1. let $A=\begin{bmatrix}0&1\\1&0\end{bmatrix}$, find $A+A^2+\cdots+A^{2002}$

sol)

$$A=\begin{bmatrix}0&1\\1&0\end{bmatrix}$$

$$A^2=\begin{bmatrix}1&0\\0&1\end{bmatrix}$$

$$A^3=\begin{bmatrix}0&1\\1&0\end{bmatrix}$$

$$\vdots$$

$$\therefore Ans=\left(\begin{bmatrix}0&1\\1&0\end{bmatrix}+\begin{bmatrix}1&0\\0&1\end{bmatrix}\right)\times 1001$$

$$=\boxed{\begin{bmatrix}1001&1001\\1001&1001\end{bmatrix}}$$

2.

Problem)

Let

$$A=\begin{bmatrix}\cos\frac{\pi}{2014}&-\sin\frac{\pi}{2014}\\ \sin\frac{\pi}{2014}&\cos\frac{\pi}{2014}\end{bmatrix}$$

Find b_{11} where

$$\begin{bmatrix}b_{11}&b_{12}\\b_{21}&b_{22}\end{bmatrix}=A+A^2+\cdots+A^{2014}$$

sol)

let

$$B=\begin{bmatrix}\cos\theta&-\sin\theta\\ \sin\theta&\cos\theta\end{bmatrix}$$

$$B^2=\begin{bmatrix}\cos^2\theta-\sin^2\theta&-2\sin\theta\cos\theta\\ 2\sin\theta\cos\theta&-\sin^2\theta+\cos^2\theta\end{bmatrix}$$

note, $\cos 2\theta=\cos^2\theta-\sin^2\theta$

$\sin 2\theta = 2\sin\theta\cos\theta$

$$\therefore B^2=\begin{bmatrix}\cos 2\theta&-\sin 2\theta\\ \sin 2\theta&\cos 2\theta\end{bmatrix}$$

in fer,

maybe $B^n=\begin{bmatrix}\cos n\theta&-\sin n\theta\\ \sin n\theta&\cos n\theta\end{bmatrix}$?

mathematical induction,

if $B^{n-1}=\begin{bmatrix}\cos(n-1)\theta&-\sin(n-1)\theta\\ \sin(n-1)\theta&\cos(n-1)\theta\end{bmatrix}$,

prove $B^n=\begin{bmatrix}\cos 2\theta&-\sin 2\theta\\ \sin 2\theta&\cos 2\theta\end{bmatrix}$

$$B^{n-1}\times B=B^n=\begin{bmatrix}\cos(n-1)\theta&-\sin(n-1)\theta\\ \sin(n-1)\theta&\cos(n-1)\theta\end{bmatrix}\times\begin{bmatrix}\cos\theta&-\sin\theta\\ \sin\theta&\cos\theta\end{bmatrix}$$

10dn

$$=\begin{bmatrix}\cos(n-1)\theta\cos\theta-\sin(n-1)\theta\sin\theta & -\cos(n-1)\theta\sin\theta-\sin(n-1)\theta\cos\theta\\ \sin(n-1)\theta\cos\theta+\cos(n-1)\theta\sin\theta & -\sin(n-1)\theta\sin\theta+\cos(n-1)\theta\cos\theta\end{bmatrix}$$

$$=\begin{bmatrix}\cos(n\theta)&-\sin(n\theta)\\ \sin(n\theta)&\cos(n\theta)\end{bmatrix}$$

$$\therefore b_{11}=\cos\left(\frac{\pi}{2014}\right)+\cos\left(\frac{2\pi}{2014}\right)+\cdots+\cos\left(\frac{2014\pi}{2014}\right)$$

since $\cos(\theta)+\cos(\pi-\theta)=0$,

$$b_{11} = \cos\left(\frac{2014\pi}{2014}\right) = -1$$

problem 3)

For a complex number $z = x + iy$,

$(x, y \in \mathbb{R})$, let $M(z) = \begin{bmatrix} x & y \\ -y & x \end{bmatrix}$

(1) prove that $M(z + z') = M(z) + M(z')$,

$M(zz') = M(z)M(z')$

-) $M(z + z')$, if we let $z = a + bi$, $z' = a' + b'i$ $(a, b, a', b' \in \mathbb{R})$

$$M(z + z') = M[(a + a') + (b + b')i]$$

$$= \begin{bmatrix} a + a' & b + b' \\ -b - b' & \end{bmatrix}$$