Homework

DPP - 2 (TRIGONOMETRY)

(1) If $\sin\alpha = 12/13$ where $0<\alpha<\pi/2$ and $\cos\beta$ $-3/5$ where $\pi<\beta<3\pi/2$ then the value of $\cos(\alpha+\beta)$ is

Solution:

$\sin\alpha = \frac{12}{13}$ $\cos\beta = \frac{-3}{5}$

$\sin\beta = \frac{4}{5}$ $\cos\alpha = \frac{5}{13}$

$$\cos(\alpha+\beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$$

$$= \left(\frac{5}{13}\right)\left(\frac{-3}{5}\right) + \left(\frac{12}{13}\right)\left(\frac{4}{5}\right)$$

$$= \frac{-15}{65} + \frac{48}{65}$$

$$= \frac{-15+48}{65}$$

$$= \frac{33}{65}$$

$$\cos(\alpha+\beta) = \frac{33}{65}$$

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② The value of $\sin 50^\circ - \sin 70^\circ + \sin 10^\circ$ is equal to

Solution:→

$\sin 50^\circ - \sin 70^\circ + \sin 10^\circ$

$\sin 50^\circ - \sin 70^\circ + \sin 10^\circ = (\sin 50^\circ + \sin 10^\circ) - \sin 70^\circ$

$= 2\sin\left(\frac{50+10}{2}\right)^\circ \cos\left(\frac{50-10}{2}\right)^\circ - \sin 70^\circ$

$= 2\sin\left(\frac{60^\circ}{2}\right)\cos\left(\frac{40}{2}\right) - \sin 70$

$= 2\sin 30^\circ \cos 20^\circ - \sin 70.$

$= \cos(90-70)^\circ - \sin 70^\circ$

$= \cancel{\sin 70^\circ} - \cancel{\sin 70^\circ}$

$\sin 50^\circ - \sin 70^\circ + \sin 10^\circ = 0$

③ The value of $(\sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ)$ is equal to

Solution: Given $\sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ$

$= \frac{\sqrt{3}}{2} \times \sin 20^\circ \sin(60^\circ - 20^\circ) \sin(60^\circ + 20^\circ)$

$= \frac{\sqrt{3}}{2} \times \sin 20^\circ (\sin^2 60^\circ - \sin^2 20^\circ$

$$= \frac{\sqrt{3}}{2} \times \sin 20^\circ \left(\frac{3}{4} - \sin^2 20^\circ\right)$$

$$= \frac{\sqrt{3}}{2} \times \left(\frac{3\sin 20^\circ - 4\sin^3 20^\circ}{4}\right)$$

$$= \frac{\sqrt{3}}{2} \times \left(\frac{\sin 3(20^\circ)}{4}\right)$$

$$= \frac{\sqrt{3}}{2} \times \sin 60^\circ \times \frac{1}{4}$$

$$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} \times \frac{1}{4}$$

$$= \frac{3}{4} \times \frac{1}{4}$$

$$= \frac{3}{16}$$

$\sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ = 3/16.$

(4) If $\sin\theta + \operatorname{cosec}\theta = 2$ then $\sin^2\theta + \operatorname{cosec}^2\theta$ is equal to.

Solution:

Given. $\sin\theta + \operatorname{cosec}\theta = 2$

squaring on both sides

$\sin^2\theta +$

$(\sin^2\theta + \operatorname{cosec}^2\theta) = 2^2$

$$\sin^2\theta + \operatorname{cosec}^2\theta + 2\sin\theta\operatorname{cosec}\theta = 4$$

$$\sin^2\theta + \operatorname{cosec}^2\theta + 2 = 4$$

$$\sin^2\theta + \operatorname{cosec}^2\theta = 4-2$$

$$\boxed{\sin^2\theta + \operatorname{cosec}^2\theta = 2}$$

(5) The expression

$$3\left[\sin^4\left(\frac{3\pi}{2}-\alpha\right)+\sin^4(3\pi+\alpha)\right]-2\left[\sin^6\left(\frac{\pi}{2}+\alpha\right)\right.$$

$$\left.+\sin^6(5\pi-\alpha)\right]$$ is equal to

Solution:

$$\sin^4\left(\frac{3\pi}{2}-\alpha\right)=\cos^4\alpha$$

$$\sin^4(3\pi-\alpha)=\sin^4\alpha$$

$$\sin^6\left(\frac{\pi}{2}+\alpha\right)=\cos^6\alpha,\ \sin^6(5\pi-\alpha)=\sin^6\alpha$$

$$=3[\cos^4\alpha+\sin^4\alpha)-2[\cos^6\alpha+\sin^6\alpha)$$

$$=3[1-2\sin^2\alpha\cos^2\alpha]-2[1-3\sin^2\alpha\cos^2\alpha]$$

$$=3-2$$

$$=1$$

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(6) If $x = \cos 10^\circ \cos 20^\circ \cos 40^\circ$ then the value of x is

Solution: -

Given: $x = \cos 10^\circ \cos 20^\circ \cos 40^\circ$

$$x = \frac{1}{2\sin 10} \, 2\sin 10 \cos 10 \cos 20 \cos 40$$

$$= \frac{2(\sin 20 \cos 20)\cos 40}{2 \times 2 \times \sin 10}$$

$$= \frac{2\sin 40 \cos 40}{2 \times 2 \times 2 \sin 10}$$

$$= \frac{\sin 80}{8 \sin 10}$$

$$= \frac{\sin(90-10)}{8\sin 10} \quad \cdots \boxed{\sin(90-\theta) = \cos\theta}$$

$$= \frac{\cos 10}{8 \sin 10} \cdots$$

$$= \frac{1}{8} \times \frac{\cos 10}{\sin 10}$$

$$= \frac{1}{8} \times \cot 10^\circ$$

$$x = \frac{1}{8} \cot 10^\circ$$

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(7) The value of $(45+\theta)-\cos(45^\circ-\theta)$ is

Solution:

$$\sin(45^\circ+\theta)-\cos(45^\circ-\theta)=(\sin 45^\circ\cos\theta+\cos 45^\circ\sin\theta)-(\cos 45^\circ\cos\theta+\sin 45^\circ\sin\theta)$$

$$=\frac{1}{\sqrt{2}}\cos\theta+\frac{1}{\sqrt{2}}\sin\theta-\left[\frac{1}{\sqrt{2}}\cos\theta+\frac{1}{\sqrt{2}}\sin\theta\right]$$

$$=\cancel{\frac{1}{\sqrt{2}}\cos\theta}+\cancel{\frac{1}{\sqrt{2}}\sin\theta}-\cancel{\frac{1}{\sqrt{2}}\cos\theta}-\cancel{\frac{1}{\sqrt{2}}\sin\theta}$$

$$=0$$

$$\sin(45^\circ+\theta)-\cos(45^\circ-\theta)=0$$

(8) If $\sin x+\cos y=a$ and $\cos x+\sin y=b$ then $\tan\frac{x-y}{2}$ is equal to

Solution:→

$$\sin x+\sin\left(\frac{\pi}{2}-y\right)=a$$

$$\cos x+\cos\left(\frac{\pi}{2}-y\right)=b$$

$$\frac{2\sin\frac{x+(\pi/2)-y}{2}\cos\frac{x-(\pi/2)+y}{2} = a}{2\cos\frac{(x+(\pi/2)-y}{2}\cos\frac{x-(\pi/2)+y}{2} = b}$$

$$\tan\left(\frac{\pi}{4}+\frac{x-y}{2}\right) = \frac{a}{b}$$

$$\frac{1+\cancel{t}an\frac{x-y}{2}}{1-\cancel{t}an\frac{x-y}{2}} = \frac{a}{b}.$$

$$\frac{x-y/2}{x-y/2} = \frac{a}{b}$$

$$\boxed{\tan\frac{x-y}{2} = \frac{a-b}{a+b}}$$

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1) (10) If $\frac{\cos x}{a} = \frac{\cos(x+\theta)}{b} = \frac{\cos(x+2\theta)}{c} = \frac{\cos(x+3\theta)}{d}$

then $\frac{a+c}{b+d}$ is equal to

Solution:→

Let $\frac{\cos x}{a} = \frac{\cos(x+\theta)}{b} = \frac{\cos(x+2\theta)}{c} = \frac{\cos(x+3\theta)}{d} = \frac{1}{K}$

$a = K\cos x, \quad b = K\cos(x+\theta) \quad c = K\cos(x+2\theta)$

$d = K\cos(x+3\theta)$

$\frac{a+c}{b+d} = \frac{K\cos x + K\cos(x+2\theta)}{K\cos(x+\theta) + K\cos(x+3\theta)}$

$= \frac{\cancel{2}\cos(x+\theta)\cancel{\cos\theta}}{\cancel{2}\cos(x+2\theta)\cancel{\cos\theta}}$

$= \frac{\cos(x+\theta)}{\cos(x+2\theta)}$

$= \frac{b}{c}$

$\frac{a+b}{b+d} = \frac{b}{c}$

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(11) If $\cos(x-y) = a\cos(x+y)$ then $\cot x \cot y$ is equal to

Solution:

$$\cos(x-y) = a\cos(x+y)$$

$$\frac{\cos(x-y)}{\cos(x+y)} = a$$

$$\cot x \cot y = \frac{\cos(x-y) + \cos(x+y)}{\cos(x-y) - \cos(x+y)}$$

$$= \frac{a+1}{a-1}$$

$$\cot x \cot y = \frac{a+1}{a-1}$$

(12) If $\cos\theta + \sin\theta = a$, $\cos 2\theta = b$ then

Solution: Given $\cos\theta + \sin\theta = a$

$= 1 + \sin 2\theta = a^2$ squaring on both side

$= \sin 2\theta = a^2 - 1$

$= \cos^2 2\theta = 1 - (a^2-1)^2$

$= b^2 = a^2(2-a^2)$

(16) Let $0 < x < \frac{\pi}{4}$ then $(\sec 2x - \tan 2x)$ equal

Solution:→

$$\sec 2x - \tan 2x = \frac{1}{\cos 2x} - \frac{\sin 2x}{\cos 2x}$$

$$= \frac{\cos^2 x + \sin^2 x - 2\sin x \cos x}{\cos^2 x - \sin^2 x}$$

$$= \frac{(\cos x - \sin)^2}{(\cos x - \sin x)(\cos x + \sin x)}$$

$$= \frac{(\cos x - \sin x)}{(\cos x + \sin x)}$$

$$= \frac{1 - \tan x}{1 + \tan x}$$

$$\sec 2x - \tan 2x \; \tan\left(\frac{\pi}{4} - x\right)$$

(18) $\gamma + \alpha + \beta = \frac{\pi}{2}$ and $\beta + \gamma = \alpha$ the $\tan\alpha$ equal

Solution:

Given $\alpha + \beta = \frac{\pi}{2}$ $\quad \beta + \gamma = \alpha \cdot \alpha = \frac{\pi}{2} - \beta$

$\tan\alpha = \cot\beta \therefore \tan\alpha \tan\beta = 1$

$\beta + \gamma = \alpha$ or $\gamma = \alpha - \beta =$

$\tan\gamma = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha \tan\beta} = \frac{\tan\alpha - \tan\beta}{2}$

$\tan\alpha = \tan\beta + 2\tan\gamma$