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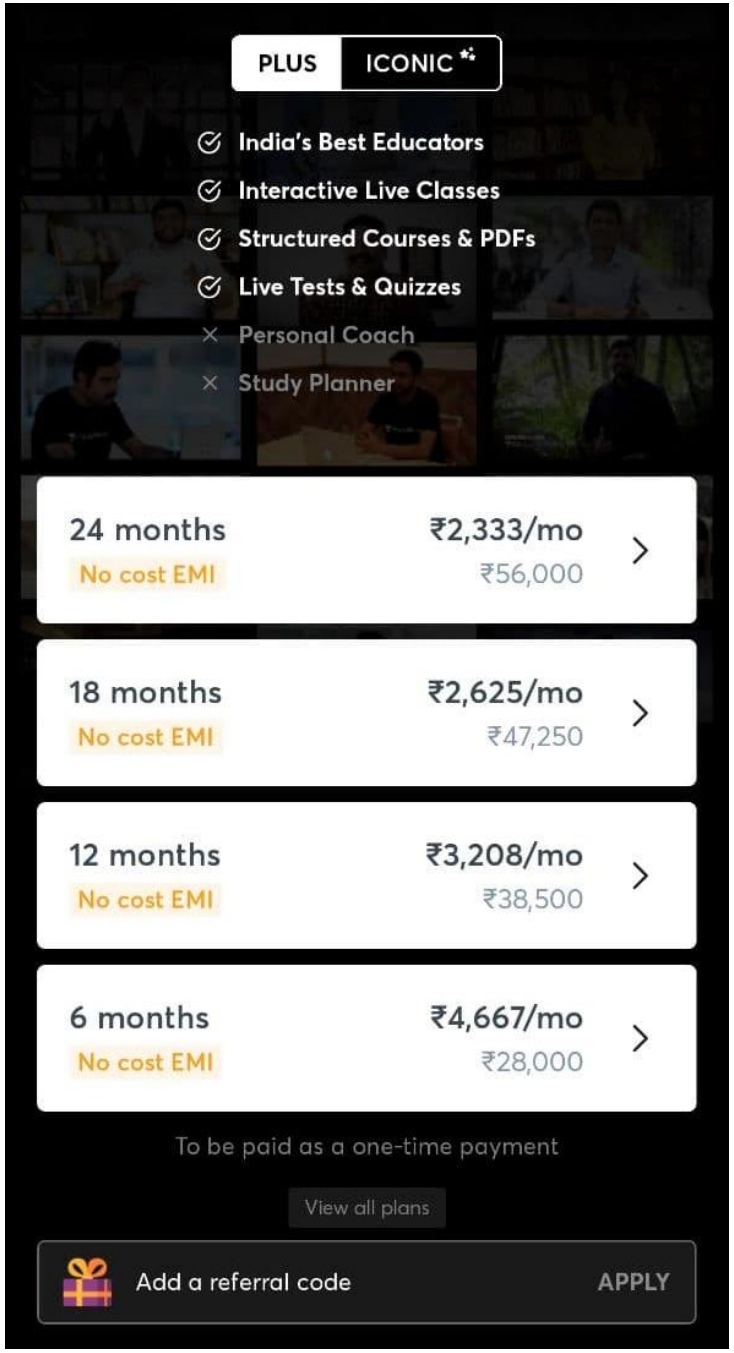

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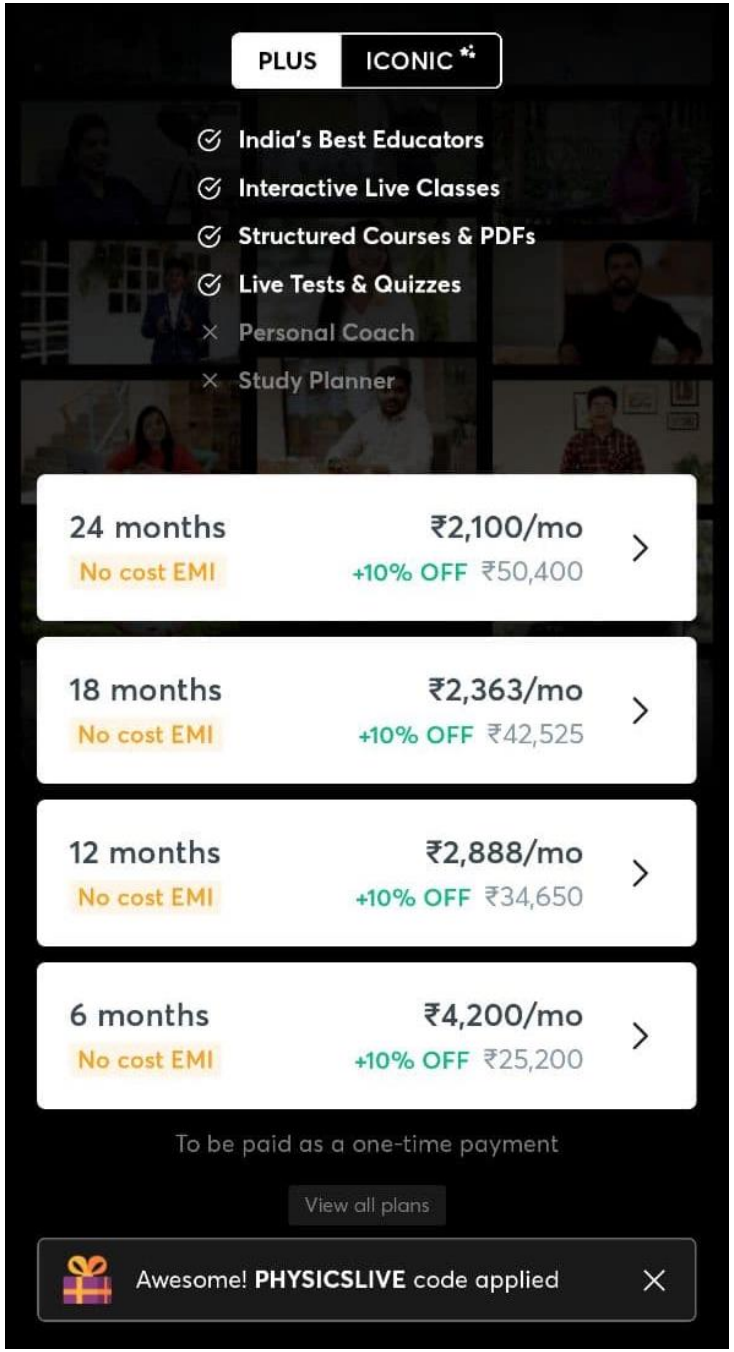

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JEE Main & Advanced
Physics DPP

DPP-4 Vectors (Cross product)

By Physicsaholics Team

Q) If $\vec{A} \times \vec{B} = \vec{C}$, then which of the following statements is wrong:

(a) $\vec{C} \perp \vec{A}$

(b) $\vec{C} \perp \vec{B}$

(c) $\vec{C} \perp (\vec{A} + \vec{B})$

(d) $\vec{C} \perp (\vec{A} \times \vec{B})$

Ans. d

$\vec{A} \times \vec{B} = \vec{C}$

Means; $\vec{C} \perp \vec{A}$
and $\vec{C} \perp \vec{B}$

and $\vec{C}$ is $\perp$ to plane of
$\vec{A}$ & $\vec{B}$.

$\because \vec{A} + \vec{B}$ is in the plane of
$\vec{A}$ & $\vec{B}$

$\therefore \vec{C} \perp (\vec{A} + \vec{B})$

& $\vec{C} = \vec{A} \times \vec{B}$

so) $\vec{C}$ can't be $\perp$ to $\vec{A} \times \vec{B}$
$\therefore \vec{C}$ is equal to $\vec{A} \times \vec{B}$

Q) If two vectors $2\hat{\imath} + 3\hat{\jmath} - \hat{k}$ and $-4\hat{\imath} - 6\hat{\jmath} - \lambda\hat{k}$ are parallel to each other then value of λ be:

(a) 0 (b) -2 (c) 2 (d) -4

Q) If two vectors $2\hat{i} + 3\hat{j} - \hat{k}$ and $-4\hat{i} - 6\hat{j} - \lambda\hat{k}$ are parallel to each other then value of λ be:

Ans. b

$\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$, $\vec{b} = -4\hat{i} - 6\hat{j} - d\hat{k}$

$\vec{a} \parallel \vec{b}$; then $\vec{a} \times \vec{b} = 0$

$\therefore \vec{a} \times \vec{b} = (2\hat{i} + 3\hat{j} - \hat{k}) \times (-4\hat{i} - 6\hat{j} - d\hat{k})$

$= \hat{i}(-3d - 6) + \hat{j}(2d + 4) + \hat{k}(-12 + 12)$

$= \hat{i}(-3d - 6) + \hat{j}(2d + 4) + \hat{k}(0)$

$\because \vec{a} \times \vec{b} = 0$

$\Rightarrow \hat{i}(-3d - 6) + \hat{j}(2d + 4) + \hat{k}(0) = 0$

$-3d - 6 = 0 \Rightarrow d = -2$

$2d + 4 = 0 \Rightarrow d = -2$

$\therefore \boxed{d = -2}$

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Q) What is the value of $(\vec{A}+\vec{B}).(\vec{A}\times\vec{B})$ =?

(a) 0

(b) $A^2 - B^2$

(c) $A^2 + B^2 + 2AB$

(d) None of these

Ans. a

$(\vec{A} + \vec{B}) \cdot (\vec{A} \times \vec{B}) = ?$

Let $\vec{C} = \vec{A} \times \vec{B}$

then $\vec{C}$ is $\perp$ to $\vec{A}$

& $\vec{C}$ is $\perp$ to $\vec{B}$

and $\vec{C}$ is $\perp$ to $(\vec{A} + \vec{B})$

$\therefore$ dot product of $\perp$ vectors is zero.

$\therefore (\vec{A} + \vec{B}) \cdot (\vec{A} \times \vec{B})$

$= (\vec{A} + \vec{B}) \cdot (\vec{C})$

$(\vec{A} + \vec{B}) \cdot (\vec{C}) =$ zero

Q) Let $\vec{A} = \hat{\imath} + \hat{\jmath} + \hat{k}$, $\vec{B} = \hat{\jmath} - \hat{k}$. If $\vec{C}$ is a vector satisfying $\vec{A} \times \vec{C} = \vec{B}$ and $\vec{A}.\vec{C} = 3$, then $\vec{C}$ is:

(a) $\frac{1}{3}(5\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$

(b) $\frac{1}{3}(5\hat{\imath} - 2\hat{\jmath} - 2\hat{k})$

(c) $3\hat{\imath} - \hat{\jmath} - \hat{k}$

(d) None of these

Ans. a

$\vec{A} = \hat{i} + \hat{j} + \hat{k}$

$\vec{B} = \hat{j} - \hat{k}$

Given; $\vec{A} \times \vec{C} = \vec{B}$

$\vec{A} \cdot \vec{C} = 3$

Let; $\vec{C} = x\hat{i} + y\hat{j} + z\hat{k}$

$\therefore \vec{A} \times \vec{C} = \vec{B}$

$(\hat{i} + \hat{j} + \hat{k}) \times (x\hat{i} + y\hat{j} + z\hat{k}) = \hat{j} - \hat{k}$

$\hat{i}(z-y) + \hat{j}(x-z) + \hat{k}(y-x) = \hat{j} - \hat{k}$

$\Rightarrow z - y = 0 \; ; \; x - z = 1 \; , \; y - x = -1$

$\boxed{y = z} \; ; \; \boxed{x = 1 + z} \; ; \; \boxed{y = x - 1}$

$\vec{A} \cdot \vec{C} = 3$

$(\hat{i} + \hat{j} + \hat{k}) \cdot (x\hat{i} + y\hat{j} + z\hat{k}) = 3$

$x + y + z = 3$

$(1+z) + (z) + z = 3$

$3z = 2 \Rightarrow \boxed{z = \frac{2}{3}}$

$\Rightarrow x = 1 + \frac{2}{3} = \frac{5}{3} \Rightarrow \boxed{x = \frac{5}{3}}$

$\boxed{y = z = \frac{2}{3}}$

$\vec{C} = \frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$

$\vec{C} = \frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$

$\boxed{\vec{C} = \frac{1}{3}(5\hat{i} + 2\hat{j} + 2\hat{k})}$

Q) The vector perpendicular to the vectors $4\hat{\imath} - \hat{\jmath} + 3\hat{k}$ and $-2\hat{\imath} + \hat{\jmath} - 2\hat{k}$ whose magnitude is 9:

(a) $3\hat{\imath} + 6\hat{\jmath} - 6\hat{k}$

(b) $3\hat{\imath} - 6\hat{\jmath} + 6\hat{k}$

(c) $-3\hat{\imath} + 6\hat{\jmath} + 6\hat{k}$

(d) None of these

Ans. c

$\vec{a} = 4\hat{i} - \hat{j} + 3\hat{k}$

$\vec{b} = -2\hat{i} + \hat{j} - 2\hat{k}$

Let $\vec{c} = \vec{a} \times \vec{b}$

$\Rightarrow \vec{c}$ is $\perp \vec{a}$

& $\vec{c} \perp \vec{b}$

$\therefore \vec{c} = (4\hat{i} - \hat{j} + 3\hat{k}) \times (-2\hat{i} + \hat{j} - 2\hat{k})$

$\vec{c} = \hat{i}(2-3) + \hat{j}(8-6) + \hat{k}(4-2)$

$\vec{c} = -\hat{i} + 2\hat{j} + 2\hat{k}$

$|\vec{c}| = \sqrt{1^2 + 2^2 + 2^2} = \sqrt{9} = 3$

$\hat{c} = \dfrac{-\hat{i} + 2\hat{j} + 2\hat{k}}{3}$

$\vec{d} = 9\hat{c}$

$\vec{d} = 9\left(\dfrac{-\hat{i} + 2\hat{j} + 2\hat{k}}{3}\right)$

$\boxed{\vec{d} = -3\hat{i} + 6\hat{j} + 6\hat{k}}$

Q) Find $\vec{a} \times \left(\vec{b} + \vec{c}\right) + \vec{b} \times (\vec{c} + \vec{a}) + \vec{c} \times \left(\vec{a} + \vec{b}\right) = ?$

(a) $\vec{a} + \vec{b} + \vec{c}$

(b) $\vec{a} \cdot \left(\vec{a} + \vec{b} + \vec{c}\right)$

(c) $\left(\vec{a} \cdot \vec{b} \cdot \vec{c}\right) \times \left(\vec{a} + \vec{b} + \vec{c}\right)$

(d) zero

Ans. d

$\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times (\vec{c} + \vec{a}) + \vec{c} \times (\vec{a} + \vec{b}) = ?$

$\vec{a} \times \vec{b} + \vec{a} \times \vec{c} + \vec{b} \times \vec{c} + \vec{b} \times \vec{a} + \vec{c} \times \vec{a} + \vec{c} \times \vec{b}$

$\because \vec{a} \times \vec{b} = -\vec{b} \times \vec{c} \Rightarrow (\vec{a} \times \vec{b}) + (\vec{b} \times \vec{c}) = 0$

& $\vec{b} \times \vec{c} = -\vec{c} \times \vec{b} \Rightarrow (\vec{b} \times \vec{c}) + (\vec{c} \times \vec{b}) = 0$

& $\vec{a} \times \vec{c} = -\vec{c} \times \vec{a} = (\vec{a} \times \vec{c}) + (\vec{c} \times \vec{a}) = 0$

$\therefore (\vec{a} \times \vec{b}) + (\vec{b} \times \vec{c}) + (\vec{b} \times \vec{c}) + (\vec{c} \times \vec{b}) + (\vec{c} \times \vec{a}) + (\vec{a} \times \vec{c})$

$= 0 + 0 + 0$

$= 0$

Q) Find $\left[2\hat{j} \times \left(3\hat{\imath} - 4\hat{k}\right)\right] \cdot \left[\left(\hat{\imath} - 2\hat{k}\right) \times \hat{k}\right] = ?$

(a) 0

(b) $\hat{\imath} + \hat{j} - \hat{k}$

(c) $2\hat{\imath} + \hat{j} - 3\hat{k}$

(d) 16

Ans. a

$$[2\hat{i} \times (3\hat{j} - 4\hat{k})] \cdot [(\hat{j} - 2\hat{k}) \times \hat{k}]$$

$$\Rightarrow [-6\hat{k} - 8\hat{j}] \cdot [-\hat{j} + 0]$$

$$= (-6\hat{k} - 8\hat{j}) \cdot (-\hat{j})$$

$$= 0$$

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Q) Find $\hat{\imath} \cdot (\hat{\jmath} \times \hat{k}) + (\hat{\imath} \times \hat{k}) \cdot \hat{\jmath} = ?$

(a) $2\hat{\imath} + \hat{\jmath}$

(b) 1

(c) 0

(d) $\hat{\imath} + \hat{\jmath} + \hat{k}$

Ans. c

$$K = \hat{j} \cdot (\vec{j} \times \hat{k}) + (\hat{j} \times \vec{k}) \cdot \vec{j} = ?$$

$$K = \hat{j} \cdot (\hat{i}) + (-\vec{j}) \cdot \vec{j}$$

$$K = 1 + (-1)$$

$$K = 1 - 1$$

$$\boxed{K = 0}$$

Q) If $|\vec{a}| = 13$, $\left|\vec{b}\right| = 5$ and $\vec{a} \cdot \vec{b} = 30$, then $\left|\vec{a} \times \vec{b}\right| =$?

(a) 30

(b) $\frac{30}{25}\sqrt{233}$

(c) $\frac{30}{33}\sqrt{193}$

(d) $\frac{65}{13}\sqrt{133}$

Ans. d

$|\vec{a}| = 13$, $|\vec{b}| = 5$; $\vec{a} \cdot \vec{b} = 30$

$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$

$30 = (13)(5) \cos\theta$

$\cos\theta = \frac{6}{13}$

$\sin\theta = \frac{\sqrt{13^2 - 6^2}}{13}$

$\sin\theta = \frac{\sqrt{133}}{13}$

$|\vec{a} \times \vec{b}| = |\vec{a}|\,|\vec{b}| \sin\theta$

~~$\sin\theta =$~~

$|\vec{a} \times \vec{b}| = (13)(5)\left(\frac{\sqrt{133}}{13}\right)$

$$\boxed{|\vec{a} \times \vec{b}| = 5\sqrt{133}}$$

or

$$\boxed{|\vec{a} \times \vec{b}| = \frac{65}{13}\sqrt{133}}$$

Q) Vector $\vec{A}$ & $\vec{B}$ have scalar product 6.00 and their vector product has magnitude +9.00 What is the angle between these two vectors?

(a) $\tan^{-1}(1.5)$

(b) $\tan^{-1}(3)$

(c) $\tan^{-1}(2)$

(d) $\tan^{-1}\left(\frac{2}{3}\right)$

Ans. a

$$\vec{A} \cdot \vec{B} = |\vec{A}|\,|\vec{B}| \cos\theta = 6 \quad \text{— (1)}$$

$$|\vec{A} \times \vec{B}| = |\vec{A}|\,|\vec{B}| \sin\theta = 9 \quad \text{— (2)}$$

$$\frac{(2)}{(1)} = \frac{\sin\theta}{\cos\theta} = \frac{9}{6}$$

$$\tan\theta = \frac{3}{2}$$

$$\tan\theta = 1.5$$

$$\boxed{\theta = \tan^{-1}(1.5)}$$

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Chalo Niklo