time-domain 和 frequency-domain 可互换

§ Review some of the Fourier Xform Pairs

rectangular wave: $A \cdot \mathrm{rect}\left(\frac{t}{W}\right) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftarrows}} A \cdot W \cdot \mathrm{sinc}(W \cdot f)$ — sinc function

impulse: $\delta(t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftarrows}} 1 \quad \forall f$ — constant

$A \cdot \delta(t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftarrows}} A \quad \forall f$

complex sinusoidal: $A \cdot e^{j2\pi f_1 t} \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftarrows}} A \cdot \delta(f - f_1)$ — delta function

real sinusoidal: $A \cos(2\pi f_1 t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftarrows}} \frac{A}{2} \cdot \delta(f - f_1) + \frac{A}{2} \delta(f + f_1)$

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signum function: $\mathrm{sgn}(t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftarrows}} \frac{1}{j\pi \cdot f}$

$x(t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftarrows}} X(f)$

delay: $x(t - \tau) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftarrows}} e^{-j2\pi f \tau} \cdot X(f)$

derivative: $\frac{dx(t)}{dt} \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftarrows}} j \cdot 2\pi \cdot f \cdot X(f)$

integral: $\int_{-\infty}^{t} x(\tau)\, d\tau \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftarrows}} X(f) \cdot \left[\frac{1}{j2\pi f} + \frac{\delta(f)}{2} \right]$

$= x(t) \circledast u(t)$ $\qquad = \frac{1}{j2\pi f} \cdot X(f) + \frac{1}{2} X(0) \cdot \delta(f)$

triangular function: $\mathrm{tri}(t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftarrows}} A \cdot W \cdot \mathrm{sinc}^2(W \cdot f)$ — sinc²

111_2 Signals and Systems HW#3 Solution

1. Calculate the Fourier transforms of the following x(t), that is X(f). Also plot their magnitude and phase responses

   (a) $x(t)=e^{-2(t-1)}u(t-1)$

   (b) $x(t)=e^{-2(t-1)}$

   (c) $x(t)=\delta(t+1)+\delta(t-1)$

   (d) $x(t)=\text{sinc}(t)$

Ans:

[法一] 由定義入手，作積分 $g(t)\ \rightarrow\ \int_{-\infty}^{\infty} g(t)e^{-j2\pi f t}dt$

[法二] 由 Fourier transform pairs 入手，套用恒等式

(a)

① $X(f)=\int_{-\infty}^{\infty} e^{-2(t-1)}u(t-1)e^{-j2\pi f t}dt$

$$=\int_{1}^{\infty} e^{-2t}e^{2}e^{-j2\pi f t}dt \quad = e^{2}\int_{1}^{\infty} e^{-2(1+j\pi f)t}dt$$

$$= e^{2}\cdot\frac{1}{-2(1+j\pi f)}\cdot\left[e^{-2(1+j\pi f)t}\right]_{t=1}^{t=\infty} = \frac{e^{2}}{-2(1+j\pi f)}\cdot\left[0 - e^{-2(1+j\pi f)}\right]$$

$$= \frac{e^{-j2\pi f}}{2(1+j\pi f)} = \frac{(1-j\pi f)e^{-j2\pi f}}{2(1+\pi^2 f^2)}$$

$z = a+bi = 1-j\pi f$

$z^2 = z\cdot z^*$

$= (a+bi)(a-bi)$

$= (1-j\pi f)(1+j\pi f)$

$= 1+\pi^2 f^2$

直

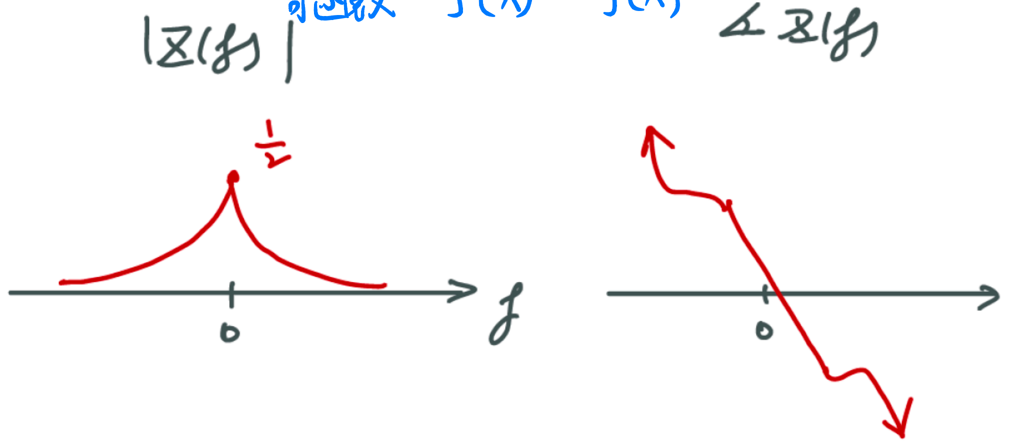
euler

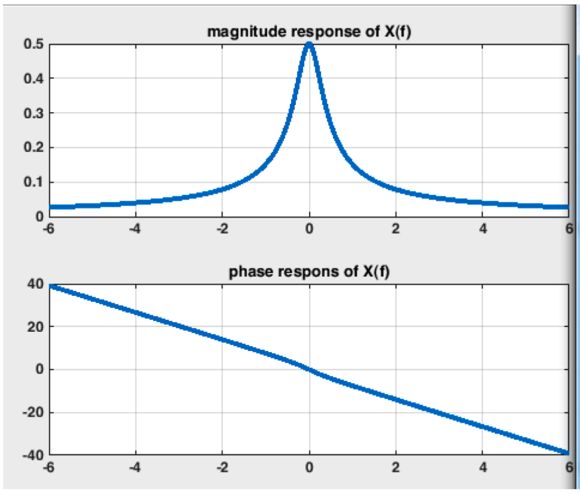
$$= \frac{\sqrt{1+\pi^2 f^2}\angle(\tan^{-1}(-\pi f))\cdot 1\angle(-2\pi f)}{2(1+\pi^2 f^2)} = \frac{\sqrt{1+\pi^2 f^2}\angle(\tan^{-1}(-\pi f)-2\pi f)}{2(1+\pi^2 f^2)}$$ #

② $|X(f)| = \dfrac{\sqrt{1+\pi^2 f^2}}{2(1+\pi^2 f^2)} = \dfrac{1}{2\sqrt{1+\pi^2 f^2}} = \dfrac{1}{\sqrt{4+4\pi^2 f^2}}$ #（振幅恒正）

③ $\angle X(f) = \tan^{-1}(-\pi f)-2\pi f = -\tan^{-1}(\pi f)-2\pi f$ #

奇函數 $f(-x)=-f(x)$






```
subplot(2, 1, 1)
f = -6: 0.01 : 6;
Xf_mag = 1./sqrt(4 + 4*pi*pi*(f.*f) );
plot(f, Xf_mag, 'LineWidth', 3)
grid on
title('magnitude response of X(f)')

subplot(2, 1, 2)
Xf_pha = -atan(pi*f) - 2*pi*f;
plot(f, Xf_pha, 'LineWidth', 3)
grid on
title('phase respons of X(f)')
```

(b)

① $X(f) = \int_{-\inf}^{\inf} e^{-2(t-1)} e^{-j2\pi f t} dt$

$= e^2 \int_{-\inf}^{\inf} e^{-2(1+j\pi f)t} dt \quad = \frac{e^2}{-2(1+j\pi f)} \left[e^{-2(1+j\pi f)t} \right]_{t=-\infty}^{t=\infty}$

$= \frac{e^2}{-2(1+j\pi f)} \left[0 - e^{+2(1+j\pi f)\infty} \right]$ 無法計算

$\Rightarrow \quad x(t)=e^{-2(t-1)}$ 的傅立葉轉換不存在 #

[註] 若 $x(t)$ 不為絕對可積（ $\int_{-\infty}^{\infty} |x(t)| dt \neq \text{finite}$ ），則 $X(f)$ "有可能"不存在

(c)

【Hint】

$$\int_{-\infty}^{\infty} \delta(t-a) \cdot g(t) = g(a)$$

振幅(絕對值) →偶對稱
頻率 →奇對稱

① $X(f) = \int_{-\inf}^{\inf} [\delta(t+1)+\delta(t-1)] e^{-j2\pi f t} dt$

$= \left[e^{-j2\pi f t} \right]_{t=-1} + \left[e^{-j2\pi f t} \right]_{t=1} \quad = e^{-j2\pi f} + e^{j2\pi f}$

$= [\cos(2\pi f) - j \cdot \sin(2\pi f)] + [\cos(2\pi f) + j \cdot \sin(2\pi f)]$

$= 2 \cdot \cos(2\pi f)$ （有負值，相角不為0。考慮：實數時域函數 → 奇對稱相位響應）

相位會隨頻率改變

½週期,考慮±π

cos(-π)

f<0
中央
f>0

$$\triangleq \begin{cases} |2\cdot\cos(2\pi f)| \angle(-\pi) \ , & \frac{-3}{4}-k < f < \frac{-1}{4}-k \\ |2\cdot\cos(2\pi f)| \angle 0 \ , & \frac{-1}{4}\pm k \le f \le \frac{1}{4}\pm k \ , \quad k=0,1,2,\cdots \ \# \\ |2\cdot\cos(2\pi f)| \angle \pi \ , & \frac{1}{4}+k < f < \frac{3}{4}+k \end{cases}$$

$2\cos(2\pi f) + i\sin\theta$

零點 (time-domain 正/餘弦波有2個f)

for $f>0$ $\quad \cos\left(\frac{\pi}{2}+n\pi\right) \rightarrow 0 = \cos(2\pi f)$

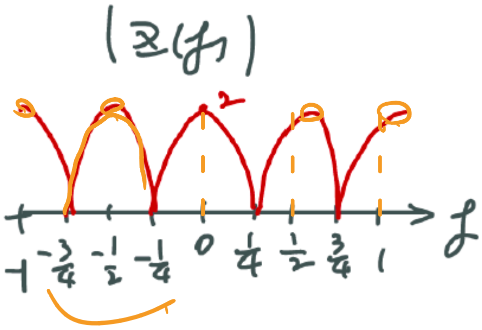
$\frac{\pi}{2}+n\pi = 2\pi f \qquad f = \frac{1}{4}+\frac{n}{2}, \quad n=0,1,2,..$

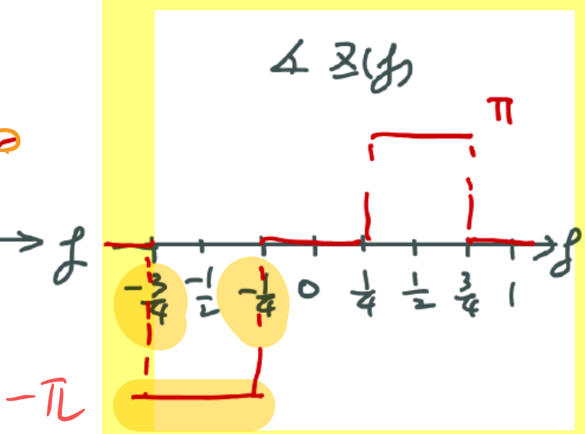
for $f<0$ $\quad \cos\left(-\frac{\pi}{2}+n\pi\right) \rightarrow 0 = \cos(2\pi f)$

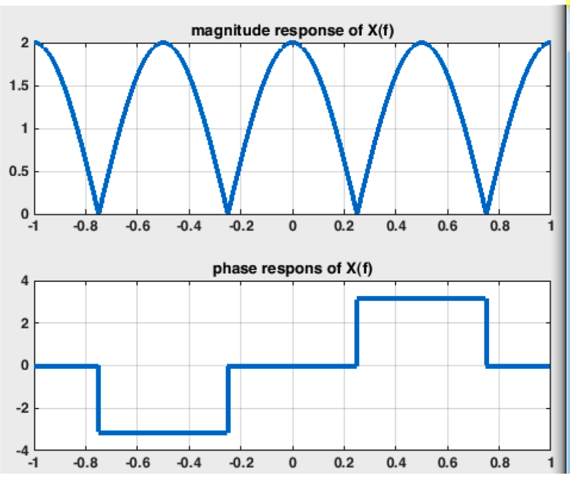
$-\frac{\pi}{2}+n\pi = 2\pi f \qquad f = -\frac{1}{4}+\frac{n}{2}, \quad n=0,-1,-2,\ldots$

② $|X(f)| = |2\cdot\cos(2\pi f)|$ # （振幅恒正）

③ $\angle X(f) = 0$ #








```
subplot(2, 1, 1)
f = -1: 0.001 : 1;
Xf_mag = abs( 2 * cos(2*pi*f) );
plot(f, Xf_mag, 'LineWidth', 3)
grid on
title('magnitude response of X(f)')

subplot(2, 1, 2)
Xf_pha = 0*f;
for i = 1 : size(f, 2)
    if (f(i)>(-3/4)) && (f(i)<(-1/4))
        Xf_pha(i) = -pi;
    elseif (f(i)>(1/4)) && (f(i)<(3/4))
        Xf_pha(i) = pi;
    end
end
plot(f, Xf_pha, 'LineWidth', 3)
grid on
title('phase respons of X(f)')
```

(d)

【提示】

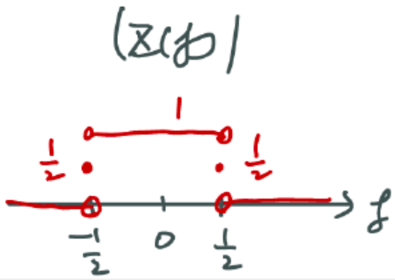
利用 Fourier transform pair： $a\cdot sinc(at) \overset{FT}{\rightarrow} rect\left(\frac{f}{a}\right)$

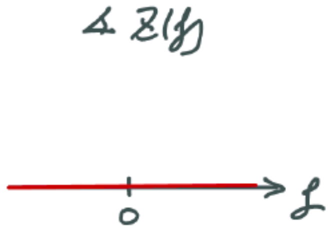
相位不隨頻率改變

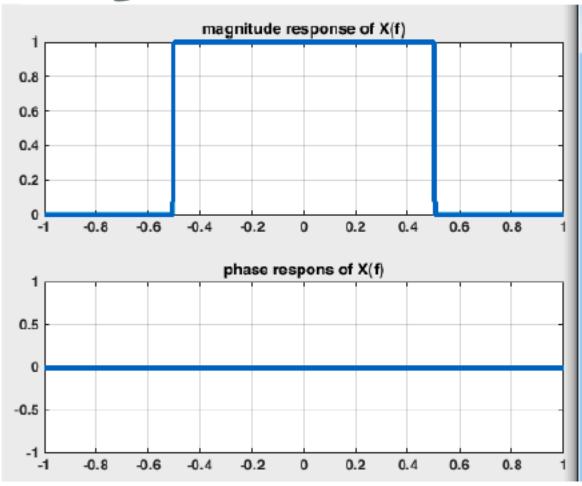
① $x(t) = sinc(t) = 1\cdot sinc(1\cdot t) \overset{FT}{\rightarrow} rect(f) = \begin{cases} 1\angle 0 & , \quad -0.5 < f < 0.5 \\ \frac{1}{2}\angle 0 & , \quad |f| = 0.5 \\ 0\angle 0 & , \quad |f| > 0.5 \end{cases} = X(f)$ #

② $|X(f)| = |rect(f)| = \begin{cases} 1 & , \quad -0.5 < f < 0.5 \\ \frac{1}{2} & , \quad |f| = 0.5 \\ 0 & , \quad |f| > 0.5 \end{cases}$ #

③ $\angle X(f) = 0$ #








```
subplot(2, 1, 1)
f = -1: 0.005 : 1;
Xf_mag = ones( size(f) );
for i = 1 : size(f, 2)
    if f(i) < -0.5
        Xf_mag(i) = 0;
    elseif f(i) > 0.5
        Xf_mag(i) = 0;
    end
end
plot(f, Xf_mag, 'LineWidth', 3)
grid on
title('magnitude response of X(f)')

subplot(2, 1, 2)
Xf_pha = 0*f;
plot(f, Xf_pha, 'LineWidth', 3)
grid on
title('phase respons of X(f)')
```

2. Consider the signal

$$x(t)=\begin{cases}0, & |t|>1 \\ (t+1)/2, & -1\le t\le 1\end{cases}.$$

$$x(t)=\begin{cases}0, & |t|>1 \\ (t+1)/2, & -1\le t\le 1\end{cases}.$$

(a) Determine $X(f)$.

(b) Take the real part of your answer to part (a), and verify that it is Fourier transform of the even part of $x(t)$.

(c) What is the Fourier transform of the odd part of $x(t)$?

Ans:

(a)

【提示】 $x(t)$ 沒有特殊形式，無法用 Fourier transform pairs，故從定義下手

$$\int u\,dv = uv - \int v\,du$$

$$uv = \int u\,dv + \int v\,du$$

$$\boxed{X(f)} = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt = \int_{-1}^{1}\frac{t+1}{2}e^{-j2\pi ft}dt$$

(by partial integration)

$$= \frac{1}{2}\left[\frac{t+1}{-j2\pi f}e^{-j2\pi ft} - \frac{1}{(-j2\pi f)^2}e^{-j2\pi ft}\right]_{t=-1}^{t=1}$$

$$= \frac{1}{2}\left[\frac{2e^{-j2\pi f}}{-j2\pi f} + \frac{-j2\sin(2\pi f)}{(2\pi f)^2}\right]$$

$$= j\left[\frac{e^{-j2\pi f}}{2\pi f} - \frac{\sin(2\pi f)}{(2\pi f)^2}\right] \quad \#$$

(by Euler's Formula)

$$= \left[\frac{\sin(2\pi f)}{2\pi f}\right] + j\cdot\left[\frac{\cos(2\pi f)}{2\pi f} - \frac{\sin(2\pi f)}{(2\pi f)^2}\right] \quad \#$$

(b)

$$x(t) = \begin{cases} 0 & , \quad |t|>1 \\ (t+1)/2 & , \quad -1\le t\le 1 \end{cases} \quad \text{and} \quad x(-t) = \begin{cases} (-t+1)/2 & , \quad -1\le t\le 1 \\ 0 & , \quad |t|>1 \end{cases}$$

① $\boxed{\text{偶函數}\ a(t) = \dfrac{x(t)+x(-t)}{2} = \begin{cases} \dfrac{1}{2} & , \quad -1\le t\le 1 \\ 0 & , \quad |t|>1 \end{cases}}$

$$\frac{1}{2}\int_{-1}^{1} e^{-j2\pi ft}dt$$

② $A(f) = \int_{-\infty}^{\infty} a(t)e^{-j2\pi ft}dt = \int_{-1}^{1}\frac{1}{2}e^{-j2\pi ft}dt = \frac{1}{2}\cdot F\left[rect\left(\frac{t}{2}\right)\right] = sinc(2f)$

×

② $A(f) = \int_{-\infty}^{\infty} a(t)e^{-j2\pi ft}dt = \int_{-1}^{1}\frac{1}{2}e^{-j2\pi ft}dt = \left[\frac{e^{-j2\pi ft}}{2(-j2\pi f)}\right]_{t=-1}^{t=1} = \frac{-j2\sin(2\pi f)}{-j4\pi f}$

$$= \frac{\sin(2\pi f)}{2\pi f} = sinc(2f)$$

ans：偶函數 $a(t)$ 的傅立葉轉換 $A(f)$ 等於 $X(f)$ 的實部，**得證**。

(c)

① $\boxed{\text{奇函數}\ b(t) = \dfrac{x(t)-x(-t)}{2} = \begin{cases} \dfrac{t}{2} & , \quad -1\le t\le 1 \\ 0 & , \quad |t|>1 \end{cases}}$

② $B(f) = \int_{-\infty}^{\infty} b(t)e^{-j2\pi ft}dt = \int_{-1}^{1}\frac{t}{2}e^{-j2\pi ft}dt = \frac{1}{2}\left[\frac{-t}{j2\pi f}e^{-j2\pi ft} - \frac{1}{(j2\pi f)^2}e^{-j2\pi ft}\right]_{t=-1}^{t=1}$

$$= \frac{1}{2}\left[\frac{-2\cos(2\pi f)}{j2\pi f} - \frac{-j2\sin(2\pi f)}{(j2\pi f)^2}\right] = j\cdot\left[\frac{\cos(2\pi f)}{2\pi f} - \frac{\sin(2\pi f)}{(2\pi f)^2}\right]$$

ans：奇函數 $b(t)$ 的傅立葉轉換 $B(f)$ 等於 $X(f)$ 的虛部，得證。

3.

Consider an LTI system described by the second-order differential equation

$$\frac{d^2y(t)}{dt^2}+2\frac{dy(t)}{dt}+y(t)=3\frac{dx(t)}{dt}+x(t) \tag{106}$$

Show that the frequency response characteristic of this system is

$$H(F)=\frac{3(j2\pi F)+1}{(j2\pi F)^2+2(j2\pi F)+1} \tag{107}$$

Hint: Apply the complex sinusoidal input $x(t)=Xe^{j2\pi Ft}$ to the differential equation and assume a solution of the form $y(t)=Ye^{j2\pi Ft}$.

Ans:

【提示】

n次微分 $g^{(n)}(t)=\dfrac{d^n g(t)}{d^n t}\rightarrow(j2\pi f)^n\cdot G(f)$

$$F[y''(t)+2y'(t)+y(t)]=F[3x'(t)+x(t)]$$
$$(j2\pi f)^2\cdot Y(f)+2(j2\pi f)\cdot Y(f)+Y(f)=3(j2\pi f)\cdot X(f)+X(f)$$
$$[\ (j2\pi f)^2+2(j2\pi f)+1\ ]\cdot Y(f)=[\ 3(j2\pi f)+1\ ]\cdot X(f)$$
$$H(f)=\frac{Y(f)}{X(f)}=\frac{3(j2\pi f)+1}{(j2\pi f)^2+2(j2\pi f)+1}$$ #

4. If the impulse response of a LTI system is $h(t)=rect(\frac{t-1}{2})$

a) Find the frequency response of h(t), i.e., H(f)

b) Plot magnitude response and phase response of H(f)

c) If LTI input $x(t)=e^{j\pi t}$, What is the LTI output $y(t)=?$

Ans:

(a) [法一] 由基本定義下手

$$H(f)=\int_{-\infty}^{\infty}h(t)e^{-j2\pi ft}dt=\int_0^2 1\cdot e^{-j2\pi ft}dt=\left[\frac{1}{-j2\pi f}e^{-j2\pi ft}\right]_{t=0}^{t=2}$$
$$=\frac{1}{j2\pi f}[1-e^{-j4\pi f}]=\frac{-j}{2\pi f}[1-\cos(4\pi f)+j\cdot\sin(4\pi f)]$$
$$=\frac{2\cdot\sin(4\pi f)}{4\pi f}-\frac{j}{2\pi f}[1-\cos(4\pi f)]$$

(a) [法二] 利用 Fourier transform pairs

【提示】

① $u(t) \rightarrow \dfrac{1}{j2\pi f}+\dfrac{\delta(f)}{2}$

② $g(t\pm a) \rightarrow e^{\pm j2\pi f a}\cdot G(f)$

$$h(t) = rect\left(\frac{t-1}{2}\right) = u(t)-u(t-2)$$

① $u(t) \rightarrow \dfrac{1}{j2\pi f}+\dfrac{\delta(f)}{2}$

② $u(t-2) \rightarrow \dfrac{e^{-j4\pi f}}{j2\pi f}+e^{-j4\pi f}\dfrac{\delta(f)}{2} = \dfrac{e^{-j4\pi f}}{j2\pi f}+e^{-j4\pi 0}\dfrac{\delta(f)}{2} = \dfrac{e^{-j4\pi f}}{j2\pi f}+\dfrac{\delta(f)}{2}$

③ $h(t) = u(t)-u(t-2) \rightarrow$

$$\begin{aligned} H(f) &= \frac{1}{j2\pi f}\left[1 - e^{-j4\pi f}\right] \\ &= \frac{-j}{2\pi f}\left[1 - \cos(4\pi f)+j\cdot\sin(4\pi f)\right] \\ &= \frac{\sin(4\pi f)}{2\pi f} - j\cdot\frac{1-\cos(4\pi f)}{2\pi f} \\ &= 2\cdot sinc(4f) - j\cdot 2sinc(2f)\cdot\sin(2\pi f) \\ &= 2\cdot sinc(2f)\cdot e^{-j2\pi f} \quad \# \end{aligned}$$

(a) [法三] 利用 Fourier transform pairs

【提示】

① $rect\left(\dfrac{t}{a}\right) \rightarrow a\cdot sinc(a\cdot f)$

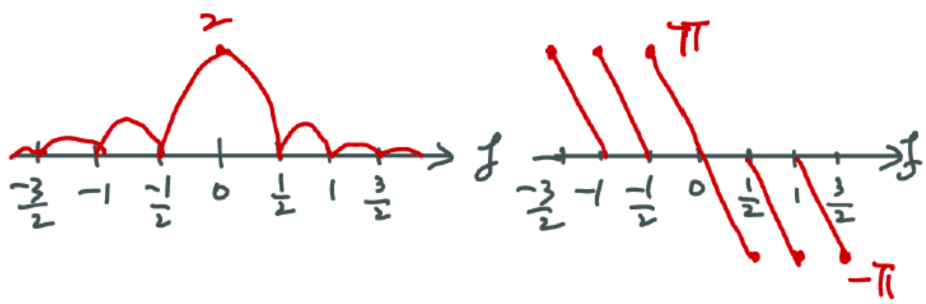
② $g(t\pm a) \rightarrow e^{\pm j2\pi f a}\cdot G(f)$

$$rect\left(\frac{t-1}{2}\right) \rightarrow 2\cdot sinc(2f)\cdot e^{-j2\pi f} \quad \#$$

(b)

① $|H(f)| = \sqrt{H(f)H^*(f)} = 2\cdot|sinc(2f)|$ #

② $\angle H(f) = \tan^{-1}\left(\dfrac{\cos(4\pi f)-1}{\sin(4\pi f)}\right)$ #



(c)

【提示】

① $y(t)=x(t)\otimes h(t) \quad \rightarrow \quad Y(f)=X(f)H(f)$

② 利用 Fourier transform pairs： $1 \quad \rightarrow \quad \delta(f)$

③ 利用 Fourier transform 特性： $e^{\pm j2\pi f_0 t} \quad \rightarrow \quad \delta(f\mp f_0)$

① $x(t)=e^{j\pi t}=e^{j2\pi\frac{1}{2}t} \quad \rightarrow \quad X(f)=\delta(f-\frac{1}{2})$

② 由(b)已得知 $H(f)$

③ 則 $$\begin{aligned} Y(f) &= [2sinc(4f) - j2sinc(2f)\sin(2\pi f)]\cdot\delta(f-\frac{1}{2}) \\ &= 2sinc(4\cdot\frac{1}{2}) - j2\,sinc(2\cdot\frac{1}{2})\sin(2\pi\cdot\frac{1}{2}) \\ &= 2sinc(2) - j2\,sinc(1)\sin(\pi) \\ &= 0 - 0 \\ &= 0 \end{aligned}$$

$Y(f)=0 \quad \overset{FT^{-1}}{\rightarrow} \quad y(t)=0$ #

5. Consider an LTI system with impulse response $h(t) = \frac{\sin(4\pi(t-1))}{\pi(t-1)}$.

a. Plot h(t) (i.e., impulse response of the system)

b. Find H(f) (i.e., frequency response of the system)

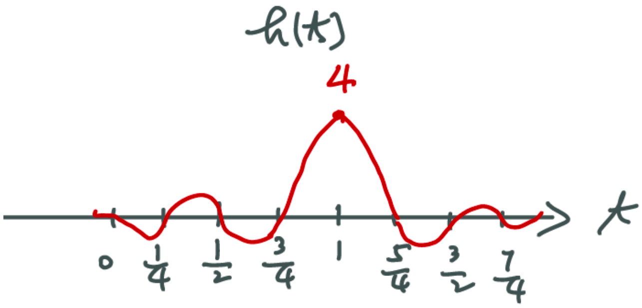
Ans:

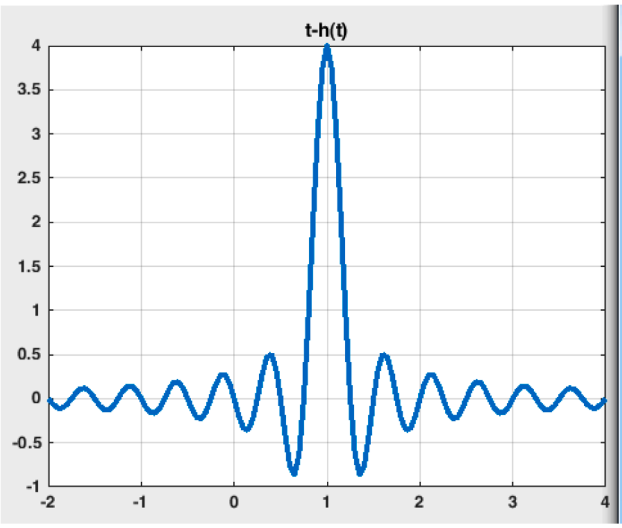
(a)

① $$h(t) = \frac{4\cdot\sin(4\pi(t-1))}{4\cdot\pi(t-1)} = 4\cdot sinc(4(t-1))$$

② 由於 $t=1$ 時， $h(t)=\frac{0}{0}$ $\Rightarrow$ [微積分]羅必達法則求極值

$$\lim_{t\to1} h(t) = \frac{\lim\limits_{t\to1} \sin'(4\pi(t-1))}{\lim\limits_{t\to1} \pi(t-1)'} = \frac{4\pi}{\pi} = 4$$






```
clear;   clc;

t = -2: 0.05 : 4;
num = sin(4*pi*(t-1));
den = pi * (t-1);
ht = num ./ den;
for i = 1 : size(ht, 2)
    if t(i)==1
        ht(i)=4;
    end
end
plot(t, ht, 'LineWidth', 3)
grid on
title('t-h(t)')
```

(b)

【提示】利用 Fourier transform pairs 及 Fourier transform 特性

① $a \cdot sinc(a \cdot t) \quad \rightarrow \quad rect(\frac{f}{a})$

② $g(t \pm a) \quad \rightarrow \quad e^{\pm j2\pi a} G(f)$

$$h(t) = 4 \cdot sinc(4(t-1)) \quad \rightarrow \quad H(f) = e^{-j2\pi f} \cdot rect(\frac{f}{4})$$ #

6. $x(t) = 2\Lambda\left(\frac{t}{2} - 3\right)$

a. Plot $x(t)$

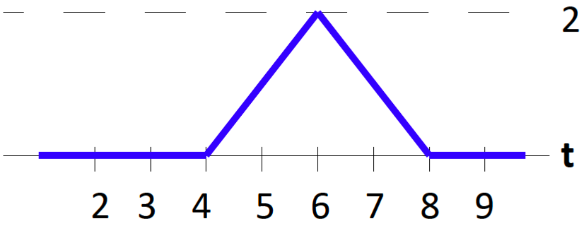
b. Find $X(f) = \mathcal{F}(x(t))$ where $\mathcal{F}(*)$ is the continuous-time Fourier Transform

c. Plot $|X(f)|$ and $\angle(X(f))$

Ans:

(a)

$$x(t) = 2\cdot tri\left(\frac{t-6}{2}\right)$$



(b)

【提示】利用 Fourier transform pairs 及 Fourier transform 特性

① $g(t\pm a) \;\rightarrow\; e^{\pm j2\pi a}G(f)$　　② $tri\left(\frac{t}{a}\right) \;\rightarrow\; a\cdot sinc^2(a\cdot f)$

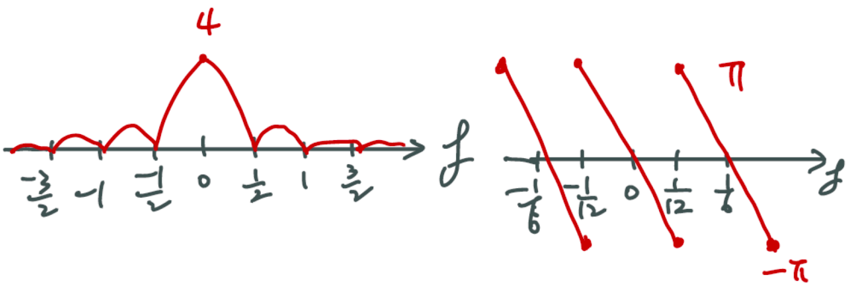
$$x(t) = 2\cdot tri\left(\frac{t-6}{2}\right) \;\rightarrow\; X(f) = 2\cdot e^{-j2\pi f6}\cdot 2\cdot sinc^2(2\cdot f)$$

$$= 4\cdot e^{-j12\pi f}\, sinc^2(2f) \quad \#$$

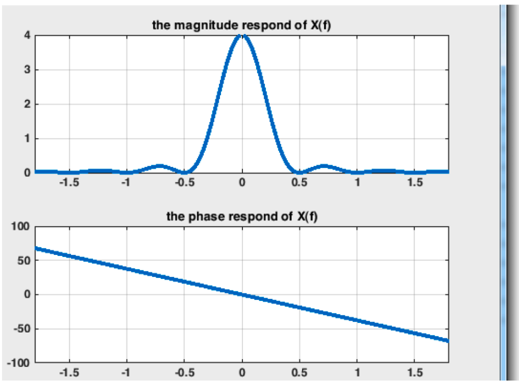
(c)

① $|X(f)| = \left|4\cdot e^{-j12\pi f}\cdot sinc^2(2f)\right| = |4|\cdot\left|e^{-j12\pi f}\right|\cdot\left|sinc^2(2f)\right| = 4\cdot 1\cdot sinc^2(2f) = 4\cdot sinc^2(2f)$

② $\angle X(f) = -12\pi f$



[註] 通常將相位響應畫在 [π,−π] 之間




```
clear;   clc;

subplot(2,1,1)
f = -1.8: 0.01 : 1.8;
factor = 2*pi*f;
Xf_mag = 4 * sin(factor) .* sin(factor) ./ ((factor).*(factor));
plot(f, Xf_mag, 'LineWidth', 3)
grid on
title('the magnitude respond of X(f)')
xlim([-1.8, 1.8])

subplot(2,1,2)
Xf_pha = -12*pi*f;
plot(f, Xf_pha, 'LineWidth', 3)
grid on
title('the phase respond of X(f)')
xlim([-1.8, 1.8])
```