

# Maths – analysis

## 1. Probability

Probability is the branch of mathematics that deals with predicting how likely events are to occur.

### Key Concepts & Formulas

- **Definition of Probability**

The probability of an event  $A$  occurring is:

$$P(A) = \frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}$$
$$P(A) = \frac{\text{total number of possible outcomes}}{\text{number of favorable outcomes}}$$

where  $0 \leq P(A) \leq 1$

- **Types of Events:**

- **Certain Event**  $P(A) = 1$  (e.g., rolling a number between 1 and 6 on a fair die).
- **Impossible Event**  $P(A) = 0$  (e.g., rolling a 7 on a fair die).
- **Independent Events:** The occurrence of one event does not affect the other.
- **Dependent Events:** The occurrence of one event affects the probability of the other.

- **Addition Rule (For Mutually Exclusive Events):**

$$P(A \cup B) = P(A) + P(B)$$

- **Multiplication Rule (For Independent Events):**

$$P(A \cap B) = P(A) \cdot P(B)$$

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## 2. Functions

Functions describe relationships between variables, usually written as  $y = f(x)$ .

### Key Types of Functions & Their Properties

- **Linear Function:**

$$f(x) = ax + b$$

- Slope:  $a$  (determines how steep the graph is).
- $b$  is the  $y$ -intercept.

- **Quadratic Function:**

$$f(x) = ax^2 + bx + c$$

- Parabola shape (opens upward if  $a > 0$ , downward if  $a < 0$ ).
- Vertex:  $x = -\frac{b}{2a}$

- **Rational Function:**

$$f(x) = \frac{P(x)}{Q(x)}$$

- Asymptotes exist where  $Q(x) = 0$ .

- **Exponential Function:**

$$f(x) = a^x, a > 0, a \neq 1$$

- Always positive and increases (if  $a > 1$ ) or decreases (if  $0 < a < 1$ ).

### 3. Systems of First- and Second-Degree Equations

A system of equations consists of two or more equations that share variables.

#### Linear System (Two First-Degree Equations)

$$\begin{cases} ax + by = c \\ dx + ey = f \end{cases}$$

#### Solving Methods:

- **Substitution** (express one variable in terms of another).
- **Elimination** (add/subtract equations to eliminate a variable).

#### Linear-Quadratic System

$$\begin{cases} y = ax^2 + bx + c \\ y = mx + n \end{cases}$$

Solutions correspond to the intersection points of a **parabola** and a **line** (0, 1, or 2 solutions).

## 4. Similar Triangles

Two triangles are similar if their corresponding angles are equal and their corresponding sides are proportional.

### Criteria for Similarity:

1. **AA (Angle-Angle)** – If two angles are equal, the triangles are similar.
2. **SAS (Side-Angle-Side)** – If two sides are proportional and the included angle is equal, the triangles are similar.
3. **SSS (Side-Side-Side)** – If all corresponding sides are proportional, the triangles are similar.

### Properties of Similar Triangles:

- **Proportionality of Sides:**  $\frac{A_1B_1}{A_2B_2} = \frac{B_1C_1}{B_2C_2} = \frac{A_1C_1}{A_2C_2}$   
 $\frac{A_1B_1}{B_1C_1} = \frac{A_2B_2}{B_2C_2}$   
 $\frac{A_1C_1}{A_2C_2} = \frac{B_1C_1}{B_2C_2}$   
 $A_1B_1 \cdot B_2C_2 = B_1C_1 \cdot A_2B_2$   
 $A_1C_1 \cdot B_2C_2 = B_1C_1 \cdot A_2C_2$   
 $A_1B_1 \cdot A_2C_2 = A_1C_1 \cdot A_2B_2$
- **Height in a Right Triangle:**  $h^2 = p \cdot q$  where  $p$  and  $q$  are the segments of the hypotenuse.

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## 5. Rational Inequalities

Inequalities involving fractions where the numerator and denominator are polynomials.

### Solving Steps:

1. Find **critical points** by setting numerator and denominator to 0.
2. Identify intervals using these points.
3. Test each interval and determine sign changes.
4. Express the final solution in **interval notation**.

Example:

$$\frac{x-2}{x+3} > 0$$

**Critical points:**  $x=2, x=-3$ , then test intervals to find the solution.

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## 6. Metric Relations Between Segments

Deals with segment relationships in triangles, especially right triangles.

### Key Formulas:

- **Right Triangle Height Relation:**  $h^2 = p \cdot q$   $h^2 = p \cdot q$
  - **Pythagorean Theorem:**  $a^2 + b^2 = c^2$   $a^2 + b^2 = c^2$
  - **Leg Theorem:**  $a^2 = c \cdot p$ ,  $b^2 = c \cdot q$   $a^2 = c \cdot p$ ,  $b^2 = c \cdot q$
  - **Projection Theorem:**  $h = a \cdot \sin B = b \cdot \sin A$   $h = a \cdot \sin B = b \cdot \sin A$
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## 7. Trigonometric Functions of an Acute Angle

Used to describe angles in right triangles.

### Basic Trigonometric Ratios:

- **Sine:**  $\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$   $\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$
- **Cosine:**  $\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$   $\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$
- **Tangent:**  $\tan A = \frac{\sin A}{\cos A} = \frac{\text{opposite}}{\text{adjacent}}$   $\tan A = \frac{\sin A}{\cos A} = \frac{\text{opposite}}{\text{adjacent}}$

### Trigonometric Identities:

$$\sin^2 A + \cos^2 A = 1 \quad \sin^2 A + \cos^2 A = 1 \quad 1 + \tan^2 A = \frac{1}{\cos^2 A} \quad 1 + \tan^2 A = \frac{1}{\cos^2 A}$$