WORK BOOK SOLUTION

OBJECTIVE QUESTIONS (ONE CORRECT ANSWER)

Level-I

1. $S_{2n} = S_n'$

$\Rightarrow \frac{2n}{2}\left[2.2+(2n-1)3\right]=\frac{n}{2}\left[2.57+(n-1)2\right]$

$\Rightarrow 5n = 55$

2. $S = \frac{1}{1-(1/2)} = 2$

$S_n = \frac{1-(1/2)^n}{1-(1/2)} = \frac{1}{2^{n-1}}$ =2-$\frac{1}{2^{n-1}}$

$S - S_n = \frac{1}{2^{n-1}} < \frac{1}{1000}$ or $2^{n-1} \geq 1000$

Now $2^{10} = 32 \times 32 = 1024$

$\therefore$ $n - 1 \geq 10$ or $n \geq 11$

Hence the least value is 11.

3. We given, $a_5 + a_{20} = a_1 + a_{24}, a_{10} + a_{15} = a_1 + a_{24}$

Hence the given relations reduce to, $3(a_{1,} + a_{24}) = 225$, giving $a_1 + a_{24} = 75$

Hence S_{24} = $\frac{n}{2}(a+l) = (24/2)(a_1 + a_{24}) = 12 \times 75 = 900$

4. $S_p = \frac{p}{2}[2A + (p-1)d] = a$

$\therefore \frac{2a}{p} = 2A + (p-1)d$ .... (i)

$\therefore \frac{2b}{q} = 2A + (q-1)d$ .... (ii)

$\therefore \frac{2c}{r} = 2A + (r-1)d$ .... (iii)

Multiply (i), (ii) and (iii) by q – r, r – p and p – q respectively and add

$\therefore \Sigma \frac{a}{p}(q-r) = 0$

5. Since the numbers are in A.P.

$\therefore\ 28 = 3^{2\sin 2\theta - 1} + 3^{4 - 2\sin 2\theta}$

or $28 = \frac{9^{\sin 2\theta}}{3} + \frac{81}{9^{\sin 2\theta}}$, where x = $9^{\sin 2\theta}$

or $x^2 - 84x + 243 = 0$

or (2.81) (x – 3) = 0 $\quad \therefore x = 81 \text{ or } 3$

$\therefore x = 9^{\sin 2\theta} = 81, 3 \text{ or } 9^2, 9^{1/2}$

$\therefore \sin 2\theta = 2 \ \text{ or } 1/2$

since sin 2θ cannot be greater than 1 so we choose sin 2θ = $\frac{1}{2}$

Hence the terms in A.P. are

3^0, 14, 27 i.e. 1, 14, 27.

$\therefore T_5 = a + 4d$ = 1 + 4.13 = 53

6. $T_m = S_m - S_{m-1}$

$\therefore$ **164 = 3 (2m – 1) + 5.1** $\therefore$ **6m = 162**

7. We can rewrite the series as

$$1 + 1 + \frac{5}{x} + \left(\frac{5}{x}\right)^2 + \left(\frac{5}{x}\right)^3 + \ldots\ldots\ldots.$$

We can sum up this series if |5/x| < 1

$\Leftrightarrow |x| > 5$

8. $b = \frac{2ac}{a+c}$ and $c = \frac{2bd}{b+d}$

$\therefore (a+c)(b+d) = \frac{2ac}{b}.\frac{2bd}{c} = 4ad$

$\Rightarrow ab + bc + cd = 3ad$

9. L.C.M. of 2 and 5 is 10.

Numbers divisible by 2 will contain numbers which are also divisible by 10. Similarly numbers divisible by 5 will contain numbers which are also divisible by 10. Thus the number divisible by 10 will occur twice. Hence we can

write S = $S_2 + S_5 - S_{10}$

Now, $S_2 = \frac{2.50.51}{2} = 2550$ by $\Sigma n = \frac{n(n+1)}{2}$

Similarly, S_5 = 1050, S_{10} = 550

$\therefore$ S = $S_2 + S_5 - S_{10}$

= 2550 + 1050 – 550 = 3050

10. $S_n = (2n-4)\frac{\pi}{2} = (n-2)180^0$ (formula for polygon)

a = 120^0, d = 5^0

$S_n = \frac{n}{2}[2a + (n-1)d]$ for A.P.

$\therefore n^2 - 25n + 144 = 0$

$\therefore$ n = 9, 16

But when n = 16 then $T_{16} = 195^0$ which is not possible

$\therefore$ n = 9 only

11. $a_1, a_2, a_3 .... a_n$ are in H.P. $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3} ...... \frac{1}{a_n}$ are in A.P.

Multiply each term by $a_1 + a_2 + a_3 + ... + a_n$ then substract 1 from each term

we get $\frac{a_2 + a_3 + ... + a_n}{a_1}, \frac{a_1 + a_3 + ... + a_n}{a_2}, ... \frac{a_1 + a_2 + ... + a_{n-1}}{a_n}$ are in A.P.

$\therefore \frac{a_1}{a_2 + a_3 + ... + a_n}, \frac{a_2}{a_1 + a_3 + ... + a_n}, ... \frac{a_n}{a_1 + a_2 + ... + a_{n-1}}$ in H.P.

12. We can write the given equation as

$$\log_2\left(x^{1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+..........}\right) = 4$$

$$\Rightarrow \log_2(x^2) = 4 \Rightarrow x^2 = 2^4 \Rightarrow x = 4$$

13. . $\frac{S_m}{S_n} = \frac{m^2}{n^2}$ $\therefore \frac{S_m}{m^2} = \frac{S_n}{n^2} = k$ (say)

$$\frac{T_m}{T_n} = \frac{S_m - S_{m-1}}{S_n - S_{n-1}} = \frac{k\left[m^2 - (m-1)^2\right]}{k\left[n^2 - (n-1)^2\right]} = \frac{2m-1}{2n-1}$$

14. $\frac{S_n}{S'_n} = \frac{(n/2)[2a + (n-1)d]}{(n/2)\left[2a' + (n-1)d'\right]} = \frac{3n+8}{7n+15}$

or, $\frac{a + \left(\frac{n-1}{2}\right)d}{a' + \left(\frac{n-1}{2}\right)d'} = \frac{3n+8}{7n+15}$ ...(i)

we get, $\frac{T_{12}}{T'_{12}} = \frac{7}{16}$

Choosing (n – 1) / 2 = 11 or n = 23 in (i)

we get, $\frac{T_{12}}{T'_{12}} = \frac{7}{16}$

15. If the numbers by x, y, z then
$1/x = \log_2 3, 1/y = \log_2 2.3 = 1 + \log_2 3$,
$1/z = \log_2 (4 \times 3) = 2 + \log_2 3$ which are in A.P.
$\therefore$ x, y, z are in H.P.

16. $T_{m+n} = ar^{m+n-1} = p \qquad T_{m-n} = ar^{m-n-1} = q$

Multiplying $a^2 r^{2m-2} = pq$

$\therefore T_m = ar^{m-1} = \sqrt{(pq)}$

17. $1, x_1, x_2 ... x_m$, 31 is an A.P. of (m + 2) terms.
$31 = T_{m+2} = a + (m + 1)d = 1 + (m + 1)d$

$\therefore d = \frac{30}{(m+1)}$

Now $\frac{x_7}{x_{m-1}} = \frac{5}{9} \qquad \therefore \frac{T_8}{T_m} = \frac{a + 7d}{a + (m-1)d} = \frac{5}{9}$

Now put for a and d and we get m = 14.

18. We have

$$\frac{\pi}{4} = \left(1 - \frac{1}{3}\right) + \left(\frac{1}{5} - \frac{1}{7}\right) + \left(\frac{1}{9} - \frac{1}{11}\right) + ... = \frac{2}{1.3} + \frac{2}{5.7} + \frac{2}{9.11} + ...$$

$$\Rightarrow \frac{1}{1.3} + \frac{1}{5.7} + \frac{1}{9.11} + ... = \frac{\pi}{8}$$

19. If t_r denotes the nth term of the series, then

$$xt_r = \frac{x}{(1+rx)(1+(r+1)x)} = \frac{1}{1+rx} - \frac{1}{1+(r+1)x}$$

$$\Rightarrow x\sum_{r=1}^{n} t_r = \sum_{r=1}^{n}\left[\frac{1}{1+rx} - \frac{1}{1+(r+1)x}\right] = \frac{1}{1+x} - \frac{1}{1+(n+1)x} = \frac{nx}{(1+x)(1+(n+1)x)}$$

$$\Rightarrow \sum_{r=1}^{n} t_r = \frac{n}{(1+x)\left[1+(n+1)x\right]}$$

20. We have

$$\frac{1}{k}(1+2+3+........+k)=\frac{1}{k}\frac{k(k+1)}{2}=\frac{k+1}{2}$$

Thus, $S=\frac{1}{2}[2+3+4+.........21]=\frac{10}{2}(2+21)=115$

21. We have $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+........$ upto ∞

$$=\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\frac{1}{5^2}+\frac{1}{6^2}...........\text{ upto }\infty$$

$$-\frac{1}{2^2}\left[1+\frac{1}{2^2}+\frac{1}{3^2}+...........\right]=\frac{\pi^2}{6}-\frac{1}{4}\left(\frac{\pi^2}{6}\right)=\frac{\pi^2}{8}$$

22. We can write S as
S = (1 – 2) (1 + 2) + (3 – 4) (3 + 4) + ... + (2001 – 2002) (2001 + 2002) + 2003^2
= [1 + 2 + 3 + 4 + ... + 2002] + 2003^2

= $-\frac{1}{2}(2002)(2003)+2003^2=2007006$

23. Using $AM \geq GM$

$$\frac{a+b+c}{3}\geq(abc)^{\frac{1}{3}}$$

$\frac{3b}{3}\geq(abc)^{\frac{1}{3}}$ (since 2b = a + c)

$$b\geq 4^{\frac{1}{3}}$$

24. Since $AM \geq HM$

$$\frac{x+y+z}{3}\geq\frac{3}{\frac{1}{x}+\frac{1}{y}+\frac{1}{z}}$$

$$\Rightarrow\frac{a}{3}\geq\frac{3}{\frac{1}{x}+\frac{1}{y}+\frac{1}{z}}\text{ or }\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\geq\frac{9}{a}$$

25. Since a and b are unequal, $\frac{a^2+b^2}{2}>\sqrt{a^2b^2}$

(A.M. > G.M. for unequal numbers)

$\Rightarrow a^2 + b^2 > 2ab$

Similarly $b^2 + c^2 > 2bc$ and $c^2 + a^2 > 2ca$

Hence $2(a^2 + b^2 + c^2) > 2(ab + bc + ca)$

$\Rightarrow ab + bc + ca < 1$

LEVEL - II

26. $x + y + z = 1 \Rightarrow 2.\frac{x}{2} + 3.\frac{y}{3} + 4.\frac{z}{4} = 1$

using weighted mean $\frac{2.\left(\frac{x}{2}\right) + 3.\left(\frac{y}{3}\right) + 4.\left(\frac{z}{4}\right)}{9} \geq \left(\left(\frac{x}{2}\right)^2 \left(\frac{y}{3}\right)^3 \left(\frac{z}{4}\right)^4\right)^{\frac{1}{9}}$

$\left(\frac{1}{9}\right)^9 \geq \frac{x^2y^3z^4}{2^{10}3^3} \Rightarrow x^2y^3z^4 \leq \frac{2^{10}}{3^{15}}$

27. $AM \geq GM$

$\frac{\frac{b}{a} + \frac{c}{a} + \frac{c}{b} + \frac{a}{b} + \frac{a}{c} + \frac{b}{c}}{6}$

$\geq (1)^{1/6}$

$\Rightarrow$ minimum value is 6

28. S = a + ar + ar^2 + ar^2 + ... + ar^{n-1}

i.e. n terms

$S = \frac{a(1 - r^n)}{(1 - r)}$ ..... (i)

$\therefore$ P = product = a. ar. ar^2....ar^{n-1} $= a^n r^{1+2+3+4+5+....n-1} = a^n.r^{n(n-1)/2}$

$\therefore p^2 = a^{2n} r^{n(n-1)}$ ..... (ii)

$$R=\frac{1}{a}+\frac{1}{ar}+\frac{1}{ar^2}+\frac{1}{ar^3}+....+\frac{1}{ar^{n-1}} \text{ (n terms)}$$

$$\therefore R=\frac{1}{a}\frac{\left(1-\frac{1}{r^n}\right)}{1-1/r}=\frac{\left(r^n-1\right)}{(r-1)}.\frac{1}{ar^{n-1}} \text{ ..... (iii)}$$

$$\therefore \frac{S}{R}=a.\frac{\left(1-r^n\right)}{1-r}.\frac{(r-1)}{\left(r^n-1\right)}.ar^{n-1}=a^2r^{(n-1)} \quad \text{by (i) and (ii).}$$

$$\therefore \left[S/R\right]^n=a^{2n}r^{n(n-1)}=p^2 \quad \text{by (ii)}$$

29. $T_p=AR^{p-1}=x$

$\log x=\log A+(p-1)\log R$

Similary write log y , log z

Multiply by q – r, r – p and p – q and add we get,

$\left(q-r\right)\log x+\left(r-p\right)\log y+\left(p-q\right)\log z=0$

30. $ar\left(1+r^3\right)=216$ and $\frac{ar^3}{ar^5}=\frac{1}{4}$

$\Rightarrow r^2=4\Rightarrow r=2,-2$

when r = 2 then 2a (9) = 216 $\therefore a=12$

when r = – 2, then – 2a (1 – 8) = 216

$\therefore a=\frac{216}{14}=\frac{108}{7}$, which is not an integer.

31. Sum of three numbers in A.P. = 3a = 12

$\therefore \left(x-4\right)\left(x^2-8x+7\right)=0$

$\therefore x=1,4,7$ or $7,4,1,\ d=\pm 3$

32. $(a^2 + b^2 + c^2)(b^2 + c^2 + d^2) \le (ab + bc + cd)^2$,

Solving this we get

$b^4+c^4+a^2c^2+a^2b^2+b^2d^2+b^2c^2-2ab^2c-2bc^2d-2abcd\le 0$

or $\left(b^2-ac\right)^2+\left(c^2-bd\right)^2+\left(ad-bc\right)^2\le 0$

$\therefore\ b^2-ac=0\ \Rightarrow\frac{b}{a}=\frac{c}{b},\ \ c^2-bd=0\ \Rightarrow\frac{c}{b}=\frac{d}{c},\ ab-bc=0\Rightarrow\frac{d}{c}=\frac{b}{a}$

$\therefore\ \frac{b}{a}=\frac{c}{b}=\frac{d}{c}$ Hence a, b, c, d are in G.P.

33. Let c.d = d

$a_p = a_1 + (p-1)d$, $a_q = a_1 + (q-1)d$, $a_r = a_1 + (r-1)d$

as a_p, a_q, a_r are in G.P.

$$\frac{a_q}{a_p} = \frac{a_r}{a_q} = \frac{a_q - a_r}{a_p - a_q} \text{ (by law of proportions)}$$

$$\text{or } \frac{a_q}{a_p} = \frac{a_r}{a_q} = \frac{a_1 + (q-1)d - a_1 - (r-1)d}{a_1 + (p-1)d - a_1 - (q-1)d} = \frac{q-r}{p-q} \qquad \text{or } \frac{a_q}{a_p} = \frac{q-r}{p-q} = \frac{r-q}{q-p}$$

34. a + ar + ar^2 = x. ar

or, $r^2 + r(1 - x) + 1 = 0$, r is real

$\Delta > 0$ i.e. $(1-x)^2 - 4 > 0$

or, $x^2 - 2x - 3 > 0$

or, $(x + 1)(x - 3) > 0$

$\Rightarrow x < -1$ or $x > 3$

35. x log a = y log b = z log c = k (say)

Also $y^2 = xz$

$$\frac{k^2}{(\log b)^2} = \frac{k^2}{\log a . \log c}$$

$$\text{or } \frac{\log a}{\log b} = \frac{\log b}{\log c} \quad \text{or } \log_b a = \log_c b$$

36. Let the common ratio be taken as k and a be the first term.

$R = T_r = ak^{r-1}$

$\therefore R^{s-t} = a^{s-t}k^{(r-1)(s-t)}$ similarly

$S^{t-r} = a^{t-r}k^{(s-1)(t-r)}$

$T^{r-s} = a^{r-s}k^{(t-1)(r-s)}$

Multiplying the above three and knowing that

$A^m.A^n.A^p = A^{m+n+p}$

$\therefore R^{s-t}S^{1-r}T^{r-s} = a^0.k^0 = 1$

$\because \Sigma(a-b) = 0, \Sigma(a+\lambda)(b-c) = 0$

37. Three numbers in G.P. are $\frac{a}{r}, a, ar$ then $\frac{a}{r}, 2a, ar$ are in A.P. as given

$\therefore\ 2(2a) = a\left(r + \frac{1}{r}\right)$

or, $r^2 - 4r + 1 = 0$ or $r = 2 \pm \sqrt{3}$

or, $r = 2 + \sqrt{3}$ as $r > 1$ for an increasing G.P.

38. Let numbers be a & b

$\Rightarrow a, g, b$ in G.P. and a, p, q, b in A.P.

$\Rightarrow g^2 = ab$ & $p - a = q - p = b - q$

we get $a = 2p - q$ & $b = 2q - p$

so $g^2 = (2p - q)(2q - p)$

39. Let T_1 & T_{2n+1} are A & B

Between there terms A + B; T_n = a, b, c when series are in A.P, G.P & H.P. respectively.

$\Rightarrow a = \frac{A+B}{2},\ b = \sqrt{AB},\ c = \frac{2AB}{A+B}$

$\Rightarrow b^2 = ac$

40. Let the two numbers a and b

given $a + b = \frac{13}{6}$

and A.M.'s are $A_1, A_2, \ldots\ldots\ldots A_{2n}$ inserted between a and b.

Here $a, A_1, A_2, \ldots\ldots\ldots, A_{2n}, b$ are in A.P. then given condition

$A_1 + A_2 + \ldots\ldots\ldots + A_{2n} = 2n + 1$

or $(a + A_1 + A_2 + \ldots\ldots\ldots + A_{2n} + b) - (a + b) = 2n + 1$

or $\frac{(2n+2)}{2}(a+b) - (a+b) = 2n+1$

or $n(a + b) = 2n + 1$

or $13n = 12n + 6$

or $n = 6$

Hence number of means are 12

41. We can write the sum upto (2n + 1) terms as

$[a + (a + d)](-d) + [(a + 2d) + (a + 3d)](-d) + \ldots\ldots\ [(a + (2n - 2)d) + (a + (2n - 1\ d]$
$(-d) + (a + 2nd)^2$

$= (-d)[a + (a + d) + (a + 2d) + \ldots\ldots + a + (2n - 1)d] + (a + 2nd)^2$

$= (-d)\frac{2n}{2}\{a + a + (2n-1)d\} + (a + 2nd)^2$

$= -2nad - n(2n - 1)d^2 + a^2 + 4n(ad) + 4n^2d^2$

$= a^2 + 2nad + n(2n + 1)d^2$

42. $T_n = \dfrac{1}{(n+1)(n+2)(n+3)......(n+k)}$

$v_{n-1} = \dfrac{1}{(n+1)(n+2)(n+3).....(n+k-2)(n+k-1)}$

$v_n - v_{n-1} = \dfrac{1}{(n+2)(n+3)....(n+k-1)}\left[\dfrac{1}{n+k} - \dfrac{1}{n+1}\right]$

$= \dfrac{1-k}{(n+1)(n+2)(n+3)......(n+k-1)(n+k)}$

$v_n - v_{n-1} = (1-k)T_n$

$\therefore (1-k)T_n = v_n - v_{n-1}$

$(1-k)T_1 = v_1 - v_0$

$(1-k)T_2 = v_2 - v_1$

$(1-k)T_3 = v_3 - v_2$

....

$(1-k)T_n = v_n - v_{n-1}$

Adding $(1 - k)\ S_n = v_n - v_0$

or $(1-k)s_n = \dfrac{1}{(n+1)(n+2).......(n+k)} - \dfrac{1}{1.2.3......k}$ $(1-k)\underset{n\to\infty}{\text{Lt}} S_n = 0 - \dfrac{1}{\underline{|k}}$

$\underset{n\to\infty}{\text{Lt}} S_n = \dfrac{1}{(k-1)\underline{|k}}$

43. Let n = 2m, then
$S_{2m} = 1^2 + 2 \times 2^2 + 3^2 + 2 \times 4^2 + ... + (2m - 1)^2 + 2\ (2m)^2$
$= 2m\ (2m + 1)^2/2 = m\ (2m + 1)$
When n = 2m – 1
$1^2 + 2 \times 2^2 + 3^2 + 2 \times 4^2 + ... + (2m -1)^2$
$= S_{2m} - 2\ (2m)^2 = m\ (2m + 1)^2 - 2\ (2m)^2$
$= m\ [4m^2 + 4m + 1 - 8m] = m\ (2m - 1)^2$
$= n^2\ (n + 1)/2$

4. We have $t_r = \sum_{k=1}^{r} t_k - \sum_{k=1}^{r-1} t_k \;= \frac{1}{12} r(r+1)(r+2) - \frac{1}{12}(r-1)(r)(r+1)$

$$= \frac{1}{4} r(r+1)$$

Now, $\frac{1}{t_r} = \frac{4}{r(r+1)} = 4\left(\frac{1}{r} - \frac{1}{r+1}\right)$

$$\Rightarrow \sum_{r=1}^{n} \frac{1}{t_r} = 4\sum_{r=1}^{n}\left(\frac{1}{r} - \frac{1}{r+1}\right) = 4\left(1 - \frac{1}{n+1}\right) = \frac{4n}{n+1}$$

45. We have $t_r = \frac{(r-1)!}{(r+4)!}$ and $t_{r+1} = \frac{r!}{(r+5)!}$

Now, $rt_r - (r+5)t_{r+1} = \frac{r!}{(r+4)!} - \frac{r!}{(r+4)!} = 0$

$$\Rightarrow rt_r - (r+1)t_{r+1} = 4t_{r+1} \Rightarrow 4\sum_{r=1}^{n-1} t_{r+1} = \sum_{r=1}^{n-1}\left[rt_r - (r+1)t_{r+1}\right]$$

$$\Rightarrow 4(t_2 + t_3 + \ldots\ldots + t_n) = 1t_1 - nt_n$$

$$\Rightarrow 4(t_1 + t_2 + \ldots\ldots\ldots t_n) = 5t_1 - nt_n$$

$$= 5\left(\frac{0!}{5!}\right) - \frac{n(n-1)!}{(n+4)}$$

$$= \frac{1}{4!} - \frac{n!}{(n+4)!}$$

$$\Rightarrow t_1 + t_2 + \ldots\ldots\ldots t_n = \frac{1}{4}\left[\frac{1}{4!} - \frac{n!}{(n+4)!}\right]$$

46. We have $t_r = \frac{2r-1}{r(r+1)(r+2)} = \frac{2}{(r+1)(r+2)} - \frac{1}{r(r+1)(r+2)}$

$$=2\left(\frac{1}{r+1} - \frac{1}{r+2}\right)\frac{-1}{2}\left(\frac{1}{r(r+1)} - \frac{1}{(r+1)(r+2)}\right)$$

Solving by using $v_n -$ method

we get sum = $\dfrac{n(3n+1)}{4(n+1)(n+2)}$

47. Let t_r denote the rth term of the series, then

$$t_r = \frac{1^3+2^3+........+r^3}{1+3+........+(2r-1)} = \frac{\frac{1}{4}r^2(r+1)^2}{r^2} = \frac{1}{4}(r+1)^2$$

$$\Rightarrow \sum_{r=1}^{n} t_r = \frac{1}{4}\sum_{r=1}^{n}(r+1)^2 = \frac{1}{4}\left[\sum_{r=1}^{n+1} r^2 - 1\right]$$

$$= \frac{1}{4}\left[\frac{(n+1)(n+2)(2n+3)}{6} - 1\right] = \frac{1}{24}\left[2n^3+9n^2+13n+6-6\right]$$

$$= \frac{n}{24}\left(2n^2+9n+13\right)$$

48. $27pqr \geq (p+q+r)^3$

$$\Rightarrow (pqr)^{1/3} \geq \left(\frac{p+q+r}{3}\right)$$

$\Rightarrow p = q = r$

Also 3p + 4q + 5r = 12 $\Rightarrow p = q = r = 1$

49. As odd number of AM, G.M and H.M. are inserted between a & b.
So, middle term of AP is AM = a_n
middle term of GP is GM = b_n
middle term of HP is HM = c_n

$\therefore$ a_n, b_n, c_n are in G.P.

$\therefore$ D = discriminant of quadratic equation < 0

$\therefore$ roots are imagnary