Homework:-

Date: 22/7/2024

DPP-② (TRIGONOMETRY)

① $\tan 5x \tan 3x \tan 2x =$

Solution:

$\tan 5x = 3x + 2x$

$\tan(5x) = \tan(3x + 2x)$

$\tan(5x) = \dfrac{\tan 3x \tan 2x}{1 - \tan 3x + \tan 2x}$

$\tan(5x) \neq (1 - \tan 3x \tan 2x) = \tan 3x \tan 2x$

$\tan 5x - \tan 3x - \tan 2x = \tan A \tan 3x \tan 2x$

$\tan 5x \tan 3x \tan 2x = \tan 5x - \tan 3x - \tan 2x$

② $\dfrac{\tan^2 2\theta - \tan^2\theta}{1 - \tan^2 2\theta \tan^2\theta}$

Solution

$\dfrac{\tan^2 2\theta - \tan^2\theta}{1 - \tan^2 2\theta \tan^2\theta} = \dfrac{(\tan 2\theta - \tan\theta)(\tan 2\theta + \tan\theta)}{(1 + \tan 2\theta \tan\theta)(1 - \tan 2\theta \tan\theta)}$

$= \dfrac{\tan(2\theta - \theta)\tan(2\theta + \theta)}{\tan\theta + \tan 3\theta}$

$= \dfrac{\tan 3\theta}{\tan\theta}$

$\dfrac{\tan^2 2\theta - \tan^2\theta}{1 - \tan^2 2\theta \tan^2\theta} = \tan 3\theta \tan\theta$

(3) If $\tan A = \frac{1}{3}$ $\tan B = \frac{1}{7}$ the the value of $2A+B =$

solution

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A + \tan B}$$

$$\tan A = \frac{1}{3} \quad A = \tan^{-1}\frac{1}{3}$$

$$\tan B = \frac{1}{7} = B = \tan^{-1}\frac{1}{7}$$

$$2A = 2 \times \tan^{-1}\frac{1}{3}$$

$$2A = \tan^{-1}\frac{2 \cdot \frac{1}{3}}{1 - 1/9}$$

$$2A = \tan^{-1} = \frac{2/3}{8/9} = \tan^{-1}\frac{3}{4}$$

$$2A+B = \tan^{-1}\frac{3}{4} + \tan^{-1}\frac{1}{7}$$

$$= \frac{\tan^{-1}(3/4 + 1/7)}{1 - 3/7 \cdot 1/7}$$

$$= \tan^{-1}\frac{25/28}{1 - 3/28}$$

$$= \tan^{-1}\frac{25/28}{25/28}$$

(4) $\frac{\cos 12^\circ - \sin 12^\circ}{\cos 12^\circ + \sin 12^\circ} + \frac{\sin 147^\circ}{\cos 147^\circ}$

Solution. $\frac{\cos 12^\circ - \sin 12^\circ}{\cos 12^\circ + \sin 12^\circ} + \frac{\sin 147^\circ}{\cos 147^\circ} = \frac{\cos 12^\circ - \sin 12^\circ}{\sin 12^\circ \; \cos 12^\circ} + \frac{\sin 147^\circ}{\cos 147^\circ}$

$= \frac{1 - \tan 12^\circ}{1 + \tan 12^\circ} + \tan 147^\circ$

$= \tan(45^\circ - 12^\circ) + \tan(180^\circ - 33^\circ)$

$= \cancel{\tan 33^\circ + (-\tan 33^\circ)}$

$\frac{\cos 12^\circ - \sin 12^\circ}{\cos 12^\circ + \sin 12^\circ} + \frac{\sin 147^\circ}{\cos 147^\circ} = 0.$

(6)

$\Rightarrow \quad 2A + B = \tan^{-1}$

$= \tan^{-1}(\tan \pi/4)$

$= \pi/4$

$\boxed{2A + B = 45^\circ}$

(5) If $\tan\alpha = \frac{m}{m+1}$ and $\tan\beta = \frac{1}{2m+1}$ the $\alpha+\beta =$

Solution: -

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha \cdot \tan\beta}$$

$$= \frac{\frac{m}{m+1} + \frac{1}{2m+1}}{1-\left(\frac{m}{m+1}\right)\left(\frac{1}{2m+1}\right)}$$

$$= \frac{\frac{2m^2+m+m+1}{\cancel{(m+1)(2m+1)}}}{\frac{2m^2+m+2m+1-m}{\cancel{(m+1)(2m+1)}}}$$

$$= \frac{2m^2+m+m+1}{2m^2+m+2m+1-m}$$

$$= \frac{\cancel{2m^2+2m+1}}{\cancel{2m^2+2m+1}}$$

$$= 1$$

$$\tan(\alpha+\beta) = 1$$

$$\tan(\alpha+\beta) = 45^0.$$

$$\tan(\alpha+\beta) = \pi/4$$

Date
Page

(5) If $\tan\alpha = \frac{m}{m+1}$ and $\tan\beta = \frac{1}{2m+1}$ the $\alpha+\beta$ =

Solution :-

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \cdot \tan\beta}$$

$$= \frac{\frac{m}{m+1} + \frac{1}{2m+1}}{1 - \left(\frac{m}{m+1}\right)\left(\frac{1}{2m+1}\right)}$$

$$= \frac{\frac{2m^2+m+m+1}{\cancel{(m+1)}\cancel{(2m+1)}}}{\frac{2m^2+m+2m+1-m}{\cancel{(m+1)}\cancel{(2m+1)}}}$$

$$= \frac{2m^2+m+m+1}{2m^2+m+2m+1-m}$$

$$= \frac{\cancel{2m^2+2m+1}}{\cancel{2m^2+2m+1}}$$

$$= 1$$

$$\tan(\alpha+\beta) = 1$$

$$\tan(\alpha+\beta) = 45^0.$$

$$\tan(\alpha+\beta) = \pi/4$$

B = 2

(6) If $\tan(A+B) = p$, $\tan(A-B) = q$ then the value of $\tan 2A$ in terms of p and q is

Solution:

Given. $\tan(A+B) = p$

$\tan(A-B) = q$

$$\tan(2A) = \frac{\tan(A+B) + \tan(A-B)}{1 - \tan(A+B)\tan(A-B)}$$

$$= \frac{p+q}{1 - p \times q}$$

$$\boxed{\tan(2A) = \frac{p+q}{1-pq}}$$

(7) If $\cos(A-B) = \frac{3}{5}$ and $\tan A \tan B = 2$ then

Solution:

Given. $\tan A \tan B = 2$

$\sin A \sin B = 2\cos A \cos B$

$\cos(A-B) = \frac{3}{5}$.

$\cos A \cos B + \sin A \sin B = \frac{3}{5}$.

$$3\cos A \cos B = \frac{3}{5}$$

$$\therefore \boxed{\cos A \cos B = \frac{1}{5}}$$