

Lecture 8

Saturday, 25 January 2020 2:42 PM

Recursion.

Mathematical Induction :-

$$\text{Sum} = n(n+1)/2.$$

$$\text{For } n = 1, \text{ Sum} = 1$$

$$\text{For } n = K, \text{ Sum} = (K)(K+1)/2 \quad \checkmark$$

$$\text{For } n = K+1, \text{ Sum} = (K+1)(K+2)/2.$$

$$\underline{1+2+3+4+\dots+K+K+1} = \text{Sum}$$

$$\frac{(K)(K+1)}{2} + (K+1) = \text{Sum}$$

$$\frac{(K+1)(K+2)}{2} = \text{Sum}$$

Q=> $\begin{array}{l} \text{p s void PD(int n) \{ } \\ \text{PD(0): } \left[\begin{array}{l} \text{if (n == 0) \{ } \\ \text{return; } \end{array} \right. \end{array}$

5
4
3
2
1

$\text{PD(5), PD(4), PD(3), PD(2), PD(1)} \rightarrow \text{xx} \left[\begin{array}{l} \text{syso(n); } \\ \text{PD(n-1); } \end{array} \right. \quad \text{--- ①}$

\Rightarrow

5
4
3
2
1

PD \rightarrow

PD(1)
PD(2)
PD(3)
PD(4)
PD(5)

$\begin{array}{l} \text{p s void PI(int n) \{ } \\ \left[\left[\left[\left[\left[\text{if (n == 0) \{ } \right. \right. \right. \right. \end{array}$

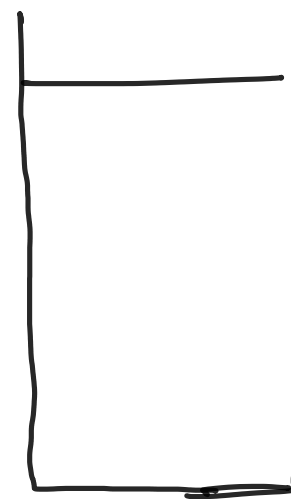
1
2
3

$$\Rightarrow \begin{array}{|c|} \hline 3 \\ \hline 4 \\ \hline 5 \\ \hline \end{array}$$

$PI(5) \rightarrow PI(4), PI(3) \rightarrow PI(2), PI(1) \rightarrow PI(0)$
 \Rightarrow $PI(5) \rightarrow PI(4) \rightarrow PI(3) \rightarrow PI(2) \rightarrow PI(1) \rightarrow PI(0)$
 \Rightarrow $PI(5) \rightarrow PI(4) \rightarrow PI(3) \rightarrow PI(2) \rightarrow PI(1) \rightarrow PI(0)$
 \Rightarrow $PI(5) \rightarrow PI(4) \rightarrow PI(3) \rightarrow PI(2) \rightarrow PI(1) \rightarrow PI(0)$

1
2
3
4
5

2 chances. $\left\{ \begin{array}{l} \text{stack is build} \\ \text{stack is finishing.} \end{array} \right.$



$\rightarrow 5 \checkmark$
 $4 \checkmark$
 $3 \checkmark$
 $2 \checkmark$
 $1 \checkmark$
 $1 \checkmark$
 $1 \checkmark$

$n=5$

p s

$\text{void PDI}(int n) \{$
 $\text{if}(n == 0) \{$
 $\text{return};$
 $\}$

$\text{syso}(n):$

2 ✓
3 ✓
4 ✓
5 ✓

POI(n-1);
→ Syso(n); ✓

int fact (int n) {

if (n == 0) {
return 1;

* int fnm1 = fact(n-1);

int fn = n * fnm1;
return fn;

}

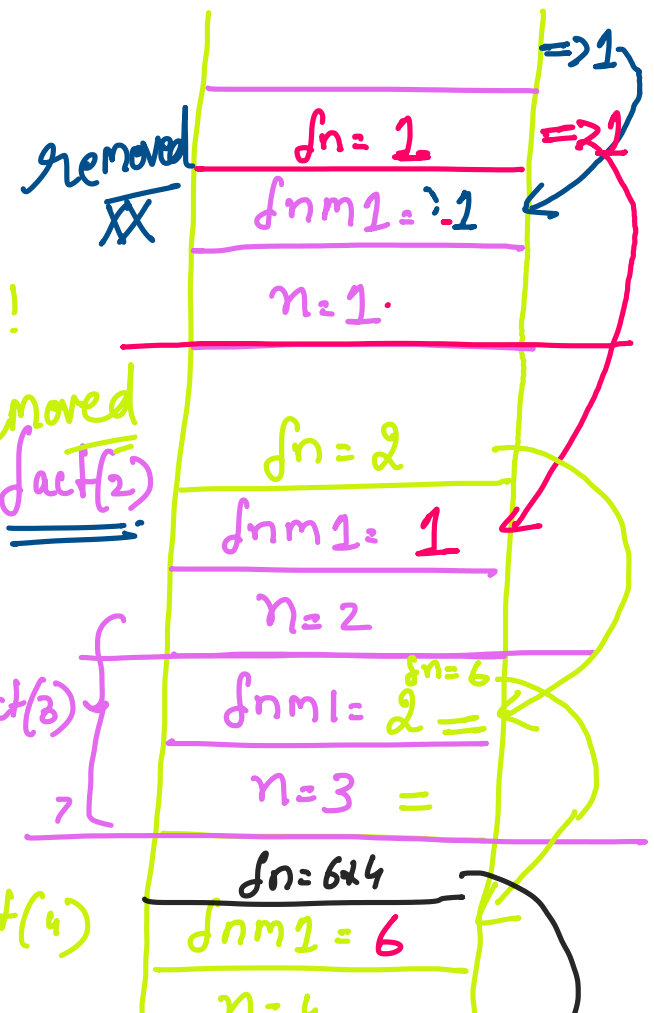
4! ⇒ 4 * 3! → Main.

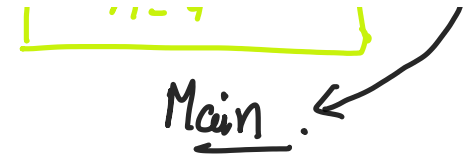
⇒ (24)

→ 3 * 2!

→ 2 * 1!

→ 1 * 0! fact(4)





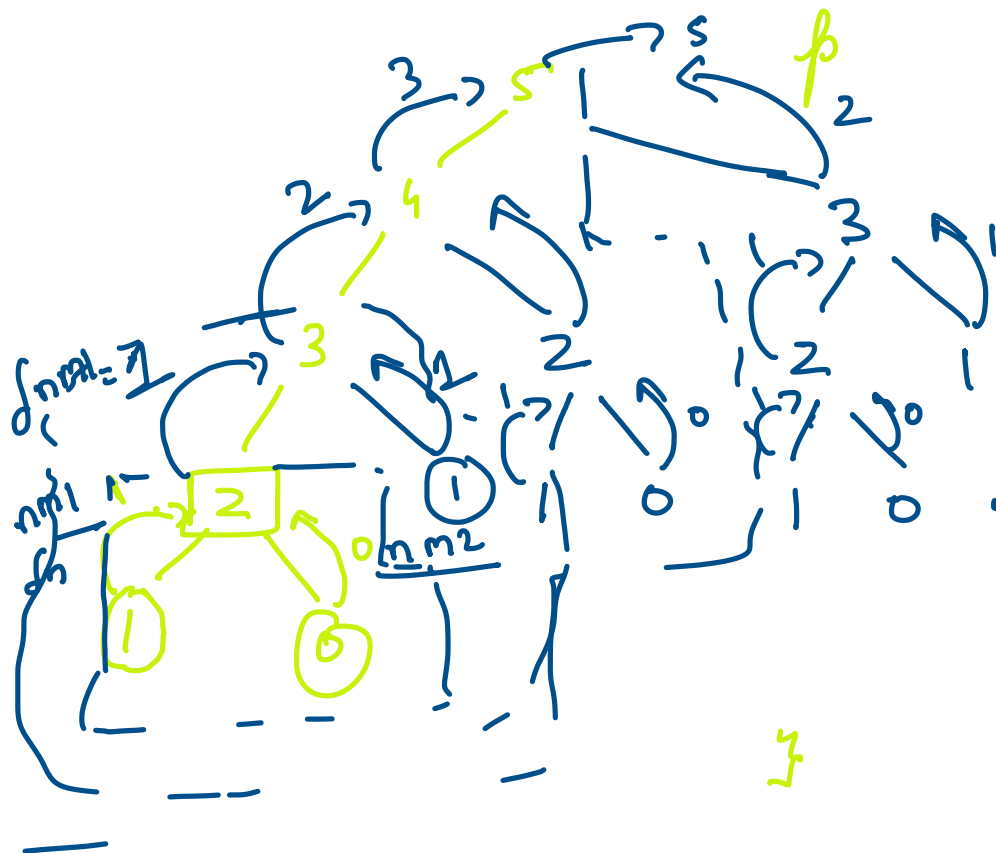
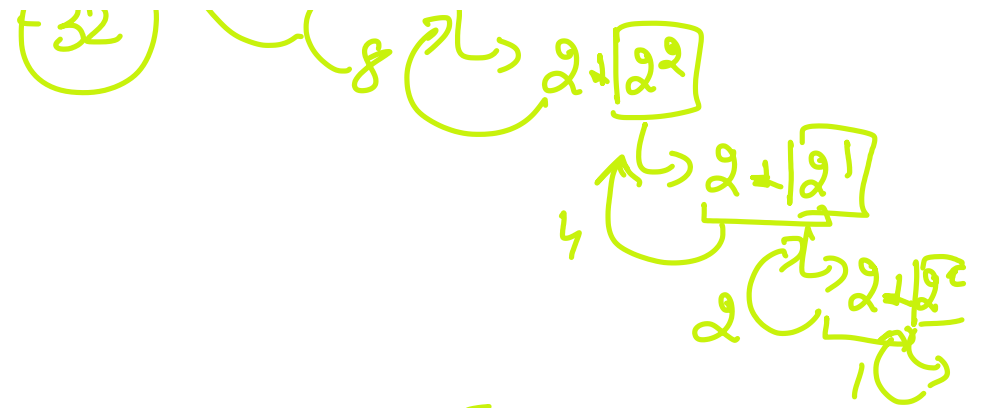
$\{$ $\{$ `int power(int n, int n) {`
 `if (n == 0) {`
 `return 1;`
 `}`
 `int pnm1 = power(n, n-1);`
 `int pn = n + pnm1;`
 `return pn;`
 $\}$

n^n

$$2^5 \Rightarrow n^n \Rightarrow n + n^{n-1}$$

$$2^5 = 2 + 2^4$$

$$16 \Rightarrow 2 + 2^3$$



```

int fib(int n) {
    if (n == 0 || n == 1) {
        return n;
    }
    int fibnm1 = fib(n-1);
    int fibnm2 = fib(n-2);
    int fibn = fibnm1 + fibnm2;
    return fibn;
}

```

Handwritten annotations for the code above:

- A yellow bracket highlights the base case condition `if (n == 0 || n == 1)`.
- A yellow arrow points from the `fib(n-1)` call to the `fibnm1` variable.
- A yellow arrow points from the `fib(n-2)` call to the `fibnm2` variable.
- A yellow box highlights the calculation `fibn = fibnm1 + fibnm2`.
- A yellow arrow points from the `return fibn;` statement to the final result 8.
- Handwritten numbers 3, 2, 1, 0, and 1 are written next to the corresponding recursive calls.
- A yellow asterisk is written next to the final result 8.

0	1	2	3	4	5	6
0	1	1	2	3	5	8

...

$n = [35, 45]$

$$\text{fib}(5) = \text{fib}(4) + \text{fib}(3)$$

$\text{fib}(5)$

$\text{fib}(4)$

Call(1)

Call(2)

