1) Find the range of the following functions :

(i) $f(x) = 3\sin x - 4\cos x$ (ii) $f(x) = x^2 - 7x + 5$

(iii) $f(x) = \log_2\left(\dfrac{\sin x - \cos x + 3\sqrt{2}}{\sqrt{2}}\right)$ (iv) $f(x) = \dfrac{1}{8 - 3\sin x}$

(v) $y = \dfrac{e^x - e^{-x}}{e^x + e^{-x}},\ x \ge 0$ (vi)

$y = \log_3\left(\log_{1/2}\left(x^2 + 4x + 4\right)\right)$

Solution : (i) $f(x) = 3\sin x - 4\cos x$

$= 5\left(\sin x \cdot \dfrac{3}{5} - \cos x \cdot \dfrac{4}{5}\right) = 5\sin(x + \theta)$, where $\tan\theta = \dfrac{3}{5}$

$\Rightarrow$ Range $(f) = [-5, 5]$

(ii) $f(x) = x^2 - 7x + 5$ $\Rightarrow f(x) = \left(x - \dfrac{7}{2}\right)^2 - \dfrac{29}{4}$ $\Rightarrow$ Range $(f) = \left[\dfrac{-29}{4}, \infty\right)$

(iii) Let $y = \log_2\left(\dfrac{\sin x - \cos x + 3\sqrt{2}}{\sqrt{2}}\right)$

$\Rightarrow 2^y = \sin\left(x - \dfrac{\pi}{4}\right) + 3 \Rightarrow -1 \le 2^y - 3 \le 1$

$\Rightarrow 2 \le 2^y \le 4 \Rightarrow y \in [1, 2]$

(iv) $f(x) = \dfrac{1}{8 - 3\sin x}$

$-1 \le \sin x \le 1$

$\Rightarrow -3 \le 3\sin x \le 3 \Rightarrow 5 \le 8 - 3\sin x \le 11 \Rightarrow \dfrac{1}{11} \le \dfrac{1}{8 - 3\sin x} \le \dfrac{1}{5}$

$\therefore$ Range $(f) = \left[\dfrac{1}{11}, \dfrac{1}{5}\right]$

(v) $y = \dfrac{e^x - e^{-x}}{e^x + e^{-x}},\ x \ge 0$

Here domain is explicitly stated so we have to consider only those values of x which are non-negative.

$y = \dfrac{e^x - e^{-x}}{e^x + e^{-x}} = \dfrac{e^{2x} - 1}{e^{2x} + 1} = \dfrac{e^{2x} + 1 - 2}{e^{2x} + 1} = 1 - \dfrac{2}{1 + e^{2x}}$ Now,

$1 \le e^{2x} < \infty \quad \forall x \in [0, \infty)$

$\Rightarrow \quad 2 \le 1+e^{2x} < \infty \qquad \Rightarrow \quad 0 < \dfrac{1}{1+e^{2x}} \le \dfrac{1}{2}$

$\Rightarrow \quad 0 < \dfrac{2}{1+e^{2x}} \le 1 \qquad \Rightarrow \quad 1 \ge \dfrac{2}{1+e^{2x}} > 0$

$\Rightarrow \quad 0 \le 1 - \dfrac{2}{1+e^{2x}} < 1 \qquad \Rightarrow \quad \text{Range}(y) = [0, 1)$

(vi) $y = \log_3\left(\log_{1/2}\left(x^2+4x+4\right)\right)$

Since $0 < \log_{1/2}\left(x^2+4x+4\right) < \infty \forall x \in$ Domain (y)

$\Rightarrow \quad -\infty < \log_3\left(\log_{1/2}\left(x^2+4x+4\right)\right) < \infty$

$\Rightarrow \quad$ Range $(y) = (-\infty, \infty)$

2) Solve the following inequalities for real values of x :

(i) $|x+1| < 2$

(ii) $|x-3| > 5$

(iii) $0 < |x-1| \le 3$

(iv) $|x-1| + |2x-3| = |3x-4|$

(v) $\left|\dfrac{x-3}{x-4}\right| \le 1$

Solution : (i) $|x+1| < 2 \quad \Rightarrow \quad -2 < x+1 < 2 \quad \Rightarrow \quad -3 < x < 1 \quad \Rightarrow \quad x \in (-3, 1)$

(ii) $|x-3| > 5$

$\Rightarrow \quad x-3 < -5$ or $\quad x-3 > 5 \quad \Rightarrow \quad x < -2$ or $x > 8 \quad \Rightarrow \quad x \in (-\infty, -2) \cup (8, \infty)$

(iii) $0 < |x-1| \le 3$

Here $|x-1| > 0 \quad \Rightarrow \quad x \ne 1$

Also, $|x-1| \le 3 \quad \Rightarrow \quad -3 \le x-1 \le 3,$

$\Rightarrow \quad -2 \le x \le 4, \quad x \ne 1 \quad \Rightarrow \quad x \in [-2,\ 1) \cup (1,\ 4]$

(iv) Since $3x-4 = x-1+2x-3,\ |3x-4| = |x-1| + |2x-3|$

$\Rightarrow \quad (x-1)(2x-3) \ge 0 \quad \Rightarrow \quad x \in (-\infty, 1] \cup [3/2, +\infty)$

(v) $\left|\dfrac{x-3}{x-4}\right| \le 1, \quad x-4 \ne 0 \quad \Rightarrow \quad x \ne 4 \quad \Rightarrow \quad -1 \le \dfrac{x-3}{x-4} \le 1$

Now, $\dfrac{x-3}{x-4} \ge -1 \Rightarrow \dfrac{x-3+x-4}{x-4} \ge 0 \quad \Rightarrow \quad \dfrac{2x-7}{x-4} \ge 0$

$\Rightarrow \quad x \in \left(-\infty, \dfrac{7}{2}\right] \cup (4, \infty) \qquad \ldots \text{(i)}$

Now let us take up, $\dfrac{x-3}{x-4} \le 1$

$\Rightarrow \dfrac{x-3-x+4}{x-4} \le 0 \quad \Rightarrow \quad \dfrac{1}{x-4} \le 0 \quad \Rightarrow \quad x-4$ … (ii)

from (i) and (ii)

$\Rightarrow \quad x \in \left(-\infty, \dfrac{7}{2}\right]$

3) Find Domain and Range of the following functions

(i) $f(x) = \dfrac{x^2-2x+9}{x^2+2x+9}$ (ii) $f(x) = \dfrac{1}{\sqrt{|x|-x}}$

Solution (i) Domain of f is the set of all real values of x for which $f(x)$ is real

Since $x^2+2x+9>0$ for all $x \in R$, therefore, the domain of f is the whole set R.

Range of f is the set of all real values of y for which x is real and a member of domain of f.

Now $y = \dfrac{x^2-2x-9}{x^2+2x+9} \Leftrightarrow yx^2+2yx+9y = x^2-2x+9$

i.e. $(y-1)x^2+(2y+2)x+9(y-1)=0$

Now if $y=1$, then the above equation reduces to $x=0$. i.e. for $x=0$, y takes the value 1. Thus $1 \in$ range.

Further if $y \ne 1$, then $(y-1)x^2+(2y+2)x+9(y-1)=0$ is a quadratic equation in x and has real roots if $(2y+2)^2-36(y-1)^2 \ge 0$

i.e. if $2y^2-5y+2 \le 0$ i.e. if $-1/2 \le y \le 2$

This gives that $[-1/2,\ 2]-\{1\}$ is another part of the range.

Hence the range is $[-1/2,\ 2]$.

(ii) $y = \dfrac{1}{\sqrt{|x|-x}}$.

The domain of f is the set of all real values of x for which y is real.

y is real $\Leftrightarrow |x|-x>0 \Leftrightarrow x<0.$ $\therefore$ the domain is $(-\infty,\ 0)$.

Range is the set of all real values of y for which x is real and $x \in (-\infty,\ 0)$.

Clearly $y>0$ … (1)

$y = \dfrac{1}{\sqrt{|x|-x}} \quad \Rightarrow \sqrt{|x|-x} = \dfrac{1}{y} \Rightarrow |x|-x = \dfrac{1}{y^2} \Rightarrow -2x = \dfrac{1}{y^2}$

Clearly x is real if $y \ne 0$ … (2)

From (1) and (2) x is real if $y>0$.

4) Find the range of each of the following function.

(a) $f(x)=1-|x-2|$ (b) $f(x)=\sqrt{16-x^2}$ (c) $f(x)=\frac{2}{3-x^2}$ (d) $f(x)=\frac{1}{\sqrt{4+3\sin x}}$

OBJECTIVE TYPE QUESTIONS

1. If $f(x+2y,\ x-2y)=xy$, then $f(x,\ y)$ equals

(a) $\frac{x^2-y^2}{8}$ (b) $\frac{x^2-y^2}{4}$ (c) $\frac{x^2+y^2}{4}$ (d) $\frac{x^2-y^2}{2}$

2. The range of the function $f(x)=6^x+3^x+6^{-x}+3^{-x}+2$ is

(a) $[-2,\ \infty)$ (b) $(-2,\ \infty)$ (c) $(6,\ \infty)$ $[6,\ \infty)$

3. If $f\left(x+\frac{1}{x}\right)=x^3+\frac{1}{x^3},\ \ x\neq 0$ then $f(x)$ is

(a) x^3 (b) x^3-3x (c) x^3+3x (d) $3x$

4. The domain of the function $f(x)=\sqrt{(2-|x|)}+\sqrt{(1+|x|)}$

(a) $[2,\ 6]$ (b) $(-2,\ 6]$ (c) $[8,\ 12]$ (d) None of these

5. The domain of the function $f(x)=\sqrt{x-[x]}$ ($[.]$ denotes the greatest integer function) is

(a) R (b) R^+ (c) $R^+\cup\{0\}$ (d) Z

6. If $f(x)=\cos(\log x)$, then the value of $f(x).f(4)-\frac{1}{2}\left[f\left(\frac{x}{4}\right)+f(4x)\right]$

(a) 1 (b) -1 (c) 0 (d) ± 1

7. If $e^{f(x)}=\frac{10+x}{10-x},\ \ x\in(-10,\ 10)$ and $f(x)=kf\left(\frac{200x}{100+x^2}\right)$, then k is equal to

(a) 0.5 (b) 0.6 (c) 0.7 (d) 0.8

8. Domain of definition of the function $f(x)=\frac{3}{4-x^2}+\log_{10}(x^3-x)$, is

(a) $(1,\ 2)$ (b) $(-1,\ 0)\cup(1,\ 2)$

(c) $(1,\ 2)\cup(2,\ \infty)$ (d) $(-1,\ 0)\cup(1,\ 2)\cup(2,\ \infty)$

9. If x is real, then value of expression $\frac{x^2+14x+9}{x^2+2x+3}$ lies between

(a) 5 and 4 (b) 5 and -4 (c) -5 and 4 (d) None of these

10. The range of the function $f(x)=4+\sqrt{x^2-16}$ is

(a) $(0,\ \infty)$ (b) $[4,\ \infty)$ (c) $(4,\ \infty)$ (d) $[-4,\ 4]$

11 The domain of definition $f(x)=\sqrt{\log_{0.4}\left(\frac{x-1}{x+5}\right)}\times\frac{1}{x^2-36}$ is

(a) $(-\infty, 0)\sim\{-6\}$ (b) $(0, \infty)\sim\{1, 6\}$ (c) $(1, \infty)\sim\{6\}$ (d) $[1,\infty)\sim\{6\}$

1. (a)	2. (d)	3. (b)	4. (D)	5. (A)
6. (C)	7. (A)	8. (D)	9. (C)	10. (B)
11. (c)				