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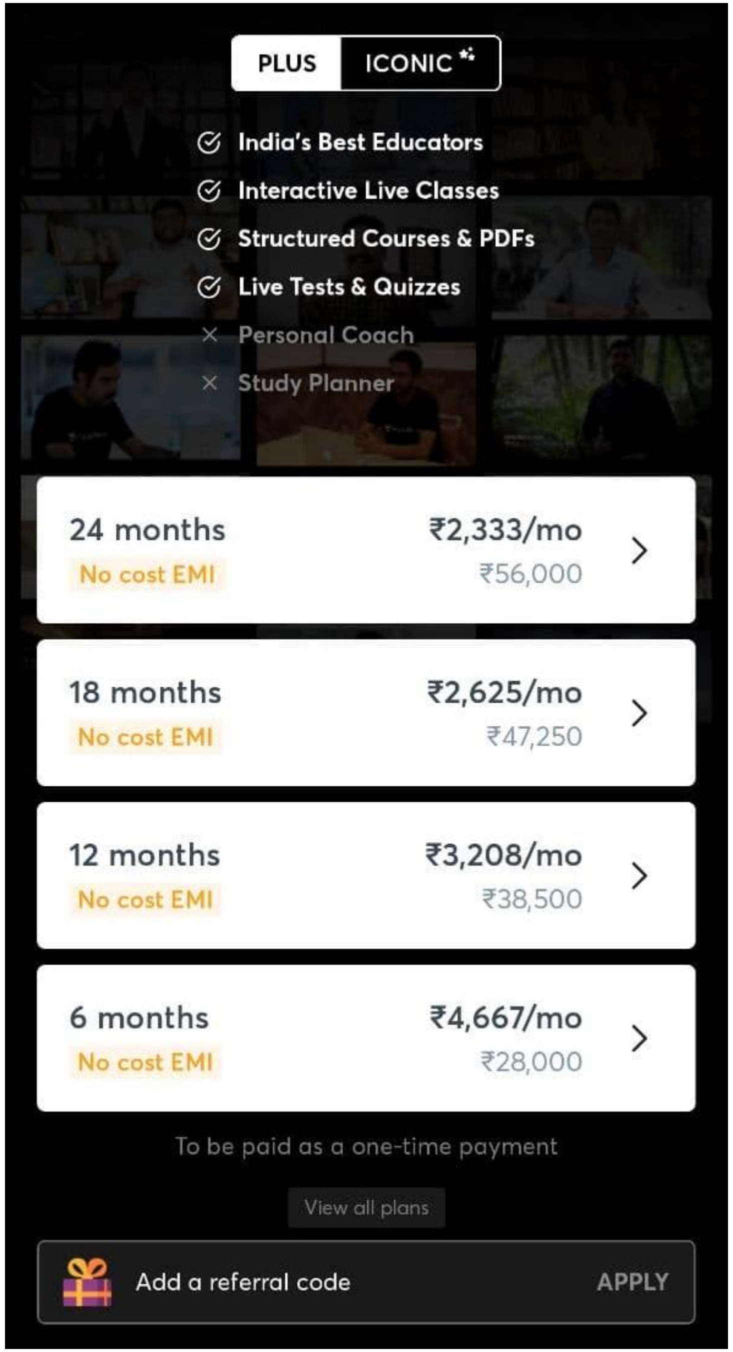

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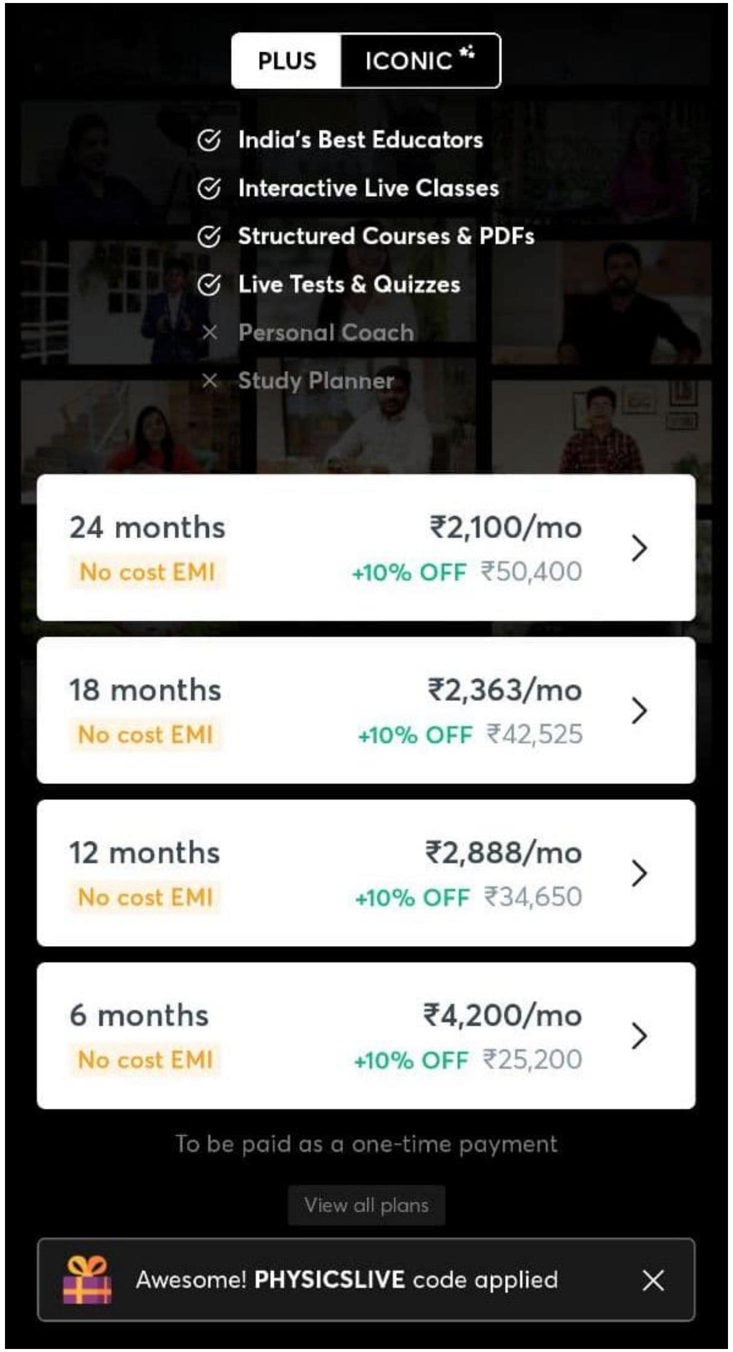

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JEE Main & Advanced
Physics DPP

DPP-3 Vectors (Dot product)

By Physicsaholics Team

Q) Find position vector of point A(2,-1,3) and its magnitude:

(a) $\vec{A} = 2\hat{\imath} - \hat{\jmath} + 3\hat{k}, |\vec{A}| = \sqrt{14}$

(b) $\vec{A} = 2\hat{\imath} + \hat{\jmath} + 3\hat{k}, |\vec{A}| = \sqrt{24}$

(c) $\vec{A} = 2\hat{\imath} - \hat{\jmath} - 3\hat{k}, |\vec{A}| = \sqrt{14}$

(d) None of these

Ans. a

$A(2, -1, 3)$

$$\vec{A} = 2\hat{i} - \hat{j} + 3\hat{k}$$

$$|\vec{A}| = \sqrt{x^2 + y^2 + z^2}$$

$$|\vec{A}| = \sqrt{(2)^2 + (-1)^2 + (3)^2}$$

$$|\vec{A}| = \sqrt{14}$$

$$\vec{A} = 2\hat{i} - \hat{j} + 3\hat{k} \;;\; |\vec{A}| = \sqrt{14}$$

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Q) If the dot product of two non-zero vectors $\overrightarrow{V_1}$ and $\overrightarrow{V_2}$ is zero. what does that tell us?

(a) $\overrightarrow{V_1} = \overrightarrow{V_2}$

(b) $\overrightarrow{V_1}$ is parallel to $\overrightarrow{V_2}$

(c) $\overrightarrow{V_1}$ is perpendicular to $\overrightarrow{V_2}$

(d) $\overrightarrow{V_1}$ is a component of $\overrightarrow{V_2}$

Ans. c

if $\vec{V_1} \cdot \vec{V_2} = 0$

$|\vec{V_1}|\ |\vec{V_2}| \cos\theta = 0$

$\cos\theta = 0$

$\boxed{\theta = 90^\circ}$

$\Rightarrow$ $\vec{V_1}$ & $\vec{V_2}$ are perpendicular to each other.

Q) Find the dot product of the pair of vectors $\vec{A} = 4\hat{\imath} + \hat{\jmath}, \vec{B} = -\hat{\imath} - \hat{\jmath}$?

(a) 5

(b) 4

(c) -5

(d) -4

Ans. c

$$\vec{A} = 4\hat{i} + \hat{j}$$
$$\vec{B} = -\hat{i} - \hat{j}$$

$$\vec{A} \cdot \vec{B} = (4\hat{i} + \hat{j}) \cdot (-\hat{i} - \hat{j})$$

$$= (4\hat{i} \cdot (-\hat{i})) + (4\hat{i} \cdot (-\hat{j})) + (\hat{j} \cdot (-\hat{i})) + (\hat{j} \cdot (-\hat{j}))$$

$$= -4 + 0 + 0 - 1$$

$$\vec{A} \cdot \vec{B} = -4 - 1$$

$$\boxed{\vec{A} \cdot \vec{B} = -5}$$

Q) If a vector $2\hat{i} - \hat{j} + 3\hat{k}$, is perpendicular to the vector $4\hat{i} - 4\hat{j} + \alpha\hat{k}$. Then the value of α is:

(a) -4 (b) $\frac{1}{4}$ (c) 4 (d) $-\frac{1}{4}$

Ans. a

if two vector are perpendicular

$\Rightarrow$ so, their dot product will be zero

$\therefore (2\hat{i} - \hat{j} + 3\hat{k}) \cdot (4\hat{i} - 4\hat{j} + \alpha\hat{k}) = 0$

$(2 \times 4) + (-1 \times -4) + (3 \times \alpha) = 0$

$8 + 4 + 3\alpha = 0 \Rightarrow \alpha = \frac{-12}{3}$

$\boxed{\alpha = 4}$

Q) The vector sum of two forces is perpendicular to their vector differences. In that case, the forces

(a) Are not equal to each other in magnitude.

(b) Are equal to each other in magnitude.

(c) Are equal to each other.

(d) Cannot be predicted.

Ans. b

given; $(\vec{a}+\vec{b}) \perp (\vec{a}-\vec{b})$

$\therefore \quad (\vec{a}+\vec{b}) \cdot (\vec{a}-\vec{b}) = 0$

$\vec{a}\cdot\vec{a} - \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{a} - \vec{b}\cdot\vec{b} = 0$

$\because \quad \vec{a}\cdot\vec{a} = a^2$

$\vec{b}\cdot\vec{b} = b^2$

$\&\quad \vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$

$\therefore \quad a^2 - ab + ab - b^2 = 0$

$a^2 - b^2 = 0$

$a^2 = b^2$

$\because a^2 = |\vec{a}|^2 \quad \& \quad b^2 = |\vec{b}|^2$

$\therefore \quad a = b$.

$\boxed{|\vec{a}| = |\vec{b}|}$

$\therefore$ $\vec{a}$ & $\vec{b}$ are equal in magnitude.

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Q) Let $\vec{A} = \hat{\imath} + \hat{\jmath}$ and, $\vec{B} = 2\hat{\imath} - \hat{\jmath}$. The magnitude of a coplanar vector $\vec{C}$ such that $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{C} = \vec{A} \cdot \vec{C}$, is given by:

(a) $\sqrt{\frac{10}{9}}$

(b) $\sqrt{\frac{5}{9}}$

(c) $\sqrt{\frac{12}{9}}$

(d) $\sqrt{\frac{9}{12}}$

Ans. b

$$\vec{A} = \hat{i} + \hat{j}, \quad \vec{B} = 2\hat{i} - \hat{j}$$

let $\vec{C} = x\hat{i} + y\hat{j}$

$$\therefore \vec{A}\cdot\vec{B} = (\hat{i}+\hat{j})\cdot(2\hat{i}-\hat{j})$$
$$= 2 - 1$$
$$\boxed{\vec{A}\cdot\vec{B} = 1}$$

$$\vec{B}\cdot\vec{C} = (\hat{i}+\hat{j})\cdot(x\hat{i}+y\hat{j}) = x+y$$
& $\vec{A}\cdot\vec{C} = (2\hat{i}-\hat{j})\cdot(x\hat{i}+y\hat{j}) = 2x-y$

$$\therefore \vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{C} = \vec{A}\cdot\vec{C}$$
$$1 = x+y = 2x-y$$

$x+y = 1$, & $2x - y = y + x$

$\Rightarrow \boxed{y = 1-x}$ $\quad \Rightarrow 2x - x = 2y$

$$\boxed{y = \frac{x}{2}}$$

$$\Rightarrow 1-x = \frac{x}{2} \Rightarrow 1 = x + \frac{x}{2} = \frac{3x}{2}$$

$$\boxed{x = \frac{2}{3}}$$

$$y = 1-x = 1-\frac{2}{3} = \frac{1}{3}$$

$$\boxed{y = \frac{1}{3}}$$

$$\vec{c} = \frac{2}{3}\hat{i} + \frac{1}{3}\hat{j}$$

$$|\vec{C}| = \sqrt{\left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2}$$

$$|\vec{C}| = \sqrt{\frac{4}{9} + \frac{1}{9}}$$

$$\boxed{|\vec{C}| = \sqrt{5/9}}$$

Q) The angle between two vectors $-2\hat{\imath} + 3\hat{\jmath} + \hat{k}$ and $\hat{\imath} + 2\hat{\jmath} - 4\hat{k}$ is:

(a) 0^o (b) 90^o (c) 180^o (d) None of these

Ans. b

$\vec{a} = -2\hat{i} + 3\hat{j} + \hat{k}$

$\vec{b} = \hat{i} + 2\hat{j} - 4\hat{k}$

for angle between $\vec{a}$ & $\vec{b}$

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\,|\vec{b}|}$$

$|\vec{a}| = \sqrt{(-2)^2 + (3)^2 + (1)^2} = \sqrt{14}$

$|\vec{b}| = \sqrt{(1)^2 + (2)^2 + (4)^2} = \sqrt{21}$

$\vec{a}\cdot\vec{b} = (-2\hat{i} + \hat{j} + \hat{k})\cdot(\hat{i} + 2\hat{j} - 4\hat{k})$

$\vec{a}\cdot\vec{b} = -2 + 6 - 4 = 0$

$$\cos\theta = \frac{0}{\sqrt{14}\cdot\sqrt{21}}$$

$\cos\theta = 0$

$\boxed{\theta = 90^\circ}$

Q) Given vector $\vec{a} = 2\hat{\imath} + 3\hat{\jmath}$, and vector $\vec{b} = \hat{\imath} + \hat{\jmath}$. What is the vector component of $\vec{a}$ in the direction of $\vec{b}$:

(a) $\frac{5}{2}\hat{\imath} + \frac{5}{2}\hat{\jmath}$

(b) $5\hat{\imath} + 5\hat{\jmath}$

(c) $2\hat{\imath} + 2\hat{\jmath}$

(d) None of these

Ans. a

$\vec{a} = 2\hat{i} + 3\hat{j}$

$\vec{b} = \hat{i} + \hat{j}$

Component of $\vec{a}$ along $\vec{b}$ is :-

$$\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} \hat{b}$$

$$\hat{b} = \frac{\vec{b}}{|\vec{b}|} = \frac{\hat{i} + \hat{j}}{\sqrt{1^2+1^2}} = \frac{\hat{i} + \hat{j}}{\sqrt{2}}$$

$$\vec{a} \cdot \vec{b} = (2\hat{i} + 3\hat{j}) \cdot (\hat{i} + \hat{j})$$
$$= 2 + 3 = 5$$

$$\text{Component} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} \hat{b}$$
$$= \frac{5}{\sqrt{2}} \times \left(\frac{\hat{i} + \hat{j}}{\sqrt{2}} \right)$$
$$= \frac{5}{2}\hat{i} + \frac{5}{2}\hat{j}$$

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Q) Find the angle between $\vec{A} = 4\hat{\imath} + \hat{\jmath} + 3\hat{k}$ and $\vec{B} = \hat{\imath} + 3\hat{\jmath} + 4\hat{k}$:

(a) $\cos^{-1}\frac{26}{19}$

(b) $\cos^{-1}\frac{19}{26}$

(c) $\cos^{-1}\frac{21}{26}$

(d) None of these

Ans. b

$$\vec{A} = 4\hat{i} + \hat{j} + 3\hat{k} \quad ; \quad \vec{B} = \hat{i} + 3\hat{j} + 4\hat{k}$$

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|} = \frac{(4\hat{i}+\hat{j}+3\hat{k})\cdot(\hat{i}+3\hat{j}+4\hat{k})}{(\sqrt{4^2+1^2+3^2})(\sqrt{1^2+3^2+4^2})}$$

$$\cos\theta = \frac{4+3+12}{(\sqrt{26})(\sqrt{26})} = \frac{19}{26}$$

$$\boxed{\theta = \cos^{-1}\left(\frac{19}{26}\right)}$$

Q) The position vectors of points A, B, C and D are $\vec{A} = 3\hat{\imath} + 4\hat{\jmath} + 5\hat{k}$, $\vec{B} = 4\hat{\imath} + 5\hat{\jmath} + 6\hat{k}$, $\vec{C} = 7\hat{\imath} + 9\hat{\jmath} + 3\hat{k}$ and $\vec{D} = 4\hat{\imath} + 6\hat{\jmath}$ then the displacement vectors AB and CD are?

(a) Perpendicular
(b) Parallel
(c) Antiparallel
(d) Inclined at an angle of 60^0

Ans. c

$\vec{A} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

$\vec{B} = 4\hat{i} + 5\hat{j} + 6\hat{k}$

$\vec{C} = 7\hat{i} + 9\hat{j} + 3\hat{k}$

$\vec{D} = 4\hat{i} + 6\hat{j}$

$\vec{AB} = \vec{B} - \vec{A}$

$\boxed{\vec{AB} = \hat{i} + \hat{j} + \hat{k}}$

$\vec{CD} = \vec{D} - \vec{C}$

$\boxed{\vec{CD} = -3\hat{i} - 3\hat{j} - 3\hat{k}}$

$\vec{CD} = -3(\hat{i} + \hat{j} + \hat{k})$

$\vec{CD} = -3(\vec{AB})$

$\therefore$ angle between $\vec{CD}$ & $\vec{AB}$ is "180°"

So, $\vec{CD}$ & $\vec{AB}$ are antiparallel.

Q) If $\vec{a},\ \vec{b},\ \vec{c}$ are vectors such that $\vec{a}+\vec{b}+\vec{c}=0$ and $|\vec{a}|=7, \left|\vec{b}\right|=5, |\vec{c}|=3.$ then the angle between c and b is:

(a) $\frac{\pi}{3}$

(b) $\frac{\pi}{6}$

(c) $\frac{\pi}{4}$

(d) π

Ans. a

$\vec{a} + \vec{b} + \vec{c} = 0$

$\vec{a} + \vec{b} = -\vec{c}$

$\vec{b} + \vec{c} = -\vec{a}$

$|\vec{b} + \vec{c}| = |\vec{a}|$

$|\vec{b} + \vec{c}|^2 = |\vec{a}|^2$

$\because (\vec{b} + \vec{c}) \cdot (\vec{b} + \vec{c}) = |\vec{b} + \vec{c}|^2$

$\therefore$

$(\vec{b} + \vec{c}) \cdot (\vec{b} + \vec{c}) = |\vec{a}|^2$

$|\vec{b}|^2 + |\vec{c}|^2 + 2|\vec{b}||\vec{c}| \cos\theta = |\vec{a}|^2$

where; θ = angle between $\vec{b}$ & $\vec{c}$

$\because |a| = 7, |b| = 5, |c| = 3$

$\Rightarrow (5)^2 + (3)^2 + 2(5)(3) \cos\theta = (7)^2$

$\cos\theta = \frac{49 - 25 - 9}{30} = \frac{15}{30} = \frac{1}{2}$

$\boxed{\theta = \frac{\pi}{3}}$

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