

Algorithm complexity (Big O notation)

Algorithm Complexity (Big O Notation) — Explained in Detail

◆ What Is Algorithm Complexity?

Algorithm complexity refers to how the **performance** of an algorithm changes relative to the size of the input data. It's measured in terms of:

- **Time Complexity**: How long it takes to run
- **Space Complexity**: How much memory it uses

We use **Big O Notation** to describe **worst-case** performance (typically), helping you reason about **scalability**.

◆ Why Big O?

Big O notation helps answer:

"If my input grows from 10 items to 10,000, how will performance scale?"

It **abstracts away hardware** and **implementation details** to focus on the **growth rate**.

◆ Common Time Complexities

Notation	Name	Example	Growth Rate
$O(1)$	Constant	Accessing array element	 Best performance
$O(\log n)$	Logarithmic	Binary search	 Fast on large input
$O(n)$	Linear	Loop through array	 Scales with size
$O(n \log n)$	Linearithmic	Merge Sort, Quick Sort avg	 Efficient sorting
$O(n^2)$	Quadratic	Nested loops (e.g. bubble sort)	 Slows fast on large n

$O(2^n)$	Exponential	Recursive Fibonacci	 Explodes quickly
$O(n!)$	Factorial	Permutations	 Very expensive

📌 Examples

1. $O(1)$ — Constant Time

```
function getFirst(arr) {
  return arr[0];
}
```

No matter how big the array is, it always does **one operation**.

2. $O(n)$ — Linear Time

```
function sum(arr) {
  let total = 0;
  for (let num of arr) {
    total += num;
  }
  return total;
}
```

Performs one operation for each element.

3. $O(n^2)$ — Quadratic Time

```
function printPairs(arr) {
  for (let i = 0; i < arr.length; i++) {
    for (let j = 0; j < arr.length; j++) {
      console.log(arr[i], arr[j]);
    }
  }
}
```

Nested loop causes exponential growth.

4. $O(\log n)$ — Logarithmic Time

```
function binarySearch(arr, target) {  
    let low = 0, high = arr.length - 1;  
    while (low <= high) {  
        let mid = Math.floor((low + high) / 2);  
        if (arr[mid] === target) return mid;  
        else if (arr[mid] < target) low = mid + 1;  
        else high = mid - 1;  
    }  
    return -1;  
}
```

Reduces the search space by half each time.

◆ Space Complexity

Describes how much **additional memory** is required:

```
function makeDouble(arr) {  
    let result = [];  
    for (let num of arr) {  
        result.push(num * 2);  
    }  
    return result;  
}  
// Space:  $O(n)$  (creates a new array)
```

If the function modifies in place (e.g., sorts in-place), space complexity can be $O(1)$.

◆ Visual Summary

Best → Worst:

$O(1) < O(\log n) < O(n) < O(n \log n) < O(n^2) < O(2^n) < O(n!)$

◆ Tips to Optimize Big O

- **Avoid nested loops** when possible
- Use **hash maps/objects** for faster lookup ($O(1)$ access)
- Choose the right **sorting/search algorithms**
- Use **memoization** or **dynamic programming** to avoid redundant work

🧠 Use Cases in Real Projects

Scenario	Desired Complexity
Search in sorted array	$O(\log n)$ (binary search)
Filter/transform data	$O(n)$
Sort items for display	$O(n \log n)$
Compare all user pairs	$O(n^2)$ (avoid if large)
Generate combinations	$O(2^n)$ or worse



Summary

- **Big O** helps you reason about algorithm **scalability**
- It describes how performance **grows with input size**
- Choose algorithms with **lower complexity** for large data
- Understand **trade-offs** (e.g., speed vs. memory)

Would you like a **cheat sheet image** or **interactive examples** to better visualize Big O complexities?