Partial Differentiation

A differential equation containing partial derivatives of a function of two or more independent variables is called <u>Partial differential equation</u> (PDE).

$z = f(x, y)$ then $\frac{\partial z}{\partial x} = p$ and $\frac{\partial z}{\partial y} = q$

$\frac{\partial^2 z}{\partial x^2} = r$, $\frac{\partial^2 z}{\partial y^2} = t$, $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial q}{\partial x} = \frac{\partial p}{\partial y} = s$

Partial differential Equation can be formed either by elimination of arbitary constant or by the elimination of arbitary function from a relation involving three or more variables.

(I) Formation of Partial Differential Equation

(a) By elimination of arbitary constant

Let function be $\phi(x, y, z, a, b) = 0$ —— (1)

where a & b are constants.

differencialing eq (1) partially w.r.t x and y.

$$\frac{\partial \phi}{\partial x} + \left(\frac{\partial z}{\partial x}\right)\frac{\partial \phi}{\partial z} = 0$$

$$\frac{\partial \phi}{\partial x} + p\frac{\partial \phi}{\partial z} = 0 \quad \text{—— (2)} \qquad \frac{\partial \phi}{\partial y} + q\frac{\partial \phi}{\partial z} = 0 \quad \text{—— (3)}$$

eliminating a & b from eq (1), (2) and (3) we get partial differential equation of first order of the form $f(x, y, z, p, q) = 0$

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$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} + \frac{\partial^2 z}{\partial x \partial y} + Py = Q \rightarrow 2^{nd}$ order PDE

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Q1. find the PDE of all sphere where centre lie on z-axis and given by equation :-
$x^2+y^2+(z-a)^2 = b^2$ where a, b are constants

$$x^2+y^2+(z-a)^2 = b^2 \quad \text{—— (1)}$$

Partially differenciating w.r.t x

$$2x + 2(z-a)\frac{\partial z}{\partial x} = 0$$

$$\cancel{2}(z-a)\frac{\partial z}{\partial x} = -\cancel{2}x$$

$$(z-a)p = -x$$

$$\boxed{z-a = \frac{-x}{p}} \quad \text{—— (2)}$$

similarly paritally diff w.r.t y

$$2y + 2(z-a)\frac{\partial z}{\partial y} = 0$$

$$(z-a)q = -y$$

$$\boxed{(z-a) = \frac{-y}{q}} \quad \text{—— (3)}$$

by putting eq (2) in (3) we get

$$\frac{-x}{p} = \frac{-y}{q}$$

$$-xq = -yp$$

$$yp - xq = 0$$

$$\boxed{y\frac{\partial z}{\partial x} - x\frac{\partial z}{\partial y} = 0} \Rightarrow \text{Required PDE.}$$

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Q2. $(x-a)^2 + (y-b)^2 + z^2 = R^2$ —— ①

Partially D w.r.t x

$$2(x-a) + 2z\frac{\partial z}{\partial x} = 0$$

$$\cancel{2}(x-a) = -\cancel{2}z\frac{\partial z}{\partial x}$$

$$(x-a) = -zp$$

$$\boxed{(x-a) + zp = 0} \quad \text{—— ②}$$

Partially D wrt y

$$2(y-b) + 2z\frac{\partial z}{\partial y} = 0$$

$$(y-b) = -zq \rightarrow (y-b) = -zq$$

$$\boxed{(y-b) + zq = 0} \quad \text{—— ③}$$

substituting values in eq ①

$$(-zp)^2 + (-zq)^2 + z^2 = R^2$$

$$z^2p^2 + z^2q^2 + z^2 = R^2$$

$$z^2(q^2 + p^2 + 1) = R^2$$

$$\boxed{z^2 = \frac{R^2}{1+p^2+q^2}} \longrightarrow \text{Required PDE.}$$

$$z^2 = \frac{R^2}{1 + \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}}.$$

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(iii) $z = (x+a)(y+b)$ — (1)

diff w.r.t x

$$\frac{\partial z}{\partial x} = y+b$$

$$p = y+b \quad \text{— (2)}$$

diff w.r.t y

$$\frac{\partial z}{\partial y} = x+a$$

$$q = x+a \quad \text{— (3)}$$

substituting (2) & (3) in eq (1)

$$z = (x+a)(y+b)$$

$$\boxed{z = pq}$$

(iv) $az+b = a^2x+y$ — (1)

diff wrt x

$$a\frac{\partial z}{\partial x} = a^2$$

$$\frac{\partial z}{\partial x} = a$$

$$p = a \quad \text{— (2)}$$

diff wrt y

$$a\frac{\partial z}{\partial y} = 1$$

$$\frac{\partial z}{\partial y} = \frac{1}{a}$$

$$q = \frac{1}{a} \quad \text{— (3)}$$

from eq (2) & (3)

$$\boxed{q = \frac{1}{p}} \Rightarrow \boxed{pq = 1}$$

(v) $ax^2+bx^2+z^2 = 1$ — (1)

Partially diff wrt x

$$2xa + 2xb + 2z\frac{\partial z}{\partial x} = 0$$

$$\cancel{2}x(a+b) = -\cancel{2}zp$$

$$x(a+b) = -zp \quad \text{— (2)} \Rightarrow \boxed{a+b = \frac{-zp}{x}}$$

Partially diff wrt y

$$2z\frac{\partial z}{\partial y} = 0$$

$$q = 0$$

from eq (1) and (2) $\quad (a+b)x^2 + z^2 = 1$

$$-\frac{zp}{\cancel{x}} \times x^{\cancel{2}} + z^2 = 1$$

$$-zpx + z^2 = 1$$

$$\boxed{z^2 - zpx - 1 = 0}$$

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(b) By eliminating arbitrary function

Q1. $z = f(x^2 - y^2)$

$z = f(x^2 - y^2)$ —— ①

Partially D wrt x

$\frac{\partial z}{\partial x} = f'(x^2 - y^2) \times 2x$

$p = 2x \cdot f'(x^2 - y^2)$ —— ②

Partially diff wrt y

$\frac{\partial z}{\partial y} = f'(x^2 - y^2) \times 2y$

$q = -2y \cdot f'(x^2 - y^2)$ —— ③

from eq ② & ③ we get

$$\frac{p}{q} = \frac{2x \cdot \cancel{f'(x^2 - y^2)}}{-2y \cdot \cancel{f'(x^2 - y^2)}}$$

$$\frac{p}{q} = -\frac{x}{y} \Rightarrow \boxed{py + qx = 0}$$

Q2. $z = yf(x) + xg(y)$ —— ①

Partially diff wrt x & y

$\frac{\partial z}{\partial x} = yf'(x) + g(y)$ —— ②

$\frac{\partial z}{\partial y} = f(x) + xg'(y)$ —— ③

now Partialy diff eq ② wrt y

$\frac{\partial^2 z}{\partial x \partial y} = f'(x) + g'(y)$ —— ④

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now eq ② $\times x$ + ③ $\times y$

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = xy f'(x) + x g(y) + y f(x) + xy g'(y)$$

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = xy\left(f'(x) + g'(x)\right) + \left[x g(y) + y f(x)\right]$$

putting values from eq ① and ④

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = xy \times \frac{\partial^2 z}{\partial x \partial y} + z$$

$$xp + yq = xy \times s + z$$

$$\boxed{xp + yq - xys = z}$$

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Q. $f(x+y+z,\ x^2+y^2+z^2) = 0$

$f(u,v) = 0$ — ①

$u = x+y+z$, $v = x^2+y^2+z^2$ — ②

since u & v are the funtc of x, y & z
diff w.r.t x & y form

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial x} + \frac{\partial u}{\partial y}\cdot\underbrace{\frac{\partial y}{\partial x}}_{0} + \frac{\partial u}{\partial z}\cdot\frac{\partial z}{\partial x}$$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial x} + \frac{\partial u}{\partial z}\cdot p$$

$$\boxed{\frac{\partial u}{\partial x} = 1 + p} \text{ — ③}$$

as $u = x+y+z$
$\frac{\partial u}{\partial x} = 1,\ \frac{\partial u}{\partial y} = 1,\ \frac{\partial u}{\partial z} = 1$

and $\boxed{\frac{\partial u}{\partial y} = 1 + q}$ — ④

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for v :-

similarly

$$\frac{\partial v}{\partial xy} = \frac{\partial u}{\partial x}\underbrace{\frac{\partial x}{\partial xy}}_{0} + \frac{\partial v}{\partial y}\frac{\partial y}{\partial y} + \frac{\partial v}{\partial z}\frac{\partial z}{\partial y}$$

$$\frac{\partial v}{\partial xy} = \frac{\partial v}{\partial y} + \frac{\partial v}{\partial z}q$$

$$\frac{\partial v}{\partial xy} = (2y) + (2z)q$$

as $v = x^2+y^2+z^2$, $\frac{\partial v}{\partial x} = 2x$, $\frac{\partial v}{\partial y} = 2y$, $\frac{\partial v}{\partial z} = 2z$

$$\boxed{\frac{\partial v}{\partial xy} = 2y + 2zq} \quad \text{and} \quad \boxed{\frac{\partial v}{\partial x} = 2x + 2zp}$$

— (5) — (6)

And from $f(u,v) = 0$

$$0 = f_x = \frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial f}{\partial v}\cdot\frac{\partial v}{\partial x} \quad \text{— (7)}$$

$$0 = f_y = \frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial y} + \frac{\partial f}{\partial v}\cdot\frac{\partial v}{\partial y} \quad \text{— (8)}$$

Putting values (3), (4), (5), (6) in eq (7) & (8)

$$0 = f_x = \frac{\partial f}{\partial u}(1+p) + \frac{\partial f}{\partial v}(2x+2zp) \quad \text{— (9)}$$

$$f_y = 0 = \frac{\partial f}{\partial u}(1+q) + \frac{\partial f}{\partial v}(2y+2zq) \quad \text{— (10)}$$

Now eliminating $\frac{\partial f}{\partial u}$ & $\frac{\partial f}{\partial v}$ from eq (9) & (10) by Jacobian's method.

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$$\begin{vmatrix} (1+p) & (2x+2zp) \\ (1+q) & (2y+2zq) \end{vmatrix} = 0$$

$$(1+p)(2y+2zq) - (1+q)(2x+2zp) = 0$$

$$2zq + 2y + \cancel{2zq} + 2py + \cancel{2zpq} - 2x - \cancel{2zp} - 2xq - 2zp - \cancel{2zpq} = 0$$

$$\cancel{2y + 2x + 2py - 2xq = 0}$$

$$\cancel{2(y-x) +} \; 2y - 2x + 2zq + 2py - 2xq - 2zp = 0$$

$$2(y-x) + 2(zq + py - xq - zp) = 0$$

$$\cancel{2}(zq + py - xq - zp) = \cancel{2}(x-y)$$

$$\cancel{z(q+p) +} \; \boxed{(y-z)p + (z-x)q = x-y}$$

Q. $z = f(x+it) + g(x-it)$ —— ①

twice diff w.r.t x & t

$$\frac{\partial z}{\partial x} = f'(x+it) + g'(x-it)$$

$$\frac{\partial^2 z}{\partial x^2} = f''(x+it) + g''(x-it) \quad \text{—— ②}$$

$$\frac{\partial z}{\partial t} = f'(x+it) \times i + g'(x-it) \times -i$$

$$\frac{\partial^2 z}{\partial t^2} = i^2 f''(x+it) + i^2 g''(x-it)$$

putting $i^2 = -1$

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$$\frac{\partial^2 z}{\partial t^2} = -g''(x-it) - g''(x-it) \quad \text{--- (3)}$$

by adding eq (2) & (3)

$$\boxed{\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial t^2} = 0}$$

(II) <u>Different Integrals of Partial differential Equation</u>

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(III) Linear Partial Differential Equation (Laguange's Method)

Any PDE, which contains p, q & z of degree one is called linear PDE where $z = f(x, y)$ and $\frac{\partial z}{\partial x} = p$ & $\frac{\partial z}{\partial y} = q$.

Solution of linear PDE of first order equation in form $\boxed{Pp + Qq = R}$

where P, Q, R are function of x, y & z.

Now, Using auxilary Equation (Subsidiary eqn)

$$\boxed{\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}}$$

Solve the auxilary equation and two solution functions will obtain as :-

$$\boxed{u(x, y, z) = c_1 \text{ and } v(x, y, z) = c_2}$$

Then solution will be written as

$$\boxed{f(u, v) = 0 \quad \text{or} \quad u = f(v)}$$

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(a) <u>Direct method</u>.

Q. $(x^2-yz)p+(y^2-zx)q = z^2-xy$

$$Pp+Qq=R$$

$P = x^2-yz$; $Q = y^2-zx$; $R = z^2-xy$

auxiliary eqn $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$

$$\Rightarrow \frac{dx}{x^2-yz} = \frac{dy}{y^2-zx} = \frac{dz}{z^2-xy}$$

$$\frac{dx-dy}{x^2-y^2-yz+zx} = \frac{dy-dz}{y^2-zx-z^2+xy} = \frac{dz-dx}{z^2-xy-x^2+yz}$$

$$\frac{dx-dy}{x^2-y^2-z(y-x)} = \frac{dy-dz}{y^2-z^2+x(y-z)} = \frac{dz-dx}{z^2-x^2+y(z-x)}$$

$$\frac{dx-dy}{x^2-y^2+z(x-y)} = \frac{dy-dz}{y^2-z^2+x(y-z)} = \frac{dz-dx}{z^2-x^2+y(z-x)}$$

$$\frac{dx-dy}{(x-y)\cancel{(x+y+z)}} = \frac{dy-dz}{(y-z)\cancel{(x+y+z)}} = \frac{dz-dx}{(z-x)\cancel{(x+y+z)}}$$

$$\frac{dx-dy}{x-y} = \frac{dy-dz}{y-z} = \frac{dz-dx}{z-x}$$

Using first two fractions

$$\frac{dx-dy}{x-y} = \frac{dy-dz}{y-z}$$

integrating both sides

$$\int \frac{dz - dy}{x - y} = \int \frac{dy - dz}{y - z}$$

$$\log(x-y) = \log(y-z) + \log c_1$$

$$\log(x-y) - \log(y-z) = \log c_1$$

$$\log\left(\frac{x-y}{y-z}\right) = \log c_1$$

$$u = \boxed{\frac{x-y}{y-z} = c_1}$$

By using last two fractions

$$\int \frac{dy - dz}{y - z} = \int \frac{dz - dx}{z - x}$$

$$\log(y-z) = \log(z-x) + \log c_2$$

$$v = \boxed{\frac{y-z}{z-x} = c_2}$$

$\therefore f(u, v) = 0$

$$\boxed{f\left(\frac{x-y}{y-z}, \frac{y-z}{z-x}\right) = 0}$$

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(b) Multiplier's Method (if integration becomes difficult we use this method)

auxiliary eqn $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$ —— (1)

Now, taking (l, m, n) as multiplier, which may be constant or function of x, y & z.

$$\frac{dx}{P} \times \frac{l}{l} = \frac{dy}{Q} \times \frac{m}{m} = \frac{dz}{R} \times \frac{n}{n}$$

$$\boxed{\frac{l\,dx}{P} = \frac{dy}{Q} = \frac{dz}{R} = \frac{l\,dx + m\,dy + n\,dz}{lP + mQ + nR}} \quad \text{—— (2)}$$

(l, m, n) may be chosen as a way that

$$lP + mQ + nR = 0 \quad \text{—— (3)}$$

Then from (2) $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R} = \frac{l\,dx + m\,dy + n\,dz}{0}$

$\Rightarrow$ Now from (3), after Integrating (3) we get

$$\boxed{u = c_1} \quad \text{—— (4)}$$

Similarly choose other multiplier we get

$$\boxed{v = c_2} \quad \text{—— (5)}$$

Then Solution will be $\boxed{f(u, v) = 0}$

Q. Solve :- $x(y^2 + z)p - y(x^2 + z)q = z(x^2 - y^2)$ (1)

$P = x(y^2 + z)$; $Q = -y(x^2 + z)$; $R = z(x^2 - y^2)$

Now auxiliary eqn :-

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R} = \frac{l\,dx + m\,dy + n\,dz}{lP + mQ + nR}$$

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as $\dfrac{dx}{x(y^2+z)} = \dfrac{dy}{-y(x^2+z)} = \dfrac{dz}{z(x^2-y^2)}$ —— (2)

choose $(x, y, -1)$ as multiplier, then

$$\frac{dx}{x(y^2+z)} = \frac{dy}{-y(x^2+z)} = \frac{dz}{z(x^2-y^2)} = \frac{x\,dx + y\,dy - dz}{x^2y^2 + x^2z - y^2x^2 - y^2z - zx^2 + zy^2}$$

$$\frac{dx}{x(y^2+z)} = \frac{dy}{-y(x^2+z)} = \frac{dz}{z(x^2-y^2)} = \frac{x\,dx + y\,dy - dz}{0} \quad (3)$$

from (3) $\quad x\,dx + y\,dy - dz = 0$

now By Integration $\int x\,dx + \int y\,dy - \int dz = 0$

$$\frac{x^2}{2} + \frac{y^2}{2} - z = \frac{c_1}{2}$$

$$\therefore \boxed{x^2 + y^2 - 2z = c_1} \quad (4)$$

choose $\left(\frac{1}{x}, \frac{1}{y}, \frac{1}{z}\right)$ as other multiplier, then

$$\frac{dx}{x(y^2+z)} = \frac{dy}{-y(x^2+z)} = \frac{dz}{z(x^2-y^2)} = \frac{\frac{1}{x}dx + \frac{1}{y}dy + \frac{1}{z}dz}{y^2 + z - x^2 - z + x^2 - y^2}$$

$$= \frac{\frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z}}{0} \quad (5)$$

we get $\dfrac{dx}{x} + \dfrac{dy}{y} + \dfrac{dz}{z} = 0$

now By Integration $\int \dfrac{dx}{x} + \int \dfrac{dy}{y} + \int \dfrac{dz}{z} = 0$

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$$\log x + \log y + \log z = \log c_2$$

$$\log(xyz) = \log c_2$$

$$\therefore \; v = \boxed{xyz = c_2} \quad \text{—— (6)}$$

Solution will be $f(u, v) = 0$

$$\boxed{f\left(x^2+y^2-2z,\; xyz\right) = 0}$$

<u>Lagrange's Method</u>

Q. $y^2 p - xy\, q = x(z-2y)$

$$P = y^2 \;;\; Q = -xy \;;\; R = x(z-2y)$$

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

$$\frac{dx}{y^2} = \frac{dy}{-xy} = \frac{dz}{x(z-2y)}$$

from first two fractions we get

$$\frac{dx}{y^2} = \frac{dy}{-xy}$$

$$\int -xy\, dx = \int y^2 dy$$

$$\int y\, dy + \int x\, dx = 0$$

$$\frac{x^2}{2} + \frac{y^2}{2} = \frac{c_1}{2} \Rightarrow \boxed{u \Rightarrow x^2+y^2 = c_1}$$

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from last two fraction we get

$$\frac{dy}{-\cancel{x}y} = \frac{dz}{\cancel{x}(z-2y)}$$

$$(z-2y)\,dy = -y\,dz$$

$$z\,dy - 2y\,dy + y\,dz = 0$$

$$-2y\,dy + d(yz) = 0$$

$$2y\,dy - d(yz) = 0$$

$$\int 2y\,dy - \int dyz = 0$$

$$\frac{2y^2}{2} - yz = C_2$$

$$\boxed{v \Rightarrow y^2 - yz = C_2}$$

$$f(u,v) = 0$$

$$\boxed{f(x^2+y^2,\ y^2-yz) = 0}$$

Q. $\left(\frac{y^2z}{x}\right)p + xzq = y^2$

$$y^2zp + x^2zq = xy^2$$

$$P = y^2z \ ; \ Q = x^2z \ , \ R = xy^2$$

$$\frac{dx}{y^2z} = \frac{dy}{x^2z} = \frac{dz}{xy^2}$$

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from (I) & (II)

$$\frac{dx}{y^2\cancel{z}} = \frac{dy}{x^2\cancel{z}}$$

$$\int x^2 dx = \int y^2 dy$$

$$\frac{x^3}{3} = \frac{y^3}{3} + \frac{C_1}{3}$$

$$u \Rightarrow \boxed{x^3 - y^3 = C_1}$$

from (II) & (3)

$$\frac{dx}{y^2 z} = \frac{dz}{xy^2}$$

$$\frac{dx}{z} = \frac{dz}{x}$$

$$\int x dx = \int z dz$$

$$\frac{x^2}{2} = \frac{z^2}{2} + \frac{c_2}{2}$$

$$v \Rightarrow \boxed{x^2 - z^2 = C_2}$$

$$f(u, v) = 0$$

$$\boxed{f(x^3 - y^3, x^2 - z^2) = 0}$$

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Q. $(mz-ny)p+(nx-lz)q=(ly-mx)$

auxiliary equation

$$\frac{dx}{mz-ny}=\frac{dy}{nx-lz}=\frac{dz}{ly-mx}$$

choose multiplier (l, m, n)

$$\frac{dx}{mz-ny}=\frac{dy}{nx-lz}=\frac{dz}{ly-mx}=\frac{ldx+mdy+ndz}{lmz-lny+nmx-lmz+nly-mnx}$$

$$\frac{dx}{mz-ny}=\frac{dy}{nx-lz}=\frac{dz}{ly-mx}=\frac{ldx+mdy+ndz}{0}$$

$$\Rightarrow \quad ldx+mdy+ndz=0$$

Now integrating

$$\int l\,dx+\int m\,dy+\int n\,dz=0$$

$$\boxed{lx+my+nz=c_1}$$

choose another multiplier (x, y, z)

$$\frac{dx}{mz-ny}=\frac{dy}{nx-lz}=\frac{dz}{ly-mx}=\frac{xdx+ydy+zdz}{mxz-nxy+nyx-lyz+lzy-mzx}$$

$$\frac{dx}{mz-ny}=\frac{dy}{nx-lz}=\frac{dz}{ly-mx}=\frac{xdx+ydy+zdz}{0}$$

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$$\Rightarrow \quad x\,dx + y\,dy + z\,dz = 0$$

Now integrating $\int x\,dx + \int y\,dy + \int z\,dz = C_2$

$$\frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{2} = \frac{C_2}{2}$$

$$\boxed{x^2+y^2+z^2 = C_2}$$

Then $f(u,v) = f(lx+my+nz,\ x^2+y^2+z^2)$

Impt

Q. $(x^2-y^2-z^2)p + 2xyq = 2xz$

auxiliary equation

$$\frac{dx}{x^2-y^2-z^2} = \frac{dy}{2xy} = \frac{dz}{2xz}$$

from last two fraction we get

$$\frac{dy}{2xy} = \frac{dz}{2xz}$$

$$\int \frac{dy}{y} = \int \frac{dz}{z}$$

$$\log y - \log z = \log C_1$$

$$\log\left(\frac{y}{z}\right) = \log C_1$$

$$\boxed{\frac{y}{z} = C_1}$$

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now choose multiplier (x, y, z)

$$\frac{dx}{x^2-y^2-z^2} = \frac{dy}{2xy} = \frac{dz}{2xz} = \frac{x\,dx + y\,dy + z\,dz}{x^3 - xy^2 - xz^2 + 2xy^2 + 2xz^2}$$

$$= \frac{x\,dx + y\,dy + z\,dz}{x^3 + xz^2 + xy^2}$$

$$\frac{dx}{x^2-y^2-z^2} = \frac{dy}{2xy} = \frac{dz}{2xz} = \frac{x\,dx + y\,dy + z\,dz}{x(x^2+z^2+y^2)}$$

from last two fraction we get

$$\frac{dz}{2\cancel{x}z} = \frac{x\,dx + y\,dy + z\,dz}{\cancel{x}(x^2+y^2+z^2)}$$

$$\frac{dz}{z}(x^2+y^2+z^2) = 2x\,dx + 2y\,dy + 2z\,dz$$

$$\frac{dz}{z} = \frac{d(x^2+y^2+z^2)}{x^2+y^2+z^2}$$

Now integrating both sides

$$\int \frac{dz}{z} = \int \frac{d(x^2+y^2+z^2)}{x^2+y^2+z^2}$$

$$\log z = \log(x^2+y^2+z^2) + \log c_2$$

$$\log(x^2+y^2+z^2) - \log z = \log c_2$$

$$\boxed{\left(\frac{x^2+y^2+z^2}{z}\right) = c_2}$$

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Q. $(y+z)p - (x+z)q = (x-y)$

auxiliary equation :

$$\frac{dx}{y+z} = \frac{dy}{-(x+z)} = \frac{dz}{x-y}$$

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choose multiplier $(1,1,1)$

$$\frac{dx}{y+z} = \frac{dy}{-(x+z)} = \frac{dz}{x-y} = \frac{dx+dy+dz}{y+z-x-z+x-y}$$

$$\text{" " "} = \frac{dx+dy+dz}{0}$$

$$\Rightarrow dx+dy+dz = 0$$

now integrating

$$\int dx + \int dy + \int dz = c_1$$

$$\boxed{x+y+z = c_1}$$

choose another multiplier $(x, y, -z)$

$$\frac{dx}{y+z} = \frac{dy}{-(x+z)} = \frac{dz}{x-y} = \frac{xdx+ydy-zdz}{xy+xz-xy-zy-zx+zy}$$

$$\text{" " "} = \frac{xdx+ydy-zdz}{0}$$

$$\Rightarrow xdx+ydy-zdz = 0$$

By integrating $\int xdx + \int ydy - \int zdz = c_2$

$$\frac{x^2}{2} + \frac{y^2}{2} - \frac{z^2}{2} = \frac{c_2}{2}$$

$$\boxed{x^2+y^2-z^2 = c_2}$$

$$f(u,v) = f(x+y+z,\ x^2+y^2-z^2)$$

(new in Syllb limited ques)

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(IV) Charpit's Method

It is a general method due to charpit for solving non-linear equations of first order when it is difficult to solve such equations under any of the standard forms then this method is used to find complete integral.

Let the given equation be $f(x, y, z, p, q) = 0$ — (1)

If we succeed to find another relation

$$F(x, y, z, p, q) = 0 \quad \text{— (2)}$$

we solve (1) & (2) for 'p' and 'q'
since z consists of two independent variable x and y.

$$\boxed{dz = p\,dx + q\,dy}$$

Auxiliary Equation of Charpit's method

$$\frac{dp}{\frac{\partial f}{\partial x} + p\frac{\partial f}{\partial z}} = \frac{dq}{\frac{\partial f}{\partial y} + q\frac{\partial f}{\partial z}} = \frac{dz}{-p\frac{\partial f}{\partial p} - q\frac{\partial f}{\partial p}} = \frac{dx}{-\frac{\partial f}{\partial p}} = \frac{dy}{-\frac{\partial f}{\partial q}} = \frac{dF}{0}$$

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Q. $pxy + pq + qy = yz$

① — $pxy + pq + qy - yz = 0 \Leftarrow f(x, y, z, p, q)$

auxiliary equation.

$$\frac{dx}{-\frac{\partial f}{\partial p}} = \frac{dy}{-\frac{\partial f}{\partial q}} = \frac{dz}{-pf_p - qf_q} = \frac{dp}{f_x + pf_z} = \frac{dq}{f_y + qf_z} = \frac{dF}{0}$$

$$\frac{dF}{0} = \frac{dx}{-(xy+q)} = \frac{dy}{-(p+y)} = \frac{dz}{-p(xy+q) - q(p+y)} = \frac{dp}{py + p(-y)} = \frac{dq}{(px+q-z) + q(-y)}$$

$$\Rightarrow \frac{dp}{\cancel{py} - \cancel{py}} = \frac{dF}{0}$$

$$dp = 0$$

integration

$$\int dp = 0$$

$$\boxed{p = a} \quad \text{② integration constant}$$

substituting $p = a$ in eq ①

$$axy + aq + qy - yz = 0$$

$$q(a+y) = yz - axy$$

$$\boxed{q = \frac{y(z-ax)}{a+y}} \quad \text{③}$$

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as we know that for $z(x,y)$

$$dz = p\,dx + q\,dy$$

substituting value from ② & ③

$$dz = a\,dx + \frac{y(z-ax)}{a+y}\,dy$$

$$dz - a\,dx = \frac{y(z-ax)}{a+y}\,dy$$

$$\frac{dz - a\,dx}{y(z-ax)} = \frac{y}{a+y}\,dy$$

$$\frac{dz - a\,dx}{z-ax} = \left(1 - \frac{a}{a+y}\right)dy$$

integrating both sides

$$\int \frac{d(z-ax)}{z-ax} = \int\left(1-\frac{a}{a+y}\right)dy$$

$$\log(z-ax) = \int dy - \int \frac{a}{a+y}$$

constant

$$\log(z-ax) = y - a\log(a+y) + \log b$$

$$\log(z-ax) = y - a\log[(a+y)] + \log b$$

$$\log(z-ax) = \log e^{y} + \log[(a+y)^{-a}] + \log b$$

$$\log(z-ax) = \log e^{y} + \log[b\cdot(a+y)^{-a}]$$

$$\boxed{(z-ax) = e^{y}\cdot b(a+y)^{-a}}$$

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Q. Solve : $(p^2+q^2)y = qz$

$$f(x,y,z,p,q) = (p^2+q^2)y - qz = 0 \quad \text{--- (1)}$$

$$\frac{\partial f}{\partial x} = 0 \qquad \frac{\partial f}{\partial y} = p^2+q^2 \qquad \frac{\partial f}{\partial z} = -q \qquad \frac{\partial f}{\partial p} = 2py$$

$$\frac{\partial f}{\partial q} = 2qy - z$$

Auxiliary equation :-

$$\frac{dF}{0} = \frac{dx}{-2py} = \frac{dy}{-(2qy-z)} = \frac{dz}{-p(2py)-q(2qy-z)} = \frac{dp}{0+p(-q)} = \frac{dq}{p^2+q^2+q(-q)} \quad \text{--- (2)}$$

now choose simplest part which includes p & q

$$\Rightarrow \frac{dp}{-pq} = \frac{dq}{p^2+\cancel{q^2}-\cancel{q^2}}$$

$$\frac{dp}{-q} = \frac{dq}{p} \Rightarrow p\,dp = -q\,dq$$

$$p\,dp + q\,dq = 0$$

integration

$$\int p\,dp + \int q\,dq = 0$$

$$\frac{p^2}{2} + \frac{q^2}{2} = \frac{a^2}{2}$$

$$\boxed{p^2+q^2 = a^2} \quad \text{--- (3)}$$

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$$p^2y + \frac{a^4y^2}{z^2}xy - \frac{a^2y}{z} = 0$$

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$$p^2y = \frac{a^2y}{z} - \frac{a^4y^3}{z^2}$$

$$p^2\cancel{y} = \frac{\cancel{y}(z^2a^2 - a^4y^2)}{z^2}$$

$$p^2 = \frac{a^2(z^2 - a^2y^2)}{z^2}$$

$$\boxed{p = \frac{a}{z}\sqrt{z^2 - a^2y^2}} \quad \text{—— (5)}$$

now substituting eq (3) in eq (1)

$$a^2y - qz = 0$$

$$qz = a^2y$$

$$\boxed{q = \frac{a^2y}{z}} \quad \text{—— (4)}$$

now we know that $z(x, y)$

$$\therefore \; dz = p\,dx + q\,dy$$

$$dz = \frac{a}{z}\sqrt{z^2 - a^2y^2}\,dx + \frac{a^2y}{z}\,dy$$

$$z\,dz = a\sqrt{z^2 - a^2y^2}\,dx + a^2y\,dy$$

$$z\,dz - a^2y\,dy = a\sqrt{z^2 - a^2y^2}\,dx$$

$$\frac{z\,dz - a^2y\,dy}{\sqrt{z^2 - a^2y^2}} = a\,dx$$

integrating both sides

$$\int \frac{z\,dz - a^2y\,dy}{\sqrt{z^2 - a^2y^2}} = \int a\,dx$$

$$\frac{1}{2} \times d\left(\frac{z^2 - a^2y^2}{\sqrt{z^2 - a^2y^2}}\right) = ax + b \quad \text{constant} \quad [t = z^2 - a^2y^2]$$

$$z^2 - a^2y^2 = ax + b$$

Soln $\boxed{z^2 - a^2y^2 = (ax + b)^2}$

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(V) Cauchy's Method of Characteristics

Let $\boxed{a u_x + b u_y = f(x,y)}$

$$a\left(\frac{\partial u}{\partial x}\right) + b\left(\frac{\partial y}{\partial y}\right) = f(x,y) \quad \text{—— (1)}$$

where $u = u(x,y)$

Then Auxiliary equation $\Rightarrow$ $\boxed{\frac{dx}{a} = \frac{dy}{b} = \frac{du}{f(x,y)}}$ —— (2)

from eqn (1) & (2) solving for 'u' gives the required solution

Q Solve by using Cauchy's Method $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = x+y$, given $u(x,0) = 0$

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = x+y \quad \text{—— (1)}$$

comparing with $a\frac{\partial u}{\partial x} + b\frac{\partial u}{\partial y} = f(x,y)$

then $a = 1$ and $b = 1$ also $\Rightarrow$ $f(x,y) = x+y$

Then from auxiliary equation $\frac{dx}{1} = \frac{dy}{1} = \frac{du}{x+y}$ —— (2)

Taking (I) & (II) fraction

$$dx = dy$$

$$\int dx - \int dy = 0$$

$$\boxed{x - y = c_1} \longrightarrow (2) \quad \boxed{x = c_1 + y} \quad \text{—— (3)}$$

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taking II & III fraction

$$\frac{dy}{1} = \frac{du}{x+y}$$

$$du = (x+y)\,dy$$

from eq ③ we know $x = c_1 + y$

$$du = (c_1 + y + y)\,dy$$

$$\int du = \int (c_1 + 2y)\,dy$$

$$\boxed{u = c_1 y + y^2 + c_2} \quad \text{—— ④}$$

let $c_2 = g(c_1) = g(x-y)$ using eq ③ → arbitary fnc

from eq ④

$$u = (x-y)y + y^2 + g(x-y) \quad \text{—— ⑤}$$

as $u(x,0) = 0$ substituting in eq ⑤

$$u = (x-0)0 + 0 + g(x-0)$$

$$= g(x) = 0 \Rightarrow \boxed{g(x-y) = 0} \quad \text{—— ⑥}$$

also

from eq ⑤ & ⑥

$$u = (x-y)y + y^2 + 0$$

$$u = xy - y^2 + y^2$$

$\boxed{u = xy}$ is the required solution.

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Q. $u_x + u_y = u$ for $u(x,0) = 1+e^x$

$u_x + u_y = u$ — (1)

comparing with standard form $a u_x + b u_y = f(x,y)$

we get $a = 1$, $b = 1$, $f(x,y) = u$

from auxiliary eqn $\frac{dx}{1} = \frac{dy}{1} = \frac{du}{u}$ — (2)

By taking I & II part of fraction in eq (2)

$\frac{dx}{1} = \frac{dy}{1} \Rightarrow \int dx = \int dy \Rightarrow \boxed{x = y + c_1}$

$\Rightarrow \boxed{x - y = c_1}$ — (3)

By taking II & III part of fraction we get

$\frac{dy}{1} = \frac{du}{u} \Rightarrow \int dy = \int \frac{du}{u}$

$\log u = y + c_2$

$\boxed{u = e^{y+c_2}}$ — (4)

Now using given condition $\boxed{u(x,0) = 1+e^x}$

let $c_2 = g(c_1)$
$\therefore c_2 = g(x-y)$ } using eq (3)

In eq (4)

$u(x,y) = u = e^{y+g(x-y)}$ — (5)

By using given condn in eq (4)

$u(x,y) = e^{y+g(x-y)}$

$u(x,0) = e^{0+g(x-0)}$

$u(x,0) = e^{g(x)}$

$1+e^x = e^{g(x)} \Rightarrow g(x) = \log(1+e^x)$

(6) — $\boxed{g(x-y) = \log(1+e^{x-y})}$

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in eq ⑤, by ~~[illegible]~~ using eq ⑥ we get

$$u = e^{y + g(x-y)}$$

$$u = e^{y + \log(1+e^{x-y})}$$

$$u = e^{y} \cdot \cancel{e}^{\log(1+e^{x-y})}$$

$$u = e^{y} \cdot \left[1 + e^{x-y}\right]$$

$$u = e^{y} + e^{y} \times e^{x-y}$$

$$u = e^{y} + \cancel{e^{y}} \times \left(\frac{e^{x}}{\cancel{e^{y}}}\right)$$

Soln $\Rightarrow$ $\boxed{u = e^{y} + e^{x}}$

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(VI) Solution of Linear Partial DE of higher order with constant coffecient

An equation of Type :-

$$\left.\begin{aligned} & a_0 \frac{\partial^n z}{\partial x^n} + a_1 \frac{\partial^n z}{\partial x^{n-1} \partial y} + \cdots + a_n \frac{\partial^n z}{\partial y^n} = Q(x,y) \\ & \text{OR} \\ & (a_0 D^n + a_1 D^{n-1} D' + \cdots + a_n D'^n) z = Q(x,y) \end{aligned}\right\} \text{—(1)}$$

is $\boxed{\text{homogeneous Linear PDE}}$ of nth order with constant cofficient where $\boxed{\frac{\partial}{\partial x} = D, \ \frac{\partial}{\partial y} = D'}$

Soln of eq (1) will be

<i> If $f(D, D') z = 0$, Then soln is
$\Rightarrow Q(x,y) = 0$

$$\boxed{Z = \text{complementry function} = CF}$$

ii> If $f(D, D') z = F(x,y)$, Then soln is
$\Rightarrow Q(x,y) \neq 0$

$$\boxed{Z = CF + PI \text{ (Particular Integral)}}$$

$\Rightarrow$ Method to find auxiliary equation
In eqn $f(D, D') z = 0$ or $f(D, D') z = F(x,y)$
using $\boxed{D = m, \ D' = 1}$

we get auxiliary eqn (AE) $= (a_0 m^n + a_1 m^{n-1} + \cdots + a_n) = 0$

$$\Rightarrow \boxed{f(m, m') = 0}$$

which is an AE.

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⇒ <u>Method to find complementry function (CF) or Complete Integral</u>

(1) If AE have roots (all different roots) as m_1, m_2, m_3

$$\boxed{CF = f_1(y+m_1x) + f_2(y+m_2x) + f_3(y+m_3x) + \cdots}$$

(2) If AE have roots (equal roots) as $m_1 = m_2 = m_3 = (m)$

$$\boxed{CF = f_1(y+mx) + x f_2(y+mx) + x^2 f_3(y+mx)}$$

(3) AE roots as $m_1 = m_2 = (m)$ & m_3

$$\boxed{CF = f_1(y+mx) + x f_2(y+mx) + f_3(y+m_3x)}$$

Q Solve $(D^3 - 4D^2D' + 3DD'^2)z = 0$ find CF.

as $f(x,y)z / f(D,D')z = 0$

put $D = m$, $D' = 1$

$$(m^3 - 4m^2 + 3m)z = 0$$
$$m(m^2 - 4m + 3)z = 0$$
$$m(m^2 - 3m - m + 3)z = 0$$
$$m[m(m-3) - 1(m-3)]z = 0$$
$$m[(m-1)(m-3)]z = 0$$
$$m(m-1)(m-3)z = 0$$
$$\boxed{m = 0, 1, 3}$$

$$CF = f_1(y+m_1x) + f_2(y+m_2x) + f_3(y+m_3x)$$

Soln $\boxed{z = f_1(y) + f_2(y+x) + f_3(y+3x)}$

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Q. $\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + 4\frac{\partial^2 z}{\partial y^2} = 0$

as $D = \frac{\partial}{\partial x}$, $D' = \frac{\partial}{\partial y}$

$(D^2 - 4DD' + 4D'^2) = 0$

putting $D = m, D' = 1$

$m^2 - 4m + 4 = 0$

$m^2 - 2m - 2m + 4 = 0$

$m(m-2) - 2(m-2) = 0$

$(m-2)(m-2) = 0$

$\boxed{m = 2, 2}$

$CF = f_1(y + mx) + x f_2(y + mx)$

Soln. $\boxed{z = f_1(y + 2x) + x f_2(y + 2x)}$

⟹ To calculate PI :

Case 1 :- When $Q(x,y) = e^{ax+by}$

Then $P.I = \frac{1}{f(D,D')} \cdot e^{ax+by}$

$$\boxed{P.I = \frac{e^{ax+by}}{f(a,b)} \quad ; D = a, D' = b \text{ if } f(a,b) \neq 0}$$

If $f(a,b) = 0$ Then

→ $P.I = \frac{x \times e^{ax+by}}{f'(D,D')} = \frac{x e^{ax+by}}{f'(a,b)}$; $D = a, D' = b$ if $f'(a,b) \neq 0$

and so on...

Partial diff^t w.r.t D i.e $\left(\frac{\partial}{\partial x}\right)$ (or diff^t wrt x)

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Case II :- When $\phi(x,y) = \sin(ax+by)$ or $\cos(ax+by)$

$$P.I = \frac{1}{f(D^2, DD', D'^2)} \times \sin(ax+by)$$

$$\boxed{P.I = \frac{\sin(ax+by)}{f(-a^2, -ab, -b^2)}}$$

$D^2 = -a^2$, $DD' = -ab$
$D'^2 = -b^2$
iff $(-a^2, -ab, -b^2) \neq 0$

If $f(-D^2, -ab, -b^2) = 0$ then 'x' multiply in Num^r and Deno^r will be diff wrt D.

$$\boxed{\frac{1}{D} = \int dx \; ; \; \frac{1}{D'} = \int dy}$$

But if the power of $D' > D$ then it will be diff w.r.t D'.

$\Rightarrow$ Diff w.r.t that which ever have greater power.

Q1. Solve $4\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = e^{x+2y}$

$$(4D^2 - 4DD' + D'^2)z = e^{x+2y}$$

$D = m$, $D' = 1$

AE $\Rightarrow$ $(4m^2 - 4m + 1) = 0$

$4m^2 - 2m - 2m + 1 = 0$

$2m(2m-1) - 1(2m-1) = 0$

$(2m-1)^2 = 0$

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$$\boxed{m = \frac{1}{2}, \frac{1}{2}.}$$

$$\boxed{CF = f_1\left(y + \frac{1}{2}x\right) + x f_2\left(y + \frac{1}{2}x\right)} \quad \text{——— (1)}$$

$$PI = \frac{1}{(4D^2 - 4DD' + D'^2)} e^{x+2y}$$

$$(4 - 4 \times 1 \times 1 + 4) \neq 0$$

$$a = 1, \; b = 2$$

$$8 - 8 = 0$$

$$PI = \frac{x}{8D - 4D'} \times e^{x+2y}$$

$$8 \times 1 - 4 \times 2 = 0$$

$$P.I = \frac{x^2}{8} \times e^{x+2y}$$

$$\boxed{P.I = \frac{x^2 \times e^{x+2y}}{8}} \quad \text{——— (2)}$$

Soln $Z = CF + P.I$

Ans $$\boxed{Z = f_1\left(y + \frac{x}{2}\right) + x f_2\left(y + \frac{x}{2}\right) + \frac{x^2}{8}\left(e^{x+2y}\right)}$$

Q2. Solve $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} = \sin x \cos 2y$

given $(D^2 - DD')z = \sin x \cos 2y$

$$(D^2 - DD')z = \frac{1}{2}\left[\sin(x + 2y) + \sin(x - 2y)\right]$$

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for AE $\rightarrow$ $D = m$, $D' = 1$

$$m^2 - m = 0$$

$$m(m-1) = 0$$

$$\boxed{m = 0, 1}$$

$$\boxed{CF = f_1(y) + f_2(y+x)} \quad \text{—— (1)}$$

$$P.I = \frac{1}{D^2 - DD'} \times \frac{1}{2}\left[\sin(x+2y) + \sin(x-2y)\right]$$

$$P.I = \underbrace{\frac{1}{2} \times \frac{\sin(x+2y)}{(D^2 - DD')}}_{I_1} + \underbrace{\frac{1}{2} \frac{\sin(x-2y)}{(D^2 - DD')}}_{I_2} \quad \text{—— (2)}$$

$$I_1 = \frac{1}{2} \times \frac{1}{D^2 - DD'} \times \sin(x+2y)$$

$a = 1$, $b = 2$, $D^2 = -a^2$, $DD' = -ab$

$$I_1 = \frac{1}{2} \times \frac{1}{-1^2 - (-1 \times 2)} \times \sin(x+2y)$$

$$= \frac{1}{2} \times \frac{1}{(-1+2)} \times \sin(x+2y)$$

$$\boxed{I_1 = \frac{1}{2} \sin(x+2y)} \quad \text{—— (3)}$$

$$I_2 = \frac{1}{2} \times \frac{1}{D^2 - DD'} \times \sin(x-2y)$$

$a = 1$, $b = -2$

$D^2 = -1$, $DD' = -(1 \times -2) = -(-2) = 2$.

$$I_2 = \frac{1}{2} \times \frac{1}{-1 - 2} \times \sin(x-2y)$$

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$$I_2 = \frac{1}{2} \times \frac{1}{-3} \times \sin(x-2y)$$

$$\boxed{I_2 = -\frac{1}{6}\sin(x-2y)} \quad \text{--- (4)}$$

$$\boxed{P.I = \frac{1}{2}\sin(x+2y) - \frac{1}{6}\sin(x-2y)} \quad \text{--- (5)}$$

$$z = CF + P.I$$

Soln

$$\boxed{z = f_1(y) + f_2(y+x) + \frac{1}{2}\sin(x+2y) - \frac{1}{6}\sin(x-2y)}$$

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$$\frac{\partial^3 z}{\partial x^3} - 7\frac{\partial^3 z}{\partial x \partial y^2} - 6\frac{\partial^3 z}{\partial y^3} = \sin(x+2y) + e^{3x+y}$$

$$D^3 - 7DD'^2 - 6D'^3 = \sin(x+2y) + e^{3x+y}$$

AE $\Rightarrow$ $m = D$ & $D' = 1$

$$m^3 - 7m - 6 = 0$$

$$(m+1)(m^2 - m - 6) = 0$$

$$(m+1)(m^2 - 3m + 2m - 6) = 0$$

$$(m+1)(m(m-3) + 2(m-3)) = 0$$

$$(m+1)(m+2)(m-3) = 0$$

$$\boxed{m = -1, -2, 3}$$

$$\boxed{CF = f_1(y-x) + f_2(y-2x) + f_3(y+3x)} \quad \text{--- (1)}$$

$$P.I = \frac{1}{D^3 - 7DD'^2 - 6D'^3} \times \sin(x+2y) + e^{3x+y}$$

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$$PI = \underbrace{\frac{\sin(x+2y)}{D^3 - 7DD'^2 - 6D'^3}}_{I_1} + \underbrace{\frac{e^{9x+y}}{D^3 - 7DD'^2 - 6D'^3}}_{I_2}$$

$$I_1 = \frac{1}{D^3 - 7DD'^2 - 6D'^3} \sin(x+2y)$$

$$a = 1, \; b = 2$$

$$I_1 = \frac{1}{D(D^2) - 7DD'(D') - 6(D'^2)D'} \sin(x+2y)$$

$$D^2 = -1, \quad DD' = -2, \quad D'^2 = -4$$

$$I_1 = \frac{1}{D(-1) - 7(-2)D' - 6(-4)D'} \sin(x+2y)$$

$$I_1 = \frac{1}{-D + 14D' + 24D'} \sin(x+2y)$$

$$\cancel{a = D, \quad b = D'}$$

$$\cancel{I_1 = \frac{1}{-(1) + 14(2) + 24(2)} \sin(x+2y)}$$

$$\cancel{I_1 = \frac{1}{75} \sin(x+2y)}$$

$$I_1 = \frac{1}{-D + 38D'} \sin(x+2y)$$

$$I_1 = -\frac{1}{(D - 38D')} \sin(x+2y)$$

Rationalisation

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$$I_1 = -\frac{(D+38D')\sin(x+2y)}{D^2-(38)^2D'^2}$$

$$I_1 = -\frac{(D+38D')\sin(x+2y)}{D^2-1444D'^2}$$

$$D^2 = -1, \quad D'^2 = -4$$

$$I_1 = -\frac{(D+38D')\sin(x+2y)}{-1-1444\times(-4)}$$

$$I_1 = -\frac{1}{5775}(D+38D')(\sin(x+2y))$$

$$I_1 = \frac{-1}{5775}\left[D\sin(x+2y) + 38D'\sin(x+2y)\right]$$

$D = \partial/\partial x$; $D' = \partial/\partial y$

$$I_1 = \frac{-1}{5775}\left[\cos(x+2y)\times 1 + 38\times\cos(x+2y)\times 2\right]$$

$$I_1 = \frac{-1}{5775}\left[\cos(x+2y) + 76\cos(x+2y)\right]$$

$$I_1 = \frac{-1}{5775}\left[77\cos(x+2y)\right]$$

$$\boxed{I_1 = -\frac{1}{75}\cos(x+2y)} \quad \text{——— (3)}$$

$$I_2 = \frac{e^{3x+y}}{D^3-7DD'^2-6D'^3}$$

$a = 3, \; b = 1$

$$I_2 = \frac{x\times e^{3x+y}}{[3D^2-7D'^2]}$$

$3^3 - 7\times 3\times 1 - 6$
$27 - 21 - 6$
$\cancel{27} - \cancel{21} = 0$

$3(3)^2 - 7(1)$
$27 - 7 = 20$

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$$I_2 = \frac{x \times e^{3x+y}}{3 \times 3^2 - 7 \times 1^2}$$

$$\boxed{I_2 = \frac{x e^{3x+y}}{20}} \quad \text{--- (4)}$$

$$P.I = I_1 + I_2 = -\frac{1}{75}\cos(x+2y) + \frac{x}{20} e^{3x+y} \quad \text{--- (5)}$$

$$z = CF + PI$$

Soln. $z = f_1(y-x) + f_2(y-2x) + f_3(y+3x) - \frac{\cos(x+2y)}{75} + \frac{x \times e^{3x+y}}{20}$

2012 Q. $(D^3 - 4D^2D' + 4DD'^2)z = 6\sin(3x+2y)$

AE $\Rightarrow$ $D = m$, $D' = 1$

$$(m^3 - 4m^2 + 4)z = 6\sin(3x+2y)$$

$$m^3 - 4m^2 + 4 = 0$$

$$m^3 - 4m^2 + (4m - 4m) + 4 = 0$$

$$[m^3 - 4m^2 + 4m] - (4m - 4) = 0$$

$$m[m^2 - 4m + 4] - 4[m-1] = 0$$

$$m[m^2 - 2m - 2m + 4] - 4(m-1) = 0$$

$$m[m(m-2) - 2(m-2)] - 4(m-1) = 0$$

$$m[(m-2)(m-2)] - 4(m-1) = 0$$

$$(m-4)(m-2)(m-2)(m-1) = 0$$

$$\boxed{m = 1, 2, 2, 4}$$

$$\boxed{CF = f_1(y+2x) + x f_2(y+2x) + f_3(y+x) + f_4(y+4x)} \quad \text{--- (1)}$$

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$$P.I = \frac{1}{D^3 - 4D^2D' + 4DD'^2} \cdot 6\sin(3x+2y)$$

$$a = 3, \quad b = 2$$

$$P.I = \frac{1}{DD^2 - 4D^2D' + 4D(D')^2} 6\sin(3x+2y)$$

$$P.I = \frac{1}{D(-9) - 4(-9)D' + 4D(-4)} 6\sin(3x+2y)$$

$$P.I = \frac{1}{-9D + 36D' - 16D} 6\sin(3x+2y)$$

$$P.I = \frac{1}{-25D + 36D'} 6\sin(3x+2y)$$

$$P.I = -\frac{1}{(25D - 36D')} 6\sin(3x+2y)$$

$$P.I = -\frac{(25D + 36D')}{(25D)^2 - (36D')^2} 6\sin(3x+2y)$$

$$P.I = -\frac{(25D + 36D')}{625D^2 - 1296D'^2} 6\sin(3x+2y)$$

$$P.I \quad -\frac{(25D + 36D')}{-5625 + 5184} 6\sin(3x+2y)$$

$$P.I = \frac{-1}{-441}\left[6 \times 25D\sin(3x+2y) + 36 \times 6D'\sin(3x+2y)\right]$$

$$P.I = \frac{1}{441}\left[150D\sin(3x+2y) + 216D'\sin(3x+2y)\right]$$

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$$P.I = \frac{1}{441}\left[150\cos(3x+2y)\times 3 + 216\cos(3x+2y)\times 2\right]$$

$$P.I = \frac{1}{441}\left[450\cos(3x+2y) + 432\cos(3x+2y)\right]$$

$$PI = \frac{882}{441}\left[\cos 3x+2y\right]$$

$$\boxed{PI = 2\cos(3x+2y)} \text{ —— } (2)$$

Soln $Z = CF + PI$

2014 Q $(D^2 - DD')Z = \sin(x+2y)$

AE $\rightarrow D = m,\ D' = 1$

$$(m^2 - m)z = \sin(x+2y)$$

$$m^2 - m = 0$$

$$m(m-1) = 0$$

$$\boxed{m = 0, 1}$$

$$\boxed{CF = f_1(y) + f_2(y+x)} \text{ —— } (1)$$

$$P.I = \frac{1}{D^2 - DD'}\sin(x+2y)$$

$a = 1,\ b = 2.$

-1

$$P.I = \frac{1}{-1-(-1\times 2)}\sin(x+2y)$$

$$P.I = \frac{1}{-1+2}\sin(x+2y) \Rightarrow \boxed{\sin(x+2y} \text{ (2)}$$

Soln $\boxed{Z = f_1(y) + f_2(y+x) + \sin(x+2y)}$

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Imp^t. **Case IV** :- When $g(x,y) = x^m y^n$

Then $\boxed{P.I = \frac{1}{f(D,D')} x^m y^n = [f(D,D')]^{-1} x^m y^n}$

<a> If $\underline{m>n} \Rightarrow \frac{1}{f(D,D')}$ is expanded in power of $\left(\frac{D'}{D}\right)$

<b> If $m<n \Rightarrow \frac{1}{f(D,D')}$ is expanded in power of $\left(\frac{D}{D'}\right)$

<c> If $m=n$, then use any one of above

formula in use :- $\left[\begin{array}{l} (1+x)^{-1} = 1 - x + x^2 - x^3 + \cdots \\ (1-x)^{-1} = 1 + x + x^2 + x^3 + \cdots \end{array}\right]$

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Imp Q 2016

$$(D^2+4D'^2)z = x^2y^4$$

AE $\Rightarrow$ $D=m,\ D'=1$

$$(m^2+4)z=0$$
$$m^2+4=0$$
$$m^2=-4$$
$$m=\sqrt{-4}$$
$$m=\pm 2i$$

$$\boxed{CF = f_1(y-2ix)+f_2(y+2ix)} \quad \text{—— (1)}$$

PI $\Rightarrow$ $\dfrac{1}{D^2+4D'^2}x^2y^4$

$m<n \therefore \left(\dfrac{D}{D'}\right)$

$$= \frac{1}{4D'^2\left(\frac{D^2}{4D'^2}+1\right)}x^2y^4$$

$$= \frac{1}{4D'^2}\left[1+\frac{D^2}{4D'^2}\right]^{-1}x^2y^4$$

using expansion of $(1+x)^{-1}$

$$= \frac{1}{4D'^2}\left[1-\frac{D^2}{4D'^2}+\frac{D^4}{16D'^4}-\cdots\right]x^2y^4$$

$$= \frac{1}{4D'^2}\left[x^2y^4-\frac{D^2}{4D'^2}(x^2y^4)+\frac{D^4}{16D'^4}(x^2y^4)-\cdots\right]$$

$$= \frac{1}{4D'^2}\left[x^2y^4-\frac{D}{4D'^2}(2xy^4)+0\right]$$

$$= \frac{1}{4D'^2}\left[x^2y^4-\frac{2y^4}{4D'^2}\right]$$

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$$= \frac{1}{4D'^2}\left[x^2y^4 - \frac{2}{4}\frac{1}{D'}\int y^4 dy\right]$$

$$= \frac{1}{4D'^2}\left[x^2y^4 - \frac{1}{2}\frac{1}{D'}\left(\frac{y^5}{5}\right)\right]$$

$$= \frac{1}{4D'^2}\left[x^2y^4 - \frac{1}{10}\int y^5 dy\right]$$

$$= \frac{1}{4D'^2}\left[x^2y^4 - \frac{1}{10}\left(\frac{y^6}{6}\right)\right]$$

$$= \frac{1}{4D'^2}\left[x^2y^4 - \frac{1}{60}y^6\right]$$

$$= \frac{1}{4}\frac{1}{D'}\left[\frac{1}{D'}x^2y^4 - \frac{1}{D'}\left(\frac{1}{60}y^6\right)\right]$$

$$= \frac{1}{4} \times \frac{1}{D'}\left[\int x^2y^4 dy - \frac{1}{60}\int y^6 dy\right]$$

$$= \frac{1}{4D'}\left[x^2\frac{y^5}{5} - \frac{1}{60}\left(\frac{y^7}{7}\right)\right]$$

$$= \frac{1}{4}\left[\frac{x^2}{5}\int y^5 dy - \frac{1}{420}\int y^7 dy\right]$$

$$= \frac{1}{4}\left[\frac{x^2}{5}\left(\frac{y^6}{6}\right) - \frac{1}{420}\left(\frac{y^8}{8}\right)\right]$$

$$= \frac{1}{4}\left[\frac{x^2y^6}{30} - \frac{y^8}{3360}\right]$$

$$P.I = \frac{1}{13440}\left(112x^2y^6 - y^8\right) \quad \text{——— (2)}$$

Soln. $Z = CF + P.I$

Sol 1 / 2014, 13

$$\left[D^2 + (a+b)DD' + abD'^2\right] z = xy$$

AE $\longrightarrow$ $D = m, \quad D' = 1$

$$m^2 + (a+b)m + ab = 0$$
$$m^2 + am + bm + ab = 0$$
$$m(m+a) + b(m+a) = 0$$
$$(m+a)(m+b) = 0$$
$$\boxed{m = -a, -b}$$

$$\boxed{CF = f_1(y - ax) + f_2(y - bx)} \quad \text{—— (1)}$$

$$P.I = \frac{1}{D^2 + (a+b)DD' + abD'^2} \cdot xy$$

$$m = n.$$

$$P.I = \frac{1}{D^2\left[1 + \frac{(a+b)DD' + abD'^2}{D^2}\right]} xy$$

$$P.I = \frac{1}{D^2}\left[1 + \frac{(a+b)DD' + abD'^2}{D^2}\right]^{-1} xy$$

by using $(1+x)^{-1}$ expansion

$$= \frac{1}{D^2}\left[1 - \left\{\frac{(a+b)DD' + abD'^2}{D^2}\right\} + \cdots\right] xy$$

$$= \frac{1}{D^2}\left[1 - \frac{(a+b)D'}{D} + \frac{(ab)D'^2}{D^2} + \cdots\right] xy$$

$$= \frac{1}{D^2}\left[xy - (a+b)\frac{D'}{D}(xy) + (ab)\frac{D'^2}{D^2}xy + \cdots\right]$$

$$= \frac{1}{D^2}\left[xy - \frac{a+b}{D} \cdot x + ab(0) + \cdots\right]$$

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$$P.I = \frac{1}{D^2}\left[xy + (a+b)\int x\,dx + 0 \right]$$

$$P.I = \frac{1}{D^2}\left[xy + (a+b)\frac{x^2}{2} \right]$$

$$P.I = \frac{1}{D}\left[\frac{1}{D}\int xy + \frac{a+b}{2}\int x^2 dx \right]$$

$$P.I = \frac{1}{D}\left[y\frac{x^2}{2} + \frac{a+b}{2}\left(\frac{x^3}{3}\right) \right]$$

$$P.I = \frac{y}{2}\int x^2 dx + \frac{a+b}{6}\int x^3 dx$$

$$P.I = \frac{y}{2}\left(\frac{x^3}{3}\right) + \frac{a+b}{6}\left(\frac{x^4}{4}\right)$$

$$P.I = \frac{yx^3}{6} + \frac{(a+b)x^4}{6\times 4}$$

$$P.I = \frac{x^3}{24}\left[4y + (a+b)x \right] \quad \text{——— (2)}$$

$$z = CF + P.I$$

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Case : IV If $Q(x,y) = \phi(ax+by)$

$$P.I = \frac{1}{f(D,D')}\phi(ax+by)$$

Now let $u = ax+by$

$$P.I = \frac{1}{f(a,b)}\int u\,du \quad ; f(a,b) \neq 0$$

$\Rightarrow$ Integration of 'u' has to be done same no of time as the no. of (highest) order of 'D'.

Q. $(D^2+DD'-6D'^2)z = x+y$

AE $\Rightarrow$ $D = m,\ D' = 1$

$$(m^2+m-6)z = x+y$$

$$m^2+m-6=0$$
$$m^2+3m-2m-6=0$$
$$m(m+3)-2(m+3)=0$$
$$\boxed{m = 2, -3}$$

$$CF = f_1(y+2x) + f_2(y-3x) \quad \text{—— (1)}$$

$$P.I = \frac{1}{(D^2+DD'-6D'^2)}(x+y)$$

$a = 1,\ b = 1$, let $u = x+y$

$$P.I = \frac{1}{1^2+1\times1-6\times1^2}\iint u\,du\,du$$

$$P.I = \frac{1}{-4}\int \frac{u^2}{2}\,du$$

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$$P.I = \frac{1}{-8}\left(\frac{u^3}{3}\right)$$

$$\boxed{P.I = \frac{1}{-24}(x+y)^3} \quad \text{—— (2)}$$

$$z = CF + PI$$

$$z = f_1(y+2x) + f_2(y-3x) - \frac{1}{24}(x+y)^3$$

2015

Q. $r + s - 2t = \sqrt{2x+y}$

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 2\frac{\partial^2 z}{\partial y^2} = (2x+y)^{1/2}$$

$$(D^2 + DD' - 2D'^2)z = (2x+y)^{1/2}$$

AE $\Rightarrow$ $D = m$, $D' = 1$

$$m^2 + m - 2 = 0$$

$$m^2 + 2m - m - 2 = 0$$

$$m(m+2) - 1(m+2) = 0$$

$$\boxed{m = 1, -2}$$

$$CF = f_1(y+x) + f_2(y-2x) \quad \text{—— (1)}$$

$$P.I = \frac{1}{D^2 + DD' - 2D'^2} \cdot (2x+y)^{1/2}$$

$$a = 2,\ b = 1,\ u = 2x+y \quad \left\{ \frac{2^2 + \cancel{2} - \cancel{2} \neq 0}{4} \right.$$

$$P.I = \frac{1}{2^2 + 2\times1 - 2\times1^2} \iint u^{1/2}\, du\, du$$

$$P.I = \frac{1}{4}\int \frac{2u^{3/2}}{3}\, du = \frac{1}{\cancel{4}} \times \frac{\cancel{2}}{3} \times \frac{\cancel{2}}{5} \times u^{5/2}$$

$$\boxed{P.I = \frac{1}{15}(2x+y)^{5/2}} \quad \text{—— (2)}$$

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2011.

Q. $4r - 4s + t = 16 \log(x+2y)$

$$4\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 16 \log(x+2y)$$

$$(4D^2 - 4DD' + D'^2) z = 16 \log(x+2y)$$

AE $\Rightarrow$ $4D^2 - 4DD' + D'^2 = 0$

$D = m, \quad D' = 1$

$4m^2 - 4m + 1 = 0$

$4m^2 - 2m - 2m + 1 = 0$

$2m(m-1) - 1(2m-1) = 0$

$$\boxed{m = 1, \; m = \frac{1}{2}}$$

$$CF = f_1(y+x) + f_2\left(y + \frac{x}{2}\right) \quad \text{——— (1)}$$

$$P.I = \frac{1}{4D^2 - 4DD' + D'^2} 16 \log(x+2y)$$

$a = 1, \; b = 2, \; u = x+2y$

$\{4(1)^2 - 4 \times 2 \times 1 + 4 = 0\}$

$$P.I. = \frac{x}{8D - 4D'} 16 \log(x+2y)$$

$\{8(1) - 4(2^2) = 0\}$

$$P.I = \frac{16x^2}{8} \log(x+2y)$$

$$\boxed{P.I = 2x^2 \log(x+2y)} \quad \text{——— (2)}$$

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Q. $(D^2 - 6DD' + 9D'^2)z = (6x+2y)$.

AE $\Rightarrow$ $m = D$, $1 = D'$

$m^2 - 6m + 9 = 0$

$m^2 - 3m - 3m + 9 = 0$

$m(m-3) - 3(m-3) = 0$

$m = 3, 3$

$$\boxed{CF = f_1(y+3x) + x f_2(y+3x)} \quad \text{--- (1)}$$

$$P.I = \frac{1}{D^2 - 6DD' + 9D'^2}(6x+2y)$$

$a = 6$, $b = 2$, $u = 6x+2y$

$\left\{ \begin{array}{l} 6^2 - 6\times6\times2 + 9\times4 \\ 36 - 72 + 36 = 0 \end{array} \right.$

$$P.I = \frac{x}{2D - 6D'}(6x+2y)$$

$\left\{ \begin{array}{l} 2(6) - 6(2) \\ = 0 \end{array} \right.$

$$\boxed{P.I = \frac{x^2}{2}(6x+2y)} \quad \text{--- (2)}.$$

Soln. $z = f_1(y+3x) + x f_2(y+3x) + \frac{x^2}{2}(6x+2y)$.

Case V :- general form.

If $Q(x,y) = y^m \times \phi(x)$ OR $(y-1) \times \phi(x)$

$$P.I = \frac{1}{f(D,D')}\phi(x)$$

always factorise

& let ~~y = c~~ $y = c + mx$

$\phi(x) = c + mx$

coefficient in $f(D,D')$

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Q. $(D^2 + DD' - 6D'^2)z = y\cos x$

AE $\Rightarrow$ $m = D$, $1 = D'$

$m^2 + m - 6 = 0$

$\boxed{m = -3, 2}$

$CF = f_1(y-3x) + f_2(y+2x)$ —— (1)

$P.I = \dfrac{1}{\underbrace{D^2+DD'-6D'^2}_{\text{always factorisation}}} y\cos x$. [Let $y = c+mx$]

$= \dfrac{1}{(D-2D')(D+3D')} y\cos x$ [

By using this let $m = 3$

$= \dfrac{1}{(D-2D')} \int \underset{u}{(c+3x)}\underset{v}{(\cos x)}\,dx$ { by Putting $y = c+3x$

Integration by Parts

$= \dfrac{1}{D-2D'} \left[(c+3x) \times \sin x + 3\cos x \right]$

$= \dfrac{1}{D-2D'} \left[y(\sin x) + 3\cos x \right]$

$= \cancel{\dfrac{1}{D-2D'}} \left[\cancel{y\sin x +} \right.$

$= \int \left[(c-2x)\sin x + 3\cos x \right] dx$ { $y = c-2x$

$= (c-2x)(-\cos x) - 2\sin x + 3\sin x$

$\boxed{P.I = -y\cos x + \sin x}$ —— (2).

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2014 Q2.(a)

Q. $(D^2 - DD' - 2D'^2)z = (y-1)e^x$

AE $\Rightarrow$ $D = m$, $D' = 1$

$m^2 - m - 2 = 0$

$m^2 - 2m + m - 2 = 0$

$m(m-2) + 1(m-2) = 0$

$\boxed{m = -1, 2}$

$CF = f_1(y-x) + f_2(y+2x)$ — (2)

$$P.I = \frac{1}{(D^2 - DD' - 2D'^2)}(y-1)e^x$$

$$P.I = \frac{1}{(D-2D')(D+D')}(y-1)e^x$$

$$P.I = \underbrace{\frac{ye^x}{(D-2D')(D+D')}}_{(I_1)} - \underbrace{\frac{e^x}{(D-2D')(D+D')}}_{(I_2)}$$

$$I_1 = \frac{1}{(D-2D')(D+D')} ye^x$$

$\left\{ y = \begin{matrix} c+mx \\ c+1x \end{matrix} \right.$

$$I_1 = \frac{1}{D-2D'} \int (c+x)e^x \, dx$$

$$I_1 = \frac{1}{D-2D'} \int ce^x + \underset{uv}{xe^x} dx$$

$$I_1 = \frac{1}{D-2D'} \left[ce^x + x \cdot e^x - \int \frac{d}{dx}(x) \cdot e^x dx \right]$$

$$I_1 = \frac{1}{D-2D'} \left[ce^x + xe^x - e^x \right]$$

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$$I_1 = \frac{1}{D-2D'}\left[(c+x)e^x - e^x\right]$$

$$I_1 = \frac{1}{D-2D'}\left[ye^x - e^x\right] \qquad \{\because y = c+x$$

$$I_1 = \int (c-2x)e^x - e^x\,dx \qquad [\ y = c-2x$$

$$I_1 = \int (c\cdot e^x - 2x\cdot e^x - e^x)\,dx$$

$$I_1 = \left[ce^x - \left[2x\cdot e^x - 2e^x\right] - e^x\right]$$

$$I_1 = \left[ce^x - 2xe^x + 2e^x - e^x\right]$$

$$I_1 = (c-2x)e^x + e^x$$

$$I_1 = ye^x + e^x$$

$$\boxed{I_1 = (y+1)e^x} \quad \text{——— (2)}$$

$$I_2 = \frac{e^x}{(D-2D')(D+D')}$$

$$I_2 = \frac{e^{(1\cdot x + 0\cdot y)}}{D^2 - DD' - 2D'^2} \qquad a = 1,\ b = 0$$

$$I_2 = \frac{e^x}{1-0-0}$$

$$\boxed{I_2 = e^x} \quad \text{——— (3)}$$

$$P.I = (y+1)e^x + e^x$$

$$\boxed{P.I = (y+2)e^x} \quad \text{——— (4)}$$

Soln. $z = CF + P.I = f_1(y-x) + f_2(y+2x) + (y+2)e^x$

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(VII) Non Homogeneous Linear with constant coffecient

Case I :- If Equation is in form of

$$(D - m_1 D' - a_1)(D - m_2 D' - a_2) \cdots (D - m_n D' - a_n) z = 0$$

Then solution is :-

$$z = e^{a_1 x} f_1(y + m_1 x) + e^{a_2 x} f_2(y + m_2 x) + \cdots$$
$$\cdots + e^{a_n x} f_n(y + m_n x)$$

Case II :- If eqn is in form $(D - mD' - a)^3 z = 0$
i.e repeated roots

Then soln is :-

$$z = e^{ax} f_1(y + mx) + x e^{ax} f_2(y + mx) + x^2 e^{ax}(y + mx)$$

Case III :- If $(\alpha D + \beta D' - \gamma) z = 0$

$$z = e^{-\left(\frac{\gamma}{\alpha}\right)x} f(\alpha y - \beta x)$$

→ Since RHS of PDE is zero then soln is $\boxed{z = CF}$

→ If αD or D is not presence then case III is not applicable.

★ Method ~~[illegible]~~ for solving P.I in Non-homo LPD is same as Homo LPD.

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2015 '14

Case I :-

Q. Solve : $r - t + p - q = 0$

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} + \frac{\partial z}{\partial p} - \frac{\partial z}{\partial q} = 0$$

$$(D^2 - D'^2 + D - D')z = 0$$

$$(D - D')(D + D' + 1)z = 0$$

comparing with standard form

$$(D - m_1 D' - a_1)(D - m_2 D' - a_2) = 0$$

$\therefore$ $m_1 = 1$, $a_1 = 0$

$m_2 = -1$, $a_2 = -1$

$$CF = e^{a_1 x} f_1(y + m_1 x) + e^{a_2 x} f_2(y + m_2 x)$$

or $z = e^{0x} f_1(y + x) + e^{-x} f_2(y - x)$

$$\boxed{z = f_1(y + x) + e^{-x} f_2(y - x)}$$

2018 Case II :-

Q. $(D + 4D' + 5)^2 z = 0$

comparing with std form $(D - mD' - a)^n z = 0$

$n = 2$, $m = -4$, $a = -5 = (a_1 = a_2)$

$(m_1 = m_2 = m)$

$$CF = e^{ax} f_1(y + mx) + x e^{ax} f_2(y + mx)$$

$$\boxed{z = e^{-5x} f_1(y - 4x) + x e^{-5x} f_2(y - 4x)}$$

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Q. $(D+D'-1)(D+2D'-3)z=0$

comparing with std form

$(D-m_1D'-a_1)(D-m_2D'-a_2)=0$

$m_1=-1,\ a_1=1$

$m_2=-2,\ a_2=3$

$CF = e^{1\cdot x} f_1(y-x) + e^{3x} f_2(y-2x)$

$$\boxed{z = e^{x} f_1(y-x) + e^{3x} f_2(y-2x)}$$

Q1. Solve $D(D-2D'-3)z = e^{x+2y}$

$(D-0D'-0)(D-2D'-3)z = e^{x+2y}$

$m_1=0,\ a_1=0$

$m_2=-2,\ a_2=-3$

$CF = e^{0x} f_1(y+0x) + e^{-3x} f_1(y-2x)$

$$\boxed{CF = f_1(y) + e^{-3x} f_1(y+2x)} \quad (1)$$

$P.I = \dfrac{1}{D(D-2D'-3)} e^{x+2y}$

$a=1,\ b=2$

$\left\{ \begin{array}{l} 1(1-4-3) \\ 1(1-7) \\ -6 \end{array} \right.$

$$\boxed{P.I = \frac{1}{-6} e^{x+2y}} \quad (2)$$

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2016

Q $(D^2 - DD' - 2D'^2 + 2D + 2D')z = \sin(2x+y)$.

$$(D^2 - DD' - 2D'^2 + 2D + 2D')z = \sin(2x+y)$$

$$(D+D')(D-2D'+2)z = \sin(2x+y)$$

comparing

$$(D - m_1D' - a_1)(D - m_2D' - a_2)z = f(x,y)$$

$$m_1 = -1, \quad a_1 = 0, \quad m_2 = 2, \quad a_2 = -2$$

$$CF = e^{0x} f_1(y - 1x) + e^{-2x} f_1(y + 2x)$$

$$CF = f_1(y-x) + e^{-2x} f_1(y+2x) \quad \text{——— (1)}$$

$$P.I = \frac{1}{(D+D')(D-2D'+2)} \sin(2x+y)$$

$$a = 2, \quad b = 1.$$

$$P.I = \frac{1}{D^2 - DD' - 2D'^2 + 2D + 2D'} \sin(2x+y) \quad \left\{ \begin{array}{l} D^2 = -a^2 = -4 \\ D'^2 = -b^2 = -1 \\ DD' = -ab = -2 \end{array} \right.$$

$$P.I = \frac{\sin(2x+y)}{-4-(-2)-2(-1)+2(D)+2(D')}$$

$$P.I = \frac{\sin(2x+y)}{2D+2D'}$$

$$P.I = \frac{\sin(2x+y)}{2(D+D')}$$

$$P.I = \frac{(D-D')\sin(2x+y)}{2(D^2-D'^2)}$$

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$$P.I = \frac{(D-D')\sin 2x+y}{2(-4-(-1))}$$

$$P.I = \frac{D-D'(\sin 2x+y)}{-6}$$

$$P.I = \frac{D(\sin 2x+y) - D'(\sin 2x+y)}{-6}$$

$$P.I = \frac{\cancel{\sin}2(\cos 2x+y) - 1(\cos 2x+y)}{-6}$$

$$\boxed{P.I = \frac{\cos(2x+y)}{-6}} \quad \text{——— (2)}$$

2011,13 Q

$$(D-D'-1)(D-D'-2)z = \sin(2x+3y)$$

comparing with

$$(D-m_1D'-a_1)(D-m_2D'-a_2)z = f(x,y)$$

$$m_1 = 1,\ a_1 = 1,\ m_2 = 1,\ a_2 = 2$$

$$CF = e^{x}f_1(y+x) + e^{2x}f_2(y+x) \quad \text{——— (1)}$$

$$P.I = \frac{1}{(D-D'-1)(D-D'-2)}\sin(2x+3y)$$

$$a = 2,\ b = 3$$

$$P.I = \frac{1}{D^2-DD'-2D-DD'+D'^2+2D'-D+D'+2}\sin(2x+3y)$$

$$P.I = \frac{1}{D^2+D'^2-2DD'-3D+3D'+2}\sin(2x+3y)$$

$$P.I \quad \frac{1}{-4-9-2(-6)-3D+3D'+2}\sin(2x+3y)$$

$$P.I \quad \frac{1}{-3D+3D'+1}\sin(2x+3y)$$

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$$P.I = \frac{1}{3(D'-D)+1} \sin(2x+3y)$$

$$P.I \quad \frac{(3D'-3D-1)\sin(2x+3y)}{3^2(D'-D)^2 - 1}$$

$$P.I = \frac{(3D'-3D-1)\sin(2x+3y)}{9(D'^2+D^2-DD') - 1}$$

$$P.I = \frac{(3D'-3D-1)\sin 2x+3y}{9(-9)+9(-4)-(-6)-1}$$

$$P.I = \frac{(3D'-3D-1)\sin 2x+3y}{-112}$$

$$P.I = \frac{3D'(\sin 2x+3y) - 3D(\sin 2x+3y) - \sin 2x+3y}{-112}$$

$$P.I = \frac{3\times3\times\cos 2x+3y - 2\times3\times\cos 2x+3y - \sin 2x+3y}{-112}$$

$$P.I = \frac{3\cos 2x+3y - \sin 2x+3y}{-112} \quad \text{--- (2)}$$

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(VIII) Equation Reducible to Partial Differential equation with constant cofficients
OR
Linear PDE with Variable cofficients.

Let $\boxed{x = e^X}$, $\boxed{y = e^Y}$

$\log x = \log e^X$ $\qquad$ $\log y = \log e^Y$

$\boxed{\log x = X}$ $\qquad$ $\boxed{\log y = Y}$

Now $\quad x\dfrac{\partial}{\partial x} = D$

$$x^2\frac{\partial^2}{\partial x^2} = D(D-1)$$

$$x^3\frac{\partial^3}{\partial x^3} = D(D-1)(D-2)$$

Similarly $\quad y\dfrac{\partial}{\partial y} = D'$

$$y^2\frac{\partial^2}{\partial y^2} = D'(D'-1)$$

$$y^3\frac{\partial^3}{\partial y^3} = D'(D'-1)(D'-2) \ldots$$

And $\quad xy\dfrac{\partial^2}{\partial x\partial y} = DD'$

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Solve.

Q. $x^2 \frac{\partial^2 z}{\partial x^2} - 4xy \frac{\partial^2 z}{\partial x \partial y} + 4y^2 \frac{\partial^2 z}{\partial y^2} + 6y \frac{\partial z}{\partial y} = x^3 y^4$

let $x = e^X$, $y = e^Y$

so $D(D-1) - 4DD' + 4D'(D'-1) + 6D' = e^{3X} \cdot e^{4Y}$

$D^2 - D - 4DD' + 4D'^2 - 4D' + 6D' = e^{3X+4Y}$

$D^2 - D - 4DD' + 4D'^2 + 2D' = e^{3X+4Y}$

$[(D^2 - 4DD' + 4D'^2) - (D - 2D')]z = e^{3X+4Y}$

$[(D-2D')^2 - (D-2D')]z = e^{3X+4Y}$

$(D-2D')(D-2D'-1)z = e^{3X+4Y}$

comparing with

$(D - m_1 D' - a_1)(D - m_2 D' - a_2) z = 0$

$m_1 = 2, a_1 = 0, m_2 = 2, a_2 = 1$

$CF = f_1(y+2x) + e^x f_2(y+2x)$ —— (1)

$P.I = \frac{1}{(D-2D')(D-2D'-1)} e^{3X+4Y}$

$a = 3, b = 4$

$P.I = \frac{1}{(3-8)(3-8-1)} e^{3X+4Y}$

$P.I = \frac{1}{(-5)(-6)} e^{3X+4Y}$

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$$P.I = \frac{1}{30} e^{3X+4Y} \quad \text{--- (2)}$$

$$z = CF + PI$$

$$z = f_1(Y+2X) + e^{X} f_2(Y+2X) + \frac{1}{30} e^{3X+4Y}$$

$$z = f_1(\log y + 2\log x) + x f_2(\log y + 2\log x) + \frac{1}{30}\left[e^{3x} \cdot e^{4y}\right]$$

$$z = f_1(\log y + \log x^2) + x f_2(\log y + \log x^2) + \frac{1}{30}(x^3 y^4)$$

$$\left\{ e^{3x} = (\underset{x}{e^{x}})^3 = x^3 \right.$$

OR

$$z = \cancel{f_1(\log x^2)}$$

$$z = f_1(\log yx^2) + x f_2(\log yx^2) + \frac{1}{30}(x^3 y^4)$$

Q $$x^2 \frac{\partial^2 z}{\partial x^2} - y^2 \frac{\partial^2 z}{\partial y^2} = xy$$

$$x = e^{X}, \quad y = e^{Y}$$

$$D(D-1) - D'(D'-1) = e^{X} e^{Y}$$

$$D^2 - D - D'^2 + D' = e^{X+Y}$$

$$(D^2 - D'^2) - (D - D') = e^{X+Y}$$

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$$(D-D')(D+D') - (D-D') = e^{x+y}$$

$$(D-D')\left[(D+D'-1)\right] = e^{x+y}$$

comparing

$$m_1 = 1,\ a_1 = 0,\ m_2 = -1,\ a_1 = 1$$

$$CF = e^{0\cdot x} f_1(y+1x) + e^{-1\cdot x} f_2(y-x)$$

$$CF = f_1(y+x) + e^{-x} f_2(y-x) \quad \text{—— (1)}$$

$$P.I = \frac{1}{(D-D')(D+D'-1)} e^{x+y}$$

$$a = 1,\ b = 1$$

$$(1-1)(1+1-1) = 0$$

$$P.I = \frac{x}{2D-1} e^{x+y}$$

$$P.I = \frac{x \cdot (e^{x+y})}{2(1)-1}$$

$$\boxed{P.I = x \cdot e^{x+y}} \quad \text{—— (2)}$$

$$z = f_1(y+x) + e^{-x} f_2(y-x) + x \cdot e^{x+y}$$

$$z = f_1(\log y + \log x) + (-x) f_2(\log y - \log x) + \log x\,(e^x \cdot e^y)$$

$$\boxed{z = f_1(\log xy) + x f_2\left(\log \frac{y}{x}\right) + \log x\,(xy)}$$

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Q3. $x^2 r - y^2 t + px - qy = \log x$

$$x^2 \frac{\partial^2 z}{\partial x^2} - y^2 \frac{\partial^2 z}{\partial y^2} + x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = \log x$$

$$x = e^X \text{ and } y = e^Y$$

$$D(D-1) - D'(D'-1) + D - D' = X$$

$$D^2 - \cancel{D} - D'^2 + \cancel{D'} + \cancel{D} - \cancel{D'} = X$$

$$D^2 - D'^2 = X$$

$$(D-D')(D+D') = X$$

$$m_1 = 1,\ a_1 = 0,\ m_2 = -1,\ a_2 = 0$$

$$CF = f_1(Y+X) + f_2(Y-X) \quad \text{——— (1)}$$

$$P.I = \frac{1}{(D-D')(D+D')} X^1 \cdot Y^0$$

$$P.I = \frac{1}{D^2\left(1-\frac{D'}{D}\right)\left(1-\frac{D'}{D}\right)} X$$

$$P.I = \frac{1}{D^2}\left[\left(1-\frac{D'}{D}\right)\left(1-\frac{D'}{D}\right)\right]^{-1} X$$

$$P.I = \frac{1}{D^2}\left[\left(1+\frac{D'}{D}+\frac{D'^2}{D^2}+\cdots\right)\left(1-\frac{D'}{D}+\frac{D'^2}{D^2}\cdots\right)\right] X$$

$$P.I = \frac{1}{D^2}\left[\left(1-\frac{D'}{D}+\cdots\right)+\left(\frac{D'}{D}-\frac{D'^2}{D^2}+\cdots\right)\right] X$$

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$$P.I = \frac{1}{D^2}\left[\left(X - \frac{D'}{D}X + 0\right) + \left(X\frac{D'}{D} - \frac{D''}{D^2}X + 0\right)\right]$$

$$P.I = \frac{1}{D^2}\left[(X - 0) + (0)\right]$$

$$P.I = \frac{1}{D^2}(X) = \frac{1}{D}\int X\,dx$$

$$= \frac{1}{D}\left(\frac{X^2}{2}\right)$$

$$= \frac{1}{2}\left(\frac{X^3}{3}\right)$$

$$= \frac{1}{6}X^3$$

$$\boxed{P.I = \frac{1}{6}(\log x)^3} \quad \text{--- (2)}$$

$$Z = CF + PI$$

$$= f_1(Y+X) + f_2(Y-X) + \frac{1}{6}X^3$$

$$= f_1(\log y + \log x) + f_2(\log y - \log x) + \frac{1}{6}(\log x)^3$$

$$\boxed{Z = f_1(\log xy) + f_2\left(\log \frac{y}{x}\right) + \frac{1}{6}(\log x)^3}$$

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