112_2 Signals and Systems HW#2 Sol

1. Consider a discrete-time LTI system initially at rest and described by the difference equation, that is, y[n] + 2y[n-1] = x[n] + 2x[n-2]
   Find and plot the impulse response of this system.

   Ans:

$y[n] + 2y[n-1] = x[n] + 2x[n-2]$

when $x[n] = \delta[n] \Rightarrow y[n] = h[n]$

$\therefore\ h[n] + 2h[n-1] = \delta[n] + 2\delta[n-2]$

when $n=0 \Rightarrow h[0] = \delta[0] = 1,\ 2h[n-1] = 2\delta[n-2] = 0$

when $n=1 \Rightarrow h[1] + 2h[0] = 0 + 0$

$\therefore h[1] = -2h[0] = -2.$

when $n=2 \Rightarrow h[2] + 2h[1] = 0 + 2$

$h[2] = 2 - 2h[1] = 2 - 2(-2) = 6$

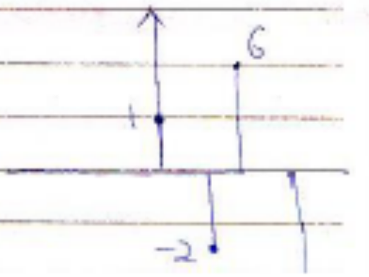
when $n=3 \Rightarrow h[3] + 2h[2] = 0 + 0$

$h[3] = -2h[2] = -6 \cdot 2 = -12$

when $n=4 \Rightarrow h[4] + 2h[3] = 0$

$h[4] = -2h[3] = 24$

$$h[n] = \begin{cases} 0, & n < 0 \\ 1, & n = 0 \\ -2, & n = 1 \\ 6 \cdot 2^{(n-2)}(-1)^n, & n \geq 2. \end{cases}$$



2. Consider the following signals

$$x[n] = \begin{cases} 1, & 0 \leq n \leq 4 \\ 0, & \text{otherwise} \end{cases},$$

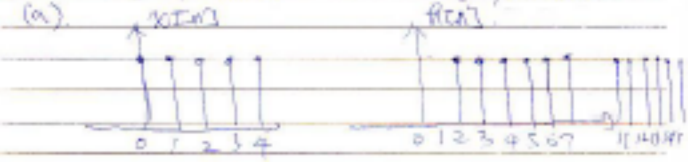
$$h[n] = \begin{cases} 1, & 2 \leq n \leq 7 \quad \textit{and} \quad 11 \leq n \leq 16 \\ 0, & \text{otherwise} \end{cases}$$

(a) Plot x[n] and h[n]

(b) Compute and plot the y[n] = x[n] * h[n], where * denotes the convolution.

$x[n] = \begin{cases} 1, & 0 \le n \le 4 \\ 0, & \text{otherwise} \end{cases}$ $h[n] = \begin{cases} 1, & 2 \le n \le 7,\ 11 \le n \le 16 \\ 0, & \text{otherwise} \end{cases}$

(a)



(b) $x[n] \circledast h[n] = \sum_{m=-\infty}^{\infty} x[n-m]\, h[m]$

$\because x[n-m] = \begin{cases} 1, & 0 \le n-m \le 4 \\ 0, & \text{otherwise} \end{cases}$ $h[m] = \begin{cases} 1, & 2 \le m \le 7 \text{ and } 11 \le m \le 16 \\ 0, & \text{otherwise} \end{cases}$

$\therefore x[n-m] = \begin{cases} 1, & n \ge m \ge -4+n \\ 0, & \text{otherwise} \end{cases}$

部份重疊 ⇒ $n \ge 2$ 且 $-4+n < 2$ ⇒ $2 \le n < 6$

⇒ 1, $n=2$

$1\cdot1+1\cdot1=2$, $n=3$

$1\cdot1+1\cdot1+1\cdot1=3$, $n=4$

$1\cdot1+1\cdot1+1\cdot1+1\cdot1=4$, $n=5$.

完全重疊 $\Rightarrow n \le 16$ 且 $-4+n \ge 2 \Rightarrow 6 \le n \le 16$

$1\cdot1+1\cdot1+1\cdot1+1\cdot1+1\cdot1=5,\ n=6$ $\quad 1\cdot1+0\cdot1+0\cdot1+0\cdot1+1\cdot1=2,\ n=11$

$1\cdot1+1\cdot1+1\cdot1+1\cdot1+1\cdot1=5,\ n=7$ $\quad 0\cdot1+0\cdot1+0\cdot1+1\cdot1+1\cdot1=2,\ n=12$

$1\cdot1+1\cdot1+1\cdot1+1\cdot1+0\cdot1=4,\ n=8$ $\quad 1\cdot1+1\cdot1+1\cdot1=3,\ n=13$

$1\cdot1+1\cdot1+1\cdot1+0\cdot1+0\cdot1=3,\ n=9$ $\quad 1\cdot1+1\cdot1+1\cdot1+1\cdot1=4,\ n=14$

$1\cdot1+1\cdot1+0\cdot1+0\cdot1+0\cdot1=2,\ n=10$ $\quad 1\cdot1+1\cdot1+1\cdot1+1\cdot1+1\cdot1=5,\ n=15$

$1\cdot1+1\cdot1+1\cdot1+1\cdot1+1\cdot1=5,\ n=16$

部分重疊 $\Rightarrow n>16$ 且 $-4+n \le 16 \Rightarrow 16<n\le 20$

$1\cdot1+1\cdot1+1\cdot1+1\cdot1=4,\ n=17$ $\quad 1\cdot1+1\cdot1=2,\ n=19$

$1\cdot1+1\cdot1+1\cdot1=3,\ n=18$ $\quad 1\cdot1=1,\ n=20$

$$x[n]*h[n] = \begin{cases} 1, n=2 & 4, n=8 & 4, n=14 & 1, n=20 \\ 2, n=3 & 3, n=9 & 5, n=15 & 0, \text{otherwise} \\ 3, n=4 & 2, n=10 & 5, n=16 & \\ 4, n=5 & 2, n=11 & 4, n=17 & \\ 5, n=6 & 2, n=12 & 3, n=18 & \\ 5, n=7 & 3, n=13 & 2, n=19 & \end{cases}$$

3. Calculate the Fourier transforms of the following x(t), that is X(f). Also plot their magnitude and phase responses

(a) $x(t) = e^{-2(t-1)}u(t-1)$

(b) $x(t) = e^{-2(t-1)}$

(c) $x(t) = \delta(t+1) + \delta(t-1)$

(d) $x(t) = \text{sinc}(t)$

Ans:

[法一] 由定義入手，作積分 $g(t) \rightarrow \int_{-\infty}^{\infty} g(t) e^{-j2\pi f t} dt$

[法二] 由 Fourier transform pairs 入手，套用恒等式

(a)

① $X(f) = \int_{-\infty}^{\infty} e^{-2(t-1)} u(t-1) e^{-j2\pi f t} dt$

$$= \int_{1}^{\infty} e^{-2t} e^{2} e^{-j2\pi f t} dt = e^{2} \int_{1}^{\infty} e^{-2(1+j\pi f)t} dt$$

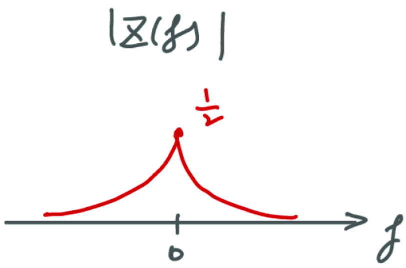
$$= e^{2} \cdot \frac{1}{-2(1+j\pi f)} \cdot \left[e^{-2(1+j\pi f)t}\right]_{t=1}^{t=\infty} = \frac{e^{2}}{-2(1+j\pi f)} \cdot \left[0 - e^{-2(1+j\pi f)}\right]$$

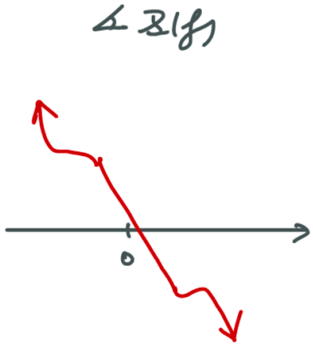
$$= \frac{e^{-j2\pi f}}{2(1+j\pi f)} = \frac{(1-j\pi f) e^{-j2\pi f}}{2(1+\pi^2 f^2)}$$

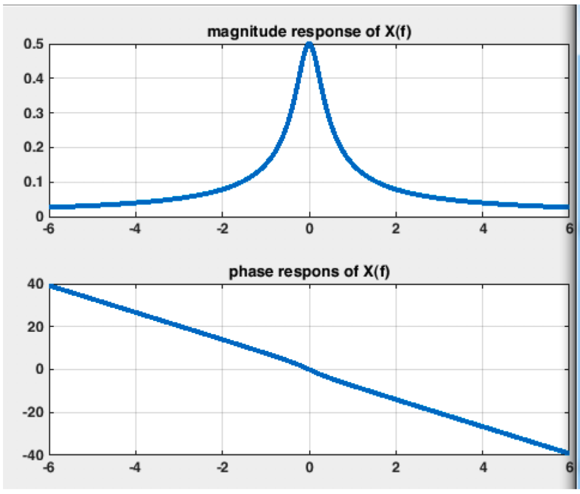
$$= \frac{\sqrt{1+\pi^2 f^2} \angle(\tan^{-1}(-\pi f)) \cdot 1\angle(-2\pi f)}{2(1+\pi^2 f^2)} = \frac{\sqrt{1+\pi^2 f^2} \angle(\tan^{-1}(-\pi f) - 2\pi f)}{2(1+\pi^2 f^2)}$$ #

② $|X(f)| = \dfrac{\sqrt{1+\pi^2 f^2}}{2(1+\pi^2 f^2)} = \dfrac{1}{2\sqrt{1+\pi^2 f^2}} = \dfrac{1}{\sqrt{4+4\pi^2 f^2}}$ # (振幅恒正)

③ $\angle X(f) = \tan^{-1}(-\pi f) - 2\pi f = -\tan^{-1}(\pi f) - 2\pi f$ #








```
subplot(2, 1, 1)
f = -6: 0.01 : 6;
Xf_mag = 1./sqrt(4 + 4*pi*pi*(f.*f) );
plot(f, Xf_mag, 'LineWidth', 3)
grid on
title('magnitude response of X(f)')

subplot(2, 1, 2)
Xf_pha = -atan(pi*f) - 2*pi*f;
plot(f, Xf_pha, 'LineWidth', 3)
grid on
title('phase respons of X(f)')
```

(b)

① $X(f) = \int_{-\inf}^{\inf} e^{-2(t-1)} e^{-j2\pi f t} dt$

$= e^2 \int_{-\inf}^{\inf} e^{-2(1+j\pi f)t} dt = \dfrac{e^2}{-2(1+j\pi f)} \left[e^{-2(1+j\pi f)t} \right]_{t=-\infty}^{t=\infty}$

$= \dfrac{e^2}{-2(1+j\pi f)} \left[0 - e^{+2(1+j\pi f)\infty} \right]$ 無法計算

$\Rightarrow$ $x(t) = e^{-2(t-1)}$ 的傅立葉轉換不存在 #

[註] 若 $x(t)$ 不為絕對可積 ($\int_{-\infty}^{\infty} |x(t)| dt \neq \text{finite}$) ，則 $X(f)$ "有可能"不存在

(c)

【Hint】

$$\int_{-\infty}^{\infty} \delta(t-a) \cdot g(t) = g(a)$$

① $X(f) = \int_{-\inf}^{\inf}[\delta(t+1)+\delta(t-1)]e^{-j2\pi f t}dt$

$= [e^{-j2\pi f t}]_{t=-1} + [e^{-j2\pi f t}]_{t=1} \quad = e^{-j2\pi f} + e^{j2\pi f}$

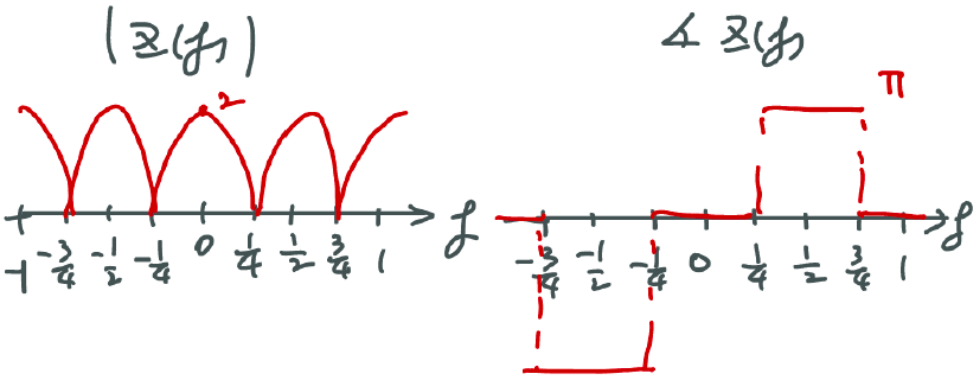
$= [\cos(2\pi f) - j\cdot\sin(2\pi f)] + [\cos(2\pi f) + j\cdot\sin(2\pi f)]$

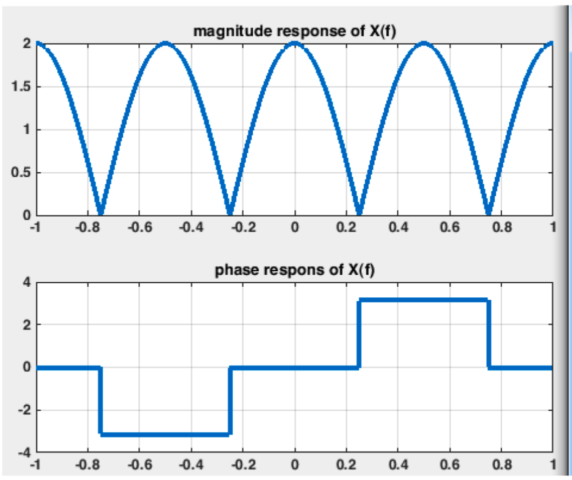
$= 2\cdot\cos(2\pi f)$ （有負值，相角不為0。考慮：實數時域函數 → 奇對稱相位響應）

$$\triangleq \begin{cases} |2\cdot\cos(2\pi f)|\angle(-\pi) \ , & \frac{-3}{4}-k < f < \frac{-1}{4}-k \\ |2\cdot\cos(2\pi f)|\angle 0 \ , & \frac{-1}{4}\pm k \le f \le \frac{1}{4}\pm k \ , \quad k=0,1,2,\cdots \ \# \\ |2\cdot\cos(2\pi f)|\angle\pi \ , & \frac{1}{4}+k < f < \frac{3}{4}+k \end{cases}$$

② $|X(f)| = |2\cdot\cos(2\pi f)|$ #（振幅恒正）

③ $\angle X(f) = 0$ #






```
subplot(2, 1, 1)
f = -1: 0.001 : 1;
Xf_mag = abs( 2 * cos(2*pi*f) );
plot(f, Xf_mag, 'LineWidth', 3)
grid on
title('magnitude response of X(f)')

subplot(2, 1, 2)
Xf_pha = 0*f;
for i = 1 : size(f, 2)
    if (f(i)>(-3/4)) && (f(i)<(-1/4))
        Xf_pha(i) = -pi;
    elseif (f(i)>(1/4)) && (f(i)<(3/4))
        Xf_pha(i) = pi;
    end
end
plot(f, Xf_pha, 'LineWidth', 3)
grid on
title('phase respons of X(f)')
```

(d)

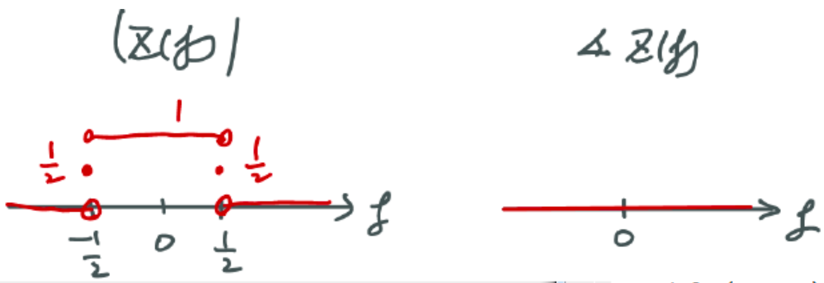
【提示】

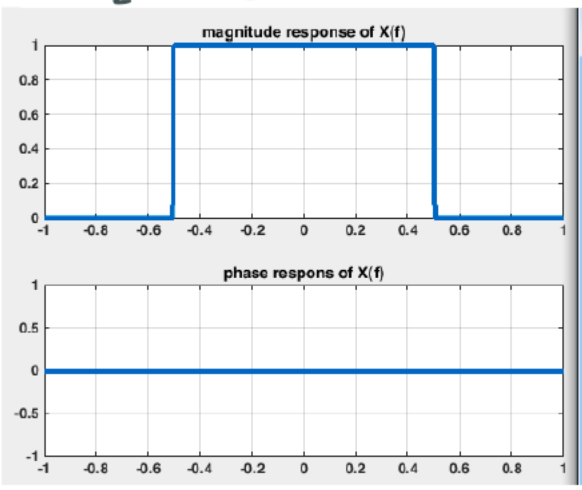
利用 Fourier transform pair： $a\cdot sinc(at) \overset{FT}{\rightarrow} rect(\frac{f}{a})$

① $x(t) = sinc(t) = 1 \cdot sinc(1 \cdot t) \overset{FT}{\rightarrow} rect(f) = \begin{cases} 1\angle 0 \ , & -0.5 < f < 0.5 \\ \frac{1}{2}\angle 0 \ , & |f| = 0.5 \\ 0\angle 0 \ , & |f| > 0.5 \end{cases} = X(f)$ #

② $|X(f)| = |rect(f)| = \begin{cases} 1 \ , & -0.5 < f < 0.5 \\ \frac{1}{2} \ , & |f| = 0.5 \\ 0 \ , & |f| > 0.5 \end{cases}$ #

③ $\angle X(f) = 0$ #






```
subplot(2, 1, 1)
f = -1: 0.005 : 1;
Xf_mag = ones( size(f) );
for i = 1 : size(f, 2)
    if f(i) < -0.5
        Xf_mag(i) = 0;
    elseif f(i) > 0.5
        Xf_mag(i) = 0;
    end
end
plot(f, Xf_mag, 'LineWidth', 3)
grid on
title('magnitude response of X(f)')

subplot(2, 1, 2)
Xf_pha = 0*f;
plot(f, Xf_pha, 'LineWidth', 3)
grid on
title('phase respons of X(f)')
```

4. Consider the signal

$$x(t) = \begin{cases} 0, & |t| > 1 \\ (t+1)/2, & -1 \le t \le 1 \end{cases}.$$

$$x(t) = \begin{cases} 0, & |t| > 1 \\ (t+1)/2, & -1 \le t \le 1 \end{cases}.$$

(a) Determine $X(f)$.

(b) Take the real part of your answer to part (a), and verify that it is Fourier transform of the even part of $x(t)$.

(c) What is the Fourier transform of the odd part of $x(t)$?

Ans:

(a)

【提示】 $x(t)$ 沒有特殊形式，無法用 Fourier transform pairs，故從定義下手

$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi f t}\,dt = \int_{-1}^{1} \frac{t+1}{2} e^{-j2\pi f t}\,dt$$ (by partial integration)

$$= \frac{1}{2}\left[\frac{t+1}{-j2\pi f}e^{-j2\pi f t} - \frac{1}{(-j2\pi f)^2}e^{-j2\pi f t}\right]_{t=-1}^{t=1}$$

$$= \frac{1}{2}\left[\frac{2e^{-j2\pi f}}{-j2\pi f} + \frac{-j2\sin(2\pi f)}{(2\pi f)^2}\right]$$

$$= j\left[\frac{e^{-j2\pi f}}{2\pi f} - \frac{\sin(2\pi f)}{(2\pi f)^2}\right]$$ # (by Euler's Formula)

$$= \left[\frac{\sin(2\pi f)}{2\pi f}\right] + j\cdot\left[\frac{\cos(2\pi f)}{2\pi f} - \frac{\sin(2\pi f)}{(2\pi f)^2}\right]$$ #

(b)

$$x(t) = \begin{cases} 0 & , \quad |t|>1 \\ (t+1)/2 & , \quad -1\le t\le 1 \end{cases} \quad \text{and} \quad x(-t) = \begin{cases} (-t+1)/2 & , \quad -1\le t\le 1 \\ 0 & , \quad |t|>1 \end{cases}$$

① 偶函數 $a(t) = \dfrac{x(t)+x(-t)}{2} = \begin{cases} \frac{1}{2} & , \quad -1\le t\le 1 \\ 0 & , \quad |t|>1 \end{cases}$

② $A(f) = \int_{-\infty}^{\infty} a(t)e^{-j2\pi f t}\,dt = \int_{-1}^{1} \frac{1}{2}e^{-j2\pi f t}\,dt = \frac{1}{2}\cdot F\left[rect(\frac{t}{2})\right] = sinc(2f)$

② $A(f) = \int_{-\infty}^{\infty} a(t)e^{-j2\pi f t}\,dt = \int_{-1}^{1} \frac{1}{2}e^{-j2\pi f t}\,dt = \left[\frac{e^{-j2\pi f t}}{2(-j2\pi f)}\right]_{t=-1}^{t=1} = \frac{-j2\sin(2\pi f)}{-j4\pi f}$

$$= \frac{\sin(2\pi f)}{2\pi f} = sinc(2f)$$

ans：偶函數 $a(t)$ 的傳立葉轉換 $A(f)$ 等於 $X(f)$ 的實部，**得證**。

(c)

① 奇函數 $b(t) = \dfrac{x(t)-x(-t)}{2} = \begin{cases} \frac{t}{2} & , \quad -1\le t\le 1 \\ 0 & , \quad |t|>1 \end{cases}$

② $B(f) = \int_{-\infty}^{\infty} b(t)e^{-j2\pi f t}\,dt = \int_{-1}^{1} \frac{t}{2}e^{-j2\pi f t}\,dt = \frac{1}{2}\left[\frac{-t}{j2\pi f}e^{-j2\pi f t} - \frac{1}{(j2\pi f)^2}e^{-j2\pi f t}\right]_{t=-1}^{t=1}$

$$= \frac{1}{2}\left[\frac{-2\cos(2\pi f)}{j2\pi f} - \frac{-j2\sin(2\pi f)}{(j2\pi f)^2}\right] = j\cdot\left[\frac{\cos(2\pi f)}{2\pi f} - \frac{\sin(2\pi f)}{(2\pi f)^2}\right]$$

ans：奇函數 $b(t)$ 的傳立葉轉換 $B(f)$ 等於 $X(f)$ 的虛部，**得證**。

5.

Consider an LTI system described by the second-order differential equation

$$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + y(t) = 3\frac{dx(t)}{dt} + x(t) \qquad (106)$$

Show that the frequency response characteristic of this system is

$$H(F) = \frac{3(j2\pi F)+1}{(j2\pi F)^2 + 2(j2\pi F)+1} \qquad (107)$$

Hint: Apply the complex sinusoidal input $x(t) = Xe^{j2\pi Ft}$ to the differential equation and assume a solution of the form $y(t) = Ye^{j2\pi Ft}$.

Ans:

【提示】

$$g^{(n)}(t) = \frac{d^n g(t)}{d^n t} \rightarrow (j2\pi f)^n \cdot G(f)$$

$$F[y''(t) + 2y'(t) + y(t)] = F[3x'(t) + x(t)]$$

$$(j2\pi f)^2 \cdot Y(f) + 2(j2\pi f)\cdot Y(f) + Y(f) = 3(j2\pi f)\cdot X(f) + X(f)$$

$$[\ (j2\pi f)^2 + 2(j2\pi f) + 1\ ]\cdot Y(f) = [\ 3(j2\pi f)+1\ ]\cdot X(f)$$

$$H(f) = \frac{Y(f)}{X(f)} = \frac{3(j2\pi f)+1}{(j2\pi f)^2 + 2(j2\pi f)+1} \quad \#$$

6. If the impulse response of a LTI system is $h(t) = rect(\frac{t-1}{2})$

(a) Find the frequency response of h(t), i.e., H(f)

(b) Plot magnitude response and phase response of H(f)

(c) If LTI input $x(t) = e^{j\pi t}$, What is the LTI output $y(t) = ?$

Ans:

(a) [法一] 由基本定義下手

$$H(f) = \int_{-\infty}^{\infty} h(t)e^{-j2\pi ft}dt = \int_0^2 1\cdot e^{-j2\pi ft}dt = \left[\frac{1}{-j2\pi f}e^{-j2\pi ft}\right]_{t=0}^{t=2}$$

$$= \frac{1}{j2\pi f}[1 - e^{-j4\pi f}] = \frac{-j}{2\pi f}[1 - \cos(4\pi f) + j\cdot\sin(4\pi f)]$$

$$= \frac{2\cdot\sin(4\pi f)}{4\pi f} - \frac{j}{2\pi f}[1 - \cos(4\pi f)]$$

(a) [法二] 利用 Fourier transform pairs

【提示】

① $u(t) \rightarrow \frac{1}{j2\pi f}+\frac{\delta(f)}{2}$

② $g(t\pm a) \rightarrow e^{\pm j2\pi f a}\cdot G(f)$

$$h(t) = rect\left(\frac{t-1}{2}\right) = u(t)-u(t-2)$$

① $u(t) \rightarrow \frac{1}{j2\pi f}+\frac{\delta(f)}{2}$

② $u(t-2) \rightarrow \frac{e^{-j4\pi f}}{j2\pi f}+e^{-j4\pi f}\frac{\delta(f)}{2} = \frac{e^{-j4\pi f}}{j2\pi f}+e^{-j4\pi 0}\frac{\delta(f)}{2} = \frac{e^{-j4\pi f}}{j2\pi f}+\frac{\delta(f)}{2}$

③ $h(t) = u(t)-u(t-2) \rightarrow H(f) = \frac{1}{j2\pi f}\left[1 - e^{-j4\pi f}\right]$

$$= \frac{-j}{2\pi f}\left[1 - \cos(4\pi f)+j\cdot\sin(4\pi f)\right]$$

$$= \frac{\sin(4\pi f)}{2\pi f} - j\cdot\frac{1-\cos(4\pi f)}{2\pi f}$$

$$= 2\cdot sinc(4f) - j\cdot 2sinc(2f)\cdot\sin(2\pi f)$$

$$= 2\cdot sinc(2f)\cdot e^{-j2\pi f} \quad \#$$

(a) [法三] 利用 Fourier transform pairs

【提示】

① $rect\left(\frac{t}{a}\right) \rightarrow a\cdot sinc(a\cdot f)$

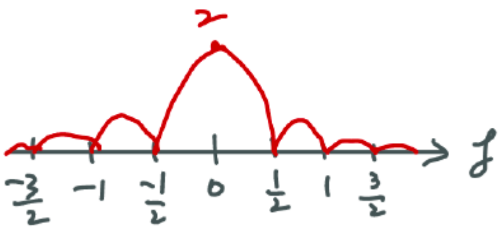
② $g(t\pm a) \rightarrow e^{\pm j2\pi f a}\cdot G(f)$

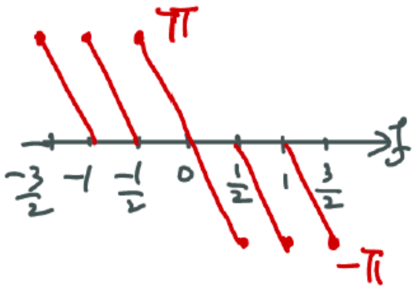
$$rect\left(\frac{t-1}{2}\right) \rightarrow 2\cdot sinc(2f)\cdot e^{-j2\pi f} \quad \#$$

(b)

① $|H(f)| = \sqrt{H(f)H^*(f)} = 2\cdot|sinc(2f)|$ #

② $\angle H(f) = \tan^{-1}\left(\frac{\cos(4\pi f)-1}{\sin(4\pi f)}\right)$ #





(c)

【提示】

① $y(t)=x(t)\otimes h(t) \quad \rightarrow \quad Y(f)=X(f)H(f)$

② 利用 Fourier transform pairs： $1 \quad \rightarrow \quad \delta(f)$

③ 利用 Fourier transform 特性： $e^{\pm j2\pi f_0 t} \quad \rightarrow \quad \delta(f \mp f_0)$

① $x(t)=e^{j\pi t}=e^{j2\pi\frac{1}{2}t} \quad \rightarrow \quad X(f)=\delta(f-\frac{1}{2})$

② 由(b)已得知 $H(f)$

③ 則 $Y(f) = [2sinc(4f) - j2sinc(2f)\sin(2\pi f)]\cdot\delta(f-\frac{1}{2})$

$$= 2sinc(4\cdot\frac{1}{2}) - j2sinc(2\cdot\frac{1}{2})\sin(2\pi\cdot\frac{1}{2})$$
$$= 2sinc(2) - j2sinc(1)\sin(\pi)$$
$$= 0 - 0$$
$$= 0$$

$Y(f)=0 \quad \overset{FT^{-1}}{\rightarrow} \quad y(t)=0$ #

7. Consider an LTI system with impulse response $h(t) = \frac{\sin(4\pi(t-1))}{\pi(t-1)}$.

a. Plot h(t) (i.e., impulse response of the system)

b. Find H(f) (i.e., frequency response of the system)

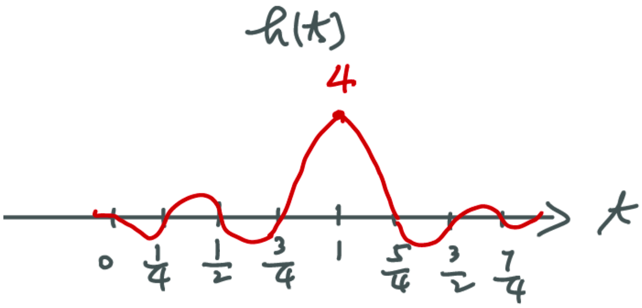
Ans:

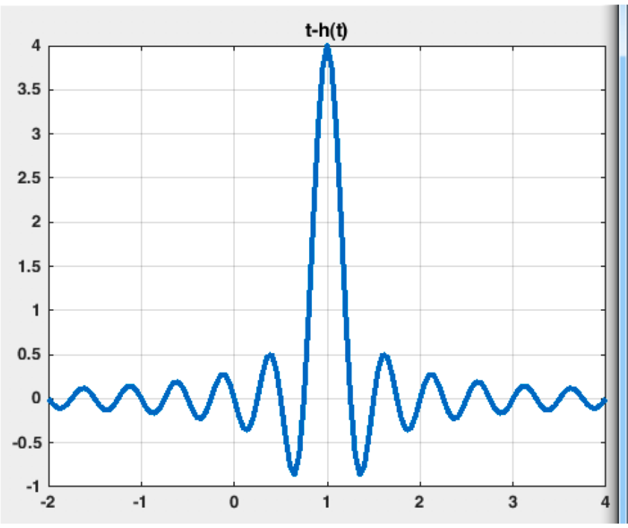
(a)

① $h(t) = \dfrac{4\cdot\sin(4\pi(t-1))}{4\cdot\pi(t-1)} = 4\cdot sinc(4(t-1))$

② 由於 $t=1$ 時， $h(t)=\frac{0}{0} \quad \Rightarrow$ [微積分]羅必達法則求極值

$$\lim_{t\to 1} h(t) = \frac{\lim_{t\to 1} \sin'(4\pi(t-1))}{\lim_{t\to 1} \pi(t-1)'} = \frac{4\pi}{\pi} = 4$$






```
clear;   clc;

t = -2: 0.05 : 4;
num = sin(4*pi*(t-1));
den = pi * (t-1);
ht = num ./ den;
for i = 1 : size(ht, 2)
    if t(i)==1
        ht(i)=4;
    end
end
plot(t, ht, 'LineWidth', 3)
grid on
title('t-h(t)')
```

(b)

【提示】利用 Fourier transform pairs 及 Fourier transform 特性

① $a \cdot sinc(a \cdot t) \quad \rightarrow \quad rect(\frac{f}{a})$

② $g(t \pm a) \quad \rightarrow \quad e^{\pm j 2\pi a} G(f)$

$$h(t) = 4 \cdot sinc(4(t-1)) \quad \rightarrow \quad H(f) = e^{-j2\pi f} \cdot rect(\frac{f}{4})$$ #

8. $x(t) = 2\Lambda\left(\frac{t}{2} - 3\right)$

a. Plot $x(t)$

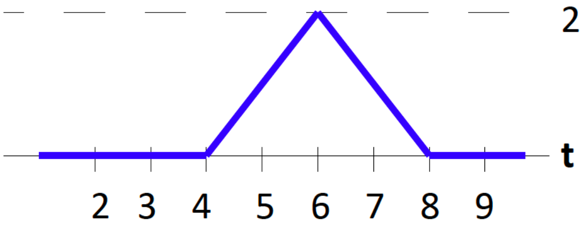
b. Find $X(f) = \mathcal{F}(x(t))$ where $\mathcal{F}(*)$ is the continuous-time Fourier Transform

c. Plot $|X(f)|$ and $\angle(X(f))$

Ans:

(a)

$$x(t) = 2\cdot tri\left(\frac{t-6}{2}\right)$$



(b)

【提示】利用 Fourier transform pairs 及 Fourier transform 特性

① $g(t\pm a) \quad \rightarrow \quad e^{\pm j2\pi a}G(f)$ ② $tri\left(\frac{t}{a}\right) \quad \rightarrow \quad a\cdot sinc^2(a\cdot f)$

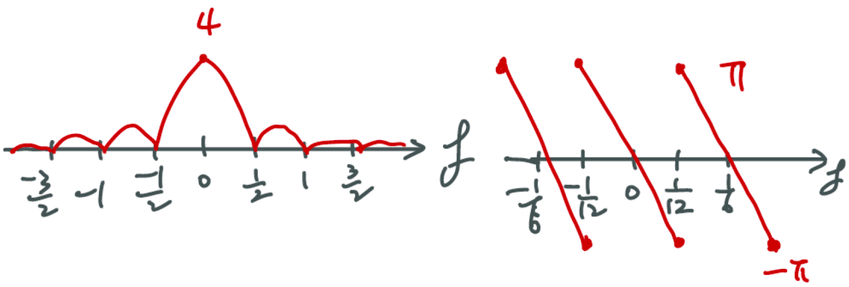
$$x(t) = 2\cdot tri\left(\frac{t-6}{2}\right) \quad \rightarrow \quad X(f) = 2\cdot e^{-j2\pi f6}\cdot 2\cdot sinc^2(2\cdot f)$$

$$= 4\cdot e^{-j12\pi f} sinc^2(2f) \quad \#$$

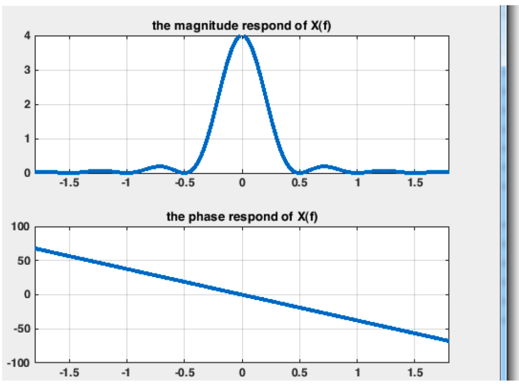
(c)

① $|X(f)| = |4\cdot e^{-j12\pi f}\cdot sinc^2(2f)| = |4|\cdot|e^{-j12\pi f}|\cdot|sinc^2(2f)| = 4\cdot 1\cdot sinc^2(2f) = 4\cdot sinc^2(2f)$

② $\angle X(f) = -12\pi f$



[註]通常將相位響應畫在 [π,−π] 之間

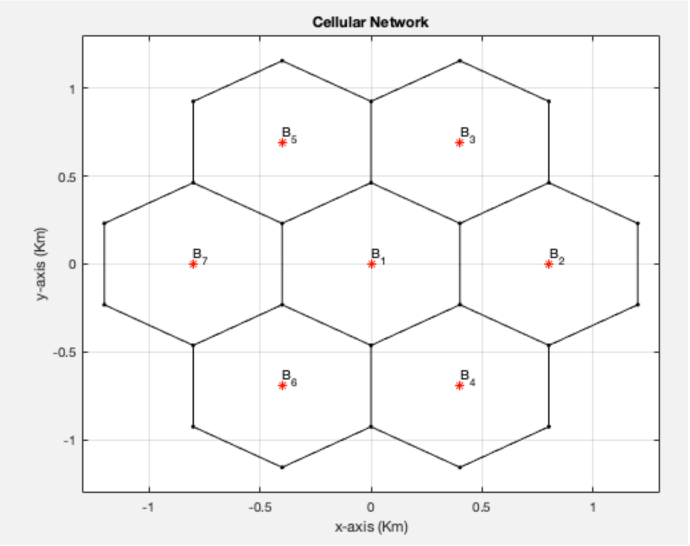



```
clear;   clc;

subplot(2,1,1)
f = -1.8: 0.01 : 1.8;
factor = 2*pi*f;
Xf_mag = 4 * sin(factor) .* sin(factor) ./ ((factor).*(factor));
plot(f, Xf_mag, 'LineWidth', 3)
grid on
title('the magnitude respond of X(f)')
xlim([-1.8, 1.8])

subplot(2,1,2)
Xf_pha = -12*pi*f;
plot(f, Xf_pha, 'LineWidth', 3)
grid on
title('the phase respond of X(f)')
xlim([-1.8, 1.8])
```

9. Matlab coding problem

A cellular network consisting of seven wrapped around hexagonal cells. Within each cell, the base station (BS) is located at the center. The BS-to-BS distance is set to be 0.8 km. Please write a Matlab script to plot the cellular network as shown below.

Some Matlab functions to be used: plot, text, hold, grid, title, xlabel, ylabel, axis



Sol:

```
close all;
clear all;
% Cell center coordinates
number_of_cell = 7;
BS_BS_distance_km = 0.8; % Km

% Location of BS in each cells
BS_horizontal = BS_BS_distance_km;
BS_vertical = BS_BS_distance_km * sin(pi/3);
cell_center_x = [0 BS_horizontal BS_horizontal/2 BS_horizontal/2 (-1*BS_horizontal/2) …
                 (-1*BS_horizontal/2) (-1*BS_horizontal)];
cell_center_y = [0 0 BS_vertical (-1*BS_vertical) BS_vertical (-1*BS_vertical) 0];
cell_center = [cell_center_x; cell_center_y]; % center cell coordinate is cell_center(1)

% Center hexagon vertices
theta = (2*pi/6)/2;
R = (BS_BS_distance_km/2)/cos(theta);
center_cell_x = cell_center_x(1);
center_cell_y = cell_center_y(1);
vertex_x = R * cos((0:5)*pi/3+pi/6) + center_cell_x;
vertex_y = R * sin((0:5)*pi/3+pi/6) + center_cell_y;
vertex = [vertex_x ; vertex_y];
```

```
figure(4);
plot(cell_center_x, cell_center_y, 'r*');
hold on;
grid on;
text(cell_center_x(1), cell_center_y(1)+0.05,'B_1');
text(cell_center_x(2), cell_center_y(2)+0.05,'B_2');
text(cell_center_x(3), cell_center_y(3)+0.05,'B_3');
text(cell_center_x(4), cell_center_y(4)+0.05,'B_4');
text(cell_center_x(5), cell_center_y(5)+0.05,'B_5');
text(cell_center_x(6), cell_center_y(6)+0.05,'B_6');
text(cell_center_x(7), cell_center_y(7)+0.05,'B_7');
plot([vertex(1,:) vertex(1,1)]+cell_center(1,1),[vertex(2,:) vertex(2,1)]+cell_center(2,1),'k.-');
plot([vertex(1,:) vertex(1,1)]+cell_center(1,2),[vertex(2,:) vertex(2,1)]+cell_center(2,2),'k.-');
plot([vertex(1,:) vertex(1,1)]+cell_center(1,3),[vertex(2,:) vertex(2,1)]+cell_center(2,3),'k.-');
plot([vertex(1,:) vertex(1,1)]+cell_center(1,4),[vertex(2,:) vertex(2,1)]+cell_center(2,4),'k.-');
plot([vertex(1,:) vertex(1,1)]+cell_center(1,5),[vertex(2,:) vertex(2,1)]+cell_center(2,5),'k.-');
plot([vertex(1,:) vertex(1,1)]+cell_center(1,6),[vertex(2,:) vertex(2,1)]+cell_center(2,6),'k.-');
plot([vertex(1,:) vertex(1,1)]+cell_center(1,7),[vertex(2,:) vertex(2,1)]+cell_center(2,7),'k.-');
title('Cellular Network');
axis([-1.3 1.3 -1.3 1.3])
xlabel('x-axis (Km)');
ylabel('y-axis (Km)');
```