Cody Kimmins

**Definition 1.1** - The *mean* of a sample of n measured responses is given by

The corresponding population mean is denoted μ.

**Definition 2.2** – The *variance* of a sample of measurements y\_1, y\_2, …, y\_n is the sume of the square of the differences between the measurements and their mean, divided by n-1. Symbolically, the sample variance is

The corresponding population variance is denoted by the symbol .

**Definition 1.3** – The *standard deviation* of a sample of measurements is the positive square root of the variance; that is,

The corresponding population standard deviation is denoted by

**Definition 2.6** – Suppose S is a sample space associated with an experiment. To every extent A in S (A is a subset of S), we assign a number, P(A), called the *probability* of A, so that the following axioms hold:

Axiom 1:

Axiom 2:

Axiom 3: If A1, A2, A3, . . . form a sequence of pairwise mutually exclusive events in S (that is, *Ai* ∩ *Aj* = ∅ if *i* ≠ *j*), then

**Definition 2.7** – An ordered arrangement of r distinct objects is called *permutation*. The number of ways of ordering n distinct objects taken r at a time will be designated by the symbol .

**Definition 2.8** – The number of combinations of n objects taken r at a time is the number of subsets, each of size r, that can be formed from the n objects. This number will be denoted by

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**Theorem 2.4** – The number of unordered subsets of size *r* chosen (without replacement) from *n* available objects is:

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**Theorem 2.9** – The *conditional probability* *of an event A*, given that an event *B* has occurred, is equal to

**Definition 2.10** - Two events A and B are said to be independent if any one of the following holds:

Otherwise, the events are said to be *dependent*.

**Theorem 2.5** – *The Multiplicative Law of Probability* – The probability of the intersection of two events *A* and *B* is

If A and B are independent, then

**Theorem 2.6** – *The Additive Law of Probability* – The probability of the union of two events *A* and *B* is

If *A* and *B* are mutually exclusive events, and

**Theorem 2.7** – If A is an event, then:

**Theorem 2.9** – *Bayes’ Rule* – Assume that is a partition of S such that , for Then

**Definition 3.4 -** Let *Y* be a discrete random variable with the probability function *p(y)*. Then the *expected value* of *Y* , *E(Y )*, is defined to be:

**Definition 3.5 -** If *Y* is a random variable with mean *E(Y ) = μ*, the variance of a random variable *Y* is defined to be the expected value of *(Y − μ)2* . That is,

The *standard deviation* of *Y* is the positive square root of *V(Y )*.

**Definition 3.7 -** A random variable *Y* is said to have a *binomial distribution* based on *n* trials with success probability *p* if and only if:

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**Theorem 3.7 -** Let *Y* be a binomial random variable based on *n* trials and success probability *p*. Then:

and

**Definition 3.8 -** A random variable *Y* is said to have a *geometric probability distribution* if and only if:

**Theorem 3.8 -** If *Y* is a random variable with a geometric distribution,

and

**Definition 3.10 -** A random variable *Y* is said to have a *hypergeometric probability distribution* if and only if:

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where *y* is an integer 0, 1, 2, . . . , *n*, subject to the restrictions *y* ≤ *r* and *n* − *y* ≤ *N* − *r*.

**Definition 3.11** - A random variable *Y* is said to have a *Poisson probability distribution* if and only if:

**Theorem 3.11 -** If *Y* is a random variable possessing a Poisson distribution with parameter *λ*, then:

and

**Theorem 3.14 -** *Tchebysheff’s Theorem -* Let *Y* be a random variable with mean *μ* and finite variance σ2. Then, for any constant *k* > 0,

or

**Definition 4.1 -** Let *Y* denote any random variable. The *distribution function* of *Y*, denoted by *F(y)*, is such that

for

**Theorem 4.1 -** If *F(y)* is a distribution function, then

1. *F(y)* is a nondecreasing function of *y*. [If *y1* and *y2* are *any* values such that , then .]

**Definition 4.2**- A random variable *Y* with distribution function *F(y)* is said to be *continuous* if *F(y)* is continuous, for .

**Definition 4.3** - Let *F(y)* be the distribution function for a continuous random variable *Y*. Then *f(y)*, given by

wherever the derivative exists, is called the *probability density function* for the random variable *Y*.

**Theorem 4.2** - *Properties of a Density Function –* If *f(y)* is a density function for a continuous random variable, then

1. for all *y*,

**Theorem 4.3** - If the random variable *Y* has density function *f(y)* and , then the probability that *Y* falls in the interval [a, b] is

**Definition 4.5** - The expected value of a continuous random variable *Y* is

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provided that the integral exists.

**Definition 4.6** - If , a random variable *Y* is said to have a continuous *uniform probability distribution* on the interval if and only if the density function of *Y* is

**Theorem 4.6** – If and Y is a random variable uniformly distributed on the interval , then

and

**Definition 5.1** - Let *Y1* and *Y2* be discrete random variables. The *joint* (or bivariate) *probability function* for *Y1* and *Y2* is given by

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**Theorem 5.1** - If *Y1* and *Y2* are discrete random variables with joint probability function *p*(*y*1, *y*2), then

1. for all
2. , where the sum is over all values that are assigned nonzero probabilities.

**Definition 5.2** – For any random variables , the joint (bivariate) distribution function *F*(*y*1, *y*2) is

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**Definition 5.3** - Let *Y1* and *Y2* be continuous random variables with joint distribution function *F*(*y*1, *y*2). If there exists a nonnegative function *f*(*y*1, *y*2), such that

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for all , then *Y1* and *Y2* are said to be *jointly continuous random variables*. The function *f*(*y*1, *y*2) is called the *joint probability density function*.

**Theorem 5.2** - If *Y1* and *Y2* are random variables with joint distribution function *F*(*y*1, *y*2), then

1. If and , then .

**Theorem 5.3** - If *Y1* and *Y2* are jointly continuous random variables with a joint density function given by *f*(*y*1, *y*2), then

1. for all .
2. .

**Definition 5.4** – a. Let be jointly discrete random variables with probability function . Then the *marginal probability functions* of and , respectively, are given by

and .

b. Let and be jointly continuous random variables with joint density function *f*(*y*1, *y*2). Then the *marginal density functions* of and , respectively, are given by

and

**Definition 5.5** – If and are jointly discrete random variables with joint probability function and marginal probability functions and , respectively, then the *conditional discrete probability function* of given is

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Provided that .

**Definition 5.6** – If and are jointly continuous random variables with joint density function , then the *conditional distribution function* of is

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**Definition 5.7** – Let and be jointly continuous random variables with joint density and marginal densities and , respectively. For any such that the conditional density of is given by

And, for any such that , the conditional density of is given by

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**Definition 5.8** – Let have distribution function have distribution function and and have joint distribution function . Then and are said to be *independent* if and only if

For every pair of real numbers .

If are not independent, they are said to be *dependent*.

**Theorem 5.4** - If and are discrete random variables with joint probability function and marginal probability functions and , respectively, then are independent if and only if

For all pairs of real numbers (.