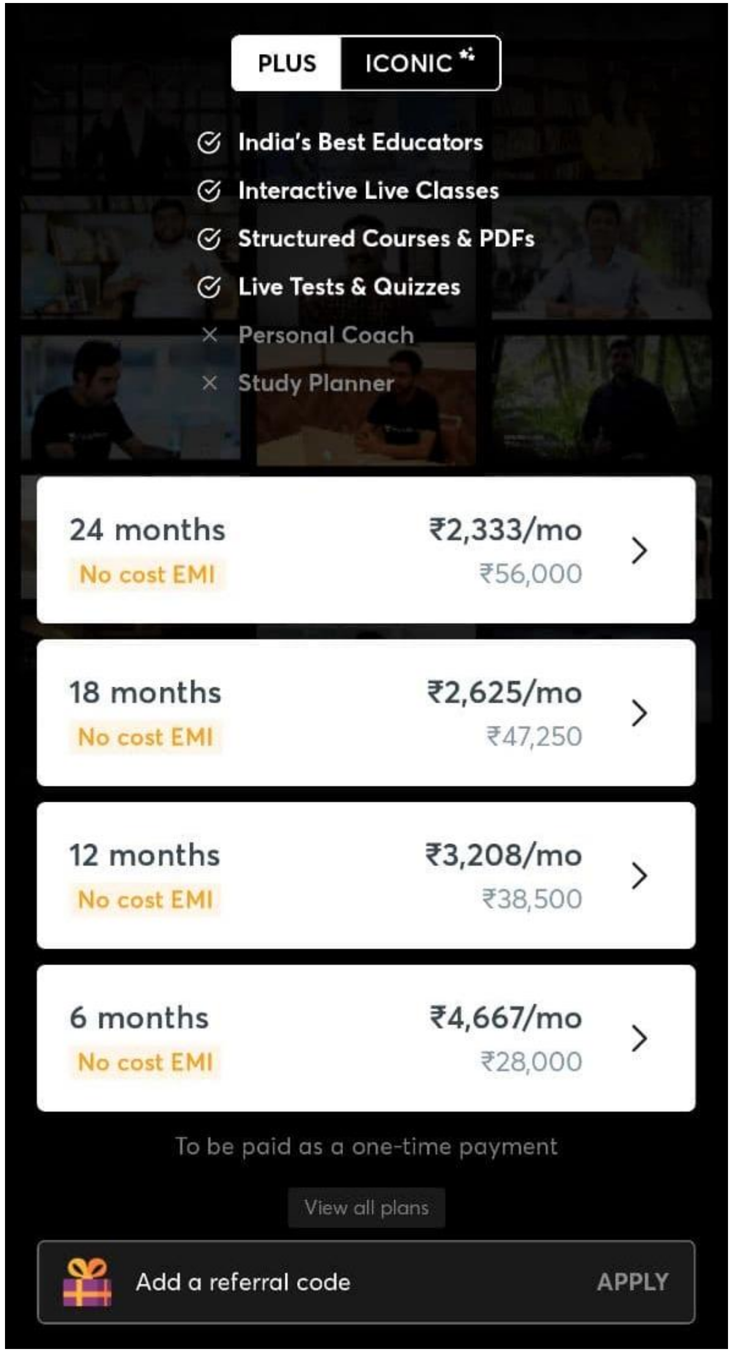
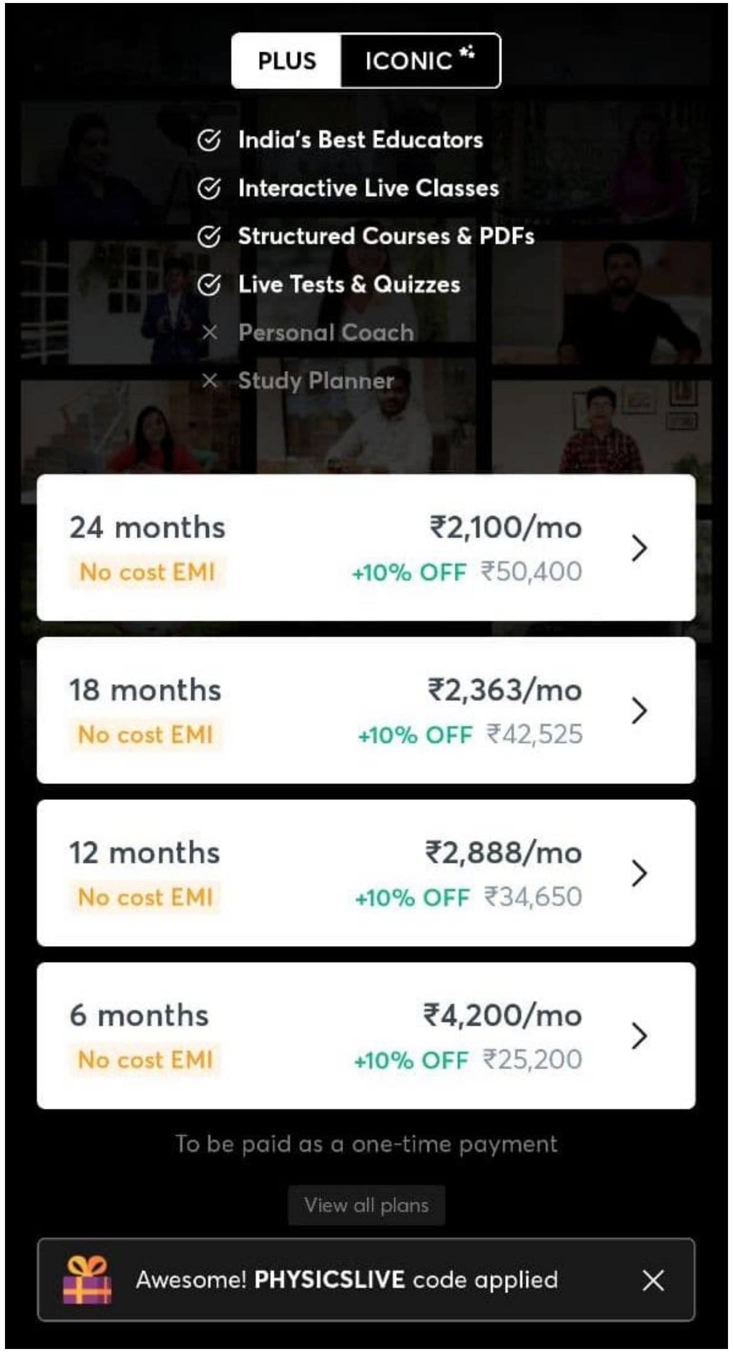
SIR PRATEEK JAIN

. IIT JEE & NEET FACULTY (KOTA)
. TOP PHYSICS FACULTY ON UNACADEMY.
. 8+ YEARS OF TEACHING EXPERIENCE
. RESEARCH WORK WITH HC VERMA SIR AT IIT KANPUR
. PRODUCED RANKS LIKE AIR 6, AIR 10 ETC.



PHYSICSLIVE



Use code PHYSICSLIVE to get 10% OFF on Unacademy PLUS.

For Video Solution of this DPP, Click on below link

Solution on Website:- https://physicsaholics.com/home/courseDetails/52

Solution on YouTube:- https://youtu.be/OJ8zVSXyEPo

JEE Main & Advanced, NSEP, INPhO, IPhO

Physics DPP

DPP-5 Projectile Motion

By Physicsaholics Team

Q) The height y and distance x along the horizontal for a body projected in the vertical plane are given by $y = 8t - 5t^2$ and $x = 6t$. The initial speed of projection is

(a) 8 m/s (b) 9 m/s (c) 10 m/s (d) (10/3) m/s

Ans. c

Solution:

$$x = 6t \Rightarrow V_x = \frac{dx}{dt} = 6$$

$$y = 8t - 5t^2 \Rightarrow V_y = \frac{dy}{dt} = 8 - 10t$$

at $t = 0$

$$V_x = 6, \quad V_y = 8$$

$$\Rightarrow V = \sqrt{6^2 + 8^2} = 10\ m/sec$$

Q) A particle is projected from the ground with an initial velocity of 20 m/s at an angle of 30° with horizontal. The magnitude of change in velocity in a time interval from t = 0 to t = 0.5s is : (g = 10 m/s^2)

(a) 5 m/s (b) 2.5 m/s (c) 2 m/s (d) 4 m/s

Ans. a

Solution:

Since acceleration is constant.

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

$$\Rightarrow \Delta \vec{v} = \vec{a}\,\Delta t$$

$$= 10 \times .5 \downarrow$$

$$= 5\ m/sec^2 \downarrow$$

Q) The velocity of a particle moving in the x-y plane is given by $\frac{dx}{dt} = 8\pi \sin 2\pi t$, $\frac{dy}{dt} = 5\pi \cos 2\pi t$. When t = 0, x = 8 and y = 0. The path of the particle is

(a) A straight line (b) A circle (c) An ellipse (d) Parabola

Ans. c

Solution:

$$\frac{dx}{dt} = 8\pi \sin 2\pi t \Rightarrow \int_8^x dx = 8\pi \int_0^t \sin 2\pi t \, dt$$

$$\Rightarrow x - 8 = \frac{\cancel{8\pi}^{\,4}}{\cancel{2\pi}}\left[-\cos 2\pi t\right]_0^t$$

$$\Rightarrow x = 8 + 4\left[(-\cos 2\pi t) - (-\cos 0)\right]$$

$$\Rightarrow x = 12 - 4\cos 2\pi t$$

$$\int_0^y dy = 5\pi \int_0^t \cos 2\pi t \, dt \Rightarrow y = \frac{5\cancel{\pi}}{2\cancel{\pi}}\left[\sin 2\pi t\right]_0^t$$

$$\Rightarrow y = \frac{5}{2}\sin 2\pi t$$

$$\sin^2 2\pi t + \cos^2 2\pi t = 1$$

$$\Rightarrow \left(\frac{2y}{5}\right)^2 + \left(\frac{12-x}{4}\right)^2 = 1$$

$\Rightarrow$ Equation of ellipse.

Q) A particle is projected at an angle of 60° above the horizontal with a speed of 10 m/s. After some time the direction of its velocity makes an angle of 30° above the horizontal. The speed of the particle at this instant is:

(a) $\frac{5}{\sqrt{3}}$m/s (b) $5\sqrt{3}$m/s (c) 5 m/s (d) $\frac{10}{\sqrt{3}}$ m/s

Ans. d

Q) A particle is projected at an angle of 60° above the horizontal with a speed of 10 m/s. After some time the direction of its velocity makes an angle of 30° above the horizontal. The speed of the particle at this instant is:

Solution:

Q) A body is thrown horizontally from a tower, 100 m high with a velocity 10 ms^{-1}. It is moving at an angle 45^0 with horizontal after:

(a) 2 sec (b) 4 sec (c) 1 sec (d) 3 sec

Ans. c

Solution:

Q) A body is thrown horizontally from a tower, 100 m high with a velocity 10 ms^{-1}. It is moving at an angle 45^0 with horizontal after:

Q) A ball is projected from origin with speed 20 m/s at an angle 30^0 with x-axis. The x-coordinate of the ball at the instant when the velocity of the ball becomes perpendicular to the velocity of projection will be

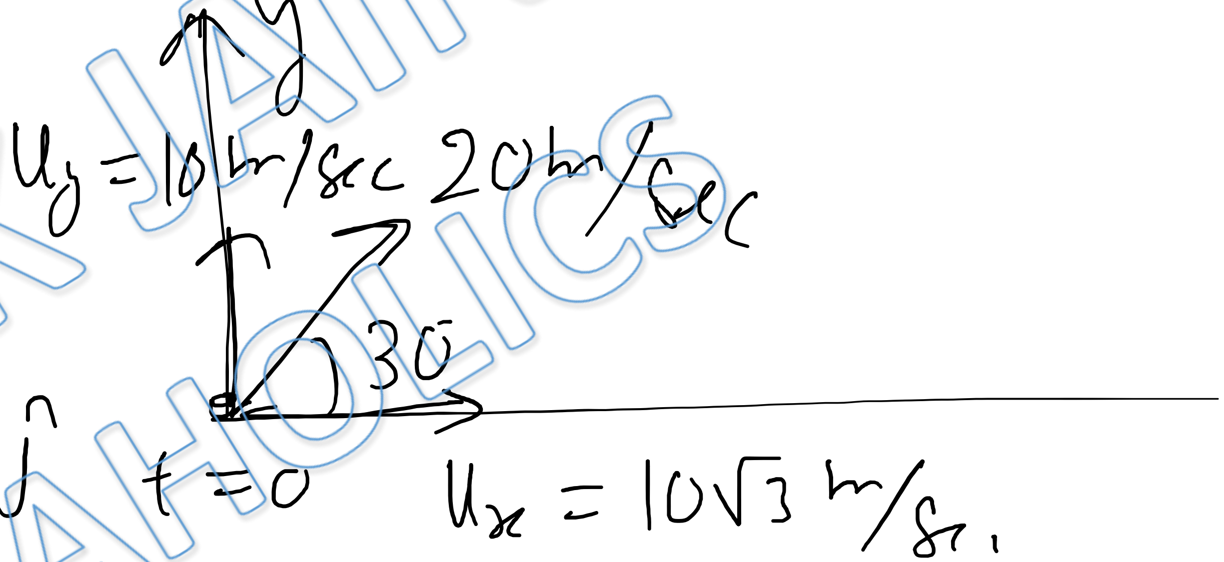
(a) $40\sqrt{3}$ m (b) 40 m (c) $20\sqrt{3}$ m (d) 20 m

Ans. a

Solution:

$$\vec{u} = 10\sqrt{3}\,\hat{i} + 10\,\hat{j}$$

at $\underline{t = t}$, $V_x = 10\sqrt{3}$



$$V_y = 10 - gt = 10 - 10t$$

$$\Rightarrow \vec{V} = 10\sqrt{3}\,\hat{i} + (10 - 10t)\,\hat{j} \qquad t = 0$$

$$\vec{V} \perp \vec{u} \Rightarrow \vec{V}\cdot\vec{u} = 0 \Rightarrow 300 + 10(10 - 10t) = 0$$

$$\Rightarrow 400 - 100t = 0 \Rightarrow t = 4\text{ sec}$$

now $x = u_x t = 10\sqrt{3} \times 4 = \underline{\underline{40\sqrt{3}}}$

Q) If the angle of projection of a particle from the horizontal is doubled keeping the speed of projection same, the particle strikes the same target on the ground, then the ratio of time of flight in the two cases will be

(a) $1 : 1$ (b) $1 : 2$ (c) $2 : \sqrt{3}$ (d) $1 : \sqrt{3}$

Ans. d

Solution:

If angle of projection in first case is θ then in second case it will be 2θ.

Since u & R are same sum of two possible angles of projection must be 90°

$$\Rightarrow 3\theta = 90^\circ \Rightarrow \theta = 30^\circ, \quad 2\theta = 60^\circ$$

$$T_1 = \frac{2u\sin 30^\circ}{g}, \quad T_2 = \frac{2u\sin 60^\circ}{g}$$

$$\frac{T_1}{T_2} = \frac{\sin 30}{\sin 60^\circ} = \frac{1}{\sqrt{3}}$$

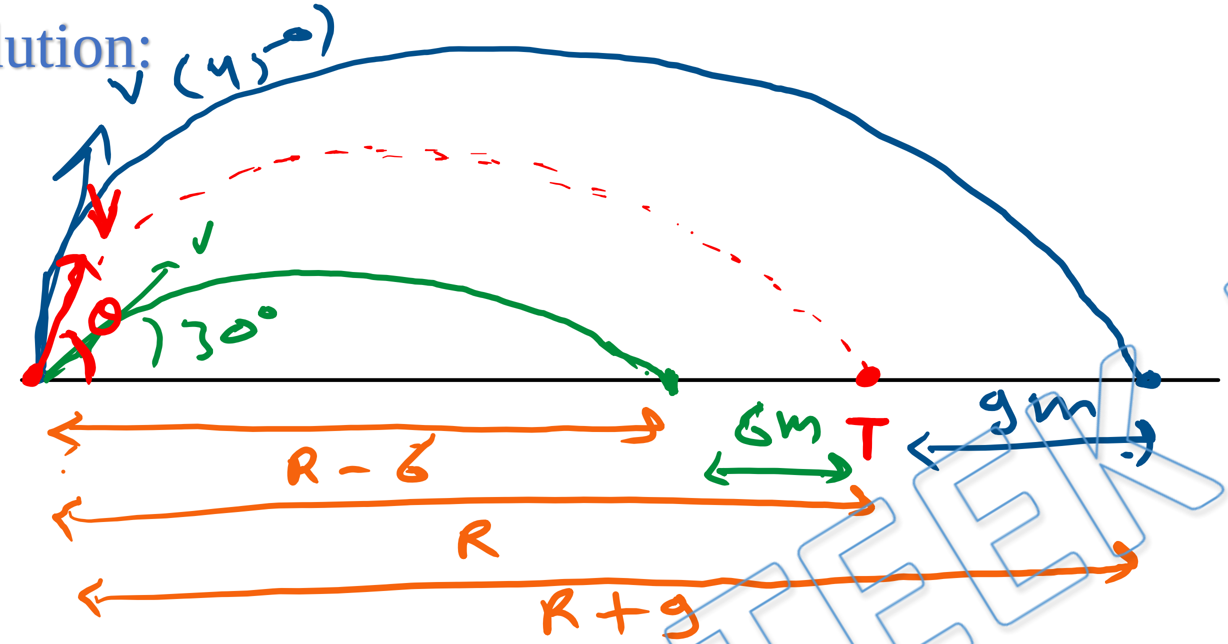
Q) A projectile is aimed at a mark on a horizontal plane through the point of projection and falls 6 m short when its elevation is 30^0 but overshoot the mark by 9 m when its elevation is 45^0. The angle of elevation of projectile to hit the target on the horizontal plane

(a) $\sin^{-1}\left[\frac{1}{5}\left(\frac{3\sqrt{3}}{2}+2\right)\right]$ (b) $\cos^{-1}\left[\frac{1}{5}\left(\frac{3\sqrt{3}}{2}+2\right)\right]$

(c) $\frac{1}{2}\cos^{-1}\left[\frac{1}{5}\left(\frac{3\sqrt{3}}{2}+2\right)\right]$ (d) $\frac{1}{2}sin^{-1}\left[\frac{1}{5}\left(\frac{3\sqrt{3}}{2}+2\right)\right]$

Ans. d

Solution:



$$R = \frac{u^2 \sin 2\theta}{g} \quad \text{—} (1)$$

$$R - 6 = \frac{u^2 \sin(2\times 30^\circ)}{g} = \frac{\sqrt{3}\,u^2}{2g} \quad \text{—} (2)$$

$$R + 9 = \frac{u^2 \sin(2\times 45^\circ)}{g} = \frac{u^2}{g} \quad \text{—} (3)$$

$$\Rightarrow \frac{(2)}{(3)} \Rightarrow \frac{R-6}{R+9} = \frac{\sqrt{3}}{2}$$

$$R = \frac{9\sqrt{3}+12}{(2-\sqrt{3})} \times \frac{(2+\sqrt{3})}{(2+\sqrt{3})}$$

$$\boxed{R = 3(3\sqrt{3}+4)(2+\sqrt{3})}$$

$$\Rightarrow \frac{(1)}{(3)} \Rightarrow \frac{R}{R+9} = \sin 2\theta$$

$$\sin 2\theta = \frac{3(3\sqrt{3}+4)(2+\sqrt{3})}{3(3\sqrt{3}+4)(2+\sqrt{3})+9}$$

$$= \frac{\cancel{3}(3\sqrt{3}+4)(2+\sqrt{3})}{\cancel{3}[(3\sqrt{3}+4)(2+\sqrt{3})+3]}$$

$$\sin 2\theta = \frac{(3\sqrt{3}+4)\cancel{(2+\sqrt{3})}}{10\cancel{(2+\sqrt{3})}}$$

$$\boxed{\theta = \frac{1}{2}\sin^{-1}\left[\frac{1}{5}\left(\frac{3\sqrt{3}}{2}+2\right)\right]}$$

Q) A batsman hits a ball at an angle of 30^0 to the horizontal with an initial speed of 15 m/s. A fielder 70 m away in the direction of the hit starts immediately to catch the ball. The speed with which the fielder should run so as to catch the ball just before it touches the ground is

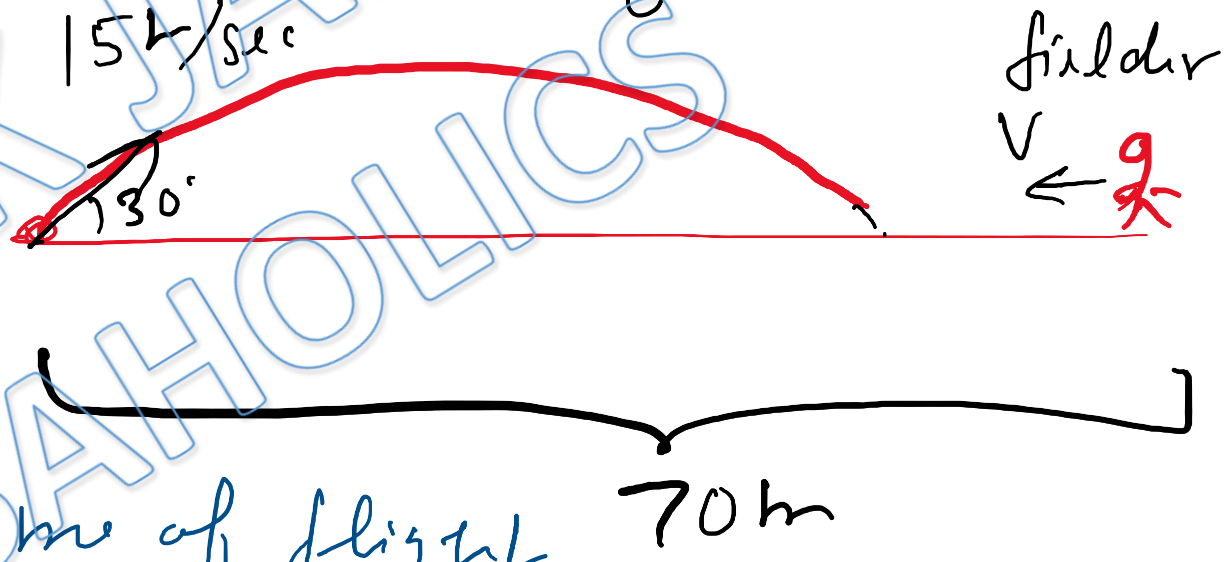
(a) 10 m/s (b) 33 m/s (c) 6.5 m/s (d)13 m/s

Ans. b

Solution:

Range of projectile $R = \frac{u^2 \sin 2\theta}{g} = \frac{15 \times 15 \times \sin 60^\circ}{g}$

$R \simeq 20\,m$



Displacement of man

$= 70 - 20 = 50\,m$

time of motion of man = time of flight

$$= \frac{2u\sin\theta}{g} = \frac{\cancel{2} \times 15 \times 1/\cancel{2}}{10} = 3/2 \text{ Sec}$$

required speed of man $= \frac{50}{3} \times 2 = \frac{100}{3}\,m/s \simeq 33\,m/s$

Q) A particle is projected from the ground with an initial speed of v at an angle θ with horizontal. The average velocity of the particle between its point of projection and highest point of trajectory is :

(a) $\frac{v}{2}\sqrt{1+2\,cos^2\theta}$

(b) $\frac{v}{2}\sqrt{1+cos^2\theta}$

(c) $\frac{v}{2}\sqrt{1+3\,cos^2\theta}$

(d) $v\cos\theta$

Ans. c

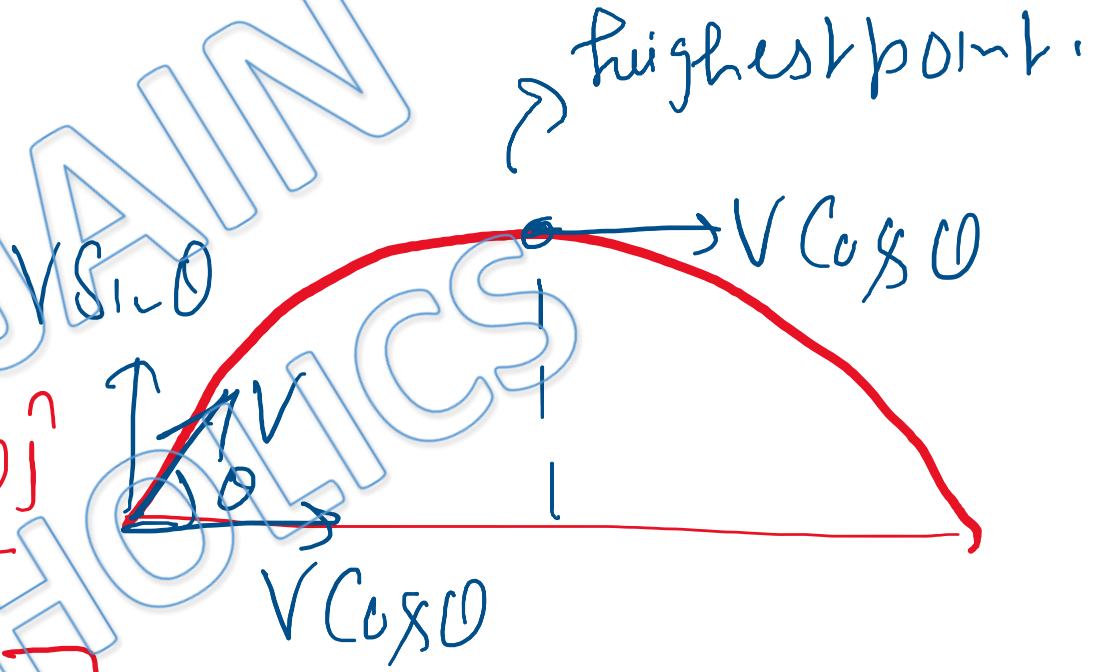
Solution:

$$\vec{u} = V\cos\theta\,\hat{i} + V\sin\theta\,\hat{j}$$

$$\vec{v} = V\cos\theta\,\hat{i}$$

$$\vec{V}_{av} = \frac{\vec{u}+\vec{v}}{2} = \frac{2V\cos\theta\,\hat{i} + V\sin\theta\,\hat{j}}{2}$$

$$= \frac{V}{2}\left[2\cos\theta\,\hat{i} + \sin\theta\,\hat{j}\right]$$



$$V_{av} = \frac{V}{2}\sqrt{4\cos^2\theta + \sin^2\theta} = \frac{V}{2}\left(1 + 3\cos^2\theta\right)$$

Q) The horizontal range and maximum height attained by a projectile are Rand H respectively. If a constant horizontal acceleration $a = \frac{g}{4}$ is imparted to the projectile due to wind, then its horizontal range and maximum height will be:

(a) (R + H), $\frac{H}{2}$

(b) $\left(R + \frac{H}{2}\right), 2H$

(c) (R + 2H), H

(d) (R + H), H

Ans. d

Solution:

Since vertical motion is same in both cases

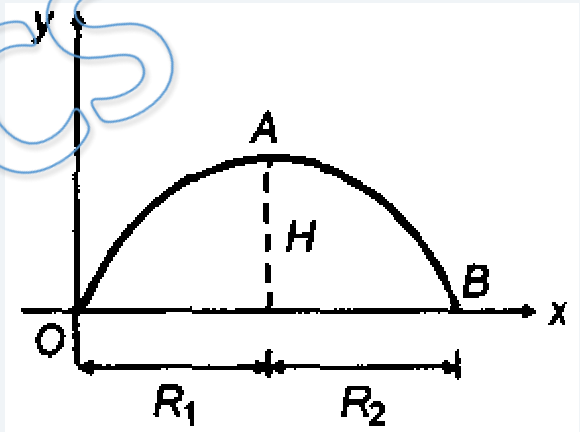
$H' = H$ & $T' = T = 2u_y/g$ $\quad R = u_x T$

In second case horizontal motion is motion with constant acceleration $(g/4)$

$$R_1 = u_x T + \frac{1}{2} \times \frac{g}{4} T^2 = R + \frac{1}{2} \times g/4 \times \left(\frac{2u_y}{g}\right)^2$$

$$= R + \frac{1}{2} \times \frac{\cancel{g}}{\cancel{4}} \times \frac{\cancel{4} u_y^2}{g^{\cancel{2}}} = R + H$$

Q) In a projectile motion let $t_{OA} = t_1$ and $t_{AB} = t_2$. The horizontal displacement from O to A is R_1 and from A to B is R_2. Maximum height is H and time of flight is T. If air drag is to be considered, then choose the correct alternative(s).



(a) t_1 will decrease while t_2 will increase

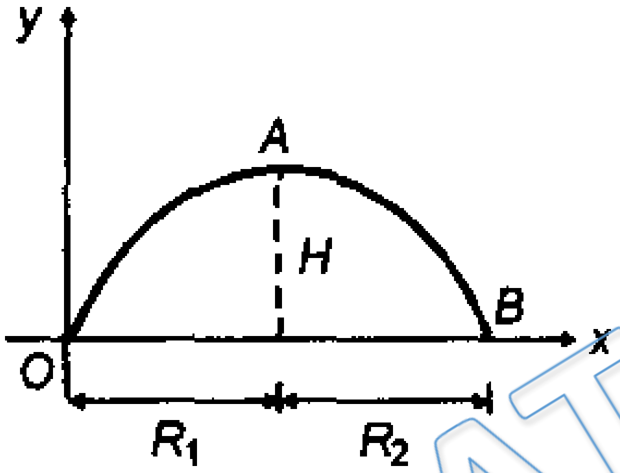
(b) H will increase

(c) R_1 will decrease while R_2 will increase

(d) None of these

Ans. a

Solution:



due to air drag, vertical acceleration of projectile will be greater than g & during descent gr will be less than g.

due to which

$$t_1 < t_2$$

Q) A ball is projected from 10 m heigh tower with initial speed 10 m/s. Find maximum possible range on ground?

(a) $10\sqrt{3}$ m

(b) $5(1+\sqrt{5})$m

(c) $5\sqrt{5}$ m

(d) none of these

Ans. a

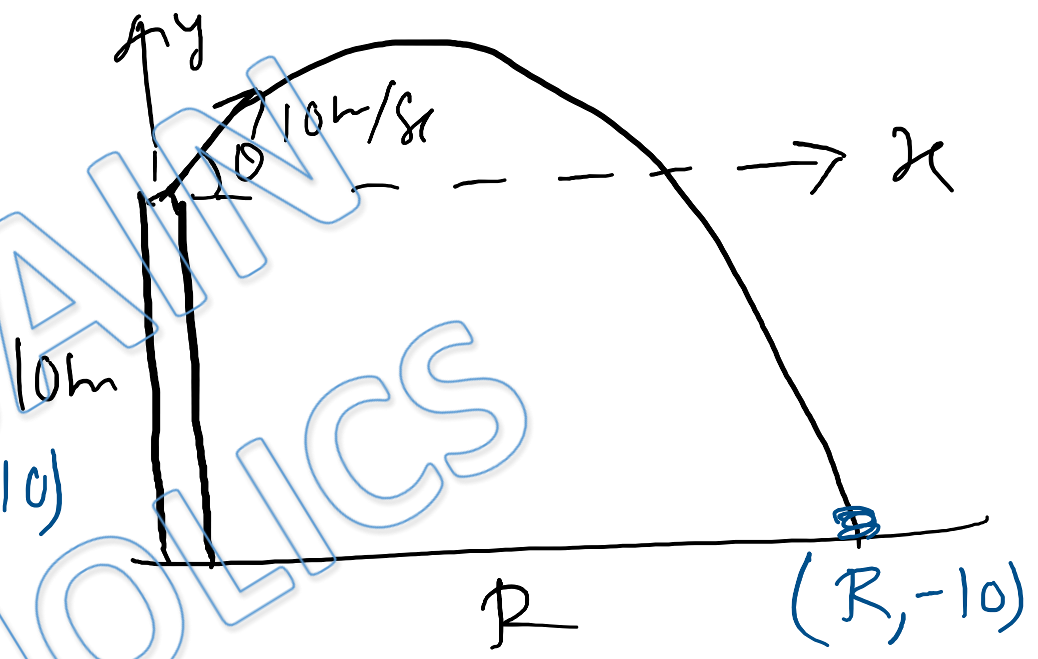
Solution:

Equation of trajectory is

$$y = x\tan\theta - \frac{\cancel{10}x^2}{2\times 10\cancel{0}}\sec^2\theta$$

Since ball is passing through $(R, -10)$

$$-10 = R\tan\theta - \frac{R^2(1+\tan^2\theta)}{20}$$



$$\Rightarrow -200 = 20R\tan\theta - R^2 - R^2\tan^2\theta$$

$$\Rightarrow R^2\tan^2\theta - 20R\tan\theta + (R^2 - 200) = 0$$

for real roots $b^2 \geq 4ac \Rightarrow \overset{100}{\cancel{400}}\cancel{R^2} \geq \cancel{4}\cancel{R^2}(R^2 - 200)$

$$\Rightarrow R^2 = 300 \Rightarrow R = 10\sqrt{3}\ m$$

For Video Solution of this DPP, Click on below link

Solution on Website:-

https://physicsaholics.com/home/courseDetails/52

Solution on YouTube:-

https://youtu.be/OJ8zVSXyEPo

Chalo Niklo