1060. Missing Element in Sorted Array

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Given a sorted array `A` of **unique** numbers, find the `K-th` missing number starting from the leftmost number of the array.

Example 1:

```
Input: A = [4,7,9,10], K = 1
Output: 5
Explanation:
The first missing number is 5.
```

Example 2:

```
Input: A = [4,7,9,10], K = 3
Output: 8
Explanation:
The missing numbers are [5,6,8,...], hence the third missing number is 8.
```

Example 3:

```
Input: A = [1,2,4], K = 3
Output: 6
Explanation:
The missing numbers are [3,5,6,7,...], hence the third missing number is 6.
```

Note:

1. `1 <= A.length <= 50000`
2. `1 <= A[i] <= 1e7`
3. `1 <= K <= 1e8`

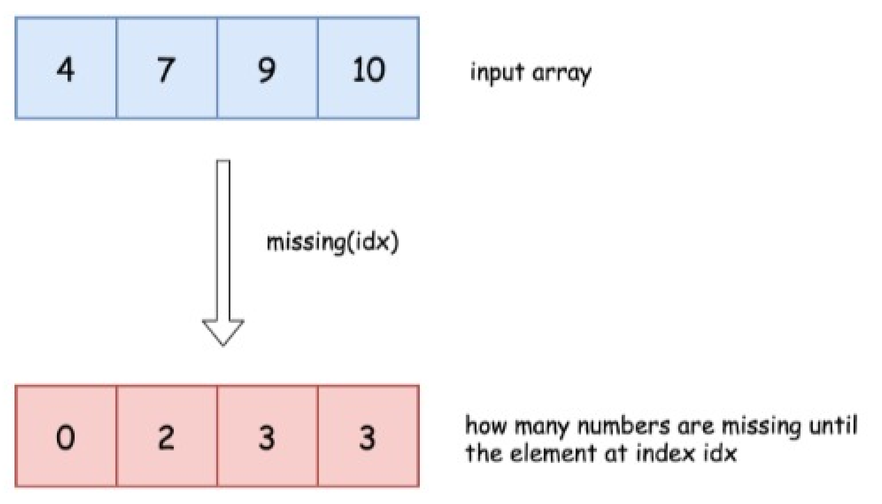
Solution

Approach 1: One Pass

Intuition

The problem is similar to First Missing Positive and the naive idea would be to solve it in a similar way by one pass approach.

Let's first assume that one has a function `missing(idx)` that returns how many numbers are missing until the element at index `idx`.

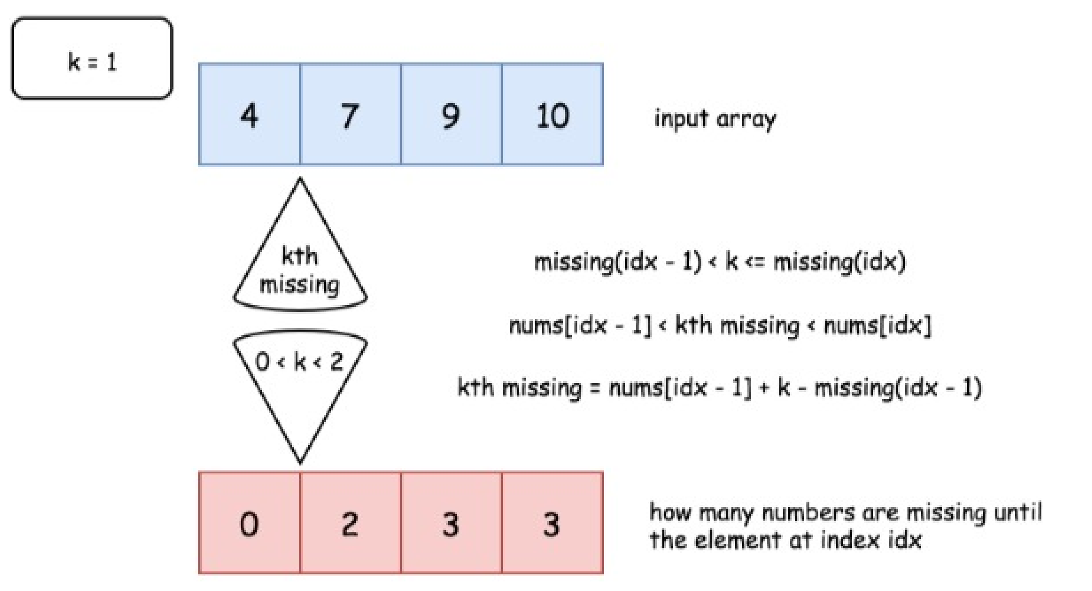


With the help of such a function the solution is straightforward :

- Find an index such that `missing(idx - 1) < k <= missing(idx)`. In other words, that means that kth missing number is in-between `nums[idx - 1]` and `nums[idx]`.

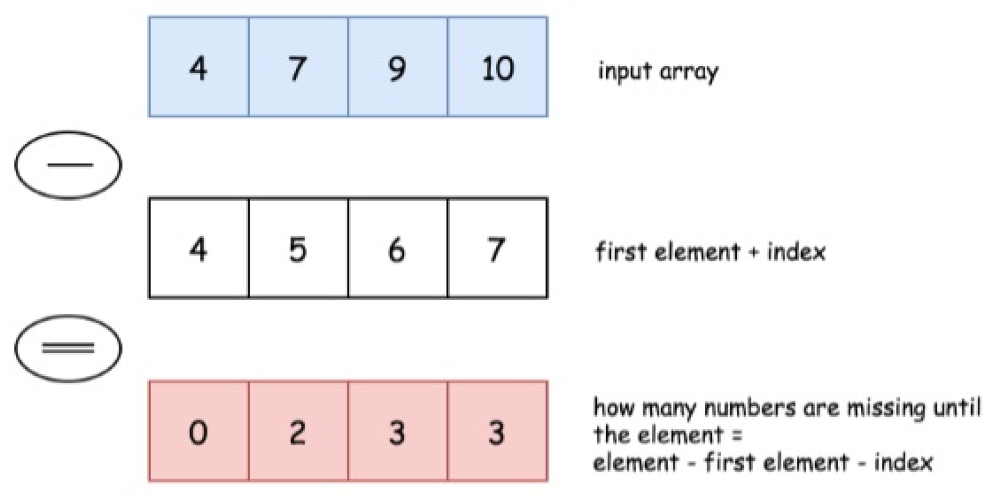
  One even could compute a difference between kth missing number and `nums[idx - 1]`. First, there are `missing(idx - 1)` missing numbers until `nums[idx - 1]`. Second, all `k - missing(idx - 1)` missing numbers from `nums[idx - 1]` to kth missing are *consecutive ones*, because all of them are less than `nums[idx]` and hence there is nothing to separate them. Together that means that kth smallest is larger than `nums[idx - 1]` by `k - missing(idx - 1)`.

- Return kth smallest `nums[idx - 1] + k - missing(idx - 1)`.



> The last thing to discuss is how to implement `missing(idx)` function.

Let's consider an array element at index `idx`. If there is no numbers missing, the element should be equal to `nums[idx] = nums[0] + idx`. If k numbers are missing, the element should be equal to `nums[idx] = nums[0] + idx + k`. Hence the number of missing elements is equal to `nums[idx] - nums[0] - idx`.



Algorithm

- Implement `missing(idx)` function that returns how many numbers are missing until array element with index `idx`. Function returns `nums[idx] - nums[0] - idx`.
- Find an index such that `missing(idx - 1) < k <= missing(idx)` by a linear search.
- Return kth smallest `nums[idx - 1] + k - missing(idx - 1)`.

Implementation

Java Python Copy

```python
class Solution:
    def missingElement(self, nums: List[int], k: int) -> int:
        # Return how many numbers are missing until nums[idx]
        missing = lambda idx: nums[idx] - nums[0] - idx

        n = len(nums)
        # If kth missing number is larger than
        # the last element of the array
        if k > missing(n - 1):
            return nums[-1] + k - missing(n - 1)

        idx = 1
        # find idx such that
        # missing(idx - 1) < k <= missing(idx)
        while missing(idx) < k:
            idx += 1

        # kth missing number is greater than nums[idx - 1]
        # and less than nums[idx]
        return nums[idx - 1] + k - missing(idx - 1)
```

Complexity Analysis

- Time complexity: $\mathcal{O}(N)$ since in the worst case it's one pass along the array.
- Space complexity: $\mathcal{O}(1)$ since it's a constant space solution.

Approach 2: Binary Search

Intuition

Approach 1 uses the linear search and doesn't profit from the fact that array is *sorted*. One could replace the linear search by a binary one and reduce the time complexity from $\mathcal{O}(N)$ down to $\mathcal{O}(\log N)$.

> The idea is to find the leftmost element such that the number of missing numbers until this element is less or equal to k.

Algorithm

- Implement `missing(idx)` function that returns how many numbers are missing until array element with index `idx`. Function returns `nums[idx] - nums[0] - idx`.
- Find an index such that `missing(idx - 1) < k <= missing(idx)` by a *binary search*.
- Return kth smallest `nums[idx - 1] + k - missing(idx - 1)`.

Implementation

Java Python Copy

```python
class Solution:
    def missingElement(self, nums: List[int], k: int) -> int:
        # Return how many numbers are missing until nums[idx]
        missing = lambda idx: nums[idx] - nums[0] - idx

        n = len(nums)
        # If kth missing number is larger than
        # the last element of the array
        if k > missing(n - 1):
            return nums[-1] + k - missing(n - 1)

        left, right = 0, n - 1
        # find left = right index such that
        # missing(left - 1) < k <= missing(left)
        while left != right:
            pivot = left + (right - left) // 2

            if missing(pivot) < k:
                left = pivot + 1
            else:
                right = pivot

        # kth missing number is greater than nums[left - 1]
        # and less than nums[left]
        return nums[left - 1] + k - missing(left - 1)
```

Complexity Analysis

- Time complexity: $\mathcal{O}(\log N)$ since it's a binary search algorithm in the worst case when the missing number is less than the last element of the array.
- Space complexity : $\mathcal{O}(1)$ since it's a constant space solution.

Analysis written by @liaison and @andvary

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