SL LONEY

EXAMPLE:- XIX

Q1 to Q9

SOLUTUONS

Q1 → Q9

① $\sin^2 72° - \sin^2 60° = \frac{\sqrt5 - 1}{8}$

$= \left(\frac{\sqrt{10+2\sqrt5}}{4}\right)^2 - \left(\frac{\sqrt3}{2}\right)^2$

$= \frac{10+2\sqrt5}{16} - \frac{3}{4}$

$= \frac{10+2\sqrt5 - 12}{16}$

$= \frac{2\sqrt5 - 2}{16} = \frac{\cancel{2}(\sqrt5-1)}{\cancel{2}\times 8}$

$= \frac{\sqrt5-1}{8}$ LHS = RHS

② $\cos^2 48° - \sin^2 12° = \frac{\sqrt5+1}{8}$

LHS

$= \cos(48-12)\cos(48+12)$

$= \cos 36° \cdot \cos 60°$

$= \frac{\sqrt5+1}{4} \times \frac{1}{2} = \frac{\sqrt5+1}{8}$ LHS = RHS

③ $\cos 12° + \cos 60° + \cos 84° = \cos 24° + \cos 48°$

LHS

$\cos 12° + (\cos 84° + \cos 60°)$

$= \cos 12° + 2\cos 72° \cdot \cos 12°$

$= (1 + 2\sin 18°)\cos 12°$

$= \left(1 + \frac{\cancel{2}(\sqrt5-1)}{\cancel{4}2}\right)\cos 12°$

$= 1 + \frac{\sqrt5-1}{2} \cos 12°$

$= \frac{\sqrt5+1}{2} \cos 12°$

RHS

$\cos 24° + \cos 48°$

$= 2\cos 36° \cdot \cos 12°$

$= 2\left(\frac{\cancel{2}(\sqrt5+1)}{\cancel{4}2}\right)\cos 12°$

$= \frac{\sqrt5+1}{2} \cdot \cos 12°$

LHS = RHS

4:) $\sin\frac{\pi}{5} \cdot \sin\frac{2\pi}{5} \cdot \sin\frac{3\pi}{5} \cdot \sin\frac{4\pi}{5} = \frac{5}{16}$

LHS

$$\left(\sin\frac{\pi}{5} \cdot \sin\frac{2\pi}{5} \cdot \sin 4\frac{\pi}{5}\right) \sin\frac{3\pi}{5}$$

$$= \left(\frac{1}{2^4} \times \frac{\sin 2^4\frac{\pi}{5}}{\sin\frac{\pi}{5}}\right) \sin\frac{3\pi}{5}$$

$$= \frac{1}{16} \times \frac{\sin\frac{16\pi}{5}}{\sin\frac{\pi}{5}}$$

4:) $\sin\frac{\pi}{5} \cdot \sin\frac{2\pi}{5} \cdot \sin\frac{3\pi}{5} \cdot \sin\frac{4\pi}{5} = \frac{5}{16}$

LHS

$$\sin 36 \cdot \sin 72 \cdot \sin 108^\circ \cdot \sin 144^\circ$$

$$= \sin 36^\circ \cdot \cos 18^\circ \cdot \cos 18^\circ \cdot \sin 36^\circ$$

$$= \sin^2 36^\circ \cdot \cos^2 18^\circ$$

$$= \left(\frac{\sqrt{10-2\sqrt{5}}}{4}\right)^2 \times \left(\frac{\sqrt{10+2\sqrt{5}}}{4}\right)^2$$

$$= \frac{10-2\sqrt{5}}{16} \times \frac{10+2\sqrt{5}}{16}$$

$$= \frac{100-20}{16 \times 16} = \frac{\cancel{80}^{5}}{\cancel{16} \times 16} = \frac{5}{16} \qquad \text{LHS} = \text{RHS}$$

5:) $\sin\frac{\pi}{10} + \sin\frac{13\pi}{10} = -\frac{1}{2}$

LHS

$$\sin 18^\circ + \sin 234^\circ$$

$$= \sin 18^\circ + \sin(270 - 36^\circ)$$

$$= \sin 18^\circ - \cos 36^\circ$$

$$= \frac{\sqrt{5}-1}{4} - \frac{\sqrt{5}+1}{4} = \frac{\cancel{\sqrt{5}} - 1 - \cancel{\sqrt{5}} - 1}{4}$$

$$= -\frac{2}{4} = \frac{-1}{2}$$

LHS = RHS

(6) $\sin\frac{\pi}{10} \cdot \sin\frac{13\pi}{10} = -\frac{1}{4}$

LHS

$= \sin\frac{\pi}{10} \cdot \sin\left(\pi + \frac{3\pi}{10}\right)$

$= -\sin\frac{\pi}{10} \cdot \sin\frac{3\pi}{10}$

$= -\sin 18^\circ \cdot \sin 54^\circ$

$= \frac{1-\sqrt{5}}{4} \times \frac{\sqrt{5}+1}{4}$

$\therefore \frac{1-5}{16} = -\frac{1}{4}$ LHS = RHS

7) $\tan 6^\circ \cdot \tan 42^\circ \cdot \tan 66^\circ \cdot \tan 78^\circ = 1$

LHS

$\tan x \cdot \tan(60-x) \cdot \tan(60+x) = \tan 3x$

$\boxed{\text{Let } x = 6^\circ}$

$\Rightarrow \tan 6^\circ \cdot \tan 54^\circ \cdot \tan 66^\circ = \tan 18^\circ$ ——— (1)

$\boxed{\text{Let } x = 18^\circ}$

$\tan 18^\circ \cdot \tan 42^\circ \cdot \tan 78^\circ = \tan 54^\circ$ ——— (2)

eq^n (1) × (2)

$\tan 6^\circ \cdot \cancel{\tan 54^\circ} \cdot \tan 66^\circ \cdot \cancel{\tan 18^\circ} \cdot \tan 42^\circ \cdot \tan 78^\circ = \cancel{\tan 18^\circ} \cdot \cancel{\tan 54^\circ}$

$\Rightarrow \tan 6^\circ \cdot \tan 66^\circ \cdot \tan 42^\circ \cdot \tan 78^\circ = 1$

LHS = RHS

8.) $\cos\frac{\pi}{15}\cdot\cos\frac{2\pi}{15}\cdot\cos\frac{3\pi}{15}\cdot\cos\frac{4\pi}{15}\cdot\cos\frac{5\pi}{15}\cdot\cos\frac{6\pi}{15}\cdot\cos\frac{7\pi}{15}=\frac{1}{2^7}$

LHS

$\cos\frac{\pi}{15}\cdot\cos\frac{2\pi}{15}\cdot\cos\frac{4\pi}{15}\left(\cos\frac{3\pi}{15}\cdot\cos\frac{6\pi}{15}\right)\cdot\left(-\cos\frac{8\pi}{15}\right)$

$=\frac{-1}{2}\left[\cos\frac{\pi}{15}\cdot\cos\frac{2\pi}{15}\cdot\cos\frac{4\pi}{15}\cdot\cos\frac{8\pi}{15}\right]\times\frac{1}{2}\left(\cos\frac{3\pi}{15}\cdot\cos\frac{6\pi}{15}\right)$

$\frac{-1}{2}\times\frac{2^3}{2^4\sin\frac{\pi}{15}}\left[2\sin\frac{\pi}{15}\cdot\cos\frac{\pi}{15}\cdot\cos\frac{2\pi}{15}\cdot\cos\frac{4\pi}{15}\cdot\cos\frac{8\pi}{15}\right]\times\frac{2}{2^2\times\sin\frac{3\pi}{15}}\left(2\sin\frac{3\pi}{15}\cdot\cos\frac{3\pi}{15}\cdot\cos\frac{6\pi}{15}\right.$

π Radian

$$\Rightarrow \frac{2^3}{2^5 \sin\frac{\pi}{15}} \left[\sin\frac{2\pi}{15}\cdot\cos\frac{2\pi}{15}\cdot\cos\frac{4\pi}{15}\cdot\cos\frac{8\pi}{15}\right] \times \frac{2}{4\sin\frac{3\pi}{15}}\left(\sin\frac{6\pi}{15}\cdot\cos\frac{6\pi}{15}\right)$$

$$= \frac{2}{32\sin\frac{\pi}{15}}\left[\sin\frac{8\pi}{15}\cdot\cos\frac{8\pi}{15}\right]\times\frac{\sin\frac{12\pi}{15}}{4\sin\frac{3\pi}{15}}$$

$$= \frac{1}{32\sin\frac{\pi}{15}}\left[\sin\frac{16\pi}{15}\right]\times\frac{\sin\frac{12\pi}{15}}{4\sin\frac{3\pi}{15}}$$

$$= \frac{\sin\left(\pi+\frac{\pi}{15}\right)}{128\sin\frac{\pi}{15}}\times\frac{\sin\left(\pi-\frac{3\pi}{15}\right)}{\sin\frac{3\pi}{15}}$$

$$= \frac{-\sin\frac{\pi}{15}}{128\sin\frac{\pi}{15}}\times\frac{\cancel{\sin\frac{3\pi}{15}}}{\cancel{\sin\frac{3\pi}{15}}}$$

$$\frac{1}{128} = \frac{1}{2^7}$$

LHS = RHS

9.) $16\cos\frac{2\pi}{15}\cdot\cos\frac{4\pi}{15}\cdot\cos\frac{8\pi}{15}\cdot\cos\frac{14\pi}{15} = 1$

LHS

$$16\times\left(\cos\frac{2\pi}{15}\cdot\cos\frac{4\pi}{15}\cdot\cos\frac{8\pi}{15}\right)\cos\frac{14\pi}{15}$$

$$= 16\times\left(\frac{1}{8}\times\frac{\sin\frac{16\pi}{15}}{\sin\frac{2\pi}{15}}\right)\cos\frac{14\pi}{15}$$

$$= 2\left(\frac{\sin\pi+\frac{\pi}{15}}{\sin\frac{2\pi}{15}}\right)\cdot\cos\frac{14\pi}{15}$$

$$= 2\times -\sin\left(\frac{\pi}{15}\right)\cdot\cos\left(\pi-\frac{\pi}{15}\right)\times\frac{1}{\sin\frac{2\pi}{15}}$$

$$= 2\times -\sin\frac{\pi}{15}\cdot -\cos\frac{\pi}{15}\times\frac{1}{\sin\frac{2\pi}{15}}$$

$$= 2\sin\frac{\pi}{15}\cdot\cos\frac{\pi}{15}\times\frac{1}{\sin\frac{2\pi}{15}}$$

$$= \cancel{\sin\frac{2\pi}{15}}\times\frac{1}{\cancel{\sin\frac{2\pi}{15}}} = 1$$

LHS = RHS