Examples – VII

1. If $A = 30^\circ$, verify that

(a) $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1$

$\Rightarrow \cos 2(30)^\circ = (\cos 30^\circ)^2 - (\sin 30^\circ)^2$

$\Rightarrow \cancel{\cos 60^\circ = \cos^2 90^\circ - \sin^2 90^\circ}$

$\Rightarrow \cancel{\frac{1}{2} = 0}$

$\Rightarrow \frac{1}{2} = \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2$

$\Rightarrow \frac{1}{2} = \frac{3}{4} - \frac{1}{4}$

$\Rightarrow \frac{1}{2} = \frac{1}{2}$

$\therefore$ L.H.S. = R.H.S.

Again,

$\cos^2 A - \sin^2 A = 2\cos^2 A - 1$

$\Rightarrow \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2 = 2\left(\frac{\sqrt{3}}{2}\right)^2 - 1$

$\Rightarrow \frac{3}{4} - \frac{1}{4} = \frac{3}{2} - 1$

$\Rightarrow \frac{1}{2} = \frac{1}{2}$

$\therefore$ L.H.S. = R.H.S.

(b) $\sin 2A = 2\sin A\cos A$

$\Rightarrow \sin 2(30^\circ) = 2\sin 30^\circ \cos 30^\circ$

$\Rightarrow \frac{\sqrt{3}}{2} = \cancel{2} \times \frac{1}{\cancel{2}} \times \frac{\sqrt{3}}{2}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$

$\therefore$ L.H.S. = R.H.S.//

(c) $\cos 3A = 4\cos^3 A - 3\cos A$

$\Rightarrow \cos 3(30^\circ) = 4(\cos 30^\circ)^3 - 3\cos 30^\circ$

$\Rightarrow 0^\circ = \cancel{4}^{2} \times \left(\frac{\sqrt{3}}{\cancel{2}}\right)^3 - 3 \times \left(\frac{\sqrt{3}}{2}\right)$

$\Rightarrow 0^\circ = \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2} = 0$

$\therefore$ L.H.S = R.H.S//

(d) $\sin 3A = 3\sin A - 4\sin^3 A$

$\Rightarrow \sin 3(30)^\circ = 3\sin 30^\circ - 4(\sin 30^\circ)^3$

$\Rightarrow 1 = 3 \times \frac{1}{2} - 4 \times \left(\frac{1}{2}\right)^3$

$\Rightarrow 1 = \frac{3}{2} - \frac{1}{2}$

$\Rightarrow 1 = 1$

$\therefore$ L.H.S = R.H.S.//

e) $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

$\Rightarrow \tan 2(30^\circ) = \dfrac{2\tan 30^\circ}{1-(\tan 30^\circ)^2}$

$\Rightarrow \sqrt{3} = \dfrac{2 \times 1/\sqrt{3}}{1-\frac{1}{3}}$

$\Rightarrow \sqrt{3} = \dfrac{\frac{2}{\sqrt{3}}}{\frac{2}{3}}$

$\Rightarrow \sqrt{3} = \dfrac{2}{\sqrt{3}} \times \dfrac{3}{2}$

$\Rightarrow 6 = 6$

$\therefore$ L.H.S. = R.H.S.

2. If $A = 45^\circ$; verify that

(a) $\sin 2A = 2\sin A \cos A$

$\Rightarrow \sin 2(45)^\circ = 2\sin 45^\circ \cdot \cos 45^\circ$

$\Rightarrow 1 = 2 \times \dfrac{1}{\sqrt{2}} \times \dfrac{1}{\sqrt{2}}$

$\Rightarrow 1 = 1$

L.H.S. = R.H.S.

(b) $\cos 2A = 1 - 2\sin^2 A$

$\Rightarrow \cos 2(45)^\circ = 1 - 2(\sin 45)^2$

$\Rightarrow 0 = 1 - 1$

$\therefore$ L.H.S = R.H.S.

(c) $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

$\Rightarrow \tan 2(45)^\circ = \dfrac{2\tan 45^\circ}{1-(\tan 45^\circ)^2}$

$\Rightarrow \infty = \dfrac{2\times 1}{1-(1)^2}$

$\Rightarrow \infty = \dfrac{2}{0}$

$\Rightarrow \infty = \infty$

$\therefore$ L.H.S = R.H.S.

verify that

3. $\sin^2 30^\circ + \sin^2 45^\circ + \sin^2 60^\circ = \dfrac{3}{2}$

L.H.S $\Rightarrow \left(\dfrac{1}{2}\right)^2 + \left(\dfrac{1}{\sqrt{2}}\right)^2 + \left(\dfrac{\sqrt{3}}{2}\right)^2$

$\Rightarrow \dfrac{1}{4} + \dfrac{1}{2} + \dfrac{3}{4} \Rightarrow \dfrac{1+2+3}{4} = \dfrac{3}{2}$

$\therefore$ R.H.S

4. $\tan^2 30° + \tan^2 45° + \tan^2 60° = 4\frac{1}{3}$

L.H.S $\Rightarrow$ $\left(\frac{1}{\sqrt{3}}\right)^2 + (1)^2 + (\sqrt{3})^2$

$\Rightarrow \frac{1+3+9}{3} \Rightarrow \frac{13}{3} = 4\frac{1}{3}$ = R.H.S.

5. $\sin 30° \cos 60° + \cos 30° \sin 60° = 1$

L.H.S $\Rightarrow$ $\frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$

$\Rightarrow \frac{1}{4} + \frac{3}{4}$

$\Rightarrow \frac{4}{4} = 1$ R.H.S.

6. $\cos 45° \cos 60° - \sin 45° \sin 60° = \frac{\sqrt{3}-1}{2\sqrt{2}}$

L.H.S. $\Rightarrow$ $\frac{1}{\sqrt{2}} \times \frac{1}{2} - \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}$

$\Rightarrow \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}} \Rightarrow \frac{-(1-\sqrt{3})}{2\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}}$

7. $\frac{4}{3}\cot^2 30^\circ + 3\sin^2 60^\circ - 2\,\mathrm{cosec}^2 60^\circ - \frac{3}{4}\tan^2 30^\circ = 3\frac{1}{3}$

L.H.S. $\Rightarrow \frac{4}{3}\times(\sqrt{3})^2 + 3\left(\frac{\sqrt{3}}{2}\right)^2 - 2\left(\frac{2}{\sqrt{3}}\right)^2 - \frac{3}{4}\left(\frac{1}{\sqrt{3}}\right)^2$

$\Rightarrow \frac{4}{\cancel{3}}\times\cancel{3} + 3\times\frac{3}{4} - \frac{8}{3} - \frac{\cancel{4}\,1}{\cancel{12}\,3}$

$\Rightarrow \frac{48 + 27 - \cancel{32} - \cancel{4}}{12} \Rightarrow \frac{\cancel{75} - \cancel{36}}{12}$

$\Rightarrow 4 + \frac{9}{4} - \frac{8}{3} - \frac{1}{3}$

$\Rightarrow \frac{48 + 27 - 32 - 3}{12} = \frac{40}{12} = 3\frac{1}{3}$

8. $\mathrm{Cosec}^2 45^\circ \sec^2 30^\circ \cdot \sin^3 90^\circ \cdot \cos 60^\circ = 1\frac{1}{3}$

L.H.S. $\Rightarrow (\sqrt{2})^2 \cdot \left(\frac{2}{\sqrt{3}}\right)^2 \cdot (1)^3 \cdot \frac{1}{2}$

$\Rightarrow \cancel{2} \cdot \frac{4}{3} \cdot 1 \cdot 1 \cdot \frac{1}{\cancel{2}} = \frac{4}{3} = 1\frac{1}{3}$ = R.H.S.

9. $4\cot^2 45^\circ - \sec^2 60^\circ + \sin^3 30^\circ = \frac{1}{8}$

$\Rightarrow 4 \times 1 - (2)^2 + \left(\frac{1}{2}\right)^3$

$\Rightarrow \cancel{4} - \cancel{4} + \frac{1}{8}$

$\Rightarrow \frac{1}{8} = R.H.S$