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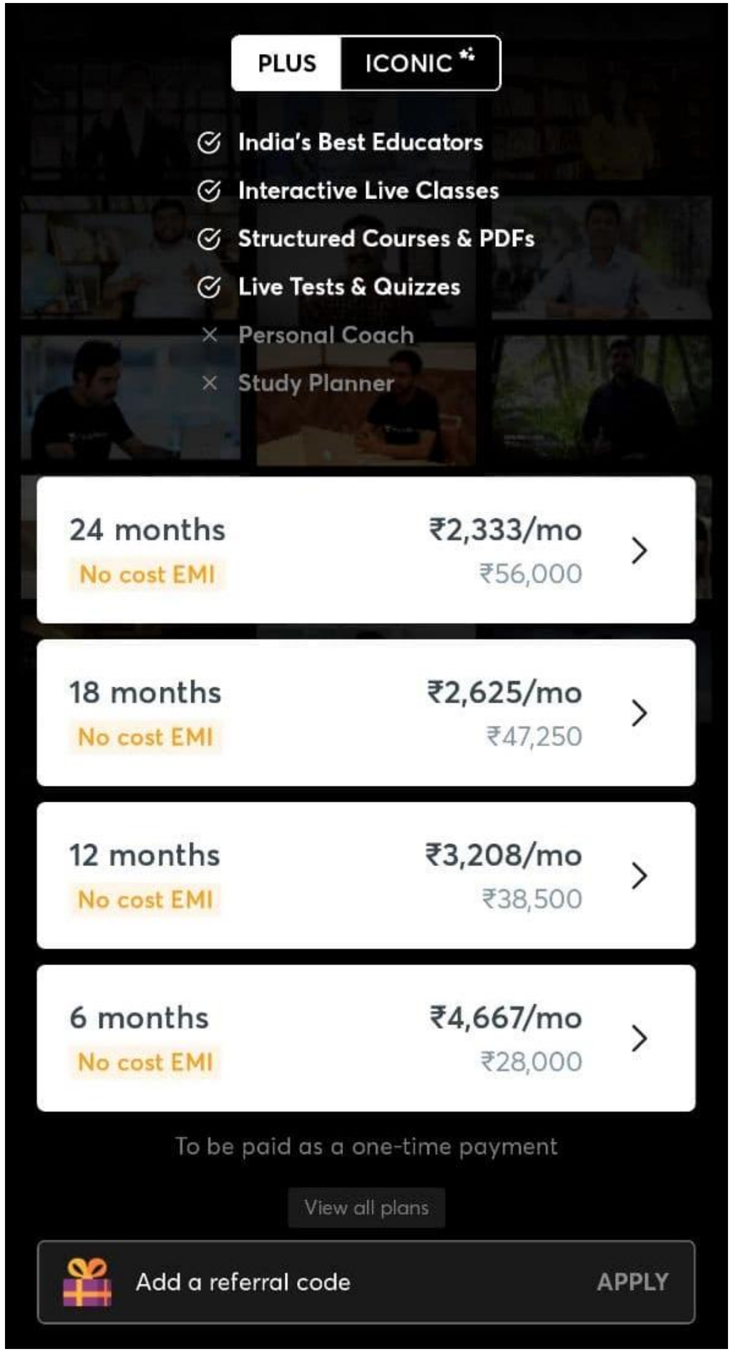

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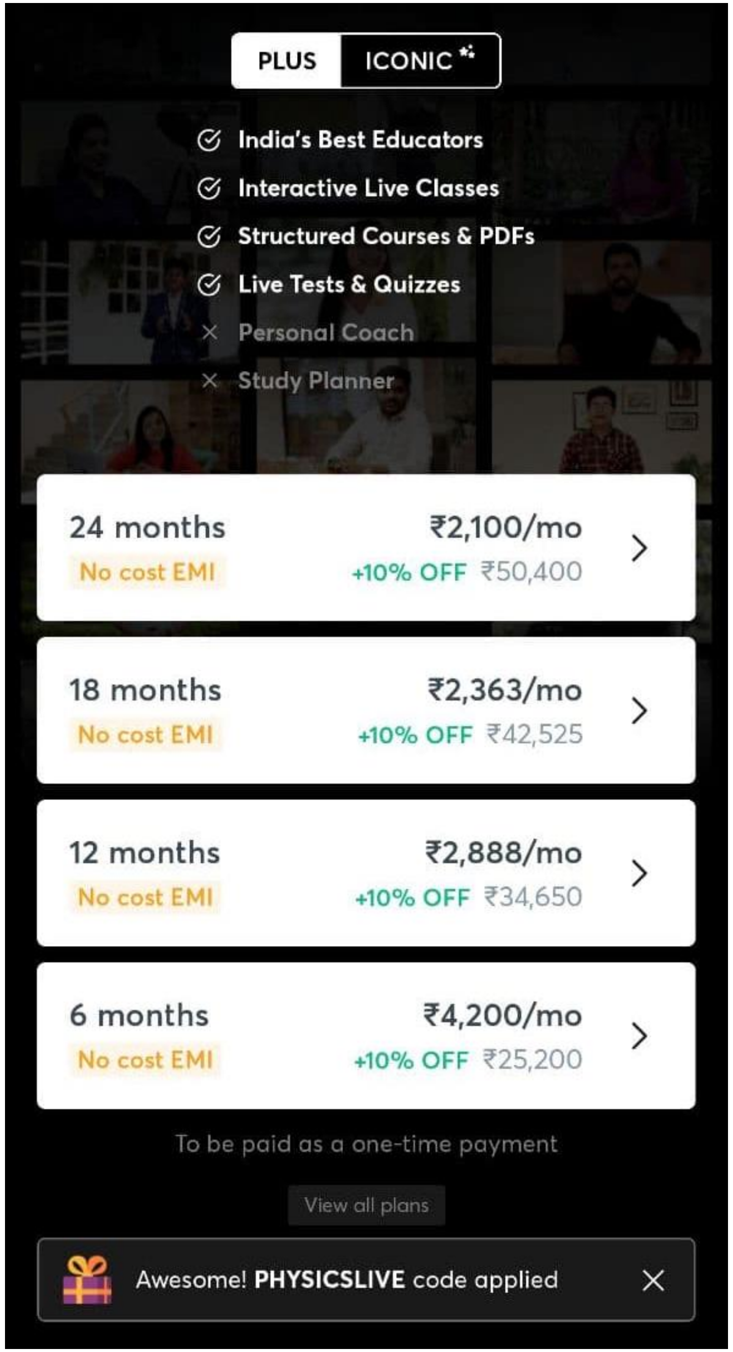

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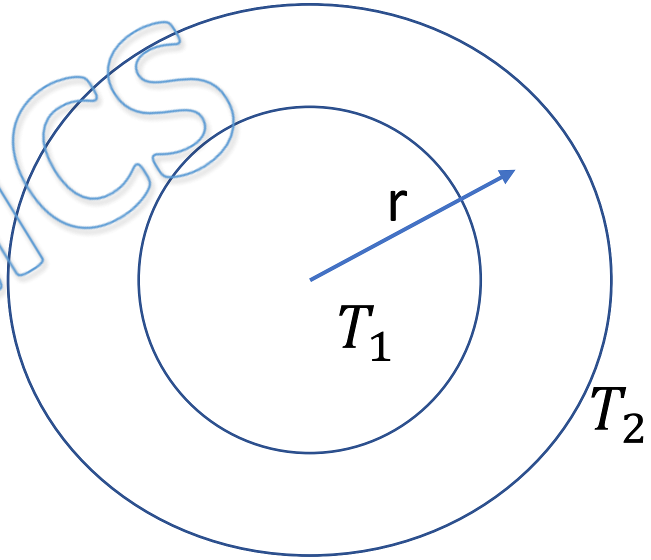
JEE Main & Advanced, NSEP, INPhO, IPhO Physics DPP

DPP- 2 Heat Transfer: Kirchhoff's law, Wheatstone bridge, Radial and cylindrical flow of heat

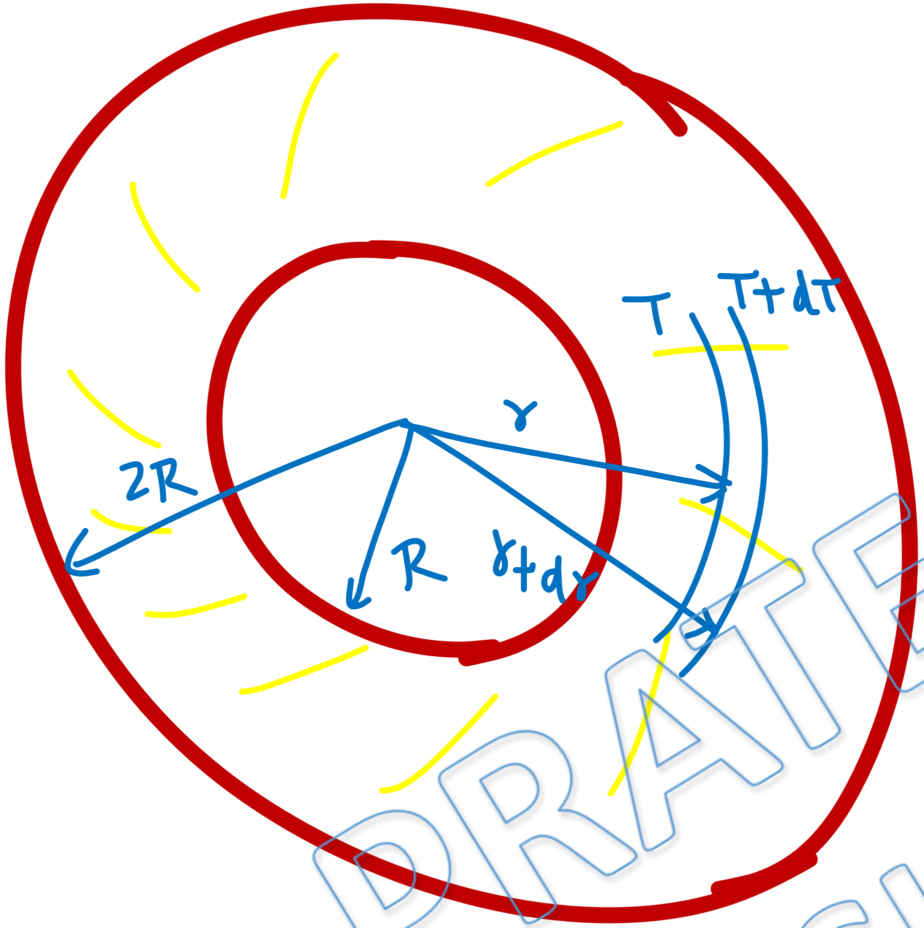
By Physicsaholics Team

Q) A hollow conducting sphere has inner radius R and outer radius 2R. Temperatures of inner cavity and surroundings are T_1 and T_2 ($T_2 < T_1$) respectively . These temperatures are not changing with time.
Temperature gradient in sphere at distance r from centre is directly proportional to



(a) r
(b) 1/r
(c) r^2
(d) None of the above

Ans. d



hollow sphere

Let Radial heat Current in sphere = i

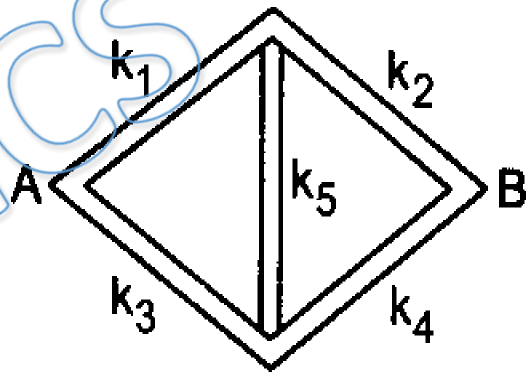
Equation of heat Current for differential spherical shell

$$\frac{dQ}{dt} = i = \frac{K(4\pi r^2)\,dT}{dr}$$

$\Rightarrow$ Temperature gradient $\frac{dT}{dr} = \frac{i}{4\pi K r^2}$

$$\Rightarrow \quad \frac{dT}{dr} \propto \frac{1}{r^2}$$

Q) Five rods of the same dimensions are arranged as shown. They have thermal conductivities k_1, k_2, k_3, k_4 and k_5. When points A and B are maintained at different temperatures, no heat flows through the central rod. It follows that



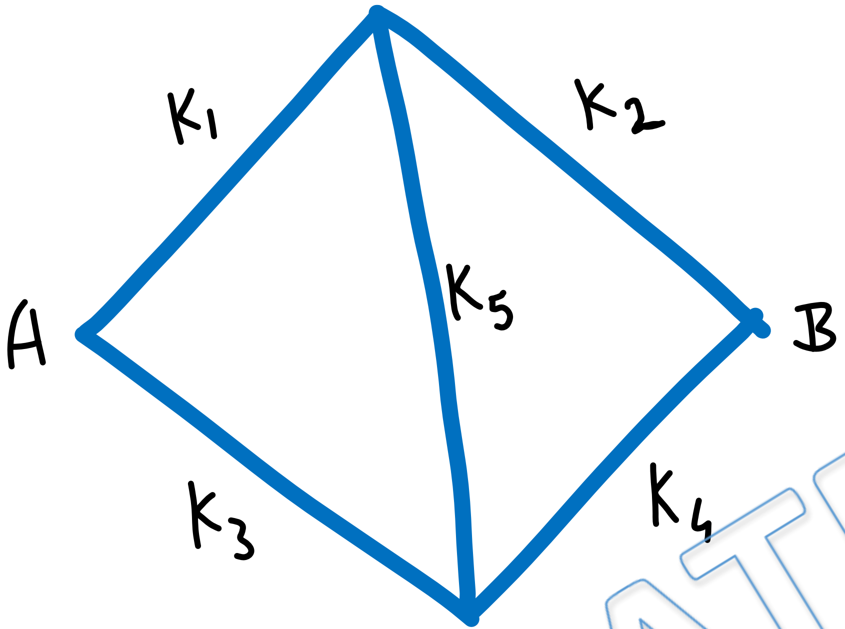
(a) $k_1 = k_4$ and $k_2 = k_3$

(b) $k_1/k_4 = k_2/k_3$

(c) $k_1k_4 = k_2k_3$

(d) $k_1k_2 = k_3k_4$

Ans. c



no heat flow in central Rod

$\Rightarrow$ ballonced wheat stone bridge.

$$\Rightarrow \frac{R_1}{R_2} = \frac{R_3}{R_4}$$ where R means thermal resistance

$$\Rightarrow \frac{K_2}{K_1} = \frac{K_4}{K_3}$$

$$\Rightarrow K_1 K_4 = K_2 K_3$$

Ans (c)

Q) Ice starts freezing in a lake with water at 0^0C when the atmospheric temperature is -10^0C. If the time taken for 1 cm of ice to be formed is 12 minutes the time taken for the thickness of the ice to change from 1 cm to 2 cm will be

(A) 12 minutes

(B) less than 12 minutes

(C) more than 12 minutes but less than 24 minutes

(D) more than 24 minutes

Ans. d

If t is time required to freeze x thickness of ice.

$$t \propto x^2 \Rightarrow t = Cx^2, \; C \text{ is Constant}$$

$$\Rightarrow \quad 12\text{ min} = C \times (1\text{cm})^2 \Rightarrow C = 12\text{ min}/\text{cm}^2.$$

$$\Rightarrow \quad t = 12x^2$$

time taken to freeze 2 cm thickness of ice.

$$t' = 12 \times 4 = 48\text{ min}.$$

time taken to increase thickness from 1 cm to 2 cm

$$= (48 - 12) = 36\text{ min} > 24\text{ min}$$

Ans (d)

Q) A pond of water at 0°C is covered with layer of ice 4 cm thick if air temperature is -10°C (constant), how long it takes ice thickness to increase to 8 cm? K_{ice} = 2 W/m°C, L_f = 80 cal/gm, ρ_{ice} = 900 kg/m^3.

Ans. 10.03 hrs.

Rate of heat flow through ice

Slab $\frac{d\theta}{dt} = \frac{KA \times 10}{x}$

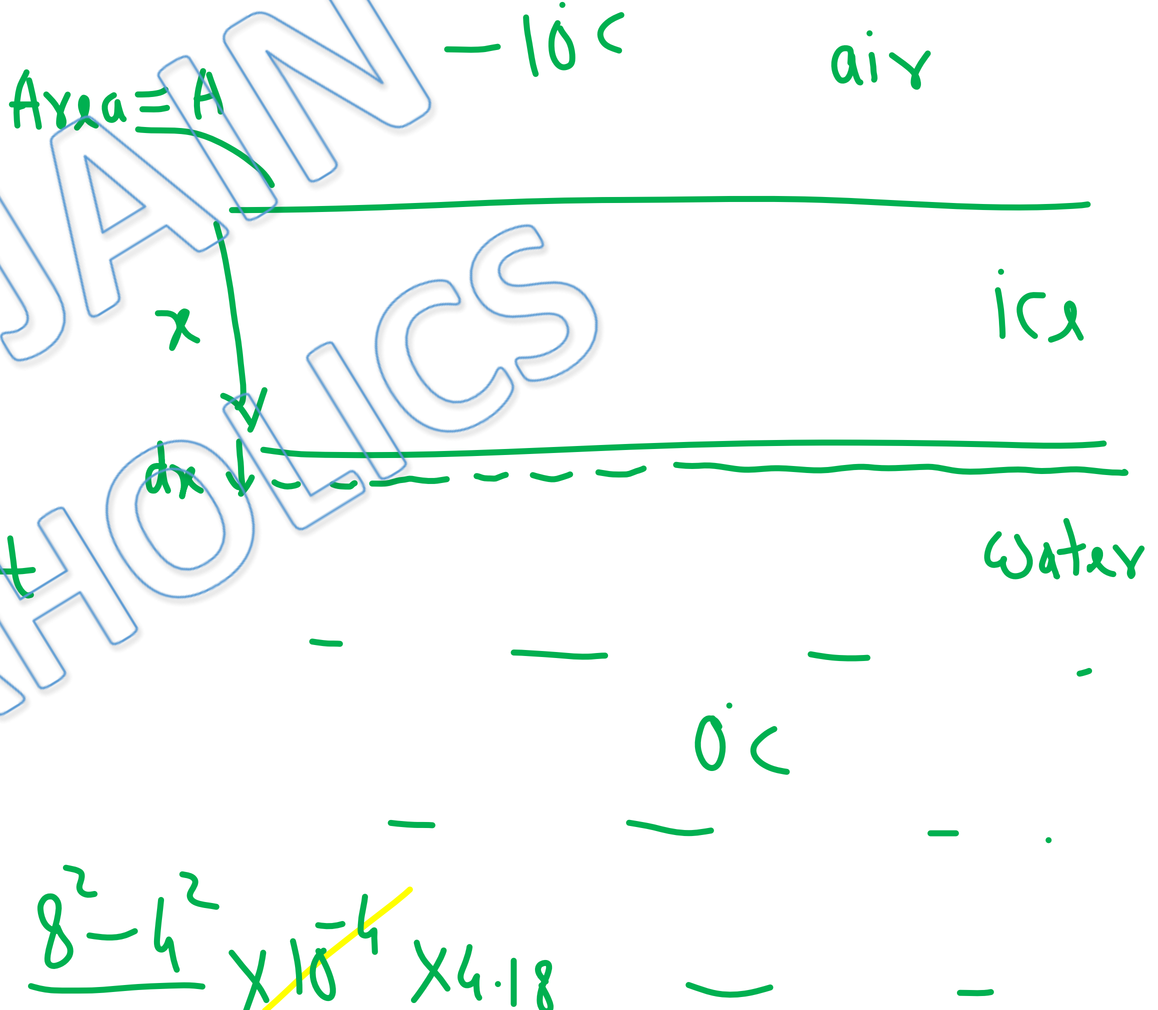
$\Rightarrow L\frac{dm}{dt} = \frac{2A\times 10}{x\times 4.18}$

$\Rightarrow 4.18\times 8\cancel{0}\times 10^3\times 900\,\cancel{A}\,x\,dx = 2\,\cancel{A}\times 1\cancel{0}\,dt$

$\Rightarrow 4.18\times 3600\times 10^3\times\int_{4\times 10^{-2}}^{8\times 10^{-2}} x\,dx = \frac{\int dt}{4.}$

$\Rightarrow t = 360\cancel{0}\times\cancel{10^3}\times\frac{8^2-4^2}{2}\times\cancel{10^{-4}}\times 4.18$

$= \frac{360\times 48}{2\times 60\times 60}\times 4.18\text{ hr} = 10.03\text{ hr}$



Q) Water in pond is at 0°C. The temperature of ambient air is constant at -20°C. Thickness x of ice film in centimeter increases with t in second according to relation (density of ice = 0.917 g/cc, conductivity of ice = 0.005 cgs and latent heat of ice = 80 cal/gm)

(a) $x = 2.73 \times 10^{-3}\, t$

(b) $x^2 = 2.73 \times 10^{-3}\, t$

(c) $t^2 = 2.73 \times 10^{-3}x$

(d) $t = 2.73 \times 10^{-3}x$

Ans. b

Rate of Heat flow through ice slab

$$\frac{dQ}{dt} = \frac{KA \times 20}{x}$$

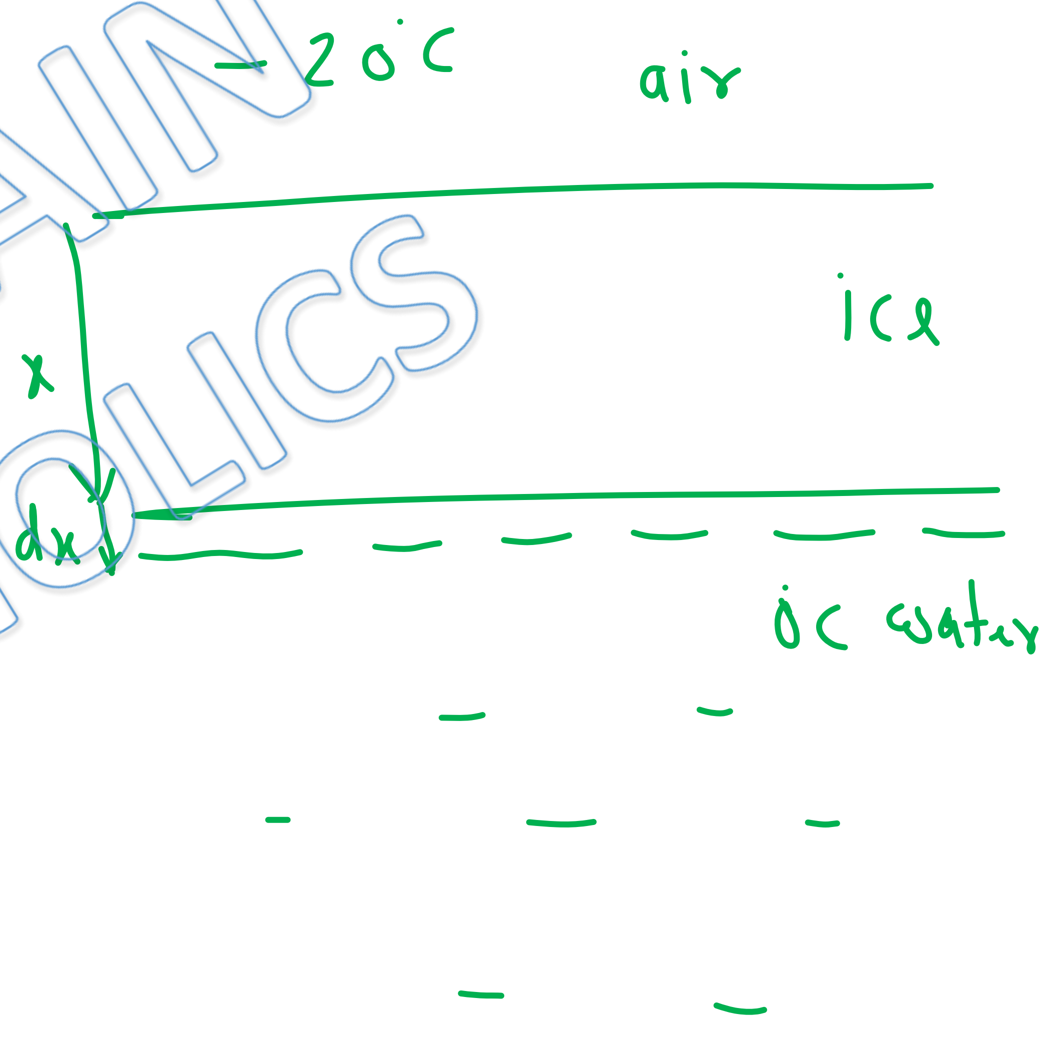
$$\Rightarrow \quad L\frac{dm}{dt} = \frac{20KA}{x}$$

$$\Rightarrow \quad \frac{L\rho A\,dx}{dt} = \frac{20KA}{x}$$

$$\Rightarrow \quad \int_0^x x\,dx = \frac{20K}{L\rho}\int_0^t dt$$

$$\Rightarrow \quad \frac{x^2}{2} = \frac{20 \times .005}{.917 \times 80}\,t$$

$$\Rightarrow \quad x^2 = 2.73 \times 10^{-3}\,t$$



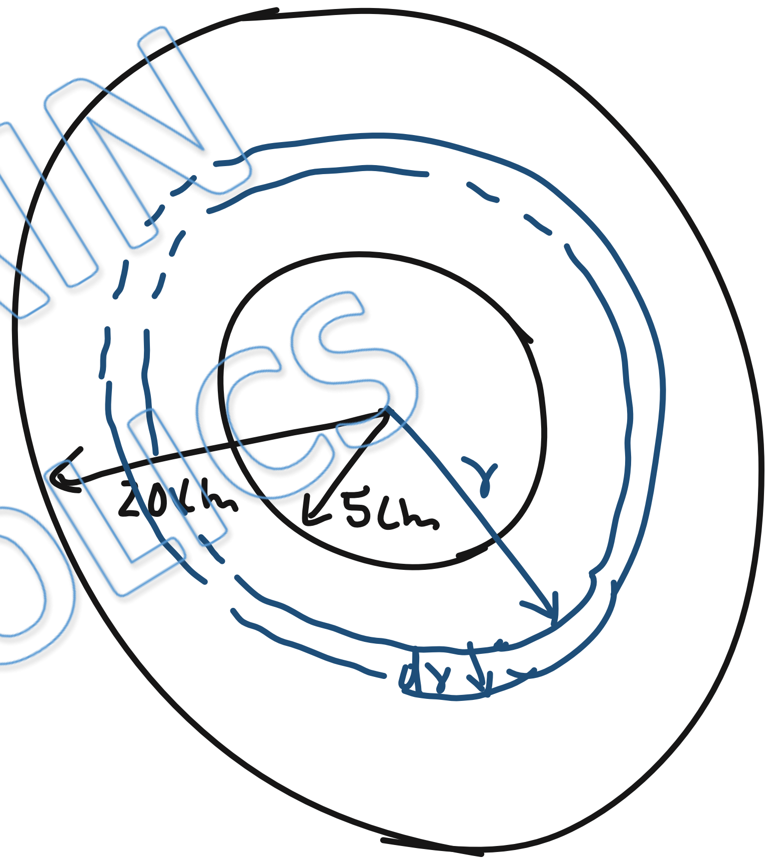
Ans(b)

Q) A hollow metallic sphere of radius 20 cm surrounds a concentric metallic sphere of radius 5 cm. The space between the two spheres is filled with a nonmetallic material. The inner and outer spheres are maintained at 50°C and 10°C respectively and it is found that 100 J of heat passes from the inner sphere to the outer sphere per second. Find the thermal conductivity of the material between the spheres.

Ans. 3

Thermal resistance of differential shell

$$dR = \frac{dr}{K\,4\pi r^2}$$



All such shells are in series.

net resistance $\quad R = \dfrac{1}{4\pi K}\displaystyle\int_{5\times10^{-2}}^{20\times10^{-2}} \frac{dr}{r^2}$

$$\Rightarrow \quad R = \frac{1}{4\pi K}\left[\frac{1}{5\times10^{-2}} - \frac{1}{20\times10^{-2}}\right]$$

$$= \frac{3}{4\pi K \times 20\times10^{-2}}$$

now $\quad \dfrac{\Delta Q}{\Delta t} = \dfrac{\Delta T}{R} \Rightarrow \dfrac{100}{1} = \dfrac{40\times4\pi K\times20\times10^{-2}}{3}$

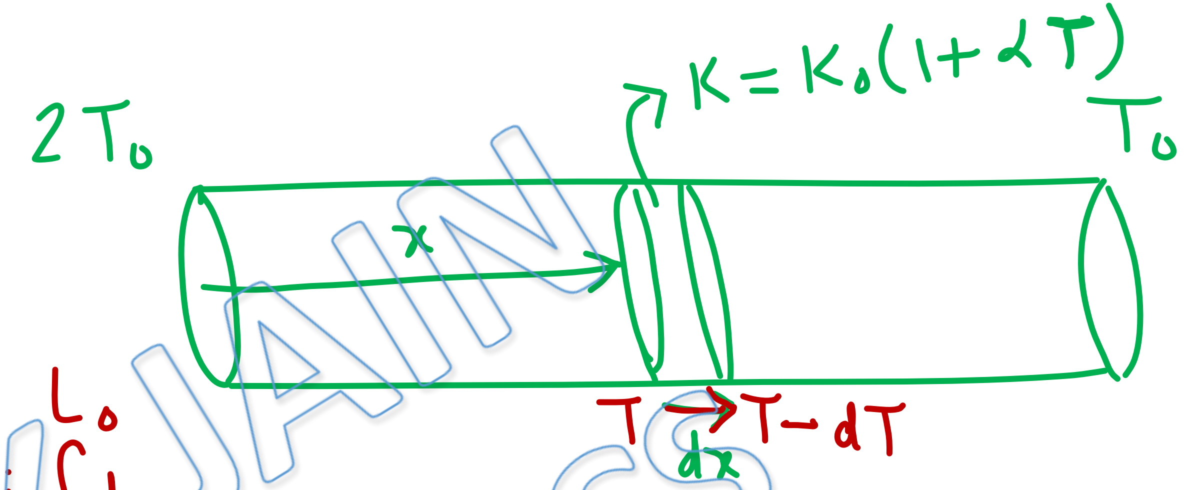
$$\Rightarrow \quad K = \frac{300}{8\times4\pi} \simeq 3$$

Ans (3)

Q) For a solid cylinder of length L_0, area A conductivity varies with temperature Tas $k = k_0(1 + \alpha T)$. If one end is at $2T_0$ and other at T_0, find rate of heat flow?

Ans. $\frac{K_o A T_o}{L_o}\left(1+\frac{3\alpha T_o}{2}\right)$

for differential disc



$$\frac{dq}{dt} = -\frac{AK_0(1+\alpha T)\,dT}{dx} = i$$

$$\Rightarrow \quad -AK_0\int_{2T_0}^{T_0}(1+\alpha T)\,dT = i\int_0^{L_0}dx$$

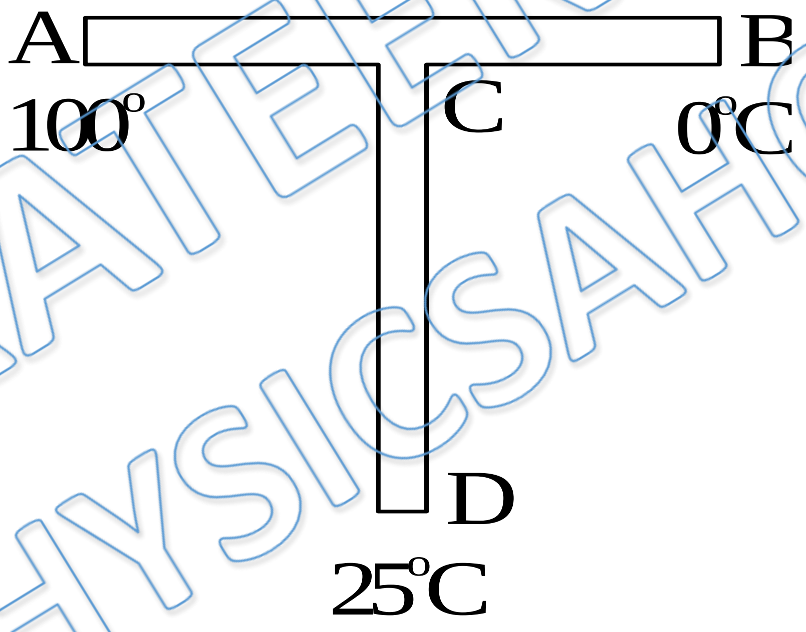
$$\Rightarrow \quad -AK_0\left[T+\frac{\alpha T^2}{2}\right]_{2T_0}^{T_0} = iL_0$$

$$\Rightarrow \quad i = \frac{AK_0}{L_0}\left[(2T_0-T_0)+\frac{\alpha}{2}(4T_0^2-T_0^2)\right]$$

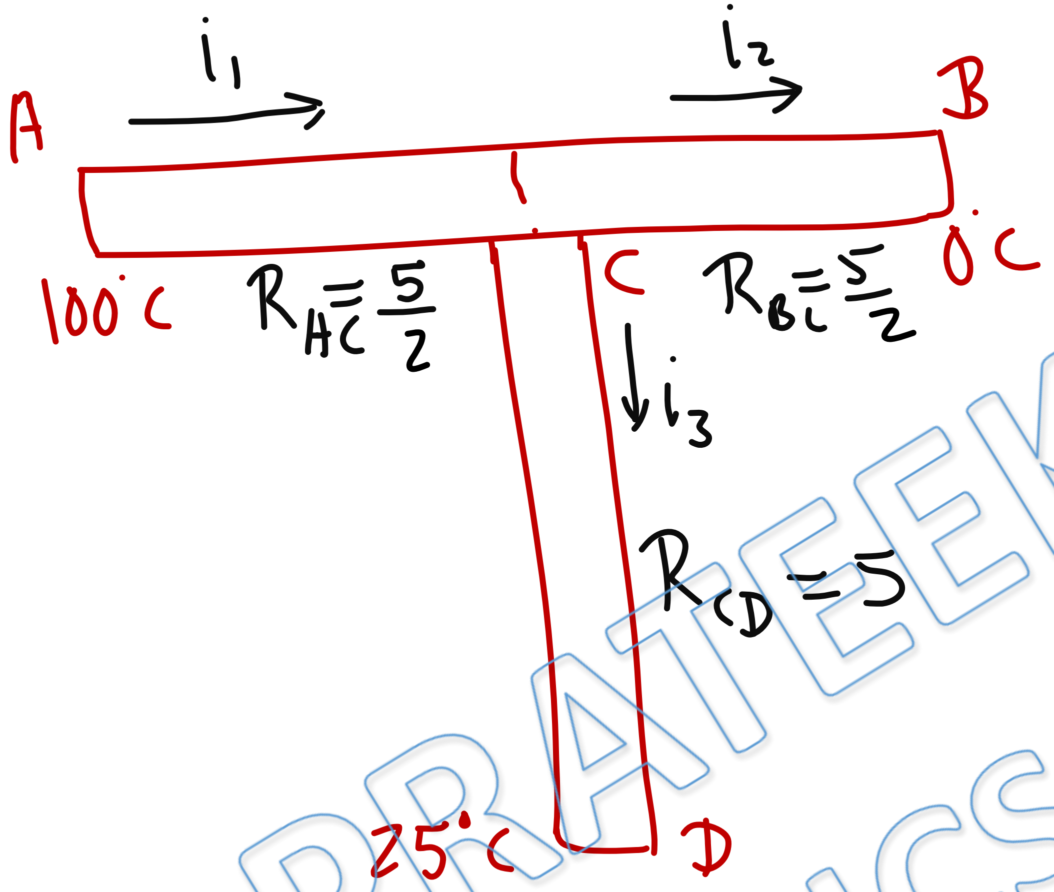
$$= \frac{AK_0T_0}{L_0}\left[1+\frac{3\alpha}{2}T_0\right]$$

1.

Q) A rod CD of thermal resistance 5.0 K/W is joined at the middle of an identical rod AB as shown in fig. The ends A, B and D are maintained at 100°C, 0°C and 25°C respectively. Find the heat current in CD in Watt.



Ans. 4



$$i_1 = i_3 + i_2$$

$$\Rightarrow \frac{100 - T_c}{5/2} = \frac{T_c - 0}{5/2} + \frac{T_c - 25}{5}$$

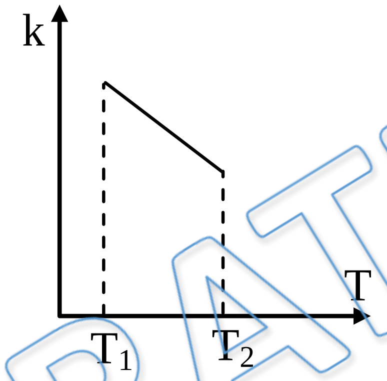
$$\Rightarrow 200 - 2T_c = 2T_c + T_c - 25$$

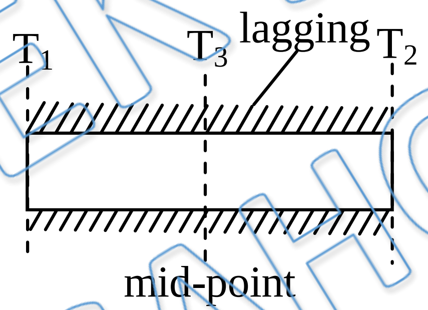
$$\Rightarrow 5T_c = 225$$

$$\Rightarrow T_c = 45$$

$$i_3 = \frac{45 - 25}{5} = 4$$

Q) Over a certain temperature range, the thermal conductivity k of a metal is not constant but varies as indicated in figure. A lagged rod of the metal has its ends maintained at temperatures T_1 and $T_2 (T_2 > T_1)$ as shown in figure. Which one of the following correctly describes how T_3, the temperature at the mid-point of the rod, compares with T_1 and T_2 ?





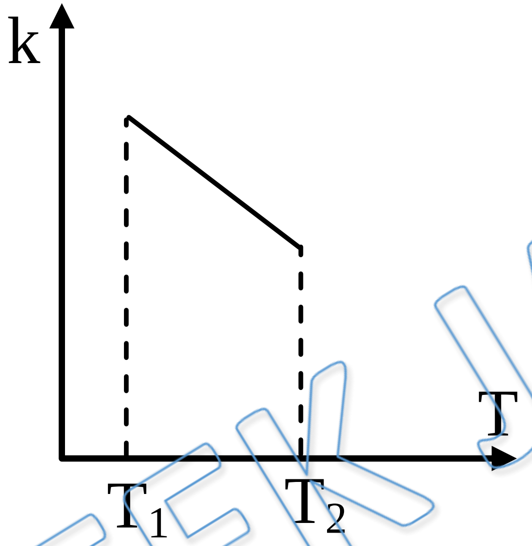
(A) $T_3 = (T_1 + T_2)/2$

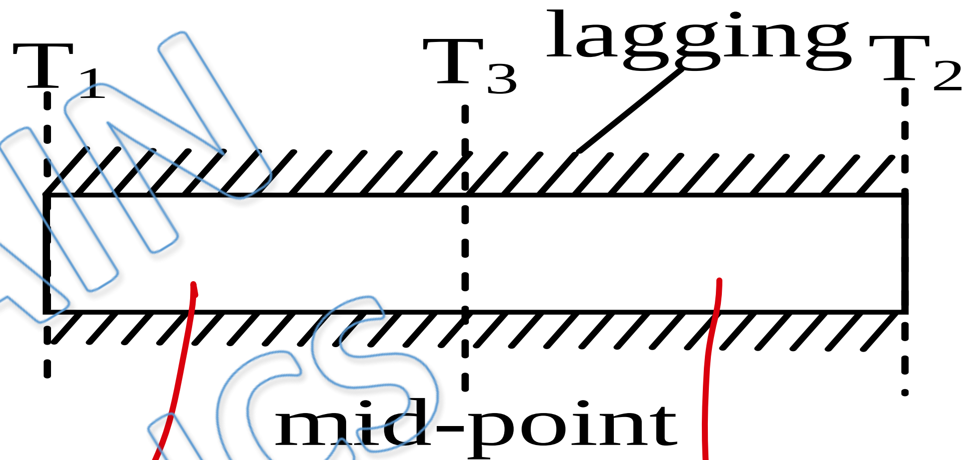
(B) $T_3 = (T_1 - T_2)/2$

(C) $T_3 > (T_1 + T_2)/2$

(D) $T_3 < (T_1 + T_2)/2$

Ans. d





temperature difference of T_3 from T_1 is low $\Rightarrow T_3 < \frac{T_1+T_2}{2}$

low Temperature	high temperature
$\Rightarrow$ high K	$\Rightarrow$ low K
$\Rightarrow$ low R	$\Rightarrow$ high R
$\Rightarrow$ low ΔT	$\Rightarrow$ high ΔT

Ans (d)

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Chalo Niklo