text{ N}$$

In CGS system, gravitational unit of force is **gram weight** (g-wt) or **gram force** (gf). It is defined as that force which produces an acceleration of 980 cm s^{-2} in a body of mass 1 g.

$$1\text{ g-wt} = 1\text{ gf} = 980\text{ cm s}^{-2} \quad \text{or} \quad 1\text{ gf} = 980\text{ dyne}$$

Relation between newton and dyne

$$1\text{ N} = 1\text{ kg} \times 1\text{ ms}^{-2}$$
$$= 1000\text{ g} \times 100\text{ cm s}^{-2}$$
$$= 10^5\text{ g cms}^{-2} \quad (1\text{ dyne} = 1\text{ g cms}^{-2})$$
$$1\text{ N} = 10^5\text{ dyne}$$

Example 5.1 *If an electron is subjected to a force of 10^{-25} N in an X-ray machine, then find out the time taken by the electron to cover a distance of 0.2 m. Take mass of the electron 10^{-30} kg.*

Sol. The acceleration of the electron

$$a = \frac{F}{m} = \frac{10^{-25}}{10^{-30}} = 10^5\text{ ms}^{-2}$$

The time taken by the electron (t) to cover the distance (s) of 0. 2m can be given by

$$s = ut + \frac{1}{2}at^2$$
$$\Rightarrow \quad 0.2 = 0 + \frac{1}{2} \times 10^5 \times t^2$$
$$\Rightarrow \quad t^2 = 0.4 \times 10^{-5} = 4 \times 10^{-6} \Rightarrow t = 2 \times 10^{-3}\text{ s}$$

Example 5.2 *A bullet of mass 0.06 kg moving with a speed of 90 ms^{-1} enters a heavy wooden block and it stopped after a distance of 60 cm. What is the average resistive force exerted by the block on the bullet?*

Sol. The retardation a of the bullet is given by

$$a = -\frac{u^2}{2s} = \frac{-90 \times 90}{2 \times 0.6} \quad (\text{From, } v^2 = u^2 + 2as, v = 0)$$

$$(\because \text{ Given, } s = 60\text{ cm} = 0.6\text{ m}, u = 90\text{ ms}^{-1})$$

$$\Rightarrow \quad a = -6750\text{ ms}^{-2}$$

From second law of motion,

Retarding force by the second law of motion is $F = ma$

where, $F = 0.06 \times 6750 \Rightarrow F = 405$ N

Example 5.3 *A stone of mass 1 kg is thrown with a velocity of 20 ms^{-1} across the frozen surface of a lake and it comes to rest after travelling a distance of 50 m. What is the magnitude of the force opposing the motion of the stone?*

Sol. Given, $u = 20\text{ ms}^{-1}$, $v = 0$, $s = 50$ m and $m = 1$ kg

To calculate force, we have the formula $F = ma$, but we have to first calculate acceleration a. Using the third equation of motion, *i.e.*,

$$v^2 = u^2 + 2as$$
$$(0)^2 = (20)^2 + 2 \times a \times 50$$
$$100a = -400$$
$$a = -\frac{400}{100} = -4\text{ ms}^{-2}$$

Acceleration $a = -4$ ms^{-2} (– ve sign shows that speed of the stone decreases, *i.e.*, retardation)

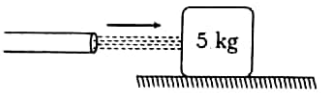
Now, $F = ma = 1\text{ kg} \times (-4)\text{ ms}^{-2} = -4\text{ kg-ms}^{-2} = -4$ N

Thus, force of friction between the stone and the ice is – 4 N.

The negative value of force shows that the frictional force acts in a direction opposite to the direction of motion.

Example 5.4 *A block of 5 kg is resting on a frictionless plane. It is struck by a jet releasing water at a rate of 3 kgs^{-1} at a speed of 4 ms^{-1}. Calculate the acceleration of the block.*

Sol. Force exerting on block $F = v\dfrac{dm}{dt} = 4 \times 3 = 12$ N



so acceleration of the block $a = \dfrac{F}{m} = \dfrac{12}{5} = 2.4$ ms^{-2}

Resolution into different components of force

If the force applied imparts acceleration a to a body, then its components in X, Y and Z-axis are $a = a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}} + a_z\hat{\mathbf{k}}$

Components of force will be F_x, F_y and F_z

$$F = ma$$

$$F_x\hat{\mathbf{i}} + F_y\hat{\mathbf{j}} + F_z\hat{\mathbf{k}} = m(a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}} + a_z\hat{\mathbf{k}})$$

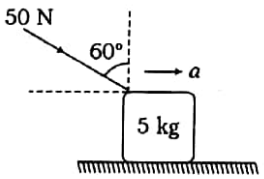
Thus,
$$F_x = ma_x = m\frac{dv_x}{dt} = m\frac{d^2x}{dt^2} = \frac{dp_x}{dt}$$

$$F_y = ma_y = m\frac{dv_y}{dt} = m\frac{d^2y}{dt^2} = \frac{dp_y}{dt}$$

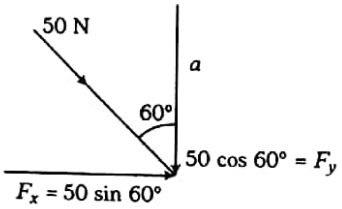
$$F_z = ma_z = m\frac{dv_z}{dt} = m\frac{d^2z}{dt^2} = \frac{dp_z}{dt}$$

The component form of Newton's second law tells that if the applied force makes some angle with the velocity of the body, it changes the component of velocity along the direction of force. The component of velocity normal to the force remains unchanged.

Example 5.5 *A force of 50 N acts in the direction as shown in figure. The block of mass 5 kg, resting on a smooth horizontal surface. Find out the acceleration of the block.*



Sol. Free body diagram



where, F_x = Horizontal component of the force
F_y = Vertical component of the force

Horizontal component of the force $= 50 \sin 60° = \dfrac{50\sqrt{3}}{2}$

Acceleration of the block a

$$= \frac{\text{component of force in the direction of acceleration}}{\text{mass}}$$

$$= \frac{50\sqrt{3}}{2} \times \frac{1}{5} = 5\sqrt{3} \text{ ms}^{-2}$$

Example 5.6 *A force* $\mathbf{F} = (6\hat{\mathbf{i}} - 8\hat{\mathbf{j}} + 10\hat{\mathbf{k}})$ *N produces acceleration of* $\sqrt{2}$ *ms*$^{-2}$ *in a body. Calculate the mass of the body.*

Sol. $\because$ Acceleration $a = \dfrac{|\mathbf{F}|}{m}$

$$\therefore \quad m = \frac{|\mathbf{F}|}{a} = \frac{\sqrt{6^2 + 8^2 + 10^2}}{\sqrt{2}} = 10 \text{ ms}^{-2}$$

Example 5.7 *A force* $\mathbf{F} = (2t\hat{\mathbf{i}} + 3t^2\hat{\mathbf{j}})N$ *acts on an object moving in XY-plane. Find magnitude of change in momentum of the object in time interval* $t = 0$ *to* $t = 2$*s.*

Sol. Given, $\mathbf{F} = (2t\hat{\mathbf{i}} + 3t^2\hat{\mathbf{j}})N \Rightarrow \dfrac{d\mathbf{p}}{dt} = 2t\hat{\mathbf{i}} + 3t^2\hat{\mathbf{j}}$

$$\Rightarrow \quad d\mathbf{p} = 2t\,dt\,\hat{\mathbf{i}} + 3t^2\,dt\,\hat{\mathbf{j}}$$

$$\Rightarrow \quad \int d\mathbf{p} = 2\int_0^2 t\,dt\,\hat{\mathbf{i}} + 3\int_0^2 t^2\,dt\,\hat{\mathbf{j}}$$

$$\Rightarrow \quad \Delta\mathbf{p} = [t^2]_0^2\hat{\mathbf{i}} + [t^3]_0^2\hat{\mathbf{j}} \Rightarrow \Delta\mathbf{p} = 4\hat{\mathbf{i}} + 8\hat{\mathbf{j}}$$

$$\Rightarrow \quad |\Delta\mathbf{p}| = \sqrt{16 + 64} = \sqrt{80} \approx 9 \text{ kg ms}^{-1}$$

Resultant force

When two or more forces act on a body simultaneously, then the single force which produces the same effect as produced by all the forces acting together is known as the resultant force.

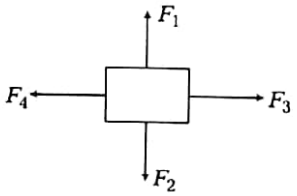


Fig. 5.2

Resultant force, $\boxed{\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_4}$

Balanced force

When a number of forces acting simultaneously on a body do not bring about any change in its state of rest or of uniform motion along a straight line, then the forces acting on the body are said to be balanced forces.

When different forces acting on a body give a zero resultant, then the forces are said to be balanced. Balanced forces do not produce any acceleration.

e.g., When two opposite forces having the same magnitude F act on a block placed on a smooth horizontal table, they fail to move the block.

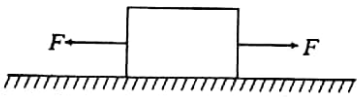


Fig. 5.3

This is because the net force is equal to zero. Similarly, two opposite forces having the same magnitude cannot change the speed of a moving body.

Unbalanced force

When a number of forces acting simultaneously on a body bring about a change in its state of rest or of uniform motion along a straight line, then these forces acting on the body are said to be unbalanced forces.

In this case, different forces acting on body do not give zero resultant. If an unbalanced force is applied on the object, there will be a change either in its speed/velocity or in the direction of its motion.

Thus, to accelerate an object, an unbalanced force is required. When net force on the body is not equal to zero, then the body at rest starts moving in the direction of resultant force.

Fig. 5.4

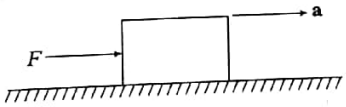
If $F_2 > F_1$, then $F_2 - F_1 > 0$ as a result the car accelerates in the direction of F_2.

Example 5.8 *Let us consider two forces F_1 and F_2 acting on a body of mass 2 kg as shown in the figure. $F_1 = 10\,N$, $F_2 = 2N$, what will be the acceleration?*

Sol. Unbalanced external force $= F_1 - F_2 = 10 - 2 = 8\text{N}$

In the direction of F_1.

So, $$F = ma \Rightarrow a = \frac{F}{m} = \frac{8}{2} = 4 \text{ ms}^{-2}$$



The body move in direction of $\mathbf{F}_1$.

Impulse

When a large force acts on a body for very small time, then product of the average of total force for that small time period and the time period itself is called impulse.

$$\boxed{\text{Impulse} = \text{Average force} \times \text{Time}}$$

$$I = F\Delta t$$

or $$\boxed{\mathbf{I} = \int_{t_1}^{t_2} \mathbf{F}\, dt}$$

It is a vector quantity and its direction is same as that of force. Dimensional formula of impulse is same as that of momentum, *i.e.*, $[MLT^{-1}]$. Its SI unit is N-s or kg ms^{-1}. In CGS system, it is dyne or g cms^{-1}.

Relation between momentum and impulse

Suppose $\mathbf{F}$ is the value of force during impact at any time and $\mathbf{p}$ is the momentum of the body at that time, then according to Newton's IInd law of motion.

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} \quad \text{or} \quad \mathbf{F} \cdot dt = d\mathbf{p} \qquad \text{...(i)}$$

Suppose that the impact lasts for a small time t and during this time, the momentum of the body changes from p_1 to p_2. Then, integrating the above equation, we get

$$\int_0^t \mathbf{F}\, dt = \int_{p_1}^{p_2} d\mathbf{p} = |\,\mathbf{p}\,|_{p_1}^{p_2}$$

$$\Rightarrow \quad \boxed{\underbrace{\int_0^t \mathbf{F}\, dt}_{\text{Impulse}} = p_2 - p_1}$$

From this equation, we found that **impulse is equal to change in momentum of the body.**

Also, if $\mathbf{F}_{av}$ is the average force (constant) during the impact, then

$$\text{Impulse, } I = \int_0^t \mathbf{F}_{av}\, dt = \mathbf{p}_2 - \mathbf{p}_1 = F_{av} \int_0^t dt = p_2 - p_1$$

or $$I = F_{av} \cdot t = \mathbf{p}_2 - \mathbf{p}_1 = \Delta p$$

Thus, **impulse is also equal to total change in momentum.** This is known as impulse-momentum theorem.

Calculation of impulse: graphical method

(i) **When applied force is constant** then the graph between this force and the time of application of this force is a straight line parallel to time axis.

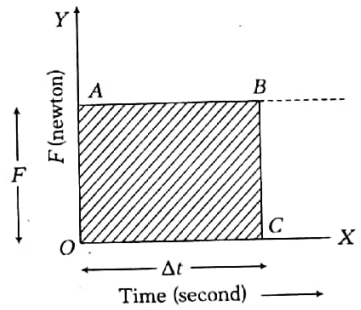


Fig. 5.5 Impulse of a constant force

Here, **impulse is given by the area covered by the graph.**

i.e., $$I = F \times \Delta t$$

where, $F = OA, \Delta t = OC$

$\therefore$ $I = OA \times OC$

$= \text{Area of rectangle } OABC.$

(ii) **When force is variable** for the time of application (Δt) then, *graph between force and time will be a curve as given in the figure below*

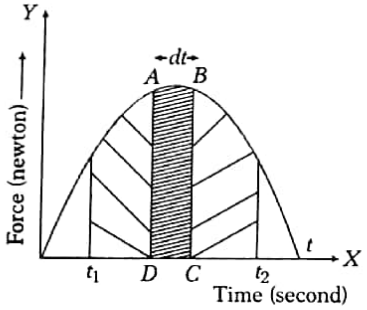


Fig. 5.6 *Impulse of a variable force*

Here, Impulse = Force × time (dt)

$= F \times dt$

= Area of shaded region

Total impulse for the force applied during period from t_1 to t_2

$$= \int_{t_1}^{t_2} F \cdot dt$$

Area under F-t curve from t_1 to t_2.

Total impulse force the force applied = Area covered between the curve and time-axis

Example 5.9 *A baseball player hits back the ball straight in the direction of the bowler without changing its initial speed of 12 ms^{-1}. If the mass of the ball is 0.15 kg. Find the impulse imparted to the ball. (Consider the ball in linear motion)*

Sol. Given, $m = 0.15$ kg, $v = 12\ \text{ms}^{-1}$ $u = -12\ \text{ms}^{-1}$

Change in momentum,

$$p_2 - p_1 = m(v - u) = 0.15[12 - (-12)] = 0.15 \times 24$$

$$p_2 - p_1 = 3.60\ \text{kg ms}^{-1}$$

Impulse, $I = p_2 - p_1 \Rightarrow I = 3.6$ N-s

Example 5.10 *A hammer of mass 1 kg moving with a speed of 6 ms^{-1} strikes a wall and comes to rest in 0.1 s. Calculate,*

(i) Impulse of the force

(ii) Average retarding force that stops the hammer.

(iii) Average retardation of the hammer.

Sol. (i) Impulse $= F \times t = m(v - u) = 1(0 - 6) = -6$ Ns

(ii) Average retarding force that stops the hammer

$$F = \frac{\text{Impulse}}{\text{Time}} = \frac{6}{0.1} = 60\ \text{N}$$

(iii) Average retardation, $a = \frac{F}{m} = \frac{60}{1} = 60\ \text{ms}^{-2}$

Example 5.11 *A cricket ball of mass 150 g is moving with a velocity of 12 ms^{-1} and is hit by a bat so that the ball is turned back with a velocity of 20 ms^{-1}. If the duration of contact between the ball and bat is 0.01 s, find the impulse and the average force exerted on the ball by the bat.*

Sol. According to given question, change in momentum of the ball

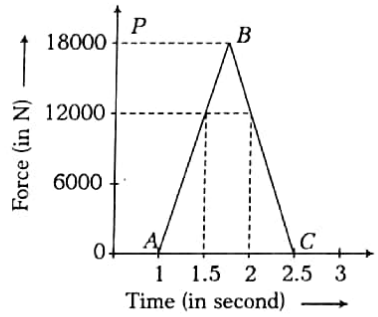
$$\Delta p = p_f - p_i = m(v - u) = 150 \times 10^{-3}\ [20 - (-12)]$$

So, by impulse-momentum theorem, impulse $I = \Delta p = 4.8$ N-s

And by time averaged definition of force in case of impulse

$$F_{av} = \frac{I}{\Delta t} = \frac{\Delta p}{\Delta t} = \frac{4.80}{0.01} = 480\ \text{N}$$

Example 5.12 *Figure shows an estimated force-time graph for a base ball struck by a bat.*



From this curve, determine

(i) Impulse delivered to the ball

(ii) Average force exerted on the ball.

Sol. (i) Impulse = Area under F-t curve

$$= \text{Area of } \Delta ABC = \frac{1}{2} \times OP \times AC = \frac{1}{2} \times 18000 \times (2.5 - 1)$$

$$= 1.35 \times 10^4\ \text{kg ms}^{-1}$$

(ii) Average force $= \frac{\text{Impulse}}{\text{Time}} = \frac{1.35 \times 10^4}{(2.5 - 1)} = 9000$ N

Newton's IIIrd law of motion

This law states that, "to every action, there is always an equal and opposite reaction or, the mutual actions of two bodies upon each other are always directed to contrary parts".

Analysis of forces

From Newton's IIIrd law, it can be analysed that we can not produce a single isolated force in nature. Thus, forces occur in equal and opposite pairs. Whenever object A exerts a force on object B, object B must also exert a force on object A. The two forces are equal in magnitude and opposite in direction.

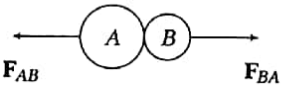


Fig. 5.7 *Forces acting on bodies A and B*

As shown in figure, if $\mathbf{F}_{BA}$ is the force exerted by body A on B and $\mathbf{F}_{AB}$ is the force exerted by B on A, then according to Newton's third law,

$$\boxed{\mathbf{F}_{AB} = -\mathbf{F}_{BA}}$$

Force on A by B = – Force on B by A

Important features of Newton's IIIrd law of motion

(*i*) **Newton's third law of motion is applicable irrespective of the nature of the forces** The forces of action and reaction may be mechanical, gravitational, electric or of any other nature.

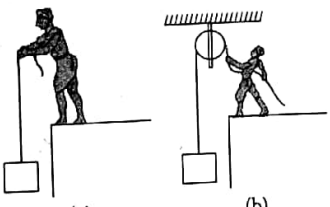
(*ii*) **Action and reaction always act on two different bodies** If they act on the same body, the resultant force would be zero and there could never be accelerated motion.

(*iii*) **The force of action and reaction cannot cancel each other** This is because action and reaction, though equal and opposite forces always act on different bodies and so cannot cancel each other.

(*iv*) **No action can occur in the absence of a reaction** In a tug of war, one team can pull the rope only if the other team is pulling the other end of the rope, no force can be exerted if the other end is free. One team exerts the force of action and the other team provides the force of reaction.

Example 5.13 *A block of mass 25 kg is raised by a 50 kg man in two different ways as shown in figure. What is the action on the floor by the man in the two cases? If the floor yields to a normal force of 700 N, which mode should the man adopt to lift the block without the floor yielding? (Take $g = 9.8\ ms^{-2}$)*

Sol. In mode (a), the man applies a force equal to 25 kg weight in upward direction. According to Newton's third law of motion, there will be a downward force of reaction on the floor.



(a) (b)

∴ Total action on the floor by the man

= 50 kg wt + 25 kg wt

= 75 kg wt

= 75× 9.8 N

= 735 N

In mode (b), the man applies a downward force on ropes equal to 25 kg wt. According to Newton's third law, the reaction will be in the upward direction by the ropes on the man so he becomes light by 25 kg wt.

∴ Total action on the floor by the man

= 50 kg wt - 25 kg wt

= 25 kg wt = 25× 9.8 N = 245 N

As the floor yields to a downward force of 700 N, so the man should adopt mode (b).

3 Law of conservation of linear momentum

The total momentum of an isolated system (a system having no external force acting on it) of constant mass remains constant or conserved and does not change with time.

If the momentum of a two particles system of masses m_1 and m_2 are p_1 and p_2 respectively, then the net momentum of whole system is given by

$$p = p_1 + p_2 = \text{Constant}$$

This principle is a consequence of Newton's second and third law of motion.

Derivation of law of conservation of linear momentum are given below.

Conservation of linear momentum for a system of two or more particles

Force applied on particle 1 by particle 2 is F_{12} and force applied on particle 2 by particle 1 is F_{21} and their respective momentum are p_1, p_2.

From Newton's IInd law

$$\mathbf{F}_{12} = \frac{dp_1}{dt} \quad \text{and} \quad \mathbf{F}_{21} = \frac{dp_2}{dt}$$

From Newton's IIIrd law

$$\mathbf{F}_{12} = -\mathbf{F}_{21} \Rightarrow \frac{d\mathbf{p}_1}{dt} = -\frac{d\mathbf{p}_2}{dt}$$

or $$\frac{dp_1}{dt} + \frac{dp_2}{dt} = 0 \Rightarrow \frac{d}{dt}(p_1 + p_2) = 0$$

Thus, $$p_1 + p_2 = \text{constant}$$

For n number of particles,

$$\boxed{\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 + \ldots + \mathbf{p}_n = \text{Constant}}$$

Conservation of linear momentum for the collision of two bodies

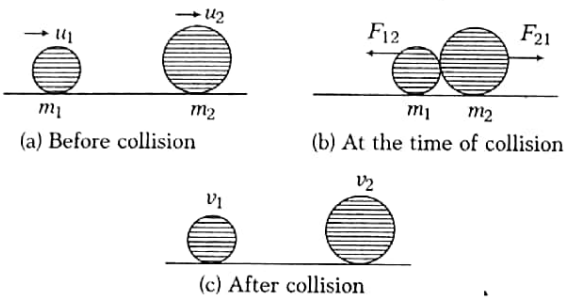
Two bodies of mass m_1 and m_2 collide on frictionless surface in the same direction with respective velocities u_1 and u_2. After collision, both the bodies separate with a variation in their velocities, *i.e.*, v_1 and v_2, respectively.

Initial momentum (before collision)

$$p_{1(\text{initial})} = m_1\mathbf{u}_1, \quad p_{2(\text{initial})} = m_2\mathbf{u}_2$$

Final momentum (After collision)

$$p_{1(\text{final})} = m_1\mathbf{v}_1, \quad p_{2(\text{final})} = m_2\mathbf{v}_2$$



(a) Before collision (b) At the time of collision

(c) After collision

Fig. 5.8

During collision, particle 1 exerts a force F_{21} on particle 2 and simultaneously particle 2 exerts a force F_{12} on particle 1.

$$\mathbf{F}_{12} = \text{Rate of change of momentum of particle 1}$$

$$= \frac{m_1v_1 - m_1u_1}{t} = \frac{m_1(v_1 - u_1)}{t}$$

Similarly, $\mathbf{F}_{21}$ = Rate of change of momentum of particle 2

$$= \frac{m_2v_2 - m_2u_2}{t} = \frac{m_2(v_2 - u_2)}{t}$$

According to Newton's IIIrd law of motion

$$\mathbf{F}_{12} = -\mathbf{F}_{21}$$

$$\frac{m_1(v_1 - u_1)}{t} = -\frac{m_2(v_2 - u_2)}{t}$$

or $$\boxed{m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2}$$

i.e., **Total momentum before collision remains same as total momentum after collision.**

Collision of ball with wall

A ball of mass m strikes a surface with velocity u at angle θ from the normal of surface and rebounds with the same speed in time t.

Here, initial momentum of the particle

$$\mathbf{p}_i = mu\cos\theta\hat{\mathbf{i}} - mu\sin\theta\hat{\mathbf{j}}$$

Final momentum of the particle

$$\mathbf{p}_f = -mu\cos\theta\hat{\mathbf{i}} - mu\sin\theta\hat{\mathbf{j}}$$

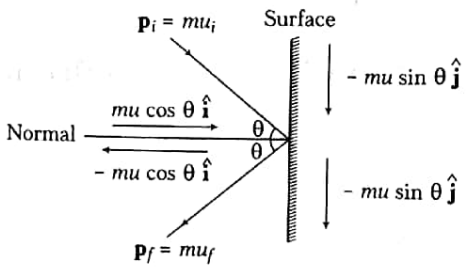


Fig. 5.9 Collision of ball with wall

Now, change in momentum, $\Delta\mathbf{p} = \mathbf{p}_f - \mathbf{p}_i$

$$= -mu\cos\theta\hat{\mathbf{i}} - mu\sin\theta\hat{\mathbf{j}} - mu\cos\theta\hat{\mathbf{i}} + mu\sin\theta\hat{\mathbf{j}}$$

$$= -2mu\cos\theta\hat{\mathbf{i}}$$

It means that momentum changes only along the normal to the surface but not along the surface. This is so because force is acting on the ball only normal to the surface (this force is reaction force of wall) and no force acts parallel to the surface.

$$|\Delta\mathbf{p}| = 2mu\cos\theta$$

$$\text{Force, } \mathbf{F} = \frac{\Delta\mathbf{p}}{\Delta t} = \frac{-2mu\cos\theta}{t}\hat{\mathbf{i}}$$

$$|\mathbf{F}| = \frac{2mu\cos\theta}{t}$$

There are two possible cases

Case I *If $\theta = 0°$, means ball is thrown perpendicular to the surface, then*

$$\Delta\mathbf{p} = -2mu\cos 0°\hat{\mathbf{i}}$$

$$\Rightarrow \quad \Delta\mathbf{p} = -2mu\hat{\mathbf{i}}$$

$$\therefore \quad \mathbf{F} = \frac{\Delta\mathbf{p}}{\Delta t} = \frac{-2mu}{t}\hat{\mathbf{i}}$$

CaseII *If θ is the angle measured from the surface, then*

$$\Delta\mathbf{p} = -2mu\sin\theta\hat{\mathbf{i}} \Rightarrow |\Delta\mathbf{p}| = 2mu\sin\theta$$

$$\mathbf{F} = \frac{-2mu\sin\theta}{t}\hat{\mathbf{i}} \Rightarrow |\mathbf{F}| = \frac{2mu\sin\theta}{t}$$

Newton's IIIrd law can be derived from principle of conservation of linear momentum

If two particles of masses m_1 and m_2 are moving under action of their mutually interacting forces with each other, such that no external force acts on the system.

Momentum of system remains constant.

i.e., $\Delta\mathbf{p}_1 + \Delta\mathbf{p}_2 = 0$

$$\Delta\mathbf{p}_1 = -\Delta\mathbf{p}_2 \Rightarrow \frac{\Delta\mathbf{p}_1}{\Delta t} = -\frac{\Delta\mathbf{p}_2}{\Delta t} \Rightarrow \mathbf{F}_{12} = -\mathbf{F}_{21}$$

Force on 1st due to 2nd = – Force on 2nd due to 1st

Example 5.14 *A bullet of mass 10 g is fired from a gun of mass 1 kg. If the recoil velocity is 5 ms^{-1}. Find the velocity of the muzzle.*

Sol. From the law of conservation of momentum

$$m_Gv_G = m_Bv_B$$

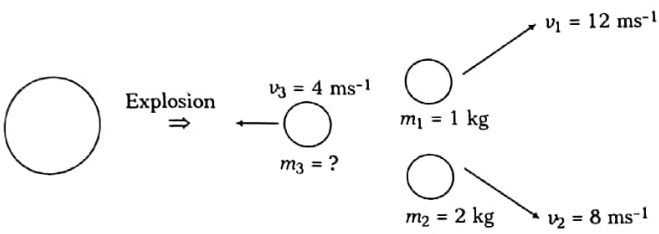
where, m_G, v_G = mass of gun and velocity of gun

m_B, v_B = mass and velocity of bullet

$$\Rightarrow \quad v_B = \frac{m_Gv_G}{m_B} = \frac{1 \times 5}{10 \times 10^{-3}} = 500 \text{ ms}^{-1}$$

Example 5.15 *On a mine site a rock is exploded. On explosion rock breaks into three parts. Two parts go off at right angles to each other. These two are, 1 kg first part moving with a velocity of 12 ms^{-1} and 2 kg second part moving with a velocity of 8 ms^{-1}. If the third part flies off with a velocity 4 ms^{-1}, what will be its mass?*

Sol.

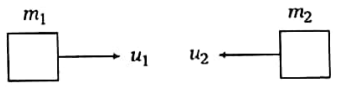


From the law of conservation of momentum,

$$p_3 = \sqrt{p_1^2 + p_2^2}$$

$$\Rightarrow \quad m_3 \times 4 = \sqrt{(1 \times 12)^2 + (2 \times 8)^2} = 20 \Rightarrow m_3 = 5 \text{ kg}$$

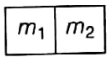
Example 5.16 *Two objects each of mass 5 kg are moving in the same straight line but in the opposite directions towards each other with same speed of 3 m/s. They stick to each other after collision. What will be the velocity of the combined object after collision?*



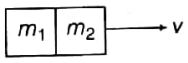
Sol. Before collision, $m_1 = m_2 = 5$ kg

$u_1 = 3$ m/s, $u_2 = -3$ m/s

At collision,



After collision,



Let the velocity of the combined object be v.

Total momentum before collision is

$$m_1u_1 + m_2u_2 = 5 \times 3 + 5 \times (-3) = 0$$

Total momentum of the system after collision is

$$m_1v + m_2v = (m_1 + m_2)v = (5 + 5)v = 10v$$

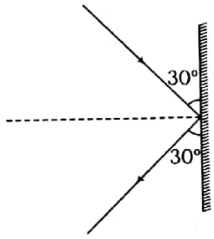
According to the conservation of momentum,

Momentum before collision = Momentum after collision

$$\therefore \quad 0 = 10v \Rightarrow v = 0$$

Hence, the velocity of the combined object after collision is zero.

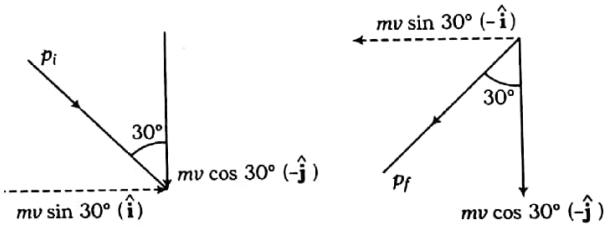
Example 5.17 *A ball of mass m strikes a rigid wall with speed v and gets reflected without any loss of speed, as shown in the figure.*



(i) *What is the magnitude of the impulse imparted to the ball by the wall?*

(ii) *What is the direction of the force on the wall due to the ball?*

Sol.



According to the question,

(i) $\mathbf{p}_i = mv \sin 30° \hat{\mathbf{i}} - mv \cos 30° \hat{\mathbf{j}}$

$\mathbf{p}_f = -mv \sin 30° \hat{\mathbf{i}} - mv \cos 30° \hat{\mathbf{j}}$

$\therefore \quad \Delta\mathbf{p} = \mathbf{p}_f - \mathbf{p}_i = -2mv \sin 30° \hat{\mathbf{i}} = -mv\hat{\mathbf{i}}$

$|\Delta\mathbf{p}| = mv$

(ii) Negative sign of the impulse shows that it is along negative x-direction. Since impulse and force are in the same direction, the force on the ball is along the negative direction of x-axis. Hence the force on the wall will be along positive x-axis.

Check point 5.1

1. Which of Newton's laws of motion explain the concept of inertia?
(a) Ist law (b) IInd law (c) IIIrd law (d) All of these

2. Inertia of an object is directly dependent on
(a) impulse (b) momentum (c) mass (d) density

3. A body of mass 6 kg is acted on by a force so that its velocity changes from 3 ms^{-1} to 5 ms^{-1}, then change in momentum is
(a) 48 Ns (b) 24 Ns (c) 30 Ns (d) 12 Ns

4. The momentum p (in kg ms^{-1}) of a particle is varying with time t (in second) as $p = 2 + 3t^2$. The force acting on the particle at $t = 3$ s will be
(a) 18 N (b) 54 N (c) 9 N (d) 15 N

5. A body is acted upon by balanced forces
(a) if it is in rest only
(b) if it is moving with constant speed
(c) if even number of forces are acting on it
(d) if it is not accelerating

6. A force $\mathbf{F} = (6\hat{i} - 8\hat{j} + 10\hat{k})$ N produces acceleration of $\sqrt{2}$ ms^{-2} in a body. Calculate the mass of the body.
(a) 10 kg (b) 8 kg (c) 12 kg (d) 9 kg

7. A force of 72 dyne is inclined to the horizontal at an angle of 60°, find the acceleration in a mass of 9 g which moves in the effect of this force in a horizontal direction.
(a) 5 cms^{-1} (b) 4 cms^{-1} (c) 2 ms^{-1} (d) 3 ms^{-1}

8. A constant force acting on a body of mass 3 kg change its speed from 2 ms^{-1} to 3.5 ms^{-1} in 25 s. In the direction of the motion of the body. What is the magnitude and direction of the force?
(a) 18 m in the direction of motion
(b) 0.32 m in the direction of motion
(c) 0.64 m in the direction of motion
(d) 0.16 m in the direciton of motion

9. A body of mass 5 kg is acted upon by two perpendicular force 8 N and 6 N, find the magnitude and direction the acceleration
(a) 3 ms^{-2}, $\theta = \cos^{-1}(0.8)$ from 8 m
(b) 2 ms^{-2}, $\theta = \cos^{-1}(0.6)$ from 6 m
(c) 3 ms^{-2}, $\theta = \cos^{-1}(0.9)$ from 6 m
(d) 5 ms^{-2}, $\theta = \cos^{-1}(0.81)$ from 8 m

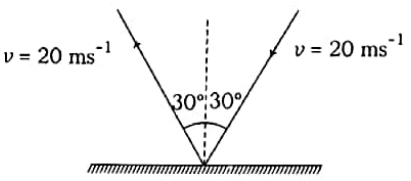
10. A bull of mass m is moving towards a player with velocity v. If player stopped it, then impulse applied by the player is
(a) $-m\mathbf{v}$ (b) $+m\mathbf{v}$ (c) $-2m\mathbf{v}$ (d) $+2m\mathbf{v}$

11. A constant retarding force of 50 N is applied to a body of mass 20 kg moving initially with a speed of 15 ms^{-1}. How long time does the body take to stop.
(a) 6 s (b) 8 s (c) 9 s (d) 10 s

12. A ball of mass 1 kg is dropped from height 9.8 m, strikes with ground and rebounds at height of 4.9 m, if the time of contact between ball and ground is 0.1 s then find impulse and average froce acting on ball.
(a) 23.52 N-s, 235.2 N (b) 235.2 N-s, 23.53 N
(c) 42.5 N-s, 525 N (d) 52.5 N-s, 525 N

13. A ball of mass 0.5 kg moving with a velocity of 2 ms^{-1} strikes a wall normally and bounces back with the same speed. If the time of contact between the ball and the wall is one millisecond, the average force exerted by the wall on the ball is
(a) 2000 N (b) 1000 N (c) 5000 N (d) 125 N

14. A ball is moving with speed 20 ms^{-1} collides with a smooth surface as shown in figure. The magnitude of change in velocity of the ball will be



(Smooth horizontal surface)

(a) $10\sqrt{3}$ ms^{-1} (b) $20\sqrt{3}$ ms^{-1} (c) $\frac{40}{\sqrt{3}}$ ms^{-1} (d) 40 ms^{-1}

15. A body of mass 5 kg is moving with velocity of $\mathbf{v} = (2\hat{i} + 6\hat{j})$ ms^{-1} at $t = 0$ s. After time $t = 2$ s, velocity of body is $(10\hat{i} + 6\hat{j})$, then change in momentum of body is
(a) $40\hat{i}$ kgms^{-1} (b) $20\hat{i}$ kgms^{-1}
(c) $30\hat{i}$ kgms^{-1} (d) $(50\hat{i} + 30\hat{j})$ kgms^{-1}

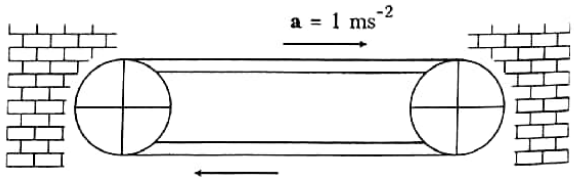
16. If impulse I varies with time t as I (kg ms^{-1}) $= 20t^2 - 40t$. The change in momentum is minimum at
(a) $t = 2$ s (b) $t = 1$ s
(c) $t = \frac{1}{2}$ s (d) $t = \frac{3}{2}$ s

17. An initially stationary device lying on a frictionless floor explodes into two peices and slides across the floor. One piece is moving in positive x-direction then other piece is moving in
(a) positive y-direction
(b) negative y- *direction*
(c) negative x-direction
(d) at angle from x-direction

18. A bullet of mass 10 g is fired from a gun of mass 1 kg with recoil velocity of gun 5 m/s. The muzzle velocity will be
(a) 30 km/min (b) 60 km/min
(c) 30 m/s (d) 500 m/s

19. A bullet of mass 0.1 kg is fired with a speed of 100 ms^{-1}. The mass of gun being 50 kg. Then, the velocity of recoil becomes
(a) 0.05 ms^{-1} (b) 0.5 ms^{-1}
(c) 0.1 ms^{-1} (d) 0.2 ms^{-1}

20. A man of mass 60 kg is standing on a horizontal conveyor belt. When the belt is given an acceleration of 1 ms^{-2}, the man remains stationary with respect to the moving belt. If $g = 10$ ms^{-2}, the net force acting on the man is



(a) zero (b) 120 N
(c) 60 N (d) 600 N

4 Forces in equilibrium

Forces which have zero linear resultant will not cause any change in the motion of the object to which they are applied. Such forces (and the object) are said to be in equilibrium. For understanding the equilibrium of an object under two or more concurrent or coplanar forces let us first discuss the resolution of force.

Resolution of a force

When a force is replaced by an equivalent set of components, it is said to be resolved. One of the most useful ways to resolve a force is to choose only two components (although a force may be resolved in three or more components also) which are at right angles also. The magnitude of these components can be very easily found using trigonometry.

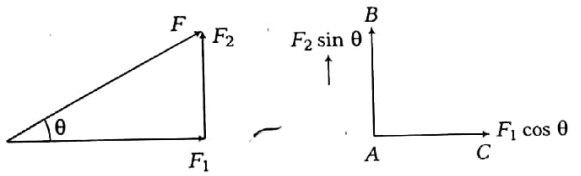


Fig. 5.10 Component of force

In the above figure,

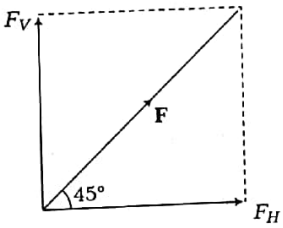
$F_1 = F\cos\theta$ = component of **F** along AC

$F_2 = F\sin\theta$ = component of **F** perpendicular to AC or along AB

The component of a force in a direction perpendicular to itself is zero.

Example 5.18 *Resolve horizontally and vertically a force $F = 8$ N which makes an angle of 45° with the horizontal.*

Sol. Horizontal component of **F** is



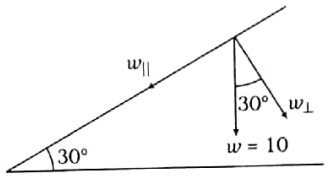
$$F_H = F\cos 45° = (8)\left(\frac{1}{\sqrt{2}}\right) = 4\sqrt{2}\text{ N}$$

and vertical component of **F** is

$$F_V = F\sin 45° = (8)\left(\frac{1}{\sqrt{2}}\right) = 4\sqrt{2}\text{ N}$$

Example 5.19 *Resolve a weight of 10 N in two directions which are parallel and perpendicular to a slope inclined at 30° to the horizontal.*

Sol. Component perpendicular to the plane



$$w_\perp = w\cos 30° = (10)\frac{\sqrt{3}}{2} = 5\sqrt{3}\text{ N}$$

and component parallel to the plane

$$w_{||} = w\sin 30° = (10)\left(\frac{1}{2}\right) = 5\text{ N}$$

Newton's first law for forces in equilibrium

When a particle is at rest or is moving with constant velocity in an inertial frame of reference, the net force acing on it, *i.e.*, the vector sum of all the forces acting on it, must be zero. $\Sigma\mathbf{F} = 0$ (Particle in equilibrium vector form)

We most often use this equation in component form:

$$\Sigma\mathbf{F}_x = 0;\ \Sigma\mathbf{F}_y = 0 \text{ and } \Sigma\mathbf{F}_z = 0$$

Equilibrium under concurrent forces (*i.e.*, those forces which act on same particle at same time) may be seen as

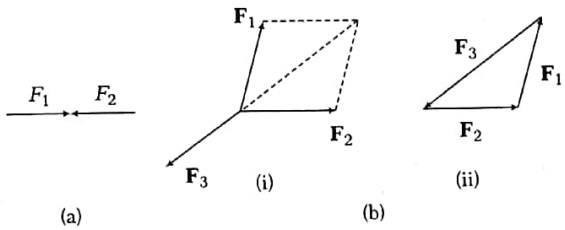


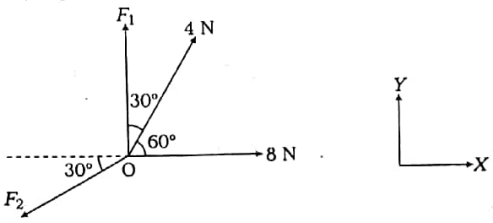
Fig. 5.11

In Fig. (a)

$$\mathbf{F}_1 + \mathbf{F}_2 = 0$$

In Fig. (b)-(i) and (b)-(ii), $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$

Example 5.20 *An object is in equilibrium under four concurrent forces in the directions shown in figure. Find the magnitude of $\mathbf{F}_1$ and $\mathbf{F}_2$.*



Sol. *The object is in equilibrium. Hence,*

(i) $$\Sigma F_x = 0$$

$$\therefore\ 8 + 4\cos 60° - F_2 \cos 30° = 0$$

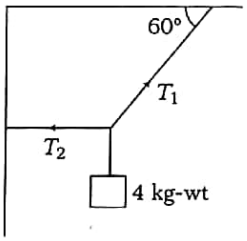
or $$8 + 2 - F_2 \frac{\sqrt{3}}{2} = 0 \text{ or } F_2 = \frac{20}{\sqrt{3}} \text{ N}$$

(ii) $$\Sigma F_y = 0 \quad \therefore \quad F_1 + 4\sin 60° - F_2 \sin 30° = 0$$

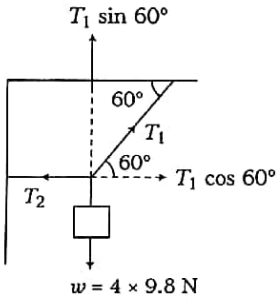
or $$F_1 + \frac{4\sqrt{3}}{2} - \frac{F_2}{2} = 0 \text{ or } F_1 = \frac{F_2}{2} - 2\sqrt{3} = \frac{10}{\sqrt{3}} - 2\sqrt{3}$$

or $$F_1 = \frac{4}{\sqrt{3}} \text{ N}$$

Example 5.21 *Determine the tensions T_1 and T_2 in the strings as shown in figure.*



Sol. Resolving the tension T_1 along horizontal and vertical directions. As the body is in equilibrium,



$T_1 \sin 60° = 4 \times 9.8$ N ...(i)

$T_1 \cos 60° = T_2$ ...(ii)

From Eq. (i)

$$T_1 = \frac{4 \times 9.8}{\sin 60°} = \frac{4 \times 9.8 \times 2}{\sqrt{3}} = 45.26 \text{ N}$$

Putting this value in Eq. (ii)

$$T_2 = T_1 \cos 60° = 45.26 \times 0.5 = 22.63 \text{ N}$$

Lami's theorem

Lami's theorem states that, "if three forces acting on a particle are in equilibrium, then each force is proportional to the sine of the angle between the other two forces".

If an object O is in equilibrium under three concurrent, coplanar forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ as shown in figure. Then,

$$\boxed{\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}}$$

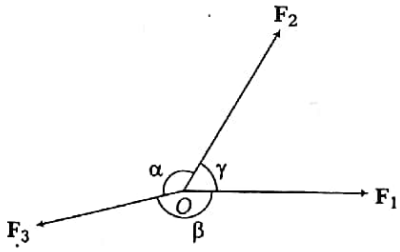


Fig. 5.12

If more than three forces are given in the problem, then solve the problem by using component approach. If three forces are in equilibrium, then the resultant of two forces is equal and opposite to the third.

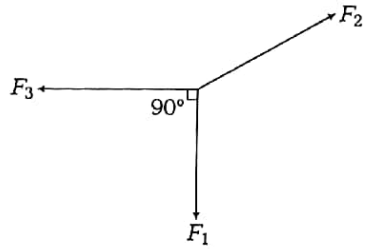


Fig. 5.13

$$F_2 = \sqrt{F_1^2 + F_3^2}$$

Working with Newton's laws

Normally any problem relating to Newton's laws is solved in following four steps

(*i*) First of all we decide the system on which the laws of motion are to be applied. The system may be a single particle, a block or a combination of two or more blocks, two blocks connected by a string, etc. The only restriction is that all parts of the system should have the same acceleration.

(*ii*) Once the system is decided, we make the list of all the forces acting on the system. Any force applied by the system on other bodies is not included in the list of the forces.

(*iii*) Then we make a free body diagram of the system and indicate the magnitude and directions of all the forces listed in step (ii) in this diagram.

(*iv*) In the last step we choose any two mutually perpendicular axes say x and y in the plane of the forces in case of coplanar forces. Choose the x-axis along the direction in which the system is known to have or is likely to have the acceleration. A direction perpendicular to it may be chosen as the y-axis. If the system is in equilibrium, any mutually perpendicular directions may be chosen. Write the components of all the forces along the x-axis and equate their sum to the product of the mass of the system and its acceleration, *i.e.*,

$$\Sigma F_x = ma \quad ...(i)$$

This gives us one equation. Now, we write the components of the forces along the y-axis and equate the sum to zero. This gives us another equation, *i.e.*,

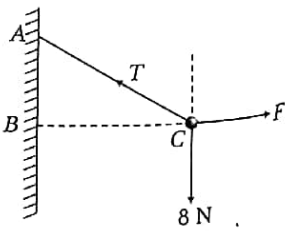
$$\Sigma F_y = 0 \quad ...(ii)$$

(a) If the system is in equilibrium, we will write the two equations as

$$\Sigma F_x = 0 \text{ and } \Sigma F_y = 0$$

(b) If the forces are collinear, the second equation, *i.e.*, $\Sigma F_y = 0$ is not needed.

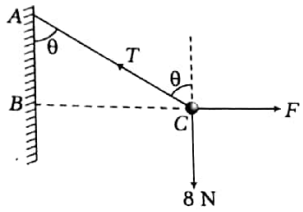
Example 5.22 *One end of a string 0.5 m long is fixed to a point A and the other end is fastened to a small object of weight 8N. The object is pulled aside by a horizontal force F, until it is 0.3 m from the vertical through A. Find the magnitudes of the tension T in the string and the force F.*



Sol. $AC = 0.5$ m, $BC = 0.3$ m,

$\therefore AB = 0.4$ m

and if $\angle BAC = \theta$



Then, $\cos\theta = \dfrac{AB}{AC} = \dfrac{0.4}{0.5} = \dfrac{4}{5}$

and $\sin\theta = \dfrac{BC}{AC} = \dfrac{0.3}{0.5} = \dfrac{3}{5}$

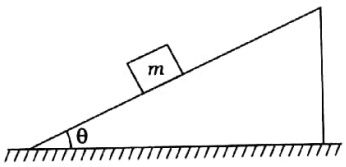
Here, the object is in equilibrium under three concurrent forces. So, we can apply Lami's theorem.

or $$\frac{F}{\sin(180° - \theta)} = \frac{8}{\sin(90° + \theta)} = \frac{T}{\sin 90°}$$

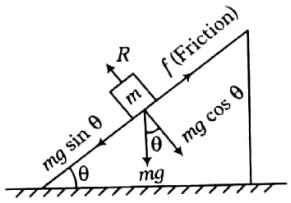
or $$\frac{F}{\sin\theta} = \frac{8}{\cos\theta} = T$$

$$\therefore T = \frac{8}{\cos\theta} = \frac{8}{4/5} = 10 \text{ N and } F = \frac{8\sin\theta}{\cos\theta} = \frac{(8)(3/5)}{(4/5)} = 6 \text{ N}$$

Example 5.23 *A block of mass m is at rest on a rough wedge as shown in figure. What is the force exerted by the wedge on the block?*



Sol.



Since, the block is permanently at rest, it is in equilibrium. Net force on it should be zero. In this case only two forces are acting on the block.

(i) Weight $= mg$ (downwards)

(ii) Contact force (resultant of normal reaction and friction force) applied by the wedge on the block.

For the block to be in equilibrium these two forces should be equal and opposite. Therefore, force exerted by the wedge on the block is mg (upwards).

Note From Newton's third law of motion-force exerted by the block on the wedge is also mg but downwards.

The result can also be obtained in a different manner. The normal force on the block is $N = mg\cos\theta$ and the friction force on the block is $f = mg\sin\theta$ (not $\mu mg\cos\theta$) because it is not the case of limiting friction.

These two forces are mutually perpendicular.

$\therefore$ Net contact force would be $\sqrt{N^2 + f^2}$ or $\sqrt{(mg\cos\theta)^2 + (mg\sin\theta)^2}$ which is equal to mg.

Check point 5.2

1. Four forces act on a point object. The object will be in equilibrium, if

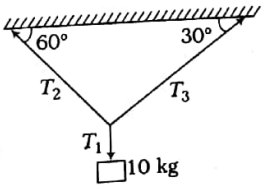
(a) all of them are in the same plane

(b) they are opposite to each other in pairs

(c) the sum of x, y and z components of forces is zero separately

(d) they can be represented by a closed figure of 4 sides by direction and magnitude.

2. A block of mass 10 kg is suspended by three strings as shown in the figure. The tension T_2 is



(a) 100 N (b) $\dfrac{100}{\sqrt{3}}$ N

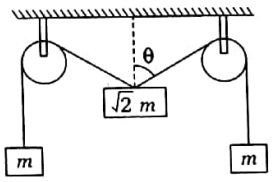
(c) $\sqrt{3} \times 100$ N (d) $50\sqrt{3}$ N

3. An object is resting at the bottom of two strings which are inclined at an angle of 120° with each other. Each string can withstand a tension of 20 N. The maximum weight of the object that can be sustained without breaking the strings is

(a) 10 N (b) 20 N

(c) $20\sqrt{2}$ N (d) 40 N

4. The pulleys and strings shown in the figure are smooth and of negligible mass. For the system to remain in equilibrium, the angle θ should be

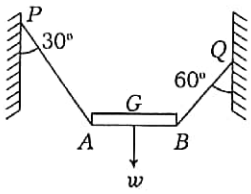


(a) 0° (b) 30°

(c) 45° (d) 60°

5. A non-uniform rod AB of weight w is supported horizontally in a vertical plane by two light strings PA and QB as shown in the figure. G is the centre of gravity of the rod. If PA and QB

make angles 30° and 60° respectively with the vertical, the ratio $\frac{AG}{GB}$ is

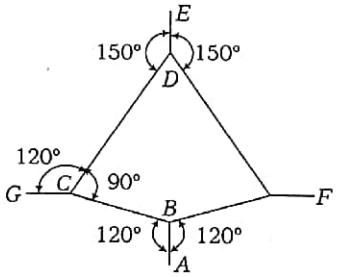


(a) $\frac{1}{2}$ (b) $\sqrt{3}$ (c) $\frac{1}{3}$ (d) $\frac{1}{\sqrt{3}}$

6. A weight w is suspended from the mid-point of a rope, whose ends are at the same level. In order to make the rope perfectly horizontal, the force applied to each of its ends must be

(a) less than w (b) equal to w
(c) equal to $2w$ (d) infinitely large

7. The below figure is the part of a horizontally stretched net. Section AB is stretched with a force of 10 N. The tensions in the sections BC and BF are

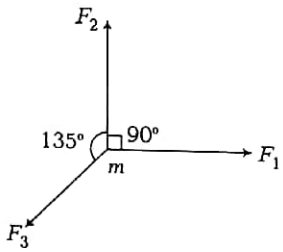


(a) 10 N, 11 N
(b) 10 N, 6 N
(c) 10 N, 10 N
(d) cannot be calculated due to insufficient data

8. Two particles of equal mass are connected to a rope AB of negligible mass, such that one is at end A and the other dividing the length of the rope in the ratio 1 : 2 from A. The rope is rotated about end B in a horizontal plane. Ratio of the tensions in the smaller part to the other is (ignore effect of gravity)

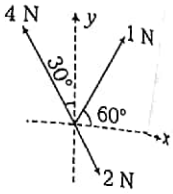
(a) 4 : 3 (b) 1 : 4 (c) 1 : 2 (d) 1 : 3

9. When a force F acts on a body of mass m, the acceleration produced in the body is a. If three equal forces $F_1 = F_2 = F_3 = F$ act on the same body as shown in figure. The acceleration produced is



(a) $(\sqrt{2} - 1)a$ (b) $(\sqrt{2} + 1)a$ (c) $\sqrt{2}\,a$ (d) a

10. Three forces acting on a body are shown in the figure. To have the resultant force only along the y-direction, the magnitude of the minimum additional force needed is

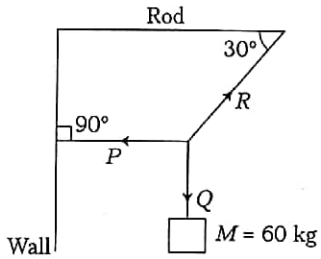


(a) 0.5 N (b) 1.5 N
(c) $\frac{\sqrt{3}}{4}$ N (d) $\sqrt{3}$ N

11. Two equals forces are acting at a point with an angle of 60° between them. If the resultant force is equal to $40\sqrt{3}$ N, the magnitude of each force is

(a) 40 N (b) 20 N
(c) 80 N (d) 30 N

12. A body of mass 60 kg suspended by means of three strings, P, Q and R as shown in the figure is in equilibrium. The tension in the string P is

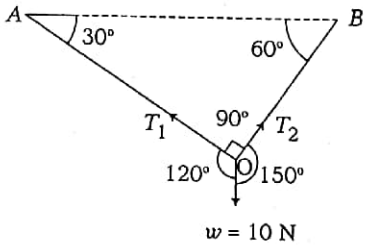


(a) 130.9 kgf (b) 60 kgf
(c) 50 kgf (d) 103.9 kgf

13. A body is under the action of two mutually perpendicular forces of 3 N and 4 N. The resultant force acting on the body is

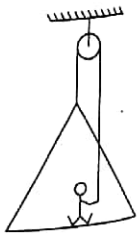
(a) 7 N (b) 1 N
(c) 5 N (d) zero

14. A ball of mass 1 kg hangs in equilibrium from a two strings OA and OB as shown in figure. What are the tensions in strings OA and OB? (Take $g = 10\,\text{ms}^{-2}$)



(a) 5 N, 5 N (b) $5\sqrt{3}$ N, $5\sqrt{3}$ N
(c) 5 N, $5\sqrt{3}$ N (d) $5\sqrt{3}$ N, 5 N

15. A man of mass 50 g stands on a frame of mass 30 g. He pulls on a light rope which passes over a pulley. The other end of the rope is attached to the frame. For the system to be in equilibrium what force man must exert on the rope?



(a) 40 g (b) 80 g
(c) 30 g (d) 50 g

5 Common forces in mechanics

Some of the common forces that we come across in mechanics are described below.

1. Weight (w)

The weight of an object is equal to the gravatitional force with which the earth pulls it downwards.

Weight of an object $w = mg$

where m = mass of the object

and g = acceleration due to gravity

2. Normal reaction (R)

Normal reaction is a contact force between two surfaces in contact, which is always perpendicular to the surfaces in contact. *The following diagrams shown normal reaction between two surfaces.*

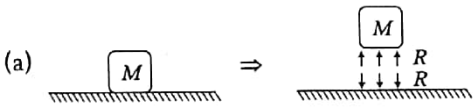


Fig. 5.14

Block pushes ground downward with force R and ground pushes the block back with force R, where R = normal reaction force.

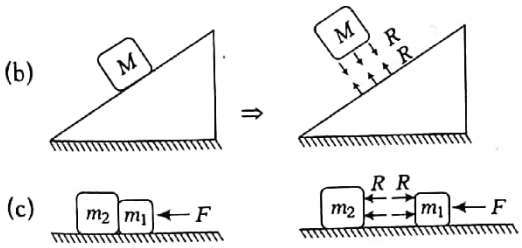


Fig. 5.15 Normal reaction between two surfaces

Here m_1 pushes m_2 towards left by force R and m_2 pushes m_1 towards right by force R.

3. Tension (T)

When a body is connected through a string or rope, a force may act on the body by the string or rope due to the tendency of extension. This force is called tension.

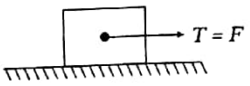


Fig. 5.16 Tension forces

Regarding tension and string, the following three points are important to remember

(i) Force of tension acts on a body in the direction away from the point of contact or tied ends of the string.

(ii) If a string is inextensible, the magnitude of acceleration of any number of masses connected through string is always same.

(iii) If a string is massless, the tension in it is same everywhere. However, if a string has a mass, tension at different points will be different.

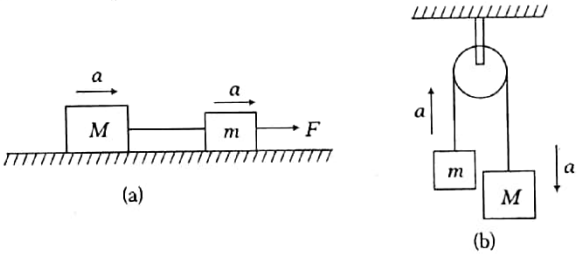


Fig. 5.17 Masses connected through string having same acceleration

(iv) If there is friction between string and pulley, tension is different on two sides of the pulley, but if there is no friction between pulley and string, tension will be same on both sides of the pulley. Last two points can be understood in diagram as follows

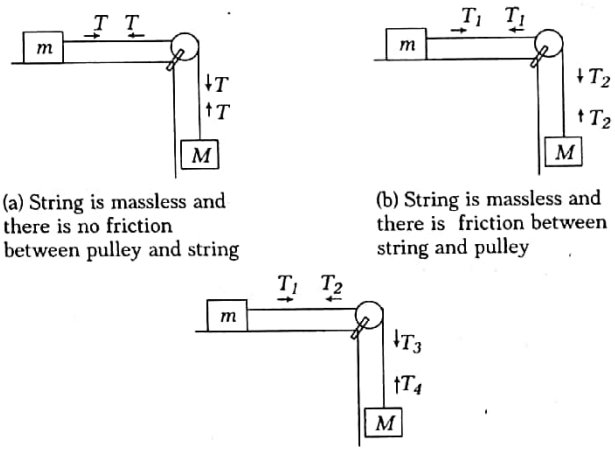


Fig. 5.18

(v) If a force is directly applied on the string, the tension will be equal to the applied force irrespective of the motion of the pulling agent.

4. Spring force

The resistive force developed in a spring, when its length is changed is called spring force.

$\therefore$ Spring force, $F = -kx$,

where, x = Change in length of the string

k = Spring constant

Free body diagram

A free body diagram (FBD) consists of a diagrammatic representation of a single body or a sub-system of bodies isolated from its surroundings showing all the forces acting on it.

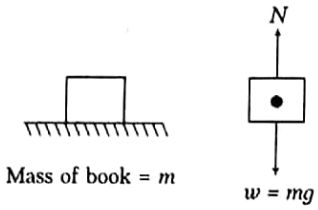
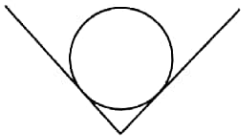


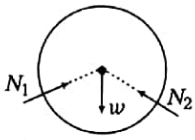
Fig. 5.19 Free body diagram

Consider, for example, a book lying on a horizontal surface. A free body diagram of the book alone would consist of its weight ($w = mg$), acting through the centre of gravity and the reaction (N) exerted on the book by the surface.

Example 5.24 *A cylinder of weight w is resting on a V-groove as shown in figure. Draw its free body diagram.*

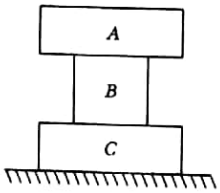


Sol. The free body diagram of the cylinder is as shown in figure.

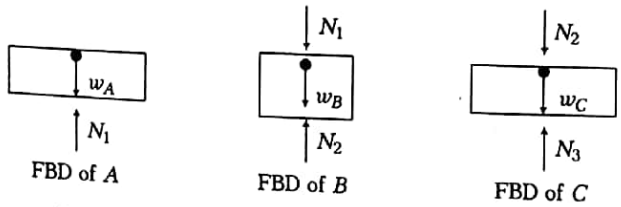


Here, w = weight of cylinder and N_1 and N_2 are the normal reactions between the cylinder and the two inclined walls.

Example 5.25 *Three blocks A, B and C are placed one over the other as shown in figure. Draw free body diagrams of all the three blocks.*

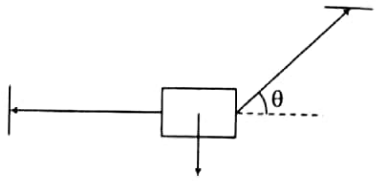


Sol. Free body diagrams of A, B and C are shown below.

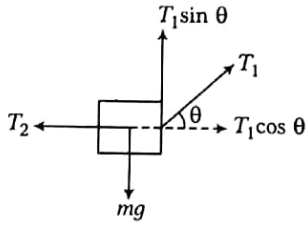


Here, N_1 = normal reaction between A and B.
N_2 = normal reaction between B and C.
N_3 = normal reaction between C and ground.

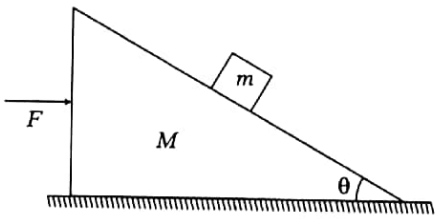
Example 5.26 *A block of mass m is attached with two string as shown in figure. Draw the free body diagram of the block.*



Sol. The free body diagram of the block is as shown in figure



Example 5.27 *All surfaces are smooth in following figure. Find F such that block remains stationary with respect to wedge.*

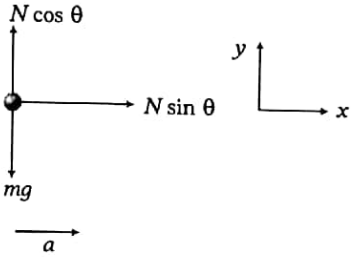


Sol. Acceleration of (block + wedge)

$$a = \frac{F}{(M+m)}$$

Let us solve the problem by both the methods.

(i) **From Inertial Frame of Reference (Ground)** FBD of block w.r.t. ground (apply real forces) with respect to ground, block is moving with an acceleration a



$\therefore \quad \Sigma F_y = 0$

$\Rightarrow \quad N\cos\theta = mg$ ...(i)

and $\quad \Sigma F_x = ma$

$\Rightarrow \quad N\sin\theta = ma$ ...(ii)

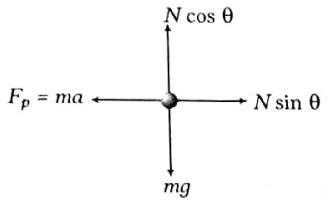
From Eqs. (i) and (ii)

$$a = g\tan\theta$$

$\therefore \quad F = (M+m)a$

$= (M+m)\,g\tan\theta$

(ii) **From Non-inertial Frame of Reference (Wedge)** FBD of block w.r.t. wedge (real forces + pseudo force)



w.r.t. wedge, block is stationary

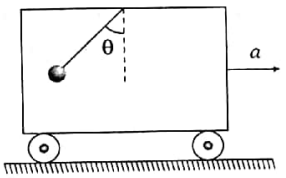
$\therefore \quad \Sigma F_y = 0 \Rightarrow N\cos\theta = mg$ ...(iii)

$\Sigma F_x = 0 \Rightarrow N\sin\theta = ma$ ...(iv)

In this way the block's net acceleration becomes zero because all the forces acting on them balance each other for an observer on the wedge and not for an observer on the ground.

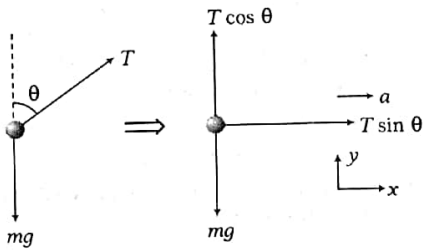
From Eqs. (iii) and (iv), we will get the same result *i.e.*, $F = (M+m)g\tan\theta$.

Example 5.28 *A bob of mass m is suspended from the ceiling of a train moving with an acceleration a as shown in figure. Find the angle θ in equilibrium position.*



Sol. This problem can also be solved by both the methods.

(i) **Inertial Frame of Reference (Ground)** FBD of bob w.r.t. ground (only real forces) bob is also moving with an acceleration a



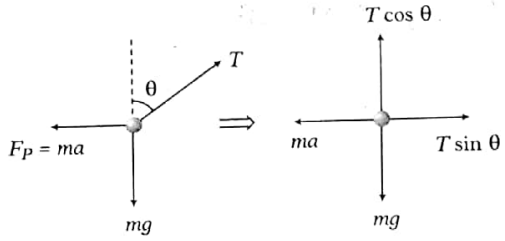
$\therefore \quad \Sigma F_x = 0 \Rightarrow T\sin\theta = ma$ ...(i)

and $\quad \Sigma F_y = 0 \Rightarrow T\cos\theta = mg$ ...(ii)

From Eqs. (i) and (ii), we get

$$\tan\theta = \frac{a}{g} \quad \text{or} \quad \theta = \tan^{-1}\left(\frac{a}{g}\right)$$

(ii) **Non-inertial Frame of Reference (Train)** FBD of bob w.r.t. train (real forces + pseudo force)



with respect to train, bob is in equilibrium

$\therefore \quad \Sigma F_x = 0 \Rightarrow T\sin\theta = ma$ ...(iii)

$\therefore \quad \Sigma F_y = 0 \Rightarrow T\cos\theta = mg$ ...(iv)

From Eqs. (iii) and (iv), we get the same result, i.e.

$$\theta = \tan^{-1}\left(\frac{a}{g}\right)$$

6 Apparent weight of a man in a lift

Let us consider a man of mass m is standing on a weighing machine placed in an elevator/lift. The actual weight mg of the man acts on the weighing machine and offers a reaction R given by the reading of the weighing machine.

This reaction R exerted by the surface of contact on the man is apparent weight of the person.

Now, we consider how R is related to mg in the different conditions.

(*i*) *When the lift moves upwards with acceleration a* as shown in figure, the net upward force on the man is

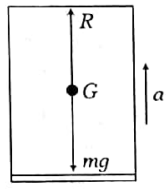


Fig. 5.20

$$R - mg = ma \Rightarrow R = ma + mg$$

Apparent weight, $R = m(g+a)$

So, **when a lift accelerates upwards, the apparent weight of the man inside it increases.**

(*ii*) *When the lift move downwards with acceleration a* as shown in figure. Then net downward force on the man is

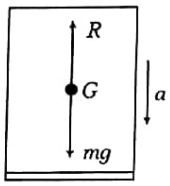


Fig. 5.21

$$mg - R = ma$$

Apparent weight, $R = m(g - a)$

So, **when a lift accelerates downwards, the apparent weight of the man inside it decreases.**

(iii) *When the lift is at rest or moving with uniform velocity v downward or upward* as shown in figure.

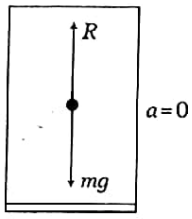


Fig. 5.22

Then, acceleration $a = 0$. So, net force on the man is

$$R - mg = m \times 0 \Rightarrow R = mg$$

or Apparent weight = actual weight

So, **when the lift is at rest, the apparent weight of the man is his actual weight.**

(iv) *When the lift falls freely under gravity* if the supporting cable of the lift breaks. Then, $a = g$.

The net downward force on the man is

$$R = m(g - g) \Rightarrow R = 0$$

Thus, **the apparent weight of the man becomes zero.** This is because both the lift and man are moving downwards with the same acceleration g and so there is no force of action and reaction existing between the man and lift. Hence, a person develops a feeling of weightlessness when the lift falls freely under gravity.

Example 5.29 *A spring balance is attached to the ceiling of an elevator. A boy hangs his bag on the spring and the spring reads 49 N, when the elevator is stationary. If the elevator moves downward with an acceleration of 5 ms^{-2}, what will be reading of the spring balance?*

Sol. When the elevator is stationary,

Then, $w = mg$

$\Rightarrow$ $49 = m \times 9.8$

$\Rightarrow$ $m = \frac{49}{9.8} = 5$

When the elevator is moving downward with an acceleration

$$R = m(9.8 - a) = 5\,(9.8 - 5) = 24 \text{ N}$$

Example 5.30 *If in an stationary elevator a man is standing with a bucket full of water. The bucket has a hole at its bottom. The rate of flow of water through this hole is R_0. If the elevator starts to move up and then down with same acceleration, then the rate of flow of water are R_u and R_d. Find the relation between R_0, R_u and R_d.*

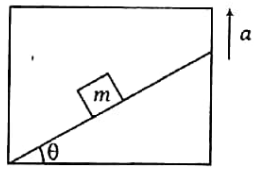
Sol. Rate of flow will be more when elevator will move in upward direction with some acceleration because the net downward pull will be more and *vice-versa.*

$$F_{\text{upward}}\ (R_u) = m\,(g + a)$$

$$F_{\text{downward}}\ (R_d) = m\,(g - a) \Rightarrow F_{\text{at rest}}\ (R_0) = mg$$

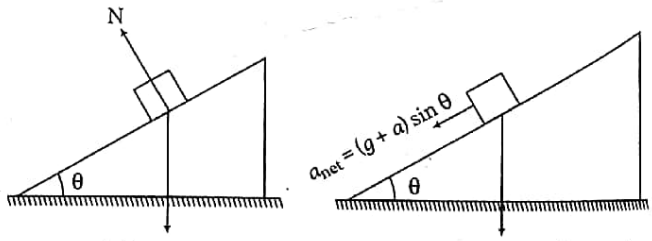
Thus, relation between R_u, R_0 and R_d is $R_u > R_0 > R_d$.

Example 5.31 *In the adjoining figure, a wedge is fixed to an elevator moving upwards with an acceleration a. A block of mass m is placed over the wedge. Find the acceleration of the block with respect to wedge. Neglect friction.*



Sol. Since, acceleration of block w.r.t. wedge (an accelerating or non-inertial frame of reference) is to be find out.

FBD of block w.r.t. wedge is shown in Fig.



$mg + F_P = mg + ma$ Pseudo-acceleration ($-a$) for non-inertial frame

The acceleration would had been $g \sin\theta$ (down the plane) if the lift were stationary or when only weight (*i.e.*, mg) acts downwards.

Here, downward force is $m\,(g + a)$

$\therefore$ Acceleration of the block (of course w.r.t. wedge) will be $(g + a) \sin\theta$ down the plane.

7 Applications of Newton's laws of motion

Consider two bodies of mass m_1 and m_2 placed in direct contact with each other on a smooth platform.

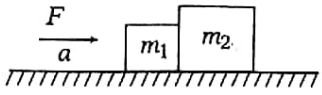


Fig. 5.23 Motion of two connected body

Suppose a horizontal force is applied and both the bodies moves with acceleration a, then

$$F = (m_1 + m_2)\, a$$

Now, we can calculate, normal reaction between the bodies, using free body diagram (FBD).

FBD of Ist body

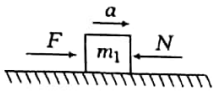


Fig. 5.24

$\therefore$ $$F - N = m_1 a \quad \text{(From Newton's IInd law)}$$

FBD of IInd body

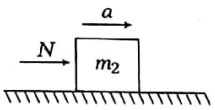
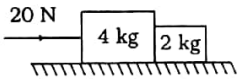


Fig. 5.25

$\therefore$ $$N = m_2 a$$

In the same way, we can calculate, the acceleration and normal reaction for three bodies in contact.

Example 5.32 *Two blocks of mass 4 kg and 2 kg are placed side by side on a smooth horizontal surface as shown in the figure. A horizontal force of 20 N is applied on 4 kg block.*



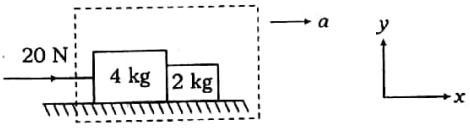
Find

(a) the acceleration of each block,

(b) the normal reaction between two blocks.

Sol. (a) Since, both the blocks will move with same acceleration (say a) in horizontal direction.

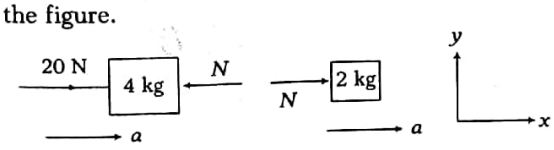
Let us take both the blocks as a system. Net external force on the system is 20 N in horizontal direction.



Using $$\Sigma F_x = ma_x$$

$$20 = (4 + 2)a = 6a \quad \text{or} \quad a = \frac{10}{3}\ \text{ms}^{-2}$$

(b) The free body diagram of both the blocks are shown in the figure.



Using $$\Sigma F_x = ma_x$$

For 4 kg block $$20 - N = 4a = 4 \times \frac{10}{3}$$

$$\Rightarrow \quad N = 20 - \frac{40}{3} = \frac{20}{3}\ \text{N}$$

For 2 kg block

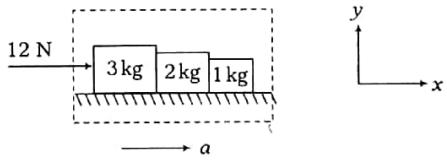
$$N = 2a = 2 \times \frac{10}{3} = \frac{20}{3}\ \text{N}$$

Here, N is the normal reaction between the two blocks.

Note In free body diagram of the blocks we have not shown the forces acting on the blocks in vertical direction, because normal reaction between the blocks and acceleration of the system can be obtained without using $\Sigma F_y = 0$.

Example 5.33 *Three blocks of mass 3 kg, 2 kg and 1 kg are placed side by side on a smooth surface as shown in figure A. Horizontal force of 12 N is applied on 3 kg block. Find the net force on 2 kg block.*

Sol. All the blocks will move with same acceleration (say a) in horizontal direction. Let us take all the blocks as a system. Net external force on the system is 12 N in horizontal direction.



Using $\Sigma F_x = ma_x$, we get

$$12 = (3 + 2 + 1)a = 6a \quad \text{or} \quad a = \frac{12}{6} = 2\ \text{ms}^{-2}$$

Now, let F be the net force on 2 kg block in x-direction, then using $\Sigma F_x = ma_x$ for 2 kg block, we get

$$F = (2)(2) = 4\ \text{N}$$

Note Here, net force F on 2 kg block is the resultant of N_1 and N_2 ($N_1 > N_2$)
where N_1 = normal reaction between 3 kg and 2 kg block
and N_2 = normal reaction between 2 kg and 1 kg block
Thus, $F = N_1 - N_2$

Motion of bodies connected through strings

Horizontal motion

When two bodies are connected through an inextensible weightless string and a force is applied to impart an acceleration 'a' in both the bodies, as given in the figure below, then equation of motion from the FBD can be given by :

For A

$$T = m_1 a \qquad \text{...(i)}$$

For B

$$F - T = m_2 a \qquad \text{...(ii)}$$

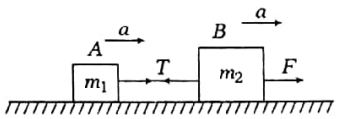


Fig. 5.26 Motion of two connected body through string

From Eqs. (i) and (ii)

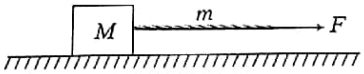
$$F = m_1 a + m_2 a \qquad \text{...(iii)}$$

From Eqs. (i) and (iii)

$$T = \frac{m_1 F}{(m_1 + m_2)}$$

In the same way we can calculate the acceleration and tension in three bodies.

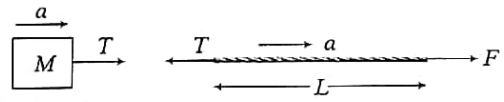
Example 5.34 *A block of mass M is pulled along a horizontal frictionless surface by a rope of mass m as shown in figure. A horizontal force F is applied to one end of the rope. Find (i) The acceleration of the rope and block (ii) The force that the rope exerts on the block. (iii) Tension in the rope at its mid-point.*



Sol. (i) Acceleration $a = \dfrac{F}{(m+M)}$

(ii) Force exerted by rope on the block is

$$T = Ma = \frac{M \cdot F}{(m+M)}$$

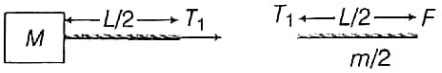


FBD of block FBD of rope

(iii) $T_1 = \left(\dfrac{m}{2} + M\right)a = \left(\dfrac{m+2M}{2}\right)\left(\dfrac{F}{m+M}\right)$

Tension in rope at mid-point $T_1 = \dfrac{(m+2M)F}{2(m+M)}$

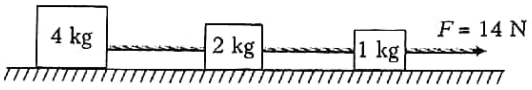
$$T = Ma = \frac{M \cdot F}{(m+M)}$$



Vertical motion

In the same manner, as discussed above we can calculate acceleration, tension in the string and common force in two or more blocks in vertical direction also.

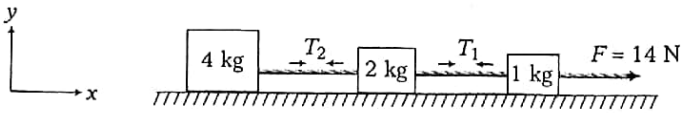
Example 5.35 *In the arrangement shown in the figure, the strings are light and inextensible. The surface over which blocks are placed is smooth. Find*



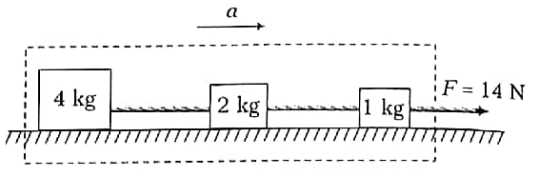
(a) the acceleration of each block

(b) the tension in each string.

Sol. (a) Let a be the acceleration of each block and T_1 and T_2 be the tensions, in the two strings as shown in the figure.



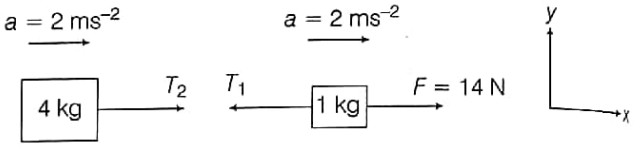
Taking the three blocks and the two strings as the system.



Using $\Sigma F_x = ma_x$

or $14 = (4+2+1)a$ or $a = \dfrac{14}{7} = 2\ \text{ms}^{-2}$

(b) Free body diagram (showing the forces in x-direction only) of 4 kg block and 1 kg block are shown in figure.



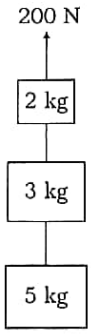
Using $\Sigma F_x = ma_x$

For 1 kg block, $F - T_1 = (1)(a)$ or $14 - T_1 = (1)(2) = (2)$

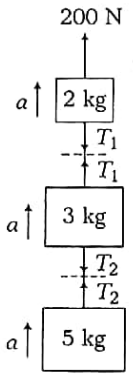
$\therefore$ $T_1 = 14 - 2 = 12$ N

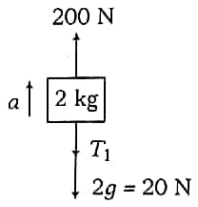
For 4 kg block, $T_2 = (4)(a)$ $\therefore$ $T_2 = (4)(2) = 8$ N

Example 5.36 *The blocks of masses 2 kg, 3 kg and 5 kg are connected by light, inextensible strings as shown. The system of blocks is raised vertically upwards by applying a force $F_0 = 200$ N. Find the common acceleration and tension in the strings.*

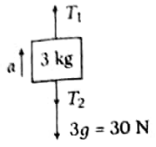


Sol.

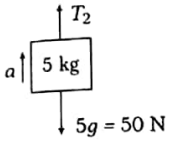




$\uparrow : 200 - T_1 - 20 = 2a$ ...(i)



$$\uparrow : T_1 - T_2 - 30 = 3a \quad \text{...(ii)}$$



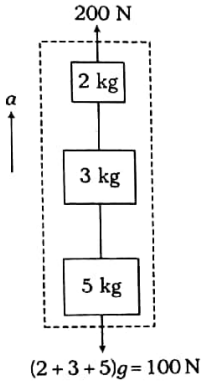
$$\uparrow : T_2 - 50 = 5a \quad \text{...(iii)}$$

Adding Eqs. (i), (ii) and (iii), we get

$$100 = 10a \quad \Rightarrow \quad a = 10\,\text{ms}^{-2}$$

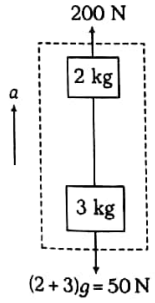
$$T_1 = 160\,\text{N} \quad \Rightarrow \quad T_2 = 100\,\text{N}$$

Alternate Method Taking three blocks together as a system



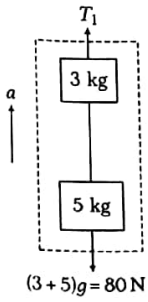
$$\uparrow : 200 - 100 = (2+3+5)a \quad \Rightarrow \quad a = 10\,\text{ms}^{-2}$$

Two blocks at a time



$$200 - 50 - T_2 = (2+3)a$$

$$150 - T_2 = 5 \times 10 \quad \Rightarrow \quad T_2 = 100\,\text{N}$$

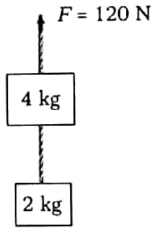


$$T_1 - 80 = (3+5)a$$

$$\Rightarrow \quad T_1 - 80 = 8 \times 10 \quad \Rightarrow \quad T_1 = 160\,\text{N}$$

Note There are various ways to take blocks as a system.

Example 5.37 *Two blocks of mass 4 kg and 2 kg are attached by an inextensible light string as shown in figure. Both the blocks are pulled vertically upwards by a force $F = 120$ N.*

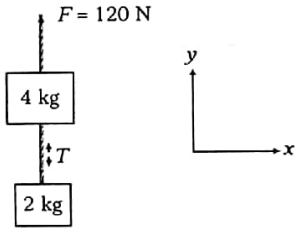


Find

(a) the acceleration of the blocks,

(b) tension in the string. (Take $g = 10$ ms^{-2})

Sol. (a) Let a be the acceleration of the blocks and T the tension in the string as shown in figure.



Taking the two blocks and the string as the system.

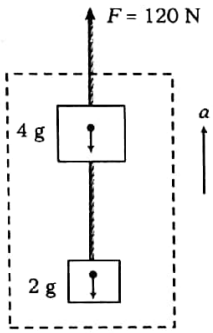
Using $\Sigma F_y = ma_y$, we get

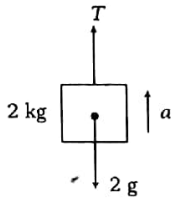
$$F - 4g - 2g = (4+2)a$$

or $\quad 120 - 40 - 20 = 6a \quad$ or $\quad 60 = 6a$

$\therefore \quad a = 10\,\text{ms}^{-2}$

(b) Free body diagram of 2 kg block is shown below.





Using $\quad \Sigma F_y = ma_y$

we get, $\quad T - 2g = 2a$

or $\quad T - 20 = (2)(10)$

$\therefore \quad T = 40$ N

Bodies attached through pulley

Case I Consider two bodies of mass m_1 and m_2 $(m_2 > m_1)$ connected to a massless and inextensible string which passes through a smooth pulleys. If they are allowed to move freely, they move with common acceleration a. The tension in the string due to mass m_1 and m_2 is T.

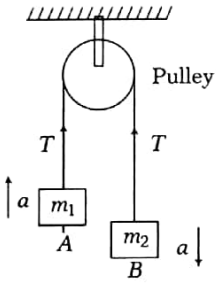


Fig. 5.27 Motion of connected bodies

From FBD,

For A,

$$T - m_1 g = m_1 a \quad \text{...(i)}$$

For B,

$$m_2 g - T = m_2 a \quad \text{...(ii)}$$

On solving Eqs. (i) and (ii), we get

$$\Rightarrow \quad \boldsymbol{a = \frac{(m_2 - m_1)}{(m_2 + m_1)} g}$$

$$\Rightarrow \quad \boldsymbol{T = \left(\frac{2m_1 m_2}{m_1 + m_2}\right) g}$$

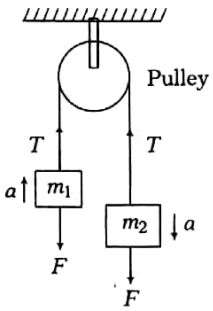
Reaction at suspension of pulley $\boldsymbol{R = 2T}$.

Example 5.38 *Two spheres of masses 2.6 kg and 4.1 kg are tied together by a light string looped over a frictionless pulley.*

(i) What will the acceleration of each mass be?

(ii) Find the value of tension in the spring.

Sol. Here, $m_1 = 2.6$ kg, $m_2 = 4.1$ kg



FBD for m_1

$$T - m_1 g = m_1 a$$

$$T - 2.6g = 2.6a \quad \text{...(i)}$$

FBD for m_2

$$m_2 g - T = m_2 a \quad \text{(downward)}$$

$$4.1g - T = 4.1a \quad \text{...(ii)}$$

Adding Eqs. (i) and (ii), we get

$$4.1g - 2.6g = (2.6 + 4.1)a$$

$$a = \frac{1.5g}{6.7} = \frac{1.5 \times 10}{6.7} = 2.2 \text{ ms}^{-2}$$

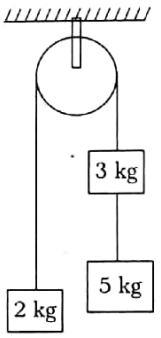
Putting the value of a in Eq. (i), we get

$$T - 2.6g = 2.6 \times 2.2$$

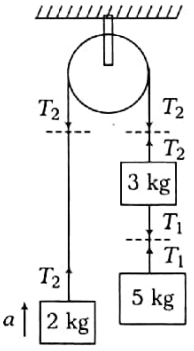
$$T = 2.6 \times 2.2 + 2.6 \times 10$$

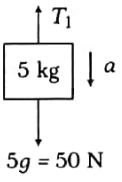
$$= 31.72 \text{ N}$$

Example 5.39 *The pulley is light and smooth; the strings are inextensible and light. The system is released from rest. Find the acceleration of each block, tensions in the strings and reaction on pulley.*

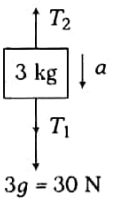


Sol.

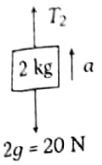




$$\downarrow : 50 - T_1 = 5a \quad \text{...(i)}$$



$$\downarrow : 30 + T_1 - T_2 = 3a \quad \text{...(ii)}$$



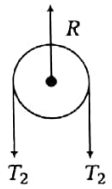
$\uparrow : T_2 - 20 = 2a$ ...(iii)

Solving Eqs. (i), (ii) and (iii), we get

$$a = 6\,\text{ms}^{-2}$$
$$T_1 = 20\,\text{N}$$
$$T_2 = 32\,\text{N}$$

Forces on pulley, $R - 2T_2 = 0$ (pulley in massless)

$$R = 2T_2 = 64\,\text{N}$$



where R is the reaction in the pulley.

Case II When two bodies are attached through a pulley as given in the figure below.

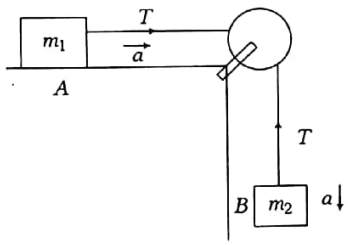


Fig. 5.54

Here, $m_2 > m_1$ then from FBD,

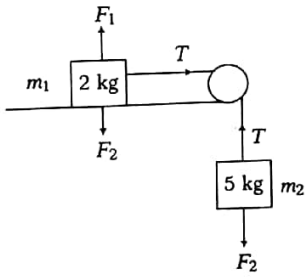
Then, for A $T = m_1 a$ ...(i)

For B, $m_2 a = m_2 g - T$ ...(ii)

Solving Eqs. (i) and (ii), we get

$$a = \frac{m_2}{(m_1 + m_2)} g \Rightarrow T = \frac{m_1 m_2}{(m_1 + m_2)} g$$

Example 5.40 *A 2 kg mass placed on a level table and is attached by a string passing over the edge of a table as illustrated in the diagram.*



(i) Calculate the magnitude of acceleration of the system.

(ii) Calculate the tension in the string.

Sol. From FBD in m_1

$$T - m_1 a = 0$$

$$\Rightarrow \quad T - 2a = 0 \quad \text{...(i)}$$

FBD in m_2

$$m_2 g - T = m_2 a$$
$$5g - T = 5a$$

Putting $T = 2a$ from Eq. (i)

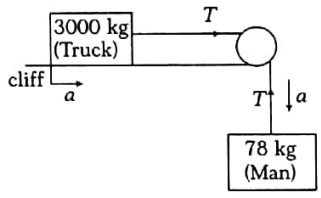
$$5g - 2a = 5a$$
$$7a = 5g$$
$$a = \frac{5}{7} \times 10 = \frac{50}{7}\,\text{ms}^{-2}$$

Again Tension in the string

$$T = 2a = 2 \times \frac{50}{7} = \frac{100}{7}\,\text{N}$$

Example 5.41 *A 3000 kg truck is on a cliff and is tied with a rope to a 78 kg person hanging over the cliff. If the cliff is very icy (frictionless), what is the magnitude of acceleration of the system?*

Sol. As the cliff is very frictionless, so it can be considered as frictionless pulles.



From FBD of truck

$$T - m_1 a = 0$$
$$T = m_1 a = 3000a \quad \text{...(i)}$$

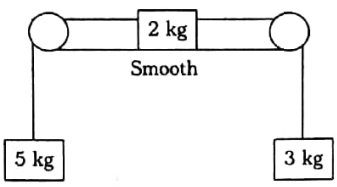
From FBD of man

$$m_2 g - T = m_2 a$$
$$78g - T = 78a$$

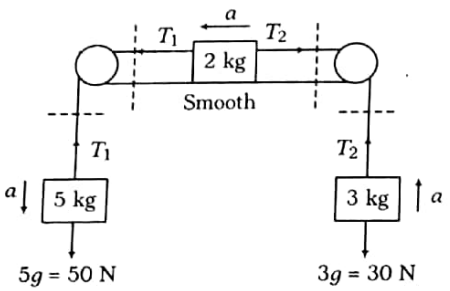
Putting the value of T from Eq. (i)

$$78g - 3000a = 78a$$
$$3078a = 78g$$
$$a = \frac{78 \times 10}{3078} = 0.25\,\text{ms}^{-2}$$

Example 5.42 *The strings are inextensible and light; the pulleys are smooth and light. Find the acceleration of each block and tensions in the strings.*



Sol.



5 kg : $\downarrow : 50 - T_1 = 5a$ ...(i)

2 kg : $\leftarrow : T_1 - T_2 = 2a$ ...(ii)

3 kg : $\uparrow : T_2 - 30 = 3a$ ...(iii)

Adding Eqs. (i), (ii) and (iii), we get

$$20 = 10a \Rightarrow a = 2\,\text{ms}^{-2}$$

$$T_1 = 40\,\text{N} \Rightarrow T_2 = 36\,\text{N}$$

Motion on inclined plane If a block is moving on an inclined plane, we choose two directions, along the plane and perpendicular to the plane. Resolve the forces along these directions and solve.

Case III When pulley is attached to the edge of an inclined plane

To understand this case, first of all we need to understand the motion of an object placed on a frictionless inclined plane.

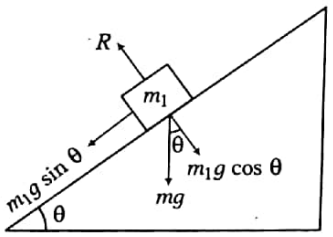


Fig. 5.28

Consider an object of mass m_1 is placed on a smooth inclined plane with angle of inclination θ.

From the FBD of object, different forces acting on it, are

(i) Normal rotation R, acting perpendicular to the plane.

(ii) Component of weight $m_1g\cos\theta$, acting perpendicular to plane.

(iii) Component of weight $m_1g\sin\theta$ downward along the inclined plane.

Here R and $m_1g\cos\theta$ will cancel each other (as here is no motion in vertically upward and vertical downward direction), so here forces can be ignored.

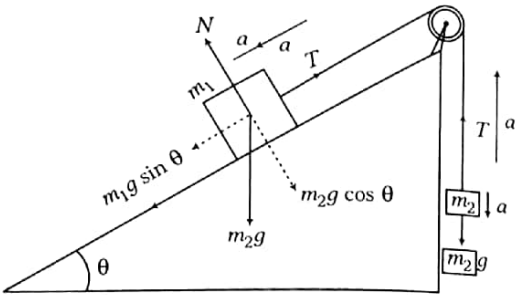


Fig. 5.29

Now a pulley is connected at the edge of inclined plane and a block of mass m_2 $(m_1 > m_2)$ is connected through a string passing over the pulley to mass m_1 as shown in Fig. 5.29.

Now equation of motion of m_2

$$T - m_2g = m_2a \quad ...(i)$$

Equation of motion in m_1

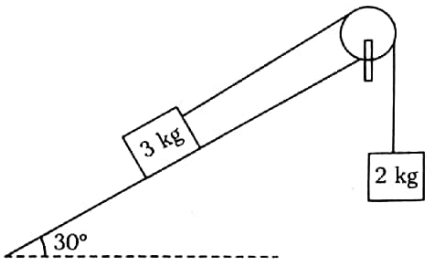
$$m_1g\sin\theta - T = m_1a \quad ...(ii)$$

From Eqs. (i) and (ii)

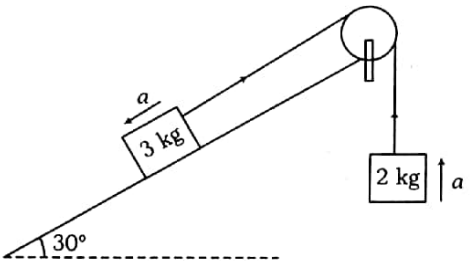
$$a = \frac{(m_1g\sin\theta - m_2g)}{(m_1 + m_2)}$$

$$\boxed{T = \frac{m_1m_2}{m_1 + m_2}(1 + \sin\theta)\,g}$$

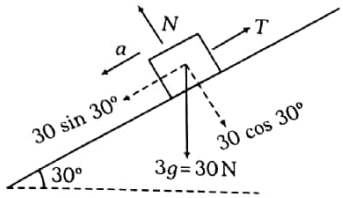
Example 5.43 *Consider the situation shown in the figure. The surface is smooth and the string and the pulley are light. Find the acceleration of each block and tension in the string.*



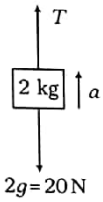
Sol.



Let a 3 kg block be moving downward.



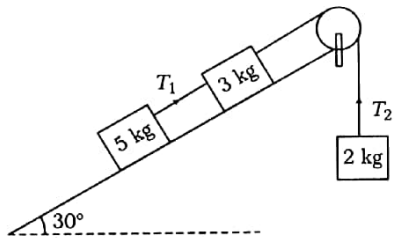
Along the plane:



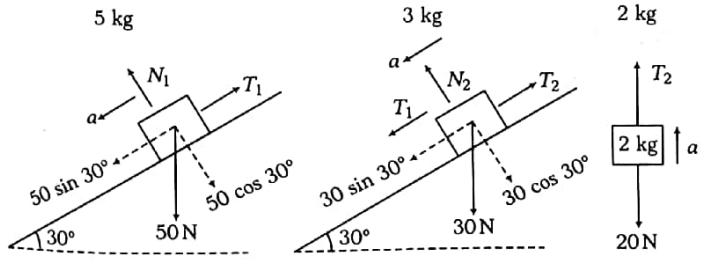
$$30 \sin 30° - T = 3a \quad ...(i)$$
$$T - 20 = 2a \quad ...(ii)$$
$$a = -1 \text{ ms}^{-2}$$
$$T = 18 \text{ N}$$

since, the acceleration is negative, *i.e.*, the block of mass 3 kg is moving upward.

Example 5.44 *In the arrangement shown, inclined plane is smooth, strings and pulleys are massless. Find* $\frac{T_1}{T_2}$.



Sol.



5 kg : along the plane : $50 \sin 30° - T_1 = 5a$ ...(i)

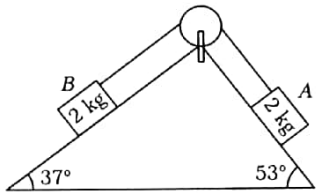
3 kg : along the plane : $30 \sin 30° + T_1 - T_2 = 3a$ ...(ii)

2 kg : $\uparrow : T_2 - 2g = 2a$ ...(iii)

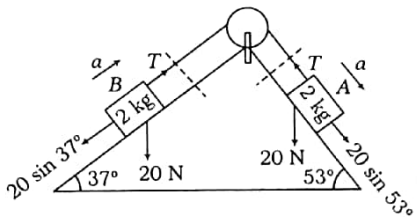
Adding Eqs. (i), (ii) and (iii), we get

$$20 = 10a \Rightarrow a = 2 \text{ ms}^{-2}$$
$$T_1 = 15 \text{ N} \Rightarrow T_2 = 24 \text{ N}$$
$$\frac{T_1}{T_2} = \frac{15}{24} = \frac{5}{8}$$

Example 5.45 *In the arrangement shown, all the surface are smooth, strings and pulleys are light. Find the tension in the string.*



Sol.



Block A : along the plane

$$20 \sin 53° - T = 2a \quad ...(i)$$

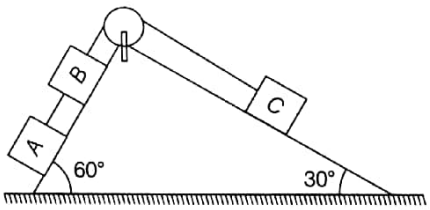
Block B : along the plane

$$T - 20 \sin 37° = 2a \quad ...(ii)$$

Adding Eqs. (i) and (ii), we get

$$20 (\sin 53° - \sin 37°) = 4a$$
$$20\left(\frac{4}{5} - \frac{3}{5}\right) = 4 = 4a \Rightarrow a = 1 \text{ ms}^{-2}$$
$$T = 20 \sin 37° + 2a = 20 \times \frac{3}{5} + 2 \times 1 = 14 \text{ N}$$

Example 5.46 *In the adjacent figure, mass of A, B and C are 1 kg, 3 kg and 2 kg, respectively. Find*



(a) the acceleration of the system, and

(b) tensions in the string.

Neglect friction. ($g = 10 \text{ ms}^{-2}$)

Sol. (a) In this case net pulling force

$$= m_A g \sin 60° + m_B g \sin 60° - m_C g \sin 30°$$
$$= (1)(10)\frac{\sqrt{3}}{2} + (3)(10)\left(\frac{\sqrt{3}}{2}\right) - (2)(10)\left(\frac{1}{2}\right)$$
$$= 24.64 \text{ N}$$

Total mass being pulled $= 1 + 3 + 2 = 6$ kg

$\therefore$ Acceleration of the system $a = \frac{24.64}{6} = 4.10 \text{ ms}^{-2}$

(b) For the tension in the string between A and B.

FBD of A

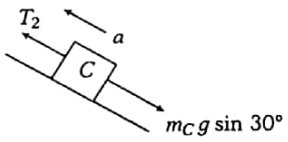
$$m_A g \sin 60° - T_1 = (m_A)(a)$$
$$\therefore \quad T_1 = m_A g \sin 60° - m_A a$$

$$= m_A (g \sin 60° - a)$$

$$\therefore \quad T_1 = (1)\left(10 \times \frac{\sqrt{3}}{2} - 4.1\right) = 4.56 \text{ N}$$

For the tension in the string between B and C.

FBD of C



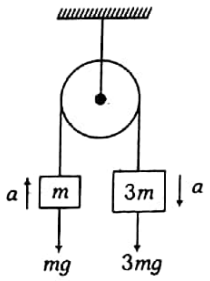
$$T_2 - m_C\, g \sin 30° = m_C\, a$$

$$\therefore \quad T_2 = m_C (a + g \sin 30°)$$

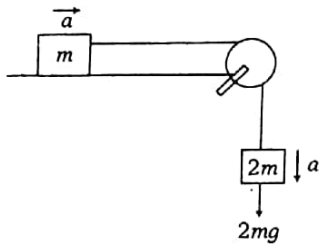
$$\therefore \quad T_2 = 2\left[4.10 + 10\left(\frac{1}{2}\right)\right] = 18.2 \text{ N}$$

Example 5.47 *Calculate the net acceleration produced in the arrangements shown below.*

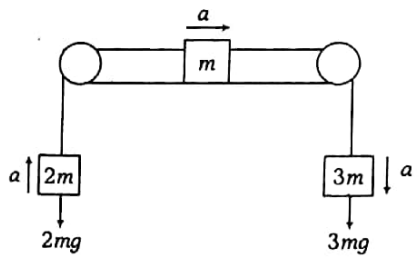
(a)



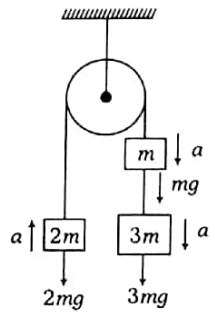
(b)



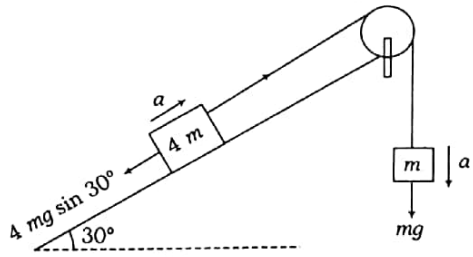
(c)



(d)



(e)



Sol.

(a) Net pulling force $= 3mg - mg = 2mg$

Total mass $= 3m + m = 4m$

$$a = \frac{2mg}{4m} = \frac{g}{2}$$

(b) Pulling force $= 2mg$

Total mass $= 3m$

$$a = \frac{2mg}{3m} = \frac{2g}{3}$$

(c) Net pulling force $= 3mg - 2mg = mg$

Total mass $= 3m + m + 2m = 6m$

$$a = \frac{mg}{6m} = \frac{g}{6}$$

(d) Net pulling force $= 3mg + mg - 2mg = 2mg$

Total mass $= 3m + m + 2m = 6m$

$$a = \frac{2mg}{6m} = \frac{g}{3}$$

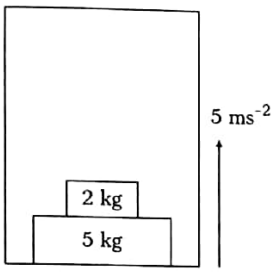
(e) Net pulling force $= 4mg \sin 30° - mg = mg$

Total mass $= 4m + m = 5m$

$$\Rightarrow \quad a = \frac{mg}{5m} = \frac{g}{5}$$

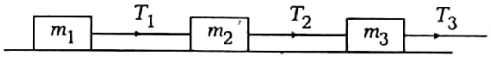
Check point 5.3

1. Find the force exerted by 5 kg block on floor of lift, as shown in figure. (Take, $g = 10\,ms^{-2}$)



(a) 100 N (b) 115 N
(c) 105 N (d) 135 N

2. Three blocks of masses m_1, m_2 and m_3 are connected by massless strings as shown on a frictionless table. They are pulled with a force $T_3 = 40$ N. If $m_1 = 10$ kg, $m_2 = 6$ kg and $m_3 = 4$ kg, the tension T_2 will be

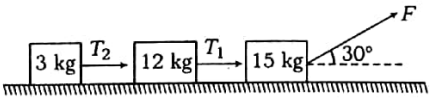


(a) 20 N (b) 40 N
(c) 10 N (d) 32 N

3. A 50 kg boy stands on a platform spring scale in a lift that is going down with a constant speed $3\,ms^{-1}$. If the lift is brought to rest by a constant deceleration in a distance of 9 m, what does the scale read during this period? (Take, $g = 9.8\,ms^{-2}$)
(a) 500 N (b) 465 N
(c) 515 N (d) zero

4. An elevator and its load have a total mass of 800 kg. If the elevator, originally moving downward at $10\,ms^{-1}$, is brought to rest with constant deceleration in a distance of 25 m, the tension in the supporting cable will be ($g = 10\,ms^{-2}$)
(a) 8000 N (b) 6400 N
(c) 11200 N (d) 9600 N

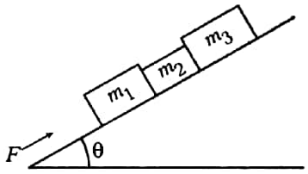
5. The surface are frictionless, the ratio of T_1 and T_2 is



(a) $\sqrt{3}:1$ (b) $1:\sqrt{3}$
(c) $1:5$ (d) $5:1$

6. Two blocks of masses 2 kg and 1 kg are in contact with each other on a frictionless table. When a horizontal force of 3 N is applied to the block of mass 2 kg, the value of the force of contact between the two blocks is
(a) 4 N (b) 3 N (c) 5 N (d) 1 N

7. Three blocks are placed at rest on a smooth inclined plane with force acting on m_1 parallel to the inclined plane. Find the contact force between m_2 and m_3.



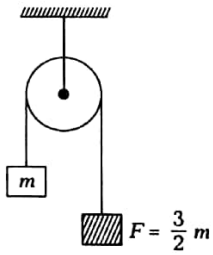
(a) $\dfrac{(m_1 + m_2 + m_3)F}{m_3}$

(b) $\dfrac{m_3 F}{m_1 + m_2 + m_3}$

(c) $F - (m_1 + m_2)g$

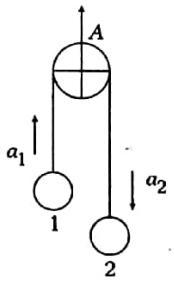
(d) None of the above

8. In the arrangement shown, the mass m will ascend with an acceleration (Pulley and rope are massless)



(a) Zero (b) $\dfrac{g}{5}$
(c) g (d) $2g$

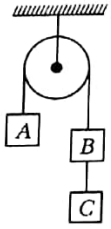
9. Two masses are connected by a string which passes over a pulley accelerating upwards at a rate A as shown. If a_1 and a_2 be the accelerations of bodies 1 and 2 respectively, then



(a) $A = a_1 - a_2$ (b) $A = a_1 + a_2$
(c) $A = \dfrac{a_1 - a_2}{2}$ (d) $A = \dfrac{a_1 + a_2}{2}$

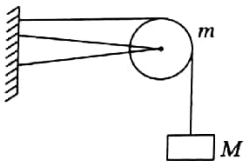
10. Three equal weights A, B and C of mass 2 kg each are hanging on a string passing over a fixed pulley which is frictionless as shown in figure.

The tension in the string connecting weight B and C is?



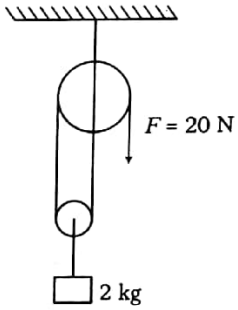
(a) zero (b) 13 N
(c) 3.3 N (d) 19.6 N

11. A light string going over a clamped pulley of mass m supports a block of mass M as shown in the figure. The force on the pulley by the clamp is given by



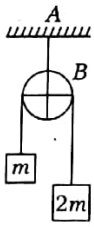
(a) $\sqrt{2}\,Mg$ (b) $\sqrt{2}\,mg$
(c) $g\sqrt{(M+m)^2+m^2}$ (d) $g\sqrt{(M+m)^2+M^2}$

12. The acceleration of the 2 kg block if the free end of string is pulled with a force of 20 N as shown is



(a) zero (b) $10\ \text{ms}^{-2}$
(c) $5\ \text{ms}^{-2}$ upward (d) $5\ \text{ms}^{-2}$ downward

13. In the arrangement shown in the figure, the pulley has a mass $3m$. Neglecting friction on the contact surface, the force exerted by the supporting rope AB on the ceiling is

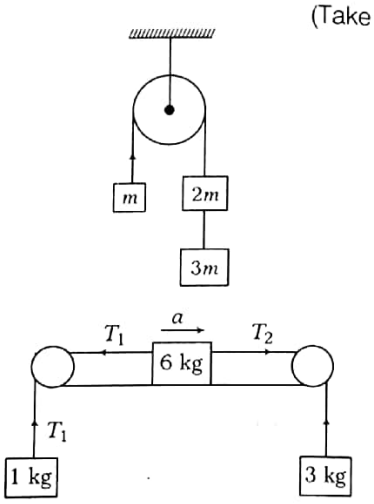


(a) $6\,mg$
(b) $3\,mg$
(c) $4\,mg$
(d) None of the above

14. In the figure given below, with what acceleration does the block of mass m will move? (Pulley and strings are massless and frictionless)

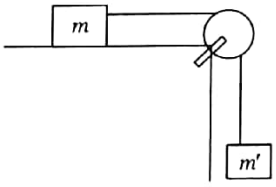
(a) $\frac{g}{3}$ (b) $\frac{2g}{5}$
(c) $\frac{2g}{3}$ (d) $\frac{g}{2}$

15. Three masses of 1 kg, 6 kg and 3 kg are connected to each other with threads and are placed on a table as shown in figure. What is the acceleration with which the system is moving? (Take $g = 10\ \text{ms}^{-2}$)



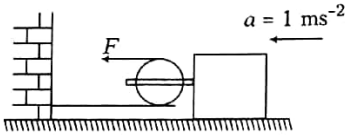
(a) Zero (b) $2\ \text{ms}^{-2}$ (c) $4\ \text{ms}^{-2}$ (d) $3\ \text{ms}^{-2}$

16. In the figure given below masses m and m' are tied with a thread passing over a pulley, m' is on a frictionless horizontal surface. If acceleration due to gravity is g, the acceleration of m' in this arrangement will be



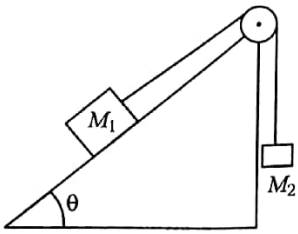
(a) g (b) $\frac{m'g}{(m+m')}$ (c) $\frac{mg}{m'}$ (d) $\frac{mg}{(m-m')}$

17. A block of mass 200 kg is set into motion on a frictionless horizontal surface with the help of frictionless pulley and a rope system as shown in figure. What horizontal force F should be applied to produce in the block an acceleration of $1\ \text{ms}^{-2}$?



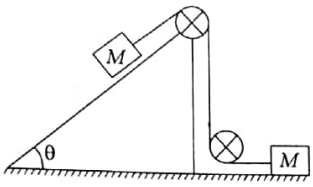
(a) 50 N (b) 100 N (c) 200 N (d) 10 N

18. Two masses M_1 and M_2 are attached to the ends of a string which passes over a pulley attached to the top of an inclined plane. The angle of inclination of the plane is 30° and $M_1 = 10$ kg, $M_2 = 5$ kg. What is the acceleration of mass M_2?



(a) $10\ \text{ms}^{-2}$ (b) $5\ \text{ms}^{-2}$
(c) Zero (d) None of these

19. Two blocks, each having a mass M, rest on frictionless surfaces as shown in the figure. If the pulleys are light and frictionless and M on the incline is allowed to move down, then the tension in the string will be



(a) $\frac{2}{3}Mg\sin\theta$ (b) $\frac{3}{2}Mg\sin\theta$ (c) $\frac{Mg\sin\theta}{2}$ (d) $2Mg\sin\theta$

20. Two bodies of masses m_1 and m_2 are connected a light string which passes over a frictionless massless pulley. If the pulley is moving upward with uniform acceleration $\frac{g}{2}$, then tension in the string will be

(a) $\frac{3m_1m_2}{m_1+m_2}g$ (b) $\frac{m_1+m_2}{4m_1m_2}g$

(c) $\frac{2m_1m_2}{m_1+m_2}g$ (d) $\frac{m_1m_2}{m_1+m_2}g$

8 Force of friction

Whenever a body moves or tends to move over the surface of another body, a force comes into play which acts parallel to the surface of contact and opposes the relative motion. This opposing force is called **friction.**

Consider a wooden block placed on a horizontal surface and is given a gentle push. The block slides through a small distance and comes to rest. According to Newton's second law a retarding force must be acting on the block. This retarding force or opposing force is called friction or friction force.

As shown in the figure, the force of friction always acts tangential to the surface in contact and in a direction opposite to the direction of (relative) motion of the body.

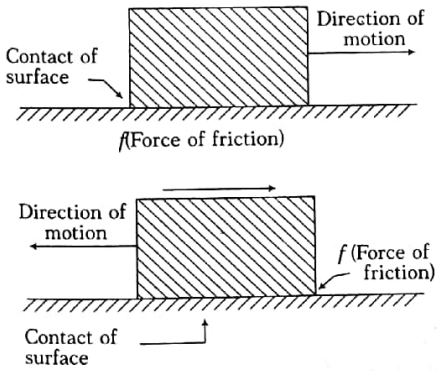


***Fig. 5.30** Friction in different directions*

If a body is at rest and no pulling force is acting on it, then force of friction on it is zero.

Friction between the two bodies depends upon

(i) Nature of medium of contact between two bodies, the roughness of medium increases the friction.

(ii) Normal reaction, as normal reaction increases, the interlocking between two surfaces in contact increases because they press harder against each other, so friction increases.

Types of friction

Friction is mainly of two types

1. Static friction

It is the friction force which comes into existence when one object tends to move over the surface of other object but the actual force has not yet started. If we keep on increasing the applied force to create movement then a stage comes when the object is on the verge of moving over the other object. Friction at this time is called **limiting friction.** Limiting friction is maximum value of static friction.

Laws of static friction

(i) The force of static friction is proportional to the normal force exerted by other surface normally on the former.

(ii) The static frictional force is given by $f_s \le \mu_s N$, where μ_s is the coefficient of static friction.

(iii) The 'less than or equal to' sign in the above equation represents the adjusting nature of the force of static friction.

(iv) The equality sign in the equation holds only when the static frictional force has its maximum value.

(v) It is independent of the area of contact between the two surfaces.

2. Kinetic friction

The friction force that comes into existence when one object is actually moving over the surface of other object, is called kinetic or dynamic friction.

Laws of kinetic friction

(i) The force of kinetic friction is proportional to the normal force, which presses the two surfaces together. Mathematically, $f_k \le \mu_k N$, where μ_k is the coefficient of kinetic friction.

(ii) It is independent of the surface area of contact.

(iii) It is independent of the speed of sliding, provided that the resulting heat does not alter the condition of the surface.

Types of kinetic friction

Kinetic friction is further divided into following two types

(i) **Sliding friction** The opposing force that comes into play when one body is actually sliding over the surface of the other body is called sliding friction. *e.g.*, A flat block is moving over a horizontal table.

(ii) **Rolling friction** The force of friction that comes into play when a body rolls over the surface of another body is called rolling friction, *e.g.*, Rolling of a wheel, circular disc or a ring over a surface.

Rolling friction is directly proportional to the normal reaction (N) and inversely proportional to the radius (r) of the rolling cylinder or wheel.

$$F_{\text{rolling}} = \mu_r \frac{N}{r}$$

μ_r is called coefficient of rolling friction. It has the dimensions of length and is measured in metre.

- Rolling friction is quite small as compared to sliding friction.
- In rolling, the surfaces at contact do not rub each other.
- The velocity of point of contact with respect to the surface remains zero all the times although the centre of the wheel moves forward.

Note If more than two blocks are placed one over the other on a horizontal ground, then normal reaction between two blocks will be equal to the weight of the blocks over the common surface.

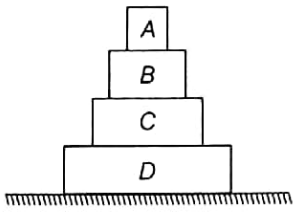


Fig. 5.31 *Normal reaction between blocks*

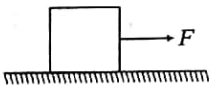
For example, N_1 = normal reaction between A and B $= m_A g$

N_2 = normal reaction between B and C

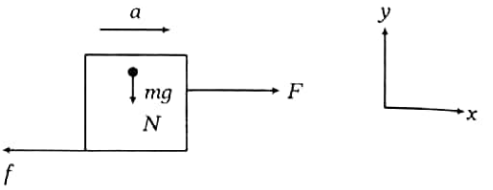
$= (m_A + m_B)g$ and so on.

The theory of static and kinetic friction can be better understood by the following simple example.

Example 5.48 *Suppose a block of mass 1 kg is placed over a rough surface and a horizontal force F is applied on the block as shown in figure. What are the values of force of friction f if the force F is gradually increased. Given, that* $\mu_s = 0.5, \mu_k = 0.4$ *and* $g = 10\,ms^{-2}$.



Sol. Free body diagram of block is



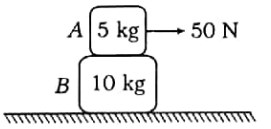
$$\Sigma F_y = 0$$

$\therefore$ $N - mg = 0$ or $N = mg = (1)(10) = 10$ N

$f_L = \mu_s N = (0.5)(10) = 5$ N

and $f_k = \mu_k N = (0.4)(10) = 4$ N

Example 5.49 *Blocks A and B of masses 5 kg and 10 kg are placed as shown in figure. If block A is pulled with 50 N. Find out acceleration of the A and B. If coefficient of friction between A and B is 0.5 and between B and ground is 0.4.*



Sol. Limiting friction between A and B,

$$f_1 = \mu_1 R_1 = 0.5 \times 5\,g = 24.5 \text{ N}$$

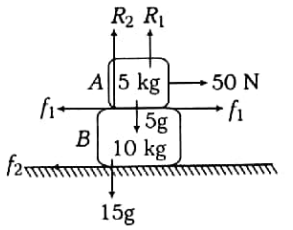
Limiting friction between B and ground

$$f_2 = \mu_2 R_2 = 0.4 \times 15g = 58.8 \text{ N}$$

For block A, $50 - f_1 = 5 \times a$

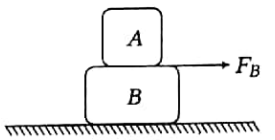
$\Rightarrow$ $50 - 24.5 = 5 \times a \Rightarrow a = 5.1 \text{ ms}^{-2}$

For block B accelerating force $f_1 = 24.5$ N is less than limiting friction $f_2 = 58.8$ N



so, block B will remain at rest.

Example 5.50 *12 N of force required to be applied on A of mass 4 kg to slip on B of mass 5 kg. Find the maximum horizontal force F to be applied on B so that A and B move together.*



Sol. Let μ be friction coefficient between A and B.

As 12 N force on A is required for slipping so

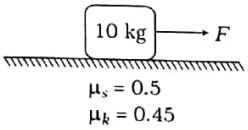
$$\mu m_A g = 12 \Rightarrow \mu = 0.3$$

maximum force (F_B) applied on B so that A and B move together.

$$F_B = (m_A + m_B)\,a$$

where $a = \mu g \Rightarrow F_B = (m_A + m_B)\mu g = (4+5)(0.3)(10) = 27$ N

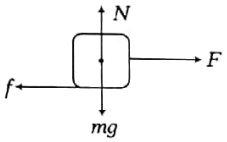
Example 5.51 *A body of mass 10 kg is kept on a horizontal floor of coefficient of static friction $\mu_s = 0.5$ and coefficient of kinetic friction $\mu_k = 0.45$ as shown in figure.*



Find the acceleration, force of friction and contact force on the body by the plane when the driving force is ($g = 10\ ms^{-2}$)

(i) 40 N *(ii)* 60 N

Sol. **FBD of the body**



Normal reaction $N = mg = 100$ N

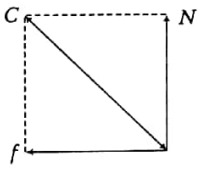
Limiting friction on the body, $f_L = \mu_s N = 0.5 \times 100$ N $= 50$ N

(i) $F = 40$ N is less than the limiting friction so the body is static. So, $a = 0$.

Force of friction acting on the body is static friction, f = Driving force = 40 N

Contact force is the resultant of force of friction and normal reaction. So,

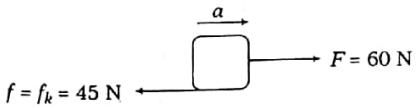
$$C = \sqrt{f^2 + N^2} = \sqrt{(40)^2 + (100)^2} = 107.7 \text{ N}$$



(ii) $F = 60$ N is greater than the limiting friction on the body, so body will start moving. Force of friction acting on the body

$= \text{kinetic friction} = \mu_k N = 0.45 \times 100 \text{ N} = 45$ N

FBD of the body



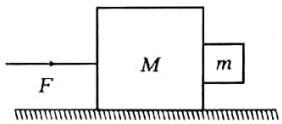
∴ Acceleration of the body is

$$a = \frac{F - f_k}{m} = \frac{60 - 45}{10} = 1.5 \text{ ms}^{-2}$$

Contact force, $C = \sqrt{f^2 + N^2} = \sqrt{f_k^2 + N^2}$

$$= \sqrt{(45)^2 + (100)^2} \approx 109.7 \text{ N}$$

Example 5.52 *In the adjoining figure, the coefficient of friction between wedge (of mass M) and block (of mass m) is μ. Find the minimum horizontal force F required to keep the block stationary with respect to wedge.*

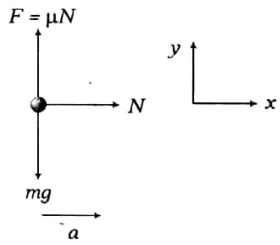


Sol. Such problems can be solved with or without using the concept of pseudo force. Let us, solve the problem by both the methods.

a = Acceleration of (wedge + block) in horizontal direction

$$= \frac{F}{M+m}.$$

(*i*) **Inertial frame of reference (ground)** FBD of block with respect to ground (only real forces have to be applied) with respect to ground block is moving with an acceleration a.



Therefore,

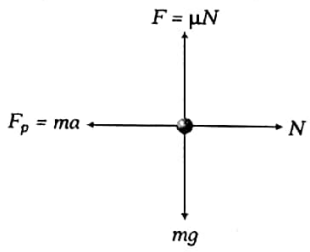
$$\Sigma F_y = 0 \text{ and } \Sigma F_x = ma$$

$$mg = \mu N \text{ and } N = ma$$

$$\therefore \quad a = \frac{g}{\mu}$$

$$\therefore \quad F = (M+m)a = (M+m)\frac{g}{\mu}$$

(*ii*) **Non-inertial frame of reference (wedge)** FBD of m with respect to wedge (real + one pseudo force) with respect to wedge block is stationary.



$$\therefore \quad \Sigma F_x = 0 = \Sigma F_y$$

$$\therefore \quad mg = \mu N$$

and $N = ma$

$$\therefore \quad a = \frac{g}{\mu} \text{ and } F = (M+m)a = (M+m)\frac{g}{\mu}$$

From the above discussion, we can see that from both the methods results are same.

Example 5.53 *If the coefficient of friction between an insect and bowl is μ and the radius of the bowl is r, find the maximum height to which the insect can crawl up in the bowl.*

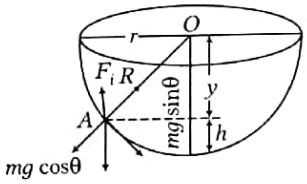
Sol. The insect will crawl up the bowl till the component of its weight along the bowl is balanced by limiting frictional force. So, resolving weight perpendicular to the bowl and along the bowl.

$$R = mg\cos\theta \quad \text{...(i)}$$

$$F_L = mg\sin\theta \quad \text{...(ii)}$$

Dividing Eq. (ii) by Eq. (i)

$$\tan\theta = \frac{F}{R} \text{ or } \tan\theta = \mu \qquad (\because F_L = \mu R)$$



or $$\frac{\sqrt{(r^2 - y^2)}}{y} = \mu \text{ or } y = \frac{r}{\sqrt{1+\mu^2}}$$

So, $$h = r - y = r\left[1 - \frac{1}{\sqrt{1+\mu^2}}\right]$$

Angle of friction (λ)

The angle which the resultant of the force of limiting friction and normal reaction makes with the direction of normal reaction is known as angle of friction.

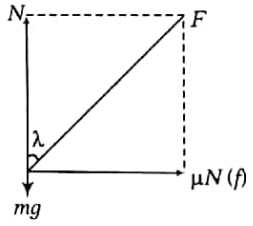


Fig. 5.32 *Angle of friction*

The resultant of these two forces is F and it makes an angle λ with the normal, where

$$\tan\lambda = \frac{\mu_s N}{N} = \mu_s = \frac{f}{N}$$

or $$\boxed{\lambda = \tan^{-1}(\mu_s)} \quad \text{...(i)}$$

or $$\boxed{\mu_s = \tan\lambda}$$

This angle λ is called the angle of friction.

Thus, the coefficient of static friction is equal to the tangent of the angle of friction.

Angle of repose (α)

The minimum angle of inclination of plane with the horizontal at which the body placed on the plane just begins to slide down, is known as angle of repose.

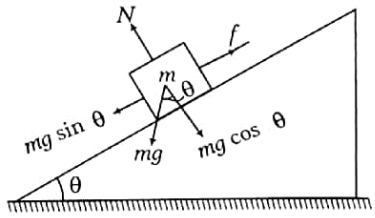


Fig. 5.33 *Angle of repose*

Suppose a block of mass m is placed on an inclined plane whose inclination θ can be increased or decreased. Let, μ be the coefficient of friction between the block and the plane. At any angle of inclination.

Normal reaction $N = mg\cos\theta$

Limiting friction $f_L = \mu_s N = \mu_s mg\cos\theta$

and the driving force (or pulling force)

$$F = mg\sin\theta \qquad \text{(Down the plane)}$$

From these three equations; we see that when θ is increased from 0° to 90°, normal reaction N and hence, the limiting friction f_L is decreased while the driving force F is increased. There is a critical angle called angle of repose (α) at which these two forces are equal. Now, if θ is further increased, then the driving force F becomes more than the limiting friction f_L and the block starts sliding.

Thus, $$f_L = F$$

at $\theta = \alpha$ or $\mu\, mg\cos\alpha = mg\sin\alpha$

or $$\tan\alpha = \mu$$

or $$\boxed{\alpha = \tan^{-1}(\mu)} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we see that angle of friction (λ) is numerically equal to the angle of repose.

or $$\lambda = \alpha$$

From the above discussion, we can conclude that

(i) if $\theta < \alpha$, $F < f_L$ the block is stationary

(ii) if $\theta = \alpha$, $F = f_L$ the block is on the verge of sliding

(iii) if $\theta > \alpha$, $F > f_L$ the block slides down with acceleration

9 Equation of motion of a body on a rough inclined plane

(a) When body is moving downward with a acceleration 'a'. f is the force of friction between inclined plane and the body.

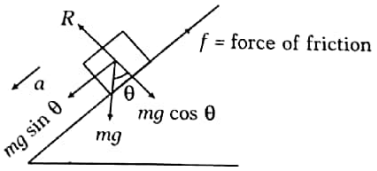


Fig. 5.34 Downward motion of a body (FBD)

From FBD,

$$\boxed{R = mg\cos\theta}$$

As the body is moving downward with acceleration a, then net force in downward direction $= ma$

$$mg\sin\theta - f = ma$$

We know that, $f = \mu R = \mu mg\cos\theta$

Thus, $$\boxed{a = (\sin\theta - \mu\cos\theta)\,g}$$

(b) When body is moving upward when a force F is applied on it as shown in the figure.

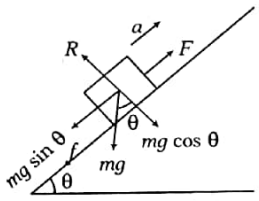


Fig. 5.35 Upward motion of body (FBD)

From FBD,

$$\boxed{R = mg\cos\theta} \quad ...(i)$$

$$F = mg\sin\theta + f \quad ...(ii)$$

$$f = \mu R = \mu mg\cos\theta \text{ [From Eq. (i)]} \quad ...(iii)$$

Putting the value of Eq (iii) in Eq. (ii)

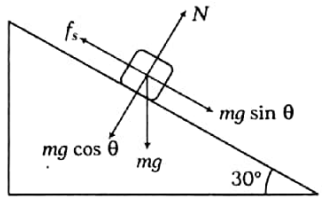
$\Rightarrow$ $$F = mg(\sin\theta + \mu\cos\theta)$$

$\Rightarrow$ $$a = g(\sin\theta + \mu\cos\theta)$$

Note To calculate the work done over a rough inclined surface, multiply force with distance travelled by the body, *i.e.*, $W = F \times S$
$\Rightarrow ma \times S mgs\,[\sin\theta + \mu\cos\theta]$ (For body moving upwards)
and $W = mgs\,(\sin\theta - \mu\cos\theta)$ (For body moving downwards)

Example 5.54 *A block of mass 5 kg rests on a inclined plane at an angle of 30° with the horizontal. If the block just begins to slide, then what is the coefficient of static friction between the block and the surface?*

Sol. In equilibrium, the resultant of these forces must be zero.



$\therefore$ $$mg\sin\theta = f_s \quad ...(i)$$

and $$mg\cos\theta = N \quad ...(ii)$$

$\therefore$ $$\tan\theta = \frac{f_s}{N} \quad \text{[Dividing Eq. (i) and (ii)]}$$

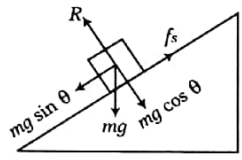
As $$f_s = \mu_s N$$

$\therefore$ $$\tan\theta = \mu_s$$

$\therefore$ $$\mu_s = \tan 30° \quad \therefore\ \mu_s = 0.577$$

Example 5.55 *A block of wood of 1 kg resting on an inclined plane of angle 30°, just starts moving down. If the coefficient of friction is 0.2, then find its velocity (in ms^{-1}) after 5 s (take, $g = 10\ ms^{-2}$)*

Sol. Acceleration of block down the plane is given by



For the motion of body along the incline,

$$a = \frac{mg\sin\theta - \mu mg\cos\theta}{m} \quad (\text{as, } R = mg\cos\theta \text{ and } f_s = \mu R)$$

$$= 10 \times \sin 30° - 0.2 \times 10\cos 30°$$

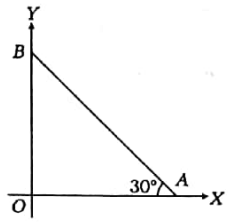
$$= 10 \times \frac{1}{2} - 2 \times \frac{\sqrt{3}}{2} = 3.278\ ms^{-2}$$

From equation of motion, $v = u - at$

Velocity after 5 s

$$v = 0 + 3.278 \times 5 = 16.35\ ms^{-1}$$

Example 5.56 *A rod AB rests with the end A on rough horizontal ground and the end B against a smooth vertical wall. The rod is uniform and of weight w. If the rod is in equilibrium in the position shown in figure.*

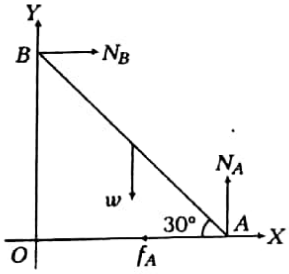


Find

(a) frictional force at A *(b) normal reaction at A,*

(c) normal reaction at B.

Sol. Let length of the rod be $2l$. Using the three conditions of equilibrium. Anti-clockwise moment is taken as positive.



(i) $\Sigma F_x = 0$

$\therefore$ $N_B - f_A = 0$

or $N_B = f_A$ ...(i)

(ii) $\Sigma F_y = 0$

$\therefore$ $N_A - w = 0$

or $N_A = w$ ...(ii)

(iii) $\Sigma \tau_o = 0$

$\therefore$ $N_A (2l \cos 30°) - N_B(2l \sin 30°) - w (l \cos 30°) = 0$

or $\sqrt{3}N_A - N_B - \frac{\sqrt{3}}{2} w = 0$ ...(iii)

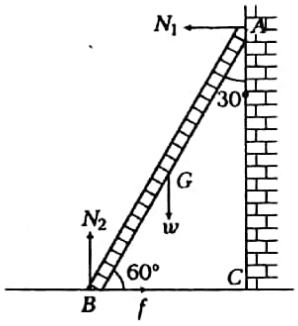
Solving these three equations, we get

(a) $f_A = \frac{\sqrt{3}}{2} w$ (b) $N_A = w$

(c) $N_B = \frac{\sqrt{3}}{2} w$

Example 5.57 *A 4 m long ladder weighing 25 kg rests with its upper end against a smooth wall and lower end on rough ground. What should be the minimum coefficient of friction between the ground and the ladder for it to be inclined at 60° with the horizontal without slipping? (Take $g = 10\ ms^{-2}$)*

Sol. In figure, AB is a ladder of weight w which acts at its centre of gravity G.



$\angle ABC = 60°$ $\therefore$ $\angle BAC = 30°$

Let N_1 be the reaction of the wall, and N_2 the reaction of the ground.

Force of friction f between the ladder and the ground acts along BC.

For horizontal equilibrium, $f = N_1$ ...(i)

For vertical equilibrium, $N_2 = w$ ...(ii)

Taking moments about B, we get for equilibrium,

$$N_1(4 \cos 30°) - w(2 \cos 60°) = 0 \quad ...(iii)$$

Here, $w = 250$ N

Solving these three equations, we get

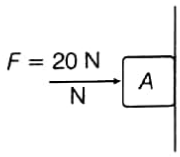
$$f = 72.17 \text{ N} \quad \text{and} \quad N_2 = 250 \text{ N}$$

$$\therefore \quad \mu = \frac{f}{N_2} = \frac{72.17}{250} = 0.288$$

Example 5.58 *A block of mass 1 kg is pushed against a rough vertical wall with a force of 20 N, coefficient of static friction being 1/4. Another horizontal force of 10 N is applied on the block in a direction parallel to the wall. Will the block move? If yes, in which direction? If no, find the frictional force exerted by the wall on the block. ($g = 10\ ms^{-2}$)*

Sol. Normal reaction on the block from the wall will be

$$N = F = 20 \text{ N}$$



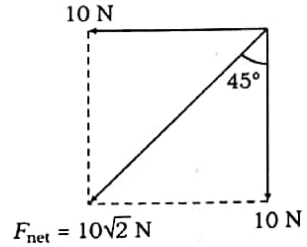
Therefore, limiting friction

$$f_L = \mu N = \left(\frac{1}{4}\right)(20) = 5 \text{ N}$$

Weight of the block is

$$W = mg = (1)(10) = 10 \text{ N}$$

A horizontal force of 10 N is applied to the block.The resultant of these two forces will be $10\sqrt{2}$ N in the direction shown in figure. Since, this resultant is greater than the limiting friction. The block will move in the direction of F_{net} with acceleration :



$$a = \frac{F_{net} - f_L}{m} = \frac{10\sqrt{2} - 5}{1} = 9.14 \text{ ms}^{-2}$$

Check point 5.4

1. Maximum value of frictional force is called
   (a) limiting frictional force
   (b) static frictional force
   (c) kinetic frictional force
   (d) rolling frictional force

2. Static friction between two surfaces
   (a) prevents the relative motion between them
   (b) opposite to the direction of motion of them
   (c) acts in opposite direction of applied force
   (d) Both (b) and (c)

3. The limiting value of static friction between two contact surfaces is
   (a) Proportional to normal force between the surfaces in contact
   (b) Independent of area of contact
   (c) Depends on the microscopic area of constant magnitude
   (d) All of the above

4. A mass placed on an inclined place is just in equilibrium. If μ is coefficient of friction of the surface, then maximum inclination of the plane with the horizontal is
   (a) $\tan^{-1}\mu$
   (b) $\tan^{-1}(\mu/2)$
   (c) $\sin^{-1}\mu$
   (d) $\cos^{-1}\mu$

5. A 30 kg block rests on a rough horizontal surface. A force of 200 N is applied on the block. The block acquires a speed of 4 ms^{-1}, starting from rest in 2 s. What is the value of coefficient of friction?
   (a) $\frac{10}{3}$ (b) $\frac{\sqrt{3}}{10}$
   (c) 0.47 (d) 0.184

6. A car having a mass of 1000 kg is moving at a speed of 30 ms^{-1}. Brakes are applied to bring the car to rest. If the frictional force between the tyres and the road surface is 5000 N, the car will come to rest in
   (a) 5 s (b) 10 s
   (c) 12 s (d) 6 s

7. A 100 N force acts horizontally on a block of mass 10 kg placed on a horizontal rough table of coefficient of friction $\mu = 0.5$. If g at the place is 10 ms^{-2}, the acceleration of the block is
   (a) zero (b) 10 ms^{-2}
   (c) 5 ms^{-2} (d) 5.2 ms^{-2}

8. A block of mass 2 kg is placed on the floor. The coefficient of static friction is 0.4. If a force of 2.8 N is applied on the block parallel to the floor, the force of friction between the block and floor is (take, $g = 10\ ms^{-2}$)
   (a) 2.8 N (b) 8 N
   (c) 2 N (d) zero

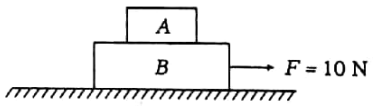
9. A body is projected along a rough horizontal surface with a velocity 6 ms^{-1}. If the body comes to rest after travelling 9 m, then coefficient of sliding friction, is (take, $g = 10\ ms^{-2}$)
   (a) 0.5 (b) 0.4
   (c) 0.6 (d) 0.2

10. The coefficient of friction between the tyres and road is 0.4. The minimum distance covered before attaining a speed of 8 ms^{-1} starting from rest is nearly (take, $g = 10\ ms^{-2}$)
    (a) 8 m (b) 4 m
    (c) 10 m (d) 16 m

11. A block is gently placed on a conveyor belt moving horizontally with constant speed. After $t = 4$ s, the velocity of the block becomes equal to the velocity of the belt. If the coefficient of friction between the block and the belt is $\mu = 0.2$, then the velocity of the conveyor belt is
    (a) 8 ms^{-1}
    (b) 4 ms^{-1}
    (c) 6 ms^{-1}
    (d) 18 ms^{-1}

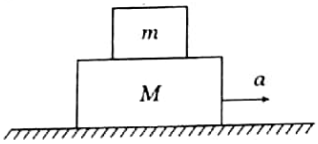
12. The breaking strength of the cable used to pull a body is 40 N. A body of mass 8 kg is resting on a table of coefficient of friction $\mu = 0.20$. The maximum acceleration which can be produced by the cable is (take, $g = 10\ ms^{-2}$)
    (a) 6 ms^{-2}
    (b) 3 ms^{-2}
    (c) 8 ms^{-2}
    (d) 8 ms^{-2}

13. In the shown arrangement mass of $A = 1$ kg, mass of $B = 2$ kg. Coefficient of friction between A and $B = 0.2$.



There is no friction between B and ground. The frictional force exerted by A on B equals.
    (a) 2 N (b) 3 N
    (c) 4 N (d) 5 N

14. A block of mass m is placed on the top of another block of mass M as shown in the figure . The coefficient of friction between them is μ.



The maximum acceleration with which the block M may move so that m also moves along with it is

(a) μg (b) $\mu \frac{M}{m} g$

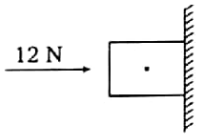
(c) $\mu \frac{m}{M} g$ (d) $\frac{g}{\mu}$

15. A block of mass 4 kg is placed on a rough horizontal plane. A time dependent force $F = kt^2$ acts on the block, where $k = 2\,\text{Ns}^{-2}$. Coefficient of friction $\mu = 0.8$. Force of friction between block and the plane at $t = 2$ s is

(a) 8 N (b) 4 N

(c) 2 N (d) 32 N

16. A block of weight 5 N is pushed against a vertical wall by a force 12 N. The coefficient of friction between the wall and block is 0.6. The magnitude of the force exerted by the wall on the block is



(a) 12 N (b) 5 N

(c) 7.2 N (d) 13 N

17. A body of mass 10 kg is placed on rough surface, pulled by a force F making an angle of 30° above the horizontal. If the angle of friction is also 30°, then the minimum magnitude of force F required to move the body is equal to (take, $g = 10\,\text{ms}^{-2}$)

(a) 100 N (b) $50\sqrt{2}$ N

(c) $100\sqrt{2}$ N (d) 50 N

18. A block of mass 2 kg rests on a rough inclined plane making an angle of 30° with the horizontal. The coefficient of static friction between the block and the plane is 0.7. The frictional force on the block is

(a) 9.8 N (b) $0.7 \times 9.8 \times \sqrt{3}$ N

(c) $9.8 \times \sqrt{3}$ N (d) 0.7×9.8 N

19. A minimum force F is applied to a block of mass 102 kg to prevent it from sliding on a plane with an inclination angle 30° with the horizontal. If the coefficients of static and kinetic friction between the block and the plane are 0.4 and 0.3 respectively, then the force F is

(a) 157 N (b) 224 N

(c) 315 N (d) zero

20. A box of mass 8 kg is placed on a rough inclined plane of inclination θ. Its downward motion can be prevented by applying an upward pull F. And it can be made to slide upwards by applying a force $2F$. The coefficient of friction between the box and the inclined plane is

(a) $\frac{1}{3}\tan\theta$ (b) $3\tan\theta$

(c) $\frac{1}{2}\tan\theta$ (d) $2\tan\theta$

Chapter exercises

A Taking it together

(Assorted questions of the chapter for advanced level practice)

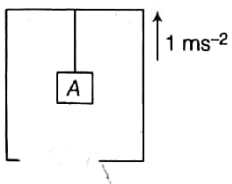
1. A ball is travelling with uniform translatory motion. This means that
(a) it is at rest
(b) the path can be a straight line or circular and the ball travels with uniform speed
(c) all parts of the ball have the same velocity (magnitude and direction) and the velocity is constant
(d) the centre of the ball moves with constant velocity and the ball spins about its centre uniformly

2. A metre scale is moving with uniform velocity. This implies
(a) the force acting on the scale is zero, but a torque about the centre of mass can act on the scale
(b) the force acting on the scale is zero and the torque acting about centre of mass of the scale is also zero
(c) the total force acting on it need not be zero but the torque on it is zero
(d) neither the force nor the torque need to be zero

3. Conservation of momentum in a collision between particles can be understood from
(a) conservation of energy
(b) Newton's first law only
(c) Newton's second law only
(d) both Newton's second and third law

4. A hockey player is moving northward and suddenly turns westward with the same speed to avoid an opponent. The force that acts on the player is
(a) frictional force along westward
(b) muscle force along southward
(c) frictional force along south-west
(d) muscle force along south-west

5. If the elevator in the shown figure is moving upwards with constant acceleration 1 ms^{-2}, the tension in the string connected to block A of mass 6 kg would be (take, $g = 10\ ms^{-2}$)



(a) 60 N (b) 66 N (c) 54 N [illegible] N

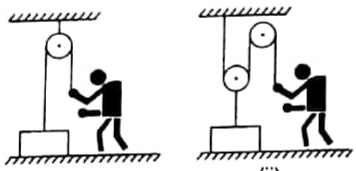
6. A block is placed on the top of a smooth inclined plane of inclination θ kept on the floor of a lift. When the lift is descending with a retardation a, the block is released. The acceleration of the block relative to the incline is
(a) $g \sin\theta$
(b) $a \sin\theta$
(c) $(g - a)\sin\theta$
(d) $(g + a)\sin\theta$

7. A car of mass m starts from rest and acquires a velocity along east, $\mathbf{v} = v\hat{\mathbf{i}}(v > 0)$ in two seconds. Assuming the car moves with uniform acceleration, the force exerted on the car is
(a) $\frac{mv}{2}$ eastward and is exerted by the car engine
(b) $\frac{mv}{2}$ eastward and is due to the friction on the tyres exerted by the road
(c) more than $\frac{mv}{2}$ eastward exerted due to the engine and overcomes the friction of the road
(d) $\frac{mv}{2}$ exerted by the engine

8. A cricket ball of mass 150 g has an initial velocity $\mathbf{u} = (3\hat{\mathbf{i}} + 4\hat{\mathbf{j}})\ ms^{-1}$ and a final velocity $\mathbf{v} = -(3\hat{\mathbf{i}} + 4\hat{\mathbf{j}})\ ms^{-1}$, after being hit. The change in momentum (final momentum-initial momentum) is (in kg ms^{-1})
(a) zero
(b) $-(0.45\hat{\mathbf{i}} + 0.6\hat{\mathbf{j}})$
(c) $-(0.9\hat{\mathbf{i}} + 1.2\hat{\mathbf{j}})$
(d) $-5(\hat{\mathbf{i}} + \hat{\mathbf{j}})\hat{\mathbf{i}}$

9. In the previous question (6), the magnitude of the momentum transferred during the hit is
(a) zero (b) 0.75 kg-ms^{-1}
(c) 1.5 kg-ms^{-1} (d) 14 kg-ms^{-1}

10. In the figure shown, a person wants to raise a block lying on the ground to a height h. In both the cases if time required is the same, then in which case he has to exert more force. Assume pulleys and strings are light



(i) (ii)

(a) (i)
(b) (ii)
(c) same in both
(d) Cannot be determined

11. A body of mass 2kg travels according to the law $x(t) = pt + qt^2 + rt^3$, where, $q = 4\text{ms}^{-2}$, $p = 3\text{ms}^{-2}$ and $r = 5\text{ms}^{-3}$. The force acting on the body at $t = 2s$ is

(a) 136 N (b) 134 N (c) 158 N (d) 68 N

12. A body with mass 5 kg is acted upon by a force $\mathbf{F} = (-3\hat{\mathbf{i}} + 4\hat{\mathbf{j}})$ N. If its initial velocity at $t = 0$ is $\mathbf{u} = (6\hat{\mathbf{i}} - 12\hat{\mathbf{j}})\text{ms}^{-1}$, the time at which it will just have a velocity along the Y-axis is

(a) never (b) 10 s (c) 2 s (d) 15 s

13. A block of mass m is placed on a smooth plane inclined at an angle θ with the horizontal. The force exerted by the plane on the block has a magnitude

(a) mg (b) $mg \sec\theta$
(c) $mg\cos\theta$ (d) $mg\sin\theta$

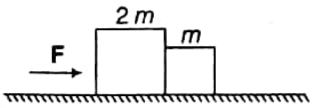
14. A block has been placed on an inclined plane. The slope angle θ of the place is such that the block slides down the plane at a constant speed. The coefficient of kinetic friction is equal to

(a) $\sin\theta$ (b) $\cos\theta$ (c) g (d) $\tan\theta$

15. A 5000 kg rocket is set for vertical firing. The exhaust speed is 800 ms^{-1}. To give an initial upward acceleration of 20 ms^{-2}, the amount of gas ejected per second to supply the needed thrust will be (Take $g = 10\text{ ms}^{-2}$)

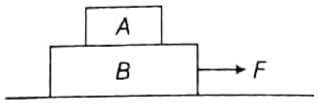
(a) 127.5 kgs^{-1} (b) 187.5 kgs^{-1}
(c) 185.5 kgs^{-1} (d) 137.5 kgs^{-1}

16. Two blocks are in contact on a frictionless table. One has mass m and the other $2m$. A force F is applied on $2m$ as shown in the figure. Now the same force F is applied from the right on m. In the two cases the ratio of force of contact between the two blocks will be



(a) same (b) 1: 2 (c) 2 : 1 (d) 1 : 3

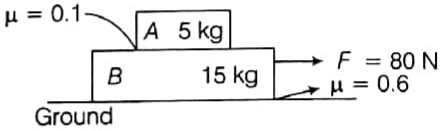
17. A 4 kg block A is placed on the top of 8 kg block B which rests on a smooth table.



A just slips on B when a force of 12 N is applied on A. Then the maximum horizontal force F applied on B to make both A and B move together, is

(a) 12 N (b) 24 N
(c) 36 N (d) 48 N

18. Find the value of friction forces between the blocks A and B and between B and ground. (Take, $g = 10\text{ ms}^{-2}$)

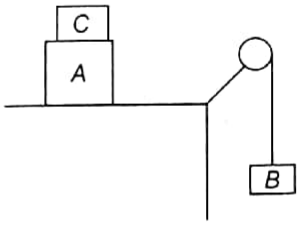


(a) 90 N, 5 N (b) 5 N, 90 N
(c) 5 N, 75 N (d) 0 N, 80 N

19. A block of mass 5 kg is kept on a horizontal floor having coefficient of friction 0.09. Two mutually perpendicular horizontal forces of 3 N and 4 N act on this block. The acceleration of the block is (take, $g = 10\text{ ms}^{-2}$)

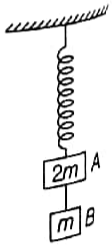
(a) zero (b) 0.1 ms^{-2}
(c) 0.2 ms^{-2} (d) 0.3 ms^{-2}

20. Two masses A and B of 10 kg and 5 kg respectively, are connected with a string passing over a frictionless pulley fixed at the corner of a table as shown in figure. The coefficient of friction of A with the table is 0.2. The minimum mass of C that may be placed on A to prevent it from moving is equal to



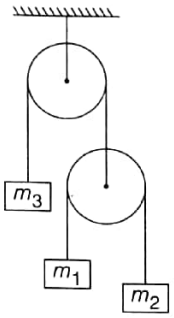
(a) 15 kg (b) 10 kg
(c) 5 kg (d) 20 kg

21. Two blocks of masses $2m$ and m are in equilibrium as shown in the figure. Now the string between the blocks is suddenly broken. The accelerations of the blocks A and B respectively, at that instant are



(a) g and g (b) g and $\frac{g}{2}$
(c) $\frac{g}{2}$ and g (d) $\frac{g}{2}$ and $\frac{g}{2}$

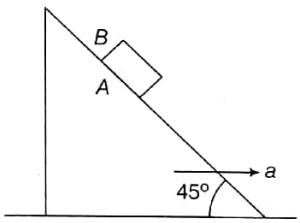
22. In the figure, pulleys are smooth and strings are massless, $m_1 = 1$ kg and $m_2 = \frac{1}{3}$ kg. To keep m_3 at rest, mass m_3 should be



(a) 1 kg (b) $\frac{2}{3}$ kg

(c) $\frac{1}{4}$ kg (d) 2 kg

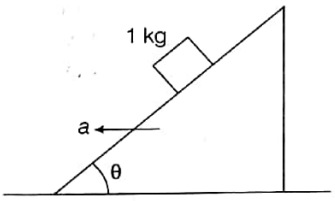
23. If the coefficient of friction between A and B is μ, the maximum acceleration of the wedge A for which B will remain at rest with respect to the wedge is



(a) μg (b) $g\left(\frac{1+\mu}{1-\mu}\right)$

(c) $g\left(\frac{1-\mu}{1+\mu}\right)$ (d) $\frac{g}{\mu}$

24. A block of mass 1 kg is at rest relative to a smooth wedge moving leftwards with constant acceleration $a = 5\,\text{ms}^{-2}$

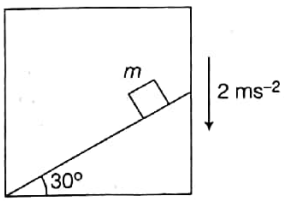


Let N be the normal reaction between the block and the wedge. Then (take, $g = 10\,\text{ms}^{-2}$)

(a) $N = 5\sqrt{5}$ N (b) $N = 15$ N

(c) $\tan\theta = \frac{1}{2}$ (d) $\tan\theta = 2$

25. A block of mass m is kept on an inclined plane of a lift moving down with acceleration of 2 ms^{-2}. What should be the coefficient of friction to let the block move down with constant velocity relative to lift.



(a) $\mu = \frac{1}{\sqrt{3}}$ (b) $\mu = 0.4$ (c) $\mu = 0.8$ (d) $\mu = \frac{\sqrt{3}}{2}$

26. Two blocks of mass 5 kg and 3 kg are attached to the ends of a string passing over a smooth pulley fixed to the ceiling of an elevator. A man inside the elevator accelerated upwards, finds the acceleration of the blocks to be $\frac{9}{32}g$. The acceleration of the elevator is

(a) $\frac{g}{3}$ (b) $\frac{g}{4}$

(c) $\frac{g}{8}$ (d) $\frac{g}{6}$

27. A balloon of weight w is falling vertically downward with a constant acceleration a ($< g$). The magnitude of the air resistance is

(a) w (b) $w\left(1+\frac{a}{g}\right)$

(c) $w\left(1-\frac{a}{g}\right)$ (d) $w\frac{a}{g}$

28. A smooth inclined plane of length L, having an inclination θ with horizontal is inside a lift which is moving down with retardation a. The time taken by a block to slide down the inclined plane from rest will be

(a) $\sqrt{\frac{2L}{\sqrt{a\sin\theta}}}$ (b) $\sqrt{\frac{2L}{g\sin\theta}}$

(c) $\sqrt{\frac{2L}{(g-a)\sin\theta}}$ (d) $\sqrt{\frac{2L}{(g+a)\sin\theta}}$

29. A body of mass M at rest explodes into three pieces, two of which of mass $M/4$ each are thrown off in perpendicular directions with velocities of 3 ms^{-1} and 4 ms^{-1} respectively. The third piece will be thrown off with a velocity of

(a) 1.5 ms^{-1} (b) 2 ms^{-1}

(c) 2.5 ms^{-1} (d) 3 ms^{-1}

30. A wooden box of mass 8 kg slides down an inclined plane of inclination 30° to the horizontal with a constant acceleration of 0.4 ms^{-2}. What is the force of friction between the box and inclined plane? (take, $g = 10\,\text{ms}^{-2}$)

(a) 36.8 N (b) 76.8 N

(c) 65.6 N (d) None of these

31. A body is projected up a 45° rough incline. If the coefficient of friction is 0.5, then the retardation of the block is

(a) $\frac{g}{2\sqrt{2}}$ (b) $\frac{g}{\sqrt{2}}$ (c) $\frac{3g}{2\sqrt{2}}$ (d) $\frac{g}{2}$

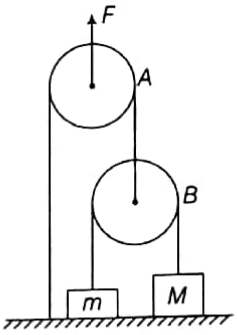
32. A mass of 3 kg descending vertically downwards supports a mass of 2 kg by means of a light string passing over a pulley. At the end of 5 s the string breaks . How much high from now the 2 kg mass will go? (take, $g = 9.8\,\text{ms}^{-2}$)

(a) 4.9 m (b) 9.8 m (c) 19.6 m (d) 2.45 m

33. The rear side of a truck is open and a box of mass 20 kg is placed on the truck 4 m away from the open end. $\mu = 0.15$ and $g = 10\,\text{ms}^{-2}$. The truck starts from rest with an acceleration of $2\,\text{ms}^{-2}$ on a straight road. The box will fall off the truck when it is at a distance from the starting point equal to

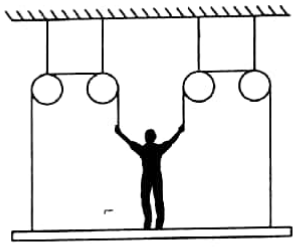
(a) 4 m (b) 8 m (c) 16 m (d) 32 m

34. Two blocks of mass $m = 5$ kg and $M = 10$ kg are connected by a string passing over a pulley B as shown. Another string connects the centre of pulley B to the floor and passes over another pulley A as shown. An upward force F is applied at the centre of pulley A. Both the pulleys are massless. The accelerations of blocks m and M, if F is 300 N are (take, $g = 10\,\text{ms}^{-2}$)



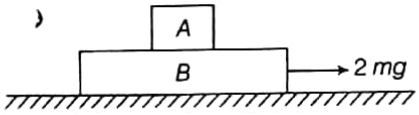
(a) $5\,\text{ms}^{-2}$, zero (b) zero, $5\,\text{ms}^{-2}$
(c) zero, zero (d) $5\,\text{ms}^{-2}$, $5\,\text{ms}^{-2}$

35. A man of mass m stands on a platform of equal mass m and pulls himself by two ropes passing over pulleys as shown in figure. If he pulls each rope with a force equal to half his weight his upward acceleration would be



(a) $\frac{g}{2}$ (b) $\frac{g}{4}$ (c) g (d) zero

36. Block A of mass m rests on the plank B of mass $3m$ which is free to slide on a frictionless horizontal surface. The coefficient of friction between the block and plank is 0.2. If a horizontal force of magnitude $2mg$ is applied to the plank B, the acceleration of A relative to the plank and relative to the ground respectively, are



(a) $0, \frac{g}{2}$ (b) $0, \frac{2g}{3}$ (c) $\frac{3g}{5}, \frac{g}{5}$ (d) $\frac{2g}{5}, \frac{g}{5}$

37. A ball is dropped on to the floor from a height of 10 m. It rebounds to a height of 2.5 m. If the ball is in contact with floor for 0.01 s. What is the average acceleration during contact?

(a) $700\,\text{ms}^{-2}$ (b) $1400\,\text{ms}^{-2}$ (c) $2100\,\text{ms}^{-2}$ (d) $2800\,\text{ms}^{-2}$

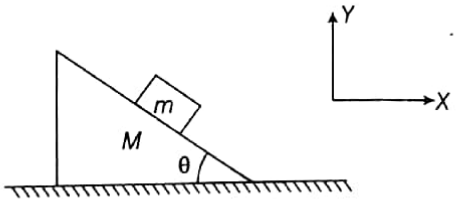
38. A rope of length L and mass M is hanging from a rigid support. The tension in the rope at a distance x from the rigid support is

(a) Mg (b) $\left(\frac{L-x}{L}\right)Mg$ (c) $\left(\frac{L}{L-x}\right)Mg$ (d) $\frac{x}{L}Mg$

39. A 40 N block supported by two ropes. One rope is horizontal and the other makes an angle of 30° with the ceiling. The tension in the rope attached to the ceiling is approximately

(a) 80 N (b) 40 N
(c) $40\sqrt{3}$ N (d) $\frac{40}{\sqrt{3}}$ N

40. Consider the shown arrangement. Assume all surfaces to be smooth. If N represents magnitudes of normal reaction between block and wedge, then acceleration of M along horizontal is equal to



(a) $\frac{N\sin\theta}{M}$ along + ve X-axis
(b) $\frac{N\cos\theta}{M}$ along − ve X-axis
(c) $\frac{N\sin\theta}{M}$ along − ve X-axis
(d) $\frac{N\sin\theta}{m+M}$ along − ve X-axis

41. In the above problem normal reaction between ground and wedge will have magnitude equal to

(a) $N\cos\theta + Mg$ (b) $N\cos\theta + Mg + mg$
(c) $N\cos\theta - Mg$ (d) $N\sin\theta + Mg + mg$

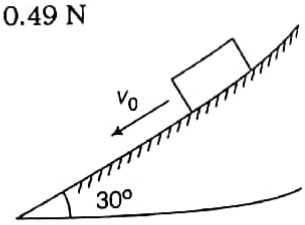
42. Starting from rest, a body slides down a 45° inclined plane in twice the time it takes to slide down the same distance in the absence of friction. The coefficient of friction between the body and the inclined plane is

(a) 0.2 (b) 0.25
(c) 0.75 (d) 0.5

43. A block of mass 0.1 kg is held against a wall applying a horizontal force of 5 N on the block. If the coefficient of friction between the block and the wall is 0.5, the magnitude of the frictional force acting the block is

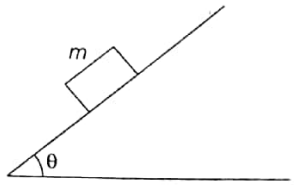
(a) 2.5 N (b) 0.98 N
(c) 4.9 N (d) 0.49 N

44. A block of mass m is given an initial downward velocity v_0 and left on an inclined place (coefficient of friction $=0.6$). The block will



(a) continue of move down the plane with constant velocity v_0
(b) accelerate downward
(c) decelerate and come to rest
(d) first accelerate downward then decelerate

45. A block of mass m is at rest on an inclined plane which is making angle θ with the horizontal. The coefficient of friction between the block and plane is μ. Then frictional force acting between the surfaces is



(a) $\mu\, mg$
(b) $\mu\, mg \sin\theta$
(c) $\mu\,(mg \sin\theta - mg\cos\theta)$
(d) $mg \sin\theta$

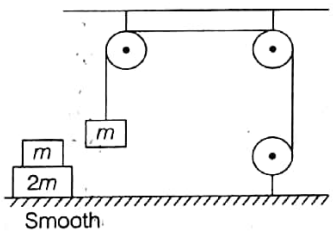
46. The upper half of an inclined plane of inclination θ is perfectly smooth while the lower half rough. A block starting from rest at the top of the plane will again come to rest at the bottom if the coefficient of friction between the block and the lower half of the plane is given by

(a) $\mu = 2\tan\theta$
(b) $\mu = \tan\theta$
(c) $\mu = \dfrac{2}{\tan\theta}$
(d) $\mu = \dfrac{1}{\tan\theta}$

47. Pushing force making an angle θ to the horizontal is applied on a block of weight w placed on a horizontal table. If the angle of friction is ϕ, the magnitude of force required to move the body is equal to

(a) $\dfrac{w\cos\theta}{\cos(\theta-\phi)}$
(b) $\dfrac{w\sin\theta}{\cos(\theta+\phi)}$
(c) $\dfrac{w\tan\phi}{\sin(\theta-\phi)}$
(d) $\dfrac{w\sin\phi}{\tan(\theta-\phi)}$

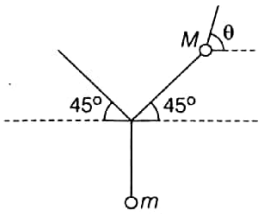
48. In the arrangement shown in figure, there is a friction force between the blocks of masses m and $2\,m$. The mass of the suspended block is m. The block of mass m is stationary with respect to block of mass $2\,m$. The minimum value of coefficient of friction between m and $2\,m$ is



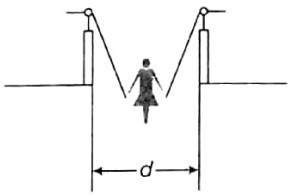
(a) $\dfrac{1}{2}$
(b) $\dfrac{1}{\sqrt{2}}$
(c) $\dfrac{1}{4}$
(d) $\dfrac{1}{3}$

49. Two masses m and M are attached with strings as shown. For the system to be in equilibrium, we have

(a) $\tan\theta = 1 + \dfrac{2M}{m}$
(b) $\tan\theta = 1 + \dfrac{2m}{M}$
(c) $\tan\theta = 1 + \dfrac{M}{2m}$
(d) $\tan\theta = 1 + \dfrac{m}{2M}$



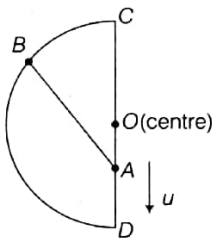
50. A man of mass m has fallen into a ditch of width d. Two of his friends are slowly pulling him out using a light rope and two fixed pulleys as shown in figure. Both the friends exert force of equal magnitudes F.



When the man is at a depth h, the value of F is

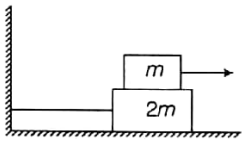
(a) $\dfrac{mg}{4h}\sqrt{d^2+4h^2}$
(b) hmg
(c) dmg
(d) $\dfrac{mg}{2h}\sqrt{h^2+d^2}$

51. Two beads A and B move along a semicircular wire frame as shown in figure . The beads are connected by an inelastic string which always remains tight. At an instant the speed of A is u, $\angle BAC = 45°$ and $\angle BOC = 75°$, where O is the centre of the semicircular arc. The speed of bead B at that instant is



(a) $\sqrt{2}u$
(b) u
(c) $\dfrac{u}{2\sqrt{2}}$
(d) $\sqrt{\dfrac{2}{3}}u$

52. Two blocks of masses m and $2m$ are placed one over the other as shown in figure.The coefficient of friction between m and $2m$ is μ and between $2m$ and ground is $\dfrac{\mu}{3}$. If a horizontal force F is applied on upper block and T is tension developed in string, then choose the incorrect alternative.

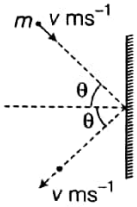


(a) If $F = \dfrac{\mu}{3}mg,\ T = 0$
(b) If $F = \mu mg,\ T = 0$
(c) If $F = 2\mu mg,\ T = \dfrac{\mu mg}{3}$
(d) If $F = 3\mu mg,\ T = 0$

53. If a body loses half of its velocity on penetrating 3 cm in a wooden block, then how much will it penetrate more before coming to rest?

(a) 1cm
(b) 2 cm
(c) 3 cm
(d) 4 cm

54. A ball is moving towards the wall as shown in diagram then momentum is conserved



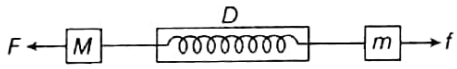
(a) along the wall
(b) along the perpendicular to the wall
(c) along any direction
(d) Both (a) and (b)

55. Two weights w_1 and w_2 are suspended from the ends of a light string passing over a smooth fixed pulley. If the pulley is pulled up at an acceleration g. The tension in the string will be

(a) $\frac{4w_1 w_2}{w_1 + w_2}$ (b) $\frac{2w_1 w_2}{w_1 + w_2}$

(c) $\frac{w_1 - w_2}{w_1 + w_2}$ (d) $\frac{w_1 w_2}{2(w_1 + w_2)}$

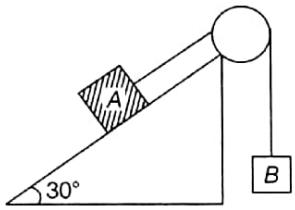
56. A dynamometer D is attached to two bodies of masses $M = 6$ kg and $m = 4$ kg. Forces $F = 20$N and $f = 10$N are applied to the masses as shown. The dynamometer reads



(a) 10N (b) 20 N

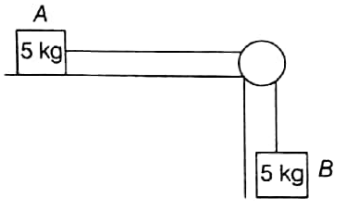
(c) 6 N (d) 14 N

57. Two blocks are connected over a massless pulley as shown in figure. The mass of block A is 10 kg and the coefficient of kinetic friction is 0.2. Block A sliders down the incline at constant speed. The mass of block B in kg is



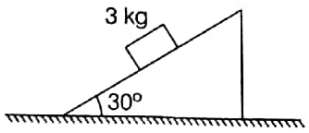
(a) 5.4 (b) 3.3 (c) 4.2 (d) 6.8

58. A block of mass 5 kg resting on a horizontal surface is connected by a cord, passing over a light frictionless pulley to a hanging block of mass 5 kg. The coefficient of kinetic friction between the block and the surface is 0.5. Tension in the cord is (take, $g = 9.8$ m s^{-2})



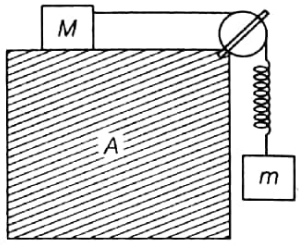
(a) 49 N (b) zero (c) 36.75 N (d) 12.75 N

59. A block of mass 3 kg is at rest on a rough inclined plane as shown in the figure. The magnitude of net force exerted by the surface on the block will be



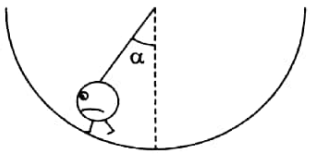
(a) $15\sqrt{3}$ N (b) 15 N (c) 10 N (d) 30 N

60. Two blocks of masses M and m are connected to each other by a massless string and spring of force constant k as shown in the figure. The spring passes over a frictionless pulley connected rigidly to the edge of a stationary block A. The coefficient of friction between block M and the plane horizontal surface of A is μ. The block M slides over the horizontal top surface of A and block m slides vertically downwards with the same speed. The mass M is equal to



(a) $\frac{m}{\mu}$ (b) $\frac{2m}{\mu}$ (c) $m\mu$ (d) μmg

61. An insect crawls up a hemispherical surface very slowly (see the figure). The coefficient of friction between the insect and the surface is 1/3. If the line joining the centre of the hemispherical surface to the insect makes an angle α with the vertical, the maximum possible value of α is given by



(a) $\cot\alpha = 3$ (b) $\tan\alpha = 3$

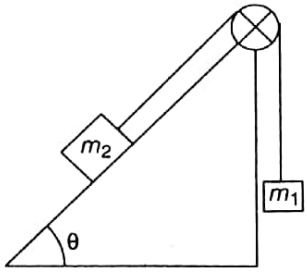
(c) $\sec\alpha = 3$ (d) $\text{cosec}\,\alpha = 3$

62. A block of mass m, lying on a rough horizontal plane, is acted upon by a horizontal force P and another force Q inclined at an angle θ to the vertical upwards. The block will remain in equilibrium, if minimum coefficient of friction between it and the surface is

(a) $\frac{(P + Q\sin\theta)}{(mg - Q\cos\theta)}$ (b) $\frac{(P\cos\theta + Q)}{(mg - Q\sin\theta)}$

(c) $\frac{(P - Q\cos\theta)}{(mg + Q\sin\theta)}$ (d) $\frac{(P\sin\theta - Q)}{(mg - Q\cos\theta)}$

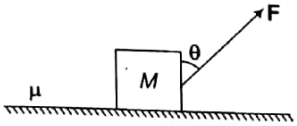
63. In the figure shown if coefficient of friction is μ, then m_2 will start moving upwards if



(a) $\frac{m_1}{m_2} > \sin\theta - \mu\cos\theta$ (b) $\frac{m_1}{m_2} > \sin\theta + \mu\cos\theta$

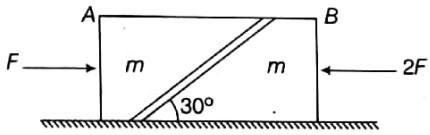
(c) $\frac{m_1}{m_2} > \mu\sin\theta - \cos\theta$ (d) $\frac{m_1}{m_2} > \mu\sin\theta + \cos\theta$

64. A block of mass M rests on a rough horizontal surface as shown. Coefficient of friction between the block and the surface is μ. A force $F = Mg$ acting at angle θ with the vertical side of the block pulls it. In which of the following cases the block can be pulled along the surface?



(a) $\tan\theta \geq \mu$
(b) $\tan\left(\frac{\theta}{2} \geq \mu\right)$
(c) $\cos\theta \geq \mu$
(d) $\cot\left(\frac{\theta}{2}\right) \geq \mu$

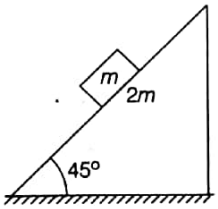
65. Two blocks A and B each of mass m are placed on a smooth horizontal surface. Two horizontal force F and $2F$ are applied on the blocks A and B respectively as shown in figure. The block A does not slide on block B.



Then the normal reaction acting between the two blocks is

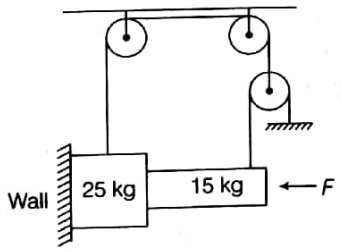
(a) F
(b) $\frac{F}{2}$
(c) $\frac{F}{\sqrt{3}}$
(d) $3F$

66. A block of mass m is placed on a wedge of mass $2m$ which rests on a rough horizontal surface. There is no friction between the block and the wedge. The minimum coefficient of friction between the wedge and the ground so that the wedge does not move is



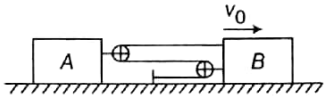
(a) 0.1
(b) 0.20
(c) 0.3
(d) 0.4

67. If coefficient of friction between all surfaces is 0.4, then find the minimum force F to have equilibrium of the system. (Take, $g = 10\,\text{ms}^{-2}$)



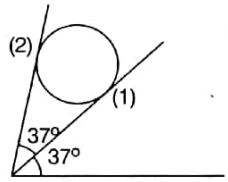
(a) 62.5 N
(b) 150 N
(c) 135 N
(d) 50 N

68. Block B moves to the right with a constant velocity v_0. The velocity of body A relative to B is



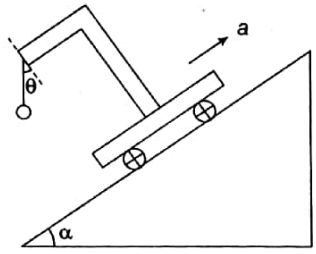
(a) $\frac{v_0}{2}$, towards left
(b) $\frac{v_0}{2}$, towards right
(c) $\frac{3v_0}{2}$, towards left
(d) $\frac{3v_0}{2}$, towards right

69. A sphere of mass m is held between two smooth inclined walls. For $\sin 37° = \frac{3}{5}$, the normal reaction of the wall (2) is equal to



(a) $\frac{16mg}{25}$
(b) $\frac{25mg}{39}$
(c) $\frac{39mg}{21}$
(d) None of these

70. A pendulum of mass m hangs from a support fixed to a trolley. The direction of the string (*i.e.*, angle θ) when the trolley rolls up a plane of inclination α with acceleration a is



(a) 0
(b) $\tan^{-1}\alpha$
(c) $\tan^{-1}\left(\frac{a + g\sin\alpha}{g\cos\alpha}\right)$
(d) $\tan^{-1}\frac{a}{g}$

71. A uniform rope of length l lies on a table. If the coefficient of friction is μ, then the maximum length l_1 of the hanging part of the rope which can overhang from the edge of the table without sliding down is

(a) l/μ
(b) $l/(\mu + 1)$
(c) $\mu l/(\mu + 1)$
(d) $\mu l/(\mu - 1)$

B) Medical entrance special format questions

Assertion and reason

Directions (Q. Nos. 1-20) *These questions consists of two statements each printed as Assertion and Reason. While answering these question you are required to choose any one of the following five responses*

(a) If both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If Assertion is false but Reason is true.
(e) If both Assertion and Reason are false.

1. **Assertion** The weighing machine measures the weight of a body.

   Reason Weightlessness means the absence of weight.

2. **Assertion** When a person walks on a rough surface, the net force exerted by surface on the person in the direction of his motion.

   Reason It is the force exerted by the road on the person that causes the motion.

3. **Assertion** Friction opposes motion of a body.

   Reason Static friction is self adjusting while kinetic friction is constant.

4. **Assertion** Friction always opposes the relative motion of two bodies.

   Reason Without friction also, one can move on a smooth surface.

5. **Assertion** A block of weight 10 N is pushed against a vertical wall by a force of 15 N. The coefficient of friction between the wall and the block is 0.6. Then, the magnitude of maximum frictional force is 9 N.

   Reason For given system block will remain stationary.

6. **Assertion** A body of mass 10 kg is placed on a rough inclined surface ($\mu = 0.7$). The surface is inclined to horizontal at angle 30°. Acceleration of the body down the plane will be zero.

   Reason Work done by friction is always negative.

7. **Assertion** During free fall of a person one feels weightlessness because his weight becomes zero.

   Reason He falls with an acceleration of g.

8. **Assertion** Earth is an inertial frame.

   Reason A frame in motion is sometimes an inertial frame and sometimes a non-inertial.

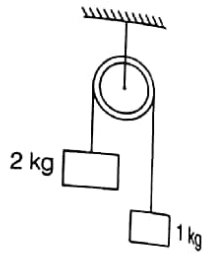
9. **Assertion** Static friction acting on a body is always greater than the kinetic friction acting on this body.

   Reason Coefficient of static friction is more than the coefficient of kinetic friction.

10. **Assertion** A string has a mass m. If it is accelerated, tension is non-uniform and if it is not accelerated, tension is uniform.

    Reason Tension force is an electromagnetic force.

11. **Assertion** In the diagram shown in figure, string is massless and pulley is smooth, Net force on 1 kg block is $\frac{10}{3}$ N.

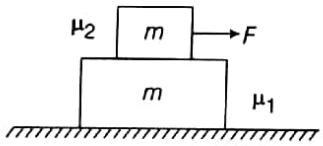
    

    Reason Net force on both the blocks will be same.

12. **Assertion** A particle is thrown vertically upwards. If air resistance is taken into consideration then retardation in upward journey is more than the acceleration in downward journey.

    Reason Some mechanical energy is lost in the form of heat due to air friction.

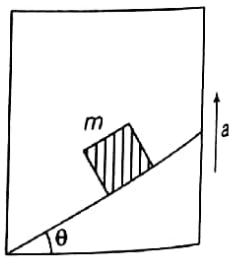
13. **Assertion** In the system of two blocks of equal masses as shown, the coefficient of friction between the blocks (μ_2) is less than coefficient of friction (μ_1) between lower block and ground.

    For all values of force F applied on upper block, lower block remains at rest.

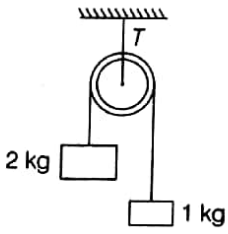
    Reason Frictional force on lower block due to upper block is not sufficient to overcome the frictional force on lower block due to ground.

14. **Assertion** In the figure shown, block of mass m is stationary with respect to lift. Force of friction acting on the block is greater than $mg \sin\theta$.

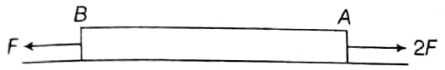
    Reason If lift moves with constant velocity, then force of friction is equal to $mg \sin\theta$.

15. **Assertion** In the shown figure, tension T connected to the ceiling is $10\text{ N} < T < 20\text{ N}$

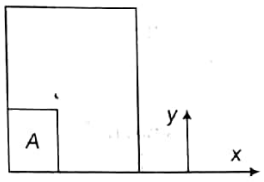
    Reason System is not stationary.

16. **Assertion** Two forces are acting on a rope lying on a smooth table as shown in figure. In moving from A to B ; tension on string decreases from $2F$ to F.



Reason Situation will becomes in determinant; if we take it a massless string.

17. **Assertion** Block A is resting on one corner of a box as shown in figure. Acceleration of box is $(2\hat{i} + 2\hat{j})\,\text{ms}^{-2}$. Let N_1 is the normal reaction on block from vertical wall and N_2 from ground of box. Then $\frac{N_1}{N_2} = \frac{1}{6}$. (Neglect friction)



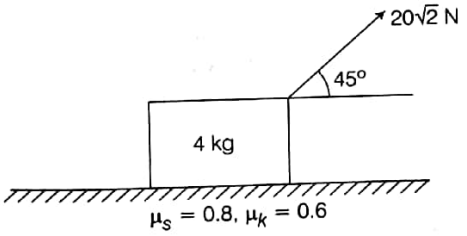
Reason $N_1 = 0$, if lift is stationary.

Match the columns

1. Match the following columns.

Column I	Column II
(A) Force of friction	(p) Opposes motion
(B) Normal reaction on a block kept on horizontal ground	(q) Opposes relative motion
	(r) Is always mg
	(s) May be equal to mg
	(t) None

2. In the diagram shown in figure, match the following columns (take $g = 10\,\text{ms}^{-2}$)

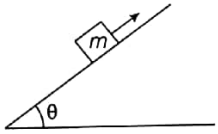


Column I	Column II
(A) Normal reaction	(p) 12 SI unit
(B) Force of friction	(q) 20 SI unit
(C) Acceleration of block	(r) zero
	(s) 2 SI unit

3. Velocity of three particles A, B and C varies with time t as, $\mathbf{v}_A = (2t\,\hat{i} + 6\hat{j})\,\text{ms}^{-1}$, $\mathbf{v}_B = (3\hat{i} + 4\hat{j})\,\text{ms}^{-1}$ and $\mathbf{v}_C = (6\hat{i} - 4t\hat{j})$. Regarding the pseudo force match the following columns.

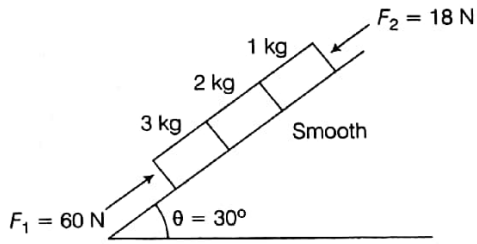
Column I	Column II
(A) On A as observed by B	(p) Along positive x-direction
(B) On B as observed by C	(q) Along negative x-direction
(C) On A as observed by C	(r) Along positive y-direction
(D) On C as observed by A	(s) Along negative y-direction
	(t) Zero

4. A block of mass m is thrown upwards with some initial velocity as shown in figure. On the block



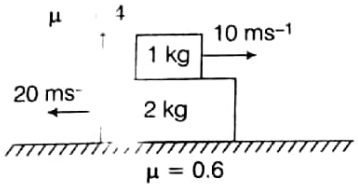
Column I	Column II
(A) Net force in horizontal direction	(p) Zero
(B) Net force in vertical direction	(q) $m(g\sin\theta + \mu g\cos\theta)$
(C) Net force along the plane	(r) $m(g\sin\theta\cos\theta + \mu g\cos^2\theta)$
(D) Net force perpendicular to plane	(s) $m(g\sin^2\theta + \mu g\sin\theta\cos\theta)$

5. In the diagram shown in figure, match the following columns. (take, $g = 10\,\text{ms}^{-2}$)



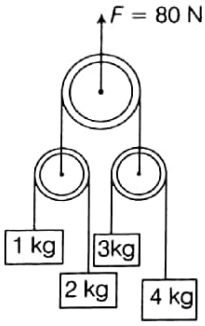
Column I	Column II
(A) Acceleration of 2 kg block	(p) 8 SI unit
(B) Net force on 3 kg block	(q) 25 SI unit
(C) Normal reaction between 2 kg and 1 kg	(r) 2 SI unit
(D) Normal reaction between 3 kg and 2 kg	(s) 45 N
	(t) None

6. In the diagram shown in figure. Match the following columns.



	Column I		Column II
(A)	Absolute acceleration of 1 kg block	(p)	11 ms^{-2}
(B)	Absolute acceleration of 2 kg block	(q)	6 ms^{-2}
(C)	Relative acceleration between the two	(r)	17 ms^{-2}
		(s)	None

7. In the diagram shown in figure, all pulleys are smooth and massless and strings are light. Match the following columns.

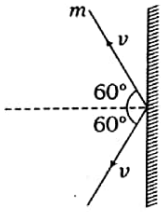


	Column I		Column II
(A)	1 kg block	(p)	will remain stationary
(B)	2 kg block	(q)	will move down
(C)	3 kg block	(r)	will move up
(D)	4 kg block	(s)	5 ms^{-2}
		(t)	10 ms^{-2}

C Medical entrances' gallery

(Collection of questions asked in NEET & various medical entrance exams)

1. A rigid ball of mass m strikes a rigid wall at 60° and gets reflected without loss of speed as shown in the figure. The value of impulse imparted by the wall on the ball will be



(a) mv (b) $2mv$
(c) $mv/2$ (d) $mv/3$

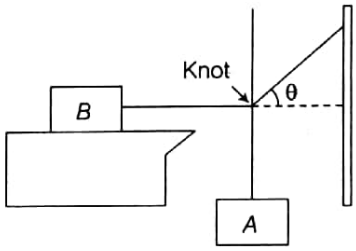
2. A body of mass 40 kg resting on rough horizontal surface is subjected to a force P which is just enough to start the motion of the body. If $\mu_s = 0.5, \mu_k = 0.4$, $g = 10\ ms^{-2}$ and the force P is continuously applied on the body, then the acceleration of the body is **[AIIMS 2015]**

(a) zero (b) 1 ms^{-2}
(c) 2 ms^{-2} (d) 2.4 ms^{-2}

3. A balloon of mass 10 kg is raising up with an acceleration of 20 ms^{-2}. If a mass of 4 kg is removed from the balloon its acceleration becomes (take, $g = 10\ ms^{-2}$) **[EAMCET 2015]**

(a) 40 ms^{-2} (b) 20 ms^{-2}
(c) 30 ms^{-2} (d) 12 ms^{-2}

4. Block B lying on a table weighs w. The coefficient of static friction between the block and the table is μ. Assume that the cord between B and the knot is horizontal. The maximum weight of the block A for which the system will be stationary is **[WB JEE 2015]**

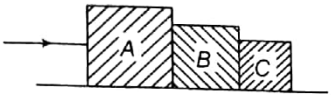


(a) $\frac{w \tan\theta}{\mu}$ (b) $\mu w \tan\theta$
(c) $\mu w\sqrt{1+\tan^2\theta}$ (d) $\mu w \sin\theta$

5. A stone of mass 0.05 kg is thrown vertically upwards. What is the direction and magnitude of net force on the stone during its upward motion? **[KCET 2015]**

(a) 0.98 N vertically downwards
(b) 0.49 N vertically downwards
(c) 9.8 N vertically downwards
(d) 0.49 N vertically upwards

6. Three blocks A, B and C of masses 4 kg, 2 kg and 1 kg respectively, are in contact on a frictionless surface, as shown. If a force of 14 N is applied on the 4 kg block, then the contact force between A and B is **[CBSE AIPMT 2015]**



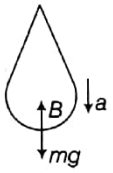
(a) 2 N (b) 6 N (c) 8 N (d) 18 N

7. A block A of mass m_1 rests on a horizontal table. A light string connected to it passes over a frictionless pulley at the edge of table and from its other end another block B of mass m_2 is suspended. The coefficient of kinetic friction between the block and the table is μ_k. When the block A is sliding on the table, the tension in the string is **[CBSE AIPMT 2015]**

(a) $\frac{(m_2 + \mu_k m_1)g}{(m_1 + m_2)}$ (b) $\frac{(m_2 - \mu_k m_1)g}{(m_1 + m_2)}$

(c) $\frac{m_1 m_2 (1 + \mu_k) g}{(m_1 + m_2)}$ (d) $\frac{m_1 m_2 (1 - \mu_k) g}{(m_1 + m_2)}$

8. A balloon with mass m is descending down with an acceleration a (where, $a < g$). How much mass should be removed from it so that it starts moving up with an acceleration a? **[CBSE AIPMT 2014]**



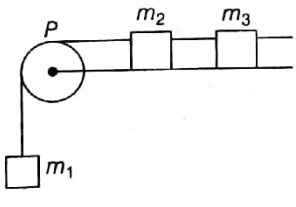
(a) $\frac{2ma}{g+a}$ (b) $\frac{2ma}{g-a}$ (c) $\frac{ma}{g+a}$ (d) $\frac{ma}{g-a}$

9. A bullet moving with a velocity of $30\sqrt{2}$ ms^{-1} is fired into a fixed target. It penetrated into the target to the extent of s metres. If the same bullet is fired into a target of thickness $\frac{s}{2}$ metres and of the same material with the same velocity, then the bullet comes out of the target with velocity **[EAMCET 2014]**

(a) 20 ms^{-1} (b) 30 ms^{-1}

(c) $20\sqrt{2}$ ms^{-1} (d) $10\sqrt{2}$ ms^{-1}

10. A system consists of three masses m_1, m_2 and m_3 connected by a string passing over a pulley P. The mass m_1 hangs freely and m_2 and m_3 are on a rough horizontal table (the coefficient of friction $= \mu$). The pulley is frictionless and of negligible mass. The downward acceleration of mass m_1 is (assume, $m_1 = m_2 = m_3 = m$) **[CBSE AIPMT 2014]**

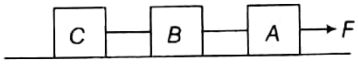


(a) $\frac{g(1-g\mu)}{9}$ (b) $\frac{2g\mu}{3}$ (c) $\frac{g(1-2\mu)}{3}$ (d) $\frac{g(1-2\mu)}{2}$

11. A block is pushed momentarily on a horizontal surface with initial velocity v. If μ is the coefficient of sliding friction between the block and surface; the block will come to rest after time (g = acceleration due to gravity) **[MHT CET 2014]**

(a) $\frac{v}{\mu g}$ (b) $\frac{vg}{g}$ (c) $\frac{v\mu}{g}$ (d) $\frac{\mu g}{v}$

12. Three identical blocks of masses $m = 2$ kg are drawn by a force 10.2 N on a frictionless surface. What is the tension (in N) in the string between the blocks B and C?



[UK PMT 2014]

(a) 9.2 (b) 8 (c) 3.4 (d) 9.8

13. A wooden block of mass 8 kg slides down an inclined plane of inclination 30° to the horizontal with constant acceleration 0.4 ms^{-2}. The force of friction between the block and inclined plane is (take, $g = 10$ ms^{-2}) **[MHT CET 2014]**

(a) 12.2 N (b) 24.4 N

(c) 36.8 N (d) 48.8 N

14. To determine the coefficient of friction between a rough surface and a block, the surface is kept inclined at 45° and the block is released from rest. The block takes a time t in moving a distance d. The rough surface is then replaced by a smooth surface and the same experiment is repeated. The block now takes a time $t/2$ in moving down the same distance d. The coefficient of friction is **[WB JEE 2014]**

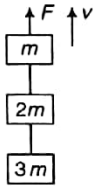
(a) 3/4 (b) 5/4 (c) 1/2 (d) $1/\sqrt{2}$

15. A body of mass m is placed on a rough surface with coefficient of friction μ inclined at θ. If the mass is in equilibrium, then **[KCET 2014]**

(a) $\theta = \tan^{-1}\mu$ (b) $\theta = \tan^{-1}\left(\frac{1}{\mu}\right)$

(c) $\theta = \tan^{-1}\frac{m}{\mu}$ (d) $\theta = \tan^{-1}\frac{\mu}{m}$

16. Three blocks with masses m, $2m$ and $3m$ are connected by strings, as shown in the figure. After an upward force F is applied on block m, the masses move upward at constant speed v. What is the net force on the block of mass $2m$? (g is the acceleration due to gravity) **[NEET 2013]**



(a) Zero (b) 2 mg (c) 3 mg (d) 6 mg

17. The upper half of an inclined plane of inclination θ is perfectly smooth while lower half is rough. A block starting from rest at the top of the plane will again come to rest at the bottom, if the coefficient of friction between the block and lower half of the plane is given by **[NEET 2013]**

(a) $\mu = \frac{1}{\tan\theta}$ (b) $\mu = \frac{2}{\tan\theta}$

(c) $\mu = 2\tan\theta$ (d) $\mu = \tan\theta$

18. Newton's second law of motion is **[J&K CET 2013]**

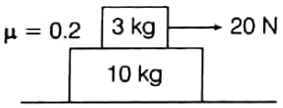
(a) $\mathbf{F} = d\mathbf{p}/dt$ (b) $\mathbf{F} = mv$

(c) $\mathbf{F} = mv^2$ (d) $\mathbf{F} = m^2 v$

19. A balloon starting from rest ascends vertically with uniform acceleration to a height of 100 m in 10 s. The force on the bottom of the balloon by a mass of 50 kg is ($g = 10\ ms^{-2}$) **[EAMCET 2013]**

(a) 100 N (b) 300 N
(c) 600 N (d) 400 N

20. A 3 kg block is placed over a 10 kg block and both are placed on a smooth horizontal surface. The coefficient of friction between the blocks is 0.2. If a horizontal force of 20 N is applied to 3 kg block, accelerations of the two blocks in ms^{-2} are (take, $g = 10\ ms^{-2}$) **[EAMCET 2013]**



(a) $\frac{13}{4}$, 0.6 (b) $\frac{14}{4}$, 3
(c) $\frac{13}{4}$, 3 (d) $\frac{14}{3}$, 0.6

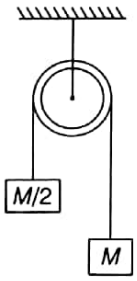
21. A 60 kg person is weighed by a balance as 54 kg in a lift which is accelerated downwards. The acceleration of the lift is **[Kerala CEE 2013]**

(a) 1.26 ms^{-2} (b) 1.76 ms^{-2}
(c) 1.98 ms^{-2} (e) 0.98 ms^{-2}
(e) None of these

22. A lift starting from rest with a constant upward acceleration moves 1.5 m in the 0.4 s. If a person standing in the lift holds a packet of 2 kg by a string then the tension in the string due to motion is **[UP CPMT 2013]**

(a) 5.89 N (b) 57.1 N
(c) 6.71 N (d) None of these

23. Two masses M and $M/2$ are joined together by means of a light in extensible string passed over a pulley as shown in figure. If the bigger mass is released, then the small one will ascend with an acceleration of **[UP CPMT 2013]**



(a) a (b) $\frac{g}{2}$
(c) $\frac{g}{3}$ (d) $\frac{3g}{2}$

24. When a body of mass m_1 moving with uniform velocity 40 ms^{-1} collides with another body of mass m_2 at rest, then the two together begin to move with uniform velocity of 30 ms^{-1}. The ratio of the masses $\left(i.e., \frac{m_1}{m_2}\right)$ of the two bodies will be **[BCECE (Mains) 2012]**

(a) 1 : 3 (b) 3 : 1
(c) 1 : 1.33 (d) 1 : 0.75

25. A lift is moving downwards with an acceleration equal to acceleration due to gravity. If a body of mass M kept on the floor of the lift is pulled horizontally (coefficient of friction $= \mu$), then the frictional resistance offered by the body is **[BCECE 2012]**

(a) μMg (b) $2\mu Mg$
(c) zero (d) Mg

26. An inclined plane of height h and length l have the angle of inclination θ. The time taken by a body to come from the top to the bottom of this inclined plane will be **[BCECE 2012]**

(a) $\sin\theta\sqrt{\frac{2h}{g}}$ (b) $\frac{1}{\sin\theta}\sqrt{\frac{2h}{g}}$
(c) $\sqrt{\frac{2h}{g}}$ (d) $\sqrt{\frac{2l}{g}}$

27. If a coin is dropped in a lift it takes t_1 time to reach the floor and takes t_2 time when lift is moving up with constant acceleration, then which one of the following relation is correct? **[BCECE (Mains) 2012]**

(a) $t_1 = t_2$ (b) $t_1 > t_2$
(c) $t_2 > t_1$ (d) $t_1 >> t_2$

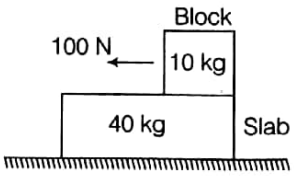
28. If a body of mass 5 g initially at rest is acted upon by a force of 50 dynes for 3 s, then the impulse will be **[BCECE (Mains) 2012]**

(a) 0.98×10^{-3} Ns (b) 1.5×10^{-3} Ns
(c) 2×10^{-3} Ns (d) 2.5×10^{-3} Ns

29. A 60 kg mass is pushed with a enough force to start it moving and the same force is continued to act afterwards. If the coefficient of static friction and sliding friction are 0.5 and 0.4 respectively, then the acceleration of the body will be **[BCECE (Mains) 2012]**

(a) $1\ ms^{-2}$ (b) $3.9\ ms^{-2}$
(c) $4.9\ ms^{-2}$ (d) $6\ ms^{-2}$

30. A 40 kg slab rests on a frictionless floor. A 10 kg block rests on the top of the slab as shown in figure. The static coefficient of friction between the block and the slab is 0.60 while the kinetic coefficient is 0.40. The 10 kg block is acted upon by a horizontal force of 100 N. If $g = 9.8\ ms^{-2}$, the resulting acceleration of the slab will be **[BHU 2012]**



(a) 6.1 ms^{-2} (b) 1.52 ms^{-2}
(c) 1.47 ms^{-2} (d) 0.98 ms^{-2}

31. A body of mass 0.25 kg is projected with muzzle velocity 100 ms^{-1} from a tank of mass 100 kg. What is the recoil velocity of the tank? **[AIIMS 2012]**

(a) 5 ms^{-1} (b) 25 ms^{-1}
(c) 0.5 ms^{-1} (d) 0.25 ms^{-1}

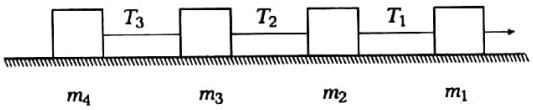
32. A rocket with a lift-off mass 3.5×10^4 kg is blast upward with an initial acceleration of 10 ms^{-2}. Then, the initial thrust of the blast is **[AIIMS 2012]**

(a) 1.75×10^5 N
(b) 3.5×10^5 N
(c) 7×10^5 N
(d) 14×10^5 N

33. A marble block of mass 2 kg lying on ice when given a velocity of 6 ms^{-1} is stopped by friction is 10 s. Then the coefficient of friction is **[AIIMS 2012]**

(a) 0.01 (b) 0.02
(c) 0.03 (d) 0.06

34. Four blocks of same mass connected by cords are pulled by a force F on a smooth horizontal surface, as shown in figure. The tensions T_1, T_2 and T_3 will be **[AFMC 2012]**

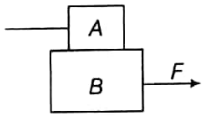


(a) $T_1=\frac{1}{4}F,\ T_2=\frac{3}{2}F,\ T_3=\frac{1}{4}F$
(b) $T_1=\frac{1}{4}F,\ T_2=\frac{1}{2}F,\ T_3=\frac{1}{2}F$
(c) $T_1=\frac{3}{4}F,\ T_2=\frac{1}{2}F,\ T_3=\frac{1}{4}F$
(d) $T_1=\frac{3}{4}F,\ T_2=\frac{1}{2}F,\ T_3=\frac{1}{2}F$

35. A body is hanging from a rigid support by an inextensible string of length l. It is struck inelastically by an identical body of mass m with horizontal velocity $v=\sqrt{2gl}$, the tension in the string increases just after striking by **[AFMC 2012]**

(a) mg (b) $3\ mg$
(c) $2\ mg$ (d) None of these

36. A block A of mass 100 kg rests on another block B of mass 200 kg and is tied to a wall as shown in the figure. The coefficient of friction between A and B is 0.2 and that between B and the ground is 0.3. The minimum force F required to move the block B is (take, $g=10\ ms^{-2}$) **[AFMC 2012]**



(a) 900 N (b) 200 N
(c) 1100 N (d) 700 N

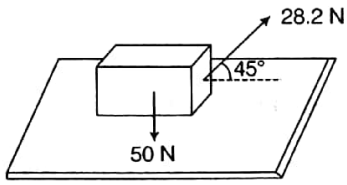
37. A machine gun fires a bullet of mass 40 g with a velocity 1200 ms^{-1}. The man holding it can exert a maximum force of 144 N on the gun. How many bullets can be fired per second at the most? **[AFMC 2012]**

(a) Only one
(b) Three
(c) Can fire any number of bullets
(d) 144×48

38. A monkey of 25 kg is holding a vertical rope. The rope does not break if a body of mass 30 kg is suspended from it, but the rope breaks if the mass of body suspended with the rope exceeds 30 kg. What will be the maximum acceleration with which the monkey can climb up along the rope? (Take, $g=10\ ms^{-2}$) **[JCECE 2012]**

(a) 2 ms^{-2} (b) 2.5 ms^{-2}
(c) 3 ms^{-2} (d) 4 ms^{-2}

39. A body of weight 50 N placed on a horizontal surface is just moved by a force of 28.2 N. The frictional force and normal reaction are **[JCECE 2012]**



(a) 2 N, 3N (b) 5 N, 6 N
(c) 10 N, 15 N (d) 20 N, 30 N

Answers

Check point 5.1

1. (a)	**2.** (c)	**3.** (d)	**4.** (a)	**5.** (a)	**6.** (a)	**7.** (b)	**8.** (a)	**9.** (b)	**10.** (a)
11. (a)	**12.** (a)	**13.** (a)	**14.** (b)	**15.** (a)	**16.** (b)	**17.** (c)	**18.** (d)	**19.** (d)	**20.** (c)

Check point 5.2

1. (b,c,d)	**2.** (d)	**3.** (b)	**4.** (c)	**5.** (c)	**6.** (d)	**7.** (c)	**8.** (d)	**9.** (a)	**10.** (a)
11. (a)	**12.** (d)	**13.** (c)	**14.** (c)	**15.** (a)					

Check point 5.3

1. (c)	**2.** (d)	**3.** (c)	**4.** (d)	**5.** (d)	**6.** (d)	**7.** (b)	**8.** (b)	**9.** (c)	**10.** (b)
11. (d)	**12.** (b)	**13.** (d)	**14.** (c)	**15.** (b)	**16.** (b)	**17.** (b)	**18.** (c)	**19.** (c)	**20.** (a)

Check point 5.4

1. (a)	**2.** (d)	**3.** (d)	**4.** (a)	**5.** (c)	**6.** (d)	**7.** (c)	**8.** (a)	**9.** (d)	**10.** (a)
11. (a)	**12.** (b)	**13.** (a)	**14.** (a)	**15.** (a)	**16.** (d)	**17.** (d)	**18.** (a)	**19.** (a)	**20.** (a)

A. Taking it together

1. (c)	**2.** (b)	**3.** (c)	**4.** (b)	**5.** (b)	**6.** (c)	**7.** (b)	**8.** (c)	**9.** (b)	**10.** (a)
11. (a)	**12.** (b)	**13.** (d)	**14.** (c)	**15.** (b)	**16.** (b)	**17.** (b)	**18.** (d)	**19.** (b)	**20.** (a)
21. (c)	**22.** (a)	**23.** (b)	**24.** (a, c)	**25.** (a)	**26.** (c)	**27.** (c)	**28.** (d)	**29.** (c)	**30.** (a)
31. (c)	**32.** (a)	**33.** (a)	**34.** (a)	**35.** (d)	**36.** (d)	**37.** (a)	**38.** (b)	**39.** (a)	**40.** (c)
41. (a)	**42.** (c)	**43.** (b)	**44.** (c)	**45.** (d)	**46.** (a)	**47.** (b)	**48.** (c)	**49.** (a)	**50.** (a)
51. (a)	**52.** (c)	**53.** (a)	**54.** (a)	**55.** (a)	**56.** (d)	**57.** (b)	**58.** (c)	**59.** (d)	**60.** (a)
61. (a)	**62.** (a)	**63.** (b)	**64.** (d)	**65.** (d)	**66.** (b)	**67.** (a)	**68.** (b)	**69.** (d)	**70.** (c)
71. (c)									

B. Medical entrance special format questions

Assertion and reason

1. (b)	**2.** (a)	**3.** (d)	**4.** (d)	**5.** (c)	**6.** (c)	**7.** (d)	**8.** (d)	**9.** (a)	**10.** (d)
11. (c)	**12.** (b)	**13.** (a)	**14.** (b)	**15.** (b)	**16.** (b)	**17.** (a)			

Match the columns

1. (A → t, B → s)
2. (A → q , B → p, C → s)
3. (A → t, B → r, C → r, D → q)
4. (A → r, B → s, C → q, D → p)
5. (A → r, B → t, C → q, D → t)
6. (A → s, B → p, C → s)
7. (A → (r, t) B → p, C → q, D → (q,s)

C. Medical entrances' gallery

1. (a)	**2.** (b)	**3.** (a)	**4.** (b)	**5.** (b)	**6.** (b)	**7.** (c)	**8.** (a)	**9.** (b)	**10.** (c)
11. (a)	**12.** (c)	**13.** (c)	**14.** (a)	**15.** (a)	**16.** (a)	**17.** (c)	**18.** (a)	**19.** (c)	**20.** (d)
21. (d)	**22.** (b)	**23.** (c)	**24.** (b)	**25.** (c)	**26.** (b)	**27.** (b)	**28.** (b)	**29.** (a)	**30.** (d)
31. (d)	**32.** (c)	**33.** (d)	**34.** (c)	**35.** (c)	**36.** (c)	**37.** (b)	**38.** (a)	**39.** (d)	

Hints and explanations

Check point 5.1

3. (*d*) Given, mass = 6 kg

velocity $\mathbf{v} = v_2 - v_1 = 5 - 3 = 2$

$\therefore$ Momentum $p = m\mathbf{v} = 6 \times 2 = 12$ N-s

4. (*a*) Given, $p = 2 + 3t^2$,

Differentiate w.r.t. t we get

$$\frac{dp}{dt} = 0 + 3 \times 2t = 6t$$

If $t = 3$ s,

then $$\frac{dp}{dt} = 6 \times 3 = 18\,\text{N}$$

$\therefore$ Force acting on the particle = 18 N

6. (*a*) $\mathbf{F} = m\mathbf{a}$ or $m = \dfrac{|\mathbf{F}|}{|\mathbf{a}|} = \dfrac{|6\hat{\mathbf{i}} - 8\hat{\mathbf{j}} + 10\hat{\mathbf{k}}|}{\sqrt{2}}$

$$= \frac{\sqrt{6^2 + 8^2 + 10^2}}{\sqrt{2}} = 10\,\text{kg}$$

7. (*b*) Given, $F = 72$ dyne

$F = 72 \times 10^{-5}\,\text{Nm}^{-2}$

$\theta = 60°$

$m = 9g = 9 \times 10^{-3}$ kg

$F' = ma$

In this case $F' = F \cos 60°$

$$a = \frac{F'}{m} = \frac{F \cos\theta}{9 \times 10^{-3}}$$

$$= \frac{72 \times 10^{-5} \times \cos 60°}{9 \times 10^{-3}} = 8 \times 10^{-2} \times \frac{1}{2}$$

$$= 4 \times 10^{-2}\,\text{ms}^{-1} = 4\,\text{cms}^{-1}$$

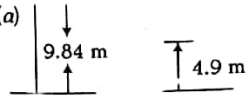
11. (*a*) Given, $F = 50$ N, $m = 20$ kg, $v = 15\,\text{ms}^{-1}$

$$\text{Impulse } F = \frac{mv}{\Delta t}$$

$$\text{Time } \Delta t = \frac{mv}{F}$$

$$\Delta t = \frac{20 \times 15}{50} = 6\,\text{s}$$

12. (*a*)



$$v = u^2 + 2gh$$

$$v^2 = 0 + 2 \times 9.8 \times 9.8$$

$$v^2 = \sqrt{2 \times 9.8 \times 9.8} \Rightarrow v = 9.8\sqrt{2}$$

Again

$$v^2 = u^2 - 2gh$$

$$0 = u^2 - 2 \times 9.8 \times 4.9$$

$$u^2 = \sqrt{(9.8)^2}$$

$$u = 9.8$$

$$\text{Impulse} = m[v - (-u)]$$

$$= 1[9.8\sqrt{2} + 9.8] = 9.8(\sqrt{2} + 1)$$

$$= 9.8 \times 2.4 = 23.52\,\text{N-s}$$

$$\text{Force} = \frac{\text{Impulse}}{\text{Time}}$$

$$F = \frac{23.52}{0.1} = 235.2\,\text{N}$$

13. (*a*) $F_{av} = ma_{av} = m\dfrac{\Delta v}{\Delta t} = \dfrac{0.5[2 - (-2)]}{10^{-3}} = 2000$ N

14. (*b*) $v_i = -v \sin 60\,\hat{\mathbf{i}} - v \cos 60\,\hat{\mathbf{j}}$

$v_f = -v \sin 60\,\hat{\mathbf{i}} + v \cos 60\,\hat{\mathbf{j}}$

$\therefore$ $\mathbf{v} = \mathbf{v}_f - \mathbf{v}_i = +2 \times 20 \times \dfrac{\sqrt{3}}{2} = 20\sqrt{3}\,\text{ms}^{-1}$

15. (*a*) Mass $m = 5$ kg

Change in velocity $v_f - v_i = [(10 - 2)\hat{\mathbf{i}} + (6 - 6)\hat{\mathbf{j}}]$

Change in momentum $= m\Delta v = 5[8\hat{\mathbf{i}}] = 40\,\hat{\mathbf{i}}\,\text{kgms}^{-1}$

16. (*b*) Impulse is defined as rate of change of momentum. For change in momentum to be minimum

$$\frac{d}{dt}(20t^2 - 40t) = 0$$

$$40t - 40 = 0$$

$$t = 1\,\text{s}$$

17. (*c*) From law of conservation of momentum,

$$p_i = p_f$$

and initial momentum, $p_i = mu = m(0) = 0$

$\therefore$ p_f should also be zero.

Hence, other piece will move in negative x-direction.

18. (*d*) Conservation of linear momentum gives

$m_1v_1 + m_2v_2 = 0$

$$m_1v_1 = -m_2v_2 \Rightarrow v_1 = \frac{-m_2v_2}{m_1}$$

Given, $m_1 = 10\,\text{g} = \left(\dfrac{10}{1000}\right)$ kg

$m_2 = 1$kg and $v_2 = -5$ m/s

∴ Velocity of muzzle,

$$v_1 = \frac{+1 \times 5}{10/1000} = 500 \text{ m/s}$$

19. (*d*) From the law of conservation of momentum,

total initial momentum = final momentum

$\Rightarrow \quad m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$\therefore \quad 0.1 \times 0 + 50 \times 0 = 0.1 \times 100 + 50(-v_2)$

$\Rightarrow \quad 0 = 10 - 50v_2$

$\therefore \quad v_2 = \frac{10}{50} = 0.2 \text{ ms}^{-1}$

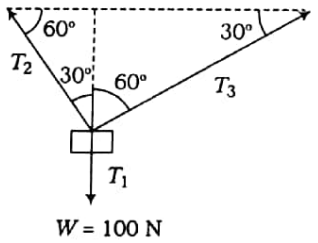
20. (*c*) F_{net} = mass × acceleration

$= 60 \times 1 = 60$ N

Check point 5.2

2. (*d*) $T_3 \sin 60° = T_2 \sin 30°$

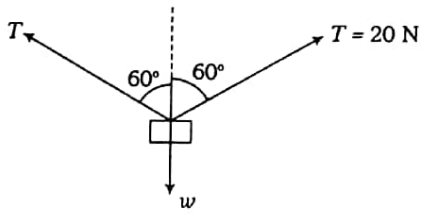
$\therefore \quad T_3 = \frac{T_2}{\sqrt{3}}$



Now, $T_2 \cos 30° + T_3 \cos 60° = T_1 = 100$

or $\frac{\sqrt{3}T_2}{2} + \frac{T_2}{\sqrt{3}} \times \frac{1}{2} = 100$ or $\frac{8T_2}{4\sqrt{3}} = 100$ or $T_2 = 50\sqrt{3}$ N

3. (*b*) $2T \cos 60° = w$



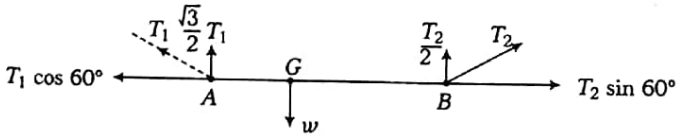
or $\quad w = T$ or $w_{max} = T_{max} = 20$ N

4. (*c*) Equilibrium of m: $\quad T = mg$ ... (i)

Equilibrium of $\sqrt{2}m$: $2T \cos \theta = \sqrt{2}mg$ ... (ii)

Solving these two equations we get $\theta = 45°$

5. (*c*) $T_1 \cos 60° = T_2 \sin 60° \Rightarrow \frac{T_1}{2} = \frac{\sqrt{3}T_2}{2}$



or $\quad T_1 \cos 60° = T_2 \sin 60°$

or $\quad T_1 = \sqrt{3}T_2$ ...(i)

$$\frac{\sqrt{3}T_1}{2} \times AG = \frac{T_2}{2} \times BG$$

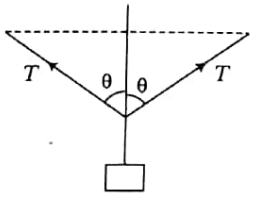
$\therefore \quad \frac{AG}{BG} = \frac{T_2}{\sqrt{3}T_1}$

But $\quad \frac{T_2}{T_1} = \frac{1}{\sqrt{3}}$ [from Eq. (i)]

Hence, $\quad \frac{AG}{BG} = \frac{1}{3}$

6. (*d*) For equilibrium of body,

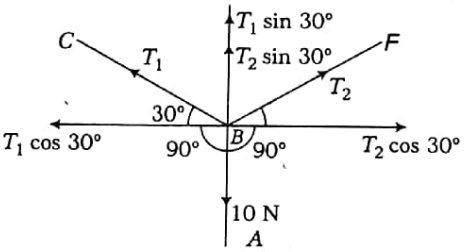
$$mg = 2T \cos \theta \Rightarrow T = \frac{mg}{2 \cos \theta}$$



For the string to be horizontal, $\theta = 90°$

$\therefore \quad T = \frac{mg}{2 \cos 90°} \Rightarrow T = \infty$

7. (*c*) As shown in figure



$T_1 \cos 30° = T_2 \cos 30°$

$\therefore \quad T_1 = T_2 = T$ (let)

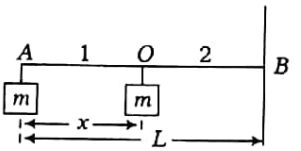
Again, $T_1 \sin 30° + T_2 \sin 30° = 10$ and $2T \sin 30° = 10$

$\Rightarrow \quad 2T \cdot \frac{1}{2} = 10 \Rightarrow T = 10$ N

Thus, the tension in section BC and BF are 10 N and 10 N respectively.

8. (*d*) Let x be distance from A to O and L the total length of string. Then, ratio of tensions is

$$\frac{T_1}{T_2} = \frac{(L - x)}{L} = \frac{3 - 2}{3} = \frac{1}{3}$$



9. (*a*) $F = ma$

Resultant of three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ will be $(\sqrt{2} - 1)F$.

Therefore, acceleration of body is also $(\sqrt{2} - 1)a$.

10. (*a*) Minimum additional force needed

$F = -(F_{resultant})_x$

$F_{resultant} = [(4 - 2)(\cos 30\hat{\mathbf{j}} - \sin 30\hat{\mathbf{i}}) + (\cos 60\hat{\mathbf{i}} + \sin 60\hat{\mathbf{j}})]$

$$= \left[2\left(\frac{\sqrt{3}}{2}\hat{\mathbf{j}} - \frac{1}{2}\hat{\mathbf{i}}\right) + \left(\frac{1}{2}\hat{\mathbf{i}} + \frac{\sqrt{3}}{2}\hat{\mathbf{j}}\right)\right]$$

$$= \left[\left(\sqrt{3} + \frac{\sqrt{3}}{2}\right)\hat{\mathbf{j}} + \left(-\hat{\mathbf{i}} + \frac{\hat{\mathbf{i}}}{2}\right)\right]$$

$$= \left[-\frac{1}{2}\hat{\mathbf{i}} + \frac{3\sqrt{3}}{2}\hat{\mathbf{j}}\right] = -\frac{\hat{\mathbf{i}}}{2} + \frac{3\sqrt{3}}{2}\hat{\mathbf{j}}$$

$$\Rightarrow \quad F_x = \left(-\frac{\hat{\mathbf{i}}}{2}\right)$$

Hence, $|F| = 0.5$ N

11. (*a*) Let equal forces $F_1 = F_2 = F$ newton

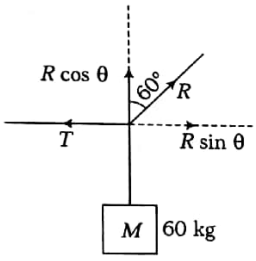
Angle between the forces, $\theta = 60°$

Resultant force, $R = 40\sqrt{3}$ N

Now, $R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos\theta}$

$\therefore \quad 40\sqrt{3} = \sqrt{F^2 + F^2 + 2FF \cos 60°}$ or $F = 40$ N

12. (*d*) The given figure can be drawn as



Taking component of forces

$R\cos\theta = Mg \Rightarrow R\cos 60° = Mg$ ...(i)

and $R\sin 60° = T$ ...(ii)

By Eqs. (i) and (ii), we get

$$\Rightarrow \quad \tan 60° = \frac{T}{Mg} \Rightarrow T = Mg\tan 60°$$

or $T = 60 \times g \times \sqrt{3} = 103.9$ kgf

13. (*c*) The resultant force will be

$$F_R = |\mathbf{F}_R| = \sqrt{3^2 + 4^2 + 2 \times 3 \times 4 \cos 90°} = 5 \text{ N}$$

14. (*c*) Apply Lami's theorem at O

$$\frac{T_1}{\sin 150°} = \frac{T_2}{\sin 120°}$$

$$= \frac{10}{\sin 90°} = \frac{10}{1} = 10$$

$$\therefore \quad T_1 = 10\sin 150° = 10 \times \frac{1}{2} = 5\text{N}$$

$$T_2 = 10\sin 120° = 10 \times \frac{\sqrt{3}}{2} = 5\sqrt{3}\text{ N}$$

15. (*a*) Force exerted by man on rope transfers to it in the form of tension.

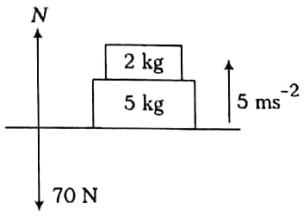
Net upwards force on the system is $2T$ or $2F$.

Net downward force is $(50 + 30)g = 80g$.

For equilibrium of system, $2F = 80g$ or $F = 40g$

Check point 5.3

1. (*c*)



$N - 70 = 7 \times 5$ ... (i)

$\therefore \quad N = 105$ N

2. (*d*) Acceleration of system

$$a = \frac{T_3}{m_1 + m_2 + m_3} = \frac{40}{10 + 6 + 4} = 2 \text{ ms}^{-2}$$

Equation of motion of m_3 is

$$T_3 - T_2 = m_3 a$$

$$\therefore \quad T_2 = T_3 - m_3 a = 40 - 4 \times 2 = 32\text{N}$$

3. (c) Given, $v = 0$, $u = 3$ ms^{-2}

So using, $v^2 = u^2 - 2as$

$$0 = (3)^2 - 2(a)(9)$$

$$\therefore \quad a = \frac{1}{2} = 0.5 \text{ ms}^{-2} \quad \text{(upwards)}$$

$$N = m(g + a) = 50(9.8 + 0.5) = 515\text{N}$$

4. (*d*) Given, $u = 10$ ms^{-1}, $v = 0$,

So using, $v^2 = u^2 - 2as$

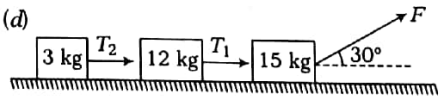
$$0 = (10)^2 - 2(a)(25)$$

$$\therefore \quad a = 2 \text{ ms}^{-2} \quad \text{(upwards)}$$

Now, $T - 800g = 800a$

$$\therefore \quad T = 800(10 + 2) = 9600 \text{ N}$$

5. (*d*)



$$T_2 = \frac{m_1 F \cos 30°}{(m_1 + m_2 + m_3)} = \frac{3F \cos 30°}{(3 + 12 + 15)}$$

$$= \frac{3F \cos 30°}{30} = \frac{F \cos 30°}{10}$$

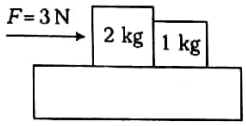
$$T_1 = \frac{(m_1 + m_2) F \cos 30°}{(m_1 + m_2 + m_3)}$$

$$= \frac{(3 + 12) F \cos 30°}{3 + 12 + 15} = \frac{15F \cos 30°}{30} = \frac{F \cos 30°}{2}$$

The ration between T_1 and T_2

$$T_1 : T_2 = \frac{F \cos 30°/2}{F \cos 30°/10} = 5:1$$

6. (*d*)



As the block are rigid under the action of a force F, both here move same acceleration.

$$a = \frac{F}{m+M} = \frac{3}{2+1} = \frac{3}{3} = 1\,\text{ms}^{-2}$$

The force is applied to 2 kg then its action on 1 kg will be

$$F_N = ma = 1 \times 1 = 1\text{N}$$

7. (*b*) Acceleration of system, $a = \dfrac{\text{Net pushing force}}{\text{Total mass}}$

or $$a = \frac{F - (m_1 + m_2 + m_3)g\sin\theta}{(m_1 + m_2 + m_3)}$$

Equation of motion for m_3

$$N - m_3 g\sin\theta = m_3 a$$

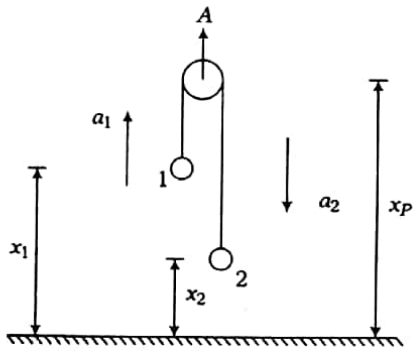
or $$N = m_3 g\sin\theta + m_3\left[\frac{F - (m_1 + m_2 + m_3)g\sin\theta}{(m_1 + m_2 + m_3)}\right]$$

$$= \frac{m_3 F}{m_1 + m_2 + m_3}$$

8. (*b*) Acceleration $a = \left[\dfrac{m_2 - m_1}{m_1 + m_2}\right] g = \dfrac{\left[\frac{3}{2}m - m\right] g}{\left[\frac{3}{2}m + m\right]} = \dfrac{g}{5}$

9. (*c*) $(x_P - x_1) + (x_P - x_2)$ = length of string = constant

Differentiating twice with respect to time, we get



$$a_P = \frac{a_1 + a_2}{2}$$

Here $a_P = A$, a_1 is positive and a_2 is negative. Hence,

$$A = \frac{a_1 - a_2}{2}$$

10. (*b*) Acceleration of system

$$a = \frac{\text{Net pulling force}}{\text{Total mass}} = \frac{4g - 2g}{6} = \frac{g}{3}$$

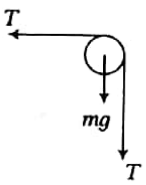
Equation of motion of block C is

$$m_C g - T_{BC} = m_C a$$

$$\therefore \quad T_{BC} = m_C(g - a)$$

$$= 2\left(9.8 - \frac{9.8}{3}\right) = 13\text{ N}$$

11. (*d*) $T = mg$



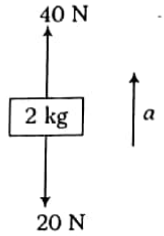
Force on the pulley (other than from clamp)

$$F_{\text{net}} = \sqrt{(T + mg)^2 + T^2}$$

$$= g\sqrt{(M + m)^2 + M^2}$$

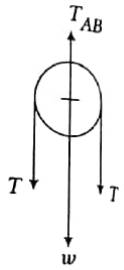
Since pulley is in equilibrium, clamp will exert the same amount of force in opposite direction. Or pulley will also exert this much force on clamp.

12. (*b*) Upward force on 2 kg block in upward direction will be 40 N (= 2*F*) in the form of tension.



$$\therefore \quad a = \frac{40 - 20}{2} = 10\text{ ms}^{-2}$$

13. (*d*) Acceleration of system,



$$a = \frac{\text{Net pulling force}}{\text{Total mass}}$$

$$= \frac{2mg - mg}{3m} = \frac{g}{3}$$

Now from equation of motion of m

$$T - mg = ma = \frac{mg}{3}$$

$$\therefore \quad T = \frac{4mg}{3}$$

For equilibrium of pulley,

$$T_{AB} = 2T + \text{weight of pulley}$$

$$= \frac{8mg}{3} + 3mg = \frac{17mg}{3}$$

14. (*c*) Acceleration $a = \left[\dfrac{m_2 - m_1}{m_2 + m_1}\right] g$

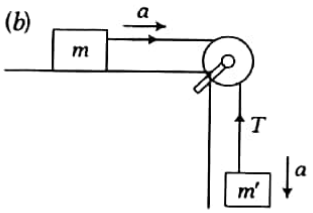
$$= \left[\frac{5m - m}{5m + m}\right] g = \frac{4m}{6m} g = \frac{2}{3} g$$

15. (*b*) The acceleration of the system

$$a = \frac{(m_2 - m_1)\,g}{(m_1 + m_2 + M)} = \frac{(3 - 1)\,g}{(1 + 3 + 6)}$$

$$= \frac{2g}{10} = \frac{g}{5} = \frac{10}{5} = 2\text{ ms}^{-2}$$

16. (*b*)



For mass m

$$T = ma \quad \text{...(i)}$$

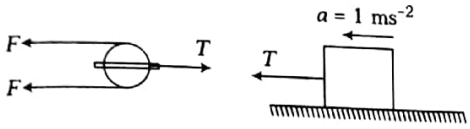
For mass m'

$$m'g - T = m'a \quad \text{...(ii)}$$

Solving the Eq. (i) and Eq. (ii)

$$\text{Acceleration } a = \frac{m'g}{(m + m')}$$

17. (*b*) As shown in figure, when force F is applied at the end of the string, the tension in the lower part of the string is also F. If T is the tension in string connecting the pulley and the block, then,



$T = 2F$

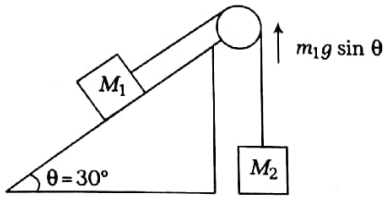
But $T = ma = (200)(1) = 200$ N

$\therefore$ $2F = 200$ N or $F = 100$ N

18. (c) Since $M_1 g \sin 30° = M_2 g$

$\therefore$ Net pulling force = 0

or acceleration of system = 0



19. (c) Acceleration of system, $a = \dfrac{\text{Net pulling force}}{\text{Total mass}} = \dfrac{Mg\sin\theta}{2M}$

$$a = \frac{1}{2} g \sin\theta$$

Now, the block on ground is moving due to tension.

Hence, $T = Ma = \dfrac{Mg\sin\theta}{2}$

20. (a) Pulley is moving upward with acceleration a_0

then Tension, $T = 2\left[\dfrac{m_1 m_2}{m_1 + m_2}\right][g + a_0]$

Here, $a_0 = \dfrac{g}{2}$

$\therefore$ $T = 2\left[\dfrac{m_1 m_2}{m_1 + m_2}\right]\left[g + \dfrac{g}{2}\right] = 3g\left[\dfrac{m_1 m_2}{m_1 + m_2}\right]$

Check point 5.4

4. (a) Maximum inclination of the plane with horizontal = angle of repose = $\tan^{-1}(\mu)$.

5. (c) $a = \dfrac{v}{t} = \dfrac{4}{2} = 2\,\text{ms}^{-2} \Rightarrow F - f = ma$

or $200 - \mu \times 30 \times 10 = 30 \times 2$

$\therefore$ $\mu = 0.47$

6. (d) Retardation, $a = \dfrac{F}{m} = 5\,\text{ms}^{-2} \Rightarrow 0 = 30 - 5t$

$\therefore$ $t = 6$s

7. (c) $a = \dfrac{F - \mu mg}{m} = \dfrac{100 - 0.5 \times 10 \times 10}{10} = 5\,\text{ms}^{-2}$

8. (a) $f_{max} = \mu mg = 0.4 \times 2 \times 10 = 8$N

Since the applied force is less than f_{max}, force of friction will be equal to the applied force or 2.8 N.

9. (d) Retardation, $a = \dfrac{\mu mg}{m} = \mu g = 10\mu$

Now, $0 = u^2 - 2as$ or $0 = (6^2) - 2(10\mu)(9)$

$\therefore$ $\mu = 0.2$

10. (a) Retardation, $a = \dfrac{\mu mg}{m} = \mu g = 4\,\text{ms}^{-2}$

Using, $v^2 = u^2 + 2as$

$(8)^2 = (0)^2 + 2(4)(s)$

$\therefore$ s = 8 m

11. (a) Due to friction ($a = \mu g$), velocity of block will become equal to velocity of belt. Relative motion between two will stop.

$\therefore$ $v = at = \mu gt = 0.2 \times 10 \times 4 = 8\,\text{ms}^{-1}$

12. (b) $a_{max} = \dfrac{T_{max} - \mu mg}{m} = \dfrac{40 - 0.2 \times 8 \times 10}{8} = 3\,\text{ms}^{-2}$

13. (a) Block A moves due to friction . Maximum acceleration of A can be $\dfrac{f_{max}}{m}$ or $\dfrac{\mu mg}{m}$ or $\mu g = 0.2 \times 10 = 2\,\text{ms}^{-2}$. If both the blocks move together, then combined acceleration of A and B can be $\dfrac{10}{3} = 3.33\,\text{ms}^{-2}$. Since, this is more than the maximum acceleration of A. Slipping between them will take place and force of friction will be maximum or $\mu m_A g = 2$N.

14. (a) m will move by friction,

$$f_{max} = \mu mg \Rightarrow (a_m)_{max} = \frac{\mu mg}{m} = \mu g$$

15. (a) $f_{max} = \mu mg = 0.8 \times 4 \times 10 = 32$N

At $t = 2$s, $F = kt^2(2)(2)^2 = 8$N

Since applied force $f < f_{max}$, force of friction will be 8 N

16. (d) N = applied force = 12 N

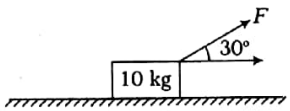
$\therefore$ $f_{max} = \mu N = 7.2$N

Since weight $w < f_{max}$

Force of friction $f = 5$ N

$\therefore$ Net contact force $= \sqrt{N^2 + f^2} = \sqrt{(12)^2 + (5)^2} = 13$N

17. (d) $\mu = \tan 30° = \dfrac{1}{\sqrt{3}}$ (30° = angle of friction)



$$N = mg - F\sin 30° = \left(100 - \frac{F}{2}\right)$$

$F\cos 30° = \mu N$

$\Rightarrow$ $\dfrac{\sqrt{3}F}{2} = \dfrac{1}{\sqrt{3}}\left(100 - \dfrac{F}{2}\right)$

$\Rightarrow$ $\dfrac{3F}{2} = 100 - \dfrac{F}{2} \Rightarrow F = 50$ N

18. (a) Angle of repose $\theta_r = \tan^{-1}(\mu_s) = \tan^{-1}(0.7)$

or $\tan\theta_r = 0.7$

Angle of plane is $\theta = 30°$, $\tan\theta = \tan 30° = 0.577$

Since $\tan\theta < \tan\theta_r$, $\theta < \theta_r$

Block will not slide or $f = mg\sin\theta \neq \mu mg\cos\theta$

or $f = (2)(9.8)\sin 30° = 9.8$N

19. (*a*) $mg\sin\theta = (102)(10)\sin 30° = 510\,\text{N}$

$\mu_s mg\cos\theta = (0.4)(102)(10)\cos 30°$

$= 353\,\text{N}$

$F = 510 - 353 = 157\,\text{N}$

20. (*a*) $F = mg\sin\theta - \mu mg\cos\theta$ ... (i)

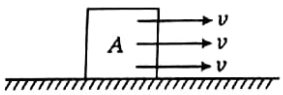
$2F = mg\sin\theta + \mu mg\cos\theta$ ... (ii)

$\Rightarrow\ 2mg\sin\theta - 2\mu mg\cos\theta = mg\sin\theta + \mu mg\cos\theta$

$\therefore\quad \mu = \frac{1}{3}\tan\theta$

A) Taking it together

1. (*c*) In a uniform translatory motion, all parts of the ball have the same velocity in magnitude and direction and this velocity is constant.



The situation is shown in adjacent diagram where a body A is in uniform translatory motion.

2. (*b*) To solve this question we have to apply Newton's second law of motion, in terms of force and change in momentum.

We known that $F = \frac{dp}{dt}$

given that meter scale is moving with uniform velocity, hence, $dp = 0$

Force $= F = 0$.

As all part of the scale is moving with uniform velocity and total force is zero, hence, torque will also be zero.

3. (*c*) We know that for a system $F_{ext} = \frac{dp}{dt}$

(from Newton's second law)

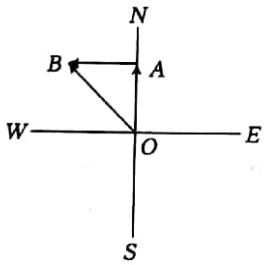
If $F_{ext} = 0$, $dp = 0 \Rightarrow p =$ constant

Hence, momentum of a system will remain conserve if external force on the system is zero.

In case of collision' between particles equal and opposite forces will act on individuel particles by Newtons third law, Hence total force on the system will be zero.

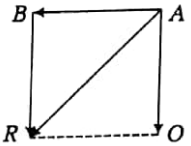
Note We should not confuse with system and individual particles. As total force on the system of both particles is zero but force acts on individual particles.

4. (*b*) Consider the adjacent diagram



Let $\mathbf{OA} = \mathbf{p}_1$

= Initial momentum of player northward



$\mathbf{AB} = \mathbf{p}_2 =$ Final momentum of player towards west.

Clearly $\mathbf{OB} = \mathbf{OA} + \mathbf{AB}$

Change in momentum $= \mathbf{p}_2 - \mathbf{p}_1$

$= \mathbf{AB} - \mathbf{OA} = \mathbf{AB} + (-\mathbf{OA})$

= Clearly resultant **AR** will be along south-west.

5. (*b*) The tension T in the string is

$T = 6(g + a) = 6(10 + 1) = 66\text{N}$

6. (*c*) If lift is accelerating, reading will be more, if it is decelerating reading will be less and if it is moving with constant velocity reading will be same.

7. (*b*) Given, mass of the car $= m$

As car starts from rest, $u = 0$.

Velocity acquired along east $= v_i$

Duration $= t = 2s$

We know that, $v = u + at \Rightarrow v\hat{\mathbf{i}} = o + a \times 2$

$\Rightarrow\quad \mathbf{a} = \frac{\mathbf{v}}{2}\hat{\mathbf{i}}$

Force, $\mathbf{F} = m\mathbf{a} = \frac{m\mathbf{v}}{2}\hat{\mathbf{i}}$

Hence, force acting on the car is $\frac{mv}{2}$ towards east. As external force on the system is only friction hence, the force $\frac{mv}{2}$ is by friction. Hence, force by engine is internal force.

8. (*c*) Given, $\mathbf{u} = (3\hat{\mathbf{i}} + 4\hat{\mathbf{j}})\ \text{ms}^{-1}$ and $\mathbf{v} = -(3\hat{\mathbf{i}} + 4\hat{\mathbf{j}})\ \text{ms}^{-1}$

Mass of the ball $= 150\,\text{g} = 0.15\,\text{kg}$

$\Delta p =$ Change in momentum

= Final momentum − Initial momentum

$= m\mathbf{v} - m\mathbf{u}$

$= m(\mathbf{v} - \mathbf{u}) = (0.15)[-(3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}) - (3\hat{\mathbf{i}} + 4\hat{\mathbf{j}})]$

$= (0.15)[-6\hat{\mathbf{i}} - 8\hat{\mathbf{j}})] = -[0.15 \times 6\hat{\mathbf{i}} + 0.15 \times 8\hat{\mathbf{j}})]$

$= -[0.9\hat{\mathbf{i}} + 120\hat{\mathbf{j}}]$

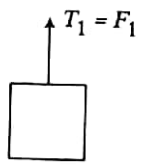
Hence, $\Delta\mathbf{p} = -[0.9\hat{\mathbf{i}} + 12\hat{\mathbf{j}}]$

9. (*b*) By previous solution $\Delta\mathbf{p} = -(0.9\hat{\mathbf{i}} + 12\hat{\mathbf{j}})$

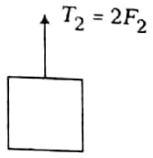
Magnitude $= |\Delta\mathbf{p}| = \sqrt{(0.9)^2 + (12)^2}$

$= \sqrt{0.81 + 144} = 15\,\text{kg}\cdot\text{ms}^{-1}$

10. (*a*) In first case,



In second case,



$\therefore$ In first case, person will have to apply more force.

11. (*a*) Given, mass = 2 kg

$$x(t) = pt + qt^2 + rt^3$$

$$v = \frac{dx}{dt} = p + 2qt + 3rt^2$$

$$a = \frac{dv}{dt} = 0 + 2q + 6rt$$

At $t = 2s$; $a = 2q + 6 \times 2 \times r = 2q + 12r$

$= 2 \times 4 + 12 \times 5 = 8 + 60 = 68\ \text{ms}^{-1}$

Force, $F = ma = 2 \times 68 = 136\ \text{N}$

12. (*b*) Given, mass, $m = 5$ kg

Acting force, $\mathbf{F} = (-3\hat{\mathbf{i}} + 4\hat{\mathbf{j}})\ \text{ms}^{-1}$

Initial velocity at $t = 0$, $\mathbf{u} = (6\hat{\mathbf{i}} - 12\hat{\mathbf{j}})\ \text{ms}^{-1}$

Retardation, $\hat{\mathbf{a}} = \frac{\mathbf{F}}{m} = \left(-\frac{3\hat{\mathbf{i}}}{5} + \frac{4\hat{\mathbf{j}}}{5}\right)\ \text{ms}^{-2}$

As final velocity is along Y-axis only, its x-component must be zero.

From $v = u + at$, for X-component only, $0 = 6\hat{\mathbf{i}} - \frac{3\hat{\mathbf{i}}}{5}t$

$$t = \frac{5 \times 6}{3} = 10\ \text{s}$$

13. (*d*) Angle of plane is just equal to the angle of repose.

or $\mu = \tan\theta$

14. (*c*) Force of friction is zero. Only contact force is normal reaction which is $mg\cos\theta$.

15. (*b*) $F = u\frac{dm}{dt} = m(g + a)$

$$\Rightarrow \quad \frac{dm}{dt} = \frac{m(g+a)}{u} = \frac{5000 \times (10 + 20)}{800} = 187.5\ \text{kgs}^{-1}$$

16. (*b*) Acceleration in both the cases will be same.

$$N_1 = ma \text{ but } N_2 = (2m)a$$

$$\therefore \quad \frac{N_1}{N_2} = \frac{1}{2}$$

17. (*b*) When force is applied on A, acceleration produced will be

$$\frac{F_A}{m_A + m_B} = \frac{\mu m_A g}{m_B} \quad \text{... (i)}$$

When force is applied on B, acceleration produced will be

$$\frac{F_B}{m_A + m_B} = \frac{\mu m_A g}{m_A} \quad \text{... (ii)}$$

Dividing these two equations, we get

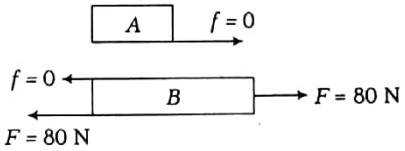
$$\frac{F_B}{F_A} = \frac{m_B}{m_A}$$

$$\therefore \quad F_B = \frac{m_B}{m_A} \cdot F_A = \frac{8}{4} \times 12 = 24\ \text{N}$$

18. (*d*) Maximum force of friction between ground and B is

$$f = (M + m)\mu g$$

$$\Rightarrow \quad (15 + 5)(0.6)(10) = 120\ \text{N}$$



As $f_{max} > F$

$\therefore$ Frictional force between B and ground will be 80 N.

19. (*b*) Net external force, $F = \sqrt{(4)^2 + (3)^2} = 5\ \text{N}$

Maximum friction $f_{max} = \mu mg$

$= (0.09)(5)(10) = 4.5\ \text{N}$

Since $F > f_{max}$, block will move with an acceleration,

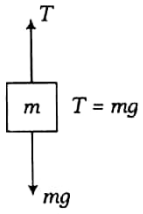
$$a = \frac{F - f_{max}}{m}$$

$$= \frac{5 - 4.5}{5} = 0.1\ \text{ms}^{-2}$$

20. (*a*) $\mu(m_A + m_C)g = m_B g$

$$\therefore \quad m_C = \frac{m_B}{\mu} - m_A = \frac{5}{0.2} - 10 = 15\ \text{kg}$$

21. (*c*) $T = mg$



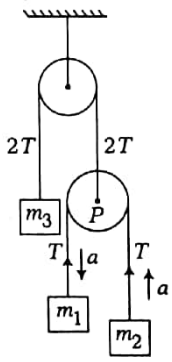
When the string breaks, T will suddenly disappear.

For B, $T - 2mg = 2ma$

$3mg - 2mg = 2ma$ $[T = 3mg]$

$a = g/2$

22. (*a*) m_3 is at rest. Therefore



$$2T = m_3 g \quad \text{...(i)}$$

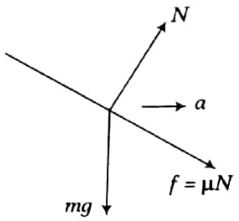
Further if m_3 is at rest, then pulley P is also at rest. Writing equations of motion.

$$m_1 g - T = m_1 a \quad \text{...(ii)}$$

$$T - m_2 g = m_2 a \quad \text{...(iii)}$$

Solving Eqs. (ii) and (iii) we get, $m_3 = 1\ \text{kg}$

23. (*b*) $N\sin\theta + \mu N\cos\theta = ma$



$$N\cos\theta - \mu N\sin\theta = mg$$

Putting $\theta = 45°$ and solving these two equations, we get

$$a = g\left(\frac{1+\mu}{1-\mu}\right)$$

24. (*a,c*) $N\sin\theta = ma = 1\times 5 = 5$ ...(i)

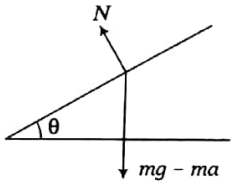
$N\cos\theta = mg = 1\times 10 = 10$ ...(ii)

From these two equations, we get

$N = 5\sqrt{5}$ N (Adding and squaring them)

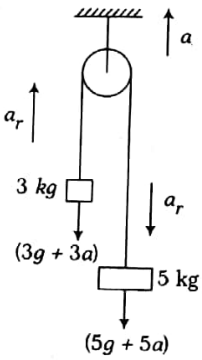
and $\tan\theta = \frac{1}{2}$

25. (*a*) $m(g-a)\sin\theta = \mu m(g-a)\cos\theta$



$\Rightarrow \quad \mu = \tan\theta = \tan 30° = \frac{1}{\sqrt{3}}$

26. (*c*) Let acceleration of lift is *a* upwards. Then with respect to lift



$$a_r = \frac{\text{Net pulling force}}{\text{Total mass}} = \frac{(5g+5a)-(3g+3a)}{8}$$

$\Rightarrow \quad \frac{9}{32}g = \frac{(g+a)}{4}$

$\therefore \quad a = \frac{g}{8}$

27. (*c*) Suppose air resistance is F (upwards), then from equation of motion of balloon, we have

$$F = \text{mass} \times \text{acceleration} = \frac{W}{g}\cdot a$$

$\therefore \quad F = W\left(1-\frac{a}{g}\right)$

28. (*d*) Moving down with retardation *a* means, lift is accelerated upwards. With respect to lift, pseudo force on the block will be *ma* in downward direction, where *m* is the mass of block. So, downward force *mg* on the block will be replaced by $m(g+a)$. Therefore, acceleration of block relative to plane will be,

$a_r = (g+a)\sin\theta$ (down the plane)

From, $L = \frac{1}{2}a_r t^2$ $\left(s = \frac{1}{2}at^2\right)$

$$t = \sqrt{\frac{2L}{a_r}} = \sqrt{\frac{2L}{(g+a)\sin\theta}}$$

29. (*c*) Momentum of one piece $= \frac{M}{4}\times 3$

Momentum of other piece $= \frac{M}{4}\times 4$

$\therefore$ Resultant momentum $= \sqrt{\frac{9M^2}{16} + M^2} = \frac{5M}{4}$

The third piece should also have the same momentum. Let its velocity be *v*, then $\frac{5M}{4} = \frac{M}{2}\times v \Rightarrow v = \frac{5}{2} = 2.5\ \text{ms}^{-1}$

30. (*a*) $mg\sin\theta - f = ma$ (f = force of friction)

or $f = m(g\sin\theta - a)$

$= 8\left(10\times\frac{1}{2} - 0.4\right) = 36.8\text{N}$

31. (*c*) Acceleration, $a = (g\sin\theta + \mu g\cos\theta)$

$$= \frac{g}{\sqrt{2}} + 0.5\times g\times\frac{1}{\sqrt{2}} = \frac{3g}{2\sqrt{2}}$$

32. (*a*) Acceleration of system before breaking the string was,

$$a = \frac{\text{Net pulling force}}{\text{Total mass}} = \frac{3g-2g}{5} = \frac{g}{5}$$

After 5 s velocity of system, $v = at = \frac{g}{5}\times 5 = g\ \text{ms}^{-2}$

Now, $h = \frac{v^2}{2g} = \frac{g^2}{2g} = \frac{g}{2} = 4.9\text{m}$

33. (*a*) Maximum acceleration of the box can be μg or $1.5\ \text{ms}^{-2}$, while acceleration of truck is $2\ \text{ms}^{-2}$. Therefore, relative acceleration of the box will be $a_r = 0.5\ \text{ms}^{-2}$ (backward). It will fall off the truck in a time.

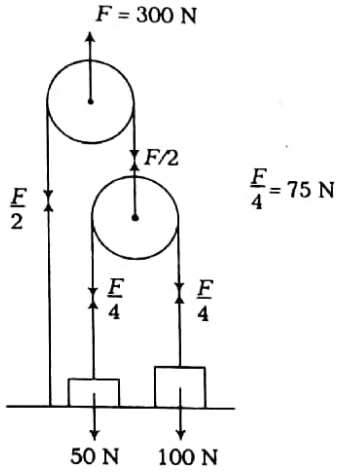
$$t = \sqrt{\frac{2l}{a_r}} \qquad \left(s = \frac{1}{2}at^2\right)$$

$$= \sqrt{\frac{2\times 4}{0.5}} = 4\ \text{s}$$

Displacement of truck upto this instant is,

$$s_r = \frac{1}{2}a_r t^2 = \frac{1}{2} \times 0.5 \times (4)^2 = 4\text{ m}$$

34. (*a*) $\frac{F}{4} = 75\text{N}$ or $\frac{F}{4} < 100\text{ N}$



Therefore, $a_M = 0$

$$a_m = \frac{75 - 50}{5} = 5\text{ m/s}^2$$

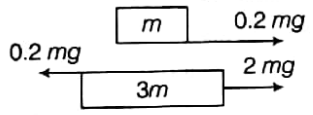
35. (*d*) Total upward force $= 2\left(\frac{mg}{2}\right) = mg.$

[Weight of man is balance by total tension acting upwards]

Total downward force is also mg

$\therefore$ $F_{net} = 0 = a_{net}$

36. (*d*) Block A moves due to friction. Maximum value of friction can be $\mu m_A g$. Therefore, maximum acceleration of A can be $\frac{\mu m_A g}{m_A}$ or $\mu g = 0.2g = \frac{g}{5}$. When force 2 mg is applied on lower block, common acceleration (if both move together) will be,



$$a = \frac{\text{Net force}}{\text{Total mass}} = \frac{2mg}{4m} = \frac{g}{2}$$

Since $a = g/2$ is greater than maximum acceleration of A which can be given to it by friction. Therefore slipping will take place.

$$a_A = 0.2g = \frac{g}{5}$$

$$a_B = \frac{2mg - 0.2mg}{3m} = 0.6g = \frac{3g}{5}$$

$$a_{AB} = a_A - a_B = +\left(\frac{g}{5} - \frac{3g}{5}\right)$$

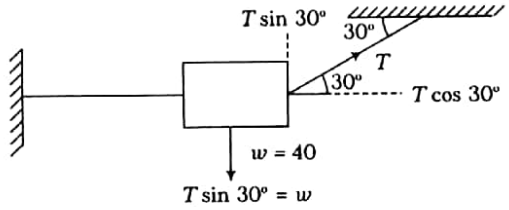
$$= \frac{-2g}{5} \text{ (in backward direction)}$$

37. (*a*) $a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t} = \frac{\sqrt{2gh_f} - \sqrt{2gh_i}}{\Delta t}$

$$= \frac{\sqrt{2 \times 9.8 \times 2.5} - \sqrt{2 \times 9.8 \times 10}}{0.01} = 700\text{ ms}^{-2}$$

38. (*b*) T_x = mass of rope of length $(L - x) \times g = \frac{M}{L}(L - x)g$

39. (*a*) $T \sin 30° = w = 40$ N

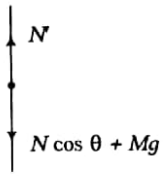


$\therefore$ $\frac{T}{2} = 40$ or $T = 80$ N

40. (*c*) Wedge moves due to horizontal component of normal reaction.

Thus, $a = \frac{N_H}{M} = \frac{N \sin\theta}{M}$ (along – ve X-axis)

41. (*a*) Net force on M in vertical direction should be zero.



In vertically downwards, two forces $N \cos\theta$ and Mg are acting. Therefore N', the normal reaction from ground should be equal to $N \cos\theta + Mg$.

42. (*c*) $s = \frac{1}{2}at^2$ or $t = \sqrt{\frac{2s}{a}}$ or $t \propto \frac{1}{\sqrt{a}}$

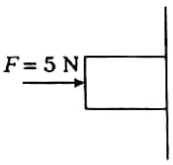
$$\frac{t_1}{t_2} = \sqrt{\frac{a_2}{a_1}}$$

or $$2 = \sqrt{\frac{g \sin\theta}{g \sin\theta - \mu g \cos\theta}} = \sqrt{\frac{1}{1 - \mu}}$$

as $\sin 45° = \cos 45°$

$\therefore$ $1 - \mu = \frac{1}{4} \Rightarrow \mu = 0.75$

43. (*b*) $N = F = 5$N

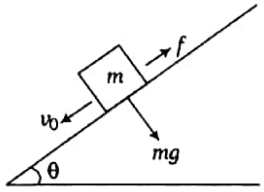


$\therefore$ $f_{max} = \mu N = 2.5$N

$w = mg = 0.1 \times 9.8 = 0.98$N

Since $w < f_{max}$, force of friction will be 0.98 N.

44. (*c*) $f_{max} = \mu mg \cos\theta = 0.6 \times mg \times \frac{\sqrt{3}}{2} = 0.52mg$



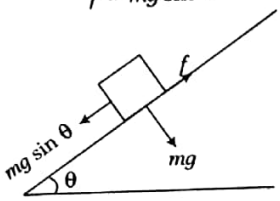
$$mg \sin\theta = \frac{mg}{2} = 0.5mg$$

Since, $f_{max} > mg \sin\theta$

Block will decelerate and come to rest.

45. (*d*) Block is a rest. Hence,

$$f = mg\sin\theta$$



46. (*a*) $v^2 = 2a_1 s \ (a_1 = g\sin\theta)$

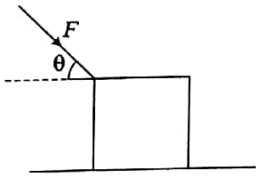
$0 = v^2 - 2a_2 s \ (a_2 = \mu g\cos\theta - g\sin\theta)$

From these two equations we see that $a_1 = a_2$

$\Rightarrow \quad g\sin\theta = \mu g\cos\theta - g\sin\theta \Rightarrow 2\tan\theta = \mu$

47. (*b*) $\quad N = w + F\sin\theta$

$\therefore \quad f_{max} = \mu N = (\tan\phi)(w + F\sin\theta)$



To move the body,

$$F\cos\theta = f_{max} = (\tan\phi)(w + F\sin\theta)$$

Solving this equation we get,

$$F = \frac{w\sin\theta}{\cos(\theta + \phi)}$$

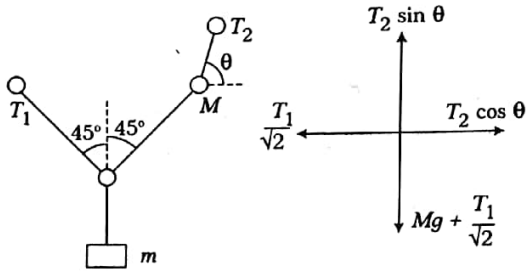
48. (*c*) Maximum acceleration due to friction of mass m over mass $2m$ can be μg. Now, for the whole system

$$a = \frac{\text{Net pulling force}}{\text{Total mass}}$$

$$\therefore \quad \mu g = \frac{mg}{4m} \text{ or } \mu = \frac{1}{4}$$

49. (*a*) $2T_1\cos 45° = mg$

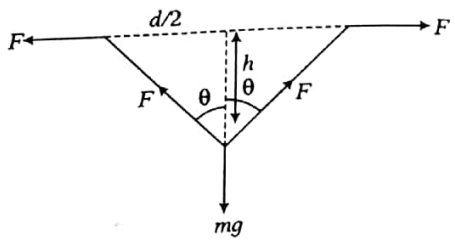
$$\therefore \quad T_1 = \frac{mg}{\sqrt{2}} \quad \text{... (i)}$$



$$T_2\cos\theta = \frac{T_1}{\sqrt{2}} = \frac{mg}{2}; \ T_2\sin\theta = mg + \frac{mg}{2}$$

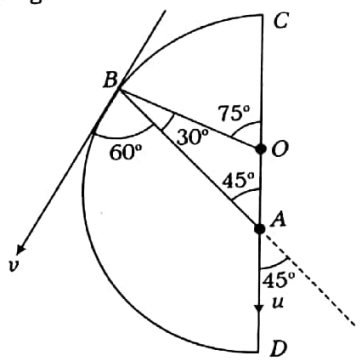
$$\tan\theta = \frac{mg + \frac{mg}{2}}{\frac{mg}{2}} = 1 + \frac{2M}{m}$$

50. (*a*) $\quad 2F\cos\theta = mg$



$$\therefore \quad F = \frac{mg}{2\cos\theta} = \frac{mg\sqrt{h^2 + \frac{d^2}{4}}}{2h} = \frac{mg}{4h}\sqrt{d^2 + 4h^2}$$

51. (*a*) To remain the string tight, component of velocities along the line joining A and B should be same.



$$\therefore \quad u\cos 45° = v\cos 60° \Rightarrow v = \sqrt{2}u$$

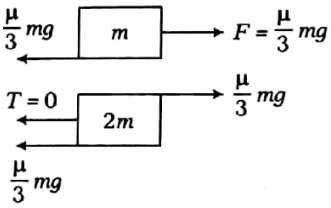
52. (*c*) f_1 = maximum value of friction between m and $2m = \mu mg$

f_2 = maximum value of friction between $2m$ and ground

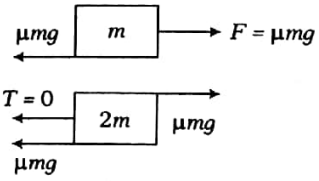
$= \frac{\mu}{3}(3m)g = \mu\, mg$

Now, let us draw free body diagram of m and $2m$ in all four cases.

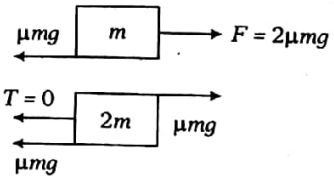
(i)



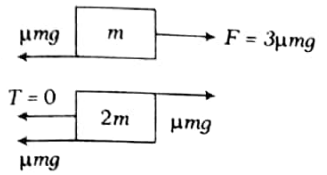
(ii)



(iii)



(iv)



Note Tension will be zero in all four cases.

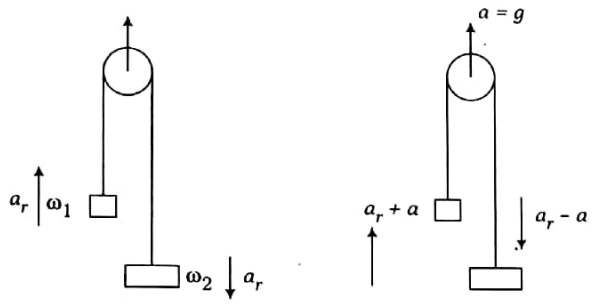
53. (*a*) Assuming the resistance force or retardation to be constant.

$$\left(\frac{v}{2}\right)^2 = v^2 - 2as_1 \quad \text{... (i)}$$

$$0 = \left(\frac{v}{2}\right)^2 - 2as_2 \quad \text{... (ii)}$$

Solving these two equations, we get, $s_2 = \frac{s_1}{3} = 1\text{cm}$

55. (*a*) Writing equation of motion for two weights.



Acceleration relative to pulley Acceleration relative to ground

$$w_1 - T = \frac{w_1}{g}(a_r - a) \quad \text{...(i)}$$

$$T - w_2 = \frac{w_2}{g}(a_r + a) \quad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get

$$T = \frac{4w_1w_2}{w_1 + w_2} \text{ with } a = g$$

56. (*d*) Acceleration of system,

$$a = \frac{F - f}{M + m} = \frac{20 - 10}{6 + 4} = 1\text{ms}^{-2} \text{ (towards left)}$$

Let F_0 be the reading of dynamometer, then equation of motion of mass m would be,

$$F_0 - f = ma$$

$$\Rightarrow \quad F_0 = f + ma = 10 + (4)(1) = 14\text{N}$$

57. (*b*) Net pulling force on the system should be zero, as velocity is constant. Hence,

$$m_Ag\sin 30° = \mu m_Ag\cos 30° + m_Bg$$

$$\therefore \quad m_B = \left(\frac{m_A}{2}\right) - \left(\frac{\mu m_A\sqrt{3}}{2}\right)$$

$$= 10\left[\frac{1}{2} - 0.2 \times \frac{\sqrt{3}}{2}\right] = 3.3\text{kg}$$

58. (*c*) $a = \frac{\text{Net pulling force}}{\text{Total mass}}$

$$= \frac{5g - 0.5 \times 5 \times g}{10} = \frac{g}{4}$$

Equation of motion for the hanging mass will be,

$$5g - T = 5a = \frac{5g}{4}$$

$$\therefore \quad T = \frac{15}{4}g = \frac{15 \times 9.8}{4} = 36.75\text{ N}$$

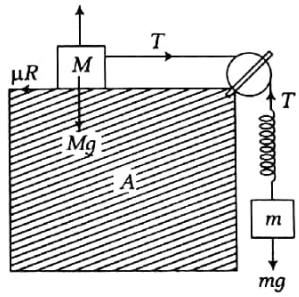
59. (*d*) Since, the block is at rest under two forces

(i) weight of block.

(ii) contact force from the plane (resultant of force of friction and normal reaction).

Contact force should be equal to weight (or 30 N) in upward direction. Because under the action of two forces, a body remains in equilibrium when both the forces are equal in magnitude but opposite in direction.

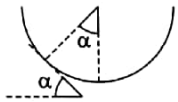
60. (*a*) When the system of two blocks of masses M and m is either at rest on moving with the same speed, no net force acts on them. In this equilibrium state.



$$T = \mu R = \mu Mg \text{ and } T = mg$$

$$\therefore \quad \mu Mg = mg \Rightarrow M = \frac{m}{\mu}$$

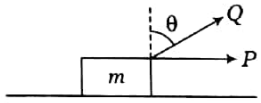
61. (*a*) α = angle of repose



$$\Rightarrow \quad \tan\alpha = \mu = \frac{1}{3}$$

$$\therefore \quad \cot\alpha = 3$$

62. (*a*) $N = mg - Q\cos\theta$



$$P + Q\sin\theta = \mu N = \mu(mg - Q\cos\theta)$$

$$\therefore \quad \mu = \frac{P + Q\sin\theta}{mg - Q\cos\theta}$$

63. (*b*) $m_1g > m_2g\sin\theta + \mu m_2g\cos\theta$

$$\Rightarrow \quad \frac{m_1}{m_2} > \sin\theta + \mu\cos\theta$$

64. (*d*) $N = Mg - F\cos\theta$

$$= Mg - Mg\cos\theta = 2Mg\sin^2\frac{\theta}{2}$$

Further block can be pulled if,

$$F\sin\theta \geq \mu N\cos\theta$$

$$\Rightarrow \quad 2Mg\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2} \geq 2\mu Mg\sin^2\frac{\theta}{2} \Rightarrow \cot\frac{\theta}{2} \geq \mu$$

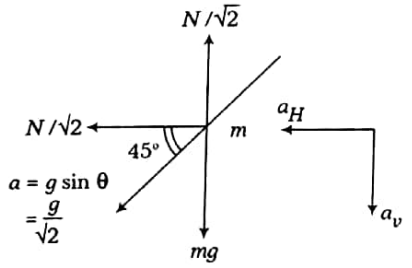
65. (*d*) $a = \dfrac{2F - F}{2m} = \dfrac{F}{2m}$ (towards left)

Writing equation of right hand side block;

$$2F - N\sin 30° = ma = \frac{F}{2}$$

$$\frac{N}{2} = 2F - \frac{F}{2} = \frac{3F}{2} \Rightarrow N = 3F$$

66. (*b*) $a = g\sin\theta = \dfrac{g}{\sqrt{2}}$



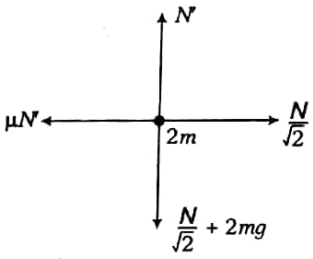
$$a_H = a_V = \frac{g}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{g}{2}$$

$$\frac{N}{\sqrt{2}} = \frac{mg}{2} \quad \text{... (i)}$$

$$mg - \frac{N}{\sqrt{2}} = \frac{mg}{2} \quad \text{... (ii)}$$

$$N' = \frac{N}{\sqrt{2}} + 2mg \quad \text{... (iii)}$$

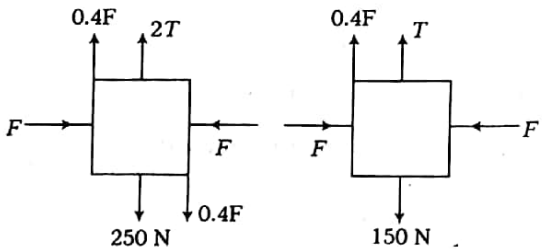
$$\mu N' = \frac{N}{\sqrt{2}} \quad \text{... (iv)}$$



Solving above four equations, we get

$$\mu = 0.2$$

67. (*a*) $2T = 250$



$\therefore \quad T = 125\text{ N} \Rightarrow T + 0.4F = 150$

$\therefore \quad F = 62.5\text{ N}$

68. (*b*) $T_B = 3N$ and $T_A = 2N$

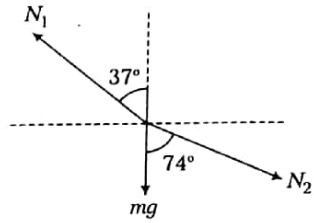
$\therefore \quad v_A = \dfrac{T_B}{T_A}\cdot v_B = \dfrac{3}{2}v_0$ (towards right)

$\therefore \quad v_{AB} = \dfrac{3v_0}{2} - v_0 = \dfrac{v_0}{2}$, towards right.

In such cases, velocity and acceleration are in the ratio of tensions.

69. (*d*) $N_1\sin 37° = N_2\sin 74°$

$\Rightarrow \quad N_1 = 2N_2\cos 37°$



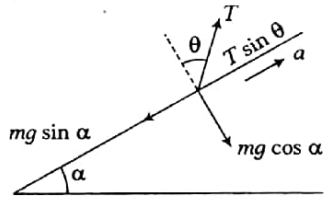
Now, $N_1\cos 37° - N_2\cos 74° = mg$

$\Rightarrow \quad 2N_2\cos^2 37° - N_2(2\cos^2 37° - 1) = mg$

$\Rightarrow \quad 2N_2\left(\dfrac{16}{25}\right) - N_2\left(\dfrac{32}{25} - 1\right) = mg$

$\Rightarrow \quad N_2 = mg$

70. (*c*) $T\sin\theta - mg\sin\alpha = ma$



$$T\cos\theta = mg\cos\alpha$$

From these two equations we get,

$$\tan\theta = \frac{a + g\sin\alpha}{g\cos\alpha} \Rightarrow \theta = \tan^{-1}\left(\frac{a + g\sin\alpha}{g\cos\alpha}\right)$$

71. (*c*) In critical case, weight of hanging part = force of friction of the part of rope lying on table.

$$\therefore \quad \frac{m}{l}\cdot l_1 g = \mu\frac{m}{l}(l - l_1)g$$

Solving this we get,

$$l_1 = \left(\frac{\mu}{1+\mu}\right)l$$

B) Medical entrance special format questions

Assertion and reason

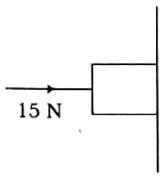
3. (*d*) Friction opposes the relative motion of the bodies in contact not the motion.

4. (*d*) Friction opposes the relative motion of the bodies in contact. By throwing something backwards you can move forwards.

5. (*c*) Normal reaction = 15 N

$f_{max} = \mu N = 9$ N

weight = 10 N



Since weight > f_{max}

∴ 9 N friction will act and block will move downwards.

6. (*c*) $mg\sin\theta = 10 \times 10 \times \frac{1}{2} = 50$ N

$\mu mg\cos\theta = 0.7 \times 10 \times 10 \times \frac{\sqrt{3}}{2} = 60.62$ N

8. (*d*) Due to rotation of earth it is non-inertial. A frame moving with constant velocity is inertial.

9. (*a*) Static friction varies between zero to a maximum limiting value.

Since $\mu mg\cos\theta$ is more, block will remain stationary.

10. (*d*) Tension is non-uniform even if string is not accelerated.

At rest

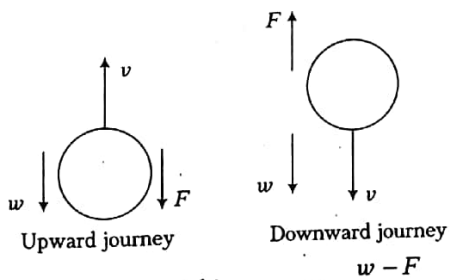
11. (*c*) $F_{net} = ma$

For 1 kg mass $\uparrow T$, $\boxed{1\text{ kg}}$ $\uparrow a$, $\downarrow g$ $\quad T - g = a$

For 2 kg mass $\uparrow T$, $\boxed{2\text{ kg}}$ $\downarrow a$, $\downarrow g$ $\quad \Rightarrow 2g - T = 2a$

Solving we get, $F_{net} = \frac{10}{3}N$

12. (*b*) Retardation in upward journey, $a_1 = \frac{w + F}{m}$



Acceleration in downward journey, $a_2 = \frac{w - F}{m}$

∴ $a_1 > a_2$

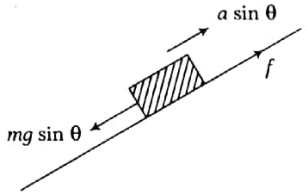
13. (*a*) $(f_2)_{max} = \mu_2 mg$

$(f_1)_{max} = \mu_1(2m)g$

Since $(f_1)_{max} > (f_2)_{max}$

Lower block will not move at all.

14. (*b*) $f - mg\sin\theta = ma\sin\theta$



∴ $f = m(g + a)\sin\theta > mg\sin\theta$

15. (*b*) Let T_0 is the tension in the string connecting the two blocks. Then

$T_0 - 10 = 1 \times a$

∴ $T_0 > 10$ N, $20 - T_0 = 2 \times a$

∴ $T_0 < 20$ N

16. (*b*) In case of massless string.

$a = \frac{2F - F}{0} = \infty$

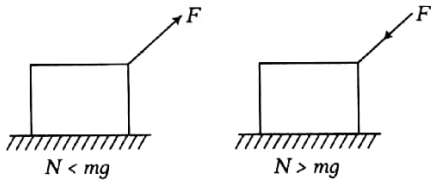
17. (*a*) $N_1 = ma_x = 2m$

$N_2 = m(g + a_y) = 12m$

∴ $\frac{N_1}{N_2} = \frac{1}{6}$

Match the columns

1. Force of friction opposes relative motion between two bodies in contact. It does not simply opposes the relative motion. Further, $N = mg$, only when no inclined force is acting on the block as shown in figure.



2. $N = mg - 20\sqrt{2}\sin 45° = 20$ N

$f = \mu_k N = 12$ N

Since, $20\sqrt{2}\cos 45° > \mu N$ block will move and kinetic friction will act,

$$a = \frac{20\sqrt{2}\cos 45° - \mu_k N}{m} = \frac{20 - 12}{4} = 2\text{ ms}^{-2}$$

3. $\mathbf{a}_A = \frac{d\mathbf{v}_A}{dt} = (2\hat{\mathbf{i}})\text{ ms}^{-2} \Rightarrow \mathbf{a}_B = \frac{d\mathbf{v}_B}{dt} = 0$

$\mathbf{a}_C = \frac{d\mathbf{v}_C}{dt} = (-4\hat{\mathbf{j}})\text{ ms}^{-2}$

Now, pseudo force acts in opposite direction to that of acceleration of frame from where object is observed.

4. Force of friction and $mg\sin\theta$ both are downwards.

∴ Acceleration of the block is

$a = (g\sin\theta + \mu g\cos\theta)$ down the plane

Now, net force in any direction is equal to,

$F = m$ (component of acceleration in that direction)

5. Acceleration of system,

$$a = \frac{60 - 18 - (m_1 + m_2 + m_3)g\sin 30°}{(m_1 + m_2 + m_3)} = 2\ \text{ms}^{-2}$$

Net force on 3 kg block $= m_3 a = 6\,\text{N}$

From free body diagram of 3 kg block, we have

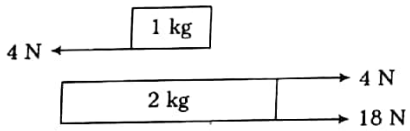
$$N_{12} - m_1 g \sin 30° - 18 = m_1 a \Rightarrow N_{12} = 25\,\text{N}$$

From free body diagram of 3 kg block, we have

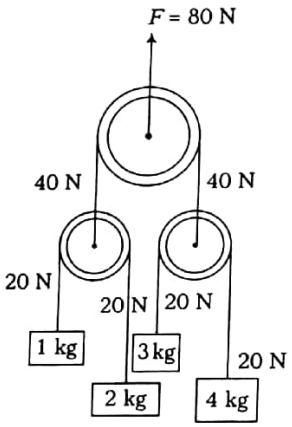
$$60 - m_3 g \sin 30° - N_{32} = m_3 a$$

$$\therefore \quad N_{32} = 39\,\text{N}$$

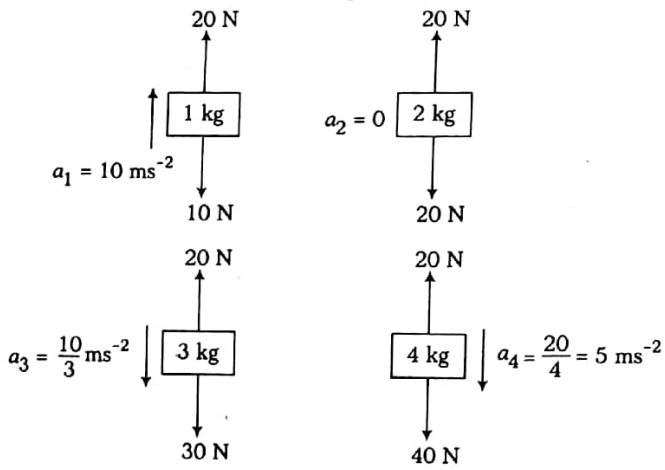
6. Force diagram on both the blocks is as shown in figure.



7. Since the pulleys are smooth, net force on each pulley should be zero. With this concept, tensions on all strings are shown below.



Now, we can draw free body diagrams of all the four blocks.



C) Medical entrances' gallery

1. (*a*) Impulse is imparted due to change in perpendicular components of momentum of ball.

$$J = \Delta p = mv_s - mv_i$$

$$= mv\cos 60° - (-mv\cos 60°)$$

$$= 2mv\cos 60° = 2mv \times \frac{1}{2}mv$$

2. (*b*) Force, $P = f_m = \mu_0 mg$ (when body is rest)

When the body starts moving with acceleration a, then

$$P - F_k = ma$$

$$\mu_0 mg - \mu_k mg = ma$$

$$a = (\mu_0 - \mu_k)g$$

$$= (0.5 - 0.4)10$$

$$= 0.1 \times 10\ \text{ms}^{-2} = 1\,\text{ms}^{-2}$$

3. (*a*) For the motion of balloon,

Let upward force is F, then $F - mg = ma$

$$\Rightarrow \quad F - 10 \times 10 = 10 \times 20$$

$$\Rightarrow \quad F - 100 = 200 \Rightarrow F = 300\ \text{N}$$

When 4 kg mass is removed then weight of balloon

$$6 \times 10 = 60\ \text{N}$$

Which is also smaller than the upward force.

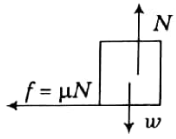
So, $\quad F - 60 = 6 \times a$

where, a = new upward acceleration

$$\Rightarrow \quad 300 - 60 = 6a \Rightarrow 240 = 6a$$

$$\therefore \quad a = \frac{240}{6} = 40\ \text{ms}^{-2}$$

4. (*b*) Let weight of A is w'. From the free body diagram, for equilibrium of the system,



$$T\cos\theta = \mu N = \mu w \quad \text{...(i)}$$

$$T\sin\theta = w' \quad \text{...(ii)}$$

where, T = tension in the thread lying between knot and the support.

On dividing Eq. (ii) by Eq. (i), we get

$$\frac{T\sin\theta}{T\cos\theta} = \frac{w'}{\mu w} \Rightarrow \tan\theta = \frac{w'}{\mu w}$$

$$\Rightarrow \quad w' = \mu w \tan\theta$$

5. (*b*) Given, $m = 0.05$ kg

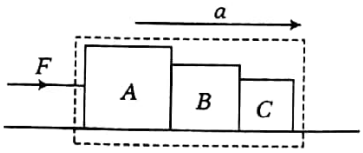
We know that, net force acting on a stone,

$$f_g = mg = 0.05 \times (-9.8) = -0.49\ \text{N}$$

$$= 0.49\,\text{N vertically downwards}$$

6. (*b*) Given, $m_A = 4$ kg, $m_B = 2$ kg

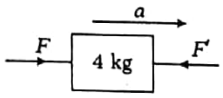
$$\Rightarrow \quad m_C = 1\,\text{kg}$$



So total mass $(M) = 4 + 2 + 1 = 7$ kg

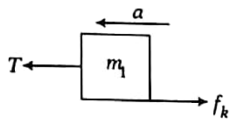
Now, $\quad F = Ma \Rightarrow 14 = 7a \Rightarrow a = 2\ \text{ms}^{-2}$

FBD of block A,



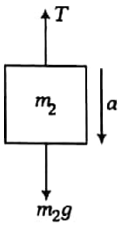
$$F - F' = 4a \Rightarrow F' = 14 - 4 \times 2 \Rightarrow F' = 6\,\text{N}$$

7. (*c*) FBD of block A,



$$T - m_1 a = f_k \quad \text{...(i)}$$

FBD of block B,



$$m_2 g - T = m_2 a \quad \text{...(ii)}$$

On adding Eqs. (i) and (ii), we get

$$m_2 g - m_1 a = m_2 a + f_k$$

$$\Rightarrow \quad m_2 g - m_1 a = m_2 a + \mu_k m_1 g$$

$$\Rightarrow \quad a = \frac{(m_2 - \mu_k m_1) g}{m_1 + m_2}$$

From Eq. (ii),

$$T = m_2(g - a) = m_2\left[1 - \frac{(m_2 - \mu_k m_1)}{m_1 + m_2}\right] g$$

$$T = \frac{m_1 m_2 (1 + \mu_k)}{m_1 + m_2} g$$

8. (*a*) When the balloon is descending down with acceleration a

So, $\quad mg - B = m \times a \quad$ ...(i)

($B \rightarrow$ Buoyant force)

Here, we should assume that while removing some mass the volume of balloon and hence buoyant force will not change.

Let the new mass of the balloon is m'.

So, $\quad$ mass removed $= m - m'$

So, $\quad B - m' g = m' \times a$...(ii)

On solving Eqs. (i) and (ii), we get

$$mg - B = m \times a$$

$$B - m'g = m' \times a$$

$$\Rightarrow \quad mg - m'g = ma + m'a$$

$$\Rightarrow \quad (mg - ma) = m'(g + a)$$

$$\Rightarrow \quad m(g - a) = m'(g + a)$$

$$\therefore \quad m' = \frac{m(g - a)}{g + a}$$

So, mass removed, $\Delta m = m - m'$

$$= m\left[1 - \frac{(g - a)}{(g + a)}\right] = m\left[\frac{(g + a) - (g - a)}{(g + a)}\right]$$

$$= m\left[\frac{g + a - g + a}{g + a}\right] = \frac{2ma}{g + a}$$

9. (*b*) Given, $u = 30\sqrt{2}\ \text{ms}^{-1}$

Distance $= s$ m

Let a be the acceleration of the bullet

According to the first condition, ($v = 0$)

Given, $\quad v^2 = u^2 - 2as$

$$0 = 900 \times 2 - 2as$$

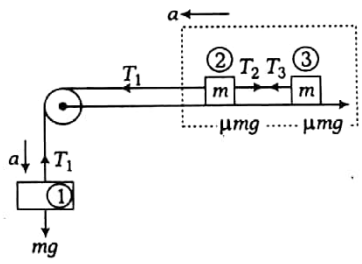
$$\Rightarrow \quad a = \frac{900}{s}\ \text{ms}^{-2}$$

For second condition,

$$v^2 = u^2 - 2as = (30\sqrt{2})^2 - 2 \times \frac{900}{s} \times \frac{s}{2}$$

$$\Rightarrow \quad v^2 = 1800 - 900 \Rightarrow v = \sqrt{900} = 30\ \text{ms}^{-1}$$

10. (*c*) First of all consider the forces on the blocks



For the Ist block, $\quad (\because m_1 = m_2 = m_3)$

$$mg - T_1 = m \times a \quad \text{...(i)}$$

$\Rightarrow$ Let us consider 2nd and 3rd blocks as a system.

So, $\quad T_1 - 2\mu mg = 2m \times a \quad$ ...(ii)

On solving Eqs. (i) and (ii), we get

$$\Rightarrow \quad mg - T_1 = m \times a$$

$$T_1 - 2\mu mg = 2m \times a$$

$$mg(1 - 2\mu) = 3m \times a \Rightarrow a = \frac{g}{3}(1 - 2\mu)$$

11. (*a*) Given, initial velocity $= v$

Final velocity $= 0$

Coefficient of friction $= \mu$

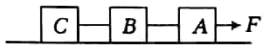
For a straight line motion, $v - u = at$

Here, $\quad 0 - v = -(\mu g)\, t \quad \left(\text{as, } a = \frac{\mu mg}{m}\right)$

Negative sign indicates retardation.

$$\therefore \quad t = \frac{v}{\mu g}$$

12. (*c*) Given, $m_1 = m_2 = m_3 = 2$ kg, $F = 10.2$ N in a frictionless surface as shown in free body diagram.



For body A, $F - T_1 = ma$ ...(i)

For body B, $T_1 - T = ma$ ...(ii)

For body C, $T = ma$ ...(iii)

Adding these three equations, $F = 3\,ma$

$$a = \frac{10.2}{3 \times 2} = 1.7 \text{ ms}^{-2}$$

Alternately acceleration can be found as net acceleration of a system, *i.e.*,

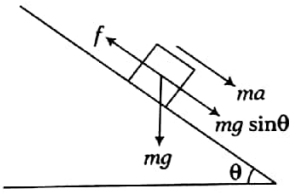
$$a = \frac{\text{Total net force}}{\text{Total mass}}$$

$$= \frac{10.2}{(2+2+2)} = \frac{10.2}{6} = 1.7 \text{ ms}^{-2}$$

So, net tension in string between the blocks B and C is

$$T = m \times a = 2 \times 1.7 = 3.4 \text{ N}$$

13. (*c*) Here, $mg \sin\theta - f = ma$



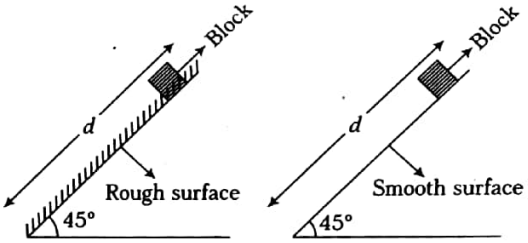
$$mg \sin\theta - ma = f$$

$\Rightarrow$ $8 \times 10 \sin(30°) - 8 \times 0.4 = f$

$\Rightarrow$ $40 - 3.2 = f$

$\Rightarrow$ $f = 36.8$ N

14. (*a*) If the same wedge is made rough, then time taken by it come down becomes n times more. The coefficient of friction.



For rough surface

$$t = \sqrt{\frac{2s}{2\sin\theta - \mu g \cos\theta}} \quad \text{...(i)}$$

For smooth surface

$$\frac{t}{n} = \sqrt{\frac{2s}{g \sin\theta}} \quad \text{...(ii)}$$

From Eqs. (i) and (ii) we have

$$\mu = \left(1 - \frac{1}{n^2}\right) \tan\theta$$

$\therefore$ $$\mu = \left[1 - \frac{1}{n^2}\right] \tan\theta = \left[1 - \frac{1}{2^2}\right] \tan 45°$$

$\Rightarrow$ $$\mu = \frac{4-1}{4} = \frac{3}{4}$$

15. (*a*) Consider a body of mass m placed on a rough inclined surface and it is just on the point of sliding down, with coefficient of friction μ inclined at angle θ, as shown in figure.

At the equilibrium point O,

In case of limiting condition, $F = mg \sin\theta$ ...(i)

Normal force, $R = mg \cos\theta$ ...(ii)

Now, comparing Eqs. (i) and (ii), we get

$$\frac{F}{R} = \frac{mg \sin\theta}{mg \cos\theta} \quad (\because \text{ force of friction, } F = \mu R)$$

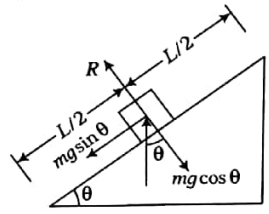
$$\mu = \tan\theta \Rightarrow \theta = \tan^{-1}(\mu)$$

This is maximum value of θ for mass m to be at rest. For smaller θ, body will be at rest, *i.e.*, in equilibrium.

So, angle of repose, i.e. $\theta = \tan^{-1}\mu$

16. (*a*) Since, all the blocks are moving with constant velocity, then the net force on the all blocks will be zero.

17. (*c*) The block may be stationary, when



$$mg \sin\theta \cdot L = \mu\, mg \cos\theta \frac{L}{2}$$

$\Rightarrow$ $$\mu = \frac{mg \sin\theta \cdot L}{mg \cos\theta \frac{L}{2}} = 2\frac{\sin\theta}{\cos\theta} = 2\tan\theta$$

$$\mu = 2\tan\theta$$

18. (*a*) From Newton's second law of motion, the rate of change of linear momentum of an object is directly proportional to the external force applied on it and change always takes place in the direction of applied force.

$$\mathbf{F} = \frac{d\mathbf{p}}{dt}$$

19. (*c*) Given that, $h = 100$ m and $t = 10$ s

From equation of motion,

$$h = \frac{1}{2}at^2$$

$$a = \frac{2h}{t^2} = \frac{2 \times 100}{(10)^2} = 2 \text{ ms}^{-2}$$

Now, $F = 50(10 + 2) \Rightarrow F = 600$ N

20. (*d*) We know that $F - f = ma$

Here for small body,

$$a_1 = \frac{20 - 0.2 \times 3 \times 10}{3} \Rightarrow a_1 = \frac{14}{3} \text{ ms}^{-2}$$

Similarly, $0.2 \times 3 \times 10 = 10 \times a_2$

$$a_2 = \frac{6}{10} = 0.6 \text{ ms}^{-2}$$

21. (*d*) Here, $M_{eff}\ g = M(g-a)$

or $\frac{g}{g-a} = \frac{M}{M_{eff}} \Rightarrow \frac{g}{g-a} = \frac{60}{54}$

$\Rightarrow \frac{g}{g-a} = \frac{10}{9} \Rightarrow 9g = 10g - 10a$

$\Rightarrow -g = -10a \Rightarrow g = 10a$

$\Rightarrow a = \frac{g}{10} \Rightarrow a = \frac{9.8}{10} = 0.98\ \text{ms}^{-2}$

22. (*b*) Given, $u = 0$, $s = 15$ m, $t = 0.4$ s

From $s = ut + \frac{1}{2}at^2 \Rightarrow 1.5 = 0 + \frac{1}{2}a(0.4)^2$

$\Rightarrow a = \frac{1.5 \times 2}{(0.4)^2} = 18.75\ \text{ms}^{-2}$

As the string is moving upwards with in acceleration

$\therefore T = m(g+a) = 2(9.8 + 18.75) = 57.1\text{N}$

23. (*c*) Given, $m_1 = M$, $m_2 = M/2$

$\therefore a = \frac{(m_1 - m_2)g}{(m_1 + m_2)} = \frac{(m - M/2)g}{(m + M/2)} = \frac{g}{3}$

24. (*b*) Initial momentum of the system

$= m_1 \times 40 + m_2 \times 0 = 40\, m_1$

Final momentum of the system $= (m_1 + m_2) \times 30$

By the law of conservation of momentum,

Initial momentum = Final momentum

$40\, m_1 = (m_1 + m_2)\, 30$

$40\, m_1 - 30\, m_1 = 30\, m_2$

$10\, m_1 = 30\, m_2$

$\frac{m_1}{m_2} = \frac{3}{1}$

25. (*c*) For downward motion of lift, $R = m(g-a)$

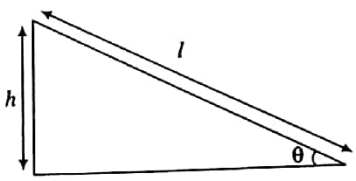
Here, $a = g$

$\therefore \quad R = 0$

So, $\quad F = \mu R = 0$

26. (*b*) Force down the plane $= mg\sin\theta$

$\therefore$ Acceleration down the plane $= g\sin\theta$



Using the relation, $h = ut + \frac{1}{2}gt^2$

$l = 0 + \frac{1}{2}(g\sin\theta)t^2$

$t^2 = \frac{2l}{g\sin\theta} = \frac{2h}{g\sin^2\theta} \qquad \left(\because \sin\theta = \frac{h}{l}\right)$

or $\quad t = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}} \qquad \left(l = \frac{h}{\sin\theta}\right)$

27. (*b*) Time t_1 for stationary lift $= \sqrt{\frac{2h}{g}}$

When lift is moving up with constant acceleration, then

$t_2 = \sqrt{\frac{2h}{g+a}}$ [Relative acceleration $= (g + a)$]

$\therefore t_1 > t_2$

28. (*b*) Impulse, $I = F \times \Delta t = 50 \times 10^{-5} \times 3 = 15 \times 10^{-3}$ Ns

29. (*a*) Kinetic friction $= \mu_k R = \mu_k mg = 0.4 \times 60 \times 10 = 240$ N

and the limiting friction $= \mu_S R = \mu_S mg = 0.5 \times 60 \times 10 = 300$ N

So, the force applied on the body is 300 N and if the body is moving afterwards with the same force, then

Net accelerating force = Applied force – Kinetic force

$ma = 300 - 240$

$a = \frac{60}{m} = \frac{60}{60} = 1\,\text{ms}^{-1}$

30. (*d*) Force of limiting friction for block $= \mu_s mg$

$= 0.60 \times 10 \times 9.8 = 58.8$ N

If the applied force is greater than 58.8 N, then the block will move over the slab.

Kinetic friction acting on the block towards right

$= \mu_K mg = 0.40 \times 10 \times 9.8 = 39.2$ N

This is also equal to the force of friction acting on slab towards left.

This is the only force acting on slab.

So, acceleration of the slab, $a = \frac{F}{m} = \frac{39.2}{40} = 0.98\ \text{ms}^{-2}$

31. (*d*) Using law of conservation of momentum, we get

$100 \times v = 0.25 \times 100$

$\Rightarrow \quad v = 0.25\ \text{ms}^{-1}$

32. (*c*) Initial thrust must be

$m(g+a) = 3.5 \times 10^4\ (10 + 10)$

$= 7 \times 10^5$ N

33. (*d*) $v = u - at$

$0 = u - \mu gt$

$\mu = \frac{u}{gt} = \frac{6}{10 \times 10} = 0.06$

34. (*c*) $T_1 = \frac{(m_2 + m_3 + m_4)F}{m_1 + m_2 + m_3 + m_4}$

$\therefore \quad T_1 = \frac{3}{4}F$

Similarly, $\quad T_2 = \frac{(m_3 + m_4)F}{m_1 + m_2 + m_3 + m_4}$

$\Rightarrow \quad T_2 = \frac{1}{2}F$

$T_3 = \frac{(m_4)F}{m_1 + m_2 + m_3 + m_4} = \frac{1}{4}F$

35. (*c*) $mv_o = (m + m)v \Rightarrow v = \frac{v_o}{2}$

$$T = \frac{2mv^2}{l} + 2mg = \frac{m(2gl)}{2l} + 2mg$$

$$\left[\because \frac{2mv^2}{l} = \text{Centrifugal force}\right]$$

$T = 3mg$

$\therefore$ Increase in tension $= 3mg - mg = 2mg$

36. (*c*) Friction force between block *A* and block *B* and between block *B* and surface will oppose the *F*

$\therefore \quad F = F_{AB} + F_{BS}$

$= \mu_{AB} m_A g + \mu_{BS}(m_A + m_B)g$

$= 0.2 \times 100 \times 10 + 0.3(100 + 200)10$

$= 200 + 900 = 1100$ N

This is the required minimum force to move the block *B*.

37. (*b*) Given that, $m_b = 40$ g, $v_b = 1200$ ms^{-1}

$F = 144$ N

$\therefore \quad F = ma = \frac{m_b v_b}{t}$

$\Rightarrow \quad 144 = \frac{40 \times 1200}{1000} \times n \Rightarrow n = 3$

38. (*a*) The maximum weight which can be suspended with the rope without breaking it = 30 kg-wt.

$= 30 \times 10 = 300$ N

$\therefore \quad 300 \text{ N} = mg + ma$

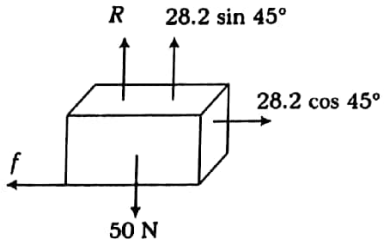
$\Rightarrow \quad ma = 300 - mg$

$= 300 - 25 \times 10 = 50$ N

$\Rightarrow \quad a = \frac{50}{m} = \frac{50}{25} = 2$ ms^{-2}

39. (*d*) Frictional force, $f = 28.2 \cos 45°$

$= 28.2 \times \frac{1}{\sqrt{2}} = 20$ N



Normal reaction, $R = 50 - 28.2 \sin 45° = 50 - 20 = 30$ N

CHAPTER 06

Work, energy and power

The terms *work* and *energy* are quite familiar to us and we use these terms in many ways. In physics work is said to be done, when point of application actually moves.

Energy is the capacity to do work and the term *power* is usually associated with the time in which the work is done.

In this chapter, we are going to develop better understanding of these three physical quantities in detail.

Inside

1) Work
- Work done by a variable force

2) Conservative forces

3) Non-conservative forces

4) Energy
- Kinetic energy
- Work-energy theorem
- Potential energy

5) Equilibrium

6) Law of conservation of energy

7) Conversion of mass and energy

8) Power
- Instantaneous power

1 Work

Work is said to be done when a force acts on a body such that the body is displaced through some distance in the direction of force.

We define the work done by a constant force on a body as the dot product of the force **F** and the displacement **s** through which the body is displaced.

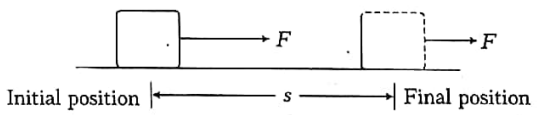


***Fig. 6.1** Work done, when force and displacement are in the same direction*

Then, the work done W is given by $\boxed{W = \mathbf{F} \cdot \mathbf{s}}$

On the other hand, in a situation when the constant force F acting on the body makes angle θ with the horizontal and body is displaced through a distance s.

Then, F can be resolved into two components

(i) $F\cos\theta$ in the direction of displacement of body.

(ii) $F\sin\theta$ in the perpendicular direction of the displacement of body.

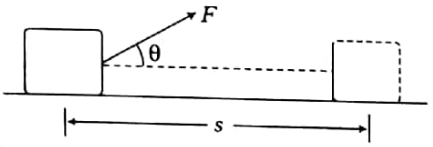


***Fig. 6.2** Work done, when force and displacement are inclined to each other*

Thus, in this case, work done by a constant force **F** is given by

$$W = \text{(component of force along the displacement)} \times \text{(displacement)}$$

or $$W = (F\cos\theta)(s)$$

or $$\boxed{W = \mathbf{F}\cdot\mathbf{s}}$$ (from the definition of dot product)

So, work done is a scalar or dot product of **F** and **s**.

Work is scalar quantity, its dimensional formula is $[ML^2T^{-2}]$. SI unit of work is **joule** and CGS unit is **erg.**

$$\boxed{1 \text{ joule} = 10^7 \text{ erg}}$$

Gravitational unit of work done is kg-m.

$$\boxed{1 \text{ kg-m} = 9.8 \text{ J}}$$

Some other convenient unit of work are eV (electron-volt), MeV (Mega electron volt) and kWh (kilo watt-hour)

- $1 \text{ eV} = 1.6\times 10^{-19}$ J, $1 \text{ J} = 6.25\times 10^{18}$ eV
- $1 \text{ MeV} = 1.6\times 10^{-13}$ J, $1 \text{ J} = 6.25\times 10^{12}$ MeV
- 1 kilowatt hour (kWh) $= 3.6\times 10^6$ J

Work done by a constant force

The work done by a constant force **F** on a particle which undergoes displacement **s** is given by

$$\boxed{W = \mathbf{F}\cdot\mathbf{s} = Fs\cos\theta}$$

where θ is the angle between **F** and **s**.

Work done is scalar or dot product of **F** and **s**.

The work done is independent of the actual path taken by the particle if force happens to be constant. It only depends on the net displacement.

(i) θ is acute, W is positive and force tries to increase the speed of the body.

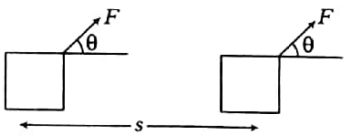


Fig. 6.3

(ii) $\theta = 90°$, W is zero and there is no change in speed of the body due the force.

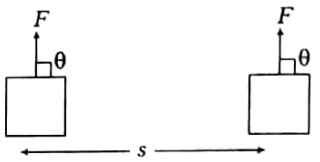


Fig. 6.4

(iii) θ is obtuse, W is negative and force tries to decrease the speed of the body.

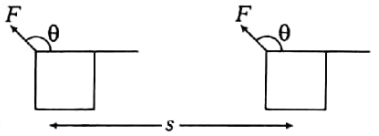


Fig. 6.5

Example 6.1 *A gas filled in a cylinder fitted with a movable piston is allowed to expand. What is the nature of the work done by the gas?*

Sol. Positive, as force due to gaseous pressure and displacement of piston are in the same direction.

Example 6.2 *A lawn roller has been pushed by a gardener through a distance of 30 m. What will be the work done by him, if he applies a force of 30 kg-wt in the direction inclined at 60° to the ground?*

Sol. Given, Displacement, $s = 30$ m

Force, $F = 30$ kg-wt $= 300$ N

$\theta = 60°$

The work done by the gardener,

$$W = F\cdot s = Fs\cos\theta$$
$$= 300\times 30\times\cos 60°$$
$$= 300\times 30\times\frac{1}{2}$$

$\Rightarrow$ $$W = 4500 \text{ J}$$

EXAMPLE 6.3 *A body moves a distance of 10 m along a straight line under an action of 5 N force. If work done is 25 J, then find the angle between the force and direction of motion of the body.*

Sol. Work is measured by the product of the applied force and the displacement of the body in the direction of the force.

$\therefore$ Work = Applied force × Displacement

$$W = (F\cos\theta)\times s = Fs\cos\theta$$

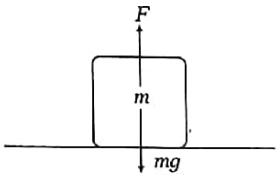
Given, $W = 25$ J, $F = 5$ N, $s = 10$ m

$\therefore$ $$\cos\theta = \frac{W}{F\cdot s} = \frac{25}{5\times 10} = \frac{1}{2}$$

$\Rightarrow$ $$\theta = \cos^{-1}\left(\frac{1}{2}\right) = 60°$$

Hence, angle between force and direction of motion of the body is 60°.

Example 6.4 *A block of mass $m = 2$ kg is pulled by a force $F = 40$ N upwards through a height $h = 2$ m. Find the work done on the block by the applied force F and its weight mg. (Take, $g = 10\ ms^{-2}$)*



Sol. Weight of the block, $mg = (2)(10) = 20$ N

Work done by the applied force,

$$W_F = Fs\cos\theta = Fh\cos 0°$$

(Here, $s = h = 2$ m and the angle between force and displacement is 0°)

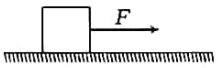
or $$W_F = (40)(2)(1) = 80 \text{ J}$$

Similarly, work done by its weight,

$$W_{mg} = (mg)(h)\cos 180°$$

or $$W_{mg} = (20)(2)(-1) = -40 \text{ J}$$

Example 6.5 *A 10 g block placed on a rough horizontal floor is being pulled by a constant force 50 N. Coefficient of kinetic friction between the block and the floor is 0.4. Find work done by each individual force acting on the block over displacement of 5 m.*



Sol. Given, mass $m = 10$ g

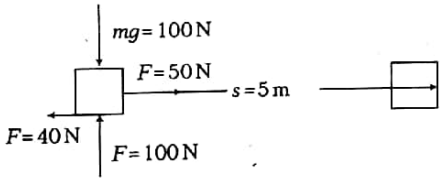
Constant force $F = 50$ N

Coefficient of kinetic friction, $\mu_k = 0.4$

Displacement, $s = 5$ m

Forces acting on the block are its weight ($mg = 100$ N).

Normal reaction ($N = 100$ N) from the ground, force due to kinetic friction ($F = \mu_k mg = 40$ N) and the applied force ($F = 50$ N) which are shown in the given figure.



Work done W_g by the gravity, *i.e.*, weight of the block

$$W_g = 0 \text{ J} \qquad (\because mg \perp s)$$

Work done W_N by the normal reaction

$$W_N = 0 \text{ J} \qquad (\because \mathbf{N} \perp \mathbf{s})$$

Work done W_F by the applied force

$$W_F = 50 \times 5 = 250 \text{ J} \qquad (\because \mathbf{F} \parallel \mathbf{s})$$

Work done W_f by the force of kinetic friction

$$W_f = 40 \times 5$$

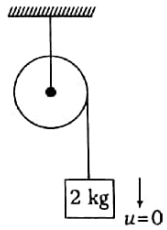
$$W_f = -200 \text{ J} \quad (\because \mathbf{F} \text{ and } \mathbf{s} \text{ are antiparallel})$$

Example 6.6 *A block of mass 2 kg is being brought down by a string. If the block acquires a speed 1 ms^{-1} in dropping down 25 cm, find the work done by the string in the process.*

Sol. Given, Distance moved by block $s = 25 \text{ cm} = \frac{1}{4}$ m

Initial velocity of block, $u = 0$

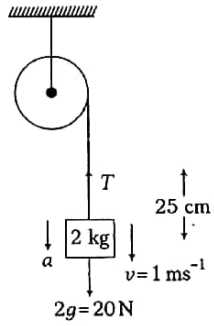
Final velocity of block, $v = 1\ ms^{-1}$



From the equation of motion for block,

$$v^2 = u^2 + 2as \Rightarrow (1)^2 = 0 + 2a \times \frac{1}{4} \Rightarrow a = 2\ ms^{-2}$$

$\therefore$ Acceleration of the block $a = 2\ ms^{-2}$



For the motion of 2 kg block in downward direction, applying Newton's law we get

$$mg - T = ma$$

$$20 - T = 2a = 2 \times 2$$

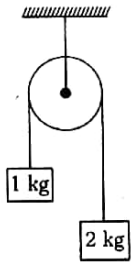
$$\Rightarrow \quad T = 16 \text{ N}$$

$\therefore$ Work done by the string,

$$W = -Ts = -16 \times \frac{1}{4} = -4 \text{ J}$$

Negative sign indicates the opposite direction of tension and displacement of the block.

Example 6.7 *Two unequal masses of 1 kg and 2 kg are attached at the two ends of a light inextensible string passing over a smooth pulley as shown in figure. If the system is released from rest, find the work done by string on both the blocks in 1 s. (Take $g = 10\ ms^{-2}$)*

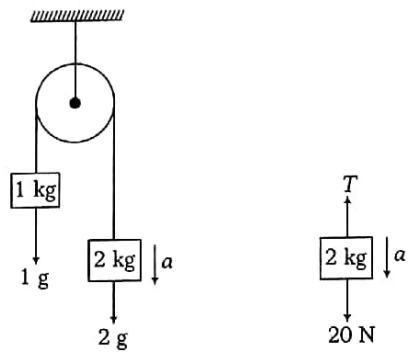


Sol. Net pulling force on the system is

$$F_{net} = 2g - 1g = 20 - 10 = 10 \text{ N}$$

Total mass being pulled

$$m = (1 + 2) = 3 \text{ kg}$$



(i) (ii)

Therefore, acceleration of the system will be

$$a = \frac{F_{net}}{m} = \frac{10}{3}\ \text{ms}^{-2}$$

Displacement of both the blocks in 1 s is

$$s = \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{10}{3}\right)(1)^2 = \frac{5}{3}\ \text{m}$$

Free body diagram of 2 kg block is shown in Fig. (ii).

Using $\Sigma F = ma$, we get

$$20 - T = 2a = 2\left(\frac{10}{3}\right)$$

or $$T = 20 - \frac{20}{3} = \frac{40}{3}\ \text{N}$$

∴ Work done by string (tension) on 1 kg block in 1 s is

$$W_1 = (T)(s)\cos 0°$$
$$= \left(\frac{40}{3}\right)\left(\frac{5}{3}\right)(1) = \frac{200}{9}\ \text{J}$$

Similarly, work done by string on 2 kg block in 1 s will be

$$W_2 = (T)(s)(\cos 180°)$$
$$= \left(\frac{40}{3}\right)\left(\frac{5}{3}\right)(-1) = -\frac{200}{9}\ \text{J}$$

Work done on a particle moving in three dimensional space

If a particle moves in space under the action of a constant force given by

$$\mathbf{F} = F_x\hat{\mathbf{i}} + F_y\hat{\mathbf{j}} + F_z\hat{\mathbf{k}}$$

Let initial position of the particle is

$$\mathbf{s}_1 = x_1\hat{\mathbf{i}} + y_1\hat{\mathbf{j}} + z_1\hat{\mathbf{k}}$$

and final position be

$$\mathbf{s}_2 = x_2\hat{\mathbf{i}} + y_2\hat{\mathbf{j}} + z_2\hat{\mathbf{k}}$$

Then, work done by the force **F** given by

$$W = \mathbf{F}\cdot\mathbf{s} = \mathbf{F}\cdot(\mathbf{s}_2 - \mathbf{s}_1)$$
$$= \mathbf{F}\cdot[(x_2 - x_1)\hat{\mathbf{i}} + (y_2 - y_1)\hat{\mathbf{j}} + (z_2 - z_1)\hat{\mathbf{k}}]$$

$$\Rightarrow \quad \boxed{W = F_x(x_2 - x_1) + F_y(y_2 - y_1) + F_z(z_2 - z_1)}$$

Example 6.8 *A constant force* $\mathbf{F} = (\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$ *N acts on a particle and displace it from* $(-1m, 2m, 1m)$ *to* $(2m, -3m, 1m)$. *Find the work done by force.*

Sol. Given, $\mathbf{F} = \hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$ and

Initial position of the particle is given by

$$\mathbf{s}_1 = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$$

Final position, $\mathbf{s}_2 = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$

Displacement of the particle,

$$\mathbf{s} = \mathbf{s}_2 - \mathbf{s}_1 = (2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \mathbf{k}) - (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}})$$
$$= 3\hat{\mathbf{i}} - 5\hat{\mathbf{j}} + 0\hat{\mathbf{k}} = 3\hat{\mathbf{i}} - 5\hat{\mathbf{j}}$$

Work done by the force **F**

$$W = \mathbf{F}\cdot\mathbf{s} = (\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})\cdot(3\hat{\mathbf{i}} - 5\hat{\mathbf{j}}) = (3 - 15) = -12\text{J}$$

Example 6.9 *A particle is shifted form point* $(0, 0, 1m)$ *to* $(1m, 1m, 2m)$, *under simultanoeus action of several forces. Two of the forces are* $\mathbf{F}_1 = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}})$ *N and* $\mathbf{F}_2 = (\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$ *N. Find work done by resultant of these two forces.*

Sol. Work done by a constant force equals to dot product of the force and displacement vectors.

$$W = \mathbf{F}\cdot\mathbf{s} \quad \text{...(i)}$$

Initial position of the particle is given by $\mathbf{s}_1 = \hat{\mathbf{k}}$

Final position, $\mathbf{s}_2 = \hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

Displacement of the particle

$$\mathbf{s} = \mathbf{s}_2 - \mathbf{s}_1 = \hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}} - \hat{\mathbf{k}} \Rightarrow \mathbf{s} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$$

and force $\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}} + \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}} = 3\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$

Substituting given values in Eq. (i), we have

$$W = (3\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})\cdot(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) = 3 + 1 + 1 = 5\ \text{J}$$

Work done by a variable force

The force is said to be variable force if it changes its direction or magnitude or both. The work done by a variable force can be calculated as

$$\boxed{W = \int_{r_1}^{r_2}\mathbf{F}\cdot d\mathbf{r}}$$

where integration is performed along the path of particle and $d\mathbf{r}$ is the position vector of the particle.

If the particle moves from $\mathbf{r}_1(x_1, y_1, z_1)$ to $\mathbf{r}_2(x_2, y_2, z_2)$ *i.e.*, when the magnitude and direction of the force vary in three dimensions, then the work done by force **F**, is given by

$$W = \int dW = \int_{x_1}^{x_2} F_x\,dx + \int_{y_1}^{y_2} F_y\,dy + \int_{z_1}^{z_2} F_z\,dz$$

where, F_x, F_y and F_z are the rectangular components of force in x, y and z-directions respectively.

- If the motion is one dimensional, $W = \int_{x_i}^{x_f} F_x\,dx$

  in which F_x is the component of force along motion.

Example 6.10 *A position dependent force $F = 7 - 2x + 3x^2$ acts on a small body of mass 2 kg and displaced it from $x = 0$ to $x = 5$ m. Calculate the work done in joule.*

Sol. Work done, $W = \int_{x_1}^{x_2} F\,dx = \int_0^5 (7 - 2x + 3x^2)\,dx$

Here, the body changes its position from $x = 0$ to $x = 5$

$$= \left[7x - \frac{2x^2}{2} + \frac{3x^3}{3}\right]_0^5 = [7(5) - (5)^2 + (5)^3 - 0] = 135 \text{ J}$$

Example 6.11 *A force $F = (2 + x)$ acts on a particle in x-direction where F is in newton and x in metre. Find the work done by this force during a displacement from $x = 1$ m to $x = 2$ m.*

Sol. As the force is variable, we shall find the work done in a small displacement from x to $x + dx$ and then integrate it to find the total work. The work done in this small displacement is

$$dW = F\,dx = (2 + x)\,dx$$

Thus, $$W = \int_1^2 dW = \int_1^2 (2 + x)\,dx$$

$$= \left[2x + \frac{x^2}{2}\right]_1^2 = \left[\left(4 + \frac{4}{2}\right) - \left(2 + \frac{1}{2}\right)\right] = 3.5 \text{ J}$$

Example 6.12 *A force $F = -\dfrac{k}{x^2}$ $(x \neq 0)$ acts on a particle in x-direction. Find the work done by this force in displacing the particle from $x = +a$ to $x = +2a$. Here, k is a positive constant.*

Sol. Work done $W = \int F\,dx = \int_{+a}^{+2a} \left(\dfrac{-k}{x^2}\right) dx = \left[\dfrac{k}{x}\right]_{+a}^{+2a} = -\dfrac{k}{2a}$

Note It is important to note that work comes out to be negative which is quite obvious as the force acting on the particle is in negative x-direction $\left(F = -\dfrac{k}{x^2}\right)$ while displacement is along positive x-direction. (from $x = a$ to $x = 2a$).

Calculation of work done by force displacement graph

The area under force-displacement curve gives work done.

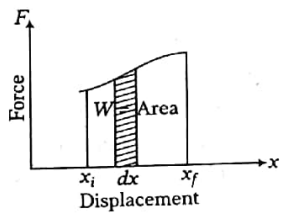


Fig. 6.6 Force displacement graph

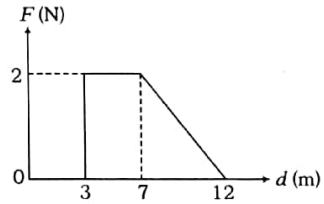
$$W = \int_{x_i}^{x_f} dW = \int_{x_i}^{x_f} F \cdot dx$$

where, x_i and x_f are the initial and final position respectively.

$$\therefore \quad W = \int_{x_i}^{x_f} (\text{Area under curve})$$

$\Rightarrow$ W = Area under curve between x_i and x_f.

Example 6.13 *Force F on a particle moving in a straight line varies with distance d as shown in the figure. Find the work done on the particle during its displacement of 12 m.*

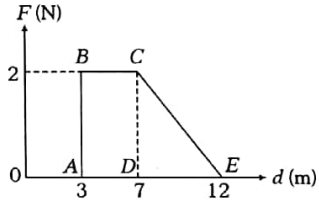


Sol. Work done = Area under (force-displacement) graph

= Area of rectangle $ABCD$ + Area of ΔDCE

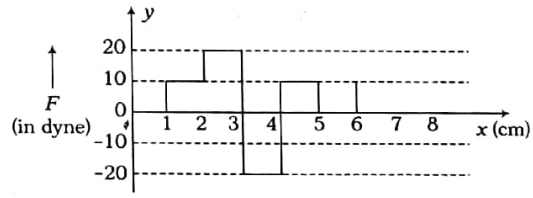
$= \text{length} \times \text{breadth} + \dfrac{1}{2} \times \text{base} \times \text{height}$

$= (AB \times AD) + \dfrac{1}{2} \times DE \times CD$



$$= 2 \times (7 - 3) + \frac{1}{2} \times 2 \times (12 - 7) = 8 + \frac{1}{2} \times 10 = 8 + 5 = 13 \text{ J}$$

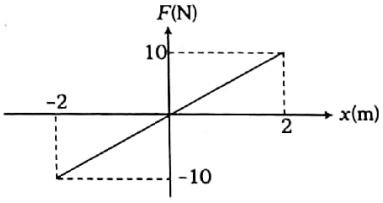
Example 6.14 *For the force displacement graph shown below. Calculate the work done by the force in displacing the body from $x = 1$ cm to $x = 5$ cm.*



Sol. Work = Area under the curve and displacement axis

$$= 10 \times 1 + 20 \times 1 - 20 \times 1 + 10 \times 1 = 20 \text{ erg}$$

Example 6.15 *A force F acting on a particle varies with the position x as shown in figure. Find the work done by this force in displacing the particle from*



(i) $x = -2$ m to $x = 0$ *(ii) $x = 0$ to $x = 2$ m.*

Sol. (i) From $x = -2$ m to $x = 0$, displacement of the particle is along positive x-direction while force acting on the particle is along negative x-direction. Therefore, work done is negative and is given by the area under F-x graph.

$$\therefore \quad W = -\frac{1}{2}(2)(10) = -10 \text{ J}$$

(ii) From $x = 0$ to $x = 2$ m, displacement of particle and force acting on the particle both are along positive x-direction. Therefore, work done is positive and given by the area under F-x graph, or

$$W = \frac{1}{2}(2)(10) = 10 \text{ J}$$

Spring block system

Consider an elastic spring of negligibly small mass having spring constant k with its one end attached to a rigid support and its other end is attached to a block of mass m that can slide over a smooth horizontal surface.

Suppose a force F is applied on the spring to stretch it from natural length to produce an elongation x in it.

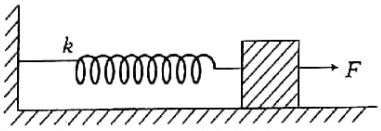


Fig. 6.7 *Spring block system*

The work done in stretching the spring, by external applied force $W = \int_0^x F\,dx = \int_0^x kx\,dx$

$$\boxed{W = \frac{1}{2}kx^2}$$

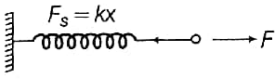
The work done by stretching or compressing force is positive. But the work done by the spring is negative because the force exerted by the spring is always opposite to elongation or contraction.

$$\boxed{\text{Work done by the spring, } W = -\frac{1}{2}kx^2}$$

When length of spring changes from $x = x_i$ to $x = x_f$

$$W = -\int_{x_i}^{x_f} F dx = -\int_{x_i}^{x_f} kx dx = \frac{1}{2}k(x_i^2 - x_f^2)$$

Example 6.16 *The work done in extending a spring by x_0 is W_0. Find the work done in further extension x_0.*



Sol. Force, $F = F_s = kx$

$$W_0 = \int F\,dx = \int_0^{x_0} kx\,dx = \frac{kx_0^2}{2}$$

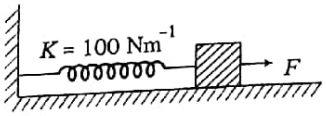
Let W be the work done in extending a spring by $2x_0$.

$$W = \int F\,dx = \int_{x_0}^{2x_0} kx\,dx$$

$$= \frac{k}{2}[(2x_0)^2 - x_0^2] = \frac{3}{2}kx_0^2$$

$$\frac{W}{W_0} = 3 \Rightarrow W = 3W_0$$

Example 6.17 *Consider a block connected to a light spring of spring constant 100 Nm^{-1}. Now the block is displaced by applying a constant force F which gives zero resultant force when spring is stretched through 10 cm.*



Evaluate

(i) *Work done by the spring force when the block attains equilibrium.*

(ii) *Net work done on the block when it attains maximum speed.*

Sol. (i) Work done by the spring force, $W = \frac{1}{2}kx^2$

$$k = 100 \text{ Nm}^{-1} \text{ and } x = 10 \text{ cm} = 0.1 \text{ m}$$

$$\therefore \quad W_S = \frac{1}{2} \times 100\,(0.1)^2 = -0.5 \text{ J}$$

Negative sign indicates that the work done by the spring force is negative.

(ii) When the block attains equilibrium, its speed is maximum.

$\therefore$ Work done on the block by external force F

$$W = F \cdot x = (10)\left(\frac{10}{100}\right) = 1 \text{ J}$$

Thus, net work done on the block

$$W_N = W_S + W = -0.5 + 1 = 0.5 \text{ J}$$

Dependence of work done on frame of reference

Work depends on frame of reference. With change of frame of reference, inertial force does not change while displacement may change.

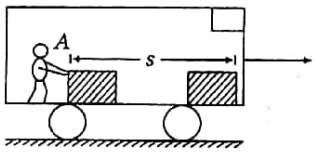


Fig. 6.8 *Dependence of work done on frame of reference*

So, the work done by a force will be different in different frames. *e.g.*, if a person A is pushing a box inside a moving bus, then work done as seen by him from the frame of reference of bus is $\mathbf{F} \cdot \mathbf{s}$ while as seen by a person B on the ground it is $\mathbf{F} \cdot (s + s_0)$. Here s_0 is the displacement of bus relative to ground.

Example 6.18 *A train is moving with a speed of 90 kmh^{-1}. A passenger X inside the train displaces his 40 kg, luggage slowly an the floor through 1 m in 10 s. Coefficient of friction of the floor of the train is 0.2. Find the work done by this passenger X and the luggage as seen by*

(i) a follow passenger Y

(ii) a person on the ground [Take, $g = 10\ ms^{-2}$].

Sol. Given, speed of the train, $v = 90\ \text{kmh}^{-1} = \dfrac{90\times1000}{60\times60} = 25\ \text{ms}^{-1}$

(i) Displacement of the luggage with respect to the train, $S = 1$ m.
As luggage is displaced slowly, the force applied on it must be same as frictional force on it by the floor

$$f = \mu\, mg = (0.2)\times 40\times 10 = 80\ \text{N}$$

Work done by the passenger X as seen by fellow passenger Y is

$$W = Fs = (80)(1) = 80\ \text{J}$$

(ii) The luggage is displaced for 10 s. Therefore distance moved by train with respect to ground during this interval is

$$s_0 = 25\times 10 = 250\ \text{m}$$

Therefore work done by passenger X on the luggage as seen by a person on the ground is

$$W_G = F(s + s_0) = (80)(1 + 250) = (251)(80) = 20.08\ \text{kJ}$$

2 Conservative forces

A force is said to be conservative if the work done by or against the force on a body is independent of path followed by the body and depends only on initial and final positions.

Work done by conservative forces

One of the following two equivalent conditions on workdone must be satisfied by conservative forces.

(i) Work done by or against a conservative force, in moving a body from one position to another, depends only on the initial and final positions of the body. It does not depend upon the nature of the path followed by the body in going from initial position to the final position.

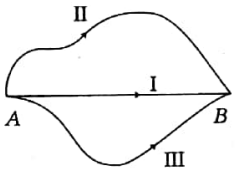


Fig. 6.9 *Work done by conservative forces*

Let W_1, W_2 and W_3 denote the net work done in moving a body from A to B along three different paths I, II and III respectively as shown in figure. If the force is conservative, then

$$W_{\text{I}} = W_{\text{II}} = W_{\text{III}} = \int \mathbf{F}\cdot d\mathbf{L}$$

$$\Rightarrow \quad \underset{\text{Path I}}{\int_A^B \mathbf{F}\cdot d\mathbf{L}} = \underset{\text{Path II}}{\int_A^B \mathbf{F}\cdot d\mathbf{L}} = \underset{\text{Path III}}{\int_A^B \mathbf{F}\cdot d\mathbf{L}}$$

(ii) Work done by or against a conservative force in moving a particle along a closed path (round trip) is zero.

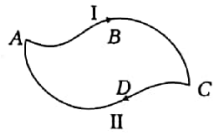


Fig. 6.10

Assume that a particle is moving along a closed path $ABCDA$ as shown in above figure.

If the force acting on the particle is conservative, then

$$W_{ABCD} = \int_{ABCD} \mathbf{F}\cdot d\mathbf{L} = \int_{ABC} \mathbf{F}\cdot d\mathbf{L} + \int_{CDA} \mathbf{F}\cdot d\mathbf{L} = 0$$

$$\Rightarrow \quad W_{ABC} + W_{CDA} = 0$$

$$\Rightarrow \quad W_{ABC} = -W_{CDA} \Rightarrow W_{\text{I}} = -W_{\text{II}}$$

Example 6.19 *An object is displaced from point A(2m, 3m, 4m) to a point B (1m, 2m, 3m) under a constant force* $\mathbf{F} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$ *N. Find the work done by this force in this process.*

Sol. Work done

$$W = \int_{\mathbf{r}_i}^{\mathbf{r}_f} \mathbf{F}\cdot d\mathbf{s} = \int_{(2m,3m,4m)}^{(1m,2m,3m)} (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})\cdot(dx\hat{\mathbf{i}} + dy\hat{\mathbf{j}} + dz\hat{\mathbf{k}})$$

$$= [2x + 3y + 4z]_{(2m,3m,4m)}^{(1m,2m,3m)} = -9\ \text{J}$$

Alternate Solution

Since, $\mathbf{F}$ = constant, we can also use.

$$W = \mathbf{F}\cdot\mathbf{s}$$

Here, $\mathbf{s} = \mathbf{s}_f - \mathbf{s}_i = (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) - (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$

$$= (-\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}})$$

$\therefore$ $W = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})\cdot(-\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}})$

$$= -2 - 3 - 4 = -9\ \text{J}$$

Example 6.20 *An object is displaced from position vector* $\mathbf{r}_1 = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$ *m to* $\mathbf{r}_2 = (4\hat{\mathbf{i}} + 6\hat{\mathbf{j}})$*m under a force* $\mathbf{F} = (3x^2\hat{\mathbf{i}} + 2y\hat{\mathbf{j}})$*N. Find the work done by this force.*

Sol. Work done, $W = \int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{F}\cdot d\mathbf{r}$

$$= \int_{\mathbf{r}_1}^{\mathbf{r}_2} (3x^2\hat{\mathbf{i}} + 2y\hat{\mathbf{j}})\cdot(dx\hat{\mathbf{i}} + dy\hat{\mathbf{j}} + dz\hat{\mathbf{k}})$$

$$= \int_{\mathbf{r}_1}^{\mathbf{r}_2} (3x^2\,dx + 2y\,dy) = [x^3 + y^2]_{(2,3)}^{(4,6)} = 83\ \text{J}$$

In the above two examples, we saw that while calculating the work done we did not mention the path through which the object was displaced.

Only initial and final coordinates were required.

It shows that in both the examples, the work done is path independent or work done will be equal to whichever path we follow.

Therefore above two forces in which work done is path independent are example of **conservative forces.**

3 Non-conservative forces

A force is said to be non-conservative if work done by or against the force in moving a body depends upon the path between initial and final positions.

e.g., Friction force, viscous force, air resistance etc.

Work done by non-conservative forces

Let W_1, W_2 and W_3 denote the net work done in moving a body from A to B along three different path 1, 2 and 3 respectively as shown in figure. If the force is non-conservative, then

$$W_1 \neq W_2 \neq W_3$$

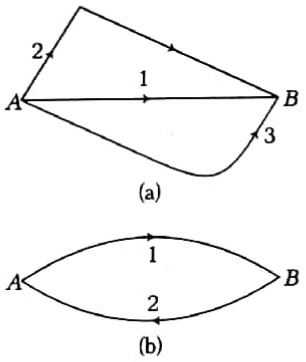


Fig. 6.11

Work done by a non-conservative force in a round trip is not zero.

$$W_1 + W_2 \neq 0$$

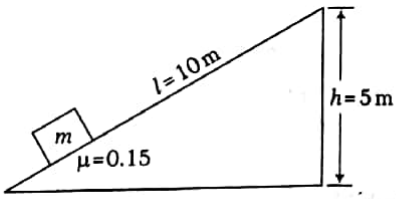
Example 16.21 *A body of mass 0.3 kg is taken up an inclined plane of length 10 m and height 5 m, and then allowed to slide down the bottom again. The coefficient of friction between the body and the plane is 0.15. What is the*

(i) work done by the applied force over the upward journey?

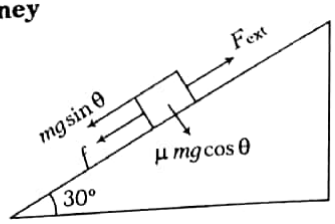
(ii) work done by gravitational force over the round trip?

(iii) work done by the frictional force over the round trip?

Which of the above forces (except applied force) is/are conservative forces?



Sol. **Upward journey**



Let us calculate work done by different forces over upward journey.

Work done by gravitational force,

$$W_1 = (mg \sin\theta)s \cos 180°$$
$$W_1 = 0.3 \times 10 \sin 30° \times 10(-1)$$
$$W_1 = -15 \text{ J}$$

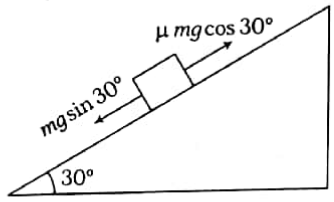
Work done by force of friction,

$$W_2 = (\mu\, mg \cos\theta)s \cos 180°$$
$$W_2 = 0.15 \times 0.3 \times 10 \cos 30° \times 10[-1]$$
$$W_2 = -3.879 \text{ J}$$

Work done by external force,

$$W_3 = F_{ext} \times s \times \cos 0°$$
$$W_3 = [mg \sin\theta + \mu\, mg \cos\theta] \times 10 \times 1$$
$$W_3 = 18.897 \text{ J}$$

Downward journey



$$mg \sin 30° > \mu\, mg \cos 30°$$

Work done by the gravitational force,

$$W_4 = mg \sin 30° \times s \cos 0°$$
$$W_4 = 0.3 \times 10 \times \frac{1}{2} \times 10 = +15 \text{ J}$$

Work done by the frictional force,

$$W_5 = \mu mg \cos 30° \times s \cos 180°$$
$$= 0.15 \times 0.3 \times \frac{10\sqrt{3}}{2} \times 10 \times (-1) = -3.897 \text{ J}$$

(*i*) Work done by applied force over upward journey,

$$W_3 = 18.897 \text{ J}$$

(*ii*) Work done by gravitational force over the round trip

$$= W_1 + W_4 = 0 \text{ J}$$

(*iii*) Work done by frictional force over the round trip,

$$W_2 + W_5 = -3.897 + (-3.897) = -7.794 \text{ J}$$

Work done by gravitational force over a closed path is zero but due to frictional force, it is non-zero. Therefore, gravitational force is conservative and frictional force is non-conservative.

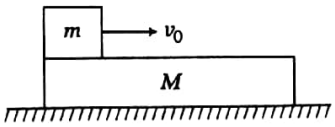
Check point 6.1

1. A gardener pushes a lawn roller through a distance 20 m. If he applies a force of 20 kg-wt in a direction inclined at 60° to the ground, the work done by him is
(a) 1960 J (b) 196 J (c) 1.96 J (d) 196 kJ

2. How much work must be done by a force on 50 kg body in order to accelerate it in the direction of force from rest to 20 ms^{-1} is 10 s?
(a) 10^{-3} J (b) 10^4 J (c) 2×10^3 J (d) 4×10^4 J

3. A horizontal force F pulls a 20 kg box at a constant speed along a rough horizontal floor. The coefficient of friction between the box and the floor is 0.25. The work done by force F on the block in displacing it by 2 m is
(a) 49 J (b) 98 J
(c) 147 J (d) 196 J

4. The coefficient of friction between the block and plank is μ and its value is such that block becomes stationary with respect to plank before it reaches the other end. Then which of the following is not correct.



(a) The work done by friction on the block is negative
(b) The work done by friction on the plank is positive
(c) The net work done by friction is negative
(d) Net work done by the friction is zero

5. A man pushes against a wall but fails to move it. He does
(a) negative work
(b) positive but not maximum work
(c) maximum positive work
(d) no work at all

6. The net work done by kinetic friction
(a) is always negative (b) is always zero
(c) may be negative or positive (d) is always positive

7. A force $(3\hat{i} + 4\hat{j})$ N acts on a body and displaces it by $(3\hat{i} + 4\hat{j})$ m. The work done by the force is
(a) 10 J (b) 12 J
(c) 16 J (d) 25 J

8. A force $\mathbf{F} = (2\hat{i} - \hat{j} + 4\hat{k})$ N displaces a particle upto $\mathbf{d} = (3\hat{i} + 2\hat{j} - \hat{k})$ m. Work done by the force is
(a) Zero (b) 8 J
(c) 4 J (d) 12 J

9. A particle moves from point P (1, 2, 3) to Q (2, 1, 4) under the action of a constant force $\mathbf{F} = (2\hat{i} + \hat{j} + \hat{k})$ N. Work done by the force is
(a) 2 J (b) 4 J
(c) 16 J (d) 8 J

10. Work done by a force $\mathbf{F} = (\hat{i} + 2\hat{j} + 3\hat{k})$ N acting on a particle in displacing it from the point $\mathbf{r}_1 = \hat{i} + \hat{j} + \hat{k}$ to the point $\mathbf{r}_2 = \hat{i} - \hat{j} + 2\hat{k}$ is
(a) – 3 J (b) – 1 J
(c) zero (d) 2 J

11. A body, constrained to move in the y-direction, is subjected to a force $\mathbf{F} = (-2\hat{i} + 15\hat{j} + 6\hat{k})$ N. What is the work done by this force in moving the body through a distance of 10 m along the Y-axis?
(a) 20 J (b) 150 J (c) 160 J (d) 190 J

12. Force acting on a particle is $(2\hat{i} + 3\hat{j})$ N. Work done by this force is zero, when a particle is moved on the line $3y + kx = 5$. Here value of k is
(a) 2 (b) 4 (c) 6 (d) 8

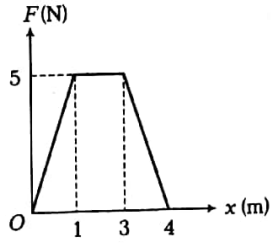
13. A particle moves under a force $F = Cx$ from $x = 0$ to $x = x_1$. The work done is
(a) Cx_1^2 (b) $\frac{Cx_1^2}{2}$ (c) zero (d) Cx_1^3

14. A particle moves along the X-axis from $x = 0$ to $x = 5$ m under the influence of a force given by $F = 10 - 2x + 3x^2$. Work done in the process is
(a) 70 units (b) 270 units
(c) 35 units (d) 150 units

15. A position dependent force, $F = 8 - 4x + 3x^2$ N acts on a small body of mass 2 kg and displaces it from $x = 0$ to $x = 5$ m. The work done in joule is
(a) 35 (b) 70 (c) 115 (d) 270

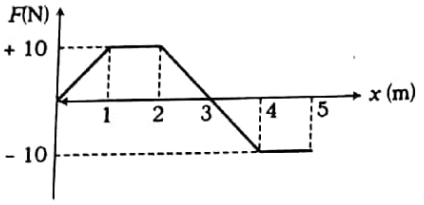
16. A force $F = Ay^2 + By + C$ acts on a body in the y-direction. The work done by this force during a displacement from $y = -a$ to $y = a$ is
(a) $\frac{2Aa^3}{3}$ (b) $\frac{2Aa^3}{3} + 2Ca$
(c) $\frac{2Aa^3}{3} + \frac{Ba^2}{2} + Ca$ (d) None of these

17. The force F acting on a particle is moving in a straight line as shown in figure. What is the work done by the force on the particle in the 4 m of the trajectory?



(a) 5 J (b) 10 J (c) 15 J (d) 2.5 J

18. A position dependent force F is acting on a particle and its force-position curve is shown in the figure. Work done on the particle, when its displacement is from 0 to 5 m is



(a) 35 J (b) 25 J (c) 15 J (d) 5 J

19. A spring of force constant 800 Nm^{-1} has an extension of 5 cm. The work done in extending it from 5 cm to 15 cm is
(a) 16 J (b) 8 J
(c) 32 J (d) 24 J

20. A spring 40 mm long is stretched by the application of a force. If 10 N force is required to stretch the spring through 1 mm, then the work done in stretching the spring through 40 mm is
(a) 84 J (b) 68 J
(c) 23 J (d) 8 J

4 Energy

The energy of a body is defined as its capacity or ability for doing work. Like work, energy is a scalar quantity having magnitude only and no direction. The dimensions of energy are the same as the dimensions of work *i.e.*, $[ML^2 T^{-2}]$. It is measured in the same unit as work *i.e.*, joule in SI and erg in CGS system.

Energy can exist in various forms such as mechanical energy (potential energy and kinetic energy), sound energy, heat energy, light energy etc.

Kinetic energy

The energy possessed by a body by virtue of its motion is called kinetic energy. Kinetic energy of a body can be calculated by the amount of work done in stopping the moving body or from the amount of work done in giving it same velocity from state of rest.

If an object of mass m has velocity **v**, *then its kinetic energy is given by,*

$$\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}m\mathbf{v}\cdot\mathbf{v}$$

The kinetic energy of a system of particles is the sum of the kinetic energies of all its constituent particles.

i.e., $$K = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{1}{2}m_3v_3^2 + \ldots$$

$$\boxed{K = \Sigma\frac{1}{2}m_iv_i^2}$$

Regarding the kinetic energy the following two points are important to note.

(*i*) Since both m and v^2 are always positive. Hence, KE is always positive and does not depend on the direction of motion of the body.

(*ii*) Kinetic energy depends on the frame of reference. For example, the kinetic energy of a person of mass m sitting in a train moving with speed v is zero in the frame of train but $1/2\, mv^2$ in the frame of earth.

Example 6.22 *When a man increases his speed by 2 ms^{-1}, he finds that his kinetic energy is doubled. Find the original speed of the man.*

Sol. Man possesses kinetic energy, because of its velocity (v). If m is mass of man, then

$$K = \frac{1}{2}mv^2$$

Given, $v_1 = v,\ m_1 = m_2 = m,$

When $v_2 = (v+2)\,\text{ms}^{-1}$

Then $K_2 = 2K_1$

$$\therefore \quad \frac{K_1}{K_2} = \frac{v_1^2}{v_2^2} \Rightarrow \frac{K_1}{2K_1} = \frac{v^2}{(v+2)^2} \Rightarrow v^2 - 4v - 4 = 0$$

This gives, $$v_1 = \frac{4+\sqrt{16+16}}{2} = \frac{4+\sqrt{32}}{2}$$

$$v_1 = 2(\sqrt{2}+1)\,\text{ms}^{-1}$$

Example 16.23 *In a ballistics demonstration, a police officer fires a bullet of mass 50 g with speed 200 ms^{-1} on soft plywood of thickness 2 cm. The bullet emerges with only 10% of its initial kinetic energy. What is the emergent speed of the bullet?*

Sol. Initial kinetic energy, $K_i = \frac{1}{2}\times\frac{50}{1000}\times 200\times 200\ \text{J} = 1000\ \text{J}$

Final kinetic energy, $K_f = \frac{10}{100}\times 1000\ \text{J} = 100\ \text{J}$

If v_f is emergent speed of the bullet, then

$$\frac{1}{2}\times\frac{50}{1000}\times v_f^2 = 100$$

$$\Rightarrow \quad v_f^2 = 4000 \Rightarrow v_f = 63.2\,\text{ms}^{-1}$$

Note that the speed is reduced by approximately 68% and not 90%.

Example 6.24 *A body of mass 0.8 kg has initial velocity $(3\hat{i} - 4\hat{j})\ ms^{-1}$ and final velocity $(-6\hat{j} + 2\hat{k})\ ms^{-1}$. Find change in kinetic energy of the body?*

Sol. Change in kinetic energy

$$\Delta KE = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

where, $v_f = \sqrt{6^2 + 2^2} = \sqrt{40}$

and $v_i = \sqrt{3^2 + 4^2} = \sqrt{25}$

$$= \frac{1}{2} \times 0.8\,[\sqrt{40})^2 - (\sqrt{25})^2]$$

$$= 0.4[40 - 25] = 0.4(15) = 6\ J$$

Relation between kinetic energy and linear momentum

The linear momentum of a body is given by $p = mv$ where, m is mass and v is the velocity of a body and kinetic energy of the body $= \frac{1}{2}mv^2 = \frac{1}{2m}(m^2v^2)$

$$K = \frac{p^2}{2m} \quad \text{or} \quad p^2 = 2m\ KE$$

$$\Rightarrow \quad \boxed{\text{Linear momentum, } p = \sqrt{2m\,KE}}$$

Example 6.25 *Two bodies A and B having masses in the ratio of 3 : 1 possess the same kinetic energy. Obtain the ratio of linear momentum of B to A.*

Sol. Kinetic energy of body is given by

$$E_K = \frac{1}{2}mv^2 \quad \text{...(i)}$$

and linear momentum $p = mv$ ...(ii)

From Eqs. (i) and (ii), we get

$$E_K = \frac{m^2v^2}{2m} = \frac{p^2}{2m}$$

When $E_{K_1} = E_{K_2}$

$$\Rightarrow \quad \frac{p_1^2}{2m_1} = \frac{p_2^2}{2m_2} \text{ or } \frac{p_1}{p_2} = \sqrt{\frac{m_1}{m_2}}$$

or $$\frac{p_1}{p_2} = \sqrt{\frac{3}{1}}$$

or $$\frac{p_2}{p_1} = \frac{1}{\sqrt{3}}$$

Example 6.26 *Kinetic energy of a particle is increased by 300%. Find the percentage increase in momentum.*

Sol. Kinetic energy $E = \frac{1}{2}mv^2$, Momentum, $p = mv$

When E is increased by 300%

$$E' = E + 3E = 4E = 4\left(\frac{1}{2}mv^2\right) = 2mv^2$$

If v' is velocity of body, then $\frac{1}{2}m(v')^2 = 2mv^2$

$$\Rightarrow \quad v' = 2v$$

So, $p' = mv' = 2mv$

hence, percentage change in momentum

$$= \frac{2mv - mv}{mv} \times 100 = 100\%$$

Work-energy theorem

This theorem states that work done by all the forces (conservative or non-conservative, external or internal) acting on a particle or an object is equal to the change in kinetic energy of it.

$$\therefore \quad W_{net} = \Delta KE = K_f - K_i$$

$$\boxed{W_{net} = \frac{1}{2}m(v_f^2 - v_i^2)}$$

$$\Rightarrow \quad W_{conservative} + W_{non\text{-}conservative} + W_{ext.force} = \Delta KE$$

If W_{net} is positive then kinetic energy will increase and *vice-versa.*

Let, $\mathbf{F}_1, \mathbf{F}_2, \ldots$ be the individual forces acting on a particle. The resultant force is $\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \ldots$ and the work done by the resultant force is

$$W = \int \mathbf{F} \cdot d\mathbf{r} = \int (\mathbf{F}_1 + \mathbf{F}_2 + \ldots) \cdot d\mathbf{r}$$

$$= \int \mathbf{F}_1 \cdot d\mathbf{r} + \int \mathbf{F}_2 \cdot d\mathbf{r} + \ldots$$

where $\int \mathbf{F}_1 \cdot d\mathbf{r}$ is the work done on the particle by $\mathbf{F}$ and so on. Thus, work energy theorem can also be written as **work done by the resultant force is equal to the sum of the work done by the individual forces.**

Note This theorem can be applied to non-inertial frames also. In a non-inertial frame, it can be written as :
Work done by all the forces (including the pseudo forces)
= Change in kinetic energy in non-inertial frame.

Example 6.27 *The position (x) of a particle of mass 1 kg moving along x-axis at time t is given by $\left(x = \frac{1}{2}t^2\right)$ metre. Find the work done by force acting on it in time interval from $t = 0$ to $t = 3$ s.*

Sol. $x = \frac{1}{2}t^2 \Rightarrow v = \frac{dx}{dt} = \frac{1}{2}(2t) = t$

$\therefore$ At $t = 0$, $v_i = 0 \Rightarrow$ At $t = 3$ s, $v_f = 3\ ms^{-1}$

According to work-energy theorem,

$$W = \Delta K = K_f - K_i$$

$$= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$= \frac{1}{2} \times 1 \times 3^2 = 4.5\ J$$

Example 6.28 *A bullet weighing 10 g is fired with a velocity 800 ms^{-1}. After passing through a mud wall 1 m thick, its velocity decreases to 100 ms^{-1}. Find the average resistance offered by the mud wall.*

Sol. According to work-energy theorem, work done by the average resistance offered by the wall = change in KE of the bullet.

$$\therefore \quad W = F \cdot s = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$\Rightarrow \quad F = \frac{m(v^2 - u^2)}{2s} = \frac{0.01(100^2 - 800^2)}{2 \times 1}$$

$$\Rightarrow \quad = -3150\,\text{N}$$

$\Rightarrow$ Resistance offered $= 3150\,\text{N}$

Example 6.29 *An object of mass 5 kg falls from rest through a vertical distance of 20 m and attains a velocity of 10 ms^{-1}. How much work is done by the resistance of the air on the object? (Take, $g = 10\,ms^{-2}$)*

Sol. Applying work-energy theorem,

work done by all the forces = change in kinetic energy

$$\text{or} \quad W_{mg} + W_{air} = \frac{1}{2}mv^2$$

$$\therefore \quad W_{air} = \frac{1}{2}mv^2 - W_{mg} = \frac{1}{2}mv^2 - mgh$$

$$= \frac{1}{2} \times 5 \times (10)^2 - (5) \times (10) \times (20) = -750\,\text{J}$$

Example 6.30 *A particle of mass m moves with velocity $v = a\sqrt{x}$ where a is a constant. Find the total work done by all the forces during a displacement from $x = 0$ to $x = d$.*

Sol. Work done by all forces, $W = \Delta KE = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$

Here, $v_1 = a\sqrt{0} = 0$, $v_2 = a\sqrt{d}$,

So, $W = \frac{1}{2}ma^2d - 0 = \frac{1}{2}ma^2d$

Example 6.31 *A vehicle of mass 10 kg is moving in x-direction with a constant speed of 10 ms^{-1}. It is subjected to a retarding force $F = -0.1x\,Jm^{-1}$ during its travel from $x = 20$ m to $x = 30$ m. Evaluate its final kinetic energy.*

Sol. According to work-energy theorem,

work done = change in kinetic energy of the vehicle

$$\therefore \quad W = K_f - K_i \text{ or } F \cdot dx = K_f - \frac{1}{2}mv_i^2$$

$$\text{or} \quad F \cdot dx = K_f - \frac{1}{2} \times 10 \times (10)^2 \text{ or } F \cdot dx = K_f - 500$$

$$\text{or} \quad \int_{x=20}^{x=30} (-0.1)x\,dx = K_f - 500$$

$$\text{or} \quad -0.1\left[\frac{x^2}{2}\right]_{x=20}^{x=30} = K_f - 500$$

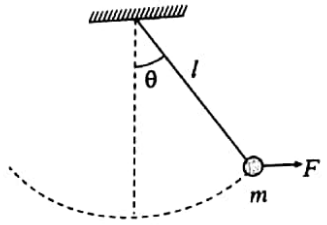
$$\text{or} \quad -0.1\left[\frac{(30)^2}{2} - \frac{(20)^2}{2}\right] = K_f - 500$$

$$\text{or} \quad K_f - 500 = -0.1\,(450 - 200)$$

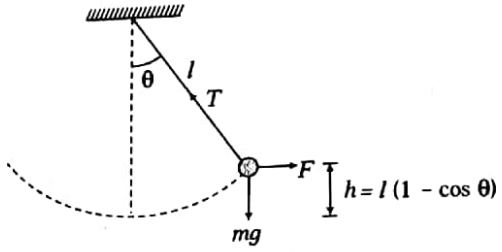
$$\text{or} \quad K_f - 500 = -25$$

$$\therefore \quad K_f = 500 - 25 = 475\,\text{J}$$

Example 6.32 *An object of mass m is tied to a string of length l and a variable force F is applied on it which brings the string gradually at angle θ with the vertical. Find the work done by the force F.*



Sol. In this case, three forces are acting on the object



1. tension (T)
2. weight (mg) and
3. applied force (F)

Using work-energy theorem

$$W_{net} = \Delta KE$$

$$\text{or} \quad W_T + W_{mg} + W_F = 0 \quad \text{...(i)}$$

$$\text{as} \quad \Delta KE = 0$$

$$\text{because} \quad K_i = K_f = 0$$

Further, $W_T = 0$, as tension is always perpendicular to displacement.

$$W_{mg} = -mgh \quad \text{or} \quad W_{mg} = -mgl\,(1 - \cos\theta)$$

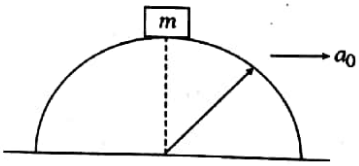
Substituting these values in Eq. (i), we get

$$W_F = mgl\,(1 - \cos\theta)$$

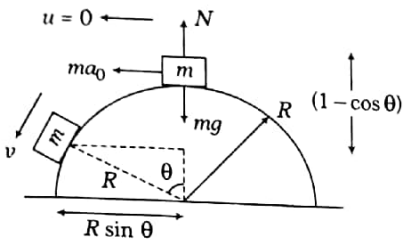
Note If applied force F is variable. We do not apply the work energy theorem, we will first find the magnitude of F at different locations and then integrate dW ($= \mathbf{F} \cdot d\mathbf{r}$) with proper limits.

Example 6.33 *A block is placed at the top of a smooth hemisphere of radius R. Now the hemisphere is given a horizontal acceleration a_0. Find the velocity of the block relative to the hemisphere as a function of θ as it slides down.*

Sol. In the reference frame of hemisphere. Apply a pseudo force on m.



From the work-energy theorem,



$$W_{total} = \Delta K \Rightarrow W_{gr} + W_{pseudo} = \Delta K$$

$$mgR(1-\cos\theta) + ma_0 R\sin\theta = \frac{1}{2}mv^2 - 0$$

$$v = \sqrt{2R\{g(1-\cos\theta) + a_0\sin\theta)\}}$$

The work done by normal reaction N is zero because it is always perpendicular to displacement.

Potential energy

The energy possessed by a body or system by virtue of its position or configuration is known as the **potential energy**. For example, a block attached to a compressed or elongated spring possesses some energy called **elastic potential energy**. This block has a capacity to do work. Similarly, a stone when released from a certain height also has energy in the form of **gravitational potential energy**. Two charged particles kept at certain distance has electric potential energy.

The units and dimensions of potential energy are same as that of kinetic energy.

There are mainly two types of potential energies as discussed below.

1. Gravitational potential energy

Gravitational potential energy of a body is the energy associated to it due to the virtue of its position above the surface of earth. If an object of mass m is placed at a height h above earth surface, then

$$\boxed{\text{Gravitational Potential Energy } U = mgh}$$

Example 6.34 *A stone of mass 0.4 kg is thrown vertically up with a speed of 9.8 ms^{-1}. Find the potential energy after half second.*

Sol. Here, mass, $m = 0.4$ kg, speed, $u = 9.8\,ms^{-1}$,

acceleration, $a = -g = -9.8\ m/s^2$ and time, $t = \frac{1}{2}$ s

From equation of motion

$$s = ut + \frac{1}{2}at^2 = \left(9.8 \times \frac{1}{2}\right) + \left(-\frac{1}{2} \times 9.8 \times \frac{1}{2} \times \frac{1}{2}\right) = 3.67\text{m}$$

$\therefore$ Potential energy $= mgh = (0.4)(9.8)(3.67) = 14.386$ J

Example 6.35 *Calculate the work done in lifting a 300 N weight to a height of 10 m with an acceleration 0.5 ms^{-2}. Take $g = 10\,ms^{-2}$.*

Sol. Given, Weight, $w = 300$ N, Height, $h = 10$ m

Acceleration, $a = 0.5\ ms^{-2}$

Here, $m = \frac{w}{g} = \frac{300}{10} = 30$ kg, $F_{net} = mg + ma$

$\therefore$ Work done, $W = mgh + mah = m(a+g)h$

$= 30(0.5 + 10)10 = 3150$ J

2. Elastic potential energy of a spring

Potential energy of a spring is the energy associated with the state of compression or expansion of an elastic spring.

When the spring is stretched or compressed by an amount x from its unstretched position.

Then, elastic potential energy of spring

$$\boxed{U = 1/2\ kx^2} \qquad (k = \text{spring constant})$$

Note that elastic potential energy is always positive.

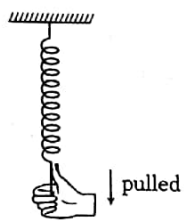
Natural length of spring indicates reference point where PE of spring is taken zero.

Example 6.36 *Two springs of spring constant 1500 Nm^{-1} and 3000 Nm^{-1}, respectively are stretched with the same force, slowly. Compute the ratio of their potential energies.*

Sol. The work done in pulling the string is stored as potential energy in the spring

$$U = \frac{1}{2}kx^2 \qquad \text{...(i)}$$

where, k is spring constant and x is distance through which it is pulled.



Also in SHM, force $\propto$ displacement

$\Rightarrow \qquad F = kx$

where, k is spring constant.

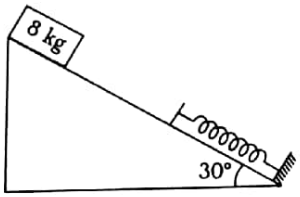
Putting, $x = \frac{F}{k}$ in Eq. (i) we get

$$U = \frac{1}{2}k\left(\frac{F}{k}\right)^2 = \frac{F^2}{2k} \qquad \text{...(ii)}$$

$$\therefore \qquad \frac{U_1}{U_2} = \frac{k_2}{k_1} = \frac{3000}{1500} = \frac{2}{1}$$

or $\qquad U_1 : U_2 = 2:1$

Example 6.37 *A block of mass 8 kg is released from the top of an inclined smooth surface as shown in figure. If spring constant of spring is 200 Nm^{-1} and block comes to rest after compressing spring by 1 m then find the distance travelled by block before it comes to rest.*



Sol. Consider d is the distance travelled by block along the plane and height it come in downward direction $h = d\sin 30°$.

Now, when block will comes to rest, the decrease in its potential energy will store in spring

$$\therefore \quad mgh = \frac{1}{2}ky^2 \quad (y \text{ is compression in spring})$$

$$\Rightarrow \quad mgd\sin 30° = \frac{1}{2}ky^2$$

$$\Rightarrow \quad 8 \times 10 \times \frac{d}{2} = \frac{1}{2} \times 200 \times 1^2$$

$$\Rightarrow \quad 40d = 100$$

$$\Rightarrow \quad d = 2.5 \text{ m}$$

Change in potential energy

Regarding the potential energy, it is important to note that it is defined for a conservative force field only.

For non-conservative forces, it has no meaning. The change in potential energy (dU) of a system corresponding to a conservative internal force is given by

$$dU = -\mathbf{F}\cdot d\mathbf{r} = -dW \qquad \left(\because F = -\frac{dU}{dr}\right)$$

or $$\int_i^f dU = -\int_{\mathbf{r}_i}^{\mathbf{r}_f} \mathbf{F}\cdot d\mathbf{r} \text{ or } U_f - U_i = -\int_{\mathbf{r}_i}^{\mathbf{r}_f} \mathbf{F}\cdot d\mathbf{r}$$

We generally choose the reference point at infinity and assume potential energy to be zero there, *i.e.*, if we take $r_i = \infty$ (infinite) and $U_i = 0$, then we can write

$$\boxed{U = -\int_{\infty}^{\mathbf{r}} \mathbf{F}\cdot d\mathbf{r} = -W}$$

or **potential energy of a body or system is the negative of work done by the conservative forces in bringing it from infinity to the present position.**

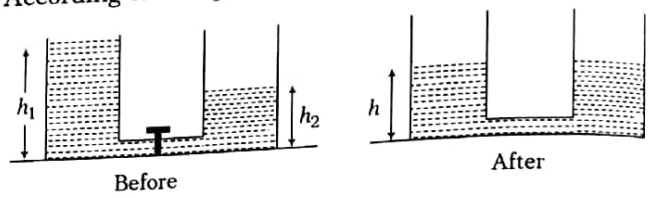
Regarding the potential energy it is worth noting that

(*i*) Potential energy should be considered to be a property of the entire system rather than assigning it to any specific particle.

(*ii*) Potential energy depends on frame of reference.

Example 6.38 *Two cylindrical vessels of equal cross-sectional area A contain water up to heights h_1 and h_2. The vessels are interconnected so that the levels in them are equal. Calculate the work done by the force of gravity during the process. The density of water is ρ.*

Sol. According to the question, we draw the following diagram.



Since the mass of water remains same

$$\rho Ah_1 + \rho Ah_2 = \rho Ah + \rho Ah \Rightarrow h = \frac{h_1 + h_2}{2}$$

Potential energy of first vessel

$$U_1 = (\rho Ah_1)g\frac{h_1}{2} + (\rho Ah_2)g\frac{h_2}{2} = \frac{\rho Ag}{2}(h_1^2 + h_2^2)$$

Potential energy of second vessel

$$U_2 = (\rho Ah)g\frac{h}{2} \times 2 = \frac{\rho Ag(h_1 + h_2)^2}{4}$$

Since the gravitational force is a conservative force, then

$$W_{gr} = U_1 - U_2 = \frac{\rho Ag}{4}(h_1 - h_2)^2$$

For three dimensional motion of the particle

$$\mathbf{F} = -\left(\frac{\partial U}{\partial x}\hat{\mathbf{i}} + \frac{\partial U}{\partial y}\hat{\mathbf{j}} + \frac{\partial U}{\partial z}\hat{\mathbf{k}}\right)$$

where, $\frac{\partial U}{\partial x}$ = partial derivative of U w.r.t. x

$\frac{\partial U}{\partial y}$ = partial derivative of U w.r.t. y

$\frac{\partial U}{\partial z}$ = partial derivative of U w.r.t. z

Example 6.39 *A particle is moving on frictionless XY-plane. It is acted on by a conservative force described by the potential-energy function.*

$$U(x, y, z) = \frac{1}{2}k(x^2 + y^2 + z^2)$$

where, $k = constant$

Derive an expression for the force acting an the particle.

Sol. Force acting an the particle in x-direction is given by

$$F_x = -\frac{\partial U}{\partial x} = -\frac{1}{2}k(2x + 0 + 0) = -kx$$

Similarly, $F_y = -ky$ and $F_z = -kz$

$\therefore$ Force acting an the particle is

$$\mathbf{F} = Fx\hat{\mathbf{i}} + Fy\hat{\mathbf{j}} + Fz\hat{\mathbf{k}} = -k(x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}) = -k\mathbf{r},$$

where $\mathbf{r}$ = position vector of the particle

$$= x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$$

Work done in pulling a chain against gravity

Consider a chain of length L and mass M is held on a frictionless table with L/x of its length hanging over the edge. We have to find the work done in pulling the hanging portion of the chain on the table.

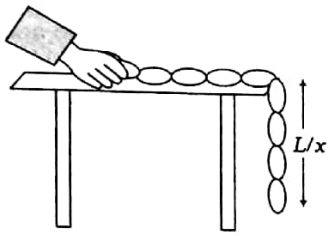


Fig. 6.9

For this, we can assume centre of mass of hanging portion of the chain at the middle.

Mass of hanging part of chain $= \frac{M}{L} \times \frac{L}{x} = \frac{M}{x}$

Now assume hanging portion of the chain as a point mass centred at its centre of mass.

Therefore work done to raise the centre of mass of the chain on the table

$$W = \frac{M}{x} \times g \times \frac{L}{2x}$$

or

$$W = \frac{MgL}{2x^2}$$

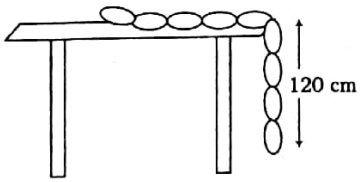
Example 6.40 *A uniform chain of length 4m is kept on a table such that a length of 120 cm hangs freely from the edge of the table. The total mass of the chain is 4 kg. What is the work done in pulling the entire chain on the table?*

Sol. Fraction of length of the chain hanging from the table

That $\frac{L}{x} = 120$ cm

$\Rightarrow$ $x = \frac{10}{3}$

Work done in pulling the chain on the table



or $v = -m'g\,(60\text{ cm})$

$$= -\left(\frac{4}{400\text{ cm}} \times 120\text{ cm}\right) g\,(60\text{ cm})$$

$W = -v = 7.2$ J

$$W = \frac{mgL}{2x^2} = \frac{4 \times 10 \times 4}{2 \times (10/3)^2} = 7.2\text{ J}$$

5 Equilibrium

If a large number of forces act on a system or on an object simultaneously in such a way that the resultant force on it is zero, then it is said to be in translation equilibrium. If the forces acting on the object are conservative and it is in equilibrium, then

$$F_{net} = 0 \Rightarrow -\frac{dU}{dr} = 0 \text{ or } \frac{dU}{dr} = 0$$

So, when force is conservative and object is in equilibrium, slope of U-r graph is zero or its potential energy is either minimum or maximum or constant. *On this basis equilibrium of object or a system can be divided into three types*

(*i*) **Stable Equilibrium** An object is said to be in stable equilibrium, if on slight displacement from equilibrium position, it has tendency to come back. Here, potential energy in equilibrium position is minimum as compared to its neighbouring points or $\frac{d^2U}{dr^2}$ = positive.

(*ii*) **Unstable Equilibrium** An object is said to be in unstable equilibrium, if on slight displacement from equilibrium position, it moves in the direction of displacement.

In unstable equilibrium potential energy is maximum or $\frac{d^2U}{dr^2}$ = negative.

(*iii*) **Neutral Equilibrium** An object is said to be in neutral equilibrium, if on displacement from its equilibrium position, it has neither the tendency to move in direction of displacement nor to come back to equilibrium position. In neutral equilibrium, potential energy of the object is constant or $\frac{d^2U}{dr^2} = 0$.

Motion using energy conservation

$E_{Mech} \rightarrow$ constant $\rightarrow$ represented by horizontal dotted line.

At x, $k = U_{Mech} - U(x)$

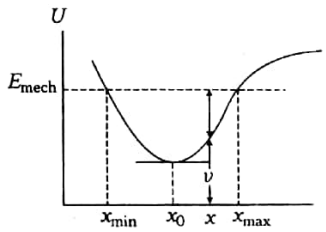


Fig. 6.12

The points $x = x_{max}$ and $x = x_{min}$ are called turning points, at these points velocity of the particle decreases to zero and reverses.

From x_{min} to x_0, slope of $U(x)$ is negative, $F = -\frac{dv}{dx}$ is positive, and acts towards x_0.

At x_0, $F = 0$, stable equilibrium point.

beyond x_0, slope is positive, indicating a negative force, towards x_0.

From the PE and position graph

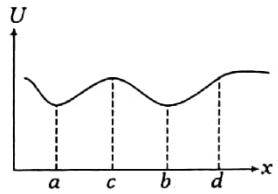


Fig. 6.13

At $x = a, b$, U is minimum, stable equilibrium

$x = c$, U is maximum, unstable equilibrium

$x = d$, U, is constant, neutral equilibrium

Example 6.41 *The potential energy of a conservative system is given by $U = ax^2 - bx$ (where a and b are positive constants). Find the equilibrium position and discuss whether the equilibrium is stable, unstable or neutral.*

Sol. In a conservative field,

$$F = -\frac{dU}{dx}$$

$$\therefore \quad F = -\frac{d}{dx}(ax^2 - bx) = b - 2ax$$

For equilibrium $\quad F = 0 \quad$ or $\quad b - 2ax = 0$

$$\therefore \quad x = \frac{b}{2a}$$

From the given equation, we can see that $\frac{d^2U}{dx^2} = 2a$ (positive), *i.e.*, U is minimum.

Therefore, $x = \frac{b}{2a}$ is the stable equilibrium position.

Example 6.42 *The potential energy for a conservative force system is given by $U = ax^2 - bx$. Where a and b are constants find out (a) The expression of force (b) Potential energy at equilibrium.*

Sol. (a) For conservative force

$$F = -\frac{dU}{dx} = -(2ax - b) = -2ax + b$$

(b) At equilibrium $F = 0$

$$\Rightarrow \quad -2ax + b = 0 \Rightarrow x = \frac{b}{2a}$$

$$\therefore \quad U = a\left(\frac{b}{2a}\right)^2 - b\left(\frac{b}{2a}\right) = \frac{b^2}{4a} - \frac{b^2}{2a} = -\frac{b^2}{4a}$$

6 Law of conservation of energy

Energy can neither be created nor be destroyed, it can only be transformed from one form to another form.

Conservation of mechanical energy

The total mechanical energy (sum of kinetic energy and potential energy) of a system is conserved if the forces acting on it, are conservative.

Example 6.43 *A body of mass 5 kg is thrown vertically up with a kinetic energy of 490 J. What will be height at which the kinetic energy of the body becomes half of the original value? (Acceleration due to gravity $= 9.8\,ms^{-2}$)*

Sol. Given, $m = 5$ kg and KE $= 490$ J

From the law of conservation of energy,

$$K_i + U_i = K_f + U_f \Rightarrow K_i + 0 = \frac{K_i}{2} + mgh$$

$$490 = 245 + 5 \times 9.8 \times h \qquad \left(\because K_f = \frac{K_i}{2}\right)$$

$$h = \frac{490 - 245}{5 \times 9.8} = \frac{245}{49} = 5 \text{ m}$$

Example 6.44 *A bullet of mass m moving with velocity v strikes a suspended wooden block of mass M and remains embeded in it. If the block rises to a height h. Find the initial velocity of the bullet.*

Sol. Initial kinetic energy of the block when the bullet strikes.

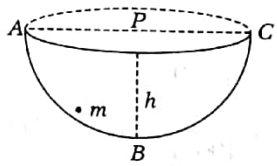
$$= \frac{1}{2}(m + M)v^2$$

Due to this kinetic energy, the block will rise to a height h. Its potential energy $= (m + M)gh$

So, from the law of conservation of energy

$$\frac{1}{2}(M + m)v^2 = (M + m)gh \Rightarrow v = \sqrt{2gh}$$

Example 6.45 *A particle of mass m makes SHM in a smooth hemispherical bowl ABC and it moves from A to C and back to A via ABC, so that PB = h. Find the speed of the ball when it just crosses the point B.*



Sol. From law of conservation of energy,

PE at A = KE at B

or $\quad mgh = \frac{1}{2}mv^2$ or $v = \sqrt{2gh}$

Example 6.46 *A child is swinging on a swing. Minimum and maximum heights of swing from the earth's surface are 0.75 m and 2 m, respectively. What will be the maximum velocity of this swing?*

Sol. From energy conservation,

Gain in kinetic energy = loss in potential energy

$\Rightarrow \quad (1/2)mv_{max}^2 = mg(H_2 - H_1)$

Here, H_1 = minimum height of swing from earth's surface = 0.75 m

H_2 = maximum height of swing from earth's surface = 2 m

$\therefore \quad (1/2)mv_{max}^2 = mg(2 - 0.75)$

or $\quad v_{max} = \sqrt{2 \times 10 \times 1.25} = \sqrt{25} = 5 \text{ ms}^{-1}$

Example 6.47 *A machine which is 75% efficient, uses 12 J of energy in lifting 1 kg mass through a certain height. The mass is then allowed to fall through the same height. Find the velocity at the end of its fall.*

Sol. Potential energy of the mass at a height above the earth's surface is given by $= \frac{75}{100} \times 12 = 9 \text{ J}$ ...(i)

Now, KE of the mass at the end of fall KE $= \frac{1}{2}mv^2$ ...(ii)

Applying law of conservation of energy,

$$\frac{1}{2}mv^2 = 9 \Rightarrow v = \sqrt{\frac{2 \times 9}{m}} = \sqrt{\frac{18}{1}} = \sqrt{18} \text{ ms}^{-1}$$

Example 6.48 *Auto manufactures study the collision of cars with mounted springs of different spring constants. Consider a car of mass 1500 kg moving with a speed of 36 kmh^{-1} on a smooth road and colliding with a horizontally mounted spring of spring constant 7.5×10^3 Nm^{-1}. Find the maximum compression of the spring.*

Sol. At maximum compression, KE of car gets converted completely into PE of the spring.

KE of car $= K = \frac{1}{2}mv^2 = \frac{1}{2} \times 1500 \times 10 \times 10$

$\left(v = 36 \times \frac{5}{18} = 10 \text{ ms}^{-1}\right)$

$= 7.5 \times 10^4 \text{ J}$

At maximum compression x_m, PE, U of the spring = KE, K of car

$$U = \frac{1}{2}kx_m^2 = 7.5 \times 10^4 \text{ J} \Rightarrow x_m = \sqrt{\frac{2 \times 7.5 \times 10^4}{7.5 \times 10^3}} = 4.47 \text{ m}$$

Example 6.49 *A spherical ball of mass 20 kg is stationary at the top of a hill of height 100 m. It rolls down a smooth surface to the ground then climbs up another hill of height 30 m and finally rolls down to a horizontal base at a height of 20 m above the ground. Find the velocity attained by the ball, when moving at horizontal base.*

Sol. According to conservation of energy.

$$mgH = \frac{1}{2}mv^2 + mgh_2 \Rightarrow mg(H - h_2) = \frac{1}{2}mv^2$$

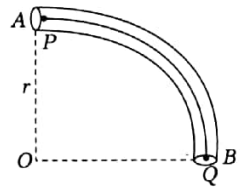
where, H = height of the first hill, h_1 = height of the second hill

h_2 = height of the horizontal base

v = velocity attained by the ball

$\Rightarrow \quad v = \sqrt{2g(100 - 20)} = \sqrt{2 \times 10 \times 80} = 40 \text{ ms}^{-1}$

Example 6.50 *A smooth narrow tube in the form of an arc AB of a circle of centre O and radius r is fixed so that A is vertically above O and OB is horizontal. Particles P of mass m and Q of mass 2 m with a light inextensible string of length ($\pi r/2$) connecting them are placed inside the tube with P at A and Q at B and released from rest. Assuming the string remains taut during motion, find the speed of particles when P reaches B.*



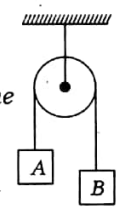
Sol. All surfaces are smooth. Therefore, mechanical energy of the system will remain conserved.

$\therefore$ Decrease in PE of both the blocks

= increase in KE of both the blocks

$$\therefore \quad (mgr) + (2mg)\left(\frac{\pi r}{2}\right) = \frac{1}{2}(m + 2m)v^2 \text{ or } v = \sqrt{\frac{2}{3}(1 + \pi)gr}$$

Example 6.51 *In the arrangement shown in figure, string is light and inextensible and friction is absent everywhere. Find the speed of both the blocks after the block A has ascended a height of 1 m. Given that $m_A = 1$ kg and $m_B = 2$ kg (Take, $g = 10$ ms^{-2}).*



Sol. Friction is absent. Therefore, mechanical energy of the system will remain conserved. From constraint relations we see that speed of both the blocks will be same. Suppose it is v. Here gravitational potential energy of 2 kg block is decreasing while gravitational potential energy of 1 kg block is increasing.

Similarly, kinetic energy of both the blocks is also increasing.

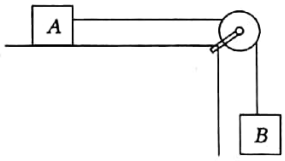
$$\therefore \quad m_B gh = m_A gh + \frac{1}{2}m_A v^2 + \frac{1}{2}m_B v^2$$

or $\quad (2)(10)(1) = (1)(10)(1) + \frac{1}{2}(1)v^2 + \frac{1}{2}(2)v^2$

or $\quad 20 = 10 + 0.5v^2 + v^2 \quad$ or $\quad 1.5v^2 = 10$

$\therefore \quad v^2 = 6.67 \text{ m}^2\text{s}^{-2}$ or $v = 2.58 \text{ ms}^{-1}$

Example 6.52 *In the arrangement shown in figure, $m_A = 1$ kg, $m_B = 4$ kg. String is light and inextensible while pulley is smooth. Coefficient of friction between block A and the table is $\mu = 0.2$. Find the speed of both the blocks when block B has descended a height h = 1 m. (Take, $g = 10$ ms^{-2})*



Sol. From constraint relation, we see that

$v_A = v_B = v$ (say)

Force of friction between block A and table will be

$f = \mu m_A g = (0.2)(1)(10) = 2 \text{ N}$

$\therefore \quad W_{nc} = \Delta U + \Delta K$

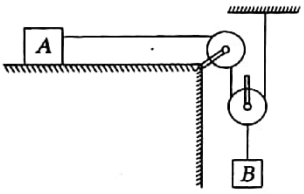
$\therefore \quad -fs = -m_B gh + (1/2)(m_A + m_B)v^2$

or $\quad (-2)(1) = -(4)(10)(1) + \frac{1}{2}(4+1)v^2$

$$-2 = -40 + 2.5\,v^2 \quad \text{or} \quad 2.5\,v^2 = 38$$

$\therefore \quad v^2 = 15.2\ \text{m}^2\text{s}^{-2} \quad \text{or} \quad v = 3.9\ \text{ms}^{-1}$

Example 6.53 *In the arrangement shown in figure $m_A = 4\,kg$ and $m_B = 1\,kg$. The system is released from rest and block B is found to have a speed 0.3 m/s after it has descended through a distance of 1 m. Find the coefficient of friction between the block and the table. Neglect friction elsewhere. (Take $g = 10\ ms^{-2}$)*



Sol. From constraint relations, we can see that $v_A = 2\,v_B$

Therefore, $\quad v_A = 2(0.3) = 0.6\ \text{ms}^{-1}$

as $\quad v_B = 0.3\ \text{ms}^{-1}$ (given)

Applying $\quad W_{nc} = \Delta U + \Delta K$

we get $\quad -\mu\, m_A g S_A = -m_B g S_B + \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$

Here $\quad S_A = 2S_B = 2\,\text{m}$

as $\quad S_B = 1\,\text{m}$ (given)

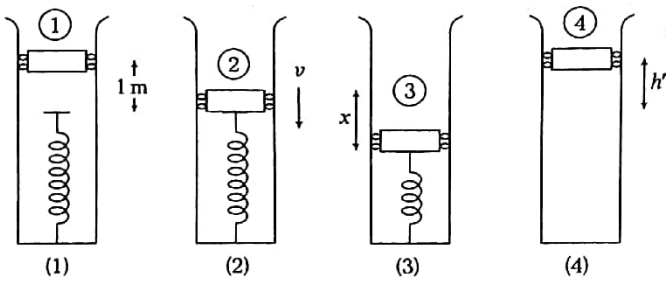
$\therefore \quad -\mu(4.0)(10)(2) = -(1)(10)(1) + \frac{1}{2}(4)(0.6)^2 + \frac{1}{2}(1)(0.3)^2$

or $\quad -80\,\mu = -10 + 0.72 + 0.045$

or $\quad 80\,\mu = 9.235 \quad \text{or} \quad \mu = 0.115$

Example 6.54 *A 20 kg body is released from rest so as to slide in between vertical rails and compresses a vertical spring ($k = 1920\,Nm^{-1}$) placed at a distance $h = 1$ m from the starting position of the body. The rails offer a frictional force of 40 N opposing the motion of body. Find (i) the velocity v of the body just before striking with the spring, (ii) the maximum compression of the spring and (iii) the distance h' through which the body is rebounded up, from maximum compressed position.*

Sol. According to the question, we draw the following diagram.



(i) Between diagram (1) and (2)

Loss in PE = gain in KE + work done against friction.

$$mgh = \frac{1}{2}mv^2 + fh \qquad (\because h = s)$$

$$20 \times 10 \times 1 = \frac{1}{2} \times 20 v^2 + 40 \times 1 \Rightarrow v = 4\ \text{m/s}$$

(ii) Between diagram (2) and (3)

Loss in PE + loss in KE = gain in spring energy + work done against friction

$$mgx + \frac{1}{2} \times mv^2 = \frac{1}{2}kx^2 + fx$$

$$20 \times 10 \times x + \frac{1}{2} \times 20 \times 4^2 = \frac{1}{2} \times 1920x^2 + 40x$$

$$960x^2 - 160x - 160 = 0 \Rightarrow 6x^2 - x - 1 = 0$$

$$\Rightarrow \quad x = \frac{1 \pm \sqrt{1 + 4 \times 6 \times 1}}{12} = \frac{1 \pm 5}{12} = \frac{1}{2}\ \text{m}$$

(c) Between diagram (3) and (4)

$$\frac{1}{2}kx^2 = mgh' + fh'$$

$$\frac{1}{2} \times 1920\left(\frac{1}{2}\right)^2 = (20 \times 10 + 40)h' \Rightarrow h' = 1\ \text{m}$$

7 Conversion of mass and energy

In 1905, Einstein discovered that mass can be converted into energy and *vice-versa*. He showed that mass and energy are equivalent and related by the relation

$$\boxed{E = mc^2}$$

where, m is mass that disappears. E is energy that appears and c is velocity of light in vacuum. Conversely, when energy E disappears, a mass m ($= E/c^2$) appears.

Thus, according to modern physics, mass and energy are not conserved separately, but are conserved as a single entity called **mass energy**.

So, the law of conservation of mass and law of conservation of energy have been unified by this relation into a single law of conservation of mass energy.

Nuclear energy

When U^{235} nucleus breaks up into lighter nuclei on being bombarded by a slow neutron, a tremendous amount of energy is released. Thus, the energy so released is called nuclear energy and this phenomenon is known as nuclear fission. Nuclear reactors and nuclear bombs are the sources of nuclear energy.

Example 6.55. *Calculate the energy in MeV equivalent to the rest mass of an electron. Given that the rest mass of an electron, $m_0 = 9.1 \times 10^{-31}$ kg, $1\,MeV = 1.6 \times 10^{-13}\,J$ and speed of light, $c = 3 \times 10^8\,ms^{-1}$.*

Sol. According to the convertion of mass and energy

$$E = m_0c^2 = 9.1 \times 10^{-31} \times (3 \times 10^8)^2 = 81.9 \times 10^{-15}\,\text{J}$$

$$= \frac{81.9 \times 10^{-15}}{1.6 \times 10^{-13}} = 0.512\,\text{MeV}$$

Example 6.56 *When slow neutrons are incident on a target containing* $^{235}_{92}U$, *a possible fission reaction is*

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{141}_{56}Ba + ^{92}_{36}Kr + 3^{1}_{0}n$$

Estimate the amount of energy released using the following data Given, mass of $^{235}_{92}U = 235.04$ *amu, mass of* $^{1}_{0}n = 1.0087$ *amu, mass of* $^{141}_{56}Ba = 140.91$ *amu, mass of* $^{92}_{36}Kr = 91.926$ *amu and energy equivalent to* 1 *amu* $= 931 MeV$.

Sol. $^{235}_{92}U + ^{1}_{0}n \rightarrow ^{141}_{56}Ba + ^{92}_{36}Kr + 3\,^{1}_{0}n$

Total initial mass $(^{235}_{92}U + ^{1}_{0}n) = 235.04 + 1.0087 = 236.0487$ amu

Total final mas $(^{141}_{56}Ba + ^{92}_{36}Kr + 3\,^{1}_{0}n) = 140.91 + 91926 + 3 \times 10087$

$= 235.8621$ amu

Decrease in mass, $\Delta m = 236.0487 - 235.8621 = 0.1866$ amu

Energy released $= \Delta m \times 931 = 0.1866 \times 931 = 173.725$ MeV

Check point 6.2

1. If the force acting on a body is inversely proportional its speed, the kinetic energy of the body is
(a) constant
(b) directly proportional to time
(c) inversely proportional to time
(d) directly proportional to square of time

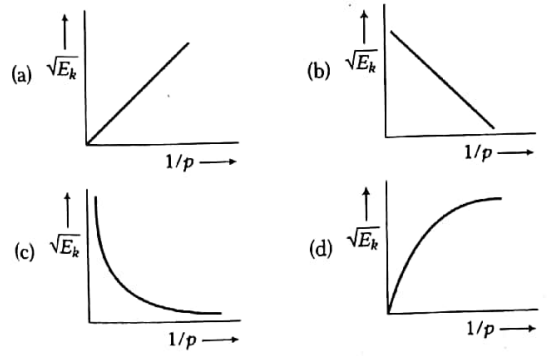
2. If the speed of a vehicle is increased by 1 ms^{-1}, its kinetic energy is doubled, then original speed of the vehicle is
(a) $(\sqrt{2} + 1)ms^{-1}$ (b) $2(\sqrt{2} - 1)ms^{-1}$
(c) $2(\sqrt{2} + 1)ms^{-1}$ (d) $\sqrt{2}(\sqrt{2} + 1)ms^{-1}$

3. A running man has half the KE that a boy of half the mass of the man. The man speeds up by 1 ms^{-1} and then has the same KE as that of boy. The original speeds of man and boy in ms^{-1} are
(a) $(\sqrt{2} + 1), (\sqrt{2} - 1)$ (b) $(\sqrt{2} + 1), 2(\sqrt{2} + 1)$
(c) $\sqrt{2}, \sqrt{2}$ (d) $(\sqrt{2} + 1), 2(\sqrt{2} - 1)$

4. Two bodies of masses m_1 and m_2 have same momentum. The ratio of their KE is
(a) $\sqrt{\frac{m_2}{m_1}}$ (b) $\sqrt{\frac{m_1}{m_2}}$ (c) $\frac{m_1}{m_2}$ (d) $\frac{m_2}{m_1}$

5. If the linear momentum is increased by 50%, then kinetic energy will be increased by
(a) 50 % (b) 100 %
(c) 125 % (d) 25 %

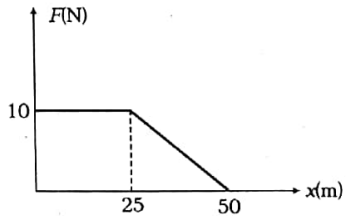
6. The graph between $\sqrt{E_k}$ and $1/p$ is
(E_k = kinetic energy and p = momentum)



7. The kinetic energy acquired by a body of mass m in travelling a certain distance starting from rest, under a constant force is
(a) directly proportional to m
(b) directly proportional to $\sqrt{m}$
(c) inversely porportional to $\sqrt{m}$
(d) independent of m

8. Under the action of a force, a 2 kg body moves such that its position x as a function of time t is given by $x = \frac{t^3}{3}$, where x is in metre and t in second. The work done by the force in the first two seconds is
(a) 1600 J (b) 160 J
(c) 16 J (d) 1.6 J

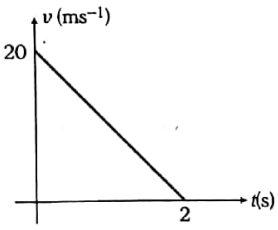
9. An object of mass 5 kg is acted upon by a force that varies with position of the object as shown. If the object starts out from rest at a point $x = 0$. What is its speed at $x = 50$ m.



(a) 12.2 ms^{-1} (b) 18.2 ms^{-1}
(c) 16.4 ms^{-1} (d) 20.4 ms^{-1}

10. A block of mass 20 kg is moving in x-direction with a constant speed of 10 ms^{-1}. It is subjected to a retarding force $F = (-0.1x)$ N during its travel from $x = 20$ m to $x = 30$ m. Its final kinetic energy will be
(a) 975 J (b) 450 J
(c) 275 J (d) 250 J

11. Velocity-time graph of a particle of mass 2 kg moving in a straight line is as shown in figure. Work done by all the forces on the particle is
(a) 400 J
(b) – 400 J
(c) – 200 J
(d) 200 J



12. A particle of mass 0.01 kg travels with velocity given by $4\hat{i} + 16\hat{k}\,ms^{-1}$. After sometime, its velocity becomes $8\hat{i} + 20\hat{k}\,ms^{-1}$. The work done on particle during this interval of time is
(a) 0.32 J (b) 6.9 J (c) 9.6 J (d) 0.96 J

13. A mass of 1 kg is acted upon by a single force $\mathbf{F} = (4\hat{i} + 4\hat{j})$N. Under this force it is displaced from (0, 0) to (1m, 1m). If initially the speed of the particle was $2\,ms^{-1}$, its final speed should be
(a) $6\,ms^{-1}$ (b) $4.5\,ms^{-1}$ (c) $8\,ms^{-1}$ (d) $4\,ms^{-1}$

14. A body of mass 5 kg is raised vertically to a height of 10 m by a force 170 N. The velocity of the body at this height will be
(a) $9.8\,ms^{-1}$ (b) $15\,ms^{-1}$
(c) $22\,ms^{-1}$ (d) $37\,ms^{-1}$

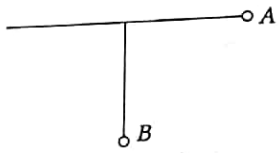
15. A body of mass 0.1 kg moving with a velocity of $10\,ms^{-1}$ hits a spring (fixed at the other end) of force constant $1000\,Nm^{-1}$ and comes to rest after compressing the spring. The compression of the spring is
(a) 0.01 m (b) 0.1 m (c) 0.2 m (d) 0.5 m

16. A mass of 2 kg falls from a height of 40 cm on a spring with a force constant of $1960\,Nm^{-1}$. The spring is compressed by (take, $g = 9.8ms^{-2}$)
(a) 10 cm (b) 1 cm (c) 20 cm (d) 5 cm

17. In which of the following cases the, potential energy is defined?
(a) both conservative and non-conservative forces
(b) conservative force only
(c) non-conservative force only
(d) Neither conservative nor-conservative forces

18. The potential energy of a system increase, if work is done
(a) by the system against a conservative force
(b) by the system against a non-conservative force
(c) upon the system by a conservative force
(d) upon the system by a non-conservative force

19. A pendulum of length 2 m left at A. When it reaches B, it looses 10% of its total energy due to air resistance. The velocity at B is



(a) $6\,ms^{-1}$ (b) $1\,ms^{-1}$ (c) $2\,ms^{-1}$ (d) $8\,ms^{-1}$

20. A body of mass m thrown vertically upwards attains a maximum height h. At what height will its kinetic energy be 75% of its initial value?
(a) $\frac{h}{6}$ (b) $\frac{h}{5}$ (c) $\frac{h}{4}$ (d) $\frac{h}{3}$

8 Power

Power is defined as the rate at which work is done or energy is transferred. If a force does work W in time t, then its average power is given by

$$\text{Average power } (P_{av}) = \text{Rate of doing work}$$

$$\boxed{\text{Average power } (P_{av}) = \frac{\text{Work done}}{\text{Time taken}} = \frac{W}{t}}$$

Power is a scalar quantity because it is the ratio of two scalar quantities work (W) and time (t). The dimensional formula of power is $[ML^2T^{-3}]$.

The SI unit of power is watt (W). The power of an agent is one watt if it does work at the rate of 1 joule per second.

$$1 \text{ watt} = \frac{1 \text{ joule}}{1 \text{ second}} = 1\,Js^{-1}$$

Another popular units of power are kilowatt and horsepower.

1 kilowatt = 1000 watt or 1 kW = $\mathbf{10^3}$ **W**

1 horsepower = 746 watt or 1 hp = **746 W**

This unit is used to describe the output of automobiles, motorbikes, engines etc.

Note

1. Kilowatt hour (kWh) or Board of Trade (BOT) is the commercial unit of electrical energy.
2. Relation between kWh and joule.

$$1\text{ kWh} = 1\text{ kW} \times 1\text{ h} = 1000\text{ W} \times 1\text{ h}$$
$$= 1000\,Js^{-1} \times 3600s \quad \text{or} \quad 1\text{ kWh} = 3.6 \times 10^6\text{ J}$$

Instantaneous power

The **instantaneous power** of an agent is defined as the limiting value of the average power of the agent in a small time interval *i.e.*, when the time interval approaches to zero. When work done by a force $\mathbf{F}$ for a small displacement $d\mathbf{r}$ is $dW = \mathbf{F} \cdot d\mathbf{r}$. Then, instantaneous power can be given as

i.e. $$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt}$$

Now, $$dW = \mathbf{F} \cdot d\mathbf{r}$$

$$\therefore \quad \boxed{P = \mathbf{F} \cdot \frac{d\mathbf{r}}{dt}}$$

Again $d\mathbf{r}/dt = \mathbf{v}$, instantaneous velocity of the agent. Therefore,

$$P = \mathbf{F} \cdot \mathbf{v} = Fv\cos\theta$$

where θ is the angle between $\mathbf{F}$ and $\mathbf{x}$.

It is zero, if force is perpendicular to velocity. *e.g.*, Power of a centripetal force in a circular motion.

Example 6.57 *A train has a constant speed of 40 ms^{-1} on a level road against resistive force of magnitude 3×10^4 N. Find the power of the engine.*

Sol. At constant speed, there is no acceleration, so the forces acting on the train are in equilibrium.

Therefore, $F = R$

$\therefore$ $F = 3 \times 10^4$ N or $P = Fv$

We have, power $= 3 \times 10^4 \times 40 = 1.2 \times 10^6$ W

Example 6.58 *A machine gun fires 240 bullets per minute. If the mass of each bullet is 10 g and the velocity of the bullets is 600 ms^{-1}, then find power (in kW) of the gun.*

Sol. Work done by the gun

$$= \text{Total kinetic energy of the bullets}$$

$$= n\frac{1}{2}mv^2 = 240 \times \frac{1}{2} \times 10 \times 10^{-3} \times (600)^2$$

$$= 120 \times 10 \times 10^{-3} \times 600 \times 600$$

$$\therefore \text{Power of gun} = \frac{\text{Work done}}{\text{Time taken}} = \frac{120 \times 10 \times 10^{-3} \times 600 \times 600}{60}$$

$$= 7.2 \text{ kW}$$

Example 6.59 *In unloading grain from the hold of a ship, an elevator lifts the grain through a distance of 12 m. Grain is discharged at the top of the elevator at a rate of 2 kg each second and the discharge speed of each grain particle is 3 ms^{-1}. Find the minimum horsepower of the motor that can elevate grain in this way. (Take $g = 10$ ms^{-2})*

Sol. The work done by the motor each second, *i.e.*,

$$\text{Power} = mgh + \frac{1}{2}mv^2, \text{ as } t = 1 \text{ s.}$$

Given, $m = 2$ kg, $v = 3$ ms^{-1} and $h = 12$ m.

$$\therefore \text{ Power} = 2 \times (10) \times (12) + \frac{1}{2}(2)(3)^2 = 249 \text{ W} = \frac{249}{746} = 0.33 \text{ hp}$$

The motor must have an output of at least 0.33 hp.

Example 6.60 *A pump can take out 7200 kg of water per hour from a well 100 m deep. Calculate the power of the pump, assuming that its efficiency is 50% ($g = 10$ ms^{-2})*

Sol. Output power $= \frac{mgh}{t} = \frac{7200 \times 10 \times 100}{3600} = 2000$ W

$$\text{Efficiency } \eta = \frac{\text{output power}}{\text{input power}}$$

$$\text{Input power} = \frac{\text{output power}}{\eta} = \frac{2000 \times 100}{50} = 4 \text{ kW}$$

Example 6.61 *A block of mass m is pulled by a constant power P placed on a rough horizontal plane. The coefficient of friction between the block and surface is μ. Find the maximum velocity of the block.*

Sol. Power $P = F \cdot v =$ constant

$$F = \frac{P}{v} \text{ or } F \propto \frac{1}{v}$$

as v increases, F decreases.

when $F = \mu mg$, net force on block becomes zero, *i.e.*, it has maximum or terminal velocity

$$\therefore \quad P = (\mu mg)\, v_{max} \text{ or } v_{max} = \frac{P}{\mu mg}$$

Example 6.62 *The force required to row a boat at constant velocity is proportional to the speed. If a speed of 4 kmh^{-1} requires 7.5 kW, how much power does a speed of 12 kmh^{-1} require?*

Sol. Let the force be $F = \alpha v$, where v is speed and α is a constant of proportionality. The power required is

$$P = Fv = \alpha v^2$$

Let P_1 be the power required for speed v_1 and P_2 be the power required for speed v_2.

$$P_1 = 7.5 \text{ kW and } v_2 = 3v_1 \Rightarrow P_2 = \left(\frac{v_2}{v_1}\right)^2 P_1$$

$$\Rightarrow \quad P_2 = (3)^2 \times 7.5 \text{ kW} = 67.5 \text{ kW}$$

Example 6.63 *An engine pumps 400 kg of water through height of 10 m in 40 s. Find the power of the engine if its efficiency is 80% (Take, $g = 10$ ms^{-2}).*

Sol. Work done by engine against gravity

$$W = mgh = 400 \times 10 \times 10 = 40 \text{ kJ}$$

$$\text{Power used by engine (output power)} = \frac{W}{\Delta t}$$

$$= \frac{40 \times 10^3}{40} W = 1 \text{ kW}$$

If power of the engine is P (input power), then

$$\eta = \frac{\text{Output power}}{\text{Input power}}$$

$$P = \frac{1 \text{ kw}}{80\%} \Rightarrow P = \frac{1000 \times 100}{80} \text{ W}$$

or $$\frac{100}{80} \text{ kW} = 1.25 \text{ kW}$$

Example 6.64 *An automobile of mass m accelerates, starting from rest. The engine supplies constant power P, show that the velocity is given as a function of time by $v = \left(\frac{2Pt}{m}\right)^{1/2}$.*

Sol. Power, $P =$ constant

Work done up to time t is $W = Pt$

From work energy theorem,

$$W = \Delta KE \text{ or } Pt = \frac{1}{2}mv^2$$

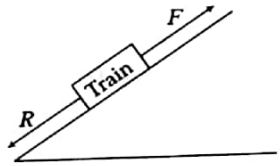
$$\therefore \quad v = \left(\frac{2Pt}{m}\right)^{1/2}$$

Example 6.65 *A train of mass 2×10^5 kg has a constant speed of 20 ms^{-1} up a hill inclined at $\theta = \sin^{-1}\left(\frac{1}{50}\right)$ to the horizontal when the engine is working at 8×10^5 W. Find the resistance to motion of the train.* (Take, $g = 9.8\ ms^{-2}$).

Sol. Since, $P = Fv$

$$F = \frac{P}{v} = \frac{8 \times 10^5}{20} = 4 \times 10^4 \text{ N}$$

At constant speed, the forces acting on the train are in equilibrium. Resolving the forces parallel to the hill



$$F = R + (2.0 \times 10^5) g \times \frac{1}{50}$$

$$4 \times 10^4 = R + 39200$$

or $$R = 800 \text{ N}$$

Therefore, the resistance is 800 N.

Example 6.66 *A small body of mass m moving with velocity v_0 on rough horizontal surface, finally stops due to friction. Find, the mean power developed by the friction force during the motion of the body, if the frictional coefficient $\mu = 0.27$, $m = 1$ kg and $v_0 = 1.5\ ms^{-1}$.*

Sol. The retardation due to friction is,

$$a = \frac{\text{force of friction}}{\text{mass}} = \frac{\mu mg}{m} = -\mu g$$

Further, $$v_0 = at$$

Therefore, $$t = \frac{v_0}{a} = \frac{v_0}{\mu g} \quad \ldots\text{(i)}$$

From work energy theorem,

work done by force of friction = change in kinetic energy

or $$W = \frac{1}{2} m v_0^2 \quad \ldots\text{(ii)}$$

$$\text{Mean power} = \frac{W}{t}$$

From Eqs. (i) and (ii), we get

$$P_{\text{mean}} = \frac{1}{2} \mu m g v_0$$

Substituting the values, we have

$$P_{\text{mean}} = \frac{1}{2} \times 0.27 \times 1.0 \times 9.8 \times 1.5 \approx 2 \text{ W}$$

Check point 6.3

1. A particle of mass M starting from rest undergoes uniform acceleration. If the speed acquired in time T is v, the power delivered to the particle is

(a) $\frac{Mv^2}{T}$ (b) $\frac{1}{2}\frac{Mv^2}{T^2}$ (c) $\frac{Mv^2}{T^2}$ (d) $\frac{1}{2}\frac{Mv^2}{T}$

2. An engine develops 10 kW of power. How much time will it take to lift a mass of 200 kg to a height of 40 m? (take, $g = 10 ms^{-2}$)

(a) 4 s (b) 5 s (c) 8 s (d) 10 s

3. An engineer claims to have made an engine delivering 10 kW power with fuel consumption of 1 gs^{-1}. The calorific value of fuel is 2 kcal g^{-1}. This claim is

(a) valid
(b) invalid
(c) dependent on engine design
(d) dependent on load

4. Water falls from a height of 60 m at the rate of 15 kg/s to operate a turbine. The losses due to frictional forces are 10% of energy. How much power is generated by the turbine ($g = 10\ ms^2$).

(a) 12.3 kW (b) 7 kW (c) 8.1 kW (d) 10.2 kW

5. A 60 kg man runs up a staircase in 12 seconds while a 50 kg man runs up the same staircase in 11 seconds. The ratio of the rate of doing their work is

(a) 6 : 5 (b) 12 : 11 (c) 11 : 10 (d) 10 : 11

6. If the heart pushes 1 cc of blood in one second under pressure 20000 Nm^{-2} the power of heart is

(a) 0.02 W (b) 400 W
(c) 5×10^{-10} W (d) 0.2 W

7. A body of mass 10 kg moves with a constant speed v of 2 ms^{-1} along a circular path of radius 8 m. The power produced by the body will be

(a) 10 Js^{-1} (b) 98 Js^{-1}
(c) 49 Js^{-1} (d) zero

8. A force of $2\hat{i} + 3\hat{j} + 4\hat{k}$ N acts on a body for 4 second, produces a displacement of $(3\hat{i} + 4\hat{j} + 5\hat{k})$m. The power used is

(a) 9.5 W (b) 7.5 W
(c) 6.5 W (d) 4.5 W

9. A particle moves with a velocity $5\hat{i} - 3\hat{j} + 6\hat{k}\ ms^{-1}$ under the influence of a constant force $\mathbf{F} = 10\hat{i} + 10\hat{j} + 20\hat{k}$N. The instantaneous power applied to the particle is

(a) 200 Js^{-1} (b) 40 Js^{-1}
(c) 140 Js^{-1} (d) 170 Js^{-1}

10. A body of mass 2 kg is projected at 20 ms^{-1} at an angle 60° above the horizontal. Power due to the gravitational force at its highest point is

(a) 200 W (b) $100\sqrt{3}$ W
(c) 50 W (d) zero

Chapter exercises

A) Taking it together

(Assorted questions of the chapter for advanced level practice)

1. An electron and a proton are moving under the influence of mutual forces. In calculating the change in the kinetic energy of the system during motion, one ignores the magnetic force of one on another. This is, because **[NCERT Exemplar]**

(a) the two magnetic forces are equal and opposite, so they produce no net effect
(b) the magnetic forces do not work on each particle
(c) the magnetic forces do equal and opposite (but non-zero) work on each particle
(d) the magnetic forces are necessarily negligible

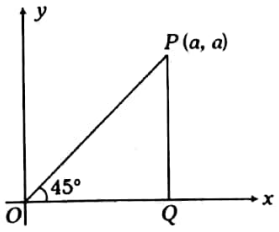
2. A man squatting on the ground gets straight up and stand. The force of reaction of ground on the man during the process is **[NCERT Exemplar]**

(a) constant and equal to mg in magnitude
(b) constant and greater than mg in magnitude
(c) variable but always greater than mg
(d) at first greater than mg and later becomes equal to mg

3. A body is falling freely under the action of gravity alone in vacuum. Which of the following quantities remain constant during the fall? **[NCERT Exemplar]**

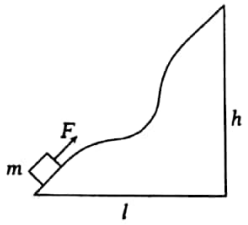
(a) Kinetic energy
(b) Potential energy
(c) Total mechanical energy
(d) Total linear momentum

4. A particle is moved from (0, 0) to (a, a) under a force $\mathbf{F} = (3\hat{i} + 4\hat{j})$ from two paths. Path 1 is OP and path 2 is OQP. Let W_1 and W_2 be the work done by this force in these two paths. Then,



(a) $W_1 = W_2$ (b) $W_1 = 2W_2$
(c) $W_2 = 2W_1$ (d) $W_2 = 4W_1$

5. A body of mass m was slowly pulled up the hill by a force F which at each point was directed along the tangent of the trajectory. All surfaces are smooth. Find the work performed by this force.

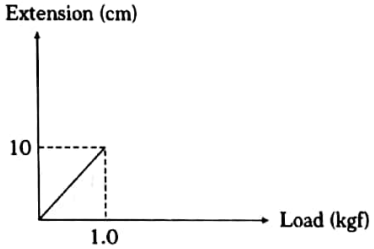


(a) mgl (b) $-mgl$ (c) mgh (d) zero

6. A rod of mass m and length l is lying on a horizontal table. Work done in making it stand on one end will be

(a) mgl (b) $\frac{mgl}{2}$ (c) $\frac{mgl}{4}$ (d) $2mgl$

7. The pointer reading *versus* load graph for a spring balance is as shown in the figure.



The spring constant is

(a) 15 kgf/cm (b) 5 kgf/cm
(c) 0.1 kgf/cm (d) 10 kgf/cm

8. The momentum of a body is p and its kinetic energy is E. Its momentum becomes $2p$. Its kinetic energy will be

(a) $\frac{E}{2}$ (b) $3E$ (c) $2E$ (d) $4E$

9. If v, p and E denote velocity, linear momentum and KE of the particle respectively, then

(a) $p = \frac{dE}{dv}$ (b) $p = \frac{dE}{dt}$

(c) $p = \frac{dv}{dt}$ (d) $p = \frac{dE}{dv} \times \frac{dE}{dt}$

10. Two masses of 1 g and 4 g are moving with equal kinetic energies. The ratio of the magnitudes of their momenta is

(a) 4 : 1 (b) $\sqrt{2}$: 1 (c) 1 : 2 (d) 1 : 16

11. When work done by force of gravity is negative, then

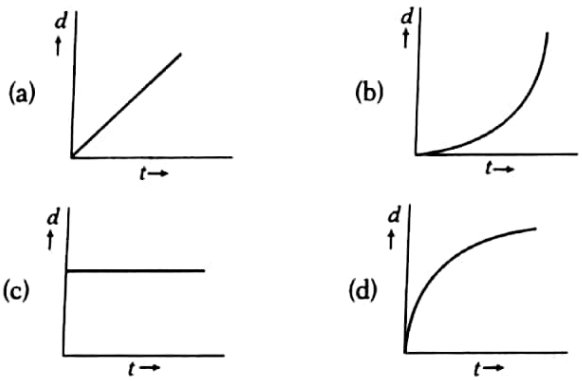
(a) PE increases (b) KE increases

(c) PE remains constant (d) PE decreases

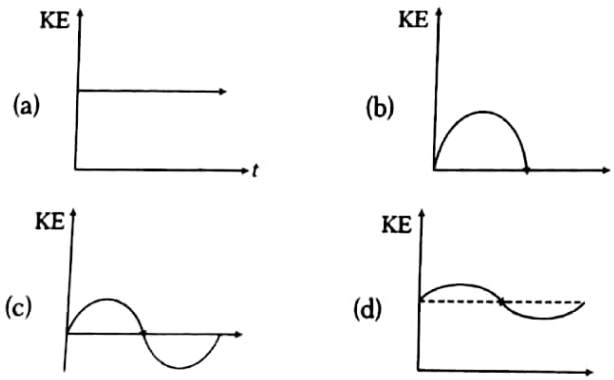
12. A bicyclist comes to a skidding stop in 10 m. During this process, the force on the bicycle due to the road is 200 N and is directly opposed to the motion. The work done by the cycle on the road is **[NCERT Exemplar]**

(a) + 2000 J (b) – 200 J (c) zero (d) – 20000 J

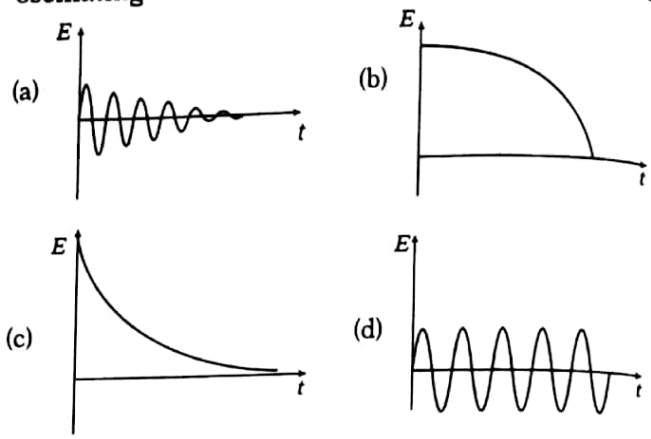
13. A body is moving unidirectionally under the influence of a source of constant power supplying energy. Which of the diagrams shown in figure correctly shown the displacement-time curve for its motion? **[NCERT Exemplar]**



14. Which of the diagrams shown in figure most closely shows the variation in kinetic energy of the earth as it moves once around the sun in its elliptical orbit? **[NCERT Exemplar]**



15. Which of the diagrams shown in figure represents variation of total mechanical energy of a pendulum oscillating in air as function of time? **[NCERT Exemplar]**



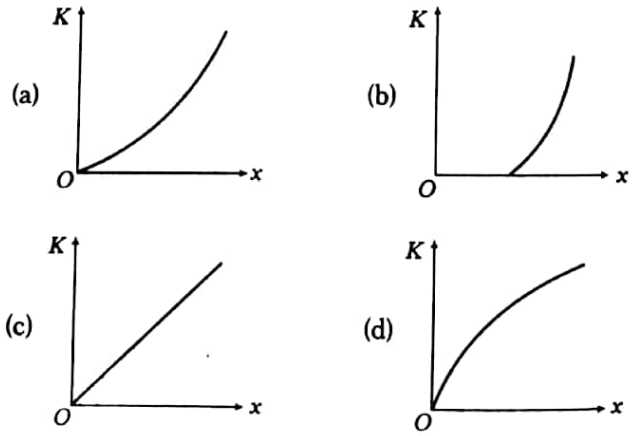
16. A mass of 5 kg is moving along a circular path of radius 1 m. If the mass moves with 300 rev min^{-1}, its kinetic energy would be **[NCERT Exemplar]**

(a) $250\pi^2$ (b) $100\pi^2$ (c) $5\pi^2$ (d) 0

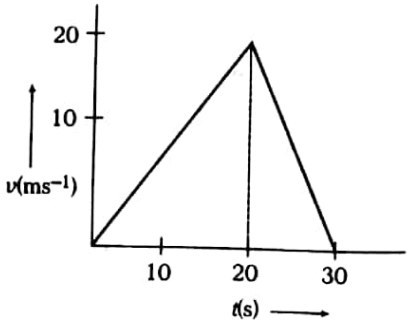
17. A particle moves from position $3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 6\hat{\mathbf{k}}$ to $14\hat{\mathbf{i}} + 13\hat{\mathbf{j}} + 9\hat{\mathbf{k}}$ due to a force $\mathbf{F} = (4\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}})$N. If the displacement is in centimeter then work done will be

(a) 1 J (b) 2 J (c) 3 J (d) 2.5 J

18. A body moves from rest with a constant acceleration. Which one of the following graphs represents the variation of its kinetic energy K with the distance travelled x?



19. v-t graph of an object of mass 1 kg is shown. Select the incorrect statement.



(a) Work done on the object in 30 s is zero
(b) The average acceleration of the object is zero
(c) The average velocity of the object is zero
(d) The average force on the object is zero

20. A toy gun uses a spring of very large value of force constant k. When charged before being triggered in the upward direction, the spring is compressed by a small distance x. If mass of shot is m, on being triggered it will go upto a height of
(a) $\frac{kx^2}{mg}$ (b) $\frac{x^2}{kmg}$ (c) $\frac{kx^2}{2mg}$ (d) $\frac{(kx)^2}{mg}$

21. A car moving with a speed of 40 kmh^{-1} can be stopped by applying brakes in 2 m. If the car is moving with a speed of 80 kmh^{-1} the minimum stopping distance under similar brake
(a) 8 m (b) 2 m
(c) 4 m (d) 6 m

22. A long spring is stretched by 2 cm. Its potential energy is U. If the spring is stretched by 10 cm, its potential energy would be
(a) $\frac{U}{25}$ (b) $\frac{U}{5}$ (c) $5U$ (d) $25U$

23. A body is falling under gravity. When it loses a gravitational potential energy by U, its speed is v. The mass of the body shall be
(a) $\frac{2U}{v}$ (b) $\frac{U}{2v}$ (c) $\frac{2U}{v^2}$ (d) $\frac{U}{2v^2}$

24. If a body of mass 200 g falls from a height 200 m and its total potential energy is converted into kinetic energy, at the point of contact of the body with the surface, then decrease in potential energy of the body at the contact is (Take, $g = 10\ ms^{-2}$)
(a) 900 J (b) 600 J (c) 400 J (d) 200 J

25. A stone of mass 2 kg is projected upwards with KE of 98 J. The height at which the KE of the body becomes half its original value, is given by (take, $g = 9.8ms^{-2}$)
(a) 5 m (b) 2.5 m (c) 1.5 m (d) 0.5 m

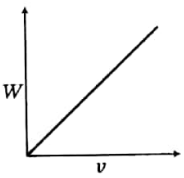
26. Mark out the correct statement(s).
(a) Total work done by internal forces on a system is always zero
(b) Total work done by internal forces on a system may sometimes be zero
(c) Total work done by friction can never be zero
(d) Total work done by friction is always zero

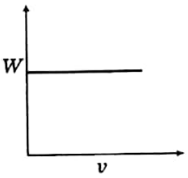
27. A vehicle needs an engine of 7500 W to keep it moving with a constant velocity of 20 ms^{-1} on a horizontal surface. The force resisting the motion is
(a) 375 dyne (b) 375 N
(c) 150000 dyne (d) 150000 N

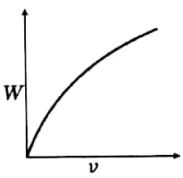
28. An engine exerts a force $\mathbf{F} = (20\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}})$ N and moves with velocity $\mathbf{v} = (6\hat{\mathbf{i}} + 20\hat{\mathbf{j}} - 3\hat{\mathbf{k}})\ ms^{-1}$. The power of the engine (in watt) is
(a) 45 (b) 75
(c) 20 (d) 10

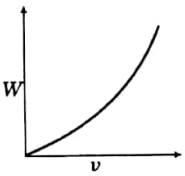
29. An object of mass m, initially at rest under the action of a constant force F attains a velocity v in time t. Then, the average power supplied to mass is
(a) $\frac{mv^2}{2t}$ (b) $\frac{Fv}{2}$
(c) Both are correct (d) Both are wrong

30. A particle at rest on a frictionless table is acted upon by a horizontal force which is constant in magnitude and direction. A graph is plotted of the work done on the particle W, against the speed of the particle v. If there are no frictional forces acting on the particle, the graph will look like

(a) 

(b) 

(c) 

(d) 

31. The energy required to accelerate a car from rest to 10 ms^{-1} is W. The energy required to accelerate the car from 10 ms^{-1} to 20 ms^{-1} is
(a) W (b) $2W$ (c) $3W$ (d) $4W$

32. A force $\mathbf{F} = -k(y\hat{\mathbf{i}} + x\hat{\mathbf{j}})$, where k is a positive constant, acts on a particle moving in the xy-plane. Starting from the origin, the particle is taken along the positive x-axis to the point $(a, 0)$ and then parallel to the y-axis to the point (a, a). The total work done by the forces $\mathbf{F}$ on the particle is
(a) $-2ka^2$ (b) $2ka^2$
(c) $-ka^2$ (d) ka^2

33. Given that the displacement of the body in metre is a function of time as follows

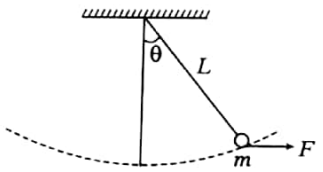
$$x = 2t^4 + 5$$

The mass of the body is 2 kg. What is the increase in its kinetic energy one second after the start of motion?
(a) 8 J (b) 16 J
(c) 32 J (d) 64 J

34. A body is at ed to the lower end of a vertical spiral spring it is gradually lowered to its equilibrium p on. This stretches the spring by a length d. If the e body attached to the same spring is allow fall suddenly, what would be the maximum stret in this case?
(a) d (b) $2d$ (c) $3d$ (d) $(1/2)d$

35. A uniform chain of length L and mass M is lying on a smooth table and one third of its length is hanging vertically down over the edge of the table . If g is acceleration due to gravity, the work required to pull the hanging part on the table is
(a) MgL (b) $\frac{MgL}{3}$ (c) $\frac{MgL}{9}$ (d) $\frac{MgL}{18}$

36. An object of mass m is tied to a string of length L and a variable horizontal force is applied on it which starts at zero and gradually increases until the string makes an angle θ with the vertical.

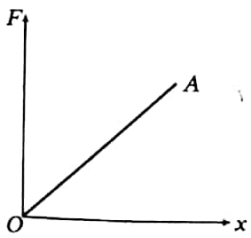


Work done by the force F is
(a) $mgL(1-\sin\theta)$ (b) mgL
(c) $mgL(1-\cos\theta)$ (d) $mgL(1+\cos\theta)$

37. A particle is moving in a conservative force field from point A to point B. U_A and U_B are the potential energies of the particle at point A and B and W_C is the work done by conservative forces in the process of taking the particle from A and B, which of the following is true?
(a) $W_C = U_B - U_A$ (b) $W_C = U_A - U_B$
(c) $U_A > U_B$ (d) $U_B > U_A$

38. Three particles A, B and C are projected from the top of a tower with the same speed. A is thrown straight upwards, B straight down and C horizontally. They hit the ground with speeds v_A, v_B and v_C, then which of the following is correct?
(a) $v_A = v_B > v_C$ (b) $v_A = v_B = v_C$
(c) $v_A > v_B = v_C$ (d) $v_B > v_C > v_A$

39. The force required to stretch a spring varies with the distance as shown in the figure . If the experiment is performed with the above spring of half the length, the line OA will



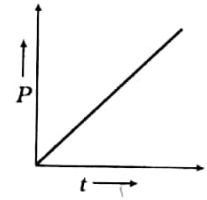
(a) shift towards F-axis
(b) shift towards X-axis
(c) remain as it is
(d) become double in length

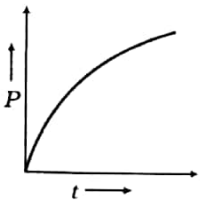
40. Kinetic energy of a particle moving in a straight line varies with time t as $K = 4t^2$. The force acting on the particle
(a) is constant
(b) is increasing
(c) is decreasing
(d) first increases and then decreases

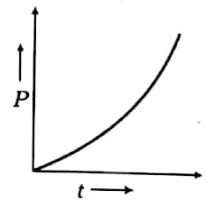
41. A particle is released from a height H. At certain height its kinetic energy is two times its potential energy. Height and speed of particle at that instant are
(a) $\frac{H}{3}, \sqrt{\frac{2gH}{3}}$ (b) $\frac{H}{3}, 2\sqrt{\frac{gH}{3}}$
(c) $\frac{2H}{3}, \sqrt{\frac{2gH}{3}}$ (d) $\frac{H}{3}, \sqrt{2gH}$

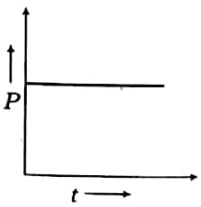
42. Power applied to a particle varies with time as $P = (3t^2 - 2t + 1)$ W, where t is in second. Find the change in its kinetic energy between time $t = 2$ s and $t = 4$ s.
(a) 32 J (b) 46 J (c) 61 J (d) 102 J

43. A motor drives a body along a straight line with a constant force. The power P developed by the motor must vary with time t as

(a)


(b)


(c)


(d)


44. Power supplied to a particle of mass 2 kg varies with time as $P = \frac{3t^2}{2}$ W. Here t is in second. If velocity of particle at $t = 0$ is $v = 0$. The velocity of particle at time $t = 2$ s will be
(a) 1 ms^{-1} (b) 4 ms^{-1} (c) 2 ms^{-1} (d) $2\sqrt{2}$ ms^{-1}

45. A force F acting on a body depends on its displacement S as $F \propto S^{-1/3}$. The power delivered by F will depend on displacement as
(a) $S^{2/3}$ (b) $S^{-5/3}$ (c) $S^{1/2}$ (d) S^0

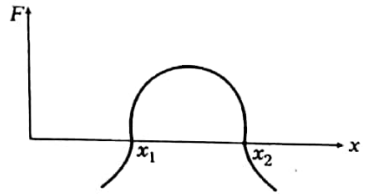
46. A ball is thrown vertically upwards with a velocity of 10 ms^{-1}. It returns to the ground with a velocity of 9 ms^{-1}. If $g = 9.8$ ms^{-2}, then the maximum height attained by the ball is nearly (assume air resistance to be uniform)
(a) 5.1 m (b) 4.1 m (c) 4.61 m (d) 5 m

47. An open knife edge of mass m is dropped from a height h on a wooden floor. If the blade penetrates upto the depth d into the wood, the average resistance offered by the wood to the knife edge is

(a) $mg\left(1+\frac{h}{d}\right)$ (b) $mg\left(1+\frac{h}{d}\right)^2$

(c) $mg\left(1-\frac{h}{d}\right)$ (d) $mg\left(1+\frac{d}{h}\right)$

48. The force acting on a body moving along X-axis varies with the position of the particle as shown in the figure. The body is in stable equilibrium at



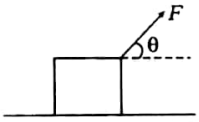
(a) $x = x_1$ (b) $x = x_2$

(c) Both x_1 and x_2 (d) Neither x_1 nor x_2

49. How much mass is converted into energy per day in Tarapur nuclear power plant operated at 10^7 kW?

(a) 9.6 g (b) 9.63 kg (c) 8.6 g (d) 7 g

50. A block of mass m is pulled along a horizontal surface by applying a force at an angle θ with the horizontal. If the block travels with a uniform velocity and has a displacement d and the coefficient of friction is μ, then the work done by the applied force is



(a) $\frac{\mu mgd}{\cos\theta + \mu\sin\theta}$ (b) $\frac{\mu mgd\cos\theta}{\cos\theta + \mu\sin\theta}$

(c) $\frac{\mu mgd\sin\theta}{\cos\theta + \mu\sin\theta}$ (d) $\frac{\mu mgd\cos\theta}{\cos\theta - \mu\sin\theta}$

51. If v be the instantaneous velocity of the body dropped from the top of a tower, when it is located at height h, then which of the following remains constant?

(a) $gh + v^2$ (b) $gh + \frac{v^2}{2}$

(c) $gh - \frac{v^2}{2}$ (d) $gh - v^2$

52. A block of mass 1 kg slides down a rough inclined plane of inclination 60° starting from its top. If coefficient of kinetic friction is 0.5 and length of the plane d = 2 m, then work done against friction is

(a) 2.45 J (b) 4.9 J (c) 9.8 J (d) 19.6 J

53. A proton is kept at rest. A positively charged particle is released from rest at a distance d in its field. Consider two experiments; one in which the charged particle is also a proton and in another, a positron. In the same time t, the work done on the two moving charged particles is **[NCERT Exemplar]**

(a) same as the same force law is involved in the two experiments

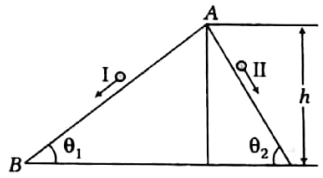
(b) less for the case of a positron, as the positron moves away more rapidly and the force on it weakens

(c) more for the case of a positron, as the positron moves away a larger distance

(d) same as the work done by charged particle on the stationary proton

54. Two inclined frictionless tracks, one gradual and the other steep meet at A from where two stones are allowed to slide down from rest, one on each track as shown in figure. **[NCERT Exemplar]**

Which of the following statement is correct?



(a) Both the stones reach the bottom at the same time but not with the same speed

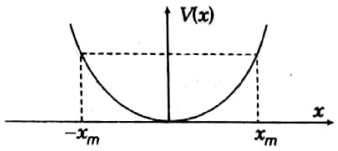
(b) Both the stones reach the bottom with the same speed and stone I reaches the bottom earlier than stone II

(c) Both the stones reach the bottom with the same speed and stone II reaches the bottom earlier than stone I

(d) Both the stones reach the bottom at different times and with different speeds

55. The potential energy function for a particle executing linear SHM is given by $V(x) = \frac{1}{2}kx^2$ where k is the force constant of the oscillator (Fig). For $k = 0.5\ \text{Nm}^{-1}$, the graph of $V(x)$ *versus* x is shown in the figure.

A particle of total energy E turns back when it reaches $x = \pm x_m$. If V and K indicate the PE and KE, respectively of the particle at $x = +x_m$, then which of the following is correct? **[NCERT Exemplar]**



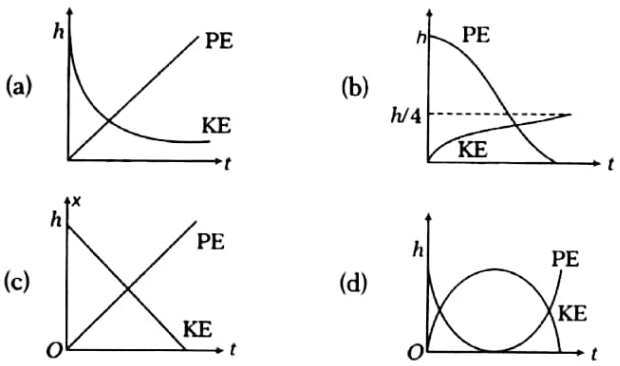
(a) $V = O, K = E$ (b) $V = E, K = O$

(c) $V < E, K = O$ (d) $V = O, K < E$

56. A body of mass 0.5 kg travels in a straight line with velocity $v = a\,x^{3/2}$ where $a = 5\ \text{m}^{-1/2}\text{s}^{-1}$. The work done by the net force during its displacement from $x = 0$ to $x = 2$ m is **[NCERT Exemplar]**

(a) 1.5 J (b) 50 J (c) 10 J (d) 100 J

57. A raindrop falling from a height h above ground, attains a near terminal velocity when it has fallen through a height $(3/4)h$. Which of the diagrams shown in figure correctly shows the change in kinetic and potential energy of the drop during its fall up to the ground? **[NCERT Exemplar]**



58. In a shotput event an athlete throws the shotput of mass 10 kg with an initial speed of 1 ms^{-1} at 45° from a height 1.5 m above ground. Assuming air resistance to be negligible and acceleration due to gravity to be 10 ms^{-2}, the kinetic energy of the shotput when it just reaches the ground will be **[NCERT Exemplar]**

(a) 2.5 J (b) 5 J (c) 52.5 J (d) 155 J

59. The potential energy between the atoms in a molecule is given by $U(x) = \dfrac{a}{x^{12}} - \dfrac{b}{x^6}$

where, a and b positive constants and x is the distance between the atoms . The atom is in equilibrium when

(a) $x = 0$ (b) $x = \left(\dfrac{a}{2b}\right)^{1/6}$

(c) $x = \left(\dfrac{2a}{b}\right)^{1/6}$ (d) $x = \left(\dfrac{11a}{5b}\right)^{1/6}$

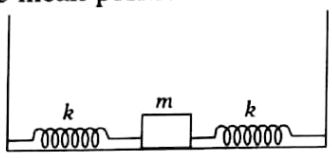
60. In position A kinetic energy of a particle is 60 J and potential energy is – 20 J. In position B, kinetic energy is 100 J and potential energy is 40 J. Then, in moving the particle from A to B

(a) work done by conservative forces is 50 J
(b) work done by external forces is 40 J
(c) net work done by all the forces is 40 J
(d) net work done by all the forces is 100 J

61. The figure shows a particle sliding on a frictionless track, which terminates in a straight horizontal section. If the particle starts slipping from the point A, how far away from the track will the particle hit the ground?

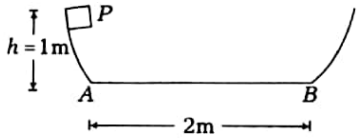
(a) 1 m (b) 2 m (c) 3 m (d) 4 m

62. A block of mass m is attached to two unstretched springs of spring constants k, each as shown. The block is displaced towards right through a distance x and is released. The speed of the block as it passes through the mean position will be



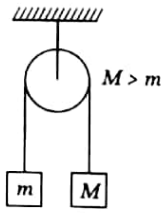
(a) $x\sqrt{\dfrac{m}{2k}}$ (b) $x\sqrt{\dfrac{2k}{m}}$ (c) $x\sqrt{\dfrac{m}{k}}$ (d) $x\dfrac{2k}{m}$

63. The curved portions are smooth and horizontal surface is rough. The block is released from P. At what distance from A it will stop (if $\mu = 0.2$)



(a) 1m (b) 2 m (c) 3 m (d) 4 m

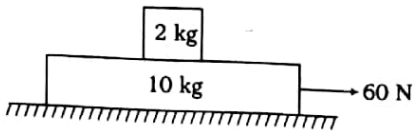
64. The system shown in the figure is released from rest. At the instant when mass M has fallen through a distance h, the velocity of m will be



(a) $\sqrt{2gh}$ (b) $\dfrac{\sqrt{2ghM}}{m}$

(c) $\sqrt{\dfrac{2gh\,(M-m)}{m+M}}$ (d) $\sqrt{\dfrac{2gh\,(M+m)}{m-M}}$

65. A plank of mass 10 kg and a block of mass 2 kg are placed on a horizontal plane as shown in the figure. There is no friction between plane and plank. The coefficient of friction between block and plank is 0.5. A force of 60 N is applied on plank horizontally. In first 2 s the work done by friction on the block is



(a) – 100 J (b) 100 J (c) zero (d) 200 J

66. A block of mass 5 kg slides down a rough inclined surface. The angle of inclination is 45°. The coefficient of sliding friction is 0.20 . When the block slides 10 cm, the work done on the block by force of friction is

(a) $-\frac{1}{\sqrt{2}}$ J (b) 1J (c) $-\sqrt{2}$ J (d) -1 J

67. A particle moves on a rough horizontal ground with some initial velocity say v_0. If $\left(\frac{3}{4}\right)$th of its kinetic energy is lost due to friction in time t_0, then coefficient of friction between the particle and the ground is

(a) $\frac{v_0}{2gt_0}$ (b) $\frac{v_0}{4gt_0}$ (c) $\frac{3v_0}{4gt_0}$ (d) $\frac{v_0}{gt_0}$

68. A 50 kg girl is swinging on a swing from rest. Then, the power delivered when moving with a velocity of $2\,\text{ms}^{-1}$ upwards in a direction making an angle 60° with the verticle is

(a) 980 W (b) 490 W (c) $490\sqrt{3}$ W (d) 245 W

69. A bead can slide on a smooth circular wire frame of radius r which is fixed in a vertical plane. The bead is displaced slightly from the highest point of the wire frame. The speed of the bead subsequently as a function of the angle θ made by the bead with the vertical line is

(a) $\sqrt{2gr}$ (b) $\sqrt{2gr(1-\sin\theta)}$

(c) $\sqrt{2gr(1-\cos\theta)}$ (d) $2\sqrt{gr}$

70. A uniform chain has a mass M and length L. It is placed on a frictionless table with length l_0 hanging over the edge. The chain begins to slide down . Then, the speed v with which the end slides down from the edge is given by

(a) $v=\sqrt{\frac{g}{L}(L+l_0)}$ (b) $v=\sqrt{\frac{g}{L}(L-l_0)}$

(c) $v=\sqrt{\frac{g}{L}(L^2-l_0^2)}$ (d) $v=\sqrt{2g(L-l_0)}$

71. A particle of mass 1 g executes an oscillatory motion on the concave surface of a spherical dish of radius 2 m placed on a horizontal plane. If the motion of the particle begins from a point on the dish at a height of 1 cm from the horizontal plane, the total distance covered by the, particle before it comes to rest, is curve surface is smooth and $\mu = 0.01$ for horizontal surface.

(a) 2 m (b) 10 m (c) 1 m (d) 20 m

72. A mass-spring system oscillates such that the mass moves on a rough surface having coefficient of friction μ. It is compressed by a distance a from its normal length and, on being released, it moves to a distance b from its equilibrium position. The decrease in amplitude for one half-cycle ($-a$ to b) is

(a) $\frac{\mu mg}{k}$ (b) $\frac{2\mu mg}{k}$ (c) $\frac{\mu g}{k}$ (d) $\frac{k}{\mu mg}$

73. A uniform flexible chain of mass m and length l hangs in equilibrium over a smooth horizontal pin of negligible diameter. One end of the chain is given a small vertical displacement so that the chain slips over the pin. The speed of chain when it leaves pin is

(a) $\sqrt{\frac{gl}{2}}$ (b) $\sqrt{gl}$ (c) $\sqrt{2gl}$ (d) $\sqrt{3gl}$

74. The potential energy of a particle of mass 1 kg is, $U = 10 + (x-2)^2$. Here U is in joule and x in metre on the positive X-axis. Particle travels upto $x = +6$ m. Choose the correct statement.

(a) On negative X-axis particle travels upto $x = -2$ m

(b) The maximum kinetic energy of the particle is 16 J

(c) Both (a) and (b) are correct

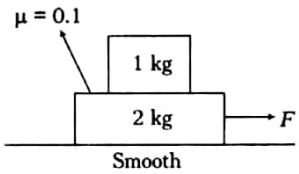
(d) Both (a) and (b) are incorrect

75. A body is moving down an inclined plane of slope 37°. The coefficient of friction between the body and plane varies as $\mu = 0.3x$, where x is the distance travelled down the plane by the body. The body will have maximum speed (Take, $g = 10\,\text{ms}^{-2}$, $\sin 37° = \frac{3}{5}$)

(a) At $x = 1.16$ m (b) At $x = 2$m

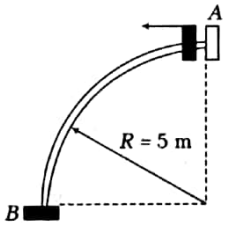
(c) At bottom of plane (d) At $x = 2.5$m

76. A force of $F = 0.5$ N is applied on lower block as shown in figure . The work done by lower block on upper block for a displacement of 3 m of the upper block with respect to ground is (Take, $g = 10\,\text{ms}^{-2}$)



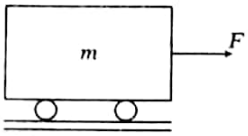
(a) -0.5 J (b) 0.5 J (c) 2 J (d) -2 J

77. A bead of mass 1/2 kg starts from rest from A to move in a vertical plane along a smooth fixed quarter ring of radius 5 m, under the action of a constant horizontal force $F = 5$N as shown in figure. The speed of bead as it reaches the point B is (Take, $g = 10\,\text{ms}^{-2}$)



(a) $14.14\,\text{ms}^{-1}$ (b) $7.07\,\text{ms}^{-1}$ (c) $5\,\text{ms}^{-1}$ (d) $25\,\text{ms}^{-1}$

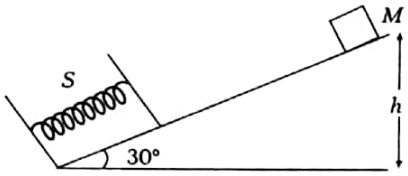
78. A car of mass m is accelerating on a level smooth road under the action of a single force F. The power delivered to the car is constant and equal to P. If the velocity of the car at an instant is v, then after travelling how much distance it becomes double?



(a) $\frac{7mv^3}{3P}$ (b) $\frac{4mv^3}{3P}$

(c) $\frac{mv^3}{P}$ (d) $\frac{18mv^3}{7P}$

79. An ideal massless spring S can be compressed 1 m by a force of 100 N in equilibrium. The same spring is placed at the bottom of a frictionless inclined plane inclined at 30° to the horizontal. A 10 kg block M is released from rest at the top of the incline and is brought to rest momentarily after compressing the spring by 2 m. If $g = 10\,\text{ms}^{-2}$, what is the speed of mass just before it touches the spring?



(a) $\sqrt{20}\ \text{ms}^{-1}$ (b) $\sqrt{30}\ \text{ms}^{-1}$

(c) $\sqrt{10}\ \text{ms}^{-1}$ (d) $\sqrt{40}\ \text{ms}^{-1}$

80. A pendulum of mass 1 kg and length $l = 1\,\text{m}$ is released from rest at angle $\theta = 60°$. The power delivered by all the forces acting on the bob at angle $\theta = 30°$ will be (Take, $g = 10\,\text{ms}^{-2}$).

(a) 13.5 W (b) 20.4 W

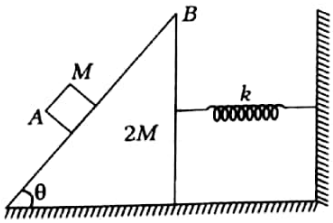
(c) 24.6 W (d) zero

81. A small block of mass m is kept on a rough inclined surface of inclination θ fixed in an elevator. The elevator goes up with a uniform velocity v and the block does not slide on the wedge. The work done by the force of friction on the block in a time t will be

(a) zero (b) $mgvt\cos^2\theta$

(c) $mgvt\sin^2\theta$ (d) $\frac{1}{2}mgvt\sin 2\theta$

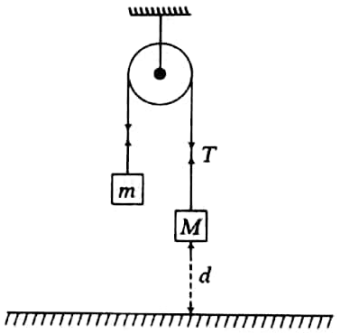
82. A block A of mass M rests on a wedge B of mass $2M$ and inclination θ. There is sufficient friction between A and B so that A does not slip on B. If there is no friction between B and ground, the compression in spring is



(a) $\frac{Mg\cos\theta}{k}$ (b) $\frac{Mg\cos\theta\sin\theta}{k}$

(c) $\frac{Mg\sin\theta}{k}$ (d) zero

83. The net work done by the tension in the figure when the bigger block of mass M touches the ground is



(a) $+Mgd$ (b) $-(M+m)gd$

(c) $-mgd$ (d) zero

B Medical entrance special format questions

Assertion and reason

Directions (Q. Nos. 1-16) *These questions consists of two statements each printed as Assertion and Reason. While answering these question you are required to choose any one of the following five responses*

(a) If both Assertion and Reason are correct and Reason is the correct explanation of Assertion.

(b) If both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.

(c) If Assertion is true but Reason is false.

(d) If Assertion is false but Reason is true.

1. Assertion Work done by constant force is equal to magnitude of force multiplied by displacement.
Reason Work done is scalar quantity. It may be positive, negative or zero.

2. Assertion Velocity of a block changes from $(2\hat{i} + 3\hat{j})\ \text{ms}^{-1}$ to $(-4\hat{i} - 6\hat{j})\ \text{ms}^{-1}$. Then, work done by all the forces during this interval of time is positive.
Reason Speed of block is increasing.

3. Assertion Spring force is a conservative force.
Reason Potential energy is defined only for conservative forces.

4. **Assertion** Consider a person of mass 80 kg who is climbing a ladder. In climbing up a vertical distance of 5 m, the contact force exerted by ladder on person's feet does 4000 J of work. [Consider $g = 10\ \text{ms}^{-2}$]

Reason Work done by a force F is defined as the ot product of force with the displacement of point of application of force.

5. **Assertion** The work done in bringing a body down from the top to the base along a frictionless inclined plane is the same as the work done in bringing it down from the vertical side.

Reason The gravitational force on the body along the inclined plane is the same as that along the vertical side.

6. **Assertion** If a force is applied on a rigid body. Body is in motion but point of application of force is stationary. Then, work done by the force is zero.

Reason If body is moving, then point of application of force should also move.

7. **Assertion** At stable equilibrium position of a body, kinetic energy cannot be zero. Because it is maximum.

Reason During oscillations of a body, potential energy is minimum at stable equilibrium position.

8. **Assertion** If work done by conservative force is negative, then potential energy associated with that force should increase.

Reason This is according to the relation

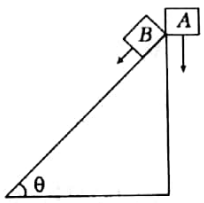
$$\Delta U = -W$$

Here ΔU is change in potential energy and W is work done by conservative force.

9. **Assertion** A body is moving along X-axis. Force $F = -2x^2$ is acting on it and work done by this force in moving the body from $x = -2$ to $x = +2$ is zero.

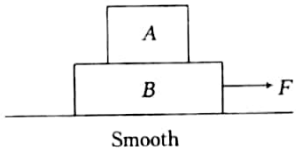
Reason From $x = -2$ to $x = 0$, work done is negative and from $x = 0$ to $x = +2$, work done is negative.

10. **Assertion** An object A is dropped from the top of an incline at $t = 0$, as shown. It will fall under gravity as indicated by the arrow. At the same time, *i.e.*, $t = 0$, another object B begins to slide down the frictionless incline. The two objects during their motion to the ground level will be travelling at equal speeds.



Reason Net force on both the objects during their motion is same.

11. **Assertion** If the surface between the blocks A and B is rough, then work done by friction on block B is always negative.



Reason Total work done by friction in both the blocks is always zero.

12. **Assertion** Force applied on a block moving in one dimension is producing a constant power, then the motion should be uniformly accelerated.

Reason This constant power multiplied with time is equal to the change in kinetic energy.

13. **Assertion** A block of mass m starts moving on a rough horizontal surface with a velocity v. It stops due to friction between the block and the surface after moving through a certain distance.

The surface is now tilted to an angle of 30° with the horizontal and the same block is made to go up on the surface with the same initial velocity v.
The decrease in the mechanical energy in the second situation is smaller than that in the first situation.

Reason The coefficient of friction between the block and the surface decreases with the increase in the angle of inclination.

14. **Assertion** Total work done by spring may be positive, negative or zero.

Reason Direction of spring force is always towards mean position.

15. **Assertion** In projectile motion, the rate of change in magnitude of potential energy of a particle first decreases and then increases during motion.

Reason In projectile motion, the rate of change in linear momentum of a particle remains constant during motion.

16. **Assertion** At any instant the magnitude of rate of change of potential energy of the projectile of mass 1 kg is numerically equal to magnitude of $a \cdot v$ (where a is acceleration due to gravity and v is velocity at that instant)

Reason The graph representing power delivered by the gravitational force acting on the projectile with time will be straight line with negative slope.

Match the Columns

1. Match the following columns.

	Column I		Column II
(A)	Work done by all forces	(p)	Negative of change in potential energy
(B)	Work done by conservative forces	(q)	Change in kinetic energy
(C)	Work done by external forces	(r)	Change in mechanical energy
		(s)	None

2. Match the appropriate sign of given potential energy is following with column II.

	Column I		Column II
(A)	Electrostatic potential energy	(p)	Positive
(B)	Gravitational potential energy	(q)	Negative
(C)	Elastic potential energy	(r)	zero
(D)	Magnetic potential energy	(s)	Not defined

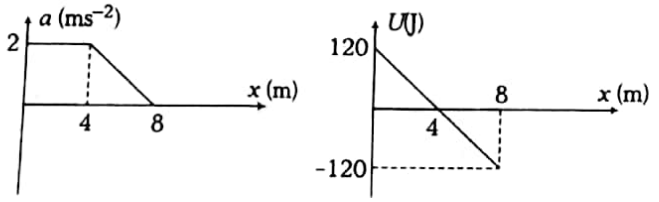
3. A force $F = kx$ (where k is a positive constant) is acting on a particle. Work done

	Column I		Column II
(A)	In displacing the body from $x = 2$ to $x = 4$	(p)	Negative
(B)	In displacing the body from $x = -4$ to $x = -2$	(q)	Positive
(C)	In displacing the body from $x = -2$ to $x = +2$	(r)	zero

4. A body is moved along a straight line by a machine delivering a power proportional to time ($P \propto t$). Then, match the following.

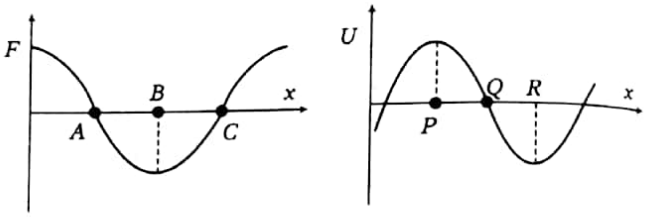
	Column I		Column II
(A)	Velocity is proportional to	(p)	t
(B)	Displacement is proportional to	(q)	t^2
(C)	Work done is proportional to	(r)	t^3

5. Acceleration *versus* x and potential energy *versus* x graph of a particle moving along X-axis is as shown in figure . Mass of the particle is 1 kg and velocity at $x = 0$ is 4 ms^{-1}. Match the following at $x = 8$ m.



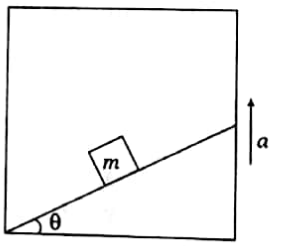
	Column I		Column II
(A)	Final kinetic energy	(p)	120 J
(B)	Work done by conservative forces	(q)	240 J
(C)	Total work done	(r)	−112 J
(D)	Work done by external forces	(s)	128 J

6. F-x and corresponding U-x graph are as shown in figures. Three points A, B and C in F-x graph may be corresponding to P, Q and R in the U-x graph. Match the following.



	Column I		Column II
(A)	A	(p)	P
(B)	B	(q)	Q
(C)	C	(r)	R
		(s)	None

7. A block of mass m is stationary with respect to a rough wedge as shown in figure. Starting from rest in time t, ($m = 1$ kg, $\theta = 30°$, $a = 2\,ms^{-2}$, $t = 4$ s) work done on block



	Column I		Column II
(A)	By gravity	(p)	144 J
(B)	By normal reaction	(q)	32 J
(C)	By friction	(r)	56 J
(D)	By all the forces	(s)	48 J
		(t)	None

Medical entrances' gallery

(Collection of questions asked in NEET & Various Medical Entrance Exams)

1. A body of mass 1 kg begins to move under the action of a time dependent force $\mathbf{F} = (2t\,\hat{i} + 3t^2\hat{j})$ N, where $\hat{i}$ and $\hat{j}$ are unit vectors along X and Y-axis. What power will be developed by the force at the time (t)? **[NEET 2016]**

(a) $(2t^2 + 4t^4)$ W (b) $(2t^3 + 3t^4)$ W
(c) $(2t^3 + 3t^5)$ W (d) $(2t + 3t^3)$ W

2. A particle of mass 10 g moves along a circle of radius 6.4 cm with a constant tangential acceleration. What is the magnitude of this acceleration, if the kinetic energy of the particle becomes equal to 8×10^{-4} J by the end of the second revolution after the beginning of the motion? **[NEET 2016]**

(a) 0.15 ms^{-2} (b) 0.18 ms^{-2} (c) 0.2 ms^{-2} (d) 0.1 ms^{-2}

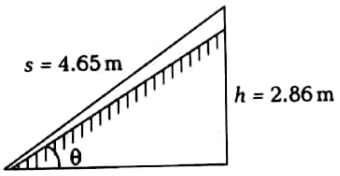
3. A block of mass 10 kg, moving in x-direction with a constant speed of 10 ms^{-1}, is subjected to a retarding force $F = 0.1\,x\,Jm^{-1}$ during its travel from $x = 20$ m to 30 m. Its final KE will be **[CBSE AIPMT 2015]**

(a) 475 J (b) 450 J (c) 275 J (d) 250 J

4. A particle of mass m is driven by a machine that delivers a constant power k watts. If the particle starts from rest, the force on the particle at time t is **[CBSE AIPMT 2015]**

(a) $\sqrt{\frac{mk}{2}}\,t^{-1/2}$ (b) $\sqrt{mk}\,t^{-1/2}$ (c) $\sqrt{2mk}t^{-1/2}$ (d) $\frac{1}{2}\sqrt{mk}t^{-1/2}$

5. A block of mass $m = 11.7$ kg is to be pushed a distance of $s = 4.65$ m along an incline and raised to a distance of $h = 2.86$ m. Assuming frictionless surface. Calculate the work done in applying a force parallel to the incline to push the block up at a constant speed. (Take, $g = 9.8\ ms^{-2}$) **[UK PMT 2015]**



(a) 328 J (b) 656 J (c) 164 J (d) 530 J

6. A force $F = (10 + 0.5x)$ acts on a particle in the x-direction. What would be the work done by this force during a displacement from $x = 0$ to $x = 2$m (F is in newton and x in metre) **[AIIMS 2015, UK PMT 2015]**

(a) 31.5 J (b) 63 J (c) 21 J (d) 42 J

7. An elevator weighing 500 kg is to be lifted up at a constant velocity of 0.20 ms^{-1}. What would be the minimum horse power of the motor to be used? **[CGPMT 2015, UK PMT 2015]**

(a) 10.30 hp (b) 5.15 hp (c) 2.62 hp (d) 1.31 hp

8. The distance of closest approach of an α-particle fired at nucleus with momentum p is d. The distance of closest approach when the α-particle is fired at same nucleus with momentum $3p$ will be **[EAMCET 2015]**

(a) $3d$ (b) $\frac{d}{3}$ (c) $9d$ (d) $\frac{d}{9}$

9. Two bodies of different masses are moving with same kinetic energy. Then, the ratio of their momenta is equal to the ratio of their **[Kerala CEE 2015]**

(a) masses (b) square of masses
(c) square root of masses (d) cube root of masses

10. Two bodies of masses 1 kg and 2 kg moving with same velocities are stopped by the same force. Then, the ratio of their stopping distances is **[Kerala CEE 2015]**

(a) 1 : 2 (b) 2 : 1 (c) $\sqrt{2}:1$ (d) $1:\sqrt{2}$

11. One moving electron when comes closer to other stationary electron, then its kinetic energy and potential energy respectively ..... and ..... . **[Guj. CET 2015]**

(a) increases, increases (b) increases, decreases
(c) decreases, increases (d) decreases, decreases

12. A bob of mass m accelerates uniformly from rest to v_1 in time t_1. As a function of t, the instantaneous power delivered to the body is **[Manipal 2015]**

(a) $\frac{mvit}{t_1}$ (b) $\frac{mv_1\,t}{t_1}$ (c) $\frac{mv_1\,t^2}{t_1}$ (d) $\frac{mv_1^2\,t}{t_1^2}$

13. A uniform chain of length l is placed on a smooth horizontal table, such that half of its length hangs over one edge. It is released from rest, the velocity with which it leaves the table is **[EAMCET 2015]**

(a) $\sqrt{\frac{3gl}{4}}$ (b) $\sqrt{\frac{3gl}{2}}$ (c) $\sqrt{\frac{2gl}{3}}$ (d) $\sqrt{\frac{gl}{3}}$

14. The kinetic energy of a body of mass 4 kg and momentum 6 N-s will be **[KCET 2015]**

(a) 4.5 J (b) 2.5 J (c) 5.5 J (d) 3.5 J

15. A body of mass $m = 3.90$ kg slides on a horizontal frictionless table with a speed of $v = 1.20\ ms^{-1}$. It is brought to rest in compressing a spring in its path. How much does spring is compressed, if its force constant k is 135 Nm^{-1}? **[UK PMT 2015]**

(a) 0.204 m (b) 0.408 m (c) 0.804 m (d) 4.04 m

16. A string of length L and force constant k is stretched to obtain extension l. It is further stretched to obtain extension l_1. The work done in second stretching is **[MHT CET 2014]**

(a) $\frac{1}{2}kl_1(2l + l_1)$ (b) $\frac{1}{2}kl_1^2$
(c) $\frac{1}{2}k(l^2 + l_1^2)$ (d) $\frac{1}{2}k(l_1^2 - l^2)$

17. A body is initially at rest. It undergoes one dimensional motion with constant acceleration. The power delivered to it at time t is proportional to **[Uttarakhand PMT 2014]**

(a) $t^{1/2}$ (b) t (c) $t^{3/2}$ (d) t^2

18. If two person A and B take 2 s and 4 s, respectively to lift an object to the same height h, then the ratio of their powers is **[Kerala CEE 2014]**

(a) 1 : 2 (b) 1 : 1
(c) 2 : 1 (d) 1 : 3

19. If, a machine gun fires n bullets per second each with kinetic energy K, then the power of the machine gun is **[Kerala CEE 2014]**

(a) nK^2 (b) $\frac{K}{n}$ (c) n^2K (d) nK

20. A uniform force of $(3\hat{i} + \hat{j})$ N acts on a particle of mass 2 kg. Hence the particle is displaced from position $(2\hat{i} + \hat{k})$ m to position $(4\hat{i} + 3\hat{j} - \hat{k})$ m. The work done by the force on the particle is **[NEET 2013]**

(a) 9 J (b) 6 J (c) 13 J (d) 15 J

21. The power (P) of an engine lifting a mass of 100 kg upto a height of 10 m in 1 min is **[J&K CET 2013]**

(a) $P = 163.3$ W (b) $P = 9800$ W
(c) $P = 10000$ W (d) $P = 5000$ W

22. A body of mass 300 kg is moved through 10 m along a smooth inclined plane of angle 30°. The work done in moving in joules is (take, $g = 9.8\ \text{ms}^{-2}$) **[EAM CET 2013]**

(a) 4900 (b) 9800
(c) 14700 (d) 2450

23. Force constants of two wires A and B of the same material are k and $2k$ respectively. If the two wires are stretched equally, then the ratio of work done in stretching $\left(\frac{W_A}{W_B}\right)$ is **[EAMCET 2013]**

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{3}{2}$ (d) $\frac{1}{4}$

24. A truck accelerates from speed v to $2v$. Work done in during this is **[KCET 2013]**

(a) three times as the work done in accelerating it from rest to v
(b) same as the work done in accelerating it from rest to v
(c) four times as the work done in accelerating it from rest to v
(d) less than the work done in accelerating it from rest to v

25. A block of 200 g mass is dropped from a height of 2 m on to a spring and compress the spring to a distance of 50 cm. The force constant of the spring is **[Kerala CET 2013]**

(a) 20 Nm^{-1} (b) 40 Nm^{-1}
(c) 30 Nm^{-1} (d) 60 Nm^{-1}

26. Work done in time t on a body of mass m which is accelerated from rest to speed v in time t_1 as a function of time t is given as **(BCECE Mains 2012)**

(a) $\frac{1}{2}m\frac{v^2}{t_1^2}t^2$ (b) $\frac{1}{2}\left(\frac{mv}{t_1}\right)^2 t^2$ (c) $m\frac{v}{t_1}t^2$ (d) $\frac{1}{2}m\frac{v}{t}t^2$

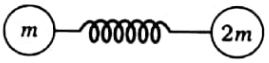
27. The slope of kinetic energy and displacement curve for a particle in motion will be **[BCECE Mains 2012]**

(a) equal to the acceleration of the particle
(b) directly proportional to the acceleration of the particle
(c) inversely proportional to the acceleration of the particle
(d) None of the above

28. The potential energy of a particle in a force field is $U = \frac{A}{r^2} - \frac{B}{r}$, where A and B are positive constants and r is the distance of particle from the centre of the field. For stable equilibrium, the distance of the particle is **[CBSE AIPMT 2012]**

(a) $B/2A$ (b) $2A/B$ (c) A/B (d) B/A

29. Two masses m and $2m$ are attached to two ends of an ideal spring as shown in figure. When the spring is in the compressed state, the energy of the spring is 60 J, if the spring is released, then at its natural length **[BHU 2012]**



(a) energy of smaller body will be 20 J
(b) energy of smaller body will be 40 J
(c) energy of smaller body will be 10 J
(d) energy of both the bodies will be same

30. If a pump pumps 200 kg of water to a height of 50 m in 10 s, then the power of pump will be **[BHU 2012]**

(a) 4×10^3 W (b) 10×10^3 W
(c) 20×10^3 W (d) None of these

31. If a body of mass M is moved from rest along a straight line by an engine which is delivering a constant power P, then the velocity of the body after time t will be **[BHU 2012]**

(a) $\frac{2Pt}{M}$ (b) $\sqrt{\frac{2Pt}{M}}$ (c) $\frac{Pt}{2M}$ (d) $\sqrt{\frac{Pt}{2M}}$

32. The particle of mass 50 kg is at rest. The work done to accelerate it by 20 ms^{-1} in 10 s is **[AIIMS 2012]**

(a) 10^3 J (b) 10^4 J (c) 2×10^3 J (d) 4×10^4 J

33. Under the action of a force $F = Cx$, the position of a body changes from 0 to x. The work done is **[AFMC 2012]**

(a) $\frac{1}{2}Cx^2$ (b) Cx^2 (c) Cx (d) $\frac{1}{2}Cx$

Answers

Check point 6.1

1. (a)	**2.** (b)	**3.** (b)	**4.** (d)	**5.** (d)	**6.** (c)	**7.** (d)	**8.** (a)	**9.** (a)	**10.** (b)
11. (b)	**12.** (a)	**13.** (b)	**14.** (d)	**15.** (c)	**16.** (b)	**17.** (c)	**18.** (d)	**19.** (b)	**20.** (d)

Check point 6.2

1. (b)	**2.** (a)	**3.** (b)	**4.** (d)	**5.** (c)	**6.** (c)	**7.** (d)	**8.** (c)	**9.** (a)	**10.** (a)
11. (b)	**12.** (d)	**13.** (b)	**14.** (c)	**15.** (b)	**16.** (a)	**17.** (b)	**18.** (a)	**19.** (c)	**20.** (c)

Check point 6.3

1. (d)	**2.** (c)	**3.** (b)	**4.** (c)	**5.** (c)	**6.** (a)	**7.** (d)	**8.** (a)	**9.** (c)	**10.** (d)

A. Taking it together

1. (b)	**2.** (d)	**3.** (c)	**4.** (a)	**5.** (c)	**6.** (b)	**7.** (c)	**8.** (d)	**9.** (a)	**10.** (c)
11. (a)	**12.** (c)	**13.** (b)	**14.** (d)	**15.** (c)	**16.** (a)	**17.** (a)	**18.** (c)	**19.** (c)	**20.** (c)
21. (a)	**22.** (d)	**23.** (c)	**24.** (c)	**25.** (b)	**26.** (b)	**27.** (b)	**28.** (a)	**29.** (c)	**30.** (d)
31. (c)	**32.** (c)	**33.** (d)	**34.** (b)	**35.** (d)	**36.** (c)	**37.** (b)	**38.** (b)	**39.** (a)	**40.** (a)
41. (b)	**42.** (b)	**43.** (a)	**44.** (c)	**45.** (d)	**46.** (c)	**47.** (a)	**48.** (b)	**49.** (a)	**50.** (b)
51. (b)	**52.** (b)	**53.** (c)	**54.** (c)	**55.** (b)	**56.** (b)	**57.** (b)	**58.** (d)	**59.** (c)	**60.** (c)
61. (a)	**62.** (b)	**63.** (a)	**64.** (c)	**65.** (b)	**66.** (a)	**67.** (a)	**68.** (c)	**69.** (c)	**70.** (c)
71. (c)	**72.** (b)	**73.** (a)	**74.** (c)	**75.** (d)	**76.** (b)	**77.** (a)	**78.** (a)	**79.** (a)	**80.** (a)
81. (c)	**82.** (d)	**83.** (d)							

B. Medical entrance special format questions

Assertion and reason

1. (d)	**2.** (a)	**3.** (a)	**4.** (d)	**5.** (b)	**6.** (c)	**7.** (d)	**8.** (a)	**9.** (d)	**10.** (c)
11. (c)	**12.** (d)	**13.** (c)	**14.** (b)	**15.** (b)	**16.** (c)				

Match the columns

1. (A $\rightarrow$ q, B $\rightarrow$ p, C $\rightarrow$ r)
2. (A $\rightarrow$ pqr, B $\rightarrow$ qr, C $\rightarrow$ pr, D $\rightarrow$ pqr)
3. (A $\rightarrow$ q, B $\rightarrow$ p, C $\rightarrow$ r)
4. (A $\rightarrow$ p, B $\rightarrow$ q, C $\rightarrow$ q)
5. (A $\rightarrow$ s, B $\rightarrow$ q, C $\rightarrow$ p, D $\rightarrow$ r)
6. (A $\rightarrow$ r, B $\rightarrow$ s, C $\rightarrow$ p)
7. (A $\rightarrow$ t, B $\rightarrow$ p, C $\rightarrow$ s, D $\rightarrow$ q)

C. Medical entrances' gallery

1. (c)	**2.** (d)	**3.** (a)	**4.** (a)	**5.** (a)	**6.** (c)	**7.** (d)	**8.** (d)	**9.** (c)	**10.** (a)
11. (c)	**12.** (d)	**13.** (a)	**14.** (a)	**15.** (a)	**16.** (a)	**17.** (b)	**18.** (c)	**19.** (d)	**20.** (a)
21. (a)	**22.** (d)	**23.** (b)	**24.** (a)	**25.** (b)	**26.** (a)	**27.** (b)	**28.** (b)	**29.** (b)	**30.** (b)
31. (b)	**32.** (b)	**33.** (a)							

Hints and explanations

Check point 6.1

1. (*a*) Given, $F = 20$ kg-wt $= 20 \times 9.8$ N and $s = 20$ m

$\therefore$ Work done $= F\, s \cos\theta$

$= 20 \times 9.8 \times 20 \times \cos 60° = 1960$ J

2. (*b*) As, equation of motion is

$$v = u + at \Rightarrow 20 = 0 + a \times 10$$

$$20 = a \times 10 \Rightarrow a = 2\ \text{m/s}^2$$

Now, distance $s = ut + \frac{1}{2}at^2$

$$\Rightarrow \quad s = 0 + \frac{1}{2} \times 2 \times 10 \times 10$$

or $\quad s = 100$ m

$$W = F \cdot s = mas = 50 \times 2 \times 100 = 10^4\ \text{J}$$

3. (*b*) Frictional force acting on the box,

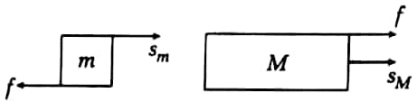
$$F = \mu mg = 0.25 \times 20 \times 9.8 = 49\ \text{N}$$

This force must be same as applied force F so that the box moves with constant speed.

Work done by the applied force, F

$$W = Fs = 49 \times 2 = 98\ \text{J}$$

4. (*d*)



$$s_m \neq s_M,\ s_m > s_M$$

Therefore, net work done by the friction cannot be zero.

5. (*d*) As, work done $(W) = \mathbf{F} \cdot \mathbf{s} = Fs\cos\theta$

When a man pushes against a wall but fails to move it, then displacement of wall, $s = 0$

$\therefore$ Work done, $W = F \times 0 \times \cos\theta = 0$

Therefore, man does no work at all.

6. (*c*) Work done by kinetic friction may be positive or negative.

7. (*d*) Given, Force $F = (3\hat{i} + 4\hat{j})$ N and

Displacement, $s = (3\hat{i} + 4\hat{j})$ m

$\because$ Work done, $W = F \cdot s = (3\hat{i} + 4\hat{j}) \cdot (3\hat{i} + 4\hat{j})$

$$W = 9 + 16$$

$$\Rightarrow \quad W = 25\ \text{J}$$

8. (*a*) Work done $W = F \cdot d$

$$= (2\hat{i} - \hat{j} + 4\hat{k})(3\hat{i} + 2\hat{j} - \hat{k})$$

$$W = 6 - 2 - 4$$

$$W = 0$$

9. (*a*) Particle moves from $P(1, 2, 3)$ to $Q(2, 1, 4)$

$\therefore$ Distance $PQ = (2\hat{i} + \hat{j} + 4\hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$

$$d = PQ = (\hat{i} - \hat{j} + \hat{k})$$

Force $F = (2\hat{i} + \hat{j} + \hat{k})$

Work done by the force

$$W = F \cdot d = (2\hat{i} + \hat{j} + \hat{k}) \cdot (\hat{i} - \hat{j} + \hat{k}) = 2 - 1 + 1 = 2\ \text{J}$$

10. (*b*) $W = \mathbf{F} \cdot \mathbf{s} = \mathbf{F} \cdot (\mathbf{s}_2 - \mathbf{s}_1)$

$$= (\hat{i} + 2\hat{j} + 3\hat{k}) \cdot [(\hat{i} - \hat{j} + 2\hat{k}) - (\hat{i} + \hat{j} + \hat{k})]$$

$$= (\hat{i} + 2\hat{j} + 3\hat{k}) \cdot (-2\hat{j} + \hat{k}) = -4 + 3 = -1\text{J}$$

11. (*b*) $W = (y$- component of force$) \times$ (displacement along Y-axis)

$$= 15 \times 10 = 150\ \text{J}$$

12. (*a*) Given, $\quad \mathbf{F} = 2\hat{i} + 3\hat{j}$

displacement $d\mathbf{s} = dx\hat{i} + dy\hat{j} + dz\hat{k}$

work done, $\quad W = \int \mathbf{F} \cdot d\mathbf{s} = \int (2dx + 3dy)$

Also $\quad 3y + kx = 5 \Rightarrow \frac{3dy}{dx} + k = 0$

$$\Rightarrow \quad 3dy = -kdx \Rightarrow W = \int (2dx - kdx) = 0$$

$$\Rightarrow \quad 2x = kx \Rightarrow k = 2$$

Alternately

$$m_1 m_2 = -1$$

$$\frac{3}{2}\left(-\frac{k}{3}\right) = -1 \Rightarrow k = 2$$

13. (*b*) Work done, $W = \int \mathbf{F} \cdot d\mathbf{x} = \int_0^{x_1} Cx \cdot dx = \frac{Cx_1^2}{2}$

14. (*d*) $W = \int_{x_1}^{x_2} F dx = \int_0^5 \mathbf{F} \cdot d\mathbf{x} = \int_0^5 (10 - 2x + 3x^2)\, dx$

$$= [10x - x^2 + x^3]_0^5 = 150 \text{ units}$$

15. (*c*) $W = \int_{x_1}^{x_2} F dx = \int_0^5 \mathbf{F} \cdot d\mathbf{x} = \int_0^5 (8 - 4x + 3x^2)\, dx$

$$= [8x - 2x^2 + x^3]_0^5 = 115\ \text{J}$$

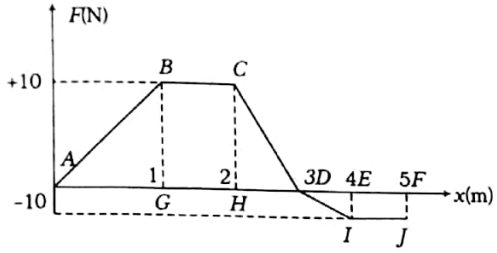
16. (*b*) $W = \int_{-a}^{+a} F dy = \left[\frac{Ay^3}{3} + \frac{By^2}{2} + Cy\right]_{-a}^{+a} = \frac{2Aa^3}{3} + 2Ca$

17. (*c*) $W =$ area under F-x graph

Area of trapezium $= \frac{1}{2} \times (4 + 2) \times 5 = 15\text{J}$

18. (*d*) Work done = Area under Fx graph

= area of ΔABG + area of rectangle of $BGHC$ + area of ΔCDH + area of ΔDEI + area of square $EFJI$



$$= \left(\frac{1}{2}\times 10\times 1\right) + (10\times 1) + \left(\frac{1}{2}\times 10\times 1\right) + \left(\frac{1}{2}(-10)\times 1\right) + (-10\times 1)$$

$W = 5\,\text{J}$

19. (*b*) $W_1 = \frac{1}{2}k(5)^2\times 10^{-4} \Rightarrow W_2 = \frac{1}{2}k(15)^2\times 10^{-4}$

where W_1 and W_2 are the work done in extending spring to 5 cm and 15 cm respectively.

$W_{1\to 2} = W_2 - W_1$

$$= \frac{1}{2}k[(15)^2 - (5)^2]\times 10^{-4} = \frac{1}{2}\times 800(200)\times 10^{-4} = 8\,\text{J}$$

20. (*d*) $k = 10\ \text{Nmm}^{-1}$

$$W_{1\to 2} = \frac{1}{2}kx^2 - 0 = \frac{1}{2}\times 10\times 10^3\,(40\times 10^{-3})^2 = 8\,\text{J}$$

Check point 6.2

1. (*b*) $F \propto \frac{1}{v}$

$F = \frac{\lambda}{v}$, λ : constant

$$ma = m\frac{dv}{dt} = \frac{\lambda}{v} \Rightarrow m\int_0^v v\,dv = \lambda\int_0^t dt$$

$$m\left|\frac{v^2}{2}\right|_0^v = \lambda\,|t|_0^t \Rightarrow \frac{1}{2}mv^2 = K = \lambda t$$

$K \propto t$

2. (*a*) It is given that, $K_f = 2K_i$

or $\frac{1}{2}m(v+1)^2 = \left(\frac{1}{2}mv^2\right)(2)$ or $v + 1 = \sqrt{2}v$

$$v = \frac{1}{\sqrt{2}-1} = \sqrt{2} + 1\,\text{ms}^{-1}$$

3. (*b*) According to the question, $\frac{1}{2}mv_1^2 = \frac{1}{2}\left[\frac{1}{2}\times\frac{m}{2}\times v_2^2\right]$

where, v_1 = speed of man and v_2 = speed of boy

Now, $\frac{1}{2}m(v_1+1)^2 = \frac{1}{2}\times\frac{m}{2}\times v_2^2$

Solving these two equations we get, $v_1 = (\sqrt{2}+1)\ \text{ms}^{-1}$ and $v_2 = 2(\sqrt{2}+1)\ \text{ms}^{-1}$

4. (*d*) Kinetic energy $K = \frac{p^2}{2m}$

$\Rightarrow$ $K_1 = \frac{p^2}{2m_1}$ and $K_2 = \frac{p^2}{2m_2}$

$\therefore$ $\frac{K_1}{K_2} = \frac{m_2}{m_1}$

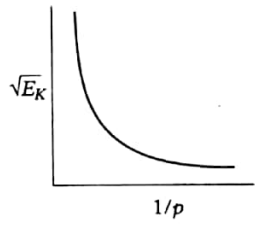
5. (*c*) $K = \frac{p^2}{2m}$, $K' = \frac{(1.5p)^2}{2m} = 2.25\frac{p^2}{2m} = 2.25\,\text{K}$

$\therefore$ Increase in kinetic energy is 125%.

6. (*c*) $p = \sqrt{2mE_K}$

It is clear that $p \propto \sqrt{E_K}$

So the graph between p and $\sqrt{E_K}$ will be straight line but graph between $1/p$ and $\sqrt{E_K}$ will be hyperbola.



7. (*d*) $K_2 - K_1 = W = Fs$

$K_2 - 0 = Fs$

K is independent of m. (Force is constant)

8. (*c*) Speed of the body, $v = \frac{dx}{dt} = \frac{d}{dt}\left(\frac{t^3}{3}\right) = t^2$

At $t = 0, v = 0$,

At $t = 2\,\text{s}, v = 4\ \text{ms}^{-1}$

From work energy theorem,

W = change in kinetic energy

$= K_f - K_i$

$$= \frac{1}{2}m(v_f^2 - v_i^2) = \frac{1}{2}\times 2\times(16-0) = 16\,\text{J}$$

9. (*a*) Change in kinetic energy = work done

= Area under F-x graph

$\therefore$ $\frac{1}{2}\times 5\times v^2 = 10\times 25 + \frac{1}{2}\times 25\times 10 = 375$

$\therefore$ $v = 12.2\ \text{ms}^{-1}$

10. (*a*) Applying work-energy theorem we get, $K_f - K_i = W$

$\therefore$ $K_f = K_i + W$

$$= \frac{1}{2}\times 20\times(10)^2 + \int_{20}^{30}(-0.1x)dx$$

$$= 1000 - \frac{0.1}{2}(x^2)_{20}^{30} = 1000 - 25 = 975\,\text{J}$$

11. (*b*) Work done by all forces = change in kinetic energy

$$= \frac{1}{2}m(v_f^2 - v_i^2) = \frac{1}{2}\times 2(0-400) = -400\,\text{J}$$

12. (*d*) Work done on the particle, $W = \Delta KE = \frac{1}{2}m(v_f^2 - v_i^2)$

$$= \frac{1}{2} \times 0.01[(64 + 400) - (16 + 256)]$$

$$= 0.96 \text{ J}$$

13. (*b*) Given, force, $F = (4\hat{i} + 4\hat{j})\text{N}$

$$\mathbf{r}_1 = 0\hat{i} + 0\hat{j}$$

$$\mathbf{r}_2 = \hat{i} + \hat{j}$$

$$\mathbf{r}_2 - \mathbf{r}_1 = \hat{i} + \hat{j}$$

Initial speed, $v_1 = 2 \text{ ms}^{-1}$

From work energy theorem we have

$$\Delta W = \Delta K$$

$$\Rightarrow \quad \mathbf{F} \cdot \Delta\mathbf{r} = \frac{1}{2}m(v_2^2 - v_1^2)$$

$$\Rightarrow \quad (4\hat{i} + 4\hat{j}) \cdot (\hat{i} + \hat{j}) = \frac{1}{2} \times 1(v_2^2 - 4)$$

$$\Rightarrow \quad 4 + 4 = \frac{1}{2}(v_2^2 - 4)$$

$$\Rightarrow \quad v_2^2 = 20 \Rightarrow v_2 = 4.5 \text{ ms}^{-1}$$

14. (*c*) Forces acting on the body are

$$W = \text{weight} = 5 \times 10 = 50 \text{ N}$$

External force, $F = 170$ N

Net upward force acting on the body

$$F_{net} = 170 - 50 = 120 \text{ N}$$

Now applying work energy theorem

we get $\quad (F_{net})\, h = \frac{1}{2}mv^2$

$$\Rightarrow \quad 120 \times 10 = \frac{1}{2} \times 5 \times v^2$$

$$\Rightarrow \quad v^2 = \frac{2 \times 12 \times 100}{5} \Rightarrow v = 22 \text{ ms}^{-1}$$

15. (*b*) Decrease in kinetic energy = increase in elastic potential energy.

$$\therefore \quad \frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

or $\quad x = \sqrt{\frac{m}{k}} \cdot v = \sqrt{\frac{0.1}{1000}} \times 10 = 0.1\text{m}$

16. (*a*) Decrease in gravitational potential energy

= increase in elastic potential energy

or $\quad mg(h + x) = \frac{1}{2}kx^2$ or $2 \times 9.8(0.4 + x) = \frac{1}{2} \times 1960 \times x^2$

Solving this equation we get, $x = 0.1$m or 10 cm.

17. (*b*) Potential energy is only associated with conservative forces.

Force, $F = -\left(\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} + \frac{\partial U}{\partial z}\hat{k}\right)$

where, F = Conservative force

$\left(\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} + \frac{\partial U}{\partial z}\hat{k}\right)$ = Partial derivative of potential energy w.r.t. x, y and z respectively.

18. (*a*) The potential energy of a system increases if work is done by the system against a conservative force.

$$-\Delta U = W_{\text{conservative force}}$$

19. (*c*) Gain in KE = Loss in PE

$$\Rightarrow \quad \frac{1}{2}mv^2 = (0.1)(mgh)$$

$$\therefore \quad v = \sqrt{0.2gh} = \sqrt{0.2 \times 10 \times 2} = 2 \text{ ms}^{-1}$$

20. (*c*) From conservation of mechanical energy, we have

$$\therefore \quad \frac{3}{4}mgh + mgh' = mgh \quad \text{or} \quad h' = \frac{h}{4} \Rightarrow \frac{1}{2}mv_i^2 = mgh$$

Check point 6.3

2. (*c*) $P = \frac{\text{Work done by the engine}}{\text{time taken}} = \frac{W}{t}$

$$\therefore \quad t = \frac{W}{P} = \frac{mgh}{P} = \frac{200 \times 10 \times 40}{10 \times 10^3} = 8 \text{ s}$$

3. (*b*) Given, $P_{output} = 10$ kW

Now, $P_{input} = 2 \times 10^3 \text{ calg}^{-1} \times 1 \text{ g s}^{-1}$

$$= 2 \times 10^3 \text{ cal s}^{-1} = 2 \times 10^3 \times 4.2 \text{ Js}^{-1} = 8.4 \text{ kW}$$

As, $P_{output} > P_{input}$, hence the claim given in question is never possible.

4. (*c*) Power given to turbine $= \frac{mgh}{t}$

$$P_{in} = \left(\frac{m}{t}\right) \times g \times h \Rightarrow P_{in} = 15 \times 10 \times 60$$

$$\Rightarrow \quad P_{in} = 9000 \text{ W} \quad \Rightarrow P_{in} = 9 \text{ kW}$$

As efficiency of turbine is 90% therefore power generated = 90 % of 9 kW

$$P_{out} = 9 \times \frac{90}{100} \quad \Rightarrow P_{out} = 8.1 \text{ kW.}$$

5. (*c*) $P = \frac{mgh}{t} \Rightarrow \frac{P_1}{P_2} = \frac{m_1}{m_2} \times \frac{t_2}{t_1}$ (As h = constant)

$$\therefore \quad \frac{P_1}{P_2} = \frac{60}{50} \times \frac{11}{12} = \frac{11}{10}$$

6. (*a*) Power $= \frac{\text{Workdone}}{\text{Time}} = \frac{\text{Pressure} \times \text{change in volume}}{\text{Time}}$

$$= \frac{20000 \times 1 \times 10^{-6}}{1} = 2 \times 10^{-2} = 0.02 \text{ W}$$

7. (*d*) Power is defined as the rate of change of energy in a system or the time rate of doing work.

$$\Rightarrow \quad P = \frac{dE}{dt} = \frac{dW}{dt}$$

Also, work = force × displacement = $F \times d$

Since, the displacement is zero,

So, $\quad P = \frac{d}{dt}(F \times d) = \frac{d}{dt} \times 0 = 0$

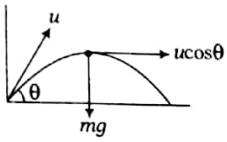
8. (a) $P = \frac{\mathbf{F}\cdot\mathbf{s}}{t} = \frac{(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})\cdot(3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 5\hat{\mathbf{k}})}{4} = \frac{38}{4} = 9.5\,\text{W}$

9. (c) $P = \mathbf{F}\cdot\mathbf{v} = (10\hat{\mathbf{i}} + 10\hat{\mathbf{j}} + 20\hat{\mathbf{k}})\cdot(5\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 6\hat{\mathbf{k}})$

$= 50 - 30 + 120 = 140\ \text{Js}^{-1}$

10. (d) $P = \mathbf{F}\cdot\mathbf{v}$

$= mg(\mu\cos\theta)\cos 90° = 0$



A Taking it together

1. (b) When electron and proton are moving under influence of their mutual forces, the magnetic forces will be perpendicular to their motion hence no work is done by these forces.

2. (d) When the man is squatting on the ground he is tilted somewhat, hence he also has to balance frictional force besides his weight in this case.

R = reactional force = friction + $mg \Rightarrow R > mg$

When the man gets straight up in that case friction ≈ 0

$\Rightarrow$ Reactional force $\approx mg$

3. (c) As the body is falling freely under gravity, the potential energy decreases and kinetic energy increases but total mechanical energy (PE + KE) of the body and earth system will be constant as external force on the system is zero.

4. (a) Given force is a constant force and work done by a constant force is always path independent.

$\therefore \quad W_1 = W_2$

5. (c) $W_F + W_g = \Delta K = 0; W_F = -W_g = mgh$

6. (b) Centre of mass of the rod moves a height, $h = \frac{l}{2}$.

$W = mgh = mgl/2$

7. (c) $F = kx$ or k = slope of F-x graph (F along Y-axis)

Here, F is along X-axis.

So, $k = \frac{1}{10} = 0.1$ kgf/cm

8. (d) $K = \frac{p^2}{2m}$ or $K \propto p^2$

When P is doubled, kinetic energy will become four times.

9. (a) Kinetic energy, $E = \frac{1}{2}mv^2$

$\therefore \quad \frac{dE}{dv} = mv$ or $p = \frac{dE}{dv}$ (as $mv = p$)

10. (c) $P = \sqrt{2Km}$ or $P \propto \sqrt{m}$

$\therefore \frac{P_1}{P_2} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

11. (a) When a body is moved upward, work done by gravity is negative but potential energy increases.

12. (c) Here, work is done by the frictional force on the cycle and is equal to $-200 \times 10 = -2000$ J. As the road is not moving, hence, work done by the cycle on the road = zero.

13. (b) Given, power = constant

We know that power (P)

$$P = \frac{dW}{dt} = \frac{\mathbf{F}\cdot d\mathbf{s}}{dt} = \frac{F\,ds}{dt}$$

($\because$ body is moving unidirectionally)

Hence, $F\cdot ds = Fds\cos 0°$

$$P = \frac{Fds}{dt} = \text{constant} \quad (\because P = \text{constant by question})$$

Now, writing dimensions

$[F][v] = \text{constant}$

$\Rightarrow \quad [MLT^{-2}][LT^{-1}] = \text{constant}$

$\Rightarrow \quad L^2T^{-3} = \text{constant}$ ($\because$ mass is constant)

$\Rightarrow \quad L \propto T^{3/2}$

$\Rightarrow$ Displacement $(d) \propto t^{3/2}$

14. (d) When the earth is closest to the sun, speed of the earth is maximum, hence, KE is maximum. When the earth is farthest from the sun speed is minimum hence, KE is minimum but never zero and negative.

This variation is correctly represented by option(d).

15. (c) When a pendulum oscillates in air, it will lose energy continuously in overcoming resistance due to air. Therefore, total mechanical energy of the pendulum decreases continuously with time.

The variation is correctly represented by curve (c).

16. (a) Given, mass = m = 5 kg, Radius = 1 m = R

Revolution per minute $\omega = 300$ rev min^{-1} = 10π rad s^{-1}

$\Rightarrow$ linear speed $= v = \omega R$ ...(1)

$= 10\pi\ \text{ms}^{-1}$

$$\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}\times 5\times(10\pi)^2$$

$$= 100\pi^2 \times 5 \times \frac{1}{2} = 250\,\pi^2\ \text{J}$$

17. (a) $\mathbf{r}_1 = 3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 6\hat{\mathbf{k}}, \hat{\mathbf{r}}_2 = 14\hat{\mathbf{i}} + 13\hat{\mathbf{j}} + 9\hat{\mathbf{k}}$

$\mathbf{d} = \mathbf{r}_2 - \mathbf{r}_1 = (11\hat{\mathbf{i}} + 11\hat{\mathbf{j}} + 15\hat{\mathbf{k}})$ cm

$\mathbf{F} = (4\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}})$N

$W = \mathbf{F}\cdot\mathbf{d} = (44 + 11 + 45)\times 10^{-2} = 1$J

18. (c) From work-energy theorem,

Work done = change in kinetic energy

$\therefore$ (as F = constant because a = constant)

Therefore K-x graph is a straight line passing through origin.

19. (c) Average velocity $= \frac{\text{Total displacement}}{\text{Time}}$

$= \frac{\text{Area under } v\text{-}t \text{ graph}}{\text{Time}}$

Since area $\neq 0$

$\therefore$ Average velocity $\neq 0$

20. (*c*) Increase in gravitational potential energy

= decrease in elastic potential energy

$\therefore \quad mgh = \frac{1}{2}kx^2$

$\therefore \quad h = \frac{kx^2}{2mg}$

$F_{brean} \times d = \frac{1}{2}mv^2$

21. (*a*) Speed is doubled. Therefore kinetic energy will become four times. Hence, minimum stopping distance will also become four times.

22. (*d*) Potential energy of the spring $U = \frac{1}{2}Kx^2$ or $U \propto x^2$

Stretch is increased to 5 times. Therefore, stored potential energy will be increased by 25 times.

23. (*c*) Applying conservation of mechanical energy we have,

$$\frac{1}{2}mv^2 = U \text{ or } m = \frac{2U}{v^2}$$

24. (*c*) Mass of the body, $m = 200\text{ g} = 200 \times 10^{-3}\text{ kg}$

Height of the body, $h = 200\text{ m}$

Potential energy, $U = mgh = 200 \times 10^{-3} \times 10 \times 200 = 400\text{ J}$

Therefore, Decrease in potential energy at the surface = 400 J.

25. (*b*) At this height, half is PE

$\therefore \quad mgh = \frac{98}{2}$ or $2 \times 9.8 \times h = 49$ or $h = 2.5\text{ m}$

26. (*b*) Work done by friction may be positive, negative and zero. Work done by conservative, internal forces may be zero in a round trip but it is non-zero for non-conservative internal forces.

27. (*b*) Power required to move the vehicle

$$P = \mathbf{F}\cdot\mathbf{v} \Rightarrow F = \frac{P}{v} = \frac{7500}{20} = 375\text{ N}$$

28. (*a*) Power of the engine,

$$P = \mathbf{F}\cdot\mathbf{v} = (20\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}})\cdot(6\hat{\mathbf{i}} + 20\hat{\mathbf{j}} - 3\hat{\mathbf{k}})$$

$$= 120 - 60 - 15 = 45\text{ W}$$

29. (*c*) $\therefore$ Average power $= \frac{W}{t} = \frac{KE}{t} = \frac{mv^2}{2t}$

$$\Rightarrow \quad P = \frac{F\left(\frac{1}{2}at^2\right)}{t = \frac{1}{2}Fv}$$

30. (*d*) Form work-energy theorem

W = change in kinetic energy

or $\quad W = \frac{1}{2}mv^2$

$\therefore$ W-v graph is a parabola.

31. (*c*) $W = \frac{1}{2} \times m \times (10)^2 = 50m$

$W' = \frac{1}{2}m \times (400 - 100) = 150\,m = 3W$

32. (*c*) $dW = \mathbf{F}\cdot d\mathbf{r} = -k(y\hat{\mathbf{i}} + x\hat{\mathbf{j}}) \times (dx\hat{\mathbf{i}} + dy\hat{\mathbf{j}})$

$= -k[y\,dx + x\,dy] = -kd(xy)$

$$W = -k\int_{(0,0)}^{(a,a)} d(xy) = -k|xy|_{(0,0)}^{(a,a)} = -ka^2$$

33. (*d*) Speed, $v = \frac{dx}{dt} = 8t^3$, $v_{0s} = 0$, $v_{1s} = 8\text{ ms}^{-1}$

$\therefore \quad \Delta KE = \frac{1}{2} \times 2 \times (64 - 0)\text{ J} = 64\text{ J}$

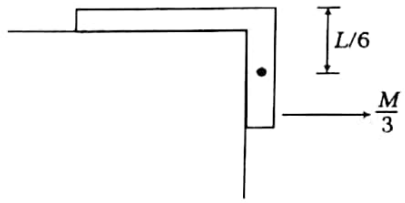
34. (*b*) In equilibrium $kd = mg$ or $d = \frac{mg}{k}$

If allowed to fall suddenly, it does not stop in its equilibrium position. In that case,

decrease in gravitational PE = increase in elastic PE

or $\quad mgd' = \frac{1}{2}kd'^2$ or $d' = \frac{2mg}{k} = 2d$

35. (*d*) Mass $\frac{M}{3}$ has its centre of mass at $\frac{L}{6}$ below the table surface.



$\therefore \quad W = mgh = \left(\frac{M}{3}\right)(g)\left(\frac{L}{6}\right) = \frac{MgL}{18}$

$W_1 = +W_g + W_T = 0, W_T = 0$

36. (*c*) W = change in potential energy $= mgh = mgL(1 - \cos\theta)$

37. (*b*) Work done by conservative force $= -\Delta U$

$\Rightarrow \quad W_C = -(U_B - U_A) \Rightarrow W_C = U_A - U_B$

Initial mechanical energy is same for all.

38. (*b*) Change in potential energy for all three particles is same. Hence, change in kinetic energy will also be same.

or $\quad v_A = v_B = v_C$

39. (*a*) $K = \frac{F}{x}$ = slope of F-x graph.

$$K \propto \frac{1}{l}$$

Length is reduced to half. Therefore, K will become two times. Slope will increase.

40. (*a*) $K = 4t^2$ or $v^2 \propto t^2$

$\therefore \quad v \propto t$

v varies linearly with time when acceleration or force is constant.

41. (*b*) Applying conservation of mechanical energy we get,

$(mgH - mgh) = 2mgh$

$\therefore \quad h = \frac{H}{3}$

$$v = \sqrt{2g(H - h)} = \sqrt{2g\left(\frac{2H}{3}\right)} = 2\sqrt{\frac{gH}{3}}$$

42. (*b*) Change in kinetic energy of the particle

$$\Delta KE = W = \int_2^4 P dt = \int_2^4 (3t^2 - 2t + 1)\, dt = [t^3 - t^2 + t]_2^4 = 46\,J$$

43. (*a*) F = constant

$\therefore$ a = constant or $v = at$

Now, $P = F \cdot v = F \cdot at$ or $P \propto t$ (as F and a both are constant)

Hence, P-t graph is a straight line passing through origin.

44. (*c*) Work done, $W = \int P dt$

or $\frac{1}{2}mv^2 = \int P dt = \int_0^2 \left(\frac{3t^2}{2}\right) dt = 8\,J \Rightarrow v = \sqrt{\frac{8}{2}} = 2\,ms^{-1}$

45. (*d*) $F \propto s^{-1/3}$ or $a \propto s^{-1/3}$

or $vdv \propto s^{-1/3} ds$ $\left(\because \quad a = v \cdot \frac{dv}{ds}\right)$

Integrating we have, $v^2 \propto s^{2/3}$ or $v \propto s^{1/3}$

Now, Power, $P \propto F \cdot v \propto s^{-1/3} \cdot s^{1/3}$ or $P \propto s^0$

46. (*c*) Loss of kinetic energy $= K_i - K_f$

$$= \frac{1}{2}m(100 - 81) = \frac{19}{2}m \qquad (m = \text{mass})$$

Half of the loss will be in upward journey.

Hence, $mgh + \frac{1}{2}\left(\frac{19}{2}m\right) = K_i = \frac{1}{2}m(100) = 50m$

$\therefore$ $h = 4.61\,m$

47. (*a*) Work done by resistive force = Change is PE

$\Rightarrow$ $F \cdot d = mg(h + d) \Rightarrow F = mg\left(1 + \frac{h}{d}\right)$

48. (*b*) When displaced from x_2 in negative direction, force is positive. So, this force is restoring in nature and will bring the body back. Hence, at x_2, body is in stable equilibrium position.

49. (*a*) $P = 10^7\,kW = 10^{10}\,W = 10^{10}\,Js^{-1}$

Time, $t = 1\,day = 24 \times 60 \times 60\,s$

Energy produced per day

$$E = Pt = 10^{10} \times 24 \times 60 \times 60 = 864 \times 10^{12}\,J$$

As $E = mc^2$

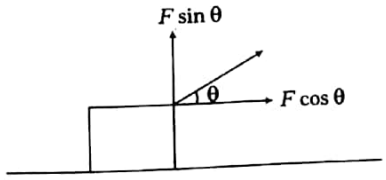
$$\therefore \quad m = \frac{E}{c^2} = \frac{864 \times 10^{12} J}{(3 \times 10^8)^2\, ms^{-1}} = 9.6 \times 10^{-3}\,kg = 9.6\,g$$

50. (*b*) Normal force on the block $N = mg - F \sin\theta$

Block moves with uniform velocity. Hence net force = 0

or, $F \cos\theta = \mu N = \mu(mg - F \sin\theta)$

$$\therefore \quad F = \frac{\mu mg}{\cos\theta + \mu \sin\theta}$$



$$W = Fs \cos\theta = \frac{\mu\, mgd \cos\theta}{\cos\theta + \mu \sin\theta}$$

51. (*b*) PE + KE $= mgh + \frac{1}{2}mv^2$ = constant

$= mgH$ (H = initial height)

or $gh + \frac{v^2}{2}$ = constant

52. (*b*) Work done against friction $= (\mu mg \cos\theta)d$

$$= 0.5 \times 1 \times 9.8 \times \cos 60° \times 2 = 4.9\,J$$

53. (*c*) Force between two protons is same as that of between proton and a positron.

As positron is much lighter than proton, it moves away through much larger distance compared to proton.

We know that work done = force × distance. As forces are same in case of proton and positron but distance moved by positron is larger, hence, work done will be more.

54. (*c*) As the given tracks are frictionless, hence, mechanical energy will be conserved. As both the tracks having common height, h.

From conservation of mechanical energy,

$\frac{1}{2}mv^2 = mgh$ (for both tracks I and II)

$v = \sqrt{2gh}$

Hence, speed is same for both stones. For stone I,

a_1 = acceleration along inclined plane $= g \sin\theta_1$

Similarly, for stone II $a_2 = g \sin\theta_2$ as $\theta_2 > \theta_1$ hence, $a_2 > a_1$.

And length for track II is also less hence, stone II reaches earlier than stone I.

55. (*b*) Total energy is E = PE + KE ...(i)

When particle is at $x = x_m$ *i.e.*, at extreme position, returns back. Hence, at $x = x_m$; $x = 0$; KE = 0

From Eq. (i)

$$E = PE + 0 = PE = V(x_m) = \frac{1}{2}kx_m^2$$

56. (*b*) Given, $v = ax^{3/2}$

$m = 0.5\,kg$, $a = 5\,m^{-1/2}\,s^{-1}$, work done $(W) = ?$

We know that

Acceleration $a_0 = \frac{dv}{dt} = v\frac{dv}{dx} = ax^{3/2}\frac{d}{dx}(ax^{3/2})$

$$= ax^{3/2} \times a \times \frac{3}{2} \times x^{1/2} = \frac{3}{2}a^2x^2$$

Now, Force $= ma_0 = m\frac{3}{2}a^2x^2$

Work done $= \int_{x=0}^{x=2} F dx = \int_0^2 \frac{3}{2} ma^2x^2 dx$

$$= \frac{3}{2}ma^2 \times (x^3/3)_0^2$$

$$= \frac{1}{2}ma^2 \times 8 = \frac{1}{2} \times (0.5) \times (25) \times 8 = 50\,J$$

57. (*b*) When drop falls first velocity increases, hence, first KE also increases. After sometime speed (velocity) is constant this is called terminal velocity, hence, KE also become constant. PE decreases continuously as the drop is falling continuously. The variation in PE and KE is best represented by (b).

58. (*d*) Given, $h = 15\,\text{m}, v = 1\,\text{ms}^{-1}, m = 10\,\text{kg}, g = 10\,\text{ms}^{-2}$

From conservation of mechanical energy,

$$(\text{PE})_i + (\text{KE})_i = (\text{PE})_f + (\text{KE})_f$$

$$\Rightarrow \quad mgh + \frac{1}{2}mv^2 = 0 + (\text{KE})_f$$

$$\Rightarrow \quad (\text{KE})_f = mgh + \frac{1}{2}mv^2$$

$$\Rightarrow \quad (\text{KE})_f = 10 \times 10 \times 15 + \frac{1}{2} \times 10 \times (1)^2$$

$$= 150 + 5 = 155\,\text{J}$$

Note We should be careful about the reference taken for PE, it may or may not be the ground.

59. (*c*) At equilibrium, $F = -\frac{dU}{dx} = 0$ or $(-12)ax^{-13} + (6bx^{-7}) = 0$

$$\therefore \quad x = \left(\frac{2a}{b}\right)^{1/6}$$

60. (*c*) Work done by conservative forces $= U_i - U_f$

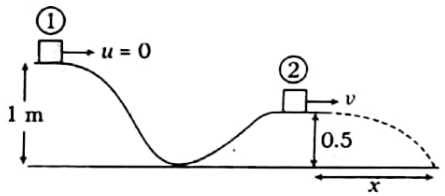
$$= -20 - 40 = -60\,\text{J}$$

Work done by external forces $= E_f - E_i = 140 - 40 = 100\,\text{J}$

and net work done by all the forces $= K_f - K_i$

$$= 100 - 60 = 40\,\text{J}$$

61. (*a*)



From law of conservation of energy

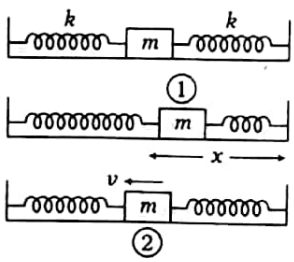
$$mg(1) = \frac{1}{2}mv^2 + mg(0.5)$$

$$v^2 = 2g(1 - 0.5) = \sqrt{g}, t = \frac{x}{v}$$

$$y = \frac{1}{2}gt^2 \Rightarrow 0.5 = \frac{gx^2}{2v^2} = \frac{10x^2}{2(10)}$$

$$x^2 = 1 \Rightarrow x = 1\,\text{m}$$

62. (*b*)



Total *QE* of spring = Total KE of block

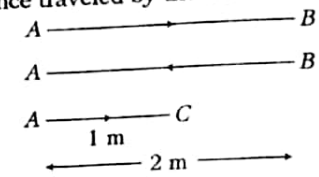
$$\frac{1}{2}kx^2 + \frac{1}{2}kx^2 = \frac{1}{2}mv^2$$

$$kx^2 = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2k}{m}}\,x$$

W_f = loss in mechanical energy

$\mu mgd = mgh \Rightarrow d = 5$ m.

63. (*a*) The distance traveled by the block on the rough surface = 5 m



The block stops at distance 1 m from *A*.

64. (*c*) From conservation of mechanical energy

$$(M - m)gh = \frac{1}{2}(M + m)v^2$$

$$\therefore \quad v = \sqrt{\frac{2gh(M - m)}{m + M}}$$

65. (*b*) Maximum acceleration of 2 kg block due to friction can be μg or 5 ms^{-2}.

Combined acceleration, if both move together with same acceleration would be,

$$a = \frac{60}{12} = 5\,\text{ms}^{-2}$$

Since, both accelerations are equal, upper block will move with acceleration $5\,\text{ms}^{-2}$ due to friction.

In first two seconds, $s = \frac{1}{2}at^2 = \frac{1}{2} \times 5 \times 4 = 10\text{m}$

and force of friction, $f = ma = 10$ N

$$\therefore \quad W_f = fs\cos 0^0 = 100\,\text{J}$$

66. (*a*) Work done by frictional force, $W_f = fs\cos 180°$

$$= (\mu mg\cos\theta)(-1)(s)$$

$$= -0.2 \times 5 \times 10 \times \frac{1}{\sqrt{2}} \times 0.1 = -\frac{1}{\sqrt{2}}\,\text{J}$$

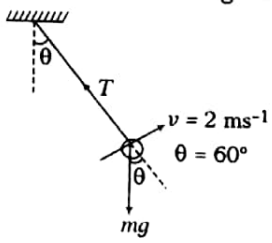
67. (*a*) $\frac{3}{4}$th of KE is lost. Hence left KE is $\frac{1}{4}$th or,

$$v^2 = \frac{v_0^2}{4}$$

$$\therefore \quad v = \frac{v_0}{2} = v_0 - at_0 = v_0 - \mu gt_0$$

or $$\mu = \frac{v_0}{2gt_0}$$

68. (*c*) Two force are acting on girl, tension and weight. Power of tension will be zero and that of weight is,

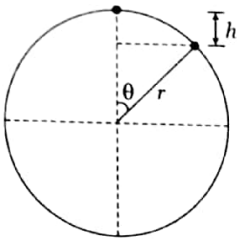


$$P = mgv\cos(90° + \theta) = -mgv\sin 60°$$

$$= -50 \times 9.8 \times 2 \times \frac{\sqrt{3}}{2} = -490\sqrt{3}\,\text{W}$$

$\therefore$ Power delivered is $490\sqrt{3}$ W.

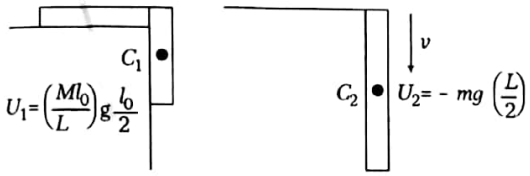
69. (*c*) $h = r(1 - \cos\theta)$



Now, decrease in PE = increase in KE

or $\quad mgr(1-\cos\theta) = \frac{1}{2}mv^2 \quad$ or $\quad v = \sqrt{2gr(1-\cos\theta)}$

70. (*c*) Decrease in gravitational PE = increase in kinetic energy



$$\frac{MgL}{2} - \frac{M}{L}\cdot l_0 g\frac{l_0}{2} = \frac{1}{2}Mv^2$$

or $\quad v = \sqrt{\frac{g}{L}(L^2 - l_0^2)}$

71. (*c*) Then,

$$\mu mgd = mgh$$

or $\quad d = \frac{h}{\mu} = \frac{10^{-2}}{0.01} = 1\text{m}$

72. (*b*) From $-a$ to b decrease in elastic potential energy

= work done against friction.

$\therefore \quad \frac{1}{2}ka^2 - \frac{1}{2}kb^2 = \mu mg(a+b)$

or $\quad (a-b) = \frac{2\mu mg}{k}$

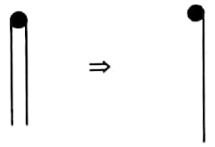
$\therefore$ Decrease in amplitude $= \frac{2\mu mg}{k}$

73. (*a*) Decrease in gravitational potential energy

= increase in kinetic energy.

Initially centre of mass of chain was at distance $\frac{l}{4}$ below the pin and in final position, it is at distance $\frac{l}{2}$ below the pin.

Hence, centre of mass has descended $\frac{l}{4}$.



$\therefore \quad mg\frac{l}{4} = \frac{1}{2}mv^2 \quad$ or $\quad v = \sqrt{\frac{gl}{2}}$

74. (*c*) At $x = 6$ m, $U = 26$ J (extreme position)

On the other side

$$U = 26 = 10 + (x-2)^2 \quad \text{or} \quad x - 2 = \pm 4$$

$\therefore \quad x = 6\text{ m} \quad$ and $\quad x = -2\text{ m}$

$U_{min} = 10$ J, at $x = 2$

$KE_{max} = E - U_{min} = 16$ J, at $x = 2$

75. (*d*) Body will have maximum speed where,

$$mg\sin\theta = \mu\, mg\cos\theta$$

or $\quad \sin 37° = (0.3x).\cos 37° \quad$ or $\quad x = 2.5$ m

76. (*b*) Maximum acceleration of 1 kg block may be

$$a_{max} = \mu g = 1\text{ms}^{-2}$$

Common acceleration, without relative motion between two blocks may be,

$$a = \frac{0.5}{3}\text{ms}^{-2}$$

Since, $\quad a < a_{max}$

There will be no relative motion and blocks will move with acceleration $\frac{0.5}{3}\text{ms}^{-2}$.

Force of friction by lower block on upper block,

$$f = ma = (1)\left(\frac{0.5}{3}\right) = \frac{1}{6}\text{N (towards right)}$$

$\therefore \quad W = f \times s = 0.5$ J

77. (*a*) From work-energy theorem

$$W_F + W_{mg} = \frac{1}{2}mv^2$$

$\Rightarrow \quad F\cdot R + mgR = \frac{1}{2}mv^2$

$\Rightarrow \quad 5\times 5 + \frac{1}{2}\times 10\times 5 = \frac{1}{2}\times\frac{1}{2}\times v^2$

$\Rightarrow \quad v = \sqrt{200} = 14.14\text{ ms}^{-1}$

78. (*a*) Power, $P = F\cdot v = m\left(v\cdot\frac{dv}{ds}\right)\cdot v$

$\therefore \quad \int_v^{2v} v^2\cdot dv = \frac{P}{m}\cdot\int_0^s ds$

$\left[\frac{v^3}{3}\right]_v^{2v} = \frac{Ps}{m} \quad$ or $\quad s = \frac{7mv^3}{3P}$

79. (*a*) $F = kx$

$\therefore \quad k = \frac{F}{x} = \frac{100}{1}\text{Nm}^{-1} = 100$ Nm

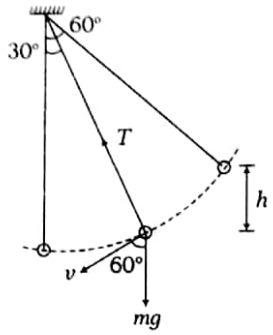
Now from energy conservation, between natural length of spring and its maximum compression state.

$$\frac{1}{2}mv^2 + mgh = \frac{1}{2}kx_{max}^2$$

$$v = \sqrt{\frac{kx_{max}^2}{m} - 2gh} = \sqrt{\frac{(100)(2)^2}{10} - (2)(10)(1)}$$

$$= \sqrt{20}\text{ ms}^{-1}$$

80. (*a*) Power due to tension = 0



Power due to $mg = (mg)(v)\cos 60°$

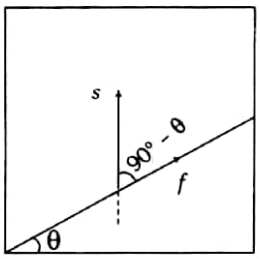
Here $\quad v = \sqrt{2gh}$

and $\quad h = l(\cos 30° - \cos 60°) = 1\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)$

$\therefore \quad v = \sqrt{2 \times 10 \times \left(\frac{\sqrt{3}-1}{2}\right)} = \sqrt{10(\sqrt{3}-1)} = 2.7\ \text{ms}^{-1}$

$\therefore$ Power, $P = \frac{mgv}{2} = \frac{1 \times 10}{2} \times 2.7 = 13.5$ W

81. (*c*) As block does not slide. Hence force of friction,



$$f = mg\sin\theta$$

In time t, displacement $s = vt$

$\therefore \quad W_f = f \cdot s \cdot \cos(90° - \theta)$

$= (mg\sin\theta)(vt)(\sin\theta)$

$= mgvt\sin^2\theta$

82. (*d*) On M horizontal components of N and f are balanced (as Mg is vertical). Hence on $2M$ also they will be balanced.

$\therefore$ Horizontal force Kx on $2M$ should be zero.

83. (*d*) Work done by tension on M is negative (force and displacement are in opposite directions). But work done by tension on m is positive. Net work done will be zero.

B) Medical entrance special format questions

Assertion and reason

1. (*d*) Work done by a constant force

= Force × displacement in the direction of force.

2. (*a*) According to work-energy theorem, work done by all the forces = change in kinetic energy.

3. (*a*) Spring force is conservative in nature. Potential energy is only associated with conservative force.

4. (*d*) $W = 0$. That feet remains stationary, which is in contact with the ladder.

5. (*b*) Change in potential energy is same.

6. (*c*) For work to be done by a force, point of application of force must be not be stationary. If a body is moving, its point of application may not move in a frame.

7. (*d*) If oscillations are not taking place, then kinetic energy may be zero at stable equilibrium position.

8. (*a*) For conservative forces, $\Delta U = -\Delta W$

where ΔU = change in PE

ΔW = work done by conservative force.

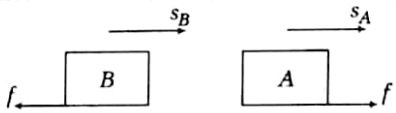
9. (*d*) Force is continuously negative. But displacement is positive (from $x = -2$ to $x = +2$)

$\therefore$ Work done is negative.

10. (*c*) **Assertion** Change in PE is same.

Reason Acceleration a is different. Hence, net force is different.

11. (*c*) Work done by friction on A is positive and on B is negative. If there is no slipping between the two blocks then $s_A = s_B$. Therefore net work done will be zero, otherwise not.



12. (*d*) $W = Pt = \frac{1}{2}mv^2$

$$v \propto t^{1/2}$$

Differentiating we get, $a \propto t^{-1/2}$

13. (*c*) **Assertion** Decrease in mechanical energy in case 1 will be

$$\Delta U_1 = \frac{1}{2}mv^2$$

But decrease in mechanical energy in case 2 will be

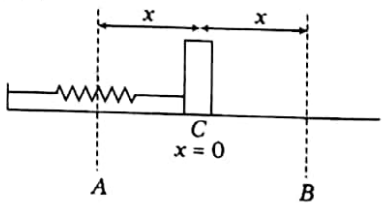
$$\Delta U_2 = \frac{1}{2}mv^2 - mgh$$

$\therefore \quad \Delta U_2 < \Delta U_1$

or assertion is correct.

Coefficient of friction is the mutual property of two surfaces. It does not depend on angle of inclination.

14. (*b*) W_{AC} = + ve



W_{CB} = − ve and $W_{ACB} = 0$

Also $\quad F \propto (-x)$

15. (*b*) **Assertion** $\frac{dU}{dt} = \frac{d}{dt}(mgh)$

$$= mg\frac{d}{dt}\left(u_y t - \frac{1}{2}gt^2\right) = mg(u_y - gt) = mgv_y$$

$$\therefore \quad \left|\frac{dU}{dt}\right| = mg|v_y|$$

$|v_y|$ first decreases, then increase with time.

Reason $\frac{dp}{dt} = F = -mg = \text{constant}$

16. (*c*) **Assertion** $\frac{dU}{dt} = \frac{d}{dt}(mgh) = mg\frac{d}{dt}\left(u_y t - \frac{1}{2}gt^2\right)$

$$= mg(u_y - gt) = mgv_y$$

$$\therefore \quad \left|\frac{dU}{dt}\right| = mg|v_y|$$

For $m = 1\text{kg}$

$$\left|\frac{dU}{dt}\right| = g(u_y - gt) \quad \text{...(i)}$$

$$a \cdot v = (-g\hat{j}) \cdot [u_x\hat{i} + (u_y - gt)\hat{j}]$$

$$= -g(u_y - gt) = g|v_y| \quad \text{...(ii)}$$

From Eqs. (i) and (ii) we can see that two magnitudes are equal.

Reason $P = \mathbf{F}.\mathbf{v} = (-mg\hat{j}) \cdot [v_x\hat{i} + (u_y - gt)\hat{j}]$

$$= -mgu_y + mg^2t$$

So, P *versus* t graph has the positive slope.

Match the Columns

1. Work done by conservative forces is negative of change in potential energy.

3. From $x = 2$ to $x = 4$, force is positive and displacement is also positive. Hence, the work done is positive. Similar logic can be applied to other cases also.

4. $P \propto t$

$$W = \int P dt = \int \alpha t\, dt \quad \text{or} \quad W \propto t^2$$

Since, work done is equal to change is KE

Hence, $v^2 \propto t^2$ or $v \propto t$

Further, $v = \frac{ds}{dt}$

$$\therefore \quad \frac{ds}{dt} \propto t \quad \text{or} \quad ds \propto t\,dt \quad \text{or} \quad s \propto t^2 \qquad \text{(by integration)}$$

5. $v_f - v_i =$ area of a-x graph $= 12\,\text{ms}^{-1}$

$$\therefore \quad v_f = 12 + 4 = 16\,\text{ms}^{-1}$$

$$\Delta\text{KE} = \frac{1}{2}m(v_f^2 - v_i^2) = 120\,\text{J}$$

Work done by all the forces $= \Delta\text{KE} = 120\,\text{J}$

$$K_f = \frac{1}{2}mv_f^2 = 128\text{J}$$

Work done by conservative forces $= U_i - U_f = 240\,\text{J}$

Work done by external force

= total work done - work done by conservative forces

$= -112\,\text{J}$

6. A is the point of stable equilibrium, so potential energy is minimum. Similarly, point C is the unstable equilibrium position, where potential energy should be maximum.

7. In $t = 4\,\text{s}$, $v = at = 8\,\text{ms}^{-1}$

and $s = \frac{1}{2}at^2 = 16\,\text{m}$

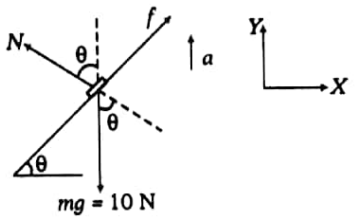
$$\text{KE} = \frac{1}{2}mv^2 = 32\,\text{J}$$

From work-energy theorem,

Work done by all the forces $= \Delta KE = 32\text{J}$

Work done by gravity, $W_g = -mgh = -(1)(10)(16) = -160\,\text{J}$

Writing equation of motion, we have, $\Sigma F_y = ma$



$$N\cos 30° + f\sin 30° - 10 = ma = 2$$

or $\sqrt{3}N + f = 24$ ...(i)

$$\Sigma F_x = 0$$

$$\therefore \quad N\sin 30° = f\cos 30°$$

or $N = \sqrt{3}f$ ...(ii)

Solving Eqs. (i) and (ii), we have

$$f = 6\text{N} \quad \text{and} \quad N = 6\sqrt{3}\,\text{N}$$

Now, $W_N = (N\cos\theta)(s)$

$$= (6\sqrt{3})\left(\frac{\sqrt{3}}{2}\right)(16) = 144\text{J}$$

$$W_f = (f\sin\theta)(s) = (6)\frac{1}{2}(16) = 48\text{J}$$

Work done by all the forces

$$W = W_g + W_N + W_F = -160 + 144 + 48 = 32\text{J}$$

C) Medical entrances' gallery

1. (*c*) According to question, a body of mass 1 kg begins to move under the action of time dependent force,

$$\mathbf{F} = (2t\,\hat{i} + 3t^2\,\hat{j})\,\text{N}$$

$$m\frac{dv}{dt} = 2t\hat{i} + 3t^2\hat{j}$$

$$\int dv = \int (2t\hat{i} + 3t^2\hat{j})dt$$

$$\mathbf{v} = t^2\hat{i} + t^3\hat{j}$$

$\therefore$ Power developed by the force at the time t will be given as

$$P = \mathbf{F}.\mathbf{v} = (2t\hat{i} + 3t^2\hat{j}).(t^2\hat{i} + t^3\hat{j}) = (2t.t^2 + 3t^2.t^3)$$

$$P = (2t^3 + 3t^5)\,\text{W}$$

2. (*d*) $W_{\text{Tangential}} = \Delta KE = K_f - K_i = K_f - 0$

$$(ma_T) \times 5 = 8 \times 10^{-4}$$

$$a_T = \frac{8 \times 10^{-4}}{10^{-2}(2\pi r \times 2)} = \frac{2 \times 10^{-2}}{\pi \times 2\pi \times 10^{-2}} = 0.1 \text{ ms}^{-2}$$

3. (*a*) From work-energy theorem,

Work done = Change in KE

$$\Rightarrow \quad W = K_f - K_i$$

$$\Rightarrow \quad K_f = W + K_i = \int_{x_1}^{x_2} Fx\,dx + \frac{1}{2}mv^2$$

$$= \int_{20}^{30} -0.1.x\,dx + \frac{1}{2} \times 10 \times 10^2$$

$$= -0.1\left[\frac{x^2}{2}\right]_{20}^{30} + 500 = -0.05\,[30^2 - 20^2] + 500$$

$$= -0.05\,[900 - 400] + 500$$

$$\Rightarrow \quad K_f = -25 + 500 = 475\,\text{J}$$

4. (*a*) As the machine delivers a constant power

So, $F \cdot v$ = constant = k (watts)

$$\Rightarrow \quad m\frac{dv}{dt} \cdot v = k \Rightarrow \int v\,dv = \frac{k}{m}\int dt$$

$$\Rightarrow \quad \frac{v^2}{2} = \frac{k}{m}t \Rightarrow v = \sqrt{\frac{2k}{m}t}$$

Now, force on the particle is given by

$$F = m\frac{dv}{dt} = m\frac{d}{dt}\left(\frac{2kt}{m}\right)^{\frac{1}{2}} = \sqrt{2km}.\left(\frac{1}{2}t^{-\frac{1}{2}}\right) = \sqrt{\frac{mk}{2}} \cdot t^{-1/2}$$

5. (*a*) Given, $m = 11.7$ kg, $s = 4.65$ m, $h = 2.86$ m

From the figure, $\sin\theta = \frac{2.86}{4.65} = 0.61$

Also, component of mg along the incline is $mg\sin\theta$.

$\therefore$ Work done by this force $= mg\sin\theta \cdot s$

$W_F + W_g = \Delta KE = 0$, $W_g = -mgh$

$$= 11.7 \times 9.8 \times 0.61 \times 4.65$$

$$= 11.7 \times 9.8 \times 2.86 = 327.9 \simeq 328\,\text{J}$$

6. (*c*) Given, force, $F = 10 + 0.5x = 10 + \frac{1}{2}x$

Let during small displacement the workdone by the force is $dW = Fdx$

So, work done during displacement from $x = 0$ to $x = 2$ is

$$W = \int_0^w dW = \int_0^2 Fdx = \int_0^2\left(10 + \frac{1}{2}x\right)dx$$

$$= 10\,[x]_0^2 + \left[\frac{x^2}{4}\right]_0^2 = 21\text{J}$$

7. (*d*) Given, mass of elevator = 500 kg

velocity = 0.20 ms^{-1}

Weight of elevator = $500 \times 9.8 = F$

Now, power, $P = Fv = 500 \times 9.8 \times 0.20 = 980$ W

Therefore, hp-rating of motor $= \frac{1}{746} \times 980 = 1.31$hp

8. (*d*) At the distance of closest approach, total kinetic energy of a particle is converted into potential energy

i.e., $\frac{p^2}{2m}$ = potential energy $= \frac{K}{d}$ ...(i)

Now, $p' = 3p$

Then, $\frac{(p')^2}{2m} = \frac{(3p)^2}{2m} = \frac{K}{d_1} \Rightarrow \frac{9p^2}{2m} = \frac{K}{d_1}$ ...(ii)

$$9\frac{k}{d} = \frac{k}{d_1} \Rightarrow d_1 = \frac{d}{9}$$

9. (*c*) Let the kinetic energy is K and masses of bodies are m_1 and m_2.

For first body, $p_1 = \sqrt{2m_1K}$

$\therefore$ $p_1^2 = 2m_1K$ ...(i)

Similarly, for second body,

$p_2^2 = 2m_2K$ ...(ii)

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{p_1^2}{p_2^2} = \frac{2m_1K}{2m_2K}, \frac{p_1}{p_2} = \sqrt{\frac{m_1}{m_2}}$$

$\frac{p_1}{p_2}$ = square root of masses.

10. (*a*) Given $m_1 = 1$ kg; $m_2 = 2$ kg

Let kinetic energy of bodies be K_1 and K_2

For first body, $K_1 = Fs_1$ ...(i)

For second body, $K_2 = Fs_2$ ...(ii)

Dividinig Eq. (i) by Eq. (ii), we get

$$\frac{s_1}{s_2} = \frac{K_1}{K_2}, \frac{s_1}{s_2} = \frac{mv_1^2}{mv_2^2}$$

Given, $v_1 = v_2$ So, $\frac{s_1}{s_2} = \frac{m_1}{m_2}$, $\frac{s_1}{s_2} = \frac{1}{2}$

11. (*c*) When electron comes closer to the other stationary electron, its kinetic energy decreases because of repulsion between them. As per conservation of energy, the potential energy increases.

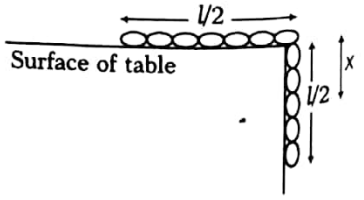
12. (*d*) From $v = u + at$, $v_i = 0 + at_i$

$$F = ma = \frac{mv_1}{t_1} \qquad \left(\because a = \frac{v_1}{t_1}\right)$$

Velocity acquired in t second $= at \Rightarrow v = \frac{v_1}{t_1}t$

$$\text{Power} = Fv = \left(\frac{mv_1}{t_1}\right)\left(\frac{v_1}{t_1}t\right) = \left(\frac{mv_1^2}{t_1^2}\right)t$$

13. (*a*) Consider the diagram.



Let mass of chain be m

$\therefore$ Mass per unit length

$$= \frac{m}{l}$$

Consider an elementary length of the chain at a depth x below the table. The potential energy of their parts is given by

$$dU = -\frac{m}{l} gx\, dx$$

Now, potential energy of hanging part,

$$U_1 = \int dU = -\int_0^{\frac{l}{2}} \frac{m}{l} gx\, dx = -\frac{m}{l} g \left[\frac{x^2}{2}\right]_0^{\frac{l}{2}} = -\frac{mgl}{8}$$

when chain is completely shipps off the table, then potential energy,

$$U_2 = -\int_0^{l} \frac{m}{l} gx\, dx = \frac{mg}{2l}[x^2]_0^l = \frac{-mgl}{2}$$

Now, loss in potential energy $= \frac{-mgl}{8} + \frac{mgl}{2}$

From conservation of energy

Loss in potential energy = gain in kinetic energy

$$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{3mgl}{8} \Rightarrow v = \sqrt{\frac{3gl}{4}}$$

14. (*a*) Given, $m = 4$ kg, $P = 6$ N-s

We know that, kinetic energy of a body,

$$K = \frac{P^2}{2m}, K = \frac{(6)^2}{2 \times 4} \Rightarrow K = \frac{36}{2 \times 4}, K = 4.5\text{ J}$$

15. (*a*) Given, $m = 3.9$ kg, $v = 1.20$ ms^{-1}

and k for spring $= 135$ Nm^{-1}

From law of conservation of energy,

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2 \Rightarrow x = \sqrt{\frac{mv^2}{k}} = \sqrt{\frac{3.9 \times 1.2 \times 1.2}{135}} = 0.204 \text{ m}$$

16. (*a*) Work done in stretching a string to obtain an extension l is

$$W_1 = \frac{1}{2}kl^2$$

Similarly, work done is stretching a string to obtain extension $(l + l_1)$

$$W_2' = \frac{1}{2}k(l_1 + l)^2$$

$$W_2 = W_2' - W_1 = \frac{1}{2}k(l_1 + l)^2 - \frac{1}{2}kl^2 = \frac{1}{2}kl_1(2l + l_1)$$

17. (*b*) So, for an instantaneous displacement dx, we can write

$$P = \frac{dW}{dt} = \mathbf{F} \cdot \left(\frac{dx}{dt}\right) = \mathbf{F} \cdot \mathbf{v}$$

$$P = (F)(at) = (ma)(at) \qquad (\because F = ma)$$

$$= (ma^2)t \Rightarrow P \propto t \qquad (\because ma^2 = \text{constant})$$

18. (*c*) Given, $t_1 = 2$s, $t_2 = 4$ s and $h_1 = h_2 = h$

As, $$P_A = \frac{mgh_1}{t_1} \qquad \text{...(i)}$$

and $$P_B = \frac{mgh_2}{t_2}$$

$$\Rightarrow \quad P_A : P_B = \frac{mgh_1/t_1}{mgh_2/t_2} = \left(\frac{h_1}{h_2}\right)\left(\frac{t_2}{t_1}\right) = \frac{t_2}{t_1} = \frac{4}{2} = \frac{2}{1} \qquad (\because h_1 = h_2)$$

$$\Rightarrow \quad P_A : P_B = 2 : 1$$

19. (*d*) The power of the machine gun

$$= \frac{\text{total work done}}{\text{time}} = \frac{n \cdot \frac{1}{2}mv^2}{t}$$

$$= n \cdot \frac{1}{2}\frac{mv^2}{t} \qquad \left(\because K = \frac{1}{2}mv^2, t = 1\text{ s}\right)$$

$\therefore$ The power of the machine gun $= nK$

20. (*a*) Given, force $F = 3\hat{\mathbf{i}} + \hat{\mathbf{j}}$

$$\mathbf{r}_1 = (2\hat{\mathbf{i}} + \hat{\mathbf{k}})\text{ m and } \mathbf{r}_2 = (4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}})\text{ m}$$

$$\therefore \quad \mathbf{s} = \mathbf{r}_2 - \mathbf{r}_1 = (4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}) - (2\hat{\mathbf{i}} + \hat{\mathbf{k}})$$

$$= (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}})\text{ m}$$

$$\therefore \quad W = \mathbf{F} \cdot \mathbf{s} = (3\hat{\mathbf{i}} + \hat{\mathbf{j}}) \cdot (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$$

$$= 3 \times 2 + 3 + 0 = 6 + 3 = 9\text{ J}$$

21. (*a*) Power $= \frac{\text{Work done}}{\text{Time}}$

Work done $= mgh = 100 \times 9.8 \times 10$

Time = 1 min = 60 s

$$\therefore \quad P = \frac{100 \times 9.8 \times 10}{60} = 163.3\text{ W}$$

22. (*d*) We know that,

$$W = mg\cos\theta \times s$$

$$W = 300 \times 9.8 \times \cos 30° \times 10$$

$$= 300 \times 9.8 \times \frac{\sqrt{3}}{2} \times 10 = 2450\text{ J}$$

23. (*b*) We know that the work done in a stretched wire

$$W = \frac{1}{2}kx^2$$

Given, $k_A = k$ and $k_B = 2k$

So that $W_A = \frac{1}{2}kx^2$ and $W_B = \frac{1}{2}(2k)x^2 = kx^2$

Hence, the ratio of work done in stretch wire

$$\frac{W_A}{W_B} = \frac{(1/2)kx^2}{kx^2} \Rightarrow \frac{W_A}{W_B} = \frac{1}{2}$$

24. (*a*) Work done to accelerate from 0 to v is, $W_1 = \frac{1}{2}mv^2$

Work done to accelerate from v to $2v$ is, $W_2 = \frac{1}{2}m(2v)^2$

$$\Rightarrow \quad \Delta W = W_2 - W_1 = \frac{1}{2}m(2v)^2 - \frac{1}{2}mv^2 = 4W_1 - W_1 = 3W_1$$

25. (*b*) Given $m = 200$ g $= 0.2$ kg, $g = 10$ ms^{-2} and

$$H_{\text{eff}} = 2 + 0.5 = 2.5\text{ m}$$

Here $\frac{1}{2}kx^2 = mg\,H_{\text{eff}} \Rightarrow k = \frac{2mg\,H_{\text{eff}}}{x^2}$

$$k = \frac{2 \times 0.2 \times 10 \times 2.5}{(0.5)^2} \Rightarrow k = \frac{10}{2.5} = 40\text{ Nm}^{-1}$$

26. (*a*) Work done = Force × displacement

$$F = ma \text{ and } s = \frac{1}{2}at^2$$

Using, $s = ut + \frac{1}{2}at^2$

$\therefore \quad W = ma \times \frac{1}{2}at^2 = \frac{1}{2}ma^2t^2$

$= \frac{1}{2}m\left(\frac{v}{t_1}\right)^2 t^2 = \frac{1}{2}m\frac{v^2}{t_1^2}t^2 \qquad \left(\text{Using } a = \frac{v}{t_1}\right)$

27. (*b*) Kinetic energy of a particle in motion $E = \frac{1}{2}mv^2$

Differentiating w.r.t. x, we get

$$\frac{dE}{dx} = \frac{1}{2}m \times 2v\frac{dv}{dx}$$

$$= mv \times \frac{dv}{dt} \times \frac{dt}{dx} = mv \times \frac{a}{v} = ma$$

So, the slope of kinetic energy-displacement curve is directly proportinal to the acceleration of the particle.

28. (*b*) Given, the potential energy of a particle in a force field

$$U = \frac{A}{r^2} - \frac{B}{r}$$

For stable equilibrium, $F = -\frac{dU}{dr} = 0$

$$0 = -\frac{2A}{r^3} + \frac{B}{r^2} \quad \text{or} \quad \frac{2A}{r} = B$$

The distance of particle from the centre of the field

$$r = \frac{2A}{B}$$

29. (*b*) According to law of conservation of momentum,

$$mv_1 + 2mv_2 = 0 \quad \text{or} \quad v_2 = -\frac{v_1}{2} \qquad \text{...(i)}$$

According to law of conservation of energy,

$$\frac{1}{2}mv_1^2 + \frac{1}{2}2mv_2^2 = 60$$

$$\Rightarrow \quad \frac{1}{2}mv_1^2 + m\left(\frac{-v_1}{2}\right)^2 = 60 \Rightarrow \frac{1}{2}mv_1^2 + \frac{1}{4}mv_1^2 = 60$$

$$\Rightarrow \quad \frac{3mv_1^2}{4} = 60 \Rightarrow \frac{1}{2}mv_1^2 = 40 \text{ J}$$

30. (*b*) We know that

power, $P = \frac{W}{t} = \frac{mgh}{t} = \frac{200 \times 10 \times 50}{10} = 10 \times 10^3$ W

31. (*b*) We have, $P = Fv =$ constant (given)

$$\Rightarrow \quad M\left(\frac{dv}{dt}\right)v = P \quad \text{or} \quad vdv = \frac{Pdt}{M}$$

Integrating both side, we get

$$\frac{v^2}{2} = \frac{P}{M}t \text{ or } v = \sqrt{\frac{2Pt}{M}}$$

32. (*b*) $W = \text{KE} = \frac{1}{2}mv^2 = 10^4$ J

33. (*a*) Work done, $W = \int_0^x F \cdot dx = \int_0^x C\,x\,dx = C\left(\frac{x^2}{2}\right)_0^x = \frac{1}{2}Cx^2$.

CHAPTER 07

Circular motion

Everyday in different activities of our daily life, we use circular motion. For example, to grind wheat from a flour mill its wheel is given circular motion, to wash cloths, parts of a washing maching performs circular motion. Similarly some natural phenomena like motion of the earth around the sun, motion of moon around the earth etc. all are examples of circular motion. In this chapter we will study circular motion in detail.

Circular motion and its types

In circular motion, an object moves along the perimeter or circumference of a circle.

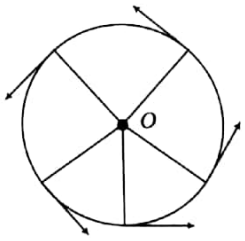


Fig. 7.1 *Circular motion*

Circular motion can be categorised into two types as follows

Uniform circular motion

If a particle moves along a circular path with a constant speed (*i.e.*, it covers equal distances along the circumference of the circle in equal intervals of time), then its motion is said to be uniform circular motion.

e.g., (i) Motion of a point on the rim of a wheel rotating uniformly.

(ii) Motion of the tip of the second hand of a clock.

Non-uniform circular motion

If the speed of the particle in a horizontal circular motion changes with respect to time, then its motion is said to be non-uniform circular motion.

e.g., A stone tied to a string moved in vertical circle.

Inside

1) Kinematics of circular motion
   - Centripetal acceleration
   - Acceleration of a particle in non-uniform circular motion
2) Dynamics of circular motion
   - Centripetal force
   - Centrifugal force
   - Examples for obtaining centripetal force in daily life
   - Motion of a particle tied to a string in verticle circle

1 Kinematics of circular motion

For a particle in circular motion, following variables are needed to describe its motion.

Radius vector (r)

When a particle moves on a circular path, its distance from the centre is fixed and it is equal to radius of the circle. If the centre of the circle is taken as origin, then the vector joining centre to the particle is called **radius vector**. It is directed from centre to the particle and magnitude is same as radius.

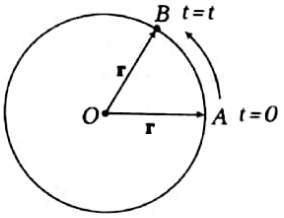


***Fig. 7.2** Radius vector in circular motion*

Angular position

Suppose a particle P is moving in a circle of radius r and centre O, as shown in figure.

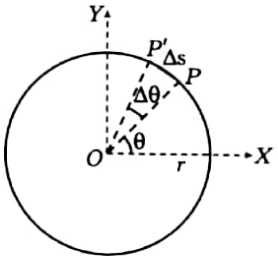


***Fig. 7.3** Moving particle on a circle*

The position of the particle P at a given instant may be described by the angle θ between OP and OX. This angle θ is called the angular position of the particle. As the particle moves on the circle its angular position θ changes. Here, P and P' are given as P (r, θ) and P' $(r, \theta + \Delta\theta)$ respectively.

Angular displacement (Δθ)

The angle traced out by the radius vector at the centre of the circular path in the given time is called angular displacement. It is denoted by Δθ and expressed in radians. In the Fig 7.3, **OP** is the initial and **OP′** is the final position of the particle.

Angular displacement

$$= \frac{\text{Linear displacement between two position}}{\text{Radius vector}}$$

$$\boxed{\Delta\theta = \frac{\Delta s}{r}}$$

Small angular displacement is taken as a vector quantity and its SI unit is radian.

Direction of angular displacement

Direction of Δθ is given by right handed (*i.e.*, the direction where screw advances) screw rule as shown in figure.

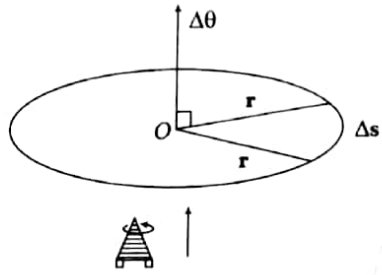


***Fig. 7.4** Direction of advancement of screw*

It is directed along a line passing through centre (O) and perpendicular to the plane of circular motion, containing **r** and **Δs**.

Note If a particle makes N revolutions, its angular displacement is

$$\boxed{\theta = 2\pi N \text{ rad}}$$

Angular velocity (ω)

Rate of change angular displacement of a particle performing circular motion is called angular velocity.

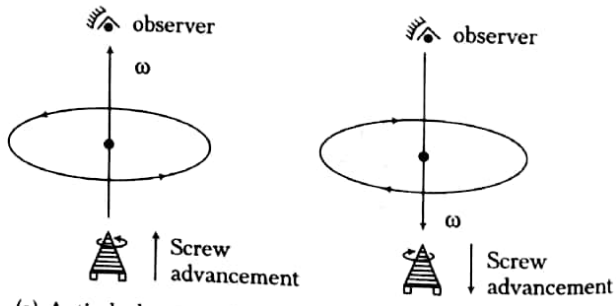
Angular velocity, $\omega = \dfrac{\Delta\theta}{\Delta t}$

Instantaneous angular velocity, $\boxed{\omega = \lim_{\Delta t \to 0} \dfrac{\Delta\theta}{\Delta t} = \dfrac{d\theta}{dt}}$

It is also a vector quantity. Its unit is rads^{-1}, rpm, rps etc., and its dimensional formula is $[M^0L^0T^{-1}]$.

Direction of angular velocity

Direction of angular velocity is given in the same way as that of angular displacement.



(a) Anti-clockwise motion (b) Clockwise motion

***Fig. 7.5** Directions of angular velocities*

Let the particle rotates in anti-clockwise direction as seen by observer angular velocity is along a line passing through the centre and perpendicular to the plane of the circle and towards the observer. This is along a direction where screw advances as shown in figure.

Similarly when the particle rotates in clockwise direction as seen by the observer, angular velocity is away from the observer.

Important points regarding the angular velocity are given below.

(i) If the particle performing circular motion completes one rotation around the circular path in T seconds, then

Angular velocity, $\omega = \frac{2\pi}{T}$ rad s^{-1}

(ii) If the particle performing circular motion makes n rotations per second around the circular path, then

Angular velocity, $\omega = 2\pi n$ rad s^{-1}

(iii) If two particles be moving on the same circle in the same direction with different constant angular speeds ω_1 and ω_2, then the angular speed of 2 with respect to 1 for an observer at center will be

$$\boxed{\omega = \omega_2 - \omega_1}$$

(iv) Angular velocity depends on axis of rotation.

$$\omega_O = \frac{\alpha}{t}$$

$$\omega_P = \frac{\theta}{t} = \frac{2\alpha}{t}$$

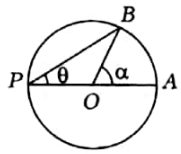


Fig. 7.6

Example 7.1 *Calculate the average angular velocity of the hour hand of a clock.*

Sol. The hour hand completes one round in 12 h. One round makes an angular displacement 2π.

$$\therefore \text{Average angular velocity } \omega_{av} = \frac{\Delta\theta}{\Delta t} = \frac{2\pi \text{ rad}}{12 \text{ h}}$$

$$= \frac{2\pi}{12 \times 3600} \text{ rad s}^{-1}$$

$$= \frac{\pi}{21600} \text{ rad s}^{-1}$$

Example 7.2 *An object revolves uniformly in a circle of diameter 0.80 m and completes 100 rev min^{-1}. Find its time period and angular velocity.*

Sol. Diameter = 0.80 m

$$\therefore \text{Radius } r = \frac{\text{Diameter}}{2} = \frac{0.80}{2} = 0.4 \text{ m}$$

$$\text{Frequency, } n = 100 \text{ rev/min} = \frac{100}{60} \text{ rev/s}$$

$$\text{Therefore, Time period, } T = \frac{1}{n} = \frac{60}{100} \text{ s} = 0.6 \text{ s}$$

$$\therefore \text{Angular velocity, } \omega = \frac{2\pi}{T} = \frac{2 \times 3.14}{0.6} \text{ rad/s} = 10.466 \text{ rad/s}$$

Example 7.3 *A threaded rod with 12 turns per cm and diameter 1.18 cm is mounted horizontally. A bar with a threaded hole to match the rod is screwed onto the rod. The bar spins at the rate of 216 rpm. Determine the velocity of the bar with which it will move the length of 1.50 cm along the rod. Also, find the time taken by it.*

Sol. Given, frequency $n = 216 \text{ rpm} = \frac{216}{60}$ rps

$$\text{Length of one turn} = \frac{1}{12} \text{ cm}$$

$\therefore$ Number of rotations required to move a distance of 1.5 cm,

$$N = \frac{\text{Distance}}{\text{Length of one turn}} = \frac{1.5}{1/12} = 18$$

Therefore, angular displacement $\theta = 2\pi N = 2\pi \times 18 = 36\,\pi$ rad

$$\therefore \text{Angular velocity of the bar, } \omega = 2\pi n = 2\pi \times \frac{216}{60}$$

$$= 7.2\,\pi \text{ rad s}^{-1}$$

$$\therefore \text{Required time, } t = \frac{\text{Angular displacement } (\theta)}{\text{Angular velocity } (\omega)} = \frac{36\pi}{7.2\pi} = 5 \text{ s}$$

Relation between linear velocity and angular velocity

A particle performing circular motion also has linear velocities (as it cover linear displacement along circular path) along with angular velocity. If linear velocity of particle performing circular motion is v and angular velocity is ω, then *both of there velocities are related as*

$$\boxed{v = r\omega}$$

where, r is radius of circular path. In vector form $\mathbf{v} = \omega \times \mathbf{r}$

Linear velocity is always along the tangent to the circular path.

Example 7.4 *A particle moves in a circle of radius 4 m with a linear velocity of 20 ms^{-1}. Find the angular velocity.*

Sol. Given, linear velocity $v = 20 \text{ ms}^{-1}$

Radius $r = 4$ m

$\because$ Linear velocity, $v = r\omega$

$$\therefore \text{Angular velocity, } \omega = \frac{v}{r} = \frac{20}{4} = 5 \text{ rad s}^{-1}$$

Example 7.5 *If the length of the second's hand in a stop clock is 3 cm, find the angular velocity and linear velocity of the tip.*

Sol. Given, radius $r = 3 \text{ cm} = 3 \times 10^{-2}$ m

Time period of stop clock $T = 60$ s

$$\text{Angular velocity } \omega = \frac{2\pi}{T} = \frac{2\pi}{60} = 0.1047 \text{ rad s}^{-1}$$

and linear velocity $v = \omega r = 0.1047 \times 3 \times 10^{-2} = 0.00314 \text{ ms}^{-1}$

Angular acceleration (α)

The rate of change of angular velocity of a particle performing circular motion is called angular acceleration.

$$\text{Angular acceleration, } \alpha = \frac{\Delta\omega}{\Delta t}$$

Instantaneous angular acceleration

$$\boxed{\alpha_{inst} = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}}$$

The SI units of angular acceleration is rad s^{-2} and its dimensions formula is $[M^0L^0T^{-2}]$. If $\alpha = 0$, circular motion is said to uniform.

It is having same characteristics as that of angular velocity.

Note Angular acceleration is an axial vector, when axis of rotation is fixed, angular acceleration and angular velocity vectors both lie along that axis.

Example 7.6 *A point on the rim of a disc starts circular motion from rest and after time t, it gains an angular acceleration given by $\alpha = 3t - t^2$. Calculate the angular velocity after 2 s.*

Sol. Angular acceleration, $\alpha = \frac{d\omega}{dt} = 3t - t^2$

$$\Rightarrow \quad \int_0^{\omega} d\omega = \int_0^t (3t - t^2)\,dt \quad \Rightarrow \quad \omega = \frac{3t^2}{2} - \frac{t^3}{3}$$

At $t = 2$ s, $\quad \omega = \frac{10}{3}$ rad s^{-1}

Relation between angular variables
(Kinematic equations of circular motion)

If angular acceleration is constant, then we have kinematic equations of circular motion as follows

(i) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ (ii) $\omega = \omega_0 + \alpha t$

(iii) $\omega^2 = \omega_0^2 + 2\alpha\theta$ (iv) $\theta_t = \omega_0 + \frac{1}{2}\alpha\,(2t - 1)$

(v) $\theta = \left(\frac{\omega + \omega_0}{2}\right)t$

Here, ω_0 and ω are the angular velocities at time $t = 0$ and t; θ and θ_t are the angular displacements in time t and t th second respectively.

Example 7.7 *The wheel of a motor rotates with a constant acceleration of 4 rad s^{-2}. If the wheel starts from rest, how many revolutions will it make in the first 20 second?*

Sol. The angular displacement in the first 20 s is given by

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = \frac{1}{2}(4 \text{ rads}^{-2})(20 \text{ s})^2$$

($\because$ Angular velocity $\omega = 0$)

$$= 800 \text{ rad}$$

As, the wheel turns by 2π radian in each revolution, the number of revolutions in 20 s is

$$N = \frac{\theta}{2\pi} = \frac{800}{2\pi} = 128$$

Example 7.8 *The wheel of a car, accelerated uniformly from rest, rotates through 1.5 rad during the first second. Find the angle rotated during the next second.*

Sol. As the angular acceleration is constant, we have

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = \frac{1}{2}\alpha t^2 \quad (\because \text{Angular velocity } \omega = 0)$$

$$\therefore \quad 1.5 \text{ rad} = \frac{1}{2}\alpha\,(1)^2$$

Angular acceleration, $\alpha = 3$ rads^{-2}

The angular displacement in first two second is given by

$$\theta_1 = \frac{1}{2} \times (3)\,(2)^2 = 6 \text{ rad}$$

Thus, the angle rotated during the 2nd second

$$= \theta_1 - \theta_2 = 6 \text{ rad} - 1.5 \text{ rad} = 4.5 \text{ rad}$$

Centripetal acceleration

When a particle is in uniform circular motion, its speed remains constant but velocity changes continuously because of change in direction of motion. Therefore motion of the particle is accelerated. This acceleration is called centripetal acceleration and it is directed along radial direction towards centre of the circle. It is given by

$$\boxed{a_r = a_c = \frac{v^2}{r}}$$

where, v = magnitude of linear velocity of the particle

and r = radius of circular path.

It is also called **radial acceleration.**

If we put, $v = r\omega$,

where, ω is angular velocity, then centripeal acceleration

$$\boxed{a_r = a_c = r\omega^2}$$

Example 7.9 *Determine the magnitude of centripetal acceleration of a particle on the tip of a fan blade, 0.30 m in diameter, rotating at 1200 rev min^{-1}.*

Sol. Given, Diameter = 0.30 m

$$\therefore \quad \text{Radius, } r = \frac{0.30}{2} = 0.15 \text{ m}$$

and Frequency, $n = 1200$ rev min^{-1}

$$= \frac{1200}{60} = 20 \text{ rps}$$

$\therefore$ Angular velocity, $\omega = 2\pi n = 2\pi \times 20 = 40\,\pi$ rad s^{-1}

Therefore, centripetal acceleration, $a_c = \omega^2 r$

$$\Rightarrow \quad a_c = 0.15 \times (40\pi)^2 \quad \Rightarrow \quad a_c = 2366.3 \text{ ms}^{-2}$$

Example 7.10 *Find the acceleration of a particle placed on the surface of the earth at the equator, due to the earth rotation. The radius of earth is 6400 km and time period of revolution of the earth about its axis is 24 h.*

Sol. Given, radius of earth $R_e = 6400 \text{ km} = 6400 \times 10^3$ m

Time period $T = 24 \text{ h} = 24 \times 60 \times 60$ s

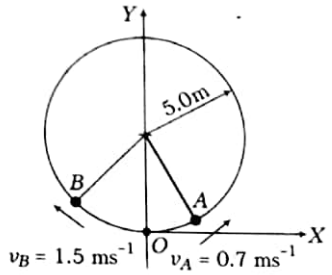
Angular speed of the earth

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{24 \times 60 \times 60} \text{ rad s}^{-1}$$

Acceleration of the particle,

$$a_c = \omega^2 R_e = \left(\frac{2\pi}{24 \times 60 \times 60}\right)^2 \times (6400 \times 10^3) = 0.0033 \text{ ms}^{-2}$$

Example 7.11 *Two particles A and B start at the origin O and travel in opposite directions along the circular path at constant speeds $v_A = 0.7\ ms^{-1}$ and $v_B = 1.5\ ms^{-1}$, respectively. Determine the time when they collide and the magnitude of the acceleration of B just before this happens.*



Sol. From the condition given in the question, it is clear that total distance (= velocity × time) will be equal to the circumference of circular path

i.e., $v_B t + v_A t$ = circumference of circular path

$$1.5\,t + 0.7\,t = 2\pi R = 10\pi$$

$$\therefore \quad \text{Time, } t = \frac{10\pi}{2.2} = 14.3 \text{ s}$$

$$\text{Acceleration, } a = \frac{v_B^2}{R} = 0.45 \text{ ms}^{-2}$$

Acceleration of a particle in non-uniform circular motion

If a particle is in non-uniform circular motion *i.e.,* its speed is not constant, then the particle has both radial and tangential components of acceleration.

Radial component (a_r)

This component of acceleration is towards the centre. This is responsible for change in direction of velocity. This is equal to $\frac{v^2}{r}$ or $r\omega^2$.

Thus, $$\boxed{a_r = \frac{v^2}{r} = r\omega^2}$$

Note The radial acceleration (a_r) is also sometimes called **normal acceleration** (a_n).

Tangential component (a_t)

This is the component of acceleration in the direction of velocity, which is responsible for change in speed of particle. It is also equal to rate of change of speed.

Hence, a_t = component of **a** along **v**

$$= \frac{dv}{dt} = \frac{d|\mathbf{v}|}{dt} \Rightarrow \mathbf{a}_t = \frac{d\omega}{dt} \times \mathbf{r} \quad (\because \mathbf{v} = \omega \times \mathbf{r})$$

$$\Rightarrow \quad \mathbf{a}_t = \alpha \times \mathbf{r} \Rightarrow \boxed{a_t = r\alpha}$$

This component is tangential.

Example 7.12 *A particle moves in a circle of radius 0.5 m at a speed that uniformly increases. Find the angular acceleration of particle if its speed changes from 2 ms^{-1} to 4 ms^{-1} in 4 s.*

Sol. The tangential acceleration of the particle is

$$a_t = \frac{dv}{dt} = \frac{4-2}{4} = 0.5 \text{ ms}^{-2}$$

The angular acceleration is $\alpha = \frac{a_t}{r} = \frac{0.5}{0.5} = 1 \text{ rad s}^{-2}$

Net acceleration

Consider the particle moving on circular path in anti-clockwise direction with increasing speed as shown in figure.

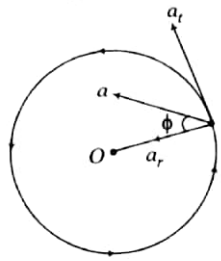


***Fig. 7.7** Non-uniform circular motion*

We know that, linear velocity, $\mathbf{v} = \omega \times \mathbf{r}$

Differentiating on both sides w.r.t. time t, we get

$$\text{Net acceleration, } \frac{d\mathbf{v}}{dt} = \frac{d\omega}{dt} \times \mathbf{r} + \omega \times \frac{d\mathbf{r}}{dt}$$

$$\Rightarrow \quad \text{Net acceleration, } \mathbf{a} = \alpha \times \mathbf{r} + \omega \times \mathbf{v}$$

$$\Rightarrow \quad \boxed{\mathbf{a} = \mathbf{a}_t + \mathbf{a}_r}$$

$$[\because \mathbf{a} = \alpha \times \mathbf{r} \text{ and } \mathbf{a}_r = \omega \times \mathbf{v}]$$

Magnitude of net acceleration $\boxed{a = \sqrt{a_r^2 + a_t^2}}$

This resultant acceleration makes an angle ϕ with the radius, where $\boxed{\tan\phi = \frac{a_t}{a_r}}$

Note

1. In accelerated circular motion dv/dt is positive and hence tangential acceleration of the particle is parallel to velocity **v**.
2. In decelerated circular motion, dv/dt is negative and hence, tangential acceleration is anti-parallel to velocity **v**.

Example 7.13 *A car is travelling along a circular curve that has a radius of 50 m. If its speed is 16 ms^{-1}, and is increasing uniformly at 8 ms^{-2}, determine the magnitude of its acceleration at this instant.*

Sol. Given, tangential acceleration $a_t = 8 \text{ ms}^{-2}$

Radius $R = 50$ m, Speed $v = 16 \text{ ms}^{-1}$

$$\therefore \text{ Radial acceleration } a_r = \frac{v^2}{R} = \frac{(16)^2}{50} = \frac{256}{50}$$

Magnitude of net acceleration of the car.

$$a = \sqrt{a_t^2 + a_r^2} = \sqrt{(8)^2 + \left(\frac{256}{50}\right)^2} = 9.5 \text{ ms}^{-2}$$

Example 7.14 *The speed of a particle moving in a circle of radius $r = 2\,m$ varies with time t as $v = t^2$, where t is in second and v in ms^{-1}. Find the radial, tangential and net acceleration at $t = 2$ s.*

Sol. Given in the question, $v = t^2$

$\therefore$ Linear speed of particle at $t = 2$s is $v = (2)^2 = 4\ ms^{-1}$

$\therefore$ Radial acceleration $a_r = \frac{v^2}{r} = \frac{(4)^2}{2} = 8\ ms^{-2}$

The tangential acceleration is $a_t = \frac{dv}{dt} = \frac{d}{dt}(t^2) = 2t$

$\therefore$ Tangential acceleration at $t = 2$ s is

$$a_t = (2)(2) = 4\ ms^{-2}$$

$\therefore$ Net acceleration of particle at $t = 2$ s is

$$a = \sqrt{(a_r)^2 + (a_t)^2} = \sqrt{(8)^2 + (4)^2} = \sqrt{64 + 16} \quad \text{or } a = \sqrt{80}\ ms^{-2}$$

Example 7.15 *A particle moves in a circle of radius 2 cm at a speed given by $v = 4t$, where v is in cms^{-1} and t in second.*

(i) Find the tangential acceleration at $t = 1$ s.

(ii) Find total acceleration at $t = 1$ s.

Sol. Given, $v = 4t$

Radial acceleration,

$$a_r = \frac{v^2}{R} = \frac{(4t)^2}{R} \quad \text{or } a_r = \frac{16t^2}{2} = 8t^2$$

At $t = 1$ s, $\quad a_r = 8\ cms^{-2}$

(i) Tangential acceleration,

$$a_t = \frac{dv}{dt}$$

or $$a_t = \frac{d}{dt}(4t) = 4\ cms^{-2}$$

i.e., a_t is constant or tangential acceleration at $t = 1$ s is $4\ cms^{-2}$.

(ii) Total acceleration, $a = \sqrt{a_t^2 + a_r^2}$

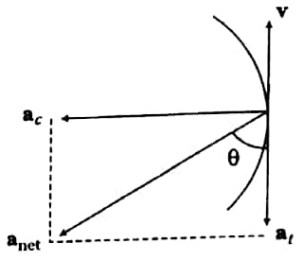
or $$a = \sqrt{(4)^2 + (8)^2} = \sqrt{80} = 4\sqrt{5}\ cms^{-2}$$

Example 7.16 *A cyclist is riding with a speed of 18 kmh^{-1}. As he approaches a circular turn on the road of radius $25\sqrt{2}$ m, he applies brakes and reduces his speed at the constant rate of 0.5 ms^{-1} every second. Determine the magnitude and direction of the net acceleration of the cyclist on the circular turn.*

Sol. Linear speed of the cyclist, $v = 18 \times \frac{5}{18} = 5\,ms^{-1}$

and Centripetal acceleration of the cyclist,

$$a_c = \frac{v^2}{R} = \frac{25}{25\sqrt{2}} = \frac{1}{\sqrt{2}}\,ms^{-2},$$



Tangential acceleration of the cyclist, $a_t = \frac{dv}{dt} = -\frac{1}{2}\,ms^{-2}$

$\therefore$ Net acceleration of two cyclist,

$$a_{net} = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2} = \frac{\sqrt{3}}{2} = 0.86\ ms^{-2},$$

$$\tan\theta = \frac{a_c}{a_t} = \frac{1/\sqrt{2}}{1/2} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

Angle made by resultant acceleration with tangential acceleration,

$\Rightarrow$ $$\theta = \tan^{-1}(\sqrt{2})$$

Check point 7.1

1. The angular speed of a flywheel making 120 rev min^{-1} is
(a) 2π rad s^{-1} (b) $4\pi^2$ rad s^{-1}
(c) π rad s^{-1} (d) 4π rad s^{-1}

2. The ratio of angular speeds of minute hand and hour hand of a watch is
(a) 1 : 12 (b) 6 : 1 (c) 12 : 1 (d) 1 : 6

3. The wheel of a toy car rotates about a fixed axis. It slows down from 400 rps to 200 rps in 2 s. Then, its angular retardation in rad s^{-2} is (rps = revolutions per second)
(a) 200π (b) 100π
(c) 400π (d) None of these

4. A rotating wheel changes angular speed from 1800 rpm to 3000 rpm in 20 s. What is the angular acceleration assuming to be uniform?
(a) 60π rad s^{-2} (b) 90π rad s^{-2}
(c) 2π rad s^{-2} (d) 40π rad s^{-2}

5. A wheel is rotating at 900 rpm about its axis. When the power is cut off, it comes to rest in 1 min. The angular retardation (in rad s^{-2}) is
(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{8}$

6. The motor of an engine is rotating about its axis with an angular velocity of 100 rev min^{-1}. It comes to rest in 15 s after being switched off, assuming constant angular deceleration. What is the number of revolutions made by it before coming to rest?
(a) 12.5 (b) 40 (c) 32.6 (d) 15.6

7. In case of a uniform circular motion, velocity and acceleration are
(a) perpendicular (b) in same direction
(c) in opposite direction (d) not related to each other

8. A particle moves with constant speed v along a circular path of radius r and completes the circle in time T. The acceleration of the particle is
(a) $\frac{2\pi v}{T}$ (b) $\frac{2\pi r}{T}$ (c) $\frac{2\pi r^2}{T}$ (d) $\frac{2\pi v^2}{T}$

9. A body is moving in a circular path with acceleration a. If its speed gets doubled, find the ratio of centripetal acceleration after and before the speed is changed
(a) 1 : 4 (b) 1 : 2 (c) 2 : 1 (d) 4 : 1

10. The circular orbit of two satellites have radii r_1 and r_2 respectively ($r_1 < r_2$). If angular velocities of satellites are same, then their centripetal accelerations are related as
(a) $a_1 > a_2$ (b) $a_1 = a_2$
(c) $a_1 < a_2$ (d) Data insufficient

11. A particle is moving on a circular track of radius 30 cm with a constant speed of 6 ms^{-1}. Its acceleration is
(a) zero (b) 120 ms^{-2} (c) 1.2 ms^{-2} (d) 36 ms^{-2}

12. A particle starts moving along a circle of radius $(20/\pi)$m with constant tangential acceleration. If velocity of the particle is 50 m/s at the end of the second revolution after motion has began, the tangential acceleration in ms^{-2} is
(a) 1.6 (b) 4 (c) 15.6 (d) 31.2

13. Let a_r and a_t represent radial and tangential accelerations. The motion of a particle may be circular, if
(a) $a_r = 0, a_t = 0$ (b) $a_r = 0, a_t \neq 0$
(c) $a_r \neq 0, a_t = 0$ (d) None of these

14. A car is circulating on a circular path of radius r. At some instant its velocity is v and rate of increase of speed is a. The resultant acceleration of the car will be
(a) $\sqrt{\frac{v^2}{a^2} + r^2}$ (b) $\sqrt{\frac{v^2}{r} + a}$ (c) $\sqrt{\frac{v^4}{r^2} + a^2}$ (d) $\left(\frac{v^2}{r} + a\right)$

15. A point starts from rest and moves along a circular path with a constant tangential acceleration. After one rotation, the ratio of its radial acceleration to its tangential acceleration will be equal to
(a) 1 (b) 2π (c) $\frac{1}{2}\pi$ (d) 4π

16. A particle is moving on a circular path of 10 m radius. At any instant of time, its speed is 5 ms^{-1} and the speed is increasing at a rate of 2 ms^{-2}. At this instant, the magnitude of the net acceleration will be
(a) 3.2 ms^{-2} (b) 2 ms^{-2} (c) 2.5 ms^{-2} (d) 4.3 ms^{-2}

17. A point on the rim of a flywheel has a peripheral speed of 10 ms^{-1} at an instant when it is decreasing at the rate of 60 ms^{-2}. If the magnitude of the total acceleration of the point at this instant is 100 ms^{-2}, the radius of the flywheel is
(a) 1.25 m (b) 12.5 m (c) 25 m (d) 2.5 m

18. A particle moves in a circular path of radius R with an angular velocity $\omega = a - bt$, where a and b are positive constants and t is time. The magnitude of the acceleration of the particle after time $\frac{2a}{b}$ is
(a) $\frac{a}{R}$ (b) a^2R (c) $R(a^2 + b)$ (d) $R\sqrt{a^4 + b^2}$

19. The distance of a particle moving on a circle of radius 12 m measured from a fixed point on the circle and measured along the circle is given by $s = 2t^3$ (in meters). The ratio of its tangential to centripetal acceleration at $t = 2$ s is
(a) 1 : 1 (b) 1 : 2 (c) 2 : 1 (d) 3 : 1

20. A body is moving on a circle of radius 80 m with a speed 20 m/s which is decreasing at the rate 5 ms^{-2} at an instant. The angle made by its acceleration with its velocity is
(a) 45° (b) 90° (c) 135° (d) 0°

2 Dynamics of circular motion

In this section, we will start with the forces in circular motion and further we will discuss their practical utilisation in the applications of circular motion.

Centripetal force

When a body moves along a circular path with an uniform speed, its direction changes continuously *i.e.*, velocity keeps on changing on account of a change in direction. According to Newton's first law of motion, a change in the direction of motion of the body can take place only if some external force acts on the body.

Thus, a particle performing circular motion is acted upon by a force directed along the radius towards the centre of the circle, this force is called the centripetal force.

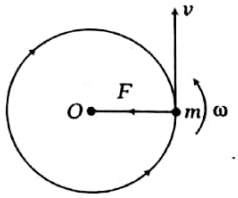


Fig. 7.8 Centripetal force on the particle

If m is mass of the particle performing circular motion as shown in Fig. 7.7, then magnitude of centripetal force is given by

Centripetal force = mass × centripetal acceleration

$$\therefore \quad F = m\left(\frac{v^2}{r}\right) \quad \text{(in magnitude)}$$

Putting $v = r\omega$

we get $\boxed{F = mr\omega^2 \text{ or } F = mv\omega}$

Putting $\omega = \frac{2\pi}{T}$ or $\omega = 2\pi n$

$$F = 2\pi mvn \Rightarrow \boxed{F = \frac{2\pi mv}{T}}$$

In vector form, centripetal force is given by

$$\boxed{\mathbf{F} = -\frac{mv^2}{r}\hat{\mathbf{r}} = -m\omega^2\mathbf{r}}$$

Centripetal Force in Different Situations

Situation	The centripetal force
A particle tied to a string and whirled in a horizontal circle	Tension in the string
Vehicle taking a turn on a level road	Frictional force exerted by the road on the tyres
A vehicle on a speed breaker	Weight of the body or a component of weight
Revolution of earth around the sun	Gravitational force exerted by the sun
Electron revolving around the nucleus in an atom	Coulomb attraction exerted by the protons on electrons
A charged particle describing a circular path in a magnetic field	Magnetic force exerted by the magnetic field

(i) In non-uniform circular motion, the particle simultaneously possesses two forces

(a) Centripetal force, $F_c = ma_c = \frac{mv^2}{r} = mr\omega^2$

(b) Tangential force, $F_t = ma_t$

$\therefore$ Net force, $\boxed{F_{net} = ma = m\sqrt{a_c^2 + a_t^2}}$

(ii) If a moving particle comes to stand still, *i.e.*, the particle will move along the radius towards the centre and if radial acceleration a_r is zero, the body will fly off along the tangent. So, a tangential velocity and a radial acceleration (hence force) is a must for uniform circular motion.

(iii) The work done by the centripetal force is always zero as it is perpendicular to velocity and displacement.

Further, by work-energy theorem

Work done = change in kinetic energy

$\therefore \quad \Delta K = 0 \quad (\because \Delta W = 0)$

i.e., K (kinetic energy) remains constant.

$$\text{Power} = \frac{\text{Work}}{\text{Time}} = \frac{0}{t} = 0$$

or Power $= \mathbf{F}_c \cdot \mathbf{v} = F_c\, v \cos 90° = 0$

(iv) Total work done $W = \mathbf{F}_t \cdot s = F_t s \cos 0° = F_t s$

$\Rightarrow$ (where, s is distance travelled by the particle)

Example 7.17 *A ball of mass 0.25 kg attached to the ends of a string of length 1.96 m is rotating in a horizontal circle. The string will break, if tension is more than 25 N. What is the maximum velocity with which the ball can be rotated?*

Sol. The centripetal force,

$$F = \frac{mv^2}{r}$$

The string will break if $F \geq 25$ N

$$v^2 \leq \frac{Fr}{m}$$

$$\Rightarrow \quad v^2 \leq \frac{25 \times 1.96}{0.25} \leq 196$$

$\therefore$ Maximum velocity of the ball, $v_{max} = \sqrt{196} = 14 \text{ ms}^{-1}$

Example 7.18 *A spaceman in training is rotated in a seat at the end of a horizontal arm of length 5 m. If he can withstand accelerations upto 9 g, then what is the maximum number of revolutions per second permissible? (Take, $g = 10 \text{ ms}^{-2}$)*

Sol. In circular motion, necessary centripetal force to the man is provided by effective weight of man.

$$\therefore \quad m \times 9g = mr\omega^2 = mr \times 4\pi^2 n^2$$

$$\Rightarrow \quad n = \sqrt{\frac{9g}{4\pi^2 r}} \Rightarrow n = \sqrt{\frac{9 \times 10}{4 \times (3.14)^2 \times 5}}$$

$$= 0.675 \text{ rev s}^{-1} \text{ or hertz (Hz)}$$

Example 7.19 *A string breaks under a load of 4.8 kg. A mass of 0.5 kg is attached to one end of the string 2 m long and is rotated in a horizontal circle determine the number of revolution per minute that the mass can make without breaking the string.*

Sol. Given, $m = 0.5$ kg, $r = 2$ m, $g = 9.8 \text{ ms}^{-2}$

The maximum tension that the string can withstand,

$$F = 4.8 \text{ kg-wt} = (4.8 \times 9.8)\text{N}$$

Let the maximum number of revolutions per second be n

$$\because \quad F = mr\omega^2 = mr(2\pi n)^2$$

$$\Rightarrow \quad n^2 = \frac{F}{4\pi^2 mr} = \frac{9.8 \times 4.8}{4 \times (3.14)^2 \times 0.5 \times 2} = 1.215$$

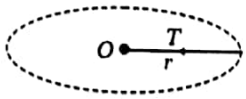
$$\Rightarrow \quad n = \sqrt{1.215} = 1.102 \text{ rps}$$

$$\Rightarrow \quad n = 1.102 \times 60 = 66.13 \text{ rpm}$$

Example 7.20 *A ball of mass 0.12 kg is being whirled in a horizontal circle at the end of a string 0.5 m long. It is capable of making 231 revolutions in one minute. Find the breaking tension of the string.*

Sol. Given, $m = 0.12$ kg, $r = 0.5$ m,

$$\omega = 231 \text{ rpm} = \frac{2\pi \times 231}{60} \text{ rps} = 24.2 \text{ rad s}^{-1}$$



When body performs circular motion, it is acted upon by a centripetal force in magnitude of which is given by

$$F = mr\omega^2 = 0.12 \times 0.5 \times (24.2)^2 = 35.1 \text{ N}$$

Centrifugal force

Centrifugal force can be defined as the radially directed outward force acting on a body in circular motion as observed by a person moving with the body.

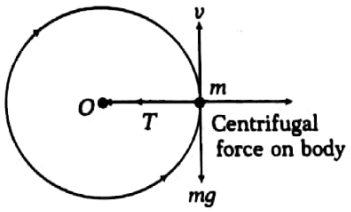


Fig. 7.9 Outward force on a body

Centrifugal force = Mass × Centrifugal acceleration

or $$F = \frac{mv^2}{r} = mr\omega^2 \quad \text{or} \quad \boxed{F = mv\omega}$$

This can be written in vector form as

$$\boxed{\mathbf{F} = \frac{mv^2}{r}\hat{\mathbf{r}}}$$

where, $\hat{\mathbf{r}}$ is the unit vector acting along $\mathbf{r}$.

(i) In an inertial frame, the centrifugal force does not act on the object.

(ii) In non-inertial frames, pseudo force arises in the form of centrifugal force.

Example 7.21 *A gramophone disc rotates at 60 rpm. If coin of mass 18 g is placed at a distance of 8 cm from the centre. Determine the centrifugal force on the coin.*

Sol. Since, angular velocity, $\omega = 60 \times \frac{2\pi}{60} = 2\pi \text{ rad s}^{-1}$.

$\therefore$ Centrifugal force, $F = m\omega^2 r = 18 \times (2\pi)^2 \times 8$

$$= 18 \times 4 \times (3.14)^2 \times 8$$

$\Rightarrow$ $$F = 5685.12 \text{ dyne}$$

$$\approx 0.06 \text{ N}$$

Examples for obtaining centripetal force in daily life

Some examples for obtaining centripetal force in daily life are given below.

1. Circular turning of roads

It is most popular example of circular motion. When vehicles go through turns, they travel along a nearly circular arc. There must be some force which will produce the required centripetal acceleration. If the vehicles travel on a horizontal circular path, this resultant force is also horizontal. *The necessary centripetal force is being provided to the vehicles by following three ways*

(i) By friction (ii) By banking of roads

(iii) By friction and banking of roads

In practical, the necessary centripetal force is provided by friction and banking of roads. Now, let us write equations of motion in each of the three cases separately and see what are the constraints in each case.

(i) By friction : motion of a car on level road

Suppose, a car of mass m is moving with a speed v in a horizontal circular arc of radius r. In this case, the necessary centripetal force to the car will be provided by force of friction f acting towards centre.

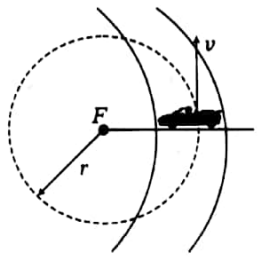


Fig. 7.10 Motion of a car on a level road

Required centripetal force, $f = \frac{mv^2}{r}$

Further, limiting value of f is μN

or $$f_L = \mu N = \mu mg \qquad (\because N = mg)$$

Here, N = normal reaction force on the car by the road.

μ = coefficient of friction between road and tyre of the car.

Therefore, for a safe turn without sliding

$$\frac{mv^2}{r} \le f_L \quad \text{or} \quad \frac{mv^2}{r} \le \mu mg \quad \text{or} \quad \mu \ge \frac{v^2}{rg}$$

or $$\boxed{v \le \sqrt{\mu rg}}$$

Here, two situations may arise. If μ and r are known to us, the speed of the vehicle should not exceed $\sqrt{\mu rg}$ and if

v and r are known to us, the coefficient of friction should be greater than v^2/rg.

Maximum velocity for no skidding or slipping

$$\boxed{v_{max} = \sqrt{\mu rg}}$$

Note You might have seen that if the speed of the car is too high, car starts skidding outwards. With this radius of the circle increases or the necessary centripetal force is reduced $\left(\text{centripetal force} \propto \frac{1}{r}\right)$.

Example 7.22 *Determine the maximum speed at which a car can turn round a curve of 30 m radius on a level road, if the coefficient of friction between the tyres and the road is 0.4.*

Sol. Given, $\mu = 0.4$, $r = 30$ m, $g = 10$ ms^{-2}

Maximum speed $v_{max} = \sqrt{\mu gr}$

$\Rightarrow$ $v_{max} = \sqrt{0.4 \times 30 \times 10} \approx 11$ ms^{-1}

Example 7.23 *A cyclist speeding at 4.5 km h^{-1} on a level road takes a sharp circular turn of radius 3 m without reducing the speed The coefficient of static friction between the road and the tyres is 0.1 will the cyclist slip while taking the turn (i) with a speed of 4.5 km h^{-1} and (ii) with a speed of 9 km h^{-1}?*

Sol. Frictional force provides the necessary centripetal force. He will slip if the turn is too sharp (*i.e.*, too small a radius) or if his speed is too large.

Maximum speed for no slipping is

$$v_{max} = \sqrt{\mu_s rg} = \sqrt{0.1 \times 3 \times 9.8} = 1.72 \text{ ms}^{-1}$$

(i) If $v = 4.5$ km h$^{-1} = 4.5 \times \frac{5}{18} = \frac{5}{4}$ ms^{-1}

$= 1.25$ ms^{-1}, he will not slip.

(ii) If $v = 9$ km h$^{-1} = 9 \times \frac{5}{18} = \frac{5}{2}$ ms$^{-1} = 2.5$ ms^{-1}, he will slip.

(ii) By banking of roads : motion of a car on banked road

Friction is not always reliable at circular turns, if high speed and sharp turns are involved. To avoid dependence on friction, the roads are banked by an angle (θ) at the turn so that the outer part of the road is somewhat raised compared to the inner part.

Applying Newton's second law along the radius and the first law in the vertical direction for the motion of a car on the banked road having mass m.

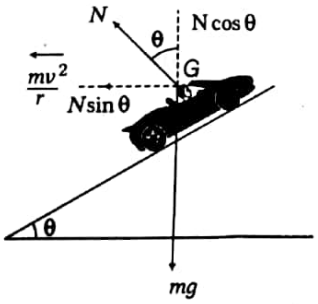


Fig. 7.11 *Motion of a car on banked road*

We get, $N\sin\theta = \frac{mv^2}{r}$

and $N\cos\theta = mg$

Here, r = radius of circular turn, θ = banking angle

From these two equations, we get

$$\boxed{\tan\theta = \frac{v^2}{rg}} \quad \text{...(i)}$$

or

$$\boxed{v = \sqrt{rg\tan\theta}} \quad \text{...(ii)}$$

Example 7.24 *A turn of radius 600 m is banked for a vehicle of mass 200 kg going with a speed of 180 kmh^{-1}. Determine the banking angle of its path.*

Sol. The turn is banked for speed,

$v = 180$ kmh$^{-1} = 180 \times \frac{5}{18}$ ms$^{-1} = 50$ ms^{-1}

and radius $r = 600$ m

$\because$ $\tan\theta = \frac{v^2}{rg} = \frac{50 \times 50}{600 \times 10}$

$\Rightarrow$ $\tan\theta = \frac{25}{60} = 0.417$

$\Rightarrow$ Banking angle $\theta = \tan^{-1}(0.417)$

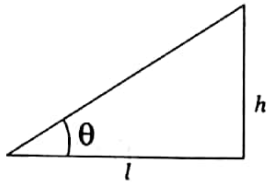
$\Rightarrow$ $\theta = 22.6°$

Example 7.25 *A train has to negotiate a curve of radius 2000 m. By how much should the outer rail be raised with respect to inner rail for a speed of 72 km h^{-1}. The distance between the rails is 1 m.*

Sol. Given, $v = 72$ km h$^{-1} = 72 \times \frac{5}{18} = 20$ ms^{-1}

$l = 1$ m, $r = 2000$ m, $g = 10$ ms^{-2}

We have, $\tan\theta = \frac{v^2}{rg}$



Also, $\tan\theta = \frac{h}{l}$

$\Rightarrow$ $\frac{v^2}{rg} = \frac{h}{l}$

$\Rightarrow$ $h = \frac{v^2 l}{rg}$

$\therefore$ $h = \frac{(20)^2 \times 1}{2000 \times 10} = \frac{1}{50}$ m

$= \frac{100}{50} = 2$ cm

(iii) By friction and banking of roads

If on a banked circular turning, there is a frictional force between car and road, then the vector sum of normal reaction force (N) and frictional force (f) provides the necessary centripetal force.

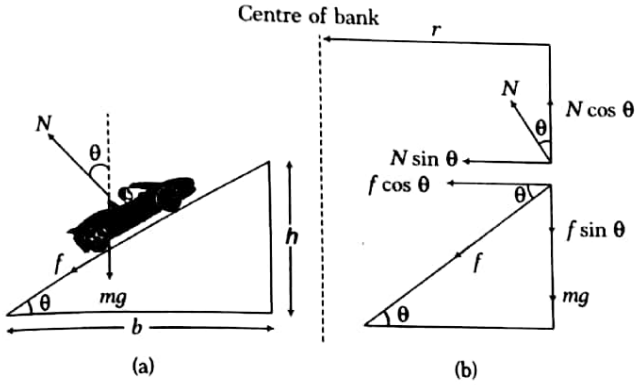


Fig. 7.12 Banked road with friction

$$\therefore \quad N\sin\theta + f\cos\theta = \frac{mv^2}{r} \quad \text{...(i)}$$

and
$$N\cos\theta = mg + f\sin\theta \quad \text{...(ii)}$$
($\because$ Vertical force is balanced)

Taking limiting condition of friction, we can write

$$f = \mu_s N \quad \text{...(iii)}$$

Here, μ_s = coefficient of static friction

To obtain the value of N, solving above three equations properly, we get,

$$N = \frac{mg}{\cos\theta - \mu_s \sin\theta}$$

After putting the value of N in Eq. (i), we get

$$v_{max} = \left[\frac{rg(\sin\theta + \mu_s\cos\theta)}{\cos\theta - \mu_s\sin\theta}\right]^{1/2}$$

$$\boxed{v_{max} = \left[\frac{rg(\mu_s + \tan\theta)}{1 - \mu_s\tan\theta}\right]^{1/2}}$$

If the vehicle is moving upward on inclined road, then we can find maximum speed for no skidding from the above formula.

If the vehicle is moving downward on inclined road, then **minimum velocity for no skidding is**

$$\boxed{v_{min} = \left[rg\frac{(\tan\theta - \mu_s)}{1 + \mu_s\tan\theta}\right]^{1/2}}$$

Note

1. For no slipping or skidding, we have, $v_{min} < v \le v_{max}$
   This speed is greater than the maximum possible speed of a car on level road ($v = \sqrt{\mu g r}$).
2. If $\mu_s = 0$, $v_0 = (gR\tan\theta)\ 1/2$
   This speed is known as **optimum speed**.

Example 7.26 *A circular race track of radius 300 m is banked at an angle of 15°. If the coefficient of friction between the wheels of a race car and the road is 0.2, then what will be the maximum permissible speed to avoid slipping?* [tan 15° = 0.26]

Sol. We know that maximum permissible speed on a banked road to avoid slipping is,

$$v_{max} = \left[\frac{rg(\mu_s + \tan\theta)}{(1 - \mu_s\tan\theta)}\right]^{1/2}$$

Now, putting the values given in the question,

$r = 300$ m, $\theta = 15°$, $g = 9.8\ (\approx 10)\ \text{ms}^{-2}$ and $\mu_s = 0.2$

We obtain, $$v_{max} = \left[\frac{300\times 9.8\,(0.2 + \tan 15°)}{1 - 0.2\tan 15°}\right]^{1/2}$$

$$= \left[\frac{300\times 9.8\,(0.2 + 0.26)}{1 - 0.2\times 0.26}\right]^{1/2}$$

After solve this, we get, $v_{max} = 38.1\ \text{ms}^{-1}$

2. Bending of a cyclist

To take a safe turn on a circular turning, cyclist bend himself inward. When the cyclist is inclined to the centre of the rounding of its path, the resultant of N, f and mg is directed horizontally to the centre of the circular path of the cycle. This resultant force imparts a centripetal acceleration to the cyclist.

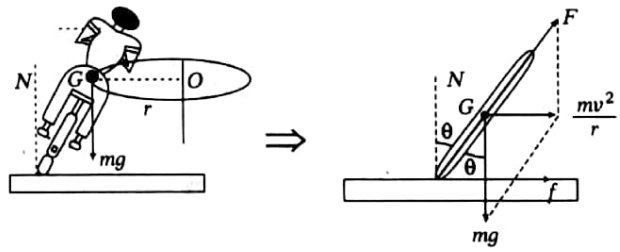


Fig. 7.13 Bending of a cyclist from vertical

From the above figure we get, resultant of normal reaction force (N) and frictional force (f),

$$F = \sqrt{N^2 + f^2}$$

This resultant of N and f, *i.e.*, F should pass through G, the centre of gravity of cyclist (for complete equilibrium, rotational as well as translational). Hence,

$$\boxed{\tan\theta = \frac{f}{N}}$$

where, $f = \dfrac{mv^2}{r}$ and $N = mg \Rightarrow \boxed{\tan\theta = \dfrac{v^2}{rg}}$

Note The angle through which cyclist should bend will be greater, if
(i) the radius of the curve is small.
(ii) the velocity of the cyclist is large.

Example 7.27 *A cyclist speeding at 6 ms^{-1} in a circle of radius 18 m makes an angle θ with the vertical. Determine the value of θ. Also, determine the minimum possible value of coefficient of friction between the tyres and the ground.*

Sol. Given, $v = 6\ \text{ms}^{-1}$, $r = 18$ m, $g = 9.8\ \text{ms}^{-2}$.

Since, $\tan\theta = \dfrac{v^2}{rg} \Rightarrow \tan\theta = \dfrac{6\times 6}{18\times 9.8} = 0.2041$

$\Rightarrow \quad \theta = \tan^{-1}(0.2041) \Rightarrow \theta = 11°32'$

Also, minimum possible value of coefficient of friction

$$\mu = \tan\theta = \frac{v^2}{rg} = 0.2041 \Rightarrow \mu = 0.2041$$

3. Conical pendulum

If a small particle of mass m tied to a string is whirled in a horizontal circle as shown in figure. The arrangement is called the conical pendulum. In case of conical pendulum the vertical component of tension balances the weight while its horizontal component provides the necessary centripetal force.

Thus, $$T\sin\theta = \frac{mv^2}{r} \quad \text{...(i)}$$

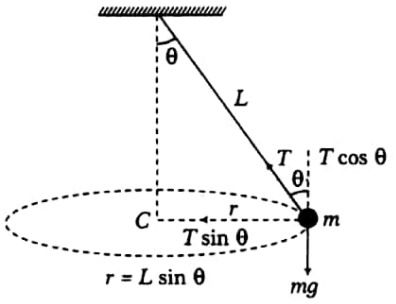


Fig. 7.14 *Conical pendulum*

and $$T\cos\theta = mg \quad \text{...(ii)}$$

From these two equations, we can find $v = \sqrt{rg\tan\theta}$

$\therefore$ Angular speed, $\omega = \dfrac{v}{r} = \sqrt{\dfrac{g\tan\theta}{r}}$

So, the time period of pendulum is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{r}{g\tan\theta}} = 2\pi\sqrt{\frac{L\cos\theta}{g}}$$

or $$\boxed{T = 2\pi\sqrt{\frac{L\cos\theta}{g}}}$$

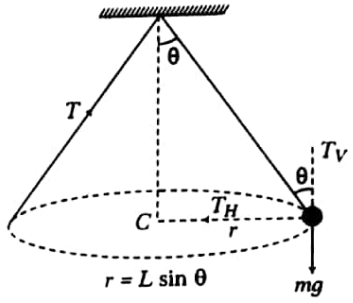
Example 7.28 *A particle of mass 200 g tied to one end of string is revolved in a horizontal circle of radius 50 cm about a vertical axis passing through the point of suspension, with angular speed 60 rev per minute (rpm). Find (a) linear speed, (b) the acceleration and (c) horizontal component of tension in the string. What will happen if string is broken? (Take, $\pi^2 = 10$)*

Sol. Angular speed, $\omega = \dfrac{2\pi n}{60} = \dfrac{2\pi\times 60}{60}$

$= 2\pi\ \text{rad s}^{-1} = 6.28\ \text{rad s}^{-1}$

Horizontal component of the tension, $T_H = T\sin\theta$,

vertical component of tension, $T_V = T\cos\theta = mg$



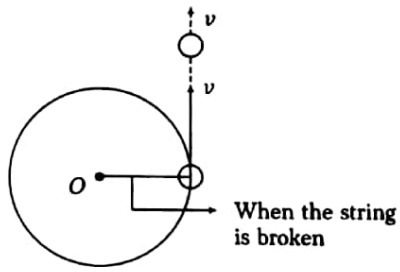
(a) Linear speed $v = r\omega = 0.5\times 2\pi = \pi\ \text{ms}^{-1} = 3.14\ \text{ms}^{-1}$

(b) Acceleration $a_c = \dfrac{v^2}{r} = \dfrac{(\pi)^2}{0.5} \approx \dfrac{10}{0.5} = 20\ \text{ms}^2$

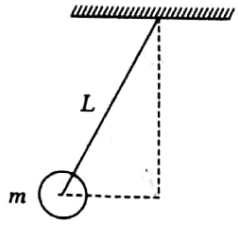
(c) Horizontal component of tension

$$T_H = \frac{mv^2}{r} = 0.2\times 20 = 2\ \text{N}$$

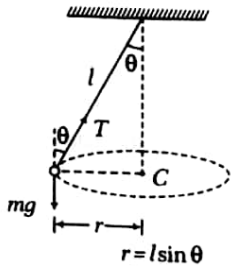
When the string is broken, tension T_H (*i.e.*, centripetal force) vanishes and body moves along the tangent in a straight line with speed 3.14 ms^{-1}.



Example 7.29 *A ball of mass (m) 0.5 kg is attached to the end of a string having length (L) 0.5 m. The ball is rotated on a horizontal circular path about vertical axis. The maximum tension that the string can bear is 324 N. Find the maximum possible value of angular velocity of ball (in rads^{-1}).*



Sol. Consider the forces acting on the ball as shown in the figure.



$T\cos\theta$ component will cancel mg.

$T\sin\theta$ component will provide necessary centripetal force to the ball towards centre C.

$\therefore \quad T\sin\theta = mr\omega^2 = m(l\sin\theta)\,\omega^2 \quad$ or $\quad T = ml\omega^2$

$\therefore$ Angular velocity $\omega = \sqrt{\dfrac{T}{ml}}$

or $$\omega_{max} = \sqrt{\frac{T_{max}}{ml}} = \sqrt{\frac{324}{0.5\times 0.5}} = 36 \text{ rads}^{-1}$$

Example 7.30 *A boy whirls a stone in a horizontal circle of radius 1.5 m and at height 2 m above level ground. The string breaks, and the stone flies off horizontally and strikes the ground after travelling a horizontal distance of 10 m. What is the magnitude of the centripetal acceleration of the stone while in circular motion?*

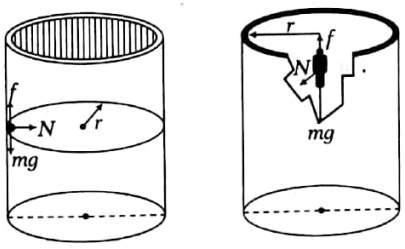
Sol. Time period $t = \sqrt{\dfrac{2h}{g}} = \sqrt{\dfrac{2\times 2}{9.8}} = 0.64\,\text{s}$

$\Rightarrow$ Linear velocity $v = \dfrac{10}{t} = 15.63 \text{ ms}^{-1}$

$\therefore$ Centripetal acceleration $a = \dfrac{v^2}{R} = 163 \text{ ms}^{-2}$

4. 'Death well' or rotor

In case of death well a person drives a bicycle on a vertical surface of a large wooden well while in case of a rotor at a certain angular speed of rotor a person hangs resting against the wall without any support from the bottom. In death well walls are at rest and person revolves while in case of rotor person is at rest and the walls rotate.



(A) Death well (B) Rotor

Fig. 7.15

In both cases friction balances the weight of person while reaction provides the centripetal force for circular motion, *i.e.*,

$$f = mg$$

and $$\boxed{N = \frac{mv^2}{r} = mr\omega^2} \qquad (v = r\omega)$$

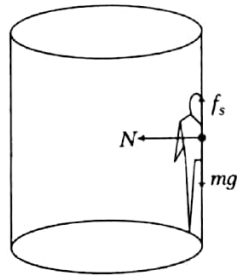
$\therefore \quad f \le \mu N \Rightarrow mg \le \mu\, mv^2/r \Rightarrow v^2 \ge \dfrac{gr}{\mu}$

$\therefore \quad$ Safe speed $v = \sqrt{\dfrac{gr}{\mu}}$

Example 7.31 *In a rotor, a hollow vertical cylinder rotates about its axis and a person rest against the inner wall. At a particular speed of the rotor, the floor below the person is removed and the person hangs resting against the wall without any floor.*

If the radius of the rotor is 2 m and the coefficient of static firction between the wall and the person is 0.2. Find the minimum speed at which the floor may be removed.

Sol. *The situation is shown in figure below*



When the floor is removed, the forces on the person are

(i) weight mg downward.

(ii) normal force N due to the wall towards the centre.

(iii) frictional force f_s parallel to the wall, upwards.

The person is moving in a circle with a uniform speed, so its acceleration is v^2/r towards the centre.

For the minimum speed when the floor may be removed, the friction is limiting one and so equals $\mu_s N$.

Theis gives

$$\mu_s N = mg \quad \text{or} \quad \frac{\mu_s mv^2}{r} = mg$$

or $$v = \sqrt{\frac{rg}{\mu_s}} = \sqrt{\frac{2\times 10}{0.2}} = 10 \text{ ms}^{-1}$$

Motion of a particle tied to a string in vertical circle

Suppose a particle of mass m is attached to an inextensible light string of length L. The particle is moving in a vertical circle of radius L about a fixed point O.

It is imparted a velocity u in horizontal direction at lowest point A. Let v be its velocity at point B, the circle as shown in Fig. 7.15.

Here we have

$$h = R(1-\cos\theta)$$

Now from conservation of mechanical energy, we have

$$\frac{1}{2}m(u^2 - v^2) = mgh \qquad \text{...(i)}$$

The necessary centripetal force is provided by the resultant of tension T and $mg\cos\theta$.

$$\therefore \quad T - mg\cos\theta = \frac{mv^2}{R} \qquad \text{...(ii)}$$

As speed of the particle decreases with height, tension in the string is maximum at the bottom. The particle will complete the circle if the string does not slack even at the highest point.

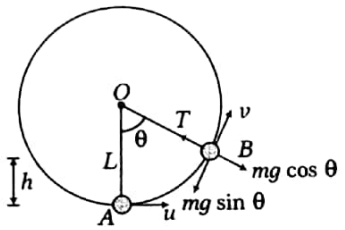


Fig. 7.16 Motion of a particle in vertical circle

Now following conclusions can be made using above Eqs. (i) and (ii).

(i) **Minimum velocity at highest point** so that particle complete the circle $v_{min} = \sqrt{gL}$, at this velocity, tension in the string is zero.

(ii) **Minimum velocity at lowest point** so that particle complete the circle, $v_{min} = \sqrt{5gL}$, at this velocity, tension in the string is 6 *mg*.

(iii) **When string is horizontal**, then minimum velocity is $\sqrt{3Rg}$ and tension in this condition is 3 *mg*.

(iv) If velocity at lowest point is less than $\sqrt{5gR}$, then tension in the string becomes zero before reaching the highest point, now the particle will leave the circle and will move on parabolic path.
In this condition, if $\sqrt{2gR} < v < \sqrt{5gR}$ then tension in the string becomes zero but velocity is not zero, the particle will leave circle at $90° < \theta < 180°$ or $h > R$

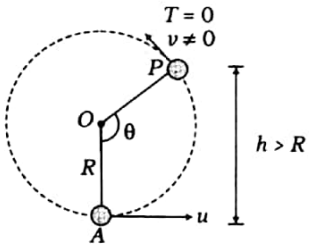


Fig. 7.17

(v) If velocity at lowest point is $0 < v \le \sqrt{2gR}$, the particle will oscillate, in this condition velocity becomes zero but tension is not zero. The particle will oscillate in lower half of circle, *i.e.*, $0° < \theta < 90°$

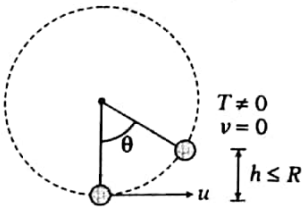


Fig. 7.18

Note The above points have been derived for a particle moving in a vertical carcle attached to a string. The same conditions apply if a particle moves inside a smooth spherical shell of radius *R*. The only difference is that the tension is replaced by the normal reaction *N*,

Example 7.32 *One end of a string of length 1 m is tied to a body of mass 0.5 kg. It is whirled in a vertical circle with angular velocity 4 rad s^{-1}. Find the tension in the string when the body is at the lower most point of its motion. (take, $g = 10\ ms^{-2}$)*

Sol. At lower most point, the tension in the string,

$$T = m\omega^2 r + mg$$
$$= 0.5 \times (4)^2 \times 1 + 0.5 \times 10$$
$$= 13\ \text{N}$$

Example 7.33 *A ball of mass 0.6 kg attached to a light inextensible string rotates in a vertical circle of radius 0.75 m such that it has speed of 5 ms^{-1} when the string is horizontal. Tension in string when it is horizontal on other side is (take, $g = 10\,ms$).*

Sol. Tension in the string when it makes angle θ with the vertical

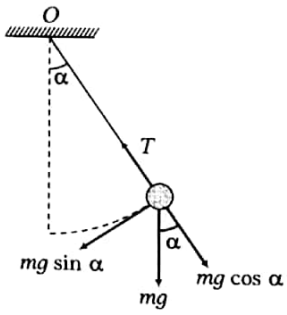
$$T = \frac{mv^2}{r} + mg\cos\theta$$

When the string is horizontal, $\theta = 90°$

$$\therefore \quad T = \frac{mv^2}{r} + mg \times 0 = \frac{mv^2}{r}$$
$$= \frac{0.6 \times (5)^2}{0.75} = 20\ \text{N}$$

Example 7.34 *A simple pendulum is constructed by attaching a bob of mass m to a string of length L fixed at its upper end. The bob oscillates in a vertical circle. It is found that the speed of the bob is v when the string makes an angle α with the vertical. Find the tension in the string and the magnitude of net force on the bob at that instant.*

Sol. (i) The force acting on the bob are
(a) the tension *T* (b) the weight *mg*



As the bob moves in a circle of radius *L* with centre at *O*. A centripetal force of magnitude $\frac{mv^2}{L}$ is required towards *O*. This force will be provided by the resultant of *T* and *mg* cos α. Thus,

$$T - mg\cos\alpha = \frac{mv^2}{L}$$

or $$T = m\left(g\cos\alpha + \frac{v^2}{L}\right)$$

(ii) $$|\mathbf{F}_{net}| = \sqrt{(mg\sin\alpha)^2 + \left(\frac{mv^2}{L}\right)^2}$$

$$= m\sqrt{g^2\sin^2\alpha + \frac{v^4}{L^2}}$$

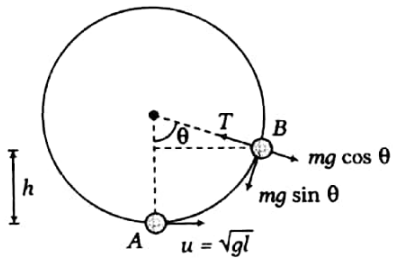
Example 7.35 *A heavy particle hanging from a fixed point by a light inextensible string of length l is projected horizontally with speed $\sqrt{gl}$. Find the speed of the particle and the inclination of the string to the vertical at the instant of the motion when the tension in the string is equal to the weight of the particle.*

Sol. Let $T = mg$ at angle θ as shown in figure.

$$h = l(1 - \cos\theta) \quad ...(i)$$

Applying conservation of mechanical energy between points A and B, we get

$$\frac{1}{2}m(u^2 - v^2) = mgh$$



Here, $$u^2 = gl \quad ...(ii)$$

and v = speed of particle in position B

$\therefore$ $$v^2 = u^2 - 2gh \quad ...(iii)$$

Further, $$T - mg\cos\theta = \frac{mv^2}{l}$$

or $$mg - mg\cos\theta = \frac{mv^2}{l} \quad (T = mg)$$

or $$v^2 = gl(1 - \cos\theta) \quad ...(iv)$$

Substituting values of v^2, u^2 and h from Eqs. (iv), (ii) and (i) in Eq. (iii), we get

$$gl(1 - \cos\theta) = gl - 2gl(1 - \cos\theta)$$

or $$\cos\theta = \frac{2}{3}$$

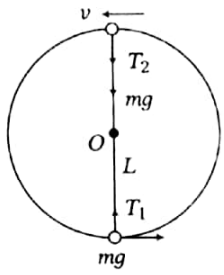
or $$\theta = \cos^{-1}\left(\frac{2}{3}\right)$$

Substituting $\cos\theta = \frac{2}{3}$ in Eq. (iv), we get

$$v = \sqrt{\frac{gl}{3}}$$

Example 7.36 *A particle of mass m is attached to a string of length L and given velocity $\sqrt{10gL}$ in the horizontal direction at the lowest point. Find tension in the string when the particle is at (a) (i) lowest position (ii) highest position; (b) when the string makes an angle 60° with (i) lower vertical and (ii) upper vertical.*

Sol. (a)



(i) At the lowest position:

$$T_1 - mg = \frac{mu^2}{L} = \frac{m}{L} \times 10gL \Rightarrow T_1 = 11\ mg$$

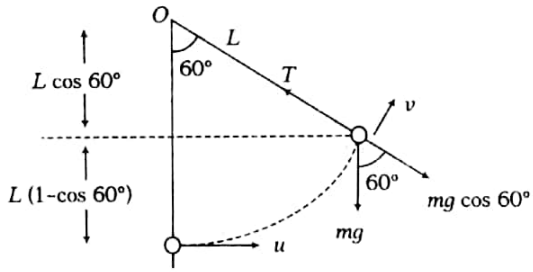
(ii) At the highest position

$$v^2 = u^2 - 2gh = u^2 - 2g \times 2L = 10gL - 4gL = 6gL$$

Also, $$T_2 + mg = \frac{mv^2}{L} = \frac{m}{L} \times 6\,gL = 6\,mg$$

$$\Rightarrow \quad T_2 = 5\,mg$$

(b) (i)



In this situation we can write

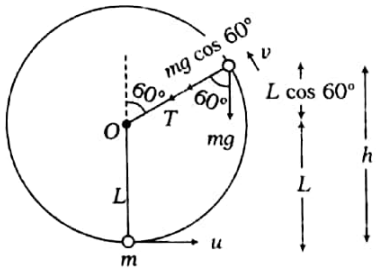
$$v^2 = u^2 - 2gh = u^2 - 2gL(1 - \cos 60°)$$

$$= 10gL - 2gL\left(1 - \frac{1}{2}\right) = 9\,gL$$

Also, $$T - mg\cos 60° = \frac{mv^2}{L} = \frac{m}{L} \times 9\,gL = 9\,mg$$

$$\Rightarrow \quad T - \frac{mg}{2} = 9\,mg \Rightarrow T = \frac{19}{2}mg$$

(ii)



In this situation we have

$$v^2 = u^2 - 2\,gh = 10\,gL - 2\,gL(1 + \cos 60°) = 7\,gL$$

Also, $T + mg\cos 60° = \frac{mv^2}{L}$

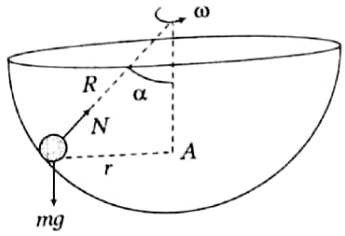
$\Rightarrow \quad T + \frac{mg}{2} = \frac{m}{L} \times 7gL = 7mg \Rightarrow T = \frac{13mg}{2}$

Example 7.37 *A hemispherical bowl of radius R is rotating about its axis of symmetry which is kept vertical. A small ball kept in the bowl rotates with the bowl without slipping on its surface. If the surface of the bowl is smooth and the angle made by the radius through the ball with the vertical is α. Find the angular speed at which the bowl is rotating.*

Sol. Let ω be the angular speed of rotation of the bowl. Two forces are acting on the ball.

(i) normal reaction N

(ii) weight mg



The ball is rotating in a circle of radius $r (= R\sin\alpha)$ with centre at A at an angular speed ω. Thus,

$$N\sin\alpha = mr\omega^2 = mR\omega^2\sin\alpha \quad \text{...(i)}$$

and $$N\cos\alpha = mg \quad \text{...(ii)}$$

Dividing Eqs. (i) by (ii), we get

$$\frac{1}{\cos\alpha} = \frac{\omega^2 R}{g} \Rightarrow \omega = \sqrt{\frac{g}{R\cos\alpha}}$$

Check point 7.2

1. A particle of mass 2 kg is moving along a circular path of radius 1 m. If its angular speed is 2π rad s^{-1}, the centripetal force on it is

(a) 4π N (b) 8π N (c) $4\pi^4$ N (d) $8\pi^2$ N

2. Two particles of equal masses are revolving in circular paths of radii r_1 and r_2 respectively with the same speed. The ratio of their centripetal forces is

(a) $\frac{r_2}{r_1}$ (b) $\sqrt{\frac{r_2}{r_1}}$ (c) $\left(\frac{r_1}{r_2}\right)^2$ (d) $\left(\frac{r_2}{r_1}\right)^2$

3. A particle of mass m is executing uniform circular motion on a path of radius r. If p is the magnitude of its linear momentum. The radial force acting on the particle is

(a) pmr (b) $\frac{rm}{p}$ (c) $\frac{mp^2}{r}$ (d) $\frac{p^2}{rm}$

4. A stone of mass of 16 kg is attached to a string 144 m long and is whirled in a horizontal circle on a smooth surface. The maximum tension the string can withstand is 16 N. The maximum velocity of revolution that can be given to the stone without breaking it, will be

(a) 20 ms^{-1} (b) 16 ms^{-1}
(c) 14 ms^{-1} (d) 12 ms^{-1}

5. If mass, speed and radius of the circle of a particle moving uniformly in a circular path are all increased by 50%, the necessary force required to maintain the body moving in the circular path will have to be increased by

(a) 225% (b) 125%
(c) 150% (d) 100%

6. A string of length 0.1 m cannot bear a tension more than 100 N. It is tied to a body of mass 100 g and rotated in a horizontal circle. The maximum angular velocity can be

(a) 100 rad s^{-1} (b) 1000 rad s^{-1}
(c) 10000 s^{-1} (d) 0.1 rad s^{-1}

7. A mass of 2 kg is whirled in a horizontal circle by means of a string at an initial speed of 5 rev min^{-1}. Keeping the radius constant the tension in the string is doubled. The new speed is nearly

(a) $\frac{5}{\sqrt{2}}$ rpm (b) 10 rpm
(c) $10\sqrt{2}$ rpm (d) $5\sqrt{2}$ rpm

8. A mass of 100 g is tied to one end of a string 2 m long. The body is revolving in a horizontal circle making a maximum of 200 rev min^{-1}. The other end of the string is fixed at the centre of the circle of revolution. The maximum tension that the string can bear is (approximately)

(a) 8.76 N (b) 8.94 N
(c) 89.42 N (d) 87.64 N

9. A car is taking turn on a circular path of radius R. If the coefficient of friction between the tyres and road is μ, the maximum velocity for no slipping is

(a) μRg (b) $2\mu Rg$
(c) $(\mu Rg)^{1/2}$ (d) $(2\mu Rg)^{1/2}$

10. A car is moving in a circular horizontal track of radius 10 m with a constant speed of 10 ms^{-1}. A plumb bob is suspended from the roof of the car by a light rigid rod of length 1 m. The angle made by the rod with track is

(a) zero (b) 30° (c) 45° (d) 60°

11. Radius of the curvedroad on national highway is R. Width of the road is b. The outer edge of the road is raised by h with respect to inner edge so that a car with velocity v can pass safely over it. The value of h is

(a) $\frac{v^2 b}{Rg}$ (b) $\frac{v}{Rgb}$
(c) $\frac{v^2 R}{bg}$ (d) $\frac{v^2 b}{R}$

12. A motor cyclist moving with a velocity of 72 km h^{-1} on a flat road takes a turn on the road at a point where the radius of curvature of the road is 20 m. The acceleration due to gravity is 10 ms^{-2}. In order to avoid skidding, he must not bend with respect to the vertical plane by an angle greater then

(a) $\theta = \tan^{-1}(6)$ (b) $\theta = \tan^{-1}(2)$

(c) $\theta = \tan^{-1}(25.92)$ (d) $\theta = \tan^{-1}(4)$

13. A car of mass 1000 kg negotiates a banked curve of radius 90 m on a frictionless road. If the banking angle is 45°, the speed of the car is

(a) 20 ms^{-1} (b) 30 ms^{-1} (c) 5 ms^{-1} (d) 10 ms^{-1}

14. Keeping the angle of banking, unchanged if the radius of curvature is made four times, the percentage increase in the maximum speed with which a vehicle can travel on a circular road is

(a) 25 % (b) 50% (c) 75% (d) 100%

15. A person wants to drive on the vertical surface of a large cylindrical wooden 'well' commonly known as 'death well' in a circus. The radius of the well is R and the coefficient of friction between the tyres of the motorcycle and the wall of the well is μ_s. The minimum speed, the motorcycle must have in order to prevent slipping, should be

(a) $\sqrt{\frac{Rg}{\mu_s}}$ (b) $\sqrt{\frac{\mu_s}{Rg}}$ (c) $\sqrt{\frac{\mu_s g}{R}}$ (d) $\sqrt{\frac{R}{\mu_s g}}$

16. A motorcyclist wants to drive on the vertical surface of wooden 'well' of radius 5 m, with a minimum speed of $5\sqrt{5}$ ms^{-1}. The minimum value of coefficient of friction between the tyres and the wall of the well must be (take, $g = 10\,ms^{-2}$)

(a) 0.10 (b) 0.20 (c) 0.30 (d) 0.40

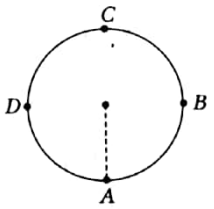
17. A block of mass m at the end of a string is whirled round in a vertical circle of radius R. The critical speed of the block at top of its swing below which the string would slacken before the block reaches the bottom is?

(a) $\sqrt{5Rg}$ (b) $\sqrt{3Rg}$ (c) $\sqrt{2Rg}$ (d) $\sqrt{Rg}$

18. A stone of mass 1 kg is tied to the end of a string 1 m long. It is whirled in a vertical circle. The velocity of the stone at the bottom of the circle is just sufficient to take it to the top of circle without slackening of the string. What is the tension in the string at the top of the circle? (Take, $g = 10\,ms^{-2}$)

(a) Zero (b) 1 N (c) $\sqrt{10}$ N (d) 10 N

19. A stone is attached to one end of a string and rotated in a vertical circle. If string breaks at the position of maximum tension, it will break at



(a) A (b) B

(c) C (d) D

20. A particle is moving in a vertical circle. The tensions in the string when passing through two positions at angles 30° and 60° from vertical (lowest position) are T_1 and T_2 respectively, then

(a) $T_1 = T_2$ (b) $T_2 > T_1$

(c) $T_1 > T_2$ (d) Data insufficient

21. A particle of mass m is being circulated on a vertical circle of radius r. If the speed of particle at the highest point be v, then

(a) $mg = \frac{mv^2}{r}$ (b) $mg > \frac{mv^2}{r}$

(c) $mg \le \frac{mv^2}{r}$ (d) $mg \ge \frac{mv^2}{r}$

22. A small sphere of mass m is suspended by a thread of length l. It is raised upto the height of suspension with thread fully stretched and released. Then, the maximum tension in thread will be

(a) mg (b) $2\,mg$

(c) $3\,mg$ (d) $6\,mg$

23. A child is swinging a swing. Minimum and the maximum heights of swing from earth's surface are 0.75 m and 2 m respectively. The maximum velocity of this swing is ($g = 10\,ms^{-2}$)

(a) 5 ms^{-1} (b) 10 ms^{-1}

(c) 15 ms^{-1} (d) 20 ms^{-1}

24. A national roadway bridge over a canal is in the form of an arc of a circle of radius 49 m. What is the maximum speed with which a car can move without leaving the ground at the highest point? (Take, $g = 9.8\,ms^{-2}$)

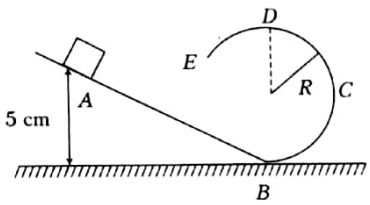
(a) 19.6 ms^{-1}

(b) 40 ms^{-1}

(c) 22 ms^{-1}

(d) None of the above

25. A frictionless track $ABCDE$ ends in a circular loop of radius R. A body slides down the track from point A which is at height h = 5 cm. Maximum value of R for a body to complete the loop successfully is



(a) 2 cm (b) $\frac{10}{3}$ cm

(c) $\frac{15}{4}$ cm (d) $\frac{18}{3}$ cm

Chapter exercises

A Taking it together

(Assorted questions of the chapter for advanced level practice)

1. A particle moves in a uniform circular motion. Choose the wrong statement.
(a) The particle moves with constant speed
(b) The acceleration is always normal to the velocity
(c) The particle moves with uniform acceleration
(d) The particle moves with variable velocity

2. A stone is tied with a string and is rotated in a circle horizontally. When the string suddenly breaks, the stone will move
(a) tangential to the motion (b) away from the centre
(c) towards the centre (d) None of these

3. A particle moving along a circular path due to a centripetal force having constant magnitude is an example of motion with
(a) constant speed and velocity
(b) variable speed and variable velocity
(c) variable speed and constant velocity
(d) constant speed and variable velocity

4. A particle moving on a circular path makes 600 rpm. In how much time it will complete one revolution?
(a) 0.2 s (b) 0.1 s (c) 0.4 s (d) 0.3 s

5. A wheel having moment of inertia 2 kgm^{-2} about its axis, rotates at 50 rpm about this axis. The angular retardation that can stop the wheel in one minute is
(a) $\frac{\pi}{36}$ rad s^{-2} (b) $\frac{\pi}{18}$ rad s^{-2} (c) $\frac{\pi}{72}$ rad s^{-2} (d) $\frac{\pi}{9}$ rad s^{-2}

6. The speed of a particle moving in a circle is increasing. The dot product of its acceleration and velocity is
(a) negative
(b) zero
(c) positive
(d) may be positive or negative

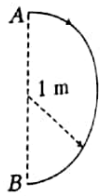
7. A particle moves with constant angular velocity in a circle. During the motion its
(a) energy is conserved
(b) momentum is conserved
(c) energy and momentum both are conserved
(d) None of the above

8. An object is moving in a circle of radius 100 m with a constant speed of 31.4 ms^{-1}. What is its average speed for one complete revolution?
(a) Zero (b) 31.4 ms^{-1}
(c) 3.14 ms^{-1} (d) $\sqrt{2} \times 31.4$ ms^{-1}

9. A bucket full of water is rotated in a vertical circle of radius R. If the water does not split out, the speed of the bucket at topmost point will be
(a) $\sqrt{Rg}$ (b) $\sqrt{5gR}$
(c) $\sqrt{2Rg}$ (d) $\sqrt{\left(\frac{R}{g}\right)}$

10. A car when passes through a convex bridge with velocity v exerts a force on it which is equal to
(a) $Mg + \frac{Mv^2}{r}$ (b) $\frac{Mv^2}{r}$
(c) $Mg - \frac{Mv^2}{r}$ (d) None of these

11. In 1 s, a particle goes from point A to point B, moving in a semicircle of radius 1 m (see figure). The magnitude of the average velocity is



(a) 3.14 ms^{-1} (b) 2 ms^{-1}
(c) 1 ms^{-1} (d) zero

12. A fan makes 2400 rpm. If after it is switched off, it comes to rest in 10 s, then find the number of times it will rotate before it comes to rest after it is switched off.
(a) 400 (b) 100 (c) 200 (d) 50

13. The distance between the rails of the track is 1.67 m. How much the outer rail be elevated for curve of 0.5 km radius, so that a train moving with speed 54 kmh^{-1} can take safe turn on track.
(a) 80 mm (b) 75 mm
(c) 60 mm (d) 75 cm

14. A car is moving on a circular path and takes a turn. If R_1 and R_2 be the reactions on the inner and outer wheels respectively, then

(a) $R_1 = R_2$ (b) $R_1 < R_2$
(c) $R_1 > R_2$ (d) $R_1 \geq R_2$

15. An unbanked curve has a radius of 60 m. The maximum speed at which a car can make a turn if the coefficient of static friction is 0.75 is

(a) $2.1\ ms^{-1}$ (b) $14\ ms^{-1}$ (c) $21\ ms^{-1}$ (d) $7\ ms^{-1}$

16. A particle is moving in a circle with uniform speed v. In moving from a point to another diametrically opposite point

(a) the momentum changes by mv
(b) the momentum changes by $2\ mv$
(c) the kinetic energy changes by $\left(\frac{1}{2}\right)mv^2$
(d) the kinetic energy changes by mv^2

17. A motor cyclist riding at $36\ kmh^{-1}$ has to turn a corner. Find the least radius of the curve, he should follow for safe travelling, if the coefficient of friction between the tyres and the road is 0.2.

(a) 10 m (b) 25 m (c) 50 m (d) 100 m

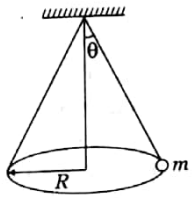
18. A circular curve of a highway is designed for traffic moving at $72\ kmh^{-1}$. If the radius of the curved path is 100 m, the correct angle of banking of the road should be

(a) $\tan^{-1}\left(\frac{2}{5}\right)$ (b) $\tan^{-1}\left(\frac{3}{5}\right)$
(c) $\tan^{-1}\left(\frac{1}{5}\right)$ (d) $\tan^{-1}\left(\frac{1}{4}\right)$

19. A particle of mass m is circulating on a circle of radius r having angular momentum L about centre. Then, the centripetal force will be

(a) $\frac{L^2}{mr}$ (b) $\frac{L^2}{mr^2}$
(c) $\frac{L^2}{mr^3}$ (d) $\frac{L}{mr^2}$

20. A conical pendulum of length L makes an angle θ with the vertical. The time period will be



(a) $2\pi\sqrt{\frac{L\cos\theta}{g}}$ (b) $2\pi\sqrt{\frac{L}{g\cos\theta}}$
(c) $2\pi\sqrt{\frac{L\tan\theta}{\gamma}}$ (d) $2\pi\sqrt{\frac{L}{g\tan\theta}}$

21. A body of mass 1 kg is moving in a vertical circular path of radius 1 m. The difference between the kinetic energies at its highest and lowest positions is

(a) 20 J (b) 10 J
(c) $4\sqrt{5}$ J (d) $10(\sqrt{5}-1)$ J

22. A particle is moving along a circular path of radius 5 m with a uniform speed $5\ ms^{-1}$. What will be the average acceleration when the particle completes half revolution?

(a) Zero (b) $10\ ms^{-2}$ (c) $10\pi\ ms^{-2}$ (d) $\frac{10}{\pi}\ ms^{-2}$

23. If the banking angle of curved road is given by $\tan^{-1}\left(\frac{3}{5}\right)$ and the radius of curvature of the road is 6 m, then the safe driving speed should not exceed (take, $g = 10\ ms^{-2}$)

(a) $86.4\ km\ h^{-1}$ (b) $43.2\ km\ h^{-1}$
(c) $21.6\ km\ h^{-1}$ (d) $30.4\ km\ h^{-1}$

24. A motor-cyclist moving with a velocity of $144\ kmh^{-1}$ on a flat road takes a turn on the road at a point where the radius of curvature of the road is 40 m. The acceleration due to gravity is $10\ ms^{-2}$. In order to avoid sliding, he must bend with respect to the vertical plane by an angle

(a) $\theta = \tan^{-1}(4)$ (b) $\theta = 45°$
(c) $\theta = \tan^{-1}(2)$ (d) $\theta = \tan^{-1}(6)$

25. A train has to negotiate a curve of radius 800 m. By how much height should the outer rail be raised with respect to inner rail for a speed of $96\ kmh^{-1}$? The distance between the rails is 1 m

(a) 4.4 cm (b) 9 cm (c) 8.8 cm (d) 3.3 cm

26. A car wheel is rotated to uniform angular acceleration about its axis. Initially its angular velocity is zero. It rotates through an angle θ_1 in the first 2 s. In the next 2 s, it rotates through an additional angle θ_2, the ratio of $\frac{\theta_2}{\theta_1}$ is

(a) 1 (b) 2 (c) 3 (d) 4

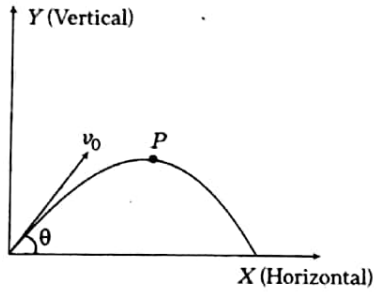
27. A pendulum bob on a 2m string is displaced 60° from the vertical and then released. What is the speed of the bob as it passes through the lowest point in its path?

(a) $\sqrt{2}\ ms^{-1}$ (b) $\sqrt{9.8}\ ms^{-1}$ (c) $4.43\ ms^{-1}$ (d) $\frac{1}{\sqrt{2}}\ ms^{-1}$

28. The bob of a 0.2 m pendulum describes an arc of circle in a vertical plane. If the tension in the cord is $\sqrt{3}$ times the weight of the bob when the cord makes an angle 30° with the vertical, the acceleration of the bob in that position is

(a) g (b) $\frac{g}{2}$ (c) $\frac{\sqrt{3}\,g}{2}$ (d) $\frac{g}{4}$

29. In a typical projectile motion, tangential acceleration at the topmost point P of the trajectory is

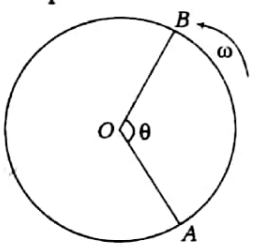


(a) g (b) $g \cos \theta$
(c) 0 (d) None of these

30. A jeep runs around a curve of radius 0.3 km at a constant speed of 60 ms^{-1}. The jeep covers a curve of 60° arc
(a) resultant change in velocity of jeep is 60 ms^{-1}
(b) instantaneous acceleration of jeep is 12 ms^{-2}
(c) average acceleration of jeep is approximately 11.5 ms^{-2}
(d) All are correct

31. A coin placed on a rotating turn-table slips, when it is placed at a distance of 9 cm from the centre. If the angular velocity of the turn-table is trippled, it will just slip, if its distance from the centre is
(a) 27 cm (b) 9 cm (c) 3 cm (d) 1 cm

32. A circular disc of radius R is rotating about its axis O with a uniform angular velocity ω rad s^{-1} as shown in the figure. The magnitude of the relative velocity of point A relative to point B on the disc is



(a) zero (b) $R\omega \sin\left(\frac{\theta}{2}\right)$
(c) $2R\omega \sin\left(\frac{\theta}{2}\right)$ (d) $\sqrt{3}\, R\omega \sin\left(\frac{\theta}{2}\right)$

33. When the angular velocity of a uniformly rotating body has increased thrice, the resultant of forces applied to it increases by 60 N. Find the accelerations of the body in the two cases. The mass of the body, $m = 3$ kg
(a) 2.5 ms^{-2}, 7.5 ms^{-2} (b) 7.5 ms^{-2}, 22.5 ms^{-2}
(c) 5 ms^{-2}, 45 ms^{-2} (d) 2.5 ms^{-2}, 22.5 ms^{-2}

34. An automobile enters a turn of radius R. If the road is banked at an angle of 45° and the coefficient of friction is 1, the minimum speed with which the automobile can negotiate the turn without skidding is
(a) $\sqrt{\frac{rg}{2}}$ (b) $\frac{\sqrt{rg}}{2}$ (c) $\sqrt{rg}$ (d) zero

35. A mass is attached to the end of a string of length l which is tied to a fixed point O. The mass is released from the initial horizontal position of the string. Below the point O at what minimum distance a peg P should be fixed so that the mass turns about P and can describe a complete circle in the vertical plane?
(a) $\left(\frac{3}{5}\right)l$ (b) $\left(\frac{2}{5}\right)l$ (c) $\frac{l}{3}$ (d) $\frac{2l}{3}$

36. A stone is rotated in a vertical circle. Speed at bottommost point is $\sqrt{8gR}$, where R is the radius of circle. The ratio of tension at the top and the bottom is
(a) 1 : 2 (b) 1 : 3
(c) 2 : 3 (d) 1 : 4

37. The string of a pendulum is horizontal. The mass of bob attached to it is m. Now the string is released. The tension in the string in the lowest positions is
(a) mg (b) $2\,mg$ (c) $3\,mg$ (d) $4\,mg$

38. A body is moving in a vertical circle of radius r such that the string is just taut at its highest point. The speed of the particle when the string is horizontal is
(a) $\sqrt{gr}$ (b) $\sqrt{2gR}$
(c) $\sqrt{3gr}$ (d) $\sqrt{4gR}$

39. A small ball is pushed from a height h along a smooth hemispherical bowl of radius R. With what speed should the ball be pushed so that it just reaches the top of the opposite end of the bowl?
(a) $\sqrt{2gh}$ (b) $\sqrt{2g(R+h)}$
(c) $\sqrt{2g(R-h)}$ (d) None of these

40. A 50 kg girl is swinging on a swing from rest. Then, the power delivered when moving with a velocity of 2 ms^{-1} upwards in a direction making an angle 60° with the vertical is
(a) 980 W (b) 490 W
(c) $490\sqrt{3}$ W (d) 245 W

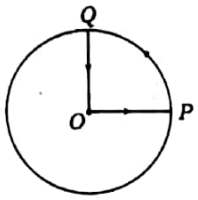
41. A simple pendulum of length l has a maximum angular displacement θ. The maximum kinetic energy of the bob of mass m will be
(a) $mgl\,(1 - \cos\theta)$ (b) $mgl \cos\theta$
(c) $mgl \sin\theta$ (d) None of these

42. Toy cart tied to the end of an unstretched string of length a, when revolved moves in a horizontal circle of radius $2a$ with a time period T. Now the toy cart is speeded up until it moves in a horizontal circle of radius $3a$ with a period T. If Hooke's law ($F = kx$) holds, then
(a) $T' = \sqrt{\frac{3}{2}}\,T$ (b) $T' = \left(\frac{\sqrt{3}}{2}\right)T$
(c) $T' = \left(\frac{3}{2}\right)T$ (d) $T' = T$

43. A sphere is suspended by a thread of length l. What minimum horizontal velocity has to be imparted to the ball for it to reach the height of the suspension?

(a) $\sqrt{5gl}$ (b) $2gl$ (c) $\sqrt{gl}$ (d) $\sqrt{2gl}$

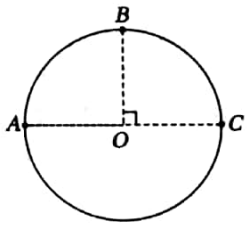
44. A cyclist starts from the centre O of a circular park of radius 1 km, reaches the edge P of the park, then cycles along the circumference and returns to the centre along QO as shown in the figure. If the round trip takes 10 min, the net displacement and average speed of the cyclist (in m and km per hour) are



(a) 0, 1 (b) $\frac{\pi+4}{2}$, 4

(c) 21.4, $\frac{\pi+4}{2}$ (d) 0, 21.4

45. Three particles A, B and C move in a circle of radius $r = \frac{1}{\pi}$ m, in anti-clockwise direction with speed $1\,ms^{-1}$, $2.5\,ms^{-1}$ and $2\,ms^{-1}$ respectively. The initial positions of A, B and C are as shown in figure.

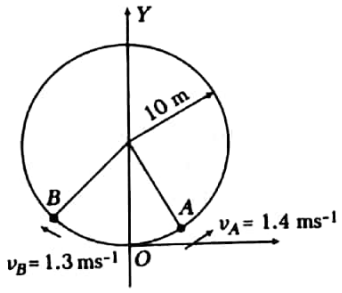


The ratio of distance travelled by B and C by the instant A, B and C meet for the first time is

(a) 3 : 2 (b) 5 : 4

(c) 3 : 5 (d) 3 : 7

46. Two particles A and B start at the origin O and travel in opposite directions along the circular path at constant speeds $v_A = 1.4\,ms^{-1}$ and $v_B = 3\,ms^{-1}$ respectively. The time when they collide and the magnitude of the acceleration of B just before this happens are



(a) 14.35, 0.45 ms^{-2} (b) 1.25, 0.2 ms^{-2}

(c) 11.25, 0.5 ms^{-2} (d) 14.55, 0.3 ms^{-2}

47. A student whirls a stone in a horizontal circle of radius 3 m and at height 8 m above level ground. The string breaks, at lowest point and the stone flies off horizontally and strikes the ground after travelling a horizontal distance of 20 m. What is the magnitude of the centripetal acceleration of the stone while breaking off.

(a) 150 ms^{-2} (b) 140 ms^{-2}

(c) 81.4 ms^{-2} (d) 163 ms^{-2}

48. A stone is tied to a string of length l and is whirled in a vertical circle with the other end of the string as the centre. At a certain instant of time, the stone is at its lowest position and has a speed u. The magnitude of the change in velocity as it reaches a position where the string is horizontal (g being acceleration due to gravity) is

(a) $\sqrt{2(u^2 - gl)}$

(b) $\sqrt{u^2 - gl}$

(c) $u - \sqrt{u^2 - 2gl}$

(d) $\sqrt{2gl}$

49. A ball suspended by a thread swings in a vertical plane so that its acceleration in the extreme position and lowest position are equal. The angle θ of thread deflection in the extreme position will be

(a) $\tan^{-1}(2)$ (b) $\tan^{-1}(\sqrt{2})$

(c) $\tan^{-1}\left(\frac{1}{2}\right)$ (d) $2\tan^{-1}\left(\frac{1}{2}\right)$

50. A body of mass m hangs at one end of a string of length l, the other end of which is fixed. It is given a horizontal velocity so that the string would just reach where it makes an angle of 60° with the vertical. The tension in the string at bottommost point position is

(a) $2mg$ (b) mg

(c) $3mg$ (d) $\sqrt{3}mg$

51. A simple pendulum oscillates in a vertical plane. When it passes through the bottommost point, the tension in the string is 3 times the weight of the pendulum bob. What is the maximum displacement of the pendulum of the string with respect to the vertical

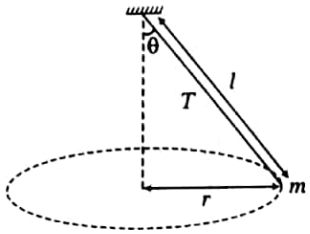
(a) 30° (b) 45° (c) 60° (d) 90°

52. A stone of mass 1 kg tied to a light inextensible string of length $L = \frac{10}{3}$ m, whirling in a circular path in a vertical plane. The ratio of maximum tension to the minimum tension in the string is 4. If g is taken to be $10\,ms^{-2}$, the speed of the stone at the highest point of the circle is

(a) 10 ms^{-1} (b) $5\sqrt{2}\,ms^{-1}$

(c) $10\sqrt{3}\,ms^{-1}$ (d) 20 ms^{-1}

53. A string of length l fixed at one end carries a mass m at the other end. The strings makes $\frac{2}{\pi}$ rev s^{-1} around the axis through the fixed end as shown in the figure, the tension in the string is



(a) 16 ml (b) 4 ml (c) 8 ml (d) 2 ml

54. A particle starts travelling on a circle with constant tangential acceleration. The angle between velocity vector and acceleration vector, at the moment when particle complete half the circular track, is

(a) $\tan^{-1}(2\pi)$ (b) $\tan^{-1}(\pi)$ (c) $\tan^{-1}(3\pi)$ (d) zero

55. A wet open umbrella is held vertical and whirled about the handle at a uniform rate of 21 rev in 44 s. If the rim of the umbrella is a circle of 1 m in diameter and the height of the rim above the floor is 4.9 m. The locus of the drop on floor is a circle of radius

(a) $\sqrt{2.5}$ m (b) 1 m (c) 3 m (d) 1.5 m

56. A heavy particle is tied to the end A of a string of length 1.6 m. Its other end O is fixed. It revolves as a conical pendulum with the string making 60° with the horizontal. Then,

(a) its period of revolution is $\frac{4\pi}{7}$ s

(b) the tension in the string is $1/\sqrt{3}$ times the weight of the particle

(c) the speed of the particle is $2.8\sqrt{3}$ ms^{-1}

(d) the centripetal acceleration of the particle is $\frac{9.8}{\sqrt{3}}$ ms^{-2}

57. A pendulum bob has a speed of 3 ms^{-1} at its lowest position. The pendulum is 0.5 m long. The speed of the bob, when string makes an angle of 60° to the vertical is (take, $g = 10\ ms^{-2}$)

(a) 2 ms^{-1} (b) $\frac{1}{2}$ ms^{-1} (c) 1 ms^{-1} (d) 2.5 ms^{-1}

58. A block is released from rest at the top of an inclined plane which later curves into a circular track of radius r as shown in figure. The minimum height h from where it should be released so that it is able to complete the circle, is

(a) r (b) 2.5 r (c) 1.5 r (d) 0.5 r

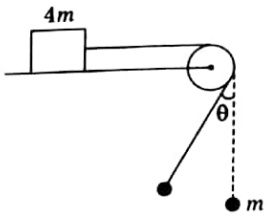
59. A small body of mass m slides without friction from the top of a hemisphere of radius r. At what height will the body be detached from the centre of the hemisphere?

(a) $h = \frac{r}{2}$ (b) $h = \frac{r}{3}$ (c) $h = \frac{2r}{3}$ (d) $h = \frac{r}{4}$

60. The maximum tension that an inextensible ring of radius 1m and mass density 0.1 kg m^{-1} can bear is 40 N. The maximum angular velocity with which it can be rotated in a circular path is

(a) 20 rad s^{-1} (b) 18 rad s^{-1}
(c) 16 rad s^{-1} (d) 15 rad s^{-1}

61. Two bodies of mass m and $4m$ are attached to a spring as shown in the figure. The body of mass m hanging from a string of length l is executing periodic motion with amplitude $\theta = 60°$ while other body is at rest on the surface.



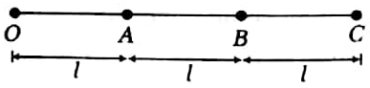
The minimum coefficient of friction between the mass $4m$ and the horizontal surface must be

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$

62. A bullet of mass m moving with a horizontal velocity u strikes a stationary wooden block of mass M suspended by a string of length $L = 50$ cm. The bullet emerges out of the block with speed $\frac{u}{4}$. If $M = 6$ m, the minimum value of u so that the block can complete the vertical circle (take, $g = 10\ ms^{-2}$)

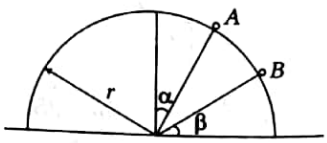
(a) 10 ms^{-1} (b) 20 ms^{-1}
(c) 30 ms^{-1} (d) 40 ms^{-1}

63. Three identical particles are joined together by a thread as shown in figure. All the three particles are moving in a horizontal plane. If the velocity of the outermost particle is v_0, then the ratio of tensions in the three sections of the string is ($T_{BC} : T_{AB} : T_{DA}$)



(a) 3 : 5 : 7 (b) 3 : 4 : 5
(c) 7 : 11 : 6 (d) 3 : 5 : 6

64. A particle moves from rest at A on the surface of a smooth circular cylinder of radius r as shown. At B it leaves the cylinder. The equation relating α and β is



(a) $3 \sin\alpha = 2 \cos\beta$ (b) $2 \sin\alpha = 3 \cos\beta$
(c) $3 \sin\beta = 2 \cos\alpha$ (d) $2 \sin\beta = 3 \cos\alpha$

65. A ball is placed on a smooth inclined plane of inclination $\theta = 30°$ to the horizontal, which is rotating at frequency 0.5 Hz about a vertical axis passing through its lower end. At what distance from the lower end does the ball remain at rest?

(a) 0.87 m (b) 0.33 m
(c) 0.5 m (d) 0.67 m

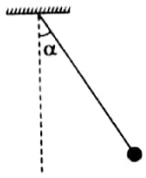
66. A particle suspended by a light inextensible thread of length l is projected horizontally from its lowest position with velocity $\sqrt{7gl/2}$. The string will slack after swinging through an angle equal to

(a) 30° (b) 90° (c) 120° (d) 150°

67. The kinetic energy K of a particle moving along a circle of radius R depends on the distance covered s as $K = as^2$. The force acting on the particle is

(a) $\frac{2as^2}{R}$ (b) $2as\left(1+\frac{s^2}{R^2}\right)^{1/2}$
(c) $as\left(1+\frac{s^2}{R^2}\right)^{1/2}$ (d) None of these

68. A simple pendulum is vibrating with an angular amplitude of 90° as shown in the figure. For what value of α, is the acceleration directed?



(i) Vertically upwards
(ii) Horizontally
(iii) Vertically downwards

(a) 0°, $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$, 90° (b) 90°, $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$, 0°
(c) 0°, $\cos^{-1}\sqrt{3}$, 90° (d) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$, 90°, 0°

69. A small block of mass m is released from the top of a smooth hemisphere of radius R with the horizontal speed u. What is the angle with vertical where it loses contact with the hemisphere?

(a) $\sin^{-1}\left(\frac{u^2}{3gR}+\frac{2}{3}\right)$ (b) $\cos^{-1}\left(\frac{u^2}{3gR}+\frac{2}{3}\right)$
(c) $\cos^{-1}\left(\frac{u^2}{6gR}+\frac{4}{3}\right)$ (d) $\sin^{-1}\left(\frac{u^2}{6gR}+\frac{2}{3}\right)$

B. Medical entrance special format questions

Assertion and reason

Directions (Q. Nos. 1-15) *These questions consists of two statements each printed as Assertion and Reason. While answering these question you are required to choose any one of the following five responses*

(a) If both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If Assertion is false but Reason is true.

1. Assertion Uniform circular motion is uniformly accelerated motion.
Reason Acceleration in uniform circular motion is always towards centre.

2. Assertion In circular motion, dot product of **v** and ω is always zero.
Reason ω is always perpendicular to the plane of the circular motion.

3. Assertion In circular motion, acceleration of particle is not always towards centre.
Reason If speed of particle is not constant, acceleration is not towards centre.

4. Assertion In circular motion, average speed and average velocity are never equal.
Reason In any curvilinear path, these two are never equal.

5. Assertion A particle is rotating in a circle of radius 1m. At some given instant, its speed is 2 ms^{-1}. Then acceleration of particle at the given instant is 4 ms^{-2}.
Reason Centripetal acceleration at this instant is 4 ms^{-2} towards centre of circle.

6. Assertion Centripetal force mv^2/R acts on a particle rotating in a circle.
Reason Summation of net forces acting on the particle is equal to mv^2/R in the above case.

7. Assertion If a particle is rotating in a circle, then centrifugal force is acting on the particle in radially outward direction.
Reason Centrifugal force is equal and opposite to the centripetal force.

8. Assertion When a car takes a circular turn on a horizontal road, then normal reaction on inner wheels is always less than the normal reaction on outer wheels.
Reason This is for rotational equilibrium of car.

9. Assertion When water in a bucket is whirled fast overhead, the water does not fall out at the top of the circular path.
Reason The centripetal force in this position on water is more than the weight of water.

10. Assertion In vertical circular motion, speed of a body cannot remain constant.
Reason In moving upwards, work done by gravity is negative.

11. Assertion Angle (θ) between **a** and **v** in circular motion is

$$0° < \theta < 180°$$

Reason Angle between any two vectors lies in the above range.

12. Assertion A ball tied by thread is undergoing circular motion (of radius R) in a vertical plane. (Thread always remains in vertical plane). The difference of maximum and minimum tension in thread is independent of speed (u) of ball at the lowest position ($u > \sqrt{5gR}$).

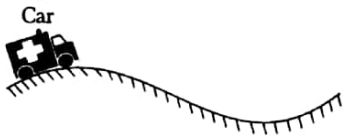
Reason For a ball of mass m tied by thread undergoing vertical circular motion (of radius R), difference in maximum and minimum magnitude of centripetal acceleration of the ball is independent of speed (u) of ball at the lowest position ($u > \sqrt{5gR}$).

13. Assertion One end of a massless rod of length l is hinged so that it is free to rotate in a vertical plane about a horizontal axis. If a particle is attached to the other end of the rod, then the minimum speed at lower most position of the particle is $\sqrt{5gl}$ to complete the circular motion.

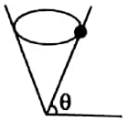
Reason Work done by centripetal force on the particle is always zero.

14. Assertion A car moves along a road with uniform speed. The path of car lies in vertical plane and is shown in figure. The radius of curvature (R) of the path is same everywhere. If the car does not loose contact with road at the highest point, it can travel the shown path without loosing contact with road anywhere else.

Reason For car to loose contact with road, the normal reaction between car and road should be zero.



15. Assertion A small block of mass m is rotating in a circle inside a smooth cone as shown in figure. In this case the normal reaction,

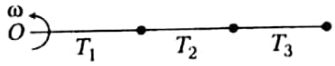


$$N \neq mg \cos \theta$$

Reason In this case, acceleration of the block is not along the surface of cone. It is horizontal.

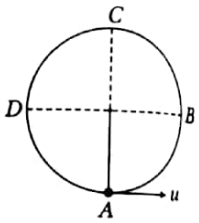
Match the columns

1. Three balls each of mass 1 kg are attached with three strings each of length 1 m as shown in figure. They are rotated in a horizontal circle with angular velocity $\omega = 4$ rads^{-1} about point O. Match the following columns.



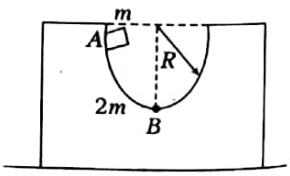
	Column I		Column II
(A)	T_1	(p)	Maximum
(B)	T_2	(q)	Minimum
(C)	T_3	(r)	80 N
		(s)	48 N
		(t)	90 N

2. A particle is suspended from a string of length R. It is given a velocity $u = 3\sqrt{gR}$ at the bottom. Match the following columns.



	Column I		Column II
(A)	Velocity at B	(p)	$7\,mg$
(B)	Velocity at C	(q)	$\sqrt{5gR}$
(C)	Tension in string at B	(r)	$\sqrt{7gR}$
(D)	Tension in string at C	(s)	$5\,mg$
		(t)	None

3. In the system shown in figure, mass m is released from rest from position A. Suppose potential energy of m at point A with respect to point B is E. Dimensions of m are negligible and all surfaces are smooth. When mass reaches at point B.

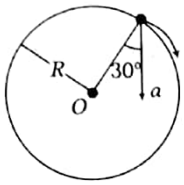


	Column I		Column II
(A)	Kinetic energy of m	(p)	$E/3$
(B)	Kinetic energy of $2m$	(q)	$2E/3$
(C)	Momentum of m	(r)	$\sqrt{4/3\,mE}$
(D)	Momentum of $2m$	(s)	$\sqrt{2/3\,mE}$
		(t)	None

C. Medical entrances' gallery

(Collection of questions asked in NEET & various medical entrance exams)

1. In the given figure, $a = 15\ m/s^2$ represents the total acceleration of a particle moving in the clockwise direction in a circle of radius $R = 2.5m$ at a given instant of time. The speed of the particle is **[NEET 2016]**



(a) 4.5 m/s (b) 5.0 m/s (c) 5.7 m/s (d) 6.2 m/s

2. A car is negotiating a curved road of radius R. The road is banked at angle θ. The coefficient of friction between the tyres of the car and the road is μ_s. The maximum safe velocity on this road is **[NEET 2016]**

(a) $\sqrt{gR\left(\frac{\mu_s + \tan\theta}{1-\mu_s \tan\theta}\right)}$ (b) $\sqrt{\frac{g}{R}\left(\frac{\mu_s + \tan\theta}{1-\mu_s \tan\theta}\right)}$

(c) $\sqrt{\frac{g}{R^2}\left(\frac{\mu_s + \tan\theta}{1-\mu_s \tan\theta}\right)}$ (d) $\sqrt{gR^2\left(\frac{\mu_s + \tan\theta}{1-\mu_s \tan\theta}\right)}$

3. A uniform circular disc of radius 50 cm at rest is free to turn about an axis which is perpendicular to its plane and passes through its centre. It is subjected to a torque which produces a constant angular acceleration of 2 rad s^{-2}. Its net acceleration in ms^{-2} at the end of 2 s is approximately **[NEET 2016]**

(a) 7 (b) 6 (c) 3 (d) 8

4. What is the minimum velocity with which a body of mass m must enter a vertical loop of radius R so that it can complete the loop? **[NEET 2016]**

(a) $\sqrt{2gR}$ (b) $\sqrt{3gR}$ (c) $\sqrt{5gR}$ (d) $\sqrt{gR}$

5. A particle of mass 10 g moves along a circle of radius 6.4 cm with a constant tangential acceleration. What is the magnitude of this acceleration, if the kinetic energy of the particle becomes equal to 8×10^{-4} J by the end of the second revolution after the beginning of the motion? **[NEET 2016]**

(a) 0.15 ms^{-2} (b) 0.18 ms^{-2}
(c) 0.2 ms^{-2} (d) 0.1 ms^{-2}

6. A fighter plane is pulling out for a dive at a speed of 900 kmh^{-1}. Assuming its path to be a vertical circle of radius 2000 m and its mass to be 16000 kg, find the upward force extended by air on it at the lowest point. **[UK PMT 2015]**

(a) 3.28×10^5 N (b) 6.56×10^5 N
(c) 9.28×10^5 N (d) 12.56×10^5 N

7. A particle is moving in a curved path. Which of the following quantities may remain constant during its motion? **[CG PMT 2015]**

(a) Acceleration (b) Velocity
(c) Magnitude of acceleration (d) None of these

8. The ratio of angular speed of a second-hand to the hour-hand of a watch is **[KCET 2015]**

(a) 3600 :1 (b) 720 : 1
(c) 72 : 1 (d) 60 : 1

9. If the length of second's hand of a clock is 10 cm, the speed of its dip (in cm s^{-1}) is nearly **[Kerala CEE 2014]**

(a) 2 (b) 0.5 (c) 1.5 (d) 3
(e) 1

10. A particle is moving uniformly in a circular path of radius r. When it moves through an angular displacement θ, then the magnitude of the corresponding linear displacement will be **[WB JEE 2014]**

(a) $2r\cos\left(\frac{\theta}{2}\right)$ (b) $2r\cot\left(\frac{\theta}{2}\right)$

(c) $2r\tan\left(\frac{\theta}{2}\right)$ (d) $2r\sin\left(\frac{\theta}{2}\right)$

11. A rotating wheel changes angular speed from 1800 rpm to 3000 rpm in 20 s. What is the angular acceleration assuming to be uniform? **[KCET 2014]**

(a) 60π rad s^{-2} (b) 90π rad s^{-2}
(c) 2π rad s^{-2} (d) 40π rad s^{-2}

12. The centripetal force is given by the expression **[J&K CET 2013]**

(a) $\frac{Mv^2}{R}$ (b) $\frac{M^2v}{R}$ (c) $\frac{Mv}{R^2}$ (d) $\frac{Mv}{R}$

13. Uniform circular motion is an example of **[J&K CET 2013]**

(a) constant speed motion (b) constant velocity motion
(c) non-accelerated motion (d) zero accelerated motion

14. A stone tied to a rope is rotated in a vertical circle with uniform speed. If the difference between the maximum and minimum tensions in the rope is 20 N, mass of the stone in kg is (take, $g = 10\ ms^{-2}$) **[EAMCET 2013]**

(a) 0.75 (b) 1.0 (c) 1.5 (d) 0.5

15. A particle moving in uniform circle makes 18 revolutions in 1 minutes. If the radius of the circle is 10 cm, the speed of the particle is **[Kerala CEE 2013]**

(a) $3\pi \times 10^{-2}\ ms^{-1}$ (b) $4\pi \times 10^{-2}\ ms^{-1}$
(c) $5\pi \times 10^{-2}\ ms^{-1}$ (d) $7\pi \times 10^{-2}\ ms^{-1}$
(e) $6\pi \times 10^{-2}\ ms^{-1}$

16. A car of mass 1000 kg negotiates a banked curve of radius 90 m on a frictionless road. If the banking angle is 45°, the speed of the car is **[CBSE AIPMT 2012]**

(a) 20 ms^{-1} (b) 30 ms^{-1} (c) 5 ms^{-1} (d) 10 ms^{-1}

17. On a railway ve, the outside rail is laid higher than the inside on that resultant force exerted on the wheels of the car by the tops of the rails will **[JCECE 2013]**

(a) equilibrate th tripetal force
(b) be vertical
(c) be decreased
(d) have a horizontal inward component

18. One end of a string of length 1 m is tied to a body of mass 0.5 kg. It is whirled in a vertical circle with angular frequency 4 rad/s. The tension in the string, when the body is at the lower most point of its motion, will be equal to (Take, $g = 10\ m/s^2$) **[AMU 2012]**

(a) 3 N (b) 5 N
(c) 8 N (d) 13 N

19. A small cone fitted with water is revolved in a vertical circle of radius 4 m and does not fall down. What must be the maximum period of revolution? **[UP CPMT 2012]**

(a) 4 s (b) 2 s
(c) 1 s (d) 6 s

Answers

Check point 7.1

1. (d)	**2.** (c)	**3.** (a)	**4.** (c)	**5.** (a)	**6.** (a)	**7.** (a)	**8.** (a)	**9.** (d)	**10.** (c)
11. (b)	**12.** (d)	**13.** (c)	**14.** (c)	**15.** (d)	**16.** (a)	**17.** (a)	**18.** (d)	**19.** (b)	**20.** (c)

Check point 7.2

1. (d)	**2.** (a)	**3.** (d)	**4.** (d)	**5.** (b)	**6.** (a)	**7.** (d)	**8.** (d)	**9.** (c)	**10.** (c)
11. (a)	**12.** (b)	**13.** (b)	**14.** (d)	**15.** (a)	**16.** (d)	**17.** (d)	**18.** (a)	**19.** (a)	**20.** (c)
21. (d)	**22.** (c)	**23.** (a)	**24.** (c)	**25.** (a)					

A. Taking it together

1. (c)	**2.** (a)	**3.** (d)	**4.** (b)	**5.** (a)	**6.** (c)	**7.** (a)	**8.** (b)	**9.** (a)	**10.** (c)
11. (b)	**12.** (c)	**13.** (b)	**14.** (b)	**15.** (c)	**16.** (b)	**17.** (c)	**18.** (a)	**19.** (c)	**20.** (a)
21. (a)	**22.** (d)	**23.** (c)	**24.** (a)	**25.** (c)	**26.** (c)	**27.** (c)	**28.** (a)	**29.** (c)	**30.** (d)
31. (d)	**32.** (c)	**33.** (d)	**34.** (d)	**35.** (a)	**36.** (b)	**37.** (c)	**38.** (c)	**39.** (c)	**40.** (c)
41. (a)	**42.** (b)	**43.** (d)	**44.** (d)	**45.** (b)	**46.** (a)	**47.** (c)	**48.** (a)	**49.** (d)	**50.** (a)
51. (d)	**52.** (a)	**53.** (a)	**54.** (a)	**55.** (a)	**56.** (d)	**57.** (a)	**58.** (b)	**59.** (c)	**60.** (a)
61. (c)	**62.** (d)	**63.** (d)	**64.** (c)	**65.** (d)	**66.** (c)	**67.** (b)	**68.** (a)	**69.** (b)	

B. Medical entrance special format questions

Assertion and reason

1. (d)	**2.** (a)	**3.** (d)	**4.** (d)	**5.** (b)	**6.** (a)	**7.** (a)	**8.** (d)	**9.** (a)	**10.** (a)
11. (c)	**12.** (a)	**13.** (d)	**14.** (c)	**15.** (a)					

Match the columns

1. (A → p, B → r, C → q, s) **2.** (A → r, B → q, C → p, D → t) **3.** (A → q, B → p, C → r, D → r)

C. Medical entrances' gallery

1. (c)	**2.** (a)	**3.** (d)	**4.** (c)	**5.** (d)	**6.** (a)	**7.** (c)	**8.** (b)	**9.** (e)	**10.** (d)
11. (c)	**12.** (a)	**13.** (a)	**14.** (b)	**15.** (e)	**16.** (b)	**17.** (d)	**18.** (d)	**19.** (a)	

Hints and explanations

Check point 7.1

1. (*d*) $\omega = 120 \text{ rev/min} = 120 \times \frac{2\pi}{60} \text{ rad s}^{-1} = 4\pi \text{ rad s}^{-1}$

2. (*c*) $\omega_{min} = \frac{2\pi}{60}$ rad/min and $\omega_{hr} = \frac{2\pi}{12 \times 60}$ rad/min

$\therefore \quad \frac{\omega_{min}}{\omega_h} = \frac{2\pi / 60}{2\pi / 12 \times 60} = \frac{12}{1}$

3. (*a*) $\alpha = \frac{\Delta\omega}{\Delta t}$ but $\omega = 2\pi f$

$\therefore$ Angular acceleration, $\alpha = \frac{2\pi\Delta f}{\Delta t} = \frac{2\pi \times 200}{2}$

$= (200\pi) \text{ rads}^{-2}$

4. (*c*) We know that $\omega = 2\pi n \quad \therefore \quad \omega_1 = 2\pi n_1$

where, $\quad n_1 = 1800 \text{ rpm}, \; n_2 = 3000 \text{ rpm}, \; \Delta t = 20 \text{ s}$

$\Rightarrow \quad \omega_1 = 2\pi \times \frac{1800}{60} = 2\pi \times 30 = 60\pi$

Similarly, $\quad \omega_2 = 2\pi n_2 = 2\pi \times \frac{3000}{60} = 2\pi \times 50 = 100\pi$

If the angular velocity of a rotating wheel about an axis changes from ω_1 to ω_2 in a time interval Δt, then the angular acceleration of rotating wheel about that axis is given by

$$\alpha = \frac{\text{change in angular velocity}}{\text{time interval}}$$

$$= \frac{\omega_2 - \omega_1}{\Delta t} = \frac{100\pi - 60\pi}{20} = \frac{40\pi}{20} = 2\pi \text{ rad s}^{-2}$$

5. (*a*) As, $\omega = \omega_0 + \alpha t$

Here, $\quad \omega_0 = 900 \text{ rpm} = \frac{(2\pi \times 900)}{60} \text{ rad s}^{-1}, \; \omega = 0$

and $\quad t = 60 \text{ s}, \; 0 = \frac{2\pi \times 900}{60} + \alpha \times 60$

$\alpha = \frac{2\pi \times 900}{60 \times 60} = \frac{\pi}{2}$

6. (*a*) From equation of motion (angular),

$0 = \omega_0 - \alpha t \Rightarrow \alpha = \frac{\omega_0}{t} = \frac{(100 \times 2\pi)/60}{15} = 0.69 \text{ rad s}^{-2}$

Now, angle rotated before coming to rest is given by

$$\theta = \frac{\omega_0^2}{2\alpha} = \frac{\left(\frac{100 \times 2\pi}{60}\right)^2}{2 \times 0.7} = 78.25 \text{ rad}$$

$\therefore$ Number of revolutions $= \frac{\theta}{2\pi} = 12.5$

7. (*a*) Velocity is tangential and acceleration is radial.

8. (*a*) Acceleration $= \omega^2 r = \frac{v^2}{r} = \omega v = \frac{2\pi}{T} v$

9. (*d*) $a = \frac{v^2}{R}$ or $a \propto v^2 \Rightarrow \frac{a_{after}}{a_{before}} = \frac{(2v)^2}{v^2} = \frac{4}{1}$

10. (*c*) $a = r\omega^2$ or $a \propto r \Rightarrow$ so if $r_1 < r_2$, then $a_1 < a_2$

11. (*b*) $a_t = 0, \; a = a_n = \frac{v^2}{R} = \frac{(6)^2}{0.3} = 120 \text{ ms}^{-2}$

12. (*d*) As $\quad v = \omega r$

$\Rightarrow \quad 50 = \omega\left(\frac{20}{\pi}\right)$

$\Rightarrow \quad \omega = \frac{50\pi}{20} = \frac{5\pi}{2}$

$\Rightarrow \quad \frac{2\pi}{T} = \frac{5\pi}{2} \Rightarrow T = \frac{4}{5} \text{ s}$

Given, $\quad t = 2T = \frac{8}{5} \text{ s}$

$\therefore \quad \omega = \alpha t$

$\Rightarrow \quad \frac{5\pi}{2} = \alpha \times \frac{8}{5} \Rightarrow \alpha = \frac{25\pi}{16} \text{ rads}^{-2}$

$\therefore \quad a_t = r(\alpha) = \left(\frac{20}{\pi}\right)\left(\frac{25\pi}{16}\right) = 31.2 \text{ rads}^{-2}$

13. (*c*) In circular motion, a_r can never be zero. So, option (c) is correct.

14. (*c*) $a = \sqrt{a_n^2 + a_t^2} = \sqrt{\left(\frac{v^2}{r}\right)^2 + a^2} = \sqrt{\frac{v^4}{r^2} + a^2}$

15. (*d*) $\frac{a_n}{a_t} = \frac{v^2 / R}{a} \qquad$ (let $a_t = a$)

Here, $v^2 = 2al = 2a\,(2\pi R) = 4\pi R$. Therefore, the ratio is $\frac{4\pi}{1}$.

16. (*a*) $a = \sqrt{a_t^2 + a_n^2}$

a_t = rate of change of speed = 2 ms^{-2}

$a_n = \frac{v^2}{R} = \frac{(5)^2}{10} = 2.5 \text{ ms}^{-2}$

$\therefore \quad a = \sqrt{a_t^2 + a_n^2} = \sqrt{(2)^2 + (2.5)^2} = 3.2 \text{ ms}^{-2}$

17. (*a*) $a_n = \sqrt{a^2 - a_t^2} = \sqrt{(100)^2 - (60)^2} = 80 \text{ ms}^{-2}$

Now, $\quad a_n = \frac{v^2}{R}$ or $R = \frac{v^2}{a_n} = \frac{(10)^2}{80} = 1.25 \text{ m}$

18. (*d*) $\alpha = \frac{d\omega}{dt} = -b$

$$a_t = R\alpha = -Rb \quad \text{...(i)}$$

At $t = \frac{2a}{b}$, $\omega = -a$

$$a_n = R\omega^2 = Ra^2 \quad \text{...(ii)}$$

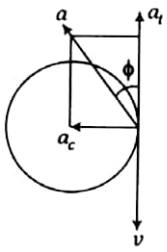
Now, $a = \sqrt{a_t^2 + a_n^2} = R\sqrt{a^4 + b^2}$

19. (*b*) The ratio of tangential to centripetal accelerations is

$$\left(\frac{a_t}{a_c}\right)_{t=t} = \frac{\frac{dv}{dt}}{\frac{v^2}{r}} = \frac{\frac{d}{dt}\left(\frac{ds}{dt}\right)}{\left(\frac{ds}{dt}\right)^2} r = \frac{\frac{d}{dt}(6t^2)}{(6t^2)^2} r = \frac{(12t)\times 12}{36t^4} = \frac{4}{t^3}$$

$$\therefore \quad \left(\frac{a_t}{a_c}\right)_{t=2s} = \frac{4}{(2)^3} = \frac{1}{2}$$

20. (*c*) $\tan\phi = \frac{\frac{v^2}{r}}{\left|\frac{dv}{dt}\right|} = \frac{\frac{20\times 20}{80}}{5} = \frac{20\times 20}{400} = 1$



$\Rightarrow \quad \phi = 45°$

angle between a (net) and v

$\theta = \pi - \phi = 135°$

Check point 7.2

1. (*d*) Centripetal force $= mR\omega^2 = (2)(1)(2\pi)^2 = 8\pi^2$ N

2. (*a*) $F = \frac{mv^2}{r}$. If m and v are constants, then $F \propto \frac{1}{r}$

$$\therefore \quad \frac{F_1}{F_2} = \left(\frac{r_2}{r_1}\right)$$

3. (*d*) Radial force $= \frac{mv^2}{r} = \frac{m}{r}\left(\frac{p}{m}\right)^2 = \frac{p^2}{mr}$

4. (*d*) Maximum tension $= \frac{mv^2}{r} = 16$ N $\Rightarrow \frac{16\times v^2}{144} = 16$

$$\therefore \quad v = 12\ \text{ms}^{-1}$$

5. (*b*) $F = \frac{mv^2}{r}$, $F' = \frac{(15m)(15v)^2}{(15r)} = 225\frac{mv^2}{r} = 225F$

Percentage increase $= \frac{225F - F}{F} \times 100 = 125\%$

Therefore, F has to be increased by 125%.

6. (*a*) $T = 100\ \text{N} = m\omega_m^2 r$

$$\Rightarrow \quad 100 = 100\times 10^{-3}\times\omega_m^2\times 0.1$$

$$\Rightarrow \quad \omega_m = 100\ \text{rads}^{-1}$$

7. (*d*) Tension in the string, $T = m\omega^2 r = 4\pi^2 n^2 mr$

$$\therefore \quad T \propto n^2 \Rightarrow \frac{n_2}{n_1} = \sqrt{\frac{T_2}{T_1}}$$

$$\therefore \quad n_2 = 5\sqrt{\frac{2T}{T}} = 5\sqrt{2}\ \text{rpm}$$

8. (*d*) Maximum tension $= m\,\omega^2 r = m\times 4\pi^2\times n^2\times r$

By substituting the values, we get $T_{max} = 87.64$ N

9. (*c*) $\frac{mv^2}{R} \le \mu mg \Rightarrow v_{max} = \sqrt{\mu Rg}$

v_{max} : maximum velocity for no slipping

10. (*c*) $\tan\theta = \frac{v^2}{Rg} = \frac{(10)^2}{(10)(10)} = 1$

$\theta = 45°$

11. (*a*) For safe turning

$$\tan\theta = \frac{v^2}{gr} \Rightarrow \frac{h}{b} = \frac{v^2}{gR} \Rightarrow h = \frac{v^2 b}{Rg}$$

12. (*b*) To avoid skidding

$$\tan\theta = \frac{v^2}{gr} = \frac{20\times 20}{10\times 20} = 2$$

$$\Rightarrow \quad \theta = \tan^{-1}(2)$$

13. (*b*) $\tan\theta = \frac{v^2}{rg} \Rightarrow \tan 45° = \frac{v^2}{(90)(10)} \Rightarrow v = 30$ m/s

14. (*d*) $\tan\theta = \frac{v^2}{Rg} = \frac{v_2^2}{4Rg} \Rightarrow v_2 = 2v_1$

$$\left(\frac{v_2}{v_1} - 1\right) = 1$$

% increase = 100%

15. (*a*) $N = \frac{mv^2}{R}$, $mg = \mu_s N = \frac{\mu_s mv^2}{R}$

$$\therefore \quad v = \sqrt{\frac{Rg}{\mu_s}}$$

16. (*d*) $N = \frac{mv^2}{R}$, $mg = \mu N = \frac{\mu mv^2}{R}$

$\therefore$ Coefficient of friction between the tyres and wall of the well will be $\mu = \frac{Rg}{v^2} = \frac{5\times 10}{(5\sqrt{5})^2} = 0.40$

17. (*d*) $mg = \frac{mv_{min}^2}{R}$ or $v_{min} = \sqrt{Rg}$ (at topmost point)

18. (*a*) In critical case, tension at topmost point is zero.

19. (*a*) Maximum tension will be at bottommost point. Therefore, the string will break at A.

20. (*c*) Tension, $T = \frac{mv^2}{r} + mg\cos\theta$

For $\theta = 30°$,

$$T_1 = \frac{mv_1^2}{r} + mg\cos 30°$$

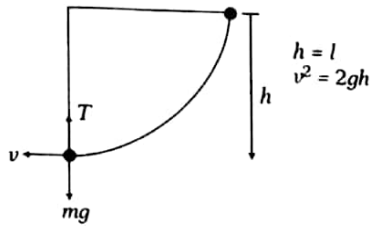
For $\theta = 60°$, $T_2 = \frac{mv_2^2}{r} + mg\cos 60°$

$\therefore \quad T_1 > T_2$ as $v_1 > v_2$ and $\cos 30° > \cos 60°$

21. (*d*) If the speed of the particle at highest point be v, then

$$mg \ge \frac{mv^2}{r}$$

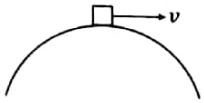
22. (*c*) $T - mg = \frac{mv^2}{l}$ or $T = 3\,mg$



Tension will be maximum at bottommost point.

23. (*a*) $v = \sqrt{2gh} = \sqrt{2g\,(h_{max} - h_{min})} = \sqrt{2 \times 10 \times 1.25} = 5\ \text{ms}^{-1}$

24. (*c*) $v_{max} = \sqrt{gR} = \sqrt{9.8 \times 49} = 22\ \text{ms}^{-1}$



25. (*a*) To complete the loop successfully $\sqrt{2gh} = \sqrt{5gR}$

$\therefore \quad R = \frac{2h}{5} = \frac{2 \times 5}{5} = 2\ \text{cm}$

A) Taking it together

4. (*b*) Here, rpm mean rotation per minute, *i.e.*, it is frequency of rotation

$$f = 600\ \text{rpm} \Rightarrow f = \frac{600}{60}\ \text{rpm} = 10\ \text{Hz}$$

$$\Rightarrow \quad T = \frac{1}{f} = \frac{1}{10} = 0.1\,\text{s}$$

5. (*a*) The initial angular velocity $= 50\ \text{rpm} = \frac{5\pi}{3}\ \text{rad s}^{-1}$

Using, $\omega = \omega_0 + \alpha t$

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{0 - \frac{5\pi}{3}}{60}\ \text{rad s}^{-2} = -\frac{\pi}{36}\ \text{rad s}^{-2}$$

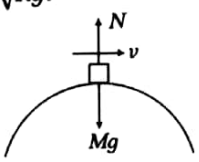
6. (*c*) Speed of particle is increasing, means tangential component of acceleration is positive.

8. (*b*) As the speed is constant throughout the circular motion, therefore its average speed is equal to instantaneous speed.

9. (*a*) Minimum velocity at topmost point is $\sqrt{Rg}$.

10. (*c*) $Mg - N = \frac{Mv^2}{r}$

or $N = Mg - \frac{Mv^2}{r}$



11. (*b*) Average velocity $= \frac{\text{Total displacement}}{\text{Total time}} = \frac{2\,\text{m}}{1\,\text{s}} = 2\ \text{ms}^{-1}$

12. (*c*) Given, $f = 2400\ \text{rpm} = \frac{2400}{60} = 40\text{Hz}$

$\therefore \quad \omega = 2\pi f = 80\pi\ \text{rad s}^{-2}$

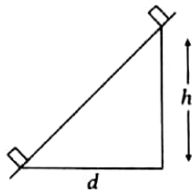
Now, $\theta = \left[\frac{\omega_0 + \omega}{2}\right] t$

$\Rightarrow \quad \theta = \left[\frac{80\pi + 0}{2}\right] \times 10 = 400\,\pi\ \text{rad}$

$\therefore$ Fan will turn angle $400\,\pi$ after it is swtiched off

$\Rightarrow$ The number of rotation $= \frac{\theta}{2\pi} = \frac{400\pi}{2\pi} = 200$

13. (*b*) h is elevation of outer rail over inner rail.



$$h = \frac{dv^2}{gr}$$

$$h = \frac{1.67 \times (15)^2}{10 \times 0.5 \times 1000}$$

$$= 0.075\,\text{m} = 75\ \text{mm}$$

17. (*c*) For safe circular turn

$$v = \sqrt{\mu\, rg}$$

$$\Rightarrow \quad r = \frac{v^2}{\mu g} \Rightarrow r = \frac{(10)^2}{0.2 \times 10} = 50\ \text{m}$$

18. (*a*) $v = 72\ \text{kmh}^{-1} = 72 \times \frac{5}{18}\ \text{ms}^{-1} = 20\ \text{ms}^{-1}$

$$\theta = \tan^{-1}\left(\frac{v^2}{Rg}\right) = \tan^{-1}\left(\frac{400}{100 \times 10}\right) = \tan^{-1}\left(\frac{2}{5}\right)$$

19. (*c*) $L = mvr \quad \therefore v = \frac{L}{mr}$

$$F = \frac{mv^2}{r} = \frac{L^2}{mr^3}$$

20. (*a*) $T\cos\theta = mg,\ T\sin\theta = mR\omega^2$

or $\tan\theta = \frac{R\omega^2}{g} = \frac{R\,(2\pi / T)^2}{g}$

$\therefore \quad T = 2\pi\sqrt{\frac{R}{g\tan\theta}}$

But $R = L\sin\theta$

$\therefore \quad T = 2\pi\sqrt{\frac{L\cos\theta}{g}}$

21. (*a*) Difference between the kinetic energies at its highest and lowest positions is

$$= 2mgr = 2 \times 1 \times 10 \times 1 = 20\ \text{J}$$

22. (*d*) $a_{av} = \frac{\Delta v}{\Delta t} = \frac{2 \times 5}{\pi} = \frac{10}{\pi}\ \text{ms}^{-2},\ T = 2\pi$

23. (*c*) $\tan\theta = \frac{v^2}{gr} \Rightarrow \frac{3}{5} = \frac{v^2}{10 \times 6}$

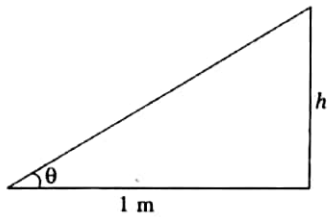
$\Rightarrow \quad v = 6\ \text{ms}^{-1} = 21.6\,\text{kmh}^{-1}$

24. (*a*) $v = 144\ \text{kmh}^{-1} = 144 \times \frac{5}{18}\ \text{ms}^{-1} = 40\ \text{ms}^{-1}$

$$\tan\theta = \frac{v^2}{Rg}$$

$$\Rightarrow \quad \tan\theta = \frac{(40)^2}{40 \times 10} = \frac{1600}{400} = 4$$

25. (*c*) $v = 96 \times \frac{5}{18}\ \text{ms}^{-1} = 26.66\ \text{ms}^{-1}$



$$\tan\theta = \frac{v^2}{Rg} = 0.088$$

$$0.088 = \frac{h}{1}$$

$\therefore$ $h = 0.088\ \text{m} = 8.8\ \text{cm}$

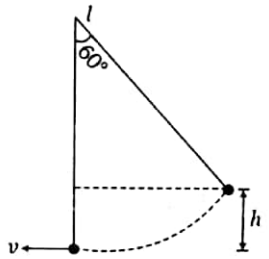
26. (*c*) α = constant

$\therefore$ $\theta = \frac{1}{2} \propto t^2$

$\therefore$ $\theta \propto t^2$

or $\frac{\theta_2 + \theta_1}{\theta_1} = \left(\frac{2+2}{2}\right)^2 = 4$ or $\frac{\theta_2}{\theta_1} = 3$

27. (*c*) From figure, $h = l\,(1 - \cos 60°) = \frac{l}{2} = 1\ \text{m}$



Applying conservation of mechanical energy, we get

$$v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 1} = 4.43\ \text{ms}^{-1}$$

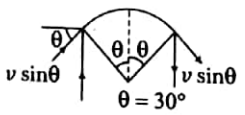
28. (*a*) $a = \sqrt{a_n^2 + a_t^2} = \sqrt{\left(\frac{T - mg\cos 30°}{m}\right)^2 + (g \sin 30°)^2}$

$$= g\sqrt{\left(\sqrt{3} - \frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{4}\right)} = g$$

29. (*c*) At highest point of a projectile, tangential component of acceleration is zero as acceleration is vertically downwards (g) and velocity is horizontal or the angle between acceleration and velocity is 90°.

30. (*d*) $\Delta t = \frac{\text{Distance travelled}}{\text{Speed}}$

$$= \frac{(2\pi R/6)}{v} = \frac{3.14 \times 300}{60 \times 3} = 5.23\ \text{s}$$



(i) $|\Delta \mathbf{v}| = |\mathbf{v}_f| - |\mathbf{v}_i|$

$$= \sqrt{v^2 + v^2 - 2v \cdot v \cos 60°} = 2v \sin 30° = 60\ \text{ms}^{-1}$$

(ii) $a_t = \frac{v^2}{R} = 12\ \text{ms}^{-2}$

(iii) $|\mathbf{a}_{av}| = \frac{|\Delta \mathbf{v}|}{\Delta t} = \frac{60}{5.23} = 11.5\ \text{ms}^{-2}$

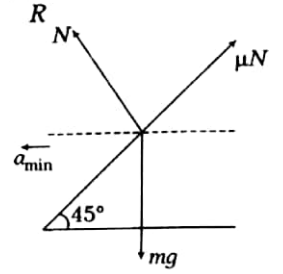
31. (*d*) Necessary centripetal force to the coin is provided by friction. Thus, $mr\,\omega_{max}^2 = \mu mg$ or $r = \frac{\mu g}{\omega_{max}^2}$

ω_{max} is made three times. Therefore, distance from centre, will remain $\frac{1}{9}$ times that is 1 cm.

32. (*c*) $|\mathbf{v}_{AB}| = |\mathbf{v}_A - \mathbf{v}_B| = \sqrt{(R\omega)^2 + (R\omega)^2 - 2(R\omega)(R\omega)\cos\theta}$

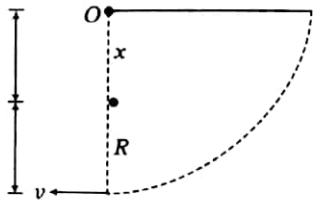
$$= 2R\,\omega \sin\frac{\theta}{2}$$

34. (*d*) $\frac{N}{\sqrt{2}} - \frac{\mu N}{\sqrt{2}} = \frac{mv_{min}^2}{R}$



$$v_{min} = \sqrt{\frac{rg(\tan\theta - \mu)}{1 + \mu\tan\theta}} \Rightarrow v_{min} = 0, \text{ as } \mu = 1$$

35. (*a*) From conservation of mechanical energy $v^2 = 2gl = 5gR$



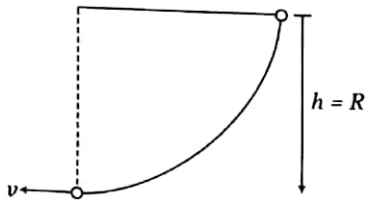
$\therefore$ $R = 0.4\,l$ or $x = l - R = 0.6\,l$

36. (*b*) $T = \frac{mu^2}{R} + mg\cos\theta$; $v^2 = u^2 - 2gh$

$T_{top} = 3mg$ and $T_{bottom} = 9\,mg$

Therefore, the ratio $\frac{T_{top}}{T_{bottom}} = \frac{3mg}{9mg} = \frac{1}{3}$

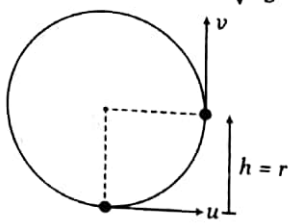
37. (*c*) $v^2 = 2gh = 2gR$



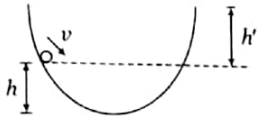
$$T - mg = \frac{mv^2}{R}$$

Substituting value of v^2, we get $T = 3\,mg$

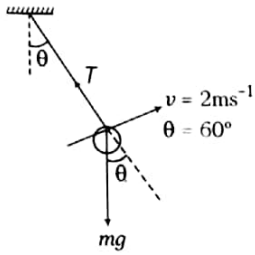
38. (*c*) $v = \sqrt{u^2 - 2gh} = \sqrt{5gr - 2gr} = \sqrt{3gr}$



39. (*c*) $h' = R - h \quad \therefore \quad v_{min} = \sqrt{2gh'} \Rightarrow v_{min} = \sqrt{2g(R-h)}$



40. (*c*) Two forces are acting on bob, tension and weight. Power of tension will be zero and that of weight is,



$$P = mgv\cos(90° + \theta)$$
$$= -mgv\sin 60°$$
$$= -50 \times 9.8 \times 2 \times \frac{\sqrt{3}}{2}$$
$$= -490\sqrt{3}\ \text{W}$$

$\therefore$ Power delivered is $490\sqrt{3}$ W.

41. (*a*) Height, $h = l(1 - \cos\theta)$

$$v_m^2 = 2gh = 2gl(1 - \cos\theta)$$

$\therefore$ Maximum kinetic energy, $K_m = \frac{1}{2}mv_m^2 = mgl(1 - \cos\theta)$

42. (*b*) In the given problem, centripetal force will be equal to

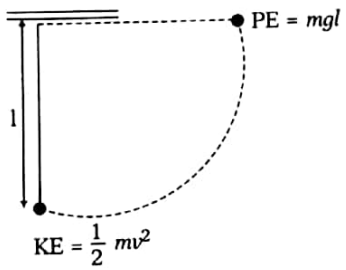
$$F = kx$$

$$\Rightarrow \quad kx = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{kxr}{m}}$$

$$\Rightarrow \quad T = \frac{2\pi r}{v} = 2\pi\sqrt{\frac{mr}{kx}} \Rightarrow T \propto \sqrt{\frac{r}{x}}$$

$$\therefore \quad T' = T\sqrt{\frac{r'x}{rx'}} = T\sqrt{\frac{3a \times a}{2a \times 2a}} = \left(\frac{\sqrt{3}}{2}\right)T$$

43. (*d*) Kinetic energy given to a sphere at lowest point= potential energy at the height of suspension



$$\Rightarrow \quad \frac{1}{2}mv^2 = mgl$$

$$\therefore \quad v = \sqrt{2gl}$$

44. (*d*) Average speed $= \dfrac{(2R + \pi R/2)\ \text{km}}{(1/6)\ \text{h}}$

$$= (12R + 3\pi R)\ \text{kmh}^{-1} = (12 + 3\pi)\frac{\text{km}}{\text{h}} \qquad (\text{as } R = 1\text{km})$$

$$= 21.4\ \text{kmh}^{-1}$$

45. (*b*) $\dfrac{d_B}{d_C} = \dfrac{v_B t}{v_C t} = \dfrac{v_B}{v_C} = \dfrac{2.5}{2} = \dfrac{5}{4}$

46. (*a*) $3t + 1.4t = 2\pi R = 20\pi$

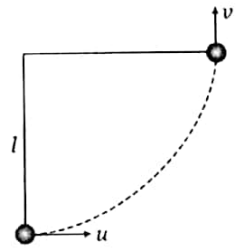
$$\therefore \quad t = \frac{10\pi}{2.2} = 14.3\ \text{s} \Rightarrow a = \frac{v_B^2}{R} = 0.45\ \text{ms}^{-2}$$

47. (*c*) $t = \sqrt{\dfrac{2h}{g}} = \sqrt{\dfrac{2 \times 8}{9.8}} = 1.28\ \text{s}$

$$v = \frac{20}{t} = 15.63\ \text{ms}^{-1}$$

$$\therefore \quad a = \frac{v^2}{R} = 81.4\ \text{ms}^{-2}$$

48. (*a*) From figure we have, $h = l$



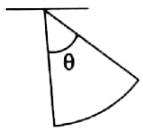
$$\therefore \quad v = \sqrt{u^2 - 2gh} = \sqrt{u^2 - 2gl}$$

$$\Rightarrow \quad |\Delta\mathbf{v}| = |\mathbf{v}_f - \mathbf{v}_i|$$

= subtraction of vectors at 90°

$$= \sqrt{v^2 + u^2 - 2vu \cdot \cos 90°}$$
$$= \sqrt{v^2 + u^2} = \sqrt{(u^2 - 2gl) + u^2} = \sqrt{2(u^2 - gl)}$$

49. (*d*) $g\sin\theta = \dfrac{v^2}{R} = \dfrac{2gh}{R} = \dfrac{2gR(1 - \cos\theta)}{R}$



or $\quad \sin\theta = 2(1 - \cos\theta)$

$$2\sin\frac{\theta}{2}\cos\frac{\theta}{2} = 2\left(2\sin^2\frac{\theta}{2}\right)$$

$$\Rightarrow \quad \tan\frac{\theta}{2} = \frac{1}{2} \Rightarrow \frac{\theta}{2} = \tan^{-1}\left(\frac{1}{2}\right) \text{ or } \theta = 2\tan^{-1}\left(\frac{1}{2}\right)$$

Note In extreme position of pendulum, only tangential component of acceleration ($a_t = g\sin\theta$) is present. In lowest position, only normal acceleration ($a_n = v^2/R$) is present.

50. (*a*) When body is released from the position p (inclined at angle θ from vertical), then velocity at mean position

$$v = \sqrt{2gl(1 - \cos\theta)}$$

$\therefore$ Tension at the lowest point $= mg + \dfrac{mv^2}{l}$

$$= mg + \frac{m}{l}[2gl(1 - \cos 60°)] = mg + mg = 2mg$$

51. (*d*) At mean position,

$$mg + \frac{mv^2}{r} = 3mg \Rightarrow v = \sqrt{2gl} \qquad \text{...(i)}$$

and if the body displace by angle θ with the vertical, then

$$v = \sqrt{2gl(1 - \cos\theta)} \qquad \text{...(ii)}$$

On comparing Eqs. (i) and (ii), we get

$$\cos\theta = 0 \Rightarrow \theta = 90°$$

52. (*a*) Minimum tension is at topmost point (speed = v) and maximum tension at bottommost point (speed = u).

$$\frac{T_{max}}{T_{min}} = \frac{mg + \frac{mu^2}{L}}{-mg + \frac{mv^2}{L}} = 4$$

$$u^2 = v^2 + 2g(2L)$$

Solving, we get $v = 10\ \text{ms}^{-1}$

53. (*a*) Balancing horizontal forces,

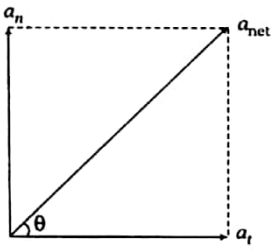
$$T \sin\theta = mr\omega^2$$

or $$T \sin\theta = m(l \sin\theta)\omega^2$$

$\therefore$ $$T = ml\omega^2 = ml(2\pi f)^2$$

$$= ml\left(2\pi \times \frac{2}{\pi}\right)^2 = 16\,ml$$

54. (*a*) $v = \sqrt{2a_t s} = \sqrt{2a_t(\pi R)}$



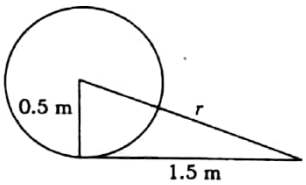
$\therefore$ $$a_n = \frac{v^2}{R} = 2\pi a_t$$

or $$\frac{a_n}{a_t} = 2\pi \Rightarrow \tan\theta = \frac{a_n}{a_t} = 2\pi$$

$\therefore$ $$\theta = \tan^{-1}(2\pi)$$

55. (*a*) $v = \frac{d}{t_1} = \frac{21 \times 2\pi \times 0.5}{44} = 1.5\ \text{ms}^{-1}$

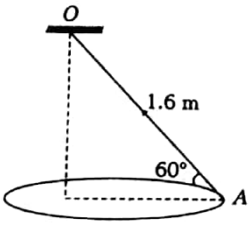
$$t_2 = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 4.9}{9.8}} = 1\text{s}$$



Horizontal distance travelled by drop $= vt_2 = 1.5$ m

$\therefore$ $$r = \sqrt{(1.5)^2 + (0.5)^2} = \sqrt{2.5}\ \text{m}$$

56. (*d*) $R = 1.6 \cos 60° = 0.8$ m

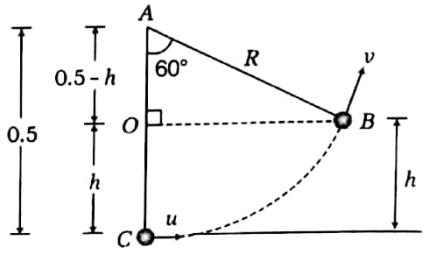


$$T \sin 60° = mg \quad \text{or} \quad T = \frac{2mg}{\sqrt{3}} \Rightarrow T \cos 60° = \frac{mv^2}{R}$$

$\therefore$ $$\frac{v^2}{R} = \frac{T}{2m} = \frac{g}{\sqrt{3}} = \frac{9.8}{\sqrt{3}}\ \text{ms}^{-2} \Rightarrow v = \sqrt{\frac{9.8}{\sqrt{3}} \times 0.8}\ \text{ms}^{-1}$$

Time period $= 2\pi \frac{R}{v}$

57. (*a*) $h = R(1 - \cos 60°) = \frac{R}{2}$



$\because$ In right angled ΔAOB,

$$\cos 60° = \frac{OA}{AB} \Rightarrow \frac{1}{2} = \frac{0.5 - h}{R} \Rightarrow R = 1 - 2h$$

$\Rightarrow$ $$2h = 1 - 2h \qquad (\because R = 2h)$$

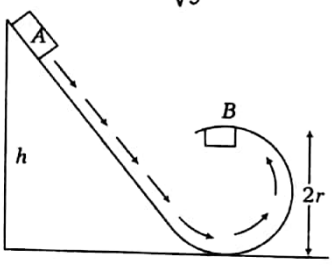
$\Rightarrow$ $$4h = 1 \Rightarrow h = \frac{1}{4}$$

$\because$ $$v = \sqrt{u^2 - 2gh} = \sqrt{9 - 2 \times 10 \times \frac{1}{4}} = \sqrt{9 - 5} = 2\ \text{ms}^{-1}$$

$\therefore$ $v = 2\ \text{ms}^{-1}$

58. (*b*) To loop the loop body should not loose contact with the track anywhere so,

$$v \geq \sqrt{gr}$$



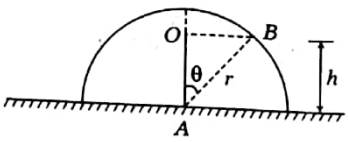
Applying law of conservation of mechanical energy between points A and B, *i.e.*, magnitude of change in kinetic energy equals the magnitude of change in potential energy.

$\Rightarrow$ $$\frac{1}{2}mv^2 - 0 = mg(h - 2r)$$

$$\frac{1}{2}m(\sqrt{gr})^2 = mg(h - 2r) \Rightarrow h = \frac{5}{2}r$$

Hence, h must be atleast equal to 2.5 r.

59. (*c*) When released from top with zero velocity block leaves contact at



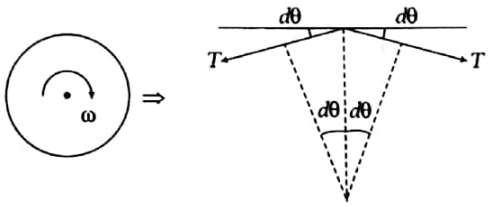
$$\cos\theta = \frac{2}{3}$$

$\because$ In ΔAOB

$$\cos\theta = \frac{OA}{AB} \Rightarrow \cos\theta = \frac{h}{r}$$

$$\therefore \quad h = r\cos\theta = \frac{2r}{3}$$

60. (*a*) To find tension in the ring, let us take an arc which subtends angle $2(d\theta)$ at centre. Tangential components of T cancel out each other, while inward components provide the necessary centripetal force. Thus,



$$2T\sin(d\theta) = (dm)R\omega^2 = (\lambda 2R\cdot d\theta)(R\omega^2)$$

Here, $\lambda \rightarrow$ linear mass density.

For small angles, $\sin d\theta \approx d\theta$

$$\therefore \quad 2Td\theta = 2\lambda R^2\omega^2 d\theta \quad \text{or} \quad T = \lambda R^2\omega^2$$

$$\omega_{max} = \sqrt{\frac{T_{max}}{\lambda}}\cdot\frac{1}{R} = \sqrt{\frac{40}{0.1}}\times\frac{1}{1} = 20 \text{ rad s}^{-1}$$

61. (*c*) $h = l(1-\cos 60°) = \frac{l}{2}, v^2 = 2gh = gl$

Now, $T_{max} - mg = \frac{mv^2}{l}$ (at bottommost point)

$$\therefore \quad T_{max} = 2mg = \mu_s(4mg)$$

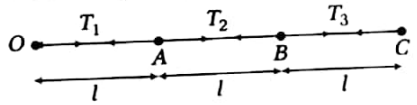
$$\therefore \quad \mu_s = 0.5$$

62. (*d*) From momentum conservation,

$$mu = m\frac{u}{4} + (6m)\sqrt{5\times 10\times 0.5}$$

Solving we get, $u = 40 \text{ ms}^{-1}$

63. (*d*) Let ω is the angular speed of revolution.



$$T_3 = m\omega^3(3l)$$

$$T_2 - T_3 = m\omega^2 2l \Rightarrow T_2 = m\omega^2(5l)$$

$$T_1 - T_2 = m\omega^2 l \Rightarrow T_1 = m\omega^2(6l)$$

$$T_3 : T_2 : T_1 = 3:5:6$$

64. (*c*) $h_{AB} = (r\cos\alpha - r\sin\beta)$

Velocity of particle at B,

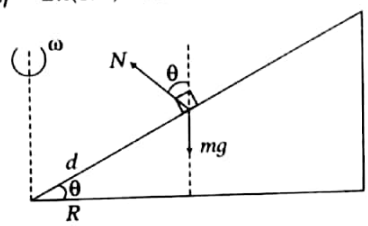
$$v = \sqrt{2gh_{AB}} = \sqrt{2g(r\cos\alpha - r\sin\beta)}$$

Particle will leave contact at B, if component of weight is just equal to centripetal force (towards centre).

$$\text{or} \quad mg\sin\beta = \frac{mv^2}{r} \quad \text{or} \quad \sin\beta = 2\cos\alpha - 2\sin\beta$$

$$\therefore \quad 3\sin\beta = 2\cos\alpha$$

65. (*d*) $\omega = 2\pi f = 2\pi(0.5) = \pi \text{ rad s}^{-1}$



From the figure,

$$N\sin\theta = mR\omega^2 \quad \text{and} \quad N\cos\theta = mg$$

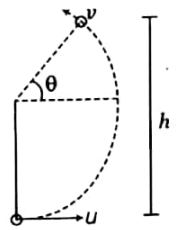
$$\text{or} \quad \tan\theta = \frac{R\omega^2}{g}$$

$$\therefore \quad R = \frac{g\tan\theta}{\omega^2}$$

$$\Rightarrow \quad R = \frac{10\cdot\tan 30°}{(\pi^2)} \approx \frac{1}{\sqrt{3}} \text{ m}$$

$$\text{Now,} \quad d = \frac{R}{\cos\theta} = \frac{1/\sqrt{3}}{\cos\theta} = \frac{1/\sqrt{3}}{\sqrt{3}/2} = \frac{2}{3} = 0.67 \text{ m}$$

66. (*c*) $h = l + l\sin\theta = l(1+\sin\theta)$



$$v^2 = u^2 - 2gh = u^2 - 2gl(1+\sin\theta)$$

String will slack where, component of weight towards centre is just equal to centripetal force or,

$$mg\sin\theta = \frac{mv^2}{l} = \frac{m}{l}[u^2 - 2gl(1+\sin\theta)]$$

Substituting $u^2 = \frac{7gl}{2}$

we get, $\sin\theta = \frac{1}{2}$ or $\theta = 30°$

$\therefore$ The desired angle is 90° + 30° or 120°.

67. (*b*) Given, $\frac{1}{2}mv^2 = as^2$

$$\text{or} \quad F_n = \frac{mv^2}{R} = \frac{2as^2}{R} \quad \text{...(i)}$$

$$\text{Further,} \quad v = \sqrt{\frac{2a}{m}}\cdot s$$

$$\text{or} \quad a_t = \frac{dv}{dt} = \sqrt{\frac{2a}{m}}\cdot\frac{ds}{dt} = \sqrt{\frac{2a}{m}}\cdot v$$

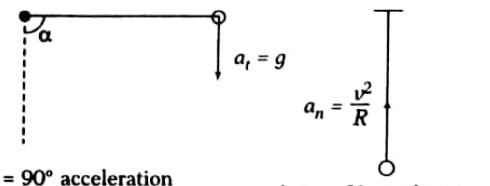
$$= \sqrt{\frac{2a}{m}}\cdot\sqrt{\frac{2a}{m}}\cdot s = \frac{2as}{m}$$

$$\therefore \quad F_t = ma_t = 2as \quad \text{...(ii)}$$

$$\therefore \quad F_{net} = \sqrt{F_n^2 + F_t^2}$$

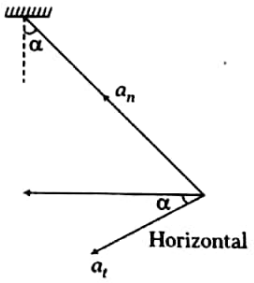
$$= 2as\sqrt{1+\frac{s^2}{R^2}}$$

68. (*a*) When a is horizontal



At $\alpha = 90°$ acceleration is downwards

At $\alpha = 0°$ acceleration is upwards



$$\tan\alpha = \frac{a_n}{a_t} = \frac{v^2/l}{g\sin\alpha} = \frac{2gh/l}{g\sin\alpha}$$

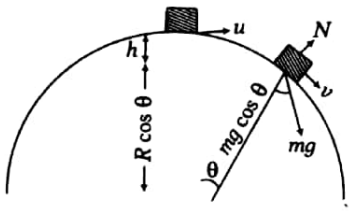
$$= \frac{2gl\cos\alpha/l}{g\sin\alpha} = 2\cot\alpha$$

$\Rightarrow$ $\tan\alpha = \sqrt{2}$

or $\cos\alpha = \frac{1}{\sqrt{3}}$

$\therefore$ $\alpha = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

69. (*b*) $h = R - R\cos\theta$



$$v^2 = u^2 + 2gh = u^2 + 2gR(1-\cos\theta) \quad ...(i)$$

$$mg\cos\theta - N = \frac{mv^2}{R} \quad ...(ii)$$

When the particle loses contact

$$N = 0 \quad ...(iii)$$

From Eqs. (i), (ii) and (iii), we have

$\Rightarrow$ $v^2 = gR\cos\theta = u^2 + 2gR(1-\cos\theta)$

$\Rightarrow$ $3gR\cos\theta = u^2 + 2gR$

$\Rightarrow$ $\cos\theta = \frac{u^2}{3gR} + \frac{2}{3}$

$\Rightarrow$ $\theta = \cos^{-1}\left(\frac{u^2}{3gR} + \frac{2}{3}\right)$

B) Medical entrance special format questions

Assertion and reason

1. (*d*) Direction of acceleration continuously changes.

In uniform circular motion, acceleration always acts towards the centre. Therefore, assertion is false but reason is true.

2. (*a*) v is always perpendicular to ω.

5. (*b*) $a_c = \frac{v^2}{R} = \frac{(2)^2}{1} = 4\ \text{ms}^{-2}$

But no information is given for tangential acceleration a_t.

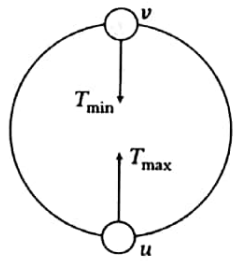
6. (*a*) $\frac{mv^2}{R}$ does not act. But this much force is required.

7. (*a*) Centripetal force acts when the particle is observed from a rotating non-inertial frame.

11. (*c*) Angle between any two vectors lies in the following range,

$$0° \le \theta \le 180°$$

12. (*a*) Let the minimum and maximum tensions be T_{min} and T_{max} and the minimum and maximum speed be u and v.



$\therefore$ $T_{max} = \frac{mu^2}{R} + mg$

$T_{min} = \frac{mv^2}{R} - mg$

$\therefore$ $\Delta T = m\left(\frac{u^2}{R} - \frac{v^2}{R}\right) + 2mg$

From conservation of energy $\frac{u^2}{R} - \frac{v^2}{R} = 4g$

$\Rightarrow$ is independent of u and $\Delta T = 6\,mg$.

$\therefore$ Reason is correct explanation of assertion.

13. (*d*) In case of massless rod, $v_{min} = \sqrt{4gl}$.

Work done by centripetal force on the particle is always zero.

14. (*c*) The normal reaction is not least at topmost point, hence assertion is false.

15. (*a*) In vertical direction, $N\cos\theta = mg$

In horizontal direction,

$$N\sin\theta = \frac{mv^2}{R}$$

Match the columns

1. $T_3 = (1)(3)(4)^2 = 48$ N $\quad (\because F = mR\omega^2)$

$T_2 - T_3 = (1)(2)(4)^2$

$\therefore \quad T_2 = 80$ N $\Rightarrow T_1 - T_2 = (1)(1)(4)^2$

$\therefore \quad T_1 = 96$ N

2. $v_B^2 = u_A^2 - 2gh_{AB} = (9gR) - (2gR) = 7gR$

$\therefore \quad v_B = \sqrt{7gR}$

Further, $\quad T_B = \dfrac{mv_B^2}{R} = 7\,mg$

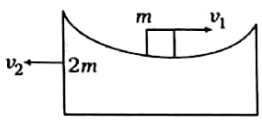
Again, $\quad v_C^2 = v_A^2 - 2gh_{AC} = (9gR) - 2(2R) = 5gR$

$\therefore \quad v_C = \sqrt{5gR}$

Further, $\quad T_C + mg = \dfrac{mv_C^2}{R}$

$\therefore \quad T_C = 4\,mg$

3.



$$mgR = E = \frac{1}{2}mv_1^2 + \frac{1}{2}2mv_2^2$$

$$2mv_2 = mv_1 \Rightarrow v_1 = 2v_2$$

$$E = \frac{1}{2}m \times 4v_2^2 + mv_2^2 = 3mv_2^2$$

$$v_2 = \sqrt{\frac{E}{3m}},\ v_1 = 2\sqrt{\frac{E}{3m}}$$

$$\text{KE of } m = \frac{1}{2}mv_1^2 + \frac{1}{2}m \times \frac{4E}{3m} = \frac{2E}{3}$$

$$\text{KE of } 2m = \frac{1}{2}2m \times v_2^2 = m \times \frac{E}{3m} = \frac{E}{3}$$

$$P_1 = mv_1 = 2m\sqrt{\frac{E}{3m}} = \sqrt{\frac{4}{3}mE}$$

$$P_2 = 2mv_2 = 2m\sqrt{\frac{E}{3m}} = \sqrt{\frac{4}{3}mE}$$

C) Medical entrances' gallery

1. (*c*) Centripetal acceleration of a particle moving on a circular path is given by

$$a_C = \frac{v^2}{R}$$

In the given figure,

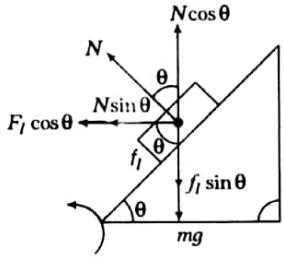
$$a_C = a\cos 30° = 15\cos 30°\ \text{ms}^{-2}$$

$$\Rightarrow \quad \frac{v^2}{R} = 15\cos 30°$$

$$\Rightarrow \quad v^2 = R \times 15 \times \frac{\sqrt{3}}{2} = 2.5 \times 15 \times \frac{\sqrt{3}}{2}$$

$$\therefore \quad v = 5.7\ \text{ms}^{-1}$$

2. (*a*) According to question, a car is negotiating a curved road of radius R. The road is banked at angle θ and the coefficient of friction between the tyres of car and the road is μ_s. So, this given situation can be drawn as shown in figure below.



Considering the case of vertical equilibrium

$$N\cos\theta = mg + f_l\sin\theta$$

$$\Rightarrow \quad mg = N\cos\theta - f_l\sin\theta \quad \text{...(i)}$$

Considering the case of horizontal equilibrium,

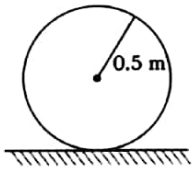
$$N\sin\theta + f_l\cos\theta = \frac{mv^2}{R} \quad \text{...(ii)}$$

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{v^2}{Rg} = \frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta} \quad [f_l \propto \mu_s]$$

$$\Rightarrow v = \sqrt{Rg\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right)} \Rightarrow v = \sqrt{Rg\left(\frac{\tan\theta + \mu_s}{1 - \mu_s\tan\theta}\right)}$$

3. (*d*) According to given question, a uniform circular disc of radius 50 cm at rest is free to turn about an axis having perpendicular to its plane and passes through its centre. This situation can be shown by the figure alongside below



$\therefore$ Angular acceleration, $\alpha = 2$ rad s^{-2} (given)

Angular speed, $\omega = \alpha t = 4$ rad s^{-1}

$\because$ Centripetal acceleration, $a_c = \omega^2 r = (4)^2 \times 0.5 = 16 \times 0.5$

$$a_c = 8\ \text{ms}^{-2}$$

$\because$ Linear acceleration at the end of 2s,

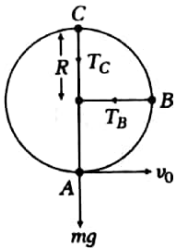
$$a_t = \alpha r = 2 \times 0.5 \Rightarrow a_t = 1\ \text{ms}^{-2}$$

Therefore, the net acceleration at the end of 2 s is given by

$$a = \sqrt{a_c^2 + a_t^2}$$

$$a = \sqrt{(8)^2 + (1)^2} = \sqrt{65} \Rightarrow a \approx 8\ \text{ms}^{-2}$$

4. (*c*) According to question, we have



Let the tension at point A be T_A. So, from Newton's second law

$$T_A - mg = \frac{mv_c^2}{R}$$

Energy at point $A = \frac{1}{2} mv_0^2$ ...(i)

Energy at point C is

$$\frac{1}{2} mv_c^2 + mg \times 2R \quad ...(ii)$$

Applying Newton's second law at point C

$$T_c + mg = \frac{mv_c^2}{R}$$

To complete the loop $T_c \geq 0$

So, $$mg = \frac{mv_c^2}{R}$$

$\Rightarrow$ $$v_c = \sqrt{gR} \quad ...(iii)$$

From Eqs. (i) and (ii) by conservation of energy

$$\frac{1}{2} mv_0^2 = \frac{1}{2} mv_c^2 + 2mgR$$

$\Rightarrow$ $$\frac{1}{2} mv_0^2 = \frac{1}{2} mgR + 2mgR \quad (\because v_c = \sqrt{gR})$$

$\Rightarrow$ $$v_0^2 = gR + 4gR$$

$\Rightarrow$ $$v_0 = \sqrt{5gR}$$

5. (*d*) Given, mass of particle, $m = 0.01$ kg.

Radius of circle along which particle is moving, $r = 6.4$ cm.

$\because$ Kinetic energy of particle, KE $= 8 \times 10^{-4}$ J

$\Rightarrow$ $$\frac{1}{2} mv^2 = 8 \times 10^{-4} \text{ J}$$

$\Rightarrow$ $$v^2 = \frac{16 \times 10^{-4}}{0.01} = 16 \times 10^{-2} \quad ...(i)$$

As it is given that KE of particle is equal to 8×10^{-4} J by the end of second revolution after the beginning of motion of particle. It means, its initial velocity (u) is 0 ms^{-1} at this moment.

$\because$ By Newton's third equation of motion,

$$v^2 = u^2 + 2a_t s$$

$\Rightarrow$ $$v^2 = 2a_t s \text{ or } v^2 = 2a_t (4\pi r)$$

($\because$ particle covers 2 revolutions)

$\Rightarrow$ $$a_t = \frac{v^2}{8\pi r} = \frac{16 \times 10^{-2}}{8 \times 3.14 \times 6.4 \times 10^{-2}}$$

[$\because$ from Eq. (i), $v^2 = 16 \times 10^{-2}$]

$\therefore$ $$a_t = 0.1 \text{ ms}^{-2}$$

6. (*a*) Given, radius of circular path = 2000 m

Mass of plane = 16000 kg

and speed = 900 kmh^{-1}

$$= \frac{900 \times 10^3}{3600} \text{ ms}^{-1} = 250 \text{ ms}^{-1}$$

The required centripetal force,

$$f_c = \frac{mv^2}{r} \quad \text{(in upward direction)}$$

$$= \frac{16000 \times 250 \times 250}{2000} = 5 \times 10^5 \text{N}$$

The gravitational force, $f_g = mg = 16000 \times 9.8$

$$= 1.568 \times 10^5 \text{ N}$$

Thus, force will act as reaction force in upward direction.

$\therefore$ Net upward force $= 5 \times 10^5 - 1568 \times 10^5$

$$= (5 - 16) \times 10^5 \text{ N} = 3.4 \times 10^5 \text{ N}$$

7. (*c*) A particle moving in a curved path is experiencing an acceleration. Even if moving around the perimeter of a circle with a constant speed, there is still a change in velocity and subsequently an acceleration. This acceleration is directed towards the centre of the circle. The quantity which may remain constant here is magnitude of acceleration.

8. (*b*) We know that, angular speed of a watch

$$\omega = \frac{\theta}{t} \Rightarrow \omega \propto \frac{1}{t}$$

Hence, $$\frac{\omega_{\text{second}}}{\omega_{\text{hour}}} = \frac{t_{\text{hour}}}{t_{\text{second}}} = \frac{12 \times 3600}{60} = 720$$

i.e. $$\frac{\omega_{\text{second}}}{\omega_{\text{hour}}} = \frac{720}{1}$$

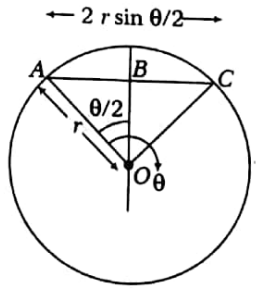
9. (*e*) We know that, ($\because r = 10$ cm)

$$\omega = \frac{\pi}{30} \text{ rads}^{-1}$$

$\therefore$ $$v = r\,\omega = 10 \times \frac{\pi}{30} = \frac{\pi}{3} \text{ rads}^{-1}$$

$$= \frac{3.14}{3} \text{ rad s}^{-1} = 1.046 \cong 1 \text{ cm s}^{-1}$$

10. (*d*) The figure is shown as below



In ΔAOB, $$\sin\frac{\theta}{2} = \frac{AB}{AO} \quad (\because AO = r)$$

$$AB = AO \sin\frac{\theta}{2} \Rightarrow AB = r \sin\frac{\theta}{2}$$

$$AC = AB + BC \quad (\because AB = BC)$$

$$r = \frac{r \sin\theta}{2} + r \sin\frac{\theta}{2}, \; AC = r \sin\frac{\theta}{2} + r \sin\frac{\theta}{2}$$

$$AC = 2r \sin\frac{\theta}{2}$$

So, the magnitude of the corresponding linear displacement will be $2r \sin\frac{\theta}{2}$.

11. (*c*) We know that, $\omega = 2\pi n \quad \therefore \quad \omega_1 = 2\pi n_1$

where, $n_1 = 1800$ rpm, $n_2 = 3000$ rpm, $\Delta t = 20$ s

$$\omega_1 = 2\pi \times \frac{1800}{60} = 2\pi \times 30 = 60\,\pi$$

Similarly, $\omega_2 = 2\pi n_2 = 2\pi \times \frac{3000}{60} = 2\pi \times 50 = 100\,\pi$

If the angular velocity of a rotating wheel about an axis is changed by change in angular velocity in a time interval Δt, then the angular acceleration of rotating wheel about that axis is given by

$$\alpha = \frac{\text{Change in angular velocity}}{\text{Time interval}}$$

$$= \frac{\omega_2 - \omega_1}{\Delta t} = \frac{100\pi - 60\pi}{20}$$

$$= \frac{40\pi}{20} = 2\pi \text{ rads}^{-2}$$

12. (*a*) Centripetal force always acts towards the centre and is given by $F_c = \frac{Mv^2}{R}$.

13. (a) For uniform circular motion, the magnitude of velocity remains unchanged but due to change in direction the velocity vector changes.

14. (*b*) We know that the difference between the maximum and minimum tensions in the rope

$$T_{max} - T_{min} = 2mg$$

Here, $T_{max} - T_{min} = 20$

Now, mass of the store $F = ma$

$$m = \frac{F}{a} = \frac{20}{2 \times 10} = 1\,\text{kg}$$

15. (*e*) Given, $n = 18$ per minute $= \frac{18 \times 2\pi}{60}$ and $r = 10$ cm

We know that $v = r\omega$

Here, $v = 0.1 \times \frac{18 \times 2\pi}{60}$

$$v = 3 \times 2\pi \times 10^{-2} \Rightarrow v = 6\pi \times 10^{-2} \text{ ms}^{-1}$$

16. (*b*) The angle of banking

$$\tan\theta = \frac{v^2}{rg}$$

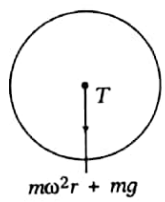
Given, $\theta = 45°$, $r = 90$ m and $g = 10$ ms^{-2}

$$\tan 45° = \frac{v^2}{90 \times 10} \Rightarrow v = \sqrt{90 \times 10 \times \tan 45°}$$

Speed of car, $v = 30$ ms^{-1}

17. (*d*) The horizontal inward component provides required centripetal force.

18. (*d*) Tension, $T = m\omega^2 r + mg$



$m\omega^2 r + mg$

$$= 0.5 \times (4)^2 \times 1 + 0.5 \times 10 = 13 \text{ N}$$

19. (*a*) Given, $R = 4$ m

$$v = \sqrt{gR} = \sqrt{9.8 \times 4}$$

Now, time period $= \frac{2\pi R}{v} = \frac{2 \times 3.14 \times 4}{\sqrt{9.8 \times 4}} = 4$ s

CHAPTER

08

Centre of mass, conservation of momemtum and collision

Inside

1) Centre of mass
2) Motion of centre of mass
   - Linear momentum of a system of particles
3) Collision
   - Head on elastic collision
   - Head on inelastic collision
   - Newton's law of restitution

1 Centre of mass

Centre of mass of system of particles or in a body can be defined as a point at which the total mass of the system is supposed to be concentrated. Centre of mass of a system represents the entire body. On application of external forces, centre of mass of the system of particles moves in the same way as a point having mass equal to that of the whole system move.

Position of centre of mass of two particles

Consider two particles of masses m_1 and m_2 located at position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$. *Then position of centre of mass* r_{CM} *is given as*

$$\mathbf{r}_{CM} = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2}{m_1 + m_2}$$

$$\boxed{\mathbf{r}_{CM} = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2}{m} = \frac{\Sigma m_i\mathbf{r}_i}{m}}$$

where, $m = m_1 + m_2 =$ total mass of system

x and y the coordinates of centre of mass can be written as

$$x_{CM} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} \text{ and } y_{CM} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2}$$

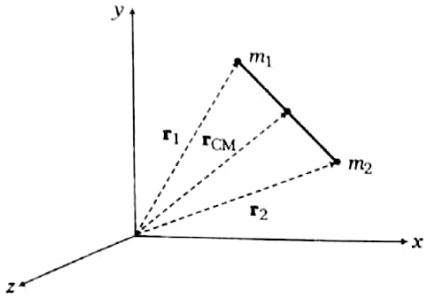


Fig. 8.1 System of two particles

Hence, **the centre of mass of two particles system lies between the two particles on the line joining them and the distance of the centre of mass from masses is in inverse ratio of masses of the particles.**

i.e., $$r \propto \frac{1}{m} \Rightarrow \boxed{\frac{r_1}{r_2} = \frac{m_2}{m_1}}$$

Consider two particles of masses m_1 and m_2 at distance r from each other. There centre of mass CM must lie in between them on the line joining them. Let distances of these particles from the centre of mass be r_1 and r_2.

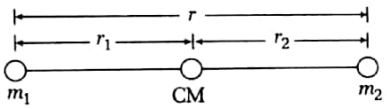


Fig. 8.2

$$\because \quad r \propto \frac{1}{m} \Rightarrow m_1r_1 = m_2r_2$$

$$\boxed{r_1 = \frac{m_2r}{m_1 + m_2}} \quad \text{and} \quad \boxed{r_2 = \frac{m_1r}{m_1 + m_2}}$$

If the two particles have the same mass $m_1 = m_2 = m$, then

$$r_{CM} = \frac{mr_1 + mr_2}{2m} = \frac{r_1 + r_2}{2}$$

Thus, **for two system particles of equal mass the centre of mass lies exactly midway between them.**

If $m_1 \neq m_2$, CM is nearer to the particle of larger mass.

Example 8.1 *Two bodies of masses 1 kg and 2 kg are located at (1, 2) and (−1, 3), respectively. Calculate the coordinates of centre of mass.*

Sol. Let the coordinates of the centre of mass be (x, y).

Given, mass, $m_1 = 1$ kg, $m_2 = 2$ kg

Coordinates, $x_1 = 1$, $x_2 = -1$ and $y_1 = 2$, $y_2 = 3$

$$\because \quad x = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

$$\Rightarrow \quad x = \frac{1 \times 1 + (2)(-1)}{1 + 2} = \frac{1 - 2}{3} = \frac{-1}{3}$$

Similarly, $$y = \frac{m_1y_1 + m_2y_2}{m_1 + m_2} = \frac{(1)(2) + (2)(3)}{1 + 2} = \frac{2 + 6}{3} = \frac{8}{3}$$

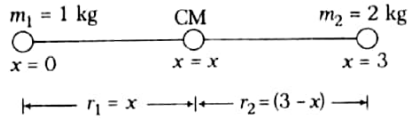
Therefore, the coordinates of centre of mass will be $\left(\frac{-1}{3}, \frac{8}{3}\right)$.

Example 8.2 *Two particles of mass 1 kg and 2 kg are located at $x = 0$ and $x = 3$ m. Find the position of their centre of mass.*

Sol. Since, both the particles lie on x-axis, the CM will also lie on x-axis. Let the CM be located at $x = x$, then

r_1 = distance of CM from the particle of mass 1 kg = x

and r_2 = distance of CM from the particle of mass 2 kg = $(3 - x)$



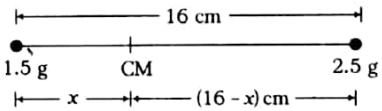
Using, $$\frac{r_1}{r_2} = \frac{m_2}{m_1}$$

we get $$\frac{x}{3 - x} = \frac{2}{1} \text{ or } x = 2 \text{ m}$$

Thus, the CM of the two particles is located at $x = 2$ m.

Example 8.3 *Two point objects of mass 1.5 g and 2.5 g respectively are at a distance of 16 cm apart, the centre of mass is at a distance x from the object of mass 1.5 g. Find the value of x.*

Sol.



We know that, centre of mass of two particles system lies between the two particles on the line joining them and divide the distance between them in inverse ratio of masses of the particles, we can write

$$\therefore \quad x = \frac{m_2r}{m_1 + m_2} \Rightarrow x = \frac{2.5 \times 16}{2.5 + 1.5} \Rightarrow x = \frac{2.5 \times 16}{4} = 10 \text{ cm}$$

Position of centre of mass of a large number of particles

If we have a system consisting of n particles, of masses $m_1, m_2, \ldots, m_n$ with $\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_n$ as their position vectors at a given instant of time. The position vector $\mathbf{r}_{CM}$ of the centre of mass of the system at that instance is given by

$$\mathbf{r}_{CM} = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2 + \ldots + m_n\mathbf{r}_n}{m_1 + m_2 + \ldots + m_n} = \frac{\sum_{i=1}^{n} m_i\mathbf{r}_i}{\sum_{i=1}^{n} m_i}$$

or $$\boxed{\mathbf{r}_{CM} = \frac{\sum_{i=1}^{n} m_i\mathbf{r}_i}{M}}$$

Here, $M = m_1 + m_2 + \ldots + m_n$ and $\Sigma m_i \mathbf{r}_i$ is called the **first moment of the mass.**

Further, $\mathbf{r}_i = x_i\hat{\mathbf{i}} + y_i\hat{\mathbf{j}} + z_i\hat{\mathbf{k}}$

and $\mathbf{r}_{CM} = x_{CM}\hat{\mathbf{i}} + y_{CM}\hat{\mathbf{j}} + z_{CM}\hat{\mathbf{k}}$

So, the cartesian coordinates of the CM will be

$$x_{CM} = \frac{m_1x_1 + m_2x_2 + \ldots + m_nx_n}{m_1 + m_2 + \ldots + m_n} = \frac{\sum_{i=1}^{n} m_ix_i}{\Sigma m_i}$$

or
$$\boxed{x_{CM} = \frac{\sum_{i=1}^{n} m_ix_i}{M}}$$

Similarly, $y_{CM} = \dfrac{\sum_{i=1}^{n} m_iy_i}{M}$ and $z_{CM} = \dfrac{\sum_{i=1}^{n} m_iz_i}{M}$

Example 8.4 *The position vectors of three particles of mass $m_1 = 1$ kg, $m_2 = 2$ kg and $m_3 = 3$ kg are $\mathbf{r}_1 = (\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + \hat{\mathbf{k}})$ m, $\mathbf{r}_2 = (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$ m and $\mathbf{r}_3 = (2\hat{\mathbf{i}} - \hat{\mathbf{j}} - 2\hat{\mathbf{k}})$ m, respectively. Find the position vector of their centre of mass.*

Sol. The position vector of CM of the three particles will be given by

$$\mathbf{r}_{CM} = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2 + m_3\mathbf{r}_3}{m_1 + m_2 + m_3}$$

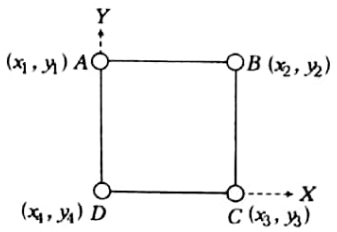
Substituting the values, we get

$$\mathbf{r}_{CM} = \frac{1(\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + \hat{\mathbf{k}}) + 2(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) + 3(2\hat{\mathbf{i}} - \hat{\mathbf{j}} - 2\hat{\mathbf{k}})}{1 + 2 + 3}$$

$$= \frac{9\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 3\hat{\mathbf{k}}}{6} \Rightarrow \mathbf{r}_{CM} = \frac{1}{2}(3\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}})\,\text{m}$$

Example 8.5 *Four particles of mass 1 kg, 2 kg, 3 kg and 4 kg are placed at the four vertices A, B, C and D of a square of side 1 m. Find the position of centre of mass of the particles.*

Sol. Assuming D as the origin, DC as X-axis and DA as Y-axis, we have



$m_1 = 1$ kg; $(x_1, y_1) = (0, 1$ m$)$

$m_2 = 2$ kg; $(x_2, y_2) = (1$ m, 1 m$)$

$m_3 = 3$ kg; $(x_3, y_3) = (1$ m, $0)$

and $m_4 = 4$ kg; $(x_4, y_4) = (0, 0)$

Coordinates of their CM are

$$x_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4}{m_1 + m_2 + m_3 + m_4}$$

$$= \frac{1(0) + 2(1) + 3(1) + 4(0)}{1 + 2 + 3 + 4} = \frac{5}{10} = \frac{1}{2}\text{ m} = 0.5\text{ m}$$

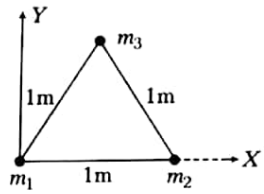
Similarly, $y_{CM} = \dfrac{m_1y_1 + m_2y_2 + m_3y_3 + m_4y_4}{m_1 + m_2 + m_3 + m_4}$

$$= \frac{1(1) + 2(1) + 3(0) + 4(0)}{1 + 2 + 3 + 4} = \frac{3}{10}\text{ m} = 0.3\text{ m}$$

$\therefore$ Centre of mass is at $(x_{CM}, y_{CM}) = (0.5$ m, 0.3 m$)$

Example 8.6 *Three point masses $m_1 = 2$ kg, $m_2 = 4$ kg and $m_3 = 6$ kg are kept at the three corners of an equilateral triangle of side 1 m. Find the location of their centre of mass.*

Sol. Assume m_1 to be at the origin and X-axis along the line joining m_1 as shown in figure.



From the figure, it is clear that the xy-coordinates of m_1 is $(x_1, y_1) = (0, 0)$ that of m_2 is $(x_2, y_2) = (1, 0)$ and that of m_3 is $(x_3, y_3) = \left(\dfrac{1}{2}, \dfrac{\sqrt{3}}{2}\right)$.

$$x_{CM} = \frac{2 \times 0 + 4 \times 1 + 6 \times 1/2}{2 + 4 + 6} = \frac{7}{12}\text{ m}$$

$$y_{CM} = \frac{2 \times 0 + 4 \times 0 + 6 \times \sqrt{3}/2}{2 + 4 + 6} = \frac{3\sqrt{3}}{12} = \frac{\sqrt{3}}{4}\text{m}$$

$\therefore$ Centre of mass is at $\left(\dfrac{7}{12}, \dfrac{\sqrt{3}}{4}\right)$.

Position of centre of mass of continuous bodies

For a real body which has a continuous distribution of matter, point masses are then differential mass elements dm and centre of mass is defined as

$$x_{CM} = \frac{\int x\,dm}{\int dm} = \frac{\int x\,dm}{M}$$

$$y_{CM} = \frac{\int y\,dm}{\int dm} = \frac{\int y\,dm}{M}$$

and
$$z_{CM} = \frac{\int z\,dm}{\int dm} = \frac{\int z\,dm}{M}$$

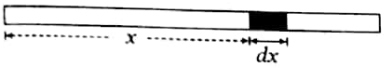
where M is total mass of that real body.

If we choose the origin of coordinates axes at centre of mass, then

$$\int x dm = \int y dm = \int z\,dm = 0$$

Example 8.7 *The linear density of a thin rod of length 1 m varies as $\lambda = (1 + 2x)$, where x is the distance from its one end. Find the distance of its centre of mass from this end.*

Sol. Let the x-axis be along the length of the rod and origin at one of its end as shown in figure.



As rod is along x-axis, for all points on it y and z will be zero so, $y_{CM} = 0$ and $z_{CM} = 0$, *i.e.*, centre of mass will be on the rod. Now consider an element of rod of length dx at a distance x from the origin,

Mass of this element $dm = \lambda dx = (1 + 2x)dx$

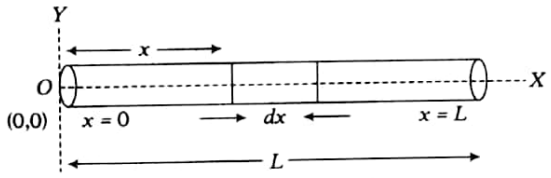
$$x_{CM} = \frac{\int x\,dm}{\int dm} = \frac{\int_0^1 x(1+2x)dx}{\int_0^1 (1+2x)dx}$$

$$= \frac{\left[\frac{x^2}{2} + \frac{2x^3}{3}\right]_0^1}{[x + x^2]_0^1}$$

$$= \left[\frac{\frac{(1)^2}{2} + \frac{2(1)^3}{3} - 0 - 0}{1 + (1)^2 - 0 - 0}\right] = \frac{\frac{1}{2} + \frac{2}{3}}{2} = \frac{7}{12}\text{ m}$$

Example 8.8 *A straight rod of length L has one of its ends at the origin and the other at x = L. If the mass per unit length of the rod is given by Ax, here A is constant, where is its mass centre?*

Sol. Let the mass of an element of length dx of the rod located at a distance x away from left end is $Axdx$.



The x-coordinate of the centre of mass is given by

$$x_{CM} = \frac{1}{M}\int x\,dm = \frac{1}{M}\int_0^L x\,(Axdx)$$

$$= \frac{\int_0^L x \cdot Axdx}{\int_0^L Axdx} = \frac{[Ax^3/3]_0^L}{[Ax^2/2]_0^L} = \frac{2}{3}L$$

The y-coordinate is $y_{CM} = \frac{1}{M}\int y\,dm = 0$

and similarly, $z_{CM} = 0$

Hence, the centre of mass is at $\left(\frac{2}{3}L, 0, 0\right)$ or at $\frac{2}{3}L$ in the middle of the rod, *i.e.*, at $\frac{2}{3}L$ from one end.

Position of centre of mass of symmetrical bodies

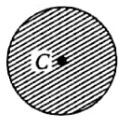
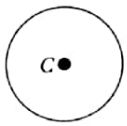
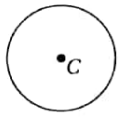
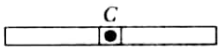
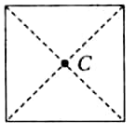
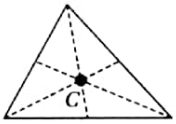
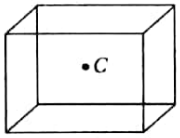
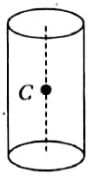
Given below are three points which are very important regarding the centre of mass of symmetrical bodies.

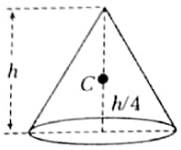
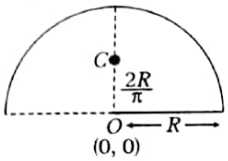
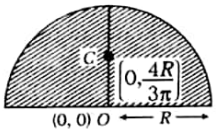
(i) For the bodies symmetrical about both the axes (x or y) or all the three axes (x, y, z), the centre of mass will lie at point of intersection of symmetrical axes. There is no need to determine CM.

(ii) The bodies which are symmetrical about one axis, centre of mass will lie on that axis. Determine only that coordinate about which there is symmetry.

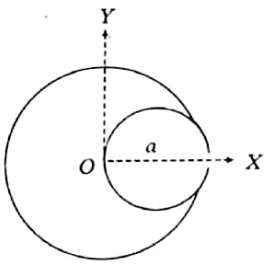
(iii) If an arrangement or body is not symmetrical about any axis, then determine all the required coordinates.

Centre of mass of some symmetric bodies are given in a table below.

Body	Figure	Position of COM
Uniform sphere (hollow or solid)		Centre of the sphere
Uniform circular ring		Centre of the ring
Unform circular disc		Centre of the disc
Uniform rod		Centre of the rod
A plane square lamina		Point of intersection of diagonals
Triangular lamina		Point of intersection of the medians
Rectangular cubical block		Point of intersection of the diagonal
Cylinder (Hollow or Solid)		Middle point of the axis of the cylinder

Body	Figure	Position of COM
Cone or pyramid		On the axis of the cone at a distant $3h/4$ from the vertex, where h is the height of the cone.
Uniform semicircular wire		Coordinates $\left(0, \frac{2R}{\pi}\right)$ or $(0, 0.64R)$ of CM
Uniform semicircular plate or disc		Coordinate of $\left(0, \frac{4R}{3\pi}\right)$ or $(0, 0.42R)$ of CM

Example 8.9 *Find the position of centre of mass of the uniform lamina shown in figure, if small disc of radius a/2 is cut from disc of radius a.*



Sol. Here, A_1 = area of complete circle $= \pi a^2$

A_2 = area of small circle

$$= \pi\left(\frac{a}{2}\right)^2 = \frac{\pi a^2}{4}$$

(x_1, y_1) = coordinates of centre of mass of large circle $= (0, 0)$

(x_2, y_2) = coordinates of centre of mass of small circle $= \left(\frac{a}{2}, 0\right)$

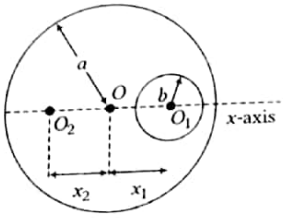
Using, 0 $\quad x_{CM} = \dfrac{A_1x_1 - A_2x_2}{A_1 - A_2}$

we get, $$x_{CM} = \frac{-\frac{\pi a^2}{4}\left(\frac{a}{2}\right)}{\pi a^2 - \frac{\pi a^2}{4}} = \frac{-\left(\frac{1}{8}\right)}{\left(\frac{3}{4}\right)} a = -\frac{a}{6}$$

and $y_{CM} = 0$ as y_1 and y_2 both are zero. Therefore, coordinates of CM of the lamina shown in figure are $\left(-\frac{a}{6}, 0\right)$.

Example 8.10 *A small disc of radius 2 cm is cut from a disc of radius 6 cm. If the distance between their centres is 3.2 cm, what is the shift in the centre of mass of the disc?*

Sol. Let radius of complete disc is a and that of small disc is b. Also, let centre of mass now shifts to O_2 at a distance x_2 from original centre.



The position of new centre of mass is given by

$$x_{CM} = \frac{-\sigma\pi b^2 x_1}{\sigma\pi a^2 - \sigma\pi b^2},$$

where, σ = mass per unit area

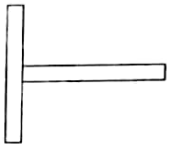
Here, $a = 6$ cm, $b = 2$ cm, $x_1 = 3.2$ cm

Hence, $$x_{CM} = \frac{-\sigma \times \pi(2)^2 \times 3.2}{\sigma \times \pi \times (6)^2 - \sigma \times \pi \times (2)^2}$$

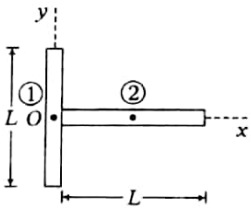
$$= -\frac{12.8\pi}{32\pi} = -0.4 \text{ cm}$$

Negative sign indicates the shift from the centre.

Example 8.11 *Two identical rods each of mass m and length L are connected as shown. Locate centre of mass.*



Sol. This system is symmetrical about the x-axis. Hence, we need to find x_{CM}. Here we will take coordinates of CM of rods.



For rod 1, mass $m_1 = m$

x-coordinate $x_1 = 0$

For rod 2, mass $m_2 = m$

x-coordinate $x_2 = L/2$

$$\therefore \quad x_{CM} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

$$x_{CM} = \frac{m \times 0 + m \times \frac{L}{2}}{m + m} = \frac{m\frac{L}{2}}{2m} = \frac{L}{4}$$

Similarly, $y_{CM} = 0$

$\therefore$ Centre of mass $\equiv \left(\frac{L}{4}, 0\right)$

Check point 8.1

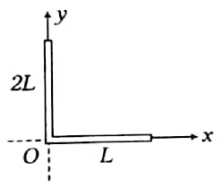
1. The centre of mass a system of particles does not depend on
   (a) masses of the particles
   (b) internal forces on the particles
   (c) position of the particles
   (d) relative distance between the particles

2. The centre of mass of a system of two particles divides the distance between them
   (a) in inverse ratio of square of masses of particles
   (b) in direct ratio of square of masses of particles
   (c) in inverse ratio of masses of particles
   (d) in direct ratio of masses of particles

3. Two bodies of masses 1 kg and 2 kg are lying in xy plane at $(-1, 2)$ and $(2, 4)$ respectively. What are the coordinates of the centre of mass?
   (a) $\left(1, \frac{10}{3}\right)$ (b) $(1, 0)$
   (c) $(0, 1)$ (d) None of these

4. In carbon monoxide molecules, the carbon and the oxygen atoms are separated by distance 1.2×10^{-10} m. The distance of the centre of mass, from the carbon atom is
   (a) 0.48×10^{-10} m (b) 0.51×10^{-10} m
   (c) 0.56×10^{-10} m (d) 0.69×10^{-10} m

5. Figure shows a composite system of two uniform rods of lengths as indicated. Then the coordinates of the centre of mass of the system of rods are

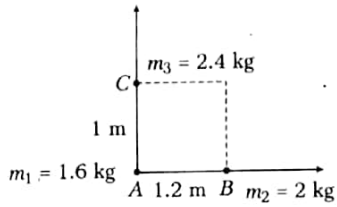


   (a) $\left(\frac{L}{2}, \frac{2L}{3}\right)$ (b) $\left(\frac{L}{4}, \frac{2L}{3}\right)$ (c) $\left(\frac{L}{6}, \frac{2L}{3}\right)$ (d) $\left(\frac{L}{6}, \frac{L}{3}\right)$

6. All the particles of a body are situated at a distance R from the origin. The distance of centre of mass of the body from the origin is
   (a) $= R$ (b) $\leq R$
   (c) $> R$ (d) $\geq R$

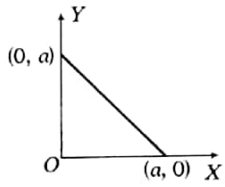
7. A uniform metal rod of length 1 m is bent at 90°, so as to form two arms of equal length. The centre of mass of this bent rod is
   (a) on the bisector of the angle, $\left(\frac{1}{\sqrt{2}}\right)$ m from vertex
   (b) on the bisector of angle, $\left(\frac{1}{2\sqrt{2}}\right)$ m from vertex
   (c) on the bisector of the angle, $\left(\frac{1}{2}\right)$ m from vertex
   (d) on the bisector of the angle, $\left(\frac{1}{4\sqrt{2}}\right)$ m from vertex

8. Three point masses m_1, m_2 and m_3 are placed at the corners of a thin massless rectangular sheet (1.2 m × 1 m) as shown. Centre of mass will be located at the point.



   (a) (0.8, 0.6) m (b) (0.6, 0.8) m
   (c) (0.4, 0.4) m (d) (0.5, 0.6) m

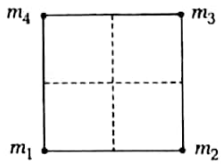
9. Three rods of the same mass are placed as shown in the figure. What will be the coordinate of centre of mass of the system?



   (a) $\left(\frac{a}{2}, \frac{a}{2}\right)$ (b) $\left(\frac{a}{\sqrt{2}}, \frac{a}{\sqrt{2}}\right)$
   (c) $\frac{2a}{3}, \frac{2a}{3}$ (d) $\left(\frac{a}{3}, \frac{a}{3}\right)$

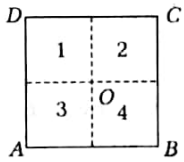
10. Centre of mass of three particles of masses 1 kg, 2 kg and 3 kg lies at the point (1, 2, 3) and centre of mass of another system of particles of total mass 3 kg lies at the point (−1, 3, −2). Where should we put a particle of mass 5 kg so that the centre of mass of the entire system lies at the centre of mass of Ist system?
   (a) (0, 0, 0) (b) (1, 3, 2)
   (c) (−1, 2, 3) (d) None of these

11. Four particles of mass $m_1 = 2m$, $m_2 = 4$ m, $m_3 = m$ and m_4 are placed at four corners of a square. What should be the value of m_4 so that the centre of mass of all the four particles are exactly at the centre of the square?



   (a) $2m$ (b) $8m$
   (c) $6m$ (d) None of these

12. Four rods AB, BC, CD and DA have mass m, $2m$, $3m$ and $4m$ respectively. The centre of mass of all the four rods
   (a) lie in region 1
   (b) lie in region 2
   (c) lie in region 3
   (d) lie at O



13. Three identical spheres, each of mass 1 kg are placed touching each other with their centres on a straight line. Their centre are marked P, Q and R respectively. The distance of centre of mass of the system from P is

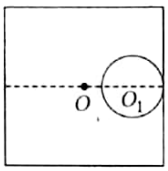
(a) $\frac{PQ + PR + QR}{3}$

(b) $\frac{PQ + PR}{3}$

(c) $\frac{PQ + QR}{3}$

(d) None of the above

14. A square plate of side 20 cm has uniform thickness and density. A circular part of diameter 8 cm is cut out symmetrically and shown in figure. The position of centre of mass of the remaining portion is



(a) at O_1

(b) at O

(c) 0.54 cm from O on the left hand side

(d) None of the above

15. A uniform metal disc of radius R is taken and out of it a disc of diameter R is cut-off from the end. The centre of mass of the remaining part will be

(a) $\frac{R}{4}$ from the centre

(b) $\frac{R}{3}$ from the centre

(c) $\frac{R}{5}$ from the centre

(d) $\frac{R}{6}$ from the centre

2 Motion of centre of mass

Let us consider the motion of a system of n particles of individual masses $m_1, m_2, ..., m_n$ and total mass M. It is assumed that no mass enters or leaves the system during its motion, so that M remains constant. Then,

Velocity of centre of mass is given by

$$\mathbf{v}_{CM} = \frac{m_1\mathbf{v}_1 + m_2\mathbf{v}_2 + .. + m_n\mathbf{v}_n}{M} \quad ...(i)$$

or

$$\boxed{\mathbf{v}_{CM} = \frac{\sum_{i=1}^{n} m_i\mathbf{v}_i}{M}}$$

Differentiating Eq. (i) with respect to time t, we get

Acceleration of centre of mass

$$\mathbf{a}_{CM} = \frac{m_1\mathbf{a}_1 + m_2\mathbf{a}_2 + ... + m_n\mathbf{a}_n}{M} \quad ...(ii)$$

or

$$\boxed{\mathbf{a}_{CM} = \frac{\sum_{i=1}^{n} m_i\mathbf{a}_i}{M}}$$

Further, in accordance with Newton's second law of motion $\mathbf{F} = m\mathbf{a}$. Hence, Eq. (ii) can be written as

$$\mathbf{F}_{CM} = \mathbf{F}_1 + \mathbf{F}_2 + ... + \mathbf{F}_n$$

Force on centre of mass

$$\boxed{\mathbf{F}_{CM} = \sum_{i=1}^{n} \mathbf{F}_i} \quad ...(iii)$$

From the expression (iii), it is clear that the **centre of mass of a system of particles moves as though it was a particle of mass equal to that of the whole system with all the external forces acting directly on it.**

There are some important points related to motion of centre of mass.

(i) If a system consists of more than one particle (or bodies) and net external force on the system in a particular direction is zero with centre of mass at rest. Then the centre of mass will not move along that particular direction even though some particles (or bodies) of the system may move along that direction.

(ii) Motion of centre of mass of a system or object is not affected by any of internal forces as they always make action-reaction pairs and so their net contribution to acceleration of centre of mass is zero.

(iii) If two particles of masses m_1 and m_2 are placed on a smooth surface at separation r and they move towards each other due to the mutual attractive force, then

(a) In a absence of any external force the acceleration of CM is zero, irrespective of the individual acceleration of particles.

(b) In a absence of any external force, the velocity of CM is also constant.

(c) If initially the centre of mass is at rest, *i.e.*, $\mathbf{v}_{CM} = 0$ and the external force is absent, *i.e.*, $\mathbf{F}_{ext} = 0$, the location of CM is fixed.

(iv) Under the external forces, the CM moves just as all the mass were concentrated at that point, *i.e.*,

$$\Sigma\mathbf{F}_{ext} = M\mathbf{a}_{ext}$$

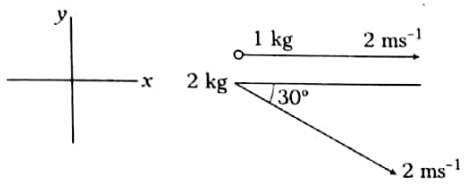
Example 8.12 *Two blocks of masses 5 kg and 2 kg are placed on a frictionless surface and connected by a spring. An external kick gives a velocity of 14 ms^{-1} to the heavier block in the direction of lighter one. Calculate the velocity gained by the centre of mass.*

Sol. Given, $m_1 = 5$ kg, $m_2 = 2$ kg, $v_1 = 14\ ms^{-1}$, $v_2 = 0$, $v_{CM} = ?$

$$\therefore \quad v_{CM} = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} = \frac{5 \times 14 + 2 \times 0}{5 + 2}$$

$$= \frac{70}{7} = 10\ ms^{-1} \text{ in the direction of lighter one.}$$

Example 8.13 *Find the velocity of centre of mass of the system shown in the figure.*



Sol. Here, $m_1 = 1$ kg, $\mathbf{v}_1 = 2\hat{\mathbf{i}}$,

$m_2 = 2$ kg, $\mathbf{v}_2 = 2\cos 30°\,\hat{\mathbf{i}} - 2\sin 30°\,\hat{\mathbf{j}}$

$$\mathbf{v}_{CM} = \frac{m_1\mathbf{v}_1 + m_2\mathbf{v}_2}{m_1 + m_2}$$

$$= \frac{1 \times 2\hat{\mathbf{i}} + 2(2\cos 30°\,\hat{\mathbf{i}} - 2\sin 30°\,\hat{\mathbf{j}})}{1 + 2}$$

$$\mathbf{v}_{CM} = \frac{2\hat{\mathbf{i}} + 2\sqrt{3}\hat{\mathbf{i}} - 2\hat{\mathbf{j}}}{3} = \left(\frac{2 + 2\sqrt{3}}{3}\right)\hat{\mathbf{i}} - \frac{2}{3}\hat{\mathbf{j}}$$

Example 8.14 *Two particles of masses 2 kg and 4 kg are approaching each other with acceleration 1 ms^{-2} and 2 ms^{-2}, respectively, on a smooth horizontal surface. Find the acceleration of centre of mass of the system.*

Sol. The acceleration of centre of mass of the system

$$\mathbf{a}_{CM} = \frac{|m_1\mathbf{a}_1 + m_2\mathbf{a}_2|}{m_1 + m_2}$$

$$\Rightarrow \quad a_{CM} = \frac{|m_1\mathbf{a}_1 + m_2\mathbf{a}_2|}{m_1 + m_2}$$

$$= \frac{2 \times 1 - 4 \times 2}{(2 + 4)} = -1\,ms^2$$

[-ve sign because direction of 4 kg is opposite to that of 2 kg]

Since $m_2a_2 > m_1a_1$ so the direction of acceleration of centre of mass will be directed towards a_1.

Example 8.15 *Two particles of mass m_1 and m_2 are projected from the top of a tower. The particle m_1 is projected vertically downward with speed u and m_2 is projected horizontally with same speed. Find acceleration of CM of system of particles by neglecting the effect of air resistance.*

Sol. As effect of air is neglected, therfore the only force acting on the particles is the gravitational force in downward direction.

Let the point of projection is taken as origin and downward direction as negative Y-axis, then

acceleration of 1st point mass $\mathbf{a}_1 = -g\hat{\mathbf{j}}$

acceleration of 2nd point mass $\mathbf{a}_2 = -g\hat{\mathbf{j}}$

$$\therefore \quad \mathbf{a}_{CM} = \frac{m_1\mathbf{a}_1 + m_2\mathbf{a}_2}{m_1 + m_2} = \frac{m_1(-g\hat{\mathbf{j}}) + m_2(-g\hat{\mathbf{j}})}{m_1 + m_2}$$

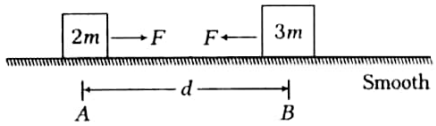
$$\Rightarrow \quad \mathbf{a}_{CM} = -g\hat{\mathbf{j}}$$

i.e., acceleration of CM is equal to acceleration due to gravity and is in downward direction.

Note If large number of particles are projected under the effect of gravity only indifferent direction, then acceleration of CM is equal to the acceleration due to gravity irrespective of directions of projection of particles.

Example 8.16 *Two particles of masses 2m and 3m are placed at separation d on a smooth surface. They move towards each other due to mutual attractive force. Find (a) acceleration of CM (b) velocity of CM. When separation between particles becomes d/3. (c) At what distance from the initial position of mass 2m, the particles collide.*

Sol.



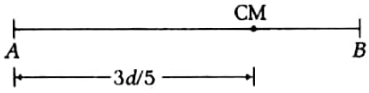
(a) In a absence of any external force the acceleration of CM is zero

$$\mathbf{F}_{ext} = 0,\ \mathbf{a}_{CM} = 0.$$

(b) Initially the particles are at rest $v_1 = v_2 = 0$, therefore, $v_{CM} = 0$. Since $\mathbf{F}_{ext} = 0$, the velocity of CM is constant and hence v_{CM} is always zero whenever the separation becomes $d/2$, $d/3$. etc.

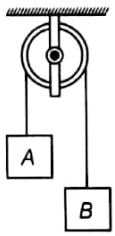
(c) First, locate the CM:

$$x_{CM} = \frac{2m \times 0 + 3m \times d}{2m + 3m} = \frac{3d}{5}$$



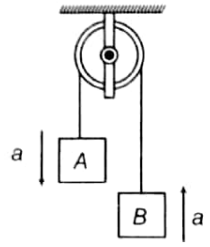
Since the CM is at rest, its position is fixed, hence particles will meet at CM *i.e.*, at distance $3d/5$ from A.

Example 8.17 *In the arrangement shown in figure, $m_A = 2$ kg and $m_B = 1$ kg. String is light and inextensible. Find the acceleration of centre of mass of both the blocks. Neglect friction everywhere.*



Sol. Net pulling force on the system is $(m_A - m_B)\,g$

or $\quad (2 - 1)\,g = g$



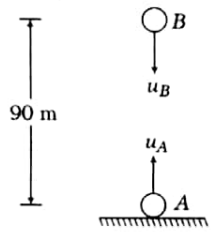
Total mass being pulled is $m_A + m_B$ or 3 kg

$$\therefore \quad a = \frac{\text{Net pulling force}}{\text{Total mass}} = \frac{g}{3}$$

Now, $$\mathbf{a}_{CM} = \frac{m_A\mathbf{a}_A + m_B\,\mathbf{a}_B}{m_A + m_B}$$

$$= \frac{2(a) - 1(a)}{1+2} = \frac{a}{3} = \frac{g}{9} \text{ (downwards)}$$

Example 8.18 *Two particles A and B of mass 1 kg and 2 kg respectively are projected in the directions shown in figure with speeds $u_A = 200\ ms^{-1}$ and $u_B = 50\ ms^{-1}$. Initially they were 90 m apart. Find the maximum height attained by the centre of mass of the particles. Assume acceleration due to gravity to be constant. (Take, $g = 10\ ms^{-2}$).*



Sol. Using, $$m_A r_A = m_B r_B \quad \text{or} \quad 1(r_A) = 2(r_B)$$

or $$r_A = 2r_B \quad \text{...(i)}$$

and $$r_A + r_B = 90 \text{ m} \quad \text{...(ii)}$$

Solving these two equations, we get

$$r_A = 60 \text{ m} \quad \text{and} \quad r_B = 30 \text{ m}$$

i.e., CM is at height 60 m from the ground at time $t = 0$.

Further, $$\mathbf{a}_{CM} = \frac{m_A\mathbf{a}_A + m_B\mathbf{a}_B}{m_A + m_B} = g = 10 \text{ ms}^{-1} \text{ (downwards)}$$

as $$\mathbf{a}_A = \mathbf{a}_B = g \quad \text{(downwards)}$$

$$\mathbf{u}_{CM} = \frac{m_A\mathbf{u}_A + m_B\mathbf{u}_B}{m_A + m_B}$$

$$= \frac{1(200) - 2(50)}{1+2} = \frac{100}{3} \text{ ms}^{-1} \quad \text{(upwards)}$$

Let h be the height attained by CM beyond 60 m. Using

$$v_{CM}^2 = u_{CM}^2 + 2a_{CM}h$$

or $$0 = \left(\frac{100}{3}\right)^2 - (2)(10)h$$

or $$h = \frac{(100)^2}{180} = 55.55 \text{ m}$$

Therefore, maximum height attained by the centre of mass is

$$H = 60 + 55.55 = 115.55 \text{ m}$$

Linear momentum of a system of particles

For a system of n-particles, total linear momentum is vector sum of linear momenta of individual particles, where, linear momentum of an individual particle is product of its mass and velocity ($\mathbf{p} = m\mathbf{v}$).

So, linear momentum of system is

$$\boxed{\mathbf{p} = \mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 + \ldots + \mathbf{p}_n}$$

or $$\mathbf{p} = m_1\mathbf{v}_1 + m_2\mathbf{v}_2 + m_3\mathbf{v}_3 + \cdots + m_n\mathbf{v}_n \quad \text{...(i)}$$

$$(\because p = mv)$$

From the concept of centre of mass, we know that

$$m_1\mathbf{v}_1 + m_2\mathbf{v}_2 + m_3\mathbf{v}_3 + \cdots + m_n\mathbf{v}_n = M \cdot \mathbf{v}_{CM} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\boxed{\text{Total linear momentum, } \mathbf{p} = M\mathbf{v}_{CM}} \quad \text{...(iii)}$$

Thus, **the total momentum of a system of particles is equal to the product of the total mass and velocity of its centre of mass.**

Note Relation between momentum and kinetic energy

$$\boxed{p = \sqrt{2mK}}$$

Conservation of linear momentum for system of particles

According to law of conservation of linear momentum, total linear momentum of a system of particles remains constant or conserved in the absence of any external force.

Total linear momentum of system of particles

$$p = Mv_{CM}$$

Differentiating both sides w.r.t. t

$$\frac{dp}{dt} = \frac{d}{dt}(Mv_{CM})$$

$$= M\frac{dv_{CM}}{dt}$$

Here, $\frac{dv_{CM}}{dt} = a_{CM}$, acceleration of centre of mass

$$\therefore \quad \frac{dp}{dt} = Ma_{CM}$$

Here, from Newton's 2nd law of motion, Ma_{CM} will be equal to the external force.

$$\therefore \quad \frac{dp}{dt} = F_{ext}$$

$\therefore$ If $$F_{ext} = 0$$

then $\frac{dp}{dt} = 0$, or p = constant

$\therefore$ $M\mathbf{v}_{CM}$ = constant $(\because \mathbf{F}_{ext} = 0)$

$\mathbf{v}_{CM}$ = constant

So, we can conclude that **if net external force on the system is zero, the linear momentum of the system is constant hence centre of mass will move with constant velocity.**

Here, rate of change of momentum is zero, *i.e.* momentum of system remains constant.

So, $\mathbf{p}_{initial} = \mathbf{p}_{final}$

Above expression represents the law of conservation of linear momentum for system of particles.

Example 8.19 *A bullet of mass 10 g is fired from a gun of mass 1 kg with recoil velocity of gun 5 m/s. Find the muzzle velocity.*

Sol. Conservation of linear momentum gives

$$m_1v_1 + m_2v_2 = 0 \Rightarrow m_1v_1 = -m_2v_2$$

$$\Rightarrow \quad v_1 = \frac{-m_2v_2}{m_1} = 500 \text{ m/s}$$

Given, $m_1 = 10 \text{ g} = \left(\frac{10}{1000}\right) \text{kg}$

$m_2 = 1 \text{ kg}$ and $v_2 = -5 \text{ m/s}$

$\therefore$ Velocity of muzzle,

$$v_1 = \frac{+1 \times 5}{10/1000} = 500 \text{ m/s}$$

Example 8.20 *The object at rest suddenly explodes into three parts with the mass ratio 2 : 1 : 1. The parts of equal masses move at right angles to each other with equal speeds. What is the speed of the third part after the explosion?*

Sol. Let the speed of the third part be v_3. Applying the law of conservation of momentum, we have

$$\sqrt{p_1^2 + p_2^2} = p \Rightarrow p_1^2 + p_2^2 = p^2$$

$$\Rightarrow \quad (m_1v_1)^2 + (m_2v_2)^2 = (m_3v_3)^2$$

$$\Rightarrow \quad (m \times v)^2 + (m \times v)^2 = (2m \times v_3)^2$$

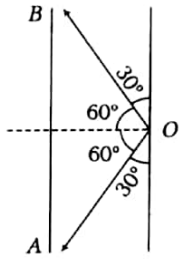
$$\Rightarrow \quad m^2v^2 + m^2v^2 = 4m^2v_3^2$$

$$\Rightarrow \quad 2m^2v^2 = 4m^2v_3^2, \; v_3 = \frac{v}{\sqrt{2}}$$

Example 8.21 *A 0.5 kg ball moving with a speed of 12 ms^{-1} strikes a hard wall at an angle of 30° with the wall. It is reflected with the same speed and at the same angle. If the ball is in contact with the wall for 0.25 s, what is the average force acting on the wall?*

Sol. The vector **OA** represents the momentum of the object before the collision and the vector **OB** that after the collision. The vector **AB** represents the change in momentum $\Delta\mathbf{p}$ of the object. As the magnitude of **OA** and **OB** are equal, the components of **OA** and **OB** along the wall are equal and in the same direction, while those perpendicular to the wall are equal and opposite.

Thus, the change in momentum is due only to the change in direction of the perpendicular components.



Hence, $\Delta p = OB \sin 30° - (-OA \sin 30°)$

$= mv \sin 30° - (-mv \sin 30°) = 2\,mv \sin 30°$

Its time rate will appear in the form of average force acting on the wall.

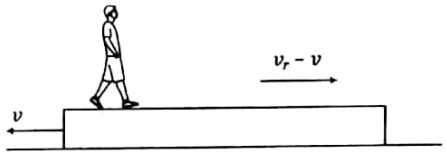
$\therefore$ $F \times t = 2\,mv \sin 30°$ or $F = \frac{2mv \sin 30°}{t}$

Given, $m = 0.5$ kg, $v = 12$ ms^{-1}, $t = 0.25$ s, $\theta = 30°$

Hence, $F = \frac{2 \times 0.5 \times 12 \times \sin 30°}{0.25} = 24 \text{ N}$

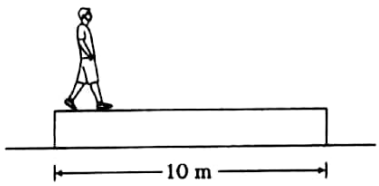
Example 8.22 *A man of mass m_1 is standing on a platform of mass m_2 kept on a smooth horizontal surface. The man starts moving on the platform with a velocity v_r relative to the platform. Find the recoil velocity of platform.*

Sol. Absolute velocity of man $= v_r - v$, where v = recoil velocity of platform. Taking the platform and the man as a system, net external force on the system in horizontal direction is zero. The linear momentum of the system remains constant. Initially both the man and the platform were at rest.



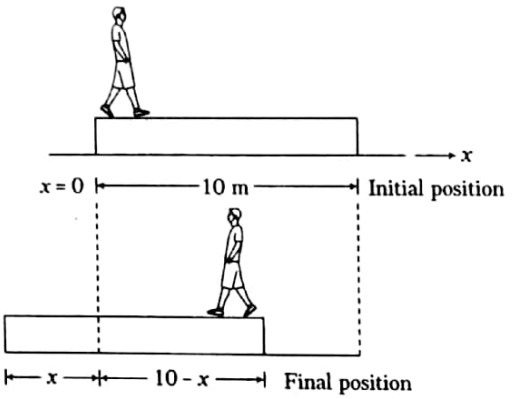
Hence, $0 = m_1(v_r - v) - m_2v \Rightarrow v = \frac{m_1v_r}{m_1 + m_2}$

Example 8.23 *A wooden plank of mass 20 kg is resting on a smooth horizontal floor. A man of mass 60 kg starts moving from one end of the plank to the other end. The length of the plank is 10 m. Find the displacement of the plank over the floor when the man reaches the other end of the plank.*



Sol. Here the system is man + plank. Net force on this system in horizontal direction is zero and initially the centre of mass of the system is at rest. Therefore, the centre of mass does not move in horizontal direction.

Let x be the displacement of the plank. Assuming the origin *i.e.*, $x = 0$ at the position shown in figure.



As we said earlier also, the centre of mass will not move in horizontal direction (X-axis). Therefore, for centre of mass to remain stationary,

$$x_i = x_f$$

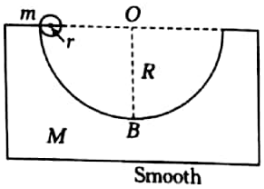
$$\frac{60(0) + 20\left(\frac{10}{2}\right)}{60 + 20} = \frac{60(10 - x) + 20\left(\frac{10}{2} - x\right)}{60 + 20}$$

or $$\frac{5}{4} = \frac{6(10 - x) + 2\left(\frac{10}{2} - x\right)}{8} = \frac{60 - 6x + 10 - 2x}{8}$$

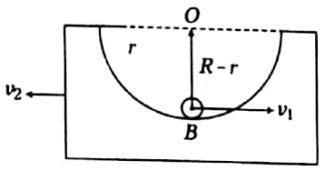
or $5 = 30 - 3x + 5 - x$ or $4x = 30$ or $x = \frac{30}{4}$ m or $x = 7.5$ m

Note The centre of mass of the plank lies at its centre.

Example 8.24 *A block of mass M with a semi-circular track of radius R rests on a smooth floor. A sphere of mass m and radius r is related from rest from A. Find the velocity of sphere and track, when the sphere reaches B.*



Sol. According to the question,



When the sphere reaches at B

Let v_1 = velocity of m

v_2 = velocity of M

By the momentum conservation in the horizontal direction

$$mv_1 = Mv_2 \quad \text{...(i)}$$

Applying the energy conservation between A and B

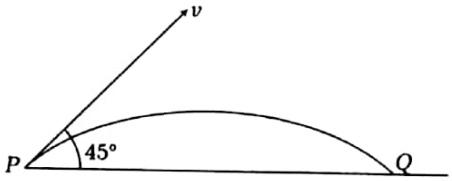
$$mg(R - r) = \frac{1}{2} mv_1^2 + Mv_2^2 \quad \text{...(ii)}$$

Solving, $$v_1 = \sqrt{\frac{2Mg(R - r)}{M + m}}$$

$$v_2 = \frac{m}{M}\sqrt{\frac{2Mg(R - r)}{M + m}}$$

Example 8.25 *A particle of mass m is projected with velocity v making an angle of 45° with the horizontal. When the particle lands on the level ground, find the magnitude of the change in its momentum.*

Sol. The situation is shown in figure.



Change in momentum,

$$\Delta\mathbf{p} = \mathbf{p}_f - \mathbf{p}_i = m(\mathbf{v}_f - \mathbf{v}_i)$$
$$= m[(v\cos 45°\,\hat{\mathbf{i}} - v\sin 45°\,\hat{\mathbf{j}}) - (v\cos 45°\,\hat{\mathbf{i}} + v\sin 45°\,\hat{\mathbf{j}})]$$

$$\Delta\mathbf{p} = m\left[\left(\frac{v}{\sqrt{2}}\hat{\mathbf{i}} - \frac{v}{\sqrt{2}}\hat{\mathbf{j}}\right) - \left(\frac{v}{\sqrt{2}}\hat{\mathbf{i}} + \frac{v}{\sqrt{2}}\hat{\mathbf{j}}\right)\right]$$
$$= -\sqrt{2}\,mv\,\hat{\mathbf{j}}$$

$\therefore \quad |\Delta\mathbf{p}| = \sqrt{2}\,mv$

which is acting along (-) ve y-direction.

Example 8.26 *A disc of mass 100 g is kept floating horizontally in air by firing bullets, each of mass 5 g with the same velocity at the same rate of 10 bullets per second. The bullets rebound with the same speed in opposite direction. Find the velocity of each bullet at the time of impact.*

Sol. From the law of conservation of momentum,

Rate of change in momentum of bullets = weight of disc

$$2m'vn = mg$$

$$\therefore \quad v = \frac{mg}{2m'n} = \frac{100 \times 980}{2 \times 5 \times 10}$$
$$= 980 \text{ m/s}$$

Check point 8.2

1. A body at rest breaks into two pieces of equal masses. The parts will move
(a) in same direction
(b) along different lines
(c) in opposite directions with equal speeds
(d) in opposite directions with unequal speeds

2. A body falling vertically downwards under gravity breaks in two parts of unequal masses. The centre of mass of the two parts taken together shifts horizontally towards
(a) heavier piece
(b) lighter piece
(c) does not shift horizontally
(d) depends on the vertical velocity at the time of breaking

3. Two balls are thrown simultaneously in air. The acceleration of the centre of mass of the two balls while in air
(a) depends on the direction of the motion of the balls
(b) depends on the masses of the two balls
(c) depends on the speeds of two balls
(d) is equal to g

4. Consider a system of two identical particles. One of the particles is at rest and the other has an acceleration **a**. The centre of mass has an acceleration
(a) zero (b) $\frac{1}{2}\mathbf{a}$
(c) **a** (d) 2**a**

5. Two particles A and B initially at rest, move towards each other under a mutual force of attraction. At the instant when the speed of A is v and the speed of B is $2v$, the speed of centre of mass of the system is
(a) zero (b) v
(c) $1.5\,v$ (d) $3v$

6. Two balls of equal mass are projected from a tower simultaneously with equal speeds. One at angle θ above the horizontal and the other at the same angle θ below the horizontal. The path of the centre of mass of the two balls is
(a) a vertical straight line
(b) a horizontal straight line
(c) a straight line at angle α ($< \theta$) with horizontal
(d) a parabola

7. One projectile moving with velocity v in space, gets burst into 2 parts of masses in the ratio 1 : 3. The smaller part becomes stationary. What is the velocity of the other part?
(a) $4\,v$ (b) v
(c) $\frac{4\,v}{3}$ (d) $\frac{3v}{4}$

8. A bomb of mass 9 kg explodes into two pieces of masses 3 kg and 6 kg. The velocity of 3 kg mass is 16 ms^{-1}. The velocity of 6 kg mass is
(a) 4 ms^{-1} (b) 8 ms^{-1}
(c) 16 ms^{-1} (d) 32 ms^{-1}

9. Two blocks of masses 10 kg and 4 kg are connected by a spring of negligible mass and placed on a frictionless horizontal surface. An impulse gives a velocity of 14 ms^{-1} to the heavier block in the direction of the lighter block. The velocity of the centre of mass is
(a) 30 ms^{-1} (b) 20 ms^{-1} (c) 10 ms^{-1} (d) 5 ms^{-1}

10. A ball kept in a closed box moves in the box making collisions with the walls. The box is kept on a smooth surface. The velocity of the centre of mass
(a) of the box remains constant
(b) of the box plus the ball system remains constant
(c) of the ball remains constant
(d) of the ball relative to the box remains constant

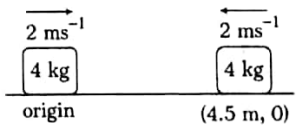
11. A metre stick is placed vertically at the origin on a frictionless surface. A gentle push in $+x$-direction is given to the top most point of the rod, when it has fallen completely x-coordinate of centre of rod is at
(a) origin (b) -0.5 m (c) -1 m (d) $+0.5$ m

12. Two bodies having masses m_1 and m_2 and velocities v_1 and v_2 collide and form a composite system. If $m_1v_1 + m_2v_2 = 0$ ($m_1 \neq m_2$), the velocity of composite system will be
(a) $v_1 - v_2$ (b) $v_1 + v_2$ (c) $\frac{v_1 + v_2}{2}$ (d) zero

13. Two particles of equal mass have coordinates ($2m$, $4m$, $6m$) and ($6m$, $2m$, $8m$). Of these one particle has a velocity $\mathbf{v}_1 = (2\,\hat{\mathbf{i}})\,ms^{-1}$ and another particle has velocity $\mathbf{v}_2 = (2\hat{\mathbf{j}})\,ms^{-1}$ at time $t = 0$. The coordinate of their centre of mass at time $t = 1$ s will be
(a) ($4m$, $4m$, $7m$) (b) ($5m$, $4m$, $7m$)
(c) ($2m$, $4m$, $6m$) (d) ($4m$, $5m$, $4m$)

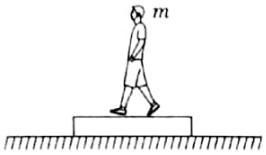
14. An isolated particle of mass m is moving in horizontal xy-plane, along the X-axis, at a certain height above the ground, it suddenly explodes into two fragments of masses $\frac{m}{4}$ and $\frac{3m}{4}$. An instant later, the smaller fragment is at $y = +15$ cm. The larger fragment at this instant is at
(a) $y = -5$ cm (b) $y = +20$ cm
(c) $y = +5$ cm (d) $y = -20$ cm

15. Blocks A and B are resting on a smooth horizontal surface given equal speeds of 2 ms^{-1} in opposite sense as shown in the figure.



At $t = 0$, the position of blocks are shown, then the coordinates of centre of mass $t = 3$ s will be
(a) (1, 0) (b) (3, 0) (c) (5, 0) (d) (2.25, 0)

16. A man of mass m is standing on a plank of equal mass m resting on a smooth horizontal surface. The man starts moving on the plank with speed u relative to the plank. The speed of the man relative to the ground is



(a) $2u$ (b) $\frac{u}{2}$

(c) zero (d) $\frac{u}{4}$

17. An object of mass $3m$ splits into three equal fragments. Two fragments have velocities $v\hat{j}$ and $v\hat{i}$. The velocity of the third fragment is

(a) $v(\hat{j} - \hat{i})$ (b) $v(\hat{i} - \hat{j})$

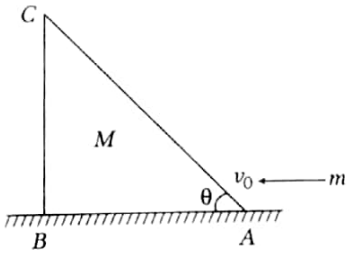
(c) $-v(\hat{i} + \hat{j})$ (d) $\frac{v(\hat{i} + \hat{j})}{\sqrt{2}}$

18. A stationary bomb explodes into two parts of masses 3 kg and 1 kg. The total KE of the two parts after explosion is 2400 J. The KE of the smaller part is

(a) 600 J (b) 1800 J

(c) 1200 J (d) 2160 J

19. A shell of mass m is moving horizontally with velocity v_0 and collides with the wedge of mass M just above point A, as shown in the figure. As a consequence, wedge starts to move towards left and the shell returns with a velocity in xy-plane. The principle of conservation of momentum can be applied for



(a) system $(m + M)$ along any direction

(b) system $(m + M)$ along vertical

(c) system $(m + M)$ horizontally

(d) None of the above

20. A shell is fired from cannon with a velocity v at an angle θ with the horizontal direction. At the highest point in its path it explodes into two pieces of equal mass. One of the pieces retraces its path to the cannon and the speed (in ms^{-1}) of the other piece immediately after the explosion is

(a) $3v \cos\theta$ (b) $2v \cos\theta$

(c) $\frac{3}{2} v \cos\theta$ (d) $\sqrt{\frac{3}{2}}\, v \cos\theta$

3 Collision

A collision is an isolated event in which two or more colliding bodies exert strong forces on each other for a retatively short time. For a collision to take place, the actual physical contact is not necessary.

Total linear momentum is conserved in all collisions, *i.e.*, the initial momentum of the system is equal to final momentum of the system.

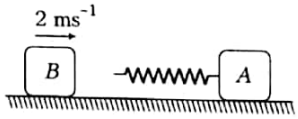
∴ Total momentum before collision = Total momentum after collision

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2' \quad \text{...(i)}$$

In the absence of any dissipative forces, the mechanical energy of the system will also remain conserved, *i.e.*,

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2 \quad \text{...(ii)}$$

Example 8.27 *Two blocks A and B of equal mass m = 1 kg are lying on a smooth horizontal surface as shown in figure. A spring of force constant k = 200 Nm^{-1} is fixed at one end of block A. Block B collides with block A with velocity v_0 = 2 ms^{-1}. Find the maximum compression of the spring.*



Sol. At maximum compression (x_m), velocity of both the blocks is same, say it is v. Applying conservation of linear momentum, we have,

$$(m_A + m_B)v = m_Bv_0$$

or $$(1 + 1)v = (1)\,v_0$$

or $$v = \frac{v_0}{2} = \frac{2}{2} = 1 \text{ ms}^{-1}$$

Using conservation of mechanical energy, we have

$$\frac{1}{2}m_Bv_0^2 = \frac{1}{2}(m_A + m_B)v^2 + \frac{1}{2}kx_m^2$$

Substituting the values, we get

$$\frac{1}{2}\times(1)\times(2)^2 = \frac{1}{2}\times(1+1)\times(1)^2 + \frac{1}{2}\times(200)\times x_m^2$$

or $$2 = 1 + 100\,x_m^2$$

or $$x_m = 0.1 \text{ m} = 10 \text{ cm}$$

Types of collisions

Collision between two bodies may be classified in two ways

Elastic and inelastic collision

A collision is said to be **elastic** if along with linear momentum kinetic energy also remains conserved before and after collision.

A collision is said to **inelastic** if only linear momentum remains conserved and not the kinetic energy. The collision is said to be perfectly inelastic if approaching particles permanently stick to each other and move with common velocity.

Head on and oblique collision

If velocity vectors of the colliding bodies are directed along the line of impact, the impact is called as direct or **Head on Collision**. And if velocity vectors of both or any of the bodies are not along the line of impact, the impact is called **oblique collision**.

Head on elastic collision

Let the two balls of masses m_1 and m_2 collide each other elastically with velocities v_1 and v_2 in the directions shown in figure below. Their velocities become v_1' and v_2' after the collision along the same line.

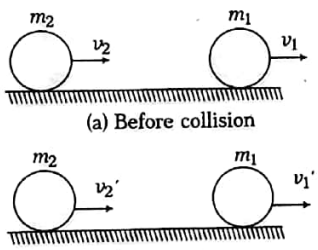


***Fig. 8.3** Head on elastic collision*

Applying conservation of linear momentum, we get

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2' \quad ...(i)$$

In an elastic collision kinetic energy before and after collision is also conserved. Hence,

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2 \quad ...(ii)$$

Solving Eqs. (i) and (ii) for v_1' and v_2', we get

$$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_1 + \left(\frac{2m_2}{m_1 + m_2}\right)v_2 \quad ...(iii)$$

and $$v_2' = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_2 + \left(\frac{2m_1}{m_1 + m_2}\right)v_1 \quad ...(iv)$$

Special cases of head on elastic collision

(i) If $m_1 = m_2$, then from Eqs. (iii) and (iv), we can see that

$$v_1' = v_2 \text{ and } v_2' = v_1$$

i.e., when two particles of equal mass collide elastically head on condition, they exchange their velocities.

(ii) If $m_1 >> m_2$ and $v_1 = 0$.

Then, $$\frac{m_2}{m_1} \approx 0$$

With these two substitutions $\left(v_1 = 0 \text{ and } \frac{m_2}{m_1} = 0\right)$

We get the following two results

$$v_1' \approx 0 \text{ and } v_2' \approx -v_2$$

i.e., If a lighter particle collides with a heavier particle at rest, then heavier particle remains at rest but lighter particle bounces back with same velocity.

(iii) If $m_2 >> m_1$ and $v_1 = 0$

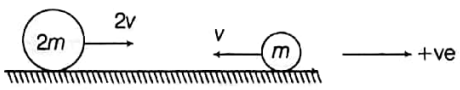
With the substitution $m_1/m_1 \approx 0$ and $v_1 = 0$, we get the results

$$v_1' \approx 2v_2 \text{ and } v_2' \approx v_2$$

i.e., If a heavier particle collides with a lighter particle at rest, then lighter particle moves with twice the velocity of heavier particle while velocity of heavier particle remains same.

Example 8.28 *Two particles of mass m and 2m moving in opposite directions collide elastically with velocities v and 2v. Find their velocities after collision.*

Sol. Here, $v_1 = -v$, $v_2 = 2v$, $m_1 = m$ and $m_2 = 2m$.



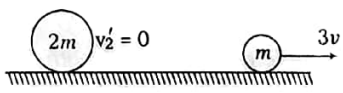
Substituting these values in Eqs. (iii) and (iv), we get

$$v_1' = \left(\frac{m - 2m}{m + 2m}\right)(-v) + \left(\frac{4m}{m + 2m}\right)(2v)$$

or $$v_1' = \frac{v}{3} + \frac{8v}{3} = 3v$$

and $$v_2' = \left(\frac{2m - m}{m + 2m}\right)(2v) + \left(\frac{2m}{m + 2m}\right)(-v)$$

or $$v_2' = \frac{2}{3}v - \frac{2}{3}v = 0$$



i.e., the second particle (of mass 2m) comes to a rest while the first (of mass m) moves with velocity 3v in the direction shown in figure given above.

Head on inelastic collision

In an inelastic collision, due to permanent deformation the kinetic energy of the particles no longer remains conserved. However, in the absence of external forces, law of conservation of linear momentum still holds good.

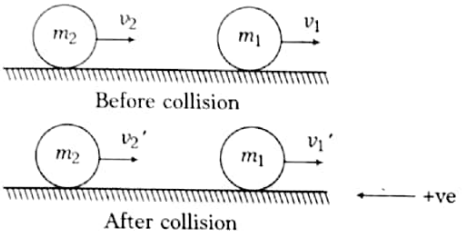


Fig. 8.4 Head on inelastic collision

Suppose the velocities of two particles of mass m_1 and m_2 before collision be v_1 and v_2 in the directions shown in figure. Let v_1' and v_2' be their velocities after collision. The law of conservation of linear momentum gives

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2' \quad ...(i)$$

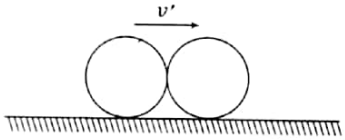


Fig. 8.5 After inelastic collision

Collision is said to be perfectly inelastic if both the particles stick together after collision and move with same velocity, say v' as shown in figure. In this case, Eq. (i) can be written as

$$m_1v_1 + m_2v_2 = (m_1 + m_2)v'$$

or
$$v' = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} \quad ...(ii)$$

Loss in kinetic energy

$$\Delta K = \left(\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2\right) - \frac{1}{2}(m_1 + m_2)v'^2$$

$$= \left(\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2\right) - \frac{1}{2}(m_1 + m_2)\left(\frac{m_1v_1 + m_2v_2}{m_1 + m_2}\right)^2$$

Solving above equation, we get

$$\boxed{\Delta K = \frac{1}{2}\left(\frac{m_1m_2}{m_1 + m_2}\right)(v_1 - v_2)^2}$$

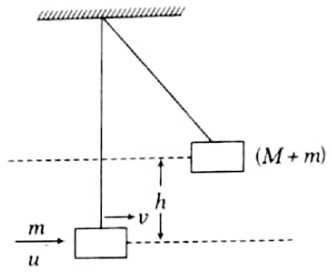
Which is positive. Therefore, **some KE is always lost in an inelastic collision.**

Example 8.29 *A simple pendulum of length 1 m has a wooden bob of mass 1 kg. It is struck by a bullet of mass 10^{-2} kg moving with a speed of 2×10^2 ms^{-1}. The bullet gets embedded into the bob. Obtain the height to which the bob rises before swinging back.*

Sol. Applying principle of conservation of linear momentum

$$mu = (M + m)v \Rightarrow 10^{-2} \times (2 \times 10^2) = (1 + 0.01)v$$

$$\Rightarrow \quad v = \frac{2}{1.01}$$



KE_1 of the block with bullet in it, is converted into PE as it rises through a height h

$$\frac{1}{2}(M + m)v^2 = (M + m)gh$$

$$\Rightarrow \quad h = \frac{v^2}{2g} = \left(\frac{2}{1.01}\right) \times \frac{1}{2 \times 9.8} = 0.2 \text{ m}$$

Example 8.30 *A body falling on the ground from a height of 10 m, rebounds to a height 2.5 m calculate*

(i) The percentage loss in KE

(ii) Ratio of the velocities of the body just before and just after the collision.

Sol. Let v_1 and v_2 be the velocity of the body just before and just after the collision

$$KE_1 = \frac{1}{2}mv_1^2 = mgh_1 \quad ...(i)$$

and
$$KE_2 = \frac{1}{2}mv_2^2 = mgh_2 \quad ...(ii)$$

$$\Rightarrow \quad \frac{v_1^2}{v_2^2} = \frac{h_1}{h_2} = \frac{10}{2.5} = 4 \Rightarrow \frac{v_1}{v_2} = 2$$

Percentage loss in KE $= \dfrac{mg(h_1 - h_2)}{mgh_1} \times 100 = \dfrac{10 - 2.5}{10} \times 100$

$= 75\%$

Example 8.31 *A pendulum bob of mass 10^{-2} kg is raised to a height 5×10^{-2} m and then released. At the bottom of its swing, it picks up a mass 10^{-3} kg. To what height will the combined mass rise?*

Sol. Velocity of pendulum bob in mean position,

$$v_1 = \sqrt{2gh} = \sqrt{2 \times 10 \times 5 \times 10^{-2}} = 1 \text{ ms}^{-1}$$

When the bob picks up a mass 10^{-3} kg at the bottom, then by conservation of linear momentum, the velocity of coalesced mass is given by

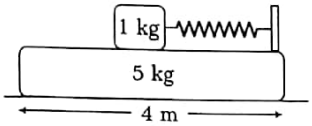
$$m_1v_1 + m_2v_2 = (m_1 + m_2)v$$

$$10^{-2} + 10^{-3} \times 0 = (10^{-2} + 10^{-3})v$$

or
$$v = \frac{10^{-2}}{1.1 \times 10^{-2}} = \frac{10}{11} \text{ ms}^{-1}$$

Now
$$h = \frac{v^2}{2g} = \frac{(10/11)^2}{2 \times 10}] = 4.1 \times 10^{-2} \text{ m}$$

Example 8.32 *A plank of mass 5 kg is placed on a frictionless horizontal plane. Further a block of mass 1 kg is placed over the plank. A massless spring of natural length 2 m is fixed to the plank by its one end. The other end of spring is compressed by the block by half of spring's natural length. The system is now released from the rest. What is the velocity of the plank when block leaves the plank ? (The stiffness constant of spring is 100 Nm^{-1})*



Sol. Let the velocity of the block and the plank, when the block leaves the spring be u and v respectively.

By conservation of energy $\frac{1}{2}kx^2 = \frac{1}{2}mu^2 + \frac{1}{2}Mv^2$

(M = mass of the plank, m = mass of the block)

$$\Rightarrow \quad 100 = u^2 + 5v^2 \quad \ldots \text{(i)}$$

By conservation of momentum

$$mu + Mv = 0 \Rightarrow u = -5v \quad \ldots \text{(ii)}$$

Solving Eqs. (i) and (ii)

$$30v^2 = 100$$

$$\Rightarrow \quad v = \sqrt{\frac{10}{3}}\ \text{ms}^{-1}$$

From this moment until block falls, both plank and block keep their velocity constant.

Thus, when block falls, velocity of plank $= \sqrt{10/3}\ \text{ms}^{-1}$.

Example 8.33 *An object of mass 40 kg and having velocity 4 ms^{-1} collides with another object of mass 60 kg having velocity 2 ms^{-1}. What is the loss of energy during this process if it is a perfectly inelastic collision?*

Sol. $m_1 = 40$ kg, $m_2 = 60$ kg, $v_1 = 4\ \text{ms}^{-1}$, $v_2 = 2\ \text{ms}^{-1}$

$$\therefore \Delta K = \frac{1}{2}\left(\frac{m_1m_2}{m_1+m_2}\right)(v_1 - v_2)^2 = \frac{1}{2}\left(\frac{40\times 60}{40+60}\right)(4-2)^2 = 48\ \text{J}$$

Example 8.34 *A particle of mass m moving with speed u collides perfectly inelastically with another particle of mass 2m at rest. Find loss of kinetic energy of system in the collision.*

Sol. Let velocity of system of particles after collision is v.

By law of momentum conservation.

$$mu = 3mv \Rightarrow v = \frac{u}{3}$$

Now, loss of kinetic energy $= K_i - K_f$

$$\Rightarrow \quad \Delta K = \frac{1}{2}mu^2 - \frac{1}{2}3m\left[\frac{u}{3}\right]^2$$

$$\Rightarrow \quad \Delta K = \frac{1}{2}mu^2 - \frac{1}{6}mu^2$$

$$\Rightarrow \quad \Delta K = \frac{1}{3}mu^2$$

Example 8.35 *The mass of the railway carriage is 8000 kg which is moving with the speed of 54 km h^{-1} collides with the another stationary carriage of same mass. Determine the loss in kinetic energy in this process.*

Sol. Given, $m_1 = 8000$ kg

$$v_1 = 54\ \text{km h}^{-1} = 54 \times \frac{5}{18}\ \text{ms}^{-1} = 15\ \text{ms}^{-1}$$

$$m_2 = 8000\ \text{kg},\ v_2 = 0$$

By conservation of momentum,

$$m_1v_1 + m_2v_2 = (m_1 + m_2)v$$

$$v = \frac{m_1v_1}{m_1 + m_2} \quad [\because v_2 = 0]$$

$$= \frac{8000\times 15}{8000 + 8000} = 7.5\ \text{ms}^{-1}$$

Loss of kinetic energy = KE before collision - KE after collision

$$= \frac{1}{2}m_1v_1^2 - \frac{1}{2}(m_1 + m_2)v^2$$

$$= \frac{1}{2}\times 8000 \times (15)^2 - \frac{1}{2}(8000 + 8000)(7.5)^2$$

$$= 400 \times 225 - 8000 \times 56.25$$

$$= 450\ \text{kJ}$$

Newton's law of restitution

When two objects are in direct (head on) impact, the speed with which they separate after impact is usually less than or equal to their speed of approach before impact.

According to Newton's law of restitution the ratio of relative velocity of separation after collision to be relative velocity of approach before collision remains constant.

$$e = \frac{\text{Relative velocity of separation after collision}}{\text{Relative velocity of approach (before collision)}}$$

The ratio e is called the coefficient of restitution and is constant for two particular objects.

$$\boxed{e = \frac{v_2 - v_1}{u_1 - u_2}}$$

where u_1 and u_2 are velocities of two bodies before collision and v_1 and v_2 are velocities after collision.

For eleastic collision, $e = 1$ and for inelastic collision $0 < e < 1$ while for perfectly inelastic collision $e = 0$.

Solving the following equation

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \text{ and } v_2 - v_1 = e(u_1 - u_2)$$

we get,

$$v_1 = \frac{m_1 - em_2}{m_1 + m_2}u_1 + \frac{(1+e)m_2}{m_1 + m_2}u_2$$

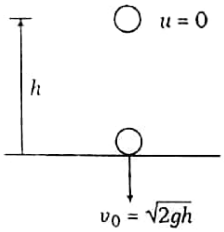
$$v_2 = \frac{m_2 - em_1}{m_1 + m_2}u_2 + \frac{(1+e)m_1}{m_1 + m_2}u_1$$

Putting $e = 1$, we will get formulas of v_1 and v_2 for an elastic collision.

Putting $e = 0$, we will get formulas of v_1 and v_2 for perfectly inelastic collision.

Note

1. If mass of one body is much greater than that of the other, then after collision, velocity of heavier body does not change appreciably. (Whether the collision is elastic or inelastic).



2. In the situation shown in figure if e is the coefficient of restitution between the ball and the ground, then after nth collision with the floor the speed of ball will remain $e^n v_0$ and it will go upto a height $e^{2n}h$ or,

$$v_n = e^n v_0 = e^n\sqrt{2gh}$$

and $$h_n = e^{2n}h$$

Total distacne travelled by the ball before it stops bouncing is

$H = h\left[\frac{1+e^2}{1-e^2}\right]$ and total time taken by the ball to stop bouncing will be

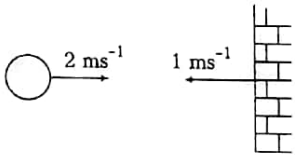
$$T = \left(\frac{1+e}{1-e}\right)\sqrt{\frac{2h}{g}}$$

3. The loss in kinetic during an inelastic collision is

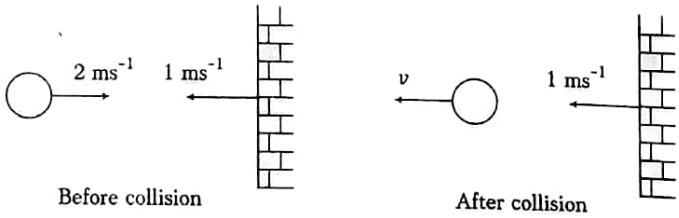
$$\Delta E = \frac{1}{2}\frac{M_1M_2}{M_1+M_2}(1-e^2)(u_1-u_2)^2$$

where, e is the coefficient of restitution. However, if the target is massive (*i.e.*, $m_2 >> m_1$) and $u_2 = 0$ then the lighter body losses all its kinetic energy.

Example 8.36 *A ball is moving with velocity 2 ms^{-1} towards a heavy wall moving towards the ball with speed 1 ms^{-1} as shown in figure. Assuming collision to be elastic, find the velocity of ball immediately after the collision.*



Sol. The speed of wall will not change after the collision. So, let v be the velocity of the ball after collision in the direction shown in figure. Since, collision is elastic ($e = 1$),

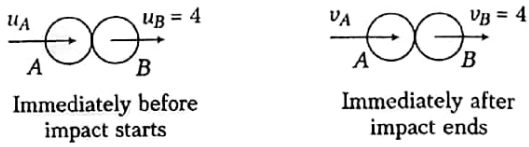


separation speed = approach speed

or $$v - 1 = 2 + 1 \quad \text{or} \quad v = 4\ \text{ms}^{-1}$$

Example 8.37 *A ball of mass 2 kg moving with speed 5 ms^{-1} collides directly with another of mass 3 kg moving in the same direction with speed 4 ms^{-1}. The coefficient of restitution is 2/3. Find the velocities after collision.*

Sol. Denoting the first ball by A and the second ball by B velocities immediately before and after the impact are shown in the figure.



Applying principle of conservation of momentum, we have

$$m_B v_B + m_A v_A = m_A u_A + m_B u_B$$

$$\Rightarrow \quad 3v_B + 2v_A = 2\times 5 + 3\times 4$$

$$3v_B + 2v_A = 22 \quad \text{...(i)}$$

Applying equation of coefficient of restitution, we have

$$v_B - v_A = e(u_A - u_B) \Rightarrow v_B - v_A = \frac{2}{3}(5-4)$$

$$3v_B - 3v_A = 2 \quad \text{...(ii)}$$

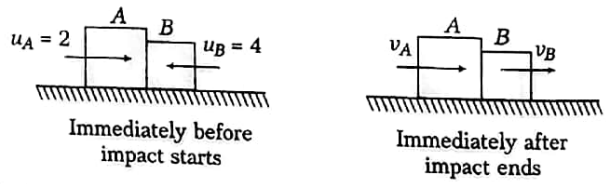
From Eqs. (i) and (ii), we have $v_A = 4\ \text{ms}^{-1}$ and $v_B = 4.67\ \text{ms}^{-1}$

Example 8.38 *A block of mass 5 kg moves from left to right with a velocity of 2 ms^{-1} and collides with another block of mass 3 kg moving along the same line in the opposite direction with velocity 4 ms^{-1}.*

(i) If the collision is perfectly elastic, determine velocities of both the blocks after their collision.

(ii) If coefficient of restitution is 0.6, determine velocities of both the blocks after their collision.

Sol. Denoting the first block by A and the second block by B velocities immediately before and after the impact are shown in the figure.



Applying principle of conservation of momentum, we have

$$m_B v_B + m_A v_A = m_A u_A + m_B u_B$$

$$\Rightarrow \quad 3v_B + 5v_A = 5\times 2 + 3\times(-4)$$

$$3v_B + 5v_A = -2 \quad \text{...(i)}$$

Applying equation of coefficient of restitution, we have

$$\Rightarrow \quad v_A - v_B = e(u_A - u_B)$$

$$v_B - v_A = e\{2-(-4)\}$$

$$v_B - v_A = 6e \quad \text{...(ii)}$$

(i) For perfectly elastic impact $e = 1$. Using this value in Eq. (ii), we have

$$v_B - v_A = 6 \quad \text{...(iii)}$$

Now, from Eqs. (i) and (iii), we obtain

$$v_A = -2.5\ \text{ms}^{-1} \quad \text{and} \quad v_B = 3.5\ \text{ms}^{-1}$$

(ii) For value $e = 0.6$, Eq. (ii) is modified as

$$v_B - v_A = 3.6 \quad \text{...(iv)}$$

Now from Eqs. (i) and (iv), we obtain

$$v_A = -1.6\ \text{ms}^{-1} \quad \text{and} \quad v_B = 2\ \text{ms}^{-1}$$

Block A reverse back with speed 1.6 ms^{-1} and B also move in opposite direction to its original direction with speed 2 ms^{-1}.

Example 8.39 *A particle of mass 2 kg moving with a velocity $5\,\hat{\mathbf{i}}\ ms^{-1}$ collides head-on with another particle of mass 3 kg moving with a velocity $-2\,\hat{\mathbf{i}}\ ms^{-1}$. After the collision the first particle has speed of $1.6\ ms^{-1}$ in negative x-direction. Find*
(i) velocity of the centre of mass after the collision
(ii) velocity of the second particle after the collision
(iii) coefficient of restitution

Sol. (i) $\mathbf{v}_c = \dfrac{m_1\mathbf{u}_1 + m_2\mathbf{u}_2}{m_1 + m_2} = 0.8\,\hat{\mathbf{i}}\ \text{ms}^{-1}$

[Velocity of CM before and after collision will be same]

(ii) $\mathbf{v}_1 = -1.6\,\hat{\mathbf{i}}\ \text{ms}^{-1}$

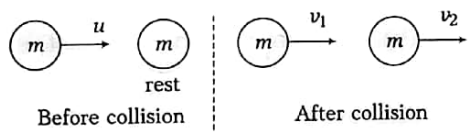
From CM, $m_1\mathbf{u}_1 + m_2\mathbf{u}_2 = m_1\mathbf{v}_1 + m_2\mathbf{v}_2 \Rightarrow v_2 = 2.4\,\hat{\mathbf{i}}\ \text{ms}^{-1}$

(iii) $e = \dfrac{v_2 - v_1}{u_1 - u_2} = \dfrac{4}{7}$

Example 8.40 *An object of mass m moving with speed u collides one dimensionally with another identical object at rest. Find their velocities after collision, if coefficient of restitution of collision is e.*

Sol. Let v_1 and v_2 be the final velocities of 1st and 2nd object respectively.

$$v_1 = \left(\frac{1-e}{2}\right)u \quad \text{and} \quad v_2 = \left(\frac{1+e}{2}\right)u$$



By law of conservation of momentum $mu + m \times 0 = mv_1 + mv_2$

$$\Rightarrow \quad v_1 + v_2 = u \quad \text{...(i)}$$

Now, $e = \dfrac{v_2 - v_1}{u_1 - u_2} \Rightarrow \dfrac{v_2 - v_1}{u} = e$

$$\Rightarrow \quad v_2 - v_1 = eu \quad \text{...(ii)}$$

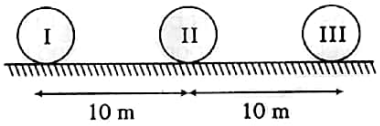
Adding Eqs. (i) and (ii)

$$2v_2 = u + eu \Rightarrow v_2 = \left[\frac{1+e}{2}\right]u$$

Subtracting Eq. (ii) from Eq. (i)

$$2v_1 = (1-e)u \Rightarrow v_1 = \left[\frac{1-e}{2}\right]u$$

Example 8.41 *Three identical balls, ball I, ball II and ball III are placed on a smooth floor on a straight line at the separation of 10 m between balls as shown in figure. Initially balls are stationary.*
Ball I is given velocity of 10 ms^{-1} towards ball II, collision between ball I and II is inelastic with coefficient of restitution 0.5 but collision between ball II and III is perfectly elastic. What is the time interval between two consecutive collisions between ball I and II ?



Sol. Let velocity of Ist ball and IInd ball after collision be v_1 and v_2

$$v_2 - v_1 = 0.5 \times 10 \quad \text{...(i)}$$

$$mv_2 + mv_1 = m \times 10 \quad \text{...(ii)}$$

$$\Rightarrow \quad v_2 + v_1 = 10$$

Solving Eqs. (i) and (ii)

$$v_1 = 2.5\ \text{ms}^{-1}, \quad v_2 = 7.5\ \text{ms}^{-1}$$

Ball II after moving 10 m collides with ball III elastically and stops. But ball I moves towards ball II. Time taken between two consecutive collisions

$$\frac{10}{7.5} + \frac{10 - 10 \times \dfrac{2.5}{7.5}}{2.5} = 4\ \text{s}$$

Check point 8.3

1. In an elastic collision
(a) Both momentum and KE are conserved
(b) only momentum is conserved
(c) only KE is conserved
(d) Neither KE nor momentum is conserved

2. If a body of mass m collides head on, elastically with velocity u with another identical body at rest. After collision velocity of the second body will be
(a) zero (b) u
(c) $2u$ (d) data insufficient

3. A ball hits the floor and rebounds after an inelastic collisions. In this case
(a) the momentum of the ball just after the collision is the same as that just before the collision
(b) the mechanical energy of the ball remains the same in the collision
(c) the total momentum of the ball and the earth is conserved
(d) the total energy of the ball and the earth is conserved

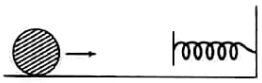
4. A body of mass M_1 collides elastically with another body of mass M_2 at rest. There is maximum transfer of energy when
(a) $M_1 > M_2$
(b) $M_1 < M_2$
(c) $M_1 = M_2$
(d) same for all values of M_1 and M_2

5. Two particles of mass m_A and m_B and their velocities are v_A and v_B respectively collides. After collision they interchanges their velocities, then ratio of $\frac{m_A}{m_B}$ is
(a) $\frac{v_A}{v_B}$ (b) $\frac{v_B}{v_A}$ (c) $\frac{v_A + v_B}{v_B - v_A}$ (d) 1

6. Two perfectly elastic particles A and B of equal mass travelling along the line joining them with velocities 15 ms^{-1} and 10 ms^{-1}. After collision, their velocities will be
(a) 10 ms^{-1}, 10 ms^{-1} (b) 15 ms^{-1}, 15 ms^{-1}
(c) 10 ms^{-1}, 15 ms^{-1} (d) 15 ms^{-1}, 10 ms^{-1}

7. The collision of two balls of equal mass takes place at the origin of coordinates. Before collision, the components of velocities are ($v_x = 50$ cms^{-1}, $v_y = 0$) and ($v_x = -40$ cms^{-1} and $v_y = 30$ cms^{-1}). The first ball comes to rest after collision. The velocity (components v_x and v_y respectively) of the second ball are
(a) 10 and 30 cms^{-1} (b) 30 and 10 cms^{-1}
(c) 5 and 15 cms^{-1} (d) 15 and 5 cms^{-1}

8. A mass of 0.5 kg moving with a a speed of 1.5 ms^{-1} on a horizontal smooth surface, collides with a nearly weightless spring of force constant $k = 50$ Nm^{-1}. The maximum compression of the spring would be



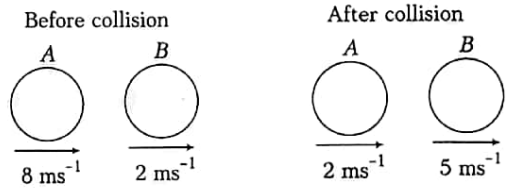
(a) 0.12 m (b) 1.5 m
(c) 0.5 m (d) 0.15 m

9. A smooth sphere of mass M moving with velocity u directly collides elastically with another sphere of mass m at rest. After collision their final velocities are v' and v respectively. The value of v is
(a) $\frac{2uM}{m}$ (b) $\frac{2um}{M}$ (c) $\frac{2u}{1+\frac{m}{M}}$ (d) $\frac{2u}{1+\frac{M}{m}}$

10. A body of mass m_1 moving with a velocity 3 ms^{-1} collides with another body at rest of mass m_2. After collision the velocities of the two bodies are 2 ms^{-1} and 5 ms^{-1} respectively along the direction of motion of m_2. The ratio $\frac{m_1}{m_2}$ is
(a) $\frac{5}{12}$ (b) 5 (c) $\frac{1}{5}$ (d) $\frac{12}{5}$

11. A body of mass m moving with velocity v collides head on with another body of mass $2m$ which is initially at rest. The ratio of KE of colliding body before and after collision will be
(a) 1 : 1 (b) 2 : 1 (c) 4 : 1 (d) 9 : 1

12. The two diagrams show the situations before and after a collision between two spheres A and B of equal radii moving along the same straight line on a smooth horizontal surface. The coefficient of restitution e is

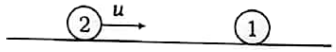


(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$

13. Two balls of equal masses have a head on collision with speed 6 ms^{-1} each. If the coefficient of restitution is 1/3, the speed of each ball after impact will be
(a) 18 ms^{-1} (b) 2 ms^{-1}
(c) 6 ms^{-1} (d) data insufficient

14. A block of mass m moving at a velocity v collides with another block of mass $2m$ at rest. The lighter block comes to rest after collision. Find the coefficient of restitution.
(a) $\frac{1}{2}$ (b) 1 (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

15. A sphere of mass m moving with a constant velocity u hits another stationary sphere of same mass. If e is the coefficient of restitution, then ratio of velocities of the two spheres $\frac{v_1}{v_2}$ after collision will be



(a) $\frac{1-e}{1+e}$ (b) $\frac{1+e}{1-e}$ (c) $\frac{e+1}{e-1}$ (d) $\frac{e-1}{e+1}$

Chapter exercises

A Taking it together

(Assorted questions of the chapter for advanced level practice)

1. A bullet of mass m and velocity v is fired into a block of mass M and sticks to it. The final velocity of the system equals

(a) $\frac{M}{m+M} \cdot v$ (b) $\frac{m}{m+M} \cdot v$

(c) $\frac{m+M}{m} \cdot v$ (d) None of these

2. A bullet of mass 5 g is fired at a velocity of 900 ms^{-1} from a rifle of mass 2.5 kg. What is the recoil velocity of the rifle?

(a) 0.9 ms^{-1} (b) 180 ms^{-1}

(c) 900 ms^{-1} (d) 1.8 ms^{-1}

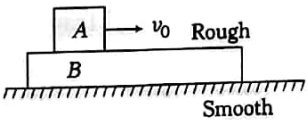
3. A body of mass a moving with a velocity b strikes a body of mass c and gets embedded into it. The velocity of the system after collision is

(a) $\frac{a+c}{ab}$ (b) $\frac{ab}{a+c}$ (c) $\frac{a}{b+c}$ (d) $\frac{a}{a+b}$

4. Two trains A and B are running in the same direction on parallel rails such that A is faster than B. Packets of equal weight are transferred between them. What will happen due to this

(a) A will be accelerated but B will be retarded
(b) B will be accelerated but A will be retarded
(c) there will be no change in A but B will be accelerated
(d) there will be no change in B, but A will be accelerated

5. In a two block system an initial velocity v_0 with respect to ground is given to block A

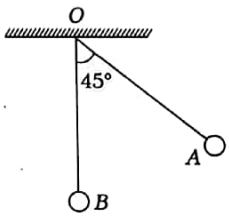


(a) the momentum of block A is not conserved
(b) the momentum of system of blocks A and B is conserved
(c) the increase in momentum of B is equal to the decrease in momentum of block A
(d) All of the above

6. If the net external forces acting on the system of particles is zero, then which of the following may vary?

(a) Momentum of the system
(b) Velocity of centre of mass
(c) Position of centre of mass
(d) None of the above

7. The bob A of a simple pendulum is released when the string makes an angle of 45° with the vertical. It hits another bob B of the same material and same mass kept at rest on a table. If the collision is elastic



(a) Both A and B rise to the same height
(b) Both A and B come to rest at B
(c) Both A and B move with the same velocity of A
(d) A comes to rest and B moves with the velocity of A

8. Consider the following two statements.

A. Linear momentum of the system remains constant.
B. Centre of mass of the system remains at rest.

(a) A implies B and B implies A
(b) A does not imply B and B does not imply A
(c) A implies B but B does not imply A
(d) B implies A but A does not imply B

9. Consider the following two statements.

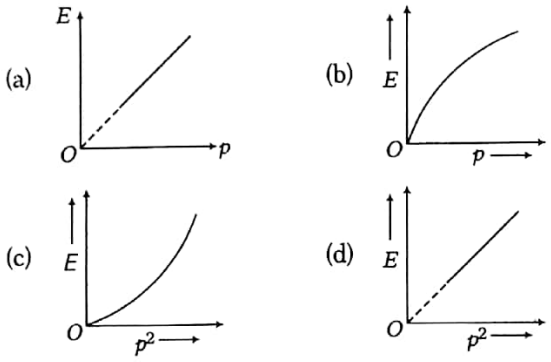
A. Linear momentum of a system of particles is zero.
B. Kinetic energy of a system of particles is zero.

(a) A implies B and B implies A
(b) A does not imply B and B does not imply A
(c) A implies B but B does not imply A
(d) B implies A but A does not imply B

10. A particle of mass m has momentum p. Its kinetic energy will be

(a) mp (b) p^2m (c) $\frac{p^2}{m}$ (d) $\frac{p^2}{2m}$

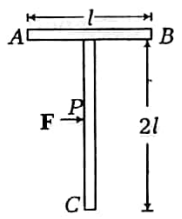
11. A particle is projected from a point at an angle with the horizontal. At any instant t, if p is the linear momentum and E the kinetic energy, then which of the following graph is/are correct?



12. A particle of mass m at rest is acted upon by a constant force F for a time t. Its kinetic energy after an interval t is

(a) $\frac{F^2t^2}{m}$ (b) $\frac{F^2t^2}{2m}$ (c) $\frac{F^2t^2}{3m}$ (d) $\frac{Ft}{2m}$

13. A 'T' shaped object with dimension shown in the figure, is lying on a smooth floor. A force **F** is applied at the point P parallel to AB, such that the object has only the translational motion without rotation. Find the location of P with respect to C

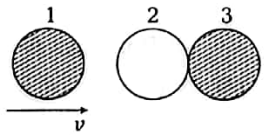


(a) $\frac{4}{3}l$ (b) l (c) $\frac{2}{3}l$ (d) $\frac{3}{2}l$

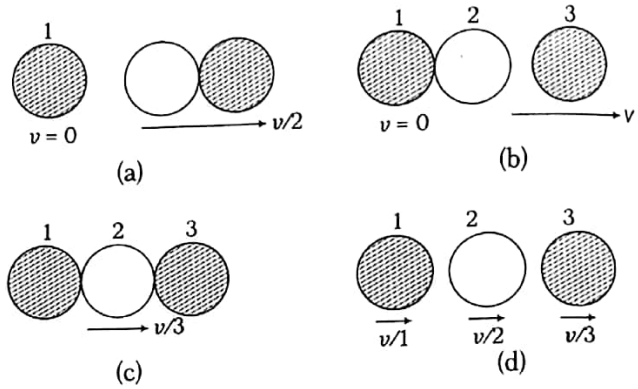
14. A cricket ball of mass 150 g moving with a speed of 126 kmh^{-1} hits at the middle of the bat, held firmly at its position by the batsman. The ball moves straight back to the bowler after hitting the bat. Assuming that collision between ball and bat is completely elastic and the two remain in contact for 0.0015 the force that the batsman had to apply to hold the bat firmly at its place would be **[NCERT Exemplar]**

(a) 10.5 N (b) 21 N
(c) 1.05×10^4 N (d) 2.1×10^4 N

15. Two identical ball bearing in contact with each other and resting on a frictionless table are hit head-on by another ball bearing of the same mass moving initially with a speed v as shown in figure.



If the collision is elastic, which of the following (figure) is a possible result after collision? **[NCERT Exemplar]**



16. During inelastic collision between two bodies which of the following quantities always remain conserved? **[NCERT Exemplar]**

(a) Total kinetic energy (b) Total mechanical energy
(c) Total linear momentum (d) Speed of each body

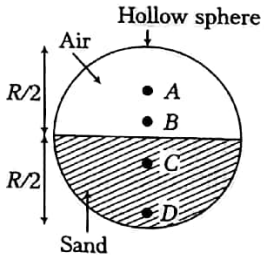
17. Conservation of momentum in a collision between particles can be understood from **[NCERT Exemplar]**

(a) Conservation of energy
(b) Newton's first law
(c) Newton's second law
(d) Both Newton's second and third law

18. For which of the following does the centre of mass lie outside the body? **[NCERT Exemplar]**

(a) A pencil (b) A shotput (c) A dice (d) A bangle

19. Which of the following points is the likely position of the centre of mass of the system as shown in figure? **[NCERT Exemplar]**



(a) A (b) B (c) C (d) D

20. A cannon ball is fired with a velocity 200 ms^{-1} at an angle of 60° with the horizontal. At the highest point of its flight, it explodes into 3 equal fragments, one going vertically upwards with a velocity 100 ms^{-1}, the second one falling vertically downwards with a velocity 100 ms^{-1}. The third fragment will be moving with a velocity

(a) 100 ms^{-1} in the horizontal direction
(b) 300 ms^{-1} in the horizontal direction
(c) 300 ms^{-1} in a direction making an angle of 60° with the horizontal
(d) 200 ms^{-1} in a direction making an angle of 60° with the horizontal

21. A particle of mass m moving with velocity u makes an elastic one dimensional collision with a stationary particle of mass m. They are in contact for a brief time T. Their force of interaction increases from zero to F_0 linearly in time $\frac{T}{2}$ and decreases linearly to zero in further time $\frac{T}{2}$. The magnitude of F_0 is

(a) $\frac{mu}{T}$ (b) $\frac{2mu}{T}$

(c) $\frac{mu}{2T}$ (d) None of these

22. Two balls of equal mass have a head on collision with speed 4 ms^{-1} each travelling in opposite directions. If the coefficient of restitution is 1/2, the speed of each ball after impact will be

(a) 1 ms^{-1} (b) 2 ms^{-1}

(c) 3 ms^{-1} (d) data insufficient

23. A metal ball falls from a height of 32 m on a steel plate. If the coefficient of restitution is 0.5, to what height will the ball rise after second bounce?

(a) 2 m (b) 4 m

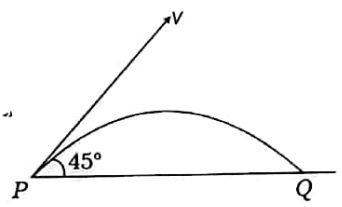
(c) 8 m (d) 16 m

24. A mass m with velocity u strikes a wall normally and returns with the same speed. What is magnitude of the change in momentum of the body when it returns

(a) 4 mu (b) mu

(c) 2 mu (d) zero

25. A projectile of mass m is fired with velocity v from a point P, as shown. Neglecting air resistance, the magnitude of the change in momentum between the points P and arriving at Q is



(a) zero (b) $\frac{mv}{\sqrt{2}}$

(c) $mv\sqrt{2}$ (d) $2mv$

26. A machine gun fires a steady stream of bullets at the rate of n per minute into a stationary target in which the bullets get embedded. If each bullet has a mass m and arrives at the target with a velocity v, the average force on the target is

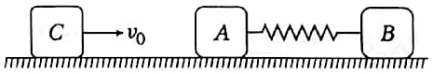
(a) 60 mnv (b) $\frac{60\,v}{mn}$ (c) $\frac{mn\,v}{60}$ (d) $\frac{m\,v}{60n}$

27. 10000 small balls, each weighing 1 g, strikes 1 cm^2 of area per second with a velocity 100 ms^{-1} in a normal direction and rebound with the same velocity. The value of pressure on the surface will be

(a) 2×10^3 Nm^{-2} (b) 2×10^5 Nm^{-2}

(c) 10^7 Nm^{-2} (d) 2×10^7 Nm^{-2}

28. A block C of mass m is moving with velocity v_0 and collides elastically with block A of mass m and connected to another block B of mass 2 m through spring of spring constant k. What is k if x_0 is compression of spring when velocity of A and B is same?



(a) $\frac{mv_0^2}{x_0^2}$ (b) $\frac{mv_0^2}{2x_0^2}$ (c) $\frac{3mv_0^2}{2x_0^2}$ (d) $\frac{2mv_0^2}{3x_0^2}$

29. A bullet of mass m is fired into a block of wood of mass M which hangs on the end of pendulum and gets embedded into it. When the bullet strikes the wooden block, the pendulum starts to swing with maximum rise R. Then, the velocity of the bullet is given by

(a) $\frac{M}{m+M}\sqrt{2gR}$ (b) $\frac{M+m}{m}\sqrt{2gR}$

(c) $\frac{M}{m}\sqrt{2gR}$ (d) None of these

30. A particle of mass m moving with speed v hits elastically another stationary particle of mass 2 m inside a smooth horizontal circular tube of radius r. The time after which the second collision will take place is

(a) $\frac{2\pi r}{v}$ (b) $\frac{4\pi r}{v}$ (c) $\frac{3\pi r}{2v}$ (d) $\frac{\pi r}{v}$

31. A bullet of mass 20 g and moving with 600 ms^{-1} collides with a block of mass 4 kg hanging with the string of length 0.4 m. What is velocity of bullet when it comes out of block, if block rises to height 0.2 m after collision?

(a) 200 ms^{-1} (b) 150 ms^{-1} (c) 400 ms^{-1} (d) 300 ms^{-1}

32. A particle of mass 1 kg is thrown vertically upward with speed 100 ms^{-1}. After 5 s, it explodes into two parts. One part of mass 400 g emerges with speed 25 ms^{-1} in downward direction, what is the velocity of other part just after explosion?

(a) 100 ms^{-1} upward (b) 600 ms^{-1} upward

(c) 100 ms^{-1} downward (d) 300 ms^{-1} upward

33. A ball falling freely from a height of 4.9 ms^{-1}, hits a horizontal surface. If $e = 3/4$, then the ball will hit the surface, second time after

(a) 1 s (b) 1.5 s
(c) 2 s (d) 3 s

34. A disc of mass 10 g is kept floating horizontally by throwing 10 marbles per second against it from below. If the mass of each marble is 5 g. What will be velocity with which the marble are striking the disc? Assume that the marble strikes the disc normally and rebound downwards with the same speed

(a) 2.98 ms^{-1} (b) 0.98 ms^{-1} (c) 0.49 ms^{-1} (d) 1.96 ms^{-1}

35. A machine gun fires a bullet of mass 40 g with a velocity 1200 ms^{-1}. The man holding it, can exert a maximum force of 144 N on the gun. How many bullets can be fired per second at the most?

(a) One (b) Four (c) Two (d) Three

36. A straight rod of length L has one of its ends at the origin and the other at $x = L$. If the mass per unit length of the rod is given by Ax, where A is constant, where is its mass centre?

(a) $\frac{L}{3}$ (b) $\frac{L}{2}$ (c) $\frac{2L}{3}$ (d) $\frac{3L}{4}$

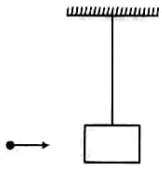
37. In a one dimensional collision between two identical particle A and B, B is stationary and A has momentum p before impact. During impact B gives an impulse J to A. Then, coefficient of restitution between the two is

(a) $\frac{2J}{p} - 1$ (b) $\frac{2J}{p} + 1$ (c) $\frac{J}{p} + 1$ (d) $\frac{J}{p} - 1$

38. A particle of mass m moving with a velocity $(3\hat{i} + 2\hat{j})$ ms^{-1} collides with a stationary body of mass M and finally moves with a velocity $(-2\hat{i} + \hat{j})$ ms^{-1}. If $\frac{m}{M} = \frac{1}{13}$, then

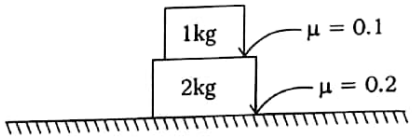
(a) the impulse is $\pm m(5\hat{i} + \hat{j})$
(b) the velocity of the M is $\frac{1}{13}(5\hat{i} + \hat{j})$
(c) Both (a) and (b) are wrong
(d) Both (a) and (b) are correct

39. A mass of 10 g moving horizontally with a velocity of 100 cms^{-1} strikes a pendulum bob of mass 10 g. Length of string is 50 cm. The two masses stick together. The maximum height reached by the system now is (Take, $g = 10$ ms^{-2})



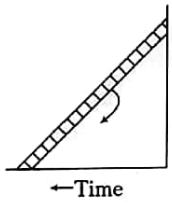
(a) 7.5 cm (b) 5 cm
(c) 2.5 cm (d) 1.25 cm

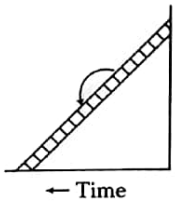
40. Both the blocks as shown in the given arrangement are given together a horizontal velocity towards right. If a_{CM} be the subsequent acceleration of the centre of mass of the system of blocks, then a_{CM} equals

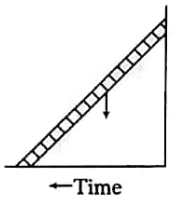


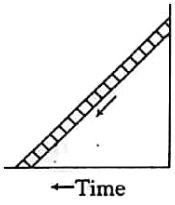
(a) zero (b) $\frac{5}{3}$ ms^{-2} (c) $\frac{7}{3}$ ms^{-2} (d) 2 ms^{-2}

41. A ladder is leaned against a smooth wall and is allowed to slip on a frictionless floor. Which figure represents trace of its centre of mass?

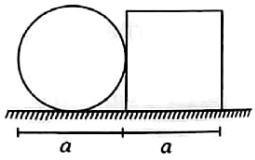
(a)


(b)


(c)


(d)


42. A circular plate of diameter a is kept in contact with a square plate of edge a as shown in figure. The density of the material and the thickness are same everywhere. The centre of mass of the composite system will be



(a) inside the circular plate (b) inside the square plate
(c) at the point of contact (d) outside the system

43. A circular ring of mass 6 kg and radius a is placed such that its centre lies at the origin. Two particles of masses 2 kg each are placed at the intersecting points of the circle with positive X-axis and positive Y-axis. Then, the angle made by the position vector of centre of mass of entire system with X-axis is

(a) 45° (b) 60° (c) $\tan^{-1}\left(\frac{4}{5}\right)$ (d) 30°

44. In a gravity free space, a man of mass M standing at a height h above the floor, throws a ball of mass m straight down with a speed u. When the ball reaches the floor, the distance of the man above the floor will be

(a) $h\left(1 + \frac{m}{M}\right)$ (b) $\left(1 + \frac{M}{m}\right)h$
(c) h (d) $\frac{m}{M}h$

45. A ball falls freely from a height of 45 m. When the ball is at a height of 25 m, it explodes into two equal pieces. One of them moves horizontally with a speed of 10 ms^{-1}. The distance between the two pieces on the ground is

(a) 20 m (b) 30 m (c) 40 m (d) 60 m

46. Two blocks of mass 10 kg and 30 kg are placed along a vertical line. The first block is raised through a height of 7 cm. By what distance should the second mass be moved to raise the centre of mass by 1 cm?

(a) 2 cm upward (b) 1 cm upward
(c) 2 cm downward (d) 1 cm downward

47. In a free space, a rifle of mass M shoots a bullet of mass m at a stationary block of mass M distance D away from it. When the bullet has moved through a distance d towards the block, the centre of mass of the bullet-block system is at a distance of

(a) $\frac{(D-d)m}{M+m}$ from the bullet

(b) $\frac{md+MD}{M+m}$ from the block

(c) $\frac{2md+MD}{M+m}$ from the block

(d) $\frac{(D-d)M}{M+m}$ from the bullet

48. A cracker is thrown into air with a velocity of 10 ms^{-1} at an angle of 45° with the vertical. When it is at a height of (1/2) m from the ground, it explodes into a number of pieces which follow different parabolic paths. What is the velocity of centre of mass, when it is at a height of 1 m from the ground?
(Takes, $g = 10\ ms^{-2}$)

(a) $4\sqrt{5}\ ms^{-1}$ (b) $2\sqrt{5}\ ms^{-1}$
(c) $5\sqrt{4}\ ms^{-1}$ (d) $5\ ms^{-1}$

49. A 2 kg block of wood rests on a long table top. A 5 g bullet moving horizontally with a speed of 150 ms^{-1} is shot into the block and sticks to it. The block then slides 2.7 m along the table top and comes to a stop. The force of friction between the block and the table is

(a) 0.052 N (b) 3.63 N
(c) 2.50 N (d) 1.04 N

50. If the kinetic energy of a body is increased by 300%, its momentum will increase by

(a) 100% (b) 150% (c) $\sqrt{300}$% (d) 175%

51. If the linear momentum is increased by 50%, the kinetic energy will increase by

(a) 50% (b) 100% (c) 125% (d) 25%

52. If the kinetic energy of a body increases by 0.1%, the per cent increase of its momentum will be

(a) 0.05% (b) 0.1%
(c) 1.0% (d) 10%

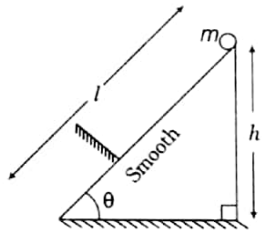
53. A bullet moving with a speed of 100 ms^{-1} can just penetrate two planks of equal thickness. Then, the number of such planks penetrated by the same bullet when the speed is doubled will be

(a) 4 (b) 8 (c) 6 (d) 10

54. A ball is projected vertically down with an initial velocity from a height of 20 m onto a horizontal floor. During the impact it loses 50% of its energy and rebounds to the same height. The initial velocity of its projection is

(a) 20 ms^{-1} (b) 15 ms^{-1} (c) 10 ms^{-1} (d) 5 ms^{-1}

55. A ball of mass m is released from the top of an inclined plane of inclination θ as shown. It strikes a rigid surface at a distances $\frac{3l}{4}$ from top elastically. Impulse imparted to ball by the rigid surface is



(a) $m\sqrt{\frac{3}{2}gh}$ (b) $m\sqrt{3gh}$
(c) $2m\sqrt{3gh}$ (d) $m\sqrt{6gh}$

56. Two identical blocks A and B, each of mass m resting on smooth floor are connected by a light spring of natural length L and spring constant k with the spring at its natural length. A third identical block C (mass m) moving with a speed v along the line joining A and B collides with A, the maximum compression in the spring is

(a) $v\sqrt{\frac{m}{2k}}$ (b) $m\sqrt{\frac{v}{2k}}$
(c) $\sqrt{\frac{mv}{k}}$ (d) $\frac{mv}{2k}$

57. A man of mass M stands at one end of a plank of length L which lies at rest on a frictionless surface. The man walks to the other end of the plank. If the mass of the plank is $\frac{M}{3}$, the distance that the man moves relative to the ground is

(a) $\frac{3L}{4}$ (b) $\frac{L}{4}$ (c) $\frac{4L}{5}$ (d) $\frac{L}{3}$

58. A man of mass m moves with a constant speed on a plank of mass M and length l kept initially at rest on a frictionless horizontal surface, from one end to the other in time t. The speed of the plank relative to ground while man is moving, is

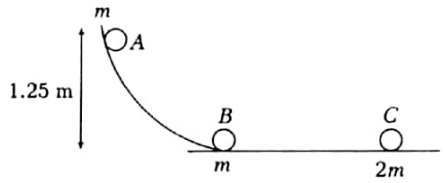
(a) $\frac{l}{t}\left(\frac{M}{m}\right)$ (b) $\frac{l}{t}\left(\frac{m}{m+M}\right)$

(c) $\frac{l}{t}\left(\frac{M}{M+m}\right)$ (d) None of these

59. At high altitude, a body at rest explodes into two equal fragments with one fragment receiving horizontal velocity of 10 ms^{-1}. Time taken by the velocity vectors of the fragments to make 90° is (Take, $g = 10\ ms^{-2}$)

(a) 0.5 s (b) 4 s
(c) 2 s (d) 1 s

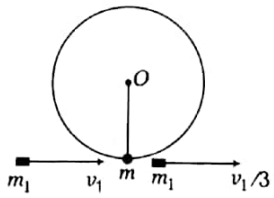
60. A particle A of mass m initially at rest slides down a height of 1.25 m on a frictionless ramp, collides with and sticks to an identical particle B of mass m at rest as shown in the figure.



Then, particles A and B together collide elastically with particle C of mass 2 m at rest. The speed of particle C after the collision with combined body $(A + B)$ would be (Take, $g = 10\ ms^{-2}$)

(a) 2 ms^{-1} (b) 1.25 ms^{-1}
(c) 2.5 ms^{-1} (d) 5 ms^{-1}

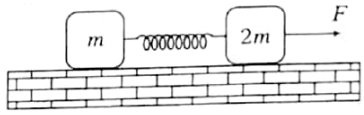
61. A pendulum consists of a wooden bob of mass m and length l. A bullet of mass m_1 is fired towards the pendulum with a speed v_1 and it emerges from the bob with speed $\frac{v_1}{3}$. The bob just completes motion along a vertical circle. Then, v_1 is



(a) $\frac{m}{m_1}\sqrt{5gl}$ (b) $\frac{3m}{2m_1}\sqrt{5gl}$

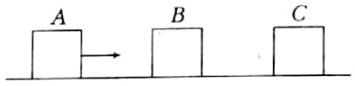
(c) $\frac{2}{3}\left(\frac{m}{m_1}\right)\sqrt{5gl}$ (d) $\left(\frac{m_1}{m}\right)\sqrt{gl}$

62. Two blocks of mass m and $2m$ are kept on a smooth horizontal surface. They are connected by an ideal spring of force constant k. Initially the spring is unstretched. A constant force is applied to the heavier block in the direction shown in figure. Suppose at time t displacement of smaller block is x, then displacement of the heavier block at this moment would be



(a) $\frac{x}{2}$ (b) $\frac{Ft^2}{6m} + \frac{x}{3}$ (c) $\frac{x}{3}$ (d) $\frac{Ft^2}{4m} - \frac{x}{2}$

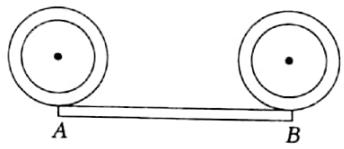
63. Three identical blocks A, B and C are placed on horizontal frictionless surface. The blocks B and C are at rest. But A is approaching towards B with a speed 10 ms^{-1}.



The coefficient of restitution for all collision is 0.5. The speed of the block C just after collision is approximately

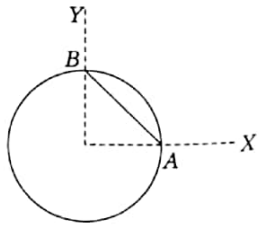
(a) 5.6 ms^{-1} (b) 6.4 ms^{-1}
(c) 3.2 ms^{-1} (d) 4.6 ms^{-1}

64. You are supplied with three identical rods of same length and mass. If the length of each rod is 2π. Two of them are converted into rings and then placed over the third rod as shown in figure. If point A is considered as origin of the coordinate system the coordinate of the centre of mass will be (you may assume AB as x-axis of the coordinate system)



(a) $\left(\frac{\pi}{2}, \frac{1}{3}\right)$ (b) $\left(\frac{\pi}{2}, \frac{2}{3}\right)$ (c) $\left(\pi, \frac{1}{3}\right)$ (d) $\left(\pi, \frac{2}{3}\right)$

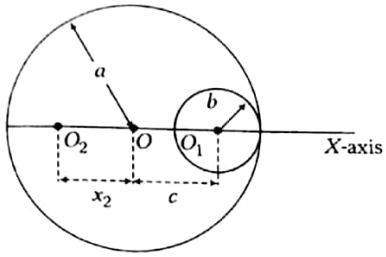
65. An object comprises a uniform ring of radius R and its uniform chord AB (not necessarily made of the same material) as shown. Which of the following cannot be the centre of mass of the object?



(a) $\left(\frac{R}{3}, \frac{R}{3}\right)$ (b) $\left(\frac{R}{\sqrt{2}}, \frac{R}{\sqrt{2}}\right)$

(c) $\left(\frac{R}{4}, \frac{R}{4}\right)$ (d) None of these

66. A uniform circular disc of radius a is taken. A circular portion of radius b has been removed from it as shown in the figure. If the centre of hole is at a distance c from the centre of the disc, the distance x_2 of the centre of mass of the remaining part from the initial centre of mass O is given by



(a) $\frac{\pi b^2}{(a^2 - c^2)}$ (b) $\frac{cb^2}{(a^2 - b^2)}$

(c) $\frac{\pi c^2}{(a^2 - b^2)}$ (d) $\frac{ca^2}{(c^2 - b^2)}$

67. From a circular disc of radius R, a square is cut out with a radius as its diagonal. The centre of mass of remaining portion is at a distance (from the centre)

(a) $\frac{R}{(4\pi - 2)}$ (b) $\frac{R}{2\pi}$ (c) $\frac{R}{(\pi - 2)}$ (d) $\frac{R}{(2\pi - 2)}$

68. A particle falls from a height h upon a fixed horizontal plane and rebounds. If e is the coefficient of restitution the total distance travelled before rebounding has stopped is

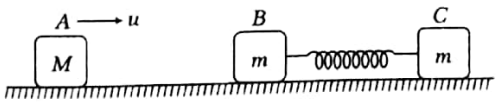
(a) $h\left(\frac{1+e^2}{1-e^2}\right)$ (b) $h\left(\frac{1-e^2}{1+e^2}\right)$

(c) $\frac{h}{2}\left(\frac{1-e^2}{1+e^2}\right)$ (d) $\frac{h}{2}\left(\frac{1+e^2}{1-e^2}\right)$

69. A train of mass M is moving on a circular track of radius R with constant speed v. The length of the train is half of the perimeter of the track. The linear momentum of the train will be

(a) πMv (b) $\frac{2Mv}{\pi}$ (c) $\frac{\pi Mv}{2}$ (d) Mv

70. A block A of mass M moving with speed u collides elastically with block B of mass m which is connected to block C of mass m with a spring.

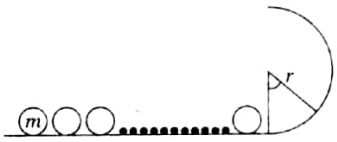


When the compression in spring is maximum, the velocity of block C with respect to block A is (Neglect the friction everywhere)

(a) zero (b) $\left(\frac{M}{M+m}\right)u$

(c) $\left(\frac{m}{M+m}\right)u$ (d) $\left(\frac{m}{M}\right)u$

71. n elastic balls are placed at rest on a smooth horizontal plane which is circular at the ends with radius r as shown in the figure. The masses of the balls are $m, \frac{m}{2}, \frac{m}{2^2}, \ldots, \frac{m}{2^{n-1}}$ respectively. What is the minimum velocity which should be imparted to the first ball of mass m such that this nth ball will complete the vertical circle?



(a) $\left(\frac{3}{4}\right)^{n-1}\sqrt{5gr}$ (b) $\left(\frac{4}{3}\right)^{n-1}\sqrt{5gr}$

(c) $\left(\frac{3}{2}\right)^{n-1}\sqrt{5gr}$ (d) $\left(\frac{2}{3}\right)^{n-1}\sqrt{5gr}$

72. A small ball rolls off the top landing of the staircase. It strikes the mid-point of the first step and then the mid-point of the second step. The steps are smooth, and identical in height and width. The coefficient of restitution between the ball and the first step is

(a) 1 (b) $\frac{3}{4}$

(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

73. A girl throws a ball with initial velocity v at an inclination of 45°. The ball strikes the smooth vertical wall at a horizontal distance d from the girl and after rebounding returns to her hand. What is the coefficient of restitution between wall and the ball?

(a) $v^2 - gd$ (b) $\frac{gd}{v^2 - gd}$

(c) $\frac{gd}{v^2}$ (d) $\frac{v^2}{gd}$

B Medical entrance special format questions

Assertion and reason

Directions (Q. Nos. 1-20) *These questions consists of two statements each printed as Assertion and Reason. While answering these question you are required to choose any one of the following five responses*

(a) If both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If Assertion is false but Reason is true.
(e) If both Assertion and Reason are false.

1. **Assertion** In head on inelastic collision, the final momentum is less than the initial momentum.

   Reason For inelastic collision, $0 \le e < 1$Hence, the magnitude of relative velocity of separation after collision is less than relative velocity of approach before collision.

2. **Assertion** In an elastic collision between two bodies, the magnitude of relative velocity of the bodies after collision is equal to the magnitude of relative velocity before collision.

   Reason In an elastic collision, the linear momentum of the system is conserved.

3. **Assertion** A given force applied in turn to a number of different masses cause the same rate of change in momentum in each but not the same acceleration to all.

   Reason $\mathbf{F} = \frac{d\mathbf{p}}{dt}$ and $\mathbf{a} = \frac{\mathbf{F}}{m}$

4. **Assertion** In inelastic collision, linear momentum of system does not remain constant during collision. But before collision and after collision, it is constant.

   Reason In elastic collision, momentum remains constant during collision also.

5. **Assertion** Two spherical bodies of mass ratio 1 : 2 travel towards each other (starting from rest) under the action of their mutual gravitational attraction. Then, the ratio of their kinetic energies at any instant is 2 : 1.

   Reason At any instant their momenta are same.

6. **Assertion** Centre of mass and centre of gravity of a body will coincide on moon.

   Reason There is no gravity on moon.

7. **Assertion** A projectile gets exploded at its highest point. All the pieces get only horizontal velocities.

   Reason The weight of the projectile is the external force for projectile.

8. **Assertion** If a projectile explodes in mid air, linear momentum of centre of mass of different fragments remains constant.

   Reason When net force on a system of particles is zero, linear momentum of the system remains constant.

9. **Assertion** When a body dropped from a height explodes in mid air, its centre of mass keeps moving in vertically downward direction.

   Reason Explosion occurs under internal forces only. External force is zero.

10. **Assertion** The centre mass of an electron and proton, when released from rest moves faster towards proton.

    Reason Proton is heavier than electron.

11. **Assertion** The relative velocity of the two particles in head on elastic collision is unchanged both in magnitude and direction.

    Reason The relative velocity is unchanged in magnitude but gets reversed in direction.

12. **Assertion** If net force on a system is zero, then momentum of every individual body remains constant.

    Reason If momentum of a system is constant, then kinetic energy of the system may change.

13. **Assertion** If a projectile explodes in mid air, then no external force acts on the projectile during explosion.

    Reason Centre of mass in this case follows the same path.

14. **Assertion** Two bodies moving in opposite directions with same magnitude of linear momentum collide each other. Then, after collision both the bodies will come to rest.

    Reason Linear momentum of the system of bodies is zero.

15. **Assertion** A body is thrown with a velocity u inclined to the horizontal at some angle. It moves along a parabolic path and falls to the ground. Linear momentum of the body, during its motion, will remain conserved.

    Reason Throughout the motion of the body, a constant force acts on it.

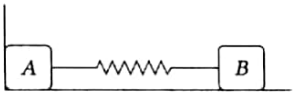
16. **Assertion** A moving ball having an inelastic collision with another moving ball can have larger kinetic energy after collision.

Reason During a collision between two bodies, transfer of energy may take place from one body to another.

17. **Assertion** A rocket launched vertically upward explodes at the highest point it reaches. The explosion produces three fragments with non-zero initial velocity. Then, the initial velocity vectors of all the three fragments are in one plane.

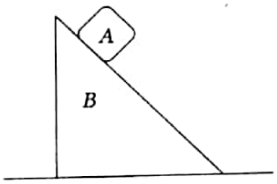
Reason For sum of momentum of three particles, all the three momentum vectors must be coplanar.

18. **Assertion** Two blocks A and B are connected at the two ends of an ideal spring as shown in figure. Initially spring was relaxed. Now block B is pressed. Linear momentum of the system will not remain constant till the spring reaches its initial natural length.



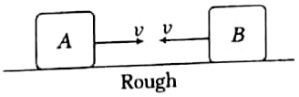
Reason An external force will act from the wall on block A.

19. **Assertion** In the figure shown, linear momentum of system (of blocks A and B) moves towards right during motion of block A over the block B.



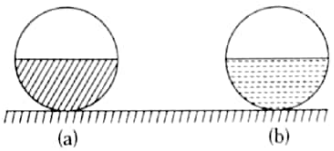
Reason Force of friction will act on B towards right (from ground).

20. **Assertion** Two blocks of masses m_A and m_B ($> m_A$) are thrown towards each other with same speed over a rough ground. The coefficient of friction of both the blocks with ground is same. Initial velocity of CM is towards left.



Reason Initial acceleration of centre of mass is towards right.

21. **Assertion** Two identical spherical spheres are half filled with two liquids of densities ρ_1 and ρ_2 ($> \rho_1$). The centre of mass of both the spheres lie at same level.



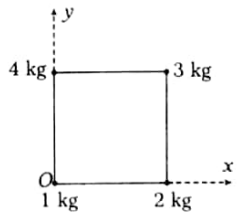
Reason The centre of mass will lie at centre of the sphere.

Match the columns

1. If net force on a system of particles is zero, then match the following columns.

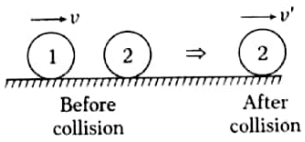
	Column I		Column II
(A)	Acceleration of centre of mass	(p)	Constant
(B)	Momentum of centre of mass	(q)	Zero
(C)	Velocity of centre of mass	(r)	May be zero
(D)	Velocity of an individual particle of the system	(s)	May be constant

2. Four point masses are placed at four corners of a square of side 4 m as shown. Match the following columns.



	Column I		Column II
(A)	x-coordinate of centre of mass of 4 kg and 2 kg	(p)	(7/2) m
(B)	x-coordinate of centre of mass of 4 kg, 2 kg and 3 kg	(q)	4/3 m
(C)	y-coordinate of centre of mass of 1 kg, 4 kg and 3 kg	(r)	3 m
(D)	y-coordinate of centre of mass of 1 kg and 3 kg	(s)	(20/9) m

3. In the diagram shown in figure mass of both the balls is same. Match the following columns.

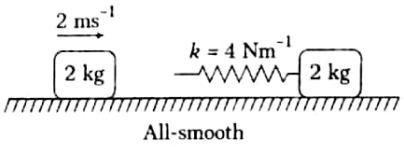


	Column I		Column II
(A)	For $v' = v$	(p)	$e = 0$
(B)	For $v' = v/2$	(q)	$e = 1$
(C)	For $v' = 3/4\ v$	(r)	$e = 1/2$
		(s)	Data is insufficient

4. A particle of mass 1 kg has velocity $\mathbf{v}_1 = (2t)\ \hat{\mathbf{i}}$ and another particle of mass 2 kg has velocity $\mathbf{v}_2 = (t^2)\ \hat{\mathbf{j}}$. Match the following columns.

	Column I		Column II
(A)	Net force on centre of mass at 2 s	(p)	$\frac{20}{9}$ unit
(B)	Velocity of centre of mass at 2 s	(q)	$\sqrt{68}$ unit
(C)	Displacement of centre of mass in 2 s	(r)	$\sqrt{80}/3$ unit
		(s)	None

5. In the arrangement shown in figure match the following



	Column I		Column II
(A)	Velocities of centre of mass	(p)	2 SI unit
(B)	Velocity of combined mass when compression in the spring is maximum	(q)	1 SI unit
(C)	Maximum compression in the spring	(r)	4 SI unit
(D)	Maximum potential energy stored in the spring	(s)	0.5 SI unit

6. A particle of mass m, kinetic energy K and momentum p collides head on elastically with another particle of mass $2\ m$ at rest. Match the following columns (after collision).

	Column I		Column II
(A)	Momentum of first particle	(p)	$3/4\ p$
(B)	Momentum of second particle	(q)	$-K/9$
(C)	Kinetic energy of first particle	(r)	$-p/3$
(D)	Kinetic energy of second particle	(s)	$8K/9$

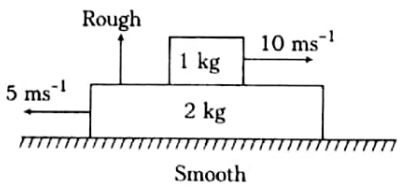
7. A particle of mass 1 kg is projected upwards with velocity 60 ms^{-1}. Another particle of mass 2 kg is just dropped from a certain height. After 2 s, match the following.
[Take, $g = 10\ ms^{-2}$]

	Column I		Column II
(A)	Acceleration of CM	(p)	Zero
(B)	Velocity of CM	(q)	10 SI unit
(C)	Displacement of CM	(r)	20 SI unit
		(s)	None

8. Match the following columns.
(p = momentum of particle, K = kinetic energy of particle)

	Column I		Column II
(A)	p is increased by 200%, corresponding change in K	(p)	800%
(B)	K is increased by 300%, corresponding change in p	(q)	100%
(C)	p is increased by 1%, corresponding change in K	(r)	0.5%
(D)	K is increased by 1%, corresponding change in p	(s)	2%

9. In a two block system shown in figure match the following.



	Column I		Column II
(A)	Velocity of centre of mass	(p)	Keep on changing all the time
(B)	Momentum of centre of mass	(q)	First decreases then become zero
(C)	Momentum of 1 kg block	(r)	Zero
(D)	Kinetic energy of 2 kg block	(s)	Constant

Medical entrances' gallery

(Collection of questions asked in NEET & various medical entrance exams)

1. A rod of weight w is supported by two parallel knife edges A and B and is in equilibrium in a horizontal position. The knives are at a distance d from each other. The centre of mass of the rod is at distance x from A. The normal reaction on A is **[CBSE AIPMT 2015]**

(a) $\frac{wx}{d}$ (b) $\frac{wd}{x}$ (c) $\frac{w(d-x)}{x}$ (d) $\frac{w(d-x)}{d}$

2. Two particles of masses m_1, m_2 move with initial velocities u_1 and u_2. On collision, one of the particles get excited to higher level, after absorbing energy ε. If final velocities of particles be v_1 and v_2, then we must have **[CBSE AIPMT 2015]**

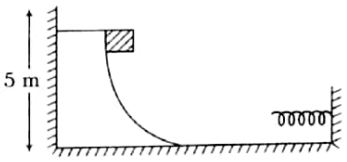
(a) $m_1^2 u_1 + m_2^2 u_2 - \varepsilon = m_1^2 v_1 + m_2^2 v_2$

(b) $\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 - \varepsilon$

(c) $\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 - \varepsilon = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

(d) $\frac{1}{2} m_1^2 u_1^2 + \frac{1}{2} m_2^2 u_2^2 + \varepsilon = \frac{1}{2} m_1^2 v_1^2 + \frac{1}{2} m_2^2 v_2^2$

3. Consider the situation shown in figure. A spring of spring constant 400 Nm^{-1} is attached at one end to a wedge fixed rigidly with the horizontal part. A 40 g mass is released from rest while situated at a height 5 cm the curved track. The minimum deformation in the spring is nearly equal to (Take, $g = 10\ ms^{-2}$) **[AIIMS 2015]**



(a) 9.8 m (b) 9.8 cm (c) 0.98 m (d) 0.009 km

4. A block having mass m collides with an another stationary block having mass 2 m. The lighter block comes to rest after collision. If the velocity of first block is v, then the value of coefficient of restitution will must be **[AIIMS 2015]**

(a) 0.5 (b) 0.4 (c) 0.6 (d) 0.8

5. A smooth curved surface of height 10 m is ended horizontally. A spring of force constant 200 Nm^{-1} is fixed at the horizontal end as shown in figure when an object of mass 10 g is released from the top, it travels along the curved path and collides with the spring. Then, the maximum compression in the spring is (Take, $g = 10\ ms^{-2}$) **[EAMCET 2015]**

(a) 10 m (b) 0.1 m (c) 1 m (d) 0.01 m

6. A frog sits on the end of a long board of length $L = 10$ cm. The board rests on a frictionless horizontal table. The frog wants to jump to the opposite end of the board. What is minimum take off speed v in ms^{-1} relative to the ground that the frog follows to do the trick? [Assume that the board and frog have equal masses.] **[UP CPMT 2015]**

(a) $2\sqrt{5}\ ms^{-1}$ (b) $5\ ms^{-1}$

(c) $5\sqrt{2}\ ms^{-1}$ (d) $10\sqrt{2}\ ms^{-1}$

7. A particle of mass m collides with another stationary particle of mass M. If the particle m stops just after collision, then the coefficient of restitution for collision is equal to **[Manipal 2015]**

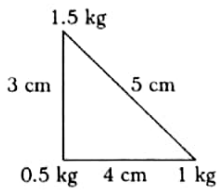
(a) 1 (b) $\frac{m}{M}$ (c) $\frac{M-m}{M+m}$ (d) $\frac{m}{M+m}$

8. A body from height h is dropped, if the coefficient of restitution is e, then calculate the height achieved after one bounce. **[Manipal 2015]**

(a) $h_1 = e^4 h$ (b) $h = e h_1$

(c) $h_1 = he^2$ (d) $h = h_1/e$

9. Three particles of masses 0.5 kg, 1 kg, 1.5 kg are placed at the three corners of a right angle triangle of sides 3, 4, 5 cm as shown in adjoining figure. What would be coordinates (x, y) of the centre of mass of system? **[UK PMT 2015, UP CPMT 2015]**



(a) (1.3, 1.5) (b) (2.3, 1.5)

(c) (1.3, 2.5) (d) (2.3, 2.5)

10. A large number of particles are placed around the origin, each at a distance R from the origin. The distance of the centre of mass of the system from the origin is **[WB JEE 2015]**

(a) equal to R (b) less than or equal to R

(c) greater than R (d) greater than or equal to R

11. A circular disc rolls on a horizontal floor without slipping and the centre of the disc moves with a uniform velocity v. Which of the following values of the velocity at a point on the rim of the disc can have? **[WB JEE 2015]**

(a) v (b) $-v$ (c) $3v$ (d) Zero

12. The linear momentum of a particle varies with time t as $p = a + bt + ct^2$. Then, which of the following is correct? **[EAMCET 2014]**
(a) Velocity of particle is inversely proportional to time
(b) Displacement of the particle is independent of time
(c) Force varies with time in a quadratic manner
(d) Force is dependent linearly on time

13. A body of mass ($4m$) is lying in xy-plane at rest. It suddenly explodes into three pieces. Two pieces each of mass (m) move perpendicular to each other with equal speeds (v). The total kinetic energy generated due to explosion is **[CBSE AIPMT 2014]**
(a) mv^2 (b) $3/2\, mv^2$ (c) $2mv^2$ (d) $4\, mv^2$

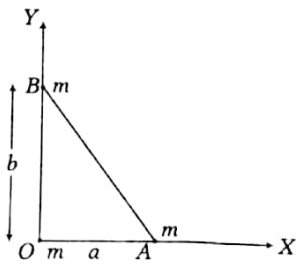
14. The position of centre of mass of a system of particles does not depend upon the **[Kerala CEE 2014]**
(a) mass of particles
(b) symmetry of the body
(c) position of the particles
(d) relative distance between the particles
(e) nature of particles

15. A gun fires a small bullet with kinetic energy K. Then kinetic energy of the gun while recoiling is **[KCET 2013]**
(a) K (b) more than K
(c) less than K (d) $\sqrt{K}$

16. An explosion breaks a rock into three parts in a horizontal plane. Two of them go off at right angles to each other. The first part of mass 1 kg moves with a speed of 12 ms^{-1} and the second part of mass 2 kg moves with speed of 8 ms^{-1}. If the third part flies off with speed of 4 ms^{-1}, then its mass is **[NEET 2013]**
(a) 3 kg (b) 5 kg (c) 7 kg (d) 17 kg

17. The linear momentum is conserved in **[J&K CET 2013]**
(a) elastic collisions (b) inelastic collisions
(c) Both (a) and (b) (d) Neither (a) nor (b)

18. Three particles, each of mass m, are placed at the vertices of a right angled triangle as shown in figure. The position vector of the centre of mass of the system is (O is the origin and $\hat{i}$, $\hat{j}$, $\hat{k}$ are unit vectors) **[EAMCET 2013]**



(a) $\frac{1}{3}(a\hat{i} - b\hat{j})$ (b) $\frac{2}{3}(a\hat{i} - b\hat{j})$
(c) $\frac{2}{3}(a\hat{i} + b\hat{j})$ (d) $\frac{1}{3}(a\hat{i} + b\hat{j})$

19. A ball of mass m moving with a horizontal velocity v strikes the bob of a pendulum at rest. Mass of the bob of the pendulum is also m. During this collision the ball sticks with the bob of the pendulum. The height to which the combined mass rises (g = acceleration due to gravity) **[EAMCET 2013]**
(a) $\frac{v^2}{4g}$ (b) $\frac{v^2}{8g}$
(c) $\frac{v^2}{g}$ (d) $\frac{v^2}{2g}$

20. In an inelastic collision **[Kerala CEE 2013]**
(a) momentum is not conserved
(b) momentum is conserved but kinetic energy is not conserved
(c) both momentum and kinetic energy are conserved
(d) neither momentum nor kinetic energy is conserved
(e) kinetic energy is conserved but not momentum

21. When a body of mass m_1 moving with uniform velocity 40 ms^{-1} collides with another body of mass m_2 at rest, then the two together begin to move with uniform velocity of 30 ms^{-1}. The ratio of the masses (*i.e.*, m_1/m_2) of the two bodies will be **[BCECE (Mains) 2012]**
(a) 1 : 3 (b) 3 : 1
(c) 1 : 1.33 (d) 1 : 0.75

22. Two spheres A and B of masses m_1 and m_2 respectively collide. A is at rest initially and B is moving with velocity v along X-axis. After collision B has a velocity $v/2$ in a direction perpendicular to the original direction. The mass A moves after collision in the direction **[CBSE AIPMT 2012]**
(a) same as that of B
(b) opposite to that of B
(c) $\theta = \tan^{-1}\left(\frac{1}{2}\right)$ to the X-axis
(d) $\theta = \tan^{-1}\left(\frac{-1}{2}\right)$ to the X-axis

23. Two persons of masses 55 kg and 65 kg respectively, are at the opposite ends of a boat. The length of the boat is 3 m and weighs 100 kg. The 55 kg man walks up to the 65 kg man and sits with him. If the boat is in still water the centre of mass of the system shifts by **[CBSE AIPMT 2012]**
(a) 3.0 m (b) 2.3 m
(c) zero (d) 0.75 m

24. A ball moving with velocity 9 ms^{-1} collides with another similar stationary ball., If after the collision, both the balls move in directions making an angle of 30° with the initial direction, then their speeds, after collision will be **[BHU 2012]**
(a) 5.2 ms^{-1} (b) 0.52 ms^{-1}
(c) 52 ms^{-1} (d) 26 ms^{-1}

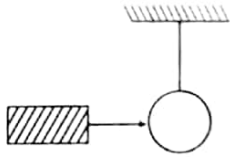
25. A body of mass 0.25 kg is projected with muzzle velocity 100 ms^{-1} from a tank of mass 100 kg. What is the recoil velocity of the tank? **[AIIMS 2012]**

(a) 5 ms^{-1} b) 25 ms^{-1}
(c) 0.5 ms^{-1} (d) 0.25 ms^{-1}

26. A body of mass $m_1 = 4$ kg moves at 5 $\hat{i}$ ms^{-1} and another body of mass $m_2 = 2$ kg moves at $10\hat{i}$ ms^{-1}. The kinetic energy of centre of mass is **[Manipal 2012]**

(a) $\frac{200}{3}$ J (b) $\frac{500}{3}$ J (c) $\frac{400}{3}$ J (d) $\frac{800}{3}$ J

27. A mass of 10 g moving horizontally with a velocity of 100 cms^{-1} strikes a pendulum bob of same mass. The two masses after collision stick together. What will be the maximum height reached by the system now? (Take, $g = 10$ ms^{-1}) **[JCECE 2012]**



(a) Zero (b) 1.25 cm
(c) 2.5 cm (d) 5 cm

Answers

Check point 8.1

1. (b)	2. (c)	3. (a)	4. (d)	5. (c)	6. (b)	7. (d)	8. (c)	9. (a)	10. (d)
11. (d)	12. (a)	13. (b)	14. (d)	15. (d)					

Check point 8.2

1. (c)	2. (c)	3. (d)	4. (b)	5. (a)	6. (d)	7. (c)	8. (b)	9. (c)	10. (b)
11. (a)	12. (d)	13. (b)	14. (a)	15. (d)	16. (b)	17. (c)	18. (b)	19. (c)	20. (a)

Check point 8.3

1. (a)	2. (b)	3. (c)	4. (c)	5. (d)	6. (c)	7. (a)	8. (d)	9. (c)	10. (b)
11. (d)	12. (b)	13. (b)	14. (a)	15. (b)					

A. Taking it together

1. (b)	2. (d)	3. (b)	4. (b)	5. (d)	6. (d)	7. (d)	8. (d)	9. (d)	10. (d)
11. (d)	12. (b)	13. (a)	14. (c)	15. (b)	16. (c)	17. (d)	18. (d)	19. (c)	20. (b)
21. (b)	22. (b)	23. (a)	24. (c)	25. (c)	26. (c)	27. (d)	28. (d)	29. (b)	30. (a)
31. (a)	32. (a)	33. (b)	34. (b)	35. (d)	36. (c)	37. (a)	38. (d)	39. (d)	40. (d)
41. (a)	42. (b)	43. (a)	44. (a)	45. (a)	46. (d)	47. (d)	48. (a)	49. (a)	50. (a)
51. (c)	52. (a)	53. (b)	54. (a)	55. (d)	56. (a)	57. (b)	58. (b)	59. (d)	60. (c)
61. (b)	62. (d)	63. (a)	64. (d)	65. (b)	66. (b)	67. (a)	68. (a)	69. (b)	70. (b)
71. (a)	72. (b)	73. (b)							

B. Medical entrance special format questions

Assertion and reason

1. (d)	2. (b)	3. (a)	4. (d)	5. (a)	6. (c)	7. (d)	8. (a)	9. (c)	10. (d)
11. (d)	12. (d)	13. (d)	14. (d)	15. (d)	16. (d)	17. (a)	18. (a)	19. (a)	20. (b)
21. (c)									

Match the columns

1. (A → q, B → r, C → r, D → s)
2. (A → q, B → s, C → p, D → r)
3. (A → q, B → p, C → r)
4. (A → q, B → r, C → p)
5. (A → q, B → q, C → q , D → p)
6. (A → r, B → t, C → r, D → s)
7. (A → q, B → p, C → r)
8. (A → p, B → q, r, C → s, D → r)
9. (A → r,s, B → r s, C → q, D → q)

C. Medical entrances' gallery

1. (d)	2. (c)	3. (b)	4. (a)	5. (b)	6. (c)	7. (b)	8. (c)	9. (a)	10. (b)
11. (d)	12. (d)	13. (b)	14. (e)	15. (c)	16. (b)	17. (c)	18. (d)	19. (b)	20. (b)
21. (b)	22. (d)	23. (c)	24. (a)	25. (d)	26. (c)	27. (b)			

Hints and explanations

Check point 8.1

1. (*b*) It does not depend on the internal forces acting on the particle.

2. (*c*) $r \propto \frac{1}{m}$

3. (*a*) Let the coordinates of the centre of mass be (x, y)

$$x = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{1\times(-1) + 2\times 2}{3} = \frac{-1+4}{3} = 1$$

$$y = \frac{m_1y_1 + m_2y_2}{m_1 + m_2} = \frac{1\times 2 + 2\times 4}{3} = \frac{2+8}{3} = \frac{10}{3}$$

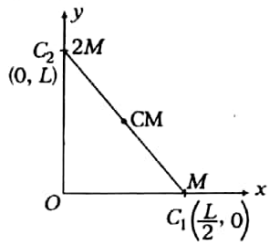
Therefore, the coordinates of centre of mass be $\left(1, \frac{10}{3}\right)$.

4. (*d*) Distance distributes in inverse ratio of masses.

Hence, $r_c = d\left(\frac{m_0}{m_0 + m_c}\right) = 1.2\times 10^{-10}\left(\frac{16}{16+12}\right)$

$= 0.69\times 10^{-10}$ m

5. (*c*) As rods are uniform, therefore centre of mass of both rods will be at their geometrical centres. The coordinates of CM of first rod C_1 are $\left(\frac{L}{2}, 0\right)$ and second rod C_2 are $(0, L)$.

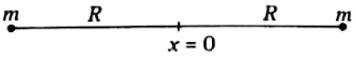


$$\therefore \quad x_{CM} = \frac{M\left(\frac{L}{2}\right) + 2M(0)}{M + 2M} = \frac{L}{6}$$

$$y_{CM} = \frac{M(0) + 2M(L)}{M + 2M} = \frac{2L}{3}$$

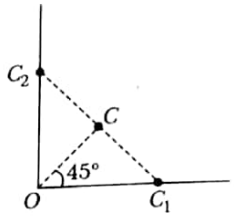
Hence, coordinates of CM are $\left(\frac{L}{6}, \frac{2L}{3}\right)$.

6. (*b*) For a single particle, distance of centre of mass from origin is R. For more than one particles distance $\leq R$.



For example, for two particles of equal mass, kept as shown in figure, distance = 0.

7. (*d*) $OC_1 = \frac{1}{4}$ m



$$\therefore \quad OC = OC_1 \cos 45° = \frac{1}{4\sqrt{2}} \text{ m}$$

8. (*c*) $m_1 = 1.6$ kg; $(x_1, y_1) = (0, 0)$

$m_2 = 2$ kg; $(x_2, y_2) = (1.2, 0)$

$m_3 = 2.4$ kg; $(x_3, y_3) = (0, 1)$

$\therefore$ Coordinates of centre of mass will be

$$x_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$= \frac{(1.6)(0) + (2)(1.2) + (2.4)(0)}{1.6 + 2 + 2.4}$$

$$x_{CM} = 0.4 \text{ m}$$

$$y_{CM} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$$

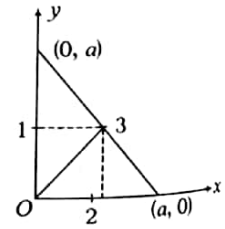
$$= \frac{(1.6)(0) + (2)(0) + (2.4)(1)}{1.6 + 2 + 2.4}$$

$$y_{CM} = 0.4 \text{ m}$$

$\therefore$ Coordinates of centre of mass = (0.4, 0.4) m

9. (*a*) $x_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$

$$= \frac{m(a/2) + (m)\left(\frac{a}{2}\right) + m\left(\frac{a}{2}\right)}{m + m + m} = \frac{a}{2}$$



Similarly, $y_{CM} = \frac{a}{2}$

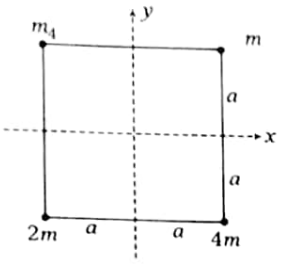
10. (*d*) Centre of mass of Ist system already lies at (1, 2, 3). Therefore centre of mass of 3 kg and 5 kg should lie at (1, 2, 3).

$$\therefore \quad \frac{3(-\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}}) + 5\mathbf{r}_5}{(3+5)} = (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$$

Solving this we get, $\mathbf{r}_5 = \frac{11}{5}\hat{\mathbf{i}} + \frac{7}{5}\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$

i.e., 5 kg mass should be kept at (11/5, 7/5, 6)

11. (*d*) $x_{CM} = 0$



or $$\frac{m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4}{m_1 + m_2 + m_3 + m_4} = 0$$

or $(2m)(-a) + 4m(a) + m(a) + m_4(-a) = 0$

or $m_4 = 3m$

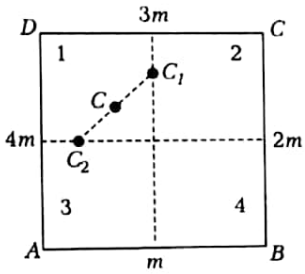
Similarly $y_{CM} = 0$

or $(2m)(-a) + 4m(-a) + m(a) + m_4(a) = 0$ or $m_4 = 5m$

Since, value of m_4 are different to satisfy by both $x_{CM} = 0$ and $y_{CM} = 0$.

Hence, it is not possible.

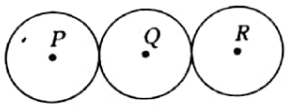
12. (*a*) $C_1 \rightarrow$ Position of centre of mass of rods *AB* and *CD* (nearer to *CD*, as it is heavy)



$C_2 \rightarrow$ Position of centre of mass of rods *BC* and *DA*.

$C \rightarrow$ Overall centre of mass of all four rods and it lies in region 1.

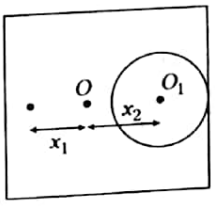
13. (*b*) X_{CM} (from P) $= \dfrac{m \times O + m \times PQ + m \times PR}{m + m + m}$ $(m = 1\,\text{kg})$



or $$X_{CM} = \frac{PQ + PR}{3}$$

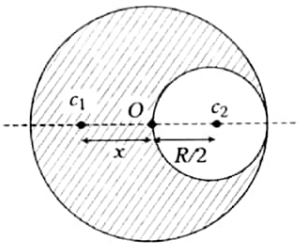
14. (*d*) $A_1x_1 = A_2x_2$ or $x_1 = \dfrac{A_2}{A_1} \cdot x_2 = \dfrac{\left(\dfrac{\pi}{4}\right)(8)^2}{(20)^2} \times 6$

$= 0.7536$ cm from *O*.



15. (*d*) Centre of mass of complete disc should lie at point *O*. C_1 is the position of centre of mass of remaining portion and C_2 the position of centre of mass of the removed disc.

$\therefore$ x (Area of remaining portion) $= \dfrac{R}{2}$ (Area of removed disc)



$$\therefore \quad x\left[\pi R^2 - \frac{\pi R^2}{4}\right] = \frac{R}{2}\left[\frac{\pi R^2}{4}\right]$$

$$\therefore \quad x = \frac{R}{6}$$

Check point 8.2

1. (*c*) $p_i = 0$

$\therefore$ $p_f = 0$, *i.e.*, both the pieces should have equal and opposite momentum.

2. (*c*) Centre of mass does not change its path during explosion. Therefore, it will keep on falling vertically and will not shift horizontally. as $F_x = 0$

3. (*d*) Both the balls in air have acceleration g downwards. Hence the acceleration of their centre of mass will also be g downwards.

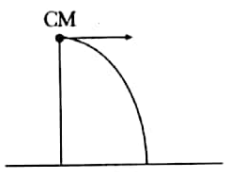
4. (*b*) Acceleration of centre of mass,

$$\mathbf{a}_{CM} = \frac{(m)(0) + (m)(\mathbf{a})}{m + m} = \frac{1}{2}\mathbf{a}$$

5. (*a*) $P_i = 0$

$\therefore$ $P_f = 0$ or centre of mass should be at rest at all instants.

6. (*d*) Vertical component of velocity of CM is zero. Horizontal component of velocity of CM is non-zero. Acceleration of CM is g downwards. Hence path is a parabola as shown in figure.



7. (*c*) $mv = \left(\dfrac{3}{4}m\right)v'$ $\quad\therefore\quad v' = \dfrac{4v}{3}$

8. (*b*) $p_3 = p_6$ or $3 \times 16 = 6 \times v$ or $v = 8\,\text{ms}^{-1}$. Therefore, the velocity of 6 kg mass is 8 ms^{-1}.

9. (*c*) Velocity of centre of mass,

$$v_{CM} = \frac{10(14) + 4(0)}{10 + 4} = 10\ \text{ms}^{-1}$$

10. (*b*) Net external force is zero. Hence, velocity of CM of the box and ball system will remain constant.

11. (*a*) In the absence of friction, centre of mass will not move in x-direction.

12. (*d*) $\mathbf{u}_{CM} = \dfrac{m_1\mathbf{u}_1 + m_2\mathbf{u}_2}{m_1 + m_2} = 0$

as $m_1\mathbf{u}_1 + m_2\mathbf{u}_2 = 0$ is given. Hence, velocity of composite system will be zero.

13. (*b*) Afer 1 s, coordinates of first particle will become $(4m, 4m, 6m)$ and coordinates of second particle will become $(6m, 4m, 8m)$.

$$\therefore \quad X_{CM} = \frac{4+6}{2} = 5\,\text{m}$$

$$Y_{CM} = \frac{4+4}{2} = 4\,\text{m} \quad \text{and} \quad Z_{CM} = \frac{6+8}{2} = 7\,\text{m}$$

Note In this problem answer is independent of the fact whether the first particle has velocity $\mathbf{v}_1$ or $\mathbf{v}_2$. Think why?

14. (*a*) Centre of mass will not move along y-axis.

or $\quad Y_{CM} = 0$ (always)

$$\because \quad Y_{CM} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2} \qquad \left\{\because m_1 = \frac{m}{4}, m_2 = \frac{3m}{4}\right\}$$

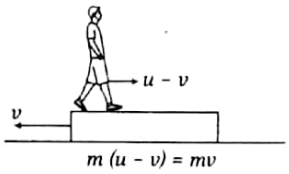
$$0 = \frac{(m/4)\,15 + (3m/4)\,(y_2)}{(m/4 + 3m/4)}$$

$$\Rightarrow \quad -\frac{15m}{4} = \frac{3m}{4}(y_2) \Rightarrow \boxed{y_2 = -5\text{ cm}}$$

15. (*d*) At $t = 0$ centre of mass is at mid-point or at (2.25m, 0). Velocity of centre of mass is zero. Hence centre of mass will remain at this position all the time.

16. (*b*) $m(u - v) = mv$

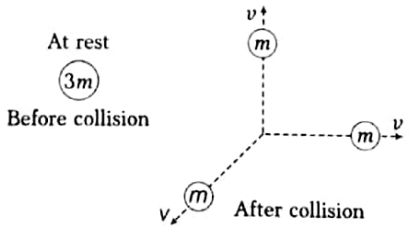
$$\therefore \quad v = \frac{u}{2}$$



$\therefore$ Velocity of man relative to ground $= u - v = \dfrac{u}{2}$

17. (*c*) Initial momentum of $3m$ mass $= 0$ ...(i)

Due to explosion, this mass splits into three fragments of equal masses.



Final momentum of system

$$= m\mathbf{v} + mv\,\hat{\mathbf{i}} + mv\,\hat{\mathbf{j}} \qquad \text{...(ii)}$$

From law of conservation of linear momentum

$$m\mathbf{v} + mv\,\hat{\mathbf{i}} + mv\,\hat{\mathbf{j}} = 0$$

$$\Rightarrow \quad \mathbf{v} = -v\,(\hat{\mathbf{i}} + \hat{\mathbf{j}})$$

18. (*b*) $\quad K_1 + K_2 = 2400 \qquad$ ...(i)

$$p_1 = p_2$$

$$\therefore \quad \sqrt{2K_1m_1} = \sqrt{2K_2m_2}$$

$$\text{or} \quad \frac{K_1}{K_2} = \frac{m_2}{m_1} = \frac{3}{1} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{K_1}{2400 - K_1} = \frac{3}{1}$$

$$\Rightarrow \quad 7200 - 3K_1 = K_1$$

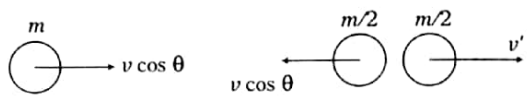
$$\Rightarrow \quad 4K_1 = 7200$$

$$\Rightarrow \quad K_1 = 1800$$

K_1 = Kinetic energy of smaller part = 1800 J

19. (*c*) In horizontal direction, net force on the system is zero. Therefore, principle of conservation of momentum can be applied for system $(m + M)$ horizontally.

20. (*a*) From conservation of linear momentum :



$$m\,(v\cos\theta) = \frac{m}{2}v' - \frac{m}{2}v\cos\theta$$

$$\therefore \quad v' = 3v\cos\theta$$

Check point 8.3

2. (*b*) Between elastic collision of two identical masses, velocities are interchanged. Therefore, after collision velocity of second body will be u.

3. (*c*) Net force on ball and earth system is zero.

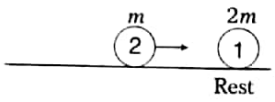
5. (*d*) By law of conservation of momentum

$$m_Av_A + m_Bv_B = m_Av_B + m_Bv_A$$

$$m_A(v_A - v_B) = m_B(v_A - v_B)$$

$$\Rightarrow \quad \frac{m_A}{m_B} = 1$$

6. (*c*) In perfectly elastic collision between two bodies of equal masses, velocities are exchanged. So, after collision, particle A will move with 10 ms^{-1} and particle B with 15 ms^{-1}



7. (*a*) x-direction $m \times 50 - m \times 40 = m \times 0 + mv_x$

$$\Rightarrow \quad v_x = 10\text{ cm/s}$$

y- direction, $m \times 0 = m \times 30 = m \times 0 + mv_y \Rightarrow v_y = 30$ cm/s

8. (*d*) From law of conservation of energy,

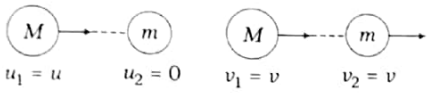
$$\frac{1}{2}mv^2 = \frac{1}{2} \times kx^2 \Rightarrow (0.5)(1.5)^2 = 50x^2$$

$$\Rightarrow \quad x^2 = \frac{1.125}{50} \Rightarrow x = 0.15$$

9. (*c*) $v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) u_2 + \frac{2m_1u_1}{m_1 + m_2}$

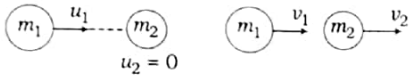
$$v = \frac{2Mu}{M + m} \qquad (\because u_2 = 0)$$

$$v = \frac{2u}{1 + \frac{m}{M}}$$



Before collision After collision

10. (*b*) If target is at rest, then final velocity of bodies are



Before collision After collision

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) u_1 \qquad ...(i)$$

and
$$v_2 = \frac{2m_1u_1}{m_1 + m_2} \qquad ...(ii)$$

From Eqs. (i) and (ii), $\frac{v_1}{v_2} = \frac{m_1 - m_2}{2m_1} = \frac{2}{5} \Rightarrow \frac{m_1}{m_2} = 5$

11. (*d*) $v'_2 = \left(\frac{m_2 - m_1}{m_2 + m_1}\right) v_2 + \left(\frac{2m_1}{m_1 + m_2}\right) v_1$

$$v_1 = 0$$

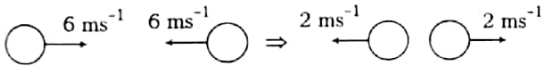
$$\therefore \quad \frac{v_2}{v'_2} = \left(\frac{m_2 + m_1}{m_2 - m_1}\right) = \left(\frac{m + 2m}{m - 2m}\right) = -3$$

$$\therefore \quad \frac{K_2}{K'_2} = \left(\frac{v_2}{v'_2}\right)^2 = 9$$

12. (*b*) $e = \frac{\text{Relative velocity of separation}}{\text{Relative velocity of approach}} = \frac{5 - 2}{8 - 2} = \frac{1}{2}$

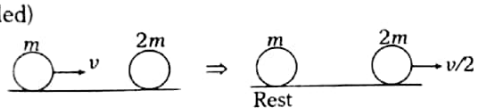
13. (*b*) Relative speed of approach is 12 ms^{-1}

$$e = \frac{1}{3}$$



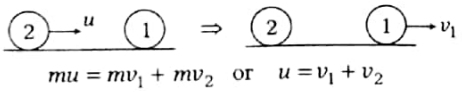
Therefore, relative speed of separation will be 4 ms^{-1}.

14. (*a*) From conservation of linear momentum, we can see that velocity of $2m$ will become $\frac{v}{2}$ after collision (as mass is doubled)



Now, $e = \frac{\text{Relative velocity of separation}}{\text{Relative velocity of approach}} = \frac{v/2}{v} = \frac{1}{2}$

15. (*b*) From conservation of linear momentum :



$$mu = mv_1 + mv_2 \quad \text{or} \quad u = v_1 + v_2 \qquad ...(i)$$

From definition of e,

$$v_1 - v_2 = eu \qquad ...(ii)$$

Solving these two equations, we get

$$v_1 = \left(\frac{1 + e}{2}\right) u \quad \text{and} \quad v_2 = \left(\frac{1 - e}{2}\right) u$$

$$\therefore \quad \frac{v_1}{v_2} = \left(\frac{1 + e}{1 - e}\right)$$

A Taking it together

1. (*b*) The situation given in the question is an example of perfectly inelastic collision in which a bullet after sticking to a block behaves as a single body

$$mv = (m + M)v'$$

$$\therefore \quad v' = \left(\frac{m}{m + M}\right) v$$

2. (*d*) $m_1v_1 = m_2v_2$ or $v_2 = \frac{m_1v_1}{m_2}$

$$v_2 = \frac{5 \times 10^{-3} \times 900}{2.5} = 1.8 \text{ ms}^{-1}$$

3. (*b*) $m_1v_1 + m_2v_2 = (m_1 + m_2)v$

$$\therefore \quad a \cdot b + c \cdot 0 = (a + c)v$$

$$\Rightarrow \quad v = \frac{ab}{a + c}$$

4. (*b*) Packet from A falls with greater momentum on B. Therefore, B is slightly accelerated.

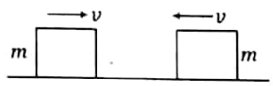
5. (*d*) Force of friction on A is backward and force of friction on B is forward. Net external force on the system is zero. Hence momentum of system will remain conserved. As the momentum of system is conserved, increase in momentum of B is equal to decrease in momentum of A.

6. (*d*) The centre of mass under the given condition may be at rest or may be moving with constant velocity.

7. (*d*) Due to the same mass of A and B as well as due to elastic collision, velocities of spheres get interchanged after the collision. So, A comes to rest and B moves with the velocity of A.

8. (*d*) If CM is at rest, it definitely means momentum of the system is constant. But momentum of the system is constant, it does not mean centre of mass is at rest.

9. (*d*) Kinetic energy of the system is zero, it definitely means momentum is zero. But momentum of the system is zero, it does not mean kinetic energy is zero.



10. (*d*) $K = \frac{p^2}{2m}$ and $p = \sqrt{2Km}$ are two standard results.

11. (*d*) $p^2 = 2Em$ or $p^2 \propto E$

i.e., p^2 *versus* E graph is a straight line passing through origin.

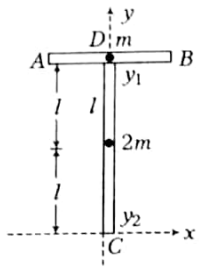
12. (*b*) Kinetic en $E = \frac{p^2}{2m} = \frac{(Ft)^2}{2m} = \frac{F^2t^2}{2m}$ (As $p = Ft$)

13. (*a*) m_1 $m_2 = 2m$

Taking D as origi

$$y_{cm} = \frac{0 + 2ml}{+ 2m} = \frac{2l}{3}$$

From C, $CP = 2l - \frac{2l}{3} = \frac{4l}{3}$



14. (*c*) $u = -v = 35$ m/s

Change in momentum of the ball

$$\Delta p = m(v - u) = \frac{3}{20}(-35 - 35)\ \text{kgms}^{-1}$$

$$= \frac{3}{20}(-70) = -\frac{21}{2}$$

We know that force $F = \frac{\Delta p}{\Delta t} = \frac{-212}{0.001}\ \text{N} = 1.05 \times 10^4$ N

Here, -ve sign shows that force will be opposite to the direction of movement of the ball before hitting.

15. (*b*) When two bodies of equal masses collides elastically, their velocities are interchanged.

When ball 1 collides with ball-2, then velocity of ball-1, v_1 becomes zero and velocity of ball-2, v_2 becomes v, *i.e.*, similarly.

$$v_1 = 0 \Rightarrow v_2 = v$$

when ball 2 collides will ball 3

$$v_2 = 0, v_3 = v$$

16. (*c*) When we are considering the two bodies as system the total external force on the system will be zero.

Hence, total linear momentum of the system remain conserved.

17. (*d*) We know that for a system

$$F_{ext} = \frac{dp}{dt} \quad \text{(from Newton's second law)}$$

If $F_{ext} = 0$, $dp = 0 \Rightarrow p = \text{constant}$

Hence, momentum of a system will remain conserve if external force on the system is zero.

In case of collision between particles equal and opposite forces will act on individual particles by Newton third law.

Hence, total force on the system will be zero.

Note We should not confuse with system and individual particles. As total force on the system of both particles is zero but force acts on individual particles.

18. (*d*) A bangle is in the form of a ring as shown in the adjacent diagram. The centre of mass lies at the centre, which is outside the body (boundary).

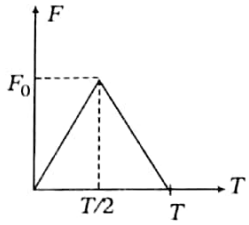
19. (*c*) Centre of mass of a system lies towards the part of the system, having bigger mass. In the above diagram, lower part is heavier, hence CM of the system lies below the horizontal diameter.

20. (*b*) $\mathbf{p}_i = \mathbf{p}_f$ (at highest point)

$$(3m)(100\hat{\mathbf{i}}) = m(100\hat{\mathbf{j}}) - m(100\hat{\mathbf{j}}) + m(\mathbf{v})$$

$\therefore$ $\mathbf{v} = 300\hat{\mathbf{i}}$ (of third part)

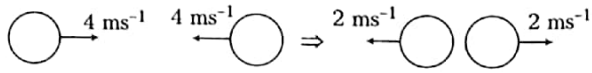
21. (*b*) In one dimensional elastic collision, between two equal masses velocities are interchanged. Therefore, change in linear momentum of any of the particle will be mu. Now impulse or area under F-T graph gives the change in linear momentum.



$\therefore$ $\frac{1}{2}F_0T = mu$

or $F_0 = \frac{2mu}{T}$

22. (*b*) Relative speed of approach is 8 ms^{-1}, $e = \frac{1}{2}$. Therefore, relative speed of separation will be 4 ms^{-1}.



23. (*a*) $h_n = he^{2n} = 32\left(\frac{1}{2}\right)^4 = \frac{32}{16} = 2$ m (here, $n = 2$, $e = 1/2$)

24. (*c*) $\Delta p = p_f - p_i = -mu - mu = -2mu$

but $|\Delta p| = 2mu$

25. (*c*) $\Delta\mathbf{p} = \mathbf{p}_f - \mathbf{p}_i = m(\mathbf{v}_f - \mathbf{v}_i)$

$$= m\left[\left(\frac{v}{\sqrt{2}}\hat{\mathbf{i}} - \frac{v}{\sqrt{2}}\hat{\mathbf{j}}\right) - \left(\frac{v}{\sqrt{2}}\hat{\mathbf{i}} + \frac{v}{\sqrt{2}}\hat{\mathbf{j}}\right)\right]$$

$$= -\sqrt{2}\,mv\,\hat{\mathbf{j}}$$

$\therefore$ $|\Delta\mathbf{p}| = \sqrt{2}\,mv$

26. (*c*) F = rate of change of linear momentum

In 1 s, $\frac{n}{60}$ bullets are embedded. Momentum of each bullet is mv

$$\therefore \quad F = \left(\frac{n}{60}\right) mv$$

27. (*d*) In 1 cm^2 area, 10^4 balls are striking per second. Therefore in 1 m^2 area, 10^8 balls will strike per unit time.

Change in momentum of each ball per second will be $2\,mu$.

$$\therefore \quad \text{Pressure} = \frac{F}{A} = \frac{(\Delta p / \Delta t)}{A}$$

$$= 10^8 \times 2 \times 10^{-3} \times 100 = 2 \times 10^7 \text{ Nm}^{-2}$$

28. (*d*) Their common velocity would be

when C strikes A, A acquires the velocity of C and C becomes stationary. A forces B to move and ultimately both acquires common velocity with spring compressed by x_0.

$$v = \frac{mv_0}{m + 2m} = \frac{v_0}{3}$$

Now applying conservation of mechanical energy

$$\frac{1}{2} mv_0^2 = \frac{1}{2} kx_0^2 + \frac{1}{2}(3m)\left(\frac{v_0}{3}\right)^2 \Rightarrow k = \frac{2}{3}\frac{mv_0^2}{x_0^2}$$

29. (*b*) $v' = \sqrt{2gR}$

From conservation of linear momentum,

$$mv = (M + m)v'$$

$$\therefore \quad v = \frac{M + m}{m} \cdot v' = \left(\frac{M + m}{m}\right)\sqrt{2gR}$$

30. (*a*) Relative speed of separation = relative speed of approach

$$= v$$

$$\therefore \text{Time of next collision} = \frac{2\pi r}{v}$$

31. (*a*) Velocity of block just after collision $= \sqrt{2gh}$

$$= \sqrt{2 \times 10 \times 0.2}$$

$$= 2 \text{ ms}^{-1}$$

Now applying conservation of linear momentum just before and just after collision.

$$0.02 \times 600 = 4 \times 2 + 0.02 \times v$$

$$\therefore \quad v = 200 \text{ ms}^{-1}$$

32. (*a*) Velocity of particle after 5 s

$$v = u - gt = 100 - 10 \times 5$$

$$= 100 - 50 = 50 \text{ ms}^{-1} \quad \text{(upwards)}$$

Conservation of linear momentum gives

$$Mv = m_1v_1 + m_2v_2 \quad \text{...(i)}$$

Taking upward direction positive,

$$v_1 = -25 \text{ ms}^{-1}, v = 50 \text{ ms}^{-1}$$

$$M = 1\text{kg}, m_1 = 400 \text{ g} = 0.4 \text{ kg}$$

$$m_2 = M - m_1 = 1 - 0.4 = 0.6 \text{ kg}$$

From Eq. (i),

$$1 \times 50 = 0.4 \times (-25) + 0.6 v_2$$

or $\quad v_2 = 100 \text{ ms}^{-1} \quad$ (upwards)

33. (*b*) Velocity on hitting the surface $= \sqrt{2 \times 9.8 \times 4.9}$

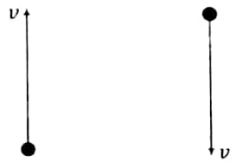
$$= 9.8 \text{ ms}^{-1}$$

Velocity after first bounce, $v = \frac{3}{4} \times 9.8$

Time taken from first bounce to the second bounce $= \frac{2v}{g}$

$$= 2 \times \frac{3}{4} \times 9.8 \times \frac{1}{9.8} = 1.5 \text{ s}$$

34. (*b*) $\mathbf{F} = \frac{\Delta \mathbf{p}}{\Delta t} = \frac{\Delta (mn\,\mathbf{v})}{\Delta t}$



$|\Delta v| = 2\,v$

Here, m = mass of one marble = 5 g

$\frac{n}{\Delta t}$ = number of molecules striking per second = 10

$$\therefore \quad Mg = m\left(\frac{n}{\Delta t}\right)|\,\Delta \mathbf{v}\,|$$

Here M = mass of disc

$$(10 \times 10^{-3})(9.80) = (5 \times 10^{-3})(10)\,2v$$

or $\quad v = 0.98 \text{ ms}^{-1}$

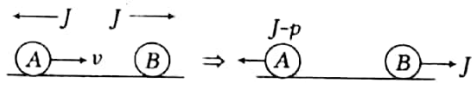
35. (*d*) $F = \frac{\Delta p}{\Delta t} = n(mv)$

Here, n = number of bullets fired per second.

$$\therefore \quad n = \frac{F}{mv} = \frac{144}{0.04 \times 1200} = 3$$

36. (*c*) $X_{CM} = \frac{\int X\,dm}{\int dm} = \frac{\int_0^L (Ax) \cdot x \cdot dx}{\int_0^L (Ax)\,dx} = \frac{2L}{3}$

37. (*a*) $e = \frac{RSOS}{RSOA} = \frac{J + (J - p)}{p} = \frac{2J}{p} - 1$



38. (*d*) $m(3\hat{\mathbf{i}} + 2\hat{\mathbf{j}}) = m(-2\hat{\mathbf{i}} + \hat{\mathbf{j}}) + M\mathbf{v}$ (Put $M = 13\,m$)

$$\therefore \quad \mathbf{v} = \frac{(5\hat{\mathbf{i}} + \hat{\mathbf{j}})}{13}$$

$$\mathbf{J}_m = \mathbf{p}_f - \mathbf{p}_i \text{ of } m$$

$$= [(-2\hat{\mathbf{i}} + \hat{\mathbf{j}}) - (3\hat{\mathbf{i}} + 2\hat{\mathbf{j}})]$$

$$\therefore \quad \mathbf{J}_m = m(5\hat{\mathbf{i}} + \hat{\mathbf{j}})$$

39. (*d*) From conservation of linear momentum, velocity of combined mass just after collision will be 50 cms^{-1} as mass has doubled.

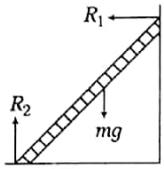
Now, $\quad H = \frac{u^2}{2g} = \frac{(0.5)^2}{20} \text{ m} = 1.25 \text{ cm}$

40. (*d*) $a_{CM} = \dfrac{\text{External force}}{\text{Total mass}}$

$= \dfrac{\text{Force of friction from ground}}{\text{Total mass}}$

$= \dfrac{0.2 \times (2+1)(10)}{1+2} = 2\ \text{ms}^{-2}$

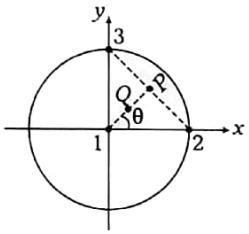
41. (*a*)



Due to net force in downward direction and towards left centre of mass will move downward and will shift horizontally.

42. (*b*) Area of circle, $\frac{\pi}{4}a^2 = A_1$, area of square $= a^2 = A_2$. Since, $A_2 > A_1$ centre of mass will lie inside the square plate.

43. (*a*) P is the position of centre of mass of particle at 2 and 3. Q is position of centre of mass of all three particles.



$$\tan\theta = \frac{y_{CM}}{x_{CM}} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1x_1 + m_2x_2 + m_3x_3}$$

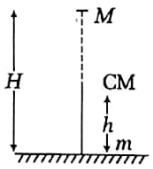
$$= \frac{6\times 0 + 2\times 0 + 2\times a}{6\times 0 + 2\times a + 2\times 0} = 1$$

or $\theta = 45°$

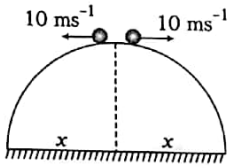
44. (*a*) Centre of mass will remain at height h.

$\therefore \quad h_{CM} = \dfrac{m\times 0 + MH}{m+M} = h$

$\therefore \quad H = h\left(1 + \dfrac{m}{M}\right)$



45. (*a*) Remaining time for the pieces to reach the ground will be,



$$t = \sqrt{\frac{2\times 45}{10}} - \sqrt{\frac{2\times 20}{10}} = 1\,\text{s}$$

$x = 10\times 1 = 10$ m

$\therefore$ Distance between two pieces will be 20 m.

46. (*d*) $y_{CM} = \dfrac{m_1y_1 + m_2y_2}{m_1 + m_2}$

or $+1 = \dfrac{(10)(7) + (30)\,y_2}{10 + 30}$

$\therefore \quad y_2 = -1$ cm

47. (*d*) Distance between bullet and block at this instant is $(D - d)$

Distance of CM from bullet $= \dfrac{M}{M+m}(D-d)$

And distance of CM from block $= \dfrac{m}{M+m}(D-d)$

48. (*a*) During explosion of a projectile, path of centre of mass does not change. At height 1 m,

$v = \sqrt{u^2 - 2gh}$ (from conservation of mechanical energy)

$= 4\sqrt{5}\ \text{ms}^{-1}$

49. (*a*) Velocity of block just after collision will be

$$v = \frac{5\times 10^{-3}\times 150}{(2 + 5\times 10^{-3})}$$

(from conservation of linear momentum)

$= 0.374\ \text{ms}^{-1}$

Let, F be the force of friction. Then, work done against friction = initial kinetic energy

or $\quad F\times 2.7 = \frac{1}{2}\times 2.005\times(0.374)^2$ or $F = 0.052$ N

50. (*a*) Let initial kinetic energy, $E_1 = E$

Final kinetic energy, $E_2 = E + 300\%$ of $E = 4E$

As $p \propto \sqrt{E}$

$\Rightarrow \quad \dfrac{p_2}{p_1} = \sqrt{\dfrac{E_2}{E_1}} = \sqrt{\dfrac{4E}{E}} = 2 \Rightarrow \pi_2 = 2\pi_1$

$\Rightarrow \quad p_2 = p_1 + 100\%$ of p_1

i.e. Momentum will increase by 100%.

51. (*c*) Let $p_1 = p$, $p_2 = p_1 + 50\%$ of $p_1 = p_1 + \dfrac{p_1}{2} = \dfrac{3p_1}{2}$

$E \propto p^2$

$\Rightarrow \quad \dfrac{E_2}{E_1} = \left(\dfrac{p_2}{p_1}\right)^2 = \left(\dfrac{3p_1/2}{p_1}\right)^2 = \dfrac{9}{4}$

$\Rightarrow \quad E_2 = 2.25E_1 = E_1 + 1.25E_1$

$\therefore \quad E_2 = E_1 + 125\%$ of E_1

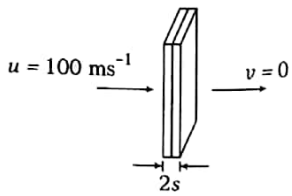
i.e. Kinetic energy will increase by 125%.

52. (*a*) $p = \sqrt{2mE} \;\therefore\; p \propto \sqrt{E}, \dfrac{dp}{p} = \dfrac{1}{2}\dfrac{dk}{k}$

Percentage increase in $p \approx \dfrac{1}{2}$ (percentage increase in E)

$= \dfrac{1}{2}(0.1\%) = 0.05\%$

53. (*b*) Let the thickness of each of plank be s. If the initial speed of bullet is 100 ms^{-1} then it stops by covering a distance $2s$.



By applying $v^2 = u^2 - 2as$

$\Rightarrow \quad 0 = u^2 - 2as$

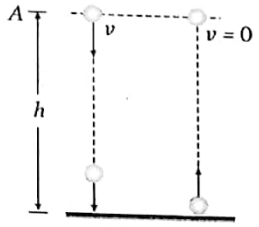
$s = \dfrac{u^2}{2a}$ or $s \propto u^2$ (If retardation is constant)

If the speed of the bullet is double, then bullet will cover four times distance before coming to rest

i.e. $s_2 = 4(s_1) = 4(2s) \Rightarrow s_2 = 8s$

So, number of planks required = 8

54. (*a*) Let ball is projected vertically downward with velocity v from height h.



Total energy at point $A = \frac{1}{2}mv^2 + mgh$

During collision, loss of energy is 50% and the ball rise up to same height. It means it possesses only potential energy at same level.

$$50\%\left(\frac{1}{2}mv^2 + mgh\right) = mgh$$

$$\frac{1}{2}\left(\frac{1}{2}mv^2 + mgh\right) = mgh$$

$$v = \sqrt{2gh} = \sqrt{2 \times 10 \times 20}$$

$$\therefore \quad v = 20 \text{ ms}^{-1}$$

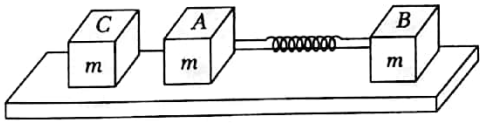
55. (*d*) Loss in PE = gain in KE

$$mgh_1 = \frac{1}{2}mv^2$$

$$mg \times \frac{3}{4}h = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{3gh}{2}}$$

Now, impulse imparted, $J = 2mv = 2m\sqrt{\frac{3gh}{2}} = m\sqrt{6gh}$

56. (*a*) After striking with A, the block C comes to rest and block A moves with velocity v, when compression in spring is maximum, both A and B will be moving with common velocity v.



From law of conservation of linear momentum

$$mv = (m + m)V \Rightarrow V = \frac{v}{2}$$

From law of conservation of energy,

KE of block C = KE of system + PE of system

$$\frac{1}{2}mv^2 = \frac{1}{2}(2m)V^2 + \frac{1}{2}kx^2$$

$$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{1}{2}(2m)\left(\frac{v}{2}\right)^2 + \frac{1}{2}kx^2$$

$$\Rightarrow \quad kx^2 = \frac{1}{2}mv^2$$

$$\Rightarrow \quad x = v\sqrt{\frac{m}{2k}}$$

57. (*b*) Let plank moves x distance in opposite direction. Then, displacement of man relative to ground will be, $(L - x)$. Applying

$$m_R x_R = m_L x_L \text{ or } M(L - x) = \frac{M}{3}x$$

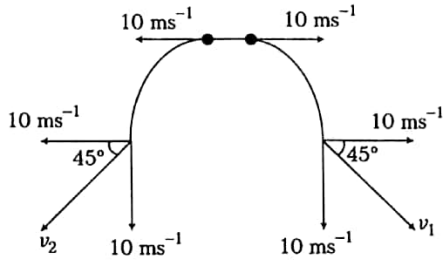
Solving this equation we get, $x = \frac{3L}{4}$

$\therefore$ Displacement of man relative to ground $= L - \frac{3L}{4} = \frac{L}{4}$

58. (*b*) $v_r = \frac{l}{t}$. Now, $m(v_r - v) = Mv$ (v = speed of plank)

$$\therefore \quad v = \frac{mv_r}{M + m} = \frac{l}{t}\left(\frac{m}{M + m}\right)$$

59. (*d*) At 1 s $\mathbf{v}_1 \cdot \mathbf{v}_2 = 0$



60. (*c*) Velocity of A just before collision

$$= \sqrt{2gh} = \sqrt{2 \times 10 \times 1.25} = 5 \text{ ms}^{-1}$$

Velocity of $(A + B)$ just after collision $= \frac{5}{2} = 2.5 \text{ ms}^{-1}$

(From conservation of linear momentum)

In elastic collision between two bodies of equal masses velocities are interchanged.

Hence, velocity of C will become 2.5 ms^{-1}.

61. (*b*) From conservation of linear momentum

$$m_1 v_1 = m\sqrt{5gl} + m_1\frac{v_1}{3} \text{ or } v_1 = \frac{3}{2}\frac{m}{m_1}\sqrt{5gl}$$

62. (*d*)

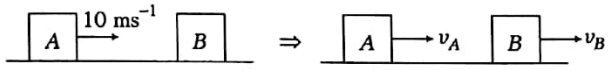
$$X_{CM} = \frac{mx + 2mx_{2m}}{m + 2m}$$

$$\therefore \quad \frac{1}{2}a_{CM}t^2 = \frac{x + 2x_{2m}}{3}$$

$$\therefore \quad \frac{3}{2}\left(\frac{F}{3m}\right)t^2 = x + 2x_{2m}$$

$$\therefore \quad X_{2m} = \frac{Ft^2}{4m} - \frac{x}{2}$$

63. (*a*) Between A and B



$$m \times 10 = mv_A + mv_B \quad \text{...(i)}$$

$$e = \frac{1}{2} = \frac{v_B - v_A}{10}$$

$$\text{or} \quad v_B - v_A = 5 \quad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get, $v_B = 7.5\ \text{ms}^{-1}$

Hence A has given 75% of its speed to B and will also transfer its 75% speed to C.

$$\therefore \quad v_C = \frac{75}{100} \times 7.5 = 5.625\ \text{ms}^{-1}$$

64. (*d*) $2\pi R = 2\pi \ \therefore \ R = 1$

$$y_{CM} = \frac{m \times 0 + m \times 1 + m \times 1}{m + m + m} = \frac{2}{3},$$

$$x_{CM} = \frac{m(\pi) + m(0) + m(2\pi)}{m + m + m} = \pi$$

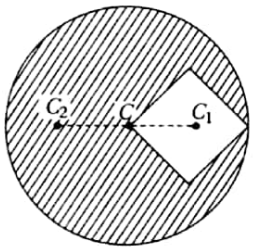
65. (*b*) The centre of mass of the object must lie on the line segment joining (0, 0) and $(R/2, R/2)$. Here (0, 0) is the centre of mass of the ring and $(R/2, R/2)$ is the centre of mass of the chord.

66. (*b*) Centre of mass of whole system was at point O. Hence,

x_2 (area of remaining portion) = c (area of removed disc)

$$\therefore \quad x_2(\pi a^2 - \pi b^2) = c(\pi b^2) \Rightarrow x_2 = \frac{cb^2}{a^2 - b^2}$$

67. (*a*) $A_1(CC_1) = A_2(CC_2)$

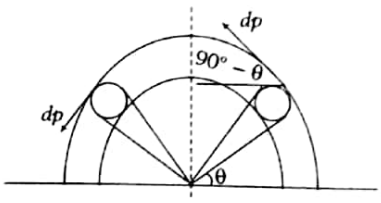


side of square will be $\frac{R}{\sqrt{2}}$

$$\therefore \quad CC_2 = \frac{A_1}{A_2}(CC_1) = \frac{(R/\sqrt{2})^2}{\pi R^2 - (R/\sqrt{2})^2}\left(\frac{R}{2}\right) = \left(\frac{R}{4\pi - 2}\right)$$

68. (*a*) Total distance travelled by the ball before it stops bouncing is $H = h\left[\frac{1+e^2}{1-e^2}\right]$.

69. (*b*) $p_{net} = \int dp \sin\theta = \int_0^\pi \left(\frac{M}{\pi} \cdot d\theta\right) v \cdot \sin\theta = \frac{2Mv}{\pi}$



From $COLM$ & $e = 1$

70. (*b*) $v_A = \left(\frac{M-m}{M+m}\right)u$ and $v_B = \frac{2Mu}{M+m}$

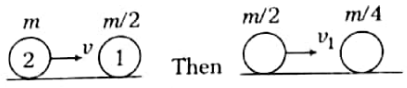
Velocity of C at maximum compression $= v_B/2$

$$v_C = \frac{1}{2}\left[\frac{2M}{M+m}\right]u = \left(\frac{M}{M+m}\right)u$$

$$\therefore \quad v_{CA} = v_C - v_A = \left(\frac{m}{M+m}\right)u$$

71. (*a*) In head on elastic collision

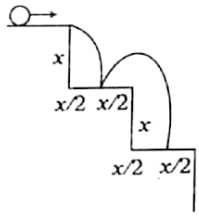
$$v'_1 = \left(\frac{m/2 - m}{m/2 + m}\right)(0) + \left[\frac{2(m)}{m/2 + m}\right](v) = \frac{4}{3}v$$

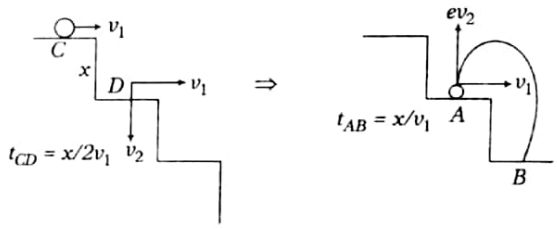


Finally, $v'_n = \left(\frac{4}{3}\right)^{n-1} \cdot v = \sqrt{5gr}$

$$\therefore \quad v = \left(\frac{3}{4}\right)^{n-1}\sqrt{5gr}$$

72. (*b*)





C to D $v_2 = \sqrt{2gx} = gt$ $\quad \therefore t = \frac{v_2}{g}$

From A to B, time will become two times.

Applying $S = ut + \frac{1}{2}at^2$ in vertical direction, we have

$$-x = (ev_2)(2t) - \frac{1}{2} \times g \times (2t)^2$$

$$-x = \frac{2ev_2^2}{g} - \frac{2v_2^2}{g} \Rightarrow -x = 2e(2x) - 2(2x)$$

$$\therefore \quad e = \frac{3}{4}$$

73. (*b*) $T = \frac{d}{v/\sqrt{2}} + \frac{d}{ev/\sqrt{2}} = \left(1 + \frac{1}{e}\right)\frac{\sqrt{2}d}{v}$

or $\frac{2v/\sqrt{2}}{g} = \left(1 + \frac{1}{e}\right)\frac{\sqrt{2}d}{v}$

or $e = \frac{gd}{v^2 - gd}$

B Medical entrance special format questions

Assertion and reason

2. (*b*) In elastic collision magnitude of relative velocity of separation = magnitude of relative velocity of approach.

4. (*d*) Momentum is always constant during any type of collision.

5. (*a*) $K = \frac{p^2}{2m}$

As momentum is equal and opposite.

$\therefore \quad K \propto \frac{1}{m}$

6. (*c*) Atmosphere is absent on the surface of moon. Gravit , present.

7. (*d*) It will fall at the same point.

8. (*a*) Path of centre of mass remains unchanged because during explosion no external force acts on COM.

9. (*c*) No horizontal force acts.

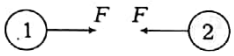
10. (*d*) Centre of mass remains stationary.

11. (*d*) $e = \frac{|\text{RVOS } i.e., \text{ Relative velocity of separation}|}{|\text{RVOA } i.e., \text{ Relative velocity of approach}|}$

In elastic collision $e = 1$

$\therefore \quad |\text{RVOS}| = |\text{RVOA}|$

12. (*d*) Two bodies are released from rest in space. Net force on the system is zero. Momentum of system is constant. But momentum of individual body is not constant. Further, kinetic energy of system is also increasing.



13. (*d*) Gravity forces act during explosion.

14. (*d*) Only in case of perfectly inelastic collision, they will come to rest.

18. (*a*) Linear momentum will not remain constant till spring will remain compressed. Therefore a force will act on block A from the wall.

19. (*a*) Force of friction will act on B towards right (from ground).

20. (*b*) $v_{CM} = \frac{m_A(+v) + m_B(-v)}{m_A + m_B} = -\text{ve}$

Both A and B will have same acceleration (μg) on A towards left and on B towards right. Since, B have more mass so acceleration of CM will be towards right.

21. (*c*) We know that centre of mass of half disc depends only on radius and not only the density of the material of disc similarly in this case centre of mass of half filled sphere will depends only on radius and not on density of liquid inside. Since both sphere are of same radius so both have CM at the same level.

Match the columns

2. Applying $X_{CM} = \frac{m_1x_1 + \ldots + m_nx_n}{m_1 + \ldots + m_n}$

And $\quad Y_{CM} = \frac{m_1y_1 + \ldots + m_ny_n}{m_1 + \ldots + m_n}$

3. When $e = 1$, collision is elastic and equal masses exchange their velocities.

For $e = 0$, collision is perfectly inelastic. Hence, velocity of each will remain half.

In the last case, when $v_2 = \frac{3}{4}v$,

Then, $v_1 = \frac{v}{4}$ (from conservation of momentum)

$$\therefore \quad e = \frac{\frac{3v}{4} - \frac{v}{4}}{v} = \frac{1}{2}$$

4. $\mathbf{F}_{CM} = \mathbf{F}_1 + \mathbf{F}_2 = m_1a_1 + m_2a_2 = (2\hat{\mathbf{i}} + 8\hat{\mathbf{j}})$

$\therefore \quad |\mathbf{F}_{CM}| = \sqrt{4 + 64}$

$\Rightarrow \quad |\mathbf{F}_{CM}| = \sqrt{68}$ unit

$$\mathbf{v}_{CM} = \frac{m_1\mathbf{v}_1 + m_2\mathbf{v}_2}{m_1 + m_2}$$

$$= \frac{(1)(4\hat{\mathbf{i}}) + (2)(4\hat{\mathbf{j}})}{3} = \frac{4\hat{\mathbf{i}} + 8\hat{\mathbf{j}}}{3}$$

$\therefore \quad |\mathbf{v}_{CM}| = \frac{1}{3}\sqrt{16 + 64} = \frac{\sqrt{80}}{3}$ unit

$$\mathbf{s}_1 = \int_0^2 \mathbf{v}_1\, dt = (4\hat{\mathbf{i}})$$

$$\mathbf{s}_2 = \int_0^2 \mathbf{v}_2\, dt = \left(\frac{8}{3}\hat{\mathbf{j}}\right)$$

Now, $\quad \mathbf{s}_{CM} = \frac{m_1\mathbf{s}_1 + m_2\mathbf{s}_2}{m_1 + m_2}$

$$= \frac{(1)(4\hat{\mathbf{i}}) + 2\left(\frac{8}{3}\hat{\mathbf{j}}\right)}{3} = \left(\frac{4}{3}\hat{\mathbf{i}} + \frac{16}{9}\hat{\mathbf{j}}\right)$$

$\therefore \quad |\mathbf{s}_{CM}| = \sqrt{\frac{16}{9} + \frac{256}{81}} = \frac{20}{9}$

5. $v_{CM} = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} = 1\,\text{ms}^{-1}$

During maximum compression also, velocity of combined mass is 1 ms^{-1}.

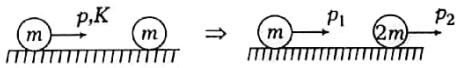
Now, $\quad U_{max} = K_i - K_f$

$$= \frac{1}{2} \times 2(2)^2 - \frac{1}{2} \times 4 \times (1)^2 = 2\text{J}$$

From $\quad \frac{1}{2}KX_{max}^2 = 2\text{J}$

We have, $\quad X_{max} = 1\,\text{m}$

6. $p_1 + p_2 = p$ ...(i)



Further, $\quad K_1 + K_2 = K$

or $\quad \frac{p_1^2}{2m} + \frac{p_2^2}{4m} = \frac{p^2}{2m}$

or $\quad 2p_1^2 + p_2^2 = 2p^2$ ...(ii)

Solving Eqs. (i) and (ii), we get

$$p_2 = \frac{4}{3}p \quad \text{and} \quad p_1 = -\frac{p}{3}, K_1 = \frac{K}{9}$$

and $\quad K_2 = \frac{8K}{9}$

7. From $a_{CM} = \dfrac{m_1a_1 + m_2a_2}{m_1 + m_2}$

$$= \frac{(1)(-10) + 2(-10)}{3} = -10 \text{ ms}^{-2}$$

$$u_{CM} = \frac{m_1u_1 + m_2u_2}{m_1 + m_2}, \text{ after } 2s$$

$$= \frac{(1)(40) + (2)(20)}{3} = \frac{40 + 40}{3} = \frac{80}{3}$$

8. (a) $K = \dfrac{p^2}{2m}$

$$\Rightarrow K' = \frac{(3p)^2}{2m} = 9\left(\frac{p^2}{2m}\right) = 9K$$

$\therefore$ % increase in $K = 800\%$

(b) $p = \sqrt{2Km}$

$$p' = \sqrt{2(4K)m} = 2\sqrt{2Km} = 2p$$

$\therefore$ % increase in $p = 100\%$

(c) $p = \sqrt{2Km} = (2Km)^{1/2}$

$$\Rightarrow K' = \left(\frac{101}{100}\right)^2 \frac{p}{2m} = 2\%$$

(d) % increase in $p = \dfrac{1}{2}$ (% increase in K) $= 0.5\%$

9. $v_{CM} = \dfrac{m_1v_1 + m_2v_2}{m_1 + m_2} = \dfrac{(1)(10) + (2)(-5)}{3} = 0$

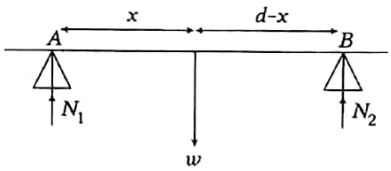
Similarly, $p_{CM} = 0$

Net force on the system is zero, hence, V_{CM} and p_{CM} will remain constant.

Velocity of 1 kg and 2 kg blocks keep on changing initially and finally both of them stop as V_{CM} was zero.

C) Medical entrances' gallery

1. (*d*) As the weight w balances the normal reactions.



So, $w = N_1 + N_2$ ...(i)

Now, balancing torque about the COM,

i.e. anti-clockwise momentum = clockwise momentum

$\Rightarrow N_1x = N_2(d - x)$

Putting the value of N_2 from Eq. (i), we get

$$N_1x = (w - N_1)(d - x)$$

$$\Rightarrow N_1x = wd - wx - N_1d + N_1x$$

$$\Rightarrow N_1d = w(d - x)$$

$$\Rightarrow N_1 = \frac{w(d - x)}{d}$$

2. (*c*) Total initial energy $= \dfrac{1}{2}m_1u_1^2 + \dfrac{1}{2}m_2u_2^2$

Since, after collision one particle absorb energy ε.

$\therefore$ Total final energy $= \dfrac{1}{2}m_1v_1^2 + \dfrac{1}{2}m_2v_1^2 + \varepsilon$

From conservation of energy,

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \varepsilon$$

$$\Rightarrow \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 - \varepsilon = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

3. (*b*) Applying the conservation of momentum to the system

$$mgh = \frac{1}{2}kx^2$$

Given, $m = 0.04$ kg, $h = 5$ m, $k = 400$ Nm^{-1}

and x = deformation (compression) in the spring

$$\Rightarrow x = \sqrt{\frac{2mgh}{k}} = \sqrt{\frac{2 \times 0.04 \times 10 \times 5}{400}}$$

$$= \frac{1}{10} \text{ m} = 10 \text{ cm} \approx 9.8 \text{ cm}$$

4. (*a*) Let the velocity of block of mass $2m$ after the collision be v', then from conservation of momentum

$$mv = 2mv' \Rightarrow v' = \frac{v}{2}$$

Now, the coefficient of restitution,

$$e = \frac{\text{Velocity of separation}}{\text{Velocity of approach}} = \frac{v'}{v} = \frac{v/2}{v} = \frac{1}{2} = 0.5$$

5. (*b*) Let spring be compressed upto a compression x then, from energy conservation, $mgh = \dfrac{1}{2}kx^2$

$$10 \times 10^{-3} \times 10 \times 10 = \frac{1}{2} \times 200 \times x^2$$

$$\Rightarrow x^2 = 10 \times 10^{-3} = 10^{-2}$$

$$\Rightarrow x = \sqrt{\frac{1}{100}} = \frac{1}{10} \text{ m} = 0.1 \text{ m}$$

Thus, compression in the spring is 0.1m

6. (*c*) Let the speed of the board be u and frog jumps with angle of inclination to the board θ, then from conservation of momentum in horizontal direction.

$$mv\cos\theta - mu = 0$$

$$u = v\cos\theta \quad \text{...(i)}$$

Let distance moved by board be x

So, $L - x = ut$ ...(ii)

and $x = v\cos\theta\, t$ ...(iii)

Solving above equations, we get

$$x = \frac{L}{2}$$

Also, $x = \dfrac{v^2 \sin 2\theta}{g}$

$$\Rightarrow \quad \frac{L}{2} = \frac{v^2 \sin 2\theta}{g} \quad \Rightarrow \quad v = \sqrt{\frac{gL}{2\sin 2\theta}}$$

Hence, v should be minimum for $\sin 2\theta = 1$ (*i.e.*, maximum)

$$\Rightarrow \quad v_{min} = \sqrt{\frac{gL}{2}} = \sqrt{10 \times \frac{10}{2}} = \sqrt{50} = 5\sqrt{2}\ \text{ms}^{-1}$$

7. (*b*) As net horizontal force acting on the system is zero, hence, momentum must remain conserved.

Hence, $mu + 0 = 0 + Mv_2$

$$v_2 = \frac{mu}{M}$$

As per definition, $e = \frac{|(v_2 - v_1)|}{|(u_2 - u_1)|} = \frac{|v_2 - 0|}{|0 - u|}$

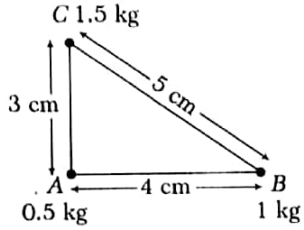
$$= \frac{v_2}{u} = \frac{\frac{mu}{M}}{u} = \frac{m}{M}$$

8. (*c*) When a body falls from height h, it strikes the ground with a velocity $v = \sqrt{2gh}$. Let it rebounced with a velocity v and rise to a height h_1.

$$v = \sqrt{2gh_1} \quad \Rightarrow \quad e = \frac{v}{u} = \sqrt{\frac{h_1}{h}}$$

Clearly, $h_1 = e^2 h$

9. (*a*) The figure given in question is



Let X and Y-axes be along AB and AC respectively, the coordinates of centre of mass would be

$$X_{CM} = \frac{0.5 \times 0 + 1 \times 4 + 1.5 \times 0}{1.5 + 1 + 0.5} = \frac{4}{3} = 1.33$$

and $$Y_{CM} = \frac{1.5 \times 3 + 0.5 \times 0 + 1 \times 0}{1.5 + 1 + 0.5} = \frac{1.5 \times 3}{3} = 1.5$$

Hence, $(X, Y) = (1.3\ ,\ 1.5)$

10. (*b*) As large number of particles is situated at a distance R from the origin. If particles are uniformly distributed and make a circular boundary around the origin, then centre of mass will be at the origin.

While, if the particles are not uniformly distributed, then centre of mass will lie between particle and origin. This implies that the distance between centre of mass and origin is always less than or equal to R.

11. (*d*) In this case (pure rolling), the velocity of point of contact is zero.

12. (*d*) Given, $p = a + bt + ct^2$

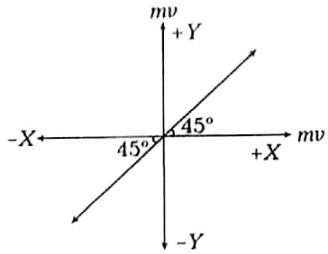
Differentiating with respect to t (time), we get

$$\frac{dp}{dt} = 0 + b + 2ct$$

From Newton's second law of motion, $F \propto \frac{dp}{dt}$

$\Rightarrow$ $F \propto t$, force is dependent linearly on time.

13. (*b*)



$\sqrt{2}(mv)$ = Resultant momentum of two small masses

$$\sqrt{2}(mv) = (2m) \times v' \Rightarrow v' = \frac{v}{\sqrt{2}}$$

So, total kinetic energy generated by the explosion

$$= \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}(2m)v'^2$$

$$= mv^2 + m \times \left(\frac{v}{\sqrt{2}}\right)^2$$

$$= mv^2 + \frac{mv^2}{2} = \frac{3}{2}mv^2$$

14. (*e*) The position of centre of mass of a system of particles does not depend upon the nature of particles.

$$r_{cm} = \frac{\Sigma m_i r_i}{\Sigma m_i}$$

15. (*c*) After firing, the momentum of gun and bullet is same. Therefore, by the relation,

$$K = \frac{p^2}{2m},$$

we have $K \propto \frac{1}{m}$ (as p is same)

Hence, as the mass of gun is greater, its kinetic energy will be loss.

16. (*b*) We have, $\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 = 0$ [$\because p = mv$]

$$\therefore \quad 1 \times 12\,\hat{\mathbf{i}} + 2 \times 8\hat{\mathbf{j}} + \mathbf{p}_3 = 0$$

$$\Rightarrow \quad 12\hat{\mathbf{i}} + 16\hat{\mathbf{j}} + \mathbf{p}_3 = 0 \Rightarrow \mathbf{p}_3 = -(12\hat{\mathbf{i}} + 16\hat{\mathbf{j}})$$

$$\therefore \quad \mathbf{p}_3 = \sqrt{(12)^2 + (16)^2} = \sqrt{144 + 256}$$

$$= 20\ \text{kg-ms}^{-1}$$

Now, $\mathbf{p}_3 = m_3 v_3 \Rightarrow m_3 = \frac{\mathbf{p}_3}{v_3} = \frac{20}{4} = 5\ \text{kg}$

17. (*c*) In all types of collision, total linear momentum of colliding particles remains conserved.

18. (*d*) Given, mass = m

and here $r_{CM} = \frac{ma\,\hat{\mathbf{i}} + mb\,\hat{\mathbf{j}} + m0}{3m}$

$$\Rightarrow \quad r_{CM} = \frac{1}{3}(a\hat{\mathbf{i}} + b\hat{\mathbf{j}})$$

19. (*b*) From the conservation of momentum

$$mv + m \times 0 = (m + m)v'$$

$$\Rightarrow \quad v' = \frac{mv}{(m+m)} \Rightarrow v' = \frac{v}{2}$$

and we know, $h = \frac{u^2}{2g}$ $\quad [\because v^2 - u^2 = 2gh]$

Here, $u = v'$

So, $h = \frac{(v')^2}{2g} \Rightarrow \frac{v^2}{8g}$ $\quad \left\{\because v' = \frac{v}{2}\right\}$

20. (*b*) In inelastic collision, kinetic energy before collision is not equal to kinetic energy after collision. But the linear momentum conserved in all collision.

21. (*b*) Initial momentum of the system

$$= m_1 \times 40 + m_2 \times 0 = 40\, m_1$$

Final momentum of the system,

$$= (m_1 + m_2) \times 30$$

By the law of conservation of momentum,

Initial momentum = Final momentum

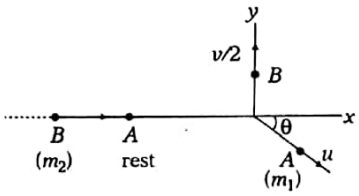
$$40m_1 = (m_1 + m_2)30$$

$$40\, m_1 - 30\, m_1 = 30\, m_2$$

$$10\, m_1 = 30\, m_2$$

$$\frac{m_1}{m_2} = \frac{3}{1}$$

22. (*d*) Here, $\mathbf{p}_i = m_2 v\hat{\mathbf{i}} + m_1 \times 0$



$$\mathbf{p}_f = m_2 v/2\,\hat{\mathbf{j}} + m_1 \times \mathbf{v}_1$$

Law of conservation of momentum

$$\mathbf{p}_i = \mathbf{p}_f$$

$$m_2 v\hat{\mathbf{i}} = m_2 \frac{v}{2}\hat{\mathbf{j}} + m_1 \times \mathbf{v}_1$$

$$v_1 = \frac{m_2}{m_1} v\hat{\mathbf{i}} - \frac{m_2}{m_1}\frac{v}{2}\hat{\mathbf{j}}$$

From this equation, we can find

$$\tan\theta = \frac{1}{2}$$

$$\theta = \tan^{-1}\left(\frac{1}{2}\right) \text{ to the } X\text{-axis.}$$

23. (*c*) Here on the entire system net external force on the system is zero, hence centre of mass remains unchanged.

24. (*a*) The given condition can be shown as

After collision,

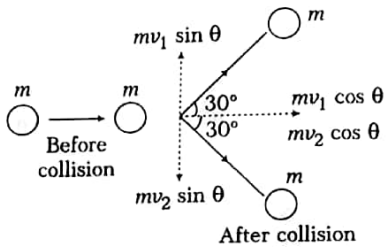
$$mv_1 \sin\theta = mv_2 \sin\theta$$

$$\Rightarrow \quad v_1 = v_2 = v$$

$$\therefore \quad 2\,mv \cos\theta = 9$$

$$\Rightarrow \quad 2mv \cos 30° = 9$$

$$\Rightarrow \quad v = \frac{9}{\sqrt{3}} = 3\sqrt{3} = 5.2 \text{ ms}^{-1}$$



25. (*d*) Using law of conservation of momentum, we get

$$100 \times v = 0.25 \times 100$$

$$\Rightarrow \quad v = 0.25 \text{ ms}^{-1}$$

26. (*c*) $v_{CM} = \dfrac{m_1 \dfrac{dr_1}{dt} + m_2 \dfrac{dr_2}{dt}}{m_1 + m_2} = \dfrac{4 \times 5\hat{\mathbf{i}} + 2 \times 10\hat{\mathbf{j}}}{4 + 2}$

$$v_{CM} = \frac{40\hat{\mathbf{i}}}{6} = \frac{20}{3}\hat{\mathbf{i}}$$

The kinetic energy,

$$K = \frac{1}{2}mv^2 = \frac{1}{2} \times (4 + 2) \times \frac{20 \times 20}{3 \times 3}$$

$$= \frac{1}{2} \times 6 \times \frac{20 \times 20}{3 \times 3} = \frac{400}{3} \text{ J}$$

27. (*b*) $mu = (m + m)v \Rightarrow v = \frac{u}{2} = \frac{1}{2}$ m/s

$$n = \frac{v^2}{2g} = \frac{1}{8g} = \frac{1}{80} \times 10 = \frac{5}{4} = 1.25$$