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478. Generate Random Point in a Circle

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Given the radius and x-y positions of the center of a circle, write a function `randPoint` which generates a uniform random point in the circle.

Note:

1. input and output values are in floating-point.
2. radius and x-y position of the center of the circle is passed into the class constructor.
3. a point on the circumference of the circle is considered to be in the circle.
4. `randPoint` returns a size 2 array containing x-position and y-position of the random point, in that order.

Example 1:

```
Input:
["Solution","randPoint","randPoint","randPoint"]
[[1,0,0],[],[],[]]
Output: [null,[-0.72939,-0.65505],[-0.78502,-0.28626],[-0.83119,-0.19803]]
```

Example 2:

```
Input:
["Solution","randPoint","randPoint","randPoint"]
[[10,5,-7.5],[],[],[]]
Output: [null,[11.52438,-8.33273],[2.46992,-16.21705],[11.13430,-12.42337]]
```

Explanation of Input Syntax:

The input is two lists: the subroutines called and their arguments. `Solution`'s constructor has three arguments, the radius, x-position of the center, and y-position of the center of the circle. `randPoint` has no arguments. Arguments are always wrapped with a list, even if there aren't any.

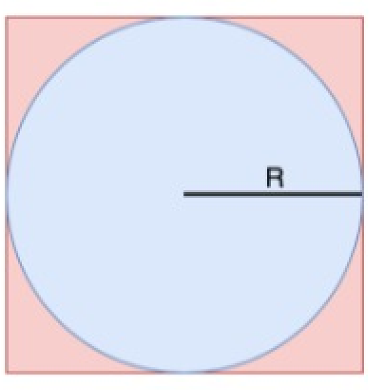
Solution

Approach 1: Rejection Sampling

Intuition

It is easy to generate a random point in a square. Could we use randomly generated points in a square to get random points in a circle? Which generated points could we use, and which ones would we need to toss away? How often would we generate points that we could use?

Algorithm



A square of size length 2R overlaid with a circle of radius R.

To get uniform random points in a circle C of radius R, we can generate uniform random points in the square S of side length $2R$, keeping all of the points which are at most euclidean distance R from the center, and rejecting all which are farther away than that. This technique is called rejection sampling. Each possible location on the circle has the same probability of being generated, so the sampling of points will be uniformly distributed.

The area of the square is $(2R)^2 = 4R$ and the area of the circle is $\pi R \approx 3.14R$. $\frac{3.14R}{4R} = \frac{3.14}{4} = .785$. Therefore, we will get a usable sample approximately 78.5% of the time and the expected number of times that we will need to sample until we get a usable sample is $\frac{1}{.785} \approx 1.274$ times.

C++ Java Copy

```
class Solution {
public:
    double rad, xc, yc;
    //c++11 random floating point number generation
    mt19937 rng{random_device{}()};
    uniform_real_distribution<double> uni{0, 1};

    Solution(double radius, double x_center, double y_center) {
        rad = radius, xc = x_center, yc = y_center;
    }

    vector<double> randPoint() {
        double x0 = xc - rad;
        double y0 = yc - rad;

        while(true) {
            double xg = x0 + uni(rng) * 2 * rad;
            double yg = y0 + uni(rng) * 2 * rad;
            if (sqrt(pow((xg - xc), 2) + pow((yg - yc), 2)) <= rad)
                return {xg, yg};
        }
    }
};
```

Complexity Analysis

- Time Complexity: $O(1)$ on average. $O(\infty)$ worst case. (per $\mathrm{randPoint}$ call)
- Space Complexity: $O(1)$.

Approach 2: Inverse Transform Sampling (Math)

Disclaimer

This solution relies on advanced math which is not expected knowledge for a coding interview. It is presented here only for educational purposes.

Algorithm

Assume that we are given a circle C of radius 1 that is centered at the origin, and our task is to sample uniform random points on this circle.

Lets imagine another circle B of radius $\frac{1}{2}$ which is also centered at the origin.

The circumference of C is twice the circumference of B, because the circumference of a circle is directly proportional to the radius. Also, the probability of sampling a point from a subregion in circle C is directly proportional to the area of the subregion. Therefore, the probability of sampling a point on the perimeter of C is twice that of sampling a point on the perimeter of B.

More generally, what is implied is that the sampling probability is directly proportional to the distance from the origin, from 0 up to R. This can be modeled as a probability density function f, where x is the distance from the origin and $f(x)$ is the relative sampling probability at x.

The area under any probability density function curve must be 1. Therefore, the equation must be $f(x) = 2x$.

Using our probability density function f, we can compute the cumulative distribution function F, where $F(x)$ is the probability of sampling a point within a distance of x from the origin.

The cumulative distribution function is the integral of the probability density function.

$$F(x) = \int f(x) = \int 2x = x^2$$

Lastly, we can use our cumulative distribution function F to compute the inverse cumulative distribution function F^{-1}, which accepts uniform random value between 0 and 1 and returns a random distance from origin in accordance with f. [†]

$$F^{-1}(F(x)) = x$$
$$F^{-1}(x^2) = x$$
$$F^{-1}(x) = \sqrt{x}$$

Now, to generate a uniform random point on C, we just need to compute a random distance D from origin using F^{-1} and a uniform random angle θ over the range $[0, 2 \cdot PI)$.

The points will be generated as polar coordinates. To convert to cartesian coordinates, we can use the following formulas.

$$X = D \cdot \cos(\theta)$$
$$Y = D \cdot \sin(\theta)$$

C++ Java Copy

```
class Solution {
public:
    double rad, xc, yc;
    //c++11 random floating point number generation
    mt19937 rng{random_device{}()};
    uniform_real_distribution<double> uni{0, 1};

    Solution(double radius, double x_center, double y_center) {
        rad = radius, xc = x_center, yc = y_center;
    }

    vector<double> randPoint() {
        double d = rad * sqrt(uni(rng));
        double theta = uni(rng) * (2 * M_PI);
        return {d * cos(theta) + xc, d * sin(theta) + yc};
    }
};
```

Complexity Analysis

- Time Complexity: $O(1)$ per $\mathrm{randPoint}$ call.
- Space Complexity: $O(1)$

Footnotes

- [†] This technique of using the inverse cumulative distribution function to sample numbers at random from the corresponding probability distribution is called inverse transform sampling.
- This solution is inspired by this answer on Stack Overflow.

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