Chapter 5
Carrier Transport Phenomena

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Carrier Drift

- Electric field is applied to electrons and holes – force
  - Drift $\Rightarrow$ Movement $\Rightarrow$ Acceleration

    漂移　位移　加速度
- Drift current　漂移電流
  - Net drift of charge　淨電荷的漂移

Drift Current Density　漂移電流密度

- Positive volume charge density ρ moving at an average velocity υ_d
  - The drift current density　体積電荷密度 ρ × 平均漂移速率 υ_d
    - $J_{drf} = \rho \upsilon_d$　(C/cm²-s or Amps/cm²)　(5.1)
  - For positively charged holes
    - $J_{p|drf} = (ep)\upsilon_{dp}$　(5.2)
      - υ_{dp}: the average velocity of the holes　電洞的平均速率 υ_{dp}
      - p : concentration of holes　電洞的濃度 p

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Carrier Drift

- In the presence of an electric filed 考慮電場
  - The equation of motion
    - $F = m_p^* a = e\mathrm{E}$ (5.3)
      - e : the magnitude of the electronic charge
      - a : acceleration
      - E : electric filed
      - m_p^*: the effective mass of hole

      若電場強度固定，電荷加速度固定
    - If the electric filed is constant $\Rightarrow$ acceleration a is a constant
      - *The velocity increases linearly with time* → $v = at \propto t$，v隨t增加
    - There are collisions in semiconductor with

      碰撞
      - Ionized impurity atoms
      - Thermally vibrating lattice atoms

      * 游離雜質原子

      * 晶格原子的熱振動

      $\Rightarrow$ *Alter the velocity characteristics of the particle* → 改變速度
    - The particles
      - Accelerates → velocity increases → collides → accelerates → …

      $\Rightarrow$ *The particle gains an average drift velocity*
    - For low electric field
      - $\upsilon_{dp} = \mu_p \mathrm{E}$ (μ_p : proportionality factor and called hole mobility) (5.4)

Carrier Drift

- Mobility μ_p 遷移率(粒子受電場影響產生的位移)
  - How well a particle will move due to an electric field
  - An important parameter (cm^2/V-s)
    - The average velocity of a carrier to the electric field (載子的平均速度)
- Drift current density 漂移電流密度
  - $J_{p|drf} = (ep)\upsilon_{dp} = e\mu_p p\mathrm{E}$ (5.5)
  - *The same direction as the applied electric field*

體積電荷密度 ρ × 平均漂移速率 v_d

- For electrons
  - $J_{n|drf} = \rho\upsilon_{dn} = (-en)\upsilon_{dn} = e\mu_n n\mathrm{E}$ (5.8)

for 電子 $\rho = -e\,n$ ($-e$: 帶電量, n: 電子濃度)

    - $J_{n|drf}$: the drift current density due to electrons
    - υ_{dn}: the average velocity of electrons
    - Since the electron is negatively charged, its motion is opposite to the electric field direction

$$\upsilon_{dn} = -\mu_n \mathrm{E}$$

    - μ_n: Electron mobility
- The total drift current density
  - $\boxed{J_{drf} = e(\mu_n n + \mu_p p)\mathrm{E}}$ (5.9)

$J_{drf} = J_{p|drif} + J_{n|drif}$

$= e\mu_p p E + e\mu_n n E = e(\mu_p p + \mu_n n)E$

$= e p v_{dp} - e n v_{dn}$

Carrier Drift

- Electron and hole mobilities are functions of temperature and doping concentrations

mobility

約2.5倍

Table 5.1 Typical mobility values at T=300K and low doping concentration

	μ_n (cm^2/V-s)	μ_p (cm^2/V-s)
Silicon	1350 3(1W) :	480 1 (3W)
Gallium Arsenide	8500	400
Germanium	3900	1900

☆ nmos 和 pmos 飽和區公式 $i_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_t)^2 \propto \mu \propto W$

所以為了使 i_D 大小相同 $i_{Dn} = i_{Dp}$

$$\frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_t)^2 = \frac{1}{2} \mu_p C_{ox} \frac{3W}{L} (V_{GS} - V_t)^2$$

$\cong \frac{\mu_n}{\mu_p}$

Carrier Drift

- **Example 5.1** Consider a gallium arsenide sample at T=300°K with doping concentration of $N_a=0$ and $N_d=10^{16}\text{cm}^{-3}$. Assume complete ionization and assume electron and hole mobilities given in Table 5.1. Calculate the drift current density if the applied electric field is E=10V/cm.
  - $N_d > N_a$
    - The majority carrier electron concentration
      - $$n=\frac{N_d-N_a}{2}+\sqrt{\left(\frac{N_d-N_a}{2}\right)^2+n_i^2}\approx 10^{16}\,\text{cm}^{-3}$$
    - The minority carrier hole concentration
      - $$p=\frac{n_i^2}{n}=\frac{(1.8\times10^{6})^2}{10^{16}}=3.24\times10^{-4}\,\text{cm}^{-3}$$
    - The drift current density for this extrinsic n-type semiconductor
      - $$J_{drf}=e(\mu_n n+\mu_p p)\mathrm{E}\approx e\mu_n N_d\mathrm{E}$$
        $$=(1.6\times10^{-19})(8500)(10^{16})(10)=136\text{A/cm}^2$$

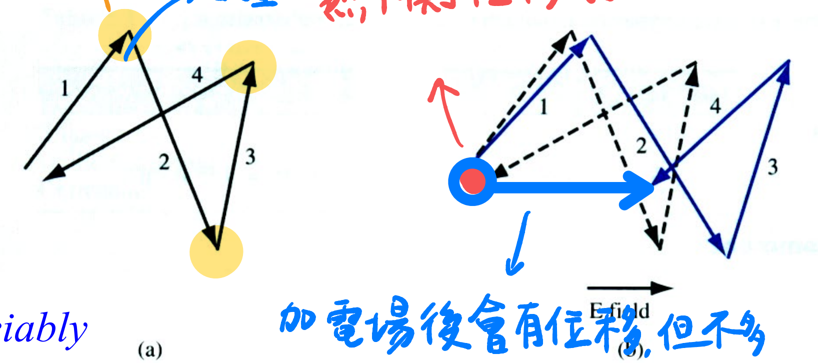
Small electric field can cause significant drift current densities

The drift current usually due primarily to the majority carrier in an extrinsic semiconductor

Carrier Drift

- Mobility Effects
  - From equ. (5.3)
    - $F = m_p^* \dfrac{d\upsilon}{dt} = e\mathrm{E}$ (5.10)
      - υ : the velocity of particle due to electric field and does not include the random thermal velocity
    - If the effective mass and electric field are $\boxed{\text{constant}}$
      - $d\upsilon = \dfrac{e\mathrm{E}}{m_p^*} dt \Rightarrow \upsilon = \dfrac{e\mathrm{E}t}{m_p^*}$ (5.11)
        - Assume the initial velocity is zero
    - (a) Random thermal velocity and motion of a hole with zero electric field
      - τ_{cp} : A mean time between collisions
    - (b) small electric field
      - A net drift of the hole in the direction of the E-field
        - A small perturbation on the random thermal velocity

小擾動



The time between collisions will not be altered appreciably

碰撞不會有可觀的變化

Carrier Drift

- The mean peak velocity
  - $$\upsilon_{d|\text{peak}} = \left(\frac{e\tau_{cp}}{m_p^*}\right)\mathrm{E} \qquad (5.12a)$$
    - Just happens prior to collision or scattering event

碰撞 散射

lattice scattering
impurity scattering

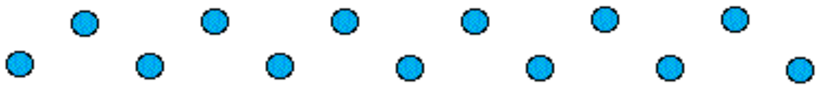
- The average drift velocity
  - $$\langle \upsilon_d \rangle = \frac{1}{2}\left(\frac{e\tau_{cp}}{m_p^*}\right)\mathrm{E} \qquad (5.12b)$$
    - Since the velocity increases linearly with time
- In a more accurate model including the effect of a **statistical distribution**
  - The factor $1/2$ disappears
    - **The hole mobility** 遷移率
      - $$\boxed{\mu_p = \frac{\upsilon_{dp}}{\mathrm{E}} = \frac{e\tau_{cp}}{m_p^*}} \qquad (5.13)$$
    - Similarly, **the electron mobility**
      - $$\boxed{\mu_n = \frac{\upsilon_{dn}}{\mathrm{E}} = \frac{e\tau_{cn}}{m_n^*}} \qquad (5.14)$$
      - τ_{cn} : A mean time between collisions for an electron 一個原子的碰撞週期

Carrier Drift

- **Phonon scattering (lattice scattering)** – the first interaction mechanism
  - The atoms in semiconductor vibrate randomly due to the thermal energy
    - **Disrupt** the perfect periodic potential function
      - Resulting in the **interaction** between electrons or holes and the vibrating lattice atoms
  - Scattering occurs is a function of **temperature**
    - First-order scattering theory predicts
    - $\boxed{\mu_L \propto T^{-3/2}}$ → 經驗結果 (5.15) μ_L只和溫度有關
      - Mobility μ_L that is only due to lattice scattering increases as the temperature decreases
      - Temperature decreases ⇒ vibration decreases ⇒ scattering effect decreases ⇒ increasing mobility
  - In **lightly doped** semiconductor $\mu \propto T^{-n}$
    - Lattice scattering dominates
      - The inserts – Mobility is proportional to T^{-n}, but n is not equal to 3/2 as the first-order scattering theory predicted

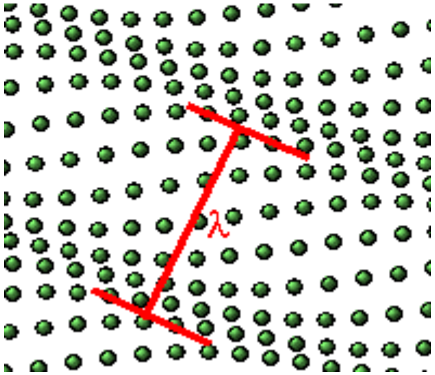
Carrier Drift

Transverse acoustical standing wave



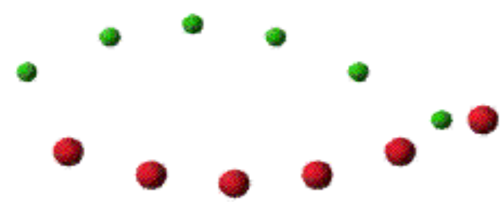
Phonons with frequency which goes to zero in the limit of small k are known as acoustical phonons

"Long" wavelength acoustical vibrations



Carrier Drift

Transverse optical mode for diatomic chain

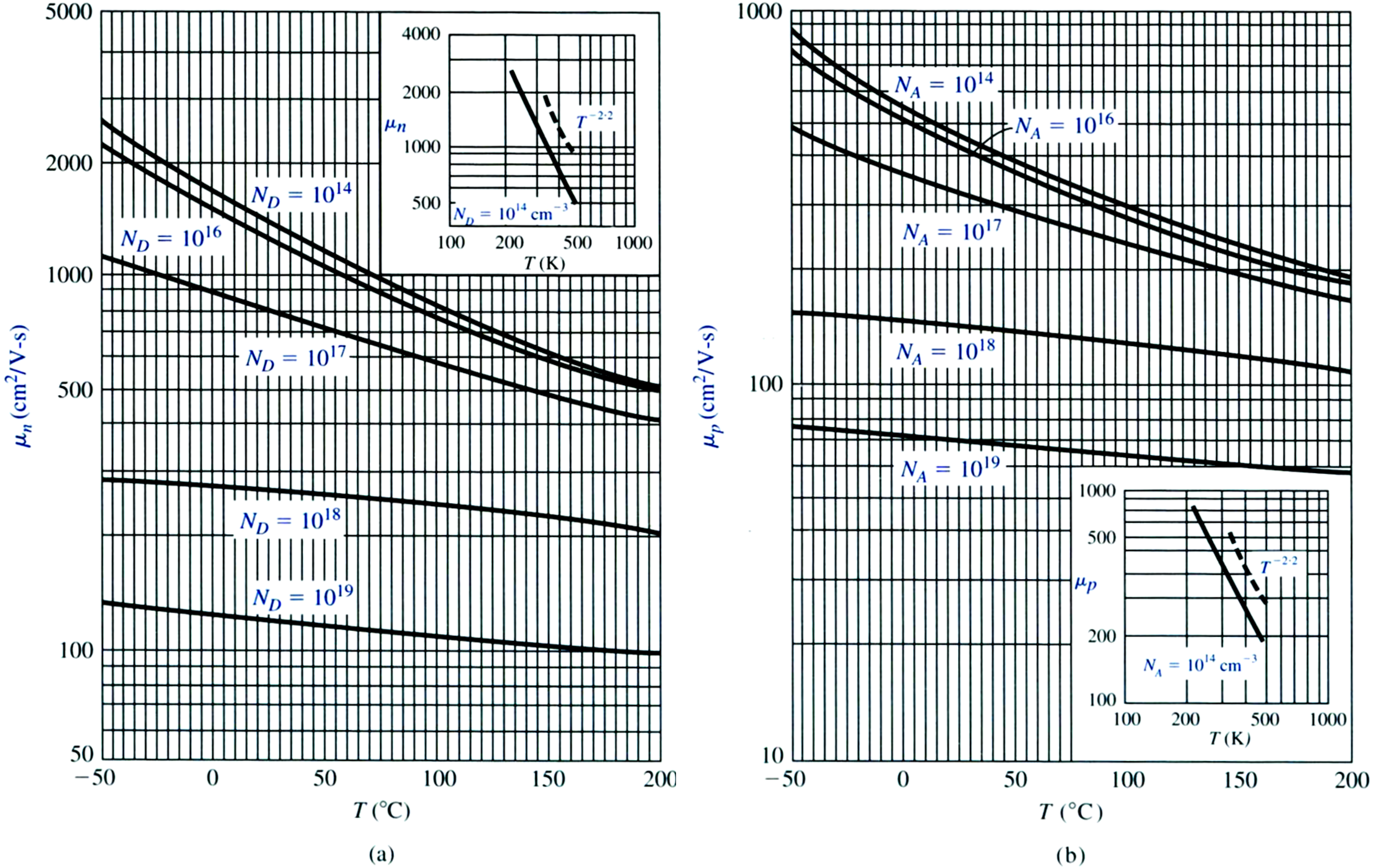


Amplitude of vibration is strongly exaggerated!

Transverse acoustical mode for diatomic chain

Carrier Drift

- Temperature dependence of electron and hole mobilities in silicon



游離雜質散射 Carrier Drift

- **Ionized impurity scattering** – the second interaction mechanism
  - At room temperature
    - The impurity atoms are ionized 游離雜質和電子電洞的庫侖作用力
      - define: The coulomb interaction exists between electrons or holes and ionized impurity
      - Produce scattering or collisions and alter the velocity of carrier
    - $\boxed{\mu_I \propto \frac{T^{+3/2}}{N_I}}$ (5.16) → 總游離雜質濃度

      游離雜質的遷移率
      - The mobility μ_I that is only due to ionized impurity scattering increases as temperature increases
      - $N_I = N_d^+ + N_a^-$: the total ionized impurity concentration in silicon
    - If temperature increases → 電子和電洞受游離雜質影響減小
      - Random thermal velocity of a carrier increases
      - Less time spent in the vicinity of a coulomb force
      - ☆ The smaller scattering effect, the larger the expected value of μ_I
    - If the number of ionized impurity increases
      - The probability of a carrier encountering an ionized impurity increases
      - A smaller value of μ_I

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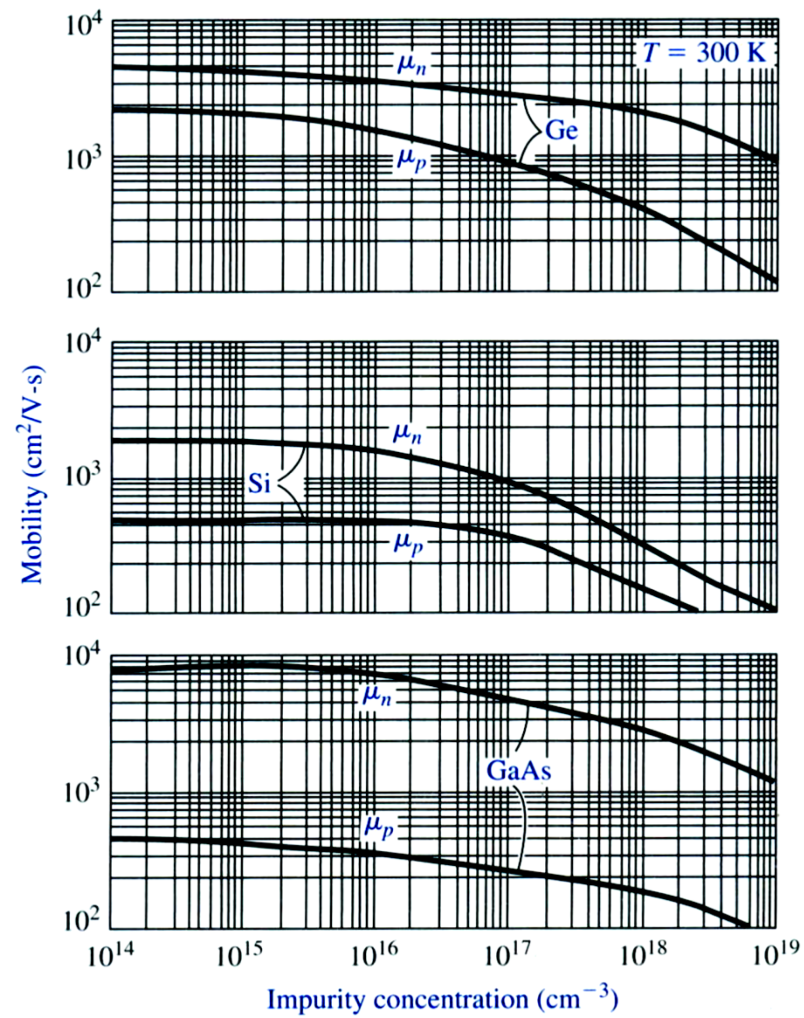
Carrier Drift

- The electron and hole mobilities in germanium, silicon, and gallium arsenide at T=300K
  - As the ionized impurity concentration N_I increases
    - The number of scattering centers increases
      - The mobility μ_I is reduced



Carrier Drift

$dt \times f$
時間 × 碰撞頻率 = 次

$\frac{dt}{\tau} = \frac{[s]}{[s/次]} = 次$

- In a differential time dt
  - The probability of a lattice scattering event occurring is dt/τ_L
    - τ_L : the mean time between collisions due to lattice scattering
      晶格散射的碰撞週期 τ_L
  - The probability of an ionized impurity scattering event occurring is dt/τ_I
    - τ_I : the mean time between collisions due to ionized impurity scattering
      游離雜質散射的碰撞週期 τ_I
  - If these two scattering process is independent
    - The total probability of a scattering event
      - $$\frac{dt}{\tau} = \frac{dt}{\tau_I} + \frac{dt}{\tau_L} \qquad (5.17)$$
        散射的機率
      - $$\boxed{\frac{1}{\mu} = \frac{1}{\mu_I} + \frac{1}{\mu_L}} \qquad (5.18)$$
      - μ_I : the mobility due to the ionized impurity scattering only
      - μ_L : the mobility due to the lattice scattering only
      - μ : the net mobility

The net mobility decreases due to two independent scattering mechanisms

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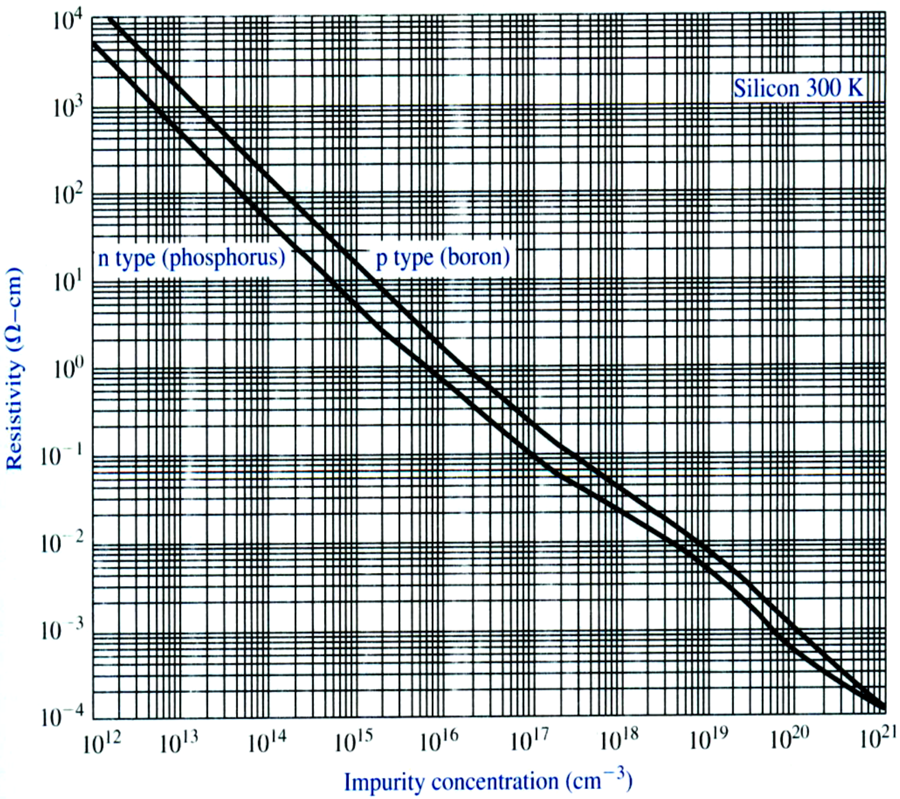
Carrier Drift

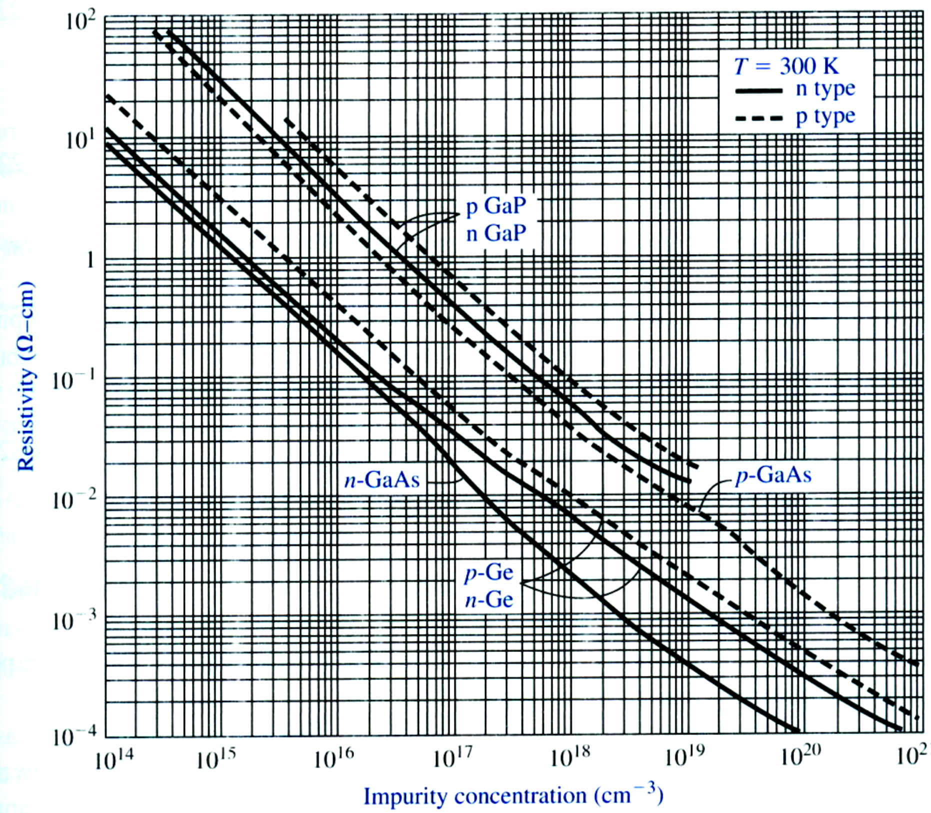
- Conductivity
  - **The drift current**
    - $J_{drf} = e(\mu_n n + \mu_p p)\mathrm{E} = \sigma\mathrm{E}$ → 導電率 (5.19)
      - σ : the conductivity of the semiconductor material
      - (A/cm²)= σ×(V/cm) ⇒ the unit of σ : (Ω-cm)$^{-1}$
    - Conductivity is a function of electron and hole concentrations and mobilities
      - Is a somewhat complicated function of impurity concentration since the mobility is related to the impurity concentrations
    - Resistivity ρ
      - Ω-cm
      - $\boxed{\rho = \frac{1}{\sigma} = \frac{1}{e(\mu_n n + \mu_p p)}}$ (5.20) 漂移電流的導電率和電阻率

導電率 電阻率

Carrier Drift

- Silicon at 300K
  - The curves are *not* linear function of N_d or N_a because of mobility effects 遷移效應





Carrier Drift

- Consider a bar of semiconductor material
  - A current I is produced

電流密度 J
    - Current density $J=\dfrac{I}{A}$ → Area (5.21a)

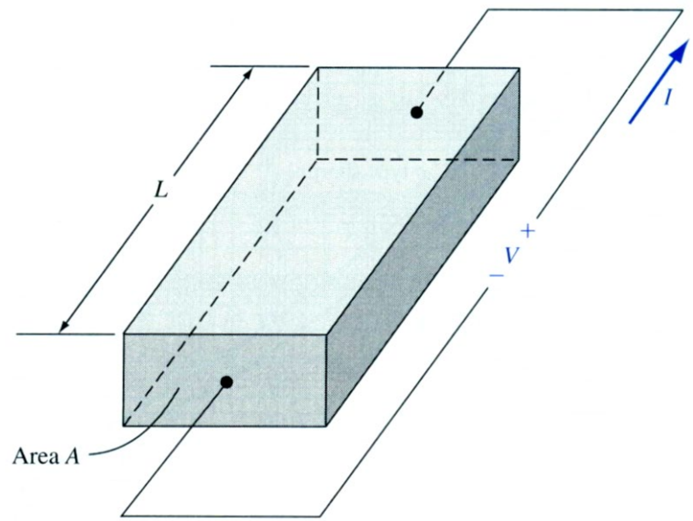
電場 E
    - The electric field $E=\dfrac{V}{L}$ $V \to \dfrac{kQ}{r}$, $L \to r$ (5.21b)
  - From (5.19)
    - $J_{drf}=\sigma E$ 電導率 (見P.16)

由漂移電流定義推導 V = I R

$$\Rightarrow \frac{I}{A}=\sigma\frac{V}{L} \quad (5.22a)$$

$$\Rightarrow V=I\frac{L}{\sigma A}=I\boxed{\frac{\rho L}{A}}=I\boxed{R} \quad (5.22b)$$

    - The resistance is a function of resistivity, conductivity and the geometry of the semiconductor



Carrier Drift

(退化型半導体)

- Consider a p-type semiconductor ($N_a >> n_i$ & $N_d=0$) 摻雜過量ⅢA元素
  - Assume the electron and hole mobilities have the same order of magnitude
    - The conductivity $\sigma = e(\mu_n n + \mu_p p) \approx e\mu_p p$ (5.23)
  - Assume complete ionization 完全游離 ($Na \gg n_i,\ p \approx Na$)
    - The conductivity $\sigma \approx e\mu_p N_a \approx \dfrac{1}{\rho}$ (5.24)
      - *The conductivity and resistivity of an extrinsic semiconductor are a function of primary of the majority carrier parameters*

假設電子&電洞遷移率相同

$\mu_n = \mu_p$ ，但 $n \approx N_d = 0$

$\Rightarrow \sigma = e(\cancel{\mu_n n} + \mu_p P)E \approx \mu_p P E$

詳見CH4筆記

https://hackmd.io/@powerful-peanut/rkPeBjd7kg/view

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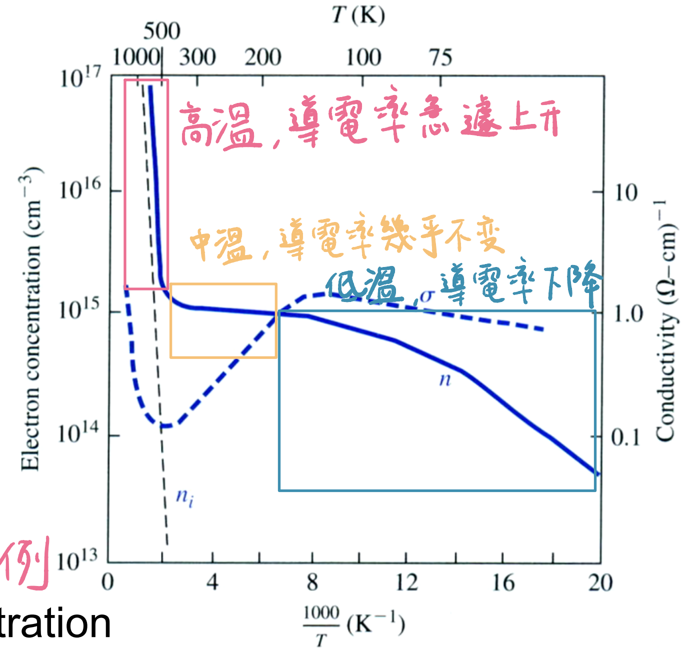
Carrier Drift

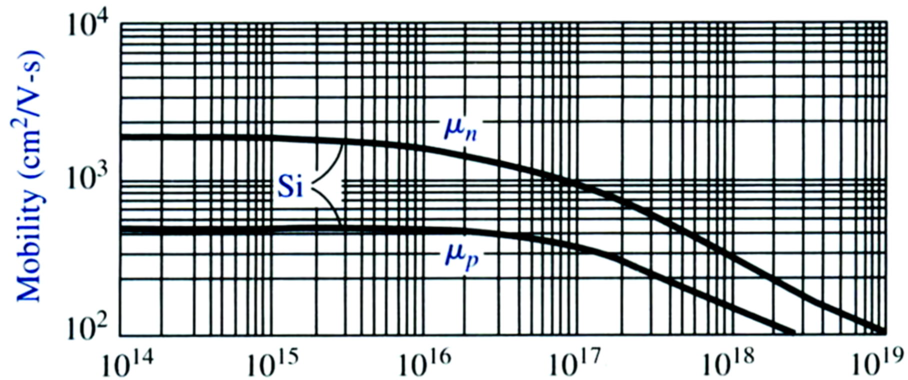
- Concentration and conductivity versus temperature
- In the midtemperature range
  - Complete ionization
    - Electron concentration remains constant
    - Conductivity varies with temperature
      - Due to temperature dependent mobility ?
- At higher temperatures
  - Intrinsic concentration dominates 由 n_i 決定 n
    - Conductivity increases 導電率和電子濃度成比例
      - That is proportional to the electron concentration
- At lower temperatures
  - Freeze-out
    - Electron concentration decreases
    - Conductivity decreases
      - That is proportional to the electron concentration



Carrier Drift

- **Example 5.2** Consider compensated n-type silicon at T=300K, with a conductivity of $\sigma = 16\ (\Omega\text{-cm})^{-1}$ and an acceptor doping concentration of 10^{17}cm^{-3}. Determine the donor concentration and the electron mobility.
  - At 300K, assume complete ionization and $N_d - N_a >> n_i$
    - $\sigma \approx e\mu_n n = e\mu_n(N_d - N_a)$

      $\Rightarrow 16 = (1.6\times10^{-19})\mu_n(N_d - 10^{17})$
    - Since mobility is a function of the ionized impurity concentration
      - From Fig. 5.3 by trial and error
        - If Choose $N_d = 2\times10^{17}$
        - $N_I = N_d^+ + N_a^- = 3\times10^{17}$
        - $\mu_n \approx 510\,\text{cm}^2/\text{V-s}$
        - $\sigma = 8.16\ (\Omega\text{-cm})^{-1}$
        - If Choose $N_d = 5\times10^{17}$
        - $N_I = N_d^+ + N_a^- = 6\times10^{17}$ and $\mu_n \approx 325\,\text{cm}^2/\text{V-s}$
        - $\sigma = 20.8\ (\Omega\text{-cm})^{-1}$



$N_d \approx 3.5\times10^{17}\,\text{cm}^{-3}$

$\mu_n \approx 400\,\text{cm}^2/\text{V-s}$

$\sigma \approx 16(\Omega\text{-cm})^{-1}$

In high-conductivity semiconductor, mobility is a strong function of carrier concentration

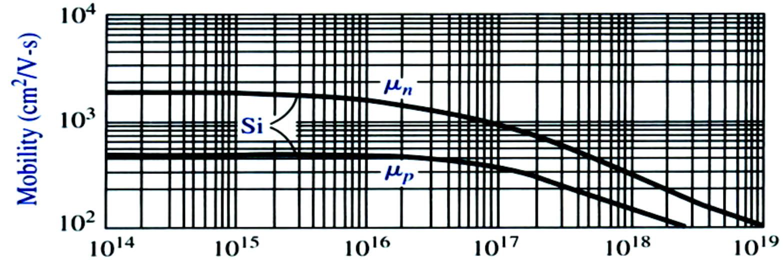
Carrier Drift

- **Example 5.3** A silicon semiconductor at T=300K is initially doped with donors at a concentration of N_d=5×10^{15}cm^{-3}. Acceptors are to be added to form a compensated p-type material. The resistor is to have a resistance of 10kΩ and handle a current density of 50 A/cm^2 where 5V is applied. Design a semiconductor resistor with a specified resistance to handle a given current density.
  - The total current
    - $I = \frac{V}{R} = \frac{5}{10k} = 0.5\text{mA}$
  - The current density is limited to 50 A/cm^2
    - The cross section area $A = \frac{I}{J} = \frac{0.5\times10^{-3}}{50} = 10^{-5}\,\text{cm}^2$
  - If we limit the electric field to E = 100V/cm
    - The length of the resistor $L = \frac{V}{\mathrm{E}} = \frac{5}{100} = 5\times10^{-2}\,\text{cm}$
    - From (5.22b), the conductivity
      - $\frac{L}{\sigma A} = R \Rightarrow \sigma = \frac{L}{RA} = \frac{5\times10^{-2}}{(10^4)(10^{-5})} = 0.50(\Omega\text{-cm})^{-1}$

Carrier Drift

- **Example 5.3 (cont')**
  - The conductivity of a compensated p-type semiconductor
    - $\sigma \approx e\mu_p p = e\mu_p (N_a - N_d)$

• The mobility is a function of the total ionized impurity concentration $N_a + N_d$



  - Trial and error
    - N_a = 1.25×10^{16}cm^{-3}
    - $N_a + N_d$ = 1.75×10^{16}cm^{-3}
    - From Fig. 5.3
      - The hole mobility is approximately μ_p = 410 cm^2/V-s
    - The conductivity is
      - $\sigma \approx e\mu_p p = e\mu_p (N_a - N_d)$

        $= (1.6\times 10^{-19})(410)(1.25\times 10^{16} - 5\times 10^{15}) = 0.492$
      - That is very close to the value we need

Since the mobility is related to the total ionized impurity concentration, the determination of impurity concentration to achieve a particular conductivity is not straightforward.

Carrier Drift

- For an intrinsic material
  - The conductivity $\sigma = e(\mu_n + \mu_p)n_i$ → n和p濃度相同 (5.25)
  - In general
    - the electron and hole mobilities are not equal, the intrinsic conductivity is *not* a minimum value possible at a given temperature

$$\sigma = e(\mu_n n + \mu_p p)n_i$$

→ n和p濃度不相同

Carrier Drift

- Velocity Saturation
  - So far the mobility is *not* a function of applied electric field
    - The drift velocity increases with applied electric field
  - The total velocity of a particle
    - Sum of the random thermal velocity and drift velocity

熱運動速度　漂移速度

  - At 300°K
    - $$\frac{1}{2}m\upsilon_{th}^{2}=\frac{3}{2}kT=\frac{3}{2}(0.0259)=0.03885\text{eV} \qquad (5.26)$$
      - This energy translates into a mean thermal velocity approximately 10^7 cm/s for an electron in silicon

矽的電子平均熱運動速率 $\approx 10^7$ cm/s

* 熱運動速率

$v_{th} \approx 10^7$

      - Assume an electron mobility μ_n = 1350 cm²/V-s in low doped silicon and the applied electric field is 75V/cm
        - A drift velocity is 10^5 cm/s roughly (1% of thermal velocity)

This electric field does not alter the energy of the electron appreciably

* 漂移電流速度

$v_{dn} = \mu_n E \approx 10^5$

$v_{dn} = \mu_n E = 1350\ cm^2/V\text{-}s \times 75\ V/cm = 101250\ cm/s \approx 10^5\ cm/s$

電場不會顯著改變電子能量，因為 $\mu_n^2 > E^2$

$\because \frac{1}{2} m v_{th}^2 = \frac{1}{2} m \mu_n^2 E^2 = \frac{1}{2} m (1350)^2 (75)^2$

$\mu_n > E$

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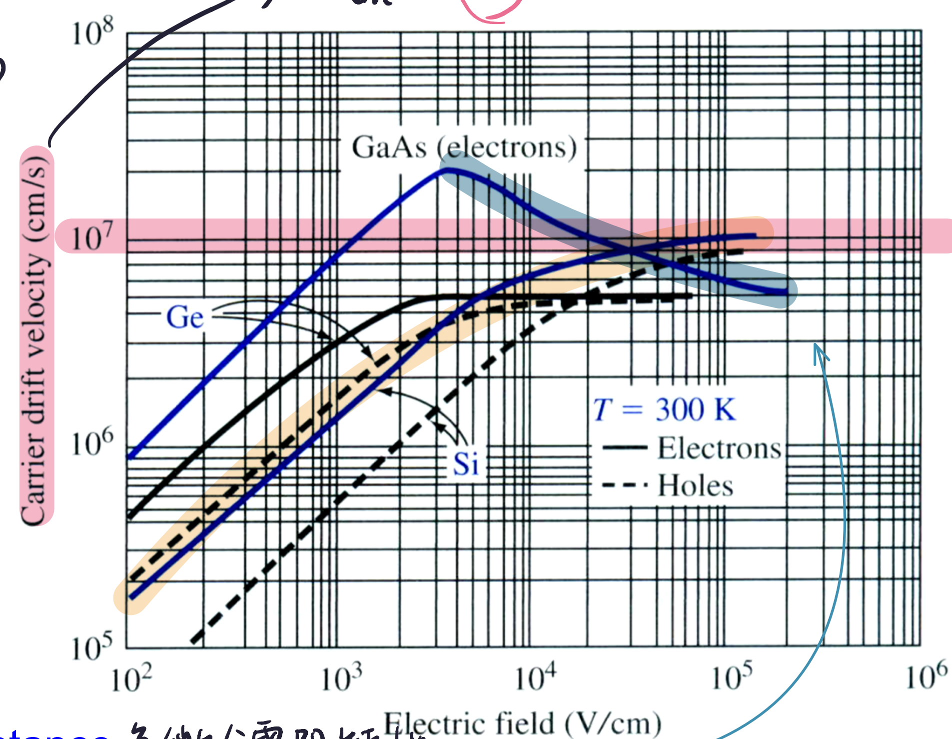
Carrier Drift

- The average drift velocity versus applied electric field 施加電場後的平均漂移速率
  - At low electric field 施加弱電場
    - The drift velocity is proportional to electric field
      - The slope is the mobility
  - At high electric field 施加強電場
    - The drift velocity deviates and saturates gradually 隨電場強度增加漂移速率脫離且逐漸飽和
    - EX. The drift velocity of electrons in silicon saturates approximately 10^7 cm/s at an applied electric field 30kV/cm
      - Independent of the applied electric field
  - Gallium arsenide 砷化鎵
    - The negative differential mobility 負微分遷移率
      - The negative differential resistance 負微分電阻特性
      - Is used in the design of oscillator 振盪器

$v_{dn} = \mu_n E$

$\mu_n < 0$



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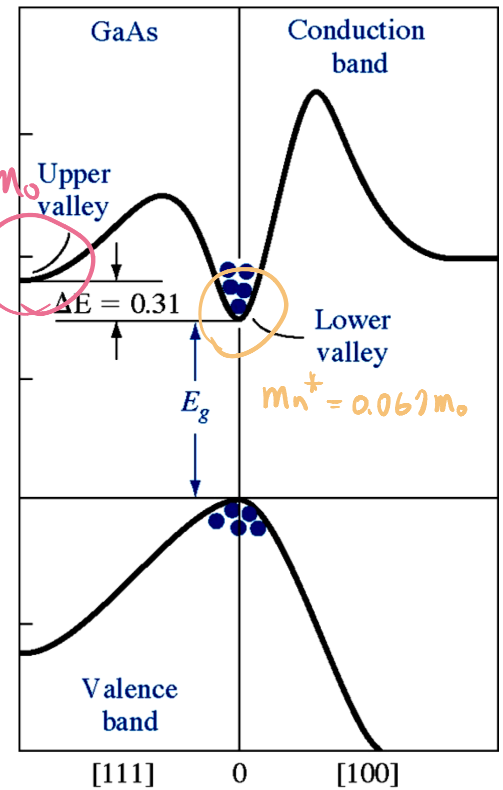
Carrier Drift

砷化鎵的 E-k 圖

- Understand the mobility from E versus k diagram of gallium arsenide
  - The small effective mass $m_n^* = 0.067 m_0$ *in the lower valley*
    - Large mobility from (5.13) and (5.14)
  - As electric field increases
    - The energy of electron increases
      - The electron can be scattered into the upper valley
  - *In the upper valley*
    - The density of states effective mass $m_n^* = 0.55 m_0$
      - Larger effective mass results in smaller mobility

$$\mu_n = \frac{\upsilon_{dn}}{\mathrm{E}} = \frac{e\tau_{cn}}{m_n^*}$$



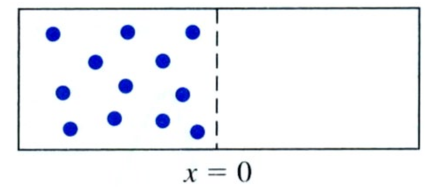
等效質量大 $m^* \uparrow$，導致遷移率變小 $\mu \downarrow$

  - The intervalley transfer mechanism results in
    - Decreasing the drift velocity of electrons with electric field
    - The negative differential mobility characteristics

μ 固定，電場下降 → 漂移速度下降

負微分遷移率

Carrier Diffusion

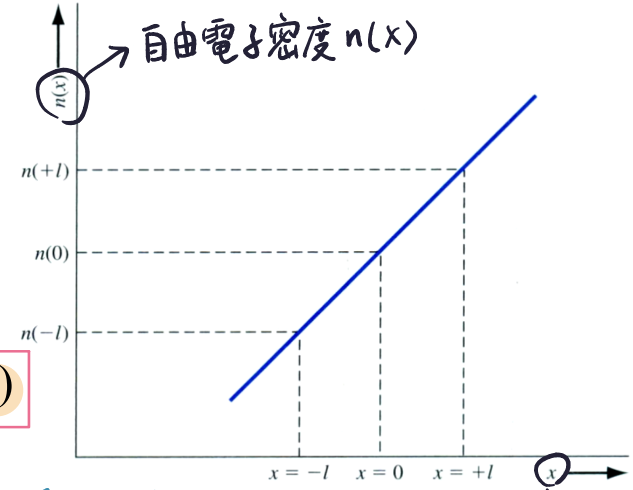


- Consider a container is divided into two parts by membrane 膜
  - Left side is full with gas molecules which is continual thermal motion at a particular temperature
- Diffusion 擴散
  - Flow from a high concentration toward low concentration
- Diffusion current 擴散電流
  - If the gas molecules were electrically charged, there is a net flow of charge

帶電的氣体分子，產生的淨電荷流動

Carrier Diffusion

- **Diffusion Current Density** 擴散電流密度
  - Consider an electron concentration varies in one dimension at a uniform temperature 均勻溫度
    - The average thermal velocity is independent of x 平均熱運動速率和x無關
  - 電子的平均自由徑→分子在兩次碰撞之間路徑長的平均
  - The distance l : the mean-free path of an electron
    - 兩次碰撞的平均距離
    - x=0 處電子碰撞 The average distance between collisions $(l = \upsilon_{th}\tau_{cn})$
    - 1/2 of electrons at $x=-l$ travel to the right ⇒ 一半電子向右
    - 1/2 of electrons at $x=+l$ travel to the left ⇒ 一半電子向左 (因為電子流和電流方向相反,要加負號)



⇒ 淨流率 Fn (電流密度)

- The net rate of electron flow F_n at $x=0$ 電子流的淨流率 Fn

⇒ $\frac{1}{2}$(向右 − 向左)

$$F_n = \underbrace{\frac{1}{2}n(-l)\upsilon_{th}}_{\text{向右}} - \underbrace{\frac{1}{2}n(+l)\upsilon_{th}}_{\text{向左}} = \frac{1}{2}\upsilon_{th}[n(-l)-n(+l)] \quad (5.27)$$

$$\Rightarrow F_n \approx \frac{1}{2}\upsilon_{th}\{[n(0)-l\frac{dn}{dx}]-[n(0)+l\frac{dn}{dx}]\} \quad (5.28)$$

硬湊的

$$\Rightarrow \boxed{F_n \approx -\upsilon_{th}l\frac{dn}{dx}} \quad (5.29)$$

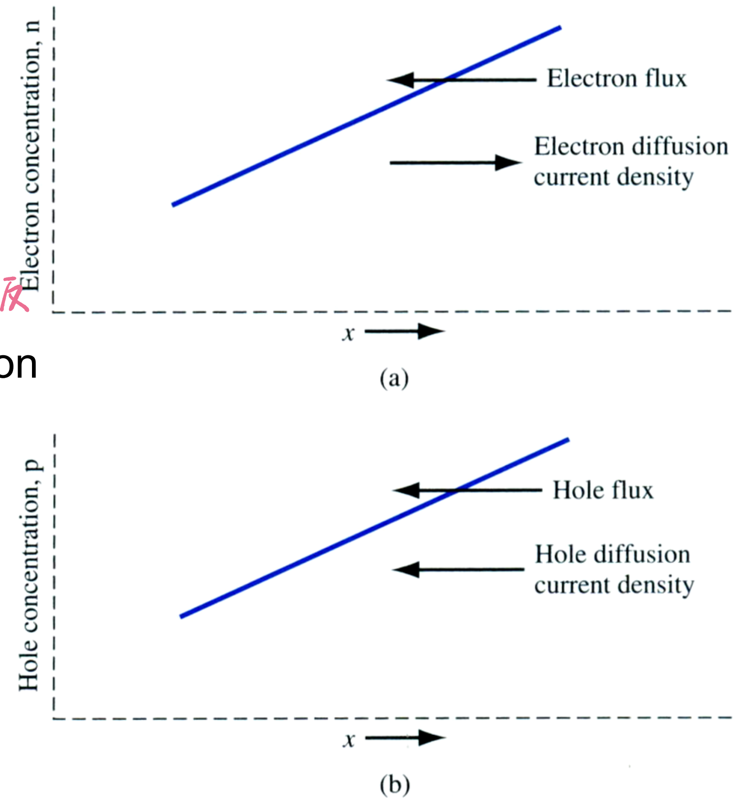
電子濃度梯度

- The electron diffusion current $\boxed{J = -eF_n \approx +e\upsilon_{th}l\frac{dn}{dx}}$ 擴散電流 (5.30)
  - Is proportional to the spatial deviation, density gradient, of the electron concentration

Carrier Diffusion

- The electron diffusion current density for one dimension
  - $$\boxed{J_{nx|dif} = eD_n \frac{dn}{dx}}$$ (5.31) 電子擴散電流密度
  - D_n : electron diffusion coefficient (cm²/s) 電子漂移係數

- For an example of a hole concentration
  - A flux of holes in the negative x direction
    - The hole diffusion current density for one dimension
      - $$\boxed{J_{px|dif} = -eD_p \frac{dp}{dx}}$$ (5.32) 電洞電子擴散方向相反
      - The negative sign denotes the $-x$ direction
      - D_p : hole diffusion coefficient (cm²/s)



Carrier Diffusion

- **Example 5.4** Assume that, in an n-type gallium arsenide semiconductor at $T=300°K$, the electron concentration varies linearly from 1×10^{18} to 7×10^{17} cm^{-3} over a distance of 0.10 cm. Calculate the diffusion current density if the electron diffusion coefficient is $D_n = 225\ cm^2/s$.

  - The diffusion current density

$$J_{nx|dif} = eD_n \frac{dn}{dx} \approx eD_n \frac{\Delta n}{\Delta x}$$

$$= (1.6\times10^{-19})(225)\left(\frac{1\times10^{18} - 7\times10^{17}}{0.10}\right) = 108\text{A/cm}^2$$

A significant diffusion current density is generated with a modest density gradient

Carrier Diffusion

- Total Current Density 總電流密度 = 總漂移電流 + 總擴散電流
  - Four possible independent mechanisms
    - Electron drift current
    - Electron diffusion current
    - Hole drift current
    - Hole diffusion current
  - For one-dimension case
    - $\boxed{J = en\mu_n \mathrm{E}_x + ep\mu_p \mathrm{E}_x + eD_n \frac{dn}{dx} - eD_p \frac{dp}{dx}}$ = 總漂移電流 + 總擴散電流
  - For three-dimension
    - $J = en\mu_n \mathrm{E} + ep\mu_p \mathrm{E} + eD_n \nabla n - eD_p \nabla p$ 密度梯度
      - Electric field
        - mobility
      - Density gradient
        - Diffusion coefficient

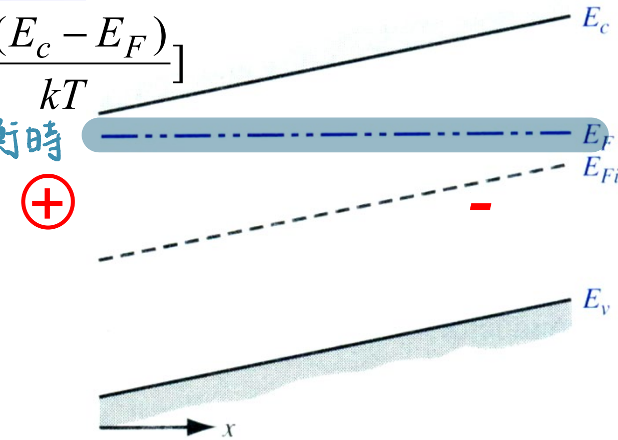
梯度雜質分佈 Graded Impurity Distribution

多摻雜

- Semiconductor is ***not*** doped uniformly
  - How to reach thermal equilibrium

$$n_0 = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

熱平衡時



- Induced Electrical Field 感應電場
  - Energy-band diagram
    - In thermal equilibrium 熱平衡時

費米能階是定值
- The Fermi energy is constant through the crystal
- The donor doping concentration decreases as x increases

令施体濃度隨x增加

- A diffusion of majority carrier from the region of high concentration to low concentration $\Rightarrow +x$ direction
- The flow of negative electrons $\Rightarrow$ positively charge donor ions left
- Induced an electric field in the direction opposite to the diffusion process

則
① 擴散電流往+X移動
② 電子擴散方向往-X軸移動

- Equilibrium is reached 擴散平衡
  - the induced electric field prevents any further separation of charge
  - The space charge by diffusion process is a small fraction of the impurity concentration
  - The mobile carrier concentration is not exactly equal to the fixed impurity concentration, but not too different from

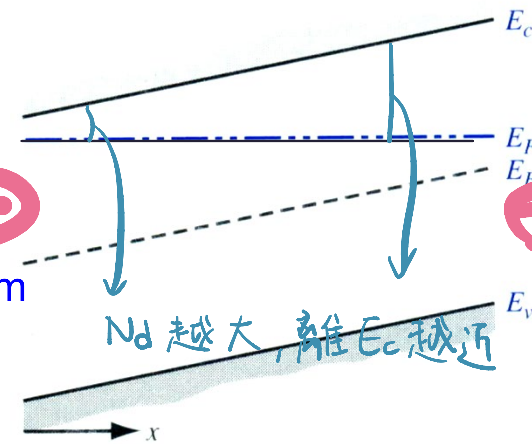
Chien-Hung Kuo

Graded Impurity Distribution

- The electric potential
  - $\phi = +\frac{1}{e}(E_F - E_{Fi})$ (5.35)
- The electric field for the one-dimension situation
  - $E_x \equiv -\frac{d\phi}{dx} = \frac{1}{e}\left(\frac{dE_{Fi}}{dx}\right)$ 能 (斜) (5.36)

電場

  - If the intrinsic Fermi level changes as a function of distance through a semiconductor in thermal equilibrium
    - The electric field exists in the semiconductor



- A quasi-neutrality condition
  - The electron concentration ≈ the donor impurity concentration
    - $n_0 = n_i \exp\left[\frac{E_F - E_{Fi}}{kT}\right] \approx N_d(x)$ (5.37)
    - $\Rightarrow E_F - E_{Fi} = kT \ln\left(\frac{N_d(x)}{n_i}\right)$ (5.38)
    - Derivative with respect to x
      - $-\frac{dE_{Fi}}{dx} = \frac{kT}{\left(\frac{N_d(x)}{n_i}\right)} \frac{1}{n_i} \frac{dN_d(x)}{dx} = \frac{kT}{N_d(x)} \frac{dN_d(x)}{dx}$ (5.39)

漂移 ⊖ → D → E_D ←

电荷 E_i → 引力 ←

⊕⊖电荷之間的电場

E之間相互抵消

→ 動態平衡

Graded Impurity Distribution

- The electric field
  - $$\boxed{E_x = \frac{1}{e}\left(\frac{dE_{Fi}}{dx}\right) = -\left(\frac{kT}{e}\right)\frac{1}{N_d(x)}\frac{dN_d(x)}{dx}} \qquad (5.40)$$
  - *The electric field as well as a potential difference are induced through the semiconductor due to nonuniform doping*

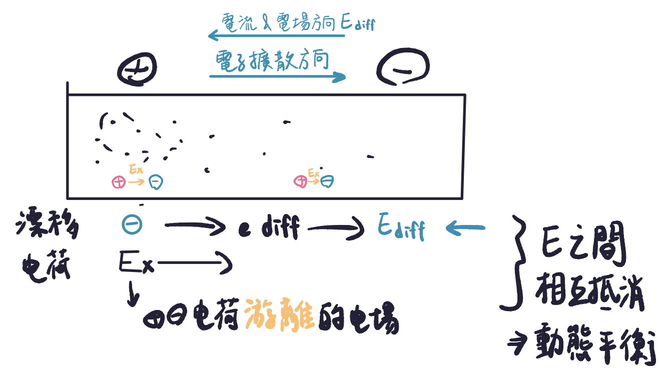
Example 5.5 Assume that the donor concentration in an n-type semiconductor at T=300°K is given by $N_d(x) = 10^{16} - 10^{19}x\ \ (\text{cm}^{-3})$ where x is given in cm and ranges between $0 \le x \le 1\mu\text{m}$

- The derivative of the donor concentration

$$\frac{dN_d(x)}{dx} = -10^{19} \qquad (\text{cm}^{-4})$$

- The electric field

$$E_x = -\left(\frac{kT}{e}\right)\frac{1}{N_d(x)}\frac{dN_d(x)}{dx} = \frac{-(0.0259)(-10^{19})}{10^{16} - 10^{19}x}$$



  - At $x=0$
    - $E_x = 25.9$ V/cm

Recall that a small electric field can cause a significant drift current density

So the induced electric field from nonuniform doping will significant influence semiconductor characteristics

Graded Impurity Distribution

- The Einstein Relation
  - **In thermal equilibrium**
    - Assume there is no electrical connections
      - The individual electron and hole currents must be zero
        - $$J_n = 0 = en\mu_n \mathrm{E}_x + eD_n \frac{dn}{dx} \quad (5.41)$$
      - Assume quasi-neutrality
        - $$n \approx N_d(x)$$

$$J_n = 0 = e\mu_n N_d(x)\mathrm{E}_x + eD_n \frac{dN_d(x)}{dx} \quad (5.42)$$

$$\Rightarrow 0 = -e\mu_n N_d(x)(\frac{kT}{e})\frac{1}{N_d(x)}\frac{dN_d(x)}{dx} + eD_n \frac{dN_d(x)}{dx}$$

$$\Rightarrow 0 = -\mu_n kT + eD_n$$

$$\Rightarrow \frac{D_n}{\mu_n} = \frac{kT}{e} \quad (5.44a)$$

      - Similarly,

$$\frac{D_p}{\mu_p} = \frac{kT}{e} \quad (5.44b)$$

Graded Impurity Distribution

- Einstein relation 擴散係數和遷移率有關
  - The diffusion coefficient and mobility are ***not*** independent parameters
    - $$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{kT}{e} \qquad (5.45)$$

Example 5.6 To determine the diffusion coefficient given the carrier mobility. Assume that the mobility of a particular carrier is 1000 cm²/V-s at T=300°K

- From Einstein relation
  - $$D = \frac{kT}{e}\mu = (0.0259)(1000) = 25.9\,\mathrm{cm}^2/s$$

It is important to keep in mind the relative orders of magnitude of the mobility and diffusion coefficient
The diffusion coefficient is approximately 40 times smaller than the mobility at room temperature

Graded Impurity Distribution

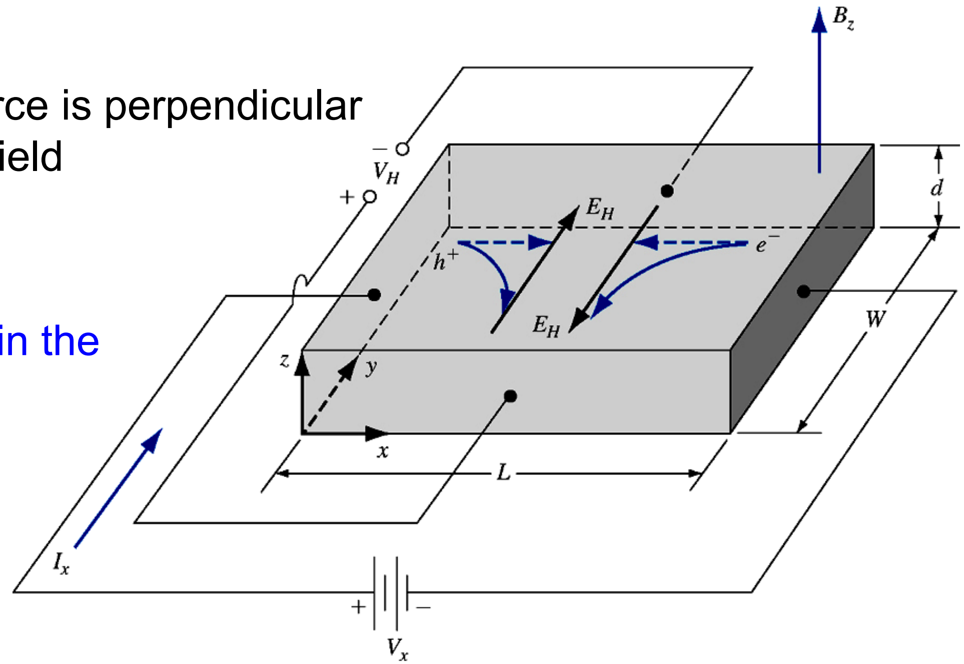
- The mobilities are strong functions of temperature because of the scattering process
  - The diffusion coefficients are also strong functions of temperature
    - It is important to keep in mind that the major temperature effects are a result of lattice scattering and ionized impurity scattering
    - The temperature dependence in equation (5.45) is a small fraction of the real temperature characteristics

$$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{kT}{e}$$

	μ_n	D_n	μ_p	D_p
Silicon	1350	35	480	12.4
Gallium arsenide	8500	220	400	10.4
Germanium	3900	101	1900	49.2

The Hall Effect

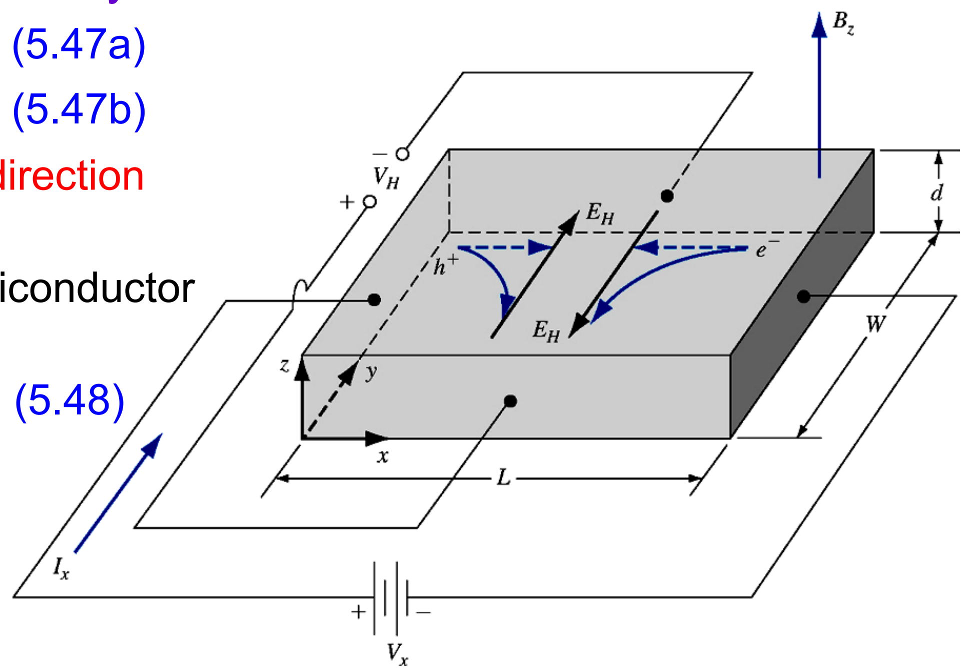
- **The Hall Effect**
  - A consequence of the forces that are exerted on moving charges by electric and magnetic fields
    - Can be used to distinguish whether a semiconductor is n type or p type
    - Can be used to measure the majority carrier concentration and mobility
    - Can be used to extensively in engineering applications as a magnetic probe
  - The force on a particle moving in a magnetic field
    - $F = q\upsilon \times B$
      - The cross product indicates the force is perpendicular to both the velocity and magnetic field
      - A current I_x
      - A magnetic field B_z
        - There is a force experienced in the (-y) direction



The Hall Effect

- For a p-type semiconductor ($p_0 > n_0$)
  - A buildup of positive charge on the y=0 surface
- For a n-type semiconductor ($n_0 > p_0$)
  - A buildup of negative charge on the y=0 surface
- An *electric field* is induced
- In steady state (finally)
  - The magnetic field is exactly balanced by the induced electric field force
    - $F = q[\mathrm{E} + \upsilon \times B] = 0$ (5.47a)
    - $q\mathrm{E}_y = q\upsilon_x B_z$ (5.47b)
    - The induced electric field in the y-direction is called the ***Hall field***
      - A *Hall voltage* across the semiconductor is produced

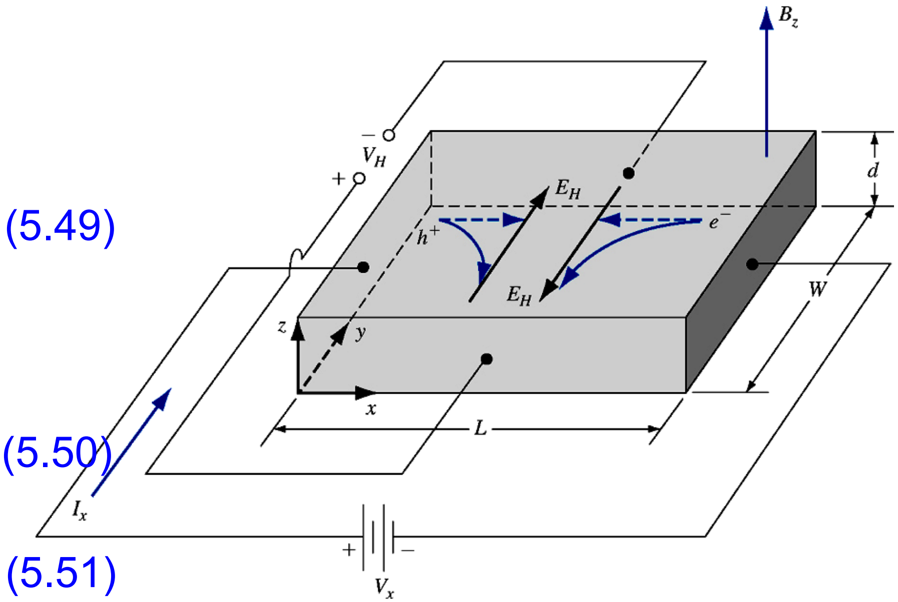
        $$\boxed{V_H = +\mathrm{E}_H W} \qquad (5.48)$$
      - E_H is assumed positive in the +y direction
      - V_H is positive with the polarity as shown



The Hall Effect

- In p-type semiconductor
  - The Hall voltage is positive as shown
- In n-type semiconductor
  - The Hall voltage will have the opposite polarity

- *The polarity of the Hall voltage can be used to determine whether an extrinsic semiconductor is n-type or p-type*



- The Hall voltage
  - $V_H = \upsilon_x W B_z$ (5.49)
  - For a p-type semiconductor
    - The drift velocity of holes

$$\upsilon_{dx} = \frac{J_x}{ep} = \frac{I_x}{(ep)(Wd)} \quad (5.50)$$

$$\Rightarrow V_H = \upsilon_x W B_z = \frac{I_x}{(ep)(Wd)} W B_z = \frac{I_x B_z}{epd} \quad (5.51)$$

    - The hole concentration

$$\boxed{p = \frac{I_x B_z}{edV_H}} \quad (5.52)$$

The majority carrier concentration is determined from the current, magnetic field, and Hall voltage

The Hall Effect

- For a n-type semiconductor
  - The Hall voltage is negative
    - $$V_H = -\frac{I_x B_z}{end} \quad (5.53)$$
  - The electron concentration
    - $$\boxed{n = -\frac{I_x B_z}{edV_H}} \quad (5.54)$$
    - The concentration is positive quantity due to negative Hall voltage

- To determine the low-field majority carrier mobility
  - For a p-type semiconductor
    - The drift current density
      - $$J_x = ep\mu_p \mathrm{E}_x \quad (5.55)$$

        $$\Rightarrow \frac{I_x}{Wd} = \frac{ep\mu_p V_x}{L} \quad (5.56)$$
    - The hole mobility
      - $$\boxed{\mu_p = \frac{I_x L}{epV_x Wd}} \quad (5.57)$$

The electron mobility

$$\boxed{\mu_n = \frac{I_x L}{enV_x Wd}} \quad (5.58)$$

The Hall Effect

- **Example 5.7** Consider the geometry shown in the above Figure. Let L = 10^{-1} cm, W=10^{-2} cm, and d=10^{-3} cm. Also assume that I_x=1.0mA, V_x=12.5V, B_z=500 gauss = 5x10^{-2} tesla, and V_H= -6.25 mV. Determine the majority concentration and mobility.
  - A negative Hall voltage
    - An n-type semiconductor

$$n=-\frac{I_xB_z}{edV_H}=\frac{-(10^{-3})(5\times10^{-2})}{(1.6\times10^{-19})(10^{-5})(-6.25\times10^{-3})}=5\times10^{21}\,\mathrm{m}^{-3}=5\times10^{15}\,\mathrm{cm}^{-3}$$

    - The electron mobility

$$\mu_n=\frac{I_xL}{enV_xWd}=\frac{(10^{-3})(10^{-3})}{(1.6\times10^{-19})(5\times10^{21})(12.5)(10^{-4})(10^{-5})}=0.10\,\mathrm{m}^2/\mathrm{V\text{-}s}=1000\,\mathrm{cm}^2/\mathrm{V\text{-}s}$$

The MKS units must be used consistently in Hall effect equation