20141218 김찬

1. (1)

p	$\to$	q	$\leftrightarrow$	$\sim p$	$\vee$	q
T	T	T	T	F	T	T
T	F	F	T	F	F	F
F	T	T	T	T	T	T
F	T	F	T	T	T	F

(2) $p \to (q \to r) \equiv p \to (\sim q \vee r)$

$\equiv \sim p \vee (\sim q \vee r)$

$\equiv (\sim p \vee \sim q) \vee r$

$\equiv \sim (p \wedge q) \vee r$

$\equiv (p \wedge q) \to r$

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$Y_1 = X_1$, $Y_{k+1} = Y_{k+1} - (X_1 \cup \cdots \cup X_k)$

show that $\forall m \neq n$, $Y_m \cap Y_n = \phi$

2. (1) ① Suppose that $m > n$, $a \in Y_m \cap Y_n$

Then. $a \in Y_m \wedge a \in Y_n$.

Since $a \in Y_m$, $a \in X_m - (X_1 \cup \cdots \cup X_{m-1})$

So, $a \in X_m \wedge a \in (X_1 \cup \cdots \cup X_{m-1})^c$

& Since $a \in Y_n$, $a \in X_n - (X_1 \cup \cdots \cup X_{n-1})$

So, $a \in X_n \wedge a \in (X_1 \cup \cdots \cup X_{n-1})^c$

Thus, $a \in X_m \wedge a \in (X_1 \cup \cdots X_{m-1})^c \wedge a \in X_n \wedge a \in (X_1 \cup \cdots X_{n-1})^c$.

This is contradiction ($\because$ $a \in X_n$, then $a \notin (X_1 \cup \cdots \cup X_{m-1})^c$).

② show that $\forall n \in \mathbb{N}$, $Y_n \subset X_n$.

$Y_n = X_n - (X_1 \cup \cdots \cup X_{n-1})$.

Let $x \in Y_n$ then $x \in X_n - (X_1 \cup \cdots \cup X_{n-1})$.

So, $x \in X_n \wedge x \in (X_1 \cup \cdots X_{n-1})^c$

$\therefore x \in X_n$.

(2) "$\supset$" Since $Y_n \subset X_n$, $\bigcup_{n \in \mathbb{N}} X_n \supset \bigcup_{n \in \mathbb{N}} Y_n$.

"$\subset$" $\bigcup_{n \in \mathbb{N}} Y_n = \{x \in X \mid \exists n \in \mathbb{N}, x \in Y_n\}$

$= \{x \in X \mid \exists n \in \mathbb{N}, x \in X_n - (X_1 \cup \cdots \cup X_{n-1})\}$

$= \{x \in X \mid \exists n \in \mathbb{N}, x \in X_n \wedge x \in (X_1 \cup \cdots \cup X_{n-1})^c\}$

Since $\forall m \neq n$ $Y_m \cap Y_n = \phi$, if $x \in Y_n$, then $x \in X_n$.

So, $\bigcup_{n \in \mathbb{N}} Y_n \supset \bigcup_{n \in \mathbb{N}} X_n$

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3. Let $x \in X$.

$x \in B \cap (\bigcup_{\gamma \in \Gamma} A_\gamma)$.

$\Leftrightarrow x \in B \wedge (\exists \gamma \in \Gamma, x \in A_\gamma)$

$\Leftrightarrow \exists \gamma \in \Gamma, (x \in B \wedge x \in A_\gamma)$ ← generalized distributive laws in logics.

$\Leftrightarrow \exists \gamma \in \Gamma, \quad x \in B \cap A_\gamma$

$\Leftrightarrow x \in \bigcup_{\gamma \in \Gamma} (B \cap A_\gamma)$

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4. "$\subset$"

Let $x \in f^{-1}(f(X) \cap B)$

Then $f(x) \in f(X) \cap B$.

Since $f(x) \in B$, $x \in f^{-1}(B)$.

"$\supset$"

Let $x \in f^{-1}(B)$.

Then $f(x) \in B$.

And since $x \in X$, $f(x) \in f(X)$.

Thus, $f(x) \in B \cap f(X)$.

So, $x \in f^{-1}(B \cap f(X))$.

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5. (1) If p is false and q is false, $p \to q$ is true (counter example)

$\therefore$ false.

(2) True

pf) Let $x_1, x_2 \in X$

Suppose that $f(x_1) = f(x_2)$

Then $g(f(x_1)) = g(f(x_2))$

$$\begin{array}{ccc} g(f(x_1)) & = & g(f(x_2)) \\ \| & & \| \\ (g \circ f)(x_1) & & g \circ f(x_2). \end{array}$$

Since $g \circ f$ is 1-1, $x_1 = x_2$.

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6. 1) reflexive

Let $x \in X$. and $x = (a, b)$.

Since $a = a+0$, $b = b+0$, $((a,b), (a,b)) = (x, x) \in R$

2) Symmetric.

Let xRy with $x = (a,b)$, $y = (c,d)$.

Then, $c = a+x$, $d = b+y$ for some $x, y \in \mathbb{Z}$

So, $a = c-x$, $b = d-y$.

Thus, $((c,d), (a,b)) = (y, x) \in R$

$\therefore yRx$

3) transitive

Let xRy and yRz with $x=(a,b)$, $y=(c,d)$, $z=(e,f)$

Then, $c = a+x_1$, $d = b+y_1$ & $e = c+x_2$, $f = d+y_2$ for some $x_1, y_1, x_2, y_2 \in \mathbb{Z}$

So, $e = a+x_1+x_2$, $f = b+y_1+y_2$

Thus, $((a,b), (e,f)) = (x, z) \in R$

$\therefore xRz$