WORK BOOK (LOG)
REF CODE MLJSIRLIVE

ASSIGMENT-1

1. **If a and b are positive real numbers other than unity, then the least value of $|\log_b a + \log_a b|$ is** [ ]

A) 0 B) 1 C) 2 D) none of these

2. **If $\log_3 x + \log_9 x^2 + \log_{27} x^3 = 9$, then x equals to** [ ]

A) 3 B) 9 C) 27 D) none of these

3. **If a, b, c are positive real numbers, then $a^{\log b - \log c} \cdot b^{\log c - \log a} \cdot c^{\log a - \log b} =$** [ ]

A) 0 B) 1 C) − 1 D) none of these

4. **If a, b, c are positive real numbers, then the value of $(ab)^{\log\left(\frac{a}{b}\right)} \cdot (bc)^{\log\left(\frac{b}{c}\right)} \cdot (ca)^{\log\left(\frac{c}{a}\right)}$ is** [ ]

A) 0 B) −1 C) 1 D) none of these

5. **If $x = \log_{2a} a$, $y = \log_{3a} 2a$ and $z = \log_{4a} 3a$, then $xyz + 1 =$** [ ]

A) 2yz B) 2xy C) 2zx D) none of these

6. **If a, b, c are positive real numbers, then $\frac{1}{\log_{ab} abc} + \frac{1}{\log_{bc} abc} + \frac{1}{\log_{ca} abc} =$** [ ]

A) 0 B) 1 C) 2 D) none of these

7. **If a, b, c are positive real numbers, then $\frac{1}{\log_a bc + 1} + \frac{1}{\log_b ca + 1} + \frac{1}{\log_c ab + 1} =$** [ ]

A) 0 B) 1 C) 2 D) −1

8. **If $y = a^{\frac{1}{1-\log_a x}}$, $z = a^{\frac{1}{1-\log_a y}}$ and $x = a^k$, then $k =$** [ ]

A) $\frac{1}{a^{1-\log_a z}}$ B) $\frac{1}{1-\log_a z}$ C) $\frac{1}{1+\log_z a}$ D) $\frac{1}{1-\log_z a}$

9. If a = log2, b = log 3, c = log 7 and $6^x = 7^{x+4}$, then x = [ ]

A) $\frac{4b}{c+a-b}$ B) $\frac{4c}{a+b-c}$ C) $\frac{4b}{c-a-b}$ D) $\frac{4a}{a+b-c}$

10. If a = 1 + $\log_x yz$, b = 1 + $\log_y zx$, c = 1 + $\log_z xy$, then ab + bc + ca = [ ]

A) 0 B) 2abc C) abc D) $a^2 + b^2 + c^2$

1. If $\frac{\log x}{a^2+ab+b^2} = \frac{\log y}{b^2+bc+c^2} = \frac{\log z}{c^2+ca+a^2}$, then x^{a-b}. y^{b-c}. Z^{c-a} = [ ]

A) 0 B) - 1 C) 1 D) 2

12. If $a^2 + b^2 - c^2 = 0$, then $\frac{1}{\log_{c+a} b} + \frac{1}{\log_{c-a} b}$ = [ ]

A) 1 B) 2 C) - 1 D) -2

13. If x = $\log_{0.1} 0.001$, y = $\log_9 81$, then $\sqrt{x-2\sqrt{y}}$ = [ ]

A) $3-2\sqrt{2}$ B) $\sqrt{3}-2$ C) $\sqrt{2}-1$ D) $\sqrt{2}-2$

14. If $5^x = (0.5)^y = 1000$, then $\frac{1}{x}-\frac{1}{y}$ = [ ]

A) 1 B) $\frac{1}{2}$ C) $\frac{1}{3}$ D) $\frac{1}{4}$

15. If x = 27 and y = $\log_3 4$, then x^y equals [ ]

A) 64 B) 16 C) $\frac{3}{7}$ D) $\frac{1}{16}$

16. If $\log_8 128$ = [ ]

A) $\frac{7}{3}$ B) 16 C) $\frac{3}{7}$ D) $\frac{1}{16}$

17. If $\frac{\log a}{2} = \frac{\log b}{3} = \frac{\log c}{5}$, then bc = [ ]

A) a B) a^2 C) a^3 D) a^4

18. If $4^x + 2^{2x-1} = 3^{x+\frac{1}{2}} + 3^{x-\frac{1}{2}}$, then x = [ ]

A) $\frac{1}{2}$ B) $\frac{3}{2}$ C) $\frac{5}{2}$ D) 1

19. If log(2a − 3b) = loga − logb, then a = [ ]

A) $\frac{3b^2}{2b-1}$ B) $\frac{3b}{2b-1}$ C) $\frac{b^2}{2b+1}$ D) $\frac{3b^2}{2b+1}$

20. If $\frac{\log x}{2a+3b-5c} = \frac{\log y}{2b+3c-5a} = \frac{\log z}{2c+3a-5b}$, then xyz = [ ]

A) 2 B) 1 C) 0 D) − 1

21. If $\log x = \frac{\log y}{2} = \frac{\log z}{5}$, then $x^4y^3z^{-2}$ = [ ]

A) 2 B) 10 C) 1 D) 0

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2. **If** $\frac{\log 3}{x-y} = \frac{\log 5}{y-z} = \frac{\log 7}{z-x}$**, then** $3^{x+y}\ 5^{y+z}\ 7^{z+x}$ **=** [ ]

A) 0 B) 2 C) 1 D) none of these

23. **If** $\frac{\log_2 a}{2} = \frac{\log_3 b}{3} = \frac{\log_4 c}{4}$ **and** $a^{1/2}\ b^{1/3}\ c^{1/4} = 24$**, then** [ ]

A) a = 24 B) b = 81 C) c = 64 D) c = 256

24. $\frac{\log_2 a}{3} = \frac{\log_2 b}{4} = \frac{\log_2 c}{5\lambda}$ **and** $a^{-3}\ b^{-4}\ c = 1$**, then** λ **=** [ ]

A) 3 B) 4 C) 5 D) −5

25. **If** $a = \log_{24}12$**,** $b = \log_{36}24$ **and** $c = \log_{48}36$**, then abc =** [ ]

A) 2bc − 1 B) 2bc + 1 C) bc − 1 D) bc + 1

26. **If a, b, c are three consecutive positive integers, then log(1 + ca) =** [ ]

A) log b B) $\log\left(\frac{b}{2}\right)$ C) $\log(2b)$ D) 2 log b

27. **If** $\frac{\log_a x}{\log_{ab} x} = 4 + k + \log_a b$**, then k =** [ ]

A) 0 B) 1 C) −2 D) − 3

28. **If** $\frac{\log a}{b-c} = \frac{\log b}{c-a} = \frac{\log c}{a-b}$**, then** $a^a\ b^b\ c^c$ **=** [ ]

A) 0 B) 1 C) abc D) none of these

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9. If $x = \log_a bc$, $y = \log_a ca$, $z = \log_c ab$, then [ ]

A) $\frac{1}{x+1}+\frac{1}{y+1}+\frac{1}{z+1}=1$

B) $\frac{1}{x-1}+\frac{1}{y-1}+\frac{1}{z-1}=1$

C) xyz = x + y + z = 1

D) xyz = 1

30. $\log_2 7$ is [ ]

A) an integer

B) a rational number

C) an irrational number

D) a prime number

Assigment -2

1. The value of $2^{\log_3 5} - 5^{\log_3 2}$ is [ ]

A) 2

B) -1

C) 1

D) 0

2. If a, b, c are the sides of a right angled triangle in which c > a, c − b ≠ 1, c + b ≠ 1, then the value of $\frac{\log_{c+b} a + \log_{c-b} a}{\log_{c+b} a . \log_{c-b} a}$ is [ ]

A) 1

B) 2

C) $\frac{1}{2}$

D) none of these

3. **If $\log_3 a . \log_a x = 4$, then x is equal to** [ ]

A) 64 B) 81 C) a^2 D) none of these

4. **If $\log_4 5 = x$ and $\log_5 6 = y$, then $\log_3 2$ is equal to** [ ]

A) $\frac{1}{2x+1}$ B) $\frac{1}{2y+1}$ C) $2xy+1$ D) $\frac{1}{2xy-1}$

5. **If $a^2 + 4b^2 = 12ab$, then $\log(a + 2b) =$** [ ]

A) $\frac{1}{2}(\log a + \log b - 2)$ B) $\log\frac{a}{2} + \log\frac{b}{2} + \log 2$

C) $\frac{1}{2}(\log a + \log b + 4\log 2)$ D) $\frac{1}{2}(\log a - \log b + 4\log 2)$

6. **The value of $3\log\frac{81}{80} + 5\log\frac{25}{24} + 7\log\frac{16}{15}$ is** [ ]

A) log 2 B) log 3 C) 1 D) 0

7. **If $\log a : \log b : \log c = (b - c) : (c - a) : (a - b)$, then** [ ]

A) $a^b . b^c . c^a = 1$ B) $a^a b^b c^c = 1$

C) $\sqrt[a]{a} . \sqrt[b]{b} . \sqrt[c]{c} = 1$ D) none of these

8. **The solution of $5^{\log_a x} + 5x^{\log_a 5} = 3\ (a > 0)$ is** [ ]

A) $2^{\log_a 5}$ B) $2^{-\log_a 5}$ C) $2^{-\log_5 a}$ D) $2^{\log_5 a}$

9. **The value of $\frac{\log 49\sqrt{7} + \log 25\sqrt{5} - \log 4\sqrt{2}}{\log 17.5}$ is** [ ]

A) 5 B) 2 C) $\frac{5}{2}$ D) $\frac{3}{2}$

10. **The value of $5^{\sqrt{\log_5 7}} - 7^{\sqrt{\log_7 5}}$ is** [ ]

A) log 2 B) 1 C) 0 D) none of these

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1. **The value of** $\dfrac{3+\log 343}{2+\frac{1}{2}\log\left(\frac{49}{4}\right)+\frac{1}{3}\log\left(\frac{1}{125}\right)}$ **is** [ ]

A) 3 B) 2 C) 1 D) $\frac{3}{2}$

12. **If log (a – b) = log a – log b, then the value of 'a' in terms of b is** [ ]

A) $\frac{b^2}{b-1}$ B) $\frac{b^2}{b+1}$ C) $\frac{b+1}{b^2}$ D) $\frac{b-1}{b^2}$

13. **If** $9a^2 + 4b^2 = 18ab$, **then log(3a + 2b) =** [ ]

A) log 5 + log 3 + log a + log 5b B) log 5 + log 3 + log 3a + log b

C) log 5 + log 3 + log b D) none of these

14. **The value of** $\log_5\left(1+\frac{1}{5}\right)+\log_5\left(1+\frac{1}{6}\right)+\log_5\left(1+\frac{1}{7}\right)+\ldots+\log_5\left(1+\frac{1}{624}\right)$ **is** [ ]

A) 5 B) 4 C) 3 D) 2

15. **If** $\log(x-y) - \log 5 - \frac{1}{2}\log x - \frac{1}{2}\log y = 0$, **then** $\frac{x}{y}+\frac{y}{x} =$ [ ]

A) 25 B) 26 C) 27 D) 28

16. **If** $2^{\log_{10} 3\sqrt{3}} = 3^{k\log_{10} 2}$, **then k =** [ ]

A) $\frac{1}{2}$ B) $\frac{3}{2}$ C) 3 D) 2

17. **If** $\log_{10}2 = 0.3010$, **then** $\log_5 64 =$ [ ]

A) $\frac{602}{233}$ B) $\frac{233}{602}$ C) $\frac{202}{633}$ D) $\frac{633}{202}$

18. **If** $4^{\log_9 3}+9^{\log_2 4}=10^{\log_x 83}$, **then x =** [ ]

A) 4 B) 9 C) 83 D) 10

9. The value of $3^{\frac{4}{\log_4 9}} + 27^{\frac{1}{\log_{36} 9}} + 81^{\frac{1}{\log_5 3}}$ **is** [ ]

A) 890 B) 860 C) 857 D) none of these

20. The value of $\log_{\sqrt{2}}\sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}}$ **is** [ ]

A) $\frac{15}{16}$ B) $\frac{7}{16}$ C) $\frac{15}{8}$ D) $\frac{31}{32}$

21. The value of $\frac{\log_a(\log_b a)}{\log_b(\log_a b)}$ **is** [ ]

A) $\log_b a$ B) $\log_a b$ C) $-\log_a b$ D) $-\log_b a$

22. If $\log_2 x + \log_4 x + \log_{16} x = \frac{21}{4}$**, then x equals to** [ ]

A) 8 B) 4 C) 2 D) 16

23. If $a^x = b^y = c^z = d^w$**, then** $\log_a(bcd)$ **equals to** [ ]

A) $\frac{1}{x}\left(\frac{1}{y}+\frac{1}{z}+\frac{1}{w}\right)$ B) $x\left(\frac{1}{y}+\frac{1}{z}+\frac{1}{w}\right)$

C) $\frac{y+z+w}{x}$ D) none of these

24. If $1.5^a = 0.15^b = 100$**, then** $\frac{1}{a}-\frac{1}{b}$ **equals to** [ ]

A) $\frac{1}{2}$ B) 1 C) 0 D) $\frac{2}{3}$

C) $a = \frac{32}{3}, b = \frac{27}{8}, c = \frac{2}{3}$ D) $a = \frac{2}{3}, b = \frac{32}{3}, c = \frac{27}{8}$

5. If $\log_2 a + \log_4 b + \log_4 c = 2$, $\log_9 a + \log_3 b + \log_9 c = 2$, $\log_{16} a + \log_{16} b + \log_4 c = 2$, then
[ ]

A) $a = \frac{2}{3}, b = \frac{27}{8}, c = \frac{32}{3}$

B) $a = \frac{27}{8}, b = \frac{2}{3}, c = \frac{32}{3}$

C) $a = \frac{32}{3}, b = \frac{27}{8}, c = \frac{2}{3}$

D) $a = \frac{2}{3}, b = \frac{32}{3}, c = \frac{27}{8}$

26. If $\log_y x = \log_z y = \log_x z$, then [ ]

A) $x < y < z$

B) $x > y \geq z$

C) $x < y \leq z$

D) $x = y = z$

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SOLUTIONS

1. We have, $\left|\log_b a + \log_a b\right| = \left|\log_b a + \frac{1}{\log_b a}\right| = \left|x + \frac{1}{x}\right|$, where $x = \log_b a$.

We know that $x + \frac{1}{x} \geq 2$ for all $x > 0$ and, $x + \frac{1}{x} \leq -2$ for all $x < 0$.

$\therefore \left|x + \frac{1}{x}\right| \geq 2$ for all $x \neq 0$

$\therefore \left|\log_b a + \log_a b\right| \geq 2$

Hence, the least value of $\left|\log_b a + \log_a b\right|$ is 2.

2. We have, $\log_3 x + \log_9 x^2 + \log_{27} x^3 = 9$

$\Rightarrow \log_3 x + \log_{3^2} x^2 + \log_{3^3} x^3 = 9$

$\Rightarrow \log_3 x + \frac{2}{2}\log_3 x + \frac{3}{3}\log_3 x = 9$

$\Rightarrow 3\log_3 x = 9 \quad \Rightarrow \log_3 x = 3 \quad \Rightarrow x = 3^3 = 27$

3. We have, $\log\{a^{\log b - \log c} . b^{\log c - \log a} . c^{\log a - \log b}\}$

$= (\log b - \log c)\log a + (\log c - \log a)\log b + (\log a - \log b)\log c = 0$

$\therefore \quad a^{\log b - \log c} . b^{\log c - \log a} . c^{\log a - \log b} = 1$

4. We have, $\log\left\{(ab)^{\log\left(\frac{a}{b}\right)} . (bc)^{\log\left(\frac{b}{c}\right)} . (ca)^{\log\left(\frac{c}{a}\right)}\right\}$

$= \log\left(\frac{a}{b}\right)\log(ab) + \log\left(\frac{b}{c}\right)\log(bc) + \log\left(\frac{c}{a}\right)\log(ca)$

$= (\log a - \log b)(\log a + \log b) + (\log b - \log c)(\log b + \log c) + (\log c - \log a)(\log c + \log a) = 0$

$\therefore (ab)^{\log\left(\frac{a}{b}\right)} . (bc)^{\log\left(\frac{b}{c}\right)} . (ca)^{\log\left(\frac{c}{a}\right)} = 1$

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. We have, $xyz + 1 = \log_{2a}a.\log_{3a}2a.\log_{4a}3a + 1$

$= \log_{4a}a + 1 = \log_{4a}a + \log_{4a}4a$

$= \log_{4a}(2a)^2 = 2\log_{4a}2a$

$= 2\log_{3a}2a.\ \log_{4a}3a = 2yz.$

6. We have, $\dfrac{1}{\log_{ab}abc} + \dfrac{1}{\log_{bc}abc} + \dfrac{1}{\log_{ca}abc}$

$= \log_{abc}ab + \log_{abc}bc + \log_{abc}ca$

$= \log_{abc}(ab.bc.ca) = \log_{abc}(abc)^2 = 2\log_{abc}abc = 2.$

7. We have, $\dfrac{1}{\log_a bc + 1} + \dfrac{1}{\log_b ca + 1} + \dfrac{1}{\log_c ab + 1}$

$= \dfrac{1}{\log_a bc + \log_a a} + \dfrac{1}{\log_b ca + \log_b b} + \dfrac{1}{\log_c ab + \log_c c}$

$= \dfrac{1}{\log_a abc} + \dfrac{1}{\log_b abc} + \dfrac{1}{\log_c abc}$

$= \log_{abc}a + \log_{abc}b + \log_{abc}c = \log_{abc}abc = 1.$

8. We have, $y = a^{\frac{1}{1-\log_a x}}$ and $z = a^{\frac{1}{1-\log_a y}}$

$\Rightarrow \log_a y = \dfrac{1}{1-\log_a x}$ and $\log_a z = \dfrac{1}{1-\log_a y}$

$\Rightarrow \log_a x = \dfrac{\log_a y - 1}{\log_a y}$ and $1-\log_a z = \dfrac{\log_a y}{\log_a y - 1}$

$\Rightarrow \log_a x = \dfrac{1}{1-\log_a z} \quad \Rightarrow k = \dfrac{1}{1-\log_a z} \quad \left[\because x = a^k, \because \log_a x = k\right]$

9. We have, $6^x = 7^{x+4}$

$\Rightarrow x\log 6 = (x+4)\log 7$

$\Rightarrow x(\log 2 + \log 3) = (x+4)\log 7$

$\Rightarrow x(a+b) = (x+4)c$

$\Rightarrow x = \dfrac{4c}{a+b-c}$

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0. We have, $a = 1 + \log_x yz = \log_x x + \log_x yz = \log_x xyz$

$b = 1 + \log_y zx = \log_y y + \log_y zx = \log_y xyz$

and, $c = 1 + \log_z xy = \log_z z + \log_z xy = \log_z xyz$

$\therefore \quad \frac{1}{a}+\frac{1}{b}+\frac{1}{c} = \log_{xyz} x + \log_{xyz} y + \log_{xyz} z = \log_{xyz} xyz = 1$

$\Rightarrow$ ab + bc + ca = abc.

1. We have, $\frac{\log x}{a^2+ab+b^2} = \frac{\log y}{b^2+bc+c^2} = \frac{\log z}{c^2+ca+a^2} = \lambda(\text{say})$

$\Rightarrow \quad \frac{(a-b)\log x}{a^3-b^3} = \frac{(b-c)\log y}{b^3-c^3} = \frac{(c-a)\log z}{c^3-a^3} = \lambda$

$\Rightarrow \quad \frac{\log x^{a-b}}{a^3-b^3} = \frac{\log y^{b-c}}{b^3-c^3} = \frac{\log z^{c-a}}{c^3-a^3} = \lambda$

$\Rightarrow \quad \log x^{a-b} = \lambda(a^3 - b^3), \quad \log y^{b-c} = \lambda(b^3 - c^3), \quad \log z^{c-a} = \lambda(c^3 - a^3)$

$\Rightarrow \quad \log(x^{a-b}.\ y^{b-c}.\ z^{c-a}) = 0.$

$\Rightarrow \quad x^{a-b}.\ y^{b-c}.\ z^{c-a} = 1.$

12. We have, $\frac{1}{\log_{c+a} b} + \frac{1}{\log_{c-a} b}$

$= \log_b(c + a) + \log_b(c - a) = \log_b(c^2 - a^2)$

$= \log_b b^2 \qquad [\because \ c^2 - a^2 = b^2]$

$= 2 \log_b b = 2.$

13. We have, $x = \log_{0.1} 0.001$ and $y = \log_9 81$

$\Rightarrow \quad x = \log_{0.1} (0.1)^3$ and $y = \log_9 9^2$

$\Rightarrow \quad x = 3$ and $y = 2$

$\therefore \quad \sqrt{x-2\sqrt{y}} = \sqrt{3-2\sqrt{2}} = \sqrt{\left(\sqrt{2}-1\right)^2} = \sqrt{2}-1$

14. We have, $5^x = (0.5)^y = 1000.$

$\Rightarrow \quad x = \log_5 1000$ and $y = \log_{0.5} 1000$

$\Rightarrow \quad \frac{1}{x}-\frac{1}{y} = \log_{1000} 5 - \log_{1000} 0.5$

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5. We have, $x = 27$ and $y = \log_3 4$

$\therefore\ x^y = 27^{\log_3 4} = \left(3^3\right)^{\log_3 4} = 3^{\log_3 4^3} = 4^3 = 64$

16. We have, $\log_8 128 = \log_{2^3} 2^7 = \frac{7}{3}\log_2 2 = \frac{7}{3}$

17. We have, $\frac{\log a}{2} = \frac{\log b}{3} = \frac{\log c}{5} = \lambda(\text{say})$

$\Rightarrow \log a = 2\lambda,\ \log b = 3\lambda,\ \log c = 5\lambda$

$\Rightarrow a = 10^{2\lambda},\ b = 10^{3\lambda},\ c = 10^{5\lambda} \Rightarrow bc = 10^{8\lambda} = \left(10^{2\lambda}\right)^4 = a^4$

18. We have, $4^x + 2^{2x-1} = 3^{x+\frac{1}{2}} + 3^{x-\frac{1}{2}}$

$\Rightarrow 2 \times 2^{2x-1} + 2^{2x-1} = 3^{x-\frac{1}{2}} \times 3 + 3^{x-\frac{1}{2}}$

$\Rightarrow 2^{2x-1}(2 + 1) = 3^{x-\frac{1}{2}}(3+1)$

$\Rightarrow 2^{2x-1} \times 3 = 3^{x-\frac{1}{2}} \times 4$

$\Rightarrow 2^{2x-3} = 3^{x-\frac{3}{2}} \quad \Rightarrow \left(2^2\right)^{x-\frac{3}{2}} = 3^{x-\frac{3}{2}}$

$\Rightarrow 4^{x-\frac{3}{2}} = 3^{x-\frac{3}{2}}$

$\Rightarrow x - \frac{3}{2} = 0 \quad \Rightarrow x = \frac{3}{2}$

19. We have, $\log(2a - 3b) = \log a - \log b$

$\Rightarrow \log(2a - 3b) = \log\left(\frac{a}{b}\right) \Rightarrow 2a - 3b = \frac{a}{b}$

$\Rightarrow 2ab - 3b^2 = a \Rightarrow a = \frac{3b^2}{2b-1}$

20. We have, $\frac{\log x}{2a+3b-5c} = \frac{\log y}{2b+3c-5a} = \frac{\log z}{2c+3a-5b}$

$\Rightarrow$ $\frac{\log x}{2a+3b-5c} = \frac{\log y}{2b+3c-5a} = \frac{\log z}{2c+3a-5b} = \frac{\log x + \log y + \log z}{0}$

$\Rightarrow$ $\log x + \log y + \log z = 0$

$\Rightarrow$ $\log (xyz) = 0$ $\Rightarrow$ $xyz = 1$

21. We have, $\frac{\log x}{1} = \frac{\log y}{2} = \frac{\log z}{5}$

$\Rightarrow$ $\frac{\log x}{1} = \frac{\log y}{2} = \frac{\log z}{5} = \frac{4\log x + 3\log y - 2\log z}{4+6-10}$

$\Rightarrow$ $4 \log x + 3 \log y - 2 \log z = 0$

$\Rightarrow$ $\log(x^4\ y^3\ z^{-2}) = 0$

$\Rightarrow$ $x^4\ y^3\ z^{-2} = 1.$

22. We have, $\frac{\log 3}{x-y} = \frac{\log 5}{y-z} = \frac{\log 7}{z-x} = \lambda$ (say)

$\Rightarrow$ $\log 3 = \lambda(x-y)$, $\log 5 = \lambda(y-z)$, $\log 7 = \lambda(z-x)$

$\Rightarrow$ $3^{x+y}.\ 5^{y+z}.\ 7^{z+x} = 10^{\lambda(x^2-y^2)}.10^{\lambda(y^2-z^2)}.10^{\lambda(z^2-x^2)}$

$\Rightarrow$ $3^{x+y}.\ 5^{y+z}.\ 7^{y+x} = 10^{\lambda(x^2-y^2+y^2-z^2+z^2-x^2)} = 10^0 = 1$

23. We have, $\frac{\log_2 a}{2} = \frac{\log_3 b}{3} = \frac{\log_4 c}{4} = \lambda(\text{say})$

$\Rightarrow$ $a = 2^{2\lambda}$, $b = 3^{3\lambda}$, $c = 4^{4\lambda}$

Now, $a^{1/2}\ b^{1/3}\ c^{1/4} = 24$

$\Rightarrow$ $2^\lambda.3^\lambda.\ 4^\lambda = 24$

$\Rightarrow$ $2^\lambda.3^\lambda.\ 4^\lambda = 2\times3\times4$ $\Rightarrow$ $\lambda = 1$

Hence, $a = 2^2 = 4$, $b = 3^3 = 27$ and $c = 4^4 = 256$.

24. We have, $\frac{\log_2 a}{3} = \frac{\log_2 b}{4} = \frac{\log_2 c}{5\lambda} = k\,(\text{say})$

$\Rightarrow$ $a = 2^{3k}$, $b = 2^{4k}$ and $c = 2^{5\lambda k}$

$\therefore$ $a^{-3}\ b^{-4}\ c = 1$ $\Rightarrow$ $2^{-9k}\times2^{-16k}\times2^{5\lambda k} = 1$

$\Rightarrow$ $2^{5\lambda k-25k} = 2^0$ $\Rightarrow$ $5\lambda k - 25k = 0$

$\Rightarrow$ $\lambda = 5$.

5. We have, $a = \log_{24}12$, $b = \log_{36}24$ and $c = \log_{48}36$

$\Rightarrow$ $abc = \log_{48}12$ and $bc = \log_{48}24$

$\Rightarrow$ $abc - 2bc = \log_{48}12 - 2\log_{48}24$

$\Rightarrow$ $abc - 2bc = \log_{48}12 - \log_{48}24^2$

$\Rightarrow$ $abc - 2bc = \log_{48}\left(\frac{12}{24^2}\right) = \log_{48}\left(\frac{1}{48}\right) = -1$

$\Rightarrow$ $abc = 2bc - 1$

26. Since a, b, c are three consecutive positive integers.

$a = b - 1$, $c = b + 1$

Now, $\log(1 + ca) = \log[1 + (b - 1)(b + 1)] = \log(1 + b^2 - 1) = \log b^2 = 2\log b$

27. We have, $\frac{\log_a x}{\log_{ab} x} = \frac{\log_x ab}{\log_x a} = \log_a ab = \log_a a + \log_a b = 1 + \log_a b$

$\therefore$ $\frac{\log_a x}{\log_{ab} x} = 4 + k + \log_a b$

$\Rightarrow$ $1 + \log_a b = 4 + k + \log_a b$

$\Rightarrow$ $1 = 4 + k \Rightarrow k = -3$

28. We have, $\frac{\log a}{b - c} = \frac{\log b}{c - a} = \frac{\log c}{a - b} = \lambda(\text{say})$

$\Rightarrow$ $a = 10^{\lambda(b-c)}$, $b = 10^{\lambda(c-a)}$, $c = 10^{\lambda(a-b)}$

$\therefore$ $a^a\, b^b\, c^c = 10^{\lambda a(b-c)}.10^{\lambda b(c-a)}.10^{\lambda c(a-b)}$

$\Rightarrow$ $a^a\, b^b\, c^c = 10^{\lambda[a(b-c)+b(c-a)+c(a-b)]} = 10^{\lambda\times 0} = 10^0 = 1$

29. We have, $x = \log_a bc$, $y = \log_b ca$, $z = \log_c ab$

$\Rightarrow$ $x + 1 = \log_a bc + \log_a a$, $y + 1 = \log_b ca + \log_b b$, $z + 1 = \log_c ab + \log_c c$

$\Rightarrow$ $x + 1 = \log_a abc$, $y + 1 = \log_b abc$, $z + 1 = \log_c abc$

$\Rightarrow$ $\frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1} = \log_{abc} a + \log_{abc} b + \log_{abc} c$

$\Rightarrow$ $\frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1} = \log_{abc} abc = 1.$

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30. Let $\log_2 7$ be a rational number equal to $\frac{m}{n}$, where $m, n \in N$ and they do not have a common factor. Then,

$$\log_2 7 = \frac{m}{n} \quad \Rightarrow \quad 7 = 2^{m/n} \quad \Rightarrow \quad 7^n = 2^m$$

Clearly, this is impossible, because 7^n is an odd natural number and 2^m is an even natural number.

Hence, $\log_2 7$ is an irrational number.

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. We have, $2^{\log_3 5} - 5^{\log_3 2} = 5^{\log_3 2} - 5^{\log_3 2} = 0$ $\left[\because\ x^{\log_a y} = y^{\log_a x}\right]$

$\therefore\ 2^{\log_3 5} - 5^{\log_3 2} = 0$

2. Since a, b, c are the sides of a right angled triangle with c as the largest side i.e., hypotenuse. Therefore, $c^2 = a^2 + b^2$

Now, $\dfrac{\log_{c+b} a + \log_{c-b} a}{\log_{c+b} a . \log_{c-b} a} = \dfrac{1}{\log_{(c-b)} a} + \dfrac{1}{\log_{(c+b)} a} = \log_a (c-b) + \log_a (c+b)$

$= \log_a(c^2 - b^2) = \log_a a^2 = 2 \log_a a = 2.$

3. We have, $\log_3 a \times \log_a x = 4$

$\Rightarrow \log_3 x = 4$

$\Rightarrow x = 3^4 = 81.$

. We have, $\log_4 5 = x$ and $\log_5 6 = y$

$\Rightarrow 5 = 4^x$ and $6 = 5^y \Rightarrow 6 = (4^x)^y \Rightarrow 6 = 4^{xy}$

$\Rightarrow 2 \times 3 = 2^{2xy} \Rightarrow 3 = 2^{2xy - 1}$

$\Rightarrow 3^{\frac{1}{2xy-1}} = 2 \Rightarrow \log_3 2 = \dfrac{1}{2xy-1}$

5. We have, $a^2 + 4b^2 = 12ab$

$\Rightarrow (a + 2b)^2 = 16ab \Rightarrow 2 \log(a + 2b) = \log 16ab$

$\Rightarrow 2 \log(a + 2b) = \log a + \log b + 4 \log 2$

$\Rightarrow \log(a + 2b) = \dfrac{1}{2}(\log a + \log b + 4 \log 2)$

6. We have, $3 \log \dfrac{81}{80} + 5 \log \dfrac{25}{24} + 7 \log \dfrac{16}{15}$

$= 3 \log\left(\dfrac{3^4}{2^4 \times 5}\right) + 5 \log\left(\dfrac{5^2}{2^3 \times 3}\right) + 7 \log\left(\dfrac{2^4}{3 \times 5}\right)$

$= \log\left\{\left(\dfrac{3^4}{2^4 \times 5}\right)^3 \times \left(\dfrac{5^2}{2^3 \times 3}\right)^5 \times \left(\dfrac{2^4}{3 \times 5}\right)^7\right\} = \log\left\{\dfrac{3^{12}}{2^{12} \times 5^3} \times \dfrac{5^{10}}{2^{15} \times 3^5} \times \dfrac{2^{28}}{3^7 \times 5^7}\right\} = \log 2.$

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7. We have, $\frac{\log a}{b-c} = \frac{\log b}{c-a} = \frac{\log c}{a-b} = \lambda(\text{say})$

$\Rightarrow \quad a = 10^{\lambda(b-c)}, \quad b = 10^{\lambda(c-a)}, \quad c = 10^{\lambda(a-b)}$

$\therefore \quad a^a\ b^b\ c^c = 10^{\lambda a(b-c) + \lambda b(c-a) + \lambda c(a-b)} = 10^0 = 1$

8. We have, $5^{\log_a x} + 5\, x^{\log_a 5} = 3$

$\Rightarrow \quad x^{\log_a 5} + 5\ x^{\log_a 5} = 3 \quad \left[\because x^{\log_a y} = y^{\log_a x}\right]$

$\Rightarrow \quad 6.\, x^{\log_a 5} = 3 \qquad \Rightarrow \quad x^{\log_a 5} = \frac{1}{2} \qquad \Rightarrow \quad x = \left(2^{-1}\right)^{\log_5 a} = 2^{-\log_5 a}$

9. We have, $\frac{\log 49\sqrt{7} + \log 25\sqrt{5} - \log 4\sqrt{2}}{\log 17.5} = \frac{\log\left(\frac{7^{5/2} \times 5^{5/2}}{2^{5/2}}\right)}{\log 17.5} = \frac{\frac{5}{2}\log 17.5}{\log 17.5} = \frac{5}{2}$

10. We have, $5^{\sqrt{\log_5 7}} - 7^{\sqrt{\log_7 5}} = 5^x - 7^{\frac{1}{x}}$, where $x = \sqrt{\log_5 7}$

$= 5^x - \left(5^{x^2}\right)^{\frac{1}{x}} \qquad [\because \quad x = \sqrt{\log_5 7} \quad \Rightarrow \quad x^2 = \log_5 7 \quad \Rightarrow \quad 7 = 5^{x^2}\ ]$

$= \ 5^x - 5^x = 0$

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1. We have, $\dfrac{3+\log 343}{2+\frac{1}{2}\log\left(\frac{49}{4}\right)+\frac{1}{3}\log\left(\frac{1}{125}\right)} = \dfrac{3+\log 7^3}{2+\frac{1}{2}\left(\log 7^2-\log 2^2\right)+\frac{1}{3}\log 5^{-3}}$

$= \dfrac{3+3\log 7}{2+\left(\log 7-\log 2\right)-\log 5} = \dfrac{3\left(1+\log 7\right)}{2+\log 7-\left(\log 2+\log 5\right)} = \dfrac{3\left(1+\log 7\right)}{1+\log 7} = 3$

12. Given log(a – b) = log a – log b

$\Rightarrow$ a – b = a/b $\Rightarrow \dfrac{a\left(b-1\right)}{b} = b \Rightarrow a = \dfrac{b^2}{b-1}$

13. We have, $9a^2 + 4b^2 = 18ab$

$\Rightarrow$ $9a^2 + 12ab + 4b^2 = 30ab$

$\Rightarrow$ $(3a + 2b)^2 = 30ab$

$\Rightarrow$ 2 log (3a + 2b) = log (5a × 3b × 2)

$\Rightarrow$ log (3a + 2b) = $\frac{1}{2}\left\{\log 5a+\log 3b+\log 2\right\}$

14. We have, $\log_5\left(1+\frac{1}{5}\right)+\log_5\left(1+\frac{1}{6}\right)+\log_5\left(1+\frac{1}{7}\right)+\ldots+\log_5\left(1+\frac{1}{624}\right)$

$= \log_5\left(\frac{6}{5}\times\frac{7}{6}\times\frac{8}{7}\times\ldots\times\frac{625}{624}\right) = \log_5\left(\frac{625}{5}\right) = \log_5 5^3 = 3$

15. We have, log(x – y) – log 5 – $\frac{1}{2}\log x - \frac{1}{2}\log y = 0$

$\Rightarrow$ 2 log(x – y) – 2 log 5 – log x – log y = 0

$\Rightarrow \dfrac{\left(x-y\right)^2}{25xy} = 1 \Rightarrow x^2 + y^2 - 2xy = 25xy \Rightarrow \dfrac{x}{y}+\dfrac{y}{x} - 2 = 25 \Rightarrow$

$\dfrac{x}{y}+\dfrac{y}{x} = 27$

16. We have, $2^{\log_{10} 3\sqrt{3}} = 3^{k\log_{10} 2}$

$\Rightarrow \left(3\sqrt{3}\right)^{\log_{10} 2} = 3^{k\log_{10} 2}$ $\left[\because\ x^{\log_a y} = y^{\log_a x}\right]$

$\Rightarrow 3^{\frac{3}{2}\log_{10} 2} = 3^{k\log_{10} 2} \Rightarrow k = \dfrac{3}{2}$

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17. We have, $\log_5 64 = \log_5 2^6 = 6 \log_5 2 = \dfrac{6}{\log_2 5} = \dfrac{6}{\log_2\left(\dfrac{10}{2}\right)}$

$$= \frac{6}{\log_2 10 - \log_2 2} = \frac{6}{\dfrac{1}{\log_{10} 2} - 1} = \frac{6}{\dfrac{1}{0.3010} - 1} = \frac{6 \times 0.3010}{1 - 0.3010}$$

$$= \frac{1.8060}{0.699} = \frac{1806}{699} = \frac{602}{233}$$

18. We have, $4^{\log_9 3} + 9^{\log_2 4} = 10^{\log_x 83}$

$\Rightarrow \quad 4^{\log_{3^2} 3} + 9^{\log_2 2^2} = 10^{\log_x 83} \quad \Rightarrow \quad 4^{1/2} + 9^2 = 10^{\log_x 83}$

$\Rightarrow \quad 83 = 10^{\log_x 83} \quad \Rightarrow \quad \log_{10} 83 = \log_x 83$

$\Rightarrow \quad x = 10$

19. We have, $3^{\frac{4}{\log_4 9}} + 27^{\frac{1}{\log_{36} 9}} + 81^{\frac{1}{\log_5 3}} = 3^{\frac{4}{\log_2 3}} + 27^{\frac{1}{\log_6 3}} + 81^{\frac{1}{\log_5 3}}$

$= 3^{4\log_3 2} + 27^{\log_3 6} + 81^{\log_3 5} = 3^{\log_3 2^4} + \left(3^3\right)^{\log_3 6} + \left(3^4\right)^{\log_3 5}$

$= 3^{\log_3 16} + \left(3^3\right)^{\log_3 6} + \left(3^4\right)^{\log_3 5} = 3^{\log_3 16} + 3^{\log_3 6^3} + 3^{\log_3 5^4}$

$= 16 + 6^3 + 5^4 = 16 + 216 + 625 = 857$

20. We have, $\log_{\sqrt{2}} \sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}} = \log_{\sqrt{2}}\left(2^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}}\right) = \log_{\sqrt{2}} 2^{\frac{15}{16}} = \dfrac{\frac{15}{16}}{\frac{1}{2}} \log_2 2 = \dfrac{15}{8}$

21. We have, $\dfrac{\log_a(\log_b a)}{\log_b(\log_a b)} = \dfrac{\log(\log_b a)}{\log a} \times \dfrac{\log b}{\log(\log_a b)}$

$$= \frac{\log\left(\dfrac{\log a}{\log b}\right)}{\log a} \times \frac{\log b}{\log\left(\dfrac{\log b}{\log a}\right)} = \frac{\log(\log a) - \log(\log b)}{\log a} \times \frac{\log b}{\log(\log b) - \log(\log a)}$$

$$= -\frac{\log b}{\log a} = -\log_a b.$$

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22. We have, $\log_2 x + \log_4 x + \log_{16} x = \frac{21}{4}$

$\Rightarrow \quad \log_2 x + \frac{1}{2}\log_2 x + \frac{1}{4}\log_2 x = \frac{21}{4}$

$\Rightarrow \quad \frac{7}{4}\log_2 x = \frac{21}{4} \quad \Rightarrow \quad \log_2 x = 3 \quad \Rightarrow \quad x = 2^3 = 8$

23. We have, $a^x = b^y = c^z = d^w$

$\Rightarrow \quad a^x = b^y, \quad a^x = c^z$ and $a^x = d^w$

$\Rightarrow \quad x \log a = y \log b, \quad x \log a = z \log c$ and $x \log a = w \log d$

$\Rightarrow \quad \frac{x}{y} = \log_a b, \quad \frac{x}{z} = \log_a c$ and $\frac{x}{w} = \log_a d$

$\Rightarrow \quad \frac{x}{y} + \frac{x}{z} + \frac{x}{w} = \log_a b + \log_a c + \log_a d$

$\Rightarrow \quad x\left(\frac{1}{y} + \frac{1}{z} + \frac{1}{w}\right) = \log_a bcd$

24. We have $1.5^a = 0.15^b = 100$

$\Rightarrow \quad a = \log_{1.5} 100$ and $b = \log_{0.15} 100$

$\Rightarrow \quad \frac{1}{a} - \frac{1}{b} = \log_{100} 1.5 - \log_{100} 0.15$

$\Rightarrow \quad \frac{1}{a} - \frac{1}{b} = \log_{100}\left(\frac{1.5}{0.15}\right) \quad = \log_{100} 10 = \frac{1}{2}$

25. We have, $\log_2 a + \log_4 b + \log_4 c = 2$

$\log_9 a + \log_3 b + \log_9 c = 2$

$\log_{16} a + \log_{16} b + \log_4 c = 2$

$\Rightarrow \quad \log_2 a + \frac{1}{2}\log_2 b + \frac{1}{2}\log_2 c = 2$

$\Rightarrow \quad \frac{1}{2}\log_3 a + \log_3 b + \frac{1}{2}\log_3 c = 2$

$\Rightarrow \quad \frac{1}{2}\log_4 a + \frac{1}{2}\log_4 b + \log_4 c = 2$

$\Rightarrow \quad \log_2 (a^2bc) = 4, \quad \log_3 (ab^2c) = 4, \quad \log_4 (abc^2) = 4$

$\Rightarrow \quad (a^2bc) = 2^4, \quad ab^2c = 3^4$ and $abc^2 = 4^4$

$\Rightarrow \quad (abc)^4 = (2 \times 3 \times 4)^4$

$\Rightarrow \quad abc = 24$