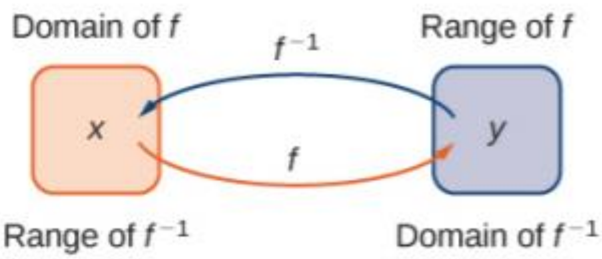
Math G180
Blank Lecture Notes
Chapter 1 – Section 1.4, 1.5

1.4 | Inverse Functions



Problem-Solving Strategy: Finding an Inverse Function

1. Solve the equation $y = f(x)$ for x.
2. Interchange the variables x and y and write $y = f^{-1}(x)$.

Definition

We say a f is a **one-to-one function** if $f(x_1) \neq f(x_2)$ when $x_1 \neq x_2$.

One way to determine whether a function is one-to-one is by looking at its graph. If a function is one-to-one, then no two inputs can be sent to the same output. Therefore, if we draw a horizontal line anywhere in the xy-plane, according to the **horizontal line test**, it cannot intersect the graph more than once. We note that the horizontal line test is different from the vertical line test. The vertical line test determines whether a graph is the graph of a function. The horizontal line test determines whether a function is one-to-one (**Figure 1.38**).

Rule: Horizontal Line Test

A function f is one-to-one if and only if every horizontal line intersects the graph of f no more than once.

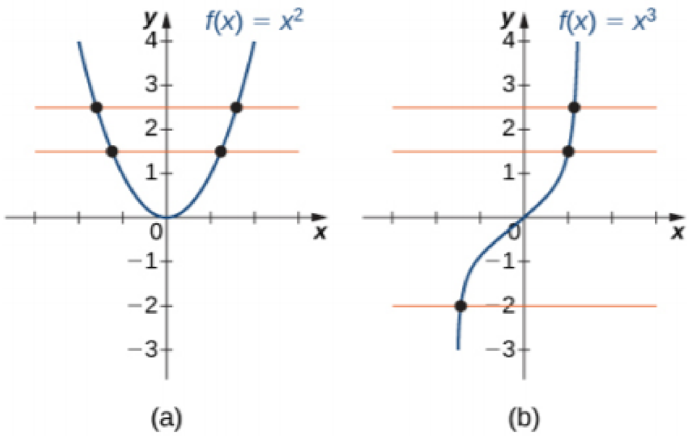
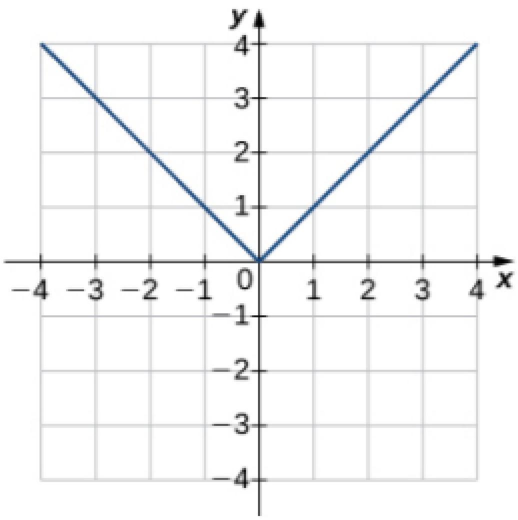


Figure 1.38 (a) The function $f(x) = x^2$ is not one-to-one because it fails the horizontal line test. (b) The function $f(x) = x^3$ is one-to-one because it passes the horizontal line test.

1.4 EXERCISES

For the following exercises, use the horizontal line test to determine whether each of the given graphs is one-to-one.

183.



For the following exercises, a. find the inverse function, and b. find the domain and range of the inverse function.

189. $f(x) = x^2 - 4,\ x \geq 0$

190. $f(x) = \sqrt[3]{x - 4}$

191. $f(x) = x^3 + 1$

1.5 | Exponential and Logarithmic Functions

Property	Example	"Proof"/Explanation
$(xy)^m = x^m \cdot y^m$	$(xy)^3 = x^3y^3$	$(xy)^3 = xy \cdot xy \cdot xy = (x \cdot x \cdot x)\cdot(y \cdot y \cdot y) = x^3y^3$
$\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$	$\left(\frac{x}{y}\right)^4 = \frac{x^4}{y^4}$	$\left(\frac{x}{y}\right)^4 = \frac{x}{y}\cdot\frac{x}{y}\cdot\frac{x}{y}\cdot\frac{x}{y} = \frac{x \cdot x \cdot x \cdot x}{y \cdot y \cdot y \cdot y} = \frac{x^4}{y^4}$
$x^m \cdot x^n = x^{m+n}$	$x^4 \cdot x^2 = x^6$	$x^4 \cdot x^2 = (x \cdot x \cdot x \cdot x)\cdot(x \cdot x) = x \cdot x \cdot x \cdot x \cdot x \cdot x = x^6$
$\frac{x^m}{x^n} = x^{m-n}$	$\frac{x^5}{x^3} = x^2$	$\frac{x^5}{x^3} = \frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x \cdot x} = \frac{x \cdot x \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x}}{\cancel{x} \cdot \cancel{x} \cdot \cancel{x}} = x \cdot x = x^2$
$\left(x^m\right)^n = x^{mn}$	$\left(x^4\right)^2 = x^8$	$(x^4)^2 = x^4 \cdot x^4 = (x \cdot x \cdot x \cdot x)\cdot(x \cdot x \cdot x \cdot x) = x^8$
$x^1 = x$	$473{,}837{,}843^1 = 473{,}837{,}843$	$x^1 = x$ multiplied by itself 1 time $= x$
$x^0 = 1$	$473{,}837{,}843^0 = 1$	$1 = \frac{x^5}{x^5} = x^{5-5} = x^0$
$\frac{1}{x^m} = x^{-m}$	$\frac{1}{3^2} = 3^{-2} = \frac{1}{8}$	$\frac{1}{2^3} = \frac{2^0}{2^3} = 2^{0-3} = 2^{-3}$
$\sqrt[n]{x} = x^{\frac{1}{n}}$	$\sqrt[3]{8} = 8^{\frac{1}{3}} = 2$	Accept that the *n*th root of a base can be written as that base raised to the reciprocal of *n*, or $\frac{1}{n}$.
$\sqrt[n]{xy} = \sqrt[n]{x} \cdot \sqrt[n]{y}$	$\sqrt{72} = \sqrt{4 \cdot 9 \cdot 2} = \sqrt{4} \cdot \sqrt{9} \cdot \sqrt{2}$ $= 2 \cdot 3 \cdot \sqrt{2} = 6\sqrt{2}$	$\sqrt{xy} = (xy)^{\frac{1}{2}} = x^{\frac{1}{2}} \cdot y^{\frac{1}{2}} = \sqrt{x} \cdot \sqrt{y}$ (Doesn't work for imaginary numbers under radicals.)
$\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$	$\sqrt[3]{\frac{27}{8}} = \frac{\sqrt[3]{27}}{\sqrt[3]{8}} = \frac{3}{2}$	$\sqrt[3]{\frac{x^3}{y^3}} = \sqrt[3]{\frac{x \cdot x \cdot x}{y \cdot y \cdot y}} = \sqrt[3]{\frac{x}{y}} \cdot \sqrt[3]{\frac{x}{y}} \cdot \sqrt[3]{\frac{x}{y}} = \frac{x}{y} = \frac{\sqrt[3]{x^3}}{\sqrt[3]{y^3}}$
$\left(\sqrt[n]{x}\right)^m = \sqrt[n]{x^m} = x^{\frac{m}{n}}$ (if *n* is even, $x \geq 0$)	$8^{\frac{2}{3}} = \sqrt[3]{8^2} = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$	Remember that the number inside the root is always on the bottom of the fraction, and the exponent is on top. (I remember that the root is in a cave so it's on the bottom).
n is **odd**: $\left(\sqrt[n]{x}\right)^n = \sqrt[n]{x^n} = x$	$\left(\sqrt[3]{-2}\right)^3 = \sqrt[3]{(-2)^3} = \sqrt[3]{-8} = -2$	$\left(\sqrt[5]{x}\right)^5 = \sqrt[5]{x^5} = x^{\frac{5}{5}} = x^1 = x$ Note that this works when *n* is even too, if $x \geq 0$.
n is **even**: $\sqrt[n]{x^n} = \|x\|$	$\sqrt{(-4)^2} = \sqrt{16} = 4$ $\sqrt{(4)^2} = \sqrt{16} = 4$	$\sqrt[4]{(\text{neg number } x)^4} = \sqrt[4]{\text{pos number } x^4}$ $= -(x) = \|x\|$ Note that for even radicals, if you move any variable to the outside, and it's raised to an odd power, you have to use the absolute value for that variable, if variables aren't assumed to be positive.
$\frac{x}{\sqrt{y}} = \frac{x}{\sqrt{y}} \cdot \frac{\sqrt{y}}{\sqrt{y}} = \frac{x\sqrt{y}}{y}$	$\frac{4}{\sqrt{2}} = \frac{4}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{{}^{2}\cancel{4}\sqrt{2}}{{}^{1}\cancel{2}} = 2\sqrt{2}$	This is called "**rationalizing**" the denominator (getting rid of the radical in the denominator) and is considered better "grammar" in math.

Properties of Logarithms and Natural Logarithms

General Logarithm Base b
$\mathbf{log}_{\mathbf{b}}\boldsymbol{x} = \boldsymbol{y}$ $\mathbf{b}^{\boldsymbol{y}} = \boldsymbol{x}$
$\log_b 1 = 0$
$\log_b b = 1$
$\log_b b^x = x \quad$ for all x
$b^{\log x} = x \qquad x > 0$
<u>Product Property</u> $\log_b AB = \log_b A + \log_b B$ and $\log_b A + \log_b B = \log_b AB$
<u>Quotient Property</u> $\log_b \frac{\mathbf{A}}{\mathbf{B}} = \log_b A - \log_b B$ and $\log_b A - \log_b B = \log_b \frac{\mathbf{A}}{\mathbf{B}}$
<u>Power Property</u> $\log_b B^t = t \log_b B$ and $t \log_b B = \log_b B^t$

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Properties of Logarithms and Natural Logarithms

Natural Logarithm Base e
$\ln x = y$ $e^y = x$
$\ln 1 = 0$
$\ln e = 1$
$\ln e^x = x \quad$ for all x
$e^{\ln x} = x \qquad x > 0$
Product Property $\ln AB = \ln A + \ln B$ and $\ln A + \ln B = \ln AB$
Quotient Property $\ln \frac{A}{B} = \ln A - \ln B$ and $\ln A - \ln B = \ln \frac{A}{B}$
Power Property $\ln B^t = t \ln B$ and $t \ln B = \ln B^t$

For the following exercises, solve the logarithmic equation exactly, if possible.

284. $\log_3 x = 0$

285. $\log_5 x = -2$

286. $\log_4(x+5) = 0$

287. $\log(2x-7) = 0$

288. $\ln\sqrt{x+3} = 2$

289. $\log_6(x+9) + \log_6 x = 2$

290. $\log_4(x+2) - \log_4(x-1) = 0$

291. $\ln x + \ln(x-2) = \ln 4$