Exercise

Units and Dimensions, Measurements & Error

(Physicsaholics)

Physicsaholics Physicsaholics_prateek Physicsaholics.com Unacademy

Exercise-4

(Subjective Type)

For JEE Main & Advanced

Q 1. Find the lengths of the following vectors: $3\hat{\imath} + 2\hat{\jmath} - \hat{k}$ *and* $5\hat{\imath} + 4\hat{\jmath} - 2\hat{k}$

Q 2. State with reasons, whether the following algebraic operations with scalar and vector physical quantities are meaningful :
(A) adding any two scalars, (B)adding a scalar to a vector of the same dimensions
(C) multiplying any vector by any scalar, (D) multiplying any two scalars,

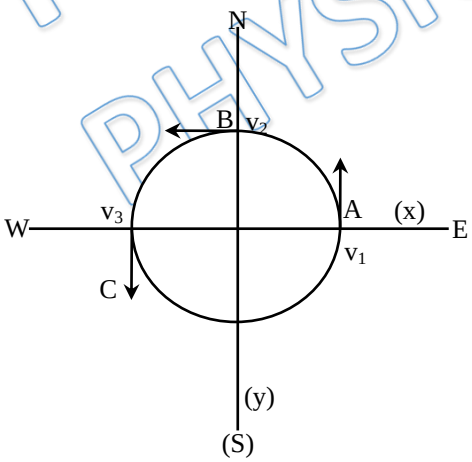
Q 3. Pick out the two scalar quantities in the following list:
Force, work, current, linear momentum, electric fields, average velocity, reaction as per Newton's third law, relative velocity.

Q 4. Prove that $(\vec{A} + \vec{B}).(2\vec{A} - 3\vec{B})$ = $2A^2 - AB\cos\theta - 3B^2$ where θ is the angle between $\vec{A}$ and $\vec{B}$

Q 5. There are three non-zero vectors $\vec{a}, \vec{b}$ and $\vec{c}$. They are related as $\vec{a} + \vec{b} = \vec{c}$ and a + b = c. Show that vector $\vec{a}$ and $\vec{b}$ are parallel.

Q 6. Find the magnitude of resultant of following three forces acting on a particle.
$\vec{F}_1 = 20N$ in eastward direction
$\vec{F}_2 = 20N$ due north east and
$\vec{F}_3 = 20N$ in southward direction

Q 7. A body is moving with uniform speed v on a horizontal circle in anticlockwise direction from A as shown in figure. What is the change in velocity in (a) half revolution (b) first quarter revolution



Q 8. Read each statement below carefully and state with reasons, if it is true or false:

(A) The magnitude of a vector is always a scalar,
(B) each component of a vector is always a scalar,
(C) the total path length is always equal to the magnitude of the displacement vector of a particle.
(D) the average speed of a particle (defined as total path length divided by the time taken to cover the path) is either greater or equal to the magnitude of average velocity of the particle over the same interval of time.
(E) Three vectors not lying in a plane can never add up to give a null vector.

Q 9. Under what condition the sum and difference of two vectors will be equal in magnitude.

Q 10. Two forces of magnitude of 10 N and 20 N are acting at 120º. What is the angle between their resultant and the smaller force.

Q 11. Angle between two coplanar vectors $\vec{r}_1$ and $\vec{r}_2$ is θ and $|\vec{r}_1| = |\vec{r}_2|$. What is the inclination of their resultant with the vectors $\vec{r}_1$ and $\vec{r}_2$.

Q 12. Find the value of p for which the vectors $\vec{a} = 3\hat{\imath} + 2\hat{\jmath} + 9\hat{k}$ and $\vec{b} = 6\hat{\imath} + 4\hat{\jmath} + p\hat{k}$ are (i) perpendicular (ii) parallel.

Q 13. Find a unit vector perpendicular to both **A** = $2\hat{\imath} + \hat{\jmath}$ and **B** = $\hat{\imath} + 2\hat{\jmath}$

Q 14. Prove that the three vectors $6\hat{\imath} + 2\hat{\jmath} + 4\hat{k}, \hat{\imath} + 5\hat{\jmath} - 4\hat{k}$ and $2\hat{\imath} - 2\hat{\jmath} - 2\hat{k}$ are at right angles to one another.

Q 15. Given that $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}; -\vec{b} = \hat{\imath} - \hat{\jmath} + \hat{k}; \vec{c} = \hat{\imath} + \hat{\jmath} - \hat{k}$, evaluate
(i) $(\vec{a}.\vec{b}) + (\vec{b}.\vec{c}) + (\vec{c}.\vec{a})$ (ii) $(\vec{a}.\vec{c})\vec{c} + (\vec{c}.\vec{b})\vec{a}$.

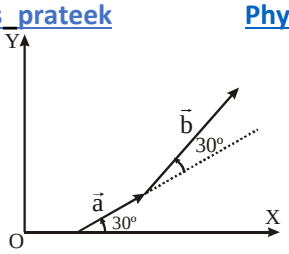
Q 16. A 50 kg block is placed on an inclined plane with an angle of 30º. Then find the components of the weight (i) perpendicular (ii) parallel to the inclined plane.

Q 17. Find the unit vector perpendicular to the pair of vectors $\hat{\imath} - \hat{\jmath} + \hat{k}, \hat{\imath} + 2\hat{\jmath} - \hat{k}$

Q 18. Verify that $\vec{b} \times \vec{a} = -(\vec{a} \times \vec{b})$ where:
(i) $\vec{a} = \hat{\imath} + \hat{\jmath}$ and $\vec{b} = 3\hat{\imath} - \hat{\jmath} + \hat{k}$ (ii) $\vec{a} = \hat{\imath} + \hat{\jmath} + 3\hat{k}$ and $\vec{b} = \hat{\imath} - \hat{\jmath} + 3\hat{k}$

Q 19. If the sum of two unit vector is a unit vector, then find the magnitude of their difference?

Q 20. Two vectors $\vec{a}$ and $\vec{b}$ of magnitude 10 unit and 20 unit respectively are shown in the figure.



(A) write the components of each vector.
(B) Write the vector in the form of their components.
(C) Write the components of $\vec{a}+\vec{b}$ and find $|\vec{a}+\vec{b}|$. Also find the angle made by $(\vec{a}+\vec{b})$ with the x-axis.

Q 21. Five equal forces of 10N are applied at one point and all are lying in one plane. If the angles between them are equal, then find the resultant of these forces.

Q 22. Given that $\vec{a}.\vec{b}=\vec{a}.\vec{c}, \vec{a}\times\vec{b}=\vec{a}\times\vec{c}$ and v is a non-zero vector. Show that $\vec{b}=\vec{c}$.

Q 23. The vector $(\vec{a}+3\vec{b})$ is perpendicular to $(7\vec{a}-5\vec{b})$ and $(\vec{a}-4\vec{b})$ is perpendicular to $(7\vec{a}-2\vec{b})$.
Find the angle between $\vec{a}$ & $\vec{b}$.

Q 24. If $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$, then show that , $(\vec{a}+\vec{b}).(\vec{a}+\vec{b})=|\vec{a}|^2+|\vec{b}|^2$.

Q 25. Find the area of a parallelogram formed from the vectors $\vec{A}=\hat{\imath}-2\hat{\jmath}+3\hat{k}$ and $\vec{B}=3\hat{\imath}-2\hat{\jmath}+\hat{k}$ as adjacent sides.

Q 26. Given that $\vec{A}+\vec{B}+\vec{C}=\vec{0}$. Out of three vectors two are equal in magnitude and the magnitude of third vector is $\sqrt{2}$ times that of either of the two having equal magnitude. Find the angle between vectors.

Q 27. $\hat{\imath}$ and $\hat{\jmath}$ are unit vectors along x–axis and y–axis respectively. What is the magnitude and direction of the vectors $\hat{\imath}+\hat{\jmath}$, and $\hat{\imath}-\hat{\jmath}$? What are the components of a vector $\vec{A}=2\hat{\imath}+3\hat{\jmath}$ along the directions of $\hat{\imath}+\hat{\jmath}$ and $\hat{\imath}-\hat{\jmath}$?

Answer Key

Ans 1. $\sqrt{14}, \sqrt{45}$.

Ans 2. Only (C) and (D) are permissible

Ans 3. Work, current

Ans 6. $\boldsymbol{20\sqrt{3}N}$

Ans 7. (a) $|\Delta \mathbf{v}|$ = 2v directed towards negative y-axis (b) $|\Delta \mathbf{v}| = \sqrt{2}$v and directed towards south-west

Ans 8. . **(A)**T **(B)** F **(C)** F **(D)** T **(E)** T

Ans 9. **A** $\perp$ **B**

Ans 10. 90º

Ans 11. $\theta/2$

Ans 12. $(\boldsymbol{i}) -\frac{\mathbf{26}}{\mathbf{9}}$, $(\boldsymbol{ii})\mathbf{18}$

Ans 15. (i) 1 (ii) $2(\hat{\imath} + \hat{\jmath})$

Ans 16. (i) $250\sqrt{3}N$ (ii) 250 N

Ans 17. $\pm\left(\frac{\mathbf{1}}{\sqrt{\mathbf{14}}}\hat{\boldsymbol{\imath}} + \frac{\mathbf{2}}{\sqrt{\mathbf{14}}}\hat{\boldsymbol{\jmath}} + \frac{\mathbf{3}}{\sqrt{\mathbf{14}}}\hat{\boldsymbol{k}}\right)$

Ans 19. $\boldsymbol{\sqrt{3}}$

Ans 20. (A) $a_x = 5\sqrt{3}$; $a_y = 5$; $b_x = 5\sqrt{3}$; $b_y = 10$

(B) $\vec{a} = 5\sqrt{3}\hat{\imath} + 5\hat{\jmath}$; $\vec{b} = 10\hat{\imath} + 10\sqrt{3}\hat{\jmath}$

(C) $\vec{r} = (5\sqrt{3} + 10)\hat{\imath} + (5 + 10\sqrt{3})\hat{\jmath}$; $r_x = (5\sqrt{3} + 10)$; $r_y = (5 + 10\sqrt{3})$; $\alpha = \tan^{-1}\left(\frac{1+2\sqrt{3}}{\sqrt{3}+2}\right)$

Ans 21. $\vec{\boldsymbol{F}}_{\boldsymbol{net}} = \mathbf{0}$

Ans 23. 60º

Ans 25. $\mathbf{4\sqrt{6}}$

Ans 26. 90°, 135°, 135°

Ans 27. $\sqrt{2}$, 45º with the x -axis ; $\sqrt{2}$, – 45º with the x-axis, $(5/\sqrt{2}, -1/\sqrt{2})$