Exercise 2.1 Page: 32

1. Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.

(i) $4x^2–3x+7$

Solution:

The equation $4x^2–3x+7$ can be written as $4x^2–3x^1+7x^0$

Since *x* is the only variable in the given equation and the powers of x (i.e., 2, 1 and 0) are whole numbers, we can say that the expression $4x^2–3x+7$ is a polynomial in one variable.

(ii) $y^2+\sqrt{2}$

Solution:

The equation $y^2+\sqrt{2}$ can be written as $y^2+\sqrt{2}y^0$

Since y is the only variable in the given equation and the powers of y (i.e., 2 and 0) are whole numbers, we can say that the expression $y^2+\sqrt{2}$ is a polynomial in one variable.

(iii) $3\sqrt{t}+t\sqrt{2}$

Solution:

The equation $3\sqrt{t}+t\sqrt{2}$ can be written as $3t^{1/2}+\sqrt{2}t$

Though, *t* is the only variable in the given equation, the powers of *t* (i.e.,1/2) is not a whole number. Hence, we can say that the expression $3\sqrt{t}+t\sqrt{2}$ is **not** a polynomial in one variable.

(iv) $y+2/y$

Solution:

The equation $y+2/y$ an be written as $y+2y^{-1}$

Though, *y* is the only variable in the given equation, the powers of *y* (i.e.,-1) is not a whole number. Hence, we can say that the expression $y+2/y$ is **not** a polynomial in one variable.

(v) $x^{10}+y^3+t^{50}$

Solution:

Here, in the equation $x^{10}+y^3+t^{50}$

Though, the powers, 10, 3, 50, are whole numbers, there are 3 variables used in the expression $x^{10}+y^3+t^{50}$. Hence, it is **not** a polynomial in one variable.

2. Write the coefficients of x^2 in each of the following:

(i) $2+x^2+x$

Solution:

The equation $2+x^2+x$ can be written as $2+(1)x^2+x$

We know that, coefficient is the number which multiplies the variable.

Here, the number that multiplies the variable x^2 is 1

$\therefore$, the coefficients of x^2 in $2+x^2+x$ is 1.

(ii) $2–x^2+x^3$

Solution:

The equation $2–x^2+x^3$ can be written as $2+(–1)x^2+x^3$

We know that, coefficient is the number (along with its sign, i.e., - or +) which multiplies the variable.

Here, the number that multiplies the variable x^2 is -1

∴ the coefficients of x^2 in $2–x^2+x^3$ is -1.

(iii) $(\pi/2)x^2+x$

Solution:

The equation $(\pi/2)x^2 +x$ can be written as $(\pi/2)x^2 + x$

We know that, coefficient is the number (along with its sign, i.e., - or +) which multiplies the variable.

Here, the number that multiplies the variable x^2 is $\pi/2$.

∴ the coefficients of x^2 in $(\pi/2)x^2 +x$ is $\pi/2$.

(iii) $\sqrt{2}x-1$

Solution:

The equation $\sqrt{2}x-1$ can be written as $0x^2+\sqrt{2}x-1$ [Since $0x^2$ is 0]

We know that, coefficient is the number (along with its sign, i.e., - or +) which multiplies the variable.

Here, the number that multiplies the variable x^2 is 0

∴, the coefficients of x^2 in $\sqrt{2}x-1$ is 0.

3. Give one example each of a binomial of degree 35, and of a monomial of degree 100.

Solution:

Binomial of degree 35: A polynomial having two terms and the highest degree 35 is called a binomial of degree 35

Eg., $3x^{35}+5$

Monomial of degree 100: A polynomial having one term and the highest degree 100 is called a monomial of degree 100

Eg., $4x^{100}$

4. Write the degree of each of the following polynomials:

(i) $5x^3+4x^2+7x$

Solution:

The highest power of the variable in a polynomial is the degree of the polynomial.

Here, $5x^3+4x^2+7x = 5x^3+4x^2+7x^1$

The powers of the variable x are: 3, 2, 1

∴ the degree of $5x^3+4x^2+7x$ is 3 as 3 is the highest power of x in the equation.

(ii) $4–y^2$

Solution:

The highest power of the variable in a polynomial is the degree of the polynomial.

Here, in $4–y^2$,
The power of the variable y is 2
∴ the degree of $4–y^2$ is 2 as 2 is the highest power of y in the equation.

(iii) $5t–\sqrt{7}$
Solution:
The highest power of the variable in a polynomial is the degree of the polynomial.
Here, in $5t–\sqrt{7}$,
The power of the variable y is: 1
∴ the degree of $5t–\sqrt{7}$ is 1 as 1 is the highest power of y in the equation.

(iv) 3
Solution:
The highest power of the variable in a polynomial is the degree of the polynomial.
Here, $3 = 3×1 = 3× x^0$
The power of the variable here is: 0
∴ the degree of 3 is 0.

5. Classify the following as linear, quadratic and cubic polynomials:
Solution:
We know that,
Linear polynomial: A polynomial of degree one is called a linear polynomial.
Quadratic polynomial: A polynomial of degree two is called a quadratic polynomial.
Cubic polynomial: A polynomial of degree three is called a cubic polynomial.

(i) x^2+x
Solution:
The highest power of x^2+x is 2
∴ the degree is 2
Hence, x^2+x is a quadratic polynomial

(ii) $x–x^3$
Solution:
The highest power of $x–x^3$ is 3
∴ the degree is 3
Hence, $x–x^3$ is a cubic polynomial

(iii) $y+y^2+4$
Solution:
The highest power of $y+y^2+4$ is 2
∴ the degree is 2
Hence, $y+y^2+4$is a quadratic polynomial

(iv) 1+x
Solution:

The highest power of 1+x is 1
$\therefore$ the degree is 1
Hence, 1+x is a linear polynomial.

(v) 3t
Solution:
The highest power of 3t is 1
$\therefore$ the degree is 1
Hence, 3t is a linear polynomial.

(vi) r^2
Solution:
The highest power of r^2 is 2
$\therefore$ the degree is 2
Hence, r^2 is a quadratic polynomial.

(vii) $7x^3$
Solution:
The highest power of $7x^3$ is 3
$\therefore$ the degree is 3
Hence, $7x^3$ is a cubic polynomial.

Exercise 2.2 Page: 34

1. Find the value of the polynomial $(x)=5x-4x^2+3$
(i) $x = 0$
(ii) $x = -1$
(iii) $x = 2$

Solution:

Let $f(x) = 5x-4x^2+3$

(iii) When $x = 0$

$f(0) = 5(0)-4(0)^2+3$

$= 3$

(ii) When $x = -1$

$f(x) = 5x-4x^2+3$
$f(-1) = 5(-1)-4(-1)^2+3$
$= -5-4+3$
$= -6$

(iii) When $x = 2$

$f(x) = 5x-4x^2+3$
$f(2) = 5(2)-4(2)^2+3$
$= 10-16+3$
$= -3$

2. Find p(0), p(1) and p(2) for each of the following polynomials:

(i) $p(y)=y^2-y+1$

Solution:

$p(y) = y^2-y+1$
$\therefore p(0) = (0)^2-(0)+1=1$
$p(1) = (1)^2-(1)+1=1$
$p(2) = (2)^2-(2)+1=3$

(ii) $p(t)=2+t+2t^2-t^3$

Solution:

$p(t) = 2+t+2t^2-t^3$
$\therefore p(0) = 2+0+2(0)^2-(0)^3=2$
$p(1) = 2+1+2(1)^2-(1)^3=2+1+2-1=4$
$p(2) = 2+2+2(2)^2-(2)^3=2+2+8-8=4$

(iii) $p(x)=x^3$

Solution:

$p(x) = x^3$
$\therefore p(0) = (0)^3 = 0$

$p(1) = (1)^3 = 1$
$p(2) = (2)^3 = 8$

(iv) P(x) = (x−1)(x+1)

Solution:

p(x) = (x–1)(x+1)
∴p(0) = (0–1)(0+1) = (−1)(1) = –1
p(1) = (1–1)(1+1) = 0(2) = 0
p(2) = (2–1)(2+1) = 1(3) = 3

3. Verify whether the following are zeroes of the polynomial, indicated against them.

(i) p(x)=3x+1, x=−1/3

Solution:

For, x = -1/3, p(x) = 3x+1
∴p(−1/3) = 3(-1/3)+1 = −1+1 = 0
∴ -1/3 is a zero of p(x).

(ii) p(x)=5x–π, x = 4/5

Solution:

For, x = 4/5, p(x) = 5x–π
∴ p(4/5) = 5(4/5)-π = 4-π
∴ 4/5 is not a zero of p(x).

(iii) p(x)=x^2−1, x=1, −1

Solution:

For, x = 1, −1;
p(x) = x^2−1
∴p(1)=1^2−1=1−1 = 0
p(−1)=$(-1)^2$−1 = 1−1 = 0
∴1, −1 are zeros of p(x).

(iv) p(x) = (x+1)(x–2), x =−1, 2

Solution:

For, x = −1,2;
p(x) = (x+1)(x–2)
∴p(−1) = (−1+1)(−1–2)
= (0)(−3) = 0
p(2) = (2+1)(2–2) = (3)(0) = 0
∴−1,2 are zeros of p(x).

(v) p(x) = x^2, x = 0

Solution:

For, x = 0 p(x) = x^2
p(0) = 0^2 = 0
∴ 0 is a zero of p(x).

(vi) $p(x) = lx+m$, $x = -m/l$

Solution:

For, $x = -m/l$; $p(x) = lx+m$

$\therefore p(-m/l) = l(-m/l)+m = -m+m = 0$

$\therefore -m/l$ is a zero of p(x).

(vii) $p(x) = 3x^2-1$, $x = -1/\sqrt{3}$, $2/\sqrt{3}$

Solution:

For, $x = -1/\sqrt{3}$, $2/\sqrt{3}$; $p(x) = 3x^2-1$

$\therefore p(-1/\sqrt{3}) = 3(-1/\sqrt{3})^2-1 = 3(1/3)-1 = 1-1 = 0$

$\therefore p(2/\sqrt{3}) = 3(2/\sqrt{3})^2-1 = 3(4/3)-1 = 4-1=3 \neq 0$

$\therefore -1/\sqrt{3}$ is a zero of p(x) but $2/\sqrt{3}$ is not a zero of p(x).

(viii) $p(x) =2x+1$, $x = 1/2$

Solution:

For, $x = 1/2$ $p(x) = 2x+1$

$\therefore p(1/2)=2(1/2)+1 = 1+1 = 2\neq 0$

$\therefore 1/2$ is not a zero of p(x).

4. Find the zero of the polynomials in each of the following cases:

(i) $p(x) = x+5$

Solution:

$p(x) = x+5$

$\Rightarrow x+5 = 0$

$\Rightarrow x = -5$

$\therefore$ -5 is a zero polynomial of the polynomial p(x).

(ii) $p(x) = x-5$

Solution:

$p(x) = x-5$

$\Rightarrow x-5 = 0$

$\Rightarrow x = 5$

$\therefore$ 5 is a zero polynomial of the polynomial p(x).

(iii) $p(x) = 2x+5$

Solution:

$p(x) = 2x+5$

$\Rightarrow 2x+5 = 0$

$\Rightarrow 2x = -5$

$\Rightarrow x = -5/2$

$\therefore x = -5/2$ is a zero polynomial of the polynomial p(x).

(iv)p(x) = 3x–2

Solution:

p(x) = 3x–2

⇒ 3x−2 = 0

⇒ 3x = 2

⇒x = 2/3

∴x = 2/3 is a zero polynomial of the polynomial p(x).

(v) p(x) = 3x

Solution:

p(x) = 3x

⇒ 3x = 0

⇒ x = 0

∴0 is a zero polynomial of the polynomial p(x).

(vi)p(x) = ax, a≠0

Solution:

p(x) = ax

⇒ ax = 0

⇒ x = 0

∴x = 0 is a zero polynomial of the polynomial p(x).

(vii)p(x) = cx+d, c ≠ 0, c, d are real numbers.

Solution:

p(x) = cx + d

⇒ cx+d =0

⇒ x = -d/c

∴ x = -d/c is a zero polynomial of the polynomial p(x).

Exercise 2.3 Page: 40

1. Find the remainder when x^3+3x^2+3x+1 is divided by

(i) x+1

Solution:

$x+1= 0$

$\Rightarrow x = -1$

$\therefore$Remainder:

$p(-1) = (-1)^3+3(-1)^2+3(-1)+1$

$= -1+3-3+1$

$= 0$

(ii) x−1/2

Solution:

$x-1/2 = 0$

$\Rightarrow x = 1/2$

$\therefore$Remainder:

$p(1/2) = (1/2)^3+3(1/2)^2+3(1/2)+1$

$= (1/8)+(3/4)+(3/2)+1$

$= 27/8$

(iii) x

Solution:

$x = 0$

$\therefore$Remainder:

$p(0) = (0)^3+3(0)^2+3(0)+1$

$= 1$

(iv) x+π

Solution:

$x+\pi = 0$

$\Rightarrow x = -\pi$

$\therefore$Remainder:

$p(0) = (-\pi)^3 +3(-\pi)^2+3(-\pi)+1$

$= -\pi^3+3\pi^2-3\pi+1$

(v) 5+2x

Solution:

$5+2x=0$

$\Rightarrow 2x = -5$

$\Rightarrow x = -5/2$

$\therefore$Remainder:

$(-5/2)^3+3(-5/2)^2+3(-5/2)+1 = (-125/8)+(75/4)-(15/2)+1$

$= -27/8$

2. Find the remainder when x^3-ax^2+6x-a is divided by x-a.

Solution:

Let $p(x) = x^3-ax^2+6x-a$

$x-a = 0$

$\therefore x = a$

Remainder:

$p(a) = (a)^3-a(a^2)+6(a)-a$

$= a^3-a^3+6a-a = 5a$

3. Check whether 7+3x is a factor of $3x^3+7x$.

Solution:

$7+3x = 0$

$\Rightarrow 3x = -7$

$\Rightarrow x = -7/3$

$\therefore$Remainder:

$3(-7/3)^3+7(-7/3) = -(343/9)+(-49/3)$

$= (-343-(49)3)/9$

$= (-343-147)/9$

$= -490/9 \neq 0$

$\therefore 7+3x$ is not a factor of $3x^3+7x$

Exercise 2.4 Page: 43

1. Determine which of the following polynomials has (x + 1) a factor:

(i) x^3+x^2+x+1

Solution:

Let $p(x) = x^3+x^2+x+1$

The zero of x+1 is -1. [x+1 = 0 means x = -1]

$p(-1) = (-1)^3+(-1)^2+(-1)+1$

$= -1+1-1+1$

$= 0$

∴By factor theorem, x+1 is a factor of x^3+x^2+x+1

(ii) $x^4+x^3+x^2+x+1$

Solution:

Let $p(x)= x^4+x^3+x^2+x+1$

The zero of x+1 is -1. . [x+1= 0 means x = -1]

$p(-1) = (-1)^4+(-1)^3+(-1)^2+(-1)+1$

$= 1-1+1-1+1$

$= 1 \neq 0$

∴By factor theorem, x+1 is not a factor of $x^4 + x^3 + x^2 + x + 1$

(iii) $x^4+3x^3+3x^2+x+1$

Solution:

Let $p(x)= x^4+3x^3+3x^2+x+1$

The zero of x+1 is -1.

$p(-1)=(-1)4+3(-1)3+3(-1)2+(-1)+1$

$=1-3+3-1+1$

$=1 \neq 0$

∴By factor theorem, x+1 is not a factor of $x^4+3x^3+3x^2+x+1$

(iv)$x^3 - x^2 - (2+\sqrt{2})x + \sqrt{2}$

Solution:

Let $p(x) = x^3-x^2-(2+\sqrt{2})x +\sqrt{2}$

The zero of x+1 is -1.

$p(-1) = (-1)^3-(-1)^2-(2+\sqrt{2})(-1) + \sqrt{2} \;\; = -1-1+2+\sqrt{2}+\sqrt{2}$

$= 2\sqrt{2} \neq 0$

∴By factor theorem, x+1 is not a factor of $x^3-x^2-(2+\sqrt{2})x +\sqrt{2}$

2. Use the Factor Theorem to determine whether g(x) is a factor of p(x) in each of the following cases:

(i) $p(x) = 2x^3+x^2-2x-1$, $g(x) = x+1$

Solution:

$p(x) = 2x^3+x^2-2x-1$, $g(x) = x+1$

$g(x) = 0$

$\Rightarrow x+1 = 0$

$\Rightarrow x = -1$

$\therefore$Zero of g(x) is -1.

Now,

$p(-1) = 2(-1)^3+(-1)^2-2(-1)-1$

$= -2+1+2-1$

$= 0$

$\therefore$By factor theorem, g(x) is a factor of p(x).

(ii) $p(x)=x^3+3x^2+3x+1$, $g(x) = x+2$

Solution:

$p(x) = x^3+3x^2+3x+1$, $g(x) = x+2$

$g(x) = 0$

$\Rightarrow x+2 = 0$

$\Rightarrow x = -2$

$\therefore$ Zero of g(x) is -2.

Now,

$p(-2) = (-2)^3+3(-2)^2+3(-2)+1$

$= -8+12-6+1$

$= -1 \neq 0$

$\therefore$By factor theorem, g(x) is not a factor of p(x).

(iii) $p(x)=x^3-4x^2+x+6$, $g(x) = x-3$

Solution:

$p(x) = x^3-4x^2+x+6$, $g(x) = x -3$

$g(x) = 0$

$\Rightarrow x-3 = 0$

$\Rightarrow x = 3$

$\therefore$ Zero of g(x) is 3.

Now,

$p(3) = (3)^3-4(3)^2+(3)+6$

$= 27-36+3+6$

$= 0$

$\therefore$By factor theorem, g(x) is a factor of p(x).

3. Find the value of k, if x–1 is a factor of p(x) in each of the following cases:

(i) $p(x) = x^2+x+k$

Solution:

If x-1 is a factor of p(x), then p(1) = 0

By Factor Theorem

$\Rightarrow (1)^2+(1)+k = 0$

$\Rightarrow 1+1+k = 0$

$\Rightarrow 2+k = 0$

$\Rightarrow k = -2$

(ii) $p(x) = 2x^2+kx+\sqrt{2}$

Solution:

If x-1 is a factor of p(x), then p(1)=0

$\Rightarrow 2(1)^2+k(1)+\sqrt{2} = 0$

$\Rightarrow 2+k+\sqrt{2} = 0$

$\Rightarrow k = -(2+\sqrt{2})$

(iii) $p(x) = kx^2-\sqrt{2}x+1$

Solution:

If x-1 is a factor of p(x), then p(1)=0

By Factor Theorem

$\Rightarrow k(1)^2-\sqrt{2}(1)+1=0$

$\Rightarrow k = \sqrt{2}-1$

(iv) $p(x)=kx^2-3x+k$

Solution:

If x-1 is a factor of p(x), then p(1) = 0

By Factor Theorem

$\Rightarrow k(1)^2-3(1)+k = 0$

$\Rightarrow k-3+k = 0$

$\Rightarrow 2k-3 = 0$

$\Rightarrow k= 3/2$

4. Factorize:

(i) $12x^2-7x+1$

Solution:

Using the splitting the middle term method,

We have to find a number whose sum = -7 and product =1×12 = 12

We get -3 and -4 as the numbers [-3+-4=-7 and -3×-4 = 12]

$$12x^2-7x+1= 12x^2-4x-3x+1$$
$$= 4x(3x-1)-1(3x-1)$$
$$= (4x-1)(3x-1)$$

(ii) $2x^2+7x+3$

Solution:

Using the splitting the middle term method,
We have to find a number whose sum = 7 and product = 2×3 = 6
We get 6 and 1 as the numbers [6+1 = 7 and 6×1 = 6]

$2x^2+7x+3 = 2x^2+6x+1x+3$
$= 2x(x+3)+1(x+3)$
$= (2x+1)(x+3)$

(iii) $6x^2+5x-6$

Solution:

Using the splitting the middle term method,
We have to find a number whose sum = 5 and product = 6×-6 = -36
We get -4 and 9 as the numbers [-4+9 = 5 and -4×9 = -36]

$6x^2+5x-6 = 6x^2+9x–4x–6$
$= 3x(2x+3)–2(2x+3)$
$= (2x+3)(3x–2)$

(iv) $3x^2–x–4$

Solution:

Using the splitting the middle term method,
We have to find a number whose sum = -1 and product = 3×-4 = -12
We get -4 and 3 as the numbers [-4+3 = -1 and -4×3 = -12]

$3x^2–x–4 = 3x^2–x–4$
$= 3x^2–4x+3x–4$
$= x(3x–4)+1(3x–4)$
$= (3x–4)(x+1)$

5. Factorize:

(i) $x^3–2x^2–x+2$

Solution:

Let $p(x) = x^3–2x^2–x+2$
Factors of 2 are ±1 and ± 2
Now,
$p(x) = x^3–2x^2–x+2$
$p(−1) = (−1)^3–2(−1)^2–(−1)+2$
$= −1 − 2 + 1 + 2$
$= 0$
Therefore, (x+1) is the factor of p(x)

$$
\begin{array}{r|l}
 & x^2-3x+2 \\
\hline
x+1 & x^3-2x^2-x+2 \\
 & x^3+x^2 \\
 & -\quad - \\
\hline
 & -3x^2-x+2 \\
 & -3x^2-3x \\
 & +\quad + \\
\hline
 & 2x+2 \\
 & 2x+2 \\
 & -\quad - \\
\hline
 & 0
\end{array}
$$

Now, Dividend = Divisor × Quotient + Remainder

$(x+1)(x^2–3x+2)$ $= (x+1)(x^2–x–2x+2)$

$= (x+1)(x(x−1)−2(x−1))$

$= (x+1)(x−1)(x+2)$

(ii) $x^3–3x^2–9x–5$

Solution:

Let $p(x) = x^3–3x^2–9x–5$

Factors of 5 are ±1 and ±5

By trial method, we find that

$p(5) = 0$

So, (x-5) is factor of p(x)

Now,

$p(x) = x^3–3x^2–9x–5$

$p(5) = (5)^3–3(5)^2–9(5)–5$

$= 125−75−45−5$

$= 0$

Therefore, (x-5) is the factor of p(x)

$$
\begin{array}{r|l}
 & x^2+2x+1 \\
\hline
x-5 & x^3-3x^2-9x-5 \\
 & x^3-5x^2 \\
 & -\quad + \\
\hline
 & \quad 2x^2-9x-5 \\
 & \quad 2x^2-10x \\
 & \quad -\quad + \\
\hline
 & \qquad x-5 \\
 & \qquad x-5 \\
 & \qquad -\ + \\
\hline
 & \qquad\quad 0
\end{array}
$$

Now, Dividend = Divisor × Quotient + Remainder

$(x-5)(x^2+2x+1) \quad = (x-5)(x^2+x+x+1)$

$= (x-5)(x(x+1)+1(x+1))$

$= (x-5)(x+1)(x+1)$

(iii) $x^3+13x^2+32x+20$

Solution:

Let $p(x) = x^3+13x^2+32x+20$

Factors of 20 are ±1, ±2, ±4, ±5, ±10 and ±20

By trial method, we find that

$p(-1) = 0$

So, $(x+1)$ is factor of $p(x)$

Now,

$p(x)= x^3+13x^2+32x+20$

$p(-1) \quad = (-1)^3+13(-1)^2+32(-1)+20$

$= -1+13-32+20$

$= 0$

Therefore, $(x+1)$ is the factor of $p(x)$

$$
\begin{array}{r|l}
 & x^2+12x+20 \\
\hline
x+1 & x^3+13x^2+32x+20 \\
 & x^3+x^2 \\
 & -\quad - \\
\hline
 & \quad 12x^2+32x+20 \\
 & \quad 12x^2+12x \\
 & \quad -\quad - \\
\hline
 & \qquad 20x+20 \\
 & \qquad 20x+20 \\
 & \qquad -\quad - \\
\hline
 & \qquad\quad 0
\end{array}
$$

Now, Dividend = Divisor × Quotient +Remainder

$(x+1)(x^2+12x+20) \quad = (x+1)(x^2+2x+10x+20)$
$= (x−5)x(x+2)+10(x+2)$
$= (x−5)(x+2)(x+10)$

(iv) $2y^3+y^2–2y–1$

Solution:

Let $p(y) = 2y^3+y^2–2y–1$

Factors = $2×(−1)$= -2 are ±1 and ±2

By trial method, we find that

$p(1) = 0$

So, (y-1) is factor of p(y)

Now,

$p(y) \quad = 2y^3+y^2–2y–1$

$p(1) \quad = 2(1)^3+(1)^2–2(1)–1$

$= 2+1−2$

$= 0$

Therefore, (y-1) is the factor of p(y)

$$
\begin{array}{r|l}
 & 2y^2+3y+1 \\
\hline
y-1 & 2y^3+y^2-2y-1 \\
 & 2y^3-2y^2 \\
 & -\quad + \\
\hline
 & \quad 3y^2-2y-1 \\
 & \quad 3y^2-3y \\
 & \quad -\quad + \\
\hline
 & \qquad y-1 \\
 & \qquad y-1 \\
 & \qquad -\ + \\
\hline
 & \qquad\quad 0
\end{array}
$$

Now, Dividend = Divisor × Quotient + Remainder

$$
\begin{aligned}
(y-1)(2y^2+3y+1) &= (y-1)(2y^2+2y+y+1) \\
&= (y-1)(2y(y+1)+1(y+1)) \\
&= (y-1)(2y+1)(y+1)
\end{aligned}
$$

Exercise 2.5

1. Use suitable identities to find the following products:

(i) (x+4)(x +10)

Solution:

Using the identity, $(x+a)(x+b) = x^2+(a+b)x+ab$

[Here, a = 4 and b = 10]

We get,

$$(x+4)(x+10) = x^2+(4+10)x+(4\times10)$$
$$= x^2+14x+40$$

(ii) (x+8)(x –10)

Solution:

Using the identity, $(x+a)(x+b) = x^2+(a+b)x+ab$

[Here, a = 8 and b = −10]

We get,

$$(x+8)(x-10) = x^2+(8+(-10))x+(8\times(-10))$$
$$= x^2+(8-10)x-80$$
$$= x^2-2x-80$$

(iii)(3x+4)(3x–5)

Solution:

Using the identity, $(x+a)(x+b) = x^2+(a+b)x+ab$

[Here, x = 3x, a = 4 and b = −5]

We get,

$$(3x+4)(3x-5) = (3x)^2+4+(-5)3x+4\times(-5)$$
$$= 9x^2+3x(4-5)-20$$
$$= 9x^2-3x-20$$

(iv)$(y^2+3/2)(y^2-3/2)$

Solution:

Using the identity, $(x+y)(x-y) = x^2-y^2$

[Here, $x = y^2$ and $y = 3/2$]

We get,

$$(y^2+3/2)(y^2-3/2) = (y^2)^2-(3/2)^2$$
$$= y^4-9/4$$

2. Evaluate the following products without multiplying directly:

(i) 103×107

Solution:

$103\times107= (100+3)\times(100+7)$

Using identity, $[(x+a)(x+b) = x^2+(a+b)x+ab$

Here, x = 100

a = 3

$b = 7$

We get, $103\times107 = (100+3)\times(100+7)$

$= (100)^2+(3+7)100+(3\times7))$

$= 10000+1000+21$

$= 11021$

(ii) 95×96

Solution:

$95\times96 = (100-5)\times(100-4)$

Using identity, $[(x-a)(x-b) = x^2-(a+b)x+ab$

Here, $x = 100$

$a = -5$

$b = -4$

We get, $95\times96 = (100-5)\times(100-4)$

$= (100)^2+100(-5+(-4))+(-5\times-4)$

$= 10000-900+20$

$= 9120$

(iii) 104×96

Solution:

$104\times96 = (100+4)\times(100–4)$

Using identity, $[(a+b)(a-b)= a^2-b^2]$

Here, $a = 100$

$b = 4$

We get, $104\times96 = (100+4)\times(100–4)$

$= (100)^2–(4)^2$

$= 10000–16$

$= 9984$

3. Factorize the following using appropriate identities:

(i) $9x^2+6xy+y^2$

Solution:

$9x^2+6xy+y^2 = (3x)^2+(2\times3x\times y)+y^2$

Using identity, $x^2+2xy+y^2 = (x+y)^2$

Here, $x = 3x$

$y = y$

$9x^2+6xy+y^2 = (3x)^2+(2\times3x\times y)+y^2$

$= (3x+y)^2$

$= (3x+y)(3x+y)$

(ii) $4y^2-4y+1$

Solution:

$4y^2-4y+1 = (2y)^2-(2\times2y\times1)+12$
Using identity, $x^2 - 2xy + y^2 = (x - y)^2$
Here, $x = 2y$
$y = 1$
$4y^2-4y+1 = (2y)^2-(2\times2y\times1)+1^2$
$= (2y-1)^2$
$= (2y-1)(2y-1)$

(iii) $x^2-y^2/100$

Solution:

$x^2-y^2/100 = x^2-(y/10)^2$
Using identity, $x^2-y^2 = (x-y)(x+y)$
Here, $x = x$
$y = y/10$

$x^2-y^2/100 = x^2-(y/10)^2$
$= (x-y/10)(x+y/10)$

4. Expand each of the following, using suitable identities:

(i) $(x+2y+4z)^2$
(ii) $(2x-y+z)^2$
(iii) $(-2x+3y+2z)^2$
(iv) $(3a-7b-c)^2$
(v) $(-2x+5y-3z)^2$
(vi) $((1/4)a-(1/2)b+1)^2$

Solution:

(i) $(x+2y+4z)^2$
Using identity, $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$

Here, $x = x$

$y = 2y$

$z = 4z$

$(x+2y+4z)^2 = x^2+(2y)^2+(4z)^2+(2\times x\times2y)+(2\times2y\times4z)+(2\times4z\times x)$
$= x^2+4y^2+16z^2+4xy+16yz+8xz$

(ii) $(2x-y+z)^2$
Using identity, $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$

Here, $x = 2x$

$y = -y$

$z = z$

$(2x-y+z)^2 = (2x)^2+(-y)^2+z^2+(2\times2x\times-y)+(2\times-y\times z)+(2\times z\times2x)$
$= 4x^2+y^2+z^2-4xy-2yz+4xz$

(iii) $(-2x+3y+2z)^2$

Solution:

Using identity, $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$

Here, $x = -2x$

$y = 3y$

$z = 2z$

$(-2x+3y+2z)^2 = (-2x)^2+(3y)^2+(2z)^2+(2\times-2x\times3y)+(2\times3y\times2z)+(2\times2z\times-2x)$
$= 4x^2+9y^2+4z^2-12xy+12yz-8xz$

(iv) $(3a-7b-c)^2$

Solution:

Using identity $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$

Here, $x = 3a$

$y = -7b$

$z = -c$

$(3a-7b-c)^2 = (3a)^2+(-7b)^2+(-c)^2+(2\times3a\times-7b)+(2\times-7b\times-c)+(2\times-c\times3a)$

$= 9a^2 + 49b^2 + c^2- 42ab+14bc-6ca$

(v) $(-2x+5y-3z)^2$

Solution:

Using identity, $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$

Here, $x = -2x$

$y = 5y$

$z = -3z$

$(-2x+5y-3z)^2 = (-2x)^2+(5y)^2+(-3z)^2+(2\times-2x\times5y)+(2\times5y\times-3z)+(2\times-3z\times-2x)$

$= 4x^2+25y^2+9z^2-20xy-30yz+12zx$

(vi) $((1/4)a-(1/2)b+1)^2$

Solution:

Using identity, $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$

Here, $x = (1/4)a$

$y = (-1/2)b$

$z = 1$

$$((1/4)a-(1/2)b+1)^2 = (\frac{1}{4}a)^2+(-\frac{1}{2}b)^2+(1)^2+(2\times\frac{1}{4}a\times-\frac{1}{2}b)+(2\times-\frac{1}{2}b\times1)+(2\times1\times\frac{1}{4}a)$$
$$=\frac{1}{16}a^2+\frac{1}{4}b^2+1^2-\frac{2}{8}ab-\frac{2}{2}b+\frac{2}{4}a$$
$$=\frac{1}{16}a^2+\frac{1}{4}b^2+1-\frac{1}{4}ab-b+\frac{1}{2}a$$

5. Factorize:

(i) $4x^2+9y^2+16z^2+12xy-24yz-16xz$

(ii) $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$

Solution:

(i) $4x^2+9y^2+16z^2+12xy-24yz-16xz$

Using identity, $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$

We can say that, $x^2+y^2+z^2+2xy+2yz+2zx = (x+y+z)^2$

$$4x^2+9y^2+16z^2+12xy-24yz-16xz = (2x)^2+(3y)^2+(-4z)^2+(2\times2x\times3y)+(2\times3y\times-4z)+(2\times-4z\times2x)$$
$$= (2x+3y-4z)^2$$
$$= (2x+3y-4z)(2x+3y-4z)$$

(iii) $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$

Using identity, $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$

We can say that, $x^2+y^2+z^2+2xy+2yz+2zx = (x+y+z)^2$

$2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$

$$= (-\sqrt{2}x)^2+(y)^2+(2\sqrt{2}z)^2+(2\times-\sqrt{2}x\times y)+(2\times y\times2\sqrt{2}z)+(2\times2\sqrt{2}\times-\sqrt{2}x)$$
$$= (-\sqrt{2}x+y+2\sqrt{2}z)^2$$
$$= (-\sqrt{2}x+y+2\sqrt{2}z)(-\sqrt{2}x+y+2\sqrt{2}z)$$

6. Write the following cubes in expanded form:

(i) $(2x+1)^3$

(ii) $(2a-3b)^3$

(iii) $((3/2)x+1)^3$

(iv) $(x-(2/3)y)^3$

Solution:

(i) $(2x+1)^3$

Using identity, $(x+y)^3 = x^3+y^3+3xy(x+y)$

$(2x+1)^3= (2x)^3+1^3+(3\times2x\times1)(2x+1)$

$= 8x^3+1+6x(2x+1)$

$= 8x^3+12x^2+6x+1$

(ii) $(2a-3b)^3$

Using identity, $(x-y)^3 = x^3-y^3-3xy(x-y)$

$(2a-3b)^3 = (2a)^3-(3b)^3-(3\times2a\times3b)(2a-3b)$

$= 8a^3-27b^3-18ab(2a-3b)$

$= 8a^3-27b^3-36a^2b+54ab^2$

(iii) $((3/2)x+1)^3$

Using identity, $(x+y)^3 = x^3+y^3+3xy(x+y)$

$((3/2)x+1)^3=((3/2)x)^3+1^3+(3\times(3/2)x\times1)((3/2)x +1)$

$$= \frac{27}{8}x^3+1+\frac{9}{2}x(\frac{3}{2}x+1)$$

$$= \frac{27}{8}x^3+1+\frac{27}{4}x^2+\frac{9}{2}x$$

$$= \frac{27}{8}x^3+\frac{27}{4}x^2+\frac{9}{2}x+1$$

(iv) $(x-(2/3)y)^3$

Using identity, $(x-y)^3 = x^3-y^3-3xy(x-y)$

$$(x-\frac{2}{3}y)^3=(x)^3-(\frac{2}{3}y)^3-(3\times x\times\frac{2}{3}y)(x-\frac{2}{3}y)$$

$$=(x)^3-\frac{8}{27}y^3-2xy(x-\frac{2}{3}y)$$

$$=(x)^3-\frac{8}{27}y^3-2x^2y+\frac{4}{3}xy^2$$

7. Evaluate the following using suitable identities:

(i) $(99)^3$
(ii) $(102)^3$
(iii) $(998)^3$

Solutions:

(i) $(99)^3$

Solution:

We can write 99 as 100–1

Using identity, $(x-y)^3 = x^3-y^3-3xy(x-y)$

$(99)^3 = (100-1)^3$

$= (100)^3-1^3-(3\times100\times1)(100-1)$

$= 1000000 -1-300(100 - 1)$

$= 1000000-1-30000+300$

$= 970299$

(ii) $(102)^3$

Solution:

We can write 102 as 100+2

Using identity, $(x+y)^3 = x^3+y^3+3xy(x+y)$

$$\begin{aligned}(100+2)^3 &= (100)^3+2^3+(3\times100\times2)(100+2)\\ &= 1000000 + 8 + 600(100 + 2)\\ &= 1000000 + 8 + 60000 + 1200\\ &= 1061208\end{aligned}$$

(iii) $(998)^3$

Solution:

We can write 99 as 1000–2

Using identity, $(x-y)^3 = x^3-y^3-3xy(x-y)$

$$\begin{aligned}(998)^3 &= (1000-2)^3\\ &= (1000)^3-2^3-(3\times1000\times2)(1000-2)\\ &= 1000000000-8-6000(1000-2)\\ &= 1000000000-8-6000000+12000\\ &= 994011992\end{aligned}$$

8. Factorise each of the following:

(i) $8a^3+b^3+12a^2b+6ab^2$
(ii) $8a^3-b^3-12a^2b+6ab^2$
(iii) $27-125a^3-135a+225a^2$
(iv) $64a^3-27b^3-144a^2b+108ab^2$
(v) $27p^3-(1/216)-(9/2)p^2+(1/4)p$

Solutions:

(i) $8a^3+b^3+12a^2b+6ab^2$

Solution:

The expression, $8a^3+b^3+12a^2b+6ab^2$ can be written as $(2a)^3+b^3+3(2a)^2b+3(2a)(b)^2$

$$\begin{aligned}8a^3+b^3+12a^2b+6ab^2 &= (2a)^3+b^3+3(2a)^2b+3(2a)(b)^2\\ &= (2a+b)^3\\ &= (2a+b)(2a+b)(2a+b)\end{aligned}$$

Here, the identity, $(x+y)^3 = x^3+y^3+3xy(x+y)$ is used.

(ii) $8a^3-b^3-12a^2b+6ab^2$

Solution:

The expression, $8a^3-b^3-12a^2b+6ab^2$ can be written as $(2a)^3-b^3-3(2a)^2b+3(2a)(b)^2$

$$\begin{aligned}8a^3-b^3-12a^2b+6ab^2 &= (2a)^3-b^3-3(2a)^2b+3(2a)(b)^2\\ &= (2a-b)^3\\ &= (2a-b)(2a-b)(2a-b)\end{aligned}$$

Here, the identity, $(x-y)^3 = x^3-y^3-3xy(x-y)$ is used.

(iii) $27–125a^3–135a+225a^2$

Solution:

The expression, $27–125a^3–135a+225a^2$ can be written as $3^3–(5a)^3–3(3)^2(5a)+3(3)(5a)^2$

$$27–125a^3–135a+225a^2 = 3^3–(5a)^3–3(3)^2(5a)+3(3)(5a)^2$$
$$= (3–5a)^3$$
$$= (3–5a)(3–5a)(3–5a)$$

Here, the identity, $(x–y)^3 = x^3–y^3-3xy(x–y)$ is used.

(iv) 64a3–27b3–144a^2b+108ab^2

Solution:

The expression, $64a^3–27b^3–144a^2b+108ab^2$ can be written as $(4a)^3–(3b)^3–3(4a)^2(3b)+3(4a)(3b)^2$

$$64a^3–27b^3–144a^2b+108ab^2=(4a)^3–(3b)^3–3(4a)^2(3b)+3(4a)(3b)^2$$
$$=(4a–3b)^3$$
$$=(4a–3b)(4a–3b)(4a–3b)$$

Here, the identity, $(x – y)^3 = x^3 – y^3 – 3xy(x – y)$ is used.

(v) $7p^3$– (1/216)−(9/2) p^2+(1/4)p

Solution:

The expression, $27p^3–(1/216)−(9/2) p^2+(1/4)p$

can be written as $(3p)^3–(1/6)^3–3(3p)^2(1/6)+3(3p)(1/6)^2$

$$27p^3–(1/216)−(9/2) p^2+(1/4)p = (3p)^3–(1/6)^3–3(3p)^2(1/6)+3(3p)(1/6)^2$$
$$= (3p–16)^3$$
$$= (3p–16)(3p–16)(3p–16)$$

9. Verify:

(i) $x^3+y^3 = (x+y)(x^2–xy+y^2)$
(ii) $x^3–y^3 = (x–y)(x^2+xy+y^2)$

Solutions:

(i) $x^3+y^3 = (x+y)(x^2–xy+y^2)$

We know that, $(x+y)^3 = x^3+y^3+3xy(x+y)$

$$\Rightarrow x^3+y^3 = (x+y)^3–3xy(x+y)$$
$$\Rightarrow x^3+y^3 = (x+y)[(x+y)^2–3xy]$$

Taking (x+y) common $\Rightarrow x^3+y^3 = (x+y)[(x^2+y^2+2xy)–3xy]$

$$\Rightarrow x^3+y^3 = (x+y)(x^2+y^2–xy)$$

(ii) $x^3–y^3 = (x–y)(x^2+xy+y^2)$

We know that, $(x–y)^3 = x^3–y^3–3xy(x–y)$

$$\Rightarrow x^3–y^3 = (x–y)^3+3xy(x–y)$$
$$\Rightarrow x^3–y^3 = (x–y)[(x–y)^2+3xy]$$

Taking (x+y) common $\Rightarrow x^3–y^3 = (x–y)[(x^2+y^2–2xy)+3xy]$

$$\Rightarrow x^3+y^3 = (x–y)(x^2+y^2+xy)$$

10. Factorize each of the following:

(i) $27y^3+125z^3$
(ii) $64m^3–343n^3$
Solutions:
(i) $27y^3+125z^3$
The expression, $27y^3+125z^3$ can be written as $(3y)^3+(5z)^3$
$27y^3+125z^3 = (3y)^3+(5z)^3$
We know that, $x^3+y^3 = (x+y)(x^2–xy+y^2)$
$\therefore 27y^3+125z^3 = (3y)^3+(5z)^3$
$= (3y+5z)[(3y)^2–(3y)(5z)+(5z)^2]$
$= (3y+5z)(9y^2–15yz+25z^2)$
(ii) $64m^3–343n^3$
The expression, $64m^3–343n^3$ can be written as $(4m)^3–(7n)^3$
$64m^3–343n^3 = (4m)^3–(7n)^3$
We know that, $x^3–y^3 = (x–y)(x^2+xy+y^2)$
$\therefore 64m^3–343n^3 = (4m)^3–(7n)^3$
$= (4m+7n)[(4m)^2+(4m)(7n)+(7n)^2]$
$= (4m+7n)(16m^2+28mn+49n^2)$

11. Factorise: $27x^3+y^3+z^3–9xyz$

Solution:
The expression $27x^3+y^3+z^3–9xyz$ can be written as $(3x)^3+y^3+z^3–3(3x)(y)(z)$
$27x^3+y^3+z^3–9xyz = (3x)^3+y^3+z^3–3(3x)(y)(z)$

We know that, $x^3+y^3+z^3–3xyz = (x+y+z)(x^2+y^2+z^2–xy –yz–zx)$

$\therefore 27x^3+y^3+z^3–9xyz = (3x)^3+y^3+z^3–3(3x)(y)(z)$
$= (3x+y+z)(3x)^2+y^2+z^2–3xy–yz–3xz$
$= (3x+y+z)(9x^2+y^2+z^2–3xy–yz–3xz)$

12. Verify that:

$x^3+y^3+z^3–3xyz = (1/2) (x+y+z)[(x–y)^2+(y–z)^2+(z–x)^2]$
Solution:
We know that,
$x^3+y^3+z^3–3xyz = (x+y+z)(x^2+y^2+z^2–xy–yz–xz)$
$\Rightarrow x^3+y^3+z^3–3xyz = (1/2)(x+y+z)[2(x^2+y^2+z^2–xy–yz–xz)]$
$= (1/2)(x+y+z)(2x^2+2y^2+2z^2–2xy–2yz–2xz)$
$= (1/2)(x+y+z)[(x^2+y^2–2xy)+(y^2+z^2–2yz)+(x^2+z^2–2xz)]$
$= (1/2)(x+y+z)[(x–y)^2+(y–z)^2+(z–x)^2]$

13. If $x+y+z = 0$, show that $x^3+y^3+z^3 = 3xyz$.

Solution:

We know that,

$x^3+y^3+z^3-3xyz = (x+y+z)(x^2+y^2+z^2–xy–yz–xz)$

Now, according to the question, let $(x+y+z) = 0$,

then, $x^3+y^3+z^3-3xyz = (0)(x^2+y^2+z^2–xy–yz–xz)$

$\Rightarrow \quad x^3+y^3+z^3–3xyz = 0$

$\Rightarrow \quad x^3+y^3+z^3 = 3xyz$

Hence Proved

14. Without actually calculating the cubes, find the value of each of the following:

(i) $(-12)^3+(7)^3+(5)^3$

(ii) $(28)^3+(-15)^3+(-13)^3$

(i) $(-12)^3+(7)^3+(5)^3$

Solution:

$(-12)^3+(7)^3+(5)^3$

Let $a = -12$

$b = 7$

$c = 5$

We know that if $x+y+z = 0$, then $x^3+y^3+z^3=3xyz$.

Here, $-12+7+5=0$

$\therefore \ (-12)^3+(7)^3+(5)^3 = 3xyz$

$= 3\times-12\times7\times5$

$= -1260$

(ii) $(28)^3+(-15)^3+(-13)^3$

Solution:

$(28)^3+(-15)^3+(-13)^3$

Let $a = 28$

$b = -15$

$c = -13$

We know that if $x+y+z = 0$, then $x^3+y^3+z^3 = 3xyz$.

Here, $x+y+z = 28–15–13 = 0$

$\therefore \ (28)^3+(-15)^3+(-13)^3 = 3xyz$

$= 0+3(28)(-15)(-13)$

$= 16380$

15. Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:

(i) Area : $25a^2–35a+12$

(ii) Area : $35y^2+13y–12$

Solution:

(i) Area : $25a^2–35a+12$

Using the splitting the middle term method,
We have to find a number whose sum = -35 and product $=25\times12=300$
We get -15 and -20 as the numbers [$-15+-20=-35$ and $-15\times-20=300$]

$$\begin{aligned} 25a^2–35a+12 &= 25a^2–15a–20a+12 \\ &= 5a(5a–3)–4(5a–3) \\ &= (5a–4)(5a–3) \end{aligned}$$

Possible expression for length = $5a–4$
Possible expression for breadth = $5a –3$

(ii) Area : $35y^2+13y–12$
Using the splitting the middle term method,
We have to find a number whose sum = 13 and product = $35\times-12 = 420$
We get -15 and 28 as the numbers [$-15+28 = 13$ and $-15\times28=420$]

$$\begin{aligned} 35y^2+13y–12 &= 35y^2–15y+28y–12 \\ &= 5y(7y–3)+4(7y–3) \\ &= (5y+4)(7y–3) \end{aligned}$$

Possible expression for length = $(5y+4)$
Possible expression for breadth = $(7y–3)$

16. What are the possible expressions for the dimensions of the cuboids whose volumes are given below?

(i) Volume : $3x^2–12x$
(ii) Volume : $12ky^2+8ky–20k$

Solution:

(i) Volume : $3x^2–12x$
$3x^2–12x$ can be written as $3x(x–4)$ by taking $3x$ out of both the terms.
Possible expression for length = 3
Possible expression for breadth = x
Possible expression for height = $(x–4)$

(ii) Volume: $12ky^2+8ky-20k$

$12ky^2+8ky-20k$ can be written as $4k(3y^2+2y-5)$ by taking 4k out of both the terms.

$12ky^2+8ky-20k = 4k(3y^2+2y-5)$

[Here, $3y^2+2y-5$ can be written as $3y^2+5y-3y-5$ using splitting the middle term method.]

$= 4k(3y^2+5y-3y-5)$

$= 4k[y(3y+5)-1(3y+5)]$

$= 4k(3y+5)(y-1)$

Possible expression for length = 4k

Possible expression for breadth = (3y +5)

Possible expression for height = (y -1)