finite range

1) Let us consider the event of rolling a die.

The $S = \{1, 2, 3, 4, 5, 6\}$

$$p(x=2) = \frac{1}{6}$$

X	1	2	3	4	5	6
$P(x=X)$	1/6	1/6	1/6	1/6	1/6	1/6

$$\text{pmf} = \begin{cases} \frac{1}{6}, & x = 1, 2, 3, 4, 5, 6 \\ 0, & \text{otherwise} \end{cases}$$

The above holds the below pmf ~~below~~ properties

* $\sum p(x) = 1$

$$\sum p(x) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = 1$$

$\therefore$ Hence proved

* $p(x=x) \geq 0 \Rightarrow$ ~~all~~ prob of all sample is $\frac{1}{6}$ (ie ~~gr~~ ≥ 0)

Infinite range:

Let A & B throw a dice alternatively till one of them gets '6' & wins the game.

$$p(\text{Success}) = \frac{1}{6}, \quad p(\text{Failure}) = \frac{5}{6}$$

$$p(A \text{ wins}) = \frac{1}{6} + \frac{1}{6} \times \left(\frac{5}{6}\right)^2 + \frac{1}{6} \times \left(\frac{5}{6}\right)^4 + \dots$$

$$= \cancel{\frac{\frac{1}{6}}{1-2}}$$

$$= \frac{1}{6}\left[1 + \left(\frac{5}{6}\right)^2 + \left(\frac{5}{6}\right)^4 + \dots\right]$$

$$= \frac{\frac{1}{6}}{1 - \frac{25}{36}} = \frac{6}{11}$$

$$p(B \text{ wins}) = 1 - \frac{6}{11} = \frac{5}{11}$$

$$\text{Total prob} = \frac{6}{11} + \frac{5}{11} = 1$$

$$\boxed{\Sigma p_i = 1}$$

$\therefore$ Hence proved

2) Variance of UD

$$Var = \sigma^2 = E(x^2) - [E(x)]^2 \rightarrow 1$$

$$f(x) = \begin{cases} \frac{1}{b-a}, & a \le x \le b \\ 0 & x < a \text{ or } x > b \end{cases} \rightarrow pdf$$

$$E(x) = \int_a^b \frac{1}{b-a} \times x \, dx \quad \hookrightarrow 2$$

$$E(x^2) = \int_a^b \frac{1}{b-a} x^2 \, dx \quad \hookrightarrow 3$$

~~that~~ Applying ② & ③ in ①

$$Var(x) = \int_a^b \frac{1}{b-a} x^2\, dx - \left[\int_a^b \frac{1}{b-a} x\, dx\right]^2$$

$$= \frac{1}{b-a}\left[\frac{x^3}{3}\right]_a^b - \left[\frac{1}{b-a}\left[\frac{x^2}{2}\right]_a^b\right]^2$$

$$= \frac{1}{b-a}\left[\frac{b^3-a^3}{3}\right] - \left[\frac{1}{b-a}\left[\frac{b^2-a^2}{2}\right]\right]^2$$

using identities

$$= \frac{\cancel{(b-a)}(a^2+ab+b^2)}{3\cancel{(b-a)}} - \left[\frac{\cancel{(b-a)}(b+a)}{2\cancel{(b-a)}}\right]^2$$

$$= \frac{a^2+ab+b^2}{3} - \frac{(b+a)^2}{4}$$

$$= \frac{4a^2+4ab+4b^2-3a^2-3b^2-6ab}{12}$$

$$= \frac{a^2-2ab+b^2}{12}$$

$$\boxed{Var(x) = \frac{(b-a)^2}{12}}$$

∴ Hence it is proved

~~that~~ Applying ② & ③ in ①

$$Var(x) = \int_a^b \frac{1}{b-a} x^2 \, dx - \left[\int_a^b \frac{1}{b-a} x \, dx \right]^2$$

$$= \frac{1}{b-a} \left[\frac{x^3}{3} \right]_a^b - \left[\frac{1}{b-a} \left[\frac{x^2}{2} \right]_a^b \right]^2$$

$$= \frac{1}{b-a} \left[\frac{b^3 - a^3}{3} \right] - \left[\frac{1}{b-a} \left[\frac{b^2 - a^2}{2} \right] \right]^2$$

using identities

$$= \frac{\cancel{(b-a)}(a^2 + ab + b^2)}{3\cancel{(b-a)}} - \left[\frac{\cancel{(b-a)}(b+a)}{2\cancel{(b-a)}} \right]^2$$

$$= \frac{a^2 + ab + b^2}{3} - \frac{(b+a)^2}{4}$$

$$= \frac{4a^2 + 4ab + 4b^2 - 3a^2 - 3b^2 - 6ab}{12}$$

$$= \frac{a^2 - 2ab + b^2}{12}$$

$$\boxed{Var(x) = \frac{(b-a)^2}{12}}$$

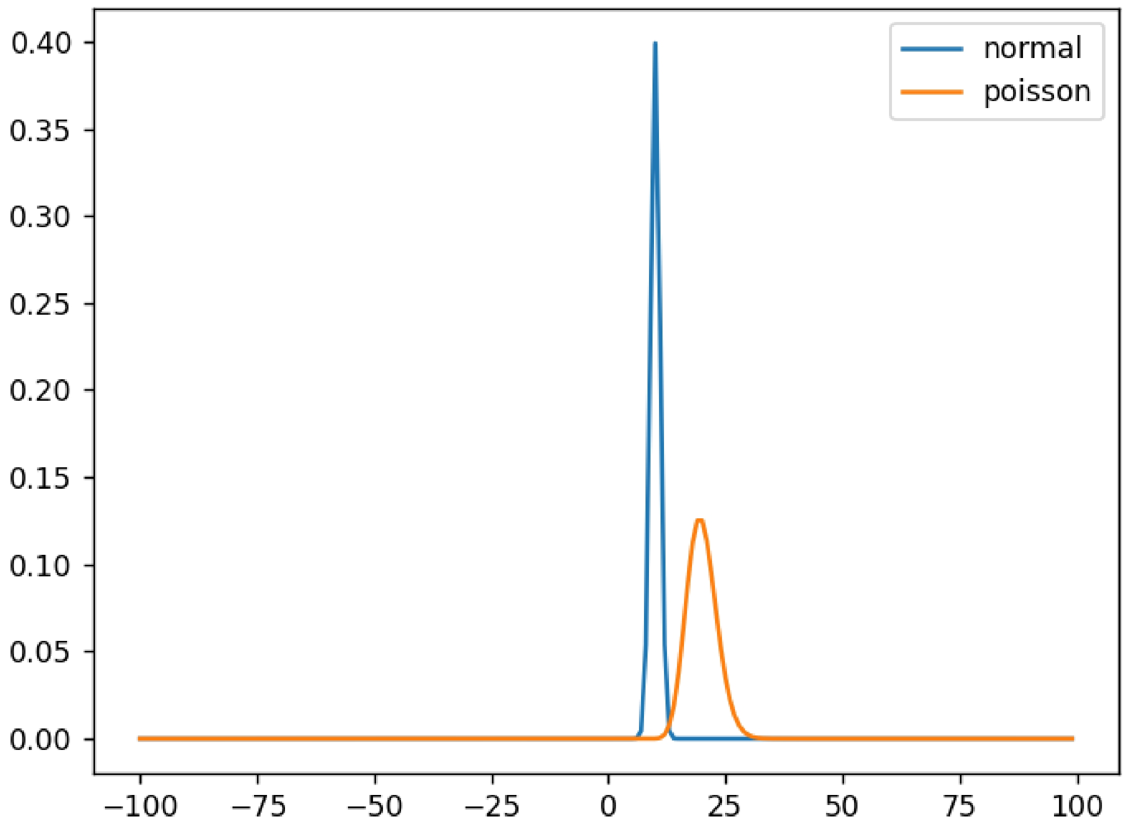
∴ Hence it is proved

3.

Code:

```
import matplotlib.pyplot as plt
import numpy as np
import scipy.stats as st
from scipy.stats import poisson

mu = 10
variance = 1

#normal
x = np.arange(-100, 100, 1)
y=st.norm.pdf(x, mu)
plt.plot(x, y)

#poisson
y = poisson.pmf(x, mu, loc=10)
plt.plot(x, y)
plt.legend(['normal','poisson'],loc="upper right")

plt.show()
```

Output:



4).

$$Var = E\left[(x-\mu)^2\right]$$

$$= \sum_{-\infty}^{\infty} (x-\mu)^2 \, p(x)$$

$$= \sum_{-\infty}^{\infty} (x^2 - 2\mu x + \mu^2) \, p(x)$$

$$= \sum_{-\infty}^{\infty} x^2 p(x) - 2\mu x p(x) + \mu^2 p(x)$$

$$= \sum_{-\infty}^{\infty} x^2 p(x) - 2\mu \sum_{-\infty}^{\infty} x p(x) + \mu^2 \sum p(x)$$

$$= E(x^2) - 2\mu E(x) + \mu^2 \cdot 1$$

~~Var = E(~~ w.r.t $\mu = E(x)$

$$= E(x^2) - 2E(x)E(x) + [E(x)]^2$$

$$= E(x^2) - 2[E(x)]^2 + [E(x)]^2$$

$$\boxed{Var = E(x^2) - [E(x)]^2}$$

$\therefore$ Hence it is proved

5) $X \sim N(\mu, \sigma^2)$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Mean

Mean $= E(X) = \int_{-\infty}^{\infty} x \, f(x)\, dx$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} x \, e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

Let $z = \frac{x-\mu}{\sigma}$

$\Rightarrow x = \sigma z + \mu$

$dx = \sigma dz$

$$E(X) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} (\sigma z + \mu) e^{-\frac{z^2}{2}} \times \sigma dz$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \sigma z \, e^{-\frac{z^2}{2}} dz + \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mu \, e^{-\frac{z^2}{2}} dp$$

Since $\frac{\sigma}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z \, e^{-\frac{z^2}{2}}$ is an odd function, we can make equal to 0

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mu \, e^{-\frac{z^2}{2}} dp$$

Let $V^2 = \frac{p^2}{2} \Rightarrow V = \frac{p}{\sqrt{2}} \Rightarrow$

$dp = \sqrt{2}\, dV$

$$= \frac{\mu}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-V^2} \sqrt{2}\, dV$$

$$= \frac{\mu}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-V^2} dV$$

By Gaussian Integral $\int e^{-u^2} du = \sqrt{\pi}$

$$= \frac{\mu}{\sqrt{\pi}} \times \sqrt{\pi}$$

$$\boxed{E(x) = \mu}$$

∴ Hence proved

Variance

$Var(x) = E(x-\mu)^2 = \cancel{\frac{1}{\sigma}\int}$

$$E(x^2) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} \cancel{(x-\mu)}\, x^2\, e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

Let $p = \frac{x-\mu}{\sigma} \Rightarrow \mu + \sigma p = x$

$\sigma\, dp = dx$

$$x = \sigma p + \mu \Rightarrow x^2 = (\sigma p + \mu)^2$$

$$= \sigma p^2 + 2\sigma p \mu + \mu^2$$

$$\frac{dp}{dx} = \frac{1}{\sigma} \Rightarrow dx = \sigma\, dp$$

$$\therefore E(x^2) = \frac{\sigma^2}{\sqrt{2\pi}} \int_{-\infty}^{\infty} p^2 e^{-\frac{p^2}{2}}\, dp \longrightarrow \sqrt{2\pi}$$

$$+ \frac{2\sigma\mu}{\sqrt{2\pi}} \int_{-\infty}^{\infty} p\, e^{-\frac{p^2}{2}}\, dp \longrightarrow 0 \; + \cancel{\mu}^2$$

$$\frac{\mu^2}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{p^2}{2}}\, dy \longrightarrow \sqrt{2\pi}$$

$$\therefore = \frac{\sigma^2}{\sqrt{2\pi}} \times \sqrt{2\pi} + 0 + \frac{\mu^2}{\sqrt{2\pi}} \times \sqrt{2\pi}$$

$$E(x^2) = \sigma^2 + \mu^2$$

$$Var(x) = E(x^2) - [E(x)]^2$$

$$= \sigma^2 + \mu^2 - \mu^2 = \sigma^2$$

$$\boxed{Var(x) = \sigma^2}$$

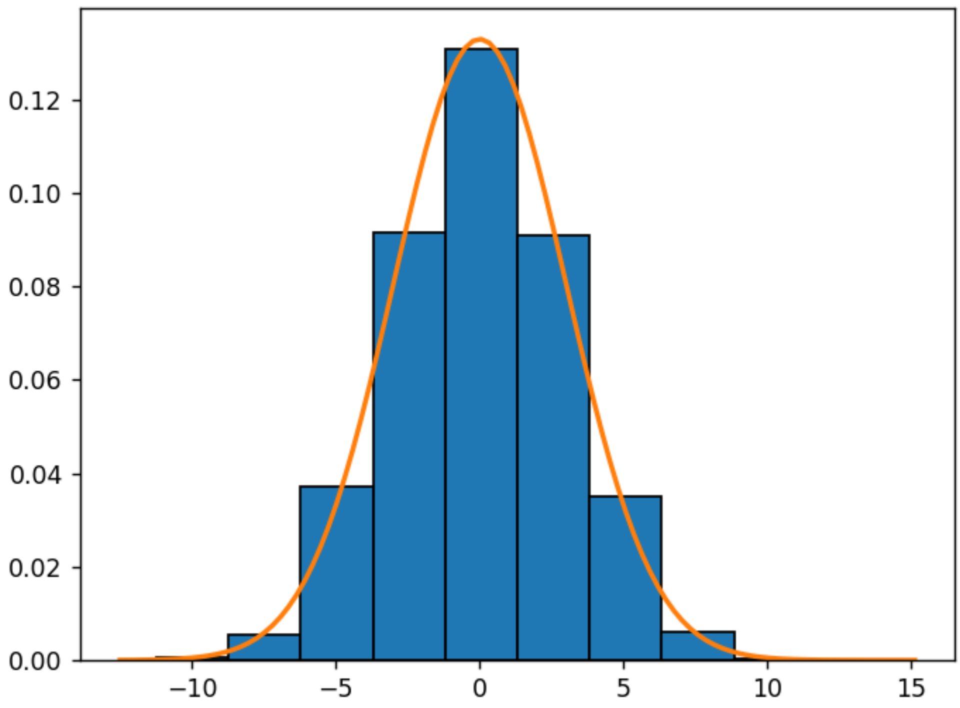
$\therefore$ Hence proved

6.

Code: (normal distribution)

```
import random
import numpy as np
import matplotlib.pyplot as plt
from statistics import NormalDist

#Normal distribution
from scipy.stats import norm

mean = 0
sigma= 3

rdn_nos = []
for i in range(10000):
    rdn_nos.append(random.random())

list = []
for i in rdn_nos:
    normInv = NormalDist(mean, sigma).inv_cdf(i)
    list.append(normInv)
plt.hist(list, bins=10, edgecolor='black',density=True)
xmin, xmax = plt.xlim()
x = np.linspace(xmin, xmax, 100)
p = norm.pdf(x, mean, sigma)
plt.plot(x, p, linewidth=2)
plt.show()
```

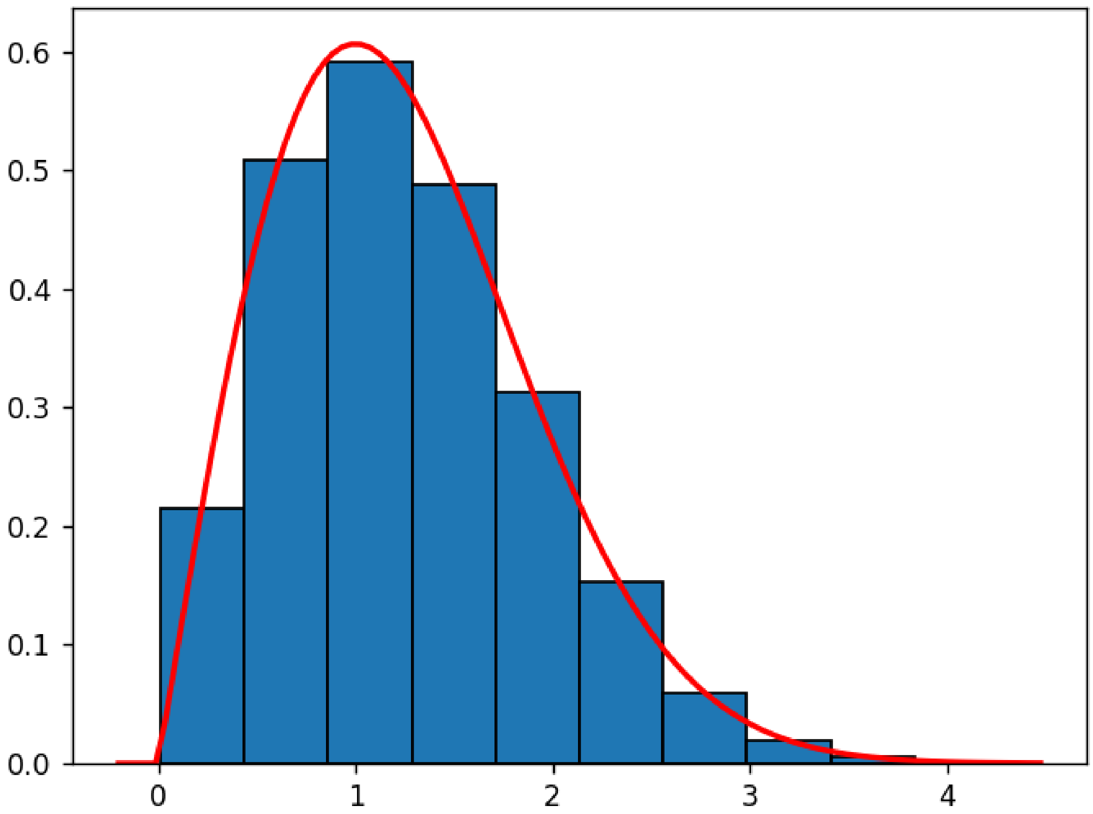
Output:



Code: Rayleigh distribution

```
import matplotlib.pyplot as plt
import random
import numpy as np
from scipy.stats import rayleigh

rdn_nos = [random.uniform(0, 1) for i in range(10000)]

x = rayleigh.ppf(rdn_nos, loc=0, scale=1)
plt.hist(x, bins=10, edgecolor='black', density=True)

xmin, xmax = plt.xlim()
x = np.linspace(xmin, xmax, 100)
pdf = rayleigh.pdf(x, loc=0, scale=1)
plt.plot(x, pdf, linewidth=2, color='red')
plt.show()
```

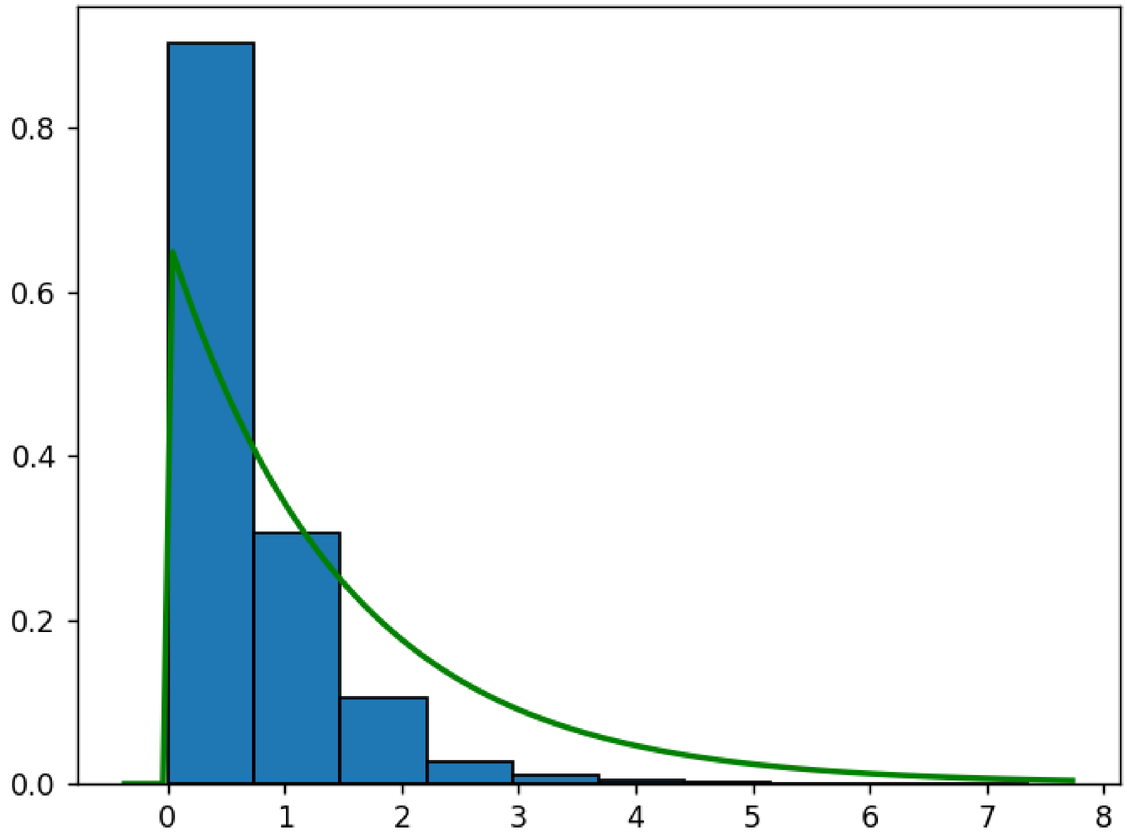
Output:



Code: (Exponential distribution)

```
import random
import math
import matplotlib.pyplot as plt
import numpy as np
import scipy.stats as st


# Exponential distribution

rand = []
for i in range(10000):
    rand.append(random.random())

expInvCDF = []
for i in rand:
    ln = (-(math.log(1 - i))) / 1.5
    expInvCDF.append(ln)
plt.hist(expInvCDF, bins=10, edgecolor='black',density=True)
xmin, xmax = plt.xlim()
x = np.linspace(xmin, xmax, 100)
exponPDF=st.expon.pdf(x,0,1.5)
plt.plot(x, exponPDF, linewidth=2,color='green')
plt.show()
```

Output:



7.

Code:

```
import matplotlib.pyplot as plt
import random
import math
import matplotlib.pyplot as plt

list = []
total = []
sum = 0
count = 0
for count in range(100):
    for i in range(500):
        sum = sum + random.random()
    total.append(sum)
    sum = 0
print(total)

max = max(total)
min = min(total)
binSize = 1
bins = (max - min) / binSize
binNew = math.floor(bins)
print(binNew)
plt.hist(total, bins=binNew, edgecolor='black', density=True)
plt.show()
```

output: