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Tutorial : LPP

Q2) Convert the following LPP to the standard form

Maximise $z = 3x_1 + 4x_2 - 2x_3$

subject $6x_1 - 4x_2 \leq 5$

$3x_1 + x_2 + 4x_3 \geq 11$

$4x_1 + 3x_2 \leq 2$

$x_1 \geq 0, x_2 \geq 0$

→ ∵ the third decision variable is unrestricted. we write $x_3 = x_3' - x_3''$ where $x_3' \geq 0, x_3'' \geq 0$. Introducing slack variables $s_1, s_2, s_3 (\geq 0)$, we write the problem as

Maximise $z = 3x_1 + 4x_2 - 2x_3' + 2x_3'' + 0s_1 + 0s_2 + 0s_3$

subject to $6x_1 - 4x_2 + 0x_3 - 0x_3'' + s_1 + 0s_2 + 0s_3 = 5$

$3x_1 + x_2 + 4x_3' - 4x_3'' + 0s_1 - s_2 + 0s_3 = 11$

$4x_1 + 3x_2 + 0x_3' - 0x_3'' + 0s_1 + 0s_2 + s_3 = 2$

$x_1, x_2, x_3', x_3'', s_1, s_2, s_3 \geq 0$

Q6. Maximise $z = x_1 + x_2 + 3x_3$

subject to $x_1 + 2x_2 + 3x_3 = 9$

$3x_1 + 2x_2 + 2x_3 = 15$

Number of variables $(n) = 3$

Number of constraints $(m) = 2$

Number of basic solutions $= {}^nC_m = {}^3C_2 = 3$

We put $n-m = 3-2 = 1$ variables to 0

~~Non basic equ~~

	Non Basic variables	Basic variables	Eqn of value of basic	BFS	Degree of freedom
1)	$x_3 = 0$	x_1, x_2	$x_1 + 2x_2 = 9$ $3x_1 + 2x_2 = 15$ Values $x_1 = 3, x_2 = 3$	Yes $x_1, x_2 \geq 0$	$x_1 \neq 0$ $x_2 \neq 0$ No	6 No.
2)	$x_2 = 0$	x_1, x_3	$x_1 + 3x_3 = 9$ $3x_1 + 2x_3 = 15$ Values. $x_1 = 27/7$ $x_2 = 12/7$	Yes $x_1, x_2 \geq 0$	$x_1 \neq 0$ $x_2 \neq 0$ No.	9 Yes
3)	$x_1 = 0$	x_1, x_3	$2x_2 + 3x_3 = 9$ $2x_2 + 2x_3 = 15$ Values $x_2 = 27/2$, $x_3 = -6$	No $x_3 \neq 0$	$x_1 \neq 0$ $x_1 \neq 0$ No	$-9/2$ No.

Answers: $x_1 = 3, x_2 = 3$; $x_1 = 27/7, x_3 = 12/7$

$x_2 = 27/2, x_3 = -6$

Q (3) Maximise $z = 100x_1 + 50x_2 + 50x_3$

subject to $4x_1 + 3x_2 + 2x_3 \leq 10$

$3x_1 + 8x_2 + x_3 \leq 8$

$4x_1 + 2x_2 + x_3 \leq 6$

$x_1, x_2, x_3 < 0$

→ We first express the problem in the standard form

$Z - 100x_1 - 50x_2 - 50x_3 + 0s_1 + 0s_2 + 0s_3 = 0$

$4x_1 + 3x_2 + 2x_3 + s_1 + 0s_2 + 0s_3 = 10$

$3x_1 + 8x_2 + x_3 + 0s_1 + s_2 + 0s_3 = 8$

$4x_1 + 2x_2 + x_3 + 0s_1 + 0s_2 + s_3 = 6$

We put this information in tabular form as follows

Simplex Table.

Iteration Number	Basic variable	x_1	x_2	x_3	s_1	s_2	s_3	RHS soln	Ration
0	Z	-100	-50	-50	0	0	0	0	
s_2 leaves	s_1	4	3	2	1	0	0	10	$10/4 = 2.5$
x_1 enters	s_2	3	8	1	0	1	0	8	$8/3 = 2.6$
	s_3	4*	2	1	0	0	1	6	$6/4 = 1.5$
1	Z	0	0	-25	0	0	25	150	
s_1 leaves	s_1	0	1	1*	-1	0	-1	4	$4/1 = 4$
x_3 enters	s_2	0	13/2	1/4	0	1	-3/4	7/2	$7/2 \times 4/1 =$
	x_1	1	1/2	1/4	0	0	1/4	7/2	$3/2 \times 4/1 =$
2	Z	0	25	0	25	0	25	250	
	x_3	0	1	1	-1	0	-1	4	
	s_2	0	25/4	0	-1/4	1	-1/2	5/2	
	x_1	1	1/4	0	-1/4	0	1/2	1/2	

$\therefore x_1 = 1/2,\ x_2 = 0,\ x_3 = 4,\ Z_{max} = 250$

Q 27) Minimise $Z = 2x_1 + x_3$

subject to $3x_1 + x_2 = 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \leq 3$

$x_1, x_2 \geq 0$

→ We have

Maximise $Z' = -Z = -2x_1 - x_2 - 0s_2 - 0s_3 - MA_1 - MA_2$

subject to $3x_1 + x_3 + 0s_2 + 0s_3 + A_1 + 0A_2 = 3$

$4x_1 + 3x_2 - s_2 + 0s_3 + 0A_1 + A_2 = 6$

$x_1 + 2x_2 + 0s_2 + s_3 + 0A_1 + 0A_2 = 3$

Multiply (2) & (3) by M and to (1)

$\therefore$ Maximise $Z' = (-2+7M)x_1 + (-1+4M)x_2 - Ms_2 + 0s_3 - A_1 - 0A_2 - 9M$

$\therefore Z' + (2-7M)x_1 + (1-4M)x_2 + Ms_2 + 0x_3 - 0A_1 + 0A_1 = -9M$

Number (Iteration)	Basic var.	Coefficients of x_1	x_2	S_2	S_3	A_1	A_2	RHS soln	Ratio
0	z'	$2-7M$	$1-4M$	M	0	0	0	$-9M$	
A_1 leaves	A_1	3*	1	0	0	1	0	3	1
x_2 enters	A_2	4	3	−1	0	0	1	6	1.5
	S_3	1	2	0	1	0	0	3	3
1	z'	0	$\frac{1}{3}-\frac{5M}{3}$	M	0	0	0	$-2-2M$	
A_2 leaves	x_1	1	1/3	0	0	0	1	3	
x_2 enters	A_2	0	5/3*	−1	0	1	2	6/5	
	S_3	0	5/3	0	1	0	2	6/5	
2	z'	0	0	1/5	0			$-12/5$	
	x_1	1	0	1/5	0			3/5	
	x_2	0	1	−3/5	0			6/5	
	S_3	0	0	1	1			0	

$\therefore x_1 = \frac{3}{5}, \quad x_2 = \frac{6}{5}, \quad Z_{min} = \frac{12}{5}$