Math G180
Blank Lecture Notes
Chapter 1 – Section 1.3

1.3 | Trigonometric Functions

Rule: Trigonometric Identities

Reciprocal identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$
$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

Pythagorean identities

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$$

Addition and subtraction formulas

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$
$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

Double-angle formulas

$$\sin(2\theta) = 2\sin\theta\cos\theta$$
$$\cos(2\theta) = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta = \cos^2\theta - \sin^2\theta$$

TRIGONOMETRY

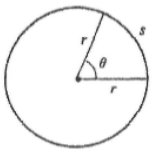
ANGLE MEASUREMENT

π radians $= 180°$

$1° = \frac{\pi}{180}$ rad $\qquad 1 \text{ rad} = \frac{180°}{\pi}$

$s = r\theta$

(θ in radians)

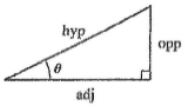


RIGHT ANGLE TRIGONOMETRY

$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}}$

$\tan\theta = \frac{\text{opp}}{\text{adj}} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$

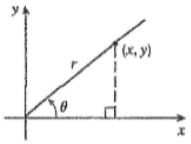


TRIGONOMETRIC FUNCTIONS

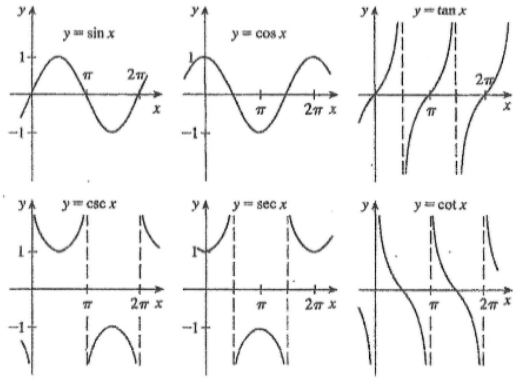
$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y}$

$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x}$

$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$



GRAPHS OF THE TRIGONOMETRIC FUNCTIONS



TRIGONOMETRIC FUNCTIONS OF IMPORTANT ANGLES

θ	radians	$\sin\theta$	$\cos\theta$	$\tan\theta$
0°	0	0	1	0
30°	$\pi/6$	$1/2$	$\sqrt{3}/2$	$\sqrt{3}/3$
45°	$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1
60°	$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
90°	$\pi/2$	1	0	—

FUNDAMENTAL IDENTITIES

$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$

$\cot\theta = \frac{1}{\tan\theta} \qquad \sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$

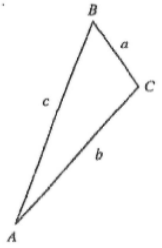
$\sin(-\theta) = -\sin\theta \qquad \cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta \qquad \sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$

$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta \qquad \tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$

THE LAW OF SINES

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$



THE LAW OF COSINES

$a^2 = b^2 + c^2 - 2bc\cos A$

$b^2 = a^2 + c^2 - 2ac\cos B$

$c^2 = a^2 + b^2 - 2ab\cos C$

ADDITION AND SUBTRACTION FORMULAS

$\sin(x + y) = \sin x\cos y + \cos x\sin y$

$\sin(x - y) = \sin x\cos y - \cos x\sin y$

$\cos(x + y) = \cos x\cos y - \sin x\sin y$

$\cos(x - y) = \cos x\cos y + \sin x\sin y$

$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x\tan y}$

$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x\tan y}$

DOUBLE-ANGLE FORMULAS

$\sin 2x = 2\sin x\cos x$

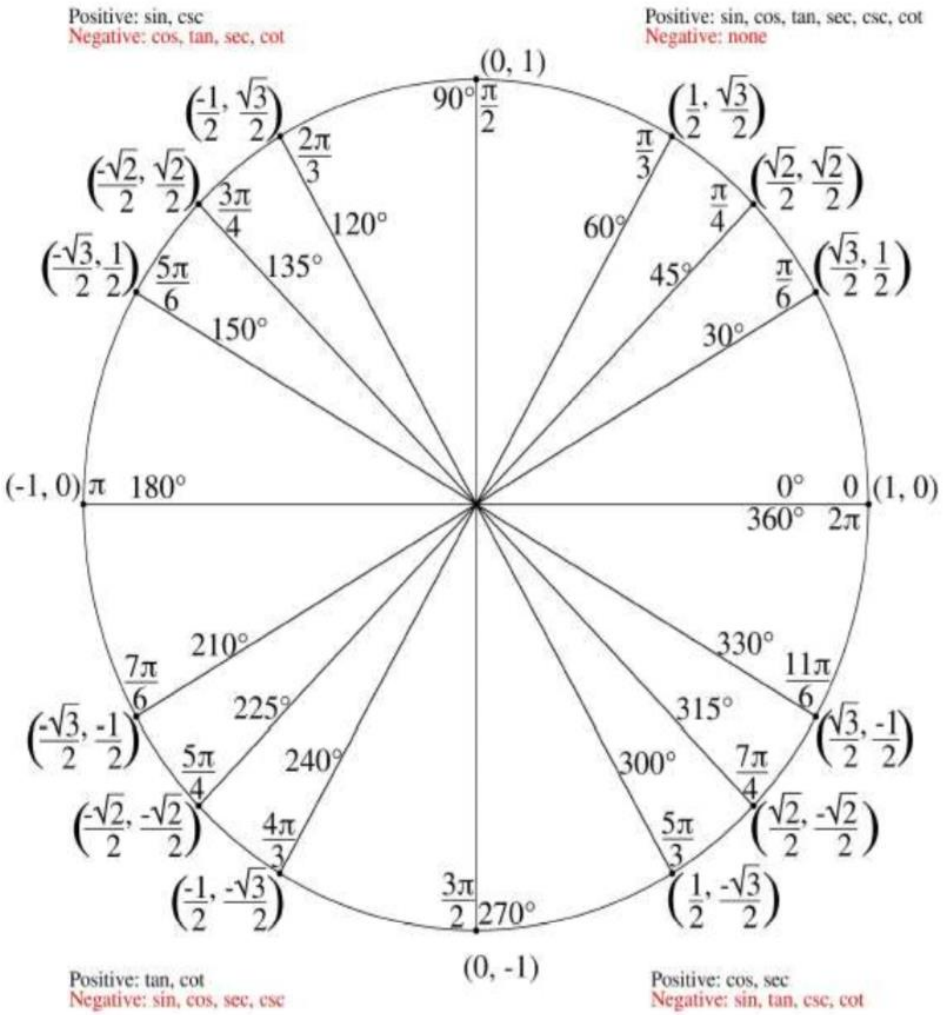
$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$

HALF-ANGLE FORMULAS

$\sin^2 x = \frac{1 - \cos 2x}{2} \qquad \cos^2 x = \frac{1 + \cos 2x}{2}$

The Unit Circle



1.3 EXERCISES

For the following exercises, convert each angle in degrees to radians. Write the answer as a multiple of π.

113. $240°$

114. $15°$

115. $-60°$

116. $-225°$

117. $330°$

For the following exercises, convert each angle in radians to degrees.

118. $\frac{\pi}{2}$ rad

119. $\frac{7\pi}{6}$ rad

120. $\frac{11\pi}{2}$ rad

121. -3π rad

122. $\frac{5\pi}{12}$ rad

Evaluate the following functional values.

123. $\cos\left(\frac{4\pi}{3}\right)$

124. $\tan\left(\frac{19\pi}{4}\right)$

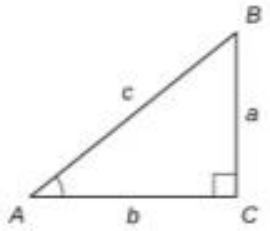
125. $\sin\left(-\frac{3\pi}{4}\right)$

126. $\sec\left(\frac{\pi}{6}\right)$

127. $\sin\left(\frac{\pi}{12}\right)$

128. $\cos\left(\frac{5\pi}{12}\right)$

For the following exercises, consider triangle ABC, a right triangle with a right angle at C. a. Find the missing side of the triangle. b. Find the six trigonometric function values for the angle at A. Where necessary, round to one decimal place.



129. $a = 4, c = 7$

130. $a = 21, c = 29$

131. $a = 85.3, b = 125.5$

132. $b = 40, c = 41$

133. $a = 84, b = 13$

134. $b = 28, c = 35$

For the following exercises, P is a point on the unit circle. a. Find the (exact) missing coordinate value of each point and b. find the values of the six trigonometric functions for the angle θ with a terminal side that passes through point P. Rationalize denominators.

135. $P\left(\frac{7}{25}, y\right), y > 0$

136. $P\left(\frac{-15}{17}, y\right), y < 0$

137. $P\left(x, \frac{\sqrt{7}}{3}\right), x < 0$

138. $P\left(x, \frac{-\sqrt{15}}{4}\right), x > 0$

For the following exercises, simplify each expression by writing it in terms of sines and cosines, then simplify. The final answer does not have to be in terms of sine and cosine only.

139. $\tan^2 x + \sin x \csc x$

140. $\sec x \sin x \cot x$

141. $\frac{\tan^2 x}{\sec^2 x}$

142. $\sec x - \cos x$

143. $(1 + \tan\theta)^2 - 2\tan\theta$

144. $\sin x(\csc x - \sin x)$

145. $\frac{\cos t}{\sin t} + \frac{\sin t}{1 + \cos t}$

146. $\frac{1 + \tan^2 \alpha}{1 + \cot^2 \alpha}$

For the following exercises, verify that each equation is an identity.

147. $\frac{\tan\theta \cot\theta}{\csc\theta} = \sin\theta$

148. $\frac{\sec^2 \theta}{\tan\theta} = \sec\theta \csc\theta$

149. $\frac{\sin t}{\csc t} + \frac{\cos t}{\sec t} = 1$

150. $\frac{\sin x}{\cos x + 1} + \frac{\cos x - 1}{\sin x} = 0$

151. $\cot\gamma + \tan\gamma = \sec\gamma \csc\gamma$

152. $\sin^2 \beta + \tan^2 \beta + \cos^2 \beta = \sec^2 \beta$

153. $\frac{1}{1 - \sin\alpha} + \frac{1}{1 + \sin\alpha} = 2\sec^2 \alpha$

154. $\frac{\tan\theta - \cot\theta}{\sin\theta \cos\theta} = \sec^2 \theta - \csc^2 \theta$

For the following exercises, solve the trigonometric equations on the interval $0 \le \theta < 2\pi$.

155. $2\sin\theta - 1 = 0$

156. $1 + \cos\theta = \frac{1}{2}$

157. $2\tan^2 \theta = 2$

158. $4\sin^2 \theta - 2 = 0$

159. $\sqrt{3}\cot\theta + 1 = 0$