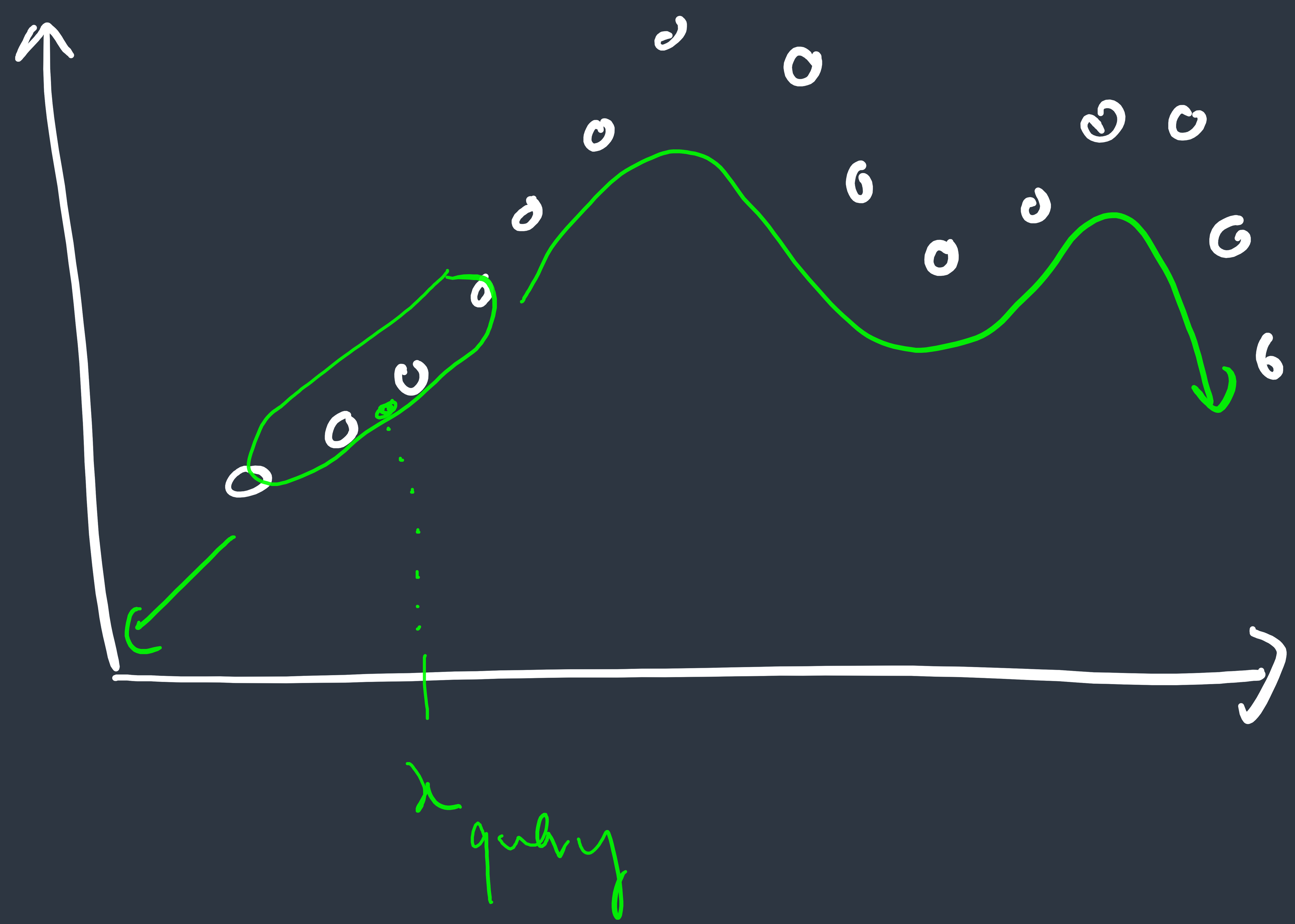
Locally weighted Regression



→ Neighbours of the query point will have more.

→ Far aways pts will have less weight

Linear Regression

$$h_\theta(x) = \theta^T x$$

$$\text{Loss} = \sum_i \left(y^i - h_\theta(x^i) \right)^2$$

more loss

0

Weighted Loss

$$\sum_i w^{(i)} \left(y^{(i)} - h_\theta(x^{(i)}) \right)^2$$

Any point

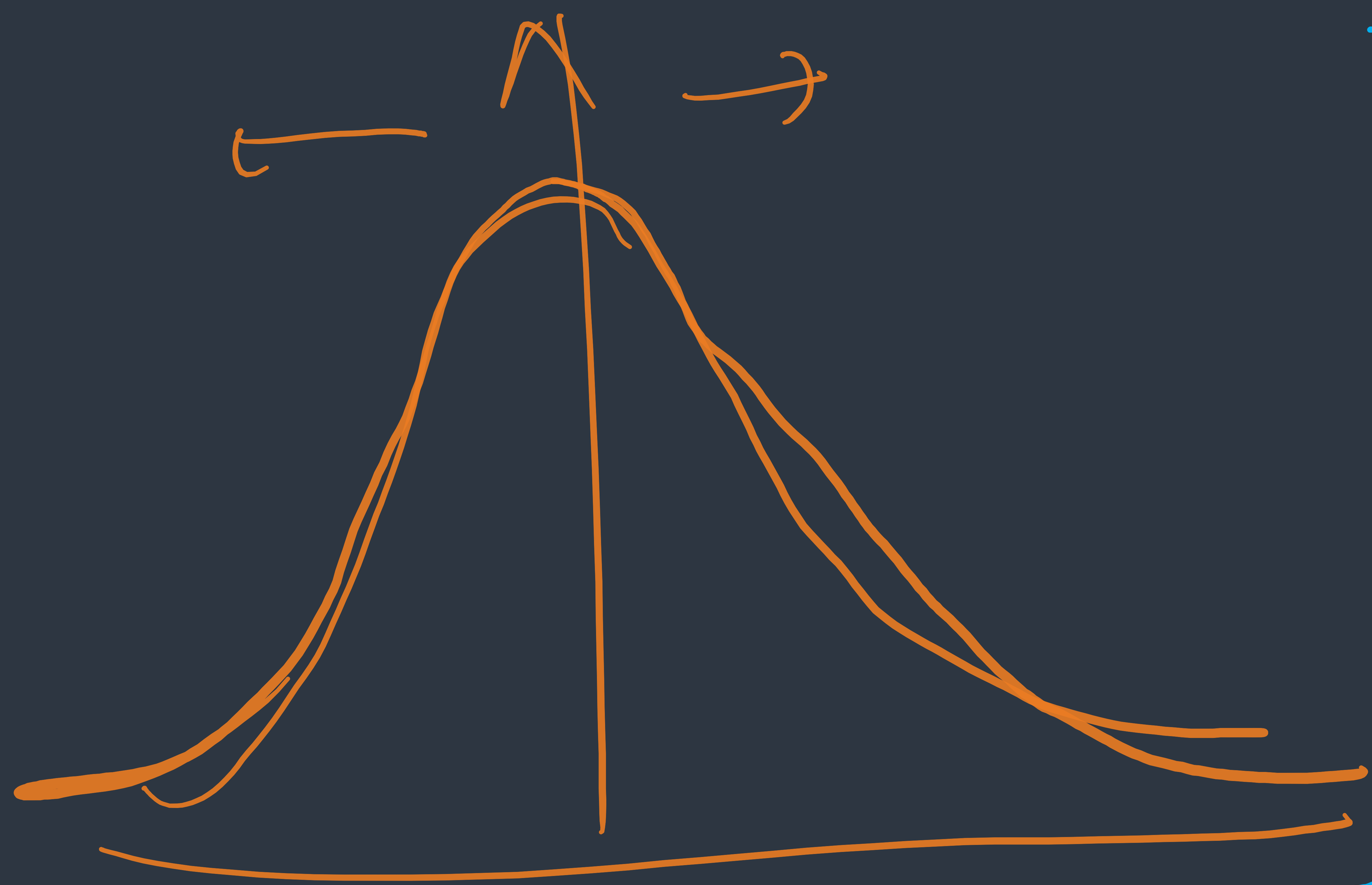
Query point

$$e^0 \approx 1$$

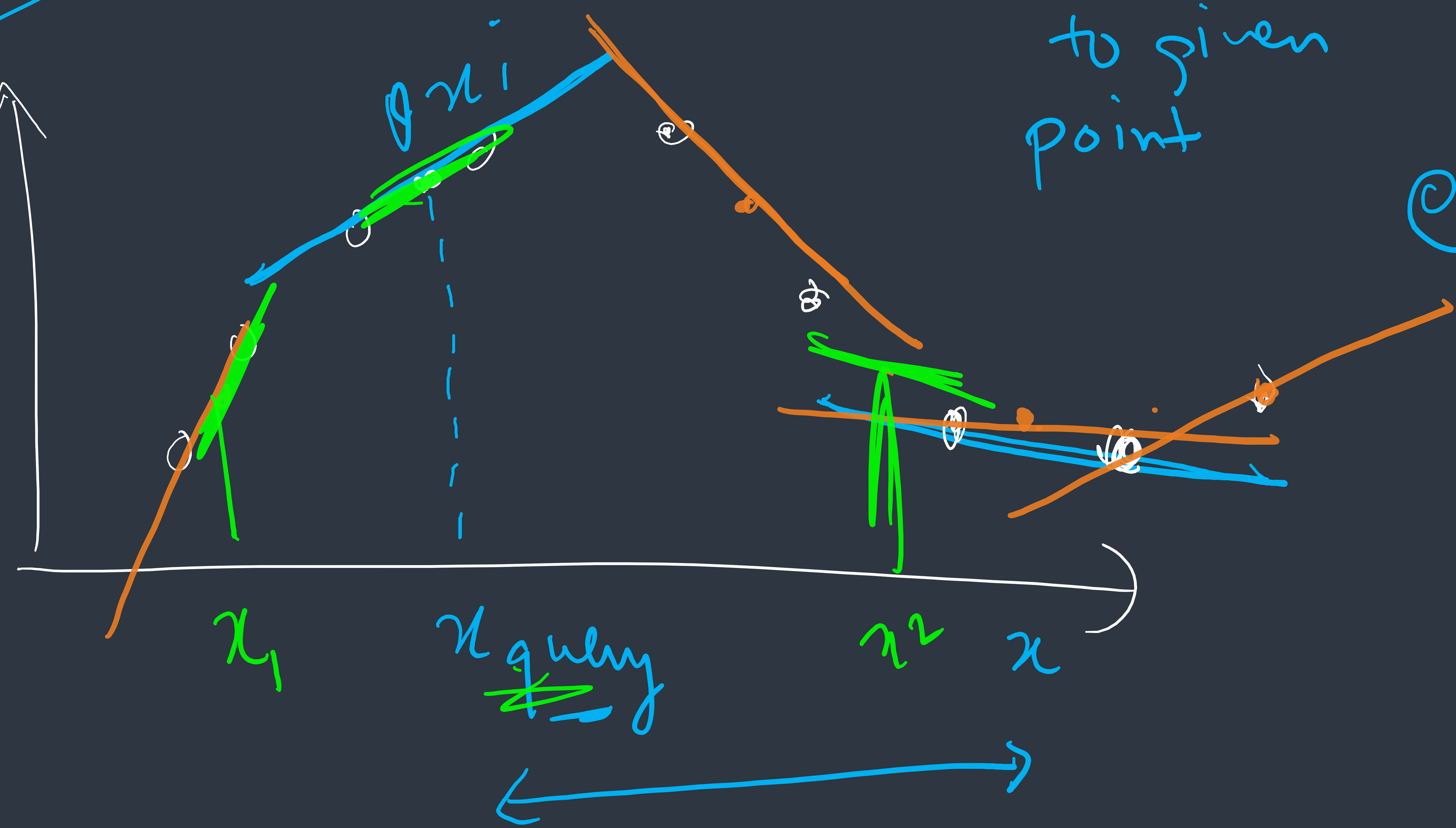
$$w^{(i)} = e^{-\frac{(x^{(i)} - x)^2}{2\tau^2}}$$

τ Bandwidth parameter.

(0, 1) if query point is close to given point



Less far away



$$e^{-\infty} = \boxed{0}$$

$$x^{(i)} \approx x$$

$$x^{(i)} - x = 0$$

$e^0 = 1$

$\Rightarrow$ No general

Non-Parametric

$\Rightarrow$ For every test point, learn θ.

↳ Closed Form Soln θ = formula

↳ Gradient Descent ✓ → 1st

↳ Newton's Descent ✓ → 2nd order

iterative

Closed Form solution for Locally weighted Regression

$$\underset{\theta}{\boxed{\min}} \text{ Loss} = \sum_{i=1}^{m} w^{(i)} \left[h_\theta(x^{(i)}) - y^{(i)} \right]^2 = (X\theta - Y)^2$$

$$= (X\theta - Y)^T (X\theta - Y)$$

$1 \times m \quad m \times 1$

scalar is different for every i
$i \in m$

(Scaling Loss)

Diagonal Matrix

$$\underbrace{\begin{bmatrix} K_1 & 0 & 0 \\ 0 & K_2 & 0 \\ 0 & 0 & K_3 \end{bmatrix}}_{\substack{3\times3 \\ n\times n}} \underbrace{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}}_{\substack{3\times1 \\ n\times1}} = \underbrace{\begin{bmatrix} K_1 \cdot 1 \\ K_2 \cdot 2 \\ K_3 \cdot 3 \end{bmatrix}}_{\substack{3\times1 \\ n\times1}}$$

scaled

Linear Algebra

$$\underbrace{\underbrace{\begin{bmatrix} (X\theta - y)^T \end{bmatrix}}_{1\times m} \underbrace{\begin{bmatrix} w_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & w_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & w_3 & 0 & 0 & 0 \\ 0 & 0 & & \ddots & & \\ & & & & & w_m \end{bmatrix}}_{m\times m} \underbrace{\overbrace{\begin{bmatrix} h_\theta x^1 - y^1 \\ h_\theta x^2 - y^2 \\ h_\theta x^3 - y^3 \\ \vdots \end{bmatrix}}^{X\theta - y}}_{m\times 1}}_{1\times 1 \,=\, \text{Scalar}}$$

Total Scalar

$$1\times 1 = \underline{\underline{\text{Scalar}}}$$

$$\boxed{\underset{m\times n}{X}\ \underset{n\times 1}{\theta} - \underset{m\times 1}{y}}$$

$$= [m\times 1]$$

$$\left[h_\theta(x^{(i)}) - y^{(i)}\right]$$

$$= w_1(h_\theta x^{(1)} - y^{(1)})$$

$$\vdots$$

$$\underbrace{w^{(i)}(h_\theta(x^{(i)}) - y^{(i)})^2}$$

$$J(\theta) = \sum_{i=1}^{m} w^{(i)} \left[h_\theta(x^{(i)}) - y^{(i)} \right]^2 = \underline{(X\theta - Y)^T \boxed{W} (X\theta - Y)}$$

$$W = \begin{bmatrix} w^1 & 0 & 0 & 0 \\ 0 & w^2 & 0 & 0 & 0 \\ 0 & 0 & \ddots & 0 & 0 \\ 0 & 0 & & w^m \end{bmatrix}$$

dependent on query point

Find $\nabla_\theta J(\theta) = 0$

$$\frac{\partial}{\partial \theta} J(\theta) = 0 = (\theta^T X^T - Y^T)(WX\theta - WY)$$

$$= \frac{\partial}{\partial \theta} \left(\theta^T X^T W X \theta - \theta^T X^T W Y - Y^T W X \theta + \cancel{Y^T W Y} \right)$$

$$= \frac{\partial}{\partial \theta}\left(\theta^T X^T W X \theta - 2\theta X^T W Y\right)$$

$$\Rightarrow \quad 2X^T W X \theta - 2X^T W Y = 0$$

$$\Rightarrow \quad \theta = \frac{X^T W Y}{X^{-T} W X}$$

$$\Rightarrow \quad \boxed{\theta = \left(X^T W X\right)^{-1}\left(\cancel{X}^T W Y\right)} \quad \checkmark$$

closed form. solution for locally weighted Regres