Previous Next

233. Number Of Digit One

June 23, 2017 | 27.8K views

Average Rating: 3.61 (36 votes)

Given an integer n, count the total number of digit 1 appearing in all non-negative integers less than or equal to n.

Example:

```
Input: 13
Output: 6
Explanation: Digit 1 occurred in the following numbers: 1, 10, 11, 12, 13.
```

Solution

Approach #1 Brute force [Time Limit Exceeded]

Intuition

Do as directed in question.

Algorithm

- Iterate over i from 1 to n:
- Convert i to string and count $'1'$ in each integer string
- Add count of $'1'$ in each string to the sum, say $countr$

C++ Copy

```cpp
int countDigitOne(int n)
{
    int countr = 0;
    for (int i = 1; i <= n; i++) {
        string str = to_string(i);
        countr += count(str.begin(), str.end(), '1');
    }
    return countr;
}
```

Complexity Analysis

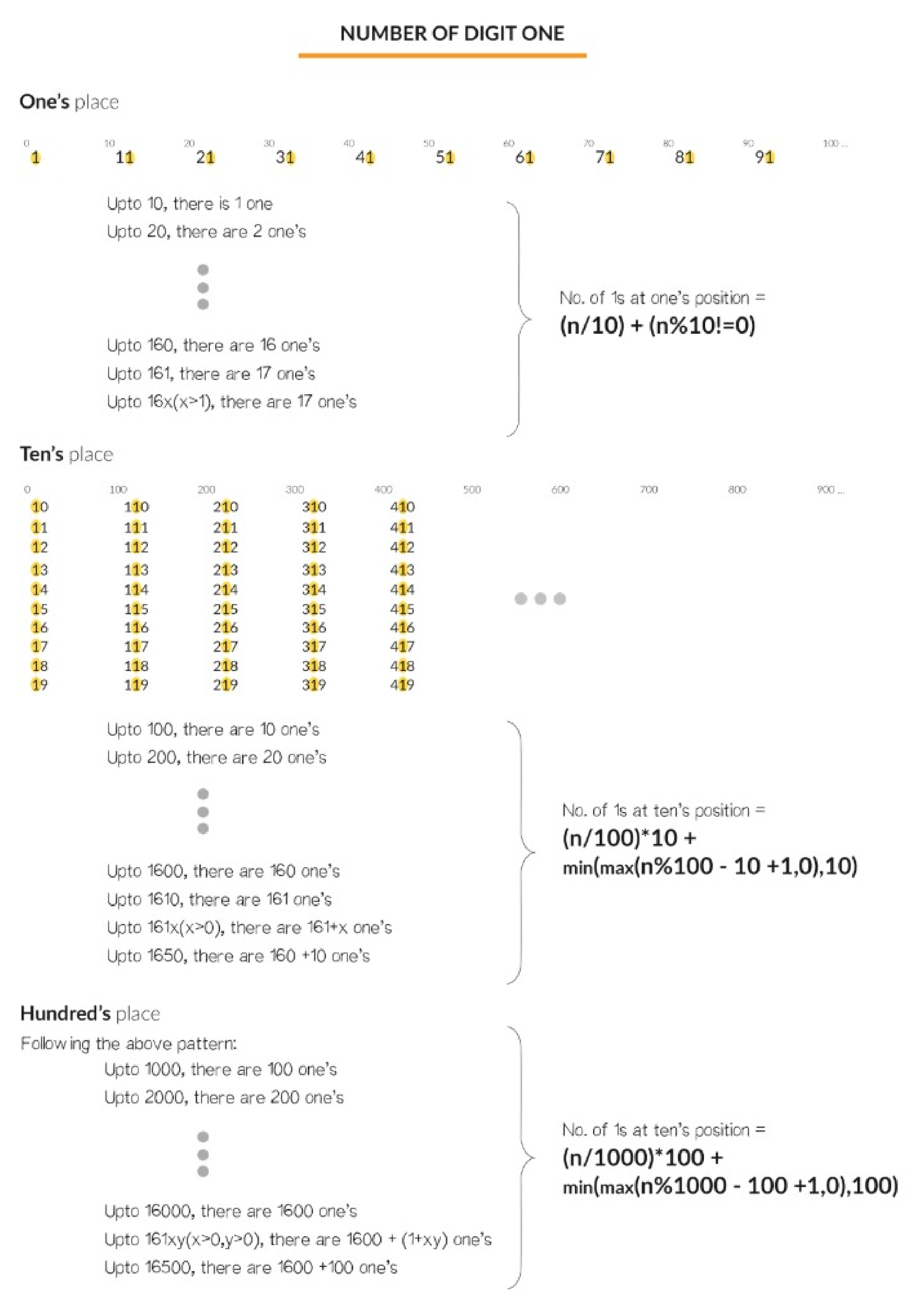
- Time complexity: $O(n * log_{10}(n))$.
- We iterate from 1 to n
- In each iteration, we convert integer to string and count '1' in string which takes linear time in number of digits in i, which is $log_{10}(n)$.
- Space complexity: $O(log_{10}(n))$ Extra space for the countr and the converted string str.

Approach #2 Solve it mathematically [Accepted]

Intuition

In Approach #1, we manually calculated the number of all the $'1's$ in the digits, but this is very slow. Hence, we need a way to find a pattern in the way $'1's$ (or for that matter any digit) appears in the numbers. We could then use the pattern to formulate the answer.

Consider the 1s in ones place , tens place, hundreds place and so on... An analysis has been performed in the following figure.



From the figure, we can see that from digit '1' at ones place repeat in group of 1 after interval of 10. Similarly, '1' at tens place repeat in group of 10 after interval of 100. This can be formulated as $(n/(i*10))*i$.

Also, notice that if the digit at tens place is $'1'$, then the number of terms with $'1's$ is increased by $x+1$, if the number is say $"\text{ab1x}"$. As if digits at tens place is greater than 1, then all the 10 occurances of numbers with $'1'$ at tens place have taken place, hence, we add 10. This is formluated as $\min(\max((\text{n mod (i*10)})-i+1,0),i)$.

Lets take an example, say $n=1234$.

No of $'1'$ in ones place = $1234/10$(corresponding to 1,11,21,...1221) + $\min(4,1)$(corresponding to 1231) = 124

No of $'1'$ in tens place = $(1234/100)*10$(corresponding to 10,11,12,...,110,111,...1919) +$\min(21,10)$(corresponding to 1210,1211,...1219)=130

No of $'1'$ in hundreds place = $(1234/1000)*100$(corresponding to 100,101,12,...,199) +$\min(135,100)$(corresponding to 1100,1101...1199)=200

No of $'1'$ in thousands place = $(1234/10000)*10000$ +$\min(235,1000)$(corresponding to 1000,1001,...1234)=235

Therefore, Total = $124+130+200+235=689$.

Herein, one formula has been devised, but many other formulae can be devised for faster implementations, but the essence and complexity remains the same. The users are encouraged to try to devise their own version of solution using the mathematical concepts.

Algorithm

- Iterate over i from 1 to n incrementing by 10 each time:
  - Add $(n/(i*10))*i$ to countr representing the repetition of groups of i sizes after each $(i*10)$ interval.
  - Add $\min(\max((\text{n mod (i*10)})-i+1,0),i)$ to countr representing the additional digits dependant on the digit in ith place as described in intuition.

C++ Copy

```cpp
int countDigitOne(int n)
{
    int countr = 0;
    for (long long i = 1; i <= n; i *= 10) {
        long long divider = i * 10;
        countr += (n / divider) * i + min(max(n % divider - i + 1, 0LL), i);
    }
    return countr;
}
```

Complexity analysis

- Time complexity: $O(log_{10}(n))$.
- No of iterations equal to the number of digits in n which is $log_{10}(n)$
- Space complexity: $O(1)$ space required.

Rate this article: ★★★★★

Previous Next

Comments: 17 Sort By

Type comment here... (Markdown is supported)

Preview Post

lelouchmatlab2018 ★ 136 July 2, 2018 12:44 AM

Wonder why would such a question be used in an interview?

76 Share Reply

SHOW 2 REPLIES

gabelois1024 ★ 33 January 11, 2019 9:47 AM

OK, I will ask interviewees to solve Fermat's Last Theorem since it has a solution now.

33 Share Reply

SHOW 3 REPLIES

onerhao ★ 31 December 26, 2017 6:26 PM

There is also a dynamic programming solution, with time complexity O(logn).

15 Share Reply

SHOW 2 REPLIES

scn7th ★ 44 March 13, 2019 5:10 PM

I have also come up with a dynammic programming solution: https://leetcode.com/problems/number-of-digit-one/discuss/254596/My-dynamic-programming-java-solution

4 Share Reply

hello_world_cn ★ 283 January 19, 2019 7:00 AM

@abhinavbansal0
For appoach 2, `min(21,10)` should be `min(25,10)`

4 Share Reply

amitkumartiwari29 ★ 1 July 27, 2019 2:14 AM

Here is my C# solution (and my first comment (:)

public int CountDigitOne(int n)
{
int count = 0;

Read More

1 Share Reply

ericqianli ★ -1 January 11, 2018 3:14 PM

java:

```
class Solution {
    public int countDigitOne(int n) {
        if (n <= 0) {
```

Read More

1 Share Reply

SHOW 1 REPLY

pranoopm ★ 0 July 20, 2019 11:47 AM

JAVA Solution:

I am not sure why it is giving me time exceeded for this solution. Could some one please explain me why
and also if you have a better solution or any changes in this solution. Thanks in advance.

Read More

0 Share Reply

unagi ★ 1 May 11, 2019 7:02 AM

How about an example in compilable code?

0 Share Reply

CatherineWong ★ 2 October 29, 2018 2:09 AM

why the figure is missing?

0 Share Reply

‹ 1 2 ›