CHAPTER 09

Rotation

In this chapter, we will discuss rotational motion, *i.e.,* the motion of an object around its own axis and the variables which describe rotational motion. Motion of wheels, gears, motors, planets, the hands of a clock, the rotor of jet engines, blades of helicopters etc are examples of rotational motion.

Rigid body

Ideally, a body is said to be a rigid body when it has a definite shape and size. For a rigid body it is considered that its constituent particles do not change their mutual distances even when an external force is applied on it.
e.g., a wheel can be considered as rigid body by ignoring a little change in its shape.

Motion of rigid body

A rigid body can possess pure **translational** motion, pure **rotational** motion or a combination of both these motions. Let us study these kinds of motions one by one.

In previous chapters, we have discussed translatory motion, *i.e.,* the motion of an object along a line, (both straight and curved).

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- Moment of inertia
- Radius of gyration
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3) Angular momentum
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- Angular momentum of a rigid body rotating about a fixed axis
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- Kinetic energy of rolling motion
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1) Rotational motion (Fixed axis of rotation)

In any kind of motion of a body there is either change in position or change in orientation or change in both. If a body changes its orientation during its motion, it is said to have rotational motion.

In pure rotational motion, every particle of the rigid body moves in circles of different radii about a fixed line, which is known as **axis of rotation**.
e.g., The motion of a ceiling fan, the vertical supporting the fan (here, axis of rotation) remains stationary and all the particles on the fan move in a circular paths. The centres of circular paths are on the axis of rotation.

Also, it is not necessary that axis of rotation pass through the body.

Physical quantities required to describe rotational motion and equations of rotational motion

All the physical quantities required to describe rotational motion are same as in circular motion. All of them have been discussed in chapter 7.

Similarly equations of rotational motion are same as that of circular motion and they have been studied in circular motion.

Moment of inertia

Moment of inertia plays the same role in rotational motion as mass plays in linear motion. It is the property of a body due to which it opposes any change in its state of rest or uniform rotation.

The moment of inertia is calculated about some axis (usually the rotational axis) and it depends on the mass as well as its distribution about that axis.

Moment of inertia of a single particle

For a very simple case, the moment of inertia of a single particle about an axis is given by

$$I = mr^2 \quad \text{...(i)}$$

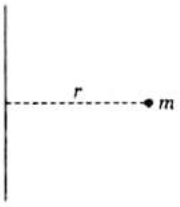


Fig. 9.1

Here, m is the mass of the particle and r its distance from the axis under consideration.

The unit of moment of inertia in SI system is kg-m^2. Its dimensions are [ML^2].

Moment of inertia of a system of particles

The moment of inertia of a system of particles about an axis is given by

$$I = \sum_i m_i r_i^2 \quad \text{...(ii)}$$

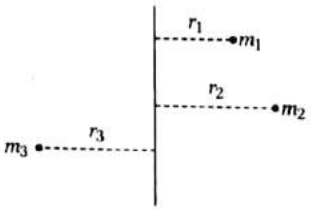


Fig. 9.2

where, r_i is the perpendicular distance from the axis of the ith particle, which has a mass m_i.

Moment of inertia of rigid bodies

For a continuous mass distribution found in a rigid body, we replace the summation of Eq. (ii) by an integral. If the system is divided into infinitesimal elements of mass dm and if r is the distance from a mass element to the axis of rotation, the moment of inertia is

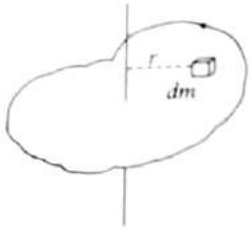


Fig. 9.3

$$I = \int r^2 \, dm$$

where, the integral is taken over the system.

Example 9.1 *A ring of diameter 0.4 m and of mass 10 kg is rotating about its geometrical axis at the rate of 35 rotation/second. Find the moment of inertia.*

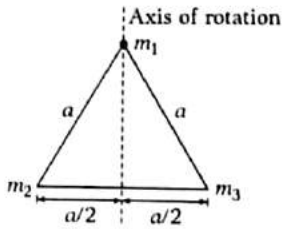
Sol. Moment of inertia of the ring about its axis is

$$I = MR^2$$

$$I = 10 \times (0.2)^2 = 10 \times 0.04 = 0.4 \text{ kg-m}^2$$

Example 9.2 *Three point of masses m_1, m_2 and m_3 are located at the vertices of an equilateral triangle of length a. Determine the moment of inertia of the system about an axis along the altitude of the triangle passing through m_1.*

Sol. Since, it can be seen in the figure given below that the axis of rotation passes through m_1. The distances of m_1, m_2 and m_3 from the axis of rotation are 0, $a/2$, $a/2$ respectively.



Therefore, moment of inertia of the system about the axis through m_1 is

$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 = m_1(0)^2 + m_2\left(\frac{a}{2}\right)^2 + m_3\left(\frac{a}{2}\right)^2$$

$$\Rightarrow \quad I = \frac{a^2}{4}(m_2 + m_3)$$

Example 9.3 *Calculate moment of inertia w.r.t. rotational axis XX′ in following figures.*

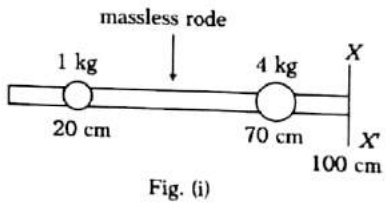


Fig. (i)

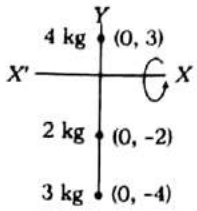


Fig. (ii)

Sol. (i) Moment of inertia about axis XX'

$$I_{XX'} = m_1r_1^2 + m_2r_2^2 = 4 \times (1 - 0.7)^2 + 1 \times (1 - 0.2)^2$$

$$I_{XX'} = 4 \times (0.3)^2 + 1 \times (0.8)^2 = 1 \text{ kg-m}^2$$

(ii) Moment of inertia about axis XX'

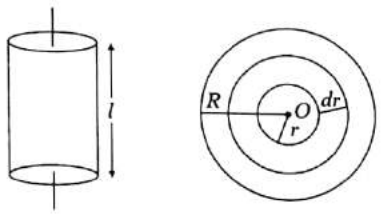
$$I_{XX'} = 4 \times (3)^2 + 2 \times (2)^2 + 3 \times (4)^2 = 92 \text{ kg-m}^2$$

Example 9.4 *Find the moment of inertia of a uniform cylinder about an axis through its centre of mass and perpendicular to its base. Mass of the cylinder is M and radius is R.*

Sol. The cylinder can be divided into annular shells of width dr and length l as shown in figure.

The moment of inertia of one of these shells

$$I = \int r^2\, dm = r^2(\rho\, dV)$$



Here, ρ = density, dV = volume of shell $= 2\pi rl\, dr$

$$\therefore \qquad I = 2\pi\rho l r^3\, dr$$

The cylinder's moment of inertia is found by integrating this expression between O and R

So, $$I = 2\pi\rho l \int_0^R r^3\, dr = \frac{\pi\rho l}{2} R^4 \qquad \text{...(i)}$$

The density ρ of the cylinder is the mass divided by the volume

$$\therefore \qquad \rho = \frac{M}{\pi R^2 l} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we have

$$I = \frac{1}{2} MR^2$$

Radius of gyration

The radius of gyration of a body about a given axis is the perpendicular distance of a point from the axis at which the whole mass of the body could be concentrated without any change in the moment of inertia of the body about that axis.

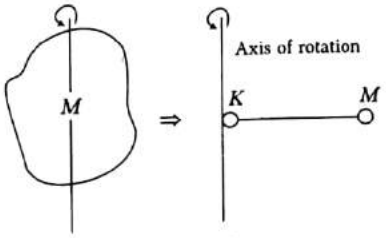


Fig. 9.4

If a body has mass M and radius of gyration is K, then moment of inertia,

$$I = MK^2 \;\Rightarrow\; K = \sqrt{I/M}$$

Here, K is radius of gyration.

Radius of gyration is also defined as the root mean square value of distance of all the particles about the axis of rotation.

i.e., $$K = \sqrt{\frac{r_1^2 + r_2^2 + r_3^2 + \ldots + r_n^2}{n}}$$

Radius of gyration depends on the shape and size of the body, position and configuration of the axis of rotation, distribution of mass of the body with respect to the axis of rotation. Radius of gyration does not depend on the mass of body.

Example 9.5 *A wheel of mass* 8 kg *has moment of inertia equals to* 0.5 kg-m^2. *Determine its radius of gyration.*

Sol. Given, mass, $M = 8$kg

Moment of Inertia, $I = 0.5$ kg-m^2

$$\because \qquad I = MK^2 \;\Rightarrow\; K^2 = \frac{I}{M} \;\Rightarrow\; K = \sqrt{\frac{I}{M}}$$

$\Rightarrow$ Radius of gyration, $K = \sqrt{\dfrac{0.5}{8}} \;\Rightarrow\; K = 0.25$ m or $K = 25$ cm

Example 9.6 *The moment of inertia of a solid cylinder about its axis of rotation is* $\dfrac{MR^2}{2}$. *What is the value of the radius of gyration of the cylinder about this axis?*

Sol. Moment of inertia of solid cylinder, $I = \dfrac{MR^2}{2}$ ...(i)

Moment of inertia, $I = MK^2$ ...(ii)

Comparing Eqs. (i) and (ii), we have

$$K^2 = \frac{R^2}{2},\; K = \frac{R}{\sqrt{2}} \text{ m}$$

Theorems on moment of inertia

There are two important theorems on moment of inertia, which enables calculation of the moment of inertia of a body about an axis, if its moment of inertia about some other axis is known. Let us now discuss both of them.

Theorem of parallel axes

It states that moment of inertia of a rigid body about any axis is equal to its moment of inertia about a parallel axis through its centre of mass plus the product of the mass of the body and the square of the perpendicular distance between the two axes.

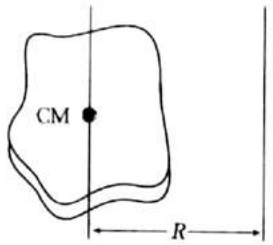


Fig. 9.5

Two such axes are shown in figure for a body of mass M. If R is the distance between the axes and I_{CM} and I are the respective moments of inertia about these axes, then

$$I = I_{CM} + MR^2$$

Note From the above theorem we can conclude that among many parallel axes moment of inertia is least about an axis which passes through centre of mass *e.g.*, I_2 is least among I_1 I_2 and I_3. Similarly, I_5 is least among I_4, I_5 and I_6.

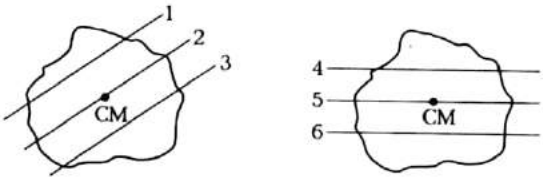


Fig. 9.6

Theorem of perpendicular axes

It states that the moment of inertia of a plane lamina about an axis perpendicular to its plane is equal to the sum of the moments of inertia of the lamina about any two mutually perpendicular axes in its plane and intersecting each other at the point, where the perpendicular axis passes through it.

$$I_Z = I_X + I_Y$$

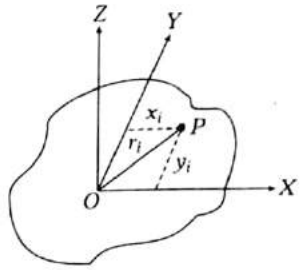


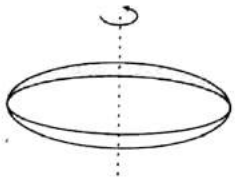
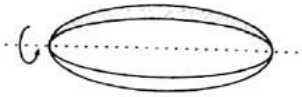
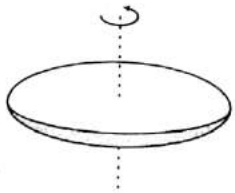
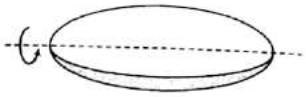
Fig. 9.7

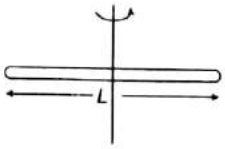
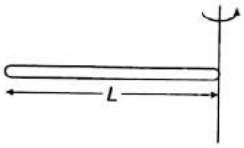
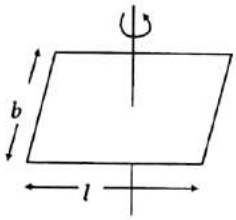
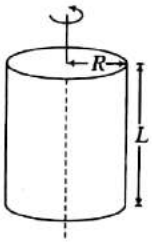
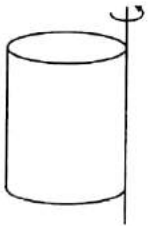
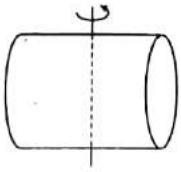
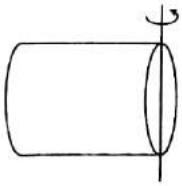
Let X and Y axes be chosen in the plane of the body and Z-axis perpendicular to this plane, three axes being mutually perpendicular, then according to the theorem

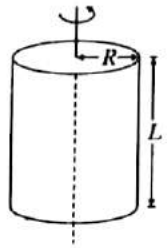
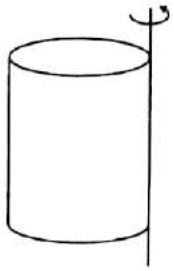
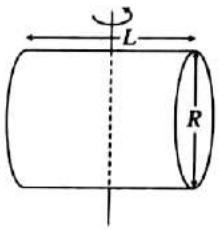
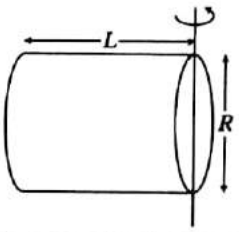
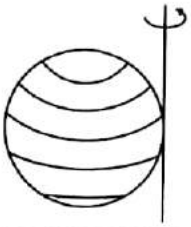
$$I_Z = I_X + I_Y$$

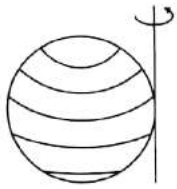
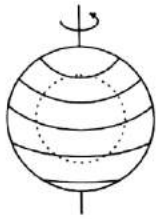
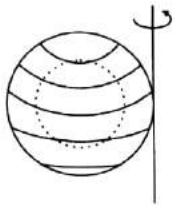
where, I_X, I_Y and I_Z are the moments of inertia about the X, Y, Z-axes respectively.

Moment of Inertia of Simple Geometrical Objects

S.No.	Body	Axis of Rotation	Figure	Moment of Inertia	K	K^2/R^2
1.	Thin circular ring, radius R	(i) About an axis passing through CG and perpendicular to its plane		MR^2	R	1
		(ii) About its diameter		$\frac{1}{2}MR^2$	$\frac{R}{\sqrt{2}}$	$\frac{1}{2}$
2.	Circular disc, radius R	(i) Perpendicular to disc at centre		$\frac{1}{2}MR^2$	$\frac{R}{\sqrt{2}}$	$\frac{1}{2}$
		(ii) About its diameter		$\frac{1}{4}MR^2$	$\frac{R}{2}$	$\frac{1}{4}$

S.No.	Body	Axis of Rotation	Figure	Moment of Inertia	K	K^2/R^2
3.	Long uniform thin rod	(i) About an axis passing through its centre of mass and perpendicular to the rod		$\frac{ML^2}{12}$	$\frac{L}{\sqrt{12}}$	
		(ii) About an axis passing through its edge and perpendicular to the rod		$\frac{ML^2}{3}$	$\frac{L}{\sqrt{3}}$	
4.	Rectangular lamina	Passing through the centre of mass and perpendicular to the plane.		$\frac{M}{12}[l^2 + b^2]$	$\sqrt{\frac{l^2 + b^2}{12}}$	
5.	Hollow cylinder, radius R	About its own axis		MR^2	R	1
6.	Cylindrical shell	(i) Tangential (Generator)		$2MR^2$	$\sqrt{2}R$	2
		(ii) About an axis passing through its CG and perpendicular to its own axis		$M\left[\frac{L^2}{12} + \frac{R^2}{2}\right]$	$\sqrt{\frac{L^2}{12} + \frac{R^2}{2}}$	
		(iii) About the diameter of one of faces of the cylinder		$M\left[\frac{L^2}{3} + \frac{R^2}{2}\right]$	$\sqrt{\frac{L^2}{3} + \frac{R^2}{2}}$	

S.No.	Body	Axis of Rotation	Figure	Moment of Inertia	K	K^2/R^2
7.	Solid cylinder	(i) About its own axis		$\frac{1}{2}MR^2$	$\frac{R}{\sqrt{2}}$	$\frac{1}{2}$
		(ii) Tangential (Generator)		$\frac{3}{2}MR^2$	$\sqrt{\frac{3}{2}}R$	$\frac{3}{2}$
		(iii) About an axis passing through its CG and perpendicular to its own axis		$M\left[\frac{L^2}{12}+\frac{R^2}{4}\right]$	$\sqrt{\frac{L^2}{12}+\frac{R^2}{4}}$	
		(iv) About the diameter of one of the faces of the cylinder		$M\left[\frac{L^2}{3}+\frac{R^2}{4}\right]$	$\sqrt{\frac{L^2}{3}+\frac{R^2}{4}}$	
8.	Spherical shell	(i) About its diametric axis		$\frac{2}{3}MR^2$	$\sqrt{\frac{2}{3}}R$	$\frac{2}{3}$
		(ii) About a tangential axis		$\frac{5}{3}MR^2$	$\sqrt{\frac{5}{3}}R$	$\frac{5}{3}$

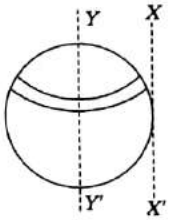
S.No.	Body	Axis of Rotation	Figure	Moment of Inertia	K	K^2/R^2
9.	Solid sphere, radius R	(i) About its diametric axis		$\frac{2}{5}MR^2$	$\sqrt{\frac{2}{5}}R$	$\frac{2}{5}$
		(ii) About a tangential axis		$\frac{7}{5}MR^2$	$\sqrt{\frac{7}{5}}R$	$\frac{7}{5}$
10.	Hollow sphere of inner radius R_1 and outer radius R_2	(i) About its diametric axis		$\frac{2}{5}M\left[\frac{R_2^5 - R_1^5}{R_2^3 - R_1^3}\right]$		
		(ii) Tangential		$\frac{2}{5}M\left[\frac{R_2^5 - R_1^5}{R_2^3 - R_1^3}\right] + MR_2^2$		

Note

1. Theorem of parallel axes is applicable for any type of rigid body whether it is a two dimensional or three dimensional, while the theorem of perpendicular axes is applicable for laminar type or two dimensional bodies only.
2. In theorem of perpendicular axes, the point of intersection of the three axes (X, Y and Z) may be any point on the plane of body (it may even lie outside the body). This point may or may not be the centre of mass of the body.

Example 9.7 *Find the moment of inertia of a sphere about a tangent to the sphere, while the mass of the sphere is M and the radius of the sphere is R.*

Sol. The moment of inertia of a sphere about its own axis (YY') is $\frac{2MR^2}{5}$



Applying the theorem of parallel axes the moment of inertia of the sphere about tangent (XX'),

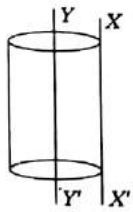
$$I_t = I + MR^2$$

$$I_t = \frac{2}{5}MR^2 + MR^2 = \frac{7}{5}MR^2$$

Example 9.8 *The mass of the cylinder is 25 kg and radius of cylinder is 5 m. Find the moment of inertia of a solid cylinder about a tangent to the cylinder.*

Sol. The moment of inertia of a cylinder aobut its own axis (YY')

$$= \frac{1}{2}MR^2$$

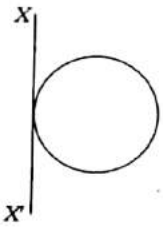


Applying the theorem of parallel axes, the moment of inertia of the cylinder about tangent (XX') is

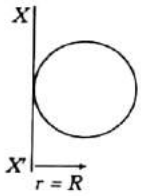
$$I_t = I + MR^2 = \frac{1}{2}MR^2 + MR^2 \Rightarrow I_t = \frac{3}{2}MR^2$$

Here, $M = 25$ kg, $R = 5$ m $\quad \therefore I_t = \frac{3}{2}(25)(5)^2 = 937.5$ kg-m^2

Example 9.9 *Find the moment of inertia of a solid sphere of mass 5 kg and radius 2 m about an axis XX' as shown in figure.*



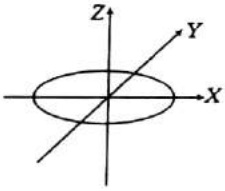
Sol. From theorem of parallel axes, the moment of inertia of solid sphere of mass is



$$I_{XX'} = I_{CM} + MR^2$$
$$= \frac{2}{5}MR^2 + MR^2 = \frac{7}{5}MR^2 = \frac{7}{5}(5)(2)^2 = 28 \text{ kg-m}^2$$

Example 9.10 *What is the moment of inertia of a*
(i) uniform circular ring of mass 2 kg about its 4 m diameter?
(ii) a thin disc about an axis coinciding with a diameter?

Sol. The moment of inertia of a uniform ring about an axis passing through its centre and perpendicular to it is given by, $I = MR^2$.
According to the theorem of perpendicular axis, $I_z = I_x + I_y$



Now x and y-axes are along the diameter of the disc and by symmetry.

$$I_x = I_y \Rightarrow I_z = 2I_x \Rightarrow I_z = MR^2$$

The moment of inertia of ring about of its diameter.

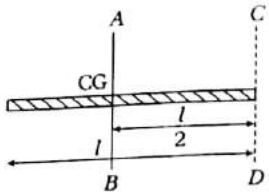
$$I_x = \frac{MR^2}{2} = \frac{(2)(2)^2}{2} = 4 \text{ kg-m}^2$$

(ii) Similarly, moment of inertia about the diameter of disc

$$I = \frac{1}{2}\left(\frac{1}{2}MR^2\right) = \frac{1}{4}MR^2$$

Example 9.11 *Calculate the moment of inertia of a rod of mass 2 kg and length 5 m about an axis perpendicular to it and passing through one of its ends.*

Sol. For the rod of mass M and length l the moment of inertia of the rod about an axis AB passing through its centre of mass is given, $I_{AB} = \frac{Ml^2}{12}$



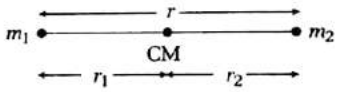
According to the parallel axes theorem,

$$I_{CD} = I_{AB} + M\left(\frac{1}{2}\right)^2 = \frac{Ml^2}{12} + \frac{Ml^2}{4}$$
$$= \frac{Ml^2 + 3Ml^2}{12} = \frac{4Ml^2}{12} = \frac{Ml^2}{3}$$
$$I_{CD} = \frac{Ml^2}{3} = \frac{2(5)^2}{3} = \frac{50}{3} \text{ kg-m}^2$$

Example 9.12 *Two masses m_1 and m_2 are placed at a distance r from each other. Find out the moment of inertia of system about an axis passing through their centre of mass.*

Sol. We have, $m_1r_1 = m_2r_2$ and $r_1 + r_2 = r$

$$\Rightarrow \quad r_1 = \frac{m_2 r}{m_1 + m_2}, \; r_2 = \frac{m_1 r}{m_1 + m_2}$$

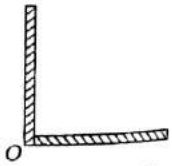


Moment of inertia of the system about an axis passing through their centre of mass is

$$I = m_1r_1^2 + m_2r_2^2 = m_1\left(\frac{m_2 r}{m_1 + m_2}\right)^2 + m_2\left(\frac{m_1 r}{m_1 + m_2}\right)^2$$
$$= \left(\frac{m_1 m_2}{m_1 + m_2}\right)r^2$$

Note Here, $I = \mu r^2$ where, μ is called reduced mass which is equal to $\frac{m_1 m_2}{m_1 + m_2}$.

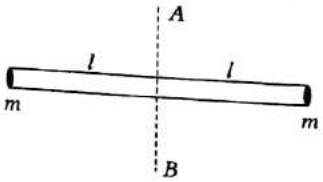
Example 9.13 *Two identical rods each of mass M and length L are kept according to figure. Find the moment of inertia of rods about an axis passing through O and perpendicular to the plane of rods.*



Sol. $\because$ Moment of inertia about an axis passing through an end $= \frac{ML^2}{3}$

$$\therefore \quad I_{\text{given system}} = \frac{ML^2}{3} + \frac{ML^2}{3} = \frac{2ML^2}{3}$$

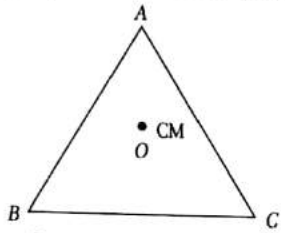
Example 9.14 *Consider a uniform rod of mass m and length 2l with two particles of mass m each at its ends. Let AB be a line perpendicular to the length of the rod and passing through its centre. Find the moment of inertia of the system about AB.*



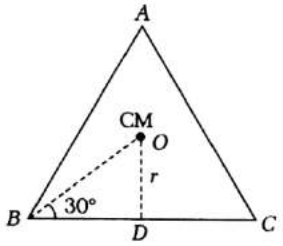
Sol. Moment of inertia of the system about AB

$$I_{AB} = I_{\text{rod}} + I_{\text{both particles}}$$
$$= \frac{m(2l)^2}{12} + 2(ml^2) = \frac{7}{3}ml^2$$

Example 9.15 *Three rods each of mass m and length l are joined together to form an equilateral triangle as shown in figure. Find the moment of inertia of the system about an axis passing through its centre of mass and perpendicular to the plane of the triangle.*



Sol. Moment of inertia of rod BC about an axis perpendicular to plane of triangle ABC and passing through the mid-point of rod BC (*i.e.*, D) is



$$I_1 = \frac{ml^2}{12} \Rightarrow AD = \sqrt{l^2 - \frac{l^2}{4}} = \frac{\sqrt{3}}{2}l$$

$$r = OD = \frac{AD}{3} = \frac{\sqrt{3}\,l}{3 \times 2} = \frac{l}{2\sqrt{3}}$$

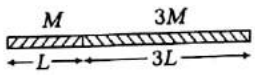
From theorem of parallel axes, moment of inertia of this rod about the axis passing through CM and perpendicular to plain ABC is

$$I_2 = I_1 + mr^2 = \frac{ml^2}{12} + m\left(\frac{l}{2\sqrt{3}}\right)^2 = \frac{ml^2}{6}$$

$\therefore$ Moment of inertia of all the three rods is

$$I = 3I_2 = 3\left(\frac{ml^2}{6}\right) = \frac{ml^2}{2}$$

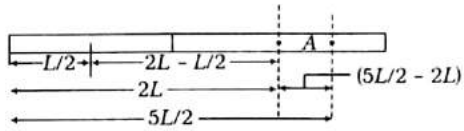
Example 9.16 *Two thin uniform rods A (M, L) and B (3M, 3L) are joined as shown. Find the MI about an axis passing through the centre of mass of system of rods and perpendicular to the length.*



Sol. Locate the centre of mass taking origin at 0.

$$x_{CM} = \frac{M \times \frac{L}{2} + 3M \times \left(L + \frac{3L}{2}\right)}{M + 3M} = 2L$$

A is CM of system of rods.

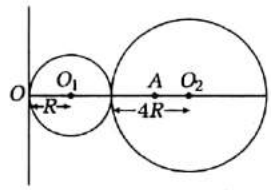


$$I_A = I_1 + I_2 = \left[\frac{ML^2}{12} + M\left(2L - \frac{L}{2}\right)^2\right] + \left[\frac{3M(3L)^2}{12} + 3M\left(\frac{5L}{2} - 2L\right)^2\right]$$

$$= ML^2\left[\left\{\frac{1}{12} + \frac{9}{4}\right\} + \left\{\frac{27}{12} + \frac{3}{4}\right\}\right]$$

$$= \frac{16ML^2}{3}$$

Example 9.17 *Two thin uniform rings made of same material and of radii R and 4R are joined as shown. The mass of smaller ring is m. Find the MI about an axis passing through the centre of mass of system of rings and perpendicular to the plane.*



Sol. Here, $m_1 = m$, $m_2 = 4m$

Location of the centre of mass from O

$$x_{cm} = \frac{m \times R + 4m(2R + 4R)}{m + 4m} = 5R$$

A is CM of system of rings

$$I_A = I_1 + I_2 = \{mR^2 + m(5R - R)^2\} + \{4m(4R)^2 + 4m(6R - 5R)^2\}$$

$$= mR^2[\{1 + 16\} + \{64 + 4\}]$$

$$I_A = 85mR^2$$

Check point 9.1

1. A car moves on a circular road. It describes equal angles about the centre in equal intervals of time. Which of the following statements about the velocity of the car is true?
(a) Magnitude of velocity is not constant
(b) Both magnitude and direction of velocity change
(c) Velocity is directed towards the centre of the circle
(d) Magnitude of velocity is constant but direction changes

2. A wheel rotates with a constant angular velocity of 300 rpm. The angle through which the wheel rotates in 1 s is
(a) π rad (b) 5π rad (c) 10π rad (d) 20π rad

3. A motor is rotating at a constant angular velocity of 500 rpm. The angular displacement per second is
(a) $\frac{3}{50\pi}$ rad (b) $\frac{3\pi}{50}$ rad
(c) $\frac{25\pi}{3}$ rad (d) $\frac{50\pi}{3}$ rad

4. The angle turned by a body undergoing circular motion depends on time as $\theta = \theta_0 + \theta_1 t + \theta_2 t^2$. Then, the angular acceleration of the body is
(a) θ_1 (b) θ_2 (c) $2\theta_1$ (d) $2\theta_2$

5. A rotating wheel changes angular speed from 1800 rpm to 3000 rpm to 20 s. What is the angular acceleration assuming it to be uniform?
(a) 60πrad s^{-2} (b) 90πrad s^{-2}
(c) 2πrad s^{-2} (d) 40πrad s^{-2}

6. A body rotating with uniform angular acceleration covers 100π (radian) in the first 5 s after the start. Its angular speed at the end of 5 s (in rad/s) is
(a) 40π (b) 30π
(c) 20π (d) 10π

7. The angular velocity of a wheel increases from 100 to 300 in 10 s. The number of revolutions made during that time is
(a) 600 (b) 1500
(c) 1000 (d) 2000

8. A wheel has angular acceleration of 3 rad s^{-2} and an initial angular speed of 2 rad s^{-1}. In a time of 2 s, it has rotated through an angle (in radian) of
(a) 6 (b) 10 (c) 12 (d) 4

9. A body rotates about a fixed axis with an angular acceleration of 3 rad / s^2. The angle rotated by it during the time when its angular velocity increases from 10 rad/s to 20 rad/s (in radian) is
(a) 50 (b) 100 (c) 150 (d) 200

10. A wheel which is initially at rest is subjected to a constant angular acceleration about its axis. It rotates through an angle of 15° in time t s. The increase in angle through which it rotates in the next $2t$ s is
(a) 90° (b) 120° (c) 30° (d) 45°

11. Analogue of mass in rotational motion is
(a) moment of inertia (b) angular momentum
(c) gyration (d) None of these

12. Moment of inertia of a body depends upon
(a) axis of rotation (b) torque
(c) angular momentum (d) angular velocity

13. A particle of mass 1 kg is kept at (1m, 1m, 1m). The moment of inertia of this particle about Z-axis would be
(a) 1 kg-m^2 (b) 2 kg-m^2
(c) 3 kg-m^2 (d) None of these

14. The radius of gyration of a solid sphere of radius R about its tangential is
(a) $\sqrt{\frac{7}{5}}R$ (b) $\sqrt{\frac{2}{5}}R$ (c) $\sqrt{\frac{5}{7}}R$ (d) R

15. What is the moment of inertia of a solid sphere of density ρ and radius R about its diameter?
(a) $\frac{105}{176}R^5\rho$ (b) $\frac{176}{105}R^5\rho$ (c) $\frac{105}{176}R^2\rho$ (d) $\frac{176}{105}R^2\rho$

16. One circular ring and one circular disc both having the same mass and radius. The ratio of their moments of inertia about the axes passing through their centres and perpendicular to planes will be
(a) 1 : 1 (b) 2 : 1 (c) 1 : 2 (d) 4 : 1

17. The moment of inertia of a solid cylinder of mass M, length $2R$ and radius R about an axis passing through the centre of mass and perpendicular to the axis of the cylinder is I_1 and about an axis passing through one end of the cylinder and perpendicular to the axis of cylinder is I_2
(a) $I_2 - I_1 = MR^2$ (b) $I_2 = I_1$
(c) $\frac{I_2}{I_1} = \frac{19}{12}$ (d) $I_1 - I_2 = MR^2$

18. If I_1 is the moment of inertia of a thin rod about an axis perpendicular to its length and passing through its centre of mass and I_2 is the moment of inertia of the ring about an axis perpendicular to plane of ring and passing through its centre formed by bending the rod, then
(a) $\frac{I_1}{I_2} = \frac{3}{\pi^2}$ (b) $\frac{I_1}{I_2} = \frac{2}{\pi^2}$ (c) $\frac{I_1}{I_2} = \frac{\pi^2}{2}$ (d) $\frac{I_1}{I_2} = \frac{\pi^2}{3}$

19. Moment of inertia of a rod of mass m and length l about its one end is I. If one-fourth of its length is cut away, then moment of inertia of the remaining rod about its one end will be
(a) $\frac{3}{4}I$ (b) $\frac{9}{16}I$ (c) $\frac{27}{64}I$ (d) $\frac{I}{16}$

20. Three thin rods each of length L and mass M are placed along X, Y and Z -axes such that one end of each rod is at origin. The moment of inertia of this system about Z-axis is
(a) $\frac{2}{3}ML^2$ (b) $\frac{4ML^2}{3}$ (c) $\frac{5ML^2}{3}$ (d) $\frac{ML^2}{3}$

2 Torque or moment of force

Torque is a quantity which measures the capability of a force to rotate a body. Torque due to a force is also known as the moment of a force. Torque is the rotational analogue of force of linear motion.

It is defined as the product of the force and the perpendicular distance between the line of action of the force and the axis of rotation. This perpendicular distance is known as the force arm.

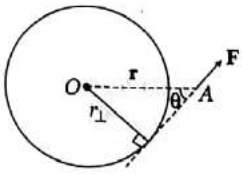


Fig. 9.8

Torque, τ = force × force arm

$$\tau = \mathbf{r} \times \mathbf{F}$$

$$\boxed{\tau = rF\sin\theta}$$

(where, θ is the angle between **r** and **F**)

The SI unit of torque is Nm. Its dimensions are $[ML^2T^{-2}]$.

Torque is an axial vector *i.e.*, its direction is always perpendicular to the plane containing vectors **r** and **F** in accordance with right-hand screw rule.

There is a mathematical relation between torque and moment of inertia and that can be given as

$$\boxed{\tau = I\alpha}$$

The above expression is similar to the expression of Newton's second law for translational motion with constant mass.

Note

1. If $\theta = 0°$ or $180°$ the torque of force is zero.
2. $\tau = \mathbf{r} \times \mathbf{F}$ is a vector product, so on reversing the direction of **F**, the direction of torque is reversed. If directions of both **r** and **F** are reversed, the direction of τ remains the same.
3. If the total torque acting on the rigid body vanishes, *i.e.*, the vector sum of the torques on the rigid body is zero.

$$\tau_1 + \tau_2 + \tau_3 \ldots \tau_n = \sum_{i=1}^{n} \tau = 0$$

Then the body is said to be in rotational equilibrium.

Example 9.18 *Find the torque of a force* $\mathbf{F} = (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}})$N *about a point O. The position vector of point of application of force about O is* $\mathbf{r} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}})$ *m.*

Sol. Torque, $\tau = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 3 & -1 \\ 1 & 2 & -3 \end{vmatrix}$

$$\tau = \hat{\mathbf{i}}(-9+2) + \hat{\mathbf{j}}(-1+6) + \hat{\mathbf{k}}(4-3)$$

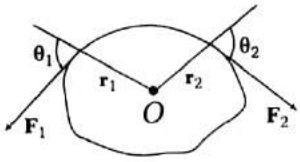
or $$\tau = (-7\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + \hat{\mathbf{k}})\text{Nm}$$

Example 9.19 *A force* $10(-\hat{\mathbf{k}})$ *N acts on the origin of the coordinate system. Find the torque about the point (1 m, −1 m, 0 m).*

Sol. Torque, $\tau = \mathbf{r} \times \mathbf{F} = [(1-0)\hat{\mathbf{i}} + (-1-0)\hat{\mathbf{j}} + (0-0)\hat{\mathbf{k}})]$

$$\times (-10)\hat{\mathbf{k}} = 10(\hat{\mathbf{i}} - \hat{\mathbf{j}}) \times (-\hat{\mathbf{k}}) = 10(\hat{\mathbf{i}} + \hat{\mathbf{j}})\text{ Nm}$$

Example 9.20 *The body in figure shown is pivoted at O and two forces act on it as shown.*



(i) Find an expression of the net torque on the body about the pivot.

(ii) If $r_1 = 1.30$ *m,* $r_2 = 2.15$ *m,* $F_1 = 4.20$ *N,* $F_2 = 4.90$ *N,* $\theta_1 = 75°$ *and* $\theta_2 = 60°$*, what is the net torque about the pivot?*

Sol. (i) Take a torque that tends to cause a counter clockwise rotation from rest to be positive and a torque that tends to cause a clockwise rotation from rest to be negative. Thus, a positive torque of magnitude $r_1F_1 \sin\theta_1$ is associated with $\mathbf{F}_1$ and a negative torque of magnitude $r_2F_2 \sin\theta_2$ is associated with $\mathbf{F}_2$. Both of these are about O. The net torque about O is $\tau = r_1F_1 \sin\theta_1 - r_2F_2 \sin\theta_2$.

(ii) Substitute the given values to obtain

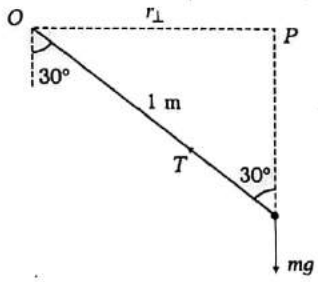
$$\tau = (1.30\text{ m})(4.20\text{ N})\sin 75° - (2.15\text{ m})(4.90\text{ N})\sin 60°$$
$$= -3.85\text{ Nm}$$

Example 9.21 *A small ball of mass 1 kg is attached to one end of a 1 m long massless string and the other end of the string is hung from a point. When the resulting pendulum is 30° from the vertical, what is the magnitude of torque about the point of suspension?*
[Take, $g = 10\text{ ms}^{-2}$*]*

Sol. Two forces are acting on the ball

(i) tension (T) (ii) weight (mg)

Torque of tension about point O is zero, as it passes through O.



$$\tau_{mg} = F \times r_\perp$$

Here, $r_\perp = OP = 1 \sin 30° = 0.5$ m

$\therefore$ $\tau_{mg} = (mg)(0.5) = (1)(10)(0.5) = 5$ Nm

Example 9.22 *A solid flywheel of 20 kg mass and 120 mm radius revolves at 600 revmin*$^{-1}$*. With what force must a brake lining be pressed against it for the flywheel to stop in 3s, if the coefficient of friction is 0.1?*

Sol. Given, revolutions per minute, $n_0 = 600$ rev min^{-1}

$$= \frac{600}{60}\text{ rev s}^{-1}$$

Revolutions per second, $n_0 = 10$ rev s^{-1}

So, initial angular velocity, $\omega_0 = 2\pi n_0 = (2\pi)(10) = 20\pi$ rad s^{-1}

Let α be the constant angular retardation, then applying

$$\omega = \omega_0 - \alpha t$$

or $$0 = (20\pi) - 3(\alpha)$$

or $$\alpha = \frac{20}{3}\pi \text{ rad s}^{-2}$$

Further, $$\alpha = \frac{\tau}{I}$$

Here, torque $\tau = \mu NR$ (R = radius)

or $\tau = \mu FR$ (F = applied force)

as $$N = F \text{ and } I = \frac{1}{2}mR^2$$

From the above equations,

$$\frac{20}{3}\pi = \frac{\mu FR}{\frac{1}{2}mR^2} = \frac{2\mu F}{mR}$$

or $$\text{Force, } F = \frac{10\pi mR}{3\mu}$$

Substituting the values, we have

$$F = \frac{10 \times 22 \times 20 \times 0.12}{3 \times 7 \times 0.1}$$

or $$F = 251.43 \text{ N}$$

Work done by torque

Consider a rigid body acted upon by a force F at a perpendicular distance r from the axis of rotation. Let under the action of this force, the body rotates through an angle $\Delta\theta$.

Work done = torque × displacement = $\tau d\theta$

The power associated with the work done by a torque acting on a rotating body is given by

$$\boxed{\text{Power} = \frac{dW}{dt} = \tau\frac{d\theta}{dt} = \tau\omega}$$

where $\tau\omega$ is also known as instantaneous rotational power.

Example 9.23 *To maintain a rotor at a uniform angular speed of 100 rads^{-1} an engine needs to transmit a torque of 200 Nm. What is the power required by engine? Assume 100% efficiency of engine.*

Sol. Work done by torque in turning rotor by an angle $d\theta$ is $= \tau d\theta$

So, power delivered by engine,

$$P = \frac{\text{Work done}}{\text{Time taken}} = \tau\frac{d\theta}{dt}, \quad (dt = \text{time taken for turning by angle } d\theta)$$

or $$P = \tau\omega$$

So, power required $= 100 \times 200 = 20000$ W

$= 20$ kW [$\because$ 1 kW = 1000]

Example 9.24 *The power output of an automobile engine is advertised to be 200 hp at 6000 rpm. What is the corresponding torque?*

Sol. $P = 200 \text{ hp} = 200 \text{ hp} \times \frac{746}{1 \text{ hp}} = 1.49 \times 10^5$ W

and $$\omega_z = 6000 \text{ rev/min} = 6000 \times \frac{2\pi}{60} = 628 \text{ rad/s}$$

$$\tau = \frac{P}{\omega} = \frac{1.49 \times 10^5}{628} = 237.5 \text{ Nm}$$

Example 9.25 *A flywheel of moment of inertia 10 kg m^2 is rotating at 50 rad/s. It must be brought to stop in 10 s.*

(i) How much work must be done to stop it?

(ii) What is the required average power?

Sol. Given, $\omega_0 = 50$ rad/s, $t = 10$ s,

From the equation, $\omega = \omega_0 - \alpha t$

$$0 = 50 - \alpha \times 10$$

$$\alpha = 5 \text{ rad/s}^2$$

Again, $$\theta = \omega_0 t - \frac{1}{2}\alpha t^2$$

$$= 50 \times 10 - \frac{1}{2} \times 5 \times (10)^2$$

$$= 500 - 250 = 250 \text{ rad}$$

Retarding torque, $\tau = I\alpha = 10 \times 5 = 50$ cm

Work done by retarding torque

$$W = \tau\theta = 50 \times 250 = 12500 \text{ J}$$

Required average power,

$$P = \frac{W}{t} = \frac{12500}{10} = 1250 \text{ W}$$

3 Angular momentum

A mass moving in a straight line has linear momentum (**p**). When a mass rotates about some point/axis, there is momentum associated with rotational motion called the **angular momentum (L)**.

Just as net external force is required to change the linear momentum of an object, a net external torque is required to change the angular momentum of an object.

Angular momentum of a particle about some point

Angular momentum of a particle (of mass m) about a point is defined as the moment of its linear momentum (**p**) about the point.

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} = \mathbf{r} \times (m\mathbf{v})$$

$$= m(\mathbf{r} \times \mathbf{v})$$

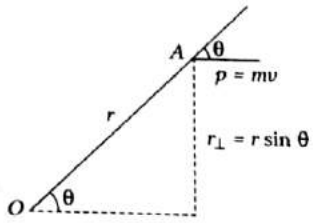


Fig. 9.9

Here, $\mathbf{r}$ is the radius vector of particle A about O at that instant of time. The magnitude of $\mathbf{L}$ is

$$L = mvr \sin\theta = mvr_{\perp}$$

Here, $r_{\perp} = r \sin\theta$ is the perpendicular distance of line of action of velocity $\mathbf{v}$ from point O. The direction of $\mathbf{L}$ is same as that of $\mathbf{r} \times \mathbf{v}$. The SI unit of angular momentum is $\text{kgm}^2\text{s}^{-1}$ and its dimensions are $[\text{ML}^2\text{T}^{-1}]$.

Important Concept Related to Angular Momentum

Angular momentum is given by $\mathbf{L} = \mathbf{r} \times \mathbf{p}$

We can obtain the three rectangular components of angular momentum, *i.e.*, L_x, L_y and L_z as we did in the case of torque.

$$L = L_x\hat{\mathbf{i}} + L_y\hat{\mathbf{j}} + L_z\hat{\mathbf{k}}$$
$$\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$$
$$\mathbf{p} = p_x\hat{\mathbf{i}} + p_y\hat{\mathbf{j}} + p_z\hat{\mathbf{k}}$$

Using determinant method, we get

$$(L_x\hat{\mathbf{i}} + L_y\hat{\mathbf{j}} + L_z\hat{\mathbf{k}}) = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ x & y & z \\ p_x & p_y & p_z \end{vmatrix}$$
$$= (yp_z - zp_y)\hat{\mathbf{i}} + (zp_x - xp_z)\hat{\mathbf{j}} + (xp_y - yp_x)\hat{\mathbf{k}}$$

Comparing the components of both sides we get

$$L_x = (yp_z - zp_y) \quad \text{...(i)}$$
$$L_y = (zp_x - xp_z) \quad \text{...(ii)}$$
$$L_z = (xp_y - yp_x) \quad \text{...(iii)}$$

And from the above three equation we get, **if the particle is moving in *yz*-plane it has the angular momentum component only in *x*-direction,**

***i.e.*, $L_x \neq 0$; $\quad L_y = L_z = 0$**

Similarly, if the particle is moving in *xz*-plane $L_y \neq 0$, $L_x = L_z = 0$ and if the particle is moving in *xy*-plane only $L_z = 0$, $L_x = L_y = 0$.

Example 9.26 *A car of mass 300 kg is travelling on a circular track of radius 100 m with a constant speed of 60 m/s. Calculate the angular momentum?*

Sol. Momentum, $\mathbf{p} = mv = 300 \times 60 = 18000$ kg m/s

(directed along the tangent to the circle)

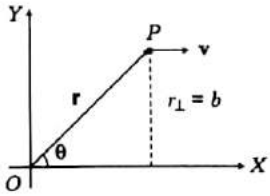
Angular momentum, $\mathbf{L} = \mathbf{r} \times \mathbf{p}$

$$\mathbf{L} = rp \sin\theta, \ \theta = 90°$$
$$\mathbf{L} = rp = 100 \times 18000 = 18 \times 10^5 \text{ kg m}^2\text{s}^{-1}$$

Example 9.27 *A particle of mass m is moving along the line $y = b$, $z = 0$ with constant speed v. State whether the angular momentum of particle about origin is increasing, decreasing or constant.*

Sol. Angular momentum of particle,

$$|\mathbf{L}| = mvr \sin\theta$$
$$= mvr_{\perp} = mvb$$



$\therefore$ $|\mathbf{L}|$ = constant as m, v and b all are constants.

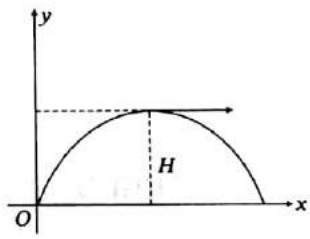
Direction of $\mathbf{r} \times \mathbf{v}$ also remains the same. Therefore, angular momentum of particle about origin remains constant with due course of time.

Note In this problem, $|\mathbf{r}|$ is increasing, θ is decreasing but $r \sin\theta$, *i.e.*, b remains constant. Hence, the angular momentum remains constant.

Example 9.28 *A particle of mass m is projected with velocity v at an angle θ with the horizontal. Find its angular momentum about the point of projection when it is at the highest point of its trajectory.*

Sol. At the highest point it has only horizontal velocity

$$v_x = v \cos\theta$$



Length of the perpendicular to the horizontal velocity from O is the maximum height, where

$$H_{max} = \frac{v^2 \sin^2\theta}{2g}$$

Angular momentum $= mvr = mv_x H_{max}$

$$= mv_x\left(\frac{v^2 \sin^2\theta}{2g}\right)$$
$$= \frac{mv^2 \sin^2\theta}{2g} v_x$$

$\Rightarrow$ Angular momentum

$$L = \frac{mv^3 \sin^2\theta \cos\theta}{2g}$$

Example 9.29 *Find the components along the X, Y, Z-axes of the angular momentum J of a particle, whose position vector is* **r** *with components x, y, z and linear momentum is* **p** *with components p_x, p_y and p_z. Show that if the particle moves only in the xy-plane, then the angular momentum has only a z component.*

Sol. The position vector **r** and the linear momentum **p** of the particle are written in terms of x, y, z components as

$$\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$$

and

$$\mathbf{r} = p_x\hat{\mathbf{i}} + p_y\hat{\mathbf{j}} + p_z\hat{\mathbf{k}}$$

By definition the angular momentum of the particle is

$$\mathbf{J} = \mathbf{r} \times \mathbf{p} = (x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}) \times (p_x\hat{\mathbf{i}} + p_y\hat{\mathbf{j}} + p_z\hat{\mathbf{k}})$$

$$= \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ x & y & z \\ p_x & p_y & p_z \end{vmatrix}$$

$$= \hat{\mathbf{i}}\,(yp_z - zp_y) + \hat{\mathbf{j}}(zp_x - xp_z) + \hat{\mathbf{k}}(xp_y - yp_x). \quad \text{...(i)}$$

The angular momentum may be written in terms of x, y, z components as

$$\mathbf{J} = J_x\hat{\mathbf{i}} + J_y\hat{\mathbf{j}} + J_z\hat{\mathbf{k}} \quad \text{...(ii)}$$

Comparing Eqs. (i) and (ii), we have

$$J_x = yp_z - zp_y,$$

$$J_y = zp_x - xp_z,$$

and

$$J_z = xp_y - yp_x.$$

If the particle moves only in the xy-plane, then $z = 0$ and $p_z = 0$. Now, Eq. (i) will be

$$\mathbf{J} = (xp_y - yp_x)\,\hat{\mathbf{k}}.$$

i.e., **J** has only z-component.

Angular momentum of a rigid body rotating about a fixed axis

The angular momentum of rigid body *i.e.*, a system of particles is the sum of angular momentum of all the particles within the system.

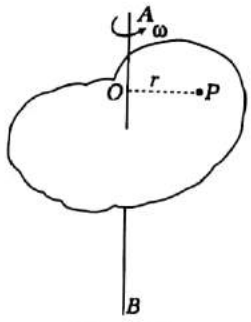


Fig. 9.10

Suppose a particle P of mass m is going in a circle of radius r and at some instant the speed of the particle is v. For finding the angular momentum of the particle about the axis of rotation, the origin may be chosen anywhere on the axis. We choose it at the centre of the circle. In this case, **r** and **p** are perpendicular to each other and $\mathbf{r} \times \mathbf{p}$ is along the axis. Thus, component of $\mathbf{r} \times \mathbf{p}$ along the axis is mvr itself. The angular momentum of the whole rigid body about AB is the sum of components of all particles, *i.e.*,

$$L = \sum_i m_i r_i v_i$$

Here, $v_i = r_i\omega$

$$\therefore \quad L = \sum_i m_i r_i^2 \omega \quad \text{or} \quad L = \omega \sum_i m_i r_i^2$$

or

$$L = I\omega \qquad \left(\text{as } \sum_i m_i r_i^2 = I\right)$$

Here, I is the moment of inertia of the rigid body about AB.

- The relation between torque and angular momentum can be given as follows.

  As angular momentum, $\mathbf{L} = \mathbf{r} \times \mathbf{p}$

$$\therefore \quad \frac{d\mathbf{L}}{dt} = \frac{d\mathbf{r}}{dt} \times \mathbf{p} + \mathbf{r} \times \frac{d\mathbf{p}}{dt}$$

$$= \mathbf{v} \times m\mathbf{v} + \mathbf{r} \times \mathbf{F} = 0 + \mathbf{r} \times \mathbf{F} = \tau_{ext}$$

$$\Rightarrow \quad \frac{d\mathbf{L}}{dt} = \tau_{ext}$$

  Thus, **rate of change of angular momentum is equal to the torque due to an external force.**

- Angular impulse $= \int \tau dt = \mathbf{L}_f - \mathbf{L}_i$

  Hence, **angular impulse of torque is equal to total change in angular momentum of the body in given time.**

Example 9.30 *A 40 kg flywheel in the form of a uniform circular disc of 1m radius is making 120 rpm. Determine the angular momentum.*

Sol. Here, $M = 40$ kg, $R = 1$m, $\nu = 120$ rpm $= \frac{120}{60}$ rps

$\Rightarrow$ Frequency, $\nu = 2$ rps

Therefore, $I = \frac{1}{2}MR^2 = \frac{1}{2} \times 40 \times (1)^2 = 20$ kgm^2.

and $\omega = 2\pi\nu = 2\pi \times 2 = 4\pi$ rads^{-1}

So, angular momentum, $L = I\omega = 20 \times 4\pi = 20 \times 4 \times 3.14$

$$L = 251.2 \text{ kgm}^2\text{s}^{-1}$$

Example 9.31 *A torque of 10 Nm is applied on a wheel having angular momentum of 2 kgm^{2}s^{-1}, calculate the angular momentum of the wheel after 4 seconds.*

Sol. Torque, $\tau = \frac{d\mathbf{L}}{dt} \Rightarrow \tau = \frac{\mathbf{L}_2 - \mathbf{L}_1}{t}$

Angular moment of the wheel,

$$\mathbf{L}_2 = \tau t + \mathbf{L}_1 = 10 \times 4 + 2 = 40 + 2 = 42 \text{ kgm}^2\text{s}^{-1}$$

Example 9.32 *The diameter of a solid disc is 0.5 m and its mass is 16 kg. What torque will increase its angular velocity from zero to 120 rotations/minute in 8 seconds?*

Sol. Moment of inertia of solid disc

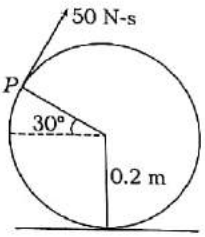
$$I = \frac{1}{2}MR^2 = \frac{1}{2} \times 16 \times \left(\frac{0.5}{2}\right)^2 = \frac{1}{2}$$

Angular velocity, $\omega = \frac{120 \times 2\pi}{60} = 4\pi$ and change in angular momentum, $\Delta L = I\omega - 0 = I\omega$

Angular impulse, $\tau \times t = \Delta L$

$$\Rightarrow \quad \tau = \frac{\Delta L}{t} = \frac{1}{8} \times \frac{1}{2} \times 4\pi = \frac{\pi}{4} \text{ N-m}$$

Example 9.33 *A solid ball of radius 0.2 m and mass 1 kg is given an instantaneous impulse of 50 N-s at point P as shown. Find the number of rotations made by the ball about its diameter before hitting the ground. The ball is kept on smooth surface initially.*



Sol. Impulse gives translational velocity

$$u = \frac{\text{Impulse}}{\text{Mass}} \text{ along impulse}$$

$$= 50 \text{ m/s}$$

$$T = \text{time of flight of projectile}$$

$$= \frac{2u \sin\theta}{g} = \frac{2 \times 50 \times \sin 60°}{10} = 5\sqrt{3} \text{ s}$$

Impulse give angular impulse also

$$\omega = \frac{\text{Impulse} \times R}{I} \quad \text{or} \quad \omega = \frac{\text{Impulse} \times R}{\frac{2}{5} mR^2} = 625 \text{ rad/s}$$

Number of rotations, $n = \frac{\omega T}{2\pi} = \frac{3125\sqrt{3}}{2\pi}$

Law of conservation of angular momentum

The law of conservation of angular momentum states that when no external torque acts on an object or isolated system of objects, no change of angular momentum can occur.

Hence, the angular momentum before an event involving only internal torques or no torques is equal to the angular momentum after the event.

We know that, $\tau_{ext} = \frac{d\mathbf{L}}{dt}$

If $\tau_{ext} = 0$, then $\frac{d\mathbf{L}}{dt} = 0$ or $\mathbf{L}$ must be a constant.

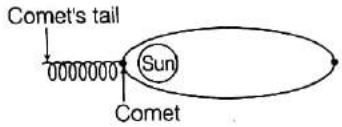
Therefore, **in the absence of any external torque, the total angular momentum of a system must remain conserved.**

As $\mathbf{L} = I\omega$, the law of conservation of momentum leads us to the following conclusion.

For an isolated system, $I\omega$ = constant or $I_1\omega_1 = I_2\omega_2$

This principle is often made use by gymnast, swimmers, circus acrobats and ballet dancers cat while catching birds etc. Constant angular momentum is extremely useful when dealing with the orbits of planets and satellites and also when analysing the Bohr's model of the atom.

Example 9.34 *The maximum and minimum distances of a comet from Sun are 1.4×10^{12} m and 7×10^{10} m. If velocity nearest to Sun is 6×10^4 ms^{-1}, what is velocity of comet when it is farthest from the Sun?*



Sol. Taking comet as a point mass and assuming path as circular

Applying conservation of angular momentum, $I_1\omega_1 = I_2\omega_2$

$$mr_1^2 \times \frac{v_1}{r_1} = mr_2^2 \times \frac{v_2}{r_2} \text{ and } v_1 r_1 = v_2 r_2$$

$$\Rightarrow \quad v_2 = \frac{v_1 r_1}{r_2} = \frac{6 \times 10^4 \times 7 \times 10^{10}}{1.4 \times 10^{12}} = 3000 \text{ ms}^{-1}$$

Example 9.35 *A wheel of moment of inertia I and radius R is rotating about its axis at an angular speed ω_0. It picks up a stationary particle of mass m at its edge. Find the new angular speed of the wheel.*

Sol. Net external torque on the system is zero. Therefore, angular momentum will remain conserved. Thus,

$$I_1\omega_1 = I_2\omega_2 \quad \text{or} \quad \omega_2 = \frac{I_1\omega_1}{I_2}$$

Here, $\quad I_1 = I, \ \omega_1 = \omega_0, \ I_2 = I + mR^2$

$$\therefore \quad \omega_2 = \frac{I\omega_0}{I + mR^2}$$

Example 9.36 *If the radius of the earth contracts to half of its present value without change in its mass, what will be the new duration of the day?*

Sol. Present angular momentum of earth, $L_1 = I\omega = \frac{2}{5} MR^2\omega$

New angular momentum because of change in radius

$$L_2 = \frac{2}{5} M\left(\frac{R}{2}\right)^2 \omega'$$

If external torque is zero, then angular momentum must be conserved $L_1 = L_2$

$$\frac{2}{5} MR^2\omega = \frac{1}{4} \times \frac{2}{5} MR^2\omega'$$

i.e., $\quad \omega' = 4\omega$

$$T' = \frac{1}{4} T = \frac{1}{4} \times 24 = 6 \text{ h}$$

4 Rotational kinetic energy of the particles

The energy due to rotational motion of a body is known as rotational kinetic energy.

Let us consider a body rotating about a fixed axis. For a rotating rigid body kinetic energy is the sum of kinetic energies of individual particles.

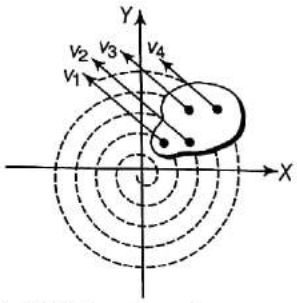


Fig. 9.11 Rotation of a rigid body

So, kinetic energy of a rotating body,

$$K = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \ldots + \frac{1}{2}m_nv_n^2$$

or
$$K = \sum_{i=1}^{n}\frac{1}{2}m_iv_i^2$$

Substituting $v = r\omega$, $\boldsymbol{K = \frac{1}{2}m_ir_i^2\omega^2}$

where, m_i is the mass of particle, v_i is the linear velocity. As, the angular velocity ω is same for all the particles of rigid body.

So,
$$\boldsymbol{K = \frac{1}{2}\omega^2\left(\sum_{i=1}^{n}m_ir_i^2\right)} \quad \text{...(i)}$$

The quantity $\sum(m_ir_i^2)$ depends on distribution of mass around axis of rotation. This quantity is called moment of inertia (I).

Kinetic energy,
$$\boxed{K = \frac{1}{2}I\omega^2} \quad \text{...(ii)}$$

When $\omega = 1$, rotational KE $= \frac{1}{2}I$

or Moment of inertia, $I = 2 \times$ rotational KE

Hence, **the moment of inertia of a rigid body about an axis of rotation is numerically equal to twice the rotational kinetic energy of the body when it is rotating with unit angular velocity about that axis.**

Rotational kinetic energy depends upon axis of rotation.

Example 9.37 *A wheel is rotating at a rate of 1000 rpm and its kinetic energy is 10^6 J. Determine the moment of inertia of the wheel about its axis of rotation.*

Sol. Given, $\nu = 1000 \text{ rpm} = \frac{1000}{60} \text{ rps} = \frac{50}{3} \text{ rps}$

$$\omega = 2\pi\nu = \frac{100\pi}{3} \text{ rads}^{-1}$$

$\because$ Rotational kinetic energy $= \frac{1}{2}I\omega^2$

$$\therefore \quad 10^6 = \frac{1}{2} \times I \times \left(\frac{100\pi}{3}\right)^2$$

$$\Rightarrow \quad I = \frac{2 \times 10^6 \times 9}{(100)^2\pi^2} = \frac{200 \times 9}{9.87}$$

$$= 182.4 \text{ kg-m}^2$$

Example 9.38 *A flywheel of mass 0.2 kg and radius 10 cm is rotating with $5/\pi$ rev/s about an axis perpendicular to its plane passing through its centre. Calculate angular momentum and kinetic energy of flywheel.*

Sol. Angular velocity, $\omega = 2\pi \times \frac{5}{\pi} = 10$ rad/s

and moment of inertia, $I = mr^2 = (0.2)(0.1)^2 = 2 \times 10^{-3}$ kg-m^2

Angular momentum $= I\omega = 2 \times 10^{-3} \times 10 = 2 \times 10^{-2}$ J-s

or $= 2 \times 10^{-2}$ kgm^{2}s^{-1}

Kinetic energy $= \frac{1}{2}I\omega^2 = \frac{1}{2} \times 2 \times 10^{-3} \times (10)^2 = 0.1$ J

Work energy theorem in rotational motion

According to work energy theorem in rotational motion, the change in the rotational kinetic energy of a rigid body is equal to the work done by external torques acting on the body.

The work done by torque = Change in rotational kinetic energy

$$\boxed{W = \frac{1}{2}I\omega_2^2 - \frac{1}{2}I\omega_1^2}$$

Example 9.39 *An energy of 484 J is spent in increasing the speed of a flywheel from 60 rpm to 360 rpm. Calculate moment of inertia of flywheel.*

Sol. Energy spent, $W = 484$ J

Initial speed, $\omega_1 = 60 \text{ rpm} = \frac{60}{60} \times 2\pi = 2\pi \text{ rad s}^{-1}$

Final speed, $\omega_2 = 360 \text{ rpm} = \frac{360}{60} \times 2\pi = 12\pi \text{ rad s}^{-1}$

Moment of inertia, $I = ?$

Energy spent, $W = E_2 - E_1 = \frac{1}{2}I\omega_2^2 - \frac{1}{2}I\omega_1^2$

$$= \frac{1}{2}I[(12\pi)^2 - (2\pi)^2] = 70I\pi^2$$

$$\Rightarrow \quad I = \frac{484}{70 \times \pi^2} = 0.7 \text{ kg-m}^2.$$

Example 9.40 *If the rotational kinetic energy of a body is increased by 300%, then determine percentage increase in its angular momentum.*

Sol. Percentage increase in momentum $= \frac{L_2 - L_1}{L_1} \times 100$

$$L \propto \sqrt{E} \Rightarrow E_1 = E$$

and $$E_2 = E + \frac{300}{100}E$$

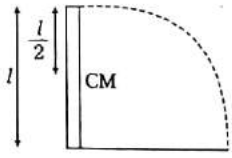
$$\Rightarrow \quad E_2 = 4E$$

Increase in momentum $= \frac{\sqrt{E_2} - \sqrt{E_1}}{\sqrt{E_1}} \times 100$

$$= \frac{\sqrt{4E} - \sqrt{E}}{\sqrt{E}} \times 100 = 100\%$$

Example 9.41 *A thin meter scale is kept vertical by placing its one end on floor, keeping the end in contact stationary, it is allowed to fall. Calculate the velocity of its upper end when it hit the floor.*

Sol. Loss in PE = gain in rotational



$$\text{KE} = \frac{mgl}{2} = \frac{1}{2}I\omega^2 = \frac{1}{2}\frac{ml^2}{3} \times \frac{v^2}{l^2} \Rightarrow v = \sqrt{3gl}$$

Check point 9.2

1. The torque of a force $\mathbf{F} = -6\hat{\mathbf{i}}$ acting at a point $\mathbf{r} = 4\hat{\mathbf{j}}$ about origin will be
(a) $-24\hat{\mathbf{k}}$ (b) $24\hat{\mathbf{k}}$
(c) $24\hat{\mathbf{j}}$ (d) $24\hat{\mathbf{i}}$

2. Moment of a force of magnitude 20 N acting along positive x-direction at point $(3m, 0, 0)$ about the point (0, 2, 0) (in N-m) is
(a) 20 (b) 60
(c) 40 (d) 30

3. The torque of force $\mathbf{F} = -3\hat{\mathbf{i}} + \hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ acting on a point $\mathbf{r} = 7\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$ about origin will be
(a) $14\hat{\mathbf{i}} - 38\hat{\mathbf{j}} + 16\hat{\mathbf{k}}$ (b) $4\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$
(c) $-14\hat{\mathbf{i}} + 38\hat{\mathbf{j}} - 16\hat{\mathbf{k}}$ (d) $-21\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$

4. The torque of a force $\mathbf{F} = -2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ acting on a point $\mathbf{r} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$ about origin will be
(a) $8\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ (b) $-8\hat{\mathbf{i}} - 5\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$
(c) $8\hat{\mathbf{i}} - 5\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ (d) $-8\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$

5. A door 1.6 m wide requires a force of 1 N to be applied at the free and to open or close it. The force that is required at a point 0.4 m distant from the hinges for opening or closing the door is
(a) 1.2 N (b) 3.6 N
(c) 2.4 N (d) 4 N

6. A flywheel of moment of inertia 2 kg-m^2 is rotated at a speed of 30 rad/s. A tangential force at the rim stops the wheel in 15 second. Average torque of the force is
(a) 4 N-m (b) 2 N-m
(c) 8 N-m (d) 1 N-m

7. A mass of 10 kg connected at the end of a rod of negligible mass is rotating in a circle of radius 30 cm with an angular velocity of 10 rad/s. If this mass is brought to rest in 10 s by a brake, what is the magnitude of the torque applied?
(a) 0.9 N-m (b) 1.2 N-m
(c) 2.3 N-m (d) 0.5 N-m

8. A disc is rotating with angular velocity ω. A force $\mathbf{F}$ acts at a point whose position vector with respect to the axis of rotation is $\mathbf{r}$. The power associated with torque due to the force is given by
(a) $(\mathbf{r} \times \mathbf{F}) \cdot \omega$ (b) $(\mathbf{r} \times \mathbf{F}) \times \omega$
(c) $\mathbf{r} \times (\mathbf{F} \cdot \omega)$ (d) $\mathbf{r} \cdot (\mathbf{F} \times \omega)$

9. Angular momentum is
(a) moment of momentum
(b) product of mass and angular velocity
(c) product of moment of inertia and velocity
(d) moment in angular motion

10. The unit mass having $\mathbf{r} = 8\hat{\mathbf{i}} - 4\hat{\mathbf{j}}$ and $\mathbf{v} = 8\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$ in its angular momentum is
(a) 64 units in $-\hat{\mathbf{k}}$ direction
(b) 64 units in $+\hat{\mathbf{k}}$ direction
(c) 64 units in $+\hat{\mathbf{j}}$ direction
(d) 64 units in $+\hat{\mathbf{i}}$ direction

11. If the earth is a point mass of 6×10^{24} kg revolving around the sun at a distance of 1.5×10^8 km and in time, $T = 3.14 \times 10^7$ s, then the angular momentum of the earth around the sun is
(a) 1.2×10^{18} kg m^2/s (b) 1.8×10^{29} kg m^2/s
(c) 1.5×10^{37} kg m^2/s (d) 2.7×10^{40} kg m^2/s

12. A particle with the position vector **r** has linear momentum **p**. Which of the following statements is true in respect of its angular momentum **L** about the origin?
(a) **L** acts along **p**
(b) **L** acts along **r**
(c) **L** is maximum when **p** and **r** are parallel
(d) **L** is maximum when **p** is perpendicular to **r**

13. By keeping moment of inertia of a body constant, if we double the time period, then angular momentum of body
(a) remains constant (b) becomes half
(c) doubles (d) quadruples

14. If torque is zero, then
(a) angular momentum is conserved
(b) linear momentum is conserved
(c) energy is conserved
(d) angular momentum is not conserved

15. Total angular momentum of a rotating body remains constant, if the net torque acting on the body is
(a) zero (b) maximum
(c) minimum (d) unit

16. The angular momentum of a rotating body changes from A_0 to $4A_0$ in 4 min. The torque acting on the body is
(a) $\frac{3}{4}A_0$ (b) $4A_0$ (c) $3A_0$ (d) $\frac{3}{2}A_0$

17. If the radius of earth contracts $1/n$ of its present day value, the length of the day will be approximately
(a) $\frac{24}{n}$ h (b) $\frac{24}{n^2}$ h
(c) $24n$h (d) $24n^2$h

18. A thin circular ring of mass M and radius R is rotating about its axis with a constant angular velocity ω. Two objects each of mass m are attached gently to the ring. The wheel now rotates with an angular velocity
(a) $\frac{\omega M}{(m+M)}$ (b) $\frac{\omega(M-2m)}{(M+2m)}$
(c) $\frac{\omega M}{(M+2m)}$ (d) $\frac{\omega(M+2m)}{M}$

19. A disc of mass 2 kg and radius 0.2 m is rotating with angular velocity 30 rad s^{-1}. What is angular velocity, if a mass of 0.25 kg is put on periphery of the disc?
(a) 24 rad s^{-1} (b) 36 rad s^{-1}
(c) 15 rad s^{-1} (d) 26 rad s^{-1}

20. Circular disc of mass 2 kg and radius 1 m is rotating about an axis perpendicular to its plane and passing through its centre of mass with a rotational kinetic energy of 8 J. The angular momentum in (Js) is
(a) 8 (b) 4
(c) 2 (d) 1

5 Rolling motion

When a body perform(s) translatory motion as well as rotational motion then this type of motion is known as rolling motion. *e.g.*, motion of football rolling on a surface. In rolling motion, the velocity of centre of mass represents linear motion while angular velocity represents rotational motion.

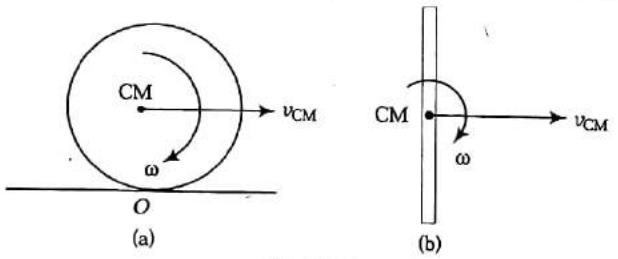


Fig. 9.12

The kinetic energy of a rolling body is equal to the sum of kinetic energies of translational and rotational motion.

$\therefore$ Total KE of a rolling body = Rotational KE + Translational KE

$$= \frac{1}{2}I\omega^2 + \frac{1}{2}mv_{CM}^2$$

Example 9.42 *A disc of mass 5 kg and radius 50 cm rolls on the ground at the rate of 10 ms^{-1}. Determine the kinetic energy of the disc.*

Sol. Given, mass of the disc, $M = 5$ kg

Radius of the disc, $R = 50$ cm $= \frac{1}{2}$ m.

Linear velocity of the disc, $v = 10$ ms^{-1}.

$\because$ $v = R\omega \Rightarrow 10 = \frac{1}{2}\omega$

$\Rightarrow$ $\omega = 10 \times 2 = 20$ rad s^{-1}

Also, moment of inertia of disc about an axis through its centre,

$$I = \frac{1}{2}MR^2$$

$\Rightarrow$ $$I = \frac{1}{2} \times 5 \times \left(\frac{1}{2}\right)^2 = \frac{5}{8} \text{kg-m}^2$$

$\because$ Kinetic energy of translation $= \frac{1}{2}mv^2$

$$= \frac{1}{2} \times 5 \times (10)^2 = 250\text{J}$$

$\because$ Kinetic energy of rotation $= \frac{1}{2}I\omega^2$

$$= \frac{1}{2} \times \frac{5}{8} \times (20)^2 = 125 \text{ J}$$

$\therefore$ Total kinetic energy $= (250 + 125)$ J $= 375$ J

Classification of rolling motion

Depending on the fact that relative velocity of point of contact of the body undergoing rolling motion, with the platform is zero or non zero, *rolling motion is classified into two categories, which are as follows.*

(i) Pure rolling or rolling without slipping/sliding or perfect rolling motion.

(ii) Impure rolling or rolling with slipping/sliding or imperfect rolling motion.

Uniform pure rolling

If the velocity of point of contact with respect to the surface is zero then it is known as pure rolling.

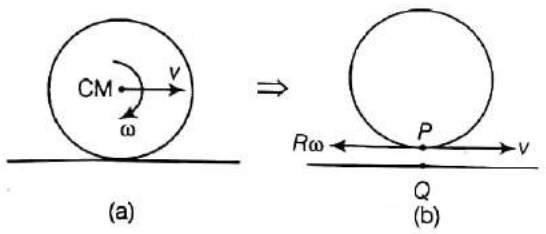


Fig. 9.13

For pure rotational motion velocity of centre of mass is zero given by the relation,

$$\therefore \qquad v = R\omega$$

In the above figure, when we observe the motion of contact point *P on the disc we find that it has two simultaneous motion.*

(i) Linear forwards velocity v because whole body is moving forward.

(ii) Linear backward velocity ωr due to rotation of the disc as a whole in clockwise direction. In case of pure rolling these two velocities become equal. In case $v > \omega r$, disc will start-slipping on the ground.

This is the condition of pure rolling on a stationary ground. Sometimes it is simply known as rolling.

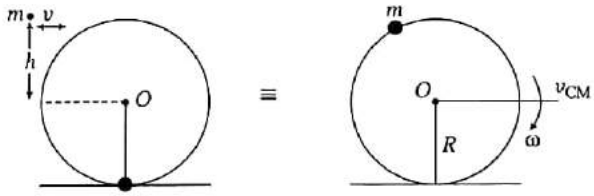
Suppose the base over which the disc in rolling, is also moving with some velocity (say v_0), then in that case condition of pure rolling is different.

e.g., in the above figure, $v_P = v_Q$ or $v - R\omega = v_0$

Thus, in this case $v - R\omega \neq 0$, but $v - R\omega = v_0$. **By uniform pure rolling we mean that v and ω are constant.** They are neither increasing nor decreasing.

Example 9.43 *A hollow sphere of mass M lies on a rough horizontal plane when a particle of mass m travelling with speed v collides and sticks with it. If line of motion of the particle is at height h, above the centre of sphere, find h if the body rolls without slipping after collision.*

Sol. Mass m moving with velocity v creating rotation in sphere.



Conservation of linear momentum

$$mv = (M + m)\, v_{CM} \;\Rightarrow\; v_{CM} = \frac{mv}{(M + m)}$$

Conservation of angular momentum about O

$$mvh = I_0\omega = \left(\frac{2}{3}MR^2 + mR^2\right)\omega$$

$$\omega = \frac{mvh}{\left(\frac{2}{3}M + m\right)R^2}$$

For pure rolling, $\quad v_{CM} = \omega R$

$$\frac{mv}{M + m} = \frac{mvh}{\left(\frac{2}{3}M + m\right)R}$$

$$h = \frac{\left(\frac{2}{3}M + m\right)R}{(M + m)} = \frac{2R}{3} \qquad \text{(Since, } m << M\text{)}$$

Kinetic energy of rolling motion

The kinetic energy of a rolling body is the sum of kinetic energy of a translation motion and kinetic energy of rotational motion.

Kinetic energy of rolling motion, $E = \frac{1}{2}I\omega^2 + \frac{1}{2}Mv_{CM}^2$

Substituting the value of $I = MK^2$, where K is radius of gyration and $v_{CM} = R\omega$, we get

$$E = \frac{1}{2}MK^2\left(\frac{v_{CM}}{R}\right)^2 + \frac{1}{2}Mv_{CM}^2$$

$$E = \frac{1}{2}Mv_{CM}^2\left(1 + \frac{K^2}{R^2}\right)$$

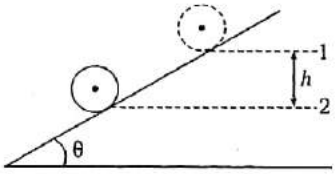
The ratio of translational energy, rotational energy and total energy can be calculated as

$$E_{\text{translational}} : E_{\text{rotation}} : E_{\text{total}}$$

$$1 : \frac{K^2}{R^2} : 1 + \frac{K^2}{R^2}$$

Example 9.44 *A solid cylinder of mass m and radius r starts rolling down an inclined plane of inclination θ. Friction is enough to prevent slipping. Find the speed of its centre of mass when its centre of mass has fallen a height h.*

Sol. Considering the two shown positions of the cylinder. As it does not slip hence total mechanical energy will be conserved.



Energy at position 1 is, $E_1 = mgh$

Energy at position 2 is

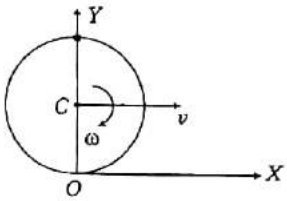
$$E_2 = \frac{1}{2} mv_{CM}^2 + \frac{1}{2} I_{CM}\,\omega^2$$

$\because$ $\frac{v_{CM}}{r} = \omega$ and $I_{CM} = \frac{mr^2}{2} \Rightarrow E_2 = \frac{3}{4} mv_{CM}^2$

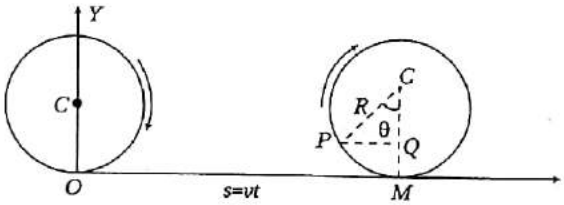
From law of conservation of energy, $E_1 = E_2$

$$\Rightarrow \quad v_{CM} = \sqrt{\frac{4}{3} gh}$$

Example 9.45 *A disc of radius R start at time t = 0 moving along the positive X-axis with linear speed v and angular speed ω. Find the x and y-coordinates of the bottommost point at any time t.*



Sol. At time t, the bottommost point will rotate an angle $\theta = \omega t$ with respect to the centre of the disc C. The centre C will travel a distance $s = vt$.



In the figure, $PQ = R \sin\theta = R \sin \omega t$

and $CQ = R \cos\theta = R \cos \omega t$

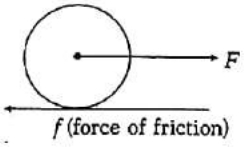
Coordinates of point P at time t are,

$$x = OM - PQ = vt - R \sin \omega t$$

and $$y = CM - CQ = R - R \cos \omega t$$

$\therefore$ $$(x, y) \equiv (vt - R \sin \omega t, R - R \cos \omega t)$$

Example 9.46 *A horizontal force F acts on the hollow sphere at its centre as shown. Coefficient of friction between ground and sphere is μ. What is maximum value of F, for which there is no slipping?*



Sol.

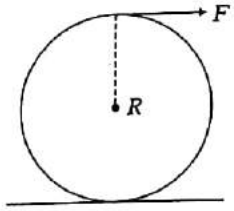
$$F - f = Ma$$

$$f \cdot R = \frac{2}{5} MR^2 \frac{a}{R}$$

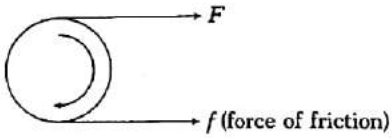
$$\Rightarrow \quad f = \frac{2}{5} Ma \Rightarrow f = \frac{2}{7} F$$

$$\frac{2}{7} F \le \mu mg \Rightarrow F \le \frac{7}{2} \mu mg$$

Example 9.47 *A tangential force F acts at the top of a thin spherical shell of mass m and radius R. Find the acceleration of the shell if it rolls without slipping.*



Sol. Let f be the force of friction between the shell and the horizontal surface.



For translational motion

$$F + f = ma \quad \text{...(i)}$$

For rotational motion,

$$FR - fR = I\alpha = I\frac{a}{R} \quad (\because a = R\alpha \text{ for pure rolling})$$

$$\Rightarrow \quad F - f = I\frac{a}{R^2} \quad \text{...(ii)}$$

Adding Eqs. (i) and (ii)

$$2F = \left(m + \frac{I}{R^2}\right)a = \left(m + \frac{2}{3}m\right)a = \frac{5}{3} ma$$

or $$F = \frac{5}{6} ma \qquad \left[\because I_{shell} = \frac{2}{3} mR^2\right]$$

$$\Rightarrow \quad a = \frac{6F}{5m}$$

Rolling down on a rough inclined plane

A body of mass m and radius R rolling down a plane inclined at an angle θ with the horizontal. If the body rolls without slipping then the centre of mass of the body moves in a straight line.

Different forces acting on the body are

(i) Friction force (f) acting upwards at the point of contact with inclined plane and parallel to the inclined plane.

(ii) Normal reaction of inclined plane (N).

(iii) Weight of the body (mg) acting vertically downwards through the centre of mass of body.

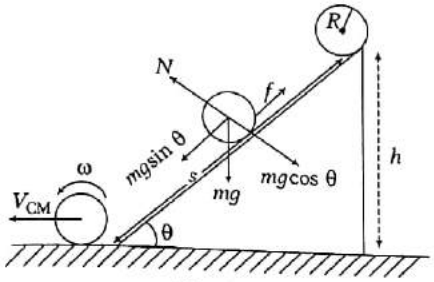


Fig. 9.14

For rolling without slipping the friction force is static and no work is done by friction force.

So, from law of conservation of mechanical energy,

$$mgh = \frac{1}{2}mv_{CM}^2 + \frac{1}{2}I\omega^2$$

Putting $I = mK^2$ where, $K =$ radius of gyration

$v_{CM} = R\omega$

we get, $$\boxed{v_{rolling} = \sqrt{\frac{2gh}{1+K^2/R^2}}}$$

From Fig. 9.14, $h = s\sin\theta$

$$\therefore \quad \boxed{v_{rolling} = \sqrt{\frac{2g\,s\sin\theta}{1+K^2/R^2}}}$$

$$= \sqrt{\frac{2g\ s\ \sin\theta}{1+n^2}} \qquad \text{(where } K = nR\text{)}$$

For disc $n = \frac{1}{\sqrt{2}}$

If body slides then, $\frac{K^2}{R^2} = 0 \Rightarrow \boxed{v_{sliding} = \sqrt{2gh} = \sqrt{2g\,s\sin\theta}}$

From the above formula we see that **velocity of rolling motion is independent of its mass (M) and radius (R). It means all uniform solid spheres or any other similar objects will reach the bottom with same speed because the ratio K^2/R^2 will be same for similar bodies.**

Acceleration, $$\boxed{a_{rolling} = \frac{g\sin\theta}{1+K^2/R^2} = \frac{g\ \sin\theta}{1+n^2}}$$

It is also independent of mass and radius.

If body slides then, $\frac{K^2}{R^2} = 0$ (for frictional less surface)

$$\therefore \quad \boxed{a_{slides} = g\sin\theta}$$

Time taken by rolling body to reach the bottom,

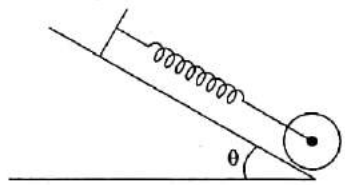
$$t = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}\left(1+\frac{K^2}{R^2}\right)}$$

A body with smaller value of K^2/R^2 will take less time to reach the bottom.

Change in kinetic energy due to rolling ($v_2 > v_1$)

$$= \frac{1}{2}m\left(1+\frac{K^2}{R^2}\right)(v_2^2 - v_1^2)$$

Example 9.48 *A sphere of mass m attached to a spring on incline plane as shown in figure is held in unstretched position of spring. Suddenly sphere is left free what is the maximum extension of spring if friction allows only rolling of sphere about horizontal diametre?*



Sol. At the point of maximum extension sphere is at rest. No work is done by frictional force in rolling.

$\Rightarrow$ Loss in gravitational PE = Gain in PE of spring

$$\Rightarrow \quad \frac{1}{2}kx^2 = mgx\sin\theta \Rightarrow x = \frac{2mg\sin\theta}{k}$$

Note

1. For pure rolling motion there should be friction on surface because it provides torque to the body, on smooth surface rolling is not possible.
2. Velocity of falling and sliding bodies from inclined plane are equal and is more than rolling body.
3. For rolling along inclined plane acceleration is maximum in case of falling and minimum in case of rolling. Falling body reaches the bottom first and rolling body in the last.
4. When ring (R), disc (D), hollow sphere (H) and solid sphere (S) rolls on same inclined plane then

$$v_S > v_D > v_H > v_R \Rightarrow a_S > a_D > a_H > a_R \Rightarrow t_S < t_D < t_H < t_R$$

Example 9.49 *Calculate the kinetic energy of rolling ring of mass 0.2 kg about an axis passing through its centre of mass and perpendicular to it, if its centre of mass is moving with a velocity of 3 m/s.*

Sol. $\text{KE} = \frac{1}{2}mv_{CM}^2\left(1+\frac{K^2}{R^2}\right)$

Moment of inertia of the ring about an axis passing through its centre of mass $= MR^2$

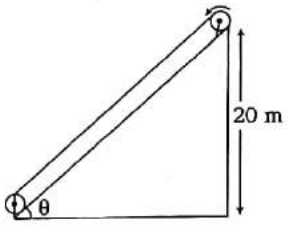
$\therefore \quad K = 1$ (radius of gyration)

$m = 0.2$ kg, $v_{CM} = 3$ m/s

$$\text{KE} = \frac{1}{2}\times 0.2\times 9(1+1) = 0.2\times 9 = 1.8\ \text{kgm}^2\text{s}^{-2}$$

Example 9.50 *A solid sphere is rolling down an inclined plane without slippping of height 20 m. Calculate the maximum velocity with which it will reach the bottom of the plane. ($g = 10\ m/s^2$)*

Sol. When the sphere rolls down, its potential energy changes to KE of rotation. Therefore, KE = PE



$$\frac{1}{2}mv^2\left(1+\frac{K^2}{R^2}\right)=mgh$$

$$\Rightarrow \qquad v=\sqrt{\frac{2gh}{(1+K^2/R^2)}}$$

Moment of inertia of solid sphere $=\frac{2}{5}MR^2$

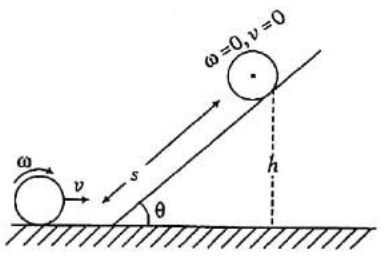
$$\therefore \qquad K^2=\frac{2}{5}R^2$$

$$\text{Hence, } v=\sqrt{\frac{2gh}{\left(1+\frac{2}{5}\right)}}=\sqrt{\frac{2gh\times 5}{7}}=\sqrt{\frac{2\times 10\times 20\times 5}{7}}$$

$$=\sqrt{285.714}=16.90\ \text{m/s}$$

Example 9.51 *A solid sphere is rolling without slipping on a level surface at a constant speed of 2 ms^{-1}. How far can it roll up a 30° ramp before it stops?*

Sol. The moment of inertia of the sphere about its axis of rotation is $I=\frac{2}{5}mr^2$. Suppose it rises to height h before stopping. Using the conservation of energy, we get



$$mgh=\frac{1}{2}mv^2+\frac{1}{2}I\omega^2=\frac{1}{2}mv^2+\frac{1}{2}\left(\frac{2}{5}mr^2\right)\omega^2$$

$$=\frac{1}{2}mv^2+\frac{1}{5}mv^2=\frac{7}{10}mv^2 \quad \text{or} \quad h=\frac{7v^2}{10g}$$

$\therefore$ The distance rolled up on the ramp is

$$s=\frac{h}{\sin\theta}=\frac{7v^2}{10g\sin\theta}=\frac{7(2\ \text{ms}^{-1})^2}{10(9.8\text{ms}^{-2})\times 1/2}=\frac{4}{7}\ \text{metre}\simeq 57\ \text{cm}$$

Example 9.52 *When a body is under pure rolling, the fraction of its total kinetic energy which is the purely rotational is 2/5. Identify the body.*

Sol. In this type of questions calculate the expression for moment of inertia I and that helps in identifying the body.

$$\text{Rotational KE}=\frac{1}{2}I\omega^2$$

$$\text{Translational KE}=\frac{1}{2}Mv^2$$

$$\therefore \qquad \text{Total KE}=\frac{1}{2}I\omega^2+\frac{1}{2}Mv^2$$

$$\text{Given,} \quad \frac{2}{5}=\frac{\text{Rotational KE}}{\text{Total KE}} \quad \text{or} \quad \frac{2}{5}=\frac{(1/2)I\omega^2}{(1/2)I\omega^2+(1/2)Mv^2}$$

$$\text{For pure rolling put } v=\omega R \text{ or } \frac{2}{5}=\frac{I\omega^2}{I\omega^2+M\omega^2R^2}$$

$$\therefore \qquad I=\frac{2MR^2}{3}$$

So, the rolling body is hollow sphere.

Example 9.53 *A uniform sphere of mass m and radius R rolls without slipping down an inclined plane set at an angle θ to the horizontal.*

Find :

(i) the friction coefficient at which slipping is absent

(ii) the kinetic energy of the sphere t seconds after the beginning of motion.

Sol. (i) If a body rolls without slipping

$$\mu_{\min}=\frac{\tan\theta}{I+\frac{R^2}{v^2}}=\frac{\tan\theta}{1+\frac{5}{2}}=\frac{2}{7}\tan\theta$$

(ii) For pure rolling

$$a=\frac{g\sin\theta}{1+\frac{k^2}{R^2}}=\frac{g\sin\theta}{1+\frac{2}{5}}=\frac{5}{7}g\sin\theta$$

Linear velocity after time t

$$v=u+at=0+\frac{5g\sin\theta t}{7}$$

In pure rolling KE

$$K=\frac{1}{2}mv^2\left(1+\frac{k^2}{R^2}\right)=\frac{1}{2}mv^2\left(1+\frac{2}{5}\right)$$

$$=\frac{7}{10}mv^2=\frac{7}{10}m\left(\frac{5g\sin\theta t}{7}\right)^2=\frac{5}{14}mg^2\sin^2\theta t^2$$

Check point 9.3

1. At any instant, a rolling body may be considered to be in pure rotation about an axis through the point of contact. This axis is translating forward with speed
(a) equal to centre of mass (b) zero
(c) twice of centre of mass (d) None of these

2. A wheel of radius R rolls on the ground with a uniform velocity v. The velocity of topmost point relative to the bottommost point is
(a) v (b) $2v$ (c) $v/2$ (d) zero

3. The centre of a wheel rolling on a plane surface moves with a speed v_0. A particle on the rim of the wheel at the same level as the centre will be moving at speed
(a) zero (b) v_0 (c) $\sqrt{2}v_0$ (d) $2v_0$

4. A uniform solid spherical ball is rolling down a smooth inclined plane from a height h. The velocity attained by the ball when it reaches the bottom of the inclined plane is v. If the ball is now thrown vertically upwards with the same velocity v, the maximum height to which the ball will rise is
(a) $\frac{5h}{8}$ (b) $\frac{3h}{5}$
(c) $\frac{5h}{7}$ (d) $\frac{7h}{9}$

5. A small object of uniform density rolls up a curved surface with an initial velocity v'. It reaches upto a maximum height of $\frac{3v^2}{4g}$ with respect to the initial position. The object is
(a) ring (b) solid sphere
(c) hollow sphere (d) disc

6. A sphere is rolling down a plane of inclination θ to the horizontal. The acceleration of its centre down the plane is
(a) $g \sin\theta$ (b) less than $g \sin\theta$
(c) greater than $g \sin\theta$ (d) zero

7. Two uniform solid spheres having unequal masses and unequal radii are released from rest from the same height on a rough incline. If the spheres roll without slipping
(a) the heavier sphere reaches the bottom first
(b) the bigger sphere reaches the bottom first
(c) the two spheres reach the bottom together
(d) the information given is not sufficient to tell which sphere will reach the bottom first

8. A solid sphere of mass m rolls without slipping on an inclined plane of inclination θ. The linear acceleration of the sphere is
(a) $\frac{7}{5} g \sin\theta$ (b) $\frac{2}{7} g \sin\theta$ (c) $\frac{3}{7} g \sin\theta$ (d) $\frac{5}{7} g \sin\theta$

9. An inclined plane makes an angle of 30° with the horizontal. A solid sphere rolling down this inclined plane from rest without slipping has a linear acceleration equal to
(a) $\frac{g}{3}$ (b) $\frac{2g}{3}$
(c) $\frac{5g}{7}$ (d) $\frac{5g}{14}$

10. A thin uniform circular ring is rolling down an inclined plane of inclination 30° without slipping. Its linear acceleration along the inclined plane is
(a) g (b) $\frac{g}{2}$
(c) $\frac{g}{3}$ (d) $\frac{g}{4}$

11. The ratio of the time taken by a solid sphere and that taken by a disc of the same mass and radius to roll down a rough inclined plane from rest, from the same height is
(a) 15 : 14 (b) $\sqrt{15} : \sqrt{14}$
(c) 14 : 15 (d) $\sqrt{14} : \sqrt{15}$

12. A solid sphere, a hollow sphere and a disc, all having same mass and radius, are placed at the top of an inclined plane and released. The friction coefficients between the objects and the incline are same and not sufficient to allow pure rolling. Least time will be taken in reaching the bottom by
(a) the solid sphere
(b) the hollow sphere
(c) the disc
(d) All will take same time

13. A solid sphere, a hollow sphere and a disc, all having same mass and radius, are placed at the top of a smooth inclined and released. Least time will be taken in reaching the bottom by
(a) the solid sphere (b) the hollow sphere
(c) the disc (d) all will take same time

14. The speed of a uniform spherical shell after rolling down an inclined plane of vertical height h from rest, is
(a) $\sqrt{\frac{10gh}{7}}$ (b) $\sqrt{\frac{6gh}{5}}$
(c) $\sqrt{\frac{4gh}{5}}$ (d) $\sqrt{2gh}$

15. A solid cylinder rolls down an inclined plane of height 3 m and reaches the bottom of plane with angular velocity, of $2\sqrt{2}$ rad s^{-1}. The radius of cylinder must be (Take $g = 10$ ms^{-2})
(a) 5 cm (b) 0.5 cm
(c) $\sqrt{10}$ cm (d) $\sqrt{5}$ m

Chapter exercises

A Taking it together

(Assorted questions of the chapter for advanced level practice)

1. The angular momentum of a system of particles is conserved
(a) when no external force acts upon the system
(b) when no external torque acts upon the system
(c) when no external impulse acts upon the system
(d) when axis of rotation remains same

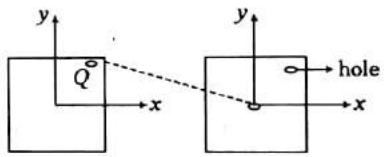
2. If a person standing on a rotating disc stretches out his hands, the angular speed will
(a) increase (b) decrease
(c) remain same (d) None of these

3. When a disc rotates with uniform angular velocity, which of the following is not true? **(NCERT Exemplar)**
(a) The sense of rotation remains same
(b) The orientation of the axis of rotation remains same
(c) The speed of rotation is non-zero and remains same
(d) The angular acceleration is non-zero and remains same

4. A ring of diameter 0.4 m and of mass 10 kg is rotating about its axis at the rate of 1200 rpm. The angular momentum of the ring is
(a) $60.28\ kgm^2s^2$
(b) $55.26\ kgm^2s^2$
(c) $40.28\ kgm^2s^2$
(d) $50.28\ kgm^2s^2$

5. A uniform disc of radius a and mass m, is rotating freely with angular speed ω in a horizontal plane about a smooth fixed vertical axis through its centre. A particle, also of mass m, is suddenly attached to the rim of the disc and rotates with it. The new angular speed is
(a) $\frac{\omega}{6}$ (b) $\frac{\omega}{3}$
(c) $\frac{\omega}{2}$ (d) $\frac{\omega}{5}$

6. A uniform square plate has a small piece Q of an irregular shape removed and glued to the centre of the plate leaving a hole behind in figure. The moment of inertia about the Z-axis is then, **(NCERT Exemplar)**



(a) increased
(b) decreased
(c) the same
(d) changed in unpredicted manner

7. A diver in a swimming pool bends his head before diving, because it
(a) decreases his moment of inertia
(b) decreases his angular velocity
(c) increases his moment of inertia
(d) decreases his linear velocity

8. A body is in pure rotation. The linear speed v of a particle, the distance r of the particle from the axis and the angular velocity ω of the body are related as $\omega = \frac{v}{r}$. Thus,
(a) $\omega \propto \frac{1}{r}$ (b) $\omega \propto r$
(c) $\omega = 0$ (d) ω is independent of r

9. The rotational kinetic energy of a body is E and its moment of inertia is I. The angular momentum is
(a) EI (b) $2\sqrt{(EI)}$
(c) $\sqrt{(2EI)}$ (d) $\frac{E}{I}$

10. A body is under the action of two equal and oppositely directed forces and the body is rotating with constant non-zero angular acceleration. Which of the following cannot be the separation between the lines of action of the forces?
(a) 1 m (b) 0.4 m (c) 0.25 m (d) zero

11. A particle performs uniform circular motion with an angular momentum L. If the frequency of the particle motion is doubled, the angular momentum becomes
(a) $2L$ (b) $4L$ (c) $\frac{L}{2}$ (d) $\frac{L}{4}$

12. Moment of a force of magnitude 10 N acting along positive y-direction at point (2m, 0, 0) about the point (0, 1m, 0) in N-m is
(a) 10 (b) 20 (c) $10\sqrt{2}$ (d) 30

13. The radius of gyration of a uniform rod of length L about an axis passing through its centre of mass is
(a) $\frac{L}{2\sqrt{3}}$ (b) $\frac{L^2}{12}$ (c) $\frac{L}{\sqrt{3}}$ (d) $\frac{L}{\sqrt{2}}$

14. Five particles of masses 2 kg each are attached to the rim of a circular disc of radius 0.1 m and negligible mass. Moment of inertia of the system about the axis passing through the centre of the disc and perpendicular to its plane is
(a) 1 kg-m^2 (b) 0.1 kg-m^2
(c) 2 kg-m^2 (d) 0.2 kg-m^2

15. A particle is moving in a circular orbit with constant speed. Select wrong alternate.
(a) Its linear momentum is conserved
(b) Its angular momentum is conserved
(c) It is moving with variable velocity
(d) It is moving with variable acceleration

16. If the equation for the displacement of a particle moving on a circular path is given by, $\theta = 2t^3 + 0.5$ where, θ is in radian and t is in second, then the angular velocity of the particle after 2 s is
(a) 8 rad s^{-1} (b) 12 rad s^{-1} (c) 24 rad s^{-1} (d) 36 rad s^{-1}

17. A wheel is at rest. Its angular velocity increases uniformly and becomes 80 rad s^{-1} after 5 s. The total angular displacement is
(a) 800 rad (b) 400 rad (c) 200 rad (d) 100 rad

18. A rigid body rotates about a fixed axis with variable angular velocity equal to $\alpha - \beta t$, at the time t, where α, β are constants. The angle through which it rotates before its stops
(a) $\frac{\alpha^2}{2\beta}$ (b) $\frac{\alpha^2 - \beta^2}{2\alpha}$ (c) $\frac{\alpha^2 - \beta^2}{2\beta}$ (d) $\frac{(\alpha - \beta)\alpha}{2}$

19. A wheel is rotating at the rate of 33 rev min^{-1}. If it comes to stop in 20 s. Then, the angular retardation will be
(a) π rad s^{-2} (b) 11π rad s^{-2}
(c) $\frac{\pi}{200}$ rad s^{-2} (d) $\frac{11\pi}{200}$ rad s^{-2}

20. A wheel is rotating at 900 rpm about its axis. When power is cut off it comes to rest in 1 min. The angular retardation in rad s^{-2} is
(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{8}$

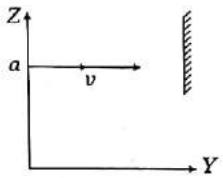
21. A wheel is subjected to uniform angular acceleration about its axis. Initially its angular velocity is zero. In the first 2 s, it rotates through an angle θ_1, in the next 2 s, it rotates through an angle θ_2. The ratio of $\frac{\theta_2}{\theta_1}$ is
(a) 1 (b) 2 (c) 3 (d) [illegible]

22. The motor of an engine is rotating about its axis with an angular velocity of 100 rpm. It comes to rest in 15 s, after being switched off. Assuming constant angular deceleration. What are the numbers of revolutions made by it before coming to rest?
(a) 12.5 (b) 40 (c) 32.6 (d) 15.6

23. If F be a force acting on a particle having the position vector r and τ be the torque of this force about the origin, then
(a) $\mathbf{r} \cdot \tau = 0$ and $\mathbf{F} \cdot \tau = 0$ (b) $\mathbf{r} \times \tau = 0$ and $\mathbf{F} \times \tau \neq 0$
(c) $\mathbf{r} \cdot \tau \neq 0$ and $\mathbf{F} \cdot \tau \neq 0$ (d) $\mathbf{r} \times \tau \neq 0$ and $\mathbf{F} \times \tau = 0$

24. A particle of mass 5 g is moving with a uniform speed of $3\sqrt{2}$ cm s^{-1} in the XY-plane along the line $y = 2\sqrt{5}$ cm. The magnitude of its angular momentum about the origin in g-cm^{2}s^{-1} is
(a) zero (b) 30 (c) $30\sqrt{2}$ (d) $30\sqrt{10}$

25. A particle of mass m is moving in YZ-plane with a uniform velocity v with its trajectory running parallel to + ve Y-axis and intersecting Z-axis at $z = a$ in figure. The change in its angular momentum about the origin as it bounces elastically from a wall at y = constant is



(NCERT Exemplar)
(a) $mva\hat{\mathbf{e}}_x$ (b) $2mva\hat{\mathbf{e}}_x$ (c) $ymv\hat{\mathbf{e}}_x$ (d) $2ymv\hat{\mathbf{e}}_x$

26. A particle of mass $m = 5$ units is moving with a uniform speed $v = 3\sqrt{2}$ units in the XY-plane along the line $y = x + 4$. The magnitude of the angular momentum about origin is
(a) zero (b) 60 unit (c) 7.5 unit (d) $40\sqrt{2}$ unit

27. A particle of mass 2 kg located at the position $(\hat{i} + \hat{j})$ m has a velocity $2(+\hat{i} - \hat{j} + \hat{k})$ ms^{-1}. Its angular momentum about Z-axis in kg-m^{2}s^{-1} is
(a) +4 (b) +8 (c) −4 (d) −8

28. A particle of mass m is projected with a velocity v making an angle of 45° with the horizontal. The magnitude of angular momentum of projectile about the point of projection when the particle is at its maximum height h is
(a) zero (b) $\frac{mvh}{\sqrt{2}}$ (c) mvh (d) $\sqrt{2}\,mvh$

29. A sphere rolls without slipping on a rough horizontal surface with centre of mass speed v_0. If mass of the sphere is M and its radius is R, then what is the angular momentum of the sphere about the point of contact?
(a) $\frac{5}{2}Mv_0R$ (b) $\frac{7}{5}Mv_0R$ (c) $\frac{3}{5}Mv_0R$ (d) $\frac{1}{2}Mv_0R$

30. A constant torque of 1000 N-m turns a wheel of moment of inertia 200 kg-m^2 about an axis through its centre. Its angular velocity after 3 s is
(a) 1 rad s^{-1} (b) 5 rad s^{-1}
(c) 10 rad s^{-1} (d) 15 rad s^{-1}

31. A merry-go-round, made of a ring-like platform of radius R and mass M, is revolving with angular speed ω. A person of mass M is standing on it. At one instant, the person jumps off the round, radially away from the centre of the round (as seen from the round). The speed of the round of afterwards is **(NCERT Exemplar)**
(a) 2ω (b) ω (c) $\frac{\omega}{2}$ (d) 0

32. A flywheel having a radius of gyration of 2 m and mass 10 kg rotates at an angular speed of 5 rad s^{-1} about an axis perpendicular to it through its centre. The kinetic energy of rotation is
(a) 500 J (b) 2000 J (c) 1000 J (d) 250 J

33. A flywheel is in the form of a uniform circular disc of radius 1 m and mass 2 kg. The work which must be done on it to increase its frequency of rotation from 5 rev s^{-1} to 10 rev s^{-1} is approximately
(a) 1.5×10^2 J (b) 3.0×10^2 J
(c) 1.5×10^3 J (d) 3.0×10^3 J

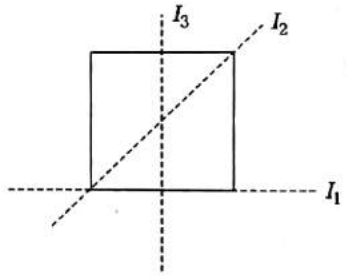
34. A rod is placed along the line, $y = 2x$ with its centre at origin. The moment of inertia of the rod is maximum about
(a) X-axis (b) Y-axis
(c) Z-axis (d) Data insufficient

35. The ratio of the radii of gyration of a circular disc and a circular ring of the same radii about a tangential axis perpendicular to plane of disc or ring is
(a) 1 : 2 (b) $\sqrt{5}:\sqrt{6}$ (c) 2 : 3 (d) $\frac{\sqrt{3}}{2}$

36. Let I_A and I_B be moments of inertia of a body about two axes A and B respectively. The axis A passes through the centre of mass of the body but B does not. Choose the correct option.
(a) $I_A < I_B$
(b) $I_A < I_B$, whether the axes are parallel or not parallel
(c) if the axes are parallel $I_A < I_B$
(d) if the axes are not parallel $I_A \geq I_B$

37. The ratio of the radii of gyration of a hollow sphere and a solid sphere of the same radii about a tangential axis is
(a) $\sqrt{\frac{7}{3}}$ (b) $\frac{5}{\sqrt{21}}$ (c) $\sqrt{\frac{21}{5}}$ (d) $\frac{25}{9}$

38. A square lamina is as shown in figure. The moment of inertia of the frame about the three axes as shown in figure are I_1, I_2 and I_3 respectively. Select the correct alternative.



(a) $I_2 = I_3 > I_1$ (b) $I_1 > I_2 > I_3$
(c) $I_2 = I_3 < I_1$ (d) $I_1 < I_2 < I_3$

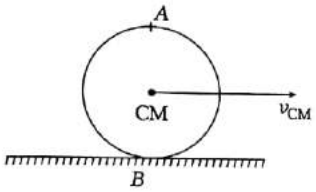
39. The ratio of the radii of gyration of a circular disc and a circular ring of the same radius about a tangential axis in the plane is
(a) $\sqrt{3}:\sqrt{4}$ (b) $\sqrt{5}:\sqrt{6}$
(c) $\sqrt{6}:\sqrt{5}$ (d) $\sqrt{4}:\sqrt{3}$

40. A thin uniform circular disc of mass M and radius R is rotating in a horizontal plane about an axis passing through its centre and perpendicular to its plane with an angular velocity ω. Another disc of same dimensions but of mass $\frac{1}{4}M$ is placed gently on the first disc co-axially. The angular velocity of the system is
(a) $\frac{2}{3}\omega$ (b) $\frac{4}{5}\omega$ (c) $\frac{3}{4}\omega$ (d) $\frac{1}{3}\omega$

41. A ring is kept on a rough inclined surface. But the coefficient of friction is less than the minimum value required for pure rolling. At any instant of time let K_T and K_R be the translational and rotational kinetic energies of the ring, then

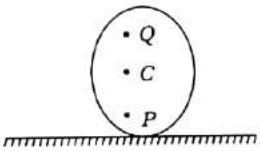
(a) $K_R = K_T$ (b) $K_R > K_T$ (c) $K_T > K_R$ (d) $K_R = 0$

42. A wheel of bicycle is rolling without slipping on a level road. The velocity of the centre of mass is v_{CM}, then true statement is



(a) the velocity of point A is $2v_{CM}$ and velocity of point B is zero
(b) the velocity of point A is zero and velocity of point B is $2v_{CM}$
(c) the velocity of point A is $2v_{CM}$ and velocity of point B is $-v_{CM}$
(d) the velocities of both A and B are v_{CM}

43. A disc is rolling without slipping on a horizontal surface with C, as its centre and Q and P the two points equidistant from C. Let v_P, v_Q and v_C be the magnitudes of velocities of points P, Q and C respectively, then



(a) $v_Q > v_C > v_P$ (b) $v_Q < v_C < v_P$
(c) $v_Q = v_P, v_C = \frac{1}{2} v_P$ (d) $v_Q < v_C > v_P$

44. A rigid body rotates with an angular momentum L. If its rotational kinetic energy is made 4 times, its angular momentum will become

(a) $4L$ (b) $16L$ (c) $\sqrt{2}\,L$ (d) $2L$

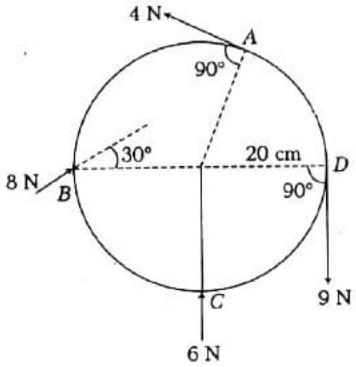
45. A ring and a disc of different masses are rotating with the same kinetic energy. If we apply a retarding torque τ on the ring, it stops after making n revolutions. After how many revolutions will the disc stop, if the retarding torque on it is also τ?

(a) $\frac{n}{2}$ (b) n
(c) $2n$ (d) Data insufficient

46. Work done by friction in case of pure rolling

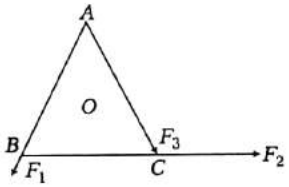
(a) is always zero
(b) is always positive
(c) is always negative
(d) may be positive, negative or zero

47. Forces are applied on a wheel of radius 20 cm as shown in the figure. The torque produced by the forces 4 N at A, 8 N at B, 6 N at C and 9 N at D at angles indicated is



(a) 5.4 N-m anti-clockwise (b) 1.80 N-m clockwise
(c) 2.0 N-m clockwise (d) 3.6 N-m clockwise

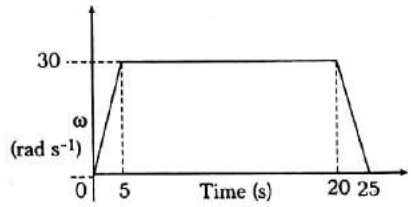
48. O is the centre of an equilateral triangle ABC. F_1, F_2 and F_3 are three forces acting along the sides AB, BC and AC as shown in figure.



What should be the magnitude of F_3 so that the total torque about O is zero?

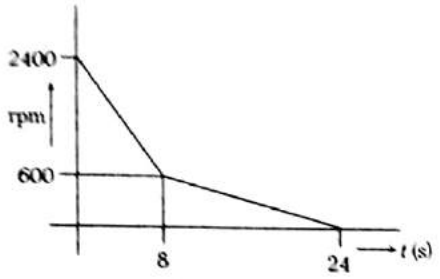
(a) $\frac{(F_1 + F_2)}{2}$ (b) $(F_1 - F_2)$
(c) $(F_1 + F_2)$ (d) $2(F_1 + F_2)$

49. The figure shows the angular velocity *versus* time graph of a flywheel. The angle, in radians through which the flywheel turns during 25 s is



(a) 120 (b) 480
(c) 600 (d) 750

50. A table fan, rotating at a speed of 2400 rpm, is switched off and the resulting variation of the rpm with time is shown in the figure. The total number of revolutions of the fan before it comes to rest is



(a) 420 (b) 190 (c) 280 (d) 380

51. A sphere can roll on a surface inclined at an angle θ, if the friction coefficient $\mu > (2/7)\, g \sin\theta$. Now suppose the friction coefficient is $(1/7)\, g \sin\theta$ and the sphere is released from rest on the incline, then
(a) it will stay at rest
(b) it will make pure translational motion
(c) it will translate and rotate about the centre
(d) the angular momentum of the sphere about its centre will remain constant

52. If a disc of mass m and radius r is reshaped into a ring of radius $2r$, the mass remaining the same, the radius of gyration about centroidal axis perpendicular to plane goes up by a factor of
(a) $\sqrt{2}$ (b) 2 (c) $2\sqrt{2}$ (d) 4

53. A disc of mass m and radius R is rolling on horizontal ground with linear velocity v. What is the angular momentum of the disc about an axis passing through bottommost point and perpendicular to the plane of motion?
(a) $\frac{3}{2} mvR$ (b) mvR
(c) $\frac{1}{2} mvR$ (d) $\frac{4}{3} mvR$

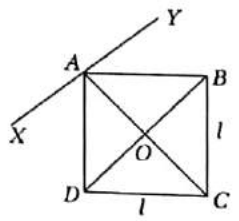
54. Two discs have same mass and thickness. Their materials are of densities d_1 and d_2. The ratio of their moments of inertia about an axis passing through the centre and perpendicular to the plane is
(a) $d_1 : d_2$ (b) $d_2 : d_1$
(c) $\left(\frac{d_1}{d_2}\right)^2$ (d) $\left(\frac{d_2}{d_1}\right)^2$

55. A ball rolls without slipping. The radius of gyration of the ball about an axis passing through its centre of mass is K. If radius of the ball be R, then the fraction of total energy associated with its rotational energy will be
(a) $\frac{K^2}{K^2 + R^2}$ (b) $\frac{R^2}{K^2 + R^2}$
(c) $\frac{K^2 + R^2}{R^2}$ (d) $\frac{K^2}{R^2}$

56. The speed of a homogeneous solid sphere after rolling down an inclined plane of vertical height h, from rest without sliding is
(a) $\sqrt{gh}$ (b) $\sqrt{\left(\frac{g}{5}\right) gh}$
(c) $\sqrt{\left(\frac{4}{3}\right) gh}$ (d) $\sqrt{\left(\frac{10}{7}\right) gh}$

57. Consider three solid spheres, sphere (i) has radius r and mass m, sphere (ii) has radius r and mass $3m$, sphere (iii) has radius $3r$ and mass m. All can be placed at the same point on the same inclined plane, where they will roll without slipping to the bottom. If allowed to roll down the incline, then at the bottom of the incline
(a) sphere (i) will have the largest speed
(b) sphere (ii) will have the largest speed
(c) sphere (iii) will have the largest kinetic energy
(d) all the spheres will have equal speeds

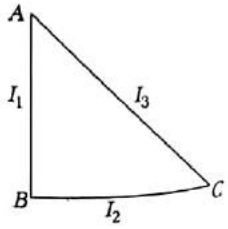
58. Four point masses each of value m, are placed at the corners of a square $ABCD$ of side l. The moment of inertia of the system about an axis passing through A and parallel to BD is



(a) $\sqrt{3}\, ml^2$ (b) $3\, ml^2$ (c) ml^2 (d) $2\, ml^2$

59. The density of a non-uniform rod of length 1 m is given by $\rho(x) = a(1 + bx^2)$ where, a and b are constants and $0 \le x \le 1$. The centre of mass of the rod will be at **(NCERT Exemplar)**
(a) $\frac{3(2+b)}{4(3+b)}$ (b) $\frac{4(2+b)}{3(3+b)}$
(c) $\frac{3(3+b)}{4(2+b)}$ (d) $\frac{4(3+b)}{3(2+b)}$

60. ABC is a right angled triangular plate of uniform thickness. The sides are such that $AB > BC$ as shown in figure. I_1, I_2 and I_3 are moments of inertia about AB, BC and AC respectively. Then, which of the following relation is correct?



(a) $I_1 = I_2 = I_3$ (b) $I_2 > I_1 > I_3$
(c) $I_3 < I_2 < I_1$ (d) $I_3 > I_1 > I_2$

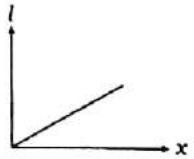
61. The moment of inertia of a cube of mass m and side a about one of its edges is equal to

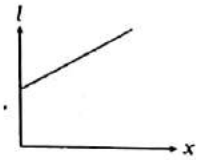
(a) $\frac{2}{3}ma^2$ (b) $\frac{4}{3}ma^2$ (c) $3ma^2$ (d) $\frac{8}{3}ma^2$

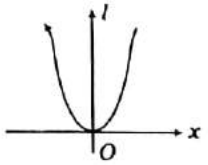
62. Two uniform, thin identical rods each of mass M and length l are joined together to form a cross. What will be the moment of inertia of the cross about an axis passing through the point at which the two rods are joined and perpendicular to the plane of the cross?

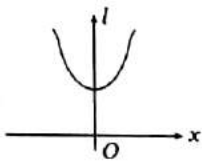
(a) $\frac{Ml^2}{12}$ (b) $\frac{Ml^2}{6}$ (c) $\frac{Ml^2}{4}$ (d) $\frac{Ml^2}{3}$

63. Figure represents the moment of inertia of the solid sphere about an axis parallel to the diameter of the solid sphere and at a distance x from it. Which one of the following represents the variations of I with x?

(a) 

(b) 

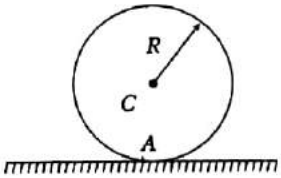
(c) 

(d) 

64. When a body is projected at an angle with the horizontal in the uniform gravitational field of the earth, the angular momentum of the body about the point of projection, as it proceeds along its path

(a) remains constant
(b) increases
(c) decreases
(d) initially decreases and after its highest point increases

65. A disc of radius R rolls on a rough horizontal surface. The distance covered by the point A in one revolution is



(a) $2\pi R$ (b) $2R$
(c) $8R$ (d) πR

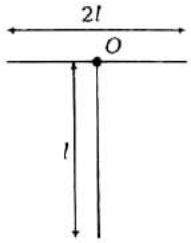
66. A uniform rod of mass 2 kg and length 1 m lies on a smooth horizontal plane. A particle of mass 1 kg moving at a speed of 2 ms^{-1} perpendicular to the length of the rod strikes it at a distance $\frac{1}{4}$ m from the centre and stops. What is the angular velocity of the rod about its centre just after the collision?

(a) 3 rad s^{-1} (b) 4 rad s^{-1}
(c) 1 rad s^{-1} (d) 2 rad s^{-1}

67. A particle P is moving in a circle of radius a with a uniform speed u. C is the centre of the circle and AB is a diameter. The angular velocities of P about A and C are in the ratio

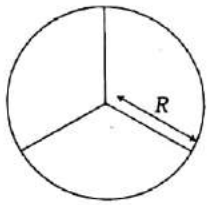
(a) 1 : 1 (b) 1 : 2 (c) 2 : 1 (d) 4 : 1

68. For the uniform T shaped structure, with mass $3M$, moment of inertia about an axis normal to the plane and passing through O would be



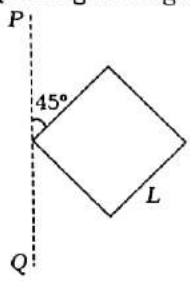
(a) $\frac{2}{3}Ml^2$ (b) Ml^2
(c) $\frac{Ml^2}{3}$ (d) None of these

69. A wheel comprises a ring of radius R and mass M and three spokes of mass m each. The moment of inertia of the wheel about its axis is



(a) $\left(M + \frac{m}{4}\right)R^2$ (b) $(M + m)R^2$
(c) $(M + 3m)R^2$ (d) $\left(\frac{M+m}{2}\right)R^2$

70. A square is made by joining four rods each of mass M and length L. Its moment of inertia about an axis PQ, in its plane and passing through one of its corner is



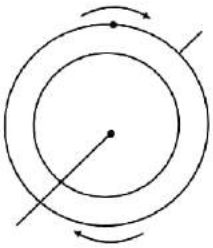
(a) $6ML^2$ (b) $\frac{4}{3}ML^2$
(c) $\frac{8}{3}ML^2$ (d) $\frac{10}{3}ML^2$

71. A solid homogeneous sphere is moving on a rough horizontal surface, partly rolling and partly sliding. During this kind of motion of the sphere
(a) total kinetic energy is conserved
(b) angular momentum of the sphere about the point of contact with the plane is conserved
(c) only the rotational kinetic energy about centre of mass is conserved
(d) angular momentum about centre of mass is conserved

72. A sphere cannot roll without applying external force on
(a) a smooth inclined surface
(b) a smooth horizontal surface
(c) a rough inclined surface
(d) a rough horizontal surface

73. A thin bar of mass m and length l is free to rotate about a fixed horizontal axis through a point at its end. The bar is brought to a horizontal position ($\theta = 90°$) and then released. The angular velocity when it reaches the lowest point is
(a) directly proportional to its length and inversely proportional to its mass
(b) independent of mass and inversely proportional to the square root of its length
(c) dependent only upon the acceleration due to gravity and the mass of the bar
(d) directly proportional to its length and inversely proportional to the acceleration due to gravity

74. A disc is free to rotate about a smooth horizontal axis passing through its centre of mass. A particle is fixed at the top of the disc. A slight push is given to the disc and it starts rotating. During the process



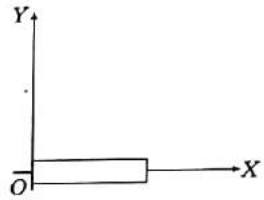
(a) only mechanical energy is conserved
(b) only angular momentum (about the axis of rotation) is conserved
(c) both mechanical energy and angular momentum are conserved
(d) Neither the mechanical energy nor the angular momentum are conserved

75. A rod of length L whose lower end is fixed along the horizontal plane starts to topple from the vertical position. The velocity of the upper end of the rod when it hits the ground is
(a) $\sqrt{3gL}$ (b) $\sqrt{2gL}$ (c) $\sqrt{gL}$ (d) $\sqrt{5gL}$

76. A solid sphere and a solid cylinder of same mass are rolled down on two inclined planes of heights h_1 and h_2 respectively. If at the bottom of the plane the two objects have same linear velocities, then the ratio of $h_1 : h_2$ is
(a) 2 : 3 (b) 7 : 5
(c) 14 : 15 (d) 15 : 14

77. A solid sphere of mass 2 kg rolls up a 30° incline with an initial speed of 10 ms^{-1}. The maximum height reached by the sphere is (g= 10 ms^{-2})
(a) 3.5 m (b) 7 m
(c) 10.5 m (d) 14 m

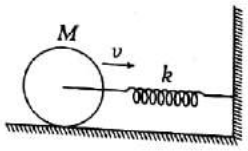
78. The figure shows a uniform rod lying along the X-axis. The locus of all the points lying on the XY-plane, about which the moment of inertia of the rod is same as that about O is



(a) an ellipse (b) a circle
(c) a parabola (d) a straight line

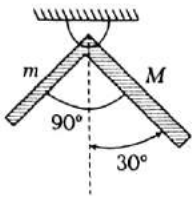
79. A cord is wound around the circumference of wheel of radius r. The axis of the wheel is horizontal and MI is I. A weight mg is attached to the end of the cord and falls from rest. After falling through a distance h, the angular velocity of the wheel will be
(a) $\sqrt{\frac{2gh}{I + mr^2}}$ (b) $\left(\frac{2mgh}{I + mr^2}\right)^{1/2}$
(c) $\left(\frac{2mgh}{I + 2mr^2}\right)^{1/2}$ (d) $\sqrt{2gh}$

80. A solid sphere rolls without slipping and presses a spring of spring constant k as shown in figure. Then, the compression in the spring will be



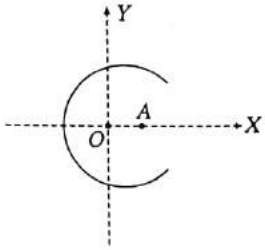
(a) $v\sqrt{\frac{2M}{3k}}$ (b) $v\sqrt{\frac{2M}{5k}}$
(c) $v\sqrt{\frac{5k}{7M}}$ (d) $v\sqrt{\frac{7M}{5k}}$

81. Two uniform rods of equal length but different masses are rigidly joined to form an L-shaped body, which is then pivoted as shown in figure. If in equilibrium the body is in the shown configuration, ratio M/m will be



(a) 2 (b) 3
(c) $\sqrt{2}$ (d) $\sqrt{3}$

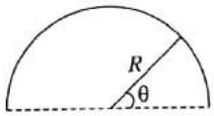
82. A portion of a ring of radius R has been removed as shown in figure. Mass of the remaining portion is m. Centre of the ring is at origin O. Let I_A and I_O be the moment of inertia passing through points



A and O are perpendicular to the plane of the ring. Then,

(a) $I_O = mR^2$ (b) $I_O = I_A$ (c) $I_O > I_A$ (d) $I_A > I_O$

83. The moment of inertia of a semicircular ring of mass M and radius R about an axis which is passing through its centre and at an angle θ with the line joining its ends as shown in figure is



(a) $\frac{MR^2}{4}$ if $\theta = 0°$ (b) $\frac{MR^2}{2}$ if $\theta = 0°$
(c) $\frac{MR^2}{2}$ if θ = any angle (d) $\frac{MR^2}{2}$ if $\theta = 90°$

B) Medical entrance special format questions

Assertion and reason

Directions (Q. Nos. 1-20) *These questions consists of two statements each printed as assertion and reason. While answering these questions you are required to choose any one of the following five responses.*

(a) If both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If Assertion is false but Reason is true.

1. Assertion Moment of inertia about an axis passing through centre of mass is always minimum.

Reason Theorem of parallel axis can be applied for 2-D as well as 3-D bodies.

2. Assertion A body is moving along a circle with a constant speed. Its angular momentum about the centre of the circle remains constant.

Reason In this situation, a constant non-zero torque acts on the body.

3. Assertion The angular velocity of a rigid body in motion is defined for the whole body.

Reason All points on a rigid body performing pure rotational motion are having same angular velocity.

4. Assertion If a particle is rotating in a circle, then angular momentum about centre is mvR.

Reason In circular motion, angular momentum about centre is always constant.

5. Assertion If we draw a circle around the centre of mass of a rigid body, then moment of inertia about all parallel axes passing through any point on this circle has a constant value.

Reason Dimensions of radius of gyration are $[M^0LT^0]$.

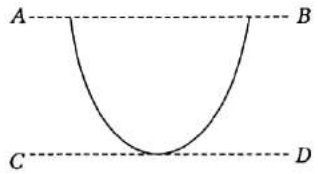
6. Assertion The condition of equilibrium for a rigid body is

Translational equilibrium $\Sigma F = 0$ and

Rotational equilibrium $\Sigma\tau = 0$

Reason A rigid body must be in equilibrium under the action of two equal and opposite forces.

7. Assertion Two axes AB and CD are as shown in figure. Given figure is of a semi-circular ring. As the axis moves from AB towards CD, moment of inertia first decreases, then increases.

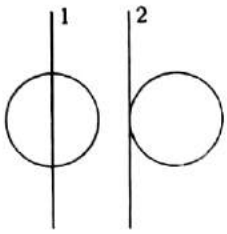


Reason Centre of mass lies somewhere between AB and CD.

8. Assertion If a particle moves with a constant velocity, then angular momentum of this particle about any point remains constant.

Reason Angular momentum has the units of Planck's constant.

9. Assertion Two identical solid spheres are rotated from rest to same angular velocity ω about two different axes as shown in figure. More work will have to be done to rotate the sphere in case-2.

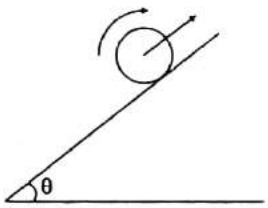


Reason Moment of inertia in case-2 is more.

10. Assertion A ring and a disc of same mass and radius begin to roll without slipping from the top of an inclined surface at $t = 0$. The ring reaches the bottom of incline in time t_1 while the disc reaches the bottom in time t_2, then $t_1 < t_2$.

Reason Disc will roll down the plane with more acceleration because of its lesser value of moment of inertia.

11. Assertion A sphere is placed in pure rolling condition over a rough inclined surface. Then, force of friction will act in downward direction.



Reason Angular acceleration (actually retardation) due to friction is anti-clockwise.

12. Assertion In rotational plus translational motion of a rigid body, different particles of the rigid body may have different velocities but they will have same accelerations.

Reason Translational motion of a particle is equivalent to the translational motion of a rigid body.

13. Assertion Angular momentum of sun and planet system about any point remains constant.

Reason Two equal and opposite forces will act on them. Net torque of these two set of forces about any point is zero.

14. Assertion Moment of inertia about an axis passing through centre of mass is minimum.

Reason Theorem of parallel axis can be applied only for two dimensional body of negligible thickness.

15. Assertion A solid sphere cannot roll without slipping on smooth horizontal surface.

Reason If the sphere is left free on smooth inclined surface, it cannot roll without slipping.

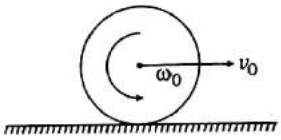
16. Assertion Speed of any point on rigid body executing rolling motion can be calculated by expression $v = r\,\omega$, where r is distance of point from instantaneous centre of rotation.

Reason Rolling motion of rigid body can be considered as a pure rotation about instantaneous centre of rotation.

17. Assertion A solid sphere and a ring of same mass and radius are released simultaneously from the top of an inclined surface. The two objects roll down the plane without slipping. They reach the bottom of the incline with equal linear speeds.

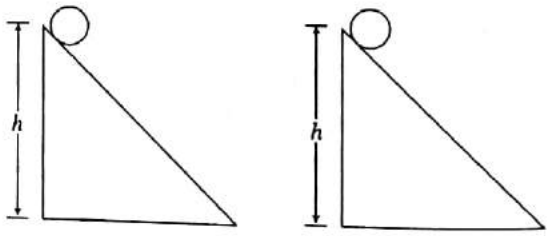
Reason Decrease in potential energy for both is the same.

18. Assertion A uniform disc of radius R is performing impure rolling motion on a rough horizontal plane as shown in figure. After some time the disc comes to rest. It is possible only when $v_0 = \frac{\omega_0 R}{2}$.



Reason For a body performing pure rolling motion, the angular momentum is conserved about any point in space.

19. Assertion Two identical spherical balls are released from two inclined plane. First is sufficiently rough and second is smooth. Both the balls will have same kinetic energy on reaching the bottom.



Reason Linear velocity of second ball will be more.

20. Assertion A solid and a hollow sphere both of equal masses and radii are put on a rough surface after rotating with some angular velocity say ω_0. Coefficient of friction for both the spheres and ground is same. Solid sphere will start pure rolling first.

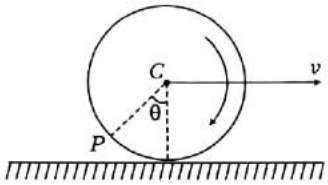
Reason Radius of gyration of hollow sphere about an axis passing through its centre of mass is more.

Match the columns

1. If radius of earth is reduced to half without changing its mass, then match the following columns.

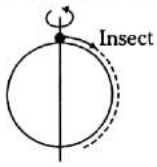
Column I	Column II
(A) Angular momentum of earth	(p) Will become two times
(B) Time period of rotation of earth	(q) Will become four times
(C) Rotational kinetic energy of earth	(r) Will remain constant
(D)	(s) None

2. A disc rolls on ground without slipping. Velocity of centre of mass is v. There is a point P on circumference of disc at angle θ. Suppose, v_P is the speed of this point. Then, match the following columns.



Column I	Column II
(A) If $\theta = 60°$	(p) $v_P = \sqrt{2}\, v$
(B) If $\theta = 90°$	(q) $v_P = v$
(C) If $\theta = 120°$	(r) $v_P = 2v$
(D) If $\theta = 180°$	(s) $v_P = \sqrt{3}\, v$

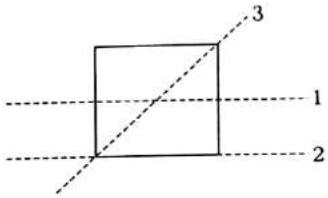
3. A solid sphere is rotating about an axis as shown in figure. An insect follows the dotted path on the circumference of sphere as shown in the figure.



Column I	Column II
(A) Moment of inertia	(p) Will remain constant
(B) Angular velocity	(q) Will first increase, then decrease
(C) Angular momentum	(r) Will first decrease then increase
(D) Rotational kinetic energy	(s) Will continuously decrease
	(t) Will continuously increase

4. Four rods of equal length l and mass m each form a square as shown in figure.

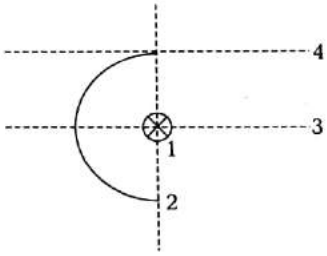
Moment of inertia about three axes 1, 2 and 3 are say I_1, I_2 and I_3. Then, match the following columns.



	Column I		Column II
(A)	I_1	(p)	$\frac{4}{3} ml^2$
(B)	I_2	(q)	$\frac{2}{3} ml^2$
(C)	I_3	(r)	$\frac{1}{2} ml^2$
		(s)	None

5. A semicircular ring has mass m and radius R as shown in figure. Let I_1, I_2, I_3 and I_4 be the moments of inertia of the four axes as shown. Axis 1 passes through centre and is perpendicular to plane of ring.

Then, match the following columns.

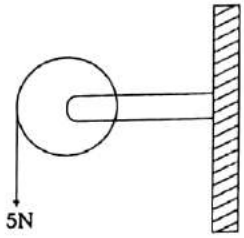


	Column I		Column II
(A)	I_1	(p)	$\frac{mR^2}{2}$
(B)	I_2	(q)	$(3/2)\, mR^2$
(C)	I_3	(r)	mR^2
(D)	I_4	(s)	Data is insufficient

C Medical entrances' gallery

(Collection of questions asked in NEET & various medical entrance exams)

1. A wheel of radius 10 cm can rotate freely about its centre as shown in figure. A string wrapped over its rim is pulled by a force of 5N. It is observed that the torque produces angular acceleration of 2rad s^{-2} in the wheel. What is the moment of inertia of the wheel? **[NEET 2016]**



(a) 0.25 kg-m^2 (b) 0.45 kg-m^2
(c) 0.15 kg-m^2 (d) 0.16 kg-m^2

2. From a disc of radius R and mass M, a circular hole of diameter R, whose rim passes through the centre is cut. What is the moment of inertia of the remaining part of the disc about a perpendicular axis, passing through the centre ? **[NEET 2016]**

(a) 13 $MR^2/32$ (b) 11 $MR^2/32$
(c) 9 $MR^2/32$ (d) 15 $MR^2/32$

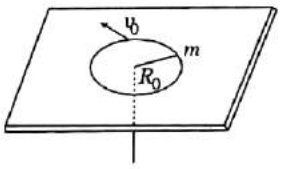
3. A disc and a sphere of same radius but different masses roll off on two inclined planes of the same altitude and length. Which one of the two objects gets to the bottom of the plane first? **[NEET 2016]**

(a) Sphere
(b) Both reach at the same time
(c) Depends on their masses
(d) Disc

4. A uniform circular disc of radius 50 cm at rest is free to turn about an axis which is perpendicular to its plane and passes through its centre. It is subjected to a torque which produces a constant angular acceleration of 2 rad s^{-2}. Its net acceleration in ms^{-2} at the end of 2 s is *a* approximately **[NEET 2016]**

(a) 7 (b) 6 (c) 3 (d) 8

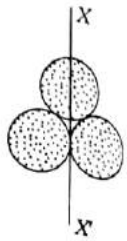
5. A mass m moves in a circle on a smooth horizontal plane with velocity v_0 at a radius R_0. The mass is attached to a string which passes through a smooth hole in the plane as shown in the figure.



The tension in the string is increased gradually and finally m moves in a circle of radius $R_0/2$. The final value of the kinetic energy is **[CBSE AIPMT 2015]**

(a) mv_0^2 (b) $\frac{1}{4}mv_0^2$ (c) $2mv_0^2$ (d) $\frac{1}{2}mv_0^2$

6. Three identical spherical shells, each of mass m and radius r are placed as shown in figure. Consider an axis XX', which is touching to two shells and passing through diameter of third shell. Moment of inertia of the system consisting of these three spherical shells about XX' axis is **[CBSE AIPMT 2015]**

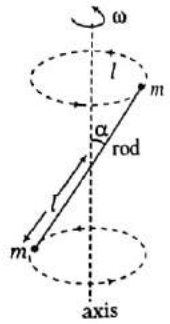


(a) $\frac{11}{5}mr^2$ (b) $3mr^2$
(c) $\frac{16}{5}mr^2$ (d) $4mr^2$

7. A uniform metallic rod rotates about its perpendicular bisector with constant angular speed. If it is heated uniformly to raise its temperature slightly, then **[AIIMS 2015]**

(a) its speed of rotation increases
(b) its speed of rotation decreases
(c) its speed of rotation remains same
(d) its speed increases because its moment of inertia increases

8. A massless rod S having length $2l$ has equal point masses attached to its two ends as shown in figure. The rod is rotating about an axis passing through its centre and making angle α with the axis. The magnitude of change of momentum of rod *i.e.*, $\left|\frac{dL}{dt}\right|$ equals **[AIIMS 2015]**



(a) $2ml^3\omega^2 \sin\theta \cdot \cos\theta$ (b) $ml^2\omega^2 \sin 2\theta$
(c) $ml^2 \sin 2\theta$ (d) $m^{1/2} l^{1/2} \omega \sin\theta \cdot \cos\theta$

9. A particle travels in a circle of radius 20 cm at a uniformly increasing speed. If the speed changes from $8ms^{-1}$ to $9ms^{-1}$ in 2s, what would be the angular acceleration in rad s^{-2}? **[UK PMT 2015]**

(a) 1.5 rad s^{-2} (b) 2.5 rad s^{-2}
(c) 3.5 rad s^{-2} (d) 4.5 rad s^{-2}

10. A solid sphere of radius r is rolling on a horizontal surface. The ratio between the rotational kinetic energy and total energy **[EAMCET 2015, AIIMS 2012, JCECE 2010]**

(a) $\frac{5}{7}$ (b) $\frac{2}{7}$
(c) $\frac{1}{2}$ (d) $\frac{1}{7}$

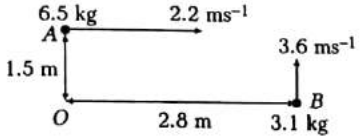
11. If two circular discs A and B are of same mass but of radii r and $2r$ respectively, then the moment of inertia of A is **[Kerala CEE 2015]**

(a) the same as that of B (b) twice that of B
(c) four times that of B (d) half that of B
(e) one-fourth that of B

12. Choose the wrong statement. **[Kerala CEE 2015]**

(a) The centre of mass of a uniform circular ring is at its geometric centre
(b) Moment of inertia is a scalar quantity
(c) Radius of gyration is a vector quantity
(d) Force in translational motion is analogous to torque in rotational motion

13. Two particles A and B are moving as shown in the figure.



Their total angular momentum about the point O is **[WB JEE 2015]**

(a) 9.8 kg m^2s^{-1} (b) zero
(c) 52.7 kg m^2s^{-1} (d) 37.9 kg m^2s^{-1}

14. A rod of mass 5 kg is connected to the string at point B. The span of rod is along horizontal. The other end of the rod is hinged at point A. If the string is massless, then the reaction of hinge at the instant when string is cut, is (Take, $g = 10$ ms^{-2}) **[UP CPMT 2015]**

(a) 10.1 N (b) 12.5 N (c) 5 N (d) 15 N

15. A particle moves with a constant velocity parallel to the X-axis. Its angular momentum with respect to the origin **[UP CPMT 2015]**

(a) is zero (b) remains constant
(c) goes on increasing (d) goes on decreasing

16. A ring of radius 0.5 m and mass 10 kg is rotating about its diameter with angular velocity of 20 rad s^{-1}. Its kinetic energy is **[Manipal 2015]**

(a) 10 J (b) 100 J (c) 500 J (d) 250 J

17. A uniform sphere of mass 500 g rolls without slipping on a plane surface so that its centre moves at a speed of 0.02 ms^{-1}. The total kinetic energy of rolling sphere would be (in J) **[AIIMS 2015]**

(a) 1.4×10^{-4} J (b) 0.75×10^{-3} J
(c) 5.75×10^{-3} J (d) 4.9×10^{-5} J

18. A rotating wheel changes angular speed from 1800 rpm to 3000 rpm in 20 s. What is the angular acceleration assuming it to be uniform? **[KCET 2014]**

(a) 60π rad s^{-2} (b) 90π rad s^{-2}
(c) 2π rad s^{-2} (d) 40π rad s^{-2}

19. Moment of inertia of ring about its diameter is I. The moment of inertia of the same ring about that axis perpendicular to its plane and passing through centre is **[KCET 2014]**

(a) $\frac{I}{2}$ (b) $2I$ (c) $\frac{I}{4}$ (d) $4I$

20. Two bodies have their moments of inertia I and $2I$, respectively about their axis of rotation. If their kinetic energies of rotation are equal, their angular velocity will be in the ratio **[UK PMT 2014]**

(a) 2 : 1 (b) 1 : 2
(c) $\sqrt{2}$: 1 (d) 1 : $\sqrt{2}$

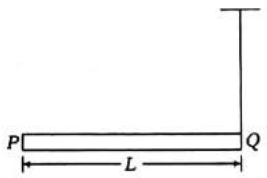
21. A body having a moment of inertia about its axis of rotation equal to 3 kg-m^2 is rotating with angular velocity of 3 rad s^{-1}. Kinetic energy of this rotating body is same as that of a body of mass 27 kg moving with a velocity v. The value of v is **[KCET 2014]**

(a) 1 ms^{-1} (b) 0.5 ms^{-1}
(c) 2 ms^{-1} (d) 1.5 ms^{-1}

22. A uniform solid spherical ball is rolling down a smooth inclined plane from a height h. The velocity attained by the ball when it reaches the bottom of the inclined plane is v. If the ball is now thrown vertically upwards with the same velocity v, the maximum height to which the ball will rise is **[WB JEE 2014]**

(a) $\frac{5h}{8}$ (b) $\frac{3h}{5}$
(c) $\frac{5h}{7}$ (d) $\frac{7h}{9}$

23. A rod PQ of mass M and length L is hinged at end P. The rod is kept horizontal by a massless string tied to point Q as shown in figure. When string is cut, the initial angular acceleration of the rod is



[NEET 2013]

(a) $\frac{3g}{2L}$ (b) $\frac{g}{L}$ (c) $\frac{2g}{L}$ (d) $\frac{2g}{3L}$

24. A small object of uniform density rolls up a curved surface with an initial velocity v'. It reaches upto a maximum height of $3v^2/4g$ with respect to the initial position. The object is **[NEET 2013]**

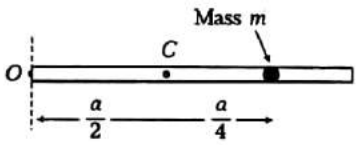
(a) ring (b) solid sphere
(c) hollow sphere (d) disc

25. The conservation of angular momentum demands that **[J&K CET 2013]**
(a) the external force on the system must be zero
(b) the external torque on the system must be zero
(c) Both the external force as well as the external torque must be zero
(d) Neither of them must be zero

26. The moment of inertia (I) and the angular momentum (L) are related by the expression **[J&K CET 2013]**
(a) $I = L\omega$ (b) $L = I\omega$
(c) $L = I^2\omega$ (d) $\omega = LI$

27. The moment of inertia (I) of a sphere of radius R and mass M is given by **[J&K CET 2013]**
(a) $I = MR^2$ (b) $I = (1/2)MR^2$
(c) $I = (4/3)MR^2$ (d) $I = (2/5)MR^2$

28. A particle mass m is attached to a thin uniform rod of length a at a distance of $\frac{a}{4}$ from the mid-point C as shown in the figure. The mass of the rod is 4 m. The moment of inertia of the combined system about an axis passing through O and perpendicular to the rod is **[EAMCET 2013]**



(a) $\frac{91}{48}ma^2$ (b) $\frac{27}{48}ma^2$ (c) $\frac{51}{48}ma^2$ (d) $\frac{64}{48}ma^2$

29. A particle moving in a circular path has an angular momentum of L. If the frequency of rotation is halved, then its angular momentum becomes **[Kerala CEE 2013]**
(a) $\frac{L}{2}$ (b) L (c) $\frac{L}{3}$ (d) $\frac{L}{4}$

30. The torque of a force $\mathbf{F} = 2\hat{i} - 3\hat{j} + 5\hat{k}$ acting at a point whose position vector $\mathbf{r} = 3\hat{i} - 3\hat{j} + 5\hat{k}$ about the origin is **[Kerala CEE 2013]**
(a) $-3\hat{i} + 5\hat{k}$ (b) $-5\hat{j} + 3\hat{k}$
(c) $-5\hat{j} - 3\hat{k}$ (d) $3\hat{i} - 5\hat{j} + 3\hat{k}$

31. Moment of inertia of a disc of radius R about a diametric axis is 25 kg m^{-2}. The MI of the disc about a parallel axis at a distance $R/2$ from the centre is **[MPPMT 2013]**
(a) 31.25 kg-m^2 (b) 37.5 kg-m^2
(c) 50 kg-m^2 (d) 62.5 kg-m^2

32. A wheel having moment of inertia 2 kg-m^2 about its vertical axis, rotates at the rate of 60 rpm about this axis. The torque which can stop the wheel's rotation in one minute would be **[UPCPMT 2012]**
(a) $\frac{\pi}{15}$ N-m (b) $\frac{2\pi}{15}$ N-m (c) $\frac{\pi}{18}$ N-m (d) $\frac{\pi}{12}$ N-m

33. What is the moment of inertia of solid sphere of density ρ and radius R about its diameter? **[UPCPMT 2012]**
(a) $\frac{105}{176}R^5\rho$ (b) $\frac{105}{176}R^2\rho$
(c) $\frac{176}{105}R^5\rho$ (d) $\frac{186}{105}R^2\rho$

34. The radius of gyration of body depends upon **[UPCPMT 2012]**
(a) shape and size of body
(b) nature of mass distribution of body
(c) choice of axis of rotation
(d) All of the above

35. If two discs have same mass and thickness but their densities are ρ_1 and ρ_2, then the ratio of their moment of inertia about central axis will be **[BCECE (Mains) 2012]**
(a) $1 : \rho_1\rho_2$ (b) $\rho_1\rho_2 : 1$ (c) $\rho_1 : \rho_2$ (d) $\rho_2 : \rho_1$

36. If a disc starting from rest acquires an angular velocity of 240 rev min^{-1} in 10 s, then its angular acceleration will be **[BCECE (Mains) 2012]**
(a) 1.52 rads^{-1} (b) 3.11 rads^{-1}
(c) 2.51 rads^{-1} (d) 1.13 rads^{-1}

37. A thin hollow sphere of mass m is completely filled with a liquid of mass m. When the sphere rolls with a velocity v, kinetic energy of the system is (neglect friction) **[AIIMS 2012]**
(a) $\frac{1}{2}mv^2$ (b) mv^2 (c) $\frac{4}{3}mv^2$ (d) $\frac{4}{5}mv^2$

38. The moment of inertia of a circular loop of radius R, at a distance of $R/2$ around a rotating axis parallel to horizontal diameter of loop is **[AIIMS 2012]**
(a) MR^2 (b) $\frac{1}{2}MR^2$ (c) $2MR^2$ (d) $\frac{3}{4}MR^2$

39. A rod of length L is composed of a uniform length $1/2\ L$ of wood whose mass is m_w and a uniform length $1/2\ L$ of brass whose mass is m_b. The moment of inertia I of the rod about an axis perpendicular to the rod and through its centre is equal to **[AMU 2012]**
(a) $(m_w + m_b)L^2/12$ (b) $(m_w + m_b)L^2/6$
(c) $(m_w + m_b)L^2/3$ (d) $(m_w + m_b)L^2/2$

Answers

Check point 9.1

1. (d)	2. (c)	3. (d)	4. (d)	5. (c)	6. (c)	7. (d)	8. (b)	9. (a)	10. (b)
11. (a)	12. (a)	13. (b)	14. (a)	15. (b)	16. (b)	17. (a)	18. (d)	19. (c)	20. (a)

Check point 9.2

1. (b)	2. (c)	3. (a)	4. (b)	5. (d)	6. (a)	7. (a)	8. (a)	9. (a)	10. (b)
11. (d)	12. (d)	13. (b)	14. (a)	15. (a)	16. (a)	17. (b)	18. (c)	19. (a)	20. (b)

Check point 9.3

1. (a)	2. (b)	3. (c)	4. (c)	5. (d)	6. (b)	7. (c)	8. (d)	9. (d)	10. (d)
11. (d)	12. (d)	13. (a)	14. (b)	15. (d)					

A. Taking it together

1. (b)	2. (b)	3. (d)	4. (d)	5. (b)	6. (b)	7. (a)	8. (d)	9. (c)	10. (d)
11. (a)	12. (b)	13. (a)	14. (b)	15. (a)	16. (c)	17. (c)	18. (a)	19. (d)	20. (a)
21. (c)	22. (a)	23. (a)	24. (d)	25. (b)	26. (b)	27. (d)	28. (b)	29. (b)	30. (d)
31. (a)	32. (a)	33. (c)	34. (c)	35. (d)	36. (c)	37. (b)	38. (c)	39. (b)	40. (b)
41. (c)	42. (a)	43. (a)	44. (d)	45. (b)	46. (a)	47. (b)	48. (c)	49. (c)	50. (c)
51. (c)	52. (c)	53. (a)	54. (b)	55. (a)	56. (d)	57. (d)	58. (b)	59. (a)	60. (b)
61. (a)	62. (b)	63. (d)	64. (b)	65. (c)	66. (a)	67. (b)	68. (b)	69. (b)	70. (c)
71. (b)	72. (b)	73. (b)	74. (a)	75. (a)	76. (c)	77. (b)	78. (b)	79. (b)	80. (d)
81. (d)	82. (d)	83. (c)							

B. Medical entrance special format questions

Assertion and reason

1. (b)	2. (c)	3. (b)	4. (a)	5. (b)	6. (c)	7. (a)	8. (b)	9. (a)	10. (d)
11. (d)	12. (d)	13. (a)	14. (c)	15. (d)	16. (c)	17. (d)	18. (c)	19. (b)	20. (a)

Match the columns

1. $(A \to q, B \to s, C \to q)$
2. $(A \to q, B \to p, C \to s, D \to r)$
3. $(A \to q, B \to r, C \to p, D \to r)$
4. $(A \to q, B \to s, C \to q)$
5. $(A \to r, B \to p, C \to p, D \to q)$

C. Medical entrance's gallery

1. (a)	2. (a)	3. (a)	4. (d)	5. (c)	6. (d)	7. (b)	8. (b)	9. (b)	10. (b)
11. (e)	12. (b)	13. (a)	14. (b)	15. (b)	16. (d)	17. (a)	18. (c)	19. (b)	20. (c)
21. (a)	22. (c)	23 (a)	24. (d)	25. (b)	26. (b)	27. (d)	28. (a)	29. (a)	30. (c)
31. (b)	32. (a)	33. (c)	34. (d)	35. (d)	36. (c)	37. (c)	38. (d)	39. (a)	

Hints and explanations

Check point 9.1

1. (*d*) As body covers equal angles in equal intervals of time, so its angular velocity and hence, magnitude of linear velocity is constant but its direction changes. $(\because v = r\omega)$

2. (*c*) Frequency of wheel, $\nu = \frac{300}{60} = 5$ rps

Angle described by wheel in one rotation = 2π rad.

Therefore, angle described by wheel in

$$1\text{s} = 2\pi \times 5 = 10\pi \text{ rad}$$

3. (*d*) Angular displacement per second

$$\theta = \omega t = \frac{500 \times 2\pi}{60} = \frac{50\pi}{3} \text{ rad}$$

4. (*d*) Angle turned by the body,

$$\theta = \theta_0 + \theta_1 t + \theta_2 t^2$$

Angular velocity, $\omega = \frac{d\theta}{dt} \Rightarrow \frac{d}{dt}(\theta_0 + \theta_1 t + \theta_2 t^2) = \theta_1 + 2\theta_2 t$

Angular acceleration, $\alpha = \frac{d\omega}{dt} \Rightarrow \frac{d}{dt}(\theta_1 + 2\theta_2 t) = 2\theta_2$

5. (*c*) We know that

$$\omega = 2\pi n \Rightarrow \omega_1 = 2\pi n_1$$

where, $n_1 = 1800$ rpm

$n_2 = 3000$ rpm

$$\omega_1 = 2\pi \times \frac{1800}{60} = 2\pi \times 30 = 60\pi$$

Similarly, $\omega_2 = 2\pi n_2 = 2\pi \times \frac{3000}{60}$

$\Rightarrow \quad = 2\pi \times 50 = 100\,\pi$

If the angular velocity of a rotating wheel about an axis changes by change in angular velocity in a time interval Δt, then the angular acceleration of rotating wheel about that axis is

$$\alpha = \frac{\text{Change in angular velociy}}{\text{Time interval}}$$

$$\alpha = \frac{\omega_2 - \omega_1}{\Delta t} \Rightarrow \alpha = \frac{100\pi - 60\pi}{20}$$

$$\alpha = \frac{40\pi}{20} = 2\pi \text{ rad s}^{-2}$$

6. (*c*) $\theta = 100\,\pi$, $t = 5$ s, $\omega = ?$

Angular speed, $\omega = \frac{\theta}{t} = \frac{100\,\pi}{5} = 20\,\pi$

7. (*d*) Angular displacement during time,

$$\theta = (\omega_2 - \omega_1)t \Rightarrow (2\pi n_2 - 2\pi n_1)t$$

$$= (600\,\pi - 200\,\pi) \times 10 = 4000\,\pi \text{ rad}$$

Therefore, number of revolutions made during this time,

$$= \frac{4000\,\pi}{2\pi} = 2000$$

8. (*b*) The kinematical equation for angular motion is

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

Here, $\alpha = 3.0$ rad s^{-2}, $\omega_0 = 2.0$ rad s^{-1}, $t = 2$ s

Hence, $\theta = 2 \times 2 + \frac{1}{2} \times 3 \times (2)^2$ or $\theta = 4 + 6 = 10$ rad

9. (*a*) Given, $\alpha = 3$ rad s^{-2}, $\omega_1 = 10$ rad s^{-1}, $\omega_2 = 20$ rad s^{-1}

Angle, $\theta = \frac{\omega_2^2 - \omega_1^2}{2\alpha} = \frac{(20)^2 - (10)^2}{2 \times 3} = 50$ rad

10. (*b*) If angular acceleration is constant, we have

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

Given, $\theta = 15°$, $\omega_0 = 0$

For the Ist condition (time = t s)

$$15° = \frac{1}{2}\alpha t^2 \quad \text{...(i)}$$

For the 2nd condition (time = $3t$ s)

$$\theta_1 = \frac{1}{2}\alpha\,(3t)^2 = \frac{1}{2}(\alpha)\,9t^2 \quad \text{...(ii)}$$

So, the increase in angle through which it rotates in the next 2 s.

$$\Delta\theta = \theta_1 - \frac{1}{2}\alpha t^2$$

$$= 9 \times \frac{1}{2}\alpha t^2 - \frac{1}{2}\alpha t^2 = 8\left(\frac{1}{2}\alpha t^2\right)$$

$$= 8 \times 15° = 120° \quad \text{[from Eq. (i)]}$$

11. (*a*) The role of moment of inertia in the study of rotational motion is analogous to that of mass in study of linear motion.

12. (*a*) Moment of inertia of a body depends on position and orientation of the axis of rotation with respect to the body.

13. (*b*) Perpendicular distance from Z-axis would be

$$\sqrt{(1)^2 + (1)^2} = \sqrt{2} \text{ m}$$

$$\therefore \quad I = Mr^2 = (1)(\sqrt{2})^2 = 2 \text{ kg-m}^2$$

14. (*a*) Radius of gyration of solid sphere

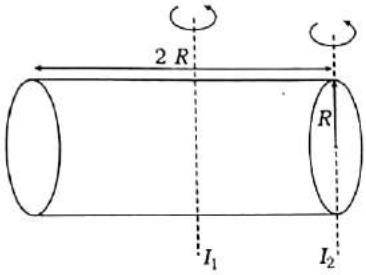
$$K = \sqrt{\frac{I}{M}} = \sqrt{\frac{\frac{7}{5}MR^2}{M}} = \sqrt{\frac{7}{5}}R$$

15. (*b*) Moment of inertia of solid sphere about its diameter,

$$I = \frac{2}{5}MR^2 = \frac{2}{5}\left[\frac{4}{3}\pi R^3\rho\right]R^2 = \frac{176}{105}\rho R^5$$

16. (*b*) Moment of inertia of circular ring and circular disc about the axes passing through their centres and perpendicular to their planes are MR^2 and $\frac{1}{2}MR^2$ respectively. So, the desired ratio is $\frac{MR^2}{(1/2)MR^2} = 2:1$

17. (*a*) From theorem of parallel axes, $I_2 = I_1 + MR^2$



18. (*d*) According to the question, $l = 2\pi R$

$$\therefore \quad R = \frac{l}{2\pi} \Rightarrow I_1 = \frac{Ml^2}{12}$$

$$I_2 = MR^2 = \frac{Ml^2}{4\pi^2} \Rightarrow \frac{I_1}{I_2} = \frac{\pi^2}{3}$$

19. (*c*) Moment of inertia, $I = \frac{ml^2}{3}$,

New moment of inertia, $I' = \frac{(3m/4)(3l/4)^2}{3} = \frac{27}{64}\left(\frac{ml^2}{3}\right) = \frac{27}{64}I$

20. (*a*) Moment of inertia of the rod lying along Z-axis will be zero. Moment of inertia of the rods along X and Y-axis will be $\frac{ML^2}{3}$ each. Hence, total moment of inertia is $\frac{2}{3}ML^2$.

Check point 9.2

1. (*b*) Torque, $\tau = \mathbf{r} \times \mathbf{F}$

$$\Rightarrow \quad \tau = 4\hat{\mathbf{j}} \times -6\hat{\mathbf{i}} = -(-24\hat{\mathbf{k}}) = 24\hat{\mathbf{k}}$$

2. (*c*) Torque, $\tau = \mathbf{r} \times \mathbf{F}$

$$= [(0-3)\hat{\mathbf{i}} + (2-0)\hat{\mathbf{j}} + (0-0)\hat{\mathbf{k}}] \times [20\hat{\mathbf{i}}]$$

$$= [-3\hat{\mathbf{i}} + 2\hat{\mathbf{j}}] \times [20\hat{\mathbf{i}}] = -40\hat{\mathbf{k}}$$

3. (*a*) Torque of the force, $\tau = \mathbf{r} \times \mathbf{F}$

$$\tau = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 7 & 3 & 1 \\ -3 & 1 & 5 \end{vmatrix} = (14\hat{\mathbf{i}} - 38\hat{\mathbf{j}} + 16\hat{\mathbf{k}})$$

4. (*b*) Torque, $\tau = \mathbf{r} \times \mathbf{F}$

$$= \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & -2 & +1 \\ -2 & 2 & +3 \end{vmatrix}$$

$$= \hat{\mathbf{i}}(-6-2) + \hat{\mathbf{j}}(-2-3) + \hat{\mathbf{k}}(2-4)]$$

Torque of force, $\tau = -8\hat{\mathbf{i}} - 5\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$

5. (*d*) τ required to open/close the door will remain constant.

$$r_1 \times F_1 = r_2 \times F_2$$

$$1.6 \times 1 = 0.4 \times F_2 \Rightarrow F_2 = 4 \text{ N}$$

6. (*a*) Given, $I = 2$ kg-m^2, $\omega = 30$ rad s^{-1} and $t = 15$ s

Average torque of the force, $\tau = I\alpha = I\frac{\omega}{t} = 2 \times \frac{30}{15} = 4$ N-m

7. (*a*) $\omega_1 = 10$ rad s^{-1}, $\omega_2 = 0$, $t = 10$ s

$$\therefore \quad \alpha = \frac{\omega_2 - \omega_1}{t} = \frac{0-10}{10} = -1 \text{ rad s}^{-2}$$

Negative sign means retardation

Now, $I = MR^2 = 10 \times (0.3)^2 = 0.9$ kg-m^2

$\therefore$ Torque, $\tau = I\alpha = 0.9 \times (1) = 0.9$ N-m

8. (*a*) Power, $P = \tau\omega$

$$P = (r \times F) \cdot \omega$$

9. (*a*) $\mathbf{L} = \mathbf{r} \times \mathbf{p}$, so angular momentum is the moment of momentum.

10. (*b*) For a body of mass m rotating with velocity v in a circle of radius r, then angular momentum is given by

$$\mathbf{L} = m(\mathbf{r} \times \mathbf{v})$$

For unit mass, $m = 1$

$$\therefore \quad |\mathbf{L}| = [(8\hat{\mathbf{i}} - 4\hat{\mathbf{j}}) \times (8\hat{\mathbf{i}} + 4\hat{\mathbf{j}})]$$

$$\Rightarrow \quad \mathbf{L} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 8 & -4 & 0 \\ 8 & 4 & 0 \end{vmatrix}$$

$$\Rightarrow \quad \mathbf{L} = \hat{\mathbf{i}}(0-0) - \hat{\mathbf{j}}(0-0) + \hat{\mathbf{k}}(32+32)$$

$$\Rightarrow \quad \mathbf{L} = 64\hat{\mathbf{k}} \text{ units}$$

11. (*d*) Angular momentum $L = mvr = m\omega r^2 = m \times \frac{2\pi}{T} \times r^2$

$$= \frac{2 \times 3.14 \times 6 \times 10^{24} \times (1.5 \times 10^{11})^2}{3.14 \times 10^7} = 2.7 \times 10^{40} \text{ kg-m}^2 \text{ s}^{-1}$$

12. (*d*) We know that, $\mathbf{L} = \mathbf{r} \times \mathbf{p}$

Angular momentum $\mathbf{L}$ is maximum when $\mathbf{p}$ is perpendicular to $\mathbf{r}$.

13. (*b*) We know that angular momentum of the body is given by

$$L = I\omega \text{ or } L = I \times \frac{2\pi}{T} \text{ or } L \propto \frac{1}{T} \Rightarrow \frac{L_1}{L_2} = \frac{T_2}{T_1}$$

$$\Rightarrow \quad \frac{L}{L_2} = \frac{2T}{T} \qquad \text{(As, } T_2 = 2T \text{ and } L_1 = L\text{)}$$

So, $L_2 = \frac{L}{2}$. Thus, on doubling the time period, angular momentum of body becomes half.

14. (*a*) Torque, $\tau = \frac{dL}{dt}$

where, L = angular momentum

If $\tau = 0$,

then $\frac{dL}{dt} = 0$

i.e. L = constant

15. (*a*) If external torque $\tau_{ext} = 0$, then

Angular momentum conserved, *i.e.*, $L_1 = L_2$.

16. (*a*) Torque = Rate of change of angular momentum $(\tau) = \frac{dL}{dt}$

Given, $dL = 4A_0 - A_0 = 3A_0$, $dt = 4$ min

$\Rightarrow$ Torque acting on the body, $\tau = \frac{3}{4}A_0$

17. (*b*) According to law of conservation of angular momentum, $I\omega$ = constant

or $\frac{R^2}{T}$ = constant $\left(\text{as } I \propto R^2 \text{ and } \omega \propto \frac{1}{T} \text{ and } I \propto \frac{1}{\omega}\right)$

$\therefore$ $T \propto R^2$

As $R' = \frac{1}{n}R$

$$\frac{T}{T'} = \left(\frac{R}{R'}\right)^2 \Rightarrow \frac{T}{T'} = \left(\frac{R \times n}{R}\right)^2$$

$$\therefore \quad T' = \frac{T}{n^2} = \frac{24}{n^2}h$$

18. (*c*) From conservation of angular momentum

$$\omega_1 I_1 = \omega_2 I_2$$

$$\therefore \quad \omega_2 = \frac{\omega_1 I_1}{I_2}$$

or $$\omega_2 = \omega\left[\frac{MR^2}{MR^2 + 2mR^2}\right] = \omega\left(\frac{M}{M + 2m}\right)$$

19. (*a*) If no external torque acts on a system of particles, then angular momentum of the system remains constant, i.e. $\tau = 0$

$$\Rightarrow \quad \frac{dL}{dt} = 0 \Rightarrow L = I\omega = \text{constant}$$

$$\Rightarrow I_1\omega_1 = I_2\omega_2 \Rightarrow \frac{1}{2}Mr^2\omega_1 = \frac{1}{2}(M + 2m)r^2\omega_2 \quad \text{...(i)}$$

Here, $M = 2$ kg, $m = 0.25$ kg, $r = 0.2$ m, $\omega_1 = 30$ rad s^{-1}

Hence, we get after putting the given values in Eq. (i)

$$\frac{1}{2} \times 2 \times (0.2)^2 \times 30 = \frac{1}{2} \times (2 + 2 \times 0.25)(0.2)^2 \times \omega_2$$

or $1.2 = 0.05\,\omega_2$ or $\omega_2 = 24$ rad s^{-1}

20. (*b*) Rotational kinetic energy $= \frac{1}{2}I\omega^2 = 8$ J

$$\Rightarrow \frac{1}{2} \times \frac{1}{2}mr^2\omega^2 = 8 \text{ or } \frac{1}{4} \times 2 \times (1)^2\omega^2 = 8$$

or $\omega^2 = 16$ or $\omega = 4$ rad s^{-1}

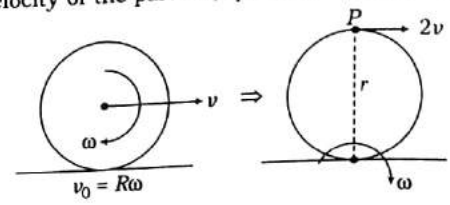
Angular momentum, $L = I\omega = \frac{1}{2}mr^2\omega$

$$= \frac{1}{2} \times 2 \times (1)^2 \times 4 = 4 \text{ J-s}$$

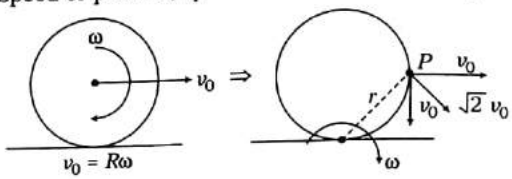
Check point 9.3

1. (*a*) Since, in this case instantaneous axis of rotation is always below the centre of mass. This is possible only when point of contact moves with a velocity equal to centre of mass.

2. (*b*) Velocity of the particle, $v_P = r\,\omega = (2R)$, $\omega = 2v$



3. (*c*) Speed of particle, $v_P = r\,\omega = (\sqrt{2}\,R)\,\omega = \sqrt{2}\,v_0$



4. (*c*) Total kinetic energy of a body which is rolling without slipping is given by $K_{total} = K_{rot} + K_{trans}$

For solid spherical ball,

$$I = \frac{2}{5}mR^2 \text{ and } v = R\omega \quad \text{(along the diameter)}$$

where, R is radius of spherical ball,

So, $$K_{total} = \frac{1}{2}\left(\frac{2}{5}mR^2\right)\omega^2 + \frac{1}{2}mR^2\omega^2$$

$$= \frac{7}{10}mR^2\omega^2;\ K = \frac{7}{10}mv^2$$

Potential energy = Kinetic energy

$$mgh = \frac{7}{10}mv^2 \Rightarrow v^2 = \frac{10}{7}gh \quad \text{...(i)}$$

For vertical projection,

$$v^2 = u^2 + 2gh' \Rightarrow \frac{10}{7}gh = 0 + 2gh' \Rightarrow h' = \frac{5}{7}h$$

5. (*d*) As, $v = \sqrt{\frac{2gh}{1 + K^2/R^2}}$

Given, $$h = \frac{3v^2}{4g} \Rightarrow v^2 = \frac{2gh}{1 + \frac{K^2}{R^2}} = \frac{2g3v^2}{4g\left(1 + \frac{K^2}{R^2}\right)} = \frac{6gv^2}{4g\left(1 + \frac{K^2}{R^2}\right)}$$

$$1 = \frac{3}{2(1 + K^2/R^2)} \text{ or } 1 + \frac{K^2}{R^2} = \frac{3}{2}$$

or $$\frac{K^2}{R^2} = \frac{3}{2} - 1 = \frac{1}{2} \Rightarrow K^2 = \frac{1}{2}R^2 \quad \text{(Equation of disc)}$$

Hence, the object is disc.

6. (*b*) The acceleration of its centre down the plane,

$$a = \frac{g\sin\theta}{1 + \frac{I}{mR^2}} \text{ or } a < g\sin\theta$$

7. (*c*) In case of pure rolling, $a = \dfrac{g\sin\theta}{1+\dfrac{I}{mR^2}}$; $\dfrac{I}{mR^2} = \dfrac{2}{5}$

$\therefore \quad a = \dfrac{5}{7} g\sin\theta$

Since, a is independent of m and R, both spheres reach the bottom together.

8. (*d*) The linear acceleration of the sphere,

$$a = \frac{g\sin\theta}{1+\dfrac{I}{mR^2}} = \frac{g\sin\theta}{1+\dfrac{2}{5}} = \frac{5}{7} g\sin\theta$$

9. (*d*) The linear acceleration,

$$a = \frac{g\sin\theta}{1+\dfrac{I}{mR^2}} \text{ and } \frac{I}{mR^2} = \frac{2}{5} \text{ (for solid sphere)}$$

$$\therefore \quad a = \frac{g\sin 30°}{1+\dfrac{2}{5}} = \frac{5g}{14}$$

10. (*d*) In case of pure rolling,

Acceleration, $a = \dfrac{g\sin\theta}{1+\dfrac{I}{MR^2}}$, for ring $\dfrac{I}{MR^2} = 1$

$\therefore \quad a = \dfrac{g}{4}$ $\quad [\because \theta = 30]$

11. (*d*) In case of pure rolling, $a = \dfrac{g\sin\theta}{1+\dfrac{I}{mR^2}}$

$\dfrac{I}{mR^2} = \dfrac{2}{5}$ for solid sphere and $\dfrac{I}{mR^2} = \dfrac{1}{2}$ for a disc.

$\therefore$ For a solid sphere, $a_1 = \dfrac{5}{7} g\sin\theta$

and for a hollow sphere, $a_2 = \dfrac{2}{3} g\sin\theta$

From $\quad s = \dfrac{1}{2}at^2, t = \sqrt{\dfrac{2s}{a}}$ or $\dfrac{t_1}{t_2} = \sqrt{\dfrac{a_2}{a_1}} = \sqrt{\dfrac{2/3}{5/7}} = \sqrt{\dfrac{14}{15}}$

12. (*d*) For all bodies, $a = g\sin\theta - \mu g\cos\theta$

Hence, all will take equal time.

13. (*a*) Time taken by rolling body to reach the bottom $t = \dfrac{1}{\sin\theta}\sqrt{\dfrac{2h}{g}\left(1+\dfrac{K^2}{R^2}\right)}$. A body with smaller value of $\dfrac{K^2}{R^2}$ will take less time to reach the bottom. Hence, solid sphere will take least time to reach bottom.

14. (*b*) In pure rolling, mechanical energy remains conserved. Therefore, at bottommost point, total kinetic energy will be mgh.

Ratio of rotational to translational kinetic energy will be $\dfrac{2}{3}$.

$\therefore \quad K_T = \dfrac{3}{5}(mgh) \Rightarrow \dfrac{3}{5}mgh = \dfrac{1}{2}mv^2$

$\therefore$ The speed of a uniform spherical shell

$$v = \sqrt{\frac{6gh}{5}}$$

15. (*d*) The relation between linear velocity (v) and angular velocity (ω) is

$$v = r\omega \Rightarrow r = \frac{v}{\omega}$$

Total kinetic energy $= \dfrac{1}{2}mv^2 + \dfrac{1}{2}I\omega^2$

$$= \frac{1}{2}mv^2 + \frac{1}{2}\times\frac{1}{2}mr^2\omega^2 = \frac{1}{2}mv^2 + \frac{1}{4}mv^2 = \frac{3}{4}mv^2$$

$$\frac{3}{4}mv^2 = mg\cdot 3 \Rightarrow v = 2\sqrt{10}$$

$\because \quad \omega r = v \Rightarrow r = \dfrac{v}{\omega} = \dfrac{2\sqrt{10}}{2\sqrt{2}} = \sqrt{5}$ m

A) Taking it together

1. (*b*) Torque, $\tau = \dfrac{d\mathbf{L}}{dt}$. If $\tau = 0$, $\mathbf{L}$ = constant

2. (*b*) We have, $I\omega = L$, constant $\Rightarrow \omega \propto 1/I$

On stretching hands, I will increase, therefore ω will decrease.

3. (*d*) We know that angular acceleration

$$\alpha = \frac{d\omega}{dt},$$

where ω is angular velocity of the disc

given ω = constant

$$\Rightarrow \quad \alpha = \frac{d\omega}{dt} = \frac{0}{dt} = 0$$

Hence, angular acceleration is zero.

4. (*d*) Here, $r = 0.2$ m, $M = 10$ kg, $v = 1200$ rpm $= 20$ rps

$$L = I\omega = (Mr^2)(2\pi v)$$

$$= 10\times(0.2)^2\times 2\times\frac{22}{7}\times 20 = 50.28 \text{ kg-m}^2\text{s}^{-2}$$

5. (*b*) From the conservation of angular momentum, $I_1\omega_1 = I_2\omega_2$

$$\therefore \quad \omega_2 = \frac{I_1}{I_2}\cdot\omega_1 = \frac{(1/2\,ma^2)}{(1/2\,ma^2)+ma^2}\cdot\omega = \frac{\omega}{3}$$

6. (*b*) In the given diagrams, when the small piece Q removed and glued to the centre of the plate, the mass comes closer to the z-axis, hence, moment of inertia decreases.

7. (*a*) On bending the head before diving in swimming pool, the moment of inertia of a person decreases.

8. (*d*) If r be the distance of point from axis will increase, then corresponding v of the point will also increase accordingly. Hence, ω is constant.

9. (*c*) We have $\quad E = \dfrac{1}{2}I\omega^2 \Rightarrow \omega = \sqrt{\dfrac{2E}{I}}$

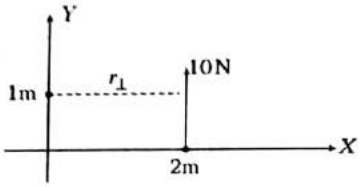
Now, angular momentumm $L = I\omega = \sqrt{2EI}$

10. (*d*) It cannot be zero, otherwise net torque or net angular acceleration will become zero.

11. (*a*) Angular momentum, $L = I\omega = I(2\pi f)$

Frequency f is doubled. Hence, angular momentum will become $2L$.

12. (*b*) Moment = Force $\times r_\perp = 10 \times 2 = 20$ N-m



13. (*a*) Since, for uniform rod, the moment of inertia about an axis passing through its centre is $mL^2/12$

$$K = \sqrt{\frac{I}{m}} = \sqrt{\frac{(mL^2/12)}{m}} = \frac{L}{2\sqrt{3}}$$

14. (*b*) Moment of inertia of the system,
$I = 5mR^2 = 5 \times 2 \times (0.1)^2 = 0.1$ kg-m^2

15. (*a*) Direction of linear velocity always keeps on changing. Hence, linear momentum is varying.

16. (*c*) Angular velocity, $\omega = \frac{d\theta}{dt} = 6t^2$, At $t = 2$ s

$$\omega = 6(2)^2 = 24 \text{ rads}^{-1}$$

17. (*c*) $\omega = \alpha t$

$$\therefore \quad \alpha = \frac{\omega}{t} = \frac{80}{5} = 16 \text{ rad s}^{-2}$$

$$\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2}(16)(5)^2 = 200 \text{ rad}$$

18. (*a*) $\omega = \alpha - \beta t$. Comparing with $\omega = \omega_0 - \alpha t$

Initial angular velocity = α

Angular retardation = β

$\therefore$ Angle rotated before it stops α^2.

$$0 = \alpha^2 - 2\beta\theta \Rightarrow \theta = \frac{\alpha^2}{2\beta}$$

19. (*d*) Angular retardation, $\alpha = \frac{\omega_0}{t}$ $\quad (\because 0 = \omega_0 - \alpha t)$

$$= \frac{2\pi \times 33/60}{20} = \frac{11\pi}{200} \text{ rad s}^{-2}$$

20. (*a*) Angular velocity, $\omega_0 = \frac{900 \times 2\pi}{60}$ rad s^{-1} $= 30\pi$ rad s^{-1}

Now, $\quad 0 = \omega_0 - \alpha t$

or Angular retardation, $\alpha = \frac{\omega_0}{t} = \frac{30\pi}{60}$ rad s^{-2} $= \frac{\pi}{2}$ rad s^{-2}

21. (*c*) We have, $\theta_1 = \frac{1}{2}(\alpha)(2)^2 = 2\alpha$,

$$\theta_2 = \frac{1}{2}\alpha(4)^2 - \frac{1}{2}\alpha(2)^2 = 6\alpha \Rightarrow \frac{\theta_2}{\theta_1} = \frac{3}{1}$$

22. (*a*) From the equation, $0 = \omega_0 - \alpha t$

$$\therefore \quad \alpha = \frac{\omega_0}{t} = \frac{(100 \times 2\pi)/60}{15} = 0.7 \text{ rad s}^{-2}$$

Now angle rotated before coming to rest, $\theta = \frac{\omega_0^2}{2\alpha}$

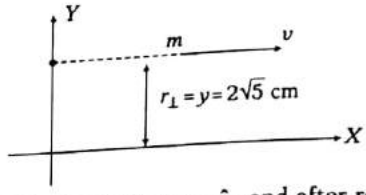
or $$\theta = \frac{(100 \times 2\pi/60)^2}{2 \times 0.7} = 78.33 \text{ rad}$$

or number of rotations, $n = \frac{\theta}{2\pi} = 12.5$

23. (*a*) Torque, $\tau = \mathbf{r} \times \mathbf{F}$ *i.e.*, τ is perpendicular to both $\mathbf{r}$ and $\mathbf{F}$

$\therefore \quad \tau \cdot \mathbf{r} = 0$ and $\tau \cdot \mathbf{F} = 0$

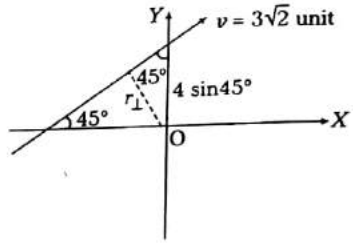
24. (*d*) The angular momentum, $L = mvr_\perp = (5)(3\sqrt{2})(2\sqrt{5})$
$= 30\sqrt{10}$ g-cm^{2}s^{-1}



25. (*b*) The initial velocity is $\mathbf{v}_i = v\hat{\mathbf{e}}_y$ and after reflection from the wall, the final velocity is $\mathbf{v}_f = -v\hat{\mathbf{e}}_y$. The trajectory is described as position vector $\mathbf{r} = y\hat{\mathbf{e}}_y + a\hat{\mathbf{e}}_x$.
Hence, the change in angular momentum is

$$\mathbf{r} \times m(\mathbf{v}_f - \mathbf{v}_i) = 2mva\hat{\mathbf{e}}_x$$

26. (*b*) From the figure, $r_\perp = 4 \sin 45° = 2\sqrt{2}$ unit



The magnitude of the angular momentum,

$$L = mvr_\perp = 5(3\sqrt{2})(2\sqrt{2}) = 60 \text{ unit}$$

27. (*d*) About origin, angular momentum will be

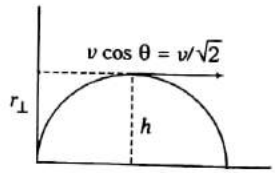
$$m(\mathbf{r} \times \mathbf{v}) = 2\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 1 & 0 \\ 2 & -2 & 2 \end{vmatrix} = 4\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 8\hat{\mathbf{k}}$$

Therefore, component of angular momentum about Z-axis would be -8 kgm^{2}s^{-1}.

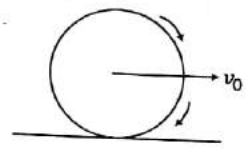
28. (*b*) The magnitude of angular momentum,

$$L = mvr_\perp$$

$$\Rightarrow \quad L = \frac{mvh}{\sqrt{2}}$$



29. (*b*) About bottommost point,



The angular momentum of sphere,

$$L = MRv_0 + \frac{2}{5}MR^2\left(\frac{v_0}{R}\right) = \frac{7}{5}MRv_0$$

30. (*d*) Angular acceleration, $\alpha = \frac{\tau}{I} = \frac{1000}{200} = 5 \text{ rad s}^{-2}$

$$\omega = \alpha t = (5)(3) = 15 \text{ rads}^{-1}$$

31. (*a*) As no external torque acts on the system, angular momentum should be conserved.

Hence, $I\omega = \text{constant}$ ...(i)

where, I is moment of inertia of the system and ω is angular velocity of the system.

From Eq. (i) $I_1\omega_1 = I_2\omega_2$

(where ω_1 and ω_2 are angular velocities before and after jumping)

$\Rightarrow$ $$I\omega = \frac{I}{2} \times \omega_2$$

(as mass reduced to half, hence, moment of inertia also reduced to half)

$\Rightarrow$ $$\omega_2 = 2\omega$$

32. (*a*) $I = mK^2 = 10(2)^2 = 40 \text{ kg-m}^2$

$$K_R = \frac{1}{2}I\omega^2 = \frac{1}{2}(40)(5)^2 = 500 \text{ J}$$

33. (*c*) W = change in rotational kinetic energy $= \frac{1}{2}I(\omega_f^2 - \omega_i^2)$

$$= \frac{1}{2}\left(\frac{1}{2}mR^2\right)[(2\pi f_f)^2 - (2\pi f_i)^2]$$

$$= \frac{1}{4} \times 2 \times (1)^2 \times 4 \times \pi^2 (100 - 25)$$

$$= 1.48 \times 10^3 \text{ J} \approx 1.5 \times 10^3 \text{ J}$$

34. (*c*) Moment of inertia depends on the distribution of mass about the Z-axis.

35. (*d*) Radius of gyration, $K = \sqrt{\frac{I}{m}}$

$$K_{disc} = \sqrt{\frac{\frac{1}{2}mR^2 + mR^2}{m}} = \sqrt{\frac{3}{2}}R$$

$$K_{ring} = \sqrt{\frac{mR^2 + mR^2}{m}} = \sqrt{2}R$$

$$\therefore \quad \frac{K_{disc}}{K_{ring}} = \frac{\sqrt{3/2}}{\sqrt{2}} = \frac{\sqrt{3}}{2}$$

36. (*c*) I_{CM} is less than I about any other axis not passing through center of mass but only when two axes are parallel.

37. (*b*) Hollow sphere, $K_H = \sqrt{\frac{I}{m}} = \sqrt{\frac{\frac{2}{3}mR^2 + mR^2}{m}} = \sqrt{\frac{5}{3}}R$

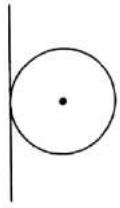
Solid sphere, $K_S = \sqrt{\frac{I}{m}} = \sqrt{\frac{\frac{2}{5}mR^2 + mR^2}{m}} = \sqrt{\frac{7}{5}}R$

$$\therefore \quad \frac{K_H}{K_S} = \sqrt{\frac{25}{21}} = \frac{5}{\sqrt{21}}$$

38. (*c*) For square lamina, $I_2 = I_3$. This value will be less than I_1 because mass is nearer to axis in this case.

39. (*b*) Radius of gyration of a circular disc about a tangential axis

$$K_1 = \sqrt{\frac{I_1}{m_1}} = \sqrt{\frac{(5/4)m_1R^2}{m_1}} = \sqrt{\frac{5}{4}}R$$



Radius of gyration of a circular ring about a tangential axis

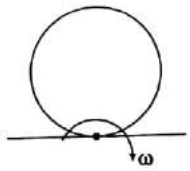
$$K_2 = \sqrt{\frac{I_2}{m_2}} = \sqrt{\frac{(3/2)m_2R^2}{m_2}} = \sqrt{\frac{3}{2}}R \Rightarrow \frac{K_1}{K_2} = \sqrt{\frac{5}{6}}$$

40. (*b*) We have, $I_1\omega_1 = I_2\omega_2$ or $\omega_2 = \frac{I_1}{I_2}\omega_1$

$$= \frac{(1/2MR^2)}{(1/2MR^2) + (1/2)(1/4MR^2)}\omega = \frac{4}{5}\omega$$

41. (*c*) Friction in this case is backward. If it is insufficient, it is a case of forward slipping or $K_T > K_R$.

43. (*a*) Velocity, $v = r\omega$ or $v \propto r$



$$r_Q > r_C > r_P \Rightarrow v_Q > v_C > v_P$$

44. (*d*) Angular momentum, $L = \sqrt{2KI}$ (Just like $p = \sqrt{2Km}$)

K is made 4 times. Hence, L will become 2 times as $L \propto \sqrt{K}$

45. (*b*) Work done by retarding torque = change in kinetic energy

Since, kinetic energy of both are equal, both will rotate with same revolutions.

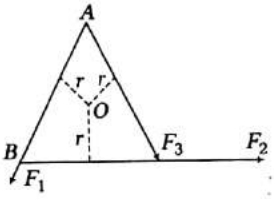
46. (*a*) Work done by pure rolling is always zero. It is because there is no displacement by force (in case of rolling it is friction). Only in case of rolling with sliding, friction does work.

47. (*b*) Force 8 N can also be resolved in tangential and radial directions.

$\tau_{net} = (9 + 8\sin 30° - 4)(0.2)$ N-m clockwise

$= 1.8$ N-m clockwise

48. (*c*) For total torque about O to be zero,

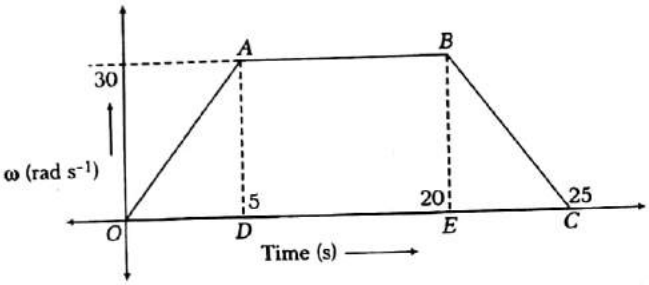


$$F_1r + F_2r - F_3r = 0 \Rightarrow \phi_3 = \phi_1 + \phi_2 \Rightarrow F_3 = F_1 + F_2$$

49. (*c*) Angle rotated, θ = area under ω-t graph.

area under ω-t graph = area of trapezium $OABCO$.

$\because$ Area of ($\square$ $OABCO$) $= \frac{1}{2}$ (Sum of parallel sides) $\times$ distance between them.



$$= \frac{1}{2}(AB + OC) \times AD = \frac{1}{2}(15 + 25) \times 30 = 600$$

50. (*c*) Area under *n-t* graph will give total number of revolution before coming to stop.

Area = Total number of revolutions

$$= \frac{1}{2} \times 8 \times \left(\frac{1800}{60}\right) + 8 \times \frac{600}{60} + \frac{1}{2} \times 16 \times \left(\frac{600}{60}\right)$$

$$= 120 + 80 + 80$$

$$= 280$$

51. (*c*) $\mu < \mu_{min}$ but $\mu \neq 0$. Therefore, sphere will roll with forward slipping.

52. (*c*) The radius of gyration, $K_D = \sqrt{\frac{I_D}{m}} = \sqrt{\frac{\frac{1}{2}mr^2}{m}} = \frac{r}{\sqrt{2}}$

$$K_R = \sqrt{\frac{I_{PR}}{m}} = \sqrt{\frac{m(2r)^2}{m}} = 2r$$

$$\therefore \quad \frac{K_R}{K_D} = 2\sqrt{2}$$

53. (*a*) $L = mvR + I_C\omega = mvR + \frac{1}{2}mvR = \frac{3}{2}mvR$

54. (*b*) We have, $(\pi R^2 td) = m$ ($\because$ mass = density $\times$ volume)

$$\therefore \quad R^2 = \frac{m}{\pi td}$$

Now, $$I = \frac{1}{2}mR^2 = \frac{m^2}{2\pi td} \quad \text{or} \quad I \propto \frac{1}{d}$$

$\therefore$ The ratio of moments of inertia, $\frac{I_1}{I_2} = \frac{d_2}{d_1}$

55. (*a*) We have, $\frac{K_R}{K_T} = \frac{I}{mR^2}$, if $v = R\omega$ in case of pure rolling $= \frac{K^2}{R^2}$

$$\therefore \quad K_R = \left(\frac{K^2}{K^2 + R^2}\right) K_{Total} \quad \text{or} \quad \frac{K_R}{K_{Total}} = \frac{K^2}{K^2 + R^2}$$

56. (*d*) At bottommost point, total kinetic will be *mgh*. Ratio of rotational to translational kinetic energy will be 2/5.

$$\therefore \quad K_T = \frac{5}{7}mgh = \frac{1}{2}mv^2$$

$$\therefore \quad v = \sqrt{\frac{10}{7}gh}$$

57. (*d*) Acceleration, $a = \frac{g \sin\theta}{1 + \frac{I}{mr^2}}$

For a sphere, $$\frac{I}{mr^2} = \frac{2}{5}$$

Hence, $$a = \frac{5}{7}g \sin\theta = \text{constant.}$$

Hence, speed of all spheres is same at the bottom. Sphere (ii) has the largest mass. Hence, it will have the maximum kinetic energy.

58. (*b*) As it is clear from the figure,

$$AC = BD = \sqrt{l^2 + l^2} = \sqrt{2}l$$

Moment of inertia of four point masses about *BD*

$$I_{BD} = m\left(\frac{l\sqrt{2}}{2}\right)^2 + m \times 0 + m\left(\frac{l\sqrt{2}}{2}\right)^2 + m \times 0$$

$$I_{BD} = ml^2$$

Applying the theorem of parallel axis,

$$I_{xy} = I_{BD} + M(AO)^2 = ml^2 + 4m\left(\frac{l}{\sqrt{2}}\right)^2 = 3ml^2$$

59. (*a*) Density is given as $\rho(x) = a(1 + bx^2)$

where *a* and *b* are constants and $0 \leq x \leq 1$

Let $b \to 0$, in this case

$$\rho(x) = a = \text{constant}$$

Hence, centre of mass will be at $x = 0.5$ m (middle of the rod)

Putting, $b = 0$ in all the options, only (a) gives 0.5.

Note We should not check options by putting $a = 0$, because $\rho = 0$ for $a = 0$.

60. (*b*) For a given body, mass is same, so it will depend only on the distribution of mass about the axis.

The mass is farthest from axis *BC*, so I_2 is maximum. Mass is nearest to axis *AC*, so I_3 is minimum.

Hence, the correct sequence will be

$$I_2 > I_1 > I_3$$

61. (*a*) From theorem of perpendicular axes, we have

$$I = I_c + m\left(\frac{a}{\sqrt{2}}\right)^2 = \left(\frac{ma^2}{12} + \frac{ma^2}{12}\right) + \frac{ma^2}{2} = \frac{2}{3}ma^2$$

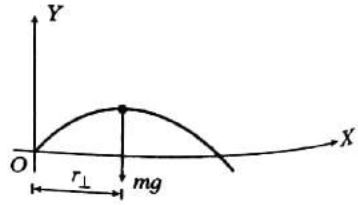
62. (*b*) Moment of inertia, $I = 4\left[\frac{M}{2}\frac{(l/2)^2}{3}\right] = \frac{Ml^2}{6}$

63. (*d*) Moment of inertia, $I = I_{CM} + mx^2$

i.e., *I-x* graph is a parabola not passing through origin.

64. (*b*) Torque, $\tau_0 = mg \times r_\perp$

As, $r_\perp$ is continuously increasing or torque is continuously increasing on the particle. Hence, angular momentum is continuously increasing.



65. (*c*) Path of *A* is cycloid and distance travelled in one rotation is 8*R*.

66. (*a*) $L_i - l_f$ (about centre of mass of the rod)

$$mvr = \frac{ML^2}{12}\cdot\omega \Rightarrow \omega = \frac{12mvr}{ML^2} = \frac{(12)(1)(2)(1/4)}{(2)(1)^2} = 3\ \text{rads}^{-1}$$

67. (*b*) From the property of a circle, if an arc subtends an angle θ at any point A on circumference, then it will subtend an angle 2θ at centre C. So in the same time interval a particle rotating in a circle turn double the angle with respect to centre point C compared to point A.

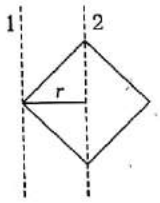
$\therefore \quad \omega_c = 2\omega_A$

68. (*b*) Moment of inertia, $I = 3\left[\frac{Ml^2}{3}\right] = Ml^2$

69. (*b*) The moment of inertia,

$$I = I_{\text{ring}} + 3I_{\text{spoke}} = MR^2 + 3\left(\frac{mR^2}{3}\right) = (M+m)R^2$$

70. (*c*)



By theorem of parallel axis, $I_1 = I_2 + (4M)r^2$

$$= 4\left[\frac{ML^2}{3}\sin^2 45°\right] + 4M\left[\frac{\sqrt{2}L}{2}\right]^2 = \frac{8}{3}ML^2$$

71. (*b*) Since, the sphere is sliding, maximum force of friction will act passing through the bottommost point. Torque of this force about point of contact is zero. Hence, angular momentum will remain conserved about the point of contact and not about the centre of mass. About centre of mass torque of friction will not be zero.

72. (*b*) On inclined surface gravitational force provides torque. On rough surface friction provides torque for rolling but on horizontal smooth surface we need external force to create torque.

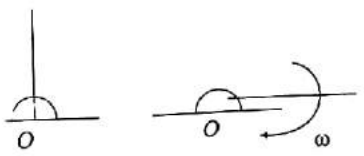
73. (*b*) Decrease in gravitational potential energy = Increase in rotational kinetic energy about an axis passing through end point.

$$\therefore \quad mg\frac{l}{2} = \frac{1}{2}\left(\frac{ml^2}{3}\right)\omega^2$$

$\Rightarrow$ Angular velocity, $\omega = \sqrt{\frac{3g}{l}}$ or $\omega \propto \sqrt{\frac{g}{l}}$

74. (*a*) Angular momentum will not remain conserved due to a torque of weight of particle about axis of rotation.

75. (*a*) Decrease in gravitational potential energy = Increase in rotational kinetic energy about point O.



or $\quad mg\frac{L}{2} = \frac{1}{2}\left(\frac{mL^2}{3}\right)\omega^2$ or $\omega = \sqrt{\frac{3g}{L}}$

Now $\quad v = r\omega \quad$ (in pure rotation)

$$\therefore \quad v = L\sqrt{\frac{3g}{L}} = \sqrt{3gL}$$

76. (*c*) According to the question,

$(K_T)_{\text{sphere}} = (K_T)_{\text{cylinder}} \quad$ (m and v are same)

$$\therefore \quad \frac{5}{7}(mgh_1) = \frac{2}{3}(mgh_2) \Rightarrow \frac{h_1}{h_2} = \frac{14}{15}$$

77. (*b*) In case of pure rolling, $\frac{K_R}{K_T} = \frac{2}{5}$

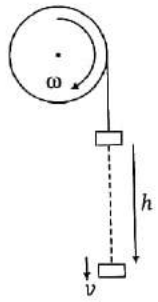
$\therefore$ Total kinetic energy = 7/5 times the translational kinetic energy. At highest point, whole of the kinetic energy will be converted into potential energy.

$$\therefore \quad mgh = \frac{7}{5}\left(\frac{1}{2}mv^2\right) \text{ or } h = \frac{7v^2}{10g} = \frac{7(10)^2}{10\times 10} = 7\text{m}$$

78. (*b*) From theorem of parallel axis, moment of inertia about an axis passing through a distance $l/2$ from centre in all directions will have moment of inertia $\frac{ml^2}{3}$. All such points at same distance from centre will lie in a circle.

79. (*b*) Velocity, $v = r\omega$

Decrease in gravitational potential energy = Increase in kinetic energy



$$\therefore \quad mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 = \frac{1}{2}I\omega^2 + \frac{1}{2}m(r\omega)^2$$

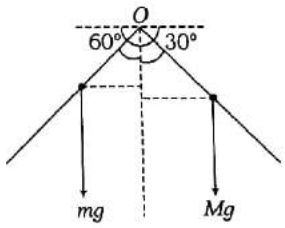
$\therefore$ Angular velocity, $\omega = \sqrt{\frac{2mgh}{I + mr^2}}$

80. (*d*) In case of pure rolling, ratio of rotational to translational kinetic energy is 2/5. Therefore, total kinetic energy is 7/5 times the translational kinetic energy. At maximum compression, whole of energy is elastic potential. Hence,

$$\frac{7}{5}\left(\frac{1}{2}Mv^2\right) = \frac{1}{2}kx_{\max}^2$$

$\therefore$ The compression of the spring, $x_{\max} = v\sqrt{\frac{7M}{5k}}$

81. (*d*) Net torque about O should be zero



$$\therefore \quad Mg\frac{l}{2}\sin 30° = mg\frac{l}{2}\sin 60°$$

$$\frac{M}{m} = \frac{\sin 60°}{\sin 30°} = \sqrt{3}$$

82. (*d*) Whole mass has equal distance from the centre *O*. Hence, $I_0 = mR^2$. Further centre of mass of the remaining portion will be to the left of point *O*. More the distance of axis from centre of mass, more is the moment of inertia. Hence, $I_A > I_0$.

83. (*c*) If we complete the ring, its mass become $2m$

$$\therefore \quad I_{\text{whole ring}} = \frac{1}{2}(2M)(R^2) = MR^2$$

($I_{\text{whole ring}}$ = MI about any diameter)

$$\therefore \quad I_{\text{Half ring}} = \frac{1}{2}(MR^2) \quad \text{(about any radius)}$$

This value is independent of angle θ.

B) Medical entrance special format questions

Assertion and reason

1. (*b*) When comparing between parallel axes moment of inertia about an axis passing through centre of mass is minimum.

2. (*c*) L about centre = mvR = constant

$\because \tau = \frac{dL}{dt}$ and derivative of any constant is zero.

$\tau_{\text{centre}} = 0$

3. (*b*) Angular velocity of particle of rigid body is same in rotational motion.

4. (*a*) Angular momentum about centre is mvR and in circular motion, angular momentum about centre is always constant.

5. (*b*) $I = I_C + mr^2$

Radius of gyration is a distance.

8. (*b*) $L = mvr\sin\theta$ or $mvr_\perp$

In case of constant velocity m, v and $r_\perp$ all are constant. Therefore, angular momentum is constant.

Further, $L = n\frac{h}{2\pi}$ (in Bohr's theory).

Therefore, L and h have same units.

9. (*a*) Moment of inertia of sphere in case 1 is $\frac{2}{5}MR^2$.

Moment of inertia of sphere in case 2 is $\frac{7}{5}MR^2$

So, more work will have to be done to rotate the sphere in case 2.

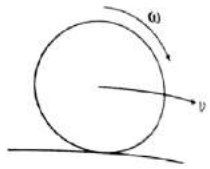
10. (*d*) $\therefore$ As acceleration, $a = \dfrac{g\sin\theta}{1+\dfrac{I}{MR^2}}$ ...(i)

Since, the moment of inertia of disc is lesser than that of ring. Therefore, the disc will roll down the plane with more acceleration and hence, it will take less time. Therefore Reason is correct but Assertion is incorrect.

11. (*d*) Linear acceleration due to $mg\sin\theta$ is downward. Hence, for pure rolling to continue, force of friction should act upwards so that angular acceleration is anti-clockwise.

14. (*c*) When compared with many parallel axes, I_{CM} is least. But it is wrong to say that parallel axis theorem can be applied to 2D body only.

15. (*d*) Sphere can roll without slipping on surface, if $v = r\omega$ on an inclined plane, it is friction which creates rotation on sphere. So smooth surface can not create rotation.



16. (*c*) In a rotating body, every particle has different linear velocity to be calculated by formula $v = \omega r$. All rolling motion can not be considered pure rotation. For pure rotation, the condition is linear velocities of body = ωr.

17. (*d*) Total energy, $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh$ (for both)

I is different for two bodies.

Hence, v and ω will be different, as $v = R\omega$ but loss of potential energy is same.

18. (*c*) Angular momentum about bottommost point will be

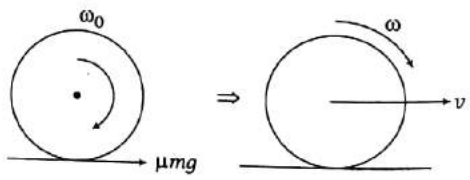
$$L = mv_0R - \frac{1}{2}mR^2\omega_0$$

If $L = 0$ or $v_0 = \frac{\omega_0 R}{2}$, disc will come to rest after some time.

In case of pure rotation, since friction force is acting therefore angular momentum is not conserved.

19. (*b*) In first case, work done by friction will be zero. That's why mechanical energy will remain conserved. Further, in second case, linear velocity will be more because this ball has only translational kinetic energy.

20. (*a*) Pure rolling will start when



$$v = R\omega, \ \mu gt = R\left[\omega_0 - \frac{\mu mgR}{I}t\right]$$

$$\therefore \quad t = \frac{R\omega_0}{\mu g + \dfrac{\mu mgR}{I}}; \ I_{\text{solid}} < I_{\text{hollow}} \ \therefore \ t_{\text{solid}} < t_{\text{hollow}}$$

Match the columns

1.

$$L_1 = I\omega = \frac{2}{5}MR^2\omega$$

$$L_2 = \frac{2}{5}M\left(\frac{R}{2}\right)^2\omega'$$

As $\quad L_1 = L_2 \Rightarrow \omega' = 4\omega$

$$\therefore \quad \frac{2\pi}{T'} = 4\left(\frac{2\pi}{T}\right) \Rightarrow T' = \frac{T}{4}$$

Further, $K = \frac{L^2}{2I}$

Since, angular momentum is constant and I has become (1/4)th. Therefore, kinetic energy will become 4 times.

2. In general as $v_P = 2v \sin\left(\frac{\theta}{2}\right)$

If $\theta = 60°$, then $v_P = 2v \sin\left(\frac{60°}{2}\right) = 2v \times \frac{1}{2} = v$

$\Rightarrow$ $(A) \rightarrow (q)$

If $\theta = 90°$, then $v_P = 2v \sin\left(\frac{90°}{2}\right) = 2v \times \frac{1}{\sqrt{2}} = \sqrt{2}v$

$\Rightarrow$ $(B) \rightarrow (p)$

If $\theta = 120°$, then $v_P = 2v \sin\left(\frac{120°}{2}\right) = 2v \times \frac{\sqrt{3}}{2} = \sqrt{3}v$

$\Rightarrow$ $(C) \rightarrow (s)$

If $\theta = 180°$, then $v_P = 2v \sin\left(\frac{180°}{2}\right) = 2v \Rightarrow (D) \rightarrow (r)$

3. Torque, $\tau = 0$

$\therefore L =$ constant, $K = \frac{L^2}{2I}$ and $I\omega =$ constant. Insect first moves away from the axis, then towards it. Hence, I will first increase and then decrease.

4. $I_1 = 2\left(\frac{ml^2}{12}\right) + 2(m)\left(\frac{l}{2}\right)^2 = \frac{2}{3}ml^2$

$I_2 = 0 + 2\left(\frac{ml^2}{3}\right) + ml^2 = \left(\frac{5}{3}\right)ml^2$

$I_3 = 4\left[\frac{ml^2}{3}\sin^2 45°\right] = \frac{2}{3}ml^2 = I_1$

Note $I_1 = I_3$ (think why?)

5. Moment of inertia of a ring about an axis perpendicular and passing through the centre of ring is MR^2. Moment of inertia of a ring about diameter is $MR^2/2$. From parallel axes theorem,

$$I_4 = \frac{MR^2}{2} + MR^2 = \frac{3}{2}MR^2$$

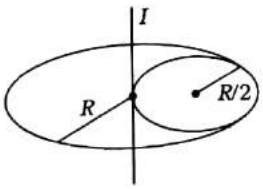
C) Medical entrances' gallery

1. (*a*) Torque, $\tau = I\alpha$, Moment of inertia of wheel, $I = \frac{\tau}{\alpha}$

Moment of inertia of wheel,

$$I = \frac{5 \times 10 \times 10^{-2}}{2} \text{ kg-m}^2 = 0.25 \text{ kg-m}^2$$

2. (*a*) Considering the information given in the question, let us draw the figure



If the above figure is considered, then moment of inertia of disc will be given as $I = I_{remain} + I_{(R/2)} \Rightarrow I_{remain} = I - I_{(R/2)}$

Putting the values, we get

$$= \frac{MR^2}{2} - \left[\frac{\frac{M}{4}\left(\frac{R}{2}\right)^2}{2} + \frac{M}{4}\left(\frac{R}{2}\right)^2\right]$$

$$= \frac{MR^2}{2} - \left[\frac{MR^2}{32} + \frac{MR^2}{16}\right] = \frac{MR^2}{2} - \left[\frac{MR^2 + 2MR^2}{32}\right]$$

$$= \frac{MR^2}{2} - \frac{3MR^2}{32} = \frac{16MR^2 - 3MR^2}{32}$$

$$I_{remain} = \frac{13MR^2}{32}$$

3. (*a*) Acceleration of an object rolling down an inclined plane is given by

$$a = \frac{g \sin\theta}{1 + I/mr^2}$$

where, θ = angle of inclination of the inclined plane

m = mass of the object

and I = moment of inertia about the axis through centre of mass

For disc, $\frac{I}{mr^2} = \frac{1/2\, mr^2}{mr^2} = \frac{1}{2}$

For solid sphere, $\frac{I}{mr^2} = \frac{2/5 mr^2}{mr^2} = \frac{2}{5}$

For hollow sphere, $\frac{I}{mr^2} = \frac{2/3 mr^2}{mr^2} = \frac{2}{3}$

$\therefore$ $a_{disc} = \frac{g \sin\theta}{1 + \frac{1}{2}} = \frac{2}{3} g \sin\theta$

$= 0.66\, g \sin\theta$

$a_{solid\ sphere} = \frac{g \sin\theta}{1 + \frac{2}{5}} = \frac{5}{7} g \sin\theta = 0.71\, g \sin\theta$

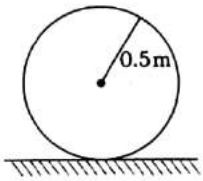
$a_{hollow\ sphere} = \frac{g \sin\theta}{1 + \frac{2}{3}} = \frac{3}{5} g \sin\theta = 0.6\, g \sin\theta$

Clearly, $a_{solid\ sphere} > a_{disc} > a_{hollow\ sphere}$

Type of sphere is not mentioned in the question. Therefore, we will assume the given sphere as solid sphere.

$\therefore$ $a_{solid\ sphere} = a_{hollow\ sphere} > a_{disc}$

4. (*d*) According to given question, a uniform circular disc of radius 50 cm at rest is free to turn about an axis having perpendicular to its plane and passes through its centre. This situation can be shown by the figure given below:



$\therefore$ Angular acceleration, $\alpha = 2$ rad s^{-2} (given)

Angular speed, $\omega = \alpha t = 4$ rad s^{-1}

$\because$ Centripetal acceleration, $a_c = \omega^2 r = (4)^2 \times 0.5 = 16 \times 0.5$

$a_c = 8$ ms^{-2}

$\because$ Linear acceleration at the end of 2 s

$$a_t = \alpha r = 2 \times 0.5 \Rightarrow a_t = 1 \text{ms}^{-2}$$

Therefore, the net acceleration at the end of 2.0 s is given by

$$a = \sqrt{a_c^2 + a_t^2}$$

$$a = \sqrt{(8)^2 + (1)^2} = \sqrt{65} \Rightarrow a \approx 8 \text{ ms}^{-2}$$

5. (*c*) According to law of conservation of angular momentum

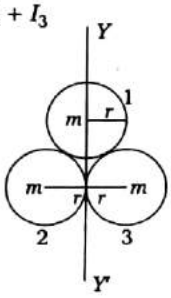
$$L_i = L_f \Rightarrow mv_0R_0 = mv'\left(\frac{R_0}{2}\right) \Rightarrow v' = 2v_0$$

So, final kinetic energy of the particle is

$$K_f = \frac{1}{2}mv'^2 = \frac{1}{2}m(2v_0)^2 = 4 \times \frac{1}{2}mv_0^2 = 2mv_0^2$$

6. (*d*) The total moment of inertia of the system is

$$I = I_1 + I_2 + I_3 \quad \text{...(i)}$$



Here, $I_1 = \frac{2}{3}mr^2$

$$I_2 = I_3 = \frac{2}{3}mr^2 + mr^2 \quad \text{(From parallel axis theorem)}$$

$$= \frac{5}{3}mr^2$$

From Eq. (i),

$$I = \frac{2}{3}mr^2 + 2 \times \frac{5}{3}mr^2 = mr^2\left(\frac{2}{3} + \frac{10}{3}\right) \Rightarrow I = 4mr^2$$

7. (*b*) When a metallic rod is heated, it expands. Its moment of inertia (I) about a perpendicular bisector increases. According to law of conservation of angular momentum, its angular speed (ω) decreases, since $\omega \propto \frac{1}{I}$ (according to law of conservation of angular momentum).

8. (*b*) The radius of the circle followed by the masses is $r = l \sin \alpha$

As, angular momentum, $\mathbf{L} = \mathbf{r} \times \mathbf{p} = \mathbf{r} \times m\mathbf{v}$

$$\Rightarrow \quad |\mathbf{L}| = l \sin\theta\,(m\,\omega\, l \sin\theta)$$

On differentiating, we get

$$\frac{d|\mathbf{L}|}{dt} = m\omega l^2 2\sin\theta \cdot \cos\theta \frac{d\theta}{dt}$$

$$\Rightarrow \quad \left|\frac{d\mathbf{L}}{dt}\right| = 2ml^2\omega^2 \sin\theta \cdot \cos\theta = ml^2\omega^2 \sin 2\theta$$

9. (*b*) Given, radius of the circle = 20 cm = 20×10^{-2} m

When speed is 8 ms^{-1} and 9 ms^{-1}, then angular speeds are respectively

$$\omega_1 = \frac{8}{20 \times 10^{-2}} = 40 \text{ rads}^{-1}$$

and $$\omega_2 = \frac{9}{20 \times 10^{-2}} = 45 \text{ rads}^{-1}$$

$$\therefore \text{ Angular acceleration} = \frac{\omega_2 - \omega_1}{t} = \frac{45 - 40}{2} = 2.5 \text{ rad s}^{-2}$$

10. (*b*) The rotation kinetic energy, $K_r = \frac{1}{2}I\omega^2$

where, I = moment of inertia of solid sphere. The total kinetic energy is the sum of rotational kinetic energy and translational kinetic energy

$$K_T = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}m\omega^2 r^2 + \frac{1}{2}I\omega^2 \quad (\because \text{ For pure rolling } v = \omega r)$$

Now, $$\frac{K_r}{K_T} = \frac{\frac{1}{2}I\omega^2}{\frac{1}{2}\omega^2(mr^2 + I)} = \frac{I}{mr^2 + I}$$

As $I = \frac{2}{5}mr^2$ (for solid sphere)

$$\Rightarrow \quad \frac{K_r}{K_T} = \frac{\frac{2}{5}mr^2}{mr^2 + \frac{2}{5}mr^2} = \frac{\frac{2}{5}mr^2}{\frac{7}{5}mr^2} = \frac{2}{5} \times \frac{5}{7} = \frac{2}{7}$$

11. (*e*) Given, radius of circular disc A, $r_1 = r$

Radius of circular disc B, $r_2 = 2r$

For disc A, $I_A = mr^2$ ...(i)

For disc B, $I_B = m(2r)^2 = 4mr^2$ ...(ii)

Dividing Eq. (i) by Eq. (ii), we get

$$\therefore \quad \frac{I_A}{I_B} = \frac{mr^2}{4mr^2} \Rightarrow \frac{I_A}{I_B} = \frac{1}{4} \Rightarrow \quad I_A = \frac{1}{4}I_B$$

So, I_A = one fourth that of B.

12. (*b*) Moment of inertia is not a scalar quantity so the option (b) is wrong.

13. (*a*) Total angular momentum about O is given as,

$$L = L_1 + L_2 = m_1v_1r_1 + m_2v_2r_2$$

$$= -6.5 \times 2.2 \times 1.5 + 3.1 \times 3.6 \times 2.8$$

$$= -21.45 + 31.248 = 9.8 \text{ kgm}^2\text{s}^{-1}$$

14. (*b*) When string is cut, the weight of the rod constitutes a torque about the hinge point A. So,

$$\tau_A = mg\frac{l}{2} \quad \text{...(i)}$$

Also from Newton's law, $\tau_A = I\alpha$ ...(ii)

where, α = angular acceleration of the rod and I = moment of inertia of rod.

From Eqs.(i) and (ii), we get

$$I\alpha = mg\frac{l}{2} \Rightarrow \alpha = mg\frac{l}{2}/I$$

As, $$I = m\frac{l^2}{3}$$

So, $$\alpha = \frac{mg(l/2)}{\frac{ml^2}{3}} = \frac{3g}{2l}$$

Now, acceleration of centre of mass of rod, $a_{CM} = \alpha \cdot r$

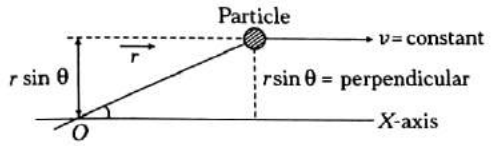
Here, r = distance between hinge point A and centre of the rod.

$$\Rightarrow \quad a_{CM} = \frac{3}{2}\frac{g}{l}\frac{l}{2} = \frac{3g}{4} \Rightarrow mg - R_A = ma_{CM}$$

where, R_A = reaction at A

$$\Rightarrow \quad mg - R_A = m \times \frac{3g}{4} \Rightarrow R_A = \frac{mg}{4} = \frac{5 \times 10}{4} = 12.5 \text{ N}$$

15. (*b*) Consider the diagram



As, the particle goes with constant linear velocity, the perpendicular distance of velocity vector from the axis is constant. So, angular momentum $mvr \sin\theta$ is constant.

16. (*d*) Moment of inertia of a ring about its diameter $I = \frac{1}{2}mr^2$

and kinetic energy is given by

$$E_k = \frac{1}{2}I\omega^2 \Rightarrow E_k = \frac{1}{4}mr^2\omega^2 = \frac{1}{4} \times 10 \times (0.5)^2 \times (20)^2 = 250 \text{ J}$$

17. (*a*) Let the velocity of centre of sphere be v.

The angular speed of the sphere about its centre is given by

$$\omega = \frac{v}{r}$$

where, r = radius of sphere.

The total kinetic energy = translational kinetic energy + rotational kinetic energy

$$= \frac{1}{2}I\omega^2 + \frac{1}{2}Mv^2 = \frac{1}{2}\cdot\frac{2}{5}Mr^2\omega^2 + \frac{1}{2}Mv^2$$

$$= \frac{1}{5}Mv^2 + \frac{1}{2}Mv^2 = \frac{7}{10}Mv^2 = \frac{7}{10} \times \frac{1}{2} \times (0.02)^2 = 1.4 \times 10^{-4} \text{ J}$$

18. (*c*) We know that, $\omega = 2\pi n \Rightarrow \omega_1 = 2\pi n_1$

where, $n_1 = 1800$ rpm, $n_2 = 3000$ rpm $\Rightarrow \Delta t = 20$ s

$$\omega_1 = 2\pi \times \frac{1800}{60} = 2\pi \times 30 = 60\pi$$

Similarly, $\omega_2 = 2\pi n_2 = 2\pi \times \frac{3000}{60} = 2\pi \times 50 = 100\pi$

If the angular velocity of a rotating wheel about an axis changes by change in angular velocity in a time interval Δt, then the angular acceleration of rotating wheel about that axis is

$$\alpha = \frac{\text{Change in angular velocity}}{\text{Time interval}}$$

$$\alpha = \frac{\omega_2 - \omega_1}{\Delta t} \Rightarrow \alpha = \frac{100\pi - 60\pi}{20}$$

$$\alpha = \frac{40\pi}{20} = 2\pi \text{ rads}^{-2}$$

19. (*b*) For a ring, $I_z = MR^2$

From perpendicular axis theorem,

$$I_x + I_y = I_z \quad \text{...(i)}$$

Given, $I_x = I_y = I$

From Eq. (i), we get

$$2I = I_z$$

20. (*c*) Consider two bodies A and B are having moment of inertia I and $2I$.

K_A = Kinetic energy of rotation for the body $A = (1/2)I\omega_A^2$

where, ω_A is angular speed of the body A

K_B = Kinetic energy of rotation for the body $B = \frac{1}{2}(2I)\omega_B^2$

According to the question, $K_A = K_B$

$$\Rightarrow \frac{1}{2}I\omega_A^2 = \frac{1}{2}(2I)\omega_B^2 \Rightarrow \omega_A^2 = 2\omega_B^2 \Rightarrow \left(\frac{\omega_A}{\omega_B}\right)^2 = 2$$

$$\Rightarrow \frac{\omega_A}{\omega_B} = \sqrt{2} \Rightarrow \omega_A : \omega_B = \sqrt{2} : 1$$

21. (*a*) We know that the kinetic energy, $K = \frac{1}{2}mv^2 = \frac{1}{2}I\omega^2$

where, $m = 27$ kg (mass of the body)

$\omega = 3 \text{ rads}^{-1}$ (angular velocity)

$I = 3 \text{ kg-m}^2$ (moment of inertia)

$v = ?$ or $mv^2 = I\omega^2$

$$\Rightarrow \quad v^2 = \frac{I\omega^2}{m} \Rightarrow v^2 = \frac{3 \times 3^2}{27} \Rightarrow v^2 = \frac{27}{27} = 1$$

$\Rightarrow$ Velocity of body, $v = \sqrt{1} = 1 \text{ ms}^{-1}$

22. (*c*) Total kinetic energy of a body which is rolling without slipping is given by

$$K_{total} = K_{rot} + K_{trans}$$

For solid spherical ball,

$$I = \frac{2}{5}mR^2 \text{ and } v = R\omega \quad \text{(along the diameter)}$$

where, R is radius of spherical ball

So, $K_{total} = \frac{1}{2}\left(\frac{2}{5}mR^2\right)\omega^2 + \frac{1}{2}mR^2\omega^2 = \frac{7}{10}mR^2\omega^2 = \frac{7}{10}mv^2$

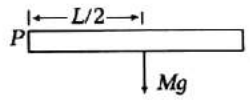
Potential energy = Kinetic energy

$$mgh = \frac{7}{10}mv^2 \Rightarrow v^2 = \frac{10}{7}gh \quad \text{...(i)}$$

For vertical projection,

$$v^2 = u^2 + 2gh' \Rightarrow \frac{10}{7}gh = 0 + 2gh' \Rightarrow h' = \frac{5}{7}h$$

23. (*a*)



Torque on the rod = Moment of weight of the rod about P,

$$\tau = Mg\frac{L}{2} \quad \text{...(i)}$$

$\because$ Moment of inertia of rod about P,

$$I = \frac{ML^2}{3} \quad \text{...(ii)}$$

As $\tau = I\alpha$

From Eqs. (i) and (ii), we get

$$Mg\frac{L}{2} = \frac{ML^2}{3}\alpha$$

$\therefore$ The initial angular acceleration of the rod

$$\alpha = \frac{3g}{2L}$$

24. (*d*) Velocity, $v = \sqrt{\dfrac{2gh}{1+K^2/R^2}}$

Given, $h = \dfrac{3v^2}{4g}$

$$v^2 = \frac{2gh}{1+\dfrac{K^2}{R^2}} = \frac{2g\,3v^2}{4g\left(1+\dfrac{K^2}{R^2}\right)} = \frac{6gv^2}{4g\left(1+\dfrac{K^2}{R^2}\right)}$$

$$1 = \frac{3}{2\left(1+\dfrac{K^2}{R^2}\right)} \text{ or } 1+\frac{K^2}{R^2} = \frac{3}{2} \text{ or } \frac{K^2}{R^2} = \frac{3}{2}-1 = \frac{1}{2}$$

$$K^2 = \frac{1}{2}R^2 \qquad \text{(Equation of disc)}$$

Hence, the object is disc.

25. (*b*) From law of conservation of angular momentum, if no external torque acts on a system, then total vector sum of angular momentum of different particles of the system remains conserved.

$$\tau_{ext.} = 0 \text{ then } \frac{d\mathbf{L}}{dt} = 0$$

26. (*b*) Angular momentum $\mathbf{L} = I\omega$

27. (*d*) Moment of inertia about its diameter is $I = \dfrac{2}{5}MR^2$

28. (*a*) In given condition, the moment of inertia of the combine system about an axis passing through O is

$$I = \frac{4}{3}ma^2 + m\left(\frac{3a}{4}\right)^2 \Rightarrow \qquad I = \frac{91}{48}ma^2$$

29. (*a*) We know that $L = I\omega = 2\pi mr^2 f$

Now, $\omega' = \omega/2$

Hence, $L' = I\omega' = mr^2\dfrac{\omega}{2} = \pi mr^2 f \Rightarrow \dfrac{L}{L'} = \dfrac{2m\pi r^2 f}{\pi mr^2 f} \Rightarrow L' = \dfrac{L}{2}$

30. (*c*) We know that, torque

$$\tau = \mathbf{r} \times \mathbf{F}$$

Given, $\mathbf{F} = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}} \Rightarrow \mathbf{r} = 3\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$

Now $\tau = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 3 & -3 & 5 \\ 2 & -3 & 5 \end{vmatrix}$

$$\tau = (-15+15)\hat{\mathbf{i}} - (15-10)\hat{\mathbf{j}}\ (-9+6)\hat{\mathbf{k}}$$

$$\tau = 0 - 5\hat{\mathbf{j}} - 3\hat{\mathbf{k}} \Rightarrow \tau = -5\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$$

31. (*b*) Moment of inertia of a disc about a parallel axis

$$I = I_v + md^2 = \frac{ma^2}{2} + md^2 \qquad (\because a = R, d = R)$$

$\Rightarrow$ Moment of inertia $= \dfrac{3}{2}ma^2 = \dfrac{3}{2}(25)$

$\therefore$ Moment of inertia $= \dfrac{75}{2} = 37.5 \text{ kg m}^{-2}$

32. (*a*) We have, $\omega_2 = \omega_1 + \alpha t \Rightarrow \omega_1 = \omega_2 - \alpha t$

Here, $\omega_1 = 0$ and $\omega_2 = 60 \text{ rpm} = \dfrac{60 \times 2\pi}{60} \text{ rad s}^{-1}$

$$\Rightarrow \qquad 0 = 2\pi \times \frac{60}{60} - \alpha \cdot 60 \text{ or } \alpha = \frac{2\pi}{60}$$

$\therefore$ Torque, $\tau = I\alpha = 2 \times \dfrac{2\pi}{60} = \dfrac{\pi}{15}$ N - m

33. (*c*) $I = \dfrac{2}{5}MR^2 = \dfrac{2}{5}\left[\dfrac{4}{3}\pi R^3\rho\right]R^2 = \dfrac{8}{15}\pi R^5\rho$

$$I = \frac{176}{105}R^5\rho \qquad \left(\because \pi = \frac{22}{7}\right)$$

34. (*d*) The radius of gyration does not depend on the total mass of a body but depends upon the shape and size of body, distribution of mass within the body and choice of rotation of axis.

35. (*d*) Moment of inertia of disc, $I = \dfrac{1}{2}MR^2 = \dfrac{1}{2}M\left(\dfrac{M}{\pi\rho t}\right)$

$$\left(\because \rho = \frac{M}{V} = \frac{M}{\pi R^2 t},\ \therefore R^2 = \frac{M}{\pi\rho t}\right)$$

$$I = \frac{1}{2}\frac{M^2}{\pi\rho t}$$

$\therefore \qquad I \propto \dfrac{1}{\rho}$ (As M and t are constants)

$$\frac{I_1}{I_2} = \frac{\rho_2}{\rho_1}$$

36. (*c*) Here, $\omega_2 = 2\pi n_2 = 2 \times \dfrac{240}{60}$ and $\omega_1 = 2\pi n_1 = 0$

The angular acceleration, $\alpha = \dfrac{\omega_2 - \omega_1}{t}$

$$\therefore \qquad \alpha = \frac{2\pi\left(\dfrac{240}{60} - 0\right)}{10} = 2.51 \text{ rads}^{-1}$$

37. (*c*) Total energy = KE + rotational KE

$$= \frac{1}{2}(2m)v^2 + \frac{1}{3}mv^2 = \frac{4}{3}mv^2$$

38. (*d*) According to theorem of parallel axis,

$$I = I_{CM} + M\left(\frac{R}{2}\right)^2$$

$$\Rightarrow \qquad I = \frac{1}{2}MR^2 + \frac{MR^2}{4}$$

Moment of inertia, $I = \dfrac{3}{4}MR^2$

39. (*a*)

Moment of inertia,

$$M_1 = m_w \times \frac{(L/2)^2}{3} = m_w\frac{L^2}{12}$$

Moment of inertia,

$$M_2 = m_b \times \frac{(L/2)^2}{3} = m_b \times \frac{L^2}{12}$$

Moment of inertia,

$$M = M_1 + M_2 = m_w\frac{L^2}{12} + m_b \times \frac{L^2}{12} = (m_w + m_b)\frac{L^2}{12}$$

CHAPTER 10

Gravitation

Each body in this universe attracts other bodies towards itself with a force known as gravitation force, thus gravitation is the phenomena of mutual attraction between any two bodies in the universe. It is the force of gravitation due to which earth attracts everything towards itself.

In this chapter, we will learn the basic laws that govern gravitational interactions.

Planets

The celestial objects which moves in fixed semicircular orbits around the sun are known as planets. Including the earth mainly we have 8 planets in our solar system. These planets move around the sun in their fixed semicircular orbits with increasing distance from the sun, *name of these planets are*

(i) Mercury	(ii) Venus
(iii) Earth	(iv) Mars
(v) Jupiter	(vi) Saturn
(vii) Uranus	(viii) Neptune

Satellites

Celestial objects which are moving around the planets are known as satellites. Obtained number of satellites are fixed according to their own planets.

For example, the moon is a natural satellite while INSAT-1B is an artificial satellite of the earth.

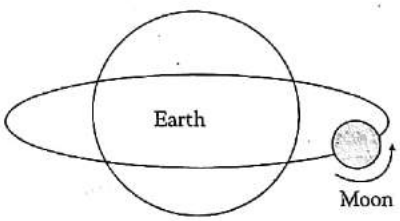


***Fig. 10.1** Earth and moon*

Inside

1) Kepler's laws of planetary motion
2) Newton's law of gravitation
3) Gravity
4) Gravitational field
5) Gravitational potential
6) Gravitational potential energy
7) Binding energy
8) Motion of satellites
9) Artificial satellite

1 Kepler's laws of planetary motion

Kepler's laws of planetary motion are three scientific laws describing motion of planets around the sun.

Kepler's first law of orbits

According to this law, "all planets move in elliptical orbits with the sun situated at one of the foci of the ellipse".

This law identifies that the distance between the sun (S) and planet (P) is constantly changing as the earth goes around its orbit *as shown in figure.*

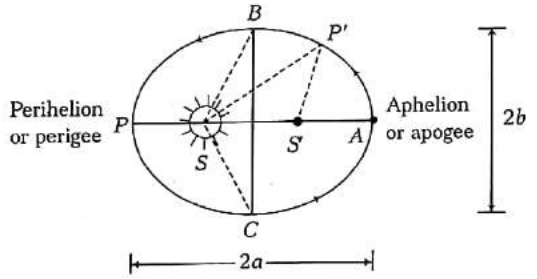


Fig. 10.2 *Law of orbit*

Above figure shows an ellipse traced out by a planet around the sun, S. The closest point P is called **perihelion** (perigee) and the farthest point A is called **aphelion** (apogee).

It is the characteristics of an ellipse that the sum of the distances of any point (P') on it from its two foci (S and S') is constant. i.e. $SP'+S'P'=$ constant.

Note Shorter the orbit of the planet around the sun, shorter is the time to take one complete revolution.

Kepler's second law of areas

According to this law, "the line joining the planet sweeps out equal areas in equal interval of time". It means that areal velocity is constant.

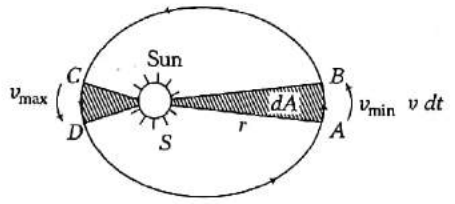


Fig. 10.3 *Law of area*

In the given figure, if the line joining the sun and the planet swept out area dA in a small time interval dt, then

$$\text{Areal velocity} = \frac{dA}{dt} = \frac{d}{dt}\left[\frac{1}{2}r(vdt)\right] = \frac{1}{2}rv = \frac{mvr}{2m} = \frac{L}{2m}$$

where L = angular momentum of the earth-planet system.

= constant,

for central gravitational force between the sun and the planet.

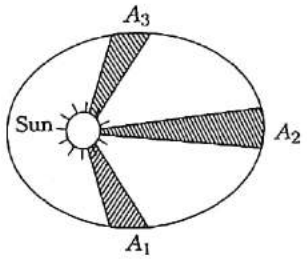
$$\therefore \quad \boxed{\text{Areal velocity} = \frac{dA}{dt} = \text{constant}}$$

Also, if the planet moves A to B in some time interval t and C to D in the same time interval t, then

$$\text{area } (ASB) = \text{area } (CSD)$$

In simple terms, this law states that the planet will move slowly (v_{min}) only when it is farthest from the sun and more rapidly (v_{max}) when it is nearest to the sun. This law is similar to law of conservation of angular momentum.

Example 10.1 *A planet moving around sun sweeps area A_1 in 2 days, A_2 in 3 days and A_3 in 6 days, then find the relation between A_1, A_2 and A_3.*



Sol. According to Kepler's second law, when a planet revolves around the sun, its areal velocity is constant.

$$\therefore \quad \frac{dA}{dt} = \text{constant}$$

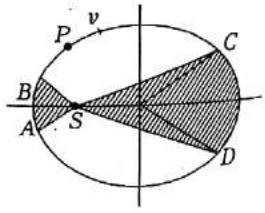
$$\therefore \quad \frac{A_1}{t_1} = \frac{A_2}{t_2} = \frac{A_3}{t_3}$$

Given, $t_1 = 2$ days, $t_2 = 3$ days and $t_3 = 6$ days

$$\therefore \quad \frac{A_1}{2} = \frac{A_2}{3} = \frac{A_3}{6} \Rightarrow 3A_1 = 2A_2 = A_3$$

This is the relation between A_1, A_2 and A_3.

Example 10.2 *The figure shows elliptical orbit of a planet P about the sun S. The shaded area CSD is twice the shaded area ASB. If t_1 is the time taken by the planet to move from C to D and t_2 is the time to move from A to B, determine the ratio t_1/t_2.*



Sol. According to Kepler's second law, the areal velocity of the planet remains constant.

$$\therefore \quad \frac{A_1}{t_1} = \frac{A_2}{t_2}$$

$$\therefore \quad \frac{t_1}{t_2} = \frac{A_1}{A_2} = \frac{\text{Area } CSD}{\text{Area } ASB} = 2 \Rightarrow t_1 = 2t_2$$

Kepler's third law of period

According to this law, "the square of the time period (T) of revolution of a planet around the sun is proportional to the cube of the semi-major axis (a) of its elliptical orbit".

i.e. $$\boxed{T^2 \propto a^3} \quad ...(i)$$

As shown in figure, we have

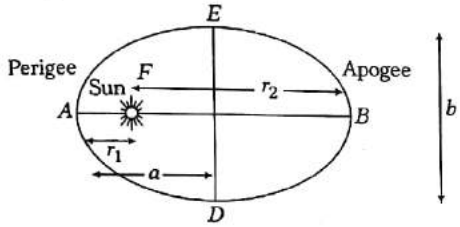


Fig. 10.4 *Law of period*

$$AB = AF + FB \Rightarrow 2a = r_1 + r_2$$

$$\therefore \quad a = \frac{r_1 + r_2}{2}$$

So, putting the value of a in Eq. (i), we get

$$\boxed{T^2 \propto \left(\frac{r_1 + r_2}{2}\right)^3}$$

where, a = semi-major axis, r_1 = shortest distance of planet from sun (perigee) and r_2 = longest distance of planet from sun (apogee).

It can shown that if e the eccentricity of an elliptical orbit of the planet then $\frac{r_2}{r_1} = \frac{1+e}{1-e}$

Note Kepler's laws are applicable not only to the solar system but also to artificial satellites as well as to the moon going around the planets.

Example 10.3 *A satellite of time period 24 h is orbiting the earth at a height 6R above the surface of earth, where R is radius of earth. What will be the time period of another satellite at a height 2.5 R from the surface of earth?*

Sol. By Kepler's third law, $T^2 \propto R^3$

$$\therefore \quad \frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3} \quad \text{or} \quad \left(\frac{T_2}{T_1}\right)^2 = \left(\frac{R_2}{R_1}\right)^3$$

$$\Rightarrow \quad T_1^2 = T_1^2\left(\frac{3.5R}{7R}\right)^3 \quad (\because R_2 = R + 2.5R \text{ and } R_1 = R + 6R)$$

$$\Rightarrow \quad T_2^2 = \frac{T_1^2}{8} \Rightarrow T_2 = \frac{T_1}{2\sqrt{2}}$$

$$\Rightarrow \quad T_2 = \frac{24}{2\sqrt{2}} \quad (\because T_1 = 24 \text{ h})$$

$$\therefore \quad T_2 = 6\sqrt{2} \text{ h}$$

Example 10.4 *The planet neptune travels around the sun with a period of 165 yr. What is the radius of the orbit approximately, if the orbit is considered as circular?*

Sol. As, $T_1 = T_{\text{Earth}} = 1$ yr and $T_2 = T_{\text{Neptune}} = 165$ yr

Let R_1 and R_2 be the radii of the circular orbits of the earth and neptune respectively.

According to Kepler's third law

$$\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3} \Rightarrow R_2^3 = \frac{R_1^3 T_2^2}{T_1^2} \Rightarrow R_2^3 = \frac{R_1^3 \times (165)^2}{1^2}$$

$$\Rightarrow \quad R_2^3 = 165^2 R_1^3 \Rightarrow R_2 \approx 30 R_1$$

2 Newton's law of gravitation

According to Newton's law of gravitation, "Every particle in the universe attracts every other particle with a force whose magnitude is directly proportional to the product of their masses and inversely proportional to the square of distances between their centres." This tendency of particles (or bodies) to attract each other is called gravitation.

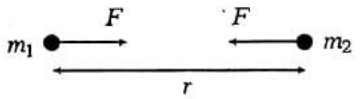


Fig. 10.5 *Force between two point masses*

Thus, the magnitude of gravitational force F between the two particles m_1 and m_2 placed at distance r can be given as

$$F \propto \frac{m_1 m_2}{r^2} \quad \text{or} \quad \boxed{F = \frac{G m_1 m_2}{r^2}} \quad ...(i)$$

where, m_1 and m_2 are the masses of two particles, r is the distance between the two particles and G is a universal gravitational constant.

The magnitude of G in MKS or SI is $6.67 \times 10^{-11} \text{Nm}^2\text{kg}^{-2}$ and in CGS, it is 6.67×10^{-8} dyne cm^2g^{-2}.

The dimensional formula of G is $[M^{-1}L^3T^{-2}]$.

Important points about force of gravitation

(i) The gravitational constant G is called a universal constant because it has same value for any two bodies anywhere in the space.

(ii) Gravitational force is always attractive in nature and is independent of nature of medium between two masses and presence or absence of other bodies.

(iii) Gravitational forces are central forces as they act along the line joining the centres of two bodies.

(iv) Gravitational forces are conservative in nature so work done by gravitational forces does not depend upon the path.

(v) For the gravitational force on a body outside a spherically symmetric body (solid spheres or spherical shells.) An entire mass of the spherical body is assumed to be concentrated at its centre.

(vi) The force of attraction due to a hollow spherical shell of uniform density on a point mass situated inside it is zero.

(vii) In calculating the gravitational force between two non-spherical bodies, we cannot assume their masses to be concentrated on the centres of mass but if the separation between them is very large in comparison to their sizes, then the bodies may be considered as particles. That means if we have to find gravitational force between a sphere and a circular ring at infinite separation then the mass of sphere can be assumed at its centre.

Generally, the gravitational force between non-spherical bodies are found by integrating the individual forces on individual particles within the bodies.

(viii) Force developed between any two masses is called gravitational force and the force between earth and any other body is called gravity force.

Example 10.5 *Two heavy particles of masses 40 kg and 60 kg attracts each other with a force of 4×10^{-5} N. If G is 6×10^{-11} N-m^2 kg^{-2}, calculate the distance between them.*

Sol. Given, $m_1 = 40$ kg, $m_2 = 60$ kg, $F = 4\times10^{-5}$ N,

$$G = 6\times10^{-11}\ \text{N-m}^2\text{kg}^{-2},\ g = 10\ \text{ms}^{-2}$$

According to Newton's law of gravitation,

$$F = \frac{Gm_1m_2}{r^2} \Rightarrow r = \sqrt{\frac{Gm_1m_2}{F}} = \sqrt{\frac{6\times10^{-11}\times40\times60}{4\times10^{-5}}}$$

$$= 0.06\ \text{m} = 6\ \text{cm}$$

Example 10.6 *If the distance between the two spherical bodies is increased to four times, then by how many times, the mass of one of the bodies to be changed to maintain the same gravitational force?*

Sol. Let $F_1 = \frac{Gm_1m_2}{r^2}$ and $F_2 = \frac{GM_1M_2}{R^2}$

Now, $R = 4r$

Also let m_1 remains unchanged, so $m_1 = M_1$

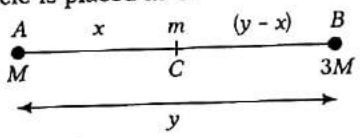
$$\therefore \quad \frac{Gm_1m_2}{r^2} = \frac{GM_1M_2}{R^2} \Rightarrow \frac{m_2}{r^2} = \frac{M_2}{(4r)^2} \Rightarrow m_2 = \frac{M_2}{16}$$

$$\therefore \quad M_2 = 16\ m_2$$

Hence, the mass of one of the bodies should be increased to 16 times so that the gravitational force remains the same.

Example 10.7 *Two particles A and B having masses M and 4M respectively are kept at a distance 2.73 m apart. Another small particle of mass m is to be placed so that the net gravitational force on it is zero. What will be its distance from body A?*

Sol. Two particles A and B are placed as shown below and a third particle is placed at C.



Now, according to the question,

$$|F_{CA}| = F_{CB}|$$

$$\therefore \quad \frac{GMm}{x^2} = \frac{G4Mm}{(y-x)^2}$$

$$\therefore \quad \frac{1}{x^2} = \frac{4}{(y-x)^2}$$

$$\therefore \quad y - x = 2x$$

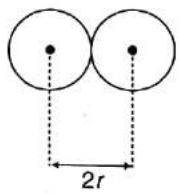
$$\therefore \quad y = 3x$$

$$\therefore \quad x = \frac{y}{3} = \frac{2.73}{3} \Rightarrow x = 0.91\ \text{m}$$

$\therefore$ A particle of mass m should be placed at 0.91 m apart from body A.

Example 10.8 *Spheres of the same material and same radius r are touching each other. Show that gravitational force between them is directly proportional to r^4.*

Sol. Two spheres are touching as shown below.



According to the question,

$$m_1 = m_2 = (\text{volume})(\text{density}) = \left(\frac{4}{3}\pi r^3\right)\rho$$

$$\therefore \text{ Gravitational force, } F = \frac{Gm_1m_2}{r^2} = \frac{G\left(\frac{4}{3}\pi r^3\right)\left(\frac{4}{3}\pi r^3\right)\rho^2}{r^2}$$

$$\Rightarrow \quad F \propto r^4$$

Example 10.9 *Force between two objects of equal masses is F. If 25% mass of one object is transferred to the other object, then find the new force.*

Sol. Suppose mass of one object $m = 100$ kg

$\because$ Masses of two objects are equal

$$\therefore \quad m_1 = m_2 = 100\ \text{kg}$$

According to Newton's law of gravitation,

$$F = G\frac{m_1m_2}{R^2}$$

$$\therefore \quad F = \frac{G \times 100 \times 100}{R^2} = \frac{G \times 10000}{R^2}$$

$$\Rightarrow \quad \frac{G}{R^2} = \frac{F}{10000} \quad \text{...(i)}$$

If 25% mass of one object is transferred to the other object, then

$$m_1 = 100 - 25 = 75 \text{ kg}$$

$$m_2 = 100 + 25 = 125 \text{ kg}$$

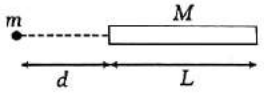
$$\therefore \quad \text{New force } F' = \frac{G \times 75 \times 125}{R^2} = \frac{G \times 9375}{R^2}$$

$$\Rightarrow \quad \frac{G}{R^2} = \frac{F'}{9375} \quad \text{...(ii)}$$

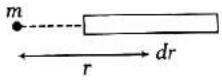
From Eqs. (i) and (ii), we get

$$F' = \frac{15}{16} F$$

Example 10.10 *A particle of mass m is placed at a distance d from one end of a uniform rod with length L and mass M as shown in the figure. Find the magnitude of the gravitational force on the particle due to the rod.*



Sol. Let us consider an elementary mass dm of length dr at a distance r from the particle of mass m. Here, $dm = M/L dr$. The gravitational force dF on m due to this elementary mass dm is



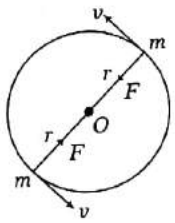
$$dF = \frac{Gm}{r^2} dm = \frac{GmM}{Lr^2} dr$$

$$\therefore \text{ Force on the particle } F = \frac{GmM}{L} \int_d^{L+d} \frac{dr}{r^2} = \frac{GmM}{L}\left[-\frac{1}{r}\right]_d^{L+d}$$

$$= -\frac{GmM}{L}\left[\frac{1}{L+d} - \frac{1}{d}\right] = \frac{GmM}{d(L+d)}$$

Example 10.11 *Two particles of equal mass m_0 are moving round a circle of radius r due to their mutual gravitational interaction. Find the time period of each particle.*

Sol. Two particles will always remain on diametrically opposite points so that the gravitational force is centripetal. Here, mutual gravitational force is



$$F = \frac{Gmm}{(2r)^2} = \frac{Gm^2}{4r^2} \quad \text{...(i)}$$

If the speed of each particle is v, then the centripetal force

$$F = \frac{mv^2}{r} \quad \text{...(ii)}$$

Equating (i) and (ii), we get $v = \sqrt{\frac{Gm}{4r}}$

$$\therefore \text{ The time period, } T = \frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{\frac{Gm}{4r}}} = \frac{4\pi}{\sqrt{Gm}} r^{3/2}$$

Vector form of newton's law of gravitation

The vector form of Newton's law of gravitation signifies that the gravitational forces acting between the two particles form action and reaction pair.

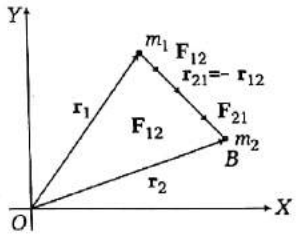


Fig. 10.6 *Vector form of gravitational force ($\mathbf{F}_{12}$) on m_1 due to m_2*

In figure, it can be seen that the two particles of masses m_1 and m_2 are placed at a distance r, therefore, according to Newton's law of gravitation, force on m_1 due to m_2.

$$\mathbf{F}_{12} = -\frac{Gm_1m_2}{|\mathbf{r}_{12}|^2} \cdot \hat{\mathbf{r}}_{12} \quad \text{...(i)}$$

$$\mathbf{F}_{12} = -Gm_1m_2 \frac{\mathbf{r}_1 - \mathbf{r}_2}{|\mathbf{r}_1 - \mathbf{r}_2|^3} \quad \left(\because \hat{\mathbf{r}}_{12} = \frac{\mathbf{r}_1 - \mathbf{r}_2}{|\mathbf{r}_1 - \mathbf{r}_2|}\right)$$

where, $\hat{\mathbf{r}}_{12}$ is a unit vector pointing from m_2 to m_1.

The negative sign in Eq.(i) indicates the direction of force $\mathbf{F}_{12}$ is opposite to that of $\hat{\mathbf{r}}_{12}$.

Similarly, force on m_2 due to m_1

$$\mathbf{F}_{21} = -\frac{Gm_1m_2}{|\mathbf{r}_{21}|^2} \cdot \hat{\mathbf{r}}_{21} \quad \text{...(ii)}$$

where, $\hat{\mathbf{r}}_{21}$ is a unit vector pointing from m_1 to m_2.

From Eqs. (i) and (ii), we get

$$\mathbf{F}_{12} = -\mathbf{F}_{21} \Rightarrow |\mathbf{F}_{12}| = |\mathbf{F}_{21}| \quad \text{...(iii)}$$

As $\mathbf{F}_{12}$ and $\mathbf{F}_{21}$ are directed towards the centres of the two particles, so gravitational force is conservative in nature.

Principle of superposition of gravitational forces

Suppose $\mathbf{F}_1, \mathbf{F}_2, \ldots, \mathbf{F}_n$ be the individual forces due to the masses $m_1, m_2, m_3, \ldots, m_n$ which are given by the universal law of gravitation, then from the principle of superposition, each of these forces acts independently and uninfluenced by the other bodies as shown in figure.

So, the resultant force **F** can be expressed in vector addition of various forces

$$\mathbf{F} = \mathbf{F}_{12} + \mathbf{F}_{13} + \mathbf{F}_{14} + \ldots + \mathbf{F}_{1n}$$

It states that, the resultant gravitational force **F** acting an a particle due to number of point masses is equal to the vector sum of forces exerted by the individual masses on the given particle.

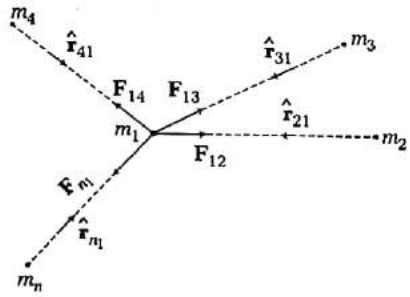


Fig. 10.7 Gravitational forces on m_1 due to $m_2, m_3, \ldots m_n$, using superposition principle of gravitational forces

Clearly, $\mathbf{F} = -G\dfrac{m_1 m_2}{r_{12}^2}\hat{\mathbf{r}}_{21} - G\dfrac{m_1 m_3}{r_{13}^2}\hat{\mathbf{r}}_{31} - \ldots\ldots - G\dfrac{m_1 m_n}{r_{1n}^2}\mathbf{r}_{n1}$

$$\boxed{\text{Resultant force, } \mathbf{F} = -Gm_1\left[\frac{m_2}{r_{12}^2}\hat{\mathbf{r}}_{21} + \frac{m_3}{r_{13}^2}\hat{\mathbf{r}}_{31} + \ldots + \frac{m_n}{r_{1n}^2}\hat{\mathbf{r}}_{n1}\right]}$$

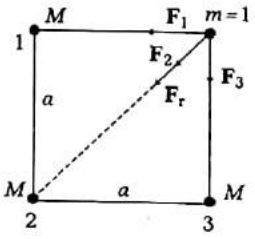
Example 10.12 *Three masses, each equal to M are placed at the three corners of a square of side a. Calculate the force of attraction on unit mass at the fourth corner.*

Sol. Force on m due to masses at 1 and 3 are $\mathbf{F}_1$ and $\mathbf{F}_3$ with

$$F_1 = F_3 = \frac{GM}{a^2}$$

Resultant of $\mathbf{F}_1$ and $\mathbf{F}_3$ is $F_r = \sqrt{2}\,\dfrac{GM}{a^2}$

and its direction is along the diagonal, *i.e.*, toward 2.



Force on m due to mass m at 2 is $F_2 = \dfrac{GM}{(\sqrt{2}a)^2} = \dfrac{GM}{2a^2}$ and F_r and F_2 act in the same direction.

Resultant of these two is net force

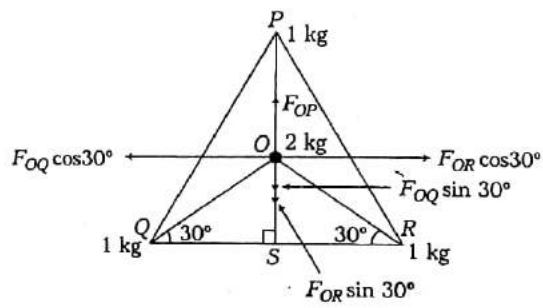
$$\mathbf{F}_{\text{net}} = \frac{\sqrt{2}GM}{a^2} + \frac{GM}{2a^2}$$

$$= \frac{GM}{a^2}\left[\sqrt{2} + \frac{1}{2}\right]$$

it is directed along the diagonal as shown in figure.

Example 10.13 *Three equal masses of 1 kg each are placed at the vertices of an equilateral ΔPQR and a mass of 2 kg is placed at the centroid O of the triangle which is at a distance of $\sqrt{2}$ m from each of the vertices of the triangle. Find the force (in newton) acting on the mass of 2 kg.*

Sol. Given, $OP = OQ = OR = \sqrt{2}$ m



The gravitational force on mass 2 kg due to mass 1 kg at P.

$$F_{OP} = G\frac{2 \times 1}{(\sqrt{2})^2} = G \text{ along } OP$$

Similarly, $$F_{OQ} = G\frac{2 \times 1}{(\sqrt{2})^2} = G \text{ along } OQ$$

and $$F_{OR} = G\frac{2 \times 1}{(\sqrt{2})^2} = G \text{ along } OR$$

$F_{OQ}\cos 30°$ and $F_{OR}\cos 30°$ are equal and acting in opposite directions, hence cancel out each other.

Force on mass 2 kg along OS

$$= F_{OQ}\sin 30° + F_{OR}\sin 30°$$

$$= 2F_{OQ}\sin 30° \qquad (\because F_{OQ} = F_{OR})$$

$$= F_{OQ}$$

The resultant force on the mass 2 kg at O,

$$F = F_{OP} - F_{OQ}$$

$$= G - G = 0 \text{ (zero)}$$

Check point 10.1

1. Kepler's second law is based on
   (a) Newton's first law
   (b) Newton's second law
   (c) special theory of relativity
   (d) conservation of angular momentum

2. When a planet moves around sun, its
   (a) areal velocity is constant
   (b) linear velocity is constant
   (c) angular velocity is constant
   (d) All of the above

3. A planet moves around the sun. It is closest to sun at a distance d_1 and have velocity v_1. At farthest distance d_2 its speed will be
   (a) $\frac{d_1^2 v_1}{d_2^2}$ (b) $\frac{d_2 v_1}{d_1}$ (c) $\frac{d_1 v_1}{d_2}$ (d) $\frac{d_2^2 v_1}{d_1^2}$

4. For a satellite in elliptical orbit which of the following quantities does not remain constant?
   (a) Angular momentum (b) Momentum
   (c) Areal velocity (d) All of these

5. The motion of planets in the solar system is an example of conservation of
   (a) mass (b) momentum
   (c) angular momentum (d) kinetic energy

6. Kepler's law states that square of the time period of any planet moving around the sun in an elliptical orbit of semi-major axis (R) is directly proportional to
   (a) R (b) $\frac{1}{R}$ (c) R^3 (d) $\frac{1}{R^3}$

7. The ratio of mean distances of three planets from the sun are 0.5 : 1 : 1 : 5, then the square of time periods are in the ratio of
   (a) 1 : 4 : 9 (b) 1 : 9 : 4
   (c) 1 : 8 : 27 (d) 2 : 1 : 3

8. The period of revolution of planet A round the sun is 8 times that of B. The distance of A from the sun is how many times greater than that of B from the sun?
   (a) 5 (b) 4 (c) 3 (d) 2

9. The distance of two planets from the sun are 10^{13} and 10^{12} m respectively. The ratio of the periods of the planet is
   (a) 100 (b) $\frac{1}{\sqrt{10}}$ (c) $\sqrt{10}$ (d) $10\sqrt{10}$

10. A satellite having time period same as that of the earth's rotation about its own axis is orbiting the earth at a height $8R$ above the surface of earth, where, R is radius of earth. What will be the time period of another satellite at a height $3.5\,R$ from the surface of earth?
    (a) $2\sqrt{2}$ h (b) $4\sqrt{2}$ h
    (c) $6\sqrt{2}$ h (d) $3\sqrt{2}$ h

11. A body is orbiting around earth at a mean radius which is two times as greater as the parking orbit of a satellite, the period of body is
    (a) 4 days (b) 16 days (c) $2\sqrt{2}$ days (d) 64 days

12. Two point masses each equal to 1 kg attract one another with a force of 10^{-9} kg-wt. The distance between the two point masses is approximately ($G = 6.6 \times 10^{-11}$MKS units)
    (a) 8 cm (b) 0.8 cm (c) 80 cm (d) 0.08 cm

13. Gravitational force between a point mass m and M separated by a distance is F. Now if a point mass $2m$ is placed next to m in contact with it, the force on M due to m and the total force on M are
    (a) $2F, F$ (b) $F, 2F$ (c) $F, 3F$ (d) F, F

14. Three equal masses of 2 kg each are placed at the vertices of an equilateral triangle and a mass of 4 kg is placed at the centroid of the triangle which is at a distance of $\sqrt{2}$ m from each of the vertices of the triangle. The force, (in newton) acting on the mass of 4 kg is
    (a) 2 (b) $\sqrt{2}$ (c) 1 (d) zero

15. The force of gravitation is
    (a) repulsive (b) attractive
    (c) conservative (d) Both (b) and (c)

16. Which of the following statements about the gravitational constant is true?
    (a) It is a force.
    (b) It has no unit.
    (c) It has same value in all systems of unit.
    (d) It does not depend on the nature of the medium in which the bodies are kept.

17. The distance of the centres of moon and earth is D. The mass of earth is 81 times the mass of the moon. At what distance from the centre of the earth, the gravitational force on a particle will be zero
    (a) $\frac{D}{2}$ (b) $\frac{2D}{3}$ (c) $\frac{4D}{3}$ (d) $\frac{9D}{10}$

18. Two balls, each of radius R, equal mass and density are placed in contact, then the force of gravitation between them is proportional to
    (a) $F \propto \frac{1}{R^2}$ (b) $F \propto R$ (c) $F \propto R^4$ (d) $F \propto \frac{1}{R}$

19. If the distance between the sun and the earth is increased by three times, then attraction between two will
    (a) remain constant (b) decrease by 63%
    (c) increase by 63% (d) decrease by 89%

20. A spherical planet far out in space has mass $2M$ and radius a. A particle of mass m is falling freely near its surface. What will be the acceleration of that particle?
    (a) $\frac{GM}{a^2}$ (b) $\frac{3GM}{a^2}$ (c) $\frac{2GM}{a^2}$ (d) $\frac{4GM}{a^2}$

3 Gravity

In Newton's law of gravitation, gravitation is the force of attraction between the two bodies. If one of the bodies is earth, then gravitation is called gravity. Hence, gravity is the force by which earth attracts the body towards its centre.

Acceleration due to gravity (g) of earth

The force of gravity acting on a body having unit mass placed on or near the surface of the earth is known as acceleration due to gravity.

Or

When a body is dropped from a certain height above the ground, it begins to fall towards the earth under the effect of gravity. The acceleration produced in the body due to gravity, is known as acceleration due to gravity.

It is denoted by g. It is a vector quantity and its average value on the earth surface at mean sea level is $9.8\ \text{ms}^{-2}$.

Relation between acceleration due to gravity (g) and gravitational constant (G)

Suppose that the mass of the earth is M, its radius is R, then the force of attraction acting on a body of mass m close to the surface of the earth is

$$F = \frac{GMm}{R^2}$$

According to Newton's second law, the acceleration due to gravity

$$\boxed{g = \frac{F}{m} = \frac{GM}{R^2}} \quad \text{...(i)}$$

This expression is free from m. It means acceleration due to gravity does not depend on the mass of the object, it is same for all. If in Eq. (i), we put the value of G as $6.67\times 10^{-11}\text{Nm}^2\ \text{kg}^{-2}$, M as 6×10^{24} kg and R as 6400 km, then on solving it, we will get the value of g as $9.8\ \text{ms}^{-2}$.

If two bodies of different masses are allowed to fall freely they will have the same acceleration, i.e. if they are allowed to fall from the same height, then they will reach the earth simultaneously. The dimension of acceleration due to gravity is $[\text{LT}^{-2}]$.

Note

1. The value of g is independent of mass, shape, size, etc of the body and depends upon the mass and radius of the earth.
2. The value of g on the surface of the moon is equal to 1/6 times the value of g on the surface of the earth.

Example 10.14 *The acceleration due to gravity at the moon's surface is $1.67\ ms^{-2}$. If the radius of the moon is 1.74×10^6 m, calculate the mass of the moon.*

Sol. As, $g = \frac{GM}{R^2}$ or $M = \frac{gR^2}{G}$

This relation is true not only to the earth but for any heavenly body which is assumed to be spherical.

Now, $g = 1.67\ \text{ms}^{-2}, R = 1.74\times 10^6$ m

$G = 6.67\times 10^{-11}\ \text{Nm}^{-2}\ \text{kg}^{-2}$

$\therefore$ Mass of the moon, $M = \frac{1.67\times(1.74\times 10^6)^2}{6.67\times 10^{-11}}$ kg

$= 7.58\times 10^{22}$ kg

Example 10.15 *Assume that if the earth were made of lead of relative density 11.3, then what would be the value of acceleration due to gravity on the earth surface?*

Sol. Since, density of earth can be given as

ρ = Relative density × density of water $= 11.3\times 10^3\ \text{kgm}^{-3}$

$\therefore$ Acceleration due to gravity on the earth's surface,

$$g = \frac{GM}{R^2} = \frac{G}{R^2}\cdot\frac{4}{3}\pi R^3\cdot\rho$$

$$= \frac{4}{3}\times\frac{22}{7}\times 6.67\times 10^{-11}\times 6.4\times 10^6\times 11.3\times 10^3$$

$$= 22.21\ \text{ms}^{-2}$$

Example 10.16 *What will be the relation between the acceleration due to gravity on the surface of the earth and on a planet respectively, whose mass and radius are four times that of the earth?*

Sol. Let the acceleration due to gravity of that planet be g_p

As, $$g_p = \frac{GM_p}{R_p^2}$$

According to the question,

$$M_p = 4M_e \text{ and } R_p = 4R_e$$

$$\therefore \quad g_p = \frac{G4M_e}{16R_e^2}$$

$$\therefore \quad g_p = \frac{1}{4}g_e \qquad \left(\because g_e = \frac{GM_e}{R_e^2}\right)$$

Variation in the value of acceleration due to gravity (g)

The value of acceleration due to gravity varies due to following factors

1. Height above the surface of earth
2. Depth below the surface of earth
3. Shape of the earth
4. Axial rotation of the earth

1. Variation in g due to height above the surface of earth

Consider a body of mass m lying on the surface of the earth of mass M and radius R. Acceleration due to gravity at the surface of the earth $g = \frac{GM}{R^2}$

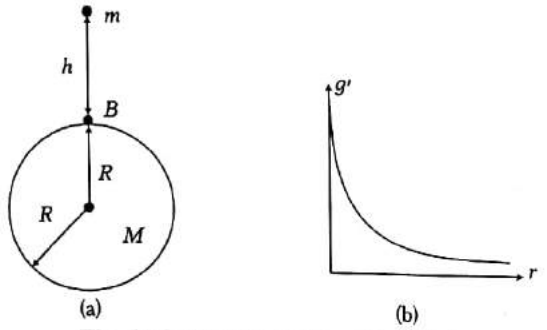


Fig. 10.8 *Variation of g with altitude*

The force of gravity on an object of mass m at a height h above the surface of the earth is

$$F = \frac{GMm}{(R+h)^2}$$

$\therefore$ Acceleration due to gravity at this height will be,

$$g' = \frac{F}{m} = \frac{GM}{(R+h)^2}$$

This can also be written as,

$$g' = \frac{GM}{R^2\left(1+\frac{h}{R}\right)^2} \Rightarrow \boxed{g' = \frac{g}{\left(1+\frac{h}{R}\right)^2}} \text{ as } \frac{GM}{R^2} = g$$

Thus, $g' < g$

i.e., **the value of acceleration due to gravity g goes on decreasing as we go above the surface of the earth.**

Further, by using Binomial theorem, we get

$$g' = g\left(1+\frac{h}{R}\right)^{-2} \Rightarrow \boxed{g' \approx g\left(1-\frac{2h}{R}\right)} \quad (\because h << R)\ldots\text{(i)}$$

Note 1. With height h, the decreases in the value of g is $g - g' = \frac{2gh}{R}$

$\therefore$ Fractional decreases in the value of g is

$$\frac{g-g'}{g} = \frac{2h}{R}$$

$\therefore$ Percentage decreases in the value of $g = \left(\frac{g-g'}{g}\right) \times 100 = \frac{2h}{R} \times 100\,\%$

2. As we move away from surface of earth, acceleration due to gravity decreases.

If $(R+h) = r$

$$g' = \frac{GM}{(R+h)^2} = \frac{gR^2}{r^2};\ g' \propto \frac{1}{r^2}$$

Example 10.17 *At what height above the Earth's surface the acceleration due to gravity will be 1/9th of its value at the Earth's surface? (Radius of Earth is 6400 km)*

Sol. Acceleration due to gravity at height h,

$$g' = \frac{g}{\left(1+\frac{h}{R_e}\right)^2} = \frac{g}{9}$$

$$\Rightarrow \quad \left(1+\frac{h}{R_e}\right) = 3 \Rightarrow h = 2R_e = 2 \times 6400 = 12800 \text{ km}$$

Example 10.18 *At what altitude, the acceleration due to gravity reduces to one fourth of its value as that on the surface of the earth? Take radius of earth as 6.4×10^6 m, g on the surface of the earth as 9.8 ms^{-2}.*

Sol. Acceleration due to gravity at height h, $g' = \frac{g}{\left(1+\frac{h}{R}\right)^2}$

If acceleration due to gravity reduced to one-fourth of its value, then

$$\Rightarrow \quad \frac{g}{4} = \frac{g}{\left(1+\frac{h}{R}\right)^2} \Rightarrow \left(1+\frac{h}{R}\right)^2 = 4 \Rightarrow 1+\frac{h}{R} = 2$$

$$\Rightarrow \quad \frac{h}{R} = 1 \Rightarrow h = R = 6.4 \times 10^5 \text{ m}$$

$\therefore$ Acceleration due to gravity is reduced to one fourth of its value on the earth's surface at an altitude of 6.4×10^6 m.

Example 10.19 *At what height the acceleration due to gravity decreases by 51% of its value on the surface of the earth?*

Sol. As, $g_h = g\left(\frac{R}{R+h}\right)^2$ and $g_h = g - 51\%$ of $g = g - \frac{51}{100}g$

$$\therefore \quad g_h = \frac{49}{100}g$$

$$\therefore \quad \frac{49}{100}g = g\left(\frac{R}{R+h}\right)^2$$

$$\therefore \quad \frac{7}{10} = \frac{R}{R+h}$$

$$\therefore \quad 7R + 7h = 10R$$

$$\therefore \quad 7h = 3R \Rightarrow h = \frac{3R}{7}$$

2. Variation in g due to depth below the surface of earth

Let an object of mass m is situated at a depth d below the earth's surface. Its distance from the centre of the earth is $(R - d)$. This mass is situated at the surface of the inner solid sphere and lies inside the outer spherical shell. The gravitational force of attraction on a mass inside a spherical

shell is always zero. Therefore, the object experiences gravitational attraction only due to inner solid sphere.

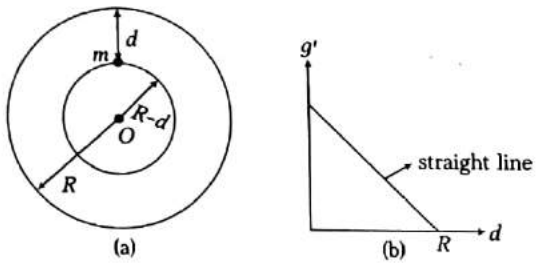


Fig. 10.9 Variation of g with depth

The mass of this sphere is, $M' = \left(\frac{M}{4/3\,\pi R^3}\right)\frac{4}{3}\pi(R-d)^3$

Here, M being the mass of whole sphere of radius R

Therefore, $$M' = \frac{(R-d)^3}{R^3}M$$

$$F = \frac{GM'm}{(R-d)^2} = \frac{GMm(R-d)}{R^3}$$

and $$g' = \frac{F}{m} = \frac{GM}{R^3}(R-d)$$

Substituting the values, we get

$$\boxed{g' = g\left(1-\frac{d}{R}\right)}$$

Since, g is constant at a given place of the earth and R is also a constant.

$\therefore$ $$(g-g')\propto d$$

Hence, **the acceleration due to gravity decreases as we move down into the surface of the earth.**

Note Decrease in the value of g with depth d is $g-g'=\frac{g}{R}d$

$\therefore$ Frictional decrease in the value of g with depth $=\frac{g-g'}{g}=\frac{d}{R}$

$\therefore$ Percentage decrease in the value of $g=\frac{g-g'}{g}\times 100=\frac{d}{R}\times 100$

We can see from this equation that $g'=0$ at $d=R$, *i.e.*, acceleration due to gravity is zero (minimum value) at the centre of the earth.

The graphical representation of change in the value of g with height and depth is as follows

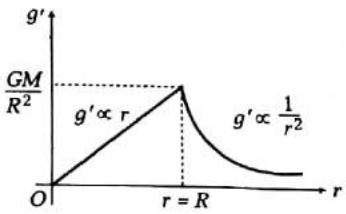


Fig. 10.10 Graphical representation for variation in g

For $r \le R$,

$$g' = g\left(1-\frac{h}{R}\right) = \frac{gr}{R}$$

as $$R-h=r \quad \text{or} \quad g' \propto r$$

For $r > R$, $$g' = \frac{g}{\left(1+\frac{h}{R}\right)^2} = \frac{gR^2}{r^2} \quad \text{or} \quad g' \propto \frac{1}{r^2}$$

Note The acceleration due to gravity is maximum (having standard value of 9.8 ms^{-2}) at the earth's surface. It decreases either we go at higher altitudes or we move below the surface.

Example 10.20 *At what depth from the surface of the earth, the acceleration due to gravity will be half the value of g on the surface of the earth?*

Sol. As $g' = g\left(1-\frac{d}{R_e}\right)$

According to the question,

$$g' = \frac{g}{2} \Rightarrow \frac{g}{2} = g\left(1-\frac{d}{R_e}\right)$$

$$\therefore \quad \frac{1}{2} = 1-\frac{d}{R_e} \Rightarrow \frac{d}{R_e} = 1-\frac{1}{2} \Rightarrow d = 0.5\,R_e$$

Example 10.21 *At which depth from Earth surface, acceleration due to gravity is decreased by 1% ?*

Sol. Frictional decrease in the value of g with depth

$$\frac{\Delta g_d}{g} = \frac{d}{R_e} \Rightarrow \frac{1}{100} = \frac{d}{6400} \Rightarrow d = 64 \text{ km}$$

Example 10.22 *Assuming earth to be a sphere of uniform mass density, how much would a body weigh half way down the centre of the earth, if it weighed 100 N on the surface?*

Sol. Given, $w = mg = 100$ N

As, $$g' = g\left(1-\frac{d}{R}\right)$$

It is also given that $\frac{d}{R} = \frac{1}{2}$

$$\therefore \quad g' = g\left(1-\frac{1}{2}\right) = \frac{g}{2}$$

Weight half way down the centre of the earth,

$$w' = mg' = \frac{mg}{2} = \frac{100}{2} = 50 \text{ N}$$

Example 10.23 *Determine the decrease in the weight of a body when it is taken 32 km below the earth surface. Take radius of the earth as 6400 km.*

Sol. Given, $R = 6400$ km, $d = 32$ km

$$\because \quad g' = g\left(1-\frac{d}{R}\right) = g\left(1-\frac{32}{6400}\right) = \left(\frac{199}{200}\right)g$$

$$\therefore \quad g-g' = g-\frac{199}{200}g = \frac{g}{200}$$

The percentage decrease in the weight of the body,

$$\frac{mg - mg'}{mg} \times 100 = \frac{g - g'}{g} \times 100 = \frac{\frac{g}{200}}{g} \times 100$$

$$= \frac{1}{200} \times 100 = 0.5\%$$

3. Variation in g due to the shape of the earth

The earth is not a perfect sphere. It is somewhat flat at the two poles. The equitorial radius (R_{eq}) is approximately 21 km more than the polar radius (R_p).

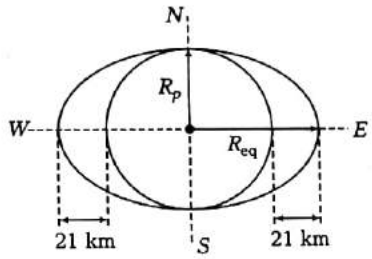


Fig. 10.11 Variation in g due to shape of the earth

Now acceleration due to gravity at poles,

$$g_p = \frac{GM}{R_p^2}$$

Acceleration due to gravity at equator,

$$g_{eq} = \frac{GM}{R_{eq}^2}$$

$\because \quad R_p < R_{eq}$

$\therefore \quad g_p > g_{eq}$

Difference in g at poles and equator due to shape

$$(\Delta g) = g_p - g_{eq} = 0.02 \text{ ms}^{-2}$$

The value of g is minimum at the equator and maximum at the poles. So, this is a reason why the weight of the body increases when it is taken from equator to the pole.

Note If weight = constant

means mg = constant, then $m \propto \frac{1}{g}$

It means that it will be profitable to buy sugar at the equator in comparison to poles because more mass (amount) will be obtained there.

Example 10.24 *Imagine a new planet having the same density as that of earth but it is 3 times bigger than the earth in size. If the acceleration due to gravity on the surface of earth is g and that on the surface of the new planet is g′, then find the relation between g and g′.*

Sol. The acceleration due to gravity on the new planet can be found using the relation

$$g = \frac{GM}{R^2} \quad \text{...(i)}$$

But $M = 4/3\ \pi R^3 \rho$, ρ being density.

Thus, Eq. (i) becomes

$$g = \frac{G \times (4/3)\pi R^3 \rho}{R^2} = G \times \frac{4}{3}\pi R\rho \Rightarrow g \propto R$$

($\because$ ρ = constant)

$$\therefore \quad \frac{g'}{g} = \frac{R'}{R} \Rightarrow \frac{g'}{g} = \frac{3R}{R}$$

$$\Rightarrow \quad \frac{g'}{g} = 3 \Rightarrow g' = 3g$$

4. Variation in g due to axial rotation of the earth

Suppose the earth is rotating on its axis with angular velocity ω. Let us consider a particle P at rest on the surface of the earth, at latitude ϕ. Then the pseudo force acting on the particle is $mr\omega^2$ in outward direction. The true acceleration due to gravity g is acting towards the centre O of the earth. Thus, the effective acceleration due to gravity g' is the resultant of g and $r\omega^2$.

After derivation we can find the following relation

$$\boxed{g' = g - R\omega^2 \cos^2 \phi}$$

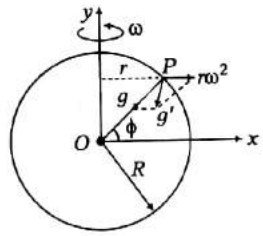


Fig. 10.12 Axial rotation of the earth

Following conclusions can be drawn from the above discussion

(i) The effective value of g is not truely vertical.

(ii) The effect of centrifugal force due to rotation of the earth is to reduce the effective value of g.

(iii) At equators $\phi = 0°$.
Therefore, $g' = g - R\omega^2$ and at poles $\phi = 90°$,
Therefore, $g' = g$
Thus, at equator g' is minimum while at poles g' is maximum.

Note

1. If the angular velocity of earth increases, the value of g will decrease at all places except at poles.
2. Since earth rotates West to East, we project our rockets also in West to East so that effective acceleration due to gravity ($g' = g - R\omega^2$) on the rocket is less.

Example 10.25 *Find the imaginary angular velocity of the earth for which the effective acceleration due to gravity at the equator shall be zero. (take, $g = 10\ m/s^2$ for the acceleration due to gravity, if the earth were at rest and radius of earth equal to 6400 km and $\phi = 60°$).*

Sol. Acceleration due to gravity,

$$g' = g - \omega^2 R \cos^2 \phi$$

$$0 = g - \omega^2 R \cos^2 60 \qquad (\because g' = 0)$$

$$0 = g - \frac{\omega^2 R}{4}$$

$$\omega = 2\sqrt{\frac{g}{R}} = 2\sqrt{\frac{10}{6400 \times 1000}}$$

$$\Rightarrow \qquad \omega = \frac{1}{400} = 2.5 \times 10^{-3} \text{rad/s}$$

Example 10.26 *Calculate the angular speed of rotation of the Earth so that the apparent g at the equator becomes half of its value at the surface. Also calculate the length of the day in this situation.*

Sol. The apparent acceleration due to gravity,

$$g = g_0 - \omega^2 R = g_0/2$$

$$\Rightarrow \qquad \omega = \sqrt{\frac{g_0}{2R}} = \sqrt{\frac{9.8\ \text{ms}^{-2}}{6.4 \times 10^6\ \text{m} \times 2}} = 8.75 \times 10^{-4}\ \text{rads}^{-1}$$

The length of the day = Time period of rotation of the Earth

$$= \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{2R}{g_0}} = 2\pi\sqrt{\frac{2 \times 6.4 \times 10^6}{9.8}} \approx 2\ \text{h}$$

Example 10.27 *Find the value of angular velocity of axial rotation of the earth, such that weight of a person at equator becomes 3/4 of its weight at pole. Radius of the earth at equator is 6400 km.*

Sol. Let acceleration due to gravity at pole is g and at equator it is g'. If mass of the person is m and angular velocity of rotation of the Earth is ω, then

$$g' = g - \omega^2 R \Rightarrow mg' = mg - m\omega^2 R$$

$$\Rightarrow \qquad \frac{3}{4} mg = mg - m\omega^2 R$$

$$\Rightarrow \qquad \omega^2 R = \frac{1}{4} g$$

$$\Rightarrow \qquad \omega = \sqrt{\frac{g}{4R}} = \sqrt{\frac{9.8}{4 \times 6400 \times 10^3}}$$

$$\Rightarrow \qquad \omega = \frac{1}{2} \times \frac{1}{80} \times \frac{1}{10^2} \times \sqrt{98} = 6.2 \times 10^{-4}\ \text{rads}^{-1}$$

Example 10.28 *Suppose the earth increases its speed of rotation. At what new time period will the weight of a body on the equator becomes zero? (Take, $g = 10\ ms^{-2}$ and radius of earth $R = 6400\ km$.)*

Sol. The weight will become zero when

$$g' = 0 \quad \text{or} \quad g - R\omega^2 = 0 \quad (\text{on the equator } g' = g - R\omega^2)$$

$$\text{or} \qquad \omega = \sqrt{\frac{g}{R}}$$

$$\therefore \qquad \frac{2\pi}{T} = \sqrt{\frac{g}{R}} \quad \text{or} \quad T = 2\pi\sqrt{\frac{R}{g}}$$

Substituting the values,

$$T = \frac{2\pi\sqrt{\frac{6400 \times 10^3}{10}}}{3600}\ \text{h} \qquad \text{or} \quad T = 1.4\ \text{hr}$$

Thus, the new time period should be 1.4 h instead of 24 h for the weight of a body to be zero on the equator.

Check point 10.2

1. The mass of a planet is twice the mass of earth and diameter of the planet is thrice the diameter of the earth, then the acceleration due to gravity on the planet's surface is
   (a) $g/2$ (b) $2g$
   (c) $2g/9$ (d) $3g/\sqrt{2}$
2. If the earth suddenly shrinks (without changing mass) to half of its present radius, then acceleration due to gravity will be
   (a) $g/2$ (b) $4g$
   (c) $g/4$ (d) $2g$
3. The diameters of two plantes are in the ratio 4:1 and their mean densities in the ratio 1:2. The acceleration due to gravity on the planets will be in ratio
   (a) 1 : 2 (b) 2 : 3
   (c) 2 : 1 (d) 4 : 1
4. If M is the mass of the earth and R its radius, then ratio of the gravitational acceleration and the gravitational constant is
   (a) $\frac{R^2}{M}$ (b) $\frac{M}{R^2}$ (c) MR^2 (d) $\frac{M}{R}$
5. If G is universal gravitational constant and g is acceleration due to gravity then the unit of the quantity $\frac{G}{g}$ is
   (a) kg-m^2 (b) kgm^{-1} (c) kgm^{-2} (d) m^2 kg^{-1}
6. If density of earth increased 4 times and its radius become half of what it is, our weight will?
   (a) be four times its present value
   (b) be doubled
   (c) remain same
   (d) be halved

7. The acceleration due to gravity g and mean density of earth ρ are related by which of the following relations? (G = gravitational constant and R = radius of earth)
(a) $\rho = \frac{4\pi g R^2}{3G}$ (b) $\rho = \frac{4\pi g R^3}{3G}$
(c) $\rho = \frac{3g}{4\pi G R}$ (d) $\rho = \frac{3g}{4\pi G R^3}$

8. If a planet consists of a satellite whose mass and radius were both half that of the earth, then acceleration due to gravity at its surface would be
(a) 4.9 ms^{-2} (b) 9.8 ms^{-2} (c) 19.6 ms^{-2} (d) 29.4 ms^{-2}

9. The height above the surface of the earth where acceleration due to gravity is 1/64 of its value at surface of the earth is approximately
(a) 45×10^6 m (b) 54×10^6 m
(c) 102×10^6 m (d) 72×10^6 m

10. If radius of earth is R, then the height h at which the value of g becomes (1/49)th of its value at the surface is
(a) $2R$ (b) $3R$ (c) $6R$ (d) $4R$

11. A body has a weight 72 N. When it is taken to a height $h = R$ = radius of earth, it would weight
(a) 72 N (b) 36 N (c) 18 N (d) zero

12. A simple pendulum has a time period T_1 when on the earth's surface and T_2 when taken to a height R above the earth's surface, where R is the radius of the earth. The value of $\frac{T_2}{T_1}$ is
(a) 1 (b) $\sqrt{2}$ (c) 4 (d) 2

13. The depth d, at which the value of acceleration due to gravity becomes $1/n$ times the value at the surface is (R = radius of the earth)
(a) $\frac{R}{n}$ (b) $R\left(\frac{n-1}{n}\right)$ (c) $\frac{R}{n^2}$ (d) $R\left(\frac{n}{n+1}\right)$

14. If the change in the value of g at a height h above the surface of earth is the same as at a depth d below it (both h and d are much smaller than the radius of the earth), then
(a) $d = h$ (b) $d = 2h$
(c) $d = h/2$ (d) $d = h^2$

15. At what depth below the surface of the earth acceleration due to gravity will be half of its value at 1600 km above the surface of the earth?
(a) 4.3×10^6 m (b) 2.4×10^6 m
(c) 3.2×10^6 m (d) 1.6×10^6 m

16. The weight of a body at the centre of the earth is
(a) zero
(b) infinite
(c) same as on the surface of earth
(d) None of the above

17. If earth is supposed to be sphere of radius R, if $g_{30°}$ is value of acceleration due to gravity at latitude of 30° and g at the equator, then value of $g - g_{30°}$ is
(a) $\frac{1}{4}\omega^2 R$ (b) $\frac{3}{4}\omega^2 R$ (c) $\omega^2 R$ (d) $\frac{1}{2}\omega^2 R$

18. Weight of a body is maximum at
(a) poles (b) equator
(c) centre of earth (d) at latitude 45°

19. The angular speed of earth in rad s^{-1}, so that the object on equator may appear weightless, is (radius of earth = 6400 km)
(a) 1.23×10^{-3} (b) 6.20×10^{-3}
(c) 1.56 (d) 1.23×10^{-5}

20. When a body is taken from the equator to the poles, its weight
(a) remains constant
(b) increases
(c) decreases
(d) increase at n-pole and decrease at s-pole

4 Gravitational field

The space around a body in which any other body experiences a force of attraction is called the gravitational field of the first body.

In the case of our planet earth since, it is surrounded by a gravitational field, so anybody brought in this field experiences a force of attraction towards the centre of earth.

Intensity of gravitational field

The intensity of gravitational field at any point in the gravitational field due to a given mass is defined as the force experienced by a unit mass placed at that point provided the presence of that unit mass does not alter the original gravitational field.

Usually, it is denoted by E or I.

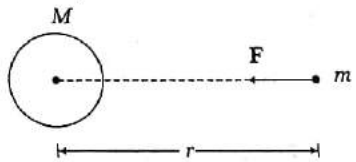


Fig. 10.13 Gravitational field intensity

Thus, if F be the gravitational force experienced by the test mass m, then the gravitational field intensity at any point will be

$$\boxed{\mathbf{E} = \frac{\mathbf{F}}{m}}$$

Its SI unit is ms^{-2} or $N\,kg^{-1}$ and dimension is $[M^0LT^{-2}]$. It is a vector quantity and is always directed towards the centre of gravity of body whose gravitational field intensity is to be considered.

Note

1. **Inertial mass** of the body can be measured as the ratio of the magnitude of the external force applied on it to the magnitude of acceleration produced in its motion.
2. **Gravitational mass** of a body is defined as the gravitational pull experienced by the body in a gravitational field of unit intensity.

Gravitational field intensity for different bodies

1. Field due to a point mass

Suppose, a point mass M is placed at point O and we want to find the intensity of gravitational field E at a point P, a distance r from O, so the magnitude of force F acting on a particle of mass m placed at P is

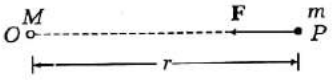


Fig. 10.14 Gravitational field due to a point mass

$$F = \frac{GMm}{r^2} \Rightarrow E = \frac{F}{m} = \frac{GM}{r^2}$$

or $$\boxed{E = \frac{GM}{r^2}}$$

In vector form, $\mathbf{E} = \frac{GM}{r^2}\hat{\mathbf{r}} = \frac{GM}{r^3}\mathbf{r}$

The direction of force $\mathbf{F}$ and hence of $\mathbf{E}$ is from P to O as shown in the figure above.

Note

1. If $r = \infty$, $E = 0$. It means the intensity of gravitational field is zero only at infinite distance from the body.
2. If a unit mass (m) is placed on the surface of earth, then the gravitational force acting on the rest mass m will be equal to the weight w of the test mass.

   Now, $E = \frac{w}{m} = \frac{mg}{m} = g$ or $E = g$
3. We can also apply principal of superposition to gravitational field intensity in the same way as that of gravitational force.

   i.e. $\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \ldots + \mathbf{E}_n$

Example 10.29 *Two point masses of mass 10 kg and 1000 kg are at a distance 1 m apart. At which points on the line joining them, will the gravitational field intensity be zero?*

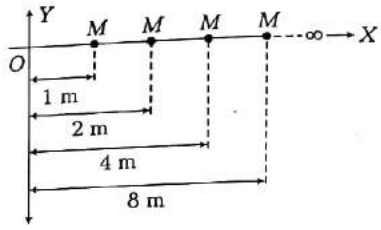
Sol. Let the resultant gravitational intensity be zero at a distance x from the mass of 10 kg on the line joining the centre of two bodies. At this point, the gravitational intensities due to the two bodies must be equal and opposite.

$$\therefore \quad \frac{G\times 10}{x^2} = \frac{G\times 1000}{(1-x)^2}$$

$$\Rightarrow \quad 100x^2 = (1-x)^2 \Rightarrow 10x = 1 - x$$

$$\Rightarrow \quad 11x = 1 \Rightarrow x = \left(\frac{1}{11}\right)\text{m}$$

Example 10.30 *Figure shows a system of point masses placed on X-axis. Find the net gravitational field intensity at the origin.*



Take sum of an infinite GP as $S = \frac{a}{1-r}$

where a = first term and r = Least common ratio.

Sol. Net gravitational field intensity at the origin,

$$\mathbf{E}_{net} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 + \mathbf{E}_4 + \ldots\infty \text{ terms}$$

$$= \frac{GM}{(1)^2}\hat{\mathbf{i}} + \frac{GM}{(2)^2}\hat{\mathbf{i}} + \frac{GM}{(4)^2}\hat{\mathbf{i}} + \ldots\infty \text{ terms}$$

$$= GM\hat{\mathbf{i}}\left(1 + \frac{1}{4} + \frac{1}{16} + \ldots\infty\right) \quad \left(\text{Here, } a = 1 \text{ and } r = \frac{1}{4}\right)$$

$$\text{So, } \mathbf{E}_{net} = GM\left(\frac{a}{1-r}\right)\hat{\mathbf{i}} = GM\hat{\mathbf{i}}\left(\frac{1}{1-1/4}\right) = GM\hat{\mathbf{i}}\left(\frac{1}{3/4}\right)$$

$$\Rightarrow \mathbf{E}_{net} = \frac{4}{3}GM\hat{\mathbf{i}}$$

2. Field due to a uniform solid sphere

Field at an external point

A uniform solid sphere may be treated as a single particle of same mass placed at its centre (O) for calculating the gravitational field at an external point (P).

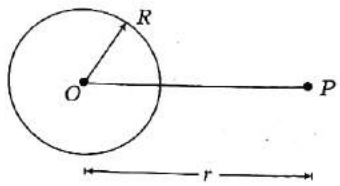


Fig. 10.15 Gravitational field at an external point P

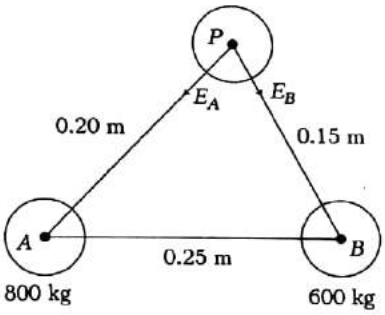
Thus, $$\boxed{E(r) = \frac{GM}{r^2}}$$

For $r \geq R$ or $E(r) \propto \frac{1}{r^2}$

Here, r is the distance of the point from the centre of the sphere and R is the radius of sphere.

Example 10.31 *Two solid spheres of radius 10 cm and masses 800 kg and 600 kg, are at a distance 0.25 m apart. Calculate the magnitude of the gravitational field intensity at a point distance 0.20 m from the 800 kg sphere and 0.15 m from the 600 kg sphere and does not lie on the line joining their centers. Given,* $G = 6.6 \times 10^{-11}\ N\text{-}m^2 kg^{-2}$.

Sol. Let E_A be the gravitational field intensity at P due to 800 kg sphere at A.



Then, $$E_A = G\frac{800}{(0.2)^2} = 2 \times 10^4 \text{ along } PA$$

Let E_B be the gravitational field intensity at P due to 600 kg sphere at B

Then, $E_B = G\dfrac{600}{(0.15)^2}$ along $PB = \dfrac{80000\,G}{3}$ along PB

The angle between E_A and E_B is 90°. $[\because\ AB^2 = AP^2 + BP^2]$

If E be the magnitude of the resultant intensity, then

$$E = \sqrt{E_A^2 + E_B^2}$$

$$= \sqrt{(2 \times 10^4\ G)^2 + \left(\frac{80000\,G}{3}\right)^2}$$

$$= G\sqrt{4 \times 10^8 + \frac{64}{9} \times 10^8}$$

$$= 6.67 \times 10^{-11} \times 2 \times 10^4 \sqrt{1 + \frac{16}{9}}$$

$$= 6.67 \times 2 \times \frac{5}{3} \times 10^{-7}\ \text{N kg}^{-1}$$

$$\Rightarrow \quad E = 2.22 \times 10^{-6}\ \text{N kg}^{-1}$$

Field at an internal point

The gravitational field due to a uniform sphere at an internal point (P) is proportional to the distance (r) of the point from the centre of the sphere.

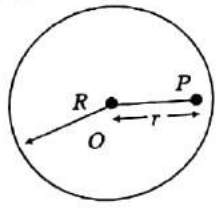


Fig. 10.16 Gravitational field at an internal point P

At the centre itself, it is zero and at surface it is $\dfrac{GM}{R^2}$, where R is the radius of the sphere.

Thus, $$E(r) = \frac{GM}{R^3} \cdot r$$

For $$r \le R$$

or $$E(r) \propto r$$

Thus, E versus r graph is as shown in figure below.

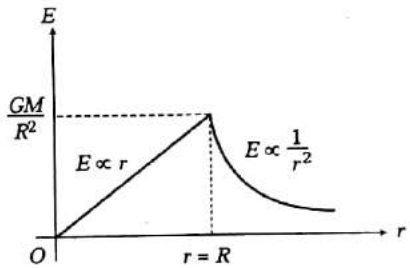


Fig. 10.17 E versus r graph for solid sphere

Example 10.32 *Gravitational field at the surface of a solid sphere is* $1.5 \times 10^{-4}\ \text{N kg}^{-1}$. *Find the gravitational field at a point situated inside the sphere at a distance equal to half of the radius of the solid sphere.*

Sol. Let mass of the solid sphere is M and radius is R.

It is given that

$$E_{\text{surface}} = \frac{GM}{R^2} = 1.5 \times 10^{-4}$$

$$\therefore \quad E(r = R/2) = \frac{GM}{R^3}(R/2)$$

$$= \frac{GM}{2R^2} = \frac{1.5 \times 10^{-4}}{2}$$

$$= 7.5 \times 10^{-4}\ \text{N kg}^{-1}$$

3. Field due to a uniform spherical shell

At an external point

For an external point (P) the shell may be treated as a single particle of same mass placed at its centre (O).

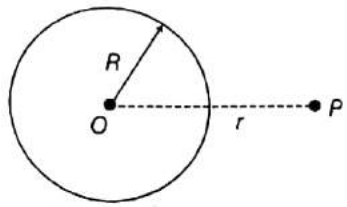


Fig. 10.18 Gravitational field at an external point P

Thus, at an external point the gravitational field is given by

$$E(r) = \frac{GM}{r^2} \qquad \text{for } r \geq R.$$

At $r = R$ (the surface of shell)

$$E = \frac{GM}{R^2}$$

and otherwise $E \propto \frac{1}{r^2}$

At an internal point

The field inside a uniform spherical shell is zero.

Thus, E versus r graph is as shown in figure below.

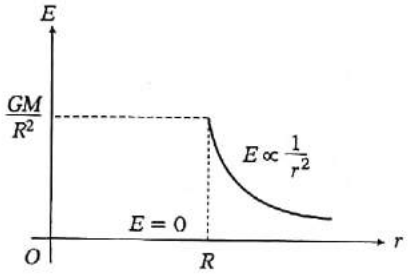
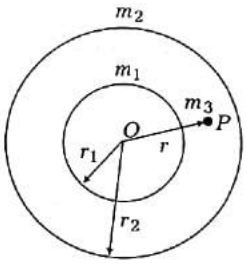


Fig. 10.19 *Eversus r graph for spherical shell*

Example 10.33 *Two concentric spherical shells have masses m_1 and m_2 and radii r_1 and r_2 ($r_2 > r_1$). What is the force exerted by this system on a particle of mass m_3, if it is placed at a distance r ($r_1 < r < r_2$) from the centre?*

Sol. The outer shell will have no contribution in the gravitational field at point P

$$\therefore \qquad E_P = \frac{Gm_1}{r^2}$$



Thus, force on mass m_3 placed at P is,

$$F = m_3 E_P$$

or

$$F = \frac{Gm_1 m_3}{r^2}$$

The field E_P and the force F both are towards centre O.

4. Field due to a uniform circular ring at a point on the axis

Gravitational field intensity at a point P on the axis of a circular ring of radius R and mass M is given by

$$E(r) = \frac{GMr}{(R^2 + r^2)^{3/2}}$$

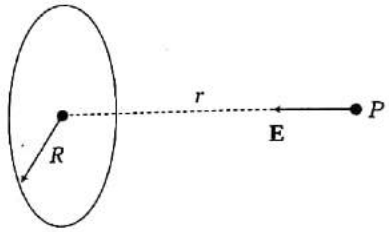


Fig. 10.20 *Electric field due to a ring*

This is directed towards the centre of the ring. It is zero at the centre of the ring and maximum at $r = R/\sqrt{2}$

(can be obtained by putting $dE/dr = 0$).

The maximum value is $E_{max} = \frac{2GM}{3\sqrt{3}R^2}$.

Thus, E versus r graph is as shown in figure below.

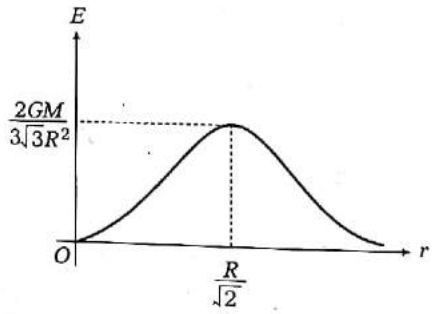


Fig. 10.21 *E versus r graph for a ring*

Example 10.34 *Mass of 20 kg is distributed uniformly over a ring of radius 2m. Find the gravitational field at a point lies an the axis of the ring at a distnace of $2\sqrt{3}$ m from the centre.*

Sol. Given, Mass of the ring, $M = 2\,\text{kg}$

Radius of the ring, $R = 2\,\text{m}$

Distance of the point, $r = 2\sqrt{3}\,\text{m}$

$\therefore$ Required gravitational field

$$E(r = 2\sqrt{3}) = \frac{GM}{(R^2 + r^2)^{3/2}} = \frac{G \times 2}{(\sqrt{4+12})^3}$$

$$= \frac{2G}{64} = \frac{G}{32} = \frac{6.67 \times 10^{-11}}{32}\,\text{Nkg}^{-1}$$

$$= 0.21 \times 10^{-11}\,\text{Nkg}^{-1} = 2.1 \times 10^{-12}\,\text{Nkg}^{-1}$$

5 Gravitational potential

The work done in bringing a unit mass slowly from reference point (infinity) to a given point in the gravitational field, is called the gravitational potential at that point.

This work is done by the external agent in bringing the mass. The gravitational potential is denoted by V.

Let W joule of work is to be done by external agent in bringing a test mass m from infinity to some point, then gravitational potential at the point will be

$$\boxed{V = \frac{W}{m}}$$

The SI unit of gravitational potential is J k g^{-1} or m^2 s^{-2} and it is a scalar quantity because it is defined as work done per unit mass. Its dimension is $[M^0L^2T^{-2}]$.

Gravitational potential for different bodies

1. Potential due to a point mass

Suppose a point mass M is situated at a point O. We want to find the gravitational potential due to this mass at a point P, a distance r from O.

For this let us find work done in taking the unit mass from P to infinity. This will be

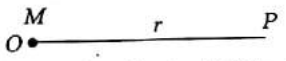


Figure 10.22 *Gravitational potential due to a point mass*

$$W = \int_r^\infty F\,dr = \int_r^\infty \frac{GM}{r^2}\,dr = \frac{GM}{r}$$

Hence, the work done in bringing unit mass from infinity to P will be $-\frac{GM}{r}$. Thus, the gravitational potential at P will be,

$$\boxed{V = -\frac{GM}{r}}$$

Gravitational potential at a point is always negative.

Note

1. When $r = \infty$ from above formula, then $V = 0$, hence gravitational potential is maximum (zero) at infinite.
2. At surface of the earth $r = R$, then $V = -\frac{GM}{R}$ at the surface of the earth.

Example 10.35 *A heavy point of mass 15 kg is placed at the origin of coordinate axis. Find the gravitational potential at a point located at $x = 5$ m on X-axis.*

Sol. Given, $M = 15$ kg

Gravitational potential at $x = 5$ m

$$V = \frac{-GM}{x} = \frac{-6.67 \times 10^{-11} \times 15}{5} = -20.01 \times 10^{-11}\ \text{J kg}^{-1}$$

2. Gravitational potential due to a system of more than one point masses

Let we have a system of point masses $M_1, M_2, M_3 \ldots M_n$. A point P is situated at distances $r_1, r_2, r_3, \ldots, r_n$ respectively from these points. Gravitational potential at this point P due to these point masses

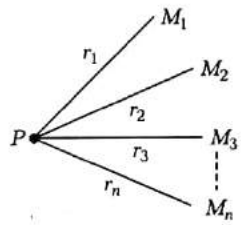


Figure 10.23 *Potential due to system of point masses*

$$\boxed{V_P = -\left[\frac{GM_1}{r_1} + \frac{GM_2}{r_2} + \frac{GM_3}{r_3} + \ldots + \frac{GM_n}{r_n}\right]}$$

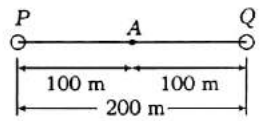
Example 10.36 *Two heavy point masses of mass 10^3 kg and 10^5 kg are separated by a distance of 200 m. What will be the potential at the mid-point of the line joining them?*

Sol. The total potential at A

$$V = V_P + V_Q = -\frac{Gm_P}{r} - \frac{Gm_Q}{r}$$

$$\therefore \quad V = \frac{-6.67 \times 10^{-11}}{100}[10^3 + 10^5]$$

$$\therefore \quad V = -6.73 \times 10^{-8}\ \text{J kg}^{-1}$$



3. Potential due to a uniform solid sphere

Potential at an external point

The gravitational potential due to a uniform solid sphere at an external point is same as that due to a single particle of same mass placed at its centre.

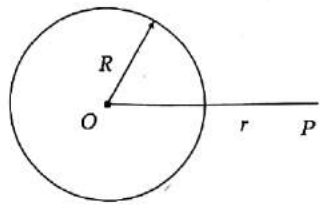


Fig. 10.24 *Potential at external point P*

Thus, $V(r) = -\dfrac{GM}{r}, \; r \geq R$

At the surface, $r = R$ and $V = -\dfrac{GM}{R}$

Example 10.37 *The radius of the earth is 6.37×10^6 m and its mean density is 5.5×10^3 kg m^{-3} and $G = 6.67 \times 10^{-11}$ N-m^2 kg^{-2}. Find the gravitational potential on the surface of the earth.*

Sol. Given, radius of the earth, $R = 6.37 \times 10^6$m

Density, $\rho = 5.5 \times 10^3$ kg m^{-3}, $G = 6.6 \times 10^{-11}$ Nm^2 kg^{-2}

$\because$ Mass of the earth, M = volume × density $= \dfrac{4}{3}\pi R^3 \rho$

$\therefore$ Gravitational potential on the earth's surface,

$$V = \frac{-GM}{R} = \frac{-G}{R} \times \frac{4}{3}\pi R^3 \rho = \frac{-4}{3}\pi G R^2 \rho$$

$$= \frac{-4}{3} \times 3.14 \times 6.66 \times 10^{-11} \times (6.37 \times 10^6)^2 \times 5.5 \times 10^3$$

$$\Rightarrow \; V = -6.22 \times 10^7 \text{ J kg}^{-1}$$

Example 10.38 *At a point above the surface of the earth, the gravitational potential is -5.12×10^7 J/kg and the acceleration due to gravity is 6.4 m/s^2. Assuming the mean radius of the earth to be 6400 km, calculate the height of the point above the earth's surface.*

Sol. If r is the distance of the given point from the centre of the earth, then gravitational potential at the point,

$$V = -\frac{GM}{r} = -5.12 \times 10^7 \text{ J/kg}$$

Acceleration due to gravity at this point,

$$g = \frac{GM}{r^2} = 6.4 \text{ m/s}^2$$

Clearly, $\dfrac{|V|}{g} = \dfrac{GM/r}{GM/r^2} = r$

Thus, $r = \dfrac{5.12 \times 10^7 \text{ J/kg}}{6.4 \text{ m/s}^2} = 8 \times 10^6 \text{ m} = 8000 \text{ km}$

Obviously, height of the point from the earth's surface

$$= (r - R) = 8000 \text{ km} - 6400 \text{ km} = 1600 \text{ km}$$

Potential at an internal point

At some internal point, potential at a distance r from the centre is given by,

$$V(r) = -\frac{GM}{R^3}(1.5R^2 - 0.5r^2), \; r \leq R$$

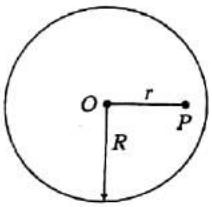


Fig. 10.25 Potential at internal point P

At $r = R$, $V = -\dfrac{GM}{R}$

while at $r = 0$, $V = -\dfrac{1.5\,GM}{R}$

i.e., at the centre of the sphere the potential is 1.5 times the potential at surface.

The variation of V versus r graph is as shown in figure below.

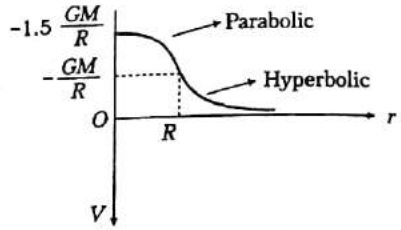


Fig. 10.26 V versus r graph for solid sphere

4. Potential due to a thin spherical shell

Potential at an external point

To calculate the potential at an external point, a uniform spherical shell may be treated as a point mass of same magnitude at its centre.

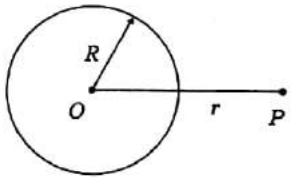


Fig. 10.27 Potential at external point P

Thus, potential at a distance r is given by

$$V(r) = \frac{-GM}{r}, \text{ where } r \geq R$$

At $r = R$, $V = -\dfrac{GM}{R}$

Example 10.39 *A particle of mass 1 kg is kept on the surface of a uniform thin spherical shell of mass 20 kg and radius 1 m. Find the work to be done against the gravitational force between them to take the particle away from the thin spherical shell.*

Sol. Potential at the surface of thin spherical shell,

$$V = -\frac{GM}{R} = -\frac{(6.67 \times 10^{-11})(20)}{1} \text{ J kg}^{-1}$$

$$= -1.334 \times 10^{-9} \text{ J kg}^{-1}$$

i.e., 1.334×10^{-9} J work is obtained to bring a mass of 1 kg from infinity to the surface of shell. Hence, the same amount of work will have to be done to take the particle away from the surface of shell. Thus,

$$W = 1.334 \times 10^{-9} \text{ J}$$

Potential at an internal point

The potential due to a uniform spherical shell is constant throughout at any point inside the shell and this is equal to $-\frac{GM}{R}$.

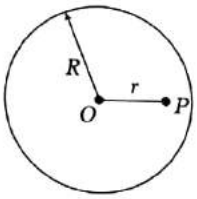


Fig. 10.28 Potential at internal point P

Thus, V versus r graph for a spherical shell is as shown in figure below.

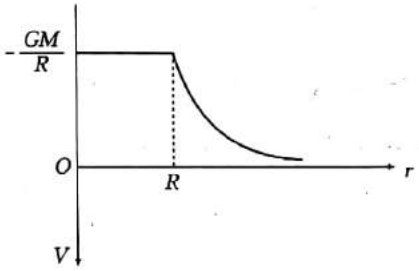
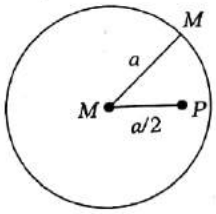


Fig. 10.29 V versus r graph for spherical shell

Example 10.40 *A particle of mass M is placed at the centre of a spherical shell of same mass and radius a. What will be the magnitude of the gravitational potential at a point situated at a / 2 distance from the centre?*

Sol. *The situation given in the question can be picturised as shown below*



Therefore, the magnitude of gravitational potential at a point situated at a / 2 distance from the centre can be given as

$$V_P = V_{\text{sphere}} + V_{\text{particle}} = \frac{-GM}{a} - \frac{GM}{a/2} = \frac{-3GM}{a}$$

5. Potential due to a uniform ring at a point on its axis

The gravitational potential at a distance r from the centre on the axis of a ring of mass M and radius R is given by

$$V(r) = -\frac{GM}{\sqrt{R^2 + r^2}}, \quad 0 \le r \le \infty$$

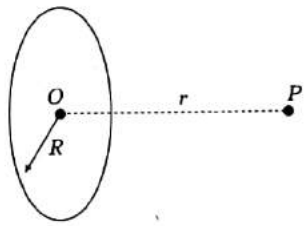


Fig. 10.30 Potential due to a ring

At $r = 0, V = -\frac{GM}{R}$, i.e. at the centre of the ring gravitational potential is $-\frac{GM}{R}$.

Thus, V versus r graph is as shown in figure below

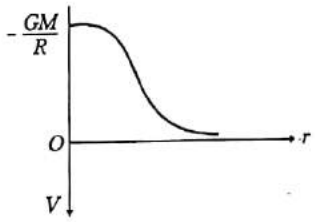


Fig. 10.31 V versus r graph for ring

Example 10.41 *Mass of 10 kg is distributed uniformly over a ring of radius 1 m. Find the gravitational potential at a point lies an the axis of the ring at a distance of 1 m from the centre.*

Sol. Mass of the ring, $M = 1$ kg

Radius of the ring, $R = 1$ m

Distance of the point, $r = 1$ m

$\therefore$ Required gravitational potential

$$V(r=1) = \frac{-GM}{\sqrt{R^2 + r^2}} = \frac{-G \times 1}{\sqrt{(1)^2 + (1)^2}}$$

$$= \frac{-G}{\sqrt{2}} = \frac{-6.67 \times 10^{-11}}{\sqrt{2}} \text{ Jkg}^{-1} = -4.716 \times 10^{-11} \text{ Jkg}^{-1}$$

Relation between gravitational field and potential

If change in gravitation potential at a points is dV, gravitational field intensity is E, then during displacement $d\mathbf{r}$ in the field

$$\boxed{dV = -\mathbf{E} \cdot d\mathbf{r}}$$

where, $\mathbf{E} = E_x \hat{\mathbf{i}} + E_y \hat{\mathbf{j}} + E_z \hat{\mathbf{k}}$

$d\mathbf{r} = dx\,\hat{\mathbf{i}} + dy\,\hat{\mathbf{j}} + dz\,\hat{\mathbf{k}}$

$\therefore \quad dV = -E_x dx - E_y dy - E_z dz$

Also we can write

$$E_x = \frac{-\partial V}{\partial x}, E_y = \frac{-\partial V}{\partial y} \text{ and } E_z = \frac{-\partial V}{\partial z}$$

where, $\frac{\partial}{\partial x}$ represents partial differentiation with respect to x treating y and z as constants. Similarly $\frac{\partial}{\partial y}$ and $\frac{\partial}{\partial z}$ are partial differentiations with respect to y and z respectively treating other variables as constants.

Note If field is given, the potential can be obtained as

$$V(\mathbf{r}_2) - V(\mathbf{r}_1) = -\int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{E} \cdot d\mathbf{r}$$

Example 10.42 *If gravitational potential is $V = xy^2$, find the gravitational field at (2, 1).*

Sol. Given, $V = xy^2$

Electric field in x-direction

$$E_x = \frac{-\partial V}{\partial x} = \frac{-\partial}{\partial x}(xy^2) = -y^2$$

Electric field in y-direction

$$E_y = \frac{-\partial V}{\partial y} = \frac{-\partial}{\partial y}(xy^2) = -2yx$$

$$\therefore \quad \mathbf{E} = E_x\hat{\mathbf{i}} + E_y\hat{\mathbf{j}} = (-y^2)\hat{\mathbf{i}} + (-2xy)\hat{\mathbf{j}}$$

$$\therefore \quad \mathbf{E}(x=2, y=1) = -(1)^2\hat{\mathbf{i}} + (-2\times 2\times 1)\hat{\mathbf{j}}$$

$$= -\hat{\mathbf{i}} - 4\hat{\mathbf{j}}$$

$$\Rightarrow \quad |\mathbf{E}| = \sqrt{(-1)^2 + (-4)^2} = \sqrt{17} \text{ unit}$$

Example 10.43 *If gravitational field is given by $\mathbf{E} = -x\hat{\mathbf{i}} - 2y^2\hat{\mathbf{j}}$. When gravitational potential is zero at (0,0), find potential cat (2, 1).*

Sol. Given, gravitational field

$$\mathbf{E} = -x\hat{\mathbf{i}} - 2y^2\hat{\mathbf{j}}, \; E_x = -x \text{ and } E_y = -2y^2$$

As we know that,

$$V(\mathbf{r}_2) - V(\mathbf{r}_1) = -\int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{E} \cdot d\mathbf{r}$$

$$\Rightarrow \quad V(2,1) - V(0,0) = -\left[\int_0^2 E_x dx + \int_0^1 E_y dy\right]$$

$$= -\left[\int_0^2 -x dx + \int_0^1 -2y^2 dy\right]$$

$$= +\left[\left(\frac{x^2}{2}\right)_0^2 + (y^2)_0^1\right] = \left[\frac{4}{2} + 1\right] = 3 \text{ unit}$$

$$\Rightarrow \quad V(2,1) = 3 + V(0,0) = 3 + 0 = 3 \text{ unit}$$

Check point 10.3

1. Three particles each of mass m are kept at vertices of an equilateral triangle of side L. The gravitational field at centre due to these particles is

(a) zero (b) $\frac{3GM}{L^2}$ (c) $\frac{9GM}{L^2}$ (d) $\frac{12GM}{\sqrt{3}L^2}$

2. At what height the gravitational field reduces by 75% the gravitational field at the surface of earth?

(a) R (b) $2R$ (c) $3R$ (d) $4R$

(where, R is the radius of earth)

3. Gravitational field due to a solid sphere

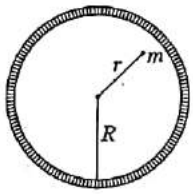
(a) remains constant throughout the sphere

(b) increases inside the sphere and decreases outside the sphere.

(c) increases throughout with distance from the center.

(d) decreases throughout with distance from the centre.

4. A mass m is placed inside a hollow sphere of mass M as shown in figure. The gravitational force on mass m is



(a) $\frac{GMm}{R^2}$ (b) $\frac{GMm}{r^2}$

(c) $\frac{GMm}{(R-r)^2}$ (d) zero

5. A uniform solid sphere of mass m and radius r is surrounded symmetrically by a uniform thin spherical shell of radius $2r$ and mass m.

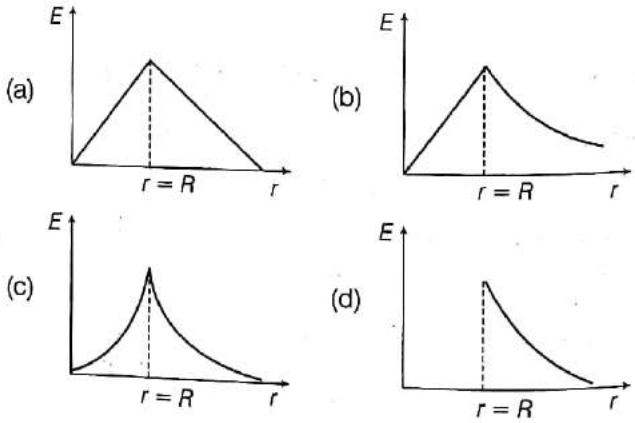
(a) The gravitational field at a distance of $1.5r$ from the centre is $\frac{2}{9}\frac{Gm}{r^2}$

(b) The gravitational field at a distance of $2.5r$ from centre is $\frac{8}{25}\frac{Gm}{r^2}$

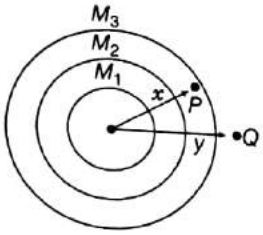
(c) The gravitational field at a distance of $1.5r$ from centre is zero

(d) The gravitational field between the sphere and spherical shell is uniform.

6. Which one of the following graphs represents correctly represent the variation of the gravitational field (E) with the distance (r) from the centre of a spherical shell of mass M radius R?



7. Consider three concentric shells of masses M_1, M_2 and M_3 having radii a, b and c respectively are situated as shown in figure



Gravitational field at a point located at Q and P is

(a) $\frac{G(M_1+M_2)}{y^2}, \frac{G(M_1+M_2)}{y^2}$

(b) $\frac{G(M_1+M_2+M_3)}{Y^2}, \frac{G(M_1+M_2)}{x^2}$

(c) $\frac{G(M_1+M_2+M_3)}{a^2}, \frac{GM_1}{a^2}$

(d) $\frac{G(M_1+M_2+M_3)}{C^2}, \frac{GM_2}{b^2}$

8. Two particles of masses m and M are placed a distance d apart. The gravitational potential at the position where the gravitional field due to them is zero is V. Then

(a) $V = -\frac{G}{d}(m+M)$ (b) $V = -\frac{Gm}{d}$

(c) $V = -\frac{GM}{d}$ (d) $V = -\frac{G}{d}(\sqrt{m}+\sqrt{M})^2$

9. A thin rod of length L is bent to form a semicircle. The mass of rod is M. What will be the gravitational potential at the centre of the circle?

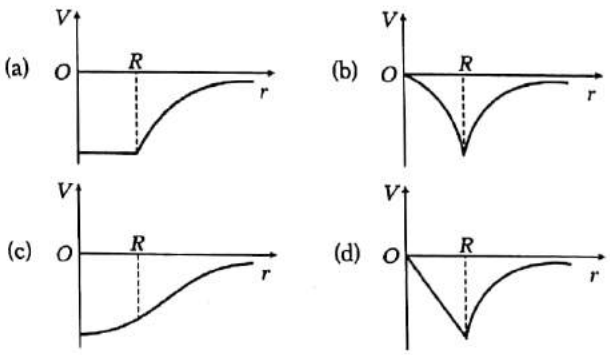
(a) $-\frac{GM}{L}$ (b) $-\frac{GM}{2\pi L}$ (c) $-\frac{\pi GM}{2L}$ (d) $-\frac{\pi GM}{L}$

10. A particle is kept on the surface of a uniform sphere of mass 100 kg and radius 10 cm. Find the work to be done per unit mass against the gravitational force between them, to take the particle far away from the sphere (you may take $h = 6.67 \times 10^{-11}\ Nm^2kg^{-2}$).

(a) 13.34×10^{-10} J (b) 3.33×10^{-10} J

(c) 6.67×10^{-9} J (d) 6.67×10^{-8} J

11. The diagram showing the variation of gravitational potential of earth with distance from the centre of earth is



12. Inside a uniform shell

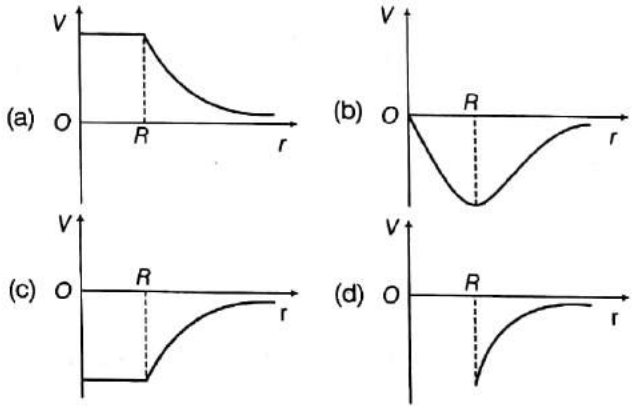
(a) potential is variable (b) potential is zero

(c) potential is constant (d) All of these

13. If V is the gravitational potential on the surface of the earth, then what is its value at the centre of the earth?

(a) $2V$ (b) $3V$ (c) $\frac{3}{2}V$ (d) $\frac{2}{3}V$

14. By which curve will be variation of gravitational potential of a hollow sphere of radius R with distance be depicted?



15. For a uniform ring of mass M and radius R at its centre

(a) field and potential both are zero

(b) field is zero but potential is $\frac{GM}{R}$

(c) field is zero but potential is $-GM/R$

(d) magnitude of field is $\frac{GM}{R^2}$ and potential $-\frac{GM}{R}$

6 Gravitational potential energy

Gravitational potential energy of a body at a point is defined as the amount of work done in bringing the given body from infinity to that point against the gravitational force. The change in potential energy (dU) of a system corresponding to a conservative internal force (F) is given by

$$dU = -\mathbf{F}\cdot d\mathbf{r}$$

or $\int_i^f dU = -\int_{\mathbf{r}_i}^{\mathbf{r}_f} \mathbf{F}\cdot d\mathbf{r}$ or $\boxed{U_f - U_i = -\int_{\mathbf{r}_i}^{\mathbf{r}_f} \mathbf{F}\cdot d\mathbf{r}}$

We generally choose the reference point at infinity and assume potential energy to be zero there, *i.e.*, if we take $r_i = \infty$ (infinite) and $U_i = 0$, then we can write

$$\boxed{U_f = U = -\int_{\infty}^{\mathbf{r}} \mathbf{F}\cdot d\mathbf{r} = -W}$$

Hence, potential energy of a body or system is negative of work done by the conservative forces in bringing it from infinity to the present position.

Gravitational potential energy of a two particle system

The gravitational potential energy of two particles of masses m_1 and m_2 separated by a distance r is given by,

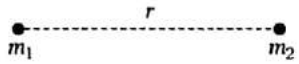


Fig. 10.32 System of two point masses separated by a distance r

$$\boxed{U = -\frac{Gm_1m_2}{r}}$$

This is actually the negative of work done in bringing these masses from infinity to a distance r, by the gravitational forces between them.

$$\text{Gravitational potential energy} = \left(-\frac{Gm_1}{r}\right) \times m_2$$

$\therefore$ Gravitational potential energy =

Gravitational potential × mass of the body

Gravitational potential energy is a scalar quantity. Its SI unit is joule and dimensional formula is $[ML^2T^{-2}]$.

Example 10.44 *Two point masses 1 kg and 4 kg are separated by a distance of 10 cm. Find gravitational potential energy of the two point masses.*

Sol. Given, $m_1 = 1$ kg, $m_2 = 4$ kg, $r = 10$ cm $= 10 \times 10^{-2}$ m

$\therefore$ Gravitational potential energy

$$U = \frac{-Gm_1m_2}{r} = \frac{-6.67 \times 10^{-11} \times 1 \times 4}{10 \times 10^{-2}} = -26.68 \times 10^{-10} \text{ J}$$

Gravitational potential energy for a system of more than two particles

The gravitational potential energy for a system of particles (say m_1, m_2, m_3 and m_4) is given by

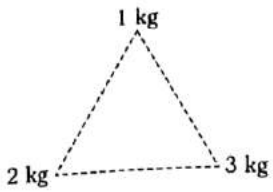
$$U = -G\left[\frac{m_4m_3}{r_{43}} + \frac{m_4m_2}{r_{42}} + \frac{m_4m_1}{r_{41}} + \frac{m_3m_2}{r_{32}} + \frac{m_3m_1}{r_{31}} + \frac{m_2m_1}{r_{21}}\right]$$

Thus, for a n particle system there are $\frac{n(n-1)}{2}$ pairs and the potential energy is calculated for each pair and added to get the total potential energy of the system.

Example 10.45 *Three masses of 1 kg, 2 kg and 3 kg are placed at the vertices of an equilateral triangle of side 1 m. Find the gravitational potential energy of this system. (Take, $G = 6.67 \times 10^{-11}$ N-m^2 kg^{-2}).*

Sol. Gravitational potential energy of the three particles system

$$U = -G\left(\frac{m_3m_2}{r_{32}} + \frac{m_3m_1}{r_{31}} + \frac{m_2m_1}{r_{21}}\right)$$



Here, $r_{32} = r_{31} = r_{21} = 1$ m

$m_1 = 1$ kg, $m_2 = 2$ kg and $m_3 = 3$ kg

Substituting in above, we get

$$U = -(6.67 \times 10^{-11})\left(\frac{3 \times 2}{1} + \frac{3 \times 1}{1} + \frac{2 \times 1}{1}\right)$$

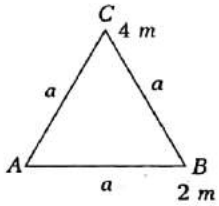
or $$U = -7.337 \times 10^{-10} \text{ J}$$

Example 10.46 *Three particles of masses m, 2m and 4m are placed at the corner of an equilateral triangle of side a. Calculate:*

(i) The potential energy of the system.

(ii) The work done on the system if the side of the triangle is changed from a to 2a. Assume the potential energy to be zero when the separation is infinity.

Sol. (i) The potential energy of the system = Sum of the potential energies of all the three possible distinct pairs.



$$\text{i.e., } U = U_{AB} + U_{BC} + U_{CA}$$

$$= \frac{-G(m)(2m)}{a} + \frac{-G(2m)(4m)}{a} + \frac{-G(4m)(m)}{a}$$

$$= \frac{-14Gm^2}{a}$$

when the side a is changed to $2a$, the potential energy

$$U' = \frac{-14Gm^2}{2a}$$

(ii) The work done on the system = The work done against the gravitational force

= The work done by the external force

$$= U' - U = \frac{14Gm^2}{2a}$$

Example 10.47 *Infinite number of bodies, each of mass 2 kg are situated on X-axis at distance 1 m, 2 m, 4 m, 8 m, respectively from the origin. What is the resulting gravitational potential due to this system at the origin?*

Sol. The resulting gravitational potential

$$V = -2G\left(\frac{1}{1} + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \cdots\right) = -2G\left(1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} \cdots\right)$$

$$\Rightarrow \quad V = \frac{-2G}{\left(1 - \frac{1}{2}\right)} = \frac{-2G}{\frac{1}{2}} = -4G$$

Gravitational potential energy of an object in different conditions

Object placed at earth's surface

Let an object of mass m is placed at a point in the gravitational field of the earth of mass M. Then, gravitational potential energy of an object placed at earth's surface

$$\boxed{U = -\frac{GMm}{R}}$$

Note

1. If the body of mass m is moved from a point at distance r_a to a point at distance r_b ($r_a > r_b$), then change in potential energy,

$$\Delta U = GMm\left[\frac{1}{r_a} - \frac{1}{r_b}\right]$$

2. Gravitational potential energy at the centre of earth relative to infinity is

$$U_{\text{centre}} = \frac{-3}{2}\left(\frac{GMm}{R}\right)$$

Object placed at a point, above the earth's surface

Let an object of mass m is placed at height h above the surface of earth, then change is gravitational potential energy when the object is taken from the centre of the earth of the height h is

$$\boxed{\Delta U = \frac{mgh}{1 + h/R}}$$

ΔU is positive, it means the potential energy of a body increases as its height above earth surface increases.

For $h << R$, $\Delta U \approx mgh$

Thus, what we read the mgh is actually the difference in potential energy (not the absolute potential energy), that too for $h << R$.

Note Gravitational potential energy of a body at height x from the earth surface is

$$\boxed{U_x = \frac{-mgR}{1 + x/R}}$$

Example 10.48 *The mass of the earth is 6×10^{24} kg. The constant of gravitation $G = 6.67 \times 10^{-11}$ N-m^2 kg^{-2}. The potential energy of the earth and moon system is -7.79×10^{28} J. Determine the mean distance between earth and moon.*

Sol. Potential energy of the earth and moon system is

$$U = \frac{-GMm}{r}$$

$$\Rightarrow \quad r = \frac{-GMm}{U} = \frac{-6.67 \times 10^{-11} \times 6 \times 10^{24} \times 7.4 \times 10^{22}}{-7.79 \times 10^{28}}$$

$$\Rightarrow \quad r = 3.8 \times 10^8 \text{ m}$$

Example 10.49 *A body of mass m is raised to a height 10R from the surface of the earth, where R is the radius of the earth. Find the increase in potential energy. (G = universal constant of gravitation, M = mass of the earth and g = acceleration due to gravity)*

Sol. Potential energy at the earth surface $= \frac{-GMm}{R}$

Potential energy at a heigh h above the earth's surface

$$= \frac{-GMm}{(R+h)}$$

$$\therefore \text{ Change in potential energy} = \frac{-GMm}{(R+h)} - \left(-\frac{GMm}{R}\right)$$

$$= \frac{GMmh}{R(R+h)}$$

Substituting, $h = 10R$

$$\Delta PE = \frac{GMm \times 10R}{R(R+10R)} = \frac{10\,GMm}{11R}$$

Example 10.50 *Find the change in the gravitational potential energy when a body of mass m is raised to a height nR above the surface of the earth. (Here, R is the radius of the earth)*

Sol. Gravitational potential energy of mass m at earth's surface

$$U_e = -\frac{GMm}{R}$$

Gravitational potential energy of same mass at a height nR from the earth's surface

$$U_h = -\frac{GMm}{(R+nR)} = -\frac{GMm}{R(n+1)}$$

Thus, magnitude of the change in gravitational potential energy

$$\Delta U = U_h - U_e$$

$$= \frac{GMm}{R}\left\{1 - \frac{1}{(n+1)}\right\}$$

$$= \left(\frac{n}{n+1}\right)\frac{GMm}{R} = \left(\frac{n}{n+1}\right)mgR \quad (\because GM = gR^2)$$

Example 10.51 *An object is dropped from height $h = 2R$ on the surface of earth. Find the speed with which it will collide with ground by neglecting effect of air. (Where R is radius of earth, take mass of earth M)*

Sol. The initial potential energy of object is $U_i = -\frac{GMm}{3R}$

Final potential energy $U_f = -\frac{GMm}{R}$

By law of conservation of energy, $\Delta KE = -\Delta PE$

$$\Rightarrow \quad \frac{1}{2}mv^2 = -(U_f - U_i) = U_i - U_f$$

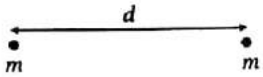
$$\Rightarrow \quad \frac{1}{2}mv^2 = -\frac{GMm}{3R} + \frac{GMm}{R}$$

$$\Rightarrow \quad \frac{1}{2}v^2 = \frac{2GM}{3R}$$

$$\Rightarrow \quad v = \sqrt{\frac{4GM}{3R}} = 2\sqrt{\frac{GM}{3R}}$$

Check point 10.4

1. Consider the two identical particles shown in the given figure. They are released from rest and may move towards each other under influence of mutual gravitational force.



Gravitational potential energy of the two particle system
(a) is zero
(b) is constant ($\neq 0$)
(c) decreases as the separation decreases
(d) increases as the separation decreases

2. In above question 1, find the speed of each particle, when the separation reduces to half its initial value.
(a) $\sqrt{\frac{Gm}{d}}$ (b) $\sqrt{\frac{2Gm}{d}}$ (c) $\sqrt{\frac{Gm}{2d}}$ (d) None of these

3. Find the potential energy of 4-particles, each of mass 1 kg placed at the four vertices of a square of side length 1 m.
(a) + 4.0 G (b) – 7.5 G (c) – 5.4 G (d) + 6.3 G

4. The magnitude of gravitational potential energy of a body at a distance r from the centre of earth is u. Its weight at a distance $2r$ from the centre of earth is
(a) $\frac{u}{r}$ (b) $\frac{u}{4r}$ (c) $\frac{u}{2r}$ (d) $\frac{4r}{u}$

5. The gravitational potential energy of a body of mass m at a distance r from the centre of the earth is U. What is the weight of the body at this distance?
(a) U (b) Ur (c) $\frac{U}{r}$ (d) $\frac{U}{2r}$

6. When a body is lifted from surface of earth height equal to radius of earth, then the change in its potential energy is
(a) $mg\,R$ (b) $2mg\,R$
(c) $\frac{1}{2}mg\,R$ (d) $4mg\,R$

7. Energy required in moving a body of mass m from a distance $2R$ to $3R$ from centre of earth of mass M is
(a) $\frac{GMm}{12R^2}$ (b) $\frac{GMm}{3R^2}$ (c) $\frac{GMm}{8R}$ (d) $\frac{GMm}{6R}$

8. If a body of mass m has to be taken from the surface to the earth to a height $h = 4R$, then the amount of energy required is (R = radius of the earth)
(a) mgR (b) $\frac{mgR}{5}$ (c) $\frac{4mgR}{5}$ (d) $\frac{mgR}{12}$

9. If an object of mass m is taken from the surface of earth (radius R) to a height $5R$, then the work done is
(a) $2mgR$ (b) mgR
(c) $\frac{5}{6}mgR$ (d) $\frac{3}{2}mgR$.

10. A body of mass m is kept at a small height h above the ground. If the radius of the earth is R and its mass is M, the potential energy of the body and earth system (with $h = \infty$ being the reference position) is
(a) $\frac{GMm}{R} + mgh$ (b) $\frac{-GMm}{R} + mgh$
(c) $\frac{GMm}{R} - mgh$ (d) $-\frac{GMm}{R} - mgh$

7 Binding energy

The minimum energy required to take a particle infinite away from the earth is called the binding energy of the earth particle system.

Binding energy of a particle on the surface of the earth

Suppose the mass m is placed on the surface of earth. The radius of the earth is R and its mass is M. Then, the kinetic energy of the particle, $K = 0$ and potential energy of the particle is $U = -\frac{GMm}{R}$.

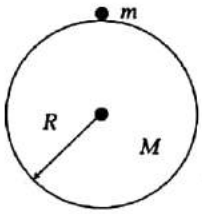


Fig. 10.33 A particle on the surface of the earth

Therefore, the total mechanical energy of the particle is,

$$E = K + U = 0 - \frac{GMm}{R}$$

or

$$E = -\frac{GMm}{R}$$

$\therefore$ Binding energy, $\boxed{BE = |E| = \frac{GMm}{R}}$

It is due to this energy, the particle is attached with the earth.

If minimum this much energy is supplied to the particle in any form (normally kinetic) the particle no longer remains bound to the earth. It goes out of the gravitational field of earth.

Example 10.52 *Find the binding energy of a satellite of mass m in a orbit of radius r. (R = radius of earth, g = acceleration due to gravity)*

Sol. The energy required to remove the satellite from its orbit around the earth to infinity is called binding energy of the satellite. It is equal to negative of total mechanical energy of satellite in its orbit.

Thus, binding energy $= -E = \frac{GMm}{2r}$

but, $g = \frac{GM}{R^2} \Rightarrow GM = gR^2$

$\therefore$ Binding energy $= |E| = \frac{gmR^2}{2r}$

Escape velocity

The minimum velocity with which a body must be projected vertically upwards in order that it may just escape the gravitational field of the earth (*i.e.*, $v = 0$) is called escape velocity.

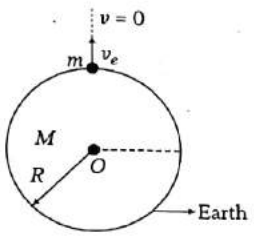


Fig. 10.34

A body projected from the surface of the earth such with speed v_e, such that $v_f = v = 0$.

As we discussed the binding energy of a particle on the surface of earth kept at rest is GMm/R. If this much energy in the form of kinetic energy is supplied to the particle, it leaves the gravitational field of the earth. So, if v_e is the escape velocity of the particle, then from conservation of mechanical energy we have,

$$\frac{1}{2}mv_e^2 = \frac{GMm}{R} \quad \text{or} \quad v_e = \sqrt{\frac{2GM}{R}}$$

as $$g = \frac{GM}{R^2}$$

$$\Rightarrow \quad \boxed{v_e = \sqrt{2gR}} \quad \text{...(i)}$$

Substituting the value of g (9.8 ms^{-2}) and R (6.4×10^6 m), we get

$$v_e \approx 11.2 \text{ kms}^{-1}$$

Thus, the minimum velocity needed to take a particle infinitely away from the earth is called the escape velocity. On the surface of earth its value is 11.2 kms^{-1}.

From Eq. (i), $$v_e = \sqrt{\frac{2G}{R} \times \frac{4}{3}\pi R^3 \times \rho}$$

where, ρ is the mean density of the earth

Escape velocity, $v_e = R\sqrt{\frac{8}{3}\pi G\rho}$

Important points related to escape velocity

(i) The escape velocity does not depend on angle of projection from the earth's surface. But as the earth rotates about its axis, so it becomes easier to attain escape velocity if the body is projected in the direction in which the earth is moving.

(ii) As the escape velocity depends on the mass and radius of the planet from the surface of which the body is projected, so value of escape velocity is different for different planets.

(iii) A planet will have atmosphere if the root mean square velocity of its atmospheric molecules is less than the escape velocity for the given planet. That is why moon has no atmosphere (v_e = 2.3 kms^{-1}) while Jupiter has a thick atmosphere (v_e = 60 kms^{-1}). Even the lightest hydrogen cannot escape from its surface.

(iv) If a body falls freely from infinite height, then it will reach the surface of the earth with a speed approximately equal to the average escape speed $v_e = 11.2$ kms^{-1}.

(v) In calculating the escape speed, the air resistance and the gravitational effect due to other celestial bodies are neglected.

(vi) For earth to become black hole, the escape speed must become equal or more than speed of light *i.e.*, $\sqrt{\frac{2GM}{R}} \geq c$. In that case radius R of the earth will become that of the size of a grape.

Note If the escape velocity of a body is equal to the velocity of light then from such bodies nothing can escape, not even light. Such bodies are called **black holes**.

Example 10.53 *Calculate the escape velocity from the surface of a planet of mass* 14.8×10^{22} *kg. It is given that radius of the planet is* 3.48×10^6 *m.*

Sol. Given, mass of planet $M_p = 14.8 \times 10^{22}$ kg

Radius of the planet $R_p = 3.48 \times 10^6$ m. Escape velocity from the surface of planet is

$$v_e = \sqrt{\frac{2GM_p}{R_p}}$$

Substituting the values, we have

$$v_e = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 14.8 \times 10^{22}}{3.48 \times 10^6}}$$

$$= 2.4 \times 10^3 \text{ ms}^{-1} \quad \text{or} \quad 2.4 \text{ kms}^{-1}$$

Example 10.54 *The ratio of the masses and radii of two planets are 4 : 6 and 8 : 18. What is the ratio of the escape speed at their surface?*

Sol. Given, $\frac{M_1}{M_2} = \frac{2}{3}$ and $\frac{R_1}{R_2} = \frac{4}{9}$

As we know escape speed $v = \sqrt{\frac{2GM}{R}}$

$$\therefore \quad \frac{v_1}{v_2} = \sqrt{\frac{M_1}{M_2} \times \frac{R_2}{R_1}} = \sqrt{\frac{2}{3} \times \frac{9}{4}} = \sqrt{\frac{3}{2}}$$

$$\therefore \quad v_1 : v_2 = \sqrt{3} : \sqrt{2}$$

Example 10.55 *Jupiter has a mass 318 times that of earth, and its radius is 11.2 times the earth's radius. Estimate the escape velocity of a body from Jupiter's surface, given that the escape velocity from the earth's surface 11.2 kms^{-1}.*

Sol. Escape velocity from the earth's surface is

$$v_e = \sqrt{\frac{2GM}{R}} = 11.2 \text{ kms}^{-1}$$

Escape velocity from Jupiter's surface will be $v_e = \sqrt{\frac{2GM'}{R'}}$

But $M' = 318\text{ M}, R' = 11.2R$

$$\therefore \quad v'_e = \sqrt{\frac{2G\,(318\,M)}{11.2R}} = \sqrt{\frac{2GM}{R} \times \frac{318}{11.2}}$$

$$= v'_e \times \sqrt{\frac{318}{11.2}} = 11.2 \times \sqrt{\frac{318}{11.2}}$$

$$\Rightarrow \quad v'_e = 59.7 \text{ kms}^{-1}$$

Example 10.56 *At what temperature, hydrogen molecules will escape from the earth's surface? (Take, radius of earth $R_e = 6.4 \times 10^6$ m, mass of hydrogen molecule $m = 0.34 \times 10^{-26}$ kg, Boltzmann constant $k = 1.38 \times 10^{-23}$ JK^{-1} and acceleration due to gravity $= 9.8$ ms^{-2}) also take rms speed of gas as $v_{rms} = \sqrt{\frac{3kT}{m}}$*

Sol. The root mean square velocity of gas is

$$v_{rms} = \sqrt{\frac{3kT}{m}} \quad \text{...(i)}$$

Escape velocity of gas molecules is $v_{es} = \sqrt{2gR_e}$ ...(ii)

As the root mean square velocity of gas molecules must be equal to the escape velocity.

$\therefore$ From Eqs. (i) and (ii),

$$\sqrt{\frac{3kT}{m}} = \sqrt{2gR_e} \Rightarrow T = \frac{2gR_e m}{3k}$$

$$\Rightarrow \quad T = \frac{2 \times 9.8 \times 6.4 \times 10^6 \times 0.34 \times 10^{-26}}{3\,(1.38 \times 10^{-23})}$$

$$= 10^4 \text{ K}$$

Therefore, 10^4K is the temperature at which hydrogen molecules will escape from earth's surface.

Example 10.57 *A space-ship is launched into a circular orbit close to the Earth's surface. What additional speed should now be imparted to the spaceship so that it overcome the gravitational pull of the Earth.*

Take kinetic energy of the space-ship,

$$K = \text{ total energy of space-ship} = \frac{GMm}{2R}$$

where m = mass of space-ship , M = mass of the earth and R = radius of the earth

Sol. Let ΔK be the additional kinetic energy imparted to the spaceship to overcome the gravitation pull then

$$\Delta K = -\text{(Total energy of spaceship)} = \frac{GMm}{2R}$$

$$\text{Total kinetic energy} = \frac{GMm}{2R} + \Delta K = \frac{GMm}{2R} + \frac{GMm}{2R} = \frac{GMm}{R}$$

then $\quad \frac{1}{2} m v_2^2 = \frac{GMm}{R} \Rightarrow v_2 = \sqrt{\frac{2GM}{R}}$

But $\quad v_1 = \sqrt{\frac{GM}{R}}$

So Additional velocity

$$= v_2 - v_1 = \sqrt{\frac{2GM}{R}} - \sqrt{\frac{GM}{R}} = (\sqrt{2} - 1)\sqrt{\frac{GM}{R}}$$

Escape energy

Minimum energy given to a particle in form of kinetic energy so that it can first escape from earth's gravitational field, is called escape energy.

$$\boxed{\text{Escape Energy} = \frac{GMm}{R}}$$

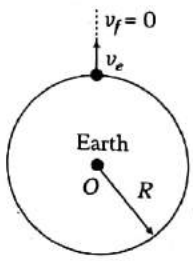
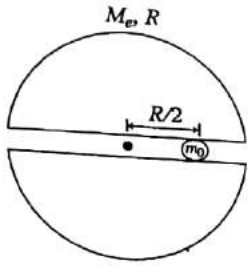


Fig. 10.35

If escape velocity is v_e, then

$$\boxed{\text{Escape energy} = \frac{1}{2} m v_e^2}$$

Example 10.58 *A very small groove is made in the earth, and a particle of mass m_0 is placed at R/2 distance from the centre as shown is figure. Find the escape energy of the particle from that place.*



Sol. Suppose we porjected the particle with speed v_e so that it just reaches at ($r \to \infty$).

Applying energy conservation

$$K_i + U_i = K_f + U_f$$

$$\Rightarrow \frac{1}{2} m_0 v_e^2 + m_0 \left[-\frac{GM_e}{2R^3} \left\{ 3R^2 - \left(\frac{R}{2}\right)^2 \right\} \right] = 0$$

$$\Rightarrow \text{Escape energy} = \frac{1}{2} m_o v_e^2 = \frac{11 G m_0 M_e}{8R}$$

Example 10.59 *The earth is assumed to be a sphere of radius R. A platform is arranged at a height R from the surface of the earth. The escape velocity of a body from this platform is $f v_e$, where v_e is its escape velocity from the surface of the earth. Find the value of f.*

Sol. For a platform at a height h,

escape energy = binding energy of sphere

$$\Rightarrow \frac{1}{2} m v'^2_e = \frac{GMm}{R+h} \Rightarrow v'_e = \sqrt{\frac{2\,GM}{R+h}} = \sqrt{\frac{2\,GM}{2R}} \quad (\because h = R)$$

But at surface of earth, $v_e = \sqrt{\frac{2GM}{R}}$

As given, $\quad v'_e = f v_e$

Hence, $\quad \sqrt{\frac{2\,GM}{2R}} = f\sqrt{\frac{2\,GM}{R}}$ or $\frac{1}{2R} = \frac{f^2}{R} \Rightarrow f = \frac{1}{\sqrt{2}}$

Example 10.60 *A body is projected upwards with a velocity of 4×11.2 km s^{-1} from the surface of earth. What will be the velocity of the body when it escapes from the gravitational pull of earth?*

Sol. Initial KE of the body $= \frac{1}{2} mv^2$

Here, $\quad v = 4 \times 11.2 \text{ kms}^{-1}$

So, $\quad \text{KE} = \frac{1}{2} \times \text{m}\,(4 \times 11.2)^2 = 16 \times \frac{1}{2} mv_e^2$

As $\frac{1}{2} mv_e^2$ energy is used up in coming out from the gravitational pull of the earth, so final KE should be $15 \times \frac{1}{2} mv_e^2$.

Hence, $\quad \frac{1}{2} mv'^2 = 15 \times \frac{1}{2} mv_e^2$

$\therefore \quad v'^2 = 15 v_e^2$

$\Rightarrow \quad v = \sqrt{15} v_e = \sqrt{15} \times 11.2 \text{ kms}^{-1}$

Maximum height attained by a particle

Suppose a particle of mass m is projected vertically upwards with a speed v and we want to find the maximum height h attained by the particle.

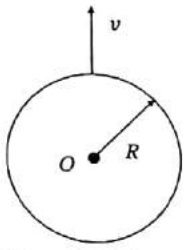


Fig. 10.36 *A particle projected vertically upwards from the surface of the earth*

Then, we can use conservation of mechanical energy, *i.e.*,
Decrease in kinetic energy = increase in gravitational potential energy of particle.

$$\therefore \quad \frac{1}{2} mv^2 = \Delta U$$

or $$\frac{1}{2} mv^2 = \frac{mgh}{1 + h/R}$$

Solving this, we get

$$\boxed{h = \frac{v^2}{2g - v^2/R}}$$

From this we can see that

(i) if $v = v_e$ or $v^2 = v_e^2 = 2gR, h = \infty$

(ii) If $v < v_e$, body will attain a certain maximum height and then may either move in an orbit around the planet or may fall down back to the planet.

(iii) If $v > v_e$, body will move in interplanetary or interstellary space with velocity $\sqrt{v^2 - v_e^2}$.

Example 10.61 *A particle is projected from the surface of earth with an initial speed of 4 km/s. Find the maximum height attained by the particle. Radius of earth = 6400 km and $g = 9.8$ m/s^2.*

Sol. The maximum height attained by the particle is,

$$h = \frac{v^2}{2g - \frac{v^2}{R}}$$

Substituting the values, we have

$$h = \frac{(4\times10^3)^2}{2\times9.8 - \frac{(4.0\times10^3)^2}{6.4\times10^6}} = 9.35\times10^5\text{m} \quad \text{or} \quad h \approx 935\,\text{km}$$

Example 10.62 *A particle is projected vertically upwards from the surface of the earth (radius R_e) with a speed equal to one fourth of escape velocity. What is the maximum height attained by it from the surface of the earth?*

Sol. From conservation of mechanical energy

$$\frac{1}{2}mv^2 = \frac{GMm}{R_e} - \frac{GMm}{R}$$

where, R = maximum height (distance) from centre of the earth

Also $$v = \frac{1}{4}v_e = \frac{1}{4}\sqrt{\frac{2GM}{R_e}}$$

$$\Rightarrow \quad \frac{1}{2}m\times\frac{1}{16}\times\frac{2GM}{R_e} = \frac{GMm}{R_e} - \frac{GMm}{R}$$

$$\Rightarrow \quad R = \frac{16}{15}R_e \Rightarrow h = R - R_e = \frac{R_e}{15}$$

Example 10.63 *With what velocity must a body be thrown from earth's surface so that it may reach a maximum height of $4R_e$ above the Earth's surface? (Radius of the Earth $R_e = 6400$ km, $g = 9.8$ ms^{-2}).*

Sol. By using conservation of mechanical energy

$$\frac{1}{2}m_0v^2 - \frac{GMm_0}{R_e} = 0 - \frac{GMm_0}{(R_e + 4R_e)}$$

$$\Rightarrow \quad \frac{1}{2}m_0v^2 = -\frac{GMm_0}{5R_e} + \frac{GMm_0}{R_e}$$

$$\Rightarrow \quad \frac{1}{2}m_0v^2 = \frac{4}{5}\frac{GMm_0}{R_e}$$

$$\Rightarrow \quad v^2 = \frac{8}{5}\frac{GM}{R_e} = \frac{8}{5}\frac{gR_e^2}{R_e}$$

$$\Rightarrow \quad v^2 = \frac{8}{5}\times9.8\times6400\times10^3 = 10^8$$

$$\Rightarrow \quad v = 10\ \text{kms}^{-1}$$

Check point 10.5

1. In a gravitational field, if a body is bound with earth, then total mechanical energy is
(a) positive (b) zero
(c) negative (d) may be positive, negative or zero

2. The binding energy of an object of mass m placed on the surface of the earth r is (R = radius of earth, g = acceleration due to gravity)
(a) $mgR/2$ (b) mgR
(c) $\frac{mgR}{4}$ (d) $\frac{mgR}{8}$

3. Escape velocity on earth is 11.2 kms^{-1} what would be the escape velocity on a planet whose mass is 1000 times and radius is 10 times that of earth?
(a) 112 kms^{-1} (b) 11.2 kms^{-1}
(c) 1.12 kms^{-1} (d) 3.7 kms^{-1}

4. When escape velocity is given to a particle on surface of earth, its total energy is
(a) zero (b) greater than zero
(c) less than zero (d) $-GMm/2R$
where, M = mass of earth, m = mass of particle and R = radius of earth.

5. The escape velocity for a body of mass 1 kg from the earth surface is 11.2 kms^{-1}. The escape velocity for a body of mass 100 kg would be
(a) 11.2×10^2 kms^{-1} (b) 112 kms^{-1}
(c) 11.2 kms^{-1} (d) 11.2×10^{-2} kms^{-1}

6. The escape velocity of a body projected vertically upward from the earth's surface is 11.2 kms^{-1}. If the body is projected in a direction making 30° angle to the vertical, its escape velocity in this case will be
(a) 11.2 kms^{-1} (b) $\frac{11.2}{2}$ kms^{-1}
(c) $11.2\times\frac{\sqrt{3}}{2}$ kms^{-1} (d) $\frac{11.2}{3}$ kms^{-1}

7. The escape velocity of a particle of mass m varies as
(a) m^2 (b) m
(c) m^0 (d) m^{-1}

8. The ratio of the radius of the earth to that of the moon is 10. The ratio of the acceleration due to gravity on the earth to that on the moon is 6. The ratio of the escape velocity from the earth's surface to that from the moon is
(a) 4 (b) 6
(c) 12 (d) None of these

9. At what angle with the horizontal should a projectile be fired with the escape velocity to enable it escape from gravitational pull of the earth?
(a) Less than 45° (b) 45°
(c) More than 45° (d) Any angle

10. The velocity with which a projectile must be fired to escape from the earth does not depend upon
(a) mass of earth (b) mass of projectile
(c) radius of earth (d) None of these

11. What will be the escape speed from a planet having mass 16 times that of earth and diameter 8 times that of the earth? ($v_e = 11.2\ \text{kms}^{-1}$)
(a) $12.82\ \text{km s}^{-1}$ (b) $15.84\ \text{km s}^{-1}$
(c) $13.85\ \text{km s}^{-1}$ (d) $10.54\ \text{km s}^{-1}$

12. There are two planets and the ratio of radius of the two planets is K but ratio of acceleration due to gravity of both planets is g. What will be the ratio of their escape velocities?
(a) $(Kg)^{1/2}$ (b) $(Kg)^{-1/2}$ (c) $(Kg)^2$ (d) $(Kg)^{-2}$

13. Escape velocity from a planet is v_e. If its mass is increased to 16 times and its radius is increased to 4 times, then the new escape velocity would be
(a) v_e (b) $\sqrt{2}v_e$ (c) $2v_e$ (d) $2\sqrt{2}\ v_e$

14. Gas escapes from the surface of a planet because it acquires an escape velocity. The escape velocity will depend on which of the following factors :
I. Mass of the planet
II. Mass of the particle escaping
III. Temperature of the planet
IV. Radius of the planet
Select the correct answer from the codes given below.
(a) I and II (b) II and IV (c) I and IV (d) I, III and IV

15. The kinetic energy needed to project a body of mass m from the earth surface (radius R) to infinity is
(a) $\frac{mgR}{2}$ (b) $2mgR$ (c) mgR (d) $\frac{mgR}{2}$

16. The escape velocity from earth is v_e. A body is projected with velocity $2v_e$. With what constant velocity will it move in the inter planetary space?
(a) v_e (b) $\sqrt{2}v_e$ (c) $\sqrt{3}v_e$ (d) $\sqrt{5}\ v_e$

17. A particle is projected vertically upwards from the surface of earth (radius R) with a kinetic energy equal to half of the minimum value needed for it to escape. The height to which it rises above the surface of earth is
(a) R (b) $2R$ (c) $3R$ (d) $4R$

18. A body is projected upwards with a velocity of $3 \times 11.2\ \text{km s}^{-1}$ from the surface of earth. What will be the velocity of the body when it escapes from the gravitational pull of earth?
(a) $11.2\ \text{km s}^{-1}$ (b) $\sqrt{2} \times 11.2\ \text{km s}^{-1}$
(c) $\sqrt{3} \times 11.2\ \text{km s}^{-1}$ (d) $\sqrt{8} \times 11.2\ \text{km s}^{-1}$

19. With what velocity should a particle be projected so that its maximum height attained becomes equal to radius of earth?
(a) $\left(\frac{GM}{R}\right)^{1/2}$ (b) $\left(\frac{8GM}{R}\right)^{1/2}$
(c) $\left(\frac{2GM}{R}\right)^{1/2}$ (d) $\left(\frac{4GM}{R}\right)^{1/2}$

20. A body is projected vertically upwards from the surface of a planet of radius R with a velocity equal to 1/3rd the escape velocity for the planet. The maximum height attained by the body is
(a) $R/2$ (b) $R/3$ (c) $R/5$ (d) $R/9$

8 Motion of satellites

Just as the planets revolve around the sun, in the same way few celestial bodies revolve around these planets. These bodies are called **Satellites**. *e.g.* Moon is a natural satellite of earth. The path of these satellites are elliptical with the centre of earth at a focus.

The difference in major and minor axes is not large, so we can assume circular orbits.

Orbital speed of satellites

The speed required to put the satellite into its orbit around the earth, is known as orbital speed of satellite.

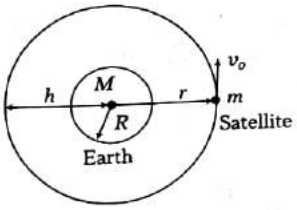


Fig. 10.37 Motion of a satellite

The necessary centripetal force to the satellite is being provided by the gravitational force exerted by the earth on the satellite. Thus,

$$\frac{mv_o^2}{r} = \frac{GMm}{r^2}$$

$$\therefore \quad \boxed{v_o = \sqrt{\frac{GM}{r}}} \quad \text{or} \quad v_o \propto \frac{1}{\sqrt{r}}$$

Here $r = R + h$, where R is radius of earth

$$\therefore \quad v_o = \sqrt{\frac{GM}{R+h}}$$

Further, we know that $GM = gR^2$

$$\therefore \quad v_o = \sqrt{\frac{gR^2}{R+h}} \Rightarrow \boxed{v_o = R\sqrt{\frac{g}{(R+h)}}}$$

Hence, the orbital speed (v_o) of the satellite decreases as the orbital radius (r) of the satellite increases. Further, the orbital speed of a satellite close to the earth's surface ($r = R + h \approx R$) is,

$$\boxed{v_o = \sqrt{\frac{GM}{R}} = \sqrt{gR}}$$

Substituting the values of g and R, we get $v_o = 7.9\ \text{kms}^{-1}$
$\approx 8\ \text{kms}^{-1}$

Note

1. Orbital velocity of a satellite is independent of mass of the satellite.
2. It decreases with increase in the radius of the orbit and with increase in the height of the satellite.
3. It depends on the mass and radius of the planet about which the satellite revolves.
4. The angular momentum of a satellite of mass m moving with velocity v_0 in an orbit of radius r $(= R + h)$ is given by $L = \sqrt{GMm^2r}$

Example 10.64 *A satellite circled around the earth at a distance of 100 km. Determine its orbital velocity, if the radius of the earth is 6400 km and $g = 9.8\ ms^{-2}$.*

Sol. Given, radius of earth, $R = 6400$ km

Distance of satellite from earth, $h = 3400$ km

$\therefore$ Orbital velocity of an artificial satellite can be given as

$$v_o = R\sqrt{\frac{g}{R+h}} = 64 \times 10^5 \sqrt{\frac{9.8}{(64+34)\times 10^5}}$$

$$\Rightarrow \quad v_o = 6400\ \text{ms}^{-1}$$

Relation between Orbital Speed and Escape Speed

The escape speed of a body from the earth's surface is

$$v_e = \sqrt{2gR}$$

The orbital speed of a satellite revolving close to earth's surface is

$$v_o = \sqrt{gR}$$

$$\therefore \quad \frac{v_e}{v_o} = \frac{\sqrt{2gR}}{\sqrt{gR}} = \sqrt{2}$$

$$\Rightarrow \quad \boxed{v_e = \sqrt{2}\, v_o} \quad \text{...(i)}$$

So, Eq. (i) represents that the escape speed of a body from the earth's surface is $\sqrt{2}$ times its orbital speed in a circular orbit just above the earth's surface.

Example 10.65 *A satellite is launched into a circular orbit close to the earth's surface. What additional velocity has new to be imparted to the satellite in the orbit to overcome the gravitational pull?*

Sol. Since, orbital velocity near earth's surface, $v_o = \sqrt{gR_e}$

Escape velocity, $v_e = \sqrt{2gR_e} = 1.414\sqrt{gR_e}$

Therefore, additional velocity required

$$= v_e - v_o = (1.414 - 1)\sqrt{gR_e}$$

$$= 0.414\sqrt{9.8 \times 6400 \times 10^3}$$

$$= 3.278 \times 10^3\ \text{ms}^{-1}$$

$$= 3.278\ \text{kms}^{-1}$$

Time period of revolution of satellite

The time taken by a satellite to complete one revolution around the earth, is known as time period of revolution of satellite.

The period of revolution (T) is given by

$$T = \frac{2\pi r}{v_o} \quad \text{or} \quad T = \frac{2\pi r}{\sqrt{\frac{GM}{r}}}$$

or $$\boxed{T = 2\pi\sqrt{\frac{r^3}{GM}}} \quad \text{or} \quad T = 2\pi\sqrt{\frac{r^3}{gR^2}}$$

or $$T = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}} \quad (\text{as } GM = gR^2 \text{ and } r = R + h)$$

Also, $$T = \frac{2\pi r}{\sqrt{\frac{GM}{r}}} = \frac{2\pi(R+h)}{v_o}$$

From this expression of T, we can make the following conclusions

(i) $T \propto r^{3/2}$ or $T^2 \propto r^3$

which is also the Kepler's third law.

(ii) Time period of a satellite very close to earth's surface $(r \approx R)$ is,

$$T = 2\pi\sqrt{\frac{R}{g}}$$

Substituting the values, we get

$$T \approx 84.6\ \text{min} \approx 1.4\ \text{h}$$

(iii) Suppose the height of a satellite is such that the time period of the satellite is 24 h and it moves in the same sense as the earth. The satellite will always be overhead a particular place on the equator. As seen from the earth, this satellite will appear to be stationary. Such a satellite is called a **geostationary satellite.** Putting $T = 24$ h in the expression of T, the radius of geostationary satellite comes out to be $r = 4.2 \times 10^4$ km. The height above the surface of earth is about 3.6×10^4 km.

Example 10.66 *A satellite revolves around the earth at a height of 1000 km. The radius of the earth is 6.38×10^3 km. Mass of the earth is 6×10^{24} kg and $G = 6.67 \times 10^{-11}$ N-m^2 kg^{-2}. Determine its orbital velocity and period of revolution.*

Sol. Given, $h = 1000\ \text{km} = 10^6$ m,

$$R = 6.38 \times 10^3\ \text{km} = 6.38 \times 10^6\ \text{m}$$

$$\Rightarrow \quad h + R = 7.38 \times 10^6\text{m}, M = 6 \times 10^{24}\ \text{kg}$$

$\therefore$ Orbital velocity, $v_0 = \sqrt{\dfrac{GM}{R+h}}$

$$= \sqrt{\frac{6.67\times10^{-11}\times6\times10^{24}}{7.38\times10^{6}}}$$

$$= 7364 \text{ ms}^{-1}$$

$\therefore$ Period of revolution, $T = \dfrac{2\pi(R+h)}{v_0} = \dfrac{2\pi\times7.38\times10^6}{7364}$

$\Rightarrow$ $T = 6297$ s

Height of satellite

As it is known that the time period of satellite,

$$T = 2\pi\sqrt{\frac{r^3}{GM}} = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}} \quad \text{...(i)}$$

By squaring on both sides of Eq. (i), we get

$$T^2 = 4\pi^2\frac{(R+h)^3}{gR^2}$$

$$\Rightarrow \quad \frac{gR^2T^2}{4\pi^2} = (R+h)^3$$

$$\Rightarrow \quad \boxed{h = \left(\frac{T^2gR^2}{4\pi^2}\right)^{1/3} - R}$$

By knowing the value of time period, the height of the satellite from the earth surface can be calculated.

Example 10.67 *A satellite constructs a circle around the earth in 90 minutes. Determine the height of the satellite above the earth's surface.*

Sol. Given, $T = 90$ min $= 5400$ s

$R = 6400$ km $= 6.4\times10^6$ m.

$\because$ Time period, $T = 2\pi\sqrt{\dfrac{(R+h)^3}{gR^2}}$

$$\Rightarrow \quad R+h = \left[\frac{gR^2T^2}{4\pi^2}\right]^{1/3}$$

$\Rightarrow$ $R + h = 6668$ km

$\Rightarrow$ $h = 6668 - R = 6668 - 6400 \Rightarrow h = 268$ km

Total energy of satellite

Assume that a satellite of mass m moving around the earth with velocity v_0 in an orbit of radius r due to gravitational pull of the earth.

The potential energy of the satellite is, $U = -\dfrac{GMm}{r}$

The kinetic energy of the satellite is

$$K = \frac{1}{2}mv_0^2 = \frac{1}{2}m\left(\frac{GM}{r}\right)$$

or $$K = \frac{1}{2}\frac{GMm}{r}$$

The total energy is, $E = K + U = -\dfrac{GMm}{2r}$

$$\Rightarrow \quad \boxed{E = -\frac{GMm}{2r}}$$

This energy is constant and negative, *i.e.*, the system is closed. The farther the satellite from the earth, the greater is its total energy.

Variation of energy with distance can be represented by graphs as shown in figure.

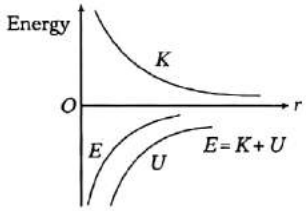


Fig. 10.38 *Variation of energy of a satellite*

Important points to be remembered

(i) The total mechanical energy of a satellite is negative if the reference level of zero potential energy is taken when separation is infinity.

(ii) For a satellite $K = -E$, $E = \dfrac{U}{2}$

and $U = -2K$

(iii) If we consider elliptical path of a satellite, *then maximum and minimum energies are shown is diagram.*

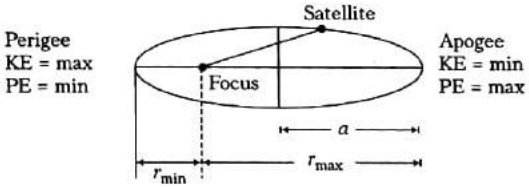


Fig. 10.39

Binding energy of a satellite

The energy required by a satellite to leave its orbit around the earth and escape to infinity, is known as binding energy of satellite.

In the last section, we have read that the total energy of the satellite is $-GMm/2r$. So, if the satellite has to leave the earth's orbit and escape to infinity, then an extra

energy equal to + $GMm/2r$ is to be supplied to it, so that its total energy becomes zero.

$$\therefore \quad \boxed{\text{Binding energy of a satellite} = \frac{GMm}{2r}}$$

Note The total energy of the satellite is negative only when it is bound to the earth. It means that its orbit is an ellipse or circle. It is positive when satellite is not bound to any star or its equivalent. In this case, the path covered by satellite will be hyperbola.

Example 10.68 *If a spaceship orbits the earth at a height of 500 km from its surface, then determine its (i) kinetic energy, (ii) potential energy, and (iii) total energy (iv) binding energy. Mass of the satellite = 300 kg, Mass of the earth = 6×10^{24} kg, radius of the earth = 6.4×10^6 m, $G = 6.67 \times 10^{-11}$ N-m^2 kg^{-2}. Will your answer alter if the earth were to shrink suddenly to half its size?*

Sol. Given, height, $h = 500$ km $= 500 \times 10^3$ m,

Mass of spaceship, $m = 300$ kg, Mass of the earth, $M = 6 \times 10^{24}$ kg

Radius of the earth, $R = 6.4 \times 10^6$ m, $G = 6.67 \times 10^{-11}$ N-m^{-2}kg^{-2}

(i) Kinetic energy $= \frac{1}{2} mv^2 = \frac{1}{2} m . \frac{GM}{R+h}$ $\left(\because v = \sqrt{\frac{GM}{R+h}}\right)$

$$= \frac{1}{2} \times \frac{300 \times 6.67 \times 10^{-11} \times 6 \times 10^{34}}{6.4 \times 10^6 + 500 \times 10^3} = 8.7 \times 10^9 \text{ J}$$

(ii) Potential energy

$$= -\frac{GMm}{(R+h)} = \frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 300}{6.4 \times 10^6 + 500 \times 10^3}$$

$$= -17.4 \times 10^9 \text{ J}$$

(iii) Total energy = KE + PE

$$= 8.7 \times 10^9 - 17.4 \times 10^9 = -8.7 \times 10^9 \text{ J}$$

(iv) Binding energy = – (Total energy) = 8.7×10^9 J

9 Artificial satellite

Those man-made objects which revolves around the earth in a fixed orbit are known as artificial satellites.

These satellites are launched from the earth. Artificial satellites are used for telecommunication, weather forecast and other applications. These satellites are of two types—Geostationary satellite and polar satellite.

Launching of an artificial satellite around the earth

Let a satellite be projected from point A with velocity v in the direction AB. For different values of v the paths are different.

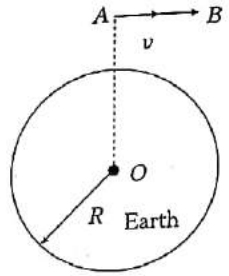


Fig. 10.40 A sattellite projected from A

Here are the possible cases

(i) If $v = 0$, path is a straight line from A to O.

(ii) If $0 < v < v_o$ path is an ellipse with centre O of the earth as a focus.

(iii) If $v = v_o$, path is a circle with O as the centre

(iv) If $v_o < v < v_e$, path is again an ellipse with O as a focus.

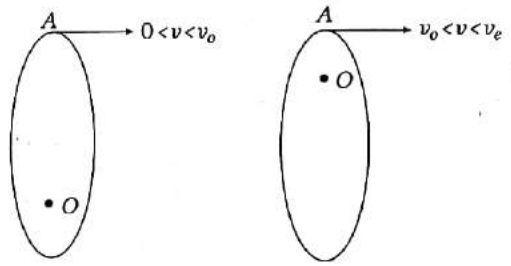


Fig. 10.41 Elliptical orbit with focus at lower side and upper side

(v) if $v = v_e$, satellite escapes from the gravitational pull of the earth and path is a parabola.

(vi) if $v > v_e$ satellite again escapes but now the path is a hyperbola.

Here, v_o = orbital speed $\left(\sqrt{\frac{GM}{r}}\right)$ at A

and v_e = escape velocity at A.

The figure given below shows the path of an artificial satellite launced from A, with different possible velocities discussed in the above cases.

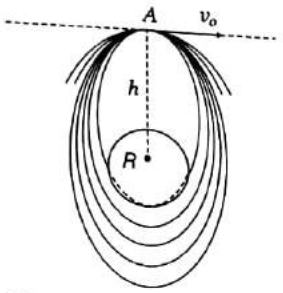


***Fig. 10.42** Path of artificial satellite*

Note

1. From case (i) to (iv) total energy of the satellite is negative. Hence, these are the closed orbits. For case (v) total energy is zero and for case (vi) total energy is positive. In these two cases orbits are open.
2. If v is not very large the elliptical orbit will intersect the earth and the satellite will fall back to earth.

Weightlessness in a Satellite

A satellite which does not produce its own gravity moves around the earth in a circular orbit under the action of gravity. The acceleration of satellite is GM/r^2 towards the centre of the earth as shown in figure.

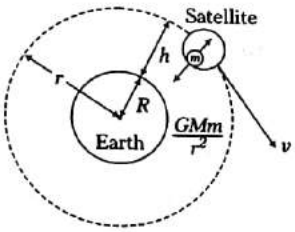


***Fig. 10.43** Weightlessness in a satellite*

If a body of mass m placed on a surface inside a satellite moving around the earth.

Then, forces on the body are

The gravitational pull of the earth $= \dfrac{GMm}{r^2}$

The reaction by the surface $= R$

By Newton's law, $\dfrac{GMm}{r^2} - R = ma$

$$\frac{GMm}{r^2} - R = m\left(\frac{GM}{r^2}\right)$$

$$\Rightarrow \quad R = 0$$

Thus, the surface does not exert any force on the body and hence, its apparent weight is zero.

Note Condition of weightlessness can be experienced only when the mass of satellite is negligible so that it does not produce its own gravity. *e.g.*, Moon is a satellite of the earth but due to its own weight, it applies gravitational force of attraction on the body placed on its surface and hence, weight of the body will not be equal to zero at the surface of the moon.

Example 10.69 *What will be the mass of an astronaut in a space satellite if at earth's surface the astronaut weighs 250 N? (Take $g = 10\ ms^{-2}$)*

Sol. In a satellite the astronaut feels weightlessness and therefore weight becomes zero but mass remains unchanged.

Therefore mass of astronaut can be calculated by using weight at the surface of the earth.

$$\therefore \quad mg = 250\,\text{N}$$

$$\Rightarrow \quad m = \frac{250}{10}$$

$$\Rightarrow \quad m = 25\,\text{kg}$$

Check point 10.6

1. The centripetal force on a satellite orbiting round the earth and the gravitational force of earth acting on the satellite both equal F. The net force on the satellite is

(a) zero (b) F (c) $F\sqrt{2}$ (d) $2F$

2. The orbital velocity of an artificial satellite in a circular orbit just above the earth's surface is v. For a satellite orbiting at an altitude of half the earth's radius the orbital velocity is

(a) $\frac{3}{2}v$ (b) $\sqrt{\frac{3}{2}}v$ (c) $\sqrt{\frac{2}{3}}v$ (d) $\frac{2}{3}v$

3. Two satellites A and B go round a planet P in circular orbits having radii $4R$ and R respectively. If the speed of the satellite A is $3v$, the speed of the satellite B will be

(a) $12v$ (b) $6v$ (c) $\frac{4}{3}v$ (d) $\frac{3}{2}v$

4. The orbital velocity of a body close to the earth's surface is

(a) $8\ \text{kms}^{-1}$
(b) $11.2\ \text{kms}^{-1}$
(c) $3 \times 10^8\ \text{ms}^{-1}$
(d) $2.2 \times 10^3\ \text{kms}^{-1}$

5. A satellite is revolving in circular orbit of radius r around the earth of mass M. Time of revolution of satellite is

(a) $T \propto \frac{r^5}{GM}$ (b) $T \propto \sqrt{\frac{r^3}{GM}}$

(c) $T \propto \sqrt{\frac{r}{\frac{GM^2}{3}}}$ (d) $T \propto \sqrt{\frac{r^3}{\frac{GM}{4}}}$

6. The period of a satellite in a circular orbit around a planet is independent of
(a) the mass of the planet
(b) the radius of the planet
(c) the mass of the satellite
(d) All the three parameters (a), (b) and (c)

7. Which of the following quantities does not depend upon the orbital radius of a satellite?
(a) $\frac{T}{R}$ (b) $\frac{T^2}{R}$ (c) $\frac{T^2}{R^2}$ (d) $\frac{T^2}{R^3}$
(where, R = radius of orbit)

8. The ratio of distance of two satellites from the centre of earth is 1:4. The ratio of their time periods of rotation will be
(a) 1:4 (b) 4:1 (c) 1:8 (d) 8:1

9. A satellite moves round the earth in a circular orbit of radius R making one revolution per day. A second satellite moving in a circular orbit,moves round the earth once in 8 days. The radius of the orbit of the second satellite is
(a) $8R$ (b) $4R$ (c) $2R$ (d) R

10. Satellite is revolving around earth. If it's radius of orbit is increased to 4 times of the radius of geostationary statellite, what will become its time period?
(a) 8 days (b) 4 days (c) 2 days (d) 16 days

11. If mean radius of earth is R, its angular velocity is ω and the acceleration due to gravity at the surface of the earth is g, then the cube of the radius of the orbit of geostationary satellite will be
(a) $\frac{R^2g}{\omega^2}$ (b) $\frac{R^2\omega^2}{g}$ (c) $\frac{R^2g}{\omega}$ (d) $\frac{Rg}{\omega^2}$

12. For a satellite orbiting very close to earth's surface, total energy is
(a) zero (b) $\frac{GMm}{R}$ (c) $-\frac{GMm}{R}$ (d) $-\frac{GMm}{2R}$

13. Two satellites A and B, ratio of masses 3 : 1 are in circular orbits of radii r and $4r$. Then ratio of total mechanical energy of A and B is
(a) 1:3 (b) 3:1 (c) 3:4 (d) 12:1

14. An artificial satellite moving in a circular orbit around the earth has total mechanical energy E_0. Its kinetic energy is
(a) $2E_0$ (b) $1.5E_0$ (c) $2E_0$ (d) E_0

15. In case of an orbiting satellite if the radius of orbit is decreased
(a) its kinetic energy decreases
(b) its potential energy increases
(c) Both (a) and (b) are correct
(d) Both (a) and (b) are wrong

16. An artificial satellite moving in a circular orbit around the earth has a total (kinetic + potential) energy E_0. Its potential energy and kinetic energy respectively are
(a) $2E_0$ and $-2E_0$ (b) $-2E_0$ and E_0
(c) $2E_0$ and $-E_0$ (d) $-2E_0$ and $-E_0$

17. Two identical satellites are orbiting at distances R and $7R$ from the surface of the earth, R being the radius of the earth. The ratio of their
(a) kinetic energies is 4 (b) potential energies is 4
(c) total energies is 4 (d) All of these

18. Two satellites P and Q ratio of masses 3 : 1 are in circular orbits of radii r and $8r$. Then ratio of total mechanical energy of A to B is
(a) 1:3 (b) 3:1
(c) 3:4 (d) 24:1

19. What is the energy required to launch a m kg satellite from earth's surface in a circular orbit at an altitude of $7R$? (R = radius of the earth)
(a) $\frac{12}{13}mgR$ (b) mgR
(c) $\frac{15}{16}mgR$ (d) $\frac{1}{9}mgR$

20. An astronaut experiences weightlessness in a space satellite. It is because
(a) the gravitational force is small at that location in space.
(b) the gravitational force is large at that location in space.
(c) the astronaut experiences no gravity
(d) the gravitational force is infinitely large at that location in space.

Chapter exercises

A) Taking it together

(Assorted questions of the chapter for advanced level practice)

1. Reason of weightlessness in a satellite is
(a) zero gravity
(b) no atmosphere
(c) zero reaction force by satellite surface
(d) None of the above

2. The acceleration due to gravity near the surface of a planet of radius R and density d is proportional to
(a) $\frac{d}{R^2}$ (b) dR^2
(c) dR (d) $\frac{d}{R}$

3. A satellite of the earth is revolving in a circular orbit with a uniform speed v. If gravitational force suddenly disappears, the satellite will
(a) continue to move with speed v along the original orbit
(b) move with the velocity v tangentially to the original orbit
(c) fall downward with increasing velocity
(d) ultimately come to rest somewhere on the original orbit

4. The weights of an object in a coal mine, at sea level, and at top of mountains are w_1, w_2 and w_3 respectively, then
(a) $w_1 < w_2 > w_3$ (b) $w_1 = w_2 = w_3$
(c) $w_1 < w_2 < w_3$ (d) $w_1 > w_2 > w_3$

5. A body is projected from earth's surface to become its satellite, its time period of revolution will not depend upon
(a) mass of earth
(b) its own mass
(c) gravitational constant
(d) radius of orbit

6. If orbital velocity of planet is given by $v = G^a M^b R^c$, then
(a) $a = \frac{1}{3}, b = \frac{1}{3}, c = -\frac{1}{3}$ (b) $a = \frac{1}{2}, b = \frac{1}{2}, c = -\frac{1}{2}$
(c) $a = \frac{1}{2}, b = -\frac{1}{2}, c = \frac{1}{2}$ (d) $a = \frac{1}{2}, b = -\frac{1}{2}, c = -\frac{1}{2}$

7. A planet is revolving round the sun in an elliptical orbit, If $\mathbf{v}$ is the velocity of the planet when its position vector from the sun is $\mathbf{r}$, then areal velocity of the planet is
(a) $|\mathbf{v} \times \mathbf{r}|$ (b) $2|\mathbf{r} \times \mathbf{v}|$
(c) $\left|\frac{1}{2}(\mathbf{r} \times \mathbf{v})\right|$ (d) None of these

8. A planet of mass m is in an elliptical orbit about the sun with an orbital period T. If A be the area of orbit, then its angular momentum would be
(a) $\frac{2mA}{T}$ (b) mAT
(c) $\frac{mA}{2T}$ (d) $2mAT$

9. The required kinetic energy of an object of mass m, so that it may escape, will be
(a) $\frac{1}{4}mgR$ (b) $\frac{1}{2}mgR$
(c) mgR (d) $2mgR$

10. What is the fractional decrease in the value of free-fall acceleration g for a particle when it is lifted from the surface to an elevation h? $(h << R)$
(a) $\frac{-h}{R}$ (b) $\frac{-2h}{R}$
(c) $\frac{-3h}{R}$ (d) $\frac{-4h}{R}$

11. The earth is an approximate sphere. If the interior contained matter which is not of the same density everywhere, then on the surface of the earth, the acceleration due to gravity **[NCERT Exemplar]**
(a) will be directed towards the centre but not the same everywhere
(b) will have the same value everywhere but not directed towards the centre
(c) will be same everywhere in magnitude directed towards the centre
(d) cannot be zero at any point

12. As observed from the earth, the sun appears to move in an approximate circular orbit. For the motion of another planet like mercury as observed from the earth, this would **[NCERT Exemplar]**

(a) be similarly true
(b) not be true because the force between the earth and mercury is not inverse square law
(c) not be true because the major gravitational force on mercury is due to the sun
(d) not be true because mercury is influenced by forces other than gravitational forces

13. Satellites orbitting the earth have finite life and sometimes debris of satellites fall to the earth. This is because **[NCERT Exemplar]**

(a) the solar cells and batteries in satellites run out
(b) the laws of gravitation predict a trajectory spiralling inwards
(c) of viscous forces causing the speed of satellite and hence height to gradually decrease
(d) of collisions with other satellites

14. Both the earth and the moon are subject to the gravitational force of the sun. As observed from the sun, the orbit of the moon **[NCERT Exemplar]**

(a) will be elliptical
(b) will not be strictly elliptical because the total gravitational force on it is not central
(c) is not elliptical but will necessarily be a closed curve
(d) deviates considerably from being elliptical due to influence of planets other than the earth

15. In our solar system, the inter-planetary region has chunks of matter (much smaller in size compared to planets) called asteroids. They **[NCERT Exemplar]**

(a) will not move around the sun, since they have very small masses compared to the sun
(b) will move in an irregular way because of their small masses and will drift away into outer space
(c) will move around the sun in closed orbits but not obey Kepler's laws
(d) will move in orbits like planets and obey Kepler's laws

16. Choose the wrong option. **[NCERT Exemplar]**

(a) Inertial mass is a measure of difficulty of accelerating a body by an external force whereas the gravitational mass is relevant in determining the gravitational force on it by an external mass
(b) That the gravitational mass and inertial mass are equal is an experimental result
(c) That the acceleration due to gravity on the earth is the same for all bodies is due to the equality of gravitational mass and inertial mass
(d) Gravitational mass of a particle like proton can depend on the presence of neighbouring heavy objects but the inertial mass cannot

17. Different points in the earth are at slightly different distances from the sun and hence experience different forces due to gravitation. For a rigid body, we know that if various forces act at various points in it, the resultant motion is as if a net force acts on the CM (centre of mass) causing translation and a net torque at the CM causing rotation around an axis through the CM. For the earth-sun system (approximating the earth as a uniform density sphere). **[NCERT Exemplar]**

(a) the torque is zero
(b) the torque causes the earth to spin
(c) the rigid body result is not applicable since the earth is not even approximately a rigid body
(d) the torque causes the earth to move around the sun

18. Two satellites of same mass are launched in the same orbit of radius r around the earth so as to rotate opposite to each other. If they collide inelastically and stick together as wreckage, the total energy of the system just after collision is

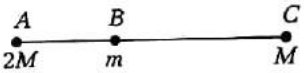
(a) $-\frac{2GMm}{r}$ (b) $-\frac{GMm}{r}$ (c) $\frac{GMm}{2r}$ (d) zero

19. Compute the additional velocity required by a satellite orbiting around earth with radius $2R$ to become free from earth's gravitational field. Mass of earth is M.

(a) $\sqrt{\frac{2GM}{R}}(\sqrt{2}-1)$ (b) $\sqrt{\frac{GM}{2R}}(\sqrt{2}-1)$

(c) $\sqrt{\frac{GM}{R}}(\sqrt{3}-1)$ (d) $\sqrt{\frac{GM}{R}}(\sqrt{2}+1)$

20. Particles of masses $2M$, m and M are respectively at points A, B and C with $AB = 1/2\,(BC)$. m is much-much smaller than M and at time $t = 0$, they are all at rest as given in figure.



At subsequent times before any collision takes place. **[NCERT Exemplar]**

(a) m will remain at rest
(b) m will move towards M
(c) m will move towards $2M$
(d) m will have oscillatory motion

21. Earth orbiting satellite will escape if

(a) its speed is increased by 41%
(b) its KE is doubled
(c) Both (a) and (b) are correct
(d) Both (a) and (b) are wrong

22. Energy required in moving a body of mass m from a distance $2R$ to $4R$ from centre of earth of mass M is

(a) $\frac{GMm}{12R^2}$ (b) $\frac{GMm}{3R^2}$ (c) $\frac{GMm}{8R}$ (d) $\frac{GMm}{4R}$

23. A body attains a height equal to the radius of the earth. The velocity of the body with which it was projected is

(a) $\sqrt{\frac{GM}{R}}$ (b) $\sqrt{\frac{2GM}{R}}$ (c) $\sqrt{\frac{1}{4}\frac{GM}{R}}$ (d) $\sqrt{\frac{GM}{2R}}$

24. Suppose the gravitational attraction varies inversely as the distance from the earth. The orbital velocity of a satellite in such a case varies as nth power of distance where n is equal to

(a) -1 (b) zero (c) $+1$ (d) $+2$

25. The rotation of the earth about its axis speeds up such that a man on the equator becomes weightless. In such a situation, what would be the duration of one day?

(a) $2\pi\sqrt{\frac{R}{g}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{R}{g}}$ (c) $2\pi\sqrt{Rg}$ (d) $\frac{1}{2\pi}\sqrt{Rg}$

26. The rotation of the earth having radius R about its axis speeds up to a value such that a man at latitude angle 60° feels weightlessness. The duration of the day in such a case is

(a) $\pi\sqrt{\frac{R}{g}}$ (b) $\frac{\pi}{2}\sqrt{\frac{R}{g}}$ (c) $\frac{\pi}{3}\sqrt{\frac{R}{g}}$ (d) $\pi\sqrt{\frac{g}{R}}$

27. Assuming the radius of the earth to be 6.4×10^6 m. What is the time period T and speed of satellite for equatorial orbit at 1.4×10^3 km above the surface of the earth.

(a) 6831 s and 7174 ms^{-1} (b) 34155 s and 3204 ms^{-1}
(c) 6831 s and 2144 ms^{-1} (d) 2431 s and 3514 ms^{-1}

28. If gravitational attraction between two points masses be given by $F = G\frac{m_1m_2}{r^n}$. Then the period of a satellite in a circular orbit will be proportional to

(a) r (b) r^2
(c) $r^{\frac{1}{2}}$ (d) independent of n

29. Suppose the gravitational force varies inversely as the nth power of distance. Then the time period of a planet in circular orbit of radius r around the sun will be proportional to

(a) $r^{\frac{1}{2}(n+1)}$ (b) $r^{\frac{1}{2}(n-1)}$ (c) r^n (d) $r^{\frac{1}{2}(n-2)}$

30. A body is projected vertically upwards from the surface of earth with a velocity equal to half the escape velocity. If R be the radius of earth, maximum height attained by the body from the surface of earth is

(a) $\frac{R}{6}$ (b) $\frac{R}{3}$ (c) $\frac{2R}{3}$ (d) R

31. A particle would take a time t to move down a straight tube from the surface of earth (supposed to be a homogeneous sphere) to its centre. If gravity were to remain constant, then the time would be t. The ratio of t/t' will be

(a) $\frac{\pi}{2\sqrt{2}}$ (b) $\frac{\pi}{\sqrt{2}}$ (c) $\sqrt{2}\pi$ (d) $\frac{\pi}{\sqrt{2}}$

32. An earth satellite of mass m revolves in a circular orbit at a height h from the surface of the earth. R is the radius of the earth and g is acceleration due to gravity at the surface of the earth. The velocity of the satellite in the orbit is given by

(a) $\frac{gR^2}{R+h}$ (b) gR (c) $\frac{gR}{R+h}$ (d) $\sqrt{\frac{gR^2}{R+h}}$

33. A body which is initially at rest at a height R above the surface of the earth of radius R, falls freely towards the earth, then its velocity on reaching the surface of the earth is

(a) $\sqrt{(2gR)}$ (b) $\sqrt{(gR)}$ (c) $\sqrt{\frac{3}{2}gR}$ (d) $\sqrt{(4gR)}$

34. A planet of mass m moves around the sun of mass M in an elliptical orbit. The maximum and minimum distance of the planet from the sun are r_1 and r_2 respectively. The time period of the planet is proportional to

(a) $r_1^{3/2}$ (b) $(r_1+r_2)^{3/2}$
(c) $(r_1-r_2)^{3/2}$ (d) $r_1^{3/2}$

35. A rocket is launched vertical from the surface of the earth of radius R with an initial speed v. If atmospheric resistance is neglected, then maximum height attained by the rocket is

(a) $h=\frac{R}{\left(\frac{2gR}{v^2}-1\right)}$ (b) $h=\frac{R}{\left(\frac{2gR}{v^2}+1\right)}$

(c) $h=\frac{R^2}{\left(\frac{2gR}{v^2}-1\right)}$ (d) $h=\frac{R^2}{\left(\frac{2gR}{v^2}+1\right)}$

36. Two particles of equal mass m go round a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is

(a) $v=\sqrt{\frac{Gm}{R}}$ (b) $v=\sqrt{\frac{Gm}{2R}}$ (c) $v=\frac{1}{2}\sqrt{\frac{Gm}{R}}$ (d) $v=\sqrt{\frac{4Gm}{R}}$

37. Suppose a smooth tunnel is dug along a straight line joining two points on the surface of the earth and a particle is dropped from rest at its one end. Assume that mass of earth is uniformly distributed over its volume. Then

(a) the particle will emerge from the other end with velocity $\sqrt{\frac{GM_e}{2R_e}}$, where M_e and R_e are earth's mass and radius respectively
(b) the overlapping will come to rest at centre of the tunnel
(c) potential energy of the particle will be equal to zero at centre of tunnel if it is along a diameter
(d) acceleration of the particle will be proportional to its distance from mid-point of the tunnel.

38. A satellite is moving in a circular orbit round the earth with a diameter of orbit $2R$. At a certain point a rocket fixed to the satellite is fired such that it

increases ti velocity of the satellite tangentially. The resulting orbit of the satellite would be

(a) same as bef
(b) circular orbi th diameter greater than $2R$.
(c) elliptical orbi th minimum distance from the earth equal to R.
(d) elliptical orbit h maximum distance from the earth equal to R.

39. If the mass of moon is $\frac{M}{81}$, where M is the mass of earth, find the distance of the point where gravitational field due to earth and moon cancel each other, from the centre of moon. Given that distance between centres of earth and moon is 60 R where R is the radius of earth

(a) $4R$ (b) $8R$
(c) $12R$ (d) $6R$

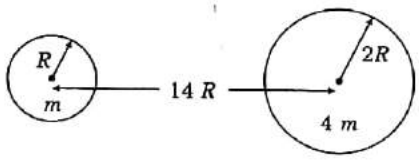
40. What is the energy required to launch a m kg satellite from earth's surface in a circular orbit at an altitude of $2R$? (R = radius of the earth)

(a) $\frac{2}{3}mgR$ (b) mgR (c) $\frac{5}{6}mgR$ (d) $\frac{1}{3}mgR$

41. The orbital angular momentum of a satellite revolving at a distance r form the centre is L. If the distance is increased to $16r$, then the new angular momentum will be

(a) $16L$ (b) $64L$
(c) $\frac{L}{4}$ (d) $4L$

42. Two spherical bodies of masses m and $4m$ and radii R and $2R$ respectively are released in free space with initial separation between their centres equal to15 R. If they attract each other due to gravitational force only, then the distance covered by smaller sphere just before collision will be



(a) $5R$ (b) $9.6R$
(c) $2.5R$ (d) $6R$

43. Two spheres of masses m and $2m$ are separated by distance d. A particle of mass $\frac{m}{5}$ is projected straight from $2m$ towards m with a velocity v_0. Which of the following statements is correct?

(a) Velocity of the particle decreases constantly
(b) Velocity of the particle increase constantly
(c) Acceleration of the particle may become momentarily zero
(d) The particle never retraces its path

44. A ring of mass m_1 and radius R is fixed in space at some location. An external agent brings a point mass m_2 from infinity to centre of the ring .Work done by the external agent will be

(a) $-\frac{Gm_1m_2}{R}$ (b) $\frac{Gm_1m_2}{R}$
(c) $\frac{G\sqrt{m_1^2+m_2^2}}{R}$ (d) $\frac{Gm_1m_2}{R(m_1^2+m_2^2)}$

45. If an artificial satellite is moving in a circular orbit around the earth with a speed equal to half the magnitude of the escape velocity from the earth, the height of the satellite above the surface of the earth is

(a) $2R$ (b) $\frac{R}{2}$ (c) R (d) $\frac{R}{4}$

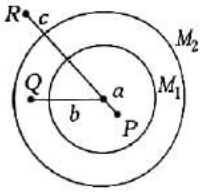
46. A person brings a mass of 1 kg infinity to a point A. Initially the mass was at rest but it moves at a speed of 2 ms^{-1} as it reached A. The work done by the person on the mass is -3 J . The potential at A is

(a) $-3\ J\ kg^{-1}$ (b) $-12\ J\ kg^{-1}$
(c) $-5\ J\ kg^{-1}$ (d) None of these

47. What impulse need to be given to a body of mass m, released from the surface of earth along a straight tunnel passsing through centre of earth, at the centre of earth, to bring it to rest. (Mass of earth M, radius of earth R)

(a) $m\sqrt{\frac{GM}{R}}$ (b) $\sqrt{\frac{GMm}{R}}$ (c) $m\sqrt{\frac{GM}{2R}}$ (d) Zero

48. Two concentric shells of masses M_1 and M_2 are concentric as shown in the figure. Which of the following is correct regarding the gravitational force on m due to M_1 and M_2 at points P, Q and R.



(a) $F_P = 0$ (b) $F_Q = \frac{GM_1m}{b^2}$
(c) $F_R = \frac{G(M_1+M_2)}{c^2}$ (d) All are correct

49. Four equal masses (each of mass M) are placed at the corners of a square of side a. The escape velocity of a body from the centre O of the square is

(a) $4\sqrt{\frac{2GM}{a}}$ (b) $\sqrt{\frac{8\sqrt{2}GM}{a}}$
(c) $\frac{4GM}{a}$ (d) $\sqrt{\frac{4\sqrt{2}GM}{a}}$

50. Energy of a satellite in circular orbit is E_0. The energy required to move the satellite to a circular orbit of 3 times the radius of the initial orbit is

(a) $\frac{2}{3}E_0$ (b) $2E_0$ (c) $\frac{E_0}{3}$ (d) $\frac{3}{2}E_0$

51. Pertaining to two planets, the ratio of escape velocities from respective surfaces is 1:2, the ratio of the time period of the same simple pendulum at their respective surfaces is 2:1 (in same order). Then the ratio of their average densities is

(a) 1:1 (b) 1:2 (c) 1:4 (d) 8:1

52. An object is released from a height twice the radius of the earth on the surface of earth. Find the speed with which it will collide with group by neglecting effect of air. (Take, R radius of earth and mass of earth as M).

(a) $2\sqrt{\frac{GM}{3R}}$ (b) $3\sqrt{\frac{GM}{2R}}$ (c) $2\sqrt{\frac{GM}{R}}$ (d) $3\sqrt{\frac{GM}{R}}$

53. A planet of mass m revolves in an elliptical orbit around the sun so that its maximum and minimum distance from the sun are equal to r_a and r_p respectively. The angular momentum of this planet relative to the sun is

(a) $m\sqrt{\frac{2GMr_pr_a}{(r_p+r_a)}}$ (b) $m\sqrt{\frac{4GMr_pr_a}{(r_p+r_a)}}$

(c) $m\sqrt{\frac{GMr_pr_a}{(r_p+r_a)}}$ (d) $m\sqrt{\frac{GMr_pr_a}{2(r_p+r_a)}}$

54. The magnitudes of the gravitational force at distances r_1 and r_2 from the centre of a uniform sphere of radius R and mass M are F_1 and F_2 respectively. Then (more than one are correct)

(a) $\frac{F_1}{F_2}=\frac{r_1}{r_2}$; if $r_1<R$ and $r_2<R$

(b) $\frac{F_1}{F_2}=\frac{r_1^2}{r_2^2}$; if $r_1>R$ and $r_2>R$

(c) $\frac{F_1}{F_2}=\frac{r_2}{r_1}$; if $r_1<R$ and $r_2<R$

(d) $\frac{F_1}{F_2}=\frac{r_2^2}{r_1^2}$; if $r_1>R$ and $r_2<R$

55. Two particles of mass m and M are initially at rest at infinite distance. Find their relative velocity of approach due to gravitational attraction when d is their separation at any instant

(a) $\sqrt{\frac{2G(M+m)}{d}}$ (b) $\sqrt{\frac{G(M+m)}{d}}$

(c) $\sqrt{\frac{G(M+m)}{2d}}$ (d) $\sqrt{\frac{G(M+m)}{4d}}$

56. The ratio of energy required to raise a satellite to a height h above the earth surface to that required to put it into the orbit is

(a) $h:2R$ (b) $2h:R$ (c) $R:h$ (d) $h:R$

57. A body of supercondense material with mass twice the mass of earth but size very small compared to the size of earth starts from rest from $h << R$ above the earth's surface. It reaches earth in time

(a) $t=\sqrt{\frac{h}{g}}$ (b) $t=\sqrt{\frac{2h}{g}}$

(c) $t=\sqrt{\frac{2h}{3g}}$ (d) $t=\sqrt{\frac{4h}{3g}}$

58. Let E be the energy required to raise a satellite to height h above earth's surface and E' be the energy required to put the same satellite into orbit at that height. Then E/E' is equal to

(a) $\frac{2h}{(R+2h)}$ (b) $\frac{2h}{(2R+3h)}$

(c) $\frac{R}{R+h}$ (d) $\frac{2R}{2h+R}$

59. A satellite is revolving round the earth with orbital speed v_o. If it stops suddenly, the speed with which it will strike the surface of earth would be (v_e = escape velocity of a particle on earth's surface.)

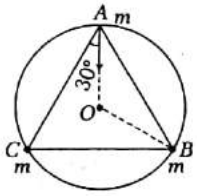
(a) $\frac{v_e^2}{v_o}$ (b) $2v_o$ (c) $\sqrt{v_e^2-v_o^2}$ (d) $\sqrt{v_e^2-2v_o^2}$

60. Four particles, each of mass M, move along a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is

(a) $\frac{GM}{R}$ (b) $\sqrt{2\sqrt{2}\frac{GM}{R}}$

(c) $\sqrt{\frac{GM}{R}(2\sqrt{2}+1)}$ (d) $\sqrt{\frac{GM}{R}\left(\frac{2\sqrt{2}+1}{4}\right)}$

61. Three particles each of mass m, are located at the vertices of an equilateral triangle of side a. At what speed must they move if they all revolve under the influence of their gravitational force of attraction in a circular orbit circumscribing the triangle while still preserving the equilateral triangle?



(a) $\sqrt{\frac{Gm}{a}}$ (b) $\sqrt{\frac{2Gm}{a}}$ (c) $\sqrt{\frac{3Gm}{a}}$ (d) $\sqrt{\frac{4Gm}{a}}$

62. Three point masses each of mass m rotate in a circle of radius r with constant angular velocity ω due to their mutual gravitational attraction. If at any instant, the masses are on the vertex of an equilateral triangle of side a, then the value of ω is

(a) $\sqrt{\frac{Gm}{a^3}}$ (b) $\sqrt{\frac{3Gm}{a^3}}$

(c) $\sqrt{\frac{Gm}{3a^3}}$ (d) None of these

63. Two identical thin rings each of radius R are coaxially placed at a distance R. If the rings have a uniform mass distribution and each has masses $2m$ and $4m$ respectively, then the work done in moving a mass m from centre of one ring to that of the other is

(a) zero (b) $\frac{\sqrt{2}Gm^2}{R}(1-\sqrt{2})$

(c) $\frac{Gm^2}{\sqrt{2}R}(\sqrt{2}-1)$ (d) $\frac{Gm^2}{\sqrt{2}R}$

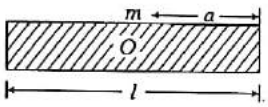
64. A solid sphere of mass M and radius R has a spherical cavity of radius $R/2$ such that the centre of cavity is at a distance $R/2$ from the centre of the sphere. A point mass m is placed inside the cavity at a distance $R/4$ from the centre of sphere. The gravitational force on mass m is

(a) $\frac{11GMm}{R^2}$ (b) $\frac{14GMm}{R^2}$ (c) $\frac{GMm}{2R^2}$ (d) $\frac{GMm}{R^2}$

65. A point $P(R\sqrt{3},0,0)$ lies on the axis of a ring of mass M and radius R. The ring is located in yz-plane with its centre at origin O. A small particle of mass m starts from P and reaches O under gravitational attraction only. Its speed at O will be

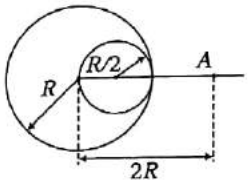
(a) $\sqrt{\frac{GM}{R}}$ (b) $\sqrt{\frac{Gm}{R}}$ (c) $\sqrt{\frac{GM}{2R}}$ (d) $\sqrt{\frac{Gm}{\sqrt{2}R}}$

66. A mass m is at a distance a from center of a uniform rod of length l and M. The gravitational force on the mass due to the rod is



(a) $\frac{4GMm}{(a+l)^2}$ (b) $\frac{4GmM}{4a^2-l^2}$ (c) $\frac{GMm}{a^2}$ (d) $\frac{GmM}{2(l+a)^2}$

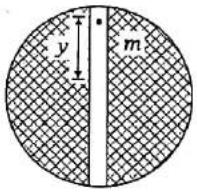
67. A solid sphere of uniform density and radius R applies a gravitational force of attraction equal to F_1 on a particle placed at a distance $2R$ from the centre of the sphere. A spherical cavity of radius $R/2$ is now made in the sphere as shown in the figure. The sphere with cavity now applies a gravitational force F_2 on the same particle. The ratio $\frac{F_2}{F_1}$ is



(a) $\frac{5}{9}$ (b) $\frac{7}{8}$

(c) $\frac{3}{4}$ (d) $\frac{7}{9}$

68. Suppose a vertical tunnel is along the diameter of earth, assumed to be a sphere of uniform mass density ρ. If a body of mass m is thrown in this tunnel, its acceleration at a distance y from the centre is given by



(a) $\frac{4\pi}{3}G\rho ym$ (b) $\frac{3}{4}\pi\rho y$

(c) $\frac{4}{3}\pi\rho y$ (d) $\frac{4}{3}\pi G\rho y$

B) Medical entrance special format questions

Assertion and reason

Directions (Q. Nos. 1-20) *These questions consists of two statements each printed as Assertion and Reason. While answering these questions you are required to choose any one of the following five responses*

(a) If both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If Assertion is false but Reason is true.

1. Assertion Gravitational force between two masses in air is F. If they are immersed in water, force will remain F.

Reason Gravitational force does not depend on the medium between the masses.

2. Assertion Angular momentum of a planet is constant about any point.

Reason Force acting on the planet is a central force.

3. Assertion Kepler's laws for planetary motion are consequence of Newton's laws.

Reason Kepler's laws can be derived by using Newton's laws.

4. **Assertion** The field strength at the centre of a ring is zero.

Reason At the centre of the ring, slope of V-r graph is zero.

5. **Assertion** The binding energy of a satellite does not depend upon the mass of the satellite.

Reason Binding energy is the negative value of total energy of satellite.

6. **Assertion** Plane of a satellite always passes through the centre of earth.

Reason Gravitational force on satellite is always towards centre of earth.

7. **Assertion** Gravitational potential and gravitational potential energy, both are related to the work done by gravitational force in the gravitational field.

Reason Gravitational field strength is related to the gravitational force in gravitational field.

8. **Assertion** On two sides of a point mass, gravitational field strength is same at same distance.

Reason As we move away from a point mass, value of gravitational potential decreases.

9. **Assertion** If gravitational potential at some point is zero, then gravitational field strength at that point will also be zero.

Reason Except at infinity gravitational potential due to a system of point masses at some finite distance can't be zero.

10. **Assertion** If radius of earth suddenly shrinks to half its present without changing its mass value, then the period of an earth's satellite will not change.

Reason Time period of a satellite does not upon the mass of earth.

11. **Assertion** The centres of two cubes of masses m_1 and m_2 are separated by a distance r. The gravitational force between these two cubes will be $\frac{Gm_1m_2}{r^2}$.

Reason According to Newton's law of gravitation, gravitational force between two point masses m_1 and m_2 separated by a distance r is $\frac{Gm_1m_2}{r^2}$.

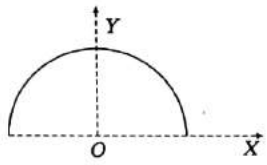
12. **Assertion** If a particle is projected from the surface of earth with velocity equal to escape velocity, then total mechanical energy is zero.

Reason Total mechanical energy of any closed system is always negative.

13. **Assertion** On earth's satellite, we feel weightlessness. Moon is also satellite of earth. But on the surface of moon, we do not feel weightlessness.

Reason Gravitational force by earth on us on the surface of moon is zero. But gravitational force by moon on us on its surface is non-zero.

14. **Assertion** The centre of semicircular ring of mass m and radius R is the origin O. The potential at origin is $-\frac{Gm}{R}$.



Reason The gravitational field strength is towards Y-axis.

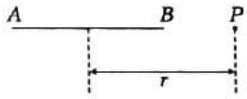
15. **Assertion** If the product of surface area and density is same for two planets, escape velocities will be same for both.

Reason Product of surface area and density is proportional to the mass of the planet per unit radius of the planet.

16. **Assertion** Areal velocity of a planet around sun will become two times if mass of planet is halved.

Reason Areal velocity $= \frac{L}{2m}$. Where L is angular momentum of planet about centre of sun.

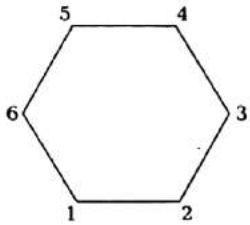
17. **Assertion** Mass of the rod AB is m_1 and of particle P is m_2. Distance between centre of rod and particle is r. Then the gravitational force between the rod and the particle is



$$F = \frac{Gm_1m_2}{r^2}$$

Reason The relation $F = \frac{Gm_1m_2}{r^2}$ can be applied directly only to find force between two particles.

18. **Assertion** Four point masses each of mass m are placed at points 1, 2, 3 and 6 of a regular hexagon of side a. Then the gravitational field at the centre of hexagon is $\frac{Gm}{a^2}$.

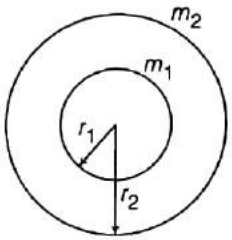


Reason The field strength due to masses at 3 and 6 are cancelled out.

19. **Assertion** Let W_1 be the work done in taking away a satellite from the surface of earth to its orbit and then W_2 the work done in rotating the satellite in circular orbit there. Then, $W_1 = W_2$

Reason $W_2 = \frac{GMm}{4R}$

20. Assertion Two spherical shells have masses m_1 and m_2. Their radii are r_1 and r_2. Let r be the distance of a point from centre. Then gravitational field strength and gravitational potential both are equal to zero for $0 < r < r_1$



Reason In the region $r_1 < r < r_2$, gravitational field strength due to m_2 is zero. But gravitational potential due to m_2 is constant (but non-zero).

Match the columns

1. Match the following columns.

Column I	Column II
(A) Kepler's first law	(p) $T^2 \propto r^3$
(B) Kepler's second law	(q) Areal velocity is constant
(C) Kepler's third law	(r) Orbit of planet is elliptical

2. Density of a planet is two times the density of earth. Radius of this planet is half. Match the followings (as compared to earth)

Column I	Column II
(A) Acceleration due to gravity on this planet's surface	(p) Half
(B) Gravitational potential on the surface	(q) Same
(C) Gravitational potential at centre	(r) Two times
(D) Gravitational field strength at centre	(s) Four times

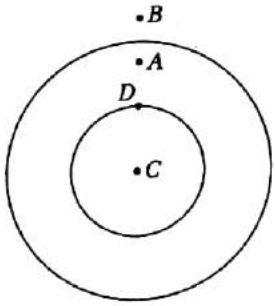
3. On the surface of earth acceleration due gravity is g and gravitational potential is V. Match the following columns.

Column I	Column II
(A) At height $h = R$, value of g	(p) Decreases by a factor $\frac{1}{4}$
(B) At depth $h = \frac{R}{2}$, value of g	(q) Decrease by a factor $\frac{1}{2}$
(C) At height $h = R$, value of V	(r) Increase by a factor $\frac{11}{8}$
(D) At depth $h = \frac{R}{2}$, value of V	(s) Increases by a factor 2
	(t) None

4. Let V and E denote the gravitational potential and gravitational field at a point. Then, match the following columns.

Column I	Column II
(A) $E = 0, V = 0$	(p) At centre of spherical shell
(B) $E \neq 0, V = 0$	(q) At centre of solid sphere
(C) $V \neq 0, E = 0$	(r) At centre of circular ring
(D) $V \neq 0, E \neq 0$	(s) At centre of two point masses of equal magnitude
	(t) None

5. Two concentric spherical shells are as shown in figure. Match the following columns.



Column I	Column II
(A) Potential at A	(p) greater than B
(B) Gravitational field at A	(q) less than B
(C) As one moves from C to D	(r) Potential remains constant
(D) As one moves from D to A	(s) Gravitational field decreases
	(t) None

6. Match the following columns.

Column I	Column II
(A) Kinetic energy of a particle in gravitational field is increasing	(p) Work done by gravitational force should be positive
(B) Potential energy of a particle in gravitational field is increasing	(q) Work done by external force should be non-zero
(C) Mechanical energy of a particle in gravitational field is increasing	(r) Work done by gravitational force should be negative
	(s) Cannot say anything

7. A particle is projected from the surface of earth of mass M and radius R with speed v. Suppose it travels a distance $x(<< R)$ when its speed becomes v to $v/2$ and $y(<< R)$ when speed changes from $v/2$ to 0. Similarly, the corresponding times are suppose t_1 and t_2. Then

	Column I		Column II
(A)	x/y	(p)	$= 1$
(B)	t_1/t_2	(q)	> 1
		(r)	< 1

8. In elliptical orbit of a planet, as the planet move from apogee position to perigee position, match the following columns.

	Column I		Column II
(A)	Speed of planet	(p)	Remains same
(B)	Distance of planet from centre of sun	(q)	Decreases
(C)	Potential energy	(r)	Increases
(D)	Angular momentum about centre of sun	(s)	Cannot say

9. Match the following columns.

	Column I		Column II
(A)	Time period of an earth satellite in circular orbit	(p)	Independent of mass of satellite
(B)	Orbital velocity of satellite	(q)	Independent of radius of orbit
(C)	Mechanical energy of satellite	(r)	Independent of mass of earth
		(s)	None

10. If earth decreases its rotational speed. Match the following columns.

	Column I		Column II
(A)	Value of g at pole	(p)	Will remain same
(B)	Value of g at equator	(q)	Will increase
(C)	Distance of geostationary satellite	(r)	Will decrease
(D)	Energy of geostationary satellite	(s)	Cannot say

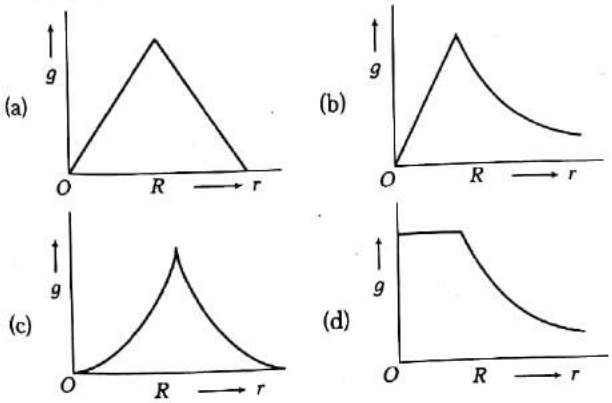
11. Match the following columns. (for a satellite in circular orbit)

	Column I		Column II
(A)	Kinetic energy	(p)	$-\frac{GMm}{2r}$
(B)	Potential energy	(q)	$\sqrt{\frac{GM}{r}}$
(C)	Total energy	(r)	$-\frac{GMm}{r}$
(D)	Orbital speed	(s)	$\frac{GMm}{2r}$

Medical entrances' gallery

(Collection of questions asked in NEET & various medical entrance exams)

1. Starting from the centre of the earth having radius R, the variation of g (acceleration due to gravity) is shown by **[NEET 2016]**



2. A satellite of mass m is orbiting the earth (of radius R) at a height h from its surface. The total energy of the satellite in terms of g_0, the value of acceleration due to gravity at the earth's surface is **[NEET 2016]**

(a) $\frac{mg_0R^2}{2(R+h)}$ (b) $-\frac{mg_0R^2}{2(R+h)}$

(c) $\frac{2mg_0R^2}{R+h}$ (d) $-\frac{2mg_0R^2}{R+h}$

3. At what height from the surface of earth, the gravitation potential and the value of g are -5.4×10^7 J kg^{-2} and 6 ms^{-2} respectively? Take, the radius of earth is 6400 km. **[NEET 2016]**

(a) 1600 km
(b) 1400 km
(c) 2000 km
(d) 2600 km

4. The ratio of escape velocity at earth (v_e) to the escape velocity at a planet (v_p) whose radius and mean density are twice as that of earth is **[NEET 2016]**

(a) $1:2\sqrt{2}$ (b) $1:4$

(c) $1:\sqrt{2}$ (d) $1:2$

5. Kepler's third law states that square of period of revolution (T) of a planet around the sun, is proportional to third power of average distance r between the sun and planet i.e. $T^2 = Kr^3$, here K is constant. If the masses of the sun and planet are M and m respectively, then as per Newton's law of gravitational force of attraction between them is $F = \frac{GMm}{r^2}$, here G is gravitational constant. The relation between G and K is described as **[CBSE AIPMT 2015]**

(a) $GK = 4\pi^2$ (b) $GMK = 4\pi^2$
(c) $K = G$ (d) $K = I/G$

6. The reading of a spring balance corresponds to 100 N while situated at the north pole and a body is kept on it. The weight record on the same scale if it is shifted to the equator, is (take, $g = 10\ \text{ms}^{-2}$ and radius of the earth, $R = 6.4 \times 10^6$ m) **[AIIMS 2015]**

(a) 99.66 N (b) 110 N
(c) 97.66 N (d) 106 N

7. The gravitational field due to an uniform solid sphere of mass M and radius a at the centre of the sphere is **[UK PMT 2015]**

(a) $\frac{GM}{a^2}$ (b) $\frac{GM}{a^3}$ (c) $\frac{GM}{2a}$ (d) zero

8. What would be the value of acceleration due to gravity at a point 5 km below the earth's surface? ($R_E = 6400$ km, $g_E = 9.8\ \text{ms}^{-2}$) **[UK PMT 2015]**

(a) $9.6\ \text{ms}^{-2}$ (b) $9.79\ \text{ms}^{-2}$
(c) $9.89\ \text{ms}^{-2}$ (d) $10\ \text{ms}^{-2}$

9. Two particles of equal masses go round a circle of radius R under the action of their mutual gravitational attraction. What would be the speed of each particle? **[UK PMT 2015]**

(a) $\sqrt{\frac{GM}{2R}}$ (b) $\sqrt{\frac{GM}{R}}$
(c) $\sqrt{\frac{GM}{4R}}$ (d) $\sqrt{\frac{2GM}{R}}$

10. What would be the escape velocity from the moon, it the mass of the moon is 7.4×10^{22} kg and its radius is 1740 km? **[UK PMT 2015]**

(a) $2.4\ \text{ms}^{-1}$ (b) $2.4\ \text{kms}^{-1}$
(c) $240\ \text{kms}^{-1}$ (d) $0.24\ \text{kms}^{-1}$

11. Two spheres of masses 16 kg and 4 kg are separated by a distance 30 m on a table. Then, the distance from sphere of mass 16 kg at which the net gravitational force becomes zero is **[EAMCET 2015]**

(a) 10 m (b) 20 m
(c) 15 m (d) 5 m

12. Orbital velocity of earth satellite does not depend on **[Kerala CEE 2015]**

(a) mass of the earth
(b) mass of the satellite
(c) radius of the earth
(d) acceleration due to gravity
(e) its height from the surface of earth

13. Gravitational potential energy of a body of mass m at a height of h above the surface of earth (M = mass of earth; R = radius of earth) is **[Kerala CEE 2015]**

(a) $\frac{GMm}{h}$ (b) $\frac{GMm}{(R+h)}$ (c) $\frac{-GM}{(R+h)}$ (d) $\frac{Gm}{(R+h)}$
(e) $-\frac{GMm}{(R+h)}$

14. According to Kepler's law of planetary motion, if T represents time period and r is orbital radius, then for two planets these are related as **[Manipal 2015]**

(a) $\left(\frac{T_1}{T_2}\right)^{3/2} = \frac{r_1}{r_2}$ (b) $\frac{T_1}{T_2} = \left[\frac{r_1}{r_2}\right]$
(c) $\left[\frac{T_1}{T_2}\right]^2 = \left[\frac{r_1}{r_2}\right]^3$ (d) $\frac{T_1^2}{T_2} = \frac{r_1^3}{r_1}$

15. If the mass of a body is M on the surface of the earth, then mass of the same body on the surface of the moon is **[KCET 2015]**

(a) $6M$ (b) $\frac{M}{6}$
(c) zero (d) M

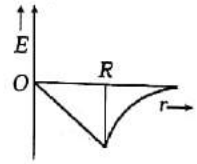
16. Two spherical bodies of masses M and $5M$ and radii R and $2R$ are released in free space with initial separation between their centres equal to $12R$. If they attract each other due to gravitational force only, then the distance covered by the smaller body before collision is **[CBSE AIPMT 2015]**

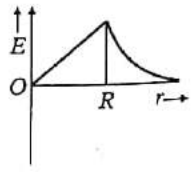
(a) $2.5R$ (b) $4.5R$ (c) $7.5R$ (d) $1.5R$

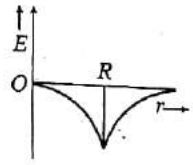
17. The force of gravitation is **[UK PMT 2014]**

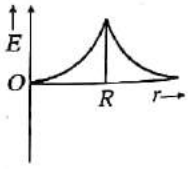
(a) repulsive (b) electrostatic
(c) conservative (d) non-conservative

18. Dependence of intensity of gravitational field (E) of earth with distance (r) from centre of earth is correctly represented by **[CBSE AIPMT 2014]**

(a) 

(b) 

(c) 

(d) 

19. Keeping the mass of the earth as constant, if its radius is reduced to 1/4 th of its initial value, then the period of revolution of the earth about its own axis and passing through the centre, (in hours) is (assume the earth to be a solid sphere and its initial period of rotation as 24 h) **[EAMCET 2014]**

(a) 12 (b) 3 (c) 6 (d) 1.5

20. A body of mass m is raised to a height $10R$ from the surface of the earth, where R is the radius of the earth. The increase in potential energy is (G = universal constant of gravitation, M = mass of the earth and g = acceleration due to gravity) **[MHT CET 2014]**

(a) $\frac{GMm}{11R}$ (b) $\frac{GMm}{10R}$ (c) $\frac{mgR}{11G}$ (d) $\frac{10GMm}{11R}$

21. An artificial satellite moves in a circular orbit around the earth. Total energy of the satellite is given by E. The potential energy of the satellite is **[WB JEE 2014]**

(a) $-2E$ (b) $2E$ (c) $\frac{2E}{3}$ (d) $-\frac{2E}{3}$

22. What is a period of revolution of the earth satellite? Ignore the height of satellite above the surface of the earth.

Given,

(i) the value of gravitational acceleration, $g = 10\ ms^{-2}$.

(ii) radius of the earth, R_g = 6400 km. (take, $\pi = 3.14$) **[KCET 2014]**

(a) 85 min (b) 156 min

(c) 83.73 min (d) 90 min

23. The time period of the earth's satellite revolving at a height of 35800 km is **[Kerala CEE 2014]**

(a) 24 h (b) 100 min (c) 12 h (d) 48 h

(e) 52 h

24. At a height H from the surface of earth, the total energy of a satellite is equal to the potential energy of a body of equal mass at a height $3R$ from the surface of the earth(R = radius of the earth). The value of H is **[EAMCET 2014]**

(a) R (b) $\frac{4R}{3}$ (c) $3R$ (d) $\frac{R}{3}$

25. A body of mass m taken from the earth's surface to the height equal to twice the radius (R) of the earth. The change is potential energy of body will be **[NEET 2013]**

(a) $mg2R$ (b) $\frac{2}{3}mgR$ (c) $3mgR$ (d) $\frac{1}{3}mgR$

26. Infinite number of bodies, each of mass 2 kg are situated on x-axis at distance 1m, 2 m, 4 m, 8 m, respectively from the origin. The resulting gravitational potential due to this system at the origin will be **[NEET 2013]**

(a) $-G$ (b) $-(8/3)G$ (c) $-(4/3)G$ (d) $-4G$

27. The universal law of gravitational is the force law known also as the **[J&K CET 2013]**

(a) triangular law (b) square law

(c) inverse square law (d) parallelogram law

28. The value of acceleration due to gravity at the surface of earth **[J&K CET 2013]**

(a) is maximum at the poles

(b) is maximum at the equator

(c) remains constant everywhere on the surface of the earth

(d) is maximum at the international time line

29. The escape velocity of a particle from the surface of the earth is given by **[J&K CET 2013]**

(a) $(gR)^{1/2}$ (b) $(2gR)^{1/2}$

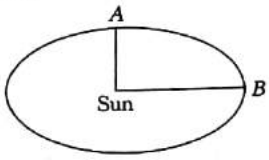
(c) $(3gR)^{1/2}$ (d) $(gR/2)^{1/2}$

30. If earth were to rotate on its own axis such that the weight of a person at the equator becomes half the weight at the poles, then its time period of rotation is (g = acceleration due to gravity near the poles and R is the radius of earth) (Ignore equatorial bulge) **[EAMET 2013]**

(a) $2\pi\sqrt{\frac{2R}{g}}$ (b) $2\pi\sqrt{\frac{R}{2g}}$

(c) $2\pi\sqrt{\frac{R}{3g}}$ (d) $2\pi\sqrt{\frac{R}{g}}$

31. Earth is moving around the sun in elliptical orbit as shown. The ratio of OB and OA and R. Then the ratio of earth's is velocities at A and B is **[KCET 2013]**



(a) R^{-1} (b) $\sqrt{R}$ (c) R (d) $R^{2/3}$

32. If two planets of radii R_1 and R_2 have densities d_1 and d_2, then the ratio of their respective acceleration due to gravity is **[Kerala CEE 2013]**

(a) $R_1d_1 : R_2d_2$ (b) $R_1^2d_1 : R_2^2d_2$

(c) $R_1^3d_1 : R_2^3d_2$ (d) $R_1d_1^2 : R_2^2d_2^2$

(e) $R_1d_1^3 : R_2^2d_2^3$

33. The weight of an object is 90 kg at the surface of the earth. If it is taken to a height equal to half of the radius of the earth, then its weight will beocme **[MPPET 2013]**

(a) 135 kg (b) 45 kg (c) 60 kg (d) 40 kg

34. The escape velocity on earth is 11.2 kms^{-1}. If the body is projected out with twice this velocity, then the speed of the body far away from the earth, ignoring the presence of any other object in universe will be **[MP PET 2013]**

(a) 11.2 kms^{-1} (b) 22.4 kms^{-1}

(c) 19.4 kms^{-1} (d) 15.2 kms^{-1}

35. A satellite of mass m is circulating around the earth with constant angular velocity. If the radius is R_0 and mass of earth is M, then the angular momentum about the centre of the earth is **[UP CPMT 2013]**

(a) $m\sqrt{GM/R_0}$ (b) $M\sqrt{GmR_0}$
(c) $m\sqrt{GMR_0}$ (d) $M\sqrt{GM/R_0}$

36. Two identical thin rings each of radius R are co-axially placed at a distance R. If mass of rings are m_1, m_2 respectively, then the work done in moving a mass m from centre of one ring to that of the other is **[UP CPMT 2012]**

(a) $\frac{Gm}{m_2R}(\sqrt{2}+1)m$ (b) $\frac{G\,m(m_1-m_2)}{\sqrt{2}R}(\sqrt{2}-1)$
(c) $\frac{G\,m\sqrt{2}}{R}(m_1+m_2)$ (d) zero

37. The distance between the sun and the earth be r, then the angular momentum of the earth around the sun is proportional to **[UP CPMT 2012]**

(a) $\sqrt{r}$ (b) $r^{3/2}$
(c) r (d) None of these

38. A spherical planet has a mass M_p and diameter D_p. A particle of mass m falling freely near the surface of this planet will experience an acceleration due to gravity, equal to **[CBSE AIPMT 2012]**

(a) $4\,GM_p/D_p^2$ (b) GM_pm/D_p^2
(c) GM_pm/D_p^2 (d) $4GM_pm/D_p^2$

39. A geostationary satellite is orbiting the earth at a height of $5R$ above that surface of the earth, R being the radius of the earth. The time period of another satellite in hours at a height of $2R$ from the surface of the earth is **[CBSE AIPMT 2012]**

(a) 5 (b) 10
(c) $6\sqrt{2}$ (d) $6/\sqrt{2}$

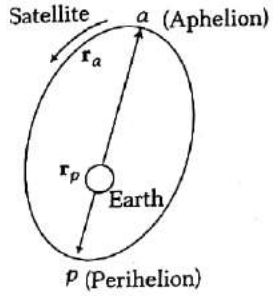
40. When a satellite is moving around the earth with velocity v, then to make the satellite escape, the minimum percentage increase in its velocity should be **[BHU 2012]**

(a) 100% (b) 82.4%
(c) 41.4% (d) None of these

41. A launching vehicle carrying an artificial satellite of mass m is set for launch on the surface of the earth of mass M and radius R. If the satellite is intended to move in a circular orbit of radius $7R$, the minimum energy required to be spent by the launching vehicle on the satellite is (Gravitational constnat = G) **[Manipal 2012]**

(a) $\frac{GMm}{R}$ (b) $-\frac{13\,GMm}{14R}$ (c) $\frac{GMm}{7R}$ (d) $\frac{GMm}{14R}$

42. Consider a satellite orbiting the earth as shown in the figure below. Let $\mathbf{L}_a$ and $\mathbf{L}_p$ represent the angular momentum of the satellite about the earth when at aphelion and perihelion respectively. Consider the following relations. **[AMU 2012]**



(i) $\mathbf{L}_a = \mathbf{L}_p$ (ii) $\mathbf{L}_a = -\mathbf{L}_p$
(iii) $\mathbf{r}_a \times \mathbf{L}_a = \mathbf{r}_p \times \mathbf{L}_p$

Which of the above relation(s) is/are true?

(a) (i) only (b) (ii) only (c) (iii) only (d) (i) and (iii)

43. A body is projected vertically upward from the surface of the earth with a velocity equal to half the escape velocity. If R is radius of the earth, then maximum height attained by the body is **[AMU 2012]**

(a) $R/6$ (b) $R/3$ (c) $2R/3$ (d) R

44. The imaginary angular velocity of the earth for which the effective acceleration due to gravity at the equator shall be zero is equal to **[AMU 2012]**

(Take, $g = 10\ \text{ms}^{-2}$ for the acceleration due to gravity if the earth were at rest and radius of earth equal to 6400 km.)

(a) 1.25×10^{-3} rad s^{-1} (b) 2.50×10^{-3} rad s^{-1}
(c) 3.75×10^{-3} rad s^{-1} (d) 5×10^{-3} rad s^{-1}

45. The mass of the moon is 1/81 of the earth but the gravitational pull is 1/6 of the earth. It is due to the fact that **[JCECE 2012]**

(a) the radius of earth is $\frac{9}{\sqrt{6}}$ of the moon
(b) the radius of moon is $\frac{81}{6}$ of the earth
(c) moon is the satellite of the earth
(d) None of the above

Answers

Check point 10.1

1. (d)	2. (a)	3. (c)	4. (b)	5. (c)	6. (c)	7. (c)	8. (b)	9. (d)	10. (c)
11. (c)	12. (a)	13. (c)	14. (d)	15. (d)	16. (d)	17. (d)	18. (c)	19. (d)	20. (c)

Check point 10.2

1. (c)	2. (b)	3. (c)	4. (b)	5. (d)	6. (b)	7. (c)	8. (c)	9. (a)	10. (c)
11. (c)	12. (d)	13. (b)	14. (b)	15. (a)	16. (a)	17. (b)	18. (a)	19. (a)	20. (b)

Check point 10.3

1. (a)	2. (a)	3. (b)	4. (d)	5. (b)	6. (d)	7. (b)	8. (d)	9. (d)	10. (d)
11. (c)	12. (c)	13. (c)	14. (c)	15. (c)					

Check point 10.4

1. (c)	2. (a)	3. (c)	4. (b)	5. (c)	6. (c)	7. (d)	8. (c)	9. (c)	10. (b)

Check point 10.5

1. (c)	2. (b)	3. (a)	4. (a)	5. (c)	6. (a)	7. (c)	8. (d)	9. (d)	10. (b)
11. (b)	12. (a)	13. (c)	14. (c)	15. (c)	16. (c)	17. (a)	18. (d)	19. (a)	20. (d)

Check point 10.6

1. (b)	2. (c)	3. (b)	4. (a)	5. (b)	6. (c)	7. (d)	8. (c)	9. (b)	10. (a)
11. (d)	12. (d)	13. (d)	14. (d)	15. (d)	16. (c)	17. (d)	18. (d)	19. (c)	20. (c)

A. Taking it together

1. (c)	2. (c)	3. (b)	4. (a)	5. (b)	6. (b)	7. (c)	8. (a)	9. (c)	10. (b)
11. (d)	12. (c)	13. (c)	14. (b)	15. (d)	16. (d)	17. (a)	18. (a)	19. (a)	20. (c)
21. (c)	22. (d)	23. (a)	24. (b)	25. (a)	26. (a)	27. (a)	28. (b)	29. (a)	30. (b)
31. (a)	32. (d)	33. (b)	34. (b)	35. (a)	36. (c)	37. (d)	38. (c)	39. (d)	40. (c)
41. (d)	42. (b)	43. (c)	44. (a)	45. (c)	46. (c)	47. (a)	48. (d)	49. (b)	50. (a)
51. (c)	52. (a)	53. (a)	54. (a)	55. (a)	56. (b)	57. (c)	58. (a)	59. (d)	60. (d)
61. (a)	62. (b)	63. (b)	64. (c)	65. (a)	66. (b)	67. (d)	68. (d)		

B. Medical entrance special format questions

Assertion and reason

1. (a)	2. (d)	3. (d)	4. (a)	5. (d)	6. (a)	7. (b)	8. (d)	9. (d)	10. (c)
11. (d)	12. (b)	13. (c)	14. (b)	15. (a)	16. (d)	17. (d)	18. (d)	19. (d)	20. (d)

Match the columns

1. (A → r, B → q, C → p)
2. (A → q, B → p, C → p, D → q)
3. (A → q, B → q, C → t, D → t)
4. (A → t, B → t, C → p,q,r,s, D → t)
5. (A → q, B → t, C → r, D → s)
6. (A → s, B → r, C → q)
7. (A → q, B → p)
8. (A → r, B → q, C → q, D → p)
9. (A → p, B → p, C → s)
10. (A → p, B → q, C → q, D → q)
11. (A → s, B → r, C → p, D → q)

C. Medical entrances' gallery

1. (b)	2. (b)	3. (d)	4. (a)	5. (b)	6. (a)	7. (d)	8. (b)	9. (c)	10. (b)
11. (b)	12. (b)	13. (e)	14. (c)	15. (d)	16. (c)	17. (c)	18. (b)	19. (a)	20. (d)
21. (a)	22. (c)	23. (a)	24. (a)	25. (b)	26. (d)	27. (c)	28. (a)	29. (b)	30. (a)
31. (c)	32. (a)	33. (d)	34. (c)	35. (c)	36. (b)	37. (a)	38. (a)	39. (c)	40. (c)
41. (b)	42. (a)	43. (b)	44. (b)	45. (a)					

Hints and explanations

Check point 10.1

1. (*d*) Kepler's second law is based on conservation of angular momentum, $\frac{dA}{dt} = \frac{L}{2m}$.

2. (*a*) Since, according to Kepler's second law, the line joining the planet to the sun sweeps out equal areas in equal interval of time. *i.e.* Areal velocity is constant.

3. (*c*) Since, Kepler's second law is same as the law of conservation of angular momentum.

$d_1v_1 = d_2v_2$ (from conservation of angular momentum at these two points)

$$\therefore \quad v_2 = \frac{d_1v_1}{d_2}$$

4. (*b*) In elliptical orbit velocity, both in magnitude and direction keep on changing. Therefore, momentum does not remain constant.

5. (*c*) From Kepler's second law of motion, a line joining any planet to the sun sweeps out equal areas in equal intervals of time.

Let L be angular momentum, I the moment of inertia and m the mass, then

$$L = I\omega = mr^2\omega$$

$$\therefore \quad \frac{dA}{dt} = \frac{L}{2m} = \text{constant}$$

Hence, angular momentum of the planet is conserved.

6. (*c*) Kepler's third law states that square of the time period of any planet about the sun is directly proportional to R^3.

7. (*c*) According to Kepler's third law,

$$T^2 \propto r^3, r_1 : r_2 : r_3 = \frac{1}{2} : 1 : \frac{3}{2}$$

$$\therefore \quad T_1^2 : T_2^2 : T_3^2 = \frac{1}{8} : 1 : \frac{27}{8} = 1 : 8 : 27$$

8. (*b*) According to Kepler's third law, $T^2 \propto r^3$

$$\because \quad T_A = 8T_B \Rightarrow \frac{T_A}{T_B} = \frac{8}{1} \quad \text{(given)}$$

$$\Rightarrow \left(\frac{T_A}{T_B}\right)^2 = \left(\frac{r_A}{r_B}\right)^3 \Rightarrow \left(\frac{8}{1}\right)^2 = \left(\frac{r_A}{r_B}\right)^3 \Rightarrow \frac{r_A}{r_B} = 4$$

$$\Rightarrow \quad r_A = 4r_B$$

9. (*d*) $T \propto r^{3/2}$

$$\therefore \quad \frac{T_1}{T_2} = \left(\frac{10^{13}}{10^{12}}\right)^{3/2} = 10\sqrt{10}$$

10. (*c*) As $T^2 \propto R^3$ (By Kepler's third law)

$$\therefore \quad \frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3} \Rightarrow \left(\frac{T_2}{T_1}\right)^2 = \left(\frac{R_2}{R_1}\right)^3$$

$$\Rightarrow \quad T_2^2 = T_1^2\left[\frac{4.5R}{9R}\right]^3 \Rightarrow T_2^2 = \frac{T_1^2}{8} \Rightarrow T_2 = \frac{T_1}{2\sqrt{2}}$$

$$T_2 = \frac{24}{2\sqrt{2}} \quad (\because T_1 = 24\text{h})$$

$$\therefore \quad T_2 = 6\sqrt{2}\text{ h}$$

11. (*c*) By Kepler's third law of planetary motion,

We have, $T^2 \propto a^3$

Given, $T_1 = 1$ day (geostationary)

$a_1 = a, a_2 = 2a$

$$\therefore \quad \frac{T_1^2}{T_2^2} = \frac{a_1^3}{a_2^3} \Rightarrow T_2^2 = \frac{a_2^3}{a_1^3}T_1^2 = \frac{(2a)^3}{a^3} \times 1 = 8$$

$$\Rightarrow \quad T_2 = 2\sqrt{2} \text{ days}$$

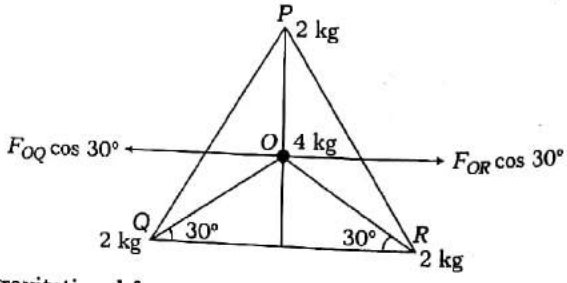
12. (*a*) $F = \frac{Gm_1m_2}{r^2}$

$$\therefore \quad r = \sqrt{\frac{Gm_1m_2}{F}} = \sqrt{\frac{6.67 \times 10^{-11} \times 1 \times 1}{9.8 \times 10^{-9}}} = 0.082 \text{ m} = 8.2 \text{ cm}$$

13. (*c*) $F = \frac{Gm_1m_2}{r^2}$ or $F \propto m_1m_2$

On M due to m force is F and due to mass $2m$ force is $2F$. Therefore, net force is $3F$.

14. (*d*) Consider the equilateral triangle as PQR with centroid at O. Given, $OP = OQ = OR = \sqrt{2}$ m



The gravitational force on mass 2 kg due to mass 1 kg at P.

$$F_{OP} = G\frac{4 \times 2}{(\sqrt{2})^2} = 4G \text{ along } OP$$

Similarly, $$F_{OQ} = G\frac{4 \times 2}{(\sqrt{2})^2} = 4G \text{ along } OQ$$

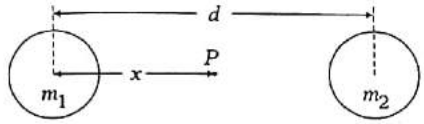
and $$F_{OR} = G\frac{4 \times 1}{(\sqrt{2})^2} = 4G \text{ along } OR$$

$F_{OQ}\cos 30°$ and $F_{OR}\cos 30°$ are equal and acting in opposite directions, hence cancel out each other. Then, the resultant force on the mass 4 kg at O, $F = 0$ (zero)

15. (*d*) Gravitational constant does not depend on the nature of the medium in which the bodies are kept.

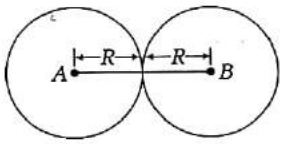
16. (*d*) As the work done by the gravitational force f in closed path is zero. So, it is conservative in nature.

17. (*d*) Force will be zero at the point P located at



$$x = \frac{\sqrt{m_1}}{\sqrt{m_1}+\sqrt{m_2}}d = \frac{\sqrt{81M}}{\sqrt{81M}+\sqrt{M}}D = \frac{9}{10}D$$

18. (*c*) Let masses of two balls are $m_1 = m_2 = m$ (given) and the density be ρ. Distance between their centres $= AB = 2R$.



Thus, the magnitude of the gravitational force F that two balls separated by a distance $2R$ exert on each other is

$$F = G\frac{(m)(m)}{(2R)^2} = G\frac{m^2}{4R^2} = G\frac{\left(\frac{4}{3}\pi R^3\rho\right)^2}{4R^2}$$

$\therefore \quad F \propto R^4$

19. (*d*) As we know, $F = \frac{G_1 m_1 m_2}{r^2}$ and $F' = \frac{G m_1 m_2}{(3r)^2} = \frac{F}{9}$

percent decrease in $F = \left(\frac{F-F'}{F}\right)\times 100 = \frac{8}{9}\times 100 = 89\%$

Check point 10.2

1. (*c*) $g = \frac{GM}{R^2}$ or $g \propto \frac{M}{R^2}$

Mass is 2 times and diameter is 3 times. Hence, $g' = \frac{2g}{9}$

2. (*b*) $g = \frac{GM}{R^2}$. If radius shrinks to half of its present value, then g will become four times.

3. (*c*) $g = \frac{4}{3}G\pi R\rho \Rightarrow \frac{g_1}{g_2} = \frac{\rho_1 R_1}{\rho_2 R_2} = \frac{1}{2}\times\frac{4}{1} = \frac{2}{1}$

4. (*b*) Acceleration due to gravity, $g = \frac{GM}{R^2} \Rightarrow \frac{g}{G} = \frac{M}{R^2}$

5. (*d*) $g = \frac{GM}{R^2}$ or $\frac{G}{g} = \frac{R^2}{m}$

$\therefore \frac{G}{g}$ will have the units $\frac{m^2}{kg}$

6. (*b*) $g \propto \rho R$. If density of earth is increased four times and its radius become half, then the value of acceleration due to gravity becomes double due to which our weight get double.

7. (*c*) $g = \frac{GM}{R^2} = \frac{G\left(\frac{4}{3}\pi R^3\right)\rho}{R^2} \quad \therefore \rho = \frac{g}{G\cdot 4\pi\frac{R}{3}} = \frac{3g}{4\pi GR}$

8. (*c*) As, $g = \frac{GM}{R^2}$ or $g \propto \frac{M}{R^2}$

Let g' be the acceleration due to gravity at the surface of earth. Since, according to question, $M' = \frac{M}{2}$ and $R = \frac{R}{2}$, where, M' and R' is the mass and radius of the planet.

$\therefore \quad g' \propto \frac{M'}{R'^2} \Rightarrow g' \propto \frac{M/2}{R^2/4} \Rightarrow g' \propto 2g$

$\Rightarrow \quad g' = 2\times 9.8 = 19.6\ \text{ms}^{-2}$

9. (*a*) Since, it is known that $g' = \frac{g}{\left(1+\frac{h}{R}\right)^2}$

Given, $\quad g' = \frac{g}{64}$

Where, g' is the value of acceleration due to gravity at height h.

$\therefore \quad 1+\frac{h}{R} = 8$ or $\frac{h}{R} = 7$

$\therefore \quad h = 7R = 44.8\times 10^6$ m

10. (*c*) With height, $g' = \frac{g}{\left(1+\frac{h}{R}\right)^2}$ ... (i)

Given, $\quad g' = \frac{g}{49}$

$\therefore \quad \frac{g}{49} = \frac{g}{\left(1+\frac{h}{R}\right)^2} \Rightarrow \frac{1}{49} = \frac{1}{\left(1+\frac{h}{R}\right)^2}$

$\Rightarrow \quad 49 = \left(1+\frac{h}{R}\right)^2$

$\Rightarrow \quad 7 = 1+\frac{h}{R} \Rightarrow h = 6R$

11. (*c*) Given, $\quad g' = \frac{g}{\left(1+\frac{h}{R}\right)^2} = \frac{g}{4}$ (at $h = R$)

$\therefore \quad W' = \frac{72}{4} = 18\text{N}$

12. (*d*) As, $\quad T = 2\pi\sqrt{\frac{l}{g}} \propto \frac{1}{\sqrt{g}}$

$\therefore \quad \frac{T_2}{T_1} = \sqrt{\frac{g_1}{g_2}} = \sqrt{\frac{g}{\frac{g}{(1+h/R)^2}}} = 2$ (at $h = R$)

13. (*b*) At depth, $g' = g\left(1-\frac{h}{R}\right)$ or $g\left(1-\frac{d}{R}\right)$

$\Rightarrow \quad \frac{g'}{g} = \frac{1}{n} = \left(1-\frac{d}{R}\right)$

or $\quad d = R\left(\frac{n-1}{n}\right)$

14. (*b*) According to the question,

$$g\left(1-\frac{2h}{R}\right)=\left(1-\frac{d}{R}\right) \qquad (\text{at } h << R)$$

or $$1-\frac{2h}{R}=1-\frac{d}{R}$$

$\therefore$ $$d=2h$$

15. (*a*) Given, $$g\left(1-\frac{d}{R}\right)=\frac{1}{2}\frac{g}{\left(1+\frac{h}{R}\right)^2}$$

or $$1-\frac{d}{R}=\frac{1}{2\left(1+\frac{1600}{6400}\right)^2}=\frac{16}{50} \Rightarrow \frac{d}{R}=\frac{34}{50}$$

$\therefore$ $$d=(6.4\times10^6\,\text{m})\left(\frac{34}{50}\right)=4.352\times10^6\,\text{m}$$

16. (*a*) At centre of earth, value of g is zero. Hence, weight is zero.

17. (*b*) Acceleration due to gravity at latitude ϕ is given by

$$g'=g-R\omega^2\cos^2\phi$$

At $30°$, $g_{30°}=g-R\omega^2\cos^2 30°=g-\frac{3}{4}R\omega^2$

$\therefore$ $$g-g_{30°}=\frac{3}{4}\omega^2R$$

18. (*a*) At poles, value of g is maximum. So, there is no effect of rotation of earth.

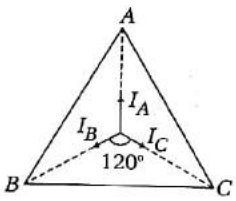
19. (*a*) At equator, $g'=g-R\omega^2 \Rightarrow 0=g-R\omega^2$

$\therefore$ $$\omega=\sqrt{\frac{g}{R}}=\sqrt{\frac{9.8}{6400\times10^3}}=1.23\times10^{-3}\text{ rad s}^{-1}$$

20. (*b*) When a body is taken from equator to poles, its weight increases, because acceleration due to gravity increases.

Check point 10.3

1. (*a*) Due to three particles, net intensity at the centre $I=I_A+I_B+I_C=0$. Because these three intensities are equal in magnitude and the angle between each other is 120°.



2. (*a*) Reduces by 75%, means 25% is left.

$$\frac{GM}{r^2}=\frac{1}{4}\left(\frac{GM}{R^2}\right) \text{ or } r=2R$$

$\therefore$ $$h=r-R=2R-R=R$$

3. (*b*) Gravitational field due to a solid sphere

$$E(r)=\frac{GM}{R^3}r \qquad (r\le R)$$

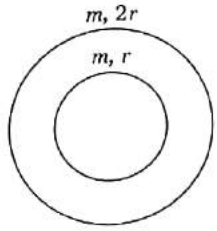
$\Rightarrow E(r)\propto r$, i.e. $E(r)$ increases inside the sphere.

$$E(r)=\frac{GM}{r^2} \qquad (r>R)$$

$\Rightarrow E(r)\propto\frac{1}{r^2}$, i.e. $E(r)$ decreases outside the sphere.

4. (*d*) Inside a shell, field strength is zero. Therefore, force on a particle is zero.

5. (*b*) At distance 2.5 r or $\frac{5}{2}r$



$$E=\frac{G(m+m)}{\left(\frac{5}{2}r\right)^2}=\frac{8Gm}{25r^2}$$

6. (*d*) Intensity will be zero inside the spherical shell.

$$E=0 \text{ upto } r=R \text{ and } E\propto\frac{1}{r^2} \text{ when } r>R$$

7. (*b*) Gravitational field at an external point due to spherical shell of mass M is $\left(\frac{GM}{r^2}\right)$ while at an internal point is zero.

(i) Point Q is external to shell M_1, M_2 and M_3

So, field at Q will be

$$E_Q=\frac{GM_1}{y^2}+\frac{GM_2}{y^2}+\frac{GM_3}{y^2}=\frac{G}{y^2}(M_1+M_2+M_3)$$

(ii) Field at P will be

$$F_p=\frac{GM_1}{x^2}+\frac{GM_2}{x^2}+0=\frac{G}{x^2}(M_1+M_2)$$

8. (*d*) $E_{net}=0$

$\therefore$ $\frac{Gm}{x^2}=\frac{GM}{(d-x)^2}$, x is distance from m

$\therefore$ $$\frac{x}{d-x}=\frac{\sqrt{m}}{\sqrt{M}} \Rightarrow x=\frac{\sqrt{m}}{\sqrt{m}+\sqrt{M}}\cdot d$$

and $$d-x=\frac{\sqrt{M}}{\sqrt{m}+\sqrt{M}}\cdot d$$

Since, gravitational potential between two bodies of masses M and m respectively, is given by

$$V=-\frac{Gm}{x}-\frac{GM}{d-x}$$

$$=-\frac{Gm(\sqrt{m}+\sqrt{M})}{\sqrt{m}\cdot d}-\frac{GM\sqrt{m}+\sqrt{M}}{\sqrt{M}\cdot d}=-\frac{G}{d}(\sqrt{m}+\sqrt{M})^2$$

9. (*d*) Since, length if rod is equal to the circumference of semicircle

$$\pi R=L \Rightarrow R=\frac{L}{\pi}$$

Therefore, the gravitational at the centre of circle will be

$$V=-\frac{GM}{R}=-\frac{\pi GM}{L}$$

10. (*d*) Gravitational potential $V_i=-\frac{GM}{r}$

$$V_i=-\frac{6.67\times10^{-11}\times100}{0.1}$$

$$V_i = -\frac{6.67 \times 10^{-9}}{0.1} = -6.67 \times 10^{-8} \text{J}$$

$\because \quad V_f = 0$

$\therefore$ Work done per unit mass,

$$W = \Delta V = (V_f - V_i) = 6.67 \times 10^{-8} \text{ J}$$

11. (c) $V_{in} = \frac{-Gm}{2R}\left[3 - \left(\frac{r}{R}\right)^2\right], V_{surface} = \frac{-GM}{R}, V_{out} = \frac{-GM}{r}$

12. (c) The value of gravitational potential is constant inside the uniform shell.

13. (c) At centre, $V_c = -\frac{3}{2}\frac{GM}{R}$, on surface $V = -\frac{GM}{R}$

$\therefore \quad V_c = \frac{3}{2}V$

14. (c) For hollow sphere

$$V_{in} = \frac{-GM}{R}, V_{surface} = \frac{-GM}{R}, V_{out} = \frac{-GM}{r}$$

i.e., potential remains constant inside the sphere and it is equal to potential at the surface and increase when the point moves away from the surface of sphere.

15. (c) For a uniform ring of mass M and radius R at it centre, the field is zero but potential is $-\frac{GM}{R}$.

Check point 10.4

1. (c) When particles are released from rest their separation decreases. Therefore gravitational potential energy of the system decreases.

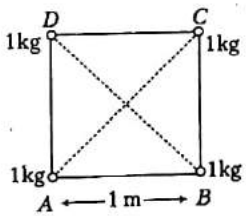
2. (a) Increase in kinetic energy of both particles

= decrease in gravitational potential energy

$$\therefore \quad 2\left(\frac{1}{2}mv^2\right) = U_i - U_f = -\frac{Gmm}{d} + \frac{Gmm}{\frac{d}{2}} = \frac{Gm^2}{d}$$

$$\therefore \quad v = \sqrt{\frac{Gm}{d}}$$

3. (c) Here, $AB = BC = CD = DA = 1\text{m}$



$$BD = AC = \sqrt{1^2 + 1^2} = \sqrt{2} \text{ m}$$

Total potential energy,

$$U = \left[\frac{-G \times 1 \times 1}{AB}\right] + \left[\frac{-G \times 1 \times 1}{BC}\right] + \left[\frac{-G \times 1 \times 1}{CD}\right] + \left[\frac{-G \times 1 \times 1}{DA}\right] + \left[\frac{-G \times 1 \times 1}{BD}\right] + \left[\frac{-G \times 1 \times 1}{AC}\right]$$

$$= 4 \times \left[\frac{-G \times 1 \times 1}{1}\right] + 2\left[\frac{-G \times 1 \times 1}{\sqrt{2}}\right] = -5.4G$$

4. (b) Potential energy $u = \frac{GMm}{r}$

At distance $2r$, $E = \frac{GM}{(2r)^2} = \frac{GM}{4r^2} = \frac{u}{4mr}$

Now, $\quad W = mE = \frac{u}{4r}$

5. (c) Magnitude of gravitational potential energy, $U = \frac{GMm}{r}$

or $\quad U = \frac{GMm}{r^2} \times r$ or $U = g \times mr \quad \left(\because g = \frac{GM}{r^2}\right)$

or $\quad U = (mg)r$ or $mg = \frac{U}{r}$

6. (c) Since, potential energy, $\Delta U = \frac{mgh}{\left(1 + \frac{h}{R}\right)}$, $h = R$ (given)

$$\therefore \quad \Delta U = \frac{mgR}{2}$$

7. (d) Change in potential energy in displacing a body from r_1 to r_2 is given by

$$\Delta U = GMm\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$= GMm\left(\frac{1}{2R} - \frac{1}{3R}\right) \quad (\because r_1 = 2R, r_2 = 3R)$$

$$= \frac{GMm}{6R}$$

8. (c) Initial gravitational potential energy,

$$U_i = -\frac{GM_e m}{R} \quad \text{...(i)}$$

Final gravitational potential energy, $U_f = -\frac{1}{5} \cdot \frac{GM_e m}{R}$ ...(ii)

$\therefore$ Energy required

$$E = U_f - U_i = -\frac{GM_e m}{5R} + \frac{GM_e m}{R}$$

$$= \frac{5GM_e m - GM_e m}{5R} = \frac{4GM_e m}{5R}$$

But, acceleration due to gravity (g) in terms of gravitational constant (G) is

$$g = \frac{GM_e}{R^2} \quad \text{...(iii)}$$

$$\therefore \quad \frac{GM_e}{R^2} \times R^2 \times \frac{4m}{5R} = g \times R^2 \times \frac{4m}{5R} \quad \text{[From Eq. (iii)]}$$

$$= \frac{4gmR}{5} = \frac{4mgR}{5}$$

9. (c) Work done, $W = \Delta U = \frac{mgh}{1 + \left(\frac{h}{R}\right)}$

Substituting $R = \frac{h}{5}$, we get

$$\Delta U = \frac{mg \times 5R}{1 + 5} = \frac{5mgR}{6}$$

10. (*b*) On surface of earth, $U = -\frac{GMm}{R}$

At height $h(<< R)$, increase in potential energy is mgh

$\therefore$ The potential energy of body and the earth is given by

$$U_h = -\frac{GMm}{R} + mgh$$

Check point 10.5

1. (*c*) Total mechanical energy of any closed system is always negative.

2. (*b*) The energy required to remove an object from the surface of the earth to infinity is called binding energy of the object. It is equal to negative of gravitational potential energy of the object.

Thus, binding energy $= -E = \frac{GMm}{R}$

but, $g = \frac{GM}{R^2} \Rightarrow GM = gR^2 \Rightarrow \text{BE} = \frac{gmR^2}{R} = gmR$

3. (*a*) $v_e = \sqrt{2gR} = \sqrt{2\frac{GM}{R^2}\cdot R}$ or $v_e \propto \sqrt{\frac{M}{R}}$

Mass is 1000 times and radius is 10 times.

$$v_e' = \sqrt{\frac{(1000M)G}{100R}}$$

$$\Rightarrow \quad v_e' = 10\sqrt{\frac{GM}{R}}$$

$$\Rightarrow \quad v_e' = 10V_e \Rightarrow v_e' = 10 \times 112 \Rightarrow v_e' = 112\,\text{kms}^{-1}$$

Therefore, escape velocity will become 10 times.

4. (*a*) At $v < v_e$, total energy is negative

$v = v_e$, total energy is zero and

$v > v_e$ total energy is positive

So, with the reference to the above given cases, it can be said that when the escape velocity is given to a particle, its total energy is zero.

5. (*c*) Escape velocity is independent of mass of projectile which is projected. Therefore, escape velocity of body of mass 100 kg will also be 11.2 kms^{-1}.

6. (*a*) Escape velocity is independent of angle at which particle is projected. Therefore, when the body is projected in a direction making 30° angle to the vertical, then its escape velocity will remain same, i.e. 11.2 kms^{-1}.

7. (*c*) Since, escape velocity is independent of mass of particle which is projected, therefore it varies as m^0.

8. (*d*) Escape velocity, $v = \sqrt{2gR}$

$$\therefore \quad \frac{v_e}{v_m} = \sqrt{\frac{g_e}{g_m} \times \frac{R_e}{R_m}} = \sqrt{\frac{6}{1} \times \frac{10}{1}} = \sqrt{60} \quad \left(\because \frac{R_e}{R_m} = \frac{10}{1}, \frac{g_e}{g_m} = \frac{6}{1}\right)$$

9. (*d*) Escape velocity is independent of the angle at which it is projected. Therefore, a projectile can be fired with escape velocity at any angle.

10. (*b*) Since, escape velocity, $v_e = \sqrt{2gR}$, therefore v_e is independent of mass of projectile.

11. (*b*) $v = \sqrt{\frac{2GM}{R}}$

$$\therefore \frac{v_p}{v_e} = \sqrt{\frac{M_p}{M_e} \times \frac{R_e}{R_p}} = \sqrt{\frac{16M_e}{M_e} \times \frac{R_e}{8R_p}}$$

$$\therefore v_p = \sqrt{2}v_e = \sqrt{2} \times 11.2 = 15.84\,\text{kms}^{-1}$$

12. (*a*) Escape velocity, $v_e = \sqrt{2gR}$

where, g is acceleration due to gravity and R is radius.

Given, $\frac{R_1}{R_2} = K, \frac{g_1}{g_2} = g$

$$\Rightarrow \quad \frac{v_1}{v_2} = \frac{\sqrt{g_1R_1}}{\sqrt{g_2R_2}} = \sqrt{Kg} \Rightarrow \frac{v_1}{v_2} = (Kg)^{1/2}$$

13. (*c*) Escape velocity of a body on earth surface is given by

$$v_e = \sqrt{\frac{2GM}{R}}$$

where, G is gravitational constant, M is mass of planet and R_e is radius.

When $M' = 16M, R' = 4R$,

then, $v_e' = \sqrt{\frac{2G(16M)}{(4R)}} = 2\sqrt{\frac{2GM}{R}} \Rightarrow v_e' = 2v_e$

14. (*c*) $v_e = \sqrt{\frac{2GM}{R}}$ *i.e.*, escape velocity depends upon the mass and radius of the planet.

15. (*c*) The kinetic energy needed to project a body of mass m from the earth surface to infinity is given by

$$\text{KE} = \frac{1}{2}mv_e^2 = \frac{1}{2}m\,2Rg$$

($\because$ Escape velocity, $v_e = \sqrt{2Rg} \Rightarrow v_e^2 = 2Rg$)

$$\Rightarrow \quad \text{KE} = mRg$$

16. (*c*) $U_i + K_i = U_f + K_f$

$$\Rightarrow \quad -\frac{GMm}{R} + \frac{1}{2}m(2v_e)^2 = 0 + \frac{1}{2}mv^2$$

or $$-\frac{GM}{R} + 2v_e^2 = \frac{1}{2}v^2$$

or $$-\frac{2GM}{R} + \frac{8GM}{R} = v^2$$

or $$v = \sqrt{\frac{6GM}{R}} = \sqrt{3\left(\frac{2GM}{R}\right)} = \sqrt{3(2gR)} = \sqrt{3}v_e$$

17. (*a*) Decrease in kinetic energy = increase in PE

$$\therefore \quad \frac{1}{2}m\left(\frac{v_e}{\sqrt{2}}\right)^2 = \frac{mgh}{1+\frac{h}{R}} \quad \text{or} \quad \frac{v_e^2}{4} = \frac{gh}{1+\frac{h}{R}}$$

or $$\frac{2gR}{4} = \frac{gh}{1+\frac{h}{R}} \quad \text{or} \quad \frac{R}{2} = \frac{h}{1+\frac{h}{R}}$$

Solving this equation, we get $h = R$

Note Kinetic energy is half the value required to escape. Therefore, speed is $1/\sqrt{2}$ times the value required to escape.

18. (*d*) Initial KE of the body $= \frac{1}{2}mv^2$

Here, $v = 3 \times 11.2\ \text{kms}^{-1}$

So, $\text{KE} = \frac{1}{2} \times \text{m}(3 \times 11.2)^2 = 9 \times \frac{1}{2}mv_e^2$

As $\frac{1}{2}mv_e^2$ energy is used up in coming out from the gravitational pull of the earth, so final KE should be $8 \times \frac{1}{2}mv_e^2$.

Hence, $\frac{1}{2}mv'^2 = 8 \times \frac{1}{2}mv_e^2$

$\therefore \quad v'^2 = 8v_e^2$

$\Rightarrow \quad v = \sqrt{8}v_e = \sqrt{8} \times 11.2\ \text{kms}^{-1}$

19. (*a*) Decrease in kinetic energy = increase in potential energy

$\therefore \quad \frac{1}{2}mv^2 = \frac{mgh}{1+\frac{h}{R}}, \quad$ (given, $h = R$)

$\therefore \quad v^2 = \frac{2gR}{2} = gR = \frac{GM}{R} \quad \left(\because g = \frac{GM}{R^2}\right)$

or $\quad v = \sqrt{\frac{GM}{R}}$

20. (*d*) Let h be the maximum height attained, then from equation of the motion

$v^2 = u^2 + 2gh$

When $\quad u = 0 \Rightarrow v = \sqrt{2gh}$

Given, $\quad v = \frac{v_e}{3}$, where, $v_e = \sqrt{2gR} \Rightarrow \sqrt{2gh} = \frac{1}{3}\sqrt{2gR}$

On squaring both sides of equation, we get

$$h = \frac{R}{9}$$

Check point 10.6

1. (*b*) Actually gravitational force provides the centripetal force. Therefore, the net force on the satellite will be F.

2. (*c*) Since, orbital velocity $v = \sqrt{\frac{GM}{r}}$ or $v \propto \frac{1}{\sqrt{r}}$

$$\frac{v_2}{v_1} = \sqrt{\frac{r_1}{r_2}}$$

$$\therefore \quad v_2 = \sqrt{\frac{r_1}{r_2}} \cdot v = \sqrt{\frac{R}{R+\frac{R}{2}}} \cdot v = \sqrt{\frac{2}{3}}v$$

3. (*b*) Orbital velocity, $v = \sqrt{\frac{GM}{R}} \Rightarrow \frac{v_A}{v_B} = \sqrt{\frac{R_B}{R_A}} = \sqrt{\frac{R}{4R}} = \frac{1}{2}$

$$\therefore \quad \frac{v_A}{v_B} = \frac{3v}{v_B} = \frac{1}{2} \Rightarrow v_B = 6v$$

4. (*a*) Since, orbital velocity of body close to earth can be given as

$$v_o = \sqrt{rg} = \sqrt{6400 \times 10^3 \times 10} = \sqrt{64 \times 10^6} = 8 \times 10^3\ \text{m s}^{-1}$$

$$v_o = 8\ \text{km s}^{-1}$$

5. (*b*) The time period of revolution of satellite is given by

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{r^3}{GM}} \Rightarrow T \propto \sqrt{\frac{r^3}{GM}}$$

6. (*c*) The time period of satellite in a circular orbit ground a planet is independent of mass of satellite.

7. (*d*) $T^2 \propto R^3$

$$\therefore \quad \frac{T^2}{R^3} = \text{constant}$$

8. (*c*) According to Kepler's third law, $T^2 \propto r^3$.

Let the distance of the two satellites from the centre of the earth be r_1 and r_2 respectively, therefore, the ratio of their time periods will be

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3 \Rightarrow \frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2} = \left(\frac{1}{4}\right)^{3/2} \quad \left(\because \frac{r_1}{r_2} = \frac{1}{4}\right)$$

$$\Rightarrow \quad \frac{T_1}{T_2} = \frac{1}{8}$$

9. (*b*) Given that, $T_1 = 1$ day and $T_2 = 8$ days

$$\therefore \quad \frac{T_2}{T_1} = \left(\frac{r_2}{r_1}\right)^{3/2}$$

$$\Rightarrow \quad \frac{r_2}{r_1} = \left(\frac{T_2}{T_1}\right)^{2/3} = \left(\frac{8}{1}\right)^{2/3} = 4 \Rightarrow r_2 = 4r_1 = 4R$$

10. (*a*) According to Kepler's third law, $T \propto r^{3/2}$

$$\therefore \quad \frac{T_2}{T_1} = \left(\frac{r_2}{r_1}\right)^{3/2} \quad \text{or} \quad T_2 = T_1(4)^{3/2} = 8T_1 = 8\ \text{days.}$$

11. (*d*) Time period of satellite, $T = \frac{2\pi r^{3/2}}{\sqrt{GM}}$ or $\sqrt{GM} = \frac{2\pi}{T} \cdot r^{3/2}$

$$\frac{2\pi}{T} = \omega$$

$$\therefore \quad \omega r^{3/2} = \sqrt{GM} \quad \text{or} \quad r^3 = \frac{GM}{\omega^2} = \frac{gR^2}{\omega^2}$$

12. (*d*) Energy of a satellite is given by $E = -\frac{GMm}{2r}$

For a satellite very close to earth, $r = R$

$$\therefore \quad E = -\frac{GMm}{2R}$$

13. (*d*) Energy of satellite is given by

$$E = -\frac{GMm}{2r} \quad \text{or} \quad E \propto \frac{m}{r}$$

$$\Rightarrow \quad \frac{E_A}{E_B} = \frac{m_A}{m_B} \times \frac{r_B}{r_A} = \frac{3}{1} \times \frac{4r}{r} \quad \left\{\because \frac{m_A}{m_B} = \frac{3}{1}, \frac{r_B}{r_A} = \frac{4r}{r}\right\}$$

$$\Rightarrow \quad \frac{E_A}{E_B} = \frac{12}{1}$$

14. (*d*) Kinetic energy is negative of the mechanical energy.

15. (*d*) As kinetic energy, $K = \frac{GMm}{2r}$

Potential energy, $U = -\frac{GMm}{r}$

Total energy, $E = -\frac{GMm}{2r}$

If r is decreased, K will increase but U and E will decrease.

16. (c) $\because$ Total energy, $E_0 = -\frac{GMm}{2r}$

Potential energy, $U = -\frac{GMm}{r} = 2E_0$

Kinetic energy, $K = +\frac{GMm}{2r} = -E_0$

17. (d) Let r_1 and r_2 be respective the distance of two satellites from the earth's surface.

$r_1 = R + R = 2R$ and $r_2 = 7R + R = 8R$

($\because$ R is the radius of the earth)

Now, $K = \frac{GMm}{2r} \Rightarrow \frac{K_1}{K_2} = \frac{r_2}{r_1} = 4$

$U = -\frac{GMm}{r}, \frac{U_1}{U_2} = \frac{r_2}{r_1} = 4$

and $E = -\frac{GMm}{2r}$ or $\frac{U_1}{U_2} = \frac{r_2}{r_1} = 4$

18. (d) Total mechanical energy of satellite

$$E = \frac{-GMm}{2r} \Rightarrow \frac{E_P}{E_Q} = \frac{m_P}{m_Q} \times \frac{r_Q}{r_P}$$

$$\Rightarrow \frac{E_P}{E_Q} = \frac{3}{1} \times \frac{8r}{r} = \frac{24}{1}$$

19. (c) E = Energy of satellite − energy of satellite on surface of earth

$$= -\frac{GMm}{2(8R)} - \left[-\frac{GMm}{R}\right] = \frac{15}{16}\frac{GMm}{R} = \frac{15}{16}mgR \quad \left(\text{as } \frac{GM}{R} = gR\right)$$

20. (c) An astronaut experiences weightlessness in a space satellite. It is because the astronaut experiences no gravity.

A) Taking it together

1. (c) Reason of weightlessness in a satellite is a zero reaction force by satellite surface.

2. (c) $g = \frac{4}{3}\pi\rho GR \Rightarrow g \propto dR$ ($\rho = d$ given in the problem)

3. (b) In circular path, if necessary centripetal force disappears, then the body moves with velocity v tangentially to original orbit tangential path.

4. (a) In a coal mine and at the top of a mountain value of g is less, hence apparent weight is less.

5. (b) $T = \frac{2\pi(r)^{3/2}}{\sqrt{GM}}$, T is independent of m, the mass of satellite.

6. (b) $v = \sqrt{\frac{GM}{R}} = G^{1/2}M^{1/2}R^{-1/2}$

7. (c) According to Kepler's second law $\frac{dA}{dt} = \frac{vr\sin\theta}{2} = \frac{1}{2}|\mathbf{r} \times \mathbf{v}|$

8. (a) Considering Kepler's second law we know that

$$\frac{A}{T} = \frac{L}{2m} \text{ or } L = \frac{2mA}{T}$$

9. (c) $\text{KE} = \frac{1}{2}mv_e^2 = \frac{1}{2}m(2gR) = mgR$

10. (b) As, $g = \frac{GM}{R^2} \Rightarrow \frac{dg}{dR} = \frac{-2GM}{R^3}$

$$\Rightarrow \frac{dg}{h} = \frac{-2GM}{R^2} \cdot \frac{1}{h}$$

$$\Rightarrow \frac{dg}{g} = -2\left(\frac{h}{R}\right)$$

11. (d) If we assume the earth as a sphere of uniform density, then it can be treated as point mass placed at its centre. In this case acceleration due to gravity $g = 0$, at the centre.

It is not so, if the earth is considered as a sphere of non-uniform density, in that case value of g will be different at different points and cannot be zero at any point.

12. (c) As observed from the earth, the sun appears to move in an approximate circular orbit. The gravitational force of attraction between the earth and the sun always follows inverse square law. Due to relative motion between the earth and mercury, the orbit of mercury, as observed from the earth will not be approximately circular, since the major gravitational force on mercury is due to the sun.

13. (c) As the total energy of the earth satellite bounded system is negative $\left(\frac{-GM}{2a}\right)$, where a is radius of the satellite and M is mass of the earth. Due to the viscous force acting on satellite, energy decreases continuously and radius of the orbit or height decreases gradually.

14. (b) As observed from the sun, two types of forces are acting on the moon one is due to gravitational attraction between the sun and the moon and the other is due to gravitational attraction between the earth and the moon. Hence, total force on the moon is not central.

15. (d) Asteroids are also being acted upon by central gravitational forces, hence they are moving in circular orbits like planets and obey Kepler's laws.

16. (d) Gravitational mass of proton is equivalent to its inertial mass and is independent of presence neighbouring heavy objects.

17. (a) As the earth is revolving around the sun in a circular motion due to gravitational attraction. The force of attraction will be of radial nature *i.e.*, angle between position vector $\mathbf{r}$ and force $\mathbf{F}$ is zero.

So, torque $= |\tau| = |\mathbf{r} \times \mathbf{F}| = rF\sin 0° = 0$

18. (a) If they collide inelastically and stick together as wreckage, then

KE = 0, only PE is present.

$$\therefore \quad E = U = 2\left(-\frac{GMm}{r}\right) = -\frac{2GMm}{r}$$

19. (a) Required additional velocity,

$$\Delta v = v_e - v_o = \sqrt{4gR} - \sqrt{2gR}$$

$$= (2 - \sqrt{2})\sqrt{gR} = (\sqrt{2} - 1)\sqrt{\frac{2GM}{R}}$$

20. (*c*) Force on B due to $A = F_{BA} = \frac{G(2Mm)}{(AB)^2}$ towards BA

Force on B due to $C = F_{BC} = \frac{GMm}{(BC)^2}$ towards BC

As, $(BC) = 2AB$

$\Rightarrow \quad F_{BC} = \frac{GMm}{(2AB)^2} = \frac{GMm}{4(AB)^2} < F_{BA}$

Hence, m will move towards BA (*i.e.*, $2M$)

21. (*c*) Esacpe velocity, $v_e = \sqrt{2}v_o = 1414v_o$

or orbital speed is to be increased by 41%

Further speed is to increase $\sqrt{2}$ times and kinetic energy is to increase two times.

22. (*d*) Change in potential energy in displacing a body from r_1 to r_2 is given by

$$\Delta U = GMm\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = GMm\left(\frac{1}{2R} - \frac{1}{4R}\right) = \frac{GMm}{4R}$$

23. (*a*) Applying conservation of mechanical energy we get,

$$\frac{1}{2}mv^2 = \frac{mgh}{1 + h/R}$$

Putting $h = R$, we get

$$v = \sqrt{gR} = \sqrt{\frac{GM}{R}}$$

24. (*b*) $F = \frac{k}{r}$ ($k =$ constant)

$\therefore \quad \frac{mv^2}{r} = \frac{k}{r}$ or $v \propto r^0$

25. (*a*) At equator, $g' = g - R\omega^2 = 0$

$\therefore \quad \omega = \sqrt{\frac{g}{R}} \Rightarrow \frac{2\pi}{T} = \sqrt{\frac{g}{R}} \Rightarrow T = 2\pi\sqrt{\frac{R}{g}}$

26. (*a*) $0 = g - R\omega^2 \cos^2 60°$

or $\quad \omega^2 = \frac{4g}{R}$ or $\omega = 2\sqrt{\frac{g}{R}}$

$$\frac{2\pi}{T} = 2\sqrt{\frac{g}{R}} \Rightarrow T = \pi\sqrt{\frac{R}{g}}$$

27. (*a*) $r = R_{earth} + h = (6.4 \times 10^6 + 14 \times 10^6)$ m

$\Rightarrow \quad r = 7.8 \times 10^6$ m

$$T = \left[\frac{4\pi^2 r^3}{GM_{earth}}\right]^{1/2} \doteq 6831\text{s}$$

Speed of satellite, $v = \sqrt{\frac{GM_{earth}}{r}} = 7174 \text{ ms}^{-1}$

28. (*b*) $\frac{mv^2}{r} = \frac{Gm_1m_2}{r^3}$ or $v \propto r^{\left(\frac{1-3}{2}\right)}$

Now, $T = \frac{2\pi r}{v}$ or $T \propto \frac{r}{v}$

$\therefore \quad T \propto \frac{r}{r^{\left(\frac{1-3}{2}\right)}} \Rightarrow T \propto r^{\frac{1+3}{2}} = r^2$

29. (*a*) As gravitational force provides the necessary centripetal force

$$\frac{mv^2}{r} \propto r^{-n} \Rightarrow v \propto r^{(1-n)/2} \Rightarrow T = \frac{2\pi r}{v} \text{ or } T \propto rv^{-1}$$

or $\quad T \propto r \cdot r^{(n-1)/2}$ or $T \propto r^{(n+1)/2}$

30. (*b*) Applying conservation of mechanical energy,

$\frac{1}{2}m\left(\frac{v_e}{2}\right)^2 = \frac{mgh}{1 + \frac{h}{R}}$ or $\frac{v_e^2}{8} = \frac{gh}{1 + \frac{h}{R}}$

or $\quad \frac{2gR}{8} = \frac{gh}{1 + h/R}$ ...(i) ($\because v_e = \sqrt{2gR}$)

On solving Eq. (i), we get $h = \frac{R}{3}$

31. (*a*) $t = \frac{T}{4} = \frac{2\pi\sqrt{R/9}}{4} = \frac{\pi}{2}\sqrt{\frac{R}{g}} \Rightarrow t' = \sqrt{\frac{2R}{g}}$

$\therefore \quad \frac{t}{t'} = \frac{\pi}{2\sqrt{2}}$

32. (*d*) Orbital velocity, $v_o = \sqrt{\frac{GM}{r}} = \sqrt{\frac{gR^2}{R + h}}$

33. (*b*) Increase in kinetic energy = decrease in potential energy

$\therefore \quad \frac{1}{2}mv^2 = \frac{mgR}{1 + R/R} = \frac{mgR}{2}$ $\left(\because \Delta U = \frac{mgh}{1 + h/R}\right)$

$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{mgR}{2} \Rightarrow v = \sqrt{gR}$

34. (*b*) Semi major axis, $a = \frac{r_1 + r_2}{2}$

Now, $\quad T^2 \propto a^3$

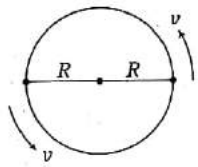
$\Rightarrow \quad T \propto a^{3/2} \propto \left(\frac{r_1 + r_2}{2}\right)^{3/2} \Rightarrow T \propto (r_1 + r_2)^{3/2}$

35. (*a*) $\frac{1}{2}mv^2 = \frac{mgh}{1 + \frac{h}{R}}$ ...(i)

On solving Eq. (i) for h, we get

$$\therefore \quad h = \frac{v^2}{2g - \frac{v^2}{R}} = \frac{R}{\left(\frac{2gR}{v^2}\right) - 1}$$

36. (*c*) As gravitational force provides the centripetal force to the satellite to revolve around the earth



$$\frac{G \cdot m \cdot m}{4R^2} = \frac{mv^2}{(R)} \Rightarrow v = \frac{1}{2}\sqrt{\frac{Gm}{R}}$$

38. (*c*) The initial position will become the perigee (nearest) position.

So, the resulting orbit of the satellite will become elliptical orbit with minimum distance from the earth equal to R.

39. (*d*) Equating gravitational field intensities,

$$\frac{\frac{GM}{81}}{r^2} = \frac{GM}{(60R - r)^2}$$

Solving, we get $60R - r = 9r \;\therefore\; r = 6R$

40. (*c*) E = Energy of satellite − energy of satellite on surface of earth

$$= -\frac{GMm}{2(3R)} - \left[-\frac{GMm}{R}\right] \qquad \left(\because h = 2R \;\therefore \text{total distance} = 3R\right)$$

$$= \frac{5}{6}\frac{GMm}{R} = \frac{5}{6}mgR \qquad \left(\text{as } \frac{GM}{R} = gR\right)$$

41. (*d*) $L = mvr = m\sqrt{\frac{GM}{r}}r = m\sqrt{GMr}$

$\therefore \quad L \propto \sqrt{r}$

$$\Rightarrow \quad \frac{t_1}{t_2} = \sqrt{\frac{r_1}{r_2}} \Rightarrow \frac{L}{L_2} = \sqrt{\frac{r_1}{r_2}} \qquad \left\{\begin{matrix}\because L_1 = L, \\ r_1 = r \\ r_2 = 16r\end{matrix}\right\}$$

$$\Rightarrow \quad \frac{L}{L_2} = \sqrt{\frac{r}{16r}} \Rightarrow \frac{L}{L_2} = \frac{1}{4} \Rightarrow L_2 = 4\,L$$

42. (*b*) Just before collision total distance travelled by both bodies $= 15R - R - 2R = 12R$. Now let smaller sphere travels a distance x. Then

$$m(x) = 4m(12R - x) \Rightarrow x = 9.6R$$

43. (*c*) Acceleration between them is zero, where force between $2m$ and $\frac{m}{5}$ is equal to the force between m and $\frac{m}{5}$ or the net force on $\frac{m}{5}$ is zero. Suppose this point is P. Now if velocity v_0 is less than value necessary to cross point P, it will retrace its path otherwise not.

44. (*a*) Work done, $W = \Delta U = U_f - U_i = U_f$ (as $U_i = 0$)

$$\Rightarrow \quad W = -(Gm_1m_2/R)$$

45. (*c*) It is given that $\sqrt{\frac{GM}{r}} = \frac{1}{2}\sqrt{2gR} = \sqrt{\frac{gR}{2}} = \sqrt{\frac{GM}{2R}}$

$\therefore \quad r = 2R \Rightarrow h = r - R = 2R - R = R$

46. (*c*) $W = E_A - E_\infty = (U_A + K_A) - (U_\infty + K_\infty)$

$$\therefore \quad -3 = \left\{(1)V_A + \frac{1}{2}\times(1)(2)^2\right\} - (0 + 0) \qquad (\because W = -3\text{J})$$

$\therefore \quad V_A = -5\text{ J kg}^{-1}$

47. (*a*) Conservation of mechanical energy gives,

$$\frac{1}{2}mv^2 = m\left[-\frac{GM}{R} + \frac{3GM}{2R}\right] = \frac{GMm}{2R}$$

$\because$ increase in KE = decrease in PE) or $v = \sqrt{\frac{GM}{R}}$

Momentum, $mv = m\sqrt{\frac{GM}{R}}$

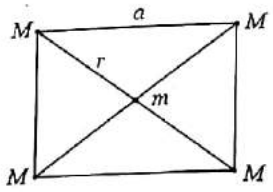
$\therefore$ Impulse required $= m\sqrt{\frac{GM}{R}}$

48. (*d*) At P, $\quad F = 0$

At Q, $\quad F = \frac{GM_1m}{b^2}$

At R, $\quad F = \frac{G(M_1 + M_2)m}{c^2}$

49. (*b*) Potential at centre $= -\frac{4GM}{r} = \frac{-4GM}{a/\sqrt{2}} = -\frac{4\sqrt{2}GM}{a}$



Potential energy $= \frac{-4\sqrt{2}GMm}{a}$

Binding energy $= \frac{4\sqrt{2}GMm}{a}$

$$\therefore \quad \frac{1}{2}mv_e^2 = \frac{4\sqrt{2}GMm}{a} \quad \text{or} \quad v_e = \sqrt{\frac{8\sqrt{2}GM}{a}}$$

50. (*a*) $E_i = E_0 = -\frac{GMm}{2r}$ or $E_f = -\frac{GMm}{2(3r)} = -\frac{GMm}{6r}$

$$\therefore \quad W = E_f - E_j = -\frac{GMm}{3r} = \frac{2}{3}E_0$$

51. (*c*) Time period of simple pendulum,

$$T = 2\pi\sqrt{\frac{l}{g}} \quad \text{or} \quad T \propto \frac{1}{\sqrt{g}}$$

$$\frac{T_1}{T_2} = \sqrt{\frac{g_2}{g_1}} = \frac{2}{1} \Rightarrow \frac{g_2}{g_1} = 4 \qquad \text{...(i)}$$

$$v_e = \sqrt{2gR} \quad \text{or} \quad v_e \propto \sqrt{gR} \Rightarrow \frac{v_{e1}}{v_{e2}} = \sqrt{\frac{g_1}{g_2}\cdot\frac{R_1}{R_2}} = 1:2$$

$$\therefore \quad \sqrt{\frac{1}{4}\times\frac{R_1}{R_2}} = \frac{1}{2} \qquad \left(\because \text{from Eq. (i) } \frac{g_2}{g_1} = 4\right)$$

$$\therefore \quad \frac{R_1}{R_2} = 1$$

Further, $\quad g = \frac{Gm}{R^2} = \frac{G\left(\frac{4}{3}\pi R^3\rho\right)}{R^2}$ or $g \propto R\rho$

$$\therefore \quad \frac{g_1}{g_2} = \frac{R_1}{R_2}\cdot\frac{\rho_1}{\rho_2} \quad \text{or} \quad \left(\frac{1}{4}\right) = (1)\frac{\rho_1}{\rho_2} \quad \text{or} \quad \frac{\rho_1}{\rho_2} = \frac{1}{4}$$

52. (*a*) The initial potential energy (U_i) of object is $U_1 = -\frac{GMm}{3R}$

Final potential energy $U_f = -\frac{GMm}{R}$

By law of conservation of energy, $\Delta\text{KE} = -\Delta\text{PE}$

$$\Rightarrow \quad \frac{1}{2}mv^2 = -[U_f - U_i] = U_i - U_f$$

$$\Rightarrow \quad \frac{1}{2}mv^2 = -\frac{GMm}{3R} + \frac{GMm}{R}$$

$$\Rightarrow \quad \frac{1}{2}v^2 = \frac{2GM}{3R} \Rightarrow v = \sqrt{\frac{4GM}{3R}} = 2\sqrt{\frac{GM}{3R}}$$

53. (*a*) Using conservation of angular momentum,

$$mv_p r_p = mv_a r_a$$

As velocities are perpendicular to the radius vectors at apogee and perigee.

$$\Rightarrow \quad v_p r_p = v_a r_a$$

Using conservation of energy,

$$-\frac{GMm}{r_p} + \frac{1}{2}mv_p^2 = \frac{-GMm}{r_a} + \frac{1}{2}mv_a^2$$

By solving, the above equations,

$$v_p = \sqrt{\frac{2GMr_a}{r_p(r_p + r_a)}} \Rightarrow L = mv_p r_p = m\sqrt{\frac{2GMr_p r_a}{(r_p + r_a)}}$$

54. (*a*) As, $g = \frac{4}{3}\pi\rho Gr$

$\therefore$ $g \propto r$ if $r < R$

$$g = \frac{GM}{r^2}$$

$$\therefore \quad g \propto \frac{1}{r^2} \text{ if } r > R$$

If $r_1 < R$ and $r_2 < R$, then $\frac{F_1}{F_2} = \frac{g_1}{g_2} = \frac{r_1}{r_2}$

If $r_1 > R$ and $r_2 > R$, then $\frac{F_1}{F_2} = \frac{g_1}{g_2} = \left(\frac{r_2}{r_1}\right)^2$

55. (*a*) $\frac{1}{2}\mu v_r^2 = U_i - U_f$. Here, μ = reduced mass $= \frac{mM}{M+m}$

$$\therefore \quad \frac{1}{2}\left(\frac{mMv_r^2}{m+M}\right) = 0 - \left(-\frac{GMm}{d}\right)$$

$\therefore$ Relative velocity, $v_r = \sqrt{\frac{2G(M+m)}{d}}$

56. (*b*) $E_1 = \Delta U = \left(\frac{mgh}{1 + h/R}\right)$

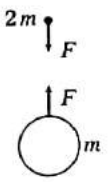
E_2 = Energy of satellite-energy of satellite on surface of earth.

$$= -\frac{GMm}{2(R+h)} + \frac{GMm}{R} = mgR\left[1 - \frac{1}{2(1+h/R)}\right]$$

$$\Rightarrow \quad E_2 = \frac{mgR\left(\frac{2h}{R} + 1\right)}{2\left(1 + \frac{h}{R}\right)}$$

$$\therefore \quad \frac{E_1}{E_2} = \frac{mgh}{1 + \frac{h}{R}} \times \frac{2\left(1 + \frac{h}{R}\right)}{mgR} = \frac{2h}{R} \qquad \left(\because h << R \quad \therefore 1 + \frac{2h}{R} \approx 1\right)$$

57. (*c*) Acceleration of super dense material $= g$



But acceleration of earth $= 2g$ (towards the mass)

$\therefore$ Relative acceleration $= 3g$

Since, we know that, $h = ut + \frac{1}{2}a_r t^2$

But the body starts from rest.

So, $\quad u = 0$

Now, $\quad h = \frac{1}{2}a_r t^2$ or $h = \frac{1}{2}(3g)t^2$ or $t = \sqrt{\frac{2h}{3g}}$

58. (*a*) $E = \Delta U = \frac{mgh}{1 + h/R} = \frac{mghR}{(R+h)}$

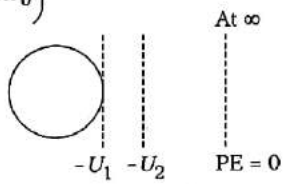
E' = energy of satellite - energy of satellite on surface of earth

$$= -\frac{GMm}{2(R+h)} - \left(-\frac{GMm}{R}\right)$$

$$= mgR^2\left[\frac{1}{R} - \frac{1}{2(R+h)}\right] = \frac{mg(R+2h)R}{2(R+h)}$$

Now, $\quad \frac{E}{E'} = \frac{2h}{(R+2h)}$

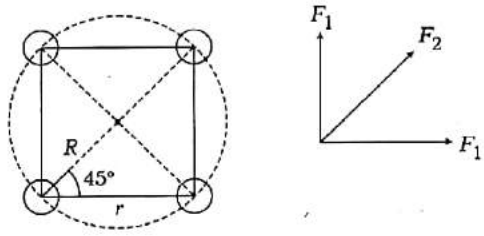
59. (*d*) $U_2 = 2\left(\frac{1}{2}mv_0^2\right)$



Now, $\quad \frac{1}{2}mv^2 = -U_2 + U_1 = U_1 - U_2 = \frac{mv_e^2}{2} - mv_o^2$

$$\therefore \quad v = \sqrt{v_e^2 - 2v_o^2}$$

60. (*d*) Side of square : $r = 2R\cos 45° = \sqrt{2}R$



Now, $\quad 2F_1\cos 45° + F_2 = \frac{Mv^2}{R}$

$$\Rightarrow \quad v = \sqrt{\frac{R}{M}(\sqrt{2}F_1 + F_2)}$$

$$\Rightarrow \quad v = \sqrt{\frac{R}{M}\left[\sqrt{2}\frac{GM.M}{(\sqrt{2}R)^2} + \frac{GMM}{(2R)^2}\right]}$$

$$= \sqrt{\frac{GM}{R}\left(\frac{2\sqrt{2}+1}{4}\right)}$$

61. (*a*) From the figure,

$$\mathbf{F}_A = \mathbf{F}_{AB} + \mathbf{F}_{AC} = 2\left[\frac{GM^2}{a^2}\right]\cos 30° = \left[\frac{Gm^2}{a^2}\cdot\sqrt{3}\right]$$

Also, $\quad r = \frac{a}{\sqrt{3}}$,

Now, $\quad \frac{mv^2}{r} = F|\mathbf{F}_A|$ or $\frac{mv^2\sqrt{3}}{a} = \frac{Gm}{a^2}\sqrt{3}$

$$\therefore \quad v = \sqrt{\frac{Gm}{a}}$$

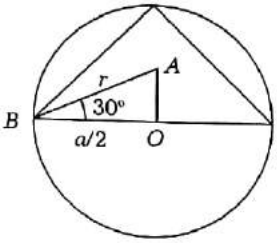
62. (*b*) In right angled ΔAOB, $\cos 30° = \frac{OB}{AB}$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{a}{2r} \Rightarrow r = \frac{a}{\sqrt{3}}$$

$$\sqrt{3}F = mr\,\omega^2$$

$$\sqrt{3}\left(\frac{Gmm}{a^2}\right) = m\frac{a}{\sqrt{3}}.\omega^2$$

$$\therefore \quad \omega = \sqrt{\frac{3Gm}{a^3}}$$



63. (*b*) Work done, $W = U_2 - U_1 = mv_2 - mv_1$

$$= m\left[\left\{-\frac{Gm_2}{R} - \frac{Gm_1}{\sqrt{2}R}\right\} - \left\{-\frac{Gm_1}{R} - \frac{Gm_2}{\sqrt{2}R}\right\}\right]$$

$$= \frac{Gm(m_1 - m_2)}{\sqrt{2}R}(\sqrt{2} - 1) = \frac{\sqrt{2}Gm^2}{R}(1 - \sqrt{2})$$

64. (*c*) Field strength is uniform inside the cavity. Let us find at its centre.

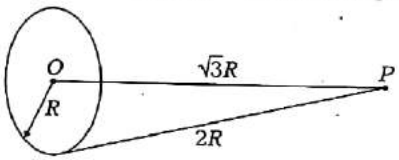
$E_T = E_R + E_C$ (T = Total, R = Remaining, C = Cavity)

$$\therefore \quad E_R = E_T - E_C$$

$$= \frac{GM}{R^3}\frac{R}{2} - 0 = \frac{GM}{2R^2}$$

$$\therefore \quad F = mF_R = \frac{GMm}{2R^2}$$

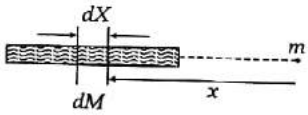
65. (*a*) Increase in kinetic energy = decrease in potential energy



or $\quad \frac{1}{2}mv^2 = U_i - U_f = m(V_i - V_f)$

or $\quad v = \sqrt{2(V_i - V_f)} = \sqrt{2\left[-\frac{GM}{2R} + \frac{GM}{R}\right]} = \sqrt{\frac{GM}{R}}$

66. (*b*) $dF = \frac{GmdM}{x^2}$



$$\therefore \quad F = \int_{x=a}^{x=a+l} dF$$

$$= \int_{a-l/2}^{a+l} \frac{G\cdot m\left(\frac{M}{l}\cdot dx\right)}{x^2} = \frac{4GmM}{4a^2 - l^2}$$

67. (*d*) $F_1 = \frac{GMm}{(2R)^2} = \frac{GMm}{4R^2}$

$$F_2 = F_1 - F_{\text{cavity}} = \frac{GMm}{4R^2} - \frac{G\left(\frac{M}{8}\right)(m)}{\left(\frac{3}{2}R\right)^2}$$

$$\Rightarrow \quad F_2 = \frac{7GMm}{36R^2} \Rightarrow \frac{F_2}{F_1} = \frac{7}{9}$$

68. (*d*) $a = g' = g\left(1 - \frac{h}{R}\right) = g\left(\frac{R-h}{R}\right) = \frac{g}{R}\cdot y \qquad \left(\because \frac{R-h}{R} = y\right)$

$$= \frac{\left(\frac{GM}{R^2}\right)}{R}\cdot y = \frac{G}{R^3}\left(\frac{4}{3}\pi R^3\rho\right)\cdot y = \frac{4}{3}\pi\rho Gy$$

B) Medical entrance special format question

Assertion and reason

2. (*d*) Angular momentum is constant about the centre of sun and the force acting on the planet is a central force.

8. (*d*) On two sides of a point mass, direction of gravitational field strength will be different. Further, gravitational potential is negative. Hence, it increase in moving away from the point mass.

9. (*d*) Gravitational potential due to a point mass at some finite distance is always negative.

10. (*c*) $T = \frac{2\pi}{\sqrt{GM}}r^{3/2}$. Time period does not depend upon the radius of earth (R) but it depends upon the mass of earth (M).

11. (*d*) In case of cube the formula cannot be applied directly.

13. (*c*) Gravitational force by earth is utilised in providing the necessary centripetal force for rotating round the earth. That's why this force in not felt to us. But gravitational force by moon is unutilised. That's why it is felt.

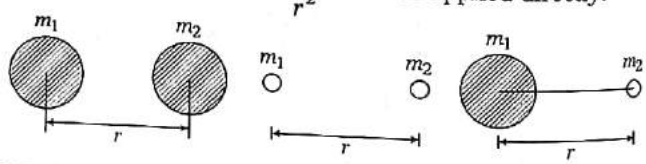
15. (*a*) $v_e = \sqrt{2gR} = \sqrt{\frac{2GM}{R}}$

$$\therefore \; v_e \propto \sqrt{\frac{M}{R}} \Rightarrow (4\pi R^2)\cdot\rho = (4\pi R^2)\frac{M}{(4/3)\pi R^3} \propto \frac{M}{R}$$

16. (*d*) $\frac{dA}{dt} = \frac{L}{2m}$ = constant $= \frac{mvr\sin\theta}{2m} = \frac{vr\sin\theta}{2}$

i.e. $\frac{dA}{dt}$ is independent of m.

17. (*d*) In the three cases $F = \frac{Gm_1m_2}{r^2}$ can be applied directly.



18. (*d*) Field strengths due to masses at 1 and 2 are acting at 60°. Therefore, resultant is $\frac{\sqrt{3}Gm}{a^2}$.

19. (*d*) $W_2 = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\sqrt{\frac{GM}{r}}\right)^2 = \frac{GMm}{2r} = \frac{GMm}{4R}$

$$W_1 = \Delta U = \frac{mgh}{1 + \frac{h}{R}} = \frac{mgRh}{R+h}$$

$$= \frac{m\left(\frac{GM}{R^2}\right)Rh}{r} = \left(\frac{GMm}{rR}\right)h = \frac{GMm}{r}$$

20. (*d*) In the region $0 < r < r_1$

$F = 0$ but v = constant

Match the columns

2. $g = \frac{Gm}{R^2} = \frac{G\left(\frac{4}{3}\pi R^3\right)\rho}{R^2}$ or $g \propto \rho R$

At surface, $V = -\frac{Gm}{R} = -\frac{G\left(\frac{4}{3}\pi R^3 \rho\right)}{R}$ or $V \propto \rho R^2$

At centre, $V = \frac{-15Gm}{R}$ or $V \propto \rho R^2$

At surface and centre potential is 0.5 times potential at surface and centre of a solid sphere. Field strength at centre is zero is both cases.

3. $g = \frac{GM}{R^2}, V = -\frac{GM}{R}$

At height $h = R$, $g' = \frac{g}{1+\frac{h}{R}} = \frac{g}{2} \Rightarrow V' = -\frac{GM}{2R}$

i.e. decrease by a factor 1/2 and V' increase by a factor of 1/2.

At depth $h = \frac{R}{2}$

$$g' = g\left(1-\frac{h}{R}\right) = g\left(1-\frac{1}{2}\right) = \frac{g}{2}$$

$$V' = -\frac{GM}{R^3}[15R^2 - 0.5r^2] = -\frac{GM}{R^3}\left[15R^2 - \frac{1}{8}R^2\right] = -\frac{11}{8}\frac{GM}{R}$$

i.e. g' decrease by a factor $\frac{1}{2}$ and V' also decreases by a factor $\frac{11}{8}$.

5. Inside a shell gravitational potential $V = -\frac{GM}{R} =$ constant

And gravitational field intensity $E = 0$

Outside the shell, $V = -\frac{GM}{r}$ and $E = \frac{GM}{r^2}$

As r increase, V increase and E decreases.

Therefore, potential at A. will be less than B. Gravitational field inside the shell will be zero. As one moves from C to D, the gravitational potential remains constant and when one moves from D to A, gravitational field decreases.

7. Given, $g = \frac{GM}{R^2} =$ constant $(\because x << R$ and $Y << R)$

When speed becomes v to $\frac{v}{2}$.

$$\left(\frac{v}{2}\right) = v - g \times t_1 \Rightarrow t_1 = \frac{v}{2g}$$

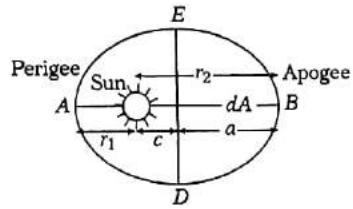
and $$\left(\frac{v}{2}\right)^2 = v^2 - 2gx \Rightarrow x = \frac{3v^2}{8g}$$

Similarly, $$0 = \frac{v}{2} - g \times t_2 \Rightarrow t_2 = \frac{v}{2g}$$

and $$0 = \left(\frac{v}{2}\right)^2 - 2gy \Rightarrow y = \frac{v^2}{8g}$$

Clearly $t_1 = t_2$, $x > y$

8. In the figure given alongside, it can be seen that at perigee position, planet is nearest to the sun. So, in the elliptical moves, as the planet orbit from opogee position to perigee position, then speed of planet increases, distance of planet from centre of sun decreases, potential energy decreases, Angular momentum about centre of sun remains the same.



9. $T = \frac{2}{\sqrt{GM}} r^{3/2}, v_0 = \sqrt{\frac{GM}{r}}$ and $E = -\frac{GMm}{2r}$

10. Due to rotation of earth, $g' = g - R\omega^2 \cos^2 \phi$

At pole there is no effect of rotation of earth.

As $\phi = 90°$

At equator g' will increase, if ω decreases.

Further, T will increase with decrease in ω.

From Kepler's third law, r should also increase.

$E = -\frac{GMm}{2r}$, so with increase in r, E also increases.

C) Medical entrance gallery

1. (*b*) Acceleration due to gravity at a depth d below the surface of the earth is given by

$$g_{\text{depth}} = g_{\text{surface}}\left(1-\frac{d}{R}\right)$$

Also, for a point at height h above surface,

$$g_{\text{height}} = g_{\text{surface}}\left[\frac{R^2}{(R+h)^2}\right]$$

Therefore, we can say that value of g increases from centre of maximum at the surface and then decreases asdepicted in graph (b).

2. (*b*) Total energy of a satellite at height h is

$$= \text{KE} + \text{PE} = \frac{GMm}{2(R+h)} - \frac{GMm}{(R+h)}$$

$$= \frac{-GMm}{2(R+h)} = \frac{-GMmR^2}{2R^2(R+h)} = \frac{-mg_0R^2}{2(R+h)} \quad \left(\because g_0 = \frac{GM}{R^2}\right)$$

3. (*d*) Gravitational potential at some height h from the surface of the earth is given by

$$V = -\frac{GM}{R+h} \quad \text{...(i)}$$

And acceleration due to gravity at some height h from the earth surface can be given as

$$g' = \frac{GM}{(R+h)^2} \quad \text{...(ii)}$$

From Eq. (i) and (ii), we get

$$\frac{|V|}{g'} = \frac{GM}{(R+h)} \times \frac{(R+h)^2}{GM} \Rightarrow \frac{|V|}{g'} = R + h \quad \text{...(iii)}$$

$\because$ $V = -5.4 \times 10^7$ J kg^{-2} and $g' = 6$ ms^{-2}

Radius of earth, $R = 6400$ km.

Substitute these values in Eq. (iii), we get

$$\frac{5.4\times10^7}{6} = R + h \Rightarrow 9\times10^6 = R + h$$

$$\Rightarrow h = (9-6.4)\times10^6 = 2.6\times10^6 \text{ m} \Rightarrow h = 2600 \text{ km}$$

4. (*a*) Since, the escape velocity of the earth can be given as

$$v_e = \sqrt{2gR} = R\sqrt{\frac{8}{3}\pi G\rho} \qquad [\rho = \text{density of the earth}]$$

$$\Rightarrow \quad v_e = R\sqrt{\frac{8}{3}\pi G\rho} \qquad \text{...(i)}$$

As it is given that the radius and mean density of planet are twice as that of the earth. So, escape velocity at planet will be

$$v_p = 2R\sqrt{\frac{8}{3}\pi G2\rho} \qquad \text{...(ii)}$$

Divide, Eq. (i) by Eq. (ii), we get

$$\frac{v_e}{v_p} = \frac{R\sqrt{\frac{8}{3}\pi G\rho}}{2R\sqrt{\frac{8}{3}\pi G2\rho}} \Rightarrow \frac{v_e}{v_p} = \frac{1}{2\sqrt{2}}$$

5. (*b*) The gravitational force of attraction between the planet and sun provide the centripetal force

i.e. $$\frac{GMm}{r^2} = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{GM}{r}}$$

The time period of planet will be

$$T = \frac{2\pi r}{v} \Rightarrow T^2 = \frac{4\pi^2 r^2}{\frac{GM}{r}} = \frac{4\pi^2 r^3}{GM} \qquad \text{...(i)}$$

Also from Kepler's third law

$$T^2 = Kr^3 \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{4\pi^2 r^3}{GM} = Kr^3 \Rightarrow GMK = 4\pi^2$$

6. (*a*) At the pole, the weight is same as the true one

Thus, $100 \text{ N} = m\,(10 \text{ ms}^{-2}) \Rightarrow m = 10 \text{ kg}$

At the equator, the apparent weight is given by

$$mg' = mg - m\,\omega^2 R$$

Also the angular speed of an equatorial point on the earth's surface is

$$\omega = (2\pi/24\times60\times60)$$

$$\Rightarrow \quad \omega = 7.27\times10^5$$

Now $mg' = 100 - 10\,(7.27\times10^{-5})^2\times6.4\times10^6 = 99.66 \text{ N}$

7. (*d*) The gravitation field at the centre of a solid sphere of mass M is zero.

8. (*b*) The value of g at a depth h from the earth's surface

$$g = g_E\left(1-\frac{d}{R}\right)$$

where, g_E acceleration due to gravity at the earth's surface.

$$\Rightarrow \quad g = 9.8\left(1-\frac{5}{6400}\right) = 9.8\left(\frac{6395}{6400}\right) = 9.79 \text{ ms}^{-2}$$

9. (*c*) The gravitational force between the particles will give the required centripetal force.

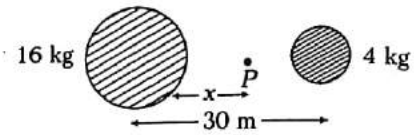
So, $$\frac{GM^2}{(2R)^2} = \frac{Mv^2}{R} \Rightarrow \frac{GM^2}{4R^2} = \frac{Mv^2}{R} \Rightarrow v = \sqrt{\frac{GM}{4R}}$$

10. (*b*) The escape velocity is given by

$$v_e = \sqrt{\frac{2GM}{R}}$$

$$v_e = \sqrt{\frac{2\times6.67\times10^{-11}\times7.4\times10^{22}}{1740\times10^3}} = 2.4 \text{ kms}^{-1}$$

11. (*b*) Consider the diagram below,



Let P be a point at a distance x from 16 kg sphere, where the net gravitation field (and hence force) due to two spheres is zero then

$$\frac{16G}{x^2} = \frac{4G}{(30-x)^2} \Rightarrow \frac{4}{x^2} = \frac{1}{(30-x)^2}$$

$$\Rightarrow \quad 4\times30\times30 + 4x^2 - 4\times2\times30x = x^2$$

$$\Rightarrow \quad 3600 + 3x^2 - 240x = 0 \Rightarrow x^2 + 1200 - 80x = 0$$

$$\Rightarrow \quad x = \frac{80 \pm \sqrt{(-80)^2 - 4\times1200\times1}}{2}$$

$$\Rightarrow \quad x = \frac{80\pm\sqrt{1600}}{2} = \frac{80\pm40}{2} = 60 \text{ or } 20$$

As $x = 60$ m in not possible so, $x = 20$ m

12. (*b*) Orbital velocity of earth, $v_0 = \sqrt{gR}$. So, that orbital velocity of earth satellite does not depend on mass of the satellite.

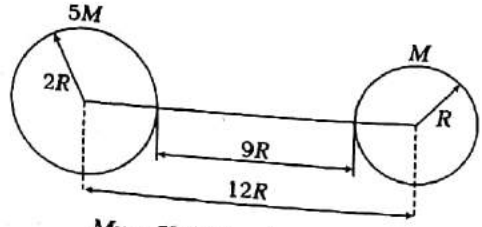
13. (*e*) Gravitational potential energy of a body of mass m at a height h above the surface of earth is given by $-\frac{GMm}{R+h}$

14. (*c*) By Kepler's third law of planetary motion, $T^2 \propto R^3$

$$\left[\frac{T_1}{T_2}\right]^2 = \left[\frac{r_1}{r_2}\right]^3$$

15. (*d*) Mass of a body is its inherent property. It is independent of location of the body. Thus, mass of the remains same (M) whether it is on the surface of the earth or on the surface of the moon.

16. (*c*) Suppose, the smaller body cover a distance x before collision, then

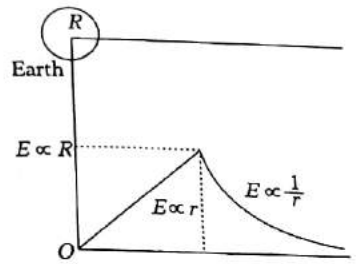


$$Mx = 5M(9R - x)$$

or, $$x = 45R - 5x \text{ or } x = \frac{45R}{6} = 7.5\,R$$

17. (*c*) As the workdone by the gravitational force f in closed path is zero, so it is conservative in nature.

18. (*b*)



19. (*a*) As we know that,

Time period $(T) = 2\pi\sqrt{\frac{R_e}{g}}$ ...(i)

So, if $R' = \frac{R_e}{4} \Rightarrow T' = 2\pi\sqrt{\frac{R_e}{4g}}$ ...(ii)

$\Rightarrow$ From Eqs. (i) and (ii), we get $\frac{T}{T'} = \frac{2\pi\sqrt{R_e/g}}{2\pi\sqrt{R_e/g}} \times 2$

$\Rightarrow$ $T' = \frac{T}{2} = \frac{24}{2} = 12$ h.

20. (*d*) Potential energy at the earth surface $= \frac{-GMm}{R}$

Potential energy at a height h above the earth's surface

$= \frac{-GMm}{(R+h)}$

$\therefore$ Change in potential energy $= \frac{-GMm}{(R+h)} - \left(-\frac{GMm}{R}\right) = \frac{GMmh}{R(R+h)}$

Substituting, $h = 10R$

$\Rightarrow$ $\Delta PE = \frac{GMm \times 10R}{R(R+10R)} = \frac{10\,GMm}{11R}$

21. (*a*) We know that, the potential energy of the satellite

$$U = -\frac{GM_em}{R_e}$$

The kinetic energy of the satellite $K = \frac{1}{2}\frac{GM_em}{R_e}$

The total energy, $E = U + K = -\frac{GM_em}{R_e} + \frac{1}{2}\frac{GM_em}{R_e} = -\frac{1}{2}\frac{GM_em}{R_e}$

$\Rightarrow$ $2E = -\frac{GM_em}{R_e} \Rightarrow -2E = U$

So, potential energy $= -2$ (total energy)

$\Rightarrow$ PE $= -2E$

22. (*c*) Given, $R_e = 6400$ km $= 6.4 \times 10^6$ m, $\pi = 3.14$, $g = 10\,\text{ms}^{-2}$

We know that the period of revolution of the earth satellite

$$T = 2\pi\sqrt{\frac{(R_e+h)^3}{gR_e^2}} \quad [\text{if } h << R_e, \text{ then } (R_e + h = R_e)]$$

So, $T = 2\pi\sqrt{\frac{R_e^3}{gR_e^2}} = 2\pi\sqrt{\frac{R_e}{g}} = 2 \times 3.14\sqrt{\frac{6.4 \times 10^6}{10}}$

$= 2 \times 3.14 \times 0.8 \times 10^3$

$= 5.024 \times 10^3 = 5024$ s $= 83.73$ min

23. (*a*) Given, $H = 35800$ km is height of geostationary orbit.

$\Rightarrow$ $T =$ time period of earth about own axis $= 24$ h

24. (*a*) We know that the total energy of the satellite is

$$E = -\frac{1}{2}\frac{GM_em}{R}$$

At a height H from the surface of the earth, the total energy of satellite,

$$E = -\frac{1}{2}\frac{GM_em}{R+H} \quad ...(i)$$

and the potential energy,

$$U = -\frac{GM_em}{R+3R} = -\frac{GM_em}{4R} \quad ...(iii)$$

According to the question,

$$E = U - \frac{1}{2}\cdot\frac{GM_em}{R+H} = -\frac{GM_em}{4R}$$

$\Rightarrow$ $\frac{1}{2R+2H} = \frac{-1}{4R} \Rightarrow 2H = 4R - 2R \Rightarrow H = R$

25. (*b*) Change in potential energy

$$\Delta U = -\frac{GMm}{R+2R} - \left(-\frac{GMm}{R}\right)$$

$$= -\frac{GMm}{3R} + \frac{GMm}{R} = \frac{2GMm}{3R} = \frac{2}{3}mgR \quad \left[\because g = \frac{GM}{R^2}\right]$$

26. (*d*) The resulting gravitational potential,

$$V = -2G\left[\frac{1}{1} + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots\right]$$

$\Rightarrow$ $V = -2G\left[1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3}\ldots\right]$

$\Rightarrow$ $V = -2G\left(1 + \frac{1}{2}\right)^{-1} \Rightarrow V = -\frac{2G}{\left(1 - \frac{1}{2}\right)} = \frac{-2G}{\left(\frac{1}{2}\right)} = -4G$

27. (*c*) Universal law of gravitation states that the gravitational force of attraction acting between two of universe is $F = \frac{Gm_1m_2}{r^2}$ where G is gravitational constant and r is the distance between them. Hence, it is also known as inverse square law.

28. (*a*) Value of acceleration due to gravity is $g = \frac{GM}{R^2}$

At poles, R is minimum and at equator R is maximum, hence g is maximum at poles.

29. (*b*) Escape velocity $v_e = \sqrt{2gR}$

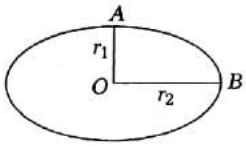
30. (*a*) Here, $g_0 = g_{90} - R\omega^2$ Given, $g_0 = \frac{g}{2}$ and $g_{90} = g$

So, $\frac{g}{2} = g - R\omega^2$, $\omega = \sqrt{\frac{g}{2R}}$

We have, $T = 2\pi\frac{r}{v} \Rightarrow T = \frac{2\pi}{\omega}$ $\left(\because \omega = \frac{v}{r}\right)$

Hence, $T = 2\pi\sqrt{\frac{2R}{g}}$

31. (*c*) In central force motion, the angular momentum of body remains conserved.



i.e., Angular momentum,

$\mathbf{L} = \mathbf{r} \times \mathbf{p} = mvr$

[at minor and major axis, **v** and **r** are perpendicular]

So if v_1 and v_2 be the speed of earth at A and B, respectively.

Then, $mv_1r_1 = mv_2r_2$

$$\Rightarrow \quad \frac{v_1}{v_2} = \frac{r_2}{r_1} = \frac{OB}{OA} \Rightarrow \frac{v_1}{v_2} = R$$

32. (*a*) We know the gravity, $g = \frac{4}{3}\pi GRd$

Here given densities d_1 and d_2 and radii R_1 and R_2

$$\text{So,} \quad \frac{g_1}{g_2} = \frac{\frac{4}{3}\pi GR_1d_1}{\frac{4}{3}\pi GP_2d_2} \Rightarrow \frac{g_1}{g_2} = \frac{R_1d_1}{R_2d_2}$$

33. (*d*) Weight of the object at height h above the surface of the earth

$$w' = \frac{w}{\left(1+\frac{h}{R}\right)^2} \Rightarrow w'(h = R/2) = \frac{w}{(1+1/2)^2}$$

$$\Rightarrow \quad w' = w \times \frac{4}{9} = 90 \times \frac{4}{9} = 40 \text{ kg}$$

34. (*c*) We know that escape velocity, $v_e = \sqrt{\frac{2GM_e}{R_e}}$. Substituting the values, we get $v_e = 112 \text{ kms}^{-1}$. But if the projected velocity is double then the speed becomes $\sqrt{3}$ times and having the value $112 \times \sqrt{3} = 19.4 \text{ kms}^{-1}$.

35. (*c*) Angular momentum = linear momentum × perpendicular distance from the axis of rotation.

= mass × orbital velocity × radius

$$= m \times \sqrt{\frac{GM}{R_0}} \times R_0 = m\sqrt{GMR_0}$$

36. (*b*) V_B = Potential at B due to A + Potential at A due to B

$$V_B = \frac{-Gm_2}{R} - \frac{-Gm_1}{\sqrt{2}R} \Rightarrow V_A = \frac{-Gm_1}{R} - \frac{-Gm_2}{\sqrt{2}R}$$

$$W_{A\to B} = m(V_B - V_A) = \frac{Gm(m_1 - m_2)}{\sqrt{2}R}(\sqrt{2}-1)$$

37. (*a*) According to Kepler's third law,

$$T^2 \propto r^3 \Rightarrow T \propto r^{3/2}$$

We have $\omega = \frac{2\pi}{T} \Rightarrow \omega \propto r^{-3/2}$

Now, $L = mr^2\omega \Rightarrow L \propto r^2$ and $l \propto \omega$, *i.e.*, $l \propto r^{-3/2}$

$$\Rightarrow \quad L \propto r^2 . r^{-3/2} \quad \therefore L \propto r^{1/2}$$

38. (*a*) Force, $F = \frac{GM_em}{R^2}$

Here, $F = \frac{GM_Pm}{(D_P/2)^2} \Rightarrow F = \frac{4GM_Pm}{D_P^2} \Rightarrow F = ma$

Acceleration due to gravity, $a = \frac{F}{m} = \frac{4GM_P}{D_P^2}$

39. (*c*) From Kepler's third law, $T^2 \propto r^3$

Hence, $T_1^2 \propto r_1^3$ and $T_2^2 \propto r_2^3$

So, $\frac{T_2^2}{T_1^2} = \frac{r_2^3}{r_1^3} = \frac{(3R)^3}{(6R)^3}$ or $\frac{T_2^2}{T_1^2} = \frac{1}{8}$

$$T_2^2 = \frac{1}{8}T_1^2 \Rightarrow T_2 = \frac{24}{2\sqrt{2}} = 6\sqrt{2} \text{ h}$$

40. (*c*) The orbital velocity of a satellite moving around the earth $v_0 = \sqrt{gr}$. The escape velocity is given by $v_e = \sqrt{2gr}$

$\therefore$ Increase in velocity $= \sqrt{2gr} - \sqrt{gr} = (\sqrt{2}-1)\sqrt{gr} = 0.414\sqrt{gr}$

So, percentage increase in velocity $= \frac{0.414}{\sqrt{gr}}\sqrt{gr} \times 100 = 414\%$

41. (*b*) The energy of artificial satellite at the surface of the earth

$$E_1 = -\frac{GMm}{R}$$

When the satellite is intended to move in a circular orbit of radius $7R$, then energy of artificial satellite, $E_2 = -\frac{1}{2}\frac{GMm}{7R}$

The minimum energy required,

$$E = E_1 - E_2 = -\frac{GMm}{R} + \frac{1}{2}\left(\frac{GMm}{7R}\right)$$

$$= \frac{-14\,GMm + GMm}{14R} = -\frac{13\,GMm}{14R}$$

42. (*a*) Using the law of conservation of angular momentum.

Angular momentum of planet at p

= Angular momentum of planet at a

i.e., $\mathbf{L}_p = \mathbf{L}_a$

But $r_a > r_p$. So, $\mathbf{r}_a \times L_a > \mathbf{r}_p \times \mathbf{L}_p$

43. (*b*) Maximum height attained by body

$$h = R\left[\frac{v^2}{v_e^2 - v^2}\right] \Rightarrow h = R\left[\frac{v_e^2/4}{v_e^2 - v_e^2/4}\right] = R\left[\frac{v_e^2/4}{3v_e^2/4}\right] \Rightarrow h = \frac{R}{3}$$

44. (*b*) Acceleration due to gravity

$$g' = g - \omega^2 R\cos^2\lambda \Rightarrow 0 = g - \omega^2 R\cos^2 60°$$

$$0 = g - \frac{\omega^2 R}{4} \Rightarrow \omega = 2\sqrt{\frac{g}{R}} = 2\sqrt{\frac{10}{6400 \times 1000}}$$

Angular velocity, $\omega' = \frac{1}{400} = 2.5 \times 10^{-3} \text{ rad s}^{-1}$

45. (*a*) Gravitational pull depends upon the acceleration due to gravity on that planet.

$$M_m = \frac{1}{81}M_e, \quad g_m = \frac{1}{6}g_e$$

By the relation, $g = \frac{GM}{R^2}$

$$\frac{R_e}{R_m} = \left(\frac{M_e}{M_m} \times \frac{g_m}{g_e}\right)^{1/2} = \left(81 \times \frac{1}{6}\right)^{1/2} \therefore R_e = \frac{9}{\sqrt{6}}R_m$$

CHAPTER 11

Simple harmonic motion

In this chapter, we are going to study a basic type of motion called simple harmonic motion and its various aspects with the help of some fundamental concepts like period, frequency, amplitude and phase. On the basis of path traced and various other properties, motion can be classified into several categories.

Two main categories of motion are given below

Periodic motion Periodic motion is a motion which repeats itself after a regular intervals of time. *e.g.* Circular motion or orbital motion, motion of a bouncing ball, motion of planets around the sun, rotation of earth about its polar axis motion of a pendulum etc.

Oscillatory motion A motion in which a particle moves to and fro (back and forth or up and down) about a fixed point after regular interval of time, is termed as oscillatory motion. The fixed point about which the body oscillates is called **mean position** or **equilibrium position.**

Oscillatory motion is also called as harmonic motion, *e.g.*, Motion of simple pendulum and spring pendulum etc.

Note Every oscillatory motion is periodic, but every periodic motion need not to be oscillatory. *e.g.*, Motion of the planet around the sun is periodic but not oscillatory.

Of all the oscillatory motions, the most important is called **simple harmonic motion** (SHM). In this type of oscillatory motion, displacement, velocity, acceleration and force vary (w.r.t. time) in a way that can be described by either the sine or the cosine function collectively called **sinusoids.** Any oscillatory motion that cannot be described, in terms of single harmonic function (*i.e.*, sine or cosine function) is called **anharmonic oscillation**.

Inside

1) Simple harmonic motion (SHM)
- Kinematics of SHM
- Force in SHM

2) Energy in SHM

3) Some systems executing simple harmonic motion
- Simple pendulum
- Spring block system
- The physical pendulum

4) SHM in some special cases
- Free oscillation
- Damping and damped oscillations
- Maintained oscillations
- Forced oscillations
- Resonance
- Vector method of combining two or more simple harmonic motions in same direction

1) Simple harmonic motion (SHM)

It is the simplest form of vibratory or oscillatory motion. The particle executing simple harmonic motion oscillates such that its acceleration is always directed towards the mean position and magnitude of acceleration is proportional to the displacement of the particle from mean position.

Mathematically, a simple harmonic motion can be expressed as

$$y = A\sin\omega t = A\sin 2\pi t/T \quad \text{...(i)}$$

or

$$y = A\cos\omega t = A\cos 2\pi t/T \quad \text{...(ii)}$$

Here, y = displacement of body from mean position at any instant t.

A = amplitude or maximum displacement of the body.

ω = angular frequency and

T = time period of SHM

Some basic terms related to SHM

(i) **Restoring force** The force acting on the particle (executing SHM) which tends to bring it towards its mean position, is called restoring force. Restoring force is always directed towards mean position or acts in the direction opposite to that of displacement.

$$\text{Restoring force, } F = -kx$$

Where, k is a constant and x is displacement about mean position.

(ii) **Time period** (T) The time taken by the particle to complete one oscillation is called time period. Its unit is second.

(iii) **Frequency** (n) The number of oscillations made by the particle (executing SHM) is called frequency.

$$\text{Frequency, } n = \frac{1}{\text{Time period}(T)}$$

Its unit is cycles per second or hertz (Hz).

(iv) **Angular frequency** (ω) The rate of change of phase angle of particle (executing SHM) with respect to time is called angular frequency. Its unit is radian/second.

$$\text{Angular frequency, } \omega = \frac{2\pi}{\text{Time period}(T)}$$

$$= 2\pi \times \text{Frequency}(n)$$

(v) **Amplitude** (A) The maximum displacement of particle from its mean position is defined as amplitude.

(vi) **Phase** (ϕ) Time varying quantity, which can fully describe displacement and velocity of particle is called phase. Its unit is radian.

In the given graph, two SHM having same amplitude A and same angular frequency ω are shown.

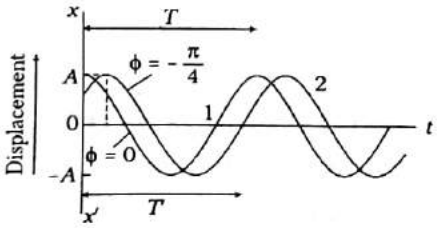


Fig. 11.1 The curves 1 and 2 are for $\phi = 0$ and $-\pi/4$ respectively.

Their phase angle ϕ are different. A phase difference of 2π radian corresponds to a path difference of 1 wavelength.

Example 11.1 *A nurse in a hospital, noted for a patient that heart was beating 75 times a minute. Find its frequency and time period.*

Sol. Number of oscillations (beats) in 60 s = 75

Number of oscillations (beats) in 1 s $= \frac{75}{60} = 1.25$

$\therefore$ Beat frequency of heart $= 1.25\ \text{s}^{-1} = 1.25$ Hz

Time period, $T = \frac{1}{n} = \frac{1}{1.25} = 0.8$ s

Example 11.2 *Two points are located at a distance of 10 m and 15 m from the source of oscillation. The period of oscillation is 0.05 s and the velocity of the wave is 300 ms^{-1}. What is the phase difference between the oscillations of two points?*

Sol. Path difference between two points

$$\Delta x = 15 - 10 = 5 \text{ m}$$

Time period, $T = 0.05$ s

Frequency, $\nu = \frac{1}{T} = \frac{1}{0.05} = 20$ Hz

Velocity, $v = 300\ \text{ms}^{-1}$

$\therefore$ Wavelength, $\lambda = \frac{v}{\nu} = \frac{300}{20} = 15$ m

Hence, phase difference, $\Delta\phi = \frac{2\pi}{\lambda} \times \Delta x = \frac{2\pi}{15} \times 5 = \frac{2\pi}{3}$

Types of SHM

SHM are of two types as follows

(i) **Linear SHM** When a particle moves to and fro about a fixed point (called equilibrium position) along a straight line, then its motion is called linear simple harmonic motion. *e.g.,* Motion of a mass connected to spring.

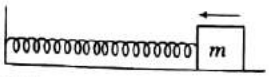


Fig. 11.2 Motion of a spring block system.

(ii) **Angular SHM** When a system oscillates angularly with respect to a fixed axis, then its motion is called angular simple harmonic motion.

e.g., Motion of a bob of simple pendulum.

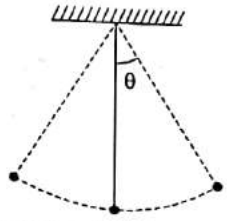


Fig. 11.3 Motion of a pendulum.

Necessary conditions to execute SHM

(i) Motion of particle should be oscillatory.

(ii) Total mechanical energy of particle should be conserved (Kinetic energy + Potential energy = constant).

(iii) Extreme position should be well defined.

(iv) **In linear SHM** The restoring force (or acceleration) acting on the particle should always be proportional to the displacement of the particle and directed towards the equilibrium position.

$\therefore \quad F \propto -y$ or $a \propto -y$

Negative sign shows that direction of force and acceleration is towards equilibrium position and y is displacement of particle from equilibrium position.

(v) **In angular SHM** The restoring torque (or angular acceleration) acting on the particle should always be proportional to the angular displacement of the particle and directed towards the equilibrium position.

$\therefore \quad \tau \propto -\theta$ or $\alpha \propto -\theta$

Kinematics of SHM

Let us consider a particle P moving along a circle of radius A with uniform angular velocity ω in anti-clockwise sense. The motion of N (foot of the perpendicular drawn from the point P) about O is simple harmonic. The particle P is called **reference particle** and the circle along which the particle P revolves is called **circle of reference.**

Thus SHM can be defined as, "the straight line motion of the foot of perpendicular drawn from the particle on the diameter of the circle".

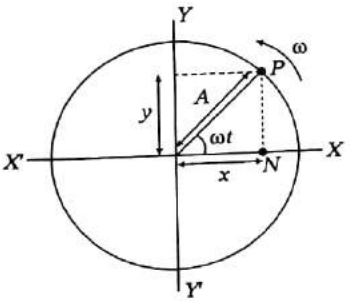


Fig. 11.4 A particle moving along a circle

1. Displacement

The displacement of a particle executing SHM at any instant (t) is given by

$$\boldsymbol{x = A\sin\omega t} \quad \text{or} \quad \boldsymbol{x = A\cos\omega t}$$

First relation is valid when the time is measured from mean position and second relation is valid when time is measured from extreme position.

As shown in figure, consider a particle P moving in anti-clockwise direction with uniform angular velocity ω along a circle of radius A and centre O. Let at time $t = 0$, it makes an angle ϕ with x-axis. In time t, it covers an angle ωt. Then at position P' it makes an angle $(\omega t + \phi)$ with positive x-axis. The position of P' on the axis as the particle moves on the circle is given by

$$\boxed{x = A\sin(\omega t + \phi)} \quad \text{or} \quad \boxed{x = A\cos(\omega t + \phi)}$$

The term $(\omega t + \phi)$ is called phase and ϕ is called initial phase or 'epoch'.

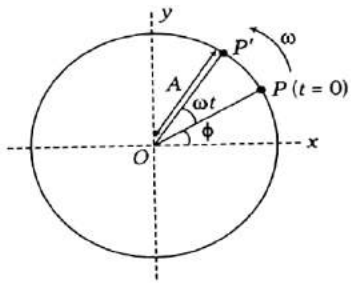


Fig. 11.5

Example 11.3 *An object performs SHM of amplitude 5 cm and time period 4 s. If timing is started when the object is at the centre of the oscillation i.e., $x = 0$ then calculate.*

(i) Frequency of oscillation.

(ii) The displacement at 0.5 s.

Sol. (i) Frequency, $n = \frac{1}{T} = \frac{1}{4} = 0.25$ Hz

(ii) The displacement equation of object is $x = A\sin\omega t$ so at $t = 0.5$ s

$$x = 5\sin(2\pi \times 0.25 \times 0.5) = 5\sin\frac{\pi}{4} = \frac{5}{\sqrt{2}} \text{ cm}$$

Example 11.4 *A particle executes SHM from extreme position and covers a distance equal to half of its amplitude in 1 s. Determine the time period of motion.*

Sol. For particle starting SHM from extreme position

$$Y = A\cos\omega t$$

$$\Rightarrow \quad \frac{A}{2} = A\cos(\omega \times 1) \qquad (\because t = 1\text{ s})$$

$$\Rightarrow \quad \cos\omega = 1/2$$

$$\Rightarrow \quad \cos\omega = \cos\frac{\pi}{3} \Rightarrow \omega = \frac{\pi}{3}$$

$$\Rightarrow \quad T = \frac{2\pi}{\omega} = \frac{2\pi \times 3}{\pi} = 6\text{ s}$$

Example 11.5 *The shortest distance travelled by a particle executing SHM from mean position in 2 s is equal to $\sqrt{3}/2$ times its amplitude. Determine its time period.*

Sol. $t = 2$ s, $x = \frac{\sqrt{3}}{2}A$, $T = ?$

Using $x = A \sin \omega t$, we get $\frac{\sqrt{3}}{2} A = A \sin \frac{2\pi}{T} \times 2$

$\Rightarrow \quad \sin \frac{4\pi}{T} = \frac{\sqrt{3}}{2} = \sin \frac{\pi}{3} \Rightarrow \frac{4\pi}{T} = \frac{\pi}{3}$

$\therefore \quad T = 12$ s

Example 11.6 *Find the equation of simple harmonic motion of a particle whose amplitude is 0.04 and whose frequency is 50 Hz. The initial phase is π/3. Assume that motion of particle is started from mean position.*

Sol. From equation of SHM, $x = A \sin(\omega t + \phi)$

Here, $A = 0.04$ m, $\nu = 50$ Hz, $\phi = \frac{\pi}{3}$

$\therefore \quad x = 0.04 \sin\left(2\pi \times 50t + \frac{\pi}{3}\right) \quad (\because \omega = 2\pi\nu)$

or $\quad x = 0.04 \sin\left(100\pi t + \frac{\pi}{3}\right)$

Example 11.7 *A harmonic oscillation is represented by $x = 0.34 \cos(3000t + 0.74)$, where x and t are in mm and s respectively. Deduce (i) amplitude (ii) angular frequency (iii) initial phase.*

Sol. $x = 0.34 \cos(3000t + 0.74)$

Comparing with general equation $x = A \cos(\omega t + \phi)$

we get, $A = 0.34$ mm, $\omega = 3000$ rads^{-1}, $\phi = 0.74$ rad

(i) Amplitude, $A = 0.34$ mm $= 0.034$ cm.

(ii) Angular frequency, $\omega = 3000$ rads^{-1}

(iii) Initial phase, $\phi = 0.74$ rad

Example 11.8 *A particle executes SHM with a time period of 4 s. Find the time taken by the particle to go from its mean position to half of its amplitude. Assume motion of particle to start from mean position.*

Sol. According to equation of SHM

$x = A \sin(\omega t)$

or $\quad \frac{A}{2} = A \sin(\omega t)$ or $\frac{1}{2} = \sin(\omega t) \Rightarrow \omega t = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$t = \frac{\pi}{6\omega} = \frac{\pi T}{6(2\pi)} \Rightarrow t = \frac{T}{12} = \frac{1}{3}$ s $\quad \left(\text{as } \omega = \frac{2\pi}{T}\right)$

2. Velocity

The velocity of a particle executing SHM at any instant, is defined as the time rate of change of its displacement at that instant.

$$v = dx/dt$$

or $$\boxed{\text{Velocity, } v = \omega\sqrt{A^2 - x^2}}$$

Special Cases

(i) When the particle is at the mean position *i.e.*, $x = 0$, then its velocity is maximum.

$\therefore \quad \boldsymbol{v_{max}} = \omega \boldsymbol{A}$ (maximum velocity)

(ii) When the particle is at the extreme position, *i.e.*, $x = \pm A$, then its velocity is zero.

$$v = \omega\sqrt{A^2 - A^2} = 0$$

Therefore, the **velocity of a particle executing SHM is zero at the either of its extreme positions.**

3. Acceleration

The acceleration of the particle executing SHM at any instant is defined as the time rate of change of its velocity at that instant.

$$a = dv/dt$$

or $$\boxed{\text{Acceleration, } a = -\omega^2 x}$$

Special Cases

(i) When the particle is at the mean position *i.e.*, $x = 0$, then acceleration is zero.

$\therefore \quad \boldsymbol{a_{\text{mean position}} = 0}$ (minimum acceleration)

(ii) When the particle is at the extreme position *i.e.*, $x = A$, then acceleration is maximum.

$\therefore \quad \boldsymbol{a_{(\text{extreme position})} = -\omega^2 A}$ (maximum acceleration)

Graphical representation of displacement, velocity and acceleration in SHM

If a particle is executing SHM and is passing through its positive extreme position ($x = +A$) at $t = 0$, then its displacement equation can be written as

$$x = A \cos \omega t$$

Velocity, $v = \frac{dx}{dt} = -\omega A \sin \omega t = \omega A \cos\left(\omega t + \frac{\pi}{2}\right)$

Acceleration, $a = \frac{dv}{dt} = -\omega^2 A \cos \omega t = \omega^2 A \cos(\omega t + \pi)$

The table gives the different values of displacement, velocity and acceleration for various values of t for one complete cycle.

Time, t	0	$\frac{T}{4}$	$\frac{T}{2}$	$\frac{3T}{4}$	T
Phase angle, $\omega t = \frac{2\pi}{T} t$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
Displacement, $x(t)$	$+A$ max	0 min	$-A$ max	0 min	$+A$ max
Velocity, $v(t)$	0 min	$-\omega A$ max	0 min	$+\omega A$ max	0 min
Acceleration, $a(t)$	$-\omega^2 A$ max	0 min	$+\omega^2 A$ max	0 min	$-\omega^2 A$ max

In the following figure, graph is plotted between x *versus* t, v *versus* t and a *versus* t for different *values* of t)

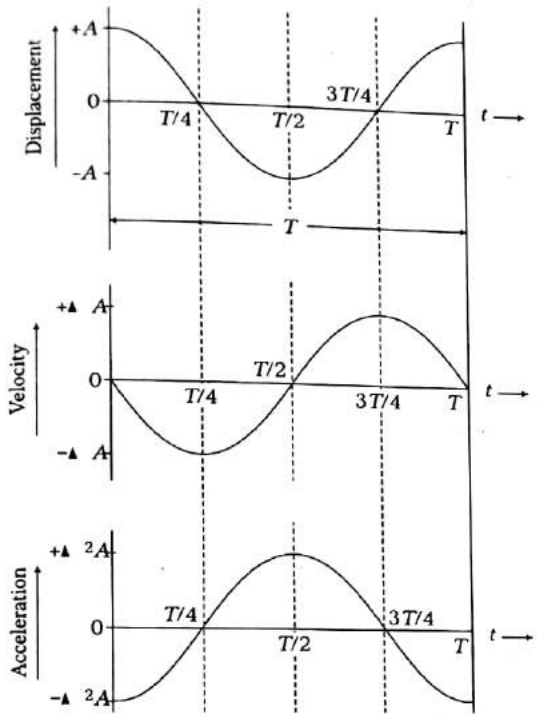


Fig. 11.6 *Relation between displacement, velocity and acceleration in SHM*

Important points regarding kinematics of SHM

(i) In linear SHM, the length of SHM path is 2 A.

(ii) In SHM, the total work done and displacement in one complete oscillation is zero but total traversed length is 4 A.

(iii) In SHM, the velocity and acceleration varies with the same frequency as displacement.

(iv) In SHM, velocity is ahead of displacement by phase angle of $\pi/2$ radian, acceleration is ahead of velocity by phase angle of $\pi/2$ radian and acceleration is ahead of displacement by phase angle of π radian.

Example 11.9 *The particle is executing SHM on a line 4 cm long. If its velocity at mean position is 12 cm/s, then determine its frequency.*

Sol. Amplitude, $A = 2$ cm

At mean position velocity is maximum $v_{max} = A\omega = 12$

$\Rightarrow \quad 2 \times 2\pi n = 12 \quad \Rightarrow \quad n = 3/\pi$ Hz

Example 11.10 *If a SHM is represented by the equation* $x = 10 \sin\left(\pi t + \frac{\pi}{6}\right)$ *in SI units. Determine its amplitude, time period and maximum velocity* v_{max} *?*

Sol. Comparing the above equation with

$$x = A \sin(\omega t + \phi)$$

we get, $\quad A = 10$ m

$$\omega = \pi \text{ s}^{-1} \quad \text{and} \quad \phi = \frac{\pi}{6}$$

$$\because \quad T = \frac{2\pi}{\omega} \quad \Rightarrow \quad T = 2 \text{ s}$$

$$v_{max} = \omega A = 10\,\pi \text{ ms}^{-1}$$

Example 11.11 *Amplitude of a harmonic oscillator is A, when velocity of particle is half of maximum velocity, then determine position of particle.*

Sol. As, $v = \omega\sqrt{A^2 - x^2}$ but it is given that $v = \frac{v_{max}}{2} = \frac{A\omega}{2}$

$$\frac{A\omega}{2} = \omega\sqrt{A^2 - x^2} \Rightarrow A^2 = 4(A^2 - x^2)$$

$$\Rightarrow \quad x^2 = \frac{4A^2 - A^2}{4} \Rightarrow x = \pm\frac{\sqrt{3}A}{2}$$

Example 11.12 *A particle executes SHM with a time period of 2 s and amplitude 10 cm. Find its (i) Displacement (ii) Velocity (iii) Acceleration after 1/6 s, starting from mean position.*

Sol. (i) $T = 2$ s, $A = 10$ cm

$$\text{Displacement, } x = A \sin\frac{2\pi}{T}t$$

$$= 10 \sin\frac{2\pi}{2} \times \frac{1}{6} = 10 \sin\frac{\pi}{6} = 5 \text{ cm}$$

(ii) $$\text{Velocity, } v = \frac{dx}{dt} = A\frac{2\pi}{T}\cos\frac{2\pi}{T}t$$

$$= 10 \times \frac{2\pi}{2}\cos\frac{2\pi}{2} \times \frac{1}{6}$$

$$= 10 \times 3.14 \times \frac{\sqrt{3}}{2} = 27.16 \text{ cm/s}$$

(iii) $$\text{Acceleration, } a = \frac{dv}{dt} = \frac{4\pi^2}{T^2}A\sin\frac{2\pi}{T}t$$

$$= \frac{4 \times (3.14)^2}{4} \times 10 \times \sin\frac{2\pi}{2} \times \frac{1}{6}$$

$$= 49.3 \text{ cm/s}^2$$

Example 11.13 *A body oscillates with SHM according to the equation,* $x = (5\ m)\cos[(2\pi\ rad\ s^{-1})\,t + \pi/4]$.

At $t = 1.5$ *s, calculate the (i) displacement, (ii) speed and (iii) acceleration of the body.*

Sol. The angular frequency ω of the body $= 2\pi$ rad s^{-1} and its time period, $\quad T = 1$ s.

At $\quad t = 1.5$ s,

(i) Displacement $= (5 \text{ m}) \cos[(2\pi \text{ rad s}^{-1}) \times 1.5 \text{ s} + \pi/4]$

$= (5 \text{ m}) \cos[(3\pi + \pi/4) \text{ rad}]$

$= -5 \times 0.707 \text{ m} = -3.535 \text{ m}$

(ii) Using $v = -\omega a \sin(\omega t + \phi)$,

Speed of the body

$$= -(5 \text{ m})(2\pi \text{ s}^{-1}) \sin\left[(2\pi \text{s}^{-1}) \times 1.5 \text{ s} + \frac{\pi}{4}\right]$$

$$= -(5\text{ m})(2\pi s^{-1})\sin\left[\left(3\pi + \frac{\pi}{4}\right)\right]$$

$$= 10\pi \times 0.707\text{ ms}^{-1} = 22\text{ ms}^{-1}.$$

(iii) Acceleration of the body

$$= -(2\pi\text{ rads}^{-1})^2 \times \text{displacement}$$

$$= -(2\pi\text{ rads}^{-1})^2 \times (-3.535\text{ m})$$

$$= 139.56\text{ ms}^{-2}$$

Example 11.14 *A particle executes simple harmonic motion about the point $x = 0$. At time $t = 0$ it has displacement $x = 2$ cm and zero velocity. If the frequency of motion is $0.25\ s^{-1}$, find (i) the period, (ii) angular frequency, (iii) the amplitude, (iv) maximum speed, (v) the displacement at $t = 3$ s and (vi) the velocity at $t = 3$ s.*

Sol. (i) Period, $T = \frac{1}{f} = \frac{1}{0.25\text{ s}^{-1}} = 4\text{ s}$

(ii) Angular frequency,

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{4} = \frac{\pi}{2}\text{ rad s}^{-1} = 1.57\text{ rad s}^{-1}$$

(iii) Amplitude is the maximum displacement from mean position. Hence, $A = 2 - 0 = 2$ cm.

(iv) Maximum speed,

$$v_{max} = A\omega = 2 \cdot \frac{\pi}{2} = \pi\text{ cms}^{-1} = 3.14\text{ cms}^{-1}$$

(v) The displacement is given by

$$x = A\sin(\omega t + \phi)$$

Initially at $t = 0$, $x = 2$ cm, then

$$2 = 2\sin\phi$$

or $\sin\phi = 1 = \sin 90°$

or $\phi = 90°$

Now at $t = 3$ s

$$x = 2\sin\left(\frac{\pi}{2} \times 3 + \frac{\pi}{2}\right) = 0$$

(vi) Velocity at $x = 0$ is v_{max}.

$$\therefore \quad v_{max} = \omega A = \left(\frac{\pi}{2}\right)(2) = 3.14\text{ cms}^{-1}$$

Example 11.15 *Two particles move parallel to x-axis about the origin with the same amplitude and frequency. At a certain instant they are found at distance A/3 from the origin on opposite sides but their velocities are found to be in the same direction. What is the phase difference between the two ?*

Sol. Let equations of two SHM be

$$x_1 = A\sin\omega t \quad \text{...(i)}$$

$$x_2 = A\sin(\omega t + \phi) \quad \text{...(ii)}$$

Given, $\frac{A}{3} = A\sin\omega t$

and $-\frac{A}{3} = A\sin(\omega t + \phi)$

Which gives $\sin\omega t = \frac{1}{3}$ ...(iii)

$$\sin(\omega t + \phi) = -\frac{1}{3} \quad \text{...(iv)}$$

From Eq. (iv), $\sin\omega t\cos\phi + \cos\omega t\sin\phi = -\frac{1}{3}$

$$\Rightarrow \quad \frac{1}{3}\cos\phi + \sqrt{1 - \frac{1}{9}}\sin\phi = -\frac{1}{3}$$

Solving this equation, we get

or $\cos\phi = -1, \frac{7}{9}$

$$\Rightarrow \quad \phi = \pi \text{ or } \cos^{-1}\left(\frac{7}{9}\right)$$

Differentiating Eqs. (i) and (ii) we obtain

$$v_1 = A\omega\cos\omega t$$

and $v_2 = A\omega\cos(\omega t + \phi)$

If we put $\phi = \pi$, we find v_1 and v_2 are of opposite signs. Hence, $\phi = \pi$ is not acceptable.

$$\therefore \quad \phi = \cos^{-1}\left(\frac{7}{9}\right)$$

Example 11.16 *A particle of mass 2 kg executing SHM has amplitude 10 cm and time period is 1 s. Find (i) the angular frequency (ii) the maximum speed (iii) the maximum acceleration (iv) the maximum restoring force (v) the speed when the displacement from the mean position is 8 cm*

(vi) the speed after $\frac{1}{12}$ s the particle was at the extreme position

(vii) the time taken by the particle to go directly from its mean position to half the amplitude (viii) the time taken by the particle to go directly from its extreme position to half the amplitude.

Sol. Given, $m = 2$ kg, amplitude $A = 10$ cm, $T = 1$ s

(i) $\omega = \frac{2\pi}{T} = 2\pi s^{-1} = 6.28\text{ s}^{-1}$

(ii) $v_{max} = A\omega = (10\text{ cm})(2\pi s^{-1}) = 0.628\text{ ms}^{-1}$

(iii) $a_{max} = A\omega^2 = (10\text{ cm})(2\pi s^{-1})^2 = 4\text{ ms}^{-2}$ (take, $\pi^2 = 10$)

(vi) $F_{max} = ma_{max} = mA\omega^2 = (2\text{ kg})(4\text{ ms}^{-2}) = 8$ N

(v) $v = \omega\sqrt{A^2 - x^2} = (6.28\text{ s}^{-1})\sqrt{(10\text{ cm})^2 - (8\text{ cm})^2}$

$= (6.28\text{ s}^{-1})(6\text{ cm})$

$= 37.68$ cm/s

(vi) Suppose $x = A\cos\omega t$, then the particle will be at the extreme position at time $t = 0$, $v = -A\omega\sin\omega t$

$\therefore$ At $t = \frac{1}{12}$ s, $v = -(10\text{ cm})(6.28\text{ s}^{-1})\sin\left(2\pi\text{s}^{-1}\cdot\frac{1}{12}\text{ s}\right)$

$$= -(62.8\text{ cms}^{-1})\sin\frac{\pi}{6} = -31.4\text{ cm s}^{-1}$$

Negative sign indicates that velocity is directed towards the mean position if the particle starts to move from the extreme right.

(vii) When time is taken from the mean position, we take

$$x = A\sin\omega t$$

Suppose, the particle reaches $x = +\frac{A}{2}$, at time t, then

$$\frac{A}{2} = A\sin\omega t \Rightarrow \sin\omega t = \frac{1}{2} \Rightarrow \omega t = \frac{\pi}{6}$$

or, $$t = \frac{\pi}{6\omega} = \frac{\pi}{6\times 2\pi/T} = \frac{T}{12} = \frac{1\text{ s}}{12} = \frac{1}{12}\text{ s}$$

(viii) When time is taken from the extreme position, we take

$$x = A\cos\omega t$$

At $t = 0$, $x = A$, *i.e.*, the particle is at the extreme right

Suppose at time t, the particle reaches $x = \frac{A}{2}$, then

$$\frac{A}{2} = A\cos\omega t$$

$$\Rightarrow \quad \cos\omega t = \frac{1}{2} \Rightarrow \omega t = \frac{\pi}{3}$$

or $$t = \frac{\pi}{3\omega} = \frac{\pi}{3\left(\frac{2\pi}{T}\right)} = \frac{T}{6} = \frac{1}{6}\text{ s}$$

Force in SHM

The acceleration of a body in SHM is $a = -\omega^2 x$. Applying the equation of motion $\mathbf{F} = m\mathbf{a}$, we have

$$F = -m\omega^2 x = -kx$$

where, $$\omega = \sqrt{\frac{k}{m}}$$

Thus, in SHM, the force is directly proportional and opposite to the displacement and is always directed towards the mean position.

This force is called **restoring force.**

Further, since $\omega = \frac{2\pi}{T} = \sqrt{\frac{k}{m}}$

$\therefore$ Time period, $\boldsymbol{T = 2\pi\sqrt{\frac{m}{k}}}$

and frequency, $\boldsymbol{n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}}$

In different types of SHM the quantities m and k will go on taking different forms and names. In general, m is called **inertia factor** and k is called **spring factor.**

In linear SHM, the general formula for time period is

$$\boldsymbol{T = 2\pi\sqrt{\frac{\text{Inertial factor}}{\text{Spring factor}}}}$$

In linear SHM, the spring factor stands for force per unit displacement and inertia factor for mass of the body executing SHM.

Thus, in linear SHM,

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{\left\{\frac{\text{Force}}{\text{Displacement}}\right\}}}$$

or $$\boxed{T = 2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}}}$$

Example 11.17 *Frequency of oscillation of a body is 6 Hz when force F_1 is applied and 8 Hz when F_2 is applied. If both forces F_1 and F_2 are applied together, then find out the frequency of oscillation.*

Sol. According to question $F_1 = -K_1x$ and $F_2 = -K_2x$

So, $$n_1 = \frac{1}{2\pi}\sqrt{\frac{K_1}{m}} = 6\text{ Hz},$$

$$n_2 = \frac{1}{2\pi}\sqrt{\frac{K_2}{m}} = 8\text{Hz}$$

Now, $$F = F_1 + F_2 = -(K_1 + K_2)x$$

Therefore, $$n = \frac{1}{2\pi}\sqrt{\frac{K_1 + K_2}{m}}$$

$$\Rightarrow \quad n = \frac{1}{2\pi}\sqrt{\frac{4\pi^2 n_1^2 m + 4\pi^2 n_2^2 m}{m}}$$

[As, $K = 4\pi^2 n^2 m$]

$$= \sqrt{n_1^2 + n_2^2} = \sqrt{8^2 + 6^2} = 10\text{ Hz}$$

Check point 11.1

1. The motion which is not simple harmonic is
(a) vertical oscillations of a spring
(b) motion of simple pendulum
(c) motion of a planet around the sun
(d) oscillation of liquid column in a U-tube
(e) vertical oscillation of a wooden plank floating in a liquid

2. Select wrong statement about simple harmonic motion.
(a) The body is uniformly accelerated
(b) The velocity of the body changes smoothly at all instants
(c) The amplitude of oscillation is symmetric about the equilibrium position
(d) The frequency of oscillation is independent of amplitude

3. The amplitude and the time period in a SHM is 0.5 cm and 0.4 s respectively. If the initial phase is $\pi/2$ rad, then the equation of SHM will be
(a) $y = 0.5 \sin 5\pi t$ (b) $y = 0.5 \sin 4\pi t$
(c) $y = 0.5 \sin 2.5\pi t$ (d) $y = 0.5 \cos 5\pi t$

4. Which of the following is a necessary and sufficient condition for SHM?
(a) Constant period
(b) Constant acceleration
(c) Proportionality between acceleration and velocity
(d) Proportionality between restoring force and displacement.

5. What is constant in SHM?
(a) Restoring force (b) Kinetic energy
(c) Potential energy (d) Periodic time

6. A body is executing SHM with an amplitude of 0.1 m. Its velocity while passing through the mean position is 3ms^{-1}. Its frequency in Hz is
(a) 15π (b) $\frac{15}{\pi}$ (c) 30π (d) 25π

7. The phase (at a time t) of a particle in simple harmonic motion tells
(a) only the position of the particle at time t
(b) only the direction of motion of the particle at time t
(c) both the position and direction of motion of the particle at time t
(d) neither the position of the particle nor its direction of motion at time t

8. A particle is moving with constant angular velocity along the circumference of a circle. Which of the following statements is true?
(a) The particle executes SHM
(b) The projection of the particle on any one of the diameters executes SHM
(c) The projection of the particle on any of the diameters executes SHM
(d) None of the above

9. The displacement of a particle in SHM is indicated by equation $y = 10 \sin(20t + \pi/3)$ where, y is in metres. The value of time period of vibration will be (in second)
(a) $10/\pi$ (b) $\pi/10$ (c) $2\pi/10$ (d) $10/2\pi$

10. The equation of a simple harmonic motion is $x = 0.34\cos(3000t + 0.74)$, where x and t are in mm and second respectively. The frequency of the motion is
(a) 3000 (b) $3000/2\pi$
(c) $0.74/2\pi$ (d) $3000/\pi$

11. The maximum velocity of a simple harmonic motion represented by $y = 3 \sin\left(100t + \frac{\pi}{6}\right)$ is given by
(a) 300 units (b) $\frac{3\pi}{6}$ units
(c) 100 units (d) $\frac{\pi}{6}$ units

12. Velocity at mean position of a particle executing SHM is v. Velocity of the particle at a distance equal to half of the amplitude will be
(a) $\frac{v}{2}$ (b) $\frac{v}{\sqrt{2}}$ (c) $\frac{\sqrt{3}}{2}v$ (d) $\frac{\sqrt{3}}{4}v$

13. A particle executes simple harmonic motion with an amplitude of 4 cm. At the mean position, the velocity of the particle is 10 cms^{-1}. The distance of the particle from the mean position when its speed becomes 5 cms^{-1} is
(a) $\sqrt{3}$ cm (b) $\sqrt{5}$ cm
(c) $2\sqrt{3}$ cm (d) $2\sqrt{5}$ cm

14. A body is executing simple harmonic motion with an angular frequency of 2 rad s^{-1}. The velocity of the body at 20 mm displacement, when the amplitude of the motion is 60 mm is
(a) 131 mms^{-1} (b) 118 mms^{-1}
(c) 113 mms^{-1} (d) 90 mms^{-1}

15. The displacement of a particle executing SHM is given by $y = 0.25 \sin 200t$ cm. The maximum speed of the particle is
(a) 200 cms^{-1} (b) 100 cms^{-1}
(c) 50 cms^{-1} (d) 5.25 cms^{-1}

16. When a particle executes SHM there is always a constant ratio between its displacement and
(a) velocity (b) acceleration
(c) mass (d) time period

17. In simple harmonic motion, the ratio of acceleration of the particle to its displacement at any time is a measure of
(a) spring constant (b) angular frequency
(c) (angular frequency)^2 (d) restoring force

18. If a particle is executing simple harmonic motion then acceleration of particle
(a) is uniform (b) varies linearly with time
(c) is non-uniform (d) Both (b) and (c)

19. What is the maximum acceleration of the particle executing the SHM?

$$y = 2\sin\left[\frac{\pi t}{2} + \phi\right], \text{ where } y \text{ is in cm.}$$

(a) $\frac{\pi}{2}$ cms^{-2} (b) $\frac{\pi^2}{2}$ cms^{-2}

(c) $\frac{\pi}{4}$ cms^{-2} (d) $\frac{\pi^2}{4}$ cms^{-2}

20. The maximum acceleration of a body moving in SHM is a_0 and maximum velocity is v_0. The amplitude is given by

(a) $\frac{v_0^2}{a_0}$ (b) $a_0 v_0$ (c) $\frac{a_0^2}{v_0}$ (d) $\frac{1}{a_0 v_0}$

21. A particle executing simple harmonic motion has an amplitude of 6 cm. Its acceleration at a distance of 2 cm from the mean position is 8 cms^{-2}. The maximum speed of the particle is

(a) 8 cms^{-1} (b) 12 cms^{-1} (c) 16 cms^{-1} (d) 24 cms^{-1}

22. For a particle in SHM, if the amplitude of the displacement is a and the amplitude of velocity is v, the amplitude of acceleration is

(a) va (b) $\frac{v^2}{a}$ (c) $\frac{v^2}{2a}$ (d) $\frac{v}{a}$

23. The average acceleration of a particle performing SHM over one complete oscillation is

(a) $\frac{\omega^2 A}{2}$ (b) $\frac{\omega^2 A}{\sqrt{2}}$ (c) zero (d) $A\omega^2$

24. In simple harmonic motion, the particle is

(a) always accelerated
(b) always retarded
(c) alternately accelerated and retarded
(d) neither accelerated nor retarded

25. In SHM, the acceleration is ahead of velocity by a phase angle

(a) 0° (b) $\frac{\pi}{2}$ (c) π (d) 2π

2 Energy in SHM

The particle executing SHM has both types of energy, potential energy and kinetic energy. When a body is displaced from its equilibrium position by doing work upon it, the body acquires potential energy. When the body is released, it begins to move back with a velocity, thus acquiring kinetic energy.

Kinetic energy

The relation for kinetic energy of a particle executing SHM can be found out by two cases, in terms of displacement and in terms of time.

(i) **In terms of displacement** If mass of the particle executing SHM is m and its displacement at any instant is x, then

$$\mathbf{KE} = \frac{1}{2} m\omega^2 (A^2 - x^2)$$

where, A = amplitude of SHM

As, $\omega = \sqrt{\frac{k}{m}} \Rightarrow m\omega^2 = k$

$\therefore \quad \text{KE} = \frac{1}{2} k (A^2 - x^2)$

KE is maximum at mean position *i.e.*, $x = 0$

$$\therefore \quad \mathbf{KE_{max}} = \frac{1}{2} m\omega^2 A^2$$

KE is minimum (zero) at extreme position *i.e.*, $x = A$

At $x = \pm A$, KE = zero

(ii) **In terms of time** Kinetic energy of a particle executing SHM at any time t is given by

$$K = \frac{1}{2} m\omega^2 A^2 \cos^2 \omega t$$

i.e., Kinetic energy of a particle executing SHM is a periodic function of time.

Kinetic energy versus time graph is given below.

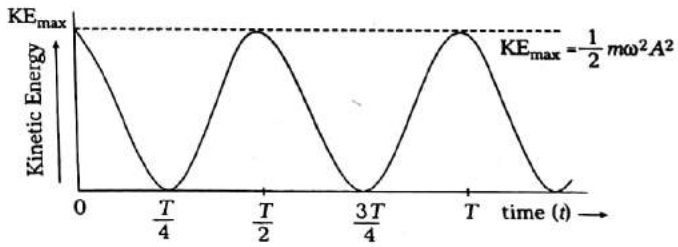


Fig. 11.7 Kinetic energy versus time graph in one oscillation

From the graph 11.7, it is clear that at $t = 0, T/2, T$, etc; KE has its maximum value. For $x = \pm A$ or $t = \frac{T}{4}, \frac{3T}{4}$, the KE has its minimum value. Clearly, twice in each cycle KE assume its peak values. It is a periodic function of time, the time period equals to $T/2$.

KE is maximum at $t = 0, \frac{T}{2}, T, \frac{3T}{2}$ and minimum at

$$t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}.$$

Potential energy

The relation for potential energy of a particle executing SHM in terms of displacement and in terms of time are

(i) **In terms of displacement** If the displacement of particle executing SHM at any instant is x, then

$$\textbf{Potential energy, } U = \frac{1}{2}m\omega^2 x^2 = \frac{1}{2}kx^2$$

Potential energy is maximum at extreme position *i.e.*, at $x = \pm A$

$$U_{max} = \frac{1}{2}m\omega^2 A^2$$

Potential energy is minimum (zero) at mean position *i.e.*, at $x = 0$, potential energy, $U = 0$

(ii) **In terms of time** Potential energy of a particle executing SHM at any time t is given by

$$U = \frac{1}{2}m\omega^2 A^2 \sin^2 \omega t$$

Potential energy versus time graph is given below

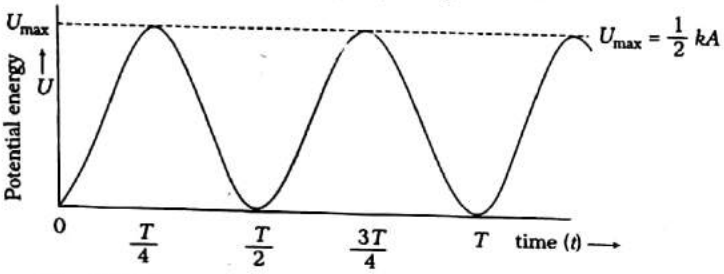


***Fig. 11.8** Potential energy versus time graph in one oscillation*

From the graph, it is clear that at mean position (*i.e.*, $x = 0$ and $t = 0, T/2, T \ldots$), PE is minimum and at extreme position
(*i.e.*, $x = \pm A$ or $t = T/4, 3T/4$), its value is maximum. Clearly, twice in each cycle, PE assume its peak values.

Hence, the potential energy is the periodic function of time with a period of $T/2$.

Total energy

Total energy can be obtained by adding potential and kinetic energies. Therefore,

$$E = K + U = \frac{1}{2}m\omega^2(A^2 - x^2) + \frac{1}{2}m\omega^2 x^2$$

$$\boxed{E = \frac{1}{2}m\omega^2 A^2} \text{ or } \boxed{E = \frac{1}{2}KA^2}$$

Which is a **constant** quantity *i.e.*, same at all instant and at all displacement.

Figure 11.9 shows the variation of kinetic energy (K), potential energy (U) and total energy (E) as a function of time [shown in Fig. 11.9(a)] and displacement [shown in Fig. 11.9(b)] of a particle in SHM

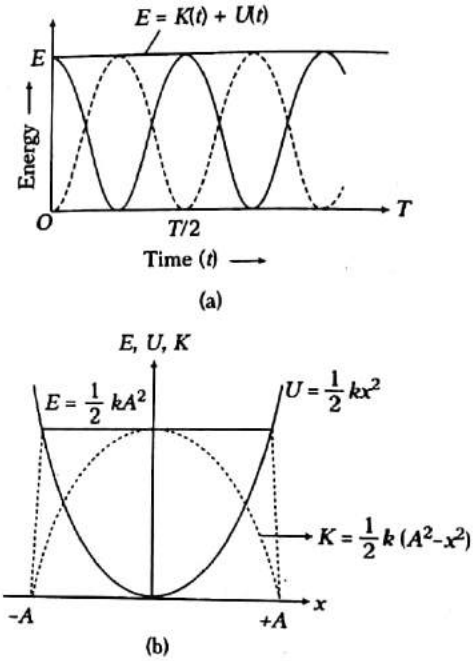


***Fig. 11.9** K, U and E as functions of time and displacement x for a harmonic oscillator*

The total energy remains constant at all t or x. So it is shown by straight line parallel to time.

Also, $U = \frac{1}{2}kx^2$ and $K = \frac{1}{2}k(A^2 - x^2)$, so the graphs of U and K *versus* x are parabolic.

Both KE and PE reach maximum value twice, during each period of SHM. In the course of motion KE increases at the expense of PE or *vice-versa*.

Therefore, we may conclude that, during an oscillation, there is a continuous exchange of kinetic and potential energies.

Important points regarding energy in SHM

(i) The frequency of oscillation of potential energy and kinetic energy is twice as that of displacement or velocity or acceleration.

(ii) Total energy of an oscillating system is constant.

(iii) The average value of KE and PE is

$$K_{av} = U_{av}$$
$$= \frac{1}{2}E = \frac{1}{4}KA^2.$$

Example 11.18 *The potential energy of a particle executing SHM is 2.5 J, when its displacement is half of the amplitude, then determine total energy of particle.*

Sol. $\text{PE} = \frac{1}{2}kx^2 \Rightarrow \frac{1}{2}k\frac{A^2}{4} = 2.5 \qquad \left(\because x = \frac{A}{2}\right)$

where, x is the displacement of the particle and A is the amplitude.

$\Rightarrow$ Total energy $= \frac{1}{2}kA^2 = 2.5 \times 4 = 10\text{ J}$

Example 11.19 *A harmonic oscillator of force constant $4 \times 10^6 Nm^{-1}$ and amplitude 0.01 m has total energy 240 J. What is maximum kinetic energy and minimum potential energy?*

Sol. $k = 4 \times 10^6$ N/m, $A = 0.01$ m, Total energy = 240 J,

Maximum kinetic energy

$$= \frac{1}{2}m\omega^2A^2 = \frac{1}{2}kA^2 = \frac{1}{2} \times 4 \times 10^6 \times (0.01)^2 = 200\text{ J}$$

Minimum potential energy

= total energy − maximum kinetic energy = 40 J

Example 11.20 *A linear harmonic oscillator has a total mechanical energy of 200 J. Potential energy of it at mean position is 50 J. Find,*

(i) the minimum potential energy,

(ii) the maximum kinetic energy,

(iii) the potential energy at extreme positions.

Sol. (i) At mean position, potential energy is minimum.

Hence, $U_{\min} = 50\text{ J}$

(ii) At mean position, KE is maximum

$\therefore \quad K_{\max} = E - U_{\min} = 200 - 50$

$= 150\text{ J}$

(iii) At extreme positions, kinetic energy is zero and potential energy is maximum

$\therefore \quad U_{\max} = E = 200\text{ J}$

(From law of conservation of energy)

Example 11.21 *A particle executes SHM, at what value of displacement are the kinetic and potential energies equal?*

Sol. We know that $\text{KE} = \frac{1}{2}m\omega^2(A^2 - x^2)$

and $U = \frac{1}{2}m\omega^2x^2$

Since, $K = U$

$$\frac{1}{2}m\omega^2(A^2 - x^2) = \frac{1}{2}m\omega^2x^2 \quad \text{or} \quad 2x^2 = A^2$$

or $x = \frac{A}{\sqrt{2}} = 0.707\text{A}$

Example 11.22 *A particle starts oscillating simple harmonically from its equilibrium position with time period T. Determine ratio of KE and PE of the particle at time $t = \frac{T}{12}$.*

Sol. At, $t = \frac{T}{12}$, $x = A\sin\frac{2\pi}{T} \times \frac{T}{12} = A\sin\frac{\pi}{6} = \frac{A}{2}$

So, $\text{KE} = \frac{1}{2}k(A^2 - x^2) = \frac{3}{4} \times \frac{1}{2}kA^2$

and $\text{PE} = \frac{1}{2}kx^2 = \frac{1}{4} \times \frac{1}{2}kA^2$

$\therefore \quad \frac{\text{KE}}{\text{PE}} = \frac{3}{1}$

Example 11.23 *A particle of mass 0.2 kg is executing SHM of amplitude 0.2 m. When it passes through the mean position, its kinetic energy is 64×10^{-3} J. Obtain the equation of motion of this particle if the initial phase of oscillation is $\pi/4$.*

Sol. Here, $A = 0.2$ m, $\phi = \frac{\pi}{4}$

As we know, $\text{KE} = \frac{1}{2}m\omega^2A^2$

where, m = mass of particle = 0.2 kg

$$64 \times 10^{-3} = \frac{1}{2} \times 0.2 \times \omega^2 \times 0.2^2$$

$$\omega^2 = \frac{128 \times 10^{-3}}{0.2 \times 0.2 \times 0.2} \quad \text{or} \quad \omega = 4\text{ s}^{-1}$$

$\therefore$ Equation of motion can be written as

$$x = A\sin(\omega t + \phi)$$

$$= 0.2\sin\left(4t + \frac{\pi}{4}\right)$$

Example 11.24 *A particle executes SHM with amplitude A and time period T. When the displacement from the equilibrium position is half the amplitude, what fractions of the total energy are kinetic and potential?*

Sol. Total energy, $E = \frac{1}{2}kA^2$

When, $x = \pm\frac{A}{2}$

The potential energy,

$$U = \frac{1}{2}kx^2 = \frac{1}{2}k \cdot \left(\pm\frac{A}{2}\right)^2$$

$$= \frac{1}{4} \cdot \frac{1}{2}kA^2 = \frac{1}{4}E = 25\% \text{ of } E$$

$\therefore$ Kinetic energy, $E - U = E - \frac{1}{4}E = \frac{3}{4}E = 75\%$ of E

Example 11.25 *The potential energy of a particle oscillating on X-axis is given as*

$$U = 20 + (x - 2)^2$$

Here, U is in joules and x in metres. Total mechanical energy of the particle is 36 J.

(i) State whether the motion of the particle is simple harmonic or not.

(ii) Find the mean position.

(iii) Find the maximum kinetic energy of the particle.

Sol. (i) $F = -\frac{dU}{dx} = -2(x-2)$

By assuming $x - 2 = X$, we have $F = -2X$

Since, $F \propto -X$

The motion of the particle is simple harmonic.

(ii) The mean position of the particle is $X = 0$ or $x - 2 = 0$, which gives $x = 2$ m

(iii) At $x = 2$ m (mean position)

$U_{min} = 20 + (2-2)^2 = 20$ J

$\therefore$ Maximum kinetic energy of the particle is,

$K_{max} = E - U_{min} = 36 - 20 = 16$ J

Note U_{min} is 20 J at mean position or at $x = 2$ m

Example 11.26 *A block whose mass is 2 kg is fastened to a spring whose spring constant is 100 Nm^{-1}. It is pulled to a distance of 0.1 m from over a frictionless surface and is released at t = 0. Calculate the kinetic energy of the block when it is 0.05 m away from its mean position.*

Sol. The block executes SHM so, its angular frequency,

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{100 \text{ Nm}^{-1}}{2 \text{ kg}}} = 7.07 \text{ rads}^{-1}$$

Its displacement at any time t is

$$x(t) = a \cos \omega t = 0.1 \cos(7.07t)$$

When the particle is 0.05m away from the mean position.

$$0.05 = 0.1 \cos(7.07t)$$

or $\cos(7.07t) = 0.5$

or $\sin(7.07t) = \frac{\sqrt{3}}{2} = 0.866$

Velocity of the block at $x = 0.05$m is

$$v = A\omega \sin \omega t$$

$$= 0.1 \times 7.07 \times 0.866 = 0.61 \text{ ms}^{-1}$$

Hence, $\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2} \times 2\,(0.61)^2 = 0.37$ J

Check point 11.2

1. The energy at the mean position of a pendulum will be
(a) zero
(b) partial PE and partial KE
(c) totally KE
(d) totally PE

2. A body executes simple harmonic motion. The potential energy (PE), the kinetic energy (KE) and total energy (TE) are measured as a function of displacement x. Which of the following statements is true.
(a) PE is maximum when $x = 0$
(b) KE is maximum when $x = 0$
(c) TE is zero when $x = 0$
(d) KE is maximum when x is maximum

3. A body of mass 1 kg is executing simple harmonic motion. Its displacement y (cm) at t seconds is given by $y = 6\sin\left(100t + \frac{\pi}{4}\right)$. Its maximum kinetic energy is
(a) 6 J (b) 18 J
(c) 24 J (d) 36 J

4. Force constant of a weightless spring is 16 Nm^{-1}. A body of mass 1 kg suspended from it is pulled down through 5 cm from its mean position and then released. The maximum kinetic energy of the body will be
(a) 2×10^{-2} J (b) 4×10^{-2} J
(c) 8×10^{-2} J (d) 16×10^{-2} J

5. The total vibrational energy of a particle in SHM is E. Its kinetic energy at half the amplitude from mean position will be
(a) $E/2$ (b) $E/3$
(c) $E/4$ (d) $3E/4$

6. A particle undergoing SHM has the equation $x = A\sin(2\omega t + \phi)$, where x represents the displacement of the particle. The kinetic energy oscillates with time period
(a) $\frac{2\pi}{\omega}$ (b) $\frac{\pi}{\omega}$
(c) $\frac{4\pi}{\omega}$ (d) None of these

7. A particle executes SHM on a line 8 cm long. Its KE and PE will be equal when its distance from the mean position is
(a) 4 cm (b) 2 cm (c) $2\sqrt{2}$ cm (d) $\sqrt{2}$ cm

8. A particle is vibrating in a simple harmonic motion with an amplitude of 4 cm. At what displacement from the equilibrium position, is its energy half potential and half kinetic?
(a) 1 cm (b) $\sqrt{2}$ cm
(c) 3 cm (d) $2\sqrt{2}$ cm

9. In SHM for how many times potential energy is equal to kinetic energy during one complete period
(a) 1 (b) 2 (c) 4 (d) 8

10. In a simple oscillating pendulum, the work done by the pendulum in one oscillation will be
(a) equal to the total energy of the pendulum
(b) equal to the KE of the pendulum
(c) equal to the PE of the pendulum
(d) zero

11. The total energy of a particle executing SHM is directly proportional to the square of the following quantity.
(a) Acceleration (b) Amplitude
(c) Time period (d) Mass

12. The total energy of a vibrating particle in SHM is E. If its amplitude and time period are doubled, its total energy will be
(a) $16E$ (b) $8E$ (c) $4E$ (d) E

13. Energy of particle executing SHM depends upon
(a) amplitude only (b) amplitude and frequency
(c) velocity only (d) frequency only

14. The total energy of a particle having a displacement x, executing simple harmonic motion is
(a) $\propto x$ (b) $\propto x^2$
(c) independent of x (d) $\propto x^{1/2}$

15. A particle of mass 0.10 kg executes SHM with an amplitude 0.05 m and frequency 20 vib/s. Its energy of oscillation is
(a) 2 J (b) 4 J (c) 1 J (d) zero

16. The total energy of a harmonic oscillator of mass 2 kg is 9 J. If its potential energy at mean position is 5 J, its KE at the mean position will be
(a) 9 J (b) 14 J (c) 4 J (d) 11 J

17. The value of total mechanical energy of a particle in SHM is
(a) Always constant (b) Depend on time
(c) $\frac{1}{2}kA^2\cos^2(\omega t+\phi)$ (d) $\frac{1}{2}mA^2\cos^2(\omega t+\phi)$

18. A particle starts SHM from the mean position. Its amplitude is a and total energy E. At one instant its kinetic energy is $\frac{3E}{4}$. Its displacement at that instant is
(a) $\frac{a}{\sqrt{2}}$ (b) $\frac{a}{2}$ (c) $\sqrt{3}\frac{a}{2}$ (d) zero

19. Amplitude of a particle in SHM is 6 cm. If instantaneous potential energy is half the total energy, then distance of particle from its mean position is
(a) 3 cm (b) 4.2 cm
(c) 5.8 cm (d) 6 cm

20. If total energy of a particle in SHM is E, then the potential energy of the particle at half the amplitude will be
(a) $E/2$ (b) $E/4$ (c) $3E/4$ (d) $E/8$

3 Some systems executing simple harmonic motion

A pure simple harmonic motion is not possible unless some conditions are not applied on it. Let us consider some of the examples of pure simple harmonic motion under certain conditions.

1. Simple pendulum

A simple pendulum consists of a point mass suspended by a massless and inextensible string tied from a rigid support.

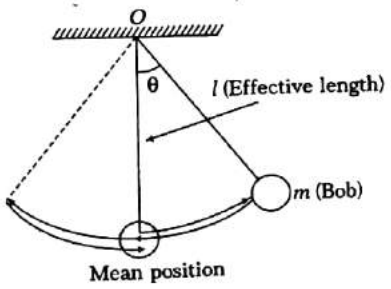


Fig. 11.10 SHM of simple pendulum is one oscillation

Consider a simple pendulum as shown in figure. A small bob of mass m is tied to an inextensible massless string of length l. The bob oscillates in a plane about the vertical line through the support. Let θ be the angle which the string makes with the vertical. At mean position $\theta = 0°$ and v is maximum. An extreme position, v is minimum for the smaller value of θ, the motion of bob is simple harmonic.

Time period of simple pendulum, $\boxed{T = 2\pi\sqrt{\frac{l}{g}}}$

Note that the time period is independent of the mass of the pendulum. This formula is valid only when radius of point mass of pendulum is negligible as compared to length of pendulum.

Variation in time period of simple pendulum in different cases

(i) Effect of acceleration

(a) Above or below the surface of earth, g decreases. Hence, time period of simple pendulum increases.

(b) At the centre of earth or in a satellite or in a freely falling lift, the value of g is zero, so time period of simple pendulum becomes infinity. That means it will not oscillate and remain as it is where it is left.

(c) If a simple pendulum is in a lift which is accelerating upwards with acceleration a, then time period of simple pendulum, $T' = 2\pi\sqrt{\frac{l}{g+a}}$, time period will decrease.

(d) If a simple pendulum is in a lift which is accelerating downwards with acceleration a, then

time period of simple pendulum, $T' = 2\pi\sqrt{\frac{l}{g-a}}$

i.e., time period will increase.

(e) If a simple pendulum is in a car which is accelerating horizontally with acceleration 'a', then time period of pendulum will be $T' = 2\pi\sqrt{\frac{l}{g_{\text{eff}}}}$

where $g_{\text{eff}} = \sqrt{g^2 + a^2}$ *i.e.*, time period will decrease.

In this case, equilibrium position will not be vertical but will make an angle $\theta = \tan^{-1}(a/g)$ to the vertical in a direction opposite to the acceleration.

(ii) Effect of buoyancy

If a simple pendulum of density ρ is made to oscillate in a liquid of density σ, then its time period will increase as compared to that of air.

$$T' = 2\pi\sqrt{\frac{l}{\left(1-\frac{\sigma}{\rho}\right)g}}$$

(iii) Effect of very large length

If the length of simple pendulum (l) is comparable to radius of earth (R) then time period

$$T = 2\pi\sqrt{\frac{1}{g\left(\frac{1}{l}+\frac{1}{R}\right)}}$$

If length becomes infinite then, time period, $T = 2\pi\sqrt{R/g}$ which is approximately equal to 84.6 min.

(iv) Effect of temperature

Suppose the thread of simple pendulum is replaced by a metal wire.

Now, $$T = 2\pi\sqrt{\frac{l}{g}}$$

and $$T' = 2\pi\sqrt{\frac{l(1+\alpha t)}{g}}$$

Here, t represents the rise in temperature and α represents the coefficient of linear expansion.

Now, $$\frac{T'}{T} = (1+\alpha t)^{1/2}$$

Using Binomial theorem,

$$(1+x)^n = 1 + nx + \ldots \text{ (neglecting higher terms)}$$

$$\frac{T'}{T} = 1 + \frac{1}{2}\alpha t \quad \text{or} \quad \frac{T'-T}{T} = \frac{1}{2}\alpha t$$

Oscillations of a simple pendulum in an electric field

(i) If the bob of simple pendulum has positive charge q and is made to oscillate in a uniform electric field acting in upward direction. Then,

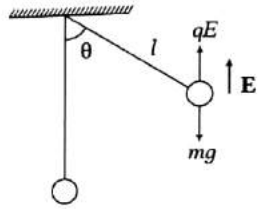


Fig. 11.11 A pendulum in upward electric field.

Time period of pendulum, $T = 2\pi\sqrt{\frac{l}{g-\frac{qE}{m}}}$.

Hence, the time period will increase.

(ii) If the bob of simple pendulum has positive charge q and pendulum is placed in uniform electric field which is in downward direction. Then,

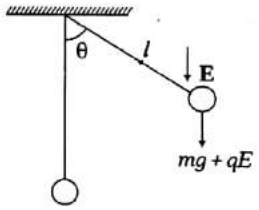


Fig. 11.12 A pendulum is downward electric field.

Time period of pendulum, $T = 2\pi\sqrt{\frac{l}{g+\frac{qE}{m}}}$

Hence, the time period will decrease.

(iii) If the direction of electric field is horizontal, then time period of pendulum

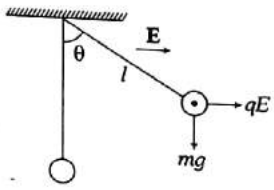


Fig. 11.13

$$T = 2\pi\sqrt{\frac{l}{\left[g^2+\left(\frac{qE}{m}\right)^2\right]^{1/2}}}$$

Hence, the time period will decrease.

Second pendulum

If the time period of a simple pendulum is 2 second, it is called second pendulum.

Time period of second pendulum, $T = 2\text{ s} = 2\pi\sqrt{\frac{l}{g}}$

On earth's surface, $g = 9.8\text{ ms}^{-2} \approx \pi^2\text{ms}^{-2}$

$\therefore$ The length of second pendulum on earth's surface is 1 metre.

Example 11.27 *Two pendulums whose lengths differ by 22 cm oscillate at the same place so that one of them makes 30 oscillations and the other 36 oscillations during same time. Find the lengths of the pendulum.*

Sol. $30 \times 2\pi\sqrt{\frac{l_1}{g}} = 36 \times 2\pi\sqrt{\frac{l_2}{g}}$ or $\frac{l_1}{l_2} = \frac{36 \times 36}{30 \times 30} = \frac{36}{25}$

Also, $l_1 - l_2 = 22\text{ cm}, \frac{l_1}{l_2} - 1 = \frac{22}{l_2}$

or $\frac{22}{l_2} = \frac{36}{25} - 1$ or $\frac{l_2}{22} = \frac{25}{11}$ or $l_2 = 50\text{ cm}$;

$l_1 = 22 + l_2 = (22 + 50)\text{ cm} = 72\text{ cm}$

Example 11.28 *The bob of a simple pendulum executes SHM in water with a period t. The period of oscillation of the bob in air is t_0. What is the relation between t and t_0 ?'*

Given, density of the bob is $\frac{4000}{3}$ kg m^{-3}. Neglect frictional force of water.

Sol. $\rho = \frac{4000}{3}\text{ kgm}^{-3}, \sigma = 1000\text{ kgm}^{-3}$

$$g' = \left(1 - \frac{\sigma}{\rho}\right)g = \left(1 - \frac{1000 \times 3}{4000}\right)g = \frac{g}{4}$$

$$t_0 = 2\pi\sqrt{\frac{L}{g}}, t = 2\pi\sqrt{\frac{L \times 4}{g}} = 2 \times 2\pi\sqrt{\frac{L}{g}}$$

or $t = 2t_0$

Example 11.29 *The length of a simple pendulum is 16 cm. It is suspended by the roof of a lift which is moving upwards with an acceleration of 6.2 ms^{-2}. Find the time period of pendulum.*

Sol. Given, length of the pendulum $(l) = 16\text{ cm} = 0.16\text{ m}$

Acceleration of the lift $(a) = 6.2\text{ ms}^{-2}$

$\because$ Time period $(T) = 2\pi\sqrt{\frac{l}{(g+a)}} = 2 \times 3.14\sqrt{\frac{0.16}{9.8 + 6.2}}$

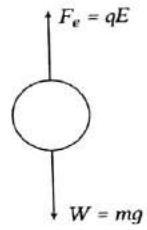
$$= 6.28 \times \sqrt{\frac{0.16}{16}}$$

$$= 6.28 \times \sqrt{\frac{1}{100}} = \frac{6.28}{10}$$

$$= 0.628\text{ s}$$

Example 11.30 *A simple pendulum consists of a small sphere of mass m suspended by a thread of length l. The sphere carries a positive charge q. The pendulum is placed in a uniform electric field of strength E directed vertically upwards. With what period will pendulum oscillate, if the electrostatic force acting on the sphere is less than the gravitational force?*

Sol. The two forces acting on the bob are shown in figure



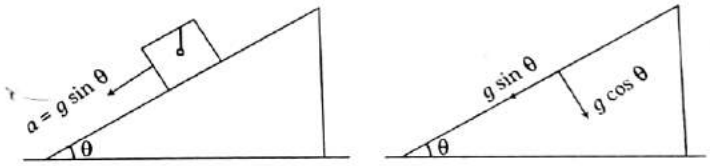
g_{eff} in this case will be $\frac{W - F_e}{m}$

or $g_{\text{eff}} = \frac{mg - qE}{m} = g - \frac{qE}{m}$

$\therefore$ $T = 2\pi\sqrt{\frac{l}{g_{\text{eff}}}} = 2\pi\sqrt{\frac{l}{g - \frac{qE}{m}}}$

Example 11.31 *A simple pendulum of length l is suspended from the ceiling of a cart which is sliding without friction on an inclined plane of inclination θ. What will be the time period of the pendulum?*

Sol. Here, point of suspension has an acceleration. $\mathbf{a} = g \sin\theta$ (down the plane). Further, **g** can be resolved into two components $g \sin\theta$ (along the plane) and $g \cos\theta$ (perpendicular to plane).



$\therefore$ $\mathbf{g}_{\text{eff}} = g \cos\theta$ (perpendicular to plane)

$\therefore$ $T = 2\pi\sqrt{\frac{l}{|\mathbf{g}_{\text{eff}}|}} = 2\pi\sqrt{\frac{l}{g\cos\theta}}$

Note If $\theta = 0°$, $T = 2\pi\sqrt{\frac{l}{g}}$ which is quiet obvious.

Example 11.32 *Determine the period of small oscillations of a simple pendulum, that is a bob suspended by a thread $l = 20$ cm in length, if it is located in a liquid whose density is $n = 3$ times less than that of the bob. The resistance of the liquid is to be neglected*

Sol. Let, V = volume of bob

ρ = density of material of the bob

g' = effective value of acceleration due to gravity

Now, effective weight = weight of bob – upthrust of liquid

$$V\rho g' = V\rho g - V\frac{\rho}{\eta}g$$

$$g' = \left(1 - \frac{1}{\eta}\right)g \quad \text{or} \quad g' = \frac{\eta - 1}{\eta}g$$

Again, $$T = 2\pi\sqrt{\frac{l}{g'}} \quad \text{or} \quad T = 2\pi\sqrt{\frac{l}{\frac{\eta-1}{\eta}g}}$$

or $$T = 2\pi\sqrt{\frac{\eta l}{(\eta - 1)g}}$$

Substituting values, we get

$$T = 2 \times \frac{22}{7}\sqrt{\frac{3 \times 20 \times 10^{-2}}{(3-1)\,9.8}}\ \text{s} = \frac{44}{7} \times 0.175\ \text{s} = 1.1\ \text{s}$$

Example 11.33 *A bob of simple pendulum is suspended by a metallic wire. If α is the coefficient of linear expansion and $d\theta$ is the change in temperature then prove that percentage change in time period is $50\alpha d\theta$.*

Sol. With change in temperature $d\theta$, the effective length of wire becomes $l' = l(1 + \alpha d\theta)$

$$T' = 2\pi\sqrt{\frac{l'}{g}} \quad \text{and} \quad T = 2\pi\sqrt{\frac{l}{g}}$$

Hence, $$\frac{T'}{T} = \sqrt{\frac{l'}{l}} = (1 + \alpha d\theta)^{1/2} = 1 + \frac{1}{2}\alpha d\theta$$

Percentage increase in time period

$$= \left(\frac{T' - T}{T}\right) \times 100 = \left(\frac{T'}{T} - 1\right) \times 100$$

$$= \left(1 + \frac{\alpha d\theta}{2} - 1\right) \times 100 = 50\,\alpha d\theta$$

2. Spring block system

Let us consider a massless spring placed on a frictionless horizontal surface. Its one end is attached to a rigid support and other end to a body of mass m. When it is pulled on one side through a small distance x and released, it moves to and fro about a mean position. Let $x = 0$ be the position of the centre of the body when the spring is at mean position.

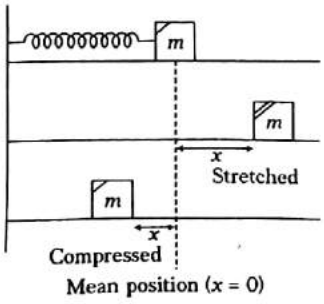


Fig. 11.14

By Hooke's law, a restoring force, F is developed in the spring on stretching and compressing it through a small displacement, x

$$F = -kx \quad \Rightarrow \quad a = -\frac{k}{m}x$$

The time period of such oscillations is given by

$$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

where, k is force constant of spring and m is mass of the object connected to the spring. If the stretch in a vertically loaded spring is x, then for equilibrium of mass m,

$$\boxed{T = 2\pi\sqrt{x/g}}$$

Note

1. If the spring has a mass M and a mass m is suspended from it, then time period is given by

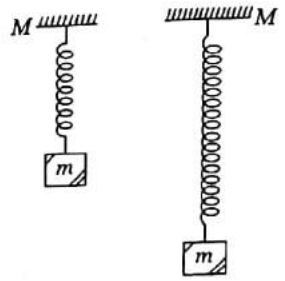


Fig. 11.15

$$T = 2\pi\sqrt{\frac{M_{\text{eff}}}{k}}, \quad \text{where} \quad M_{\text{eff}} = M + \frac{M}{3}$$

2. If two masses m_1 and m_2 are connected by a spring of spring constant k and made to oscillate on horizontal surface. Then, time period is given by

$$T = 2\pi\sqrt{\frac{\mu}{k}}$$

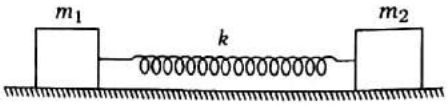


Fig. 11.16 *Two blocks connected by a spring*

Here, μ = reduced mass = $\dfrac{m_1 m_2}{m_1 + m_2}$

Example 11.34 *A block with a mass of 3 kg is suspended from an ideal spring having negligible mass and stretches the spring 0.2 m.*

(i) What is the force constant of the spring?

(ii) What is the period of oscillation of the block, if it is pulled down and released?

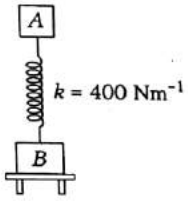
Sol. (i) In equilibrium, $kl = mg \quad \therefore \quad k = \dfrac{mg}{l}$

Substituting the proper values, we have

$$k = \frac{(3)(9.8)}{0.2} = 147\ \text{Nm}^{-1}$$

(ii) $$T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{0.2}{9.8}} = 0.897\ \text{s}$$

Example 11.35 *Two rigid bodies A and B of masses 1 kg and 2 kg respectively are rigidly connected to a spring of force constant 400 Nm^{-1}. The body B rests on a horizontal table. From the rest position, the body A is compressed by 2 cm and then released. Deduce (i) the frequency of oscillation, (ii) total oscillation energy, (iii) the amplitude of the harmonic vibration of the reaction of the table on body B.*



Sol. Since it is the body A which is oscillating, $m = 1$ kg

(i) Frequency of oscillation, $\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

or $\nu = \frac{1}{2\pi}\sqrt{\frac{400}{1}}\text{ Hz} = \frac{10}{\pi}\text{ Hz} = 3.182\text{ Hz}$

(ii) Total oscillation energy, $E = \frac{1}{2}mA^2\omega^2$

or $E = \frac{1}{2}mA^2\left(\frac{k}{m}\right) = \frac{1}{2}kA^2$

$= \frac{1}{2} \times 400 \times (0.02)^2\text{ J} = 0.08\text{ J}$ $(\because a = 2\text{ cm} = 0.02\text{ m})$

(iii) Total force acting on the table

$= (1 + 2)\text{ kg wt} = 3\text{ kg wt} = 3 \times 9.8\text{ N} = 29.4\text{ N}$

This is the mean upward reaction. Now, due to oscillations, the maximum tension developed in the spring is given by

$F = kA = 400 \times 0.02\text{ N} = 8\text{ N}$

Thus, the amplitude of the vibration of reaction on the table is 8 N. In other words, the net reaction of the table will vary from (29.4 + 8) N to (29.4) N.

Example 11.36 *A block with mass M attached to a horizontal spring with force constant k is moving with simple harmonic motion having amplitude A_1. At the instant when the block passes through its equilibrium position a lump of putty with mass m is dropped vertically on the block from a very small height and sticks to it.*

(i) Find the new amplitude and period.

(ii) Repeat part (i) for the case in which the putty is dropped on the block when it is at one end of its path.

Sol. (i) Before the lump of putty is dropped the total mechanical energy of the block and spring is

$$E_1 = \frac{1}{2}kA_1^2$$

Since, the block is at the equilibrium position, $U = 0$, and the energy is purely kinetic. Let v_1 be the speed of the block at the equilibrium position, we have

$$E_1 = \frac{1}{2}Mv_1^2 = \frac{1}{2}kA_1^2 \Rightarrow v_1 = \sqrt{\frac{k}{M}}A_1$$

During the process momentum of the system in horizontal direction is conserved. Let v_2 be the speed of the combined mass, then

$$(M + m)v_2 = Mv_1$$

$$\therefore \quad v_2 = \frac{M}{M + m}v_1$$

Now, let A_2 be the amplitude afterwards. Then,

$$E_2 = \frac{1}{2}kA_2^2 = \frac{1}{2}(M + m)v_2^2$$

Substituting the proper values, we have

$$A_2 = A_1\sqrt{\frac{M}{M + m}}$$

Note (i) $E_2 < E_1$, as some energy is lost into heating up the block and putty.

Further, $T_2 = 2\pi\sqrt{\frac{M + m}{k}}$

(ii) When the putty drops on the block, the block is instantaneously at rest. All the mechanical energy is stored in the spring as potential energy. Again the momentum in horizontal direction is conserved during the process. But now it is zero just before and after putty is dropped. So, in this case, adding the extra mass of the putty has no effect on the mechanical energy, *i.e.*,

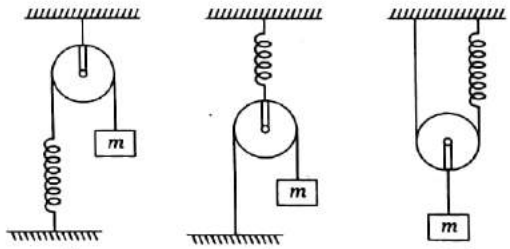
$$E_2 = E_1 = \frac{1}{2}kA_1^2$$

and the amplitude is still A_1.

Thus, $A_2 = A_1$

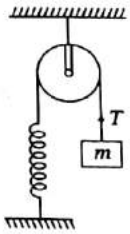
and $T_2 = 2\pi\sqrt{\frac{M + m}{k}}$

Example 11.37 In the following arrangements, block is slightly displaced vertically down from its equilibrium position and released. Find time period of vertical oscillations. Assume the pulley to be light.



(i) (ii) (iii)

Sol. (i) Let at some instant extension of spring is x, tension in the string $T = kx$

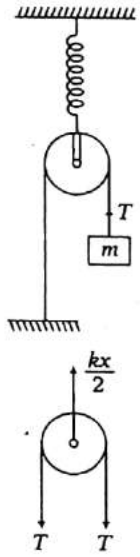


Restoring force, $F = T = -kx$ (Hooke's law)

where, k = spring constant

Time period, $T_0 = 2\pi\sqrt{\frac{m}{k}}$

(ii) Let block move down by x, pulley will move by $x/2$.



$$2T = \frac{kx}{2} \Rightarrow T = \frac{kx}{4}$$

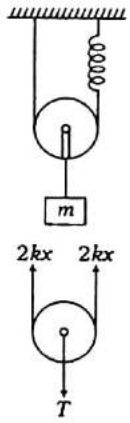
Restoring force, $F = -\frac{k}{4}x$

Comparing it with equation $F = -kx$

we get, $k' = k/4$

$$\therefore \quad T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow T_0 = 2\pi\sqrt{\frac{m}{k/4}} = 4\pi\sqrt{\frac{m}{k}}$$

(iii) If block moves by x, pulley also moves x and hence spring is stretched by $2x$.



Tension produced in spring, $T = 4kx$

Restoring force, $F = -4kx$

Comparing it with equation $F = -k'x$

We get $\quad k' = +4k$

$$\therefore \quad T = 2\pi\sqrt{\frac{m}{k}}$$

$$\Rightarrow \quad T_0 = 2\pi\sqrt{\frac{m}{4k}} = \pi\sqrt{\frac{m}{k}}$$

Oscillations of spring combination

Series combination

If two springs of spring constant k_1 and k_2 are joined in series (as shown in figure) the resultant spring constant of the combination is given by

$$\frac{1}{k_s} = \frac{1}{k_1} + \frac{1}{k_2} = \frac{k_1 + k_2}{k_1 k_2}; \; k_s = \frac{k_1 k_2}{k_1 + k_2}$$

and hence, $$T = 2\pi\sqrt{\frac{m}{k_s}} = 2\pi\sqrt{\frac{(k_1 + k_2)m}{k_1 k_2}}$$

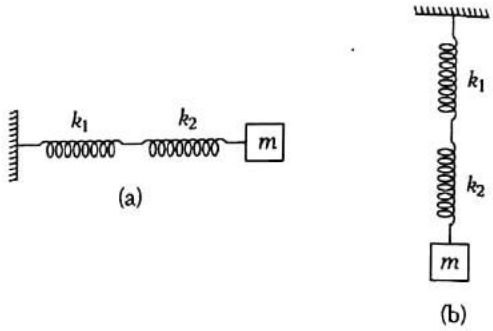


Fig. 11.17 Series combination of springs

Parallel combination

If two springs of spring constant k_1 and k_2 are joined in parallel as shown in figure.

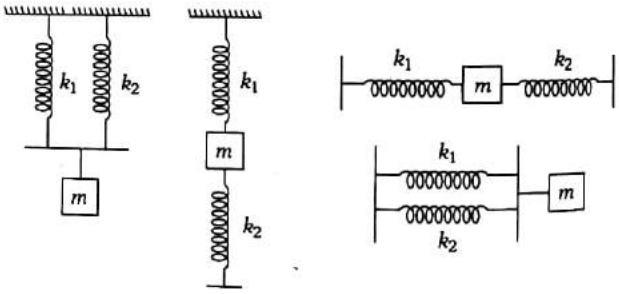


Fig. 11.18 Parallel combination of springs

Then spring constant, $k_p = k_1 + k_2$

Time period, $T = 2\pi\sqrt{\frac{m}{k_p}} = 2\pi\sqrt{\frac{m}{(k_1 + k_2)}}$

Important points regarding spring block system

(i) If the length of the spring is made n times then the effective force constant becomes $1/n$ times and the time period becomes $\sqrt{n}$ times.

(ii) If a spring of spring constant k is divided into n equal parts, the spring constant of each part becomes nk and time period becomes $\frac{1}{\sqrt{n}}$ times.

(iii) The force constant of a stiffer spring is higher than that of a soft spring.

Example 11.38 *One body is suspended from a spring of length l, spring constant k and has time period T. Now if spring is divided in two equal parts which are joined in parallel and the same body is suspended from this arrangement, then determine new time period.*

Sol. When a spring of spring constant k is divided into 2 parts, the spring constant of each part becomes $2k$.

$\therefore$ Spring constant in parallel combination $k' = 2k + 2k = 4k$

$$\therefore \quad T = 2\pi\sqrt{\frac{m}{k'}} = 2\pi\sqrt{\frac{m}{4k}} = 2\pi\sqrt{\frac{m}{k}} \times \frac{1}{\sqrt{4}} = \frac{T}{\sqrt{4}} = \frac{T}{2}$$

Example 11.39 *A 2.5 kg collar is attached to a spring of spring constant 250 Nm^{-1}. It slides without friction over a horizontal surface. It is displaced from its equilibrium position by 20 cm and released. Calculate the period of oscillation and the maximum speed.*

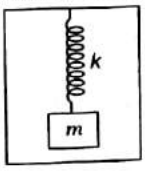
Sol. The period of oscillation, $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{2.5}{250}}$

$$= \frac{2 \times 3.14}{10} = 0.63 \text{ s}$$

Maximum speed,

$$v_m = A\omega = 0.2\sqrt{\frac{k}{m}} = 0.2 \times \sqrt{\frac{250}{2.5}} = 0.2 \times 10 = 2 \text{ ms}^{-1}$$

Example 11.40 *A spring mass system is hanging from the ceiling of an elevator in equilibrium. The elevator suddenly starts accelerating upwards with acceleration a, find*



(i) the frequency and

(ii) the amplitude of the resulting SHM.

Sol. (i) Frequency $= 2\pi\sqrt{m/k}$

(Frequency is independent of g in spring)

(ii) Initial extension in spring in equilibrium $= \frac{mg}{k}$

Extension in spring in equilibrium when lift starts accelerating upwards $= \frac{m(g+a)}{k}$

$$\therefore \text{ Amplitude} = \frac{m(g+a)}{k} - \frac{mg}{k} = \frac{ma}{k}$$

Example 11.41 *A body of mass m attached to a spring which is oscillating with time period 4 s. If the mass of the body is increased by 4 kg, its time period increases by 2 s. Determine value of initial mass m.*

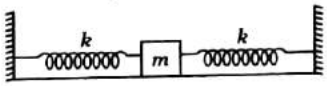
Sol. In 1st case, $T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow 4 = 2\pi\sqrt{\frac{m}{k}}$ ...(i)

2nd case, $6 = 2\pi\sqrt{\frac{m+4}{k}}$ ...(ii)

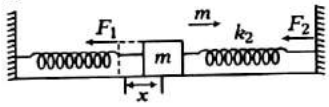
Divide Eq. (i) by Eq. (ii)

$$\frac{4}{6} = \sqrt{\frac{m}{m+4}} \Rightarrow \frac{16}{36} = \frac{m}{m+4} \Rightarrow m = 3.2 \text{ kg}$$

Example 11.42 *Two identical springs of spring constant k are attached to a block of mass m and to fixed supports as shown in figure. Show that when the mass is displaced from its equilibrium position on either side, it executes a simple harmonic motion. Find the period of oscillations.*



Sol. Let the mass be displaced by a small distance x to the right side of the equilibrium position, as shown in figure. Under this situation, the spring on the left side gets elongated by a length equal to x and that on the right side gets compressed by the same length. The forces acting on the mass are then,



$F_1 = -kx$ (force exerted by the spring on the left side, trying to pull the mass towards the mean position)

$F_2 = -kx$ (force exerted by the spring on the right side, trying to push the mass towards the mean position)

The net force F acting on the mass is then given by, $F = -2kx$. Hence, the force acting on the mass is proportional to the displacement and is directed towards the mean position therefore, the motion executed by the mass is simple harmonic. The time period of oscillations is

$$T = 2\pi\sqrt{\frac{m}{2k}}$$

Example 11.43 *Two springs have their force constants k_1 and k_2 ($k_1 > k_2$). Determine the spring on which more work is done when (i) their length are increased by the same amount, and (ii) they are stretched by the same force.*

Sol. (i) Let each spring be stretched through a distance y.

Work done on second spring $= \frac{1}{2}k_2y^2$

Work done on first spring $= \frac{1}{2}k_1y^2$. But $k_1 > k_2$ (given)

$$\therefore \quad \frac{1}{2}k_1y^2 > \frac{1}{2}k_2y^2$$

So, more work is done on the first spring.

(ii) Let each spring be stretched by the same force F. Let y_1 and y_2 be the extension in the first and second springs respectively.

Then, $y_1 = \frac{F}{k_1}$

and $y_2 = \frac{F}{k_2}$

Work done on first spring $= \frac{1}{2}k_1y_1^2 = \frac{1}{2}k_1\left(\frac{F}{k_1}\right)^2 = \frac{1}{2}\frac{F^2}{k_1}$

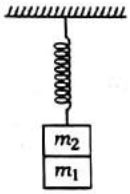
Similarly, the work done on the second spring $= \frac{1}{2}\frac{F^2}{k_2}$

But $\quad k_1 > k_2$ (given)

$\therefore \quad \frac{1}{2}\frac{F^2}{k_1} < \frac{1}{2}\frac{F^2}{k_2}$

So, more work is done on the second spring.

Example 11.44 *Two masses m_1 and m_2 are suspended together by a massless spring of spring constant k as shown in figure. When the masses are in equilibrium, m_1 is removed without disturbing the system. Calculate the amplitude of oscillation and the angular frequency of m_2.*



Sol. Let l_1 and l_2 be the extension produced by masses m_1 and m_2 respectively.

Then, $\quad (m_1 + m_2)\, g = k\,(l_1 + l_2) \quad$ ...(i)

After m_1 is removed, $\quad m_2 g = k l_2 \quad$ ...(ii)

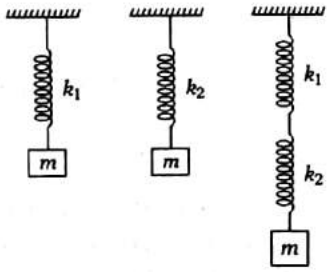
Eqs. (i) – (ii) gives $\quad m_1 g = k l_1 \;$ or $\; l_1 = \frac{m_1 g}{k}$

Now, angular frequency of m_2, $\omega = \frac{2\pi}{T} = 2\pi\sqrt{\frac{k}{m_2}}$

Example 11.45 *Periodic time of oscillation T_1 is obtained when a mass is suspended from a spring and if another spring is used with same mass then periodic time of oscillation is T_2. Now if this mass is suspended from series combination of above springs then calculate the time period.*

Sol. $T_1 = 2\pi\sqrt{\frac{m}{k_1}} \Rightarrow T_1^2 = 4\pi^2\frac{m}{k_1} \Rightarrow k_1 = \frac{4\pi^2 m}{T_1^2}$ and $T_2 = 2\pi\sqrt{\frac{m}{k_2}}$

$\Rightarrow \quad T_2^2 = 4\pi^2\frac{m}{k_2} \Rightarrow k_2 = \frac{4\pi^2 m}{T_2^2}$



Now, $\quad T = 2\pi\sqrt{\frac{m}{k'}}$

where, $\quad \frac{1}{k'} = \frac{1}{k_1} + \frac{1}{k_2}$

$$\Rightarrow \quad k' = \frac{k_1 k_2}{k_1 + k_2} = \frac{\left(\frac{4\pi^2 m}{T_1^2}\right)\left(\frac{4\pi^2 m}{T_2^2}\right)}{\frac{4\pi^2 m}{T_1^2} + \frac{4\pi^2 m}{T_2^2}}$$

$$k' = \frac{4\pi^2 m\left(\frac{4\pi^2 m}{T_1^2 T_2^2}\right)}{4\pi^2 m\left(\frac{1}{T_1^2} + \frac{1}{T_2^2}\right)} = \frac{4\pi^2 m}{T_1^2 + T_2^2}$$

$$\therefore \quad T = 2\pi\sqrt{\frac{m}{k'}} = 2\pi\sqrt{\frac{m}{\frac{4\pi^2 m}{T_1^2 + T_2^2}}} = \sqrt{T_1^2 + T_2^2}$$

3. The physical pendulum

The physical pendulum is just a rigid body, of whatever shape, capable of oscillating about a horizontal axis passing through it. For small oscillations the motion of a physical pendulum is almost as easy as for a simple pendulum. Figure shows a body of irregular shape pivoted at O so, that it can oscillate without friction about an axis passing through O.

In equilibrium the centre of gravity (G) in directly below O. *In the position shown in figure, the body is displaced from equilibrium by an angle* θ. *The distance from* O to the centre of gravity is l. The moment of inertia of the body about the axis of rotation through O is I and the total mass is m. In the displaced position, the weight mg causes a restoring torque,

$$\tau = -(mg)(l \sin\theta)$$

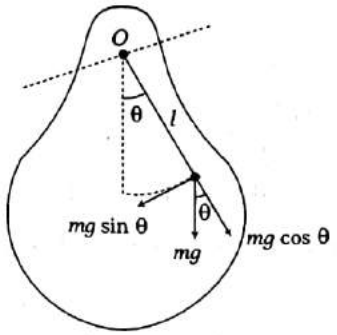


Fig. 11.19

The negative sign shows that the restoring torque is clockwise when the displacement is counterclockwise and *vice-versa*. For small oscillations,

$$\sin\theta \approx \theta \quad \text{and} \quad \Sigma\tau = I\alpha \;\Rightarrow\; -(mgl)\,\theta = I\alpha$$

As α is proportional to $-\theta$, the motion is simple harmonic, the time period of which is

$$\boxed{T = 2\pi\sqrt{\left|\frac{\theta}{\alpha}\right|}} \quad \text{or} \quad \boxed{T = 2\pi\sqrt{\frac{I}{mgl}}}$$

Example 11.46 *A uniform circular disc of radius R oscillates in a vertical plane about a horizontal axis. Find the distance of the axis of rotation from the centre for which the period is minimum. What is the value of this period?*

Sol. The time period of a compound pendulum is the minimum when its length is equal to the radius of gyration about its centre of gravity, *i.e.*, $l = K$.

Since, the moment of inertia of a disc about an axis perpendicular to its plane and passing through its centre is equal to,

$$I = MK^2 = \frac{1}{2}MR^2 \Rightarrow K = \frac{R}{\sqrt{2}}$$

Thus, the disc will oscillate with the minimum time period when the distance of the axis of rotation from the centre is $\frac{R}{\sqrt{2}}$.

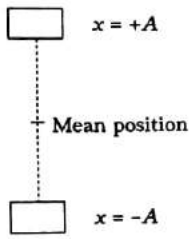
And the value of this minimum time period will be

$$T_{min} = 2\pi\sqrt{\frac{2R/\sqrt{2}}{g}} = 2\pi\sqrt{\frac{\sqrt{2}R}{g}} \quad \text{or} \quad T_{min} \approx 2\pi\sqrt{\frac{1.414R}{g}}$$

Example 11.47 *Find the period of small oscillations of a uniform rod with length l, pivoted at one end.*

Sol. Time period of uniform rod which is same as physical pendulum

$$T = 2\pi\sqrt{\frac{I_0}{mg(OG)}}$$

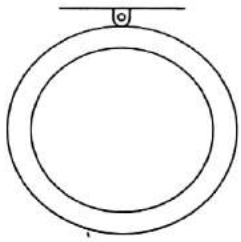


Here, $I_0 = \frac{1}{3}ml^2$ and $OG = \frac{l}{2}$

$$\therefore \quad T = 2\pi\sqrt{\frac{\left(\frac{1}{3}ml^2\right)}{(m)(g)\left(\frac{l}{2}\right)}}$$

or $$T = 2\pi\sqrt{\frac{2l}{3g}}$$

Example 11.48 *A ring of radius r is suspended from a point on its circumference. Determine its angular frequency of small oscillations.*



Sol. It is a physical pendulum, the time period of which is,

$$T = 2\pi\sqrt{\frac{I}{mgl}}$$

Here, I = moment of inertia of the ring about point of suspension

$= mr^2 + mr^2 = 2mr^2$

and l = distance of point of suspension from centre of gravity

$= r$

$$\therefore \quad T = 2\pi\sqrt{\frac{2mr^2}{mgr}} = 2\pi\sqrt{\frac{2r}{g}}$$

$\therefore$ Angular frequency $\omega = \frac{2\pi}{T}$ or $\omega = \sqrt{\frac{g}{2r}}$

Check point 11.3

1. If the metal bob of a simple pendulum is replaced by a wooden bob, then its time period will
   (a) increase (b) decrease
   (c) remain the same (d) may increase or decrease
2. The length of a simple pendulum is $39.2/\pi^2 m$. If $g = 9.8\ m/s^2$, the value of time period is
   (a) 4 s (b) 8 s
   (c) 2 s (d) 3 s
3. The length of a simple pendulum is increased four times of its initial value, its time period with respect to its previous value will
   (a) become twice (b) not be different
   (c) be halved (d) be $\sqrt{2}$ times
4. The length of a simple pendulum executing simple harmonic motion is increased by 21%. The percentage increase in the time period of the pendulum of increased length is
   (a) 11% (b) 21% (c) 42% (d) 10.5%
5. A cabin is falling freely under gravity, what is the time period of a pendulum attached to its ceiling?
   (a) zero (b) ∞ (c) 1 s (d) 2 s
6. A simple pendulum is suspended from the roof of a trolley which moves, in a horizontal direction with an acceleration a, then the time period T is given by, $T = 2\pi\sqrt{l/g'}$, where g' is equal to
   (a) g (b) $g - a$
   (c) $g + a$ (d) $\sqrt{g^2 + a^2}$

7. A man measures the period of a simple pendulum inside a stationary lift and finds it to be T second. If the lift accelerates upwards with an acceleration $g/4$, then the period of the pendulum will be

(a) T (b) $\frac{T}{4}$ (c) $\frac{2T}{\sqrt{5}}$ (d) $2T\sqrt{5}$

8. A simple pendulum is made of a body which is a hollow sphere containing mercury suspended by means of a wire. If a little mercury is drained off, the period of pendulum will

(a) remains unchanged (b) increase
(c) decrease (d) become erratic

9. The time period of a simple pendulum of infinite length is (R = radius of earth).

(a) infinite (b) $2\pi\sqrt{\frac{R}{g}}$ (c) $2\pi\sqrt{\frac{g}{R}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{R}{g}}$

10. The length of a second pendulum is

(a) 99.8 cm (b) 99 cm
(c) 100 cm (d) None of these

11. An object of mass m is suspended from a spring and it executes SHM with frequency n. If the mass is increased 4 times, the new frequency will be

(a) $2n$ (b) $n/2$ (c) n (d) $n/4$

12. A mass m is suspended from a spring. Its frequency of oscillation is f. The spring is cut into two halves and the same mass is suspended from one of the two pieces of the spring. The frequency of oscillation of the mass will be

(a) $\sqrt{2}f$ (b) $\frac{f}{2}$ (c) f (d) $2f$

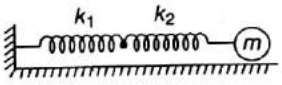
13. Three masses 0.1 kg, 0.3 kg and 0.4 kg are suspended at end of a spring. When the 0.4 kg mass is removed, the system oscillates with a period 2 s. When the 0.3 kg mass is also removed, the system will oscillate with a period

(a) 1 s (b) 2 s (c) 3 s (d) 4 s

14. Time period of a spring mass system is T. If this spring is cut into two parts whose lengths are in the ratio 1 : 3 and the same mass is attached to the longer part, the new time period will be

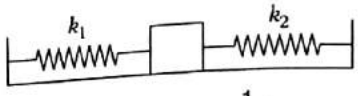
(a) $\sqrt{\frac{3}{2}}T$ (b) $\frac{T}{\sqrt{3}}$
(c) $\frac{\sqrt{3}T}{2}$ (d) $\sqrt{3}T$

15. As shown in the figure, two light springs of force constant k_1 and k_2 oscillate a block of mass m. Its effective force constant will be



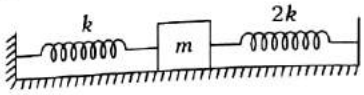
(a) k_1k_2 (b) $k_1 + k_2$ (c) $\frac{1}{k_1} + \frac{1}{k_2}$ (d) $\frac{k_1k_2}{k_1 + k_2}$

16. If both spring constants k_1 and k_2 are increased to $4k_1$ and $4k_2$ respectively, what will be the new frequency, if f was the original frequency?



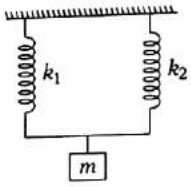
(a) f (b) $2f$ (c) $\frac{1}{2}f$ (d) $4f$

17. Frequency of oscillation is proportional to



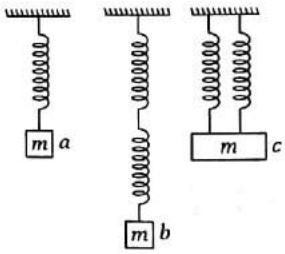
(a) $\sqrt{\frac{3k}{m}}$ (b) $\sqrt{\frac{k}{m}}$ (c) $\sqrt{\frac{2k}{m}}$ (d) $\sqrt{\frac{m}{3k}}$

18. The spring constants of two springs of same length are k_1 and k_2 as shown in figure. If an object of mass m is suspended and set in vibration, the time period will be



(a) $2\pi\sqrt{\frac{mk_1}{k_2}}$ (b) $2\pi\sqrt{\frac{m}{k_1k_2}}$
(c) $2\pi\sqrt{\frac{m}{k_1 - k_2}}$ (d) $2\pi\sqrt{m/(k_1 + k_2)}$

19. Five identical springs are used in the three configurations as shown in figure. The time periods of vertical oscillations in configurations (a), (b) and (c) are in the ratio



(a) $1 : \sqrt{2} : \frac{1}{\sqrt{2}}$ (b) $2 : \sqrt{2} : \frac{1}{\sqrt{2}}$
(c) $\frac{1}{\sqrt{2}} : 2 : 1$ (d) $2 : \frac{1}{\sqrt{2}} : 1$

20. A thin uniform rod of length l is pivoted at its upper end. It is free to swing in a vertical plane. Its time period for oscillations of small amplitude is

(a) $2\pi\sqrt{\frac{l}{g}}$ (b) $2\pi\sqrt{\frac{2l}{3g}}$ (c) $2\pi\sqrt{\frac{3l}{2g}}$ (d) $2\pi\sqrt{\frac{l}{3g}}$

4 SHM in some special cases

Here we will discuss motion of some objects being reduced to simple harmonic motion under specific condition.

(i) **Vertical motion of a straight wire** If a mass m is suspended from a wire of length l, cross-section A and Young's modulus Y and is pulled along the length of the wire then restoring force will be developed by the elasticity of the wire.

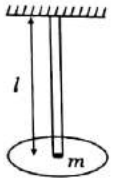


Fig. 11.20 Mass suspended from a wire.

Here restoring force is linear so motion is simple harmonic motion where time period is given by

$$T = 2\pi\sqrt{\frac{mL}{YA}}$$

(ii) **Motion of a liquid in U-shape tube** If a incompressible and non-viscous liquid is filled in a U-tube of uniform cross section. If the liquid is pressed in one limb then it starts to perform SHM along limb of the tube, time period of this SHM is given by

$$T = 2\pi\sqrt{\frac{h}{g}}$$

where, h is height of liquid in one limb.

(iii) **Motion of ball in bowl** If a small steel ball of mass m is placed at a small distance from O inside a smooth concave surface of radius R and released, it will oscillate about O.

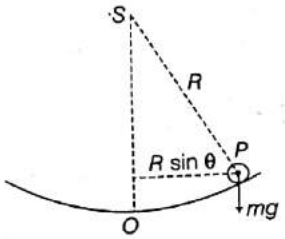


Fig. 11.21 A ball of mass is moving in a bowl.

Time period of such oscillations is

$$T = 2\pi\sqrt{\frac{R}{g}}$$

(iv) **Motion of a ball in a tunnel through the earth** If a ball moves through a tunnel along a diameter of earth, then due to gravitational force between ball and earth a restoring force is set up, due to which the ball performs SHM, whose time period is given by

$$T = 2\pi\sqrt{R/g}$$

where, R = radius of earth

Example 11.49 *If the earth were a homogeneous sphere and a straight hole was bored in it through its centre, so when a body is dropped in the hole, it will execute SHM. Determine the time period of its oscillation. Radius of the earth is 6.4×10^5 m and $g=9.8$ ms^{-2}*

Sol. Given, $R = 6.4\times10^5$ m, $g = 9.8$ ms^{-2}

$\therefore$ Time period of oscillation of body will be

$$T = 2\pi\sqrt{\frac{R}{g}} = 2\pi\sqrt{\frac{6.4\times10^5}{9.8}} = 1605.67 \text{ s}$$

Example 11.50 *A liquid of mass m is set into oscillations in a U-tube of cross section A. Its time period recorded is T, where $T = 2\pi\sqrt{l/2g}$, here l is the length of liquid column. If the liquid of same mass is set into oscillations in U-tube of cross-section A/16 then determine time period of oscillation.*

Sol. As mass is constant

$\therefore$ Volume × density = constant $\left(\because \text{Density} = \frac{\text{Mass}}{\text{Volume}}\right)$

$\Rightarrow$ $V_1 d = V_2 d$

where, V_1 and V_2 are the volume of liquid in U-tube in two cases respectively.

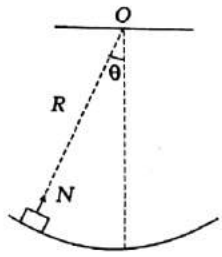
$$(Al)d = \left|\frac{A}{16}l'\right|d \Rightarrow l' = 16\,l$$

$$\because \quad T = 2\pi\sqrt{\frac{l}{2g}}$$

$$\therefore \quad \frac{T'}{T} = \sqrt{\frac{l'}{l}} = \sqrt{\frac{16\,l}{l}} = 4 \Rightarrow T' = 4T$$

Example 11.51 *A small block oscillates back and forth on a smooth concave surface of radius R. Find the time period of small oscillation.*

Sol. For the oscillation of small block on a smooth concave surface, following figure can be drawn, where R is the radius of the concave surface.



$\therefore$ Restoring torq τ $= -mgR\sin\theta$

As $I = mR^2 \Rightarrow \tau = -mgR\theta$ (since $\sin\theta \simeq \theta$)

$$\therefore \quad I\alpha = -mgR\theta \Rightarrow \alpha = -\frac{mgR}{I}\theta = -\omega^2\theta$$

Time period of such lation is

$$T = 2\pi\sqrt{\frac{I}{mgR}} = 2\pi\sqrt{\frac{mR^2}{mgR}} \Rightarrow T = 2\pi\sqrt{\frac{R}{g}}$$

Free oscillation

When a body oscillates with its own natural frequency, it is said to execute free oscillations. The frequency and the time period of free oscillations depends on the dimensions of the body and the force constant, i.e. the inertia factor and the spring factor.

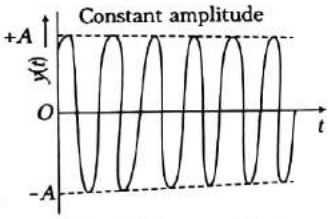


Fig. 11.22 Free oscillations

The frequency of free oscillations (or natural oscillations) is given by

$$\boxed{\nu_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}}$$

e.g., The vibrations of the prongs of tunning fork struck against a rubber pad, the vibrations of string of a sitar when pulled aside and released etc.

Damping and damped oscillations

Damping is an influence within or upon an oscillatory system, which has the effect of reducing, restricting and preventing its oscillations. In physical systems, damping is produced by processes that dissipates the energy stored in the oscillation.

The oscillations in which the amplitude decreases gradually with the passage of time are called damped oscillations. Most of the oscillations occuring in viscous media like water, air etc are damped oscillations.

So, part of the energy is dissipated in overcoming resistive forces.

Consequently, the amplitude of oscillations goes on decreasing exponentially with time and eventually, these oscillations die out. *e.g.*, Oscillations of simple pendulum in air, a swinging ball left to itself will eventually stop etc.

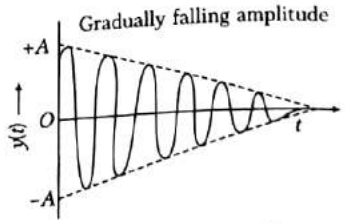


Fig. 11.23 Damped oscillations

Amplitude of damped oscillations $A = A_0 e^{-\gamma t}$

Energy of damped oscillation $E = E_0 e^{-2\gamma t}$

where, γ = damping coefficient $= \dfrac{b}{2m}$

Here, m is the mass of body undergoing damped oscillations.

Damping force $F_d = -bv$

where, b = damped constant, v = velocity of oscillation.

Equation of damped oscillation

$$\frac{md^2y}{dt^2} + b\frac{dy}{dt} + ky = 0$$

Displacement of damped oscillations

$$\boxed{y = y_m e^{-bt/2m}\sin(\omega' t + \phi)}$$

where, ω' = angular frequency of damped oscillations

$$= \sqrt{\omega_0^2 - (b/2m)^2}$$

Maintained oscillations

The oscillations of a system are said to be maintained or sustained oscillations if the system vibrates with its own natural frequency and with constant amplitude even in the presence of damping. If energy is supplied to the oscillator at the same rate at which it is dissipated, the amplitude of oscillations remains unchanged.

e.g., Balance wheel of watch, electronic oscillator, electrically maintained tunning fork.

Forced oscillations

When a body is maintained in a state of oscillation by a strong periodic force of frequency other than the natural frequency of the body, the oscillations are called forced oscillations. The external agent which exerts the periodic force is called the driver and the oscillating system under consideration is called the driven body.

e.g., (i) When the stem of a vibrating tunning fork is pressed against a table, a loud sound is heard. This is

because the particles of table are forced to vibrate with the frequency of the tunning fork.

(ii) When the free end of the string of a simple pendulum is held in hand and the pendulum is made to oscillate by giving jerks by the hand, the pendulum executes forced oscillations.

Resonance

When a body oscillates with its own natural frequency, with the help of an external periodic force whose frequency is equal to the natural frequency of the body, the oscillations of the body are called resonance.

Condition for resonance: $\nu_0 = \nu_d$

where, ν_0 = natural frequency

ν_d = frequency of external periodic force.

Example 11.52 *The amplitude of a damped oscillator becomes half in one minute. The amplitude after 3 minutes will be $1/x$ times of the original. Determine the value of x.*

Sol. Amplitude of damped oscillations is $A = A_0 e^{-\gamma t}$

(From $x = x_m e^{-\gamma t}$)

As $A = A_0/2$ at $t = 1$ minute,

So $\frac{A_0}{2} = A_0 e^{-\gamma}$ or $e^{\gamma} = 2$

After 3 minutes the amplitude will be A_0/x

So $\frac{A_0}{x} = A_0 e^{-\gamma 3}$

or $x = e^{3\gamma} = (e^{\gamma})^3 = 2^3 = 8$

Vector method of combining two or more simple harmonic motions in same direction

A simple harmonic motion is produced when a force (called restoring force) proportional to the displacement acts on a particle. If a particle is acted upon by two such forces the resultant motion of the particle is a combination of two simple harmonic motions. Suppose the two individual motions are represented by,

$$x_1 = A_1 \sin \omega t$$

and

$$x_2 = A_2 \sin(\omega t + \phi)$$

Both the simple harmonic motions have same angular frequency ω.

The resultant displacement of the particle is given by

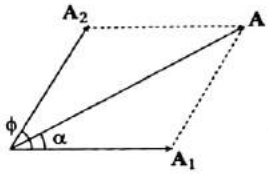


Fig. 11.24

$$x = x_1 + x_2 = A_1 \sin \omega t + A_2 \sin(\omega t + \phi)$$
$$= A \sin(\omega t + \alpha)$$

Here, $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos \phi}$

and $\boxed{\tan \alpha = \frac{A_2 \sin \phi}{A_1 + A_2 \cos \phi}}$

Thus, we can see that this is similar to the vector addition. The same method of vector addition can be applied to the combination of more than two simple harmonic motions.

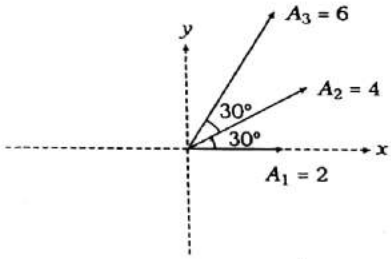
Example 11.53 *Find the displacement equation of the simple harmonic motion obtained by combining the motions.*

$$x_1 = 2 \sin \omega t, \quad x_2 = 4 \sin\left(\omega t + \frac{\pi}{6}\right)$$

and $x_3 = 6 \sin\left(\omega t + \frac{\pi}{3}\right)$

Sol. The resultant equation is,

$$x = A \sin(\omega t + \phi)$$
$$\Sigma A_x = 2 + 4 \cos 30° + 6 \cos 60° = 8.46$$



and $\Sigma A_y = 4 \sin 30° + 6 \sin 60° = 7.2$

$\therefore$ $A = \sqrt{(\Sigma A_x)^2 + (\Sigma A_y)^2}$

$= \sqrt{(8.46)^2 + (7.2)^2} = 11.25$

and $\tan \phi = \frac{\Sigma A_y}{\Sigma A_x} = \frac{7.2}{8.46} = 0.85$

or $\phi = \tan^{-1}(0.85) = 40.4°$

Thus, the displacement equation of the combined motion is,

$$x = A \sin(\omega t + \phi)$$
$$x = 11.25 \sin(\omega t + \phi)$$

where $\phi = 40.4°$

Check point 11.4

1. A point mass m is suspended at the end of a massless wire of length L and cross sectional area A. If Y is the Young's modulus of the wire. Then the frequency of the oscillation for the simple harmonic oscillation along the vertical direction is
(a) $\frac{1}{2\pi}\sqrt{\frac{LA}{mY}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{LAm}{Y}}$
(c) $\frac{1}{2\pi}\sqrt{\frac{YA}{mL}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{mY}{AL}}$

2. A rectangular block of mass m and area of cross-section A floats in a liquid of density ρ. If it is given small vertical displacement from equilibrium, it undergoes oscillation with a time period T, then
(a) $T \propto m$ (b) $T \propto \rho$
(c) $T \propto \frac{1}{A}$ (d) $T \propto \sqrt{\frac{1}{\rho}}$

3. Imagine a narrow tunnel between the two diametrically opposite points of the earth. A particle of mass m is released in this tunnel. The time period of oscillation is
(a) $\pi\sqrt{\frac{R}{g}}$ (b) $\frac{\pi}{2}\sqrt{\frac{R}{g}}$ (c) $2\pi\sqrt{\frac{R}{g}}$ (d) $\frac{2}{\pi}\sqrt{\frac{R}{g}}$

4. In damped oscillations damping force is directly proportional to speed of oscillator. If amplitude becomes half of its maximum value in 1s, then after 2 s amplitude will be (A_0 – initial amplitude)
(a) $\frac{1}{4}A_0$ (b) $\frac{1}{2}A_0$ (c) A_0 (d) $\frac{\sqrt{3}A_0}{2}$

5. In case of damped oscillation frequency of oscillation is
(a) greater than natural frequency
(b) less than natural frequency
(c) equal to natural frequency
(d) Both (a) and (b)

6. In damped oscillations, the amplitude of oscillations is reduced to one-third of its initial value a_0 at the end of 100 oscillations. When the oscillator completes 200 oscillations, its amplitude must be
(a) $a_0/2$ (b) $a_0/4$
(c) $a_0/6$ (d) $a_0/9$

7. A weakly damped harmonic oscillator of frequency n_1 is driven by an external periodic force of frequency n_2. When the steady state is reached, the frequency of the oscillator will be
(a) n_1 (b) n_2 (c) $\frac{n_1+n_2}{2}$ (d) (n_1+n_2)

8. In the case of sustained forced oscillations the amplitude of oscillations
(a) decreases linearly
(b) decreases sinusoidally
(c) decreases exponentially
(d) always remains constant

9. Amplitude of vibrations remains constant in case of
(i) free vibrations
(ii) damped vibrations
(iii) maintained vibrations
(iv) forced vibrations
(a) (i), (iii), and (iv) (b) (ii) and (iii)
(c) (i), (ii) and (iii) (d) (ii) and (iv)

10. In forced oscillations, a particle oscillates simple harmonically with a frequency equal to
(a) frequency of driving force
(b) natural frequency of body
(c) difference of frequency of driving force and natural frequency
(d) mean of frequency of driving force and natural frequency

11. In case of a forced vibration, the resonance wave becomes very sharp when the
(a) applied periodic force is small
(b) quality factor is small
(c) damping force is small
(d) restoring force is small

12. Two sources of sound are in resonance when
(a) they look alike
(b) they are situated at a particular distance from each other
(c) they produce the sound of same frequency
(d) they are excited by the same exciting device

13. When a tuning fork is vibrated, another in the neighbourhood begins to vibrate. This is due to the phenomenon of
(a) gravitation (b) Newton's III law
(c) resonance (d) consonance

14. Resonance is a special case of
(a) forced oscillations (b) damped oscillations
(c) undamped oscillations (d) coupled oscillations

15. During the phenomenon of resonance
(a) the amplitude of oscillation becomes large
(b) the frequency of oscillation becomes large
(c) the time period of oscillation becomes large
(d) All of the above

Chapter exercises

A Taking it together

(Assorted questions of the chapter for advanced level practice)

1. Which of the following quantities is always negative in SHM?

(a) $\mathbf{F} \cdot \mathbf{a}$ (b) $\mathbf{v} \cdot \mathbf{s}$ (c) $\mathbf{a} \cdot \mathbf{s}$ (d) $\mathbf{F} \cdot \mathbf{v}$

Here, s is displacement from mean position.

2. In a simple pendulum, the period of oscillation T is related to length of the pendulum l as

(a) $\frac{l}{T}$ = constant (b) $\frac{l^2}{T}$ = constant

(c) $\frac{l}{T^2}$ = constant (b) $\frac{l^2}{T^2}$ = constant

3. A particle executing simple harmonic motion along Y-axis has its motion described by the equation $y = A \sin(\omega t) + B$. The amplitude of the simple harmonic motion is

(a) A (b) B (c) $A + B$ (d) $\sqrt{A+B}$

4. Two particles are executing SHMs. The equations of their motions are

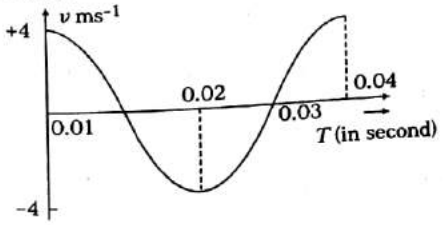
$$y_1 = 10 \sin\left(\omega t + \frac{\pi}{4}\right) \text{ and } y_2 = 5 \sin\left(\omega t + \frac{\sqrt{3}\,\pi}{4}\right)$$

What is the ratio of their amplitudes.

(a) 1 : 1 (b) 2 : 1

(c) 1 : 2 (d) None of these

5. The velocity-time diagram of a harmonic oscillator is shown in the adjoining figure. The frequency of oscillation is



(a) 25 Hz (b) 50 Hz

(c) 12.25 Hz (d) 33.3 Hz

6. The ratio of frequencies of two pendulums are 2 : 3, then their lengths are in ratio

(a) $\sqrt{\frac{2}{3}}$ (b) $\sqrt{\frac{3}{2}}$

(c) $\frac{4}{9}$ (d) $\frac{9}{4}$

7. A body oscillates with SHM according to the equation (in SI unit), $x = 5 \cos\left(2\pi t + \frac{\pi}{4}\right)$. Its instantaneous displacement at $t = 1$ s is

(a) $\frac{\sqrt{2}}{5}$ m (b) $\frac{1}{\sqrt{3}}$ m (c) $\frac{5}{\sqrt{2}}$ m (d) $\frac{1}{2}$ m

8. A body is vibrating in simple harmonic motion. If its acceleration is 12 cms^{-2} at a displacement 3 cm from the mean position, then time period is

(a) 6.28 s (b) 3.14 s (c) 1.57 s (d) 2.57 s

9. A horizontally placed spring mass system has time period T. The same system is now placed on a car moving with acceleration a in horizontal direction. Then,

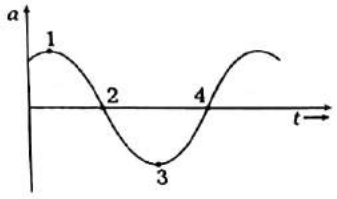
(a) time period will increase

(b) time period will decrease

(c) time period will remain constant

(d) no conclusion can be drawn

10. The acceleration a of a particle undergoing SHM is shown in the figure. Which of the labelled points corresponds to the particle being at $-x_{max}$



(a) 4 (b) 3

(c) 2 (d) 1

11. Two particles are executing simple harmonic motion. At an instant of time t their displacements are $y_1 = a\cos(\omega t)$ and $y_2 = a\sin(\omega t)$
Then the phase difference between y_1 and y_2 is
(a) 120° (b) 90° (c) 180° (d) zero

12. A particle executing SHM of amplitude 4 cm and $T = 4$ s. The time taken by it to move from positive extreme position to half the amplitude is
(a) 1 s (b) $\frac{1}{3}$ s (c) $\frac{2}{3}$ s (d) $\sqrt{\frac{3}{2}}$ s

13. The maximum acceleration of a particle in SHM is made two times keeping the maximum speed to be constant. It is possible when
(a) amplitude of oscillation is doubled while frequency remains constant
(b) amplitude is doubled while frequency is halved
(c) frequency is doubled while amplitude is halved
(d) frequency of oscillation is doubled while amplitude remains constant

14. Under the action of a force $F = -kx^3$, the motion of a particle is (k = a positive constant)
(a) simple harmonic motion
(b) uniformly accelerated motion
(c) not periodic
(d) periodic but not simple harmonic

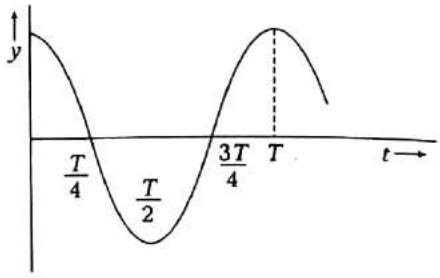
15. A light spring of force constant $8\,\text{Nm}^{-1}$ is cut into two equal halves and the two are connected in parallel; the equivalent force constant of the system is
(a) $16\,\text{Nm}^{-1}$ (b) $32\,\text{Nm}^{-1}$
(c) $8\,\text{Nm}^{-1}$ (d) $24\,\text{Nm}^{-1}$

16. In order that the resultant path on superimposing two mutually perpendicular SHM be a circle, the conditions are that
(a) the amplitudes on both SHM should be equal and they should have a phase difference of $\frac{\pi}{2}$
(b) the amplitudes should be in the ratio 1 : 2 and the phase difference should be zero
(c) the amplitudes should be in the ratio 1 : 2 and the phase difference should be $\frac{\pi}{2}$
(d) the amplitudes should be equal and the phase difference should be zero

17. The period of a particle executing SHM is 8 s. At $t = 0$ it is at the mean position. The ratio of the distances covered by the particle in the 1st second to the 2nd second is
(a) $\frac{1}{\sqrt{2}+1}$ (b) $\sqrt{2}$
(c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{2}+1$

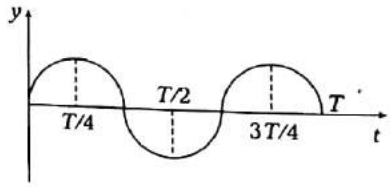
18. Force constant of a weightless spring is $16\,\text{Nm}^{-1}$. A body of mass 1 kg suspended from it is pulled down through 5 cm from its mean position and then released. The maximum kinetic energy of the system (spring + body) will be
(a) 2×10^{-2} J (b) 4×10^{-2} J (c) 8×10^{-2} J (d) 16×10^{-2} J

19. The displacement-time graph of a particle executing SHM is shown in figure. Which of the following statements is false?



(a) The acceleration is maximum at $t = T$
(b) The force is zero at $t = \frac{3T}{4}$
(c) The potential energy equals the total oscillation energy at $t = \frac{T}{2}$
(d) None of the above

20. The graph shows the variation of displacement of a particle executing SHM with time. We infer from this graph that



(a) the force is zero at time $\frac{3T}{4}$
(b) the velocity is maximum at time $\frac{T}{2}$
(c) the acceleration is maximum at time T
(d) the PE is equal to total energy at time $\frac{T}{2}$

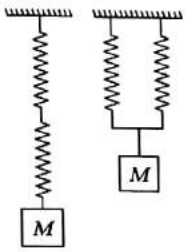
21. The time period of a simple pendulum inside a stationary lift is $\sqrt{5}$ s. What will be the time period when the lift moves upward with an acceleration $\frac{g}{4}$?
(a) $\sqrt{5}$ s (b) $2\sqrt{5}$ s (c) $(2+\sqrt{5})$ s (d) 2 s

22. A linear harmonic oscillator of force constant $2\times10^6\ \text{Nm}^{-1}$ and amplitude 0.01 m has a total mechanical energy of 160 J. Its
(a) maximum potential energy is 160 J
(b) maximum potential energy is 100 J
(c) minimum potential energy is zero
(d) minimum potential energy is 100 J

23. A second pendulum is moved to moon where acceleration due to gravity is 1/6 times that of the earth, the length of the second pendulum on moon would be

(a) 6 times (b) 12 times
(c) $\frac{1}{6}$ times (d) $\frac{1}{12}$ times

24. Two identical springs of spring constant k each are connected in series and parallel as shown in figure. A mass M is suspended from them. The ratio of their frequencies of vertical oscillation will be



(a) 1 : 2 (b) 2 : 1
(c) 4 : 1 (d) 1 : 4

25. The relation between acceleration and displacement of four particles are given below **[NCERT Exemplar]**

(a) $a_x = +2x$
(b) $a_x = +2x^2$
(c) $a_x = -2x^2$
(d) $a_x = -2x$

Which, one of the particle is exempting simple harmonic motion?

26. A particle executing SHM has a maximum speed of 30 cm s^{-1} and a maximum acceleration of 60 cm s^{-2}. The period of oscillation is **[NCERT Exemplar]**

(a) π s (b) $\frac{\pi}{2}$ s
(c) 2π s (d) $\frac{\pi}{t}$ s

27. A disc of radius R and mass M is pivoted at the rim and is set for small oscillations about an axis perpendicular to plane of disc. If a simple pendulum has to have the same time period as that of the disc, the length of the pendulum should be

(a) $\frac{5}{4}R$ (b) $\frac{2}{3}R$
(c) $\frac{3}{4}R$ (d) $\frac{3}{2}R$

28. If the length of second's pendulum is decreased by 2%, how many seconds it will lose per day

(a) 3927s (b) 3727s
(c) 3427s (d) 864s

29. A clock which keeps correct time at 20°C, is subjected to 40°C. If coefficient of linear expension of the pendulum is 12×10^{-6} /°C. How much will it gain or loose in time?

(a) 10.3 second/day (b) 20.6 second/day
(c) 5 second/day (d) 20 minute/day

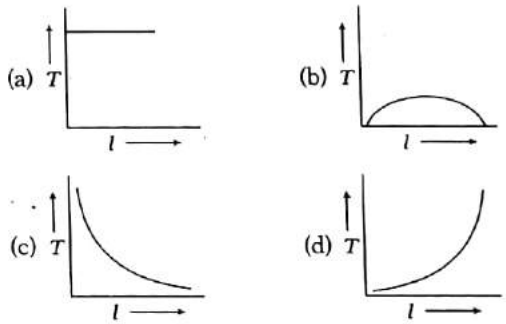
30. The equation of a damped simple harmonic motion is $m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$. Then the angular frequency of oscillation is

(a) $\omega = \left(\frac{k}{m} - \frac{b^2}{4m^2}\right)^{1/2}$ (b) $\omega = \left(\frac{k}{m} - \frac{b}{4m}\right)^{1/2}$
(c) $\omega = \left(\frac{k}{m} - \frac{b^2}{4m}\right)^{1/2}$ (d) $\omega = \left(\frac{k}{m} - \frac{b^2}{4m^2}\right)^{-1/2}$

31. The motion of a particle is given $x = A \sin \omega t + B \cos \omega t$. The motion of the particle is

(a) not simple harmonic
(b) simple harmonic with amplitude $A + B$
(c) simple harmonic with amplitude $\frac{(A+B)}{2}$
(d) simple harmonic with amplitude $\sqrt{A^2 + B^2}$

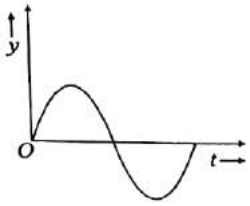
32. In case of a simple pendulum, time period *versus* length is depicted by



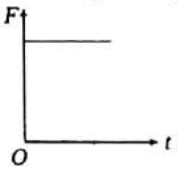
33. The displacement equation of a particle is $x = 3 \sin 2t + 4 \cos 2t$. The amplitude and maximum velocity will be respectively

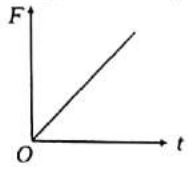
(a) 5, 10 (b) 3, 2
(c) 4, 2 (d) 3, 4

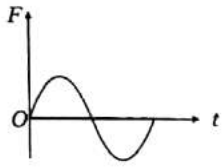
34. The displacement-time graph of a particle executing SHM is as shown in the figure.

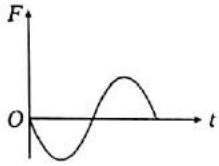


The corresponding force-time graph of the particle is

(a) 

(b) 

(c) 

(d) 

35. A tunnel has been dug through the centre of the earth and a ball is released in it. It will reach the other end of the tunnel after
(a) 84.6 min
(b) 42.3 min
(c) 1 day
(d) will not reach the other end

36. Two bodies M and N of equal masses are suspended from two separate massless springs of spring constants k_1 and k_2 respectively. If the two bodies oscillate vertically such that their maximum velocities are equal, the ratio of the amplitude of vibration of M to that on N is
(a) $\frac{k_1}{k_2}$ (b) $\sqrt{\frac{k_1}{k_2}}$ (c) $\frac{k_2}{k_1}$ (d) $\sqrt{\frac{k_2}{k_1}}$

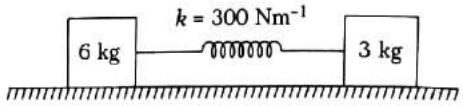
37. A particle is executing simple harmonic motion with a period of T seconds and amplitude in metre. The shortest time it takes to reach a point $\frac{a}{\sqrt{2}}$ m from its mean position in seconds is
(a) T (b) $\frac{T}{4}$ (c) $\frac{T}{8}$ (d) $\frac{T}{16}$

38. A mass M is suspended from a massless spring. An additional mass m stretches the spring further by a distance x. The combined mass will oscillate with a period
(a) $2\pi\sqrt{\left\{\frac{(M+m)x}{mg}\right\}}$ (b) $2\pi\sqrt{\left\{\frac{mg}{(M+m)x}\right\}}$
(c) $2\pi\sqrt{\left\{\frac{(M+m)}{mgx}\right\}}$ (d) $\frac{\pi}{2}\sqrt{\left\{\frac{mg}{(M+m)x}\right\}}$

39. A particle executes linear simple harmonic motion with an amplitude of 2 cm. When the particle is at 1 cm from the mean position the magnitude of its velocity is equal to that of its acceleration. Then its time period in seconds is
(a) $\frac{1}{2\pi\sqrt{3}}$ (b) $2\pi\sqrt{3}$ (c) $\frac{2\pi}{\sqrt{3}}$ (d) $\frac{\sqrt{3}}{2\pi}$

40. A spring has a natural length of 50 cm and a force constant of 2×10^3 Nm^{-1}. A body of mass 10 kg is suspended from it and the spring is stretched. If the body is pulled down to a length of 58 cm and released, it executes simple harmonic motion. What is the net force on the body when it is at its lowermost position of its oscillation? (Take, $g = 10$ ms^{-2}).
(a) 20 N (b) 40 N (c) 60 N (d) 80 N

41. Two point masses of 3 kg and 6 kg are attached to opposite ends of horizontal spring whose spring constant is 300 Nm^{-1} as shown in the figure. The natural vibration frequency of the system is approximately

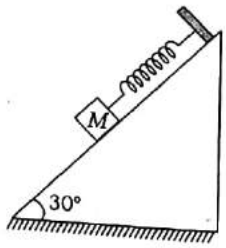


(a) 4 Hz (b) 3 Hz (c) 2 Hz (d) 1 Hz

42. In a spring-mass system, the length of the spring is L, and it has a mass M attached to it and oscillates with an angular frequency ω. The spring is then cut into two parts, one (i) with relaxed length αL and the other (ii) with relaxed length $(1-\alpha)L$. The force constants of the two springs A and B are
(a) $\frac{k}{1-\alpha}$ and $\frac{k}{\alpha}$ (b) $\frac{k}{\alpha}$ and $\frac{k}{1-\alpha}$
(c) $\alpha k, (1-\alpha)k$ (d) k and k

43. A particle is attached to a vertical spring and is pulled down a distance 0.04 m below its equilibrium position and is released from rest. The initial upward acceleration of the particle is 0.30 ms^{-2}. The period of the oscillation is
(a) 4.08 s (b) 1.92 s (c) 3.90 s (d) 2.29 s

44. A block of mass 0.2 kg, which slides without friction on a 30° incline, is connected to the top of the incline by a massless spring of force constant 80 Nm^{-1} as shown in figure.
If the block is pulled slightly from its mean position, what is the period of oscillations?



(a) π s (b) $\frac{\pi}{10}$ s (c) $\frac{2\pi}{5}$ s (d) $\frac{\pi}{2}$ s

45. Two simple harmonic motions are represented by the following equations

$$y_1 = 40 \sin \omega t$$

and $$y_2 = 10(\sin \omega t + c \cos \omega t)$$

If their displacement amplitudes are equal, then the value of c (in appropriate units) is

(a) $\sqrt{13}$ (b) $\sqrt{15}$
(c) $\sqrt{17}$ (d) 4

46. The potential energy of a particle of mass 2 kg in SHM is $(9x^2)$ J. Here, x is the displacement from mean position. If total mechanial energy of the particle is 36 J. Then maximum speed of the particle is

(a) 4 ms^{-1} (b) 2 ms^{-1}
(c) 6 ms^{-1} (d) 10 ms^{-1}

47. A particle of mass 0.1 kg is executing SHM of amplitude 0.1m. When the particle passes through the mean position, its KE is 8×10^{-3} J. Find the equation of motion of the particle if the initial phase of oscillation is 45°

(a) $y = 0.1 \cos\left(3t + \frac{\pi}{4}\right)$ (b) $y = 0.1 \sin\left(6t + \frac{\pi}{4}\right)$
(c) $y = 0.1 \sin\left(4t + \frac{\pi}{4}\right)$ (d) $y = 0.1 \cos\left(4t + \frac{\pi}{4}\right)$

48. Molten-wax of mass m drops on a block of mass M, which is oscillating on a frictionless table. Select the incorrect option.

(a) amplitude does not change
(b) amplitude increases
(c) time period decreases
(d) time period increases

49. The displacement of a particle is represented by the equation $y = 3 \cos\left(\frac{\pi}{4} - 2\omega t\right)$. The motion of the particle is **[NCERT Exemplar]**

(a) simple harmonic with period $2\pi / \omega$
(b) simple harmonic with period π / ω
(c) periodic but not simple harmonic
(d) non-periodic

50. The displacement of a particle is represented by the equation $y = \sin^3 \omega t$. The motion is **[NCERT Exemplar]**

(a) non-periodic
(b) periodic but not simple harmonic
(c) simple harmonic with period $2\pi/\omega$
(d) simple harmonic with period π / ω

51. Motion of an oscillating liquid column in a U-tube is **[NCERT Exemplar]**

(a) periodic but not simple harmonic
(b) non-periodic
(c) simple harmonic and time period is independent of the density of the liquid
(d) simple harmonic and time period is directly proportional to the density of the liquid

52. A particle is acted simultaneously by mutually perpendicular simple harmonic motion $x = a \cos\omega t$ and $y = a \sin \omega t$. The trajectory of motion of the particle will be **[NCERT Exemplar]**

(a) an ellipse
(b) a parabola
(c) a circle
(d) a straight line

53. The displacement of a particle varies with time according to the relation $y = a \sin \omega t + b \cos \omega t$. **[NCERT Exemplar]**

(a) The motion is oscillatory but not SHM
(b) The motion is SHM with amplitude $a + b$
(c) The motion is SHM with amplitude $a^2 + b^2$
(d) The motion is SHM with amplitude $\sqrt{a^2 + b^2}$

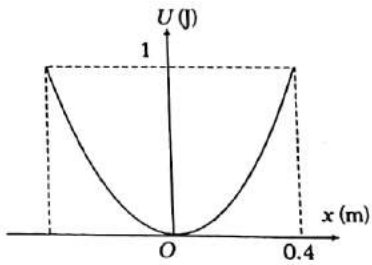
54. The equation of motion of a particle is $x = a \cos (\alpha t)^2$. The motion is **[NCERT Exemplar]**

(a) periodic but not oscillatory
(b) periodic and oscillatory
(c) oscillatory but not periodic
(d) Neither periodic nor oscillatory

55. A mass of 0.2 kg is attached to the lower end of a massless spring of force constant 200 Nm^{-1}, the upper end of which is fixed to a rigid support. Which of the following statement is true?

(a) In equilibrium, the spring will be stretched by 1 cm
(b) If the mass is raised till the spring becomes unstretched and then released, it will go down by 2 cm before moving upwards
(c) The frequency of oscillation will be nearly 5 Hz
(d) All of the above

56. A particle of mass 2 kg moves in simple harmonic motion and its potential energy U varies with position x as shown. The period of oscillation of the particle is



(a) $\frac{2\pi}{5}$ s (b) $\frac{2\sqrt{2}\,\pi}{5}$ s
(c) $\frac{\sqrt{2}\,\pi}{5}$ s (d) $\frac{4\pi}{5}$ s

57. The speed (v) of a particle moving along a straight line, when it is at a distance (x) from a fixed point on the line, is given by $v^2 = 144 - 9x^2$. Select correct alternate.
(a) The magnitude of acceleration at a distance 3 units from the fixed point is 27 units
(b) The motion is simple harmonic with $T = \frac{2\pi}{3}$ units
(c) The maximum displacement from the fixed point is 4 units
(d) all are correct

58. Maximum kinetic energy of a particle of mass 1 kg in SHM is 8 J. Time period of SHM is 4 s. Maximum potential energy during the motion is 10 J. Then
(a) amplitude of oscillations is approximately 2.53 m
(b) minimum potential energy of the particle is 2 J
(c) maximum acceleration of the particle is approximately 6.3 ms^{-2}
(d) minimum kinetic energy of the particle is 2 J

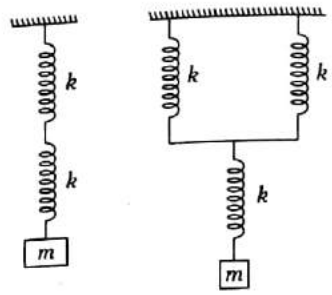
59. The potential energy of a particle of mass 0.1 kg, moving along the X-axis, is given by $U = 5x(x - 4)$ J, where x is in metres. Choose the wrong option.
(a) The speed of the particle is maximum at $x = 2$ m
(b) The particle executes simple harmonic motion
(c) The period of oscillation of the particle is $\frac{\pi}{5}$ s
(d) All are correct

60. A simple pendulum has a time period T in vacuum. Its time period when it is completely immersed in a liquid of density one-eight of the density of material of the bob is
(a) $\sqrt{\frac{7}{9}}T$ (b) $\sqrt{\frac{5}{8}}T$
(c) $\sqrt{\frac{3}{8}}T$ (d) $\sqrt{\frac{8}{7}}T$

61. A simple pendulum has time period T. The bob is given negative charge and surface below it is given positive charge. The new time period will be
(a) less than T
(b) greater than T
(c) equal to T
(d) infinite

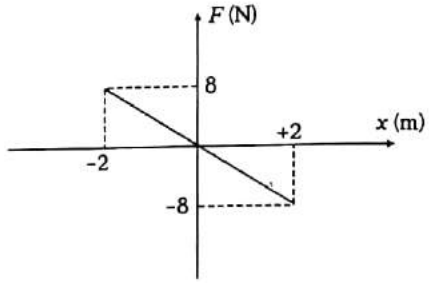
62. Two simple pendulums whose lengths are 100 cm and 121 cm are suspended side by side. Their bobs are pulled together and then released. After how many minimum oscillations of the longer pendulum, will the two be in phase again.
(a) 11 (b) 10
(c) 21 (d) 20

63. Some springs are combined in series and parallel arrangement as shown in the figure and a mass m is suspended from them. The ratio of their frequencies will be



(a) 1 : 1 (b) 2 : 1 (c) $\sqrt{3}$: 2 (d) 4 : 1

64. A body of mass 0.01 kg executes simple harmonic motion (SHM) about $x = 0$ under the influence of a force shown in figure. The period of the SHM is



(a) 1.05 s (b) 0.52 s
(c) 0.25 s (d) 0.31 s

65. The vertical motion of a ship at sea is described by the equation $\frac{d^2x}{dt^2} = -4x$, where x is the vertical height of the ship (in metre) above its mean position. If it oscillates through a height of 1 m
(a) its maximum vertical speed will be 1 ms^{-1}
(b) its maximum vertical speed will be 2 ms^{-1}
(c) its greatest vertical acceleration is 2 ms^{-2}
(d) its greatest vertical acceleration is 1 ms^{-2}

66. A simple pendulum is suspended from the ceiling of a car and its period of oscillation is T when the car is at rest. The car starts moving on a horizontal road with a constant acceleration g (equal to the acceleration due to gravity, in magnitude) in the forward direction. To keep the time period same, the length of the pendulum
(a) will have to be increased by $\sqrt{2}\,l$
(b) will have to be increased by $(\sqrt{2} - 1)\,l$
(c) will have to be decreased by $\sqrt{2}\,l$
(d) will have to be decreased by $(\sqrt{2} - 1)\,l$

67. A block of mass m, attached to a spring of spring constant k, oscillates on a smooth horizontal table. The other end of the spring is fixed to a wall. The block has a speed v when the spring is at its natural length. Before coming to an instantaneous rest. If the block moves a distance x from the mean position, then

(a) $x=\sqrt{\frac{m}{k}}$ (b) $x=\frac{1}{v}\sqrt{\frac{m}{k}}$ (c) $x=v\sqrt{\frac{m}{k}}$ (d) $x=\sqrt{\frac{mv}{k}}$

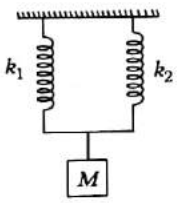
68. A small ball is dropped from a certain height on the surface of a non-viscous liquid of density less than the density of ball. The motion of the ball is

(a) SHM
(b) periodic but not SHM
(c) not periodic
(d) SHM for half a period and non-periodic for rest half of the period

69. Two particles executes SHM of the same amplitude and frequency along the same straight line. They pass one another when going in opposite directions each time their displacement is half their amplitude. What is the phase difference between them?

(a) 60° (b) 30° (c) 90° (d) 120°

70. A mass is suspended separately by two springs of spring constants k_1 and k_2 in successive order. The time periods of oscillations in the two cases are T_1 and T_2 respectively. If the same mass be suspended by connecting the two springs in parallel, (as shown in figure) then the time period of oscillations is T. The correct relation is



(a) $T^2=T_1^2+T_2^2$ (b) $T^{-2}=T_1^{-2}+T_2^{-2}$
(c) $T^{-1}=T_1^{-1}+T_2^{-1}$ (d) $T=T_1+T_2$

71. A horizontal platform with an object placed on it is executing SHM in the vertical direction. The amplitude of oscillation is 4×10^{-3} m. What must be least period of these oscillations, so that the object is not detached from the platform? (take, $g=10\ \text{ms}^{-2}$)

(a) $\frac{\pi}{25}$ s (b) $\frac{\pi}{5}$ s (c) $\frac{\pi}{10}$ s (d) $\frac{\pi}{50}$ s

72. Two masses 8 kg and 4 kg are suspended together by a massless spring of spring constant 1000 Nm^{-1}. When the masses are in equilibrium 8 kg is removed without disturbing the system. The amplitude of oscillation is

(a) 0.5 m (b) 0.08 m (c) 0.4 m (d) 0.04 m

73. In SHM, potential energy of a particle at mean position is E_1 and kinetic energy is E_2, then

(a) $E_1=E_2$
(b) total potential energy at $x=\frac{\sqrt{3}A}{2}$ is $E_1+\frac{3E_2}{4}$
(c) total kinetic energy at $x=\frac{\sqrt{3}A}{2}$ is $\frac{3E_2}{4}$
(d) total kinetic energy at $x=\frac{A}{\sqrt{2}}$ is $\frac{E_2}{4}$

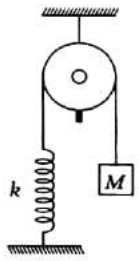
74. A particle executes SHM with amplitude of 20 cm and time period of 12 s. What is the minimum time required for it to move between two points 10 cm on either side of the mean position?

(a) 1 s (b) 2 s
(c) 3 s (d) 4 s

75. A uniform spring whose unstressed length is l has a force constant k. The spring is cut into two pieces of unstressed lengths l_1 and l_2, where $l_1=nl_2$, where n being an integer. Now a mass m is made to oscillate with first spring. The time period of its oscillation would be

(a) $T=2\pi\sqrt{\frac{mn}{k(n+1)}}$ (b) $T=2\pi\sqrt{\frac{m}{k(n+1)}}$
(c) $T=2\pi\sqrt{\frac{m}{nk}}$ (d) $T=2\pi\sqrt{\frac{m(n+1)}{nk}}$

76. A mass $M=5$ kg is attached to a spring as shown in the figure and held in position so that the spring remains unstretched. The spring constant is 200 Nm^{-1}. The mass M is then released and begins to undergo small oscillations. The amplitude of oscillation is

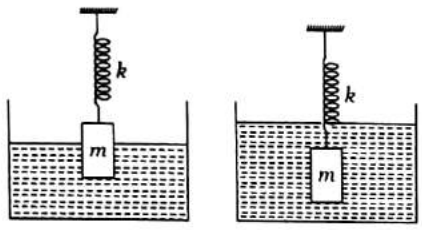


(a) 0.5 m (b) 0.25 m
(c) 0.2 m (d) 0.1 m

77. A rectangular block of mass m and area of cross-section A floats in a liquid of density ρ. If it is given a small vertical displacement from equilibrium it undergoes oscillation with a time period T. Then,

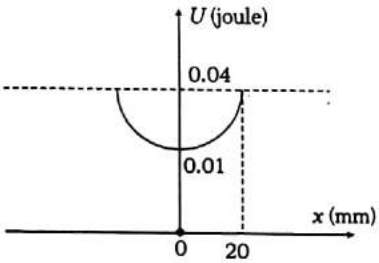
(a) $T\propto\sqrt{\rho}$ (b) $T\propto\rho^0$
(c) $T\propto\frac{1}{\rho}$ (d) $T\propto\frac{1}{\sqrt{\rho}}$

78. Period of small oscillations in the two cases shown in figure is T_1 and T_2 respectively. Assume fluid does not have any viscosity, then



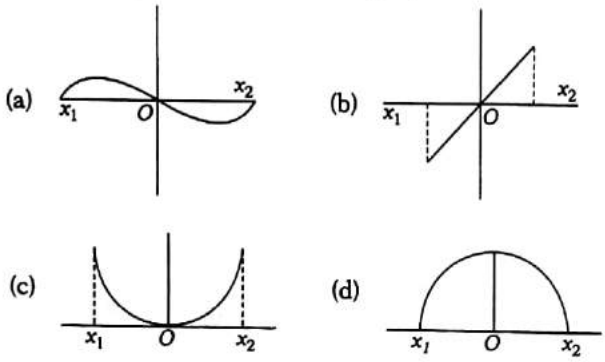
(a) $T_1 = T_2$ (b) $T_1 < T_2$
(c) $T_1 > T_2$ (d) Cannot say anything

79. The variation of PE of harmonic oscillator is as shown in figure. The spring constant is



(a) 1×10^2 Nm^{-1} (b) 1.5×10^2 Nm^{-1}
(c) 2×10^2 Nm^{-1} (d) 3×10^2 Nm^{-1}

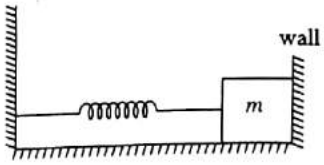
80. A particle of mass m oscillates with simple harmonic motion between points x_1 and x_2, the equilibrium position being at O. Its potential energy is plotted. It will be as given below in the graph



81. A solid cube floats in water half immersed and has small vertical oscillations of time period $\frac{\pi}{5}$ s. Its mass (in kg) is (take, $g = 10$ ms^{-2})

(a) 4 (b) 2 (c) 1 (d) 0.5

82. In the figure, the block of mass m, attached to the spring of stiffness k is in contact with the completely elastic wall, and the compression in the spring is e. The spring is compressed further by e by displacing the block towards left and is then released. If the collision between the block and the wall is completely elastic then the time period of oscillations of the block will be



(a) $\frac{2\pi}{3}\sqrt{\frac{m}{k}}$ (b) $2\pi\sqrt{\frac{m}{k}}$ (c) $\frac{\pi}{3}\sqrt{\frac{m}{k}}$ (d) $\frac{\pi}{6}\sqrt{\frac{m}{k}}$

83. A cubical block of mass M vibrates horizontally with amplitude of 4 cm and a frequency of 2 Hz. A small block of mass m is palced on the bigger block. In order that the smaller block does not slide on the bigger block, the minimum value of the coefficient of static friction between the two blocks is

(a) 0.36 (b) 0.40
(c) 0.64 (d) 0.72

84. Two pendulums of time periods 3 s and 7 s respectively start oscillating simultaneously from two opposite extreme positions. After how much time they will be in same phase?

(a) $\frac{21}{8}$ s (b) $\frac{21}{4}$ s (c) $\frac{21}{2}$ s (d) $\frac{21}{10}$ s

85. A particle under the action of a SHM has a period of 3 s and under the effect of another it has a period 4 s. What will be its period under the combined action of both the SHM's in the same direction?

(a) 7 s (b) 5 s (c) 2.4 s (d) 0.4 s

86. A particle performs SHM in a straight line. In the first second, starting from rest, it travels a distance a and in the next second it travels a distance b in the same side of mean position. The amplitude of the SHM is

(a) $a - b$ (b) $\frac{2a - b}{3}$
(c) $\frac{2a^2}{3a - b}$ (d) None of these

87. A particle is in linear SHM of amplitude A and time period T. If v refers to its average speed during any interval of $T/3$, then the maximum possible value of v is

(a) $\frac{3\sqrt{3}}{T}A$ (b) $\frac{\sqrt{3}A}{T}$
(c) $\frac{2\sqrt{3}}{T}A$ (d) $\frac{3A}{T}$

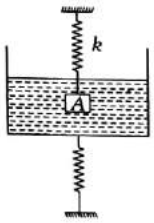
88. A particle of mass m is dropped from a great height h above the hole in the earth dug along its diameter.

(a) The motion of the particle is simple harmonic
(b) The motion of the particle is periodic

(c) The speed of the particle at the centre of earth equals $\sqrt{\frac{2GM}{(R+h)}}$, where R and M are the radius and mass of the earth respectively

(d) The speed of the particle at the centre of earth equals $\sqrt{\frac{GM(R+3h)}{R(R+h)}}$, where R and M are the radius and mass of the earth respectively

89. The system shown in figure is in equilibrium. The mass of the container with liquid is M, density of liquid in the container is ρ and the volume of the block is V. If the container is now displaced downwards through a distance x_0 and released such that the block remains well inside the liquid then during subsequent motion



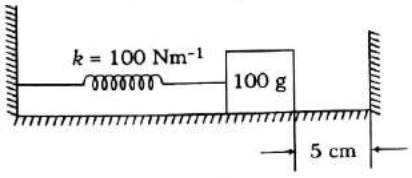
(a) time period of SHM of the container will be $2\pi\sqrt{\frac{M}{k}}$

(b) time period of SHM of the container will be $2\pi\sqrt{\frac{M+\rho V}{k}}$

(c) amplitude of SHM of the container is x_0

(d) amplitude of SHM of the container is $2x_0$

90. A block of mass 100 g attached to a spring of spring constant 100 Nm^{-1} is lying on a frictionless floor as shown. The block is moved to compress the spring by 10 cm and then released. If the collisions with the wall in front are elastic, then the time period of the motion is



(a) 0.2 s (b) 0.1 s (c) 0.15 s (d) 0.132 s

91. One end of a long metallic wire of length L is tied to the ceiling. The other end is tied to massless spring of spring constant k. A mass m hangs freely from the free end of the spring. The area of cross-section and Young's modulus of the wire are A and Y respectively. If the mass is slightly pulled down and released, it will oscillate with a time period T equal to

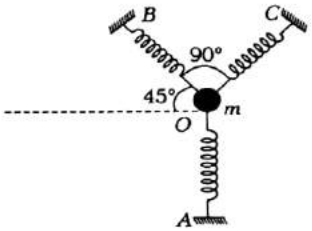
(a) $2\pi\sqrt{\frac{m}{k}}$ (b) $2\pi\sqrt{\frac{m(YA+kL)}{YAk}}$

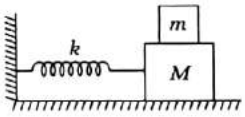
(c) $2\pi\sqrt{\frac{m(YA+kL)}{Ak}}$ (d) $2\pi\sqrt{\frac{m(Y+kL)}{YAk}}$

92. A particle of mass m is attached to three identical springs A,B and C each of force constant k as shown in figure. If the particle of mass m is pushed slightly against the spring A and released, then the time period of oscillation is

(a) $2\pi\sqrt{\frac{2m}{k}}$ (b) $2\pi\sqrt{\frac{m}{2k}}$ (c) $2\pi\sqrt{\frac{m}{k}}$ (d) $2\pi\sqrt{\frac{m}{3k}}$

93. A mass M is attached to a horizontal spring of force constant k fixed one side to a rigid support as shown in figure. The mass oscillates on a frictionless surface with time period T and amplitude A. When the mass M is in equilibrium position, another mass m is gently placed on it. What will be the new amplitude of oscillations?

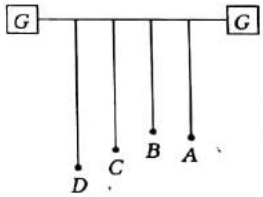




(a) $\sqrt{\frac{(M+m)}{M}}A$ (b) $\sqrt{\frac{(M-m)}{M}}A$

(c) $\sqrt{\frac{M}{(M+m)}}A$ (d) $\sqrt{\frac{M}{(M-m)}}A$

94. Four pendulums A, B, C and D are suspended from the same **[NCERT Exemplar]**



elastic support as shown in figure. A and C are of the same length, while B is smaller than A and D is larger than A. If A is given a transverse displacement,

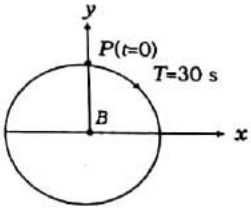
(a) D will vibrate with maximum amplitude

(b) C will vibrate with maximum amplitude

(c) B will vibrate with maximum amplitude

(d) All the four will oscillate with equal amplitude

95. Figure shows the circular motion of a particle. The radius of the circle, the period, sense of revolution and the initial position are indicated on the figure. The simple harmonic motion of the x-projection of the radius vector of the rotating particle P is **[NCERT Exemplar]**



(a) $x(t) = B\sin\left(\frac{2\pi t}{30}\right)$ (b) $x(t) = B\cos\left(\frac{\pi t}{15}\right)$

(c) $x(t) = B\sin\left(\frac{\pi t}{15}+\frac{\pi}{2}\right)$ (d) $x(t) = B\cos\left(\frac{\pi t}{15}+\frac{\pi}{2}\right)$

B Medical entrance special format questions

Assertion and reason

Directions (Q. Nos. 1-20) *These questions consists of two statements each printed as Assertion and Reason. While answering these question you are required to choose any one of the following five responses*

(a) If both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If Assertion is false but Reason is true.
(e) If both Assertion and Reason are false.

1. **Assertion** In SHM to find time taken in moving from one point to another point we can not apply the relation,

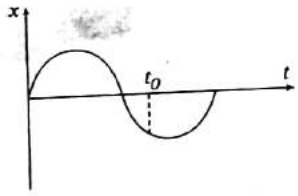
$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

Reason In SHM speed is not constant.

2. **Assertion** Mean position of SHM is the stable equilibrium position.

Reason In stable equilibrium, position potential energy is minimum.

3. **Assertion** In the x-t graph of a particle in SHM acceleration of particle at time t_0 is positive but velocity is negative.



Reason $a \propto -x$ and velocity is slope of x-t graph.

4. **Assertion** In $x = A\cos\omega t$ equation of SHM, X is measured from the extreme position, $x = +A$.

Reason At time $t = 0$, particle is at $x = +A$.

5. **Assertion** If a pendulum is suspended in a lift and lift accelerates upwards, then its time period will decrease.

Reason Effective value of g will be

$$g_e = g + a$$

6. **Assertion** Simple harmonic motion is an example of one dimensional motion with non-uniform acceleration.

Reason In simple harmonic motion, acceleration varies with displacement linearly.

7. **Assertion** If amplitude of SHM is increased, time period of SHM will increase.

Reason If amplitude is increased, body have to travel more distance in one complete oscillation.

8. **Assertion** x-t equation of a particle in SHM is given as:

$$x = A\cos\omega t$$

At time $t = \frac{\pi}{2\omega}$, potential energy is minimum.

Reason In the given equation the minimum potential energy is zero.

9. **Assertion** In $x = 3 + 4\cos\omega t$, amplitude of oscillation is 4 units.

Reason Mean position is at $x = 3$.

10. **Assertion** In SHM v-x graph is an ellipse, where v is velocity and x is displacement from mean position.

Reason Equation between v and x is $\frac{v^2}{\omega^2} + \frac{x^2}{1} = A^2$, which is the equation of an ellipse.

11. **Assertion** In $x = 5 - 4\sin\omega t$, motion of body is SHM about the mean position $x = 5$.

Reason Amplitude of oscillations is 9.

12. **Assertion** Displacement-time equation of a particle moving along X-axis is $x = 4 + 6\sin\omega t$. Under this situation, motion of particle is not simple harmonic.

Reason $\frac{d^2x}{dt^2}$ for the given equation is proportional to $-x$.

13. **Assertion** Time period of a spring-block system is T. If length of spring is decreased, time period will decrease.

Reason If length is decreased, then the block will have to travel less distance and it will take less time.

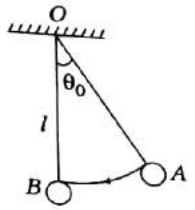
14. **Assertion** A particle performing SHM at certain instant is having velocity v. It again acquires a velocity v for the first time after a time interval of T second. Then the time period of oscillation is T second.

Reason A particle performing SHM can have same velocity at two instants in one cycle.

15. **Assertion** Average kinetic energy in one oscillation during SHM of a body is $\frac{1}{4}m\omega^2A^2$.

Reason Maximum kinetic energy is $\frac{1}{2}m\omega^2A^2$.

16. **Assertion** Bob is released from rest from position A. Given θ_0 very small. Angular velocity of bob about point O is maximum and $\left(\sqrt{\frac{g}{l}}\right)\theta_0$ at point O.



Reason For small angular amplitudes, motion of bob is simple harmonic.

17. **Assertion** In a spring block system if length of spring and mass both are halved, time period of oscillation will remain unchanged.

Reason Angular frequency of SHM is $\omega = \sqrt{\frac{k}{m}}$ where, k is spring constant and m is mass of block.

Match the Columns

1. In SHM match the following.

	Column I		Column II
(A)	Displacement and velocity	(p)	Phase difference of zero
(B)	Displacement and acceleration	(q)	Phase difference of $\frac{\pi}{2}$
(C)	Velocity and acceleration	(r)	Phase difference of π

2. In SHM match the following columns.

	Column I		Column II
(A)	Acceleration-displacement graph	(p)	Parabola
(B)	Velocity-acceleration graph	(q)	Straight line
(C)	Velocity-time graph	(r)	Circle
(D)	Acceleration-time graph	(s)	None

3. In case of second's pendulum, match the following columns. (Consider shape of earth also).

	Column I		Column II
(A)	At pole	(p)	$T > 2s$
(B)	On a satellite	(q)	$T < 2s$
(C)	At mountain	(r)	$T = 2s$
(D)	At centre of earth	(s)	$T = 0$
		(t)	$T = \infty$

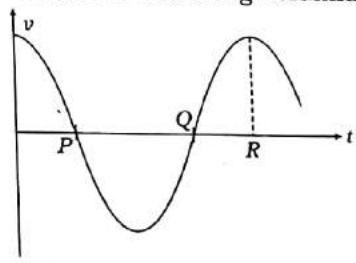
4. In the equation $y = A\sin\left(\omega t + \frac{\pi}{4}\right)$ match the following. For $x = \frac{A}{2}$

	Column I		Column II
(A)	Kinetic energy	(p)	Half the maximum value
(B)	Potential energy	(q)	3/4 times the maximum value
(C)	Acceleration	(r)	1/4 times the maximum value
		(s)	Cannot say anything

5. In spring-block system match the following columns.

	Column I		Column II
(A)	If k (the spring constant) is made 4-times	(p)	Speed will become 16 times
(B)	If m (the mass of block) is made 4-times	(q)	Potential energy will become 4 times
(C)	If k and m both are made 4-times	(r)	Kinetic energy will remain unchanged
		(s)	None

6. Velocity-time graph of a particle in SHM is as shown in figure. Match the following columns.



	Column I		Column II
(A)	At P	(p)	Particle is at $x = -A$
(B)	At Q	(q)	Acceleration of particle is maximum
(C)	At R	(r)	Displacement of particle is zero
		(s)	Acceleration of particle is zero
		(t)	None

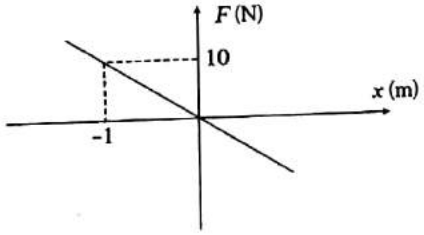
7. In $y = A\sin\omega t + A\sin\left(\omega t + \frac{2\pi}{3}\right)$, match the following.

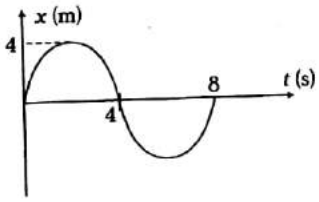
	Column I		Column II
(A)	Motion	(p)	Is periodic but not SHM
(B)	Amplitude	(q)	Is SHM
(C)	Initial phase	(r)	A
(D)	Maximum velocity	(s)	$\pi/3$
		(t)	$\omega A/2$
		(u)	None

8. x-t equation of a particle in SHM is given as $x = 1 \sin(12\pi t)$ in SI units. Potential energy at mean position is zero. Mass of particle is 1/4 kg. Match the following columns.

Column I	Column II
(A) Frequency with which kinetic energy oscillates	(p) 1/2 SI unit
(B) Speed of particle is maximum at time t	(q) $18\pi^2$ SI unit
(C) Maximum potential energy	(r) 12 SI unit
(D) Force constant k	(s) $36\pi^2$ SI unit

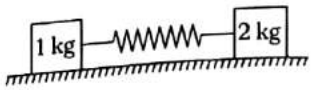
9. F-x and x-t graph of a particle in SHM are as shown in figure. Match the following columns.





Column I	Column II
(A) Mass of the particle	(p) $\pi/2$ SI unit
(B) Maximum kinetic energy of particle	(q) $(160/\pi^2)$ SI unit
(C) Angular frequency of particle	(r) (8×10^{-3}) SI unit
	(s) None

10. In the two block spring system, force constant of spring is $k = 6\ \text{Nm}^{-1}$. Spring is stretched by 12 cm towards left. Match the following columns.



Column I	Column II
(A) Angular frequency of oscillation	(p) 4.8×10^{-3} SI unit
(B) Maximum kinetic energy of 1 kg	(q) 3 SI unit
(C) Maximum kinetic energy of 2 kg	(r) 2.4×10^{-3} SI unit
	(s) None

11. A uniform rod of length l is suspended from a point P and the rod and is made to undergo small oscillations. Match the following for the time period.

Column I	Column II
(A) If P is the centre of mass	(p) Zero
(B) If P is the end point	(q) $2\pi\sqrt{\frac{l}{3g}}$
(C) Length of simple pendulum having the time period equal to that of the rod when P is end point	(r) $\frac{l}{3}$
	(s) $\frac{2l}{3}$
	(t) None

C Medical entrances' gallery

(Collection of questions asked in NEET & Various Medical Entrance Exams)

1. A body of mass m is attached to the lower end of a spring whose upper end is fixed. The spring has negligible mass. When the mass m is slightly pulled down and released, it oscillates with a time period of 3 s. When the mass m is increased by 1 kg, the time period of oscillations becomes 5 s. The value of m in kg is **[NEET 2016]**

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{16}{9}$ (d) $\frac{9}{16}$

2. Two similar springs P and Q have spring constants k_P and k_Q, such that $k_P > k_Q$. They are stretched, first by the same amount (case a), then by the same force (case b). The work done by the springs W_P and W_Q are related as, in case (a) and case (b), respectively **[CBSE AIPMT 2015]**

(a) $W_P = W_Q$; $W_P > W_Q$ (b) $W_P = W_Q$; $W_P = W_Q$
(c) $W_P > W_Q$; $W_Q > W_P$ (d) $W_P < W_Q$; $W_Q < W_P$

3. A particle is executing SHM along a straight line. Its velocities at distances x_1 and x_2 from the mean position are v_1 and v_2, respectively. Its time period is **[CBSE AIPMT 2015; EAMCET 2013]**

(a) $2\pi\sqrt{\frac{x_1^2+x_2^2}{v_1^2+v_2^2}}$ (b) $2\pi\sqrt{\frac{x_2^2-x_1^2}{v_1^2-v_2^2}}$

(c) $2\pi\sqrt{\frac{v_1^2+v_2^2}{x_1^2+x_2^2}}$ (d) $2\pi\sqrt{\frac{v_1^2-v_2^2}{x_1^2-x_2^2}}$

4. The velocity vector v and displacement vector x of a particle executing SHM are related as $\frac{v\,dv}{dx} = -\omega^2 x$ with the initial condition $v = v_0$ at $x = 0$. The velocity v, when displacement is x, is **[AIIMS 2015]**

(a) $v = \sqrt{v_0^2 + \omega^2 x^2}$ (b) $v = \sqrt{v_0^2 - \omega^2 x^2}$

(c) $v = \sqrt[3]{v_0^3 + \omega^3 x^3}$ (d) $v = v_0 - (\omega^3 x^3 e^{x^3})^{1/3}$

5. A particle hanging from a spring stretches it by 1 cm at earth's surface. How much will same particle stretch the same spring at a place 800 km above the earth's surface? **[UK PMT 2015]**

(a) 0.79 cm (b) 1.79 cm

(c) 0.21 cm (d) None of these

6. A particle of mass 200 g executes simple harmonic motion. The restoring force is provided by a spring of spring constant 80 Nm^{-1}. What would be the time period? **[UK PMT 2015]**

(a) 0.93 s (b) 0.62 s

(c) 0.31 s (d) None of these

7. A body of mass 40 g executing simple harmonic motion of amplitude 2 cm. If the time period is 0.20 s, what would be the total mechanical energy of the system? **[UK PMT 2015]**

(a) 14.8×10^{-3} J (b) 7.9×10^{-3} J

(c) 23.7×10^{-3} J (d) 3.9×10^{-3} J

8. A copper sphere attached to the bottom of a vertical spring is oscillating with time period 10 s. If the copper sphere is immersed in a fluid (assume the viscosity of the fluid is negligible) of specific gravity $\frac{1}{4}$ of that of the copper, then time period of the oscillation is **[EAMCET 2015]**

(a) 5 s (b) 10 s

(c) 2.5 s (d) 20 s

9. The physical quantity which remains constant in simple harmonic motion is **[Kerala CEE 2015]**

(a) kinetic energy (b) potential energy

(c) restoring force (d) displacement

(e) frequency

10. Out of the following functions representing motion of a particle, which functions/function represent SHM?

A. $y = \sin\omega t - \cos\omega t$

B. $y = \sin^3 \omega t$

C. $y = 5\cos\left(\frac{3\pi}{4} - 3\omega t\right)$

D. $y = 1 + \omega t + \omega^2 t^2 + \omega^3 t^3$ **[CG PMT 2015]**

(a) A and B (b) A and C

(c) A only (d) A, B and C not D

11. The ratio of kinetic energy to the potential energy of a particle executing SHM at a distance equal to half its amplitude, the distance being measured from its equilibrium position is **[KCET 2015; Kerala CEE 2014]**

(a) 2 : 1 (b) 3 : 1 (c) 8 : 1 (d) 4 : 1

(e) 1:1

12. A body oscillates with SHM according to the equation (in SI unit), $x = 5\cos\left(2\pi t + \frac{\pi}{4}\right)$. Its instantaneous displacement at $t = 1$ s is **[Kerala CEE 2014]**

(a) $\frac{\sqrt{2}}{5}$ m (b) $\frac{1}{\sqrt{3}}$ m

(c) $\frac{1}{\sqrt{2}}$ m (d) $\frac{1}{2}$ m

(e) $\frac{5}{\sqrt{2}}$ m

13. Two pendulums have time period T and $5T/4$. They starts SHM at the same time from the mean position. What will be the phase difference between them after the bigger pendulum completed one oscillation? **[UP CPMT 2014]**

(a) 45° (b) 90° (c) 60° (d) 30°

14. The amplitude of a particle executing SHM is 4 cm. At the mean position, the speed of the particle is 16 cms^{-1}. The distance of the particle from the mean position at which the speed of the particle becomes $8\sqrt{3}$ cms^{-1} will be **[AFMC 2014]**

(a) $2\sqrt{3}$ cm (b) $\sqrt{3}$ cm

(c) 1 cm (d) 2 cm

15. What is the phase difference between two simple harmonic motions represented by $x_1 = A\sin\left(\omega t + \frac{\pi}{6}\right)$ and $x_2 = A\cos\omega t$? **[WB JEE 2014]**

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{2}$ (d) $\frac{2\pi}{3}$

16. The oscillation of a body on a smooth horizontal surface is represented by the equation, $x = A \cos \omega t$

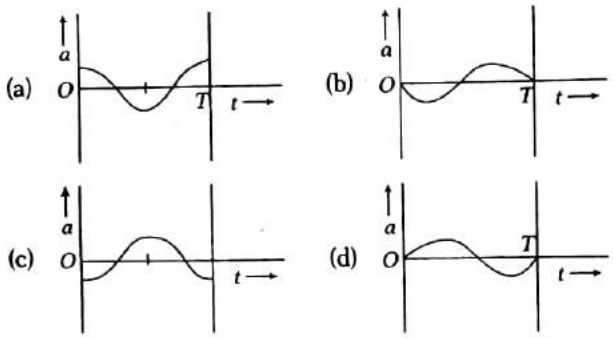
where, x = displacement at time t

ω = frequency of oscillation

Which one of the following graphs shows correctly the variation of a with t? **[CBSE AIPMT 2014]**

Here, a = acceleration at time t

T = time period



17. When a wave travels in a medium, displacement of a particle is given by $y = a \sin 2\pi (bt - cx)$, where a, b and c are constants. The maximum particle velocity will be twice the wave velocity, if **[MHT CET 2014]**

(a) $b = ac$ (b) $b = \frac{1}{ac}$ (c) $c = \pi a$ (d) $c = \frac{1}{\pi a}$

18. When a particle executing SHM oscillates with a frequency ν, then the kinetic energy of the particle **[WB JEE 2014]**

(a) changes periodically with a frequency of ν
(b) changes periodically with a frequency of 2ν
(c) changes periodically with a frequency of ν/2
(d) remains constant

19. If a body is executing simple harmonic motion and its current displacement is $\sqrt{3}/2$ times the amplitude from its mean position, then the ratio between potential energy and kinetic energy is **[EAMCET 2014]**

(a) 3 : 2 (b) 2 : 3 (c) $\sqrt{3}$: 1 (d) 3 : 1

20. A 10 kg metal block is attached to a spring of spring constant 1000 Nm^{-1}. A block is displaced from equilibrium position by 10 cm and released. The maximum acceleration of the block is **[KCET 2014]**

(a) 10 ms^{-2} (b) 100 ms^{-2} (c) 200 ms^{-2} (d) 0.1 ms^{-2}

21. A block resting on the horizontal surface executes SHM in horizontal plane with amplitude A. The frequency of oscillation for which the block just starts to slip is (μ = coefficient of friction, g = gravitational acceleration) **[MHT CET 2014]**

(a) $\frac{1}{2\pi}\sqrt{\frac{\mu g}{A}}$ (b) $\frac{1}{4\pi}\sqrt{\frac{\mu g}{A}}$ (c) $2\pi\sqrt{\frac{A}{\mu g}}$ (d) $4\pi\sqrt{\frac{A}{\mu g}}$

22. The displacement of a particle in a periodic motion is given by $y = 4 \cos^2 (t/2) \sin (1000t)$. This displacement may be considered as the result superposition of n independent harmonic oscillations. Here, n is **[WB JEE 2014]**

(a) 1 (b) 2 (c) 3 (d) 4

23. The time period of a simple pendulum of length 9.8 m is **[JK CET 2013]**

(a) 0.159 s (b) 3.14 s (c) 6.5 s (d) 6.28 s

24. A particle executing simple harmonic motion covers a distance equal to half its amplitude in one second. Then the time period of the simple harmonic motion is **[Kerala CEE 2013]**

(a) 4 s (b) 6 s (c) 8 s (d) 12 s
(e) 20 s

25. The displacement, velocity and acceleration in a simple harmonic motion are related as the **[JK CET 2013]**

(a) displacement, velocity and acceleration all act in the same direction
(b) displacement and velocity act in the same direction but acceleration in the opposite direction
(c) velocity and acceleration are parallel and both are perpendicular to the displacement
(d) displacement and acceleration are anti-parallel and both perpendicular to the velocity

26. If T_1 and T_2 are the time-periods of oscillation of a simple pendulum on the surface of earth (of radius R) and at a depth d, the d is equal to **[Kerala CEE 2013]**

(a) $\left(1 - \frac{T_1^2}{T_2^2}\right)R$ (b) $\left(1 - \frac{T_2^2}{T_2^2}\right)R$

(c) $\left(1 - \frac{T_2}{T_2}\right)R$ (d) $\left(1 - \frac{T_2}{T_1}\right)R$

(e) $\frac{T_1^2}{T_2^2}$

27. A particle executes SHM in accordance with $x = A \sin \omega t$. If t_1 is the time taken by it to reach from $x = 0$ to $x = \sqrt{3}\,(A/2)$ and t_2 is the time taken by it to reach from $x = (\sqrt{3}/2)\,A$ to $x = A$, the value of t_1/t_2 is **[MP PMT 2013]**

(a) 2 (b) $\frac{1}{2}$ (c) 3 (d) $\frac{1}{3}$
(e) None of the these

28. In case of a forced vibration, the resonance wave becomes very sharp when the **[UP CPMT 2013]**

(a) restoring force is small
(b) damping force is small
(c) quality factor is small
(d) applied periodic force is small

29. When a boy is playing on a swing in the sitting position, the time period of oscillations of the swing is T. If the boy stands up, the time period of oscillation of the spring will be **[UP CPMT 2013]**

(a) more than T (b) less than T
(c) equal to T (d) Cannot be predicted

30. To make the frequency of an oscillator double, one have to **[UP CPMT 2013]**

(a) half the mass
(b) quadruple the mass
(c) double the mass
(d) reduce the mass to one-fourth

31. An SHM is given by $y = 5[\sin(3\pi t) + \sqrt{3}\cos(3\pi t)]$. What is the amplitude of the motion of y in metre? **[UP CPMT 2012]**

(a) 10 (b) 20 (c) 1 (d) 5

32. The equations of displacement of two waves are

$$y_1 = 10\sin(3\pi t + \pi/3)$$

and $$y_2 = 5[\sin 3\pi t + \sqrt{3}\cos 3\pi t]$$

What is the ratio of their amplitude? **[UP CPMT 2012]**

(a) 1 : 2 (b) 2 : 1
(c) 1 : 1 (d) None of these

33. If a simple pendulum is taken to a place where g decreases by 2%, then time period **[UP CPMT 2012]**

(a) increase by 5% (b) increase by 1%
(c) increase by 2% (d) decrease by 5%

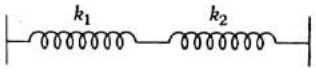
34. If a simple pendulum executing SHM falls freely along with the support, then **[BCECE 2012]**

(a) it does not oscillate at all
(b) its periodic time increases
(c) its periodic time decreases
(d) None of the above

35. If the equation $y_1 = A\sin\omega t$ and $y_2 = \frac{A}{2}\sin\omega t + \frac{A}{2}\cos\omega t$ represents SHM, then the ratio of the amplitudes of the two motions is **[BCECE 2012]**

(a) 0.5 (b) $\sqrt{2}$
(c) 1 (d) 2

36. If the two springs with spring constants k_1 and k_2 are arranged as shown in figure, then the effective spring constant of two spring system will be **[BCECE 2012]**



(a) $k_1 - k_2$ (b) $k_1 + k_2$
(c) $k_1k_2 / k_1 - k_2$ (d) $k_1k_2 / k_1 + k_2$

37. A simple harmonic oscillator consists of a particle of mass m and an ideal spring with spring constant k. The particle oscillates with a time period T. The spring is cut into two equal parts. If one part oscillates with the same particle, the time period will be **[AIIMS 2012]**

(a) $2T$ (b) $\sqrt{2}\,T$ (c) $\frac{T}{\sqrt{2}}$ (d) $\frac{T}{2}$

38. **Assertion** If the bob of a simple pendulum is kept in a horizontal electric field, its period of oscillation will remain same.

Reason If bob is charged and kept in horizontal electric field, then the time period will be decreased.

(a) If both assertion and reason are correct but reason is the correct explanation of assertion.
(b) If both assertion and reason are correct but reason is not the correct explanation of assertion.
(c) If assertion is correct and reason is incorrect.
(d) If assertion is incorrect and reason is correct.

39. The displacements of two particles of same mass executing SHM are represented by the equations

$$x_1 = 4\sin\left(10t + \frac{\pi}{6}\right) \text{ and } x_2 = 5\cos(\omega t).$$

The value of ω for which the energies of both the particles remain same is **[Manipal 2012]**

(a) 16 unit (b) 6 unit (c) 4 unit (d) 8 unit

40. The amplitude and the periodic time of a SHM are 5 cm and 6 s respectively. At a distance of 2.5 cm away from the mean position, the phase will be **[JCECE 2012]**

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{5\pi}{12}$

Answers

Check point 11.1

1. (c)	2. (a)	3. (d)	4. (d)	5. (d)	6. (b)	7. (c)	8. (c)	9. (b)	10. (b)
11. (a)	12. (c)	13. (c)	14. (c)	15. (c)	16. (b)	17. (c)	18. (c)	19. (b)	20. (a)
21. (b)	22. (b)	23. (c)	24. (c)	25. (b)					

Check point 11.2

1. (c)	2. (b)	3. (b)	4. (a)	5. (d)	6. (b)	7. (c)	8. (d)	9. (c)	10. (d)
11. (b)	12. (d)	13. (b)	14. (c)	15. (a)	16. (c)	17. (a)	18. (b)	19. (b)	20. (b)

Check point 11.3

1. (c)	2. (a)	3. (a)	4. (d)	5. (b)	6. (d)	7. (c)	8. (b)	9. (b)	10. (c)
11. (b)	12. (a)	13. (a)	14. (b)	15. (d)	16. (b)	17. (a)	18. (d)	19. (a)	20. (b)

Check point 11.4

1. (c)	2. (d)	3. (c)	4. (a)	5. (b)	6. (d)	7. (b)	8. (d)	9. (a)	10. (a)
11. (c)	12. (c)	13. (c)	14. (a)	15. (a)					

A. Taking it together

1. (c)	2. (c)	3. (a)	4. (b)	5. (a)	6. (d)	7. (c)	8. (b)	9. (c)	10. (d)
11. (b)	12. (c)	13. (c)	14. (d)	15. (b)	16. (a)	17. (c)	18. (a)	19. (d)	20. (d)
21. (d)	22. (a)	23. (c)	24. (a)	25. (d)	26. (a)	27. (d)	28. (d)	29. (a)	30. (a)
31. (d)	32. (b)	33. (a)	34. (d)	35. (b)	36. (d)	37. (c)	38. (a)	39. (c)	40. (c)
41. (c)	42. (b)	43. (d)	44. (b)	45. (b)	46. (c)	47. (c)	48. (d)	49. (b)	50. (b)
51. (c)	52. (c)	53. (d)	54. (c)	55. (d)	56. (d)	57. (d)	58. (a,c)	59. (d)	60. (d)
61. (a)	62. (b)	63. (c)	64. (d)	65. (b)	66. (a)	67. (c)	68. (b)	69. (d)	70. (b)
71. (a)	72. (b)	73. (b)	74. (b)	75. (a)	76. (b)	77. (b)	78. (b)	79. (b)	80. (c)
81. (a)	82. (a)	83. (c)	84. (a)	85. (c)	86. (c)	87. (a)	88. (d)	89. (b)	90. (d)
91. (b)	92. (b)	93. (c)	94. (b)	95. (a)					

B. Medical entrance special format questions

Assertion and reason

1. (a)	2. (a)	3. (a)	4. (d)	5. (a)	6. (d)	7. (d)	8. (c)	9. (b)	10. (a)
11. (c)	12. (d)	13. (c)	14. (d)	15. (b)	16. (b)	17. (d)			

Match the columns

1. $(A \to q, B \to r, C \to q)$
2. $(A \to q, B \to s, C \to s, D \to s)$
3. $(A \to q, B \to t, C \to p, D \to t)$
4. $(A \to q, B \to s, C \to p)$
5. $(A \to q, B \to s, C \to q)$
6. $(A \to q, B \to p,q, C \to r,s)$
7. $(A \to q, B \to r, C \to s, D \to u)$
8. $(A \to r, B \to p, C \to q, D \to s)$
9. $(A \to q, B \to r, C \to s)$
10. $(A \to q, B \to s, C \to p)$
11. $(A \to t, B \to t, C \to s)$

C. Medical entrances' gallery

1. (d)	2. (c)	3. (b)	4. (b)	5. (a)	6. (c)	7. (d)	8. (b)	9. (e)	10. (b)
11. (b)	12. (e)	13. (b)	14. (d)	15. (b)	16. (c)	17. (d)	18. (b)	19. (d)	20. (a)
21. (a)	22. (c)	23. (d)	24. (d)	25. (b)	26. (a)	27. (e)	28. (b)	29. (b)	30. (d)
31. (a)	32. (c)	33. (b)	34. (a)	35. (b)	36. (b)	37. (c)	38. (b)	39. (d)	40. (c)

Hints and explanations

Check point 11.1

1. (c) The motion of planets around the sun is periodic but not simple harmonic motion.

2. (a) The body is not uniformly accelerated in case of simple harmonic motion.

3. (d) $y = a \sin(\omega t + \phi) = a \sin\left(\frac{2\pi}{T} t + \phi\right)$

$$\Rightarrow \quad y = 0.5 \sin\left(\frac{2\pi}{0.4} t + \frac{\pi}{2}\right)$$

$$y = 0.5 \sin\left(5\pi t + \frac{\pi}{2}\right) = 0.5 \cos 5\pi t$$

4. (d) $F = -kx$

$\Rightarrow F \propto -x$, so this is a necessary and sufficient condition for SHM.

5. (d) Periodic time remains constant in SHM.

6. (b) $\omega A = 3 \text{ ms}^{-1}$ or $\omega = \frac{3}{A} = \frac{3}{0.1} = 30 \text{ rad s}^{-1}$

$$f = \frac{\omega}{2\pi} = \frac{30}{2\pi} = \left(\frac{15}{\pi}\right) \text{Hz}$$

7. (c) The phase at any instant tells us both about the displacement and direction of motion of the particle.

8. (c) When the particle moves along the circumference of a circle with constant angular velocity, the projection of the particle on any of the diameters executes SHM.

9. (b) $y = 10 \sin(20t + \pi / 3)$ ...(i)

Comparing Eq. (i) with general equation of motion of simple harmonic, we get

$$A = 10 \text{ m} \qquad \omega = 20 \text{ rad s}^{-1}$$

As $\omega = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{\omega} = \frac{\pi}{10}$

10. (b) $x = 0.34 \cos(3000t + 0.74)$

Comparing with general equation $x = A\cos(\omega t + \phi)$

We get, $A = 0.34$ mm, $\omega = 3000 \text{ rad s}^{-1}$

$\therefore$ Frequency, $\nu = \frac{\omega}{2\pi} = \frac{3000}{2\pi}$ Hz

11. (a) $v_{max} = a\omega = 3 \times 100 = 300$ units

12. (c) Velocity at mean position $v = a\omega$, velocity at a distance of half amplitude

$$v' = \omega\sqrt{a^2 - y^2} = \omega\sqrt{a^2 - \frac{a^2}{4}} = \frac{\sqrt{3}}{2} a\omega = \frac{\sqrt{3}}{2} v$$

13. (c) $v_{max} = a\omega \Rightarrow \omega = \frac{v_{max}}{a} = \frac{10}{4}$

Now, $v = \omega\sqrt{a^2 - y^2}$

$\Rightarrow \quad v^2 = \omega^2(a^2 - y^2)$

$\Rightarrow \quad y^2 = a^2 - \frac{v^2}{\omega^2}$

$\Rightarrow \quad y = \sqrt{a^2 - \frac{v^2}{\omega^2}} = \sqrt{4^2 - \frac{5^2}{(10/4)^2}} = 2\sqrt{3}$ cm

14. (c) $v = \omega\sqrt{A^2 - x^2} = 2\sqrt{(60)^2 - (20)^2} = 113 \text{ mms}^{-1}$

15. (c) $v_{max} = \omega A = (200)(0.25) = 50 \text{ cm s}^{-1}$

16. (b) $a = -\omega^2 x$

$\therefore \quad \frac{x}{a} = -\frac{1}{\omega^2} = \text{constant}$

17. (c) $a = -\omega^2 x \Rightarrow \left|\frac{a}{x}\right| = \omega^2$

18. (c) The acceleration of the particle is non-uniform in case of simple harmonic motion.

19. (b) $\omega = \frac{\pi}{2} \text{ rad s}^{-1}$ and $A = 2$ cm

$\therefore$ Maximum acceleration $= \omega^2 A = \frac{\pi^2}{2} \text{ cms}^{-2}$

20. (a) $\omega^2 A = a_0$ ...(i)

$\omega A = v_0$ ...(ii)

On solving, we get $A = \frac{v_0^2}{a_0}$

21. (b) $a = \omega^2 y \Rightarrow \omega = \sqrt{\frac{a}{y}} = \sqrt{\frac{8}{2}} = 2 \text{ rads}^{-1}$

Now $v_{max} = A\omega = 6 \times 2 = 12 \text{ cms}^{-1}$

22. (b) We know that, $v_{max} = a\omega$ and

Maximum acceleration $= \omega^2 a$ $[\because x = a]$

$$= \left(\frac{v}{a}\right)^2 a = \frac{v^2}{a}$$

23. (c) The average acceleration of a particle performing SHM over one complete oscillation is zero. Because acceleration is a function of sine and average value of sin for 1 complete oscillation = 0

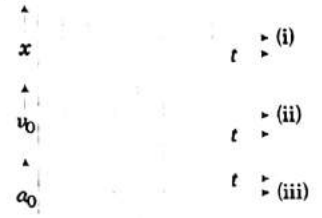
24. (c) From mean position to extreme position body is decelerated and from extreme position to mean position body is accelerated.

25. (*b*) As $x = A\sin\omega t$

Velocity, $v = A\omega \sin\left(\omega t + \frac{\pi}{2}\right)$ and the acceleration,

$a = A\omega^2 \sin(\omega t + \pi)$

Here, A = amplitude



$\therefore$ In SHM, the acceleration is ahead of velocity by a phase angle $\frac{\pi}{2}$.

Check point 11.2

1. (*c*) At mean positions. KE is maximum and PE is zero.

2. (*b*) KE is maximum at mean position, *i.e.*, at $x = 0$

3. (*b*) $a = 6$ cm, $\omega = 100$ rad s^{-1}

$$K_{max} = \frac{1}{2} m\omega^2 a^2 = \frac{1}{2} \times 1 \times (100)^2 \times (6 \times 10^{-2})^2 = 18 \text{ J}$$

4. (*a*) $\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{16}{1}} = 4$ rad s^{-1}

Now, $K_{max} = \frac{1}{2} m\omega^2 A^2$

$$= \frac{1}{2} \times 1 \times (4)^2 \times (5 \times 10^{-2})^2 = 2 \times 10^{-2} \text{ J}$$

5. (*d*) $\text{KE} = \frac{1}{2} m\,\omega^2 (A^2 - x^2) \Rightarrow \frac{1}{2} m\,\omega^2 \left(A^2 - \frac{A^2}{4}\right)$ $\left(\because x = \frac{A}{2}\right)$

$$= \frac{3}{4} \times \frac{1}{2} m\,\omega^2 A^2 = \frac{3}{4} E$$

6. (*b*) Frequency of oscillation of kinetic energy is doubled, *i.e.*, 2ω

$\therefore$ $T = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$

7. (*c*) Amplitude, $A = 4$ cm

For $\text{KE} = \text{PE} \Rightarrow \frac{1}{2} m\,\omega^2 (A^2 - x^2) = \frac{1}{2} m\,\omega^2 x^2$

$\Rightarrow$ $x^2 = \frac{A^2}{2} \Rightarrow x = \frac{A}{\sqrt{2}}$

$\therefore$ $x = \frac{4}{\sqrt{2}} = 2\sqrt{2}$ cm

8. (*d*) Let x be the point, where KE = PE

Hence, $\frac{1}{2} m\omega^2 (a^2 - x^2) = \frac{1}{2} m\omega^2 x^2$

$$2x^2 = a^2,\ x = \frac{a}{\sqrt{2}}$$

$\Rightarrow$ $x = \frac{4}{\sqrt{2}} = 2\sqrt{2}$ cm

9. (*c*) $U = K$ or $U = \frac{E}{2}$

$\therefore$ $\frac{1}{2} Kx^2 = \frac{1}{2}\left(\frac{1}{2} KA^2\right)$

i.e., at $x = \pm \frac{A}{\sqrt{2}}, U = K$

and this situation will occur for four times in one complete period.

10. (*d*) The work done by the pendulum in one oscillation will be zero because displacement is zero.

11. (*b*) Total Energy, $E = \frac{1}{2} m\,\omega^2 A^2$

12. (*d*) As $E = \frac{1}{2} m\,\omega^2 A^2$

or $E = \frac{1}{2} m \left(\frac{2\pi}{T}\right)^2 A^2$ $\left(\because \omega = \frac{2\pi}{T}\right)$

As $T' = 2T$ $A' = 2A$

$\therefore$ $E' = \frac{1}{2} m \times \frac{1}{4}\left(\frac{2\pi}{T}\right)^2 \times 4\,(A)^2 = E$

13. (*b*) Energy in SHM, $E = U_{mean} + \frac{1}{2} m\omega^2 A^2$

i.e., E depends on ω and A both.

14. (*c*) Total energy, $E = \frac{1}{2} m\,\omega^2 A^2$

$\Rightarrow$ It is independent of x

15. (*a*) $E = \frac{1}{2} m\omega^2 A^2 = \frac{1}{2} m(2\pi f)^2 A^2 = 2\pi^2 f^2 m A^2$

$= 2\pi^2 (20)^2 (0.1)(0.05)^2 = 2$ J (approximately)

16. (*c*) From law of conservation of energy,

$\text{KE} = E - U = 4$ J

17. (*a*) The value of total mechanical energy of a particle in SHM always remains constant.

18. (*b*) $\frac{K}{E} = \frac{\frac{1}{2} m\omega^2 (a^2 - y^2)}{\frac{1}{2} m\omega^2 a^2} = \frac{a^2 - y^2}{a^2} = 1 - \frac{y^2}{a^2}$

So, $\frac{\left(\frac{3E}{4}\right)}{E} = 1 - \frac{y^2}{a^2} \Rightarrow \frac{y^2}{a^2} = 1 - \frac{3}{4} = \frac{1}{4} \Rightarrow y = \frac{a}{2}$

19. (*b*) If at any instant displacement is y, then it is given that

$$U = \frac{1}{2} \times E$$

$\Rightarrow$ $\frac{1}{2} m\omega^2 y^2 = \frac{1}{2} \times \left(\frac{1}{2} m\omega^2 A^2\right)$

$\Rightarrow$ $y = \frac{A}{\sqrt{2}} = \frac{6}{\sqrt{2}} = 4.2$ cm

20. (*b*) $U = \frac{1}{2} m\,\omega^2 x^2 = \frac{1}{2} m\omega^2 \left(\frac{A^2}{4}\right) = \frac{1}{4} E$

Check point 11.3

1. (c) Time period will remains the same because time period of simple pendulum T is independent of mass of the bob.

2. (a) $T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow T = 2\pi\sqrt{\frac{39.2}{\pi^2 \times 9.8}} = 4$ s

3. (a) $L' = 4L$

$$T' = 2\pi\sqrt{\frac{4L}{g}} = 2T$$

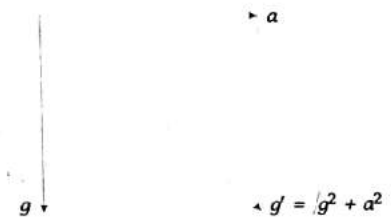
4. (d) As, $T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow \frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta l}{l}$

Given, $\frac{\Delta l}{l} = 21\%$

$\Rightarrow \frac{\Delta T}{T} = \frac{1}{2} \times 21\% = 10.5\%$

5. (b) $T = 2\pi\sqrt{\frac{l}{g_e}}$. Here, g_e = effective value of g, which is zero in a freely falling lift. Hence, $T = \infty$

6. (d) $g' = \sqrt{g^2 + a^2}$



7. (c) In stationary lift, $T = 2\pi\sqrt{\frac{l}{g}}$

In upward motion of lift,

$$T' = 2\pi\sqrt{\frac{l}{(g+a)}} \qquad (a = \text{acceleration of lift})$$

$$\Rightarrow \frac{T'}{T} = \sqrt{\frac{g}{g+a}} = \sqrt{\frac{g}{\left(g+\frac{g}{4}\right)}} = \sqrt{\frac{4}{5}} \Rightarrow T' = \frac{2T}{\sqrt{5}}$$

8. (b) When a little mercury is drained off, the position of centre of mass of ball falls (with respect to fixed end) so that effective length of pendulum increases hence T increases.

9. (b) $T = 2\pi\sqrt{\frac{R_e}{g}} = 84.6$ min [If l is infinite]

10. (c) The length of second pendulum is 100 cm.

11. (b) $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \Rightarrow f \times \frac{1}{\sqrt{m}}$

$\therefore f' = f/2$ or $n/2$ $(\because \sqrt{m'} = \sqrt{4m})$

12. (a) $k \propto \frac{1}{l}$ and $f \propto \sqrt{k}$

Spring constant of each part will be $2k$

$\therefore f' = \sqrt{2}f$

13. (a) $T = 2\pi\sqrt{\frac{m}{k}}$ or $T \propto \sqrt{m}$

Mass has reduced to $\frac{1}{4}$th of the initial mass. Hence, time period will remain half.

So, according to question, if 0.3 kg mass is also removed, then the system will oscillate with a period of $\frac{1}{2} \times 2 = 1$ s.

14. (b) The spring mass system oscillates in SHM, its time period is given by

$$T = 2\pi\sqrt{\frac{m}{k}}$$

When spring is cut into ratio 1 : 3, the new time constant is $k' = 3k$

$$\therefore \frac{T}{T'} = \sqrt{\frac{3k}{k}} \Rightarrow T' = \frac{T}{\sqrt{3}}$$

15. (d) The springs are corrected in series. So, the effective force constant will be

$$\frac{1}{k_e} = \frac{1}{k_1} + \frac{1}{k_2} \Rightarrow k_e = \frac{k_1 k_2}{k_1 + k_2}$$

16. (b) $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ or $f \propto \sqrt{k}$

$k = k_1 + k_2$ and $k' = 4(k_1 + k_2) = 4k$

So, $f \propto \sqrt{k'} \Rightarrow f' \propto \sqrt{4k}$

$f' \propto 2\sqrt{k} \Rightarrow f' \propto 2f$

17. (a) $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ or $f \propto \sqrt{\frac{k}{m}}$

Here value of k_e will become $3k$

($\because$ Springs are connected in parallel)

$$\therefore f \propto \sqrt{\frac{3k}{m}}$$

18. (d) The springs are corrected in parallel. So, the effective force constant will be $k_e = k_1 + k_2$

$$\therefore T = 2\pi\sqrt{\frac{m}{k_e}} = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$$

19. (a) $T_a = 2\pi\sqrt{\frac{m}{k}}$

$T_b = 2\pi\sqrt{\frac{m}{(k/2)}}$ ($\because$ Springs are connected in series)

$T_c = 2\pi\sqrt{\frac{m}{2k}}$ ($\because$ Springs are connected in parallel)

20. (b) $T = 2\pi\sqrt{\frac{I}{mgl/2}} = 2\pi\sqrt{\frac{(ml^2/3)}{(mgl/2)}} \left(\because I_{rod} = \frac{ml^2}{3}\right) = 2\pi\sqrt{\frac{2l}{3g}}$

Check point 11.4

1. (c) $k = \frac{YA}{L}$

$$\therefore f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{YA}{mL}}$$

2. (*d*) $T = 2\pi\sqrt{\frac{m}{\rho A g}} \Rightarrow T \propto \frac{1}{\sqrt{\rho}}$

3. (*c*) The time period in this case is given by $T = 2\pi\sqrt{\frac{R}{g}}$

where, R is the radius of earth.

4. (*a*) Amplitude of damped oscillation is $A = A_0 e^{-\gamma t}$

$$A = \frac{A_0}{2}$$

where $t = 1$ s

So, $\frac{A_0}{2} = A_0 e^{-\gamma 1}$ or $e^{\gamma} = 2$

After 2 s, $A = A_0 e^{-\gamma 3} \Rightarrow A = \frac{A_0}{4}$

5. (*b*) Frequency is less with damping.

6. (*d*) In damped oscillation, amplitude goes on decaying exponentially. $a = a_0 e^{-bt}$ where b = damping coefficient.

Initially, $\frac{a_0}{3} = a_0 e^{-b \times 100 T}$, T = time of one oscillation

or $\frac{1}{3} = e^{-100 bT}$ ...(i)

Finally, $a = a_0 e^{-b \times 200 T}$

or $a = a_0 [e^{-100 bT}]^2$

or $a = a_0 \times \left[\frac{1}{3}\right]^2$ [From Eq. (i)]

or $a = a_0 / 9$

7. (*b*) With weak damping frequency of system reaches frequency of driving force.

8. (*d*) Amplitude always remains constant in case of sustained forced oscillations.

9. (*a*) Amplitude remains constant in free vibrations, maintained vibrations and forced vibrations.

10. (*a*) In forced oscillations, particle oscillates with a frequency equal to the frequency of driving force.

11. (*c*) In resonant vibrations of a body, the frequency of external force applied on the body is equal to its natural frequency. If on increasing and decreasing the frequency of external force from the natural frequency by a factor, the amplitude of vibrations reduces very much. In this case, sharp resonance will take place.

But if it reduces by a small factor, then flat resonance will take place. The sharp and flat resonance will depend on damping present in the body executing resonant vibrations. Less the damping, greater will be sharpness.

12. (*c*) Two sources of sound are in resonance when they produce the sound of same frequency.

13. (*c*) It is due to resonance.

14. (*a*) Resonance occurs when an external driving force is present.

15. (*a*) During the phenomenon of resonance, the amplitude of oscillation becomes large. Because applied frequency is equal to natural frequency.

A) Taking it together

1. (*c*) a and s are always in opposite directions.

2. (*c*) $T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow \frac{l}{T^2} = \frac{g}{4\pi^2}$ = constant

3. (*a*) The amplitude is a maximum displacement from the mean position.

4. (*b*) There is no effect of phase angle on amplitude. Hence

$$\frac{A_1}{A_2} = \frac{10}{5} = \frac{2}{1}$$

5. (*a*) $f = \frac{1}{T} = \frac{1}{0.04} = 25$ Hz

6. (*d*) Frequency, $n \propto \frac{1}{\sqrt{l}}$

$$\Rightarrow \frac{n_1}{n_2} = \sqrt{\frac{l_2}{l_1}} \Rightarrow \frac{l_2}{l_1} = \frac{n_1^2}{n_2^2} \Rightarrow \frac{l_2}{l_1} = \frac{(2)^2}{(3)^2}$$

$$\frac{l_2}{l_1} = \frac{4}{9} \Rightarrow \frac{l_1}{l_2} = \frac{9}{4}$$

7. (*c*) Given, $x = 5\cos\left(2\pi t + \frac{\pi}{4}\right)$ (where, x is a displacement)

$\Rightarrow$ $x = 5\cos\left(2\pi + \frac{\pi}{4}\right)$ (at $t = 1$ s)

[$\because \cos(360° + \theta) = \cos\theta$]

$\Rightarrow$ $x = 5\cos\frac{\pi}{4} \Rightarrow x = 5 \times \frac{1}{\sqrt{2}}$

$\therefore$ $x = \frac{5}{\sqrt{2}}$ m

8. (*b*) $|a| = \omega^2 x$

$$12 = \omega^2(3)$$

$\therefore$ $\omega = 2 \text{ rad s}^{-1} = \frac{2\pi}{T}$

$\therefore$ $T = (\pi)$ s or 3.14 s

9. (*c*) In case of spring-block system, time period does not change because restoring force is still kx.

10. (*d*) Using acceleration, $A = -\omega^2 x$

At $-x_{max}$, A will be maximum and positive.

11. (*b*) $y_1 = a\cos\omega t = a\sin\left(\omega t + \frac{\pi}{2}\right)$

$y_2 = a\sin\omega t$

$\therefore$ $\Delta\phi = \omega t + \frac{\pi}{2} - \omega t = \frac{\pi}{2} = 90°$

12. (*c*) Equation of motion, $y = a\cos\omega t$

$\Rightarrow$ $\frac{a}{2} = a\cos\omega t \Rightarrow \omega t = \frac{\pi}{3}$

$\Rightarrow$ $\frac{2\pi t}{T} = \frac{\pi}{3} \Rightarrow t = \frac{\frac{\pi}{3} \times T}{2\pi} = \frac{4}{3 \times 2} = \frac{2}{3}$ s

13. (*c*) ωA = constant and $\omega^2 A$ is made two times.

This is possible when ω is doubled and A is halved.

14. (*d*) At $x = 0$, $F = 0$. For $x > 0$, force is in negative direction and for $x < 0$, force is in positive x-direction. Therefore, motion of the particle is periodic about mean position $x = 0$.

15. (*b*) $k \propto \frac{1}{l}$ hence, k of individual halves will be 16 Nm^{-1}. When they are connected in parallel, effective value of k will become $32\,Nm^{-1}$.

16. (*a*) Suppose $x = A \sin \omega t$ and $y = A \cos \omega t$

Then by squaring and adding these two equations, we get

$$x^2 + y^2 = A^2$$

17. (*c*) $\omega = \frac{2\pi}{T} = \frac{\pi}{4}$ s. Therefore $y = A \sin \frac{\pi}{4} t$. Now put $x = 1$s and then $x = 2$ s.

18. (*a*) $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{16}{1}} = 4 \text{ rad s}^{-1}$

Now, $K_{max} = \frac{1}{2} m\omega^2 A^2$

$$= \frac{1}{2} \times 1 \times (4)^2 (5 \times 10^{-2})^2$$

$$= 2 \times 10^{-2} \text{ J}$$

19. (*d*) Acceleration is maximum when displacement is maximum. Force is zero when displacement is zero and potential energy is maximum, when displacement is maximum.

20. (*d*) At time $\frac{T}{2}$; $v = 0$

$\therefore$ Total energy = Potential energy

21. (*d*) $T = 2\pi \sqrt{\frac{l}{g}}$ and $T' = 2\pi \sqrt{\frac{l}{\left(g + \frac{g}{4}\right)}}$

or $T' = \frac{2}{\sqrt{5}} T = 2$ s

22. (*a*) $\frac{1}{2} kA^2 = \frac{1}{2} \times 2 \times 10^6 \times (0.01)^2 = 100$ J

At mean position,

$U = 60$ J and $K = 100$ J

At extreme positions,

$U = 160$ J and $K = 0$

23. (*c*) For second's pendulum at the surface of earth

$$\Rightarrow \quad 2 = 2\pi \sqrt{\frac{l_e}{g_e}} \quad \text{...(i)}$$

For second's pendulum at the surface of moon

$$= 2\pi \sqrt{\frac{l_m}{g_m}} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), $\frac{l_e}{g_e} = \frac{l_m}{g_m} \Rightarrow l_m = \left(\frac{g_m}{g_e}\right) l_e$

As $g_m = \frac{1}{6} g_e \Rightarrow l_m = \frac{l_e}{6} = \frac{1}{6}$ times ($\because l_e = 1$m)

24. (*a*) $k_s = \frac{k \times k}{k + k} = \frac{k}{2}$

$$k_p = k + k = 2k$$

$$\therefore \frac{n_s}{n_p} = \frac{\frac{1}{2\pi}\sqrt{\frac{k_s}{M}}}{\frac{1}{2\pi}\sqrt{\frac{k_p}{M}}} = \sqrt{\frac{k_s}{k_p}} = \sqrt{\frac{k/2}{2k}} = \frac{1}{2}$$

25. (*d*) For motion to be SHM acceleration of the particle must be proportional to negative of displacement.

i.e., $a \propto -(y \text{ or } x)$

We should be clear that y has to be linear.

26. (*a*) Let equation of an SHM is represented by $y = a \sin \omega t$

$$v = \frac{dy}{dt} = a\,\omega \cos \omega t$$

$$\Rightarrow \quad (v)_{max} = a\omega = 30 \quad \text{...(i)}$$

Acceleration $(A) = \frac{dy^2}{dt^2} = -a\omega^2 \sin \omega t$

$$A_{max} = \omega^2 a = 60 \quad \text{...(ii)}$$

Eqs. (i) and (ii), we get

$$\omega\,(\omega a) = 60 \Rightarrow \omega\,(30) = 60$$

$$\Rightarrow \quad \omega = 2 \text{ rad s}^{-1}$$

$$\Rightarrow \quad \frac{2\pi}{T} = 2 \text{ rad s}^{-1}$$

$$\Rightarrow \quad T = \pi \text{ s}^{-1}$$

27. (*d*) $T = 2\pi \sqrt{\frac{I}{MgL}} = 2\pi \sqrt{\frac{l}{g}}$

or $l = \frac{I}{ML} = \frac{(3/2\,MR^2)}{MR} = \frac{3}{2} R$

28. (*d*) $T \propto \sqrt{l} \Rightarrow \frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta l}{l} = \frac{0.02}{2} = 0.01$

$$\Rightarrow \quad \Delta T = 0.01T$$

Loss of time per day $= 0.01 \times 24 \times 60 \times 60 = 864$ s

29. (*a*) Time period $T \propto \sqrt{l}$

$$\Rightarrow \quad \frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta l}{l} = \frac{1}{2}\alpha\Delta\theta$$

Also according to thermal expansion $l' = l(1 + \alpha\Delta\theta)$

$\frac{\Delta l}{l} = \alpha\Delta\theta$.

Hence, $\frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta l}{l} = \frac{1}{2}\alpha\,\Delta\theta$

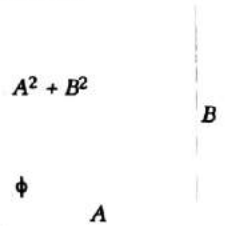
$$= \frac{1}{2} \times 12 \times 10^{-6} \times (40 - 20) = 12 \times 10^{-5}$$

$$\Rightarrow \quad \Delta T = 12 \times 10^{-5} \times 86400 \text{ second/day}$$

$$\therefore \quad \Delta T \approx 10.3 \text{ second/day}$$

30. (*a*) $\omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$

31. (*d*) $x = A\sin\omega t + B\cos\omega t$



$$= \sqrt{A^2+B^2}\left(\frac{A}{\sqrt{A^2+B^2}}\sin\omega t + \frac{B}{\sqrt{A^2+B^2}}\cos\omega t\right)$$

$$= \sqrt{A^2+B^2}\,\sin(\omega t + \phi)$$

where, $\tan\phi = \dfrac{B}{A}$

32. (*b*) $T = 2\pi\sqrt{\dfrac{l}{g}}$ or $T^2 \propto l$

i.e., T-l graph is a parabola

33. (*a*) $x = 3\sin 2t + 4\cos 2t$. From given equation

$a_1 = 3,\ a_2 = 4$ and $\phi = \dfrac{\pi}{2}$

$\therefore \quad a = \sqrt{a_1^2 + a_2^2} = \sqrt{3^2+4^2} = 5$

$\Rightarrow \quad v_{max} = a\omega = 5\times 2 = 10$

34. (*d*) Acceleration $= -\omega^2 y$. So, $F = -m\omega^2 y$

y is sinusoidal function.

So, F will also be sinusoidal function with phase difference π.

35. (*b*) Ball will execute SHM inside the tunnel with time period

$$T = 2\pi\sqrt{R/g} = 84.63 \text{ min}$$

Hence, time to reach the ball from one end to the other end of the tunnel $t = \dfrac{84.63}{2} = 42.3$ min.

36. (*d*) $\omega_1 A_1 = \omega_2 A_2$

$\Rightarrow \quad \dfrac{A_1}{A_2} = \dfrac{\omega_2}{\omega_1} = \sqrt{\dfrac{k_2}{k_1}} \qquad \left(\text{as } \omega = \sqrt{\dfrac{k}{m}}\right)$

37. (*c*) $y = a\sin\dfrac{2\pi}{T}t$

$\Rightarrow \quad \dfrac{a}{\sqrt{2}} = a\sin\dfrac{2\pi}{T}\cdot t$

$\Rightarrow \quad \sin\dfrac{2\pi}{T}t = \dfrac{1}{\sqrt{2}} = \sin\dfrac{\pi}{4}$

$\Rightarrow \quad \dfrac{2\pi}{T}t = \dfrac{\pi}{4}$

$\therefore \quad t = \dfrac{T}{8}$

38. (*a*) $mg = kx$

$\therefore \quad k = \dfrac{mg}{x}$

$\therefore \quad T = 2\pi\sqrt{\dfrac{M+m}{k}} = 2\pi\sqrt{\dfrac{(M+m)x}{mg}}$

39. (*c*) Velocity, $v = \omega\sqrt{A^2 - x^2}$ and acceleration $= \omega^2 x$

Now given, $\omega^2 x = \omega\sqrt{A^2 - x^2}$

$\Rightarrow \quad \omega^2\cdot 1 = \omega\sqrt{2^2 - 1^2}$

$\Rightarrow \quad \omega = \sqrt{3}$

$\therefore \quad T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{\sqrt{3}}$

40. (*c*) At its lowermost point spring is stretched by 8 cm or 8×10^{-2} m.

$\therefore \quad F_{net} = kx - mg$

$= (2\times 10^3 \times 8\times 10^{-2}) - (10\times 10) = 60$ N

41. (*c*) Reduced mass of two blocks,

$$\mu = \frac{m_1 m_2}{m_1 + m_2} = 2 \text{ kg}$$

Now, $\quad f = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{\mu}} = \dfrac{1}{2\pi}\sqrt{\dfrac{300}{2}} \approx 2$ Hz

42. (*b*) Force constant, $k \propto \dfrac{1}{\text{Length of spring}}$

$\therefore \quad \dfrac{k_A}{k} = \dfrac{L}{\alpha L}$

$\therefore \quad k_A = \dfrac{k}{\alpha}$

Similarly, $\quad k_B = \dfrac{k}{1-\alpha}$

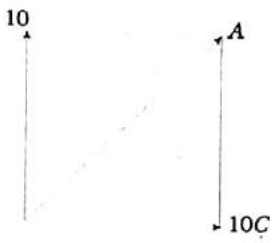
43. (*d*) $A = 0.04$ m, $\omega^2 A = 0.3$ ms^{-2}

$\therefore \quad \omega = 2.74$ rads^{-1}

Now $\quad T = \dfrac{2\pi}{\omega} = 2.29$ s

44. (*b*) $T = 2\pi\sqrt{\dfrac{m}{k}} = 2\pi\sqrt{\dfrac{0.2}{80}} = \dfrac{\pi}{10}$ s

45. (*b*) For y_2



$\therefore \quad A = \sqrt{(10)^2 + (10C)^2}$

or $\quad 10\sqrt{1 + C^2} = 40$

$\therefore \quad C = \sqrt{15}$

46. (*c*) At $x = 0$, $U = 0$. Therefore, total mechanical energy is equal to the maximum kinetic energy.

$$\therefore \quad \frac{1}{2} m v_{max}^2 = 36$$

or $$v_{max} = \sqrt{\frac{72}{m}} = \sqrt{\frac{72}{2}} = 6 \text{ ms}^{-1}$$

47. (*c*) $$\frac{1}{2} m\omega^2 A^2 = 8 \times 10^{-3} \text{ J}$$

$$\therefore \quad \frac{1}{2} \times 0.1 \times \omega^2 \times (0.1)^2 = 8 \times 10^{-3}$$

or $$\omega = 4 \text{ rads}^{-1}$$

Therefore, the equation of motion of the particle if the initial phase of the oscillation is 45°, will be

$$y = 0.1 \sin\left[4t + \frac{\pi}{4}\right]$$

48. (*d*) $T = 2\pi\sqrt{\frac{m}{K}}$ or $T \propto \sqrt{m}$

Mass is increasing, therefore time period will increase.

49. (*b*) Given, $y = 3\cos\left(\frac{\pi}{4} - 2\omega t\right)$

Velocity of the particle

$$v = \frac{dy}{dt} = \frac{d}{dt}\left[3\cos\left(\frac{\pi}{4} - 2\omega t\right)\right]$$

$$= 3(-2\omega)\left[-\sin\left(\frac{\pi}{4} - 2\omega t\right)\right] = 6\omega \sin\left(\frac{\pi}{4} - 2\omega t\right)$$

Acceleration, $$a = \frac{dv}{dt} = \frac{d}{dt}\left[6\omega \sin\left(\frac{\pi}{4} - 2\omega t\right)\right]$$

$$= 6\omega \times (-2\omega)\cos\left(\frac{\pi}{4} - 2\omega t\right)$$

$$= -12\omega^2 \cos\left(\frac{\pi}{4} - 2\omega t\right)$$

$$= -4\omega^2\left[3\cos\left(\frac{\pi}{4} - 2\omega t\right)\right]$$

$$\Rightarrow \quad a = -4\omega^2 y$$

$\Rightarrow$ As acceleration, $a \propto -y$

Hence, due to negative sign motion is SHM.

Clearly, from the equation

$$\omega' = 2\omega \quad [\because \text{Standard equation } y = a\cos(\omega t + \phi)]$$

$$\Rightarrow \quad \frac{2\pi}{T'} = 2\omega \Rightarrow T' = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$$

[and given equation $y = 3\cos(-2\omega t + \frac{\pi}{4})$]

So, motion is SHM with period $\frac{\pi}{\omega}$.

50. (*b*) Given, equation of motion is

$$y = \sin^3 \omega t = (3\sin\omega t - 4\sin 3\omega t)/4$$

$$[\because \sin 3\theta = 3\sin\theta - 4\sin^3\theta]$$

$$\Rightarrow \quad \frac{dy}{dt} = \left[\frac{d}{dt}(3\sin\omega t) - \frac{d}{dt}(4\sin 3\omega t)\right]/4$$

$$\Rightarrow \quad 4\frac{dy}{dt} = 3\omega\cos\omega t - 4 \times [3\omega\cos 3\omega t]$$

$$\Rightarrow \quad 4 \times \frac{d^2y}{dt^2} = -3\omega^2\sin\omega t + 12\omega^2\sin 3\omega t$$

$$\Rightarrow \quad \frac{d^2y}{dt^2} = -\frac{3\omega^2\sin\omega t + 12\omega^2\sin 3\omega t}{4}$$

$\Rightarrow$ $\frac{d^2y}{dt^2}$ is not proportional to y.

Hence, motion is not SHM.

As the expression is involving sine function, hence it will be periodic.

51. (*c*) Consider the diagram in which a liquid column oscillates. In this case, restoring force acts on the liquid due to gravity. Acceleration of the liquid column, can be calculated in terms of restoring force.

Restoring force $f =$ Weight of liquid column of height $2y$

$\Rightarrow$ $f = -(A \times 2y \times \rho) \times g = -2A\rho g y$ $[\because m = \rho v]$

$\Rightarrow$ $f \propto -y \Rightarrow$ Motion is SHM with force constant $k = 2A\rho g$.

$\Rightarrow$ Time period $$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{A \times 2h \times \rho}{2A\rho g}} = 2\pi\sqrt{\frac{h}{g}}$$

$$T = 2\pi\sqrt{\frac{l}{g}}, \text{ where } l = h$$

Which is independent of the density of the liquid.

52. (*c*) Given, $x = a\cos\omega t$ ...(i)

$y = a\sin\omega t$ ... (ii)

Squaring and adding Eqs. (i) and (ii),

$$x^2 + y^2 = a^2 \quad (\cos^2\omega t + \sin^2\omega t)$$

$$= a^2 \Rightarrow x^2 + y^2 = a^2 \quad [\because \cos^2\omega t + \sin^2\omega t = 1]$$

This is the equation of a circle,

Clearly, the locus is a circle of constant radius a.

53. (*d*) According to the question, the displacement

$$y = a\sin\omega t + b\cos\omega t$$

Let $a = A\sin\phi$ and $b = A\cos\phi$

Now, $a^2 + b^2 = A^2\sin^2\phi + A^2\cos^2\phi$

$$= A^2 \Rightarrow A = \sqrt{a^2 + b^2}$$

$$\therefore \quad y = A\sin\phi\sin\omega t + A\cos\phi\cos\omega t$$

$$= A\sin(\omega t + \phi)$$

$$\Rightarrow \quad \frac{dy}{dt} = A\omega\cos(\omega t + \phi)$$

$$\Rightarrow \quad \frac{d^2y}{dt^2} = -A\omega^2\sin(\omega t + \phi) = -Ay\omega^2 = (-A\omega^2)y$$

$$\Rightarrow \quad \frac{d^2y}{dt^2} \propto (-y)$$

Hence, it is an equation of SHM with amplitude, $A = \sqrt{a^2 + b^2}$.

54. (*c*) As the given equation is

$$x = a\cos(\alpha t)^2$$

is a cosine function. Hence, it is an oscillatory motion.

Now, putting $t + T$ in place of t

$$x(t+T) = a\cos[\alpha(t+T)]^2 \qquad [\because x(t) = a\cos(\alpha t)^2]$$

$$= a\cos[\alpha t^2 + \alpha T^2 + 2\alpha t T] \neq x(t)$$

where, T is supposed as period of the function $\omega(t)$.

Hence, it is not periodic.

55. (*d*) At equilibrium, $x = \frac{mg}{k} = \frac{0.2 \times 10}{200} = 0.01\text{ m} = 1\text{ cm}$

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}} = \frac{1}{2\pi}\sqrt{\frac{200}{0.2}} \approx 5\text{ Hz}$$

56. (*d*) $\frac{1}{2}kA^2 = 1\text{ J}$

$$\therefore \quad k = \frac{2}{A^2} = \frac{2}{(0.4)^2} = \frac{25}{2}\text{ Nm}^{-1}$$

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{2}{(25/2)}} = \frac{4\pi}{5}\text{ s}$$

57. (*d*) $v = \omega\sqrt{A^2 - x^2}$ (in SHM)

or $v^2 = \omega^2A^2 - \omega^2x^2$

Comparing the given equation with this equation we get,

$$\omega^2 = 9$$

$$\therefore \quad \omega = 3 = \frac{2\pi}{T}$$

$$\therefore \quad T = \frac{2\pi}{3}\text{ units}$$

Also, $\omega^2A^2 = 144$

$\therefore$ $A = 4$ units, $|a| = \omega^2x = (9)(3) = 27$ units

Displacement $\leq$ distance

58. (*a, c*) Maximum kinetic energy = energy of oscillation in SHM

$$\therefore \quad 8 = \frac{1}{2}kA^2$$

$$\therefore \quad kA^2 = 16 \qquad \text{...(i)}$$

Further, $2\pi\sqrt{\frac{m}{k}} = 4 \qquad (\because T = 4\text{ s})$

$$\therefore \quad \frac{1}{k} = \frac{4}{\pi^2}$$

or $k = \frac{\pi^2}{4}$ ...(ii)

From Eqs. (i) and (ii) we get,

$$k \approx 2.5\text{ Nm}^{-1}$$

and $A \approx 2.53\text{ m}$

Maximum acceleration of the particle will be

$$a_{max} = \omega^2A = \frac{k}{m}A$$

$$= \left(\frac{2.5}{1}\right)(2.53) \approx 6.3\text{ ms}^{-2}$$

59. (*d*) $U = 5x^2 - 20x$, $\frac{dU}{dx} = 10x - 20$,

$$F = -\frac{dU}{dx} = (-10x + 20)$$

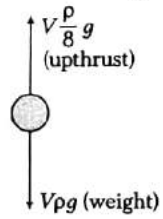
Let us suppose, $x = (X + 2)$. Then $F = -10X$, $F = 0$ at $X = 0$ or $x = 2$, *i.e.*, $x = 2$m is the mean position about which particle is in SHM. $k = 10$

$\therefore \quad T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.1}{10}} = \frac{\pi}{5}$ s. At mean position, kinetic energy is maximum and force acting on particle in SHM is variable.

60. (*d*) In vacuum, $T = 2\pi\sqrt{\frac{l}{g}}$

Let V be the volume and T be the density of the mass of the bob. Net downward force acting on the bob in side the liquid

$$= \text{Weight} - \text{upthrust} = V\rho g - V\cdot\frac{T}{8}g = \frac{7}{8}V\rho g$$

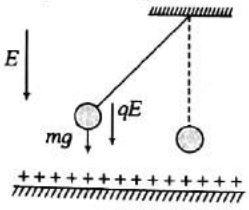


i.e., effective value of g is $\frac{7}{8}g$

So, time period of the bob inside the liquid

$$\therefore \quad T_1 = 2\pi\sqrt{\frac{1}{(7/8)g}} = 2\pi\sqrt{\frac{l}{g}} \times \sqrt{\frac{8}{7}} = \sqrt{\frac{8}{7}}T.$$

61. (*a*) In this case time period of pendulum becomes



$$T' = 2\pi\sqrt{\frac{1}{\left(g + \frac{qE}{m}\right)}} \Rightarrow T' < T$$

62. (*b*) Let T_1 and T_2 be the time period of the two pendulums

$$T_1 = 2\pi\sqrt{\frac{100}{g}} \text{ and } T_2 = 2\pi\sqrt{\frac{121}{g}} \qquad (T_1 < T_2 \text{ because } l_1 < l_2)$$

Let at $t = 0$, they start swinging together. Since their time periods are different, the swinging will not be in unision always. Only when number of completed oscillation differs by an integer, the two pendulum will again begin to swing together.

Let longer length pendulum complete n oscillation and shorter length pendulum complete $(n + 1)$ oscillation, for the unision swinging, then $(n + 1)T_1 = nT_2$.

$$(n+1) \times 2\pi\sqrt{\frac{100}{g}} = n \times 2\pi\sqrt{\frac{121}{g}} \Rightarrow n = 10$$

63. (*c*) For series arrangement, $f_s = \frac{1}{2\pi}\sqrt{\frac{k_s}{m}} = \frac{1}{2\pi}\sqrt{\frac{k}{2m}}$

$$\left(\because k_s = \frac{k_1 k_2}{k_1 + k_2} = \frac{k}{2}\right)$$

For parallel and series arrangement,

$$f_p = \frac{1}{2\pi}\sqrt{\frac{2k}{3m}} \qquad \left(\because k_p = \frac{2k\,k}{2k + k} = \frac{2k}{3}\right)$$

$$\Rightarrow \quad \frac{f_s}{f_p} = \sqrt{\frac{k}{2m}} \times \sqrt{\frac{3m}{2k}} = \frac{\sqrt{3}}{2} \Rightarrow \sqrt{3}:2$$

64. (*d*) From graph, slope $K = \frac{F}{x} = \frac{8}{2} = 4$

$$T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow T = 2\pi\sqrt{\frac{0.01}{4}} = 0.31\text{s}$$

65. (*b*) Comparing with $\frac{d^2x}{dt^2} = -\omega^2 x$

we have, $\omega = 2\ \text{rads}^{-1}$

$$v_{max} = \omega A = 2\ \text{ms}^{-1}$$

$$a_{max} = \omega^2 A = 4\ \text{ms}^{-2}$$

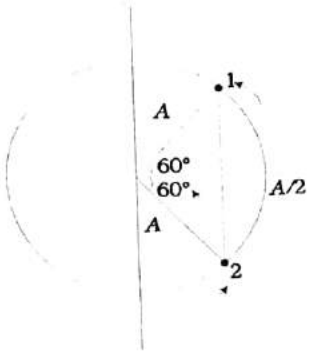
66. (*a*) To keep time period same, $l' = \sqrt{2}\,l$

67. (*c*) $\frac{1}{2}mv^2 = \frac{1}{2}kx^2$

$$\therefore \quad x = v\sqrt{\frac{m}{k}}$$

68. (*b*) After attaining terminal velocity, ball will move with constant velocity. Therefore, the motion of the ball is periodic but not SHM.

69. (*d*)



70. (*b*) $T = 2\pi\sqrt{\frac{m}{k}}$ or $T \propto \frac{1}{\sqrt{k}}$ or $k = \frac{\alpha}{T^2}$

Now, $\quad k_e = k_1 + k_2$

$$\therefore \quad \frac{\alpha}{T^2} = \frac{\alpha}{T_1^2} + \frac{\alpha}{T_2^2}$$

or $\quad T^{-2} = T_1^{-2} + T_2^{-2}$

71. (*a*) $\omega^2 A = g$

As, $\quad \omega = \sqrt{\frac{g}{A}}$ or $T = 2\pi\sqrt{\frac{A}{g}}$

$$= 2\pi\sqrt{\frac{4 \times 10^{-3}}{10}} = 2\pi \times 2 \times 10^{-2} \Rightarrow T = \frac{4\pi}{100} = \frac{\pi}{25}\ \text{s}$$

72. (*b*) $x_{12} = \frac{12 \times 10}{1000} = 0.12$ m $\qquad \left(x = \frac{mg}{k}\right)$

$$x_4 = \frac{4 \times 10}{1000} = 0.04\ \text{m}$$

$$\therefore \quad A = x_{12} - x_4 = 0.08\ \text{m}$$

73. (*b*) $U = U_{mean} + \frac{1}{2}kX^2$

Given, $U_{mean} = E_1$ and $\frac{1}{2}kA^2 = E_2$ = maximum kinetic energy at mean position

$$\therefore \quad X = \frac{\sqrt{3}A}{2}, U = E_1 + \frac{1}{2}k\left(\frac{\sqrt{3}A}{2}\right)^2 = E_1 + \frac{3}{4}E_2$$

74. (*b*) $X = A\sin\omega t$

or $\quad 10 = 20\sin\omega t$

$$\therefore \quad \omega t = \left(\frac{\pi}{6}\right) \Rightarrow t = \frac{\pi}{6\omega}$$

The desired time will be $2t$

or $\quad \frac{\pi}{3\omega} = \frac{\pi}{3(2\pi/T)} = \frac{T}{6} = \frac{12}{6} = 2\ \text{s}$

75. (*a*) $\quad k \propto \frac{1}{l}$

$$\frac{l_1}{l_2} = n$$

$$\therefore \quad l_1 = \left(\frac{n}{n+1}\right)l \text{ or } k' = \left(\frac{n+1}{n}\right)k$$

$$\therefore \quad T = 2\pi\sqrt{\frac{mn}{k(n+1)}}$$

76. (*b*) Amplitude of oscillation, $A = \frac{mg}{k} = \frac{5 \times 10}{200} = 0.25$ m

77. (*b*) $T = 2\pi\sqrt{\frac{l}{g}}$, where l is immersed length.

78. (*b*) $T_1 = 2\pi\sqrt{\frac{m}{k + \rho Ag}}$ but $T_2 = 2\pi\sqrt{\frac{m}{k}}$.

Hence, $T_1 < T_2$

79. (*b*) $\frac{1}{2}kA^2 = (0.04 - 0.01)\ \text{J} = 0.03\ \text{J}$

$$\therefore \quad k = \frac{0.06}{A^2} = \frac{0.06}{(0.02)^2} = 150\ \text{Nm}^{-1}$$

80. (*c*) Potential energy graph is parabolic with its minimum value at mean position.

81. (*a*) Since, the cube is half immersed. The density of cube should be half the density of water, *i.e.*, 500 kgm^{-3}

$$T = 2\pi\sqrt{\frac{m}{\rho_w A g}} \qquad (k = \rho_w A g)$$

$$\frac{\pi}{5} = 2\pi\sqrt{\frac{a^3 \times \rho}{\rho_w \times a^2 \times g}}$$

$$\therefore \quad \frac{a\rho}{g\rho_w} = \frac{1}{100}$$

$$\therefore \quad \frac{a}{20} = \frac{1}{100} \text{ or } a = (0.2)\text{ m}$$

Now, $m = a^3\rho = 4$ kg $\quad$ ($\because$ Mass = Density × Volume)

82. (*a*) From e to $2e$ or $\frac{A}{2}$ to A, $t = \frac{T}{6}$

$\therefore$ Time period of oscillation $= 2t = \frac{T}{3} = \frac{2\pi}{3}\sqrt{\frac{m}{k}}$

83. (*c*) $\mu g >$ maximum acceleration in SHM $> \omega^2 A$

$$\therefore \quad \mu > \frac{\omega^2 A}{g} \Rightarrow \mu > \frac{(2\pi f)^2 \times A}{g}$$

$$\Rightarrow \quad \mu > \frac{(2 \times 3.14 \times 2)^2 \times 4 \times 10^{-2}}{10}$$

$$\Rightarrow \quad \mu > 0.631 \approx 0.64$$

84. (*a*) $\quad y_1 = A\sin\left(\omega_1 t + \frac{\pi}{2}\right) \Rightarrow y_2 = A\sin\left(\omega_2 t - \frac{\pi}{2}\right)$

Now, $\omega_1 t + \frac{\pi}{2} = \omega_2 t - \frac{\pi}{2}$

or $$t = \frac{\pi}{\omega_2 - \omega_1} = \frac{\pi}{(2\pi/T_2) - (2\pi/T_1)}$$

$$= \frac{T_1 T_2}{2(T_1 - T_2)} = \frac{3 \times 7}{2(7-3)} = \frac{21}{8}\text{ s}$$

85. (*c*) $T = 2\pi\sqrt{\frac{m}{k}}$ or $T \propto \frac{1}{\sqrt{k}}$

T has increased $\frac{4}{3}$ times. Hence, $k' = \frac{9}{16}k$

When both are combined,

$$k_{net} = k + k' = \frac{25}{16}k \text{ or } \frac{25}{16}\text{ times}$$

Hence, new time period will become $\frac{4}{5}$ times of 3 s or 2.4 s.

86. (*c*) Particle starts from rest. Hence, $x = A\cos\omega t$

$$a = A - A\cos(\omega \times 1)$$

or $$\cos\omega = \frac{A-a}{A} = \left(1 - \frac{a}{A}\right)$$

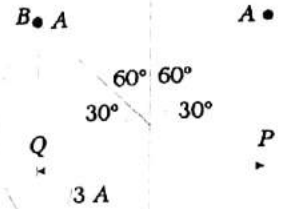
$$\Rightarrow \quad a + b = A - A\cos(\omega \times 2)$$

$$= A - A(2\cos^2\omega - 1)$$

$$= 2A - 2A\left(1 - \frac{a}{A}\right)^2$$

On solving, we get $A = \frac{2a^2}{3a - b}$

87. (*a*) $\frac{T}{3}$ means 120° in reference circle. Maximum possible value of V is near mean position between P and Q



$$\therefore \quad V = \frac{\text{Distance}}{\text{Time}} = \frac{\sqrt{3}A}{(T/3)} = \frac{3\sqrt{3}A}{T}$$

88. (*d*) The motion is simple harmonic only inside earth. Further

$$\frac{1}{2}mv^2 = -\frac{GMm}{(R+h)} + \frac{3}{2}\frac{GMm}{R}$$

$$\Rightarrow \quad v = \sqrt{\frac{GM(R+3h)}{R(R+h)}}$$

89. (*b*) M_e = Mass of container + $\frac{\text{Upthrust}}{g}$

$$= M + \frac{\rho V g}{g} = M + \rho V$$

90. (*d*) $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.1}{100}} = 0.198$ s

From $x = A\sin\omega t$ or $5 = 10\sin\omega t$

$$\omega t = \frac{\pi}{6} \text{ or } \frac{2\pi}{T}t = \frac{\pi}{6} \text{ or } 2t = \frac{T}{6}$$

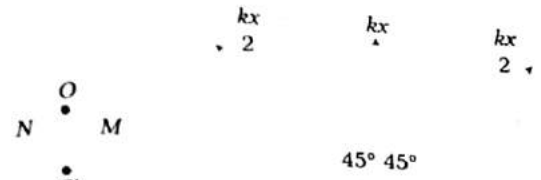
The desired time in the question will be $\frac{T}{2} + 2t$

or $$\frac{T}{2} + \frac{T}{6} = \frac{2T}{3} = \left(\frac{2}{3}\right)(0.198) = 0.132\text{ s}$$

91. (*b*) $$k_{eq} = \frac{k_1 k_2}{k_1 + k_2} = \frac{\frac{YAk}{L}}{\frac{YA}{L} + k}$$

$$= \frac{YAk}{YA + Lk} = 2\pi\sqrt{\frac{m(YA + Lk)}{YAk}}$$

92. (*b*)



$$OO' = x \text{ (say)}$$

Then $\quad O'M = O'N \approx \dfrac{x}{\sqrt{2}}$

i.e., elongation in spring B and C is $x/\sqrt{2}$, while compression in spring A is x.

Net restarting force,

$$F = -\left[kx + \frac{2kx}{\sqrt{2}}\cos 45°\right] = -2kx$$

$$\therefore \quad a = \frac{F}{m} = -\frac{2k}{m}x$$

$$T = 2\pi\sqrt{\left|\frac{x}{a}\right|} = 2\pi\sqrt{\frac{m}{2k}}$$

93. (*c*) $\quad (P_i = P_f)$

Law of conservation of momentum

$$Mv_{max} = (m + M)v$$

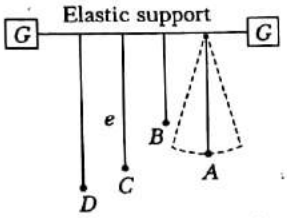
$$\Rightarrow \quad v_{max} = \frac{Mv_{max}}{(m + M)}$$

$$\because \quad v_{max} = A'\omega'$$

$$\Rightarrow \quad \frac{M \cdot A}{(m + M)}\sqrt{\frac{k}{M}} = A'\sqrt{\frac{k}{(m + M)}}$$

$$\Rightarrow \quad A' = A\sqrt{\frac{M}{(m + M)}}$$

94. (*b*) According to the question, A is given a transverse displacement.



Through the elastic support the disturbance is transferred to all the pendulums.

A and C are having same length, hence they will be in resonance, because their time period of oscillation.

$$T = 2\pi\sqrt{\frac{l}{g}} \text{ and hence, frequency is same.}$$

So, amplitude of A and C will be maximum.

Note In this problem, we have assumed that the support is perfectly elastic and there is no damping. Hence, oscillation is considered as undamped.

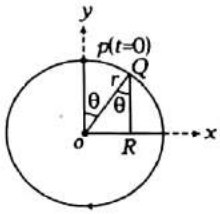
95. (*a*) Let angular velocity of the particle executing circular motion is ω and when it is at Q makes and angle θ as shown in the diagram.

Clearly, $\theta = \omega t$

Now, we can write OR

$= OQ\cos(90 - \theta) = OQ\sin\theta = OQ\sin\omega t$

$= r\sin\omega t \qquad [\because OQ = r]$



$$\Rightarrow \quad x = r\sin\omega t = B\sin\omega t \qquad [\because r = B]$$

$$= B\sin\frac{2\pi}{T}t = B\sin\left(\frac{2\pi}{30}t\right)$$

Clearly, this equation represents SHM.

B) Medical entrance special format questions

Assertion and reason

1. (*a*)

2. (*a*) For stable equilibrium, PE is minimum.

3. (*a*) At given time x is negative. Therefore, acceleration is positive ($a \propto -x$). Further slope is negative. Therefore, velocity is negative.

5. (*a*) $T = 2\pi\sqrt{\dfrac{l}{g_e}}$

Here $g_e = g + a$, if the lift accelerates upwards.

6. (*d*) Simple harmonic motion is not always one-dimensional. In case of angular simple harmonic motion, it is 2-D also.

7. (*d*) Time period does not depend on amplitude of oscillation.

8. (*c*) At given time $x = 0$, *i.e.*, body is at mean position. At mean position, potential energy is always minimum. But this minimum potential energy may or may not be zero.

9. (*b*) $X = x - 3 = 4\cos\omega t \Rightarrow X = 0$ at $x = 3$

10. (*a*) $v = \omega\sqrt{A^2 - x^2}$

$$\therefore \quad \frac{v^2}{\omega^2} + \frac{x^2}{1} = A^2$$

11. (*c*) Amplitude is 4.

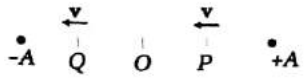
12. (*d*) Motion of particle is simple harmonic, but mean position is at $x = 4$. Amplitude is 6. The particle will oscillate between $x = 10$ and $x = -2$.

13. (*c*) Force constant of a spring, $k \propto \dfrac{1}{l}$ and $T \propto \dfrac{1}{\sqrt{k}}$

$$\therefore \quad T \propto \sqrt{l}$$

14. (*d*) $\quad t_{PQ} = T \qquad$ (given)

But t_{PQ} is not the time period of oscillation.



15. (*b*) If $v = \omega A \sin \omega t$

Then, $v_{max} = \omega A$

$\therefore \quad K_{max} = \frac{1}{2} m v_{max}^2 = \frac{1}{2} m\omega^2 A^2$

Further, $< K > = < \frac{1}{2} m\omega^2 A^2 \sin^2 \omega t >$

$= \frac{1}{2} m\omega^2 A^2 < \sin^2 \omega t >$

But $< \sin^2 \omega t > = \frac{1}{2}$ in one oscillation

$\therefore \quad < K > = \frac{1}{4} m\omega^2 A^2$

16. (*b*) Maximum angular velocity $= \omega\theta_0$

$= (2\pi f)\,\theta_0 = (2\pi)\left(\frac{1}{2\pi}\sqrt{\frac{g}{l}}\right)\theta_0$

$= \left(\sqrt{\frac{g}{l}}\right)\theta_0$

17. (*d*) $k \propto \frac{1}{\text{length of spring}}$

If length is halved k will become two times.

From, $\quad T = 2\pi\sqrt{\frac{m}{k}}$

Time period T will remain half for the given conditions.

Match the columns

1. Suppose $x = A \sin \omega t$

Then, $\quad v = \frac{dx}{dt} = \omega A \cos \omega t$

and $\quad a = \frac{dv}{dt} = -\omega^2 A \sin \omega t$

2. $a = -\omega^2 x$, *i.e.*, a-x graph is straight line passing through origin.

If $\quad v = v_0 \sin \omega t$

then, $\quad a = \frac{dv}{dt} = v_0\omega \cos \omega t = v_0\omega\sqrt{1 - \sin^2 \omega t}$

$= v_0\omega\sqrt{1 - \frac{v^2}{v_0^2}}$

$\Rightarrow \quad a = \omega\sqrt{v_0^2 - v^2}$

So, a-v graph is neither a straight line nor a parabola. Further, acceleration and velocity-time graphs are sine or cosine functions.

3. On a satellite and at centre of earth $g' = 0$

$\therefore \quad T = \infty$

At pole, value of g is more than the normal value. Hence, $T < 2$ s

At mountain, or at some height, value of g is less,

Hence, $\quad T > 2$ s

4. $v = \omega\sqrt{A^2 - \frac{A^2}{4}} = \frac{\sqrt{3}}{2}\omega A = \frac{\sqrt{3}}{2} v_{max}$

$\therefore \quad K = \left(\frac{\sqrt{3}}{2}\right)^2$ times

$K_{max} = \frac{3}{4} K_{max}$

About PE, it will be $\frac{1}{4}$ times its maximum value only when potential energy at mean position is zero.

5. $\omega = \sqrt{\frac{k}{m}}$, $v = \omega\sqrt{A^2 - X^2}$, $\text{PE} = \frac{1}{2}kx^2$

6. At P, velocity is zero, but just after few seconds velocity is negative. Therefore, at P particle is at $x = +A$. At this point acceleration of the particle will be maximum. On similar ground we can say that

At Q, $x = -A$

At R, v is maximum, *i.e.*, particle is at mean position, where acceleration is also zero.

7. The given equation,

$y = A \sin \omega t + A \sin\left(\omega t + \frac{2\pi}{3}\right)$ can also be written as.

$y = 2A \sin\left(\omega t + \frac{\pi}{3}\right) \cdot \cos\left(\frac{\pi}{3}\right) = A \sin\left(\omega t + \frac{\pi}{3}\right)$

Now, we can see that this is SHM with amplitude A and initial phase $\frac{\pi}{3}$.

9. $T = 8$ s

$\therefore \quad \omega = \frac{2\pi}{T} = \frac{\pi}{4}\ \text{rads}^{-1}$

$K = -$ slope of F-x graph $= 10\ \text{Nm}^{-1}$

From, $T = 2\pi\sqrt{\frac{m}{k}}$

We have, $8 = 2\pi\sqrt{\frac{m}{10}}$

$\therefore \quad m = \frac{160}{\pi^2}$ kg

$K_{max} = \frac{1}{2} m\omega^2 A^2 = \frac{1}{2} \times \frac{160}{\pi^2} \times \frac{\pi^2}{16} \times (4 \times 10^{-2})^2$

$= 8.0 \times 10^{-3}$ J

10. $\omega = \sqrt{\frac{k}{\mu}}$

Here, $\quad \mu =$ reduced mass

$= \frac{m_1 m_2}{m_1 + m_2} = \frac{2}{3}$ kg

$\therefore \quad \omega = \sqrt{\frac{6}{2/3}} = 3\ \text{rads}^{-1}$

Amplitude 12 cm distributes in inverse ratio of mass.

$\therefore \quad A_1 = 8$ cm and $A_2 = 4$ cm

Now, maximum kinetic energy

$$K_1 = \frac{1}{2} m_1 \omega A_1^2$$

$$= \frac{1}{2} \times 1 \times 3 \times (8 \times 10^{-2})^2 = 9.6 \text{ mJ}$$

$$K_2 = \frac{1}{2} \times 2 \times 3 \times (4 \times 10^{-2})^2 = 4.8 \text{ mJ}$$

11. $T = 2\pi \sqrt{\frac{I}{mgl}}$. Here, l is distance of point P from centre of mass.

When P is the centre, $l = 0$. Therefore, $T = \infty$, *i.e.*, rod will not oscillate.

When P is end point, $I = \frac{ml^2}{3}$

$$\therefore \quad T = 2\pi \sqrt{\frac{l}{3g}} \Rightarrow T_{\text{rod}} = T_{\text{pendulum}}$$

or $$2\pi \sqrt{\frac{2l}{3g}} = 2\pi \sqrt{\frac{L}{g}}$$

or $$L = \frac{2l}{3}$$

Medical entrances' gallery

1. (*d*) Time period,

$$T = 2\pi \sqrt{\frac{m}{k}}$$

Case I $$T_1 = 2\pi \sqrt{\frac{m}{k}} \quad \text{...(i)}$$

Case II When the mass m is increased by 1 kg, then $= m + 1$

$$T_2 = 2\pi \sqrt{\frac{m+1}{k}} \quad \text{...(ii)}$$

From Eqs. (ii) and (i), we get

$$\frac{T_2}{T_1} = \sqrt{\frac{m+1}{m}} \Rightarrow \frac{5}{3} = \sqrt{\frac{m+1}{m}}$$

$$\Rightarrow \quad \frac{25}{9} = \frac{m+1}{m} \Rightarrow \frac{25}{9} = 1 + \frac{1}{m}$$

$$\Rightarrow \quad \frac{1}{m} = \frac{16}{9}$$

$$\therefore \quad m = \frac{9}{16} \text{kg}$$

2. (*c*) Given, $K_P > K_Q$

In case (*a*), the elongation is same

i.e., $x_1 = x_2 = x$

So, $W_P = \frac{1}{2} K_P x^2$ and $W_Q = \frac{1}{2} K_Q x^2$

$$\therefore \quad \frac{W_P}{W_Q} = \frac{K_P}{K_Q} > 1 \quad (K_P > K_Q)$$

$$\Rightarrow \quad W_P > W_Q$$

In case (*b*), the spring force is same

i.e., $F_1 = F_2 = F$

So, $x_1 = \frac{F}{K_P}, x_2 = \frac{F}{K_Q}$

$$\therefore \quad W_P = \frac{1}{2} K_P x_1^2 = \frac{1}{2} K_P \frac{F^2}{K_P^2} = \frac{1}{2} \frac{F^2}{K_P}$$

and $$W_Q = \frac{1}{2} K_Q x_2^2 = \frac{1}{2} K_Q \cdot \frac{F^2}{K_Q^2} = \frac{1}{2} \frac{F^2}{K_Q}$$

$$\therefore \quad \frac{W_P}{W_Q} = \frac{K_Q}{K_P} < 1 \Rightarrow W_P < W_Q$$

3. (*b*) Let A be the amplitude of oscillation then

$$v_1^2 = \omega^2 (A^2 - x_1^2) \quad \text{...(i)}$$

$$v_2^2 = \omega^2 (A^2 - x^2) \quad \text{...(ii)}$$

On subtracting Eq. (ii) from Eq. (i), we get

$$v_1^2 - v_2^2 = \omega^2 (x_2^2 - x_1^2)$$

$$\Rightarrow \quad \omega = \sqrt{\frac{v_1^2 - v_2^2}{x_2^2 - x_1^2}} \Rightarrow \frac{2\pi}{T} = \sqrt{\frac{v_1^2 - v_2^2}{x_2^2 - x_1^2}} \Rightarrow T = 2\pi \sqrt{\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}}$$

4. (*b*) Given, $v \frac{dv}{dx} = -\omega^2 x$

On integrating within the limit $\int_{v_0}^{v} v dv = \int_0^x -\omega^2 x \, dx$

$$\Rightarrow \quad \left[\frac{v^2}{2}\right]_{v_0}^{v} = -\omega^2 \left[\frac{x^2}{2}\right]_0^x$$

$$\Rightarrow \quad v^2 - v_0^2 = -\omega^2 x^2 \Rightarrow v = \sqrt{v_0^2 - \omega^2 x^2}$$

5. (*a*) As per question, we can write $mg = kx$

$$m \times 9.8 = k \times 1 \times 10^{-2}$$

$$\Rightarrow \quad k = \frac{9.8\, m}{10^{-2}} = 980 \text{ m}$$

Now, value of g at height 800 km above the earth's surface is g. Hence,

$$g' = g\left(1 - \frac{2h}{R}\right) = 9.8\left(1 - \frac{2 \times 800}{6400}\right) = 9.8\left(1 - \frac{1}{4}\right)$$

$$= 9.8 \times \frac{3}{4} = \frac{29.4}{4}$$

Again, $m \times \frac{29.4}{4} = kx$

$$\Rightarrow \quad x = \frac{m \times 29.4}{k \times 4} = \frac{m \times 29.4}{4 \times 980\, m} = 0.79 \text{ cm}$$

6. (*c*) Time period, $T = 2\pi \sqrt{\frac{m}{k}}$

$$\Rightarrow \quad T = 2 \times 3.14 \sqrt{\frac{200 \times 10^{-3}}{80}}$$

or, $$T = 2 \times 3.14 \sqrt{\frac{1}{400}} \text{ or, } T = \frac{2 \times 3.14}{20}$$

or, $$T = 0.315$$

7. (*d*) Total mechanical energy of the system,

$$E = \frac{1}{2} m\omega^2 a^2 = \frac{1}{2} \times 40 \times 10^{-3} \left(\frac{2\pi}{T}\right)^2 \times (2 \times 10^{-2})^2$$

$$= \frac{1}{2} \times 40 \times 10^{-3} \left(\frac{3.14 \times 2}{20}\right)^2 \times (2 \times 10^{-2})^2$$

$$= 3.94 \times 10^{-3} \text{ J}$$

8. (*b*) Given, time period, $T = 10$ s

$$\therefore \quad T = 2\pi \sqrt{\frac{m}{k}}$$

when viscous the system is immersed in liquid with specific gravity $\frac{1}{4}$th of the copper, the liquid is non-viscous, so the time period would not be changed because, there is no change in m and k.

9. (*e*) A harmonic oscillation of constant amplitude and a single frequency under a restoring force whose magnitude is proportional to the displacement and always acts towards mean position is known as simple harmonic motion.

10. (*b*) For a simple harmonic motion, $\frac{d^2Y}{dt^2} \propto (-Y)$

Equations, $Y = \sin \omega t - \cos \omega t$

and $\quad Y = 5\cos\left(\frac{3\pi}{4} - 3\omega t\right)$

are satisfying this condition.

Equation, $Y = 1 + \omega t + \omega^2 t^2 + \omega^3 t^3$

is not periodic and $Y = \sin^3 \omega t$ is periodic but not SHM.

11. (*b*) As, potential energy of a particle executing simple harmonic motion also periodic with period $T/2$. So, potential energy is zero at the mean position and maximum at the extreme displacements.

Let the amplitude of SHM be A.

Now, potential energy of SHM $= \frac{1}{2} kx^2$

Here, $x = \frac{A}{2} \Rightarrow U = \frac{1}{2} k \frac{A^2}{4}$ ...(i)

Kinetic energy, $K = \frac{1}{2} kA^2 - \frac{1}{2} k \frac{A^2}{4} \Rightarrow K = \frac{3}{8} kA^2$ ...(ii)

On dividing Eq. (ii) by Eq. (i), we get

$$\frac{K}{U} = \frac{3}{8} \times \frac{8}{1} = \frac{K}{U} = \frac{3}{1}$$

12. (*e*) Given, $x = 5\cos\left(2\pi t + \frac{\pi}{4}\right)$ (where, x is a displacement)

$$\Rightarrow \quad x = 5\cos\left(2\pi + \frac{\pi}{4}\right) \quad \text{(at } t = 1 \text{ s)}$$

$[\because \cos(360° + \theta) = \cos\theta]$

$$\Rightarrow \quad x = 5\cos\frac{\pi}{4} \Rightarrow x = 5 \times \frac{1}{\sqrt{2}}$$

$$\therefore \quad x = \frac{5}{\sqrt{2}} \text{ m}$$

13. (*b*) When bigger pendulum of time period ($5T/4$) completes one vibration, the smaller pendulum will complete ($5/4$) vibrations. It means the smaller pendulum will be leading, the bigger pendulum by phase $T/4$ s $= \pi/2$ rad $= 90°$.

14. (*d*) At mean position, velocity is maximum

$$v_{max} = \omega a \Rightarrow \omega = \frac{v_{max}}{a} = \frac{16}{4} = 4$$

$$\therefore \quad v = \omega\sqrt{a^2 - y^2} \Rightarrow 8\sqrt{3} = 4\sqrt{4^2 - y^2}$$

$$192 = 16(16 - y^2) \Rightarrow 12 = 16 - y^2 \Rightarrow y = 2 \text{ cm}$$

15. (*b*) Given, $x_1 = A\sin\left(\omega t + \frac{\pi}{6}\right)$ and $x_2 = A\cos\omega t$

$$x_2 = A\sin\left(\omega t + \frac{\pi}{2}\right)$$

Phase difference, $\Delta\phi = \phi_2 - \phi_1 \Rightarrow \Delta\phi = \frac{\pi}{2} - \frac{\pi}{6}$

$$\Rightarrow \quad \Delta\phi = \frac{3\pi - \pi}{6} = \frac{\pi}{3}$$

16. (*c*) $a = -\omega^2 x = -\omega^2 A\cos\omega t$

$\therefore$ a-t graph is $-\cos\omega t$ graph

17. (*d*) Given, $y = a\sin 2\pi(bt - cx) = a\sin(2\pi bt - 2\pi cx)$

Comparing it with general equation,

$$y = r\sin\left[\frac{2\pi t}{T} - \frac{2\pi}{\lambda}x\right]$$

We get, $\frac{2\pi}{T} = \omega = 2\pi b$ and $r = a$

$$\lambda = \frac{1}{c} \text{ and } T = \frac{1}{b}$$

Maximum particle velocity,

$$\omega r = 2\pi ba$$

Wave velocity, $v = \frac{\lambda}{T} = \frac{b}{a}$

Given, maximum particle velocity $= 2 \times$ wave velocity

$$2\pi ba = 2 \times \frac{b}{c} \Rightarrow c = \frac{1}{\pi a}$$

18. (*b*) We know that kinetic energy, $K = \frac{1}{2} mv^2$

where, $u = \frac{dy}{dt} = \omega A\cos\omega t$

So, $\quad K = \frac{1}{2} m\omega^2 A^2 \cos^2 \omega t$

Hence, kinetic energy varies periodically with double the frequency of SHM. So, when a particle executing SHM oscillates with a frequency ν, then the kinetic energy of particle changes periodically with a frequency of 2ν.

19. (*d*) The current displacement is $\frac{\sqrt{3}}{2}$ times the amplitude.

i.e., $\quad y = \frac{\sqrt{3}}{2} A$

We know that potential energy of a body in SHM is given by where, m and ω are constant.

$$U = \frac{1}{2} m\omega^2 y^2$$

$$U = \frac{1}{2} m\omega^2 \left(\frac{\sqrt{3}}{2} A\right)^2 = \frac{1}{2} m\omega^2 \times \frac{3A^2}{4} \quad ...(i)$$

Similarly,

Kinetic energy of a body in SHM is given by

$$K = \frac{1}{2} m\omega^2 (A^2 - y^2) = \frac{1}{2} m\omega^2 \left(A^2 - \frac{3A^2}{4}\right) = \frac{1}{2} m\omega^2 \frac{A^2}{4} \quad ...(ii)$$

So, the ratio of PE and KE

$$\text{PE} : \text{KE} = \frac{1}{2} m\omega^2 \times \frac{3A^2}{4} : \frac{1}{2} m\omega^2 \frac{A^2}{4} = 3 : 1$$

20. (*a*) We know that in spring SHM, the restoring force is proportional to displacement

i.e., $F = -m\omega^2 y$ ...(i)

$F = -ky$ ...(ii)

where, k = force constant of the spring

$m = 10 \text{ kg} \Rightarrow A = 10 \text{ cm} = 0.1 \text{ m} \Rightarrow k = 1000 \text{ Nm}^{-1}$

Comparing the both equations, we get

$$\omega^2 = \frac{k}{m} \Rightarrow \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{1000}{10}} \Rightarrow 10 \text{ rads}^{-1}$$

and acceleration in SHM.

$$a_{max} = -\omega^2 \cdot y = -10^2 \times (0.1) = -10 \text{ ms}^{-2} = 10 \text{ ms}^{-2}$$

21. (*a*) When restoring force will become equal to the frictional force, block will start to slip.

$\therefore$ Restoring force = Friction force

$\Rightarrow$ $kA = \mu mg$ $[\because F_{rest} = -kx]$ ...(i)

Now, frequency, $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

From Eq. (i), we get $f = \frac{1}{2\pi}\sqrt{\frac{\mu g}{A}}$

22. (*c*) The radius of particle is 3 cm which is maximum, so the amplitude of simple harmonic motion is 3 cm.

23. (*d*) Time period $T = 2\pi\sqrt{\frac{l}{g}}$

Given, $g = 9.8 \text{ ms}^{-2}$, $l = 9.8 \text{ m}$

$$T = 2\pi\sqrt{\frac{9.8}{9.8}} = 2\pi = 2 \times 3.14 = 6.28\text{s}$$

24. (*d*) We know that equation of SHM $y = a \sin \omega t$

Here in given condition, $\frac{a}{2} = a \sin \omega \times 1$

$$\frac{1}{2} = \sin \omega \Rightarrow \sin \omega = \frac{1}{2} \Rightarrow \omega = \frac{\pi}{6}$$

Here time period of SHM

$$\frac{2\pi}{T} = \frac{\pi}{6}, T = 6 \times 2 \Rightarrow T = 12\text{s}$$

25. (*b*) For SHM $\alpha = -\omega^2 y$, where α is acceleration, ω is angular velocity, y is displacement. Hence, it is clear that acceleration is directly proportional to displacement and is always opposite to displacement.

26. (*a*) We know that the time period of a simple pendulum

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$\Rightarrow \quad T \propto \frac{l}{\sqrt{g}}$$

Here in given condition,

$$T_1 = \frac{k}{\sqrt{g}} \quad ...(i)$$

and $$T_2 = \frac{K}{\sqrt{g\left(1 - \frac{d}{R}\right)}} \quad ...(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{T_1}{T_2} = \frac{K/\sqrt{g}}{K\Big/\sqrt{g\left(1 - \frac{d}{R}\right)}}$$

or, $$\frac{T_1}{T_2} = \sqrt{\left(1 - \frac{d}{R}\right)}$$

or, $$\frac{T_1^2}{T_2^2} = 1 - \frac{d}{R}$$

$$\Rightarrow \quad d = \left[1 - \frac{T_1^2}{T_2^2}\right] R$$

27. (*e*) Given, $x = A \sin \omega t$

if $x = 0, t = 0$

if $x = \frac{\sqrt{3}}{2} A, \omega t_1 = \frac{\pi}{3}$

if $x = \frac{\sqrt{3}}{2} A, \omega t_2 = \frac{\pi}{3}$

if $x = A, \omega t_2 = \frac{\pi}{2}$

$$\therefore \quad \frac{\omega t_1}{\omega t_2} = \frac{\pi/3}{\pi/2} = \frac{2}{3} \Rightarrow \frac{t_1}{t_2} = \frac{2}{3}$$

28. (*b*) The resonance wave becomes very sharp when damping force is small.

29. (*b*) As the boy stands up, the centre of gravity of the pendulum is raised up, decreasing the effective length of the pendulum of the swing and hence the time period T decreases.

30. (*d*) Frequency of oscillator, $n = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$

We get, $$\frac{n_2}{n_1} = \sqrt{\frac{m_1}{m_2}}$$

$$\therefore \quad 2 = \sqrt{\frac{m_1}{m_2}} \text{ or } m_2 = \frac{1}{4} m_1$$

31. (*a*) Given equation is

$$y = 5[\sin 3\pi t + \sqrt{3} \cos 3\pi t]$$

or $$y = 5 \times 2\left[\frac{1}{2} \sin 3\pi t + \frac{\sqrt{3}}{2} \cos 3\pi t\right]$$

or $$y = 10\left[\sin 3\pi t \cos\frac{\pi}{3} + \cos 3\pi t \sin\frac{\pi}{3}\right]$$

or $$y = 10 \sin\left(3\pi t + \frac{\pi}{3}\right)$$

32. (*c*) $a_1 = 10, a_2 = \sqrt{25+75} = 10$

$$a_1 : a_2 = 1:1$$

33. (*b*) $T = 2\pi\sqrt{\frac{l}{g}}$, $\log T = \log 2\pi + \frac{1}{2}\log l - \frac{1}{2}\log g$

Differentiating, $\frac{dT}{T} = 0 + \frac{dl}{2l} - \frac{1}{2}\frac{dg}{g}$

$\because$ $l = \text{constant}$

$\therefore$ $dl = 0$

$$100 \times \frac{dT}{T} = \frac{-1}{2}\frac{dg}{g} \times 100$$

$$= \frac{-1}{2} \times \frac{(-2)\times 100}{g \times 100} = +1\% \text{ increase}$$

34. (*a*) Time period of a simple pendulum,

$$T = 2\pi\sqrt{\frac{l}{g}}$$

For freely falling system effective, $g = 0$

So, $T = \infty$

$\therefore$ Pendulum does not oscillate at all.

35. (*b*) Given, $y_1 = A\sin\omega t$

and $$y_2 = \frac{A}{2}\sin\omega t + \frac{A}{2}\cos\omega t$$

or $$y_2 = \frac{A}{2}(\sin\omega t + \cos\omega t)$$

$$= \frac{A}{2}\sqrt{2}[\sin(\omega t + 45°)] = \frac{A}{\sqrt{2}}\sin(\omega t + 45°)$$

$\therefore$ The ratio of amplitudes of two motions

$$\frac{A_1}{A_2} = \frac{A}{A/\sqrt{2}} = \sqrt{2}$$

36. (*b*) When external force is applied, one spring gets extended and another gets contracted by the same distance, hence force due to two springs act in same direction.

i.e., $$F = F_1 + F_2$$

$$-kx = -k_1x - k_2x$$

$$k = k_1 + k_2$$

37. (*c*) Mass of the particle $= m$

Spring constant $= k$

The time period of oscillator, $T = 2\pi\sqrt{\frac{m}{k}}$

As $k \propto \frac{1}{l}$ (where, l is the length of spring)

$\because$ $k' = 2k$

$\therefore$ $$T' = 2\pi\sqrt{\frac{m}{2k}} = \frac{1}{\sqrt{2}}T$$

38. (*b*) Assertion and Reason both are true but Reason is not correct explanation of Assertion.

When the bob is placed in an electric field, the time period of simple pendulum will remain same as the bob is not charged. If simple pendulum having charged bob is placed in a horizontal electric field, then the period will be decreased because there will be an increase in restoring force.

39. (*d*) The equation of displacement,

$$x_1 = 4\sin\left(10t + \frac{\pi}{6}\right)$$

The energy of this equation,

$$E_1 = \frac{1}{2}m\omega_1^2 a_1^2 = \frac{1}{2}m \times 10 \times 10 \times 4 \times 4$$

The second equation of displacement

$$x_2 = 5\cos(\omega t)$$

The energy of this equation

$$E_2 = \frac{1}{2}m\omega_2^2 a_2^2 = \frac{1}{2}m\omega_2^2 \times 5 \times 5$$

According to question

$\because$ $E_1 = E_2$

$\therefore$ $$\frac{1}{2}m\omega_2^2 \times 5 \times 5 = \frac{1}{2}m \times 10 \times 10 \times 4 \times 4$$

$$\omega_2^2 = \frac{10 \times 10 \times 4 \times 4}{5 \times 5}$$

$$\omega_2^2 = 2 \times 2 \times 4 \times 4$$

or $$\omega_2 = \sqrt{2 \times 2 \times 4 \times 4}$$

$$= 2 \times 4 = 8 \text{ unit}$$

40. (*c*) The equation of motion is given as

$$y = 5\sin\frac{2\pi t}{6}$$

Here, $y = 2.5$ cm

$\therefore$ $$2.5 = 5\sin\frac{2\pi t}{6}$$

$\Rightarrow$ $$\frac{\pi}{6} = \frac{2\pi t}{6}$$

$\Rightarrow$ $$t = \frac{1}{2}\text{ s}$$

$\therefore$ The phase $= \frac{2\pi t}{6} = \frac{2\pi}{6} \times \frac{1}{2} = \frac{\pi}{6}$

CHAPTER 12

Elasticity

A solid has a definite shape and size. In order to change the shape or size of a body, a force is required, such force is called **deforming force**. Whenever a load is attached to a thin hanging wire, it elongates and the load moves downwards. When load is removed, the wire attains its original size and shape. The property of a body, by virtue of which it tends to regain its original size and shape, when applied force is removed, is known as **elasticity** and the deformation caused is known as **elastic deformation.** In this chapter we will study about the elastic behaviour and mechanical properties of solids.

Some terms related to elasticity are discussed below

(i) **Perfectly Elastic Body** A body is said to be perfectly elastic, if it returns back completely to its original size on removing the external force(s).

(ii) **Plastic Body** If a body remains in the deformed shape and does not even partially regain its original shape, after removal of external force, is called plastic body or perfectly inelastic body.

(iii) **Elastic Limit** Elastic limit is the upper limit of deforming force up to which, if deforming force is removed, the body regains its original form completely and beyond which if deforming force is increased, the body looses its property of elasticity and gets permanently deformed.

Note

1. Elastic limit is the property of body whereas elasticity is the property of material of a body.
2. All rigid bodies are elastic to some extent which means we can change their dimensions slightly by pulling or pushing them.
3. No body is perfectly elastic or perfectly plastic. All the bodies found in nature lie between these two limits. When the elastic behaviour of body decreases, its plastic behaviour increases.

Inside

1) Stress and strain
2) Hooke's law
3) Poisson's ratio
- Stress-strain curve
- Workdone or potential energy Stored in a stretched wire
- Thermal stresses and strains

1) Stress and strain

When an external force is applied to a body, then at each cross-section of the body, an internal restoring force is developed which tends to restore the body back to its original position. **The internal restoring force acting per unit area of cross-section of the deformed body is called stress.**

It is usually denoted by σ (sigma).

Thus,

$$\boxed{\text{Stress } (\sigma) = \frac{\text{Restoring force}}{\text{Area}}}$$

If there is no permanent change in configuration of the body, the restoring force is equal and opposite to the external deforming force applied. So stress can be given as stress $(S) = \dfrac{\text{External deforming force}}{\text{Area}}$

In SI system, the unit of stress is Nm^{-2} or pascal (Pa) and in CGS system, it is dyne cm^{-2}.

The dimensions of stress are $[ML^{-1}T^{-2}]$

Note The minimum value of stress required to break a wire is called breaking stress. It depends on nature of material, temperature and impurities.

Due to external force, shape of the body changes and it is said to have strain.

Strain is defined as the ratio of change in shape of an object to the original shape.

$$\text{Strain} = \frac{\text{Change in configuration of the object}}{\text{Original configuration of the object}}$$

Strain is a pure number, it has no unit. *There are three types of stress resulting in three types of strain.*

1. Longitudinal stress

When the stress is normal to the surface of object, then it is known as longitudinal stress.

It is of two types.

(i) **Tensile Stress** If the stress produced in an object is due to increase in its length, then it is called tensile stress.

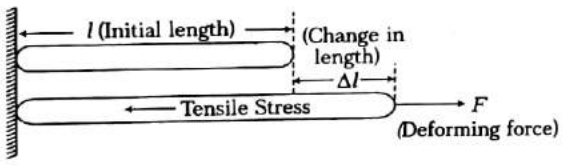


Fig. 12.1 An object under tensile stress

(ii) **Compressive Stress** If the stress produced in an object is due to decrease in its length, then it is called compressive stress.

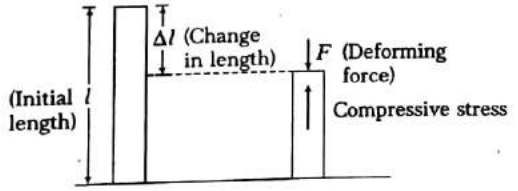


Fig. 12.2 An object under compressive stress

Longitudinal strain

It is defined as the change in length per unit length of the object on application of deforming force.

$$\text{Longitudinal strain} = \frac{\text{Change in length } (\Delta l)}{\text{Original length } l}$$

2. Volumetric stress

If equal normal forces are applied on an object all over its surface then its volume changes. The restoring force acting per unit area inside the object opposing change in volume is called volume stress.

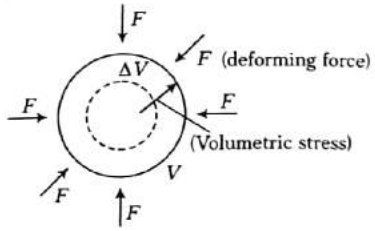


Fig. 12.3 An object under effect of normal deforming forces

Volumetric strain

It is defined as the change in volume per unit volume of the object on application of deforming force.

$$\text{Volumetric strain} = \frac{\text{Change in volume } (\Delta V)}{\text{Original volume } (V)}$$

3. Shearing stress

When a force is applied on an object along the tangential direction of the surface of the object, then stress produced in the object is called shearing stress. Such force tends to change the shape of object. For this, the opposite surface is kept fixed.

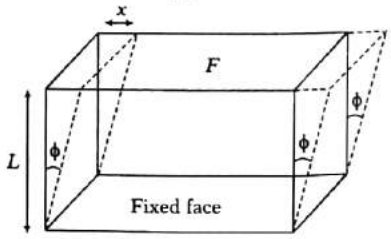


Fig. 12.4 An object under tangential deforming forces

Tangential or shearing strain

It is defined as the ratio of displacement (x) of the upper surface to the distance (L) between two faces on the application of deforming force.

$$\text{Shearing strain} = \frac{x}{L}$$

Example 12.1 *Find the greatest length of steel wire that can remain hanged vertically without breaking. Breaking stress of steel = $8 \times 10^8 Nm^{-2}$. Density of steel = 8×10^3 kgm^{-3}. (Take, $g = 10\ ms^{-2}$)*

Sol. Let l be the length of the wire that can hang vertically without breaking. The stretching force on it is equal to its own weight.

If A is the area of cross-section and ρ is the density, then

$$\text{Maximum stress } (\sigma_m) = \frac{\text{weight}}{A} \qquad \left(\because \text{ Stress} = \frac{\text{force}}{\text{area}}\right)$$

$$\text{Weight} = mg = V\rho g \quad (\because \text{ Mass} = \text{volume} \times \text{density})$$

$$= Al\rho g$$

$$(\because \text{ Volume} = \text{area of cross-section} \times \text{length})$$

or

$$\sigma_m = \frac{(Al\rho)g}{A} \qquad \left(\because \rho = \frac{m}{V}\right)$$

$$\therefore \text{ Length of the steel wire, } l = \frac{\sigma_m}{\rho g}$$

$$l = \frac{8 \times 10^8}{(8 \times 10^3)(10)} = 10^4 \text{ m}$$

Example 12.2 *A rod has a radius of 100 mm and a length of 10 cm. A 100 N force compresses along its length. Calculate the longitudinal stress developed in the rod.*

Sol. Given, radius of the rod, $r = 100 \text{ mm} = 100 \times 10^{-3}$ m,

Length of the rod, $l = 10 \text{ cm} = 10 \times 10^{-2}$m

and force, $F_n = 100$ N

Longitudinal stress (developed in the rod)

$$= \frac{F_n}{A} = \frac{100}{\pi (100 \times 10^{-3})^2}$$

$$= 3183 \text{ Nm}^{-2}$$

Example 12.3 *Consider a rod of steel having radius of 8 mm and the length of 2 m. If a force of 150 kN stretches it along its length, then calculate the stress, percentage strain in the rod if the elongation in length is 7.46 mm.*

Sol. Radius of rod, $r = 8 \text{ mm} = 8 \times 10^{-3}$ m, Length, $L = 2$ m

Applied force, $F = 150 \text{ kN} = 15 \times 10^4$ N

Cross-section area of wire,

$$A = \pi r^2 = \pi \times (8 \times 10^{-3})^2 = 201 \times 10^{-6} \text{ m}^2$$

$\Delta L = 7.46 \text{ mm} = 7.46 \times 10^{-3}$ m, Percentage strain = ?

$$\text{Stress in rod} = \frac{F}{A} = \frac{15 \times 10^4}{201 \times 10^{-6}}$$

$$= 0.0746 \times 10^{10} \text{ Nm}^{-2}$$

$$= 7.46 \times 10^8 \text{ Nm}^{-2}$$

$$\text{Longitudinal strain} = \frac{\Delta L}{L} = \frac{7.46 \times 10^{-3}}{2} = 3.73 \times 10^{-3}$$

$$\text{Percentage strain} = 3.73 \times 10^{-3} \times 100 = 0.37\%$$

Example 12.4 *If the angle of shear is 30° for a cubical body and the change in length is 250 cm, then what must be the volume of this cubical body.*

Sol. Given, angle of shear = 30°

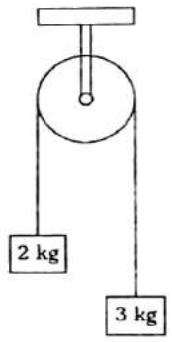
and change in length, $\Delta L = 250 \text{ cm} = 2.5$ m

$$\therefore \text{ Shear strain, } \tan\phi = \frac{\Delta L}{L} \Rightarrow \tan 30° = \frac{2.5}{L}$$

$$L = \frac{2.5}{\tan 30°} = \frac{2.5}{0.577} = 4.332$$

Volume, $V = L^3 = 81.309 \text{ m}^3$

Example 12.5 *Two blocks of masses 2 kg and 3 kg are connected by a metal wire going over a smoother pulley as shown in figure. The breaking stress of the metal is $2 \times 10^9 Nm^{-2}$. What would be the minimum radius of the wire used if it is not to break? Take, $g = 10 ms^{-2}$.*

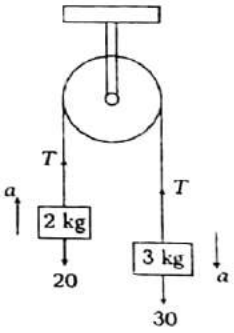


Sol. Equation of motion for 3 kg mass

$$30 - T = 3a \qquad \text{...(i)}$$

Equation of motion for 2 kg mass

$$T - 20 = 2a \qquad \text{...(ii)}$$



From Eqs. (i) and (ii),

$$\frac{30 - T}{T - 20} = 3/2$$

$$2(30 - T) = 3(T - 20)$$

$$60 - 2T = 3T - 20$$

$$5T = 80 \Rightarrow T = 16\text{N}$$

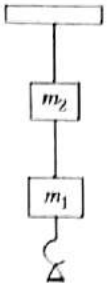
$$\text{Breaking stress, } \sigma = \frac{T}{A} \Rightarrow 2 \times 10^9 = \frac{16}{A} \qquad \left(\because \text{ Stress} = \frac{\text{force}}{\text{area}}\right)$$

Area, $A = 8 \times 10^{-9} m^2 \Rightarrow \pi r^2 = 8 \times 10^{-9}$ $(\because A = \pi r^2)$

Minimum radius of the wire

$$r = \left(\frac{8 \times 10^{-9}}{3.14}\right)^{1/2} = 5 \times 10^{-5} m$$

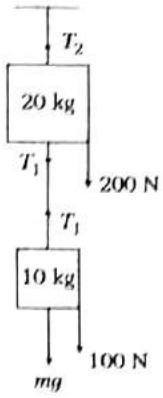
Example 12.6 *The two wires shown in figure are made of the same material which has a breaking stress of 8×10^8 Nm^{-2}. The area of cross-section of the upper wire is $0.006 cm^2$ and that of the lower wire is $0.003 cm^2$. The mass $m_1 = 10$ kg $m_2 = 20$ kg and the hanger is light. Find the maximum load that can be put on the hanger without breaking a wire. Which wire will break first if the load is increased.*



Sol. Maximum tension that the upper wire tolerate

= Breaking stress × area

$$T_2 = 8 \times 10^8 \times 0.006 \times 10^{-4} = 480 N$$



Maximum tension on that the lower wire has

$$T_1 = 8 \times 10^8 \times 0.003 \times 10^{-4} = 240 N$$

we have, $T_2 = T_1 + 200$ and $T_1 = 100 + mg$

$\therefore$ $T_2 = 300 + mg$

$\because$ $T_2 \le 480$

$\therefore$ $300 + mg \le 480$ or $mg \le 180$ or $m \le 18 kg$

Again $T_1 \le 240$ or $100 + mg \le 240$

or $m \le 14$ kg

Maximum mass $m_{max} = 14$ kg, when load is increased, T_1 increases hence, lower wire will break.

Example 12.7 *Consider a solid cube which is subjected to a pressure of $6 \times 10^5 Nm^{-2}$. Due to this pressure, the each side of the cube is shortened by 2%. Find out the volumetric strain of the cube.*

Sol. Let L be the initial length of the each side of the cube.

Volume, $V = L \times L \times L = L^3$ = Initial volume (V_i say)

If each side of the cube is shortened by 2%, then final length of the cube $= L - 2\%$ of L

$$= \left(L - \frac{2L}{100}\right) = L\left(1 - \frac{2}{100}\right)$$

$\therefore$ Final volume, $V_f = L^3\left(1 - \frac{2}{100}\right)^3 = V\left(1 - \frac{2}{100}\right)^3$

Change in volume, $\Delta V = V_f - V_i$

$$= V\left(1 - \frac{2}{100}\right)^3 - V = V\left[\left(1 - \frac{2}{100}\right)^3 - 1\right]$$

$$\frac{\Delta V}{V} = \left(1 - \frac{2}{100}\right)^3 - 1 \simeq \left[1 - \frac{2 \times 3}{100}\right] - 1$$

$[\because (1 - x)^n \simeq 1 - nx$ for $x << 1]$

$\therefore$ Volumetric strain, $\frac{\Delta V}{V} = 1 - 0.06 - 1 = 0.06$

[take positive sign]

Check point 12.1

1. In solids interatomic forces are
(a) totally repulsive (b) totally attractive
(c) Both (a) and (b) (d) None of these

2. The property of matter by virtue of which it does not regain its original shape and size after the removal of deforming force is called
(a) plasticity (b) elasticity
(c) elastic fatigue (d) None of these

3. Breaking stress depends on
(a) length of wire
(b) area of cross-section of wire
(c) Both (a) & (b)
(d) independent of length and area of cross-section

4. The maximum deforming force upto which a body retains its property of elasticity is called
(a) elastic limit of the material (b) elastic fatigue
(c) elastic after effect (d) None of these

5. The internal restoring force acting per unit area of cross-section of the deformed body is called
(a) strain (b) stress
(c) elastic limit (d) volumetric strain

6. The ratio of change in configuration to the original configuration is called
(a) stress (b) force
(c) strain (d) volumetric strain

7. The deforming force produces a change in the shape of the body without changing its volume, strain produced is called
(a) longitudinal strain (b) shearing strain
(c) volumetric strain (d) normal stress

8. The property of matter by virtue of which it regains its original shape and size, when the deforming forces have been removed is called
(a) deforming force (b) plasticity
(c) elasticity (d) electricity

9. The lower surface of a cube is fixed. On its upper surface, force is applied at an angle of 30° from its surface. The change will be in its
(a) shape (b) size
(c) volume (d) Both shape and size

10. When a spiral spring is stretched by force, strain produced is
(a) longitudinal strain (b) volumetric strain
(c) shear strain (d) Both (a) and (c)

11. A metallic cube whose each side is 10 cm is subjected to a shearing force of 100 kgf. Calculate the shearing produced.
(a) 9.8×10^4 Nm^{-2} (c) 10 Nm^{-2}
(c) 9.8 Nm^{-2} (d) 9.8×10^2 Nm^{-2}

12. The length of the wire is reduced by half. What will be the effect on the increase in its length under a given load?
(a) Reduced by half (b) Increase by half
(c) Remains same (d) None of these

13. A wire of length 2.5 m has a percentage strain of 0.012% under a tensile force. The extension produced in the wire will be
(a) 0.03 mm (b) 0.3 mm (c) 0.3 m (d) 0.03 m

14. A cube is subjected to a uniform volume compression. If the side of the cube decreases by 2%, the bulk strain is
(a) 0.02 (b) 0.03 (c) 0.04 (d) 0.06

15. A metallic cube of side 10 cm is subjected to a shearing force of 300 kgf. The top face is displaced through 0.25 cm with respect to the bottom? Calculate the shearing strain produced.
(a) 0.25 (b) 2.5 (c) 0.025 (d) 0.08

2) Hooke's law

According to Hooke's law

"For small deformation, the stress in a body is proportional to the corresponding strain." *i.e.,*

$$\text{stress} \propto \text{strain} \quad \text{or} \quad \text{strees} = (E)\ (\text{strain})$$

Here, $E = \dfrac{\text{stress}}{\text{strain}}$ is a constant called the **modulus of elasticity.**

The SI unit of modulus of elasticity is Nm^{-2} and its dimensions are $[\text{ML}^{-1}\text{T}^{-2}]$.

The modulus of elasticity depends on the type of material and temperature. It does not depend upon the values of stress and strain.

Depending upon the nature of force applied on the body, *the modulus of elasticity is classified in the following three types*

1. Young's modulus of elasticity (*i.e.,* modulus of elasticity of length)
2. Bulk modulus of elasticity (*i.e.,* modulus of elasticity of volume)
3. Modulus of rigidity (*i.e.,* modulus of elasticity of shape)

1. Young's modulus of elasticity (Y)

When a wire is acted upon by two equal and opposite forces in the direction of its length, the length of the body is changed. The change in length per unit length $\left(\dfrac{\Delta l}{l}\right)$ is called the longitudinal strain and the restoring force (which is equal to the applied force in equilibrium) per unit area of cross-section of the wire is called the longitudinal stress.

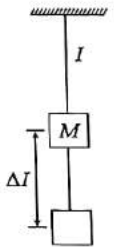


Fig. 12.5

For small change in the length of the wire, the ratio of the longitudinal stress to the corresponding strain is called the Young's modulus of elasticity (Y) of the wire.

Thus,
$$Y = \frac{\dfrac{F}{A}}{\dfrac{\Delta l}{l}} \quad \text{or} \quad Y = \frac{Fl}{A\Delta l}$$

Let there be a wire of length l and radius r. Its one end is clamped to a rigid support and a mass M is attached at the other end. Then

$$F = Mg \text{ and } A = \pi r^2$$

Substituting in above equation, we have

$$\boxed{Y = \frac{Mgl}{(\pi r^2)\Delta l}}$$

Young's modulus is only defined for solids, and not for liquids and gases. Larger the value of Y for a material, more elastic it would be. For this reason, steel is more elastic than rubber.

The material which has smaller value of Y is more ductile, *i.e.*, it offers less resistance in framing it into wire. Thus, for making wire we choose a material having less value of Y.

Note For a perfectly rigid body, value of Y is infinite.

Force constant of wire

Force required to produce unit elongation in a wire is called force constant of material of wire. It is denoted by k.

$$k = \frac{F}{\Delta l} = \frac{YA}{l} \qquad \left(\because \frac{F}{\Delta l} = \frac{YA}{l}\right)$$

If a spring is stretched or compressed by an amount Δl, the restoring force produced in it is,

$$F_s = k\,\Delta l \qquad \text{...(i)}$$

Here, k = force constant of spring.

Similarly, if a wire is stretched by an amount Δl, the restoring force produced in it is,

$$F = \left(\frac{YA}{l}\right)\Delta l \qquad \text{...(ii)}$$

Comparing Eqs. (i) and (ii), we can see that force constant of a wire is,

$$\boxed{k = \frac{YA}{l}} \qquad \text{...(iii)}$$

i.e., **a wire is just like a spring of force constant $\frac{YA}{l}$.**

So, all formulae which we use in case of a spring can be applied to a wire also.

Example 12.8 *If a wire of length 4 m and cross-sectional area of 2 m^2 is stretched by a force of 3 kN, then determine the change in length due to this force. Given, Young's modulus of material of wire is 110×10^9 Nm^{-2}.*

Sol. Given, area of cross-section, $A = 2\text{m}^2$

Applied force, $F = 3\text{ kN} = 3 \times 10^3$ N, Length, $L = 4$ m

Young's modulus, $Y = 110 \times 10^9\ \text{Nm}^{-2}$

$$Y = \frac{FL}{A\Delta L}$$

$\therefore$ Change in length,

$$\Rightarrow \quad \Delta L = \frac{FL}{AY} = \frac{3 \times 10^3 \times 4}{2 \times 110 \times 10^9} = 0.0545 \times 10^{-6}\ \text{m}$$

$$\Delta L = 54.5 \times 10^{-6}\ \text{mm}$$

Example 12.9 *A steel wire of length 4 m and diameter 5 mm is stretched by 5 kg-wt. Find the increase in its length, if the Young's modulus of steel is 2×10^{12} dyne cm^{-2}.*

Sol. Hence, length of the wire, $l = 4$ m $= 400$ cm,

Diameter of wire, $2r = 5$ mm

or radius of wire, $r = 2.5$ mm $= 0.25$ cm

Force, $F = 5$ kg-wt $= 5000$ g-wt

$= 5000 \times 980$ dyne

Young's modulus, $Y = 2.4 \times 10^{12}$ dyne cm^{-2}

As we have, $Y = \frac{F}{\pi r^2} \times \frac{l}{\Delta l}$

Increase in length, $\Delta l = \frac{Fl}{\pi r^2 Y}$

$$= \frac{(5000 \times 980) \times 400}{(22/7) \times (0.25)^2 \times 2.4 \times 10^{12}} = 0.0041\ \text{cm}$$

Example 12.10 *Determine the elongation of the steel bar 1 m long and 1.5 cm^2 cross-sectional area when subjected to a pull of 1.5×10^4 N. (Take, $Y = 2 \times 10^{11}\ Nm^{-2}$).*

Sol. Young's modulus, $Y = \frac{F/A}{\Delta l / l}$

$\therefore$ Elongation of the steel wire, $\Delta l = \frac{Fl}{AY}$

$$\Delta l = \frac{(1.5 \times 10^4)(1)}{(1.5 \times 10^{-4})(2 \times 10^{11})} = 0.5 \times 10^{-3}\ \text{m}$$

or $\quad \Delta l = 0.5$ mm

Example 12.11 *A cable is replaced by another one of same length and material but twice the diameter. How will this effect the elongation under a given load? How does this effect the maximum load it can support without exceeding the elastic limit?*

Sol. Young's modulus,

$$Y = \frac{Mgl}{\pi r^2 \cdot \Delta l} = \frac{Mgl}{\pi\left(\frac{D}{2}\right)^2 \Delta l} = \frac{4Mgl}{\pi D^2 \cdot \Delta l}$$

where, D is the diameter of the wire.

$\therefore$ Elongation, $\Delta l = \frac{4Mgl}{\pi D^2 \cdot Y}$ *i.e.* $\Delta l \propto \frac{1}{D^2}$

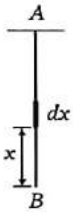
Clearly, if the diameter is doubled, the elongation will become one-fourth.

Also, maximum stress, $\sigma_m = \frac{M_m g}{(\pi D^2/4)}$ or $M_m \propto D^2$

Clearly, if the diameter is doubled, the wire can support four times the original load.

Example 12.12 *A bar of mass m and length l is hanging from point A as shown in figure. Find the increase in its length due to its own weight. The Young's modulus of elasticity of the wire is Y and area of cross-section of the wire is A.*

Sol. Consider a small section dx of the bar at a distance x from B. The weight of the bar for a length x is,



$$w = \left(\frac{mg}{l}\right)x$$

Elongation in section dx will be, $dl = \left(\frac{w}{AY}\right)dx = \left(\frac{mg}{lAY}\right)x\,dx$

Total elongation in the bar can be obtained by integrating this expression for $x = 0$ to $x = l$.

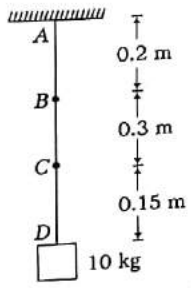
$$\therefore \quad \Delta l = \int_{x=0}^{x=l} dl = \left(\frac{mg}{lAY}\right)\int_0^l x\,dx$$

or Increase in length, $\Delta l = \frac{mgl}{2AY}$

Example 12.13 *A rod AD consisting of three segments AB, BC and CD joined together is hanging vertically from a fixed support at A. The lengths of the segments are respectively 0.2m, 0.3 m and 0.15 m. The cross-section of the rod is uniformly 10^{-4} m^2. A weight of 10 kg is hung from D. Calculate the displacements of points B, C and D if $Y_{AB} = 3.5 \times 10^{10}\,Nm^{-2}$, $Y_{BC} = 5 \times 10^{10}$, $Y_{CD} = 2 \times 10^{10}\,Nm^{-2}$. (Neglect the weight of the rod)*

Sol. Given, area, $A = 10^{-14}\ m^2$

$$Y_{AB} = 3.5 \times 10^{10}\,Nm^{-2}$$
$$Y_{BC} = 5 \times 10^{10}\,Nm^{-2}$$
$$Y_{CD} = 2 \times 10^{10}\,Nm^{-2}$$



Increase in length,

$$\Delta L = \frac{FL}{AY} = \frac{MgL}{AY} = \frac{10 \times 10L}{10^{-4}Y} = 10^6\frac{L}{Y}$$

Now, increase in length of AB segment

$$(\Delta L_{AB}) = 10^6 \times \frac{L_{AB}}{Y_{AB}} = 10^6 \times \frac{0.2}{3.5 \times 10^{10}} = 5.7 \times 10^{-6}\text{m}$$

Increase in length of BC segment

$$(\Delta L)_{BC} = 10^6 \times \frac{L_{BC}}{Y_{BC}}$$
$$= 10^6 \times \frac{0.3}{5 \times 10^{10}} = 6 \times 10^{-6}\text{m}$$

Increase in length of CD segment

$$(\Delta L)_{CD} = 10^6\frac{L_{CD}}{Y_{CD}} = 10^6 \times \frac{0.15}{2 \times 10^{10}} = 7.5 \times 10^{-6}\text{ m}$$

So, Displacement of $B = \Delta L_{AB} = 5.7 \times 10^{-6}$ m

Displacement of $C = \Delta L_{AB} + \Delta L_{BC} = 11.7 \times 10^{-6}$m

Displacement of $D = \Delta L_{AB} + \Delta L_{BC} + \Delta L_{CD} = 26.7 \times 10^{-6}$ m

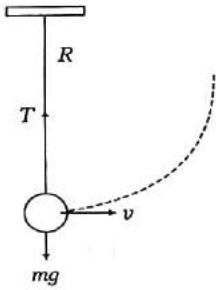
Example 12.14 *A block of weight 10 N is fastened to one end of wire of cross-sectional area 4 mm^2 and is rotated in a vertical circle of radius 30 cm. The speed of the block at the bottom of the circle is 3 ms^{-1}. Find the elongation of the wire when the block is at the bottom of the circle. Young's modulus of the material of the wire $= 2 \times 10^{11}\ Nm^{-2}$. ($g = 10\ ms^{-2}$)*

Sol. Given, mass $m = 100$ N

Area $A = 4\ \text{mm}^2$

Radius $R = 30$ cm

Speed of block $v = 3\ \text{ms}^{-1}$



Tension of the wire

$$T - mg = \frac{mv^2}{R} \quad \text{or} \quad T = mg + \frac{mv^2}{R} = 100 + \frac{10 \times 3^2}{0.3} = 400\text{N}$$

The elongation of the wire when the block is at the bottom of the circle

$$Y = \frac{FR}{A\Delta L}$$

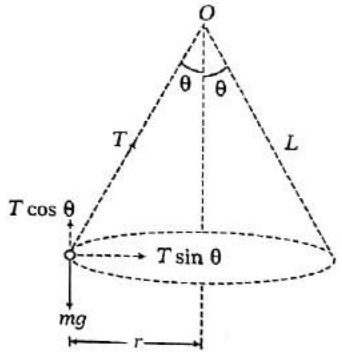
Here force (F) is equal to the tension in the wire.

So, $$Y = \frac{TR}{A\Delta L}$$

$$\Delta L = \frac{TR}{AY} = \frac{400 \times 0.3}{4 \times 10^{-6} \times 2 \times 10^{11}} = 1.5 \times 10^{-4}\text{ m}$$

Example 12.15 *A 2 kg mass is suspended by a rubber cord 2 m long and of cross-section 0.5 cm. It is made to describe a horizontal circle of radius 50 cm in 4 times a second. Find the extension of the cord. (Young's modulus $Y = 7 \times 10^8\ Nm^{-2}$)*

Sol. Given, $m = 2$ kg, $\omega = 2\pi f = 2\pi \times 4 = 8\pi$ rad s^{-1}, $r = 0.5$ m



From the figure

$$T\cos\theta = mg \quad \text{and} \quad T\sin\theta = \frac{mv^2}{r} = m\omega^2$$

Tension in cord

$$T = \sqrt{(T\sin\theta)^2 + (T\cos\theta)^2}$$
$$= \sqrt{(m\omega^2 r)^2 + (mg)^2}$$
$$T = \sqrt{[2 \times (8\pi)^2 \times 0.5]^2 + [2 \times 10]^2} = 631\text{ N}$$

The extension of the cord,

$$\Delta L = \frac{TL}{AY} = \frac{631 \times 2}{0.5 \times 10^{-4} \times 7 \times 10^8}$$
$$= 360 \times 10^{-4}\text{ m} = 3.60\text{ cm}$$

Example 12.16 *A light rod of length 2 m is suspended from the ceiling horizontally by means of two vertical wires of equal length tied to its ends. One of the wires is made of steel and is of cross-section $10^{-3}\ m^2$ and the other is of brass of cross-section $2 \times 10^{-3} m^2$. Find out the position along the rod at which a weight may be hang to produce,*

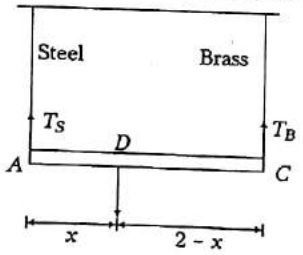
(i) equal stresses in both wires,

(ii) equal strains on both wires.

Young's modulus for steel is $2 \times 10^{11} Nm^{-2}$ and for brass is $10^{11} Nm^{-2}$.

Sol. (i) Given,

Stress in steel = stress in brass



$$\therefore \quad \frac{T_S}{A_S} = \frac{T_B}{A_B} \Rightarrow \frac{T_S}{T_B} = \frac{A_S}{A_B} = \frac{10^{-3}}{2 \times 10^{-3}} = \frac{1}{2} \quad \text{...(i)}$$

As the system is in equilibrium, taking moments about D, we have

$$T_S \cdot x = T_B\,(2 - x)$$
$$\therefore \quad \frac{T_S}{T_B} = \frac{2 - x}{x} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get, $x = 1.33$ m

(ii) Strain $= \dfrac{\text{Stress}}{Y}$

Given, strain in steel = strain in brass

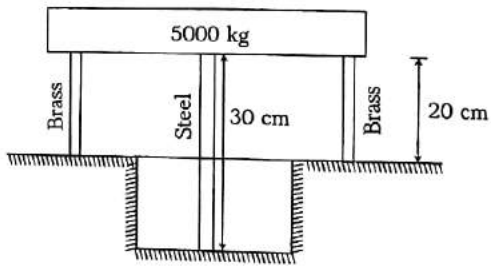
$$\therefore \quad \frac{T_S/A_S}{Y_S} = \frac{T_B/A_B}{Y_B}$$

$$\therefore \quad \frac{T_S}{T_B} = \frac{A_S Y_S}{A_B Y_B} = \frac{(1 \times 10^{-3})(2 \times 10^{11})}{(2 \times 10^{-3})(10^{11})} = 1 \quad \text{...(iii)}$$

From Eqs. (ii) and (iii), we have

$$x = 1\text{ m}$$

Example 12.17 *A steel rod of cross-sectional area 16 cm^2 and two brass rods each of cross-sectional area 10 cm^2 together support a load of 5000 kg as shown in figure. Find the stress in the rods. Take Young's modulus for steel $= 2 \times 10^6$ kg cm^{-2} and for brass $= 1 \times 10^6$ kg cm^{-2}.*



Sol. Given, area of steel rod, $A_S = 16\text{ cm}^2$

Area of two brass rods, $A_B = 2 \times 10 = 20\text{ cm}^2$

Load, $F = 5000$ kg

Young's modulus for the steel, $Y_S = 2 \times 10^6$ kg cm^{-2}

Young's modulus for the brass, $Y_B = 1 \times 10^6$ kg cm^{-2}

Length of the steel rod, $l_S = 30$ cm

Length of the brass rod, $l_B = 20$ cm

Let σ_S = stress in steel and σ_B = stress in brass

Decrease in length of the steel rod

= decrease in length of the brass rod

or $$\frac{\sigma_S}{Y_S} \times l_S = \frac{\sigma_B}{Y_B} \times l_B$$

or $$\sigma_S = \frac{Y_S}{Y_B} \times \frac{l_B}{l_S} \times \sigma_B = \frac{2 \times 10^6}{1 \times 10^6} \times \frac{20}{30} \times \sigma_B$$

$$\therefore \quad \sigma_S = \frac{4}{3}\sigma_B \quad \text{...(i)}$$

Now, using the relation,

$$F = \sigma_S A_S + \sigma_B A_B \quad \text{or} \quad 5000 = \sigma_S \times 16 + \sigma_B \times 20 \quad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get

$$\sigma_B = 120.9\text{ kg cm}^{-2}$$

2. Bulk modulus of elasticity (B)

When a uniform pressure (normal force) is applied all over the surface of a body, the volume of the body changes. The change in volume per unit volume of the body is called the 'volume strain' and the normal force acting per unit area of the surface (pressure) is called the normal stress or volume stress. For small strains, the ratio of the volume stress to the volume strain is called the '**Bulk modulus**' of the material of the body. It is denoted by B.

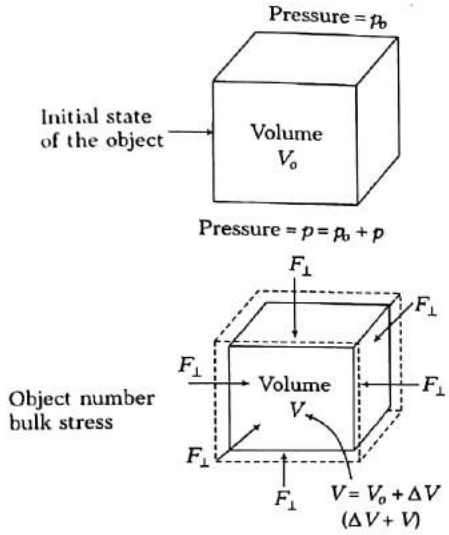


Fig. 12.6

Consider a solid cube body of volume V and surface area A. In order to compress the body. Let a force F be applied normally on the entire surface of the body and its volume decreases by ΔV then volumetric strain $= -\dfrac{\Delta V}{V}$ here negative sign shows the volume is decreasing when force is applied normal stress $= F/A$

$$\therefore \quad B = \frac{F/A}{-\Delta V/V}$$

Then,
$$\boxed{B = \frac{-p}{\Delta V/V}}$$

Here, negative sign implies that when the pressure increases volume decreases and *vice-versa*.

The SI unit of bulk modulus is $\mathrm{Nm^{-2}}$ and its dimensions are $[\mathrm{ML^{-1}T^{-2}}]$. The relation between density ρ, pressure Δp and bulk modulus (B) is given by

$$\boxed{\rho' = \rho\left(1 + \frac{\Delta p}{B}\right)}$$

Note For a perfectly rigid body, $B = \infty$

In case of gas The isothermal bulk modulus of the gas (B_T) is equal to the pressure of the gas at that instant.

$$B_T = p$$

The adiabatic bulk modulus of gas (B_s) is equal to γ times the pressure of the gas at that instant time $B_s = \gamma p$

Example 12.18 *How much should the pressure on a litre of water be changed to compress it by 0.10%? Bulk modulus of water* $= 2.2 \times 10^9$ *Pa*

Sol. Bulk modulus, $B = -\dfrac{p}{\dfrac{\Delta V}{V}}$

Here, $\dfrac{\Delta V}{V} = 0.10\% = -\dfrac{0.10}{100} = -1 \times 10^{-3}$

Change in pressure, $p = B\left(-\dfrac{\Delta V}{V}\right)$

$$= (2.2 \times 10^9\ \mathrm{Pa}) \times 1 \times 10^{-3} = 2.2 \times 10^6\ \mathrm{Pa}$$

Compressibility

The reciprocal of the bulk modulus of the material of a body is called the compressibility of that material. Thus,

Compressibility $= \dfrac{1}{B} = \dfrac{\Delta V}{pV} = k$. The SI unit of compressibility is $\mathrm{N^{-1}m^2}$ and its dimensions are $[\mathrm{M^{-1}LT^2}]$.

The bulk modulus is defined for all solids, liquids and gases. The value of B for solids are much larger than for liquids, and that for liquids are much larger than for gases. Thus, solids are least compressible while gases are most compressible.

Example 12.19 *What will be the decrease in volume of* $100\ \mathrm{cm^3}$ *of water under pressure of* 100 *atm if the compressibility of water is* 4×10^{-5} *per unit atmospheric pressure?*

Sol. Bulk modulus (B) $= \dfrac{1}{\text{Compressibility}} = \dfrac{1}{k}$

$$= \frac{1}{4 \times 10^{-5}} = 0.25 \times 10^5\ \mathrm{atm}$$
$$= 0.25 \times 10^5 \times 1.013 \times 10^5\ \mathrm{Nm^{-2}}$$
$$= 2.533 \times 10^9\ \mathrm{Nm^{-2}}$$

Volume, $V = 100\ \mathrm{cm^3} = 10^{-4}\ \mathrm{m^3}$

Pressure, $p = 100\ \mathrm{atm} = 100 \times 1.013 \times 10^5\ \mathrm{Nm^{-2}}$

$$= 1.013 \times 10^7\ \mathrm{Nm^{-2}}$$

Now, apply $\dfrac{1}{B} = k = \dfrac{\Delta V}{pV}$

$$\therefore \quad \Delta V = \frac{pV}{B} = \frac{1.013 \times 10^7 \times 10^{-4}}{2.533 \times 10^9}$$

Decrease in volume,

$$\Delta V = 0.4 \times 10^{-6}\ \mathrm{m^3} = 0.4\ \mathrm{cm^3}$$

Example 12.20 *Find the decrease in the volume of a sample of water from the following data. Initial volume* $= 2000\ cm^2$, *initial pressure* $= 10^5\ Nm^{-2}$, *final pressure* $= 10^6\ Nm^{-2}$, *compressibility of water* $= 50 \times 10^{-11}\ N^{-1}\ m^2$.

Sol. Bulk modulus

$$B = \frac{1}{\text{Compressibility}} = \frac{1}{25 \times 10^{-11}} = 4 \times 10^9 \text{ and } B = \frac{-\Delta p}{\Delta V / V}$$

∴ Decrease in the volume

$$\Delta V = -\frac{\Delta p V}{B} = \frac{(10^6 - 10^5)(2000 \times 10^{-6})}{4 \times 10^9}$$

$$= 4.5 \times 10^{-7} m^3 = 4.5 \times 10^{-7}\ 10^6\ cm^3 = 0.45\ cm^3$$

Example 12.21 *What is the density of lead under a pressure of* $2 \times 10^8\ Nm^{-2}$, *if the bulk modulus of lead is* $8 \times 10^9 Nm^{-2}$ *and initially the density of lead is* $11.4\ g\ cm^{-3}$?

Sol. The changed density, $\rho' = \dfrac{\rho}{1 - \dfrac{dp}{B}}$

Substituting the value, we have

$$\rho' = \frac{11.4}{1 - \dfrac{2 \times 10^8}{8 \times 10^9}} \quad \text{or} \quad \rho' = 11.69\ g\ cm^{-3}$$

Example 12.22 *The bulk modulus of water is* $2.3 \times 10^9\ Nm^{-2}$.

(i) Find its compressibility.

(ii) How much pressure in atmospheres is needed to compress a sample of water by 0.1%.

Sol. Here, $B = 2.3 \times 10^9\ Nm^{-2} = \dfrac{2.3 \times 10^9}{1.01 \times 10^5} = 2.27 \times 10^4$ atm

(i) Compressibility $= \dfrac{1}{B} = \dfrac{1}{2.27 \times 10^4} = 4.4 \times 10^{-5}\ atm^{-1}$

(ii) Here, $\dfrac{\Delta V}{V} = -0.1\% = -0.001$

Required increase in pressure,

$$\Delta p = B \times \left(-\frac{\Delta V}{V}\right) = 2.27 \times 10^4 \times 0.001 = 22.7 \text{ atm}$$

3. Modulus of rigidity (η)

When a body is acted upon by an external force tangential to a surface of the body, the opposite surface being kept fixed, it suffers a change in shape but the its volume remains unchanged. Then, the body is said to be sheared.

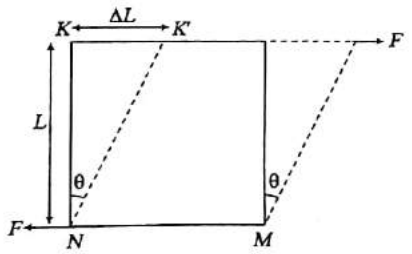


Fig. 12.7

The ratio of the displacement of a layer in the direction of the tangential force and the distance of the layer from the fixed surface is called the shearing strain and the tangential force acting per unit area of the surface is called the **shearing stress.**

For small strain, the ratio of the shearing stress to the shearing strain is called the **modulus of rigidity** of the material of the body. It is denoted by η.

Thus, $$\eta = \frac{F/A}{KK'/KN}$$

Here, $$\frac{KK'}{KN} = \tan\theta \approx \theta$$

∴ $$\eta = \frac{F/A}{\theta}$$

or $$\boxed{\eta = \frac{F}{A\theta}} \quad \text{...(i)}$$

Since, $\theta = \tan\theta = \dfrac{KK'}{KN} = \dfrac{\Delta L}{L}$

Therefore, Eq. (i) will become,

$$\boxed{\eta = \frac{F}{A} \cdot \frac{L}{\Delta L}}$$

The SI unit of modulus of rigidity is Nm^{-2} and its dimensions are $[ML^{-1}T^{-2}]$.

Shear modulus (or modulus of rigidity) is generally less than Young's modulus (from table). For most material $\eta \approx Y/3$. Shear modulus is defined for solids only.

A solid will have all the three modulii of elasticity Y, B and η. But in case of liquid or a gas only B can be defined as liquid or gas cannot be framed into a wire or no shear force can be applied on them.

Example 12.23 *A 4 cm cube has its upper face displaced by 0.1 mm by a tangential force of 8 kN. Calculate the shear modulus of the cube.*

Sol. Here, each side of the cube $L = 4$ cm

Area of the face over which the force is applied,

$$A = L^2 = 16 \text{ sq. cm}$$

Displacement, $\Delta L = 0.1$ mm $= 0.01$ cm

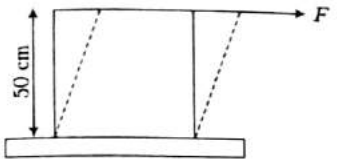
Force applied, $F = 8$ kN $= 8000 \times 10^5 = 8 \times 10^8$ dyne

As, $$\eta = \frac{FL}{A\Delta L}$$

Shear modulus of the cube,

$$\eta = \frac{8 \times 10^8 \times 4}{16 \times 0.01} = 2 \times 10^{10} \text{ dyne/sq. cm}$$

Example 12.24 *A square lead slab of side 50 cm and thickness 100 cm is subjected to a shearing force (on its narrow face) of magnitude 9.0×10^4 N. The lower edge is riveted to the floor as shown in figure. How much is the upper edge displaced, if the shear modulus of lead is 5.6×10^8 Pa?*



Sol. Here, $L = 50 \text{ cm} = 50 \times 10^{-2}\text{m}$,

$$\eta = 5.6 \times 10^9 \text{ Pa}, F = 9.0 \times 10^4 \text{ N}$$

Area of the face on which force is applied,

$$A = 50 \times 10 = 500 \text{ sq. cm} = 0.05 \text{ m}^2$$

If ΔL is the displacement of the upper edge of the slab due to tangential force F applied, then

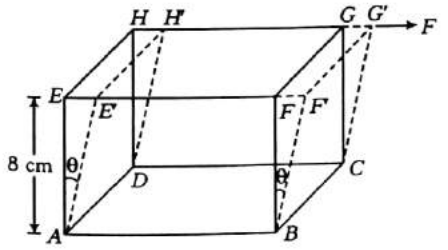
$$\eta = \frac{Fla}{\Delta L/L}$$

or $$\frac{FL}{\eta A} = \frac{9 \times 10^4 \times 50 \times 10^{-2}}{5.6 \times 10^9 \times 0.05}$$

$\Rightarrow$ $$\Delta L = 1.6 \times 10^{-4} \text{ m}$$

Example 12.25 *Consider an Indian rubber cube having modulus of rigidity of 2×10^7 dyne cm^{-2} and of side 8 cm. If one side of the rubber is fixed, while a tangential force equal to the weight of 300 kg is applied to the opposite face, then find out the shearing strain produced and distance through which the strain side moves.*

Sol. Given, modulus of rigidity, $\eta = 2 \times 10^7$ dyne cm^{-2}



Side of the cube, $l = 8$ cm

Area, $A = l^2 = 64 \text{ cm}^2$

Force or load, $F = 300 \text{ kgf} = 300 \times 1000 \times 981$ dyne

Modulus of rigidity, $\eta = \dfrac{F}{A\theta} \Rightarrow \theta = \dfrac{F}{A\eta}$

Shearing strain, $\theta = \dfrac{300 \times 1000 \times 981}{64 \times 2 \times 10^7} = 0.23$ rad

As, $$\eta = \frac{F}{A}\frac{l}{\Delta l} \Rightarrow \frac{\Delta l}{l} = \theta$$

Distance through which the strain side moves

$\Rightarrow$ $$\Delta l = l\theta = 7 \times 0.23; \Delta l = 1.84 \text{ cm}$$

Factors affecting elasticity

(i) **Effect of temperature** Almost for all materials, the modulus of elasticity decreases with the rise in temperature, but the elasticity of invar remains unchanged with the change in temperature.

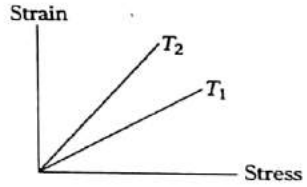
(ii) **Effect of impurities** The addition of impurities affects the elastic properties depending on whether impurities are themselves more or less elastic. When carbon is added to iron and potassium to gold, their elasticities are strengthened.

(iii) **Effect of annealing** By annealing (*i.e.*, heating and the cooling gradually) large crystal grains are formed, and hence the elasticity of the material decreases.

(iv) **Effect of hammering and rolling** By hammering and rolling, crystal grains break up into smaller units, and hence the elasticity of the material increases.

Example 12.26 *The stress-strain graph for a metallic wire is shown at two different temperatures, T_1 and T_2 which temperature is high T_1 or T_2 ?*

Sol. The slope of stress-strain curve with strain at X-axis gives the value of Young's modulus.



In the above graph strain is taken along y-axis. Therefore, the slope of graph at temperature T_1 is less than the slope of graph at temperature T_2.

Now as we know with increase in temperature the value of modulus of elasticity decreases. Therefore temperature T_1 is less than temperature T_2.

Check point 12.2

1. Write Copper, Steel, Glass and Rubber in order at increasing coefficient of elasticity
   (a) Steel, Rubber, Copper, Glass
   (b) Rubber, Copper, Glass, Steel
   (c) Rubber, Glass, Steel, Copper
   (d) Rubber, Glass, Copper, Steel

2. The Young's modulus of the material of a wire is equal to the
   (a) stress required to increase its length four times
   (b) stress required to produce unit strain
   (c) strain produced in it
   (d) half the strain produced in it

3. Which one of the following statements is wrong?
   (a) Young's modulus for a perfectly rigid body is zero
   (b) Bulk modulus is relevant for solids, liquids and gases
   (c) Rubber is less elastic than steel
   (d) The Young's modulus and shear modulus are relevant for solids
   (e) The stretching of a coil spring is determined by its shear modulus

4. Two wires are made of the same material and have the same volume. However wire 1 has cross-sectional area A and wire 2 has cross-sectional area $3A$. If the length of wire 1 increases by Δx on applying force F, how much force is needed to stretch wire 2 by the same amount
   (a) F (b) $4F$
   (c) $6F$ (d) $9F$

5. A rubber cord 10 m is suspended vertically. How much does it stretch under its own weight (density of rubber is $1500\ \text{kg m}^{-3}$, $Y = 5\times10^{8}\ \text{Nm}^{-2}$, $g = 10\ \text{ms}^{-2}$)
   (a) 15×10^{-4} m
   (b) 7.5×10^{4} m
   (c) 12×10^{-4} m
   (d) 25×10^{-4} m

6. Two similar wires under the same load yield elongation of 0.1 mm and 0.05 mm, respectively. If the area of cross-section of the first wire is $4\ \text{mm}^2$, then the area of cross-section of the second wire is
   (a) $6\ \text{mm}^2$ (b) $8\ \text{mm}^2$
   (c) $10\ \text{mm}^2$ (d) $12\ \text{mm}^2$

7. A 5 m long aluminium wire ($Y = 7\times10^{10}\ \text{Nm}^{-2}$) of diameter 3 mm supports a 40 kg mass. In order to have the same elongation in a copper wire ($Y = 12\times10^{10}\ \text{Nm}^{-2}$) of the same length under the same weight, the diameter should now be in mm.
   (a) 1.75 (b) 1.5
   (c) 2.5 (d) 5.0

8. Choose the wrong statement.
   (a) The bulk modulus for solids is much larger than for liquids.
   (b) Gases are least compressible.
   (c) The incompressibility of the solids is due to the tight coupling.
   (d) The reciprocal of the bulk modulus is called compressibility.

9. The bulk modulus of an ideal gas at constant temperature
   (a) is equal to its volume V
   (b) is equal to $p/2$
   (c) is equal to its pressure p
   (d) Cannot be determined

10. A uniform cube is subjected to volume compression. If each side is decreased by 1%, then bulk strain is
    (a) 0.01 (b) 0.06
    (c) 0.02 (d) 0.03

11. The average depth of Indian ocean is about 3000 m. The fractional compression, $\frac{\Delta V}{V}$ of water at the bottom the ocean (Given that the bulk modulus of the water $= 2.2\times10^{9}\ \text{Nm}^{-2}$ and $g = 10\ \text{ms}^{-2}$ is
    (a) 0.82% (b) 0.91%
    (c) 1.36% (d) 1.24%

12. To what depth below the surface of sea should a rubber ball be taken as to decrease its volume by 0.1%. (Take, density of sea water $= 1000\ \text{kgm}^{-3}$, Bulk modulus of rubber $= 9\times10^{8}\ \text{Nm}^{-2}$, acceleration due to gravity $= 10\ \text{ms}^{-2}$).
    (a) 9 m (b) 18 m
    (c) 180 m (d) 90 m

13. A 2 m long rod of radius 1 cm which is fixed from one end is given a twist of 0.8 radius. The shear strain developed will be
    (a) 0.002 (b) 0.004
    (c) 0.008 (d) 0.016

14. Modulus of rigidity of a liquid
    (a) Non-zero constant (b) Infinite
    (c) Zero (d) Cannot be predicted

15. Mark the wrong statement
    (a) Sliding of molecular layer is much easier than compression or expansion.
    (b) Reciprocal of bulk modulus of elasticity is called compressibility.
    (c) It is difficult to twist a long rod as compared to small rod.
    (d) Hollow shaft is much stronger than a solid rod of same length and same mass.

3 Poisson's ratio

When a wire is loaded, its length increases but its diameter decreases. The strain produced in the direction of applied force is called **longitudinal strain** and strain produced in the perpendicular direction is called lateral strain. Within the elastic limit, the ratio of lateral strain to the longitudinal strain is called **Poisson's ratio**. It is represented by (σ).

Let the length of the loaded wire increases from L to $L + \Delta L$ and its diameter decreases from D to $D - \Delta D$.

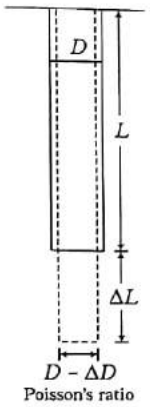


Fig. 12.8

Longitudinal strain $= \dfrac{\Delta L}{L}$. Lateral strain $= \dfrac{-\Delta D}{D}$

Poisson's ratio is $\sigma = \dfrac{\text{Lateral strain}}{\text{Longitudinal strain}} = \dfrac{-\Delta D / D}{\Delta L / L}$

$$\boxed{\text{Poisson's ratio, } \sigma = \frac{-L}{D} \cdot \frac{\Delta D}{\Delta L}}$$

The negative sign indicates that longitudinal and lateral strain are in opposite sense, it has no units and dimensions.

$$\begin{cases} -1 < \sigma < 0.5 & \text{for theoretical purpose} \\ 0 < \sigma < 0.5 & \text{for practical purpose} \end{cases}$$

Relation between γ, β and η and σ

There are four relations between Y, B, η and σ, which can be given as follows :

(i) $B = \dfrac{Y}{3(1 - 2\sigma)}$ (ii) $\eta = \dfrac{Y}{(1 + \sigma)2}$

(iii) $\sigma = \dfrac{3B - 2\eta}{2\eta + 6B}$ (iv) $\dfrac{9}{Y} = \dfrac{1}{B} + \dfrac{3}{\eta}$

Example 12.27 *Determine the poisson's ratio of the material of a wire whose volume remains constant under an external normal stress.*

Sol. Volume of a wire, $V = \dfrac{\pi D^2}{4} l$

As volume remains constant, the differentiation of above equation gives

$$0 = \frac{\pi l}{4} \cdot 2D \cdot dD + \frac{\pi D^2}{4} \cdot dl$$

$$-2ldD = Ddl$$

$$\Rightarrow \quad \frac{dD}{D} = -\frac{1}{2}\frac{dl}{l}$$

$\because$ By definition, Poisson's ratio

$$\sigma = \frac{-dD / D}{dl / l} = \frac{-\frac{1}{2}\frac{dl}{l}}{\frac{dl}{l}} = \frac{1}{2} = 0.5$$

Example 12.28 *A material is having Poisson's ratio 0.2. A load is applied on it due to which it suffers the longitudinal strain 3×10^{-3}, then find out the percentage change in its volume.*

Sol. Given, Poisson's ratio, $\sigma = 0.2$

Longitudinal strain, $\dfrac{\Delta L}{L} = 3 \times 10^{-3}$

$\because$ Poisson's ratio, $\sigma = \dfrac{-\Delta D / D}{\Delta L / L}$

$$\Rightarrow \quad \sigma = \frac{-\Delta R / R}{\Delta L / L}$$

$$\Rightarrow \quad \frac{\Delta R}{R} = -\sigma \times \frac{\Delta L}{L} = -0.6 \times 10^{-3}$$

$\because$ Volume, $V = \pi R^2 L$

Therefore, percentage change in volume,

$$\left(\frac{\Delta V}{V} \times 100\right) = \left(\frac{2\Delta R}{R} + \frac{\Delta L}{L}\right) \times 100$$

$$= [2 \times (-0.6 \times 10^{-3}) + 3 \times 10^{-3}] \times 100$$

$$= 0.18\%$$

Stress-strain curve

When a wire is stretched by a load as in Fig. (*a*), it is seen that for small value of load the extension produced in the wire is proportional to the load.

Hence, stress $\propto$ strain

(*i*) When a graph between stress and strain is plotted, then the stress is found to be proportional to strain (% elongation) up to point A. Thus, Hooke's law is fully obeyed in this region, the point A is known as point of **proportional limit**.

(*ii*) When stres increased beyond *A*, then for small stress, there large strain in the wire upto point *B*.

(*iii*) When the loa gradually removed between points *O* to *B*, the wi turn to its original length.

The wire regain s original dimension only when load applied is less than or equal to certain limit. This limit is called **elastic limit**. The point *B* on stress-strain curve is known as **elastic limit** or **yield point.** The material of the wire in the region *OB* shows the elastic behaviour hence known as **elastic region.**

(*iv*) If the stress or load increases beyond point *B*, the strain further increases. This increase in strain represented by *BC* part of the curve. Now, if the load is removed, the wire does not regain its original length. But the increase in the length of the wire is permanent.

In other words, there is permanent strain equal *OO'* in the wire even when the stress is zero. This permanent strain in the wire is known as permanent set.

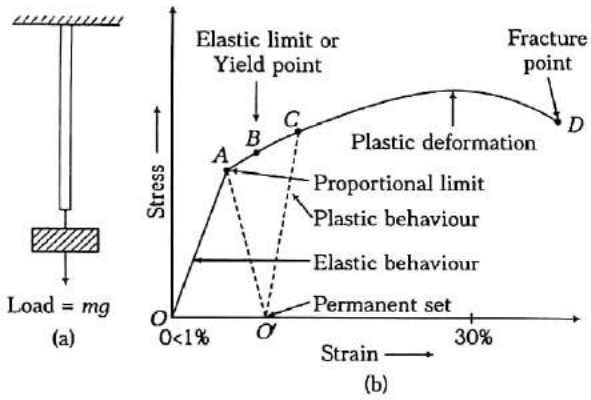


Fig. 12.9 A typical stress-strain curve

(*v*) Now, as the stress beyond *C* is increased, there is large strain in the wire. This large increase in the strain for small stress is represented by *CD* part of the curve. The wire breaks at point *D* which is also known as **fracture point.**

The material of the wire from point *C* to point *D* shows the **plastic behaviour** or **plastic deformation**. The stress needed to cause the actual fracture of the material is known as **breaking stress** or **ultimate tensile strength.**

Note

1. Elastic limit and limit of proportionality are very close to each other so that Hooke's law is nearly applicable upto elastic limit.
2. In the yield region, strains 15 to 20 times those that takes place upto the proportional limit occur during yielding.

Breaking Stress of Some Materials

Material	Breaking stress (in Nm^{-2})
Aluminium	2.2×10^8
Iron	3.0×10^8
Brass	4.7×10^8
Phosphor bronze	5.6×10^8
Steel	5 to 20×10^8
Glass	10×10^8

Different nature of the stress-strain curve

The stress-strain curve differ considerably for different materials. The steeper curve indicates a stiffer material, line parallel to the strain axis indicates a ductile material and in the absence of yield point or plastic behaviour refer to a brittle material.

Ductile and brittle materials

On the basis of elastic and plastic properties, materials can be classified in two ways ductile materials and brittle materials.

The materials which have large plastic range of extension are called **ductile materials.** As shown in the stress-strain curve, the fracture point is widely separated from the elastic limit. Such materials undergo an irreversible increase in length before snapping. So, they can be drawn into thin wires *e.g.*, copper, silver, iron, aluminium, etc.

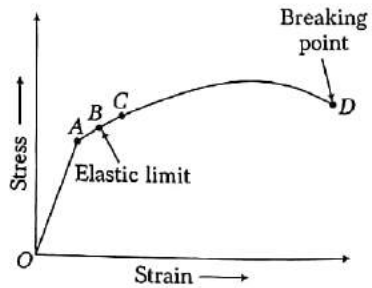


Fig. 12.10 Stress-strain curve for a ductile material

The materials which have very small range of plastic extension are called **brittle materials**. Such materials break as soon as the stress is increased beyond the elastic limit.

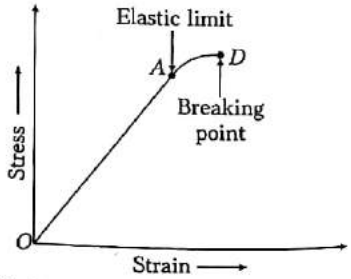


Fig. 12.11 Stress-strain curve for a brittle material

Their breaking point lies just close to their elastic limit, as shown in figure *e.g.*, cast iron, glass, ceramics etc.

Malleability

When a solid is compressed, a stage is reached beyond which it cannot regains its original shape after the deforming force is removed.

This is the elastic limit point (A') for compression. The solid then behaves like a plastic body.

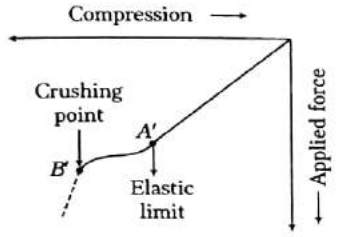


***Fig. 12.12** Load compression curve for a metal*

The yield point (B') obtained under compression is called **crushing point.**

After this stage, metals are said to be **malleable** *i.e.* they can be hammered or rolled into thin sheets. *e.g.*, gold, silver, lead etc.

Elastomers

The materials which can be elastically stretched to large values of strain are called **elastomers**. *e.g.*, rubber can be stretched to several times its original length but still it can regain its original length when the applied force is removed. There is no well defined plastic region, rubber just breaks when pulled beyond a certain limit.

Elastic region in such cases is very large, but the material does not obey Hooke's law.

In our body, the elastic tissue of aorta (the large blood vessel carrying blood from the heart) is an elastomer, for which the stress-strain curve is shown in figure.

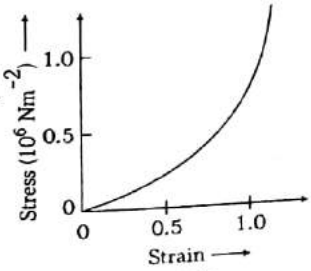


Fig. 12.13

Elastic hysteresis

As a natural consequence of the elastic after-effect, the strain in the body tends to lag behind the stress applied to the body so that during a rapidly changing stress, the strain is greater for the same value of stress. This lag of strain behind the stress is called elastic hysteresis.

Due to elastic hysteresis, the original curve (OAB) is not retraced when the deforming force is removed, although finally acquires natural length. The figure clearly indicates that the work done by the material in returning to its original shape is less than the work done by the deforming force. Hence, some amount of energy is absorbed by the material in the cycle which appears as heat.

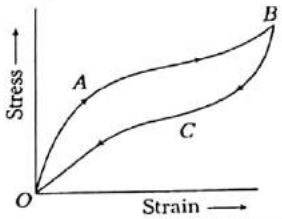
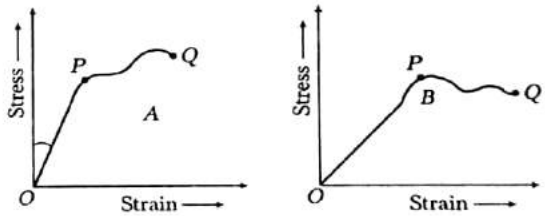


***Fig. 12.14** Vulcanised rubber*

The magnitude of the energy absorbed is proportional to the area of the loop. The material having low elastic hysteresis have low elastic relaxation time.

Example 12.29 *The stress-strain graphs for two materials A and B are shown in the figure. The graphs are drawn to the same scale.*



(a) Which material has a greater Young's modulus?

(b) Which material is more ductile?

(c) Which material is more brittle?

(d) Which of the two is the stronger material?

Sol. (a) Material A has a greater Young's modulus because the slope of the linear portion of the stress-strain curve is greater for the material A.

(b) Material A is more ductile, because it has a greater plastic region (from elastic limit P to breaking point Q) than material B.

(c) Material B is more brittle, because it has a smaller plastic region.

(d) Material A is stronger, because it can withstand a greater stress before it breaks.

Work done or potential energy stored in a stretched wire

When a wire is stretched, work is done against the interatomic forces. This work is stored in the wire in the form of elastic potential energy.

Thus, stored elastic energy is

$$U = \frac{1}{2}\text{Stress} \times \text{strain} \times \text{volume}$$

$$U = \frac{1}{2}Y(\text{Strain})^2 \times \text{volume}$$

$$\boxed{U = \frac{1}{2}\text{Load} \times \text{elongation}}$$

Example 12.30 *Calculate the work done in stretching a steel wire of Young's modulus of $2 \times 10^{11}\ Nm^{-2}$, length of 200 cm and area of cross-section is 0.06 cm^2 slowly applied without the elastic limit being reached.*

Sol. Here, force $F = mg = 40 \times 9.8$ kg,

Length of wire, $l = 200$ cm $= 2$ m

Area of cross section, $A = 0.06\ \text{cm}^2 = 0.06 \times 10^{-4}\ \text{m}^2$,

Young's modulus, $Y = 2 \times 10^{11}\ \text{Nm}^{-2}$

$$\text{Work done} = \frac{1}{2}\text{stretching force} \times \text{extension} = \frac{1}{2}F\Delta l$$

$$= \frac{1}{2}F \times \frac{Fl}{AY}$$

$$W = \frac{F^2 l}{2AY} = \frac{(40 \times 9.8)^2 \times 2}{2 \times 0.06 \times 10^{-4} \times 2 \times 10^{11}} = 0.128\ \text{J}$$

Example 12.31 *A steel wire 4 m in length is stretched through 2 mm. The cross-sectional area of the wire is 2.0 mm^2. If Young's modulus of the steel is $2 \times 10^{11}\ Nm^{-2}$. Find*

(i) the energy density of wire,

(ii) the elastic potential energy stored in the wire.

Sol. Here, length of wire, $l = 4.0$ m, $\Delta l = 2 \times 10^{-3}$ m

Area, $A = 2.0 \times 10^{-6}\ \text{m}^2$

Young's modulus, $Y = 2.0 \times 10^{11}\ \text{Nm}^{-2}$

(i) The energy density of stretched wire

$$U = \frac{1}{2} \times \text{stress} \times \text{strain} = \frac{1}{2} \times Y \times (\text{strain})^2$$

$$= \frac{1}{2} \times 2.0 \times 10^{11} \times \left[\frac{(2 \times 10^{-3})^2}{4}\right] = 0.25 \times 10^5$$

$$= 2.5 \times 10^4\ \text{Jm}^{-3}$$

(ii) Elastic potential energy = energy density × volume

$$= 2.5 \times 10^4 \times (2.0 \times 10^{-6}) \times 4.0\ \text{J}$$

$$= 20 \times 10^{-2} = 0.20\ \text{J}$$

Example 12.32 *A 45 kg boy whose leg bones are 5 cm^2 in area and 50 cm long falls through a height of 2 m without breaking his leg bones. If the bones can stand a stress of $0.9 \times 10^8\ Nm^{-2}$, then what will be the Young's modulus for the material of the bone?*

Sol. Here, mass $m = 45$ kg, height of leg bones $h = 2$ m,

Length $L = 0.50$ m, Area of bones $A = 5 \times 10^{-4}\ \text{m}^2$

Loss in gravitational energy

= gain in elastic energy in both leg bones

So, $$mgh = 2 \times \left(\frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}\right)$$

Here, volume $= AL = 5 \times 10^{-4} \times 0.50 = 2.5 \times 10^{-4}\ \text{m}^3$

$$\therefore\ 45 \times 10 \times 2 = 2 \times \left[\frac{1}{2} \times 0.9 \times 10^8 \times 2.5 \times \text{strain} \times 10^{-4}\right]$$

or $$\text{Strain} = \frac{45 \times 10 \times 2}{0.9 \times 2.5 \times 10^4} = 0.04$$

∴ Young's modulus of bones material

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{0.9 \times 10^8}{0.04} = 2.25 \times 10^9\ \text{Nm}^{-2}$$

Example 12.33 *A rubber cord has a cross-sectional area 1 mm^2 and total unstretched length 10.0 cm. It is stretched to 12.5 cm and then released to project a missile of mass 5.0 g. Taking, Young's modulus Y for rubber as $5.0 \times 10^8\ Nm^{-2}$. Calculate the velocity of projection.*

Sol. Equivalent force constant of rubber cord,

$$k = \frac{YA}{l} = \frac{(5.0 \times 10^8)(1.0 \times 10^{-6})}{(0.1)} = 5.0 \times 10^3\ \text{Nm}^{-1}$$

Now, from conservation of mechanical energy, elastic potential energy of cord = kinetic energy of missile

$$\therefore\quad \frac{1}{2}k(\Delta l)^2 = \frac{1}{2}mv^2$$

$$\therefore\quad v = \left(\sqrt{\frac{k}{m}}\right)\Delta l$$

$$= \left(\sqrt{\frac{5.0 \times 10^3}{5.0 \times 10^{-3}}}\right)(12.5 - 10.0) \times 10^{-2} = 25\ \text{ms}^{-1}$$

Note *Following assumptions have been made in this example*

(i) k has been assumed constant, even though it depends on the length (l).

(ii) The whole of the elastic potential energy is converting into kinetic energy of missile.

Example 12.34

(a) A wire 4 m long and 0.3 mm in diameter is stretched by a force of 100 N. If extension in the wire is 0.3 mm, calculate the potential energy stored in the wire.

(b) Find the work done in stretching a wire of cross-section 1 mm^2 and length 2 m through 0.1 mm. Young's modulus for the material of wire is $2.0 \times 10^{11}\ Nm^{-2}$.

Sol. (a) Energy stored

$$U = \frac{1}{2}(\text{stress})(\text{strain})(\text{volume})$$

or $$U = \frac{1}{2}\left(\frac{F}{A}\right)\left(\frac{\Delta l}{l}\right)(Al) = \frac{1}{2}F \cdot \Delta l$$

$$= \frac{1}{2}(100)(0.3 \times 10^{-3}) = 0.015 \text{ J}$$

(b) Work done = Potential energy stored

$$= \frac{1}{2}k(\Delta l)^2 = \frac{1}{2}\left(\frac{YA}{l}\right)(\Delta l)^2 \qquad \left(\text{as } k = \frac{YA}{l}\right)$$

Substituting the values, we have

$$W = \frac{1}{2}\frac{(2.0 \times 10^{11})(10^{-6})}{(2)}(0.1 \times 10^{-3})^2 = 5.0 \times 10^{-4} \text{ J}$$

Thermal stresses and strains

Whenever there is some increase or decrease in the temperature of the body, it causes the body to expand or contract. If the body is allowed to expand or contract freely, with the rise or fall in the temperature, no stresses are induced in the body. But, if the deformation of the body is prevented, some stresses are induced in the body. Such stresses are called thermal stresses or **temperature stresses.**

The corresponding strains are called **thermal strains** or **temperature strains.**

Consider a rod AB fixed at two supports as shown in figure.

Let l = length of the rod

A = area of cross-section of the rod

Y = Young's modulus of elasticity of the rod

and α = thermal coefficient of linear expansion of the rod.

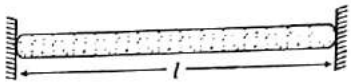


Fig. 12.15

Let the temperature of the rod be increased by an amount t. The length of the rod would had increased by an amount Δl, if it were not fixed at two supports. Here,

$$\Delta l = l\alpha t$$

But since, the rod is fixed at the supports, so a compressive strain will be produced in the rod due to which a force is applied on the supports, given

$$\boxed{F = YA\alpha\Delta t}$$

Example 12.35 *A steel wire of length 20 cm and uniform cross-section 1 mm^2 is tied rigidly at both the ends. If temperature of the wire is altered from 40°C to 20°C, calculate the change in tension. Given coefficient of linear expansion of steel is $1.1 \times 10^{-5}\,°C^{-1}$ and Young's modulus for steel is $2.0 \times 10^{11}\,Nm^{-2}$.*

Sol. The change in the wire of length l when wire is cooled by temperature Δt is given by

$$\Delta l = l\alpha\Delta t$$

or $$\Delta l/l = \alpha\Delta t$$

where, α is the coefficient of linear expansion. Change in the tension of the wire when it is cooled is given by

$$F = YA\Delta l/l = YA\alpha\Delta t$$

$$= (2.0 \times 10^{11}) \times (10^{-6}) \times (1.1 \times 10^{-5}) \times (40° - 20°)$$

$$= 44 \text{ N}$$

Check point 12.3

1. There is no change in the volume of a wire due to change in its length on stretching. The poission's ratio of the material of the wire is

(a) + 0.50 (b) – 0.50

(c) 0.25 (d) – 0.25

2. Which statement is true for a metal?

(a) $Y < \eta$ (b) $Y = \eta$

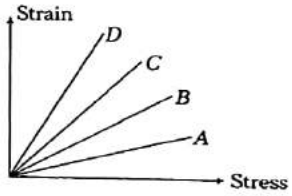
(c) $Y > \eta$ (d) $Y < 1/\eta$

3. For a given material, the Young's modulus is 2.4 times that of rigidity modulus. Its poission's ratio is

(a) 2.4 (b) 1.2

(c) 0.4 (d) 0.2

4. Stress-strain curve for four metals are shown in figure. The maximum Young modulus of elasticity is for metal



(a) *A* (b) *B* (c) *C* (d) *D*

5. Substances which can be elastically stretched to large value of strain are called

(a) Isomers (b) Isodiapheres

(c) Plastomers (d) Elastomers

6. A stretched rubber has
(a) increased kinetic energy
(b) increased potential energy
(c) decreased kinetic energy
(d) decreased potential energy

7. A metallic rod of length l and cross-sectional area A is made of a material of Young modulus Y. If the rod is elongated by an amount y, then the work done is proportional to
(a) y (b) $1/y$
(c) y^2 (d) $1/y^2$

8. The work per unit volume to stretch the length by 1% of a wire with cross-sectional area of 1 mm^2 will be ($Y = 9\times10^{11}\ Nm^{-2}$)
(a) 9×10^{11} J
(b) 4.5×10^{7} J
(c) 9×10^{7} J
(d) 4.5×10^{11} J

9. When a load of 10 kg is hung from the wire, then extension of 2 m is produced. Then work done by restoring force is
(a) 200 J (b) 100 J
(c) 50 J (d) 25 J

10. A wire suspended vertically from one of its ends is stretched by attaching a weight of 200 N to the lower end. The weight stretches the wire by 1 mm. Then the energy stored in the wire is
(a) 0.1 J (b) 0.2 J
(c) 10 J (d) 20 J

11. If a spring extends by x on loading, then the energy stored by the spring is (if T is tension in the spring and k is spring constant).
(a) $\frac{T^2}{2x}$ (b) $\frac{T^2}{2k}$
(c) $\frac{2x}{T^2}$ (d) $\frac{2T^2}{k}$

12. Wires A and B are made from the same material. A has twice the diameter and three times the length of B. If the elastic limits are not reached, when each is stretched by the same tension, the ratio of energy stored in A to that in B is
(a) 2 : 3 (b) 3 : 4 (c) 3 : 2 (d) 6 : 1

13. A wire of length 50 cm and cross-sectional area of 1 sq mm is extended by 1 mm. The required work will be ($Y = 2\times10^{10}\ Nm^{-2}$)
(a) 6×10^{-2} J (b) 4×10^{-2} J
(c) 2×10^{-2} J (d) 1×10^{-2} J

14. When a force is applied on a wire of uniform cross-sectional area 3×10^{-6} m and lengh 4 m, the increase in length is 1 mm. Energy stored in it will be ($Y = 2\times10^{11}\ Nm^{-2}$)
(a) 6250 J (b) 0.177 J
(c) 0.075 J (d) 0.150 J

15. When a 4 kg mass is hung vertically on a light spring that obey's Hooke's law, the spring stretches by 2 cms. The work required to be done by an external agent in stretching this spring by 5 cm will be ($g = 9.8\ ms^{-2}$)
(a) 4. 9 J (b) 2.4 J
(c) 0.495 J (d) 0.24 J

Chapter exercises

A Taking it together

(Assorted questions of the chapter for advanced level practice)

1. The Young's modulus of a perfectly rigid body
(a) is zero
(b) is unity
(c) is infinity
(d) may have any finite non-zero value

2. In practice, Poisson's ratio σ lies between
(a) $-\infty$ to $+\infty$ (b) 0 and $+\infty$
(c) 0 and 0.5 (d) 0.5 and 1.0

3. The only elastic modulus that applies to fluids is
(a) Young's modulus
(b) Shear modulus
(c) Modulus of rigidity
(d) Bulk modulus

4. Modulus of rigidity of ideal liquids is **[NCERT Exemplar]**
(a) infinity
(b) zero
(c) unity
(d) some finite small non-zero constant value

5. Strain energy per unit volume in a stretched string is
(a) $\frac{1}{2}$ stress × strain (b) stress × strain
(c) (stress × strain)^2 (d) stress/strain

6. The maximum load a wire can withstand without breaking, when its length is reduced to half of its original length, will **[NCERT Exemplar]**
(a) be double (b) be half
(c) be four times (d) remain same

7. For steel $Y = 2 \times 10^{11}\,\text{Nm}^{-2}$. The force required to double the length of a steel wire of area 1 cm^2 is
(a) 2×10^7 N (b) 2×10^6 N
(c) 2×10^8 N (d) 2×10^5 N

8. Longitudinal stress of 1 kg mm^{-2} is applied on a wire. The percentage increase in length is ($Y = 10^{11}$ Nm^{-2})
(a) 0.002 (b) 0.001
(c) 0.003 (d) 0.01

9. The Young's modulus of a wire is Y, if the energy per unit volume is E, then the strain will be
(a) $\sqrt{\frac{2E}{Y}}$ (b) $E\sqrt{2Y}$
(c) EY (d) $\frac{E}{Y}$

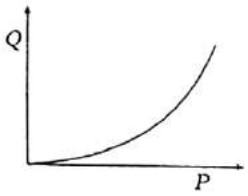
10. When a force is applied at one end of an elastic wire, it produces a strain E in the wire. If Y is the Young's modulus of the material of the wire, the amount of energy stored per unit volume of the wire is given by
(a) $Y \times E$ (b) $\frac{1}{2}(Y \times E)$
(c) $Y \times E^2$ (d) $\frac{1}{2}(Y \times E^2)$

11. Young's modulus of the material of a wire of length L and radius r is Y Nm^{-2}. If the length is reduced to $\frac{L}{2}$ and radius to $\frac{r}{2}$, the Young's modulus will be
(a) Y (b) $2Y$ (c) $\frac{Y}{4}$ (d) $\frac{Y}{2}$

12. The Young's modulus of a wire is numerically equal to the stress which will
(a) not change the length of the wire
(b) double the length of the wire
(c) increase the length by 50%
(d) change the radius of the wire to half

13. The length of an iron wire is L and area of cross-section is A. The increase in length is l on applying the force F on its two ends. Which of the statements is correct?
(a) Increase in length is inversely proportional to its length L
(b) Increase in length is proportional to area of cross-section A
(c) Increase in length is inversely proportional to A
(d) Increase in length is proportional to Young's modulus

14. The graph shows the behaviour of a length of wire in the region for which the substance obeys Hooke's law. P and Q represent.



(a) P = applied force, Q = extension
(b) P = extension, Q = applied force
(c) P = extension, Q = stored elastic energy
(d) P = stored elastic energy, Q = extension

15. A metal block is experiencing an atmospheric pressure of 10^5 Nm^{-2}. When the same block is placed in a vacuum chamber, the fractional change in its volume (the bulk modulus of metal is 1.25×10^{11} Nm^{-2})

(a) 4×10^{-7} (b) 2×10^{-7}
(c) 8×10^{-7} (d) 1×10^{-7}

16. The bulk modulus of water is 2.1×10^9 Nm^{-2}. The pressure required to increase the density of water by 0.1% is

(a) 2.1×10^3 Nm^{-2} (b) 2.1×10^6 Nm^{-2}
(c) 2.1×10^5 Nm^{-2} (d) 2.1×10^7 Nm^{-2}

17. If the Young's modulus of the material is 3 times its modulus of rigidity, then its volume elasticity will be

(a) zero (b) infinity
(c) 2×10^{10} N/m^2 (d) 3×10^{10} N/m^2

18. For a given material, the Young's modulus is 2.4 times that of rigidity modulus. Its Poisson's ratio is

(a) 2.4 (b) 1.2
(c) 0.4 (d) 0.2

19. Mark the wrong statement.

(a) Sliding of molecular layer is much easier than compression or expansion
(b) Reciprocal of bulk modulus of elasticity is called compressibility
(c) It is difficult to twist a long rod as compared to small rod.
(d) Hollow shaft is much stronger than a solid rod of same length and same mass

20. The increase in length on stretching a wire is 0.05%. If its Poisson's ratio is 0.4, then its diameter

(a) reduce by 0.02% (b) reduce by 0.1%
(c) increase by 0.02% (d) decrease by 0.4%

21. A spring is stretched by applying a load to its free end. The strain produced in the spring is

(a) volumetric (b) shear
(c) longitudinal and shear (d) longitudinal

22. An elastic material with Young's modulus Y is subjected to a tensile stress S, elastic energy stored per unit volume of the material is

(a) $\frac{YS}{2}$ (b) $\frac{S^2}{Y}$ (c) $\frac{S^2}{2Y}$ (d) $\frac{S}{2Y}$

23. The temperature of a wire is doubled. The Young's modulus of elasticity **[NCERT Exemplar]**

(a) will also double (b) will become four times
(c) will remain same (d) will decrease

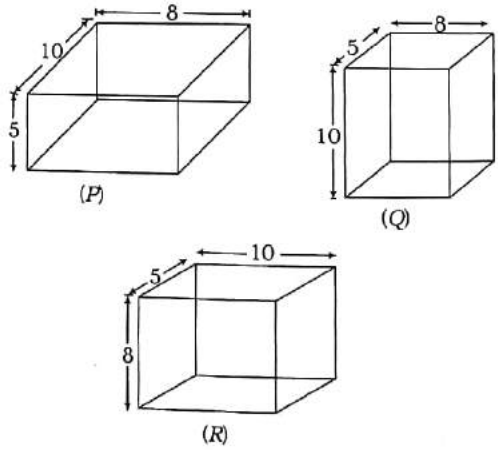
24. When a pressure of 100 atm is applied on a spherical ball, then its volume reduces to 0.01%. The bulk modulus of the material of the rubber in dyne cm^{-2} is

(a) 10×10^{12} (b) 100×10^{12}
(c) 1×10^{12} (d) 20×10^{12}

25. A 5 m long wire is fixed to the ceiling. A weight of 10 kg is hang at the lower end and is 1 m above the floor. The wire was elongated by 1 mm. The energy stored in the wire due to stretching is

(a) 0.01 J (b) 0.05 J (c) 0.02 J (d) 0.04 J

26. A rectangular block of size 10 cm × 8 cm × 5 cm is kept in three different positions P, Q and R in turn as shown in the figure. In each case, the shaded area is rigidly fixed and a definite force F is applied tangentially to the opposite face to deform the block. The displacement of the upper face will be



(a) same in all the three cases
(b) maximum in P position
(c) maximum in Q position
(d) maximum in R position

27. A uniform cube is subjected to volume compression. If each side is decreased by 2%, then bulk strain is

(a) 0.01 (b) 0.03 (c) 0.02 (d) 0.06

28. The longitudinal extension of any elastic material is very small. In order to have an appreciable change, the material must be in the form of

(a) long thick wire (b) short thick wire
(c) long thin wire (d) short thin wire

29. A cable that can support a load W is cut into two equal parts. The maximum load that can be supported by either part is

(a) $\frac{W}{4}$ (b) $\frac{W}{2}$

(c) W (d) $2W$

30. When a metal wire elongates by hanging a load Mg on it, the gravitational potential energy of mass M decreases by Mgl. This energy appears

(a) as elastic potential energy completely

(b) as thermal energy completely

(c) half as elastic potential energy and half as thermal energy

(d) as kinetic energy of the load completely

31. A ball falling in a lake of depth 200 m shows 0.1% decrease in its volume at the bottom. What is the bulk modulus of the material of the ball

(a) 19.6×10^{8} Nm^{-2}

(b) 19.6×10^{9} Nm^{-2}

(c) 19.6×10^{10} Nm^{-2}

(d) 19.6×10^{12} Nm^{-2}

32. A heavy mass is attached to a thin wire and is whirled in a vertical circle. The wire is most likely to break

(a) when the mass is at the highest point

(b) when the mass is at the lowest point

(c) when the wire is horizontal

(d) at an angle of $\cos^{-1}(1/3)$ from the upward vertical

33. Two wires of copper having the length in the ratio 4 : 1 and their radii ratio as 1 : 4 are stretched by the same force. The ratio of longitudinal strain in the two will be

(a) 1 : 16 (b) 16 : 1

(c) 1 : 64 (d) 64 : 1

34. The interatomic distance for a metal is 3×10^{-10} m. If the interatomic force constant is 3.6×10^{-9} N/Å, then the Young's modulus in Nm^{-2} will be

(a) 1.2×10^{11} (b) 4.2×10^{11}

(c) 10.8×10^{8} (d) 2.4×10^{10}

35. Young's modulus of rubber is 10^{4} Nm^{-2} and area of cross-section is 2 cm^{2}. If force of 2×10^{5} dynes is applied along its length, then its initial length l becomes

(a) $3l$ (b) $4l$

(c) $2l$ (d) None of these

36. Two wires of same diameter of the same material having the length l and $2l$. If the force F is applied on each, the ratio of the work done in the two wires will be

(a) 1 : 2 (b) 1 : 4 (c) 2 : 1 (d) 1 : 1

37. A force of 20 N is applied at one end of a wire of length 2 m and having area of cross-section 10^{-2} cm^{2}. The other end of the wire is rigidly fixed. If the coefficient of linear expansion of the wire $\alpha = 8\times 10^{-6}$ /°C and Young's modulus $Y = 2.2\times 10^{11}$ Nm^{-2} and its temperature is increased by 5°C, then the increase in the tension of the wire will be

(a) 4.2 N (b) 4.4 N

(c) 2.4 N (d) 8.8 N

38. When a steel wire fixed at one end is pulled by a constant force F at its other end, its length increases by l. Which of the following statements is not correct?

(a) Work done by the external force is Fl

(b) Some heat is produced in the wire in the process

(c) The elastic potential energy of the wire is $\frac{Fl}{2}$

(d) The heat produced is equal to half of the elastic potential energy stored in the wire

39. The temperature of a wire of length 1 m and area of cross-section 1 cm^{2} is increased from 0°C to 100°C. If the rod is not allowed to increase in length, the force required will be ($\alpha = 10^{-5}$/°C and $Y = 10^{11}$ Nm^{-2})

(a) 10^{3} N (b) 10^{4} N

(c) 10^{5} N (d) 10^{9} N

40. What is the approximate change in density of water in a lake at a depth of 400 m below the surface? The density of water at the surface is 1030 kgm^{-3} and bulk modulus of water is 2×10^{9} Nm^{-2}.

(a) 4 kgm^{-3} (b) 2 kgm^{-3}

(c) 6 kgm^{-3} (d) 8 kgm^{-3}

41. A load suspended by a massless spring produces an extension of x cm in equilibrium. When it is cut into two unequal parts, the same load produces an extension of 7.5 cm when suspended by the larger part of length 60 cm. When it is suspended by the smaller part, the extension is 5.0 cm. Then

(a) $x = 12.5$ cm

(b) $x = 3.0$ cm

(c) the length of the original spring is 90 cm

(d) the length of the original spring is 80 cm

42. A steel wire of cross-sectional area 3×10^{-6} m^{2} can withstand a maximum strain of 10^{-3}. Young's modulus of steel is 2×10^{11}Nm^{-2}. The maximum mass this wire can hold is

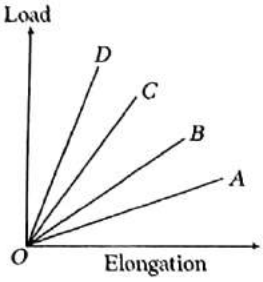
(a) 40 kg

(b) 60 kg

(c) 80 kg

(d) 100 kg

43. The load *versus* elongation graph for four wires of the same material is shown in the figure. The thickest wire is represented by the line



(a) *OD* (b) *OC* (c) *OB* (d) *OA*

44. Two wires of same material and length but diameter in the ratio 1 : 2 are stretched by the same force. The potential energy per unit volume for the two wires when stretched will be in the ratio

(a) 16 : 1 (b) 4 : 1 (c) 2 : 1 (d) 1 : 1

45. A stress of 10^6 Nm^{-2} is required for breaking a material. If the density of the material is $3\times 10^3 kgm^{-3}$, then what should be the length of the wire made of this material so that it breaks under its own weight?

(a) 10 m (b) 33.3 m
(c) 5 m (d) 66.6 m

46. A cable is replaced by another one of the same length and material but of twice the diameter. The maximum load that the new wire can support without exceeding the elastic limit, as compared to the load that the original wire could support, is

(a) half (b) double
(c) four times (d) one-fourth

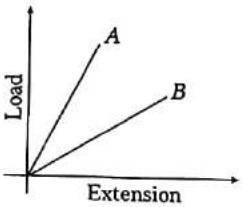
47. A uniform steel bar of cross-sectional area A and length L is suspended so that it hangs vertically. The stress at the middle point of the bar is (ρ is the density of steel)

(a) $\frac{L}{2A}\rho g$ (b) $\frac{L\rho g}{2}$ (c) $\frac{LA}{\rho g}$ (d) $L\rho g$

48. A thick rope of rubber of length 8 m and density $1.5\times 10^3 kg\ m^{-3}$ has Young's modulus 5×10^6 Nm^{-2}. When hung from ceiling of a room, the increase in length due to its own weight is

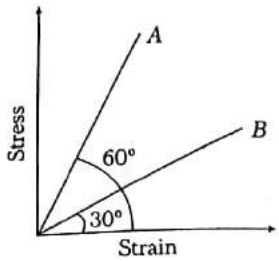
(a) 96×10^{-3} m (b) 19.2×10^{-5} m
(c) 9.4 cm (d) 9.6 mm

49. In the given figure, if the dimensions of the wires are the same and materials are different, Young's modulus is more for



(a) *A* (b) *B*
(c) Both (d) None of these

50. The stress *versus* strain graphs for wires of two materials A and B are as shown in the figure. If Y_A and Y_B are the Young's modulii of the materials, then



(a) $Y_B = 2Y_A$ (b) $Y_A = Y_B$
(c) $Y_B = 3Y_A$ (d) $Y_A = 3Y_B$

51. An elevator cable is to have a maximum stress of 7×10^7 Nm^{-2} to allow for appropriate safety factors. Its maximum upward acceleration is 1.5 ms^{-2}. If the cable has to support the total weight of 2000 kg of a loaded elevator, the area of cross-section of the cable should be

(a) 3.22 cm^2 (b) 2.38 cm^2
(c) 0.32 cm^2 (d) 8.23 cm^2

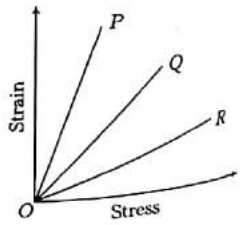
52. Four wires of the same material are stretched by the same load. The dimensions of the wires are as given below. The one which has the maximum elongation is of

(a) diameter 1 mm and length 1 m
(b) diameter 2 mm and length 2 m
(c) diameter 0.5 mm and length 0.5 m
(d) diameter 3 mm and length 3 m

53. A copper wire ($Y = 10^{11}$ Nm^{-2}) of length 8 m and a steel wire ($Y = 2\times 10^{11}$ Nm^{-2}) of length 4 m, each of 0.5 cm^2 cross- section are fastened end to end and stretched with a tension of 500 N. Choose the correct option.

(a) Elongation in copper wire is 0.8 mm
(b) Elongation in steel is $\frac{1}{4}$th the elongation in copper wire
(c) Total elongation is 1.0 mm
(d) All of the above

54. The strain-stress curves of three wires of different materials are shown in the figure. P, Q and R are elastic limits of the wires. The figure shows that

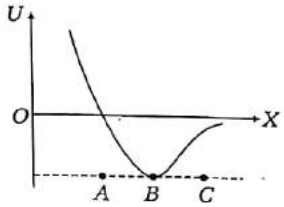


(a) elasticity of wire P is maximum
(b) elasticity of wire Q is maximum
(c) tensile strength of R is maximum
(d) None of the above

55. The extension in a string obeying Hooke's law is x. The speed of sound in the stretched string is v. If the extension in the string is increased to 1.5 x, the speed of sound will be

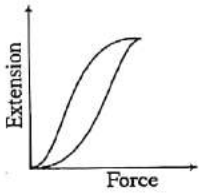
(a) 1.22 v (b) 0.61 v (c) 1.50 v (d) 0.75 v

56. The potential energy U between two molecules as a function of the distance X between them has been shown in the figure. The two molecules are



(a) attracted when x lies between A and B and are repelled when X lies between B and C
(b) attracted when x lies between B and C and are repelled when X lies between A and B
(c) attracted when they reach B
(d) repelled when they reach B

57. The diagram shows a force extension graph for a rubber band. Consider the following statements:



(i) It will be easier to compress this rubber than expand it
(ii) Rubber does not return to its original length after it is stretched
(iii) The rubber band will get heated if it is stretched and released

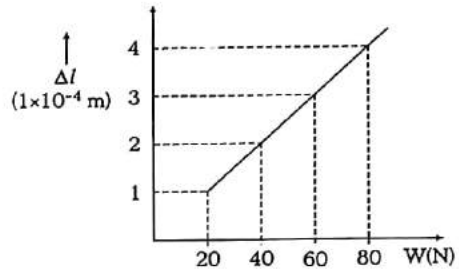
Which of the following can be deduced from the graph.

(a) Only III (b) II and III (c) I and III (d) Only I

58. Consider two cylindrical rods of identical dimensions, one of rubber and the other of steel. Both the rods are fixed rigidly at one end to the roof. A mass M is attached to each of the free ends at the centre of the rods. **[NCERT Exemplar]**

(a) Both the rods will elongate but there shall be no perceptible change in shape
(b) The steel rod will elongate and change shape but the rubber rod will only elongate
(c) The steel rod will elongate without any perceptible change in shape, but the rubber rod will elongate and the shape of the bottom edge will change to an ellipse
(d) The steel rod will elongate, without any perceptible change in shape, but the rubber rod will elongate with the shape of the bottom edge tapered to a tip at the centre

59. The graph shown below gives the extension (Δl) of a wire of length 1 m suspended from the top of a roof at one end with a load W connected to ther other end. If the cross-sectional area of the wire is 10^{-6} m^2, calculate the Young's modulus of the material of the wire.



(a) 2×10^{11} N/m^2 (b) 2×10^{-11} N/m^2
(c) 3×10^{-12} N/m^2 (d) 2×10^{-13} N/m^2

60. A brass rod of length 2 m and cross-sectional area 2 cm^2 is attached end to end to a steel rod of length L and cross-sectional area 1 cm^2. The compound rod is subjected to equal and opposite pulls of magnitude 5×10^4 N at its ends. If the elongations of the two rods are equal, the length of the steel rod (L) is ($Y_{brass} = 1 \times 10^{11}$ Nm^{-2} and $Y_{steel} = 2 \times 10^{11}$ Nm^{-2})

(a) 1.5 m (b) 1.8 m
(c) 1 m (d) 2 m

61. One end of a uniform wire of length L and weight w is attached rigidly to a point in roof and a weight w_1 is suspended from its lower end. If S is the area of cross-section of the wire, the stress in the wire at a height of $\frac{3L}{4}$ from its lower end is

(a) $\frac{w_1}{S}$ (b) $\frac{w_1 + \frac{w}{4}}{S}$ (c) $\left(\frac{w_1 + \frac{3w}{4}}{S}\right)$ (d) $\frac{w_1 + w}{S}$

62. A wire is elongated by 2 mm when a brick is suspended from it. When the brick is immersed in water, the wire contracts by 0.6 mm. What is the density of brick?

(a) 3333 kgm^{-3} (b) 4210 kgm^{-3}
(c) 5000 kgm^{-3} (d) 2000 kgm^{-3}

63. A rigid bar of mass M is supported symmetrically by three wires each of length l. Those at each end are of copper and the middle one is of iron. The ratio of their diameters, if each is to have the same tension, is equal to

(a) $\frac{\sqrt{Y_{copper}}}{Y_{iron}}$ (b) $\sqrt{\frac{Y_{iron}}{Y_{copper}}}$ (c) $\frac{Y^2_{iron}}{Y^2_{copper}}$ (d) $\frac{Y_{iron}}{Y_{copper}}$

64. A uniform rod of mass m, length L, area of cross-section A and Young's modulus Y hangs from the ceiling. Its elongation under its own weight will be

(a) zero (b) $\frac{mgL}{2AY}$ (c) $\frac{mgL}{AY}$ (d) $\frac{2mgL}{AY}$

65. The density of a metal at normal pressure is ρ. Its density when it is subjected to an excess pressure p is ρ'. If B is the bulk modulus of the metal, the ratio $\frac{\rho'}{\rho}$ is

(a) $\frac{1}{1-\frac{p}{B}}$ (b) $1+\frac{B}{p}$ (c) $\frac{1}{1-\frac{B}{p}}$ (d) $1+\frac{p}{B}$

66. One end of a long metallic wire of length L is tied to the ceiling, the other end is tied to a massless spring of spring constant k. A mass m hangs freely from the free end of the spring. The area of cross-section and Young's modulus of the wire are A and Y, respectively. If the mass is slightly pulled down and released, it will oscillate with a time period T equal to

(a) $2\pi\sqrt{\frac{m}{k}}$ (b) $2\pi\sqrt{\frac{m(YA+kL)}{YAk}}$

(c) $2\pi\sqrt{\frac{mYA}{kL}}$ (d) $2\pi\sqrt{\frac{mL}{YA}}$

67. The length of an elastic string is a metre when the tension is 4 N, and b metre when the tension is 5 N. The length, in metre, when the tension is 9 N, is

(a) $(a+b)$ (b) $(4b-5a)$

(c) $(5b-4a)$ (d) $(9b-9a)$

68. A uniform elastic plank moves due to a constant force F placed over a smooth surface. The area of end face is S and Young's modulus of the material is E. What is average strain produced in the direction of the force?

(a) $\frac{F}{SE}$ (b) $\frac{F}{2SE}$

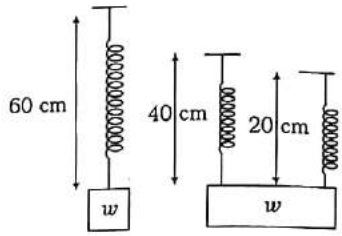
(c) $\frac{F}{4SE}$ (d) Zero

69. A uniform pressure p is exerted on all sides of a solid cube at temperature $t°$C. By what amount should the temperature be raised in order to bring the volume back to that it had been before the pressure was applied.

(Linear expansivity of the material of the cube is α and the bulk modulus of elasticity is β)

(a) $\frac{p\alpha}{\beta}$ (b) $\frac{3p\alpha}{\beta}$ (c) $\frac{p}{\alpha\beta}$ (d) $\frac{p}{3\alpha\beta}$

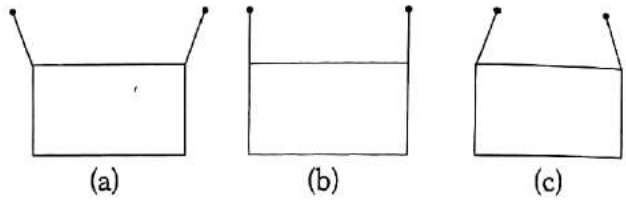
70. A block of weight w produces an extension of 9 cm when it is hung by an elastic spring of length 60 cm and is cut into two parts one of length 40 cm and the other of length 20 cm. The same load w hangs in equilibrium supported by both parts as shown in figure. The extension in cm now is



(a) 9 (b) 6

(c) 5 (d) 2

71. A rectangular frame is to be suspended symmetrically by two strings of equal length on two supports (figure). It can be done in one of the following three ways;

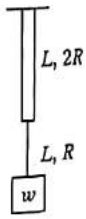


The tension in the strings will be **[NCERT Exemplar]**

(a) the same in all cases (b) least in (a)

(c) least in (b) (d) least in (c)

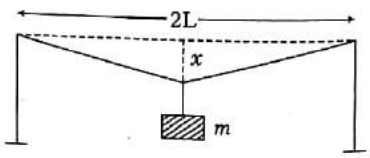
72. Two wires of the same material (Young's modulus = Y) and same length L but radii R and $2R$ respectively are joined end to end and a weight w is suspended from the combination as shown in the figure.



The elastic potential energy in the system is

(a) $\frac{3w^2L}{4\pi R^2Y}$ (b) $\frac{3w^2L}{8\pi R^2Y}$ (c) $\frac{5w^2L}{8\pi R^2Y}$ (d) $\frac{w^2L}{\pi R^2Y}$

73. A mild steel wire of length $2L$ and cross-sectional area A is stretched, well within elastic limit, horizontally between two pillars (figure). A mass m is suspended from the mid-point of the wire. Strain in the wire is **[NCERT Exemplar]**



(a) $\frac{x^2}{2L^2}$ (b) $\frac{x}{L}$

(c) x^2/L (d) $x^2/2L$

B Medical entrance special format questions

Assertion and reason

Directions (Q. Nos. 1-14) *These questions consists of two statements each printed as assertion and reason. While answering these questions you are required to choose any one of the following five responses.*

(a) If both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If Assertion is false but Reason is true.
(e) If both Assertion and Reason are false.

1. **Assertion** Young's modulus of elasticity is not defined for liquids.
   Reason Liquids cannot be stretched as wires.
2. **Assertion** Stress and modulus of elasticity have the same dimensions.
   Reason Strain is dimensionless.
3. **Assertion** Modulus of elasticity does not depend upon the dimensions of material.
   Reason Modulus of elasticity is a material property.
4. **Assertion** Upto the elastic limit, strain $\propto$ stress.
   Reason Upto elastic limit, material returns to its original shape and size, when external force is removed.
5. **Assertion** Incompressible liquids have finite value of bulk modulus of elasticity.
   Reason Compressibility is inverse of bulk modulus of elasticity.
6. **Assertion** If length of a wire is halved, its Young's modulus of elasticity will become two times.
   Reason The ratio of longitudinal stress and longitudinal strain is called Young's modulus of elasticity.
7. **Assertion** If a wire is stretched, only half of the work done in stretching the wire remains stored as elastic potential energy.
   Reason Potential energy stored in the wire is 1/2 (stress) $\times$ (strain)
8. **Assertion** The materials having low value of Young's modulus of elasticity are more ductile.
   Reason If Young's modulus is less, then they can be easily stretched as wires.
9. **Assertion** Bulk modulus of elasticity of gases is pressure dependent.
   Reason More the pressure of gas, more is the Bulk modulus of elasticity of gas.
10. **Assertion** Bulk modulus of an incompressible fluid is infinite.
    Reason Density of incompressible fluid remains constant.
11. **Assertion** Upto elastic limit of a stress-strain curve, the steel wire tends to regain its original shape when stress is removed.
    Reason Within elastic limit, the wire follows Hooke's law.
12. **Assertion** Adiabatic bulk modulus of an ideal gas is more than its isothermal bulk modulus.
    Reason Both the modulii are proportional to the pressure of gas at that moment.
13. **Assertion** If radius of cylindrical wire is doubled, then this wire can bear four times stress.
    Reason By doubling the radius, area of cross-section will become four times.

Match the columns

1. Match the following columns.

Column I	Column II
(A) Steel	(p) Young's modulus of elasticity
(B) Water	(q) Bulk modulus of elasticity
(C) Hydrogen gas filled in a chamber	(r) Shear modulus of elasticity

	A	B	C
(a)	p	q	r
(b)	p	p	q
(c)	p	q	q
(d)	p	r	r

2. Match the following columns.

Column I	Column II
(A) Longitudinal stress	(p) Volume changes
(B) Shear stress	(q) Shape changes
(C) Volumetric stress	(r) Volume does not changes
(D) Tensile stress	(s) Shapes does not change

	A	B	C	D
(a)	p,s	q,r	p,s	p,s
(b)	q,r	p,s	p,s	p,s
(c)	p,s	p,s	q,r	p,s
(d)	p,s	p,s	p,s	q,r

3. Match the following columns.

	Column I		Column II
(A)	Stress × strain	(p)	J
(B)	$\frac{YA}{l}$	(q)	Nm^{-1}
(C)	Yl^3	(r)	Jm^{-3}
(D)	$\frac{Fl}{AY}$	(s)	m

	A	B	C	D
(a)	r	q	p	s
(b)	q	r	p	s
(c)	p	s	r	q
(d)	q	r	s	p

4. With regard to dependence of quantities given in Column I and II, match the following.

	Column I		Column II
(A)	Young's modulus of a substance	(p)	Depends on temperature
(B)	Bulk modulus of a substance	(q)	Depends on length
(C)	Modulus of rigidity of a substance	(r)	Depends on area of cross-section
(D)	Volume of a substance	(s)	Depends on the nature of material

	A	B	C	D
(a)	p,s	q,s	p,s	p,q
(b)	p,s	p,s	p,s	p,q
(c)	p,s	p,s	p,s	p,q,r
(d)	p,q,r	p,s	p,s	p,r

C Medical entrances' gallery

(Collection of questions asked in NEET & various medical entrance exams)

1. The length of a metal wire is l_1 when the tension in it is T_1 and is l_2 when the tension is T_2. The natural length of the wire is **[AIIMS 2015, WBJEE 2015, Manipal 2015]**

(a) $\frac{l_1 + l_2}{2}$ (b) $\sqrt{l_1 l_2}$

(c) $\frac{l_1T_2 - l_2T_1}{T_2 - T_1}$ (d) $\frac{l_1T_2 + l_2T_1}{T_1 + T_2}$

2. A load of 4 kg is suspended from a ceiling through a steel wire of length 2 m and radius 2 mm. It is found that the length of the wire increases by 0.031 mm as equilibrium is achieved. What would be the Young's modulus of steel? (Take, $g = 3.1\,\pi\,ms^{-2}$) **[UK PMT 2015]**

(a) $2.0 \times 10^{11}\ Nm^{-2}$

(b) $2.82 \times 10^{11}\ Nm^{-2}$

(c) $0.20 \times 10^{11}\ Nm^{-2}$

(d) $0.028 \times 10^{11}\ Nm^{-2}$

3. Two wires of same length and same material but of radii r and $2r$ are stretched by forces F and f respectively to produce equal elongation. The ratio F to f is **[Kerala CEE 2015]**

(a) 1 : 1 (b) 1 : 2

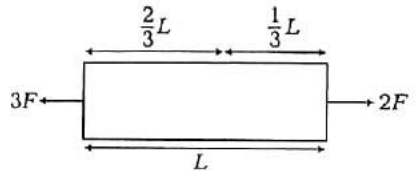
(c) 2 : 1 (d) 1 : 4

4. The elastic potential energy of a stretched wire is given by **[CG PMT 2015]**

(a) $U = \frac{AL}{2Y} l^2$ (b) $U = \frac{AY}{2L} l^2$

(c) $U = \frac{1}{2}\left(\frac{ALl}{Y}\right) l$ (d) $U = \frac{1}{2} \cdot \frac{YL}{2A} \cdot l$

5. A uniform cylindrical rod of length L and cross-sectional area A having Young's modulus Y is acted upon by forces as shown in figure. **[CG PMT 2015]**



The elongation produced in the rod is

(a) $\frac{3FL}{8AY}$ (b) $\frac{3FL}{5AY}$ (c) $\frac{8FL}{3AY}$ (d) $\frac{5FL}{3AY}$

6. The ratio of hydraulic stress to the corresponding strain is known as **[KCET 2015]**

(a) Young's modulus (b) compressibility

(c) rigidity modulus (d) Bulk modulus

7. The pressure on an object of bulk modulus B undergoing hydraulic compression due to a stress exerted by surrounding fluid having volume strain $\left(\frac{\Delta V}{V}\right)^2$ **[J&K CET 2014]**

(a) $B^2\left(\frac{\Delta V}{V}\right)$ (b) $B\left(\frac{\Delta V}{V}\right)^2$

(c) $\frac{1}{B}\left(\frac{\Delta V}{V}\right)$ (d) $B\left(\frac{\Delta V}{V}\right)$

8. For most of the material, Young's modulus (Y) and rigidity modulus (G) are related as **[UK PMT 2014]**

(a) $G = 3Y$ (b) $G = \frac{Y}{3}$ (c) $G = \frac{3}{2}Y$ (d) $G = \frac{Y}{8}$

9. The following four wires are made of the same material. Which of these will have the largest extension when the same tension is applied? **[NEET 2013]**
(a) Length = 50 cm, diameter = 0.5 mm
(b) Length = 100 cm, diameter = 1 mm
(c) Length = 200 cm, diameter = 2 mm
(d) Length = 300 cm, diameter = 3 mm

10. The Young's modulus of a rope of 10 m length and having diameter of 2 cm is 20×10^{11} dyne cm^{-2}. If the elongation produced in the rope is 1 cm, the force applied on the rope is **[J&K CET 2013]**
(a) 6.28×10^5 N (b) 6.28×10^4 N
(c) 6.28×10^4 dyne (d) 6.28×10^5 dyne

11. A steel wire of length l and cross-section area A is stretched by 1 cm under a given load. When the same load is applied to another steel wire of double its length and half of its cross-section area, the amount of stretching is **[Kerala CEE 2013]**
(a) 0.5 cm (b) 2 cm
(c) 4 cm (d) 1.5 cm

12. Theoretically the value of Poisson's ratio σ lies between **[MP PMT 2013]**
(a) $0 < \sigma < 1$ (b) $-1 < \sigma < 0.5$
(c) $0.2 < \sigma < 0.4$ (d) $-1 < \sigma < 1$

13. A steel ring of radius r and cross-sectional area A is fitted on to a wooden disc of radius R $(R > r)$. If Young's modulus of the steel is Y, then the force with which the steel ring is expanded is **[MP PMT 2013]**
(a) $\frac{AYr}{(R-r)}$ (b) $\frac{Y(R-r)}{r}$
(c) $\frac{AY(R-r)}{R}$ (d) $\frac{AY(R-r)}{r}$

14. A rubber cord L metre long and having A m^2 area of cross-section is suspended vertically. If the wire extends 1 m under its own weight, then change in length (l) is (Density of ruber = D kgm^{-3} and Young's modulus of rubber = E Nm^{-2}). **[UP CPMT 2013]**
(a) $\frac{L^2Dg}{2E}$ (b) $\frac{L^2Dg}{E}$
(c) $\frac{L^2Dg}{4E}$ (d) L

15. The density of a metal at normal pressure is ρ. Its density when it is subjected to an excess pressure p is ρ'. If B is the bulk modulus of the metal, then the ratio ρ'/ρ is **[UP CPMT 2013]**
(a) $1 + p/B$ (b) $1 + B/p$ (c) $\frac{1}{(1 - B/p)}$ (d) $\frac{1}{(1 - p/B)}$

16. If to break a wire of 1 m length, minimum 40 kg-wt is required, then to break a wire of same material 6 m in length and double in radius, the breaking weight required will be **[BCECE (Mains) 2012, Haryana PMT 2010]**
(a) 40 kg-wt (b) 80 kg-wt (c) 160 kg-wt (d) 320 kg-wt

17. If Poisson's ratio σ for a material is $-\frac{1}{2}$, then the material is **[BCECE (Mains) 2012]**
(a) elastic fatigue (b) incompressible
(c) compressible (d) None of these

18. Two block of masses of 1 kg and 2 kg are connected by a metal wire going over a smooth pulley. The breaking stress of metal is $\frac{40}{3\pi} \times 10^6$ Nm^{-2}. What should be the minimum radius of wire used if it should not break? (Take, $g = 10$ ms^{-2}). **[Manipal 2012]**
(a) 0.5 mm (b) 1 mm (c) 1.5 mm (d) 2 mm

19. Which one of the following is not a unit of Young's modulus? **[AFMC 2012]**
(a) Nm^{-1} (b) Nm^{-2}
(c) Dyne cm^{-2} (d) Mega pascal

20. There is some change in length when a 33000 N tensile force is applied on a steel rod of area of cross-section 10^{-3} m^{-2}. The change of temperature required to produce the same elongation, if the steel rod is heated is (The modulus of elasticity is 3×10^{11} Nm^{-2} and coefficient of linear expansion of steel is 11×10^{-5} 1°C). **[AFMC 2012]**
(a) 20°C (b) 15°C (c) 10°C (d) 0°C

21. In designing, a beam for its use to support a load. The depression at centre is proportional to (where, Y is Young's modulus). **[AIIMS 2012]**
(a) Y^2 (b) Y
(c) $\frac{1}{Y}$ (d) $\frac{1}{Y^2}$

Answers

Check point 12.1

1. (c)	2. (a)	3. (d)	4. (a)	5. (b)	6. (c)	7. (b)	8. (c)	9. (d)	10. (a)
11. (d)	12. (a)	13. (b)	14. (d)	15. (c)					

Check point 12.2

1. (d)	2. (b)	3. (a)	4. (b)	5. (c)	6. (c)	7. (c)	8. (b)	9. (d)	10. (c)
11. (d)	12. (d)	13. (a)	14. (b)	15. (c)					

Check point 12.3

1. (a)	2. (c)	3. (d)	4. (a)	5. (d)	6. (b)	7. (c)	8. (b)	9. (b)	10. (b)
11. (b)	12. (c)	13. (c)	14. (a)	15. (b)					

A. Taking it together

1. (c)	2. (c)	3. (d)	4. (b)	5. (a)	6. (d)	7. (a)	8. (b)	9. (a)	10. (d)
11. (a)	12. (a)	13. (c)	14. (c)	15. (c)	16. (b)	17. (b)	18. (d)	19. (c)	20. (a)
21. (c)	22. (c)	23. (d)	24. (c)	25. (b)	26. (c)	27. (d)	28. (c)	29. (c)	30. (c)
31. (a)	32. (b)	33. (b)	34. (a)	35. (c)	36. (a)	37. (d)	38. (d)	39. (b)	40. (b)
41. (a)	42. (b)	43. (a)	44. (a)	45. (b)	46. (c)	47. (b)	48. (c)	49. (a)	50. (d)
51. (a)	52. (c)	53. (d)	54. (d)	55. (a)	56. (b)	57. (a)	58. (d)	59. (a)	60. (d)
61. (c)	62. (a)	63. (b)	64. (b)	65. (a)	66. (b)	67. (c)	68. (b)	69. (d)	70. (d)
71. (c)	72. (c)	73. (a)							

B. Medical entrance special format questions

Assertion and reason

1. (a)	2. (a, b)	3. (a)	4. (d)	5. (d)	6. (d)	7. (c)	8. (a)	9. (a)	10. (b)
11. (c)	12. (b)	13. (d)							

Match the columns

1. (c)	2. (a)	3. (a)	4. (a)

C. Medical entrances' gallery

1. (c)	2. (a)	3. (d)	4. (b)	5. (c)	6. (d)	7. (d)	8. (b)	9. (a)	10. (b)
11. (c)	12. (b)	13. (d)	14. (a)	15. (d)	16. (c)	17. (b)	18. (c)	19. (a)	20. (c)
21. (c)									

Hints and explanations

Check point 12.1

1. (*c*) The interatomic forces in solids are both attractive and repulsive.
2. (*a*) The property of matter by virtue of which it does not regain its origional shape and size after the removal of deforming force is called plasticity.
3. (*d*) Breaking stress is independent of length and area of cross-section.
4. (*a*) The maximum deforming force upto which a body retains its property of elasticity is called elastic limit of the material.
5. (*b*) The internal restoring force acting per unit area of cross-section of the deforming body is called stress.
6. (*c*) The ratio of change in configuration of the original configuration is called strain
7. (*b*) The deforming force produces a change in the shape of the body without changing its volume, strain produced is called shearing strain.
8. (*c*) The property of matter by virtue of which it regains its original shape and size, when the deforming forces have been removed is called elasticity.
9. (*d*) On stretching a spiral spring, both types of strains are produced, *i.e.*, longitudinal and shear strain.
10. (*a*) Increase in length will be reduced to half since $\Delta l \propto l_1$, where l_1 = initial length of the wire.
11. (*d*) This is the case of shear strain in which both shape and size changes.
12. (*a*) Here, $L = 10 \text{ cm} = 10 \times 10^{-2} \text{ m}$

$$F = 100 \text{ kgf} = 100 \times 9.8 \text{ N}$$

$$\therefore \text{ Shearing stress} = \frac{F}{L^2} = \frac{100 \times 9.8}{(10 \times 10^{-2})^2} = 9.8 \times 10^4 \text{ Nm}^{-2}$$

13. (*b*) Here, original length, $L = 2.5$ m

$$\text{Strain} = \frac{\Delta L}{L} = 0.012\% = \frac{0.012}{100}$$

$$\therefore \quad \Delta L = \text{Strain} \times L$$

$$\text{or} \quad \Delta L = \text{Extension} = \frac{0.012}{100} \times L$$

$$= \frac{0.012 \times 2.5}{100} = 3 \times 10^{-4} \text{ m} = 0.3 \text{ mm}$$

14. (*d*) We have $V = l^3$

$$\frac{\Delta V}{V} = \frac{3\Delta l}{l} = 3\left(\frac{2}{100}\right) = 0.06$$

15. (*c*) Here, $L = 10 \text{ cm} = 10 \times 10^{-2} \text{ m}$

$$\Delta L = 0.25 \text{ cm} = 0.25 \times 10^{-2} \text{ m}$$

$$\therefore \text{ Shearing strain} = \frac{\Delta L}{L} = \frac{0.25 \times 10^{-2}}{10 \times 10^{-2}} = 0.025$$

Check point 12.2

1. (*d*) Rubber < Glass < Copper < Steel
2. (*b*) Young's modulus of material

$$Y = \frac{\text{Linear stress}}{\text{Longitudinal strain}}$$

If longitudinal strain is equal to unity then, Y = linear stress produced.

3. (*a*) For a perfectly rigid body, Young's modulus is infinite.
4. (*b*) Solids are least compressible whereas gases are most compressible.
5. (*c*) Isothermal bulk modulus = Pressure of gas
6. (*c*) Modulus of rigidity is defined for solids only. Hence, for liquid it is zero.
7. (*c*) For twisting, angle of shear $\phi \propto \frac{1}{L}$

i.e., if L is more, then ϕ will be small.

8. (*b*) The shear strain $\phi = \frac{10}{L} = \frac{10^{-2} \times 8}{2} = 0.004$
9. (*d*) Bulk modulus, $B = \frac{p}{-\frac{\Delta V}{V}} \Rightarrow B = \frac{h\rho g}{0.1 \times 10^{-2}}$

$$\Rightarrow \text{ Depth, } h = \frac{B \times 0.1 \times 10^{-2}}{\rho g} = \frac{9 \times 10^8 \times 0.1 \times 10^{-2}}{10^3 \times 10} = 90 \text{ m}$$

10. (*c*) The pressure exerted by a 3000 m column of water on the bottom layer is

$$P = h\rho g = 3000 \times 1000 \times 10 = 3 \times 10^7 \text{ N/m}^2$$

Fractional compression $\frac{\Delta V}{V}$ is

$$\frac{\Delta V}{V} = \frac{P}{B} = \frac{3 \times 10^7}{2.2 \times 10^9} = 1.36 \times 10^{-2} = 1.36\%$$

11. (*d*) If side of the cube is L then

$$V = L^3 = \frac{dV}{V} = 3\frac{dL}{L}$$

$$\therefore \quad \text{\% change in volume} = 3 \times (\text{\% change in length})$$
$$= 3 \times 1\% = 3\%$$

$$\therefore \quad \text{Bulk strain} = \frac{\Delta V}{V} = 0.03$$

12. (*d*) Elongation $\Delta l = \frac{FL}{AY} = \frac{FV}{A^2Y}$ $\left(V = A \times L \Rightarrow L = \frac{V}{A}\right)$

or $F = \frac{A^2 \Delta l}{V}$

$\therefore$ $F \propto A^2$

As cross-sectional area of 2nd wire is 3 times therefore $9F$ force is required for same elongation.

13. (*a*) Elongation in rubber cord,

$$l = \frac{L^2 dg}{2Y} = \frac{(10)^2 \times 1500 \times 10}{2 \times 5 \times 10^8} = 15 \times 10^{-4} \text{ m}$$

14. (*b*) Elongation $\Delta l = \frac{FL}{AY}$ $\therefore \Delta l \propto \frac{1}{A}$ [F, L and Y are constant]

or $\frac{A_2}{A_1} = \frac{\Delta l_1}{\Delta l_2}$ (Area of cross-section of second wire)

$\Rightarrow$ $A_2 = A_1\left(\frac{0.1}{0.05}\right) = 2A_1 = 2 \times 4 = 8 \text{ mm}^2$

15. (*c*) $\Delta l = \frac{FL}{\pi r^2 Y}$

$\Rightarrow$ $r^2 \propto \frac{1}{Y}$ [F, L and Δl are constant]

$$\frac{r_2}{r_1} = \left(\frac{Y_1}{Y_2}\right)^{1/2} = \left(\frac{7 \times 10^{10}}{1.2 \times 10^{10}}\right)^{1/2}$$

$\Rightarrow$ Radius $r_2 = 1.5 \times \left(\frac{7}{12}\right)^{1/2} = 1.145$ mm

$\therefore$ $d = 2.29$ mm

Check point 12.3

1. (*a*) We know that $\frac{dV}{V} = (1 - 2\sigma)\frac{dL}{L}$

If $\frac{dV}{V} = 0 \Rightarrow 1 - 2\sigma = 0$ or $\sigma = \frac{1}{2}$

$\therefore$ $\sigma = +0.50$

2. (*c*) $Y = 2\eta(1 + \sigma)$

$\therefore$ $Y > \eta$

3. (*d*) We have

$$Y = 2\eta(1 + \sigma)$$
$$2.4\eta = 2\eta(1 + \sigma)$$
$$12 = (1 + \sigma)$$

Poisson's ratio, $\sigma = 0.2$

4. (*a*) As, $Y = \frac{\text{Stress}}{\text{Strain}}$

$\therefore$ Slope of strain and stress graph $= \frac{1}{Y}$

$\therefore$ A has maximum young modulus of elasticity as its slope is minimum.

5. (*d*) Elastomers can be elastically stretched to large value of strain.

6. (*b*) A stretched rubber has increased potential energy.

7. (*c*) Volume V = cross-sectional $A \times$ length l or $V = Al$

$$\text{Strain} = \frac{\text{Elongation}}{\text{Original length}} = \frac{y}{l}$$

Young's modulus, $Y = \frac{\text{stress}}{\text{strain}}$

Work done, $W = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}$

$$W = \frac{1}{2} \times Y \times (\text{strain})^2 \times Al$$
$$= \frac{1}{2} \times Y \times \left(\frac{y}{1}\right)^2 \times Al = \frac{1}{2}\left(\frac{YA}{l}\right) y^2$$

$\Rightarrow$ $W \propto y^2$

8. (*b*) The work per unit volume

$$U = \frac{1}{2} \times Y \times (\text{Strain})^2 = \frac{1}{2} \times 9 \times 10^{11} \times \left(\frac{1}{100}\right)^2$$
$$= 4.5 \times 10^7 \text{ J}$$

9. (*b*) $W = \frac{1}{2}$ (Maximum stretching force) $\times$ (Elongation produced)

$$= \frac{1}{2} F \Delta l$$

Here, $F = mg = 10 \times 10$

$\Delta l = 2$ m

$\therefore$ $W = \frac{1}{2}(10 \times 10) \times 2 = 100$ J

10. (*b*) Energy stored by the spring

$$U = \frac{F^2}{2k} = \frac{T^2}{2k}$$

11. (*b*) $U = \frac{1}{2} Fl = \frac{F^2 l}{2AY}$ [F and Y are constant]

$$U \propto \frac{l}{r^2}$$

$\therefore$ $\frac{U_A}{U_B} = \left(\frac{l_A}{l_B}\right) \times \left(\frac{r_B}{r_A}\right)^2 = (3) \times \left(\frac{1}{2}\right)^2 = \frac{3}{4}$

12. (*c*) Energy stored, $U = \frac{1}{2} \times \frac{YAl^2}{L}$

$$= \frac{1}{2} \times \frac{2 \times 10^{11} \times 3 \times 10^{-6} \times (1 \times 10^{-3})^2}{4}$$
$$= 0.075 \text{ Joule}$$

13. (*c*) Required work $W = \frac{YAl^2}{2L}$

$$= \frac{2 \times 10^{10} \times 10^{-6} \times (10^{-3})^2}{2 \times 50 \times 10^{-2}}$$
$$= 2 \times 10^{-2} \text{ J}$$

14. (*a*) The energy stored in the wire

$$U = \frac{1}{2} \times F \times l = \frac{1}{2} \times 200 \times 10^{-3} = 0.1 \text{ J}$$

15. (b) We have $k = \frac{F}{x} = \frac{mg}{x}$

$$= \frac{4 \times 9.8}{2 \times 10^{-2}} = 19.6 \times 10^2$$

Work done $= \frac{1}{2} \times 19.6 \times 10^2 \times (0.05)^2 = 2.45$ Joule

A Taking it together

1. (c) For perfectly rigid body, $\Delta l = 0$

$\therefore$ Strain $= 0$ or $Y = \infty$

4. (b) No frictional (viscous) force exists in case of ideal fluid, hence, tangential forces are zero so there is no stress developed.

5. (a) The energy stored in unit volume of wire

$$U_V = \frac{1}{2} \times \text{stress} \times \text{strain}$$

6. (d) We know that,

$$\text{Breaking stress} = \frac{\text{Breaking force}}{\text{Area of cross - section}} \quad \text{...(i)}$$

When length of the wire changes, area of cross-section remains same.

Hence, breaking force will be same when length changes.

7. (a) Stress required to double the length is called Young's modulus.

$\therefore \quad Y = \frac{F}{A}$ or $F = Y \cdot A = 2 \times 10^{11} \times 10^{-4} = 2 \times 10^7$ N

8. (b) Longitudinal strain $= \frac{\text{stress}}{Y} = \frac{10^6}{10^{11}} = 10^{-5}$

Percentage increase in length $= 10^{-5} \times 100 = 0.001\%$

9. (a) Energy per unit volume $= \frac{1}{2} \times Y \times (\text{strain})^2$

$\therefore \quad \text{strain} = \sqrt{\frac{2E}{Y}}$

10. (d) Energy per unit volume $= \frac{1}{2} \times \text{Stress} \times \text{strain}$

$$= \frac{1}{2} \times (Y \times \text{strain}) \times \text{strain} = \frac{1}{2} \times Y \times E^2$$

11. (a) Young's modulus is property of material. So, it is independent of the length.

12. (a) $Y = \frac{\text{Stress}}{\text{Strain}}$, $Y =$ Stress for unit strain or $\Delta l = l$

13. (c) Elongation, $\Delta l = \frac{FL}{YA} \Rightarrow \Delta l \propto \frac{1}{A}$

14. (c) Graph between applied force and extension will be straight line because in elastic range,

Applied force $\propto$ extension

but the graph given is parabolic in nature

So, it will be between extension and stored elastic energy

As $U = 1/2 kx^2$ or $U \propto x^2$

15. (c) As $B = \frac{-\Delta p}{\Delta V / V}$

$$\therefore \quad \frac{\Delta V}{V} = \frac{\Delta p}{B} = \frac{10^5}{1.25 \times 10^{11}} = 8 \times 10^{-7}$$

16. (b) We have $d\rho = \frac{\rho}{B} \cdot dp$ or $\frac{d\rho}{\rho} = \frac{dp}{B} \Rightarrow \frac{0.1}{100} = \frac{dp}{2.1 \times 10^9}$

$\therefore$ The pressure required to increase the density

$$dp = 2.1 \times 10^6 \text{ Nm}^{-2}$$

17. (b) $Y = 2\eta(1 + \sigma)$

$\Rightarrow \quad 3\eta = 2\eta(1 + \sigma)$

$\Rightarrow \quad \sigma = \frac{3}{2} - 1 = \frac{1}{2}$

Now substituting the value of σ in the following expression.

$$Y = 3K(1 - 2\sigma) \Rightarrow K = \frac{Y}{3(1 - 2\sigma)} = \infty$$

18. (d) $Y = 2\eta(1 + \sigma)$

$$2.4\eta = 2\eta(1 + \sigma) \Rightarrow 1.2 = 1 + \sigma \Rightarrow \sigma = 0.2$$

19. (c) For twisting, Angle of shear $\phi \propto \frac{1}{L}$

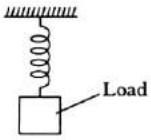
i.e., if L is more, then ϕ will be small.

20. (a) Poisson's ratio $= \frac{\text{Lateral strain}}{\text{Longitudinal strian}}$

$$\text{Lateral strain} = 0.4 \times \frac{0.05}{100}$$

So, it will be reduced by 0.025.

21. (c) Consider the diagram where a spring is stretched by applying a load to its free end. Clearly the length and shape of the spring changes.



The change in length corresponds to longitudinal strain and change in shape corresponds to shearing strain.

22. (c) Strain $= \frac{\text{Stress}}{Y} = \frac{S}{Y}$

Now, energy stored per unit volume $= \frac{1}{2} \times \text{stress} \times \text{strain}$

$$= \frac{1}{2} \times S \times \frac{S}{Y} = \frac{S^2}{2Y}$$

23. (d) We know that with increase in temperature length of a wire changes as

$$L_t = L_o (1 + \alpha \Delta T)$$

where, ΔT is change in the temperature, L_o is original length, α is coefficient of linear expansion and L_t is length at temperature T.

Now we can write, $\Delta L = L_t - L_o = L_o \alpha \Delta T$

Young's modulus $(Y) = \frac{\text{Stress}}{\text{Strain}} = \frac{FL_o}{A \times \Delta L} = \frac{FL_o}{AL_o \alpha \Delta T} \propto \frac{1}{\Delta T}$

As, $Y \propto \frac{1}{\Delta T}$

When temperature increases ΔT increases, hence, Y decreases.

24. (*c*) $\because$ Bulk modulus of the material of the rubber

$$B = \frac{\Delta p}{\Delta V / V} = \frac{100}{0.01/100} = 10^6 \text{ atm} = 10^{11} \text{ Nm}^{-2}$$
$$= 10^{12} \text{ dyne cm}^{-2}$$

25. (*b*) The energy stored in the wire.

$$U = \frac{1}{2} \times F \times l = \frac{1}{2} mg\, l = \frac{1}{2} \times 10 \times 10 \times 1 \times 10^{-3} = 0.05 \text{ J}$$

26. (*c*) $\eta = \frac{F/A}{x/L} \Rightarrow x = \frac{L}{\eta} \times \frac{F}{A}$

If η and F are constant then $x \propto \frac{L}{A}$

For maximum displacement area at which force applied should be minimum and vertical side should be maximum, this is given in the Q position of rectangular block.

27. (*d*) If side of the cube is L, then $V = L^3$

$$\Rightarrow \quad \frac{dV}{V} = 3\frac{dL}{L}$$

$\therefore$ % change in volume = 3 × (% change in length) = 3 × 2% = 6%

$\therefore$ Bulk strain, $\frac{\Delta V}{V} = 0.06$

28. (*c*) We have $\Delta l = \frac{Fl}{AY}$ or $\Delta l \propto \frac{l}{A}$ ...(i)

As per the information given in the question, the longitudinal extension of any material of wire is small.

$\therefore$ From Eqn. (i), $\Delta l \propto \frac{1}{A}$

It means that if Δl is small, then A will be large, which indicates that inorder to have an appreciable change, the material must be in the form of a long thin wire.

29. (*c*) $(\text{Stress})_1 = (\text{Stress})_2$

Maximum load does not depend on the length.

The maximum load that can be supported by either part is W.

31. (*a*) Bulk modulus of the material, $B = \frac{\Delta p}{\Delta V / V} = \frac{h\rho g}{0.1/100}$

$$= \frac{200 \times 10^3 \times 9.8}{1/1000} = 19.6 \times 10^8 \text{ Nm}^{-2}$$

32. (*b*) At lowest point, tension in the string is maximum.

33. (*b*) Strain $\propto$ stress $\propto \frac{F}{A}$

Ratio of strain $= \frac{A_2}{A_1} = \left(\frac{r_2}{r_1}\right)^2 = \left(\frac{4}{1}\right)^2 = \frac{16}{1}$

34. (*a*) Young Modulus $Y = \frac{3.6 \times 10^{-9} \text{ N/Å}}{3 \times 10^{-10} \text{ m}}$

$$= \frac{3.6 \times 10^{-9}}{3 \times 10^{-10} \times 10^{-10}} = 1.2 \times 10^{11} \text{ Nm}^{-2}$$

35. (*c*) Elongation of the wire

$$\Delta l = \frac{Fl}{AY} = \frac{(2)l}{(10^4)(2 \times 10^{-4})} = l \quad [\because 1 \text{ dyne} = 10^{-5} \text{ N}]$$

$\therefore$ New length will become $2l$.

36. (*a*) Work done, $W = \frac{1}{2} F\, l$

$\therefore$ Work done $W \propto l$ (F is constant)

$$\therefore \quad \frac{W_1}{W_2} = \frac{l_1}{l_2} = \frac{l}{2l} = \frac{1}{2}$$

37. (*d*) Increase in tension of wire $= YA\alpha\, \Delta\theta$

$$= 8 \times 10^{-6} \times 2.2 \times 10^{11} \times 10^{-2} \times 10^{-4} \times 5 = 8.8 \text{ N}$$

38. (*d*) The heat produced is equal to half of the elastic potential energy stored in the wire.

39. (*b*) F = Force developed $F = YA\alpha\ (\Delta\theta)$

$$= 10^{11} \times 10^{-4} \times 10^{-5} \times 100 = 10^4 \text{ N}$$

40. (*b*) The density of water $d\rho = \frac{\rho}{B} \cdot dp$

$$dp = \rho gh = (1030)(10)(400) = 4.12 \times 10^6 \text{ Nm}^{-2}$$

$$\therefore \quad d\rho = \frac{1030}{2 \times 10^9} \times 4.12 \times 10^6 = 2.12 \text{ kgm}^{-3}$$

41. (*a*) Elongation of the wire $\Delta l = \frac{Fl}{AY}$ (can also be applied for a spring)

$\therefore \quad \Delta l \propto l$

$$\frac{7.5}{5.0} = \frac{60}{l_2}$$

$\therefore \quad l_2 = 40$ cm

$\therefore$ Length of original spring is (60 + 40) cm = 100 cm

Now $\frac{x}{7.5} = \frac{100}{60}$

$\therefore \quad x = 12.5$ cm

42. (*b*) Maximum stress $= Y \times$ (maximum strain)

$$\frac{Mg}{A} = 2 \times 10^{11} \times 10^{-3} = 2 \times 10^8$$

Maximum mass of this wire

$$\therefore \quad M = \frac{2 \times 10^8 \times 3 \times 10^{-6}}{10} = 60 \text{ kg}$$

43. (*a*) Elongation of the wire

$$\Delta l = \frac{FL}{AY}$$

$\therefore \quad \Delta l \propto \frac{1}{r^2}$ (Y, L and F are constants)

i.e., for the same load, thickest wire will show minimum elongation. So, line OD represent the thickest wire.

44. (*a*) $\Delta l = \frac{Fl}{AY} \Rightarrow \frac{\Delta l}{l} = \left(\frac{F}{AY}\right)$

Potential energy per unit volume $= \frac{1}{2} \times$ stress $\times$ strain

$$u = \frac{1}{2} \times \frac{F}{A} \times \frac{F}{AY} \text{ or } u \propto \frac{1}{A^2} \text{ or } u \propto \frac{1}{d^4}$$

$$\therefore \quad \frac{u_1}{u_2} = \left(\frac{d_2}{d_1}\right)^4 = \frac{16}{1}$$

45. (*b*) Maximum breaking stress $= \frac{F}{A} = \frac{mg}{A}$ or $\frac{(Al\rho)\,g}{A} = \sigma_{max}$

$$\therefore \quad l_{max} = \frac{\sigma_{max}}{\rho g} = \frac{10^6}{(3 \times 10^3)(10)} = 33.33 \text{ m}$$

46. (*c*) We have $\sigma = \frac{W}{\pi r^2} = \frac{W'}{\pi (2r)^2}$

Load, $W' = 4$ W

47. (*c*) At middle, T = weight of half the length of steel bar

$$= \left(\frac{L}{2}\right) A\rho g$$

$$\therefore \quad \text{Stress} = \frac{T}{A} = \frac{L\rho g}{2}$$

48. (*c*) Due to own weight, $\Delta l = \frac{mgl}{2AY} = \frac{(lA\rho)\,gl}{2AY}$

$$= \frac{l^2\rho g}{2Y} = \frac{(8)^2\,(1.5 \times 10^3)\,(9.8)}{2 \times 5 \times 10^6}$$

$$= 9.4 \times 10^{-2} \text{m} = 9.4 \text{ cm}$$

49. (*a*) $F = \left(\frac{YA}{l}\right) \cdot \Delta l$ *i.e.* $F - \Delta l$ graph is a straight line with slope $\frac{YA}{l}$ or slope is proportional to Y.

$$(\text{Slope})_A > (\text{Slope})_B$$

$$\therefore \quad Y_A > Y_B$$

50. (*d*) $\frac{Y_A}{Y_B} = \frac{\tan\theta_A}{\tan\theta_B} = \frac{\tan 60°}{\tan 30°} = \frac{\sqrt{3}}{1/\sqrt{3}} = 3 \Rightarrow Y_A = 3Y_B$

51. (*a*) Tension, $T_{max} = m\,(g + a) = (2000)(9.8 + 1.5) = 22600$ N

$$\text{Maximum stress} = \frac{T_{max}}{\text{Area}}$$

$$\therefore \quad \text{Area} = \frac{T_{max}}{\text{Maximum stress}} = \frac{22600}{7 \times 10^7}$$

$$= 3.22 \times 10^{-4} \text{ m}^2 = 3.22 \text{ cm}^2$$

52. (*c*) Elongation of the wire

$$\Delta l = \frac{Fl}{AY} \quad \text{or} \quad \Delta l \propto \frac{l}{d^2} \qquad \left(\because A = \frac{\pi}{4} d^2\right)$$

Therefore, Δl will be maximum for that wire for which $\frac{l}{d^2}$ is maximum.

53. (*d*) Elongation in the copper wire

$$(\Delta l)_C = \left(\frac{Fl}{AY}\right)_C = \frac{500 \times 8}{0.5 \times 10^{-4} \times 10^{11}}$$

$$= 0.8 \times 10^{-3} \text{ m} = 0.8 \text{ mm}$$

Elongation in the steel wire

$$(\Delta l)_S = \left(\frac{Fl}{AY}\right)_S = \frac{500 \times 4}{0.5 \times 10^{-4} \times 2 \times 10^{11}}$$

$$= 0.2 \times 10^{-3} \text{ m} = 0.2 \text{ mm}$$

$$(\Delta l)_S = \frac{1}{4}(\Delta l)_C$$

and total elongation $\Delta l = 0.8 + 0.2 = 1.0$ mm

54. (*d*) As stress is shown on X-axis and strain on Y-axis.

So, we can say that $Y = \cot\theta = \frac{1}{\tan\theta} = \frac{1}{\text{slope}}$

So, elasticity of wire P is minimum and of wire R is maximum.

55. (*a*) According to the question, $T \propto x$, $T' \propto (1.5\,x)$

We have, $v_s \propto \sqrt{T}$ $\qquad \left(\therefore v_s = \sqrt{\frac{T}{\mu}}\right)$

$$\therefore \quad \frac{v_s'}{v_s} = \sqrt{\frac{T'}{T}} = \sqrt{\frac{1.5\,x}{x}} = \sqrt{1.5}$$

The speed of sound $v_s' = \sqrt{1.5}\,v_s = \sqrt{1.5}\,v = 1.22\,v$

56. (*b*) Force $F = \frac{-dU}{dx}$

BC : slope is +ve, F is attractive

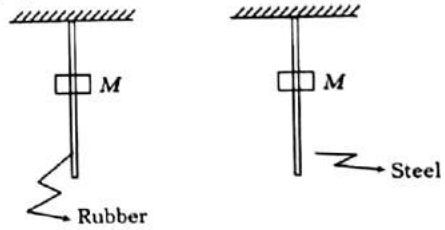
AB : slope is −ve, F is repulsive

57. (*a*) Energy loss $\propto$ area of hysteris loop

Energy loss appear as heat

58. (*d*) Consider the diagram.

A mass M is attached at the centre. As the mass is attached to both the rods, both rod will be elongated, but due to different elastic properties of material rubber changes shape also.



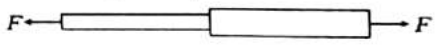
59. (*a*) We have, $\frac{\Delta L}{W} = \frac{(4-1) \times 10^{-4}}{80 - 20} = \frac{10^{-4}}{20}$

Elongation, $\Delta L = \frac{WL}{AY} \Rightarrow Y = \frac{L}{A}\frac{W}{\Delta L}$

Young's Modulus, $Y = \frac{1}{10^{-6}} \times \frac{20}{10^{-4}} = 2 \times 10^{11}\,\text{Nm}^{-2}$

60. (*d*) $\therefore$ Since the elongations of the two rods are equal

$$(\Delta l)_b = (\Delta l)_s$$



or $\quad \left(\frac{Fl}{AY}\right)_b = \left(\frac{Fl}{AY}\right)_s \qquad (F_b = F_s)$

or $\quad \left(\frac{l}{AY}\right)_b = \left(\frac{l}{AY}\right)_s$

$\therefore$ The length of the steel rod

$$l_s = \left(\frac{A_s Y_s}{A_b Y_b}\right) l_b = \left(\frac{1.0 \times 2.0 \times 10^{11}}{2.0 \times 1.0 \times 10^{11}}\right)(2\text{ m}) = 2\text{ m}$$

61. (*c*) At length $\frac{3L}{4}$ from lower end, tension in the wire,

$$T = \text{suspended load} + \frac{3}{4} \times \text{weight of wire} = w_1 + \frac{3w}{4}$$

$$\therefore \quad \text{Stress} = \frac{T}{S} = \frac{w_1 + \frac{3w}{4}}{S}$$

62. (*a*) We have $\Delta l \propto F$

$$\frac{\Delta l_1}{\Delta l_2} = \frac{F_1}{F_2} = \frac{\text{Weight}}{\text{Weight} - \text{upthrust}} \Rightarrow \frac{2}{1.4} = \frac{V\rho g}{V\rho g - V\rho_w g}$$

or Density of brick, $\rho = \frac{2}{0.6}\rho_w = \frac{2}{0.6} \times 1000 = 3333\text{ kg m}^{-3}$

63. (*b*) We know that Young's modulus

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{F/A}{\Delta L/L} = \frac{F}{A} \times \frac{L}{\Delta L} = \frac{F}{\pi (D/2)^2} \times \frac{L}{\Delta L} = \frac{4FL}{\pi D^2 \Delta L}$$

$$\Rightarrow \quad D^2 = \frac{4FL}{\pi \Delta L Y} \Rightarrow D = \sqrt{\frac{4FL}{\pi \Delta L Y}}$$

As F and $\frac{L}{\Delta L}$ are constants.

Hence, $D \propto \sqrt{\frac{1}{Y}}$

Now, we can find ratio as $\frac{D_{\text{copper}}}{D_{\text{iron}}} = \sqrt{\frac{Y_{\text{iron}}}{Y_{\text{copper}}}}$

64. (*b*) Tension at distance x from the bottom

$$T = \left(\frac{mg}{L}\right)x$$

Now, $\quad dl = \frac{T \cdot dx}{AY}$

$\therefore$ The elongation of the wire

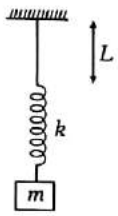
$$\Delta l = \int_0^L dl = \frac{mgL}{2AY}$$

65. (*a*) Density, $\rho = \frac{m}{V}$ or $\rho \propto \frac{1}{V}$ (as m = constant)

$$\therefore \quad \frac{\rho'}{\rho} = \frac{V}{V'} = \frac{V}{V + dV} \quad \left(\text{as } B = -\frac{p}{dV/V}\right)$$

$$\therefore \quad \frac{\rho'}{\rho} = \frac{V}{V - \frac{Vp}{B}} = \frac{1}{1 - \frac{p}{B}}$$

66. (*b*) Wire is equivalent to a spring of spring constant, $k' = \frac{AY}{L}$



k' and k are in series

$$k_{eq} = \frac{k'k}{k' + k} = \frac{(AY/L)k}{\left(\frac{AY}{L}\right) + k} = \frac{AYk}{AY + kL}$$

$$T = 2\pi\sqrt{\frac{m}{k_{eq}}} = 2\pi\sqrt{\frac{m(AY + kL)}{AYk}}$$

67. (*c*) Let l be the natural length and $K\left(= \frac{YA}{l}\right)$ be the force constant of wire. Then

$$a = l + \frac{4}{K} \quad \text{...(i)} \quad \left(F = K\,\Delta l \text{ or } \Delta l = \frac{F}{K}\right)$$

and $\quad b = l + \frac{5}{K} \quad$ ...(ii)

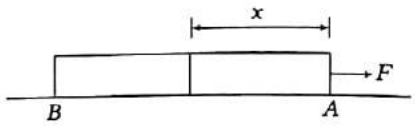
On solving Eqs. (i) and (ii), we get

or $\quad \frac{1}{K} = (b - a)$ and $l = (5a - 4b)$

Now, when $T = 9\text{ N}$

$$l' = l + \frac{9}{K} = (5a - 4b) + 9(b - a) = (5b - 4a)$$

68. (*b*) Tension at distance x from end A, $T_x = F - \frac{F}{L}x$



Total change in length, $\Delta l = \int_0^L \frac{Tx\,dx}{ES} = \frac{FL}{2ES}$

$\therefore$ Average strain $= \frac{\Delta l}{L} = \frac{F}{2ES}$

69. (*d*) We have $\left(\frac{\Delta V}{V}\right)_1 = \gamma \Delta t = 3\alpha\,(\Delta t)$ (by change in temperature)

and $\quad \left(\frac{\Delta V}{V}\right)_2 = \frac{p}{\beta} \quad$ (by pressure)

$$\left(\frac{\Delta V}{V}\right)_1 = \left(\frac{\Delta V}{V}\right)_2 \quad \text{or} \quad 3\alpha\,(\Delta t) = \frac{p}{\beta}$$

$$\therefore \quad \Delta t = \frac{p}{3\alpha\beta}$$

70. (*d*) Force constant, $k \propto \frac{1}{l}$

Let k be the force constant of original spring.

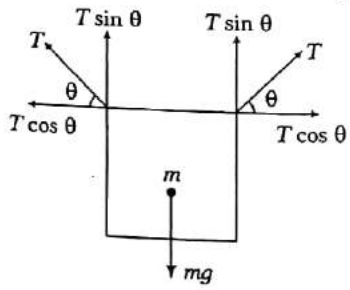
Then, $\quad k_{40} = \frac{60}{40}k = \frac{3}{2}k$ and $k_{20} = \frac{60}{20}k = 3k$

$$k_{net} = k_{40} + k_{20} = \frac{9k}{2}$$

Now, $\quad \Delta l = \frac{F}{k} \propto \frac{1}{k}$

$$\therefore \quad \Delta l' = \left(\frac{k}{9k/2}\right)(9\text{ cm}) = 2\text{ cm}$$

71. (*c*) Consider the FBD diagram of the rectangular frame



Balancing vertical forces $2T \sin\theta - mg = 0$

[T is tension in the string]

$\Rightarrow$ $$2T \sin\theta = mg \quad ...(i)$$

Total horizontal force $= T\cos\theta - T\cos\theta = 0$

Now from Eq. (i), $T = \dfrac{mg}{2\sin\theta}$

As mg is constant

$$\Rightarrow \quad T \propto \frac{1}{\sin\theta}$$

$$\Rightarrow \quad T_{max} = \frac{mg}{2\sin\theta_{min}}$$

$$\sin\theta_{min} = 0 \Rightarrow \theta_{min} = 0$$

No option matches with $\theta = 0°$

$$T_{min} = \frac{mg}{2\sin\theta_{max}} \quad (\text{since, } \sin\theta_{max} = 1)$$

$\sin\theta_{max} = 1 \Rightarrow \theta = 90°$

Matches with option (b).

Hence, tension is least for the case (b).

Note We should be careful when measuring the angle, it must be in the direction as given in the diagram.

72. (*c*) We have $\Delta l_1 = \dfrac{wL}{(4\pi R^2)Y}$, and $\Delta l_2 = \dfrac{wL}{\pi R^2 Y}$

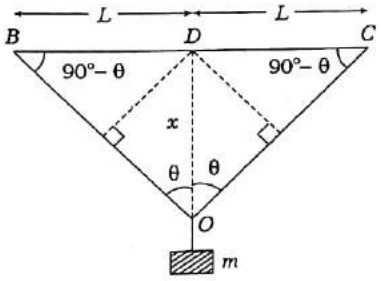
The elastic potential energy in the system

$$\therefore U = \frac{1}{2}K_1(\Delta l_1)^2 + \frac{1}{2}K_2(\Delta l_2)^2 \quad \left(K = \frac{YA}{L}\right)$$

$$= \frac{1}{2}\times\frac{Y(4\pi R^2)}{L}\times\left[\frac{wL}{4\pi R^2 Y}\right]^2 + \frac{1}{2}\times\frac{Y(\pi R^2)}{L}\times\left[\frac{wL}{\pi R^2 Y}\right]^2$$

$$= \frac{5w^2L}{8\pi R^2 Y}$$

73. (*a*) Consider the diagram below



Hence, change in length

$$\Delta L = BO + OC - (BD + DC)$$

$$= 2BO - 2BD \quad (\because BO = OC, BD = DC)$$

$$= 2[BO - BD] = 2[(x^2 + L^2)^{1/2} - L]$$

$$= 2L\left[\left(1 + \frac{x^2}{L^2}\right)^{1/2} - 1\right] \approx 2L\left[1 + \frac{1}{2}\frac{x^2}{L^2} - 1\right] = \frac{x^2}{L} \quad [\because x << L]$$

$$\therefore \text{Strain} = \frac{\Delta L}{2L} = \frac{x^2/L}{2L} = \frac{x^2}{2L^2}$$

B) Medical entrance special format questions

Assertion and reason

5. (*d*) Bulk modulus is $B = \dfrac{-\Delta p}{\Delta V/V}$

For incompressible fluids, $\Delta V = 0$

$\therefore \quad B = \infty$

Apart form this, it is true to say that compressibility is inverse of bulk modulus of elasticity.

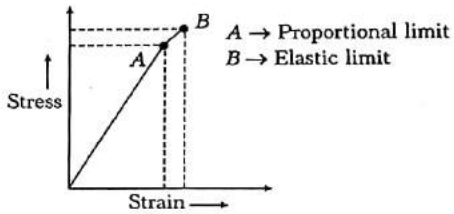
6. (*d*) Young's modulus is the property of material. It does not depend on length of the wire.

7. (*c*) Potential energy per unit volume is $\dfrac{1}{2}$ (stress) × strain.

9. (*a*) If pressure of gas is more, it is more difficult to compress a gas. Therefore, bulk modulus is high.

10. (*b*) Incompressible fluid is that which cannot be compressed ($\Delta V = 0$) by applying pressure on it.

11. (*c*) Hooke's law states that Stress ∝ Strain



It is obeyed upto proportional limit.

12. (*b*) $B_S = \gamma p$ = Adiabatic bulk modulus of elasticity.

$B_T = p$ = Isothermal bulk modulus of elasticity

$B_S > B_T$ and both are proportional to p.

13. (*d*) Stress $= \dfrac{\text{Restoring force}}{\text{Area}}$

$$S = \frac{\text{Force}}{\pi r^2} = \frac{F}{\pi r^2} \quad ...(i)$$

When, the radius of cylindrical wire is doubled

$$S' = \frac{F}{\pi(2r)^2}$$

$$\Rightarrow \quad S' = \frac{F}{4\pi r^2} = \frac{S}{4}$$

Now, $A = \pi r^2$

By doubling the radius, area of cross-section

$$A' = \pi(2r)^2 = 4\pi r^2 \Rightarrow A' = 4A$$

Therefore, area of cross-section will become four times.

Match the columns

1. (*c*) Young's modulus of elasticity and shear modulus is only defined for solids. Bulk modulus of elasticity is defined for all, *i.e.*, solids, liquids and gases.

2. (*a*) Volume changes in case of longitudinal and tensile stress whereas shape gets changed in shear stress.

3. (*a*) (a) Stress × Strain $= \text{Nm}^{-2} = \text{Nm}^{-2} \cdot \frac{\text{m}^3}{\text{m}^3} = \text{Jm}^{-3}$

(b) $\frac{YA}{l} = \frac{\text{Nm}^{-2}\text{m}^2}{m} = \text{Nm}^{-1}$

(c) $Yl^3 = \text{Nm}^{-2}\text{m}^3 = \text{Nm} = \text{J}$

(d) $\frac{Fl}{AY} = \frac{\text{Nm}}{\text{m}^2\text{Nm}^{-2}} = \text{m}$

4. (*a*) Modulus of elasticity depends on the temperature and nature of material. Volume of a substance depends on temperature, length and area of cross-section.

C) Medical entrances' gallery

1. (*c*) As, Young's modulus of elasticity, $Y = \frac{\text{Stress}}{\text{Strain}}$

$$\Rightarrow \quad Y = \frac{T_1 \times L}{A \times (l_1 - L)} \quad \text{... (i)}$$

where, L is the original length of the wire and also

$$Y = \frac{T_2 \times L}{A \times (l_2 - L)} \quad \text{... (ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{T_1 L}{A(l_1 - L)} = \frac{T_2 L}{A(l_2 - L)} \Rightarrow \quad L = \frac{T_2 l_1 - T_1 l_2}{T_2 - T_1}$$

2. (*a*) Given, mass of load = 4 kg

Length of wire, $l = 2$ m

Radius of wire (cross-section) = 2 mm = 2×10^{-3} m

and change in length, $\Delta l = 0.031$ mm = 0.031×10^{-3} m

Young's modulus,

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{\frac{mg}{A}}{\left(\frac{\Delta l}{l}\right)} = \frac{\frac{4 \times 3.1\pi}{\pi(2 \times 10^{-3})^2}}{(\Delta l / l)}$$

$$\Rightarrow \quad Y = \frac{4 \times 3.1\pi}{\pi \times 4 \times 10^{-6}} \times \frac{2}{0.031 \times 10^{-3}}$$

$$= \frac{3.1 \times 2}{0.031} \times 10^9 = 2 \times 10^{11}\ \text{Nm}^{-2}$$

3. (*d*) Given, $r_1 = r, r_2 = 2r$

Now, we know that, $Y = \frac{F}{A} \cdot \frac{l}{\Delta l} = \frac{F}{\pi r^2} \cdot \frac{l}{\Delta l}$

So, $\frac{F}{\pi r_1^2} = \frac{f}{\pi r_2^2}$ [$\because$ l, Δl and Y are same]

$$\frac{F}{f} = \frac{r_1^2}{r_2^2} \Rightarrow \frac{F}{f} = \frac{r^2}{4r^2} \Rightarrow \frac{F}{f} = \frac{1}{4}$$

4. (*b*) Elastic potential energy of a stretched wire is given by

$$E = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume} = \frac{1}{2} \times \frac{F}{A} \times \frac{l}{L} \times AL$$

$$= \frac{1}{2} \times Y \times \text{strain} \times \text{strain} \times AL = \frac{1}{2} \times Y \times \left(\frac{l}{L}\right)^2 \times AL$$

$$\therefore \quad E = \frac{AYl^2}{2L}$$

5. (*c*) Elongation produced if the $\frac{2}{3}$ part of length L of rod is stretched by a force of $3F$, is given by

$$\Delta L_1 = \frac{F_1 L_1}{AY} = \frac{3F \times \frac{2}{3}L}{AY} = 2\frac{FL}{AY}$$

Elongation produced if the remaining 1/3 part of length L of rod is stretched by a force of $2F$, is given by

$$\Delta L_2 = \frac{F_2 L_2}{AY} = \frac{2F \times \frac{1}{3}L}{AY} = \frac{2}{3}\frac{FL}{AY}$$

Total elongation produced in the rod is

$$\Delta L = \Delta L_1 + \Delta L_2 = 2\frac{FL}{AY} + \frac{2}{3}\frac{FL}{AY}$$

$$\therefore \quad \Delta L = \frac{8}{3}\frac{FL}{AY}$$

6. (*d*) The ratio of hydraulic stress to the corresponding strain is known as bulk modulus.

i.e., Bulk modulus $[B] = \frac{\text{hydraulic stress}}{\text{strain}}$

7. (*d*) $B = \frac{\text{Normal stress}}{\text{Volume strain}} = \left|\frac{p}{-\left(\frac{\Delta V}{V}\right)}\right| = \left|-\frac{pV}{V}\right|$

$$p = V\frac{\Delta V}{V} = B\left(\frac{\Delta V}{V}\right)$$

where, normal stress = p, volume strain $= -\frac{\Delta V}{V}$

8. (*b*) We know that,

Relation between Y and G

$$Y = 2G(1+\sigma) \Rightarrow G = \frac{Y}{2(1+\sigma)} \Rightarrow G \cong \frac{Y}{3}$$

9. (*a*) $\Delta L = \frac{FL}{AY}$

or $\Delta L \propto \frac{L}{d^2}$ $\left[\because A = \frac{\pi d^2}{4}\right]$

Therefore, ΔL will be maximum for that wire for which $\frac{L}{d^2}$ is maximum.

10. (*b*) Young's modulus of a rope, $Y = \frac{FL}{A\Delta l}$

Given, $L = 10$ m, $A = \pi r^2 = \pi(1)^2 = \pi$

$Y = 20 \times 10^{11}$ dyne cm^{-2}, $\Delta l = 1$ cm,

$$F = \frac{Y \cdot A \cdot \Delta l}{L} \Rightarrow F = \frac{20 \times 10^{11} \times \pi \times 1}{10 \times 10^2}$$

$F = 6.28 \times 10^9$ dyne

$F = 6.28 \times 10^4$ N

11. (*c*) We know that, $\Delta l = \frac{FL}{AY}$

Here, $\Delta l \propto \frac{L}{A}$

According to question

$$\Rightarrow \quad \frac{\Delta l_1}{\Delta l_2} = \frac{L_1}{L_2} \times \frac{A_2}{A_1}$$

$$\Rightarrow \quad \frac{\Delta l_1}{\Delta l_2} = \frac{1}{2} \times \frac{1}{2} \Rightarrow \frac{\Delta l_1}{\Delta l_2} = \frac{1}{4}$$

$$\Delta l_2 = 4\,\Delta l_1,\ \Delta l_2 = 4 \times 1,\ \Delta l_2 = 4 \text{ cm}$$

12. (*b*) Theoretically, the value of Poisson's ratio σ lies between $-1 < \sigma < 0.5$.

13. (*d*) Change in length of ring $= 2\pi R - 2\pi r$

Longitudinal strain $= \dfrac{2\pi R - 2\pi r}{2\pi r} = \dfrac{R-r}{r}$

Young modulus, $Y = \dfrac{F/A}{\dfrac{R-r}{r}}$

$$\Rightarrow \quad Y = \frac{Fr}{A(R-r)}$$

$$\Rightarrow \quad F = \frac{YA(R-r)}{r}$$

14. (*a*) Volume of the rubber cord $= AL$

Weight of the rubber cord $= ALDg$.

$$\therefore \quad \text{Stress} = \frac{ALDg}{A} = LDg$$

The weight of the rubber cord acts at the centre of gravity which is $\dfrac{L}{2}$ from the top.

$$\therefore \quad \text{Strain} = \frac{l}{L/2} = \frac{2l}{L}$$

So, Young's modulus, $E = \dfrac{\text{Stress}}{\text{Strain}} = \dfrac{LDg}{2l/L} = \dfrac{L^2Dg}{2l}$

or $\quad l = \dfrac{L^2Dg}{2E}$

15. (*d*) Bulk modulus $B = \dfrac{p}{dV/V}$ or $dV = \dfrac{pV}{B}$

$$\therefore \quad \rho' = \frac{M}{V - dV} = \frac{M}{V - (pV/B)}$$

$$= \frac{M}{V(1 - p/B)} = \frac{\rho}{(1 - p/B)}$$

$$\therefore \quad \frac{\rho'}{\rho} = \frac{1}{(1 - p/B)}$$

16. (*c*) Breaking force = Breaking stress × area of cross-section of wire

$\therefore$ Breaking force $\propto r^2$

If radius becomes doubled, then breaking force becomes 4 times *i.e.* $40 \times 4 = 160$ kg-wt

17. (*b*) We know that compressibility,

$$C = \frac{1}{K}$$

Also, $\quad \dfrac{dV}{V} = (1 + 2\sigma)\dfrac{dL}{L}$

Given, $\quad \sigma = -\dfrac{1}{2}$,

then $\quad \dfrac{dV}{V} = 0$, *i.e.*, volume is constant

or $\quad K = \infty$ *i.e.*, the material is incompressible.

18. (*c*) Breaking stress $= \dfrac{F}{A} = \dfrac{40}{3\pi} \times 10^6 \text{ Nm}^{-2}$

According to question,

$$\frac{40}{3\pi} \times 10^6 = \frac{3 \times 10}{\pi r^2}$$

$$r^2 = \frac{9}{4 \times 10^6}$$

$$r = 1.5 \times 10^{-3} \text{ m} = 1.5 \text{ mm}$$

19. (*a*) $Y = \dfrac{\text{Stress}}{\text{Strain}} = \text{Nm}^{-2}$ or pascal (in SI system)

and $\quad Y = \dfrac{\text{dyne}}{\text{cm}^2}$ (in CGS system)

Thus, Nm^{-1} is not the unit of Young's modulus.

20. (*c*) Modulus of elasticity $= \dfrac{\text{Force}}{\text{Area}} \times \dfrac{l}{\Delta l}$

$$3 \times 10^{11} = \frac{33000}{10^{-3}} \times \frac{l}{\Delta l}$$

$$\frac{\Delta l}{l} = \frac{33000}{10^{-3}} \times \frac{1}{3 \times 10^{11}} = 11 \times 10^{-5}$$

Change in length, $\dfrac{\Delta l}{l} = \alpha\,\Delta T$

$$11 \times 10^{-5} = 1.1 \times 10^{-5} \times \Delta T$$

$$\Rightarrow \quad \Delta T = 10 \text{ K or } 10°\text{C}$$

21. (*c*) We have $\delta = \dfrac{wl^3}{4Ybd^3}$ or $\delta \propto \dfrac{1}{Y}$

CHAPTER 13

Fluid mechanics

The term 'fluid' refers to a substance which cannot withstand shear so, it has no definite shape of its own and it has the ability to flow. Thus, the term fluid applies to both liquids and gases. In this chapter, we will study some important properties of fluids like pressure, viscosity, surface tension etc.

Inside

1) Pressure
- Pressure in a fluid
- Density of a liquid
- Pascal's law
- Archimedes' principle

2) Flow of fluids
- Steady flow
- Turbulent flow
- Equation of continuity

3) Bernoulli's theorem
- Applications based on Bernoulli's theorem

4) Viscosity

5) Stoke's law

6) Surface tension
- Surface energy
- Angle of contact
- Capillarity

1) Pressure

Normal force acting per unit area of a surface is called pressure. It is generally represented by p.

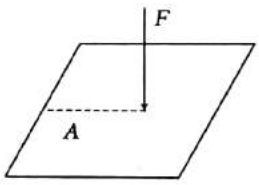


Fig. 13.1

If surface area is A and normal force acting on the surface is F (shown in figure), then pressure on the surface is

$$p = \frac{\text{Normal force } (F)}{\text{Area } (A)}$$

If area is very small, then

$$p = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} = \frac{dF}{dA}$$

It is clear from the above formula that, pressure acting on a surface depends upon the normal force acting on the surface and its area. Thus, force of same magnitude exerts different pressure on different area that is for the same force if area is lesser then pressure will be greater.

SI unit of pressure is Nm^{2} or pascal (Pa). Pressure is a scalar quantity and its dimensional formula is $[ML^{-1}T^{-2}]$.

Pressure in a fluid

When a fluid (either liquid or gas) is at rest, it exerts a force perpendicular to any surface in contact with it, such as a container wall or a body immersed in the fluid.

While the fluid as a whole is at rest, the molecules or atoms that makes up the fluid are in motion. These molecules or atoms collides continuously with walls of container. Pressure of a fluid is due to molecules colliding with the container walls.

The pressure applied by a fluid can be defined as the magnitude of the normal force (applied by fluid) acting per unit surface area.

If the pressure is the same at all points of a finite plane surface with area A, then

$$\boxed{p = \frac{F_\perp}{A}}$$

where, $F_\perp$ is the normal force on the surface.

Unit of fluid pressure French scientist Pascal contributed many researches in the field of fluid pressure. Thus, in his honour SI unit of pressure is known as pascal which is denoted by Pa.

$$1\,\text{Pa} = 1\,\text{N}\,\text{m}^{2}$$

One unit used practically in meterology is the bar which is equal to 10^5 Pa.

$$1\ \text{bar} = 10^5\ \text{Pa}$$

Atmospheric pressure (p_{atm})

It is pressure of the earth's atmosphere. This changes with weather and elevation. Normal atmospheric pressure at sea level (an average value) is 1.013×10^5 Pa. Thus,

$$1\ \text{atm} = 1.013\times 10^5\ \text{Pa}$$

Other practical units of pressure are bar and torr (mm of Hg)

$$1\ \text{atm} = 1.01\ \text{bar} = 760\ \text{torr}$$

Note Fluid pressure acts perpendicular to any surface in the fluid no matter how that surface is oriented. Pressure has no unique direction of its own, so it is taken as a scalar.

Absolute pressure and gauge pressure

The excess pressure above atmospheric pressure is usually called gauge pressure and the total pressure is called absolute pressure. Thus,

$$\boxed{\text{Gauge pressure} = \text{absolute pressure} - \text{atmospheric pressure}}$$

Absolute pressure is always greater than or equal to zero. While gauge pressure can be negative also.

Example 13.1 *An open container has dimensions of $4.0\,m \times 5.0\,m$ and height of 3.0 m.*

(i) Find the weight of the air in the container at 20 °C.

(ii) What is the weight of an equal volume of water? Also find pressure at the base of container due to this weight of water.

(iii) What is the total downward force on the base of the container due to air pressure of 1.0 atm?

Take the densities of air and water as 1.2 kg/m^3 and 10^3 kg/m^3 respectively.

Sol. It is given that density of air $\rho_{air} = 1.2\ \text{kg/m}^3$

and density of water $\rho_{water} = 10^3\ \text{kg/m}^3$

(i) The volume of air in the container

$$= (4.0)(5.0)(3.0) = 60\ \text{m}^3$$

$\therefore$ The mass of the air is $m_{air} = \rho_{air} \cdot V_{air}$

$$= (1.2)\,(60) = 72\ \text{kg}$$

The weight of the air is $w_{air} = m_{air}g = (72)(9.8)$

$$= 705.6\ \text{N}$$

(ii) The weight of water $w_{water} = m_{water}g = \rho_{water} \cdot V_{water}g$

$$= (10^3)(60)(9.8)$$

$$= 5.9 \times 10^5\ \text{N}$$

Pressure at the base due to this weight

$$p = \frac{F\ (= w)}{A} = \frac{5.9 \times 10^5}{20} = 2.95 \times 10^4\ \text{Nm}^{-2}$$

(iii) The downward force on the base is

$$= (\text{Air pressure})\ (\text{surface area})$$

$$= (1.013 \times 10^5)(4.0 \times 5.0) \simeq 2.0 \times 10^6\ \text{N}$$

Note The above force has large magnitude. This force is enough to collapse the base. Why does it not collapse? It is due to the upward force on the underside of the base. If we neglect the thickness of the base, this upward force is exactly equal in magnitude to downward force on the base and hence the net force due to air pressure is zero.

Density of a liquid

Density (ρ) of a liquid or any substance is defined as the mass per unit volume.

$$\therefore \quad \text{Density,}\ \ \rho = \frac{\text{Mass}}{\text{Volume}} = \lim_{\Delta V \to 0} \frac{\Delta m}{\Delta V} = \frac{dm}{dV}$$

SI unit of density is kgm^{-3} and its dimension is $[\text{ML}^{-3}\text{T}^0]$ and CGS unit is gcc^{-1} with $1\text{gcc}^{-1} = 10^3\ \text{kg m}^{-3}$.

Relative density (RD)

In case of a liquid (or a substance), sometimes an another term **relative density** (RD) or **specific gravity** is defined. It is the ratio of density of the substance (or a liquid) to the density of water at 4°C.

Hence,

$$\boxed{\text{RD} = \frac{\text{Density of liquid or substance}}{\text{Density of water at } 4°\text{C}}}$$

Relative density is a pure ratio so, it has no units. It is also sometimes referred as specific gravity.

Density of water at 4°C in CGS is 1 gcm^{-3}. Therefore, numerically the RD and density of a substance in CGS are equal. In SI unit the density of water at 4°C is 1000 kgm^{-3}.

Example 13.2 *Relative density of an oil is 0.8. Find the absolute density of oil in CGS and SI units.*

Sol. Density of oil (in CGS) = (RD) gcm^{-3}

$$= 0.8\ gcm^{-3} = 800\ kgm^{-3}$$

Density of a mixture of two or more liquids

Case 1 Suppose two liquids of densities ρ_1 and ρ_2 having masses m_1 and m_2 are mixed together. Then the density of the mixture will be

$$\rho = \frac{\text{Total mass}}{\text{Total volume}} = \frac{(m_1 + m_2)}{(V_1 + V_2)} = \frac{(m_1 + m_2)}{\left(\frac{m_1}{\rho_1} + \frac{m_2}{\rho_2}\right)}$$

If $\quad m_1 = m_2$, then $\rho = \dfrac{2\rho_1\rho_2}{\rho_1 + \rho_2}$

Case 2 If two liquids of densities ρ_1 and ρ_2 having volumes V_1 and V_2 are mixed, then the density of the mixture is,

$$\rho = \frac{\text{Total mass}}{\text{Total volume}} = \frac{m_1 + m_2}{V_1 + V_2} = \frac{\rho_1 V_1 + \rho_2 V_2}{V_1 + V_2}$$

If $\quad V_1 = V_2$, then $\rho = \dfrac{\rho_1 + \rho_2}{2}$

Example 13.3 *Two liquids of densities ρ and 3ρ having volumes $3V$ and V are mixed together. Find density of the mixture.*

Sol. Given, density of first liquid, $\rho_1 = \rho$

Density of second liquid, $\rho_2 = 3\rho$

Volume of first liquid, $V_1 = 3V$

Volume of second liquid, $V_2 = V$

$\therefore$ Density of the mixture

$$\rho_m = \frac{\rho_1 V_1 + \rho_2 V_2}{V_1 + V_2} = \frac{\rho \times 3V + 3\rho \times V}{3V + V} = \frac{6\rho}{4} = \left(\frac{3}{2}\right)\rho$$

Effect of temperature on density

When the temperature of a liquid is increased, the mass remains the same while the volume is increased and hence, the density of the liquid decreases $\left(\text{as } \rho \propto \dfrac{1}{V}\right)$.

Thus, $\dfrac{\text{Density after increase of temperature by } \Delta\theta}{\text{Initial volume of density}} = \dfrac{\rho'}{\rho}$

$$= \frac{V}{V'} = \frac{V}{V + dV} = \frac{V}{V + V\gamma\Delta\theta}$$

or $\quad \dfrac{\rho'}{\rho} = \dfrac{1}{1 + \gamma\Delta\theta}$

Here, γ = thermal coefficient of volume expansion

and $\Delta\theta$ = rise in temperature

Therefore, $\quad \boxed{\rho' = \dfrac{\rho}{1 + \gamma\Delta\theta}}$

Effect of pressure on density

As pressure is increased, volume decreases and hence density will increase. Thus, $\rho \propto \dfrac{1}{V}$

$$\therefore \quad \frac{\text{Density after increase of pressure}}{\text{Initial density}} = \frac{\rho'}{\rho}$$

$$= \frac{V}{V'} = \frac{V}{V + dV} = \frac{V}{V - \left(\frac{dp}{B}\right)V}$$

or $\quad \dfrac{\rho'}{\rho} = \dfrac{1}{1 - \frac{dp}{B}}$

Here, dp = change in pressure,

and B = bulk modulus of elasticity of the liquid

Therefore, $\quad \boxed{\rho' = \dfrac{\rho}{1 - \frac{dp}{B}}}$

Variation of fluid pressure with depth

Consider a fluid of density ρ is kept at rest; in a cylindrical vessel of height h as shown in figure.

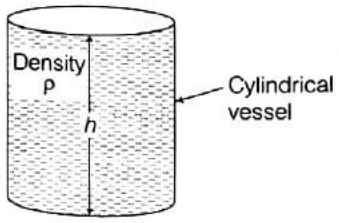


Fig. 13.2

Pressure applied by liquid at the bottom of the container is

$$\boxed{p = h\rho g} \quad \text{...(i)}$$

From the above formula, we can say that pressure applied by a liquid column is proportional to height of liquid column.

If the container is open, then atmospheric pressure will also work on the top surface, so total pressure on the bottom will be equal to

$$\boxed{p_{\text{Total}} = p_0 (\text{atmospheric pressure}) + \rho g h}$$

From Eq. (i), we can also find the difference of pressure between any two points in the liquid column.

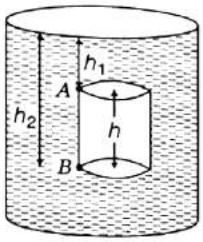


Fig. 13.3

Consider two points A and B in a liquid column situated at h_1 and h_2 respectively below free surface of liquid.

Pressure due to liquid column at point A, $p_A = h_1\rho g$

Pressure due to liquid column at point B, $p_B = h_2\rho g$

$\therefore$ Pressure difference $p_B - p_A = h_2\rho g - h_1\rho g = (h_2 - h_1)\rho g$

$$\boxed{p_B - p_A = h\rho g}$$

Hence, the **pressure difference depends on the vertical height (*i.e.*, distance between point *A* and point *B*), density of the fluid and the acceleration due to gravity.**

According to Eq. (i) pressure increases linearly with depth, if ρ and g are uniform.

A graph between p and h is shown below.

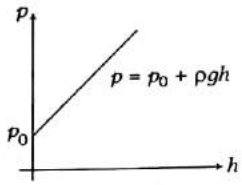


Fig. 13.4 Graph between pressure and height

Important points related with fluid pressure

Important points related with fluid pressure are given below

(i) At a point in the liquid column, the pressure applied on it is same in all directions.

(ii) In a liquid, pressure will be same at all points at the same level.

(iii) The pressure exerted by a liquid depends only on the height of fluid column and is independent of the shape of the containing vessel.

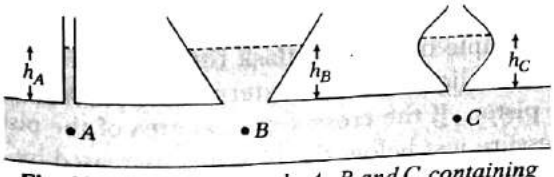


Fig. 13.5 The three vessels A, B and C containing different amounts of liquids

If $h_A = h_B = h_C$, then $p_A = p_B = p_C$

(iv) Consider following shapes of vessels

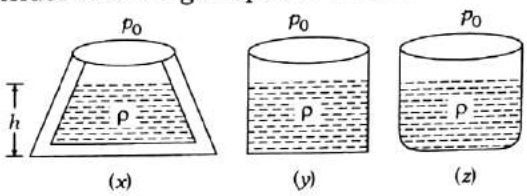


Fig. 13.6

Pressure at the base of each vessel

$p_x = p_y = p_z = p_0 + \rho gh$ but $w_x \neq w_y \neq w_z$

where ρ = density of liquid in each vessel

h = height of liquid in each vessel

p_0 = atmospheric pressure

(v) In the figure, a block of mass 'm' floats over a fluid surface

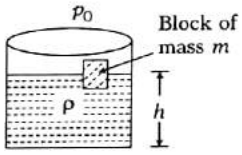


Fig. 13.7

If ρ = density of the liquid

A = area of the block

Pressure at the base of the vessel in

$$\boxed{p = p_0 + \rho gh + \frac{mg}{A}}$$

Example 13.4 *Find the pressure exerted below a column of water, open to the atmosphere, at depth*

(i) 10 *m* *(ii)* 30 *m*

(Given, density of water $= 1 \times 10^3 \text{kgm}^{-3}$, $g = 10\text{ms}^{-2}$*)*

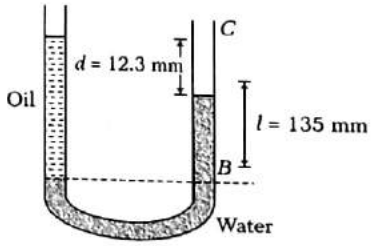
Sol. (i) Pressure at a depth of 10 m

$$p = p_a + \rho gh$$
$$= 1.013 \times 10^5 \text{ Pa} + (1 \times 10^3 \text{ kgm}^{-3})(10\text{ms}^{-2})(10\text{m})$$
$$= 1.013 \times 10^5 \text{ Pa} + 1 \times 10^5 \text{ Pa} = 2.013 \times 10^5 \text{ Pa}$$

(ii) Pressure at a depth of 30 m

$$p = p_a + \rho gh$$
$$= 1.013 \times 10^5 \text{ Pa} + (1 \times 10^3 \text{ kgm}^{-3})(10\text{ms}^{-2})(30\text{m})$$
$$= 1.013 \times 10^5 \text{ Pa} + 3 \times 10^5 \text{ Pa}$$
$$= 4.013 \times 10^5 \text{ Pa} \approx 4 \text{ atm}$$

Example 13.5 *For the arrangement shown in the figure, what is the density of oil?*



Sol. According to the figure, for the equilibrium of liquid

Net downward force = Net upward force

$$p_0 + \rho_w gl = p_0 + \rho_{oil}(l + d)g$$

$$\Rightarrow \quad \rho_{oil} = \frac{\rho_w l}{l + d} = \frac{1000 \times (135)}{(135 + 12.3)} = 916 \text{ kgm}^{-3}$$

Example 13.6 *A curved glass vessel full of water upto a height of 10 cm has a bottom of area 10 cm^2, top of area 30 cm^2 and volume 1 L.*

(i) *Find the force exerted by the water on the bottom.*

(ii) *Find the resultant force exerted by the sides of the glass on the water.*

(iii) *If the glass vessel is covered by a jar and the air inside the jar is completely pumped out, what will be the answers to parts (a) and (b).*

(iv) *If a glass vessel of different shape is used provided the height, the bottom area and the volume are unchanged, will the answers to parts (a) and (b) change.*

(Take, $g = 10 \text{ ms}^{-2}$, density of water $= 10^3 \text{ kgm}^{-2}$ and atmospheric pressure $= 1.01 \times 10^5 \text{ Nm}^{-2}$)

Sol. (i) Force exerted by the water on the bottom

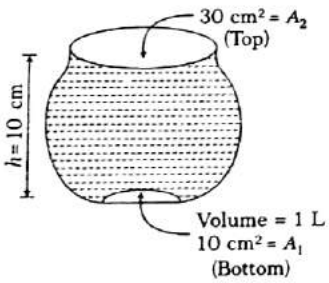
$$F_1 = (p_0 + \rho gh)A_1 \quad \text{...(i)}$$

Here, p_0 = atmospheric pressure = $1.01 \times 10^5 \text{ Nm}^{-2}$

ρ = density of water = 10^3 kgm^{-3}

$g = 10 \text{ ms}^{-2}$, $h = 10 \text{ cm} = 0.1 \text{ m}$

and A_1 = area of base = $10 \text{ cm}^2 = 10^{-3} \text{ m}^2$



Substituting these values in Eq. (i), we get

$$F_1 = (1.01 \times 10^5 + 10^3 \times 10 \times 0.1) \times 10^{-3}$$

or $F_1 = 102$ N (downwards)

(ii) Force exerted by atmosphere on water

$$F_2 = (p_0)A_2$$

Here, A_2 = area of top = $30 \text{ cm}^2 = 3 \times 10^{-3} \text{ m}^2$

$$\therefore \quad F_2 = (1.01 \times 10^5)(3 \times 10^{-3})$$

$$= 303 \text{ N (downwards)}$$

Force exerted by bottom on the water

$$F_3 = -F_1 \quad \text{or} \quad F_3 = 102 \text{ N (upwards)}$$

Weight of water, w = (volume)(density)(g)

$$= (10^{-3})(10^3)(10)$$

$$= 10 \text{ N (downwards)}$$

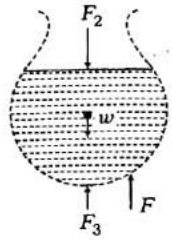
Let F be the force exerted by side walls on the water (upwards). Then for equilibrium of water (FBD)

Net upward force = net downward force

or $F + F_3 = F_2 + w$

$\therefore$ $F = F_2 + w - F_3 = 303 + 10 - 102$

or $F = 211$ N (upwards)



(FBD of water)

(iii) If the air inside the jar is completely pumped out,

$$F_1 = (\rho gh)A_1 \quad (\text{as } p_0 = 0)$$

$$= (10^3)(10)(0.1)(10^{-3})$$

$$= 1 \text{ N (downwards)}$$

In this case, $F_2 = 0$

and $F_3 = 1$ N (upwards)

$\therefore$ $F = F_2 + w - F_3$

$= 0 + 10 - 1$

$= 9$ N (upwards)

(iv) No, the answer will remain the same. Because the answers depend upon p_0, ρ, g, h, A_1 and A_2.

Pascal's law

It states that, "pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and the walls of the containing vessel".

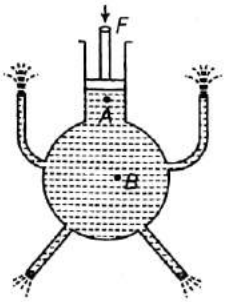


***Fig. 13.8** Pressure applied to an enclosed fluid*

As an example in which a flask fitted with a piston is filled with a liquid. Let an external force F is be applied on the piston. If the cross-sectional area of the piston is A, the pressure just below the piston is increased by F/A. By Pascal's law the pressure applied by piston will be transmitted equally at all points in the flask. If we make some holes in the flask then the liquid from all the holes will emerge out with same intensity.

Hydraulic lift

A well known application of Pascal's law is the hydraulic lift used to support or lift heavy objects. It is schematically illustrated in figure.

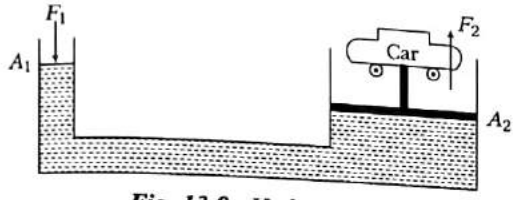


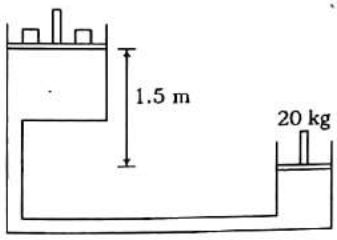
Fig. 13.9 *Hydraulic lift*

A piston with small cross-section area A_1 exerts a force F_1 on the surface of a liquid such as oil. The applied pressure $p = F_1/A_1$ is transmitted through the connecting pipe to a larger piston of area A_2. It pushes larger piston with force F_2. The applied pressure is the same in both cylinders, so

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \text{ or } F_2 = \frac{A_2}{A_1} \cdot F_1$$

Now, since $A_2 >> A_1$, therefore, $F_2 >> F_1$. Thus, hydraulic lift is a force multiplying device with a multiplication factor equal to the ratio of the areas of the two pistons. Dentist's chairs, car lifts and jacks, hydraulic elevators and hydraulic brakes all use this principle.

Example 13.7 *Figure shows a hydraulic press with the larger piston of diameter 35 cm at a height of 1.5 m relative to the smaller piston of diameter 10 cm. The mass on the smaller piston is 20 kg. What is the force exerted on the load by the larger piston? The density of oil in the press is 750 kgm^{-3}. Take, $g = 9.8\ ms^{-2}$)*



Sol. Pressure on the smaller piston $= \dfrac{20 \times 9.8}{\pi \times (5 \times 10^{-2})^2}$ Nm^{-2}

Pressure on the larger piston $= \dfrac{F}{\pi \times (17.5 \times 10^{-2})^2}$ N m^{-2}

The difference between the two pressures $= h\rho g$

where, $h = 1.5$ m

and $\rho = 750$ kgm^{-3}

Thus,
$$= \frac{20 \times 9.8}{\pi \times (5 \times 10^{-2})^2} - \frac{F}{\pi \times (17.5 \times 10^{-2})^2}$$
$$= 1.5 \times 750 \times 9.8$$

which gives, $F = 1.3 \times 10^3$ N

Note Atmospheric pressure is common to both pistons and has been ignored.

Example 13.8 *Two pistons of a hydraulic machine have diameters 20 cm and 2 cm. Find the force exerted on the larger piston when 50 kg wt is placed on the smaller piston. When the smaller piston moves is through 50 cm, by what distance the other piston moves out?*

Sol. For smaller piston, area $A_1 = \pi \times (1\text{ cm})^2$

For larger piston, area $A_2 = \pi \times (10\text{ cm})^2$

$\therefore$ Force exerted on the larger piston,

$$F_2 = \frac{A_2}{A_1} \times F_1 = \frac{\pi(10\text{ cm})^2}{\pi(1\text{ cm})^2} \times 50 \times 9.8$$
$$= 100 \times 50 \times 9.8 \simeq 5 \times 10^4 \text{N}$$

This is the force exerted on the larger piston. The liquids are considered incompressible. Therefore, volume covered by movement of smaller piston inwards equal to the outward movement of larger piston.

$$\therefore \quad L_1A_1 = L_2A_2 \Rightarrow \quad L_2 = \left(\frac{A_1}{A_2}\right)L_1 = \frac{\pi(1\text{ cm})^2}{\pi(10\text{ cm})^2} \times 5\text{ cm}$$
$$= \frac{1}{100} \times 5\text{ cm} = 0.05\text{ cm}$$

So, the distance moved out by the larger piston is 0.05 cm.

Measurement of pressure

Pressure can be measured by using following two devices

1. Barometer

It is a device used to measure atmospheric pressure. In principle, any liquid can be used to fill the barometer, but mercury is the substance of choice because its great density makes possible an instrument of reasonable size.

A barometer is an inverted evacuated tube, put over a mercury volume. Outside pressure pushes mercury into tube till the weight of liquid column equalises the force due to external pressure. In given diagram in equilibrium

$$p_1 = p_2$$

Here, p_1 = atmospheric pressure (p_0)

and $p_2 = 0 + \rho gh = \rho gh$

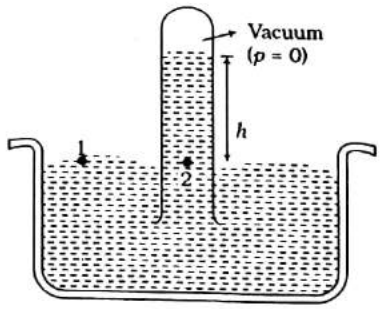


Fig. 13.10 *Barometer*

Also, ρ = density of mercury

$\therefore \quad p_0 = \rho g h$

Thus, the mercury barometer reads the atmospheric pressure (p_0) directly from the height of the mercury column.

e.g., if the height of mercury in a barometer is 760 mm, then atmospheric pressure will be,

$$p_0 = \rho g h = (13.6 \times 10^3)(9.8)(0.760) = 1.01 \times 10^5 \text{ Nm}^{-2}$$

Example 13.9 *What will be the length of mercury column in a barometer tube, when the atmospheric pressure is 76 cm of mercury and the tube is inclined at an angle of 30° with the horizontal direction?*

Sol. Here, $h = 76$ cm, $\theta = 30°$

If l is the length of mercury column is the barometer tube then

$$\frac{h}{l} = \sin 30° \Rightarrow \frac{76 \text{ cm}}{l} = \frac{1}{2}$$

$$\Rightarrow \quad l = 2 \times 76 = 152 \text{ cm}$$

2. Manometer

It is a device used to measure the pressure of a gas inside a container.

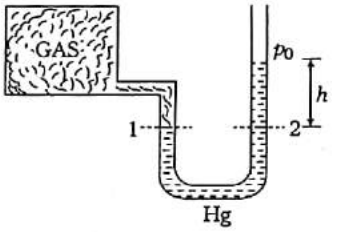


Fig. 13.11 *Manometer*

The U-shaped tube often contains mercury.

Let 1 and 2 are points on same horizontal level then,

$$p_1 = p_2$$

Here, p_1 = pressure of the gas in the container (p)

and p_2 = atmospheric pressure (p_0) + $\rho g h$

$\therefore \quad p = p_0 + \rho g h$

This can also be written as

$$p - p_0 = \text{gauge pressure} = \rho g h$$

Here, ρ is the density of the liquid used in U-tube.

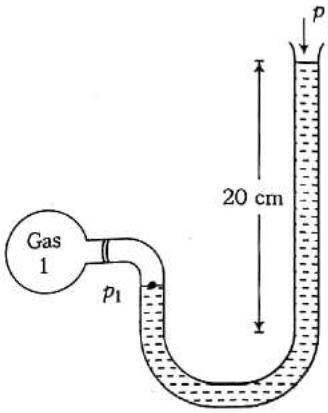
Thus, by measuring h we can find absolute (or gauge) pressure in the vessel.

Example 13.10 *A manometer tube contains a liquid of density 4×10^3 kg m^{-3}. When connected to a vessel containing a gas, the liquid level in the other arm of the tube is higher by 20 cm. When connected to another sample of enclosed gas, the liquid level in the other arm of the manometer tube falls 8 cm below the liquid level in the first arm. Which of the two samples exerts more pressure and by what amount?*

Sol. **For sample 1**

Difference in level of liquids, $h_1 = 20$ cm $= 0.2$ m

Pressure of the gas in the containing, $p_1 = p_a + \rho g h_1$ ...(i)



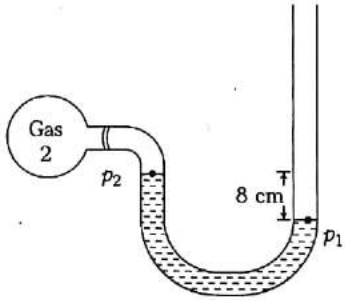
For sample 2

In this case, level of the liquid in the left arm is higher than that in the right arm by 8 cm.

$\therefore$ Atmospheric pressure p_a is greater than the pressure exerted by the sample

$$\Rightarrow \quad p_a = p_2 + \rho g h_2 \Rightarrow p_2 = p_a - \rho g h_2$$

Comparing equations (i) and (ii), it is clear that $p_1 > p_2$.



Therefore the gas in sample 1 exerts greater pressure than that in sample 2.

The difference in the two pressures is

$$\begin{aligned} p_1 - p_2 &= (p_a + \rho g h_1) - (p_a - \rho g h_2) \\ &= \rho g (h_1 + h_2) \\ &= \rho g (28 \text{ cm}) \\ &= (3 \times 10^3 \text{ kg m}^{-3}) \times (9.8 \text{ ms}^{-2})(0.28 \text{ m}) \\ &= 8.23 \times 10^3 \text{ Pa} \approx 8 k \text{ Pa} \end{aligned}$$

Archimedes' principle

If a heavy object is immersed in water, it seems to weigh less than when it is in air. This is because the water exerts a net upward normal force called **buoyant force**.

The Archimedes principle gives the magnitude of buoyant force on a body. It states that, "A body wholly or partially submerged in a fluid experiences an upward force which is equal to the weight of the displaced fluid."

Thus, the magnitude of buoyant force (F) which is also called upthrust is given by,

$$\boxed{F = V_i \rho_L g}$$

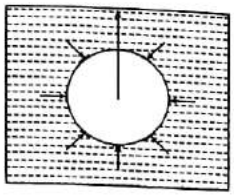


Fig. 13.12 Net upward force on immersed object

where, V_i = immersed volume of solid

ρ_L = density of liquid

and g = acceleration due to gravity

The upthrust act vertically upwards through the centre of gravity of displaced fluid.

Apparent weight of a body inside a liquid

If a body is completely immersed in a liquid its effective weight gets decreased. The decrease in its weight is equal to the upthrust on the body.

Hence, Apparent weight of body = $W_{app} = W_{actual}$ − upthrust

or $W_{app} = V\rho_S g - V\rho_L g$

Here, V = total volume of the body

ρ_S = density of body

and ρ_L = density of liquid

Thus, $$\boxed{W_{app} = Vg(\rho_S - \rho_L)}$$

or $$W_{app} = V\rho_S g\left(1 - \frac{\rho_L}{\rho_S}\right)$$

or $$W_{app} = W_{actual}\left(1 - \frac{\rho_L}{\rho_S}\right)$$

$$\boxed{W_{app} = W_{actual}\left(1 - \frac{\rho_L}{\rho_S}\right)}$$

Special cases

Three possibilities may now arise.

(i) **If** $\rho_S < \rho_L$, then in this condition the upthrust applied by the liquid will be greater than the weight of the body. That means if the body is completely immersed in liquid it will experience a net upward force. When released the body comes up to the fluid surface till the upthrust becomes equal to the weight of the body, at this point the body floats partially immersed in the fluid.

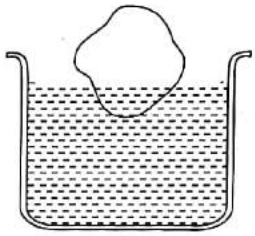


Fig. 13.13

(ii) **If** $\rho_S = \rho_L$, then $W_{app} = 0$ in this condition upthrust of the liquid balances the weight of the body. The body floats completely submerged just below the surface of the fluid.

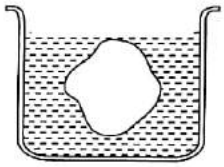


Fig. 13.14

(iii) **If** $\rho_S > \rho_L$, then in this condition the upthrust of the liquid is less than the weight of the liquid. *i.e.*, it is not sufficient to balance the weight of the body, so the body sinks.

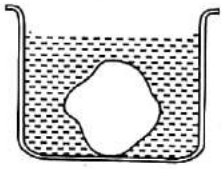


Fig. 13.15

Example 13.11 *An ornament weighing 50 g in air weights only 46 g is water. Assuming that some copper is mixed with gold to prepare the ornament. Find the amount of copper in it. Specific gravity of gold is 20 and that of copper is 10.*

Sol. Let m be the mass of the copper in ornament.

Then mass of gold in it is $(50 - m)$.

Volume of copper $V_1 = \frac{m}{10}$ $\left(\because \text{Volume} = \frac{\text{Mass}}{\text{Density}}\right)$

and volume of gold $V_2 = \frac{50 - m}{20}$

When immersed in water ($\rho_w = 1\ \text{gcm}^{-3}$)

Decrease in weight = upthrust

$\therefore$ $(50 - 46)g = (V_1 + V_2)\rho_w g$

or $4 = \frac{m}{10} + \frac{50 - m}{20}$ or $80 = 2m + 50 - m$

$\therefore$ $m = 30\ g$

Example 13.12 *Density of ice is 900 kg m^{-3}. A piece of ice is floating in water (of density 1000 kg m^{-3}). Find the fraction of volume of the piece of ice outside the water.*

Sol. Let V be the total volume and V_i the volume of ice piece immersed in water. For equilibrium of ice piece,

weight = upthrust ...(i)

$\therefore$ $V\rho_i g = V_i \rho_w g$

Here, ρ_i = density of ice = 900 kgm^{-3}

and ρ_w = density of water = 1000 kgm^{-3}

Substituting in Eq. (i), we get

$$\frac{V_i}{V} = \frac{\rho_i}{\rho_w} = \frac{900}{1000} = 0.9$$

i.e., the fraction of volume outside the water, $f = 1 - 0.9 = 0.1$

Example 13.13 *A piece of ice is floating in a glass vessel filled with water. How will the level of water in the vessel change when the ice melts?*

Sol. Let m be the mass of ice piece floating in water.

In equilibrium, weight of ice piece = upthrust

or $mg = V_i \rho_w g$ or $V_i = \frac{m}{\rho_w}$ ...(i)

Here, V_i is the volume of ice piece immersed in water.

When the ice melts, let V be the volume of water formed by m mass of ice.

Then, $V = \frac{m}{\rho_w}$ ...(ii)

From Eqs. (i) and (ii), we see that, $V_i = V$

Hence, the level will not change.

Example 13.14 *A piece of ice having a stone frozen in it floats in a glass vessel filled with water. How will the level of water in the vessel change when the ice melts?*

Sol. Let, m_1 = mass of ice, m_2 = mass of stone,

ρ_S = density of stone, and ρ_w = density of water.

In equilibrium, when the piece of ice floats in water,

weight of (ice + stone) = upthrust

$$(m_1 + m_2)g = V_i \rho_w g$$

$\therefore$ $V_i = \frac{m_1}{\rho_w} + \frac{m_2}{\rho_w}$ ...(i)

Here, V_i = Volume of ice immersed

When the ice melts, m_1 mass of ice converts into water and stone of mass m_2 is completely submerged.

Volume of water formed by m_1 mass of ice, $V_1 = \frac{m_1}{\rho_w}$

Volume of stone (which is also equal to the volume of water displaced)

$$V_2 = \frac{m_2}{\rho_S} \quad \text{Since, } \rho_S > \rho_w$$

Therefore, $V_1 + V_2 < V_i$

or, the level of water will decrease.

Buoyant force in accelerating fluids

If a body is dipped inside a liquid of density ρ_L placed in an elevator moving upward with an acceleration **a**, as shown in figure.

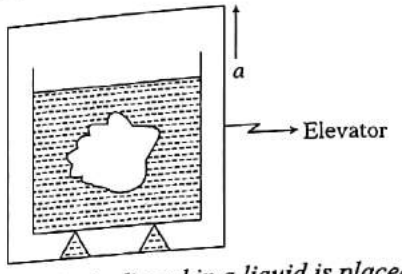


Fig. 13.16 A body dipped in a liquid is placed inside an elevator

Now consider the force acting on the liquid replaced by the body. For upward motion of the replaced liquid, we can write

$$F - w = ma$$

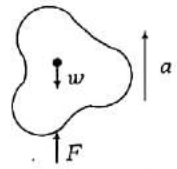


Fig. 13.17 Force acting on the displaced liquid

where, w = weight of the displaced liquid

F = buoyant force acting on the body

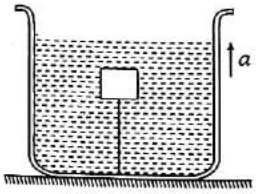
$\therefore$ $F = w + ma = m(g + a) = \rho_L V(g + a)$

$$\boxed{F = V\rho_L g_{eff}}$$

Here, $g_{eff} = |\mathbf{g} + \mathbf{a}|$

e.g., if the lift is **moving upwards** with an acceleration a, the value of g_{eff} is $\boldsymbol{g} + \boldsymbol{a}$ and if it is **moving downwards** with acceleration a, the g_{eff} is $\boldsymbol{g} - \boldsymbol{a}$. In a **freely falling** lift $\boldsymbol{g}_{eff}$ **is zero** (as $a = g$) and hence, net buoyant force is zero. This is why, in a freely falling vessel filled with some liquid, the air bubbles do not rise up (which otherwise move up due to buoyant force).

Example 13.15 *The tension in a string holding a solid block below the surface of a liquid (of density greater than that of solid) as shown in figure is T_0 when the system is at rest. What will be the tension T in the string, if the system has an upward acceleration a?*



Sol. Let m be the mass of block.

Initially for the equilibrium of block,

$$F = T_0 + mg \quad \text{...(i)}$$

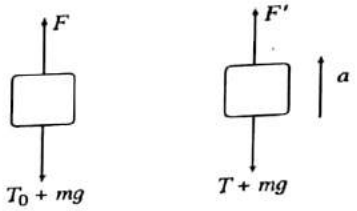
Here, F is the upthrust on the block.

When the lift is accelerated upwards, g_{eff} becomes $g + a$ instead of g.

Hence,
$$F' = F\left(\frac{g+a}{g}\right) \quad \text{...(ii)}$$

From Newton's second law,

$$F' - T - mg = ma \quad \text{...(iii)}$$



Solving Eqs. (i), (ii) and (iii), we get

$$T = T_0\left(1 + \frac{a}{g}\right)$$

Check point 13.1

1. A fluid comparises of
   (a) liquid only (b) gases only
   (c) Neither liquid nor gases (d) Both (a) and (b)

2. Dimensional formula for pressure is
   (a) $[M^0L^0T^0]$ (b) $[ML^{-1}T^{-2}]$ (c) $[M^0L^1T^{-2}]$ (d) $[M^0L^1T^0]$

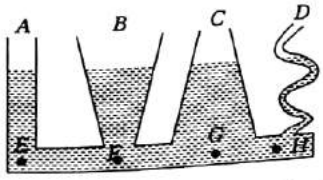
3. Pins and nails are made to have pointed end because
   (a) it transmits very small pressure
   (b) it transmits a large pressure
   (c) it provide a large area
   (d) None of the above

4. The two thigh bones each of cross-sectional area 15 cm^2 support the upper part of a person of mass 70 kg. The pressure sustained by these thigh bones is
   (a) 2.5×10^5 N m^2 (b) 1.33×10^5 N m^2
   (c) $4.66\times10^5\times10^5$ Nm^2 (d) 2.33×10^5 N m^2

5. Which of the following is correct.
   (a) Gauge pressure = absolute pressure + atmospheric pressure
   (b) Absolute pressure = gauge pressure – atmospheric pressure
   (c) Gauge pressure = absolute pressure – atmospheric pressure
   (d) Absolute pressure = atmospheric pressure – gauge pressure

6. At a depth of 500 m in an ocean, what is the absolute pressure? Given that the density of sea water is 1.03×10^3 kg m^3 and $g = 10\,ms^{-2}$.
   (a) 40 atm (b) 52 atm
   (c) 32 atm (d) 62 atm

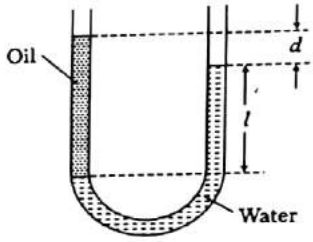
7. Four vessels A, B, C and D have different shapes and hold different amount of water. Which of the following is correct.



   (a) $P_E > P_F > P_G > P_H$ (b) $P_E < P_F < P_G < P_H$
   (c) $P_E = P_F = P_G = P_H$ (d) $P_E = P_F > P_G = P_H$

8. Two liquids of densities 2ρ and ρ having their volumes in the ratio 3 : 2 are mixed together. Density of the mixture will be
   (a) $\frac{2\rho}{3}$ (b) $\frac{\rho}{2}$
   (c) $\frac{8\rho}{5}$ (d) $\frac{4\rho}{5}$

9. The U-tube in figure contains two different liquids in static equilibrium, water in the right arm and oil of unknown density ρ_x in the left. If l = 135 mm and d = 15 mm. Density of the oil is



   (a) 1000 kg m^3 (b) 920 kg m^3
   (c) 895 kg m^3 (d) 900 kg m^3

10. Increase in pressure at one point of the enclosed liquid in equilibrium at rest is transmitted equally to all other points. This is as per
    (a) impulse
    (b) Pascal's law
    (c) conservation of momentum
    (d) None of the above

11. In a vehicle lifter the enclosed gas exerts a force F on a small piston having a diameter of 8 cm. This pressure is transmitted to a second piston of diameter 24 cm. If the mass of the vehicle to be lifted is 1400 kg then value of F is
    (a) 1200 N (b) 1800 N (c) 1600 N (d) 700 N

12. What will be the length of mercury column in a barometer tube when the atmospheric pressure is x cm of mercury and the tube is inclined at an angle ϕ with the vertical direction?
    (a) $\frac{x}{\sin\phi}$ (b) $\frac{x}{\cos\phi}$ (c) $\frac{x}{\tan\phi}$ (d) x

13. A boat floating in a water tank is carrying a number of stones. If the stones were unloaded into water, then the water level
(a) increases
(b) remains same
(c) decreases
(d) data is insufficient

14. An iron casting containing a number of cavities weights 6000 N in air and 4000 N in water. What is the volume of the cavities in the casting? Density of iron is 7.87 gcm^{-3} (Take, $g = 9.8\ ms^{-2}$ and density of water $= 10^3\ kgm^{-3}$)
(a) 0.16 m^3
(b) 0.2 m^3
(c) 0.12 m^3
(d) 0.14 m^3

15. A body floats in water with 40% of its volume outside water. When the same body floats in an oil. 60% of its volume remains outside oil. The relative density of oil is
(a) 0.9
(b) 1.0
(c) 1.2
(d) 1.5

2 Flow of fluids

Till now we have studied about the fluids at rest. In this section we will study fluids in motion. When a fluid is in flow, its motion can either be smooth or irregular depending on its velocity of flow.

Steady flow

Consider a liquid flowing in a tube as shown in the figure given below. If the velocity of fluid particles at any point does not vary with time, the flow is said to be steady. Steady flow is also called streamlined or laminar flow. The velocity at different points may be different. Hence, in the figure,

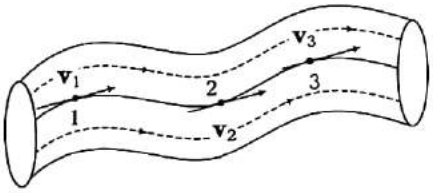


Fig. 13.18 Steady flow of liquid through a tube

we can say that velocity of fluid particles at different points 1, 2 and 3 remains same with time.

i.e., $\mathbf{v}_1 = \text{constant}, \mathbf{v}_2 = \text{constant}, \mathbf{v}_3 = \text{constant}$.

but $\mathbf{v}_1 \neq \mathbf{v}_2 \neq \mathbf{v}_3$

Streamlines

The path followed by a fluid particle in steady flow is called **streamline.** Velocity of fluid particle at any point of the streamline is along tangent to the curve at that point.

Properties of streamline are given below

(i) In streamline flow, no two streamlines can cross each other. If they do so, the particles of the liquid at the point of intersection will have two different directions for their flow, which will decrease the steady nature of the liquid flow.

(ii) The greater is the crowding of streamlines at a place greater is the velocity of the liquid particles at that place and *vice-versa.*

(iii) Streamline flow is possible only if the liquid velocity does not exceed a limiting value called critical velocity.

(iv) The bundle of stream lines forming a tubular region is called a tube of flow.

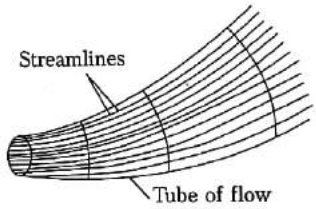


Fig. 13.19 Tube of flow formed by streamlines

Turbulent flow

In rivers and canals where speed of water is quite high or the boundary surfaces cause abrupt changes in velocity of the flow, then the flow becomes irregular. Such flow of liquid is known as turbulent flow.

Thus, the flow of fluid in which velocity of all particles crossing a given point is not same and the motion of the fluid becomes irregular or disordered. This is called turbulent flow.

A few examples of turbulent flow are

(i) a jet of air striking a flat plate.

(ii) the smoke rising from a burning stock of wood.

(iii) when a fast flowing streames encounters rocks, foamy whirlpool like regions are formed.

Reynolds number

Reynolds number (R_e) is a dimensionless number, whose value gives us an idea whether the flow would be laminar (streamlined) or turbulent.

$$R_e = \frac{\text{Inertial force}}{\text{Viscous force}}$$

Thus, R_e represents the ratio of inertial force of moving fluid to viscous force offered by the fluid.

The expression for R_e is

$$\boxed{R_e = \frac{\rho v D}{\eta}}$$

Here, ρ = density of fluid,

v = speed of fluid

D = diameter of tube

η = viscosity of fluid (later on it is discussed in detail)

- For $R_e < 1000$, flow is streamline or laminar.
- For $R_e \rightarrow 1000$ to 2000, flow is unsteady.
- For $R_e > 2000$, flow is turbulent.

Example 13.16 *Water is flowing in a pipe of diameter 6 cm with an average velocity 7.5 cms^{-1}. s^{-1} and its density is 10^3 $kg\ m^{-3}$. What is the nature of flow? Given coefficient of viscosity of water is 10^{-3} $kgm^{-1}\ s^{-1}$.*

Sol. Reynolds number for the given situation is given as

$$R_e = \frac{\rho v D}{\eta}$$

Here ρ, density $= 10^3 kgm^{-3}$

Coefficient of viscosity, $\eta = 10^{-3} kgm^{-1}s^{-1}$

Average velocity of water, $v = 7.5\ cms^{-1} = 0.075\ ms^{-1}$

Diameter of pipe, $D = 0.075 = 6\ cm = 0.06\ m$

Hence, $$R_e = \frac{10^3 \times 0.075 \times 0.06}{10^{-3}}$$

$$= 10^6 \times 0.0045 = 4500$$

$\because$ $R_e > 2000$

Therefore, the flow is turbulent

Equation of continuity

When an incompressible and non-viscous liquid flows in a streamlined motion through a tube of non-uniform cross-section, then mass flow rate, is same at every section of the tube

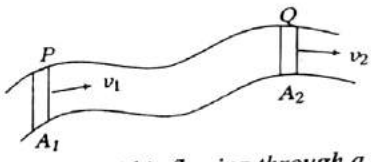


Fig. 13.20 *A liquid is flowing through a tube of non-uniform cross-section.*

Thus, $$\boxed{A_1 v_1 = A_2 v_2}$$

or $$Av = \text{constant} \quad \text{or} \quad v \propto \frac{1}{A}$$

This is called equation of continuity which is basically derived from the law of conservation of mass.

We can generalise the continuity equation for the case in which the fluid is not incompressible. If ρ_1 and ρ_2 are the densities at sections 1 and 2, then

$$\boxed{\rho_1 A_1 v_1 = \rho_2 A_2 v_2}$$

So, **this is the continuity equation for an incompressible fluid.**

Note

The product Av is the volume flow rate $\frac{dV}{dt}$, the rate at which volume crosses a section of the tube. Hence,

$$\frac{dV}{dt} = \text{volume flow rate} = Av$$

The mass flow rate is the mass flow per unit time through a cross-section. This is equal to

$$\frac{dm}{dt} = \rho \cdot \frac{dV}{dt}$$

Example 13.17 *Water is flowing through a horizontal tube of non-uniform cross-section. At a place the radius of the tube is 1.0 cm and the velocity of water is 2 ms^{-1}. What will be the velocity of water where the radius of the pipe is 2.0 cm?*

Sol. Using equation of continuity,

$$A_1 v_1 = A_2 v_2 \quad \text{or,} \quad v_2 = \left(\frac{A_1}{A_2}\right) v_1$$

or, $$v_2 = \left(\frac{\pi r_1^2}{\pi r_2^2}\right) v_1 = \left(\frac{r_1}{r_2}\right)^2 v_1$$

Given, $r_1 = 1\ cm = 1 \times 10^{-2}\ m$, $r_2 = 2\ cm = 2 \times 10^{-2}\ m$

$$v_1 = 2\ ms^{-1}$$

Substituting the above values, we get

$$v_2 = \left(\frac{1.0 \times 10^{-2}}{2.0 \times 10^{-2}}\right)^2 (2) \quad \text{or} \quad v_2 = 0.5\ ms^{-1}$$

3 Bernoulli's theorem

Bernoulli's theorem is based on the law of conservation of energy and applied to ideal fluids.

It states that the sum of pressure energy per unit volume, kinetic energy per unit volume and potential energy per unit volume of an incompressible, non-viscous fluid in a streamlined irrotational flow remains constant at every cross-section throughout the liquid flow.

Mathematically, it can be expressed as

$$\boxed{p + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}}$$

where, p represents for pressure energy per unit volume (or pressure), $\frac{1}{2}\rho v^2$ for kinetic energy per unit volume and $\rho g h$ for potential energy per unit volume.

Proof Consider an ideal fluid having streamline flow through a pipe of varying area of cross-section as shown in figure.

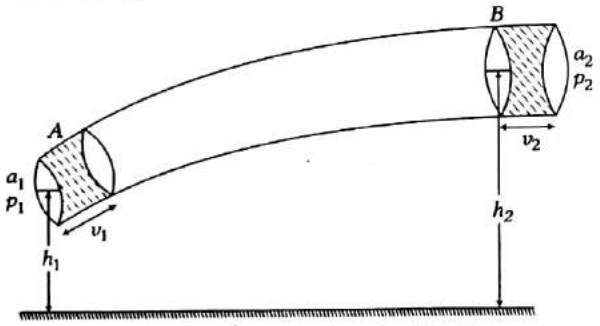


Fig. 13.21 Flow of an ideal fluid

If p_1 and p_2 are the pressures at two ends of the tube respectively, the work done by pressure difference in pushing the volume ΔV of fluid from the point A to B through the tube

$$W = p\Delta V = (p_1 - p_2)\,\Delta V \quad \text{...(i)}$$

The change in potential energy of mass Δm (of volume ΔV)

$$\Delta U = \Delta m g (h_2 - h_1) \quad \text{...(ii)}$$

The change in kinetic energy

$$\Delta K = \frac{1}{2}\Delta m\,(v_2^2 - v_1^2) \quad \text{...(iii)}$$

By conservation of energy

$$W = \Delta K + \Delta U$$

Putting the values from Eqs. (i), (ii) and (iii)

$$(p_1 - p_2)\,\Delta V = \frac{1}{2}\Delta m\,(v_2^2 - v_1^2) + \Delta m g\,(h_2 - h_1)$$

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2 \quad \left(\text{as } \rho = \frac{\Delta m}{\Delta V}\right)$$

$$\boxed{p + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}} \quad \text{...(i)}$$

This is called Bernoulli's equation.

Dividing both sides of Eq. (i) by ρg, we get

$$\frac{p}{\rho g} + h + \frac{v^2}{2g} = \frac{\text{constant}}{\rho g} = \text{new constant} \quad \text{...(ii)}$$

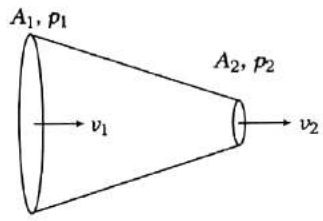
Here, $p/\rho g$ is called **pressure head,** h is called **gravitational head** and $v^2/2g$ is called **velocity head.**

Note **Bernoulli's equation for the fluid at rest** When a fluid is at rest *i.e.*, the velocity is zero everywhere, then the Bernoulli's equation becomes

$$p_1 + \rho g h_1 = p_2 + \rho g h_2 \Rightarrow p_1 - p_2 = \rho g\,(h_2 - h_1)$$

Example 13.18 *Calculate the rate of flow of glycerine of density $1.25\times 10^3\ kgm^{-3}$ through the conical section of a horizontal pipe, if the radii of its ends are 0.1 m and 0.04 m and the pressure drop across its length is 10 Nm^{-2}.*

Sol. According to the question, we draw the following diagram.



From continuity equation,

$$A_1 v_1 = A_2 v_2$$

or

$$\frac{v_1}{v_2} = \frac{A_2}{A_1} = \frac{\pi r_2^2}{\pi r_1^2}$$

$$= \left(\frac{r_2}{r_1}\right)^2 = \left(\frac{0.04}{0.1}\right)^2 = \frac{4}{25} \quad \text{...(i)}$$

From Bernoulli's equation,

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2 \quad (\text{As } h_1 = h_2)$$

or

$$v_2^2 - v_1^2 = \frac{2(p_1 - p_2)}{\rho}$$

or

$$v_2^2 - v_1^2 = \frac{2\times 10}{1.25\times 10^3} = 1.6\times 10^{-2}\ \text{m}^2\text{s}^{-2} \quad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get

$$v_2 \approx 0.128\ \text{ms}^{-1}$$

$\therefore$ Rate of volume flow through the tube

$$Q = A_2 v_2 = (\pi r_2^2) v_2 = \pi\,(0.04)^2 (0.128)$$

$$= 6.43\times 10^{-4}\ \text{m}^3\text{s}^{-1}$$

Applications based on Bernoulli's theorem

Applications based on Bernoulli's theorem are given below

1. Venturimeter

Figure shows a venturimeter used to measure flow speed in a pipe. We apply Bernoulli's equation to the wide (point 1) and narrow (point 2) parts of a horizontal pipe, with $h_1 = h_2$, $p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$

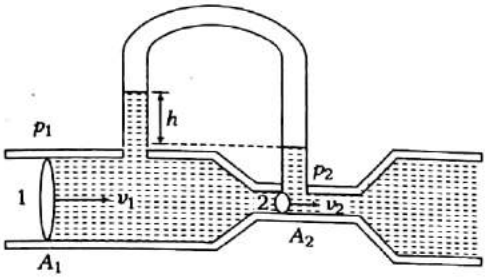


***Fig. 13.22** Venturimeter*

From the continuity equation, $v_2 = \frac{A_1 v_1}{A_2}$

substituting and rearranging, we get

$$p_1 - p_2 = \frac{1}{2}\rho v_1^2 \left(\frac{A_1^2}{A_2^2} - 1\right) \quad ...(i)$$

Because A_1 is greater than A_2, v_2 is greater than v_1 and hence the pressure p_2 is less than p_1. A net force to the right accelerates the fluid as it enters the narrow part of the tube (called throat) and a net force to the left slows as it leaves. The pressure difference is also equal to ρgh, where h is the difference in liquid level in the two limbs of a manometer tube. Substituting in Eq. (i), we get

$$\boxed{v_1 = \sqrt{\frac{2gh}{\left(\frac{A_1}{A_2}\right)^2 - 1}}}$$

Note

1. The discharge or volume flow rate can be obtained as,

$$\frac{dV}{dt} = A_1 v_1 = A_1 \sqrt{\frac{2gh}{\left(\frac{A_1}{A_2}\right)^2 - 1}}$$

2. The venturi effect can be used to give a qualitative understanding of the lift of an air plane wing and the path of a pitcher's curve ball. An airplane wing is designed, so that air moves faster over the top of the wing that it does under the wing, thus, making the air pressure less on top, than underneath. This difference in pressure results in a net upward lifting force on the wing.

Example 13.19 *The flow of blood in a large artery of an anesthetised dog is diverted through a venturimeter. The wider part of the meter has a cross-sectional area equal to that of the artery, $A = 16\,mm^2$. The narrower part has an area $a = 8mm^2$. The pressure drop in the artery is 24 Pa. What is the speed of the blood in the artery?*

Sol. Bernoulli's equation for the horizontal flow of blood is

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

By equation of continuity,

$$A v_1 = a v_2 \quad \text{or } v_2 = A v_1 / a$$

$$\therefore \quad p_1 - p_2 = \frac{1}{2}\frac{\rho A^2 v_1^2}{a^2} - \frac{1}{2}\rho v_1^2 = \frac{1}{2}\rho v_1^2\left[\frac{A^2}{a^2} - 1\right]$$

Here, $p_1 - p_2 = 24\,\text{Pa}$

ρ (blood) $= 1.06 \times 10^3$ kg m^{-3}, $A/a = 16/8 = 2$

$$\therefore \quad v_1 = \sqrt{\frac{2(p_1 - p_2)}{\rho\left(\frac{A^2 - 1}{a^2}\right)}} = \sqrt{\frac{2 \times 24}{1.06 \times 10^3 \times (2^2 - 1)}} = 0.125 \text{ ms}^{-1}$$

2. Speed of efflux

The outflow of a fluid is called efflux and the speed of the liquid coming out is called speed of efflux.

Consider a closed vessel filled with a liquid upto height H and a small hole is made in the wall of the vessel at a depth h below the surface of liquid. Then, the speed of efflux is,

$$\boxed{v = \sqrt{2gh}}$$

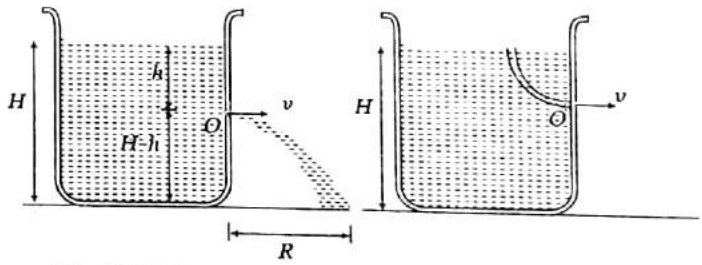


***Fig. 13.23** Liquid coming out from the orifice of a tank*

From the above formula it is clear that

"The velocity of efflux of a liquid issuing out of an orifice is the same as it would attain, if allowed to fall freely through the vertical height between the liquid surface and orifice."

The above statement is also known as **Torricelli's theorem.**

Range

The range of liquid (R) *i.e.*, the horizontal distance covered by liquid coming out of the hole is given by

or $$\boxed{R = 2\sqrt{h(H - h)}}$$

R is maximum at $h = \frac{H}{2}$ and $R_{max} = H$.

$$R^2 = 4(Hh - h^2)$$

For R to be maximum, $\frac{dR^2}{dh} = 0$

or $$H - 2h = 0 \text{ or } h = \frac{H}{2}$$

That is, R is maximum at $h = \frac{H}{2}$

and $$R_{max} = 2\sqrt{\frac{H}{2}\left(H - \frac{H}{2}\right)}$$

$\Rightarrow$ $$\boxed{R_{max} = H}$$

i.e., the maximum horizontal distance covered by liquid coming out of a hole is equal to the height of the liquid column.

- Time taken to empty a tank is given by the formula

$$\boxed{t = \frac{A}{a}\sqrt{\frac{2H}{g}}}$$

Example 13.20 *If the water emerge from an orifice in a tank in which the gauge pressure is 4×10^5 Nm^{-2} before the flow starts, then what will be the velocity of the water emerging out? Take density of water is 1000 kg m^{-3}.*

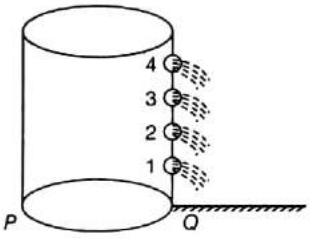
Sol. Here, $p = 4 \times 10^5$ N m^{-2} and $\rho = 1000$ kg m^{-3}, $g = 10$ m s^{-2}

Apply, $p = h\rho g \Rightarrow h = \frac{p}{\rho g} = \frac{4 \times 10^5}{1000 \times 10}$

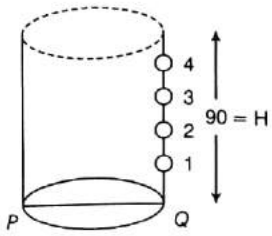
Velocity of efflux, $v = \sqrt{2gh} = \sqrt{\frac{2 \times 10 \times 4 \times 10^5}{1000 \times 10}}$

$$= \sqrt{800} = 28.28 \text{ ms}^{-1}$$

Example 13.21 *A cylindrical vessel of 90 cm height is kept filled with water upto the rim. It has four holes 1, 2, 3, 4 which are respectively at heights of 20 cm, 45 cm and 50 cm from the horizontal floor PQ. Through which of the holes water is falling at the maximum horizontal distance?*



Sol. Different holes of the cylindrical vessel are shown in the figure. Water comes out from different holes with different speeds. Therefore range will be different corresponding to different holes.



Horizontal range will be maximum

when $$h = \frac{H}{2} = \frac{90}{2} = 45 \text{ cm}$$

This height corresponds to hole 3.

Example 13.22 *A tank is filled with a liquid upto a height H. A small hole is made at the bottom of this tank. Let t_1 be the time taken to empty first half of the tank and t_2 the time taken to empty rest half of the tank.*

Then find the value of $\frac{t_1}{t_2}$.

Sol. As we know that, the time in which level of liquid in a tank falls from H_1 to H_2 is

$$t = \frac{A}{a}\sqrt{\frac{2}{g}}(\sqrt{H_1} - \sqrt{H_2})$$

where, A = area of cross-section of the tank,

a = area of hole

It is given that $H_1 = H$ and $H_2 = \frac{H}{2}$

$\therefore$ $$t_1 = \frac{A}{a}\sqrt{\frac{2}{g}}\left(\sqrt{H} - \sqrt{\frac{H}{2}}\right) \quad \text{...(i)}$$

Similarly time taken to empty the rest half of the tank

$$t_2 = \frac{A}{a}\sqrt{\frac{2(H/2)}{g}} = \frac{A}{a}\sqrt{\frac{H}{g}} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

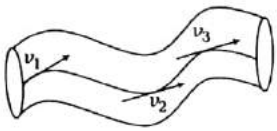
$$\frac{t_1}{t_2} = \sqrt{2} - 1$$

or $$\frac{t_1}{t_2} = 0.414$$

Note From here we see that $t_1 < t_2$. This is because initially the pressure is high and the liquid comes out with greater speed.

Check point 13.2

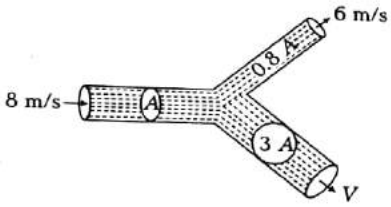
1. Consider streamline flow of a liquid flowing through a tube as shown in the figure which of the following is correct regarding velocities of liquid at different points?



(a) v_1 = constant, v_2 = constant, v_3 = constant
(b) $v_1 \neq v_2 \neq v_3$.
(c) $v_1 = v_2 = v_3$
(d) Both (a) and (b) are current

2. If R_e is the Reynold's number, then which of the following is incorrect
(a) For R_e flow is laminar
(b) For $1000 < R_e < 2000$, flow is steady
(c) For $R_e > 2000$ flow is turbulent
(d) All are incorrect

3. An incompressible liquid is flowing through a horizontal pipe as shown in figure. The value of speed v is



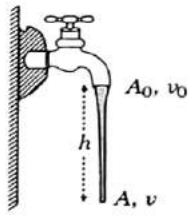
(a) 1.1 ms^{-1}
(b) 2.1 ms^{-1}
(c) 3.1 ms^{-1}
(d) 5.1 ms^{-1}

4. Bernoulli's theorem is a consequence of
(a) conservation of mass
(b) conservation of energy
(c) conservation of linear momentum
(d) conservation of angular momentum

5. Water is in streamline flows along a horizontal pipe with non-uniform cross-section. At a point in the pipe where the area of cross-section is 10 cm^{-2}, the velocity of water is 1 ms^{-1} and the pressure is 2000 Pa. The pressure at another point where the cross-sectional area is 5 cm^2 is
(a) 4000 Pa (b) 2000 Pa
(c) 1000 Pa (d) 500 Pa

6. If the velocity head of a stream of water is equal to 10 cm, then its speed of flow is approximately
(a) 1.0 ms^{-1} (b) 1.4 ms^{-1}
(c) 140 ms^{-1} (d) 10 ms^{-1}

7. Water falls from a tap with $A_0 = 4\,m^2$, $A = 1\,m^2$ and $h = 2$ m, then velocity v is



(a) 2.5 ms^{-1}
(b) 6.5 ms^{-1}
(c) 4.5 ms^{-1}
(d) 1.5 ms^{-1}

8. The velocity of efflux of a liquid through an orifice in the bottom of a tank does not depend upon
(a) density of liquid
(b) height of the liquid column above orifice
(c) acceleration due to gravity
(d) None of the above

9. The lift of an aeroplane is based on
(a) Torricelli's theorem (b) Bernoulli's principle
(c) law of gravitation (d) continuity equation

10. A tank is filled to a height H. The range of water coming out of a hole which is a depth $H/4$ from the surface of water level is
(a) $\frac{2H}{\sqrt{3}}$ (b) $\frac{\sqrt{3}H}{2}$
(c) $\sqrt{3}H$ (d) $\frac{3H}{4}$

4 Viscosity

The characteristic of fluid by virtue of which relative motion between different layers is opposed is known as viscosity.

Viscosity is internal friction in a fluid it occurs due to diffusion of molecules of one layer into another. Viscous forces opposes the motion of one portion of a fluid relative to the other. The simplest example of viscous flow is motion of a fluid between two parallel plates.

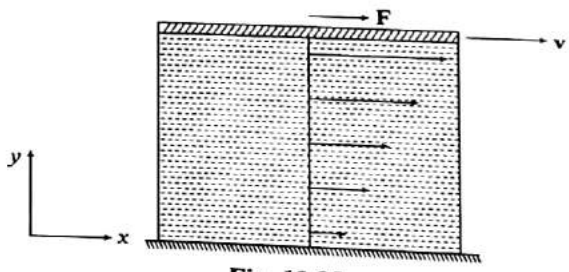


Fig. 13.25

The bottom plate is stationary and the top plate moves with constant velocity **v**. The fluid in contact with each surface has same velocity as that of the surface. The flow

speeds of intermediate layers of fluid increase uniformly from bottom to top, as shown by arrows. So, the fluid layers slide over one another.

According to Newton, the frictional force F (or viscous force) between *two layers depends upon the following factors,*

(i) Force F is directly proportional to the area (A) of the layers in contact, *i.e.,*

$$F \propto A$$

(ii) Force F is directly proportional to the velocity gradient $\left(\frac{dv}{dy}\right)$ between the layers. Combining these two, we have

$$F \propto A\frac{dv}{dy} \quad \text{or} \quad \boxed{F = -\eta A\frac{dv}{dy}}$$

Here, η is constant of proportionality and is called **coefficient of viscosity.** Its value depends on the nature of the fluid. The negative sign in the above equation shows that the direction of viscous force F is opposite to the direction of relative velocity of the layer.

The SI unit of η is Nsm^{-2}. It is also called decapoise or pascal second.

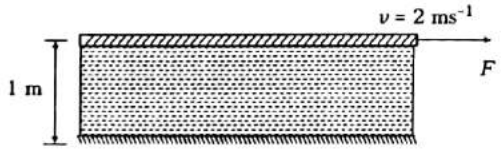
Thus, 1 decapoise = 1 Nsm^{-2} = 1 Pa-s = 10 poise

Dimensions of η are $[ML^{-1}T^{-1}]$

Coefficient of viscosity of water at 10°C is $\eta = 1.3 \times 10^{-3}\ Nsm^{-2}$. Experiments show that coefficient of viscosity of a liquid decreases as its temperature rises.

Example 13.23 *A plate of area $2m^2$ is made to move horizontally with a speed of $2\ ms^{-1}$ by applying a horizontal tangential force over the free surface of a liquid. The depth of the liquid is 1 m and the liquid in contact with the bed is stationary. Coefficient of viscosity of liquid is 0.01 poise. Find the tangential force needed to move the plate.*

Sol. Velocity gradient $= \frac{2-0}{1-0} = 2\ s^{-1}$



From Newton's law of viscous force,

$$|F| = \eta A\frac{\Delta v}{\Delta y} = (0.01 \times 10^{-1})(2)(2) = 4 \times 10^{-3}\ N$$

So, to keep the plate moving a force of 4×10^{-3} N must be applied.

Flow of liquid through a cylindrical pipe

Figure shows the flow speed profile for laminar flow of a viscous fluid in a long cylindrical pipe.

The speed is greatest along the axis and zero at the pipe walls.

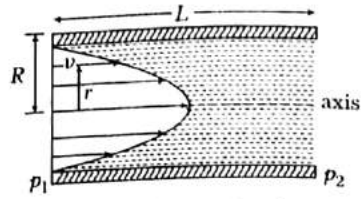


Fig. 13.24 A viscous fluid flowing in a long cylindrical pipe

The flow speed v at a distance r from the axis of a pipe of radius R is,

$$\boxed{v = \frac{p_1 - p_2}{4\eta L}(R^2 - r^2)}$$

where, p_1 and p_2 are the pressure at the two ends of a pipe with length L. The flow is always in the direction of decreasing pressure.

At $r = R$ (along the walls) $v = 0$,

The velocity of viscous fluid along the wall of the pipe is zero. and At $r = 0$ (along the axis)

$$v = \frac{(p_1 - p_2)R^2}{4\eta L} = v_{max}$$

The velocity of viscous fluid flowing along a pipe is maximum along its axis.

Volume flow rate $\left(Q \text{ or } \frac{dV}{dt}\right)$: Poiseuille equation

To find the total volume flow rate through the pipe, we consider a ring with inner radius r, outer radius $r + dr$ and cross-sectional area $dA = 2\pi r\, dr$. The volume flow rate through this element is $v\, dA$. The total volume flow rate is found by integrating from $r = 0$ to $r = R$.

The result is, $$\boxed{Q = \frac{dV}{dt} = \frac{\pi}{8}\left(\frac{R^4}{\eta}\right)\left(\frac{p_1 - p_2}{L}\right)}$$

The relation was first derived by Poiseuille and is called Poiseuille's equation.

Some important points related to Poiseuille equation are given below

(i) Poiseuille's equation can also be written as,

$$Q = \frac{p_1 - p_2}{\left(\frac{8\eta L}{\pi R^4}\right)} = \frac{\Delta p}{X} \quad \text{Here,} \quad X = \frac{8\eta L}{\pi R^4}$$

This equation can be compared with the current equation through a resistance, *i.e.,* $i = \frac{\Delta V}{R}$

Here, ΔV = potential difference and
R = electrical resistance

For current flow through a resistance, potential difference is a requirement similarly for flow of liquid through a pipe pressure difference is must.

(ii) Problems of series and parallel combination of pipes can be solved in the similar manner as is done in case of an electrical circuit. *The only difference is,*

(a) Potential difference (ΔV) is replaced by the pressure difference (Δp).

(b) The electrical resistance $R\left(=\rho\dfrac{L}{A}\right)$ is replaced by Flow resistance $X\left(=\dfrac{8\eta L}{\pi R^4}\right)$.

(c) The electrical current i is replaced by volume flow rate Q or dV/dt.

Combination of capillary (thin) tubes

(i) Series combination

(a) The rate of flow (Q) will be same.

(b) Pressure differece across each tube is different.

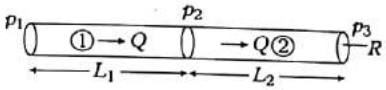


Fig. 13.26

By analogy, total flow resistance is,

$$X_{eq} = X_1 + X_2 = \frac{8\eta}{R^4}(L_1 + L_2)$$

Also, $p_1 - p_2 = QX_1$, $p_2 - p_3 = QX_2$

and $p_1 - p_3 = QX_{eq}$

(ii) Parallel combination

(a) The pressure difference across each tube is same.

(b) The rate of flow across each tube is different.

Flow resistance of tubes are $X_1 = \dfrac{8\eta L}{\pi R_1^4}$

and $X_2 = \dfrac{8\eta L}{\pi R_2^4}$

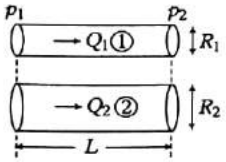


Fig. 13.27

By electrical analogy, total flow resistance of parallel tubes is given by, $\dfrac{1}{X_{eq}} = \dfrac{1}{X_1} + \dfrac{1}{X_2}$, $p_1 - p_2 = Q_1X_1 = Q_2X_2$

Also, $p_1 - p_2 = (Q_1 + Q_2)X_{eq}$

Following examples illustrates above discussion.

Example 13.24 *Water is flowing through a horizontal tube 8 cm in diameter and 4 km in length at the rate of 20 litre/s. Assuming only viscous resistance. Find the pressure required to maintain the flow in terms of mercury column. (Coefficient of viscosity of water is 0.001 Pa-s) is*

Sol. Here, $2r = 8$ cm $= 0.08$ m or $r = 0.04$m, $l = 4$ km $= 4000$ m,

$V = 20$ litre/s $= 20 \times 10^{-3}$ m^{3}s^{-1}, $\eta = 0.001$ Pa-s, $p = ?$

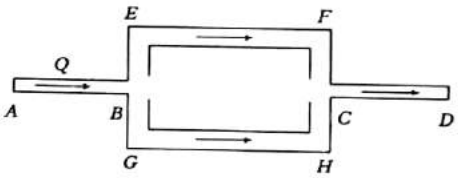
As, $$V = \frac{\pi p r^4}{8\eta l} \text{ or } p = \frac{8V\eta l}{\pi r^4}$$

$$= \frac{8 \times (20 \times 10^{-3}) \times 0.001 \times 4000}{\left(\frac{22}{7}\right) \times (0.04)^4} = 7.954 \times 10^4 \text{ Pa}$$

$\therefore$ Height of mercury column for pressure difference p will be,

$$h = \frac{p}{\rho g} = \frac{7.954 \times 10^4]}{(13.6 \times 10^3) \times 9.8} = 0.5968\text{m} = 59.68\text{ cm}$$

Example 13.25 *A liquid is flowing through horizontal pipes as shown in figure.*



Length of different pipes has the following ratio

$$L_{AB} = L_{CD} = \frac{L_{EF}}{2} = \frac{L_{GH}}{2}$$

Similarly, radii of different pipes has the ratio,

$$R_{AB} = R_{EF} = R_{CD} = \frac{R_{GH}}{2}$$

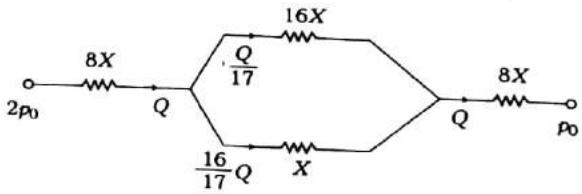
Pressure at A is $2p_0$ and pressure at D is p_0. The volume flow rate through the pipe AB is Q. Find,

(a) volume flow rates through EF and GH

(b) pressure at E and F.

Sol. The equivalent electrical circuit can be drawn as under,

$$X \propto \frac{L}{R^4} \qquad \left(\text{as } X = \frac{8\eta L}{\pi R^4}\right)$$



$$\therefore \quad X_{AB} : X_{CD} : X_{EF} : X_{GH}$$

$$= \frac{\frac{1}{2}}{\left(\frac{1}{2}\right)^4} : \frac{\left(\frac{1}{2}\right)}{\left(\frac{1}{2}\right)^4} : \frac{(1)}{\left(\frac{1}{2}\right)^4} : \frac{(1)}{(1)^4} = 8 : 8 : 16 : 1$$

(a) As the current is distributed in the inverse ratio of the resistance (in parallel). The Q will be distributed in the inverse ratio of X.

Thus, volume flow rate through EF will be $\frac{Q}{17}$ and that from GH will be $\frac{16}{17}Q$.

(b) $$X_{net} = 8X + \left[\frac{(16X)(X)}{(16X)+(X)}\right] + 8X = \frac{288}{17}X$$

$$\therefore \quad Q = \frac{\Delta p}{X_{net}} \qquad \left(\text{as } i = \frac{\Delta V}{R}\right)$$

$$= \frac{(2p_0 - p_0)}{\frac{288}{17}X} = \frac{17p_0}{288X}$$

Now, let p_1 be the pressure at E, then

$$2p_0 - p_1 = 8QX = \frac{8 \times 17p_0}{288}$$

$$\therefore \quad p_1 = \left(2 - \frac{17 \times 8}{288}\right)p_0 = 1.53\,P_0$$

Similarly, if p_2 be the pressure at F, then

$$p_2 - p_0 = 8QX$$

$$\therefore \quad p_2 = p_0 + \frac{8 \times 17}{288}p_0 \quad \text{or} \quad p_2 = 1.47\,p_0$$

5 Stoke's law

When an object moves through a fluid, it experiences a viscous force which acts in opposite direction of its velocity.

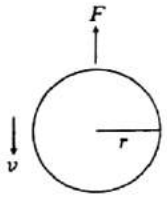


Fig. 13.28 An object is moving through a fluid

The formula for the viscous force on a sphere was first derived by the English physicist G. Stokes in 1843. According to him, a spherical object of radius r moving at velocity v experiences a viscous force given by

$$\boxed{F = 6\pi\eta r v} \quad (\eta = \text{coefficient of viscosity})$$

This law is called **Stoke's law.**

Terminal velocity (v_T)

Consider a small sphere falling from rest through a large column of viscous fluid.

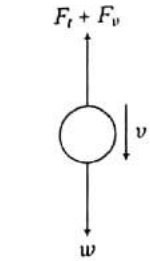


Fig. 13.29 Force acting on a small sphere falling through a liquid

The forces acting on the sphere are,

(i) weight w of the sphere acting vertically downwards.

(ii) upthrust buoyant force F_t acting vertically upwards.

(iii) viscous force F_v acting vertically upwards, *i.e.*, in a direction opposite to velocity of the sphere.

Initially, $F_v = 0$ (as $v = 0$)

and $w > F_t$

and the sphere accelerates downwards. As the velocity of the sphere increases, F_v increases. Eventually a stage in reached when

$$w = F_t + F_v \quad \text{...(i)}$$

After this net force on the sphere is zero and it moves downwards with a constant velocity called **terminal velocity (v_T).**

It can be defined as the maximum constant velocity acquired by a body while falling through a viscous medium.

Substituting proper values in Eq. (i) we have,

$$\frac{4}{3}\pi r^3 \rho g = \frac{4}{3}\pi r^3 \sigma g + 6\pi\eta r v_T \quad \text{...(ii)}$$

Here, ρ = density of sphere, σ = density of fluid

and η = coefficient of viscosity of fluid

From Eq. (ii), we get

$$\boxed{v_T = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta}}$$

Figure shows the variation of the velocity v of the sphere with time

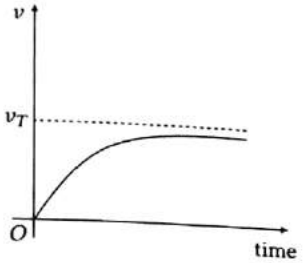


Fig. 13.30 *Variation of velocity of a sphere falling through a liquid, with time*

Note From the above expression we can see that terminal velocity of a spherical body is directly proportional to the difference in the densities of the body and the fluid ($\rho - \sigma$). If the density of fluid is greater than that of body (*i.e.*, $\sigma > \rho$), the terminal velocity is negative. This means that the body instead of falling, moves upward. This is why air bubbles rise up in water.

Example 13.26 *With what terminal velocity will an air bubble 0.8 mm in diameter rise in a liquid of viscosity 0.15 Nsm^{-2} and specific gravity 0.9. Density of air is 1.293 $kg\,m^{-3}$.*

Sol. The terminal velocity of the bubble is given by,

$$v_T = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta}$$

Here, $r = 0.4 \times 10^{-3}$ m,

$$\sigma = 0.9 \times 10^3 \text{ kgm}^{-3} \quad \rho = 1.293 \text{ kgm}^{-3}$$

$$\eta = 0.15, \text{Nsm}^{-2} \text{ and } g = 9.8 \text{ ms}^{-2}$$

Substituting the values, we have,

$$v_T = \frac{2}{9} \times \frac{(0.4 \times 10^{-3})^2 (1.293 - 0.9 \times 10^3) \times 9.8}{0.15}$$

$$= -0.0021 \text{ ms}^{-1}$$

or $$v_T = -0.21 \text{ cms}^{-1}$$

Note Here negative sign implies that the bubble will rise up.

Example 13.27 *Two spherical raindrops of equal size are falling vertically through air with a terminal velocity of 1 ms^{-1}. What would be the terminal speed, if these two drops were to coalesce to form a large spherical drop.*

Sol. As, $$v_T \propto r^2 \quad \text{...(i)}$$

Let r be the radius of small raindrops and R the radius of large drop.

Equating the volumes, we have

$$\frac{4}{3}\pi R^3 = 2\left(\frac{4}{3}\pi r^3\right)$$

$\therefore$ $$R = (2)^{1/3}.r \text{ or } \frac{R}{r} = (2)^{1/3}$$

$\therefore$ $$\frac{v_T'}{v_T} = \left(\frac{R}{r}\right)^2 = (2)^{2/3}$$

$\therefore$ $$v_T' = (2)^{2/3} v_T = (2)^{2/3}(1.0) \text{ ms}^{-1} = 1.587 \text{ ms}^{-1}$$

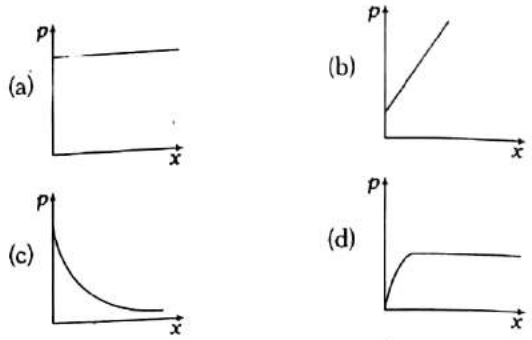
Check point 13.3

1. Units of coefficient of viscosity are
   (a) Nms^{-1} (b) Nm^2s^{-1}
   (c) $Nm^{-2}s$ (d) None of these
2. As the temperature of water increases, its viscosity
   (a) remains unchanged
   (b) decreases
   (c) increases
   (d) increases or decreases depending on the external pressure
3. The rate of flow of liquid in a tube of radius r, length l, whose ends are maintained at a pressure difference p is $V = \frac{\pi N p r^4}{\eta l}$, where η is coefficient of the viscosity and N is
   (a) 8 (b) $\frac{1}{8}$
   (c) 16 (d) $\frac{1}{16}$
4. Two capillary tubes of the same length but different radii r_1 and r_2 are fitted in parallel to the bottom of a vessel. The pressure head is p. What should be the radius of a single tube that can replace the two tubes so that the rate of flow is same as before
   (a) $r_1 + r_2$ (b) $\frac{r_1 r_2}{r_1 + r_2}$
   (c) $\frac{r_1 + r_2}{2}$ (d) None of these
5. The terminal velocity v of a small steel ball of radius r falling under gravity through a column of viscous liquid of coefficient of viscosity η depends on mass of the ball m, acceleration due to gravity g, coefficient of viscosity η and radius r. Which of the following relations is dimensionally correct?
   (a) $v \propto \frac{mgr}{\eta}$ (b) $v \propto mg\,\eta r$
   (c) $v \propto \frac{mg}{\eta r}$ (d) $v \propto \frac{\eta mg}{r}$
6. The ratio of the terminal velocities of two drops of radii R and $R/2$ is
   (a) 2 (b) 1
   (c) 1/2 (d) 4

7. An air bubble rises from the bottom of a lake of large depth. The rising speed of air bubble will
(a) go on increasing till it reaches surface
(b) go on decreasing till it reaches surface
(c) increases in two beginning, then will become constant
(d) be constant all throughout

8. Two equal drops of water are falling through air with a steady velocity v. If the drops coalesce, the new velocity will be
(a) $2v$ (b) $\sqrt{2}v$
(c) $2^{2/3}v$ (d) $\frac{v}{\sqrt{2}}$

9. From amongst the following curves, which one shows the variation of the velocity v with time t for a small sized spherical body falling vertically in a long column of a viscous liquid



10. A spherical ball of radius 3.0×10^{-4} m and density 10^4 kgm^{-3} falls freely uner gravity through a distance h before enternig a tank of water. If after entering the water the velocity of the ball does not change, what is the value of h (Viscosity of water is 9.8×10^{-6} Nsm^{-2}).
(a) 1.65×10^3 m (b) 2.65×10^2 m
(c) 3.65×10^4 m (d) 1.45×10^2 m

6 Surface tension

Surface tension is the property of liquid at rest by virtue of which a liquid surface tends to occupy a minimum surface area and behaves like stretched membrane.

A small drop of liquid is spherical in shape, because for given volume, sphere has minimum surface area.

This stretched behaviour is due to net downward force acting on a molecule on the surface of liquid as shown in figure. A molecule well inside the liquid experiences no net force.

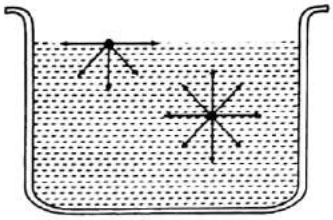


Fig. 13.31 Forces between molecules of a liquid

Now consider a line AB on the free surface of the liquid. The small elements of the surface on this line are in equilibrium because they are acted upon by equal and opposite forces, acting perpendicular to the line from either side as shown in figure.

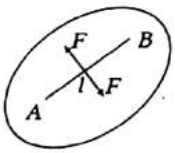


Fig. 13.32 Force on a line at the surface of a liquid

The force acting on this line is proportional to the length of this line. If l is the length of imaginary line and F the total force on either side of the line, then

$$F \propto l$$

$$\Rightarrow \quad F = Sl$$

or $$\boxed{\text{Surface tension, } S = \frac{\text{Force}}{\text{Length}}}$$

From this expression, **surface tension can be defined as the force acting per unit length of an imaginary line drawn on the liquid surface, the direction of force being perpendicular to this line and tangential to the liquid surface.** It is denoted by S and it is a **scalar quantity.**

Note The surface tension of a particular liquid usually decreases as temperature increases. To wash clothing throughly, water must be forced through the tiny spaces between the fibres. This requires increasing the surface area of the water, which is difficult to do because of surface tension. Hence, hot water and soapy water is better for washing.

Units and dimension of surface tension

SI units of surface tension = Nm^{-1}

CGS unit of surface tension = dyne cm^{-1}

Dimension of surface tension

$$= \frac{\text{Force}}{\text{Length}} = \frac{[MLT^{-2}]}{[L]}$$

$$= [ML^0T^{-2}]$$

Example 13.28 *A liquid is kept in a beaker of radius 4 cm. Consider a diameter of the beaker on the surface of the water. Find the force by which the surface on one side of the diameter pulls the surface on the other side. Surface tension of liquid = 0.075 Nm^{-1}*

Sol. The length of the diameter is

$$l = 2r = 8 \text{ cm} = 0.08 \text{ m}$$

The surface tension is $S = F / l$. Thus,

$$F = Sl = (0.075 \text{Nm}^{-1}) \times (0.08 \text{m}) = 6 \times 10^{-3} \text{ N}$$

Surface energy

The free surface of a liquid always has a tendency to contract and possess minimum surface area. To increase the surface area of the liquid work has to be done. This work done is stored in the surface film of the liquid as its potential energy. This potential energy per unit area of the surface film is called the surface energy.

Hence, **the surface energy may be defined as the amount of work done in increasing the area of the surface film through unity**. Thus,

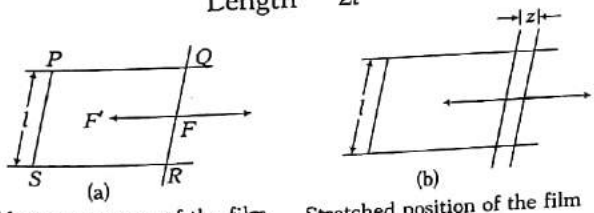
$$\text{Surface energy} = \frac{\text{Work done in increasing the surface area}}{\text{Increase in surface area}}$$

The SI unit of surface energy is Jm^{-2}.

Relation between surface energy and surface tension

Consider a rectangular frame *PQRS*. Here, wire *QR* is movable. A soap film is formed on the frame. The film pulls the movable wire *QR* inward due to surface tension.

As, surface tension $= \dfrac{\text{Force}}{\text{Length}} = \dfrac{F'}{2l} \Rightarrow F' = S \times 2l$



Equilibrium position of the film Stretched position of the film

Fig. 13.33

If *QR* is moved through a distance *z* by an external force *F* very slowly, then some work has to be done against this force.

$\therefore$ External work done = Force × Distance

$$= S \times 2l \times z \qquad [\because F' = F]$$

Increase in surface area of film $= 2l \times z$

[As soap film has two sides]

$$\text{Surface energy} = \frac{\text{Work done}}{\text{Surface area}} = \frac{S2lz}{2lz} = S$$

So, **value of surface energy of liquid is numerically equal to the value of surface tension.**

Note

1. Work done in forming a drop $W = S \times 4\pi r^2$ (as $\Delta A = 4\pi r^2 - 0 = 4\pi r^2$)
2. Work done in forming a bubble $W = s \times 4\pi r^2 \times 2$ (As bubble in air has two surfaces)

Example 13.29 *How much work will be done in increasing the diameter of a soap bubble from 2 cm to 5 cm. Surface tension of soap solution is 3.0×10^{-1} Nm^{-1}.*

Sol. Soap bubble has two surfaces.

Hence, $W = T\,\Delta A$

Here, $\Delta A = 2[4\pi\{(2.5 \times 10^{-2})^2 - (1.0 \times 10^{-2})^2\}]$

$$= 1.32 \times 10^{-2} \text{ m}^2$$

$\therefore$ Work done $W = (3.0 \times 10^{-2})(1.32 \times 10^{-2})$ J $= 3.96 \times 10^{-4}$ J

Example 13.30 *Calculate the energy released when 1000 small water drops each of radius 10^{-7} m coalesce to form one large drop. The surface tension of water is 7.0×10^{-2} Nm^{-1}.*

Sol. Let *r* be the radius of smaller drops and *R* of bigger one. Equating the initial and final volumes, we have

$$\frac{4}{3}\pi R^3 = (1000)\left(\frac{4}{3}\pi r^3\right)$$

or $R = 10r = (10)(10^{-7})\text{m}$

or $R = 10^{-6}$ m

Further, the water drops have only one free surface. Therefore,

$$\Delta A = 4\pi R^2 - (1000)(4\pi r^2)$$
$$= 4\pi\,[(10^{-6})^2 - (10^3)(10^{-7})^2]$$
$$= -36\pi(10^{-12})\text{m}^2$$

Here, negative sign implies that surface area is decreasing. Hence, energy released in the process.

$$U = T|\Delta A|$$
$$= (7 \times 10^{-2})(36\pi \times 10^{-12})\text{ J} = 7.9 \times 10^{-12}\text{ J}$$

Excess pressure

Due to surface tension, a drop or bubble tends to contract and so compresses the matter enclosed. This in turn increases the internal pressure which prevents further contraction and equilibrium is achieved. So, in equilibrium the pressure inside a bubble or drop is greater than outside and the difference of pressure between two sides of the liquid surface is called excess pressure.

1. Excess pressure inside soap bubble

A soap bubble consists of two spherical surface films with a thin layer of liquid between them. Because of surface tension, the film tend to contract in an attempt to minimize their surface area. But as the bubble contracts, it compresses

the inside air, eventually increasing the interior pressure to a level that prevents further contraction.

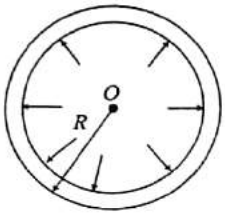


Fig. 13.34 A soap bubble of radius R

Excess pressure inside the soap bubble in terms of its radius R and the surface tension T of the liquid is given by

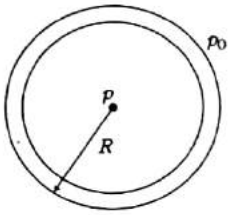


Fig. 13.35 Pressures inside and outside of a soap bubble

or
$$\boxed{\Delta p = p - p_0 = \frac{4T}{R}}$$

Example 13.31 *0.04 cm liquid column balances the excess pressure inside a soap bubble of radius 6 mm. Evaluate density of the liquid. Surface tension of soap solurtion = 0.03 Nm^{-1}.*

Sol. The excess pressure inside a soap bubble is

$$\Delta p = 4T / R = \frac{4 \times 0.03\,\text{Nm}^{-1}}{6 \times 10^{-3}\,\text{m}} = 20\,\text{Nm}^{-2}$$

The pressure due to 0.04 cm of the liquid column is

$$\Delta p = h\rho g = (0.04 \times 10^{-2}\text{m})\rho\,(10\,\text{ms}^{-2})$$

Thus, $20\,\text{Nm}^{-2} = (0.04 \times 10^{-2}\text{m})\,\rho(10\,\text{ms}^{-2})$

or, $\rho = 5 \times 10^3\,\text{kgm}^{-3}$

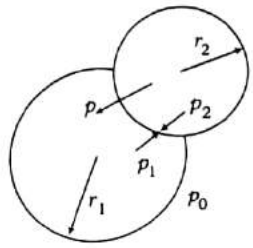
Example 13.32 *Two separate soap bubbles (radii 0.004 m and 0.002 m) formed of the same liquid (surface tension 0.07 Nm^{-1}) come together to form a double bubble. Find the radius and the sense of curvature of the internal film surface common to both the bubbles.*

Sol. Excess pressure inside first soap bubble

$$p_1 = p_0 + \frac{4T}{r_1}$$

Excess pressure inside second soap bubble

$$p_2 = p_0 + \frac{4T}{r_2}$$



$\because$ $r_2 < r_1$ $\therefore$ $p_2 > p_1$

i.e., pressure inside the smaller bubble will be more. The excess pressure

$$p = p_2 - p_1 = 4T\left(\frac{r_1 - r_2}{r_1 r_2}\right) \quad \text{...(i)}$$

This excess pressure acts from concave to convex side, the interface will be concave towards smaller bubble and convex towards larger bubble. Let R be the radius of interface then,

$$p = \frac{4T}{R} \quad \text{...(ii)}$$

From Eqs. (i) and (ii),

$$R = \frac{r_1 r_2}{r_1 - r_2} = \frac{(0.004)(0.002)}{(0.004 - 0.002)} = 0.004\text{ m}$$

2. Excess pressure inside a liquid drop

A liquid drop has only one surface film excess pressure inside the liquid drop is given by

or
$$\boxed{\Delta p = \frac{2T}{R}}$$

Some Important points regarding excess pressure

(i) If we have an air bubble inside a liquid, a single surface is formed. There is air on the concave side and liquid on the convex side. The pressure in the concave side (that is in the air) is greater than the pressure in the convex side (that is in the liquid) by an amount $2T/R$.

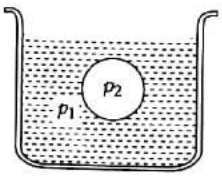


Fig. 13.36 Excess pressure in an air bubble inside a liquid

$$\therefore \quad p_2 - p_1 = \frac{2T}{R}$$

The above expression has been written by assuming p_1 to be constant from all sides of the bubble. For small size bubbles this can be assumed.

(ii) From the above discussion, we can make a general statement. The pressure on the concave side of a spherical liquid surface is greater than the convex side by $2T/R$ as shown below.

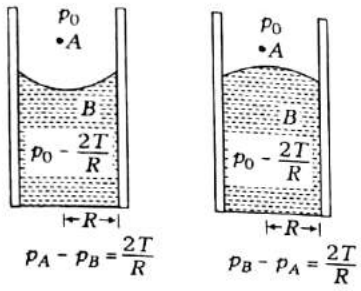


Fig. 13.37

(iii) Excess pressure inside a soap bubble can also be understood in terms of excess pressure inside a curved surface a shown below.

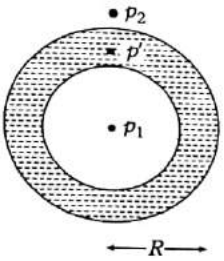


Fig. 13.38

$$p_1 - p' = \frac{2T}{R} \Rightarrow p' - p_2 = \frac{2T}{R}$$

Excess pressure $\Delta p = p_1 - p_2 = \frac{4T}{R}$

(iv) If two bubbles of different sizes are connected with each other through a thin tube (as shown in figure) then the air will rush from smaller to larger bubble (pressure inside smaller bubble will be greater than pressure inside larger bubble), so that the smaller will shrink while the larger will expand till the smaller bubble reduces to a droplet.

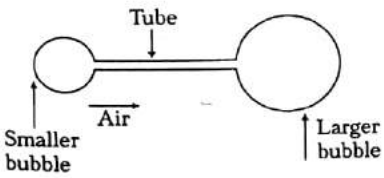


Fig. 13.39

Example 13.33 *What should be the pressure inside a small air bubble of 0.1 mm radius situated just below the water surface? Surface tension of water* $= 7.2 \times 10^{-2}\ Nm^{-1}$ *and atmospheric pressure* $= 1.013 \times 10^5\ Nm^{-2}$.

Sol. Surface tension of water $T = 7.2 \times 10^{-2}\ \text{Nm}^{-1}$

Radius of air bubble $R = 0.1\ \text{mm} = 10^{-4}\ \text{m}$

The excess pressure inside the air bubble is given by,

$$p_2 - p_1 = \frac{2T}{R}$$

$\therefore$ Pressure inside the air bubble, $p_2 = p_1 + \frac{2T}{R}$

Substituting the values, we have

$$p_2 = (1.013 \times 10^5) + \frac{(2 \times 7.2 \times 10^{-2})}{10^{-4}} = 1.027 \times 10^5\ \text{Nm}^{-2}$$

Angle of contact

The surface of liquid near the plane of contact with another medium is in curved shape.

The angle of contact is defined as the angle that the tangent to the liquid surface at the point of contact makes with the solid surface. It is denoted by θ.

The value of contact angle determines whether a liquid will spread on the surface of a solid or it will form droplets on it.

When a liquid is put on a solid surface, then there are three interfacial tensions at all the three interfaces; liquid-air, solid-air and solid-liquid denoted by S_{la}, S_{sa} and S_{sl}. At the line of contact, the surfaces between the three media must be in equilibrium.

The following cases arises

(i) If the surface tension at the solid-liquid S_{sl}, interface is greater than the surface tension at the liquid-air S_{la} interface, *i.e.*, $S_{sl} > S_{la}$, then $\theta > 90°$ (the angle of contact is **obtuse angle**).

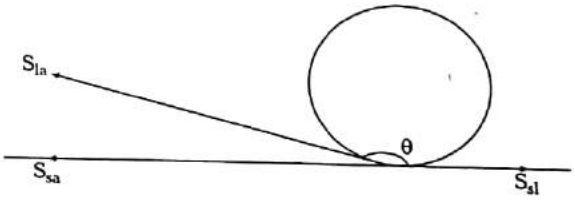


Fig. 13.40

The molecules of a liquid are attracted strongly to themselves and weakly to those of solid. It costs a lot of energy to create a liquid- solid surface. The liquid then does not wet the solid.

e.g., Water-leaf or glass-mercury interface.

(ii) If the surface tension at the solid-liquid S_{sl} interface is less than the surface tension at the liquid-air S_{la} interface, *i.e.*, $S_{sl} < S_{la}$, then $0 < 90°$ (the angle of contact is **acute angle**).

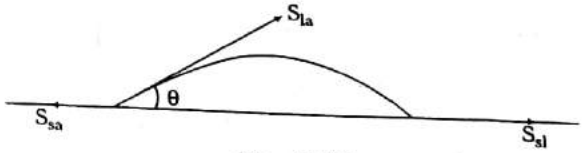


Fig. 13.41

The molecules o liquid are strongly attracted to those of solid and weak ttracted to themselves. It costs less energy to create a id-solid surface and liquid wets the solid.

e.g., When soap or gent is added to water, the angle of contact becomes s .

Shape of liquid surface

The curved surface of the liquid is called meniscus. The shape of the meniscus (convex or concave) is determined by the relative strengths of cohesive and adhesive forces. The force between the molecules of the same material is known as **cohesive force** and the force between the molecules of different kinds of material is called **adhesive force**.

When the adhesive force between solid and liquid molecules is more than the cohesive force between liquid-liquid molecules (as with water and glass), shape of the meniscus is concave and the angle of contact θ is less than 90°. In this case the liquid wets or adheres to the solid surface.

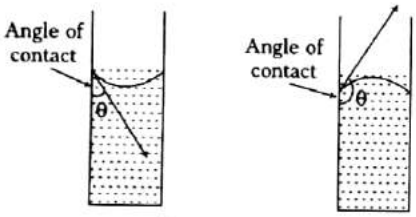


Fig. 13.42

When the adhesive force between solid and liquids molecules is less than the cohesive force between liquid-liquid molecules than shape of meniscus in convex.

Note The angle of contact between water and clean glass is zero and that between mercury and clean glass is 137°.

Capillarity

The term capilla means hair which is Latin word. A tube of very fine (hair-like) bore is called a **capillary tube.**

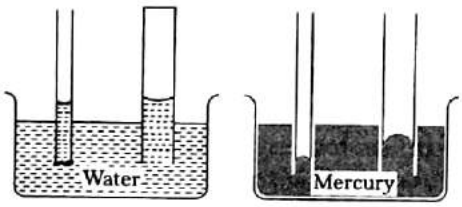


Fig. 13.43 Elevation and depression liquid in a capillary tube

If a capillary tube of glass is dipped in liquid like water, the liquid rises in the tube, but when the capillary tube is dipped in a liquid like mercury, the level of liquid falls in the tube. This phenomenon of rise or fall of a liquid in the capillary is called **capillarity.**

Some examples are

(i) Small capillaries in fibers of towels rocks water from our skin.

(ii) In trees sap rises in stem due to capillary action.

Formula of capillary rise

When a capillary tube is dipped in a liquid then the level of liquid in capillary tube rises or falls w.r.t. free surface of liquid outside the capillary.

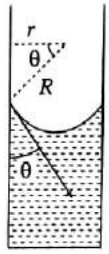


Fig. 13.44 Shape of meniscus

This phenomena of rises or fall of liquid is called capillary action.

The formula in capillary motion is

$$\boxed{h = \frac{2T\cos\theta}{r\rho g}}$$

where h = height of liquid column rises or falls

r = radius of capillary tube

ρ = density of liquid

g = acceleration due to gravity

θ = angle of contact and T = surface tension

The result has following notable features,

(i) If the contact angle θ is greater than 90°, the term cos θ is negative and hence, h is negative. The expression, then gives the depression of the liquid in the tube.

(ii) Suppose a capillary tube is held vertically in a liquid which has a concave meniscus, then capillary rise is given by,

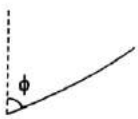
$$h = \frac{2T\cos\theta}{r\rho g} = \frac{2T}{R\rho g} \qquad \left(\text{as } R = \frac{r}{\cos\theta}\right)$$

or $$\boxed{hR = \frac{2T}{\rho g}} \qquad \text{...(ii)}$$

Note

1. If the tube is of a length l less than $h(l < h)$, the liquid does not overflow. The angle made by liquid surface with the tube adjusts in such a way that rise will stop when it is equal to length of tube.
2. If the tube makes an angle ϕ units vertical, rise will be

$$\boxed{h' = \frac{h}{\cos\phi}}$$



Example 13.34 *A capillary tube whose inside radius is 0.5 mm is dipped in water having surface tension 7.0×10^{-2} Nm^{-1}. To what height is the water raised above the normal water level? Angle of contact of water with glass is 0°. Density of water is 10^3 kg m^{-3} and $g = 9.8$ ms^{-2}.*

Sol. Height raised, $h = \dfrac{2T\cos\theta}{r\rho g}$

Substituting the proper values, we have

$$h = \frac{(2)(7.0\times 10^{-2})\cos 0°}{(0.5\times 10^{-3})(10^3)(9.8)}$$
$$= 2.86\times 10^{-2}\ \text{m}$$
$$= 2.86\ \text{cm}$$

Example 13.35 *A glass tube of radius 0.4 mm is dipped vertically in water. Find upto what height the water will rise in the capillary? If the tube in inclined at an angle of 60° with the vertical, how much length of the capillary is occupied by water. Surface tension of water = 7.0×10^{-2} Nm^{-1}, density of water = 10^3 kgm^{-3}.*

Sol. For glass-water, angle of contact $\theta = 0°$.

Now, height of water in capillary

$$h = \frac{2T\cos\theta}{r\rho g} = \frac{(2)(7.0\times 10^{-2})\cos 0°}{(0.4\times 10^{-3})(10^3)(9.8)}$$
$$= 3.57\times 10^{-2}\ \text{m} = 3.57\ \text{cm}$$

Length of capillary occupied after tilting it is,

$$l = \frac{h}{\cos 60°} = \frac{3.57}{1/2} = 7.14\ \text{cm}$$

Check point 13.4

1. Which of the following is not the unit of surface tension?
   (a) newton/metre (b) joule/(metre)^2
   (c) kg/(second)^2 (d) watt/metre
2. A square frame of side L is dipped in a liquid. On taking out, a membrane is formed. If the surface tension of the liquid is T, the force acting on one side of the frame will be
   (a) $2TL$ (b) $4TL$ (c) TL (d) $\frac{TL}{2}$
3. A 10 cm long wire is placed horizontal on the surface of water and is gently pulled up with a force of 2×10^2 N to keep the wire in equilibrium. The surface tension, in Nm^{-1} of water is
   (a) 0.1 (b) 0.2 (c) 0.001 (d) 0.002
4. If temperature increases, the surface tension of a liquid
   (a) increases
   (b) decreases
   (c) remains the same
   (d) first increases then decreases
5. If work W is done is blowing a bubble of radius R from soap solution, then the work done in blowing a bubble of radius $2R$ from the same solution is
   (a) $W/2$ (b) $2W$ (c) $4W$ (d) $2\frac{1}{3}W$
6. If two soaps bubble of equal radii r coalesce, then the radius of curvature of interface between two bubble will be
   (a) r (b) 0 (c) infinity (d) $\frac{r}{2}$
7. The surface tensionof soap solution is 25×10^{-3} Nm^{-1}. The excess pressure inside a soap bubble of diameter 1 cm is
   (a) 10 Pa (b) 20 Pa
   (c) 5 Pa (d) None of these
8. A liquid does not wet the sides of a solid, if the angle of contact is
   (a) zero (b) obtuse
   (c) acute (d) 90°
9. Surface tension is due to
   (a) friction forces between molecules
   (b) cohesive force between molecules
   (c) adhesive forces between molecules
   (d) gravitational forces
10. Water rises in a capillary tube to a height of 2.0 cm. In another capillary tube whose radius is one-third of it, how much the water will rise?
   (a) 6.0 cm (b) 2.0 cm
   (c) 4.0 cm (d) 8.0 cm

Chapter exercises

A Taking it together

(Assorted Questions of the Chapter for Advanced Level Practice)

1. The property of surface tension is predominantly obtained in
(a) solids, liquids and gases (b) liquids
(c) gases (d) matter

2. The water droplets in free fall are spherical due to
(a) gravity (b) viscosity
(c) surface tension (d) inter-molecular attraction

3. Due to capillary action, a liquid will rise in a tube, if the angle of contact is
(a) acute (b) obtuse (c) 90° (d) zero

4. A liquid wets a solid completely. The meniscus of the liquid in a sufficiently long tube is
(a) flat (b) concave
(c) convex (d) cylindrical

5. A glass plate is partly dipped vertically in the mercury and angle of contact is measured. If the plate is inclined, then the angle of contact will
(a) increase (b) remain unchanged
(c) increase or decrease (d) decrease

6. Hair of shaving brush cling together when it is removed from water due to
(a) force of attraction between hair
(b) surface tension
(c) viscosity of water
(d) characteristic property of hair

7. Coatings used on raincoat are waterproof because they
(a) increases angle of contact
(b) decreases angle of contact
(c) does not alters angle of contact
(d) form a smooth surface

8. Small droplets of liquid are usually more spherical in shape than larger drops of the same liquid because
(a) force of surface tension is equal and opposite to the force of gravity
(b) force of surface tension predominates the force of gravity
(c) force of gravity predominates the force of surface tension
(d) force of gravity and force of surface tension act in the same direction and are equal

9. Water does not wet an oily glass because
(a) cohesive force of oil > adhesive force between oil and glass
(b) cohesive force of oil > cohesive force of water
(c) oil repels water
(d) cohesive force of water > adhesive force between water and oil molecules

10. Which of the fact is not due to surface tension?
(a) Dancing of a camphor piece over the surface of water
(b) Small mercury drop itself becomes spherical
(c) A liquid surface comes at rest after stirring
(d) Mercury does not wet the glass vessel

11. On mixing the salt in water, the surface tension of water will
(a) increase (b) decrease
(c) remain unchanged (d) None of these

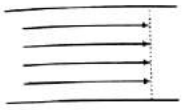
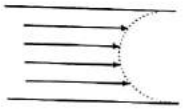
12. If two identical mercury drops are combined to form a single drop, then its temperature will
(a) decrease (b) increase
(c) remains the same (d) None of these

13. A water proofing agent changes the angle of contact
(a) from acute to 90°
(b) from obtuse to 90°
(c) from an acute to obtuse value
(d) from an obtuse to acute value

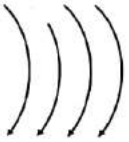
14. Along a streamline,
(a) the velocity of a fluid particle remains constant
(b) the velocity of all fluid particles crossing a given position is constant
(c) the valocity of all fluid particles at a given instant is constant
(d) the speed of a fluid particle remains constant

15. In a streamline flow
(a) the speed of a particle always remains same
(b) the velocity of a particle always remains same
(c) the kinetic energies of all particles arriving at a given point are the same
(d) the potential energies of all the particles arriving at a given point are the same

16. A viscous fluid is flowing through a cylindrical tube. The velocity distribution of the fluid is best represented by the diagram

(a) 
(b) 
(c)
(d) None of these

17. Which of the following diagrams does not represent a streamline flow?

(a)
(b)
(c) 
(d)

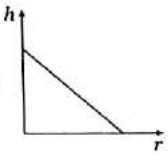
18. A body measures 5 N in air and 2 N when put in water. The buoyant force is
(a) 7 N (b) 9 N
(c) 3 N (d) None of these

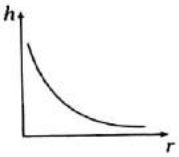
19. The reading of spring balance when a block is suspended from it in air, is 60 N. This reading is changed to 40 N when the block is immersed in water. The specific gravity of the block is
(a) 3 (b) 2
(c) 6 (d) 3/2

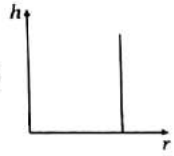
20. A hole is made at the bottom of the tank filled with water (density $= 1000\ \text{kgm}^{-3}$). If the total pressure at the bottom of the tank is three atmospheres (1 atmosphere $= 10^5\ \text{Nm}^{-2}$), then the velocity of efflux is nearest to
(a) $\sqrt{400}\ \text{ms}^{-1}$
(b) $\sqrt{200}\ \text{ms}^{-1}$
(c) $\sqrt{600}\ \text{m s}^{-1}$
(d) $\sqrt{500}\ \text{m s}^{-1}$

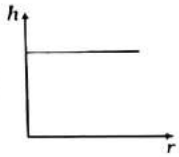
21. An iron block is on a boat which floats in a pond. The block is thrown into the water. The level of water in the pond will be
(a) equal to the earlier level
(b) less than the earlier level
(c) more than the earlier level
(d) depends on how large the block is

22. The correct curve between the height or depression h of liquid in a capillary tube and its radius r is

(a) 

(b) 

(c) 

(d) 

23. When two capillary tubes of different diameters are dipped vertically, the rise of the liquid is
(a) same in both the tubes
(b) more in the tube of larger diameter
(c) less in the tube of smaller diameter
(d) more in the tube of smaller diameter

24. The liquid in the capillary tube will rise if the angle of contact is
(a) 120° (b) 90° (c) obtuse (d) acute

25. The surface tension of liquid at its boiling point
(a) becomes zero
(b) becomes infinity
(c) is equal to the value at room temperature
(d) is half to the value at the room temperature

26. When the temperature is increased the angle of contact of a liquid?
(a) increases
(b) decreases
(c) remains the same
(d) first increases and then decreases

27. The surface tension of a soap solution is $2 \times 10^{-2}\ \text{Nm}^{-1}$. To blow a bubble of radius 1 cm, the work done is
(a) $4\pi \times 10^{-6}\ \text{J}$ (b) $8\pi \times 10^{-6}\ \text{J}$
(c) $12\pi \times 10^{-6}\ \text{J}$ (d) $16\pi \times 10^{-6}\ \text{J}$

28. A liquid rises in a vertical tube. The relation between the weight of the liquid in the tube, surface tension of the liquid T and radius of the tube r is given by, if the angle of contact is zero
(a) $W = \pi r^2 T$ (b) $W = 2\pi r T$
(c) $W = 2r^2 \pi T$ (d) $W = \frac{3}{4}\pi r^3 T$

29. When two soap bubbles of radius r_1 and $r_2 (r_2 > r_1)$ coalesce, the radius of curvature of common surface is

(a) $r_2 - r_1$ (b) $\frac{r_2 - r_1}{r_1 r_2}$

(c) $\frac{r_1 r_2}{r_2 - r_1}$ (d) $r_2 + r_1$

30. The excess pressure due to surface tension in a spherical liquid drop of radius r is directly proportional to

(a) r (b) r^2

(c) r^{-1} (d) r^{-2}

31. If two soap bubble of different radii are in communication with each other

(a) air flows from larger bubble into the smaller one

(b) the size of the bubbles remains the same

(c) air flows from the smaller bubble into the large one and the larger bubble grows at the expense of the smaller one

(d) None of the above

32. Assuming that the atmosphere has the same density anywhere as at sea level ($\rho = 1.3$ kgm^{-3}) and g to be constant ($g = 10$ ms^{-2}). What should be the approximate height of atmosphere? ($p_0 = 1.01 \times 10^5$ Nm^{-2})

(a) 6 km (b) 8 km

(c) 12 km (d) 18 km

33. If the angle of contact is less than 90° the pressure just inside the surface of a meniscus

(a) is less than atmospheric pressure

(b) is greater than atmospheric pressure

(c) is same as the atmospheric pressure

(d) None of the above

34. The spiders and insects move and run on the surface of water without sinking because

(a) elastic membrane is formed on water due to property of surface tension

(b) spiders and insects are lighter

(c) spiders and insects swim on water

(d) spiders and insects experience upthrust

35. If two glass plates are quite nearer to each other in water, then there will be force of

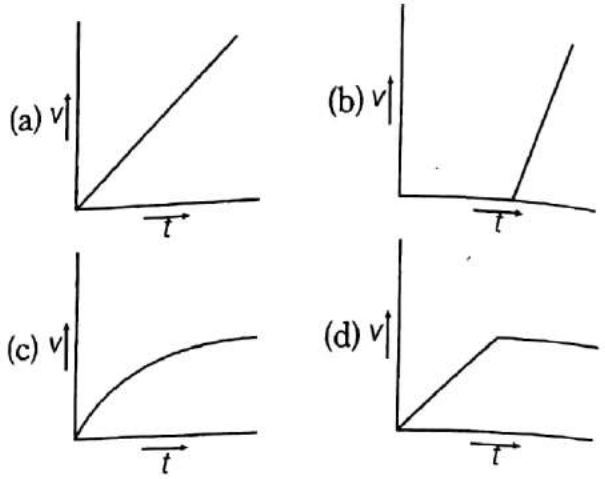
(a) attraction

(b) repulsion

(c) attraction or repulsion

(d) Neither attraction nor repulsion

36. A tall cylinder is filled with viscous oil. A round pebble is dropped from the top with zero initial velocity. From the plot shown in figure, indicate the one that represents the velocity (v) of the pebble as a function of time (t).



37. A tank if filled with water upto height H. When a hole is made at a distance h below the level of water. What will be the horizontal range of water jet?

(a) $2\sqrt{h(H-h)}$ (b) $4\sqrt{h(H+h)}$

(c) $4\sqrt{h(H-h)}$ (d) $2\sqrt{h(H+h)}$

38. A tank is filled to a height H. The range of water coming out of a hole which is a depth $H/4$ from the surface of water level is

(a) $\frac{2H}{\sqrt{3}}$ (b) $\frac{\sqrt{3}H}{2}$ (c) H (d) $\frac{3H}{4}$

39. The density of ice is x g cm^{-3} and that of water is y g cm^{-3} when m gram of ice melts, then the change in volume is

(a) $m(y-x)$ (b) $\frac{y-x}{m}$

(c) $my(y-x)$ (d) $\frac{m}{y} - \frac{m}{x}$

40. Two capillary tubes P and Q are dipped in water. The height of water level in capillary P is 2/3 to the height in Q capillary. The ratio of their diameters is

(a) 2 : 3 (b) 3 : 2 (c) 3 : 4 (d) 4 : 3

41. In a surface tension experiment with capillary tube water rises upto 0.1 m. If the same experiment is repeated on an artificial satellite, while is revolving around the earth, water will rise in the capillary tube upto a height of

(a) 0.1 m

(b) 0.2 m

(c) 0.98 m

(d) full length of the capillary tube

42. Two parallel glass plates are dipped partly in the liquid of density d keeping them vertical. If the distance between the plates is x surface tension for the liquid is T and angle of contact θ, then rise of liquid between the plates due to capillary will be

(a) $\frac{T\cos\theta}{xd}$ (b) $\frac{2T\cos\theta}{xdg}$ (c) $\frac{2T}{xdg\cos\theta}$ (d) $\frac{T\cos\theta}{xdg}$

43. A balloon has volume of 2000 m^3. It is filled with hydrogen ($\rho = 0.009\ gL^{-1}$). If the density of air is $129\ gL^{-1}$, it can lift a total weight of

(a) 600 kg (b) 2400 kg (c) 300 kg (d) 1800 kg

44. A raft of mass $M = 600$ kg floats in calm water with 7 cm submerged. When a man stands on the raft, 8.4 cm are submerged, the man's mass is

(a) 30 kg (b) 60 kg (c) 90 kg (d) 120 kg

45. A cylinder of mass M and density d_1 hanging from a string, is lowered into a vessel of cross-sectional area A, containing a liquid of density d_2 ($d_2 < d_1$) until it is fully immersed. The increase in pressure at the bottom of the vessel is

(a) $\frac{Md_2g}{d_1A}$ (b) $\frac{Mg}{A}$ (c) $\frac{Md_1g}{d_2A}$ (d) zero

46. Which of the following statements are true in case when two water drops coalesce and make a bigger drop?

(a) energy is released
(b) energy is absorbed
(c) the surface area of the bigger drop is greater than the sum of the surface areas of both the drops
(d) the surface area of the bigger drop is same that of the sum of the surface areas of both the drops

47. A piston of cross-sectional area 100 cm^2 is used in a hydraulic pressure to exert a force of 10^7 dyne on the water. The cross-sectional area of the other piston which support a truck of mass 2000 kg is

(a) $9.8\times 10^2\ cm^2$ (b) $9.8\times 10^3\ cm^2$
(c) $1.96\times 10^3\ cm^2$ (d) $1.96\times 10^4\ cm^2$

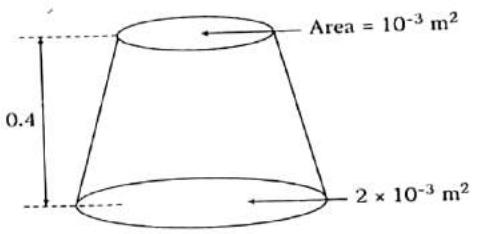
48. An ideal fluid flows through two pipes of circular cross-section with diameters 2.5 cm and 3.75 cm connected one after another. The ratio of the velocities in the two pipes is

(a) 9 : 4 (b) 3 : 2
(c) $\sqrt{3} : \sqrt{2}$ (d) $\sqrt{2} : \sqrt{3}$

49. The angle of contact at the interface of water-glass is 0°, ethyl alchol-glass is 0°, mercury-glass is 140° and methyliodide-glass is 30°. A glass capillary is put in a trough containing one of these four liquids. It is observed that the meniscus is convex. The liquid in the trough is

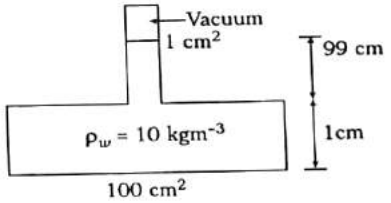
(a) water (b) ethylalcohol
(c) mercury (d) methyliodide

50. A uniformly tapering vessel is filled with a liquid of density 900 kgm^{-3}. The force that acts on the base of the vessel due to the liquid (excluding atmospheric force) is ($g = 10\ ms^{-2}$)



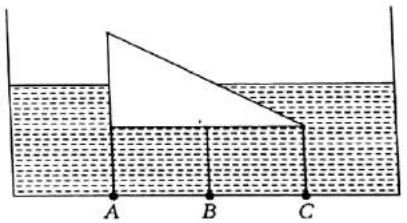
(a) 3.6 N (b) 7.2 N
(c) 9.0 N (d) 12.6 N

51. For the arrangement shown in the figure, the force at the bottom of the vessel is



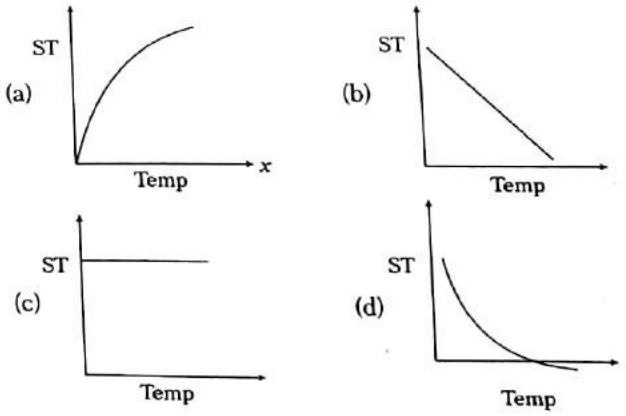
(a) 200 N (b) 100 N (c) 20 N (d) 2 N

52. An object of uniform density is allowed to float in water kept in a beaker. The object has triangular cross-section as shown in the figure. If the water pressure measured at the three points A, B and C below the object are p_A, p_B and p_C respectively. Then



(a) $p_A > p_B > p_C$ (b) $p_A > p_B < p_C$
(c) $p_A = p_B = p_C$ (d) $p_A = p_B < p_C$

53. Which graph represents the variation of surface tension with temperature over small temperature ranges for water?



54. A cylindrical vessel of 100 cm height is kept filled upto the brim. It has four holes 1, 2, 3, 4 which are respectively at heights of 27 cm, 30 cm, 50 cm and 80 cm from the horizontal floor. The water falling at the maximum horizontal distance from the vessel comes from
(a) hole number 4 (b) hole number 3
(c) hole number 2 (d) hole number 1

55. A solid shell loses half, its weight in water. Relative density of shell is 0.5 what fraction of its volume is hollow?
(a) $\frac{3}{5}$ (b) $\frac{2}{5}$ (c) $\frac{1}{5}$ (d) $\frac{4}{5}$

56. A tank full of water has a small hole at its bottom. Let t_1 be the time taken to empty the first half of the tank and t_2 be the time needed to empty the rest half of the tank, then
(a) $t_1 = t_2$ (b) $t_1 > t_2$
(c) $t_1 < t_2$ (d) $t_1 = 0.523 t_2$

57. An open U-tube contains mercury. When 11.2 cm of water is poured into one of the arms of the tube, how high does the mercury rise in the other arm from its initial level?
(a) 0.82 cm (b) 1.35 cm (c) 0.41 cm (d) 2.32 cm

58. A small ball (mass m) falling under gravity in a viscous medium experiences a drag force proportional to the instantaneous speed u such that $F_{drag} = ku$. Then the terminal speed of ball within viscous medium is
(a) $\frac{k}{mg}$ (b) $\frac{mg}{k}$
(c) $\sqrt{\frac{mg}{k}}$ (d) None of these

59. If a capillary tube of radius r is immerse in water, the mass of water risen in capillary is M. If the radius of capillary be doubles, the mass of water risen in the capillary will be
(a) $M/2$ (b) M
(c) $2M$ (d) $4M$

60. The pressure of water in a pipe when tap is closed is 5.5×10^5 Nm^{-2}. When tap gets open, pressure reduces to 5×10^5 Nm^{-2}. The velocity with which water comes out on opening the tap is
(a) 10 ms^{-1} (b) 5 ms^{-1}
(c) 20 ms^{-1} (d) 15 ms^{-1}

61. The level of water in a tank is 5 m high. A hole of area 1 cm^2 is made at the bottom of the tank. The rate of leakage of water from the hole is ($g = 10$ ms^{-2})
(a) 10^{-3} m^{3}s^{-1} (b) 10^{-4} m^{3}s^{-1}
(c) 10 m^{3}s^{-1} (d) 10^{-2} m^{3}s^{-1}

62. Water is flowing through two horizontal pipes of different diameters which are connected together. The diameters of the two pipes are 3 cm and 6 cm respectively. If the speed of water in the narrower tube is 4 ms^{-1}. Then the speed of water in the wider tube is
(a) 16 ms^{-1} (b) 1 m s^{-1}
(c) 4 m s^{-1} (d) 2 m s^{-1}

63. A block of wood floats in water with (4/5) th of its volume submerged. If the same block just floats in a liquid, the density of the liquid is (in kgm^{-3})
(a) 1250 (b) 600
(c) 400 (d) 800

64. Water flows along a horizontal pipe of non-uniform cross-section. The pressure is 1 cm of Hg where the velocity is 35 cms^{-1}. At a point where the velocity is 65 cms^{-1}, the pressure will be
(a) 0.89 cm of Hg (b) 0.62 cm of Hg
(c) 0.5 cm of Hg (d) 1 cm of Hg

65. For a body immersed in a liquid, when the weight of the body is less than the upthrust then the body will
(a) float partially immersed (b) sink
(c) float full immersed (d) both (a) and (c)

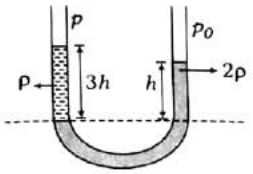
66. Three liquids of equal masses are taken in three identical cubical vessels A, B and C. Their densities are ρ_A, ρ_B and ρ_C respectively but $\rho_A < \rho_B < \rho_C$. The force exerted by the liquid on the base of the cubical vessel is
(a) maximum in vessel C (b) minimum in vessel C
(c) the same in all the vessels (d) maximum in vessel A

67. A solid of density D is floating in a liquid of density d. If v is the volume of solid submerged in the liquid and V is the total volume of the solid, then v/V equal to
(a) $\frac{d}{D}$ (b) $\frac{D}{d}$
(c) $\frac{D}{(D+d)}$ (d) $\frac{D+d}{D}$

68. An object weights m_1 in a liquid of density d_1 and that in liquid of density d_2 is m_2. The density of the object is
(a) $\frac{m_2 d_2 - m_1 d_1}{m_2 - m_1}$ (b) $\frac{m_1 d_1 - m_2 d_2}{m_2 - m_1}$
(c) $\frac{m_2 d_1 - m_1 d_2}{m_1 - m_2}$ (d) $\frac{m_1 d_2 - m_2 d_1}{m_1 - m_2}$

69. An ice-cube of density 900 kgm^{-3} is floating in water of density 1000 kg m^{-3}. The percentage of volume of ice-cube outside the water is
(a) 20% (b) 80% (c) 10% (d) 90%

70. In the figure shown,



(a) $p_0 > p$ (b) $p > p_0$ (c) $p = p_0$ (d) $p = 0$

71. A balloon has volume of $1000\ m^3$. It is filled with hydrogen ($\rho = 0.09 gL^{-1}$). If the density of air is $1.29\ gL^{-1}$, it can lift a total weight of

(a) 600 kg (b) 1200 kg
(c) 300 kg (d) 1800 kg

72. A boat having a length 3 m and breadth 2 m is floating on a lake. The boat sinks by 1 cm when a man gets on it. The mass of the man is

(a) 60 kg (b) 72 kg (c) 52 kg (d) 65 kg

73. A small block of wood of relative density 0.5 is submerged in water. When the block is released, it stats moving upwards, the acceleration of the block is ($g = 10\ ms^{-2}$)

(a) $5\ ms^{-2}$ (b) $10\ ms^{-2}$ (c) $7.5\ ms^{-2}$ (d) $15\ ms^{-2}$

74. A raft of wood of mass 120 kg floats in water. The weight that can be put on the raft to make it just sink,should be ($d_{raft} = 600\ kgm^{-3}$)

(a) 80 kg (b) 50 kg (c) 60 kg (d) 30 kg

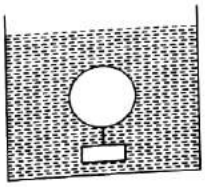
75. An object of w and density ρ is submerged in liquid of density σ, its apparent weight will be

(a) $(\rho - \sigma)$ (b) $(\rho - \sigma)/w$
(c) $w\left(1 - \frac{\sigma}{\rho}\right)$ (d) $w\left(1 - \frac{\rho}{\sigma}\right)$

76. A stone of relative density K is released from rest on the surface of a lake. If viscous effects are ignored, the stone sinks in water with an acceleration of

(a) $g(1 - K)$ (b) $g(1 + K)$
(c) $g\left(1 - \frac{1}{K}\right)$ (d) $g\left(1 + \frac{1}{K}\right)$

77. A body floats in a liquid contained in a beaker. The whole system shown in figure is falling under gravity, the upthrust on the body due to the liquid is



(a) zero
(b) equal to weight of the body in air
(c) equal to weight of liquid displaced
(d) equal to the weight of the immersed part of the body

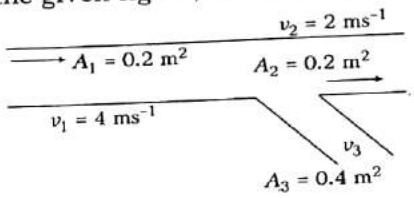
78. The relative density of ice is 0.9 and that of sea water is 1.125. What fraction of the whole volume of an iceberg appears above the surface of the sea?

(a) 1/5 (b) 2/5
(c) 3/5 (d) 4/5

79. A metallic sphere floats in immiscible mixture of water (density $10^3\ kgm^{-3}$) and a liquid (density $8\times 10^3\ kgm^{-3}$) such that its (2/3)part is in water and (1/3) part in the liquid. The density of the metal is

(a) $\frac{5000}{3} kgm^{-3}$ (b) $\frac{10000}{3} kgm^{-3}$
(c) $5000\ kgm^{-3}$ (d) $2000\ kgm^{-3}$

80. In the given figure, the velocity v_3 will be



(a) $2\ ms^{-1}$ (b) $4\ ms^{-1}$
(c) $1\ ms^{-1}$ (d) $3\ ms^{-1}$

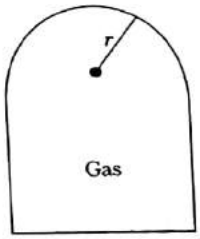
81. A water tank standing on the floor has two small holes punched in the vertical wall one above the other. The holes are 2.4 cm and 7.6 cm above the floor. If the jest of water from the holes hit the floor at the same point, then the height of water in the tank is

(a) 10 cm (b) 5 cm
(c) 20 cm (d) 4.8 cm

82. A body of volume V and density ρ is initially submerged in a non-viscous liquid of density σ ($> \rho$). If it is rises by itself through a height h in the liquid. Its kinetic energy will

(a) increase by $hV(\sigma - \rho)g$ (b) increase by $hV(\rho + \sigma)g$
(c) increase by $\frac{hV\rho g}{\sigma}$ (d) decrease by $\frac{hV\rho g}{\sigma}$

83. The pressure of the gas in a cylindrical chamber is p_0. The vertical force exerted by the gas on its hemispherical end is



(a) $p_0 r^2$
(b) $4p_0\pi r^2$
(c) $2p_0\pi r^2$
(d) $p_0\pi r^2$

84. Water rises in a capillary tube to a certain height such that the upward force due to surface tension is balanced by 75×10^{-4} N force due to the weight of the liquid. If the surface tension of water is $6\times 10^{-2}\ N\ m^{-1}$, the inner circumference of the capillary must be

(a) 1.25×10^{-2} m (b) 0.50×10^{-2} m
(c) 6.5×10^{-2} m (d) 12.5×10^{-2} m

85. Water rises upto 10 cm height in a long capillary tube. If this tube is immersed in water so that the height above the water surface is only 8 cm, then
(a) water flows out continuously from the upper end
(b) water rises upto upper end and forms a spherical surface
(c) water only rises upto 6 cm height
(d) water does not rise at all

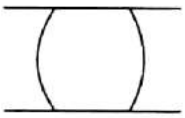
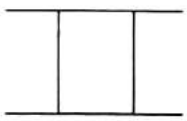
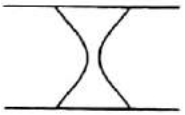
86. If a capillary tube is dipped into liquid and the levels of the liquid inside and outside are same, then the angle of contact is
(a) 120° (b) 90° (c) 45° (d) 30°

87. Water rises to a height of 30 mm in a capillary tube. If the radius of the capillary tube is made 3/4 of its previous value. The height to which the water will rise in the tube is
(a) 30 mm (b) 20 mm (c) 40 mm (d) 10 mm

88. A vessel, whose bottom has round holes with diameter of 1.0 mm is filled with water. The maximum height to which the water can be filled without leakage is (Surface tension of water = 75 dyne cm^{-1}, $g = 1000\ cms^{-2}$)
(a) 100 cm (b) 75 cm
(c) 50 cm (d) 30 cm

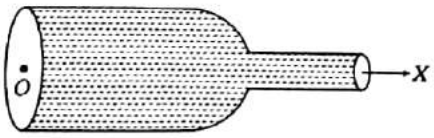
89. A long cylindrical glass vessel has a small hole of radius r at its bottom. The depth to which the vessel can be lowered vertically in the deep water bath (surface tension T) without any water entering inside is
(a) $4T/\rho rg$ (b) $3T/\rho rg$
(c) $2T/\rho rg$ (d) $T/\rho rg$

90. If a water drop is kept between two glass plates, then its shape is

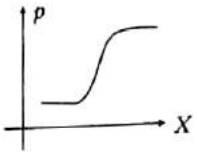
(a) 
(b) 
(c) 
(d) None of these

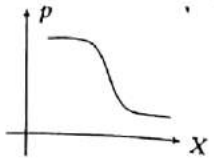
91. Two small drops of mercury each of radius r form a single large drop. The ratio of surface energy before and after this change is
(a) $2 : 2^{2/3}$ (b) $2^{2/3} : 1$ (c) $2 : 1$ (d) $1 : 2$

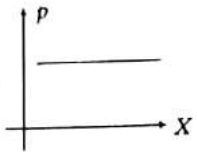
92. A non viscous liquid is flowing through a frictionless duct, cross-section varying as shown in figure.

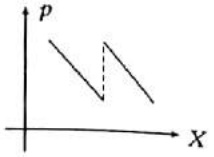


Which of the following graph represents the variation of pressure p along the axis of tube?

(a) 

(b) 

(c) 

(d) 

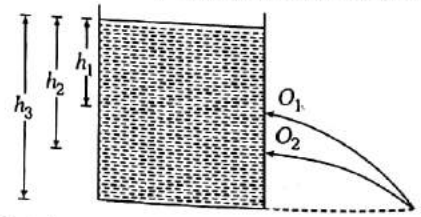
93. There is hole of area a at the bottom of a cylindrical of area A. Water is filled upto a height h and water flows out in t second. If water is filled to a height $4h$, it will flow out in time
(a) $\frac{t}{4}$ (b) $2t$ (c) $4t$ (d) $\frac{t}{2}$

94. A container has a small hole at its bottom. Area of cross-section of the hole is A_1 and that of the container is A_2. Liquid is poured in the container at a constant rate $Q\ m^3s^{-1}$. The maximum level of liquid in the container will be
(a) $\frac{Q^2}{2g A_1A_2}$ (b) $\frac{Q^2}{2g A_1^2}$ (c) $\frac{Q}{2g A_1A_2}$ (d) $\frac{Q^2}{2g A_2^2}$

95. Air stream flows horizontally past an aeroplane wing of surface area 4 m^2. The speed of air over the top surface is 60 ms^{-1} and under the bottom surface is 40 ms^{-1} The force of lift on the wing is (density of air = 1 kgm^{-3})
(a) 800 N (b) 1000 N (c) 4000 N (d) 3200 N

96. Water from a tap emerges vertically down with an initial speed of 1.0 ms^{-1}. The cross-sectional area of tap is $10^{-4}\ m^2$. Assume that the pressure is constant throughout the stream of water, and that the flow is steady. The cross-sectional area of the stream 0.15 m below the tap is
(a) $5.0\times 10^{-4}\ m^2$ (b) $1.0\times 10^{-5}\ m^2$
(c) $5.0\times 10^{-5}\ m^2$ (d) $2.0\times 10^{-5}\ m^2$

97. There are two holes O_1 and O_2 in a tank of height H. The water emerging from O_1 and O_2 strikes the ground at the same points, as shown in figure. Then



(a) $H = h_1 + h_2$ (b) $H = h_2 - h_1$
(c) $H = \sqrt{h_1h_2}$ (d) None of these

98. Two rain drops of same radius r falling with terminal velocity v merge and from a bigger drop of radius R. The terminal velocity of the bigger drop is

(a) $v\frac{R}{r}$ (b) $v\frac{R^2}{r^2}$ (c) v (d) $2v$

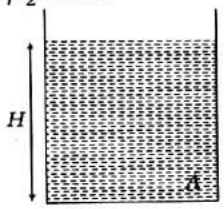
99. A cubical vessel of height 1m is full of water. What is the amount of work done in pumping water out of the vessel? (Take, $g = 10\text{ms}^{-2}$)

(a) 1250 J (b) 5000 J
(c) 1000 J (d) 2500 J

100. A cubical block of steel of each side equal to 1 is floating on mercury in vessel. The densities of steel and mercury are ρ_s and ρ_m. The height of the block above the mercury level is given by

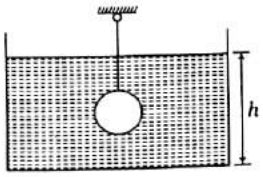
(a) $l\left(1+\frac{\rho_s}{\rho_m}\right)$ (b) $l\left(1-\frac{\rho_s}{\rho_m}\right)$
(c) $l\left(1+\frac{\rho_m}{\rho_s}\right)$ (d) $l\left(1-\frac{\rho_m}{\rho_s}\right)$

101. A cylindrical tank contains water up to a height H. If the tank is accelerated upwards with acceleration a, the pressure at the point A is p_1. If the tank is accelerated downwards with acceleration a the pressure at A is p_2. Then



(a) $p_1 < p_2$ (b) $p_1 = p_2$
(c) $p_1 > p_2$ (d) Data insufficient

102. A metal sphere connected by a string is dipped in a liquid of density ρ as shown in figure. The pressure at the bottom of the vessel will be, (p_0 = atmospheric pressure)



(a) $p = p_0 + \rho gh$ (b) $p > p_0 + \rho gh$
(c) $p < p_0 + \rho gh$ (d) p_0

103. An air bubble of radius 1 mm is formed inside water at a depth 10 m below free surface (where air pressure is $10^5\ \text{Nm}^{-2}$). The pressure inside the bubble is (surface tension of water = $2\times10^{-7}\ \text{Nm}^{-1}$).

(a) $2.28\times10^5\ \text{Nm}^{-2}$ (b) $2.0028\times10^5\ \text{Nm}^{-2}$
(c) $214\times10^5\ \text{Nm}^{-2}$ (d) $2.0014\times10^5\ \text{Nm}^{-2}$

104. A block of mass 4 kg and volume $5\times10^{-4}\ \text{m}^3$ is suspended by a spring balance in a lift which is accelerating. The apparent weight shown by the spring balance is 3 kg. Now the block is immersed in water in a container inside the lift. The apparent weight in kg shown by the spring balance is

(a) 2.375 (b) 2.625
(c) 2.5 (d) 3.125

105. A cubical block of wood of specific gravity 0.5 and chunk of concrete of specific gravity 2.5 are fastened together. The ratio of the mass of wood to the mass of concrete, which makes the combination to float with its entire volume submerged under water is

(a) 1/5 (b) 1/3
(c) 3/5 (d) 2/3

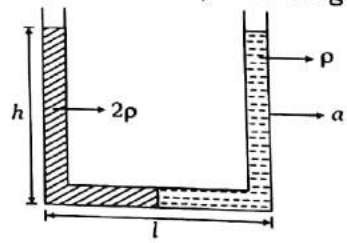
106. Two identical cylindrical vessels, each of base area A, have their bases at the same horizontal level. They contain a liquid of density ρ. In one vessel the height of the liquid is h_1 and in the other h_2 ($> h_1$). When the two vessels are connected, the work done by gravity in equalising the levels is

(a) $2\rho Ag\,(h_2-h_1)^2$ (b) $\rho Ag\,(h_2-h_1)^2$
(c) $\frac{1}{2}\rho Ag\,(h_2-h_1)^2$ (d) $\frac{1}{4}\rho Ag\,(h_2-h_1)^2$

107. A small ball of density ρ is immersed in a liquid of density $\sigma(\sigma > \rho)$ to a depth h and then released. The height above the surface of water up to which the ball will jump is

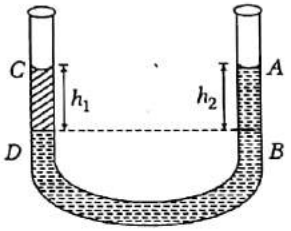
(a) $\frac{\sigma h}{\rho}$ (b) $\left(\frac{\sigma}{\rho}-1\right)h$
(c) $\left(1-\frac{\sigma h}{\rho}\right)h$ (d) $\frac{\rho h}{\sigma}$

108. A U-tube of base length l filled with same volume of two liquids of densities ρ and 2ρ is moving with an acceleration a on the horizontal plane. If the height difference between the two surfaces (open to atmosphere) becomes zero, then height h is given by



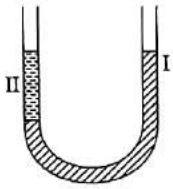
(a) $\frac{al}{g}$ (b) $\frac{3al}{2g}$
(c) $\frac{2al}{3g}$ (d) $\frac{al}{2g}$

109. In a U-tube experiment, a column AB of water is balanced by a column CD of paraffin. The relative density of paraffin is



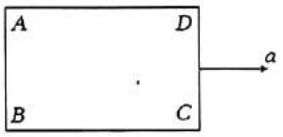
(a) $\frac{h_2}{h_1}$ (b) $\frac{h_1}{h_2}$
(c) $\frac{h_2 - h_1}{h_1}$ (d) $\frac{h_2}{h_1 + h_2}$

110. A U-tube of uniform cross-section shown in figure is partially filled with liquid I. Another liquid II which does not mix with I is poured into one side. The liquid levels of the two sides is found the same, while the level of liquid I has risen by 2 cm. If the specific gravity of liquid I is 1.1, then specific gravity of liquid II must be



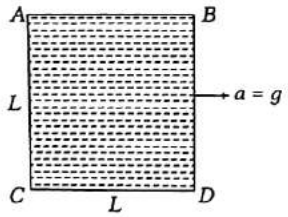
(a) 1.2 (b) 1.1 (c) 1.3 (d) 1.0

111. A closed rectangular tank is completely filled with water and is accelerated horizontally with an acceleration towards right. Pressure is



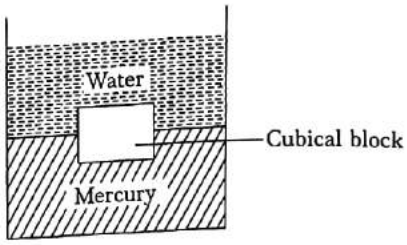
(i) maximum at, and (ii) minimum at
(a) (i)B (ii)D (b) (i)C (ii)D
(c) (i)B (ii)C (d) (i)B (ii)A

112. The liquid inside the container has density ρ. Choose the correct option.



(a) $p_A - p_C = 2\rho gL$ (b) $p_C - p_B = \sqrt{2}\rho gL$
(c) $p_C - p_D = \rho gL$ (d) $p_A - p_D = 0\,\rho gL$

113. A tank contains water on top of mercury as shown in figure. A cubical block of side 10 cm is in equilibrium inside the tank. The depth of the block inside mercury is (RD of the material of block = 8.56, RD of mercury = 13.6)

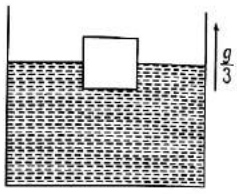


(a) 6 cm (b) 5 cm (c) 7 cm (d) 8 cm

114. If T is the surface tension of a liquid, the energy needed to break a liquid drop of radius R into 64 drops is
(a) $6\pi R^2 T$ (b) $\pi R^2 T$
(c) $12\pi R^2 T$ (d) $8\pi R^2 T$
(e) $4\pi R^2 T$

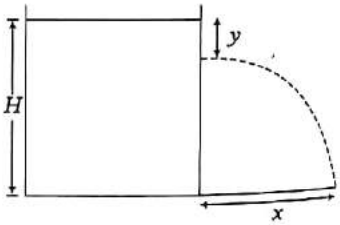
115. Two solid spheres of same metal but of mass M and $8M$ fall simultaneously on a viscous liquid and their terminal velocities are v and nv, then value of n is
(a) 16 (b) 8 (c) 4 (d) 2

116. A cubical block is floating in a liquid with half of its volume immersed in the liquid. When the whole system accelerates upwards with acceleration of $g/3$, the fraction of volume immersed in the liquid will be



(a) 1/2 (b) 3/8 (c) 2/3 (d) 3/4

117. A cylindrical vessel is filled with a liquid up to a height H. A small hole is made in the vessel at a distance y below the liquid surface as shown in figure. The liquid emerging from the hole strike the ground at distance x

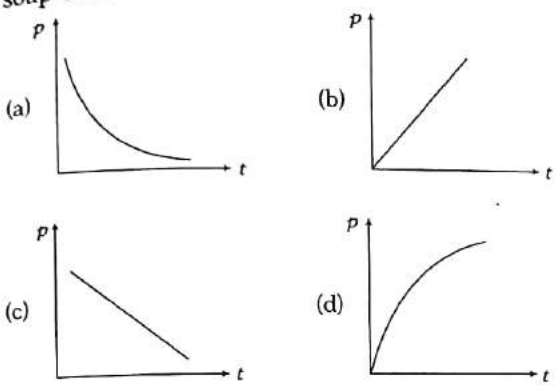


(a) x is equal if hole is at depth y or $H - y$
(b) x is maximum for $y = H/2$
(c) Both (a) and (b) are correct
(d) Both (d) and (d) are wrong

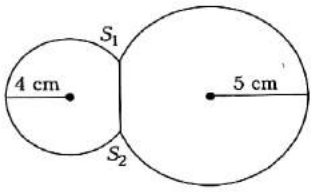
118. Water flows along a horizontal pipe whose cross-section in not constant. The pressure is 1 cm of Hg where the velocity is 35 cms^{-1}. At a point where the velocity is 65 cms^{-1}, the pressure will be
(a) 0.89 cm of Hg (b) 8.9 cm of Hg
(c) 0.5 cm of Hg (d) 1cm of Hg

119. Two capillary of lengths L and $2L$ and of radius R and $2R$ are connected in series. The net rate of flow of fluid through them will be (given rate of the flow through single capillary, $X = \pi p R^4 / 8\eta L$)
(a) $\frac{8}{9}X$ (b) $\frac{9}{8}X$ (c) $\frac{5}{7}X$ (d) $\frac{7}{5}X$

120. A soap bubble is blown with the help of a mechanical pump at the mouth of a tube. The pump produces a constant increase per minute in the radius of the bubble, irrespective of its internal pressure. The graph between the excess pressure inside the soap bubble and time t will be

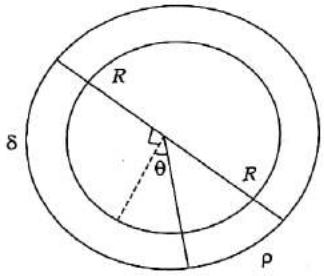


121. Two soap bubbles of radii r_1 and r_2 equal to 4 cm and 5 cm are touching each other over a common surface S_1S_2 (shown in figure). Its radius will be



(a) 4 cm (b) 20 cm (c) 5 cm (d) 4.5 cm

122. A uniform long tube is bent into a circle of radius R and it lies in vertical plane. Two liquids of same volume but densities ρ and δ fill half the tube. The angle θ is



(a) $\tan^{-1}\left(\frac{\rho-\delta}{\rho+\delta}\right)$ (b) $\tan^{-1}\left(\frac{\rho}{\delta}\right)$

(c) $\tan^{-1}\left(\frac{\delta}{\rho}\right)$ (d) $\tan^{-1}\left(\frac{\rho+\delta}{\rho-\delta}\right)$

123. Two substances of relative densities ρ_1 and ρ_2 are mixed in equal volume and the relative density of mixture is 4. When they are mixed in equal masses, the relative density of the mixture is 3. The values of ρ_1 and ρ_2 are

(a) $\rho = 6$ and $\rho_2 = 2$ (b) $\rho_1 = 3$ and $\rho_2 = 5$

(c) $\rho_1 = 12$ and $\rho_2 = 4$ (d) None of these

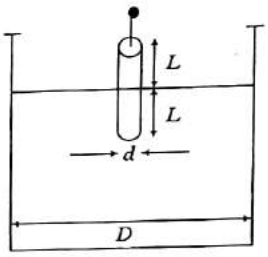
124. A large block of ice 10 cm thick with a vertical hole drilled through it is floating in a lake. The minimum length of the rope required to scoop out a bucket full of water through the hole is (density of ice $= 0.9\,\text{g cm}^{-3}$)

(a) 0.5 m (b) 1.0 m (c) 1.2 m (d) 1.8 m

125. A body of density ρ is dropped from rest from a height h into a lake of density $\sigma(\sigma > \rho)$. The maximum depth the body sinks inside the liquid is (neglect viscous effect of liquid)

(a) $\frac{h\rho}{\sigma-\rho}$ (b) $\frac{h\sigma}{\sigma-\rho}$ (c) $\frac{h\rho}{\sigma}$ (d) $\frac{h\sigma}{\rho}$

126. A candle of diameter d is floating on a liquid in a cylindrical container of diameter $D(D >> d)$ as shown in figure. If it is burning at the rate of 2 cmh^{-1}, then the top of the candle will



(a) remain at the same height

(b) fall at the rate 1 cmh^{-1}

(c) fall at the rate of 2 cmh^{-1}

(d) go up at the rate of 1 cmh^{-1}

127. A spherical object of mass 1kg and radius 1m is falling vertically downward inside a viscous liquid in a gravity free space. At a certain instant the velocity of the sphere is 2 m s^{-1}. If the coefficient of viscosity of the liquid is $\frac{1}{6\pi}$ SI units, then velocity of ball will become 0.5 ms^{-1} after a time.

(a) ln 4 s (b) 2 ln 4 s (c) 3 ln 4 s (d) 3 ln 2s

128. A wooden block of mass 8 kg is tied to a string attached to the bottom of the tank. In the equilibrium the block is completely immersed in water. If relative density of wood is 0.8 and $g = 10\ \text{ms}^{-2}$, the tension T, in the string is

(a) 120 N (b) 100 N (c) 80 N (d) 20 N

129. A metal ball immersed in alcohol weighs w_1 at 0°C and w_2 at 59°C. The coefficient of cubical expansion of the metal is less than that of alcohol. Assuming that the density of the metal is large compared to that of alcohol, it can be shown that

(a) $w_1 > w_2$ (b) $w_1 = w_2$

(c) $w_1 < w_2$ (d) $w_1 = (w_1/2)$

130. A barometer kept in an elevator reads 76 cm when it is at rest. If the elevator goes up with increasing speed, the reading will be

(a) zero (b) 76 cm

(c) > 76 cm (d) <76 cm

131. The surface energy of a liquid drop is S. It is sprayed into 1000 equal droplets. Then its surface energy becomes

(a) S (b) $10\,S$ (c) $100\,S$ (d) $1000\,S$

132. An open tank containing non-viscous liquid to a height of 5 m is placed over the ground. A heavy spherical ball falls from height 40 m over the ground in the tank. Ignoring air resistance find the height to which ball will go back. Collision between ball and bottom of tank is perfectly elastic

(a) 45 m (b) 35 m (c) 40 m (d) 20 m

133. A large open tank has two holes in the wall. One is a square hole of side L at a depth y from the top and the other is a circular hole of radius R at a depth $4y$ from the top. When the tank is completely filled with water, the quantities of water flowing out per second from holes are the same. Then R is equal to

(a) $L/\sqrt{2\pi}$ (b) $2\pi L$ (c) L (d) $L/2\pi$

134. A piece of steel has a weight w in air, w_1 when completely immersed in water and w_2 when completely immersed in an unknown liquid. The relative density (specific gravity) of liquid is

(a) $\frac{w-w_1}{w-w_2}$ (b) $\frac{w-w_2}{w-w_1}$ (c) $\frac{w_1-w_2}{w-w_1}$ (d) $\frac{w_1-w_2}{w-w_2}$

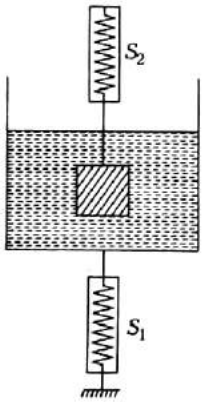
135. Two cylinders of same cross-section and length L but made of two materials of densities d_1 and d_2 are connected together to form a cylinder of length $2L$. The combination floats in a liquid of density d with a length $L/2$ above the surface of the liquid. If $d_1 < d_2$, then

(a) $d_1 < \frac{3}{4}d$ (b) $\frac{d}{2} > d_1$ (c) $\frac{d}{4} > d_1$ (d) $d_1 > \frac{d}{4}d$

136. A block of wood is floating on the surface of water in a beaker. The beaker is covered with a bell jar and the air is evacuated. What will happen to the block?

(a) Sink a little (b) Rise a little
(c) Remain unchanged (d) Sink completely

137. A beaker containing water is kept on a spring scale. The mass of water and beaker is 5 kg. A block of mass 2 kg and specific gravity 10 is suspended by means of thread from a spring balance as shown. The readings of scales S_1 and S_2 are respectively



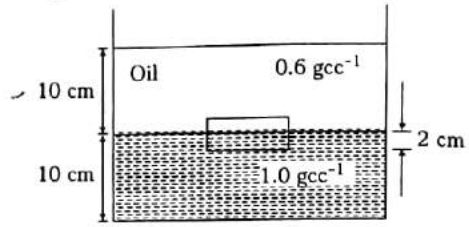
Take, $g = 10\,\text{ms}^{-2}$

(a) 52 N and 20 N
(b) 50 N and 18 N
(c) 52 N and 18 N
(d) 52 N and 22 N

138. A sphere of solid material of specific gravity 8 has a concentric spherical cavity and just sinks in water. Then the ratio of the radius of the cavity to the outer radius of the sphere must be

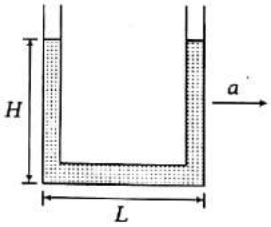
(a) $\frac{\sqrt[3]{3}}{2}$ (b) $\frac{\sqrt[3]{5}}{2}$
(c) $\frac{\sqrt[3]{7}}{2}$ (d) $\frac{2}{\sqrt[3]{7}}$

139. A cubical block of side 10 cm floats at the interface of an oil and water. The pressure above that of atmosphere at the lower face of the block is



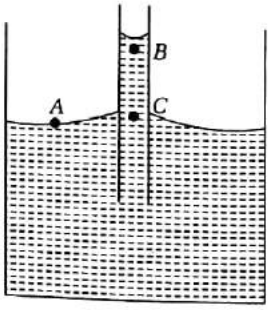
(a) 200 Nm^{-2} (b) 680 N m^{-2}
(c) 400 Nm^{-2} (d) 800 Nm^{-2}

140. A liquid stands at the plane level in U-tube when at rest. If areas of cross-section of both the limbs are equal, what will be the difference in heights h of the liquid in the two limbs of U-tube, when the system is given an acceleration a in horizontal direction towards right as shown?



(a) $\frac{g}{a}\frac{L^2}{H}$ (b) $\frac{La}{g}$ (c) $\frac{L^2}{H}\frac{a}{g}$ (d) $\frac{Lg}{a}$

141. A capillary tube is dipped in a liquid. Let pressure at points A, B and C be p_A, p_B, p_C respectively, then

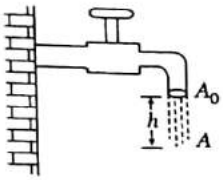


(a) $p_A = p_B = p_C$ (b) $p_A = p_B < p_C$
(c) $p_A = p_C < p_B$ (d) $p_A = p_C > p_B$

142. The volume of an air bubble becomes three times as it rises from the bottom of a lake to its surface. Assuming temperature to be constant and atmospheric pressure to be 75 cm of Hg and the density of water to be 1/10 of the density of the mercury, the depth of the lake is

(a) 5 m (b) 10 m (c) 15 m (d) 20 m

143. Figure shows how the stream of water emerging from a faucet necks donw as it falls. The area changes from A_0 to A through a fall of h. At what rate does the water flow from the tap?



(a) $A_0\sqrt{\frac{2ghA^2}{A_0^2 - A^2}}$ (b) $2A_0\sqrt{\frac{ghA^2}{A_0^2 - A^2}}$

(c) $A_0\sqrt{\frac{gh}{2}}$ (d) $2A\sqrt{\frac{ghA_0^2}{A_0^2 - A^2}}$

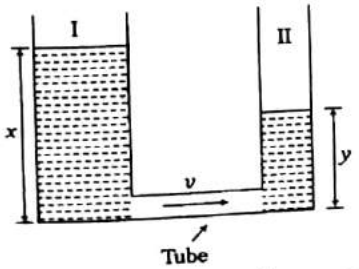
144. A ball of relative density 0.8 falls into water from a height of 2 m. The depth to which the ball will sink is (neglect viscous forces)

(a) 8 m (b) 2 m (c) 6 m (d) 16 m

145. A pump is designed as a horizontal cylinder with a piston area A and an outlet orifice arranged near the axis of the cylinder. Find the velocity of outflow of liquid from pump, if the piston moves with a constant velocity under the action of a constant force F. The density of liquid is ρ

(a) $\sqrt{\frac{F}{A\rho}}$ (b) $\sqrt{\frac{2F}{A\rho}}$ (c) $\sqrt{\frac{A\rho}{F}}$ (d) $\sqrt{\frac{A\rho}{2F}}$

146. If cross- sectional area of limb I is A_1 and that of limb II is A_2 then velocity of the liquid in the tube will be, (cross- sectional area of tube is very small)



(a) $\sqrt{2g(x-y)}$ (b) $\frac{A_1}{A_2}\sqrt{2g(x-y)}$

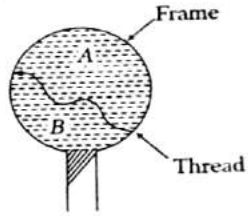
(c) $\frac{A_2}{A_1}\sqrt{2g(x-y)}$ (d) None of these

147. Two capillaries of same length and radii in the ratio 1:2 are connected in series. A liquid flows through them in streamlined condition. If the pressure across the two extreme ends of the combination is 1 m of water, then pressure difference across first capillary is

(a) 9.4 m (b) 4.9 m

(c) 0.49 m (d) 0.94 m

148. A thread is tied slightly loose to a wire frame as in figure and the frame is dipped into a soap solution and taken out. The frame is completely covered with the film.



When A is pricked

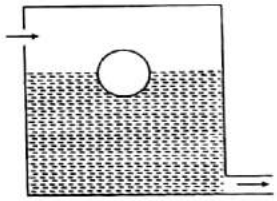
(a) thread will become concave on seeing from side A

(b) thread will become concave on seeing from side B

(c) thread will become straight

(d) thread will remain as it is

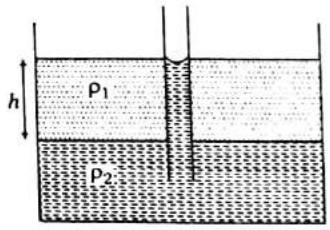
149. A tank filled with water has two taps to exhaust and pour. A hollow spherical ball is half submerged in water. Through one tap, water is taken out and through another tap, a liquid of density double the density of water is poured in tank such that volume of liquid in tank remains constant. Sphere will



(a) go down (b) go up

(c) maintain same height (d) sink to bottom

150. A container has two immiscible liquids of densities ρ_1 and ρ_2 ($> \rho_1$). A capillary tube of radius r is inserted in the liquid so that its bottom reaches upto the denser liquid. The denser liquid rises in the capillary and attains a height h from the interface of the liquids, which is equal to the column length of the lighter liquid. Assuming angle of contact to be zero, the surface tension of heavier liquid is



(a) $2\pi r\rho_2 gh$ (b) $\frac{\rho_2 rgh}{2}$

(c) $\frac{r}{2}(\rho_2 - \rho_1)gh$ (d) $2\pi r(\rho_2 - \rho_1)gh$

151. A ball of mass 1 kg falls from a height of 5m above the free surface of water. The relative density of the solid ball is $s = 2/3$. The ball travels a distance of 2m under water and becomes stationary. The work done by the resistive forces of water is

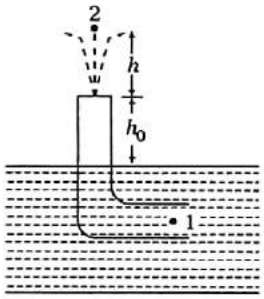
(a) −50 J (b) −20 J
(c) −40 J (d) −30 J

152. Under isothermal condition two soap bubbles of radii r_1 and r_2 coalesce to form a single bubble of radius r. The external pressure is p_0. The surface tension of the soap in terms of the given parameters is

(a) $\frac{2p_0(r^3 - r_1^3 - r_2^3)}{4(r_1^2 + r_2^2 - r^2)}$ (b) $\frac{p_0(r^3 - r_1^3 - r_2^3)}{4(r_1^2 + r_2^2 - r^2)}$

(c) $\frac{p_0(r^3 + r_1^2 + r_2^3)}{4(r_1^2 + r_2^2 + r^2)}$ (d) None of these

153. A bent tube is lowered into the stream as shown in the figure below. The velocity of the stream relative to the tube equal to v. The closed upper end of the tube located at the hight h_0. To what height h will the water jet spurt?



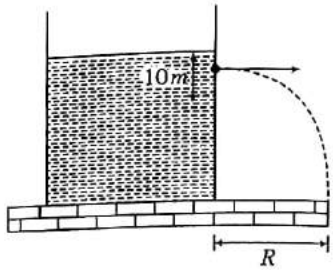
(a) $\frac{v^2}{2g}$ (b) $\frac{v^2}{2g} + h_0$

(c) $\frac{v^2}{2g} - h_0$ (d) $\frac{v^2}{4g} - h_0$

154. A solid ball of density half that of water falls freely under gravity from a height of 19.6 m and then enters water. Upto what depth will the ball go. How much time will it take to come again to the water surface. Neglect air resistance and viscosity effects in water. ($g = 9.8\ \text{ms}^{-2}$)

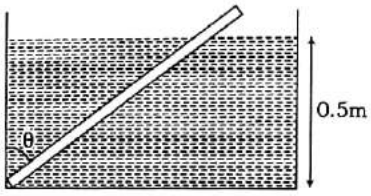
(a) 4 s (b) 8 s
(c) 6 s (d) 2 s

155. A large tank is filled with water (density $= 10^3\ \text{kgm}^{-3}$). A small hole is made at a depth 10 m below water surface. The range of water issuing out of the hole is R on ground. What extra pressure must be applied on the water surface so that the range become $2R$ (take 1 atm = 10^5 Pa and $g = 10\ \text{ms}^{-2}$)



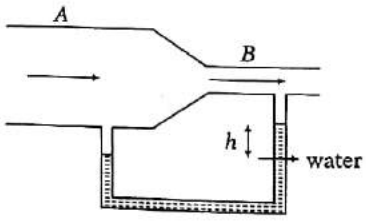
(a) 1 atm (b) 2 atm (c) 4 atm (d) 3 atm

156. A wooden plank of length 1m and uniform cross-section is hinged at one end to the bottom of a tank as shown. The tank is filled with water upto a height of 0.5 m. The specific gravity of the plank is 0.5. The angle θ made by the plank in equilibrium position is



(a) 30° (b) 45° (c) 60° (d) 90°

157. Air is blown through a pipe AB at a rate of 15 Lmin.$^{-1}$ The cross-sectional area of the broad portion of the pipe AB is 2 cm^2 and that of the narrow portion is 0.5 cm^2. The difference in water level h is (density of air = 1.32 kg m^{-3})



(a) 16 mm (b) 1.5 mm
(c) 10 mm (d) 3.2 mm

B Medical entrance special format questions

Assertion and reason

Direction (Q. Nos. 1-20) *These questions consists of two statements each printed as assertion and reason. While answering these questions you are required to choose any one of the following five responses.*

(a) If both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If Assertion is false but Reason is true.
(e) If both Assertion and Reason are false.

1. **Assertion** Density of an incompressible liquid is constant.

   Reason An ideal fluid is incompressible.

2. **Assertion** In a freely falling liquid container, upthrust force is zero.

   Reason In freely falling case value of effective value of g is zero.

3. **Assertion** Bulk modulus of an incompressible liquid is infinite.

   Reason Compressibility is inverse of bulk modulus.

4. **Assertion** If angle of contact is 0°, then liquid will neither rise nor fall in a capillary.

   Reason When angle of contact is 0°, surface is neither convex nor concave inside the capillary. It is flat.

5. **Assertion** Deep inside a liquid density is more than the density on surface.

   Reason Density of liquid increases with increase in depth.

6. **Assertion** Small water drops are spherical while bigger water drops are not.

   Reason In small water drops surface tension forces dominate while in bigger water drops gravity forces dominate.

7. **Assertion** When an ideal fluid flows through a pipe of non-uniform cross-section, then pressure is more at that section where area is more if the pipe is horizontal.

   Reason According to Bernoulli's theorem speed at broader cross-section will be less.

8. **Assertion** If ice is floating in water and it melts, then level of water remains unchanged.

   Reason When the ice is floating, weight of liquid displaced is equal to the weight of ice.

9. **Assertion** An ice ball is floating in water. Some stone pieces are embedded inside the ice. When ice will melt, level of water will fall.

   Reason In floating condition, stone pieces will displace more liquid compared to the condition when they sink.

10. **Assertion** A solid is floating in a liquid of density ρ_1. When the solid melts its density becomes ρ_2 in liquid state. If $\rho_1 > \rho_2$ level of liquids will increase after melting.

    Reason In liquid state volume always increases after a solid melts.

11. **Assertion** A solid sphere and a hollow sphere both of same material are immersed in a liquid, then change in weight in both the spheres will be same.

    Reason Upthrust depends upon the volume of the solid immersed not the mass.

12. **Assertion** If water is filled in a balloon and this is immersed in water itself. Then volume of water displaced is equal to the volume of water filled in the balloon.

    Reason Volume of a liquid displaced is equal to the volume of solid immersed in that liquid.

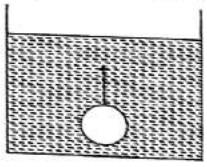
13. **Assertion** At same level of same liquid pressure is always same.

    Reason When any fluid travels from a region of higher pressure to lower pressure (at same levels) it gains same speed.

14. **Assertion** A solid is floating in a liquid. If temperature is increased and expansion of solid is ignored, then fraction of volume immersed will increase.

    Reason By increasing the temperature density of liquid will decrease.

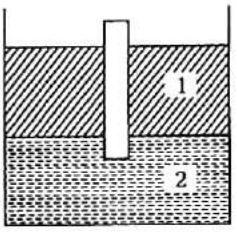
15. **Assertion** A ball is released from the bottom of a tank filled with a liquid. It moves upwards. In moving upwards upthrust will decrease.

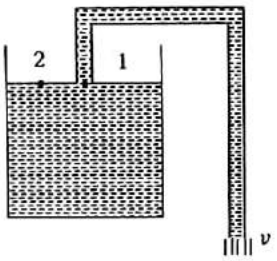
    Reason Density of ball is less than the density of liquid.

16. Assertion A wooden plank is floating in two liquids as shown. Net force applied by liquid-1 on plank is zero.

Reason Contribution in upthrust due to liquid -1 is $V_1\rho_1 g$. Where V_1 = volume immersed in liquid-1 and ρ_1 is the is the density of liquid -1



17. Assertion In the given figure shown, $p_1 = p_2$.

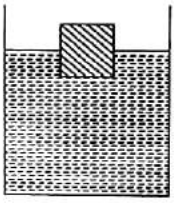


Reason Pressure at 1 is less than the atmospheric pressure.

18. Assertion A solid object of iron is dipped in water, both are at same temperature of 2°C. If the temperature of water is increased by 2°C, then the buoyancy force action of the object will increase.

Reason If we increase the temperature of water from 2°C to 4°C, then density of water will increase. Ignore expansion of solid sphere.

19. Assertion Mass of solid floating in liquid is m_1 and mass of liquid is m_2. Base area is A. Then pressure at bottom is $p_0 + \frac{(m_1 + m_2)g}{A}$.



Reason Upward force on liquid from base of vessel is pA, where p is pressure at bottom.

Match the columns

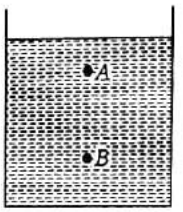
1. A liquid is flowing through a pipe of non-uniform cross-section. At a point where area of cross-section of the pipe is less, match the following columns.

	Column I		Column II
(A)	Volume of liquid flowing per second	(p)	Is less
(B)	Speed of liquid	(q)	Is more
(C)	Pressure of liquid	(r)	Is same

2. Two soap bubbles coalesce to form a single large drop. Match the following columns.

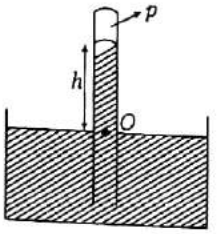
	Column I		Column II
(A)	Surface energy in the process will	(p)	Increase
(B)	Temperature of the drop will	(q)	Decrease
(C)	Pressure inside the soap bubble will	(r)	Remain same

3. There are two points A and B inside a liquid as shown in figure. Now the vessel starts moving upwards with an acceleration a. Match the following columns.



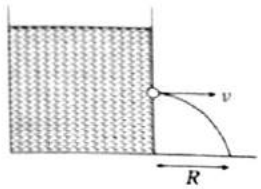
	Column I		Column II
(A)	Pressure at B will	(p)	Increase
(B)	Pressure difference between A and B will	(q)	Decrease
(C)	Upthrust on an object inside the vessel will	(r)	Remain same

4. A tube is inverted in a mercury vessel as shown in figure. If pressure p is increased, then



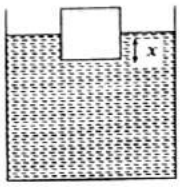
	Column II		Column II
(A)	Height h	(p)	Will increase
(B)	Pressure at O	(q)	Will decrease
(C)	Pressure at 1 cm above O	(r)	Will remain same

5. In the figure shown velocity of liquid which comes out is v, time of liquid to fall to ground is t and range on ground is R. If the vessel is taken to a mountain, match the following, (consider all cases which might possible)



	Column I		Column II
(A)	v	(p)	Will increase
(B)	t	(q)	Will decrease
(C)	R	(r)	Will remain same

6. A cube is floating in liquid as shown in figure. Match the following columns.



	Column I		Column II
(A)	If density of liquid decreases x will	(p)	Increase
(B)	If size of cube is increased x will	(q)	Decrease
(C)	If the whole system is accelerated upwards x will	(r)	Remain same

Medical entrances' gallery

(Collection of questions asked in NEET & various medical entrance exams)

1. A rectangular film of liquid is extended from (4 cm × 2cm) to (5cm × 4 × cm). If the work done is 3×10^{-4} J, the value of the surface tension of the liquid is **[NEET 2016]**

(a) $0.250\ Nm^{-1}$ (a) $0.125\ Nm^{-1}$
(c) $0.2\ Nm^{-1}$ (d) $8.0\ Nm^{-1}$

2. Three liquids of densities ρ_1, ρ_2 and ρ_3 (with $\rho_1 > \rho_2 > \rho_3$), having the same value of surface tension T, rise to the same height in three identical capillaries. The angles of contact θ_1, θ_2 and θ_3 obey **[NEET 2016]**

(a) $\frac{\pi}{2} > \theta_1 > \theta_2 > \theta_3 \geq 0$ (b) $0 \leq \theta_1 < \theta_2 < \theta_3 < \frac{\pi}{2}$
(c) $\frac{\pi}{2} < \theta_1 < \theta_2 < \theta_3 < \pi$ (d) $\pi > \theta_1 > \theta_2 > \theta_3 > \frac{\pi}{2}$

3. Two non-mixing liquids of densities ρ and $n\rho (n > 1)$ are put in a container. The height of each liquid is h. A solid cylinder of length L and density d is put in this container. The cylinder floats with its axis vertical and length pL $(p < 1)$ in the denser liquid. The density d is equal to **[NEET 2016]**

(a) $\{2 + (n + 1)p\}\rho$ (b) $\{2 + (n - 1)p\}\rho$
(c) $\{1 + (n - 1)p\}\rho$ (d) $\{1 + (n + 1)p\}\rho$

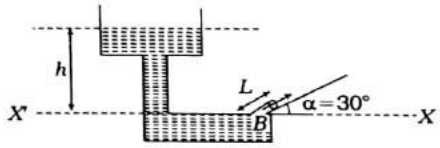
4. A wind with speed 40 ms^{-1} blows parallel to the roof of a house. The area of the roof is 250 m^2. Assuming that the pressure inside the house is atmospheric pressure, the force exerted by the wind on the roof and the direction of the force will be ($\rho_{air} = 1.2\ kgm^{-3}$) **[CBSE AIPMT 2015]**

(a) 4.8×10^5 N, downwards (b) 4.8×10^5 N, upwards
(c) 2.4×10^5 N, upwards (d) 2.4×10^5 N, downwards

5. The approximate depth of an ocean is 2700 m. The compressibility of water is $45.4 \times 10^{-11}\ Pa^{-1}$ and density of water is $10^3 kgm^{-3}$. What fractional compression of water will be obtained at the bottom of the ocean? **[CBSE AIPMT 2015]**

(a) 0.8×10^{-2} (b) 1×10^{-2} (c) 1.2×10^{-2} (d) 1.4×10^{-2}

6. Determine the height above the dashed line XX' attained by the water stream coming out through the hole is situated at point B in the diagram given below. Given that $h = 10$ m, $L = 2$ m and $\alpha = 30°$. **[AIIMS 2015]**



(a) 10 m (b) 7.1 m (c) 5 m (d) 3.2 m

7. A water drop of radius 1 cm is broken into 1000 equal droplets. What would be the gain in surface energy if the surface tension of water is $0.075\ Nm^{-1}$? **[UK PMT 2015]**

(a) 16.96×10^{-4} J (b) 8.48×10^{-4} J
(c) 4.24×10^{-4} J (d) 2.12×10^{-4} J

8. The lower end of a capillary tube is dipped into water and it is seen that the water rises through 7.5 cm in capillary tube. What would be the radius of capillary tube, if surface tension of water is 7.5×10^{-2} Nm^{-1}? The contact angle between water and glass is 0° and $g = 10\ ms^{-2}$. **[UK PMT 2015]**

(a) 0.2 cm (b) 0.1 cm (c) 0.4 mm (d) 0.2 mm

9. A soap bubble of diameter a is produced using the soap solution of surface tension T. Find the energy required to double the radius of the bubble without change of temperature. **[EAMCET 2015]**

(a) $2\pi a^2T$ (b) $6\pi a^2T$ (c) $3\pi a^2T$ (d) $12\pi a^2T$

10. A boat carrying a few number of big stones floats in a water tank. If the stones are unloaded into water, the water level **[Kerala CEE 2015]**

(a) rises till half the number of stones are unloaded and then begins to fall
(b) remains unchanged
(c) rises
(d) fall till half the number of stones are unloaded and then begins to rise
(e) falls

11. Choose the correct statement. **[Kerala CEE 2015]**

(a) Terminal velocities of rain drops are proportional to square of their radii
(b) Water proof agents decrease the angle of contact between water and fibres
(c) Detergents increase the surface tension of water
(d) Hydraulic machines work on the principle of Torricelli's law
(e) Venturimeter measures the flow speed of compressible fluids

12. A 20 cm long capillary tube is dipped vertically in water and the liquid rises upto 10 cm. If the entire system is kept is a freely falling platform, the length of the water column in the tube will be **[WB JEE 2015]**

(a) 5 cm (b) 10 cm (c) 15 cm (d) 20 cm

13. By sucking through a straw, a student can reduce the pressure in his lungs to 750 mm of Hg (density 13.6 gcm^{-3}) using the straw, he can drink water from a glass up to a maximum depth of **[UP CPMT 2015]**

(a) 10.2 cm (b) 75.3 cm (c) 13.6 cm (d) 1.96 cm

14. If 1000 drops are combined to form a larger drop, then the ratio of surface energy of smaller drop to the larger drop will be **[UP CPMT 2015]**

(a) 1 : 100 (b) 1 : 1000 (c) 1 : 10 (d) 0·1:1

15. A solid floats such that its 1/3 part is above the water surface. Then, the density of solid is **[UP CPMT 2015]**

(a) 744 kgm^{-3} (b) $\frac{1000}{3}$ kgm^{-3}
(c) $\frac{2000}{3}$ kgm^{-3} (d) 910 kgm^{-3}

16. The amount of work done in blowing a soap bubble such that its diameter increases from d to D is (S = surface tension of the solution) **[Manipal 2015]**

(a) $2\pi\ (D^2 - d^2)S$ (b) $\pi(D^2 - d^2)S$
(c) $4\pi(D^2 - d^2)S$ (d) $8\pi(D^2 - d^2)S$

17. With the increase in temperature, the angle of contact **[Manipal 2015]**

(a) increases
(b) decreases
(c) remains constant
(d) first increases and then decreases

18. A bubble is at the bottom of the lake of depth h. As the bubble comes to sea level, its radius increases three times. If atmospheric pressure is equal to l metre of water column, then h is equal to **[UK PMT 2014]**

(a) $26\,l$ (b) l (c) $25\,l$ (d) $30\,l$

19. A wooden block is floating on water kept in a beaker. 40% of the block is above the water surface. Now the beaker is kept inside a lift that starts going upward with acceleration equal to $g/2$. The block will then **[WB JEE 2014]**

(a) sink
(b) float with 10% above the water surface
(c) float with 40% above the water surface
(d) float with 70% above the water surface

20. A small metal sphere of radius a is falling with a velocity v through a vertical column of a viscous liquid. If the coefficient of viscosity of the liquid is η, then the sphere encounters an opposing force of **[WB JEE 2014]**

(a) $6\pi\eta a^2v$ (b) $\frac{6\eta v}{\pi a}$
(c) $6\pi\eta av$ (d) $\frac{\pi\eta v}{6a^3}$

21. A flow of liquid is streamline, if the Reynolds' number is **[CBSE AIPMT 2014]**

(a) less than 1000 (b) greater than 1000
(c) between 2000 to 3000 (d) between 4000 to 5000

22. A certain number of spherical drops of a liquid of radius r coalesce to form a single drop of radius R and volume V. If T is the surface tension of the liquid, then **[CBSE AIPMT 2014]**

(a) energy $= 4VT\left(\frac{1}{r} - \frac{1}{R}\right)$ is released
(b) energy $= 3VT\left(\frac{1}{r} + \frac{1}{R}\right)$ is absorbed
(c) energy $= 3VT\left(\frac{1}{R} - \frac{1}{r}\right)$ is released
(d) energy is neither released nor absorbed

23. A drop of some liquid of volume 0.04 cm^3 is placed on the surface of a glass slide. Then, another glass slide is placed on it in such a way that the liquid forms a thin layer of area 20 cm^2 between the surfaces of the two slides. To separate the slides a force of 16×10^5 dyne has to be applied normal to the surfaces. The surface tension of the liquid is (in dyne cm^{-1}) **[WB JEE 2014]**

(a) 60 (b) 70 (c) 80 (d) 90

24. Under isothermal condition, energy E is supplied to a soap bubble of surface tension σ and radius r, to double the radius of the soap bubble. The value of E is **[EAMCET 2014]**

(a) $16\pi r^2\sigma$ (b) $24\pi r^2\sigma$ (d) $8\pi r^2\sigma$ (d) $12\pi r^2\sigma$

25. A 20 cm long capillary tube is dipped in water. The water rises upto 8 cm. If the entire arrangement is put in a freely falling elevator, the length of water column in the capillary tube will be **[UK PMT 2014]**

(a) 4 cm (b) 20 cm (c) 8 cm (d) 10 cm

26. The wettability of a surface by a liquid depends primarily on **[NEET 2013]**

(a) viscosity

(b) surface tension of liquid and air

(c) density

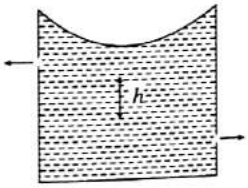
(d) angle of contact between the surface and the liquid

27. "The pressure exerted at any point in an enclosed fluid is transmitted equally in all directions". This is known as **[J&KCET 2013]**

(a) Archimedes' principle (b) Law of floatation

(c) Pascal's law (d) Bernoulli's principle

28. There are two identical small holes on the opposite sides of a tank containing a liquid. The tank is open at the top. The difference in height between the two holes is h. As the liquid comes out of the two holes, the tank will experience a net horizontal force proportional to **[J&KCET 2013]**



(a) $\sqrt{h}$ (b) h (c) $h^{3/2}$ (d) h^2

29. Water rises in a capillary tube upto a height of 10 cm whereas mercury depresses in it by 3.42 cm. If the angle of contact and density of mercury are 135° and 13.6 gcc^{-1} respectively, then the ratio of the surface tension of water and mercury will be nearly **[EAMCET 2013]**

(a) 13 : 2 (b) 5 : 16 (c) 16 : 5 (d) 2 : 13

30. Two capillary tubes of lengths in the ratio 2 : 1 and radii in the ratio 1 : 2 are connected in series. Assume the flow of the liquid through the tube is steady. Then, the ratio of pressure difference across the tubes is **[EAMCET 2013]**

(a) 1 : 8 (b) 1 :16 (c) 32 : 1 (d) 1 : 1

31. Two spherical soap bubbles of diameters 10 cm and 6 cm are formed, one at each end of a narrow horizontal glass tube. If the surface tension of the soap solution is 0.03 Nm^{-1}, then the pressure difference in pascal between the two ends of the tube is **[Kerala CEE 2013]**

(a) 16 (b) 1.6 (c) 0.016 (d) 0.08

(e) 0.16

32. The ratio of inertial force to viscous force of a fluid is called **[Kerala CEE 2013]**

(a) coefficient of viscosity (b) surface tension

(c) Reynolds number (d) specific gravity

(e) Lorentz number

33. The excess pressure inside one soap bubble is three times that inside a second bubble. The ratio of the volume of first bubble to that of the second **[MPPMT 2013]**

(a) 1:27 (b) 27:1 (c) 1:9 (d) 9:1

34. Water rises to a height of 20 mm in a capillary. If the radius of the capillary is made one-third of its previous value, then the new value of capillary rise will be **[MPPMT 2013]**

(a) $\frac{20}{3}$ mm (b) 60 mm (c) $\frac{20}{9}$ mm (d) 180 mm

35. The unit of viscosity in the CGS system is poise (P) and that in SI is poiseuille (Pl). Which of the following statement is correct? **[UP CPMT 2013]**

(a) 1 P = 1 Pl (b) 1 P = 10 Pl

(c) 10 P = 1 Pl (d) None of these

36. Liquid rises to a height of 2 cm in a capillary tube and the angle of contact between the solid and the liquid is zero. If the tube is depressed more now so that top of capillary is only 1 cm above the liquid, then the apparent angle of contact between the solid and the liquid is **[UP CPMT 2013]**

(a) 0° (b) 30° (c) 60° (d) 90°

37. The ratio of radii of two bubbles is 2 : 1. What is the ratio of excess pressures inside them? **[UP CPMT 2012]**

(a) 1 : 2 (b) 1 : 4 (c) 2 : 1 (d) 4 : 1

38. A solid of density D is floating in a liquid of density d. If V is the volume of solid submerged in the liquid and V' is the total volume of the solid, then V/V' is equal to **[UP CPMT 2012]**

(a) $\frac{d}{D}$ (b) $\frac{D}{d}$ (c) $\frac{D}{d+D}$ (d) $\frac{D+d}{D}$

39. 10 min are taken to emptied a rectangular vessel of height h through an orifice in its bottom. How much time will it take to be emptied the vessel when half filled? **[BCECE (Mains) 2012]**

(a) 3 min (b) 5 min (c) 7 min (d) 9 min

40. Water is moving with a speed of 5.18 ms^{-1} through a pipe with a cross-sectional area of 4.20 cm^2. The water gradually descends 9.66 m as the cross-sectional area of pipe is increased to 7.60 cm^2. The speed of the flow of water at lower level is **[BCECE (Mains) 2012]**

(a) 2.86 ms^{-1} (b) 3.0 ms^{-1}
(c) 3.82 ms^{-1} (d) 5.7 ms^{-1}

41. A block of ice in which a piece of stone is embedded is floating on water contained in a beaker. When all the ice melts the level of water in the beaker **[BHU 2012]**

(a) rises (b) falls
(c) remains unchanged (d) None of the above

42. **Assertion** The water rises higher in a capillary tube of small diameter than in the capillary tube of large diameter.

Reason Height through which liquid rise in capillary tube inversely proportional to the capillary tube. **[AIIMS 2012]**

(a) If both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If both Assertion and Reason are false.

43. Water rises in plant fibres due to **[Manipal 2012]**

(a) capillarity (b) viscosity
(c) fluid pressure (d) osmosis

44. The excess pressure inside a spherical soap bubble of radius 1 cm is balanced by a column of oil (specific gravity = 0.8), 2 mm high, the surface tension of the bubble is **[Manipal 2012]**

(a) 3.92 Nm^{-1} (b) 0.0392 Nm^{-1}
(c) 0.392 Nm^{-1} (d) 0.00392 Nm^{-1}

45. Water from a tap emerges vertically downwards with initial velocity 4 ms^{-1}. The cross-sectional area of the tap is A. The flow is steady and pressure is constant throughout the stream of water. The distance h vertically below the tap, where the cross-sectional area of the stream becomes (2/3) A, is ($g = 10\ ms^{-2}$) **[Manipal 2012]**

(a) 2 m (b) 1 m
(c) 0.5 m (d) 4 m

46. The terminal speed attained by an aluminium sphere of radius 1 mm falling through water at 20°C will be close to **[AMU 2012]**

(a) 9.2 ms^{-1} (b) 6.9 ms^{-1}
(c) 4.6 ms^{-1} (d) 2.3 ms^{-1}

(Assume laminar flow, specific gravity of Al = 2.7 and $\eta_{water} = 8 \times 10^{-4}$ Pl)

47. Water flows in a streamlined manner through a capillary tube of radius a the pressure difference being p and the rate of flow Q. If the radius is reduced to $a/2$ and the pressure increased to $2p$. The rate of flow becomes **[AMU 2012]**

(a) $4Q$ (b) Q
(c) $\frac{Q}{4}$ (d) $\frac{Q}{8}$

48. A raindrop of radius 0.3 mm has a terminal velocity in air 1 ms^{-1}. The viscosity of air is 18×10^{-3} poise. The viscous force on it is **[AFMC 2012]**

(a) 101.37×10^{-4} dyne (b) 101.73×10^{-2} dyne
(c) 16.95×10^{-5} dyne (d) 16.95×10^{-4} dyne

49. If two soap bubbles of different radii are connected by a tube, then **[Manipal 2012]**

(a) air flow from bigger bubble to the smaller bubble till sizes becomes equal
(b) air flow from bigger bubble to the smaller bubble till sizes are interchanged
(c) air flow from smaller bubble to bigger bubble
(d) there is no flow of air

Answers

Check point 13.1

1. (d)	2. (b)	3. (b)	4. (d)	5. (c)	6. (b)	7. (c)	8. (c)	9. (d)	10. (b)
11. [illegible]	12. (b)	13. (c)	14. (c)	15. (d)					

Check point 13.2

1. (d)	2. (b)	3. (a)	4. (b)	5. (b)	6. (b)	7. (b)	8. (a)	9. (b)	10. (b)

Check point 13.3

1. (c)	2. (b)	3. (b)	4. (d)	5. (c)	6. (d)	7. (c)	8. (c)	9. (d)	10. (a)

Check point 13.4

1. (d)	2. (a)	3. (a)	4. (b)	5. (c)	6. (c)	7. (b)	8. (b)	9. (b)	10. (a)

A. Taking it together

1. (b)	2. (c)	3. (a)	4. (b)	5. (b)	6. (b)	7. (a)	8. (b)	9. (d)	10. (c)
11. (a)	12. (b)	13. (c)	14. (b)	15. (c)	16. (c)	17. (d)	18. (c)	19. (a)	20. (a)
21. (b)	22. (b)	23. (d)	24. (d)	25. (a)	26. (b)	27. (d)	28. (b)	29. (c)	30. (c)
31. (c)	32. (b)	33. (a)	34. (a)	35. (a)	36. (c)	37. (a)	38. (c)	39. (d)	40. (b)
41. (d)	42. (b)	43. (b)	44. (d)	45. (a)	46. (a)	47. (d)	48. (a)	49. (c)	50. (b)
51. (b)	52. (c)	53. (b)	54. (b)	55. (a)	56. (b)	57. (c)	58. (d)	59. (c)	60. (a)
61. (a)	62. (b)	63. (d)	64. (a)	65. (a)	66. (c)	67. (b)	68. (d)	69. (c)	70. (a)
71. (b)	72. (a)	73. (b)	74. (a)	75. (c)	76. (c)	77. (a)	78. (a)	79. (b)	80. (c)
81. (a)	82. (a)	83. (d)	84. (d)	85. (b)	86. (b)	87. (c)	88. (d)	89. (c)	90. (c)
91. (a)	92. (b)	93. (b)	94. (b)	95. (c)	96. (c)	97. (a)	98. (b)	99. (b)	100. (b)
101. (c)	102. (a)	103. (d)	104. (b)	105. (c)	106. (d)	107. (b)	108. (b)	109. (a)	110. (b)
111. (a)	112. (c)	113. (a)	114. (c)	115. (c)	116. (a)	117. (c)	118. (a)	119. (a)	120. (a)
121. (b)	122. (a)	123. (a)	124. (b)	125. (a)	126. (b)	127. (a)	128. (d)	129. (c)	130. (d)
131. (b)	132. (c)	133. (a)	134. (b)	135. (a)	136. (c)	137. (c)	138. (c)	139. (d)	140. (b)
141. (d)	142. (c)	143. (a)	144. (a)	145. (b)	146. (a)	147. (d)	148. (a)	149. (b)	150. (c)
151. (c)	152. (b)	153. (c)	154. (a)	155. (d)	156. (b)	157. (b)			

B. Medical entrance special format questions

Assertion and reason

1. (b)	2. (a)	3. (b)	4. (e)	5. (a)	6. (b)	7. (d)	8. (b)	9. (d)	10. (c)
11. (a)	12. (a)	13. (d)	14. (a)	15. (a)	16. (d)	17. (d)	18. (a)	19. (b)	

Match the columns

1. $(A \to r, B \to q, C \to p)$
2. $(A \to q, B \to p, C \to q)$
3. $(A \to p, B \to r\ C \to p)$
4. $(A \to q, B \to r, C \to r)$
5. $(A \to q, B \to p, C \to r)$
6. $(A \to p, B \to p, C \to r)$

C. Medical entrances' gallery

1. (b)	2. (b)	3. (c)	4. (c)	5. (c)	6. (d)	7. (b)	8. (d)	9. (a)	10. (e)
11. (a)	12. (d)	13. (c)	14. (a)	15. (c)	16. (a)	17. (b)	18. (a)	19. (b)	20. (c)
21. (a)	22. (c)	23. (c)	24. (a)	25. (b)	26. (d)	27. (c)	28. (b)	29. (d)	30. (c)
31. (b)	32. (c)	33. (a)	34. (a)	35. (c)	36. (c)	37. (a)	38. (b)	39. (c)	40. (a)
41. (b)	42. (a)	43. (a)	44. (b)	45. (b)	46. (c)	47. (d)	48. (b)	49. (c)	

Hints and explanations

Check point 13.1

1. (*d*) A fluid comparises of both liquid and gases.

2. (*b*) Dimensional formula for pressure is $[ML^{-1}T^{-2}]$.

3. (*b*) Pointed ends have very small area therefore transmits a large pressure.

4. (*d*) Pressure, $p = \frac{F}{A} = \frac{70 \times 10}{15 \times 10^{-4} \times 2} = \frac{7 \times 10^4}{30} \times 10^2$

$= 2.33 \times 10^5 \text{ m}^{-2}$

5. (*c*) Gauge pressure = absolute pressure − atmospheric pressure

6. (*b*) Absolute pressure $p = p_a + \rho g h$

here $p_a = 1.01 \times 10^5$ Pa

$\rho = 1.03 \times 10^3 \text{ kgm}^{-3}$

$\therefore \quad p = 1.01 \times 10^5 \text{Pa} + 51.5 \times 10^5 \text{Pa}$

$= 1.01 \times 10^5 \text{Pa} + 51.5 \times 10^5 \text{Pa}$

$= 52.5 \times 10^5 \text{Pa} = 52 \text{ atm}$

7. (*c*) The shape of the container does not affect the pressure. The four vessels *A B C* and *D* have different shapes and hold different amount of water. The free surface of water in all vessels are in same horizontal plane. The pressure at the four points *E, F, G* and *H* lying in the same horizontal plane will be same.

8. (*c*) $\rho = \frac{\rho_1 V_1 + \rho_2 V_2}{V_1 + V_2} = \frac{(2\rho) + (\rho)(2/3)}{1 + 2/3} = \frac{8\rho}{5}$

9. (*d*) If the pressure at the oil-water interface in the left-arm is p, then the pressure in the right arm at the level of the interface will also be p.

In the left arm,

$$p = p_0 + \rho_x g(l + d) \quad \text{...(i)}$$

in the right arm,

$$p = p_0 + \rho_w g l \quad \text{...(ii)}$$

Equating (i) and (ii)

$$\rho_x = \frac{l}{l + d}\rho_w = \frac{135 \text{ mm}}{135 \text{ mm} + 15 \text{ mm}} \times 1000 \text{ kgm}^{-3} = 900 \text{ kgm}^{-3}$$

10. (*b*) It is due to Pascal's law.

11. (*c*) Here, $r_1 = 4$ cm and $r_2 = 12$ cm

$F_1 = F$ and $F_2 = mg = 1400 \times 10$ N

According to pascal's law

$$F_1 = F_2 \times \frac{r_1^2}{r_2^2} = 14 \times 10^3 \times \left(\frac{4}{12}\right)^2$$

$$= 14 \times 10^3 \times \frac{1}{9} \simeq 1600 \text{ N}$$

12. (*b*) Given, $h = x$ cm, $\theta = \phi$

let, e = length of mercury column

$\therefore \quad \frac{\lambda}{l} = \sin(90^\circ - \phi) \Rightarrow \frac{x}{l} = \cos\phi$

$\Rightarrow \quad l = x / \cos\phi = x \sec\phi$

13. (*c*) Let weight of boat = W and weight of stone = w

Assuming density of water = 1gcc^{-1}]

Volume of water displaced initially = $(w + W)$

Later, volume displaced = $\left(W + \frac{w}{\rho}\right)$ (ρ = density of stones)

$\Rightarrow$ Water level comes down.

14. (*c*) Let v be the volume of cavities and V the volume of solid iron.

Then $V = \frac{\text{mass}}{\text{density}} = \left(\frac{6000/9.8}{7.87 \times 10^3}\right) = 0.078 \text{ m}^3$

Further, decrease in weight = upthrust

$\therefore \quad (6000 - 4000) = (V + v)\rho_w g$

or $\quad 2000 = (0.078 + v) \times 10^3 \times 9.8$

or $\quad 0.078 + v \simeq 0.2$

$\therefore \quad v = 0.12 \text{ m}^3$

15. (*d*) Using the law of buoyancy, we have $V\sigma g = 0.6 V\sigma_1 g$, for the part of body outside oil and $V\sigma g = 0.4 V\sigma_2 g$, for the part of body outside water, hence we get

$$1 = \frac{0.6\sigma_1}{0.4\sigma_2}$$

So, we have, $\frac{\sigma_1}{\sigma_2} = \frac{6}{4} = \frac{3}{2} = 15.$

Check point 13.2

1. (*d*) In the streamline flow

V_1 = constant, V_2 = constant, V_3 = constant

$V_1 \neq V_2 \neq V_3$

2. (*b*) For $1000 < \text{Re} < 2000$, flow is unsteady.

3. (*a*) By equation of continuity

AV = constant

$\therefore \quad A_1V_1 = A_2V_2 + A_3V_3$

$\Rightarrow \quad A \times 8 = 0.8A \times 6 + 3 \times v$

$\Rightarrow \quad 4A = 2.4A + 1.5Av \Rightarrow 4 = 2.4 + 1.5v$

$\Rightarrow \quad v = \frac{1.6}{1.5} = 1.1 \text{ms}^{-1}$

4. (*b*) Bernoulli's theorem is based upon conservation of mechanical energy.

5. (*d*) According to equation of continuity

$$A_1 v_1 = A_2 v_2$$

$$10 \times 1 = 5 \times v_2 \Rightarrow v_2 = 2 \text{ ms}^{-1}$$

Now applying Bernoulli's equation we get,

$$\therefore \quad \frac{p_1}{\rho} + \frac{v_1^2}{2} = \frac{p_2}{\rho} + \frac{v_2^2}{2}$$

$$\Rightarrow \quad \frac{2000}{1000} + \frac{1^2}{2} = \frac{p_2}{1000} + \frac{2^2}{2} \Rightarrow p_2 = 500 \text{ Pa}$$

6. (*b*) As water falls from a tap, the velocity of water goes on increasing hence from the equation of continuity, the cross section of water-stream goes on decreasing. If A_0 = area of cross-section of the mouth of the tap and v_0 is the velocity of water at this point, and A, v are corresponding quantities at depth h, then

$$A_0 v_0 = Av = V(\text{volume rate of flow})$$

and $$v_2 - v_0^2 = 2gh$$

$$\therefore \quad v = A_0 A\sqrt{\frac{2gh}{A_0^2 - A^2}} = 4 \times 1\sqrt{\frac{2 \times 10 \times 2}{16 - 1}}$$

$$= 8\sqrt{\frac{2}{3}} \text{ ms}^{-1} = 6.5 \text{ ms}^{-1}$$

7. (*b*) Given, $\frac{v^2}{2g} = h = 10$ cm

$$\therefore \quad v = \sqrt{2gh} = \sqrt{2 \times 10 \times 0.1} = \sqrt{2} \text{ ms}^{-1}$$

8. (*a*) Velocity of efflux = $\sqrt{2gh}$, which is independent of density of liquid.

9. (*b*) The lifting of an aeroplane is based on Bernoulli's principle. When airplane moves through air, pressure difference is created due to higher speed of air in the region above the wings.

10. (*b*) Range of water = $2\sqrt{H(H-h)} = 2\sqrt{\frac{H}{4}} \times \frac{3H}{4} = \frac{\sqrt{3}H}{2}$

Check point 13.3

1. (*c*) Unit of η is Nm^{-2} s.

2. (*b*) Viscosity of water decreases with rise of temperature.

3. (*d*) Rate of flow through a tube is given by $V = \frac{\pi p r^4}{8\eta l}$

4. (*d*) When the tubes are fitted in parallel

$$V = V_1 + V_2$$

$$\Rightarrow \quad \frac{\pi p r^4}{8\eta l} = \frac{p r_1^4}{8\eta l} + \frac{\pi p r_2^4}{8} \Rightarrow r^4 = r_1^4 + r_2^4$$

$$\therefore \quad r = (r_1^4 + r_2^4)^{1/4}$$

5. (*c*) $F = 6\pi\eta r v$ *i.e.*, v has the dimensions of $\frac{F}{\eta r}$ or $\frac{mg}{\eta r}$

6. (*d*) Terminal velocity, $v_T \propto r^2$

$$\therefore \quad \frac{(v_T)_1}{(v_T)_2} = \left(\frac{R}{R/2}\right)^2 = 4$$

7. (*c*) Velocity becomes constant terminal velocity is attained.

8. (*c*) Let r be the radius of smaller drop and R the radius of bigger drop. By equating the volume we have,

$$2\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi R^3 \text{ or } R = (2)^{1/3}.r$$

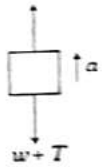
Now terminal velocity $\propto$ (radius)^2

$$\therefore \quad v' = (2)^{2/3} v$$

9. (*d*) When the small sized spherical body's falling through the viscous liquid first its velocity increases and then becomes constant which is called terminal velocity.

10. (*a*) Before entering the water the velocity of ball is $\sqrt{2gh}$. If after entering the water this velocity does not change, then this value should be equal to the terminal velocity. Therefore,

$$\sqrt{2gh} = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta}$$



$$\therefore \quad h = \frac{\left\{\frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta}\right\}^2}{2g} = \frac{2}{81} \times \frac{r^4(\rho - \sigma)^2 g}{\eta^2}$$

$$= \frac{2}{81} \times \frac{(3 \times 10^{-4})^4 (10^4 - 10^3)^2 \times 9.8}{(9.8 \times 10^{-6})^2} = 1.65 \times 10^3 \text{m}$$

Check point 13.4

1. (*d*) Surface tension $T = F/l$ and $\frac{\Delta W}{\Delta A}$ *i.e.*, unit of surface tension is Nm^{-1} or Jm^{-2}

$$\frac{\text{N}}{\text{m}} = \frac{\text{kg - ms}^2}{\text{m}} = \frac{\text{kg}}{\text{s}^2}$$

2. (*a*) Force on each side = $2TL$ (due to two surface).

3. (*a*) Surface tension $T = \frac{F}{2l} = \frac{2 \times 10^{-2}}{2 \times 10 \times 10^{-2}} = 0.1 \text{Nm}^{-1}$.

5. (*c*) $W = 8\pi R^2 T$

$$\therefore \quad W \propto R^2$$

If radius double, then work done will become four times.

6. (*c*) Radius of curvature, $r = \frac{r_1 r_2}{r_2 - r_1} = \infty$, *i.e.*, a straight plane.

7. (*b*) Express pressure, $\Delta p = \frac{4T}{r} = \frac{4 \times 2 \times 25 \times 10^{-2}}{1 \times 10^{-2}}$

$$= 20 \text{ Nm}^{-2} = 20 \text{ Pa} \qquad (\text{as } r = d/2)$$

10. (*a*) $h = \frac{2T\cos\theta}{r\rho g}$

$$\therefore \quad hr = \frac{2T\cos q}{rg} = \text{constant}$$

$$\Rightarrow \quad h_1 r_1 = h_2 r_2 \text{ or } h_2 = \frac{h_1 r_1}{r_2}$$

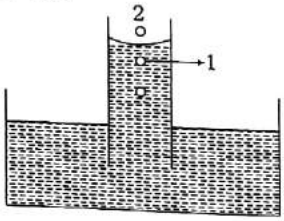
Substituting the values,

$$h_2 = (2.0)(3) \qquad \left(\frac{r_2}{r_1} = \frac{1}{3}\right)$$

$$= 6.0 \text{ cm}$$

A Taking it together

1. (*b*) Surface tension is found mainly in liquids.
2. (*c*) Water droplets assumes minimum area due to surface tension and becomes spherical.
3. (*a*) $h = \frac{2T\cos\theta}{rdg}$. If θ is less than 90°, then h will be positive.
4. (*b*) A wetting liquid has a concave meniscus.
5. (*b*) For a body immersed in liquid, when the weight of the body is less than the upthrust then the body will float partially immersed.
6. (*b*) As brush is withdrawn, water surface formed over bristle tends to take smallest area due to surface tension.
7. (*a*) Water proof surface coatings are made up of materials which increases angle of contact and water does not wets the surface.
8. (*b*) For small drops, force of surface tension predominates gravitational force and so, they are more spherical.
9. (*d*) Cohesive force of water is greater than adhesive force and hence water does not wets the surface.
10. (*c*) This happens due to viscosity.
11. (*a*) Soluble impurities increases the surface tension.
12. (*b*) Surface energy of combined drop will be lowered, so excess surface energy will raise the temperature of the drop
13. (*c*) Water proofing agents increases angle of contact.
14. (*b*) As we know for a streamline flow of a liquid velocity of each particle at a particular cros-section is constant, because Av = constant (law of continuity) between two cross-section of a tube of flow.
15. (*c*) In streamlined flow, velocity of all the particles arriving at a common point remains same with time (both in magnitude and direction). So, the KE $= \frac{1}{2}mv^2$ = a constant for all the particles as all fluid particles have identical mass also.
16. (*c*) Velocity of different layers of a flowing fluid decreases with distance from the axis of the tube. It is maximum at centre and minimum at walls.
17. (*d*) In a streamline flow at any given point, the velocity of each passing fluid particles remains constant. If we consider a cross-sectional area, then a point on the area cannot have different velocities at the same time, hence two streamlines of flow cannot cross each other.
18. (*c*) $F = W_{air} - W_{water} = 5 - 2 = 3\,N$
19. (*a*) Specific gravity or relative density1
$$= \frac{\text{weight in air}}{\text{change of weight in water}} = \frac{60}{20} = 3$$
20. (*a*) Neglecting gravitational head, difference in pressure energy = difference in kinetic energy
$$\therefore \quad (p_0 - p_0) = \frac{1}{2}\rho v^2 \quad \text{or} \quad v = 2\sqrt{\frac{p_0}{\rho}} = 2\sqrt{\frac{10^5}{10^3}}$$
$$= 20\ \text{m s}^{-1} \text{ or } \sqrt{400}\ \text{m s}^{-1}$$
21. (*b*) When iron block was floating, it displaces V_1 volume of water such that weight of this V_1 volume of water is equal to weight of block. But since density of iron is more than density of water, this volume V_1 will be greater than volume V_2 of iron block. When iron block sinks in the pond, it displaces water of volume equal to its own volume V_2.
Since $V_1 > V_2$, displaced volume in first case was more than displaced volume in second case. Hence, level will fall.
22. (*b*) $h = \frac{2T\cos\theta}{rdg}$ $\therefore h \propto \frac{1}{r}$. If θ is less than h will be more.
23. (*d*) $h \propto \frac{1}{r}$, so for less r, h will be more.
24. (*d*) With acute angle, of contact, liquid will rise in capillary.
25. (*a*) Surface tension decreases with temperature and becomes zero at boiling point.
26. (*b*) Cohesive force decreases. So angle of contact decreases.
27. (*d*) Work done, $W = 8\pi R^2 T$
$$= 8 \times \pi \times (10^{-2})^2 \times 2 \times 10^{-2} = 16\pi \times 10^{-6}\ \text{J}$$
28. (*b*) weight is balanced by force of surface tension.
$$\therefore \quad w = 2\pi r T$$
29. (*c*) Radius of curvature of common surface is,
$$R_{Common} = \frac{r_1 r_2}{r_2 - r_1}$$
30. (*c*) $\Delta p = \frac{4T}{r}$
31. (*c*) Since, $\Delta p \propto 1/R$
32. (*b*) 1 atmosphere = $1.01 \times 10^5\ \text{N m}^{-2} = \rho g h$
$$\therefore \quad h = \frac{1.01 \times 10^5}{1.3 \times 10} = 7769\ \text{m} \approx 8\ \text{km}$$
33. (*a*) $p_1 < p_2$ for $\theta < 90°$



34. (*a*) Weight of spiders or insects can be balanced by vertical component of force due to surface tension.
35. (*a*) Due to force of attraction it is not easier to separate the two glass plates.
36. (*c*) When the pebble is falling through the viscous oil the viscous force is
$$F = 6\pi\eta r v$$
where, r is radius of the pebble, v is instantaneous speed, η is coefficient of viscosity. As the force is variable, hence acceleration is also variable so v-t graph will not be straight line. First velocity increases and then becomes constant known as terminal velocity.
37. (*a*) Range, $R = 2\sqrt{h_{top} \times h_{bottom}} = 2\sqrt{h(H-h)}$

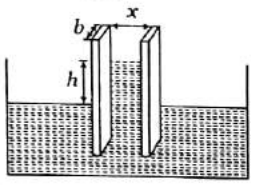
38. (*c*) Range, $R = 2\sqrt{H(H-h)} = 2\sqrt{\frac{H}{2}\times\frac{H}{2}} = H$

39. (*d*) Volume of ice > Volume of water $\Delta V = V_w - V_i = \frac{m}{y} - \frac{m}{x}$

40. (*b*) $r \propto \frac{1}{h} \Rightarrow \frac{r_P}{r_Q} = \frac{h_Q}{h_P} = \frac{h}{\frac{2}{3}h} = \frac{3}{2}$

41. (*d*) In artificial satellite, gravity is absent so capillary will be fully filled.

42. (*b*) Let the width of each plate is b and due to surface tension liquid will rise upto height h, then upward force due to surface tension $= 2Tb\cos\theta$ ...(i)



Weight of the liquid rises in between the plates

$= Vdg = (bxh)dg$ ...(ii)

Equating Eqs. (i) and (ii), we get

$2Tb\cos\theta = bxhdg$

$\therefore \quad h = \frac{2T\cos\theta}{xdg}$

43. (*b*) $1\,gc^{-1} = 1\,kgm^{-3}$

Upthrust = total downward force

$\therefore \quad 2000 \times 129 \times g = (2000 \times 0.09 \times g + mg)$

$\therefore \quad m = 2400$ kg

44. (*d*) Upthrust on 7 cm = 600 kg

$\therefore$ Upthrust on further 1.4 cm $= \frac{600}{7} \times 1.4 = 120$ kg

45. (*a*) Increase in pressure

$= \frac{\text{Force exerted by the cylinder on the liquid}}{A} = \frac{\text{Upthrust}}{A}$

46. (*a*) Energy is released in process.

47. (*d*) $\frac{F_1}{A_1} = \frac{F_2}{A_2}$ (Pascal's law)

$\therefore \quad A_2 = \frac{F_2}{F_1}.A_1 = \left(\frac{2000\times 9.8}{10^2}\right)(100) = 1.96\times 10^4\ cm^2$

48. (*a*) As given

d_1 = Diameter of Ist pipe is 2.5.

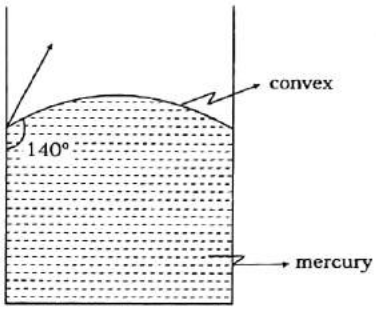
d_2 = Dimeter of IInd pipe is 3.75.

Applying equation of continuity for cross-sections A_1 and A_2.

$\Rightarrow \quad A_1v_1 = A_2v_2 \Rightarrow \frac{v_1}{v_1} = \frac{A_2}{A_1} = \frac{\pi(r_2^2)}{\pi(r_1^2)} = \left(\frac{r_2}{r_1}\right)^2$

$= \left(\frac{\frac{3.75}{2}}{\frac{2.5}{2}}\right)^2 = \left(\frac{3.75}{2.5}\right)^2 = \frac{9}{4}\left[r_2 = \frac{d_2}{2},\ r_1 = \frac{d_1}{2}\right]$

49. (*c*) According to the question,the observed meniscus is of convex figure shape. Which is only possible when angle of contact is obtuse. Hence, the combination will be of mercury-glass (140°).



50. (*b*) Force on the base

$F = p\,ghA$

$= 900 \times 10 \times 0.4 \times 2 \times 10^{-3} = 7.2$ N

51. (*b*) Force at the bottom

$F = p \times A = (\rho g h_{total})A$

$= (10^3 \times 10 \times 10)(100 \times 10^{-4}) = 100$ N

52. (*c*) In horizontal direction, pressure remains constant unless the fluid volume is accelerated.

53. (*b*) $T_c = T_0(1-\alpha t)$ *i.e.*, surface tension decreases linearly with increase in temperature.

54. (*b*) When hole is made at centre of vessel, maximum range is obtained.

55. (*a*) Loss of weight = Upthrust

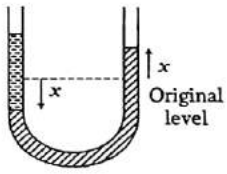
$\frac{W}{2}$ = total volume $\times \rho_w \times g$

or $\frac{(V - V_C)\,5 \times \rho_w \times g}{2} = V \times \rho_w \times g$ (V_C = volume of cavity)

or $V - V_C = \frac{2}{5}V \Rightarrow V_C = \frac{3}{5}V$

56. (*b*) Initially due to more pressure at the hole velocity of efflux will be more. So, the liquid drains out more quickly.

57. (*c*) [Equating pressure] (112) (1) = (13.6) (2*x*)



$\therefore \quad x = 0.41$ cm

58. (*d*) Weight – upthrust = drag force.

But no information is given regarding the upthrust.

59. (*c*) $h = \frac{2T\cos\theta}{r\rho g}$ or $h \propto \frac{1}{r}$

$m = (\pi r^2 h)\rho$ or $m \propto r^2 h$ or $m \propto r$

60. (*a*) Decrease in pressure energy = increase in kinetic energy

or $\Delta p = \frac{1}{2}\rho v^2$

$\therefore \quad v = \sqrt{\frac{2(\Delta p)}{\rho}} = \sqrt{\frac{2 \times 0.5 \times 10^5}{10^3}} = 10 \text{ ms}^{-1}$

61. (*a*) Rate of leakage of water from the hole

$= Av = A\sqrt{2gh} = 10^{-4}\sqrt{2 \times 10 \times 5} = 10^{-3} \text{ m}^3\text{s}^{-1}$

62. (*b*) From continuity equation $v \propto \frac{1}{A}$ or $v \propto \frac{1}{d^2}$

(d = diameter of pipe)

$\frac{v_{\text{narrow}}}{v_{\text{wider}}} = \frac{d_w}{d_N} = \left(\frac{6}{3}\right) = 4 \Rightarrow v_{\text{wider}} = \frac{v_{\text{narrow}}}{4} = \frac{4}{4} = 1 \text{ms}^{-1}$

63. (*d*) Submerge part = Replaced water

$\frac{4}{5}h\rho_w = h \times \rho_\lambda$

$\therefore \quad \rho_\lambda = \frac{4}{5} \times \rho_w = \frac{4}{5} \times 1000$

$\rho_w = 800 \text{ kgm}^{-3}$

64. (*a*) Applying Bernoulli's equation, we get

$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$

or $(10^{-2} \times 13.6 \times 10^3 \times 10) + \frac{1}{2} \times 10^3 \times (0.35)^2$

$= p_2 + \frac{1}{2} \times 10^3 \times (0.65)^2$

or $p_2 = 1210 \text{ Nm}^{-2} = \frac{1210 \times 10^2}{13.6 \times 10^3 \times 10}$ cm of Hg

= 0.89 cm of Hg

65. (*a*) For a body immersed in liquid when the weight of the body is less than the upthrust then the body will float partially immersed.

66. (*c*) Force exerted by the liquid on the base of the vessel is

$F = mg$

Here, $m_A = m_B = m_C$

$\therefore \quad F_A = F_B = F_C$

67. (*b*) W = Upthrust or $VDg = vdg$ or $\frac{v}{V} = \frac{D}{d}$

68. (*d*) Given that an object weights m_1 in a liquid of density d_1 and m_2 in a liquid of density d_2, so when the density of the object is d, then we get

$V(d - d_1) = m_1$ and $V(d - d_2) = m_2$

Thus, $\frac{d - d_1}{d - d_2} = \frac{m_1}{m_2}$

So, we get $d = \frac{m_1 d_2 - m_2 d_1}{m_1 - m_2}$

69. (*c*) Percentage volume outside the water

$= \frac{\rho_{\text{water}} - \rho_{\text{ice}}}{\rho_{\text{water}}} \times 100 = \frac{1000 - 900}{1000} \times 100 = 10\%$

70. (*a*) Pressure at same level in two limbs will be same.

$\therefore \quad p + \rho g(3h) = p + 2\rho gh$

$\therefore \quad p = p_0 - \rho gh$ or $p < p_0$

71. (*b*) $1 \text{ gL}^{-1} = 1 \text{ kgm}^{-3}$

Upthrust = total downward force

$\therefore \quad 1000 \times 1.29 \times g = (1000 \times 0.09 \times g + mg)$

$\therefore \quad m = 1200 \text{ kg}$

72. (*a*) Weight of man = extra upthrust

$\therefore \quad mg = (3 \times 2 \times 10^{-2})(10)^3(10)$

or $m = 60$ kg

73. (*b*) $a = \frac{\text{Upthrust} - \text{Weight}}{\text{Mass}} = \frac{V\rho_w g - V\rho g}{V\rho}$

$= \left(\frac{\rho_w - \rho}{\rho}\right)g = \left(\frac{1 - 0.5}{0.5}\right)(10) = 10 \text{ ms}^{-2}$

74. (*a*) For just sink, $(m_{\text{raft}} + m)g = V_{\text{raft}}\rho_w g$

$\therefore \quad m = V_{\text{raft}}\rho_w - m_{\text{raft}}$

$= \frac{120}{600} \times 1000 - 120 = 80$ kg

75. (*c*) $W_{\text{app}} = W - \text{upthrust} = W - \left(\frac{W/g}{\rho}\right)\sigma g = W\left(1 - \frac{\sigma}{\rho}\right)$

76. (*c*) The net force acting on the stone in a dwonward direction would be equal to $F = v\sigma g - v\rho g$, where m is the mass of the stone, the force acting downwards would be equal to $V\sigma g$ due to its weight and the force acting upwards would be equal to $V\rho g$ due to the buoyant force.

Hence, we get $F = V\sigma g - V\rho g = V\sigma g\left(1 - \frac{\rho}{\sigma}\right)$

$= mg\left(1 - \frac{p}{\sigma}\right) = mg\left(1 - \frac{1}{K}\right)$

Thus, $a = g\left(1 - \frac{1}{K}\right)$

77. (*a*) In a freely falling vessel upthrust is zero, because effective value of g is zero.

78. (*a*) Fraction immersed $= \frac{\rho_{\text{solid}}}{\rho_{\text{liquid}}} = \frac{0.9}{1.125} = 0.8$

Therefore, fraction outside the water = 0.2 = 1 / 5

79. (*b*) Weight of sphere = upthrust from water + upthrust from liquid

$\therefore \quad V\rho g = \left(\frac{2}{3}V \times 10^3 \times g\right) + \left(\frac{V}{3} \times 8 \times 10^3 \times g\right)$

or $\rho = \frac{10000}{3} \text{ kgm}^{-3}$

80. (*c*) $A_1 v_1 = A_2 v_2 + A_3 v_3$

$\therefore \quad v_3 = \frac{A_1 v_1 - A_2 v_2}{A_3} = \frac{(0.2 \times 4) - (0.2 \times 2)}{0.4} = 1 \text{ m s}^{-1}$

81. (*a*) $R_h = R_{H-h}$. Since, range from both the holes is same

$h = 2.4$ cm and $H - h = 7.6$ cm

Adding these two, we get $H = 10$ cm

82. (*a*) Work done by all the forces W = change in kinetic energy
Two forces are acting, weight and upthrust.

$$\therefore \quad W = (\sigma - \rho)g(hV)$$

83. (*d*) Force exerted by gas on its hemispherical end = Pressure of gas × Projected area = $p_0\ (\pi r^2)$.

84. (*d*) 6×10^{-2} × circumference = Force

$$\therefore \quad \text{Circumference} = \frac{75 \times 10^{-4}}{6 \times 10^{-2}} = 12.5 \times 10^{-2}\text{m}$$

85. (*b*) In a capillary of insufficient height, liquid fills the capillary and shape of meniscus is changed.

86. (*b*) Angle of contact will be 90°.

87. (*c*) $h_1 r_1 = h_2 r_2 \Rightarrow h_2 = 30 \times 4/3 = 40$ mm

88. (*d*) $h = \dfrac{2T}{rdg} \Rightarrow h = \dfrac{2 \times 75}{0.005 \times 1 \times 10^3} = 30$ cm

89. (*c*) $h\rho g = \dfrac{2T}{r} \Rightarrow h = \dfrac{2T}{r\rho g}$

90. (*c*) Angle of contact is acute.

91. (*a*) Suppose R be the radius of bigger drop. Then by equating volumes, we have

$$2(4/3\pi r^3) = (4/3)\pi R^3 \quad \text{or} \quad R = (2)^{1/3} r$$

Now, surface energy ∝ surface area

$$\therefore \quad \frac{U_1}{U_2} = \frac{A_1}{A_2} = \frac{2r^2}{R^2} = \frac{2}{2^{2/3}}$$

92. (*b*) As we know according to equation of continuity, when cross-section of duct decreases, the velocity of flow of liquid increases and in accordance with Bernoullis theorem in a horizontal pipe, graph (b) correctly represents the variation of pressure.

93. (*b*) Time to empty a tank $t = \dfrac{A}{a}\sqrt{\dfrac{2H}{g}}$

$$\text{or} \quad t \propto \sqrt{H} \Rightarrow t_2/t_1 = 2$$

94. (*b*) Level in the container will become maximum when rate of inflow = rate of outflow.

$$Q = A_1 v = A_1\sqrt{2gh_{max}}$$

$$\therefore \quad h_{max} = \frac{Q^2}{2gA_1^2}$$

95. (*c*) Pressure difference $\Delta p = \Delta\ \text{KE} = \dfrac{1}{2} \times \rho(v_1^2 - v_2^2)$

$$\text{or} \quad \Delta p = \frac{1}{2} \times 1.0 \times (3600 - 1600)$$

$$= 1000\ \text{Nm}^{-2}$$

$$\Rightarrow \quad F = (\Delta p) \times A = 4000\ \text{N}$$

96. (*c*) Decrease in potential energy = increase in kinetic energy

$$\therefore \quad \rho gh = \frac{1}{2}\rho\,(v_f^2 - v_i^2)$$

$$\text{or} \quad 2\,(10)\,(0.15) = v_f^2 - (10)^2$$

$$\text{or} \quad v_f = 2\ \text{ms}^{-1}$$

Now, from continuity equation,

$$A_1 v_1 = A_2 v_2 \text{ or } A \propto \frac{1}{v}$$

Velocity has become two times. Hence, area of cross-section will remain half.

97. (*a*) $R_h = R_{H-h}$ or $h_1 = H - h_2$ or $h_1 + h_2 = H$

98. (*b*) Terminal velocity, $v = \dfrac{2}{9}\dfrac{r^2(\rho - \sigma)g}{\eta}$

$$\therefore \quad v \propto r^2 \Rightarrow \frac{v}{V} = \frac{r^2}{R^2} \Rightarrow V = \frac{vR^2}{r^2}$$

99. (*b*) The volume of water contained in the vessel would be equal to $V = l^3 = (1\text{m})^3 = 1\text{m}^3$. Assuming water to be pure so the density of water would be equal to $d = 1000\ \text{kgm}^{-3}$, hence the mass of water contained in the vessel would be equal to $m = Vd = 1000$ kg.

Thus, the work done would be equal to

$$W = mgh = 1000 \times 10 \times \frac{1}{2} = 5000\ \text{J}$$

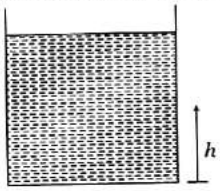
100. (*b*) $f_i = \dfrac{\rho_{solid}}{\rho_{liquid}} = \dfrac{\rho_s}{\rho_m}$

$$\therefore \quad h_i = \frac{\rho_s}{\rho_m}.\,l$$

$$\therefore \text{ Height above the mercury level} = \frac{\rho_s}{\rho_m}\,l = \left(1 - \frac{\rho_s}{\rho_m}\right)l$$

101. (*c*) When moving upwards $\dfrac{dp}{dh} = -\rho(g + a)$ and when moving downwards $\dfrac{dp}{dh} = -\rho(g - a)$

In first case, pressure decreases with h more rapidly.



102. (*a*) Upthrust on sphere from the liquid makes equal and opposite pair of forces. Or $p = p_0 + \rho gh$

103. (*d*) Pressure inside the bubble

$$= \text{Pressure outside it} + \frac{2T}{r} = (p_0 + \rho gh) + \frac{2T}{r}$$

$$= 10^5 + (10^3 \times 10 \times 10) + \frac{2 \times 7 \times 10^{-2}}{10^{-3}}$$

$$= 2.0014 \times 10^5\ \text{Nm}^{-2}$$

104. (*b*) $3g = 4\,(g - a) \Rightarrow g - a = \dfrac{3g}{4}$

$$\text{Upthrust} = (5 \times 10^{-4})\,(10^3)\left(\frac{3g}{4}\right)\text{N} = 0.375\ \text{kg}$$

Apparent weight = (3 − 0.375) = 2.625 kg

105. (*c*) W = Upthrust

or $(m_1 + m_2)\,g = [(m_1/0.5 \times \rho_w) + (m_2/2.5 \times \rho_w)]\,\rho_w g$

or $m_1 + m_2 = 2m_1 + 0.4\,m_2$

or $m_1 = 0.6\,m_2$

$\therefore \quad \dfrac{m_1}{m_2} = 0.6 = \dfrac{3}{5}$

106. (*d*) Common height after they are connected can be determined by equating the volumes. Hence

$$(A_1 + A_2)\,h = A_1h_1 + A_2h_2 \qquad (A_1 = A_2 = A)$$

$$\therefore \quad h = \frac{h_1 + h_2}{2}$$

Work done by gravity $= -\Delta U = U_i - U_f$

$$= \left\{Ah_1\rho.\,g.\,\frac{h_1}{2} + Ah_2\rho.\,g.\,\frac{h_2}{2}\right\} - A\,(h_1 + h_2)\,\rho\,.\,g.\,\frac{(h_1 + h_2)}{4}$$

$$= \frac{\rho Ag}{4}\,(h_2 - h_1)^2$$

107. (*b*) $2a_1S_1 = 2a_2S_2$

Here a_1 = acceleration inside the liquid

$$= \frac{\text{upthrust} - \text{weight}}{\text{mass}} = \frac{V\sigma g - V\rho g}{\rho} = \left(\frac{\sigma}{\rho} - 1\right) g$$

a_2 = retardation in air $= g$

$$\therefore \quad S_2 = \frac{a_1}{a_2}.\,S_1 = \left(\frac{\sigma}{\rho} - 1\right) h$$

108. (*b*) Writing pressure equation from one end to other end of tube.

$$p_0 + 2\rho gh - 2\rho\,.\,\frac{l}{2}\,.\,a - \rho\,\frac{l}{2}\,.\,a - \rho gh = p_0$$

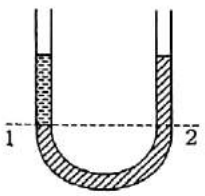
$$\therefore \quad h = \frac{3al}{2g}$$

109. (*a*) Pressure is same in a horizontal level, $p_D = p_B$

$\therefore \quad p_0 + \rho_1 gh_1 = p_0 + \rho_w gh_2$

or $\dfrac{\rho_1}{\rho_w}$ = relative density of paraffin $= \dfrac{h_2}{h_1}$

110. (*b*) Equating pressure at 1 and 2, we see that $\rho_1 = \rho_2$.



111. (*a*) $\dfrac{dp}{dx} = -\rho a$ (along horizontal towards right) *i.e.*, in the direction of acceleration along horizontal pressure will decrease of $p_{AB} > p_{CD}$

$$\frac{dp}{dy} = -\rho g$$

(along vertically upwards direction pressure will decrease)

Hence, pressure at B is maximum and pressure at D is minimum.

112. (*c*) $\dfrac{\Delta p}{\Delta x} = \rho a = \rho g$ (in horizontal direction)

Pressure decreases in the direction of acceleration in horizontal.

$\therefore \quad p_D < p_C \;\Rightarrow\; p_C - p_D = \rho gh$

$\dfrac{\Delta p}{\Delta x} = \rho g$ (in vertical direction)

Pressure increases with depth in vertical direction.

$\therefore \quad p_A - p_C = \rho gh$

113. (*a*) Weight = upthrust from mercury + upthrust from water

$\therefore \quad 10 \times 8.56 = (x \times 13.6) + (10 - x) \times 10$

Here x = depth inside mercury

Solving, we get

$$x = 6 \text{ cm}$$

114. (*c*) Total volume of 64 drops = Volume of big drop

$$64 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3 \;\Rightarrow\; R^3 = 64r^3$$

$$\Rightarrow \quad R = 4r \quad \Rightarrow \quad r = \frac{R}{4}$$

Energy of big drop $= T \cdot 4\pi R^2$

Energy of 64 drops $= 64(T \times 4\pi r^2) = 64\left[T \times 4\pi\left(\dfrac{R}{4}\right)^2\right]$

$$= 16\pi TR^2$$

$\therefore$ Energy needed to break the drop into 64 droplets.

$$\Delta W = E_2 - E_1 = 12\pi R^2 T$$

115. (*c*) The terminal velocity is given by v, where $v \propto r^2$ (r = radius of the sphere).

The mass of the sphere can be given by $m = \dfrac{4}{3}\pi r^3\rho$.

Thus $\quad m \propto r^3$

So, according to the question,

$$\frac{m_1}{m_2} = \frac{1}{8} = \left(\frac{r_1}{r_2}\right)^2 \Rightarrow \frac{r_1}{r_2} = \frac{1}{2}$$

and since, $\dfrac{v_1}{v_2} = \left(\dfrac{r_1}{r_2}\right)^2 = \left(\dfrac{1}{2}\right)^2 = \dfrac{1}{4} = \dfrac{v}{nv} = \dfrac{1}{4} \Rightarrow n = 4$

116. (*a*) Let x be the fraction of volume immersed in first case and y the fraction in second case.

Then in first case, W = Upthrust

or $\quad V\rho_b g = (xV)\rho_l g \;\text{ or }\; x = \dfrac{\rho_b}{\rho_l}$ ...(i)

In second case,

Upthrust − weight = mass × acceleration

or $\quad (yV)\,\rho_l\,(g + g/3) - (V\rho_b)g = (V\rho_b)\left(\dfrac{g}{3}\right) \;\text{ or }\; y = \dfrac{\rho_b}{\rho_l}$ ...(ii)

From Eqs. (i) and (ii), we see that $x = y$ or fraction of volume immersed does not change by the acceleration of vessel.

117. (c) At $h = \frac{H}{2}$, maximum range is obtained, which is equal to H. Further $R_h = R_{H-h}$.

118. (a) In horizontal pipe,

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2 \quad ...(i)$$

Here, $p_1 = \rho_m g h_1 = 13600 \times 9.8 \times 10^{-2}$

$$p_2 = 13600 \times 9.8 \times h \Rightarrow \rho = 1000\,\text{kgm}^{-3}$$

$$v_1 = 35 \times 10^{-2}\text{ms}^{-1} \Rightarrow v_2 = 65 \times 10^{-2}\text{ms}^{-1}$$

$\therefore$ From Eq. (i), we get

$$13600 \times 9.8 \times 10^{-2} = \frac{1}{2} \times 1000 \times (0.35)^2$$

$$= 13600 \times 9.8 \times h + \frac{1}{2} \times 1000 \times (0.65)^2$$

After solving, 0.89 cm of Hg.

119. (a) Fluid resistance is given by $R = \frac{8\eta l}{\pi r^4}$.

When two capillary tubes of same size are joined in series, then equivalent fluid resistance is

$$R_e = R_1 + R_2 = \frac{8\eta l}{\pi r^4} + \frac{8\eta \times 2l}{\pi (2R)^4} = \left(\frac{8\eta l}{\pi r^4}\right) \times \frac{9}{8}$$

Equivalent resistance becomes 9/8 times so rate of flow will be (8/9)X.

120. (a) $\Delta p = \frac{4T}{r} \Rightarrow \Delta p \propto \frac{1}{r}$

As radius of soap bubble increases with time.

$$\therefore \quad \Delta p \propto \frac{1}{t}$$

121. (b) $r = \frac{r_1 r_2}{r_1 - r_2} = \frac{5 \times 4}{5 - 4} = 20$ cm

122. (a) From the figure, we have

$$\delta g R(\cos\theta + \sin\theta) = \rho g R(\cos\theta - \sin\theta)$$

$$\Rightarrow \quad \delta\cos\theta + \delta\sin\theta = \rho\cos\theta - \rho\sin\theta$$

$$\Rightarrow \quad \sin\theta(\delta + \rho) = \cos\theta(\rho - \delta) \Rightarrow \tan\theta = \frac{\rho - \delta}{\rho + \delta}$$

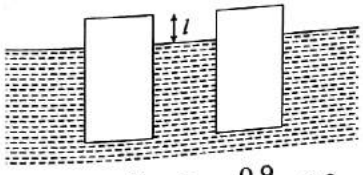
123. (a) Density of the mixture

$$4 = \frac{\rho_1 V + \rho_2 V}{2V} \text{ or } \rho_1 + \rho_2 = 8 \quad ...(i)$$

$$3 = \frac{m + m}{(m/\rho_1) + (m/\rho_2)} \text{ or } \frac{\rho\rho_2}{\rho_1 + \rho_2} = \frac{3}{2} \quad ...(ii)$$

Solving these two equations, we get $\rho_1 = 6$ and $\rho_2 = 2$

124. (b) In equilibrium, weight = Upthrust *i.e.*, $V\rho_i g = V_i \rho_w g$



or $$\frac{V_i}{V} = \frac{\rho_i}{\rho_w} = \frac{0.9}{1.0} = 0.9$$

i.e., 90% or 9 m of ice block is inside the water. Therefore minimum length l required to scoop out a bucket of water is $10 - 9 = 1.0$ m

125. (a) $v = \sqrt{2gh}$

Retardation in liquid $a = \frac{V\sigma g - V\rho g}{V} = \left(\frac{\sigma - \rho}{\rho}\right) g$

$$\therefore \quad d_{max} = \frac{v^2}{2a} = \frac{2gh}{2 \times \left(\frac{\sigma - \rho}{\rho}\right) g} = \frac{\rho h}{\sigma - \rho}$$

126. (b) Fraction of length will remain unchanged.

$$\frac{y_i}{y} = \text{constant} = \frac{1}{2} \text{ in the given question.}$$

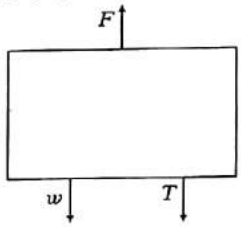
$\therefore$ If total melting rate is 2 cmh^{-1}, then top of candle will fall at the rate of 1 cmh^{-1}.

127. (a) $F = (6\pi\eta r v) = 6\pi\left(\frac{1}{6\pi}\right)(1)(v) = v$

$$\therefore \quad \text{Retardation } a = -\frac{F}{m} = -v \text{ or } \frac{dv}{dt} = -v$$

$$\Rightarrow \quad \int_0^t dt = -\int_2^{0.5} \frac{dv}{v} \text{ or } t = \ln(4)\text{ s}$$

128. (d) Upthrust $F = w + T$



$$T = F - w = V\rho_w g - V\rho_b g$$

$$= (\rho_w - \rho_b)Vg$$

$$= (1000 - 800)\left(\frac{8}{800}\right)(10) = 20\text{ N}$$

129. (c) Upthrust = (volume of metal ball) × (density of liquid) × g

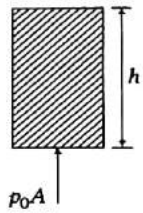
with increase in temperature volume of ball will increase and density of liquid will decrease.

But coefficient of cubical expansion of liquid is more.

Hence second effect is more dominating.

Therefore upthrust, at higher temperatures will be less or apparent weight will be more.

130. (d) Initially $p_0 A$ supports the weight of mercury column. While in second case it provides acceleration too.



131. (*b*) Let r be the radius of smaller drops and R the radius of bigger drop. Then by equating volume, we have

$$1000\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi R^3 \quad \text{or} \quad r = \frac{R}{10}$$

Surface energy $\propto$ surface area

$$S_i = 4\pi R^2 \quad \text{or} \quad S_f = 1000\,(4\pi r^2) = 10\,(4\pi R^2) = 10\,S_i$$

As surface area has increased 10 times, surface energy will also become ten times.

132. (*c*) From conservation of mechanical energy it will return to same height.

133. (*a*) $A_1 v_1 = A_2 v_2$ or $A_1 = \sqrt{2gh_2} = A_2\sqrt{2gh}$

or $A_1^2 h_1 = A_2^2 h_2 \Rightarrow (L^2)^2\,(y) = (\pi R^2)^2\,(4y)$

or $R = \dfrac{L}{\sqrt{2\pi}}$

134. (*b*) Relative density of steel $= \dfrac{\text{weight in air}}{\text{change in weight in water}}$

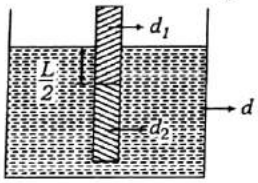
$$= \frac{w}{w - w_1}$$

Now change in weight in the given liquid = upthrust in this liquid

or $$w - w_2 = \left\{\frac{w/g}{(w/w - w_1)\,\rho_w}\right\} d_2 \times \rho_w \times g$$

$\therefore$ $$d_2 = \frac{w - w_2}{w - w_1} = \text{relative density of given liquid.}$$

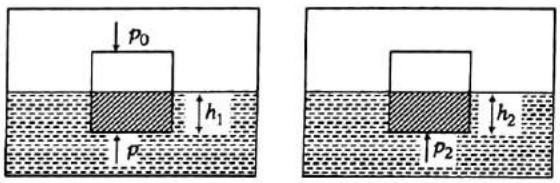
135. (*a*) Weight of combined cylinder = upthrust



$$\therefore \quad d_1 LAg + d_2 LAg = d\left(\frac{3L}{2}\right)Ag \Rightarrow \quad d_1 + d_2 = \frac{3}{2}d$$

But since $d_1 < d_2 \Rightarrow d_1 < \dfrac{3}{4}d$

136. (*c*)



In the first case,

Initially $(p - p)A$ = weight of block

or $$[(p_0 + \rho g h_1) - p_0]A = w \quad \text{or} \quad h_1 = \frac{w}{A\rho g} \qquad \text{...(i)}$$

In the second case,

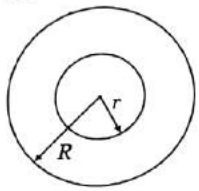
$$p_2 A = w \quad \text{or} \quad (\rho g h_2)A = w \quad \text{or} \quad h_2 = \frac{w}{\rho A g} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we see that $h_1 = h_2$

137. (*c*) Upthrust $= \dfrac{2}{10} \times 1 \times 10 = 2$ N

Reading of S_1 will increase by 2N *i.e.*, if will become $50 + 2 = 52$ N while that of S_2 will decrease by 2N *i.e.*, $20 - 2 = 18$ N.

138. (*c*) Weight = upthrust



$$\left(\frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3\right)(8 \times \rho_w \times g) = \left(\frac{4}{3}\pi R^3\right)\rho_w g$$

or $$8R^3 - 8r^3 = R^3$$

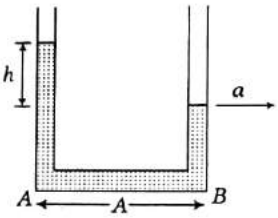
$\therefore$ $$8r^3 = 7R^3 \quad \text{or} \quad \frac{r}{R} = \left(\frac{7}{8}\right)^{1/3} = \frac{\sqrt[3]{7}}{2}$$

139. (*d*) Pressure, $p = p_0 + \rho_1 g h_1 + \rho_2 g h_2$

or $p - p_0 = \rho_1 g h_1 + \rho_2 g h_2$

$= (600)\,(10)\,(0.1) + (1000)\,(10)\,(0.02) = 800\ \text{N m}^{-2}$

140. (*b*) Change in pressure along horizontal direction, $\dfrac{dp}{dX} = \rho a$



$\therefore$ $$\Delta p_{AB} = \rho a L = \rho g h$$

$\therefore$ $$h = \frac{La}{g}$$

141. (*d*) $p_A = p_C = p_0$ and $p_B = p_0 - \dfrac{2T}{R}$

142. (*c*) If temperature remains constant, then

$$p_1 V_1 = p_2 V_2$$

$$(75 + h)V = (75)\,(3V)$$

$\therefore$ $h = 150$ cm of Hg

$= 1.5$ m of Hg or 15 m of water

143. (*a*) As, $A_0 v_0 = Av \Rightarrow v^2 = v_0^2 + 2gh$

$$\Rightarrow \quad v_0 = \sqrt{\frac{2ghA^2}{A_0^2 - A^2}} \quad \text{or} \quad R = A_0 v_0 = A_0\sqrt{\frac{2ghA^2}{A_0^2 - A^2}}$$

144. (*a*) Velocity of ball just before striking water surface, $v = \sqrt{2gh}$ $(h = 2\text{m})$

or $$v = \sqrt{4g}\ \text{m s}^{-1}$$

Retardation in water, $a = \dfrac{\text{Upthrust} - \text{Weight}}{\text{Mass}}$

or $$a = \frac{V\rho_w g - V\rho g}{V\rho} = \left(\frac{1 - 0.8}{0.8}\right)g = g/4$$

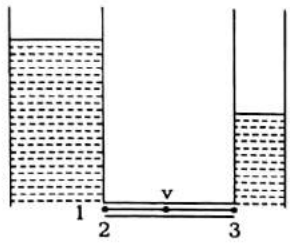
Now, depth to which ball will sink is

$$d = \frac{v^2}{2a} = \frac{4g}{2g/4} = 8\,\text{m}$$

145. (*b*) $\Delta p = \frac{F}{A}$. Difference in pressure energy is equal to difference in kinetic energy.

$$\therefore \quad \frac{F}{A} = \frac{1}{2}\rho v^2 \quad \text{or} \quad v = \sqrt{\frac{2F}{\rho A}}$$

146. (*a*) $p_2 = p_3$ and $p_1 > p_2$



$$p_1 = p_0 + \rho g x \Rightarrow p_2 = p_0 + \rho g y$$

$$\therefore \quad \Delta p = p_1 - p_2 = \rho g (x - y)$$

Between 1 and 2

Difference in pressure energy = difference in kinetic energy

$$\text{or} \quad \rho g (x - y) = \frac{1}{2}\rho v^2 \quad \text{or} \quad v = \sqrt{2g(x - y)}$$

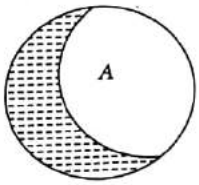
147. (*d*) Given, $l_1 = l_2 = 1\text{m}$ and $\frac{r_1}{r_2} = \frac{1}{2}$

$$V = \frac{\pi p_1 r_1^4}{8\eta l} = \frac{\pi p_2 r_2^4}{8\eta l} \Rightarrow \frac{p_1}{p_2} = \left(\frac{r_2}{r_1}\right)^4 = 16 \Rightarrow p_1 = 16 p_2$$

Since, both tubes are connected in series, hence pressure difference across combination,

$$p = p_1 + p_2 \Rightarrow p_1 = \frac{p_1}{16} \Rightarrow p_1 = \frac{16}{17} = 0.94\text{ m}$$

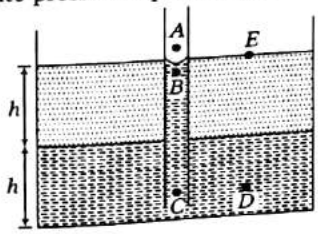
148. (*a*) *B* will keep its area as minimum as possible.



149. (*b*) $W = \text{Upthrust} = V_i \rho l g$ or $V_i = \frac{W}{\rho l g}$

If density of liquid ρl is increased, immersed volume V_i will decrease or the ball will go up.

150. (*c*) Let us write pressure equation in path *ABCDE*.



$$p_0 - \frac{2T}{r} + \rho_2 g (h + h') - \rho_2 g h' - \rho_1 g h = p_0$$

$$\therefore \quad T = \frac{r}{2}(\rho_2 - \rho_1) g h$$

151. (*c*) From work-energy theorem,

work done by all the forces = change in kinetic energy.

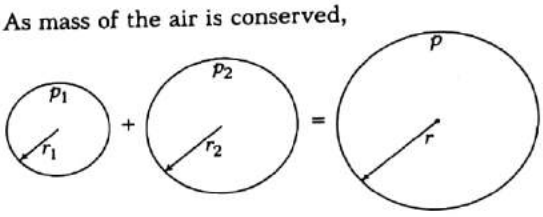
∴(Work done by gravity for 7 m) + (work done by upthrust for 2 m) + (work done by resistive forces) = 0 as Δ KE = 0

$$\therefore \quad (1)(10)(7)\cos 0° + \left(\frac{1}{2/3 \times \rho_w}\right)(\rho_w g)$$

(2) cos 80° + work done by resistive forces = 0

∴ Work done by resistive forces = – 40 J

152. (*b*) As mass of the air is conserved,



$$\therefore \quad n_1 + n_2 = n \qquad (\text{as } pV = nRT)$$

$$\therefore \quad \frac{p_1 V_1}{RT_1} + \frac{p_2 V_2}{RT_2} = \frac{pV}{RT}$$

As temperature is constant,

$$T_1 = T_2 = T$$

$$\therefore \quad p_1 V_1 + p_2 V_2 = pV$$

$$\therefore \left(p_0 + \frac{4S}{r_1}\right)\left(\frac{4}{3}\pi r_1^3\right) + \left(p_0 + \frac{4S}{r_2}\right)\left(\frac{4}{3}\pi r_2^3\right) = \left(p_0 + \frac{4S}{r}\right)\left(\frac{4}{3}\pi r^3\right)$$

Solving, this we get

$$S = \frac{p_0 (r^3 - r_1^3 - r_2^3)}{4(r_1 r^2 + r_2^2 - r^2)}$$

Note To avoid confusion with the temperature surface tension here is represented by *S*.

153. (*c*) Let tube's entrance be a depth *y* below the surface. Take point 1 at entry, 2 at the maximum height of the fountain. Applying Bernoulli's theorem,

$$p_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

Taking, $h_1 = 0$, $h_2 = (y + h_0 + h)$, $v_1 = v$, $v_2 = 0$

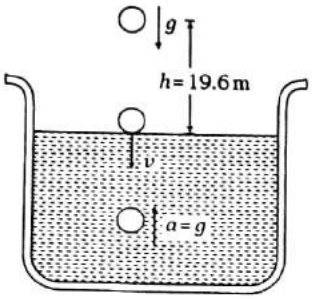
substituting, $p_0 + \rho g y + \frac{1}{\rho} v^2 = p_0 + \rho g (y + h_0 + h)$

$$\Rightarrow \quad \frac{1}{2}\rho v^2 = \rho g (h_0 + h)$$

$$\Rightarrow \quad h = \left(\frac{v^2}{2g} - h_0\right)$$

154. (*a*) $v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 19.6} = 19.6\text{ ms}^{-1}$

Let ρ be the density of ball and 2ρ the density of water. Net retardation inside the water,



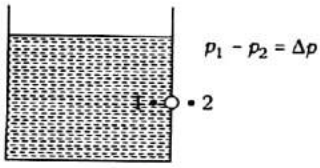
$$a = \frac{\text{upthrust} - \text{weight}}{\text{mass}}$$

$$= \frac{V(2\rho)g - V(\rho)(g)}{V(\rho)g} \qquad (V = \text{volume of ball})$$

$$= g = 9.8 \text{ ms}^{-2}$$

Hence, the ball will go upto the same depth 19.6 m below the water surface. Further, time taken by the abll to come back to water surface is, $t = 2\left(\frac{v}{a}\right) = 2\left(\frac{19.6}{9.8}\right) = 4$ s

155. (*d*) $\Delta p = \frac{1}{2}\rho v^2$ or $v = \sqrt{\frac{2(\Delta p)}{\rho}}$



$$R = vt \quad \text{or} \quad R \propto \sqrt{\Delta p}$$

To make R two times Δp should be 4 times. Initially

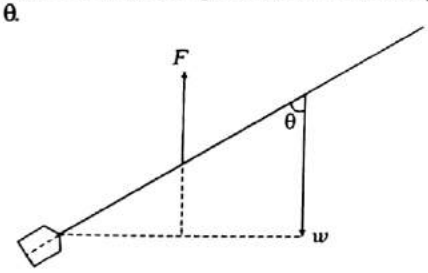
$$\Delta p = \rho_w gh$$
$$= 10^3 \times 10 \times 10$$
$$= 10^5 \text{ Nm}^{-2} = 1 \text{ atm}$$

In second case, $\rho_w gh$ + extra pressure = 4 (Δp) = 4 atm

or 1 atm + extra pressure = 4 atm

∴ Extra pressure = 3 atm.

156. (*b*) About hinge clockwise torque of hinge should be equal to anti-clockwise torque of upthrust.Immersed length will be 0.5 sec θ.



$$\therefore \quad w = \left(\frac{1}{2}\sin\theta\right) = F\left(\frac{0.5 \sec\theta}{2}\right)\sin\theta$$

or $w = 0.5 F \sec\theta$ or $(1 \times A \times 0.5 \times \rho_w \times g)$

$$= 0.5\,(0.5 \sec\theta) \times A \times 1.0 \times \rho_w \times g \times \sec\theta$$

$\sec^2\theta = 2$ or $\theta = 45°$

157. (*b*) $Q = Av_1 = A_2v_2 = 15 \text{ Lmin}^{-1} = \frac{15 \times 10^{-3}}{60} = 2.5 \times 10^{-4} \text{ m}^3 \text{ s}^{-1}$

$$v_1 = \frac{2.5 \times 10^{-4}}{2 \times 10^{-4}} = 1.25 \text{ ms}^{-1} \Rightarrow v_2 = \frac{2.5 \times 10^{-4}}{0.5 \times 10^{-4}} = 5 \text{ ms}^{-1}$$

Now difference in pressure = difference in kinetic energy

$$\rho_w gh = \frac{1}{2}\rho_a (v_2^2 - v_1^2)$$

$$\therefore \quad h = \frac{\rho_a (v_2^2 - v_1^2)}{2\rho_w g} = 1.5 \times 10^{-3} \text{ m} \approx 1.5 \text{ mm}$$

or $h\rho_w g = \frac{1}{2}\rho_{air}(v_2^2 - v_1^2)$

or $h = \frac{\rho_{air}}{2\rho_w g}(v_2^2 - v_1^2) = \frac{1.32}{2 \times 10^3 \times 10}[(5)^2 - (1.25)^2]$

$$= 1.5 \times 10^{-3} \text{ m} \approx 1.5 \text{ mm}$$

B) Medical entrance special format questions

Assertion and reason

1. (*b*) For an incompressible fluid, $\Delta V = 0$ and $\rho = \frac{m}{V} = a$ constant

2. (*a*) upthrust = mass of fluid displaced × $g_{effective}$

And in case of free fall, $g_{effective} = 0$

3. (*b*) $\beta = \frac{pV}{\Delta v}$ and $\Delta V = 0$ for an incompressible fluid

$\Rightarrow \quad \beta = \infty$

4. (*e*) $k = \frac{2T\cos\theta}{r\rho g} \Rightarrow h \propto \cos\theta$

and $h = 0$ when $\theta = 90°$

Also angle of contact can be 0° in any practical case.

So, both A and R are incorrect.

6. (*b*) In small drops force of surface tension is much larger. So, they colasces to a smaller radii spheres.

8. (*b*) Both *A* and *R* are correct but '*R*' is not an explanation for *A*.

9. (*d*) Water level rises as stone pieces displaces more liquid compared to condition when they sinks.

11. (*a*) upthrust = weight of fluid displaced

= volume of body × ρ_{fluid} × g.

12. (*a*) Volume of fluid displaced = volume of body immersed.

16. (*d*) Upthrust by liquid = weight of liquid displaced.

Match the columns

1. From, $A_1v_1 = A_2v_2$ or $v \propto \frac{1}{A}$

At a point where area of cross-section is less, volume of liquid flowing per second is same but speed is more. Therefore, here kinetic energy is more or the pressure will be less.

2. When two soap bubbles coalesce to form a larger drop, radius increases. Hence, the excess pressure $4T/R$ will decrease. Further, surface area will decrease. Hence, the surface energy will decrease or the temperature will increase.

3. At depth h $p = p_0 + \rho(g + a)h \Rightarrow \Delta p = \rho(g + a)\Delta h$

Upthrust will also increase because upthrust

$$F = V_i(g + a)\rho$$

Here, V_i is the immersed volume.

4. Pressure at O is the atmospheric pressure which will remain same.

5. $v = \sqrt{2gh}$, $t = \sqrt{\frac{2h}{g}}$ and $R = 2\sqrt{h(H-h)}$

At mountain, value of g will be less. Hence, v will decrease, t will increase and R will remain unchanged.

6. In equilibrium upthrust = weight

If density of liquid is decreasing, more volume should be immersed in the liquid, so that upthrust remains unchanged and it balances the weight.

Medical entrances' gallery

1. (b) Increase in surface area,

$$\Delta A = (5 \times 4 - 4 \times 2) \times 2 \quad (\because \text{ film has two surfaces})$$
$$= (20 - 8) \times 2 \text{ cm}^2 = 24 \text{ cm}^2 = 24 \times 10^{-4} \text{ m}^2$$

So, work done, $W = T\Delta A$

$$3 \times 10^{-4} = T \times 24 \times 10^{-4}$$

$$\therefore \quad T = \frac{1}{8} = 0.125 \text{ N m}^{-1}$$

2. (b) Ascent formula for capillary tube,

$$h = \frac{2T\cos\theta}{\rho g r}$$

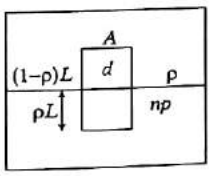
$$\therefore \quad \frac{\cos\theta_1}{\rho_1} = \frac{\cos\theta_2}{\rho_2} = \frac{\cos\theta_3}{\rho_3}$$

Thus, $\cos\theta \propto \rho$ $[\because \rho_1 > \rho_2 > \rho_3]$

$$\therefore \quad \cos\theta_1 > \cos\theta_2 > \cos\theta_3$$

$$0 \le \theta_1 < \theta_2 < \theta_3 < \frac{\pi}{2}$$

3. (c) According to question, the situation can be drawn as following.



Applying Archimedes principle,

Weight of cylinder = $(\text{upthrust})_1 + (\text{upthrust})_2$

i.e., $ALdg = (1-p)LA\rho g + (pLA)n\rho g s$

$$\Rightarrow \quad d = (1-p)\rho + pn\rho = \rho - p\rho + n\,p\rho$$
$$= \rho + (n-1)p\rho = \rho[1 + (n-1)p]$$

4. (c) From Bernoulli's theorem

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

where, p_1, p_2 are pressure inside and outside the roof and v_1, v_2 are velocities of wind inside and outside the roof. Neglect the width of the roof. Pressure difference is

$$p_1 - p_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) = \frac{1}{2} \times 12(40^2 - 0) = 960 \text{ Nm}^{-2}$$

Force acting on the roof is given by

$$F = (p_1 - p_2)A = 960 \times 250$$
$$= 24 \times 10^4 \text{N} = 2.4 \times 10^5 \text{ N}$$

As the pressure inside the roof is more than outside to it. So, the force will act in the upward direction.

i.e., $F = 2.4 \times 10^5$ N (upward).

5. (c) Given, $d = 2700$ m $\Rightarrow \rho = 10^3$ kgm^{-3}

Compressibility $= 45.4 \times 10^{-11}$ per pascal

The pressure at the bottom of ocean is given by

$$p = \rho g d = 10^3 \times 10 \times 2700 = 27 \times 10^6 \text{ Pa}$$

So, fractional compression = compressibility × pressure

$$= 45.4 \times 10^{-11} \times 27 \times 10^6 = 1.2 \times 10^{-2}$$

6. (d) Let the velocity at point B is v_B.

From conservation of total mechanical energy,

$$mg[h - L\sin\alpha] = \frac{1}{2}mv_B^2$$

$$\Rightarrow \quad v_B^2 = 2g(h - L\sin\alpha) = 2g\left(10 - 2 \times \frac{1}{2}\right)$$

$$\Rightarrow \quad v_B^2 = 18g$$

Now, let maximum height attained by water stream, be (H).

$$\therefore \quad H = L\sin\theta + \frac{v_B^2 \sin^2\alpha}{2g}$$

$$\Rightarrow \quad H = 2 \times \frac{1}{2} + \frac{18g\left(\frac{1}{2}\right)^2}{2g} = 1 + \frac{18}{8} = 3.25 \text{ m}$$

7. (b) Gain in surface energy

$$E = 4\pi R^2(n^{1/3} - 1)T$$
$$= 4 \times 3.14 \times (10^{-2})^2[(1000)^{1/3} - 1] \times 0.075$$
$$= 8.48 \times 10^{-4} \text{ J}$$

8. (d) Given, height raised by liquid,

$$h = 7.5 \text{ cm} = 7.5 \times 10^{-2} \text{ m}$$

Surface tension, $S = 7.5 \times 10^{-2}$ Nm^{-1}

and contact angle, $\theta = 0°$

As, $r = \frac{2S\cos\theta}{h\rho g}$

where, ρ = density of water = 1000 kgm^{-3}

$$\Rightarrow \quad r = \frac{2 \times 7.5 \times 10^{-2} \times \cos 0°}{7.5 \times 10^{-2} \times 1000 \times 10}$$

$$= \frac{2}{10^4} = 2 \times 10^{-4} \text{ m}$$

$$= \frac{2}{10} \times 10^{-3} \text{ m} = 0.2 \text{ mm}$$

9. (*a*) The area of soap bubble $= 4\pi\left(\frac{a}{2}\right)^2 = 4\pi\frac{a^2}{4} \Rightarrow A = \pi a^2$

As surface energy $= \frac{T}{A}$

and surface energy is equal to surface tension.

Total energy associated with surface, $U = TA$

As soap bubbles are of two layers so

$$U = T \times 2A = T \times 2\pi a^2 = 2\pi a^2 T$$

10. (*e*) When the stones are unloaded into water, the water level falls because the volume of the water displaced by stones in water will be less than the volume of water displaced when stones are in the boat.

11. (*a*) Terminal velocities of raindrops are proportional to square of their radii.

So, option (a) is correct.

12. (*d*) In freely falling platform a body experience weight lessness. So, the liquid will rise upto to length of the capillary. *i.e.*, height raised by the liquid will be 20 cm.

13. (*c*) Pressure difference between atmosphere and lungs = pressure difference at a depth h of fluid.

$$\Rightarrow \quad 10 \text{ mm} \times \rho_{Hg} \times g = h\rho_w g$$

$$h = 10 \text{ mm} \times \frac{\rho_{Hg}}{\rho_w} = 13.6 \text{ cm}$$

14. (*a*) Let, R = radius of bigger drop

and r = radius of smaller drop

Then, $\frac{4}{3}\pi R^3 = 1000 \times \frac{4}{3}\pi r^3 \Rightarrow R = 10r$

The surface energy of smaller drop, $E_1 = s \times 4\pi r^2$ and the surface energy of bigger drop,

$$E_2 = s \times 4\pi (10r)^2 \Rightarrow \frac{E_1}{E_2} = \frac{1}{100}$$

15. (*c*) Volume of part above the water surface $= \frac{V}{3}$

($\because V$ = Volume of solid)

Volume of solid inside the water surface $= V - \frac{V}{3} = \frac{2V}{3}$

Weight of displaced water

$$W = \frac{2V}{3} \times 10^3 \times g$$

Now, weight of body = Weight of displaced water

$$\Rightarrow \quad V\rho g = \frac{2V}{3} \times 10^3 g$$

$$\Rightarrow \quad \rho = \frac{2}{3} \times 1000 = \frac{2000}{3} \text{ kg m}^{-3}$$

16. (*a*) Change in surface area

$$= 2 \times 4\pi\left[\left(\frac{D}{2}\right)^2 - \left(\frac{d}{2}\right)^2\right] = 2\pi(D^2 - d^2)$$

$\therefore$ Work done = surface tension × change in area

$$= 2\pi S(D^2 - d^2)$$

17. (*b*) With the increase in temperature, the surface tension of liquid decreases and angle of contact also decreases.

18. (*a*) Assuming temperature constant

As from the Boyle's law pV = constant

So, from the two cases at h depth and at the surface.

$$p_1V_1 = p_2V_2 \quad \text{...(i)}$$

where, p_1 = pressure at h depth, V_1 = volume at h depth

p_2 = pressure at surface, V_2 = volume at surface.

$$\Rightarrow \quad p_1 = (h + l)\rho g, \; V_1 = \frac{4}{3}\pi r^3$$

$$p_2 = l\rho g, V_2 = \frac{4}{3}\pi(3r)^3$$

Putting these values in Eq. (i), we get

$$(h + l)\rho g\left(\frac{4}{3}\right)\pi r^3 = l\rho g \times \left(\frac{4}{3}\right)\pi(3r)^3$$

So, $\quad (h + l) = 27l$

$\Rightarrow \quad h = 26l$

19. (*b*) According to Archimedes' principle, weight of the wooden block = weight of the liquid displaced

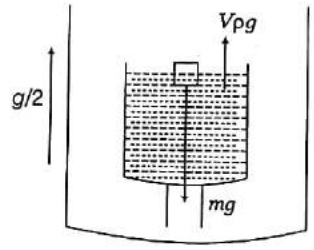
$$\Rightarrow \quad mg = \left(\frac{60}{100}\right)V \times \rho_l \times g = \frac{3}{5}V \times \rho_l \times g \quad \text{...(i)}$$

[Here, V = total volume of block, ρ_l = density of liquid]

When beaker is kept in a lift,

Net weight of the block $= mg + mg/2$

$$= \frac{3mg}{2} \quad \left[\because R - mg = \frac{mg}{2}\right]$$



Again for float condition

Net weight of the block = Buoyant force

$$\Rightarrow \quad \frac{3mg}{2} = V_i \times \rho_l \times g \quad \text{...(ii)}$$

Here, V_i = volume of block inside liquid from Eqs. (i) and (ii), we get, $\left(\frac{3}{2}\right) \times \frac{3}{5}(V \times \rho_l \times g) = V_i \times \rho_l \times g$

or $\quad V_i = \frac{9}{10}V$

So, volume of the block above liquid

$$= \frac{1}{10}V = 10\% \text{ of total volume.}$$

20. (*c*) Stoke established that if a sphere of radius a moves with velocity v through a fluid of viscosity η, the viscous force opposing the motion of the sphere is $F = 6\pi\eta av$.

21. (*a*) $R_e \le 1000$: Streamlined flow

$1000 < R_e < 2000$;Unstable flow

$R_e \ge 2000$; Turbulent flow

22. (*c*) Here, if the surface area changes, it will change the surface energy as well. If the surface area decreases, it means that energy is released and *vice-versa*.

Change in surface energy $\Delta A \times T$ ...(i)

Let we have n number of drops initially.

So, $\Delta A = 4\pi R^2 - n(4\pi r^2)$ ...(ii)

Volume is constant

So, $n\frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3 = V$ ...(iii)

From Eqs. (ii) and (iii), we get

$$\Delta A = \frac{3}{R}\frac{4\pi}{3}\times R^3 - \frac{3}{r}\left(n\frac{4\pi}{3r^3}\right) = \frac{3}{R}\times V - \frac{3}{r}V$$

$$\Rightarrow \quad \Delta A = 3V\left(\frac{1}{R} - \frac{1}{r}\right) = \text{negative value}$$

As $R > r$, so ΔA is negative. It means that surface area is decreased, so energy must be released.

Energy released $= \Delta A \times T = -3VT\left(\frac{1}{r} - \frac{1}{R}\right)$

23. (*c*) Let thickness of layer be x.

So, volume V = Area $\times x$

$\Rightarrow \quad V = A \times x$ $(\because x = 2r)$

$x = V/A \Rightarrow 2r = \frac{V}{A} \Rightarrow r = \frac{V}{2A}$ ...(i)

and $\Delta p = \frac{T}{r}$

We know that $F = \Delta p \times A = \frac{T}{r}\times A$

$$F = \frac{T}{\left(\frac{V}{2A}\right)}\times A \quad \text{[from Eq. (i)]}$$

$$T = \frac{F \times V}{2A^2}$$

where, $F = 16\times10^5$ dyne, $V = 0.04$ cm^3,

$$A = 20 \text{ cm}^2 = \frac{16\times10^5\times0.04}{2\times20^2}$$

$$= \frac{8\times10^5\times4}{20^2\times100} = \frac{8\times10^5\times4}{400\times100}$$

$$= 8\times10^5\times10^{-4} = 80 \text{ dyne cm}^{-1} = 80 \text{ dyne cm}^{-1}$$

24. (*a*) The energy of soap bubbles (when radius = r)

E = surface area of soap bubble × surface tension

$$E = 4\pi r^2\sigma$$

Now, radius becomes a double.

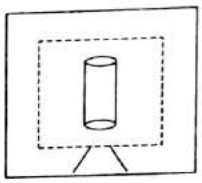
Then, the value of $E = 4\pi(2r)^2\sigma = 16\pi r^2\sigma$

25. (*b*) Consider the situation where a capillary tube is dipped in water of density ρ.

Net force on the liquid due to surface tension will be upward, and is balanced by the downward force of the weight

$w = \pi r^2 h\rho g$ = weight of liquid

So, $2\pi rS\cos\theta$ = Net force due to surface tension



$\Rightarrow \quad \pi r^2 h\rho g = 2\pi rS\cos\theta$

where, S is surface tension and θ is the contact angle.

$$\Rightarrow \quad h = \frac{2S\cos\theta}{r\rho g} \Rightarrow h \propto \frac{1}{g}$$

where, g is acceleration due to gravity.

When the entire arrangement is falling with an acceleration g.

$$g_{\text{eff}} = g - g = 0$$

Hence, $h' \propto \frac{1}{g_{\text{eff}}} = \frac{1}{0} = \infty$

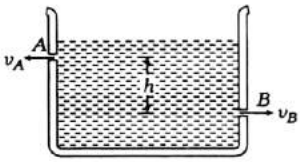
Hence, there is a overflow in the capillary tube. As length of the capillary tube is l.

$\Rightarrow \quad h' = l = 20$ cm

26. (*d*) The value of angle of contact determines whether a liquid will spread on the surface.

27. (*c*) From Pascal's law of transmission of pressure, the increase in pressure at any one point of the enclosed liquid in equilibrium of rest is transmitted equally to all other points of the liquid and also to the container walls.

28. (*b*)



Net horizontal force (reaction) $= F = F_B - F_A = \frac{dp_B}{dt} - \frac{dp_A}{dt}$

$= av_B p \times v_B - av_A p \times v_A$

$\therefore \quad F = ap(v_B^2 - v_B^2)$ ...(i)

According to Bernoulli's theorem

$$p_A + \frac{1}{2}pv_A^2 + pgh = p_B + \frac{1}{2}pv_B^2 + (p_A - p_B)$$

$$\Rightarrow \quad \frac{1}{2}(v_B^2 - v_B^2) = pgh \Rightarrow v_B^2 - v_A^2 = 2gh$$

From equation (i), $F = 2apgh \Rightarrow F \propto h$

29. (*d*) Given, $h_1 = 10$ cm, $h_2 = 3.42$ cm

$\theta_2 = 135$ and $d_2 = 13.6$ gcc^{-1}

Here, $\frac{T_1}{T_2} = \frac{h_1 d_1}{h_2 d_2}\frac{\cos\theta_2}{\cos\theta_1} \Rightarrow \frac{T_1}{T_2} = \frac{10\times1}{3.43\times13.6}\frac{\cos135°}{\cos45°} \Rightarrow \frac{T_1}{T_2} = \frac{2}{13}$

30. (*c*) We know that the pressure $p \propto \frac{r^4}{l}$

Given, $l_1 : l_2 = 2:1$ and $r_1 : r_2 = 1:2$

Let $l_1 = 2l$, $l_2 = l$ and $r_1 = r$, $r_2 = 2r$

Now, $\frac{p_1}{p_2} = \frac{r_1^4}{l_1}\times\frac{l_2}{r_2^4} \Rightarrow \frac{p_1}{p_2} = \frac{r^4}{2l}\times\frac{l}{(2r)^4}$

$$\frac{p_1}{p_2} = \frac{1}{2\times(2)^4} \Rightarrow \frac{p_1}{p_2} = \frac{1}{32}$$

31. (*b*) We know that the pressure at the end of tube $p = \frac{4T}{r}$

Here the pressure difference at the both end of tube

$$\Delta p = \Delta p_1 - \Delta p_2 = \frac{4T}{r_1} - \frac{4T}{r_2} = 4T\left[\frac{r_2 - r_1}{r_1 r_2}\right]$$

Given $r_1 = 5\text{ cm} = 0.05\text{ m}$ and $r_2 = 3\text{ cm} = 0.03\text{ m}$

$$\Delta p = 4 \times 0.03\left[\frac{0.05 - 0.03}{0.05 \times 0.03}\right]$$

$$= \frac{4 \times 0.03 \times 0.02}{0.05 \times 0.03} = \frac{0.0024}{0.0015} = 1.6\text{ Pa}$$

32. (*c*) The ratio of inertial force to viscous force of a fluid is called Reynolds number.

33. (*a*) We know that, for soap bubble, $p = \frac{4T}{r}$

Given, $p_1 = 3p_1$

Here, $\frac{4T}{r_1} = 3\left(\frac{4T}{r_2}\right) \Rightarrow r_2 = 3r_1$ ...(i)

$$V_1 = \frac{4}{3}\pi r_1^3 \text{ and } V_2 = \frac{4}{3}\pi r_2^3 = \frac{4}{3}\pi (3r_1)^3 = \frac{4}{3}\pi (27 r_1^3)$$

$$\Rightarrow \quad \frac{V_1}{V_2} = \frac{(4/3)\pi r_1^3}{(4/3)\pi 27 r^3} = \frac{1}{27}$$

34. (*a*) We know that capillary rises

$$h = \frac{2\sigma\cos\theta}{\rho g r} \Rightarrow 20 \times \frac{1}{3} = \frac{20}{3}\text{ mm}$$

35. (*c*) We have, $1\text{ P} = 1\text{ g cm}^{-1}\text{s}^{-1}$

$\therefore \quad 1\text{ Pl} = 10\text{ gcm}^{-1}\text{s}^{-1} = 10\text{ P}$

36. (*c*) If a liquid can rise to a height h, but the tube has insufficient height h', then the angle of contact increases from θ to θ' given by $\frac{h}{\cos\theta} = \frac{h'}{\cos\theta'}$

Here, $h = 2\text{ cm}, h' = 1\text{ cm}, \theta = 0°$

$$\therefore \frac{2}{\cos 0} = \frac{1}{\cos\theta'} \Rightarrow \frac{2}{1} = \frac{1}{\cos\theta'} \text{ or } \cos\theta' = \frac{1}{2} \Rightarrow \theta' = 60°$$

37. (*a*) The excess pressure inside the bubble

$$p = \frac{4T}{r}, \text{ then } p_1 = \frac{4T}{r_1} \quad \text{...(i)}$$

$$p_2 = \frac{4T}{r_2} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{p_1}{p_2} = \frac{4T/r_1}{4T/r_2} = \frac{r_2}{r_1} = \frac{1}{2}$$

38. (*b*) A solid is floating in liquid, so weight of solid body = weight of liquid displaced by immersed part of the body

$$VDg = vdg \Rightarrow \frac{v}{V} = \frac{D}{d}$$

39. (*c*) Time taken to empty a vessel of height h, $t = \sqrt{2h/g}$

$\therefore$ Time taken in emptying a vessel of height $h/2$,

$$t' = \sqrt{\frac{2h/2}{g}} = \sqrt{\frac{h}{g}} \therefore \frac{t'}{t} = \frac{1}{\sqrt{2}} \text{ or } t' = \frac{t}{\sqrt{2}} = \frac{10}{\sqrt{2}} \approx 7\text{ min}$$

40. (*a*) From the principle of continuity, $A_1 v_1 = A_2 v_2$

$$v_2 = \frac{A_1 v_1}{A_2} = \frac{4.20 \times 5.18}{7.60} = 2.86\text{ ms}^{-1}$$

41. (*b*) Volume of liquid displaced is more than the volume of water formed. So, the level of water in the beaker falls.

42. (*a*) The height of capillary rise is inversely proportional to radius (or diameter) of capillary tube $h \propto \frac{1}{r}$

So, for smaller r the value of h is higher.

43. (*a*) The rise or fall of liquids in vertical capillary tubes is called capillarity. Water in plant fibres rises due to same phenomenon.

44. (*b*) The excess pressure of soap bubble

$$p = \frac{4T}{R} \text{ or } h\rho g = \frac{4T}{R} \quad (\because p = h\rho g)$$

$$\therefore \quad T = \frac{Rh\rho g}{4} = \frac{1 \times 10^{-2} \times 2 \times 10^{-3} \times 0.8 \times 10^3 \times 9.8}{4}$$

$$= 0.0392\text{ Nm}^{-1}$$

45. (*b*) The equation of continuity

$$A_1 v_1 = A_2 v_2 \Rightarrow A \times 4 = \frac{2}{3} A \times v_2 \Rightarrow v_2 = 6\text{ ms}^{-1}$$

From Bernoulli's theorem,

$$p + \rho g h_1 + \frac{1}{2}\rho v_1^2 = p + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

$$\text{or } g(h_1 - h_2) = \frac{1}{2}(v_2^2 - v_1^2) \text{ or } g \times h = \frac{1}{2}[(6)^2 - (4)^2] \quad (\because h_1 - h_2 = h)$$

$$\text{or} \quad 10 \times h = \frac{1}{2}[36 - 16] \text{ or } h = \frac{20}{20} = 1\text{ m}$$

46. (*c*) Specific gravity of Al = 2.7

Density of air $= 2.7 \times 10^3\text{ kg m}^{-3}$

Terminal velocity, $v = \frac{2r^2(\rho - \sigma)g}{9\eta}$

$$v = \frac{2 \times (1 \times 10^{-3})^2 (2.7 \times 10^3 - 1000) \times 9.8}{8 \times 9 \times 10^{-4}}$$

Terminal velocity $v = 4.6\text{ ms}^{-1}$.

47. (*d*) As, $V = \frac{\pi p r^4}{8\eta l}$

$\therefore \quad V \propto pr^4$ (η and l are constants)

$$\therefore \quad \frac{V_2}{V_1} = \left(\frac{p_2}{p_1}\right)\left(\frac{r_2}{r_1}\right)^4 = 2 \times \left(\frac{1}{2}\right)^4 = \frac{1}{8} \Rightarrow V_2 = \frac{Q}{8}$$

48. (*b*) Given, terminal velocity $v = 1\text{ ms}^{-1} = 100\text{ cms}^{-1}$ radius of the raindrop, $r = 0.3\text{ mm} = 0.3 \times 10^{-1}\text{ cm}$

Viscosity of air $\eta = 18 \times 10^{-3}$ Poise

$\therefore$ Viscous force, $F = 6\pi\eta r v$

$$= 6 \times 3.14 \times 18 \times 10^{-3} \times 0.3 \times 10^{-1} \times 100$$

$$= 1017.36 \times 10^{-3}\text{ dyne} = 101.73 \times 10^{-2}\text{ dyne.}$$

49. (c) Air flow from smaller bubble to bigger as the pressure in smaller bubble is higher.

CHAPTER 14

Thermometry, thermal expansion and kinetic theory of gases

Inside

1) Temperature
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- Thermometers

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4) Ideal gas laws
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The branch of science which deals with the measurement of temperature of a substance is known as *thermometry.*

Almost all substances (solid, liquid and gases) expand on heating and contract on cooling. Few exceptions are : water from 0°C to 4°C and silver iodide from 80°C to 140°C which contract on heating and expand on cooling. The expansion of a substance on heating is called *thermal expansion* of substance.

Further, in this chapter, we will study average distance between the molecules increases. *kinetic theory of gases*, that relates the macroscopic properties (pressure temperature etc.) of gases to the microscopic properties (velocity, speed) of gas molecules.

1) Temperature

The degree of hotness or coldness of body, is known as the temperature of the body. An object at higher temperature is said to be hotter than the one at lower temperature.

Scales of temperature

The measurement of temperature is done by some specified scales. In any scale of temperature two fixed points are considered; one is ice point (Lower Fixed Point—LFP) and other is steam point (Upper Fixed Point—UFP). Generally these points are **freezing point** and **boiling point** of water at 1 atm.

Some of the commonly used temperature scales are discussed below.

1. The celsius scale (°C)

This scale was designed by Andre celsius. It defines ice point temperature as 0°C and the steam point temperature as 100°C. The space between 0°C and 100°C marks is equally divided into 100 intervals and each interval is known as 1°C.

$$0°\text{ C} \xleftrightarrow{100°\text{C}} 100°\text{ C (100 equal parts)}$$

2. The fahrenheit scale (°F)

The scale was designed by Fahrenheit. It defines the ice point temperature as 32°F and the steam point as 212°F. The space between 32°F and 212°F is divided into 180 equal intervals and each interval represents 1°F.

$$32°\text{ F} \xleftrightarrow{180°\text{F}} 212°\text{ F (180 equal parts)}$$

3. The kelvin scale (K)

The scale was designed by Kelvin. It defines lower ice point temperature as 273.15 K and steam point temperature as 373.15 K. The space between 273.15 K and 373.15 K is equally divided into 100 intervals and each interval represents 1 K. Kelvin is the SI unit of temperature.

$$273\text{ K} \xleftrightarrow{100\text{ K}} 373.15\text{ K (100 equal parts)}$$

Conversion of temperature from one scale to another

In order to convert the temperature from one scale to another, the following relation should be kept in mind, *i.e.*,

$$\frac{\text{Temperature on one scale} - \text{LFP (ice point)}}{\text{UFP (steam point)} - \text{LFP (ice point)}} = \frac{\text{Temperature on other scale} - \text{LFP (ice point)}}{\text{UFP (steam point)} - \text{LFP (ice point)}}$$

In simple terms, *the temperature measured by different scales is given as*

$$\boxed{\frac{C-0}{100} = \frac{F-32}{180} = \frac{K-273}{100}}$$

Temperature difference measured by different scales is given as

$$\boxed{\Delta C = \Delta K = 5/9\,\Delta F}$$

Example 14.1 *Express a temperature of 60°F in degrees celsius and in kelvins.*

Sol. Temperature in degree celsius, $\frac{C-0}{100} = \frac{F-32}{180}$

$$C = \frac{5}{9}(F-32) = \frac{5}{9}(60° - 32°) = 15.55°\text{C}$$

Temperature in kelvin, $K = C + 273.15$

$$= 15.55°\text{C} + 273.15 = 288.7\text{ K}$$

Example 14.2 *The temperature of an iron piece is raised from 30°C to 90°C. What is the change in its temperature on the fahrenheit scale and on the kelvin scale?*

Sol. Given, $\Delta C = 90° - 30° = 60°\text{C}$

Temperature difference on Fahrenheit scale,

$$\Delta F = \frac{9}{5}\Delta C = \frac{9}{5}(60°\text{C}) = 108°\text{F}$$

Temperature difference on Kelvin scale, $\Delta K = \Delta C = 60\text{ K}$

Example 14.3 *On a new scale of temperature (which is linear) and called the W scale, the freezing and boiling points of water are 39°W and 239°W, respectively. What will be the temperature on the new scale, corresponding to a temperature of 39°C on the Celsius scale?*

Sol. In general, whenever we are to go from any known scale to any unknown scale, then we follow the equation

$$= \frac{\text{(Temperature on one scale)} - \text{(LFP for known scale)}}{(\text{UFP} - \text{LFP})_{\text{known}}}$$

$$= \frac{\text{(Temperature on other scale)} - \text{(LFP for unknown scale)}}{(\text{UFP} - \text{LFP})_{\text{unknown}}}$$

$$\Rightarrow \quad \frac{39-0}{100-0} = \frac{t-39}{239-39} \Rightarrow t = 117°\text{W}$$

Example 14.4 *An arbitrary scale has the ice point at −20° and the steam point at 180°. When the thermometer reads 5°, then find the read of centigrade thermometer.*

Sol. In order of convert the temperature from one scale to another, then

$$\frac{C-0}{100-0} = \frac{t-(-20)}{180-(-20)}$$

Here, $t = 5°$

$$\Rightarrow \quad \frac{C}{100} = \frac{5+20}{200}$$

∴ Reading of centigrade scale.

$$C = 12.5°\text{C}$$

Example 14.5 *A faulty scale has its fixed points marked as 5°C and 95°C. The temperature of a body as measured by the faulty scale is 59°. Find the correct temperature of the body on Celsius scale.*

Sol. Let θ_0 be the lower fixed point of the faulty scale and n be the number of divisions between its lower and the upper fixed points. If a temperature C on Celsius scale corresponds to temperature θ on the scale of faulty scale, then

$$\frac{C-0}{100} = \frac{\theta - \theta_0}{n} \quad \text{...(i)}$$

Here, $\theta_0 = 5°$, $n = 95 - 5 = 90$ and $\theta = 59°$

Therefore, the Eq. (i) becomes

$$\frac{C-0}{100} = \frac{59-5}{90}$$

or

$$C = \frac{54}{90} \times 100 = 60°\text{C}$$

Example 14.6 *The steam point and the ice point of a mercury thermometer are marked as 80° and 10°. At what temperature on centigrade scale the reading of this thermometer will be 59°?*

Sol. Let the relation between the thermometer reading and centigrade be $y = ax + b$

(x represents °C and y represents temperature in new scale)

Given at $x = 100$, $y = 80$ and at $x = 0$, $y = 10$

$$80 = 100a + b, 10 = b \Rightarrow a = 0.7$$

Now, we have to find x when $y = 59$

$$59 = 0.7x + b \Rightarrow x = 70$$

$\therefore$ The answer is 70° C.

Thermometers

The thermometers work on the thermometric property, *i.e.*, the property which changes with temperature like any physical quantity such as length, volume, pressure and resistance etc., which varies linearly with a certain range of temperature. Let X denote the thermometric physical quantity and X_0, X_{100} and X_t be its values at 0°C, 100°C and t°C respectively. Then,

$$\text{Temperature, } t = \left(\frac{X_t - X_0}{X_{100} - X_0}\right) \times 100° \text{ C}$$

Different types of thermometers

Some different types of thermometers are given below.

1. Mercury thermometer

In this thermometer, the length of a mercury column from some fixed point is taken as thermometric property. If length of mercury column at 0° and 100° are l_0 and l_{100} respectively and at $t°$ the length of mercury is l_t. Then

$$t = \left(\frac{l_t - l_0}{l_{100} - l_0}\right) \times 100°\text{C}$$

2. Constant volume gas thermometer

It works on the principle of change in pressure with temperature when the volume is kept constant. If p_0, p_{100} and p_t are the pressures of gas at temperatures 0° C, 100°C, and unknown temperature (t°C) respectively keeping the volume constant, then

$$t = \left(\frac{p_t - p_0}{p_{100} - p_0} \times 100\right)°\text{C}$$

Note For constant pressure gas thermometes, $t = \frac{V_t - V_0}{V_{100} - V_0} \times 100°\text{C}$

3. Platinum resistance thermometer

It works on the principle of variation of resistance of metals with temperature.

If R_0, R_{100} and R_t are the resistances of a platinum wire at temperature 0°C, 100°C and unknown temperature (t°C) respectively.

Then, $$t = \left(\frac{R_t - R_0}{R_{100} - R_0} \times 100\right)°\text{C} = \left(\frac{R_t}{R_{tr}} \times 273.16\right)\text{K}$$

Here *temperature coefficient of resistance* (α) *is given by*

$$\boxed{\alpha = \frac{R_{100} - R_0}{R_0 \times 100}}$$

4. Pyrometers

Pyrometers are the devices used to measure the temperature by measuring the intensity of radiations received from the body. They are based on the fact that the amount of radiation emitted from a body per unit area per second is directly proportional to the fourth power of temperature, *i.e.*, Stefan's law. These can be used to measure temperature ranging from 800°C to 6000°C.

Example 14.7 *The pressures of the gas filled in the bulb of a constant volume gas thermometer are 66 cm and 88 cm of mercury column at 0°C and 100°C respectively. When its bulb is immersed in a liquid placed in a vessel, its pressure is 82.5 cm of mercury column. Calculate the temperature of the liquid.*

Sol. Given, pressure at 0°C is $p_0 = 66$ cm,

Pressure at 100°C is $p_{100} = 88$ cm

Pressure at unknown temperature t is $p_t = 82.5$ cm

According to formula, the unknown temperature is

$$t = \left(\frac{p_t - p_0}{p_{100} - p_0}\right) \times 100°\text{C} = \left(\frac{82.5 - 66}{88 - 66}\right) \times 100°\text{C}$$

$$= \frac{16.5}{22} \times 100°\text{C} = 75°\text{C}$$

Example 14.8 *The following observation were recorded on a platinum resistance thermometer. Resistance at melting point of ice = 3.70 Ω, resistance at boiling point of water at normal pressure = 4.71 Ω and resistance at t°C = 5.29 Ω. Calculate*

(i) Temperature coefficient of resistance of platinum.

(ii) Value of temperature t.

Sol. Given, resistance at melting point of ice, $R_0 = 3.70\,\Omega$

Resistance at boiling point of water at normal pressure,

$$R_{100} = 4.71\,\Omega$$

Resistance at $t°$C, $R_t = 5.29\,\Omega$

(i) According to the formula, temperature coefficient of resistance is given by

$$\alpha = \frac{R_{100} - R_0}{R_0 \times 100} = \frac{4.71 - 3.70}{3.70 \times 100} = \frac{1.01}{370} = 2.73 \times 10^{-3}/°\text{C}$$

(ii) According to the formula, for temperature t, we have

$$t = 100°\text{C} \times \frac{R_t - R_0}{R_{100} - R_0}$$

$$= 100°\text{C} \times \frac{5.29 - 3.70}{4.71 - 3.70} = 100°\text{C} \times \frac{1.59}{1.01} = 157.4°\text{C}$$

Check point 14.1

1. A device used to measure temperature is
   (a) Hygrometer (b) Thermometer
   (c) Barometer (d) Calorimeter
2. On which of the following scales of temperature, the temperature is never negative?
   (a) Celsius (b) Fahrenheit (c) Reaumur (d) Kelvin
3. A difference of temperature of 25°C is equivalent to a difference of
   (a) 45°F (b) 72°F (c) 32°F (d) 25°F
4. The absolute zero temperature in Fahrenheit scale is
   (a) – 273°F (b) – 32°F (c) – 460 °F (d) – 132 °F
5. The freezing point on a thermometer is marked as – 20° and the boiling point as 130°. A temperature of human body (34°C) on this thermometer will be read as
   (a) 31° (b) 51° (c) 20° (d) None of these
6. At what temperature does the temperature in Celsius and Fahrenheit equalise?
   (a) – 40° (b) 40° (c) 36.6° (d) 38°
7. A faulty thermometer has its fixed points marked 5 and 95. If the temperature of a body as shown on the Celsius scale is 40, then its temperature shown on this faulty thermometer is
   (a) 39° (b) 40° (c) 41° (d) 44.4°
8. The temperature of a body on kelvin scale is found to be xK. When it is measured by Fahrenheit thermometer, it is found to be x°F, then the value of x is
   (a) 40 (b) 313 (c) 574.25 (d) 301.25
9. The reading of air thermometer at 0°C and 100°C are 50 cm and 75 cm of mercury column respectively. The temperature at which its reading is 80 cm of mercury column is
   (a) 105°C (b) 110°C
   (c) 115°C (d) 120°C
10. The readings of a bath on Celsius and Fahrenheit thermometers are in the ratio 2 : 5. The temperature of the bath is
   (a) – 26.66°C (b) 40°C
   (c) 45.71°C (d) 26.66°C

2 Thermal expansion

When matter is heated without any change in its state, it usually expand. This phenomena of expansion of matter on heating it, is called thermal expansion.

On heating the matter, the energy of atoms increases and hence the average distance between them increases, which results in thermal expansion. Thermal expansion is minimum in solids and maximum in gases because intermolecular forces are maximum in solids and minimum in gases.

Types of thermal expansion

Solids can expand in one dimension (linear expansion), two dimensions (superficial expansion) and three dimensions (volume expansion) while liquid and gases usually expand only in volume.

Thermal expansion in solids

There are three types of thermal expansion.

1. Linear expansion

Suppose the temperature of a thin uniform rod of length l is changed from T to $T + \Delta T$, then its length becomes $l + \Delta l$.

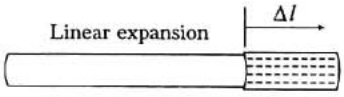


Fig. 14.1 Thermal expansions in rod

Then, increase in length $\Delta l = l_0 \alpha \Delta T \Rightarrow \boxed{\alpha = \frac{\Delta l}{l_0 \Delta T}}$

Therefore, final length, $l = l_0 + \Delta l$

$$\boxed{l = l_0\,[1 + \alpha\ \Delta T]}$$

Here, the constant α is called the **coefficient of linear expansion** of the material of the rod and its units are K^{-1} or $[(°C)^{-1}]$. Its dimension is $[\theta^{-1}]$.

Note Test tubes, beakers and cubicles are made up of pyrex-glass or silica because they have very low value of coefficient of linear expansion.

Example 14.9 *A steel ruler exactly 20 cm long is graduated to give correct measurements at 20°C.*

(i) Will it give readings that are too long or too short at lower temperatures?

(ii) What will be the actual length of the ruler be when it is used in the desert at a temperature of 40°C?
$(\alpha_{steel} = 1.2 \times 10^{-5}\ °C^{-1})$

Sol. (i) If the temperature decreases, the length of the ruler also decreases through thermal contraction. Below

20°C, each centimetre division is actually somewhat shorter than 1.0 cm, so the steel ruler gives readings that are too long.

(ii) Given, temperature $\Delta T = T_2 - T_1 = 40 - 20 = 20°C$

length $l = 20$ cm

At 40°C, the increase in length of the ruler is

$$\Delta l = l\alpha\Delta T = (20)(1.2\times 10^{-5})\, 20°C$$

$$= 0.48\times 10^{-2} \text{ cm}$$

$\therefore$ The actual length of the ruler is,

$$l' = l + \Delta l = 20 + 0.48\times 10^{-2} = 20.0048 \text{ cm}$$

Example 14.10 *The length of a steel rod is 5 cm longer than that of a brass rod. If this difference in their lengths is to remain the same at all temperatures, then find the length of brass rod. (Coefficient of linear expansion for steel and brass are 12×10^{-6}/°C and 18×10^{-6}/°C respectively)*

Sol. Given, $\Delta l_{St} - \Delta l_{Br} = \Delta l$,

Let, $l_{St} = l \Rightarrow l_{Br} = (l-5)$ cm

($\because$ steel rod is 5 cm longer than that of a brass rod)

$$\alpha_{St} = 12\times 10^{-6}/°C \text{ and } \alpha_{Br} = 18\times 10^{-6}/°C$$

We know that, $\Delta l_{St} = l_{St}\,\alpha t_1$

$$\therefore \quad \alpha_{St} = \frac{\Delta l_{St}}{l_{St}\times t_1}$$

$$\Rightarrow \quad 12\times 10^{-6} = \frac{\Delta l}{l\times t} \quad \text{...(i)}$$

Similarly, $\Delta l_{Br} = l_{Br}\,\alpha t_2$

$$\alpha_{Br} = \frac{\Delta l_{Br}}{l_{Br}\times t_2}$$

$$\Rightarrow \quad 18\times 10^{-6} = \frac{\Delta l}{(l-5)\times t} \quad \text{...(ii)}$$

Dividing Eq. (i) by Eq, (ii), we get

$$\frac{12\times 10^{-6}}{18\times 10^{-6}} = \frac{\Delta l/l\times t}{\Delta l/(l-5)\times t} \Rightarrow \frac{2}{3} = \frac{l-5}{l}$$

$$2l = 3l - 15 \Rightarrow l = 15$$

So, $l_{Br} = l - 5 = 15 - 5 = 10$ cm

2. Superficial expansion

Suppose the temperature of a metal sheet of an initial surface area A_0 is changed from T to $T + \Delta T$, then its area becomes $A + \Delta A$.

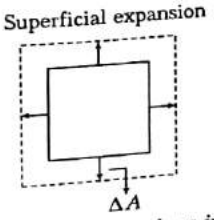


Fig. 14.2 Thermal expansions in thin sheet

Then, Increase in area, $\Delta A = A_0\beta\Delta T$

Therefore, final area, $A = A_0 + \Delta A$

$$\boxed{A = A_0\,[1 + \beta\Delta T]}$$

where, β is the **coefficient of superficial expansion** and its unit is $[(°C)^{-1}]$ or K^{-1}.

Example 14.11 *A metal ball having a diameter of 0.4 m is heated from 273 to 360 K. If the coefficient of areal expansion of the material of the ball is 0.000034 K^{-1}, then determine the increase in surface area of the ball.*

Sol. Given, diameter = 0.4 m and radius, $r = \frac{0.4}{2} = 0.2$ m

$\therefore$ Area of ball $A_0 = 4\pi r^2 = 4\times\pi\times(0.2)^2 = 0.5024 \text{ m}^2$

Temperature, $\Delta T = T_2 - T_1 = 360 \text{ K} - 273 \text{ K} = 87 \text{ K}$

Coefficient of area expansion, $\beta = 0.000034 \text{ K}^{-1}$

Apply, $\Delta A = \beta A_0 \Delta T$

$$\Rightarrow \quad \Delta A = 0.000034\times 0.5024\times 87 = 0.001486$$

$$= 1.486\times 10^{-3} \text{ m}^2$$

3. Volume expansion

Suppose the temperature of a metal block of volume V_0 is increased from T to $T + \Delta T$, if its volume becomes $V + \Delta V$.

Then, increase in volume, $\Delta V = V\gamma\Delta T$

Therefore, new volume $V = V_0 + \Delta V = V_0\,[1 + V\Delta T]$

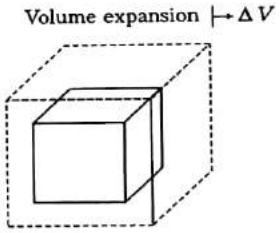


Fig. 14.3 Thermal expansions in solid cube

Here, γ is called the **coefficient of volume expansion** and its unit is K^{-1} or $[(°C)^{-1}]$.

Relation between α, β and γ

For solid material α, β and γ are related as

$$\boxed{\alpha : \beta : \gamma = 1 : 2 : 3}$$

i.e. $\gamma = 3\alpha, \beta = 2\alpha, 2\gamma = 3\beta$

Percentage change in area = 2 × percentage change in length.

Percentage change in volume = 3 × percentage change in length

Example 14.12 *On heating a glass block of 10000 cm^3, from 25°C to 40°C, its volume increases by 4 cm^3. Determine the coefficient of linear expansion of glass.*

Sol. Given, Volume $V = 10000 \text{ cm}^3$

Temperature, $\Delta T = (40 - 25) = 15°C$, $\Delta V = 4 \text{ cm}^3$

∴ Coefficient of cubical expansion is given by

$$\gamma = \frac{\Delta V}{V \cdot \Delta T} = \frac{4}{10000 \times 15}$$
$$= 26.67 \times 10^{-6}\ ^\circ C^{-1}$$

∴ Coefficient of linear expansion,

$$\alpha = \frac{\gamma}{3} = \frac{26.67 \times 10^{-6}}{3} = 8.89 \times 10^{-6}\ ^\circ C^{-1}$$

Example 14.13 *The volume of mercury in the bulb of a thermometer is $10^{-6} m^3$. The area of cross-section of the capillary tube is $2 \times 10^{-7}\ m^2$. If the temperature is raised by 100° C, then find the increase in the length of the mercury column. ($\gamma_{Hg} = 18 \times 10^{-5}/^\circ C$)*

Sol. By cubical expansion relation, $\Delta V = V \times \gamma \Delta T$ ...(i)

where $\gamma = 18 \times 10^{-5}/^\circ C$,

and initial volume, $V = 10^{-6} m^3$

Temperature, $\Delta T = 100^\circ C$

Putting these values in Eq. (i), we get

$$\Delta V = 10^{-6} \times 18 \times 10^{-5} \times 10^2 = 18 \times 10^{-9}\ m^3$$

Since, $\Delta V = A \times \Delta l$

∴ $18 \times 10^{-9} = 2 \times 10^{-7} \times \Delta l$

or $9 \times 10^{-2} = \Delta l$ or $\Delta l = 9$ cm

Some effects of thermal expansion in solids

Some effects of thermal expansion in solids are given below.

(i) If a **bimetallic strip** (strip made up of two strips of different materials placed one over another and rigidly joint) is heated then it bends such that the strip of material having high value of α will be on convex side.

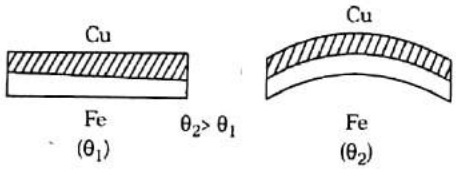


Fig. 14.4 *A bimetallic strip is heated from θ_1 to θ_2*

Since $\alpha_{Cu} > \alpha_{Fe}$, increase in length of copper strip will be greater, hence, strip with higher value of α will be on convex side.

(ii) If the **temperature of a pendulum** increases then its length increases, hence its time period changes. We know that,

$$T = 2\pi\sqrt{l/g} \Rightarrow \frac{\Delta T}{T} = \frac{\Delta l}{2l} = \frac{t\alpha\Delta\theta}{2t}$$

Change in time period $\boxed{\Delta T = \frac{1}{2}\alpha\Delta\theta T}$

Time lost by the clock in a day (t = 86400 s)

$$\Delta t = \frac{1}{2} \times \alpha\Delta\theta \times t = \frac{1}{2} \times \alpha\Delta\theta \times 86400 = 43200\alpha\Delta\theta\ s$$

where, T is initial time period, $\Delta\theta$ = change in temperature and α is coefficient of linear expansion of material of pendulum.

Change in time (shown by pendulum clock)

$$\boxed{\Delta t = \frac{\Delta T}{T} t}$$

If temperature increases then time period increases and *vice-versa*. On increasing temperature pendulum clock runs slowly and *vice-versa*.

(iii) If a **metal rod** fixed at both the ends with rigid support, is heated then due to thermal expansion it exerts force on rigid supports.

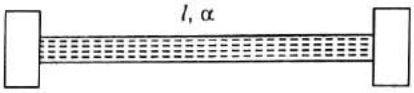


Fig. 14.5 *A rod with a rigid support*

We know that $Y = \frac{F/A}{\Delta l/l} = \frac{F}{A} \times \frac{l}{\Delta l} = \frac{F}{A} \times \frac{l}{l\alpha\Delta t}$

⇒ Force on rigid supports

$$\boxed{F = YA\alpha\Delta t}$$

where, Δt is rise in temperature, α is coefficient of linear expansion, Y is Young's modulus of material of rod and A is area of cross-section of rod.

Example 14.14 *A clock which keeps correct time at 20°C is subjected to 40°C. If coefficient of linear expansion of the pendulum is 12×10^{-6} /°C. How much will it gain or loss in time?*

Sol. Given, Coefficient of linear expansion $\alpha = 12 \times 10^{-6}/^\circ C$

Change in temperature $\Delta\theta = \theta_2 - \theta_1 = 40 - 20 = 20$

Initial time period $T = 24 \times 60 \times 60 = 24 \times 3600 = 86400$

We know that, change in time period,

$$\Delta T = \frac{1}{2} T\alpha\Delta\theta = \frac{1}{2} \times 86400 \times 12 \times 10^{-6} \times 20$$
$$= 12 \times 10^{-5} \times 86400 = 10368 \times 10^{-3}$$
$$= 10.3\ s\ days^{-1}$$

Example 14.15 *A second's pendulum clock has a steel wire. The clock is calibrated at 20°C. How much time does the clock loss or gain in one weak when the temperature is increased to 30°C? ($\alpha_{steel} = 1.2 \times 10^{-5}\ ^\circ C^{-1}$)*

Sol. The time period of second's pendulum is 2 s. As the temperature increases length and hence, time period increases. Clock becomes slow and it loses the time. The change in time period is

$$\Delta T = \frac{1}{2} T\alpha\Delta\theta = \left(\frac{1}{2}\right)(2)(1.2 \times 10^{-5})(30^\circ - 20^\circ)$$
$$= 1.2 \times 10^{-4}\ s$$

$\therefore$ New time period is,

$$T' = T + \Delta T = (2 + 1.2 \times 10^{-4}) = 2.00012 \text{ s}$$

$\therefore$ Time lost in one week

$$\Delta t = \left(\frac{\Delta T}{T'}\right) t = \frac{(1.2 \times 10^{-4})}{(2.00012)} (7 \times 24 \times 3600) = 36.28 \text{ s}$$

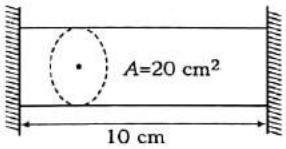
Example 14.16 *An aluminium cylinder 10 cm long, with a cross-sectional area of 20 cm², is to be used as a spacer between two steel walls. At 17.2°C it just slips in between the walls. When it warms to 22.3°C, calculate the stress in the cylinder and the total force it exerts on each wall, assuming that the walls are perfectly rigid and a constant distance apart.*

Sol. Given, For aluminium, $Y = 7.0 \times 10^{10}$ Pa

and $\alpha = 2.4 \times 10^{-5} \text{K}^{-1}$.

Change in temperature, $\Delta T = 5.1°\text{C} = 5.1 \text{ K}$

Area $A = 20 \text{ cm}^2 = 20 \times 10^{-4} \text{ m}^2$



$$\therefore \text{ Stress} = \frac{F}{A} = Y\alpha\Delta T = (0.70 \times 10^{11}\text{Pa})(2.4 \times 10^{-5} \text{ K}^{-1})(5.1 \text{ K})$$

$$= 1.68 \times 5.1 \times 10^6 = 8.6 \times 10^6 \text{ Pa}$$

The total force F is the cross-sectional times the stress.

$$F = \text{Stress} \times \text{Area} = (8.6 \times 10^6 \text{ Pa})(20 \times 10^{-4} \text{m}^2) = 1.7 \times 10^4 \text{N}$$

Thermal expansion of liquids

Liquids do not have linear and superficial expansion but these only have volumetric expansion.

Since, liquids are always heated in a vessel, so initially on heating the system (liquid + vessel), the level of liquid in vessel falls (as vessel expands more since it absorbs heat and liquid expands less) but later on, it starts rising due to faster expansion of the liquid. *Thus, liquids have two coefficients of volume expansion.*

(i) **Coefficient of real expansion** γ_r, which is due to the actual increase in volume of liquid due to heating.

(ii) **Coefficient of apparent expansion** γ_a, which is apparent increase in the volume of liquid if expansion of vessel containing the liquid is also taken into account.

$$\gamma_a = \frac{\text{observed (or apparent) increase in volume}}{\text{original volume} \times \text{change in temperature}}$$

$$\gamma_r = \frac{\text{real increase in volume}}{\text{Orginal volume} \times \text{change in temperature}}$$

$$\therefore \quad \boxed{\gamma_r = \gamma_a + \gamma_g}$$

Here, γ_g = expansion of vessel

Anomalous behaviour of water

Generally, with increasing temperature the volume expansion coefficient of liquids is about ten times greater than that of solids. Water is an exception to this rule. From 0°C to 4°C water contracts and beyond 4°C it expands. Thus, density of water reaches a maximum value of 1000 kgm^{-3} at 4°C.

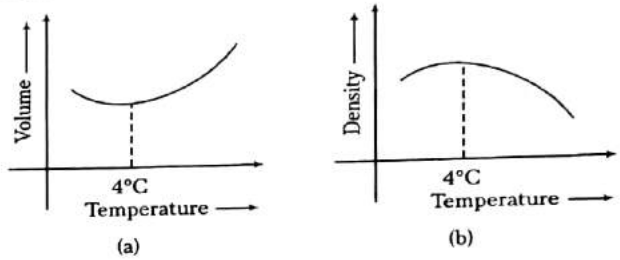


Fig. 14.6 *(a) Thermal expansion of water*
(b) Variation of density of water with temperature

Example 14.17 *A non-conducting body floats in a liquid at 25° C with 1/3 of its volume immersed in the liquid. When liquid temperature is increased to 105°C, 1/2 of body's volume is immersed in the liquid. Then, find the coefficient of real expansion of the liquid (neglecting the expansion of container of the liquid).*

Sol. Coefficient of real expansion,

$$\gamma_R = \frac{V_2 - V_1}{V_1(t_2 - t_1)} \quad \text{...(i)}$$

Here, $V_2 = \frac{1}{2}, V_1 = \frac{1}{3}$

and $(t_2 - t_1) = (105 - 25) = 80°\text{C}$

$\therefore$ Putting thesev values, in Eq. (i) we get

$$\gamma_R = \frac{\left(\frac{1}{2} - \frac{1}{3}\right)}{1/3\,(80)} = \frac{1}{160}$$

$$= 62.5 \times 10^{-4} \circ \text{C}^{-1}$$

Example 14.18 *The coefficient of apparent expansion of a liquid when determined using two different vessels A and B are γ_1 and γ_2, respectively. If the coefficient of linear expansion of vessel A is α. Find the coefficient of linear expansion of the vessel B.*

Sol. From thermal expansion in liquid,

$$\gamma_{\text{real}} = \gamma_{\text{app}} + \gamma_{\text{vessel}}$$

Given, $\gamma_{\text{vessel}} = 3\alpha$

For vessel A, $\gamma_{\text{real}} = \gamma_1 + 3\alpha_A$

For vessel B, $\gamma_{\text{real}} = \gamma_2 + 3\alpha_B$

Hence, $\gamma_1 + 3\alpha_A = \gamma_2 + 3\alpha_B$

$$\Rightarrow \quad \alpha_B = \frac{\gamma_1 - \gamma_2}{3} + \alpha_A$$

Example 14.19 *A glass vessel of volume V_0 is completely filled with a liquid and its temperature is raised by ΔT. What volume of the liquid will overflow ? Coefficient of linear expansion of glass $= \alpha_g$ and coefficient of volume expansion of the liquid $= \gamma_l$.*

Sol. Volume of the liquid overflown

= Increase in the volume of the liquid

− Increase in the volume of the container

$$= [V_0(1 + \gamma_l \Delta T) - V_0] - [V_0(1 + \gamma_g \Delta T) - V_0]$$

$$= V_0 \Delta T (\gamma_l - \gamma_g) = V_0 \Delta T (\gamma_l - 3\alpha_g) \quad (\because \gamma_g \approx 3\alpha_g)$$

Example 14.20 *Assume that one early morning when the temperature is 10°C, a driver of an automobile gets his gasoline tank which is made of steel, filled with 75 L of gasoline, which is also at 10°C. During the day, the temperature rises to 30°C, how much gasoline will overflow?*

(Given, α for steel $= 1.2 \times 10^{-5}\,°C^{-1}$, γ for gasoline $= 9.5 \times 10^{-4}\,°C^{-1}$)

Sol. Change in volume of gasoline, $\Delta V_g = \gamma_g V \Delta T$

$$= 9.5 \times 10^{-4} \times V \times 20 = 190 \times 10^{-4} V$$

Change in volume of steel tanks, $\Delta V_s = \gamma_s V \Delta T$

$$= 3\alpha_s V \Delta T = 3 \times 1.2 \times 10^{-5} \times V \times 20 = 7.2 \times 10^{-4}\ V$$

∴ Volume of gasoline that overflows

$$= \Delta V_g - \Delta V_s = (190 \times 10^{-4} - 7.2 \times 10^{-4}) V$$

$$= 182.8 \times 10^{-4} \times 75 = 1.37\,L.$$

Thermal expansion of gases

On heating gases expand more than solids or liquids and equal volumes of different gases expand equally, when heated by the same amount.

All gases have coefficient of volume expansion γ_V *with volume variation given by*

$$\boxed{V = V_0\ (1 + \gamma_V \Delta\theta)}$$

and pressure variation given by $\boxed{p = p_0\ (1 + \gamma_p \Delta\theta)}$

Variation of density with temperature

Most substances expand when they are heated, *i.e.*, volume of a given mass of a substance increases on heating, so the density should decrease $\left(\text{as } \rho \propto \frac{1}{V}\right)$.

Let us see how the density (ρ) varies with increase in temperature.

$$\rho = \frac{m}{V} \quad \text{or} \quad \rho \propto \frac{1}{V} \quad \text{(for a given mass)}$$

$$\therefore \quad \frac{\rho'}{\rho} = \frac{V}{V'} = \frac{V}{V + \Delta V} = \frac{V}{V + \gamma V \Delta T} = \frac{1}{1 + \gamma \Delta T}$$

$$\therefore \quad \rho' = \frac{\rho}{1 + \gamma \Delta T}$$

This expression can also be written as,

$$\rho' = \rho\ (1 + \gamma \Delta T)^{-1}$$

As γ is small, $(1 + \gamma \Delta T)^{-1} \approx 1 - \gamma \Delta T$

$$\therefore \quad \boxed{\rho' \approx \rho\ (1 - \gamma \Delta T)}$$

Example 14.21 *A vertical column whose density is 50 g cm^{-3} long at 50°C balances another column of same liquid whose density is 60 g cm^{-3} long at 100°C. Find the coefficient of absolute expansion of the liquid.*

Sol. Given, density of vertical column $\rho = 50$ g cm^{-3}

Temperature $T_1 = 50$

Density of another column $\rho_1 = 60$ g cm^{-3}

Temperature $T_2 = 100°$ C

∴ Variation of density with temperature

$$\rho' = \rho(1 - \gamma \Delta T)$$

$$60 = 50[1 - \gamma(100 - 50)]$$

$$\frac{60}{50} = [1 - \gamma(50)] \Rightarrow 50\gamma = 1 - \frac{6}{5}$$

$$\gamma = -\frac{1}{5 \times 50} = -\frac{1}{250}$$

$$\gamma = 0.005\ /°\,C$$

Example 14.22 *A sphere of diameter 7 cm and mass 266.5 g floats in a bath of liquid. As the temperature is raised, the sphere just sinks at a temperature of 35°C. If the density of the liquid at 0°C is 1.527 gm/cm^3, find the coefficient of cubical expansion of the liquid.*

Sol. The sphere will sink in the liquid at 35°C, when its density becomes equal to the density of liquid at 35° C.

The density of sphere,

$$\rho_{35} = \frac{\text{Mass}}{\text{Volume}} = \frac{266.5}{\frac{4}{3} \times \left(\frac{22}{7}\right) \times \left(\frac{7}{2}\right)^3} = 1.483\ \text{g/cm}^3$$

Now,

$$\rho_0 = \rho_{35}\,[1 + \gamma \Delta T]$$

$$1.527 = 1.483[1 + \gamma \times 35]$$

$$1.029 = 1 + \gamma \times 35$$

$$\gamma = \frac{1.029 - 1}{35} = 0.00083\ /°\,C$$

Example 14.23 *The coefficient of volume expansion of glycerine is 49×10^{-5} °C^{-1}. What is the fractional change in its density for a 30°C rise in temperature?*

Sol. Given, coefficient of volume expansion $\gamma = 49 \times 10^{-5}\ C^{-1}$

Temperature $T = 30°$ C

Let ρ_0 its density at 0°C, ρ_t its density at t°C.

∴ Fractional change in density,

$$\frac{\rho_0 - \rho_t}{\rho_0} = \gamma \Delta T = 49 \times 10^{-5} \times 30 = 0.0147$$

Check point 14.2

1. A solid ball of metal has a spherical cavity inside it. If the ball is heated, the volume of the cavity will
(a) increase (b) decrease
(c) remain unchanged (d) Data insufficient

2. Ratio among linear expansion coefficient (α), area expansion coefficient (β) and volume expansion coefficient (γ) is
(a) 1:2:3 (b) 3:2:1
(c) 4:3:2 (d) All of these

3. Bimetal strips are used for
(a) metal thermometers
(b) opening or closing electrical circuits
(c) thermostats
(d) All of the above

4. The temperature of a physical pendulum, whose time period is T, is raised by $\Delta\theta$. The change in its time period is
(a) $\frac{1}{2}\alpha T\Delta\theta$ (b) $2\alpha T\Delta\theta$
(c) $\frac{1}{2}\alpha\Delta\theta$ (d) $2\alpha\Delta\theta$

5. On heating a liquid having coefficient of volume expansion α in a container having coefficient of linear expansion $\alpha/2$, the level of the liquid in the container would
(a) rise
(b) fall
(c) remains almost stationary
(d) Cannot be predicted

6. $ABCD$ is a thin iron sheet. A hole with radius r is made in it in the middle. If we heat up the sheet, what will happen to the hole.
(a) Radius will increase
(b) Radius will decrease
(c) It will remains constant
(d) It can increase or decrease depending upon size

7. A beaker is completely filled with water at 4°C. It will overflow if
(a) heated above 4°C
(b) cooled below 4°C
(c) both heated and cooled above and below 4°C respectively
(d) None of the above

8. A bar of iron is 10 cm at 20°C. At 19°C it will be ($\alpha_{Fe} = 11\times10^{-6}/°C$)
(a) 11×10^{-6} cm longer (b) 11×10^{-6} cm shorter
(c) 11×10^{-5} cm shorter (d) 11×10^{-5} cm longer

9. A steel tape gives correct measurement at 20°C. A piece of wood is being measured with the steel tape at 0°C. The reading is 25 cm on the tape, the real length of the given piece of wood must be:
(a) 25 cm (b) <25 cm
(c) >25 cm (d) can not say

10. Two rods of lengths l_1 and l_2 are made of materials whose coefficient of linear expansions are α_1 and α_2 respectively. If the difference between two lengths is independent of temperature, then
(a) $\frac{l_1}{l_2} = \frac{\alpha_1}{\alpha_2}$ (b) $\frac{l_1}{l_2} = \frac{\alpha_2}{\alpha_1}$
(c) $l_2^2\,\alpha_1 = l_1^2\,\alpha$ (d) $\frac{\alpha_1^2}{l_1} = \frac{\alpha_2^2}{l_2}$

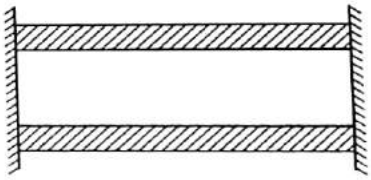
11. The radius of a ring is R and its coefficient of linear expansion is α. If the temperature of ring increases by θ then its circumference will increase by
(a) $\pi R\alpha\theta$ (b) $2\pi R\alpha\theta$ (c) $\pi R\alpha\frac{\theta}{2}$ (d) $\pi R\alpha\frac{\theta}{4}$

12. If two rods of lengths L and $2L$ having coefficient of linear expansion α and 2α respectively are connected end-on-end, the average coefficient of linear expansion of the composite rod, equals
(a) $\frac{3}{2}\alpha$ (b) $\frac{5}{2}\alpha$
(c) $\frac{5}{3}\alpha$ (d) None of these

13. A metre rod of silver at 0°C is heated to 100°C. It's length is increased by 0.19 cm. Coefficient of volume expansion of the silver rod is
(a) $5.7\times10^{-5}/°C$ (b) $0.63\times10^{-5}/°C$
(c) $1.9\times10^{-5}/°C$ (d) $16.1\times10^{-5}/°C$

14. A uniform metal rod is used as a bar pendulum. If the room temperature rises by 10° C, and the coefficient of linear expansion of the metal of the rod is 2×10^{-6} per° C the period of the pendulum will have percentage increase of
(a) -2×10^{-3} (b) -1×10^{-3}
(c) 2×10^{-3} (d) 1×10^{-3}

15. Two rods of different materials having coefficients of thermal expansions α_1, α_2 and Young's modulli Y_1, Y_2 respectively are fixed between two rigid massive walls. The rods are heated such that they undergo the same increase in temperature. There is no bending of the rods. If $\alpha_1 : \alpha_2 = 2 : 3$, the thermal stresses developed in the two rods are equal provided $Y_1 : Y_2$ is equal to



(a) 2 : 3 (b) 1 : 1 (c) 3 : 2 (d) 4 : 9

16. Coefficient of volume expansion of mercury is $0.18\times10^{-3}/°C$. If the density of mercury at 0°C is 13.6 g/cc, then its density at 200°C is
(a) 13.11 g/cc (b) 52.11 g/cc
(c) 16.11 g/cc (d) 26.11 g/cc

17. A glass flask of volume 200 cm^3 is just filled with mercury at 20°C. The amount of mercury that will overflow when the temperature of the system is raised to 100°C is
($\gamma_{glass} = 1.2 \times 10^{-5}$°/C , $\gamma_{mercury} = 1.8 \times 10^{-4}$/°C)
(a) 2.15 cm^3 (b) 2.69 cm^3
(c) 2.52 cm^3 (d) 2.25 cm^3

18. Solids expand on heating because
(a) Kinetic energy of atom increases
(b) Potential energy of atom increases
(c) Total energy of atom increases
(d) The potential energy curve is asymmetric about the equilibrium distance between neighbouring atoms

19. The coefficient of linear expansion of crystal in one direction is α_1 and that in other two directions perpendicular to it is α_2. The coefficient of cubical expansion is
(a) $\alpha_1 + \alpha_2$ (b) $2\alpha_1 + \alpha_2$
(c) $\alpha_1 + 2\alpha_2$ (d) None of these

20. A steel rod of diameter 1 cm is clamped firmly at each end when its temperature is 25°C so that it cannot contract on cooling. The tension in the rod at 0°C is approximately
($\alpha = 10^{-5}$/°C, $Y = 2 \times 10^{11}$ Nm^{-2})
(a) 4000 N (b) 7000 N
(c) 7400 N (d) 4700 N

3 Kinetic theory of ideal gases

Kinetic theory of gases explains the behavior of gases based on the idea that the gas consists of rapidly moving atoms of molecules. It correlates the macroscopic properties of gases *e.g.*, Pressure, temperature etc., to the microscopic properties of gas molecules *e.g.*, Speed, momentum, kinetic energy of molecules etc.

Postulates of kinetic theory

The kinetic theory of an ideal gas is based on the following postulates.

(*i*) A gas consists of large number of tiny particles, either atoms or molecules, moving around randomly.

(*ii*) The volume of the particles themselves is negligible compared with total volume of the gas, most of the volume of a gas is empty space.

(*iii*) The gas particles act independently of one another; there are no attractive or repulsive forces between particles.

(*iv*) Collisions of the gas particles, either with other particles or with the walls of the container, are elastic; *i.e.*, the total kinetic energy of the gas particles is constant at constant temperature. Particles have no energy in the form of potential energy because particles do not have mutual attraction or repulsion except gravitational attraction which is very feeble considering their masses.

(*v*) The average kinetic energy of the gas particles is proportional to the kelvin temperature of the sample.

(*vi*) The pressure exerted by a gas is the result of collisions of the particles with the walls of the container.

(*vii*) At any particular time, different particles in a gas have different speeds hence, different kinetic energy.
However average kinetic energy the molecules remain constant at a particular temperature.

Pressure of an ideal gas (Concept of pressure)

On the basis of kinetic theory of gases, we can do mathematical calculations to find expression for pressure exerted by a gas.

Consider an ideal gas consisting of N molecule each of mass m in a container of volume V having edges of length l.

Due to their continuous movement gas momecules collide with the walls of the container, thus applying pressure on the walls.

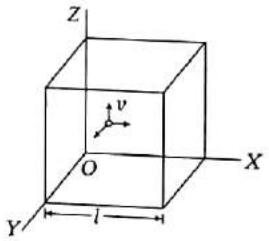


***Fig. 14.7** A cubical box with sides of length d containing an ideal gas. The molecule shown moves with velocity v.*

The pressure exerted by gas molecules on the *wall of container is given by*

$$p = \frac{1}{3}\frac{mN}{V}\bar{v}^2 = \frac{1}{3}\rho\bar{v}^2$$

Here m is mass of one gas molecule, N is total number of gas molecules, V is volume of gas, $\bar{v}$ is root mean square velocity of gas molecular and ρ is density of gas.

Writing the above expression as

$$p = \frac{1}{3}m\left(\frac{N}{V}\right)\bar{v}^2 = \frac{2}{3}\left(\frac{1}{2}m\bar{v}^2\right)\frac{N}{V}$$

Here, $\frac{1}{2}m\bar{v}^2$ is kinetic energy of one gas molecule.

$$\therefore \quad \boxed{p = \frac{2}{3}\frac{N}{V}(\text{KE})} \quad \text{or} \quad p = \frac{2}{3}\left(\frac{N}{2}\frac{m\bar{v}^2}{V}\right)$$

or

$$\boxed{p = \frac{2}{3}\left(\frac{1}{2}\rho\bar{v}^2\right)}$$

Here, $\frac{1}{2}\rho\bar{v}^2$ **is average kinetic energy of gas molecules per unit volume.**

$$\therefore \quad p = \frac{2}{3} \times \text{average kinetic energy per unit volume}$$

$$\therefore \quad \boxed{p = \frac{2}{3}E}$$

If E_{mol} be the energy of one mole of gas then

$$\boxed{p = \frac{2}{3}\frac{E_{mol}}{V}}$$

where, V is volume of one mole gas which is equal to 22.4 litres.

$$pV = \frac{2}{3}E_{mol} = RT \Rightarrow E_{mol} = \frac{3}{2}RT \Rightarrow E_{mol} \propto T$$

Example 14.24 *If the mass of each molecule of a gas is halved and speed is doubled. Find the ratio of initial and final pressure.*

Sol. Given, $m_1 = m$, $v_1 = v$, $V_1 = V$, $m_2 = m/2$, $v_2 = 2v$, $V_2 = V_1 = V$

$\therefore$ Pressure exerted by first gas molecule $p_1 = \frac{1}{3}\frac{m_1}{V_1}v_1^2$

and pressure exerted by second gas molecule

$$p_2 = \frac{1}{3}\frac{m_2}{V_2}v_2^2$$

$$\therefore \quad \frac{p_1}{p_2} = \frac{m_1}{m_2} \times \frac{V_2}{V_1} \times \frac{v_1^2}{v_2^2}$$

Putting the values, we get

$$\frac{p_1}{p_2} = \frac{m}{m/2} \times \frac{V}{V} \times \left(\frac{v}{2v}\right)^2 = 2 \times \frac{1}{4} = \frac{1}{2}$$

Ideal gas equation

Pressure of an ideal gas is given by

$$p = \frac{1}{3}\frac{mN}{V}\bar{v}^2 \quad \text{or} \quad pV = \frac{2N}{3}\left(\frac{1}{2}m\bar{v}^2\right) \quad \text{...(i)}$$

For ideal gas kinetic energy of gas molecules is proportional to temperature of the gas.

$$\therefore \quad \frac{1}{2}m\bar{v}^2 \propto T \quad \text{or} \quad \bar{v}^2 \propto T \Rightarrow \bar{v}^2 = kT$$

where, k is a constant.

From Eq. (i), $\quad pV = \frac{2}{3}N\left(\frac{1}{2}mkT\right) = RT = N\frac{R}{N}T$

We can write the equation

$$\boxed{pV = NkT} \quad \text{where, } k = \frac{R}{N}$$

where, k is called **Boltzmann's constant.**

Example 14.25 *Oxygen is filled in a closed metal jar of volume 10^{-3} m^3 at a pressure of 1.5×10^5 Pa and temperature 400 K. The jar has a small leak in it. The atmospheric pressure is 10^5 Pa and the atmospheric temperature is 300 K. Find the mass of the gas that leaks out by the time the pressure and the temperature inside the jar equalize with the surrounding.*

Sol. Given, volume $V = 10^{-3}$ m^3,

Pressure $p_1 = 1.5 \times 10^5$ Pa, $p_2 = 10^5$ Pa

Temperature $T_1 = 400$ K, $T_2 = 300$ K,

From ideal gas equation, $p_1V = n_1RT_1$

$$\Rightarrow \quad n_1 = \frac{p_1V}{RT_1}$$

Similarly, $\quad p_2V = n_2RT_2 \Rightarrow n_2 = \frac{p_2V}{RT_2}$

$\therefore$ Number of mole leaked

$$\Delta n = n_1 - n_2 = \left(\frac{p_1}{T_1} - \frac{p_2}{T_2}\right)\frac{V}{R}$$

$$= \left(\frac{1.5}{400} - \frac{1}{300}\right) \times \frac{10^5 \times 10^{-3}}{8.3}$$

$$= 5.02 \times 10^{-3}$$

Mass of gas leaked $= \Delta n\, M_0 = 5.02 \times 10^{-3} \times 32 = 0.16$ g

Example 14.26 *Find the coefficient of volume expansion for an ideal gas at constant pressure.*

Sol. For an ideal gas, $pV = nRT$

As P is constant, we have

$$p \cdot dV = nRdT$$

$$\therefore \quad \frac{dV}{dT} = \frac{nR}{p}$$

According to volume expansion, $dV = V\gamma dt$

$$\text{or} \quad \gamma = \frac{1}{V} \cdot \frac{dV}{dT} = \frac{nR}{pV} = \frac{nR}{nRT} = \frac{1}{T}$$

$$\therefore \quad \gamma = \frac{1}{T}$$

Molecular velocities

In gaseous state, molecules tend to move in different directions with different speeds.

Based upon the calculations, there are three main kinds of molecular velocities.

1. Average velocity (v or v_{av})

It is the arithmetic mean of the velocities of the different molecules of a gas.

Mathematically, $\bar{v} = \dfrac{v_1 + v_2 + v_3 + \ldots + v_n}{n}$

where, $v_1\ v_2\ v_3 \ldots v_n$ are the velocities of the individual molecules $n = n_1 + n_2 + n_3 \ldots$, total number of molecules.

This related to temperature T of the gas as

$$\boxed{\bar{v} = \sqrt{\frac{8RT}{\pi M}}} = \sqrt{\frac{8p}{\pi \rho}} = \sqrt{\frac{8kT}{\pi M}}$$

where M is molar mass in kg/mol, where ρ is density.

Example 14.27 *Find the average speed of nitrogen molecules at 27° C.*

Sol. Mass of 1 mole of nitrogen gas

$$M = 14 \text{ g} = 14 \times 10^{-3} \text{ kg}$$

Average speed $\bar{v} = \sqrt{\dfrac{8RT}{\pi M}}$

where, T = temperature of the gas (in kelvin)

$$= 273 + 27° = 300 \text{ K}$$

R = gas constant = 8.3143 J mol^{-1} K^{-1}

$$\therefore \quad \bar{v} = \sqrt{\frac{8 \times 8.314 \times 300}{3.14 \times 14 \times 10^{-3}}} = 673.72 \text{ ms}^{-1}$$

2. Root mean square velocity (u or v_{rms})

It is the square root of the mean of the squares of the velocities of the molecules of gas.

Mathematically, $v_{rms} = \sqrt{\dfrac{v_1^2 + v_2^2 + v_3^2 + \ldots + v_n^2}{n}}$

From kinetic gas equation,

$$\boxed{v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3pV}{M}} = \sqrt{\frac{3p}{M/V}} = \sqrt{\frac{3p}{d}}}$$

where, p = pressure (RT/V)

and d = density (M/V)

Example 14.28 *In a container, two gases neon and argon are filled. Find the ratio of the root mean square speed of the molecules of the two gases. $M_{Ne} = 20.2\ \mu$ and $M_{Ar} = 39.9\ \mu$. The temperature of the system is 30°C.*

Sol. From kinetic gas equation

$$v_{rms} = \sqrt{\frac{3kT}{M}}$$

$$\therefore \quad v_{rms}^2 \propto \frac{1}{M}$$

$$\therefore \quad \frac{(v_{rms}^2)_{Ne}}{(v_{rms}^2)_{Ar}} = \frac{(M)_{Ar}}{(M)_{Ne}} = \sqrt{\frac{39.9}{20.2}} \simeq 1.42$$

Example 14.29 *What is rms velocity of O_2 gas at 127° C? The molecular weight of oxygen is 32.*

Sol. We know $v_{rms} = \sqrt{\dfrac{3RT}{M}}$ ...(i)

Given, $R = 8.3143$ J mol^{-1} K^{-1},

$T = 127°\text{C} = (273 + 127)\text{ K} = 300$ K

$M = 32$ g mol^{-1} = 32×10^{-3} kg mol^{-1}

Putting these values in Eq. (i), we get

$$v_{rms} = \sqrt{\frac{3 \times 8.3143 \times 300}{32 \times 10^{-3}}}$$

$$= 4.8357 \times 10^2 \text{ ms}^{-1}$$

Example 14.30 *Determine the rms velocity of oxygen molecules at STP. The molecular weight of oxygen is 32.*

Sol. At STP, pressure, $p = 1$ atm $= 1.013 \times 10^5$ Nm^{-2}.

Mass, $M = 32g = 32 \times 10^{-3}$ kg.

Molar volume $V = 22.4$ L $= 22.4 \times 10^{+3}$ m^3.

Root mean square velocity of oxygen molecules at STP

$$v_{rms} = \sqrt{\frac{3p}{d}} = \sqrt{\frac{3pV}{M}} = \sqrt{\frac{3 \times 1.013 \times 10^{-5} \times 22.4 \times 10^3}{32 \times 10^{-3}}}$$

$$v_{rms} = 461.23 \text{ ms}^{-1}$$

Example 14.31 *The molecules of a given mass of a gas have root mean square speeds of 100 ms^{-1} at 27 °C and 1.00 atm pressure. What will be the root mean square speed of the molecules of the gas at 127 °C and 2.0 atm pressure?*

Sol. Given, At 27°C, root mean square speed

$$v_{rms_1} = 100 \text{ ms}^{-1}$$

$$(v_{rms_1}) = \sqrt{\frac{3p_1}{d_1}} = \sqrt{\frac{3p_1V_1}{M}} \quad \text{...(i)}$$

According to ideal gas equation

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$$

$$\Rightarrow \quad \frac{V_1}{V_2} = \frac{p_2T_1}{p_1T_2} = \frac{2 \times 300}{400} = \frac{3}{2}$$

$\therefore$ At 127°C, root mean square speed

$$(v_{rms})^2 = \sqrt{\frac{3p_2}{d_2}} = \sqrt{\frac{3p_2V_2}{M}} \quad ...(ii)$$

From Eqs. (i) and (ii)

$$\therefore \quad (v_{rms})_2^2 = (v_{rms})_1^2 \times \frac{V_2}{V_1} \times \frac{p_2}{p_1} = (100)^2 \times \frac{2}{3} \times 2$$

or $$(v_{rms})_2 = \frac{200}{\sqrt{3}} \text{ ms}^{-1}$$

3. Most probable velocity (v_{mp} or U_{mp})

It is the velocity, possessed by the largest number of molecules of gas. (Maxwell speed distribution law)

From kinetic gas equations,

$$\boxed{v_{mp} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2pV}{M}} = \sqrt{\frac{2p}{d}}}$$

Note The ratio in different molecular velocities is

$$v_{rms} : v_{av} : v_{mp} = \sqrt{3} : \sqrt{\frac{8}{\pi}} : \sqrt{2} = \sqrt{3} : \sqrt{2.5} : \sqrt{2}$$

Example 14.32 *Calculate the root mean square, average and most probable speeds of oxygen molecules at 27° C.*

Sol. Given, Molar mass of oxygen

$$M = 32 \text{ g mol}^{-1} = 0.032 \text{kg mol}^{-1}$$

Temperature $T = 27° \text{ C} = (27 + 273) \text{ K} = 300 \text{ K}$

(i) Root mean square speed, $v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3\,(8.314)(300)}{(0.032)}}$

$$= 483.6 \text{ ms}^{-1}$$

(ii) Average speed,

$$\bar{v} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{8(8.314)(300)}{(3.14)(0.032)}} = 445.6 \text{ ms}^{-1}$$

(iii) Most probable speed,

$$v_{mp} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2(8.314)(300)}{(0.032)}} = 394.8 \text{ ms}^{-1}$$

Kinetic interpretation of temperature

According to kinetic theory of gases, $pV = (1/3)\, mN\bar{v}^2$

but according to equation of state for an ideal gas

$$pV = nRT$$

$$\therefore \quad \frac{1}{3} mN\bar{v}^2 = nRT \text{ to } \frac{1}{3} m\bar{v}^2 = \frac{3}{2}\frac{RT}{n} = \frac{3}{2}kT,$$

where, k is the Boltzmann's constant. Its value is 1.38×10^{-23} J mol^{-1} K^{-1}.

$$\boxed{\text{Mean translational kinetic energy } E = \frac{3}{2}kT}$$

i.e. the mean translational kinetic energy of a gas molecule depends only on its temperature and is independent of its nature or mass etc.

On this basis, we can say that **absolute zero temperature is the temperature at which translational kinetic energy of a gas molecule becomes zero *i.e.*, at which the molecular motion ceases altogether.**

Example 14.33 *At what temperature, the kinetic energy of a gas molecule is half of the value at 27 °C?*

Sol. Given, $T_1 = 27° \text{ C} = 27 + 273 = 300 \text{ K}$

Kinetic energy of a gas molecule $E = \frac{3}{2}kT$

where, k is Boltzmann's constant.

$$\therefore \quad E \propto T$$

$$\Rightarrow \quad \frac{E_1}{E_2} = \frac{T_1}{T_2}$$

$$\Rightarrow \quad \frac{E}{(E/2)} = \frac{300}{T_2}$$

When $T_2 = 150$ K

then $T_2 = 150 - 273 = -123° \text{ C}$

Example 14.34 *Prove that the pressure of an ideal gas is numerically equal to two-third of the mean translational kinetic energy per unit volume of the gas.*

Sol. Translational KE per unit volume

$$E = \frac{1}{2}(\text{mass perunit volume})(\bar{v}^2) = \frac{1}{2}(\rho)\left(\frac{3p}{\rho}\right) = \frac{3}{2}p$$

or $$p = \frac{2}{3}E$$

Example 14.35 *A balloon has 5.0 g mole of helium at 7°C. Calculate*

(i) the number of atoms of helium in the balloon.

(ii) the total internal energy of the system.

(Avogadro's number $N_A = 6.02 \times 10^{23}$

and Boltzmann's constant $k = 1.38 \times 10^{-23}$)

Sol Given, $n = 5.0$, $T = 7° \text{ C} = 7 + 273 = 280 \text{ K}$

(i) Number of atoms $= nN_A = 5.0 \times 6.02 \times 10^{23}$

$$\approx 30 \times 10^{23}$$

(*ii*) Average kinetic energy per molecule $= \frac{3}{2}kT$

$\therefore$ Total internal energy $= \frac{3}{2}kT \times N$

$$= \frac{3}{2} \times 1.38 \times 10^{-23} \times 280 \times 30 \times 10^{23}$$

$$= \frac{3}{2} \times 30 \times 280 \times 1.38 = 1.74 \times 10^4 \text{ J}$$

Example 14.36 *A tank used for filling helium balloons has a volume of $3.0\,m^3$ and contains 2.0 mol of helium gas at 20.0°C. Assuming that the helium behaves like an ideal gas.*

(i) What is the total translational kinetic energy of the molecules of the gas ?

(ii) What is the average kinetic energy per molecule ?

Sol. (i) Using $(KE)_{trans} = \frac{3}{2} nRT$

Given $n = 2.0$ mol and $T = 293$ K

$$\therefore \quad (KE)_{trans} = \frac{3}{2}(2.0)(8.31)(293) = 7.3 \times 10^3 \text{ J}$$

(ii) The average kinetic energy per molecule is $\frac{3}{2}kT$.

$$(KE)_{av} = \frac{3}{2}kT = \frac{3}{2}(1.38 \times 10^{-23})(293)$$
$$= 6.07 \times 10^{-21} \text{ J}$$

Mean free path (λ)

Every gas consists of a large number of molecules undergoing frequent collision, therefore they cannot move straight or unhindered, these molecules are in a state of continuous and random motion. Also, they undergo perfectly elastic collision against each other.

The average distance travelled by a molecule between two successive collisions is known as the mean free path of the molecule.

$$\boxed{\text{Mean free path, } \lambda = \frac{1}{\sqrt{2}\, n\pi d^2}} \quad \text{...(i)}$$

where, d = diameter of each molecule.

n = number of molecules per unit volume.

Relation Between Mean Free Path (λ), Temperature (T) and Pressure (p)

For the derviation of Eq. (i) we have assumed the other moleculer to be at rest. Taking into consideration the motion of all the gas molecules, the mean free path comes out to be

$$\lambda = \frac{1}{\sqrt{2}\, n\pi d^2}$$

If m is the mass of each gas molecule, then the density of the gas is

$$\rho = mn \quad \text{or} \quad n = \frac{\rho}{m}$$

$$\therefore \quad \lambda = \frac{m}{\sqrt{2}\pi d^2 \rho}$$

For one mole of a gas, $pV = RT$

$$\text{or} \quad p = \frac{RT}{V} = \frac{N}{V} \times \frac{R}{N} \times T = nk_B T$$

$$\text{or} \quad n = \frac{p}{k_B T} \Rightarrow \boxed{\lambda = \frac{k_B T}{\sqrt{2}\pi d^2 p}}$$

Example 14.37 *Calculate the ratio of the mean free path of the molecules of two gases if the ratio of the numbers density per cm^3 of the gases is 5 : 3 and the ratio of the diameters of the molecules of the gases is 4 : 5.*

Sol. Given, ratio of the numbers density $\frac{n_1}{n_2} = \frac{5}{3}$

Ratio of the diameters, $\frac{d_1}{d_2} = \frac{4}{5}$

Mean free path of the molecules of first gas, $\lambda_1 = \frac{1}{\sqrt{2}\pi n_1 d_1^2}$

Similarly, mean free path of the second gas, $\lambda_2 = \frac{1}{\sqrt{2}\pi n_2 d_2^2}$

$\therefore$ Ratio of λ_1 and λ_2

$$\frac{\lambda_1}{\lambda_2} = \frac{n_2 d_2^2}{n_1 d_1^2} = \frac{3}{5} \times \frac{5}{4} \times \frac{5}{4} = \frac{15}{16}$$

$$\Rightarrow \quad \lambda_1 : \lambda_2 = 15 : 16$$

Example 14.38 *For a molecule of an ideal gas $n = 3 \times 10^8\ cm^{-3}$ and mean free path is 10^{-2} cm. Calculate the diameter of the molecule.*

Sol. Given, $n = 3 \times 10^8\ \text{cm}^{-1}$, $\lambda = 10^{-2}$ cm

Mean free path is given by

$$\lambda = \frac{1}{\sqrt{2}\pi n d^2}$$

$$\Rightarrow \quad d^2 = \frac{1}{\sqrt{2}\pi n \lambda}$$

$$\Rightarrow \quad d^2 = \frac{1}{\sqrt{2} \times 3.14 \times 3 \times 10^8 \times 10^{-2}}$$

$$= \frac{10^{-6}}{1.414 \times 3.14 \times 3}$$

$$\Rightarrow \quad d = \sqrt{7.5 \times 10^{-6}} = \sqrt{7.5} \times 10^{-4}$$

$$\Rightarrow \quad d = 2.7 \times 10^{-4} \text{ cm}$$

Hence, the diameter of the molecule of the gas is 2.7×10^{-4} cm.

Check point 14.3

1. Which one of the following is not an assumption in the kinetic theory of gases?
(a) The volume occupied by the molecules of the gas is negligible
(b) The force of attraction between the molecules is negligible
(c) The collision between molecules are elastic
(d) All molecules have same speed

2. A closed vessel is maintained at a constant temperature. It is first evacuated and then vapour is injected into it continuously. The pressure of the vapour in the vessel
(a) increases continuously
(b) first increases and then remains constant
(c) first increases and then decreases
(d) None of the above

3. The average speed of molecules in a gas is given by
(a) $\sqrt{\frac{3p}{\rho}}$ (b) $\sqrt{\frac{\gamma p}{\rho}}$ (c) $\sqrt{\frac{2p}{\rho}}$ (d) $\sqrt{\frac{8p}{\pi\rho}}$

4. If Maxwell distribution is valid and v_p denotes the most probable speed, v the average speed and v_{rms} the root-mean-square speed, then
(a) $v < v_p < v_{rms}$ (b) $v < v_{rms} < v_p$
(c) $v_p < v < v_{rms}$ (d) $v_p < v_{rms} < v$

5. The average kinetic energy of a gas molecule is
(a) proportional to pressure of gas
(b) inversely proportional to volume of gas
(c) inversely proportional to absolute temperature of gas
(d) directly proportional to absolute temperature of gas

6. KE per unit volume is E. The pressure exerted by the gas is given by
(a) $\frac{E}{3}$ (b) $\frac{2E}{3}$ (c) $\frac{3E}{2}$ (d) $\frac{E}{2}$

7. If at the same temperature and pressure, the densities of two diatomic gases are d_1 and d_2, respectively. The ratio of mean kinetic energy per molecule of gases will be
(a) 1 : 1 (b) $d_1 : d_2$ (c) $\sqrt{d_1} : \sqrt{d_2}$ (d) $\sqrt{d_2} : \sqrt{d_1}$

8. Two vessels A and B having equal volume contain equal masses of hydrogen in A and helium in B at 300 K. Then, mark the correct statement?
(a) The pressure exerted by hydrogen is half that exerted by helium
(b) The pressure exerted by hydrogen is equal to that exerted by helium
(c) Average KE of the molecule of hydrogen is half the average KE of the molecules of helium
(d) The pressure exerted by hydrogen is twice that exerted by helium

9. A vessel contains 1 mole of O_2 gas (molar mass 32) at a temperature T. The pressure of the gas is p. An identical vessel containing one mole of He gas (molar mass 4) at temperature $2T$ has a pressure of
(a) $p/8$ (b) p (c) $2p$ (d) $8p$

10. What will be the temperature when the rms velocity is double of that at 300 K?
(a) 300 K (b) 600 K (c) 900 K (d) 1200 K

11. By what factor the rms velocity will change, if the temperature is raised from 27°C to 327°C?
(a) $\sqrt{2}$ (b) 2 (c) $2\sqrt{2}$ (d) 1

12. The velocities of three molecules are $3v$, $4v$ and $5v$ respectively. Their rms speed will be
(a) $\sqrt{\frac{50}{3}}\,v$ (b) $\sqrt{\frac{5}{2}}\,v$ (c) $\frac{7}{2}v$ (d) $\frac{5}{2}v$

13. The temperature at which the root mean square speed of a gas will be half its value at 0°C is (assume the pressure remains constant)
(a) −86.4°C (b) −204.75°C (c) −104.75°C (d) −68.25°C

14. Four molecules of a gas have speeds 1, 2, 3 and 4 km s^{-1}. The value of the root-mean-square speed of the gas molecules is
(a) $\frac{1}{2}\sqrt{15}$ kms^{-1} (b) $\frac{1}{2}\sqrt{10}$ kms^{-1}
(c) 2.5 kms^{-1} (d) $\sqrt{15/2}$ kms^{-1}

15. A sealed container with negligible coefficient of volumetric expansion contains helium (a monatomic gas). When it is heated from 300 K to 600 K, the average KE of helium atoms is
(a) halved (b) unchanged
(c) doubled (d) increased by factor $\sqrt{2}$

16. The gases carbon monoxide (CO) and nitrogen at the same temperature have translational kinetic energies E_1 and E_2, respectively. Then,
(a) $E_1 = E_2$ (b) $E_1 > E_2$
(c) $E_1 < E_2$ (d) None of these

17. Pressure of an ideal gas is increased by keeping temperature constant. What is the effect on kinetic energy of molecules?
(a) Increases (b) Decreases
(c) No change (d) Cannot be determined

18. Some gas at 300 K is enclosed in a container. Now, the container is placed on a fast moving train. While the train is in motion, the temperature of the gas
(a) rises above 300 K (b) falls below 300 K
(c) remains unchanged (d) becomes unsteady

19. The root-mean-square (rms) speed of oxygen molecules (O_2) at a certain absolute temperature is v. If the temperature is doubled and the oxygen gas dissociates into atomic oxygen, the rms speed would be
(a) v (b) $\sqrt{2}\,v$ (c) $2v$ (d) $2\sqrt{2}\,v$

20. The molecules of a given mass of gas have rms speed 200 ms^{-1} at 27°C and 10^5 Nm^{-2} pressure. When the absolute temperature is doubled and the pressure is halved, the rms speed of the molecules of the same gas is
(a) 200 ms^{-1} (b) 400 ms^{-1}
(c) $200\sqrt{2}$ ms^{-1} (d) $400\sqrt{2}$ ms^{-1}

4 Ideal gas laws

Ideal gases irrespective of their nature obey the following laws

Boyle's law

According to this law, for a given mass of a gas the volume of a gas at constant temperature (called isothermal process) is inversely proportional to its pressure, *i.e.*,

$$V \propto \frac{1}{p} \qquad (T = \text{constant})$$

or $$\boxed{pV = \text{constant}} \text{ or } \boxed{p_i V_i = p_f V_f}$$

Thus, p-V graph in an isothermal process is a rectangular hyperbola. Or pV *versus* p or V graph is a straight line parallel to p or V-axis.

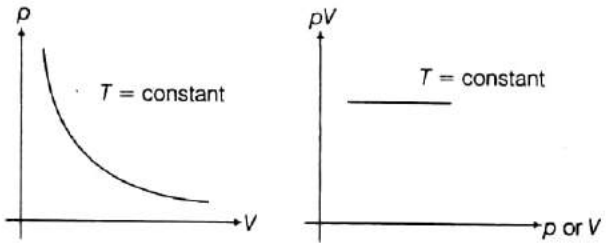


Fig. 14.8 *p versus V graph and pV versus p or V graph*

Example 14.39 *A gas at 27 °C in a cylinder has a volume of 4 L and pressure 100 Nm^{-2}. If the gas is first compressed at constant temperature so that the pressure is 150 Nm^{-2}. Estimate the change in volume.*

Sol. Given, $V_1 = 4$ L, $V_2 = ?$, $p_1 = 100$ Nm^{-2},

$p_2 = 150$ Nm^{-2}, $\Delta V = ?$

Using Boyle's law for constant temperature, we have

$$p_1V_1 = p_2V_2$$

$$\Rightarrow \quad V_2 = \frac{p_1V_1}{p_2} = \frac{100 \times 4}{150} = 2.667 \text{ L}$$

$\therefore$ Change in volume, $\Delta V = V_1 - V_2$

$= 4 - 2.667 = 1.33$ L

Example 14.40 *The pressure of a given mass of a gas filled in a vessel of volume V at constant temperature is reduced to 1/3 rd of its initial value. Calculate the percentage change in its volume.*

Sol. According to the question, $p_2 = \frac{1}{3}p_1$

Applying Boyle's law, $p_1V_1 = p_2V_2$

$$\therefore \quad p_1V_1 = \frac{1}{3}p_1V_2$$

then, $V_2 = 3V_1$

Change in volume $\Delta V = V_2 - V_1 = 2V_1$

Percentage change $= \frac{\Delta V}{V} \times 100 = \frac{2V_1}{V_1} \times 100 = 200\%$

Charles' law

According to this law, for a given mass of a gas the volume of a gas at constant pressure (called **isobaric** process) is directly proportional to its absolute temperature, *i.e.*,

$$V \propto T \qquad (p = \text{constant})$$

or $$\boxed{\frac{V}{T} = \text{constant}}$$

or $$\boxed{\frac{V_i}{T_i} = \frac{V_f}{T_f}}$$

Thus, V-T graph is an isobaric process is a straight line passing through origin. Or V/T *versus* V or T graph is a straight line parallel to V or T axis.

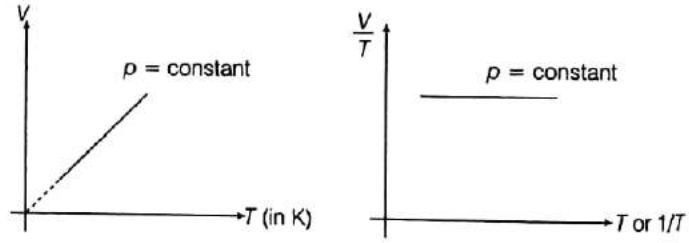


Fig. 14.9 *V versus T graph and V/T versus p or T graph*

If volume of ideal gas at 0°C is V_0 and at t°C is V_t. Then

$$V_t = V_0 (1 + \alpha t)$$

where, α is volume coefficient of the gas, for all gases its value is nearly $\frac{1}{273}$/°C.

$$\therefore \quad V_t = V_0\left(1 + \frac{t}{273}\right)$$

Example 14.41 *Air is filled in a container at 333 K. Calculate the temperature upto which it should be heated so that 1/3 rd of air may escape out of the vessel.*

Sol Given, $T_1 = 333$ K, $T_2 = ?$, $V_1 = V$, $V_2 = V + \frac{V}{3}$

($\because$ According to question, 1/3rd of air may escape out the vessel)

$$= \frac{4}{3}V$$

According to Charles' law, we get

$$\frac{V_2}{V_1} = \frac{T_2}{T_1}$$

$$\Rightarrow \quad T_2 = T_1 \times \frac{V_2}{V_1} = 333 \times \frac{4}{3} = 444 \text{ K}$$

Gay lussacs' law or pressure law

According to this law, for a given mass of a gas the pressure of a gas at constant volume (called **isochoric** process) is directly proportional to its absolute temperature

i.e., $\quad p \propto T \quad (V = \text{constant})$

or $$\boxed{\frac{p}{T} = \text{constant}}$$

or $$\boxed{\frac{p_i}{T_i} = \frac{p_f}{T_f}}$$

Thus, p-T graph in an isochoric process is a straight line passing through origin or p/T *versus* p or T graph is a straight line parallel to p or T-axis.

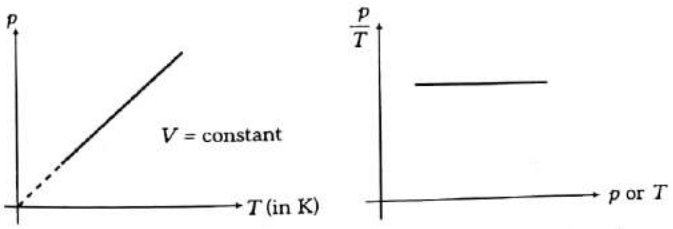


Fig. 14.10 *p versus T graph and p/T versus p or T graph*

If pressure of an ideal gas at 0°C is p_0 and at t°C is p_t. Then

$$p_t = p_0 (1 + \beta t)$$

where, β is pressure coefficient of gas. For all gases its volume is nearly $\frac{1}{273}$/°C.

$$\therefore \quad \boxed{p_t = p_0 \left(1 + \frac{t}{273}\right)}$$

Example 14.42 *The pressure of a gas filled in a closed container increases by 1% when temperature is increased by 1 °C. Find the initial temperature of the gas.*

Sol. Given, $\quad p_1 = p, T_1 = T$ K

Increased pressure, $p_2 = p + \frac{1}{100} p = \frac{101}{100} p$

Increased temperature, $T_2 = (T + 1)$ K

According to the Gay Lussacs' law, we get

$$\frac{p_1}{p_2} = \frac{T_1}{T_2}$$

$$\Rightarrow \quad \frac{p}{\frac{101}{100} p} = \frac{T}{(T+1)}$$

$$\Rightarrow \quad \frac{100}{101} = \frac{T}{T+1}$$

$$\Rightarrow 100T + 100 = 101\,T \Rightarrow T = 100 \text{ K}$$

Dalton's law of partial pressure

It states that the total pressure of a mixture of non-interacting ideal gases is the sum of partial pressures exerted by individual gases in the mixture.

i.e., $\quad p = p_1 + p_2 + p_3 + \cdots$

Consider a mixture of non-interacting ideal gases. Let gas 1 has μ_1 moles, gas 2 has μ_2 moles and so on.

Suppose the net volume of the mixture is V, temperature of mixture is T and pressure of mixture is p.

Equation of state for the mixture, $pV = (\mu_1 + \mu_2 + \cdots) RT$

$$p = \frac{\mu_1 RT}{V} + \frac{\mu_2 RT}{V} + \cdots$$

$$\boxed{\text{Dalton's law, } p = p_1 + p_2 + p_3 + \cdots}$$

Here, $p_1 = \frac{\mu_1 RT}{V}$ is the pressure of gas 1 would exert at the same condition of volume and temperature if no other gases are present.

Example 14.43 *One mole each of hydrogen, oxgen and nitrogen are mixed in a close container of volume 10 L and temperature 27 °C. Calculate the pressure exerted by the mixture.* ($R = 8.314\, J\, mol^{-1}\, K^{-1}$)

Sol. Given, $R = 8.314$ J mol^{-1} K^{-1}

Temperature, $T = 27°\text{C} = 27 + 273 = 300$ K

Volume $V = 10$ L $= 10 \times 10^{-3}$ m^3

According to Dalton's law of partial pressure,

$$p = p_1 + p_2 + p_3$$

$$\Rightarrow \quad p = \frac{\mu RT}{V} + \frac{\mu RT}{V} + \frac{\mu RT}{V} = \frac{3\mu RT}{V}$$

where, $\quad \mu = 1$ mole

$$\therefore \quad p = \frac{3RT}{V} = \frac{3 \times 8.314 \times 300}{10 \times 10^{-3}}$$

$$= 7.48 \times 10^5 \text{ Nm}^{-2}$$

Example 14.44 *4 g Hydrogen is mixed with 11.2 L of He at STP in a container of volume 20 L. If the final temperature is 300 K find the pressure.*

Sol. $\because$ 4 g hydrogen = 2 moles hydrogen.

$\therefore$ 11.2 L He at STP $= \frac{1}{2}$ mole of He

According to Dalton's law of partial pressure

$$p = p_H + p_{He} = (n_H + n_{He})\frac{RT}{V} = \left(2 + \frac{1}{2}\right)\frac{8.31 \times (300\text{ K})}{(20 \times 10^{-3})\text{m}^3}$$

$$= 3.12 \times 10^5 \text{ N/m}^2$$

Grahm's law of diffusion

It states that the rate of diffusion of a gas is inversely proportional to the square root of its density $r \propto \frac{1}{\sqrt{\rho}}$.

where, r = rate of diffusion and ρ = density of the gas. Let r_1 and r_2 be the rates of diffusion of gases 1 and 2 respectively, then

$$\boxed{\text{Grahm's law, } \frac{r_1}{r_2} = \sqrt{\frac{\rho_2}{\rho_1}}}$$

where, ρ_1 is density of gas 1 and ρ_2 is density of gas 2.

$$\frac{r_1}{r_2} = \sqrt{\frac{\rho_2}{\rho_1}} = \sqrt{\frac{M_2}{M_1}}$$

If V is the volume of gas diffused in t second then

$$r = \frac{v}{t} = \frac{r_1}{r_2} = \frac{v_1}{v_2} \times \frac{t_2}{t_1}$$

Behaviour of real gases

If a gas obeys the gas law at all condition is called an **ideal gas** and if it does not, it is called the **real gas.**

(i) All the existing gases are real and the ideal gases are hypothetical.

(ii) Real gases obey the gas laws under the moderate conditions of temperature and pressure. At low temperature and high pressure, the real gases deviate from the ideal gas behaviour significantly (*i.e.* start disobeying gas laws)

(iii) Real gases behave ideally at high temperature and low pressure.

Reason for deviation of real gas from ideal gas behaviour

Real gases deviate from ideal gas behaviour because some of assumption of kinetic theory of gases do not hold good for real gases.

(i) For ideal gas it has been considered that there is no force of attraction between gas molecules. But this assumption is not valid for all pressure and temperature, at low temperature and high pressure gas molecules are close to each other so intermolecular attraction cannot be ignored between them.

(ii) For ideal gas volume of gas molecules is negligible as compared to volume of gas. But at high pressure and low temperature total volume of gas decreases in this condition (total volume changes but volume of molecules remains same) volume of gas molecules is not negligible as compared to volume of gas.

van der Waals' equation

Based on the deviation in behaviour of real gas from ideal gas, van der waals' modified ideal gas by taking into consideration the intermolecular atom forces and the volume occupied by the gas molecules. Introduced a new equation known as van der waals' equation which explains behaviour of real gas convertly.

van der Waals' equation is

$$\boxed{\left(p + \frac{an^2}{V^2}\right)(V - nb) = nRT}$$

Here, a and b are called van der Waals' constants. In this equation, p, V, T and n represent the measured values of pressure, volume, temperature (expressed on the absolute scale) and number of moles, respectively, just as in the ideal gas equation.

Here, the term $\frac{an^2}{V^2}$ is called pressure correction for molecular attraction and term nb is called volume correction for finite size of molecules.

Significance of van der Waals' constants

The quantities a and b are experimentally derived constants that differ for different gases. When a and b are both zero, the van der Waals' equation reduces to the ideal gas equation ($pV = nRT$).

(i) Larger molecules have greater value of b. For instance H_2 (a diatomic molecule) has greater value of b than that of He (a monoatomic smaller molecules).

(ii) $\frac{n^2a}{V^2}$ affects internal pressure of the gas and 'a' is a measure of force of attraction between gas molecules.

(iii) Greater the value of a, greater the liquefaction of the gas. That is why, H_2 and He having smaller value of 'a' are difficult to liquify.

(iv) 'b' is also called co-volume or excluded volume $b = 4VN_0$

where V is the volume of a spherical molecule of radius r, $V = 4/3\ \pi r^3$

(v) a is expressed in atm $L^2\ mol^{-2}$, b is expressed in $L\ mol^{-1}$.

Example 14.45 *If pressure of CO_2 (real gas) in a container is given by $p = \frac{RT}{2V-b} - \frac{9}{4b^2}$, then compute the mass of the gas in container*

Sol. *van der Waals' gas equation for n mole of real gas*

$$\left(p + \frac{n^2a}{V^2}\right)(V - nb) = nRT$$

$$P + \frac{n^2a}{V^2} = \frac{nRT}{V - nb}$$

$$\therefore \quad p = \left(\frac{nRT}{V-nb}\right) - \frac{n^2a}{V^2}$$

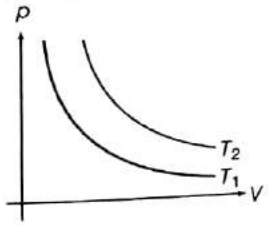
Given equation, $$p = \left(\frac{RT}{2V-b} - \frac{a}{4b^2}\right)$$

On comparing the given equation with this standard equation, we get, $n = 1/2$

Hence, $n = \frac{m}{M}$

$\Rightarrow$ mass of gas, $m = nM = \frac{1}{2} \times 44 = 22$ g

Example 14.46 *The p-V diagrams of same mass of a gas are drawn at two different temperatures T_1 and T_2. Explain whether $T_1 > T_2$ or $T_2 > T_1$.*



Sol. The ideal gas equation is,

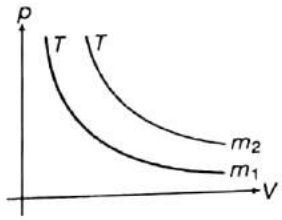
$$pV = nRT \quad \text{or} \quad T = \frac{pV}{nR}$$

$T \propto pV$, if number of moles of the gas are kept constant. Here mass of the gas is constant, which implies that number of moles are constant, *i.e.*, $T \propto pV$.

In the given diagram product of p and V for T_2 is more than T_1 at all points (keeping either p or V same for both graphs).

Hence, $T_2 > T_1$

Example 14.47 *The p-V diagram of two different masses m_1 and m_2 are drawn (as shown) at constant temperature T. State whether $m_1 > m_2$ or $m_2 > m_1$?*



Sol. The ideal gas equation is

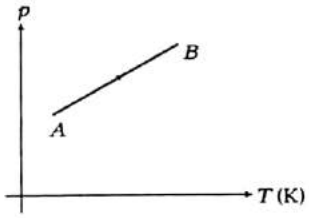
$$pV = nRT = \frac{m}{M}RT$$

$$\therefore \quad m = (pV)\left(\frac{M}{RT}\right)$$

or $\quad m \propto pV$ if $T =$ constant

From the graph we can see that $p_2V_2 > p_1V_1$ (for same p or V). Therefore, $m_2 > m_1$

Example 14.48 *The p-T graph for the given mass of an ideal gas is shown in figure. What inference can be drawn regarding the change in volume (whether it is constant, increasing or decreasing)?*



Sol. From the given graph we can write the p-T equation as,

$$p = aT + b \qquad (y = mx + c)$$

Here a and b are positive constants. Further,

$$\frac{p}{T} = a + \frac{b}{T}$$

Now, $\quad T_B > T_A$

$$\therefore \quad \frac{b}{T_B} < \frac{b}{T_A}$$

or $$\left(\frac{p}{T}\right)_B < \left(\frac{p}{T}\right)_A$$

or $$\left(\frac{T}{p}\right)_B > \left(\frac{p}{T}\right)_A$$

or $\quad V_B > V_A$

Thus, as we move from A to B, volume of the gas is increasing.

5 Degree of freedom

The number of independent ways in which a molecule or an atom can exhibit motion or have energy is called, its degree of freedom. A rigid body of finite size (gas molecule) can have both translatory and rotatory motions. Like translatory motion, rotatory motion can be resolved into mutually perpendicular components.

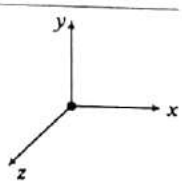
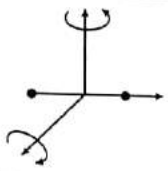
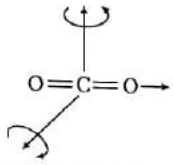
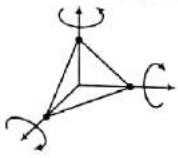
Hence, a rigid body has 6 degrees of freedom; three for translatory motion and three for rotatory motion. Out of three possible rotational degrees, only two are applicable for linear molecules. For non-linear third rotational is allowed. Apart from 3 rotational and 3 translatory, there are 3 N-6 degree of freedom which are vibrational.

A very important relation regarding degree of freedom is

$$\gamma = 1 + \frac{2}{f}; \quad \gamma = C_p / C_V$$

$$f = \text{degree of freedom}$$

Degrees of freedom for different gases according to atomicity of gas at low temperature is given below.

Atomicity of Gas	Translational	Rotational	Total	
Monoatomic Ex. Ar, Ne, Ideal gas etc	3	0	3	
Diatomic Ex, O_2, Cl_2, N_2, etc	3	2	5	
Triatomic (linear) Ex. CO_2, C_2H_2	3	2	5	
Triatomic (Non-linear) or Polyatomic Ex. H_2O, NH_3, CH_4	3	3	6	

Note Degree of freedom for different gases can also be given by the formula, $f = 3NK$

where, N = number of particles in the system

K = number of independent relations between the particles.

Maxwell's law of equipartition energy

The total kinetic energy of a gas molecule is equally distributed among its all degrees of freedom and the energy associated with each degree of freedom at absolute temperature T is $\frac{1}{2}kT$, where, k is Boltzmann's constant.

For a gas in thermal equilibrium at temperature T, the average value of translational energy of the molecule is

$$\langle E_t \rangle = \left\langle \frac{1}{2} m v_x^2 \right\rangle + \left\langle \frac{1}{2} m v_y^2 \right\rangle + \left\langle \frac{1}{2} m v_z^2 \right\rangle$$

Translation energy of the molecules, (for monoatomic gas)

$$\boxed{\langle E_t \rangle = \frac{3}{2} kt}$$

Same is the case with an ideal gas. It distributes its internal energy equally in all degrees of freedom. In each degree of freedom energy of one mole of an ideal gas is $\frac{1}{2}RT$, where T is the absolute temperature of the gas. Thus, if f be the number of degrees of freedom, the internal energy of 1 mole of the gas will be $\frac{f}{2}RT$ or internal energy of n moles of the gas will be $\frac{n}{2}fRT$. Thus,

$$\boxed{\text{Internal energy, } U = \frac{n}{2} fRT}$$

Equipartition of energy of gases is given below

Types of Gases	f	Translation energy of one molecule	Internal energy of one mole
Monoatomic gas	3	$\frac{3}{2}kT$	$\frac{3}{2}RT$
Diatomic gas			
(a) excluding vibration	5	$\frac{5}{2}kT$	$\frac{5}{2}RT$
(b) including vibration	7	$\frac{7}{2}kT$	$\frac{7}{2}RT$
Polyatomic gas like CO_2, H_2O	6	$\frac{6}{2}kT = 3kT$	$\frac{6}{2}RT = 3RT$

Different energies of a system of degree of freedom fare as follows.

(i) Total energy associated with each molecule $= \frac{f}{2}kT$

(ii) Total energy associated with N molecule $= \frac{f}{2}NkT$

(iii) Total energy associated with 1 mole $= \frac{f}{2}RT$

(iv) Total energy associated with μ moles $= \frac{f}{2}\mu RT$

(v) Total enenrgy associated with each gram $= \frac{f}{2}rT$

(vi) Total energy associated with in gram $= \frac{f}{2}mrT$

Example 14.49 *Calculate the ratio of KE of molecule of oxygen and neon gas at 27 °C.*

Sol. For oxygen gas number of degree of freedom is 5

$$\therefore \quad (\text{KE})_{\text{oxygen}} = \frac{5}{2}kT$$

Neon is monatomic, therefore it has 3 degrees of freedom

$$(\text{KE})_{\text{neon}} = \frac{3}{2}kT$$

Hence, the ratio of their KE $= \frac{(\text{KE})_{\text{oxygen}}}{(\text{KE})_{\text{neon}}} = \frac{5/2\,kT}{3/2\,kT} = \frac{5}{3}$

Example 14.50 *Calculate the average kinetic energy of oxygen molecule at 0°C.* ($R = 8.314\ J\ mol^{-1}\ K^{-1}, N_A = 6.02 \times 10^{23}$)

Sol. Given, $T = 0°\text{C} = 273$ K

Oxygen is diatomic molecule, therefore it has 5 degrees of freedom, 3 translational 2 rotational.

$$\therefore \quad \text{KE} = \frac{5}{2}\frac{RT}{N_A} = \frac{5}{2} \times \frac{8.314}{6.023 \times 10^{23}} \times 273 = 9.4 \times 10^{-21}\ \text{J}$$

Example 14.51 *Find the average kinetic energy per molecule at temperature T for an equimolar mixture of two ideal gases A and B, where A is monoatomic and B is diatomic.*

Sol. Number of degrees of freedom per molecule for $A = 3$

Number of degrees of freedom per molecule for $B = 5$

Since, the mixture is equimolar, the average kinetic energy per molecule will be the simple average of the two values *i.e.*, $\left(\frac{3+5}{2}\right)kT = 4kT$. Where k is Boltzmann's constant.

Example 14.52 *Calculate the change in internal energy of 3.0 moles of helium gas when its temperature is increased by 2.0 K.*

Sol. Helium is a monoatomic gas.

Internal energy of n moles of the gas is,

$$U = \frac{3}{2}nRT \quad \therefore \quad \Delta U = \frac{3}{2}nR(\Delta T)$$

Substituting the values, $\Delta U = \left(\frac{3}{2}\right)(3)(8.31)(2.0) = 74.8$ J

Example 14.53 *Given : Avogadro's number $N = 6.02 \times 10^{23}$ and Boltzmann's constant $k = 1.38 \times 10^{-23}$ J/K. Calculate*

(i) The average kinetic energy of translation of the molecules of an ideal gas at 0°C and at 100°C.

(ii) Also calculate the corresponding energies per mole of the gas.

Sol. (i) According to the kinetic theory, the average kinetic energy of translation per molecule of an ideal monoatomic gas at kelvin temperature T is $\left(\frac{3}{2}\right)kT$,

where k is Boltzmann's constant.

At 0° C ($T = 273$ K),

the kinetic energy of translation $= \frac{3}{2}kT$

$$= \frac{3}{2} \times (1.38 \times 10^{-23}) \times 273$$

$$= 5.65 \times 10^{-21}\ \text{J/molecule}$$

At 100° C ($T = 373$ K), the energy is

$$\frac{3}{2} \times (1.38 \times 10^{-23}) \times 373 = 7.72 \times 10^{-21}\ \text{J/molecule}$$

(ii) 1 mole of gas contains N ($\doteq 6.02 \times 10^{23}$) mol

Therefore, at 0° C, the kinetic energy of translation of 1 mole of the gas is

$$= (5.65 \times 10^{-21})(6.02 \times 10^{23})$$

$$\approx 3401\ \text{J/mol and at } 100°\text{ C.}$$

The kinetic energy of translation of 1 mol of gas is

$$= (7.72 \times 10^{-21})(6.02 \times 10^{23})$$

$$\approx 4647\ \text{J/mol}$$

Example 14.54 *One mole of an ideal monoatomic gas is taken at a temperature of 300 K. Its volume is doubled keeping its pressure constant. Find the change in internal energy.*

Sol. Since, pressure is constant

$$\therefore \quad V \propto T \quad \therefore \quad \frac{V_i}{T_i} = \frac{V_f}{T_f}$$

$$\therefore \quad T_f = \frac{V_f}{V_i}T_i \quad \Rightarrow \quad T_f = 2T_i = 600\ \text{K}$$

$$\therefore \quad \Delta U = \frac{f}{2}n \cdot R\Delta T = \frac{3}{2}R(600 - 300) = 450R$$

Critical temperature, pressure and volume

Gases can't be liquified above a temperature called critical temperature (T_C) however large the pressure may be.

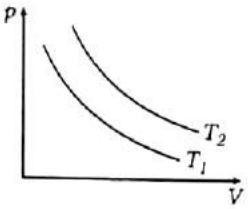
The pressure required to liquify the gas at critical temperature is called critical pressure (p_C) and the volume of the gas at critical temperature and pressure is called critical volume (V_C). Value of critical constants in terms of van der Waals' constants a and b are as under

$$V_C = 3b, \quad p_C = \frac{a}{27b^2} \quad \text{and} \quad T_C = \frac{8a}{27Rb}$$

Further, $\frac{RT_C}{p_C V_C} = \frac{8}{3}$ is called critical coefficient and is same for all gases.

Check point 14.4

1. The adjoining figure shows graph of pressure and volume of a gas at two temperatures T_1 and T_2. Which of the following is correct?



(a) $T_1 > T_2$
(b) $T_1 = T_2$
(c) $T_1 < T_2$
(d) Nothing can be said about temperatures

2. In the gas equation $pV = RT$, V refers to the volume of
(a) any amount of gas (b) one gram of gas
(c) one gram mole of gas (d) one litre of the gas

3. What is the degree of freedom in case of a monoatomic gas?
(a) 1 (b) 3
(c) 5 (d) None of these

4. The number of translational degree of freedom for a diatomic gas is
(a) 2 (b) 3 (c) 5 (d) 6

5. The degrees of freedom of a molecule of a non-linear triatomic gas is (ignore vibrational motion)
(a) 2 (b) 4 (c) 6 (d) 8

6. The mean kinetic energy of one mole of gas per degree of freedom is
(a) $\frac{1}{2}kT$ (b) $\frac{3}{2}kT$ (c) $\frac{3}{2}RT$ (d) $\frac{1}{2}RT$

7. The average translational kinetic energy of O_2 (molar mass 32) at a particular temperature is 0.048 eV. The average translational kinetic energy of N_2 (molar mass 28) molecules in eV at the same temperature is
(a) 0.0015 (b) 0.003 (c) 0.048 (d) 0.768

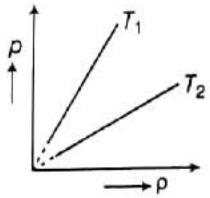
8. A perfect gas at 27°C is heated at constant pressure so as to triple its volume. The temperature of the gas will be
(a) 81°C (b) 900°C
(c) 627°C (d) 450°C

9. 16 g of oxygen, 14 g of nitrogen and 11 g of carbon dioxide are mixed in an enclosure of volume 5 L and temperature 27°C. The pressure exerted by the mixture is
(a) 4×10^5 Nm^{-2} (b) 5×10^5 Nm^{-2}
(c) 6×10^5 Nm^{-2} (d) 9×10^5 Nm^{-2}

10. A balloon is filled at 27 °C and 1 atm pressure by 500 m^3 He. At –3°C and 0.5 atm pressures, the volume of He-gas contained in balloon will be
(a) 700 m^3 (b) 900 m^3
(c) 1000 m^3 (d) 500 m^3

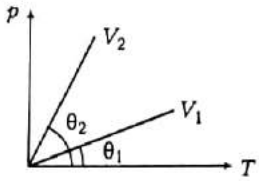
11. A perfect gas at 27°C is heated at constant pressure so as to double its volume. The increase in temperature of the gas will be
(a) 600°C (b) 327°C (c) 54°C (d) 300°C

12. The figure shows graphs of pressure *versus* density for an ideal gas at two temperatures T_1 and T_2, then



(a) $T_1 > T_2$ (b) $T_1 = T_2$
(c) $T_1 < T_2$ (d) None of these

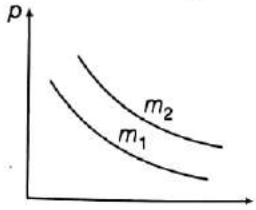
13. From the p-T graph what conclusion can be drawn?



(a) $V_2 = V_1$ (b) $V_2 < V_1$
(c) $V_2 > V_1$ (d) None of these

14. A cylinder contains 20 kg of N_2 gas ($M = 28$ kg k^{-1}mol^{-1}) at a pressure of 5 atm. The mass of hydrogen ($M = 2$ kg k^{-1}mol^{-1}) at a pressure of 3 atm contained in the same cylinder at same temperature is
(a) 1.08 kg (b) 0.86 kg (c) 0.68 kg (d) 1.68 kg

15. Two different isotherms representing the relationship between pressure p and volume V at a given temperature of the same ideal gas are shown for masses m_1 and m_2, then



(a) $m_1 > m_2$
(b) $m_1 = m_2$
(c) $m_1 < m_2$
(d) Nothing can be predicted

16. A gas is found to obey the law p^2V = constant. The initial temperature and volume are T_0 and V_0. If the gas expands to a volume $3V_0$, its final temperature becomes
(a) $\frac{T_0}{3}$ (b) $\frac{T_0}{\sqrt{3}}$
(c) $3T_0$ (d) None of these

17. A gas has volume V and pressure p. The total translational kinetic energy of all the molecules of the gas is
(a) $\frac{3}{2}pV$ only if the gas in monoatomic
(b) $\frac{3}{2}pV$ only if the gas is diatomic
(c) $\frac{3}{2}pV$ in all cases
(d) None of the above

18. A vessel contains a mixture of one mole of oxygen and two moles of nitrogen at 300 K. The ratio of the average rotational kinetic energy per O_2 molecule to that per N_2 molecules is

(a) 1 : 1 (b) 1 : 2
(c) 2 : 1 (d) 8 : 7

19. Two gases are at absolute temperatures 300 K and 350 K respectively. Ratio of average kinetic energy of their molecules is

(a) 7 : 6 (b) 6 : 7
(c) 36 : 49 (d) 49 : 36

20. At 27°C temperature, the kinetic energy of an ideal gas is E_1. If the temperature is increased to 327°C, then the kinetic energy will be

(a) $\frac{E_1}{\sqrt{2}}$ (b) $\sqrt{2}\,E_1$
(c) $2E_1$ (d) $\frac{E_1}{2}$

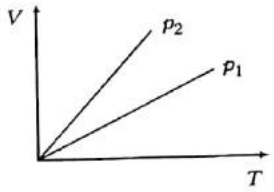
21. Temperature remaining constant, the pressure of gas is decreased by 20%. The percentage change in volume

(a) increases by 20%
(b) decreases by 20%
(c) increases by 25%
(d) decreases by 25%

22. One litre of an ideal gas at 27°C is heated at a constant pressure to the 297°C. Then, the final volume is the approximately

(a) 1.2 L (b) 1.9 L
(c) 19 L (d) 2.4 L

23. In the adjacent V-T diagram what is the relation between p_1 and p_2?

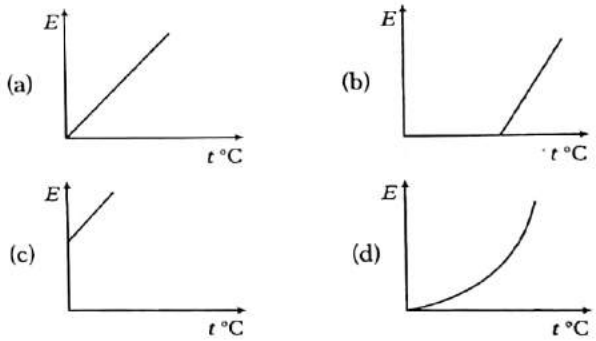


(a) $p_2 = p_1$ (b) $p_2 > p_1$
(c) $p_2 < p_1$ (d) Cannot be predicted

24. The gas in a vessel is subjected to a pressure of 20 atm at a temperature 27°C. The pressure of the gas in a vessel after one-half of the gas is released from the vessel and the temperature of the remainder is raised by 50°C is

(a) 8.5 atm (b) 10.8 atm (c) 11.7 atm (d) 17 atm

25. The graph which represents the variation of mean kinetic energy of molecules with temperature t°C is?



Chapter exercises

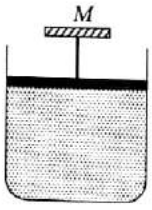
A Taking it together

(Associated questions of the chapter for advanced level practice)

1. Heat is associated with **[NCERT Exemplar]**
(a) kinetic energy of random motion of molecules
(b) kinetic energy of orderly motion of molecules
(c) total kinetic energy of random and orderly motion of molecules
(d) kinetic energy of random motion in some cases and kinetic energy of orderly motion in other

2. Boyle's law is applicable for an **[NCERT Exemplar]**
(a) adiabatic process (b) isothermal process
(c) isobaric process (d) isochoric process

3. A cylinder containing an ideal gas is in vertical position and has a piston of mass M that is able to move up or down without friction (figure). If the temperature is increased **[NCERT Exemplar]**



(a) both p and V of the gas will change
(b) only p will increase according to Charles' law
(c) V will change but not p
(d) p will change but not V

4. The absolute zero temperature in Fahrenheit scale is
(a) $-273°F$ (b) $-32°F$
(c) $-460°F$ (d) $-132°F$

5. On which of the following scales of temperature, the temperature is never negative?
(a) Celsius (b) Fahrenheit
(c) Reaumur (d) Kelvin

6. In a mercury thermometer the ice point (lower fixed point) is marked as 10° and the steam point (upper fixed point) is marked as 130°. At 40°C temperature, what will this thermometer read?
(a) 78° (b) 66° (c) 62° (d) 58°

7. On a new scale of temperature (which is linear) and called the W scale, the freezing and boiling points of water are 39° W and 239° W respectively. What will be the temperature on the new scale, corresponding to a temperature of 39°C on the celsius scale?
(a) 200° W (b) 139° W
(c) 78° W (d) 117° W

8. A centigrade and a Fahrenheit thermometer are dipped in boiling water. The water temperature is lowered until the Fahrenheit thermometer registers 140°. What is the fall in temperature as registered by the centigrade thermometer?
(a) 30° (b) 40° (c) 60° (d) 80°

9. A constant volume gas thermometer shows pressure reading of 50 cm and 99 cm of mercury at 0°C and 100°C respectively. When the pressure reading is 60 cm of mercury, the temperature is
(a) 25°C (b) 40°C (c) 15°C (d) 12.5°C

10. A rod of silver at 0°C is heated to 100°C. It's length is increased by 0.19 cm. Coefficient of cubical expansion of the silver rod is (original length = 1 m)
(a) $5.7 \times 10^{-5}/°C$ (b) $0.63 \times 10^{-5}/°C$
(c) $1.9 \times 10^{-5}/°C$ (d) $16.1 \times 10^{-5}/°C$

11. The volume of gas at 20°C is 100 cm^3 at normal pressure. If it is heated to 100°C, its volume becomes 125 cm^3 at the same pressure, then volume coefficient of the gas at normal pressure is
(a) 0.0015/°C (b) 0.0045/°C
(c) 0.0025/°C (d) 0.0033/°C

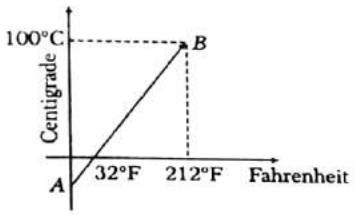
12. Coefficient of real expansion of mercury is $0.18 \times 10^{-3}/°C$. If the density of mercury at 0°C is 13.6 g/cc, its density at 173 K is
(a) 13.11 g/cc (b) 26.22 g/cc
(c) 52.11 g/cc (d) None of these

13. A clock with a metal pendulum beating seconds keeps correct time at 0°C. If it loses 12.5 s a day at 25°C, the coefficient of linear expansion of metal pendulum is

(a) $\frac{1}{86400}$ /°C (b) $\frac{1}{43200}$ /°C

(c) $\frac{1}{14400}$ /°C (d) $\frac{1}{28800}$ /°C

14. The graph AB shown in figure is a plot of temperature of a body is degree celsius and degree Fahrenheit. Then



(a) Slope of line AB is 9/5
(b) Slope of line AB is 5/9
(c) Slope of line AB is 1/9
(d) Slope of line AB is 3/9

15. The value of densities of two diatomic gases at constant temperature and pressure are d_1 and d_2, then the ratio of the speed of sound in these gases will be

(a) $d_1 d_2$ (b) $\sqrt{d_2/d_1}$

(c) $\sqrt{d_1/d_2}$ (d) $\sqrt{d_1 d_2}$

16. A gas at the temperature 250 K is contained in a closed vessel. If the gas is heated through 1 K, then the percentage increase in its pressure will be

(a) 0.4% (b) 0.2%
(c) 0.1% (d) 0.8%

17. An electron tube was sealed off during manufacture at a pressure of 1.2×10^{-7} mm of mercury at 27°C. Its volume is 100 cm^3. The number of molecules that remain in the tube is

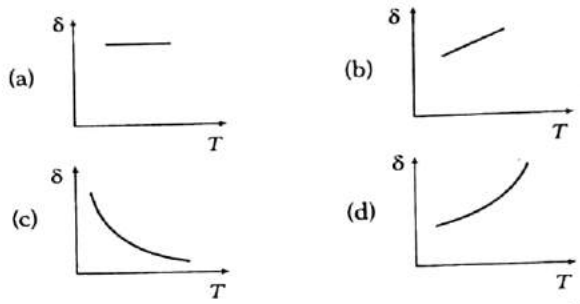
(a) 2×10^{16} (b) 3×10^{15}

(c) 3.86×10^{11} (d) 5×10^{11}

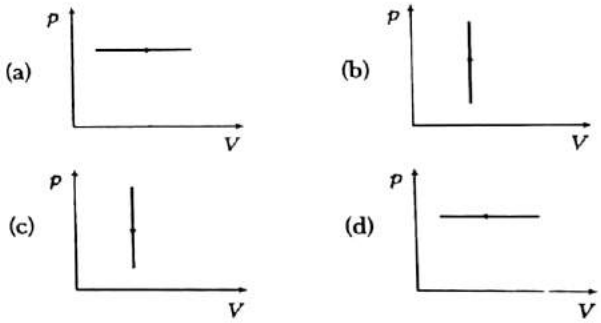
18. At what temperature is the root mean square velocity of the gaseous hydrogen molecules equal to that of oxygen molecules at 47°C.

(a) 20 K (b) 80 K
(c) – 73 K (d) 3 K

19. An ideal gas is initially at temperature T and volume V. Its volume is increased by ΔV due to an increase in temperature ΔT, pressure remaining constant. The quantity $\delta = \Delta V/(V\ \Delta T)$ varies with temperature as



20. Variation of internal energy with density of one mole of monoatomic gas is depicted in the adjoining figure. Corresponding variation of pressure with volume can be depicted as (Assuming the curve is rectangular hyperbola)



21. A chamber containing a gas was evacuated till the vacuum attained was 10^{-14} m of Hg. If the temperature of the chamber was 30°C, the number of molecules that remains in it per cubic metre is

(a) 3.2×10^{11} (b) 3.2×10^{12}

(c) 2.3×10^{12} (d) 2.3×10^{10}

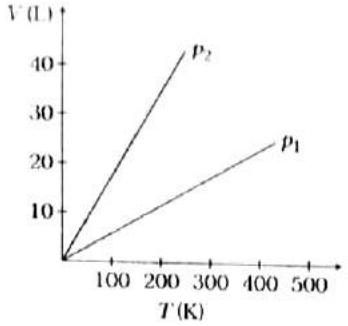
22. A bimetallic strip is made of aluminium and steel ($\alpha_{Al} > \alpha_{steel}$). On heating, the strip will **[NCERT Exemplar]**

(a) remains straight
(b) get twisted
(c) will bend with aluminium on concave side
(d) will bend with steel on concave side

23. A uniform metallic rod rotates about its perpendicular bisector with constant angular speed. If it is heated uniformly to raise its temperature slightly **[NCERT Exemplar]**

(a) its speed of rotation increases
(b) its speed rotation decreases
(c) its speed of rotation remains same
(d) its speed increase because its moment of inertia increases

24. Volume *versus* temperature graphs for a given mass of an ideal gas are shown in figure. At two different values of constant pressure. What can be inferred about relation between p_1 and p_2? **[NCERT Exemplar]**



(a) $p_1 > p_2$ (b) $p_1 = p_2$
(c) $p_1 < P_2$ (d) Data is insufficient

25. As the temperature is increased, the period of a pendulum **[NCERT Exemplar]**
(a) increases as its effective length increases even though its centre of mass still remains at the centre of the bob
(b) decreases as its effective length increases even though its centre of mass still remains at the centre of the bob
(c) increases at its effective length increases due to shifting to centre of mass below the centre of the bob
(d) decreases as its effective length remains same but the centre of mass shifts above the centre of the bob

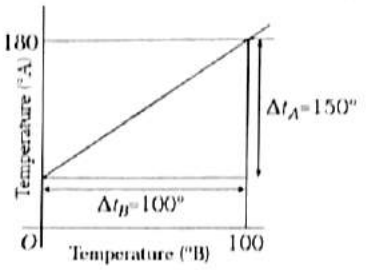
26. An aluminium sphere is dipped into water. Which of the following is true? **[NCERT Exemplar]**
(a) Buoyancy will be less in water at 0°C than that in water at 4°C
(b) Buoyancy will be more in water at 0°C than that in water at 4°C
(c) Buoyancy in water at 0°C will be same as that in water at 4°C
(d) Buoyancy may be more or less in water at 4°C depending on the radius of the sphere

27. The radius of a metal sphere at room temperature T is R and the coefficient of linear expansion of the metal is α. The sphere heated a little by a temperature ΔT so that its new temperature is $T + \Delta T$. The increase in the volume of the sphere is approximately. **[NCERT Exemplar]**
(a) $2\pi R\alpha\Delta T$ (b) $\pi R^2\alpha\Delta T$
(c) $4\pi R^3\alpha\Delta T/3$ (d) $4\pi R^3\alpha\Delta T$

28. A cubic vessel (with face horizontal + vertical) contains an ideal gas at NTP. The vessel is being carried by a rocket which is moving at a speed of 500 ms^{-1} in vertical direction.

The pressure of the gas inside the vessel as observed by us on the ground **[NCERT Exemplar]**
(a) remains the same because 500 ms^{-1} is very much smaller than v_{rms} of the gas
(b) remains the same because motion of the vessel as a whole does not affect the relative motion of the gas molecules and the walls
(c) will increase by a factor equal to $(v_{rms}^2 + (500)^2)/v_{rms}^2$ where v_{rms} was the original mean square velocity of the gas.
(d) will be different on the top wall an bottom wall of the vessel

29. The graph between two temperature scales A and B is shown in figure between upper fixed point and lower fixed point there are 150 equal division and scale A and 100 on scale. B. The relationship for conversion between the two scales is given by **[NCERT Exemplar]**



(a) $\frac{t_A - 180}{100} = \frac{t_B}{150}$ (b) $\frac{t_A - 30}{150} = \frac{t_B}{100}$
(c) $\frac{t_B - 180}{150} = \frac{t_A}{100}$ (d) $\frac{t_B - 40}{100} = \frac{t_A}{180}$

30. 1 mole of H_2 gas is contained in a box of volume $V = 100\ m^3$ at $T = 300\ K$. The gas is heated to a temperature of $T = 3000$ K and the gas gets converted to a gas of hydrogen atoms. The final pressure would be (considering all gases to be ideal) **[NCERT Exemplar]**
(a) same as the pressure initially
(b) 2 times the pressure initially
(c) 10 times the pressure initially
(d) 20 times the pressure initiallly

31. An inflated rubber balloon contains one mole of an ideal gas, has a pressure p, volume V and temperature T. If the temperature rises to 1.1 T, and the volume is increased to 1.05 V, the final pressure will be **[NCERT Exemplar]**
(a) 1.1 p (b) p
(c) less than p (d) between p and 1.1

32. A glass vessel of volume V_0 is completely filled with a liquid and its temperature is raised by ΔT. What volume of the liquid will overflow? Coefficient of linear expansion of glass $= \alpha_g$ and coefficient of volume expansion of the liquid $= \gamma_l$.
(a) $2V_0AT(\gamma_l - 3\alpha_g)$ (b) $V_0\Delta T(\gamma_l - 3\alpha_g)$
(c) $V_0\Delta T(\gamma_l - \alpha_g)$ (d) $\frac{V_0\Delta T}{2}(\gamma_l - 3\alpha_g)$

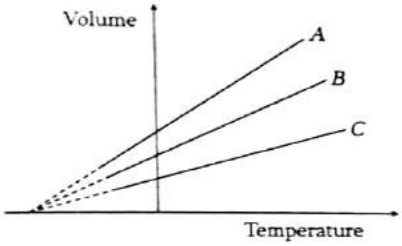
33. The steam point and the ice point of a mercury thermometer are marked as 80° and 10°. At what temperature on centigrade scale the reading of this thermometer will be 59°?

(a) 70°C (b) 60°C (c) 40°C (d) 50°C

34. Two uniform brass rods A and B of length l and $2l$ and radii $2r$ and r respectively are heated to the same temperature. The ratio of the increase in the volume of A to that of B is

(a) 1 : 1 (b) 1 : 2 (c) 2 : 1 (d) 1 : 4

35. The expansion of an ideal gas of mass m at a constant pressure p is given by the straight line B. Then, the expansion of the same ideal gas of mass $2m$ at a pressure $2p$ is given by the straight line.



(a) C (b) A
(c) B (d) None of these

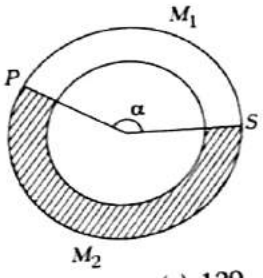
36. The root mean square velocity of the molecules in a sample of helium is 5/7th that of the molecules in a sample of hydrogen. If the temperature of hydrogen sample is 0°C, then the temperature of the helium sample is about

(a) 0°C (b) 0 K
(c) 273°C (d) 100°C

37. A room temperature, the rms speed of the molecules of certain diatomic gas is found to be 1930 ms^{-1}. The gas is

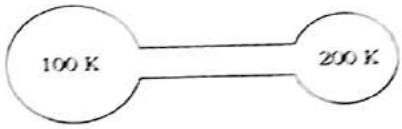
(a) H_2 (b) F_2 (c) O_2 (d) Cl_2

38. A ring shaped tube contains two ideal gases with equal masses and relative molar masses $M_1 = 32$ and $M_2 = 28$. The gases are separated by one fixed partition P and another movable stopper S which can move freely without friction inside the ring. The angle α in equilibrium as shown in the figure (in degrees) is



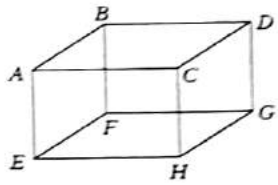
(a) 291 (b) 219 (c) 129 (d) 192

39. Figure shows two flasks connected to each other. The volume of the flask 1 is twice that of flask 2. The system is filled with an ideal gas at temperature 100 K and 200 K respectively. If the mass of the gas in 1 be m then what is the mass of the gas in flask 2



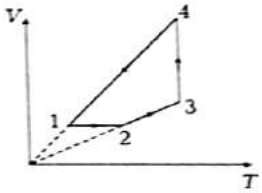
(a) m (b) $m/2$
(c) $m/4$ (d) $m/8$

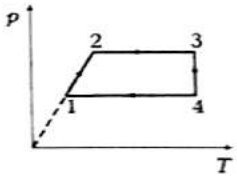
40. 1 mole of an ideal gas is contained in a cubical volume V, $ABCDEFGH$ at 300 K (figure). One face of the cube ($EFGH$) is made up of a material which totally absorbs any gas molecule incident on it. At any given time. **[NCERT Exemplar]**

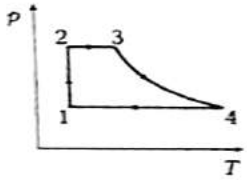


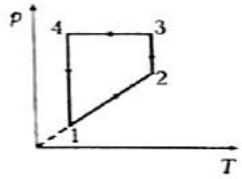
(a) the pressure on $EFGH$ would be zero
(b) the pressure on all the faces will the equal
(c) the pressure of $EFGH$ would be double the pressure on $ABCD$
(d) the pressure on $EFGH$ would be half that on $ABCD$

41. A cyclic process 1-2-3-4-1 is depicted on V-T diagram. The p-T diagram and p-V diagrams for this cyclic process are given below. Select the incorrect choices.



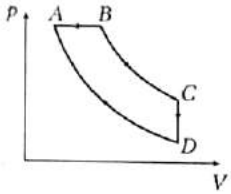
(a)


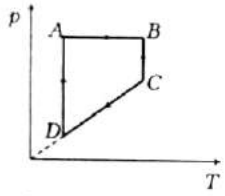
(b)


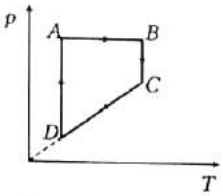
(c)


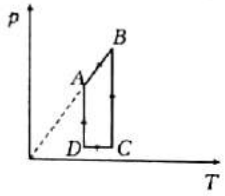
(d) None of these

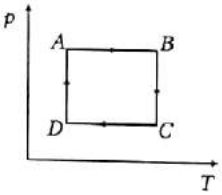
42. A cyclic process *ABCDA* is shown in the *p-V* diagram. Which of the following curves represent the same process?



(a)


(b)


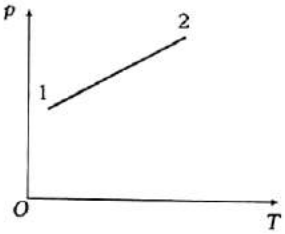
(c)


(d)


43. The average kinetic energy per molecule at temperature T for an equimolar mixture of two ideal gases A and B, where A is monoatomic and B is diatomic is

(a) $2\,kT$ (b) $4\,kT$ (c) $3\,kT$ (d) $8\,kT$

44. *p-T* diagram was obtained when a given mass of gas was heated. During the heating process from the state 1 to state 2 the volume

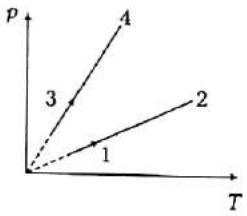


(a) remained constant
(b) decreased
(c) increased
(d) changed erratically

45. A sphere of diameter 7 cm and mass 266.5 g floats in a both of liquid. As the temperature is raised, the sphere just sinks at a temperature of 35°C. If the density of the liquid at 0°C is 1.527 g cm^{-3}, then the coefficient of cubical expansion of the liquid is

(a) 0.0043/°C (b) 0.00083/°C
(c) 0.00025/°C (d) 0.00010/°C

46. Pressure *versus* temperature graph of an ideal gas of equal number of moles of different volumes are plotted as shown in figure. Choose the correct alternative.



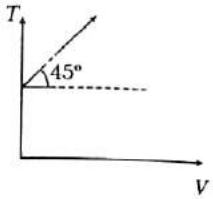
(a) $V_1 = V_2$, $V_3 = V_4$ and $V_2 > V_3$
(b) $V_1 = V_2$, $V_3 = V_4$ and $V_2 < V_3$
(c) $V_1 = V_2 = V_3 = V_4$
(d) $V_4 > V_3 > V_2 > V_1$

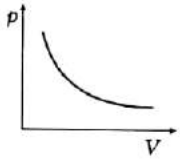
47. A cylindrical steel plug is inserted into a circular hole of diameter 2.60 cm in a brass plate. When the plug and the plates are at a temperature of 20°C, the diameter of the plug is 0.010 cm smaller than that of the hole. The temperature at which the plug will just fit in it is

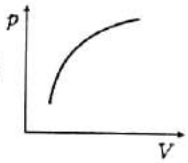
$$\left(\text{Given, } \alpha_{\text{steel}} = \frac{11\times10^{-6}}{°\text{C}} \text{ and } \alpha_{\text{brass}} = \frac{19\times10^{-6}}{°\text{C}}\right)$$

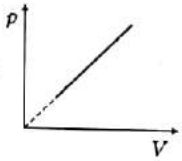
(a) −48°C (b) −20°C (c) −10°C (d) −458°C

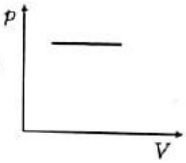
48. The given curve represents the variations of temperature as a function of volume for one mole of an ideal gas. Which of the following curves best represents the variation of pressure as a function of volume?



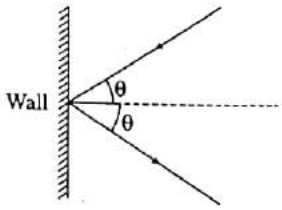
(a)


(b)


(c)


(d)


49. The mass of hydrogen molecule is 3.32×10^{-27} kg. If 10^{23} hydrogen molecules strike per second at 2 cm^2 area of a rigid wall at an angle of 45° from the normal and rebound back with a speed of 1000 ms^{-1}, then the pressure exerted on the wall is



(a) 2.34×10^3 Pa (b) 0.23×10^6 Pa
(c) 0.23×10^3 Pa (d) 23.4×10^3 Pa

50. The apparent coefficient of expansion of a liquid when heated in a copper vessel is C and when heated in a silver vessel it is S. If A is the linear coefficient of expansion of copper, then the linear coefficient of expansion of silver is

(a) $\frac{C+S-3A}{3}$ (b) $\frac{C-S+3A}{3}$
(c) $\frac{S+3A-C}{3}$ (d) $\frac{C+S+3A}{3}$

51. Two identical containers joined by a small pipe initially contain the same gas at pressure p_0 and absolute temperature T_0. One container is now maintained at the same temperature while the other is heated to $2T_0$. The common pressure of the gases will be

(a) $\frac{3}{2}p_0$ (b) $\frac{4}{3}p_0$ (c) $\frac{5}{3}p_0$ (d) $2p_0$

52. A piece of metal weighs 46 g in air. When it is immersed in a liquid of specific gravity 1.24 at 27°C, it weighs 30 g. When the temperature is raised to 42°C, the metal piece weighs 30.5 g. If the specific gravity of the liquid at 42°C is 1.20, the coefficient of linear expansion of the metal is

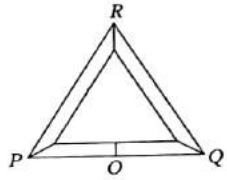
(a) $\frac{1.4 \times 10^{-5}}{°C}$ (b) $\frac{2.3 \times 10^{-5}}{°C}$

(c) $\frac{4.3 \times 10^{-5}}{°C}$ (d) $\frac{3.4 \times 10^{-5}}{°C}$

53. A vertical cylinder closed at both ends is fitted with a smooth piston dividing the volume into two parts each containing one mole of air. At the equilibrium temperature of 320 K, the upper and lower parts are in the ratio 4 : 1. The ratio will become 3 : 1 at a temperature of

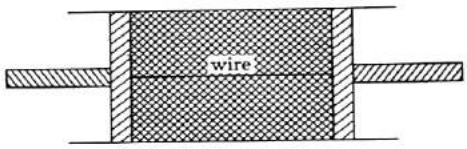
(a) 450 K (b) 228 K
(c) 420 K (d) 570 K

54. Three rods of equal length l are joined to form an equilateral triangle PQR. O is the mid-point of PQ. Distance OR remains same for small change in temperature. Coefficient of linear expansion for PR and RQ is same *i.e.*, α_2 but that for PQ is α_1. Then



(a) $\alpha_2 = 3\alpha_1$ (b) $\alpha_2 = 4\alpha_1$ (c) $\alpha_1 = 3\alpha_2$ (d) $\alpha_1 = 4\alpha_2$

55. A cylindrical tube of uniform cross-sectional area A is fitted with two air tight frictionless pistons.



The pistons are connected to each other by a metallic wire. Initially the pressure of the gas is p_0 and temperature is T_0, atmospheric pressure is also p_0. Now, the temperature of the gas is increased to $2T_0$, the tension in the wire will be

(a) $2p_0A$ (b) p_0A (c) $\frac{p_0A}{2}$ (d) $4p_0A$

B) Medical entrance special format questions

Assertion and reason

Directions (Q. Nos. 1-14) *These questions consists of two statements each printed as Assertion and Reason. While answering these question you are required to choose any one of the following five responses.*

(a) If both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If Assertion is false but Reason is true.

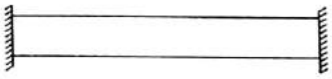
1. **Assertion** At the same temperature and pressure, equal volumes of all gases contain equal number of molecules.

Reason In 1 L at NTP total number of molecules are 6.02×10^{23}.

2. **Assertion** In isochoric process p-V graph is straight line parallel to p-axis.

Reason In isochoric process density ρ remains constant.

3. **Assertion** A metallic rod is fixed from two ends as shown in figure. When the temperature is increased compressive stresses are developed in the rod.



Reason At higher temperature, natural length of the rod will be more.

4. **Assertion** Pressure of a gas is given as $p = \frac{2}{3}E$.

Reason In the above expression, E represents kinetic energy of the gas per unit volume.

5. **Assertion** An actual gas behaves as an ideal gas most closely at low pressure and high temperature.

Reason At low pressure and high temperature, real gases obey the gas laws .

6. **Assertion** In summers, a metallic scale will read more than the actual.

Reason In summers, length of metallic scale will increase.

7. **Assertion** Total kinetic energy of any gas at temperature T would be $\frac{1}{2} m v_{rms}^2$.

Reason Translational kinetic energy of any type of gas at temperature T would be $\frac{3}{2} RT$ of one mole.

8. **Assertion** If a gas chamber containing a gas is moved translationally, then temperature of gas will increase.

Reason Total kinetic energy of the gas molecules will increase by the translational motion of gas chamber.

9. **Assertion** Degree of freedom of a monoatomic gas is always three, whether we consider vibrational effects or not.

Reason At all temperatures (low or high), vibrational kinetic energy of an ideal gas is zero.

10. **Assertion** Pressure of a gas is $\frac{2}{3}$ times translational kinetic energy of gas molecules.

Reason Translational degree of freedom of any type of gas is three, whether the gas is monoatomic, diatomic or polyatomic.

11. **Assertion** Total internal energy of oxygen gas at a given temperature is E of this energy $\frac{3}{5}E$ is translational kinetic energy and $\frac{2}{5}E$ is rotational kinetic energy.

Reason Potential energy of an ideal gas is zero.

12. **Assertion** Straight line on V-T graph represents isobaric process.

Reason If $V \propto T$, then p = constant *i.e.*, process is isobaric.

13. **Assertion** Any straight line on V-T diagram represents isobaric process.

Reason In isobaric process, if V is doubled, then T will also become two times.

14. **Assertion** At triple point, three states (solid,liquid and gas) may co-exist simultaneously.

Reason For water, the values of pressure and temperature corresponding to triple point are 10 mm of Hg and 273.16 K.

Match the columns

1. For a monoatomic gas at temperature T, match the following columns.

Column I		Column II	
(A)	Speed of sound	(p)	$\sqrt{2RT/M}$
(B)	RMS speed of gas molecules	(q)	$\sqrt{8RT/\pi M}$
(C)	Average speed of gas molecules	(r)	$\sqrt{3RT/M}$
(D)	Most probable speed of gas molecules	(s)	$\sqrt{5RT/3M}$

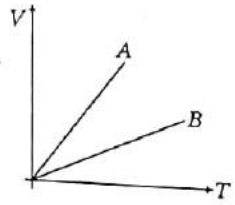
2. Match the following columns.

Column I		Column II	
(A)	In $p = \frac{2}{3}E$, E is	(p)	isochoric
(B)	In $U = 3RT$ for an monoatomic gas U is	(q)	Translational kinetic energy of unit volume
(C)	$\frac{V}{T}$ = constant is valid for	(r)	Internal energy of one mole
(D)	$\frac{p}{T}$ = constant is associated with	(s)	isobaric process
		(t)	None

3. In the process $T \propto 1/V$, pressure of the gas increases from p_0 to $4p_0$. Match the following columns.

Column I		Column II	
(A)	Temperature of the gas	(p)	Positive
(B)	Volume of the gas	(q)	Negative
(C)	Work done by the gas	(r)	Two times
(D)	Heat supplies to the gas	(s)	Cannot say anything
		(t)	None

4. In the V-T graph shown in the figure match the following columns.



Column I		Column II	
(A)	Gas A is ... and gas B is ...	(p)	Monoatomic, diatomic
(B)	p_A/p_B is	(q)	Diatomic, monoatomic
(C)	n_A/n_B is	(r)	>1
		(s)	<1
		(t)	Cannot say anything

Medical entrances' gallery

(Collection of questions asked in NEET & various medical entrance exams)

1. A given sample of an ideal gas occupies a volume V at a pressure p and absolute temperature T. The mass of each molecule of the gas is m. Which of the following gives the density of the gas? **[NEET 2016]**
(a) $p/(kT)$ (b) $pm/(kT)$ (c) $p/(kTV)$ (d) mkT

2. Coefficient of linear expansion of brass and steel rods are α_1 and α_2. Lengths of brass and steel rods are l_1 and l_2 respectively. If $(l_2 - l_1)$ is maintained same at all temperatures, which one of the following relations holds good ? **[NEET 2016]**
(a) $\alpha_1 l_2^2 = \alpha_2 l_1^2$ (b) $\alpha_1^2 l_2 = \alpha_2^2 l_1$
(c) $\alpha_1 l_1 = \alpha_2 l_2$ (d) $\alpha_1 l_2 = \alpha_2 l_1$

3. The molecules of a given mass of a gas have rms velocity of 200 ms^{-1} at 27°C and 10×10^5 Nm^{-2} pressure. When the temperature and pressure of the gas are respectively, 127°C and 0.05×10^5 Nm^{-2}, the rms velocity of its molecules in ms^{-1} is **[NEET 2016]**
(a) $\frac{400}{\sqrt{3}}$ (b) $\frac{100\sqrt{2}}{3}$ (c) $\frac{100}{3}$ (d) $100\sqrt{2}$

4. The rms speed of oxygen molecule in a gas at 27°C would be given by **[UK PMT 2015]**
(a) 483 ms^{-1} (b) 966 ms^{-1} (c) 4.83 ms^{-1} (d) 9.66 ms^{-1}

5. The pressure of an ideal gas is directly proportional to **[UK PMT 2015]**
(a) total kinetic energy (b) translational kinetic energy
(c) rotational kinetic energy (d) vibrational kinetic energy

6. Two metal rods of lengths L_1 and L_2 and coefficients of linear expansion α_1 and α_2 respectively are welded together to make a composite rod of length $(L_1 + L_2)$ at 0°C. Find the effective coefficient of linear expansion of the composite rod. **[EAMCET 2015]**
(a) $\frac{L_1\alpha_1^2 - L_2\alpha_2^2}{L_1^2 + L_2^2}$ (b) $\frac{L_1^2\alpha_1 - L_2^2\alpha_2}{L_1^2 + L_2^2}$
(c) $\frac{L_1\alpha_1 + L_2\alpha_2}{L_1 - L_2}$ (d) $\frac{L_1\alpha_1 + L_2\alpha_2}{L_1 + L_2}$

7. The ratio of rms speed of an ideal gas molecules at pressure p to that at pressure $2p$ is **[Kerala CEE 2015]**
(a) 1 : 2 (b) 2 : 1 (c) $1 : \sqrt{2}$ (d) $\sqrt{2} : 1$

8. The rms speed of oxygen is v at a particular temperature. If the temperature is doubled and oxygen molecules dissociate into oxygen atoms, the rms speed becomes **[WB JEE 2015]**
(a) v (b) $\sqrt{2}v$ (c) $2v$ (d) $4v$

9. The deviation of a real gas from the ideal one is minimum at **[UP CPMT 2015]**
(a) low pressure and high temperature
(b) low temperature and low pressure
(c) high pressure and low temperature
(d) high pressure and high temperature

10. The density of hydrogen gas at STP is 0.9 kg m^{-3}. The mean kinetic energy of one gram molecule of hydrogen gas is approx **[UP CPMT 2015]**
(a) 3403 T (b) 3500 J (c) 3704 J (d) 3207 J

11. A substance having coefficient of critical expansion 4.9×10^{-4} °C^{-1}. The fractional change in the density for the rise in temperature 30 K is **[UP CPMT 2015]**
(a) 0.0012 (b) 0.1280 (c) 0.0235 (d) 0.0145

12. At constant pressure, the ratio of increase in volume of an ideal gas per degree rise in kelvin temperature to its volume is **[Manipal 2015]**
(a) $1/T$ (b) $1/T^2$ (c) T (d) T^2

13. Water is heated from 0° C to 10° C, then its volume **[KCET 2015]**
(a) does not change (b) decreases
(c) first decreases and then increases
(d) increases

14. The length of a steel rod is 5 cm longer than that of a brass rod. If this difference in their lengths is to remain the same at all temperatures, then the length of brass rod will be (coefficient of linear expansion for steel and brass are 12×10^{-6} /°C and 18×10^{-6} /°C respectively) **[EMACET 2015]**
(a) 20 cm (b) 15 cm (c) 5 cm (d) 10 cm

15. In anomalous expansion of water, at what temperature, the density of water is maximum? **[KCET 2014]**
(a) 4°C (b) < 4° C
(c) > 4° C (d) 10° C

16. A metal rod is fixed rigidly at two ends so as to prevent its thermal expansion. If L, α and Y respectively denote the length of the rod, coefficient of linear thermal expansion and Young's modulus of its material, then for an increase in temperature of the rod by ΔT, the longitudinal stress developed in the rod is **[AMU 2014]**
(a) inversely proportional to α
(b) inversely proportional to Y
(c) directly proportional to $\Delta T / Y$
(d) independent of L

17. Which one of the following is a wrong statement in kinetic theory of gases? **[Kerala CEE 2014]**
(a) The gas molecules are in random motion.
(b) The gas molecules are perfect elastic spheres.
(c) The volume occupied by the molecules of a gas is negligible.
(d) The force of attraction between the molecules is negligible.
(e) The collision between molecules is inelastic.

18. The mean free path of molecules of a gas (radius) is inversely proportional to **[CBSE AIPMT 2014]**
(a) r^3 (b) r^2 (c) r (d) $\sqrt{r}$

19. The average pressure of an ideal gas is **[J&K CET 2013]**
(a) $p = (1/3)\, mnV_{av}^2$ (b) $p = (1/2)\, mnV_{av}$
(c) $p = (1/4)\, mnV_{av}^2$ (d) $p = (1/3)\, mnV_{av}$

20. According to equipartition law of energy each particle in a system of particles have thermal energy E equal to **[J&K CET 2013]**
(a) $E = k_BT$ (b) $E = (1/2)\, k_BT$
(c) $E = 3\, k_BT$ (d) $E = (3/2)\, k_BT$

21. When a liquid is heated in a glass vessel, its coefficient of apparent expansion is $1.03\times 10^{-3}/°C$. When the same liquid is heated in a copper vessel, its coefficient of apparent expansion is $1.006\times 10^{-3}/°C$. If the coefficient of linear expansion of copper is $17\times10^{-6}/°C$, then the coefficient of linear expansion of glass is **[EAMCET 2013]**
(a) $8.5\times10^{-4}/°C$ (b) $9\times10^{-6}/°C$
(c) $27\times10^{-6}/°C$ (d) $10\times10^{-4}/°C$

22. A horizontal uniform tube, open at both ends is containing a liquid of certain length at some temperature. When the temperature is changed, the length of the liquid in the tube is not changed. If α is the coefficient of linear expansion of the material of the tube and γ is the coefficient of volume expansion of the liquid, then **[EAMCET 2013]**
(a) $\gamma = 2\alpha$ (b) $\gamma = 3\alpha$ (c) $\gamma = 4\alpha$ (d) $\gamma = \alpha$

23. The average kinetic energy of a gas molecule at absolute temperature T is **[Kerala CEE 2013]**
(a) directly proportional to T^2
(b) inversely proportional to T^2
(c) directly proportional to T
(d) inversely proportional to T
(e) directly proportional to $\sqrt{T}$

24. At what temperature, the rms velocity of gas molecules would be double of its value at NTP, if pressure is remaining constant? **[MP PMT 2013]**
(a) 819 °C (b) 819 K (c) 546 K (d) 546°C

25. The average kinetic energy per mole of hydrogen at given temperature is **[MP PMT 2013]**
(a) equal of that of oxygen (b) 16 times that of oxygen
(c) $\frac{1}{16}$ times that of oxygen (d) $\frac{1}{8}$ times that of oxygen

26. In kinetic theory of gases, it is assumed that molecular collisions are **[UP CPMT 2013]**
(a) for negligible duration
(b) inelastic
(c) one-dimensional (head on)
(d) unable to exert mutual force

27. The number of molecules in a litre of a gas at temperature of 27°C and a pressure of 10^6 dyne cm^{-2} is **[UP CPMT 2013]**
(a) 2.4×10^{20} (b) 2.4×10^{21}
(c) 2.4×10^{22} (d) 2.4×10^{23}

28. At standard temperature and pressure, the density of a gas is 1.3 kgm^{-3} and the speed of sound in gas is 330 ms^{-1}. Then the degree of freedom of the gas will be **[BCECE (Mains) 2012]**
(a) 5 (b) 6
(c) 7 (d) 8

29. A non-conducting body floats in a liquid at 20°C with $\frac{2}{3}$ of its volume immersed in the liquid. When liquid temperature is increased to 100°C, $\frac{3}{4}$ of body's volume is immersed in the liquid. Then the coefficient of real expansion of the liquid is (neglecting the expansion of container of the liquid) **[AIIMS 2012]**
(a) 15.6×10^{-4}/°C (b) 156×10^{-4}/°C
(c) 1.56×10^{-4}/°C (d) 0.156×10^{-4}/°C

30. During an experiment an ideal gas is found to obey an additional law Vp^2 = constant. The gas is initially at temperature T and volume V, when it expands to volume $2V$, the resulting temperature is **[UP CPMT 2012]**
(a) $\frac{T}{2}$ (b) $2T$
(c) $\sqrt{2}\,T$ (d) $\frac{T}{\sqrt{2}}$

Answers

Check point 14.1

1. (b) 2. (d) 3. (a) 4. (c) 5. (a) 6. (a) 7. (c) 8. (c) 9. (d) 10. (c)

Check point 14.2

1. (a) 2. (a) 3. (d) 4. (a) 5. (b) 6. (a) 7. (c) 8. (c) 9. (b) 10. (d)
11. (c) 12. (c) 13. (a) 14. (d) 15. (c) 16. (a) 17. (b) 18. (d) 19. (c) 20. (a)

Check point 14.3

1. (d) 2. (b) 3. (b) 4. (c) 5. (d) 6. (b) 7. (a) 8. (d) 9. (c) 10. (d)
11. (a) 12. (a) 13. (b) 14. (d) 15. (c) 16. (a) 17. (c) 18. (c) 19. (c) 20. (c)

Check point 14.4

1. (c) 2. (c) 3. (b) 4. (b) 5. (c) 6. (d) 7. (c) 8. (c) 9. (c) 10. (b)
11. (d) 12. (a) 13. (c) 14. (b) 15. (c) 16. (d) 17. (c) 18 (a) 19. (b) 20. (c)
21. (c) 22. (b) 23. (c) 24. (c) 25. (c)

A. Taking it together

1. (a) 2. (b) 3. (c) 4. (c) 5. (d) 6. (d) 7. (d) 8. (b) 9. (a) 10. (a)
11. (d) 12. (a) 13. (a) 14. (b) 15. (b) 16. (a) 17. (c) 18. (a) 19. (c) 20. (d)
21. (a) 22. (d) 23. (b) 24. (a) 25. (a) 26. (a) 27. (d) 28. (b) 29. (b) 30. (d)
31. (a) 32. (b) 33. (a) 34. (c) 35. (c) 36. (a) 37. (a) 38. (d) 39. (c) 40. (a)
41. (c) 42. (a) 43. (b) 44. (c) 45. (b) 46. (a) 47. (d) 48. (a) 49. (a) 50. (b)
51. (b) 52. (b) 53. (a) 54. (d) 55. (b)

B. Medical entrance special format questions

Assertion and reason

1. (c) 2. (b) 3. (a) 4. (c) 5. (a) 6. (d) 7. (b) 8. (d) 9. (c) 10. (c)
11. (b) 12. (d) 13. (d) 14. (c)

Match the columns

1. (A→r, B→t, C→q, D→s)
2. (A→s, B→r, C→q, D→p)
3. (A→q, B→t, C→s, D→t)
4. (A→t, B→t, C→t)
5. (A→r, B→p, C→q, D→s)

C. Medical entrances' gallery

1. (b) 2. (c) 3. (a) 4. (a) 5. (a) 6. (d) 7. (c) 8. (c) 9. (a) 10. (a)
11. (d) 12. (a) 13. (c) 14. (d) 15. (a) 16. (c) 17. (e) 18. (b) 19. (a) 20. (b)
21. (b) 22. (a) 23. (c) 24. (a) 25. (a) 26. (a) 27. (c) 28. (a) 29. (a) 30. (c)

Hints and explanations

Check point 14.1

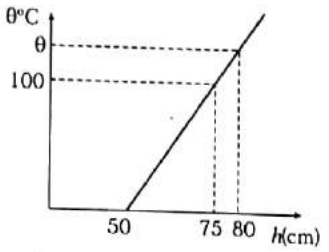
1. (*b*) Pyrometer is used to measure very high temperature.
2. (*d*) Zero kelvin = – 273° C (absolute temperature). As no body can attain this temperature, hence temperature can never be negative on kelvin scale.
3. (*a*) $\Delta T_F = \frac{9}{5}\Delta T_C = \frac{9}{5} \times 25°\,C = 45°F$
4. (*c*) Since, $\frac{F-32}{9} = \frac{K-273}{5} \Rightarrow \frac{F-32}{9} = \frac{0-273}{5}$

   $\Rightarrow \quad F = -459.4°F \approx -460°F$
5. (*a*) On this thermometer 34°C will read as,

   $$-20 + \frac{150}{100} \times 34 = 31°$$
6. (*a*) Let the temperature is θ

   $\therefore \quad \frac{\theta - 0}{100} = \frac{\theta - 32}{180} \Rightarrow \theta = -40°$
7. (*c*) Temperature shown on faulty thermometer will be

   $$t = 5 + \left(\frac{95-5}{100}\right) \times 40 = 41°$$
8. (*c*) $\frac{F-32}{9} = \frac{K-273}{5} \Rightarrow \frac{x-32}{9} = \frac{x-273}{5} \Rightarrow x = 574.25$
9. (*d*) $\frac{\theta}{80-50} = \frac{100}{75-50}$



   $\therefore \quad \theta = 120°\,C$
10. (*c*) Given, $\frac{T_C}{T_F} = \frac{2}{5} \Rightarrow T_F = \frac{5}{2}T_C$

    $\therefore \quad \frac{T_C}{100} = \frac{(5/2)T_C - 32}{180} \Rightarrow 9T_C = \frac{25}{2}T_C - 160$

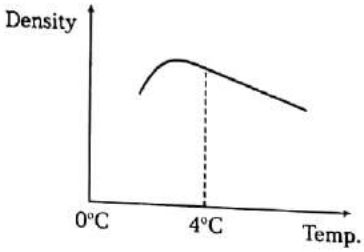
    $\Rightarrow \quad T_C = 45.71°\,C$

Check point 14.2

1. (*a*) Volumetric expansion takes place. So, if the ball is heated, then the volume of the cavity will increase.
3. (*d*) When bimetallic strip is heated, the brass expands more than the steel and the strip curves with the brass on the outside. If the strip is cooled, it curves with the steel on outside and can be used in thermostat, as a switch or in metal thermometers.
4. (*a*) $T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow T \propto \sqrt{l}$

   $\therefore \quad \frac{T'}{T} = \left(\frac{l'}{l}\right)^2 = 1 + \frac{1}{2}\frac{\Delta l}{l} = 1 + \frac{1}{2}\alpha\Delta\theta$

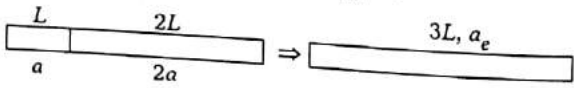
   $\therefore \quad \Delta T = T' - T = \frac{1}{2}\alpha T\Delta\theta$
5. (*b*) Coefficient of volume expansion of container will become $3\left(\frac{\alpha}{2}\right)$, which is greater than coefficient of volume expansion of liquid. Hence; container expands more.
6. (*a*) The hole will increase in size.
7. (*c*) Water has maximum density at 4°C so, if the water is heated above 4°C or cooled below 4°C density decreases *i.e.*, volume increases. In other words, it expands so it overflows in both the cases.



8. (*c*) Change is length, $\Delta l = l_0\,\alpha\,\Delta T = 10 \times 11 \times 10^{-6}(19-20)$

   $= -11 \times 10^{-5}$ cm
10. (*d*) $\Delta l_1 = \Delta l_2$ or $l_1\alpha_1\Delta\theta = l_2\alpha_2\Delta\theta$ or $\frac{l_1}{l_2} = \frac{\alpha_2}{\alpha_1}$
11. (*c*) Original value of circumference, $l = 2\pi R$

    $\therefore \quad \Delta l = l\alpha\theta = (2\pi R)\,\alpha\theta$
12. (*c*) $L\alpha\Delta\theta + 2L\,(2\alpha)\,(\Delta\theta) = (3L)\,(\alpha_{av})\,\Delta\theta$



    or $\quad \alpha_{av} = \frac{5}{3}\alpha$
13. (*a*) As, $\Delta L = L\,\alpha\,\Delta T \Rightarrow 0.19 = 100 \times \alpha \times 100$

    $\Rightarrow \quad \alpha = 0.19 \times 10^{-4} = 1.9 \times 10^{-5}\,°C^{-1}$

    $\Rightarrow \quad \gamma = 3\alpha = 5.7 \times 10^{-5}/°C$

14. (d) Fractional change in period,

$$\frac{\Delta T}{T} = \frac{1}{2}\alpha\Delta\theta = \frac{1}{2}\times 2\times 10^{-6}\times 10 = 10^{-5}$$

$$\% \text{ change} = \frac{\Delta T}{T}\times 100 = 10^{-5}\times 100 = 10^{-3}\%$$

15. (c) Strain $= \frac{\Delta l}{l} = \frac{l\alpha\Delta\theta}{l} = \alpha\Delta\theta$

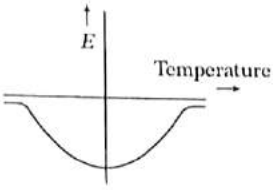
Stress $= Y \times$ Strain $= Y\alpha\Delta\theta$

$$(\text{Stress})_1 = (\text{Stress})_2$$

$\therefore$ $Y_1\alpha_1 = Y_2\alpha_2$ $\quad(\Delta\theta \rightarrow \text{same})$

or $\frac{Y_1}{Y_2} = \frac{\alpha_2}{\alpha_1} = \frac{3}{2}$

18. (d) Potential energy curve of atoms is shown in figure. It is assymetric so when temperature increases vibration about mean position becomes longer in one side so average distance increases between the atoms.



19. (c) $V = V_0(1+\gamma\Delta\theta)$

$$L^3 = L_0(1+\alpha_1\Delta\theta)\,L_0^2(1+\alpha_2\Delta\theta)^2$$
$$= L_0^3(1+\alpha_1\Delta\theta)(1+\alpha_2\Delta\theta)^2$$

Since $L_0^3 = V_0$ and $L^3 = V$,

Hence $1+\gamma\Delta\theta = (1+\alpha_1\Delta\theta)(1+\alpha_2\Delta\theta)^2$

$$\cong (1+\alpha_1\Delta\theta)(1+2\alpha_2\Delta\theta)$$
$$\cong (1+\alpha_1\Delta\theta+2\alpha_2\Delta\theta)$$

$\Rightarrow$ $\gamma = \alpha_1 + 2\alpha_2$

20. (a) Strain $= \frac{\Delta l}{l} = \alpha\Delta\theta$

Stress $= Y \times$ strain $= Y\alpha\Delta\theta$

$\therefore$ Force or tension, $T =$ Stress $\times$ area $= YA\alpha\Delta\theta$

$$= \frac{\pi Y\alpha d^2\Delta\theta}{4}\left(A = \frac{\pi d^2}{4}\right)$$

or $T = \frac{\pi\times 2\times 10^{11}\times 10^{-5}\times 10^{-4}\times 25}{4}$

$$= 3926\text{ N} \approx 4000\text{ N}$$

Check point 14.3

1. (d) All molecules have same speed. This statement is true but this is not an assumption of kinetic theory of gases.

2. (b) Total pressure inside the container is given as

$$p = \frac{1}{3}\frac{mN}{V}v^2_{rms} \Rightarrow p \propto m \quad \text{...(i)}$$

(when, volume and temperature are constant)

From Eq. (i),

Pressure increases first and then become constant.

3. (b) Average speed of molecule is the arithmetic mean of the speed of molecules in a gas at given temperature is

$$v_{av} = \frac{v_1+v_2+v_3+\ldots}{N}$$

From kinetic theory of gases,

$$v_{av} = \sqrt{\frac{8p}{\pi\rho}} = \sqrt{\frac{8}{\pi}\frac{RT}{M}} = \sqrt{\frac{8}{\pi}\frac{KT}{M}}$$

4. (c) $v_{rms} \propto \sqrt{\frac{3RT}{M}}$, $v_{av} = \sqrt{\frac{8RT}{\pi M}}$ and $v_p = \sqrt{\frac{2RT}{M}}$

$\therefore$ $v_{rms} > v > v_p$

5. (d) Average kinetic energy, $E = \frac{3}{2}kT \Rightarrow E \propto T$

6. (b) The pressure exerted by the gas,

$$p = \frac{1}{3}\rho\bar{v}^2 = \frac{1}{3}\frac{m}{V}\bar{v}^2 = \frac{2}{3}\left(\frac{1}{2}m\bar{v}^2\right)$$

$\left(\because \frac{1}{2}m\bar{v}^2 = \frac{E}{V} = \text{energy per unit volume, i.e. } V = 1\right)$

$\Rightarrow$ $p = \frac{2}{3}E$

7. (a) At a given temperature (T) all the ideal gas molecules no matter what their masses have the same average translational kinetic energy.

i.e., $E = \frac{3}{2}kT$

So, E does not depend upon density $\quad$ (as mass $= \rho V$)

$$\frac{E_1}{E_2} = 1:1$$

8. (d) As, $pV = nRT$

$\Rightarrow$ $p_{H_2} = \frac{m}{M_{H_2}}\cdot\frac{RT}{V}$ and $p_{He} = \frac{m}{M_{He}}\cdot\frac{RT}{V}$

$\therefore$ $\frac{p_{H_2}}{p_{He}} = \frac{M_{He}}{M_{H_2}} = \frac{4\times 10^{-3}}{2\times 10^{-3}} = 2 \Rightarrow p_{H_2} = 2p_{He}$

9. (c) $pV = \mu RT \Rightarrow p \propto \mu T$ $\quad(\because V$ and $R =$ constant)

$\Rightarrow$ $\frac{p_2}{p_1} = \frac{\mu_2}{\mu_1}\times\frac{T_2}{T_1} \Rightarrow \frac{p_{He}}{p} = \frac{1}{1}\times\frac{2T}{T} \Rightarrow p_{He} = 2p$

10. (d) $v_{rms} \propto \sqrt{T}$, v_{rms} will become two times when temperature is made four times.

11. (a) $T_1 = 273 + 27 = 300$ K $\Rightarrow T_2 = 273 + 327 = 600$ K

$$T_2 = 2T_1 \text{ and } v_{rms} \propto \sqrt{T}$$

$$\frac{v'_{rms}}{v''_{rms}} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{T_1}{2T_1}}$$

$\Rightarrow$ $\frac{v'_{rms}}{v''_{rms}} = \frac{1}{\sqrt{2}} \Rightarrow v''_{rms} = \sqrt{2}v'_{rms}$

12. (a) $v_{rms} = \sqrt{\frac{(3v)^2+(4v)^2+(5v)^2}{3}} = \sqrt{\frac{50}{3}}v$

13. (b) $v_{rms} \propto \sqrt{T}$

rms speed will remain half if temperature become (1/4) th or $\left(\frac{273}{4} - 273\right)°$C or $-204.75°$C

14. (d) $v_{rms} = \sqrt{\frac{(1)^2+(2)^2+(3)^2+(4)^2}{4}} = \sqrt{\frac{15}{2}}$ kms^{-1}

15. (*c*) As, kinetic energy of gas is given as

$$K = \frac{1}{2} m v_{rms}^2$$

Also, $$v_{rms} = \sqrt{\frac{3KT}{M}}$$

Now, $$v_{rms} \propto \sqrt{T} \Rightarrow \frac{K_2}{K_1} = \left[\frac{(v_{rms})_2}{(v_{rms})_1}\right]^2 = \left[\frac{\sqrt{T_2}}{\sqrt{T_1}}\right]^2$$

$$\Rightarrow \quad K_2 = K_1\left(\frac{T_2}{T_1}\right) \Rightarrow K_2 = K_1\left(\frac{600}{300}\right)$$

$$\therefore \quad K_2 = 2K_1$$

16. (*a*) The gases carbon monoxide (CO) and nitrogen (N_2) have equal translational kinetic energy $\frac{3}{2}kT$, *i.e.*, $E_1 = E_2$.

17. (*c*) Kinetic energy of ideal gas depends only on its temperature. Hence, it remains constant whether its pressure is increased or decreased.

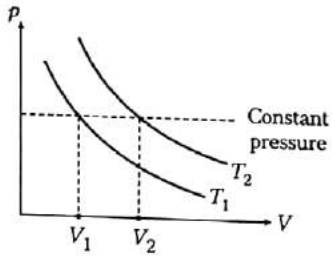
19. (*c*) $v_{rms} \propto \sqrt{\frac{T}{M}}$

T is doubled and M has become half. Therefore v_{rms} will become two times.

20. (*c*) $v_{rms} \propto \sqrt{T}$. If temperature is doubled, rms speed will become $\sqrt{2}$ times is $v_{rms} = 200\sqrt{2}\ ms^{-1}$

Check point 14.4

1. (*c*) For a given pressure, volume will be more if temperature is more (Charle's law)



From the graph, it is clear that $V_2 > V_1 \Rightarrow T_2 > T_1$

2. (*c*) In $pV = nRT$, $pV = RT$ if $n = 1$

So, in gas equation $pV = RT$, volume V refers to one gram mole of gas.

3. (*b*) $f = 3N - k$

For monoatomic gas,

$$N = 1, k = 0$$

$$\therefore \quad f = 3 \times 1 - 0 = 3$$

where, N = number of particles in the system

K = number of independent relations between the particles.

4. (*b*) Translational degree of freedom for any gas is three.

5. (*c*) $f = 3N - K$

For non-linear triatomic gas, $f = 3 \times 3 - 3 = 6$

6. (*d*) The mean kinetic energy per molecule per degree of freedom is $(1/2)\,k_B T$. For one mole of gas

$$U_{av} = \frac{1}{2} k_B N_A T = \frac{1}{2} RT \qquad [\because k_B N_A = R]$$

7. (*c*) Average translational kinetic energy per molecule is $3/2\,kT$ which depends only on temperature and not on molecular mass.

8. (*c*) $V \propto T \Rightarrow \frac{V_1}{V_2} = \frac{T_1}{T_2} \Rightarrow \frac{V}{3V} = \frac{(273+27)}{T_2}$

$$\Rightarrow \quad T_2 = 900\ K = 627°\ C$$

9. (*c*) $$p = p_1 + p_2 + p_3 = \left(\frac{nRT}{V}\right)_1 + \left(\frac{nRT}{V}\right)_2 + \left(\frac{nRT}{V}\right)_3$$

$$= (n_1 + n_2 + n_3)\frac{RT}{V}$$

$$= \frac{(0.5 + 0.5 + 0.25)(8.31)(300)}{5 \times 10^{-3}}$$

$$= 6.23 \times 10^5\ Nm^{-2}$$

10. (*b*) As, $\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$ or $V_2 = \frac{p_1V_1T_2}{p_2T_1}$

$\therefore$ Volume of He, $V_2 = \frac{1 \times 500 \times (273-3)}{0.5 \times (273+27)}$

$$= \frac{1 \times 500 \times 270}{0.5 \times 300} = 900\ m^3$$

11. (*d*) We know for perfect gas, $V \propto T$

Here $$\frac{V_1}{V_2} = \frac{T_1}{T_2} \qquad ...(i)$$

According to the question,

$V_1 = V$, then $V_2 = 2V$ and $T_1 = 300$ K

$$\therefore \quad \frac{1}{2} = \frac{300}{T_2} \Rightarrow T_2 = 600\ K \Rightarrow T_2 = 327°\ C$$

So, $\Delta t = 327 - 27 = 300°C$

12. (*a*) As, $pV = nRT = \frac{m}{M}RT$

where, M = molecular weight

$$\Rightarrow \quad \frac{p}{m/V} = \frac{RT}{M} \Rightarrow \frac{p}{\rho} = \frac{RT}{M} \Rightarrow \frac{p}{\rho} \propto T$$

($\because$ for a given gas M is constant)

Temperature is directly proportional to the slope of p-ρ graph. So, $T_1 > T_2$.

13. (*c*) As $\theta_2 > \theta_1 \Rightarrow \tan\theta_2 > \tan\theta_1$

$$\Rightarrow \quad \left(\frac{T}{p}\right)_2 > \left(\frac{T}{p}\right)_1 \Rightarrow \frac{T}{p} \propto V \Rightarrow V_2 > V_1$$

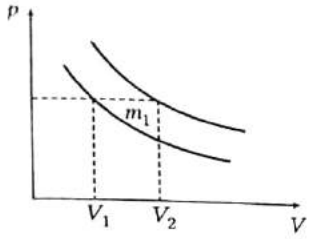
14. (*b*) $\frac{V}{T} = \frac{nR}{p}$

V and T for both cases in same. Hence

$$\frac{n_1}{p_1} = \frac{n_2}{p_2} \text{ or } \frac{m_1}{p_1M_1} = \frac{m_2}{p_2M_2}$$

or $$m_2 = \frac{p_2M_2}{p_1M_1} \cdot m_1 = \frac{(3)(2)}{(5)(28)} \cdot 20 = 0.86\ kg$$

15. (c) $pV = \mu RT = \frac{m}{M}RT$



For 1st graph, $p = \frac{m_1}{M}\frac{RT}{V_1}$ ...(i)

For 2nd graph, $p = \frac{m_2}{M}\frac{RT}{V_2}$ ...(ii)

From Eqs. (i) and (ii), we get

$$\frac{m_1}{m_2} = \frac{V_1}{V_2} \Rightarrow m \propto V$$

As $V_2 > V_1 \Rightarrow m_1 < m_2$

16. (d) p^2V = constant

$$p = \frac{nRT}{V} \text{ or } p = \frac{CT}{V} \quad (C = \text{constant})$$

$$\therefore \quad \frac{T^2}{V} = \text{constant or } T \propto \sqrt{V}$$

V is increased to 3 times. Hence T will increase $\sqrt{3}$ times.

17. (c) Translational degree of freedom for any type of gas is three.

$\therefore$ Total translational kinetic energy $= 3\left(\frac{1}{2}nRT\right) = \frac{3}{2}pV$

18. (a) Both O_2 and N_2 are diatomic gases. Rotational degree of freedom in both cases is two. Therefore, average rotational kinetic energy per molecule for each of them is $2\left(\frac{1}{2}kT\right)$ or the ratio is 1 : 1.

19. (b) In monoatomic gases, only translational kinetic energy is present, which is proportional to temperature given by $(3/2)k_BT$.

20. (c) $E = \frac{3}{2}kT \Rightarrow E \propto T$

$$\therefore \quad \frac{E_2}{E_1} = \frac{T_2}{T_1} = \frac{600}{300} = 2 \Rightarrow E_2 = 2E_1$$

21. (c) According to Boyle's law, $p_1V_1 = p_2V_2$

As, the pressure is decreased by 20%, therefore

$$p_2 = \frac{80}{100}p_1 \Rightarrow p_1V_1 = \frac{80}{100}p_1V_2 \Rightarrow V_1 = \frac{80}{100}V_2$$

$\therefore$ Percentage increase in volume

$$= \frac{V_2 - V_1}{V_1} \times 100 = \frac{100 - 80}{80} \times 100 = 25\%$$

22. (b) By Charles' law, $\frac{V_1}{V_2} = \frac{T_1}{T_2}$

So, $V_2 = \frac{V_1T_2}{T_1}$

$$\therefore \quad V_2 = 1 \times \frac{(297 + 273)}{(27 + 273)} = \frac{570}{300} = 1.9 \text{ L}$$

23. (c) V-T graph is a straight line passing through origin. Hence process is isobaric.

$$V = \left(\frac{nR}{p}\right)T \Rightarrow \text{Slope} = \frac{nR}{p}$$

Slope of 2 > slope of 1

$\therefore \quad p_2 < p_1$

24. (c) As, $pV = \mu RT = \frac{m}{M}RT \Rightarrow 20 \times V = \frac{m}{M} \times R \times 300$ ...(i)

When $m = m/2$ and $p = p'$, then

As, $p' \times V = \left(\frac{m/2}{M}\right)R \times 350$ ...(ii)

From Eqs. (i) and (ii), we get

$$p' = \frac{140}{12} \approx 11.7 \text{ atm}$$

25. (c) Mean kinetic energy of gas molecules

$$E = \frac{f}{2}kT = \frac{f}{2}k(t + 273) = \left(\frac{f}{2}k\right)t + \frac{f}{2} \times 273k$$

Comparing it with standard equation of straight line $y = mx + c$. We get $m = \frac{f}{2}k$ and $c = \frac{f}{2}273k$. So, the graph between E and t will be straight line with positive intercept on E-axis and positive slope with t-axis.

A Taking it together

1. (a) We know that as temperature increases vibration of molecules about their mean position increases hence, kinetic associated with random motion of molecules increases.

2. (b) Boyle's law is applicable when temperature is constant

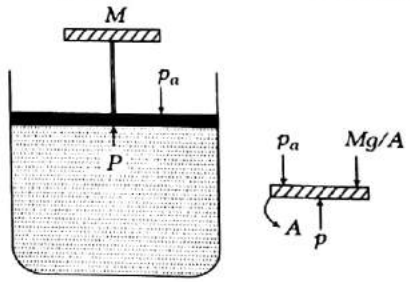
i.e., $pV = nRT$ = constant

$\Rightarrow$ pV = constant (at constant temperature)

i.e., $p \propto \frac{1}{V}$ [where, p = pressure, V = volume]

So, this process can be called as isothermal process.

3. (c) Consider the diagram where an ideal gas is contained in a cylinder, having a piston of mass M. Friction is absent.



The pressure inside the gas will be

$$p = p_a + Mg/A$$

where, p_a = atmospheric pressure

A = area of cross-section of the piston.

Mg = weight of piston

Hence, p = constant

When temperature increases

as $pV = nRT \Rightarrow$ Volume (V) increases at constant pressure.

4. (*c*) $\frac{F-32}{9} = \frac{K-273}{5} \Rightarrow \frac{F-32}{9} = \frac{0-273}{5}$

$$F = -459.4°F \approx -460°F$$

5. (*d*) Zero kelvin = – 273° C (absolute temperature). As no matter can attain this temperature, hence temperature can never be negative on kelvin scale.

6. (*d*) Using $\frac{T_c - 0}{100-0} = \frac{T-10}{130-10}$

$$\frac{40-0}{100-0} = \frac{T-10}{130-10}$$

$$\frac{40}{100} = \frac{T-10}{120} \text{ or } \frac{2}{5} = \frac{T-10}{120}$$

$$5T - 50 = 240 \Rightarrow 5T = 290$$

$$\Rightarrow \quad T = \frac{290}{5} = 58°$$

7. (*d*) $\frac{X - LFP}{UFP - LFP} = \text{constant}$

where, X = Any given temperature on that scale

LFP = Lower fixed point (Freezing point)

UFP = Upper fixed point (Boiling point)

$$\frac{W-39}{239-39} = \frac{39-0}{100-0} \Rightarrow \frac{W-39}{200} = \frac{39}{100}$$

$$\Rightarrow \quad W = 78 + 39 \Rightarrow W = 117°W$$

8. (*b*) $\frac{\Delta T_C}{100} = \frac{\Delta T_F}{180} = \frac{212-140}{180}$

i.e., $\Delta T_C = 100 \times \frac{72}{180} = 40°\,C$

$\therefore$ Fall in temperature = 40°.

9. (*a*) $t = \frac{(P_t - P_0)}{(P_{100} - P_0)} \times 100°\,C = \frac{(60-50)}{(90-50)} = 25°\,C$

10. (*a*) $\alpha = \frac{\Delta L}{L_0(\Delta\theta)} = \frac{0.19}{100(100-0)} = 1.9 \times 10^{-5}/°C$

Now, $\gamma = 3\alpha = 3 \times 1.9 \times 10^{-5}/°C = 5.7 \times 10^{-5}/°C$

11. (*d*) $\frac{V_1}{V_2} = \frac{1+\gamma t_1}{1+\gamma t_2} \Rightarrow \frac{100}{125} = \frac{1+\gamma \times 20}{1+\gamma \times 100}$

$\Rightarrow \gamma = 0.0033/°C$

12. (*a*) $\rho = \rho_0(1 - \gamma \cdot \Delta\theta) = 13.6[1 - 0.18 \times 10^{-3}(473-273)]$

$= 13.6[1 - 0.036] = 13.11 \text{gm/cc}$

13. (*a*) Number of seconds lost in a day

$$\Delta t = \frac{1}{2}\alpha\Delta\theta \times 86400$$

The coefficient of linear expansion of metal pendulum

$$\alpha = \frac{2\Delta t}{\Delta\theta \times 86400} = \frac{2 \times 12.5}{25 \times 86400} \Rightarrow \alpha = \frac{1}{86400}/°C$$

14. (*b*) Relation between celsius and Fahrenheit scale of temperature is $\frac{C}{5} = \frac{F-32}{9}$

$$\Rightarrow \quad C = \frac{5}{9}F - \frac{160}{9}$$

Equating above equation with standard equation of the line $y = mx + c$ we get slope of the line AB is $m = \frac{5}{9}$.

15. (*b*) Speed of sound in gases is given by

$$v_{\text{sound}} = \sqrt{\frac{\gamma p}{\rho}} \Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{\rho_2}{\rho_1}} = \sqrt{\frac{d_2}{d_1}}$$

16. (*a*) $p \propto T \Rightarrow \frac{p_1}{p_2} = \frac{T_1}{T_2} \Rightarrow \frac{p_2 - p_1}{p_1} = \frac{T_2 - T_1}{T_1}$

$$\Rightarrow \quad \left(\frac{\Delta p}{p}\right)\% = \left(\frac{251-250}{250}\right) \times 100 = 0.4\%$$

17. (*c*) Gas equation for N molecules $pV = NkT$

$$\Rightarrow \quad N = \frac{pV}{kT} = \frac{1.2 \times 10^{-10} \times 13.6 \times 10^3 \times 10 \times 10^{-4}}{1.38 \times 10^{-23} \times 300}$$

$$= 3.86 \times 10^{11}$$

18. (*a*) $v_{rms} = \sqrt{\frac{3RT}{M}} \Rightarrow T \propto M$ $\quad (\because v_{rms}, R \rightarrow \text{constant})$

$$\Rightarrow \quad \frac{T_{H_2}}{T_{O_2}} = \frac{M_{H_2}}{M_{O_2}} \Rightarrow \frac{T_{H_2}}{(273+47)} = \frac{2}{32} \Rightarrow T_{H_2} = 20\text{ K}$$

19. (*c*) From ideal gas equation, $pV = RT$ ...(i)

or $p\Delta V = R\Delta T$ ...(ii)

Dividing Eqs. (ii) by (i), we get

$$\frac{\Delta V}{V} = \frac{\Delta T}{T} \Rightarrow \frac{\Delta V}{V\Delta T} = \frac{1}{T} = \delta \quad \text{(given)}$$

$\therefore \delta = \frac{1}{T}$. So, the graph between δ and T will be rectangular hyperbola.

20. (*d*) $U \propto \frac{1}{\rho}$ or $T \propto \frac{1}{(1/V)} \Rightarrow T \propto V$

Hence p = constant

Further ρ is increasing. Therefore V should decrease.

21. (*a*) $pV = nRT$

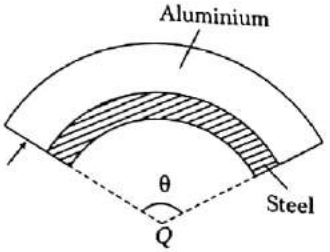
$$n = \frac{pV}{RT} \text{ or}$$

$$N = n \times 6.02 \times 10^{23} = \frac{pV}{RT} \times 6.02 \times 10^{23}$$

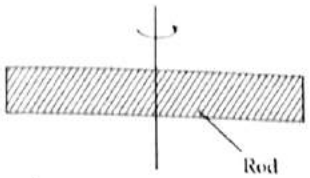
$$\text{or} \quad N = \frac{(10^{-14} \times 13.6 \times 10^3 \times 10)(1)}{(8.31)(273+30)} \times 6.02 \times 10^{23}$$

$$= 3.2 \times 10^{11}$$

22. (*d*) As $\alpha_{Al} > \alpha_{steel}$, aluminium will expand more. So, it should have larger radius of curvature. Hence, aluminium will be on convex side.



23. (*b*) As the rod is heated, it expands. No external torque is acting on the system so angular momentum should be conserved.



L = Angular momentum

$= I\omega$ = constant

$\Rightarrow \quad I_1\omega_1 = I_2\omega_2$

Due to expansion of the rod $I_2 > I_1$

$$\Rightarrow \quad \frac{\omega_2}{\omega_1} = \frac{I_1}{I_2} < 1 \Rightarrow \omega_2 < \omega_1$$

So, angular velocity (speed of rotation) decreases.

24. (*a*) We know for an ideal gas,

$$pV = nRT \Rightarrow V = \left(\frac{nR}{p}\right)T$$

Slope of the V-T graph, $m = \frac{dV}{dT} = \frac{nR}{p}$ [m = slope of V-t graph]

$$\Rightarrow \quad m \propto \frac{1}{p} \qquad [\because nR = \text{constant}]$$

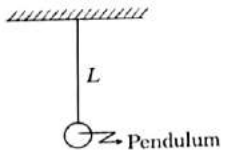
$$\Rightarrow \quad p \propto \frac{1}{m}$$

Hence, $\frac{p_1}{p_2} = \frac{m_2}{m_1} < 1$ $\left[\begin{array}{l} p = \text{pressure}, V = \text{volume} \\ n = \text{number of moles of gases} \\ R = \text{gas constant} \\ T = \text{temperature} \end{array}\right]$

where, m_1 is slope of the graph corresponding to p_1 and similarly m_2 is slope corresponding to p_2.

$$\Rightarrow \quad p_2 < p_1 \text{ or } p_1 > p_2$$

25. (*a*) As the temperature is increased length of the pendulum increases. We know that time period of pendulum



$$T = 2\pi\sqrt{\frac{L}{g}} \Rightarrow T \propto \sqrt{L}, \text{ as } L, \text{ increases}$$

So, time period (T) also increases.

26. (*a*) Let volume of the sphere is V and ρ is its density, then we can write buoyant force

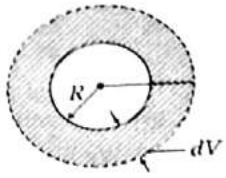
$F = V\rho G$ (g = acceleration due to gravity)

$\Rightarrow \quad F \propto \rho$ ($\because V$ and g are almost constant)

$\Rightarrow \quad \frac{F_{4°C}}{F_{0°C}} = \frac{\rho_{4°C}}{\rho_{0°C}} > 1$ ($\because \rho_{4°C} > \rho_{0°C}$)

Hence, buoyancy will be less in water at 0° C than that in water at 4° C.

27. (*d*) Let the radius of the sphere is R. As the temperature increases radius of the sphere increases as shown.



Original volume, $V_0 = \frac{4}{3}\pi R^3$

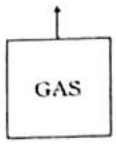
Coefficient of linear expansion = α

$\therefore$ Coefficient of volume expansion = 3α

$$\therefore \quad \frac{1}{V}\frac{dV}{dT} = 3\alpha \Rightarrow dV = 3V\alpha dT \approx 4\pi R^3\alpha\Delta T$$

= Increase in the volume

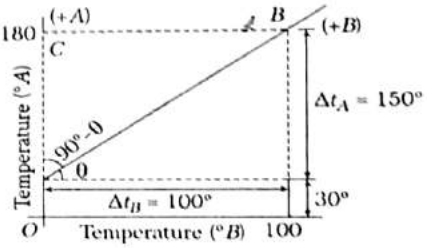
28. (*b*) As the motion of the vessel as a whole does not effect the relative motion of the gas molecules with respect to the walls of the vessel, hence pressure of the gas inside the vessel, as observed by us, onthe ground remains the same.



$v = 500 \text{ ms}^{-1}$

29. (*b*) It is clear from the graph that lowest point for scale A is 30° and lowest point for scale B is 0°. Highest point for the scale A is 180° and for scale B is 100°. Hence, correct relation is

$$\frac{t_A - (\text{LFP})_A}{(\text{UFP})_A - (\text{LFP})_A} = \frac{t_B - (\text{LFP})_B}{(\text{UFP})_B - (\text{LFP})_B}$$



$$\Rightarrow \quad \frac{t_A - 30}{180 - 30} = \frac{t_B - 0}{100 - 0} \Rightarrow \frac{t_A - 30}{150} = \frac{t_B}{100}$$

where, LFP → Lower fixed point

UFP → Upper fixed point

30. (*d*) Consider the diagram, when the molecules breaks into atoms, the number of moles would become twice.

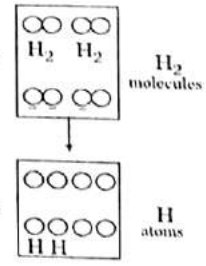
Now, by ideal gas equation

p = pressure of gas, n = number of moles

R = gas constant, T = temperature

$pV = nRT$

As volume (V) of the container is constant,



As gases breaks number of moles becomes twice of initial, so $n_2 = 2n_1$ So, $p \propto nT$

$$\Rightarrow \quad \frac{p_2}{p_1} = \frac{n_2T_2}{n_1T_1} = \frac{(2n_1)(3000)}{n_1(300)} = 20 \Rightarrow p_2 = 20\,p_1$$

Hence, final pressure of the gas would be 20 times the pressure initially.

31. (*a*) We know for an ideal gas, $pV = nRT$ (Ideal gas equation)

$\Rightarrow$ n = number of moles, p = pressure, V = volume
R = gas constant, T = temperature

$$= \frac{pV}{RT}$$

As number of moles of the gas remains fixed, hence we can write

$$\frac{p_1V_1}{RT_1} = \frac{p_2V_2}{RT_2} \Rightarrow p_2 = (p_1V_1)\left(\frac{T_2}{V_2T_1}\right)$$

$$= \frac{(p)(V)(11T)}{(105)V(T)} \qquad \begin{bmatrix} p_1 = p \\ V_2 = 105 \text{ and } T_2 = 11T \end{bmatrix}$$

$$= p \times \left(\frac{1.1}{1.05}\right) = p(1.0476) = 1.05\,p$$

Hence, final pressure p_2 lies between p and $1.1\,p$

32. (*b*) Volume of the liquid overflow

= Increase in the volume of the liquid
− Increase in the volume of the container

$$= [V_0(1+\gamma_l\Delta T) - V_0] - [V_0(1+\gamma_g\Delta T) - V_0]$$

$$= V_0\Delta T(\gamma_l - \gamma_g) = V_0\Delta T(\gamma_l - 3\alpha_g) \qquad (\because \gamma_g \approx 3\alpha_g)$$

33. (*a*) Let the relation between the thermometer reading and centigrade be $y = ax + b$.

Given, at $x = 100$, $y = 80$ and at $x = 0$, $y = 10$

$\therefore$ $80 = 100a + b$, $10 = b \Rightarrow a = 0.7$

Now, we have to find x when $y = 59$

$\therefore$ $59 = 0.7x + b \Rightarrow x = 70$

$\therefore$ The answer is 70° C.

34. (*c*) Let the original temperature be 0°C;

Volume of $A = l \times \pi(2r)^2 = V_1$; $V_1' = V_1(1+\gamma\Delta T)$

$$\frac{(V_1' - V_1)}{V_1} = \gamma\Delta T \Rightarrow V_1' - V_1 \propto V_1$$

$$\frac{(V_2' - V_2)}{V_2} = \gamma\Delta T \Rightarrow V_2' - V_2 \propto V_2$$

$$\therefore \quad \frac{\Delta V_1}{\Delta V_2} = \frac{l(2r)^2}{2lr^2} = \frac{2}{1}$$

35. (*c*) $V = \left(\frac{nR}{p}\right)T = \left(\frac{mR}{Mp}\right)T$

Slope of V-T graph $\propto \frac{m}{p}$

Since $\frac{m}{p}$ ratio in both cases is same.

Straight line will be B.

36. (*a*) $v_{rms} = \sqrt{\frac{3RT}{M}} \Rightarrow v_{rms} \propto \sqrt{\frac{T}{M}}$

$$\frac{v_{He}}{V_{H_2}} = \frac{5}{7} = \sqrt{\frac{T_{He}}{M_{He}} \times \frac{M_{H_2}}{T_{H_2}}} \Rightarrow T_{He} = \frac{25}{49} \times \frac{4}{2} \times 273 \approx 0°\,C$$

37. (*a*) $v_{rms} = \sqrt{\frac{3RT}{M}} \Rightarrow M = \frac{3RT}{v_{rms}^2}$

$$\therefore \quad M = \frac{3 \times 8.3 \times 300}{(1920)^2} = 2 \times 10^{-3}\,\text{kg} = 2\,\text{g}$$

$\Rightarrow$ Gas is hydrogen.

38. (*d*) Pressure on both sides should be same or,

$$\frac{n_1RT}{V_1} = \frac{n_2RT}{V_2} \quad \text{or} \quad \frac{m}{32(360° - \alpha)} = \frac{m}{28\,\alpha}$$

or $\alpha = 192°$

39. (*c*) $pV = \frac{m}{M}RT \Rightarrow V \propto mT \Rightarrow \frac{V_1}{V_2} = \frac{m_1}{m_2} \cdot \frac{T_1}{T_2}$

$$\Rightarrow \quad \frac{2V}{V} = \frac{m}{m_2} \times \frac{100}{200} \Rightarrow m_2 = \frac{m}{4}.$$

40. (*a*) In an ideal gas, when a molecule collides elastically with a wall, the momentum transferred to each molecule will be twice the magnitude of its normal momentum. For the face *EFGH*, it transfers only half of that on other faces.

41. (*c*) **Process 1-2** V = constant, $p \propto T$

T is increasing. Therefore pressure will be increase.

Process 2-3 p = constant, $V \propto T$

V and T both are increasing.

Process 3-4 T = constant, $p \propto \frac{1}{V}$

V is increasing. Therefore p will decrease.

Process 4-1 Inverses of process 2-3.

42. (*a*) ***AB*** Isobaric process. Both volume and temperature are increasing.

BC Isothermal process. Volume is increasing. So, pressure should decrease. (T = constant, $p \propto \frac{1}{V}$)

CD Isochoric process. Pressure and temperature both are decreasing. (V = constant, $p \propto T$)

DA Again isothermal process. Volume is deceasing and pressure is increasing (T = constant, $p \propto \frac{1}{V}$)

43. (*b*) Number of degrees of freedom per molecule for $A = 3$.

Number of degrees of freedom per molecule for $B = 5$. Since, the mixture is equimolar, the average kinetic energy per molecule will be the simple average of the two values *i.e.*, $\left(\frac{3+5}{2}\right)kT = 4kT$ where, k is Boltzmann's constant.

44. (*c*) p-T equation from the graph should be like.

$p = aT + b$ (a and b are positive constants)

$$\therefore \quad \frac{p}{T} = a + \frac{b}{T} \Rightarrow T_2 > T_1$$

$$\therefore \quad \left(\frac{p}{T}\right)_2 < \left(\frac{p}{T}\right)_1 \quad \text{or} \quad \left(\frac{T}{p}\right)_2 > \left(\frac{T}{p}\right)_1$$

or $V_2 > V_1$ as $V \propto \frac{T}{p}$

45. (*b*) The sphere will sink in the liquid at 35°C, when its density becomes equal to the density of liquid at 35°C. The density of sphere,

$$\rho_{35} = \frac{266.5}{\frac{4}{3}\times\left(\frac{22}{7}\right)\times\left(\frac{7}{2}\right)^3} = 1.483\,\text{g cm}^{-3}$$

Now, $\rho_0 = \rho_{35}[1+\gamma\Delta T] \Rightarrow 1.527 = 1.483[1+\gamma\times 35]$

$$1.029 = 1+\gamma\times 35$$

$$\gamma = \frac{1029-1}{35} = 0.00083/^\circ\text{C}$$

46. (*a*) From ideal gas equation $pV = \mu RT$

$\Rightarrow$ Slope of p-T curve, $\frac{p}{T} = \frac{\mu R}{V} \Rightarrow$ Slope $\propto \frac{1}{V}$

It means line of smaller slope represent greater volume of gas. In the given problems 1 and 2 are on the same line, so they will represent same volume *i.e.*, $V_1 = V_2$.

Similarly points 3 and 4 are on the same line, so they will represent same volume *i.e.*, $V_3 = V_4$.

Also slope of line 1-2 is less than 3-4.

Hence $(V_1 = V_2) > (V_3 = V_4)$

47. (*d*) Diameter of brass plate = 2.6 cm and diameter of steel plate $= (2.6 - 0.01)\text{cm} = 2.59$ cm

Now, $(d+\Delta d)_b = (d+\Delta d)_s$

or $d_b\,(1+\alpha\Delta\theta)_b = d_s\,(1+\alpha\Delta\theta)_s$

$\therefore$ $2.6\,(1+19\times10^{-6}\,\Delta\theta) = 2.59\,(1+11\times10^{-6}\Delta\theta)$

$\therefore$ $\Delta\theta = -478^\circ\text{C}$ or $\theta_f = -458^\circ\text{C}$

48. (*a*) Slope of line is 1. Therefore T-V equation can be written as

$$T = V + T_0$$

$\therefore$ $\frac{pV}{nR} = V + T_0 \;\therefore\; p = (nR) + \frac{nRT_0}{V}$

$\therefore$ p *versus* V graph is a sort of rectangular hyperbole.

49. (*a*) $F = \frac{\Delta p}{\Delta t} = (nm)\,(2v\cos\theta)$

Pressure, $p = \frac{F}{A} = \frac{(nm)\,(2v\cos\theta)}{A}$

$$= \frac{10^{23}\times 3.32\times10^{-27}\times 2\times 1000\times\cos 45^\circ}{2\times10^{-4}}$$

$$= 2.34\times10^3\ \text{Nm}^{-2}$$

50. (*b*) $\gamma_r = \gamma_a + \gamma_V$, where γ_r = coefficient of real expansion,

γ_a = coefficient of apparent expansion and

γ_V = coefficient of expansion of vessel.

For copper, $\gamma_r = C + 3\alpha_{Cu} = C + 3A$

For silver, $\gamma_r = C + 3\alpha_{Ag} = \frac{C - S + 3A}{3}$

$\Rightarrow$ $C + 3A = S + 3\alpha_{Ag} \Rightarrow \alpha_{Ag} = \frac{C - S + 3A}{3}$

51. (*b*) $(n_1 + n_2)_i = (n_1 + n_2)_f$

$$\frac{p_0V}{RT_0} + \frac{p_0V}{RT_0} = \frac{pV}{RT_0} + \frac{pV}{2RT_0}$$

$\therefore$ $p = (4/3)\,p_0$

52. (*b*) Change in weight = Upthrust (F)

where $F = V_s\rho_l\,g$

$\Rightarrow$ $F' = V_s'\rho_l'\,g$

$$\frac{F'}{F} = \frac{V_s'}{V_s}\cdot\frac{\rho_l'}{\rho_l} \text{ or } \frac{F'}{F} = (1+\gamma_s\,\Delta\theta)\frac{\rho_l'}{\rho_l}$$

$$\left(\frac{46-30.6}{46-30}\right) = (1+3\times\alpha_s\times15)\left(\frac{1.20}{1.24}\right)$$

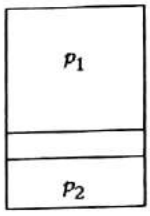
or $\alpha_s = 2.3\times10^{-5}/^\circ\text{C}$

53. (*a*) $(p_2 - p_1)\,A = mg$

or $\frac{mg}{A} = \frac{RT_i}{V_1} - \frac{RT_i}{4V_1} = \frac{3RT_i}{4V_1}$ ...(i)

Similarly in second case,

$$\frac{mg}{A} = \frac{RT_f}{V_2} - \frac{RT_f}{3V_2} = \frac{3RT_f}{3V_2} \quad ...(ii)$$



Further $5V_1 = 4V_2$

Equating Eqs. (i) and (ii), we get

$$\frac{3T_i}{4V_1} = \frac{2T_f}{3V_2}$$

or $T_f = \frac{9}{8}\times\frac{V_2}{V_1}\times T_i$

$$= \frac{9}{8}\times\frac{5}{4}\times 320 = 450\ \text{K}$$

54. (*d*) $(OR)^2 = (PR)^2 - (PO)^2 = l^2 - \left(\frac{l}{2}\right)^2$

$$= [l\,(1+\alpha_2 t)]^2 - \left[\frac{l}{2}\,(1+\alpha_1 t)\right]^2$$

$$l - \frac{l^2}{4} = l^2\,(1+\alpha_2^2 t^2 + 2\alpha_2 t) - \frac{l^2}{4}\,(1+\alpha_1^2 t^2 + 2\alpha_1 t)$$

Neglecting $\alpha_2^2 t^2$ and $\alpha_1^2 t^2$

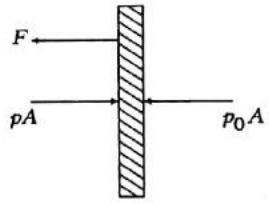
$$0 = l^2\,(2\alpha_2 t^2) - \frac{l^2}{4}\,(2\alpha_1 t)$$

$\Rightarrow$ $2\alpha_2 = \frac{2\alpha_1}{4}$

$\Rightarrow$ $\alpha_1 = 4\alpha_2$

55. (*b*) Volume of the gas is constant

$\therefore$ V = constant



$\therefore$ $p \propto T$ *i.e.*, pressure will be doubled, temperature is doubled

$\therefore$ $p = 2p_0$

Now let F be the tension in the wire. Then equilibrium of any one piston gives.

$$F = (p - p_0)\,A = (2p_0 - p_0)\,A = p_0A$$

B Medical entrance special format question

Assertion and reason

1. (*c*) In 22.4 L at NTP, number of molecules are 6.02×10^{23}.
2. (*b*) In isochoric process, we increase pressure by increasing temperature so assertion is true. Since mass and volume does not change so density is constant.
3. (*a*) At higher temperature natural length is more. It means rod is compressed.
4. (*c*) E represents translational kinetic energy per unit volume.
5. (*a*) At the given conditions, the intermolecular force between the gas molecules is almost zero.
6. (*d*) It will read less than the actual.
7. (*b*) Only translational kinetic energy is $\frac{1}{2} m v_{rms}^2$.

   Further, translational degree of freedom of any type of gas is three. Therefore, translational kinetic energy of any type of gas (of one mole) is $\frac{3}{2} RT$.
8. (*d*) Translational motion of the gas chamber increases the total kinetic energy but it does not contribute in internal energy. Therefore, temperature of gas does not increase by translational motion of the gas chamber.
9. (*c*) In ideal gases, there is no inter-particle interaction. If the gas is monoatomic so no vibrational energy is there.

   $\therefore$ Only the kinetic energy contribute to the internal energy.
10. (*c*) $K_T = \frac{1}{2}(m)v_{rms}^2 = \frac{1}{2} m \left(\frac{3RT}{M}\right) = \frac{3}{2} nRT = \frac{3}{2} pV$

    $\therefore \quad p = \frac{2}{3} \cdot \frac{K_T}{V} = \frac{2}{3}$

    (translational kinetic energy per unit volume).
11. (*b*) Translational degree of freedom of a diatomic gas (O_2) is three and rotational kinetic energy is two. Therefore, according to law of equipartion of energy, $\frac{K_T}{K_R} = \frac{3}{2}$.

    Further, intermolecular force between ideal gas molecules is zero. Hence, potential energy is zero.
12. (*d*) Only that straight line which passes through origin represents isobaric process.
13. (*d*) Only that straight line passing through origin represents isobaric process.
14. (*c*) For water triple point temperature and pressure are 273.16 K and 4.58 mm of mercury (or 6.11×10^{-3} Pa).

Match the columns

1. For monoatomic gas, $\gamma = \frac{5}{3}$

   Speed of sound in a gas

   $$v_{sound} = \sqrt{\frac{\gamma RT}{M}}$$
2. Internal energy of n moles of an ideal gas,

   $$U = n\left(\frac{f}{2} RT\right) \quad (f = \text{degree of freedom})$$

   For monoatomic gas, $f = 3$

   $$\therefore \quad U = \frac{3nRT}{2}$$

   or $U = 3RT$ for $n = 2$ is valid for any gas.
3. $T \propto \frac{T}{V}$ or $T \propto \frac{1}{T/p}$ or $T \propto \sqrt{p}$

   p has become four times. Therefore, T has become two times or V will have become $\frac{1}{2}$ times.

   Since, V is decreasing, work done by the gas is negative. Further, T is increasing, hence ΔU will be positive. W is negative and ΔU is positive. Hence, we cannot say about ΔQ.
4. From the given V-T graph we cannot tell nature of gas

   $$V = \frac{nRT}{p} = \left(\frac{nR}{p}\right) T$$

   If p is constant, V-T graph is a straight line. Slope of this line is $\frac{nR}{p}$.

   $$(\text{Slope})_A > (\text{Slope})_B$$

   or $\frac{n_A}{n_B}$ may be greater than 1 or $\frac{p_A}{p_B}$ may be less than 1.

   But is certain that $\left(\frac{n}{p}\right)_A > \left(\frac{n}{p}\right)_B$.

C Medical entrance gallery

1. (*b*) Pressure, $p = \frac{1}{3} \cdot \frac{nm}{V} v_{rms}^2$

   $\because$ nm = mass of the gas, V = volume of the gas

   $\therefore \frac{mn}{V}$ = density of the gas. Thus,

   $$p = \frac{1}{3} \rho v_{rms}^2 = \frac{1}{3} \rho \frac{3RT}{M_0} = \frac{\rho RT}{M_0} \quad \left(\therefore v_{rms} = \sqrt{\frac{3RT}{M_0}}\right)$$

   $$\rho = \frac{pM_0}{RT} = \frac{pmN_A}{kN_A T} \quad [\because R = N_A k \text{ and } M_0 = mN_A]$$

   $$\rho = \frac{pm}{kT}$$
2. (*c*) According to question,

   Coefficient of linear expression of brass = α_1

   Coefficient of linear expression of steel = α_2

   Length of brass and steel rods are l_1 and l_2 respectively.

   As given difference increase in length $(l_2' - l_1')$ is same for all temperature.

   So, $\quad l_2' - l_1' = l_2 - l_1$

   $\Rightarrow l_2(1 + \alpha_2 \Delta t) - l_1(1 + \alpha_1 \Delta t) = l_2 - l_1 \Rightarrow l_2 \alpha_2 = l_1 \alpha_1$
3. (*a*) It is given that

   $v_{rms} = 200 \text{ ms}^{-1}$, $T_1 = 300$ K, $P_1 = 10^5 \text{ Nm}^{-2}$

   $T_2 = 400$ K, $P_2 = 0.05 \times 10^5 \text{ Nm}^{-2}$

As, rms velocity of gas molecules,

$$\because \quad v_{rms} \propto \sqrt{T} \qquad \left(\because v_{rms} = \sqrt{\frac{3RT}{m}}\right)$$

For two different cases,

$$\Rightarrow \quad \frac{(v_{rms})_1}{(v_{rms})_2} = \sqrt{\frac{T_1}{T_2}} \Rightarrow \frac{200}{(v_{rms})_2} = \sqrt{\frac{300}{400}} = \sqrt{\frac{3}{4}}$$

$$\Rightarrow \quad (v_{rms})_2 = \frac{2}{\sqrt{3}} \times 200 = \frac{400}{\sqrt{3}} \text{ ms}^{-1}$$

4. (*a*) The rms speed is given by

$$v_{rms} = \sqrt{\frac{3RT}{\text{Mol. wt}}} = \sqrt{\frac{3 \times 8.3 \times (273 + 27)}{2 \times 16 \times 10^{-3}}} = \sqrt{\frac{3 \times 8.3 \times 300}{32 \times 10^{-3}}}$$

$$\Rightarrow \quad v_{rms} = 483 \text{ ms}^{-1}$$

5. (*a*) As, kinetic energy (translational)

$$K = \frac{1}{2} M v_{rms}^2 \qquad \text{...(i)}$$

and

$$v_{rms} = \sqrt{\frac{3pV}{M}} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$K = \frac{1}{2} M \frac{(3pV)}{M} = \frac{3}{2} pV$$

$$\Rightarrow \quad p = \frac{2K}{3V} \Rightarrow p \propto K \qquad [\text{if } V = \text{constant}]$$

6. (*d*) Given, the length of rods are L_1 and L_2.

Coefficients of linear expansions are α_1 and α_2

As we know that

$$\Delta L_1 = \alpha_1 L_1 \Delta t \Rightarrow \Delta L_2 = \alpha_2 L_2 \Delta t$$

and for combined rod

$$\Delta L = \alpha (L_1 + L_2) \Delta t$$

Now,

$$\Delta L = \Delta L_1 + \Delta L_2$$

$$\Rightarrow \quad \alpha (L_1 + L_2) \Delta t = \alpha_1 L_1 \Delta t + \alpha_2 L_2 \Delta t$$

$$\Rightarrow \quad \alpha (L_1 + L_2) = \alpha_1 L_1 + \alpha_1 L_2$$

$$\Rightarrow \quad \alpha = \frac{\alpha_1 L_1 + \alpha_2 L_2}{L_1 + L_2}$$

7. (*c*) According to the question,

In the first condition,

$$p = \frac{1}{3} \rho C_{rms}^2 \qquad \text{...(i)}$$

In the second condition,

$$2p = \frac{1}{3} \rho C_{rms}'^2 \qquad \text{...(ii)}$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{C_{rms}^2}{C_{rms}'^2} = \frac{1}{2} \Rightarrow \frac{C_{rms}}{C_{rms}'} = \frac{1}{\sqrt{2}}$$

8. (*c*) As the rms speed is given by

$$v_{rms} = \sqrt{\frac{3RT}{M}} \Rightarrow v_{rms} \propto \sqrt{\frac{T}{M}}$$

When temperature is doubled and molecules dissociates into atoms, then

$$\frac{v_{rms_1}}{v_{rms_2}} = \frac{\sqrt{\frac{T}{M}}}{\sqrt{\frac{2T}{M/2}}} = \frac{\sqrt{\frac{T}{M}}}{\sqrt{\frac{4T}{M}}} = \frac{\sqrt{T}}{\sqrt{4T}} = \frac{1}{2}$$

If v_{rms_1} is v, then v_{rms_2} will be $2v$.

9. (*a*) A gas behaves more closely as an ideal gas at low pressure and high temperature.

10. (*a*) Given, $\rho = 0.09 \text{ kg m}^{-3}$

Pressure, p (at STP) $= 1.01 \times 10^5$ Pa

According to kinetic theory of gas, $p = (1/3) \rho c^2$

$$\Rightarrow \quad C = \sqrt{\frac{3p}{\rho}} = \sqrt{\frac{3 \times 1.01 \times 10^5}{0.09}} = 1837.5 \text{ ms}^{-1}$$

Now, volume occupied by one mole of hydrogen at STP

$$= 22.4 \text{ L} = 22.4 \times 10^{-3} \text{ m}^3$$

$\therefore$ Mass of hydrogen = volume × density

$$= 22.4 \times 10^{-3} \times 0.09$$

$$= 2.016 \times 10^{-3} \text{ kg}$$

$\therefore$ Average KE per mol $= \frac{1}{2} mc^2 = \frac{1}{2} \times 2.016 \times 10^{-3} \times (1837.5)^2$

$$= 3403.4 \text{ J}$$

11. (*d*) Here, $\gamma = 4.9 \times 10^{-4} \, ^\circ\text{C}^{-1}$, $\Delta T = 30 \text{ K} = 30^\circ \text{C}$

As $\quad V' = V + \Delta V = V(1 + \gamma \Delta T)$

$$= V(1 + 4.9 \times 10^{-4} \times 30) = 1.0147 \text{ V}$$

$$\Rightarrow \quad \frac{V'}{V} = 1.0147$$

Now, $\quad \rho = \frac{m}{V} \Rightarrow V \propto \frac{1}{\rho}$

$$\therefore \quad \rho' = \frac{\rho - 0.9855\rho}{\rho} = 0.0145$$

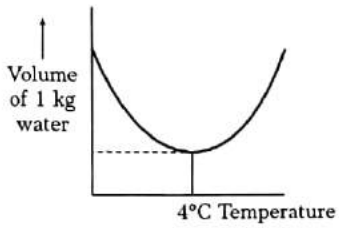
12. (*a*) According to the ideal gas law,

$$\Rightarrow \quad pV = RT \Rightarrow V = \left[\frac{R}{p}\right] T$$

$$\Rightarrow \quad V \propto T \Rightarrow \frac{V_1}{V_2} = \frac{T_1}{T_2} \text{ or } \frac{V_2}{V_1} - 1 = \frac{T_2}{T_1} - 1$$

$$\Rightarrow \quad \frac{V_2 - V_1}{V_1} = \frac{T_2 - T_1}{T_1} \Rightarrow \frac{V_2 - V_1}{V_1} = \frac{1}{T_1} = \frac{1}{T}$$

13. (*c*) Water is heated from 0°C to 10°C, then its volume first decreases and then increases.



14. (*d*) $\Delta L_S - \Delta L_B = \Delta L$

$$L_S = L,\ L_B = (L-5)\text{ cm},\ \alpha_S = 12\times10^{-6}/^\circ\text{C}$$

$$\alpha_B = 18\times10^{-6}/^\circ\text{C}$$

As, $\alpha_S = \dfrac{\Delta L_S}{L_3\times t_1} \Rightarrow 12\times10^{-6} = \dfrac{\Delta L_S}{L_3\times t_1}$ ...(i)

$\Rightarrow$ $\alpha_B = \dfrac{\Delta L_B}{L_B\times t_2} \Rightarrow 18\times10^{-6} = \dfrac{\Delta L}{(L-5)\times t}$ ...(ii)

Dividing Eqs. (i) and (ii), we get

$$\frac{2}{3} = \frac{L-5}{L} \text{ or } L = 15 \Rightarrow L_B = 15-5 = 10\text{ cm}$$

15. (*a*) When cooled from room temperature liquid water becames dense, as compared with other substances, but at approximately 4° C (39° F), pure water reaches its maximum density. If it is cooled further, it expands to become less dense.

16. (*c*) Strain $= \alpha\cdot\Delta T \Rightarrow$ Stress $Y \propto \Delta T$

where, L = length of the rod

α = coefficient of linear thermal expansion

and Y = Young's modulus of its material

So, the longitudinal stress developed in the rod is directly proportional to $\dfrac{\Delta T}{Y}$.

17. (*e*) The kinetic theory of gases assume the collision between molecules as perfectly elastic.

18. (*b*) Mean free path, $\lambda = \dfrac{1}{\sqrt{2}\pi n d^2} \Rightarrow \lambda \propto \dfrac{1}{r^2}$

where, d = diameter of the molecule

n = number of molecules per unit volume.

19. (*a*) Pressure exerted by a perfect gas on the walls of the container is

$$p = \frac{1}{3}mnV_{av}^2$$

20. (*b*) From law of equipartition of energy for any system in thermal equilibrium, the total energy is equally distributed among its various degree of freedom and each degree of freedom is associated with energy $(1/2)k_BT$.

21. (*b*) Here, $\gamma_r = \gamma_{ag} + 3\alpha_g = \gamma_{ac} + 3\alpha_c$

$\Rightarrow$ $1.03\times10^{-3} + 3\alpha_g = 1.006\times10^{-3} + 3\times(17\times10^{-6})$

$\Rightarrow$ $\alpha_g = 9\times10^{-6}/^\circ\text{C}$

22. (*a*) The coefficient of linear expansion (α) of the material of the tube and the coefficient of volume expansion (γ) of the liquid, then, $\gamma = 2\alpha$

23. (*c*) We know that, KE $= \dfrac{3}{2}kT$

where $\dfrac{3}{2}k$ is constant, so KE $\propto T$

Now we can say that the average kinetic energy of a gas molecule at absolute temeperature T is directly proportional to T.

24. (*a*) From gas equation, we know's $C = \sqrt{\dfrac{3RT}{m}}$

In given condition, $\dfrac{C_T}{C_0} = \sqrt{\dfrac{T}{T_0}} \Rightarrow \dfrac{2C_0}{C_0} = \sqrt{\dfrac{T}{T_0}}$

$\Rightarrow$ $4 = \dfrac{T}{T_0}$

$\Rightarrow$ $T = 4T_0 = 4\times273 = 1092\text{ K} = 819^\circ\text{C}$

25. (*a*) For an ideal gas, the mean KE is proportional to the absolute temperature. It does not depend on the kind of gas.

26. (*a*) In kinetic theory of gases, it is assumed that time taken in a collision is negligible compared to the average time taken in free travel between any two collisions, to be precise.

27. (*c*) The number of molecules (n) in a volume (V) at pressure p and temperature T is given by $n = \dfrac{pV}{KT}$

Here, $p = 10^6$ dyne cm^{-2}, $V = 1000$ cm^3,

$T = 273 + 27 = 300$ K

$K = 1.38\times10^{-6}$ ergs per molecule per K.

$$\therefore \quad n = \frac{(10^6)(1000)}{1.38\times10^{-16}\times300} \approx 2.4\times10^{22}$$

28. (*a*) We know that, $v = \sqrt{\dfrac{\gamma p}{\rho}}$

$$\gamma = \frac{v^2\rho}{p} = \frac{(330)^2\times1.3}{1.015\times10^5} = 1.4$$

$\Rightarrow$ $1 + \dfrac{2}{n} = 1.4 \Rightarrow n = 5$

29. (*a*) Coefficient of real expansion

$$\gamma_R = \frac{V_2 - V_1}{V_1(t_2 - t_1)}$$

Here, $V_2 = \dfrac{3}{4}, V_1 = \dfrac{2}{3}$

and $(t_2 - t_1) = (100 - 20) = 80$ °C

$$\therefore \quad \gamma_R = \frac{\left(\frac{3}{4} - \frac{2}{3}\right)}{\frac{2}{3}(80)} = \frac{1}{640} = 15.6\times10^{-4}\,^\circ\text{C}^{-1}$$

30. (*c*) $pV = RT$ = constant

$$Vp^2 = \text{constant}$$

$$p = \frac{RT}{V} \Rightarrow V\left(\frac{RT}{V}\right)^2 = \text{constant}$$

$\Rightarrow$ $\dfrac{R^2T^2}{V} = \text{constant}$

$\Rightarrow$ $\dfrac{T^2}{V} = \text{constant} \Rightarrow \dfrac{T^2}{V} = \dfrac{T'^2}{2V}$

$\Rightarrow$ $T'^2 = 2T^2 \Rightarrow T' = \sqrt{2}\,T$

CHAPTER 15

Laws of thermodynamics

Thermodynamics is the branch of Physics that deals with the concepts of heat, work and interconversion of heat and other forms of energy. In this chapter, we will study the laws of thermodynamics, various thermodynamic processes, basic theory of heat engines, refrigerators and Carnot engine.

Inside

1 Thermodynamic terms
- Zeroth law of thermodynamics

2 First law of thermodynamics
- Further explanation of the terms used in first law
- Applications of first law of thermodynamics

3 Heat capacity
- Thermodynamic parameters for a mixture of gases

4 Thermodynamic processes
- Isobaric process
- Isochoric process
- Isothermal process
- Cyclic process

5 Heat engine

6 Refrigerator

7 Second law of thermodynamics
- Reversible and irreversible process
- Carnot engine
- Entropy

1 Thermodynamic terms

Thermodynamics studies transformation of energy taking place with physical and chemical changes. In order to understand these transformations we need to understand the terms given below.

1. Thermodynamic system

It is an assembly of an extremely large number of particles (atoms or molecules) which is capable of exchange of energy with its surroundings. A thermodynamic system may contains matter in solid, liquid or gaseous form or a combination of two or more of these states.

Everything outside the system which has a direct effect on the system is called its **surroundings**. All space in universe outside the system is surroundings. *e.g.*, Environment.

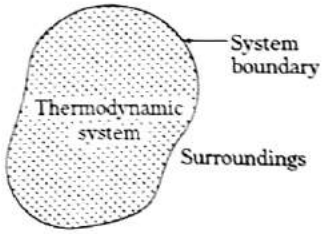


Fig. 15.1 A thermodynamic system having boundary and surroundings

A system is separated from its surrounding by a **boundary**. It is a real or imaginary two-dimensional closed surface that enclose the volume or region that a thermodynamic system occupies.

A thermodynamic system can be classified by the nature of the transfer of heat that are allowed to occur across its boundary.

(*i*) **Open system** It exchanges both heat (or energy) and matter with its surroundings.

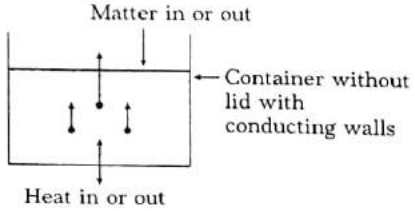


Fig. 15.2 Open system

(*ii*) **Closed system** It exchanges only heat (or energy) no matter with its surroundings.

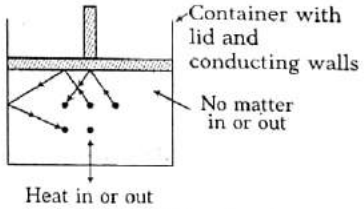


Fig. 15.3 Closed system

(*iii*) **Isolated system** It exchanges neither heat (or energy) nor matter with its surroundings.

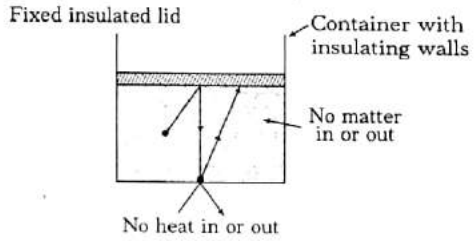


Fig. 15.4 Isolated system

2. Thermodynamic or state variables

The quantities like pressure (p), volume (V) and temperature (T) which help us to completely, specify state of thermodynamic system are called thermodynamic variables. Theromodynamic variables may be extensive or intensive in nature.

Extensive variables are those which depends on the size of the system and **intensive variables** are those which do not depend on the size of the system. *e.g.*, Internal energy (U), volume (V) and total mass (M) are extensive variables and pressure (p), temperature (T) and density (ρ) are intensive variables.

3. Equation of state

The equation which represents the relationship between the state variables of a system is called its equation of state. *e.g.*, For an ideal gas, the equation of state is

$$pV = \mu RT$$

For a fixed amount of gas given, there are thus only two independent variables, say p and V or T and V.

4. Thermodynamic process

When state of a system changes, then thermodynamic variables associated with the system like pressure p, volume V and temperature T of the system also change. This process is known as thermodynamic process.

5. Thermal equilibrium

Two systems are said to be in thermal equilibrium if there is no net flow of heat between them when they are brought into thermal contact.

6. Adiabatic and diathermic wall

Adiabatic wall is an insulating wall between two thermodynamic systems which does not allow flow of energy (or heat) from one system to another system. diathermic wall is a conducting wall between two thermodynamic systems which allows energy flow (or heat) from one system to another system.

Zeroth law of thermodynamics

The Zeroth law of thermodynamics states that if two systems A and B are separately in thermal equilibrium with a third system C, then A and B are in thermal equilibrium with each other.

Hence, from Zeroth law if systems A and B are separately in thermal equilibrium with C, then

$$T_A = T_C \quad \text{and} \quad T_B = T_C$$

$$\Rightarrow \quad T_A = T_B$$

So, systems A and B will also be in thermal equilibrium with each other.

Now, to understand zeroth law of thermodynamic, let us consider two systems A and B separated by a fixed adiabatic wall.

The two systems A and B are in contact with a third system C through diathermic wall. The macroscopic variables of A and B will vary until both A and B come in thermal equilibrium with the third system C.

This shows that two systems A and B are separately in thermal equilibrium with a third system C.

This condition is shown in Fig. (a).

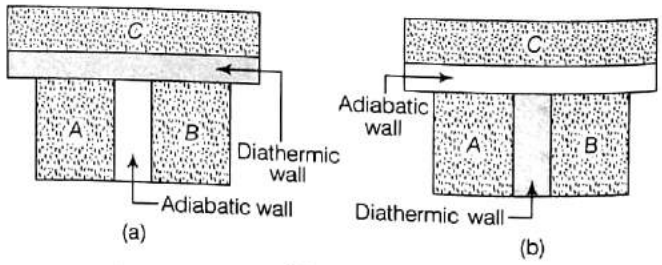


Fig. 15.5

Let the adiabatic wall between A and B is replaced by a diathermic wall and an adiabatic wall insulates C from A and B. A and B are found in thermal equilibrium with each other. This condition is shown in Fig. (b). This experiment prove the zeroth law of thermodynamics.

2 First law of thermodynamics

First law of thermodynamics is a statement of conservation of energy applied to any system in which energy transfer from or to the surroundings is taken into account.

It states that if some quantity of heat is supplied to a system capable of doing external work, then the quantity of heat absorbed by the system is equal to the sum of the increase in the internal energy of the system and the external work done by the system.

i.e., $$\boxed{\text{Heat supplied, } \Delta Q = \Delta U + \Delta W} \quad \text{...(i)}$$

where, ΔQ = Heat supplied to the system by the surroundings. It is a path dependent function.

ΔW = Work done by the system on the surroundings. It is a path dependent function.

ΔU = Change in internal energy of the system. It depends only on the initial and final states.

Often the first law is used in its differential form, which is

$$\boxed{dU = dQ - dW}$$

This can also be written as

$$\boxed{dQ = dU + dW}$$

Thermodynamic Sign Conventions for Heat and Work

Process	Convention
Heat added to the system	$\Delta Q > 0$ *i.e.*, positive
Heat removed from the system	$\Delta Q < 0$ *i.e.*, negative
Work done by the system (volume increases)	$\Delta W > 0$ *i.e.*, positive
Work done on the system (volume decreases)	$\Delta W < 0$, *i.e.*, negative
When temperature of the system rises	$\Delta U > 0$ *i.e.*, positive
When temperature of the system falls	$\Delta U < 0$ *i.e.*, negative

- In the Eq. (i), all three quantities ΔQ, ΔU and ΔW must be expressed in same unit either in joule or in calorie.
- dQ and dW depend not only on the initial and final states but also on the path of thermodynamic process, dU depends only on the initial and final states.

Further explanation of the terms used in first law

First law of thermodynamics basically revolves round the three terms ΔQ, ΔU and ΔW. If you substitute these three terms correctly with proper signs in the equation $\Delta Q = \Delta U + \Delta W$, then you are able to solve most of the problems of first law. Let us understand each term one by one, for an ideal gas system.

1. Work done by a thermodynamic system

By work, we mean work done by the system or on the system. Suppose a gas is confined in a cylinder with a movable piston, if p is pressure on the piston and A is area of piston, then force exerted by the gas on the piston of cylinder, $F = pA$.

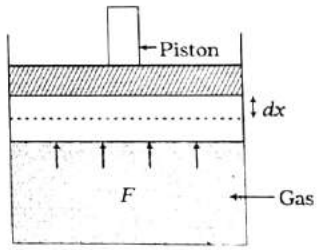


Fig. 15.6 *Work done during expansion*

When the piston is pushed outward by the gas through an infinitesimal distance dx, then the work done by the gas

$$dW = Fdx = pA\,dx = pdV$$

and for a finite volume change from V_i to V_f,

$$\text{Work done, } W = \int_{V_i}^{V_f} dW$$

$$\boxed{\text{Work done, } W = \int_{V_i}^{V_f} pdV}$$

Here, p is constant.

From the equation of state

$$pV = RT$$

$$\therefore \quad p\Delta V = nR\Delta T$$

$$\therefore \quad \text{Work done } W = p\Delta V = nR\Delta T$$

or $$\boxed{\text{Work done } W = nR\int_{T_i}^{T_f} \Delta T = nR(T_f - T_i)}$$

When variation of pressure with the volume is known to us then the area bounded by p-V graph (indicator diagram) represents the work done.

$$\boxed{\text{Work} = \int_{V_i}^{V_f} p\,dV = \text{area under } p\text{-}V \text{ graph.}}$$

- Work done is a path dependent quantity. So, work done is not a state function.
- If **volume is constant** in a thermodynamic process then **work done is zero.**
- If volume increases then work done is **positive** otherwise **negative.**
- For a cyclic process work done is **positive** if cycle is **clockwise** and **negative** if cycle is **anti-clockwise.**

Example 15.1 *Two moles of an ideal gas are in a rubber balloon at 30°C in equilibrium at 1 atm pressure. The balloon is fully expanded and can be assumed to require no energy in its expansion. The temperature of the balloon increases slowly to 35°C. What is the work done of expansion of balloon?* $[R = 8.31\ J\ mol^{-1}\text{-}K^{-1}]$.

Sol. If the balloon is fully expanded (*i.e.*, does not develop significant tension as it expands), the pressure inside the balloon is always nearly equal to the pressure outside the balloon. And the pressure outside the balloon is 1 atm (constant). So the pressure inside the balloon is constant at about 1 atm. According to the question, the limits of temperature are 30°C and 35°C.

$$\therefore \quad T_1 = 30°\,C = 30 + 273 = 303\ K$$
$$T_2 = 35°\,C = 35 + 273 = 308\ K$$

We know that, when the limits of temperature are given and pressure is constant, then

Work done, $W = nR\Delta T = nR[T_2 - T_1]$

$$= 2 \times 8.31\,[308 - 303] = 2 \times 8.31 \times [5] = 83.1\ J$$

Example 15.2 *(i) 2 moles of an ideal gas is heated at constant pressure so that its temperature increases from 27 °C to 127 °C. Find work done by the gas.*

(ii) In a process, the volume of gas increases from 1000 cc to 2000 cc at constant pressure 100 kPa. Find work done by the gas.

Sol. Given, pressure $p = 100$ kPa $= 100 \times 10^3$ Pa,

Temperature $T_1 = 27°\,C$, $T_2 = 127°\,C$

Volume $V_1 = 1000$ cc, $V_2 = 2000$ cc

According to the question, pressure is constant

$$\therefore \quad \Delta W = p\Delta V = nR\Delta T$$

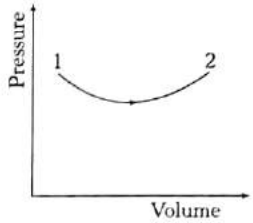
(i) Work done by the gas

$$\Delta W = nR\Delta T = nR(T_2 - T_1)$$
$$= 2 \times 8.3 \times (127 - 27)$$
$$(\because \text{ Number of moles of the gas, } n = 2)$$
$$= 1660\ J$$

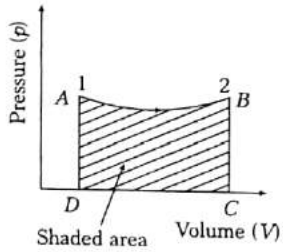
(ii) Work done by the gas

$$\Delta W = p\Delta V = p(V_2 - V_1)$$
$$= 100 \times 10^3\,(2000 - 1000) \times 10^{-6}$$
$$(\because \text{ Pressure } p = 100\,kPa)$$
$$= 100\ J$$

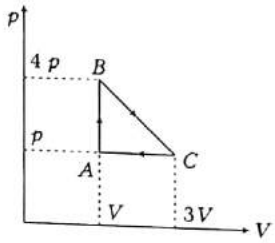
Example 15.3 *Consider the process on a system shown in figure below. During the process, predict the nature of work done by the system.*



Sol. We said that the area under the p-V diagram is equal to the work done. So, if we move from 1 to 2 the shaded area ($ABCD$) under the graph is continuously increasing so work done by the system continuously increases.



Example 15.4 *An ideal gas is taken around the cycle ABCA as shown in the p-V diagram.*



Evaluate the total work done by the gas during the cycle.

Sol. Work done = Area enclosed by triangle under the p-V diagram.

$$= \frac{1}{2} AB \times AC$$

Here side AB of the triangle is parallel to pressure (p) axis therefore

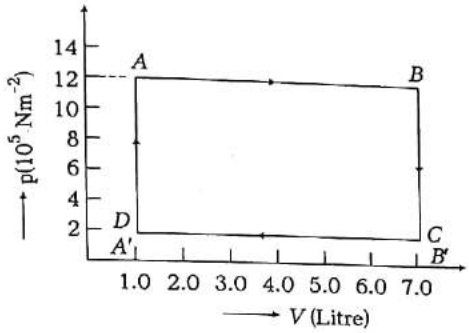
AB = Change in pressure $\Delta p = p_B - p_A = 4p - p = 3p$

Similarly side AC of the triangle is parallel to volume (V) axis, therefore

$$AC = \text{Change in volume } \Delta V$$
$$= V_C - V_A = 3V - V = 2V$$

$\therefore$ Work done, $W = \frac{1}{2} 3p \times 2V = 3pV$

Example 15.5 *The figure shows a p-V graph of the thermodynamic behaviour of an ideal gas. Find out from this graph (i) work done by the gas in the process $A \to B$, $B \to C$, $C \to D$ and $D \to A$, (ii) work done by the gas in complete cycle $A \to B \to C \to D \to A$.*



Sol. (i) The work done in a thermodynamic process is equal to the area enclosed between the p-V curve and the volume axis.

Work done by the gas in the process $A \to B$ is

$$W_1 = \text{area } ABB'A' = AB \times A'A$$
$$= (7-1)\text{ litre} \times (12 \times 10^5)\text{ Nm}^{-2}$$
$$= 6 \times 10^{-3}\text{ m}^3 \times (12 \times 10^5)\text{ Nm}^{-2}$$
$$= 7200\text{ N-m} = 7200\text{ J}$$

Work done in the process $B \to C$ is zero since volume remains constant.

Work done on the gas in the process $C \to D$ is

$$W_2 = \text{area } DCB'A'$$
$$= DC \times A'D$$
$$= (-6 \times 10^{-3}) \times (2 \times 10^5)$$
$$= -1200\text{ J}.$$

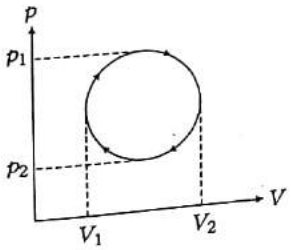
(Negative sign is taken because volume decreases)

Work done in the process $D \to A$ is also zero, because volume remains constant.

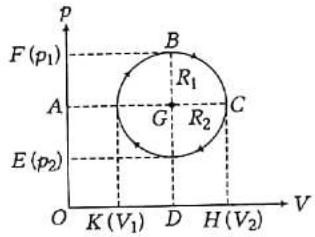
(ii) Work W_1 is positive, while W_2 is negative. Hence, the net work done in the whole cycle by the gas is

$$W = W_1 - W_2$$
$$= 7200 - 1200$$
$$= 6000\text{ joule}$$

Example 15.6 *Find the work done during the perfectly circular cyclic process as shown in the diagram.*



Sol. As we know that area of a circle of radius R is πR^2.



From the diagram it is clear that

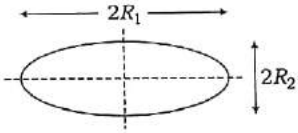
Vertical radius of the circle

$$R_1 = GB$$
$$= AF = \frac{EF}{2} = \frac{OF - OE}{2}$$
$$= \frac{p_1 - p_2}{2}$$

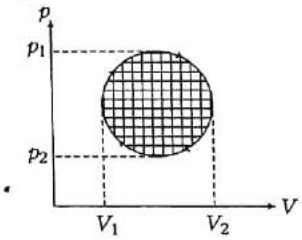
Horizontal radius of the circle

$$R_2 = GC = DH = \frac{KH}{2} = \frac{OH - OK}{2} = \frac{V_2 - V_1}{2}$$

We know that the area of an ellipse is $\pi R_1 R_2$, where R_1 and R_2 are semi-major and semi-minor axis respectively, as shown in figure.



If $R_1 = R_2$, the ellipse becomes a circle as shown in figure.



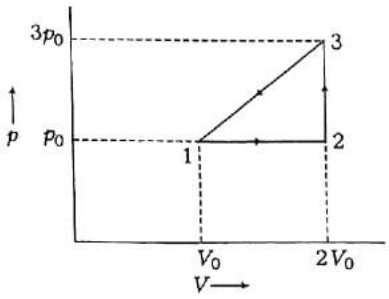
So, here the work done = area of the circle

= area of shaded portion

$$= \pi R_1 R_2 = \pi\left(\frac{p_1 - p_2}{2}\right)\left(\frac{V_2 - V_1}{2}\right)$$

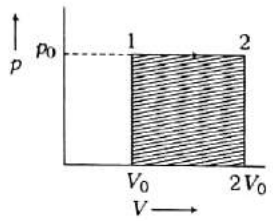
The positive sign is due to the fact that the cycle is clockwise.

Example 15.7 *Calculate the work done by a gas, when it is taken from 1 to 2, 2 to 3 and 3 to 1 as shown. Also calculate the work done in cyclic process $1 \to 2 \to 3 \to 1$*



Sol. The work done in a process is the area below the process in p-V diagram.

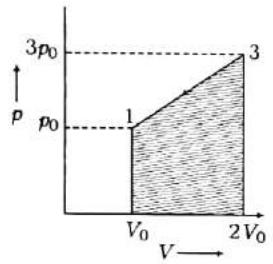
Process $1 \rightarrow 2$



$$\Delta W_{1\rightarrow 2} = p_0(2V_0 - V_0) = p_0V_0$$

In the process $2 \rightarrow 3$, volume remains same, $\Delta W_{2\rightarrow 3} = 0$

Process $3 \rightarrow 1$



$$\Delta W_{3\rightarrow 1} = \text{area below the process } 3 \rightarrow 1$$
$$= -\frac{1}{2}(p_0 + 3p_0)(2V_0 - V_0) = -2p_0V_0$$

Work done is negative because volume is decreasing.

Work done in cyclic process:

$$\Delta W_{\text{cyclic}} = \Delta W_{1\rightarrow 2\rightarrow 3\rightarrow 1}$$
$$= \Delta W_{1\rightarrow 2} + \Delta W_{2\rightarrow 3} + \Delta W_{3\rightarrow 1}$$
$$= p_0V_0 + 0 - 2p_0V_0 = -p_0V_0$$

2. Internal energy of a thermodynamic system

Internal energy of a system is defined as the total energy possessed by the system due to molecular motion and molecular configuration.

It is represented by U. The energy due to molecular motion is called **internal kinetic energy** U_K. The molecular motion includes translation, rotation and vibrational motion of the molecules.

The energy due to molecular configuration is called **internal potential energy** U_P.

Then, internal energy, $\boxed{U = U_K + U_P}$

- Internal energy is sum of the kinetic energies and potential energies of all the constituent particles of the system.
- Internal energy is a state variable it does not depend on the path taken to arrive at that state.

Internal energy of an ideal gas

In an ideal gas, there are no inter-molecular forces hence the gas does not possess intermolecular potential energy, thus $U_P = 0$.

So, internal energy of an ideal gas is just the sum of kinetic energies associated with various random motions (*i.e.*, rotational, translational and vibrational) of its molecules.

Therefore, internal energy of an ideal gas depends only on its temperature and is given by $U = U_K = \frac{3}{2}nRT$

$\therefore$ $\boxed{\text{Change in internal energy, } \Delta U = \frac{3}{2}nR\Delta T}$

Note For any gas, $\Delta U = nC_V\Delta T = n\left(\frac{f}{2}\right)R\Delta T = \frac{nR}{r-1}\Delta T$

$$f = \text{degree of freedom}, r = \frac{C_p}{C_V}$$

Change in internal energy is independent of path followed by a thermodynamic system.

$\therefore$ $\boxed{\text{Change in internal energy, } \Delta U = U_f - U_i}$

Example 15.8 *1 kg of water at 373 K is converted into steam at same temperature. Volume of 1 cm^3 of water becomes 1671 cm^3 on boiling. What is the change in the internal energy of the system if the latent heat of vaporisation of water is 5.4×10^5 cal kg^{-1}?*

Sol. Volume of 1 kg of water $= 1000\ \text{cm}^3 = 10^{-3}\ \text{m}^3$,

Volume of 1 kg of steam $= 10^3 \times 1671\ \text{cm}^3 = 1.671\ \text{m}^3$

Change in volume, $\Delta V = (1.671 - 10^{-3})\ \text{m}^3 = 1.670\ \text{m}^3$,

Pressure, $p = 1\ \text{atm} = 1.01 \times 10^5\ \text{Nm}^{-2}$

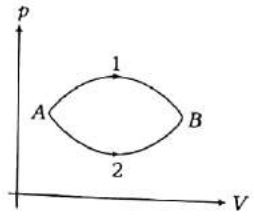
In expansion work done, $\Delta W = p\Delta V = 1.01 \times 10^5 \times 1.67\ \text{J}$

$$= \frac{1.686 \times 10^5}{4.2}\ \text{cal} = 4.015 \times 10^4\ \text{cal}$$

But $\Delta U = \Delta Q - \Delta W$ (first law of thermodynamics)

or $\Delta U = (5.4 \times 10^5 - 0.4015 \times 10^5)\ \text{cal} = 4.9985 \times 10^5\ \text{cal}$

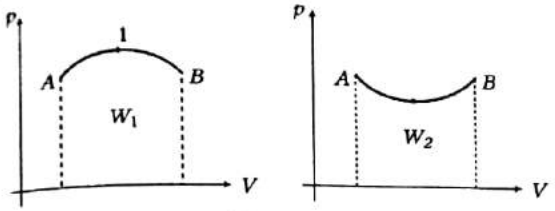
Example 15.9 *A certain amount of an ideal gas passes from state A to B first by means of process 1, then by means of process 2. In which of the process is the amount of heat absorbed by the gas greater?*



Sol. From the first law of thermodynamics.

$$\Delta Q_1 = \Delta W_1 + \Delta U_1 \text{ and } \Delta Q_2 = \Delta W_2 + \Delta U_2$$

U is a state function. Hence, ΔU depends only on the initial and final positions. Therefore,



$$\Delta U_1 = \Delta U_2$$

But $$\Delta W_1 > \Delta W_2$$

as the area under 1 is greater than area under 2. Hence,

$$\Delta Q_1 > \Delta Q_2$$

Example 15.10 *When a system goes from state A to state B, it is supplied with 400 J of heat and it does 100 J of work.*

(i) For this transition, what is the system's change in internal energy?

(ii) If the system moves from B to A, what is the change in internal energy?

(iii) If in moving from A to B along a different path in which $W'_{AB} = 400$ J of work is done on the system, how much heat does it absorb?

Sol. (i) From the first law of the thermodynamics,

$$\Delta U_{AB} = Q_{AB} - W_{AB} = (400 - 100)\text{ J} = 300\text{ J}$$

(ii) Consider a closed path that passes through the state A and B. Internal energy is a state function, so ΔU is zero for a closed path.

Thus, $$\Delta U = \Delta U_{AB} + \Delta U_{BA} = 0$$

or $$\Delta U_{BA} = -\Delta U_{AB} = -300\text{ J}$$

(iii) The change in internal energy is the same for any path, so

$$\Delta U_{AB} = \Delta U'_{AB} = Q'_{AB} - W'_{AB}$$

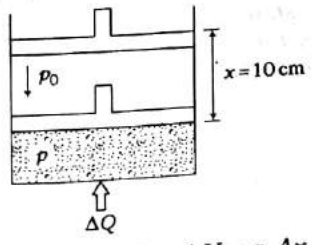
$$300\text{ J} = Q'_{AB} - (-400\text{ J})$$

and the heat exchanged is, $Q'_{AB} = -100$ J

The negative sign indicates that the system loses heat in this transition.

Example 15.11 *A gas is enclosed in a cylindrical vessel fitted with a frictionless piston. The gas is slowly heated for some time. During the process, 10 J of heat is supplied and the piston is found to move out 10 cm. Find the increase in the internal energy of the gas. The area of cross-section of the cylinder $= 4\text{ cm}^2$ and the atmospheric pressure $= 100$ kPa.*

Sol. In given process piston moves against atmospheric process so;



Work done by the gas $= \Delta W = p\Delta V = p_0 Ax$

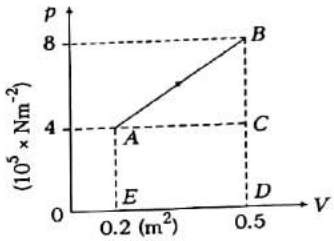
$$= 100 \times 10^3 \times 4 \times 10^{-4} \times 0.1 = 4\text{ J}$$

From first law of thermodynamics

$\Rightarrow$ $$\Delta Q = \Delta U + \Delta W \Rightarrow 10 = \Delta U + 4\text{J}$$

$\therefore$ Increase in internal energy, $\Delta U = 6$ J

Example 15.12 *The pressure in monoatomic gas increases linearly from $4 \times 10^5\ Nm^{-2}$ to $8 \times 10^{+5}\ Nm^{-2}$ when its volume increases from $0.2\ m^3$ to $0.5\ m^3$. Calculate*



(i) Work done by the gas.

(ii) Increase in the internal energy.

(iii) Amount of heat supplied.

Sol. Given, $p_1 = 4 \times 10^5\ \text{Nm}^{-2}$, $p_2 = 8 \times 10^{+5}\ \text{Nm}^{-2}$, $V_1 = 0.2\ \text{m}^3$, $V_2 = 0.5\ \text{m}^3$

(i) Work done by the gas (ΔW) = Area under p-V graph (Area $ABCDEA$)

$$= \frac{1}{2}(AE + BD) \times AC$$

$$= \frac{1}{2}(4 \times 10^5 + 8 \times 10^5) \times (0.5 - 0.2)$$

$$= \frac{1}{2} \times 12 \times 10^5 \times 0.3 = 1.8 \times 10^5\text{ J}$$

(ii) Increase in internal energy

$$\Delta U = C_V(T_2 - T_1) = \frac{C_V}{R}R(T_2 - T_1) = \frac{C_V}{R}(p_2V_2 - p_1V_1)$$

For monoatomic gas

$$C_V = \frac{3}{2}R$$

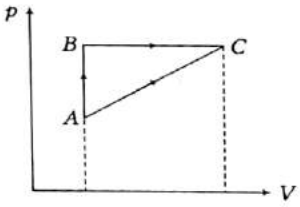
$$\therefore \quad \Delta U = \frac{3}{2}[(8 \times 10^5 \times 0.5) - (4 \times 10^5 \times 0.2)]$$

$$= \frac{3}{2}[4 \times 10^5 - 0.8 \times 10^5] = 4.8 \times 10^5\text{ J}$$

(iii) Amount of heat supplied,

$$\Delta Q = \Delta U + \Delta W = 4.8 \times 10^5 + 1.8 \times 10^5 = 6.6 \times 10^5\text{ J}$$

Example 15.13 *A thermodynamical process is shown in figure with $p_A = 3 \times 10^4\ Pa; V_A = 2 \times 10^{-3}\ m^3$; $p_B = 8 \times 10^4\ Pa, V_C = 5 \times 10^{-3}\ m^3$. In the processes AB and BC, 600 J and 200 J of heat is added to the system respectively. Find the change in internal energy of the system in process AC :*



Sol. Since internal energy is a state function so the change in internal energy during the process AC will be same as that during the total processes AB plus BC. Calculate the total

work ΔW during AB plus BC and total heat ΔQ during these processes. Then calculate $\Delta U = \Delta Q - \Delta W$.

During the process AB the volume doesn't change so work done is zero.

During BC the pressure is constant, so total work done

$$= p_B(V_C - V_B) = p_B(V_C - V_A) \qquad [\because V_B = V_A]$$
$$= 8 \times 10^4 \text{ Nm}^{-2} (5 \times 10^{-3} \text{ m}^3 - 2 \times 10^{-3} \text{ m}^3)$$
$$= 240 \text{ J}$$

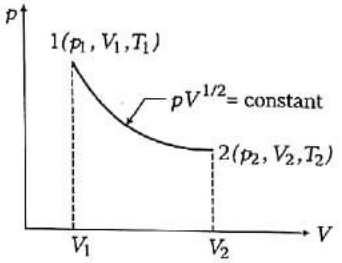
$\therefore$ ΔW during process AB plus $BC = 240$ J

Now, given ΔQ during the same whole process

$$= 600 + 200 = 800 \text{ J}$$

$$\therefore \qquad \Delta U = \Delta Q - \Delta W = 800 - 240 = 560 \text{ J}$$

Example 15.14 *Consider a p-V diagram in which the path followed by one mole of perfect gas in a cylindrical container is shown in figure.*



(i) Find the work done when the gas is taken from state 1 to state 2.

(ii) What is the ratio of temperatures T_1/T_2, if $V_2 = 2V_1$?

(iii) Given the internal energy for one mole of gas at temperature T is (3/2) RT, find the change in internal energy and work done by the gas when it is taken from states 1 to 2, with $V_2 = 2V_1$.

Sol. (i) It is given that

$$pV^{\frac{1}{2}} = \text{constant}$$

Let this constant is A

$$\therefore \qquad pV^{1/2} = A$$

$$\Rightarrow \qquad p = \frac{A}{V^{1/2}} = \frac{A}{\sqrt{V}}$$

Also we can write

$$p_1V_1^{1/2} = p_2V_2^{1/2} = A$$

$\therefore$ Work done by the gas,

$$\Delta W = \int_{V_1}^{V_2} p\,dV = A\int_{V_1}^{V_2} \frac{dV}{\sqrt{V}} = A\left[\frac{\sqrt{V}}{1/2}\right]_{V_1}^{V_2}$$
$$= 2A(\sqrt{V_2} - \sqrt{V_1})$$
$$= 2p_1V_1^{1/2}[V_2^{1/2} - V_1^{1/2}]$$

(ii) Since, $T = pV/nR = \dfrac{A}{nR} \cdot \sqrt{V}$

Thus, $\dfrac{T_2}{T_1} = \sqrt{\dfrac{V_2}{V_1}} = \sqrt{2}$

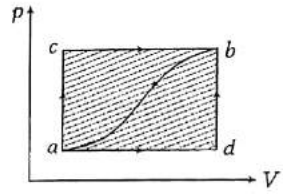
(iii) Then, the change in internal energy

$$\Delta U = U_2 - U_1 = \frac{3}{2}R(T_2 - T_1) = \frac{3}{2}RT_1(\sqrt{2} - 1)$$

Work done by the gas,

$$\Delta W = 2A\sqrt{V_1}(\sqrt{2} - 1) = 2RT_1(\sqrt{2} - 1)$$

Example 15.15 *When a system is taken from state a to state b, in figure along the path*



$a \to c \to b$, 60 J of heat flow into the system, and 30 J of work is done

(i) How much heat flows into the system along the path $a \to d \to b$ if the work is 10 J.

(ii) When the system is returned from b to a along the curved path, the work done by the system is 20 J. Does the system absorb or liberate heat, and how much?

Sol. For the path a, c, b, $\Delta U = \Delta Q - \Delta W = 60 - 30 = 30$ J

or $U_b - U_a = 30$ J

(i) Along the path a, d, b, $\Delta Q = \Delta U + \Delta W = 30 + 10 = 40$ J

(ii) Along the curved path b, a,

$\Delta Q = (U_a - U_b) + \Delta W = (-30) + (-20) = -50$ J, heat flows out of the system

Example 15.16 *In the previous example if $U_a = 0$ and $U_d = 22$ J then find the heat absorbed in the process $a \to d$ and $d \to b$.*

Sol. For the path $a \to d \to b$

$$W_{adb} = W_{a\to d} + W_{d\to b} = W_{a\to d} + 0$$

$$\Rightarrow \qquad W_{a\to d} = W_{adb} = 10\text{J}$$

Now, applying first low of thermodynamics for the path $a \to d$

$$\Delta Q_{a\to d} = \Delta U_{a\to d} + \Delta W_{a\to d}$$
$$= U_d - U_a + 10 = 22 - 0 + 10 = 32\text{J}$$

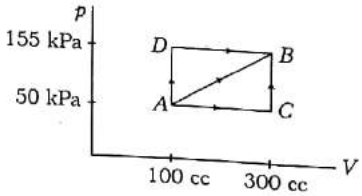
Similarly for the path $d \to b$

$$\Delta Q_{d\to b} = \Delta U_{d\to b} + \Delta W_{d\to b} = U_b - U_d + 0 = U_b - U_d$$

Given, $U_b - U_a = 30\text{J} \Rightarrow U_b - 0 = 30 \Rightarrow U_b = 30\text{J}$

$$\therefore \qquad Q_{d\to b} = 30 - 22 = 8\text{J}$$

Example 15.17 *50 cal of heat should be supplied to take a system from the state A to the state B through the path ACB as shown in figure. Find the quantity of heat to be supplied to take it from A to B via ADB.*



Sol. In process ACB

$$\Delta W_{ACB} = \Delta W_{AC} + \Delta W_{CB}$$
$$= 50 \times 10^3 \times (300 - 100) \times 10^{-6} + 0$$
$$= 10 \text{ J} = \frac{10}{4.2} = 2.4 \text{ cal}$$

Given, $\Delta Q = 50$ cal

From the first law of thermodynamics

$$\Delta Q = \Delta U + \Delta W = U_B - U_A + \Delta W$$
$$\Rightarrow \quad 50 = (U_B - U_A) + 2.4$$
$$\Rightarrow \quad U_B - U_A = 47.6 \text{ cal}$$

In process ADB

$$\Delta W_{ADB} = \Delta W_{AD} + \Delta W_{DB}$$
$$= 0 + 155 \times 10^3 \times (300 - 100) \times 10^{-6}$$
$$= 31 \text{ J} = \frac{31}{4.2} = 7.4 \text{ cal}$$

$\therefore$ Quantity of heat supplied,

$$\Delta Q = \Delta U + \Delta W = U_B - U_A + 7.4$$
$$= 47.6 + 7.4 = 55 \text{ cal}$$

Applications of first law of thermodynamics

Applications of first law of thermodynamics are given below.

Melting process

When a substance melts, the change in volume (dV) is very small and can, therefore, be neglected. The temperature of a substance remains unchanged during melting process.

Heat absorbed during melting process, $\Delta Q = mL$, where L is latent heat of fusion.

By the first law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W$$
$$\Rightarrow \quad \boxed{mL = \Delta U} \quad [\because \Delta W = p\Delta V = p \times 0 = 0]$$

So, the internal energy increases by mL during the melting process.

Example 15.18 *At normal pressure and 0°C temperature when 2 kg of ice melts its volume decreases by 182 cm^3. Latent heat of ice for melting is 3.4×10^5 J kg^{-1}. Find out by what value internal energy of the ice changes when its melts at normal pressure and 0°C temperature? (Normal pressure $= 1.01 \times 10^5$ Nm^{-2})*

Sol. Heat required to melt 2 kg of ice, $Q = mL$

$$= 2 \text{ kg} \times (3.4 \times 10^5) \text{ J kg}^{-1} = 6.8 \times 10^5 \text{ J}$$

Decrease in volume of ice $= \Delta V = 182$ cm^3

Therefore, work done by external force on the ice

$$W = -p \times \Delta V = (1.01 \times 10^5 \text{ Nm}^{-2}) \times (-182 \times 10^{-6} \text{ m}^3)$$
$$= -18.4 \text{ J}$$

Because work is done on the ice therefore it is negative.

According to first law of thermodynamics change in internal energy of ice,

$$\Delta U = Q - W = 6.8 \times 10^5 \text{ J} - (-18.4 \text{ J})$$
$$= (6.8 \times 10^5 + 18.4) \text{ J} \approx 6.8 \times 10^5 \text{ J}$$

ΔU is positive. Therefore, internal energy of ice increases when it melts.

Boiling process

When a liquid is heated, it changes into vapour at constant temperature (called boiling point) and pressure. Let us consider the vaporisation of liquid of mass m. Let V_l and V_v be the liquid and vapours respectively.

The work done in expanding at constant temperature and pressure p,

$$\Delta W = p\Delta V = p(V_v - V_l)$$

Let the latent heat of vaporisation $= L$

$\therefore$ Heat absorbed during boiling process, $\Delta Q = mL$

Let U_l and U_v be the internal energies of the liquid and vapours respectively then $\Delta U = U_v - U_l$

According to the first law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W$$
$$\therefore \quad \boxed{mL = (U_v - U_l) + p(V_v - V_l)}$$

Example 15.19 *When water is boiled at 2 atm pressure then latent heat of vaporization is 2.2×10^6 Jkg^{-1} and boiling point is 120°C. At this temperature volume of 1 kg of water is 10^{-3} m^3 and that of 1 kg vapour's it is 0.824 m^3. When 1 kg water is converted to the vapour at 120°C temperature then find the work done and increase in internal energy. (1 atm pressure $= 1.013 \times 10^5$ Nm^{-2})*

Sol. Heat required to convert water into vapour at 120°C is

$$\Delta Q = m \times L = 1 \text{ kg} \times 2.2 \times 10^6 \text{ Jkg}^{-1} = 2.2 \times 10^6 \text{ J}$$

Change in volume of water $= (0.824 - 0.001)$ m^3 $= 0.823$ m^3

Therefore, work done against atmospheric pressure

$$\Delta W = p\Delta V$$
$$= (2 \times 1.013 \times 10^5 \text{ Nm}^{-2}) \times (0.823 \text{ m}^3)$$
$$= 166739.8 \text{ Nm} = 1.67 \times 10^5 \text{ J}$$

From first law of thermodynamics, $\Delta Q = \Delta W + \Delta U$

or $\quad \Delta U = \Delta Q - \Delta W = 2.2 \times 10^6 \text{ J} - 166739.8 \text{ J}$

$$= 2033260.2 = 2.03 \times 10^6 \text{ J}$$

$\therefore$ Required work $= 1.67 \times 10^5$ J

and changes in internal energy $= 2.03 \times 10^6$ J

Check point 15.1

1. First law of thermodynamics corresponds to
(a) conservation of energy
(b) conservation of momentum
(c) law of conservation of angular momentum
(d) conservation of mass

2. In a given process for an ideal gas, $\Delta W = 0$ and $\Delta Q < 0$. Then for the gas
(a) the temperature will decrease
(b) the volume will increase
(c) the pressure will remain constant
(d) the temperature will increase

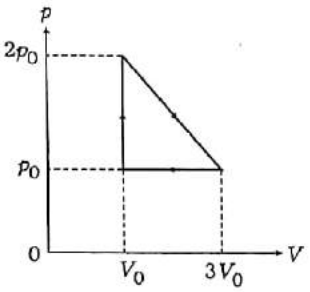
3. If ΔQ and ΔW represent the heat supplied to the system and the work done on the system respectively, then the first law of thermodynamics can be written as
(a) $\Delta Q = \Delta U + \Delta W$
(b) $\Delta Q = \Delta U - \Delta W$
(c) $\Delta Q = \Delta W - \Delta U$
(d) $\Delta Q = -\Delta W - \Delta U$

where, ΔU is the change in internal energy.

4. A thermo-dynamic system is taken from state (p_1, V_1) to (p_2, V_2) by two different process. The quantity which will remain same will be?
(a) ΔQ (b) ΔW
(c) $\Delta Q + \Delta W$ (d) $\Delta Q - \Delta W$

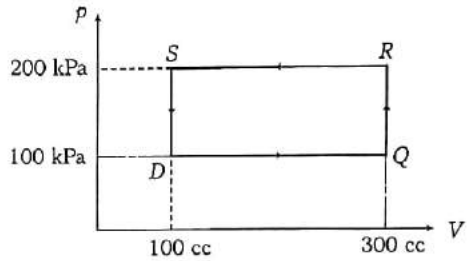
5. A system is taken from state A to state B along two different paths 1 and 2. The heat absorbed and work done by the system along these two paths are Q_1 and Q_2 and W_1 and W_2 respectively. Then
(a) $Q_1 = Q_2$
(b) $W_1 = W_2$
(c) $Q_1 - W_1 = Q_2 - W_2$
(d) $Q_1 + W_1 = Q_2 + W_2$

6. An ideal monoatomic gas is taken around the cycle as shown in p-V diagram. The work done during the cycle is given by



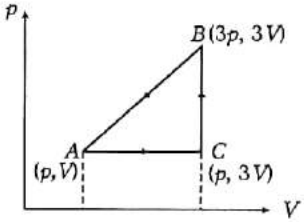
(a) $\frac{1}{2} p_0V_0$ (b) p_0V_0
(c) $2p_0V_0$ (d) None of these

7. A thermodynamic system is taken through the cyclic process $PQRSP$. The net work done by the system is



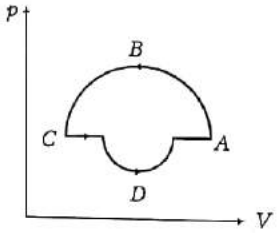
(a) 20 J (b) −20 J (c) 400 J (d) −400 J

8. An ideal gas is taken around $ABCA$ as shown in the p-V diagram. The work done during the cycle is



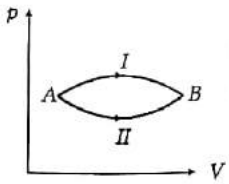
(a) $2pV$ (b) pV
(c) $\frac{pV}{2}$ (d) zero

9. Work done by the gas in the process shown in figure is



(a) positive
(b) negative
(c) zero
(d) may be positive or negative

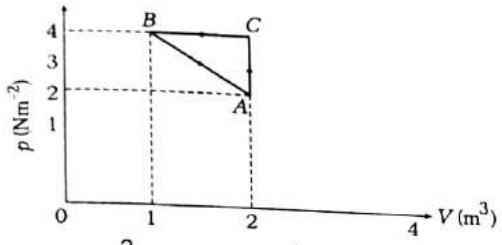
10. A certain amount of an ideal gas is taken from state A to state B, one time by process I and another time by process II. If the amount of heat absorbed by the gas are Q_1 and Q_2 respectively, then



(a) $Q_1 = Q_2$ (b) $Q_1 < Q_2$
(c) $Q_1 > Q_2$ (d) Data insufficient

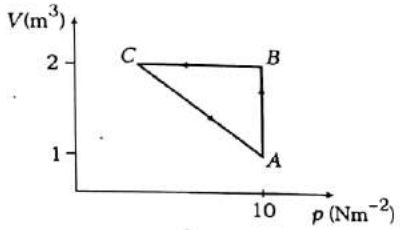
11. In thermodynamic process, 200 J of heat is given to a gas and 100 J of work is also done on it. The change in internal energy of the gas is
(a) 100 J (b) 300 J (c) 419 J (d) 24 J

12. Corresponding to the process shown in figure, what is the heat given to the gas in the process *ABCA*?



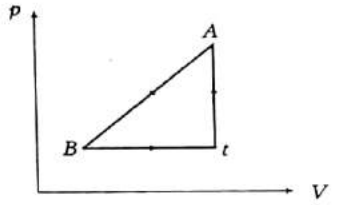
(a) 1 J (b) $\frac{3}{2}$ J (c) $\frac{1}{2}$ J (d) 0

13. An ideal gas is taken through the cycle $A \rightarrow B \rightarrow C \rightarrow A$, as shown in the figure. If the net heat supplied to the gas in the cycle is 5 J, the work done by the gas in the process $C \rightarrow A$ is



(a) −5 J (b) −10 J (c) −15 J (d) −20 J

14. The *p-V* diagram of a system undergoing thermodynamic transformation is shown in figure. The work done on the system in going from $A \rightarrow B \rightarrow C$ is 50 J and 20 cal heat is given to the system. The change in internal energy between *A* and *C* is



(a) 34 J
(b) 70 J
(c) 84 J
(d) 134 J

15. A closed system undergoes a change of state by process $1 \rightarrow 2$ for which $Q_{12} = 10$ J and $W_{12} = -5$ J. The system is now returned to its initial state by a different path $2 \rightarrow 1$ for which Q_{21} is −3 J. The work done by the gas in the process $2 \rightarrow 1$ is
(a) 8 J
(b) zero
(c) −2 J
(d) +5 J

3 Heat capacity

If an amount of heat ΔQ is needed to change the temperature of a body by ΔT, then heat capacity of the material of the body is given by

$$\text{Heat capacity, } S = \frac{\Delta Q}{\Delta T}$$

Heat capacity is equal to the heat energy required to change the temperature of a body by unity. Its unit is JK^{-1}.

Note Heat capacity of a body is not constant over a large temperature range. Different amounts of heat may be needed for a unit rise in temperature at different temperature values.

Specific heat capacity

The amount of heat required to change the temperature of unit mass of a substance by unity is known as specific heat capacity of the substance.

$$\text{Specific heat capacity, } s = \frac{S}{m} = \frac{1}{m}\frac{\Delta Q}{\Delta T}$$

$$\text{Specific heat capacity, } s = \frac{1}{m}\frac{\Delta Q}{\Delta T}$$

It depends on the nature of substance and its temperature. Its unit J/kg-K.

Molar specific heat capacity

The amount of heat required to raise the temperature of one gram mole of a substance by unity is called molar specific heat capacity of the substance.

Heat capacity per mole is known as molar specific heat capacity or molar heat capacity.

$$C = \frac{S}{n} = \frac{1}{n}\left(\frac{\Delta Q}{\Delta T}\right)$$

$$\text{Molar specific heat capacity, } C = \frac{1}{n}\frac{\Delta Q}{\Delta T}$$

where, n = number of moles. It depends on the nature of the substance, its temperature and the conditions under which heat is supplied. Its unit is J/mol-K.

On the basis of condition under which the heat is supplied to a gas, *motion specific heat capacity is of two types*

(i) **Molar heat capacity at constant pressure**

$$C_p = \left(\frac{\Delta Q}{n\Delta T}\right)_{\text{constant pressure}}$$

(ii) Molar heat capacity at constant volume

$$C_V = \left(\frac{\Delta Q}{n\Delta T}\right)_{\text{constant volume}}$$

Note In case of solids and liquids, volume expansion is small (*i.e.*, $\Delta W = 0$) therefore we define only one specific heat for solids and liquids.

Relation between C_p and C_V

For an ideal gas, the relation between C_p and C_V is

$$C_p - C_V = R$$

Here, C_p and C_V are molar specific heat capacities of an ideal gas at constant pressure and volume respectively and R is the universal gas constant. This relation is known as **Mayer's formula.**

The ratio of C_p and C_V is notified by γ.

$$\gamma = C_p / C_V$$

It is known as **heat capacity ratio.**

From the above two relation between C_p and C_V, we get

$$C_V = \frac{R}{\gamma - 1} \quad \text{and} \quad C_p = \frac{\gamma R}{\gamma - 1}$$

The ratio in terms of degrees of freedom is $\gamma = 1 + \frac{2}{f}$

where, f = degrees of freedom.

Values of f, C_V, C_p and γ for different types of gases are given below.

Atomicity of gas	f	C_V	C_p	γ
Monoatomic	3	$\frac{3}{2}R$	$\frac{5}{2}R$	$\frac{5}{3} = 1.67$
Diatomic, triatomic and Triatomic linear (at normal temperature)	5	$\frac{5}{2}R$	$\frac{7}{2}R$	$\frac{7}{5} = 1.4$
Poly atomic Triangular (Non-linear)	6	$\frac{6}{2}R = 3R$	$\frac{8}{2}R = 4R$	$\frac{4}{3} = 1.33$

For n moles of a gas, internal energy at temperature T is

$$U = \frac{nfRT}{2} \quad \text{and} \quad U = nC_VT$$

and change in internal energy $\Delta U = nC_V\Delta T$.

Example 15.20 *5 moles of oxygen is heated at constant volume from 10°C to 20°C. What will be change in the internal energy of the gas? The gram molecular specific heat of oxygen at constant pressure is (C_p = 8 cal mole^{-1} and R = 8.36 joule/mole °C)*

Sol. According to the Mayor's formula $C_V = C_p - R$

$$= 8 - 2 = 6 \text{ cal/mol °C}$$

$\therefore$ Heat absorbed by 5 moles of oxygen at constant volume

$$(= nC_V\Delta T = 5 \times 6\,(20 - 10) = 30 \times 10 = 300 \text{ cal}$$

At constant volume $\Delta V = 0$.

$\therefore \quad \Delta W = 0$

$\therefore$ From first law of thermodynamics $Q = \Delta U + \Delta W$

$\Rightarrow \quad 300 = \Delta U + 0 \Rightarrow \Delta U = 300$ cal

Example 15.21 *At normal temperature and pressure (NTP) density of air is 1.293 kg m^{-3} and specific heat at constant volume is 169 cal kg^{-1} - K^{-1}. Find the value of specific heat at constant pressure.*

Sol. Density $= \frac{\text{Mass}}{\text{Volume}}$ or Volume $= \frac{\text{Mass}}{\text{Density}}$

Volume of 1 kg of air, $V = \frac{1}{1.293}$

Normal pressure that is 76 cm mercury column pressure

$$p = h\rho g = 76 \times 10^{-2} \times 13.6 \times 10^3 \times 9.8 \text{ Nm}^{-2}$$

Normal temperature, $T = 273$ K

For 1 kg air, $pV = RT$

$$R = \frac{pV}{T} = \frac{(76 \times 10^{-2} \times 13.6 \times 10^3 \times 9.8)\left(\frac{1}{1.293}\right)}{273}$$

$$= \frac{76 \times 136 \times 98 \times 10^{-1}}{273 \times 1.293} = 286.96 \text{ Jkg}^{-1}\text{- K}^{-1}$$

From Mayor's formula,

$$C_p = C_V + \frac{R}{J} \qquad (\text{where, } J = 4.2 \text{ J cal}^{-1})$$

$$= 169 + \frac{286.96}{4.2} = 169 + 68.32$$

$$= 273.32 \text{ cal kg}^{-1}\text{- K}^{-1}$$

Example 15.22 *Find the work done in increasing temperature of nitrogen at constant pressure from 37°C to 337°C. C_p = 7 cal/mol °C, in this process what is the work done by the gas?*

Sol. Heat given to increases temperature of n mole of a gas by dT

$$\Delta Q = nC_pdT$$

$$= 0.2 \times 7(337 - 37)$$

$$= 0.2 \times 7 \times 300 = 420 \text{ cal}$$

Equivalent work done on nitrogen

$$\Delta W = J\Delta Q = 4.2 \times 420 = 1764 \text{ J}$$

Change in internal energy,

$$\Delta U = nC_V\Delta T$$

$$\Rightarrow \quad \Delta U = 0.2 \times 5 \times (337 - 37)$$

$$(\because C_V = C_p - R = 7 - 2 = 5 \text{ cal/mol °C})$$

$$= 0.2 \times 5 \times 300 = 300 \text{ cal}$$

Work done by the gas,

$$\Delta W = \Delta Q - \Delta U = 420 - 300 = 120 \text{ cal}$$

$$= 120 \times 4.2 = 504 \text{ J}$$

Example 15.23 *An ideal gas has a specific heat or molar heat capacity at constant pressure $C_p = 5R/2$. The gas is kept in a closed vessel of volume $0.0083\ m^3$ at a temperature of 300 K and a pressure of $1.6 \times 10^6\ Nm^{-2}$. An amount of 2.49×10^4 J of heat energy is supplied to the gas. Calculate the final temperature and pressure of the gas.*

Sol. According to the Mayor's formula

$$C_V = C_p - R = \frac{5R}{2} - R = \frac{3R}{2},\ \Delta V = 0,\ T_1 = 300\text{ K},$$

$$V = 0.0083\text{ m}^3,\ p_1 = 1.6\times 10^6\text{ Nm}^{-2}$$

From first law of thermodynamics $Q = \Delta U + p\Delta V$

$$\Rightarrow \quad \Delta U = Q = 2.49 \times 10^4\text{ J}$$

From gas equation $n = \frac{pV}{RT} = \frac{1.6\times 10^6 \times 0.0083}{8.3\times 300} = \frac{16}{3}$

$$\therefore \quad \Delta U = nC_V\Delta T$$

$$\Rightarrow \quad \Delta T = \frac{\Delta U}{nC_V} = \frac{2.49\times 10^4 \times 6}{3\times 8.3\times 16} = 375\text{ K}$$

Final temperature $= 300 + 375 = 675$ K

As volume remains constant, $p \propto T \Rightarrow \frac{p_2}{p_1} = \frac{T_2}{T_1}$

$$\Rightarrow \quad p_2 = \frac{T_2}{T_1} \times p_1 = \frac{1.6 \times 10^6 \times 675}{300}$$

$$= 3.6 \times 10^6\text{ Nm}^{-2}$$

Example 15.24 *Find molar specific heat for the process $p = \frac{a}{T}$ for a monoatomic gas. $\left(a = \text{constant}, C_V = \frac{3}{2}R\right)$.*

Sol. From first law of thermodynamics,

$$\Delta Q = \Delta W + \Delta U$$

$$\Rightarrow \quad C = \frac{\Delta Q}{\Delta T} = \frac{\Delta W}{\Delta T} + \frac{\Delta U}{\Delta T}$$

or $$C = \frac{\Delta W}{\Delta T} + C_V \qquad (\because \Delta U = C_V \Delta T)$$

$$= \frac{p\Delta V}{\Delta T} + C_V$$

From gas equation,

$$pV = RT$$

$$\Rightarrow \quad V = \frac{RT}{p} = \frac{RT^2}{a} \qquad \left(\because p = \frac{a}{T}\right)$$

$$\Rightarrow \quad \frac{\Delta V}{\Delta T} = \frac{dV}{dT} = \frac{2RT}{a}$$

So, $$C = \frac{p\Delta V}{\Delta T} + C_V = p\left(\frac{2RT}{a}\right) + C_V$$

$$= \frac{a}{T}\left(\frac{2RT}{a}\right) + C_V$$

$$\Rightarrow \quad C = 2R + C_V = 2R + \frac{3}{2}R = \frac{7}{2}R$$

Thermodynamic parameters for a mixture of gases

Thermodynamic parameters for a mixture of gases are given below

(*i*) **Equivalent molar mass** When n_1 moles of a gas with molar mass M_1 are mixed with n_2 moles of a gas with molar mass M_2, the equivalent molar mass of the mixture is given by $M = \frac{n_1M_1 + n_2M_2}{n_1 + n_2}$

(*ii*) **Internal energy of the mixture** The total energy of the mixture is $U = U_1 + U_2$ ...(i)

(*iii*) **C_V of the mixture**

As, $U = U_1 + U_2 \Rightarrow \Delta U = \Delta U_1 + \Delta U_2$

or $nC_V\Delta T = n_1 C_{V_1}\Delta T + n_2 C_{V_2}\Delta T$ ...(ii)

or $(n_1 + n_2)\, C_V = n_1\, C_{V_1} + n_2 C_{V_2}$ (as $n = n_1 + n_2$)

$$\therefore \quad C_V = \frac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2}$$

(*iv*) **C_p of the mixture**

As, $C_p = C_V + R$ or $C_p = \frac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2} + R$

$$= \frac{n_1\,(C_{V_1} + R) + n_2\,(C_{V_2} + R)}{n_1 + n_2} = \frac{n_1 C_{p_1} + n_2 C_{p_2}}{n_1 + n_2}$$

Thus, $C_p = \frac{n_1 C_{p_1} + n_2 C_{p_2}}{n_1 + n_2}$

(*v*) **γ of the mixture** From Eq. (i),

$$(n_1 + n_2)\, C_V = n_1 C_{V_1} + n_2 C_{V_2}$$

or $$\frac{(n_1 + n_2)\, R}{\gamma - 1} = \frac{n_1 R}{\gamma_1 - 1} + \frac{n_2 R}{\gamma_2 - 1}$$

or $$\frac{n_1 + n_2}{\gamma - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1}$$

Thus, γ of the mixture is given by above equation.

Example 15.25 *A gaseous mixture enclosed in a vessel consists of 1g mole of a gas A $\left(\gamma = \frac{5}{3}\right)$ and some amount of gas B $\left(\gamma = \frac{7}{5}\right)$ at a temperature T. The gases A and B do not react with each other and are assumed to be ideal. Find number of gram moles of the gas B, if γ for the gaseous mixture is $\left(\frac{19}{13}\right)$.*

Sol. Using formula for C_V of a mixture of two gases, we have

$$\Rightarrow (C_V)_{mix} = \frac{n_A(C_V)_A + n_B(C_V)_B}{n_A + n_B} \Rightarrow \frac{13}{6}R = \frac{1 \times \frac{3}{2}R + n_B \times \frac{5}{2}R}{1 + n_B}$$

$$\Rightarrow 13 + 13\,n_B = 9 + 15\,n_B \Rightarrow n_B = 2\text{ g-mol}$$

Check point 15.2

1. If 1000 cal of heat is added to 4 mole of a monoatomic gas without changing its volume, then change in temperature of the gas is
   (a) 233 K (b) 833 K
   (c) 123 K (d) 433 K
2. The specific heat at constant pressure is greater than that of the same gas at constant volume because
   (a) at constant valume work is done in expanding the gas
   (b) at constant pressure work is done in expanding the gas
   (c) the molecular attraction is more at constant pressure
   (d) the molecular vibration is more at constant pressure
3. For hydrogen gas $C_p - C_V = a$ and for oxygen gas $C_p - C_V = b$ then the relation between a and b is (where C_p and C_V are gram specific heats)
   (a) $a = 16b$ (b) $b = 16a$
   (c) $a = b$ (d) None of these
4. The molar specific heats of an ideal gas at constant pressure and volume are denoted by C_p and C_V respectively. If $\gamma = C_p/C_V$ and R is the universal gas constant, then C_V is equal to
   (a) $\frac{1+\gamma}{1-\gamma}$ (b) $\frac{R}{(\gamma-1)}$ (c) $\frac{(\gamma-1)}{R}$ (d) γR
5. The molar heat capacity of a gas at constant volume is C_V. If n moles of the gas undergo ΔT change in temperature, its internal energy will change by $nC_V\,\Delta T$
   (a) only if the change of temperature occurs at constant volume
   (b) only if the change of temperature occurs at constant pressure
   (c) in any process which is not adiabatic
   (d) in any process
6. Two moles of a gas at temperature T and volume V are heated to twice its volume at constant pressure. If $C_p/C_V = \gamma$ then increase in internal energy of the gas is
   (a) $\frac{RT}{\gamma-1}$ (b) $\frac{2RT}{\gamma-1}$
   (c) $\frac{2RT}{3(\gamma-1)}$ (d) $\frac{2T}{\gamma-1}$
7. A gas mixture consists of 2 moles of oxygen and 4 moles of argon at temperature T. Neglecting all vibrational modes, the total internal energy of the system is
   (a) $4RT$ (b) $15RT$
   (c) $9RT$ (d) $11RT$
8. Five moles of helium are mixed with two moles of hydrogen to form a mixture. Take molar mass of helium $M_1 = 4$ g and that of hydrogen $M_2 = 2$ g. The equivalent molar mass of the mixture is
   (a) 6g (b) $\frac{13g}{7}$
   (c) $\frac{18g}{7}$ (d) $\frac{24g}{7}$
9. The equivalent value of γ in the above question is
   (a) 1.59 (b) 1.53
   (c) 1.56 (d) None of these
10. A gaseous mixture enclosed in a vessel consists of 3 g mole of a gas X with $\gamma = (5/3)$ and some amount of gas Y with $\gamma = (7/5)$ at a temperature T. The gases X and Y do not react with each other and are assumed to be ideal. If γ for the gaseous mixture is $\left(\frac{29}{23}\right)$ than the number of gram moles of the gas Y is
    (a) 0.5 (b) 0.4 (c) 0.8 (d) 1

4 Thermodynamic processes

When state of a thermodynamic system changes or the state variables changes with time, then the process is known as **thermodynamic process**.

Few of thermodynamic processes, are discussed below.

1. Isobaric process

A thermodynamic process occurring at constant pressure, is known as isobaric process. *e.g.* Boiling of water in an open container.

Equation of state

In this process, volume V and temperature T changes but pressure p remains constant.

$$\frac{V}{T} = \text{constant} \qquad \text{(Charles' law)}$$

Graphs

Graphs related to isobaric process are given below.

p-V graph

In an isobaric process, pressure of the gas remains constant.

Therefore, p-V graph associated with this process is a straight line parallel to V-axis (having slope, $\frac{dp}{dV} = 0$) as shown below.

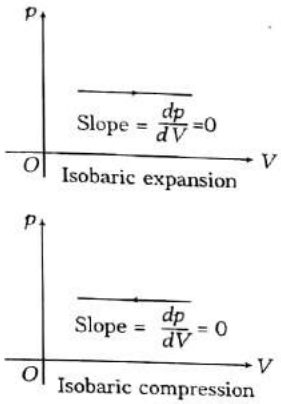


Fig. 15.7 p-V graph for isobaric expansion and compression

V-T graph

In isobaric process, $\frac{V}{T} = \text{constant}$

$\therefore \quad V \propto T$

i.e., V-T graph is a straight line passing through origin as shown below.

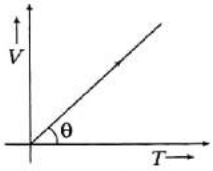


Fig. 15.8 V-T graph for isobaric process

Note Slope of the V-T graph, $\tan\theta = \frac{V}{T} = \frac{nR}{p} = \text{constant}$.

p-T graph

In isobaric process, pressure remains constant so volume changes resulting in change of temperature, as shown below.

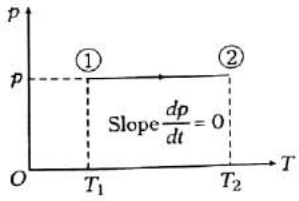


Fig. 15.9 p-T graph for isobaric process

Work done

Suppose the pressure p of a gas remains constant and its volume changes from V_i to V_f, then the work done by the gas is

$$\text{Work done, } \boldsymbol{W = p(V_f - V_i)}$$

or

$$\text{Work done, } \boldsymbol{W = nR(T_f - T_i)}$$

First law of thermodynamics for isobaric process

For isobaric process, $\Delta W = p\Delta V$

or $\quad \Delta W = nR\Delta T$

From $\quad \Delta Q = \Delta U + \Delta W$

So, $\quad \boldsymbol{\Delta Q = \Delta U + nR\Delta T}$

Example 15.26 *Find the ratio of $\frac{\Delta Q}{\Delta U}$ and $\frac{\Delta Q}{\Delta W}$ in an isobaric process. The ratio of molar heat capacities $\frac{C_p}{C_V} = \gamma$.*

Sol. In an isobaric process, p = constant. Therefore, $C = C_p$.

Now,
$$\frac{\Delta Q}{\Delta U} = \frac{nC_p\Delta T}{nC_V\Delta T} = \frac{C_p}{C_V} = \gamma$$

and
$$\frac{\Delta Q}{\Delta W} = \frac{\Delta Q}{\Delta Q - \Delta U}$$
$$= \frac{nC_p\Delta T}{nC_p\Delta T - nC_V\Delta T}$$
$$= \frac{C_p}{C_p - C_V} = \frac{C_p / C_V}{C_p / C_V - 1}$$
$$= \frac{\gamma}{\gamma - 1}$$

Example 15.27 Suppose 1.0 *g of water vaporises isobarically at atmospheric pressure* (1.01×10^5 Pa). *Its volume in the liquid state is* $V_i = V_{\text{liquid}} = 1.0\ \text{cm}^3$ *and its volume in vapour state is* $V_f = V_{\text{vapour}} = 1671\ cm^3$. *Find the work done in the expansion and the change in internal energy of the system. Ignore any heat transfer of the steam and the surrounding air. Take latent heat of vaporisation*

$$L_v = 2.26 \times 10^6\ Jkg^{-1}$$

Sol. Because the expansion takes place at constant pressure, the work done is

$$W = \int_{V_i}^{V_f} p_0 dV = p_0 \int_{V_i}^{V_f} dV = p_0 (V_f - V_i)$$
$$= (1.01 \times 10^5)(1671 \times 10^{-6} - 1.0 \times 10^{-6})$$
$$= 169\ \text{J}$$

Heat of vaporisation

$$Q = mL_v = (1.0 \times 10^{-3})(2.26 \times 10^6) = 2260\ \text{J}$$

Hence, from the first law, the change in internal energy

$$\Delta U = Q - W = 2260 - 169 = 2091\ \text{J}$$

Note The positive value of ΔU indicates that the internal energy of the system increases. We see that most $\left(\frac{2091\ \text{J}}{2260\ \text{J}} = 93\%\right)$ of the energy transferred to the liquid goes into increasing the internal energy of the system only $\frac{169\ \text{J}}{2260\ \text{J}} = 7\%$ leaves the system by work done by the steam in pushing the surrounding atmosphere.

2. Isochoric process

A thermodynamic process occurring at constant volume, is known as isochoric process. *e.g.*, Heat given to a system with fixed walls.

Equation of state

In this process, p and T changes but V = constant,

Gay-Lussac's law is obeyed, $\boldsymbol{p \propto T \Rightarrow \frac{p}{T} = \text{constant}}$

Graph

Graphs related to isobaric process are given below.

p-V Graph

In an isochoric process, volume of the gas remains constant. Therefore, p-V graph is a straight line parallel to p-axis with slope, $\frac{dp}{dV} = \tan\frac{\pi}{2} = \infty$ as shown below.

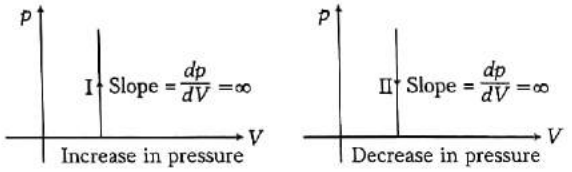


Fig. 15.10 Graph for isochoric process

p-T Graph

For isochoric process, $\frac{p}{T}$ = constant.

$\therefore \quad p \propto T$

i.e., p-T graph is a straight line passing through origin.

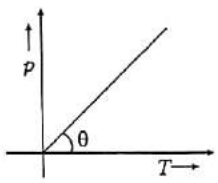


Fig. 15.11 p-T Graph for isochoric process

Note Slope of the p-T graph $\tan\theta = \frac{p}{T} = \frac{nR}{V}$ = constant.

V-T Graph

In isochoric process volume remains constant. So pressure changes resulting in change of temperature as shown below.

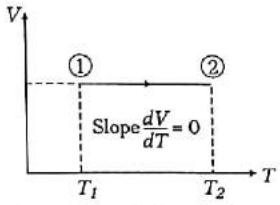


Fig. 15.12 V-T Graph for isochoric process

Work done

As volume remains constant

i.e., $\quad V$ = constant

$\Rightarrow \quad \Delta V = 0$

$\therefore$ Work done, $\boldsymbol{\Delta W = p\Delta V = 0}$

First law of thermodynamics for isochoric process

$$\Delta Q = nC_V\Delta T$$

From first law of thermodynamics

We get, $\quad \Delta Q = \Delta U + \Delta W$

As, $dW = 0$ (for isochoric process),

$$\Delta Q = \Delta U = nC_V\Delta T$$

For one mole of a gas $n = 1$,

$$\Rightarrow \quad \boldsymbol{\Delta Q = C_V\Delta T = \Delta U}$$

Example 15.28 *What is the heat input needed to raise the temperature of 2 moles of helium gas from 0°C to 100°C (i) at constant volume? (ii) at constant pressure? (iii) What is the work done by the gas in part (ii) Give your answer in terms of R?*

Sol. Helium is monoatomic gas. Therefore,

$$C_V = \frac{3R}{2} \text{ and } C_p = \frac{5R}{2}$$

(i) At constant volume,

$$Q = nC_V\Delta T = (2)\left(\frac{3R}{2}\right)(100) = 300R$$

(ii) At constant pressure,

$$Q = nC_p\,\Delta T = (2)\left(\frac{5R}{2}\right)(100) = 500R$$

(iii) At constant pressure,

$$\begin{aligned} W &= Q - \Delta U \\ &= nC_p\Delta T - nC_V\Delta T \\ &= nR\Delta T = (2)(R)(100) \\ &= 200R \end{aligned}$$

3. Isothermal process

A thermodynamic process occuring at constant temperature, is known as isothermal process. *e.g.* Freezing of water at 0°C to form ice at 0°C.

Equation of state

For an ideal gas, the equation of state for isothermal process is given by

$$\boldsymbol{pV = \text{constant}}$$

So, gas follows Boyle's law

$$p_iV_i = p_fV_f, \text{ for isothermal process}$$

Note Slow processes are treated as isothermal.

Graph

Graphs related to isothermal process are given below.

p-V Graph

For isothermal process p-V = constant.

Therefore, p-V graph is a rectangular hyperbola as shown below.

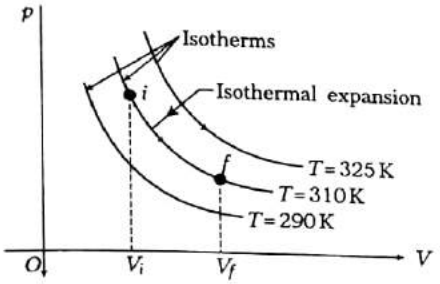


Fig. 15.13 An isothermal expansion process. An isotherm is a curve that connects points of same temperature

V-T Graph

Temperature of the gas remains constant during isothermal process. Thus, V-T graph is parallel to V-axis as shown in figure.

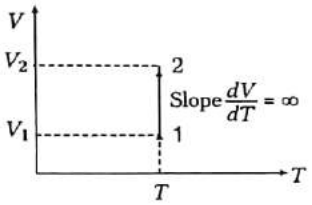


Fig. 15.14 V-T Graph for isothermal process

p-T Graph

Similar to V-T graph, p-T graph is parallel to p-axis.

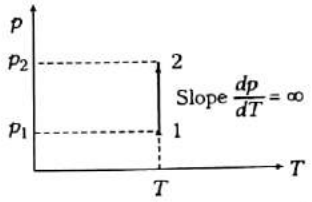


Fig. 15.15 p-T graph for isothermal process

Slope of p-V graph

For isothermal process, pV = constant

Differentiating, we get

$$pdV + Vdp = 0$$

$$\Rightarrow \quad \frac{dp}{dV} = -p/V$$

$$\text{Slope of graph} = -\tan\phi = \frac{dp}{dV} = -\frac{p}{V}$$

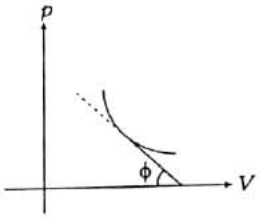


Fig. 15.16 Slope of Isothermal Curve

Specific and molar heat capacity

During isothermal process, $\Delta T = 0$

$\therefore$ Specific heat capacity, $C = \dfrac{\Delta Q}{m\Delta T} = \dfrac{\Delta Q}{m(0)} = \infty$

$\therefore$ Molar heat capacity, $C_{\text{iso}} = \dfrac{\Delta Q}{n\Delta T} = \infty$

Work done

Work done $W = \int_{V_i}^{V_f} p\, dV = \int_{V_i}^{V_f} \left(\dfrac{nRT}{V}\right) dV \quad \left(\text{as } p = \dfrac{nRT}{V}\right)$

$$= nRT \int_{V_i}^{V_f} \frac{dV}{V} \qquad (\text{as } T = \text{constant})$$

$$\boldsymbol{W = nRT \ln\left(\frac{V_f}{V_i}\right)}$$

or $$\boldsymbol{W = nRT \ln\left(\frac{p_i}{p_f}\right)} \qquad (\text{as } p_iV_i = p_fV_f)$$

Note • If $V_f > V_i$, then $W > 0$, hence $Q > 0$ i.e., gas absorbs heat and work is done by the gas on the surrounding during expansion.

• If $V_f < V_i$, then $W < 0$, hence $Q < 0$ i.e, during compression, work is done on the gas by the surrounding and heat is relased.

First law of thermodynamics for isothermal process

As temperature remains constant in isothermal process, so.

$\therefore \qquad \Delta U = 0$

So, by first law of thermodynamics,

$$\Delta Q = \Delta W + \Delta U$$

$$\Rightarrow \quad \Delta Q = \Delta W = nRT \ln\left(\frac{V_f}{V_i}\right)$$

$$= 2.3026\, nRT \log_{10}\left(\frac{V_f}{V_i}\right)$$

where, n = number of moles

R = gas constant, T = temperature

Example 15.29 *Five moles of an ideal gas at constant temperature 1200K is compressed from 200 L to 20L. Evaluate the work done in the process.*

Sol. It is given that, number of moles, $n = 5$

Temperature of the gas, $T = 1200$ K

$\therefore$ Work done in an isothermal process,

$$W = 2.3026\, nRT \log_{10}\left(\frac{V_2}{V_1}\right)$$

$$= 2.3026 \times 5 \times 8.3 \times 1200 \log_{10}\left(\frac{20}{200}\right)$$

$$= -11.4 \times 10^4 \text{ J}$$

Example 15.30 *If 1 g mole of a gas is expanded isothermally at 127 °C such that its volume increases two times, then find the work done and heat absorbed by the gas.*

Sol. Given, temperature of the gas

$T = 127°\text{C} = 127 + 273 = 400$ K

Initial volume, $V_1 = V$ (assume)

Final volume $V_2 = 2V$

Work done, $W = 2.303\, nRT \log_{10}\left(\frac{V_2}{V_1}\right)$

$$= 2.303 \times 1 \times 8.3 \times 400 \log_{10}\left(\frac{2V}{V}\right)$$

$$= 2.303 \times 8.3 \times 400 \times 0.3010 = 2301 \text{ J}$$

For isothermal process, $dU = 0$

$$\Delta Q = dU + \Delta W = 0 + 2301 = 2301 \text{ J}$$

$$\Rightarrow \quad \Delta Q = \frac{2301}{4.2} = 547.9 \text{ cal}$$

Example 15.31 *2 g mole ideal gas is expanded isothermally at 27 °C. Its volume increases three times of initial volume. Find the work done and heat absorbed by the gas. Take, $R = 8.3\ J\ mol^{-1}\text{-}K^{-1}$*

Sol. Work done by n mole of an ideal gas at isothermal temperature T, when its volume changes from V_i to V_f is

$$W = 2.302\, nRT \log_{10} \frac{V_f}{V_i}$$

According to the question

$$n = 2 \text{ g-mole}, T = 273 + 27 = 300 \text{ K}$$

and $\quad V_f / V_i = 3$

$$W = 2.3026 \times 2 \text{ g-mole} \times 8.314 \text{ J mol}^{-1}\text{-K} \times 300 \times \log_{10} 3$$

$$= 2.3026 \times 2 \times 8.31 \times 300 \times 0.4771 = 5.48 \times 10^3 \text{ J}$$

Now, $W = JQ$

$\therefore$ Absorbed heat $Q = \frac{W}{J}$

$$= \frac{5.48 \times 10^3 \text{ J}}{4.18 \text{ J cal}^{-1}} = 1.31 \times 10^3 \text{ cal}$$

4. Adiabatic process

A thermodynamic process which takes place in such a way that there is no transfer of heat into or out of a system.

e.g., Rapid compression like filling of a cycle tube by a hand pump, rapid expansion.

Equation of state

For an ideal gas, equation of state for adiabatic process is

$$\boldsymbol{pV^\gamma = \text{constant}}$$

Proof of the *p-V* relation

In adiabatic process, $dQ = 0$

and $\quad dW = -dU$

$\therefore \quad pdV = -C_V dT \quad$ (for $n = 1$)

$\therefore \quad dT = -\frac{pdV}{C_V} \quad$ ...(i)

Also for 1 mole of an ideal gas,

$$d(pV) = d(RT) \quad \text{or} \quad pdV + Vdp = RdT$$

or $\quad dT = \frac{pdV + Vdp}{R} \quad$ ...(ii)

From Eqs. (i) and (ii), we get

$$C_V Vdp + (C_V + R)\, pdV = 0$$

or $\quad C_V Vdp + C_p pdV = 0$

Dividing this equation by pV, we are left with

$$C_V \frac{dp}{p} + C_p \frac{dV}{V} = 0$$

or $\quad \frac{dp}{p} + \gamma \frac{dV}{V} = 0 \quad \text{or} \quad \int \frac{dp}{p} + \gamma \int \frac{dV}{V} = 0$

or $\quad \ln(p) + \gamma \ln(V) = \text{constant}$

or $\quad \ln(p) + \ln(V^\gamma) = \text{constant}$

or $\quad \ln(pV^\gamma) = \text{constant}$

We can write this in the form,

$$\boldsymbol{pV^\gamma = \text{constant}}$$

This equation is the condition that must be obeyed by an ideal gas in an adiabatic process. *e.g.*, if an ideal gas makes an adiabatic transition from a state with pressure and volume p_i and V_i to a state with p_f and V_f, then

$$\boldsymbol{p_i V_i^\gamma = p_f V_f^\gamma}$$

The equation pV^γ = constant can be written in terms of other pairs of thermodynamic variables by combining it with the ideal gas law ($pV = nRT$).

Putting $p = \frac{RT}{V}$ (From ideal gas equation)

we get $TV^{\gamma-1} = \text{constant}$

Again putting $V = \frac{RT}{p}$ (From ideal gas equation)

we get $T^{\gamma}p^{1-\gamma} = \text{constant}$

Graphs

Graphs related to adiabatic process are given below.

p-V graph

For no heat change in adiabatic process, pressure decreases exponentially with increase in volume as shown below.

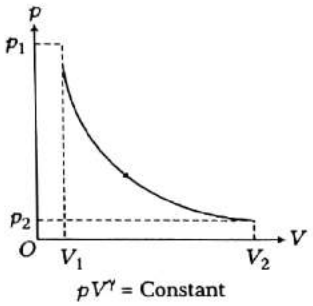


***Fig. 15.17** p-V Graph for adiabatic process.*

V-T graph

In adiabatic process, with increase in temperature volume decreases exponentially as shown below.

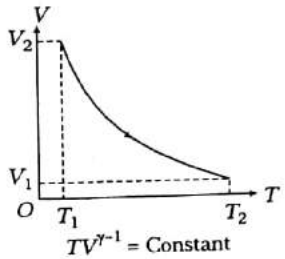


***Fig. 15.18** V-T Graph for adiabatic process*

p-T graph

For adiabatic process, with decrease in temperature pressure decreases as shown below.

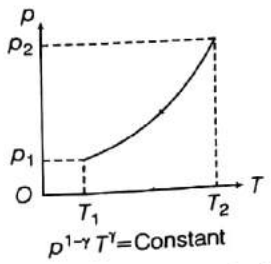


***Fig. 15.19** p-T Graph for adiabatic process*

Slope of p-V graph

In an adiabatic process (pV^{γ} = constant), the slope of p-V diagram at any point is

$$\frac{dp}{dV} = -\tan\phi = \frac{d}{dV}\left(\frac{\text{constant}}{V^{\gamma}}\right) = -\gamma\left(\frac{p}{V}\right)$$

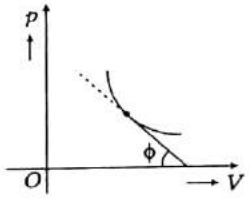


Fig. 15.20

Thus, $$\boxed{(\text{Slope})_{\text{adiabatic}} = -\gamma\left(\frac{p}{V}\right)}$$

Also we know that in an isothermal process $(\text{Slope})_{\text{isothermal}} = -\frac{p}{V}$. Because $\gamma > 1$, the isothermal curve is not as steep as that for the adiabatic expansion.

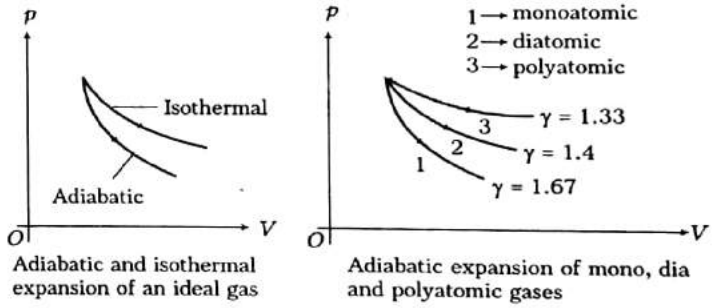


***Fig. 15.21** Adiabatic and isothermal expansion*

Note In similar way for adiabatic and isothermal compression of an ideal gas p-V diagram is as shown, the curve which is more vertical is adiabatic.

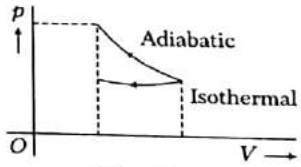


Fig. 15.22

Example 15.32 *Initial pressure of a gas is 5×10^5 Nm^{-2}. Under an adiabatic process its volume becomes 1/9 of its initial value. What is the pressure of the gas in this situation? (Take, $\gamma = 3/2$)*

Sol. For adiabatic process of the gas, $p_1V_1^{\gamma} = p_2V_2^{\gamma} \Rightarrow p_2 = p_1\left(\frac{V_1}{V_2}\right)^{\gamma}$

Given, $\gamma = 3/2$, $V_1 = V$, $V_2 = \frac{V}{9}$ and $p_1 = 5 \times 10^9$ Nm^{-2}

$$\therefore \quad p_2 = 5 \times 10^5 \times \left(\frac{V}{V/9}\right)^{3/2}$$

$$= 5 \times 10^5 \times (9)^{3/2} = 5 \times 10^5 \times (3^2)^{3/2}$$
$$= 5 \times 10^5 \times 3^3 = 135 \times 10^5 \text{ Nm}^{-2}$$

Example 15.33 *Two samples of a gas initially at same temperature and pressure are compressed from a volume V to V/2. One sample is compressed isothermally and the other adiabatically. In which sample is the pressure greater?*

Sol. Let initial volume, $V_1 = V$ and pressure, $p_1 = p$,

Final volume, $V_2 = \frac{V}{2}$ and final pressure, $p_2 = ?$

For isothermal compression $p_2V_2 = p_1V_1$ or $p_2 = \frac{p_1V_1}{V_2} = \frac{pV}{V/2} = 2p$

For adiabatic compression, $p_2' = p_1\left[\frac{V_1}{V_2}\right]^\gamma$

$$\Rightarrow \quad p_2' = p\left[\frac{V}{V/2}\right]^\gamma = 2^\gamma p \Rightarrow p_2' = 2^\gamma p,$$

$\because \quad \gamma > 1 \therefore 2^\gamma > 2$ and $p_2' > p_2$

Pressure during adiabatic compression is greater than the pressure during isothermal compression.

Example 15.34 *An ideal monoatomic gas at 300 K expands adiabatically to twice its volume. What is the final temperature?*

Sol. For an ideal monoatomic gas, $\gamma = 5/3$

In an adiabatic process, $TV^{\gamma-1} = \text{constant}$

$$\therefore \quad T_f V_f^{\gamma-1} = T_i V_i^{\gamma-1}$$

$$\text{or} \quad T_f = T_i\left(\frac{V_i}{V_f}\right)^{\gamma-1} = (300)\left(\frac{1}{2}\right)^{\frac{5}{3}-1} \approx 189 \text{ K}$$

Specific and molar heat capacity

During adiabatic process, $\Delta Q = 0$

$\therefore$ Specific heat capacity, $C = \frac{\Delta Q}{n\Delta T} = 0$

$\therefore$ Molar heat capacity, $C_{\text{adi}} = \frac{\Delta Q}{n\Delta T} = 0$

Work done

Let initial state of gas is (p_1, V_1, T_1) and after adiabatic change final state of gas is (p_2, V_2, T_2), then work done

$$W = \frac{p_1V_1 - p_2V_2}{\gamma - 1}$$

On putting $pV = nRT$, we get

$$\Rightarrow \quad W = \frac{nR}{(\gamma - 1)}(T_1 - T_2)$$

Note

1. If work is done by the gas in an adiabatic process ($W > 0$), then $T_2 < T_1$.
2. If work is done on the gas ($W < 0$), we get $T_2 > T_1$, i.e., the temperature of the gas rises.

First law of thermodynamics for adiabatic process

Now, by first law of thermodynamics, $\Delta Q = \Delta U + \Delta W$

As, $\quad \Delta Q = 0 \Rightarrow \Delta U = -\Delta W$

So, if work is done by the system, on the expense of internal energy and so the temperature of system falls *i.e.*, $W > 0$, $T_f < T_i$. Conversely, if work is done on the system, the internal energy and so the temperature of system increases. *i.e.*, $W < 0$, $T_f > T_i$

Bulk modulus of gas

Bulk modulus of gas is given by

$$B = -\frac{dp}{(dV/V)} \quad \text{or} \quad B = -V\left(\frac{dp}{dV}\right)$$

where $\frac{dp}{dV}$ is slope of p-V graph.

For isothermal process, $\frac{dp}{dV} = \frac{p}{V}$

$\therefore$ Bulk modulus of gas $B = -V\left(\frac{p}{V}\right)$

$$\boxed{B = -p}$$

For adiabatic process, $\frac{dp}{dV} = \frac{\gamma p}{V}$

$\therefore$ Bulk modulus of gas $B = -V\left(\frac{\gamma p}{V}\right)$ or $B = -\gamma p$

Therefore, isothermal bulk modulus is equal to pressure of the gas and adiabatic bulk modulus is γ times the pressure of gas.

p-V Diagram of Different Processes

p-V diagram of different processes is shown as follows. We can compare the work done in each process from area under p-V curve.

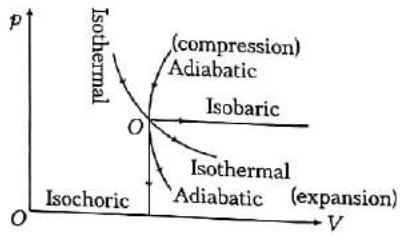


Fig. 15.23 *Combined p-V diagram of different processes*

We know that in an isothermal process, $\frac{\Delta p}{\Delta V} = -\frac{p}{V}$. For adiabatic process, $\frac{\Delta p}{\Delta V} = -\frac{\gamma p}{V}$ it means that at a particular

point, slope (value) of adiabatic curve is more than that for isotherm or we can say adiabatic curve is more steep than an isotherm for expansion and just reverse for compression. It is clear from the figure that for expansion that occurs within same limits.

$$W_{\text{isobaric}} > W_{\text{isothermal}} > W_{\text{adiabatic}} > W_{\text{isochoric}}$$

Note In general many thermodynamic processes follows a rule of type.

pV^n = constant.

These processes are called polytropic process and 'n' is called polytropic index.

- Work done in a polytropic $(1 \to 2)$ process $\Delta W = \int p\,dv = \frac{p_1V_1 - p_2V_2}{n-1}$
- Heat transfer in a polytropic process $Q = \frac{R(T_1 - T_2)}{n-1} - C_V(T_1 - T_2)$

  where, $C_V = \frac{R}{r-1} \Rightarrow Q = \left(\frac{r-n}{r-1}\right) W = C_n(T_2 - T_1)$
- Specific heat capacity of gas in a polytropic process

  $C_n = C_V + \frac{R}{1-n} = C_V\left(\frac{r-n}{1-n}\right)$

Process	n
p = constant	0
V = constant	∞
T = constant	1
$\Delta Q = 0$	r

Summary of ΔQ, ΔU and ΔW for different processes

Name of the Process	ΔQ	ΔU	ΔW
Isothermal	$\Delta Q = \Delta W$	0	$nRT \ln\left(\frac{V_f}{V_i}\right) = nRT \ln\left(\frac{p_i}{p_f}\right)$
Adiabatic	0	$nC_V\Delta T$	$\frac{p_iV_i - p_fV_f}{\gamma - 1} = -\Delta U$
Isobaric	$nC_p\,\Delta T$	$nC_V\Delta T$	$p(V_f - V_i)$
Isochoric	$\Delta Q = \Delta U = nC_V\Delta T$	$nC_V\Delta T$	0

Example 15.35 *14 g of nitrogen gas is compressed in an adiabatic process such that its temperature increases by 60°C. Find the work done on the gas.*

Sol. Given, $m = 14$ g, $T_2 - T_1 = 60°C$, $\gamma = 1.5$

Work done on the gas,

$$W = \frac{nR}{\gamma - 1}[T_1 - T_2]$$

$$\because \quad n = \frac{m}{M} = \frac{14}{28} = \frac{1}{2}$$

$$\therefore \quad W = \frac{1}{2} \times \frac{8.3}{1 - 1.5} \times 60 = -498 \text{ J}$$

Negative sign shows that work is done on the gas.

Example 15.36 *Temperature of a 5 mole of hydrogen gas decreases by 260 K while expanded through adiabatic process. Find the work done by the gas. What is the change in internal energy of the gas?*

Sol. Work done by the gas, $W = \frac{nR}{\gamma - 1}(T_i - T_f)$

According to the question,

$$n = 5 \text{ mole}, R = 8.13 \text{ J/mol-K}$$

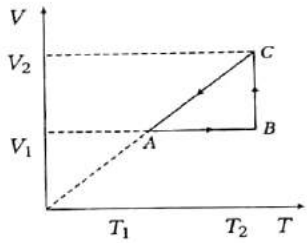
$$\gamma = 1.4,\ T_i = 273 \text{ K}, T_f = 260 \text{ K}$$

$$\text{Work done, } W = \frac{5 \times 8.31}{1.4 - 1}(273 - 260) \text{ K} \approx 1350 \text{ J}$$

According to first law of thermodynamics

$$\Delta U = Q - W = 0 - (1350) = -1350 \text{ J}$$

Example 15.37 *Figure shows a process ABCA performed on an ideal gas. Find the net heat given to the system during the process.*



Sol. Since the process is cyclic, hence the change in internal energy is zero.

The heat given to the system is then equal to the work done by it.

The work done in part AB is $W_1 = 0$ (the volume remains constant). The part BC represents an isothermal process so that the work done by the gas during this part is

$$W_2 = nRT_2 \ln \frac{V_2}{V_1}$$

During the part $CA : V \propto T$. So, V/T is constant and hence, $p = \frac{nRT}{V}$ is constant

The work done by the gas during the part CA is

$$W_3 = p(V_1 - V_2) = nRT_1 - nRT_2$$
$$= -nR(T_2 - T_1).$$

The net work done by the gas in the process $ABCA$ is

$$W = W_1 + W_2 + W_3$$
$$= nR\left[T_2 \ln \frac{V_2}{V_1} - (T_2 - T_1)\right]$$

The same amount of heat is given to the gas.

Example 15.38 *$2m^3$ volume of a gas at a pressure of 4×10^5 Nm^{-2} is compressed adiabaticaly so that its volume becomes 0.5 m^3. Find the new pressure. Compare this with the pressure that would result if the compression was isothermal. Calculate work done in each process. (Given, $\gamma = 1.4$)*

Sol. Given, $V_1 = 2 \text{ m}^3$, $p_1 = 4 \times 10^5 \text{ Nm}^{-2}$, $V_2 = 0.5 \text{ m}^3$

In adiabatic process $p_1V_1^\gamma = p_2V_2^\gamma$

$$\Rightarrow p_2 = 4\times 10^5\left[\frac{2}{0.5}\right]^{1.4} = 4\times 10^5\,(4)^{1.4} = 2.8\times 10^6\ \text{Nm}^{-2}$$

In isothermal process $p_1V_1 = p_2V_2$

$$\Rightarrow \qquad p_2 = \frac{p_1V_1}{V_2} = \frac{4\times 10^5\times 2}{0.5} = 1.6\times 10^6\ \text{Nm}^{-2}$$

Now work done in adiabatic process

$$W = \frac{p_1V_1 - p_2V_2}{\gamma - 1} = \frac{(4\times 10^5\times 2 - 2.8\times 10^6\times 0.5)}{1.4-1}$$

$$= -1.5\times 10^6\ \text{J}$$

Work done in isothermal process

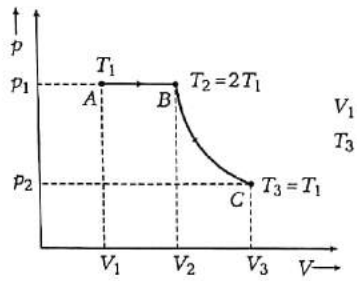
$$W = 2.3026\,RT\log\frac{V_2}{V_1} = 2.3026\,p_1V_1\log\frac{V_2}{V_1}$$

$$= 2.3026\times 4\times 10^5\times 2\times\log\left[\frac{0.5}{2.0}\right]$$

$$= 2.3026\times 4\times 10^5\times 2\log\left(\frac{1}{4}\right) = -1.1\times 10^6\ \text{J}$$

Example 15.39 *Two moles of helium gas ($\gamma = 5/3$), assumed ideal, are initially at 27 °C and occupy a volume of 20 litres. The gas is first expanded at constant pressure till its volume is doubled. It then undergoes an adiabatic change until the temperature returns to its initial value. Determine the final pressure and volume of the gas. Also, calculate the work done under isobaric and adiabatic processes. [$R = 8.3\ J\ mol^{-1}\ K^{-1}$]*

Sol.



$V_1 = 20\times 10^{-3}\ \text{m}^3$,
$T_3 = T_1 = 300$ K

For process AB (p = constant)

$$T_2 = 2T_1 = 600\ \text{K} \qquad (\because V_2 = 2V_1)$$

For process BC $\quad TV^{\gamma-1} = \text{constant}$

Therefore, $(2T_1)(V_2)^{5/3-1} = (T_1)(V_3)^{5/3-1}$

$$\Rightarrow \qquad V_3 = (V_2)(2)^{3/2} = 2\sqrt{2}V_2$$

$$= 80\sqrt{2}\times 10^{-3}\ \text{m}^3 = 8\sqrt{2}\ \text{litre}$$

$$\Rightarrow \qquad p_3 = \frac{nRT_3}{V_3} = \frac{(2)(8.3)(300)}{80\sqrt{2}\times 10^{-3}} = 0.44\times 10^5\ \text{Nm}^{-2}$$

Work done under isobaric process $= p\Delta V = p_1(V_2 - V_1)$

$$= \frac{nRT_1}{V_1}(V_2 - V_1) = \frac{2\times 8.3\times 300}{20\times 10^{-3}}(40\times 10^{-3} - 20\times 10^{-3}) = 4980\ \text{J}$$

Work done under adiabatic process $= \dfrac{nR(T_2 - T_3)}{(\gamma - 1)}$

$$= \frac{2\times 8.3\times (300)}{(5/3 - 1)} = 7470\ \text{J}$$

5. Cyclic process

A single process or a series of processes in which is after certain interchanges of heat and work the system is restored to its initial state are known as a **cyclic process**.

As both initial and final states are same in a cyclic process,

$$\Delta U = U_f - U_i = 0$$

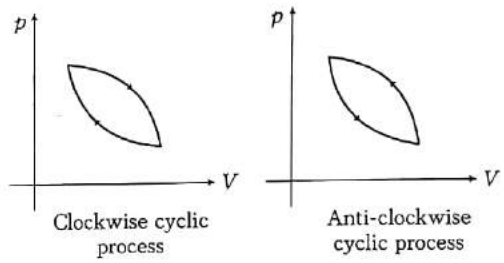


Fig. 15.24 *Cyclic processes*

For a cyclic process, p-V graph is a closed curve and area enclosed by the curve is equal to the work done.

From first law of thermodynamics, $\Delta Q = \Delta U + \Delta W$

For cyclic process, $\Delta U = 0$ or $\Delta Q = \Delta W$

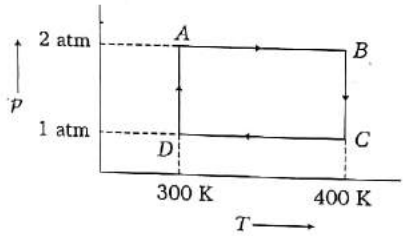
So, **heat supplied to the system is converted into work done and *vice-versa*.** Heat is supplied to the system for clockwise cycle and heat is taken from the system for anti-clockwise cycle.

Example 15.40 *Two moles of helium gas undergo a cyclic process as shown in figure. Assuming the gas to be ideal calculate the following quantities in the process:*

(i) the net work done

(ii) the net change in internal energy

(iii) the net change in heat energy [$R = 8.32\ J\ mol^{-1}K^{-1}$]



Sol. Given, $n = 2$, $C_V = 3R/2$, $C_p = 5R/2$,

$$T_A = T_D = 300\ \text{K},\ T_B = T_C = 400\ \text{K}$$

(i) $A \to B$ (isobaric process) :

$$\Delta W_{A\to B} = p\Delta V = nR\Delta T = 2R(400 - 300) = 200R$$

$B \to C$ (isothermal expansion):

$$\Delta W_{B\to C} = nRT_B\ln(p_B/p_C) = 2R\times 400\ln(2/1) = 800\,R\ln 2$$

$C \to D$ (isobaric process) :

$$\Delta W_{C\to D} = nR\Delta T = 2R(300 - 400) = -200R$$

$D \to A$ (isothermal compression):

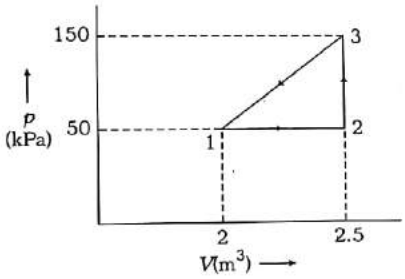
$$\Delta W_{D\to A} = nRT_A \ln(p_D/p_A)$$
$$= 2R \times 300 \ln(1/2) = -600\,R\ln 2$$
$$\Delta W_{\text{cyclic}} = \Delta W_{AB} + \Delta W_{BC} + \Delta W_{CD} + \Delta W_{DA}$$
$$= 200R + 800R\ln 2 - 200R - 600\,R\ln 2$$
$$= 200R\ln 2 = 200 \times 8.3 \times 0.693 = 1150\text{ J}$$

(ii) For cyclic process, $\Delta T = 0$, $\Delta U = 0$

(iii) For cyclic process, $\Delta Q = \Delta W = 1150$ J

Example 15.41 *A gas is initially at a pressure of 50 kPa. Its pressure is kept constant and the volume is changed from 2 m^3 to 2.5 m^3. Its volume is now kept constant and the pressure is increased from 50 kPa to 150 kPa. The gas is brought back to its initial state via a process in which, the pressure varies linearly with its volume. Whether the heat is supplied to or extracted from the gas in the complete cycle?*

Sol. First draw p-V diagram as shown below.



According to the p-V diagram, work done

$$\Delta W_{\text{cyclic}} = -\frac{1}{2}(150-50)\times 10^3 \times (2.5-2.0)$$
$$= -25000\text{ J}$$

In the cyclic process

$$\Delta Q = \Delta W = -25000\text{ J}$$

Negative sign shows that heat is extracted from the gas.

Efficiency of a cycle

Efficiency (η) of a cycle can be defined as

$$\boxed{\eta = \left(\frac{\text{Work done by the working substance during a cycle}}{\text{Heat supplied to the gas during the cycle}}\right) \times 100}$$

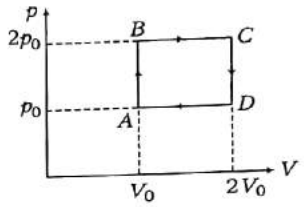
$$= \frac{W_{\text{total}}}{|Q_{\text{absorbed}}|} \times 100 = \frac{|Q_{\text{absorbed}}| - |Q_{\text{relased}}|}{|Q_{\text{absorbed}}|} \times 100$$
$$= \left\{1 - \left|\frac{Q_{\text{relased}}}{Q_{\text{absorbed}}}\right|\right\} \times 100$$

Thus, $$\eta = \frac{W_{\text{total}}}{|Q_{\text{absorbed}}|} \times 100 = \left\{1 - \left|\frac{Q_{\text{relased}}}{Q_{\text{absorbed}}}\right|\right\} \times 100$$

There cannot be a cycle whose efficiency is 100%. Hence, η is always less than 100%.

Thus, $$W_{\text{Total}} \neq Q_{\text{absorbed}}$$

Example 15.42 *N moles of a monoatomic gas is carried round the rectangular cycle ABCDA as shown in the diagram. The temperature at A is T_0. Find the thermodynamic efficiency of the cycle.*



Sol. First of all note that point C is at highest temperature and A is at lowest temperature. So during process A to B and B to C heat is being added to the cycle. Work output can be calculated by calculating the area under the cycle of p-V diagram. Then efficiency can be calculated as work output upon heat input. Note that the temperature at A is T_0, that at B is $2T_0$ and that at C is $4T_0$. Also note C_V for monoatomic gas is $3R/2$. For A-B

$$\Delta Q = NC_V\Delta T \qquad (\because \text{ constant volume process})$$
$$= N\frac{3R}{2}(2T_0 - T_0) = \frac{N}{2}3RT_0$$

For B-C, $$\Delta Q = NC_p\Delta T \qquad (\because \text{ constant pressure process})$$
$$= N\frac{5R}{2}(4T_0 - 2T_0) = 5NRT_0$$

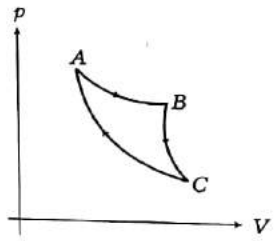
Now, Total heat input $Q = \left(\frac{3}{2} + 5\right)NRT_0 = \frac{13}{2}NRT_0$

Work output = Area under the cycle

$$= (2p_0 - p_0)(2V_0 - V_0) = p_0V_0 = NRT_0$$

$$\therefore \text{ Efficiency} = \frac{W}{Q} = \frac{NRT_0}{\frac{13}{2}NRT_0} = \frac{2}{13} \text{ or } 15.38\%$$

Example 15.43 *An ideal gas expands isothermally along AB and does 700 J of work.*



(i) How much heat does the gas exchange along AB?

(ii) The gas then expands adiabatically along BC and does 400 J of work. When the gas returns to A along CA, it exhausts 100 J of heat to its surroundings. How much work is done on the gas along this path?

Sol. (i) AB is an isothermal process. Hence,

$$\Delta U_{AB} = 0 \text{ and } Q_{AB} = W_{AB} = 700\text{ J}$$

(ii) BC is an adiabatic process. Hence,

$$Q_{BC} = 0 \Rightarrow W_{BC} = 400\text{ J} \qquad \text{(given)}$$
$$\therefore \quad \Delta U_{BC} = -W_{BC} = -400\text{ J}$$

$ABCA$ is a cyclic process and internal energy is a state function.

Therefore, $(\Delta U)_{\text{whole cycle}} = 0 = \Delta U_{AB} + \Delta U_{BC} + \Delta U_{CA}$ and from first law of thermodynamics,

$$Q_{AB} + Q_{BC} + Q_{CA} = W_{AB} + W_{BC} + W_{CA}$$

Substituting the values, $700 + 0 - 100 = 700 + 400 + \Delta W_{CA}$

$$\therefore \quad \Delta W_{CA} = -500 \text{ J}$$

Negative sign implies that work is done on the gas.

Different values in different processes

Process	Q (J)	W (J)	ΔU (J)
AB	700	700	0
BC	0	400	– 400
CA	– 100	– 500	400
For complete cycle	600	600	0

Note Total work done is 600 J, which implies that area of the closed curve is also 600 J.

Example 15.44 *In example* 15.43 *find the efficiency of the given cycle.*

Sol. From table of the previous example, we can see that Q_{absored} during the cycle is 700 J, while the total work done in the cycle is 600 J.

$$\therefore \quad \eta = \frac{W_{\text{total}}}{|Q_{\text{absored}}|} \times 100$$

$$= \left(\frac{600}{700}\right) \times 100$$

$$= 85.71\%$$

Check point 15.3

1. Choose the incorrect statement related to an isobaric process
(a) $\frac{V}{T}$ = constant (b) $W = p\,\Delta V$
(c) Heat given to a system is used up in raising the temperature only
(d) None of the above

2. A gas expands under constant pressure p from volumes V_1 and V_2. The work done by the gas is
(a) $p(V_2 - V_1)$ (b) $p(V_1 - V_2)$ (c) $p(V_1^\gamma - V_2^\gamma)$ (d) $p\frac{V_1V_2}{V_2 - V_1}$

3. When 1 g of water at 0°C and 1×10^5 Nm^{-2} pressure is converted into ice of volume 1.092 cm^3, the work done will be
(a) 0.0091 J (b) 0.0182 J
(c) 0.091 J (d) Data insufficient

4. An ideal gas of volume 1.5×10^{-3} m^3 and at pressure 1.0×10^5 Pa is supplied with 70 J of energy. The volume increases to 1.7×10^{-3} m^3, the pressure remaining constant. The internal energy of the gas is
(a) increased by 90 (b) increased by 70 J
(c) increased by 50 J (d) decreased by 50 J

5. In an isochoric process if $T_1 = 27°$C and $T_2 = 127°$C, then $\frac{p_1}{p_2}$ will be equal to
(a) $\frac{9}{59}$ (b) $\frac{2}{3}$
(c) $\frac{3}{4}$ (d) None of these

6. Which of the following is a slow process?
(a) Isothermal (b) Adiabatic
(c) Isobaric (d) None of these

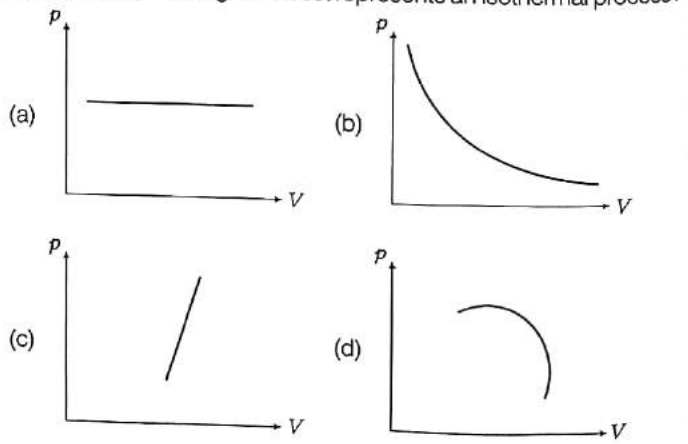
7. When an ideal gas is compressed isothermally then its pressure
(a) increases (b) decreases
(c) remains same
(d) first increases then decreases

8. In an isothermal expansion of an ideal gas, select wrong statement
(a) there is no change in the temperature of the gas
(b) there is no change in the internal energy of the gas
(c) the work done by the gas is equal to the heat supplied to the gas
(d) the work done by the gas is equal to the change in its internal energy

9. During an isothermal expansion of an ideal gas
(a) its internal energy decreases
(b) its internal energy does not change
(c) the work done by the gas is equal to the quantity of heat absorbed by it
(d) Both (b) and (c) are correct

10. Which is incorrect?
(a) In an isobaric process, $\Delta p = 0$
(b) In an isochoric process, $\Delta W = 0$
(c) In an isothermal process, $\Delta T = 0$
(d) In an isothermal process, $\Delta Q = 0$

11. Which of the p-V, diagrams best represents an isothermal process?



12. A cycle tyre bursts suddenly. This represents an
(a) isothermal process (b) isobaric process
(c) isochoric process (d) adiabatic process

13. When an enclosed perfect gas is subjected to an adiabatic process
(a) its total internal does not change
(b) its temperature does not change
(c) its pressure varies inversely as a certain power of its volume
(d) the product of its pressure and volume is directly proportional to its absolute temperature

14. A gas for which $\gamma = 1.5$ is suddenly compressed to (1/4)th of the initial volume. Then the ratio of the final to the initial pressure is
(a) 1 : 16 (b) 1 : 8 (c) 1 : 4 (d) 8 : 1

15. For a gas $\gamma = \frac{5}{3}$ and 640 cc of this gas is suddenly compressed to 80 cc. If the initial pressure is p, then the final pressure will be
(a) $8p$ (b) $32p$ (c) $16p$ (d) $64p$

16. The volume of a gas is reduced adiabatically to 1/4 of its volume at 27°C. If $\gamma = 1.4$, the new temperature will be
(a) $(300)\,2^{0.4}$ K (b) $(300)\,4^{1.4}$ K
(c) $(300)\,4^{0.4}$ K (d) $(300)\,2^{1/4}$ K

17. An ideal gas at a pressure of 1 atm and temperature of 27°C is compressed adiabatically until its pressure becomes 8 times the initial pressure, then the final temperature is ($\gamma = 3/2$)
(a) 627°C (b) 527°C (c) 427°C (d) 327°C

18. During an adiabatic process, the pressure of a gas is found to be proportional to the cube of its absolute temperature. The ratio $\frac{C_p}{C_v} = \gamma$ for the gas is
(a) 2 (b) $\frac{3}{2}$ (c) $\frac{5}{3}$ (d) $\frac{4}{3}$

19. Specific heat of gas undergoing adiabatic changes is
(a) zero (b) infinite
(c) positive (d) negative

20. The work done in which of the following processes is zero?
(a) Isothermal process (b) Adiabatic process
(c) Adiabatic (d) Isochoric

21. A given system undergoes a change in which the work done by the system equals the decrease in its internal energy. The system must have undergone an
(a) isothermal change (b) adiabatic change
(c) isobaric change (d) isochoric change

22. In an adiabatic expansion of one mole of gas, initial and final temperature are T_1 and T_2 respectively, then the change in internal energy of the gas is
(a) $\frac{R}{\gamma - 1}(T_2 - T_1)$ (b) $\frac{R}{\gamma - 1}(T_1 - T_2)$
(c) $R(T_1 - T_2)$ (d) zero

23. In an adiabatic expansion, a gas does 25 J of work while in an adiabatic compression 100 J of work is done on a gas. The change of internal energy in the two processes respectively are
(a) 25 J and −100 J (b) −25 J and 100 J
(c) −25 J and −100 J (d) 25 J and 100 J

24. A given quantity of monoatomic gas is at pressure p and absolute temperature T. The isothermal bulk modulus of the gas is
(a) $\frac{2}{3}p$ (b) p
(c) $\frac{3}{2}p$ (d) $2p$

25. Two identical samples of a gas are allowed to expand (i) isothermally (ii) adiabatically. Work done is
(a) more in the isothermal process
(b) more in the adiabatic process
(c) zero in both of them
(d) equal in both processes

5 Heat engine

A heat engine is a device which converts heat energy into mechanical energy.

The essential parts of a heat engine are as follows

Source It is a reservoir of heat at high temperature and infinite thermal capacity. Any amount of heat can be extracted from it at constant temperature.

Sink It is a reservoir of heat at low temperature and infinite thermal capacity. Any amount of heat can be given to the sink at constant temperature.

Working Substance A material which absorbs heat energy from the source and converts it into mechanical energy (useful work) by rejecting some of the heat to sink is called working substance. *e.g.*, Steam, petrol, etc.

Working

The working substance absorbs heat Q_1 (as shown in figure) from the source, does an amount of work W, returns the remaining amount of heat Q_2 to the sink and comes back to its original state and there occurs no change in its internal energy.

Source is always at higher temperature than sink.

By repeating the same cycle over and over again, work can be continuously obtained from an engine,

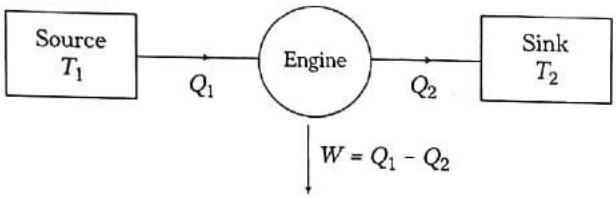


Fig. 15.25 Working of heat engine

Thermal efficiency

Thermal efficiency of a heat engine is defined as the ratio of net work done per cycle by the engine to the total amount of heat absorbed per cycle by the working substance from the source. It is denoted by η. Thus,

$$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

$$\boxed{\text{Thermal efficiency, } \eta = 1 - \frac{Q_2}{Q_1}}$$

where,

Q_1 = heat absorbed by the working substance

Q_2 = heat released by the working substance to the sink

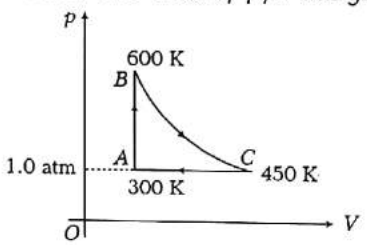
η = thermal efficiency of heat engine

W = work done

For an ideal heat engine $Q_2 = 0$

So, $\eta = 1$ *i.e.*, 100% (practically not possible)

Example 15.45 *The p-V diagram of 0.2 mol of an ideal diatomic gas used as working substance in heat engine is shown in figure. Process BC is adiabatic. The value of γ for this gas is 1.4.*



(i) Find the pressure and volume at points A, B and C.

(ii) Calculate ΔQ, ΔW and ΔU for each of the three processes.

(iii) Find the thermal efficiency of the cycle.

(Take, 1 atm = 1.0×10^5 Nm^2)

Sol. (i) Given, $p_A = p_C = 1$ atm $= 1.01 \times 10^5$ Nm^{-2}

Process AB is an isochoric process.

$$\therefore \quad p \propto T \text{ or } \frac{p_B}{p_A} = \frac{T_B}{T_A}$$

$$\therefore \quad p_B = \left(\frac{T_B}{T_A}\right) p_A = \left(\frac{600}{300}\right)(1 \text{ atm}) = 2 \text{ atm}$$

$$= 2.02 \times 10^5 \text{ Nm}^{-2}$$

From ideal gas equation, $V = \dfrac{nRT}{p}$

$$\therefore \quad V_A = V_B = \frac{nRT_A}{p_A} = \frac{(0.2)(8.31)(300)}{(1.01 \times 10^5)}$$

$$\approx 5.0 \times 10^{-3} \text{ m}^3 = 5 \text{ L}$$

$$\text{and } V_C = \frac{nRT_C}{p_C} = \frac{(0.2)(8.31)(455)}{(1.01 \times 10^5)} = 7.6 \times 10^{-3} \text{m}^3 \approx 7.6 \text{ L}$$

State	p	V
A	1 atm	5 L
B	2 atm	5 L
C	1 atm	7.6 L

(ii) **Process *AB*** is an isochoric process.

Hence, $\Delta W_{AB} = 0$

$$\Delta Q_{AB} = \Delta U_{AB} = nC_V \Delta T = n\left(\frac{5}{2}R\right)(T_B - T_A)$$

$$= (0.2)\left(\frac{5}{2}\right)(8.31)(600 - 300) \approx 1246 \text{ J}$$

Process *BC* is an adiabatic process.

Hence, $\Delta Q_{BC} = 0$

$$\therefore \quad \Delta W_{BC} = -\Delta U_{BC}$$

$$\Delta U_{BC} = nC_V \Delta T = nC_V (T_C - T_B)$$

$$= (0.2)\left(\frac{5}{2}R\right)(455 - 600)$$

$$= (0.2)\left(\frac{5}{2}\right)(8.31)(-145) \text{ J} \approx -602 \text{ J}$$

$$\therefore \quad \Delta W_{BC} = -\Delta U_{BC} = 602 \text{ J}$$

Process *CA* is an isobaric process.

$$\text{Hence, } \Delta Q_{CA} = nC_p \Delta T = n\left(\frac{7}{2}R\right)(T_A - T_C)$$

$$= (0.2)\left(\frac{7}{2}\right)(8.31)(300 - 455) \approx -902 \text{ J}$$

$$\Delta U_{CA} = nC_V \Delta T = \frac{\Delta Q_{CA}}{\gamma} \qquad \left(\text{as } \gamma = \frac{C_p}{C_V}\right)$$

$$= -\frac{902}{1.4} \approx -644 \text{ J}$$

$$\therefore \quad \Delta W_{CA} = \Delta Q_{CA} - \Delta U_{CA} = -902 \text{ J} + 644 \text{ J} = -258 \text{ J}$$

Process	ΔQ (in J)	ΔW (in J)	ΔU (in J)
AB	1246	0	1246
BC	0	602	−602
CA	−902	−258	−644
Total	344	344	0

(iii) Efficiency of the cycle

$$\eta = \frac{W_{\text{total}}}{|Q_{+\text{ve}}|} \times 100 = \frac{344}{1246} \times 100 = 27.6\%$$

6 Refrigerator

It is a device which works in the reverse direction of heat engine. Refrigerator takes heat Q_2 (as shown in figure) from a cold body (sink), work (W) is done on it and the work done together with the heat absorbed (*i.e.*, Q_1) is rejected to the source.

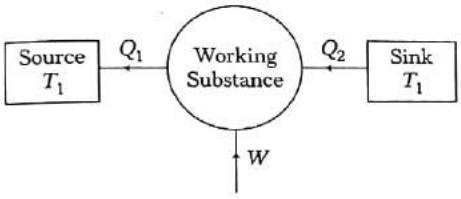


***Fig. 15.26** Working of refrigerator*

Coefficient of performance (COP)

The performance of a refrigerator is expressed by means of coefficient of performance β which is defined as the ratio of the heat extracted from the cold body to the work needed to transfer it to the hot body.

i.e., $$\beta = \frac{\text{Heat extracted}}{\text{Work done}} = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2}$$

$$\boxed{\text{Coefficient of performance, } \beta = \frac{Q_2}{Q_1 - Q_2}}$$

- A perfect refrigerator is one which transfers heat from cold to hot body without doing work.
  i.e., $W = 0$, so that $Q_1 = Q_2$ and hence $\beta = \infty$
- For an ideal refrigerator or heat engine we can take,
  $$\frac{Q_2}{Q_1} = \frac{T_2}{T_1}.$$
- Air conditioners are basically refrigerators whose refrigerated space is a room or a building.
- In domestic refrigerator, inside portion acts as a cold reservoir and the surrounding act as hot reservoir.

Relation between coefficient of performance and efficiency of heat engine

We know $$\beta = \frac{Q_2}{Q_1 - Q_2} = \frac{Q_2/Q_1}{1 - Q_2/Q_1} \quad ...(i)$$

But $$\eta = 1 - \frac{Q_2}{Q_1} \quad \text{or} \quad \frac{Q_2}{Q_1} = 1 - \eta \quad ...(ii)$$

From Eqs. (i) and (ii), we get $\boxed{\beta = \frac{1-\eta}{\eta}}$

Example 15.46 *An ideal refrigerator runs between $-23°C$ and $27°C$. Find the heat rejected to atmosphere for every joule of work input.*

Sol. Let the heat rejected to the atmosphere $Q_1 = x$

and given $W = 1$ J. Now $Q_2 = Q_1 - W = x - 1$.

Given, hot temperature $T_1 = 273 + 27 = 300$ K

and cold temperature $T_2 = 273 - 23 = 250$ K

For an ideal refrigerator we know that, $\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$

$$\therefore \quad \frac{x}{x-1} = \frac{300}{250} \quad \text{or,} \quad x = 6 \text{ J}$$

Example 15.47 *Calculate the amount of work that must be done to freeze one gram of water at 0°C by means of an ideal refrigerator. Temperature of surroundings is 27°C. How much heat in passed on the surroundings in this process. Latent heat of fusion $L = 80$ cal g^{-1}.*

Sol. Heat released during freezing,

$$Q_2 = mL = 1 \times 80 = 80 \text{ cal}, \; T_2 = 0° \text{C} = 273 \text{ K}$$

and $$T_1 = 27° \text{C} = 300 \text{ K}$$

Since, $$\frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} = \frac{T_2}{T_1 - T_2}$$

$$\therefore \text{ Work done, } W = \frac{Q_2(T_1 - T_2)}{T_2}$$

$$= \frac{80\,(300 - 273)}{273} = 7.91 \text{ cal}$$

$\Rightarrow$ Heat passed to the surrounding,

$$Q_1 = Q_2 + W = (80 + 7.91) = 87.91 \text{cal}$$

7 Second law of thermodynamics

The second law of thermodynamics gives a fundamental limitation to the efficiency of a heat engine and the coefficient of performance of a refrigerator . It says that efficiency of a heat engine can never be unity (or 100%). This implies that heat released to the cold reservoir can never be made zero.

For a refrigerator, the second law says that the coefficient to performance can never be infinite. This implies that external work can never be zero.

Second law of thermodynamics may be stated in various ways, but two statements are worth mentioning which are as follows

Kelvin-Planck statement

No process is possible which can completely convert heat absorbed from a reservoir into work.

Clausius statement

No process is possible which can transfer heat from a cold body to a hot body without doing any external work.

Reversible and irreversible processes

Reversible process

A process which could be reserved in such a way that the system and its surrounding returns exactly to their initial states with no other changes in the surroundings is known as **reversible process**.

e.g., If heat is absorbed in the direct process, then same amount of heat should be given out in the reverse process. If work is done on the working substance in the direct process, then the same amount of work should be done by the working substance in the reverse process.

The conditions for reversibility are as follows

(i) There must be complete absence of dissipative forces such as friction, viscosity, electric resistance etc.

(ii) The direct and reverse processes must take place infinitely slowly.

(iii) The temperature of the system must not differ appreciably from its surroundings.

Irreversible process

Any process which is not reversible exactly is an **irreversible process**. All natural processes such as conduction, radiation, radioactive decay etc., are irreversible processes.

Carnot engine

The reversible engine which operates between two temperatures of source and sink is known as **Carnot heat engine**. The designed engine is a theoretical engine which is free from all the defects of a practical engine. This engine cannot be realised in actual practice, however, this can be taken as a standard against which the performance of an actual engine can be judged.

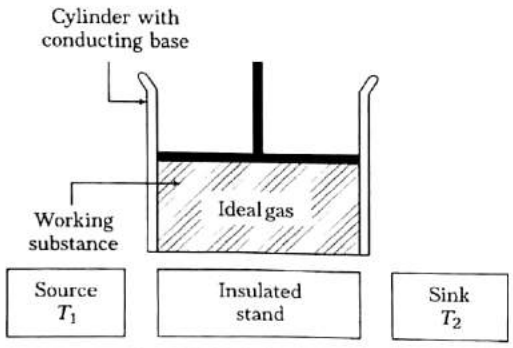


Fig. 15.27 A Carnot engine

The main parts of Carnot engine are as follows

Cylinder The cylinder has conducting base and insulating walls. It consists the gas as a working substance, insulating and frictionless piston is attached with it as shown in the figure.

Source It is a hot reservoir at a temperature T_1 with conducting walls. It has infinite thermal capacity. Any amount of heat can be taken from it without changing the temperature.

Sink It is a cold reservoir at temperature T_2. It has an infinite thermal capacity so any amount of heat can be rejected to it without changing the temperature of the sink.

Working substance We use an ideal gas as a working substance in the cylinder.

Insulating stand The base of the cylinder could be placed on the insulated stand, to isolate it completely from the surroundings.

Carnot cycle

As the engine works, the working substance of the engine undergoes a cycle known as **Carnot cycle**.

The Carnot cycle consists of the following four strokes

First stroke (Isothermal expansion) (Curve *AB*)

The cylinder containing one mole of an ideal gas as working substance is allowed to expand slowly at the constant temperature T_1 by putting it on the source.

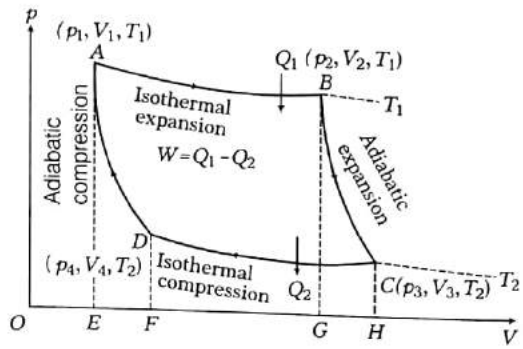


Fig. 15.28 Various processes in carnot cycle

Work done = heat absorbed by the gas

$$W_1 = Q_1 = \int_{V_1}^{V_2} p dV$$

$$= RT_1 \log_e \left(\frac{V_2}{V_1}\right)$$

= area of $ABGE$

Second stroke (Adiabatic expansion) (Curve *BC*)

The cylinder is then placed on the non-conducting stand and the gas is allowed to expand adiabatically till the temperature falls from T_1 to T_2.

$$W_2 = \int_{V_2}^{V_3} pdV = \frac{R}{(\gamma - 1)}(T_1 - T_2)$$

$$= \text{area of } BCHG$$

Third stroke (Isothermal compression) (Curve *CD*)

The cylinder is placed on the sink and the gas is compressed at constant temperature T_2.

Work done = heat released by the system

$$W_3 = Q_2 = -\int_{V_3}^{V_4} pdV = -RT_2 \log_e \frac{V_4}{V_3}$$

$$= RT_2 \log_e \frac{V_3}{V_4} = \text{area of } CDFH$$

Fourth stroke (Adiabatic compression) (Curve *DA*)

Finally, the cylinder is again placed on non-conducting stand and the compression is continued so that gas returns to its initial stage.

$$W_4 = -\int_{V_4}^{V_1} pdV = -\frac{R}{\gamma - 1}(T_2 - T_1)$$

$$= \frac{R}{\gamma - 1}(T_1 - T_2) = \text{area of } ADFE$$

Efficiency of the carnot cycle

The efficiency of engine is defined as the ratio of net work done to the heat supplied *i.e.*,

$$\eta = \frac{\text{Net work done}}{\text{Heat input}} = \frac{W_{net}}{Q_1}$$

$$= \frac{W_1 + W_2 + W_3 + W_4}{Q_1}$$

$$= 1 - \frac{Q_2}{Q_1}$$

On further calculation, we get

$$\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$$

$\therefore$ Efficiency of Carnot engine, $\boxed{\eta = 1 - \frac{T_2}{T_1}}$

- The efficiency of the Carnot engine depends only on the temperatures of source and sink.
- The efficiency of Carnot engine does not depend on the nature of the working substance.
- The efficiency of Carnot engine will be 100%, if the temperature of the sink is 0 K. As practically, we cannot attain a sink at 0 K so it is not possible to have 100% efficiency.

Carnot theorem

According to Carnot theorem,

(i) A heat engine working between the two given temperatures T_1 of hot reservoir *i.e.*, source and T_2 of cold reservoir *i.e.*, sink cannot have efficiency more than that of the Carnot engine.

(ii) The efficiency of the Carnot engine is independent of the nature of working substance.

Example 15.48 *A Carnot engine takes in 3000 kcal of heat from a reservoir at 627 °C and gives a part of it to a sink at 27 °C. Find the work done by the engine.*

Sol. Here, $T_1 = 273 + 627 = 900$ K

and $T_2 = 273 + 27 = 300$ K

Now, $\eta = \frac{W}{Q_1} = 1 - \frac{T_2}{T_1}$

$\therefore$ $\frac{W}{3000 \text{ kcal}} = 1 - \frac{300}{900}$

$\therefore$ Work done by the engine, $W = 2000$ kcal

$= 2000 \times 4.2$ kJ $= 8.4 \times 10^6$ J

Example 15.49 *Carnot's engine takes in a thousand kilo calories of heat from a reservoir at 827 °C and exhausts it to a sink at 27° C. How much work does it perform? What is the efficiency of the engine?*

Sol. Given, $Q_1 = 10^6$ cal, $T_1 = (827 + 273) = 1100$ K

and $T_2 = (27 + 273) = 300$ K

as, $\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$

$\therefore$ $Q_2 = \frac{T_2}{T_1} \cdot Q_1 = \left(\frac{300}{1100}\right)(10^6) = 2.72 \times 10^5$ cal

Efficiency of the cycle, $\eta = \left(1 - \frac{T_2}{T_1}\right) \times 100$

or $\eta = \left(1 - \frac{300}{1100}\right) \times 100 = 72.72\%$

Example 15.50 *The efficiency of a Carnot's engine at a particular source and sink temperature is 1/2. When the sink temperature is reduced by 100 °C, the engine efficiency becomes 2/3. Find the source temperature.*

Sol. According to Carnot's engine

$$1-\frac{T_2}{T_2}=\frac{1}{2} \quad ...(i)$$

(T_2 : sink temperature, T_1 : source temperature)

and $$1-\frac{(T_2-100)}{T_1}=\frac{2}{3} \quad ...(ii)$$

or, $$\frac{T_2}{T_1}=\frac{1}{2} \quad \text{[From Eq. (i)]}$$

and $$\frac{(T_2-100)}{T_1}=\frac{1}{3} \quad \text{[From Eq. (ii)]}$$

Dividing these two equations, we get

$$\frac{T_2}{T_2-100}=\frac{3}{2} \text{ or, } T_2=300\text{ K}$$

So, $$T_1=600\text{ K}$$

Example 15.51 *If a Carnot engine whose heat sink is at 27°C has an efficiency of 40%. By how many degrees should the temperature of the source be changed to increase the efficiency by 10% of the original efficiency?*

Sol. Given, $T_2=27+273=300\text{ K}, \eta=40\%$

As $$\eta=1-\frac{T_2}{T_1}$$

$\therefore$ $$\frac{T_2}{T_1}=1-\eta=1-\frac{40}{100}=\frac{60}{100}=\frac{3}{5}$$

$\Rightarrow$ $$T_1=\frac{5}{3}\times T_2=\frac{5}{3}\times 300=500\text{ K}$$

$\therefore$ New efficiency, $\eta'=40+4=44\%$

($\because$ Increase in efficiency = 10% of 40 = 4%)

Let the new temperature of the source be T_1' K, then

$$\eta'=1-\frac{T_2}{T_1'} \Rightarrow \frac{44}{100}=\frac{1-300}{T_1'}$$

$\Rightarrow$ $$\frac{300}{T_1'}=1-\frac{44}{100}=\frac{56}{100}$$

$\Rightarrow$ $$T_1'=\frac{100\times 300}{56}=535.7\text{ K}$$

$\therefore$ Increase in the temperature of the source,

$$=535.7-500=35.7\text{ K or } 35.7°\text{ C}.$$

Example 15.52 *A scientist claims to have developed 60% efficient engine while working between 27°C and 327°C. Does he claim right?*

Sol. Here sink temperature $T_2=273+27=300$ K.

(low temperature happens to be sink temperature)

Given source temperature $T_1=327+273=600$ K

So, within this temperature difference the maximum possible efficiency is

$$\eta_{max}=1-\frac{T_2}{T_1}=1-\frac{300}{600}=0.5=50\%$$

But the scientist is claiming more efficiency. So, as per Carnot's theorem his claim is not correct.

Entropy

Like pressure, volume, temperature, internal energy, etc. We have another thermodynamic variable of a system, named entropy. It is related to the disorder of molecular motion of the system. Greater the randomness or disorder, greater is the entropy.

The change in entropy

$$i.e., \Delta S=\frac{\text{Heat absorbed by the system}}{\text{Absolute temperature}}=\frac{\Delta Q}{T}$$

This relation is also called mathematical form of second law of thermodynamics.

SI unit of ΔS is J/K and dimensional formula is $[ML^2T^{-2}K^{-1}]$

Note We always define change in entropy rather than its absolute value.

Change in entropy for solids and liquids

(i) When heat is supplied to a solid and its state changes such that temperature remains constant, then

Change in entropy, $\Delta S=\frac{\Delta Q}{T}$

$\Rightarrow$ $$dS=\frac{dQ}{T}=\frac{\pm|mL|}{T}$$

Positive sign is used for heat absorption and negative sign is used for heat rejection.

(ii) When temperature of a substance changes from T_1 to T_2, then

$$dS=\int\frac{dQ}{T}=mS\int_{T_1}^{T_2}\frac{dT}{T}=ms\ln\left(\frac{T_2}{T_1}\right)$$

$$=2.303\,mS\log_{10}\left(\frac{T_2}{T_1}\right)$$

where, m = Mass of the substance

s = Specific heat of the substance

Example 15.53 *200 gram of ice at 0°C is converted into water vapour at 100°C. Evaluate the change in entropy.*

Sol. Consider conversion of ice into water

$$\underset{0°C}{\text{ice}} \longrightarrow \underset{0°C}{\text{water}}$$

Change in entropy during this conversion

$$\Delta S_1=\frac{\Delta Q}{T_1}=\frac{mL_{ice}}{T_1}=\frac{200\times 80}{273}=58.6\text{ cal/K}$$

Again consider rise of temperature of water from 0°C to 100°C.

$$\underset{0°C}{\text{water}} \longrightarrow \underset{100°C}{\text{water}}$$

Change in entropy for this increase in temperature of the water

$$\Delta S_2 = ms_w \ln \frac{T_2}{T_1} = 200 \times 1 \times \ln\left(\frac{373}{273}\right)$$

$$= 200 \ln (1.4) \text{ cal/K}$$

Again consider conversion of water to vapour

$$\underset{100°C}{\text{water}} \longrightarrow \underset{100°C}{\text{vapour}}$$

Change in entropy for this conservation is

$$\Delta S_3 = \frac{\Delta Q}{T_1} = \frac{mL_{\text{steam}}}{T_2} = \frac{200 \times 540}{373} = 289.5 \text{ cal/K}$$

Net change in entropy

$$\Delta S = \Delta S_1 + \Delta S_2 + \Delta S_3$$
$$= 58.6 + 200 \ln(1.4) + 289.5$$
$$= 348.1 + 200 \times 2.303 \log_{10} (1.4)$$
$$= 415.4 \text{ cal/K}$$

Check point 15.4

1. A system undergoes a cyclic process in which it absorbs Q_1 heat and gives out Q_2 heat. The efficiency of the process is η and work done is W. Select the correct statement.

(a) $W = \frac{Q_1 - Q_2}{Q_2}$

(b) $\eta = \frac{W}{Q_1}$

(c) Both (a) and (b) are correct

(d) Both (a) and (b) are wrong

2. An ideal heat engine is opening between 227°C and 127°C. It absorbs 10^4 J amount of heat at the higher temperature. The amount of heat converted into work is

(a) 2000 J (b) 4000 J

(c) 8000 J (d) 5600 J

3. The inside and outside temperature of a refrigerator are 273 K and 303 K respectively. Assuming that refrigerator cycle is reversible, for every joule of work done the heat delivered to the surroundings will be

(a) 10 J (b) 20 J (c) 30 J (d) 50 J

4. A Carnot engine has an efficiency of 50% at sink temperature 50°C. Calculate the temperature of source.

(a) 133°C (b) 143°C (c) 100°C (d) 373°C

5. A Carnot's engine is made to work between 200°C and 0°C first and then between 0° and –200°C. The ratio of efficiencies $\left(\frac{\eta_2}{\eta_1}\right)$ of the engine in the two cases is

(a) 1 : 1.5 (b) 1 : 1 (c) 1 : 2 (d) 1.73 : 1

6. If a Carnot's engine functions at source temperature = 127°C and sink temperature = 87°C, what is its efficiency?

(a) 10% (b) 25%

(c) 40% (d) 50%

7. A Carnot's engine, with its cold body at 17°C has 50% efficiency. If the temperature of its hot body is now increased by 145°C, the efficiency becomes

(a) 55% (b) 60%

(c) 40% (d) 45%

8. The heat reservoir of an ideal Carnot engine is at 800 K and its sink is at 400 K. The amount of heat taken in it in one second to produce useful mechanical work at the rate of 750 J is

(a) 2250 J

(b) 1125 J

(c) 1500 J

(d) 750 J

9. For which combination of temperatures the efficiency of Carnot's engine is highest?

(a) 80 K, 60 K

(b) 100 K, 80 K

(c) 60 K, 40 K

(d) 40 K, 20 K

10. Entropy of a system is related with

(a) disorder of molecular motion of the system

(b) volume of the system

(c) pressure of the system

(d) None of the above

Chapter exercises

A Taking it together

(Assorted questions of the chapter for advanced level practice)

1. In an adiabatic process, the state of a gas is changed from p_1, V_1, T_1 to p_2, V_2, T_2. Which of the following relation is correct?

(a) $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$ (b) $p_1V_1^{\gamma-1} = p_2V_2^{\gamma-1}$

(c) $T_1p_1^{\gamma} = T_2p_2^{\gamma}$ (d) $T_1V_1^{\gamma} = T_2V_2^{\gamma}$

2. Identify the incorrect statement related to a cyclic process

(a) the initial and final conditions always coincide

(b) $Q = W$

(c) $W > 0$

(d) None of the above

3. p-V diagram of a diatomic gas is a straight line passing through origin. The molar heat capacity of the gas in the process will be

(a) $4R$ (b) $2.5R$ (c) $3R$ (d) $\frac{4R}{3}$

4. One mole of helium is adiabatically expanded from its initial state (p_i, V_i, T_i) to its final state (p_f, V_f, T_f). The decrease in the internal energy associated with this expansion is equal to

(a) $C_V(T_i - T_f)$ (b) $C_p(T_i - T_f)$

(c) $\frac{1}{2}(C_p + C_V)(T_i - T_f)$ (d) $(C_p - C_V)(T_i - T_f)$

5. Two identical containers joined by a small pipe initially contain the same gas at pressure p_0 and absolute temperature T_0. One container is now maintained at the same temperature while the other is heated to $2T_0$. The common pressure of the gases will be

(a) $\frac{3}{2}p_0$ (b) $\frac{4}{3}p_0$ (c) $\frac{5}{3}p_0$ (d) $2p_0$

6. An ideal monoatomic gas is compressed (no heat being added or removed in the process) so that its volume is halved. The ratio of the new pressure to the original pressure is

(a) $(2)^{3/5}$ (b) $(2)^{4/3}$ (c) $(2)^{3/4}$ (d) $(2)^{5/3}$

7. If a Carnot's engine functions at source temperature $= 137°C$ an sink temperature $= 97°C$, what is its efficiency?

(a) 9.75% (b) 10.5%

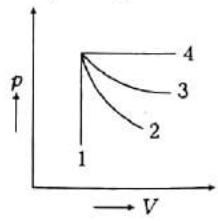
(c) 40% (d) 50%

8. A gas is compressed at a constant pressure of 50 Nm^{-2} from a volume of 10 m^3 to a volume of 4 m^3. Energy of 10 J then added to the gas by heating. Its internal energy is

(a) increased by 400 J (b) increased by 200 J

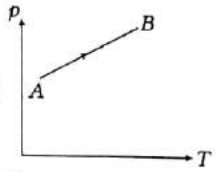
(c) increased by 100 J (d) decreased by 200 J

9. An ideal gas undergoes four different processes from the same initial state (figure). Four processes are adiabatic, isothermal, isobaric and isochoric. Out of 1, 2, 3 and 4 which one is adiabatic?



(a) 4 (b) 3 (c) 2 (d) 1

10. The process $\Delta U = 0$, for an ideal gas can be best represented in the form of a graph

(a)


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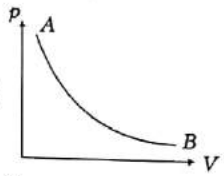

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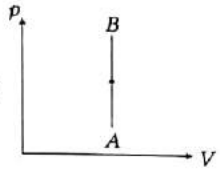
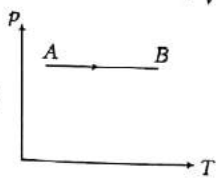
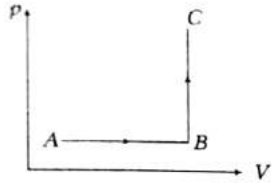


(d)


11. The p-V diagram of a system undergoing thermodynamic transformation is shown in figure. The work done by the system in going from $A \to B \to C$ is 30 J and 40 J heat is given to the

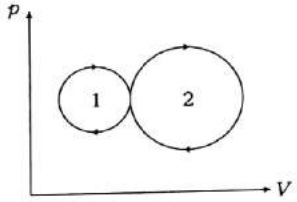
system. The change in internal energy between A and C is



(a) 10 J (b) 70 J
(c) 84 J (d) 134 J

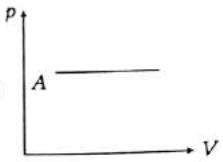
12. A thermally insulated rigid container contain an ideal gas heated by a filament of resistance 100 Ω through a current of 1 A for 5 min, then change in internal energy is
(a) 0 kJ (b) 10 kJ
(c) 20 kJ (d) 30 kJ

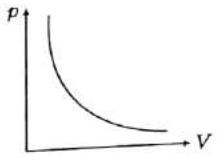
13. In the p-V diagram, shown in figure the net amount of work done will be



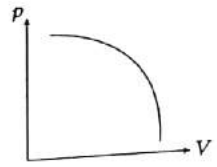
(a) positive
(b) negative
(c) zero
(d) infinity

14. Which of the accompanying pV diagram best represents an isothermal process

(a)


(b)


(c)

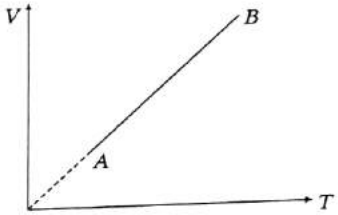
(d)


15. An ideal gas is allowed to expand freely against vacuum in a rigid insulated container. The gas undergoes
(a) an increase in its internal energy
(b) a decrease in its internal energy
(c) Neither an increase nor decrease in temperature or internal energy
(d) an increase in temperature

16. An ideal gas mixture filled inside a balloon expands according to the relation $pV^{2/3}$ = constant. The temperature inside the balloon is
(a) increasing
(b) decreasing
(c) constant
(d) Cannot be defined

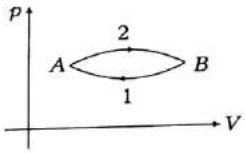
17. The internal energy of a gas is given by $U = 2pV$. It expands from V_0 to $2V_0$ against a constant pressure p_0. The heat absorbed by the gas in the process is
(a) $2p_0V_0$ (b) $4p_0V_0$ (c) $3p_0V_0$ (d) p_0V_0

18. An ideal monoatomic gas undergoes the process AB as shown in the figure. If the heat supplied and the work done in the process are ΔQ and ΔW respectively. The ratio $\Delta Q : \Delta W$ is



(a) 2.50 (b) 1.67 (c) 1.25 (d) 0.40

19. The figure shown two paths for the change of state of a gas from A to B. The ratio of molar heat capacities in path 1 and path 2 is



(a) < 1 (b) > 1
(c) 1 (d) data insufficient

20. An ideal gas expands in such a manner that its pressure and volume can be related by equation pV^2 = constant. During this process, the gas is
(a) heated
(b) cooled
(c) neither heated nor cooled
(d) first heated and then cooled

21. The equation of a state of a gas is given by $p(V - b) = nRT$. 1 mol of a gas is isothermally expanded from volume V to $2V$, the work done during the process is
(a) $RT \ln\left|\frac{2V-b}{V-b}\right|$ (b) $RT \ln\left|\frac{V-b}{V}\right|$
(c) $RT \ln\left|\frac{V-b}{2V-b}\right|$ (d) $RT \ln\left|\frac{V}{V-b}\right|$

22. A closed system undergoes a change of state by process $1 \rightarrow 2$ for which $Q_{12} = 10$ J and $W_{12} = -5$ J. The system is now returned to its initial state by a different path $2 \rightarrow 1$ for which Q_{21} is -3 J. The work done by the gas in the process $2 \rightarrow 1$ is
(a) 8 J (b) zero (c) -2 J (d) $+5$ J

23. A monoatomic gas is supplied the heat Q very slowly keeping the pressure constant. The work done by the gas will be
(a) $\frac{2}{3}Q$ (b) $\frac{3}{5}Q$ (c) $\frac{2}{5}Q$ (d) $\frac{1}{5}Q$

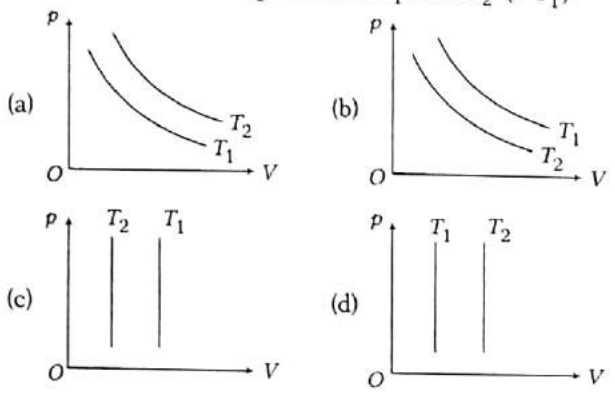
24. 70 cal of heat are required to raise the temperature of 2 moles of an ideal gas at constant pressure from 30°C to 35°C. The amount of heat required in calories to raise the temperature of the same gas through the same range (30°C-35°C) at constant volume is

(a) 30 cal (b) 50 cal
(c) 40 cal (d) 90 cal

25. The temperature of a hypothetical gas increases to $\sqrt{2}$ times when compressed adiabatically to half the volume. Its equation can be written as

(a) $pV^{3/2}$ = constant (b) $pV^{5/2}$ = constant
(c) $pV^{7/3}$ = constant (d) $pV^{4/3}$ = constant

26. Identity the graph (s) which correctly represents an isotherm at two temperatures T_1 and T_2 ($> T_1$)



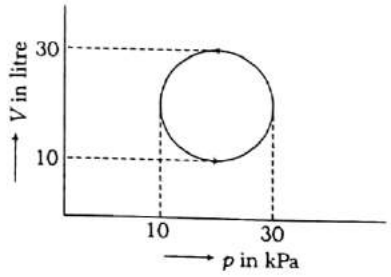
27. One mole of a perfect gas in a cylinder fitted with a piston has a pressure p, volume V and temperature T. If the temperature is increased by 1 K keeping pressure constant, the increase in volume is

(a) $\frac{2V}{273}$ (b) $\frac{V}{91}$ (c) $\frac{V}{273}$ (d) V

28. Ideal monoatomic gas is taken through a process $dQ = 2dU$. The molar heat capacity for the process is (where dQ is heat supplied and dU is change in internal energy)

(a) 2.4 R (b) 3R (c) R (d) 2R

29. Heat energy absorbed by a system in going through a cyclic process shown in figure is



(a) $10^7\ \pi$ J (b) $10^4\ \pi$ J (c) $10^2\ \pi$ J (d) $10^2\ \pi$ J

30. Unit mass of a liquid with volume V_1 is completely changed into a gas of volume V_2 at a constant external pressure p at temperature T. If the latent heat of evaporation is L, then the increase in the internal energy of the system is

(a) zero (b) $p(V_2 - V_1)$
(c) $L - p(V_2 - V_1)$ (d) L

31. A gas mixture consists of 4 moles of oxygen and 6 moles argon at temperature T. Neglecting all vibrational modes, the total internal energy of the system is

(a) 4RT (b) 14RT (c) 8RT (d) 19RT

32. One mole of an ideal monoatomic gas is at 360 K and a pressure of 10^5 Pa. It is compressed at constant pressure until its volume is halved. Taking R as 8.3 J mol^{-1} K^{-1} and the initial volume of the gas as 3.0×10^{-2} m^3, the work done on the gas is

(a) −1500 J (b) +1500 J (c) −3000 J (d) +3000 J

33. A monoatomic ideal gas, initially at temperature T_1, is enclosed in a cylinder fitted with a frictionless piston. The gas is allowed to expand adiabatically to a temperature T_2 by releasing the piston suddenly. If L_1 and L_2 are lengths of the gas column before and after expansion respectively, then T_1/T_2 is given by

(a) $\left(\frac{L_1}{L_2}\right)^{2/3}$ (b) $\left(\frac{L_1}{L_2}\right)^{5/3}$ (c) $\left(\frac{L_2}{L_1}\right)^{5/3}$ (d) $\left(\frac{L_2}{L_1}\right)^{2/3}$

34. If an average person jogs, he produces 14.5×10^3 cal/min. This is removed by the evaporation of sweat. The amount of sweat evaporated per minute (assuming 1 kg requires 580×10^3 cal for evaporation) is

(a) 0.25 kg (b) 2.25 kg (c) 0.05 kg (d) 0.20 kg

35. An ideal monoatomic gas undergoes a process in which its internal energy U and density ρ vary as $U\rho$ = constant. The ratio of change in internal energy and the work done by the gas is

(a) $\frac{3}{2}$ (b) $\frac{-2}{3}$ (c) $\frac{1}{3}$ (d) $\frac{3}{5}$

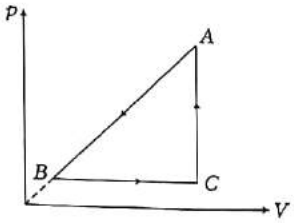
36. The molar heat capacity in a process of a diatomic gas if it does a work of $Q/4$ when a heat of Q is supplied to it is

(a) $\frac{2}{5}R$ (b) $\frac{5}{2}R$ (c) $\frac{10}{3}R$ (d) $\frac{6}{7}R$

37. The amount of heat required to raise the temperature of 1 mole of a monoatomic gas from 20°C to 30°C at constant volume is H. Then the amount of heat required to raise the temperature of 2 mole of a diatomic gas from 20°C to 25°C at constant pressure is

(a) 2H (b) $\frac{4}{3}H$ (c) $\frac{5}{3}H$ (d) $\frac{7}{3}H$

38. p-V diagram of a cyclic process $ABCA$ is as shown in figure. Choose the correct statement.

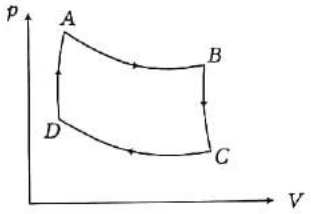


(a) $\Delta Q_{A \to B}$ = negative
(b) $\Delta U_{B \to C}$ = positive
(c) ΔW_{CAB} = negative
(d) All of the above

39. Two different ideal diatomic gases A and B are initially in the same state. A and B are then expanded to same final volume through adiabatic and isothermal process respectively. If p_A, p_B and T_A, T_B represents the final pressure and temperatures at A and B resepectively, then

(a) $p_A < p_B$ and $T_A < T_B$ (b) $p_A > p_B$ and $T_A > T_B$
(c) $p_A > p_B$ and $T_A < T_B$ (d) $p_A < p_B$ and $T_A > T_B$

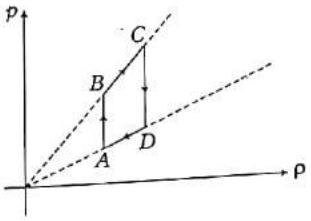
40. In the cyclic process $ABCDA$ shown in the figure, consider the following statements.



I. Area $ABCD$ = Work done on the gas
II. Area $ABCD$ = Net heat absorbed
III. Change in the internal energy in cycle = 0
Which of these are correct?

(a) I only (b) II only
(c) II and III (d) I, II and III

41. Pressure *versus* density graph of an ideal gas is shown in figure

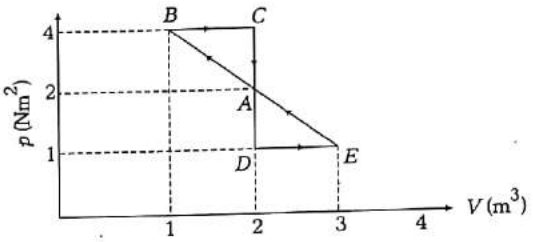


(a) during the process AB work done by the gas is positive
(b) during the process AB work done by the gas is negative
(c) during the process BC internal energy of the gas is increasing
(d) None of the above

42. A refrigerator, whose coefficient of performance β is 4, extracts heat from the cooling compartment at the rate of 400 J per cycle. How much work per cycle is required to operate the refrigerator?

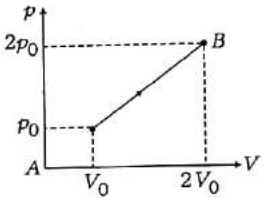
(a) 100 J (b) 200 J
(c) 50 J (d) 25 J

43. One mole of a monoatomic gas is carried along process $ABCDEA$ as shown in the diagram. Find the net work done by gas



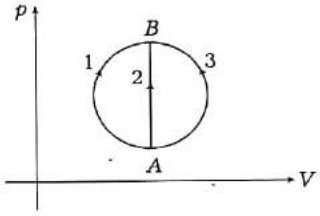
(a) $\frac{3}{2}$ J (b) 1 J (c) $\frac{1}{2}$ J (d) 0 J

44. The p-V diagram of 2 g of helium gas for a certain process $A \to B$ is shown in the figure. What is the heat given to the gas during the process $A \to B$?



(a) $4p_0V_0$ (b) $6p_0V_0$
(c) $3p_0V_0$ (d) $2p_0V_0$

45. A gas undergoes A to B through three different processes 1, 2, and 3 as shown in the figure. The heat supplied to the gas is Q_1, Q_2 and Q_3 respectively, then

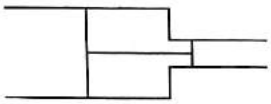


(a) $Q_1 = Q_2 = Q_3$ (b) $Q_1 < Q_2 < Q_3$
(c) $Q_1 > Q_2 > Q_3$ (d) $Q_1 = Q_3 > Q_2$

46. A perfect gas goes from state A to another state B by absorbing 8×10^5 J of heat and doing 6.5×10^5 J of work (by the gas). It is now transferred between the same two states in another process in which it absorbs 10^5 J of heat. Then, in the second process

(a) work done on gas is -0.5×10^5 J
(b) work done by gas is 0.5×10^5 J
(c) work done on gas is 10^5 J
(d) work done by gas is 10^5 J

47. Two pistons can move freely inside a horizontal cylinder having two sections of unequal cross-sections. The pistons are joined by an inextensible, light string and some gas is enclosed between the pistons. On heating the system, the piston will



(a) move to the left
(b) move to the right
(c) remain stationary
(d) (a) or (c) depending upon the initial pressure of the gas

48. When an air bubble rises from the bottom to the surface of a lake, its radius becomes double. Find the depth of the lake. Given that the atmospheric pressure is equal to the pressure due to a column of water 10 m high. Assume constant temperature and disregard surface tension

(a) 30 m (b) 40 m
(c) 70 m (d) 80 m

49. A container of volume $1\,m^3$ is divided into two equal compartments by a partition. One of these compartments contains an ideal gas at 300 K. The other compartment is vacuum. The whole system is thermally isolated from its surroundings. The partition is removed in the gas container. Its temperature now would be

(a) 300 K (b) greater than 300 K
(c) less than 300 K (d) Data insufficient

50. Starting with the same initial conditions, an ideal gas expands from volume V_1 to V_2 in three different ways. The work done by the gas is W_1 if the process is isothermal, W_2 if isobaric and W_3 if adiabatic, then

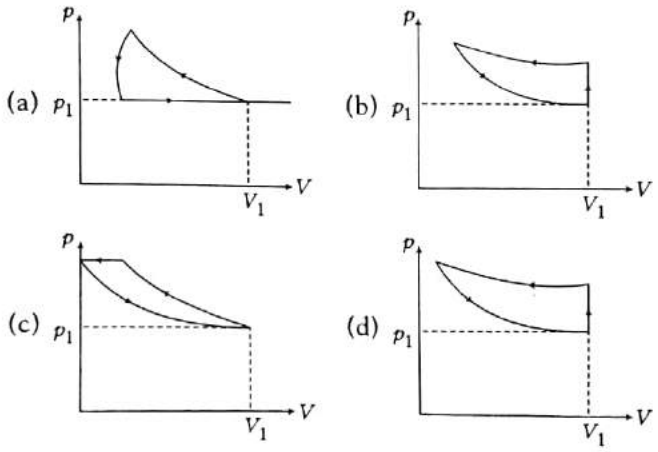
(a) $W_2 > W_1 > W_3$ (b) $W_2 > W_3 > W_1$
(c) $W_1 > W_2 > W_3$ (d) $W_1 > W_3 > W_2$

51. A carnot's engine whose low temperature reservoir is at 7° C has efficiency of 50%. It is desired to increase the efficiency to 70%. By how many degrees should the temperature of the high temperature reservoir be increased.

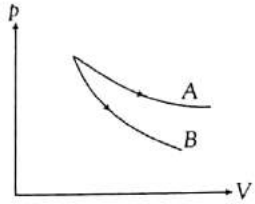
(a) 933 K (b) 432 K
(c) 373 K (d) 267 K

52. A certain mass of an ideal gas is at pressure p_1 and volume V_1. It is compressed isothermally and then allowed to expand adiabatically until its pressure returns to p_1. The gas is then allowed to expand its original volume.

Upon which of the following p-V graphs are these processes correctly shown?



53. A and B are two adiabatic curves for two different gases. Then A and B corresponds to



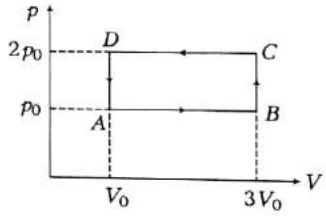
(a) Ar and He respectively (b) He and H_2 respectively
(c) O_2 and H_2 respectively (d) H_2 and He respectively

54. The pressure ($1 \times 10^5 Nm^{-2}$) of the air filled in a vessel is decreased adiabatically so much as to increase its volume three times.

The air pressure is (γ for air = 14, $\log_{10} 3 = 0.4771$, $\log_{10} 2.148 = 0.33206$)

(a) $2.14 \times 10^4\ Nm^{-2}$ (b) $2.19 \times 10^5\ Nm^{-2}$
(c) $1.14 \times 10^5\ Nm^{-2}$ (d) $3.14 \times 10^4\ Nm^{-2}$

55. An ideal gas undergoes cyclic process $ABCDA$ as shown in given p-V diagram. The amount of work done by the gas is

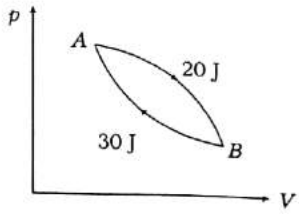


(a) $6p_0V_0$ (b) $-2p_0V_0$
(c) $+2p_0V_0$ (d) $+4p_0V_0$

56. Two different masses of a gas m and $2m$ are heated separately in vessels of equal volume. The T-p curve for mass $2m$ makes angle α with T-axis and that for mass m takes angle β with T-axis then

(a) $\tan\alpha = \tan\beta$ (b) $\tan\alpha = 2\tan\beta$
(c) $\tan\beta = 2\tan\alpha$ (d) None of these

57. In a cyclic process shown in the figure, an ideal gas is adiabatically taken from B to A, the work done on the gas during the process $B \to A$ is 30 J, when the gas is taken from $A \to B$ the heat absorbed by the gas is 20 J. The change in internal energy of the gas in the process $A \to B$ is



(a) 20 J (b) −30 J (c) 50 J (b) −10 J

58. n moles of an ideal gas undergo a process in which the temperature changes with volume as $T = KV^2$. The work done by the gas as the temperature changes from T_0 to $4T_0$ is

(a) $3nRT_0$ (b) $\left(\frac{5}{2}\right)nRT_0$

(c) $\left(\frac{3}{2}\right)nRT_0$ (d) zero

59. One mole of a gas expands with temperature T such that its volume, $V = kT^2$, where k is a constant. If the temperature of the gas changes by 60°C, then the work done by the gas is

(a) $120R$ (b) $R \ln 60$ (c) $kR \ln 60$ (d) $60kR$

60. An ideal monoatomic gas undergoes a process in which the gas volume relates to temperature as VT = constant. Then molar specific heat of gas in this process is

(a) $\frac{R}{2}$ (b) R

(c) $\frac{3R}{2}$ (d) None of these

61. One mole of an ideal gas with heat capacity at constant pressure C_p undergoes the process $T = T_0 + \alpha V$, where T_0 and α are constants. If its volume increases from V_1 to V_2 the amount of heat transferred to the gas is

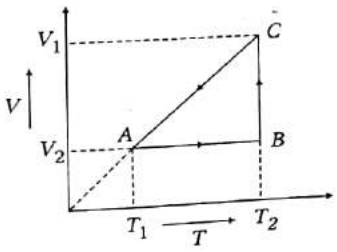
(a) $C_p RT_0 \ln\left|\frac{V_2}{V_1}\right|$

(b) $\alpha C_p(V_2 - V_1) - RT_0 \ln\left|\frac{V_2}{V_1}\right|$

(c) $\alpha C_p(V_2 - V_1) + RT_0 \ln\left|\frac{V_2}{V_1}\right|$

(d) $RT_0 \ln\left|\frac{V_2}{V_1}\right| - \alpha C_p(V_2 - V_1)$

62. A cyclic process for 1 mol of an ideal is shown in the V-T diagram. The work done in AB, BC and CA respectively is



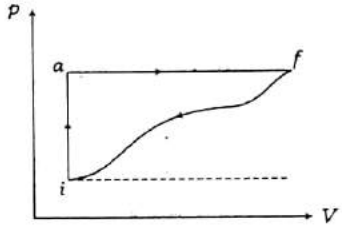
(a) 0, $RT_2 \ln\left|\frac{V_1}{V_2}\right|$, $R(T_2 - T_1)$

(b) $R(T_1 - T_2)$, 0, $RT_1 \ln\left|\frac{V_1}{V_2}\right|$

(c) 0, $RT_2 \ln\left|\frac{V_1}{V_2}\right|$, $R(T_1 - T_2)$

(d) 0, $RT_2 \ln\left|\frac{V_2}{V_1}\right|$, $R(T_2 - T_1)$

63. When a system is taken from state i to a state f along path iaf, $Q = 50$ J and $W = 20$ J. If $W = -13$ J for the curved return path fi, Q in this path is



(a) 33 J (b) 23 J

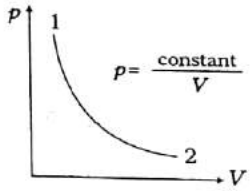
(c) −7 J (d) −43 J

64. Two Carnot's engines A and B are operated in succession. The first one, A receives heat from a source at T_1 = 800 K and rejects to sinks at T_2 K. The second engine B receives heat rejected by the first engine and rejects to another sink at T_3 = 300 K. If the work outputs of two engines are equal, then the value of T_2 is

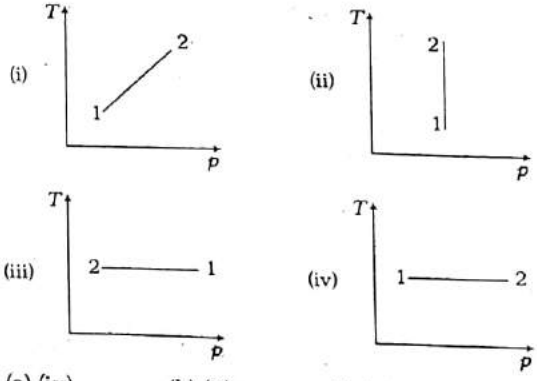
(a) 100 K (b) 300 K

(c) 550 K (d) 700 K

65. Consider p-V diagram for an ideal gas shown in figure.



Out of the following diagrams, which figure represents the T-p diagram?



(a) (iv) (b) (ii) (c) (iii) (d) (i)

66. A sample of 100 g water is slowly heated from 27°C to 87°C. What is the change in the entropy of the water? Specific heat capacity of water = 4200) J/kg-K.

(a) 420 ln (1.2) JK^{-1} (b) 220 ln (1.4) JK^{-1}
(c) 120 ln (1.5) JK^{-1} (d) 320 ln (1.2) JK^{-1}

67. An ideal gas is taken through a cyclic thermodynamic process through four steps. The amounts of heat involved in these steps are $Q_1 = 5960$ J, $Q_2 = -5585$ J, $Q_3 = 2980$ J and $Q_4 = 3645$ J respectively. The corresponding works involved are $W_1 = 2200$J, $W_2 = -825$J, $W_3 = -1100$ J and W_4 respectively.

What is the efficiency of the cycle

(a) 10.82 % (b) 11.42 %
(c) 9.32 % (d) 5.82 %

68. At 27°C two moles of an ideal monoatomic gas occupy a volume V. The gas expands adiabatically to a volume 2V. Which of the following option correct.

(a) Final temperature of the gas is 189 K
(b) Change in its internal energy, −2767.235
(c) The work done by the gas during the process is 2767.235
(d) All are correct

69. Consider two containers A and B containing identical gases at the same pressure, volume and temperature. The gas in container A is compressed to half of its original volume isothermally while the gas in container B is compressed to half of its original value adiabatically. The ratio of final pressure of gas in B to that of gas in A is

(a) $2^{\gamma-1}$ (b) $\left(\frac{1}{2}\right)^{\gamma-1}$
(c) $\left(\frac{1}{1-\gamma}\right)^2$ (d) $\left(\frac{1}{\gamma-1}\right)^2$

70. Three copper blocks of masses M_1, M_2 and M_3 kg respectively are brought into thermal contact till they reach equilibrium. Before contact, they were at temperatures T_1, T_2 and T_3 ($T_1 > T_2 > T_3$). Assuming there is no heat loss to the surroundings, the equilibrium temperature T is (s is specific heat of copper)

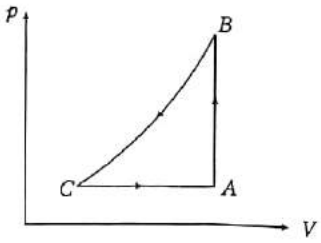
(a) $T = \frac{T_1 + T_2 + T_3}{3}$

(b) $T = \frac{M_1T_1 + M_2T_2 + M_3T_3}{M_1 + M_2 + M_3}$

(c) $T = \frac{M_1T_1 + M_2T_2 + M_3T_3}{3(M_1 + M_2 + M_3)}$

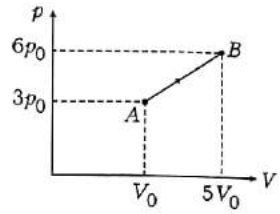
(d) $T = \frac{M_1T_1s + M_2T_2s + M_3T_3s}{M_1 + M_2 + M_3}$

71. A sample of an ideal gas is taken through a cycle as shown in figure. It absorbs 50 J of energy during the process AB, no heat during BC, rejects 70 J during CA. 40 J of work is done on the gas during BC. Internal energy of gas at A is 1500 J, the internal energy at C would be



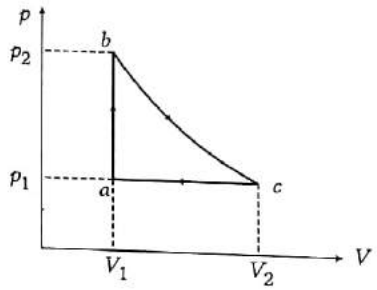
(a) 1590 J (b) 1620 J (c) 1540 J (d) 1570 J

72. One mole of a monoatomic ideal gas undergoes the process $A \rightarrow B$ in the given p-V diagram. The molar heat capacity for this process is



(a) $\frac{3R}{2}$ (b) $\frac{13R}{6}$
(c) $\frac{5R}{2}$ (d) $2R$

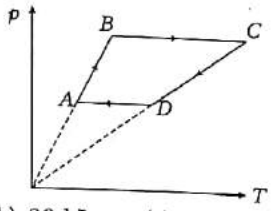
73. Carbon monoxide is carried around a closed cycle $abca$ in which bc is an isothermal process as shown in the figure. The gas absorbs 7000 J of heat as its temperature increases from 300 K to 1000 K in going from a to b.



The quantity of heat rejected by the gas during the process ca is approximately

(a) 4200 J (b) 5000 J
(c) 9000 J (d) −9800 J

74. Six moles of an ideal gas performs a cycle shown in figure. If the temperatures are $T_A = 600$ K, $T_B = 800$ K, $T_C = 2200$ K and $T_D = 1200$ K, the work done per cycle is approximately



(a) 20 kJ (b) 30 kJ (c) 40 kJ (d) 60 kJ

B) Medical entrance special format questions

Assertion and reason

Directions (Q. Nos. 1-15) *These questions consists of two statements each printed as Assertion and Reason. While answering these questions you are required to choose any one of the following five responses*

(a) If both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If Assertion is false but Reason is true.

1. **Assertion** In isobaric process, $\frac{\Delta Q}{\Delta W}$ for helium gas is $\frac{5}{2}$.
   Reason In isobaric process, work done by the gas is $nR\Delta T$.
2. **Assertion** In adiabatic expansion, product of pV always decreases.
   Reason In adiabatic expansion, work done by the gas is positive.
3. **Assertion** In isochoric process, ρ-V graph is straight line parallel to ρ-axis.
   Reason In isochoric process, density (ρ) remains constant.
4. **Assertion** For given source and sink temperatures, efficiency of Carnot's engine is 100%.
   Reason For given source and sink temperatures, efficiency of Carnot's engine is maximum.
5. **Assertion** Efficiency of any heat engine cannot be greater than the heat engine of Carnot's engine.
   Reason Heat flow never takes place from a body at lower temperature to a body at higher temperature.
6. **Assertion** First law of thermodynamics can be applied only for an ideal or real gas system.
   Reason First law of thermodynamics is nothing but law of conservation of energy.
7. **Assertion** Any process taking place in atmosphere is considered as an isobaric process.
   Reason Work done by the system in that case is $p_0\ \Delta V$. Where, p_0 is atmospheric pressure.
8. **Assertion** During melting of ice work done by surrounding on (ice + water) system is positive.
   Reason Volume of the given system decreases on melting of ice.
9. **Assertion** Molar heat capacity cannot be defined for isothermal process.
   Reason In isothermal process, $(p-V)$ *versus* T graph is a dot.
10. **Assertion** If initial and final volumes are equal, then work done by gas is zero.
    Reason In isochoric process, initial and final volumes are equal and work done by gas is zero.
11. **Assertion** In isobaric process, $\frac{Q}{\Delta U}$ is equal to $\gamma\ (= C_p\ /\ C_V)$ of the gas.
    Reason For monoatomic gas, ratio $\frac{Q}{\Delta U}$ in isobaric process is equal to $\frac{5}{3}$.
12. **Assertion** A system of ideal gas is heated by constant pressure. If temperature *versus* time graph is a straight line, the process must be isochoric.
    Reason For isochoric process.
    $$\Delta Q = nC_V\,\Delta T$$
    $$\Rightarrow \quad \frac{\Delta Q}{\Delta T} = p = nC_V\ \frac{\Delta T}{\Delta t}$$
    $$\Rightarrow \quad \frac{\Delta T}{\Delta t} = \frac{p}{nC_V} = \text{constant}$$
13. **Assertion** If volume of a gas is increasing but temperature of the gas is decreasing, then heat given to the gas may be positive, negative or zero.
    Reason Heat given to a gas is a state function. It is not path function.
14. **Assertion** In adiabatic expansion, the product of P and V always decreases.
    Reason In adiabatic expansion process, work is done by the gas at the cost of internal energy of gas.
15. **Assertion** There are two processes : Process-1 is pV = constant and process-2 is pV^2 = constant. In both the processes, volume of gas is increased from V_1 to V_2. Initial co-ordinates (p_1, V_1) of the gas were same. Then more work is done by the gas in process-1.
    Reason In second process, pressure drops more rapidly with increase in volume.

Match the columns

1. Match the following columns.

	Column I		Column II
(A)	Adiabatic expansion	(p)	No work done
(B)	Isobaric expansion	(q)	Constant internal energy
(C)	Isothermal expansion	(r)	Increase in internal energy
(D)	Isochoric process	(s)	Decrease in internal energy

2. Match the following columns.

	Column I		Column II
(A)	Cyclic process	(p)	$\Delta U < 0$
(B)	Isobaric process	(q)	$\Delta Q = \Delta W$
(C)	Isochoric process	(r)	$\Delta W = \Delta nR\Delta T$
(D)	Adiabatic process	(s)	$\Delta Q = \Delta U$

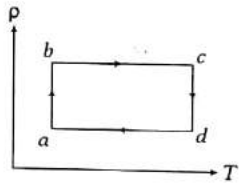
3. Match the following columns.

	Column I		Column II
(A)	Adiabatic bulk modulus	(p)	$-\frac{p}{V}$
(B)	Slope of p-V graph in isothermal process	(q)	$\frac{2}{\gamma - 1}$
(C)	Degree of freedom	(r)	γp
(D)	Molar heat capacity at constant pressure divided by R	(s)	$\frac{\gamma}{\gamma - 1}$

4. For one mole of a monoatomic gas match the following columns.

	Column I		Column II
(A)	Isothermal bulk modulus	(p)	$-\frac{RT}{V^2}$
(B)	Adiabatic bulk modulus	(q)	$-\frac{5P}{3V}$
(C)	Slope of p-V graph in isothermal process	(r)	$\frac{T}{V}$
(D)	Slope of p-V graph in adiabatic process	(s)	$\frac{4T}{3V}$
		(t)	None

5. In the ρ-T graph shown in figure, match the following.



	Column I		Column II
(A)	Process a-b	(p)	Isochoric process
(B)	Process b-c	(q)	$\Delta U = 0$
(C)	Process c-d	(r)	p increasing
(D)	Process d-a	(s)	p decreasing

C) Medical entrances' gallery

(Collection of questions asked in NEET & Various Medical Entrance Exams)

1. One mole of an ideal monatomic gas undergoes a process described by the equation $pV^3 =$ constant. The heat capacity of the gas during this process is **[NEET 2016]**

(a) $\frac{3}{2}R$ (b) $\frac{5}{2}R$ (c) $2R$ (d) R

2. The temperature inside a refrigerator is t_2 °C and the room temperature is t_1 °C. The amount of heat delivered to the room for each joule of electrical energy consumed ideally will be **[NEET 2016]**

(a) $\frac{t_1}{t_1 - t_2}$ (b) $\frac{t_1 + 273}{t_1 - t_2}$

(c) $\frac{t_2 + 273}{t_1 - t_2}$ (d) $\frac{t_1 + t_2}{t_1 + 273}$

3. A gas is compressed isothermally to half its initial volume. The same gas is compressed separately through an adiabatic process until its volume is again reduced to half. Then **[NEET 2016]**

(a) compressing the gas through adiabatic process will require more work to be done

(b) compressing the gas isothermally or adiabatically will require the same amount of work

(c) which of the case (whether compression through isothermal or through adiabatic process) requires more work will depend upon the atomicity of the gas

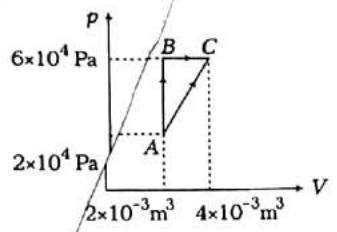
(d) compressing the gas isothermally will require more work to be done

4. A refrigerator works between 4°C and 30°C. It is required to remove 600 calories of heat every second

in order to keep the temperature of the refrigerated space constant. The power required is (Take, 1 cal = 4.2 J) **[NEET 2016]**

(a) 23.65 W (b) 236.5 W
(c) 2365 W (d) 2.365 W

5. Figure below shows two paths that may be taken by a gas to go from a state A to a state C.



In process AB, 400 J of heat is added to the system and in process BC, 100 J of heat is added to the system. The heat absorbed by the system in the process AC will be **[CBSE AIPMT 2015]**

(a) 380 J (b) 500 J (c) 460 J (d) 300 J

6. A carnot engine, having an efficiency of $\eta = 1/10$ as heat engine, is used as a refrigerator. If the work done on the system is 10 J, the amount of energy absorbed from the reservoir at lower temperature is **[CBSE AIPMT 2015]**

(a) 100 J (b) 99 J
(c) 90 J (d) 1 J

7. If ΔU represents the increases in internal energy and W the work done by the thermodynamic system, then **[Kerala CEE 2015]**

(a) $\Delta U = -W$ is an adiabatic process
(b) $\Delta U = W$ is an isothermal process
(c) $\Delta U = W$ is an adiabatic process
(d) $\Delta U = W$ is an isochoric process

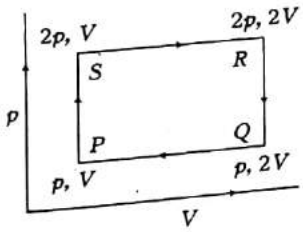
8. If the energy input to a Carnot engine is thrice the work it performs, then the fraction of energy rejected to the sink is **[Kerala CEE 2015]**

(a) 1/3 (b) 1/4 (c) 2/5 (d) 2/3
(e) 3/4

9. A gas with $\gamma = 1.4$ undergoes the adiabatic process of compression, if the pressure is increased by 0.5%, then the volume decreases by (about) **[UP CPMT 2015]**

(a) 0.25% (b) 0.1% (c) 1% (d) 0.5%

10. An ideal monoatomic gas is taken round the cycle $PQRS$ as shown p-V diagram.



The work done during the cycle is **[CG PMT 2015]**

(a) pV (b) $2pV$ (c) p (d) zero

11. The efficiency of a Carnot engine which operates between the two temperatures $T_1 = 500$ K and $T_2 = 300$ K is **[KCET 2015]**

(a) 75 % (b) 50%
(c) 40% (d) 25%

12. An ideal diatomic gas is heated at constant pressure. What fraction of heat energy is utilised to increase its internal energy? **[EAMCET 2015]**

(a) $\frac{5}{7}$ (b) $\frac{2}{5}$
(c) $\frac{3}{5}$ (d) $\frac{3}{7}$

13. How much heat (in joules) is required to raise the temperature of 1g of water from 14.5°C to 15.5°C at 1 atmospheric pressure? **[UK PMT 2015]**

(a) 41.86 J (b) 4.186 J
(c) 418.6 J (d) None of these

14. The increase in internal energy of 1kg of water at 100°C when it is converted into steam at the same temperature and at 1 atmospheric pressure will be (density of steam is 0.6 kg m^{-3}, latent heat of vaporisation of water = 2.25×10^6 Jkg^{-1}) **[UK PMT 2015]**

(a) 1.04×10^6 J (b) 2.08×10^6 J
(c) 4.16×10^6 J (d) 8.32×10^6 J

15. The ratio of the specific heats $\frac{C_p}{C_V} = \gamma$ in terms of degrees of freedom (n) is given by **[CBSE AIPMT 2015]**

(a) $\left(1+\frac{1}{n}\right)$ (b) $\left(1+\frac{n}{3}\right)$
(c) $\left(1+\frac{2}{n}\right)$ (d) $\left(1+\frac{n}{2}\right)$

16. The molar specific heat of a gas as given from the kinetic theory is $\frac{5}{2}R$. If it is not specified whether it is C_p or C_V, one could conclude that the molecules of the gas **[AIIMS 2015]**

(a) are definitely monoatomic
(b) are definitely rigid diatomic
(c) are definitely non-rigid diatomic
(d) can be monoatomic or rigid diatomic

17. Specific heats of one mole of hydrogen at constant pressure and at constant volume are 450 JK^{-1} and 300JK^{-1}, respectively. Then, what is the density of the gas at STP? ($p_{atm} = 1.013 \times 10^5$Nm^{-2}) **[EAMCET 2015]**

(a) 2.47 kg m^{-3} (b) 3 kg m^{-3}
(c) 3.58 kg m^{-3} (d) 4 kg m^{-3}

18. If for hydrogen $C_p - C_V = m$ and for nitrogen $C_p - C_V = n$ where, C_p and C_V refer to specific heat per unit mass respectively at constant pressure and constant volume, the relation between m and n is (Molecular weight of hydrogen = 2 and molecular weight of nitrogen = 28) **[AIIMS, UP CPMT 2014]**

(a) $n = 14\,m$ (b) $n = 7m$ (c) $m = 7n$ (d) $m = 14n$

19. Assertion For monoatomic gas atom the number of degrees of freedom is 3.

Reason $\frac{C_p}{C_V} = \gamma = \frac{5}{3}$ **[AIIMS 2014]**

(a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion
(b) Both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
(c) Assertion is correct but Reason is incorrect
(d) Assertion is incorrect but Reason is correct

20. The change in internal energy of a thermodynamic system which has absorbed 2 kcal of heat and done 400 J of work is (Take, 1 cal = 4.2 J) **[Kerala CEE 2014]**

(a) 2 kJ (b) 8 kJ (c) 3.5 kJ (d) 5.5 kJ
(e) 4.2 kJ

21. A monoatomic gas at a pressure p, having a volume V expands isothermally to a volume $2V$ and then adiabatically to a volume $16\,V$. The final pressure of the gas is (take, $\gamma = \frac{5}{3}$) **[CBSE AIPMT 2014]**

(a) $64\,p$ (b) $32\,p$ (c) $\frac{p}{64}$ (d) $16p$

22. The $p\,T$ relation for an adiabatic expansion is **[UK PMT 2014]**

(a) $p^{\gamma}T^{\gamma-1} = \text{constant}$ (b) $p^{\gamma-1}T^{\gamma} = \text{constnat}$
(c) $p^{\gamma}T^{1-\gamma} = \text{constant}$ (d) $p^{1-\gamma}T^{\gamma} = \text{constant}$

23. In isochoric process,

(a) $\Delta W = 0$ (b) $\Delta U = 0$
(c) $\Delta Q = 0$ (d) $\Delta T = 0$

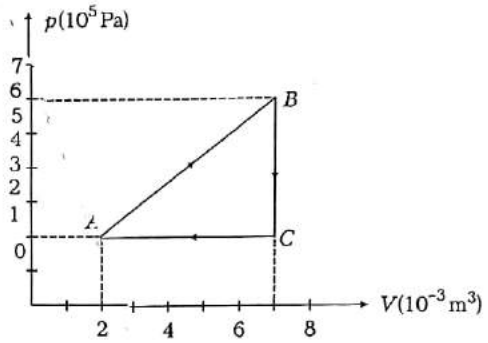
24. A Carnot refrigerator extracts heat from water at 0°C and rejects it to room at 24.4°C. The work required by the refrigerator for every 1 kg of water converted into ice (latent heat of ice = 336) **[EAMCET 2014]**

(a) 30 kJ (b) 336 kJ (c) 11.2 kJ (d) 24.4 kJ

25. A Carnot engine working between 300 K and 600 K has work output of 800 J cycle^{-1}. The amount of heat energy supplied from the source of engine in each cycle is **[WB JEE 2014]**

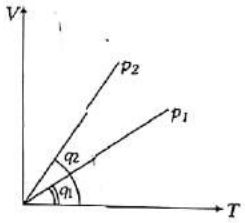
(a) 800 J (b) 1600 J (c) 3200 J (d) 6400 J

26. A gas is taken through they cycle $A \to B \to C \to A$, as shown. What is the net work done by the gas? **[NEET 2013]**



(a) 2000 J (b) 1000 J (c) Zero (d) −2000 J

27. During an adiabatic process, the pressure of a gas is found to be proportional to the cube of its temperature. The ratio of $\frac{C_p}{C_V}$ for the gas is **[NEET 2013]**



(a) $\frac{4}{3}$ (b) 2 (c) $\frac{5}{3}$ (d) $\frac{3}{2}$

28. In the given (V-T) diagram, what is the relation between pressures p_1 and p_2? **[NEET 2013]**

(a) $p_2 = p_1$ (b) $p_2 > p_1$
(c) $p_2 < p_1$ (d) Cannot be predicted

29. The amount of heat energy required to raise the temperature of 1 g of helium at NTP, from T_1 K to T_2 K is **[NEET 2013]**

(a) $\frac{3}{8}N_AK_B\,(T_2 - T_1)$ (b) $\frac{3}{2}N_AK_B\,(T_2 - T_1)$
(c) $\frac{3}{4}N_AK_B\,(T_2 - T_1)$ (d) $\frac{3}{4}N_AK_B\left(\frac{T_2}{T_1}\right)$

30. The zeroth law of thermodynamics for three systems A, B and C in contact demands that **[J & K CET 2013]**

(a) A and B are in thermal equilibrium
(b) B and C are in thermal equilibrium
(c) A and C are in thermal equilibrium
(d) A, B and C are in thermal equilibrium

31. The efficiency of a Carnot engine kept at the temperatures of 27°C and 127°C is **[J & K CET 2013]**

(a) 20% (b) 25% (c) 30% (d) 40%

32. Match the following from table A, in the case of gases, with those in table B. **[EAMCET 2013]**

Table A	Table B
(a) Work done in isobaric process	(d) $nRT \log_e (V_2/V_1)$
(b) Work done in isothermal process	(e) $p(V_2 - V_1)$
(c) Work done in adiabatic process	(f) $\frac{nR(T_1 - T_2)}{\gamma - 1}$
	(g) zero

(a) $a \to e,\ b \to d,\ c \to f$ (b) $a \to h,\ b \to g,\ c \to e$
(c) $a \to e,\ b \to d,\ c \to g$ (d) $a \to g,\ b \to h,\ c \to e$

33. Two cylinders A and B fitted with pistons contain equal amounts of an ideal diatomic gas at 300 K. Piston A is free to move and piston of B is fixed. Same amount of heat is given to the gases in the two cylinders. Temperature of the gas in cylinder A increases by 30 K. Then, increase in temperature of the gas in the cylinder B is ($\gamma = 14$ for diatomic gas) **[EAMCET 2013]**

(a) 24 K (b) 36 K (c) 54 K (d) 42 K

34. For which combination of working temperatures of source and sink, the efficiency of Carnot's heat engine is maximum? **[KCET 2013]**

(a) 600 K, 400 K (b) 400 K, 200 K
(c) 500 K, 300 K (d) 300 K, 100 K

35. During an adiabatic process of an ideal gas, if p is proportional to $\frac{1}{V^{1.5}}$, then the ratio of specific heat capacities at constant pressure to that at constant volume for the gas is **[Kerala CEE 2013]**

(a) 1.5 (b) 0.25 (c) 0.75 (d) 0.4
(e) 0.45

36. In a reversible cyclic process of a gaseous system, **[Kerala CEE 2013]**

(a) $\Delta Q = \Delta U$ (b) $\Delta U = \Delta W$
(c) $\Delta W = 0$ (d) $\Delta Q = 0$
(e) $\Delta U = 0$

37. One litre of a gas $\left(\text{with } \gamma = \frac{5}{3}\right)$ at NTP is compressed adiabatically to one cubic centimetre, then the resulting pressure is **[MP PMT 2013]**

(a) 10 Atm (b) 10^3 Atm
(c) 10^5 Atm (d) 100 Atm

38. The specific heat of an ideal gas for an isothermal process is **[MP PMT 2013]**

(a) zero (b) $\frac{3}{2}R$
(c) $\frac{5}{2}R$ (d) infinite

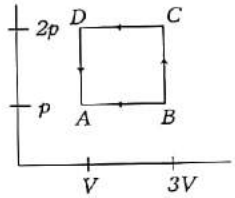
39. One mole of an ideal gas is heated at a constant pressure of 1 atm from 0°C to 100°C. Work done by the gas is **[UP CPMT 2012]**

(a) 8.31×10^3 J (b) 8.31×10^{-3} J
(c) 8.31×10^{-3} J (d) 8.31×10^2 J

40. The efficiency of a Carnot engine is 60%. If the temperature of source is 127°C. The sink must be maintained at **[UP CPMT 2012]**

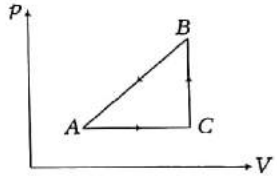
(a) 113 K (b) +113°C
(c) −113°C (d) −113 K

41. A thermodynamic system is taken through the cycle $ABCD$ as shown in figure. Heat rejected by the gas during the cycle is **[CBSE AIPMT 2012]**



(a) $2pV$ (b) $4pV$ (c) $\frac{1}{2}pV$ (d) pV

42. If an ideal gas changes from state 1 to state 2 as shown in figure, then the work done by the gas in the process will be **[BHU (Screening) 2012]**



(a) positive (b) negative
(c) zero (d) infinite

43. The molar specific heats of an ideal gas at constant pressure and volume are denoted by C_p and C_V respectively. If $\frac{C_p}{C_V} = \gamma$ and R is the gas constant for 1 g mole of a gas, then C_p is equal to **[BHU 2012]**

(a) γR (b) $\frac{R}{\gamma - 1}$ (c) $\frac{\gamma R}{\gamma - 1}$ (d) R

44. If a gas expands, with temperature according to the relation $V = KT^{2/3}$, then what will be work done, when the temperature changes by 30°C? **[BCECE (Mains) 2012]**

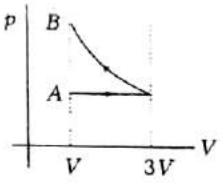
(a) $40R$ (b) $30R$ (c) $20R$ (d) $10R$

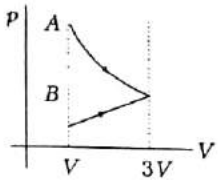
45. When heat is supplied to a diatomic gas at constant pressure, then the ratio of $\Delta Q : \Delta U : \Delta W$ will be **[BCECE (Mains) 2012]**

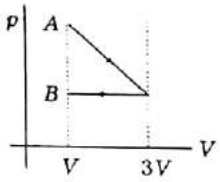
(a) 5 : 2 : 3 (b) 7 : 2 : 5
(c) 5 : 3 2 (d) 7 : 5 : 2

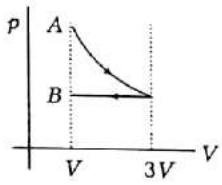
46. One mole of an ideal gas goes from an initial state A to final state B *via* two processes. It first undergoes isothermal expansion from volume V to $3V$ and then its volume is reduced from $3V$ to V at constant

pressure. The correct p-V diagram representing the two processes is **[CBSE AIPMT 2012]**

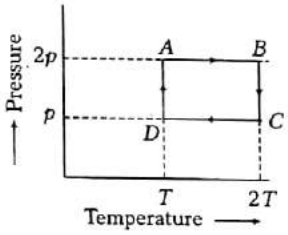
(a) 

(b) 

(c) 

(d) 

47. An ideal monoatomic gas is taken through the theromodynamic states $A \to B \to C \to D$ *via* the paths shown in the figure. If U_A, U_B, U_C and U_D represent the internal energy of the gas in states A, B, C and D respectively, then which of the following is not true? **[Mainipal 2012]**



(a) $U_A = U_D$ (b) $U_B < U_A$
(c) $U_B = U_C$ (d) $U_C > U_D$

48. During an adiabatic process, the cube of the pressure is found to be inversely proportional to the fourth power of the volume. Then, the ratio of specific heats is **[Manipal 2012]**

(a) 1 (b) 1.33
(c) 1.67 (d) 1.4

49. A Carnot's engine has an efficiency of 50% at sink temperature 50°C. Calculate the temperature of source. **[Manipal 2012]**

(a) 133°C
(b) 143°C
(c) 100°C
(d) 373°C

50. The water of volume 4 m^3 at the height 20 m is pressed by 2×10^5 N pressure. The work done by motor is **[Manipal 2012]**

(a) 8×10^5 J (b) 16×10^5 J
(c) 12×10^5 J (d) 32×10^5 J

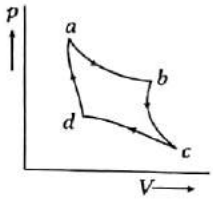
51. In adiabatic process, the work done by system is 50 J, then **[Manipal 2012]**

(a) the temperature of the system will be increase by 50 J
(b) the temperature of the system will be constant
(c) the internal energy of the system will be increase by 50 J
(d) the internal energy of the system will be decrease by 50 J

52. 100 mol of an ideal gas is heated from 10 to 20°C keeping its (i) volume constant (ii) pressure constant. Let ΔU_a and ΔU_b denote the change in the internal energy of the gas due to process (i) and (ii) respectively. Then, which of the following shall be true? **[AMU 2012]**

(a) $\Delta U_a = \Delta U_b$ (b) $\Delta U_a > \Delta U_b$
(c) $\Delta U_a < \Delta U_b$ (d) $\Delta U_a = \Delta U_b = 0$

53. Refer to the Carnot cycle of an ideal gas shown in the figure. Let $W_{a\to b}, W_{b\to c}, W_{c\to d}$ and $W_{d\to a}$ represent the work done by the system during the processes $a \to b$, $b \to c$, $c \to d$ and $d \to a$ respectively. Consider the following relations **[AMU 2012]**



(i) $W_{a\to b} + W_{b\to c} + W_{c\to d} + W_{d\to a} > 0$
(ii) $W_{a\to b} + W_{b\to c} + W_{c\to d} + W_{d\to a} < 0$
(iii) $W_{a\to b} + W_{c\to d} > 0$
(iv) $W_{b\to c} + W_{d\to a} = 0$

Which of the above relation(s) is/are true?

(a) (i) and (iii) (b) (iii) and (iv)
(c) (i) (iii) and (iv) (d) (ii), (iii) and (iv)

54. An ideal gas undergoes a reversible isothermal expansion from a state i to f. The change in entropy $\Delta S(i \to f)$ is **[AMU 2012]**

(a) 0 (b) > 0
(c) < 0 (d) not defined for the process

Answers

Check point 15.1

1. (a)	2. (a)	3. (b)	4. (d)	5. (c)	6. (b)	7. (b)	8. (a)	9. (b)	10. (c)
11. (b)	12. (a)	13. (a)	14. (d)	15. (a)					

Check point 15.2

1. (b)	2. (b)	3. (a)	4. (b)	5. (d)	6. (b)	7. (d)	8. (d)	9. (c)	10. (a)

Check point 15.3

1. (c)	2. (a)	3. (a)	4. (c)	5. (c)	6. (a)	7. (a)	8. (d)	9. (d)	10. (d)
11. (b)	12. (d)	13. (c)	14. (d)	15. (b)	16. (c)	17. (d)	18. (b)	19. (a)	20. (d)
21. (b)	22. (a)	23. (b)	24. (b)	25. (a)					

Check point 15.4

1. (b)	2. (a)	3. (a)	4. (d)	5. (d)	6. (a)	7. (b)	8. (c)	9. (d)	10. (a)

A. Taking it together

1. (a)	2. (c)	3. (c)	4. (a)	5. (b)	6. (d)	7. (a)	8. (a)	9. (c)	10. (b)
11. (a)	12. (d)	13. (b)	14. (b)	15. (c)	16. (a)	17. (c)	18. (a)	19. (a)	20. (b)
21. (a)	22. (a)	23. (c)	24. (b)	25. (a)	26. (a)	27. (c)	28. (b)	29. (c)	30. (c)
31. (d)	32. (b)	33. (d)	34. (a)	35. (a)	36. (c)	37. (c)	38. (d)	39. (a)	40. (c)
41. (d)	42. (a)	43. (c)	44. (b)	45. (c)	46. (a)	47. (a)	48. (c)	49. (a)	50. (a)
51. (c)	52. (a)	53. (d)	54. (a)	55. (b)	56. (b)	57. (b)	58. (c)	59. (a)	60. (a)
61. (c)	62. (c)	63. (d)	64. (c)	65. (c)	66. (a)	67. (a)	68. (a)	69. (a)	70. (b)
71. (a)	72. (b)	73. (d)	74. (c)						

B. Medical entrance special format questions

Assertion and reason

1. (b)	2. (b)	3. (d)	4. (d)	5. (c)	6. (d)	7. (b)	8. (a)	9. (b)	10. (d)
11. (b)	12. (d)	13. (c)	14. (b)	15. (b)					

Match the columns

1. (A $\rightarrow$ s, B $\rightarrow$ r, C $\rightarrow$ q, D $\rightarrow$ p)
2. (A $\rightarrow$ q, B $\rightarrow$ r, C $\rightarrow$ s, D $\rightarrow$ p)
3. (A $\rightarrow$ r, B $\rightarrow$ p, C $\rightarrow$ q, D $\rightarrow$ s)
4. (A $\rightarrow$ t, B $\rightarrow$ t, C $\rightarrow$ p, D $\rightarrow$ q)
5. (A $\rightarrow$ q, r; B $\rightarrow$ p, r; C $\rightarrow$ q, s; D $\rightarrow$ p, s)

C. Medical entrances' gallery

1. (d)	2. (b)	3. (a)	4. (b)	5. (c)	6. (c)	7. (a)	8. (d)	9. (d)	10. (a)
11. (c)	12. (a)	13. (b)	14. (b)	15. (c)	16. (d)	17. (a)	18. (d)	19. (a)	20. (b)
21. (c)	22. (d)	23. (a)	24. (a)	25. (b)	26. (b)	27. (d)	28. (c)	29. (a)	30. (a)
31. (b)	32. (a)	33. (d)	34. (d)	35. (a)	36. (e)	37. (c)	38. (d)	39. (d)	40. (c)
41. (a)	42. (c)	43. (c)	44. (c)	45. (d)	46. (d)	47. (d)	48. (b)	49. (d)	50. (b)
51. (d)	52. (b)	53. (c)	54. (a)						

Hints and explanations

Check point 15.1

1. (*a*) First case of thermodynamics is another form of case of conservation of energy.

2. (*a*) $\Delta U = \Delta Q - \Delta W \Rightarrow \Delta U = \Delta Q\ (< 0)$ $\quad (\because \Delta W = 0)$

$\Delta U < 0$, So temperature will decrease.

3. (*b*) $\Delta Q = \Delta U + \Delta W$

$\because$ Heat is supplied to the system so $\Delta Q \rightarrow$ Positive and work is done on the system so $\Delta W \rightarrow$ Negative

Hence, $\quad \Delta Q = \Delta U - \Delta W$

4. (*d*) Change in internal energy does not depend upon path, so $\Delta U = \Delta Q - \Delta W$ remain constant.

5. (*c*) Internal energy of gas is state function, *i.e.*

$$\Delta U_1 = \Delta U_2 \text{ or } Q_1 - W_1 = Q_2 - W_2$$

6. (*b*) W = area under p-V diagram.

$$W = \frac{1}{2} \times (3V_0 - V_0) \times (2p_0 - p_0) = \frac{1}{2} \times 2V_0 \times p_0 = p_0 V_0$$

7. (*b*) Work done by the system = Area of shaded portion on p-V diagram = $(300 - 100)\,10^{-6} \times (200 - 100) \times 10^3 = 20$ J cycle is anti-clockwise. Hence, work done is negative.

8. (*a*) Work done = Area enclosed by triangle

$$ABC = \frac{1}{2} AC \times BC$$

$$= \frac{1}{2} \times (3V - V) \times (3p - p) = 2pV$$

9. (*b*) In cyclic process, work done is area enclosed by the cycle. It is positive when cycle is clockwise and negative when cycle is anti-clockwise. (With p on Y-axis and V on X-axis).

10. (*c*) ΔU in both cases will be same, as potential energy is state function. Area under p-V graph in case 1 is more. Hence $W_1 > W_2$. Therefore, $Q_1 > Q_2$.

11. (*b*) From first law of thermodynamics $\Delta Q = \Delta U + \Delta W$,

$\Delta Q = 200$ J and $\Delta W = -100$ J

$\Rightarrow \quad \Delta U = \Delta Q - \Delta W = 200 - (-100) = 300$ J

12. (*a*) Cycle is clockwise. Hence, heat will be given to the gas, as work done will be positive.

$$Q = W = \text{area of cycle} = 1\text{J}$$

13. (*a*) For cyclic process, Total work done

$$= W_{AB} + W_{BC} + W_{CA}$$

$$\Delta W_{AB} = p\,\Delta V = 10\,(2 - 1) = 10\text{ J}$$

and $\quad \Delta W_{BC} = 0 \quad$ (as V = constant)

From first law of thermodynamics, $\Delta Q = \Delta U + \Delta W$

$$\Delta U = 0 \text{ (Process } ABCA \text{ is cyclic)}$$

$$\Rightarrow \quad \Delta Q = \Delta W_{AB} + \Delta W_{BC} + \Delta W_{CA}$$

$$\Rightarrow \quad 5 = 10 + 0 + \Delta W_{CA} \Rightarrow \Delta W_{CA} = -5\text{ J}$$

14. (*d*) Heat given, $\Delta Q = 20$ cal $= 20 \times 4.2 = 84$ J

Work done, $\Delta W = -50$ J (As process is anti-clockwise)

By first law of thermodynamics,

$$\Rightarrow \quad \Delta U = \Delta Q - \Delta W = 84 - (-50) = 134\text{J}$$

15. (*a*) In a cyclic process, $\Delta U = 0$

$$\therefore \quad Q_{\text{total}} = W_{\text{total}} \text{ or } 10 - 5 = -3 + W_{21} \text{ or } W_{21} = 8\text{J}$$

Check point 15.2

1. (*b*) For monoatomic gas, $C_V = \frac{3}{2}R$

Thus, $\quad C_V = \frac{3}{2} \times 8.31\text{Jmol-K}^{-1}$

$$= \frac{3}{2} \times \frac{8.31}{4.18} \approx 3 \text{ cal (mol-K)}^{-1}$$

Let ΔT be the rise in temperature when n mole of the gas is given Q cal of heat at constant volume. Then,

$$Q = nC_V \Delta T$$

or $\quad \Delta T = \frac{Q}{nC_V} = \frac{10000 \text{ cal}}{4.0 \text{ mol} \times 3 \text{ cal (mol-K)}} = 833\text{ K}$

2. (*b*) When a gas is heated at constant volume, all the heat supplied is used to increase the temperature or internal energy of the gas. At constant pressure, an additional amount of heat equivalent to work done is utilised. Hence, $C_p > C_V$.

3. (*a*) Given, $C_p - C_V = a$ (For hydrogen gas)

$C_p - C_V = b$ (For oxygen gas)

$$\therefore \quad C_p - C_V = a = \frac{R}{1}\,(H_2) \quad \text{...(i)}$$

$$C_p - C_V = b = \frac{R}{16}\,(O_2) \quad \text{...(ii)}$$

Comparing Eqs. (i) and (ii)

$$a = 16\,b$$

4. (*b*) As, we know $\gamma = \frac{C_p}{C_V}$

then value of molar specific heat of an ideal gas at constant volume, *i.e.*, $C_V = \frac{R}{\gamma - 1}$

6. (*b*) Increase is internal energy

$$\Delta U = \mu C_V\,\Delta T = \mu \left(\frac{R}{\gamma - 1}\right) \Delta T$$

$$\Rightarrow \quad \Delta U = \frac{p\Delta V}{\gamma - 1} = \frac{p\,(2\,V - V)}{\gamma - 1} = \frac{pV}{\gamma - 1}$$

As $pV = nRT = 2RT$

$\therefore \quad \Delta U = \dfrac{2RT}{\gamma - 1}$ $[\because n = 2]$

7. (*d*) For, 2 moles of Oxygen and 4 moles of Argon

$$\Delta U = 2C_{V_O}T + 4C_{V_{Ar}}T$$
$$= 2T(C_{V_O} + 2C_{V_{Ar}})$$
$$= 2T\left[\frac{R}{2/5} + 2\times\frac{R}{2/3}\right]$$
$$= 2TR\left[\frac{5+6}{2}\right] = 11\,RT$$

8. (*d*) Molar mass of mixture $= \dfrac{n_1M_1 + n_2M_2}{n_1 + n_2}$

$$= \frac{5(4) + 2(2)}{5+2} = \frac{24}{7}\text{ g}$$

9. (*c*) $C_{V_{(mix)}} = \dfrac{n_1C_V + n_2C_V}{n_1 + n_2} = \dfrac{5\times\frac{3}{2}R + 2\times\frac{5}{2}R}{5+2}$

$$= \frac{\frac{15}{2}R + 5R}{7} = \frac{15R + 10R}{2\times 7} = \frac{25R}{14}$$

$$\gamma_{mix} = 1 + \frac{R}{C_{V_{(mix)}}} = 1 + \frac{R}{25R}\times 14 = 1.56$$

10. (*a*) As, $C_V = \dfrac{R}{(\gamma - 1)}$

$\therefore \quad (C_V)_1 = \dfrac{R}{\left(\frac{5}{3}\right) - 1} = \dfrac{3}{2}R$

and $\quad (C_V)_2 = \dfrac{R}{\left(\frac{7}{5}\right) - 1} = \dfrac{5}{2}R$

$\Rightarrow \quad (C_V)_{mix} = \dfrac{R}{\left(\frac{29}{23}\right) - 1} = \dfrac{23}{6}R$

$\because \quad C_{V_{mix}} = \dfrac{n_1(C_V)_1 + n_2(C_V)_2}{n_1 + n_2}$

We have $\quad \dfrac{23}{6}R = \dfrac{3\times\frac{3}{2}R + n_2\frac{5}{2}R}{1 + n_2} = \dfrac{(9 + 5n_2)R}{2(1 + n_2)}$

or $\quad 23 + 23n_2 = 27 + 15n_2$

i.e., $\quad n_2 = 0.5$ g-mol

Check point 15.3

1. (*c*) In isobaric process heat is used in both, in raising the temperature and in doing work.

2. (*a*) Work done $= p\,\Delta V = p\,(V_2 - V_1)$

3. (*a*) Work done $= p\,(V_2 - V_1)$

4. (*c*) Work done by gas,

$$W = p\,(V_f - V_i)$$
$$= 1.0\times 10^5(1.7\times 10^{-3} - 1.5\times 10^{-3}) = 20\text{ J}$$
$$\Delta U = Q - W = 70 - 20 = 50\text{ J}$$

5. (*c*) At constant volume, $p \propto T$

$\Rightarrow \quad \dfrac{p_1}{p_2} = \dfrac{T_1}{T_2} \Rightarrow \dfrac{p_1}{p_2} = \dfrac{300}{400} = \dfrac{3}{4}$

6. (*a*) Slow process is isothermal process with $\Delta U = 0$

8. (*d*) During isothermal change T = constant

$\Rightarrow \Delta U = 0$ also from first law of thermodynamics, $\Delta Q = \Delta W$.

11. (*b*) In isothermal process $p \propto \dfrac{1}{V}$

Hence, graph between p and V is a hyperbola.

12. (*d*) The process is very fast, so the gas fails to gain or lose heat. Hence, this process is adiabatic.

13. (*c*) In adiabatic process, pV^γ = constant or $p \propto \dfrac{1}{V^\gamma}$

14. (*d*) For adiabatic compression,

$$p_1V_1^\gamma = p_2V_2^\gamma \Rightarrow \frac{p_2}{p_1} = \left[\frac{V_1}{V_2}\right]^\gamma = \left[\frac{4}{1}\right]^{3/2} = \frac{8}{1}$$

15. (*b*) Fast processes are adiabatic. Hence,

$$p_1V_1^\gamma = p_2V_2^\gamma$$

$\therefore \quad p_2 = p_1\left(\dfrac{V_1}{V_2}\right)^\gamma = p\left(\dfrac{640}{8}\right)^{5/3} = p\,(8)^{5/3} = 32p$

16. (*c*) In adiabatic process, $TV^{\gamma-1}$ = constant

or $\quad T_2V_2^{\gamma-1} = T_1V_1^{\gamma-1}$

or $\quad T_2 = \left(\dfrac{V_1}{V_2}\right)^{\gamma-1} T_1$

$$= (4)^{1.4-1}(300)$$
$$= (300)(4)^{0.4}\text{ K}$$

17. (*d*) Using relation $\dfrac{T_2}{T_1} = \left(\dfrac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = (8)^{\frac{3/2-1}{3/2}} = 2$

$\Rightarrow \quad T_2 = 2T_1$

$\Rightarrow \quad T_2 = 2(273 + 27) = 600\text{ K} = 327°\text{ C}$

18. (*b*) $p \propto T^3$

In adiabatic process, $p^{1-\gamma}\,T^\gamma$ = constant

or $\quad p \propto T^{\frac{\gamma}{\gamma-1}}$ *i.e.*, $\dfrac{\gamma}{\gamma - 1} = 3$

Solving this equation we get, $\gamma = \dfrac{3}{2}$

19. (*a*) Heat given to the gas, $\Delta Q = mS\,\Delta\theta$

$\therefore \quad S = \dfrac{\Delta Q}{m\cdot\Delta\theta}$

In adiabatic process, $\Delta Q = 0$

$\therefore \quad S = 0$

21. (*b*) In adiabatic change, $\Delta Q = 0$

So, $\Delta W = -\Delta U$ $(\because \Delta Q = \Delta U + \Delta W)$

23. (*b*) In adiabatic process, $\Delta U = -\Delta W_1$

In the first process, $\Delta W_1 = +25$ J

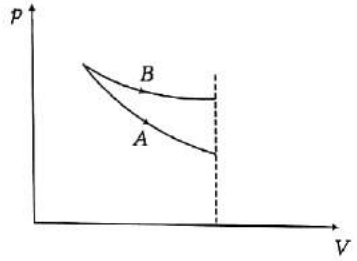
$\therefore$ $\Delta U_1 = -25$ J

In second process, $\Delta W_2 = -100$ J

$\therefore$ $\Delta U_2 = +100$ J

24. (*b*) Isothermal bulk modulus, $B = p$

25. (*a*) Work done = Area under p-V diagram.



$(\text{Area})_{iso} > (\text{Area})_{adi} \Rightarrow W_{iso} > W_{adi}$

Check Point 15.4

1. (*b*) $\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1}$

2. (*a*) $\frac{W}{Q_1} = \left(1 - \frac{T_2}{T_1}\right)$ or $W = Q_1\left(1 - \frac{T_2}{T_1}\right)$

$= 10000\left(1 - \frac{400}{500}\right) = 2000$ J

3. (*a*) Coefficient of performance, $\beta = \frac{Q_2}{W} = \frac{T_L}{T_H - T_L}$

$Q_2 = \frac{273 \times 1}{303 - 273} = \frac{273}{30} = 9$J

Heat delivered to the surroundings,

$Q_1 = Q_2 + W = 9 + 1 = 10$J

4. (*d*) Thermal efficiency of a Cornot engine is given by

$\eta = 1 - \frac{T_2}{T_1}$ ...(i)

where, T_2 is temperature of sink and T_1 is the temperature of the source.

Here, $\eta = 50$ %, $T_2 = 50°$ C $= 273 + 50 = 323$K

Putting these values in Eq. (i), we get

$\frac{50}{100} = 1 - \frac{323}{T_1}$ or $\frac{323}{T_1} = 1 - \frac{1}{2} = \frac{1}{2}$

$\therefore$ $T_1 = 646$ K $= 373°$

6. (*a*) $\eta = \left(1 - \frac{T_2}{T_1}\right) \times 100 = \left\{1 - \frac{(87 + 273)}{(127 + 273)}\right\} \times 100 = 10\%$

7. (*b*) Efficiency of the engine, $\eta = 1 - \frac{T_2}{T_1} = 0.5$ or $T_1 = 2T_2$

$= 2(17 + 273) = 580$ K

Temperature of hot body is increased by 145°C or 145 K.

$\therefore$ $T_1' = (580 + 145) = 725$ K

and $T_2 = (17 + 273) = 290$ K

$\therefore$ $\eta = \left(1 - \frac{290}{725}\right) \times 100 = 60\%$

8. (*c*) Given, $\frac{W}{Q} = 1 - \frac{T_2}{T_1} = 1 - \frac{800}{400} = 0.5$

or Amount of heat taken, $Q = \frac{W}{0.5} = \frac{750}{0.5} = 1500$ J

9. (*d*) $\eta = 1 - \frac{T_2}{T_1}$; for η to be maximum ratio, $\frac{T_2}{T_1}$ should be minimum.

10. (*a*) Entropy is related to disorder of molecular motion.

A) Taking it together

1. (*a*) pV^γ = constant $\Rightarrow TV^{\gamma-1}$ = constant

2. (*c*) In cyclic process; work done may be greater than or less than zero.

3. (*c*) p-V diagram of the gas is a straight line passing through origin. Hence, $p \propto V$ or pV^{-1} = constant

Molar heat capacity in the process pV^x = constant is

$C = \frac{R}{\gamma - 1} + \frac{R}{1 - x}$; Here $\gamma = 14$ (For diatomic gas)

$\Rightarrow$ $C = \frac{R}{1.4 - 1} + \frac{R}{1 + 1} \Rightarrow C = 3R$

4. (*a*) Decrease in internal energy, $\Delta U = C_V\,\Delta T = C_V(T_f - T_i)$

$= -C_V\,(T_i - T_f) \Rightarrow |\Delta U| = C_V(T_i - T_f)$

5. (*b*) $(n_1 + n_2)_i = (n_1 + n_2)_f$

or $\frac{p_0V}{RT_0} + \frac{p_0V}{RT_0} = \frac{pV}{RT_0} + \frac{pV}{2RT_0}$ or $\frac{2p_0V}{RT_0} = \frac{3pV}{2RT_0}$

$\therefore$ $p = \frac{4}{3}p_0$

6. (*d*) Process is adiabatic, as $\Delta Q = 0$

$p_1V_1^\gamma = p_2V_2^\gamma \Rightarrow \frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^\gamma = (2)^{5/3}$

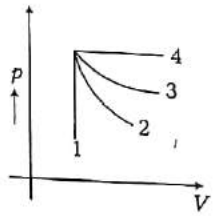
7. (*a*) Efficiency, $\eta = 1 - \frac{T_2}{T_1} = \left(\frac{273 + 97}{273 + 137}\right) = 9.75\%$

8. (*a*) From first law of thermodynamics,

$\Delta Q = \Delta U + \Delta W = \Delta U + p\,\Delta V$

$\Rightarrow$ $100 = \Delta U + 50 \times (4 - 10) \Rightarrow \Delta U = 400$ J

9. (*c*) For the curve 4 pressure is constant, so this is an isobaric process.



For the curve 1, volume is constant, so it is isochoric process. Between curves 3 and 2, curve 2 is steeper, so it is adiabatic and 3 is isothermal.

10. (*b*) $\Delta U = 0$

$\therefore \quad T = \text{constant or } p \propto \frac{1}{V}$

11. (*a*) Heat given, $\Delta Q = 40$ J and work done, $\Delta W = 30$ J

$\Rightarrow \quad \Delta U = \Delta Q - \Delta W = 40 - 30 = 10$ J

12. (*d*) Since, Volume of the ideal gas is constant so $W = 0$

Using first law of thermodynamics of

$$\Delta Q = \Delta U \Rightarrow \Delta U = i^2 rt$$
$$= 1^2 \times 100 \times 5 \times 60 = 30 \times 10^3 = 30 \text{ kJ}$$

13. (*b*) Cyclic process 1 is clockwise whereas process 2 is anti-clockwise. Since, negative area (2) > positive area (1), hence net work done is negative.

14. (*b*) It isothermal process $p \propto \frac{1}{V}$

Hence, graph between p and V is a hyperbola.

15. (*c*) Since, vessel is insulated. Therefore, $Q = 0$

Against vacuum, $W = 0$

$\therefore \quad \Delta U \text{ or } \Delta T = 0$

16. (*a*) $pV^{2/3} = \text{constant}$

$\therefore \left(\frac{T}{V}\right) \cdot V^{2/3} = \text{constant} \quad \text{or} \quad TV^{-1/3} = \text{constant} \quad \text{or} \quad T \propto V^{1/3}$

with increase in V, temperature T will also increase.

17. (*c*) Heat absorbed, $Q = W + \Delta U$

$$= p_0(2V_0 - V_0) + [2p_0 \cdot 2V_0 - 2p_0V_0]$$
$$= p_0V_0 + 2p_0V_0 = 3p_0V_0$$

18. (*a*) V-T graph is a straight line passing through origin. Or $V \propto T$. Hence, the given process is isobaric.

$$\frac{\Delta Q}{\Delta W} = \frac{nC_p\,\Delta T}{nR\Delta T} = \frac{C_p}{R} = \frac{5}{2} = 2.5$$

19. (*a*) Using first law of thermodynamics, we have

$$\Delta Q_1 < \Delta Q_2$$

$\therefore \quad \frac{\Delta Q_1}{\Delta T} < \frac{\Delta Q_2}{\Delta T} \quad \text{or} \quad C_1 < C_2 \quad \text{or} \quad \frac{C_1}{C_2} < 1$

20. (*b*) $pV^2 = \text{constant}$

$\therefore \quad pV \propto \frac{1}{V} \text{ or } T \propto \frac{1}{V}$

21. (*a*) $p = \frac{nRT}{V - b}$

$$W = \int_V^{2V} p\,dV = nRT \int_V^{2V} \frac{dV}{V - b} \quad (\text{as } T = \text{constant})$$

$$= nRT \ln\left(\frac{2V - b}{V - b}\right) = RT \ln\left(\frac{2V - b}{V - b}\right) \quad (\text{as } n = 1)$$

22. (*a*) In a cyclic process, $\Delta U = 0$

$\therefore \quad Q_{\text{total}} = W_{\text{total}}$

or $\quad 10 - 5 = -3 + W_{21} \quad \text{or} \quad W_{21} = 8$ J

23. (*c*) $\Delta Q = \Delta U + \Delta W$

$$\Rightarrow \Delta W = (\Delta Q)_p - \Delta U = (\Delta Q)_p \left[1 - \frac{(\Delta Q)_V}{(\Delta Q)_p}\right]$$

$$= (\Delta Q)_p \left[1 - \frac{C_V}{C_p}\right] = Q = \left[1 - \frac{3}{5}\right] = \frac{2}{5} Q$$

$\because \quad (\Delta Q)_p = Q$ and $\gamma = \frac{5}{3}$ for monoatomic gas.

24. (*b*) $Q_1 = nC_p\Delta T \Rightarrow \quad Q_2 = nC_V\Delta T$

$\therefore \quad Q_1 - Q_2 = nR\Delta T$

or $\quad Q_2 = Q_1 - nR\Delta T = (70) - (2)\left(\frac{8.31}{4.18}\right)(5) = 50$ cal

25. (*a*) For adiabatic process,

$$TV^{\gamma - 1} = \text{constant}$$

$\therefore \quad \frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma - 1} \quad \text{or} \quad \left(\frac{1}{2}\right)^{\gamma - 1} = \sqrt{\frac{1}{2}}$

$\therefore \quad \gamma - 1 = \frac{1}{2} \text{ or } \gamma = \frac{3}{2}$

$\therefore \quad pV^{3/2} = \text{constant}$

26. (*a*) In isothermal process p-V diagram is a rectangular hyperbola. $T_2 > T_1$. Hence, $(pV)_2 > (pV)_1$.

27. (*c*) For isobaric process $\frac{V_2}{V_1} = \frac{T_2}{T_1} \Rightarrow V_2 = V \times \frac{274}{273}$

Increase $= \frac{274\,V}{273} - V = \frac{V}{273}$

28. (*b*) $\Delta Q = 2 \cdot \Delta U$ (given)

$\therefore \; nC \cdot \Delta T = 2\,(nC_V\,\Delta T)$

or $\quad C = 2C_V = 2 \times \frac{3}{2} R = 3R$

29. (*c*) In cyclic process, $\Delta U = 0$

$\therefore$ Heat absorbed = work done

= area under p-V graph

$= (\pi)\,(R_x)\,(R_y)$

$= \pi\,(10 \times 10^3)\,(10 \times 10^{-3}) = (10^2\pi)$ J

30. (*c*) First law of thermodynamics given

$$\Delta Q = \Delta U + p\,\Delta V$$

$\Rightarrow \quad mL = \Delta U + p\,(V_2 - V_1)$

$\Rightarrow \quad \Delta U = L - p\,(V_2 - V_1) \quad (\because m = 1)$

31. (*d*) Oxygen is diatomic gas, hence its energy of two moles

$$= 4 \times \frac{5}{2} RT = 10\,RT$$

Argon is a monoatomic gas, hence its internal energy of 6 moles $= 6 \times (3/2)\,RT = 9RT$

Total internal energy $= (9 + 10)\,RT = 19\,RT$

32. (*b*) Work done by the gas should be negative, but work done on the gas should be positive.

$W = p\,(V_i - V_f)$ (on the gas)

$= 10^5\,(3.0 \times 10^{-2} - 1.5 \times 10^{-2}) = +1500$ J

33. (*d*) In adiabatic process, $TV^{\gamma-1}$ = constant

$$\therefore \quad \frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{AL_2}{AL_1}\right)^{5/3-1} = \left(\frac{L_2}{L_1}\right)^{2/3}$$

34. (*a*) Amount of sweat evaporated/minute

$$= \frac{\text{Sweat produced/minute}}{\text{Number of calories required for evaporation/kg}}$$

$$= \frac{\text{Amount of heat produced per minute in jogging}}{\text{Latent heat (in cal / kg)}}$$

$$= \frac{14.5 \times 10^3}{580 \times 10^3} = \frac{145}{580} = 0.25 \text{ kg}$$

35. (*a*) $U\rho$ = constant

$$\therefore \quad \frac{T}{V} = \text{constant}$$

$$\therefore \quad p = \text{constant}$$

$$\frac{\Delta U}{\Delta W} = \frac{\Delta U}{\Delta Q - \Delta W} = \frac{C_V}{C_p - C_V}$$

$$= \frac{1}{C_p / C_V - 1} = \frac{1}{(5/3) - 1} = \frac{3}{2}$$

36. (*c*) $\Delta U = C_V \Delta T = \left(\frac{5}{2}R\right)\Delta T$ or $\Delta T = \frac{2(\Delta U)}{5R}$

From first law of thermodynamics,

$$\Delta U = \Delta Q - \Delta W = Q - \frac{Q}{4} = \frac{3Q}{4}$$

Now molar heat capacity,

$$C = \frac{\Delta Q}{\Delta T} = \frac{Q}{\frac{2(\Delta U)}{5R}} = \frac{5RQ}{2\left(\frac{3Q}{4}\right)} = \frac{10}{3}R$$

37. (*c*) At constant volume $\Delta Q_V = C_V \Delta T_V$

At constant pressure $\Delta Q_p = 2C_p \Delta T_p$

$$\therefore \quad \frac{\Delta Q_p}{\Delta Q_V} = 2\frac{C_p}{C_V} \times \frac{\Delta T_p}{\Delta T_c} = 2 \times \frac{5}{3} \times \frac{5}{10} = \frac{5}{3}$$

$$\Rightarrow \quad \Delta Q_p = \frac{5}{3}(\Delta Q_V) = \frac{5}{3}(H)$$

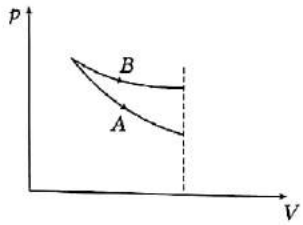
38. (*d*) During process *A* to *B*, pressure and volume both are decreasing. Therefore, temperature and hence, internal energy of the gas will decrease ($T \propto pV$) or $\Delta U_{A \to B}$ = negative. Further $\Delta W_{A \to B}$ is also negative as the volume of the gas is decreasing. Thus, $\Delta Q_{A \to B}$ is negative.

In process *B* to *C* pressure of the gas is constant, while volume is increasing. Hence, temperature should increase or $\Delta U_{B \to C}$ = positive. During process *CAB*, volume of the gas is decreasing. Hence, work done by the gas is negative.

39. (*a*) In adiabatic process, pressure will be less. Further temperature is the product of p and V.

Hence $p_A < p_B$

and $T_A < T_B$.



40. (*c*) Work is done by the gas (as cyclic process is clockwise)

$$\therefore \quad \Delta W = \text{Area } ABCD$$

So, from the first law of thermodynamics ΔQ (net heat absorbed), ΔW = Area *ABCD*

As change in internal energy in cycle $\Delta U = 0$.

41. (*d*) In process *AB*, density is constant. Hence, volume is constant. Therefore, work done will be zero.

In process *BC*, $p \mu \rho$ but $\rho \mu \frac{1}{V}$

$$\therefore \quad p \propto \frac{1}{V}$$

Or the process is isothermal. Hence internal energy is constant.

42. (*a*) $\beta = \frac{Q_2}{W} \Rightarrow 4 = \frac{400}{W} \Rightarrow W = 100 \text{ J}$

43. (*c*) Net work done = Area *ABC* − Area *AED*

$$= \frac{1}{2} \times 2 \times 1 - \frac{1}{2} \times 1 \times 1 = \frac{1}{2} \text{ J}$$

44. (*b*) Number of moles $= \frac{2}{4} = \frac{1}{2}$

$$W_{A \to B} = \text{Area under } p\text{-}V \text{ graph}$$

$$= \frac{1}{2} \times 3p_0 \times V_0 = \frac{3}{2} p_0 V_0$$

$$\Delta U_{A \to B} = nC_V \Delta T$$

$$= \frac{1}{2}\left(\frac{3}{2}R\right)\left(\frac{8p_0V_0}{R} - \frac{2p_0V_0}{R}\right) = \frac{9}{2} p_0 V_0$$

$$Q_{A \to B} = W_{A \to B} + \Delta U_{A \to B} = 6p_0V_0$$

45. (*c*) ΔU = same in all processes.

W_1 = + ve, $W_2 = 0$

and W_3 = − ve

$$\therefore \quad Q_1 > Q_2 > Q_3$$

46. (*a*) $\Delta U_1 = \Delta U_2$ as U is state function.

$$\therefore \quad Q_1 - W_1 = Q_2 - W_2$$

or $W_2 = Q_2 + W_1 - Q_1$

$$= (10^5) + (6.5 \times 10^5) - (8 \times 10^5)$$

$$= -0.5 \times 10^5 \text{ J}$$

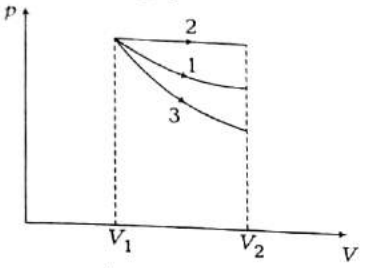
47. (*a*) On heating, volume will increase. Hence, piston will move to the left.

48. (*c*) $p_1V_1 = p_2V_2$ or $p \propto \frac{1}{V}$

Radius has become two times. Hence, volume will become eight times, *i.e.* pressure at surface is $\frac{1}{8}$ times the pressure at depth *h*. On surface, pressure is equal to 10 m of water. Hence, pressure at depth *h* is equal to 80 m of water. Of this 10 m is atmospheric pressure. Therefore, *h* is 70 m.

49. (*a*) This is the case of free expansion and in this case $\Delta W = 0$, $\Delta U = 0$ so temperature remains same, *i.e.* 300 K.

50. (*a*) W = area under p-V graph



$$A_2 > A_1 > A_3$$

$\therefore$ $$W_2 > W_1 > W_3$$

51. (*c*) Initially, $$\eta = \frac{T_1 - T_2}{T_1}$$

$\Rightarrow$ $$0.5 = \frac{T_1 - (273 + 7)}{T_1}$$

$\Rightarrow$ $$\frac{1}{2} = \frac{T_1 - 280}{T_1} \Rightarrow T_1 = 560 \text{ K}$$

Finally, $$\eta_1' = \frac{T'_1 - T_2}{T'_1}$$

$\Rightarrow$ $$0.7 = \frac{T'_1 - (273 + 7)}{T'_1}$$

$\Rightarrow$ $$T_1' = 933 \text{ K}$$

$\therefore$ Increase in temperature = 933 – 560 = 373 K.

52. (*a*) Correct graph is a.

53. (*d*) In adiabatic process as γ increase slope of p-V graph increases. Slope of B is more. Therefore, $\gamma_B > \gamma_A$. B should be monoatomic and A diatomic.

54. Using $p_1V_1^\gamma = p_2V_2^\gamma$ we have

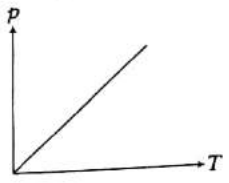
$$p_2 = 2.14 \times 10^4 \text{ Nm}^{-2}$$

55. (*b*) Consider the p-V diagram given in the question.
Work done in the process $ABCD$ = area of rectangle $ABCDA$

$$= (AB) \times BC = (3V_0 - V_0) \times (2p_0 - p_0)$$
$$= 2V_0 \times p_0 = 2\, p_0\, V_0$$

As the process is going anti-clockwise, hence there is a net compression in the gas. So, work done by the gas = $-2\, p_0V_0$.

56. (*b*) At constant volume,



$$p \propto T$$

or $$p = \left(\frac{nR}{V}\right) T = \left(\frac{mR}{MV}\right) T$$

i.e., p-T graph is a straight line passing through origin. Slope of this line is proportional to m, mass of gas.

$\therefore$ $$\tan\alpha = 2\tan\beta$$

57. (*b*) In adiabatic process, $\Delta U = -W$

$\therefore$ $$U_A - U_B = -(-30) = 30 \text{ J} \quad \text{or} \quad U_B - U_A = -30 \text{ J}$$

58. (*c*) Given, $T = kV^2$

$\therefore$ $$dT = (2kV) \cdot dV$$

or $$dV = \frac{dT}{2kV}, \; p = \frac{nRT}{V}$$

$\Rightarrow$ $$p\, dV = \frac{nRt}{2kV^2} \cdot dT = \frac{nRT}{2(T)} \cdot dT = \frac{nR}{2} \cdot dT$$

$\Rightarrow$ $$W = \int p\, dV = \int_{T_0}^{4T_0} \frac{nR}{2} \cdot dT = \frac{3}{2} nRT_0$$

59. (*a*) Given, $V = kT^2$

$\therefore$ $$2kT \cdot dT$$

and $$p = \frac{RT}{V} = \frac{RT}{kT^2} = \frac{R}{kT}$$

or $$p\, dV = 2R\, dT$$

$$W = \int p\, dV = 2R \int dT = 2R\,(60) = 120R$$

60. (*a*) VT = constant, $V(pV)$ = constant (as $T \propto pV$)

$\therefore$ $$pV^2 = \text{constant}$$

Molar heat capacity of a gas in the process

$$pV^x = \text{constant is}$$

$$C = C_V + \frac{R}{1 - x}$$

Here, $$x = 2$$

$$C = \frac{3}{2}R + \frac{R}{1 - 2} = \frac{R}{2}$$

61. (*c*) $p = \dfrac{RT}{V} = \dfrac{R(T_0 + \alpha V)}{V}$

$\therefore$ $$W = \int_{V_1}^{V_2} p \cdot dV = \int_{V_1}^{V_2} \frac{R\,(T_0 + \alpha V)}{V} \cdot dV$$

$$= RT_0 \ln\frac{V_2}{V_1} + R\alpha\,(V_2 - V_1)$$

$$\Delta U = nC_V\,\Delta T = (C_p - R)\,[(T_0 + \alpha V_2) - (T_0 + \alpha V_1)]$$
$$= \alpha\,(C_p - R)\,(V_2 - V_1)$$

Now $$Q = W + \Delta U$$

$$= RT_0 \ln\frac{V_2}{V_1} + \alpha C_p (V_2 - V_1)$$

62. (*c*) Process AB is isochoric. Hence, $W_{AB} = 0$

Process BC is isothermal. Hence, $W_{BC} = nRT_2 \ln\left(\dfrac{V_f}{V_i}\right)$

or $$W_{BC} = RT_2 \ln\left(\frac{V_1}{V_2}\right)$$

Process CA is isobaric (as $V \propto T$)

$\therefore$ $$W_{CA} = p\,(V_f - V_i) = nR(T_f - T_i)$$
$$= R\,(T_1 - T_2)$$

63. (*d*) ΔU, remains same of both path

For path *iaf* $\quad \Delta U = \Delta Q - \Delta W = 50 - 20 = 30$ J

For path *fi* $\quad \Delta U = -30$ J and $\Delta W = -13$ J

$\Rightarrow$ $$\Delta Q = -30 - 13 = -43 \text{ J}$$

64. (c) $\eta_A = \frac{T_1 - T_2}{T_1} = \frac{W_A}{Q_1} \Rightarrow \eta_B = \frac{T_2 - T_3}{T_2} = \frac{W_B}{Q_2}$

$\therefore \quad \frac{Q_1}{Q_2} = \frac{T_1}{T_2} \times \frac{T_2 - T_3}{T_1 - T_2} = \frac{T_1}{T_2}$

$\therefore \quad W_A = W_B$

$\therefore \quad T_2 = \frac{T_1 + T_3}{2} = \frac{800 + 300}{2} = 550\text{ K}$

65. (c) According to the question given that pV = constant

Hence, we can say that the gas is going through an isothermal process.

Clearly, from the graph that between process 1 and 2 temperature is constant and the gas expands and pressure decreases *i.e.*, $p_2 < p_1$ which corresponds to diagram (iii).

66. (a) $\Delta Q = ms\Delta T \Rightarrow \Delta S = \frac{\Delta Q}{T} = \frac{ms\Delta T}{T}$

The total change in entropy as temperature rises from T_1 to T_2

$$S_2 - S_1 = \int_{T_1}^{T_2} ms\frac{dT}{T} = ms\ln\left(\frac{T_2}{T_1}\right)$$

$$= 0.1 \times 4200 \ln\left(\frac{87 + 273}{27 + 273}\right)$$

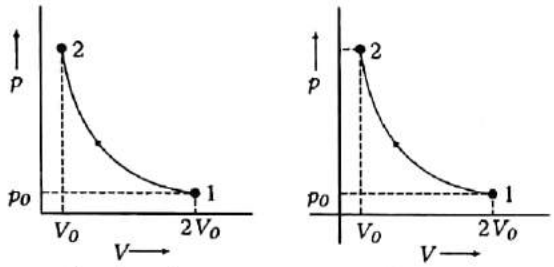
$$= 420 \ln(1.2)\text{ JK}^{-1}$$

67. (a) $\eta = \frac{\Sigma\Delta W}{\Sigma\Delta Q} = 10.82\%$

68. (a) Using $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$, we get

$T_2 = 189\text{ K}$

69. (a) Consider the p-V diagram shown for the container A (isothermal) and for container B (adiabatic).



Container A (Isothermal) Container B (Adiabatic)

Both the process involving compression of the gas.

For isothermal process (gas A) (during $1 \to 2$)

$$p_1V_1 = p_2V_2 \Rightarrow p_0(2V_0) = p_2(V_0) \Rightarrow p_2 = 2p_0$$

For adiabatic process, (gas B) (during $1 \to 2$)

$$p_1V_1^{\gamma} = p_2V_2^{\gamma} \Rightarrow p_0(2V_0)^{\gamma} = p_2(V_0)^{\gamma}$$

$$\Rightarrow \quad p_2 = \left(\frac{2V_0}{V_0}\right)^{\gamma} p_0 = (2)^{\gamma} p_0$$

Hence, $\frac{(p_2)_B}{(p_2)_A}$ = Ratio of final pressure

$$= \frac{(2)^{\gamma} p_0}{2p_0} = 2^{\gamma - 1}$$

where, γ is ratio of specific heat capacities for the gas.

70. (b) Let the equilibrium temperature of the system is T.

Let us assume that $T_1, T_2 < T < T_3$.

According to question, there is no net loss to the surroundings.

Heat lost by M_3 = Heat gained by M_1 + Heat gained by M_2

$\Rightarrow \quad M_3 s(T_3 - T) = M_1 s(T - T_1) + M_2 s(T - T_2)$

(where, s is specific heat of the copper material)

$\Rightarrow \quad T[M_1 + M_2 + M_3] = M_3T_3 + M_1T_1 + M_2T_2$

$\Rightarrow \quad T = \frac{M_1T_1 + M_2T_2 + M_3T_3}{M_1 + M_2 + M_3}$

71. (a) $\Delta W_{AB} = 0$ as V = constant

$\therefore \quad \Delta Q_{AB} = \Delta U_{AB} = 50\text{ J}$ (given)

$U_A = 1500\text{ J}$

$\therefore \quad U_B = (1500 + 50)\text{ J} = 1550\text{ J}$

$\Delta W_{BC} = -\Delta U_{BC} = -40\text{ J}$ (given)

$\therefore \quad \Delta U_{BC} = 40\text{ J}$

$\therefore \quad U_C = (1550 + 40)\text{ J} = 1590\text{ J}$

72. (b) $W_{A \to B}$ = Area under p-V diagram $= 18p_0V_0$

$$\Delta U_{A \to B} = nC_V\Delta T$$

$$= (1)\left(\frac{3}{2}R\right)\left(\frac{30p_0V_0}{R} - \frac{3p_0V_0}{R}\right) = \frac{81}{2}p_0V_0$$

$$\therefore \quad Q_{A \to B} = W_{A \to B} + \Delta U_{A \to B} = \frac{117}{2}p_0V_0$$

Molar heat capacity, $C = \frac{Q}{n\Delta T}$

$$= \frac{\frac{117}{2}p_0V_0}{\left(\frac{30p_0V_0}{R}\right) - \left(\frac{3p_0V_0}{R}\right)} = \frac{13}{6}R$$

73. (d) For path ab $(\Delta U)_{ab} = 7000\text{ J}$

By using $\Delta U = nC_V\,\Delta T$

$$7000 = n \times \frac{5}{2}R \times 700 \Rightarrow n = 0.48 \text{ For path } ca$$

$$(\Delta Q)_{ca} = (\Delta U)_{ca} + (\Delta W)_{ca} \quad \text{...(i)}$$

$\because \quad (\Delta U)_{ab} + (\Delta U)_{bc} + (\Delta U)_{ca} = 0$

$\therefore \quad 7000 + 0 + (\Delta U)_{ca} = 0$

$\Rightarrow \quad (\Delta U)_{ca} = -7000\text{ J} \quad \text{...(ii)}$

Also $\quad (\Delta W)_{ca} = p_1(V_1 - V_2) = nR(T_1 - T_2)$

$= 0.48 \times 8.31 \times (300 - 1000)]$

$= -2792.16\text{ J} \quad \text{...(iii)}$

By solving Eqs. (i), (ii) and (iii)

$(\Delta Q)_{ca} = -7000 - 2792.16$

$= -9792.16\text{ J} = -9800\text{ J}$

74. (c) Processes A to B and C to D are parts of straight line graphs passing through origin.

$p \propto T$. So, volume remains constant for the graphs AB and CD.

So, no work is done during processes for A to B and C to D

i.e. $\quad W_{AB} = W_{CD} = 0$

and $\quad W_{BC} = p_2(V_C - V_B) = nR(T_C - T_B)$

$$= 6R\,(2200 - 800)$$
$$= 6R \times 1400 \text{ J}$$

Also, $$W_{DA} = p_1(V_A - V_D) = nR\,(T_A - T_B)$$
$$= 6R(600 - 1200)$$
$$= -6R \times 600 \text{ J}$$

Hence, work done in complete cycle
$$W = W_{AB} + W_{BC} + W_{CD} + W_{DA}$$
$$= 0 + 6R \times 1400 + 0 - 6R \times 600$$
$$= 6 \times 8.3 \times 800 = 40 \text{ kJ}$$

B) Medical entrance special format questions

Assertion and reason

1. (*b*) $\frac{\Delta Q}{\Delta W} = \frac{nC_p\Delta T}{nR\Delta T} = \frac{C_p}{R} = \frac{5/2R}{R} = \frac{5}{2}$

2. (*b*) In adiabatic expansion, gas is cooled down. Therefore, temperature decreases or product of p and V will decrease. Because $T \propto pV$

3. (*d*) In isochoric process ρ and V both are constants. Therefore ρ-V graph is a dot.

4. (*d*) For given source and sink temperatures, efficiency of Carnot engine is maximum but not 100%.

5. (*c*) For given temperatures T_1 and T_2, efficiency of carnot engine is maximum.

6. (*d*) First law can be applied for any type of system.

9. (*b*) $C = \frac{Q}{n\Delta T}$

In isothermal process, $\Delta T = 0$

Therefore, C is not defined. Further, in isothermal process pV and T both are constants.

Therefore, pV *versus* T graph is a dot.

10. (*d*) If initial and final volumes are equal, then work done may or may not be zero. For work done by the gas to be zero volume should remain constant throughout the process. For example in isochoric process.

12. (*d*) For isobaric process also T-t graph under given condition may be a straight line. Replace C_V by C_p.

13. (*c*) Volume is increasing. Therefore, ΔW is positive. Therefore, ΔU is negative.

Now, $$\Delta Q = \Delta W + \Delta U$$

Therefore, Q may be positive, negative or zero.

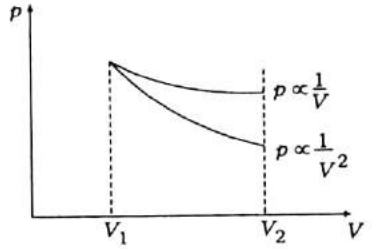
Further, heat given to gas is path function.

14. (*b*) In adiabatic expansion,
$$Q = 0 \Rightarrow W = +\text{ve}$$
$$\therefore \quad \Delta U = -\text{ve}$$

Therefore U, T or product of pV will decrease.

Because, $$U \propto T \propto pV$$

15. (*b*) Area under p-V graph in process-1 is more.



Match the columns

1. Process a-b T = constant

$$\therefore \quad \Delta T = 0 \text{ or } \Delta U = 0 \Rightarrow p \propto \frac{1}{V}$$

ρ is increasing. Therefore, V is decreasing $V \propto \frac{1}{\rho}$ or p will increase.

Process b-c ρ = constant

$\therefore$ V = constant

Therefore, $p \propto T$

Since, temperature is increasing. Hence, pressure should also increase.

2. In cyclic process $\Delta U = 0, \Delta Q = \Delta W$

In isobaric process
$$\Delta W = \Delta Q - \Delta U = nC_p\Delta T - nC_V\Delta T = nR\Delta T$$

In isochoric process $\Delta W = 0, \Delta Q = \Delta U$

In adiabatic expansion $\Delta Q = 0$

$$\therefore \quad \Delta U = -\Delta W \text{ or } \Delta U < 0$$

As work done in expansion is positive.

3. Isothermal bulk modulus,
$$B_T = p = \frac{RT}{V} \qquad (\text{as } n = 1)$$

and adiabatic bulk modulus
$$B_S = \gamma p = \frac{5}{3}\frac{RT}{V}$$

Slope of p-V graph in isothermal process $= -\frac{p}{V} = \frac{-RT}{V^2}$

and Slope of p-V graph in adiabatic process
$$= -\frac{\gamma p}{V} = -\frac{5}{3}\frac{RT}{V^2} = -\frac{5}{3}\frac{p}{V}$$

5. For adiabatic process bulk modules $B = \gamma p$

[where, B is constant]

For isothermal process, temperature is constant

$$\therefore \quad \text{Slope} = -\frac{p}{V}$$
$$\gamma = 1 + \frac{2}{f} \Rightarrow f = \frac{2}{\gamma - 1}$$
$$\Rightarrow \quad C_p = \frac{\gamma}{\gamma - 1}$$

C Medical entrances' gallery

1. (*d*) For polytropic process of index α

specific heat capacity $= C_V + \dfrac{R}{1-\alpha}$

$\because$ Process, $pV^3 = \text{constant} \Rightarrow \alpha = 3$

$\therefore \quad C = C_V + \dfrac{R}{1-\alpha} = \dfrac{fR}{2} + \dfrac{R}{1-3}$

For monatomic gas, $f = 3$

So, $\quad C_V = \dfrac{fR}{2} = \dfrac{3R}{2} \Rightarrow C = \dfrac{3R}{2} - \dfrac{R}{2} = R$

2. (*b*) For a refrigerator, we know that

$$\frac{Q_1}{W} = \frac{Q_1}{Q_1 - Q_2} = \frac{T_1}{T_1 - T_2}$$

where, Q_1 = amount of heat delivered to the room

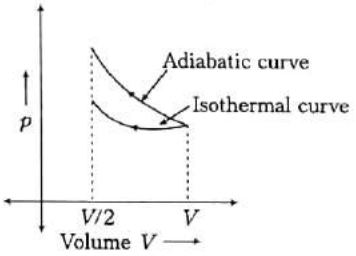
W = electrical energy consumed

T_1 = room temperature $= t_1 + 273$

T_2 = temperature of sink $= t_2 + 273$

$\therefore \quad \dfrac{Q_1}{1} = \dfrac{t_1 + 273}{t_1 + 273 - (t_2 + 273)} \Rightarrow Q_1 = \dfrac{t_1 + 273}{t_1 - t_2}$

3. (*a*) The solution of this question can be understood by plotting a p-V graph for the compression of a gas isothermally and adiabatically simultaneously to half of its initial volume. *i.e.*



Since, the isothermal curve is less steeper than the adiabatic curve. So, area under the p-V curve for adiabatic process has more magnitude than isothermal curve. Hence, work done in adiabatic process will be more than that of isothermal process.

4. (*b*) Given, temperature of source, $T = 30°\text{C} = 30 + 273$

$$T_1 = 303\text{ K}$$

Temperature of sink, $T_2 = 4°\text{C} = 4 + 273 \Rightarrow T_2 = 277\text{ K}$

As, we know that

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2} \Rightarrow \frac{Q_2 + W}{Q_2} = \frac{T_1}{T_2} \qquad (\because W = Q_1 - Q_2)$$

where Q_2 is the amount of heat drawn from the sink (at T_2), W is work done on working substance, Q_1 is amount of heat rejected to source (at room temperature T_1).

$\Rightarrow \quad WT_2 + T_2Q_2 = T_1Q_2 \Rightarrow WT_2 = T_1Q_2 - T_2Q_2$

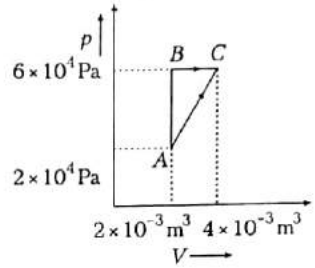
$\Rightarrow \quad WT_2 = Q_2(T_1 - T_2)$

$\Rightarrow \quad W = Q_2\left(\dfrac{T_1}{T_2} - 1\right) \Rightarrow W = 600 \times 4.2 \times \left(\dfrac{303}{277} - 1\right)$

$\Rightarrow \quad W = 600 \times 4.2 \times \left(\dfrac{26}{277}\right) \Rightarrow W = 236.5\text{ J}$

$\therefore \quad \text{Power} = \dfrac{\text{Work done}}{\text{Time}} = \dfrac{W}{t} = \dfrac{236.5}{1} = 236.5\text{ W}$

5. (*c*) Since, initial and final points are same



So, $\quad \Delta U_{A\to B\to C} = \Delta U_{A\to C} \quad$ ...(i)

Also $A \to B$ is isochoric process

So $\quad dW_{A\to B} = 0$ and $dQ = dU + dW$

So, $\quad dQ_{A\to B} = dU_{A\to B} = 400\text{ J}$

Next $B \to C$ is isobaric process

So, $\quad dQ_{B\to C} = dU_{B\to C} + dW_{B\to C} = dU_{B\to C} + p\Delta V_{B\to C}$

$\Rightarrow \quad 100 = dU_{B\to C} + 6\times10^4(2\times10^{-3})$

$\Rightarrow \quad dU_{B\to C} = 100 - 120 = -20\text{ J}$

From Eq. (i),

$\because \quad \Delta U_{A\to B\to C} = \Delta U_{A\to C}$

$\Rightarrow \quad \Delta U_{A\to B} + \Delta U_{B\to C} = dQ_{A\to C} - dW_{A\to C}$

$\Rightarrow \quad 400 + (-20) = dQ_{A\to C} - (p\Delta V_A + \text{Area of } \Delta ABC)$

$\Rightarrow \quad dQ_{A\to C} = 380 + \left(2\times10^4 \times 2\times10^{-3} + \dfrac{1}{2}\times2\times10^{-3}\times4\times10^4\right)$

$= 380 + (40 + 40)$

$\Rightarrow \quad dQ_{A\to C} = 460\text{ J}$

6. (*c*) As, $\quad Q_1 + W = Q_2$

Given, $\quad \eta = \dfrac{1}{10}$

Now, using $\quad \eta = 1 - \dfrac{T_1}{T_2}$

So, $\quad \dfrac{1}{10} = 1 - \dfrac{T_1}{T_2} \Rightarrow \dfrac{T_1}{T_2} = \dfrac{9}{10}$

Now $\quad \dfrac{Q_1}{Q_2} = \dfrac{T_1}{T_2} \Rightarrow \dfrac{Q_1}{Q_1 + W} = \dfrac{9}{10}$

$\Rightarrow \quad 10Q_1 = 9Q_1 + 9W$

$\Rightarrow \quad Q_1 = 9W = 9\times10 = 90\text{ J}$

7. (*a*) From the first law of thermodynamics,

$$Q = \Delta U + W$$

In an isothermal change, $\Delta U = 0$ as temperature remains constant.

In an adiabatic change, $Q = 0$

$\therefore \quad \Delta U = -W$

8. (*d*) We know that $\eta = \dfrac{W\text{ (output)}}{\text{Heat (input)}} = \dfrac{W}{3W} = \dfrac{1}{3}$

Now, efficiency of engine,

$$\eta = 1 - \frac{Q_2}{Q_1} \Rightarrow \frac{1}{3} = 1 - \frac{Q_2}{Q_1} \Rightarrow \frac{Q_2}{Q_1} = 1 - \frac{1}{3} \Rightarrow \frac{Q_2}{Q_1} = \frac{2}{3}$$

9. (*d*) As for adiabatic process, pV^{γ} = constant

$$(p_1V_1)^{\gamma} = (p_2V_2)^{\gamma} \Rightarrow p_1V_1 = p_2V_2, \frac{p_1}{p_2} = \frac{V_2}{V_1}$$

So, percentage of increase in pressure is equal to the percentage of decrease of the volume.

10. (*a*) The work done during the given cyclic process *PSRQ* is

$$\Delta W = \text{area } PQRS = (2V - V)(2p - p) = pV$$

11. (*c*) Given, $T_1 = 500$ K, $T_2 = 300$ K

We know that, efficiency of a Carnot engine,

$$= \left[1 - \frac{T_2}{T_1}\right] \times 100 = \left[1 - \frac{300}{500}\right] \times 100 = 0.4 \times 100 = 40\%$$

12. (*a*) The $\gamma = \frac{C_p}{C_V}$ for an ideal diatomic gas is $1.40 = \frac{7}{5}$

The fraction of heat energy is utilised to increase the internal energy of gas is $\frac{5}{7}$.

13. (*b*) By definition of calorie, we know that heat required to raise 1g of water from 14.5°C to 15.5 °C is called 1 calorie.

1 calorie = 4.186 J

14. (*b*) Internal energy of stream

$$\Delta U = \Delta Q - W = mL - pV$$

$$= 1 \times 2.25 \times 10^6 - 1.01 \times \frac{10^5 \times 1}{0.6}$$

$$= 2.25 \times 10^6 - \frac{1.01 \times 10^6}{6}$$

$$= 2.25 \times 10^6 - 0.168 \times 10^6 = 2.08 \times 10^6 \text{ J}$$

15. (*c*) The specific heat of gas at constant volume in terms of degree of freedom n is

$$C_V = \frac{n}{2}R$$

Also $\quad C_p - C_V = R$

So $\quad C_p = \frac{n}{2}R + R = R\left(1 + \frac{n}{2}\right)$

Now, $\quad \gamma = \frac{C_p}{C_V} = \frac{R\left(1 + \frac{n}{2}\right)}{\frac{n}{2}R} = \frac{2}{n} + 1$

16. (*d*) A monoatomic gas has 3 degrees of freedom and diatomic gas has 5 degrees of freedom.

For monoatomic gas,

$$C_V = \frac{3}{2}R \text{ and } C_p = C_V + R = R + \frac{3}{2}R = \frac{5}{2}R$$

For a diatomic gas,

$$C_V = \frac{5}{2}R \text{ and } C_p = C_V + R = R + \frac{5}{2}R = \frac{7R}{2}$$

Hence, we can conclude that the molecules of the gas can be monoatomic or rigid diatomic.

17. (*a*) Given,

$$C_p = 450 \text{ JK}^{-1} \text{ and } C_V = 300 \text{ JK}^{-1}$$

$$C_p - C_V = R$$

$$R = 450 - 300 = 150 \text{ JK}^{-1}$$

At STP, $\quad T = 273$ K

$$p = p_{atm} = 1013 \times 10^5 \text{ Nm}^{-2}$$

From ideal equation of gas,

$$pV = nRT$$

$$p\frac{M}{\rho} = \frac{M}{m}RT \Rightarrow \rho = \frac{pm}{RT}$$

or $\quad \rho = \frac{1013 \times 10^5 \times 1}{150 \times 273}$ or $\rho = 2.47 \text{ kg/m}^3$

18. (*d*) For hydrogen, $C_p - C_V = \frac{R}{M_{H_2}} = \frac{R}{2}$,

but given that $C_p - C_V = m \Rightarrow m = R/2$

For nitrogen, $C_p - C_V = n$

$\therefore \quad n = R/28$

$\therefore \quad \frac{m}{n} = \frac{R/2}{R/28} = 14$

Hence, $\quad m = 14n$

19. (*a*) Using the relation, $\gamma = 1 + \frac{2}{f}$,

(where, f is the number of degrees of freedom for gas atom)

For monoatomic gas, $f = 3$

Hence, $\quad \gamma = \frac{C_p}{C_V} = 1 + \frac{2}{3} = \frac{5}{3}$

20. (*b*) According to first law of thermodynamics

i.e., $\quad Q = \Delta U + W, \quad Q = 2 \text{ kcal} = 2000 \times 4.2 = 8400$ J

$W = 400$ J $\Rightarrow 8400 = \Delta U + 400$

$\Delta U = 8400 - 400$

$\Delta U = 8000$ J $\Rightarrow$ change in internal energy,

i.e., $\quad \Delta U = 8$ kJ

21. (*c*) For isothermal expansion,

i.e., $\quad pV = p' \times 2V \qquad [\because V' = 2V]$

$$p' = \frac{p}{2}$$

For adiabatic expansion, pV^{γ} = constant

$$p'V'^{\gamma} = p''V''^{\gamma}$$

$$\frac{p}{2}[2V]^{5/3} = p''[16V]^{5/3} \qquad \left[\because \gamma = \frac{5}{3}\right]$$

$$p'' = \frac{p}{2}\left[\frac{2V}{16V}\right]^{5/3} = \frac{p}{2}\left[\frac{1}{8}\right]^{5/3}$$

Final pressure of the gas $= \frac{p}{2}[0.03125] = 0.0156\,p = p/64$

22. (*d*) For an adiabatic process, pressure (p) and volume (V) of the system are related as

$$pV^{\gamma} = \text{constant} \qquad \text{...(i)}$$

where γ is the ratio of specific heat capacities at constant pressure and constant volume.

For an ideal gas, $pV = nRT$ (ideal gas equation)

$$\Rightarrow \quad V = \frac{nRT}{p} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$p\left(\frac{nRT}{p}\right)^{\gamma} = \text{constant} \Rightarrow p^{1-\gamma}T^{\gamma} = \text{constant.}$$

[$\because$ Number of moles (n), gas constant (R) are constant]

23. (*a*) Isochoric process takes place at constant volume. We know that work done by a system $W = p \times \Delta V$ where, p is pressure and ΔV the change in volume.
In an isochoric process change in volume $\Delta V = 0$, therefore, work done $\Delta W = 0$.
While $\Delta Q = 0$ is an adiabatic process, ΔU or $\Delta T = 0$ in an isothermal process.

24. (*a*) Given, $m = 1\,\text{kg}$

$$T_1 = 24.4 + 273 = 297.4\,\text{K},\ T_2 = 0 + 273 = 273\,\text{K}$$
$$L = 336\,\text{kJ kg}^{-1} = 336 \times 10^3\,\text{kJ}$$

Total heat produced per kg of mass

i.e. $Q_2 = mL = 1 \times 336\,\text{kJ}$

We know that

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2} \Rightarrow Q_1 = \left(\frac{T_1}{T_2}\right)Q_2 = \frac{297.4}{273} \times 336$$
$$= \frac{99926.4}{273}\,\text{kJ} = 366.0\,\text{kJ}$$

Work required by refrigerator,

$$W = Q_1 - Q_2 = 366 - 336 = 30\,\text{kJ}$$

25. (*b*) The efficiency of Carnot engine is given by

$$\eta = 1 - \frac{T_2}{T_1}$$

Given, $T_1 = 600\,\text{K},\ T_2 = 300\,\text{K}$

$\therefore$ $\eta = 1 - \frac{300}{600}$

$\Rightarrow$ $\eta = 1 - \frac{1}{2} \Rightarrow \eta = \frac{1}{2} = 50\%$

Also, $\eta = \frac{W}{Q} = \frac{800}{Q} = \frac{1}{2}$

Thus, amount of heat supplied from source to engine is

$$Q = 800 \times 2 = 1600\,\text{J}$$

26. (*b*) Net work done = Area enclosed in p-V curve, *i.e.* ΔABC

$$= \frac{1}{2} \times 5 \times 10^{-3} \times 4 \times 10^5\,\text{J} = 10^3\,\text{J} = 1000\,\text{J}$$

27. (*d*) According to question,

$$p \propto T^3$$
$$pV = nRT \text{ and } p \propto T^3$$
$$p \propto (pV)^3 \Rightarrow p^2V^3 = \text{constant}$$
$$pV^{3/2} = \text{constant}$$

$\because$ $pV^{\gamma} = \text{constant}$

$\therefore$ $\gamma = \frac{3}{2}$

28. (*c*) The slope of the graph $\propto \frac{1}{p}$. Hence, $p_2 < p_1$.

29. (*a*) We know that, $Q = \frac{F}{2} nR\,\Delta T$

$\therefore$ Amount of heat required,

$$Q = \frac{3}{2} \times \frac{1}{4} \times K_B N_A \Delta T = \frac{3}{8} N_A K_B\,(T_2 - T_1)$$

30. (*a*) From zeroth law of thermodynamics, if two systems A and B are separated by an adiabatic wall, are seperately and independently in thermal equilibrium with a third system C, then the systems A and B are also in a state of thermal equilibrium with each other.

31. (*b*) Efficiency, $\eta = 1 - \frac{T_2}{T_1}$

Given, $T_2 = 27°\,\text{C} = 27 + 273 = 300\,\text{K}$
$T_1 = 127°\,\text{C} = 127 + 273 = 400\,\text{K}$

$$\eta = 1 - \frac{3}{4} = 25\%$$

32. (*a*) We know that

(i) The work done in isobaric process is $p\,(V_2 - V_1)$

(ii) The work done in isothermal process is $nRT \log\left(\frac{V_2}{V_1}\right)$

(iii) The work done in adiabatic process is $\frac{nR\,(T_1 - T_2)}{\gamma - 1}$

33. (*d*) We know that, $dQ = nC_p dT_A = nC_V \alpha T_B$

$$nC_p \alpha T_A = nC_V \alpha T_B$$
$$\frac{C_p}{C_V} = \frac{\alpha T_B}{\alpha T_A} \Rightarrow \gamma = \frac{dT_B}{dT_A} \qquad \left(\because \gamma = \frac{C_p}{C_V}\right)$$
$$\alpha T_B = Y \propto T_A = 1.4 \times 30 = 42\,\text{K}$$

34. (*d*) The efficiency of Carnot's heat engine, $\eta = \frac{T_1 - T_2}{T_1}$

For given combinations $\eta_1 = \frac{1}{3},\ \eta_2 = \frac{1}{2},\ \eta_3 = \frac{2}{5},\ \eta_4 = \frac{2}{3}$

So, it is maximum for, $T_1 = 300\,\text{K}$ and $T_2 = 100\,\text{K}$

35. (*a*) Given $p \propto \frac{1}{V^{1.5}}$

So, $pV^{1.5} \propto \text{constant}$

We know, the value of $\gamma = 1.5$ for adiabatic process, so

$$\frac{C_p}{C_V} = \gamma = 1.5$$

36. (*e*) In a reversible cyclic process of a gaseous system, $\Delta U = 0$.

37. (*c*) We know that the gas equation in adiabatic process,

$$pV^{\gamma} = \text{constant}$$

Here in given condition,

or, $p \propto \frac{I}{V^{\gamma}} \Rightarrow \frac{p_1}{p_2} = \left(\frac{V_2}{V_1}\right)^{\gamma}$

$$\frac{1}{p_2} = \left(\frac{1}{10^3}\right)^{\frac{5}{3}} = \frac{1}{10^5} \Rightarrow p_2 = 10^5\,\text{atm}$$

38. (*d*) Specific heat, $C = \frac{dQ}{dT} = \frac{dQ}{0} = \infty$ [$\because T = \text{constant}$]

39. (*d*) $dU = dQ - dW \Rightarrow dW = dQ - dU$

For, ideal gas, $dU = 0$

$\therefore \quad dW = dQ = R(T_2 - T_1) = 8.31 \times 100 = 8.31 \times 10^2 \text{J}$

40. (*c*) $n = 1 - \frac{T_2}{T_1}$

where, T_2 = sink temperature and T_1 = source temperature.

$$0.6 = 1 - \frac{T_2}{400} \Rightarrow \frac{T_2}{400} = 0.4 \Rightarrow T_2 = 160 \text{ K}$$

$$T_2 = 160 \text{ K} = -113°\text{C}$$

41. (*a*) For given cyclic process, $\Delta U = 0$

$\therefore \quad Q = W$

Also, $W = -$ area enclosed by the curve

$= AB \times AD = (2p - p)(3V - V) = -p \times 2V$

$\therefore$ Heat rejected $= 2pV$

42. (*c*) From the figure,

As $\quad p \propto T$

So, the volume of the gas remains constant. Hence, work done is zero.

43. (*c*) We know that, $C_p - C_V = R$

Given, $\quad \frac{C_p}{C_V} = \gamma \Rightarrow C_V = \frac{C_p}{\gamma}$

$\therefore \quad C_p - \frac{C_p}{\gamma} = R$ or $C_p\left(1 - \frac{1}{\gamma}\right) = R$

or $\quad C_p\left(\frac{\gamma - 1}{\gamma}\right) = R \Rightarrow C_p = \frac{\gamma R}{\gamma - 1}$

44. (*c*) Work done,

$$W = \int p dV = \int \frac{RT}{V} dV$$

Since, $\quad V = KT^{2/3} \Rightarrow dV = \frac{2}{3} KT^{-1/3} dT$

Eliminating K, we find $\frac{dV}{V} = \frac{2}{3}\frac{dT}{T}$

Hence, $W = \int_{T_1}^{T_2} \frac{2}{3}\frac{RT}{T} dT = \frac{2}{3} R(T_2 - T_1) = \frac{2}{3} R(30) = 20R$

45. (*d*) We know that, $\Delta Q = \mu C_p \Delta T = \frac{7}{2}\mu R \Delta T \quad \left[\because C_p = \frac{7}{2}R\right]$

and $\Delta U = \mu C_V \Delta T = \frac{5}{2}\mu R \Delta T \quad \left[\because C_V = \frac{5}{2}R\right]$

and $\Delta W = \Delta Q - \Delta U = \mu R \Delta T$

$\therefore \quad \Delta Q : \Delta U : \Delta W = 7 : 5 : 2$

46. (*d*) According to question first gas goes from volume V to $3V$ and after this volume is reduced from $3V$ to V at constant pressure.

In isothermal expansion, p-V curve is rectangular hyperbola.

47. (*d*) Internal energy of a gas,

$$U = \frac{3}{2} nRT$$

where, R is constant.

Thus, for a given number of moles of the same gas

$$U \propto T$$

From the graphs, we conclude that

$$U_A = U_D \Rightarrow U_B = U_C \Rightarrow U_B > U_A \Rightarrow U_C > U_D \quad \text{...(i)}$$

48. (*b*) Equation of an adiabatic process is

Given, $\quad p^3 = \frac{k}{V^4} \Rightarrow p^3 V^4 = k \quad$ (constant)

Comparing Eqs. (i) and (ii), we get

$$\gamma = \frac{4}{3} = 133$$

49. (*d*) $\eta = \frac{\text{Net work done per cycle}}{\text{Total amount of heat absorbed per cycle}}$

or $\quad \eta = \frac{W}{Q_1}$

$\therefore \quad \eta = \frac{Q_1 - Q_2}{Q_1}$ or $\eta = 1 - \frac{Q_2}{Q_1}$

as $\quad \frac{Q_2}{Q_1} = \frac{T_2}{T_1}$

$\therefore \quad \eta = 1 - \frac{T_2}{T_1} \quad \text{...(i)}$

where, T_2 is temperature of sink and T_1 is the temperature of the sources.

Here, $\eta = 50\%$, $T_2 = 50°\text{C} = 273 + 50 = 323$ K

Putting these values in Eq. (i), we get

$$\frac{50}{100} = 1 - \frac{323}{T_1} \Rightarrow T = 646 \text{ K} = 373°\text{C}$$

50. (*b*) Work, $W = mgh + \Delta pV = V\rho gh + \Delta pV$

$= 4 \times 10^3 \times 10 \times 20 + (2 \times 10^5) \times 4 = 16 \times 10^5$ J

51. (*d*) From first law of thermodynamics, $dQ = dU + dW$

From adiabatic process, $dQ = 0$

$\therefore \quad dU + dW = 0 \Rightarrow dU = -50$ J

Therefore, the internal energy of system will decrease.

52. (*b*) Volume is constant

i.e., $\quad \Delta V = 0 \Rightarrow \Delta U = Q - p\Delta V \Rightarrow \Delta U_a = Q$

(ii) Pressure is constant,

i.e., $\quad \Delta U_b = Q - p\Delta V$

$\therefore \quad \Delta U_a > \Delta U_b$

53. (*c*) W_{a-b} is isothermal

W_{b-c} is adiabatic

W_{c-d} is isothermal (Negative)

W_{d-a} is adiabatic (Negative)

Net work done during the complete cycle,

$$W = W_{a-b} + W_{b-c} + (-W_{c-d}) + (-W_{d-a})$$

$$W = W_{a-b} - W_{c-d} \quad [\text{As } W_{b-c} = W_{d-a}]$$

i.e., $\quad W_{a-b} + W_{c-d} > 0$

and $\quad W_{a-b} + W_{b-c} + W_{c-d} + W_{d-a} > 0$

$\because \quad W_{b-c} = W_{d-a}$

$\therefore \quad W_{b-c} + (-)W_{d-a} = 0$

54. (*a*) Change in entropy, $dS = \frac{dQ}{T}$

Entropy remains constant for reversible process

$$dS = 0$$

CHAPTER 16

Calorimetry and heat transfer

In our day-to-day life, we have noticed that when we touch a utensil placed on fire, it appears hot. When steam is passed through water, the water becomes warm. On the other hand, when we touch a piece of ice, it appears cold. To understand the cause of these phenomena, let us study in detail about heat and temperature. Further, we will also study about the various processes by which the heat flows from one body to another.

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1) Heat

Practically all the bodies around us either release or absorb energy often in the form of heat. The form of energy which is exchanged among various bodies or system on account of temperature difference is defined as **heat**. Heat is a scalar quantity. Its SI unit is joule (J). Other practical units of heat are erg, cal, BTU etc.

$$1 \text{ joule} = 10^7 \text{ erg}$$

$$1 \text{ cal} = 4.18 \text{ joule}$$

$$1 \text{ BTU} = 252 \text{ cal} = 1055 \text{ J}$$

Calorimetry is the measurement of quantity of heat exchanged.

Heat capacity

The heat capacity of substances is defined as the amount of heat required to raise its temperature by unity, *i.e.*, 1°C (or 1K). It is denoted by S. If temperature of the substance changes from T to $T + \Delta T$, when it exchanges an amount of heat ΔQ with its surroundings, then its,

$$\boxed{\text{Heat capacity, } S = \frac{\Delta Q}{\Delta T}}$$

The SI unit of heat capacity is JK^{-1} and its dimensional formula is $[ML^2T^{-2}K^{-1}]$.

Specific heat capacity

The amount of heat required to raise the temperature of unit mass of a substance by unity is known as specific heat capacity of the substance. It is denoted by c.

If the temperature of a substance of mass m changes from T to $T + \Delta T$, when it exchanges an amount of heat ΔQ with its surroundings, then its

$$\boxed{\text{Specific heat capacity, } c = \frac{1}{m}\frac{\Delta Q}{\Delta T}}$$

$\therefore$ Heat $Q = mc\Delta T$

The SI unit of specific heat capacity is $\text{Jkg}^{-1}\text{K}^{-1}$ and its dimensional formula is $[\text{M}^0\text{L}^2\text{T}^{-2}\text{K}^{-1}]$.

In differential form, we may write $\boxed{c = \frac{1}{m}\frac{dQ}{dT}.}$

Therefore, if c varies with temperature over an interval, then

$$\boxed{\text{Heat } Q = m\int_{T_1}^{T_2} c\,dT}$$

Important points related to specific heat capacity are given below

(i) The specific heat capacity of water is approximately 1 cal/g-°C.

(ii) Specific heat capacity is constant for a particular material.

(iii) If a substance is undergoing change of state which takes place at constant temperature, *i.e.*, $\Delta T = 0$, then specific heat $c = \frac{\Delta Q}{m\Delta T} = \frac{Q}{m \times 0} = \infty$

(iv) If the temperature of a body changes without transfer of heat with the surrounding (*i.e.*, $Q = 0$), then specific heat $c = \frac{\Delta Q}{m\Delta T} = \frac{0}{m \times \Delta T} = 0$

(v) Specific heat of a substance can be negative.

(vi) Specific heat of a substance depends on temperature, nature of material of body, state of substance, *i.e.*, solid, liquid or gas.

Note Specific heat of water is very high in comparison to other liquids and solids. For this reason water is used as a heater in hot water bags as well as a coolant in automobile radiators. Owing to its high specific heat, the water warms up much more slowly than the land during summer and consequently wind from the sea has a cooling effect. This is the reason for temperate climate of coastal areas.

Relation between heat capacity and specific heat capacity

As we know that

Heat capacity, $S = \frac{\Delta Q}{\Delta T}$

and specific heat capacity, $c = \frac{1}{m}\frac{\Delta Q}{\Delta T}$

$\therefore$ $\boxed{S = m \times c}$

i.e., Heat capacity = mass × specific heat capacity

Example 16.1 *When 400 J of heat are added to a 0.1 kg sample of metal, its temperature increases by 20°C. What is the specific heat of the metal?*

Sol. Given, Heat $\Delta Q = 400$ J, Mass $m = 0.1$ kg

Temperature $\Delta T = 20°\text{C}$

$\therefore$ Specific heat, $c = \frac{1}{m}\cdot\frac{\Delta Q}{\Delta T} = \left(\frac{1}{0.1}\right)\left(\frac{400}{20}\right) = 200\ \text{J kg}^{-1}\,°\text{C}^{-1}$

Example 16.2 *A 10 kW drilling machine is used to drill a bore in a small aluminium block of mass 8.0 kg. How much is the rise in temperature of the block in 2.5 min, assuming 50% of power is used up in heating the machine itself or lost to the surroundings. Specific heat of aluminium = 0.91 $Jg^{-1}\,°C^{-1}$.*

Sol. Given, Power, $P = 10\ \text{kW} = 10 \times 10^3\ \text{W}$

Time, $t = 2.5\ \text{min} = 2.5 \times 60$ s

As power is defined as rate at which energy is consumed,

i.e., $P = \frac{E}{t}$

$\therefore$ Total energy used $E = Pt = 10 \times 10^3 \times 2.5 \times 60 = 1.5 \times 10^6$ J

Energy absorbed by aluminium block,

$Q = 50\%$ of the total energy $= 50\%$ of $1.5 \times 10^6 = 0.75 \times 10^6$ J

Also, $m = 8.0\ \text{kg} = 8.0 \times 10^3$ g, $c = 0.91\ \text{Jg}^{-1}\,°\text{C}^{-1}$

As, $Q = mc\,\Delta T$

$\therefore$ $\Delta T = \frac{Q}{mc} = \frac{0.75 \times 10^6}{8.0 \times 10^3 \times 0.91} = 103.02°\text{C}$

Example 16.3 *A geyser heats water flowing at the rate of 3.0 L per minute from 27°C to 77°C. If the geyser operates on a gas burner, what is the rate of combustion of fuel if its heat of combustion is 4.0×10^4 Jg^{-1}?*

Sol. Given, Volume of water heated,

$V = 3.0\ \text{L min}^{-1} = 3 \times 10^{-3}\ \text{m}^3\ \text{min}^{-1}$

As Density, $\rho = \frac{\text{Mass } (m)}{\text{Volume } (V)}$

$\therefore$ Mass of water heated,

$m = \rho V = 10^3 \times 3 \times 10^{-3}\ \text{kg min}^{-1} = 3\ \text{kg min}^{-1}$

Rise in temperature of water,

$\Delta T = T_2 - T_1 = 77 - 27 = 50°\text{C}$

Specific heat of water, $c = 4.2\ \text{Jg}^{-1}\text{C}^{-1} = 4.2 \times 10^3\ \text{J kg}^{-1}\text{C}^{-1}$

Heat required by water, $\Delta Q = mc\Delta T$

$= 3 \times 4.2 \times 10^3 \times 50\ \text{J min}$

$= 63 \times 10^4\ \text{J min}^{-1}$ ...(i)

Heat of combustion of fuel $= 4.0\times 10^4\ \text{Jg}^{-1} = 4.0\times 10^7\ \text{J kg}^{-1}$

Let m kg min^{-1} be the rate of combustion of fuel.

$\therefore$ Heat supplied by gas burner or combustion of fuel

$$= m \times 4 \times 10^7\ \text{J min}^{-1} \quad \text{...(ii)}$$

$\therefore$ From Eqs. (i) and (ii), we get

$$63 \times 10^4 = m \times 4 \times 10^7$$

or $$m = \frac{63 \times 10^4}{4 \times 10^7} = 15.75\times 10^{-3}\ \text{kg min}^{-1}$$

or rate of combustion of fuel ≈ 16 g min^{-1}

Molar specific heat capacity

The amount of heat required to raise the temperature of one mole of a substance (usually a gas) by unity is known as the **molar heat capacity** of that substance. It is denoted by C. Its SI unit is J mol^{-1}K^{-1}.

$\therefore$ $$\boxed{\text{Molar specific heat capacity } C = \frac{S}{n} = \frac{\Delta Q}{n\Delta T}}$$

where n = number of moles of substance (gas).

There are two types of molar specific heat capacities for a gas.

(i) **Molar specific heat capacity at constant pressure (C_p)** It is molar heat capacity of a gas at constant pressure. The amount of heat required to raise the temperature of 1 mole of a gas by unity at constant pressure is known as molar heat capacity at constant pressure.

$\therefore$ $$\boxed{C_p = \left[\frac{\Delta Q}{\Delta T}\right]_{p=\text{constant}}}$$

$\Rightarrow$ $$\Delta Q = C_p \Delta T$$

For n moles of a gas $\Delta Q = nC_p\Delta T$

(ii) **Molar specific heat capacity at constant volume** It is molar heat capacity of a gas at constant volume. The amount of heat required to raise the temperature of 1 mole of gas by unity at constant volume is known as molar specific heat capacity at constant volume.

$\therefore$ $$\boxed{C_V = \left(\frac{\Delta Q}{\Delta T}\right)_{V=\text{constant}}}$$

$\Rightarrow$ $$\Delta Q = C_V \Delta T$$

For n moles of a gas $\Delta Q = nC_V\Delta T$

Note When a solid substance is heated in a small range of temperature, its volume almost remains constant. Thus specific heat capacity of a solid can be taken as specific heat at constant volume, C_V.

Relation between specific heat capacity and molar specific heat capacity

As, number of moles, $n = \dfrac{m}{M}$

$\Rightarrow$ $$m = nM$$

where, m = mass of the substance and M = molecular mass

$\Rightarrow$ $$C = \frac{1}{n}\left[\frac{\Delta Q}{\Delta T}\right] = M\left[\frac{\Delta Q}{m\Delta T}\right]$$

But, $$\frac{\Delta Q}{m\Delta T} = c$$

$\Rightarrow$ $$\boxed{C = Mc}$$

where, c = specific heat capacity

M = molecular mass of the substance

C = molar specific heat capacity.

Hence, molar specific heat capacity

= molecular mass × specific heat capacity.

Note

1. If a gas is compressed or expanded suddenly such that no exchange of heat takes place with the surrounding

   i.e., $\Delta Q = 0$

   Then, $C = \dfrac{\Delta Q}{n\Delta T} = 0$

2. If a gas is allowed to expand or compressed very slowly, such that $\Delta T = 0$

   Then, $C = \dfrac{\Delta Q}{n\Delta T} = \infty$

Example 16.4 *0.64 g of oxygen is kept in a rigid container and is heated. Find the amount of heat required to raise the temperature from 20°C to 30°C. The molar heat capacity of oxygen at constant volume is 20 JK^{-1} mol^{-1}.*

Sol. Mass of the oxygen $m = 0.64$ g

The molecular weight of oxygen $M = 32$ g mol^{-1}.

$\therefore$ Number of moles of the gas is

$$n = \frac{\text{Mass } (m)}{\text{Molecular weight } (M)} = \frac{0.64}{32} = 0.02$$

Molar heat capacity at constant volume $C_V = 20$ JK^{-1} mol^{-1}

$\therefore$ The amount of heat required,

$$\Delta Q = nC_V\Delta T = (0.02)(20)(30-20) = 4\ \text{J}$$

Example 16.5 *A certain substance has a mass of 50 g/mol. When 300 J of heat is added to 25 g of sample of this material, its temperature rises from 25 to 45°C. Calculate (i) thermal capacity. (ii) specific heat and (iii) molar heat capacity of the sample.*

Sol. Given, Mass $m = 25$ g,

Temperature $\Delta T = 45 - 25 = 20°$ C,

Heat $\Delta Q = 300$ J

Molar mass $M = 50$ g/mol

(i) Thermal capacity, $S = \frac{\Delta Q}{\Delta T} = \frac{300}{20} = 15$ J/°C

(ii) Specific heat $c = \frac{\Delta Q}{\Delta T \times m} = 15 \times \frac{1}{25} = 0.6$ J/g °C

$= 0.6 \times 10^3$ J/kg-°C $= 600$ J/kg °C

(iii) Number of moles $n = \frac{\text{Mass } (m)}{\text{Molecular mass } (M)} = \frac{25}{50} = \frac{1}{2}$

$\because$ Heat $\Delta Q = nC\Delta T$

$\therefore$ Molar heat capacity, $C = \frac{\Delta Q}{n\Delta T} = \frac{300}{(1/2)(20)} = 30$ J/mole °C

Calorimetry

The branch of physics that deals with the measurement of heat is called calorimetry.

Principle of calorimetry

Whenever two bodies at different temperatures are placed in contact with each other, heat flows from the body at higher temperature to the body at lower temperature till the two bodies attains the same temperature.

Thus, the principle of calorimetry states that total heat lost by the hotter body equals to the total heat gained by colder body, provided there is no exchange of heat with the surroundings.

i.e., heat lost by hotter body = heat gained by colder body.

The principle of calorimetry is based on the law of conservation of energy.

Calculations based on principle of calorimetry

If there are two bodies of masses m_1 and m_2 having specific heat capacities as c_1 and c_2 respectively. Let these bodies be at temperatures T_1 and T_2 respectively such that $T_1 > T_2$.

When they are placed in contact with each other, they attain common temperature T. Then according to principle of calorimetry,

Heat lost by first body = Heat gained by second body

$\Rightarrow \quad m_1c_1(T_1 - T) = m_2c_2(T - T_2)$

$\therefore$ Common temperature of the mixture at equilibrium

$$T = \frac{m_1c_1T_1 + m_2c_2T_2}{m_1c_1 + m_2c_2}$$

Example 16.6 *10 g of water at 70°C is mixed with 5 g of water at 30°C. Find the temperature of the mixture in equilibrium.*

Sol. Given, Mass $m_1 = 10$ g, $m_2 = 5$ g,

Temperature $T_1 = 70°$ C, $T_2 = 30°$ C

Let $T°$ C be the temperature of the mixture.

From principle of calorimetry, common temperature of the mixture at equilibrium is given by

$$T = \frac{m_1c_wT_1 + m_2c_wT_2}{m_1c_w + m_2c_w} = \frac{c_w(m_1T_1 + m_2T_2)}{c_w(m_1 + m_2)}$$

$$= \frac{m_1T_1 + m_2T_2}{m_1 + m_2} = \frac{10 \times 70 + 5 \times 30}{10 + 5} = \frac{700 + 150}{15}$$

$$= \frac{850}{15} = 56.67° \text{ C}$$

Example 16.7 *Liquids A and B are at 30°C and 20°C. When mixed in equal masses, the temperature of the mixture is found to be 26°C. Find the ratio of their specific heat capacities.*

Sol. Let the specific heat of A and B be c_A and c_B respectively.

Now heat lost by A in cooling from 30°C to 26°C will be same as liquid B warming up from 20°C to 26°C. Since, the masses are same then from the principle of calorimetry,

Heat lost by A = Heat gained by B

$\therefore \quad m.c_A(30° \text{ C} - 26° \text{ C}) = m.c_B(26° \text{ C} - 20° \text{ C})$

or $\quad 4c_A = 6c_B$ or $\frac{c_A}{c_B} = \frac{6}{4} = \frac{3}{2}$

Example 16.8 *The temperature of equal masses of three different liquid A, B and C are 12°C, 19°C and 28°C respectively. The temperature when A and B are mixed is 16°C, and when B and C are mixed, it is 23°C. What will be the temperature when A and C are mixed?*

Sol. Given, temperature $T_A = 12°$ C, $T_B = 19°$ C and $T_C = 28°$ C

Let specific heat of liquids A, B and C are c_A, c_B and c_C respectively.

When A and B are mixed, then from principle of calorimetry

$$mc_B(19 - 16) = mc_A(16 - 12)$$

$$3c_B = 4c_A \Rightarrow c_A = \frac{3}{4}c_B$$

When B and C are mixed, then from principle of calorimetry

$$mc_C(28 - 23) = mc_B(23 - 19)$$

$$5c_C = 4c_B \Rightarrow c_C = \frac{4}{5}c_B$$

When A and C are mixed, let equilibrium temperature be T. From principle of calorimetry common temperature of the mixture at equilibrium

$$T = \frac{mc_C\,T_1 + mc_AT_2}{mc_C + mc_A}$$

$$= \frac{\frac{4}{5}c_B(28) + \frac{3}{4}c_B \times 12}{\frac{4}{5}c_B + \frac{3}{4}c_B}$$

$$= \frac{c_B\left(\frac{4}{5} \times 28 + \frac{3}{4} \times 12\right)}{\left(\frac{4}{5} + \frac{3}{4}\right)c_B}$$

$$= \frac{\frac{112}{5} + 9}{\frac{16+15}{20}} = \frac{(112+45)20}{(31)\times 5}$$

$$= \frac{157 \times 20}{155} = \frac{3140}{155} = 20.3°\,C$$

Water equivalent of a substance

Water equivalent of an object is the amount of water that absorbs or gives out the same amount of heat as is done by the substance when heated or cooled through same temperature difference as that of object.

To attain the same temperature as that of water, the calorimeter gains some heat.

Water equivalent of an object = Mass of the object × specific heat of material of object

$$\boxed{W = mc}$$

Water equivalent is usually expressed in gram.

Example 16.9 *A piece of iron of mass 100 g is kept inside a furnace for a long time and then put in a calorimeter of water equivalent 10 g containing 240 g of water at 20°C. The mixture attains an equilibrium temperature of 60°C. Find the temperature of the furnace specific heat of iron = 470 J/kg-°C.*

Sol. Let temperature of furnace be t.

Given, $m_i = 100$ g, $m_w = 240$ g,

water equivalent $m'_w = 10$ g,

Equilibrium temperature = 60° C

Then from principle of calorimetery,

Heat given = Heat taken

$$m_i s_i\,(t - 60) = (m'_w + m_w)\,s_w\,(60 - 20)$$

$$100 \times 470\,(t - 60) = (10 + 240) \times 4200 \times 40$$

$$t - 60 = \frac{250 \times 4200 \times 40}{100 \times 470} = 893.6$$

$$t = 953.6°\,C$$

Check point 16.1

1. Water is used to cool radiators of engine because
(a) of its lower density (b) it is easily available
(c) it is cheap (d) it has high specific heat

2. If specific heat of a substance is infinite it means
(a) heat is given out
(b) heat is taken in
(c) no change in temperature takes place whether heat is taken in a or given out
(d) All of the above

3. How much heat energy is gained when 5 kg of water at 20°C is brought to its boiling point (Specific heat of water = 4.2 kJ kg^{-1}°C^{-1})
(a) 1680 kJ (b) 1700 kJ (c) 1720 kJ (d) 1740 kJ

4. A metal block is made from a mixture of 2.4 kg of aluminium 1.6 kg of brass and 0.8 kg of copper. The amount of heat required to raise the temperature of this block from 20°C to 80°C is (specific heats of aluminium, brass and copper are 0.216, 0.0917 and 0.0931 cal kg^{-1}°C^{-1} respectively)
(a) 96.2 cal (b) 44.4 cal
(c) 86.2 cal (d) 62.8 cal

5. 50 g of copper is heated to increase its temperature by 10°C. If the same quantity of heat is given to 10 g of water, the rise in its temperature is (Specific heat of copper = 420 J kg^{-1}°C^{-1})
(a) 5°C (b) 6°C (c) 7°C (d) 8°C

6. A bubble of 8 mole of helium is submerged at a certain depth in water. The temperature of water increases by 30°C. How much heat is added approximately to helium during expansion
(a) 4000 J (b) 3000 J
(c) 3500 J (d) 5000 J

7. A beaker contains 200 g of water. The heat capacity of the beaker is equal to that of 20 g of water. The initial temperature of water in the beaker is 20°C. If 440 g of hot water at 92°C is poured in it, the final temperature (neglecting radiation loss) will be nearest to
(a) 58°C (b) 68°C (c) 73°C (d) 78°C

8. 100 g of ice at 0°C is mixed with 100 g of water at 100°C. What will be the final temperature of the mixture?
(a) 10°C (b) 20°C (c) 30°C (d) 0°C

9. A liquid of mass m and specific heat c is heated to a temperature $2T$. Another liquid of mass $m/2$ and specific heat $2c$ is heated to a temperature T. If these two liquids are mixed, the resulting temperature of the mixture is
(a) $\left(\frac{2}{3}\right)T$ (b) $\left(\frac{8}{5}\right)T$ (c) $\left(\frac{3}{5}\right)T$ (d) $\left(\frac{3}{2}\right)T$

10. 0.1 m^3 of water at 80°C is mixed with 0.3 m^3 of water at 60°C. The final temperature of the mixture is
(a) 70°C (b) 65°C
(c) 60°C (d) 75°C

2 Change of state (or phase)

Matter exists in three states: solid, liquid and gas. These states (or phases) can be changed into one another by heating or cooling. This process of conversion of one state of a substance into the another state, is known as change of state.

The common changes of states of matter are as follows

(i) Solid to liquid (and *vice-versa*)

(ii) Liquid to gas (and *vice-versa*)

During the change of state of matter, the exchange of heat takes place between the substance and its surroundings.

Latent heat

The amount of heat required to change the state of a unit mass of a substance without change in its temperature and pressure is known as latent heat.

Latent heat is denoted by L. SI unit of latent heat is J kg^{-1}. If a mass m of a substance undergoes a change from one state to the another, then the quantity of heat, Q required is given by

$$Q = mL$$

i.e., $$\boxed{\text{Latent heat, } L = Q/m}$$

Hence, during the phase change, the heat required by the substance depends on the mass m of the substance and heat of transformation Q.

There are two types of latent heat of materials:

Latent heat of fusion or melting It is the quantity of heat required to change the state of a substance from solid to liquid state at its melting point. It is denoted by L_f.

Latent heat of vaporisation It is the quantity of heat required to change the state of unit mass of a substance from liquid to vapour state at its boiling point. It is denoted by L_V.

Note For water at 1 atm, latent heat of fusion is 80.0 $calg^{-1}$. This simply means 80.0 cal of heat are required to melt 1.0 g of water or 80.0 cal heat is liberated when 1.0 g of water freezes at 0°C. Similarly, latent heat of vaporisation for water at 1 atm is 540 cal/g.

Example 16.10 *At 1 atm pressure, 1.000 g of water having a volume of 1.000 cm^3 becomes 1671 cm^3 of steam when boiled. The heat of vaporisation of water at 1 atm is 539 $calg^{-1}$. What is the change in internal energy during the process ?*

Sol. Heat spent during vaporisation

$$Q = mL = 1.000 \times 539 = 539 \text{ cal}$$

Work done $$W = p(V_V - V_l)$$
$$= 1.013 \times 10^5 \times (1671 - 1.000) \times 10^{-6}$$
$$169.2 \text{ J} = \frac{169.2}{4.18} \text{ cal} = 40.5 \text{ cal}$$

∴ Change in internal energy,

$$U = 539 \text{ cal} - 40.5 \text{ cal}$$
$$= 498.5 \text{ cal}$$

Example 16.11 *Find the quantity of heat required to convert 40 g of ice at -20°C into water at 20°C. Given $L_{ice} = 0.336 \times 10^6$ J/kg. Specific heat of ice = 2100 J/kg-K. Specific heat of water = 4200 J/kg-K*

Sol. Given, mass $m = 40 \text{ g} = 40 \times 10^{-3} \text{ kg} = 0.04 \text{ kg}$

Temperature $T_1 = -20°\text{C}$ and $T_2 = 20°\text{C}$

Latent heat of ice $L_{ice} = 0.336 \times 10^6$ J/kg

Specific heat of ice $c_i = 2100$ J/(kg-K)

Specific heat of water $c_w = 4200$ J/kg-K

Heat required to raise the temperature of ice from – 20°C to 0°C $= mc_i T_1 = 0.04 \times 2100 \times 20 = 1680$ J

Heat required to convert the ice into water at 0°C

$$= mL_{ice} = 0.04 \times 0.336 \times 10^6 = 13440 \text{ J}$$

Heat required to heat water from 0°C to 20°C

$$= mc_w T_2 = 0.04 \times 4200 \times 20 = 3360 \text{ J}$$

Total heat required $= 1680 + 13440 + 3360 = 18480$ J

Note While doing numericals, keep in mind whenever there is a change in temperature, use $Q = mc\Delta T$ or $Q = nC\Delta T$. If there is a change in state, use $Q = mL$.

Example 16.12 *In a container of negligible mass 30 g of steam at 100°C is added to 200 g of water that has a temperature of 40°C. If no heat is lost to the surroundings, what is the final temperature of the system? Also, find masses of water and steam in equilibrium. (Take, $L_V = 539$ cal g^{-1} and $c_{water} = 1$ cal g^{-1} °C^{-1})*

Sol. Given, mass $m = 200$ g, $m_1 = 30$ g

Temperature $T_1 = 100°\text{C}$, $T_2 = 40°\text{C}$

Latent heat $L_V = 539$ cal g^{-1}

Specific heat $c_w = 1$ cal g^{-1} °C^{-1}

Let Q be the heat required to convert 200 g of water at 40°C into 100°C, then

$$Q = mc\Delta T$$
$$= (200)(1.0)(100 - 40) = 12000 \text{ cal}$$

Now, suppose m_0 mass of steam converts into water to liberate this much amount of heat, then

$$m_0 = \frac{Q}{L} = \frac{12000}{539} = 22.26 \text{ g}$$

Since, it is less than 30 g, the temperature of the mixture is 100°C.

Mass of steam in the mixture $= m_1 - m_0 = 30 - 22.26 = 7.74$ g

and mass of water in the mixture $= m + m_0$

$$= 200 + 22.26 = 222.26 \text{ g}$$

Example 16.13 *5 g of water at 30° C and 5 g of ice at −20°C are mixed together in a calorimeter. Find the final temperature of mixture. Water equivalent of calorimeter is negligible, specific heat of ice = 0.5 cal g^{-1} °C^{-1} and latent heat of ice = 80 cal g^{-1}.*

Sol. In this case heat is given by water and taken by ice.

Heat available with water to cool from 30° C to 0° C

$$= mc\Delta T = 5 \times 1 \times 30 = 150 \text{ cal}$$

Heat required by 5 g ice to increase its temperature up to 0° C

$$mc\Delta T = 5 \times 0.5 \times 20 = 50 \text{ cal}$$

Out of 150 cal heat available, 50 cal is used for increasing temperature of ice from − 20° C to 0° C. The remaining heat 100 cal is used for melting the ice.

If mass of ice melted is mg then

$$m \times 80 = 100 \Rightarrow m = 1.25 \text{ g}$$

Thus, 1.25 g ice out of 5 g melts and mixture of ice and water is at 0° C.

Example 16.14 *5 g ice at 0°C is mixed with 1 g steam at 100°C. Find the final temperature and composition of the mixture.*

Sol. Given, mass of the steam, $m = 1$ g

Temperature $\Delta T = T_2 - T_1 = 100 - 0 = 100°$ C

Heat required to melt ice at

$$0°C = Q_1 = m_i L_f \text{ (where } m_i = \text{mass of ice)}$$
$$= 5 \times 80 = 400 \text{ cal}$$

Heat required to raise the temperature of water from 0 to 100°C

$$= Q_2 = m_i c_w \Delta T = 5 \times 1 \times 100 = 500 \text{ cal}$$

Maximum heat ice can absorb for steam to reach boiling point is

$$Q_1 + Q_2 = 900 \text{ cal}$$

Heat rejected by steam on complete condensation

$$= Q_3 = m_s L_V = 1g \times 540 \text{ cal}$$

This heat rejected can melt the ice completely but cannot raise the temperature of water from 0 to 100°C as there is a deficiency of heat $= Q_1 + Q_2 - Q_3 = (900 - 540)$ cal $= 360$ cal

∴ Resulting temperature

$$= 100°\text{C} - \frac{\text{Heat deficient}}{\text{Thermal capacity of system } i.e., \text{ 6 g water}}$$

$$= 100°\text{C} - \frac{360 \text{ cal}}{6\text{g} \times 1 \text{ cal g}^{-1}\,°\text{C}^{-1}} = 100°\text{C} - \frac{360°\text{C}}{6}$$

$$= 100°\text{C} - 60°\text{C} = 40°\text{C}$$

Example 16.15 *A lead bullet penetrates into a solid object and melts. Assuming that 25% of its kinetic energy was used to heat it, calculate the initial speed of the bullet. The initial temperature of the bullet is 27 °C and its melting point is 327 °C. Latent heat of fusion of lead = 2.5×10^4 J kg^{-1} and specific heat capacity of lead = 125 J $kg^{-1}K^{-1}$.*

Sol. Let the mass of the bullet = m.

Heat required to take the bullet from 27°C to 327°C

$$Q = ms(T_2 - T_1) = m \times 125 \times (327 - 27)$$
$$= m \times (125)(300) = m \times (3.75 \times 10^4 \text{Jkg}^{-1})$$

Heat required to melt the bullet $= mL = m \times (2.5 \times 10^4 \text{J kg}^{-1})$

If the initial speed be v, the kinetic energy is $\frac{1}{2}mv^2$ and hence the heat developed is $\frac{1}{4}\left(\frac{1}{2}mv^2\right) = \frac{1}{8}mv^2$.

Thus, $$\frac{1}{8}mv^2 = Q = m(3.75 + 2.5) \times 10^4 \text{ J kg}^{-1}$$

or $$v = 707.11 \text{ ms}^{-1}$$

Example 16.16 *Suppose you want to cool 0.25 kg of cola (mostly water), at 25°C by adding ice initially at −20° C. How much ice should you add so that the final temperature will be 0°C with all the ice melt? Neglect the heat capacity of the container. Specific heat of ice is 2000 J kg^{-1} K^{-1}. [Take specific heat of cola 4160 Jkg^{-1} K^{-1}]*

Sol. Heat lost by cola is

$$Q_{\text{cola}} = m_{\text{cola}} c_{\text{cola}} \Delta T_{\text{cola}}$$
$$= (0.25\text{kg})(4160 \text{ Jkg}^{-1} \text{ K}^{-1})(25°\text{C} - 0°\text{C})$$
$$= 26000 \text{ J}$$

Let the mass of the required ice be m_{ice}, then the heat needed to warm it from −20° C to 0°C is

$$Q_1 = m_{\text{ice}}\, c_{\text{ice}} \Delta T_{\text{ice}}$$
$$= m_{\text{ice}}(2000 \text{ J kg}^{-1} \text{ K}^{-1})(0°\text{C} - (-20°\text{C}))$$
$$= m_{\text{ice}}\,(4.0 \times 10^4 \text{ J kg}^{-1})$$

The heat needed to melt this mass of ice is

$$Q_2 = m_{\text{ice}} L_f = m_{\text{ice}}(3.34 \times 10^5 \text{ J kg}^{-1})$$

∴ Total energy gained by ice is

$$Q_1 + Q_2 = m_{\text{ice}}\,(3.74 \times 10^5 \text{ J kg}^{-1})$$

Using principle of calorimetry, heat lost = heat gained, we get

$$26000 \text{ J} = m_{\text{ice}}(3.74 \times 10^5 \text{ J kg}^{-1}) \text{ or, } m_{\text{ice}} = 0.07 \text{ kg} = 70 \text{ g}$$

Example 16.17 *An ice cube of mass 0.1 kg at 0° C is placed in an isolated container which is at 227° C. The specific heat c of the container varies with temperature T according to the empirical relation c = A + BT, where A = 100 cal/kg-K and B = 2×10^{-2} cal/kg-K^2. If the final temperature of the container is 27° C, determine the mass of the container. (Latent heat of fusion of water = 8×10^4 cal/kg, specific heat of water = 10^3 cal/kg-K)*

Sol. Given

Initial temperature of container, $T_1 = 227 + 273 = 500$ K

Final temperature of container, $T_2 = 27 + 273 = 300$ K

Specific heat of container, $c = A + BT = 100 + 2 \times 10^{-2}T$

Let m is mass of container.

∴ Heat lost by container

$$\Delta Q = \int_{T_1}^{T_2} mcdT = m \int_{500}^{300} (100 + 2 \times 10^{-2}T)\, dT$$

$$= m \left| 100\,T + 2 \times 10^{-2} \frac{T^2}{2} \right|_{500}^{300}$$

$= m\,[100(300 - 500) + 10^{-2}\,(300^2 - 500^2)]$

$= m\,[-2 \times 10^4 + 10^{-2}\,(300 + 500)\,(300 - 500)]$

$= m\,[-2 \times 10^4 - 1600]$

$= -21600\,m$

Heat gained by ice $= mL + mc_w\Delta T$

$= 0.1 \times 8 \times 10^4 + 0.1 \times 10^3\,(27 - 0)$

$= 10700$ cal

Heat lost by container = Heat gained by ice

$21600\,m = 10700$

$m = 0.495$ kg

Heating curve

The graph between temperature and time is called heating curve.

Suppose the heat is supplied to the solid at constant rate P, then a graph is plotted between temperature and time, as shown.

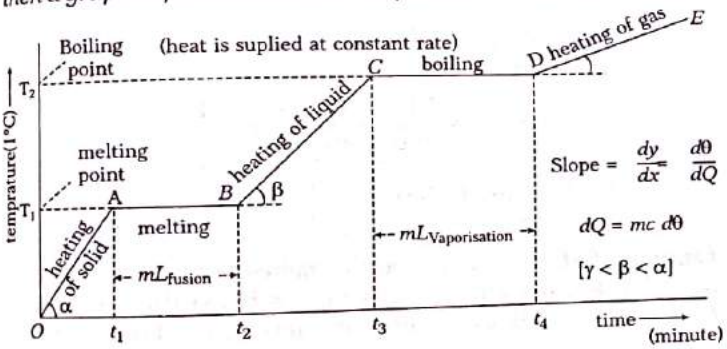


Fig. 16.1 Heating curve

We can study this graph by dividing it into different regions.

Region *OA* (Temperature increases)

From the graph it is clear that in this region temperature rises with time.

Heat given $\quad Q = mc\,\Delta T$

If here rate at which heat is supplied is P, then heat supplied in time Δt is $Q = P\Delta t$

$\therefore \quad P\Delta t = mc\Delta T$

$$\therefore \quad \frac{\Delta T}{\Delta t} = \frac{P}{mc} = \frac{P}{\text{Heat capacity}}$$

Here, $\frac{\Delta T}{\Delta t}$ is slope of temperature-time graph. So slope of temperature-time graph is inversely proportional to heat capacity or specific heat capacity.

In this region, the solid absorbs heat till it reaches melting point temperature T_1.

Region *AB* (Temperature constant)

In this region, temperature is constant so change of state is taking place here. This change is indicated by the horizontal line AB. At point A solid starts melting and at point B all solids gets converted into liquid. If latent heat of fusion is L_f, then heat supplied in this state is

$$Q = mL_f = P\Delta t$$

$\therefore$ Latent heat of fusion is proportional to the time taken in fusion of heat. It means latent heat of fusion is directly proportional to the length of line AB (line of zero slope).

Region *BC* (Temperature increases)

In this region, temperature rises again (now the substance is in liquid state), so

Slope of temperature-time graph (line BC) $\propto 1/$ (Heat Capacity or specific heat of liquid)

Greater the heat capacity, lesser will be the slope and hence there is a little change in temperature in this region. The point C corresponds to the boiling point of a substance.

Region *CD* (Temperature constant)

In this region, change in temperature is constant so change of state from liquid to gas takes place. This change is indicated by horizontal the length of line CD.

So (like region AB) latent heat of vapourisation is proportional to the length of line CD.

At point C liquid starts to convert into vapour and at point D whole of the liquid is converted into vapour.

Region *DE* (Temperature increases)

Line DE represents absolute vapour state of the substance.

Here, also the specific heat is inversely proportional to the slope of temperature *versus* time graph.

From the graph, it can be concluded that the total heat required to convert a solid into vapour form will be the sum of all the heats given from O to D.

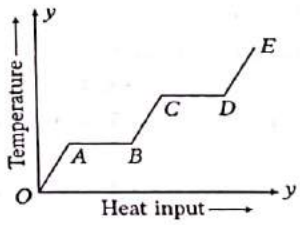
Example 16.18 *A solid material is supplied with heat at a constant rate. The temperature of the material is changing with the heat input as shown in figure. Study the graph carefully and answer the following question:*

(i) What do the horizontal regions AB and CD represent?

(ii) If CD = 2 BA, what do you infer?

(iii) What does slope DE represent?

(iv) The slope of OA > the slope of BC. What does this indicate?



Sol. (i) In horizontal portions AB and CD, temperature remains same hence these portions show change of state.

AB : conversion from solid to liquid state

CD : conversion from liquid to vapours

(ii) When change of state takes place, $\Delta Q = mL$

length of line $\propto$ Latent heat

$$CD = 2AB \Rightarrow \Delta Q_{CD} = 2\Delta Q_{AB}$$

$$mL_V = 2mL_l \Rightarrow L_V = 2L_l$$

where L_l and L_V are the latent heat of fusion and vaporisation respectively.

(iii) Heat $\Delta Q = mc\,\Delta T$

$$\frac{\Delta T}{\Delta Q} = \text{Slope of inclined line}$$

$$= \frac{1}{mc} = \frac{1}{\text{Thermal capacity}}$$

The slope of line DE represents reciprocal of heat capacity.

(iv) Slope of OA > slope of BC

$$\frac{1}{(mc)_s} > \frac{1}{(mc)_l} \Rightarrow c_l > c_s$$

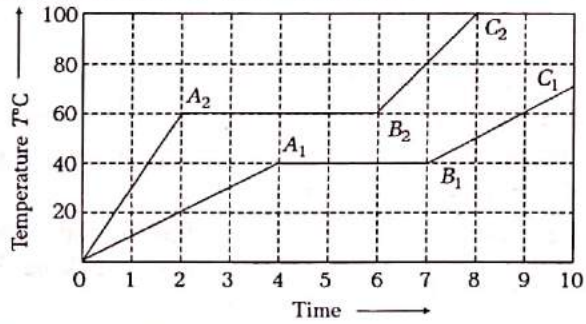
The specific heat of liquid is greater than that of specific heat of solid.

Example 16.19 *Two bodies of equal mass m are heated at a uniform rate under identical conditions. Their change in temperature are shown graphically in figure.*

(i) What are their melting points?

(ii) What is the ratio of their latent heats?

(iii) What is the ratio of their specific heats?



Sol. (i) A_1B_1 and A_2B_2 represent change of state for substances 1 and 2. During change of state, temperature remains same.

Melting point of substance 1 : 40° C

Melting point of substance 2 : 60° C

(ii) Heat $\Delta Q \propto$ time

Since heat is supplied at constant rate

$\Delta Q = kt$, k : constant rate (J/s)

$\Delta Q = mL$

$mL = kt \Rightarrow L \propto t$

$$\frac{L_1}{L_2} = \frac{t_1}{t_2} = \frac{\text{time during change of state from } A_1 \text{ to } B_1}{\text{time during } A_2 \text{ to } B_2}$$

$$= \frac{(7-4)}{(6-2)} = \frac{3}{4}$$

The ratio of latent heat of fusion of two substances are in the ratio 3 : 4

(iii) $\Delta Q = mc\Delta T \Rightarrow kt = mc(T - T_0)$

$T = \frac{kt}{mc} + T_0$, T *versus* t is straight line

So, $\frac{k}{mc}$ represents slope of inclined line

In solid state, $$\frac{\left(\frac{k}{mc}\right)_1}{\left(\frac{k}{mc}\right)_2} = \frac{\text{slope of } OA_1}{\text{slope of } OA_2}$$

$$= \frac{40/4}{60/2} = \frac{1}{3}$$

$\therefore$ Ratio of specific heat $\frac{c_1}{c_2} = \frac{1}{3}$

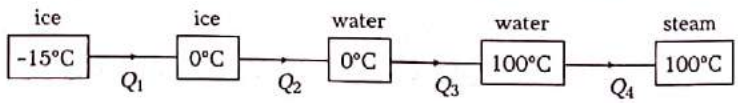
In liquid state,

$$\frac{\left(\frac{k}{mc}\right)_1}{\left(\frac{k}{mc}\right)_2} = \frac{\text{slope of } B_1C_1}{\text{slope of } B_2C_2}$$

$$= \frac{(70-40)/(10-7)}{(100-60)/(8-6)} = \frac{1}{2}$$

Ratio of specific heat $\frac{c_2}{c_1} = \frac{1}{2}$

Example 16.20 *How much heat is required to convert 8.0 g of ice at −15° C to steam at 100°C? (Given, $c_{ice} = 0.53\ cal\ g^{-1}\ °C^{-1}$, $L_f = 80\ cal\ g^{-1}$ and $L_V = 539\ cal\ g^{-1}$ and $c_{water} = 1\ cal\ g^{-1}\ °C^{-1}$)*



Sol. $Q_1 = mc_{ice}(T_f - T_i)$

$= (8.0)(0.53)[0 - (-15)] = 63.6$ cal

$Q_2 = mL_f = (8)(80) = 640$ cal

$Q_3 = mc_{water}(T_f - T_i)$

$= (8.0)(1.0)[100 - 0] = 800$ cal

$Q_4 = mL_V = (8.0)(539) = 4312$ cal

$\therefore$ Net heat required

$$Q = Q_1 + Q_2 + Q_3 + Q_4 = 5815.6 \text{ cal}$$

Example 16.21 100 g *ice at* 0°C *is mixed with* 10 *g steam at* 100°C. *Find the final temperature and composition.*

Sol. Heat required to melt ice at 0°C,

$$Q_1 = m_iL_f = 100 \times 80 \text{ cal g}^{-1}$$

$$= 8000 \text{ cal.}$$

Heat required to raise the temperature of water from 0°C to 100°C

$$= Q_2 = m_ic_w\Delta t.$$

$$= 100 \text{ g} \times 1 \text{ calg°C} \times 100°\text{C} = 10000 \text{ cal}$$

Maximum heat ice can absorb from steam to reach boiling point

$$= Q_1 + Q_2 = 18000 \text{ cal}$$

Heat rejected by steam on complete condensation,

$$Q_3 = m_s L_v = 10 \text{ g} \times 540 \text{ cal g}^{-1}$$
$$= 5400 \text{ cal}$$

Heat rejected by water at 100°C to cooled to 0°C

$$Q_4 = m_s c_w \Delta t = 10 \text{ g} \times 100 \text{ cal g}^{-1}{}^\circ\text{C}^{-1}$$
$$= 1000 \text{ cal}$$

Maximum heat can be supplied to steam to ice at 0°C

$$= Q_3 + Q_4 = (5400 + 1000) \text{ cal}$$
$$= 6400 \text{ cal}$$

To melt the ice, 8000 cal heat is required, but maximum heat supplied by steam, 6400 cal is in sufficient to melt the ice. So resulting temperature of mixture is = 0°C.

Phase diagram and triple point

The temperature of a substance remains constant during its phase change (or change of state) and this temperature depends on the pressure.

The graph between temperature and pressure of substance is called **phase diagram** or p-T diagram. The phase diagram (drawn for water and CO_2) shows p-T plane is divided into three regions *i.e.*, solid region, liquid region and vapour region which are separated by **sublimation curve, fusion curve and vaporisation curve.**

These curves are described below

(i) A curve which shows the variation of boiling point with pressure is called **vaporisation curve** (*OC*). The points on this curve represents the states in which liquid and vapour phases coexist. The vaporisation line is called **steam line** in case of water.

(ii) A curve which shows the variation of melting point with pressure is called the **fusion curve** (*OA*). The points on this curve represents the states in which solid and liquid phases coexist. The fusion line is called **ice line** in case of water.

(iii) A curve which shows the variation of pressure with temperature at which a solid changes directly into vapour state is called **sublimation curve** (*OB*). The points on this curve represents the state in which solid and vapour phases coexist. The sublimation line is called the **hoar-frost line** in case of water.

(iv) The temperature and pressure at which these three curves co-exist simultaneously is known as the **triple point** of the substance.

e.g., The triple point for water is represented by temperature 273.16 K and pressure 6.11×10^{-3} Pa.

Two figures below show the phase diagram with triple point for (a) water (b) CO_2.

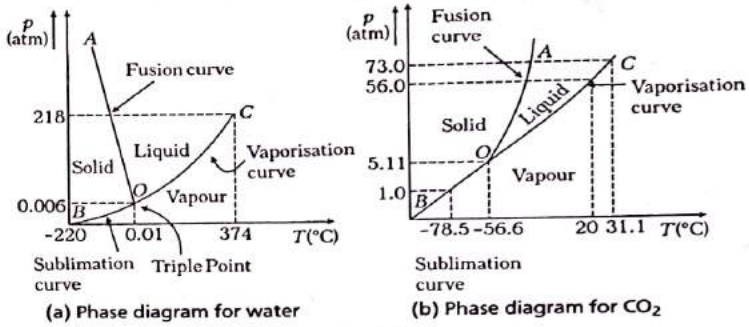
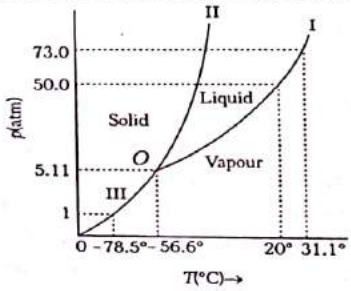


Fig. 16.2

(v) At pressure below that of its triple point, no substances can exist as a liquid.

Example 16.22 *Answer the following questions based on the p-T phase diagram of carbon dioxide as shown in the figure.*



(i) At what temperature and pressure can the solid, liquid and vapour phases of CO_2 co-exist in equilibrium?

(ii) What is the effect of decrease of pressure on the fusion and boiling point of CO_2?

(iii) What are the critical temperature and pressure for CO_2? What is their significance?

(iv) Is CO_2 solid, liquid or gas at (a) – 70° C under 1 atm (b) – 60° C under 10 atm (c) 15° C under 56 atm?

Sol. (i) The solid, liquid and vapour phases of CO_2 co-exist in equilibrium at its triple point O for which

$$p_{tr} = 5.11 \text{ atm and } T_{tr} = -56.6^\circ \text{C}$$

(ii) The vaporisation curve I and fusion curve II show that both the boiling point and fusion point of CO_2 decrease with decrease in pressure.

(iii) For CO_2, $p_c = 73.0$ atm and $T_c = 31.1^\circ$ C.
Above its critical temperature, CO_2 gas cannot be liquefied, however large pressure may be applied.

(iv) (a) – 70° C under 1 atm. This point lies in vapour region. Therefore, at –70° C under 1 atm, CO_2 is vapour.
(b) – 60° C under 10 atm. This point lies in solid region. Therefore, CO_2 is solid at – 60° C under 10 atm.
(c) 15°C under 56 atm. This point lies in liquid region. Therefore, CO_2 is liquid at 15°C under 56 atm.

Check point 16.2

1. A substance of mass m kg requires a power input of P watts to remain in the molten state at its melting point. When the power is turned off, the sample completely solidifies in time t second. What is the latent heat of fusion of the substance?
(a) $\frac{Pm}{t}$ (b) $\frac{Pt}{m}$ (c) $\frac{m}{Pt}$ (d) $\frac{t}{Pm}$

2. 50 g of ice at 0°C is mixed with 50 g of water at 60°C, final temperature of mixture will be
(a) 0°C (b) 40°C (c) 10°C (d) 15°C

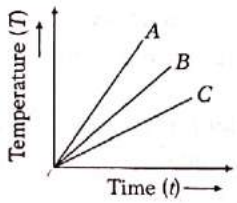
3. 80 g of water at 30°C are poured on a large block of ice at 0°C. The mass of ice that melts is
(a) 30 g (b) 80 g (c) 1600 g (d) 150 g

4. An iron ball of mass 0.2 kg is heated to 10°C and put into a block of ice at 0°C. 25 g of ice melts. If the latent heat of fusion of ice is 80 cal g^{-1}, then the specific heat of iron in cal g^{-1} °C is
(a) 1 (b) 0.1 (c) 0.8 (d) 0.08

5. Steam at 100°C is passed into 1.1 kg of water contained in a calorimeter of water equivalent 0.02 kg at 15°C till the temperature of calorimeter and its contents rises to 80°C. The mass of the steam condensed in kilogram is ($c_w = 10^3$ cal kg^{-1} °C^{-1} and $L_V = 540 \times 10^3$ cal kg^{-1})
(a) 0.130 (b) 0.065 (c) 0.260 (d) 0.135

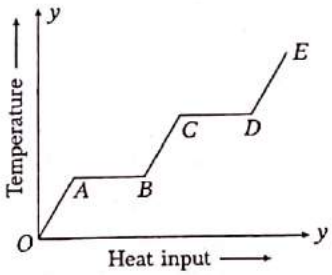
6. A lead bullet of 10 g travelling at 300 ms^{-1} strikes against a block of wood and comes to rest. Assuming 50% of heat is absorbed by the bullet, the increase in its temperature is
(a) 100°C (b) 125°C (c) 150°C (d) 200°C

7. The temperatures *versus* time graph is shown in figure. Which of the substances *A*, *B* and *C* has the lowest heat capacity if heat is supplied to all of them at equal rates.



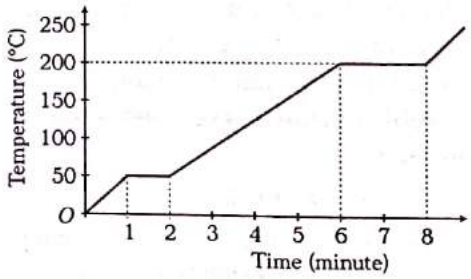
(a) *A*
(b) *B*
(c) *C*
(d) All have equal specific heat

8. A solid material is supplied with heat at a constant rate. The temperature of material is changing with heat input as shown in the figure. What does slope DE represent.



(a) latent heat of liquid
(b) latent heat of vapour
(c) heat capacity of vapour
(d) inverse of heat capacity of vapour

9. A student takes 50 g wax (specific heat = 0.6 kcal/kg°C) and heats it till it boils. The graph between temperature and time is as follows. Heat supplied to the wax per minute and boiling point are respectively



(a) 500 cal, 50°C (b) 1000 cal, 100°C
(c) 1500 cal, 200°C (d) 1000 cal, 200°C

10. 4 kg of ice at −15°C are added to 5 kg of water at 15°C. The temperature of the resulting mixture equals
(a) −15°C (b) 0°C (c) 5°C (d) 15°C

3 Heat transfer

As we have already studied, heat is a form of energy which transfers from a body at higher temperature to a body at lower temperatures. The transfer of heat from one body to the other may take place by any of three possible ways namely
(i) Conduction (ii) Convection (iii) Radiation

In the first two processes, a medium is necessary for the heat transfer. Radiation, however, does not have this restriction. This is also the fastest mode of heat transfer, in which heat is transferred from one place to the other in the form of electromagnetic radiations.

1. Conduction

The transfer of heat taking place due to molecular vibrations (*i.e.*, molecular collisions) is known as heat conduction. In this

process, there is no change in average position of the molecule and hence there is no mass movement of matter. Through in solids, heat is transferred by the process of conduction.

2. Convection

Convection is the process in which heat is transferred om one point to another by the actual motion of matter fi n a region of high temperature to a region of lower temperature. This process of heat transfer takes place only in fluids.

3. Radiation

It is a mode of heat transfer from one place to another without heating the intervening medium. The heat is transferred by the means of thermal radiations, radiant energy or simply radiation. The heat from the sun reaches to the earth by radiation after travelling millions of kilometers of empty space (*i.e.* without any material medium).

Conduction of heat in conducting rod

Figure shows a rod whose ends are in thermal contact with a hot reservoir at temperature T_1 and a cold reservoir at temperature T_2. The sides of the rod are covered with insulating medium, so the transport of heat is along the rod and not through the sides.

Heat is transferred from the hot end to the cold end due to molecular collisions. Molecules at hot end transfer their kinetic energy to neighbouring molecules by collision. In this way, heat transfers from the one end to other. During heat transfer, the molecules do not change their equilibrium position.

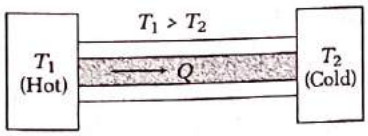


Fig. 16.3

Steady state

Consider a section *ab* of a rod as shown in figure.Suppose Q_1 heat enters into the section at *a* and Q_2 leaves at *b*, then $Q_2 < Q_1$. Part of the energy $Q_1 - Q_2$ is utilised in raising the temperature of section *ab* and the remaining is lost to atmosphere through *ab*.

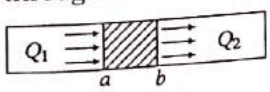


Fig. 16.4

If heat is continuously supplied from the left end of the rod, a stage comes when temperature of the section becomes constant. This is called the steady state condition. In that case, $Q_1 = Q_2$ if rod is insulated from the surroundings (or heat loss through *ab* is zero).

In steady state, temperature of different sections of the rod remains constant with time.

In the given figure,

$$T_1 = \text{constant},\ T_2 = \text{constant etc.}$$

and $$T_1 > T_2 > T_3 > T_4$$

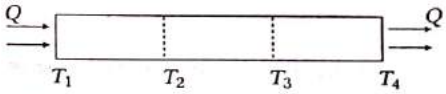


Fig. 16.5 Insulated rod in steady state

For a conductor in steady state neither absorption nor emission takes place.

Rate of heat transfer and temperature gradient

When a quantity of heat dQ is transferred through the rod in a time dt, the rate of heat flow is $\frac{dQ}{dt}$.

This rate is also called the **heat current**, denoted by H. Experiments show that the **heat current is proportional to the cross-section area A of the rod and to the temperature gradient dT/dx,** which is the rate of change of temperature with distance along the bar as shown in the figure. In general

$$H = \frac{dQ}{dt} = -KA\frac{dT}{dx} \quad \text{...(i)}$$

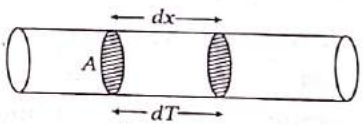


Fig. 16.6

The negative sign indicates that temperature decreases along the direction of flow of heat.

SI unit of temperature gradient is Km^{-1} or $°Cm^{-1}$.

Thermal conductivity

The constant K (in the above expression) is called the thermal conductivity, which is a measure of the ability of a material to conduct heat.

A substance with a large thermal conductivity K is a good heat conductor. Let us apply Eq. (i) to a rod of length L and constant cross sectional area A in which a steady state

has been reached. In a steady state the temperature at each point is constant in time. Hence,

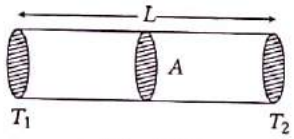


Fig. 16.7

$$\frac{dT}{dx} = \frac{T_1 - T_2}{L}$$

Therefore, the heat ΔQ transferred in time Δt is

$$\boxed{\Delta Q = KA\left(\frac{T_1 - T_2}{L}\right)\Delta t} \quad \text{...(ii)}$$

If $A = 1, T_1 - T_2 = 1, L = 1, \Delta t = 1$, then $\Delta Q = k$

Hence, the coefficient of thermal conductivity of a material may be defined as the quantity of heat that flows per unit time through a unit cube of the material when its opposite faces are kept at a temperature difference of one degree.

The SI unit of K is $Js^{-1}\,m^{-1}K^{-1}$ or $Wm^{-1}\,K^{-1}$ and its dimension is $[MLT^{-3}K^{-1}]$.

Note

1. A substance which is a good conductor of heat (*e.g.*, silver) is also a good conductor of electricity. For such substances at a given temperature T, the ratio of thermal conductivity to electrical conductivity ($K/\sigma T$) is constant. This is known as Wiedmann-Franz Law.
2. The ratio of the coefficient of thermal conductivity (K) to the thermal capacity per unit volume (ρ_c) of a material is called thermometric conductivity or diffusivity.

Thermal resistance (R)

In differential form, rate of heat flow can be written as

$$\frac{dQ}{dt} = H = \frac{\Delta T}{l/KA} = \frac{\Delta T}{R} \quad \text{...(iii)}$$

Here, ΔT = temperature difference (TD) and

$R = \dfrac{l}{KA}$ = thermal resistance of the rod.

Also, we can write $R = \dfrac{\Delta T}{H}$

Thus thermal resistance is the the ratio of the temperature difference between the ends of conductors to the heat current.

SI unit of thermal resistance is KW^{-1} (Kelvin per watt) and dimensional formula of R is $[M^{-1}L^{-2}T^{3}K]$.

Example 16.23 *Thickness of ice on a lake is 5 cm and the temperature of air is −20°C. If the rate of cooling of water inside the lake be 20000 cal min^{-1} through each square metre surface, find K for ice.*

Sol. Thickness of ice on a lake, $L = 5\text{ cm} = 5 \times 10^{-2}$ m

Change in temperature, $\Delta T = 0 - (-20) = 20°\text{C}$

Rate of cooling of water $= \dfrac{\Delta Q}{\Delta t} = 20000\text{ cal min}^{-1}$

$$= \frac{20000}{60}\text{ cal s}^{-1}$$

$$= \frac{20000}{60} \times 4.2 = 1400\text{ Js}^{-1}$$

As $$\frac{\Delta Q}{\Delta t} = KA\left(\frac{\Delta T}{L}\right)$$

Where K is thermal conductivity

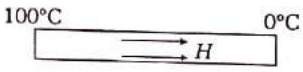
$$\therefore \quad K = \frac{\Delta Q}{\Delta T} \cdot \frac{L}{\Delta TA}$$

On putting values, we get

$$K = \frac{1400 \times 5 \times 10^{-2}}{1 \times 20} = 3.5\text{ Wm}^{-1}\,°\text{C}^{-1} \quad (\text{where } A = 1\text{ m}^2)$$

Example 16.24 *One end of a copper rod of length 1 m and area of cross-section 4.0×10^{-4} m^2 is maintained at 100°C. At the other end of the rod ice is kept at 0°C. Neglecting the loss of heat from the surroundings find the mass of ice melted in 1 h. (Given, $K_{Cu} = 401\,Wm^{-1}K^{-1}$ and $L_f = 3.35 \times 10^5\,Jkg^{-1}$)*

Sol. Thermal resistance of the rod,



$$R = \frac{l}{KA} = \frac{1.0}{(401)(4 \times 10^{-4})} = 6.23\text{ KW}^{-1}$$

$$\therefore \quad \text{Heat current, } H = \frac{\text{Temperature difference}}{\text{Thermal resistance}}$$

$$= \frac{(100 - 0)}{6.23} = 16\text{ W}$$

Heat transferred in 1 h,

$$Q = Ht \qquad \left(\because H = \frac{Q}{t}\right)$$

$$= (16)(3600) = 57600\text{ J}$$

Now, let m be the mass of ice that melts in 1 h, then

$$m = \frac{Q}{L} \qquad (Q = mL)$$

$$= \frac{57600}{3.35 \times 10^5} = 0.172\text{ kg or } 172\text{ g}$$

Example 16.25 *An iron boiler is 1 cm thick and has a heating area 2.5 m^2. The two surfaces of the boiler are at 230°C and 100°C respectively. If the latent heat of the steam is 540 kcal kg^{-1} and thermal conductivity of iron is 1.6×10^{-2} kcal s^{-1} m^{-1}K^{-1}, then how much water will be evaporated into steam per minute?*

Sol. Here, thickness of iron boiler, $\Delta x = 1\text{ cm} = 0.01\text{ m}$

Heating area, $A = 2.5\text{ m}^2$

Temperature $\Delta T = T_1 - T_2 = 230 - 100 = 130°\text{ C}$

Latent heat of steam = 540 kcal kg^{-1}

Thermal conductivity of iron, $K = 1.6 \times 10^{-2}$ kcal $s^{-1}m^{-1}K^{-1}$

$$\Delta t = 1 \text{ min} = 60 \text{ s}$$

Let m kg of the mass be evaporated in 1 min, then

Heat conducted, $\Delta Q = \dfrac{KA(T_1 - T_2)\Delta t}{\Delta x} = mL$

$$\Rightarrow \quad m = \frac{KA(T_1 - T_2)\Delta t}{\Delta x L}$$

$$= \frac{(1.6 \times 10^{-2}) \times (2.5) \times 130 \times 60}{0.01 \times 540} = 57.78 \text{ kg}$$

Example 16.26 *A copper rod 2 m long has a circular cross-section of radius 1 cm. One end is kept at 100°C and the other at 0°C, and the surface is insulated so that negligible heat is lost through the surface. Find*

(i) the thermal resistance of the bar

(ii) the thermal current H.

(iii) the temperature gradient dT/dx and

(iv) the temperature at 25 cm from the hot end.

Thermal conductivity of copper is 401 $Wm^{-1}K^{-1}$.

Sol. (i) Thermal resistance, $R = \dfrac{l}{KA} = \dfrac{l}{K(\pi r^2)}$

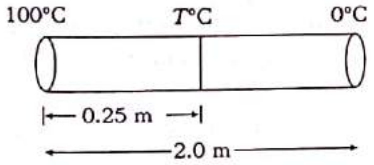
or $$R = \frac{(2)}{(401)(\pi)(10^{-2})^2} = 15.9 \text{ kW}^{-1}$$

(ii) Thermal current, $H = \dfrac{\Delta T}{R} = \dfrac{100}{15.9}$ or $H = 6.3$ W

(iii) Temperature gradient

$$\frac{\Delta T}{dx} = \frac{0 - 100}{2} = -50 \text{ Km}^{-1} = -50° \text{Cm}^{-1}$$

(iv)



Let T° C be the temperature at 25 cm from the hot end, then

$$(T - 100) = (\text{temperature gradient}) \times (\text{distance})$$

or $$T - 100 = (-50)(0.25) \quad \text{or} \quad T = 87.5°\text{C}$$

Connection of rods with different thermal conductivities

If two or more rods or thermal conductors are connected with one another, the equivalent thermal conductivity as a whole may be altered. *Now we will discuss different combinations of rods and heat flow associated with them.*

1. Series combination

Consider a series combination of rods with sides insulated so that heat flow across length of each rod. In series when steady state is reached, heat current is same and temperature difference across each rod is different.

Let us assume that n slabs each of cross-sectional area A, lengths $L_1, L_2, L_3 \ldots . L_n$ and thermal conductivities $K_1, K_2, K_3 \ldots K_n$ respectively are connected in series as shown in the figure.

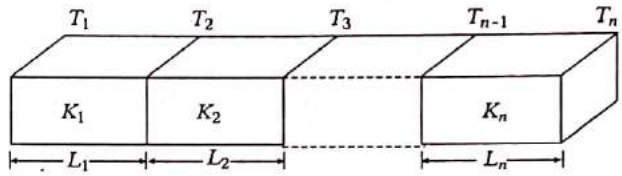


Fig. 16.8

(a) **Heat current** As heat current is the same in all the conductors in steady state,

We have $\dfrac{Q}{t} = H_1 = H_2 = H_3 \ldots = H_n = H$

$$\Rightarrow \quad \frac{K_1 A(T_1 - T_2)}{L_1} = \frac{K_2 A(T_2 - T_3)}{L_2} = \frac{K_n A(T_{n-1} - T_n)}{L_n}$$

(b) **Equivalent thermal resistance** Equivalent thermal resistance of the combination is given by

$$R_{eq} = R_1 + R_2 + \ldots + R_n$$

(c) **Equivalent thermal conductivity**

As $$R_{eq} = R_1 + R_2 + \ldots + R_n$$

$$\Rightarrow \frac{L_1 + L_2 + L_3 + \ldots + L_n}{K_{eq} A} = \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} + \ldots + \frac{L_n}{K_n A}$$

Equivalent thermal conductivity is given by

$$K_{eq} = \frac{L_1 + L_2 + \ldots + L_n}{\dfrac{L_1}{K_1} + \dfrac{L_2}{K_2} + \ldots + \dfrac{L_n}{K_n}} = \frac{\sum_{i=1}^{n} L_i}{\sum_{i=1}^{n} \dfrac{L_i}{K_i}}$$

Case I If slabs are of equal length

i.e. $L_1 = L_2 = L_3 = \ldots = L_n = L$

$$\therefore \quad \boxed{K_{eq} = \frac{n}{\dfrac{1}{K_1} + \dfrac{1}{K_2} + \ldots + \dfrac{1}{K_n}}}$$

Case II If two slabs of equal lengths are combined in series

$$\boxed{K_{eq} = \frac{2K_1K_2}{K_1 + K_2}}$$

(d) **Temperature of common interface of junction of two composite rods** Let temperature at two ends be T_1 and T_2 and temperature of the junction be T.

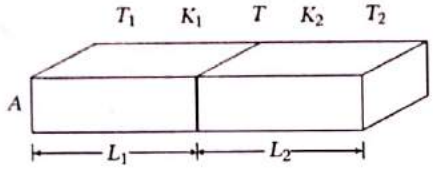


Fig. 16.9

As the two rods are connected in series, heat current must be same through both the rods

$$\therefore \quad \frac{K_1A(T_1 - T)}{L_1} = \frac{K_2A(T - T_2)}{L_2}$$

Solving we get

$$T = \frac{\frac{K_1}{L_1}T_1 + \frac{K_2}{L_2}T_2}{\frac{K_1}{L_1} + \frac{K_2}{L_2}}$$

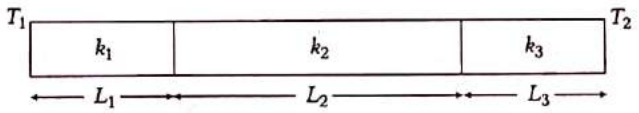
Case I If $L_1 = L_2 = L$, then

$$\boxed{T = \frac{K_1T_1 + K_2T_2}{K_1 + K_2}}$$

Case II If $K_1 = K_2 = K$ and $L_1 = L_2 = L_1$, then

$$\boxed{T = \frac{T_1 + T_2}{2}}$$

Example 16.27 *Three rods of same cross-section but different length and conductivity are joined in series. If the temperature of the two extreme ends are T_1 and T_2 ($T_1 > T_2$) find the rate of heat transfer H.*



Sol. Total thermal resistance

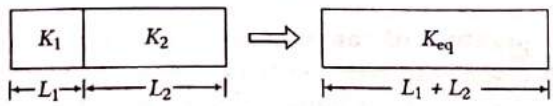
$$R = R_1 + R_2 + R_3 = \frac{L_1}{K_1A} + \frac{L_2}{K_2A} + \frac{L_3}{K_3A}$$

Now, Heat transfer $H = \dfrac{T_1 - T_2}{R}$

$$\therefore \quad H = \frac{(T_1 - T_2)}{\frac{L_1}{K_1A} + \frac{L_2}{K_2A} + \frac{L_3}{K_3A}}$$

Example 16.28 *A composite slab is prepared by pasting two plates of thicknesses L_1 and L_2 and thermal conductivities K_1 and K_2. The slabs have equal cross-sectional area. Find the equivalent conductivity of the slab.*

Sol.



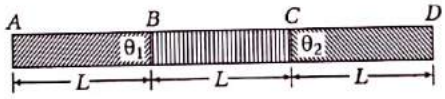
$$R_{eq} = R_1 + R_2$$

$$\frac{(L_1 + L_2)}{K_{eq}A} = \frac{L_1}{K_1A} + \frac{L_2}{K_2A}$$

$$K_{eq} = \frac{L_1 + L_2}{\frac{L_1}{K_1} + \frac{L_2}{K_2}}$$

Example 16.29 *The three bars of equal lengths and of equal areas of cross-section are connected in series. Their thermal conductivities are in the ratio of 2 : 4 : 3. If the open ends of the first and the last bars are at temperatures 200°C and 18°C respectively in the steady state, calculate the temperatures of both the junctions.*

Sol. Let the temperature of the junctions B and C be θ_1 and θ_2, respectively. In the steady state, the rate of flow of heat through all rods will be the same.



Therefore , Heat transfer $H = (2K)A\dfrac{(200 - \theta_1)}{L}$

$$= (4K)A\frac{(\theta_1 - \theta_2)}{L} = (3K)A\frac{\theta_2 - 18}{L}$$

or $\quad 2(200 - \theta_1) = 4(\theta_1 - \theta_2) = 3(\theta_2 - 18)$

Solving, $\theta_1 = 116°\text{C}$, $\theta_2 = 74°\text{C}$

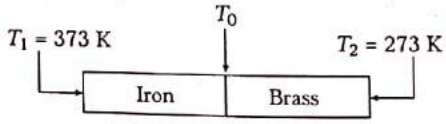
Example 16.30 *An iron bar having length $L_1 = 0.1$ m, area of cross-section $0.02\,m^2$ thermal conductivity $K_1 = 79\,Wm^{-1}K^{-1}$ and brass bar having length $L_2 = 0.1$ m area of cross-section, $A_2 = 0.02\,m^2$ and thermal conductivity $K_2 = 109\,Wm^{-1}K^{-1}$ are soldered end to end as shown in figure.*

The terminal ends of two rods are maintained at 373 K and 273 K respectively. Find the expression and compute

(i) the temperature of the junction of two bars.

(ii) equivalent thermal conductivity of composite bar and

(iii) the heat current through the composite bar



Sol. Given, Length $L_1 = L_2 = L = 0.1$ m

Area, $A_1 = A_2 = A = 0.02\ \text{m}^2$

Thermal conductivity, $K_1 = 79\ \text{Wm}^{-1}\text{K}^{-1}$, $K_2 = 109\ \text{Wm}^{-1}\text{K}^{-1}$

Temperature, $T_1 = 373$ K and $T_2 = 273$ K

At steady state, heat transferred from any section of thermal conductor is same *i.e.* $H_1 = H_2 = H$

$$\Rightarrow \quad \frac{K_1A_1(T_1 - T_0)}{L_1} = \frac{K_2A_2(T_0 - T_2)}{L_2} \quad \text{...(i)}$$

For $A_1 = A_2 = A$ and $L_1 = L_2 = L$, Eq. (i) becomes

$$K_1(T_1 - T_0) = K_2(T_0 - T_2) \Rightarrow T_0 = \frac{K_1T_1 + K_2T_2}{K_1 + K_2}$$

Therefore, heat current through other bar

$$H = \frac{K_1 A(T_1 - T_0)}{L} = \frac{K_2 A(T_0 - T_2)}{L} = \frac{A(T_1 - T_2)}{L\left(\frac{1}{K_1} + \frac{1}{K_2}\right)}$$

Now, heat current through composite bar of length $L_1 + L_2 = 2L$ and equivalent thermal conductivity K', can be given by

$$H' = \frac{K' A(T_1 - T_2)}{2L} = H \Rightarrow K = \frac{2K_1K_2}{K_1 + K_2}$$

(i) So, the temperature of the junction of two bars is

$$T_0 = \frac{K_1T_1 + K_2T_2}{K_1 + K_2}$$

$$= \frac{79\text{Wm}^{-1}\text{K}^{-1} \times 373\text{ K} + 109\text{Wm}^{-1}\text{K}^{-1} \times 273\text{ K}}{79\text{ Wm}^{-1}\text{K}^{-1} + 109\text{Wm}^{-1}\text{K}^{-1}} = 315\text{ K}$$

(ii) Equivalent thermal conductivity $K' = \frac{2K_1K_2}{K_1 + K_2}$

$$= \frac{2 \times 79\text{ Wm}^{-1}\text{K}^{-1} \times 109\text{ Wm}^{-1}\text{K}^{-1}}{79\text{ Wm}^{-1}\text{K}^{-1} + 109\text{ Wm}^{-1}\text{K}^{-1}}$$

$$= 91.6\text{ Wm}^{-1}\text{K}^{-1}$$

(iii) Heat current through the composite bar

$$H' = H = \frac{K' A(T_1 - T_2)}{2L}$$

$$= \frac{91.6\text{ Wm}^{-1}\text{K}^{-1} \times 0.02\text{ m}^2 \times (373 - 273)\text{ K}}{2 \times 0.1\text{ m}} = 916\text{ W}$$

2. Parallel combination

In a parallel combination of rods, one end of each of rod is kept at higher temperature and other end of each of rod is kept at lower temperature. In this way rods, provide parallel path for flow of heat energy. In parallel, heat currents are different but temperature difference across each rod is same.

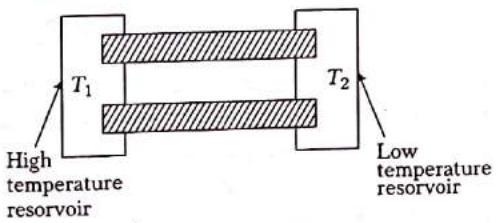


Fig. 16.10

(a) **Heat current** Heat current will be different in each rod. Net heat current will be given by sum of heat currents through individual rod.

i.e., $H = H_1 + H_2 + \ldots + H_n$

(b) **Equivalent resistance**

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$$

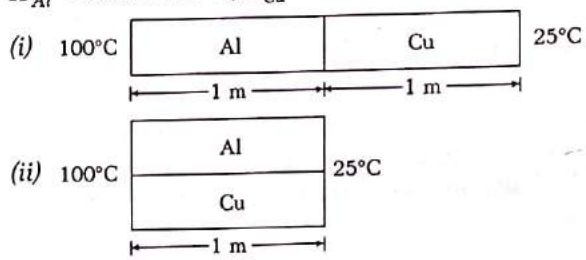
For two rods, $R_{eq} = \frac{R_1R_2}{R_1 + R_2}$

(c) **Temperature gradient** It is same across each rod and is given by $\frac{T_1 - T_2}{L}$.

(d) **Equivalent thermal conductivity**

$$\boxed{K_{eq} = \frac{K_1A_1 + K_2A_2 + \ldots + K_nA_n}{A_1 + A_2 + \ldots + A_n}}$$

Example 16.31 *An aluminium rod and copper rod of equal length 1 m and cross-sectional area 1 cm^2 are welded as shown. $K_{Al} = 200\ W/m°C$, $K_{Cu} = 400\ W/m°C$. Find heat current.*



Sol. Resistance, $R_{Al} = \frac{l}{K_{Al}A} = \frac{1}{200 \times 10^{-4}} = 50°\text{C/W}$

$$R_{Cu} = \frac{l}{K_{Cu}A} = \frac{1}{400 \times 10^{-4}} = 25°\text{C/W}$$

(i) Equivalent resistance, $R_{eq} = R_{Al} + R_{Cu} = 50 + 25 = 75°\text{C/W}$

Heat current $i = \frac{100 - 25}{R_{eq}} = \frac{75}{75} = 1\text{ W}$

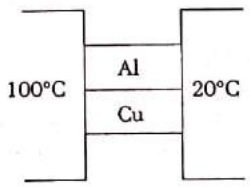
(ii) Equivalent resistance,

$$R_{eq} = \frac{R_{Al} \cdot R_{Cu}}{R_{Al} + R_{Cu}} = \frac{50 \times 25}{50 + 25} = \frac{50}{3}\ °\text{C/W}$$

Heat current $i = \frac{100 - 25}{50/3} = 4.5\text{ W}$

Example 16.32 *Two metal cubes with 3 cm edges of copper and aluminium are arranged as shown in figure. Find*

(i) The total thermal current from one reservoir to the other.



(ii) The ratio of the thermal current carried by the copper cube to that carried by the aluminium cube. Thermal conductivity of copper is 401 $Wm^{-1}K$ and that of aluminium is 237 $Wm^{-1}K$.

Sol. (i) Thermal resistance of aluminium cube, $R_1 = \frac{l}{KA}$

or $$R_1 = \frac{(3.0 \times 10^{-2})}{(237)(3.0 \times 10^{-2})^2}$$

$$= 0.14\text{ KW}^{-1}$$

and thermal resistance of copper cube, $R_2 = \frac{l}{KA}$

or $$R_2 = \frac{(3.0\times 10^{-2})}{(401)(3.0\times 10^{-2})^2} = 0.08\ \text{KW}^{-1}$$

As these two resistances are in parallel, their equivalent resistance will be

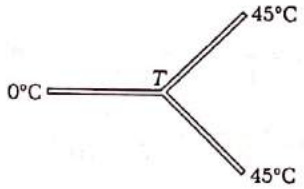
$$R = \frac{R_1R_2}{R_1+R_2} = \frac{(0.14)(0.08)}{(0.14)+(0.08)} = 0.05\ \text{KW}^{-1}$$

$\therefore$ Thermal current, $H = \dfrac{\text{Temperature difference}}{\text{Thermal resistance}}$

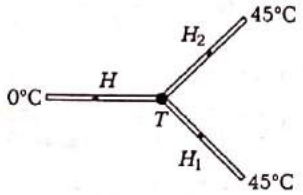
$$= \frac{(100-20)}{0.05} = 1.6\times 10^3\ \text{W}$$

(ii) In parallel, thermal current distributes in the inverse ratio of resistance. Hence, $\dfrac{H_{Cu}}{H_{Al}} = \dfrac{R_{Al}}{R_{Cu}} = \dfrac{R_1}{R_2} = \dfrac{0.14}{0.08} = 1.75$

Example 16.33 *Three identical rods have been joined at a junction to make it a Y shape structure. If two free ends are maintained at 45°C and the end is at 0°C, then what is the junction temperature T?*



Sol. According to the question,



Rod are in steady state

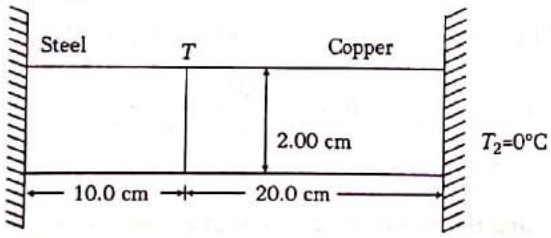
So, $$H = H_1 + H_2$$

$$\Rightarrow \quad \frac{KA(T-0)}{L} = \frac{KA(45-T)}{L} + KA\frac{(45-T)}{L}$$

$$\Rightarrow \quad T = 45 - T + 45 - T \ \Rightarrow\ 3T = 90 \ \Rightarrow\ T = 30°\text{C}$$

Example 16.34 *A steel bar 10.0 cm long is welded end to end to a copper bar 20.0 cm long. Both bars are insulated perfectly on their sides. Each bar has a square cross-section, 2.00 cm on a side. The free end of the steel bar is maintained at 100°C by placing it in contact with ice. Find the temperature at the junction of the two bars and the total rate of heat flow. Thermal conductivity of steel = 50.2 $Wm^{-1}K^{-1}$. Thermal conductivity of copper = 385 $Wm^{-1}K^{-1}$*

Sol.



In the steady state, the two heat currents are equal. We have

$$H_{steel} = \frac{K_{steel}A(100°\text{C} - T)}{L_{steel}}$$

and $$H_{copper} = \frac{K_{copper}A(T - 0°\text{C})}{L_{copper}}$$

Substituting $L_{steel} = 0.100$ m, $L_{copper} = 0.200$ m

$$K_{steel} = 50.2\ \text{Wm}^{-1}\text{K}^{-1},$$

$$K_{copper} = 385\ \text{Wm}^{-1}\text{K}^{-1}$$

Also, both rods are same area.

So, $$\frac{(50.2)(100°\text{C} - T)}{0.100} = \frac{(385)(T - 0°\text{C})}{0.200}$$

Solving for T, we obtain $T = 20.7°C$

We can find the total heat current by substituting this value for T,

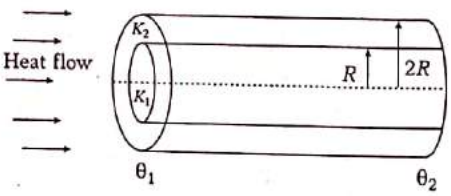
$$H_{steel} = \frac{(50.2)(0.0200)(100 - 20.7)}{0.100}$$

$$H_{steel} = 796.17\ \text{W}$$

$\because$ $$H_{steel} = H_{copper}$$

$\therefore$ $$H_{copper} = 796.17\ \text{W}$$

Example 16.35 *A cylinder of radius R made of a material of thermal conductivity $K_1 = K$ is surrounded by cylindrical shell of inner radius R and outer radius 3R made of a material of thermal conductivity $K_2 = 2K$. The two ends of the combined system are maintained at two different temperatures. There is no loss of heat across the cylindrical surface and system is in steady state. What is the effective thermal conductivity of the system?*



Sol. In this situation, a rod of length L and area of cross-section πR^2 and another of same length L and area of cross-section $\pi[(3R)^2 - R^2] = 8\pi R^2$ will conduct heat simultaneously, so total heat flowing per second will be

$$\frac{dQ}{dt} = \frac{dQ_1}{dt} + \frac{dQ_2}{dt}$$

$$= \frac{K_1\pi R^2(T_1 - T_2)}{L} + \frac{K_2 8\pi R^2(T_1 - T_2)}{L} \quad \text{...(i)}$$

Now, if the equivalent conductivity is K.

Then, $$\frac{dQ}{dt} = K\frac{9\pi R^2(T_1 - T_2)}{L} \quad [\text{As } A = \pi(3R)^2] \quad \text{...(ii)}$$

So, from Eqs. (i) and (ii), we have

$$9K = K_1 + 8K_2$$

i.e., $$K = \frac{(K_1 + 8K_2)}{9} = \frac{K + 8\times 2K}{9} = \frac{17K}{9}$$

Check point 16.3

1. The layers of atomsphere are heated through
   (a) Convection (b) Conduction
   (c) Radiation (d) Both (a) and (c)

2. Mud houses are cooler in summer and warmer in winter because
   (a) Mud is superconductor of heat
   (b) Mud is good conductor of heat
   (c) Mud is bad conductor of heat
   (d) None of the above

3. Snow is more heat insulating than ice, because
   (a) Air is filled in pores of snow
   (b) Ice is more bad conductor than snow
   (c) Air is filled in pores of ice
   (d) Density of ice is more

4. On heating one end of a rod, the temperature of whole rod will be uniform when
   (a) $K = 1$ (b) $K = 0$
   (c) $K = 100$ (d) $K = \infty$

5. If the temperature difference on the two sides of a wall increases from 100°C to 200°C, its thermal conductivity
   (a) remains unchanged (b) is doubled
   (c) is halved (d) becomes four times

6. The thermal conductivity of a rod depends on
   (a) length (b) mass
   (c) area of x-section (d) material of the rod

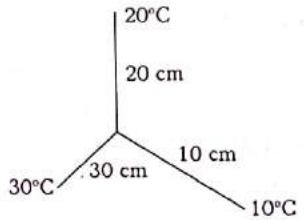
7. The SI unit of thermal conductivity is
   (a) $Js^{-1}K$ (b) $Js^{-1}m^2K$
   (c) $Jm^{-1}K$ (d) $Js^{-1}m^{-1}K^{-1}$

8. Wires A and B have identical lengths and have circular cross-sections. The radius of A is twice the radius of B, *i.e.*, $r_A = 2r_B$. For a given temperature difference between the two ends, both wires conduct heat at the same rate. The relation between the thermal conductivities is given by
   (a) $K_A = 4K_B$ (b) $K_A = 2K_B$
   (c) $K_A = \frac{K_B}{2}$ (d) $K_A = \frac{K_B}{4}$

9. The ends of two rods of different materials with their thermal conductivities, radii of cross-section and lengths all in the ratio 1 : 2 are maintained at the same temperature difference. If the rate of flow of heat in the larger rod is 4 cal s^{-1}, then that in the shorter rod in cals^{-1} will be
   (a) 1 (b) 2
   (c) 8 (d) 16

10. The length of the two rods made up of the same metal and having the same area of cross-section are 0.6 m^2 and 0.8 m^2 respectively. The temperature between the ends of first rod is 90°C and 60°C and that for the other rod is 150°C and 110°C. For which rod the rate of conduction will be greater?
    (a) First (b) Second
    (c) Same for both (d) None of these

11. Three rods made of the same material and having same cross-sectional area but different length 10 cm, 20 cm and 30 cm are joined as shown. The temperature of the junction is



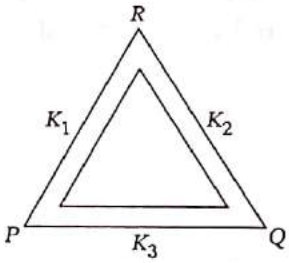
    (a) 10.8°C (b) 14.6°C (c) 16.4°C (d) 18.2°C

12. Equal temperature differences exist between the ends of two metallic rods 1 and 2 of equal lengths. Their thermal conductivities are K_1 and K_2 and area of cross-section are A_1 and A_2, respectively. The condition of equal rates of heat transfer is
    (a) $K_1A_2 = K_2A_1$ (b) $K_1A_1 = K_2A_2$
    (c) $K_1A_1^2 = K_2A_2^2$ (d) $K_1^2A_2 = K_2^2A_1$

13. Consider a compound slab consisting of two different materials having equal thickness and thermal conductivities K and $2K$ in series. The equivalent conductivity of the slab is
    (a) $\frac{2}{3}K$ (b) $\sqrt{2}K$ (c) $3K$ (d) $\left(\frac{4}{3}\right)K$

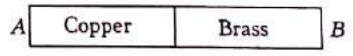
14. In a steady state, the temperature at the end A and B of 20 cm long rod AB are 100°C and 0°C. The temperature of a point 9 cm from A is
    (a) 45°C (b) 55°C (c) 5°C (d) 65°C

15. Three rods of same dimensions are arranged as shown in figure. They have thermal conductivities K_1, K_2 and K_3. The points P and Q are maintained at different temperatures. For the heat flow at the same rate along PRQ and PQ which of the following option is correct?



    (a) $K_3 = \frac{1}{2}(K_1 + K_2)$ (b) $K_3 = K_1 + K_2$
    (c) $K_3 = \frac{K_1K_2}{K_1 + K_2}$ (d) $K_3 = 2(K_1 + K_2)$

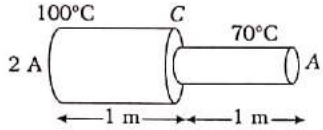
16. Two rods of copper and brass ($K_C > K_B$) of same length and area of cross-section are joined as shown. End A is kept at 100°C and end B at 0°C. The temperature at the junction



    (a) will be more than 50°C
    (b) will be less than 50°C
    (c) will be 50°C
    (d) may be more or less than 50°C depending upon the size of rods

17. A wall has two layers A and B, each made of different material. Both the layers have the same thickness. The thermal conductivity for A is twice that of B and, under steady condition, the temperature difference across the wall is 36°C. The temperature difference across the layer A is
(a) 6°C (b) 12°C (c) 24°C (d) 18°C

18. A metal rod of length $2m$ has cross-sectional areas $2A$ and A as shown in figure. The ends are maintained at temperatures 100°C and 70°C. The temperature at middle point C is



(a) 80°C (b) 85°C (c) 90°C (d) 95°C

19. A slab consists of two parallel layers of two different materials of same thickness having thermal conductivities K_1 and K_2. The equivalent conductivity of the combination is
(a) $K_1 + K_2$ (b) $\frac{K_1 + K_2}{2}$
(c) $\frac{2K_1 + K_2}{K_1 + K_2}$ (d) $\frac{K_1 + K_2}{2K_1K_2}$

20. Two rods of same length and material transfer a given amount of heat in 12 s, when they are joined end to end. But when they are joined lengthwise, then they will transfer same heat in same conditions in
(a) 24 s (b) 3 s (c) 1.5 s (d) 48 s

4 Radiation

Radiation is the transfer of heat energy in form of electromagnetic waves. In this mode transfer of heat takes place without heating the intervening medium. All bodies even at lower temperature emit energy in the form of the electromagnetic radiation. Heat energy is carried by infrared waves with wavelength greater than 750 nm.

Absorption, emission, reflection and transmission of heat radiations

When heat radiations are incident on a surface, then part of radiations is absorbed, part of it is reflected and part of radiation is transmitted as shown.

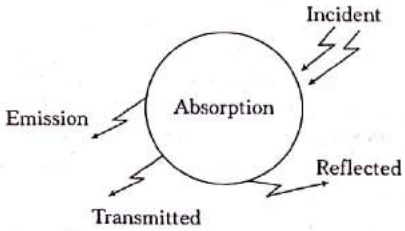


Fig. 16.11

Absorption

The process of absorbing the incident radiation by the surface of a body is called absorption. Due to absorption; temperature of surface of the body increases.

Absorptive power (a)

It is defined as the ratio of the radiant energy absorbed by a body in a given time to the total radiant energy incident on it in the same interval of time.

$$\text{Absorptive Power } a = \frac{\text{energy absorbed } (Q_a)}{\text{energy incident } (Q_i)}$$

It is a dimensionless quantity.

Spectral absorptive power a_λ The absorptive power a refers to radiations of all wavelengths (or the total energy) while the spectral absorptive power is the ratio of radiant energy absorbed by a surface to the radiant energy incident on it for a particular wavelength λ. It may have different values for different wavelengths for a given surface.

Let us take an example, suppose $a = 0.6$, $a_\lambda = 0.4$ for 1000 Å and $a_\lambda = 0.7$ for 2000 Å for a given surface. Then, it means that this surface will absorb only 60% of the total radiant energy incident on it. Similarly, it absorbs 40% of the energy incident on it corresponding to 1000 Å and 70% corresponding to 2000 Å.

The spectral absorptive power a_λ is related to absorptive power a through the relation

$$a = \int_0^\infty a_\lambda \, d\lambda$$

Emission

The process of emitting radiation by the surface of a body is called emission. Due to emission, temperature of surface of the body decreases.

Radiation energy emitted by a surface depends upon following factors.

(i) Wavelength of radiation
(ii) Nature of the surface
(iii) Surface area
(iv) Temperature of the surface

Emissive power (e)

For a given surface it is defined as the radiant energy emitted per second per unit area of the surface.

It is the total amount of energy radiated by a body per second per unit area of surface.

$$\boxed{\text{Emissive Power } e = \frac{1}{A}\frac{\Delta Q}{\Delta t}}$$

SI unit of emissive power is $Js^{-1}m^{-2}$ or Wm^{-2}.

Object having lesser emissive power is called bad emitter and object having greater emissive power is called good emitter.

Spectral emissive power e_λ It is emissive power for a particular wavelength λ and is defined as the total amount of thermal energy emitted per unit time, per unit area of the body for a particular wavelengths.

Thus, $$\boxed{e = \int_0^\infty e_\lambda d\lambda}$$

Its SI unit is $\frac{\text{Joule}}{m^2 s}$ or $\frac{\text{watt}}{m^2}$ and dimensional formula is $[MT^{-3}]$.

Emissivity (ε)

The ratio of emissive power (e) of a body to the emissive power (E) of a black body (a body that absorbs all the radiations falling on it) at the same temperature is called emissivity. It is denoted by ε.

Thus, emissivity, $$\boxed{\varepsilon = \frac{e}{E}}$$

It is a dimensionless quantity.

(i) For highly polished body $\varepsilon_1 = 0$

(ii) For practical bodies $0 < \varepsilon < 1$

Transmittance (Transmittivity)

The ratio of heat energy transmitted by a body in a given time to the total radiation energy incident on it in the same interval of time is called transmittance or transmissivity. It is denoted by t.

$$\therefore \quad \boxed{\text{Transmittance } t = \frac{Q_{\text{transmitted}}}{Q_{\text{incident}}} = \frac{Q_t}{Q_i}}$$

It is a dimensionless quantity.

Reflectivity

The ratio of heat energy reflected by a body in a given time to the total radiation energy incident on it in the same interval of time is called reflectivity.

It is denoted by r

$$\therefore \quad \boxed{\text{Reflectivity } r = \frac{Q_{\text{reflected}}}{Q_{\text{incident}}} = \frac{Q_r}{Q_i}}$$

It is dimensionless quantity.

Relation between Absorptive Power (a) Transmittance (t) and Reflectivity (r)

$\because \quad Q_a + Q_t + Q_r = Q$

Dividing both sides by Q, we get

$$\frac{Q_a}{Q} + \frac{Q_t}{Q} + \frac{Q_r}{Q} = 1$$

$\Rightarrow \quad a + t + r = 1$

For a perfect reflector, $a = t = 0, r = 1$

for a perfect transmitter, $a = r = 0, t = 1$

Black body radiation

A body that absorbs almost all the radiations falling on it is known as a black body. It emits the radiations at the fastest rate. The radiations emitted by a black body is known as black body radiation. The black body is also called as ideal radiator.

Perfectly Black Body

A body that completely absorbs the radiation of all the wavelengths incident on it is called perfectly black body.

Properties of perfectly black body

Properties of perfectly black body are given below

(i) It does not reflect any radiation

$\therefore$ Reflectivity $r = 0$

(ii) It does not transmit any radiation

$\therefore$ Transmitty $t = 0$

(iii) A perfectly black body absorbs all radiations incident on it, the absorptive power of a perfectly black body is maximum and unity, *i.e.*, $a = 1$.

(iv) A perfectly black body does not reflect any wavelength, so it appears black; whatever be the colour of radiations incident on it.

(v) When heated to a suitable high temperature, it emits radiation of all possible wavelengths.

e.g., Sun is at very high temperature ($\approx 6 \times 10^3$ K) it emits all possible radiations so it is an example of black body.

(vi) For perfectly black body $\varepsilon = 1$

Materials like black velvet or lamp black come close to being ideal black bodies, but the best practical realisation of an ideal black body is a small hole leading into a cavity, as this absorbs 98% of the radiation incident on them called ferry's black body.

Kirchhoff's radiation law

According to this law, "the ratio of emissive power (e) to absorptive power (a) is same for all surfaces at the same temperature and is equal to emissive power (E) of a black body at that temperature."

Then, $$\left(\frac{e}{a}\right)_{\text{for any surface}} = \text{constant} = E$$

Similarly for a particular wavelength λ,

$$\boxed{\left(\frac{e_\lambda}{a_\lambda}\right)_{\text{for any body}} = E_\lambda}$$

Here, E_λ = emissive power of black body

Stefan's law

All objects radiate energy continuously in the form of electromagnetic waves. The rate at which an object radiates energy is proportional to the fourth power of its absolute temperature. This is known as the Stefan's law and is expressed in equation form as

$$\boxed{P = \sigma A e T^4}$$

Here P is the power in watts (Js^{-1}) radiated by the object, A is the surface area in m^2, T is the absolute temperature in kelvin (K), e is a fraction between 0 and 1 called the emissivity of the object and σ is a universal constant called Stefan's constant, which has the value

$$\sigma = 5.67 \times 10^{-8}\ Wm^{-2}\text{-}K^{-4}$$

This law is also known as Stefan-Boltzmann Law.

Example 16.36 *A black body at 227°C radiates heat at a rate of 7 cal cm^{-2}s. At a temperature of 727°C, what will be the rate of heat radiated in the same units.*

Sol. According to stefan's law, $E = \sigma T^4$

$\therefore$ $$7 = \sigma(227 + 273)^4 = \sigma \times (500)^4$$

and $$x = \sigma(727 + 273)^4 = \sigma \times (1000)^4$$

Hence, $$\frac{x}{7} = \frac{(1000)^4}{(500)^4} = 16$$

$\Rightarrow$ $$x = 16 \times 7 = 112 \text{ cal cm}^{-2}\text{s}$$

Cooling by radiation

Consider a hot body at temperature T placed in an environment at a lower temperature T_0. The body emits more radiation than it absorbs and cools down while the surroundings absorb radiation from the body and warm up. The body is losing energy by emitting radiations at a rate.

$$P_1 = eA\sigma T^4$$

and is receiving energy by absorbing radiations at a rate

$$P_2 = aA\sigma T_0^4$$

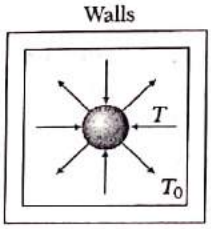


Fig. 16.12 *Diagram showing cooling by radiation*

Here a is a pure number between 0 and 1 indicating the relative ability of the surface to absorb radiation from its surroundings. Note that this a is different from the absorptive power a. In thermal equilibrium, both the body and the surrounding have the same temperature (say T_c) and,

$$P_1 = P_2 \quad \text{or} \quad eA\sigma T_c^4 = aA\sigma T_c^4 \quad \text{or} \quad e = a$$

Thus, when $T > T_0$, the net rate of heat transfer from the body to the surroundings is, $\frac{dQ}{dt} = eA\sigma\ (T^4 - T_0^4)$

or $$mc\left(-\frac{dT}{dt}\right) = eA\sigma\ (T^4 - T_0^4) \quad (\because dQ = mcdT)$$

Rate of cooling,

$$\left(-\frac{dT}{dt}\right) = \frac{eA\sigma}{mc}(T^4 - T_0^4)$$

or $$\boxed{-\frac{dT}{dt} \propto (T^4 - T_0^4)}$$

So, **we can say that rate of cooling of a body by radiation is proportional to the difference of fourth power of temperature of body and its environment.**

Example 16.37 *A cubical block of mass 1.0 kg and edge 5.0 cm is heated to 227°C. It is kept in an evacuated chamber maintained at 27°C. Assuming that the block emits radiation like a blackbody, find the rate at which the temperature of the block will decrease. Specific heat capacity of the material of the block is 400 J/kg-K. ($\sigma = 6 \times 10^{-8}\ Wm^{-2}\text{-}K^4$.)*

Sol. Rate of cooling

$$\frac{dT}{dt} = \frac{e\sigma A}{ms}(T^4 - T_0^4)$$

Given, $\sigma = 1$, Area,

$$A = 6a^2 = 6 \times (5 \times 10^{-2})^2 = 150 \times 10^{-4}\ m^2$$

Temperature, $T = 227 + 273 = 500$ K

$T_0 = 27 + 273 = 300$ K

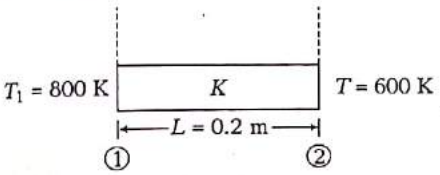
Given, $m = 1$ kg, $s = 400$ J/kg-K

$$= \frac{6 \times 10^{-8} \times 150 \times 10^{-4}[(500)^4 - (300)^4]}{1 \times 400}$$

$$= 2.25 \times 10^{-12} \times (625 - 81) \times 10^8 = 0.12°\text{C/s}$$

Example 16.38 *One end of a rod of length 0.2 m is inserted in a furnace at 800 K. The sides of the rod are covered with an insulating material and the other end emits the radiation like a blackbody. The temperature of this end is 600 K in the steady state. The temperature of the surrounding air is 300 K. Assuming energy is lost only by radiation from the open end of the rod, find the thermal conductivity of the material of the rod. Stefan's constant $\sigma = 6 \times 10^{-8}$ W/m^2-K^4.*

Sol. According to the question,



Energy received per second at end (1)

$$Q_1 = \frac{KA(T_1 - T)}{L} \quad \text{...(i)}$$

Energy radiated per second at end (2)

$$Q_2 = \sigma A(T^4 - T_0^4) \quad \text{...(ii)}$$

In steady state

$$Q_1 = Q_2$$

$$\frac{KA(T_1 - T)}{L} = \sigma A(T^4 - T_0^4)$$

$$K = \frac{\sigma L(T^4 - T_0^4)}{(T_1 - T)}$$

$$= 6 \times \frac{6 \times 10^{-8} \times 0.2[(600)^4 - (300)^4]}{(800 - 600)}$$

$$= \frac{1.2 \times 10^{-8} \times (6^4 - 3^4) \times 10^8}{200}$$

$$= 6 \times 10^{-3}\,(1296 - 81) = 7.29\ \text{W/m°C}$$

Newton's law of cooling

Newton's law of cooling states that the rate of cooling of a body (R) is directly proportional to the temperature difference between the body and its surroundings, provided the temperature difference is small

i.e. Rate of cooling $(R) \propto$ Temperature difference between the body and its surroundings.

$$-\frac{dT}{dt} \propto (T - T_0)$$

$$\boxed{\text{Rate of cooling, } (R) - \frac{dT}{dt} = k(T - T_0)}$$

Where, k is a constant which depends upon surface area and nature of the surface. It can also be written as

$$\left(\frac{T_1 - T_2}{t}\right) = k\left[\left(\frac{T_1 + T_2}{2}\right) - t_0\right]$$

Here T_1 and T_2 are temperatures of the body in time interval t.

Example 16.39 *A body cools in 10 min from 60 °C to 40 °C. What will be its temperature after next 10 min? The temperature of the surroundings is 10 °C.*

Sol. According to Newton's law of cooling,

$$\left(\frac{T_1 - T_2}{t}\right) = k\left[\left(\frac{T_1 + T_2}{2}\right) - T_0\right]$$

For the given conditions,

$$\frac{60 - 40}{10} = k\left[\frac{60 + 40}{2} - 10\right] \quad \text{...(i)}$$

Let T be the temperature after next 10 min. Then,

$$\frac{40 - T}{10} = k\left[\frac{40 + T}{2} - 10\right] \quad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get $T = 28°$C

Example 16.40 *A body cools down from 52.5°C to 47.5°C in 5 min and to 42.5°C in 7.5 min. Find the temperature of the surroundings.*

Sol. Using Newton's law of cooling,

$$\frac{T_1 - T_2}{t} = k\left(\frac{T_1 + T_1}{2} - T_0\right)$$

we get $$\frac{52.5°\text{C} - 47.5°\text{C}}{5\ \text{min}} = k\left(\frac{52.5°\text{C} + 47.5°\text{C}}{2} - T_0\right)$$

or $$\frac{5°\text{C}}{5\ \text{min}} = k(50°\text{C} - T_0) \quad \text{...(i)}$$

and $$\frac{47.5°\text{C} - 42.5°\text{C}}{7.5\ \text{min}} = k\left(\frac{47.5°\text{C} + 42.5°\text{C}}{2} - T_0\right)$$

or $$\frac{5°\text{C}}{7.5\ \text{min}} = k(45°\text{C} - T_0) \quad \text{...(ii)}$$

From (i) and (ii), we get $\dfrac{50°\text{C} - T_0}{45°\text{C} - T_0} = \dfrac{7.5}{5} = \dfrac{3}{2}$

or $$T_0 = 35°\text{C}$$

Cooling curves

(i) **Curve of log $(T - T_0)$ *versus* time**

$$\because \quad \frac{dT}{dt} = -K(T - T_0)$$

$$\Rightarrow \quad \int \frac{dT}{T - T_0} = -\int K dt$$

$$\Rightarrow \quad \log_e(T - T_0) = -Kt + \log_e A$$

where, $\log_e A$ is a constant

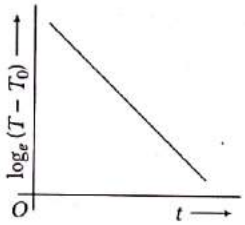


Fig. 16.13

Thus **graph is a straight line with negative slope.** It means rate of cooling of the body decreases with time.

(ii) **Curve of body temperature *versus* time**

We know that temperature of the body at any time t is given by $T = T_0 + (T_i - T_0)e^{-\alpha t}$. From this expression, we can say that $T = T_i$ at $t = 0$ and $T = T_0$ at $t = \infty$, *i.e.*, **temperature of the body varies exponentially with time from T_i to T_0 ($< T_i$).** The temperature *versus* time graph is as shown in figure.

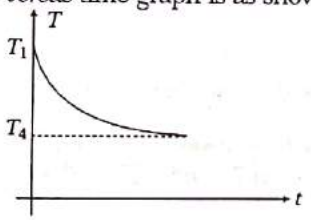


Fig. 16.14

(iii) **Curve of rate of cooling *versus* body temperature**

Rate of cooling $(R) = -\dfrac{dT}{dt} = -K(T - T_0)$

Graph of R *versus* T is a straight line with intercept $-KT_0$ as shown in figure.

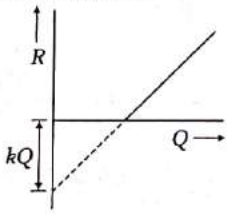


Fig. 16.15

(iv) **Curve of rate of cooling *versus* temperature difference between body temperature (T) and surrounding (T_0).**

Rate of cooling $(R) = -\dfrac{dT}{dt} = K(T - T_0)$

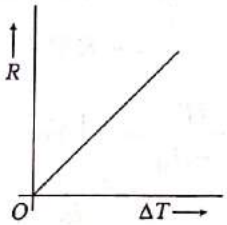


Fig. 16.16

Clearly $R \propto (T - T_0)$

i.e Rate of cooling $(R) \propto$ temperature difference (ΔT)

Therefore, graph between R and ΔT is a straight line passing origin as shown in figure.

Wien's displacement law

At ordinary temperatures (below 600°C) the thermal radiation emitted by a body is not visible, most of it is concentrated in wavelengths much longer than those of visible light as when temperature increases wavelength shift to shorter wavelength. According to Wein's displacement law, **the wavelength corresponding to maximum emission of radiation decreases with increasing temperature.**

$$\lambda_{max} \propto \frac{1}{T}$$

or

$$\boxed{\lambda_{max} T = b}$$

Here, b is a constant called Wien's constant. The value of this constant in SI unit is 2.898×10^{-3} m-K. *Thus, the graph between intensity, I versus λ can be drawn as*

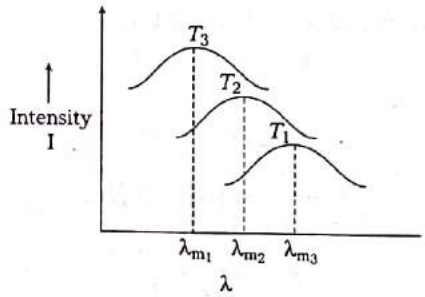


Fig. 16.17

This law is used to find temperature of the sun and stars. For example; in the spectrum of the sun,

$\lambda_m = 4753 \times 10^{-10}$ m

∴ Temperature of the surface of sun

$$T = \frac{b}{\lambda_m} = \frac{2.892 \times 10^{-3}}{4753 \times 10^{-10}} = 6050 \text{ K}$$

Example 16.41 *The spectral energy distribution of the sun has maxima at 4753 Å. Find the temperature of a star for which spectral distribution has maxima at 10350 Å. [Temperature of sun is 6000 K]*

Sol. According to Wien's displacement law

$$\lambda_m \propto \frac{1}{T} \quad \therefore \frac{\lambda_2}{\lambda_1} = \frac{T_1}{T_2} \Rightarrow T_2 = T_1 \frac{\lambda_1}{\lambda_2}$$

$$\Rightarrow \quad T_2 = 6000 \times \frac{4753}{10350}$$

$$\Rightarrow \quad T_2 = 2755.4 \text{ K}$$

Example 16.42 *Two bodies A and B have thermal emissivities of 0.01 and 0.81 respectively. The outer surface areas of the two bodies are same. The two bodies emit total radiant power at the same rate. The wavelength λ_B corresponding to maximum spectral radiancy from B is shifted from the wavelength corresponding to maximum spectral radiancy in the radiation from A by 1.0 μm. If the temperature of A is 5802 K, calculate (i) the temperature of B, (ii) wavelength λ_B.*

Sol. (i) $p_A = p_B$

$$\therefore \quad e_A \sigma A_A T_A^4 = e_B \sigma A_B T_B^4$$

$$\therefore \quad T_B = \left(\frac{e_A}{e_B}\right)^{1/4} T_A \qquad (\text{as } A_A = A_B)$$

Substituting the values

$$T_B = \left(\frac{0.01}{0.81}\right)^{1/4} (5802) = 1934 \text{ K}$$

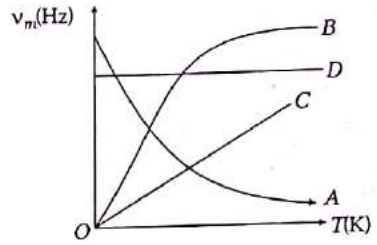
(ii) According to Wein's displacement law,

$$\lambda_A T_A = \lambda_B T_B \Rightarrow \lambda_B = \left(\frac{5802}{1934}\right)\lambda_A \quad \text{or} \quad \lambda_B = 3\lambda_A$$

Also, $\lambda_B - \lambda_A = 1\ \mu\text{m}$

or $\lambda_B - \left(\frac{1}{3}\right)\lambda_B = 1\ \mu\text{m}$ or $\lambda_B = 1.5\ \mu\text{m}$

Example 16.43 *Which of the following is ν_m-T graph for perfectly black body? ν_m is the frequency of radiations with maximum intensity and T is the absolute temperature.*



Sol. According to Wien's displacement law,

$$\lambda_m T = b = \text{Wien's constant}$$

If ν_m is the frequency corresponding to wavelength λ_m, then

$$\left(\frac{C}{\nu_m}\right)T = b$$

or $$\nu_m = \frac{C}{b}T$$

i.e., $$\nu_m \propto T$$

$\therefore \nu_m$-T graph is straight line shown by the curve C.

Check point 16.4

1. In a room containing air, heat can go from one place to another mainly by
   (a) conduction (b) convection
   (c) radiation (d) All the three
2. Ice formed over lakes
   (a) has very high thermal conductivity and helps in further ice formation
   (b) has very low conductivity and retards further formation of ice
   (c) permits quick convection and retards further formation of ice
   (d) is very good radiator
3. Air is bad conductor of heat, still vacuum is preferred between the walls of the thermo flask because
   (a) it is difficult to fill the air between the walls of thermo flask
   (b) due to more pressure of air, the flask can crack
   (c) by convection, heat can flow through air
   (d) None of the above
4. Two thin blankets keep more hotness than one blanket of thickness equal to these two. The reason is
   (a) their surface area increases
   (b) a layer of air is formed between these two blankets which is bad conductor
   (c) these have more wool
   (d) they absrob more heat from outside
5. One likes to sit under sunshine in winter seasons, because
   (a) the air surrounding the body is hot by which body gets heat
   (b) we get heat by radiation from Sun
   (c) we get heat by conduction by Sun
   (d) we get heat by conduction from Sun.
6. A hot and a cold body are kept in vacuum separated from each other. Which of the following causes decrease in temperature of the hot body?
   (a) Radiation (b) Convection
   (c) Conduction (d) Temperature remains same
7. A body, which emits radiations of all possible wavelengths, is known as
   (a) Good conductor (b) Partial radiator
   (c) Absorber of photons (d) Perfectly black body
8. Distribution of energy in the spectrum of a black body can be correctly represented by
   (a) Wein's law (b) Stefan's law
   (c) Planck's law (d) Kirchhoff's law
9. Which of the following law states that "good absorbes of heat are good emmitters"?
   (a) Stefan's law (b) Kirchhoff's law
   (c) Planck's law (d) Wien's law

10. The ratio of the emissive power to the absorptive power of all substances for a particular wavelength is the same at given temperature. The ratio is known as
(a) the emissive power of a perfectly black body
(b) the emissive power of any type of body
(c) the Stefan's constant
(d) the Wein's constant

11. If e_λ and a_λ be the emissive and absorptive power of a body, then according to Kirchhoff's law, which is true
(E_λ = emissive power of perfectly black body)
(a) $e_\lambda = a_\lambda = E_\lambda$
(b) $e_\lambda E_\lambda = a_\lambda$
(c) $e_\lambda = a_\lambda E_\lambda$
(d) $e_\lambda a_\lambda E_\lambda$ = constant

12. There is a black spot on a body. If the body is heated and carried in dark room, then it glows more. This can be explained on the basis of
(a) Newton's law of cooling
(b) Wein's law
(c) Kirchhoff's law
(d) Stefan's law

13. In MKS system, Stefan's constant is denoted by σ. In CGS system multiplying factor of σ will be
(a) 1 (b) 10^3
(c) 10^5 (d) 10^2

14. If a black body radiates 10 calories per second at 227°C, it will radiate at 727°C
(a) 10 cals^{-1} (b) 80 cals^{-1}
(c) 160 cals^{-1} (d) None of these

15. Two spherical black bodies of radii r_1 and r_2 and with surface temperatures T_1 and T_2 respectively radiate the same power. Then, $\frac{r_1}{r_2}$ must be equal to
(a) $\left(\frac{T_1}{T_2}\right)^2$ (b) $\left(\frac{T_2}{T_1}\right)^2$ (c) $\left(\frac{T_1}{T_2}\right)^4$ (d) $\left(\frac{T_2}{T_1}\right)^4$

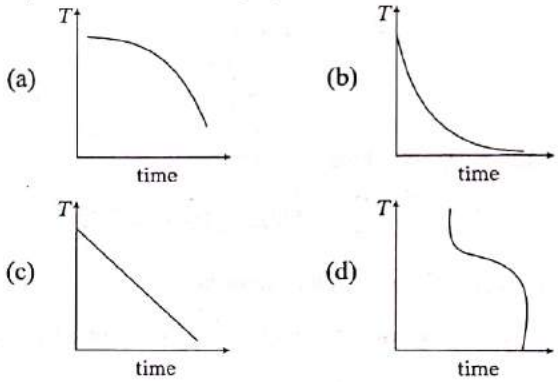
16. A sphere has a surface area of 1.0 m^2 and a temperature of 400 K and the power radiated from it is 150 W. Assuming the sphere is black body radiator, the power in kilowatt radiated when the area expands to 2.0 m^2 and the temperature changes to 800 K.
(a) 6.2 (b) 9.6 (c) 4.8 (d) 16

17. Two spheres of the same material have radii 1 m and 4 m, temperatures 4000 K and 2000 K respectively. Then the ratio of energy radiated per second by the first sphere as compared to that by the second is
(a) 4 : 1 (b) 2 : 1
(c) 1 : 1 (d) 1 : 4

18. The area of the hole of heat furnace is 10^{-4}m^2. It radiates 1.58×10^5 cal of heat per hour. If the emissivity of the furnace is 0.80, then its temperature is
(a) 1500 K (b) 2000 K
(c) 2500 K (d) 3000 K

19. If a body cools down from 80°C to 60°C in 10 min when the temperature of the surrounding is 30°C. Then, the temperature of the body after next 10 min will be
(a) 50°C (b) 48°C
(c) 30°C (d) None of these

20. A block of metal is heated to a temperature much higher than the room temperature and allowed to cool in a room free from air currents. Which of the following curves correctly represents the cooling? (T : Temerature of block)



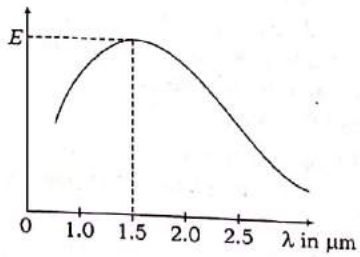
21. If wavelength of maximum intensity of radiation emitted by sun and moon are 0.5×10^{-6} m and 10^{-4} m respectively then the ratio of their temperatures is
(a) 2000 (b) 1000 (c) 100 (d) 200

22. The maximum wavelength of radiations emitted at 900 K is 4 μm. What will be the maximum wavelength of radiations emitted at 1200 K?
(a) 3 μm (b) 0.3 μm
(c) 1 μm (d) None of these

23. The maximum energy in thermal radiation from a source occurs at wavelength 4000 Å. The effective temperature of the source is approximately
(a) 7000 K (b) 80000 K (c) 10^4 K (d) 10^6 K

24. The intensity of radiation emitted by the sun has its maximum value at a wavelength of 510 nm and that emitted by the North star has the maximum value at 350 nm. If these stars behave like black bodies, then the ratio of the surface temperature of the sun and North star is
(a) 1.46 (b) 0.69 (c) 1.21 (d) 0.83

25. In the figure, the distribution of energy density of the radiation emitted by a black body at a given temperature is shown. The possible temperature of the black body is



(a) 1500 K (b) 2000 K (c) 2500 K (d) 3000 K

Chapter exercises

A) Taking it together

(Assorted questions of the chapter for advanced level practice)

1. The temperature of body is increased from 27°C to 127°C. The radiation emitted by it increases by a factor of
(a) $\frac{256}{81}$ (b) $\frac{15}{9}$ (c) $\frac{4}{5}$ (d) $\frac{12}{27}$

2. The calories of heat developed in 200 W heater in 7 min is estimated
(a) 15000 (b) 100 (c) 1000 (d) 20000

3. The thickness of a metallic plate is 0.4 cm. The temperature between its two surface is 20°C. The quantity of heat flowing per second is 50 cal from 5 cm^2 area. In CGS system, the coefficient of thermal conductivity will be
(a) 0.4 (b) 0.6 (c) 0.2 (d) 0.5

4. A spherical black body with radius 12 cm radiates 640 W power at 500 K. If the radius is halved and the temperature doubled, the power radiated in watts would be
(a) 5120 W (b) 640 W
(c) 2560 W (d) 1280 W

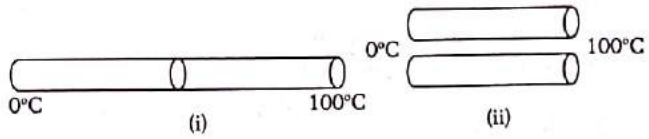
5. Rate of heat flow through a cylindrical rod is H_1. Temperatures of ends of rod are T_1 and T_2. If all the dimensions of rod become double and temperature difference remains same and rate of heat flow becomes H_2. Then
(a) $H_2 = 2H_1$ (b) $H_2 = H_1$
(c) $H_2 = \frac{H_1}{4}$ (d) $H_2 = 4H_1$

6. A sphere, a cube and a thin circular plate are heated to the same temperature. All are made of the same material and have the equal masses. If t_1, t_2 and t_3 are the respective time taken by the sphere, cube and the circular plate in cooling down to a common temperature, then
(a) $t_1 > t_2 > t_3$ (b) $t_1 < t_2 < t_3$
(c) $t_2 > t_1 > t_3$ (d) $t_1 = t_2 = t_3$

7. Certain amount of heat is given to 100 g of copper to increase its temperature by 21°C. If the same amount of heat is given to 50 g of water, then the rise in its temperature is (specific heat capacity of copper = 400 $Jkg^{-1}K^{-1}$ and that for water = 4200 $J\,kg^{-1}K^{-1}$)
(a) 4°C (b) 5.25°C (c) 8°C (d) 6°C

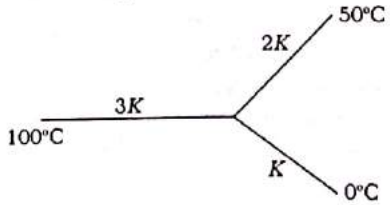
8. 2 g of steam condenses when passed through 40 g of water initially at 25°C. The condensation of steam raises the temperature of water to 54.3°C. What is the latent heat of steam?
(a) 540 cal/g (b) 536 cal/g (c) 270 cal/g (d) 480 cal/g

9. Two identical square rods of metal are welded end to end as shown in Fig. (i), 20 calories of heat flows through it in 4 min. If the rods are welded as shown in Fig. (ii), the same amount of heat will flow through the rods is



(a) 1 min (b) 2 min
(c) 4 min (d) 16 min

10. Three rods of same dimensions have thermal conductivities 3*K*, 2*K* and *K*. They are arranged as shown, with their ends at 100°C, 50°C and 0°C. The temperature of their junction is

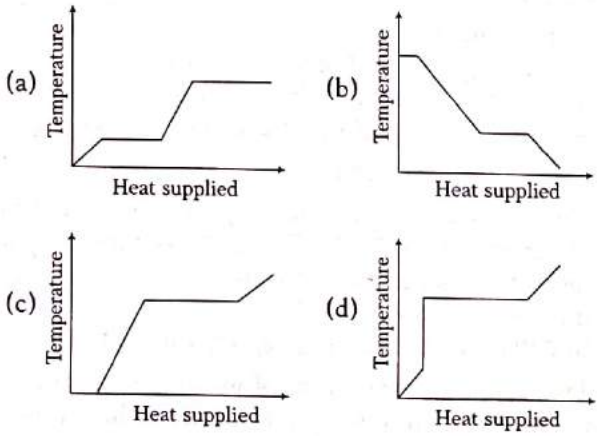


(a) 75°C (b) $\frac{200}{3}$°C (c) 40°C (d) $\frac{100}{3}$°C

11. A wall has two layers A and B each made of different materials. The layer A is 10 cm thick and B is 20 cm thick. The thermal conductivity of A is thrice that of B. Under thermal equilibrium, temperature difference across the wall is 35°C. The difference of temperature across the layer A is
(a) 20°C (b) 10°C (c) 15°C (d) 5°C

12. A body cools from 50°C to 40°C in 5 min. The surrounding temperature is 20°C. In what further time (in minute) will it cool to 30°C?
(a) 5 (b) $\frac{15}{2}$ (c) $\frac{25}{3}$ (d) 10

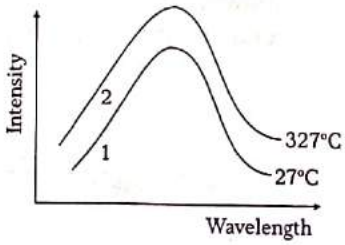
13. A block of ice at −10°C is slowly heated and converted to steam at 100°C. Which of the following curves represent the phenomenon qualitatively?



14. If one kilogram water at 100°C is vaporised in open atmosphere. The correct statement is
(a) increase in internal energy is equal to L (L is latent heat of vaporisation for 1 kg)
(b) increase in internal energy is zero
(c) increase in internal energy is less than L
(d) None of the above

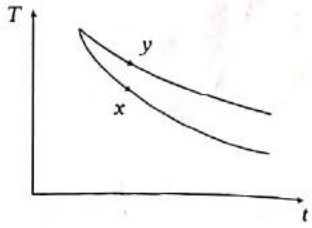
15. A liquid cools from 50°C to 45°C in 5 min and from 45°C to 41.5°C in the next 5 min. The temperature of the surrounding is
(a) 27°C (b) 40.3°C (c) 23.3°C (d) 33.3°C

16. The spectrum of a black body at two temperatures 27°C and 327°C is shown in the figure. Let A_1 and A_2 be the areas under the two curves respectively. The value of A_2/A_1 is



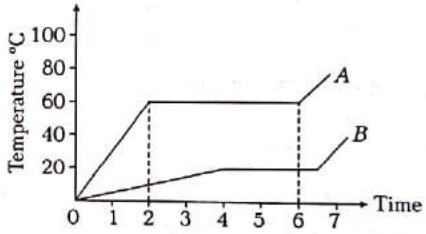
(a) 1 : 16 (b) 4 : 1
(c) 2 : 1 (d) 16 : 1

17. The graph, shown in the adjacent diagram, represents the variation of temperature (T) of two bodies, x and y having same surface area, with time (t) due to the emission of radiation. Find the correct relation between the emissivity (e) and absorptivity (a) of two bodies



(a) $e_x > e_y$ and $a_x < a_y$ (b) $e_x < e_y$ and $a_x > a_y$
(c) $e_x > e_y$ and $a_x > a_y$ (d) $e_x < e_y$ and $a_x < a_y$

18. Two substances A and B of equal mass m are heated at uniform rate of 6 cal s^{-1} under similar conditions. A graph between temperature and time is shown in figure. Ratio of heat absorbed $\frac{H_A}{H_B}$ by them for complete fusion is



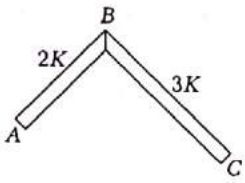
(a) $\frac{9}{4}$ (b) $\frac{4}{9}$ (c) $\frac{24}{5}$ (d) $\frac{5}{8}$

19. Sunrays are allowed to fall on a lens of diameter 20 cm. They are then brought to focus on a calorimeter containing 20 g of ice. If the absorption by the lens is negligible, the time required to melt all the ice is (solar constant = 1.9 cal $min^{-1}cm^{-2}$ and L = 80 $calg^{-1}$)
(a) 6.4 min (b) 3.2 min
(c) 7.2 min (d) 2.7 min

20. If a body coated black at 600 K surrounded by atmosphere at 300 K has cooling rate r_0, the same body at 900 K, surrounded by the same atmosphere, will have cooling rate equal to
(a) $\frac{16}{3}r_0$ (b) $\frac{8}{16}r_0$
(c) $16r_0$ (d) $4r_0$

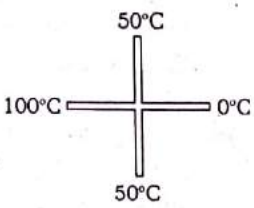
21. A black body radiates power P and maximum energy is radiated by it around a wavelength λ_0. The temperature of the black body is now changed such that it radiates maximum energy around the wavelength $3\lambda_0/4$. The power radiated by it now is
(a) $\frac{256}{81}P$ (b) $\frac{27}{64}P$
(c) $\frac{64}{27}P$ (d) $\frac{81}{256}P$

22. In the figure ABC is a conducting rod whose lateral surfaces are insulated. The length of the section AB is one-half of that of BC, and the respective thermal conductivities of the two sections are as given in the figure. If the ends A and C are maintained at 0°C and 70°C respectively, the temperature of junction B in the steady state is



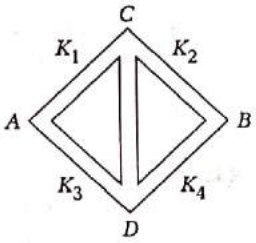
(a) 30°C (b) 40°C (c) 50°C (d) 60°C

23. Two similar rods are joined as shown in figure. Then temperature of junction is (assume no heat loss through lateral surface of rod and temperatures at the ends are shown in steady state)



(a) 50°C (b) 75°C (c) 66.6°C (d) 33.3°C

24. Five rods of same dimensions are arranged as shown in figure. They have thermal conductivities K_1, K_2, K_3, K_4 and K_5. When points A and B are maintained at different temperatures, no heat flows through the central rod if



(a) $K_1 = K_4$ and $K_2 = K_3$
(b) $K_1K_4 = K_2K_3$
(c) $K_1K_2 = K_3K_4$
(d) None of the above

25. Three identical rods A, B and C are placed end to end. A temperature difference is maintained between the free ends of A and C. The thermal conductivity of B is thrice that of C and half of that of A. The effective thermal conductivity of the system will be (K_A is the thermal conductivity of rod A)

(a) $\frac{1}{3}K_A$ (b) $3K_A$ (c) $2K_A$ (d) $\frac{2}{3}K_A$

26. 0.3 kg of hot coffee, which is at 70°C, is poured into a cup of mass 0.12 kg. Find the final equilibrium temperature. Take room temperature as 20°C. (s_{coffee} = 4080 J/kg-K and s_{cup} = 1020 J/kg-K.)

(a) 45.5°C (b) 55.5°C
(c) 65.5°C (d) 40.5°C

27. A calorimeter contains 10 g of water at 20°C. The temperature falls to 15°C in 10 min. When calorimeter contains 20 g of water at 20°C, it takes 15 min for the temperature to become 15°C. The water equivalent of the calorimeter is

(a) 5 g (b) 10 g (c) 25 g (d) 50 g

28. 19 g of water at 30°C and 5 g of ice at −20°C are mixed together in a calorimeter. What is the final temperature of the mixture? Given specific heat of ice = 0.5 cal g^{-1} (°C^{-1}) and latent heat of fusion of ice = 80 cal g^{-1}

(a) 0°C (b) −5°C (c) 5°C (d) 10°C

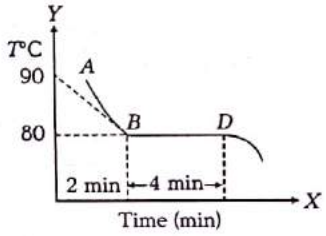
29. Work done in converting one gram of ice at −10°C into steam at 100°C is

(a) 3045 J (b) 6056 J (c) 721 J (d) 616 J

30. Two rigid boxes containing different ideal gases are placed on a table. Box A contains one mole of nitrogen at temperature T_0, while Box B contains one mole of helium at temperature (7/3) T_0. The boxes are then put into thermal contact with each other and heat flows between them until the gases reach a common final temperature (Ignore the heat capacity of boxes). Then, the final temperature of the gases, T_f, in terms of T_0 is

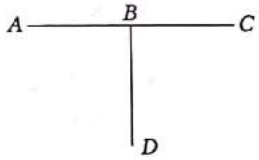
(a) $T_f = \frac{7}{3}T_0$ (b) $T_f = \frac{3}{2}T_0$ (c) $T_f = \frac{5}{2}T_0$ (d) $T_f = \frac{3}{7}T_0$

31. The figure given below shows the cooling curve of pure wax material after heating. It cools from A to B and solidifies along BD. If L and C are respectively values of latent heat and the specific heat of the liquid wax, the ratio L/C is



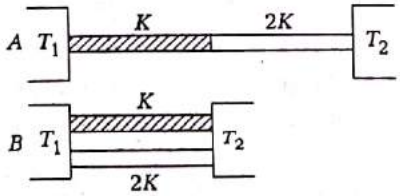
(a) 40 (b) 80 (c) 100 (d) 20

32. Three conducting rods of same material and cross-section are shown in figure. Temperatures of A, D and C are maintained at 20°C, 90°C and 0°C. The ratio of lengths BD and BC if there is no heat flow in AB is



(a) 2/7 (b) 7/2 (c) 9/2 (d) 2/9

33. Two rods with the same dimensions have thermal conductivities in the ratio 1 : 2. They are arranged between heat reservoirs with the same temperature difference, in two different configurations, A and B. The rates of heat flow in A and B are I_A and I_B respectively. The ratio $\frac{I_A}{I_B}$ is equal to



(a) 1 : 2 (b) 1 : 3 (c) 2 : 5 (d) 2 : 9

34. Two identical conducting rods are first connected independently to two vessels, one containing water at 100°C and the other containing ice at 0°C. In the second case, the rods are joined end to end and connected to the same vessels. Let q_1 and q_2 gs^{-1} be the rate of melting of ice in two cases respectively. The ratio of $\frac{q_1}{q_2}$ is

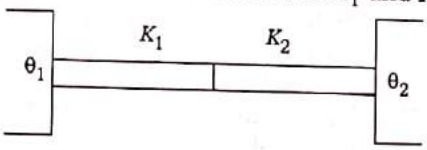
(a) $\frac{1}{2}$ (b) $\frac{2}{1}$ (c) $\frac{4}{1}$ (d) $\frac{1}{4}$

35. Three rods of identical area of cross-section and made from the same metal from the sides of an isosceles triangle *ABC*, right angled at *B*. The points *A* and *B* are maintained at temperatures T and $\sqrt{2}T$ respectively. In the steady state the temperature of the point *C* is T_C.

Assuming that only heat conduction takes place, T_C/T is equal to

(a) $\frac{1}{(\sqrt{2}+1)}$ (b) $\frac{3}{(\sqrt{2}+1)}$ (c) $\frac{1}{2(2\sqrt{2}-1)}$ (d) $\frac{1}{\sqrt{3}(\sqrt{2}-1)}$

36. Two identical rods are made of different materials whose thermal conductivities are K_1 and K_2.

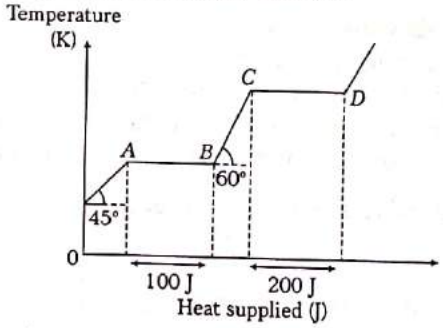


They are placed end to end between two heat reservoirs at temperatures θ_1 and θ_2. The temperature of the junction of the rod is

(a) $\frac{\theta_1+\theta_2}{2}$ (b) $\frac{K_1\theta_1+K_2\theta_2}{K_1+K_2}$

(c) $\frac{K_1\theta_2+K_2\theta_1}{K_1+K_2}$ (d) $\frac{K_1\theta_1+K_2\theta_2}{|K_1-K_2|}$

37. The temperature change *versus* heat supplied curve is given for 1 kg of a solid block. Then, which of the following statement is/are correct?



(a) Specific heat capacity of the solid is 2 J kg^{-1} K^{-1}
(b) Specific heat capacity of liquid phase is $\sqrt{3}$ J kg^{-1} K^{-1}
(c) Latent heat of vaporisation is 100 J kg^{-1}
(d) Latent heat of vaporisation is 200 J kg^{-1}

38. 2 kg of ice at −20°C is mixed with 5 kg of water at 20°C in an insulating vessel having a negligible heat capacity. Calculate the final mass of water remaining in the container. It is given that the specific heats of water and ice are 1 kcal kg^{-1} °C^{-1} and 0.5 kcal kg^{-1} per °C while the latent heat of fusion of ice is 80 kcal kg^{-1}.

(a) 7 kg (b) 6 kg (c) 4 kg (d) 2 kg

39. 10 g of ice cubes at 0°C are released in a tumbler (water equivalent 55 g) at 40°C. Assuming that negligible heat is taken from the surroundings, the temperature of water in the tumbler becomes nearly (L = 80 cal/g)

(a) 31°C (b) 22°C (c) 19°C (d) 15°C

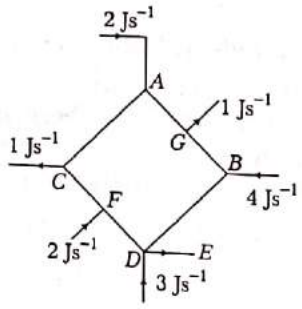
40. A cylindrical metallic rod in thermal contact with two reservoirs of heat at its two ends conducts an amount of heat Q in time t. The metallic rod is melted and the material is formed into a rod of half the radius of the original rod. What is the amount of heat conducted by the new rod when placed in thermal contact with the two reservoirs in time t?

(a) $\frac{Q}{4}$ (b) $\frac{Q}{16}$ (c) $2Q$ (d) $\frac{Q}{2}$

41. If 1 g of steam is mixed with 1 g of ice, then the resultant temperature of the mixture is

(a) 270°C (b) 230°C (c) 100°C (d) 50°C

42. The rate of flow of heat through 12 identical conductors made of same material is as shown in the figure. Then, which of the following is correct?



(a) The rate of flow of heat through rod *DE* is 8 Js^{-1}
(b) Junctions *C* and *F* are at the same temperature
(c) Junction *A* and *G* are at the same temperature
(d) The rate of flow of heat through *CF* is 5 Js^{-1}

43. Equal masses of three liquids *A*, *B* and *C* have temperatures 10°C, 25°C and 40°C respectively. If *A* and *B* are mixed, the mixture has a temperature of 15°C. If *B* and *C* are mixed, the mixture has a temperature of 30°C. If *A* and *C* are mixed, the mixture will have a temperature of

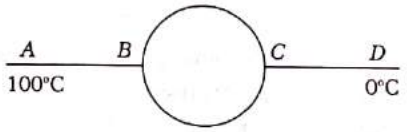
(a) 16°C (b) 20°C
(c) 25°C (d) 29°C

44. In a industrical process 10 kg of water per hour is to be heated from 20°C to 80°C. To do this, steam at 150°C is passed from aboiler into a copper coil immersed in water. The steam condenses in the coil and is returned to the boiler as water at 90°C, how many kg of steam is required per hour.

(Specific heat of steam = 1 cal per g°C, latent heat of vaporisation = 540 cal g^{-1})

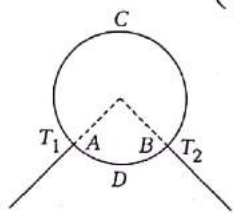
(a) 1 g (b) 1 kg
(c) 10 g (d) 10 kg

45. Two identical conducting rods AB and CD are connected to a circular conducting ring at two diametrically opposite points B and C. The radius of the ring is equal to the length of rods AB and CD. The area of cross-section, thermal conductivity of the rod and ring are equal. Points A and D are maintained at temperatures of 100°C and 0°C. Temperature at point C will be



(a) 62°C (b) 37°C
(c) 28°C (d) 45°C

46. A ring consisting of two parts ADB and ACB of same conductivity K carries an amount of heat H. The ADB part is now replaced with another metal keeping the temperatures T_1 and T_2 constant. The heat carried increases to $2H$. What should be the conductivity of the new ADB part? $\left(\text{Given, } \frac{ACB}{ADB} = 3\right)$.



(a) $\frac{7}{3}K$ (b) $2K$ (c) $\frac{5}{2}K$ (d) $3K$

47. Water is being boiled in a flat bottomed kettle placed on a stove. The area of the bottom is 300 cm^2 and the thickness is 2 mm. If the amount of steam produced is 1 g min^{-1}, then the difference of the temperature between the inner and the outer surfaces of the bottom is (thermal conductivity of the material of the kettle = 0.5 cal cm^{-1}°C^{-1} s^{-1} and latent heat of the steam is equal to 540 cal g^{-1})

(a) 12°C (b) 1.2°C (c) 0.2°C (d) 0.012°C

B) Medical entrance special format questions

Assertion and reason

Directions (Q. Nos. 1-17) *These questions consists of two statements each printed as Assertion and Reason. While answering these question you are required to choose any one of the following five responses.*

(a) If both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If Assertion is false but Reason is true.
(e) If both Assertion and Reason are false.

1. Assertion A body that is a good radiator is also a good absober of radiation at a given wavelength.

Reason According to Kirchhoff's law, the absorptivity of a body is equal to its emissivity at a given wavelength.

2. Assertion All black coloured objects are considered black bodies.

Reason Black colour is a good absorber of heat.

3. Assertion Absorptive power of any substance is temperature independent. But emissive power depends on the temperature.

Reason Emissive power $\propto T^4$.

4. Assertion A normal body cannot radiate energy more than a perfectly black body.

Reason Perfectly black body is always black in colour.

5. Assertion For higher temperature, the peak emission wavelength of a black body shifts to lower wavelengths.

Reason Peak emission wavelength of a blackbody is proportional to the fourth power of temperature.

6. Assertion Temperature near the sea coast are moderate.

Reason Water has a high thermal conductivity.

7. Assertion Air is hotter over the top of a fire than at the same distance on the sides.

Reason Air surrounding the fire conducts more heat upwards.

8. **Assertion** Blue star is at high temperature than red star.

Reason Wein's displacement law states that

$$T \propto \left(\frac{1}{\lambda_m}\right).$$

9. **Assertion** Snow is better insulator than ice.

Reason Snow contain air packet and air is good insulator of heat.

10. **Assertion** Good conductors of electricity are also good conductors of heat.

Reason In good conductors of electricity, there are large numbers of free electrons.

11. **Assertion** If temperature of a body is increased, more number of photons of small wavelengths are radiated.

Reason By increasing the temperature, total energy radiation will increase.

12. **Assertion** Emissive power of a perfectly black body is one.

Reason Absorptive power of perfectly black body is one.

13. **Assertion** If a body is good absorber of green light then it will be good reflector of red light.

Reason At a given temperature, the ratio of emissive power to absorptive power is same for all substances.

14. **Assertion** Water vapours at 100°C will burn you more than water at 100°C.

Reason Heat required to convert total mass of any substance from one state to another state is called latent heat.

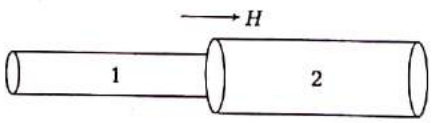
15. **Assertion** Heat required to convert 1 g ice at 0°C into vapour at 100°C is 720 cal.

Reason Conversion of solid state directly into vapour state is called vaporisation.

16. **Assertion** Gravity plays very important role in the process of natural convection.

Reason Convection mainly takes place in liquids and gases.

17. **Assertion** Two conducting rods of same material and same lengths are joined end to end as shown in figure. Heat current H is flowing through them as shown. Temperature difference across rod-1 is more than the temperature difference across rod-2.



Reason Thermal resistance of rod-1 is less compared to rod-2.

Match the columns

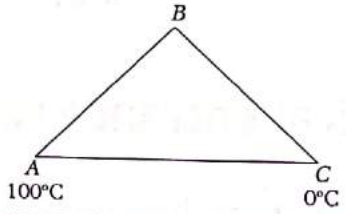
1. Match the following columns.

	Column I		Column II
(A)	Specific heat	(p)	$[MLT^{-3}K^{-1}]$
(B)	Coefficient of thermal conductivity	(q)	$[MT^{-3}K^{-4}]$
(C)	Boltzmann constant	(r)	$[L^2T^{-2}K^{-1}]$
(D)	Stefan's constant	(s)	$[ML^2T^{-2}K^{-1}]$

2. Match the following columns.

	Column I		Column II
(A)	Thermal resistance	(p)	$[MT^{-3}K^{-4}]$
(B)	Stefan's constant	(q)	$[M^{-1}L^{-2}T^3K]$
(C)	Wien's constant	(r)	$[ML^2T^{-3}]$
(D)	Heat current	(s)	[LK]

3. Three rods of equal length of same material are joined to form an equilateral triangle ABC as shown in figure. Area of cross-sectional of rod AB is S, of rod BC is $2S$ and that of AC is S. Then, match the following columns.



	Column I		Column II
(A)	Temperature of junction B	(p)	Greater than 50°C
(B)	Heat current in AB	(q)	Less than 50°C
(C)	Heat current in BC	(r)	Is equal to heat current in BC
		(s)	Is $\frac{2}{3}$ times heat current in AC
		(t)	None

4. Three liquids A, B and C having same specific heat and mass m, $2m$ and $3m$ have temperatures 20°C, 40°C and 60°C respectively. Temperature of the mixture when

	Column I		Column II
(A)	A and B are mixed	(p)	33.3°C
(B)	A and C are mixed	(q)	52°C
(C)	B and C are mixed	(r)	50°C
(D)	A, B and C all three are mixed	(s)	45°C
		(t)	None

Medical entrances' gallery

(Collection of questions asked in NEET & various medical entrance exams)

1. A black body is at a temperature of 5760 K. The energy of radiation emitted by the body at wavelength 250 nm is U_1, at wavelength 500 nm is U_2 and that at 1000 nm is U_3. Wien's constant, $b = 2.88 \times 10^6$ nmK. Which of the following is correct? **[NEET 2016]**

(a) $U_3 = 0$ (b) $U_1 > U_2$ (c) $U_2 > U_1$ (d) $U_1 = 0$

2. A piece of ice falls from a height h so that it melts completely. Only one-quarter of the heat produced is absorbed by the ice and all energy of ice gets converted into heat during its fall. The value of h is (Latent heat of ice is 3.4×10^5 J/kg and $g = 10$ N/kg) **[NEET 2016]**

(a) 544 km (b) 136 km (c) 68 km (d) 34 km

3. The two ends of a metal rod are maintained at temperatures 100°C and 110°C. The rate of heat flow in the rod is found to be 4 Js^{-1}. If the ends are maintained at temperatures 200°C and 210°C, the rate of heat flow will be **[CBSE AIPMT 2015]**

(a) 44 Js^{-1} (b) 16.8 Js^{-1} (c) 8 Js^{-1} (d) 4 Js^{-1}

4. The black body spectrum of an object O_1 is such that its radiant intensity (*i.e.*, intensity per unit wavelength interval) is maximum at a wavelength of 200 nm. Another object O_2 has the maximum radiant intensity at 600 nm. The ratio of power emitted per unit area by source O_1 to that of source O_2 is **[AIIMS 2015]**

(a) 1 : 81 (b) 1 : 9 (c) 9 : 1 (d) 81 : 1

5. Two plates of same area are placed in contact. Their thickness as well as thermal conductivities are in the ratio 2:3. The outer surface of one plate is maintained at 10 °C and that of the other at 0 °C. What is the temperature at the common surface? **[CG PMT 2015]**

(a) 0 °C (b) 2.5 °C (c) 5 °C (d) 6.5 °C

6. For maximum radiant energy, from the moon the corresponding wavelength is 14 micron. If Wien constant is $b = 2892 \times 10^{-6}$ mK, then temperature of the moon is **[CG PMT 2015]**

(a) 207 K (b) 270 K
(c) 207 °C (d) 270 °C

7. A solid maintained at t°_1C is kept in an evacuated chamber at temperature t°_2C ($t_2 >> t_1$). The rate of heat absorbed by the body is proportional to **[WB JEE 2015]**

(a) $t_2^4 - t_1^4$ (b) $(t_2^4 + 273) - (t_1^4 + 273)$
(c) $t_2 - t_1$ (d) $t_2^2 - t_1^2$

8. A black body with surface area 0.001 m^2 is heated upto a temperature 400 K and is suspended in a room temperature 300 K. The initial rate of loss of heat from the body to room is **[UP CPMT 2015]**

(a) 10 W (b) 1 W
(c) 0.1 W (d) 0.5 W

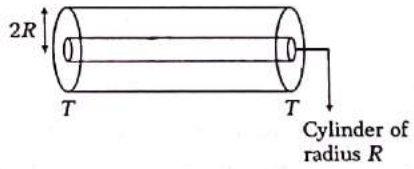
9. Identify the meaningless **[UP CPMT 2015]**

(a) temperature of the surface of a body
(b) the heat of a body
(c) the calorific value of fuel
(d) the heat transferred to a body

10. Water is boiled in flat bottom kettle placed in a stove. The area of the bottom is 3000 cm^2 and the thickness is 2 mm. If the amount of steam produced is 1g min^{-1}. Then, the difference of temperature between inner and outer surfaces of the bottom is [K, for the material of kettle is 0.5 cal°C^{-1} s^{-1}1 cm^{-1}] **[UP CPMT 2015]**

(a) 0.12×10^{-5} K (b) 1.9×10^{-3} K
(c) 1.3×10^{-4} °C (d) 1.2×10^{-3} K

11. A cylinder of radius R of thermal conductivity K_1 is surrounded by a cylindrical shell of inner radius R and outer radius $2R$ made of thermal conductivity K_2. The two ends of the whole system are maintained at two different temperatures T_1 and T_2 ($T_1 < T_2$). The system is in steady state and there is no loss of heat across cylinderical surface. The electrical thermal conductivity of the system is **[CG PMT 2015, KCET 2013]**



(a) $K = K_1 + K_2$ (b) $K = \frac{K_1 K_2}{K_1 + K_2}$

(c) $K = \frac{K_1 + 3K_2}{4}$ (d) $K = \frac{3K_1 + K_2}{4}$

12. On observing light from three different stars P, Q and R, it was found that intensity of violet colour is maximum in the spectrum of P, the intensity of green colour is maximum in the spectrum of R and the intensity of red colour is maximum in the spectrum of Q. If T_P, T_Q and T_R are the respective

absolute temperatures of P, Q and R, then it can be concluded from the above observations that [CBSE AIPMT 2015]

(a) $T_P > T_Q > T_R$ (b) $T_P > T_R > T_Q$
(c) $T_P < T_R < T_Q$ (d) $T_P < T_Q < T_R$

13. A piece of ice of mass 100 g and at temperature 0°C is put in 200 g of water at 25°C. How much ice will melt as the temperature of water reaches 0°C ?
(Latent heat of fusion of ice $= 3.4 \times 10^5$ Jkg^{-1})

(a) 128 g (b) 185.4 g [UK PMT 2015]
(c) 92.8 g (d) 61.8 g

14. A pan filled with hot food cools from 94°C to 86°C in 2 min. When the room temperature is at 20°C, the time taken to cool from 71°C to 69°C is [EAMCET 2015]

(a) 50 s (b) 40 s (c) 38 s (d) 42 s

15. 1 g of ice is mixed with 1 g of steam. At thermal equilibrium, the temperature of the mixture is [KCET 2015]

(a) 50° C (b) 0° C
(c) 55° C (d) 100° C

16. Steam at 100°C is passed into 20 g of water at 10°C. When water acquires a temperature of 80°C, the mass of water present will be [Take, specific heat of water 1 calg^{-1} °C^{-1} and latent heat of steam = 540 calg^{-1}] [CBSE AIPMT 2014]

(a) 24 g (b) 31.5 g (c) 42.5 g (d) 22.5 g

17. A 10 W electric heater is used to heat a container filled with 0.5 kg of water. It is found that the temperature of water and container rises by 3°K in 15 min. The container is then emptied, dried and filled with 2 kg of oil. The same heater now raises the temperature of container oil system by 2 K in 20 min. Assume there is no heat loss in the process and the specific heat of water is 4200 J kg^{-1}K^{-1}, the specific heat of oil in the same limit is equal to

(a) 1.50×10^3 (b) 2.55×10^3 [WB JEE 2014]
(c) 3.00×10^3 (d) 2.10×10^3

18. A block of ice of mass 50 kg is sliding on a horizontal plane. It starts with speed 5 ms^{-1} and stop 3 after moving through some distance. The mass of ice that has melted due to friction between the block and the surface is (assuming that no energy is lost and latent heat of fusion of ice is 80 calg^{-1}, J = 4.2 J cal^{-1}) [WB JEE 2014]

(a) 2.86 g (b) 3.86 g
(c) 0.86 g (d) 1.86 g

19. Same quantity of ice is filled in each of the two metal containers P and Q having the same size, shape and wall thickness but made of different materials. The containers are kept in identical surroundings. The ice in P melts completely in time t_1, whereas in Q takes a time t_2. The ratio of thermal conductivities of the materials of P and Q is [WB JEE 2014]

(a) $t_2 : t_1$ (b) $t_1 : t_2$
(c) $t_1^2 : t_2^2$ (d) $t_2^2 : t_1^2$

20. Two identical rods are connected between two containers. One of them is at 100°C. If rods are connected in parallel, then the rate of melting of ice is g_1 gs^{-1}. If they are connected in series then the rate is g_2 gs^{-1}. The ratio g_2/g_1 is [UK PMT 2014]

(a) 2 (b) 4 (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

21. Two rods of lengths d_1 and d_2 and coefficients of thermal conductivities K_1 and K_2 are kept in end to end contact with each other. The equivalent thermal conductivity is [UK PMT 2014]

(a) $K_1 d_1 + K_2 d_2$ (b) $K_1 + K_2$
(c) $\frac{K_1 d_1 + K_2 d_2}{d_1 + d_2}$ (d) $\frac{d_1 + d_2}{\left(\frac{d_1}{K_1} + \frac{d_2}{K_2}\right)}$

22. Certain quantity of water cools from 70°C to 60°C in the first 5 min and to 54°C in the next 5 min. The temperature of the surrounding is [CBSE AIPMT 2014]

(a) 45°C (b) 20°C
(c) 42°C (d) 10°C

23. A piece of iron is heated in a flame. If at first it becomes dull red, then becomes reddish yellow and finally turns to white hot. The correct explanation for the above observation is possible by using [NEET 2013]

(a) Stefan's law (b) Wien's displacement law
(c) Kirchoff's law (d) Newton's law of cooling

24. The temperature at the two ends A and B of a rod of length 25 cm and circular cross-section are 100°C and 0°C respectively. In the steady state the temperature at a point 10 cm from the end B is (Ignore loss of heat from curved surface of the rod) [EAMCET 2013]

(a) 60° C (b) 80° C
(c) 90° C (d) 40° C

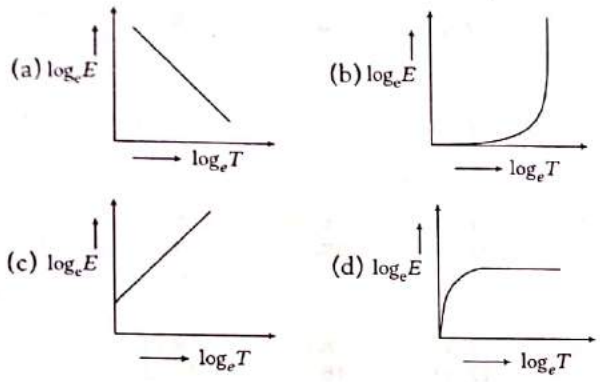
25. Two stars A and B radiate maximum energy at the wavelength of 360 nm and 480 nm respectively. Then the ratio of the surface temperatures of A and B is [KCET 2013]

(a) 3 : 4 (b) 81 : 256
(c) 4 : 3 (d) 256 : 81

26. In a room, where the temperature is 30°C a body cools from 61°C to 59°C in 4 minutes. The time taken by the body to cool from 51°C to 49°C will be [MP PMT 2013]

(a) 4 min (b) 5 min (c) 6 min (d) 8 min

27. The emissive power of a body at temperature T(K) is E. Then, the graph between $\log_e E$ and $\log_e T$ will be **[MP PMT 2013]**



28. When 500 kg of water is heated from 20°C to 100°C, then the increase in mass of the water will be **[UP CPMT 2013]**

(a) 3.2×10^{-9} kg (b) 1.87×10^{-9} kg
(c) 0.96×10^{-9} kg (d) 2.8×10^{-9} kg

29. To cool the car engines in radiators, water is used because it is having **[UP CPMT 2012]**

(a) high value of specific heat
(b) high density
(c) low surface tension
(d) low density

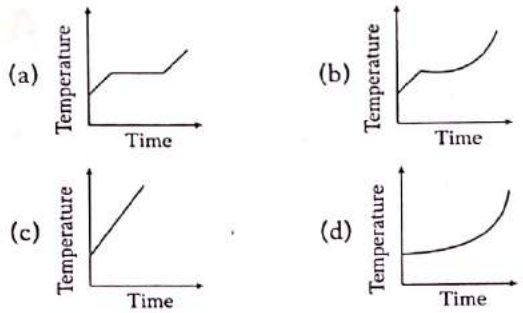
30. A body cools from 60°C to 50°C in 10 min. If room temperature is 95°C. Temperature of body at the end of next 10 min, will be **[UP CPMT 2012]**

(a) 38.5°C (b) 40°C
(c) 45°C (d) 42.85°C

31. If the radius of a star is R and it acts as a black body, what would be the temperature of the star, in which the rate of energy production is Q? **[CBSE AIPMT 2012]**

(a) $Q/4\pi R^2\sigma$
(b) $(Q/4\pi R^2\sigma)^{-1/2}$
(c) $(4\pi R^2 Q/\sigma)^{1/4}$
(d) $(Q/4\pi R^2\sigma)^{1/4}$
(σ stands for stefan's constant)

32. Liquid oxygen at 50 K is heated to 300 K at constant pressure of 1 atm. The rate of heating is constant. Which one of the following graphs represents the variation of temperature with time? **[CBSE AIPMT 2012]**



33. The temperature at which a black body of unit area losses its energy at the rate of 1 Js^{-1} will be **[BHU 2012]**

(a) 65°C (b) - 65°C (c) 65 K (d) 72 K

34. A rod AB is 1m long. The temperature of its one end A is maintained at 100°C and other end B at 10°C, the temperature at a distance of 60 cm from point B is **[AIIMS 2012]**

(a) 64°C (b) 36°C (c) 46°C (d) 72°C

35. Two slabs A and B of different materials but of the same thickness are joined end to end to form a composite slab. The thermal conductivities of A and B are K_1 and K_2 respectively. A steady temperature difference of 12°C is maintained across the composite slab. If $K_1 = \frac{K_2}{2}$, the temperature difference across slabs A is **[AIIMS 2012]**

(a) 4°C (b) 6°C (c) 8°C (d) 10°C

36. A piece of blue glass heated to a high temperature and a piece of red glass at room temperature, are taken inside a dimly lit room, then **[AFMC 2012]**

(a) the blue piece will look blue and red will look as usual
(b) red looks brighter red and blue looks ordinary blue
(c) blue shines like brighter red compared to the red piece
(d) Both the pieces will look equally red

37. The temperature gradient in a rod of 0.5 m long is 80°Cm^{-1}. If the temperature of hotter end of the rod is 30°C, then the temperature of the cooler end is **[JCECE 2012]**

(a) 0°C (b) – 10°C (c) 10°C (d) 40°C

38. The thickness of a metallic plate is 0.4 cm. The temperature between its two surfaces is 20°C. The quantity of heat flowing per second is 50 calories from 5 cm^2 area. The coefficient of thermal conductivity (in CGS system) will be **[BCECE (Mains) 2012]**

(a) 0.2 (b) 0.3 (c) 0.4 (d) 0.5

Answers

Check point 16.1

1. (d)	2. (c)	3. (a)	4. (b)	5. (a)	6. (d)	7. (b)	8. (a)	9. (d)	10. (b)

Check point 16.2

1. (b)	2. (a)	3. (a)	4. (b)	5. (a)	6. (c)	7. (a)	8. (d)	9. (c)	10. (b)

Check point 16.3

1. (d)	2. (c)	3. (a)	4. (d)	5. (a)	6. (d)	7. (d)	8. (d)	9. (a)	10. (c)
11. (c)	12. (b)	13. (d)	14. (b)	15. (c)	16. (a)	17. (b)	18. (c)	19. (b)	20. (d)

Check point 16.4

1. (b)	2. (b)	3. (c)	4. (b)	5. (b)	6. (a)	7. (d)	8. (c)	9. (b)	10. (a)
11. (c)	12. (c)	13. (b)	14. (c)	15. (b)	16. (c)	17. (c)	18. (c)	19. (d)	20. (b)
21. (d)	22. (a)	23. (a)	24. (b)	25. (b)					

A. Taking it together

1. (a)	2. (d)	3. (c)	4. (c)	5. (a)	6. (a)	7. (a)	8. (a)	9. (a)	10. (b)
11. (d)	12. (c)	13. (a)	14. (c)	15. (d)	16. (d)	17. (c)	18. (c)	19. (d)	20. (a)
21. (a)	22. (a)	23. (a)	24. (b)	25. (a)	26. (c)	27. (b)	28. (c)	29. (a)	30. (b)
31. (d)	32. (b)	33. (d)	34. (c)	35. (b)	36. (b)	37. (d)	38. (b)	39. (b)	40. (b)
41. (c)	42. (b)	43. (a)	44. (b)	45. (c)	46. (a)	47. (d)			

B. Medical entrance special format questions

Assertion and reason

1. (a)	2. (d)	3. (a)	4. (e)	5. (b)	6. (c)	7. (c)	8. (a)	9. (a)	10. (a)
11. (b)	12. (d)	13. (d)	14. (c)	15. (c)	16. (b)	17. (c)			

Match the columns

1. (A → r, B → p, C → s, D → q)
2. (A → q, B → p, C → s, D → r)
3. (A → p, B → r, C → t)
4. (A → p, B → r, C → q, D → t)

C. Medical entrances' gallery

1. (c)	2. (b)	3. (d)	4. (d)	5. (c)	6. (a)	7. (c)	8. (b)	9. (b)	10. (d)
11. (c)	12. (b)	13. (d)	14. (d)	15. (d)	16. (d)	17. (b)	18. (d)	19. (a)	20. (d)
21. (d)	22. (a)	23. (b)	24. (d)	25. (c)	26. (b)	27. (c)	28. (b)	29. (a)	30. (d)
31. (d)	32. (a)	33. (c)	34. (a)	35. (c)	36. (c)	37. (b)	38. (a)		

Hints and explanations

Check point 16.1

1. (*d*) Due to large specific heat of water, it releases large heat with very small temperature change.

2. (*c*) $Q = mcQT \Rightarrow c = \frac{Q}{m\Delta T}$

when, $\Delta T = 0 \Rightarrow c = \infty$

3. (*a*) Heat energy gained

$$Q = mc\,\Delta T = 5 \times (1000 \times 4.2) \times (100 - 20)$$
$$= 1680 \times 10^3 \text{ J} = 1680 \text{ kJ}$$

4. (*b*) Heat required

$$Q = m_1c_1\Delta T + m_2c_2\Delta T + m_3c_3\Delta T$$
$$= (2.4 \times 0.216 + 1.6 \times 0.0917 + 0.8 \times 0.0931)\,(60)$$
$$= 44.376 \text{ cal} \approx 44.44 \text{ cal}$$

5. (*a*) Same amount of heat is supplied to copper and water.

So $\quad m_Cc_C\Delta T_C = m_Wc_W\Delta T_W$

$$\Rightarrow \quad \Delta T_W = \frac{m_Cc_C(\Delta T)_C}{m_Wc_W}$$
$$= \frac{50 \times 10^{-3} \times 420 \times 10}{10 \times 10^{-3} \times 4200} = 5°\text{C}$$

6. (*d*) $n = 8$ mole, $\Delta t = 30°\text{C}$

$$Q = nc_p\Delta T \Rightarrow Q = 8 \times \frac{5}{2} \times 8.31 \times 30 \approx 5000.$$

7. (*b*) Heat lost by hot water = Heat gained by cold water in beaker + Heat absorbed by beaker

$$\Rightarrow \quad 440(92 - T) = 200 \times (T - 20) + 20 \times (T - 20)$$
$$\Rightarrow \quad T = 68°\text{C}$$

8. (*a*) Heat lost = Heat gained

$\therefore (100 \times 80) + (100 \times 1 \times T) = 100 \times 1 \times (100 - T)$

or $\quad T = 10°\text{C}$

9. (*d*) Temperature of mixture,

$$\theta_{mix} = \frac{m_1c_1T_1 + m_2c_2T_2}{m_1c_1 + m_2c_2} = \frac{m \times c \times 2T + \frac{m}{2}(2c)\,T}{m \times c + \frac{m}{2}(2c)} = \frac{3}{2}T$$

10. (*b*) Density of water $= 10^3 \text{ kg m}^{-3}$

Let the final temperature of the mixture be t.

Assuming no heat transfer to or from container.

Heat lost by water $= 0.1 \times 10^3 \times S_{water} \times (80 - t)$

Heat gained by water $= 0.3 \times 10^3 \times S_{water}\,(t - 60)$

According to principle of calorimetry

Heat lost = heat gain

$$0.1 \times 10^3 \times S_{water} \times (80 - t) = 0.3 \times 10^3 \times S_{water} \times (t - 60)$$
$$\Rightarrow \quad 1 \times (80 - t) = 3 \times (t - 60) \Rightarrow t = 65°\text{C}$$

Check point 16.2

1. (*b*) Heat lost in t second $= mL$

or heat lost per second $= \frac{mL}{t}$

This must be the heat supplied for keeping the substance in matter state per second

$$\therefore \quad \frac{mL}{t} = P \quad \text{or} \quad \frac{Pt}{m}$$

2. (*a*) Heat liberated when 50 g of water at 60°C converts into water at 0°C.

$$Q = ms\Delta\theta = 50 \times 1 \times 60 = 3000 \text{ cal}$$

From this heat, mass of ice which can be melted is

$$m = \frac{Q}{L} = \frac{3000}{80} = 37.5 \text{ g}$$

i.e., whole ice is not melted and temperature of mixture will be 0°C.

3. (*a*) If m g ice melts, then

Heat lost = Heat gain

$$80 \times 1 \times (30 - 0) = m \times 80 \Rightarrow m = 30 \text{ g}$$

4. (*b*) Heat taken = Heat given

or $\quad mc\Delta\theta = mL$

$\therefore \quad (200)\,(s)\,(100) = 25 \times 80$

$\therefore \quad c = 0.1$ cal/g°C

5. (*a*) Heat required,

$$Q_1 = (1.1 + 0.02) \times 10^3 \times (80 - 15) = 72800 \text{ cal}$$

Suppose m kg of steam is condensed and it also reaches to 80°C. Then heat rejected,

$$Q_2 = mL + mc\Delta\theta$$
$$= m(540 \times 10^3 + 20 \times 10^3)$$
$$= 560 \times 10^3 \text{ m}$$

Equating Q_1 and Q_2 we get,

$$m = 0.13 \text{ kg}$$

6. (*c*) Heat absorbed, $mc\Delta T = \frac{1}{2}\left(\frac{1}{2}mv^2\right)$

$\therefore \quad \Delta T = \frac{v^2}{4c} = \frac{(300)^2}{4 \times 150} = 150°C$

7. (*a*) Slope $\propto \frac{1}{\text{(Heat capacity)}}$

8. (*d*) $\Delta Q = mc\Delta T$

Slope of inclined line $\frac{\Delta Q}{\Delta T} = \frac{1}{mc} = \frac{1}{\text{Thermal capacity}}$

9. (*c*) From $t = 0$ to $t = 1$ min

$\Delta Q = ms\,\Delta T = \frac{50}{100} \times 0.6 \times 1000 \times (50 \times 0) = 1500$ cal

10. (*b*) Heat released when 5 kg of water at 15°C converts into 0°C of water is, $Q = mc\Delta\theta = (5000)(1)(15) = 75000$ cal. This heat can melt mass of water $m = \frac{Q}{L} = \frac{75000}{80} = 937.5$ g or 0.9375 kg. Since, whole of ice is not melting, temperature of mixture will be 0°C.

Check point 16.3

1. (*d*) Radiation from sun and convection currents, are responsible for heating.

2. (*c*) Mud is bad conductor of heat. So it prevents the flow of heat between surrounding and inside.

3. (*a*) Snow contains air pockets.

4. (*d*) $\frac{dQ}{dT} = -KA\frac{d\theta}{dx}$; when $K = \infty$, $\frac{d\theta}{dx} = 0$

i.e., θ is independent of x, *i.e.*, constant or uniform.

5. (*a*) Thermal conductivity is independent of temperatures of the wall, it is a constant for the material, so it will remain unchanged.

6. (*d*) Thermal conducting is a material property.

7. (*d*) $Q = \frac{KA(\theta_1 - \theta_2)t}{l}$

$\therefore \quad K = \frac{Q \times l}{A(\theta_1 - \theta_2)t}$

Unit of $K = \frac{\text{Jm}}{\text{m}^2\text{K-s}} = \frac{\text{J}}{\text{s-mK}}$

8. (*d*) Rate of heat flow, $\frac{Q}{t} = \frac{KA\Delta\theta T}{l}$

$\Rightarrow \quad \frac{K_A}{K_B} = \frac{A_B}{A_A} = \left(\frac{r_B}{r_A}\right)^2 = \frac{1}{4} \Rightarrow K_A = \frac{K_B}{4}$

9. (*a*) $\because \quad \frac{dQ}{dt} = \frac{K(\pi r^2)\Delta\theta}{l}$

$\Rightarrow \quad \frac{\left(\frac{dQ}{dt}\right)_2}{\left(\frac{dQ}{dt}\right)_1} = \frac{K_2 \times r_2^2 \times l_1}{K_1 \times r_1^2 \times l_2} = \frac{1}{2} \times \frac{1}{4} \times \frac{2}{1}$

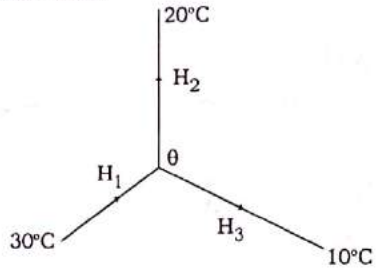
$\Rightarrow \quad \left(\frac{dQ}{dt}\right)_2 = \frac{\left(\frac{dQ}{dt}\right)_1}{4} = \frac{4}{4} = 1$

10. (*c*) Rate of heat transfer,

$\frac{Q_1}{t} = \frac{KA(90 - 60)}{0.6} = 50\,KA$

and $\frac{Q_2}{t} = \frac{KA(150 - 110)}{0.8} = 50\,KA$

11. (*c*) Let θ be the temperature of junction and H_1, H_2 and H_3 the heat currents. Then



$H_1 = H_2 + H_3$

or $\frac{30 - \theta}{(30/KA)} = \frac{\theta - 20}{(20/KA)} + \frac{\theta - 10}{(10/KA)}$

or $2(30 - \theta) = 3(\theta - 20) + 3(2\theta - 20)$

or $\theta = 16.36°\,C \approx 16.4°C$

12. (*b*) $\because \quad H_1 = H_2$

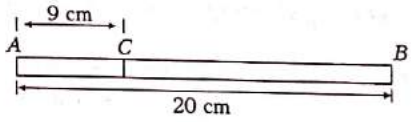
$\therefore \quad \left(\frac{TD}{R}\right)_1 = \left(\frac{TD}{R}\right)_2$ (where TD = Temperature difference)

or $R_1 = R_2$

$\therefore \quad \frac{l}{K_1A_1} = \frac{l}{K_2A_2}$ or $K_1A_1 = K_2A_2$

13. (*d*) $K_{eq} = \frac{2K_1K_2}{K_1 + K_2} = \frac{2(K)(2K)}{K + 2K} = \frac{4}{3}K$

14. (*b*) $H = \frac{100 - 0}{R_{AB}} = \text{Heat current} = \frac{100}{R_{AB}}$



$100 - \theta_C = H \cdot R_{AC} = \frac{100}{R_{AB}} \cdot R_{AC} = \frac{100 \times 9}{20} = 45$

$\therefore \quad \theta_C = 55°C$

15. (*c*) $\because \quad R_{PR} + R_{RQ} = R_{PQ}$

$\therefore \quad \frac{l}{K_1A} + \frac{l}{K_2A} = \frac{l}{K_3A}$ or $K_3 = \frac{K_1K_2}{K_1 + K_2}$

16. (*a*) $K_C > K_B$

$\therefore \quad R_C < R_B$

Temperature difference across copper $= HR_C = (TD)_C$

And temperature difference across brass $= HR_B = (TD)_B$

Heat current H will be same in both, as they are in series.

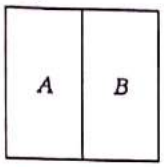
Since $R_C < R_B$

$\therefore \quad (TD)_C < (TD)_B$

[illegible] of junction is more than 50°C

17. (*b*) $\because$ $K_A = 2K_B$

$\therefore$ $$R_A = \frac{R_B}{2} \qquad \left(R = \frac{l}{KA}\right)$$



Suppose, $R_A = R$, then $R_B = 2R$

Heat current, $$H = \frac{36}{R + 2R} = \frac{36}{3R} = \frac{12}{R}$$

$\therefore$ Temperature difference across $A = H \times R_A = \frac{12}{R} \times R = 12°C$

18. (*c*) Let T be temperature of middle point C and in series rate of heat flow is same

$\Rightarrow$ $K(2A)(100 - T) = KA(T - 70)$

$\Rightarrow$ $200 - 2T = T - 70 \Rightarrow 3T = 270 \Rightarrow T = 90°\,C$

19. (*b*) $(K_{eq})_{11} = \frac{k_1 + k_2}{2}$

20. (*d*) Let the heat transferred be Q.

When rods are joined end to end. Heat transferred by each rod

$$Q = \frac{K(2A)\,\Delta\theta}{l} \times 12 \qquad ...(i)$$

When rods are joined lengthwise,

$$Q = \frac{KA\Delta\theta}{2l}\,t \qquad ...(ii)$$

As the same amount of heat is transferred

$\therefore$ From Eqs. (i) and (ii), we get $t = 48$ s

Check point 16.4

1. (*b*) In gases conduction, is most prominent mode.

2. (*b*) Ice is non-conducting.

3. (*c*) No flow of heat by convection in vacuum.

5. (*b*) Heat is received from sun by radiation

6. (*a*) Radiation occurs without any medium.

7. (*d*) Black bodies emits all possible wavelengths.

8. (*c*) Planck's law explains the distribution of energy correctly at low temperature as well as at high temperature.

10. (*a*) According to Kirchhoff's law,

$$\left(\frac{e_\lambda}{a_\lambda}\right)_{\text{Any body}} = \left(\frac{e_\lambda}{a_\lambda}\right)_{\text{Perfectly black body}} = E_\lambda$$

where E_λ is emissive power of a perfectly black body.

11. (*c*) According to Kirchhoff's law, the ratio of emissive power to absorptive power is same for all bodies, which is equal to the emissive power of a perfectly black body, *i.e.*,

$$\left(\frac{e}{a}\right)_{\text{body}} = E_{\text{Black body}} \text{ for a particular wavelength}$$

$$\left(\frac{e_\lambda}{a_\lambda}\right)_{\text{body}} = (E_\lambda)_{\text{Black body}} \Rightarrow e_\lambda = a_\lambda E_\lambda$$

12. (*c*) According to Kirchhoff's law, a good emitter is also a good absorber.

13. (*b*) In MKS system, unit of σ is $\frac{\text{J}}{\text{m}^2 \times \text{sec} \times \text{K}^4}$

$$\Rightarrow \frac{\text{J}}{\text{m}^2 \times \text{sec} \times \text{K}^4} = \frac{10^7 \text{ erg}}{10^4 \text{ cm}^2 \times \text{sec} \times \text{K}^4} = 10^3 \frac{\text{erg}}{\text{cm}^2 \times \text{sec} \times \text{K}^4}$$

14. (*c*) As, $E \propto T^4$,

$T_1 = 500$ K and $T_2 = 1000$ K $\Rightarrow T_2 = 2T_1$

$\therefore$ $E_2 = (2)^4 E_1 = 16E_1 = 160 \text{ cals}^{-1}$

15. (*b*) According to Stefan's law,

$$4\pi r_1^2 T_1^4 = 4\pi r_2^2 T_2^4$$

$$\therefore \quad \frac{r_1}{r_2} = \frac{T_2^2}{T_1^2}$$

16. (*c*) $P \propto AT^4$

Area and temperature both are doubled. Hence, power will become $(2)^5$ or 32 times.

$$\therefore \quad P' = 32P = \frac{32 \times 150}{1000} \text{ kW} = 4.8 \text{ kW}$$

17. (*c*) $\frac{P_1}{P_2} = \frac{A_1T_1^4}{A_2T_2^4} = \frac{4\pi\,(1)^2\,(4000)^4}{4\pi\,(4)^2\,(2000)^4} = \frac{1}{1}$

or $P_1 = P_2$

18. (*c*) According to Stefan's law, $E = \sigma\varepsilon AT^4$

$$\Rightarrow \frac{1.58 \times 10^5 \times 4.2}{60 \times 60} = 5.67 \times 10^{-8} \times 10^{-4} \times 0.8 \times T^4$$

$$T = 2500 \text{ K}$$

19. (*b*) Applying, $\frac{\theta_1 - \theta_2}{t} = \alpha\left[\frac{\theta_1 + \theta_2}{2} - \theta_0\right]$ two times we get,

where α is constant.

$$\frac{80 - 60}{10} = \alpha\left[\frac{80 + 60}{2} - 30\right] \qquad ...(i)$$

$$\frac{60 - \theta}{10} = \alpha\left[\frac{60 + \theta}{2} - 30\right] \qquad ...(ii)$$

Solving these two equations, we get

$$\theta = 48°C$$

21. (*d*) According to Wein's law, $\lambda_m T = \text{constant}$

$\therefore$ $(\lambda_m T)_s = (\lambda_m T)_m$

or $$\frac{T_s}{T_m} = \frac{(\lambda_m)_m}{(\lambda_m)_s} = \frac{10^{-4}}{0.5 \times 10^{-6}} = 200$$

22. (*a*) As, $\frac{\lambda_2}{\lambda_1} = \frac{T_1}{T_2} \Rightarrow \lambda_2 = \frac{T_1}{T_2} \times \lambda_1 = \frac{900}{1200} \times 4 = 3\,\mu\text{m}$

23. (*a*) Applying Wein's law, we have, $\lambda_m = \frac{b}{T}$

$$\Rightarrow \quad T = \frac{b}{\lambda_m} = \frac{2.89 \times 10^{-3}}{4000 \times 10^{-10}}$$

$$= 7225 \text{ K} \approx 7000 \text{ K}$$

24. (*b*) From Wein's law,

$$\frac{T_S}{T_N} = \frac{(\lambda_N)_{max}}{(\lambda_S)_{max}} = \frac{350}{510} = 0.69$$

25. (*b*) $\lambda_m T = b$, where $b = 2.89 \times 10^{-3}$ mK

$$\Rightarrow \quad T = \frac{b}{\lambda_m} = \frac{2.89 \times 10^{-3}}{1.5 \times 10^{-6}} = 2000 \text{ K}$$

A) Taking it together

1. (*a*) $E \propto T^4$, $T_1 = 300$ K, $T_2 = 400$ K

$$\therefore \quad \frac{E_2}{E_1} = \left(\frac{T_2}{T_1}\right)^4 = \left(\frac{400}{300}\right)^4 = \frac{256}{81}$$

2. (*d*) $H = \frac{200 \times 7 \times 60}{4.18}$ cal $= 20095.7 \approx 20000$ cal

3. (*c*) Heat flowing per second,

$$\frac{Q}{t} = \frac{KA(\Delta T)}{l} \Rightarrow 50 = \frac{5 \times 20K}{0.4} \Rightarrow \quad K = \frac{1}{5} = 0.2$$

4. (*c*) Ratio of powers, $\frac{P_1}{P_2} = \frac{A_1 T_1^4}{A_2 T_2^4}$

$$P_2 = \frac{A_2}{A_1}\left(\frac{T_2}{T_1}\right)^4 P_1 = \left(\frac{1}{2}\right)^2 (2)^4 (640) = 2560 \text{ W}$$

5. (*a*) Rate of heat flow, $H = \frac{Q}{t} = \frac{KA(T_1 - T_2)}{l}$

or $\quad H \propto \frac{A}{l}$ or $H_2 = 2H_1$

6. (*a*) A body having maximum surface area will cool at fastest rate.

7. (*a*) The amount of heat supplied is given by the relation

$$Q = mc\Delta T$$

Here, $\quad m = 100$ g $= 0.1$kg, $c = 400$ J/kg-K,

$\Delta T = 21$K for copper.

Thus, $\quad Q = 0.1 \times 400 \times 21 = 840$ J

Hence, $\quad 840 = 0.05 \times 4200 \times \Delta T \Rightarrow \Delta T = 4°$C

8. (*a*) Let L be the latent heat and using principle of calorimetry.

$$2L + 2(100 - 54.3) = 40 \times (54.3 - 25.3)$$

$$\Rightarrow \quad L = 540.3 \text{ cal/g.}$$

9. (*a*) Rate of heat flow, $\frac{Q}{t} = \frac{KA\Delta T}{l} = \frac{\Delta T}{(l/KA)} = \frac{\Delta T}{R}$

(R = Thermal resistance)

$\Rightarrow \quad t \propto R \quad$ ($\because Q$ and ΔT are same)

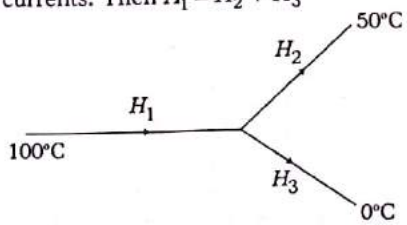
$$\Rightarrow \quad \frac{t_p}{t_s} = \frac{R_p}{R_s} = \frac{R/2}{2R} = \frac{1}{4}$$

$$\Rightarrow \quad t_p = \frac{t_s}{4} = \frac{4}{4} = 1\text{min.}$$

(Series resistance, $R_s = R_1 + R_2$ and parallel resistance,

$$R_p = \frac{R_1 R_2}{R_1 + R_2})$$

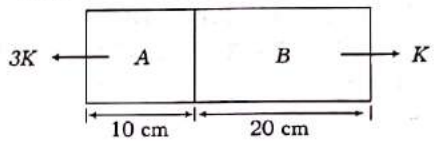
10. (*b*) Suppose T be the temperature of junction. H_1, H_2 and H_3 are heat currents. Then $H_1 = H_2 + H_3$



or $\frac{100 - T}{(l/3KA)} = \frac{T - 50}{(l/2KA)} + \frac{T - 0}{(l/KA)}$ or $3(100 - T) = 2(T - 50) + T$

Solving we get, $\quad T = \frac{200}{3}$°C

11. (*d*) $R_A = \frac{10}{3KA}$ $\quad \left(\because R = \frac{l}{KA}\right)$



$$R_B = \frac{20}{KA} \Rightarrow \frac{R_A}{R_B} = \frac{1}{6}$$

So let $\quad R_A = R$, then $R_B = 6R$

Heat current, $H = \frac{\text{Temperature difference}}{\text{Total resistance}} = \frac{35}{7R} = \frac{5}{R}$

Now temperature difference across A

$$= H \times R_A = \frac{5}{R} \times R = 5°\text{C}$$

12. (*c*) Applying, $\frac{T_1 - T_2}{t} = \alpha\left[\frac{T_1 - T_2}{2} - T_0\right]$ two times we have,

or $\quad \frac{50 - 40}{5} = \alpha\left[\frac{50 + 40}{2} - 20\right]$ ...(i)

and $\quad \frac{40 - 30}{t} = \alpha\left[\frac{40 + 30}{2} - 20\right]$ ...(ii)

Solving these two equations, we get

$$t = \frac{25}{3} \text{ min}$$

13. (*a*) Temperature first increases from −10°C to 0°C. Then remains constant at 0°C till whole ice is melted. Then it increases from 0°C to 100°C. Again it remains constant at 100°C till whole water converts into steam.

14. (*c*) Heat of vaporisation in given by,

$$Q = mL = L \quad (\because m = 1\text{kg})$$

$$\Delta U = Q - W = L - W$$

i.e., $\quad \Delta U < L$

15. (*d*) Applying $\frac{\theta_1 - \theta_2}{t} = \alpha\left[\frac{\theta_1 + \theta_2}{2} - \theta_0\right]$ two times, we get

where α is constant.

$$\frac{50-45}{5} = \alpha\left[\frac{50+45}{2} - \theta_0\right] \quad ...(i)$$

$$\frac{45-41.5}{5} = \alpha\left[\frac{45+41.5}{2} - \theta_0\right] \quad ...(ii)$$

Solving these two equations, we get

θ_0 = temperature of atmosphere $= 33.3°C$

16. (*d*) Area $\propto$ Total energy radiated $\propto T^4$

$T_1 = 300$ K and $T_2 = 600$ K

Temperature has doubled. Therefore area will become 16 times.

17. (*c*) Rate of cooling $\left(-\frac{dT}{dt}\right) \propto$ emissivity (e)

From graph, $\left(-\frac{dT}{dt}\right)_x > \left(-\frac{dT}{dt}\right)_y$

$\Rightarrow$ $e_x > e_y$

Further emissivity $e \propto$ absorptive power (a).

$\Rightarrow$ $a_x > a_y$ ($\because$ good absorbers are good emitters)

18. (*c*) From given curve,

Melting point for $A = 60°C$

And melting point for $B = 20°C$

Time taken by A for fusion $= (6-2) = 4$ min

Time taken by B for fusion $= (6.5-4) = 2.5$ min

Then $\frac{H_A}{H_B} = \frac{6\times4\times60}{6\times2.5\times20} = \frac{24}{5}$

19. (*d*) Heat required to melt whole ice is

$Q = mL = 20\times80 = 1600$ cal

Energy incident per minute is

$$P = 1.9\times\frac{\pi}{4}\times d^2$$

$$= 1.9\times\frac{\pi}{4}\times(20)^2 = 597 \text{ cal min}^{-1}$$

Therefore, time required to melt whole ice is $t = \frac{Q}{P} \approx 2.7$ min

20. (*a*) Cooling rate $\propto T^4 - T_0^4$

$$r = \left\{\frac{(900)^4-(300)^4}{(600)^4-(300)^4}\right\} r_0 = \frac{16}{3} r_0$$

21. (*a*) According to Wien's law,

$\lambda_m T$ = constant

or $T \propto \frac{1}{\lambda_m}$

λ_m has become $\frac{3}{4}$ times.

$\therefore$ Temperature will have become $\frac{4}{3}$ times.

Now, according to Stefan's law, power $P \propto T^4$.

Therefore new power will be

$$\left(\frac{4}{3}\right)^4 P = \frac{256}{81} P$$

22. (*a*) Heat currents in both the rods are equal.

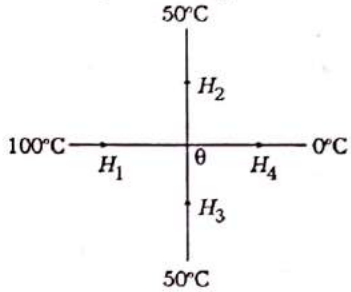
or $H_{CB} = H_{BA}$

or $\frac{\theta_C - \theta_B}{(2l/3KA)} = \frac{\theta_B - \theta_A}{(l/2KA)}$

$\therefore$ $\frac{3}{2}(70-\theta_B) = 2(\theta_B - 0)$

or $\theta_B = 30°C$

23. (*a*) Let θ be the temperature of junction.



H_1, H_2, H_3 and H_4 be the heat currents. Then

and $H_1 = H_2 + H_3 + H_4$

or $\frac{100-\theta}{R} = \frac{\theta-50}{R} + \frac{\theta-50}{R} + \frac{\theta-0}{R}$

(R = Thermal resistance of each part of two rods)

Solving these two equations, we get

$\theta = 50°C$

24. (*b*) For no current flow between C and D

$$\left(\frac{Q}{t}\right)_{AC} = \left(\frac{Q}{t}\right)_{CB}$$

$\Rightarrow$ $\frac{K_1A(\theta_A-\theta_C)}{l} = \frac{K_2A(\theta_C-\theta_B)}{l}$

$\Rightarrow$ $\frac{\theta_A-\theta_C}{\theta_C-\theta_B} = \frac{K_2}{K_1}$ ...(i)

Also $\left(\frac{Q}{t}\right)_{AD} = \left(\frac{Q}{t}\right)_{DB}$

$\Rightarrow$ $\frac{K_3A(\theta_A-\theta_D)}{l} = \frac{K_4A(\theta_D-\theta_B)}{l}$

$\Rightarrow$ $\frac{\theta_A-\theta_D}{\theta_D-\theta_B} = \frac{K_4}{K_3}$ ...(ii)

It is given that $\theta_C = \theta_D$, hence from Eqs. (i) and (ii),

we get $\frac{K_2}{K_1} = \frac{K_4}{K_3} \Rightarrow K_1K_4 = K_2K_3$

25. (*a*) Given, $K_B = K_A/2$ and $K_B = 3K_C$

$\therefore$ $K_C = K_A/6$

Rods are in series form so

$$\frac{L}{K} = \frac{l_1}{K_A} + \frac{l_2}{K_B} + \frac{l_3}{K_C} \quad (\because l = l_1 = l_2 = l_3$$

$$\frac{3l}{K} = \frac{l}{K_A} + \frac{l}{K_A/2} + \frac{l}{K_A/6}$$

or $\frac{3l}{K} = \frac{9l}{K_A}$ or $K = \frac{K_A}{3}$

26. (*c*) Let T be the final equilibrium temperature.

Heat lost by coffee = Heat gained by cup

$$0.3 \times c_{\text{coffee}} \times (70 - T) = 0.12 \times c_{\text{cup}} \times (T - 20)$$

$$\Rightarrow \quad 0.3 \times 4080(70 - T) = 0.12 \times 1020 \times (T - 20)$$

$$\Rightarrow \quad 4 \times 70 - 4T = 0.4T - 8 \Rightarrow T = \frac{288}{4.4} = 65.5°\text{C}$$

27. (*b*) Let water equivalent be x

$$\therefore \quad (10 + x)\, c \frac{\Delta T}{\Delta t} = -k\left(\frac{T_1 + T_2}{2} - T_0\right)$$

Where c is the specific heat of water

ΔT = change in temperature

Δt = time at which temperature falls

$$\therefore (10 + x)\, c \frac{(20 - 15)}{10} = -k\left(\frac{20 + 15}{2} - T_0\right) \quad \text{...(i)}$$

$$(20 + x)\, c \frac{(20 - 15)}{15} = -k\left(\frac{20 + 15}{2} - T_0\right) \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{10 + x}{10} = \frac{20 + x}{15}$$

$$150 + 15x = 200 + 10x$$

$$5x = 50 \Rightarrow x = 10 \text{ g}$$

28. (*c*) Here, specific heat of ice, $c_{\text{ice}} = 0.5 \text{ cal g}^{-1}\,°\text{C}^{-1}$

Specific heat of water, $c_{\text{water}} = 1 \text{ cal g}^{-1}\,°\text{C}^{-1}$

Latent heat of fusion of ice, $L_{\text{ice}} = 80 \text{ cal g}^{-1}$

Here ice will absorb heat while hot water will release it.

Let T be the final temperature of the mixture.

Assuming water equivalent of calorimeter to be neglected.

Heat given by water, $Q_1 = m_{\text{water}} c_{\text{water}} \Delta T$

$$= 19 \times 1 \times (30 - T) = 570 - 19T \quad \text{...(i)}$$

Heat absorbed by ice,

$$Q_2 = m_{\text{ice}} \times c_{\text{ice}} \times [0 - (-20)] + m_{\text{ice}} \times L_{f\,\text{ice}} + m_{\text{ice}} \times c_{\text{water}} \times (T - 0)$$

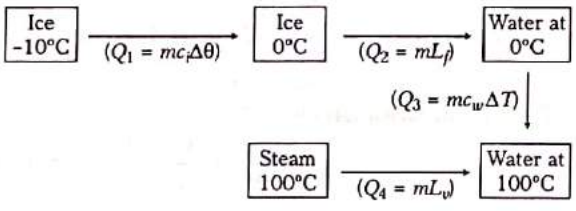
$$= 5 \times 0.5 \times 20 + 5 \times 80 + 5 \times 1 \times T = 450 + 5T \quad \text{...(ii)}$$

According to principle of calorimetry, $Q_1 = Q_2$

i.e., $570 - 19T = 450 + 5T \Rightarrow T = \frac{120}{24} = 5°\text{C}$

29. (*a*) Ice (−10° C) converts into steam as follows

(c_i = Specific heat of ice, c_w = Specific heat of water)



Total heat required $Q = Q_1 + Q_2 + Q_3 + Q_4$

$$\Rightarrow Q = 1 \times 0.5(10) + 1 \times 80 + 1 \times 1 \times (100 - 0) + 1 \times 540$$

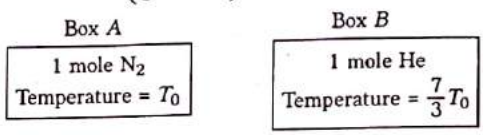
$$= 725 \text{ cal}$$

Hence, work done $W = JQ = 4.2 \times 725 = 3045 \text{ J}$

30. (*b*) When two gases are mixed together then

Heat lost by the Helium gas = Heat gained by the Nitrogen gas

$$n_B \times (c_v)_{\text{He}} \times \left(\frac{7}{3} T_0 - T_f\right) = n_A \times (c_V)_{N_2} \times (T_f - T_0)$$



$$\Rightarrow \quad 1 \times \frac{3}{2} R \times \left(\frac{7}{3} T_0 - T_f\right) = 1 \times \frac{5}{2} R \times (T_f - T_0)$$

By solving we get $T_f = \frac{3}{2} T_0$

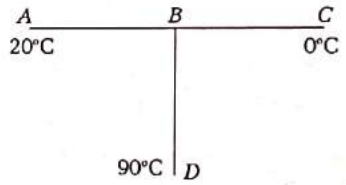
31. (*d*) Let the quantity of heat supplied per minute be Q, then quantity of heat supplied in 2 min = $mc(90 - 80)$

In 4 min, heat supplied = $2mc(90 - 80)$

$$\therefore \quad 2mc(90 - 80) = mL$$

$$\Rightarrow \quad \frac{L}{c} = 20.$$

32. (*b*) There will be no heat flow through AB, if temperature of B is also 20°C and heat current in DB is equal to heat current in BC.



$$\therefore \quad H_{DB} = H_{BC}$$

or $$\frac{70}{(l_{BD}/KA)} = \frac{20}{(l_{BC}/KA)} \quad \text{or} \quad \frac{l_{BD}}{l_{BC}} = \frac{7}{2}$$

33. (*d*) Rate of heat flow $I \propto \frac{\text{TD}}{R}$

(H = Temperature difference between two points / R)

As TD = temperature difference is same.

$$\therefore \quad I \propto \frac{1}{\text{Thermal resistance}}$$

$$\frac{I_A}{I_B} = \frac{R_B}{R_A} = \frac{(R)(2R)/R + 2R}{(R + 2R)} = \frac{2}{9}$$

34. (*c*) Initially the rods are in parallel

$$\frac{Q}{t} = \frac{(\theta_1 - \theta_2)}{R}$$

$$\Rightarrow \quad \left(\frac{Q}{t}\right)_1 = \frac{mL}{t} = q_1 L = \frac{(100 - 0)}{R/2} \quad \text{...(i)}$$

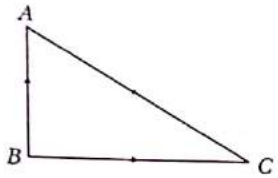
Finally when rods are in series

$$\Rightarrow \quad \left(\frac{Q}{t}\right)_2 = \frac{mL}{t} = q_2 L = \frac{(100 - 0)}{2R} \quad \text{...(ii)}$$

From Eqs. (i) and (ii),

$$\frac{q_1}{q_2} = \frac{4}{1}$$

35. (*b*) $\because T_B > T_A \Rightarrow$ Heat will flow B to A *via* two paths (i) B to A (ii) along BCA as shown.



Rate of flow of heat in path BCA will be same

i.e., $$\left(\frac{Q}{t}\right)_{BC} = \left(\frac{Q}{t}\right)_{CA}$$

$$\Rightarrow \quad \frac{K(\sqrt{2}T - T_C)A}{a} = \frac{K(T_C - T)A}{\sqrt{2}a} \Rightarrow \frac{T_C}{T} = \frac{3}{1+\sqrt{2}}$$

36. (*b*) Heat currents in the two rods will be same.

$\therefore$ $$H_1 = H_2$$

or $$\frac{\theta_1 - \theta}{(l/K_1A)} = \frac{\theta - \theta_2}{(l/K_2A)} \quad (\theta = \text{temperature of junction})$$

or $$K_1(\theta_1 - \theta) = K_2(\theta - \theta_2)$$

$\therefore$ $$\theta = \frac{K_1\theta_1 + K_2\theta_2}{K_1 + K_2}$$

37. (*d*) Heat supplied, $dQ = ms \cdot dT$

$$m = 1 \text{ kg}$$

$\therefore$ $$s = \frac{dQ}{dT} = \text{Slope of } Q\text{-}T \text{ graph}$$

$$S_{\text{solid}} = \tan 45° = 1 \text{ J kg}^{-1}\text{K}^{-1}$$

$$S_{\text{liquid}} = \tan 30° = \frac{1}{\sqrt{3}} \text{ J kg}^{-1}\text{K}^{-1}$$

Latent heat of vaporisation

$$= \text{heat required to melt complete 1 kg} = 200 \text{ J kg}^{-1}$$

38. (*b*) Initially ice will absorb the heat to raise its temperature at 0°C, then its melting takes places.

If m_i = Initial mass of ice, m'_i = Mass of ice that melts and m_W = Initial mass of water

By law of mixture, heat gained by ice = heat lost by water

$\Rightarrow$ $$m_1 \times c \times (20) + m'_i \times L = m_W c_W (20)$$

$\Rightarrow$ $$2 \times 0.5(20) + m_i \times 80 = 5 \times 1 \times 20 \Rightarrow m'_i = 1\text{kg}$$

So final mass of water = Initial mass of water + Mass of ice that melts

$$= 5 + 1 = 6 \text{ kg}$$

39. (*b*) Let θ be the temperature when thermal equilibrium has reached.

heat gained by ice to be converted to water at θ° C

$$= mL + m \times c \times (T - 0) = 10 \times 80 + 10 \times 1 \times T$$

Heat lost by tumbler and its contents $= 55 \times (40 - T)$

Using principle of calorimetry, heat gained = heat lost

$$10 \times 80 + 10 \times 1 \times T = 55 \times (40 - T)$$

$$65\,T = 2200 - 800 = 1400$$

$$T = \frac{1400}{65} \approx 21.5°\text{C} \approx 22°\text{C}$$

40. (*b*) In steady state the amount of heat flowing from one face to the other face in time t is given by $Q = \frac{KA(\theta_1 - \theta_2)t}{l}$ where K is coefficient of thermal conductivity of material of rod

$\Rightarrow$ $$\frac{\alpha}{l} \propto \frac{A}{l} \propto \frac{r^2}{l} \quad \text{...(i)}$$

As the metallic rod is melted and the material is formed into a rod of half the radius

$$V_1 = V_2 \Rightarrow \pi r_1^2 l_1 = \pi r_2^2 l_2 \Rightarrow l_1 = \frac{l_2}{4} \quad \text{...(ii)}$$

Now, from Eqs. (i) and (ii), we get

$$\frac{Q_1}{Q_2} = \frac{r_1^2}{l_1} \times \frac{l_2}{r_2^2} = \frac{r_1^2}{l_1} \times \frac{4l_1}{(r_2/2)^2} \Rightarrow Q_1 = 16\,Q_2 \Rightarrow Q_2 = \frac{Q}{16}$$

41. (*c*) Heat required by 1 g ice at 0° C to ment into 1 g water at 0° C.

$$Q_1 = mL \quad \text{(latent heat of fusion } L = 80 \text{ cal/g)}$$
$$= 1 \times 80 = 80 \text{ cal}$$

Heat required by 1g of water of 0° C to boil at 100° C,

$$Q_2 = mc\Delta T \quad \text{(specific heat of water } c = 1 \text{cal/g°C)}$$
$$= 1 \times 1(100 - 0)$$
$$= 100 \text{ cal}$$

Thus, total heat required by 1g of ice to reach a temperature of 100° C,

$$Q = Q_1 + Q_2 = 80 + 100 = 180 \text{ cal}$$

Heat available with 1 g of steam to condense into 1g ofwater at 100° C,

$$Q' = mL' \quad \text{(latent of vaporisation } L' = 536 \text{ cal/g)}$$
$$= 1 \times 536 \text{ cal}$$
$$= 536 \text{ cal}$$

Obviously, the whole steam will not be condensed and ice will attain temperature of 100° C. Thus, the temperature of mixture is 100° C.

42. (*b*) No heat is flowing through C and F. Therefore, they are at same temperatures.

43. (*a*) When A and B are mixed

$$mc_A(15 - 10) = mc_B(25 - 15) \text{ or } \frac{c_A}{c_B} = 2 \quad \text{...(i)}$$

When B and C are mixed

$$mc_B(30 - 25) = mc_C(40 - 30) \text{ or } \frac{c_B}{c_C} = 2 \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we have

$$\frac{c_A}{c_C} = 4$$

Now let θ be the equilibrium temperature when A and C are mixed. Then

$$mc_A(\theta - 10) = mc_C(40 - \theta)$$

or $$4(\theta - 10) = 40 - \theta \quad \left(\text{Using } \frac{c_A}{c_C} = 4\right)$$

or $$\theta = 16°\text{C}$$

44. (*b*) Suppose m kg steam is required per hour

Heat is released by steam in following three steps:

(i) When steam at 150°C $\xrightarrow[Q_1]{}$ steam at 100 °C

$$Q_1 = mc_{\text{Steam}}\,\Delta\theta$$
$$= m \times 1(150 - 100) = 50\,m \text{ cal}$$

(ii) When steam at 100°C $\xrightarrow[Q_2]{}$ water at 100°C

$$Q_2 = mL_V = m \times 540 = 540m \text{ cal}$$

(iii) When water at 100°C $\xrightarrow[Q_2]{}$ water at 90°C

$$Q_3 = mc_W\Delta\theta$$
$$= m \times 1 \times (100 - 90) = 10m \text{ cal}$$

Hence, total heat given by the steam

$$Q = Q_1 + Q_2 + Q_3 = 600m \text{ cal} \quad \text{...(i)}$$

Heat taken by 10 kg water

$$Q' = mc_W\Delta\theta$$
$$= 10 \times 10^3 \times 1 \times (80 - 20)$$
$$= 600 \times 10^3 \text{ cal}$$

Hence $\quad Q = Q'$

$\Rightarrow \quad 600m = 600 \times 10^3$

$\Rightarrow \quad m = 10^3 \text{ g} = 1 \text{ kg}$

45. (*c*) $R_{AB} = R_{CD} = R$ (say)

Length of semicircle $= \pi l$

$\therefore$ Resistance of semicircle $= \pi R$ as $R \propto l$

$$R_{BC} = \frac{\pi R}{2} \text{ (in parallel)}$$

Heat current, $H = \dfrac{100 - 0}{R + R + \frac{\pi}{2}\cdot R} = \dfrac{28}{R}$ units

Now $\quad \theta_C - 0° = HR = 28°\text{C}$

$\therefore \quad \theta_C = 28°\text{C}$

46. (*a*) Heat carried increases to 2 times. Therefore, new net thermal resistance will remain half.

$$\therefore \quad R' = \frac{R}{2} \Rightarrow \frac{R'_1 R'_2}{R'_1 + R'_2} = \frac{R_1 R_2}{2(R_1 + R_2)}.$$

or $$\frac{(3l/KA)(l/K'A)}{(3l/KA) + (l/K'A)} = \frac{(3l/KA)(l/KA)}{2\left\{\frac{3l}{KA} + \frac{l}{KA}\right\}}$$

Solving, this we get $\quad K' = \dfrac{7}{3}K$

47. (*d*) $\dfrac{d\theta}{dt} = L\cdot\dfrac{dm}{dt}$ = Heat transfer from bottom $= \dfrac{\text{TD}}{(l/KA)}$

Heat TD = desired temperature difference.

$$\therefore \quad \text{TD} = \frac{L\cdot l\cdot\frac{dm}{dt}}{KA} = \frac{(540)(2\times10^{-1})\left(\frac{1}{60}\right)}{0.5 \times 300}$$
$$= 0.012°\text{C}$$

B) Medical Entrance Special Format Questions

Assertion and reason

1. (*a*) Good radiations are good absorbers.

2. (*d*) By black body we mean an object which can absorb all radiations falling over it.

3. (*a*) $E = \int E_\lambda d\lambda \propto T^4$

4. (*e*) A normal body can also radiate energy more than a perfectly black body if its temperature is very high.

5. (*b*) $\lambda_m \propto 1/T$

6. (*c*) Water has high specific heat due to which it does not losses heat easily.

7. (*c*) Rearon is incorrect,

8. (*a*) $T \propto \dfrac{1}{\lambda_m} \propto f \Rightarrow$ As $f_{\text{blue}} > f_{\text{red}}$. So, blue stars are hotter.

9. (*a*) Both Assertion and Reason are correct and reason explains Assertion.

10. (*a*) both heat and electricity are wndurted by free electing

11. (*b*) According to Wien's displacement law,

$$\lambda_m \propto \frac{1}{T}$$

12. (*d*) Emissive power is energy radiated per unit area per unit time which is not one.

13. (*d*) Statement of reason is Kirchhoff's law.

14. (*c*) Water in vapour state has more heat content.

15. (*c*) $Q = mL_1 + ms\Delta\theta + mL_2$

$$= 1 \times 80 + 1 \times 1 \times 100 + 1 \times 540$$
$$= 720 \text{ cal}$$

16. (*b*) Due to gravity hot (but lighter) material rises up and cold (but heavier) material comes down.

17. (*c*) $R = \dfrac{l}{KA}$ or $R \propto \dfrac{1}{A}$

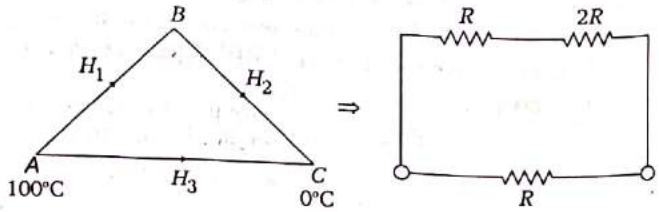
$\therefore \quad R_1 > R_2$

Further, $\quad \text{TD} = H \times R$ or $\text{TD} \propto R$

$\therefore \quad (\text{TD})_1 > (\text{TD})_2$

Match the columns

3. Thermal resistance $\propto \dfrac{1}{\text{Area}}$



So let, $R_{BC} = R$

Then $R_{AB} = R_{AC} = 2R$

$H_1 = H_2$

$\therefore \quad \frac{100 - \theta_B}{R} = \frac{\theta_B - 0}{2R}$

$\therefore \quad \theta_B = 66.7°C$

$R_{ABC} = 3R$ and $R_{AC} = R$

$\therefore \quad H_1 = H_2 = \frac{H_3}{3}$

4. When A and B are mixed together

$$(m)(c)(\theta - 20) = (2m)(c)(40° - \theta)$$

or $\theta = \frac{100}{3} = 33.3°C$

When A and C are mixed together

$$(m)(c)(\theta - 20) = (3m)(c)(60° - \theta) \text{ or } \theta = 50°C$$

When B and C are mixed together

$$(2m)(c)(\theta - 40) = (3m)(c)(60° - \theta) \Rightarrow \theta = 52°C$$

When A, B and C all are mixed

$$(m)(c)(\theta - 20) + (2m)(c)(60° - \theta) = (3m)(c)(60° - \theta)$$

or $\theta = 40°C$

Medical entrances' gallery

1. (*c*) Given, temperature, $T_1 = 5760$ K

Since, it is given that energy of radiation emitted by the body at wavelength 250 nm is U_1, at wavelength 500 nm is U_2 and that at 1000 nm is U_3.

$\because$ According to Wien's law, we get

$$\lambda_m T = b$$

where, b = Wien's constant = 2.88×10^6 nmK

$$\Rightarrow \lambda_m = \frac{b}{T} \Rightarrow \lambda_m = \frac{2.88 \times 10^6 \text{ nmK}}{5760 \text{ K}} \Rightarrow \lambda_m = 500 \text{ nm}$$

$\because \lambda_m$ = wavelength corresponding to maximum energy, so, $U_2 > U_1$.

2. (*b*) According to question as conservation of energy, energy gained by the ice during its fall from height h is given by

$$E = mgh$$

As given, only one quarter of its energy is absorbed by the ice.

So, $\frac{mgh}{4} = mL_f$

$$\Rightarrow h = \frac{mL_f \times 4}{mg} = \frac{L_f \times 4}{g} = \frac{3.4 \times 10^5 \times 4}{10} = 13.6 \times 10^4$$

$$= 13600 \text{ m} = 136 \text{ km}$$

3. (*d*) Here, $\Delta T_1 = 110 - 100 = 10°C$

$$\frac{dQ_1}{dt} = 4 \text{ Js}^{-1} \Rightarrow \Delta T_2 = 210 - 200 = 10°C, \frac{dQ_2}{dt} = ?$$

As the rate of heat flow is directly proportional to the temperature difference and the temperature difference in both the cases is same, *i.e.*, 10°C. So, the same rate of heat will flow in the second case.

Hence, $\frac{dQ_2}{dt} = 4 \text{ Js}^{-1}$

4. (*d*) From Wien's displacement law,

$$\lambda_m T = b \Rightarrow \lambda_m = \frac{b}{T} \quad \text{...(i)}$$

From Stefan's law,

$$E \propto T^4$$

$$\Rightarrow \frac{E_1}{E_2} = \left(\frac{T_1}{T_2}\right)^4 = \left(\frac{\lambda_{m_2}}{\lambda_{m_1}}\right)^4 \quad \text{[By Eq. (i)]}$$

$$\Rightarrow \frac{E_1}{E_2} = \left(\frac{600}{200}\right)^4 \Rightarrow E_1 : E_2 = 81 : 1$$

5. (*c*) If T is the temperature of the interface of two plates of same area, then

$$\frac{dQ}{dt} = \frac{K_1 A(10 - T)}{\Delta x_1} = \frac{K_2 A(T - 0)}{\Delta x_2}$$

or $\frac{K_1 A(10 - T)}{\Delta x_1} = \frac{\frac{3}{2} K_1 A(T - 0)}{3/2\, \Delta x_1} \quad \left[\because \frac{K_1}{K_2} = \frac{2}{3} \text{ and } \frac{\Delta x_1}{\Delta x_2} = \frac{2}{3}\right]$

or $10 - T = T - 0$ or $T + T = 10$ or $2T = 10$

$\therefore \quad T = 5° C$

6. (*a*) Given, $\lambda_m = 14$ micron $= 14 \mu m = 14 \times 10^{-6}$ m

and Wien constant, $b = 2892 \times 10^{-6}$ mK

According to Wien's displacement law,

$$\lambda_m = \frac{b}{T} \text{ or } T = \frac{b}{\lambda_m} = \frac{2892 \times 10^{-6} \text{ mK}}{14 \times 10^{-6} \text{ m}}$$

$\therefore \quad T = 206.6 \text{ K} \approx 207 \text{ K}$

7. (*c*) As we know that, the rate of cooling is

$$\frac{d\theta}{dt} = bA(\theta - \theta_0)$$

where, bA = constant

$$\Rightarrow \frac{d\theta}{dt} \propto \theta - \theta_0$$

Also $-\frac{dT}{dt} \propto T - T_0$

where, T = temperature and t = time

So, the rate of heat absorbed by cooled body is proportional to $t_2 - t_1$.

8. (*b*) The rate of heat emission, $u = \sigma A T^4$.

At room temperature to the rate of absorption is

$$u_0 = \sigma A T_0^4$$

The net rate of heat loss.

$$u - u_0 = \sigma A (T^4 - T_0^4)$$

Given that $A = 0.001 \text{ m}^2$

$T = 400$ K and $T_o = 300$ K

Thus, $u - u_0 = 5.67 \times 10^{-8} \times 0.001 \times [(400)^4 - (300)^4] = 0.99$ W

9. (*b*) If we talk about heat of a body, it is meaningless.

10. (*d*) Rate of production of mass of steam $= \frac{dm}{dt} = \frac{1}{60}\,\text{gs}^{-1}$

Rate of heat transformed by stove,

$$\frac{d\theta}{dt} = L\frac{dm}{dt} = 540 \times \frac{1}{60} = 9\,\text{cal}\,^\circ\text{C}^{-1}\text{s}^{-1}\text{cm}^{-1}$$

Also, $$\frac{d\theta}{dt} = \frac{KA\,d\theta}{dx} \Rightarrow L\frac{dm}{dt} = \frac{KA\,d\theta}{dx}$$

$$\Rightarrow \quad 9 = \frac{0.5 \times 3000 \times d\theta}{0.2}$$

$$\Rightarrow \quad d\theta = 1.2 \times 10^{-3}\,^\circ\text{C} = 1.2 \times 10^{-3}\,\text{K}$$

11. (*c*) At steady state, $\frac{dQ}{dt} = \frac{dQ_1}{dt} + \frac{dQ_2}{dt}$

or $$\frac{K(4\pi R^2)(T_2 - T_1)}{L} = \frac{K_1(\pi R^2)(T_2 - T_1)}{L} + \frac{K_2(3\pi R^2)(T_2 - T_1)}{L}$$

or $$4K = K_1 + 3K_2$$

$\therefore$ $$K = \frac{K_1 + 3K_2}{4}$$

12. (*b*) We know from Wien's displacement law,

$$\lambda_m T = \text{constant}$$

So, $$T \propto \frac{1}{\lambda_m}$$

As, $$\lambda_r > \lambda_g > \lambda_v$$

So, $$T_r < T_g < T_v$$

Given, $P \to v_{max}$, $Q \to r_{max}$, $R \to g_{max}$

Hence, $$T_Q < T_R < T_P$$

i.e., $$T_P > T_R > T_Q$$

13. (*d*) Given that, mass of ice-piece, $m_1 = 100\text{ g} = 0.1\text{kg}$

Mass of water, $m_2 = 200\text{ g} = 0.2\text{ kg}$

When temperature of mixture is 0°C, then heat lost by water

$$Q = m_2 c(\Delta\theta) = 0.2 \times 4200(25 - 0)$$

[$\because$ specific heat of water = 4200 J]

$$\Rightarrow \quad Q = 21000\text{ J}$$

Now, mass of melted ice,

$$m' = \frac{Q}{L} = \frac{21000}{3.4 \times 10^5} \simeq 61.8\text{ g}$$

14. (*d*) From the Newton's law of cooling,

$$\frac{\theta_1 - \theta_2}{t} \propto \left(\frac{\theta_1 + \theta_2}{2} - \theta_0\right)$$

where, θ_0 = temperature of surrounding

$\therefore$ $$\frac{94 - 86}{t_1} \propto \left(\frac{94 + 86}{2} - 20\right)$$

$$\frac{8}{t_1} \propto (70) \quad \text{...(i)}$$

and $$\frac{71 - 69}{t_2} \propto \left(\frac{71 + 69}{2} - 20\right)$$

$$\frac{2}{t_2} \propto 50 \quad \text{...(ii)}$$

Dividing Eq (ii) by Eq (i), we get

$$\frac{t_2}{t_1} = \frac{70}{50 \times 4} \Rightarrow t_2 = \frac{70 \times 2}{50 \times 4}$$

$$t_2 = 0.7\text{ min} \Rightarrow t_2 = 42\text{ s}$$

15. (*d*) According to principle of calorimetry, total heat given by a hotter body is equal to the total heat received by colder body *i.e.*, heat lost by hotter body = heat gained by colder body

Heat required to melt 1g of ice at 0°C into 1g of water at 0°C is 80 cal.

Heat required to convert 1 g of water at 0°C into 1 g of water at 100°C = 1 × 1 × 100 = 100 cal

Heat required to condense 1g of steam = 1 × 540 cal = 540 cal

Clearly, whole of steam is not condensed. So, temperature of the mixture is 100°C.

16. (*d*) Heat lost by system = Heat gain by water.

Let m' be amount of heat converted into water

$$m' \times L = mc\Delta t$$

$$m' \times 540 = 20 \times 1 \times (80 - 10)$$

$$m' = \frac{20 \times 70}{540} = 2.5\text{ g}$$

Now, net mass of water = 20 + 2.5 = 22.5 g

17. (*b*) As we know, work done by an electric heater,

i.e., $m_1c_1\,\Delta t + m_2c_2\,\Delta t$ = work done

So, $m_1c_1\Delta t + m_2c_2\Delta t = P_1t_1$

where, $m_1 = 0.5$ kg, specific heat $c_1 = 4200\text{ Jkg}^{-1}\text{K}^{-1}$,

$$\Delta t = \Delta t_1 = \Delta t_2 = 3\text{ K},$$

$$P_1 = P_2 = 10\text{ W}$$

$$t_1 = 15 \times 60 = 900\text{ s}$$

c_2 = specific heat capacity of container.

So, $0.5 \times 4200 \times (3 - 0) + m_2c_2 \times (3 - 0) = 10 \times 15 \times 60$

$$2100 \times 3 + m_2c_2 \times 3 = 9000$$

$$m_2c_2 = \frac{9000 - 6300}{3} = 900$$

Similarly, in the case of oil,

$$m_1c_o\Delta t + m_2c_2\Delta t = P_2t_2$$

where, c_o = specific heat capacity of oil

$$P_1 = P_2 = 10\text{ W}$$

$$2 \times c_o \times 2 + 900 \times 2 = 10 \times 20 \times 60$$

$$4c_o + 1800 = 12000$$

$$c_o = 2550 = 2.55 \times 10^3\text{ J kg}^{-1}\text{K}^{-1}$$

18. (*d*) Given, $m = 50$ kg, $v = 5\text{ ms}^{-1}$, $L = 80\text{ calg}^{-1}$

$$J = 4.2\text{ J cal}^{-1} \text{ and } m = ?$$

Heat lost *i.e.*, Q_1 = work done, K = kinetic energy

i.e., $$\frac{1}{2}mv^2 = \frac{1}{2} \times 50 \times 5^2 = 625\text{ J}$$

and heat gained *i.e.*, $Q_2 = m' \times L = 80 \times m' \times 4.2$

$\because$ $$Q_1 = Q_2$$

$$625 = m' \times 8 \times 42 \Rightarrow m' = 1.86\text{ g}$$

19. (*a*) We know that, relation between the temperature gradient (TG) and thermal conductivity (K),

$$\frac{dQ}{dt} = -KA\frac{d\theta}{dx} = -KA \times (\text{TG})$$

i.e., $\text{TG} \propto \frac{1}{K}$ $\left(\frac{dQ}{dt} = \text{constant}\right)$

or $K \propto \frac{1}{t} \Rightarrow K_1 \propto \frac{1}{t_1}$ ...(i)

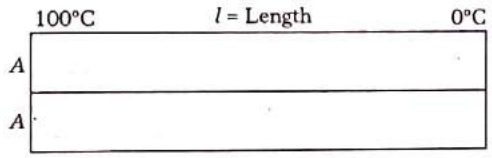
$K_2 \propto \frac{1}{t_2}$ ...(ii)

From the Eqs. (i) and (ii), we get

$$\frac{K_1}{K_2} = \frac{1/t_1}{1/t_2} \Rightarrow K_1 : K_2 = t_2 : t_1$$

20. (*d*) According to the question,

When two identical rods are connected in parallel,



So, rate of heat transfer is given by

$$\frac{dQ}{dT} = \frac{2KA(T_1 - T_2)}{l} = \frac{2KA(100-0)}{l} = \frac{200KA}{l}$$

where, net area = $2A$

K = thermal conductivity constant

T_1 = temperature at one end

T_2 = temperature at other end

l = length of rod.

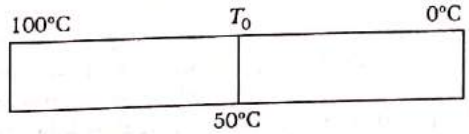
As, $\frac{dQ}{dT}$ is rate of heat transfer $= \frac{mL}{T}$

So, $g_1 = \frac{dQ}{dT} = \frac{Q}{T} \Rightarrow \left(\frac{m}{T}\right)L = \frac{200\,KA}{l}$

So, for parallel connection,

$$\frac{m}{T} = g_1 \text{ g/s} = \frac{200KA}{Ll} \quad ...(i)$$

Similarly, for series connection



So, $100 - T_0 = T_0 - 0 \Rightarrow 2T_0 = 100$

$\Rightarrow$ $T_0 = 50$

So, $\frac{dQ}{dT} = \left(\frac{m}{l}\right)L = \frac{KA(50-0)}{l}$

and it can be written as,

$$g_2(\text{g/s}) = \frac{KA\,50}{lL} \quad ...(ii)$$

Dividing Eqs. (i) and (ii), we get

$$\frac{g_1}{g_2} = \frac{200}{50} = 4 \Rightarrow \frac{g_2}{g_1} = \frac{1}{4}$$

21. (*d*) Consider the diagram where two rods are connected end to end (series).

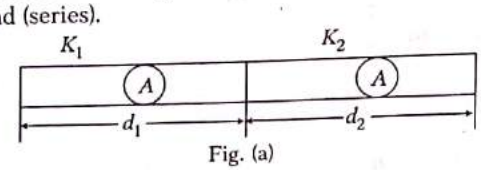


Fig. (a)

Thermal resistance (R) of a rod of length l, thermal conductivity K and area of cross-section A is given by

$$R = \frac{l}{KA}$$

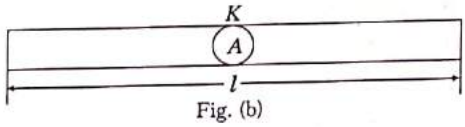


Fig. (b)

When rods are connected in series

R_{eq} = equivalent thermal resistance $= R_1 + R_2$ ...(i)

where, R_1 = thermal resistance of first rod

R_2 = thermal resistance of second rod

For Fig. (a),

Thermal resistance of first rod, $R_1 = \frac{d_1}{K_1A}$

where, A is area of cross-section of each rod. Similarly, thermal resistance of second rod, $R_2 = \frac{d_2}{K_2A}$

Now, from Fig. (a), $R_{eq} = \frac{d_1}{K_1A} + \frac{d_2}{K_2A}$

$$\Rightarrow \frac{L_{eq}}{K_{eq}A_{eq}} = \frac{d_1}{K_1A} + \frac{d_2}{K_2A} \quad ...(ii)$$

where, L_{eq} = equivalent length of rod

K_{eq} = equivalent thermal conductivity

A_{eq} = equivalent area of cross-section.

For series connection,

L_{eq} = equivalent length $= d_1 + d_2$

A_{eq} = equivalent cross-sectional area $= A$

Now, from Fig. (b)

$$\frac{d_1 + d_2}{(K_{eq})(A)} = \frac{d_1}{K_1A} + \frac{d_2}{K_2A} \Rightarrow \frac{d_1 + d_2}{K_{eq}} = \frac{d_1}{K_1} + \frac{d_2}{K_2}$$

$$\Rightarrow K_{eq} = \left(\frac{K_1K_2}{K_2d_1 + K_1d_2}\right)(d_1 + d_2) = \frac{d_1 + d_2}{\frac{d_1}{K_1} + \frac{d_2}{K_2}}$$

[Dividing numerator and denominator by K_1K_2]

Equivalent thermal conductivity, $K_{eq} = \frac{d_1 + d_2}{\left(\frac{d_1}{K_1} + \frac{d_2}{K_2}\right)}$

22. (*a*) Let temperature of surrounding be Q_0, then from Newton's law of cooling

***Case* I** $\frac{Q_2 - Q_1}{t} = K\left(\frac{Q_2 + Q_1}{2} - Q_0\right)$

$$\frac{70 - 60}{5} = (65 - Q_0) \quad ...(i)$$

Case II $\frac{60-54}{5} = (57 - Q_0)K$ ...(ii)

From Eqs. (i) and (ii)

$$\frac{10}{6} = \frac{65 - Q_0}{57 - Q_0} \Rightarrow Q_0 = 45°\,C$$

23. (*b*) The equation of Wien's displacement law, *i.e.*, $\lambda_m T$ = constant.

24. (*d*) From the equation, $\frac{\theta}{x} = \frac{\Delta t}{l}$

Given, $t_1 = 100°\,C$, $t_2 = 0°\,C$, $l = 25$ and $\pi = 10$ cm

$$\frac{\theta}{10} = \frac{100-0}{25} \Rightarrow \frac{\theta}{10} = 4 \Rightarrow \theta = 40°\,C$$

25. (*c*) According to Wien's displacement law,

$$\lambda T = \text{constant}$$

$$\therefore \quad \frac{T_1}{T_2} = \frac{\lambda_2}{\lambda_1} = \frac{480}{360} = \frac{4}{3}$$

26. (*b*) We know that according to Newton's law of cooling

$$\log\left(\frac{\theta_1 - \theta_2}{\theta_2 - \theta_0}\right) = -Kt \Rightarrow \log\left(\frac{61-59}{59 - \frac{61+59}{2}}\right) = -K \times 4 \quad ...(i)$$

$$\Rightarrow \quad \log\left(\frac{51-49}{49 - \frac{51+49}{2}}\right) = -K \times t \quad ...(ii)$$

Solving Eqs. (i) and (ii), we get

$$t = 5\text{ min}$$

27. (*c*) According to Stefan-Boltzmann law, the emissive power is directly proportional to the fourth power of temperature. So, it will give a straight line.

28. (*b*) Given, $m = 500$ kg

Rise in temperature, $dT = (100° - 20°)\,C = 80°\,C$

Specific heat of water, $c = 10^3$ cal kg^{-1} C^{-1}

Heat energy gained by water, $E = mcdT$

$$= 500 \times 10^3 \times 80 \text{ cal} = 4 \times 10^7 \times 4.2\text{ J} = 16.8 \times 10^7\text{ J}$$

From, $E = mc^2$

Increase in mass of water,

$$m = \frac{E}{c^2} = \frac{16.8 \times 10^7}{(3 \times 10^8)^2} = 187 \times 10^{-9}\text{ kg}$$

29. (*a*) The heat absorbed by a substance is directly proportional to its specific heat.Therefore, a coolant having high specific heat will absorb a large amount of heat from the engine. So, water is used as a coolant to cool the car engines in radiators.

30. (*d*) Newton's cooling law is given as

$$\frac{\theta_1 - \theta_2}{t} = k\left[\frac{\theta_1 + \theta_2}{2} - \theta_0\right]$$

Given, $\theta_0 = 25°\,C$, $\theta_1 = 50°\,C$ and $\theta_2 = 60°\,C$

By putting values, $k = 1/3$

Again, $\theta_2 = \theta$, $\theta_1 = 50°\,C$, $\theta_0 = 25°\,C$ $\therefore$ $\theta = 42.8°\,C$

31. (*d*) From Stefan' law, $E = \sigma T^4$

So, the rate of energy production,

$$Q = E \times A \Rightarrow Q = \sigma T^4 \times 4\pi R^2$$

Temperature of star, $T = \left(\frac{Q}{4\pi R^2 \sigma}\right)^{1/4}$

32. (*a*) Graph (a) shows the variation of temperature with time. At first, temperature will increase, then there will be state change from liquid to gas.

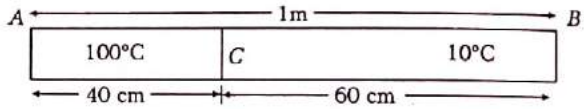
33. (*c*) According to Stefan's law,

$$E = \sigma T^4$$

$$\Rightarrow \quad 1 = 5.6 \times 10^{-8} \times T^4 \text{ or } T^4 = \frac{1}{5.6 \times 10^{-8}}$$

$$\Rightarrow \quad T = \left[\frac{1}{5.6 \times 10^{-8}}\right]^{1/4} = 65\text{ K}$$

34. (*a*) Let the temperature at distance 60 cm from point B be θ



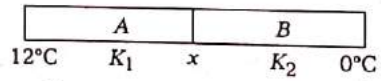
where, K = coefficient of thermal conductivity, A = area of rod

$$\frac{KA(100-\theta)}{40} = \frac{KA(\theta - 10)}{60}$$

or $$\frac{100-\theta}{2} = \frac{\theta - 10}{3}$$

or $$300 - 3\theta = 2\theta - 20 \text{ or } 5\theta = 320 \text{ or } \theta = 64°\,C$$

35. (*c*) The given situation can be shown as



Rate of flow of heat will be equal in both the slabs

$$\therefore \quad (12 - x)K_1 = K_2(x - 0)$$

$$12 - x = 2x \Rightarrow x = 4°\,C \qquad \left(\because K_1 = \frac{K_2}{2}\right)$$

The temperature difference across slab

$$A = (12 - x) = (12 - 4) = 8\,°C$$

36. (*c*) According to Stefan's law, the energy emitted by a body per second is directly proportional to the fourth power of the temperature of the body. Here, the temperature of blue glass is more than that of red glass, so it will look brighter.

37. (*b*) Temperature gradient $= \frac{\theta_1 - \theta_2}{l}$

Here, $\theta_1 = 30°C \Rightarrow l = 0.5$ m

$$\therefore \quad 80 = \frac{30 - \theta_2}{0.5}$$

$$\theta_2 = 30 - 80 \times 0.5 \Rightarrow \theta_2 = 30 - 40 = -10°C$$

38. (*a*) We know that

$$\frac{Q}{t} = \frac{KA(\Delta\theta)}{t}$$

$$50 = \frac{5 \times 20K}{0.4} \text{ or } K = \frac{50 \times 0.4}{5 \times 20} = 0.2$$