1(a)

Step1: Probability of selling computer A $= P(A) = 0.6$
Probability of selling computer B $= P(B) = 0.4$

Step2: Probability of selling either one of computer $P(A \cup B)$

$= P(A) + P(B) - P(A) \times P(B) = 0.6 + 0.4 - 0.6 \times 0.4$

or $= 1 - P(\bar{A}) \times P(\bar{B})$

$= 0.76$

Step3: $P(A \mid (A \cup B)) = \dfrac{P(A \cap (A \cup B))}{P(A \cup B)}$

$= \dfrac{P(A)}{0.76} = \dfrac{0.60}{0.76} = 0.79$

1(b)

(i) Suppose that an event 'A' can occur if and only if one of the set of mutually exclusive events $B_1, B_2, \ldots B_K$ occurs. Given the apriori probability $P(B_1), P(B_2) \ldots P(B_K)$ and the conditional probability $P(A|B_1), P(A|B_2) \ldots P(A/B_K)$ the posteriori probability

$$P(B_i|A) = \frac{P(B_i)\,P(A|B_i)}{\sum_{j=1}^{K} P(B_j)\,P(A|B_j)}$$

1 Marks

(iii)

Let the number of items produces by

$X \rightarrow 3n$

$Y \rightarrow n$

$Z \rightarrow n$

$E_1 \rightarrow$ produce by factory X

$E_2 \rightarrow$ produce by factory Y

$E_3 \rightarrow$ produce by factory Z

$P(E_1) = \frac{3n}{3n+n+n} = 0.6 \quad P(E_2) = \frac{n}{5n} = 0.2$

$P(E_3) = \frac{n}{5n} = 0.2$

Also $P(A|E_1) = P(A|E_3) = 0.03$

$P(A|E_2) = 0.05$

$P(A) = \sum_{i=1}^{3} P(A \cap E_i) = \sum_{i=1}^{3} P(E_i)P(A|E_i)$

$= 0.6 \times 0.03 + 0.2 \times 0.05 + 0.2 \times 0.03$

$= 0.034$

2 Marks.

Using Bayes' Rule

$$P(E_1|A) = \frac{P(E_1)\,P(A|E_1)}{P(A)} = \frac{0.6 \times 0.03}{0.034}$$

$$= \frac{9}{17}$$

1 Marks

$$P(E_2|A) = \frac{0.2 \times 0.05}{0.034} = \frac{5}{17}$$

1 Marks

$$P(E_3|A) = \frac{0.2 \times 0.03}{34} = \frac{3}{17}$$

1 Marks

2(a)

A continous random variable X is said to follow uniform distribution if its pdf is given by:

$$f(x) = \frac{1}{b-a}, \quad a < x < b$$
$$= 0, \quad \text{elsewhere.}$$

It is also called rectangular distribution or uniform distribution

[2]

Expectation

$$E(X) = \int_{-\infty}^{\infty} x f(x)\, dx = \int_a^b x \frac{1}{b-a}\, dx$$

$$= \frac{1}{b-a}\left[\frac{x^2}{2}\right]_a^b$$

$$= \frac{1}{2(b-a)}(b^2 - a^2) = \frac{a+b}{2}$$

[2]

Variance

$$Var = E(x^2) - (E(x))^2$$

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx$$

$$= \int_a^b x^2 \frac{1}{b-a}\, dx$$

$$= \frac{1}{b-a}\left[\frac{x^3}{3}\right]_a^b = \frac{b^3 - a^3}{3(b-a)}$$

$$= \frac{a^2 + b^2 + ab}{3}$$

$$Var = \frac{a^2 + b^2 + ab}{3} - \left(\frac{a+b}{2}\right)^2$$

$$= \frac{(b-a)^2}{12}$$

$\equiv 2$

2(b)

$$f(x) = \begin{cases} e^{-x}, & x \geq 0 \\ 0, & \text{elsewhere} \end{cases}$$

$$E(X) = \int_0^{\infty} x e^{-x} \, dx$$

$$= -\left[\frac{x}{e^x} + e^{-x}\right]_0^{\infty} = 1$$

$\Rightarrow$ 1 Mar

Markov's inequality suggests that

$$P(x \geq a) \leq \frac{E(x)}{a}$$

$$\Rightarrow \int_a^{\infty} f(x) \, dx \leq \frac{1}{a}$$

$$\Rightarrow \int_a^{\infty} e^{-x} \, dx \leq \frac{1}{a}$$

$$\Rightarrow e^{-a} \leq \frac{1}{a}$$ — 1 Mark

$$\Rightarrow \frac{1}{e^a} \leq \frac{1}{a} \Rightarrow a \leq e^a$$

1 Mark

$\Rightarrow \ln a \leq a$ is true for $\forall a > 0$

3(a)

Exponential distribution is define as

$$f(x) = \lambda e^{-\lambda x}, \quad x \geq 0, \lambda > 0$$
$$= 0, \quad \text{otherwise.}$$

1 Marks.

Moment Generating function is define as.

$$M(t) = E(e^{tx}) = \int_0^\infty e^{tx} \lambda . e^{-\lambda x} dx$$

$$= \lambda \int_0^\infty e^{(t-\lambda)x} dx$$

1 Marks

$$= \lambda \left[\frac{e^{(t-\lambda)x}}{t-\lambda} \right]_0^\infty$$

$$= \frac{\lambda}{\lambda - t} = \left(1 - \frac{t}{\lambda}\right)^{-1}$$

1 Marks

3(b)

Given: $\mu = 3750$

$\sigma = 500$

$x = 4500$

$$z = \frac{x-\mu}{\sigma} = \frac{4500-3750}{500}$$

$$= \frac{3}{2} = 1.5$$ 1 Marks

Percentage of posts survive up to $z = 1.5$

$= 1 - P(z \le 1.5)$

$= P(z > 1.5)$

Due to symmetry of normal curve

$P(z > 1.5) = P(z < -1.5)$

2 Marks

From the given value of standard Normal value

$P(z > 1.5) = 0.0668$ 1 Marks

$\Rightarrow$ 6.68 % posts will survive.

3(C)

The exhaustive numbers of cases for the birthday of n persons is $\underline{\underline{365^n}} \rightarrow C$

The number of favourable cases that the birthday of all n persons fall on different days

$$C_1 = 365(365-1)\ldots[365-(n-1)]$$

1 Marks

the probability (P) that birthdays of all the n persons are different is

$$P = \frac{C1}{C}$$

Hence the required probability is

$$1-P = 1-\left(1-\frac{1}{365}\right)\left(1-\frac{2}{365}\right)\cdots\left(1-\frac{n-1}{365}\right)$$

1 Marks.

Q4(a) Fit the curve $y = ax^b$ for the following data

x:	1	2	3	4	5	6
y:	1200	900	600	200	110	50

Solution Given curve is $y = ax^b$

$\log_e y = \log_e a + b \log_e x$ — (01)

i.e. $z = \alpha_1 + \alpha_2 t$

we have $\sum z_i = n\cdot\alpha_1 + \alpha_2 \sum t_i$

$\sum z_i t_i = \alpha_1 \sum t_i + \alpha_2 \sum t_i^2$ (A) (02)

as $t = \log_e x$; & $z = \log_e y$;

x	1	2	3	4	5	6
y	1200	900	600	200	110	50
$\log x$	log 1	log 2	log 3	log 4	log 5	log 6
$\log y$	log 1200	log 900	log 600	log 200	log 110	log 50

we have (A) as. $6\alpha_1 + 6.5793\,\alpha_2 = 34.2002$ (02)

& $6.5793\,\alpha_1 + 9.4099\,\alpha_2 = 33.6624$

Solving these we have $\alpha_1 = 7.6180$; $\alpha_2 = -1.7491$ 01

So $b = -1.7491$; $a = \text{anti}\log(7.6180)$

$= 2034.4891$

$\Rightarrow$ $\boxed{y = 2034.4891 \cdot x^{-1.7491}}$ Ans (01)

Q4(b) As per chebychev's inequality $P\{|X-\mu| \geq c\} \leq \frac{\sigma^2}{c^2}$

or $P\{|X-\mu| < c\} \geq 1 - \frac{\sigma^2}{c^2}$; (01)

Here $P\{|X-\mu_x| < 2\sigma\}$ will be $\geq 1 - \frac{\cancel{\sigma^2}}{4\cancel{\sigma^2}} = 0.75$

since lower bound is 0.75. so there DOES NOT EXIST a r.v. X for which $P\{\mu_x - 2\sigma \leq X \leq \mu_x + 2\sigma\} = 0.6$ holds. (01)

~~One More Method~~ way

Q4(a) Fit the curve $y = ax^b$ for following data

x:	1	2	3	4	5	6
y:	1200	900	600	200	110	50

Solution:- Given curve is $y = ax^b$

If we take log $\log_{10} y = \log_{10} a + b \log_{10} x$. (1)

$z = \alpha_1 + \alpha_2 t$

Now

x	y	$t = \log_{10} x$	$z = \log_{10} y$	t_i^2	$z_i t_i$
1	1200	$\log_{10} 1$	$\log_{10} 1200$		
2	900	$\log_{10} 2$	$\log_{10} 900$		
3	600	$\log_{10} 3$	$\log_{10} 600$		
4	200	$\log_{10} 4$	$\log_{10} 200$		
5	110	$\log_{10} 5$	$\log_{10} 110$		
6	50	$\log_{10} 6$	$\log_{10} 50$		
		$\Sigma t_i = 2.8573325$	$\Sigma z_i = 14.852968$	$\Sigma t_i^2 = 1.7748184$	$\Sigma z_i t_i = 6.349116$

Normal eqn are

$\Sigma z_i = n\alpha_1 + \alpha_2 \Sigma t_i$

$\Sigma z_i t_i = \alpha_1 \Sigma t_i + \alpha_2 \Sigma t_i^2$

02

02

our Normal eqn are

$$6\alpha_1 + 2.8573325\,\alpha_2 = 14.852968 \quad (1)$$

$$2.8573325\,\alpha_1 + 1.7748184\,\alpha_2 = 6.349116 \quad (2)$$

Solving them

$$6 \times 2.8573325\,\alpha_1 + 8.164349\,\alpha_2 = 42.439868$$

$$6 \times 2.8573325\,\alpha_1 + 10.64891\,\alpha_2 = 38.0946996$$

$$-2.484561\,\alpha_2 = 4.3451684$$

$$\alpha_2 = -1.74887$$

01

Substituting in (1) $\alpha_1 = 3.3083452$; $\alpha_1 = \log_{10} a \Rightarrow a = 10^{3.3083452}$

$= 2033.97$

So our eqn is

$\Rightarrow y = 2033.97\, x^{-1.7489}$ 01 Ans

Solution (1)

Q.5 a/ $Y = 1/X$

$G(y) = P(Y \le y)$

$= P\left(\frac{1}{X} \le y\right)$

$= P\left(X \ge \frac{1}{y}\right)$

$\int_{1/y}^{1} 1 \cdot dx = [x]_{1/y}^{1} = 1 - \frac{1}{y}$ → (2.5 mark)

$g(y) = G'(y) = \frac{d}{dy}\left(1 - \frac{1}{y}\right)$

$= 0 + \frac{1}{y^2} = \frac{1}{y^2} \quad \therefore y > 1$ → (2.5 mark)

Q.5b. The moment about mean

$\mu_1 = 0$ —— 1 marks

$\mu_2 = \mu_2' - \mu_1'^2$

$= 17 - (-1.5)^2 = 14.75$ —— 1 marks

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$ —— 1 marks

$= 39.75$

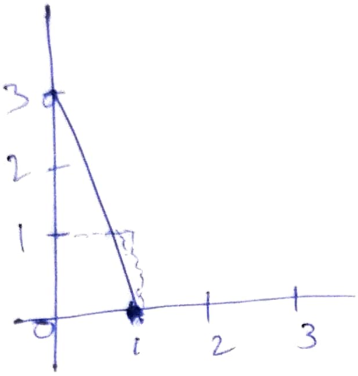
$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$

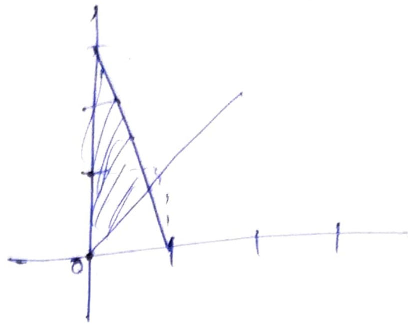
$= 142.3125$ —— 1 marks

Pg 2

Q.6

i)





Intersection Point $y = x$ & $x + 3y = 3$

$x = \frac{3}{4}$ & $y = 3/4$

$$P(X<Y) = \int_0^{3/4} \left[\int_x^{3-3x} \frac{x+y}{2}\, dy \right] dx$$

$$= \int_0^{3/4} \left(\frac{9}{4} - 3x \right) dx = \left[\frac{9}{4}x - \frac{3x^2}{2} \right]_0^{3/4}$$

$$= \left[\frac{9}{4} \times \frac{3}{4} - 3 \times \frac{9}{32} \right]$$

$$= \frac{27}{16} - \frac{27}{32} = \frac{27}{32}$$

03 Marks.

Q.6 ii) (Pg 3)

$$g(x) = \int_0^{3-3x} \frac{x+y}{2}\,dy$$

$$= \frac{1}{2}\left[xy + \frac{y^2}{2}\right]_0^{3-3x}$$

$$= \frac{1}{2}\left[3x - 3x^2 + \frac{(3-3x)^2}{2}\right]$$

$$= \frac{1}{2}\left[3x - 3x^2 + \frac{9 + 9x^2 - 18x}{2}\right]$$

$$= \frac{1}{2}\left[\frac{6x - 6x^2 + 9 + 9x^2 - 18x}{2}\right]$$

$$= \frac{1}{4}\left[3x^2 - 12x + 9\right]$$

$$= \frac{9}{4} - 3x + \frac{3}{4}x^2 ; 0 < x < 1$$

1.5 Marks

Q.6 iii) $h(y) = \int_0^{1-y/3} \frac{x+y}{2}\,dx = \frac{1}{2}\left[\frac{x^2}{2} + yx\right]_0^{1-y/3}$

$$= \cancel{\frac{1}{2}\left[1 + \frac{y^2}{9} - \frac{2y}{3} + 2y - \frac{2y}{3}\right]}$$

$$= \frac{1}{4} + \frac{1}{3}y - \frac{5}{36}y^2 \quad 0 < y < 3$$

1.5 Marks

6. iv) $f_{XY}(x,y) \neq g(x) * h(y) \Rightarrow$ not independent. → 1 mark

3 Marks

$$E(X) = \int_{-\infty}^{\infty} x \cdot g(x)\,dx = \int_0^1 x \cdot \left(\frac{9}{4} - 3x + \frac{3}{4}x^2\right) dx$$

$$= \int_0^1 \left(\frac{9}{4}x - 3x^2 + \frac{3}{4}x^3\right) dx = \frac{9}{8} - 1 + \frac{3}{16}$$

$$= 5/16$$

$$E(Y) = \int_{-\infty}^{\infty} y \cdot h(y)\,dy = \int_0^3 y\left(\frac{1}{4} + \frac{1}{3}y - \frac{5}{36}y^2\right) dy$$

$$= \int_0^3 \left(\frac{1}{4}y + \frac{1}{3}y^2 - \frac{5}{36}y^3\right) dy = 21/16. \Rightarrow$$

$$E(XY) = \int_0^1 \left(\int_0^{3-3x} xy \cdot \frac{x+y}{2}\,dy\right) dx = \int_0^1 \left(\frac{x^2}{4}(3-3x)^2 + \frac{x}{6}(3-3x)^3\right) dx$$

$$= \int_0^1 \left(\frac{9}{2}x - \frac{45}{4}x^2 + 9x^3 - \frac{9}{4}x^4\right) dx$$

$$= \frac{9}{4} - \frac{15}{4} + \frac{9}{4} - \frac{9}{20} = \frac{6}{20} = 3/10$$

$$\mathrm{Cov}(XY) = E(XY) - E(X) * E(Y) = \frac{3}{10} - \frac{5}{16} \times \frac{21}{16} = -\frac{141}{1280}$$

$$= -0.11016$$

Pg 5

Q.7 a) $\bar{x} = \frac{1614}{30} = 53.8$ } 2 marks

$P\left(\bar{x} - \frac{2\sigma}{\sqrt{n}} < \mu < \bar{x} + 2\sigma/\sqrt{n}\right) = 0.9544$

$= P\left(53.8 - \frac{30}{\sqrt{30}} < \mu < 53.8 + \frac{30}{\sqrt{3}}\right) = 0.9544$

$= P\left(48.322, 59.277\right) \Rightarrow 95.44$ Confidence. } 3 marks

Q.7 b Definition of Sample & Sample mean - 2 Marks.

$E(\bar{x}) = E\left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)$

$= \frac{1}{n} E\left(\sum_{i=1}^{n} x_i\right)$

$= \frac{1}{n}\{E(x_1) + E(x_2) + \cdots E(x_n)\}$

$= \frac{1}{n}\sum_{i=1}^{n} E(x_i)$

$= \frac{1}{n}\sum_{i=1}^{n} \mu = \frac{1}{\cancel{n}} \times \cancel{n}\mu$

$= \mu$

3 marks

$\therefore$ Sample mean $\bar{x}$ is an unbiased Statistic of the Population mean μ