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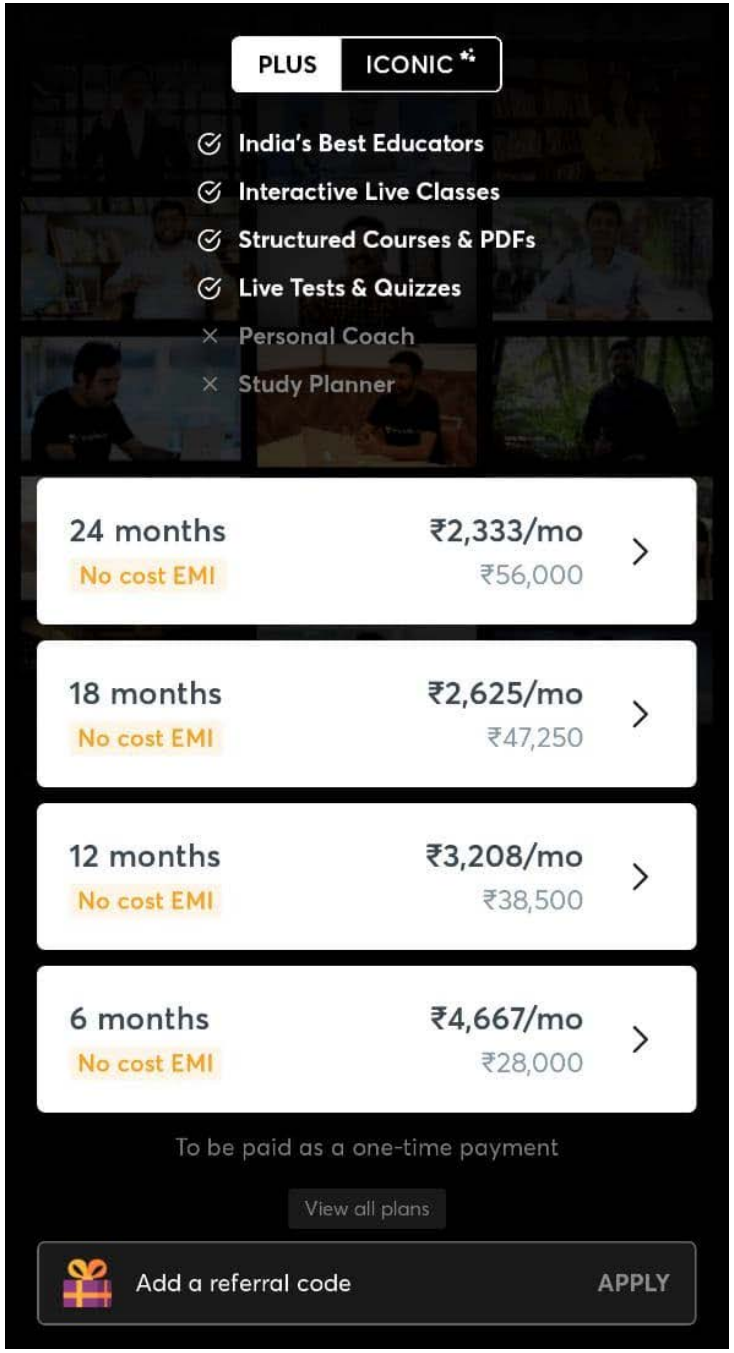

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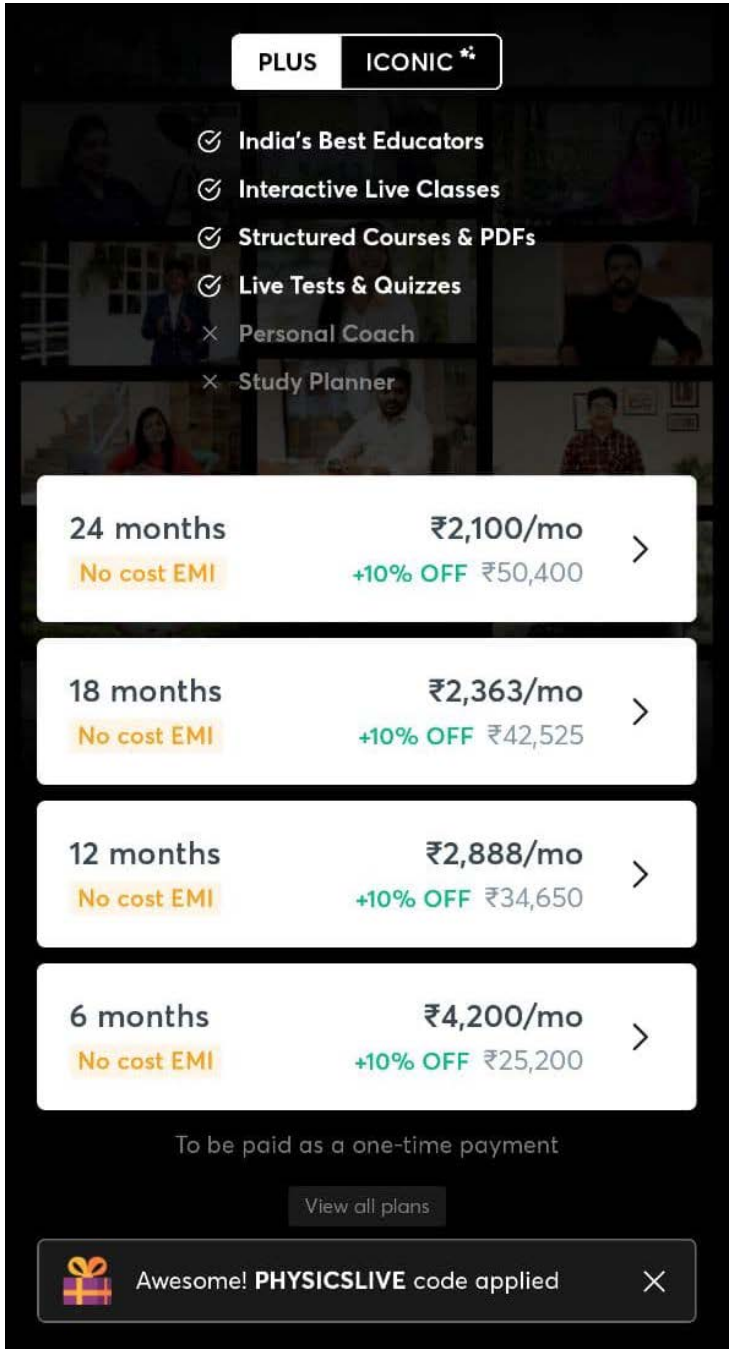

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IIT JEE
Physics DPP

DPP-5 Basic Math: Differentiation (Product rule, Quotient rule, Chain rule, Double Derivatives)

By Physicsaholics Team

Q) Differentiate $y = \ln x^2$ w.r.t. 'x':

(a) $\frac{dy}{dx} = \frac{1}{x}$

(b) $\frac{dy}{dx} = 2$

(c) $\frac{dy}{dx} = \frac{2}{x}$

(d) None of these

Ans. c

$$y = \ln(x^2)$$

$$\frac{dy}{dx} = \frac{1}{x^2}\left(\frac{d}{dx}(x^2)\right)$$

$$\frac{dy}{dx} = \frac{1}{x^2}(2x)$$

$$\boxed{\frac{dy}{dx} = \frac{2}{x}}$$

Q) Find the value of k at x= 2, where $k = \frac{dy}{dx}$, and $y = \ln x^2$:

(a) $k = 2$

(b) $k = 1$

(c) $k = \frac{2}{x}$

(d) None of these

Ans. b

$$y = \ln(x^2)$$

$$\frac{dy}{dx} = \frac{1}{x^2}(2x)$$

$$\frac{dy}{dx} = \frac{2}{x}$$

$$\text{So, } k = \left(\frac{dy}{dx}\right)_{x=3} = \frac{2}{2}$$

$$\boxed{k = 1}$$

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Q) Differentiate $y = e^{x^2}$ w.r.t. 'x':

(a) $\frac{dy}{dx} = 2xe^{x^2}$

(b) $\frac{dy}{dx} = e^{x^2}$

(c) $\frac{dy}{dx} = 2e^{x}$

(d) None of these

Ans. a

$$y = e^{x^2}$$

$$\frac{dy}{dx} = e^{x^2} \frac{d}{dx}(x^2)$$

$$\frac{dy}{dx} = e^{x^2}(2x)$$

$$\boxed{\frac{dy}{dx} = 2x e^{x^2}}$$

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Q) Differentiate $y = ae^x$ w.r.t. 'x'(where a = constant):

(a) $\frac{dy}{dx} = axe^x$

(b) $\frac{dy}{dx} = a$

(c) $\frac{dy}{dx} = ae^x$

(d) None of these

Ans. c

$$y = a e^x$$

$$\frac{dy}{dx} = a\left(\frac{d}{dx}(e^x)\right) + e^x\left(\frac{d}{dx}(a)\right)$$

$$\frac{dy}{dx} = a e^x + 0$$

$$\boxed{\frac{dy}{dx} = a e^x}$$

Q) Differentiate $F(x) = (x^2 - 1)(x + 5)$, w.r.t. 'x':

(a) $F'(x) = 3x^2 + 10x - 1$

(b) $F'(x) = x^2 - 10x - 1$

(c) $F'(x) = (2x)(x)$

(d) None of these

Ans. a

$$F(x) = (x^2-1)(x+5)$$

$$F'(x) = (2x)(x+5) + (x^2-1)(1)$$

$$= (2x)(x+5) + x^2-1$$

$$= 2x^2 + 10x + x^2 - 1$$

$$\boxed{F'(x) = 3x^2 + 10x - 1}$$

Q) Differentiate $F(x) = \sin x \cos x$, w.r.t. 'x':

(a) $F'(x) = 1$

(b) $F'(x) = \cos^2 x - \sin^2 x$

(c) $F'(x) = \cos x - \sin x$

(d) None of these

Ans. b

$$F(x) = \sin x \cdot \cos x$$

$$F'(x) = \frac{d}{dx}(\sin x)\cdot(\cos x) + \sin x \cdot \frac{d}{dx}(\cos x)$$

$$= \cos x \cdot \cos x + \sin x\,(-\sin x)$$

$$\boxed{F'(x) = \cos^2 x - \sin^2 x}$$

Q) Differentiate $y = x^2 \ln x$ w.r.t. 'x':

(a) $\frac{dy}{dx} = x(2 \ln x + 1)$

(b) $\frac{dy}{dx} = \ln x + 2x$

(c) $\frac{dy}{dx} = x^2 \ln x + 1$

(d) None of these

Ans. a

$$y = x^2 \ln x$$

$$\frac{dy}{dx} = (2x) \ln x + x^2 \left(\frac{1}{x}\right)$$

$$\boxed{\frac{dy}{dx} = x(2 \ln x + 1)}$$

Q) Differentiate $y = \frac{e^x}{x}$, w.r.t. 'x':

(a) $\frac{dy}{dx} = -\frac{e^x}{x^2}$

(b) $\frac{dy}{dx} = \frac{e^x}{x^2}(x+1)$

(c) $\frac{dy}{dx} = \frac{e^x}{x^2}(x-1)$

(d) None of these

Ans. c

$$y = \frac{e^x}{x}$$

$$\frac{dy}{dx} = \frac{x\left(\frac{d}{dx}e^x\right) - e^x\left(\frac{d}{dx}(x)\right)}{x^2}$$

$$\frac{dy}{dx} = \frac{xe^x - e^x}{x^2}$$

$$\boxed{\frac{dy}{dx} = \frac{e^x}{x^2}(x-1)}$$

Q) Differentiate $y = \frac{\sin x}{\cos x}$, w.r.t. 'x':

(a) $\frac{dy}{dx} = \cos^2 x$

(b) $\frac{dy}{dx} = \frac{\cos^2 x - \sin^2 x}{\cos^2 x}$

(c) $\frac{dy}{dx} = \sec^2 x$

(d) None of these

Ans. c

$$y = \frac{\sin x}{\cos x}$$

$$\frac{dy}{dx} = \frac{\cos x\left(\frac{d}{dx}\sin x\right) - \sin x\left(\frac{d}{dx}\cos x\right)}{(\cos x)^2}$$

$$\frac{dy}{dx} = \frac{\cos x\,(\cos x) - \sin x\,(-\sin x)}{(\cos x)^2}$$

$$\frac{dy}{dx} = \frac{\cos^2 x + \sin^2 x}{(\cos x)^2}$$

$$\boxed{\frac{dy}{dx} = \frac{1}{\cos^2 x} = \sec^2 x}$$

Q) Differentiate $y = \frac{x}{\ln x}$, w.r.t. 'x':

(a) $\frac{dy}{dx} = 1$

(b) $\frac{dy}{dx} = \frac{\ln x - 1}{(\ln x)^2}$

(c) $\frac{dy}{dx} = \frac{1}{(\ln x)^2}$

(d) None of these

Ans. b

$$y = \frac{x}{\ln x}$$

$$\frac{dy}{dx} = \frac{\ln(x)\left(\frac{d}{dx}(x)\right) - x\left(\frac{d}{dx}(\ln x)\right)}{(\ln x)^2}$$

$$\frac{dy}{dx} = \frac{\ln x - x\left(\frac{1}{x}\right)}{(\ln x)^2}$$

$$\boxed{\frac{dy}{dx} = \frac{\ln x - 1}{(\ln x)^2}}$$

Q) Differentiate $y = \frac{6x^2}{2-x}$, w.r.t. 'x':

(a) $\frac{dy}{dx} = \frac{24x-6x^2}{(2-x)^2}$

(b) $\frac{dy}{dx} = \frac{6x^3-12x^2+24x}{(2-x)^2}$

(c) $\frac{dy}{dx} = \frac{24x}{(2-x)^2}$

(d) None of these

Ans. a

$$y = \frac{6x^2}{2-x}$$

$$\frac{dy}{dx} = \frac{(2-x)(12x) - 6x^2(0-1)}{(2-x)^2}$$

$$\frac{dy}{dx} = \frac{24x - 12x^2 + 6x^2}{(2-x)^2}$$

$$\boxed{\frac{dy}{dx} = \frac{24x - 6x^2}{(2-x)^2}}$$

Q) Find double derivative of $y = x^3 - x^2 + x - 1$, w.r.t. 'x'

(a) $\frac{d^2y}{dx^2} = 3x^2 - 2x + 1$

(b) $\frac{d^2y}{dx^2} = 6x - 2$

(c) $\frac{d^2y}{dx^2} = 6$

(d) None of these

Ans. b

$$y = x^3 - x^2 + x - 1$$

$$\frac{dy}{dx} = 3x^2 - 2x + 1$$

$$\boxed{\frac{d^2y}{dx^2} = 6x - 2}$$

Q) Find value of $\frac{d^2y}{dx^2}$ at $x = \frac{\pi}{2}$, if $y = \sin x$:

(a) $\frac{d^2y}{dx^2} = -1$

(b) $\frac{d^2y}{dx^2} = 1$

(c) $\frac{d^2y}{dx^2} = zero$

(d) $\frac{d^2y}{dx^2} = 2$

Ans. a

$$y = \sin x$$

$$\frac{dy}{dx} = \cos x$$

$$\frac{dy}{dx} = -\sin x$$

at, $x = \pi/2$

$$\left(\frac{dy}{dx}\right)_{x=\pi/2} = -\sin\left(\frac{\pi}{2}\right)$$

$$\boxed{\left(\frac{dy}{dx}\right)_{x=\pi/2} = -1}$$

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Q) Find $\frac{d^2y}{dx^2}$, if $y = e^x$:

(a) $\frac{d^2y}{dx^2} = xe^x$

(b) $\frac{d^2y}{dx^2} = e^x + 1$

(c) $\frac{d^2y}{dx^2} = e^x$

(d) None of these

Ans. c

$$y = e^x$$

$$\frac{dy}{dx} = \frac{d(e^x)}{dx} = e^x$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(e^x) = e^x$$

$$\therefore \boxed{\frac{d^2y}{dx^2} = e^x}$$

Q) Find $\frac{d^2y}{dx^2}$, if $y = \ln x$:

(a) $\frac{d^2y}{dx^2} = -x^2$

(b) $\frac{d^2y}{dx^2} = -\frac{1}{x^2}$

(c) $\frac{d^2y}{dx^2} = \frac{1}{x^2}$

(d) None of these

Ans. b

$$y = \ln x$$

$$\frac{dy}{dx} = \frac{1}{x} = x^{-1}$$

$$\frac{d^2y}{dx^2} = (-1)\,x^{-1-1}$$

$$\frac{d^2y}{dx^2} = (-1)x^{-2}$$

$$\boxed{\frac{d^2y}{dx^2} = \frac{-1}{x^2}}$$

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Chalo Niklo