SINGLE CORRECT ANSWER

LEVEL - 1

Basic Concepts

1. The complex numbers $\sin x + i\cos 2x$ and $\cos x - i\sin 2x$ are conjugate of each other for $(n \in I)$, x =

(A) $n\pi$ (B) 0 (C) $(2n+1)\frac{\pi}{2}$ (D) no value of x

The Conjugate of a Complex Number

2. The conjugate of $\frac{2-i}{(1-2i)^2}$ is

(A) $\frac{2}{25}+\frac{11}{25}i$ (B) $\frac{2}{25}-\frac{11}{25}i$ (C) $-\frac{2}{25}+\frac{11}{25}i$ (D) $-\frac{2}{25}-\frac{11}{25}i$

3. If $\frac{1-i\alpha}{1+i\alpha}$ = A + iB, then $A^2 + B^2$ equals to

(A) 1 (B) α^2 (C) –1 (D) $-\alpha^2$

4. If $\omega = \frac{z}{\bar{z}}$, then $|\omega| =$

(A) 0 (B) 1 (C) |z| (D) $\frac{1}{|z|}$

The Modulus of a Complex Number

5. The modulus of the complex number $\frac{(1+7i)}{(2-i)^2}$ is

(A) $\frac{7}{4}$ (B) $\sqrt{2}$ (C) $\frac{4}{7}$ (D) $\frac{1}{\sqrt{2}}$

6. The modulus of the complex number $\frac{(1+2i)}{(1-3i)}$ is

(A) $\frac{2}{3}$ (B) $\sqrt{2}$ (C) $\frac{3}{2}$ (D) $\frac{1}{\sqrt{2}}$

7. The modulus of $\sqrt{2}i - \sqrt{-2}i$ is

(A) 2 (B) $\sqrt{2}$ (C) 0 (D) $2\sqrt{2}$

8. If z is a complex number satisfying $z^4 + z^3 + 2z^2 + z + 1 = 0$, then the set of possible values of |z| is

(A) {1, 2} (B) {1} (C) {1, 2, 3} (D) {1, 2, 3, 4}

Geometrical Representation of Complex Number

9. Let A, B and C represent the complex numbers z_1, z_2 and z_3 on the Argand plane. If circumcentre of the triangle ABC is at the origin, then the complex number corresponding to orthocentre is

(A) $\frac{1}{4}(z_1+z_2+z_3)$ (B) $\frac{1}{3}(z_1+z_2+z_3)$

(C) $\frac{1}{2}(z_1+z_2+z_3)$ (D) $z_1+z_2+z_3$

Argument of a Complex Number

10. $\arg\left(-\frac{2}{5}\right)$ equals

(A) 0 (B) $\frac{\pi}{2}$ (C) $\frac{-\pi}{2}$ (D) π

Polar form of a Complex Number

11. If $z_k = \cos\frac{\pi}{2^k} + i\sin\frac{\pi}{z^k}, k = 1, 2, 3....$ then the value of $z_1 z_2$ ..... ∞

(A) 0 (B) 1 (C) –1 (D) 2

De-Moivre's Theorem

12. If $\theta = \frac{\pi}{6}$, then the 10th term of the series $1+(\cos\theta+i\sin\theta)+(\cos\theta+i\sin\theta)^2+....$ is

(A) –i (B) i (C) $\frac{1+i\sqrt{3}}{2}$ (D) $\frac{1-i\sqrt{3}}{2}$

13. $\left(\frac{1+i\tan\theta}{1-i\tan\theta}\right)^n - \frac{1+i\tan n\theta}{1-i\tan n\theta} =$

(A) 0 (B) 1 (C) –1 (D) i

14. If α, β are the roots of $x^2-2x+4=0$, then $\alpha^n+\beta^n$ is equal to

(A) $2^n\cos\frac{n\pi}{3}$ (B) $2^n\cos\frac{(n+1)\pi}{3}$

(C) $2^{n+1}\cos\frac{n\pi}{3}$ (D) $2^{n+1}\cos\frac{(n+1)\pi}{3}$

Concept of Rotation

15. The complex number 3 - 4i is rotated by 90° in the anticlockwise direction about origin. The complex number in new position is

(A) –3 + 4i (B) 4 + 3i (C) -4 + 3i (D) -4 - 3i

16. A necessary and sufficient condition for the points z_1, z_2 and z_3 to be collinear is that the complex number $\frac{z_3-z_1}{z_2-z_1}$ is

(A) real (B) imaginary

(C) purely imaginary (D) of the form $\lambda(1+i)$, $\lambda \in R - \{0\}$

ection Formula

17. The points z_1, z_2, z_3 and z_4 in the Argand plane are the vertices of a parallelogram if and only if

(A) $z_1 + z_4 = z_2 + z_3$ (B) $z_1 + z_3 = z_2 + z_4$

(C) $z_1 + z_2 = z_3 + z_4$ (D) $z_1 + z_2 + z_3 + z_4 = 0$

The n^{th} roots of unity (Specially cube roots of unity)

18. Let $\alpha_o, \alpha_1, \alpha_2, ..., \alpha_{n-1}$ be n, n^{th} roots of unity. Value of $(1-\alpha_1).....(1-\alpha_{n-1})$ is

(A) n (B) n – 1 (C) $(-n)^n$ (D) 0

19. $\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7} =$

(A) 1 (B) $\frac{1}{2}$ (C) –1 (D) $-\frac{1}{2}$

20. Let $\alpha_o, \alpha_1, \alpha_2, ..., \alpha_{n-1}$ be n, n^{th} roots of unity. Value of $(1+\alpha_0)(1+\alpha_1).....(1+\alpha_{n-1})$ [n is even] is

(A) 3 (B) 1 (C) 0 (D) 2

21. If $x = a + b$, $y = a\gamma + b\beta$ and $z = a\beta + b\gamma$, where γ and β are the imaginary cube roots of unity, then $xyz =$

(A) $(a^2 + b^2)(a + b)$ (B) $a^3 + b^3$ (C) $(a + b)^2 (a - b)$ (D) $(a + b)^3$

22. The equation, whose roots are i, $\frac{1+i}{\sqrt{2}}, \frac{i-1}{\sqrt{2}}$, their conjugates, – 1 and 1 is

(A) $x^8 = 1$ (B) $x^8 = -1$ (C) $x^8 + x^4 - 2 = 0$ (D) $x^8 = x^4$

23. If α is the angle which each side of a regular polygon of n sides subtends at its centre, then $1+\cos\alpha+\cos 2\alpha+\cos 3\alpha+..........+\cos(n-1)\alpha$ is equal to

(A) n (B) 0 (C) 1 (D) –1

Equation of Circle in Complex Form

24. If $z = x + iy$ then the equation $\left|\frac{2z-i}{z+1}\right| = m$ does not represent a circle when

(A) $m = \frac{1}{2}$ (B) m = 1 (C) m = 2 (D) m = 3

Equation of Circle

25. If $|z| = 1$, then $\frac{z}{\bar{z}}$ lies on a

(A) circle (B) parabola (C) straight line (D) hyperbola

LEVEL - 2

Basic Concepts

26 If one root of $z^2 + (a + i)z + b + ic = 0$ is real, where $a, b, c \in R$, then $c^2 + b - ac =$

(A) 0 (B) 1 (C) –1 (D) a + b + c

27. If $(1 + 2i)$ is a root of the equation $x^2 + bx + c = 0$, where b and c are real then (b, c) is given by

(A) (2, –5) (B) (–3, 1) (C) (–2, 5) (D) (3, 1)

28. if $x = 2 + 5i$, then the value of $x^3 - 5x^2 + 33x - 19$ is

(A) 4 (B) 6 (C) 8 (D) 10

29. The smallest positive integer n, for which $\left(\frac{1+i}{1-i}\right)^n = 1$ is

(A) 4 (B) 8 (C) 12 (D) 16

The Conjugate of a Complex Number

30. If $z = x + iy$ lies in IIIrd quadrant then $\frac{\bar{z}}{z}$ also lies in the III quadrant if

(A) $x > y > 0$ (B) $x < y < 0$ (C) $y < x < 0$ (D) $y > x > 0$

The Modulus of a Complex Number

31. If $\left|\frac{z_1 - iz_2}{z_1 + iz_2}\right| = 1$, then $\frac{z_1}{z_2}$ is

(A) purely imaginary (B) real (C) imaginary (D) of unit modulus

32. If $|z| = 4$, then the minimum value of $|z + 3 + 4i|$ is

(A) 1 (B) 3/2 (C) 2 (D) 0

33. If $\left|\frac{1 - iz}{z - i}\right| = 1$, then z lies on

(A) imaginary axis (B) real axis

(C) a unit circle (D) a line not passing through origin

34. For $x_1, x_2, y_1, y_2 \in R$, if $0 < x_1 < x_2$, $y_1 = y_2$ and $z_1 = x_1 + iy_1$, $z_2 = x_2 + iy_2$ and $z_3 = \frac{1}{2}(z_1 + z_2)$, then z_1, z_2 and z_3 satisfy :

(A) $|z_1| = |z_2| = |z_3|$ (B) $|z_1| < |z_2| < |z_3|$

(C) $|z_1| > |z_2| > |z_3|$ (D) $|z_1| < |z_3| < |z_2|$

35. For any complex number z, the minimum value of $|z| + |z - 2i|$ is

(A) 1 (B) 2 (C) 3 (D) does not exist

36. If z_1 and z_2 are two complex numbers such that $|z_1| = |z_2| + |z_1 - z_2|$, $|z_1| > |z_2|$, then

(A) $\text{Im}\left(\frac{z_1}{z_2}\right) = 0$ (B) $\text{Re}\left(\frac{z_1}{z_2}\right) = 0$

(C) $\text{Re}\left(\frac{z_1}{z_2}\right) = \text{Im}\left(\frac{z_1}{z_2}\right)$ (D) $\text{Im}\left(\frac{z_1}{z_2}\right) = 1$

Argument of a Complex Number

37. If $\left|\frac{z_1}{z_2}\right| = 1$ and $\arg(z_1 z_2) = 0$, then

(A) $z_1 = z_2$ (B) $|z_2|^2 = z_1 z_2$ (C) $z_1 z_2 = 1$ (D) $z_1 + z_2 = 0$

38. If $|z_1 + z_2| = |z_1 - z_2|$, then the difference of the arguments of z_1 and z_2 is $(z_1, z_2 \neq 0)$

(A) 0 (B) $\pi/2$ (C) π (D) $3\pi/2$

39. If z_1 and z_2 are two non-zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then $\text{Arg}z_1 - \text{Arg}z_2$ is equal to

(A) $-\pi$ (B) $-\frac{\pi}{2}$ (C) 0 (D) π

Polar form of a Complex Number

40. The polar form of the complex number $\frac{-16}{1 + i\sqrt{3}}$ is

(A) $\frac{8}{\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}}$ (B) $8\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

(C) $\frac{8}{\cos\frac{2\pi}{3} - i\sin\frac{2\pi}{3}}$ (D) $8\left(\cos\frac{2\pi}{3} - i\sin\frac{2\pi}{3}\right)$

41. If $|z| = 2$ and $\arg(z) = -\frac{\pi}{6}$, then z =

(A) $\sqrt{3} - i$ (B) $\frac{\sqrt{3}}{2} - \frac{i}{2}$ (C) 2 (D) 0

De-Moivre's Theorem

42. If $z=\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^{15}$, then

(A) Re(z) = 1 (B) Re(z) = -1 (C) Im(z) = 1 (D) Im(z) = -1

43. If $x+\frac{1}{x}=1$, then $x^{2009}+\frac{1}{x^{2009}}$ is equal to

(A) 0 (B) 1 (C) –1 (D) 2

Concept of Rotation

44 If A and B are the points on the Argand plane corresponding to –3+5i and –5–3i, then $\angle AOB$ is (O is the origin)

(A) $\frac{5\pi}{3}$ (B) $\frac{3\pi}{5}$ (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{4}$

45. If z, – z and 1–z are the vertices of an equilateral triangle, then Re(z) =

(A) 1 (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{1}{4}$

46. If z is rotated about origin by a an angle of 90° in the anticlockwise direction, then the new complex number is

(A) -z (B) $\bar{z}$ (C) $-iz$ (D) iz

47. If a, b, c and u, v, w are complex numbers representing the vertices of two triangles such that c = (1 – r) a + rb and w = (1 – r)u + rv, where r is a complex number, then the two triangles

(A) have the same area (B) are similar

(C) are congruent (D)have the same parameter

48. Let z_1 and z_2 be the complex roots of the equation $3z^2 + 3z + b = 0$. If the origin, together with the points represented by z_1 and z_2 form an equilateral triangle, then the value of b

(A) is –1 (B) is 0

(C) is 1 (D) can not be decided

49. A necessary and sufficient condition for the points z_1, z_2 and z_3 to be collinear is that the complex number $\frac{z_3-z_1}{z_2-z_1}$ is

(A) real (B) imaginary

(C) purely imaginary (D) of the form $\lambda(1+i),\ \lambda \in R - \{0\}$

The nth roots of unity

50. If the cube roots of unity are 1, w, w^2, then the roots of the equation $(x-1)^3 + 8 = 0$ are

(A) –1, 1 + 2w, 1 + 2w^2 (B) –1, 1 –2w,1 –2w^2 (C) –1,–1,–1 (D) 1, w, w^2

SINGLE CORRECT ANSWER

LEVEL -1

1. (D)	2. (C)	3. (A)	4. (B)
5. (B)	6. (D)	7. (A)	8. (B)
9. (D)	10. (D)	11. (C)	12. (A)
13. (A)	14. (C)	15. (B)	16. (A)
17. (B)	18. (A)	19. (D)	20. (C)
21. (B)	22. (A)	23. (B)	24. (C)
25. (A)			

LEVEL- 2

26. (A)	27. (C)	28. (D)	29. (A)
30. (C)	31. (B)	32. (A)	33. (B)
34. (D)	35. (B)	36. (A)	37. (B)
38. (B)	39. (C)	40. (B)	41. (A)
42. (D)	43. (B)	44. (C)	45. (D)
46. (D)	47. (B)	48. (C)	49. (A)
50. (B)			

SINGLE CORRECT ANSWER

LEVEL 1

1. $\sin x + i\cos 2x = \cos x - i\sin 2x \Rightarrow \sin x = \cos x$ and $\cos 2x = -\sin 2x$

which is not true for any value of x.

2. The given number is $\frac{2-i}{-3-4i} = \frac{(2+i)(-3-4i)}{25} = -\frac{2}{25} + i\frac{11}{25}$.

3. $A + iB = \frac{1-i\alpha}{1+i\alpha} \Rightarrow A - iB = \frac{1+i\alpha}{1-i\alpha}$

$\Rightarrow (A+iB)\,(A-iB) = \frac{(1-i\alpha)(1+i\alpha)}{(1+i\alpha)(1-i\alpha)} = 1 \Rightarrow A^2 + B^2 = 1$

Hence (A) is the correct alternative.

4. $|\omega| = \frac{|z|}{|\bar{z}|} = 1$ as $|\bar{z}| = |z|$

5. $\left|\frac{1+7i}{(2-i)^2}\right| = \frac{|1+7i|}{|2-i|^2} = \frac{\sqrt{50}}{\left(\sqrt{5}\right)^2} = \sqrt{2}$

6. $\left|\frac{1+2i}{1-3i}\right| = \frac{|1+2i|}{|1-3i|} = \frac{\sqrt{5}}{\sqrt{10}} = \frac{1}{\sqrt{2}}$

7. The given number is $\sqrt{2}i + \sqrt{2}$, as $\sqrt{-2} = \sqrt{2}i$

8. The given equation is $(z^2 + z + 1)(z^2 + 1) = 0$

$\Rightarrow$ $z = \pm i, \omega, \omega^2$, ω being an imaginary cube root of unity. Thus $|z| = 1$.

9. Centroid of ΔABC is at $\frac{z_1 + z_2 + z_3}{3}$.

It is a known fact that orthocenter divides the join of centroid and circum centre in 2: 3 externally. So orthocentre is given by $z_1 + z_2 + z_3$.

10. $\arg\left(-\frac{2}{5}\right) = \pi$, as $\frac{-2}{5}$ lies on negative real axis.

11. $z_1z_2z_3......\infty = \cos\left(\frac{\pi}{2}+\frac{\pi}{4}+\frac{\pi}{8}+......\right)+i\sin\left(\frac{\pi}{2}+\frac{\pi}{4}+\frac{\pi}{8}+......\right) = -1$

12. 10^{th} term $= (\cos\theta + i\sin\theta)^9 = \cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2} = -i$

13. The given expression is $\left(\frac{\cos\theta + i\sin\theta}{\cos\theta - i\sin\theta}\right)^n - \frac{\cos n\theta + i\sin n\theta}{\cos n\theta - i\sin n\theta} = 0,$

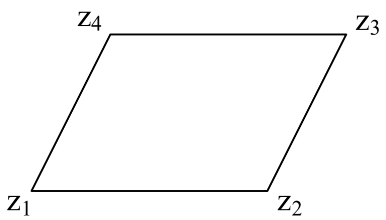
as $(\cos\theta - i\sin\theta)^n = (\cos(-\theta) + i\sin(-\theta))^n = \cos n\theta - i\sin n\theta$

14. $\alpha = 1+\sqrt{3}\,i = 2e^{i\pi/3}$, $\beta = 1-\sqrt{3}\,i = 2e^{-i\pi/3}$

15. $i(3 - 4i) = 4 + 3i$

16. z_1, z_2 and z_3 are collinear if and only if $\arg\left(\frac{z_3 - z_1}{z_2 - z_1}\right) = 0$ or π i.e. $\frac{z_3 - z_1}{z_2 - z_1}$ is real.

17. Points are the vertices of a parallelogram if and only if mid points of the join of z_1 and z_3 is the same as the mid point of the join of z_2 and z_4 i.e., $\frac{z_1 + z_3}{2} = \frac{z_2 + z_4}{2}$.



18. $x^n - 1 \equiv (x-1)(x-\alpha_1)(x-\alpha_2)....(x-\alpha_{n-1})$

$\Rightarrow (x-\alpha_1)(x-\alpha_2)....(x-\alpha_{n-1}) = 1 + x + x^2 + ... + x^{n-1}$.Put $x = 1$

19. Roots of $z^7 = 1$ are $e^{2k\pi i/7}$, where k = 0, 1, 2, ..., 6.

Sum of roots = 0 $\Rightarrow$ Real part of sum of roots = 0.

$$\Rightarrow 1 + \cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7} + \cos\frac{8\pi}{7} + \cos\frac{10\pi}{7} + \cos\frac{12\pi}{7} = 0$$

$\Rightarrow \cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}=-\frac{1}{2}$

20. $x^n-1\equiv(x-\alpha_o)(x-\alpha_1)....(x-\alpha_{n-1})$. Put x = -1.

21. $xyz=(a+b)(a\omega+b\omega^2)(a\omega^2+b\omega)=(a+b)(a+b\omega)(a+b\omega^2)=a^3+b^3$

22. There are eight roots satisfying $x^8 = 1$. Also all of them are different.

23. $\sum_{r=0}^{n-1}\cos r\alpha = \text{Re}\sum_{r=0}^{n-1} e^{\frac{ir\pi}{n}}$ = Re(sum of the n roots of unity) = 0

Hence (B) is the correct answer.

24. The given equation is $\left|\frac{z-\frac{i}{2}}{z+1}\right|=\frac{m}{2}$, which does not represent a circle when $\frac{m}{2}=1$.

25. Let $\omega=\frac{z}{\bar{z}}$, then $|\omega|=1$

Hence $\frac{z}{\bar{z}}$ lies on a circle.

LEVEL 2

26. Let $\alpha\in R$ be a root. Then $\alpha^2+(a+i)\alpha+b+ic=0$

$\Rightarrow \alpha^2+a\alpha+b=0$ and $\alpha+c=0$

$\Rightarrow c^2-ac+b=0$

27. 1 + 2i is a root $\Rightarrow (1+2i)^2+b(1+2i)+c=0 \Rightarrow b=-2, c=5$

28. $(x-2)^2 = -25 \Rightarrow x^2-4x+29=0$

Thus, $x^3-5x^2+33x-19=(x-1)(x^2-4x+29)+10=10$.

29. $\left(\frac{1+i}{1-i}\right)^n=i^n=1$. The least value of n is 4.

30. Given x < 0 and y < 0.

$\frac{\bar{z}}{z}=\frac{\bar{z}^2}{|z|^2}$. Thus $\text{Re}\left(\frac{\bar{z}}{z}\right)=\frac{x^2-y^2}{x^2+y^2}$, $\text{Im}\left(\frac{\bar{z}}{z}\right)=\frac{-2xy}{x^2+y^2}$

$\Rightarrow x^2-y^2<0$ and $-2xy<0 \Rightarrow y<x<0$.

31. $\left|\dfrac{\dfrac{z_1}{z_2}-i}{\dfrac{z_1}{z_2}+i}\right| = 1 \Rightarrow \dfrac{z_1}{z_2}$ is equidistant from -i and i $\Rightarrow$ $\dfrac{z_1}{z_2}$ is a real number.

32. $|z+3+4i| = |z-(-3-4i)| \geq ||z|-5| = 1$

33. $|1-iz| = |z-i| \Rightarrow |z+i| = |z-i|$

$\Rightarrow$ z is equidistant from i and -i.

$\Rightarrow$ z lies on the real axis.

34. Represent z_1, z_2 and z_3 on the Argand plane.

35. The minimum value is obtained when z lies in between 0 and 2i.

36. $|z_1 - z_2| = |z_1| - |z_2| = ||z_1| - |z_2|| \Rightarrow \arg z_1 = \arg z_2 \Rightarrow \arg\left(\dfrac{z_1}{z_2}\right) = 0 \Rightarrow \operatorname{Im}\left(\dfrac{z_1}{z_2}\right) = 0.$

37. $\left|\dfrac{z_1}{z_2}\right| = 1 \Rightarrow |z_1| = |z_2|$

Now $\arg(z_1\, z_2) = 0 \Rightarrow \arg(z_1) + \arg(z_2) = 0$ $\Rightarrow \arg(z_2) = -\theta_1$, where $\arg(z_1) = \theta$

Therefore, $z_2 = |z_1|(\cos(-\theta_1) + i\sin(-\theta_1) = \bar{z}_1$

Hence (B) is the correct alternative.

38. z_1 is equidistant from z_2 and $-z_2$

$\Rightarrow$ origin is the mid point of a side (join of $-z_2$ and z_2) of a triangle having vertices z_1, z_2 and $-z_2$.

39. Let $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$

where $r_1 = |z_1|$, $r_2 = |z_2|$, $\theta_1 = \operatorname{Arg} z_1$, $\theta_2 = \operatorname{Arg} z_2$

$\therefore\ z_1 + z_2 = r_1(\cos\theta_1 + i\sin\theta_1) + r_2(\cos\theta_2 + i\sin\theta_2)$

$= (r_1\cos\theta_1 + r_2\cos\theta_2) + i(r_1\sin\theta_1 + r_2\sin\theta_2)$

$\Rightarrow |z_1 + z_2|^2 = r_1^2 + r_2^2 + 2r_1r_2\cos(\theta_1 - \theta_2)$ (1)

and $|z_1| + |z_2| = r_1 + r_2$

$\Rightarrow (|z_1| + |z_2|)^2 = r_1^2 + r_2^2 + 2r_1r_2$ (2)

Since $|z_1 + z_2| = |z_1| + |z_2|$

From (1) and (2) we get $\cos(\theta_1 - \theta_2) = 1$ or $\theta_1 - \theta_2 = 0$

40. $\frac{-16}{1+i\sqrt{3}} = -4 + i(4\sqrt{3})$, whose modulus value is 8 and argument is $\frac{2\pi}{3}$.

41. $z = 2\left(\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right) = \sqrt{3} - i$

42. $\left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^{15} = \left(\cos\left(\frac{-\pi}{6}\right) + i\sin\left(\frac{-\pi}{6}\right)\right)^{15} = \cos\frac{5\pi}{2} - i\sin\frac{5\pi}{2} = -i$

43. $x + \frac{1}{x} = 1 \Rightarrow x^2 - x + 1 = 0 \Rightarrow x = -\omega \text{ or } -\omega^2$

44. –5–3i = i(–3+5i), so required angle is $\frac{\pi}{2}$, as $i = e^{i\pi/2}$.

45. $z^2 + z^2 + (1 - z)^2 = -z^2 - z + z^2 + z - z^2$

$\Rightarrow \quad 4z^2 - 2z + 1 = 0$

$\Rightarrow \quad z = \frac{2 \pm \sqrt{4-16}}{8}$. Thus Re(z) = $\frac{1}{4}$

46. The new complex number is $e^{i\pi/2} = iz$

47. $r = \frac{c-a}{b-a} = \frac{w-u}{v-u} \Rightarrow \arg\left(\frac{c-a}{b-a}\right) = \arg\left(\frac{w-u}{v-u}\right)$ and $\left|\frac{c-a}{b-a}\right| = \left|\frac{w-u}{v-u}\right|$.

48. z_1, z_2, 0 will be the vertices of an equilateral triangle if $z_1^2 + z_2^2 + 0^2 = z_1z_2 + 0z_2 + 0z_1$

$\Rightarrow 1 - \frac{2b}{3} = \frac{b}{3} \Rightarrow b = 1$

49. z_1, z_2 and z_3 are collinear if and only if $\arg\left(\frac{z_3 - z_1}{z_2 - z_1}\right) = 0$ or π i.e. $\frac{z_3 - z_1}{z_2 - z_1}$ is real.

50. $x = 1 - 2(1)^{1/3}$

$\Rightarrow x = 1 - 2,\ 1 - 2\omega,\ 1 - 2\omega^2$.