342. Power of Four

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Given an integer (signed 32 bits), write a function to check whether it is a power of 4.

Example 1:

```
Input: 16
Output: true
```

Example 2:

```
Input: 5
Output: false
```

Follow up: Could you solve it without loops/recursion?

Solution

Overview

Prerequisites

This bitwise trick will be used as something already known:

- How to check if the number is a power of two : `x > 0 and x & (x - 1) == 0`.

  Please check the article Power of Two for the detailed explanation.

Intuition

There is an obvious $\mathcal{O}(\log N)$ time solution and we're not going to discuss it here.

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```
class Solution(object):
    def isPowerOfTwo(self, n):
        if n == 0:
            return False
        while n % 4 == 0:
            n /= 4
        return n == 1
```

Let's discuss $\mathcal{O}(1)$ time and $\mathcal{O}(1)$ space solutions only.

Approach 1: Brute Force + Precomputations

Let's precompute all possible answers, as we once did for the problem Nth Tribonacci Number.

Input number is known to be signed 32 bits integer, i.e. $x \leq 2^{31} - 1$. Hence the max power of four to be considered is $\lfloor \log_4 (2^{31} - 1) \rfloor = 15$. Voila, here is all 16 possible answers: $4^0, 4^1, 4^2, ..., 4^{15}$. Let's precompute them all, and then during the runtime just check if input number is in the list of answers.

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```
class Powers:
    def __init__(self):
        max_power = 15
        self.nums = nums = [1] * (max_power + 1)
        for i in range(1, max_power + 1):
            nums[i] = 4 * nums[i - 1]

class Solution:
    p = Powers()
    def isPowerOfFour(self, num: int) -> bool:
        return num in self.p.nums
```

Complexity Analysis

- Time complexity: $\mathcal{O}(1)$.
- Space complexity: $\mathcal{O}(1)$.

Approach 2: Math

If num is a power of four $x = 4^a$, then $a = \log_4 x = \frac{1}{2} \log_2 x$ is an integer. Hence let's simply check if $\log_2 x$ is even number.

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```
from math import log2
class Solution:
    def isPowerOfFour(self, num: int) -> bool:
        return num > 0 and log2(num) % 2 == 0
```

Complexity Analysis

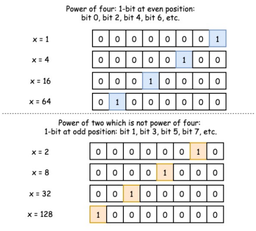
- Time complexity : $\mathcal{O}(1)$.
- Space complexity : $\mathcal{O}(1)$.

Approach 3: Bit Manipulation

Let's first check if num is a power of two: `x > 0 and x & (x - 1) == 0`.

Now the problem is to distinguish between even powers of two (when x is a power of four) and odd powers of two (when x is not a power of four). In binary representation both cases are single 1-bit followed by zeros.

> What is the difference? In the first case (power of four), 1-bit is at even position: bit 0, bit 2, bit 4, etc. In the second case, at odd position.



Hence power of four would make a zero in a bitwise AND with number $(101010...10)_2$:

$4^a \wedge (101010...10)_2 == 0$

> How long should be $(101010...10)_2$ if x is a signed integer? 32 bits. To write shorter, in 8 charaters instead of 32, it's common to use hexadecimal representation: $(101010...10)_2 = (aaaaaaaa)_{16}$.

$x \wedge (aaaaaaaa)_{16} == 0$

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```
class Solution:
    def isPowerOfFour(self, num: int) -> bool:
        return num > 0 and num & (num - 1) == 0 and num & 0xaaaaaaaa == 0
```

Complexity Analysis

- Time complexity : $\mathcal{O}(1)$.
- Space complexity : $\mathcal{O}(1)$.

Approach 4: Bit Manipulation + Math

Let's first check if x is a power of two: `x > 0 and x & (x - 1) == 0`. Now one could be sure that $x = 2^a$. Though x is a power of four only if a is even.

Next step is to consider both cases $a = 2k$ and $a = 2k + 1$, and to compute x modulo after division by three:

$(2^{2k} \mod 3) = (4^k \mod 3) = ((3+1)^k \mod 3) = 1$

$((2^{2k+1}) \mod 3) = ((2 \times 4^k) \mod 3) = ((2 \times (3+1)^k) \mod 3) = 2$

> If x is a power of two and `x % 3 == 1`, then x is a power of four.

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```
class Solution:
    def isPowerOfFour(self, num: int) -> bool:
        return num > 0 and num & (num - 1) == 0 and num % 3 == 1
```

How this works: mod arithmetic

To show the idea, let's compute $x = 2^{2k} \mod 3$.

First, $2^{2k} = 2^{2k} = 4^k$. Second, since $4 = 3 + 1$, x could be rewritten as

$x = ((3+1)^k \mod 3)$

Let's decompose

$(3+1)^k = (3+1) \times (3+1)^{k-1} = 3 \times (3+1)^{k-1} + (3+1)^{k-1}$.

The first term is divisible by 3, i.e. $(3 \times (3+1)^{k-1}) \mod 3 = 0$. Hence

$x = ((3+1)^{k-1} \mod 3)$

One could continue like this k -> k - 1 -> k - 2 -> ... -> 1 and finally rewrite x as

$x = ((3+1)^1 \mod 3) = 1$.

The job is done. Now $y = 2^{2k+1} \mod 3$ is simple, because if $x \mod 3 = 1$, then $y \mod 3 = 2x \mod 3 = 2$.

Complexity Analysis

- Time complexity : $\mathcal{O}(1)$.
- Space complexity : $\mathcal{O}(1)$.

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magetron ★ 3 October 27, 2019 12:35 AM

might not be a very decent approach, we can also use __builtin_popcount

```
class Solution {
public:
    bool isPowerOfFour(int num) {
```

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nmaryala ★ 3 September 6, 2019 4:55 AM

Some refresher formulas for mod arithmetic would be helpful in case of approach 4 !!

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bohebohe00 ★ 1 November 16, 2019 3:17 AM

Can someone tell me why we can't use `return Math.log(num) / Math.log(4) % 1 == 0;` ?

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takenpilot ★ 2 May 7, 2020 1:23 AM

An Accepted JavaScript solution that converts the number to base4 and then uses regex.

```
var isPowerOfFour = function(num) {
  return num > 0 && /^10*$/.test(num.toString(4));
```

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