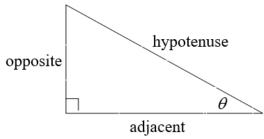
Definition of the Trig Functions

Right triangle definition

For this definition we assume that

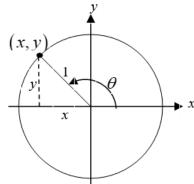
$$0<\theta<\frac{\pi}{2} \text{ or } 0^{\circ}<\theta<90^{\circ}.$$



$$\begin{split} &\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} & \csc(\theta) = \frac{\text{hypotenuse}}{\text{opposite}} \\ &\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}} & \sec(\theta) = \frac{\text{hypotenuse}}{\text{adjacent}} \\ &\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}} & \cot(\theta) = \frac{\text{adjacent}}{\text{opposite}} \end{split}$$

Unit Circle Definition

For this definition θ is any angle.



$$\begin{split} \sin(\theta) &= \frac{y}{1} = y & \csc(\theta) &= \frac{1}{y} \\ \cos(\theta) &= \frac{x}{1} = x & \sec(\theta) &= \frac{1}{x} \\ \tan(\theta) &= \frac{y}{x} & \cot(\theta) &= \frac{x}{y} \end{split}$$

Facts and Properties

Domain

The domain is all the values of θ that can be plugged into the function.

 $\sin(\theta)$, θ can be any angle

 $cos(\theta)$, θ can be any angle

$$\tan(\theta),\, \theta
eq \left(n+\frac{1}{2}\right)\pi,\,\, n=0,\pm 1,\pm 2,\ldots$$

$$\csc(\theta),\,\theta\neq n\pi,\,\,n=0,\,\pm1,\,\,\pm2,\dots$$

$$\sec(\theta), \ \theta \neq \left(n + \frac{1}{2}\right)\pi, \ n = 0, \pm 1, \pm 2, \dots$$

$$\cot(\theta), \ \theta \neq n\pi, \ n = 0, \pm 1, \pm 2, \dots$$

Period

The period of a function is the number, T, such that $f\left(\theta+T\right)=f\left(\theta\right)$. So, if ω is a fixed number and θ is any angle we have the following periods.

$$\begin{array}{cccc} \sin{(\omega\,\theta)} & \to & T = \frac{2\pi}{\omega} \\ \cos{(\omega\,\theta)} & \to & T = \frac{2\pi}{\omega} \\ \tan{(\omega\,\theta)} & \to & T = \frac{\pi}{\omega} \\ \csc{(\omega\,\theta)} & \to & T = \frac{2\pi}{\omega} \\ \sec{(\omega\,\theta)} & \to & T = \frac{2\pi}{\omega} \\ \cot{(\omega\,\theta)} & \to & T = \frac{\pi}{\omega} \end{array}$$

Range

The range is all possible values to get out of the function.

$$\begin{aligned} -1 & \leq \sin(\theta) \leq 1 & -1 \leq \cos(\theta) \leq 1 \\ -\infty & < \tan(\theta) < \infty & -\infty & < \cot(\theta) < \infty \\ \sec(\theta) & \geq 1 \text{ and } \sec(\theta) \leq -1 & \csc(\theta) \geq 1 \text{ and } \csc(\theta) \leq -1 \end{aligned}$$

Formulas and Identities

Tangent and Cotangent Identities

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$
 $\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$

Reciprocal Identities

$$\begin{aligned} & \csc(\theta) = \frac{1}{\sin(\theta)} & \sin(\theta) = \frac{1}{\csc(\theta)} \\ & \sec(\theta) = \frac{1}{\cos(\theta)} & \cos(\theta) = \frac{1}{\sec(\theta)} \\ & \cot(\theta) = \frac{1}{\tan(\theta)} & \tan(\theta) = \frac{1}{\cot(\theta)} \end{aligned}$$

Pythagorean Identities

$$\sin^{2}(\theta) + \cos^{2}(\theta) = 1$$

$$\tan^{2}(\theta) + 1 = \sec^{2}(\theta)$$

$$1 + \cot^{2}(\theta) = \csc^{2}(\theta)$$

Even/Odd Formulas

$$\begin{aligned} \sin(-\theta) &= -\sin(\theta) & \csc(-\theta) &= -\csc(\theta) \\ \cos(-\theta) &= \cos(\theta) & \sec(-\theta) &= \sec(\theta) \\ \tan(-\theta) &= -\tan(\theta) & \cot(-\theta) &= -\cot(\theta) \end{aligned}$$

Periodic Formulas

If n is an integer then,

$$\sin(\theta + 2\pi n) = \sin(\theta)$$
 $\csc(\theta + 2\pi n) = \csc(\theta)$
 $\cos(\theta + 2\pi n) = \cos(\theta)$ $\sec(\theta + 2\pi n) = \sec(\theta)$
 $\tan(\theta + \pi n) = \tan(\theta)$ $\cot(\theta + \pi n) = \cot(\theta)$

Degrees to Radians Formulas

If x is an angle in degrees and t is an angle in radians then

$$\frac{\pi}{180} = \frac{t}{x}$$
 \Rightarrow $t = \frac{\pi x}{180}$ and $x = \frac{180t}{\pi}$

Double Angle Formulas

$$\begin{aligned} \sin(2\theta) &= 2\sin(\theta)\cos(\theta) \\ \cos(2\theta) &= \cos^2(\theta) - \sin^2(\theta) \\ &= 2\cos^2(\theta) - 1 \\ &= 1 - 2\sin^2(\theta) \\ \tan(2\theta) &= \frac{2\tan(\theta)}{1 - \tan^2(\theta)} \end{aligned}$$

Half Angle Formulas

$$\begin{split} &\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos(\theta)}{2}} \\ &\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos(\theta)}{2}} \\ &\tan\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos(\theta)}{1+\cos(\theta)}} \end{split}$$

Half Angle Formulas (alternate form)

$$\begin{aligned} &\sin^2(\theta) = \frac{1}{2} \left(1 - \cos(2\theta)\right) \\ &\cos^2(\theta) = \frac{1}{2} \left(1 + \cos(2\theta)\right) \end{aligned} \\ &\tan^2(\theta) = \frac{1 - \cos(2\theta)}{1 + \cos(2\theta)} \end{aligned}$$

Sum and Difference Formulas

$$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$$

$$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$$

$$\tan(\alpha \pm \beta) = \frac{\tan(\alpha) \pm \tan(\beta)}{1 \mp \tan(\alpha)\tan(\beta)}$$

Product to Sum Formulas

$$\begin{aligned} &\sin(\alpha)\sin(\beta) = \frac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right] \\ &\cos(\alpha)\cos(\beta) = \frac{1}{2}\left[\cos(\alpha-\beta) + \cos(\alpha+\beta)\right] \\ &\sin(\alpha)\cos(\beta) = \frac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right] \\ &\cos(\alpha)\sin(\beta) = \frac{1}{2}\left[\sin(\alpha+\beta) - \sin(\alpha-\beta)\right] \end{aligned}$$

Sum to Product Formulas

Degrees to Radians Formulas
$$\sin(\alpha) + \sin(\beta) = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$
 If x is an angle in degrees and t is an angle in radians then
$$\sin(\alpha) - \sin(\beta) = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

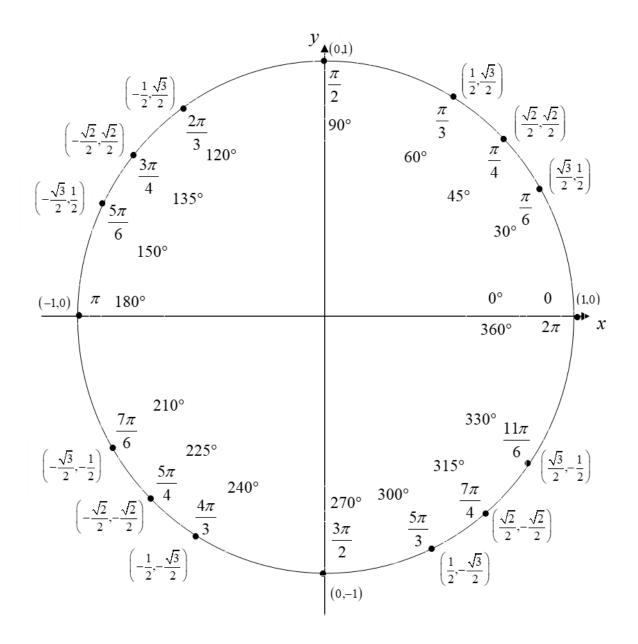
$$\sin(\alpha) - \sin(\beta) = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos(\alpha) + \cos(\beta) = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$
 Double Angle Formulas
$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

$$\cos(\alpha) - \cos(\beta) = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

Cofunction Formulas

$$\begin{split} \sin\left(\frac{\pi}{2} - \theta\right) &= \cos(\theta) & \cos\left(\frac{\pi}{2} - \theta\right) &= \sin(\theta) \\ \csc\left(\frac{\pi}{2} - \theta\right) &= \sec(\theta) & \sec\left(\frac{\pi}{2} - \theta\right) &= \csc(\theta) \\ \tan\left(\frac{\pi}{2} - \theta\right) &= \cot(\theta) & \cot\left(\frac{\pi}{2} - \theta\right) &= \tan(\theta) \end{split}$$



For any ordered pair on the unit circle (x, y): $cos(\theta) = x$ and $sin(\theta) = y$

Example

$$\cos\left(\frac{5\pi}{3}\right) = \frac{1}{2} \qquad \qquad \sin\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

Inverse Trig Functions

Definition

$$y = \sin^{-1}(x)$$
 is equivalent to $x = \sin(y)$

$$y = \cos^{-1}(x)$$
 is equivalent to $x = \cos(y)$

$$y = \tan^{-1}(x)$$
 is equivalent to $x = \tan(y)$

Domain and Range

$$\begin{array}{lll} & \text{Function} & \text{Domain} & \text{Range} & \sin^{-1}(x) = \arcsin(x) \\ y = \sin^{-1}(x) & -1 \leq x \leq 1 & -\frac{\pi}{2} \leq y \leq \frac{\pi}{2} & \cos^{-1}(x) = \arccos(x) \\ y = \cos^{-1}(x) & -1 \leq x \leq 1 & 0 \leq y \leq \pi & \tan^{-1}(x) = \arctan(x) \\ y = \tan^{-1}(x) & -\infty < x < \infty & -\frac{\pi}{2} < y < \frac{\pi}{2} \end{array}$$

Inverse Properties

$$\cos\left(\cos^{-1}(x)\right) = x \quad \cos^{-1}\left(\cos(\theta)\right) = \theta$$

$$\sin\left(\sin^{-1}(x)\right) = x \qquad \sin^{-1}\left(\sin(\theta)\right) = \theta$$

$$\tan\left(\tan^{-1}(x)\right) = x \quad \tan^{-1}\left(\tan(\theta)\right) = \theta$$

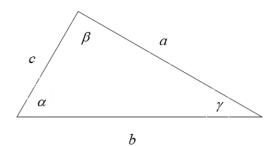
Alternate Notation

$$\sin^{-1}(x) = \arcsin(x)$$

$$\cos^{-1}(x) = \arccos(x)$$

$$tan^{-1}(x) = arctan(x)$$

Law of Sines, Cosines and Tangents



Law of Sines

$$\frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b} = \frac{\sin(\gamma)}{c}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos(\alpha)$$

$$b^2 = a^2 + c^2 - 2ac\cos(\beta)$$

$$c^2 = a^2 + b^2 - 2ab\cos(\gamma)$$

Law of Tangents

$$\frac{a-b}{a+b} = \frac{\tan\left(\frac{1}{2}(\alpha-\beta)\right)}{\tan\left(\frac{1}{2}(\alpha+\beta)\right)}$$

$$\frac{b-c}{b+c} = \frac{\tan\left(\frac{1}{2}(\beta - \gamma)\right)}{\tan\left(\frac{1}{2}(\beta + \gamma)\right)}$$

$$\frac{a-c}{a+c} = \frac{\tan\left(\frac{1}{2}(\alpha-\gamma)\right)}{\tan\left(\frac{1}{2}(\alpha+\gamma)\right)}$$

Mollweide's Formula

$$\frac{a+b}{c} = \frac{\cos\left(\frac{1}{2}(\alpha-\beta)\right)}{\sin\left(\frac{1}{2}\gamma\right)}$$