

Table 4.1 | Effective density of states function and effective mass values

	$N_c \text{ (cm}^{-3}\text{)}$	$N_v \text{ (cm}^{-3}\text{)}$	m_n^*/m_0	m_p^*/m_0
Silicon	2.8×10^{19}	1.04×10^{19}	1.08	0.56
Gallium arsenide	4.7×10^{17}	7.0×10^{18}	0.067	0.48
Germanium	1.04×10^{19}	6.0×10^{18}	0.55	0.37

Table 4.2 | Commonly accepted values of n_i at $T = 300 \text{ K}$

Silicon	$n_i = 1.5 \times 10^{10} \text{ cm}^{-3}$
Gallium arsenide	$n_i = 1.8 \times 10^6 \text{ cm}^{-3}$
Germanium	$n_i = 2.4 \times 10^{13} \text{ cm}^{-3}$

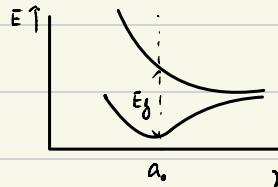
4.1

Calculate the intrinsic carrier concentration, n_i , at $T = 200, 400$, and 600 K for (a) silicon, (b) germanium, and (c) gallium arsenide.

$$n_i^2 = N_0 P_0$$

$$= N_c N_v \exp\left(-\frac{E_g}{kT}\right), \quad N_c = N_{c0} \left(\frac{T}{300}\right)^{\frac{3}{2}}$$

$$= N_{c0} N_v \left(\frac{T}{300}\right)^3 \exp\left(-\frac{E_g}{kT}\right) \quad 300k, \quad a_0, \quad E_g = 1.12 \text{ eV}$$



- 4.7 Assume the Boltzmann approximation in a semiconductor is valid. Determine the ratio of $n(E) = g_c(E)f_F(E)$ at $E = E_c + 4kT$ to that at $E = E_c + kT/2$.

$$\frac{n(E_1)}{n(E_2)} = \frac{g_c(E_1)}{g_c(E_2)} \frac{\exp\left(\frac{E_1 - E_F}{kT}\right)}{\exp\left(\frac{E_2 - E_F}{kT}\right)}$$

$$= \frac{\frac{4\pi(2m_n^*)^{\frac{3}{2}}}{h^3} \sqrt{E_1 - E_c}}{\frac{4\pi(2m_n^*)^{\frac{3}{2}}}{h^3} \sqrt{E_2 - E_c}} \exp\left[\frac{(E_1 - E_F) - (E_2 - E_F)}{kT}\right]$$

$$= \frac{\sqrt{E_1 - E_c}}{\sqrt{E_2 - E_c}} \exp\left(\frac{E_1 - E_2}{kT}\right)$$

$$= \frac{\sqrt{4kT}}{\sqrt{\frac{kT}{2}}} \exp\left[-(4 - \frac{1}{2})\right]$$

$$= 2\sqrt{2} \exp(-3.5) = 0.085$$

4.11 Calculate E_{Fi} with respect to the center of the bandgap in silicon for $T = 200, 400$, and 600 K.

$$n_o = p_o$$

$$N_c \exp\left(\frac{(E_c - E_F)}{kT}\right) = N_v \exp\left(\frac{-(E_{Fi} - E_v)}{kT}\right)$$

$$\ln(N_c) - \frac{E_c - E_F}{kT} = \ln(N_v) - \frac{(E_{Fi} - E_v)}{kT}$$

$$kT \ln(N_c) - E_c + E_F = kT \ln(N_v) - E_{Fi} + E_v$$

$$2E_{Fi} - (E_c + E_v) = kT \ln\left(\frac{N_v}{N_c}\right)$$

$$E_{Fi} - \frac{1}{2}(E_c + E_v) = \frac{1}{2}kT \ln\left(\frac{N_v}{N_c}\right)$$

$E_{\text{edge}}^{\text{gap}}$

4.17 Silicon at $T = 300$ K is doped with arsenic atoms such that the concentration of electrons is $n_0 = 7 \times 10^{15} \text{ cm}^{-3}$. (a) Find $E_c - E_F$. (b) Determine $E_F - E_v$. (c) Calculate p_0 . (d) Which carrier is the minority carrier? (e) Find $E_F - E_{Fi}$.

$$(a) n_o = N_c \exp\left(\frac{(E_c - E_F)}{kT}\right) \quad N_c = 2.8 \times 10^{19}$$

$$\ln(n_o) = \ln(N_c) - \frac{E_c - E_F}{kT} \quad N_v = 1.04 \times 10^{19}$$

$$E_c - E_F = kT \ln\left(\frac{N_c}{n_o}\right) \\ = 0.0259 \cdot \ln\left(\frac{2.8 \times 10^{19}}{7 \times 10^{15}}\right)$$

$$= 0.2148 \text{ eV}$$

$$(b) E_g - (E_c - E_F) = E_F - E_v \quad 1.12 - 0.2148 = 0.9052 \text{ eV}$$

$$(c) p_0 = N_v \exp\left(-\frac{(E_F - E_v)}{kT}\right) \\ = 1.04 \times 10^{19} \exp\left(-\frac{-0.9052}{0.0259}\right) = 6894.8534 \approx 6.9 \times 10^3$$

(d) Holes

$$(e) n_o = N_i \exp\left(\frac{E_F - E_{Fi}}{kT}\right) = \ln(n_o) = \ln(N_i) + \frac{E_F - E_{Fi}}{kT}$$

$$\Rightarrow E_F - E_{Fi} = kT \ln\left(\frac{n_o}{N_i}\right) = 0.0259 \cdot \ln\left(\frac{7 \times 10^{15}}{1.5 \times 10^{10}}\right) = 0.338 \text{ eV}$$

- 4.20** (a) If $E_c - E_F = 0.28$ eV in gallium arsenide at $T = 375$ K, calculate the values of n_0 and p_0 . (b) Assuming the value of n_0 in part (a) remains constant, determine $E_c - E_F$ and p_0 at $T = 300$ K.

$$(a) n_0 = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

$$kT = (0.0259) \frac{375}{300} = 0.032375$$

$$N_{c_0} = 4.7 \times 10^{17}$$

$$= 6.56 \times 10^{17} \exp\left(\frac{-0.28}{0.032375}\right)$$

$$= 1.15 \times 10^{14}$$

$$N_{v_0} = 7 \times 10^{18}$$

$$N_c = 4.7 \times 10^{17} \left(\frac{375}{300}\right)^{\frac{3}{2}}$$

$$P_0: E_F - E_V = E_g - (E_n - E_F)$$

$$= 6.56 \times 10^{17}$$

$$1.42 - 0.28 = 1.14$$

$$N_V = 7 \times 10^{18} \left(\frac{375}{300}\right)^{\frac{3}{2}}$$

$$P_0 = N_V \exp\left(\frac{-1.14}{0.032375}\right) =$$

$$(b) E_c - E_F = kT \ln\left(\frac{N_c}{N_{v_0}}\right)$$

$$= 0.0259 \ln\left(\frac{4.7 \times 10^{17}}{1.15 \times 10^{14}}\right) = 0.2154 \text{ eV}$$

$$E_F - E_V = 1.42 - 0.2154 = 1.2046 \text{ eV}$$

$$P_0 = 7 \times 10^{18} \cdot \exp\left(\frac{-1.2046}{0.0259}\right) = 4.42 \times 10^{-2} \text{ cm}^{-3}$$

4.24

Silicon at $T = 300$ K is doped with boron atoms such that the concentration of holes is $p_0 = 5 \times 10^{15} \text{ cm}^{-3}$. (a) Find $E_F - E_v$. (b) Determine $E_c - E_F$. (c) Determine n_0 . (d) Which carrier is the majority carrier? (e) Determine $E_{F_i} - E_F$.

$$(a) E_F - E_v = kT \ln\left(\frac{N_v}{p_0}\right) = 0.0259 \cdot \ln\left(\frac{1.04 \times 10^{19}}{5 \times 10^{15}}\right) = 0.1979 \text{ eV}$$

$$(b) E_F - E_F = E_g - (E_F - E_v) = 1.12 - 0.1979 = 0.922 \text{ eV}$$

$$(c) n_0 = N_c \exp\left(-\frac{(E_c - E_F)}{kT}\right) = 2.8 \times 10^{19} \cdot \exp\left(-\frac{0.922}{0.0259}\right) = 9.66 \times 10^3 \text{ cm}^{-3}$$

(d) Holes

$$(e) E_{F_i} - E_F = kT \ln\left(\frac{p_0}{n_i}\right)$$

$$= kT \ln\left(\frac{5 \times 10^{15}}{1.5 \times 10^{10}}\right) = 0.329 \text{ eV}$$

Repeat Problem 4.20 for silicon.

4.28 (a) Assume that $E_F = E_c + kT/2$ at $T = 300$ K in silicon. Determine n_0 . (b) Repeat part (a) for GaAs.

$$(a) E_c - E_F < 3 \quad n_F = \frac{E_c - E_F}{kT} = 0.5$$

$$n_0 = \frac{2}{\pi} N_c F_{1/2} (\eta_F)$$

$F_{1/2}(0.5) \approx 1.0$, 有考的話應該會給, 沒給要算 $F_{1/2}$

$$n_0 = \frac{2}{\pi} 2.8 \times 10^{19} \cdot 1 = 3.16 \times 10^{19} \text{ cm}^{-3} \quad F_{1/2} = \int_{E_c}^{\infty} \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

(b)

沒辦法手算

- 4.30** (a) In silicon at $T = 300$ K, we find that $E_F - E_C = 4 kT$. Determine the electron concentration. (b) Repeat part (a) for GaAs.

[Ans] 4.28

- 4.36** (a) Consider a germanium semiconductor at $T = 300$ K. Calculate the thermal equilibrium electron and hole concentrations for (i) $N_d = 2 \times 10^{15} \text{ cm}^{-3}$, $N_a = 0$, and (ii) $N_a = 10^{16} \text{ cm}^{-3}$, $N_d = 7 \times 10^{15} \text{ cm}^{-3}$. (b) Repeat part (a) for GaAs. (c) For the case of GaAs in part (b), the minority carrier concentrations are on the order of 10^{-3} cm^{-3} . What does this result mean physically?

$$(i) \quad n_o = \frac{N_d - N_a}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2} \quad \text{Ge : } n_i = 2.4 \times 10^{13} \text{ cm}^{-3} \\ = \frac{2 \times 10^{15}}{2} + \sqrt{\left(\frac{2 \times 10^{15}}{2}\right)^2 + (2.4 \times 10^{13})^2} \\ = 2 \times 10^{15}$$

$$P_o = \frac{n_i^2}{n_o} = \frac{(2.4 \times 10^{13})^2}{2 \times 10^{15}} = 2.88 \times 10^{11}$$

$$(ii) \quad P_o = \frac{N_a - N_d}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2} \\ = \frac{3 \times 10^{15}}{2} + \sqrt{\left(\frac{3 \times 10^{15}}{2}\right)^2 + (2.4 \times 10^{13})^2} \\ = 3 \times 10^{15} \text{ cm}^{-3}$$

$$n_o = \frac{n_i^2}{P_o} = 1.92 \times 10^{11}$$

- 4.39 A silicon semiconductor material at $T = 300$ K is doped with arsenic atoms to a concentration of $2 \times 10^{15} \text{ cm}^{-3}$ and with boron atoms to a concentration of $1.2 \times 10^{15} \text{ cm}^{-3}$. (a) Is the material n type or p type? (b) Determine n_0 and p_0 . (c) Additional boron atoms are to be added such that the hole concentration is $4 \times 10^{15} \text{ cm}^{-3}$. What concentration of boron atoms must be added and what is the new value of n_0 ?

(a) $N_d > N_a$ n-type

(b)

$$n_0 = \frac{(N_d - N_a)}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2}$$

$$= 4 \times 10^{14} + \sqrt{(4 \times 10^{14})^2 + (1.5 \times 10^{15})^2}$$

$$= 8 \times 10^{14} \text{ cm}^{-3}$$

$$P_0 = \frac{n_i^2}{n_0} = \frac{(1.5 \times 10^{15})^2}{8 \times 10^{14}} = 2.81 \times 10^5 \text{ cm}^{-3}$$

$$(c) P_0 = 4 \times 10^{15} = \frac{N_a - N_d}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2}$$

$$4 \times 10^{15} = \frac{N_a - 2 \times 10^{15}}{2} + \sqrt{\left(\frac{N_a - 2 \times 10^{15}}{2}\right)^2 + (1.5 \times 10^{15})^2}$$

$$= N_{a\text{new}} - 2 \times 10^{15}$$

$$N_{a\text{new}} = 6 \times 10^{15}$$

$$N_{a\text{add}} = 4.8 \times 10^{15} \text{ cm}^{-3}$$

$$n_0 = \frac{n_i^2}{P_0} = 5.625 \times 10^4 \text{ cm}^{-3}$$

4.47 In silicon at $T = 300$ K, it is found that $N_a = 7 \times 10^{15} \text{ cm}^{-3}$ and $p_0 = 2 \times 10^4 \text{ cm}^{-3}$.

- (a) Is the material n type or p type? (b) What are the majority and minority carrier concentrations? (c) What must be the concentration of donor impurities?

(a) n-type

$$(b) n_i = \frac{n_i^2}{p_0} = \frac{(1.5 \times 10^{10})^2}{2 \times 10^4} = 1.125 \times 10^{16} \text{ cm}^{-3}$$

$$p_0 = 2 \times 10^4 \text{ cm}^{-3}$$

(c) $n_i \approx N_d - N_a$

$$1.125 \times 10^{16} = N_d - 7 \times 10^{15}$$

$$N_d = 1.825 \times 10^{16}$$

- 4.49** Consider silicon at $T = 300$ K with donor concentrations of $N_d = 10^{14}, 10^{15}, 10^{16}$, and 10^{17} , cm^{-3} . Assume $N_a = 0$. (a) Calculate the position of the Fermi energy level with respect to the conduction band for these donor concentrations. (b) Determine the position of the Fermi energy level with respect to the intrinsic Fermi energy level for the donor concentrations given in part (a).

$$E_c - E_f = kT \left(\frac{N_c}{n_0} \right)$$

$$n_0 = \frac{N_d - N_i}{2} + \sqrt{\left(\frac{N_d - N_i}{2}\right)^2 + N_i^2}$$

$$N_d \gg N_i$$

$$n_0 \approx N_d$$

$$E_c - E_f = kT \ln \left(\frac{N_c}{N_d} \right)$$

$$N_c = 2.8 \times 10^{15} \text{ from Table 1}$$

$$(a) E_c - E_f = 0.0259 \ln \left(\frac{2.8 \times 10^{15}}{10^{14}} \right) = 0.3249 \text{ eV}$$

$$\vdots \quad 10^{15} = 0.2652 \text{ eV}$$

$$\vdots \quad 10^{16} = 0.2056 \text{ eV}$$

$$\vdots \quad 10^{17} = 1.459 \text{ eV}$$

(b)

$$E_F - E_{F_i} = kT \ln \left(\frac{N_d}{n_i} \right)$$

$$= 0.0259 \ln \left(\frac{N_d}{1.5 \times 10^{16}} \right)$$

4.50 A silicon device is doped with donor impurity atoms at a concentration of 10^{15} cm^{-3} . For the device to operate properly, the intrinsic carriers must contribute no more than 5 percent to the total electron concentration. (a) What is the maximum temperature that the device may operate? (b) What is the change in $E_c - E_F$ from the $T = 300 \text{ K}$ value to the maximum temperature value determined in part (a). (c) Is the Fermi level closer or further from the intrinsic value at the higher temperature?

$$n_0 = \frac{N_d}{2} + \sqrt{\left(\frac{N_d}{2}\right)^2 + n_i^2}$$

↓
是多 5%

$$n_{0\max} = 1.05 N_d \quad 0.05 \text{ 來自 } n_i \text{ 貢獻}$$

$$n_{0\max} = \frac{N_d}{2} + \sqrt{\left(\frac{N_d}{2}\right)^2 + n_{i\max}^2}$$

$$1.05 N_d = \frac{N_d}{2} + \sqrt{\left(\frac{N_d}{2}\right)^2 + n_{i\max}^2}$$

$$0.55 N_d = \sqrt{\left(\frac{N_d}{2}\right)^2 + n_{i\max}^2}$$

$$(0.55 N_d)^2 = \left(\frac{N_d}{2}\right)^2 + n_{i\max}^2 \quad N_d = 10^{15}$$

$$n_{i\max}^2 = (0.55 N_d)^2 - \left(\frac{N_d}{2}\right)^2 = 5.25 \times 10^{28}$$

$$n_{i\max} = N_c N_v \left(\frac{T}{300}\right)^3$$

$$n_i^2 = N_c N_v \exp\left(-\frac{E_g}{kT}\right)$$

$$5.25 \times 10^{28} = 2.8 \times 10^{19} \times 1.04 \times 10^{17} \left[\frac{T}{300}\right]^3 e^{-\frac{-12972.973}{T}} \quad E_g \text{ 也會隨溫度變化}$$

α_0 會變化

$$T = \dots$$

$$(b) E_c - E_F = kT \left(\frac{N_c}{n_0}\right) \Rightarrow n_{0\max} = 1.5 n_0 \text{ in } 300 \text{ K}$$

簡單理解 热胀冷缩？

這裡忽略

(c) closer

