

# Algorithm complexity (Big O notation)

## Algorithm Complexity (Big O Notation) — Explained in Detail

### ◆ What Is Algorithm Complexity?

Algorithm complexity refers to how the **performance** of an algorithm changes relative to the size of the input data. It's measured in terms of:

- **Time Complexity:** How long it takes to run
- **Space Complexity:** How much memory it uses

We use **Big O Notation** to describe **worst-case** performance (typically), helping you reason about **scalability**.

### ◆ Why Big O?

Big O notation helps answer:

| "If my input grows from 10 items to 10,000, how will performance scale?"

It **abstracts away hardware** and **implementation details** to focus on the **growth rate**.

### ◆ Common Time Complexities

Notation	Name	Example	Growth Rate
$O(1)$	Constant	Accessing array element	🟢 Best performance
$O(\log n)$	Logarithmic	Binary search	⚡ Fast on large input
$O(n)$	Linear	Loop through array	📈 Scales with size
$O(n \log n)$	Linearithmic	Merge Sort, Quick Sort avg	🔄 Efficient sorting
$O(n^2)$	Quadratic	Nested loops (e.g. bubble sort)	🐌 Slows fast on large $n$

$O(2^n)$	Exponential	Recursive Fibonacci	💣 Explodes quickly
$O(n!)$	Factorial	Permutations	🔥 Very expensive

## 📌 Examples

### 1. $O(1)$ — Constant Time

```
function getFirst(arr) {
  return arr[0];
}
```

No matter how big the array is, it always does **one operation**.

### 2. $O(n)$ — Linear Time

```
function sum(arr) {
  let total = 0;
  for (let num of arr) {
    total += num;
  }
  return total;
}
```

Performs one operation for each element.

### 3. $O(n^2)$ — Quadratic Time

```
function printPairs(arr) {
  for (let i = 0; i < arr.length; i++) {
    for (let j = 0; j < arr.length; j++) {
      console.log(arr[i], arr[j]);
    }
  }
}
```

Nested loop causes exponential growth.

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## 4. $O(\log n)$ — Logarithmic Time

```
function binarySearch(arr, target) {  
  let low = 0, high = arr.length - 1;  
  while (low <= high) {  
    let mid = Math.floor((low + high) / 2);  
    if (arr[mid] === target) return mid;  
    else if (arr[mid] < target) low = mid + 1;  
    else high = mid - 1;  
  }  
  return -1;  
}
```

Reduces the search space by half each time.

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## ◆ Space Complexity

Describes how much **additional memory** is required:

```
function makeDouble(arr) {  
  let result = [];  
  for (let num of arr) {  
    result.push(num * 2);  
  }  
  return result;  
}  
// Space:  $O(n)$  (creates a new array)
```

If the function modifies in place (e.g., sorts in-place), space complexity can be  $O(1)$ .

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## ◆ Visual Summary

Best → Worst:

$O(1) < O(\log n) < O(n) < O(n \log n) < O(n^2) < O(2^n) < O(n!)$

## ◆ Tips to Optimize Big O

- **Avoid nested loops** when possible
- Use **hash maps/objects** for faster lookup (  $O(1)$  access)
- Choose the right **sorting/search algorithms**
- Use **memoization** or **dynamic programming** to avoid redundant work

## 🧠 Use Cases in Real Projects

Scenario	Desired Complexity
Search in sorted array	$O(\log n)$ (binary search)
Filter/transform data	$O(n)$
Sort items for display	$O(n \log n)$
Compare all user pairs	$O(n^2)$ (avoid if large)
Generate combinations	$O(2^n)$ or worse



## Summary

- **Big O** helps you reason about algorithm **scalability**
- It describes how performance **grows with input size**
- Choose algorithms with **lower complexity** for large data
- Understand **trade-offs** (e.g., speed vs. memory)

Would you like a **cheat sheet image** or **interactive examples** to better visualize Big O complexities?