1. If $C^2 + S^2 = 1$, then $\frac{1+C+iS}{1+C-iS}$ is equal to :

(a) C + iS (b) C - iS (c) S + iC (d) S - iC

2. If (1 + i) (1 + 2i) (1 + 3i) ..... (1 + ni) = $\alpha + i\beta$, then 2 .5 . 10 ...... $(1 + n^2)$ =

(a) $\alpha - i\beta$ (b) $\alpha^2 - \beta^2$ (c) $\alpha^2 + \beta^2$ (d) None of these

3. $(z - 1)(\bar{z} - 1)$ can be written as

(a) $z\bar{z} + 1$ (b) $|z|^2 + 1$ (c) $|z - 1|^2$ (d) $|z|^2 + 2$

4. The argument of $\frac{1+i\sqrt{3}}{\sqrt{3}+1}$ is equal to

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) None of these

5. If |z| = 1 and p = $\frac{z-1}{z+1}$ (where $z \neq -1$) then Re(p) equals -

(a) 0 (b) $-\frac{1}{|z+1|^2}$ (c) $\left|\frac{z}{z+1}\right|\frac{1}{|z+1|^2}$ (d) $\frac{\sqrt{2}}{|z+1|^2}$

6. If $iz^3 + z^2 - z + i = 0$, then | z | equals -

(a) 4 (b) 1 (c) 3 (d) None of these

7. If x + iy = $\frac{u+iv}{u-iv}$, then $x^2 + y^2$ =

(a) 1 (b) -1 (c) 0 (d) None of these

8. The square root of i is-

(a) $\pm\frac{1}{\sqrt{2}}(1 + i)$ (b) $\pm\frac{1}{\sqrt{2}}(1 - i)$

(c) $\pm\frac{1}{\sqrt{2}}(-1 + i)$ (d) None of these

9. A square root of 3 + 4i is

(a) $\sqrt{3}+i$ (b) 2 - I (c) 2 + I (d) None of these

---

10. The argument of the complex number $\sin\frac{6\pi}{5} + i\left(1+\cos\frac{6\pi}{5}\right)$ is

(a) $\frac{6\pi}{5}$ (b) $\frac{5\pi}{6}$ (c) $\frac{9\pi}{10}$ (d) $\frac{2\pi}{5}$

11. A square root of 3 + 4i is

(a) $\sqrt{3}+i$ (b) 2 - I (c) 2 + I (d) None of these

12. If z is a complex number such that | z | = 4 and arg(z) = $\frac{5\pi}{6}$ then z is equal to

(a) -2$\sqrt{3}$ + 2i (b) 2$\sqrt{3}$ + 2i (c) 2$\sqrt{3}$ - 2i (d) -$\sqrt{3}$ + i

13. If i = $\sqrt{-1}$ and n is a positive integer, then $i^n + i^{n+1} + i^{n+2} + i^{n+3}$ is equal to

(a) 1 (b) I (c) i^n (d) 0

14. The modulus of $\frac{1+7i}{(2-i)^2}$ is -

(a) $\sqrt{5}$ (b) 0 (c) $\sqrt{2}$ (d) 1

15. The value of the sum $\sum_{n=1}^{13}\left(i^n + i^{n+1}\right)$, where i = $\sqrt{-1}$ is

(a) I (b) i - 1 (c) -I (d) 0

16. If (1 + i)z = (1 - i) $\bar{z}$, then z is -

(a) x(1 - i), x ∈ R (b) x (1 + i), x ∈ R

(c) $\frac{x}{1+i}$, x ∈ R^+ (d) None of these

17. If $\frac{1-i\alpha}{1+i\alpha}$ = A + iB, then $A^2 + B^2$ equals -

(a) 1 (b) α^2 (c) -1 (d) $-\alpha^2$

18. If α, β are two different complex number such that |α| = 1, |β| = 1 then the expression $\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|$ equals :

(a) $\frac{1}{2}$ (b) 1 (c) 2(d) None of these

19. If (1 + i) (1 + 2i) (1 + 3i) .....(1 + ni) = α + iβ, then 2.5, 10, ...(1 + n^2) =

(a) α - iβ (b) $\alpha^2 - \beta^2$ (c) $\alpha^2 + \beta^2$ (d) None of these

20. If z_1, z_2, z_3 are the complex numbers, such that $|z_1| = |z_2| = |z_3| = \left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right| = 1$, then

$|z_1 + z_2 + z_3|$ is -

(a) Equal to 1 (b) Less than 1

(c) Greater than 1 (d) Equal to 3

21. If z_1 and z_2 are two non-zero complex numbers such that

$|z_1 + z_2| = |z_1| + |z_2|$, then Arg z_1 - Arg z_2 =

(a) $-\pi$ (b) $-\pi/2$ (c) 0 (d) $\pi/2$

22. The value of $\sum_{r=1}^{102} i^r$ is -

(a) 0 (b) I (c) - I (d) -1

23. The modulus of the complex number $\frac{(1-i\sqrt{3})(\cos\theta+i\sin\theta)}{2(1-i)(\cos\theta-i\sin\theta)}$ is

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2\sqrt{2}}$

(c) $\frac{1}{\sqrt{3}}$ (d) None of these

24. If $i=\sqrt{-1}$ and n is a positive integer, than

$i^n + i^{n+1} + i^{n+2} + i^{n+3} =$

(a) 1 (b) I (c) i^n (d) 0

25. If $\left(\frac{1+i}{1-i}\right)^x = 1$ then

(a) x = 4n, where n is any positive integer

(b) x = 2n, where n is any positive integer

(c) x = 4n +1, where n is any positive integer

(d) x = 2n +1, where n is any positive integer

26. If $x = 3+i$, then $x^3 - 3x^2 - 8x + 15 =$

(a) 6 (b) 10 (c) - 18 (d) - 15

27. $\frac{1-2i}{2+i}+\frac{4-i}{3+2i}=$

(a) $\frac{24}{13}+\frac{10}{13}i$ (b) $\frac{24}{13}-\frac{10}{13}i$

(c) $\frac{10}{13}+\frac{24}{13}i$ (d) $\frac{10}{13}-\frac{24}{13}i$

28. $\left(\frac{1}{1-2i}+\frac{3}{1+i}\right)\left(\frac{3+4i}{2-4i}\right)$

(a) $\frac{1}{2}+\frac{9}{2}i$ (b) $\frac{1}{2}-\frac{9}{2}i$

(c) $\frac{1}{4}-\frac{9}{4}i$ (d) $\frac{1}{4}+\frac{9}{4}i$

29. The real value of θ for which the expression $\frac{1+i\cos\theta}{1-2i\cos\theta}$ is a real number, is

(a) $n\pi+\pi/2$ (b) $n\pi-\pi/2$

(c) $n\pi\pm\pi/2$ (d) None of these

30. Which of the following is correct

(a) $6+i>8-i$ (b) $6+i>4-i$

(c) $6+i>4+2i$ (d) None of these

31. The complex numbers $\sin x+i\cos 2x$ and $\cos x-i\sin 2x$ are conjugate to each other for

(a) $x=n\pi$ (b) $x=\left(n+\frac{1}{2}\right)\pi$

(c) $x=0$ (d) No value of x

32. The conjugate of complex number $\frac{2-3i}{4-i}$ is

(a) $\frac{3i}{4}$ (b) $\frac{11+10i}{17}$ (c) $\frac{11-10i}{17}$ (d) $\frac{2+3i}{4i}$

33. The real part of $(1-\cos\theta+2i\sin\theta)^{-1}$ is

(a) $\frac{1}{3+5\cos\theta}$ (b) $\frac{1}{5-3\cos\theta}$

(c) $\frac{1}{3-5\cos\theta}$ (d) $\frac{1}{5+3\cos\theta}$

34. The reciprocal of $3+\sqrt{7}i$ is

(a) $\frac{3}{4}-\frac{\sqrt{7}}{4}i$ (b) $3-\sqrt{7}i$

(c) $\frac{3}{16}-\frac{\sqrt{7}}{16}i$ (d) $\sqrt{7}+3i$

35. $\left|(1+i)\frac{(2+i)}{(3+i)}\right| =$

(a) -1/2 (b) ½ (c) 1 (d) - 1

36. If z and ω are to non-zero complex numbers such that $|z\omega|=1$ and arg (z) - arg $(\omega)=\frac{\pi}{2}$, then $\bar{z}\omega$ is equal to

(a) 1 (b) - 1 (c) I (d) - i

37. The square root of $3-4i$ are

(a) $\pm(2-i)$ (b) $\pm(2+i)$

1. **(a)**

Let Z = C + iS $\Rightarrow$ 1/Z = C - iS

Al, $\frac{1+C+iS}{1+C-iS}=\frac{1+Z}{1+1/Z}$ = Z = C + iS

2. **(c)**

Taking modulus and squaring on both sides

$(1 + i)^2 . (1 + 2i)^2$ .... $|1 + ni|^2 = |\alpha + i\beta|^2$

(1 + 1) . (1 + 4) .... $(1 + n^2) = \alpha^2 + \beta^2$ 2 . 5 . 10 . ... $(1 + n^2) = \alpha^2 + \beta^2$

3. **(c)**

(Z - 1) . ($\bar{Z}$ - 1) = (Z - 1) . ($\overline{Z-1}$) = $|(Z - 1)|^2$

(□ $Z\bar{Z}$ = $|Z|^2$)

4. **(c)**

Since arg (x + iy) = $\tan^{-1}(y/x)$

we have, arg $\left(\frac{1+i\sqrt{3}}{\sqrt{3}+1}\right)=\tan^{-1}\left[\frac{\sqrt{3}}{(\sqrt{3}+1)}\Big/\frac{1}{(\sqrt{3}+1)}\right]$

= $\tan^{-1}(\sqrt{3})$ = $\pi/3$

5. **(a)**

|z| = 1 $\Rightarrow z\bar{z}$ = 1

2Re (P) = p + $\bar{p}$ = $\frac{z-1}{z+1}$ + $\frac{\bar{z}-1}{\bar{z}+1}$ = $\frac{z-1}{z+1}$ + $\frac{\frac{1}{z}-1}{\frac{1}{z}+1}$ = $\frac{z-1}{z+1}$ + $\frac{1-z}{1+z}$ = 0

$\Rightarrow$ Re (p) = 0

6. (b)

$i\bar{Z}^3 + \bar{Z}^2 = \bar{Z} - i \Rightarrow \bar{Z}^2\,(i\bar{Z}+1) = \bar{Z} - i$

$\Rightarrow i\,\bar{Z}^2\,(\bar{Z}-i) = (\bar{Z}-i)$

$\bar{Z}-i = 0$ $\quad$ $i\bar{Z}^2 = 1$

$|\bar{Z}| = 1$ $\quad$ $|\bar{Z}| = 1$

7. (a)

Taking modulus on both sides,

$\sqrt{x^2+y^2} = \dfrac{\sqrt{u^2+v^2}}{\sqrt{u^2+v^2}} \Rightarrow x^2 + y^2 = 1$

8. (a)

$\sqrt{i} = x + iy$; $i = x^2 - y^2 + 2ixy$; $x^2 - y^2 = 0$, $xy = 1/2$,

□ $x^2 = y^2$ $\quad \therefore x = y$

on solving,

$x = \dfrac{1}{\sqrt{2}}$, $y = \dfrac{1}{\sqrt{2}}$ and $x = -\dfrac{1}{\sqrt{2}}$, $y = -\dfrac{1}{\sqrt{2}}$

$\sqrt{i} = \pm \dfrac{1}{\sqrt{2}}(1 + i)$

9. (c)

$3 + 4i = 4 - 1 + 2 \times 2 \times i = (2)^2 + (i)^2 + 2 \times 2 \times i$

$= (2 \times i)^2$

A square root of (3 + 4i) is (2 + i)

10. (c)

$3 + 4i = 4 - 1 + 2 \times 2 \times i = (2)^2 + (i)^2 + 2 \times 2 \times i$

$= (2 \times i)^2$

A square root of (3 + 4i) is (2 + i)

11. (a)

z = 1 = 1 (cos (arg(z) + i sin (arg(z))

$= 4\left(\cos\dfrac{5\pi}{6} + i\sin\dfrac{5\pi}{6}\right)$

$= -2\sqrt{3} + 2i$

12. (d)

$i^n + i^{n+1} + i^{n+2} + i^{n+3} = i^n [1 + i + i^2 + i^3]$

$= i^n[1 + i + (-1) + (-i)] = 0$

13. (c)

$\frac{1+7i}{4+i^2-4i} = \frac{1+7i}{3-4i}$

$\frac{|1+7i|}{|3-4i|} = \frac{\sqrt{1^2+7^2}}{\sqrt{3^2+(-4)^2}} = \frac{\sqrt{50}}{5} = \sqrt{2}$

14. (b)

$\sum_{n=1}^{13} i^n + \sum_{n=1}^{13} i^{n+1}$

$= (i + i^2 + \ldots\ldots i^{13}) + (i^2 + i^3 + \ldots\ldots\ldots i^{14})$

$= (i + 0) + (i^2 + 0)$

$= i - 1$

15. (a)

$(1 + i)(x + iy) = (1 - i)(x - iy)$

$(x - y) + i(x + y) = (x - y) - i(x + y)$

$\therefore x + y = 0$

$\therefore z = x - ix = x(1 - i)$

16. (a)

$A + iB = \frac{1-i\alpha}{1+i\alpha} \Rightarrow A - iB = \frac{1+i\alpha}{1-i\alpha}$

$\Rightarrow \quad (A + iB)(A - iB) = \left(\frac{1-i\alpha}{1+i\alpha}\right)\left(\frac{1+i\alpha}{1-i\alpha}\right) = 1$

$\Rightarrow \quad A^2 + B^2 = 1$

17. (b)

$\frac{|\beta-\alpha|}{|\beta\bar{\beta}-\bar{\alpha}\beta|} = \frac{|\beta-\alpha|}{|\beta||\bar{\beta}-\bar{\alpha}|} = \frac{|\beta-\alpha|}{|\beta||\overline{\beta-\alpha}|} = 1$

18. (c)

Taking modulus both sides

$|1+ i|\ |1 + 2i|\ |1 + 3i| \ldots\ldots\ldots |1 + ni|\ |\alpha + i\beta|$

$\sqrt{2}\,.\,\sqrt{5}\,.\,\sqrt{10}$ .... $\sqrt{1+n^2} = \sqrt{\alpha^2+\beta^2}$

i.e. 2.5.10 .... $(1 + n^2) = \alpha^2 + \beta^2$

19. (a)

$|z_1| = |z_2| = |z_3| = 1 \Rightarrow \bar{z}_1 = \frac{1}{z_1}, \bar{z}_2 = \frac{1}{z_2}, \bar{z}_3 = \frac{1}{z_3}$

$|\overline{z_1+z_2+z_3}| = \left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right| = 1$

□ $|z_1 + z_2 + z_3| = |\overline{z_1+z_2+z_3}|\ |\bar{z}_1+\bar{z}_2+\bar{z}_3|$

$= \left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right| = 1$ (given)

20. (c)

$|z_1 + z_2| = |z_1| + |z_2|$

$|z_1|^2 + |z_2|^2 + 2|z_1|\ |z_2| \cos(\theta_1 - \theta_2) = |z_1|^2 + |z_2|^2 + 2|z_1|\ |z_2|$

$\Rightarrow \cos(\theta_1 - \theta_2) = 1 \Rightarrow \theta_1 - \theta_2 = 0^0$

$\arg(z_1) - \arg(z_2) = 0^0$

21. (a)

$\sum_{r=1}^{102} i^r = i + i^2 + i^3 \text{........} + i^{102}$

$= \frac{i(i^{102}-1)}{i-1} = \frac{i(1-1)}{i-1} = 0$

22. (a)

$\frac{|1-i\sqrt{3}|.|\cos\theta+i\sin\theta|}{|2|.|1-i|.|\cos\theta-i\sin\theta|} = \frac{\sqrt{1+3}.\sqrt{\cos^2\theta+\sin^2\theta}}{2\sqrt{1+1}.\sqrt{\cos^2\theta+\sin^2\theta}} = \frac{1}{\sqrt{2}}$

23. (d)

$i^n + i^{n+1} + i^{n+2} + i^{n+3} = i^n(1+i+i^2+i^3) = i^n(1+i-1-i) = o.$ **Trick:** Since the sum of four consecutive powers of *i* is always zero.

$\Rightarrow i^n + i^{n+1} + i^{n+2} + i^{n+3} = 0,\ \ n \in I.$

24. (a)

$\frac{1+i}{1-i} = \frac{1+i}{1-i}.\frac{1+i}{1+i} \Rightarrow \left(\frac{1+i}{1-i}\right)^x = 1 \Rightarrow i^x = 1 \Rightarrow x = 4n,\ n \in I^+.$

25. (d)

Given that; $x-3=i \Rightarrow (x-3)^2 = i^2 \Rightarrow x^2-6x+10=0$

$$x^3-3x^2-8x+15 = x(x^2-6x+10)+3(x^2-6x+10)-15$$

Now, $= 0+0-15=-15.$

26 (d)

$$\frac{1-2i}{2+i}+\frac{4-i}{3+2i} = = \frac{(1-2i)(3+2i)+(4-i)(2+i)}{(2+i)(3+2i)} = \frac{50-120i}{65} = \frac{10}{13}-\frac{24}{13}i.$$

27d)

$$\left(\frac{1}{1-2i}+\frac{3}{1+i}\right)\left(\frac{3+4i}{2-4i}\right) = \left[\frac{1+2i}{1^2+2^2}+\frac{3-3i}{1^2+1^2}\right]\left[\frac{6-16+12i+8i}{2^2+4^2}\right]$$

$$=\left(\frac{2+4i+15-15i}{10}\right)\left(\frac{-1+2i}{2}\right)$$

$$=\frac{(17-11i)(-1+2i)}{20} = \frac{5+45i}{20} = \frac{1}{4}+\frac{9}{4}i.$$

28. (c)

Given that $\frac{1+i\cos\theta}{1-2i\cos\theta} = \frac{(1+i\cos\theta)(1+2i\cos\theta)}{(1-2i\cos\theta)(1+2i\cos\theta)}$

$$=\left[\frac{(1-2\cos^2\theta)}{(1+4\cos^2\theta)}\right]+i\left[\frac{3\cos\theta}{1+4\cos^2\theta}\right]$$

Since $\text{Im}(z)=0$, then $3\cos\theta=0 \Rightarrow \theta = n\pi \pm \pi/2.$

29 (d)

Because, inequality is not applicable for a complex number.

30 (d)

$\sin x + i\cos 2x$ and $\cos x - i\sin 2x$ are conjugate to each other if $\sin x = \cos x$ and $\cos 2x = \sin 2x$

or $\tan x = 1 \Rightarrow x = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \ldots\ldots$ (i) and

$\tan 2x = 1 \Rightarrow 2x = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \ldots\ldots$ or $x = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \ldots\ldots$ (ii) There exists no value of *x* common in (i) and (ii). Therefore there is no value of *x* for which the given complex numbers are conjugate.

31. (b)

$$\frac{2-3i}{4-i} = \frac{(2-3i)(4+i)}{(4-i)(4+i)} = \frac{8+3-12i+2i}{16+1} = \frac{11-10i}{17}$$

$\Rightarrow$ Conjugate $= \frac{11+10i}{17}.$

32. (c)

$$\{(1-\cos\theta)+i.2\sin\theta\}^{-1}=\left\{2\sin^2\frac{\theta}{2}+i.4\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right\}^{-1}=\left(2\ \sin\frac{\theta}{2}\right)^{-1}\ \left\{\sin\frac{\theta}{2}+i.2\cos\frac{\theta}{2}\right\}^{-1}$$

$$=\left(2\sin\frac{\theta}{2}\right)^{-1}.\frac{1}{\sin\frac{\theta}{2}+i.2\cos\frac{\theta}{2}}\times\frac{\sin\frac{\theta}{2}-i.2\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}-i.2\cos\frac{\theta}{2}}=\frac{\sin\frac{\theta}{2}-i.2\cos\frac{\theta}{2}}{2\ \sin\frac{\theta}{2}\left(\sin^2\frac{\theta}{2}+4\cos^2\frac{\theta}{2}\right)}$$

Hence, real part

$$=\frac{\sin\frac{\theta}{2}}{2\sin\frac{\theta}{2}\left(1+3\cos^2\frac{\theta}{2}\right)}=\frac{1}{2\left(1+3\cos^2\frac{\theta}{2}\right)}=\frac{1}{5+3\cos\theta}.$$

33. (c)

$$\frac{1}{3+\sqrt{7}i}=\frac{1}{3+\sqrt{7}i}.\frac{3-\sqrt{7}i}{3-\sqrt{7}i}=\frac{3-\sqrt{7}i}{9+7}=\frac{3-\sqrt{7}i}{16}=\frac{3}{16}-\frac{\sqrt{7}}{16}i.$$

34. (c)

$$z=\frac{(1+i)(2+i)}{(3+i)}=\frac{1+3i}{3+i}\times\frac{3-i}{3-i}=\frac{3+4i}{5}\Rightarrow|z|=1$$

Trick : $|z|=\frac{|z_1||z_2|}{|z_3|}=\frac{\sqrt{2}.\sqrt{5}}{\sqrt{10}}=1$

35. (d)

$|z||\ \omega|=1$ .....(i) and *arg* $\left(\frac{z}{\omega}\right)=\frac{\pi}{2}$

$\Rightarrow\ \frac{z}{\omega}=i\ \Rightarrow\ \left|\frac{z}{\omega}\right|=1$ .....(ii)

From equation (i) and (ii),

$|z|=|\ \omega|=1$ and $\frac{z}{\omega}+\frac{\bar{z}}{\bar{\omega}}=0;\quad z\bar{\omega}+\bar{z}\omega=0$

$\Rightarrow\ \bar{z}\omega=-z\bar{\omega}=\frac{-z}{\omega}\bar{\omega}\omega\ \Rightarrow\ \bar{z}\omega=-i|\ \omega|^2=-i.$

36. (a)

$$|z|=5,\therefore\sqrt{3-4i}=\pm\left(\sqrt{\frac{5+3}{2}}-i\sqrt{\frac{5-3}{2}}\right)=\pm\ (2-i)$$