Inverse matrices

1) let the Identity function
$F(x)$
$F(a+b) = a+b = F(a) + F(b)$
$F(cx) = cx = c \times x = c \times F(x) = cF(x)$
$\therefore$ F is linear.

2)
(1) $|A| = xz - y \neq 0$
(2) $A^{-1} = \frac{1}{xz-y}\begin{pmatrix} z & -y \\ 0 & x \end{pmatrix} = \begin{pmatrix} x & y \\ 0 & z \end{pmatrix}$

$x = z \times \frac{1}{xz-y}$

$y = -y \times \frac{1}{xz-y}$

$z = x \times \frac{1}{xz-y}$

i) $y = 0$

$x = \frac{1}{x}, \quad x \neq 0$

$z = 0$

ii) $y \neq 0$

$\frac{1}{xz - y} = -1$

$y - xz = 1$

$y = xz + 1 = -x^2 + 1$

$x = -z$

$\therefore \boxed{\begin{array}{l} y = -x^2 + 1 \\ z = -x \end{array}}$

3) $B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ $(BA)^2 = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$

what is $(AB)^2$?

given:

$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \times A \times \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \times A = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$$

multiply

$$\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$$

$$A \times \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \times A = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

multiply B.

$$A \times \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \times A \times \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$$

$$= \boxed{\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}}$$