CSCM10 Research Methodology
Research in Theoretical Computer Science
Specification and Verification

Anton Setzer

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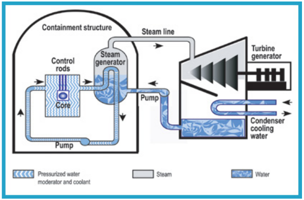
Definition

Definition: A **critical system** is a

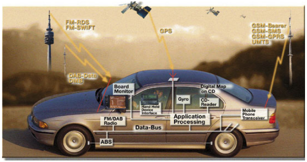
- computer, electronic or electromechanical system
- the failure of which may have serious consequences, such as
  - substantial financial losses,
  - substantial environmental damage,
  - injuries or death of human beings.

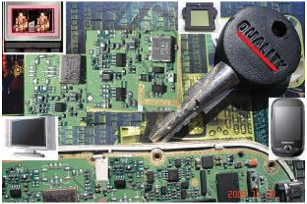
Example 1: Nuclear Power



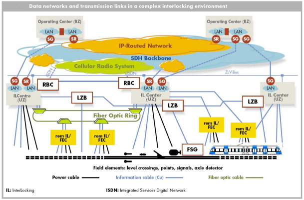
Example: Medical Devices

Example: Embedded Systems in Automobile Industry





Example: Railways



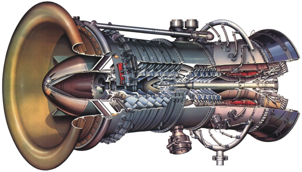
Failure of a Critical System

Failure of a Critical System

Industrial Partners of Swansea Group in Theoretical Computer Science

irg invensys™ rail group

SIEMENS

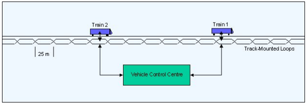


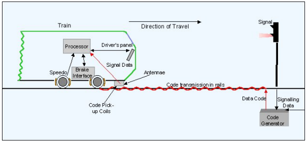
Group in Theoretical Computer Science (Theory Group)

- The department of Computer Science has a big group working on logic, theoretical computer science and applications to verification of software and hardware.
- Long experience in working with verification of software and hardware.
- Industrial connections with companies such as Rolls Royce, Developers of Electronic Payment Systems, Siemens.

Theory Group

- Well established collaboration with Siemens Rail Automation (Chippenham, formerly Invensys Railsystems) on modelling and verification of new generations of railway interlocking systems.
  - Currently working on radio controlled moving block systems (ERTMS).





Expertise of Theory Group

- Verification in the Railway Domain
  - Ulrich Berger
  - Phil James
  - Faron Moller
  - Liam O'Reilly
  - Markus Roggenbach
  - Monika Seisenberger
  - Anton Setzer
- Embedded Systems and Testing
  - Arnold Beckmann,
  - Markus Roggenbach.

Why Formal Specification?

- Natural language specification can be ambiguous.
  - "The output is a red light or a green light".
    - Do you mean "either or" or "inclusive or"?

Why Formal Specification?

- Formal specification enforce precision.
  - Example: If the level of the water in the tank is above a certain level, the plug valve must be closed.
    Do you mean
    - maximum level,
    - average,
    - medium,
    - or . . . (lots of other possibilities)?

Why Formal Specification?

- Natural language specifications don't allow formal verification.

Challenges in Specification

- Finding a suitable language which is
  - expressive
  - and simple enough for the user to understand it.
- Describe the meaning of specification languages (semantics).
- For specifying a formal system, determine the right
  - notions,
  - level of abstraction

Example

- Distant signals and main signal in railways.
  Is
  - the main signal a function of the distant signal,
  - or the distant signal a function of the main signal,
  - or are main signal and distant signal in a relation.
- During specification, often need to switch between different choices.
- General problem of modelling systems.

Expertise of Swansea Theory Group

- Algebraic Specification.
  - Markus Roggenbach (CASL)
  - John Tucker (theory of algebraic specification)
- Process Algebras
  - Faron Moller (CCS),
  - Markus Roggenbach (CSP-CASL),
  - Anton Setzer (CSP-Agda).

Verification

- Verification is the process of determining whether a software product coincides with its specification.
- Many methods.
- Main method is testing.
- Testing usually not complete.
- In order to guarantee that a program is guaranteed to be correct, one needs prove that the output of software coincides with the specification.
    - Necessary especially for critical systems.
    - Increasingly used for general systems, e.g. by Microsoft, to guarantee security of its software.
- Done using theorem proving techniques.

4 Ways of Proving Theorems

1. **Theorem proving by hand.**
    - What mathematicians do all the time.
    - Will remain in the near future the main way for proving theorems.
    - Problem: Errors.
        - As in programs after a certain amount of lines there is a bug, after a certain amount of lines a proof has a bug.
        - The problem can only be reduced by careful proof checking, but not eliminated completely.
    - Unsuitable for verifying large software and hardware systems.
        - Data usually too large.
        - Likely that one makes the same mistakes as in the software.

4 Ways of Proving Theorems

2. **Theorem proving with some machine support.**
    - Machine checks the syntax of the statements, creates a good layout, translates it into different languages.
    - Theorem proving still to be done by hand.
    - **Example:** most systems for specification of software.
    - **Advantages:**
        - Less errors.
        - User is forced to obey a certain syntax.
        - Specifications can be exchanged more easily.
    - **Disadvantage:** Similar to 1.

3. **Interactive Theorem Proving.**
    - Proofs are fully checked by the system.
    - Proof steps have to be carried out by the user.
    - **Advantages:**
        - Correctness guaranteed (provided the theorem prover is correct).
        - Everything which can be proved by hand, should be possible to be proved in such systems.

4 Ways of Proving Theorems

- (Interactive theorem proving)
  - **Disadvantages:**
    - It takes much longer than proving by hand.
    - Similar to programming:
      To say in words what a program should do, doesn't take long.
      To write the actual program, can take a long time, since much more details are involved than expected.
    - Requires experts in theorem proving.

4 Ways of Proving Theorems

4. Automated Theorem Proving.

- The theorem is shown by the machine.
- It is the task of the user to
  - state the theorem,
  - bring it into a form so that it can be solved,
  - usually adapt certain parameters so that the theorem proving solves the problem within reasonable amount of time.

4 Ways of Proving Theorems

- (Automated theorem proving)
  - **Advantages**
    - Less complicated to “feed the theorem into the machine” rather than actually proving it.
      Might be done by non-specialists.
    - Sometimes faster than interactive theorem proving.

4 Ways of Proving Theorems

- (Automated theorem proving)
  - **Disadvantages**
    - Many problems cannot be proved automatically.
    - Can often deal only with finite problems.
    - We can show the correctness of one particular processor.
    - But we cannot show a theorem, stating the correctness of a parametric unit (like a generic n-bit adder for arbitrary n.
    - In some cases this can be overcome.
    - Limits on what can be done (some hardware problems can be verified as 32 bit versions, but not as 64 bit versions).

Verification in Industry

- Most verification done using testing.
- Some theorem proving by hand and with some machine support done.
- Increasingly theorem proving using automated theorem proving done.
  - Investment of Microsoft in various automated theorem provers.
  - Package management in Linux became much faster due to use of SAT solvers (Automated Theorem Provers).

Verification in Industry

- Interactive theorem proving on its way into industry.
  - Typical scenario:
  - General properties of a system proved used interactive theorem proving
    - E.g. signalling principles formally expressed safety.
  - That a concrete installation is in accordance with those general principles done using automated theorem proving.
    - E.g. show that a railway interlocking system fulfils signalling principles.

Expertise of Verification

- Verification using automated theorem provers (ATP).
  - Oliver Kullmann (SAT solvers, e.g. OK-Solver)
- Verification using interactive theorem provers (ITP).
  - Markus Roggenbach (Isabelle),
  - Ulrich Berger (Minlog, Coq),
  - Monika Seisenberger (Minlog),
  - Anton Setzer (Agda).

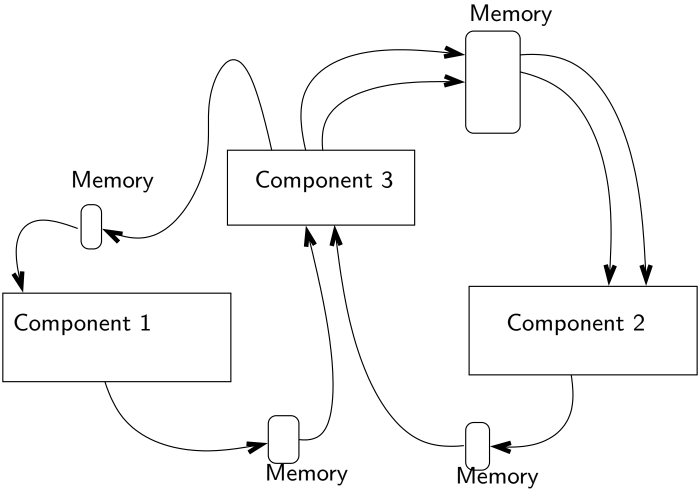
Agda

- Agda is a theorem prover which is as well a prototype of a dependently typed programming language.
- In Agda proofs and programs are the same.
- A proof of a theorem A is a program p of type A written as

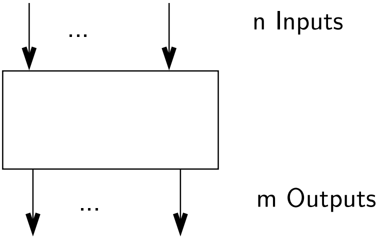
$$p : A$$

- Relatively easy for programmers, since they don't need to learn a different activity.
- Agda uses the novel concept of dependent types.
- In Swansea Anton Setzer is expert in Agda.

Example: Boolean Circuits



What is a Component?



A Boolean Component can be represented by a

$$f : \mathrm{Bool}^n \to \mathrm{Bool}^m$$

What is the type $\mathrm{Bool}^n \to \mathrm{Bool}^m$?

- $\mathrm{Bool}^n \to \mathrm{Bool}^m$ is a type depending on $n, m : \mathbb{N}$.
- In most languages you don't have any dependent type.
  You need to replace this by $\mathrm{List}(\mathrm{Bool}) \to \mathrm{List}(\mathrm{Bool})$.
- In C++ you can define

  $$\mathrm{Bool}^n \to \mathrm{Bool}^m$$

  but only, if n, m are known at compile time.
  - Disallows dynamic dependencies, e.g. depending on user input.
- In Agda we can directly use $\mathrm{Bool}^n \to \mathrm{Bool}^m$ as a dependent type.

Example 2: Grammars

- Assume you want to write programs which manipulate Java programs.
  - E.g. change a variable not using brute query replace.
- One way of doing this:
  - Define a data type of Java programs.
  - Translate strings into this data type and back again.
  - Write programs which work on this data type of Java programs.

Example 2: Grammars

- An oversimplified grammar for Java might start as follows:

$$
\begin{array}{lcl}
\text{JavaProg} & \longrightarrow & \text{“}\mathsf{class}\text{”}\,\text{identifier}\,\text{“}\{\text{”}\,\text{JavaProgBody}\ \ \text{“}\}\ \text{”} \\
\text{JavaProgBody} & \longrightarrow & (\ \text{VariableDecl}\ )^{*}\ (\ \text{MethodDecl}\ )^{*} \\
\text{VariableDecl} & \longrightarrow & \text{TypeDecl VariableName “;”} \\
\ \ \ldots & &
\end{array}
$$

Transformers of Java Programs

- Let Grammarsymbol be the set of terminals and non-terminals (JavaProg, JavaProgbody, ...).
- For each Grammarsymbol S we define the type $[\![S]\!]$ of entities of this type, e.g.
  - $[\![\mathrm{TypeDecl}]\!] = \mathrm{String}$.
  - $[\![\mathrm{VariableName}]\!] = \mathrm{String}$.
  - $[\![\mathrm{VariableDecl}]\!] = \mathrm{String} \times \mathrm{String}$.
- $[\![S]\!]$ is a **dependent type** depending on $S : \mathrm{GrammarSymbol}$.

Type of the Parser

$$
\begin{aligned}
\text{Parser} \quad &: \quad (\text{GrammarSymbol} \times \text{String}) \to \text{Bool} \\[1em]
\text{Transformer} \quad &: \quad (S : \text{GrammarSymbol}) \\
&\quad\ \to (s : \text{String}) \\
&\quad\ \to \text{Parser}(S, s) == \text{true} \\
&\quad\ \to [\![S]\!]
\end{aligned}
$$

- Makes heavy use of the dependent type $[\![S]\!]$.
- Parser Libraries in C++, Haskell, Agda have been built based on this idea.

Generative Programming

- These are examples of generative programming.
- In generative programming you want to build highly generic programs, which generate and manipulate programs from elements of data types.

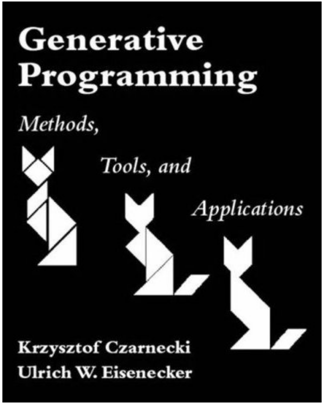
Generative Programming

- So we have
  - a base data type $\mathrm{BaseType}$ (like $\mathrm{GrammarSymbol}$ before),
  - a type of programs $\mathrm{Program}(S)$ based on $S : \mathrm{BaseType}$ (like $[\![S]\!]$ before),
  - operations which manipulate $\mathrm{Program}(S)$, e.g.

$$
\begin{array}{lcl}
\mathrm{transform1} & : & ((S : \mathrm{BaseType1}) \times \mathrm{Program1}(S)) \\
 & & \to \mathrm{BaseType2} \\
\mathrm{transform2} & : & ((S : \mathrm{BaseType1}) \times \mathrm{Program1}(S)) \\
 & & \to \mathrm{Program2}(\mathrm{transform1}(S, s))
\end{array}
$$

Generative Programming

- Now we can create factories for generating programs.
- Replace handcrafted programs by generated programs.
- Similar to step from pre-industrial to industrial age.



Dependent Types for Writing Verified Programs

- Assume we want to assign a type to a sorting function sort on lists of natural numbers.
- In most programming language, the type of it is essentially

$$\mathrm{sort} : \mathrm{NatList} \to \mathrm{NatList}$$

  for the type of lists of natural numbers $\mathrm{NatList}$.
- In dependent type theory, we can demand more correctness, namely that its type is

$$\mathrm{sort} : \mathrm{NatList} \to \mathrm{SortedList} \quad .$$

  - We assume some notion of $\mathrm{NatList}$ (list of natural numbers).

SortedList

- What is SortedList?
  - An element of SortedList is a list which is sorted.
  - It is a pair $\langle l, p \rangle$ s.t.
    - l is a NatList.
    - p is a proof or verification that l is sorted:
    - $p : \mathrm{Sorted}(l)$.

Sorted Lists

- For the moment, ignore what is meant by $\mathrm{Sorted}(l)$ as a type.
- Only important: $\mathrm{Sorted}(l)$ depends on l.
  - $\mathrm{Sorted}(l)$ is a predicate expressed as a type.
- Elements of SortedList are pairs $\langle l, p \rangle$ s.t.
  - $l : \mathrm{NatList}$.
  - $p : \mathrm{Sorted}(l)$.
- $\mathrm{Sorted}(l)$ is a dependent type.

Sorted Lists (Cont.)

- An element of $\mathrm{Sorted}(l)$ will be a **proof** that l is sorted.
- If l **is sorted**, then $\mathrm{Sorted}(l)$ will be provable, and therefore **will have an element.**
  - It is possible to write a program which computes an element of $\mathrm{Sorted}(l)$.
- If l is **not sorted**, then $\mathrm{Sorted}(l)$ will have no proof and it will therefore **no element**.
  - Then it is not possible to write a program which computes an element of $\mathrm{Sorted}(l)$.

The Dependent Product

- Then the pair $\langle l, p\rangle$ will be an element of

$$\mathrm{SortedList} := (l : \mathrm{NatList}) \times \mathrm{Sorted}(l) \ \ .$$

- $\mathrm{SortedList}$ is the type of pairs $\langle l, p\rangle$ s.t.
  - $l : \mathrm{NatList}$,
  - $p : \mathrm{Sorted}(l)$.

  called the **dependent product**
- $\mathrm{sort} : \mathrm{NatList} \to ((l : \mathrm{NatList}) \times \mathrm{Sorted}(l))$ expresses:
  - sort converts lists into sorted lists.

The Dependent Function Type

- From a sorting function we know more:
  - It takes a list and converts it into a sorted list **with the same elements**.
- Assume a type (or predicate) $\mathrm{EqElements}(l, l')$ standing for
  - l and l' have the same elements.

The Dependent Function Type

- A refined version of sort has type

  $$(l : \mathrm{NatList}) \to ((l' : \mathrm{NatList}) \times \mathrm{Sorted}(l') \times \mathrm{EqElements}(l, l'))$$

- "$\mathrm{sort}(l)$ is a list, which is sorted and has the same elements".
- "sort is a program, which takes a list and returns a sorted list with the same elements."
- The type of sort is an instance of the **dependent function type**:
  - The result type depends on the arguments.

Exerts in Dependent Type Theory and Generative Programming

- Dependent Type Theory
  - Ulrich Berger (using Coq), Anton Setzer (using Agda).
- Generative Programming
  - Oliver Kullmann, Anton Setzer

Theoretical Topics

- Computability Theory and Limits of Computation
  - Ulrich Berger, Jens Blanck, Arno Pauly, Monika Seisenberger, John Tucker
- Computable Analysis and Exact Real Number Computation
  - Ulrich Berger, Jens Blanck, Arno Pauly Monika Seisenberger.
- Program Extraction
  - Ulrich Berger, Monika Seisenberger
- Proof Theory
  - Arnold Beckmann, Ulrich Berger, Monika Seisenberger, Anton Setzer

Theoretical Topics

- Complexity Theory
  - Arnold Beckmann, Oliver Kullmann, Faron Moller, Jean Razafindrakoto.
- Formal Argumentation, Reasoning, knowledge graphs
  - Xiuyi Fan, Adam Wyner
- AI and law
  - Adam Wyner.

Lecturers in Cyber Security

- Jingjing Deng
  - Data mining
- Phillip James
  - Penetration Testing
  - Education Principles for Security
  - Formal modelling and analysis of security principles
- Mark Jones
  - Data Analysis
  - Combining user intelligence and machine intelligence in security
- Pardeep Kumar
  - Authentication protocols for low-powered devices
  - Privacy issues and solutions in smart metering networks
  - 5G Security
- Siyuan Liu
  - Soft security
  - Recommender systems
  - Serious Games

Lecturers in Cyber Security

- Bertie Müller
  - Multi-agent systems verification
  - AI methods for anomaly detection
  - data privacy
  - federated learning
- Markus Roggenbach
  - Formal Methods to guarantee the safety & security of Computer Systems
- Harold Thimbleby
  - Security of Medical Devices
  - Usable security
  - Protocol design
- John V Tucker
  - Data collection and linkage
  - monitoring and surveillance
  - digital identity

Lecturers in Cyber Security

- Monika Seisenberger
  - Cyberterrorism
  - Formal methods in cyber security.
- Arnold Beckmann, Anton Setzer.
  - Cryptocurrencies, Blockchain
- Xianghua Xie
  - Big cata, pattern recognition, machine learning.

Conclusion

- Critical Systems require more formal specification and verification.
- Expertise in Swansea in specification and verification.
- Problems of natural language specification can be overcome by formal specification.
- Verification techniques – from proving by hand to interactive and automated theorem proving.
- Agda as an example of a programming language based on dependent types.
- Use of dependent types for generative programming.
- Wide range of theoretical topics covered in Swansea.
- Research related to Cyber Security.