Math G180
Blank Lecture Notes
Chapter 4 – Sections 4.10

Selected Integral Formulas

INTEGRATION PROPERTIES

$$\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$$

$$\int_a^b f(x) \pm g(x)\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

$$\int_a^a f(x)\,dx = 0 \text{ and } \int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

$$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$$

COMMON INTEGRALS

$\int k\,dx = kx + C$	$\int \sec^2 x\,dx = \tan x + C$
$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C, n \neq -1$	$\int \sec x \tan x\,dx = \sec x + C$
$\int x^{-1}\,dx = \int \frac{1}{x}\,dx = \ln\lvert x\rvert + C$	$\int \csc x \cot x\,dx = -\csc x + C$
$\int \frac{1}{ax+b}\,dx = \frac{1}{a}\ln\lvert ax+b\rvert + C$	$\int \csc^2 x\,dx = -\cot x + C$
$\int \ln(x)\,dx = x\ln(x) - x + C$	$\int \tan x\,dx = \ln\lvert\sec x\rvert + C$
$\int e^x\,dx = e^x + C$	$\int \sec x\,dx = \ln\lvert\sec x + \tan x\rvert + C$
$\int \cos x\,dx = \sin x + C$	$\int \frac{1}{a^2+u^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$
$\int \sin x\,dx = -\cos x + C$	$\int \frac{1}{\sqrt{a^2-u^2}}\,dx = \sin^{-1}\left(\frac{u}{a}\right) + C$

Definition

Given a function f, the **indefinite integral** of f, denoted

$$\int f(x)dx,$$

is the most general antiderivative of f. If F is an antiderivative of f, then

$$\int f(x)dx = F(x) + C.$$

The expression $f(x)$ is called the *integrand* and the variable x is the *variable of integration*.

$$\int 2x dx = x^2 + C.$$

The collection of all functions of the form $x^2 + C$, where C is any real number, is known as the *family of antiderivatives of* $2x$. **Figure 4.85** shows a graph of this family of antiderivatives.

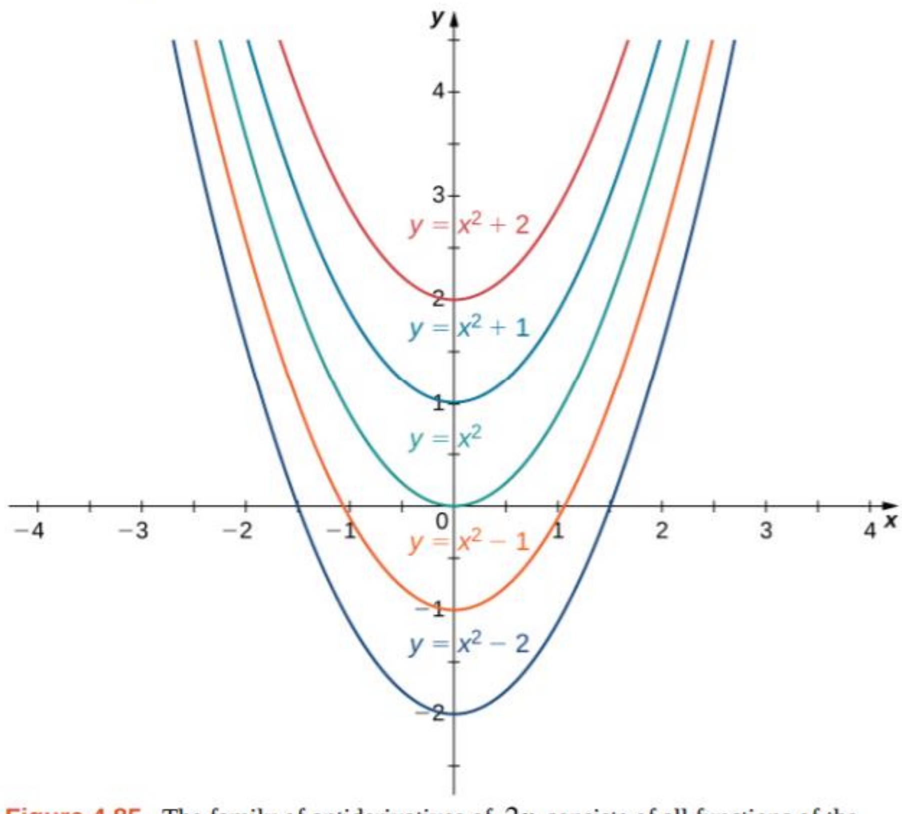

Figure 4.85 The family of antiderivatives of $2x$ consists of all functions of the form $x^2 + C$, where C is any real number.

Example 4.50

Finding Antiderivatives

For each of the following functions, find all antiderivatives.

a. $f(x) = 3x^2$

b. $f(x) = \frac{1}{x}$

c. $f(x) = \cos x$

d. $f(x) = e^x$

Differentiation Formula	Indefinite Integral
$\frac{d}{dx}(k) = 0$	$\int k dx = \int kx^0 dx = kx + C$
$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dn = \frac{x^{n+1}}{n+1} + C$ for $n \neq -1$
$\frac{d}{dx}(\ln\lvert x\rvert) = \frac{1}{x}$	$\int \frac{1}{x} dx = \ln\lvert x\rvert + C$
$\frac{d}{dx}(e^x) = e^x$	$\int e^x dx = e^x + C$
$\frac{d}{dx}(\sin x) = \cos x$	$\int \cos x dx = \sin x + C$
$\frac{d}{dx}(\cos x) = -\sin x$	$\int \sin x dx = -\cos x + C$
$\frac{d}{dx}(\tan x) = \sec^2 x$	$\int \sec^2 x dx = \tan x + C$
$\frac{d}{dx}(\csc x) = -\csc x \cot x$	$\int \csc x \cot x dx = -\csc x + C$
$\frac{d}{dx}(\sec x) = \sec x \tan x$	$\int \sec x \tan x dx = \sec x + C$
$\frac{d}{dx}(\cot x) = -\csc^2 x$	$\int \csc^2 x dx = -\cot x + C$
$\frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{1-x^2}} = \sin^{-1} x + C$
$\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}$	$\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$
$\frac{d}{dx}\left(\sec^{-1}\lvert x\rvert\right) = \frac{1}{x\sqrt{x^2-1}}$	$\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1}\lvert x\rvert + C$

Table 4.13 Integration Formulas

Evaluate each of the following indefinite integrals:

a. $\int \left(5x^3 - 7x^2 + 3x + 4\right) dx$

b. $\int \frac{x^2 + 4\sqrt[3]{x}}{x} dx$

c. $\int \frac{4}{1 + x^2} dx$

d. $\int \tan x \cos x \, dx$

Example 4.53

Solving an Initial-Value Problem

Solve the initial-value problem

$$\frac{dy}{dx} = \sin x, \; y(0) = 5.$$

4.10 EXERCISES

For the following exercises, show that $F(x)$ are antiderivatives of $f(x)$.

465. $F(x) = 5x^3 + 2x^2 + 3x + 1,\ f(x) = 15x^2 + 4x + 3$

466. $F(x) = x^2 + 4x + 1,\ f(x) = 2x + 4$

467. $F(x) = x^2 e^x,\ f(x) = e^x\left(x^2 + 2x\right)$

468. $F(x) = \cos x,\ f(x) = -\sin x$

469. $F(x) = e^x,\ f(x) = e^x$

For the following exercises, find the antiderivative of the function.

470. $f(x) = \frac{1}{x^2} + x$

471. $f(x) = e^x - 3x^2 + \sin x$

472. $f(x) = e^x + 3x - x^2$

473. $f(x) = x - 1 + 4\sin(2x)$

For the following exercises, find the antiderivative $F(x)$ of each function $f(x)$.

474. $f(x) = 5x^4 + 4x^5$

475. $f(x) = x + 12x^2$

476. $f(x) = \frac{1}{\sqrt{x}}$

477. $f(x) = (\sqrt{x})^3$

478. $f(x) = x^{1/3} + (2x)^{1/3}$

479. $f(x) = \frac{x^{1/3}}{x^{2/3}}$

480. $f(x) = 2\sin(x) + \sin(2x)$

481. $f(x) = \sec^2(x) + 1$

482. $f(x) = \sin x \cos x$

483. $f(x) = \sin^2(x)\cos(x)$

484. $f(x) = 0$

485. $f(x) = \frac{1}{2}\csc^2(x) + \frac{1}{x^2}$

486. $f(x) = \csc x \cot x + 3x$

487. $f(x) = 4\csc x \cot x - \sec x \tan x$

488. $f(x) = 8\sec x(\sec x - 4\tan x)$

489. $f(x) = \frac{1}{2}e^{-4x} + \sin x$

For the following exercises, evaluate the integral.

490. $\int(-1)dx$

491. $\int \sin x\, dx$

492. $\int(4x + \sqrt{x})dx$

493. $\int \frac{3x^2 + 2}{x^2}dx$

494. $\int(\sec x \tan x + 4x)dx$

495. $\int\left(4\sqrt{x} + \sqrt[4]{x}\right)dx$

496. $\int\left(x^{-1/3} - x^{2/3}\right)dx$

497. $\int \frac{14x^3 + 2x + 1}{x^3}dx$

498. $\int(e^x + e^{-x})dx$

For the following exercises, solve the initial value problem.

499. $f'(x) = x^{-3},\ f(1) = 1$

500. $f'(x) = \sqrt{x} + x^2,\ f(0) = 2$

501. $f'(x) = \cos x + \sec^2(x),\ f\left(\frac{\pi}{4}\right) = 2 + \frac{\sqrt{2}}{2}$

502. $f'(x) = x^3 - 8x^2 + 16x + 1,\ f(0) = 0$