* Elementry row operations / Gaussian elemination

let,

$17x_1 + 3x_2 = 9$
$10x_1 - 2x_2 = 0$

=) translate to matrix

$$\left(\begin{array}{cc|c} 17 & 3 & 9 \\ 10 & -2 & 0 \end{array}\right)$$

=) divide by the first coofficient

$$\left(\begin{array}{cc|c} 1 & \frac{3}{17} & \frac{9}{17} \\ 1 & -\frac{1}{5} & 0 \end{array}\right)$$

=) add -② to ①

$$\left(\begin{array}{cc|c} 0 & \frac{3}{17}+\frac{1}{5} & \frac{9}{17} \\ 1 & -\frac{1}{5} & 0 \end{array}\right)$$

=) divide by first coefficient

$$\left(\begin{array}{cc|c} 0 & 1 & \frac{45}{22} \\ 1 & -\frac{1}{5} & 0 \end{array}\right)$$

=) swap

$$\left(\begin{array}{cc|c} 1 & -\frac{1}{5} & 0 \\ 0 & 1 & \frac{45}{22} \end{array}\right)$$

=) add $\frac{1}{5}$ ② to ①

$$\left(\begin{array}{cc|c} 1 & 0 & \frac{9}{22} \\ 0 & 1 & \frac{45}{22} \end{array}\right)$$

$\therefore \; x_1 = \frac{9}{22}$

$x_2 = \frac{45}{22}$

Operate by 3 Elementry row operations

(1) you may multiply a row, by a value k $(\neq 0)$
(2) You may add a row to row
(3) you can swap two diffrent rows.

=) these operation doesn't change the solutions.

* The gaussian elimination

=) make it into this form

(1) have ones in the main diagnal
(2) have zeros below it

=) like this

$$\left(\begin{array}{ccc|c} 1 & ? & ? & ? \\ 0 & 1 & ? & ? \\ 0 & 0 & 1 & ? \end{array}\right)$$

* How many solution?

ex 1)

$$\left(\begin{array}{cc|c} 1 & 1 & 1 \\ 1 & 1 & 0 \end{array}\right)$$

$R_2 = R_2 - R_1$

$$\left(\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 0 & -1 \end{array}\right)$$

=> no solution

ex 2)

$$\left(\begin{array}{cc|c} 1 & 1 & 1 \\ 1 & 1 & 1 \end{array}\right)$$

$R_2 = R_2 - R_1$

$$\left(\begin{array}{cc|c} 1 & 1 & 1 \\ \boxed{0 \;\; 0} & & \boxed{0} \end{array}\right)$$

No meaning

=) infinite solutions

* REF / RREF

=) REF : Leading ones have to make some kind of stairs.

$$\left(\begin{array}{cccccccccc|c} 0 & 1 & ? & ? & ? & ? & ? & ? & ? & & 7 \\ 0 & 0 & 0 & 0 & 1 & ? & ? & ? & & & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & & & 9 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & 10 \end{array}\right)$$

=) RREF

=) REF, But every thing except leading zeros are 0.

$$\left(\begin{array}{cccccccccc|c} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & 7 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & & & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & & & 9 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & 10 \end{array}\right)$$

Theorm! RREF is unique.

* Vectors

(1) Vector addition

i) geometrically

⇒ make a triangle, tip-to-tail

ii) algebraically

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} a_1 + b_1 \\ a_2 + b_2 \end{pmatrix}$$

(2) finding points.

can we solve

$$x_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} + x_2 \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} ?$$

$$\Rightarrow \begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \end{pmatrix}$$

inverse of

$$\begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix} \text{ is } \frac{1}{3} \begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{3} & \frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{pmatrix}$$

multiply both side,

$$\begin{pmatrix} \frac{1}{3} & \frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{pmatrix} \times \begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{pmatrix} \times \begin{pmatrix} 0 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -\frac{2}{3} \\ -\frac{4}{3} \end{pmatrix}$$