Math G180
Blank Lecture Notes
Chapter 1 – Section 1.1

1.1 | Review of Functions

Learning Objectives

- 1.1.1 Use functional notation to evaluate a function.
- 1.1.2 Determine the domain and range of a function.
- 1.1.3 Draw the graph of a function.
- 1.1.4 Find the zeros of a function.
- 1.1.5 Recognize a function from a table of values.
- 1.1.6 Make new functions from two or more given functions.
- 1.1.7 Describe the symmetry properties of a function.

In this section, we provide a formal definition of a function and examine several ways in which functions are represented—namely, through tables, formulas, and graphs. We study formal notation and terms related to functions. We also define composition of functions and symmetry properties. Most of this material will be a review for you, but it serves as a handy reference to remind you of some of the algebraic techniques useful for working with functions.

Functions

Given two sets A and $B,$ a set with elements that are ordered pairs $(x, y),$ where x is an element of A and y is an element of $B,$ is a relation from A to $B.$ A relation from A to B defines a relationship between those two sets. A function is a special type of relation in which each element of the first set is related to exactly one element of the second set. The element of the first set is called the *input*; the element of the second set is called the *output*. Functions are used all the time in mathematics to describe relationships between two sets. For any function, when we know the input, the output is determined, so we say that the output is a function of the input. For example, the area of a square is determined by its side length, so we say that the area (the output) is a function of its side length (the input). The velocity of a ball thrown in the air can be described as a function of the amount of time the ball is in the air. The cost of mailing a package is a function of the weight of the package. Since functions have so many uses, it is important to have precise definitions and terminology to study them.

Definition

A **function** f consists of a set of inputs, a set of outputs, and a rule for assigning each input to exactly one output. The set of inputs is called the **domain** of the function. The set of outputs is called the **range** of the function.

Figure 1.2 A function can be visualized as an input/output device.

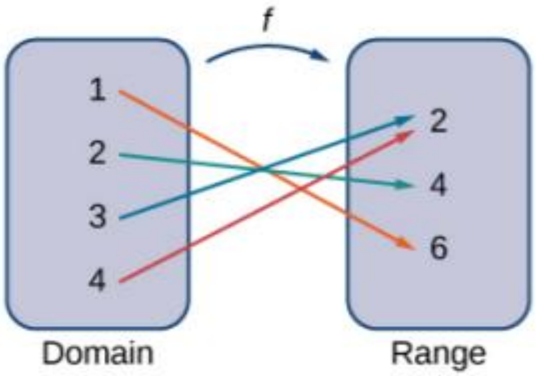


Figure 1.3 A function maps every element in the domain to exactly one element in the range. Although each input can be sent to only one output, two different inputs can be sent to the same output.

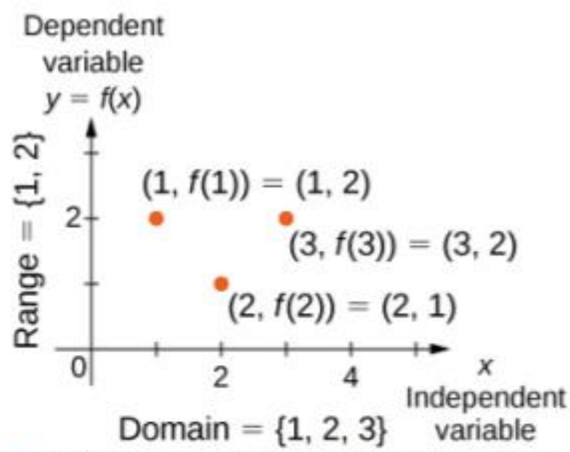


Figure 1.4 In this case, a graph of a function f has a domain of $\{1, 2, 3\}$ and a range of $\{1, 2\}$. The independent variable is x and the dependent variable is y.

Rule: Vertical Line Test

Given a function f, every vertical line that may be drawn intersects the graph of f no more than once. If any vertical line intersects a set of points more than once, the set of points does not represent a function.

Example 1.1

Evaluating Functions

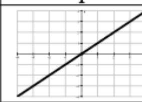
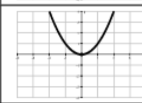
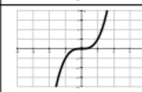
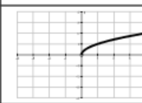
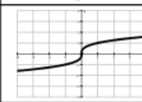
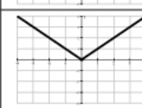
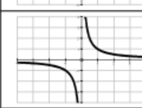
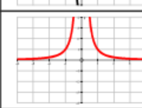
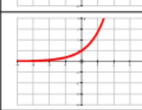
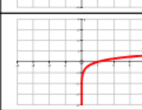
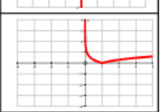
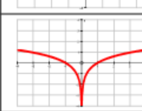
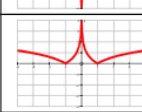
For the function $f(x) = 3x^2 + 2x - 1,$ evaluate

a. $f(-2)$

b. $f(\sqrt{2})$

c. $f(a + h)$

Important reference functions and their properties

Function	X-intercept	Y-intercept	Domain	Range	End Behavior	Other	Graph
$y = x$	(0,0)	(0,0)	$(-\infty,\infty)$	$(-\infty,\infty)$	$\lim\limits_{x\to\infty} y = \infty$ $\lim\limits_{x\to-\infty} y = -\infty$	$\lim\limits_{x\to0^+} y = 0$ $\lim\limits_{x\to0^-} y = 0$	
$y = x^2$	(0,0)	(0,0)	$(-\infty,\infty)$	$[0,\infty)$	$\lim\limits_{x\to\infty} y = \infty$ $\lim\limits_{x\to-\infty} y = \infty$	$\lim\limits_{x\to0^+} y = 0$ $\lim\limits_{x\to0^-} y = 0$	
$y = x^3$	(0,0)	(0,0)	$(-\infty,\infty)$	$(-\infty,\infty)$	$\lim\limits_{x\to\infty} y = \infty$ $\lim\limits_{x\to-\infty} y = -\infty$	$\lim\limits_{x\to0^+} y = 0$ $\lim\limits_{x\to0^-} y = 0$	
$y = \sqrt{x}$	(0,0)	(0,0)	$[0,\infty)$	$[0,\infty)$	$\lim\limits_{x\to\infty} y = \infty$ $\lim\limits_{x\to0^+} y = 0$	$\lim\limits_{x\to0^+} y = 0$ $\lim\limits_{x\to0^-} y = DNE$	
$y = \sqrt[3]{x}$	(0,0)	(0,0)	$(-\infty,\infty)$	$(-\infty,\infty)$	$\lim\limits_{x\to\infty} y = \infty$ $\lim\limits_{x\to-\infty} y = -\infty$	$\lim\limits_{x\to0^+} y = 0$ $\lim\limits_{x\to0^-} y = 0$	
$y = \|x\|$	(0,0)	(0,0)	$(-\infty,\infty)$	$[0,\infty)$	$\lim\limits_{x\to\infty} y = \infty$ $\lim\limits_{x\to-\infty} y = \infty$	$\lim\limits_{x\to0^+} y = 0$ $\lim\limits_{x\to0^-} y = 0$	
$y = \frac{1}{x}$	None	None	$\{X \mid X \neq 0\}$	$\{Y \mid Y \neq 0\}$	$\lim\limits_{x\to\infty} y = 0$ $\lim\limits_{x\to-\infty} y = 0$	$\lim\limits_{x\to0^+} y = \infty$ $\lim\limits_{x\to0^-} y = -\infty$	
$y = \frac{1}{x^2}$	None	None	$\{X \mid X \neq 0\}$	$(0,\infty)$	$\lim\limits_{x\to\infty} y = 0$ $\lim\limits_{x\to-\infty} y = 0$	$\lim\limits_{x\to0^+} y = \infty$ $\lim\limits_{x\to0^-} y = \infty$	
$y = e^x$	None	(0,1)	$(-\infty,\infty)$	$(0,\infty)$	$\lim\limits_{x\to\infty} y = \infty$ $\lim\limits_{x\to-\infty} y = 0$	$\lim\limits_{x\to0^+} y = 1$ $\lim\limits_{x\to0^-} y = 1$	
$y = \log(x)$	(1,0)	None	$(0,\infty)$	$(-\infty,\infty)$	$\lim\limits_{x\to\infty} y = \infty$ $\lim\limits_{x\to0^+} y = -\infty$	$\lim\limits_{x\to0^+} y = -\infty$ $\lim\limits_{x\to0^-} y = DNE$	
$y = \|\log(x)\|$	(1,0)	None	$(0,\infty)$	$[0,\infty)$	$\lim\limits_{x\to\infty} y = \infty$ $\lim\limits_{x\to0^+} y = \infty$	$\lim\limits_{x\to0^+} y = \infty$ $\lim\limits_{x\to0^-} y = DNE$	
$y = \log(x^2)$	(-1,0),(1,0)	None	$\{X \mid X \neq 0\}$	$(-\infty,\infty)$	$\lim\limits_{x\to\infty} y = \infty$ $\lim\limits_{x\to-\infty} y = \infty$	$\lim\limits_{x\to0^+} y = -\infty$ $\lim\limits_{x\to0^-} y = -\infty$	
$y = \|\log(x^2)\|$	(-1,0),(1,0)	None	$\{X \mid X \neq 0\}$	$[0,\infty)$	$\lim\limits_{x\to\infty} y = \infty$ $\lim\limits_{x\to-\infty} y = \infty$	$\lim\limits_{x\to0^+} y = \infty$ $\lim\limits_{x\to0^-} y = \infty$	

Definition

If $f(x) = f(-x)$ for all x in the domain of f, then f is an **even function**. An even function is symmetric about the y-axis.

If $f(-x) = -f(x)$ for all x in the domain of f, then f is an **odd function**. An odd function is symmetric about the origin.

For the following exercises, find the values for each function, if they exist, then simplify.

a. $f(0)$ b. $f(1)$ c. $f(3)$ d. $f(-x)$ e. $f(a)$ f. $f(a+h)$

7. $f(x) = 5x - 2$

9. $f(x) = \frac{2}{x}$

10. $f(x) = |x - 7| + 8$

11. $f(x) = \sqrt{6x + 5}$

12. $f(x) = \frac{x - 2}{3x + 7}$

13. $f(x) = 9$

For the following exercises, find the domain, range, and all zeros/intercepts, if any, of the functions.

14. $f(x) = \frac{x}{x^2 - 16}$

15. $g(x) = \sqrt{8x - 1}$

16. $h(x) = \frac{3}{x^2 + 4}$

17. $f(x) = -1 + \sqrt{x + 2}$

18. $f(x) = \frac{1}{\sqrt{x - 9}}$

19. $g(x) = \frac{3}{x - 4}$

20. $f(x) = 4|x + 5|$

21. $g(x) = \sqrt{\frac{7}{x - 5}}$

For the following exercises, use the vertical line test to determine whether each of the given graphs represents a function. **Assume that a graph continues at both ends if it extends beyond the given grid.** If the graph represents a function, then determine the following for each graph:

a. Domain and range

b. x-intercept, if any (estimate where necessary)

c. y-Intercept, if any (estimate where necessary)

d. The intervals for which the function is increasing

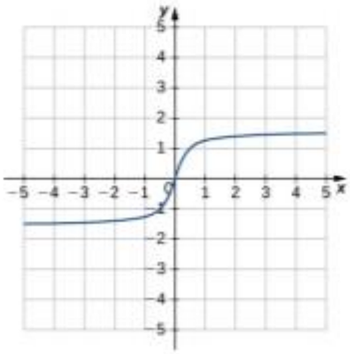
e. The intervals for which the function is decreasing

f. The intervals for which the function is constant

g. Symmetry about any axis and/or the origin

h. Whether the function is even, odd, or neither

31.



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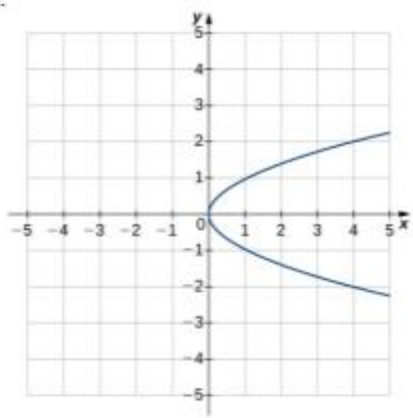

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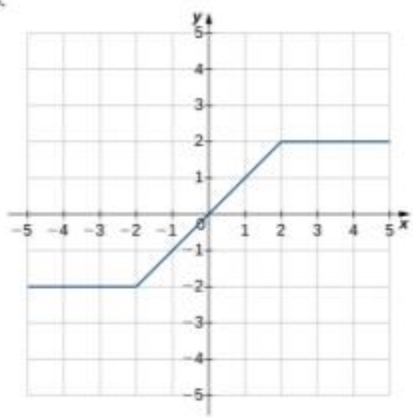

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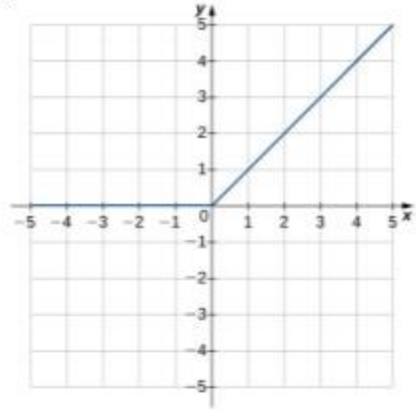

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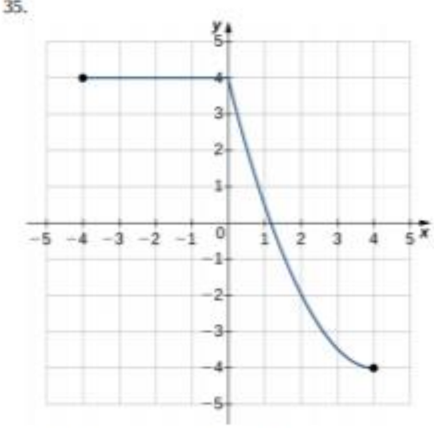

For the following exercises, for each pair of functions, find a. $f + g$ b. $f - g$ c. $f \cdot g$ d. f/g. Determine the domain of each of these new functions.

36. $f(x) = 3x + 4,\ g(x) = x - 2$

37. $f(x) = x - 8,\ g(x) = 5x^2$

38. $f(x) = 3x^2 + 4x + 1,\ g(x) = x + 1$

39. $f(x) = 9 - x^2,\ g(x) = x^2 - 2x - 3$

40. $f(x) = \sqrt{x},\ g(x) = x - 2$

41. $f(x) = 6 + \frac{1}{x},\ g(x) = \frac{1}{x}$